Research Notes

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Last updated June 22, 2020

1 Monday, June 15

1. Preliminary meeting
2. We will begin with ϕ^4 term due to lower energy bound.
3. Beginning with code in Python, switch to C/C++ if necessary.

2 Tuesday, June 16

1. Preliminary concepts to understand:
   (a) scalar field theory on lattice
   (b) Markov chains and Monte Carlo
   (c) the gradient flow
   (d) $O(n)$ symmetry
2. General components to research, executed in parallel
   (a) Reading
   (b) Mathematical analysis
   (c) Writing code
   (d) Present (writing, plot generation,...)
3. Things to get out of dissertation [9]
   (a) Markov chain
   (b) Cluster algorithms
      i. Metropolis is a method for narrowing possibilities by accepting only some changes. It can get stuck in a local mininum, loss of ergodicity. We solve these with cluster algoriths. Wolff grows clusters probabilistically and flips, while Swenson and Yang identifies clusters and flips them probabilistically.
      ii. Near a phase transition, correlation length grows and changes become less likely to be accepted: need clusters. Clusters dont work far from the phase transition. This is manifested as a sequence of a few metropolis steps and a cluster step.

4. Research plan
   (a) Start with 2D ϕ^4
      i. Set up lattice with sign flip for reflection
      ii. Use Markov chain Monte Carlo to simulate.
      iii. Measure, magnetization and suseptibility, Binder cumulant
   (b) Transition to 3D, then maybe transition to C/C++.
   (c) Implement the gradient flow
   (d) move to 2-3d nonlinear sigma model.
   (e) Motivation: the nonlinear sigma model works for QCD given the asymtotic freedom. We may also want to explore topology.

3 Wednesday, June 17

1. Code tips:
   (a) Try Swendsen-Wang algorithm in addition to Wolff
   (b) Print out time taken
   (c) Optimize Hamiltonian
   (d) Save every tenth measurement or store configurations to calculate path integral. Exclude thermaliztion (first 200)
   (e) Write in terms of sweeps, not iterations
   (f) Parallelize (look into checkerboard algorithm)
   (g) Implement Binder cumulant and suseptibility
   (h) Store every few states
   (i) Look into multigrid algorithm
2. Reading on Monte Carlo Markov chain and cluster algorithms.

4 Friday, June 19

1. Looking over code
   (a) Might be too slow, move to C/C++ eventually?
   (b) Why is the energy increasing with metropolis algorithm?
   (c) Shift to using action instead of Hamiltonian.
   (d) Transition from Broken Phase, look at μ_0^2 term.
   (e) Profile code for possible optimizations.

5 Monday, June 22

1. Coding
   (a) Add plots to `.gitignore`.
   (b) Try parallelizing code, using either multigrid or checkerboard algorithm
2. Reading
   (a) Start to focus more on understanding the theory behind research.
   (b) Read dissertation [9] Chap. 6.5 and 6.6, take notes on questions.
   (c) Newman [8] (Main textbook for Monte Carlo in Statistical Physics)
3. Just some things to remember
   (a) Correlation functions correlate values in statistical systems and relate to propagators in QFT.
   (b) The problem of renormalization: $a \to 0$ leads to unbounded correlation function. As real physical lengths are measured in terms of the lattice constant, these sometimes tend to infinity.

References

[1] *The gradient flow in simple field theories*, Trieste, Italy, 2016. Sissa Medialab.

[2] S. Aoki, K. Kikuchi, and T. Onogi. Gradient flow of o(n) nonlinear sigma model at large n. *arXivJHEP 1504 (2015) 156*, page 1412.8249v3, 2014.

[3] R. R. Horgan. *Statistical Field Theory.* 2014.

[4] V. Kleinert, H Schulte-Frohlind. *Critical Properties of ?4-Theories.* Freie Uni.

[5] H. Makino, F. Sugino, and H. Suzuki. Large-n limit of the gradient flow in the 2D o(n) nonlinear sigma model. *Progress of Theoretical and Experimental Physics*, 2015(4):43B07–0, 2015.

[6] H. Makino and H. Suzuki. Renormalizability of the gradient flow in the 2D $o(n)$ non-linear sigma model. *arXiv*, page 1410.7538v3, 2014.

[7] C. Monahan and K. Orginos. Locally smeared operator product expansions in scalar field theory. *Physical Review D*, 91(7), 2015.

[8] M. E. J. Newman and G. T. Barkema. *Monte Carlo Methods in Statistical Physics.* Oxford University Press, Oxford, 1999.

[9] David A. Schaich and William Loinaz. *Lattice Simulations of Nonperturbative Quantum Field Theories.* PhD thesis, Amherst College, Amherst, MA, 2006.

[10] D. Tong. *Quantum Field Theory.* 2007.

[11] D. Tong. *Statistical Field Theory.* University of Cambridge, 2017.