Contents

Q1

```
P = [ 8/10 3/10 3/10; 1/10 6/10 1/10; 1/10 1/10 6/10 ];


[V, D] = eig(P);

eigenvalues = diag(D);

precision = 10^(-10);
rounded_eigenvalues = round(eigenvalues / precision) * precision;

unique_eig = unique(rounded_eigenvalues);

occurances = {};


for i=1:size(unique_eig)
    aRow = {unique_eig(i), 0};
    occurances = [occurances; aRow];

end

occurances = cell2mat(occurances);


% algebraic multiplicity
for i=1:size(eigenvalues)

    index = find(unique_eig == rounded_eigenvalues(i));

    occurances(index, 2) = occurances(index, 2) + 1;

end

% geometric multiplicity

identity = eye(size(P));
for i= 1:size(unique_eig)

    formula = P - unique_eig(i)*identity;

    nullity = size(P, 1) - rank(formula);

    occurances(i, 3) = nullity;
end


for i=1:size(unique_eig)
    disp(["eig: " + occurances(i,1)])
```

```
    disp("algebraic multiplicity")
    disp(occurances(i,2));
    disp("geometric multiplicity")
    disp(occurances(i,3));
end

iminusp = identity - P;

iminuspRREF = rref(iminusp);

nullSpace = iminuspRREF(:,3)*-1 + [0; 0; 1];

t = 1 / sum(nullSpace);

steady_vector = t*nullSpace;
disp('steady_vect:')
disp(steady_vector)
```

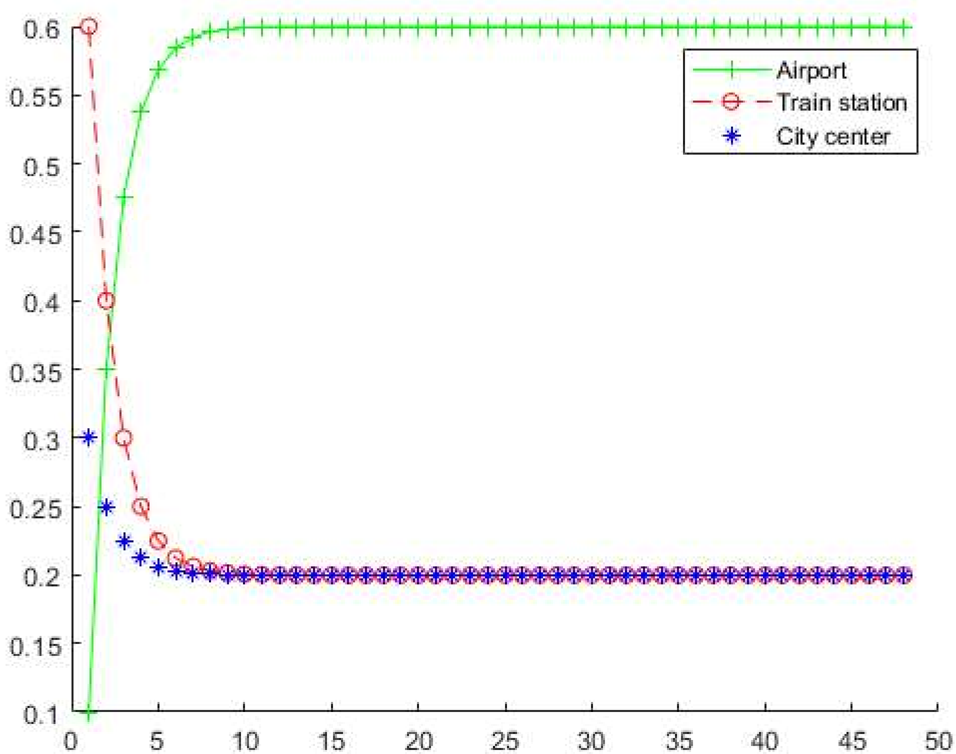
```
eig: 0.5
algebraic multiplicity
     2

geometric multiplicity
     2

eig: 1
algebraic multiplicity
     1

geometric multiplicity
     1

steady_vect:
    0.6000
    0.2000
    0.2000
```

Q2

```
% This is a Matlab code to determine the steady-state distribution for the cars
% in Markov process in Problem A16 (Practice Problems of Section 6.3)
% see page 378 of text book


%x0 stands for the states of system at time t=0 (it can be chosen in a diffrent way)
x0=[0.1;0.6;0.3];

%P is the transition matrix for the system, here P is a 3x3 matrix
P=[0.8 0.3 0.3;
   0.1 0.6 0.1;
   0.1 0.1 0.6];

v1=[];
v2=[];
```

```
v3=[];

v1=[v1 x0(1)];
v2=[v2 x0(2)];
v3=[v3 x0(3)];


powersofD = V*D*inv(V);
for i=1:500
    x1=(powersofD^i)*x0; % Finding the i-th state vector is computed
    if (x1-powersofD*x1)<1e-15
        disp('Steady state found')
        break;
    end
v1=[v1 x1(1)];
v2=[v2 x1(2)];
v3=[v3 x1(3)];
end
figure(1)
hold on
plot(v1,'g+-')
plot(v2,'ro--')
plot(v3,'b*')
legend('Airport','Train station','City center')
```

```
Steady state found
```



