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Topology of the $\mathrm{O}(3)$ non-linear sigma model under the gradient flow

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The $\mathrm{O}(3)$ non-linear sigma model (NLSM) is a prototypical field theory for QCD and ferromagnetism, featuring topological qualities. Though the topological susceptibility χ_t should vanish in physical theories, lattice simulations of the NLSM find that χ_t diverges in the continuum limit. We study the effect of the gradient flow on this quantity using a Markov Chain Monte Carlo method, finding that a logarithmic divergence persists. This result supports a previous study and indicates that either the definition of topological charge is problematic or the NLSM has no well-defined continuum limit. We also introduce a θ-term and analyze the topological charge as a function of θ under the gradient flow.

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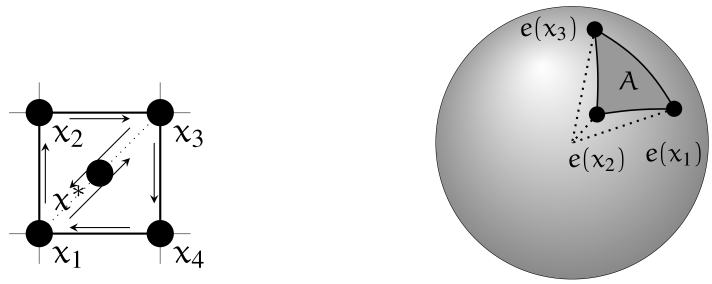


(a) a plaquette broken up into two triangles **(b)** signed area of triangle in target space

Figure 1

1. The Non-Linear Sigma Model

We study the O(3) non-linear sigma model (NLSM) in 1+1 dimensions, defined by the Euclidean action

$$S_E = \frac{\beta}{2} \, d^2x \, \left[(\partial_t e)^2 + (\partial_x e)^2 \right]$$

where, e is 3-component real vector constrained by $|e| = 1$ and β is the inverse coupling constant. In solid-state systems, this model describes Heisenberg ferromagnets [1] and in nuclear physics, it acts as a prototype for quantum chromodynamics (QCD), the gauge theory that describes the strong nuclear force. In general, the NLSM shares key features with non-Abelian gauge theories such as QCD, including a mass gap and asymptotic freedom [2]. Therefore, the NLSM is a useful model for exploring the effect of these properties in a simpler system.

Following [3], we define topological charge density $q(x^*)$ for each plaquette x^* such that

$$Q = {}_{x^*} q(x^*) \tag{1}$$

where

$$q(x^*) = \frac{1}{4\pi} \left[A\Big(e(x_1), e(x_2), e(x_3)\Big) + A\Big(e(x_1), e(x_3), e(x_4)\Big) \right]. \tag{2}$$

The topology of the NLSM is visualized in Fig. 1

$$S[e] \to S[e] - i\theta Q[e].$$

A nonzero θ implies nonzero $\langle Q \rangle$. Furthermore,

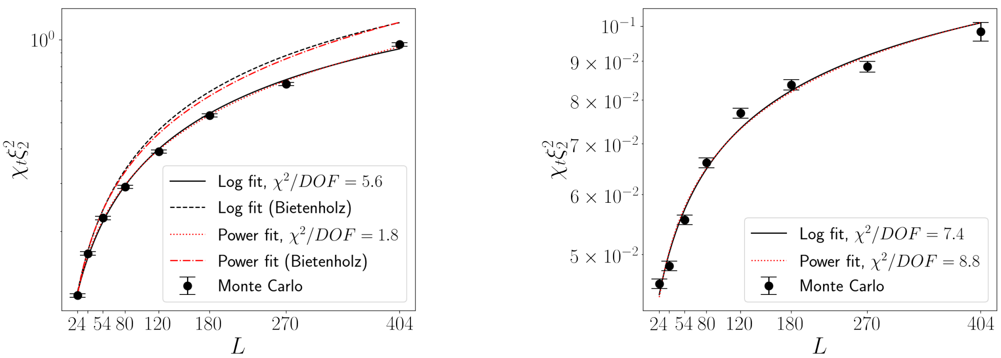
$$\chi_t \propto \left. \frac{d \, \mathrm{Im}\langle Q \rangle}{d\theta} \right|_{\theta=0} \tag{3}$$

2. Computational Methods

Our study of the gradient flow in the NLSM is based on a computational system that simulates quantum fields numerically. We begin by implementing a numerical Monte Carlo method to simulate the lattice in two dimensions and verify our program with the well-studied ϕ^4 scalar field theory. We then generalize our model to a vector field to simulate the NLSM and implement a numerical solution to the gradient flow. We use the data produced by this program to study the topological charge and susceptibility under the gradient flow.

We first implement the ϕ^4 model in Python. Afterwards, we transition to the NLSM, using C++ for increased efficiency . To compile the C++ simulation, we use the `gcc` compiler with the highest level of optimization.

3. Results



(a) $\tau = 0$ **(b)** $\tau = 5t_0$

Figure 2: $\chi_t \xi_2^2$ as a function of L. ξ_2 is the second moment of the correlation function and t_0 is a scale-independent unit of flow time. We fit the data with both a logarithmic and power fit. Simulation run with 10,000 measurements every 50 sweeps, 1,000 sweep thermalization. In the $\tau = 0$ case, we have compared our result with the curve fits found in [4].

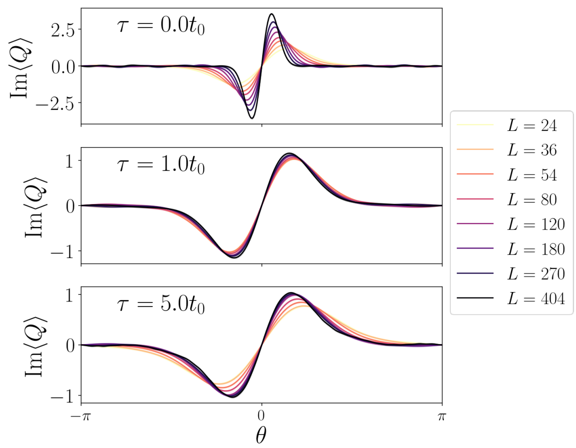


Figure 3: Nontrivial $\mathrm{Im}\langle Q\rangle$ as a function of θ. Simulation run with 10,000 measurements, every 50 sweeps, 1,000 sweep thermalization. Note the different scaling of the y-axis.

References

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