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Decision Support for the Physician Scheduling Process at a German Hospital

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Abstract. The process of manually constructing monthly working schedules for physicians in medium-sized and large departments at hospitals is a very time-consuming and error-prone task. The scheduler, typically a senior physician, is an expensive resource and oftentimes almost irreplaceable because of his acquired expertise in the scheduling process. We develop a mathematical model that formalizes every rule and regulation necessary to generate lawful schedules in the anesthesiology department of a 626-bed hospital in Berlin, Germany. We embed our detailed and complex mixed-integer programming formulation, which generates schedules superior to the ones currently in use, in an Excel environment to ensure ease of use, maximum flexibility with respect to changing all relevant inputs, and a visual output representation for practitioners. The presented approach reduces the workload for the scheduler dramatically, thereby increasing his availability for medical services. Our generated schedules outperform manually created schedules by significantly reducing the number of rule and regulation violations, while also improving key performance measures such as assigned overtime, granted employee-preferred shifts, and fairness considerations. Our approach also highlights important aspects in modeling the physician scheduling problem for practical implementation that have been widely ignored in the existing literature.

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Keywords: workforce management • physician scheduling • health services • hospital

1. Introduction

Demand and supply planning is a central and important aspect of ensuring adequate healthcare services for a population. Rising life expectancy and declining birth rates in many countries lead to aging populations, causing the demand for medical care services and corresponding budgets to increase (Rais and Viana 2011). Furthermore, the prevalent shortage of skilled physicians in most industrialized countries, including Germany, has been a major driver of staffing problems for many years. In 2013, 58% of hospitals in Germany were affected by this problem (Blum et al. 2013). Moreover, physicians are usually seen as the scarce resource in a hospital system (Schall et al. 2004). Ultimately, physician scheduling is extremely important to ensuring a high quality of medical care (Santos and Eriksson 2014). Physician scheduling in general is one of the most complex topics in personnel scheduling because of the high heterogeneity among the individual employees with respect to qualifications and contractual agreements as well as the demand for very specific workforce compositions.

High quality rosters are required to efficiently match supply and demand and provide an adequate work balance for the personnel. Schedules with unbalanced workloads and high stress levels are not conducive to employee satisfaction and physician retainment (Bruni and Detti 2014). In addition, shift design and assignment policies have an impact on the physical, mental, and social well-being of the staff (Knauth 1993). In practice, physician schedules in specialty departments are often created manually by experienced and highly qualified physicians, deducting their valuable time from performing value-adding medical services (Brunner and Edenharter 2011). The resulting schedules of this difficult, error-prone, and time-consuming task are frequently criticized among colleagues (Cervesato and Righini 2014), partially because the perception of the quality of a schedule is very subjective (Carter and Lapierre 2001). Without an automated approach, it is very problematic to ensure the quality dimensions of a roster beyond meeting contractually agreed working hour restrictions. Besides integrating multiple facets of roster quality in the scheduling process and providing better information on the quality of the resulting rosters, an automated application enables the user to generate multiple alternative solutions to choose from. Last but not least, the notable reduction of the time required to generate schedules grants the planner more time to spend on medical services. In total, an automated scheduling approach yields

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great potential to improve this laborious process (Burke et al. 2004). Operations research methods have been shown to be very helpful in automating the scheduling process in a hospital (Ovchinnikov and Milner 2008).

In our work, we apply mathematical programming in the form of a mixed integer program (MIP) to automate and optimize the scheduling of physicians within the anesthesiology department of a hospital in Berlin, Germany. The investigated personnel scheduling problem deals with the assignment of predefined shifts, stints, and days off to a given workforce of physicians. The generated schedules over monthly planning horizons are acyclical, because the physicians are heterogeneous in terms of their qualifications and contractual agreements. Moreover, planning horizons vary from month to month in terms of length and distribution of work days, weekends, and holidays. In addition, we consider personal preferences that change monthly with respect to day-off requests, preferred shift requests, and evenly distributed on-call shift assignments.

While the proposed model is very complex with its large number of sets, variables, and constraints, and specific for German laws, rules, and regulations, it also provides a blueprint on how to model the physician shift assignment problem for practical implementation in other settings. The way the problem is formulated facilitates adaptations to other countries/settings, as many types of additional time- or task-related constraints could be formulated based on the presented basic modeling framework. At the same time, constraints that are irrelevant in other settings could simply be dropped from the model. Moreover, we propose an interface for practitioners to make the necessary adjustments of all required inputs prior to the schedule optimization. These involve the workforce composition, the actual time horizon for the specific schedule, preference parameters, and changes to the shift design. Moreover, we transform the mathematical solution output into a readily accessible visual representation of the resulting schedule.

