Parameterized Post-Einsteinian Gravitational Waveforms in Various Modified Theories of Gravity

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I. INTRODUCTION

General relativity (GR) is one of the cornerstones of modern physics, and so far the most successful theory of gravitation. Along with the elegant mathematical structure and solid conceptual foundation, GR has passed all the tests with high accuracy [1]. However, there are theoretical and observational motivations which lead to the demand of a modified theory of gravitation. Regarding the former, GR is a purely classical theory and incompatible with quantum mechanics. Strong gravitational fields at Planck scale where quantum effects cannot be ignored [2, 3], such as black holes and very early universe, require a consistent theory of quantum gravity for complete description. Regarding the latter, puzzling observations such as accelerated expansion of the universe [4–11] and anomalous kinematics of the galaxies [12–18] also suggest that one may need to go beyond GR to explain such cosmological phenomena if one does not wish to introduce dark energy or dark matter that are currently unknown.

Before gravitational waves (GWs) were directly detected by LIGO and Virgo, tests of gravity mainly focused on using solar system experiments and observations of radio pulsars and cosmology. Each of these cover different ranges of length scale and curvature strength. Solar system experiments constrain gravity in the weak field and slow motion environment. In terms of relativistic equations of motion, such experiments give access mostly to first order corrections to Newtonian dynamics [1, 19]. Pulsar timing observations of neutron stars offer us both weak-field and strong-field tests of gravity [20–29]. On one hand, separations of binaries are wide and the relative motion of two stars in a binary is slow (and thus weak-field). On the other hand, binary pulsars consist of neutron stars which are compact and are strong-field sources of gravity. Cosmological observations constrain gravity in the weak field regime but at length scales which are many orders of magnitude larger compared to other tests [19, 30–33]. Cosmological tests of gravity include observations of cosmic microwave background radiation (CMB) [34–39], study of big bang nucleosynthesis (BBN) [40–46], weak gravitational lensing [47–52], and observations of galaxies [19]. Other tests include using orbital motion of stars near the Galactic Center [53–55].

Until now, six GW sources have been found (five of them are from binary black holes [56–60] and the remaining one is from a binary neutron star [61]), which opened completely new ways of testing GR. GWs provide the opportunity to probe gravity in the strong-field and highly dynamical regime. Binary black hole events have been used to carry out a model-independent test of gravity by estimating the amount of residuals in the detected signals of GW150914 from the best-fit waveform [62]. GW150914 has also been used to perform a consistency test of GR between the inspiral and post-inspiral phases [62]. An addition of Virgo allowed one to look for non-tensorial polarization modes of GWs [60]. Meanwhile, the arrival time difference between gravitons and photons in the binary neutron star merger event GW170817 can be used to constrain the deviation in the propagation speed of the former from the latter to one part in 10^{15}, to place bounds on the violation of Lorentz invariance and to carry out a new test of the equivalence principle via the Shapiro time delay [63]. Such a constraint on the propagation speed of GWs has led one to rule out many of modified theories of gravity that can explain current accelerating expansion of our Universe without introducing dark energy [64–68]. So far, no evidence has been found that indicates non-GR effects.

One can carry out yet another type of tests of GR by directly measuring or constraining non-GR parameters in the waveform. One can derive modifications to GR waveforms by choosing specific modified theories of gravity, though perhaps a more efficient approach is to perform the test in a model-independent way. A pioneering work along this line has been carried out in [69–71], where the authors treat each post-Newtonian (PN) term in the waveform independently and look for consistency among them. Based on this, a data analysis pipeline (TIGER) was developed [72, 73]. One drawback of such a formalism is that one can only treat PN terms in non-GR theories that are also present in GR, and one cannot capture e.g. scalar dipole radiation effect entering at a negative PN order that is absent in GR. To over come this, Yunes and Pretorius [74] proposed a new framework called *parameterized post-Einsteinian* formalism, where they introduced new parameters that can capture non-GR effects in waveforms in a generic way. The original work focused on tensorial polarizations for a quasi-circular binaries and introduced only the leading PN non-

GR corrections in Fourier domain. Such an analysis was later extended to include non-tensorial polarizations [75] and multiple PN correction terms [76], and for time domain waveforms [77], eccentric binaries [78] and a sudden turn on of non-GR effects [79, 80]. The LIGO Scientific Collaboration and Virgo Collaboration developed a generalized IMRPhenom model [63] that is similar to the PPE formalism [81]. Generic non-GR parameters in the waveform phase have been constrained in [58, 63, 81, 82].

In this paper, we derive PPE waveforms in various modified theories of gravity. Many of previous literature focused on deriving phase corrections since matched filtering is more sensitive to such phase corrections than to amplitude corrections. Having said this, there are situations where amplitude corrections are more useful to probe, such as amplitude birefringence in parity violating theories of gravity [83–86]. We first derive PPE amplitude and phase corrections in terms of generic modifications to the frequency evolution and Kepler's third law that determines the waveform in Fourier domain. For our purpose, this formalism is more useful than that in [75], which derives the amplitude and phase corrections in terms of generic modifications to the binding energy of a binary and GW luminosity. We follow the original PPE framework and focus on tensorial modes only [74, 87]. At least in scalar-tensor theories, amplitude of a scalar polarization is of higher PN order than amplitude corrections to tensor modes [75, 88].

Non-GR corrections can enter in the GW waveform through activation of different theoretical mechanisms, which can be classified as generation mechanisms and propagation mechanisms [81]. Generation mechanisms take place close to the source (binary), while propagation mechanisms occur in the far-zone and accumulate as the waves propagate. In this paper, we focus on the former. PPE parameters in various modified theories of gravity are summarized in Table I and Table II. Some of amplitude corrections were derived here for the first time. We also correct some of errors in previous literature. Corrections to propagation effects have been used to place bounds on the mass of the graviton [63]. PPE waveforms due to modifications in the propagation sector can be found in Refs. [81, 89, 90], which have been used for GW150914, GW151226 [81] and GW170104 [58].

II. PPE WAVEFORM

We begin by reviewing the PPE formalism. The original formalism (that we explain in detail in App. A) was developed by considering non-GR corrections to the binding energy E and GW luminosity $\dot{E}$ [74, 75]. The former (latter) correspond to conservative (dissipative) corrections. Here, we take a slightly different approach and consider corrections to the GW frequency evolution $\dot{f}$ and the Kepler's law $r(f)$, where r is the orbital separation while f is the GW frequency. This is because these two quantities directly determine the amplitude and phase corrections away from GR, and hence, the final expressions are simpler than the original ones. Moreover, non-GR corrections to $\dot{f}$ and $r(f)$ have already been derived in the literature for many modified theories of gravity.

PPE gravitational waveform for a compact binary inspiral in the Fourier domain is given by [74]

$$\tilde{h}(f) = \tilde{h}_{\rm GR}(1+\alpha u^a)e^{i\delta\Psi}\,, \tag{1}$$

where $\tilde{h}_{\rm GR}$ is the gravitational waveform in GR. αu^a corresponds to the non-GR correction to the GW amplitude while $\delta\Psi$ is that to the GW phase with

$$u = (\pi \mathcal{M} f)^{\frac{1}{3}}\,. \tag{2}$$

$\mathcal{M} = (m_1 m_2)^{3/5}/(m_1+m_2)^{1/5}$ is the chirp mass for the binary with component masses m_1 and m_2. u is proportional to the relative velocity of the binary components. α represents the overall magnitude of the amplitude correction while a gives the velocity dependence of the correction term. In a similar manner, one can rewrite the phase correction as

$$\delta\Psi = \beta u^b\,, \tag{3}$$

α, β, a, and b are called PPE parameters. When $(\alpha,\beta) = (0,0)$, Eq. (1) reduces to the waveform in GR.

As we mentioned earlier, the PPE modifications in Eq. (1) enter through corrections to the orbital separation and the frequency evolution. We parameterize the former as

$$r = r_{\rm GR}(1+\gamma_r u^{c_r})\,, \tag{4}$$

where γ_r and c_r are non-GR parameters which show the deviation of the orbital separation r away from the GR contribution $r_{\rm GR}$. To leading PN order, $r_{\rm GR}$ is simply given by the Newtonian Kepler's law as $r_{\rm GR} = \left(m/\Omega^2\right)^{1/3}$. Here $m \equiv m_1+m_2$ is the total mass of the binary while $\Omega \equiv \pi f$ is the orbital angular frequency. The above correction to the orbital separation comes purely from conservative corrections (namely corrections to the binding energy).

Similarly, we parameterize the GW frequency evolution with non-GR parameters $\gamma_{\dot{f}}$ and $c_{\dot{f}}$ as

$$\dot{f} = \dot{f}_{\rm GR}\left(1+\gamma_{\dot{f}} u^{c_{\dot{f}}}\right)\,. \tag{5}$$

Here $\dot{f}_{\rm GR}$ is the frequency evolution in GR which, to leading PN order, is given by [99, 100]

$$\dot{f}_{\rm GR} = \frac{96}{5}\pi^{8/3}\mathcal{M}^{5/3}f^{11/3} = \frac{96}{5\pi\mathcal{M}^2}u^{11}\,. \tag{6}$$

Unlike the correction to the orbital separation, the one to the frequency evolution originates corrections from both the conservative and dissipative sectors.

