

Thesis Notes

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Monday, June 15

- Preliminary meeting
- We will begin with ϕ^4 term due to lower energy bound.
- Beginning with code in Python, switch to C/C++ if necessary.

Tuesday, June 16

- Preliminary concepts to understand:
 - scalar field theory on lattice
 - Markov chains and Monte Carlo
 - the gradient flow
 - $O(n)$ symmetry
- General components to research, executed in parallel
 - Reading
 - Mathematical analysis
 - Writing code
 - Present (writing, plot generation,...)
- Things to get out of dissertation (Schaich and Loinaz 2006)
 - Markov chain
 - Cluster algorithms
 - * Metropolis is a method for narrowing possibilities by accepting only some changes. It can get stuck in a local minimum, loss of ergodicity. We solve these with cluster algorithms. Wolff grows clus-

ters probabilistically and flips, while Swenson and Yang identifies clusters and flips them probabilistically.

- * Near a phase transition, correlation length grows and changes become less likely to be accepted: need clusters. Clusters don't work far from the phase transition. This is manifested as a sequence of a few metropolis steps and a cluster step.

- Research plan

- Start with 2D ϕ^4
 - * Set up lattice with sign flip for reflection
 - * Use Markov chain Monte Carlo to simulate.
 - * Measure, magnetization and susceptibility, Binder cumulant
- Transition to 3D, then maybe transition to C/C++.
- Implement the gradient flow
- move to 2-3d nonlinear sigma model.
- Motivation: the nonlinear sigma model works for QCD given the asymptotic freedom. We may also want to explore topology.

Wednesday, June 17

- Code tips:
 - Try Swendsen-Wang algorithm in addition to Wolff
 - Print out time taken
 - Optimize Hamiltonian
 - Save every tenth measurement or store configurations to calculate path integral. Exclude thermalization (first 200)
 - Write in terms of sweeps, not iterations
 - Parallelize (look into checkerboard algorithm)
 - Implement Binder cumulant and susceptibility
 - Store every few states
 - Look into multigrid algorithm
- Reading on Monte Carlo Markov chain and cluster algorithms.

Friday, June 19

- Looking over code
 - Might be too slow, move to C/C++ eventually?
 - Why is the energy increasing with metropolis algorithm?
 - Shift to using action instead of Hamiltonian.
 - Transition from Broken Phase, look at μ_0^2 term.
 - Profile code for possible optimizations.

Monday, June 22

- Coding
 - Add plots to `.gitignore`.
 - Try parallelizing code, using either multigrid or checkerboard algorithm
- Reading
 - Start to focus more on understanding the theory behind research.
 - Read dissertation (Schaich and Loinaz 2006) Chap. 6.5 and 6.6, take notes on questions.
 - Newman (Newman and Barkema 1999) (Main textbook for Monte Carlo in Statistical Physics)
- Just some things to remember
 - Correlation functions correlate values in statistical systems and relate to propagators in QFT.
 - The problem of renormalization: $a \rightarrow 0$ leads to unbounded correlation function. As real physical lengths are measured in terms of the lattice constant, these sometimes tend to infinity.

Wednesday, June 24

- Code

- Parallelize
- Transition to numpy
- Theory Question
 - renormalize and regularize: what do they mean?
 - Look at LePage

Tuesday, June 30

- Code
 - Try `mpi4py`.
 - Move to 3D (this may decrease parallel overhead)
- Reading
 - Continue Reading Collins, others.

Thursday, July 2

- Coding
 - Continue implementing MPI
 - Parallelization may be more apparent in 3D
- Reading
 - Dirac fermions, represented by 4D spinor field (spin up/down, electron/positron)
 - Look at Tong (Chap. 4)
 - Charge (See Tong, Noether's Theorem)
- Next week back: start gradient flow on linear ϕ^4 model

Monday, July 13

- Update from last week
 - Implemented MPI, slowdown may be due to thermal throttling?
 - Questions:

- * Noether's Theorem, 4-current?
- * Star vs. dagger
- Invariance vs covariance
 - Q (charge) is invariant, not covariant. Derivative is 0 (conserved) so no effect of boost. See (StackExchange)[<https://physics.stackexchange.com/questions/270296/what-is-the-difference-between-lorentz-invariant-and-lorentz-covariant>]
- Operator product expansion
 - Taylor (Laurent in reality) series for operators
 - Used to expand nonlocal (slightly) operators using local operators
- TODO:
 - Read conference proceedings, then paper with Orginos
 - Define ρ field (Eq. 2.4), implement it using the exact solution (Eq. 2.5). This will require a FFT

