Jack Stubblefield Final Project

The Power of Buying Put Options Significantly Below Stock Price

Overview

Many investment strategies rely heavily on stocks, some would say too heavily. Individual stocks can have significant risk associated with them. That is why it is very important to diversify a portfolio with stock from different market sectors and other financial instruments. One alternative investors can consider for their portfolio is options. Options are a type of financial instrument in which one can buy or sell calls or puts on an underlying security such as a stock. Call options are bought in anticipation that a particular stock price will increase over time in order to make a profit while put options are a bet that a particular stock price will drop over time. Options can be bought and sold with very distant expiration dates such as a year or two into the future and can also be bought with an expiration date only a few weeks into the future. American options can be bought or sold at any point in the life of the option until the expiration date. The stock fund and options that I will be analyzing are the S&P SDPR 500 ETF (SPY) and SPY put options. The SPY, like the Dow Jones and NASDAQ, is a very good indicator of overall trends in the market because it is a stock fund that invests in S&P 500 stocks to track the S&P 500 Index. The particular strategy that I will be analyzing in this project is buying put options with a strike price 30% below the current stock price. This is an uncommon strategy since these put option strike prices are so far below actual stock price that there is little chance of these ever being "in the money". However, these puts are so volatile that during certain market crashes (for example October 2008), the option price can increase by as much as 4500%. The idea behind this strategy is to eliminate the downside that results from a crash, recession, or depression. To begin my analysis, my goal is to first find the Kelly Criterion strategy to determine the percentage of the portfolio that should be invested in put options with a strike price 30% below the actual stock price. Next, I will compare the returns of an investment strategy with that Kelly Criterion amount invested in put options and the rest of the portfolio invested in SPY stock with an investment strategy based on investing solely in SPY stock over the same period.

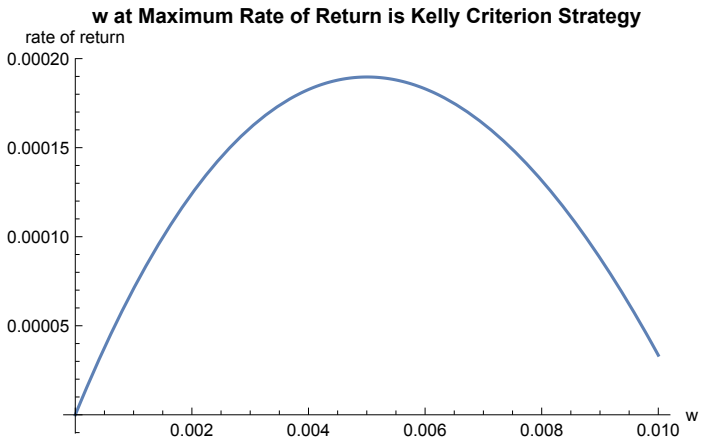
Assumptions

To simplify this model, I will be making a number of assumptions. First, all of the put options analyzed will have a strike price 30% below the actual stock fund price and will have an expiration date two months into the future. Secondly, all of the put options will be sold one month in the future (one

month remaining left until expiration). Next, all of return rates will be on a per month basis as each time period will be measured in months. In comparison of the two strategies, I will look at returns during a 100 month period (8 1/3 years). Next, since the SPY stock price averages around a 10-15% growth rate per year, I will assume a normally distributed growth rate with a mean monthly growth rate of 1% (since 1.01^12=1.1268)and a monthly standard deviation for growth rate of 0.03 [N~(0.01, 0.03)]. For the 100 month period, I will take a random sample of 100 monthly growth rates based on the distribution given. However, to highlight the major effect of having put options with a strike price so far below the current price, for one of the months I chose a normal distribution for monthly growth rate with a mean of -15% and a standard deviation of 0.05. My last major assumption involves the returns on the put options bought in this portfolio. Although I have the data from the last ten years, it was acquired from a vendor during a work internship so the exact results are proprietary. As a result, to simplify the model, the put options are estimated to give a return of 0 about 67% of the time, a return of 0.5 about 30% of the time, a return of 10 for 2% of the time, and a return of 40 for 1% of the time. For our random sample of 100 months, I will make sure the return of 40 lines up with the huge drop in growth rate that is N ~ (-.15, .05). In addition to the 100 month sample, I will also include a 65 month sample starting right before the large decrease in stock price to compare the returns of the two types of investment strategies. Lastly, I will repeat all of the steps with an example of a catastrophic drop in stock price (40% decrease) to see how much more that affects the differences between the returns on the two strategies.

First, we will solve for the Kelly Criterion strategy for percentage of portfolio devoted to our put options using our four possible outcomes for returns:

```
Plot[Log[1 + 40 w] / 100 + Log[1 + 10 w] / 50 + 30 * Log[1 + .5 w] / 100 + 67 * Log[1 - w] / 100,
 {w, 0, 0.01}, PlotLabel →
  Style["w at Maximum Rate of Return is Kelly Criterion Strategy", Bold, Black],
 AxesLabel → {"w", "rate of return"}]
```



```
FindRoot[
 D[Log[1 + 40 w] / 100 + Log[1 + 10 w] / 50 + 30 * Log[1 + .5 w] / 100 + 67 * Log[1 - w] / 100, w] ==
  0, {w, 0.005}]
```

```
{w → 0.00500502}
```

It has been determined from the graph that the optimal percentage we should invest in the 30% below stock price put options should be 0.500502%.

Next we will make our 100 month sample of a random arrangement of the four possible return values for the put options:

```
onereturn :=
  If[Random[] < 1 / 100, 40., If[Random[] < 1 / 50, 10., If[Random[] < 30 / 100, 0.5, 0]]];
```

```
returntable = Table[onereturn, {100}]
```

```
{0, 0.5, 0, 0, 0, 0, 0.5, 0, 0.5, 0.5, 0, 0, 0, 0, 0, 0, 0.5, 0.5, 0.5, 0, 0, 0, 0.5, 0, 0.5,
 0.5, 0, 0, 0.5, 0, 0, 0, 0, 0.5, 0, 0, 40., 0, 0.5, 0, 0, 0, 0, 0, 0.5, 0, 0, 10., 0, 0.5,
 0, 0.5, 0, 0.5, 0.5, 0.5, 0, 0.5, 0.5, 0.5, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0.5, 0, 0, 0,
 0, 0.5, 0, 0, 0, 0, 0, 0.5, 0.5, 0, 0, 0, 0, 0.5, 0, 0.5, 0, 0, 0, 0.5, 0, 0, 0, 0, 0, 0}
```

100 monthly returns on put options:

```
tableof100returns = returntable
```

```
{0, 0.5, 0, 0, 0, 0, 0.5, 0, 0.5, 0.5, 0, 0, 0, 0, 0, 0, 0.5, 0.5, 0.5, 0, 0, 0, 0.5, 0, 0.5,
 0.5, 0, 0, 0.5, 0, 0, 0, 0, 0.5, 0, 0, 40., 0, 0.5, 0, 0, 0, 0, 0, 0.5, 0, 0, 10., 0, 0.5,
 0, 0.5, 0, 0.5, 0.5, 0.5, 0, 0.5, 0.5, 0.5, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0.5, 0, 0, 0,
 0, 0.5, 0, 0, 0, 0, 0, 0.5, 0.5, 0, 0, 0, 0, 0.5, 0, 0.5, 0, 0, 0, 0.5, 0, 0, 0, 0, 0, 0}
```

This table shows the 37th month gives a return of 40. I am highlighting this now because that means that a significant drop in stock price must have occurred.

```
tableof100returns[[37]]
```

```
40.
```

100 month time period example

Now, using our assumptions, we create a function that gives a random monthly return on our portfolio given our assumptions on stock price distribution. “w” is the variable for amount invested in put options so we will make w=0.00500502.

```
returnfracoptions[w_] :=
  (1 - w) * Random[NormalDistribution[1.01, .03]] + w * onereturn;
```

Now creating a table with 100 random monthly returns on our portfolio which will be used for our cumulative 100 month returns later:

```
SPYreturntable = Table[returnfracoptions[0.00500502], {100}]
```

```
{1.00551, 1.06842, 0.978089, 0.98783, 0.966592, 1.0451, 1.06065, 0.976408, 1.07958,
 0.982223, 1.00946, 0.995549, 0.974303, 0.99814, 1.01303, 1.0337, 0.984317,
 0.998077, 1.05573, 0.98382, 0.961505, 0.955621, 1.05534, 1.00456, 0.97805, 1.02042,
 1.01214, 1.00074, 0.997545, 0.998887, 1.02103, 1.06826, 1.01988, 0.990559,
 1.00491, 1.04388, 1.01012, 1.04728, 1.02083, 1.01884, 1.00837, 1.0645, 1.00512,
 0.976926, 1.02482, 1.00243, 0.981114, 0.999129, 0.997229, 1.02697, 1.02892,
 1.02206, 0.969096, 1.02518, 1.00399, 1.0187, 0.949893, 0.958971, 0.982896, 1.02855,
 1.06749, 1.00579, 0.979592, 0.931272, 0.996132, 1.03225, 1.02609, 0.968608,
 1.00127, 1.00526, 1.01969, 0.959053, 0.998383, 0.978948, 1.05179, 0.984008,
 0.962797, 0.986045, 1.01537, 0.9972, 1.02591, 1.03082, 1.02049, 0.968722,
 0.950808, 1.01121, 1.02324, 1.01478, 1.09096, 0.962998, 0.995037, 1.02287,
 1.0319, 0.974142, 0.997233, 0.998943, 0.972103, 0.994841, 1.00634, 1.02345}
```

