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MATH 232 D100

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### Assignment 3

Q1:

To find if we can diagonalize, we found our eigenvalues using the eig function of our transition matrix:  $P = \begin{bmatrix} 8/10 & 3/10 & 3/10 \\ 1/10 & 6/10 & 1/10 \\ 1/10 & 1/10 & 6/10 \end{bmatrix}$ ,  $[V, D] = \text{eig}(P)$ . Using D, we found our unique eigenvalues and putting it into a matrix using unique(). We then indexed the eigenvalues array and counted the number of occurrences. For every unique eigenvalue we compared to the number of occurrences of eigenvalues in D. We saw for each eigenvalue resulted in algebraic multiplicity being equal to the algebraic multiplicity. **Eigenvalue of 0.5 had the same number of 2 for both algebraic and geometric. Eigenvalue of 1 has the same number of 1 for the algebraic and geometric multiplicity. What this means is this matrix can be diagonalized.** To find our ready state vector, we used  $\text{null}(I - P)$  and solved for  $3t + 1t + 1t = 1$  and  $t \cdot \text{null}(I - P)$  and gives us a steady state vector of  $[0.6; 0.2; 0.2]$ . I verified this result with the graph provided and it matches.

Q2:

To compute the graph using the powers of D instead of P we can use the eig function like we did above,  $[V, D] = \text{eig}(P)$ . Then we put it in the formula:  $\text{powersofD} = V \cdot D^k \cdot \text{inv}(V)$ ; This gives up the equivalence of P. We then replaced reference of P matrix with the powersofD matrix we calculated. A reason this could be better than using P itself is because D and V are easier to computer because when multiplying matrices with D, only the diagonal is considered which greatly reduces computation complexities for every power k of D.

Q3:

By doing the similar process above and finding our transition matrix,  $P = \begin{bmatrix} 0.8 & 0.2 & 0.3 \\ 0.1 & 0.7 & 0.2 \\ 0.1 & 0.1 & 0.5 \end{bmatrix}$ , we discovered that it can be diagonalized with the same algebraic and geometric multiplicity of 1 and 1 for all eigenvalues of 0.4, 0.6, 1. We got the steady state vector of  $[0.54; 0.29; 0.17]$  with the graph:

