EM425 Assignment #2

Problem Statements

1. (Based on problem 3.5) Determine the fourth root of 200 by finding the numerical solution of the equation $x^4 - 200 = 0$. Use Newton's method (by hand). Start at x = 8 and carry out the first five iterations.

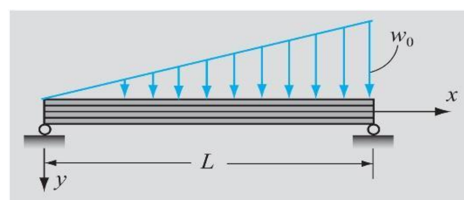
2. (Based on problem 3.22) Create a function to carry out Newton's method for finding a root for a nonlinear equation. The function should have the signature: `Xs = NewtonSol(Fun, FunDer, Xest)` where, `Xs` is a root, `Fun` and `FunDer` are handles to the function to be solved and its derivative, and `Xest` is an initial starting guess for the root. Your function should use the estimated relative error as a stopping criteria with a threshold value of 10^{-9}. The number of iterations should be limited to 100; if the solution is not found in 100 iterations the program should stop and display an error message. Use your function to solve the equation given in problem #1.

3. (Based on problem 3.24) Steffensen's method is a scheme for finding a numerical solution of an equation of the form $f(x)=0$ that is like Newton's method but does not require the derivative of $f(x)$. The solution process starts by choosing a point x_1, near the solution, as the first estimate of the solution. The subsequent estimates of the solution x_{i+1} are calculated by:

$$x_{i+1} = x_i - \frac{f(x_i)^2}{f(x_i + f(x_i)) - f(x_i)}$$

Write a MATLAB function that solves a nonlinear equation with Steffensen's method. The function signature should be: Xs = SteffensenRoot(Fun,Xest) where Xs is a root, Fun is a handle to the function you wish to solve, and Xest is the estimated root. Use your function to find the root of $f(x) = x - 2e^{-x}$. Use 2 as an initial starting point. As termination criteria, use the estimated relative error with 1×10^{-9} as a tolerance, and maximum iterations with a limit of 100.

4. (Based on problem 3.32) A simply supported I-beam is loaded with a distributed load, as shown in the figure. The deflection, y, of the centerline of the beam as a function of the position, x, is given by the equation:



$$y = \frac{w_o x}{360LEI}\left(7L^4 - 10L^2x^2 + 3x^4\right)$$

Where L = 4m is the length, E = 70 GPa is the elastic modulus, I = 52.9x10^{-6}m^4 is the moment of intertia, and w_0 = 20 kN/m is the distributed load.

Find the position x where the deflection of the beam is maximum, and determine the deflection at this point. (The maximum deflection is at the point where $\frac{dy}{dx} = 0$.)

a) Write a MATLAB function that solves this nonlinear equation using the Secant method. The function signature should be Xs = SecantRoot(Fun,Xa,Xb,Err,imax) where Err is the estimated relative error which you should set at 1×10^{-9}. Use 1.5 for Xa and 2.5 for Xb.

b) Repeat the solution of this equation with NewtonSol and SteffensonRoot.

Numeric Solutions

Problem #1:

If $x_1 = 8$, x_6 is approximately 3.7902

Problem #2:

```
Root from NewtonSol is: 3.760603093086393
```

Problem #3:

```
Root from SteffensenRoot is: 0.852605502013726
```

Problem #4:

The solution for all three methods should be similar. For my implementations:

```
Root from SecantRoot is: 2.077318489436912
Position of maximum deflection is x = 2.077318489436912
Maximum deflection y = 0.00901798
Root from NewtonSol is: 2.077318489436913
Position of maximum deflection is x = 2.077318489436913
Maximum deflection y = 0.00901798
Root from SteffensenRoot is: 2.077318489436934
Position of maximum deflection is x = 2.077318489436934
Maximum deflection y = 0.00901798
```