This return rate will have to be changed because our distribution for month 37 will have to be completely different from the other 99 months.

```
SPYreturntable[[37]]
```

```
1.01012
```

To get large random drop in price that would simulate the 2008 recession, I will create another Random normal distribution with mean of -15% and sd=0.05 for month 37 since that is when the put option gives a return of 40. We need a large random drop in price, like in October of 2008, for a put option to realistically give a return of 40.

```
Random[NormalDistribution[-.15, .05]]
```

```
-0.210383
```

As a result, we will use a -21.0383% drop in stock price to simulate when our put option return is 40.

```
returnfracmarketcrash[w_] = 1 + (1 - w) * (-.210383) + w * tableof100returns[[37]]
```

```
1 - 0.210383 (1 - w) + 40. w
```

Using the same type function with our new stock price change, we will calculate the return on our total portfolio.

```
SPYreturntable[[37]] = returnfracmarketcrash[.00500502]
```

```
0.990871
```

Even with a significant drop in stock price of over 21%, our total investment value decreases by less than 1%.
The next line checks to make sure our actual return for month 37 is correctly input into the data:

```
SPYreturntable
```

```
{1.00551, 1.06842, 0.978089, 0.98783, 0.966592, 1.0451, 1.06065, 0.976408, 1.07958,
 0.982223, 1.00946, 0.995549, 0.974303, 0.99814, 1.01303, 1.0337, 0.984317,
 0.998077, 1.05573, 0.98382, 0.961505, 0.955621, 1.05534, 1.00456, 0.97805, 1.02042,
 1.01214, 1.00074, 0.997545, 0.998887, 1.02103, 1.06826, 1.01988, 0.990559,
 1.00491, 1.04388, 0.990871, 1.04728, 1.02083, 1.01884, 1.00837, 1.0645, 1.00512,
 0.976926, 1.02482, 1.00243, 0.981114, 0.999129, 0.997229, 1.02697, 1.02892,
 1.02206, 0.969096, 1.02518, 1.00399, 1.0187, 0.949893, 0.958971, 0.982896, 1.02855,
 1.06749, 1.00579, 0.979592, 0.931272, 0.996132, 1.03225, 1.02609, 0.968608,
 1.00127, 1.00526, 1.01969, 0.959053, 0.998383, 0.978948, 1.05179, 0.984008,
 0.962797, 0.986045, 1.01537, 0.9972, 1.02591, 1.03082, 1.02049, 0.968722,
 0.950808, 1.01121, 1.02324, 1.01478, 1.09096, 0.962998, 0.995037, 1.02287,
 1.0319, 0.974142, 0.997233, 0.998943, 0.972103, 0.994841, 1.00634, 1.02345}
```

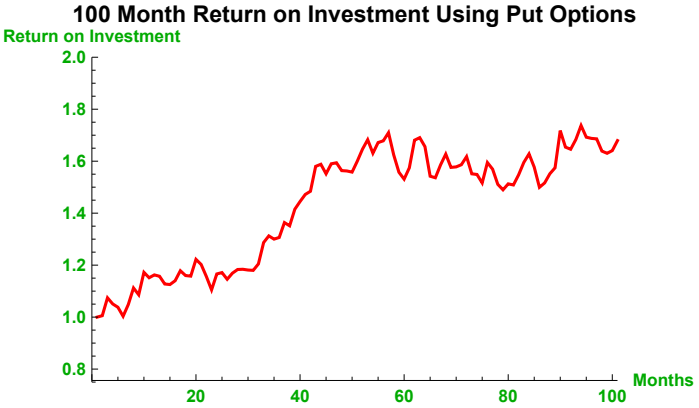
The next line of code makes the random sample of 100 monthly returns cumulative so the last number in this list is the total return on investment since the beginning of the period. As a result, the first number in the list is 1 since because that is where we start.

```
SPYaccumulatedchange = FoldList[Times, 1, %]
```

```
{1, 1.00551, 1.0743, 1.05076, 1.03798, 1.0033, 1.04855, 1.11215, 1.08591,
 1.17232, 1.15148, 1.16238, 1.1572, 1.12747, 1.12537, 1.14003, 1.17845,
 1.15997, 1.15774, 1.22226, 1.20248, 1.15619, 1.10488, 1.16602, 1.17134,
 1.14563, 1.16902, 1.18322, 1.18409, 1.18119, 1.17987, 1.20468, 1.28691,
 1.3125, 1.30011, 1.3065, 1.36383, 1.35138, 1.41528, 1.44475, 1.47198, 1.48429,
 1.58003, 1.58812, 1.55148, 1.58999, 1.59385, 1.56375, 1.56239, 1.55806,
 1.60007, 1.64634, 1.68266, 1.63066, 1.67173, 1.67839, 1.70978, 1.62411,
 1.55747, 1.53083, 1.57454, 1.68081, 1.69054, 1.65604, 1.54222, 1.53626,
 1.5858, 1.62718, 1.5761, 1.57811, 1.58641, 1.61765, 1.55141, 1.5489, 1.51629,
 1.59482, 1.56931, 1.51093, 1.48985, 1.51275, 1.50851, 1.54761, 1.59531,
 1.628, 1.57708, 1.4995, 1.51631, 1.55155, 1.57448, 1.7177, 1.65414, 1.64593,
 1.68358, 1.73729, 1.69237, 1.68769, 1.6859, 1.63887, 1.63042, 1.64076, 1.67923}
```

This plot shows the 100 month return on our investment with roughly 99.5% of our investment in SPY stock and roughly 0.5% of our investment in put options with strike price 30% below stock fund price:

```
p1 = ListLinePlot[SPYaccumulatedchange,
  PlotRange → {{0, 101}, {0.75, 2}}, PlotStyle → Red, PlotLabel →
   Style["100 Month Return on Investment Using Put Options", Larger, Black],
  AxesLabel → {"Months", "Return on Investment"},
  LabelStyle → Directive[Darker[Green], Bold]]
```



The graph above shows that the total 100 month return on this investment strategy is 1.67923. This means that an initial $1,000,000 portfolio will return $1,679,230 after 100 months. This portfolio increases by 67.923% during the 100 month period.

Naming a return table for a alternate investment strategy solely based on investing in SPY stock:

```
SPYreturntablenoputs = SPYreturntable
```

```
{1.00551, 1.06842, 0.978089, 0.98783, 0.966592, 1.0451, 1.06065, 0.976408, 1.07958,
 0.982223, 1.00946, 0.995549, 0.974303, 0.99814, 1.01303, 1.0337, 0.984317,
 0.998077, 1.05573, 0.98382, 0.961505, 0.955621, 1.05534, 1.00456, 0.97805, 1.02042,
 1.01214, 1.00074, 0.997545, 0.998887, 1.02103, 1.06826, 1.01988, 0.990559,
 1.00491, 1.04388, 0.990871, 1.04728, 1.02083, 1.01884, 1.00837, 1.0645, 1.00512,
 0.976926, 1.02482, 1.00243, 0.981114, 0.999129, 0.997229, 1.02697, 1.02892,
 1.02206, 0.969096, 1.02518, 1.00399, 1.0187, 0.949893, 0.958971, 0.982896, 1.02855,
 1.06749, 1.00579, 0.979592, 0.931272, 0.996132, 1.03225, 1.02609, 0.968608,
 1.00127, 1.00526, 1.01969, 0.959053, 0.998383, 0.978948, 1.05179, 0.984008,
 0.962797, 0.986045, 1.01537, 0.9972, 1.02591, 1.03082, 1.02049, 0.968722,
 0.950808, 1.01121, 1.02324, 1.01478, 1.09096, 0.962998, 0.995037, 1.02287,
 1.0319, 0.974142, 0.997233, 0.998943, 0.972103, 0.994841, 1.00634, 1.02345}
```

Now to account for the fact that this investment strategy has no put options, when the return on the put options is 0 we must add 0.00500502 to the return, when the return is 0.5 we add 0.00250251, when the return is 10 we subtract (9*0.00500502).

```
SPYreturntablenoputs[[1]] = 1.00551 + .00500502
```

```
1.01052
```

