Thesis Notes

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Monday, June 15

- Preliminary meeting
- We will begin with ϕ^4 term due to lower energy bound.
- Beginning with code in Python, switch to C/C++ if necessary.

Tuesday, June 16

- Preliminary concepts to understand:
  - scalar field theory on lattice
  - Markov chains and Monte Carlo
  - the gradient flow
  - $O(n)$ symmetry
- General components to research, executed in parallel
  - Reading
  - Mathematical analysis
  - Writing code
  - Present (writing, plot generation,. . .)
- Things to get out of dissertation (Schaich and Loinaz 2006)
  - Markov chain
  - Cluster algorithms
    * Metropolis is a method for narrowing possibilities by accepting only some changes. It can get stuck in a local mininum, loss of ergodicity. We solve these with cluster algoriths. Wolff grows clus-

ters probabilistically and flips, while Swenson and Yang identifies clusters and flips them probabilistically.

    * Near a phase transition, correlation length grows and changes become less likely to be accepted: need clusters. Clusters dont work far from the phase transition. This is manifested as a sequence of a few metropolis steps and a cluster step.

- Research plan
  - Start with 2D ϕ^4
    * Set up lattice with sign flip for reflection
    * Use Markov chain Monte Carlo to simulate.
    * Measure, magnetization and suseptibility, Binder cumulant
  - Transition to 3D, then maybe transition to C/C++.
  - Implement the gradient flow
  - move to 2-3d nonlinear sigma model.
  - Motivation: the nonlinear sigma model works for QCD given the asymtotic freedom. We may also want to explore topology.

Wednesday, June 17

- Code tips:
  - Try Swendsen-Wang algorithm in addition to Wolff
  - Print out time taken
  - Optimize Hamiltonian
  - Save every tenth measurement or store configurations to calculate path integral. Exclude thermaliztion (first 200)
  - Write in terms of sweeps, not iterations
  - Parallelize (look into checkerboard algorithm)
  - Implement Binder cumulant and suseptibility
  - Store every few states
  - Look into multigrid algorithm
- Reading on Monte Carlo Markov chain and cluster algorithms.

Friday, June 19

- Looking over code
  - Might be too slow, move to C/C++ eventually?
  - Why is the energy increasing with metropolis algorithm?
  - Shift to using action instead of Hamiltonian.
  - Transition from Broken Phase, look at μ_0^2 term.
  - Profile code for possible optimizations.

Monday, June 22

- Coding
  - Add plots to `.gitignore`.
  - Try parallelizing code, using either multigrid or checkerboard algorithm
- Reading
  - Start to focus more on understanding the theory behind research.
  - Read dissertation (Schaich and Loinaz 2006) Chap. 6.5 and 6.6, take notes on questions.
  - Newman (Newman and Barkema 1999) (Main textbook for Monte Carlo in Statistical Physics)
- Just some things to remember
  - Correlation functions correlate values in statistical systems and relate to propagators in QFT.
  - The problem of renormalization: $a \to 0$ leads to unbounded correlation function. As real physical lengths are measured in terms of the lattice constant, these sometimes tend to infinity.

Wednesday, June 24

- Code

- Parallelize
- Transition to numpy

- Theory Question
  - renormalize and regularlize: what do they mean?
  - Look at LePage

Tuesday, June 30

- Code
  - Try `mpi4py`.
  - Move to 3D (this may decrease parallel overhead)
- Reading
  - Continue Reading Collins, others.

