Learning to Generate Lumped Hydrological Models

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Abstract

In a lumped hydrological model, a catchment's hydrologic function is characterized by only a few parameters. This is due to

1 Introduction

Deep learning methods have been widely adopted in rainfall-runoff modeling, where they are commonly used to learn functions that map climate forcing time series to catchment runoff hydrographs from observation data (Nearing et al., 2021; Shen & Lawson, 2021). A hydrological model of a catchment can be derived by fitting parameters of a neural network to minimize errors in runoff prediction. In many of the current studies, the climate forcings are averaged over the catchment area (S. Anderson & Radić, 2022). This is similar to conventional lumped hydrological models, where the average responses of a catchment to the catchment-average climate forcings are modeled using empirical or physically based equations (Beven, 2011; M. P. Anderson et al., 2015; Yu, 2015; Coron et al., 2017).

In conventional lumped hydrological modeling, a unique model representation can be defined by a combination of a model structure and a set of parameter values (Spieler et al., 2020; Knoben et al., 2020). While the terms "model representation", "model structure", and "parameters" have been used without definition in many previous studies, this study provides simple definitions to avoid ambiguity and confusion in presentation and discussion. In this study, a unique model representation is referred to as a "model instance", which is a numerical model that can produce some outputs for given inputs. A model structure is referred to as a "model class", which is a function that can generate a model instance for a given set of settings, called "parameters". These definitions follow computer science conventions (Stefik & Bobrow, 1985; Wickham, 2019; Yang & Chui, 2023).

From these definitions, it is easy to compare the differences between the current deep learning methods and the conventional lumped hydrological modeling methods: the deep learning methods typically aim to learn model instances directly from the data, and the conventional modeling methods focus on developing different model classes that are capable of producing good model instances (Weiler & Beven, 2015). The difference between the goals of the two modeling approaches is similar to the difference between "discriminative modeling" and "generative modeling" approaches in machine learning. Discriminative modeling aims to learn good predictive models, and generative modeling aims to learn the underlying generative process that defines a joint distribution over random variables or simply how the observational data is generated (Kingma & Welling, 2019; Tomczak, 2022; Foster, 2022). A conventional lumped model class can be considered generative from this perspective because it provides a way to model how runoffs are generated under climate forcing for different parameter settings. In other words, for a model class, by sampling parameters from a certain distribution, we expect that the corresponding model instances under climate forcing can resemble the observed runoffs of all catchments in the world or selected groups of catchments.

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Discriminative models can be created from generative models (Kingma & Welling, 2019). For a given catchment and lumped model class, a discriminative model can be created by searching through the parameter space and selecting the one that is most likely to produce the observed runoffs under climate forcing. This discriminative is considered as an optimal model instance for the catchment, and the process of finding the optimal model known as model calibration in hydrological modeling (Beven, 2011; Efstratiadis & Koutsoyiannis, 2010; Pechlivanidis et al., 2011). This can be done because for a lumped model class there exists a mapping from the parameter space to a specific runoff generation function. The parameters used in a lumped model class usually have some physical meaning and are often of low dimension. Therefore, it is easy to determine a reasonable range for each parameter and sample from it.

Are the neural networks used in current hydrological studies generative? Yes, for a given network structure, different parameter values (i.e., weights and biases) can correspond to different model instances representing various fictional catchments. However, they have rarely been used in generative tasks. This is because the total number of parameters used in a network can easily exceed hundreds or thousands (Botterill & McMillan, 2023; Shen & Lawson, 2021; Kratzert, Klotz, Shalev, et al., 2019; Song et al., 2020; Solgi et al., 2021), which is much larger than the number of parameters in a lumped model. The high dimensionality of the parameter space makes it difficult to sample meaningful model instances. In addition, the parameters usually do not have an explicit physical meaning. Thus, it is not clear how to define a subspace of the high-dimensional parameter space from which one can sample meaningful model instances.

Attempts have been made to facilitate the generation of a meaningful model instance from a neural network structure. This is done by using other information to infer the parameter values that represent a catchment, and by reducing the number of parameters needed to generate a model instance (i.e., defining a smaller and meaningful subspace of the parameter space). For example, in the regional modeling methods (Kratzert, Klotz, Herrnegger, et al., 2019; Xu & Liang, 2021), a general rainfall-runoff model is learned for different catchments, and a unique model instance is created by feeding the general model a set of numerical values. Here, a general model and its required numerical values can be considered as a model class and parameters, respectively. The parameter values are usually catchment attributes that describe the physical characteristics of the catchment or their derivatives (Kratzert, Klotz, Shalev, et al., 2019; Kratzert, Klotz, Herrnegger, et al., 2019; Feng et al., 2020). Therefore, ideally, we can feed the general model with modified catchment attributes to create a new model instance.

However, the approach to generating new model instances in regional modeling methods has several limitations. First, by using catchment attributes to create model instances, it is assumed that the differences between the runoff generation functions of different catchments can be explained by these attributes. However, the degree of truth of this assumption can be easily affected by the choice of attributes, and the relationship between a catchment attribute and a catchment's runoff generation function captured by the general model may not be universally true for catchments from different parts of the world. Second, some catchment attributes may only serve the same purpose as purely random values for distinguishing between the catchments in a network, and the true connection between a catchment attribute and a catchment's runoff generation function may not be learned by the model. It has been shown in Li et al. (2022) that, in a regional modeling setting, the random values assigned to each catchment can be a good substitute for catchment attributes for distinguishing between catchments. Similarly, using random values as a surrogate for catchment attributes also does not ensure that the general model learns a useful mapping from the parameter space to the runoff generation function of a catchment.

There are other methods that can facilitate the generation of model instances from neural networks. For example, Botterill and McMillan (2023) used a catchment's rainfall-runoff time series to infer the parameters required to create a model instance, and Ma et al. (2021) used a weight freezing technique from transfer learning (Weiss et al., 2016) to reduce the number of parameters required to create a model instance.

It is worth noting that in a lumped hydrological model, the mapping from parameters to hydrological functions of model instances is explicitly defined, i.e., changing the parameter value is expected to change the hydrological function of the model instance.

Such a mapping, however, is not learned directly in many of the current deep learning methods. Instead, these methods try to learn a mapping between catchment-specific variables (such as catchment attributes) and different hydrological functions. However, there is no guarantee that there is a meaningful mapping from catchment identity to hydrological functions.

and such a mapping is not learned explicitly in most of the current deep learning methods used in hydrology.

In summary, in most of the current deep learning methods, each catchment is assigned a fixed vector (such as catchment attributes) or catchment specific inputs representing catchment identity, and the mapping from the identity variable to different hydrological function of the corresponding model instances is learned from data. However, there is no guarantee that there exist a meaningful mapping from catchment identity to hydrological functions.

2 Methods and materials

3 Numerical experiments and results

3.1 Using generative models in regional hydrological modeling

3.1.1 Experiments performed

3.1.2 Results

3.1.3 Discussions

3.2 Using generative models as conventional lumped model classes

3.2.1 Experiments performed

3.2.2 Results

3.2.3 Discussions

4 Conclusions

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