Changing SPY return table to the results in a portfolio that solely invests in SPY and puts no money in

put options:

```
SPYreturntablenoputs[[3]] = SPYreturntablenoputs[[3]] + .00500502;
SPYreturntablenoputs[[4]] = SPYreturntablenoputs[[4]] + .00500502;
SPYreturntablenoputs[[5]] = SPYreturntablenoputs[[5]] + .00500502;
SPYreturntablenoputs[[6]] = SPYreturntablenoputs[[6]] + .00500502;
SPYreturntablenoputs[[8]] = SPYreturntablenoputs[[8]] + .00500502;
SPYreturntablenoputs[[11]] = SPYreturntablenoputs[[11]] + .00500502;
SPYreturntablenoputs[[12]] = SPYreturntablenoputs[[12]] + .00500502;
SPYreturntablenoputs[[13]] = SPYreturntablenoputs[[13]] + .00500502;
SPYreturntablenoputs[[14]] = SPYreturntablenoputs[[14]] + .00500502;
SPYreturntablenoputs[[15]] = SPYreturntablenoputs[[15]] + .00500502;
SPYreturntablenoputs[[16]] = SPYreturntablenoputs[[16]] + .00500502;
SPYreturntablenoputs[[20]] = SPYreturntablenoputs[[20]] + .00500502;
SPYreturntablenoputs[[21]] = SPYreturntablenoputs[[21]] + .00500502;
SPYreturntablenoputs[[22]] = SPYreturntablenoputs[[22]] + .00500502;
SPYreturntablenoputs[[24]] = SPYreturntablenoputs[[24]] + .00500502;
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SPYreturntablenoputs[[28]] = SPYreturntablenoputs[[28]] + .00500502;
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SPYreturntablenoputs[[31]] = SPYreturntablenoputs[[31]] + .00500502;
SPYreturntablenoputs[[32]] = SPYreturntablenoputs[[32]] + .00500502;
SPYreturntablenoputs[[33]] = SPYreturntablenoputs[[33]] + .00500502;
SPYreturntablenoputs[[35]] = SPYreturntablenoputs[[35]] + .00500502;
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```
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SPYreturntablenoputs[[100]] = SPYreturntablenoputs[[100]] + .00500502;

SPYreturntablenoputs[[2]] = SPYreturntablenoputs[[2]] + 0.00250251;
SPYreturntablenoputs[[7]] = SPYreturntablenoputs[[7]] + 0.00250251;
SPYreturntablenoputs[[9]] = SPYreturntablenoputs[[9]] + 0.00250251;
SPYreturntablenoputs[[10]] = SPYreturntablenoputs[[10]] + 0.00250251;
SPYreturntablenoputs[[17]] = SPYreturntablenoputs[[17]] + 0.00250251;
SPYreturntablenoputs[[18]] = SPYreturntablenoputs[[18]] + 0.00250251;
SPYreturntablenoputs[[19]] = SPYreturntablenoputs[[19]] + 0.00250251;
SPYreturntablenoputs[[23]] = SPYreturntablenoputs[[23]] + 0.00250251;
SPYreturntablenoputs[[25]] = SPYreturntablenoputs[[25]] + 0.00250251;
SPYreturntablenoputs[[26]] = SPYreturntablenoputs[[26]] + 0.00250251;
SPYreturntablenoputs[[29]] = SPYreturntablenoputs[[29]] + 0.00250251;
SPYreturntablenoputs[[34]] = SPYreturntablenoputs[[34]] + 0.00250251;
SPYreturntablenoputs[[39]] = SPYreturntablenoputs[[39]] + 0.00250251;
SPYreturntablenoputs[[45]] = SPYreturntablenoputs[[45]] + 0.00250251;
SPYreturntablenoputs[[50]] = SPYreturntablenoputs[[50]] + 0.00250251;
SPYreturntablenoputs[[52]] = SPYreturntablenoputs[[52]] + 0.00250251;
SPYreturntablenoputs[[54]] = SPYreturntablenoputs[[54]] + 0.00250251;
```

```
SPYreturntablenoputs[[55]] = SPYreturntablenoputs[[55]] + 0.00250251;
SPYreturntablenoputs[[56]] = SPYreturntablenoputs[[56]] + 0.00250251;
SPYreturntablenoputs[[58]] = SPYreturntablenoputs[[58]] + 0.00250251;
SPYreturntablenoputs[[59]] = SPYreturntablenoputs[[59]] + 0.00250251;
SPYreturntablenoputs[[60]] = SPYreturntablenoputs[[60]] + 0.00250251;
SPYreturntablenoputs[[71]] = SPYreturntablenoputs[[71]] + 0.00250251;
SPYreturntablenoputs[[76]] = SPYreturntablenoputs[[76]] + 0.00250251;
SPYreturntablenoputs[[82]] = SPYreturntablenoputs[[82]] + 0.00250251;
SPYreturntablenoputs[[83]] = SPYreturntablenoputs[[83]] + 0.00250251;
SPYreturntablenoputs[[88]] = SPYreturntablenoputs[[88]] + 0.00250251;
SPYreturntablenoputs[[90]] = SPYreturntablenoputs[[90]] + 0.00250251;
SPYreturntablenoputs[[94]] = SPYreturntablenoputs[[94]] + 0.00250251;
```

```
SPYreturntablenoputs[[48]] = SPYreturntablenoputs[[48]] - (9 * .00500502);
```

Lastly, for month 37 with the huge drop we just subtract the 21.0383% stock price drop since this investment strategy is purely based in stock.

```
SPYreturntablenoputs[[37]] = (1 - .210383)
```

```
0.789617
```

The table of 100 monthly returns on a pure SPY stock investment strategy:

```
SPYreturntablenoputs
```

```
{1.01052, 1.07092, 0.983094, 0.992835, 0.971597, 1.0501, 1.06316, 0.981413, 1.08208,
 0.984725, 1.01447, 1.00055, 0.979308, 1.00314, 1.01804, 1.03871, 0.98682,
 1.00058, 1.05823, 0.988825, 0.96651, 0.960626, 1.05784, 1.00957, 0.980552,
 1.02292, 1.01715, 1.00574, 1.00005, 1.00389, 1.02603, 1.07327, 1.02489, 0.993062,
 1.00992, 1.04889, 0.789617, 1.05229, 1.02333, 1.02385, 1.01337, 1.06951, 1.01013,
 0.981931, 1.02732, 1.00743, 0.986119, 0.954084, 1.00223, 1.02947, 1.03392,
 1.02456, 0.974101, 1.02768, 1.00649, 1.0212, 0.954898, 0.961474, 0.985399,
 1.03105, 1.07249, 1.01079, 0.984597, 0.936277, 1.00114, 1.03725, 1.0311,
 0.973613, 1.00628, 1.01027, 1.02219, 0.964058, 1.00339, 0.983953, 1.05679,
 0.986511, 0.967802, 0.99105, 1.02038, 1.0022, 1.03092, 1.03332, 1.023, 0.973727,
 0.955813, 1.01622, 1.02825, 1.01728, 1.09597, 0.965501, 1.00004, 1.02788,
 1.03691, 0.976645, 1.00224, 1.00395, 0.977108, 0.999846, 1.01135, 1.02845}
```

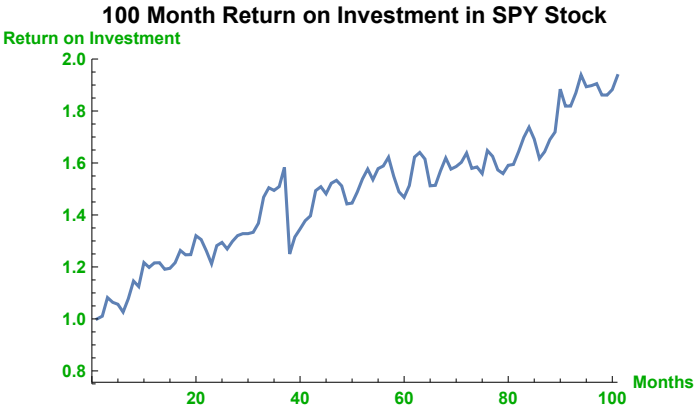
This table shows the accumulated monthly returns on a pure SPY stock fund investment strategy through a 100 month (8 1/3 years) period. The last number is the total return on investment through the 100 month period.

```
SPYaccumulatedchangenoputs = FoldList[Times, 1, %]
```

```
{1, 1.01052, 1.08218, 1.06389, 1.05627, 1.02626, 1.07768, 1.14575, 1.12445,
 1.21674, 1.19816, 1.21549, 1.21617, 1.191, 1.19475, 1.2163, 1.26337, 1.24672,
 1.24744, 1.32008, 1.30533, 1.26162, 1.21194, 1.28204, 1.29431, 1.26913,
 1.29822, 1.32048, 1.32807, 1.32813, 1.3333, 1.36801, 1.46824, 1.50478, 1.49434,
 1.50916, 1.58294, 1.24992, 1.31527, 1.34596, 1.37805, 1.39648, 1.49355,
 1.50867, 1.48141, 1.52189, 1.5332, 1.51192, 1.4425, 1.44572, 1.48832, 1.53881,
 1.57661, 1.53578, 1.5783, 1.58854, 1.62222, 1.54906, 1.48938, 1.46763, 1.51321,
 1.62291, 1.64042, 1.61516, 1.51223, 1.51395, 1.57036, 1.61919, 1.57647,
 1.58637, 1.60265, 1.63822, 1.57934, 1.58469, 1.55926, 1.64781, 1.62559,
 1.57325, 1.55916, 1.59094, 1.59445, 1.64375, 1.69852, 1.73758, 1.69193,
 1.61717, 1.64339, 1.68981, 1.71902, 1.88398, 1.81899, 1.81906, 1.86978,
 1.93879, 1.89351, 1.89774, 1.90524, 1.86162, 1.86134, 1.88246, 1.93602}
```

Plot of SPY return over 100 month period with no investment in put options:

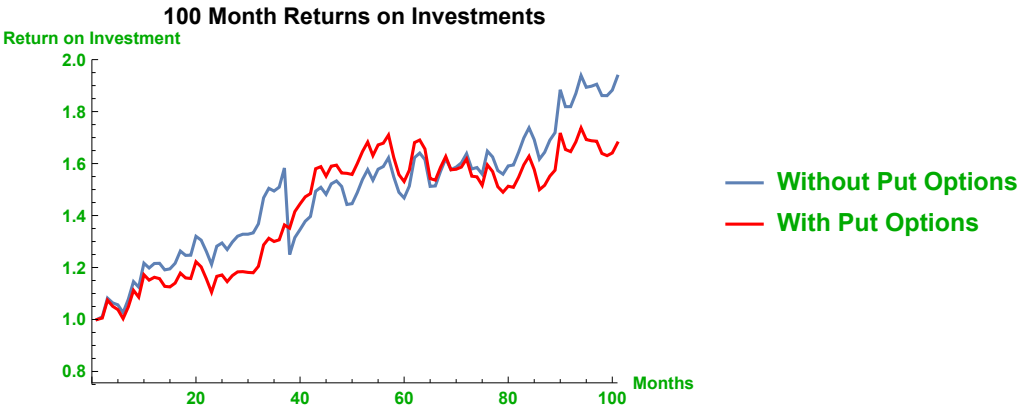
```
p2 = ListLinePlot[SPYaccumulatedchangenoputs, PlotRange → {{0, 101}, {.75, 2}},
  PlotLabel → Style["100 Month Return on Investment in SPY Stock", Larger, Black],
  AxesLabel → {"Months", "Return on Investment"},
  LabelStyle → Directive[Darker[Green], Bold]]
```



This graph above shows that the total 100 month return on this investment strategy is 1.93602. This means that an initial $1,000,000 portfolio will return $1,936,020 after 100 months. This portfolio experiences a 93.602% increase during this 100 month period.