Thursday, July 2

- Coding
  - Continue implementing MPI
  - Parallelization may be more apparent in 3D
- Reading
  - Dirac fermions, represented by 4D spinor field (spin up/down, electron/positron)
  - Look at Tong (Chap. 4)
  - Charge (See Tong, Noether's Theorem)
- Next week back: start gradient flow on linear ϕ^4 model

Monday, July 13

- Update from last week
  - Implemented MPI, slowdown may be due to thermal throttling?
  - Questions:

    * Noether's Theorem, 4-current?
    * Star vs. dagger
- Invariance vs covariance
  - Q (charge) is invariant, not covariant. Derivative is 0 (conserved) so no effect of boost. See (StackExchange)[https://physics.stackexchange.com/questions/270296/what-is-the-difference-between-lorentz-invariant-and-lorentz-covariant]
- Operator product expansion
  - Taylor (Laurent in reality) series for operators
  - Used to expand nonlocal (slightly) oeprators using local operators
- TODO:
  - Read conference proceedings, then paper with Orginos
  - Define ρ field (Eq. 2.4), implement it using the exact solution (Eq. 2.5). This will require a FFT

Wednesday, July 15

- Code organization
  - Perhaps implement the nonlinear σ model (at a later date) and the ϕ^4 model as subclasses of a generic lattice class with abstract methods of action, etc.
- Gradient flow
  - Splitting momentum at half of lattice is a-ok.
  - Next, implement observable!
- New terms:
  - autocorrelation times (see Schaich -> MC Textbook -> Wolff): not a QFT concept!
  - Gamma analysis: used to calculate autocorrelation
  - summation window: part of the Gamma analysis
- Three classes of QFTs
  - renormalizable: infinities can be absorbed by a finite number of counterterms

- nonrenormalizeable: requires infinitely many counterterms
- super-renormalizeable: there is only one parameter that is divergent

Friday, July 17

- Critical mass problem
  - Check multiple measures (Binder Cumulant, bimodality, etc.)
  - https://journals.aps.org/prd/abstract/10.1103/PhysRevD.58.076003
  - Infinite volume limit?
  - Turn off coupling?
  - Behavior for one node? Two nodes?
  - Metropolis checkerboard? (see (Schaich and Loinaz 2006, pg. 79))
  - Look at Schaich's calculations
  - Note: No true phase transition in finite system since phase transitions are defined by correlation lengths going to infinity

Monday, July 20

- Remarks on critical point graph
  - I should be averaging the measurements from run.
  - Probably will need many more measurements.
  - Spend more time thermalizing
  - Calculate autocorrelation time, use to determine the `record_rate`
  - Production quality: try 10,000 sweeps, with 1000 thermalization
  - Run with slightly larger lattice to verify critical point
- Todo:
  - Flow time dependence (with action)
  - Average all measurements (incorporate this into `recorder.py`)
    - Plot mean and standard error as error bars
  - Autocorrelation times using Gamma analysis
    - Paper has Mathematica notebook link, though it may be broken.

  - Double check that large lattice volumes show correct critical mass.

Wednesday, July 22

- Things I did:
  - Reorganized the `recorder.py` to include measurements and means/errors.
  -
- Questions:
  - Implementing the gradient flow: how does this play into the Monte Carlo simulation?
- TODO:
  - Critical mass
    * Plot histogram for two points of single lattice (thermalized)
    * See if this is affected by a large thermalization, cold start
    * Binder cumulant should be step function
  - Autocorrelation times
    * See Eq. 41
    * Get MatLab code
  - Gradient flow
    * Each measured lattice should be evolved in flow time before recording

Friday, July 24

- Things to do:
  - Gradient flow
  - Try $m_0^2 = -0.8$, see if it become bimodal with large thermalization.
  - Move towards critical value: does τ_{int} increase? (This is what we expect)
  - Perhaps change recording rate?

- Is it time to transcibe the code?
  - Explore efficiency boost
  - C++?
  - Cython?
  - Local cluster?

Monday, July 27

- Shift to utilizing the 0th node for computation.
- Things to note:
  - Histogram in conference proceedings bins single lattice, not average over Markov chain.
- Questions:
  - collecting plots
- To-Do:
  - Run a proper histogram for different thermalizaitons, see if 10^4 is necessary.

Wednesday, July 29

- Plots look good, but are a little wide and not localized on the convergent point
  - Try a cold start
  - Look at 256
  - Average a couple trials
- To-Do:
  - See if I can find the width issue
  - Check autocorrelation times (with/without cluster)
    - close to critical value

- wait on gradient flow until we are more confident in Markov chain.

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