Wednesday, July 15

- Code organization
 - Perhaps implement the nonlinear σ model (at a later date) and the ϕ^4 model as subclasses of a generic lattice class with abstract methods of action, etc.
- Gradient flow
 - Splitting momentum at half of lattice is a-ok.
 - Next, implement observable!
- New terms:
 - autocorrelation times (see Schaich -> MC Textbook -> Wolff): not a QFT concept!
 - Gamma analysis: used to calculate autocorrelation
 - summation window: part of the Gamma analysis
- Three classes of QFTs
 - renormalizable: infinities can be absorbed by a finite number of counterterms

- nonrenormalizeable: requires infinitely many counterterms
- super-renormalizeable: there is only one parameter that is divergent

Friday, July 17

- Critical mass problem
 - Check multiple measures (Binder Cumulant, bimodality, etc.)
 - <https://journals.aps.org/prd/abstract/10.1103/PhysRevD.58.076003>
 - Infinite volume limit?
 - Turn off coupling?
 - Behavior for one node? Two nodes?
 - Metropolis checkerboard? (see (Schaich and Loinaz 2006, pg. 79))
 - Look at Schaich's calculations
 - Note: No true phase transition in finite system since phase transitions are defined by correlation lengths going to infinity

Monday, July 20

- Remarks on critical point graph
 - I should be averaging the measurements from run.
 - Probably will need many more measurements.
 - Spend more time thermalizing
 - Calculate autocorrelation time, use to determine the `record_rate`
 - Production quality: try 10,000 sweeps, with 1000 thermalization
 - Run with slightly larger lattice to verify critical point
- Todo:
 - Flow time dependence (with action)
 - Average all measurements (incorporate this into `recorder.py`)
 - * Plot mean and standard error as error bars
 - Autocorrelation times using Gamma analysis
 - * Paper has Mathematica notebook link, though it may be broken.

- Double check that large lattice volumes show correct critical mass.

Wednesday, July 22

- Things I did:
 - Reorganized the `recorder.py` to include measurements and means/errors.
 -
- Questions:
 - Implementing the gradient flow: how does this play into the Monte Carlo simulation?
- TODO:
 - Critical mass
 - * Plot histogram for two points of single lattice (thermalized)
 - * See if this is affected by a large thermalization, cold start
 - * Binder cumulant should be step function
 - Autocorrelation times
 - * See Eq. 41
 - * Get MatLab code
 - Gradient flow
 - * Each measured lattice should be evolved in flow time before recording

Friday, July 24

- Things to do:
 - Gradient flow
 - Try $m_0^2 = -0.8$, see if it become bimodal with large thermalization.
 - Move towards critical value: does τ_{int} increase? (This is what we expect)
 - Perhaps change recording rate?

- Is it time to transcribe the code?
 - Explore efficiency boost
 - C++?
 - Cython?
 - Local cluster?

Monday, July 27

- Shift to utilizing the 0th node for computation.
- Things to note:
 - Histogram in conference proceedings bins single lattice, not average over Markov chain.
- Questions:
 - collecting plots
- To-Do:
 - Run a proper histogram for different thermalizations, see if 10^4 is necessary.

Wednesday, July 29

- Plots look good, but are a little wide and not localized on the convergent point
 - Try a cold start
 - Look at 256
 - Average a couple trials
- To-Do:
 - See if I can find the width issue
 - Check autocorrelation times (with/without cluster)
 - * close to critical value

- wait on gradient flow until we are more confident in Markov chain.

Friday, July 31

- Calculate autocorrelation times (as a function of mass), binder cumulant
- Check Wolff algorithm, try Swendsen Wang
- Multigrid algorithm?
- Focus on metropolis algorithm. We don't want to study a cluster algorithm on the backdrop of a broken metropolis algorithm.
- Literature search: must the metropolis algorithm have randomly included sites?

Thursday, August 6

- Progress:
 - Autocorrelation: see page 6 from Schaich
 - * Affected by the measurement rate?
 - Wolff implemented in C, large speed-up
 - Fixed neighbor bug in Wolff algorithm
 - Histogram issue still persists

References

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