A Modified Covariance Matrix Adaptation Evolution Strategy for Real-World Constrained Optimization Problems

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ABSTRACT

Most of the real-world black-box optimization problems are associated with multiple non-linear as well as non-convex constraints, making them difficult to solve. In this work, we introduce a variant of the Covariance Matrix Adaptation Evolution Strategy (CMA-ES) with linear timing complexity to adopt the constraints of Constrained Optimization Problems (COPs). CMA-ES is already well-known as a powerful algorithm for solving continuous, non-convex, and black-box optimization problems by fitting a second-order model to the underlying objective function (similar in spirit, to the Hessian approximation used by Quasi-Newton methods in mathematical programming). The proposed algorithm utilizes an ϵ-constraint-based ranking and a repair method to handle the violation of the constraints. The experimental results on a group of real-world optimization problems show that the performance of the proposed algorithm is better than several other state-of-the-art algorithms in terms of constraint handling and robustness.

CCS CONCEPTS

• **Computer systems organization** → **Embedded systems**; *Redundancy*; Robotics; • **Networks** → Network reliability.

KEYWORDS

Constrained Optimization problem, Covariance Matrix Adaptation Evolution Strategy, ϵ-constrained, Constraint Handling

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1 INTRODUCTION

Covariance Matrix Adaptation Evolution Strategy (CMA-ES) [1] is a powerful and time-tested method for non-convex and derivative-free optimization over continuous parameter spaces. The algorithm uses only ranking among the candidate solutions to learn the distribution of samples over the fitness landscape without requiring any gradient information about the function to be optimized. Despite the reported success of CMA-ES for black-box optimization, the applicability of the algorithm for constrained optimization problems is yet to be investigated at a large scale. In this work, we investigate CMA-ES as a base optimizer for solving non-linearly constrained optimization problems. Hereupon, it is believed that the robust features of CMA-ES can be exploited for designing a powerful tool for constrained optimization. To deal with the restrictions of CMA-ES on constrained optimization problems we propose the following modifications:

(1) the basic framework of CMA-ES is modified by implementing a few changes in its sampling scheme and stopping criteria and
(2) a ranking scheme and a repair method are integrated with the framework of the algorithm to efficiently handle the constraint violations.

Recently, a simplified CMA-ES algorithm, ϵMAgES, [2] is proposed for constrained fitness landscapes. The performance of this algorithm is very competitive with the state-of-the-art algorithms and ranked second in the IEEE CEC (Congress on Evolutionary Computation) 2018 competition on real-parameter constrained optimization. In this work, we put forth an advanced variant of the ϵMAgES. We modify the basic steps of sampling new solutions in CMA-ES to reduce the time-complexity and we employ restart schemes to improve the performance for multi-modal constrained optimization problems. The steps of the proposed algorithm are discussed in the next section.

2 THE PROPOSED ALGORITHM

The pseudo-code of the proposed algorithm, referred as ϵsCMAgES, is reported in Algorithm-1.

2.1 Proposed Modifications to the ϵMAgES

The framework of ϵMAgES [2] is modified by the following steps:

2.1.1 Sampling of new solutions. In the proposed algorithm, a new solution is sampled by using a single-variate normal distribution with mean $\bar{x}$ and a standard deviation of $\bar{\sigma}$. However, to boost up the performance of the algorithm, the parameter $\bar{\sigma}$ should be adapted effectively in each iteration. Thus, we adapt $\bar{\sigma}^g$. The proposed equations are shown in line no 40-45 of Algorithm-1.

2.1.2 Restart Scheme. At the end of each iteration, the following stopping criteria are examined.

- Maximum number of consecutive unsuccessful generations: The optimization process is reinitialized when the number of consecutive iterations, where the best solution is not updated, reaches its maximum value.