Below, we will derive how the PPE parameters (α,β,a,b) are given in terms of (γ_r, c_r) and $(\gamma_{\dot{f}}, c_{\dot{f}})$. We

Theories	PPE Phase Parameters		Binary Type
	Magnitude (β)	Exp. (b)	
Scalar-Tensor Theories [91, 92]	$-\frac{5}{7168}\eta^{2/5}(\alpha_1-\alpha_2)^2$	-7	Any
EdGB Gravity [93]	$-\frac{5}{7168}\zeta_{\rm EdGB}\frac{\left(m_1^2\tilde{s}_2^{\rm EdGB}-m_2^2\tilde{s}_1^{\rm EdGB}\right)^2}{m^4\eta^{18/5}}$	-7	Any
dCS Gravity [81, 94]	$\frac{1549225}{11812864}\eta^{-14/5}\zeta_{\rm dCS}\left[-2\delta_m\chi_a\chi_s+\left(1-\frac{16068\eta}{61969}\right)\chi_a^2+\left(1-\frac{231808\eta}{61969}\right)\chi_s^2\right]$	-1	BH/BH
Einstein-Æther Theory [95]	$-\frac{5}{3584}\eta^{2/5}\frac{(s_1^{\rm EA}-s_2^{\rm EA})^2}{[(1-s_1^{\rm EA})(1-s_2^{\rm EA})]^{4/3}}\frac{(c_{14}-2)w_0^3-w_1^3}{c_{14}w_0^3w_1^3}$	-7	Any
Khronometric Theory [95]	$-\frac{5}{96}\eta^{2/5}\frac{(s_1^{\rm kh}-s_2^{\rm kh})^2}{[(1-s_1^{\rm kh})(1-s_2^{\rm kh})]^{4/3}}\sqrt{\bar{\alpha}_{\rm kh}}\left[\frac{(\bar{\beta}_{\rm kh}-1)(2+\bar{\beta}_{\rm kh}+3\bar{\lambda}_{\rm kh})}{(\bar{\alpha}_{\rm kh}-2)(\bar{\beta}_{\rm kh}+\bar{\lambda}_{\rm kh})}\right]^{3/2}$	-7	Any
Noncommutative Gravity [96]	$-\frac{75}{256}\eta^{-4/5}(2\eta-1)\Lambda^2$	-1	BH/BH
Varying-G Theories [97]	$-\frac{\mathbf{75\mathcal{M}_0}}{\mathbf{851968G_0}^{\mathbf{10/3}}}\left(\frac{\mathbf{11\dot{G}_0}}{\mathbf{3G_0}}+\frac{\mathbf{17\dot{m}_0}}{\mathbf{3m_0}}-\frac{\mathbf{m_{1,0}\dot{m}_{2,0}+\dot{m}_{1,0}m_{2,0}}}{\mathbf{m_0}^{\mathbf{2}}\eta\mathbf{0}}\right)$	$\mathbf{-13}$	Any

TABLE I. PPE corrections to the GW phase $\delta\Psi\equiv\beta u^b$ in Fourier space in various modified theories of gravity, where β is the magnitude correction (second coulmn) and b is the exponent correction (third column). $u\equiv(\pi\mathcal{M}f)^{1/3}$, where $\mathcal{M}$ and η are the chirp mass and the symmetric mass ratio of the binary respectively. The expressions in dCS and noncommutative gravity only applies to binary BHs, while those in other theories apply to any compact binaries (last column). The mass, sensitivity, and scalar charge of the Ath binary component are reperesented by m_A, s_A, and α_A respectively. $\zeta_{\rm EdGB}$ and $\zeta_{\rm dCS}$ are the dimensionless coupling constants in EdGB and dCS gravity respectively. $\tilde{s}_A^{\rm EdGB}$ are the spin-dependent factors of the scalar charges in EdGB gravity, given below Eq. (29) for BHs while 0 for ordinary stars. $\chi_{s,a}$ are the symmetric and antisymmetric combinations of dimensionless spin parameters and δ_m is the fractional difference in masses relative to the total mass m. The amount of Lorentz violation in Einstein-Æther theory and khronometric gravity is controlled by (c_1,c_2,c_3,c_4) and $(\bar{\alpha}_{\rm kh},\bar{\beta}_{\rm kh},\bar{\lambda}_{\rm kh})$ respectively. w_s is the propagation speed of spin-s mode in EA theory given by Eqs. (36)-(38), and $c_{14}\equiv c_1+c_4$. The representative parameter in noncommutative gravity is Λ. Subscript 0 in varying-G theories denotes the quantity measured at the time of coalescence t_0, while a dot refers to a time derivative.

will also show how the amplitude PPE parameters (α,a) can be related to the phase PPE ones (β,b) in certain cases. We will assume that non-GR corrections are always smaller than the GR contribution and keep only to leading order in such corrections at the leading PN order.

A. Amplitude Corrections

Let us first look at the amplitude corrections. Within the stationary phase approximation [101, 102] the waveform amplitude for the dominant quadrupolar radiation in Fourier domain is given by

$$\tilde{\mathcal{A}}(f)=\frac{\mathcal{A}(\bar{t})}{\sqrt{\dot{f}}}\,. \tag{7}$$

Here $\mathcal{A}$ is the waveform amplitude in the time domain while $\bar{t}(f)$ represents time at the stationary point. $A(\bar{t})$ can be obtained by using the quadrupole formula for the metric perturbation in the transverse-traceless gauge given by [103]

$$h^{ij}(t)\propto\frac{G}{D_L}\frac{d^2}{dt^2}Q^{ij}\,. \tag{8}$$

Here D_L is the source's luminosity distance and Q^{ij} is the source's quadruple moment tensor.

For a quasi-circular compact binary, $\tilde{\mathcal{A}}$ in Eq. (7) then becomes

$$\tilde{\mathcal{A}}(f)\propto\frac{1}{\sqrt{\dot{f}}}\frac{G}{D_L}\mu r^2f^2\propto\frac{r^2}{\sqrt{\dot{f}}}\,, \tag{9}$$

where μ is the reduced mass of the binary. Substituting Eqs. (4) and (5) into Eq. (9) and keeping only to leading order in non-GR corrections, we find

$$\tilde{\mathcal{A}}(f)=\tilde{\mathcal{A}}_{\rm GR}\left(1+2\gamma_r u^{c_r}-\frac{1}{2}\gamma_{\dot{f}}u^{c_{\dot{f}}}\right)\,, \tag{10}$$

where $\tilde{\mathcal{A}}_{\rm GR}$ is the amplitude of Fourier waveform in GR. Notice that this expression is much simpler than that in the original formalism in Eq. (A7).

Let us now show the expressions for the PPE parameters α and a for three different cases using Eq. (10):

- *Dissipative-dominated Case*

  When dissipative corrections dominate, we can neglect corrections to the binary separation ($\gamma_r=0$) and Eq. (10) reduces to

$$\tilde{\mathcal{A}}(f)=\tilde{\mathcal{A}}_{\rm GR}\left(1-\frac{1}{2}\gamma_{\dot{f}}u^{c_{\dot{f}}}\right)\,. \tag{11}$$

Theories	PPE Amplitude Parameters	
	Magnitude (α)	Exponent (a)
Scalar-Tensor Theories [75, 88, 98]	$-\frac{5}{192}\eta^{2/5}(\alpha_1-\alpha_2)^2$	-2
EdGB Gravity	$-\frac{\mathbf{5}}{\mathbf{192}}\zeta_{\rm EdGB}\frac{\left(\mathbf{m_1^2\tilde{s}_2^{\rm EdGB}-m_2^2\tilde{s}_1^{\rm EdGB}}\right)^{\mathbf{2}}}{\mathbf{m^4}\eta^{\mathbf{18/5}}}$	$\mathbf{-2}$
dCS Gravity	$\frac{\mathbf{185627}}{\mathbf{1107456}}\eta^{\mathbf{-14/5}}\zeta_{\rm dCS}\left[\mathbf{-2}\delta_{\mathbf{m}}\chi_{\mathbf{a}}\chi_{\mathbf{s}}+\left(\mathbf{1}-\frac{\mathbf{53408}\eta}{\mathbf{14279}}\right)\chi_{\mathbf{a}}^{\mathbf{2}}+\left(\mathbf{1}-\frac{\mathbf{3708}\eta}{\mathbf{14279}}\right)\chi_{\mathbf{s}}^{\mathbf{2}}\right]$	$\mathbf{+4}$
Einstein-Æther Theory [95]	$-\frac{\mathbf{5}}{\mathbf{96}}\eta^{\mathbf{2/5}}\frac{(\mathbf{s_1^{\rm EA}-s_2^{\rm EA}})^{\mathbf{2}}}{[(\mathbf{1-s_1^{\rm EA}})(\mathbf{1-s_2^{\rm EA}})]^{\mathbf{4/3}}}\frac{(\mathbf{c_{14}-2})\mathbf{w_0^3-w_1^3}}{\mathbf{c_{14}w_0^3w_1^3}}$	$\mathbf{-2}$
Khronometric Theory [95]	$-\frac{\mathbf{5}}{\mathbf{96}}\eta^{\mathbf{2/5}\frac{(\mathbf{s_1^{\rm kh}-s_2^{\rm kh}})^{\mathbf{2}}}{[(\mathbf{1-s_1^{\rm kh}})(\mathbf{1-s_2^{\rm kh}})]^{\mathbf{4/3}}}}\sqrt{\bar{\alpha}_{\rm kh}}\left[\frac{(\bar{\beta}_{\rm kh}\mathbf{-1})(\mathbf{2}+\bar{\beta}_{\rm kh}+\mathbf{3}\bar{\lambda}_{\rm kh})}{(\bar{\alpha}_{\rm kh}\mathbf{-2})(\bar{\beta}_{\rm kh}+\bar{\lambda}_{\rm kh})}\right]^{\mathbf{3/2}}$	$\mathbf{-2}$
Noncommutative Gravity	$-\frac{\mathbf{3}}{\mathbf{8}}\eta^{\mathbf{-4/5}}(\mathbf{2}\eta-\mathbf{1})\mathbf{\Lambda^2}$	$+4$
Varying-G Theories [97]	$-\frac{\mathbf{5}\mathcal{M}_{\mathbf{0}}}{\mathbf{512G_0}^{\mathbf{5/3}}}\left(\frac{\mathbf{7\dot{G}_0}}{\mathbf{G_0}}+\frac{\mathbf{5\dot{m}_0}}{\mathbf{m_0}}+\frac{\mathbf{m_{1,0}\dot{m}_{2,0}+\dot{m}_{1,0}m_{2,0}}}{\mathbf{m_0}^{\mathbf{2}}\eta\mathbf{0}}\right)$	$\mathbf{-8}$

TABLE II. PPE corrections to the GW amplitude $|\tilde{h}|=|\tilde{h}_{\rm GR}|(1+\alpha u^a)$ in Fourier space in various modified theories of gravity with the magnitude α (second column) and the exponent a (third column), and $|\tilde{h}_{\rm GR}|$ representing the amplitude in GR. The meaning of other parameters are the same as in Table I. The expressions in boldface correspond to either those derived here for the first time, corrected expressions from previous literature, or an extension of a previous analysis.

