post-Einsteinian Parameters for Different Modified Gravity Theories

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1 Introduction

2 ppE Paramters

Fourier waveform for inspiral phase in standard ppE framework [7],

$$\tilde{h}(f) = \tilde{h}_{GR}(1 + \alpha u^a)e^{\delta\Psi} \tag{1}$$

$$\delta\Psi = i\beta u^b \tag{2}$$

$$u = (\mathcal{M}_c\Omega)^{\frac{1}{3}} \tag{3}$$

In above equations α, β, a, and b are ppE parameters. Ω is the orbital angular frequency and $\mathcal{M}_c$ is the chirp mass. In order to obtain the expression of these parmeteres for different modified gravity theories we consider correction to binding energy E and rate of change of binding energy $\dot{E}$. These corrections lead to the modification of Kepler's law and frequency evolution $\dot{f}$.

For conservative correction we need to consider modification to binding energy which can be parametrized as follows [1]

$$E = E_{GR}[1 + A(\frac{m}{r})^p] \tag{4}$$

where we assume A is small, such that the correction represents a small deformation away from GR. Such a binding energy modifies Keplers third law as

$$\Omega^2 = \frac{m}{r^3}[1 + \frac{1}{2}Ap(\frac{m}{r})^p] \tag{5}$$

Above equation gives us orbital seperation as

$$a(t) = a_{GR}[1 + \frac{1}{6}Apu^{2p}\eta^{-\frac{2p}{5}}] \tag{6}$$

Dissipative correction to waveform comes from the modification of rate of change of binding energy. Let us assume a modification to the rate of change of binding energy of the form [1]

$$\dot{E} = \dot{E}_{GR}[1 + B(\frac{m}{r})^q] \tag{7}$$

where $\dot{E}_{GR}$ is the GR energy flux and the second term is assumed small relative to the first, as we are interested in small deformations away from GR. By invoking energy balance law equation (7) gives us the frequency evolution as

$$\dot{f} = \dot{f}_{GR}[1 + B\eta^{\frac{-2q}{5}}u^{2q} + \frac{1}{3}A(2p^2 - 2p - 3)\eta^{\frac{-2p}{5}}u^{2p}] \tag{8}$$

From stationary phase approximation [6],

$$\tilde{h}(f) = \frac{\mathcal{A}(t_0)}{\sqrt{l\dot{F}}}e^{-i\Psi} \tag{9}$$

Correction to the Amplitude

We consider correction to the quadruple radiation only. So, we will take $l = 2$ in (9) which gives the amplitude of Fourier waveform as

$$\tilde{\mathcal{A}}(f) = \frac{\mathcal{A}(t_0)}{\sqrt{\dot{f}}} \tag{10}$$

$\dot{f}$ is the frequency evolution of gravitational wave. $\mathcal{A}(t_0) \sim \ddot{Q}$ where Q is the quadruple moment which is proportional to $a(t_0)^2$. So we can write

$$\mathcal{A}(f) \sim \frac{a(t)^2}{\sqrt{\dot{f}}} \tag{11}$$

Using equation (8) and (6) in (11) and keeping only leading order terms,

$$\tilde{\mathcal{A}}(f) = \tilde{\mathcal{A}}_{GR}[1 - \frac{B}{2}\eta^{\frac{-2q}{5}}u^{2q} - \frac{1}{6}A(2p^2 - 2p - 3)\eta^{\frac{-2p}{5}}u^{2p} + \frac{1}{3}Ap\eta^{\frac{-2p}{5}}u^{2p}] \tag{12}$$

When dissipative correction dominates ($q > p$), comparing above equation with the ppE waveform in (1),

$$\alpha = -\frac{1}{2}B\eta^{\frac{-2q}{5}}, \qquad a = 2q \tag{13}$$

When conservative correction dominates ($q < p$),

$$\alpha = \frac{1}{6}A(4p - 2p^2 + 3)\eta^{\frac{-2p}{5}}, \qquad a = 2p \tag{14}$$