Comparing the returns of each strategy through 100 months:

```
p12 = ListLinePlot[{SPYaccumulatedchangenoputs, SPYaccumulatedchange},
  PlotRange → {{0, 101}, {.75, 2}},
  PlotLabel → Style["100 Month Returns on Investments", Larger, Black],
  PlotStyle → {, Red} , AxesLabel → {"Months", "Return on Investment"},
  LabelStyle → Directive[Darker[Green], Bold],
  PlotLegends → {"Without Put Options", "With Put Options"}]
```



Finding when the strategy with puts has a greater cumulative return than the strategy with no puts:

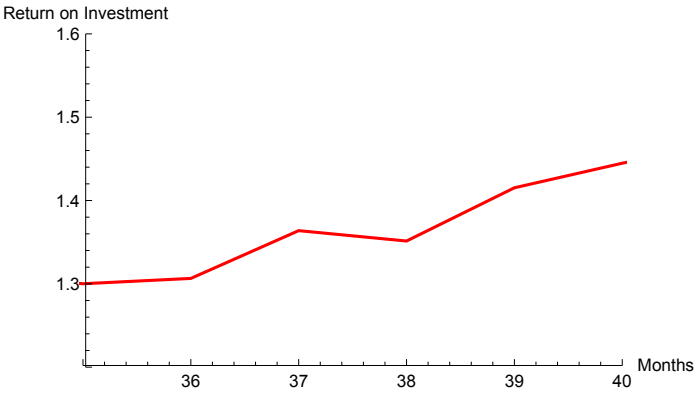
```
Thread[SPYaccumulatedchange >= SPYaccumulatedchangenoputs]
```

```
{True, False, False, False, False, False, False, False, False, False, False,
 False, False, False, False, False, False, False, False, False, False, False,
 False, False, False, False, False, False, False, False, False, False, False,
 False, False, False, False, True, True, True, True, True, True, True, True,
 True, True, True, True, True, True, True, True, True, True, True, True,
 True, True, True, True, True, True, True, True, True, True, False,
 False, False, False, False, False, False, False, False, False, False,
 False, False, False, False, False, False, False, False, False, False, False,
 False, False, False, False, False, False, False, False, False, False, False}
```

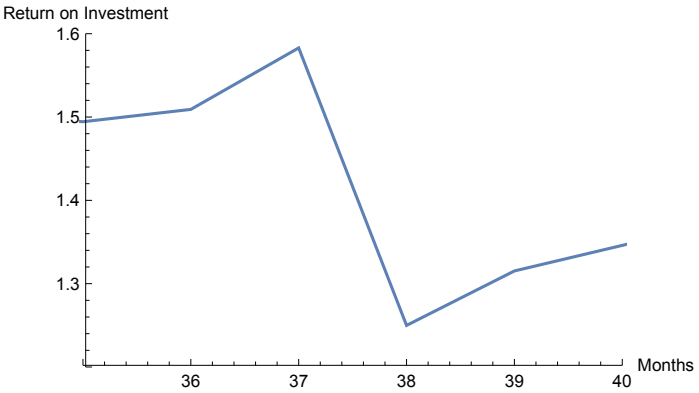
Based on this, in the graph above, we see that starting between months 37 and 38, the strategy with puts has a higher cumulative return than the strategy with just SPY stock fund investment. It takes another 31 months, between months 68 and 69, for a portfolio with no put options to catch up to the return of one with put options.

The next two plots highlight the difference in monthly return on the portfolios with and without put options. The first plot shows the return on the portfolio with put options while the second plot shows the portfolio with no put options.

```
crash1plot = ListLinePlot[SPYaccumulatedchange, PlotRange → {{35, 40}, {1.2, 1.6}},
  PlotStyle → {Red}, AxesLabel → {"Months", "Return on Investment"}]
```




```
crash2plot = ListLinePlot[SPYaccumulatedchangenoputs,
  PlotRange → {{35, 40}, {1.2, 1.6}}, AxesLabel → {"Months", "Return on Investment"}]
```



65 month time period example starting right before the crash

Next, I will track the returns on both types of strategy starting in the month before the major drop in stock price (month 36). Below is table for the monthly return for an investment strategy purely investing in SPY from the 36th month to the 100th month (65 months).

```
SPYreturntablenoputssincecrash = SPYreturntablenoputs[[36 ;; 100]]
```

```
{1.04889, 0.789617, 1.05229, 1.02333, 1.02385, 1.01337, 1.06951, 1.01013, 0.981931,
 1.02732, 1.00743, 0.986119, 0.954084, 1.00223, 1.02947, 1.03392, 1.02456,
 0.974101, 1.02768, 1.00649, 1.0212, 0.954898, 0.961474, 0.985399, 1.03105,
 1.07249, 1.01079, 0.984597, 0.936277, 1.00114, 1.03725, 1.0311, 0.973613,
 1.00628, 1.01027, 1.02219, 0.964058, 1.00339, 0.983953, 1.05679, 0.986511,
 0.967802, 0.99105, 1.02038, 1.0022, 1.03092, 1.03332, 1.023, 0.973727,
 0.955813, 1.01622, 1.02825, 1.01728, 1.09597, 0.965501, 1.00004, 1.02788,
 1.03691, 0.976645, 1.00224, 1.00395, 0.977108, 0.999846, 1.01135, 1.02845}
```

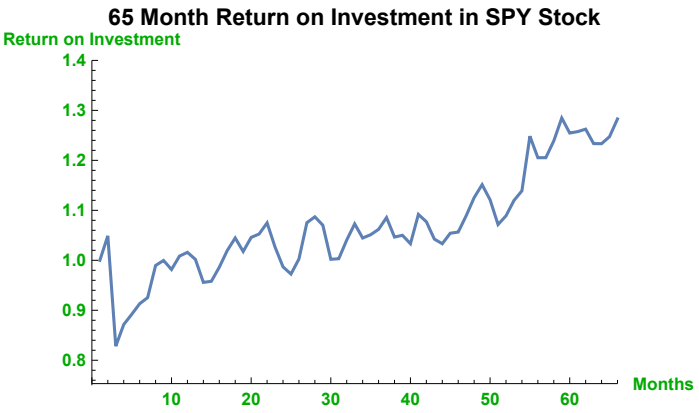
This is the table of the accumulated monthly returns for a pure SPY stock investment strategy from the 36th month through the 100th month:

```
SPYaccumulatedchangenoputssincecrash = FoldList[Times, 1, %]
```

```
{1, 1.04889, 0.828218, 0.871524, 0.891856, 0.913126, 0.925335, 0.989652,
 0.999675, 0.981611, 1.00843, 1.01593, 1.00183, 0.955828, 0.957963, 0.986193,
 1.01965, 1.04469, 1.01764, 1.04581, 1.0526, 1.07492, 1.02643, 0.98689,
 0.972481, 1.00268, 1.07537, 1.08698, 1.07023, 1.00204, 1.00318, 1.04055,
 1.07291, 1.0446, 1.05116, 1.06195, 1.08551, 1.0465, 1.05004, 1.03319, 1.09187,
 1.07714, 1.04246, 1.03313, 1.05419, 1.05651, 1.08918, 1.12547, 1.15136,
 1.12111, 1.07157, 1.08895, 1.1197, 1.13905, 1.24836, 1.2053, 1.20535, 1.23895,
 1.28468, 1.25467, 1.25748, 1.26245, 1.23355, 1.23336, 1.24735, 1.28284}
```

Graph showing the return on investment through this 65 month period:

```
p4 = ListLinePlot[SPYaccumulatedchangenoputssincecrash,
   PlotRange → {{0, 66}, {.75, 1.4}},
   PlotLabel → Style["65 Month Return on Investment in SPY Stock", Larger, Black],
   AxesLabel → {"Months", "Return on Investment"},
   LabelStyle → Directive[Darker[Green], Bold]]
```



The graph above shows that the total 65 month return on this investment strategy is 1.28284. This means that an initial $1,000,000 portfolio will return $1,282,840 after 65 months. This portfolio experiences a 28.284% increase during this 65 month period.

