Math 232 - Computing Assignment 1

Due Date: February 16th, at 10:55pm.

You must upload to Crowdmark both your code (i.e., the Matlab or Python or other programming app commands you used) and your report (i.e, answers to the problems here). The assignment is due at 10:55pm. The due time is set in Crowdmark to 11:00pm and if Crowdmark indicates that you submitted late, you will be given 0 on the assignment.

- Please read the Guidelines for Computing Assignments in Canvas first.
- Keep in mind that Canvas discussions are open forums but feel free to discuss this assignment on the Canvas discussion board.
- Acknowledge any collaborations and assistance from colleagues/TAs/instructor.

Programming Preamble:

Matlab: R = rand$(5, 7)$ produces a 5×7 matrix with random entries.
Matlab: Use either A' or Matlab command transpose(A) to produce A^T.
Matlab: Given the matrix A, the Matlab function rref (A) produces the reduced row echelon form of A.
Matlab: Given A and B , simply set $[A \quad B]$ to produce an augmented matrix in Matlab. [a]
Python: import numpy
R = numpy .random.rand $(5, 7)$

[a] You may click here to see sample runs of the above Matlab functions. Notice that the matrices are random and different numbers will be produced in each run.

Computing Assignment

Required submission: 1 page **ONLY** PDF report with your answers to the problems here, and PDF document with your Matlab or Python code, both uploaded to Crowdmark (so you will upload 2 different things). Please note that if you upload a computing report of more than one page, the TA will not read anything beyond the first page and you will have marks deducted.

Part 1 - Solutions of systems of linear equations

Observation

Let $m = 10$. Choose randomly n column vectors $\mathbf{y}_1, \mathbf{y}_2, \ldots, \mathbf{y}_n$ in $\mathbb{R}^m$ in following cases:

- Case I. $m = n$;
- Case II. $m > n$;
- Case III. $m < n$.

For each case, generate $\mathbf{y}_1, \mathbf{y}_2, \ldots, \mathbf{y}_n$ at least five times and monitor the number of times, the set $\{\mathbf{y}_1, \mathbf{y}_2, \ldots, \mathbf{y}_n\}$ is a linearly independent set of vectors and the number of times it is a linearly dependent set.

1. Now let b be a random vector of size m. Using the above observation, justify a typical solution set of $Ax = b$ in each of the cases I, II and III where A is an $m \times n$ random matrix. (In this part, m and n are both integers and both are greater than 10 or larger).
2. Now give examples of exceptions for each of the above cases (but for this part, you can use integers m and n that need only be larger than 2, so 3 or larger).

Part 2 - Linear independence, Intersection of subspaces

Consider the following two subspaces in $\mathbb{R}^4$:

$$\mathbb{S}_1 = \text{span}\,\{\mathbf{w}_1, \mathbf{w}_2, \mathbf{w}_3, \mathbf{w}_4\}$$

where

$$\mathbf{w}_1 = \begin{bmatrix} 1 \\ 2 \\ -1 \\ 3 \end{bmatrix}, \mathbf{w}_2 = \begin{bmatrix} 4 \\ 1 \\ 1 \\ 8 \end{bmatrix}, \mathbf{w}_3 = \begin{bmatrix} 1 \\ 0 \\ 2 \\ 2 \end{bmatrix}, \mathbf{w}_4 = \begin{bmatrix} -1 \\ 1 \\ 2 \\ -1 \end{bmatrix}$$

and

$$\mathbb{S}_2 = \text{span}\,\{\mathbf{z}_1, \mathbf{z}_2, \mathbf{z}_3\}$$

where

$$\mathbf{z}_1 = \begin{bmatrix} 2 \\ 4 \\ -2 \\ a \end{bmatrix}, \quad \mathbf{z}_2 = \begin{bmatrix} 1 \\ 0 \\ 2 \\ 2 \end{bmatrix}, \quad \mathbf{z}_3 = \begin{bmatrix} 3 \\ 4 \\ 0 \\ 8 \end{bmatrix}.$$

1. Show that the subspace $\mathbb{S}_1$ is not equal to $\mathbb{R}^4$. Find a maximal linearly independent set of vectors that spans the subspace $\mathbb{S}_1$ and determine whether the subspace $\mathbb{S}_1$ is a hyperplane or a plane.

2. Are either of vectors $\mathbf{z}_2$ and $\mathbf{z}_3$ in $\mathbb{S}_1$? Explain why or why not.

3. Let $a = 6$ in $\mathbf{z}_1$ and determine the dimension of $\mathbb{S}_2$.

4. Let $a = 5$ in $\mathbf{z}_1$ and find the dimension of subspace $\mathbb{S}_1 \cap \mathbb{S}_2$.

Hint on finding an intersection between subspaces.

Let $\mathbb{B}_1$ and $\mathbb{B}_2$ be two subspaces of $\mathbb{R}^k$ such that $\dim(\mathbb{B}_1) = n$ and $\dim(\mathbb{B}_2) = m$. First we need to find the bases $\{\mathbf{v}_1, \mathbf{v}_2, ...\mathbf{v}_n\}$ and $\{\mathbf{w}_1, \mathbf{w}_2, ..., \mathbf{w}_m\}$ for $\mathbb{B}_1$ and $\mathbb{B}_2$, respectively. Then, we form the matrices

$$M_1 = [\mathbf{v}_1 \ \mathbf{v}_2 \ ... \ \mathbf{v}_n]$$

and

$$M_2 = [\mathbf{w}_1 \ \mathbf{w}_2 \ ... \ \mathbf{w}_m].$$

Now we need to determine for which vectors $\mathbf{x}$ and $\mathbf{y}$, the following relation holds

$$M_1\mathbf{x} = M_2\mathbf{y},$$

or equivalently,

$$M_1\mathbf{x} - M_2\mathbf{y} = \mathbf{0}.$$

We can rewrite the above equation as a homogeneous

$$[M_1 \ \ -M_2]\begin{bmatrix} \mathbf{x} \\ \mathbf{y} \end{bmatrix} = \mathbf{0}.$$

Hence to find a basis for $\mathbb{B}_1 \cap \mathbb{B}_2$, we need to determine a basis for the nullspace of the matrix $[M_1 \ \ -M_2]$.