

Thesis Notes

Stuart Thomas

Chris Monahan

Monday, June 15

- Preliminary meeting
- We will begin with ϕ^4 term due to lower energy bound.
- Beginning with code in Python, switch to C/C++ if necessary.

Tuesday, June 16

- Preliminary concepts to understand:
 - scalar field theory on lattice
 - Markov chains and Monte Carlo
 - the gradient flow
 - $O(n)$ symmetry
- General components to research, executed in parallel
 - Reading
 - Mathematical analysis
 - Writing code
 - Present (writing, plot generation,...)
- Things to get out of dissertation (Schaich and Loinaz 2006)
 - Markov chain
 - Cluster algorithms
 - * Metropolis is a method for narrowing possibilities by accepting

only some changes. It can get stuck in a local minimum, loss of ergodicity. We solve these with cluster algorithms. Wolff grows clusters probabilistically and flips, while Swenson and Yang identifies clusters and flips them probabilistically.

- * Near a phase transition, correlation length grows and changes become less likely to be accepted: need clusters. Clusters don't work far from the phase transition. This is manifested as a sequence of a few metropolis steps and a cluster step.

- Research plan

- Start with 2D ϕ^4
 - * Set up lattice with sign flip for reflection
 - * Use Markov chain Monte Carlo to simulate.
 - * Measure, magnetization and susceptibility, Binder cumulant
- Transition to 3D, then maybe transition to C/C++.
- Implement the gradient flow
- move to 2-3d nonlinear sigma model.
- Motivation: the nonlinear sigma model works for QCD given the asymptotic freedom. We may also want to explore topology.

Wednesday, June 17

- Code tips:
 - Try Swendsen-Wang algorithm in addition to Wolff
 - Print out time taken
 - Optimize Hamiltonian
 - Save every tenth measurement or store configurations to calculate path integral. Exclude thermalization (first 200)
 - Write in terms of sweeps, not iterations
 - Parallelize (look into checkerboard algorithm)
 - Implement Binder cumulant and susceptibility

- Store every few states
- Look into multigrid algorithm
- Reading on Monte Carlo Markov chain and cluster algorithms.

Friday, June 19

- Looking over code
 - Might be too slow, move to C/C++ eventually?
 - Why is the energy increasing with metropolis algorithm?
 - Shift to using action instead of Hamiltonian.
 - Transition from Broken Phase, look at μ_0^2 term.
 - Profile code for possible optimizations.

Monday, June 22

- Coding
 - Add plots to `.gitignore`.
 - Try parallelizing code, using either multigrid or checkerboard algorithm
- Reading
 - Start to focus more on understanding the theory behind research.
 - Read dissertation (Schaich and Loinaz 2006) Chap. 6.5 and 6.6, take notes on questions.
 - Newman (Newman and Barkema 1999) (Main textbook for Monte Carlo in Statistical Physics)
- Just some things to remember
 - Correlation functions correlate values in statistical systems and relate to propagators in QFT.
 - The problem of renormalization: $a \rightarrow 0$ leads to unbounded correlation

function. As real physical lengths are measured in terms of the lattice constant, these sometimes tend to infinity.

Wednesday, June 24

- Code
 - Parallelize
 - Transition to numpy
- Theory Question
 - renormalize and regularize: what do they mean?
 - Look at LePage

Tuesday, June 30

- Code
 - Try `mpi4py`.
 - Move to 3D (this may decrease parallel overhead)
- Reading
 - Continue Reading Collins, others.

Thursday, July 2

- Coding
 - Continue implementing MPI
 - Parallelization may be more apparent in 3D
- Reading
 - Dirac fermions, represented by 4D spinor field (spin up/down, electron/positron)
 - Look at Tong (Chap. 4)
 - Charge (See Tong, Noether's Theorem)

- Next week back: start gradient flow on linear ϕ^4 model

Monday, July 13

- Update from last week
 - Implemented MPI, slowdown may be due to thermal throttling?
 - Questions:
 - * Noether's Theorem, 4-current?
 - * Star vs. dagger
- Invariance vs covariance
 - Q (charge) is invariant, not covariant. Derivative is 0 (conserved) so no effect of boost. See (StackExchange)[<https://physics.stackexchange.com/questions/270296/what-is-the-difference-between-lorentz-invariant-and-lorentz-covariant>]
- Operator product expansion
 - Taylor (Laurent in reality) series for operators
 - Used to expand nonlocal (slightly) operators using local operators
- TODO:
 - Read conference proceedings, then paper with Orginos
 - Define ρ field (Eq. 2.4), implement it using the exact solution (Eq. 2.5). This will require a FFT

Wednesday, July 15

- Code organization
 - Perhaps implement the nonlinear σ model (at a later date) and the ϕ^4 model as subclasses of a generic lattice class with abstract methods of action, etc.
- Gradient flow
 - Splitting momentum at half of lattice is a-ok.
 - Next, implement observable!

