№1

(a)

¬D, (D ∨ F), ¬F – not consistent

Construct a proof for the argument: $\neg D, D \vee F, \neg F \therefore \bot$

1	$\neg D$	
2	$D \vee F$	
3	$\neg F$	
4	D	
5	$\bot$	¬E 1, 4
6	F	
7	$\bot$	¬E 3, 6
8	$\bot$	∨E 2, 4–5, 6–7

NEW LINE NEW SUBPROOF

☺ Congratulations! This proof is correct.

(b)

(T → K), ¬K, (K ∨ ¬T)

K	T	T → K	¬K	K ∨ ¬T
0	0	1	1	1
0	1	0	1	0
1	0	1	0	1
1	1	1	0	1

✓ is consistent

(c)

¬(A → (¬C → B)), ((B ∨ C) ∧ A) – not consistent

Construct a proof for the argument: $\neg[A \rightarrow (\neg C \rightarrow B)], (B \vee C) \wedge A \therefore \bot$

1	$\neg[A \rightarrow (\neg C \rightarrow B)]$	
2	$(B \vee C) \wedge A$	
3	A	
4	$\neg C$	
5	$B \vee C$	∧E 2
6	B	
7	B	R 6
8	C	
9	$\bot$	¬E 4, 8
10	B	X 9
11	B	∨E 5, 6–7, 8–10
12	$\neg C \rightarrow B$	→I 4–11
13	$A \rightarrow (\neg C \rightarrow B)$	→I 3–12
14	$\bot$	¬E 1, 13

NEW LINE NEW SUBPROOF

☺ Congratulations! This proof is correct.

(d)

(C → B), (D ∨ C), ¬B, (D → B) – not consistent

Construct a proof for the argument: $C \rightarrow B, D \vee C, \neg B, D \rightarrow B \therefore \bot$

1	$C \rightarrow B$	
2	$D \vee C$	
3	$\neg B$	
4	$D \rightarrow B$	
5	D	
6	B	→E 4, 5
7	$\bot$	¬E 3, 6
8	C	X 7
9	C	
10	C	R 9
11	C	∨E 2, 5–8, 9–10
12	B	→E 1, 11
13	$\bot$	¬E 3, 12

NEW LINE NEW SUBPROOF

☺ Congratulations! This proof is correct.

№2

(a)

Construct a proof for the argument: $H \rightarrow (R \wedge C), \neg R \vee \neg C \therefore \neg H$

1	$H \rightarrow (R \wedge C)$	
2	$\neg R \vee \neg C$	
3	$\neg(R \wedge C)$	DeM 2
4	$\neg H$	MT 1, 3

NEW LINE NEW SUBPROOF

☺ Congratulations! This proof is correct.

(b)

Construct a proof for the argument: $K \wedge S, \neg K \therefore \neg S$

1	$K \wedge S$	
2	$\neg K$	
3	K	∧E 1
4	$\bot$	¬E 2, 3
5	$\neg S$	X 4

NEW LINE NEW SUBPROOF

☺ Congratulations! This proof is correct.

(c)

Construct a proof for the argument: $A \rightarrow \neg A \therefore \neg A$

1	$A \rightarrow \neg A$	
2	$\quad A$	
3	$\quad \neg A$	→E 1, 2
4	$\quad \neg A$	
5	$\quad \neg A$	R 4
6	$\neg A$	LEM 2–3, 4–5

NEW LINE NEW SUBPROOF

☺ Congratulations! This proof is correct.

(d)

Construct a proof for the argument: $(P \wedge Q) \vee (P \wedge R) \therefore P$

1	$(P \wedge Q) \vee (P \wedge R)$	
2	$\quad P \wedge Q$	
3	$\quad P$	∧E 2
4	$\quad P \wedge R$	
5	$\quad P$	∧E 4
6	P	∨E 1, 2–3, 4–5

NEW LINE NEW SUBPROOF

☺ Congratulations! This proof is correct.