Next, the table for the monthly return for the investment strategy of 0.5% of portfolio in put options from month 36 to month 100:

```
SPYreturntablesincecrash = SPYreturntable[[36 ;; 100]]
```

```
{1.04388, 0.990871, 1.04728, 1.02083, 1.01884, 1.00837, 1.0645, 1.00512, 0.976926,
 1.02482, 1.00243, 0.981114, 0.999129, 0.997229, 1.02697, 1.02892, 1.02206,
 0.969096, 1.02518, 1.00399, 1.0187, 0.949893, 0.958971, 0.982896, 1.02855,
 1.06749, 1.00579, 0.979592, 0.931272, 0.996132, 1.03225, 1.02609, 0.968608,
 1.00127, 1.00526, 1.01969, 0.959053, 0.998383, 0.978948, 1.05179, 0.984008,
 0.962797, 0.986045, 1.01537, 0.9972, 1.02591, 1.03082, 1.02049, 0.968722,
 0.950808, 1.01121, 1.02324, 1.01478, 1.09096, 0.962998, 0.995037, 1.02287,
 1.0319, 0.974142, 0.997233, 0.998943, 0.972103, 0.994841, 1.00634, 1.02345}
```

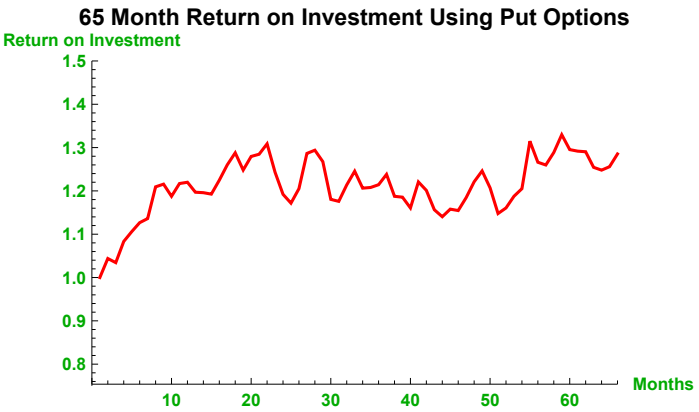
This table shows the accumulated monthly return for this 65 month period:

```
SPYaccumulatedchangesincecrash = FoldList[Times, 1, %]
```

```
{1, 1.04388, 1.03435, 1.08326, 1.10582, 1.12666, 1.13608, 1.20936, 1.21556,
 1.18751, 1.21698, 1.21994, 1.1969, 1.19586, 1.19254, 1.2247, 1.26012, 1.28792,
 1.24812, 1.27955, 1.28465, 1.30867, 1.2431, 1.1921, 1.17171, 1.20516,
 1.2865, 1.29395, 1.26754, 1.18042, 1.17586, 1.21378, 1.24545, 1.20635,
 1.20789, 1.21425, 1.23815, 1.18745, 1.18553, 1.16058, 1.22068, 1.20116,
 1.15647, 1.14033, 1.15787, 1.15462, 1.18455, 1.22105, 1.24608, 1.2071,
 1.14772, 1.16059, 1.18757, 1.20512, 1.31473, 1.26609, 1.2598, 1.28862,
 1.32973, 1.29535, 1.29176, 1.2904, 1.2544, 1.24793, 1.25584, 1.28529}
```

Graph showing return on investment with 0.5% in put options:

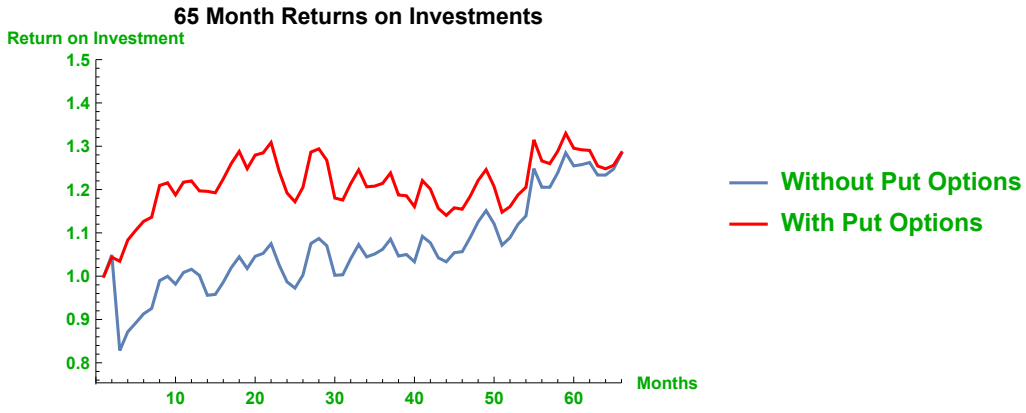
```
p3 = ListLinePlot[SPYaccumulatedchangesincecrash,
  PlotRange → {{0, 66}, {.75, 1.5}}, PlotStyle → Red, PlotLabel →
   Style["65 Month Return on Investment Using Put Options", Larger, Black],
  AxesLabel → {"Months", "Return on Investment"},
  LabelStyle → Directive[Darker[Green], Bold]]
```



The graph above shows that the total 65 month return on this investment strategy is 1.28529. This means that an initial $1,000,000 portfolio will return $1,285,290 after 65 months. This portfolio experiences a 28.529% increase during this 65 month period.

Comparing both investment strategies' returns on investment:

```
p34 = ListLinePlot[{SPYaccumulatedchangenoputssincecrash,
    SPYaccumulatedchangesincecrash}, PlotRange → {{0, 66}, {.75, 1.5}},
  PlotLabel → Style["65 Month Returns on Investments", Larger, Black],
  PlotStyle → {, Red} , AxesLabel → {"Months", "Return on Investment"},
  LabelStyle → Directive[Darker[Green], Bold],
  PlotLegends → {"Without Put Options", "With Put Options"}]
```



Comparing when the strategy with put options has a greater cumulative return than the strategy without put options:

```
Thread[SPYaccumulatedchangesincecrash >= SPYaccumulatedchangenoputssincecrash]
```

```
{True, False, True, True, True, True, True, True, True, True, True, True, True, True,
 True, True, True, True, True, True, True, True, True, True, True, True, True,
 True, True, True, True, True, True, True, True, True, True, True, True, True,
 True, True, True, True, True, True, True, True, True, True, True, True, True,
 True, True, True, True, True, True, True, True, True, True, True, True, True}
```

This, along with the graph above, shows that at the end of the second month with the huge crash, the strategy with put options has a greater cumulative return on investment than the strategy with no put options. As a result, if starting right before a crash, it takes an investment strategy of just SPY stock fund over 64 months (5 1/3 years) to catch up to the cumulative return of an investment strategy with these specific put options.

Next, an alternate example for a much more catastrophic drop in the stock market such as 40%. This is an even greater monthly drop in price than SPY experienced in 2008.

100 month time period example

With stock price drop of 40%, put options at a strike price 30% below the stock price will now be in the money, and the price could increase so much that we could get a return of 70. As a result, we will use 70 as our return on a put during that month.

This is the function for the monthly return on an investment strategy including puts with w as the fraction of the investment in put options.

```
returnfracmarkethugecrash[w_] = 1 + (1 - w) * (-.4) + w * 70
```

```
1 - 0.4 (1 - w) + 70 w
```

This new table will be used for the monthly return on our investment with put options:

```
SPYreturntablecrash = SPYreturntable
```

```
{1.00551, 1.06842, 0.978089, 0.98783, 0.966592, 1.0451, 1.06065, 0.976408, 1.07958,
 0.982223, 1.00946, 0.995549, 0.974303, 0.99814, 1.01303, 1.0337, 0.984317,
 0.998077, 1.05573, 0.98382, 0.961505, 0.955621, 1.05534, 1.00456, 0.97805, 1.02042,
 1.01214, 1.00074, 0.997545, 0.998887, 1.02103, 1.06826, 1.01988, 0.990559,
 1.00491, 1.04388, 0.990871, 1.04728, 1.02083, 1.01884, 1.00837, 1.0645, 1.00512,
 0.976926, 1.02482, 1.00243, 0.981114, 0.999129, 0.997229, 1.02697, 1.02892,
 1.02206, 0.969096, 1.02518, 1.00399, 1.0187, 0.949893, 0.958971, 0.982896, 1.02855,
 1.06749, 1.00579, 0.979592, 0.931272, 0.996132, 1.03225, 1.02609, 0.968608,
 1.00127, 1.00526, 1.01969, 0.959053, 0.998383, 0.978948, 1.05179, 0.984008,
 0.962797, 0.986045, 1.01537, 0.9972, 1.02591, 1.03082, 1.02049, 0.968722,
 0.950808, 1.01121, 1.02324, 1.01478, 1.09096, 0.962998, 0.995037, 1.02287,
 1.0319, 0.974142, 0.997233, 0.998943, 0.972103, 0.994841, 1.00634, 1.02345}
```

Now, using above function, solving for the monthly return of this type of portfolio during the month with a 40% drop in SPY stock fund price:

```
SPYreturntablecrash[[37]] = returnfracmarkethugecrash[.00500502]
```

```
0.952353
```