If the modifications to the binding energy enter at the same PN order as the modifications to the energy flux, ie. $q = k = p$, then

$$\alpha = (\frac{1}{6}A(4k - 2k^2 + 3) - \frac{B}{2})\eta^{\frac{-2k}{5}}, \qquad a = 2k \tag{15}$$

Correction to the Phase

Modification to the fourier phase in leading order can be calculated as [1]

$$\delta\Psi = -\frac{5}{32}A\frac{2p^2 - 2p - 3}{(4-p)(5-2p)}\eta^{\frac{-2p}{5}}u^{2p-5} - \frac{15}{32}B\frac{1}{(4-q)(5-2q)}\eta^{\frac{-2q}{5}}u^{2q-5} \tag{16}$$

Comparing with (1) we find ppE parameters β and b as

$$q > p, \qquad \beta = -\frac{15}{32}B\frac{1}{(4-q)(5-2q)}\eta^{\frac{-2q}{5}}, \qquad , b = 2q - 5 \tag{17}$$

$$q < p, \qquad \beta = -\frac{5}{32}A\frac{2p^2 - 2p - 3}{(4-p)(5-2p)}\eta^{\frac{-2p}{5}}, \qquad b = 2p - 5 \tag{18}$$

$$q = p = k, \qquad \beta = -(\frac{5}{32}A\frac{2k^2 - 2k - 3}{(4-k)(5-2k)} + \frac{15}{32}B\frac{1}{(4-k)(5-2k)})\eta^{\frac{-2k}{5}} \tag{19}$$

Alternative

We want to write orbital seperation as,

$$a = a_{GR}(1 + \gamma_a u^{c_a}) \tag{20}$$

and frequency evolution as,

$$\dot{f} = \dot{f}_{GR}(1 + \gamma_{\dot{f}} u^{c_{\dot{f}}}) \tag{21}$$

Comparing with (6),

$$c_a = 2p, \qquad \gamma_a = \frac{1}{6} Ap\eta^{-2p/5} \tag{22}$$

$\gamma_{\dot{f}}$ and $c_{\dot{f}}$ can take different forms depending on whether the dominative correction is dissipative or conservative.

Dissipative $(p < q)$

Comparing with (8),

$$\gamma_{\dot{f}} = B\eta^{\frac{-2q}{5}}, \qquad c_{\dot{f}} = 2q \tag{23}$$

ppE parameters,

$$\beta = -\frac{15\gamma_{\dot{f}}}{16(c_{\dot{f}} - 8)(c_{\dot{f}} - 5)} \tag{24}$$

$$b = c_{\dot{f}} - 5, \qquad a = c_{\dot{f}} \tag{25}$$

$$\alpha = -\frac{\gamma_{\dot{f}}}{2} \tag{26}$$

Conservative$(p > q)$

$$\gamma_{\dot{f}} = \frac{1}{3} A(2p^2 - 2p - 3)\eta^{\frac{-2p}{5}}, \qquad c_{\dot{f}} = c_a = 2p \tag{27}$$

ppE parameters,

$$\beta = -\frac{15\gamma_{\dot{f}}}{16(c_{\dot{f}} - 8)(c_{\dot{f}} - 5)} \tag{28}$$

$$b = c_{\dot{f}} - 5, \qquad a = c_a \tag{29}$$

$$\alpha = -\frac{\gamma_a}{c_a}(c_a^2 - 4c_a - 6) \tag{30}$$

Dissipative=Conservative $(p = q = k)$

$$\gamma_{\dot{f}} = B\eta^{\frac{-2k}{5}} + \frac{1}{3} A(2k^2 - 2k - 3)\eta^{\frac{-2k}{5}}, \qquad c_{\dot{f}} = 2k \tag{31}$$

ppE parameters,

$$\beta = -\frac{15\gamma_{\dot{f}}}{16(c_{\dot{f}} - 8)(c_{\dot{f}} - 5)} \tag{32}$$

$$b = c_{\dot{f}} - 5, \qquad a = c_{\dot{f}} \tag{33}$$

$$\alpha = 2\gamma_a - \frac{\gamma_{\dot{f}}}{2} \tag{34}$$

3 Example Theories

3.1 EdGB Theory

Dissipative correction dominates. From [8] and [4],

$$\beta_{EdGB} = \frac{-5}{7168}\zeta_{EdGB}\frac{(m_1^2 s_2^{EdGB} - m_2^2 s_1^{EdGB})^2}{m^4\eta^{\frac{18}{5}}} \tag{35}$$

$$\alpha_{EdGB} = \frac{-5}{192}\zeta_{EdGB}\frac{(m_1^2 s_2^{EdGB} - m_2^2 s_1^{EdGB})^2}{m^4\eta^{\frac{18}{5}}} \tag{36}$$

