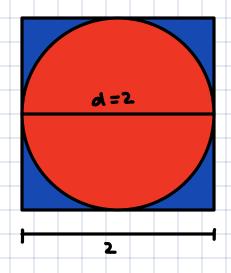
Monte Carlo				
Classical Statist	ic S			
1. Identify wh	ect you wont t	o measure		
2. Create Test s	Statistic and	determine i	ts distributi	<b>6</b> 0
3. Determine cr	rifical value o	not/or p-va	ilve	
Issue: Requires	large sample	c Coffee n	_ 00 ~ n	230)
2350E · Nequires	idige sampi	- Cotten M		
Q: Is there a w	ay to estimat	e the p-va	lve?	
-estimating is	fine as loca			
est mating is	tine or rong	as p-value	13 1101 400	C1085 40 00
Cox = cutoff for	significance)			
Exact: P=0.	045	Exact: (	0=0.045	
Estimate: p=	0.042	Estimate:	P= 0.049	
CSTITUTE PA	0.012		, , , , , ,	
Cutoff: $x = 0$	05	Cutoff: d	k = 0.05	
			×	
Monte Carlo utili	izes repeated	random sa	indina to p	redict
probabilities of	various outcon	nes		
Idea # 1: Law o	L laga Muse	hece (114)		
TORK HIL. FON	- Large 100M	OCTS CLLIV		
Average conver	ges to Expect	ation as sa	mple size be	ecomes large
Idea #2: Utilize	computer bomer	to generate	large sampl	es
Take advantage	of dockanias			
inac aavairage	. J. Technology			