Reproducing a table with new monthly return in month 37 just to ensure it is in the table:

```
SPYreturntablecrash
```

```
{1.00551, 1.06842, 0.978089, 0.98783, 0.966592, 1.0451, 1.06065, 0.976408, 1.07958,
 0.982223, 1.00946, 0.995549, 0.974303, 0.99814, 1.01303, 1.0337, 0.984317,
 0.998077, 1.05573, 0.98382, 0.961505, 0.955621, 1.05534, 1.00456, 0.97805, 1.02042,
 1.01214, 1.00074, 0.997545, 0.998887, 1.02103, 1.06826, 1.01988, 0.990559,
 1.00491, 1.04388, 0.952353, 1.04728, 1.02083, 1.01884, 1.00837, 1.0645, 1.00512,
 0.976926, 1.02482, 1.00243, 0.981114, 0.999129, 0.997229, 1.02697, 1.02892,
 1.02206, 0.969096, 1.02518, 1.00399, 1.0187, 0.949893, 0.958971, 0.982896, 1.02855,
 1.06749, 1.00579, 0.979592, 0.931272, 0.996132, 1.03225, 1.02609, 0.968608,
 1.00127, 1.00526, 1.01969, 0.959053, 0.998383, 0.978948, 1.05179, 0.984008,
 0.962797, 0.986045, 1.01537, 0.9972, 1.02591, 1.03082, 1.02049, 0.968722,
 0.950808, 1.01121, 1.02324, 1.01478, 1.09096, 0.962998, 0.995037, 1.02287,
 1.0319, 0.974142, 0.997233, 0.998943, 0.972103, 0.994841, 1.00634, 1.02345}
```

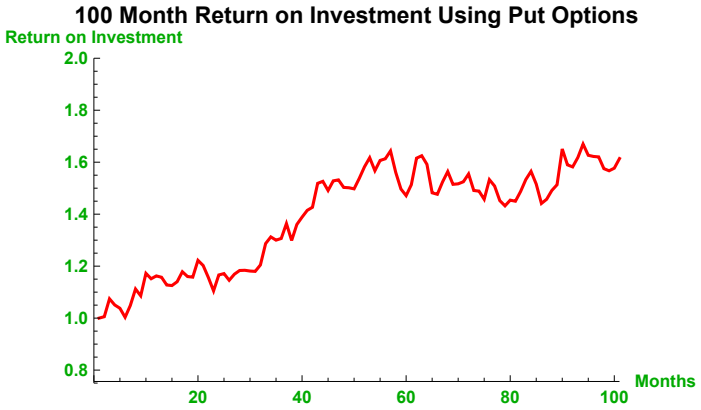
Now producing a table with the accumulated monthly returns on investment. The last number is the return on an investment with put options through the 100 month period.

```
SPYaccumulatedchangecrash = FoldList[Times, 1, %]
```

```
{1, 1.00551, 1.0743, 1.05076, 1.03798, 1.0033, 1.04855, 1.11215, 1.08591, 1.17232,
 1.15148, 1.16238, 1.1572, 1.12747, 1.12537, 1.14003, 1.17845, 1.15997, 1.15774,
 1.22226, 1.20248, 1.15619, 1.10488, 1.16602, 1.17134, 1.14563, 1.16902,
 1.18322, 1.18409, 1.18119, 1.17987, 1.20468, 1.28691, 1.3125, 1.30011, 1.3065,
 1.36383, 1.29885, 1.36026, 1.38859, 1.41476, 1.42659, 1.51861, 1.52639,
 1.49117, 1.52818, 1.5319, 1.50296, 1.50165, 1.49749, 1.53787, 1.58235, 1.61725,
 1.56728, 1.60674, 1.61315, 1.64332, 1.56097, 1.49693, 1.47133, 1.51334,
 1.61547, 1.62482, 1.59166, 1.48227, 1.47654, 1.52416, 1.56393, 1.51483,
 1.51676, 1.52474, 1.55476, 1.4911, 1.48869, 1.45735, 1.53282, 1.50831,
 1.4522, 1.43193, 1.45395, 1.44988, 1.48745, 1.53329, 1.56472, 1.51578,
 1.44121, 1.45737, 1.49124, 1.51328, 1.65093, 1.58984, 1.58195, 1.61814,
 1.66976, 1.62658, 1.62208, 1.62037, 1.57517, 1.56704, 1.57698, 1.61396}
```

Plot shows 100 month return on investment with put options as a part of the strategy:

```
pcrash1 = ListLinePlot[SPYaccumulatedchangecrash,
  PlotRange → {{0, 101}, {0.75, 2}}, PlotStyle → Red, PlotLabel →
   Style["100 Month Return on Investment Using Put Options", Larger, Black],
  AxesLabel → {"Months", "Return on Investment"},
  LabelStyle → Directive[Darker[Green], Bold]]
```



The graph above shows that the total 100 month return on this investment strategy is 1.61396. This means that an initial $1,000,000 portfolio will return $1,613,960 after 100 months. This portfolio experiences a 61.396% increase during this 100 month period.

Setting up table for 100 month return on investment with no put options:

```
SPYreturntablecrashnoputs = SPYreturntablecrash
```

```
{1.00551, 1.06842, 0.978089, 0.98783, 0.966592, 1.0451, 1.06065, 0.976408, 1.07958,
 0.982223, 1.00946, 0.995549, 0.974303, 0.99814, 1.01303, 1.0337, 0.984317,
 0.998077, 1.05573, 0.98382, 0.961505, 0.955621, 1.05534, 1.00456, 0.97805, 1.02042,
 1.01214, 1.00074, 0.997545, 0.998887, 1.02103, 1.06826, 1.01988, 0.990559,
 1.00491, 1.04388, 0.952353, 1.04728, 1.02083, 1.01884, 1.00837, 1.0645, 1.00512,
 0.976926, 1.02482, 1.00243, 0.981114, 0.999129, 0.997229, 1.02697, 1.02892,
 1.02206, 0.969096, 1.02518, 1.00399, 1.0187, 0.949893, 0.958971, 0.982896, 1.02855,
 1.06749, 1.00579, 0.979592, 0.931272, 0.996132, 1.03225, 1.02609, 0.968608,
 1.00127, 1.00526, 1.01969, 0.959053, 0.998383, 0.978948, 1.05179, 0.984008,
 0.962797, 0.986045, 1.01537, 0.9972, 1.02591, 1.03082, 1.02049, 0.968722,
 0.950808, 1.01121, 1.02324, 1.01478, 1.09096, 0.962998, 0.995037, 1.02287,
 1.0319, 0.974142, 0.997233, 0.998943, 0.972103, 0.994841, 1.00634, 1.02345}
```

Changing SPY return table to the results in a portfolio that solely invests in SPY and doesn't invest money in put options:

```
SPYreturntablecrashnoputs = SPYreturntablecrash + .00500502;
```

```
SPYreturntablecrashnoputs[[2]] = SPYreturntablecrashnoputs[[2]] - 0.00250251;
SPYreturntablecrashnoputs[[7]] = SPYreturntablecrashnoputs[[7]] - 0.00250251;
SPYreturntablecrashnoputs[[9]] = SPYreturntablecrashnoputs[[9]] - 0.00250251;
SPYreturntablecrashnoputs[[10]] = SPYreturntablecrashnoputs[[10]] - 0.00250251;
SPYreturntablecrashnoputs[[17]] = SPYreturntablecrashnoputs[[17]] - 0.00250251;
SPYreturntablecrashnoputs[[18]] = SPYreturntablecrashnoputs[[18]] - 0.00250251;
SPYreturntablecrashnoputs[[19]] = SPYreturntablecrashnoputs[[19]] - 0.00250251;
SPYreturntablecrashnoputs[[23]] = SPYreturntablecrashnoputs[[23]] - 0.00250251;
SPYreturntablecrashnoputs[[25]] = SPYreturntablecrashnoputs[[25]] - 0.00250251;
SPYreturntablecrashnoputs[[26]] = SPYreturntablecrashnoputs[[26]] - 0.00250251;
SPYreturntablecrashnoputs[[29]] = SPYreturntablecrashnoputs[[29]] - 0.00250251;
SPYreturntablecrashnoputs[[34]] = SPYreturntablecrashnoputs[[34]] - 0.00250251;
SPYreturntablecrashnoputs[[39]] = SPYreturntablecrashnoputs[[39]] - 0.00250251;
SPYreturntablecrashnoputs[[45]] = SPYreturntablecrashnoputs[[45]] - 0.00250251;
SPYreturntablecrashnoputs[[50]] = SPYreturntablecrashnoputs[[50]] - 0.00250251;
SPYreturntablecrashnoputs[[52]] = SPYreturntablecrashnoputs[[52]] - 0.00250251;
SPYreturntablecrashnoputs[[54]] = SPYreturntablecrashnoputs[[54]] - 0.00250251;
SPYreturntablecrashnoputs[[55]] = SPYreturntablecrashnoputs[[55]] - 0.00250251;
SPYreturntablecrashnoputs[[56]] = SPYreturntablecrashnoputs[[56]] - 0.00250251;
SPYreturntablecrashnoputs[[58]] = SPYreturntablecrashnoputs[[58]] - 0.00250251;
SPYreturntablecrashnoputs[[59]] = SPYreturntablecrashnoputs[[59]] - 0.00250251;
SPYreturntablecrashnoputs[[60]] = SPYreturntablecrashnoputs[[60]] - 0.00250251;
SPYreturntablecrashnoputs[[71]] = SPYreturntablecrashnoputs[[71]] - 0.00250251;
SPYreturntablecrashnoputs[[76]] = SPYreturntablecrashnoputs[[76]] - 0.00250251;
SPYreturntablecrashnoputs[[82]] = SPYreturntablecrashnoputs[[82]] - 0.00250251;
SPYreturntablecrashnoputs[[83]] = SPYreturntablecrashnoputs[[83]] - 0.00250251;
SPYreturntablecrashnoputs[[88]] = SPYreturntablecrashnoputs[[88]] - 0.00250251;
SPYreturntablecrashnoputs[[90]] = SPYreturntablecrashnoputs[[90]] - 0.00250251;
SPYreturntablecrashnoputs[[94]] = SPYreturntablecrashnoputs[[94]] - 0.00250251;

SPYreturntablecrashnoputs[[48]] = SPYreturntablecrashnoputs[[48]] - (10 * .00500502);

SPYreturntablecrashnoputs[[37]] = (1 - .4)
0.6
```

