# Geometric Comb Waveforms for Reverberation Suppression

Henry Cox
Hung Lai

ORINCON Corporation
4350 North Fairfax Drive, Suite 470
Arlington, VA 22203

## Abstract

*The major problem for sonar systems operating in shallow water is reverberation from the ocean bottom. A new class of waveforms is described which combines high range resolution with excellent Doppler properties for detecting moving targets in stationary reverberation. The waveforms consist of multiple simultaneously transmitted CW tone pulses. The tones are non-uniformly spaced in frequency to break up range ambiguities. A special subclass is the geometric comb in which the frequency spacing between the adjacent components follows a geometric progression. Analytic approximations can be used to obtain the important properties of this class, resulting in a practical design methodology. The resulting waveforms have excellent Doppler properties and an unambiguous high resolution range estimation capability. Techniques for reducing the peak-to-average power ratio are discussed.*

## 1 Introduction

It is sometimes said that the three most important problems for active sonars operating in shallow water are reverberation, reverberation and reverberation. There are two standard signal design approaches for fighting reverberation. The first exploits range resolution. The second exploits target Doppler to shift the echo into a valley of the reverberation spectrum.

In the first approach, a spectrally flat wideband pulse is used. This spreads the reverberation power over the bandwidth B of the pulse. When coherent matched filter processing is used, the pulse is compressed into an equivalent duration of $B^{-1}$. This provides a gain of 10 log BT against reverberation over that which would be experienced for a CW pulse of the same duration T against a zero Doppler echo. The performance of the wideband pulse is independent of the Doppler shift of the echo, whereas the CW pulse performance increases with increasing echo Doppler shift. Multipath time-spreading, which is usually severe in shallow water environments, limits the amount of echo compression which can be achieved by matched filtering and thereby limits the effectiveness of this approach.

Doppler exploitation typically involves the use of a long shaded CW pulse of duration T in order to produce a reverberation spectrum with a large narrow peak at zero Doppler and a rapid falloff with increasing Doppler shift. The idea is that target motion will shift the echo away from the peak into a spectral region in which the reverberation power is low. The spectral or Doppler resolution of the CW pulse is $T^{-1}$, making a long pulse desirable. However, the range resolution is proportional to T so that good Doppler rejection of reverberation is achieved at the expense of range resolution. The reverberation spectrum of the CW pulse can be smeared by source or receiver motion so that performance is best for stationary, slowly moving or highly directional systems. The weaknesses of the long CW pulse are the large peak of zero Doppler reverberation and the poor range resolution. The combination of a wideband pulse and a CW pulse in the same transmission is commonly used in active sonar systems to take advantage of the complementary nature of their characteristics.

The goal of our new waveform is to achieve significant improvement over a long CW pulse for Doppler rejection of reverberation and for tracking. To motivate our approach, consider a signal which is the sum of N simultaneously transmitted CW pulses with a uniform spacing s between adjacent tones. The spacing s is chosen so that it is large compared to the Doppler shifts of interest. We call this signal a uniform comb because of its uniformly spaced spectral peaks. The reverberation spectrum would have N peaks, and near each peak there would be a spectral component of the echo. The signal-to-reverberation spectral ratio of each component would be nearly the same as for a single CW pulse. Coherent addition of all these components provides a gain of 10 log N over the CW pulse. There are two severe problems with the uniform comb which make it unattractive as a sonar waveform, although it is useful for combating fading in communication. These problems are illustrated in Fig. 1 which shows the signal, its spectrum and its



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autocorrelation function for a uniform Hanning shaded comb with 34 components. First, the signal has large narrow peaks separated by wide valleys resulting in a very high peak-to-average power ratio. Since most sonar transmitters are peak power limited, with this type of waveform one must greatly lower the average power or suffer peak distortion. Second, the autocorrelation function has multiple peaks of nearly equal height causing severe range ambiguity. The geometric comb waveform overcomes both of these problems while retaining good spectral properties.

## 2 The Geometric Comb

The geometric comb is a multi-tone pulse in which the frequency spacings between successive components are a geometric progression. The basic signal is described by

$$x(t) = w(t)\sum_{n=1}^{N}\cos 2\pi f_n(t+\alpha), \quad 0 \le t \le T \qquad (1)$$

where $w(t)$ is a shading function used to reduce spectral sidelobes and $\alpha$ is a time shift parameter. All the components add in phase at $t = -a$. The frequencies $f_n$ satisfy the following relationships:

$$f_2 = f_1 + s \qquad (2a)$$

$$\vdots$$

$$f_N = f_{N-1} + r^{N-2}s \qquad (2b)$$

so that r is the ratio of adjacent spacings and is assumed to be slightly larger than one. The maximum spacing is

$$f_N - f_{N-1} = r^{N-2}s = Rs \qquad (3)$$

where R is the ratio of the maximum to the minimum spacing.