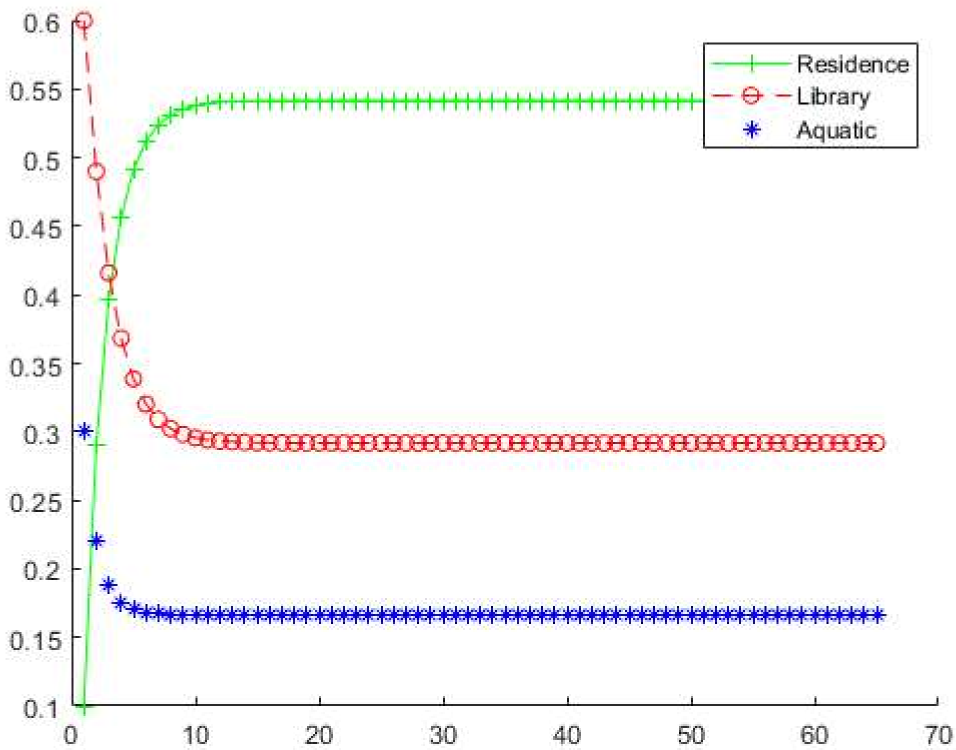
Q3

```
PQ3 = [0.8, 0.1, 0.1; 0.2, 0.7, 0.1; 0.3, 0.2, 0.5];
PQ3 = transpose(PQ3);

[VQ3, DQ3] = eig(PQ3);

eigenvaluesQ3 = diag(DQ3);

unique_eigQ3 = unique(eigenvaluesQ3);

occurancesQ3 = {};


for i=1:size(unique_eigQ3)
    aRow = {unique_eigQ3(i), 0};
    occurancesQ3 = [occurancesQ3; aRow];

end

occurancesQ3 = cell2mat(occurancesQ3);


% algebraic multiplicity
for i=1:size(eigenvaluesQ3)

    index = find(unique_eigQ3 == eigenvaluesQ3(i));

    occurancesQ3(index, 2) = occurancesQ3(index, 2) + 1;

end

% geometric multiplicity

identity = eye(size(PQ3));
for i= 1:size(unique_eigQ3)

    formulaQ3 = PQ3 - unique_eigQ3(i)*identity;

    nullityQ3 = size(PQ3, 1) - rank(formulaQ3);

    occurancesQ3(i, 3) = nullityQ3;
end


for i=1:size(unique_eigQ3)
    disp(["eig: " + occurancesQ3(i,1)])
    disp("algebraic multiplicity")
    disp(occurancesQ3(i,2));
    disp("geometric multiplicity")
    disp(occurancesQ3(i,3));
end

iminuspQ3 = identity - PQ3;

iminuspRREFQ3 = rref(iminuspQ3);

nullSpaceQ3 = iminuspRREFQ3(:,3)*-1 + [0; 0; 1];
```

```
t = 1 / sum(nullSpaceQ3);

steady_vector = t*nullSpaceQ3;
disp('steady_vect:')
disp(steady_vector)


%x0 stands for the states of system at time t=0 (it can be chosen in a diffrent way)
x0=[0.1;0.6;0.3];


v1=[];
v2=[];
v3=[];

v1=[v1 x0(1)];
v2=[v2 x0(2)];
v3=[v3 x0(3)];


for i=1:500
    x1 = PQ3^i*x0; % Finding the i-th state vector is computed

    if (x1-PQ3^i*x1)<1e-15
        disp('Steady state found')
        break;

    end



v1=[v1 x1(1)];
v2=[v2 x1(2)];
v3=[v3 x1(3)];
end

figure(3)
hold on
plot(v1,'g+-')
plot(v2,'ro--')
plot(v3,'b*')
legend('Residence','Library','Aquatic')
```

```
eig: 0.4
algebraic multiplicity
     1

geometric multiplicity
     1

eig: 0.6
algebraic multiplicity
     1

geometric multiplicity
     1
```

```
eig: 1
algebraic multiplicity
     1

geometric multiplicity
     1

steady_vect:
    0.5417
    0.2917
    0.1667

Steady state found
```



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