Comparing this with the PPE waveform in Eq. (1), we find

$$\alpha=-\frac{\gamma_{\dot{f}}}{2}, \quad a=c_{\dot{f}}\,. \tag{12}$$

- *Conservative-dominated Case*

  When conservative corrections dominate, $c_r=c_{\dot{f}}$ and there is an explicit relation between γ_r and $\gamma_{\dot{f}}$. Though finding such a relation is quite involved and one needs to go back to the original PPE formalism as explained in App. A. Non-GR corrections to the GW amplitude in such a formalism is shown in Eq. (A14). Setting the dissipative correction to zero, one finds

$$\alpha=-\frac{\gamma_r}{a}(a^2-4a-6)\,, \quad a=c_r=c_{\dot{f}}\,. \tag{13}$$

- *Comparable Dissipative and Conservative Case*

  If dissipative and conservative corrections enter at the same PN order, we can set $c_r=c_{\dot{f}}$ in Eq. (10), which gives

$$\alpha=2\gamma_r-\frac{\gamma_{\dot{f}}}{2}\,, \quad a=c_r=c_{\dot{f}}\,. \tag{14}$$

B. Phase Corrections

Next, let us study corrections to the GW phase. The phase in the Fourier domain Ψ is related to the frequency evolution as [104]

$$\frac{d^2\Psi}{d\Omega^2}=2\frac{dt}{d\Omega}\,, \tag{15}$$

which can be rewritten as

$$\frac{d^2\Psi}{d\Omega^2}=\frac{2}{\pi\dot{f}}\,. \tag{16}$$

Substituting Eq. (5) to the right hand side of the above equation and keeping only to leading non-GR correction, we find

$$\frac{d^2\Psi}{d\Omega^2}=\frac{2}{\pi\dot{f}_{\rm GR}}(1-\gamma_{\dot{f}}u^{c_{\dot{f}}})\,. \tag{17}$$

Using further Eq. (6) to Eq. (17) gives

$$\frac{d^2\Psi}{d\Omega^2}=\frac{5}{48}\mathcal{M}^2u^{-11}(1-\gamma_{\dot{f}}u^{c_{\dot{f}}})\,. \tag{18}$$

We are now ready to derive Ψ and extract the PPE parameters β and b. Using $\Omega=\pi f$, we can integrate Eq. (18) twice to find

$$\Psi=\Psi_{\rm GR}-\frac{15\gamma_{\dot{f}}}{16(c_{\dot{f}}-8)(c_{\dot{f}}-5)}u^{c_{\dot{f}}-5}\,, \tag{19}$$

for $c_{\dot{f}}\neq 5$ and $c_{\dot{f}}\neq 8$. Here we only keep to leading non-GR correction and $\Psi_{\rm GR}$ is the GR contribution given in Eq. (A12) to leading PN order. Similar to the amplitude case, the above expression is much simpler than that in the original formalism in Eq. (A11). Comparing this with Eqs. (1) and (3), we find

$$\beta=-\frac{15\gamma_{\dot{f}}}{16(c_{\dot{f}}-8)(c_{\dot{f}}-5)}\,, \quad b=c_{\dot{f}}-5\,. \tag{20}$$

The above relation is valid for all three types of corrections considered for the GW amplitude case.

In App. A, we review $\delta\Psi$ derived in the original PPE formalism, where we show dissipative and conservative contributions explicitly. In particular, one can use Eq. (A15) to find β for all three cases separately.

C. Relations among ppE Parameters

Finally, we study relations among PPE parameters. From Eqs. (12)–(14) and (20), one can easily see

$$b = a - 5\,, \tag{21}$$

which holds in all three cases considered previously. Let us consider such three cases in turn below to derive relations between α and β.

- *Dissipative-dominated Case*

  When dissipative corrections dominate, we can use Eqs. (12) and (20) to find α in terms of β and a as

$$\alpha = \frac{8}{15}(a-8)(a-5)\,\beta\,. \tag{22}$$

- *Conservative-dominated Case*

  When conservative corrections dominate, we can set the dissipative correction to vanish in Eq. (A15) to find

$$\beta = -\frac{15}{8}\frac{\gamma_r}{c_r}\frac{c_r^2 - 2c_r - 6}{(8-c_r)(5-c_r)}\,, \quad b = c_r - 5\,. \tag{23}$$

  Using this equation together with Eq. (13), we find

$$\alpha = \frac{8}{15}\frac{(8-a)(5-a)(a^2-4a-6)}{a^2-2a-6}\beta\,. \tag{24}$$

- *Comparable Dissipative and Conservative Case*

  When dissipative and conservative corrections enter at the same PN order, there is no explicit relation between α and β. This is because α depends both on γ_r and $\gamma_{\dot f}$ (see Eq. (14)) while β depends only on the latter (see Eq. (19)), and there is no relation between the former and the latter. Thus, one can rewrite $\gamma_{\dot f}$ in terms of β and substitute into Eq. (14) but cannot eliminate γ_r from the expression for α.

III. EXAMPLE THEORIES

In this section we consider some viable modified theories of gravity where non-GR corrections arise from generation mechanisms. We briefly discuss each theory, describing differences from GR and its importance. We derive the PPE parameters for each theory following the formalism in Sec. II. Among the various example theories we present here, dissipative corrections dominate in scalar-tensor theories, EdGB gravity, Einstein-Æther theory, and khronometric gravity. On the other hand, dissipative and conservative corrections enter at the same PN order in dCS gravity, noncommutative gravity, and varying-G theories. We do not consider any theories where conservative corrections dominate dissipative ones, though such a situation can be realized for e.g. equal-mass and equal-spin binaries in dCS gravity, where the scalar quadrupolar radiation is suppressed and dominant corrections arise from the scalar dipole interaction and quadrupole moment corrections in the conservative sector.

A. Scalar-Tensor Theories

Scalar-tensor theories are one of the most well-established modified theories of gravity where at least one scalar field is introduced through a non-minimal coupling to gravity [19, 105, 106]. Such theories arise naturally from the dimensional reduction of higher dimensional theories, such Kaluza-Klein theory [107, 108] and string theories [109, 110]. Scalar-tensor theories have implications in cosmology as well since they are viable candidates for accelerating expansion of our Universe [111–115], structure formation [116], inflation [30, 117–119], and primordial nucleosynthesis [41, 42, 120, 121]. Such theories also offer simple ways to self-consistently model possible variations in Newton's constant [30]. One of the simplest scalar-tensor theories is Brans-Dicke (BD) theory, where a non-canonical scalar field is non-minimally coupled to the metric with an effective strength inversely proportional to the coupling parameter $\omega_{\rm BD}$ [91, 122]. So far the most stringent bound on the theory has been placed by the Cassini-Huygens satellite mission via Shapiro time delay measurement, which gives $\omega_{\rm BD} > 4\times10^4$ [123]. Another class of scalar-tensor theories that has been studied extensively is Damour-Esposito-Farèse (DEF) gravity, which has two coupling constants (α_0, β_0) and α_0 is directly related to $\omega_{\rm BD}$. This theory predicts nonperturbative spontaneous or dynamical scalarization phenomena for neutron stars [124, 125].

When scalarized NSs form compact binaries, these systems emit scalar dipole radiation that changes the orbital evolution from that in GR. Such an effect can be used to place bounds on scalar-tensor theories. For example, combining observational orbital decay results from multiple binary pulsars, the strongest upper bound on β_0 that controls the magnitude of scalarization in DEF gravity has been obtained as $\beta_0 \gtrsim -4.38$ at 90% confidence level [126]. More recently, observations of a hierarchical stellar triple system PSR J0337+1715 placed strong bounds on the Strong Equivalence Principle (SEP) violation parameter[1] as $|\Delta| \lesssim 2\times10^{-6}$ at 95% confidence level [127]. This bound stringently constrained

[1] SEP violation parameter is defined as $\Delta = m_G/m_I - 1$, where

the parameter space (α_0, β_0) of DEF gravity [124, 128–131], which reproduces BD theory as a special case when $\beta_0 = 0$.

Can BHs also possess scalar hair like NSs in scalar-tensor theories? BH no-hair theorem can be applied to many of scalar-tensor theories that prevents BHs to acquire scalar charges [132–136] including BD and DEF gravity, though exceptions exist, such as EdGB gravity [137–141] that we explain in more detail in the next subsection. On the other hand, if the scalar field cosmologically evolves as a function of time, BHs can acquire scalar charges, known as the BH miracle hair growth [142, 143] (see also [144, 145] for related works).

Scalar hair of compact objects in a binary produce scalar dipole radiation, which changes the evolution of the binary and modifies the gravitational waveform from that in GR. Using the orbital decay rate of compact binaries in scalar-tensor theories in [20, 28], one can read off non-GR corrections to $\dot{f}$ as

$$\gamma_{\dot{f}} = \frac{5}{96}\eta^{2/5}(\alpha_1 - \alpha_2)^2\,, \tag{25}$$

with $c_{\dot{f}} = -2$. Given that the leading correction to the waveform is the dissipative one in scalar-tensor theories, one can use Eq. (20) to derive the PPE phase correction as

$$\beta_{\rm ST} = -\frac{5}{7168}\eta^{2/5}(\alpha_1 - \alpha_2)^2\,, \tag{26}$$

with $b = -7$. Here α_A represents the scalar charge of the Ath binary component. Using further Eq. (22), one finds the amplitude correction as

$$\alpha_{\rm ST} = -\frac{5}{192}\eta^{2/5}(\alpha_1 - \alpha_2)^2\,, \tag{27}$$

with $a = -2$.

