Problem Formulation

The primary objective of this project is to reconstruct a reference electrocardiogram (ECG) given an ECG distorted by magnetohydrodynamics (MHD) effects. The available data are the reference ECG, the distorted ECG, and the magnetic field strength. The magnetic field strength is constant, and no hemodynamic data are available. While other features are available (e.g., patient demographics, breathing patterns, position), we start a simple general model and assume that those features are not informative in reconstructing the reference ECG. The data can be found in the following link:

https://www.physionet.org/content/mhd-effect-ecg-mri/1.0.0/

Physics-Informed Neural Network (PINN)

Physics Equations

We assume the following model:

$$d(t) = s(t) + m(t) + \varepsilon(t) \tag{1}$$

where $d(t)$ is the distorted ECG signal, $s(t)$ is the reference ECG signal, $m(t)$ is the induced voltage due to MHD effects, and $\varepsilon(t)$ is random noise. The induced electric field is

$$\vec{E}_{induced}(t) = \vec{E}_{external} + \vec{v}(t) \times \vec{B}(t) \tag{2}$$

where $\vec{v}(t)$ is the blood velocity and $\vec{B}(t)$ is the magnetic field. We assume that the external electrical field is negligible and the magnetic field is constant. We reduce (2) to

$$\vec{E}_{induced} = \vec{v}(t) \times \vec{B} \tag{3}$$

By the Faraday Law of Induction, the induced voltage due to MHD effects is

$$m(t) = \oint \vec{E}_{induced} \cdot d\vec{l} = \oint (\vec{v}(t) \times \vec{B}) \cdot d\vec{l} \tag{4}$$

For the velocity profile, we use Ohm's law to determine the current density. This is given as

$$\vec{J} = \sigma(\vec{E}_{external} + \vec{v} \times \vec{B}) = \sigma(\vec{v} \times \vec{B}) \tag{5}$$

where $\vec{J}$ is the current density. Neglecting the external electric field, the Lorentz force is given by

$$\vec{F}_{Lorentz} = \vec{J} \times \vec{B} \tag{6}$$

To model blood flow, we assume that blood is incompressible and non-Newtonian. We use the Navier-Stokes equation, which is given as

$$\rho(\frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \nabla)\vec{v}) = -\nabla p + \nabla \cdot \vec{\vec{\tau}} + \vec{F}_{Lorentz} \tag{7}$$

where ρ is the blood density, p is the pressure, and $\vec{\vec{\tau}}$ is the stress tensor. We assume that only the Lorentz force is the only significant external body force.

Model Equations

Under the constraint that there is no fluid velocity data, we assume that the fluid velocity takes the form $\vec{v} \propto f(s(t))$. Additionally, since the magnetic field is constant, $\vec{B} = B_0\hat{z}$. We rewrite (4) as

$$m(t) \propto B_0 f(s(t)) \tag{8}$$

and substitute this equation into (1). This yields

$$d(t) = s(t) + \alpha B_0 f(s(t)) + \varepsilon(t) \tag{9}$$

where α is a hyperparameter that weighs the strength of the MHD effect. Since we are interested in reconstructing the reference ECG signal, the PINN seeks to compute

$$\hat{s}(t) = d(t) - \alpha B_0 f(\hat{s}(t)) \tag{10}$$

where $\hat{s}(t)$ is the reconstructed reference ECG signal.

Loss Functions

We define the data loss as

$$Loss_{data} = \frac{1}{N}\Sigma_{i=1}^{N}(\hat{s}(t_i) - s_{obs}(t_i))^2 \tag{11}$$

We define the physics loss as

$$Loss_{physics} = \frac{1}{N}\Sigma_{i=1}^{N}(d(t_i) - \hat{s}(t_i) - \alpha B_0 f(\hat{s}(t_i)))^2 \tag{12}$$

We define the total loss as

$$Loss = \lambda_{data} Loss_{data} + \lambda_{physics} Loss_{physics} \tag{13}$$

where λ_{data} and $\lambda_{physics}$ are hyperparameters used to weigh the two losses.