- New terms:
 - autocorrelation times (see Schaich -> MC Textbook -> Wolff): not a QFT concept!
 - Gamma analysis: used to calculate autocorrelation
 - summation window: part of the Gamma analysis
- Three classes of QFTs
 - renormalizable: infinities can be absorbed by a finite number of counterterms
 - nonrenormalizable: requires infinitely many counterterms
 - super-renormalizable: there is only one parameter that is divergent

Friday, July 17

- Critical mass problem
 - Check multiple measures (Binder Cumulant, bimodality, etc.)
 - <https://journals.aps.org/prd/abstract/10.1103/PhysRevD.58.076003>
 - Infinite volume limit?
 - Turn off coupling?
 - Behavior for one node? Two nodes?
 - Metropolis checkerboard? (see (Schaich and Loinaz 2006, pg. 79))
 - Look at Schaich's calculations
 - Note: No true phase transition in finite system since phase transitions are defined by correlation lengths going to infinity

Monday, July 20

- Remarks on critical point graph
 - I should be averaging the measurements from run.
 - Probably will need many more measurements.
 - Spend more time thermalizing

- Calculate autocorrelation time, use to determine the `record_rate`
 - Production quality: try 10,000 sweeps, with 1000 thermalization
 - Run with slightly larger lattice to verify critical point
- Todo:
 - Flow time dependence (with action)
 - Average all measurements (incorporate this into `recorder.py`)
 - * Plot mean and standard error as error bars
 - Autocorrelation times using Gamma analysis
 - * Paper has Mathematica notebook link, though it may be broken.
 - Double check that large lattice volumes show correct critical mass.

References

- Aoki, Sinya, Kengo Kikuchi, and Tetsuya Onogi. 2015. “Gradient Flow of $O(N)$ Nonlinear Sigma Model at Large N .” *Journal of High Energy Physics* 2015 (4): 156. doi:10.1007/JHEP04(2015)156.
- Horgan, R R. 2014. *Statistical Field Theory*.
- Introduction to QCD and the Standard Model*. n.d.
- Kleinert, Hagen. 2016. *Particles and Quantum Fields*.
- Kleinert, Hagen, and Verena Schulte-Frohlinde. 2001. *Critical Properties of ϕ^4 -Theories*.
- Loinaz, Will, and R. S. Willey. 1998. “Monte Carlo Simulation Calculation of Critical Coupling Constant for Continuum ϕ_2^4 .” *Physical Review D* 58 (7): 076003. doi:10.1103/PhysRevD.58.076003.
- Makino, H., F. Sugino, and H. Suzuki. 2015. “Large- N Limit of the Gradient Flow in the 2d $O(N)$ Nonlinear Sigma Model.” *Progress of Theoretical and Experimental Physics* 2015 (4): 43B07–0. doi:10.1093/ptep/ptv044.
- Makino, Hiroki, and Hiroshi Suzuki. 2015. “Renormalizability of the Gradient Flow in the 2d $O(N)$ Non-Linear Sigma Model.” *Progress of Theoretical and Experimental Physics* 2015 (3): 33B08–0. doi:10.1093/ptep/ptv028.
- Mélin, R. 2000. “Spin-Resolved Andreev Reflection in Ferromagnet-Superconductor Junctions with Zeeman Splitting.” *EPL (Europhysics Letters)* 51 (2): 202. doi:10.1209/epl/i2000-00532-1.
- Monahan, Christopher. 2016. “The Gradient Flow in Simple Field Theories.” In *Proceedings of the 33rd International Symposium on Lattice Field Theory — PoS(LATTICE 2015)*, 052. Kobe International Conference Center, Kobe, Japan:

Sissa Medialab. doi:10.22323/1.251.0052.

Monahan, Christopher, and Kostas Orginos. 2015. “Locally Smeared Operator Product Expansions in Scalar Field Theory.” *Physical Review D* 91 (7): 074513. doi:10.1103/PhysRevD.91.074513.

Newman, M. E. J., and G. T. Barkema. 1999. *Monte Carlo Methods in Statistical Physics*. Oxford: Oxford University Press.

Schaich, David A, and William Loinaz. 2006. “Lattice Simulations of Nonperturbative Quantum Field Theories.” Amherst College.

Tong, David. 2007. *Quantum Field Theory*. University of Cambridge Part III Mathematical Tripos.

———. 2017. *Statistical Field Theory*. University of Cambridge Part III Mathematical Tripos.

Wolff, Ulli. 2007. “Monte Carlo Errors with Less Errors.” *Computer Physics Communications* 176 (5): 383. doi:10.1016/j.cpc.2006.12.001.