The scalar charges α_A depend on specific theories and compact objects. For example, in situations where the BH no-hair theorem [132–134] applies, $\alpha_A = 0$. On the other hand, if the scalar field is evolving cosmologically, BHs undergo *miracle hair growth* [142] and acquire scalar charges given by [143]

$$\alpha_A = 2\, m_A\, \dot{\phi}\, [1 + (1 - \chi_A^2)^{1/2}]\,, \tag{28}$$

where $\dot{\phi}$ is the growth rate of the scalar field, m_A is individual masses and χ_A is the magnitude of the spin angular momentum of the Ath body normalized by its mass squared. The PPE phase parameter β for binary BHs in such a situation was derived in [81]. Another well-studied example is Brans-Dicke theory, where one can replace $(\alpha_1 - \alpha_2)^2$ in Eqs. (26) and (27) as $2(s_1 - s_2)^2/(2 + \omega_{\rm BD})$ [20]. Here s_A is the sensitivity of the Ath body and roughly equals to its compactness (0.5 for BHs and ~ 0.2 for NSs). PPE parameters in this theory has been found in [75]. Scalar charges and PPE parameters in generic screened modified gravity have recently been derived in [98, 146].

The phase correction in Eq. (26) has been used to derive current and future projected bounds with GW interferometers. Regarding the former, GW150914 and GW151226 do not place any meaningful bounds on $\dot{\phi}$ [81]. On the other hand, by detecting GWs from BH-NS binaries, aLIGO and Virgo with their design sensitivities can place bounds that are stronger than the above binary pulsar bounds from dynamical scalarization for certain equations of state and NS mass range [80, 126, 147, 148][2]. Einstein Telescope, a third generation ground-based detector, can yield constraints on BD theory from BH-NS binaries that are 100 times stronger the current bound [149]. Projected bounds with future space-borne interferometers, such as DECIGO, can be as large as four orders of magnitude stronger than current bounds [150], while those with LISA may not be as strong as the current bound [92, 151].

Up until now, we have focused on theories with a massless scalar field, but let us end this subsection by commenting on how the above expressions for the PPE parameters change if one considers massive a scalar field instead. In such a case, the scalar dipole radiation is present only when the mass of the scalar field m_s is smaller than the orbital angular frequency $\Omega = \pi f$. Then, if the Yukawa-type correction to the binding energy is subdominant, Eqs. (26) and (27) simply acquire an additional factor of $\Theta(\Omega - m_s/\hbar)$, where Θ is the Heaviside function. For example, the gravitational waveform phase in massive BD theory is derived in [152]. The situation is similar if massive pseudo-scalars are present, such as axions [153].

B. EdGB Gravity

EdGB gravity is a well-known extension of GR, which emerges naturally in the framework of low-energy effective string theories and gives one of the simplest viable high-energy modifications to GR [154, 155]. It also arises as a special case of Horndeski gravity [19, 156], which is the most generic scalar-tensor theory with at most second-order derivatives in the field equations. One obtains the EdGB action by adding a quadratic-curvature term to the Einstein-Hilbert action, where the scalar field (dilaton) is non-minimally coupled to the Gauss-Bonnet term with a coupling constant $\bar{\alpha}_{\rm EdGB}$ [157]. A stringent upper bound on such a coupling constant has been placed using the orbital deca measurement of a BH low-mass X-ray binary (LMXB) as $\sqrt{|\bar{\alpha}_{\rm EdGB}|} < 1.9 \times 10^5$ cm [158].

m_G and m_I are respectively the gravitational and inertial mass of the pulsar [127].

[2] One needs to multiply Eq. (26) by a step-like function to capture the effect of dynamical scalarization.

A similar upper bound has been placed from the existence of BHs [155]. Equation-of-state-dependent bounds from the maximum mass of neutron stars have also been derived in [159].

BHs in EdGB theory are of particular interest since they are fundamentally different from their GR counterparts. Perturbative but analytic solutions are available for static [137, 139, 160, 161] and slowly rotating EdGB BHs [162–164] while numerical solutions have been found for static [157, 165, 166] and rotating [155, 167, 168] BHs. One of the important reasons for considering BHs in EdGB is that BHs acquire scalar monopole charge [93, 139, 169, 170] while ordinary stars such as neutron stars do not if the scalar field is coupled linearly to the Gauss-Bonnet term in the action [93, 171]. This means that binary pulsars are inefficient to constrain the theory, and one needs systems such as BH-LMXBs [158] or BH/pulsar binaries [171] to have better probes on the theory.

We now show the expressions of PPE parameters for EdGB gravity. The scalar monopole charge of EdGB BHs in generates scalar dipole radiation, which leads to an earlier coalescence of BH binaries compared to GR. Such scalar radiation modifies the GW phase with the PPE parameter β as [81, 93]

$$\beta_{\rm EdGB} = -\frac{5}{7168}\zeta_{\rm EdGB}\frac{(m_1^2\tilde{s}_2^{\rm EdGB} - m_2^2\tilde{s}_1^{\rm EdGB})^2}{m^4\eta^{18/5}}\,. \qquad (29)$$

Here, $\zeta_{\rm EdGB} \equiv 16\pi\bar{\alpha}_{\rm EdGB}^2/m^4$ is the dimensionless EdGB coupling parameter and $\tilde{s}_A^{\rm EdGB}$ are the spin-dependent factors of the BH scalar charges given by $\tilde{s}_A^{\rm EdGB} \equiv 2(\sqrt{1-\chi_A{}^2} - 1 + \chi_A{}^2)/\chi_A{}^2$ [169, 170][3]. In EdGB gravity, the leading order correction to the phase enters through the correction of GW eneregy flux, and hence the theory corresponds to a dissipative-dominated case. We can then use Eq. (22) to calculate the amplitude PPE parameter $\alpha_{\rm EdGB}$ from $\beta_{\rm EdGB}$ as

$$\alpha_{\rm EdGB} = -\frac{5}{192}\zeta_{\rm EdGB}\frac{(m_1^2\tilde{s}_2^{\rm EdGB} - m_2^2\tilde{s}_1^{\rm EdGB})^2}{m^4\eta^{18/5}}\,. \qquad (30)$$

One can use the phase correction in Eq. (29) to derive bounds on EdGB gravity with current [81] and future [158] GW observations. Similar to the scalar-tensor theory case, current binary BH GW events do not allow us to place any meaningful bounds on the theory. Future second- and third-generation ground-based detectors and LISA can place bounds that are comparable to current bounds from low-mass X-ray binaries [158]. On the other hand, DECIGO has the potential to go beyond the current bounds by three orders of magnitude.

[3] $\tilde{s}_A^{\rm EdGB}$ are zero for ordinary stars like neutron stars [93, 171].

C. dCS Gravity

DCS gravity is described by Einstein-Hilbert action with a dynamical (pseudo)scalar field which is non-minimally coupled to the Pontryagin density with a coupling constant $\bar{\alpha}_{\rm dCS}$ [172, 173]. Similar to EdGB gravity, dCS gravity arises as an effective field theory from the compactification of heterotic string theory [174, 175]. Such a theory is also important in the context of particle physics [172, 176–178], loop quantum gravity [179, 180], and inflationary cosmology [181]. Demanding that the critical length scale (below which higher curvature corrections beyond quadratic order cannot be neglected in the action) has to be smaller than the scale probed by table-top experiments, one finds $\sqrt{|\bar{\alpha}_{\rm dCS}|} < \mathcal{O}(10^8{\rm km})$ [182]. Similar constraints have been placed from measurements of the frame-dragging effect by Gravity Probe B and LAGEOS satellites [183].

We now derive the expressions of PPE parameters for dCS gravity. While BHs in EdGB gravity possess scalar monopole charges, BHs in dCS gravity possess scalar dipole charges which induce scalar quadrupolar emission [93]. On the other hand, scalar dipole charges induce a scalar interaction force between two BHs. Each BH also acquires a modification to the quadrupole moment away from the Kerr value. All of these modifications result in both dissipative and conservative corrections entering at the same order in gravitational waveforms. For spin-aligned binaries[4], corrections to the Kepler's law and frequency evolution in dCS gravity are given in [94] within the slow-rotation approximation for BHs, from which we can derive

$$\gamma_r = \frac{25}{256}\eta^{-9/5}\zeta_{\rm dCS}\chi_1\chi_2 - \frac{201}{3584}\eta^{-14/5}\zeta_{\rm dCS}\left(\frac{m_1^2}{m^2}\chi_2^2 + \frac{m_2^2}{m^2}\chi_1^2\right)\,, \qquad (31)$$

with $c_r = 4$, and

$$\gamma_{\dot f} = \frac{38525}{39552}\eta^{-9/5}\zeta_{\rm dCS}\chi_1\chi_2 - \frac{309845}{553728}\eta^{-14/5}\zeta_{\rm dCS}\left(\frac{m_1^2}{m^2}\chi_2^2 + \frac{m_2^2}{m^2}\chi_1^2\right)\,, \qquad (32)$$

with $c_{\dot f} = 4$. Here $\zeta_{\rm dCS} = 16\pi\bar{\alpha}_{\rm dCS}^2/m^4$ is the dimensionless coupling constant. Using Eqs. (31) and (32) in Eqs. (14) and (20) respectively, one finds

$$\alpha_{\rm dCS} = \frac{185627}{1107456}\eta^{-14/5}\zeta_{\rm dCS}\left[-2\delta_m\chi_a\chi_s + \left(1 - \frac{53408\eta}{14279}\right)\chi_a^2 + \left(1 - \frac{3708\eta}{14279}\right)\chi_s^2\right]\,, \qquad (33)$$

[4] See recent works [184, 185] for precession equations in dCS gravity.

with $a = 4$, and

$$\beta_{\rm dCS} = \frac{1549225}{11812864}\eta^{-14/5}\zeta_{\rm dCS}\left[-2\delta_m\chi_a\chi_s + \left(1-\frac{16068\eta}{61969}\right)\chi_a^2 + \left(1-\frac{231808\eta}{61969}\right)\chi_s^2\right], \tag{34}$$

with $b = -1$. Here $\chi_{s,a} = (\chi_1 \pm \chi_2)/2$ are the symmetric and antisymmetric combinations of dimensionless spin parameters and $\delta_m = (m_1 - m_2)/m$ is the fractional difference in masses relative to the total mass.