3.2 Scalar-Tensor Theories

Dissipative correction dominates. From [8],

$$\beta_{SC} = \frac{-5}{1792}\dot{\phi}^2\eta^{\frac{2}{5}}(m_1 s_1^{ST} - m_2 s_2^{ST})^2 \tag{37}$$

$$\alpha_{ST} = \frac{-5}{48}\dot{\phi}^2\eta^{\frac{2}{5}}(m_1 s_1^{ST} - m_2 s_2^{ST})^2 \tag{38}$$

3.3 DCS Theory

Dissipative and conservative correction enters at same order. From [5],

$$\beta_{DCS} = \frac{1549225\zeta_{DCS}}{11812864\eta^{14/5}}(-2\delta_m\chi_a\chi_s + \left(1 - \frac{16068\eta}{61969}\right){\chi_a}^2 + \left(1 - \frac{231808\eta}{61969}\right){\chi_s}^2) \tag{39}$$

$$\alpha_{DCS} = \frac{185627\zeta_{DCS}}{1107456\eta^{14/5}}(-2\delta_m\chi_a\chi_s + \left(1 - \frac{53408\eta}{14279}\right){\chi_a}^2 + \left(1 - \frac{3708\eta}{14279}\right){\chi_s}^2) \tag{40}$$

3.4 Einstein-Æther Theory

Dissipative correction dominates. From [2],

$$\beta_{Æ} = -\frac{5\eta^{2/5}\left(s_1 - s_2\right)^2\left(\left(c_{14}-2\right)w_0^3 - w_1^3\right)}{3584c_{14}w_0^3w_1^3\left(G_N\left(s_1-1\right)\left(s_2-1\right)\right)^{4/3}} \tag{41}$$

$$\alpha_{Æ} = -\frac{5\eta^{2/5}\left(s_1 - s_2\right)^2\left(\left(c_{14}-2\right)w_0^3 - w_1^3\right)}{96c_{14}w_0^3w_1^3\left(G_N\left(s_1-1\right)\left(s_2-1\right)\right)^{4/3}} \tag{42}$$

3.5 KG Theory

Dissipative correction dominates. From [2],

$$\alpha_{KG} = -\frac{5\sqrt{\alpha^{KG}}\left(\frac{(\beta^{KG}-1)(2+\beta^{KG}+3\lambda^{KG})}{(\alpha^{KG}-2)(\beta^{KG}+\lambda^{KG})}\right)^{3/2}\eta^{2/5}(s_1-s_2)^2}{96(G_N(s_1-1)(s_2-1))^{4/3}} \tag{43}$$

$$\beta_{KG} = -\frac{5\sqrt{\alpha^{KG}}\left(\frac{(\beta^{KG}-1)(2+\beta^{KG}+3\lambda^{KG})}{(\alpha^{KG}-2)(\beta^{KG}+\lambda^{KG})}\right)^{3/2}\eta^{2/5}(s_1-s_2)^2}{3584(G_N(s_1-1)(s_2-1))^{4/3}} \tag{44}$$

