Resource Allocation for Mini-batch SGD via Adaptive Batch Sizes

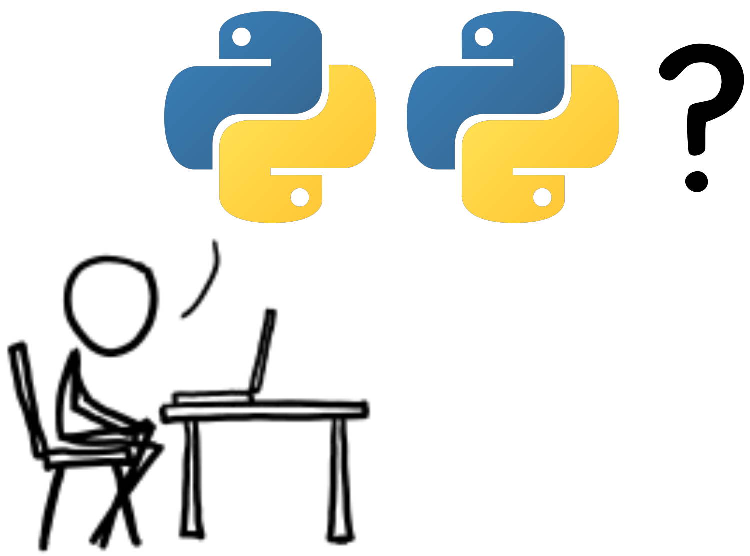
Scott Sievert

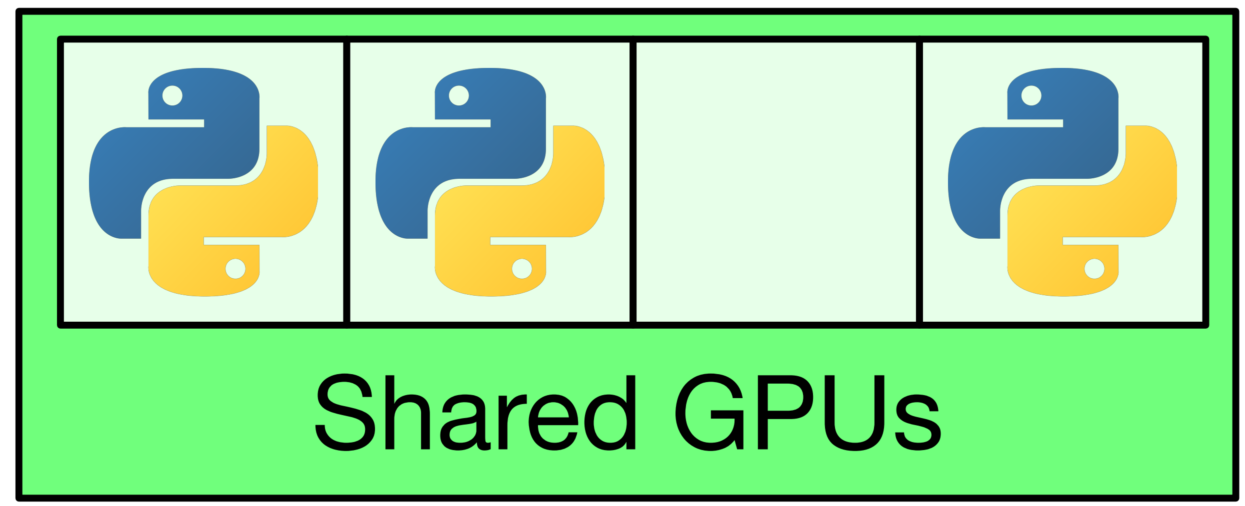
Agenda

- Motivation
- Results
- Practical implementation, with experiments

Joint work with
Zachary Charles

Motivation





Motivation: SGD

$$\min_{\boldsymbol{w}\in\mathbb{R}^d} F(\boldsymbol{w}) := \frac{1}{n}\sum_{i=1}^{n} f(\boldsymbol{w};\boldsymbol{z}_i)$$

In deep learning, typically solved via SGD (or a variant):

$$\boldsymbol{w}_{k+1} = \boldsymbol{w}_k - \frac{\gamma}{B}\sum_{i=1}^{B} \nabla f(\boldsymbol{w}_k;\boldsymbol{z}_{i_j})$$