№3

(a)

A = philosophers ponder profound problems

B = their quandaries quell quotidian quibbles

C = right reasoning reveals reality

(A → B) ∧ (¬B ∨ C) ∧ A ∴ C

A	B	C	A → B	¬B ∨ C	A	C
0	0	0	1	1	0	0
0	0	1	1	1	0	1
0	1	0	1	0	0	0
0	1	1	1	1	0	1
1	0	0	0	1	1	0
1	0	1	0	1	1	1
1	1	0	1	0	1	0
1	1	1	1	1	1	1

✓ valid

Construct a proof for the argument: $A \to B, \neg B \vee C, A \therefore C$

1	$A \to B$	
2	$\neg B \vee C$	
3	A	
4	$(A \to B) \wedge (\neg B \vee C)$	∧I 1, 2
5	$((A \to B) \wedge (\neg B \vee C)) \wedge A$	∧I 4, 3
6	B	→E 1, 3
7	$\neg B$	
8	$\bot$	¬E 6, 7
9	C	X 8
10	C	
11	C	R 10
12	C	∨E 2, 7–9, 10–11

NEW LINE NEW SUBPROOF

☺ Congratulations! This proof is correct.

(b)

A = aardvarks are adorable

B = baby baboons don't beat bongos

C = crocodiles can't consume cute capybaras

(A → (B ∨ C)) ∧ ¬B ∧ (¬A ∨ C) ∴ ¬A

A	B	C	A → (B ∨ C)	¬B	¬A ∨ C	¬A	
0	0	0	1	1	1	1	✓ valid
0	0	1	1	1	1	1	✓ valid
0	1	0	1	0	1	1	
0	1	1	1	0	1	1	
1	0	0	0	1	0	0	
1	0	1	1	1	1	0	✗ invalid
1	1	0	1	0	0	0	
1	1	1	1	0	1	0	

Counterexample valuation: A = 1, B = 0, C = 1

(c)

A = discipline doesn't defeat deficiency

B = geniuses generally get good grades

C = homework has harmed humanity

(A → B) ∧ (¬A → C) ∴ B ∨ C

A	B	C	A → B	¬A → C	B ∨ C	
0	0	0	1	0	0	
0	0	1	1	1	1	✓ valid
0	1	0	1	0	1	
0	1	1	1	1	1	✓ valid
1	0	0	0	1	0	
1	0	1	0	1	1	
1	1	0	1	1	1	✓ valid
1	1	1	1	1	1	✓ valid

Construct a proof for the argument: $A \to B, \neg A \to C \therefore B \vee C$

1	$A \to B$	
2	$\neg A \to C$	
3	$\neg(B \vee C)$	
4	B	
5	$B \vee C$	∨I 4
6	$\bot$	¬E 3, 5
7	$\neg B$	¬I 4–6
8	$\neg(A \vee C)$	
9	A	
10	$A \vee C$	∨I 9
11	$\bot$	¬E 8, 10
12	$\neg A$	¬I 9–11
13	C	
14	$A \vee C$	∨I 13
15	$\bot$	¬E 8, 14
16	$\neg C$	¬I 13–15
17	C	→E 2, 12
18	$\bot$	¬E 16, 17
19	$A \vee C$	IP 8–18
20	A	
21	$\neg(\neg A \vee B)$	
22	B	→E 1, 20
23	$\bot$	¬E 7, 22
24	$\neg A \vee B$	IP 21–23
25	B	
26	$\bot$	¬E 7, 25
27	C	X 26
28	$B \to C$	→I 25–27
29	B	→E 1, 20
30	C	→E 28, 29
31	C	
32	C	R 31
33	C	∨E 19, 20–30, 31–32
34	$B \vee C$	∨I 33
35	$\bot$	¬E 3, 34
36	$B \vee C$	IP 3–35

NEW LINE NEW SUBPROOF

☺ Congratulations! This proof is correct.

(d)

A = crocodiles consume cute capybaras

B = incarcerating iguanas isn't illegal

C = mad monkeys make mayhem

D = dinosaurs do disco dance

(A → B) ∧ ((C ∧ D) ∨ A) ∧ ¬B ∴ D ↔ C

A	B	C	D	A → B	(C ∧ D) ∨ A	¬B	D ↔ C
0	0	0	0	1	0	1	1
0	0	0	1	1	0	1	0
0	0	1	0	1	0	1	0
0	0	1	1	1	1	1	1
0	1	0	0	1	0	0	1
0	1	0	1	1	0	0	0
0	1	1	0	1	0	0	0
0	1	1	1	1	1	0	1
1	0	0	0	0	1	1	1
1	0	0	1	0	1	1	0
1	0	1	0	0	1	1	0
1	0	1	1	0	1	1	1
1	1	0	0	1	1	0	1
1	1	0	1	1	1	0	0
1	1	1	0	1	1	0	0
1	1	1	1	1	1	0	1

✓ valid

Construct a proof for the argument: $A \rightarrow B, (C \wedge D) \vee A, \neg B \therefore D \leftrightarrow C$

