STA690 Final Project

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This document presents the mathematical formulation, assumptions, and summary of the class scheduling optimization problem. Full class scheduling optimal information is provided in the results excel file, and the code used to produce the optimal solution is provided.

Indices:

- i: $\{1, ..., i, .., |i|\}$ Professor Index
- j: $\{1, ..., j, .., |j|\}$ Class Index
- k: $\{1, ..., k, .., |k|\}$ Day/Time Index
- g: Class Groupings, by bins of 100 except for the 500-level group
- t: Day/Time Groupings (e.x. M/W 8-9 W/F 8-9 is one group)
- p: Prime Time Groupings (e.x. 10:05 to 2:40)
- o: Tuesday/Thursday Day Groupings

Decision Variables:

- $x_{i,j}$: Binary, 1 if professor i is teaching the j^{th} class, 0 o.w.
- $y_{j,k}$: Binary, 1 if j^{th} class is assigned to the k^{th} time, 0 o.w.
- $z_{i,k}$: Binary, 1 if professor i is assigned the k^{th} time, 0 o.w.
- $l_{i,j,k}$: Binary, 1 if professor i is teaching lab for class j is assigned to the k^{th} time, 0 o.w.

Pre-determined Variables:

- a_i: The maximal course credit load that the i^{th} professor can be assigned
- b_j: The total credits for the j^{th} class, (in a graduate school aligned basis)
- $c_{i,j}$: Binary, 1 if the i^{th} professor can teach the j^{th} class
- $d_{i,k}$: Binary, 1 if the i^{th} professor can teach during the k^{th} time slot
- e_j: Binary, 1 if Labs/Sections for the j^{th} class exist

Optimization Model

$$\text{Maximize:} \sum_{\forall i,j} x_{i,j} b_j \text{(Course credit maximization)}$$

$$\text{s.t.} \sum_{\forall i} x_{i,j} = 1 \quad \forall j \qquad \text{(Only one professor for each class, } |j| \text{ \# of constraints)} \tag{1}$$

$$\sum_{\forall j} x_{i,j} b_j \leq a_i \quad \forall i \qquad \text{(Professors maximal course load, } |i| \text{ \# of constraints)} \tag{2}$$

$$x_{i,j} \leq c_{i,j} \quad \forall i,j \qquad \text{(Proper course assignment, } |i| * |j| \text{ \# of constraints)} \tag{3}$$

$$\sum_{\forall k} y_{j,k} = 1 \quad \forall j \qquad \text{(Class can only be one time, } |j| \text{ \# of constraints)} \tag{4}$$

$$\sum_{\forall k} z_{i,k} \leq \lceil \frac{a_i}{3} \rceil \quad \forall i \qquad \text{(Limit professor time slots, } |i| \text{ \# of constraints)} \tag{5}$$

$$\sum_{\forall k \in t} z_{i,k} + \sum_{\forall k \in t, j} l_{i,j,k} \leq 1 \quad \forall i \qquad \text{(Limit professor overlap, } |i| \text{ \# of constraints)} \tag{6}$$

$$z_{i,k} \leq d_{i,k} \quad \forall i,k \qquad \text{(Proper professor time slots, } |i| * |k| \text{ \# of constraints)} \tag{7}$$

$$x_{i,j} + y_{j,k} - 1 \leq z_{i,k} \quad \forall i,j,k \qquad \text{(Link professor and class, } |i| * |j| * |k| \text{ \# of constraints)} \tag{8}$$

$$\sum_{\forall j} x_{i,j} * y_{j,k} \leq 1 \quad \forall i,k \qquad \text{(No double booking, } |i| * |k| \text{ \# of constraints)} \tag{9}$$

$$\sum_{\forall i,k} l_{i,j,k} = \sum_{\forall k} y_{j,k} e_j \quad \forall j \qquad \text{(Lab existence, } |j| \text{ \# of constraints)} \tag{10}$$

$$\frac{1}{3}(x_{i,j} + (1 - y_{j,k}) + d_{i,k}) \geq l_{i,j,k} \quad \forall i,j,k \qquad \text{(Link prof with lab, } |i| * |j| * |k| \text{ \# of constraints)} \tag{11}$$

$$\sum_{\forall j \in g, \forall k \in t} y_{j,k} + \sum_{i, \forall j \in g, \forall k \in t} l_{i,j,k} \leq 1 \quad \forall g,t \qquad \text{(Groupings not in same time, } g * t \text{ \# of constraints)} \tag{12}$$

$$y_{22,8} + y_{22,10} = 1 \qquad \text{(Grad research seminar soft constraint, 1 constraint)} \tag{13}$$

$$\sum_{j \in g, g \neq 6, k \in p} y_{j,k} * 2 \leq \sum_{j \in g, g \neq 6, \forall k} y_{j,k} \quad \forall g \qquad \text{(50\% prime time limit, g \# of constraints)} \tag{14}$$

$$\sum_{j \in g, g \neq 6, k \in o} y_{j,k} * 2 \leq \sum_{j \in g, g \neq 6, \forall k} y_{j,k} \quad \forall g \qquad \text{(50\% Tuesday, Thursday limit, g \# of constraints)} \tag{15}$$

$$x_{i,j}, y_{j,k}, z_{i,k}, c_{i,j}, d_{i,k}, e_j, l_{i,j,k} = \{0,1\} \quad a_i, b_j \geq 0 \qquad \text{(Domain)} \tag{16}$$

Considerations

Throughout creating the class schedule some assumptions and considerations were taken to ensure a solution, and these assumptions will be listed below.

1. Notice constraint #9, is non-linear in its formulation. Gurobi takes this and automatically linearizes it to allow for this to be an LP, this shortcut is taken in our document for brevity on notation.

2. Labs are modeled with a binary variable, with the assumption that when a class has multiple labs, the professor will split the sections between the two days that the lab gets assigned to. Labs in reality only one day and time slot, so when the optimal solution is shown, it should be noted that the professor will "split" their labs into the two days. If there is only one lab for a class, then a professor can choose the day to put their lab on. Note, no 500+ level classes have more than two lab sections, therefore there with the dual option there is no situation where a 500+ level class has two lab sections on one day and time.

3. We take a strong assumption that all 500 and 600 level classes cannot be scheduled during the same day/time, and labs of course. The solution is feasible with some looser assumptions as well, but we know that 500 shouldn't conflict with 600 so the other way is assumed as well.

4. Some items in the excel file we are assuming say constant, for example the time index, if new times get added or removed or the order changes, there will be a break down in the code.

5. We have a strong assumption that at most 50% of classes will be in the T/Th day pattern, therefore forcing the at least 40% requirement on M/W/F schedule.

Summary

Refer to the excel results document for the full schedule. The solution is optimal at assigning all 74 class credits (in graduate basis). In general, the results reflect just one of many optimal solutions for the problem. Further enhancements can put restrictions on assigning too many classes on the "outside" time blocks, and also spreading out the lab sections out to more time blocks when there is more than two labs.