EM425 Assignment #5

Problem Statements

1. (Based on 6.19) Create a user-defined function for linear regression. The signature should be [a,Er] = LinReg(x,y). In addition to determining the constants a_0 and a_1 for a linear least-squares fit to the data, the function should also calculate the squared residual:

$$\text{Er} = \sum_{i=1}^{n}\left[y_i - \left(a_1 x_i + a_0\right)\right]^2$$

The input arguments x and y are vectors with the values of the data points. Use the function to find the coefficients for a linear least square fit to the following data and find the error.

x	1	3	4	6	9	12	14
y	2	4	5	6	7	9	11

2. (Based on 6.40) The following are measurements of the rate coefficient, k, for the reaction $CH_4 + O \rightarrow CH_3 + OH$ at different temperatures T.

T (K)	595	623	761	849	989	1076	1146	1202	1382	1445	1562
$k \times 10^{20}$ (m^3/s)	2.12	3.12	14.4	30.6	80.3	131	186	240	489	604	868

Use the method of least squares to best fit a function of the form $\ln(k) + C + b\ln(T) - \dfrac{D}{T}$ to the data. Determine the constants C, b and D by curve fitting a linear combination of the functions $f_1(T) = 1$, $f_2(T) = \ln(T)$, and $f_3(T) = \dfrac{1}{T}$ to the given data. Usually, the rate coefficient is expressed in the form of an Arrhenius equation $k = AT^b e^{-E_a/(RT)}$ where A and b are constants, R=8.314 J/mole/K is the universal gas constant, and E_a is the activation energy for the reaction. Having determined C, b, and D, deduce the values of A (m^3/s) and E_a (J/mole) in the Arrhenius expression.

3. (Based on 6.4) The following data is given:

x	0.2	0.5	1	2	3
y	3	2	1.4	1	0.6

By hand, determine the coefficients m and b in the function $y = \dfrac{1}{mx+b}$ that best fit the data using linear least-squares fit.

m = ____________, b = ______________

4. (Based on 6.27) The resistance R of a tungsten wire as a function of temperature can be modeled with the equation $R = R_0\left[1+\alpha\left(T-T_0\right)\right]$ where R_0 is the resistance corresponding to temperature T_0, and α is the temperature coefficient of resistance. Determine R_0 and α such that the equation will best fit the data. Use T_0 = 20°C.

T (°C)	20	100	180	260	340	420	500
R (Ω)	500	676	870	1060	1205	1410	1565

5. (Based on 6.30) In a uniaxial tension test, a dog-bone-shaped specimen is pulled in a machine. During the test, the force applied to the specimen, F, and the length of a gage section, L, are measured. The true stress, σ_t, and the true strain, ϵ_t, are defined by:

$$\sigma_t = \frac{F}{A_0}\frac{L}{L_0} \quad \text{and} \quad \epsilon_t = \ln\frac{L}{L_0}$$

Where A_0 and L_0 are the initial cross-sectional area and gage length, respectively. The true stress-strain curve in the region beyond the yield stress is often modeled by:

$$\sigma_t = K\epsilon_t^m$$

The following are values of F and L measured in an experiment. Determine the values of the coefficients K and m that best fit the data. The initial cross-sectional area and gage length are $A_0 = 1.25\times10^{-4}$ m², and $L_0 = 0.0125$ m. [**Note:** be careful with units.]

F (kN)	24.6	29.3	31.5	33.3	34.8	35.7	36.6	37.5	38.8	39.6	40.4
L (mm)	12.58	12.82	12.91	12.95	13.05	13.21	13.35	13.49	14.08	14.21	14.48

Numerical Answers

1.
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Coefficients:
    1.9357
    0.6214

Square Residual (error): 1.36429
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2. My advice is for the third function just use 1/T and not -1/T. Let the sign be provided by your curve fit function. Answer: c_1 = -52.4427, c_2 = 2.1216, c_3 = -3815.342 (check your result with a plot of the data and the curve fit)

3. m = 0.45, b = 0.24

4.
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Ro = 505.357
alpha = 0.00441519
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5.
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K = 5.49476e+08
m = 0.208528
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