...with a static batch size *B* (another hyperparameter)

Computer Science > Distributed, Parallel, and Cluster Computing arXiv:1711.04325

Extremely Large Minibatch SGD: Training ResNet-50 on ImageNet in 15 Minutes

Takuya Akiba, Shuji Suzuki, Keisuke Fukuda

(Submitted on 12 Nov 2017)

We demonstrate that training ResNet-50 on ImageNet for 90 epochs can be achieved **in 15 minutes with 1024 Tesla P100 GPUs**. This was made possible by using **a large minibatch size of 32k.** ...

Computer Science > Computer Vision and Pattern Recognition arXiv:1706.02677

Accurate, Large Minibatch SGD: Training ImageNet in 1 Hour

Priya Goyal, Piotr Dollár, Ross Girshick, Pieter Noordhuis, Lukasz Wesolowski, Aapo Kyrola, Andrew Tulloch, Yangqing Jia, Kaiming He

(Submitted on 8 Jun 2017 (v1), last revised 30 Apr 2018 (this version, v2))

… trains ResNet- 50 with **a minibatch size of 8192 on 256 GPUs in one hour**, …

It'd be nice to train quickly
without having to buy 100's of GPUs

To do that, let's *grow* the batch size:

$$\boldsymbol{w}_{k+1} = \boldsymbol{w}_k - \frac{\gamma}{B_k} \sum_{i=1}^{B_k} \nabla f(\boldsymbol{w}_k; \boldsymbol{z}_{i_j})$$

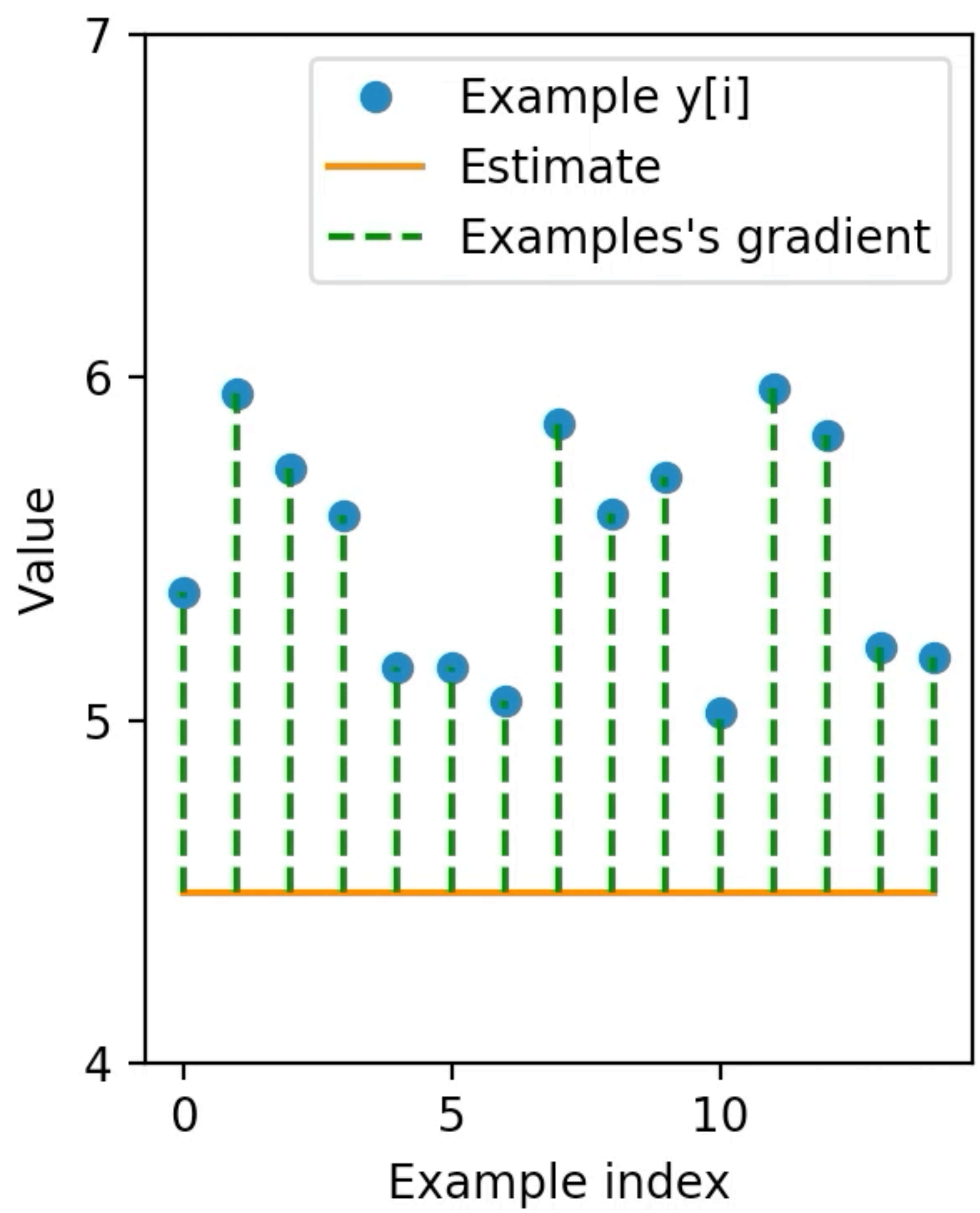
Why?

