Intrinsic Verification of Parsers and Formal Grammar Theory in Dependent Lambek Calculus

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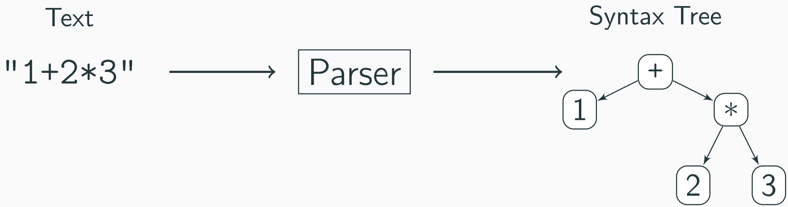
Parsing is Everywhere



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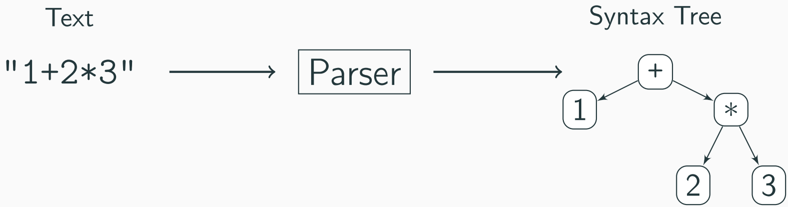

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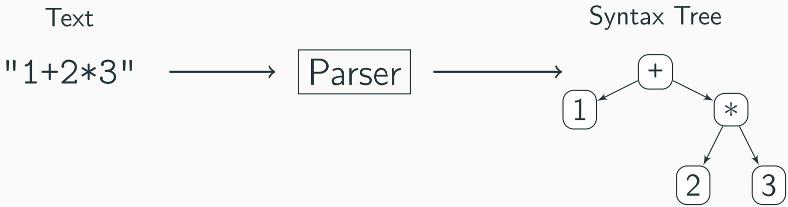

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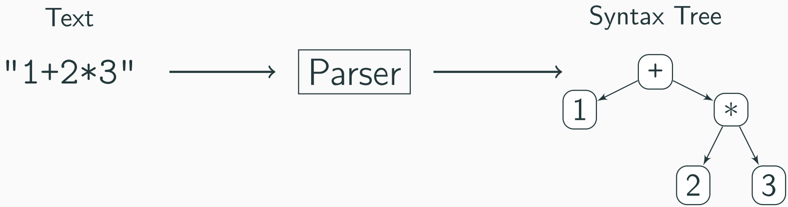

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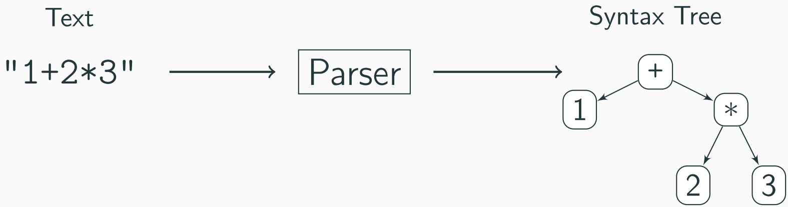

The Cost of Incorrect Parsers

- Critical real-world vulnerabilities, including remote code execution **(Equifax, 2017)**
- The guarantees of other software verification can be invalidated
- Early versions of CompCert
  - 0 middle or backend bugs, 6 parser bugs **(Yang et al., 2011)**

Ensuring Parser Correctness

For a formal grammar $\mathtt{A}$, a parser is a function $\mathtt{f} : \textbf{String} \rightharpoonup \textbf{Parse}(\mathtt{A})$

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Completeness Parser returns a parse tree whenever one exists

Verified Parsing Implementations

Existing verified parsers often implement a single formalism/algorithm

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- **LL**, **LR**, **CYK**, parser combinators, Brzozowski derivatives, etc
- Each formalism requires its own implementation

Dependent Lambek Calculus (Lambek$^{\mathbb{D}}$)

Lambek$^{\mathbb{D}}$: DSL for *intrinsically* verified parsing

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Universal formalism that expresses grammars as types

