Defining condensates in low-dimensional spin models through the gradient flow

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Non-Linear σ Model

- a model in Quantum Field Theory which provides a good toy model for Quantum Chromodynamics
- characterized by the action

$$S = \frac{1}{2g} \int d^2x \, \partial_\mu \vec{\phi}(x) \cdot \partial^\mu \vec{\phi}(x)$$

  where the $\vec{\phi}(x)$ is a 3-component vector with norm 1 and g is a coupling constant.
- We use a Monte Carlo method to simulate this model.

Problem

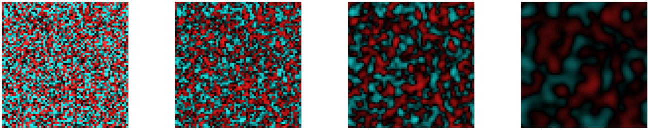
When simulating this and other theories on the lattice, results diverge as $N \to \infty$.

Possible solution: the Gradient Flow

- Introduces a new dimension, τ, called the "flow time," through which the field evolves as follows:

$$\rho(x,\tau) = \frac{1}{4\pi\tau}\int d^2y\, e^{-(x-y)^2/4\tau}\phi(y)$$

- Suppresses infinities while keeping observables constant.
- To compare with existing results we first start with the ϕ^4 model.



(a) $\tau = 0$ (b) $\tau = 0.5$ (c) $\tau = 2$ (d) $\tau = 10$

Figure: Effect of flow time evolution on a random lattice in the symmetric phase. Red and blue indicate positive and negative values of the field in 2D spacetime.