

# Research Notes

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## 1 Monday, June 15

1. Preliminary meeting
2. We will begin with  $\phi^4$  term due to lower energy bound.
3. Beginning with code in Python, switch to C/C++ if necessary.

## 2 Tuesday, June 16

1. Preliminary concepts to understand:
  - (a) scalar field theory on lattice
  - (b) Markov chains and Monte Carlo
  - (c) the gradient flow
  - (d)  $O(n)$  symmetry
2. General components to research, executed in parallel
  - (a) Reading
  - (b) Mathematical analysis
  - (c) Writing code
  - (d) Present (writing, plot generation,...)
3. Things to get out of dissertation [9]
  - (a) Markov chain
  - (b) Cluster algorithms
    - i. Metropolis is a method for narrowing possibilities by accepting only some changes. It can get stuck in a local minimum, loss of ergodicity. We solve these with cluster algorithms. Wolff grows clusters probabilistically and flips, while Swenson and Yang identifies clusters and flips them probabilistically.
    - ii. Near a phase transition, correlation length grows and changes become less likely to be accepted: need clusters. Clusters don't work far from the phase transition. This is manifested as a sequence of a few metropolis steps and a cluster step.

#### 4. Research plan

- (a) Start with 2D  $\phi^4$ 
  - i. Set up lattice with sign flip for reflection
  - ii. Use Markov chain Monte Carlo to simulate.
  - iii. Measure, magnetization and susceptibility, Binder cumulant
- (b) Transition to 3D, then maybe transition to C/C++.
- (c) Implement the gradient flow
- (d) move to 2-3d nonlinear sigma model.
- (e) Motivation: the nonlinear sigma model works for QCD given the asymptotic freedom. We may also want to explore topology.

### 3 Wednesday, June 17

#### 1. Code tips:

- (a) Try Swendsen-Wang algorithm in addition to Wolff
- (b) Print out time taken
- (c) Optimize Hamiltonian
- (d) Save every tenth measurement or store configurations to calculate path integral. Exclude thermalization (first 200)
- (e) Write in terms of sweeps, not iterations
- (f) Parallelize (look into checkerboard algorithm)
- (g) Implement Binder cumulant and susceptibility
- (h) Store every few states
- (i) Look into multigrid algorithm

#### 2. Reading on Monte Carlo Markov chain and cluster algorithms.

### 4 Friday, June 19

#### 1. Looking over code

- (a) Might be too slow, move to C/C++ eventually?
- (b) Why is the energy increasing with metropolis algorithm?
- (c) Shift to using action instead of Hamiltonian.
- (d) Transition from Broken Phase, look at  $\mu_0^2$  term.
- (e) Profile code for possible optimizations.

## 5 Monday, June 22

1. Coding
  - (a) Add plots to `.gitignore`.
  - (b) Try parallelizing code, using either multigrid or checkerboard algorithm
2. Reading
  - (a) Start to focus more on understanding the theory behind research.
  - (b) Read dissertation [9] Chap. 6.5 and 6.6, take notes on questions.
  - (c) Newman [8] (Main textbook for Monte Carlo in Statistical Physics)
3. Just some things to remember
  - (a) Correlation functions correlate values in statistical systems and relate to propagators in QFT.
  - (b) The problem of renormalization:  $a \rightarrow 0$  leads to unbounded correlation function. As real physical lengths are measured in terms of the lattice constant, these sometimes tend to infinity.

## 6 Wednesday, June 24

1. Code
  - (a) Parallelize
  - (b) Transition to numpy
2. Theory Question
  - (a) renormalize and regularize: what do they mean?
  - (b) Look at LePage

## References

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