Printing table of monthly returns on investment with no puts, confirming that the correct changes were made from original table:

```
SPYreturntablecrashnoputs
```

```
{1.01051, 1.07092, 0.983094, 0.992835, 0.971597, 1.0501, 1.06316, 0.981413, 1.08208,
 0.984725, 1.01447, 1.00055, 0.979308, 1.00314, 1.01804, 1.03871, 0.98682,
 1.00058, 1.05823, 0.988825, 0.96651, 0.960626, 1.05784, 1.00957, 0.980552,
 1.02292, 1.01715, 1.00574, 1.00005, 1.00389, 1.02603, 1.07327, 1.02489, 0.993062,
 1.00992, 1.04889, 0.6, 1.05229, 1.02333, 1.02385, 1.01337, 1.06951, 1.01013,
 0.981931, 1.02732, 1.00743, 0.986119, 0.954084, 1.00223, 1.02947, 1.03392,
 1.02456, 0.974101, 1.02768, 1.00649, 1.0212, 0.954898, 0.961474, 0.985399,
 1.03105, 1.07249, 1.01079, 0.984597, 0.936277, 1.00114, 1.03725, 1.0311,
 0.973613, 1.00628, 1.01027, 1.02219, 0.964058, 1.00339, 0.983953, 1.05679,
 0.986511, 0.967802, 0.99105, 1.02038, 1.0022, 1.03092, 1.03332, 1.023, 0.973727,
 0.955813, 1.01622, 1.02825, 1.01728, 1.09597, 0.965501, 1.00004, 1.02788,
 1.03691, 0.976645, 1.00224, 1.00395, 0.977108, 0.999846, 1.01135, 1.02845}
```

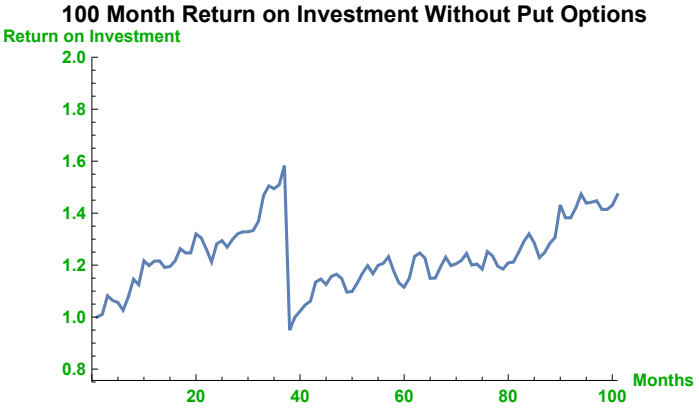
This is a table of accumulated monthly returns through a 100 month (8 1/3 years) period for an investment strategy purely involving SPY stock. The last number is the total return on investment through the 100 month period.

```
SPYaccumulatedchangecrashnoputs = FoldList[Times, 1, %]
```

```
{1, 1.01051, 1.08218, 1.06388, 1.05626, 1.02626, 1.07768, 1.14574, 1.12445,
 1.21674, 1.19816, 1.21549, 1.21616, 1.191, 1.19474, 1.21629, 1.26337, 1.24672,
 1.24744, 1.32008, 1.30533, 1.26161, 1.21194, 1.28204, 1.2943, 1.26913,
 1.29822, 1.32048, 1.32806, 1.32813, 1.3333, 1.368, 1.46823, 1.50478, 1.49434,
 1.50916, 1.58293, 0.94976, 0.999421, 1.02274, 1.04713, 1.06113, 1.13488,
 1.14638, 1.12566, 1.15642, 1.16502, 1.14885, 1.0961, 1.09855, 1.13092,
 1.16928, 1.198, 1.16698, 1.19928, 1.20707, 1.23266, 1.17706, 1.13172, 1.11519,
 1.14982, 1.23318, 1.24649, 1.22729, 1.14909, 1.15039, 1.19325, 1.23036,
 1.19789, 1.20541, 1.21779, 1.24482, 1.20007, 1.20414, 1.18482, 1.25211,
 1.23522, 1.19545, 1.18475, 1.20889, 1.21156, 1.24902, 1.29064, 1.32032,
 1.28563, 1.22882, 1.24875, 1.28402, 1.30621, 1.43156, 1.38218, 1.38223,
 1.42077, 1.47321, 1.4388, 1.44202, 1.44771, 1.41457, 1.41435, 1.4304, 1.4711}
```

Plot of SPY return over 100 month period with no part of investment in put options:

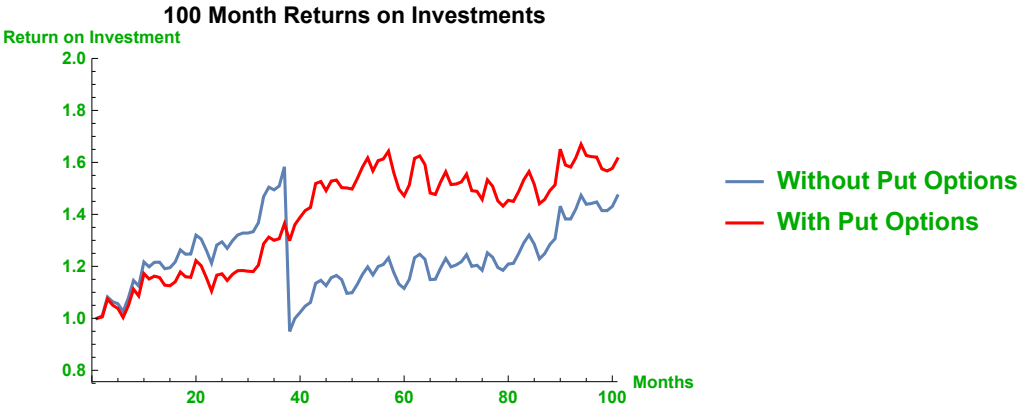
```
pcrash2 = ListLinePlot[SPYaccumulatedchangecrashnoputs,
  PlotRange → {{0, 101}, {.75, 2}}, PlotLabel →
   Style["100 Month Return on Investment Without Put Options", Larger, Black],
  AxesLabel → {"Months", "Return on Investment"},
  LabelStyle → Directive[Darker[Green], Bold]]
```



The graph above shows that the total 100 month return on this investment strategy is 1.4711. This means that an initial $1,000,000 portfolio will return $1,471,100 after 100 months. This portfolio experiences a 47.11% increase during this 100 month period.

Comparing 100 month return on each type of investment strategy:

```
pcrash12 = ListLinePlot[{SPYaccumulatedchangecrashnoputs,
    SPYaccumulatedchangecrash}, PlotRange → {{0, 101}, {.75, 2}},
  PlotLabel → Style["100 Month Returns on Investments", Larger,  Black],
  PlotStyle → {, Red} , AxesLabel → {"Months", "Return on Investment"},
  LabelStyle → Directive[Darker[Green], Bold],
  PlotLegends → {"Without Put Options", "With Put Options"}]
```



The graph above shows that with such a large drop in SPY stock fund price, the investment strategy with no put options is affected so greatly that it cannot catch up to the total return on the portfolio with put options for the next sixty two months.

65 month example with time period starting just before the crash

Now, as in the previous example, will look at the return on investments starting the month before the huge SPY stock fund price drop. Below is the table for the 65 month period for the investment strategy with no put options.

```
SPYreturntablenoputssincehugecrash = SPYreturntablecrashnoputs[[36 ;; 100]]
```

```
{1.04889, 0.6, 1.05229, 1.02333, 1.02385, 1.01337, 1.06951, 1.01013, 0.981931,
 1.02732, 1.00743, 0.986119, 0.954084, 1.00223, 1.02947, 1.03392, 1.02456,
 0.974101, 1.02768, 1.00649, 1.0212, 0.954898, 0.961474, 0.985399, 1.03105,
 1.07249, 1.01079, 0.984597, 0.936277, 1.00114, 1.03725, 1.0311, 0.973613,
 1.00628, 1.01027, 1.02219, 0.964058, 1.00339, 0.983953, 1.05679, 0.986511,
 0.967802, 0.99105, 1.02038, 1.0022, 1.03092, 1.03332, 1.023, 0.973727,
 0.955813, 1.01622, 1.02825, 1.01728, 1.09597, 0.965501, 1.00004, 1.02788,
 1.03691, 0.976645, 1.00224, 1.00395, 0.977108, 0.999846, 1.01135, 1.02845}
```