Procedure:  B = large integer  For 1,2,, B  Generate sample of size in from some distribution  Compute any interesting values and test statistic.  Estimate p-value $p$ -value $\simeq$ $\frac{1}{8+1}$ B+1  Ex: Goodness-of-Fit test  You want to see if a dice is fair or not. After 60 ralls, you record:  1: 10 2:3 3:12  4:24 5:6 6:5  Compute the exact and estimated p-value.  Test: $\chi^2$ goodness of fit test $\chi^2 = \xi_1^2 \frac{(O_1 - E_1)^2}{E_1^2} = 29$																															
B = large integer  For 1,2,, B  Generate sample of size n from some distribution  Compute any interesting values and test statistic.  Estimate p-value  p-value =	) DO	ed	UN	2:																											
For 1, 2,, B  Generate sample of size n from some distribution  Compute any interesting values and test statistic.  Estimate p-value $ \begin{array}{c} \text{He more extreme test statistics} + 1 \\ \text{P-value} \simeq \\ \hline \text{B+1} \\ \hline \text{St.: Goodness-of-Fit test} \\ \text{You want to see if a dice is fair or not. After 60 rolls, you record:} \\ 1: 10     2:3     3: 12     4: 24     5: 6     6: 5     6: 5     6  Compute the exact and estimated p-value.  Test: \chi^2 goodness of fit test  \chi^2 = \sum_{i=1}^{3} \frac{(o_i - E_i)^2}{E_i} = 29     \text{ df = k-1 = 5} $																															
For 1, 2,, B  Generate sample of size n from some distribution  Compute any interesting values and test statistic.  Estimate p-value $ \begin{array}{c} \text{He more extreme test statistics} + 1 \\ \text{P-value} \simeq \\ \hline \text{B+1} \\ \hline \text{St.: Goodness-of-Fit test} \\ \text{You want to see if a dice is fair or not. After 60 rolls, you record:} \\ 1: 10     2:3     3: 12     4: 24     5: 6     6: 5     6: 5     6  Compute the exact and estimated p-value.  Test: \chi^2 goodness of fit test  \chi^2 = \sum_{i=1}^{3} \frac{(o_i - E_i)^2}{E_i} = 29     \text{ df = k-1 = 5} $	В	=	la	M	e	int	ea	e.r																							
Generate sample of size n from some distribution  Compute any interesting values and test statistic.  Estimate p-value $ \begin{array}{cccccccccccccccccccccccccccccccccc$				J			J	<u> </u>																							
Generate sample of size n from some distribution  Compute any interesting values and test statistic.  Estimate p-value $ \begin{array}{cccccccccccccccccccccccccccccccccc$	F	DC	-	. 2			R																								
Compute any interesting values and test statistic.  Estimate p-value  It more extreme test statistics + 1  B+1  Ex: Goodness-of-Fit test  You want to see if a dice is fair or not. After 60 rolls, you record:  1: 10 2:3 3:12  4:24 5:6 6:5  Compute the exact and estimated p-value.  Test: $\chi^2$ goodness of fit test $\chi^2 = \xi^2 \frac{(O_i - E_i)^2}{E_i} = 29$ Af=k-1=5	- '	<b>U</b> 1	•	,	,	.,																									H
Compute any interesting values and test statistic.  Estimate p-value  It more extreme test statistics + 1  B+1  Ex: Goodness-of-Fit test  You want to see if a dice is fair or not. After 60 rolls, you record:  1: 10 2:3 3:12  4:24 5:6 6:5  Compute the exact and estimated p-value.  Test: $\chi^2$ goodness of fit test $\chi^2 = \xi^2 \frac{(O_i - E_i)^2}{E_i} = 29$ Af=k-1=5			G	م	0	o k		50/	MΩ	le	of	Si	<b>2</b> e	Λ	f	OM	Sc	ME	2 6	eis.	trit	sut	ion								
Estimate p-value $ \begin{array}{cccccccccccccccccccccccccccccccccc$									7					ļ ·																	
Estimate p-value $ \begin{array}{cccccccccccccccccccccccccccccccccc$						مد	40		iat	0 64	ــــــ		300		9 6	200		<del>ل</del> ەر	1	c.	Lie	ti.									
p-value $\simeq$   State   State			_	-OI	Ψ		411	7		-	.37	נייי	, ,,	-,,						314	1110	•									L
p-value $\simeq$   State   State				- 1																											
P-value $\approx$ B+1  You want to see if a dice is fair or not. After 60 rolls, you record:  1: 10  2:3  3:12  4:24  5:6  6:5  Compute the exact and estimated p-value.  Test: $\chi^2$ goodness of fit test $\chi^2 = \frac{(0i - E_i)^2}{E_i} = 29$ df = k-1 = 5	F	:54	M	at	e	P-	val	ve																							
P-value $\approx$ B+1  You want to see if a dice is fair or not. After 60 rolls, you record:  1: 10  2:3  3:12  4:24  5:6  6:5  Compute the exact and estimated p-value.  Test: $\chi^2$ goodness of fit test $\chi^2 = \frac{(0i - E_i)^2}{E_i} = 29$ df = k-1 = 5																															
B+1  Ex: Goodness-of-Fit test  You want to see if a dice is fair or not. After 60 rolls, you record:  1: 10  2:3  3:12  4:24  5:6  6:5  Compute the exact and estimated p-value.  Test: $\chi^2$ goodness of fit test $\chi^2 = 5! \frac{(0i - E_i)^2}{E_i} = 29$ df = k-1 = 5								1	# n	nor	æ	ex	tre	me	1	2st	sta	tis	tic	5	+ l										
Ex: Goodness-of-Fit test  You want to see if a dice is fair or not. After 60 rolls, you record:  1: 10  2:3  3:12  4:24  5:6  6:5  Compute the exact and estimated p-value.  Test: $\chi^2$ goodness of fit test $\chi^2 = \frac{(0i - E_i)^2}{E_i} = 29$ df = k-1 = 5			F	)—(	vq\	ve	=						•	-	•																
You want to see if a dice is fair or not. After 60 rolls, you record:  1: 10  2:3  3:12  4:24  5:6  6:5  Compute the exact and estimated p-value.  Test: $\chi^2$ goodness of fit test $\chi^2 = \xi^2 \frac{(0i - E_i)^2}{E_i} = 29$ df = k-1 = 5													7	ד נ	-																
You want to see if a dice is fair or not. After 60 rolls, you record:  1: 10  2:3  3:12  4:24  5:6  6:5  Compute the exact and estimated p-value.  Test: $\chi^2$ goodness of fit test $\chi^2 = \xi^2 \frac{(0i - E_i)^2}{E_i} = 29$ df = k-1 = 5																															
You want to see if a dice is fair or not. After 60 rolls, you record:  1: 10  2:3  3:12  4:24  5:6  6:5  Compute the exact and estimated p-value.  Test: $\chi^2$ goodness of fit test $\chi^2 = \xi^2 \frac{(0i - E_i)^2}{E_i} = 29$ df = k-1 = 5	+																														
You want to see if a dice is fair or not. After 60 rolls, you record:  1: 10  2:3  3:12  4:24  5:6  6:5  Compute the exact and estimated p-value.  Test: $\chi^2$ goodness of fit test $\chi^2 = \xi^2 \frac{(0i - E_i)^2}{E_i} = 29$ df = k-1 = 5				•			r			La	. L																				
1: 10 2:3 3:12  4:24 5:6 6:5  Compute the exact and estimated p-value.  Test: $\chi^2$ goodness of fit test $\chi^2 = \sum_{i=1}^{3} \frac{(O_i - E_i)^2}{E_i} = 29$	<u> </u>	G	00	a۸	67	-0	t -	<b>P</b> i	Ť '	TE,	ST																				
1: 10 2:3 3:12  4:24 5:6 6:5  Compute the exact and estimated p-value.  Test: $\chi^2$ goodness of fit test $\chi^2 = \sum_{i=1}^{3} \frac{(O_i - E_i)^2}{E_i} = 29$																		_					_								
4:24 S:6 6:5  Compute the exact and estimated p-value.  Test: $\chi^2$ goodness of fit test $\chi^2 = 5 \frac{(0i - Ei)^2}{E_i} = 29 \qquad df = k-1 = 5$	Tov	) (	NA	nt	*	, 5	ee	iŧ	0	A G	dic	e '	īS	fa	ir	Dſ	n	ot.	P	144	ور	6	9	oll	S,	yο·	. 1	දෙක	rd	:	
4:24 S:6 6:5  Compute the exact and estimated p-value.  Test: $\chi^2$ goodness of fit test $\chi^2 = 5 \frac{(0i - Ei)^2}{E_i} = 29 \qquad df = k-1 = 5$																															
Compute the exact and estimated p-value.  Test: $\chi^2$ goodness of fit test $\chi^2 = 5 \frac{(0_i - E_i)^2}{E_i} = 29$ af = k-1 = 5		1:	I				2	: 3	3			;	3:	12																	
Compute the exact and estimated p-value.  Test: $\chi^2$ goodness of fit test $\chi^2 = 5 \frac{(0_i - E_i)^2}{E_i} = 29$ af = k-1 = 5								_						_																	
Test: $\chi^2$ goodness of fit test $\chi^2 = 5 \frac{(0i - E_i)^2}{E_i} = 29 \qquad df = k-1 = 5$		4	. 2	4			5	: 6	•				<b>5</b> :	5																	
Test: $\chi^2$ goodness of fit test $\chi^2 = 5 \frac{(0i - E_i)^2}{E_i} = 29 \qquad df = k-1 = 5$																															
Test: $\chi^2$ goodness of fit test $\chi^2 = 5 \frac{(0i - E_i)^2}{E_i} = 29 \qquad df = k-1 = 5$	Con	·ρυ	te	. •	tne	. e	Xa	et	Ov	٦d	€	st	m	nte	?d	P	-VC	alv	٤.												
$\chi^2 = 5 \frac{(0_i - E_i)^2}{E_i} = 29$																															
$\chi^2 = 5 \frac{(0_i - E_i)^2}{E_i} = 29$	Tes	<b>.</b>	7	2	ac	od	AP.	22	of.	Li	+	to	<b>x</b> +																		
					J				•	-	•																				
							_	. 2		-																					
	ス	2	= 1	5.	(0	<u>:                                    </u>	Eį	)_	= ;	29			ل	\ <b>f</b> :	k	-1	= 5														
Calculate p-value: 2.32 × 10 <sup>-5</sup>	+ -			J		E				<u> </u>						·															
Calculate p-value: 2.32×10 <sup>-5</sup>																															
		۱.	لما	-0	O-	<b>.</b> ~	fc		2 2	2	2 I F	- 5																			
	La	121	AUT	~	۲.	- vu		•	۲٠:	، عر ا	- 10																				
	+																														