Can GW observations place stronger bounds on the theory? Current GW observations do not allow us to put any meaningful bounds on dCS gravity [81] (see also [86]). However, future observations have potential to place bounds on the theory that are six to seven orders of magnitude stronger than current bounds [94]. Such stronger bounds can be realized due to relatively strong gravitational field and large spins that source the pseudoscalar field. Measuring GWs from extreme mass ratio inspirals with LISA can also place bounds that are three orders of magnitude stronger than current bounds [186].

D. Einstein-Æther and Khronometric Theory

In this section, we study two example theories that break Lorentz invariance in the gravity sector, namely EA and khronometric theory. Lorentz-violating theories of gravity are candidates for low-energy descriptions of quantum gravity [187, 188]. Although Lorentz-violation in the gravity sector has not been as stringently constrained as that in the matter sector [189–191], and several mechanisms exist that prevents percolation of the former to the latter [191, 192].

Einstein-Æther (EA) theory is a vector-tensor theory of gravity, where along with the metric, a spacetime is endowed with a dynamical timelike unit vector (Æther) field [193, 194]. Such a vector field specifies a particular rest frame at each point in spacetime, and hence breaks the local Lorentz symmetry. The amount of Lorentz violation is controlled by four coupling parameters (c_1, c_2, c_3, c_4). EA theory preserves diffeomorphism invariance and hence is a Lorentz-violating theory without abandoning the framework of GR [194]. Along with the spin-2 gravitational perturbation of GR, the theory predicts the existence of spin-1 and spin-0 perturbations [195–197]. Such perturbation modes propagate at speeds that are functions of coupling parameters c_i, and in general differ from the speed of light [196].

Khronometric theory is a variant of EA theory, where the Æther field is restricted to be hypersurface-orthogonal. Such a theory arises as a low-energy limit of Hořava gravity, a power-counting renormalizable quantum gravity model with only spatial diffeomorphism invariance [19, 188, 198–200]. The amount of Lorentz violation in the theory is controlled by three parameters, $(\bar{\alpha}_{\rm kh}, \bar{\beta}_{\rm kh}, \bar{\lambda}_{\rm kh})$[5]. Unlike EA theory, spin-1 propagating modes are absent in khronometric theory.

Most of parameter space in EA and khronometric theory have been constrained stringently from current observations and theoretical requirements. Using the measurement of the arrival time difference between GWs and electromagnetic waves in GW170817, the difference in the propagation speed of GWs away from the speed of light has been constrained to be less than $\sim 10^{-15}$ [61, 63]. Such a bound can be mapped to bounds on Lorentz-violating gravity as $|c_1 + c_3| \lesssim 10^{-15}$ [201] and $|\bar{\beta}_{\rm kh}| \lesssim 10^{-15}$ [202][6]. Imposing further constraints from solar system experiments [204–206], Big Bang nucleosynthesis [207] and theoretical constraints such as the stability of propagating modes, positivity of their energy density [208] and the absence of gravitational Cherenkov radiation[209], allowed regions in the remaining parameter space have been derived for EA [201] and khronometric [202] theory. Binary pulsar bounds on these theories were studied in [210, 211] before the discovery of GW170817, within a parameter space that is different from allowed regions in [201, 202].

Let us now derive PPE parameters in EA and khronometric theories. The propagation of scalar and vector modes is responsible for dipole radiation and loss of angular momentum in binary systems, which increase the amount of orbital decay rate. Regarding EA theory, the PPE phase correction is given by [95]

$$\beta_{\rm EA} = -\frac{5}{3584}\eta^{2/5}\frac{(s_1^{\rm EA} - s_2^{\rm EA})^2}{[(1-s_1^{\rm EA})(1-s_2^{\rm EA})]^{4/3}} \times \frac{(c_{14}-2)w_0^3 - w_1^3}{c_{14}w_0^3 w_1^3}, \tag{35}$$

with $b = -7$. Here w_s is the propagation speed of the spin-s mode in EA theory given by [194]

$$w_0^2 = \frac{(2-c_{14})c_{123}}{(2+3c_2+c_+)(1-c_+)c_{14}}, \tag{36}$$

$$w_1^2 = \frac{2c_1 - c_+c_-}{2(1-c_+)c_{14}}, \tag{37}$$

$$w_2^2 = \frac{1}{1-c_+}, \tag{38}$$

with

$$c_{14} \equiv c_1 + c_4, \quad c_\pm \equiv c_1 \pm c_3, \quad c_{123} \equiv c_1 + c_2 + c_3. \tag{39}$$

s_A in Eq. (35) is the sensitivity of the A-th body and has been calculated only for NSs [210, 211]. Given that the leading order correction in EA theory arises from the

[5] We use barred quantities for coupling constants and $\bar{\alpha}_{\rm kh}, \bar{\beta}_{\rm kh}$ are not to be confused with PPE parameters $(\alpha_{\rm kh}, \beta_{\rm kh})$.

[6] Such bounds are consistent with the prediction in [95] based on [203].

dissipative sector [95], we can use Eq. (22) to find the PPE amplitude correction as[7]

$$\alpha_{\text{EA}} = -\frac{5}{96}\eta^{2/5}\frac{(s_1^{\text{EA}} - s_2^{\text{EA}})^2}{[(1-s_1^{\text{EA}})(1-s_2^{\text{EA}})]^{4/3}} \times \frac{(c_{14}-2)w_0^3 - w_1^3}{c_{14}w_0^3 w_1^3}\,, \tag{40}$$

with $a=-2$. Similar to EA theory, PPE parameters in khronometric theory is given by [95]

$$\beta_{\text{kh}} = -\frac{5}{3584}\eta^{2/5}\frac{(s_1^{\text{kh}} - s_2^{\text{kh}})^2}{[(1-s_1^{\text{kh}})(1-s_2^{\text{kh}})]^{4/3}} \times \sqrt{\bar{\alpha}_{\text{kh}}}\left[\frac{(\bar{\beta}_{\text{kh}}-1)(2+\bar{\beta}_{\text{kh}}+3\bar{\lambda}_{\text{kh}})}{(\bar{\alpha}_{\text{kh}}-2)(\bar{\beta}_{\text{kh}}+\bar{\lambda}_{\text{kh}})}\right]^{3/2}\,, \tag{41}$$

with $b=-7$, and

$$\alpha_{\text{kh}} = -\frac{5}{96}\eta^{2/5}\frac{(s_1^{\text{kh}} - s_2^{\text{kh}})^2}{[(1-s_1^{\text{kh}})(1-s_2^{\text{kh}})]^{4/3}} \times \sqrt{\bar{\alpha}_{\text{kh}}}\left[\frac{(\bar{\beta}_{\text{kh}}-1)(2+\bar{\beta}_{\text{kh}}+3\bar{\lambda}_{\text{kh}})}{(\bar{\alpha}_{\text{kh}}-2)(\bar{\beta}_{\text{kh}}+\bar{\lambda}_{\text{kh}})}\right]^{3/2}\,, \tag{42}$$

with $a=-2$.

Above corrections to the gravitational waveform can be used to compute current and projected future bounds on the theories with GW observations, provided one knows what sensitivities are for compact objects in binaries. Unfortunately, such sensitivities have not been calculated for BHs, and hence, one cannot derive bounds on the theories from recent binary BH merger events. Instead, Ref. [81] used the next-to-leading 0PN correction that is independent of the sensitivities and derived bounds from GW150914 and GW151226, though such bounds are weaker than those from binary pulsar observations [210, 211]. On the other hand, Ref. [95] includes both the leading and next-to-leading corrections to the waveform and estimate projected future bounds with GWs from binary NSs. The authors found that bounds from second-generation ground-based detectors are less stringent that existing bounds even with their design sensitivities. ***st: I added this sentence-*** However, third-generation ground-based ones and space-borne interferometers can place constraints that are comparable, and in some cases, two orders of magnitude stronger compared to the current bounds [95, 255].

E. Noncommutative Gravity

Although the concept of nontrivial commutation relations of spacetime coordinates is rather old [212, 213], the idea has revived recently with the development of noncommutative geometry [214–218], and the emergence of noncommutative structure of spacetime in a specific limit of string theory [219, 220]. Quantum field theories on noncommutative spacetime have been studied extensively as well [221–223]. In the simplest model of noncommutative gravity, spacetime coordinates are promoted to operators, which satisfy a canonical commutation relation:

$$[\hat{x}^\mu, \hat{x}^\nu] = i\theta^{\mu\nu}\,, \tag{43}$$

where $\theta^{\mu\nu}$ is a real constant antisymmetric tensor. In ordinary quantum mechanics, Planck's constant $\hbar$ measures the quantum fuzziness of phase space coordinates. In a similar manner, $\theta^{\mu\nu}$ introduces a new fundamental scale which measures the quantum fuzziness of spacetime coordinates [96].