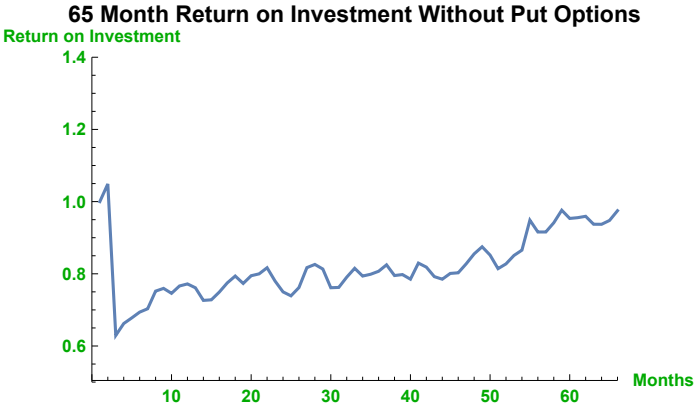
Below is the table of accumulated monthly returns for a pure SPY stock fund investment strategy from the 36th month through the 100th month. The last number is the total return on investment in this 65 month period.

```
SPYaccumulatedchangenoputssincehugecrash = FoldList[Times, 1, %]
```

```
{1, 1.04889, 0.629332, 0.662238, 0.677688, 0.693849, 0.703127, 0.751999,
 0.759615, 0.745889, 0.76627, 0.771967, 0.761251, 0.726297, 0.72792, 0.74937,
 0.774792, 0.793823, 0.773264, 0.794671, 0.799829, 0.816787, 0.779949, 0.7499,
 0.738951, 0.761899, 0.817132, 0.825953, 0.813231, 0.761409, 0.762275,
 0.790674, 0.815263, 0.793751, 0.798734, 0.806935, 0.824841, 0.795195,
 0.797889, 0.785085, 0.829673, 0.818482, 0.792128, 0.785038, 0.801037,
 0.802803, 0.827625, 0.855205, 0.874871, 0.851886, 0.814243, 0.827448,
 0.850821, 0.865523, 0.948585, 0.915859, 0.915898, 0.941432, 0.976179,
 0.95338, 0.955513, 0.959286, 0.937326, 0.937182, 0.947817, 0.974784}
```

Below is a plot showing the return on investment through this 65 month period for a pure SPY stock investment strategy:

```
pcrash4 = ListLinePlot[SPYaccumulatedchangenoputssincehugecrash,
  PlotRange → {{0, 66}, {.5, 1.4}}, PlotLabel →
   Style["65 Month Return on Investment Without Put Options", Larger, Black],
  AxesLabel → {"Months", "Return on Investment"},
  LabelStyle → Directive[Darker[Green], Bold]]
```



The graph above shows that the total 65 month return on this investment strategy is 0.974784. This means that an initial $1,000,000 portfolio will return $974,784 after 65 months. This portfolio experiences a 2.5216% decrease during this 65 month period.

This table shows the monthly returns for the investment strategy of roughly 0.5% of the investment in put options with strike price 30% below stock price during this 65 month period:

```
SPYreturntablesincehugecrash = SPYreturntablecrash[[36 ;; 100]]
```

```
{1.04388, 0.952353, 1.04728, 1.02083, 1.01884, 1.00837, 1.0645, 1.00512, 0.976926,
 1.02482, 1.00243, 0.981114, 0.999129, 0.997229, 1.02697, 1.02892, 1.02206,
 0.969096, 1.02518, 1.00399, 1.0187, 0.949893, 0.958971, 0.982896, 1.02855,
 1.06749, 1.00579, 0.979592, 0.931272, 0.996132, 1.03225, 1.02609, 0.968608,
 1.00127, 1.00526, 1.01969, 0.959053, 0.998383, 0.978948, 1.05179, 0.984008,
 0.962797, 0.986045, 1.01537, 0.9972, 1.02591, 1.03082, 1.02049, 0.968722,
 0.950808, 1.01121, 1.02324, 1.01478, 1.09096, 0.962998, 0.995037, 1.02287,
 1.0319, 0.974142, 0.997233, 0.998943, 0.972103, 0.994841, 1.00634, 1.02345}
```

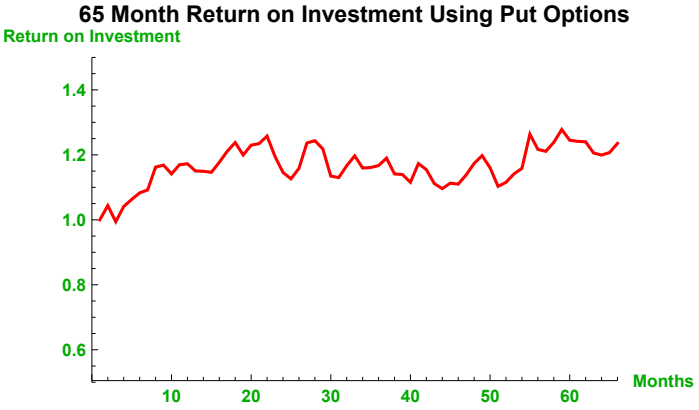
Table for the accumulated monthly return for the investment strategy with 0.500502% of the portfolio invested in put options. The last number in this table is the total return on investment through the 65 month period.

```
SPYaccumulatedchangesincehugecrash = FoldList[Times, 1, %]
```

```
{1, 1.04388, 0.994144, 1.04115, 1.06283, 1.08286, 1.09192, 1.16235, 1.1683,
 1.14135, 1.16968, 1.17252, 1.15037, 1.14937, 1.14619, 1.1771, 1.21114,
 1.23785, 1.1996, 1.22981, 1.23471, 1.2578, 1.19478, 1.14576, 1.12616, 1.15831,
 1.23649, 1.24365, 1.21827, 1.13454, 1.13015, 1.1666, 1.19704, 1.15946,
 1.16094, 1.16705, 1.19002, 1.1413, 1.13945, 1.11546, 1.17323, 1.15447,
 1.11152, 1.09601, 1.11286, 1.10974, 1.1385, 1.17359, 1.19764, 1.16018,
 1.10311, 1.11548, 1.1414, 1.15827, 1.26363, 1.21687, 1.21083, 1.23853,
 1.27804, 1.24499, 1.24155, 1.24024, 1.20564, 1.19942, 1.20703, 1.23533}
```

Below is graph showing return on investment with put options:

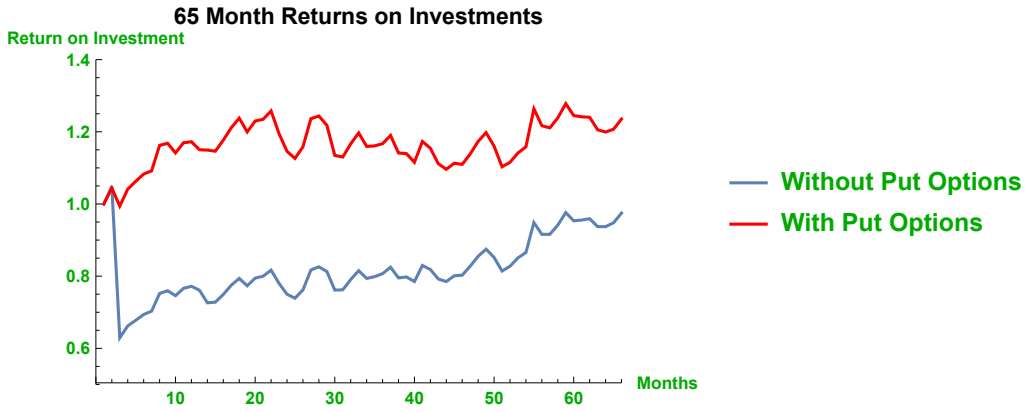
```
pcrash3 = ListLinePlot[SPYaccumulatedchangesincehugecrash,
  PlotRange → {{0, 66}, {.5, 1.5}}, PlotStyle → Red, PlotLabel →
   Style["65 Month Return on Investment Using Put Options", Larger, Black],
  AxesLabel → {"Months", "Return on Investment"},
  LabelStyle → Directive[Darker[Green], Bold]]
```



The graph above shows that the total 65 month return on this investment strategy is 1.23533. This means that an initial $1,000,000 portfolio will return $1,235,330 after 65 months. This portfolio experiences a 23.533% increase during this 65 month period.

Comparing returns on investment for both strategies through this 65 month period:

```
pcrash34 = ListLinePlot[{SPYaccumulatedchangenoputssincehugecrash,
    SPYaccumulatedchangesincehugecrash}, PlotRange → {{0, 66}, {.5, 1.4}},
  PlotLabel → Style["65 Month Returns on Investments", Larger, Black],
  PlotStyle → {, Red} , AxesLabel → {"Months", "Return on Investment"},
  LabelStyle → Directive[Darker[Green], Bold],
  PlotLegends → {"Without Put Options", "With Put Options"}]
```



The graph above shows that similar to the 100 month period, the investment strategy with no put options cannot catch up to the return on the strategy with put options after the huge drop in stock price for the rest of the tested period. It takes over 64 months for an investment in solely SPY stock to catch up.

Conclusion

The purpose of this project is to show the benefits of investing in put options significantly below the actual price of the stock. Put options eliminate the downside risk of a particular stock. While the market is trending upward, the return on an investment strategy with these put options will not be as great as an investment strategy purely in that stock (but will be lower by only about half a percent each month in this analysis or less depending on monthly returns of the put options). However, when the market has a significant downturn, this type of put option (30% below the current stock price) will provide insurance and reduce the impact of the downturn on the portfolio as we witnessed in these examples. A strategy without these put options takes a huge hit on its return which takes years to overcome. Although no one can accurately and consistently predict market changes, if there is a sense of volatility and instability in the market place or other conditions that usually signal large drops in stock prices, it might be wise to set aside a small percentage of your portfolio to invest in 30% below stock price put options.