WHQV401 Assignment [50]

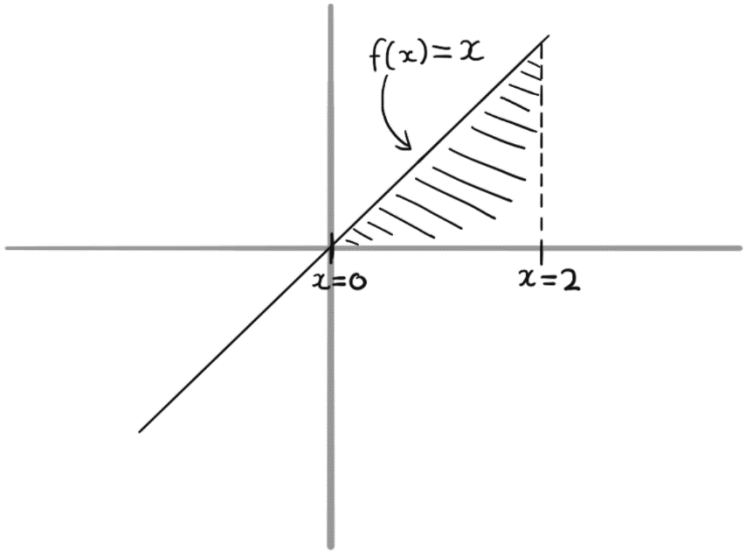
Notes

i. Copying of assignments is, of course, strictly prohibited.
ii. Marks will be deducted for unnecessary algorithm inefficiency.
iii. Due date: 28th October 2019 11:59 PM. Submit your code on Moodle.
iv. When it is stated that you must create a function to do X. You must, of course, write this function, but you are also allowed to write helper functions that the main function calls.
v. Working code will be considered far superior to *"almost"* working code. Try your best to get your code to compile and run.
vi. Please clearly separate your code into specific questions.
vii. All questions must be answered using F#.

Question 1a – Definite Integral Approximation [10]

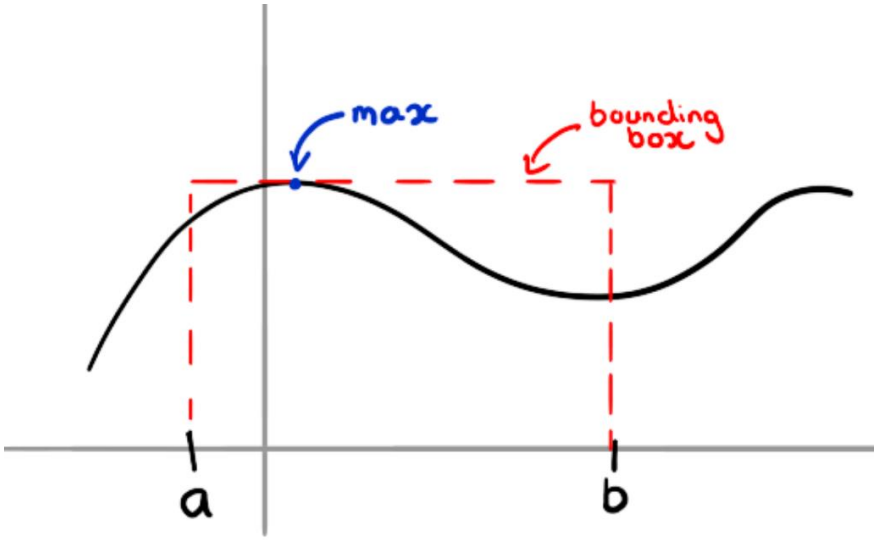
You may be familiar with calculating the definite integral of a graph. You may not be. Fear not. The definite integral of a graph is simply the area underneath the graph between two given points. So, let us say you're given the graph defined by the function $f(x) = x$ and the two points that we wish to integrate between are 0 and 2.

Let's start by drawing the information we're given.



So, the definite integral, in this case, is equal to the area of the shaded part. Which, if you recall the formula for the area of a triangle, would be: $0.5 \times 2 \times 2 = 2$. But what if I gave you the function $f(x) = 4x^3 - 2x^2 + 9x - 2$? Drawing the graph and manually calculating the area wouldn't be as easy. You could of course apply a bit of calculus and find the answer. Or. You could approximate the answer with brute force and computing power.

Perhaps you're familiar with the following method of integral approximation. Let's start by simplifying the problem slightly and making the assumption that all functions we're going to deal with will lie above the x-axis. Now, if you are given a function f and two bounds to integrate between, a and b, then we need to find the maximum value of that function on the interval $[a, b]$. This will allow us to draw a bounding box around the part of the graph that we are concerned with. For example:



Now, it is pretty easy to calculate the area of the bounding box, $A = \max \times (b - a)$. Now imagine that we threw a number of darts, let's say N darts, randomly this at bounding box. If we count the number of darts below the graph, B, then an approximate definite integral would be equal to: $\frac{B}{N} \times A$.