Intuition for batch size growth

Why have a large batch size with poor initialization?

Let's fit the simplest linear model:

$$\min_{x \in \mathbb{R}} \frac{1}{2} \sum_{i=1}^{n} (x - y_i)^2$$



Batch size growth

Let's grow the batch size via

a constant

best possible loss
(possibly unknown)

$$B_k = \left\lceil \frac{c}{F(\boldsymbol{w_k}) - F^\star} \right\rceil$$

Poor initialization: $B_k = 1$
At optimum: $B_k = \infty$

Main result: summary

Feature of SGD: no dependence on number of examples in training set

Feature of gradient descent (GD): few model updates

When the batch size is $B_k = \left\lceil \dfrac{c}{F(\boldsymbol{w_k}) - F^\star} \right\rceil$, then

the same number of

- model updates as GD is required
- examples as SGD is required

I have formal theorems backing this statement.

Main result

To achieve a model $\boldsymbol{x}$ with training loss $F(\boldsymbol{x}_k) - F^\star \in [\epsilon/2, \epsilon]$,

Function class	SGD	Adaptive batch sizes	Gradient descent
Smooth and convex	$\mathcal{O}\left(1/\epsilon^2\right)$	$\mathcal{O}\left(1/\epsilon\right)$	$\mathcal{O}\left(1/\epsilon\right)$
Strongly convex*	$\mathcal{O}\left(1/\epsilon\right)$	$\mathcal{O}\left(\log(1/\epsilon)\right)$	$\mathcal{O}\left(\log(1/\epsilon)\right)$

model updates need to be completed and

Function class	SGD	Adaptive batch sizes	Gradient descent
Smooth and convex	$\mathcal{O}\left(1/\epsilon^2\right)$	$\mathcal{O}\left(1/\epsilon^2\right)$	$\mathcal{O}\left(n/\epsilon\right)$
Strongly convex*	$\mathcal{O}\left(1/\epsilon\right)$	$\mathcal{O}\left(\log(1/\epsilon)/\epsilon\right)$	$\mathcal{O}\left(n\log(1/\epsilon)\right)$

examples need to be processed

Similar results for reaching a saddle point with non-convex function

*actually a generalization of strongly convex [3]

These theorems are confirmed with simulations.

1. Bubeck et al. *Convex optimization: Algorithms and complexity*. 2015.
2. Hamed Karimi et al. *Linear convergence of gradient and proximal-gradient methods under the polyak-łojasiewicz condition*. 2016.
3. Yurii Nesterov. *Introductory lectures on convex optimization: A basic course*. 2013.