1	$A \rightarrow B$	
2	$(C \wedge D) \vee A$	
3	$\neg B$	
4	$\quad D$	
5	$\quad\quad C \wedge D$	
6	$\quad\quad C$	∧E 5
7	$\quad\quad A$	
8	$\quad\quad B$	→E 1, 7
9	$\quad\quad \bot$	¬E 3, 8
10	$\quad\quad C$	X 9
11	$\quad C$	∨E 2, 5–6, 7–10
12	$\quad C$	
13	$\quad\quad C \wedge D$	
14	$\quad\quad D$	∧E 13
15	$\quad\quad A$	
16	$\quad\quad B$	→E 1, 15
17	$\quad\quad \bot$	¬E 3, 16
18	$\quad\quad D$	X 17
19	$\quad D$	∨E 2, 13–14, 15–18
20	$D \leftrightarrow C$	↔I 4–11, 12–19

№4

(a)

Construct a proof for the argument: $\neg\neg A \therefore A$

1	$\neg\neg A$	
2	$\neg A$	
3	$\bot$	¬E 1, 2
4	A	IP 2–3

NEW LINE NEW SUBPROOF

☺ Congratulations! This proof is correct.

(b)

Construct a proof for the argument: $(A \to B) \to A \therefore A$

1	$(A \to B) \to A$	
2	$\neg(\neg(A \to B) \vee A)$	
3	$\neg(A \to B)$	
4	$\neg(A \to B) \vee A$	∨I 3
5	$\bot$	¬E 2, 4
6	$A \to B$	IP 3–5
7	A	
8	$\neg(A \to B) \vee A$	∨I 7
9	$\bot$	¬E 2, 8
10	$\neg A$	¬I 7–9
11	A	→E 1, 6
12	$\bot$	¬E 10, 11
13	$\neg(A \to B) \vee A$	IP 2–12
14	$\neg(A \to B)$	
15	$\neg A$	
16	A	
17	$\bot$	¬E 15, 16
18	B	X 17
19	$A \to B$	→I 16–18
20	$\bot$	¬E 14, 19
21	A	IP 15–20
22	$\neg(A \to B) \to A$	→I 14–21
23	$\neg(\neg(A \to B) \vee (A \to B))$	
24	$A \to B$	
25	$\neg(A \to B) \vee (A \to B)$	∨I 24
26	$\bot$	¬E 23, 25
27	$\neg(A \to B)$	¬I 24–26
28	$\neg(A \to B) \vee (A \to B)$	∨I 27
29	$\bot$	¬E 23, 28
30	$\neg(A \to B) \vee (A \to B)$	IP 23–29
31	$\neg(A \to B)$	
32	A	→E 22, 31
33	$(A \to B)$	
34	A	→E 1, 33
35	A	∨E 30, 31–32, 33–34

NEW LINE NEW SUBPROOF

☺ Congratulations! This proof is correct.

(c)

Construct a proof for the argument: $\neg B \rightarrow \neg A \therefore A \rightarrow B$

1	$\neg B \rightarrow \neg A$	
2	A	
3	$\neg B$	
4	$\neg A$	→E 1, 3
5	$\bot$	¬E 2, 4
6	B	IP 3–5
7	$A \rightarrow B$	→I 2–6

NEW LINE NEW SUBPROOF Start a new sub

☺ Congratulations! This proof is correct.

(d)

Construct a proof for the argument: $\neg(A \vee B) \therefore \neg A \wedge \neg B$

1	$\neg(A \vee B)$	
2	A	
3	$A \vee B$	∨I 2
4	$\bot$	¬E 1, 3
5	$\neg A$	¬I 2–4
6	B	
7	$A \vee B$	∨I 6
8	$\bot$	¬E 1, 7
9	$\neg B$	¬I 6–8
10	$\neg A \wedge \neg B$	∧I 5, 9

NEW LINE NEW SUBPROOF

☺ Congratulations! This proof is correct.

(e)

Construct a proof for the argument: $\neg A \wedge \neg B \therefore \neg(A \vee B)$

1	$\neg A \wedge \neg B$	
2	$A \vee B$	
3	A	
4	$\neg A$	∧E 1
5	$\bot$	¬E 3, 4
6	B	
7	$\neg B$	∧E 1
8	$\bot$	¬E 6, 7
9	$\bot$	∨E 2, 3–5, 6–8
10	$\neg(A \vee B)$	¬I 2–9

NEW LINE NEW SUBPROOF

Add a new line at end.