The series (2) may be summed to obtain the following expression for $f_N$

$$f_N = f_1 + s(r^{N-1} - 1)/(r-1) \qquad (4)$$

An approximate expression for the average spacing may be obtained from (3) and (4)

$$s_{AV} \approx s(R-1)/\ln R \qquad (5)$$

### 2.1 Autocorrelation Function

The attractiveness of the geometric comb as a signal waveform is primarily due to its well behaved autocorrelation function.

$$R_x(\tau) = \frac{1}{2}\int w(t)w(t+\tau)\left[\sum_{n=1}^{N}\cos 2\pi f_n\tau + \textit{oscillating sum and difference frequency terms}\right]dt \qquad (6)$$

Hence for a smooth positive window function $w(t)$

$$R_x(\tau) \approx \frac{1}{2}R_w(\tau)\sum_{n=1}^{N}\cos 2\pi f_n\tau \qquad (7)$$

Comparing (7) and (1), the autocorrelation is seen to be simply a shifted version of $x(t)$ in which the window function $w(t)$ is replaced by its autocorrelation function and multiplied by 0.5, the average power of a sine wave. The autocorrelation is important because it is the range ambiguity function at zero Doppler. The expression (7) does not depend on the geometric spacing and is approximately valid for any multi-tone pulse in which $T^{-1}$ is much smaller than the minimum frequency spacing. The peak at $\tau = 0$ has width $(f_N - f_1)^{-1}$.

The effect of non-uniform spacing is to spread and reduce the ambiguous peaks of the autocorrelation function of the uniform comb shown in Fig. 1. The important result for geometric spacing is that the ambiguous peaks are spread into plateaux with envelopes of constant level. The properties of these plateaux were derived by Chow [1] for the beampatterns of geometrically spaced arrays. The derivation involves the Poisson sum representation of a finite sum of cosines, used by Ishimaru [2], combined with the stationary phase approximation for evaluating the resulting integrals. The $k-th$ ambiguity plateau's envelope may be characterized as follows:

$$\text{Start} \quad : \quad k/Rs \qquad (8a)$$

$$\text{Stop} \quad : \quad k/s \qquad (8b)$$

$$\text{Relative Level} \quad : \quad (Nk\ln R)^{-\frac{1}{2}} \qquad (8c)$$

The basic properties of the autocorrelation function are simply described in terms of N, R, s, and T, which greatly simplifies waveform design. We have found that 1.6 is a good value for R. In the region where plateaux overlap, the components can sometimes add in phase giving a local peak amplitude equal to the sum of the overlapping envelope amplitudes. Overlap of multiple plateaux at large values of $\tau/s$ is countered by the shading function's autocorrelation, $R_w(\tau)$.

Fig. 2 shows a Hanning shaded geometric comb ($R = 1.59, N = 34$, $s = 2.227$, $\alpha = 0$) which may be directly compared with the uniform comb of Fig. 1 to see the effects of geometric spacing. The ambiguous peaks in the autocorrelation function have all been suppressed by more than 10 dB. The envelope is well described by (8). Another useful property of the autocorrelation function is the deep valleys adjacent to the peak at $\tau = 0$. This permits the use of split window normalization in range without being limited by the plateaux regions.

### 2.2 The Peak-to-Average Power Ratio

It was noted earlier that the autocorrelation function was essentially a shifted version of the basic waveform. Hence, the suppression of ambiguous peaks in the autocorrelation function also suppresses unwanted peaks in the waveform, thereby improving the peak-to-average power ratio. The role of the parameter $\alpha$ in (1) is to shift the peak at $t = -\alpha$ and the adjacent deep valleys so they are not part of the transmitted waveform. For Hanning weighting a shift may not be necessary since the values of the weighting function are very small near $t = 0$. These effects are easily seen in comparing the waveforms of Figs. 1 and 2. The peak amplitude in the geometric comb is smaller by more than 10 dB when compared to the uniform comb.

In judging peak-to-average power ratio it is useful to define a factor F, the ratio of the peak-to-average power ratio of the waveform of interest to that of a sine wave with the same shading. For a geometric comb

$$F \approx (\text{signal peak})^2 / N \tag{9}$$

For the signal of Fig. 2, F is approximately 2.5. This compares with a value of 34 for the uniform comb for which F=N.