Limitations

1. Using the *entire* train dataset *every* model update takes a long time
2. How does the model perform on unseen data?
3. GPU memory is finite

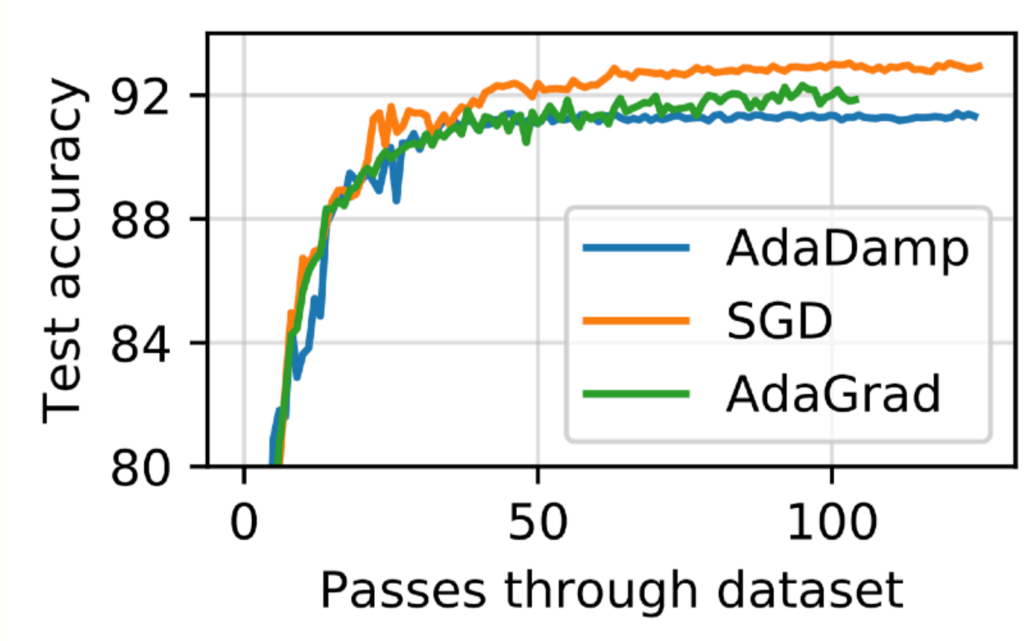
Algorithm 1 Mini-batch SGD with dampening of noise in gradient approximation

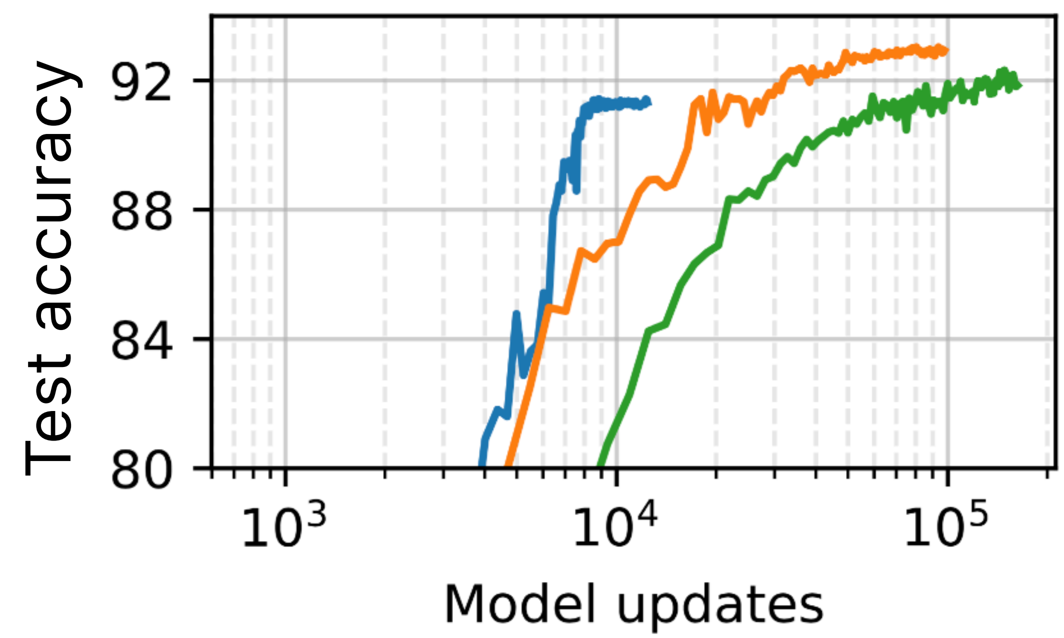
procedure ADADAMP(initial batch size B_0, initial model x_0, step size γ, maximum batch size $B_{\max}$)
    for $k \in [0, 1, 2, \ldots]$ **do**
        if $k = 0$ **then**
            $c \leftarrow B_0(F(x_0) - F^\star)$
        $B_k \leftarrow \lceil c/(F(x_k) - F^\star) \rceil$
        $\gamma' = \gamma$
        if $B_k > B_{\max}$ **then**
            $\gamma' \leftarrow \gamma B_{\max}/B_k$
            $B_k = B_{\max}$
        $x_{k+1} \leftarrow$ TRAIN(x_k, γ', B_k) ▷ TRAIN computes n gradients
    return x_k