3.6 Non-Commutative Gravity

Dissipative and conservative correction enters at same order. [3]

$$\alpha_{NC} = -\frac{3(2\eta-1)\Lambda^2}{8\eta^{4/5}} \tag{45}$$

$$\beta_{NC} = -\frac{75(2\eta-1)\Lambda^2}{256\eta^{4/5}} \tag{46}$$

3.7 Varying-G Theory

$$\alpha_{\dot{G}} = \frac{-5M_0{\eta_0}^{3/5}}{512{G_0}^{5/3}}\left(\frac{7\dot{G}}{G_0} + \frac{5\dot{M}}{M_0} + \frac{M_{1,0}\dot{M}_2 + M_{2,0}\dot{M}_1}{{M_0}^2\eta 0}\right) \tag{47}$$

$$\beta_{\dot{G}} = \frac{-75M_0{\eta_0}^{3/5}}{851968{G_0}^{10/3}}\left(\frac{11\dot{G}}{3G_0} + \frac{17\dot{M}}{3M_0} - \frac{M_{1,0}\dot{M}_2 + M_{2,0}\dot{M}_1}{{M_0}^2\eta 0}\right) \tag{48}$$

4 Table

ppE Parameters				
Theories	β	b	α	a
EdGB	$\frac{-5}{7168}\zeta_{EdGB}\frac{(m_1^2 s_2^{EdGB}-m_2^2 s_1^{EdGB})^2}{m^4\eta^{\frac{18}{5}}}$	-7	$\frac{-5}{192}\zeta_{EdGB}\frac{(m_1^2 s_2^{EdGB}-m_2^2 s_1^{EdGB})^2}{m^4\eta^{\frac{18}{5}}}$	-2
Scalar-Tensor	$\frac{-5}{1792}\dot{\phi}^2\eta^{\frac{2}{5}}(m_1 s_1^{ST}-m_2 s_2^{ST})^2$	-7	$\frac{-5}{48}\dot{\phi}^2\eta^{\frac{2}{5}}(m_1 s_1^{ST}-m_2 s_2^{ST})^2$	-2
DCS	$\frac{1549225\zeta_{DCS}}{11812864\eta^{14/5}}\left(-2\delta_m\chi_a\chi_s+\left(1-\frac{16068\eta}{61969}\right)\chi_a{}^2+\left(1-\frac{231808\eta}{61969}\right)\chi_s{}^2\right)$	-1	$\frac{185627\zeta_{DCS}}{1107456\eta^{14/5}}\left(-2\delta_m\chi_a\chi_s+\left(1-\frac{53408\eta}{14279}\right)\chi_a{}^2+\left(1-\frac{3708\eta}{14279}\right)\chi_s{}^2\right)$	4
KG	$\frac{-5\sqrt{\alpha^{KG}}\left(\frac{(\beta^{KG}-1)(2+\beta^{KG}+3\lambda^{KG})}{(\alpha^{KG}-2)(\beta^{KG}+\lambda^{KG})}\right)^{3/2}\eta^{2/5}(s_1-s_2)^2}{3584(G_N(s_1-1)(s_2-1))^{4/3}}$	-7	$\frac{112}{3}\beta_{KG}$	-2
NC	$-\frac{75(2\eta-1)\Lambda^2}{256\eta^{4/5}}$	-1	$-\frac{3(2\eta-1)\Lambda^2}{8\eta^{4/5}}$	4
Einstein-Æther	$-\frac{5\eta^{2/5}(s_1-s_2)^2\left((c_{14}-2)w_0^3-w_1^3\right)}{3584c_{14}w_0^3w_1^3(G_N(s_1-1)(s_2-1))^{4/3}}$	-7	$-\frac{5\eta^{2/5}(s_1-s_2)^2\left((c_{14}-2)w_0^3-w_1^3\right)}{96c_{14}w_0^3w_1^3(G_N(s_1-1)(s_2-1))^{4/3}}$	-2
Varying-G Theory	$-\frac{75M_0\eta_0{}^{3/5}}{851968G_0{}^{10/3}}\left(\frac{11\dot{G}}{3G_0}+\frac{17\dot{M}}{3M_0}-\frac{M_{1,0}\dot{M}_2+M_{2,0}\dot{M}_1}{M_0{}^2\eta 0}\right)$	-13	$\frac{-5M_0\eta_0{}^{3/5}}{512G_0{}^{5/3}}\left(\frac{7\dot{G}}{G_0}+\frac{5\dot{M}}{M_0}+\frac{M_{1,0}\dot{M}_2+M_{2,0}\dot{M}_1}{M_0{}^2\eta 0}\right)$	-8

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