☺ Congratulations! This proof is correct.

(f)

Construct a proof for the argument: $(A \to B) \land (\neg A \to B) \therefore B$

1	$(A \to B) \land (\neg A \to B)$	
2	$(A \to B)$	∧E 1
3	$(\neg A \to B)$	∧E 1
4	$\neg(A \lor \neg A)$	
5	A	
6	$A \lor \neg A$	∨I 5
7	$\bot$	¬E 4, 6
8	$\neg A$	¬I 5–7
9	$A \lor \neg A$	∨I 8
10	$\bot$	¬E 4, 9
11	$A \lor \neg A$	IP 4–10
12	A	
13	B	→E 2, 12
14	$\neg A$	
15	B	→E 3, 14
16	B	∨E 11, 12–13, 14–15

NEW LINE NEW SUBPROOF

☺ Congratulations! This proof is correct.

№5

(a)

Construct a proof for the argument: $\therefore (A \to B) \lor (B \to A)$

1	$\neg((A \to B) \lor (B \to A))$	
2	$A \to B$	
3	$(A \to B) \lor (B \to A)$	∨I 2
4	$\bot$	¬E 1, 3
5	$\neg(A \to B)$	¬I 2–4
6	B	
7	$\neg A$	
8	A	
9	$\bot$	¬E 7, 8
10	B	X 9
11	$A \to B$	→I 8–10
12	$\bot$	¬E 5, 11
13	A	IP 7–12
14	$B \to A$	→I 6–13
15	$(A \to B) \lor (B \to A)$	∨I 14
16	$\bot$	¬E 1, 15
17	$(A \to B) \lor (B \to A)$	IP 1–16

NEW LINE NEW SUBPROOF

☺ Congratulations! This proof is correct.

(b)

Construct a proof for the argument: $\therefore A \rightarrow (B \rightarrow A)$

1	A	
2	$\quad B$	
3	$\quad A$	R 1
4	$B \rightarrow A$	→I 2–3
5	$A \rightarrow (B \rightarrow A)$	→I 1–4

NEW LINE NEW SUBPROOF

☺ Congratulations! This proof is correct.

(c)

Construct a proof for the argument: $\therefore (\neg B \rightarrow \neg A) \rightarrow [(\neg B \rightarrow A) \rightarrow B]$

1	$\neg B \rightarrow \neg A$	
2	$\quad \neg B \rightarrow A$	
3	$\quad\quad \neg(B \vee \neg A)$	
4	$\quad\quad\quad B$	
5	$\quad\quad\quad B \vee \neg A$	∨I 4
6	$\quad\quad\quad \bot$	¬E 3, 5
7	$\quad\quad \neg B$	¬I 4–6
8	$\quad\quad\quad \neg A$	
9	$\quad\quad\quad B \vee \neg A$	∨I 8
10	$\quad\quad\quad \bot$	¬E 3, 9
11	$\quad\quad A$	IP 8–10
12	$\quad\quad \neg A$	→E 1, 7
13	$\quad\quad \bot$	¬E 11, 12
14	$\quad B \vee \neg A$	IP 3–13
15	$\quad\quad B$	
16	$\quad\quad B$	R 15
17	$\quad\quad \neg A$	
18	$\quad\quad\quad \neg B$	
19	$\quad\quad\quad A$	→E 2, 18
20	$\quad\quad\quad \bot$	¬E 17, 19
21	$\quad\quad B$	IP 18–20
22	$\quad B$	∨E 14, 15–16, 17–21
23	$(\neg B \rightarrow A) \rightarrow B$	→I 2–22
24	$(\neg B \rightarrow \neg A) \rightarrow ((\neg B \rightarrow A) \rightarrow B)$	→I 1–23

NEW LINE NEW SUBPROOF

☺ Congratulations! This proof is correct.

(d)

Construct a proof for the argument: $\therefore [A \to (B \to C)] \to [(A \to B) \to (A \to C)]$

1	$A \to (B \to C)$	
2	$A \to B$	
3	A	
4	B	→E 2, 3
5	$B \to C$	→E 1, 3
6	C	→E 5, 4
7	$A \to C$	→I 3–6
8	$(A \to B) \to (A \to C)$	→I 2–7
9	$(A \to (B \to C)) \to ((A \to B) \to (A \to C))$	→I 1–8

NEW LINE NEW SUBPROOF

☺ Congratulations! This proof is correct.