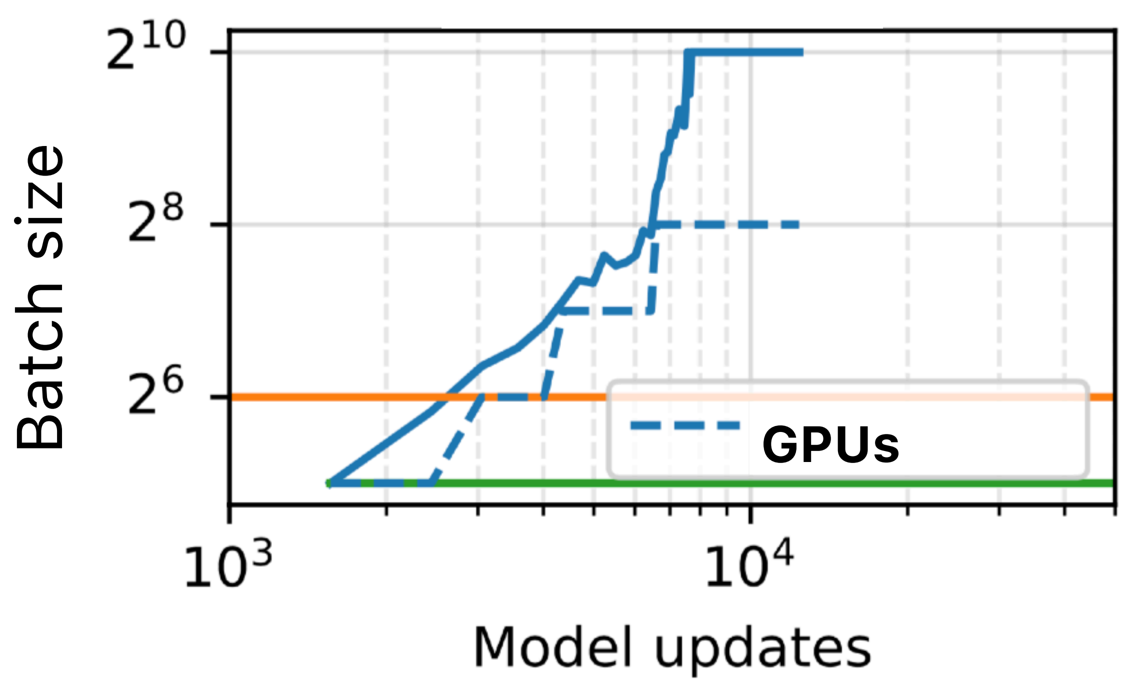
Smith et. al. *A bayesian perspective on generalization and stochastic gradient decent.* 2017

