

EM486A Numerical Methods

Example Iterative Solution

Carry out the first three iterations of the solution to the following system of equations using the Jacobi iterative method. For the first guess of the solution, take the value of all unknowns to be zero.

$$A = \begin{bmatrix} 7 & 3 & -1 \\ 3 & 8 & 1 \\ -1 & 1 & 4 \end{bmatrix}, b = \begin{bmatrix} 3 \\ -4 \\ 2 \end{bmatrix}$$

Solution: $\begin{bmatrix} 1 & -1 & 1 \end{bmatrix}^T$

$$x^0 = [0, 0, 0]^T$$

$$x_i^{k+1} = \frac{1}{a_{ii}} \left[b_i - \sum_{j=1, j \neq i}^n a_{ij} x_j^{(k)} \right]$$

$$x_1^1 = \frac{1}{7} [3 - (3(0) + -1(0))] = 0.4286$$

$$x_2^1 = \frac{1}{8} [-4 - (3(0) + 1(0))] = -0.5$$

$$x_3^1 = \frac{1}{4} [2 - [-1(0) + 1(0)]] = 0.5$$

$$x_1^2 = \frac{1}{7} [3 - (3(-0.5) + -1(0.5))] = 0.7143$$

$$x_2^2 = \frac{1}{8} [-4 - [3(0.4286) + 1(0.5)]] = -0.7232$$

$$x_3^2 = \frac{1}{4} [2 - [-1(0.4286) + 1(-0.5)]] = \cancel{0.7322} 0.7322$$

$$x_1^3 = \frac{1}{7} [3 - (3(-0.7232) - 1(0.7322))] = \cancel{0.8967} 0.8431$$

$$x_2^3 = \frac{1}{8} [-4 - [3(0.7143) + 1(0.7322)]] = \cancel{-0.8594} -0.8594$$

$$x_3^3 = \frac{1}{4} [2 - [-1(0.7143) + 1(-0.7232)]] = 0.8594$$

2. Carry out the first three iterations of the solution to the following system of equations using the Gauss-Seidel iterative method. For the first guess of the solution, take the value of all unknowns to be zero.

Solution: $[1 \ -1 \ 1]^T$

$$A = \begin{bmatrix} 7 & 3 & -1 \\ 3 & 8 & 1 \\ -1 & 1 & 4 \end{bmatrix}, b = \begin{bmatrix} 3 \\ -4 \\ 2 \end{bmatrix}$$

$$x^0 = [0 \ 0 \ 0]$$

$$x_1^1 = \frac{1}{7} [3 - [3(0) + -1(0)]] = 0.4286$$

$$x_2^1 = \frac{1}{8} [-4 - [3(0.4286) + 1(0)]] = -0.6607$$

$$x_3^1 = \frac{1}{4} [2 - [-1(0.4286) + 1(-0.6607)]] = 0.7723$$

$$x_1^2 = \frac{1}{7} [3 - [3(-0.6607) + -1(0.7723)]] = 0.8221$$

$$x_2^2 = \frac{1}{8} [-4 - [3(0.8221) + 1(0.7723)]] = -0.9048$$

$$x_3^2 = \frac{1}{4} [2 - [-1(0.8221) + 1(-0.9048)]] = 0.9317$$

$$x_1^3 = \frac{1}{7} [3 - [3(-0.9048) + -1(0.9317)]] = 0.9494$$

$$x_2^3 = \frac{1}{8} [-4 - [3(0.9494) + 1(0.9317)]] = -0.9725$$

$$x_3^3 = \frac{1}{4} [2 - [-1(0.9494) + 1(-0.9725)]] = 0.9805$$