EM425 Assignment #7

Problem Statements

1. (Based on 8.17) Write a MATLAB user-defined function that determines the first derivative of a function that is given by a set of discrete points with equal spacing. For the function name use yd = FirstDeriv(x,y). The input arguments x and y are vectors with the coordinates of the points, and the output argument yd is a vector with the values of the derivative at each point. At the first and last points, the function should calculate the derivative with three-point forward and backward difference formulas, respectively. At all other points FirstDeriv should use the two-point central difference formula. Use FirstDeriv to calculate the derivative of the function given in the following tabulated data:

x	1.1	1.2	1.3	1.4	1.5
$f(x)$	0.6133	0.7822	0.9716	1.1814	1.4117

Output the value of $\frac{df}{dx}$ at $x = 1.3$

2. (Based on 8.18) Write a MATLAB user-defined function that calculates the second derivative of a function that is given by a set of discrete data points with equal spacing. For the function name and arguments use ydd = SecDeriv(x,y), where the input arguments x and y are vectors with the coordinates of the points, and ydd is a vector with the values of the second derivative at each point. For calculating the second derivative, the function SecDeriv should use the finite difference formulas that have a truncation error of $\mathcal{O}(h^2)$. Use SecDeriv for calculating the second derivative of the function that is given by the tabulated data below:

x	−1	−0.5	0	0.5	1	1.5	2	2.5	3	3.5	4	4.5
$f(x)$	−3.632	−0.3935	1	0.6487	−1.282	−4.518	−8.611	−12.82	−15.91	−15.88	−9.402	9.017

Output the value of $\frac{d^2 f}{dx^2}$ at $x = 2.5$.

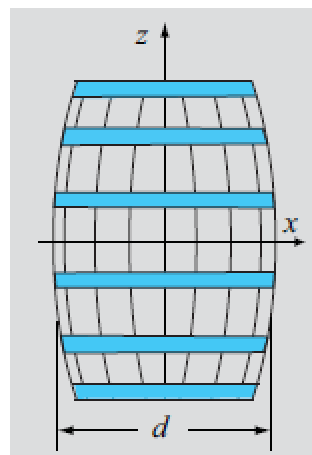
3. (Based on 8.37) A radar station is tracking the motion of an aircraft. The recorded distance to the aircraft, r, and the angle θ during a period of 60s is given in the table below. The magnitude of the instantaneous velocity and acceleration of the aircraft can be calculated by:

$$v = \sqrt{\left(\frac{dr}{dt}\right)^2 + \left(r\frac{d\theta}{dt}\right)^2}, \quad a = \sqrt{\left[\frac{d^2r}{dt^2} - r\left(\frac{d\theta}{dt}\right)^2\right]^2 + \left[r\frac{d^2\theta}{dt^2} + 2\frac{dr}{dt}\frac{d\theta}{dt}\right]^2}$$

Determine the magnitudes of the velocity and acceleration at the times given in the table. Plot the velocity and acceleration versus time on the same plot with two different y-axes using the built-in MATLAB tool yyaxis. Calculate the derivatives using the FirstDeriv and SecDeriv functions you developed for the previous two problems.

t (s)	0	4	8	12	16	20	24	28
r (km)	18.803	18.861	18.946	19.042	19.148	19.260	19.376	19.495
θ(rad)	0.7854	0.7792	0.7701	0.7594	0.7477	0.7350	0.7215	0.7073
t (s)	32	36	40	44	48	52	56	60
r (km)	19.617	19.741	19.865	19.990	20.115	20.239	20.362	20.484
θ(rad)	0.6925	0.6771	0.6612	0.6448	0.6280	0.6107	0.5931	0.5750

4. (Based on 9.19) Write a user-defined MATLAB function for integration with the trapezoidal method of a function $f(x)$ that is given in a set of n discrete points. The points do not have to be spaced equally. For the function name and arguments use I=IntPointsTrap(x,y), where the input arguments x and y are vectors with the values of x and the corresponding values of $f(x)$, respectively. The output argument I is the value of the integral.



Use the function to estimate the surface area and volume of a wine barrel. The diameter of the barrel is measured at the points provided in the table below. The surface area, S, and volume, V, can be determined by:

$$S = 2\pi\int_0^L r\, dz \quad \text{and} \quad V = \pi\int_0^L r^2\, dz$$

z (in.)	−18	−12	−6	0	6	12	18
d (in.)	28	30.2	31.5	32	31.5	30.2	28

Output the surface area and volume of the barrel.

5. (Based on 9.20) Write a user-defined MATLAB function for integration with Simpson's Rule of a function $f(x)$ that is given in a set of n discrete points that are spaced equally. For the function name and arguments use I=SimpsonPoints(x,y), where the input arguments x and y are vectors with the values of x and the corresponding values of $f(x)$, respectively. The output argument I is the value of the integral. Use the function to compute $\int_0^{1.8} f(x)\,dx$ with tabulated data below:

x	0	0.3	0.6	0.9	1.2	1.5	1.8
$f(x)$	0.5	0.6	0.8	1.3	2	3.2	4.8

Output the value of the integral in MATLAB.

6. (Based on 9.25) Write a user-defined MATLAB function for integration of a function $f(x)$ in the domain $[a,b]$ with five-point Gauss quadrature. For function name and arguments use I=GaussQuad5ab(Fun,a,b) where Fun is a name for the function that is being integrated, a and b are the lower and upper bounds of the integral and I is the value of the integral. Use the function to calculate the value of the following integral:

$$\int_0^3 e^{-x^2}\,dx$$

Output the value of the integral in MATLAB.

Solutions:

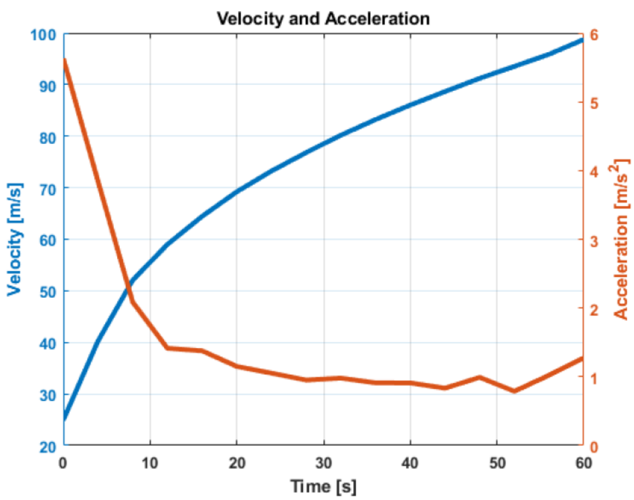
Problem #1

The first derivative at x = 1.3 is 1.996

Problem #2

The second derivative at x = 2.5 is 4.476

Problem #3



Problem #4

Surface Area: 3457.01 in^2
Volume: 26467.5 in^3

Problem #5

The integral is: 3.13

Problem #6

The integral is: 0.886529