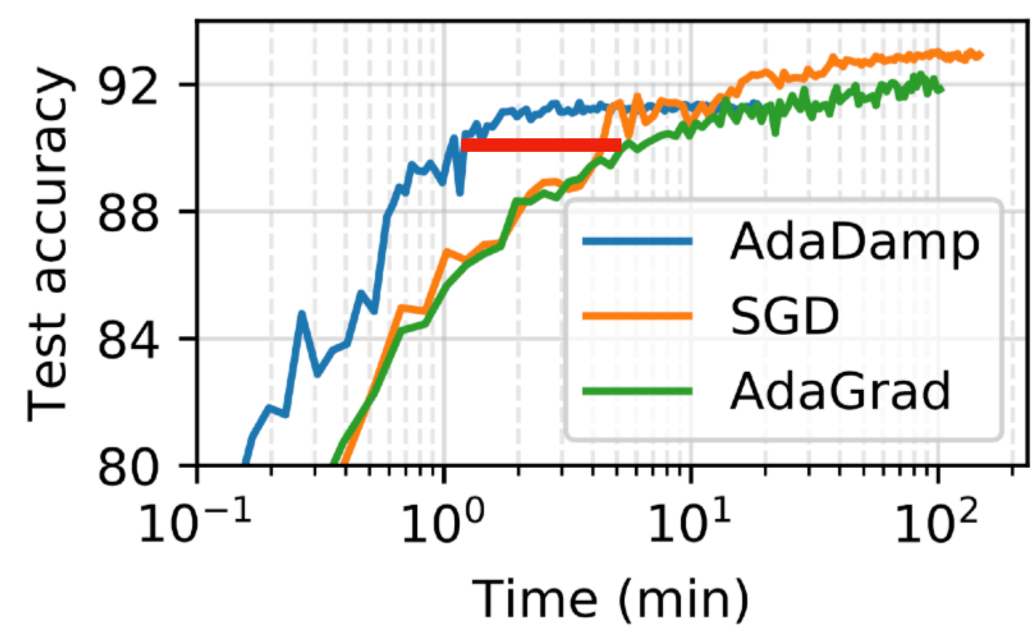
Experiment

ResNet-34 + CIFAR-10
Brief hyperparameter tuning









All plots assume oracle provides $F(\boldsymbol{x}_k) - F^\star$

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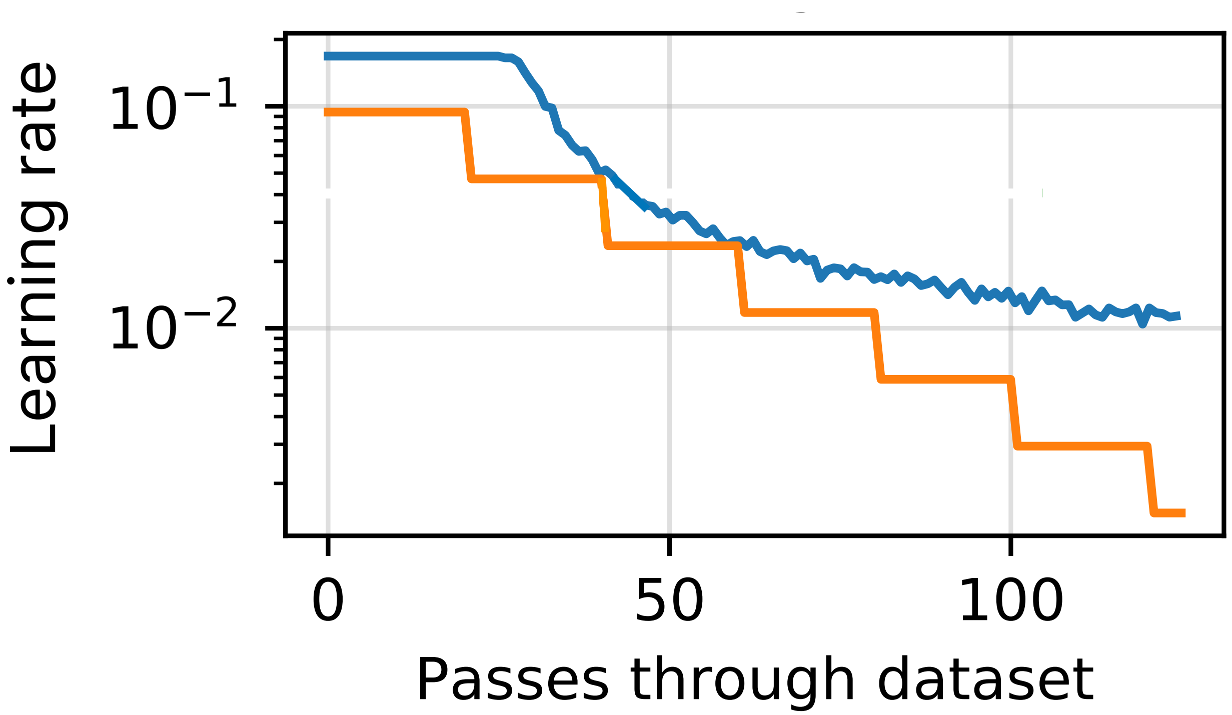
How can the batch size be estimated?

