## EM424 Analytical Methods

Lecture 31 - Polar Coordinates with Radial Symmetry

## Objectives:

- Carry out separation of variables process to solve the wave equation in polar coordinates.
- Quantitative example with MATLAB

Wave Equation in Polar Coordinates

PDE:

Apply radial symmetry to simplify the PDE. (be careful in making this assumption)

Example: Radial Vibrations of a Circular Membrane

BCs:

Step 1 - Assume a product solution

Step 2 - Insert into PDE

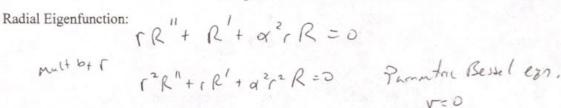
Step 3 - Separate Variables

e Variables
$$\frac{d^{2}(R'' + + + R' + )}{d^{2}(R'' + + + + R' + )} = \frac{R'' + + + R'}{R} = \frac{T''}{2T} = -\lambda$$

$$R'' + R' + \lambda (R = 0)$$

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Take an analysis short cut by assuming: 051.11 + 1.50 = 1.50. Therefore  $\lambda = \sqrt{\frac{2}{3}} = 1.50 = 1.50$ 



General Solution:

Menli: Y diwges to -00 as r-> of Boundary condition for r = 0: lim R(1) < 00 => (2=0

Boundary condition for r = c

Resulting eigenvalues and eigenfunctions:

Solution for T(t):

Product solution:

Apply initial position:

Apply initial position:
$$u(r, o) = \underbrace{\mathcal{E}}_{\Lambda=1} \left( A_{\Lambda} \cdot 1 + B_{\Lambda} \cdot o \right) J_{\sigma}(\alpha_{\Lambda} r) = \underbrace{\mathcal{E}}_{\Lambda=1} A_{\Lambda} J_{\sigma}(\alpha_{\Lambda} r) = f(r)$$

mult both sizes by Soldars and weight furtin place -

$$\int_{0}^{\infty} A_{n} J_{0}(\alpha_{n}r) J_{0}(\alpha_{n}r) r dr = \int_{0}^{\infty} f(r) J_{0}(\alpha_{n}r) r dr$$

$$= \int_{0}^{\infty} f(r) J_{0}(\alpha_{n}r) r dr = \int_{0}^{\infty} f(r) J_{0}(\alpha_{n}r) r dr$$

$$= \int_{0}^{\infty} f(r) J_{0}(\alpha_{n}r) r dr$$

Apply initial velocity:

E ada Ba Jo (dar) = 9(r) Some dell as above...

Be = 
$$\frac{1}{ada} \int_{0}^{c} g(r) J_{o}(dar) r dr$$

B examples

See MATLAB examples