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Universal formalism that expresses grammars as types

Implemented as an embedded DSL in Cubical Agda

- Verified parsers for selected context-free grammars
- Verified parser generator for regular expressions

1. Lambek$^{\mathtt{D}}$

2. Dyck Grammar: An Extended Example

3. Conclusion

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Lambek Calculus: The Basis for Lambek$^{\mathbb{D}}$

Parsing is a deductive problem

$$\dfrac{\dfrac{\cdots}{\texttt{"a"} \vdash \texttt{'a'} \oplus \texttt{'b'}} \qquad \dfrac{\cdots}{\texttt{"b"} \vdash (\texttt{'a'} \oplus \texttt{'b'})^*}}{\texttt{"ab"} \vdash (\texttt{'a'} \oplus \texttt{'b'})^*}$$

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Lambek$^{\mathtt{D}}$ extends the expressivity of Lambek Calculus by adding dependent types

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data A* : LinTy where
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- Characters `'c'`
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- Kleene Star $\cdot^*$
  - Inductive linear type of lists
  - Arbitrary inductive types generalize to context-free grammars and beyond

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A Linear, Ordered Theory

$\boxed{\text{Parse trees of "cat"}}$

$$x : \texttt{'c'}, y : \texttt{'a'}, z : \texttt{'t'} \vdash (x, (y, z)) : \texttt{'c'} \otimes \texttt{'a'} \otimes \texttt{'t'} \quad \checkmark$$

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Variables used in order (no exchange)

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These restrictions ensure parsers are sound-by-construction

Terms as Grammar Reductions

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All strings parsed by $\mathtt{(A} \otimes \mathtt{A)}^*$ are also parsed by $\mathtt{A}^*$

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```
flatten : (A ⊗ A)* ⊸ A*
flatten nil = nil
flatten (cons (a , a') aas) =
   cons a (cons a' (flatten aas))
```

Dyck Grammar as a Type in Lambek$^{\mathrm{D}}$

Context-free grammar of balanced parentheses

$$D \to \varepsilon \mid \mathtt{(} D \mathtt{)} D$$

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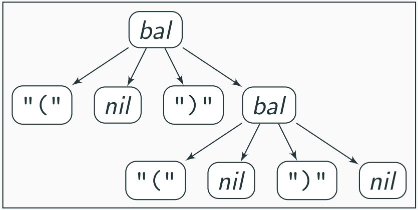
$$l : \texttt{'('}, r : \texttt{')'}, l' : \texttt{'('}, r' : \texttt{')'} \vdash bal\ l\ nil\ r\ (bal\ l'\ nil\ r') : \texttt{Dyck}$$



A Parser for the Dyck Grammar

Dyck Parser

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Completeness Not guaranteed, as the parser can choose to always fail

A Parser for the Dyck Grammar

Complete Dyck Parser

A *complete* parser for $\texttt{Dyck}$ is a choice of type $\texttt{Dyck}_{\neg}$ and function

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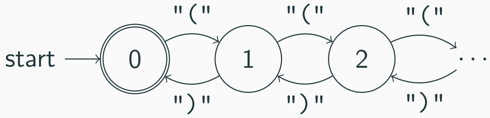
To define the parser we will use an automaton

Deterministic Automaton for the Dyck Grammar



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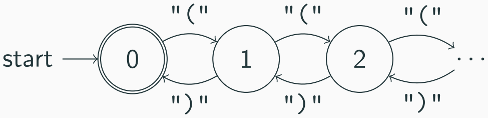


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data Trace : Bool → ℕ → LinTy where
 eof : Trace true 0
 leftovers : ∀ n → Trace false (suc n)
 push : ∀ b n → '(' ⊸ Trace b (suc n) ⊸ Trace b n
 pop : ∀ b n → ')' ⊸ Trace b n ⊸ Trace b (suc n)
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Trace b n denotes a path through the automaton from state n ending in some state with acceptance criteria b

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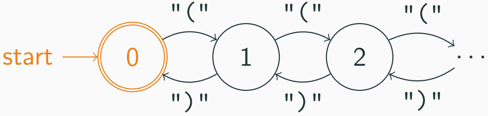


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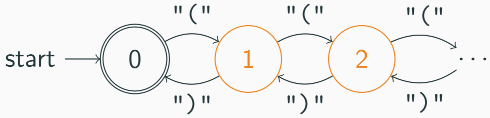


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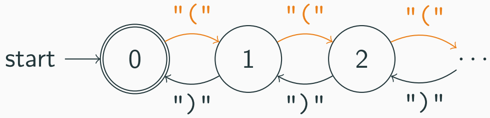


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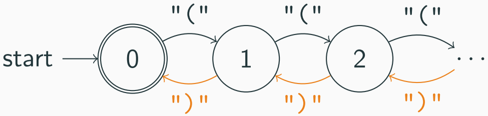


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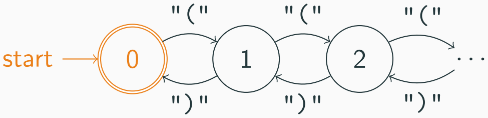


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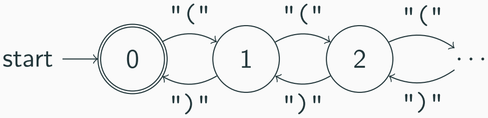


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A Parser for Dyck Traces

Looks like an ordinary functional program