Our upper bounds suggest $B_{\text{epoch}} \propto r \cdot \text{epoch}$

Similar method uses $B_{\text{epoch}} \propto r^{\text{epoch}}$



This needs more investigation

Smith et. al., *Don't decay the learning rate, increase the batch size.* 2017

*for smooth+convex and non-convex functions

Thanks!

Questions?

Future work

after this work is finished

How does this method generalize?

Bounds

Convex and smooth	$F(\boldsymbol{x}_k) - F^\star \leq \mathcal{O}\left(r^k\right)$
α-PL (generalization of strongly convex)	$F(\boldsymbol{x}_k) - F^\star \leq \mathcal{O}\left(\frac{1}{k}\right)$
Non-convex	$\min\limits_{k=0,\ldots,T-1} \\|\nabla F(\boldsymbol{x}_k)\\|_2^2 \leq \mathcal{O}\left(\frac{1}{k}\right)$

Max batch size?

Computer Science > Machine Learning

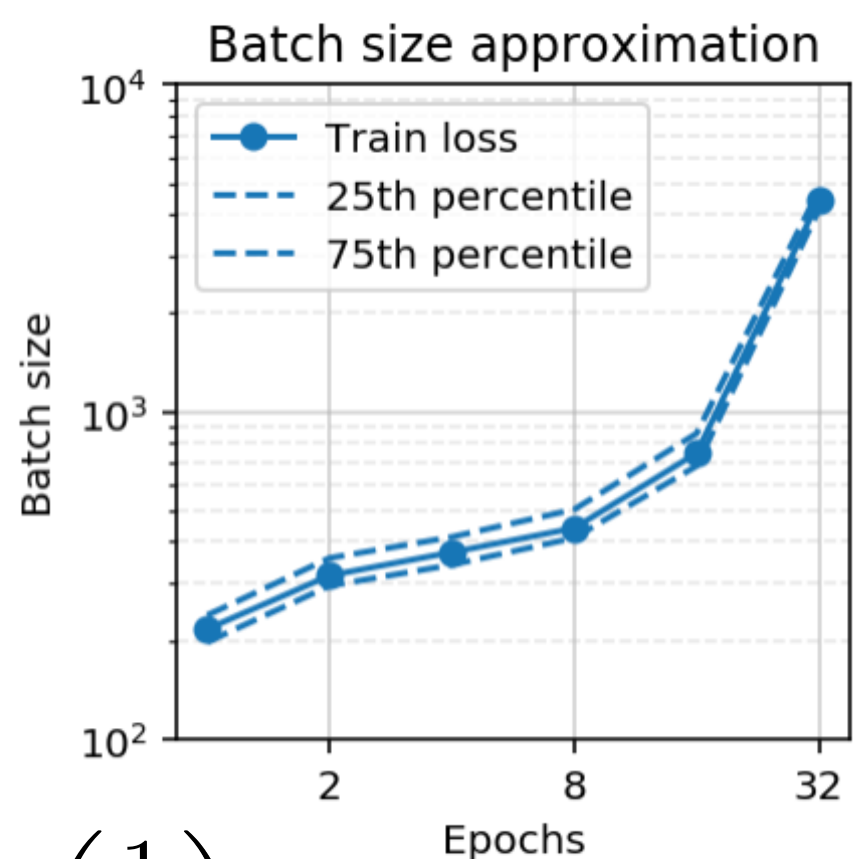
A Bayesian Perspective on Generalization and Stochastic Gradient Descent

Samuel L. Smith, Quoc V. Le

(Submitted on 17 Oct 2017 (v1), last revised 14 Feb 2018 (this version, v3))

… We also demonstrate that, when one holds the learning rate fixed, **there is an optimum batch size which maximizes the test set accuracy.** …

Entire train dataset?



$$F(\boldsymbol{x}_k) - F^\star \leq \mathcal{O}\left(\frac{1}{k}\right)$$

*For convex and non-convex functions. For convex, relies on

Rule of thumb can likely be developed