The remainder of this paper is structured as follows. In Section 2, we present a review of the most relevant related literature. Section 3 provides a general problem characterization and a detailed description of the scheduling problem in the anesthesiology department. We discuss our mathematical formulation, which is shown in detail in the appendix, and possible extensions in Section 4. In Section 5, we present our results, including comparisons of original schedules generated by the planner and the ones resulting from our optimization approach. Finally, we summarize our findings and provide an outlook on further research in Section 6.

2. Related Literature and Contribution

While there has been extensive research on the related topic of nurse scheduling, which has been reviewed by Burke et al. (2004), Cheang et al. (2003), Van den Bergh et al. (2013), and Benazzouz et al. (2015), less attention had been given to physician scheduling until more recently. As shown by Erhard et al. (2018), researchers have focused increasingly on addressing the physician scheduling problem during the last decade. In this section, we provide a brief overview of the physician scheduling literature most closely related to our work after introducing the concept of hard and soft constraints in modeling, which is an important part of our modeling approach.

A mathematical model for the physician scheduling problem typically consists of hard and soft constraints (Gendreau et al. 2007). In a feasible solution, hard constraints must be satisfied, whereas soft constraints may be violated at a penalty to obtain a practical and executable schedule (Bard and Purnomo 2007). A hard constraint, for instance, could be used to ensure that the demand for personnel with the appropriate qualifications is satisfied in every shift (Burke et al. 2004). Soft constraints may be used to model, for example, ergonomic aspects, which contribute to improving the quality of the schedules (Beaulieu et al. 2000). Note that a hard constraint in one hospital can represent a soft constraint in another hospital and vice versa (Carter and Lapierre 2001). In our study, replacing what would normally be considered hard constraints (e.g., legislative regulations on qualification levels for particular shift and task assignments) with soft constraints whose violations are heavily penalized proved to be very useful. First, doing so allows for finding feasible solutions in problem settings where the physicians on staff cannot cover all necessary shifts. Second, it also permits the evaluation of finalized schedules from the past via the mathematical model, which may well have violated such constraints.

Beaulieu et al. (2000) present a mixed integer programming formulation with multiple objectives. Every day, three 8-hour shifts must be occupied by physicians in the emergency room. To solve their model, they decompose the planning horizon of six months into four-week cycles, which are solved sequentially, and each cycle builds on the previous horizon. Bruni and Detti (2014) claim that their MIP permits an adaption to different problem instances and is more flexible than other models by its very general formulation and its generic structure. In their case study, each day is divided into two 12-hour shifts, and the planning horizon is four months. Ovchinnikov and Milner (2008) develop an integer program in an Excel spreadsheet environment for scheduling medical residents. In a radiology department, on-call and weekend shifts must be assigned for a whole year. The authors advocate the use of a spreadsheet because of its understandability for users without operations research knowledge. However, this is usually true only for small problems. Ferrand et al. (2011)

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formulate a model to introduce cyclic schedules for emergency room physicians in a hospital in the United States. The cycle spans eight weeks, wherein a day is divided into three 8-hour shifts. The model is solved to optimality with a standard solver. After implementing their approach in the hospital, they also conduct a survey among physicians with regard to their satisfaction with the new schedules. Cervesato and Righini (2014) present an integer program to automatically schedule physicians in a cardiology department in Italy. Their problem consists of two shift types and a monthly planning horizon. Brunner et al. (2009) introduce a new and flexible approach for scheduling physicians in which shifts with variable starting times and durations are generated implicitly. The MIP allows consideration of individual preferences as well as an integration of breaks. Their case study in an anesthesiology department with a two-week planning horizon requires an enormous computational effort. Therefore, the authors apply a heuristic, in which the model is solved sequentially with several week-long problem instances. Brunner et al. (2011) develop a branch-and-price algorithm for solving up to six-week instances. Stolletz and Brunner (2012) reformulate this model as a reduced set-covering model. This allows solving optimally the same two-week instances using standard software. Additionally, the results are better both in terms of computational time and quality. Furthermore, they present how fairness aspects, namely, an equal distribution of working time and standby shifts among physicians, can be involved. Carter and Lapierre (2001) analyze the physician scheduling problems in emergency rooms of six institutions in Canada. Subsequently, they derive important rules for creating schedules and present a collection of mathematical constraints, some of which are represented in our model. In a recent publication, Fügener et al. (2015) presents a scheduling approach for physicians in a department of anesthesiology. The introduced MIPs comprise a line of work construction and a subsequent workstation assignment model. Additionally, they incorporate fairness aspects and individual preferences. The problem they address is rather big, with 20 shift types, 25 workstations, and 120 physicians. Nevertheless, their models are solved within seconds.