Examining the waveform of Fig. 2, it appears that the peak-to-average power ratio could be further reduced by passing the waveform through a limiter which clipped off the peaks. This will cause a distortion of the signal spectrum and partially fill in the valleys between the spectral peaks. Adjusting the limiting threshold provides a tradeoff between peak-to-average power ratio and spectral distortion. This effect is illustrated in Fig. 3 in which the limiting threshold is set at 7. Then $F \approx 7^2 / 34 = 1.44$ which is remarkably close to that of a sine wave. Limiting of the signal peaks has almost no effect on the autocorrelation function. The spectral fill-in is about 40 dB down from the peaks.

## 2.3 Doppler Ambiguity

The basic concept is to choose the frequency spacings so that they are larger than the Doppler shifts of interest. This avoids Doppler ambiguity. This may not always be possible so that it is useful to examine Doppler ambiguity.

Consider an echo from a target with radial velocity v so that each frequency $f_k$ is shifted by $2f_k v / c$. In the special case in which

$$r = 1 + s / f_1 \tag{10}$$

the frequencies themselves, not just the spacings, form a geometric progression, and

$$f_{k+1} = r f_k \tag{11}$$

In this case, the ratio of the Doppler shift to the spacing is the same for each frequency component. That is

$$\left(\frac{2v}{c}\right) f_k / r^{k-1} s = \frac{2v}{c} f_1 / s \tag{12}$$

When $2(v/c)f_1 = s$, all frequency components would be simultaneously shifted to the frequency of the next higher component. Usually we choose $r > 1 + s/f_1$, so that this does not occur. The Doppler ambiguity is then reduced and smeared over the region $s / f_1 \le \left|\frac{2v}{c}\right| \le Rs / f_{N-1}$.

## 3 Alternative Processing Strategies

An attractive feature of the geometric comb waveform is that it offers a number of alternative processing strategies. These include matched filtering, incoherent sum across frequency, M of N across frequency and various combinations. The reverberation spectrum will be far from white, hence there is significant potential gain by preceding any of these processors with an adaptive prewhitening filter. Prewhitening is also helpful for a long CW pulse.

In conditions of severe time-spread, the performance of the matched filter can be seriously degraded. Even in these conditions, by incoherently combining frequencies the geometric comb can provide approximately 5 log N improvement over the standard CW pulse. This compares to 10 log N improvement by matched filtering in non-time-spread conditions. Thus, for a 30 component geometric comb we expect 7.5 to 15 dB improvement in signal-to-reverberation ratio over a CW pulse of the same duration.

## 4 Conclusion

Geometric comb waveforms are very attractive for shallow water reverberation-limited situations. They have excellent Doppler properties for reverberation suppression. They can provide a unique high range resolution peak for tracking, with range ambiguities more than 10 dB down. They have a good peak-to-average power ratio. This can be further improved by peak limiting at the expense of some spectral distortion. Alternative processing schemes make it possible to achieve 7.5 to 15 dB gain over a CW pulse depending on the severity of the multipath time-spread.

## 5 References


[1] Y.L. Chow, "*On Grating Plateaux of Nonuniformly Spaced Arrays*", IRE Trans Ant and Prop, (AP-13) pp 208-215, Mar 1965.

[2] A. Ishimaru, "*Theory of Unequally-Spaced Arrays*", IRE Trans Ant and Prop, (AP-10) pp 691-702, Nov 1963.