In order to obtain stringent constraints on the scale of noncommutativity, low-energy experiments are advantageous over high-energy ones [224, 225]. Low-energy precision measurements such as clock-comparison experiments with nuclear-spin-polarized $^9\text{Be}^+$ ions [226] give a constraint on noncommutative scale as $1/\sqrt{\theta} \gtrsim 10$ TeV [224], where θ refers to the magnitude of the spatial-spatial components of $\theta^{\mu\nu}$[8]. A similar bound has been obtained from the measurement of Lamb shift [227]. Another speculative bound is derived from the analysis of atomic experiments which is 10 orders of magnitude stronger [225, 228]. Study of inflationary observables using cosmic microwave background data from Planck gives the lower bound on the energy scale of noncommutativity as 19 TeV [229, 230].

Let us now review how the binary evolution is modified from that in GR in this theory. Several formulations of noncommutative gravity exist [231–236], though the first order noncommutative correction vanishes in all of them [237, 238] and the leading order correction enters at second order. On the other hand, first order corrections can arise from gravity-matter interaction [238, 239]. Thus one can neglect corrections to the pure gravity sector and focus on corrections to the matter sector (i.e., energy-momentum tensor) [96]. Making corrections to classical matter source and following an effective field theory approach, expressions of energy and GW luminosity for quasi-circular black hole binaries have been derived in Ref. [96], which give the correction to frequency evolution in Eq. (5) as

$$\gamma_{\dot{f}} = \frac{5}{4}\eta^{-4/5}(2\eta-1)\Lambda^2\,, \tag{44}$$

with $c_{\dot{f}} = 4$ and $\Lambda^2 = \theta^{0i}\theta_{0i}/(l_p^2 t_p^2)$ with l_p and t_p representing the Planck length and time respectively. On

[7] Eqs. (40) and (42) correct errors in the first version of [95].

[8] The corresponding bound on the time-spatial components of $\theta^{\mu\nu}$ is roughly six orders magnitude weaker than that on the spatial-spatial components.

the other hand, modified Kepler's law in Eq. (4) can be found as [96]

$$\gamma_r = \frac{1}{8}\eta^{-4/5}(2\eta-1)\Lambda^2\,, \tag{45}$$

with $c_r = 4$.

We are now ready to derive PPE parameters in noncommutative gravity. Given that the dissipative and conservative leading corrections enter at the same PN order, one can use Eqs. (44) and (45) in Eq. (14) to find the PPE amplitude correction as

$$\alpha_{\rm NC} = -\frac{3}{8}\eta^{-4/5}(2\eta-1)\Lambda^2\,, \tag{46}$$

with $a = 4$. Similarly, substituting Eq. (44) into Eq. (20) gives the PPE phase correction as

$$\beta_{\rm NC} = -\frac{75}{256}\eta^{-4/5}(2\eta-1)\Lambda^2\,, \tag{47}$$

with $b = -1$. $\beta_{\rm NC}$ can also be read off from the phase correction derived in [96]. Notice that the above leading noncommutative corrections in the amplitude and phase enter at 2PN order.

The above phase correction has already been used to derive bounds on noncommutative gravity from GW150914 as $\sqrt{\Lambda} \lesssim 3.5$ [96], which means that the energy scale of noncommutativity to be the order of Planck scale. Such a bound, so far, is the most stringent constraint on noncommutative scale and is 15 orders of magnitude stronger compared to the bounds coming from particle physics and low-energy precision measurements[9].

IV. VARYING-G THEORY

Many of modified theories of gravity that violate the strong equivalence principle [1, 240, 241] predict that locally measured gravitational constant (G) might vary with time [242]. Since the gravitational self-energy of a body is a function of gravitational constant, in a theory where G is time dependent, masses of compact bodies are also time dependent [243]. The rate at which the mass of an object varies with time is proportional to the rate of change of gravitational coupling constant [243]. Such a variation of mass causes the compact bodies experience anomalous acceleration, which results in a time-evolution of angular momentum [243]. Existing experiments that search for variation in G at present time (i.e., at very small redshift) include lunar laser ranging observations [244], pulsar timing observations [245, 246], radar observation of planets and spacecraft [247], and surface temperature observation of PSR J0437-4715 [248].

[9] Notice that the GW bound is on the time-spatial components of $\theta^{\mu\nu}$, while most of particle physics and low-energy precision experiments place bounds on its spatial-spatial components.

Another class of constraints on long-term variation of G come from Big Bang Nucleosynthesis (BBN) calculations [249, 250] and helioseismology [251]. Most restrictive bound on $|\dot{G}/G|$ coming from either kind of experiments is of the order $\lesssim 10^{-13}\,\mathrm{yr}^{-1}$ [97].

More than one gravitational constants can appear in different arenas of a gravitational theory. Here we introduce two different kinds of gravitational constant, one that arises in the dissipative sector and another in the conservative sector. The constant which enters in the GW luminosity through Einstein equations, i.e. the constant in Eq. (8), is the one we refer to as dissipative gravitational constant (G_D), while that enters in Kepler's law or binding energy of the binary is what we refer as the conservative one (G_C). These two constants are the same in GR, but they can be different in some modified theories. An example of such a theory is Brans-Dicke theory with a cosmologically evolving scalar field [252].

PPE parameters for varying-G theories have previously been derived in [97] for $G_D = G_C$. Here, we improve the analysis by considering the two different types of gravitational constant and including variation in masses, which is inevitable for strongly self-gravitating objects when G varies [243]. We also correct small errors in [97]. We follow the analysis of [253] that derives gravitational waveforms from BH binaries with varying mass effects using the specific angular momentum. We present another derivation using the energy balance argument used in [97].

The formalism presented in Sec. II assumes that G and the masses to be constant, and hence are not applicable to varying-G theories. Thus, we will derive PPE parameters in varying-G theories by promoting the PPE formalism to admit time variation in the gravitational constants and masses as

$$m_A(t) \approx m_{A,0} + \dot{m}_{A,0}(t-t_0)\,, \tag{48}$$
$$G_C(t) \approx G_{C,0} + \dot{G}_{C,0}(t-t_0)\,, \tag{49}$$
$$G_D(t) \approx G_{D,0} + (1+\delta)\dot{G}_{C,0}(t-t_0)\,, \tag{50}$$

where t_0 is the time of coalescence. Here we assumed that the spatial variations of G_C and G_D are small compared to the variation in time. δ gives the fractional difference between the rates at which G_C and G_D vary in time, and could be a function of parameters in a theory. Subscript 0 denotes that the quantity is measured at time $t = t_0$. Other time variations to consider are those in the specific angular momentum j_0 and the total mass m:

$$j(t) \approx j_0 + \dot{j}_0(t-t_0)\,, \tag{51}$$
$$m(t) = m_0 + \dot{m}_0(t-t_0)\,. \tag{52}$$

st: About omitting the following sentence adding [253] as a reference, actually anomalous acceleration should be present even in [253] and specific angular momentum is not supposed to be conserved, although the analysis in [253] is based on the conservation of specific angular momentum in the absence of gravitational wave. This is

something I wanted to discuss with you few days ago and then I forgot. Let me know your opinion. $\dot{j}_0$ and $\dot{m}_0$ can be written in terms of binary masses and sensitivities defined by

$$s_A = -\frac{G_C}{m_A}\frac{\delta m_A}{\delta G_C}. \tag{53}$$

as [243]

$$\dot{j}_0 = \frac{m_{1,0}s_{1,0} + m_{2,0}s_{2,0}}{m_{1,0}+m_{2,0}}\frac{\dot{G}_{C,0}}{G_{C,0}} j_0 \,, \tag{54}$$

$$\dot{m}_0 = -\frac{m_{1,0}s_{1,0} + m_{2,0}s_{2,0}}{m_{1,0}+m_{2,0}}\frac{\dot{G}_{C,0}}{G_{C,0}} m_0 \,, \tag{55}$$

respectively.

Next, we explain how the binary evolution is affected by the variation of the above parameters. First, GW emission makes the orbital separation r decay with the rate given by [254]

$$\dot{r}_{\rm GW} = -\frac{64}{5}\frac{G_D G_C^2 \mu m^2}{r^3} \,. \tag{56}$$

Second, time variation of mass, gravitational constant and specific angular momentum changes r at a rate of

$$\dot{r}_H = -\left(\frac{\dot{G}_{C,0}}{G_C} + \frac{\dot{m}_0}{m} - 2\frac{\dot{j}_0}{j}\right) r \,, \tag{57}$$

which is derived by taking a time derivative of the specific angular momentum $j \equiv \sqrt{G_C m r}$. Having the evolution of r at hand, one can derive the evolution of the orbital angular frequency using Kepler's third law as

$$\dot{\Omega} = \frac{1}{2\Omega r^3}\left(m\dot{G}_{C,0} + \dot{m}_0 G_C - 3mG_C\frac{\dot{r}}{r}\right) . \tag{58}$$

Using the evolution of the binary separation $\dot{r} \equiv \dot{r}_{\rm GW} + \dot{r}_H$, together with Eqs. (54) and (55), in Eq. (58), we can find the GW frequency evolution as

$$\begin{aligned}\dot{f} &= \frac{\dot{\Omega}}{\pi} \\ &= \frac{96}{5}\pi^{8/3} G_C^{2/3} G_D \mathcal{M}^{5/3} f^{11/3}\left\{1 \right. \\ &\left. + \frac{5}{96}\frac{\dot{G}_{C,0}G_C}{G_D \eta}[2m - 5(m_{1,0}s_{1,0} + m_{2,0}s_{2,0})]x^{-4}\right\} ,\end{aligned} \tag{59}$$

where $x = v^2 = (\pi G_C m f)^{2/3}$ is the squared velocity of the relative motion. Here we only considered the leading correction to the frequency evolution entering at -4PN order. Using Eqs. (48)- (50) and Eq. (55) to Eq. (59) one finds

$$\begin{aligned}\dot{f} &= \frac{96}{5}\pi^{8/3} f^{11/3}\eta_0 G_{C,0}^{2/3} G_{D,0} m_0^{5/3}\left\{1\right. \\ &-\frac{5G_{C,0}\dot{G}_{C,0}}{768\eta_0 G_{D,0}^2}[3(1+\delta)G_{C,0}m_0 - (3s_{1,0} + 3s_{2,0} \\ &\left.+14)G_{D,0}m_0 + 41(m_{1,0}s_{1,0} + m_{2,0}s_{2,0})G_{D,0}]\, x_0^{-4}\right\} .\end{aligned} \tag{60}$$

Notice that $G_{C,0}$ and $G_{D,0}$ differ only by a constant quantity, and such a difference will enter in $\dot{f}$ at 0PN order which is much higher than the -4PN corrections. We will thus ignore such corrections and simply use $G_{D,0} = G_{C,0} \equiv G_0$ from now on.