All of the aforementioned publications have in common that the planning horizon is not quite suitable for a straightforward practical implementation with regard to the considered planning horizon. In most of them, it is based on multiples of full weeks or months. Others assume that the first day of planning is always a Monday (e.g., Beaulieu et al. 2000, Carter and Lapierre 2001), usually to ensure that their weekend shift constraints can be modeled correctly. A practitioner who has to make sure that monthly and weekly requirements with respect to working time, days off, overtime, etc., are fulfilled, however, needs to find a schedule that works even if the first day of the month is a Saturday; if the month consists of 28, 29, 30, or 31 days; and if there are a few holidays sprinkled throughout. Hence, we develop a model that can be universally applied for monthly planning horizons of any manifestation. Following are a few publications that have also tackled the problem of finding appropriate ways to model the planning horizon of physician scheduling problems.

Bard et al. (2013) schedule on a monthly basis within their MIP, and in particular define a function that assigns a weekday to every day. However, they focus on a different problem (interns and residents), which differs strongly from the physician scheduling problem in our work (e.g., no weekend shifts). Bruni and Detti (2014) do not plan with a set of days, but rather with a set that contains all shifts. Subsets include shifts on weekdays and rest days. Their advantage is that each day is divided into two disjoint 12-hour shifts, and thus they plan in half days. Similar to our model is the work of Cervesato and Righini (2014). The authors plan on a monthly basis, but need to define a set for all days of the entire year, linked to the corresponding weekday. In contrast, our studied problem necessitates integrating only a few days of the previous and following months to ensure a consistent monthly transition.

To our knowledge, our proposed model is the first one in the literature to incorporate real-world constraints such as the distinction between standard and standby time and day-off compensation into a scheduling model. Additionally, only Fügener et al. (2015) consider more shift types than we do, and just two publications schedule more than 30 physicians (Bruni and Detti 2014, Fügener et al. 2015). The embedding of the model into an Excel spreadsheet to achieve improved usability for the practitioner has been prominently mentioned only by Ovchinnikov and Milner (2008). Compared to Ovchinnikov and Milner (2008), we also provide a user interface for the scheduler in the form of a spreadsheet, to change input data and modify the results in an easy and straightforward way. Our main advantage is the generic model formulation, which allows modifications in staff composition (number of physicians, contractual agreements, requests, availability, qualifications) or shift types to be easily added in the spreadsheet and flexibly handled in the model. Furthermore, our model formulation incorporates an unprecedented degree of detail with respect to the incorporated rules and regulations.

3. Problem Description

Currently, the creation of a monthly schedule is carried out manually by a senior physician at the collaborating hospital in Berlin, Germany—in the following, also referred to as the scheduler. Every physician has login data