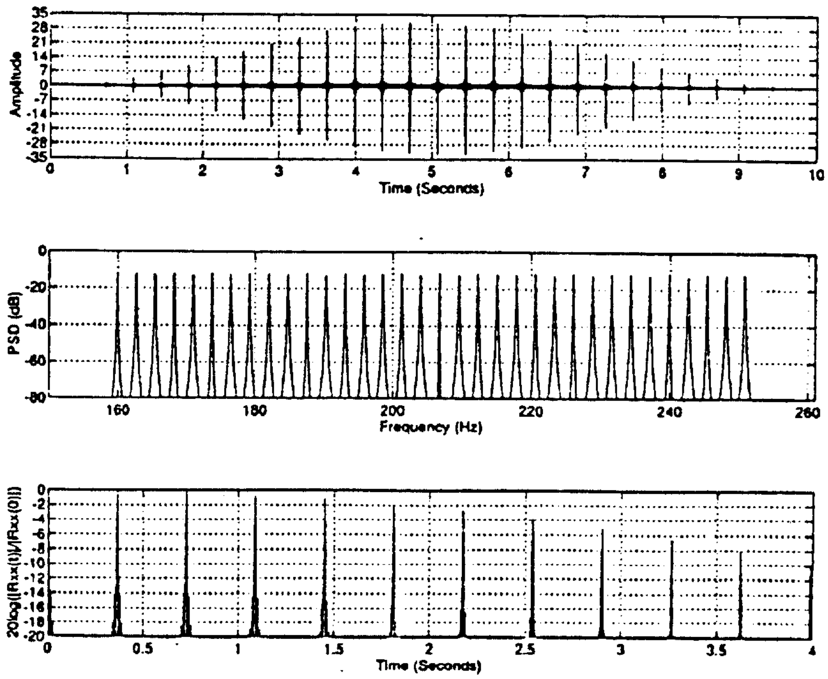


**Figure 1 Uniform Comb with Hanning: Waveform, Spectrum, Autocorrelation (N=34, R=1.0, s=2.758, $\alpha = 0$)**

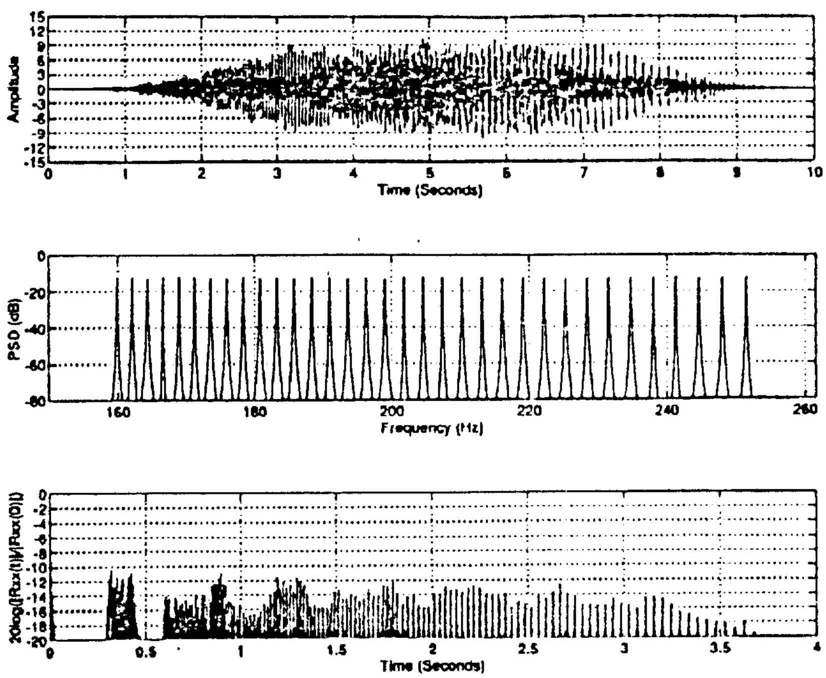


**Figure 2 Geometric Comb with Hanning: Waveform, Spectrum, Autocorrelation (N=34, R=1.59, s=2.227, $\alpha = 0$)**

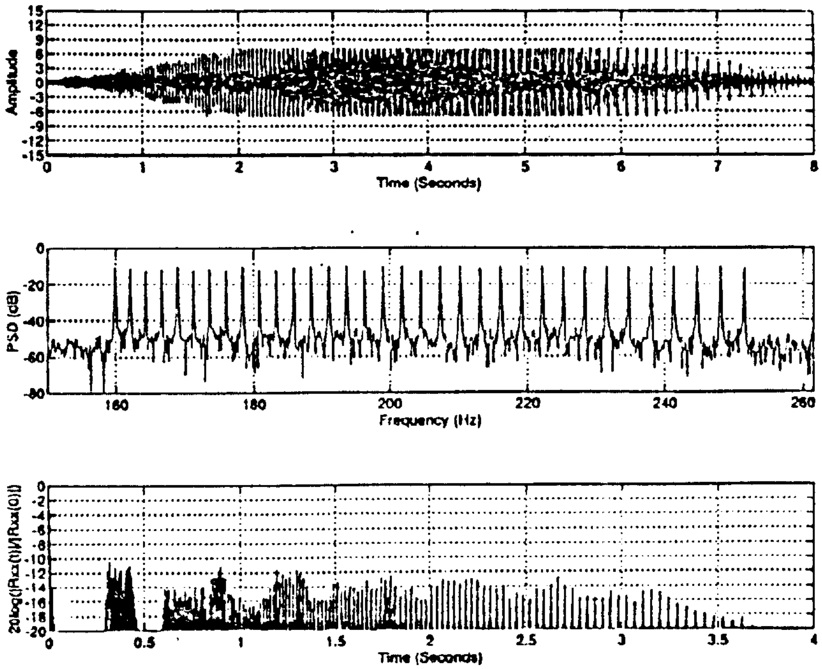


**Figure 3 Geometric Comb with Peak Limiting at 7: Waveform, Spectrum, Autocorrelation (N=34, R=1.59, s=2.227, $\alpha=0$)**