So, here's what you need to do. Write a function that will accept a mathematical function (like the ones above) as well as two integration bounds, a and b. You will then first need to find the maximum value of this function on the domain $[a, b]$. Instead of doing this algebraically, you may break up the interval $[a, b]$ into 10,000 equally spaced points. Check the value of the function at each of these points and select the maximum of these values. With that information, you can determine the bounding box. Now, write another function to generate 100,000 random points within the bounding box and count how many fall below the graph. Finally, produce an estimate integral value based on description above.

Question 1b - Multithreaded Definite Integral Approximation [4]

Extend your previous program such that the processing is distributed between five threads.

DO NOT EDIT QUESTION 1a – DUPLICATE AND CHANGE IT.

The process of finding the maximum value of the graph need not be multithreaded. i.e Only the part of generating random points and counting how many are below the graph needs to be multithreaded.

Question 2a – Successive Multiples [5]

Imagine you are given a list of integers, for example: [1;5;9;-2;10;1;-2;3]. Let use define a 'successive multiple of length 4' to be the multiple of 4 numbers that are next to one another. For example, $1 \times 5 \times 9 \times -2$ or $10 \times 1 \times -2 \times 3$.

You must create a function that will accept a list of integers and return the tuple (A, B) where A is the four integers that form the largest successive multiple of length 4 and B is the value. For example, for the list: [1;2;3;4;5] your function should return ("2*3*4*5", 120). *[The first item in the tuple is of type 'string', the second of type 'int'.]*

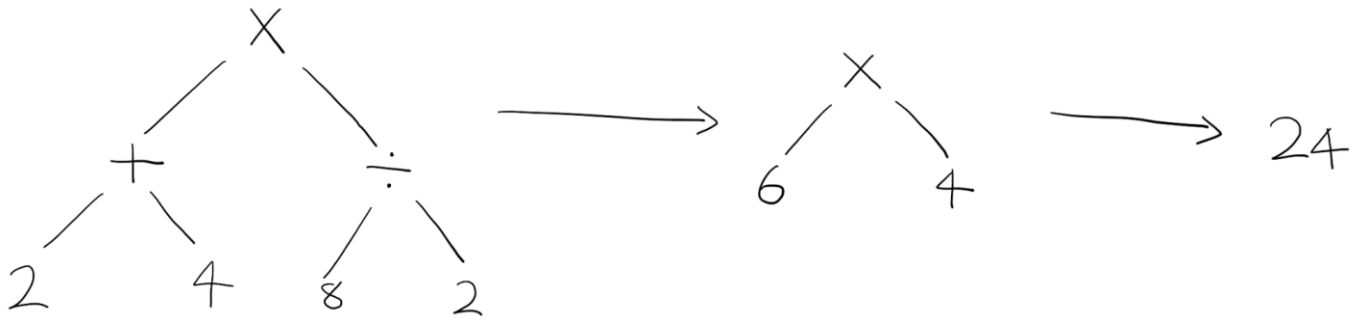
Hint: You may assume that any list will have at least 4 numbers in it.

Question 2b – Concatenated List Sum [4]

Consider the numbers [100; 101; … ; 999]. Now consider the list of numbers where each of the previous numbers were concatenated with themselves, i.e. D = [100100; 101101; 102102; … ; 998998; 999999]. You must write a function that will generate the list D and sum together all the numbers in the list that are **not** divisible by the number 7.

Question 3a – Expression Tree Nodes [8]

It is possible to express basic algebra in the form of a binary tree. Consider the following example:



The leaf nodes of the tree are values and the internal nodes are operations. Each operation has two children that it applies itself to. The tree is calculated from the bottom up.