## Estimating TT

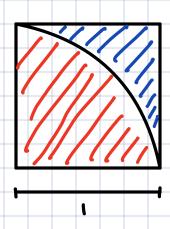
- -classic coding problem
- -recall dart board problem



Area of Circle = 
$$\Pi \left(\frac{d}{Z}\right)^2 = \Pi$$

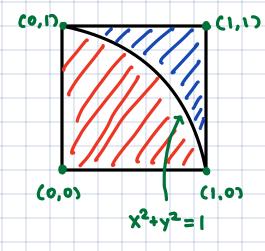
Total Area = 4

Use Quarter Circle



$$P(red) = \frac{Red Area}{Total Area} = \frac{\frac{1}{4}\pi(1^2)}{1^2} = \frac{\pi}{4}$$

Use whichever scheme is easier to program



Idea:

Generate many random points to see if
they fall inside the red or blue area  $IP(red) \approx \frac{\text{# red points}}{\text{# points}}$ 

By Law of Large Numbers, as # points grows large, the probability approaches expected value

C	im	ا ا			P.		٠	ro '	•																				
	1.6	er	.er	nt€	٤ ١	۸ (	poi	nts	i	1	۱×۱	S	qv	ar(	2														
				•	_	• •					•	ماد:																	
	2.	Def	err	Nir	16	(t	P	۸íO	t i	S	IVZ	(aę	. 1	TA	. •	eg.	01												
			di.	sta	лCE	2	:d	=	Ţ	2,	Y		د ا																
									ΛV	•	1		<i>-</i> '																
	3. (	Esł	in	nat	e	끈	·	sir	ر,	Ħ	ρo	ini	i	1	ed.	/ ‡	t p	nic	ls-										
						•																							
	۲ <u>. ۱</u>	401	tip	<b>/</b> \	Ьу	. L	1-	0	es+	im	ate	2	Ц																
E	Xþ	<b>&amp;</b> L	mj	.6~	ts	•																							
	C	la	<b>55</b> '	Τ		di	iff	ورو	yt.	<b>V</b> (	alu	<b>62</b>	οf	•															
	M	lini	i– f	roj	ec	<del>t :</del>	Tr	Y	ha	lf-	Ci	rele	2	)(	wh	ole	-	eir	cle	• •	Aρ	ply	fo	•	the	<b>ar</b> e	?as	·	