

## Topology of the $O(3)$ non-linear sigma model under the gradient flow

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The  $O(3)$  non-linear sigma model (NLSM) is a prototypical field theory for QCD and ferromagnetism, featuring topological qualities. Though the topological susceptibility  $\chi_t$  should vanish in physical theories, lattice simulations of the NLSM find that  $\chi_t$  diverges in the continuum limit. We study the effect of the gradient flow on this quantity using a Markov Chain Monte Carlo method, finding that a logarithmic divergence persists. This result supports a previous study and indicates that either the definition of topological charge is problematic or the NLSM has no well-defined continuum limit. We also introduce a  $\theta$ -term and analyze the topological charge as a function of  $\theta$  under the gradient flow.

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## 1. The Non-Linear Sigma Model

We study the  $O(3)$  non-linear sigma model (NLSM) in 1+1 dimensions, defined by the Euclidean action

$$S_E = \frac{\beta}{2} \int d^2x \left[ (\partial_t \vec{e})^2 + (\partial_x \vec{e})^2 \right]$$

where,  $\vec{e}$  is 3-component real vector constrained by  $|\vec{e}| = 1$  and  $\beta$  is the inverse coupling constant. In solid-state systems, this model describes Heisenberg ferromagnets [1] and in nuclear physics, it acts as a prototype for quantum chromodynamics (QCD), the gauge theory that describes the strong nuclear force. In general, the NLSM shares key features with non-Abelian gauge theories such as QCD, including a mass gap and asymptotic freedom [2]. Therefore, the NLSM is a useful model for exploring the effect of these properties in a simpler system.

## References

- [1] C. Callan, D. Friedan, E. Martinec and M. Perry, *Strings in background fields*, .
- [2] A. Polyakov, *Interaction of goldstone particles in two dimensions. Applications to ferromagnets and massive Yang-Mills fields*, .