You will be constructing something similar, but slightly more complicated. Using discriminated unions, you are to create the following types:

Operator, with subtypes:

i. Plus
ii. Minus
iii. Multiply
iv. Divide

BooleanOperator, with subtypes:

i. GreaterThan
ii. GreaterThanEqual
iii. LessThan
iv. LessThanEqual
v. Equal

UnaryOperator, with subtypes:

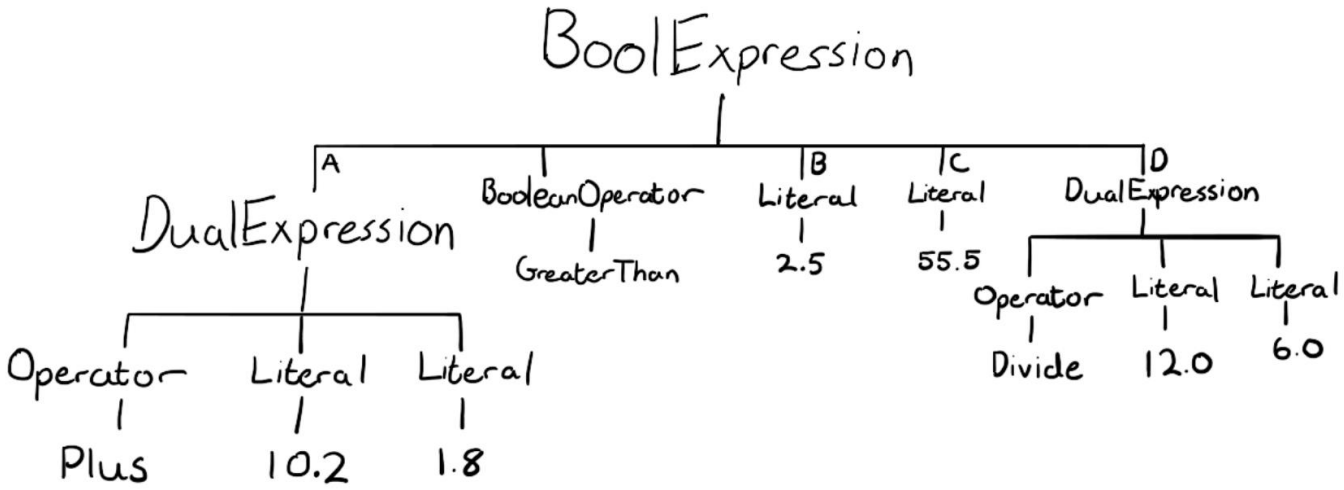
i. Negate
ii. Increment
iii. Decrement

Expression, with subtypes:

i. Literal
   - A '**Literal'** is simply a floating-point number, such as '7.5' or '-12.3'.
ii. UnaryExpression
   - A '**UnaryExpression'** applies a **UnaryOperator** to an **Expression**.
iii. DualExpression
   - A '**DualExpression'** applies an **Operator** to two **Expressions**.
iv. BoolExpression
   - A '**BoolExpression'** considers 4 **Expressions** A, B, C and D. It applies a **BooleanOperator** to **Expressions** A and B. If the result is true, **Expression** C is returned as the answer, else **Expression** D is returned.

Question 3b – Expression Tree Evaluation [10]

Using the types you created in 3a), you can build expression trees. These are trees that represent algebraic calculations. For example:



The root of this tree is a BooleanExpression that evaluate its child nodes A and B. A, in turn, is a DualExpression that, once evaluated, returns 12. B is the simplest type of expression, a plain floating-point number. A is then checked if it is GreaterThan B. Which it is. Thus child node C will be evaluated and returned as the answer, i.e. 55.5. Keep in mind, this is a basic example. This entire tree could be the sub-tree of a much larger tree!

You are to write a function that can evaluate any tree that can be created by the nodes in 3a). This function will accept a tree and will return a single float value, i.e. the final expression of the tree.

Note for Questions 3c-e

The functions you create in the following questions ***must generate Expression Trees.*** They must not simply give the answer to the expression. For instance, you should be able to use answer from 3b) to evaluate your trees and find the answer.

Question 3c – Absolute Value Tree [2]

Using the tree nodes that you created in 3a), you must write a function that will accept a number A and ***generate an Expression Tree*** that, when evaluated, will return the absolute value of A.

Question 3d – Maximum Value Tree [3]

Let us say that you wish to determine which number, A or B, is the larger of the two. You could do so with a simple if statement. However, there is an algebraic way of doing so. Consider the following equation:

$$\max(x, y) = \frac{x + y + |x - y|}{2}$$

Note: $|x|$ represents the absolute value of x.

You must write a function that will accept two numbers, A and B, and will ***generate an Expression Tree*** that, when evaluated, will return the maximum of A and B using the provided formula. Furthermore, you must make use of your answer in 3c) to generate the absolute value part of this question.

Question 3e – Maximum Absolute Value in List Tree [4]

The maximum absolute value of a list is equivalent to applying the absolute value function to the list and selecting the largest value from the resulting list. You must write a function that will accept a list of numbers and ***generate an Expression Tree*** that, when evaluated, will return the maximum absolute value the list. You must make use of your answers in 3c) and 3d) in your tree generation.

Hint: Generating this tree requires traversal of the list. You may assume that the list has at least 1 value in it.