Based on the above binary evolution, we now derive corrections to the GW phase. We integrate Eq. (60) to obtain time before coalescence $t(f) - t_0$ and the GW phase $\phi(f) \equiv \int 2\pi f dt = \int (2\pi f/\dot{f}) df$ as

$$\begin{aligned}t(f) = t_0 &- \frac{5}{256}G_0\mathcal{M}_0 {u_0}^{-8}\left\{1\right. \\ &-\frac{5}{1536}\frac{\dot{G}_{C,0}}{\eta_0}[11m_0 + 3(s_{1,0} + s_{2,0} - \delta)m_0 \\ &\left.-41(m_{1,0}s_{1,0} + m_{2,0}s_{2,0})]\, x_0^{-4}\right\} ,\end{aligned} \tag{61}$$

$$\begin{aligned}\phi(f) = \phi_0 &- \frac{1}{16}{u_0}^{-5}\left\{1\right. \\ &-\frac{25}{9984}\frac{\dot{G}_{C,0}}{\eta_0}[11m_0 + 3(s_{1,0} + s_{2,0} - \delta)m_0 \\ &\left.-41(m_{1,0}s_{1,0} + m_{2,0}s_{2,0})]\, x_0^{-4}\right\} ,\end{aligned} \tag{62}$$

with $u_0 \equiv (\pi G_0 \mathcal{M}_0 f)^{\frac{1}{3}}$. The GW phase in the Fourier space is then given by

$$\begin{aligned}\Psi(f) =& 2\pi f t(f) - \phi(f) - \frac{\pi}{4} \\ =& 2\pi f t_0 - \phi_0 - \frac{\pi}{4} + \frac{3}{128}u_0^{-5}\left\{1 - \frac{25}{19968}\frac{\dot{G}_{C,0}}{\eta_0}[11m_0 + 3(s_{1,0}+s_{2,0}-\delta)m_0 - 41(m_{1,0}s_{1,0} + m_{2,0}s_{2,0})]\, x_0^{-4}\right\} .\end{aligned} \tag{63}$$

From Eq. (63), one finds the PPE phase parameters as $b = -13$ and

$$\beta_{\dot{G}} = -\frac{25}{851968}\dot{G}_{C,0}\,\eta_0^{3/5}\,[11m_0 + 3(s_{1,0} + s_{2,0} - \delta)m_0$$

$$-41(m_{1,0}s_{1,0} + m_{2,0}s_{2,0})] \,. \tag{64}$$

Next, we derive the PPE amplitude parameters. Using Kepler's law to Eq. (9), one finds

$$\begin{aligned}\tilde{\mathcal{A}}(f) &\propto \frac{1}{\sqrt{\dot{f}}}\frac{G_D(t)}{D_L}\mu(t)r(t)^2 f^2 \\ &\propto \frac{1}{\sqrt{\dot{f}}} G_D(t) G_C(t)^{2/3}\mu(t)m(t)^{2/3} \,. \end{aligned} \tag{65}$$

Using furhter Eqs. (48)-(50) in Eq. (65), we find the amplitude PPE parameters as $a = -8$ and

$$\begin{aligned}\alpha_{\dot{G}} = \frac{5}{512}\eta_0^{3/5}\dot{G}_{C,0}\,&[-7m_0 + (s_{1,0} + s_{2,0} - \delta)m_0 \\ &+13(m_{1,0}s_{1,0} + m_{2,0}s_{2,0})] \,. \end{aligned} \tag{66}$$

Let us comment on how the above new PPE parameters in varying G theories differ from those obtained previously in [97]. The latter considers $G_D = G_C$ (which corresponds to $\delta = 0$) and $s_A = 0$ (which is only true for weakly-gravitating objects). However, the above expressions for PPE parameters do not reduce to those in [97] under these limits. This is because Ref. [97] did not take into account the fact that the binding energy is not conserved in the absence of GW emission in varying-G theories. In Appendix B, we show that the correct application of the energy balance law does indeed lead to the above PPE expressions.

Eqs. (64) and (66) can be used to constrain varying-G theories with GW observations. Recent GW events (GW150914 and GW151226) place constraints on variation of G which are much weaker than the current constraints [81]. Projected GW bounds have been calculated in Ref. [97] (see [255] for an updated forecast of future GW bounds on $\dot{G}$) which gives $|\dot{G}_0/G_0| \lesssim 10^{-11}\,\mathrm{yr}^{-1}$, considering a single merger event. Although GW bound is less restrictive compared to the existing bounds [252], it is unique in the sense that it can provide constraints at intermediate redshifts, while the existing bounds are for very small and large redshifts [97]. Furthermore, GW constraints give $\dot{G}_0/G_0$ at the location of merger events, which means that a sufficient number of GW observations can be used to construct a 3D constraint map of $\dot{G}_0/G_0$ as a function of sky locations and redshifts [97].

V. CONCLUSIONS

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We derived non-GR corrections to GW phase and amplitude in various modified theories of gravity. In the aim of doing so, we revisited the standard PPE formalism considering generic corrections to GW frequency evolution and Kepler's third law. Such a formalism yields the expressions of PPE parameters which are simpler compared to the original formalism [81]. We calculated PPE amplitude parameters for the first time in EdGB gravity, dCS gravity, and noncommutative gravity. Whereas some errors in the expressions of PPE amplitude parameters in Einstein- Æther and khronometric theories in previous literature [95] were corrected.

The formulation of PPE parameters as described above does not account for theories with a variable gravitational constant. We derive PPE parameters in such theories independently, arguing that there are two different gravitational constants entering in PPE corrections. We refer to the gravitational constant that eneters in GW luminosity through Einstein equation as dissipative gravitational constant, while the gravational constant appearing in Kepler's law is the conservative gravitational constant. While the latter could be a function of the former, they are not necessarily same in the context of modified theories of gravity. The time-variation in conservative gravitational constant leads to a variation in mass and angular momentum of compact objects. Hence we promoted the two gravitatinal constants, binary masses, and their angular momentum to time-dependent forms in order to derive PPE waveform. Our work extends the previous work in Ref. [97] where dissipatve and conservative gravitational constants were considered to be same, and also fixes an error in their energy-balance equation.

st: The main results of our paper are the amplitude corrections, except from varying-G where we calculated both phase and amplitude corrections. I think only in EdGB, noncommutative, and varying-G theories amplitude corrections have not been used to cosntrain the theories. I mention only those in the follwoing paragraph. Feel free to modify this paragraph. The analytic expressions of PPE amplitude corrections derived in this paper can be used to place projected constraints on different modified theories of gravity using data analysis techniques. Restricted PN waveforms (i.e., the waveforms considering non-GR modifications to GW phase only) have been used to place bounds on EdGB gravity [81] and noncommutative gravity [96]. A semi-restricted waveform allowing modifications to GW amplitudes can be used to further constrain those theories. As for varying-G theories, the newly derived expressions of phase and amplitude corrections in this paper can be used to place bounds on variability of gravitational constants which also gives the validity of strong equivalence principle. Also GW events provide the measurements of $\dot{G}$ at the locations of merger events and at various redshifts. This opens the possibility that a sufficient number of future GW observations can be used to construct a constraint map as a function of sky angles and redshifts [97]. All calculations of this paper can be further generalized by including eccentricities of the binaries. While the calculations of Einstein-Æther theory, khronometric gravity, noncommutative gravity, and varying-G theories can be extended by including the effects of spins as well. Although the recent GW events [56–61] do not heavily constrain any of the theories discussed in this paper, future GW events detected by third generation ground-based and space-borne detectors [256–258] are supposed to place more stringent constraints on them.

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Appendix A: Original PPE Formalism

In this appendix, we review the original PPE formalism. In particular, we will show how the amplitude and phase corrections depend on the conservative and dissipative corrections, where the former are corrections to the effective potential while the latter are those to the GW luminosity. We will mostly follow [75].

First, let us introduce the conservative corrections. We modify the reduced effective potential as

$$V_{\rm eff} = \left(-\frac{m}{r} + \frac{L_z^2}{2\mu^2 r^2}\right)\left[1 + A\left(\frac{m}{r}\right)^p\right], \tag{A1}$$

where m is the total mass of the binary and L_z is the z-component of the angular momentum. A and p show the magnitude and exponent of the non-GR correction term respectively. Such a modification to the effective potential also modifies Kepler's law. Taking the radial derivative of $V_{\rm eff}$ in Eq. (A1) and equating it to zero gives modified Kepler's law as

$$\Omega^2 = \frac{m}{r^3}\left[1 + \frac{1}{2}A\,p\left(\frac{m}{r}\right)^p\right]. \tag{A2}$$

The above equation further gives the orbital separation as

$$r(t) = r_{\rm GR}\left[1 + \frac{1}{6}A\,p\,\eta^{-\frac{2p}{5}}u^{2p}\right], \tag{A3}$$

where to leading PN order, $r_{\rm GR}$ is given by Kepler's law as $r_{\rm GR} = (m/\Omega^2)^{1/3}$. For a circular orbit, radial kinetic energy does not exist and the effective potential energy is same as the binding energy of the binary. Using Eq. (A3) in Eq. (A1) and keeping only to leading order in non-GR corrections, the binding energy becomes

$$E = -\frac{1}{2}\eta^{-2/5}u^2\left[1 - \frac{1}{3}A(2p-5)\eta^{-2p/5}u^{2p}\right], \tag{A4}$$

where $u = (\pi\mathcal{M}f)^{1/3}$.