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Algorithm 1: ϵsCMAgES

Result: $y_{best}, f(y_{best}), \nu(y_{best})$;
Initialization: $\mu, \lambda, \sigma^0, P_\sigma \leftarrow 0, P_c \leftarrow 0\ C^0 \leftarrow 0, S^{-1} \leftarrow [\,], FEs \leftarrow 0$;
while $(FEs \leq FE_{max})$ **do**
  for $i = 1 : \lambda$ **do**
    for $j = 1 : D$ **do**
      $y_{ij} \leftarrow l_j + (u_j - l_j) \cdot u(0,1)$;
    end
    $\bar{y}_i \leftarrow [y_{i1}, y_{i2}, ..., y_{iD}]$;
    $S^0 \leftarrow S^0 \cup \{\bar{y}_i\}$;
  end
  $\epsilon^0 \leftarrow \sum_{i=1}^{\lfloor \theta_t \lambda \rfloor} \frac{\nu(\bar{y}_{i:\lambda})}{\lfloor \theta_t \lambda \rfloor}$;
  $\gamma \leftarrow max\left(\gamma_{min}, \frac{(-5-log(\epsilon^0))}{log(0.05)}\right)$;
  $FEs \leftarrow FEs + \lambda$;
  $\bar{x}^0 \leftarrow \sum_{i=1}^{\mu} w_i \bar{y}_{i:\lambda}$ according to $\leq_\epsilon$;
  $y_{best} \leftarrow \bar{y}_{1:\lambda}$;
  $g \leftarrow 0$;
  $Restart \leftarrow 1$;
  while $(FEs \leq FE_{max} \wedge Restart == 1)$ **do**
    for $i = 1 : \lambda$ **do**
      $z_i^g \leftarrow N(0, I)$;
      $d_i^g \leftarrow \sqrt{diag\,(C^g)} \cdot z_i^g$;
      $\bar{y}_i^g \leftarrow \bar{x}^g + \sigma^g \cdot d_i^g$;
      $FEs \leftarrow FEs + 1$;
      $\hat{y}_i^g \leftarrow$ BoxConstRepair($\bar{y}_i^g$);
      if $mod(g, D) == 0 \wedge u(0,1) < \theta_p$ **then**
        $h \leftarrow 1$;
        while $h \leq \theta_r \wedge \nu(\hat{y}_i) > 0$ **do**
          $\hat{y}_i^g \leftarrow$ GradientRepair($\hat{y}_i^g$);
          $FEs \leftarrow FEs + 1$;
        end
      end
      if $\hat{y}_i^g \neq \bar{y}_i^g$ **then**
        $\bar{y}_i^g \leftarrow \hat{y}_i^g$;
        $d_i^g \leftarrow \frac{\hat{y}_i^g - \bar{x}^g}{\sigma^g}$;
        $z_i^g \leftarrow diag\,(C^g)^{-1} \cdot d_i^g$;
      end
    end
    if $\bar{y}_{1:\lambda}^g \leq_\epsilon y_{best}$ **then**
      $y_{best} \leftarrow \bar{y}_{1:\lambda}^g$;
    end
    $\bar{x}^{g+1} \leftarrow \bar{x}^g + \sigma^g \sum_{i=1}^{\mu} w_i d_{i:\lambda}^g$;
    $P_\sigma^{g+1} \leftarrow (1 - c_\sigma) P_\sigma^g + \sqrt{\mu_w c_\sigma (2 - c_\sigma)} \sum_{i=1}^{\mu} w_i z_{i:\lambda}^g$;
    $h_\sigma^g = \frac{||P_\sigma||^2}{D\left(1-(1-c_\sigma)^{\frac{2FEs}{\lambda}}\right)} < 2 + \frac{4}{D+1}$;
    $P_c^g = (1 - c_c) P_c^{g-1} + h_\sigma \sqrt{\mu_{eff} c_c (2 - c_c)} \sum_{i=1}^{\mu} w_i d_{i:\lambda}^g$;
    $C^g =$
    $\left(1 - c_\mu \left(1 - \frac{1}{\mu_{eff}}\right) \sum_{i=1}^{\mu} w_i z_{i:\lambda}^g \left(z_{i:\lambda}^g\right)^T\right) C^{g-1} + \frac{1}{\mu_{eff}} P_c^g \left(P_c^g\right)^T$;
    $\sigma^{g+1} \leftarrow min\left(\sigma^g exp\left[\frac{c_\sigma}{2}\left(\frac{||P_\sigma^{g+1}||^2}{D} - 1\right)\right], \sigma_{max}\right)$;
    $g \leftarrow g + 1$;
    if $g < T$ **then**
      $\epsilon^g \leftarrow \epsilon^0 \left(1 - \frac{g}{T}\right)^\gamma$;
    else
      $\epsilon^g \leftarrow 0$;
    end
    if *Stopping Criteria is satisfied* **then**
      $\lambda \leftarrow 1.5 \cdot \lambda$;
      $Restart \leftarrow 0$;
    end
  end
end

Table 1: Ranking Statistics of all algorithms on real-world constrained benchmark suite.

Algorithm	Best	Mean	Median	Weighted	Rank
IUDE	0.6606	0.6322	0.6190	0.6437	4
ϵMAgES	**0.1663**	0.3301	0.2484	0.2318	2
ILSHADE$_\epsilon$	0.6140	0.4974	0.5460	0.5654	3
ϵsCMAgES	0.2629	**0.1713**	**0.1791**	**0.2186**	**1**

- Convergence of solution: The optimization process is reinitialized when the solutions are converged to a fixed point that may not be the optimum value.

If one or more above-mentioned stopping criteria are met, the optimization process is reinitialized with following parameter setting.

- $\lambda = 1.5 \cdot \lambda$,
- with new random distribution of population,
- all parameters are reset to their default value.

2.1.3 Gradient-based Repair Method. In addition to ϵ-constrained based ranking scheme, the proposed algorithm also utilizes the gradient-based repair method approach to handle the complex non-linear equality constraints.

3 RESULTS AND DISCUSSION

To investigate the robustness of the proposed algorithm, 57 real-world constrained optimization problems (suggested in [3]) are considered as the constituents of our experiment. A comparative analysis is done on the obtained results of the proposed algorithm versus the obtained results of state-of-the-art algorithms provided in [3]. To perform comparative analysis, ranking statistics of all algorithms are prepared as suggested in [3] and shown in Table 1. It is clearly observed from this table that the proposed algorithm outperforms other algorithms and it secures the first rank in the statistic table.

4 CONCLUSION

In this paper, a simplified variant of the Covariance Matrix Adaptation Evolution Strategy is proposed to solve global constrained optimization problems efficiently. As compared to standard CMA-ES, the proposed algorithm have simple steps with improved performance. In addition to that, an efficient constraint handling technique, ϵ-constraint handling technique, is employed to handle the non-linear constraints of the given problems. Moreover, to handle the non-linear equality constraints, a gradient-based repair method is also utilized within the basic steps of proposed algorithms. This repair method transforms the infeasible solutions to feasible ones if solutions are near to the boundary of the feasible region.

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