```
parse : String ⊸ &[ n ∈ ℕ ] (Trace true n ⊕ Trace false n)
parse nil zero = σ true eof
parse nil (suc n) = σ false leftovers
parse (cons (σ '(' a) w) n =
  let σ b tr = parse w (suc n) in
  σ b (push a tr)
parse (cons (σ ')' a) w) zero =
  σ false (unexpected a _)
parse (cons (σ ')' a) w) (suc n) =
  let σ b tr = parse w n in
  σ b (pop a tr)
```

A Parser for the Dyck Grammar

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$$\texttt{String} \cong \texttt{Trace}\ \texttt{true}\ 0 \oplus \texttt{Trace}\ \texttt{false}\ 0$$

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$$\begin{aligned} \mathtt{String} &\multimap \mathtt{Trace}\ \mathtt{true}\ 0 \oplus \mathtt{Trace}\ \mathtt{false}\ 0 \\ &\multimap \mathtt{Dyck} \oplus \mathtt{Trace}\ \mathtt{false}\ 0 \end{aligned}$$

Strategy for Building a Parser

1. Describe a grammar A and the traces of an automaton each as *linear types*
2. In the equational theory of $\text{Lambek}^{\mathtt{D}}$, prove A is equivalent to the type of traces
3. Use the equivalence to create a parser for A

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- Parser for an $\mathbf{LL}(1)$ grammar of arithmetic expressions and a lookahead automaton
- *Parser generator* for regular expressions and finite automata
  - Thompson's construction and Powerset construction

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In the Paper

Proving unambiguity of grammars in $\mathsf{Lambek}^{\mathtt{D}}$

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Weak equivalence v. strong equivalence of grammars

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Key to describing grammars of unrestricted complexity

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Key to describing grammars of unrestricted complexity

Denotational Semantics

Interpret grammars in the category **String** $\to$ **Set**

Ongoing and Future Work

- Encode richer parsing algorithms (e.g. **LL**, **LR**, and forms of context-sensitivity)

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- Encode richer parsing algorithms (e.g. **LL**, **LR**, and forms of context-sensitivity)
- Verified semantic actions to integrate with language backends
- Verified frontends (typechecking and semantic analysis) in a modified version of Lambek$^{\mathtt{D}}$

Dependent Lambek Calculus (Lambek$^{\mathtt{D}}$)

Lambek$^{\mathtt{D}}$: DSL for *intrinsically* verified parsing

- Functional programming language for writing verified parsers
- Ordered, linear type system guarantees parser soundness for free
  - Completeness with a little effort
- Extensible framework for implementing grammar formalisms, their parsers, and their parser generators

Universal formalism that expresses grammars as types

Implemented as an embedded DSL in Cubical Agda

- Verified parsers for selected context-free grammars
- Verified parser generator for regular expressions