Next, let us introduce dissipative corrections. Such corrections to the GW luminosity can be parameterized by

$$\dot{E} = \dot{E}_{\rm GR}\left[1 + B\left(\frac{m}{r}\right)^q\right], \tag{A5}$$

where $\dot{E}_{GR}$ is the GR luminosity which is proportional to $v^2(m/r)^4$ with $v = \Omega r = (\pi m f)^{1/3}$ is the relative velocity of binary components[10].

Let us now derive the amplitude corrections. First, using Eqs. (A4) and (A5) and applying the chain rule, the GW frequency evolution is given by

$$\begin{aligned}\dot{f} &= \frac{df}{dE}\frac{dE}{dt}\\ &= \dot{f}_{GR}\left[1 + B\eta^{-\frac{2q}{5}}u^{2q} + \frac{1}{3}A(2p^2-2p-3)\eta^{\frac{-2p}{5}}u^{2p}\right],\end{aligned} \tag{A6}$$

where $\dot{f}_{\rm GR}$ is given by Eq. (6). Next, using Eqs. (A3) and (A6) to Eq. (9) and keeping only to leading order in non-GR corrections, the GW amplitude in Fourier domain becomes

$$\tilde{A}(f) = \tilde{A}_{GR}\left[1 - \frac{B}{2}\eta^{\frac{-2q}{5}}u^{2q} - \frac{1}{6}A(2p^2-4p-3)\eta^{\frac{-2p}{5}}u^{2p}\right]. \tag{A7}$$

Next, we move onto deriving phase corrections. One can derive the GW phase in the Fourier domain by integrating Eq. (18) twice. Equivalently, one can use the following expression

$$\Psi(f) = 2\pi f t(f) - \phi(f) - \frac{\pi}{4}, \tag{A8}$$

where $t(f)$ gives the relation between time and frequency and can be obtained by integrating (A6) as

$$\begin{aligned}t(f) &= \int \frac{dt}{df}\,df\\ &= t_0 - \frac{5\mathcal{M}}{256u^8}\left[1 + \frac{4}{3}A\frac{(2p^2-2p-3)}{(p-4)}\eta^{-\frac{2p}{5}}u^{2p}\right.\\ &\quad\left. + \frac{4}{q-4}B\eta^{-\frac{2q}{5}}u^{2q}\right],\end{aligned} \tag{A9}$$

with t_0 representing the time of coalescence and keeping only the Newtonian term and leading order non-GR corrections. On the other hand, $\phi(f)$ in Eq. (A8) corresponds to the GW phase in the time domain and can be calculated from Eq. (A6) as

$$\begin{aligned}\phi(f) &= \int 2\pi f dt = \int \frac{2\pi f}{\dot{f}}df\\ &= \phi_0 - \frac{1}{16u^5}\left[1 + \frac{5}{3}A\frac{(2p^2-2p-3)}{(2p-5)}\eta^{-\frac{2p}{5}}u^{2p}\right.\\ &\quad\left. + \frac{5}{2q-5}B\eta^{-\frac{2q}{5}}u^{2q}\right],\end{aligned} \tag{A10}$$

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[10] If we assume $\dot{E}_{\rm GR}$ to be proportional to $r^4\Omega^6$, we will find slightly different expressions for $\dot{f}$ and the waveform [75]

with ϕ_0 representing the coalescence phase. Using Eqs. (A9) and (A10) into (A8) and writing $\Psi(f)$ as $\Psi_{\rm GR}(f)+\delta\Psi(f)$, non-GR modifications to the phase can be found as

$$\delta\Psi(f) = -\frac{5}{32}A\frac{2p^2-2p-3}{(4-p)(5-2p)}\eta^{\frac{-2p}{5}}u^{2p-5} - \frac{15}{32}B\frac{1}{(4-q)(5-2q)}\eta^{-\frac{2q}{5}}u^{2q-5}\,, \tag{A11}$$

with $\Psi_{\rm GR}$ to leading PN order is given by [100]

$$\Psi_{\rm GR} = 2\pi f t_0 - \phi_0 - \frac{\pi}{4} + \frac{3}{128}u^{-5}\,. \tag{A12}$$

We can easily rewrite the above expressions using γ_r and c_r. Comparing Eq. (A3) with Eq. (4) we find

$$A = \frac{12\gamma_r}{c_r}\eta^{\frac{c_r}{5}}\,, \quad p = \frac{c_r}{2}\,. \tag{A13}$$

Using this, we can rewrite the GW amplitude in Eq. (A7) as

$$\tilde{\mathcal{A}}(f) = \tilde{\mathcal{A}}_{GR}\left[1 - \frac{B}{2}\eta^{\frac{-2q}{5}}u^{2q} - \frac{\gamma_r}{c_r}(c_r^2-4c_r-6)u^{c_r}\right]\,. \tag{A14}$$

Similarly, one can rewrite the correction to the GW phase in Eq. (A11) as

$$\delta\Psi(f) = -\frac{15}{8}\frac{\gamma_r}{c_r}\frac{c_r^2-2c_r-6}{(8-c_r)(5-c_r)}u^{c_r-5} - \frac{15}{32}B\frac{1}{(4-q)(5-2q)}\eta^{-\frac{2q}{5}}u^{2q-5}\,. \tag{A15}$$

Appendix B: GW Frequency Evolution From Energy-Balance Law in Varying-G Theories

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In this appendix, we show an alternative approach to find $\dot{f}$ in Eq. (59) by applying and correcting the energy balance law used in Ref. [97]. We begin by considering the total energy of a binary given by $E=-(G_C\mu m)/2r$. In order to calculate the leading order correction to the frequency evolution due to $\dot{G}$, we use Kepler's law to rewrite the binding energy as

$$E(f,G_C,m_1,m_2) = -\frac{1}{2}\mu(G_C m\Omega)^{2/3}\,, \tag{B1}$$

where $\Omega=\pi f$ is the orbital angular frequency. Taking a time derivative of the above expression and using Eqs. (48)-(50) in Eq. (B1), the rate of change of the binding energy becomes

$$\frac{dE}{dt} = \frac{\pi^{2/3}}{6f^{1/3}G_C^{1/3}m^{4/3}}\left[-3fG_Cm(\dot{m}_1m_2+m_1\dot{m}_2) - 2m^3\eta(G_C\dot{f}+f\dot{G}_C)+m^2fG_C\eta\dot{m}\right]\,. \tag{B2}$$

We can use the following energy balance argument to derive $\dot{f}$. In GR, the time variation in the binding energy is balanced with the GW luminosity $\dot{E}_{\rm GW}$ emitted from the system given by

$$\dot{E}_{\rm GW} = \frac{1}{5}G_D\left\langle \dddot{Q}_{ij}\dddot{Q}_{ij} - \frac{1}{3}(\dddot{Q}_{kk})^2\right\rangle = \frac{32}{5}r^4G_D\mu^2\Omega^6\,. \tag{B3}$$

In varying-G theories, there is an additional contribution $\dot{E}_{\dot{G}}$ due to the variation in G, masses, and the angular momentum. Namely, the binding energy is not conserved even in the absence of GW emission and the energy balance law is modified as

$$\frac{dE}{dt} = -\dot{E}_{\rm GW} + \dot{E}_{\dot{G}}\,. \tag{B4}$$

To estimate such additional contribution, we rewrite the binding energy in terms of the specific angular momentum as

$$E(G_C,m_1,m_2,j) = -\frac{G_C^2\,\mu\, m^2}{2j^2}\,. \tag{B5}$$

Taking the time variation of this leads to

$$\dot{E}_{\dot{G}} = \frac{\partial E}{\partial j}\dot{j}_0 + \frac{\partial E}{\partial m_1}\dot{m}_{10} + \frac{\partial E}{\partial m_2}\dot{m}_{20} + \frac{\partial E}{\partial G_C}\dot{G}_{C0}\,, \tag{B6}$$

where $\dot{j}_0$, purely due to the variation of G_C, is given by Eq. (54).

We are now in a position to derive the frequency evolution. Using Eqs. (B3), (B5) and (B6) in Eq. (B4), one finds ***ky: Can you please put this in a form of leading $\times$ (1+correction)?***

$$\frac{dE}{dt} = -\frac{32}{5}\pi^{10/3}f^{10/3}\eta^2G_C^{4/3}G_Dm^{10/3} + \frac{1}{j^3}G_C^2\eta m^3\dot{j}_0 - \frac{1}{j^2}G_Cm^3\eta\dot{G}_{C,0} - \frac{G_C^2}{2j^2}\left[m(\eta m+m_1)\dot{m}_{2,0} - m(\eta m+m_2)\dot{m}_{1,0}\right] \tag{B7}$$

Substituting this further into Eq. (B2) and solving for $\dot{f}$, one finds the frequency evolution as

$$\dot{f} = \frac{96}{5}\pi^{8/3}G_C^{2/3}G_D\mathcal{M}^{5/3}f^{11/3}\left\{1+\frac{5}{96}\frac{G_C}{G_D}\dot{G}_{C,0}\eta^{3/5}[2m - 5(m_{1,0}s_{1,0}+m_{2,0}s_{2,0})]u^{-8}\right\}\,, \tag{B8}$$

where $u=(\pi G_C\mathcal{M}f)^{1/3}$, in agreement with Eq. (59).

Along with the constancy of masses, the last term in Eq. (B4) was also missing in [97]. Consequently, our PPE parameters in Eqs. (64) and (66) do not agree with Ref. [97] even when we take the limit of no time variation in masses. Difference in $\beta_{\dot{G}}$ is smaller than 20% while $\alpha_{\dot{G}}$ differs by a factor of 7. Despite the discrepancy, we expect the projected bounds on $\dot{G}_0/G_0$ calculated in Ref. [97] to be qualitatively correct.

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