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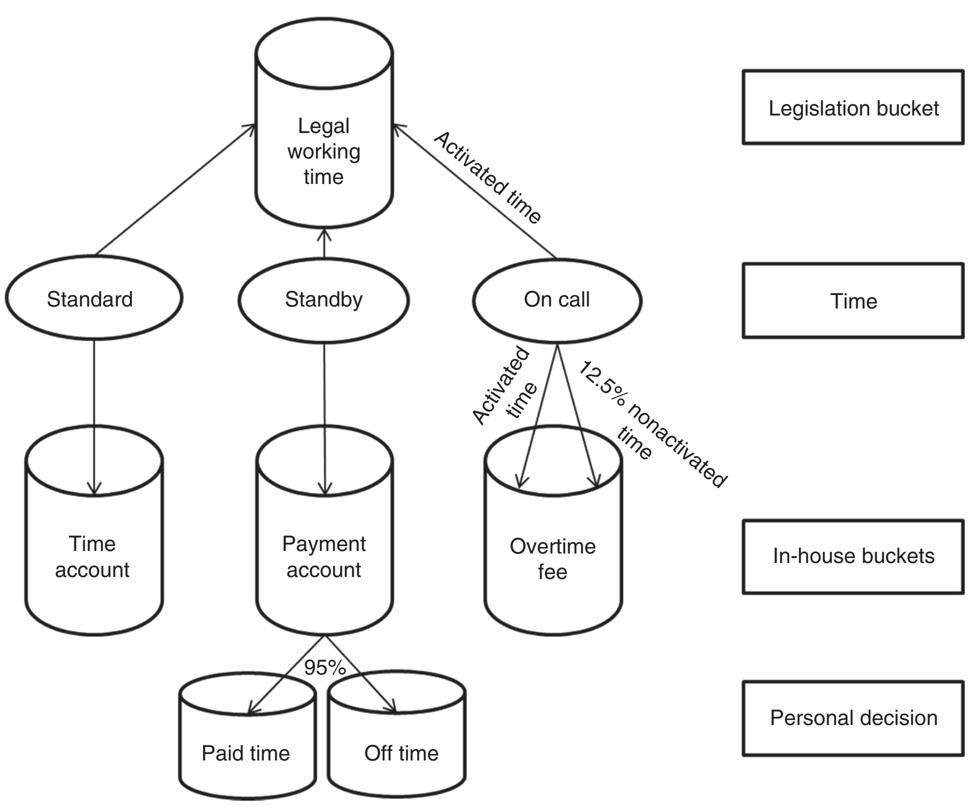
for the software that lets them access their personal schedule and enter requests for vacation and, if applicable, compensatory days off well ahead of time, more than three months before the concerned time period. In the next step, the scheduler approves or rejects these requests depending on projected schedule feasibility. Additionally, a handwritten list for further, nonbinding shift and day-off requests is posted 12 weeks before the specific monthly schedule starts. After a 4-to-5-week time span, the scheduler tries to accommodate most of these requests, to assign every shift except for the standard shift in the operating room, and to follow the many laws and guidelines—all in a large handwritten template. Subsequently, all remaining available weekdays of the physicians are assigned the standard shift in the operating theater, denoted OP, in the completed template, which is transferred manually into a standard personnel management software. Although the software supports checking against violations of excessive weekly legal working time, excessive time account levels, and prohibited workload averages, additional manual and laborious examination is necessary. The preliminary schedule is then submitted for examination to the employee council approximately six weeks before the start of the scheduled time horizon. After a one- to two-week admissibility check—particularly working time restrictions are considered—the scheduler has up to one week to correct violations. The final schedule is published for all physicians at least four weeks in advance.

A finalized physician schedule provides information on the assignments for each physician on every day of the one-month scheduling horizon. It is important to note that assignments can consist of a specific shift type or an absence (e.g., a day off or medical education). Hence, in our model, time off is also explicitly assigned rather than resulting from a lack of a shift assignment. Therefore, we will sometimes call assignments "icons," in reference to their graphical representation in the planning phase, whenever we do not explicitly talk about shifts. Physicians can be assigned to work in the operating room, the acute care departments, the intensive care unit (ICU), the emergency department, and on ambulances. Furthermore, it is crucial to distinguish between the type of required availability in different shifts—standard, standby, on call, or a combination of two of them—as each of them has specific monetary and legislative implications, which makes reality more complicated than typically assumed in theoretical papers on physician scheduling. Figure 1 illustrates the complex distinction. The "standard time" shifts count toward the contractual working time per week of each physician and add up to the so-called "time account." "Standby time" is considered as legal working hours (as defined per law) and does not fill the time account. The standby time accumulates on a separate time account, the so-called "payment

Figure 1. Logic Behind the Work Time Differences



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account," 95% of which may be paid out at a fixed hourly rate (set by a collective agreement), and the remaining 5% forfeited at the end of the month. Alternatively, the payment account can be granted as compensatory time off. Moreover, the day following a night shift, which contains standby time, is always a day off for a physician, and if it is a weekday, eight hours are transferred from the payment account to the time account. The on-call time does not count against the legal working time limit, only the activated time. The activated time and 12.5% of the nonactivated time are compensated with an overtime fee as laid out in the collective agreement.

The medical team comprised between 31 and 34 physicians with four different experience levels (head, chief, senior, and assistant physician) during the time of the project. The size of the team and the personnel structure changed slightly from month to month. The workforce typically consists of around 50% assistant, 33% senior, and 15% chief physicians, as well as one head physician. Furthermore, physicians differ with regard to their contracts. In addition to full-time employees, about 30% of the team works part time, with four different part-time contract choices (80%, 66.7%, 50%, or 40% of the full-time equivalent). Additionally, each physician can consent to an opt-out agreement, which allows hospital management to assign them shifts in excess of their maximum weekly work time, provided that the long-term average stays under that limit.

The overall objective of our work is to automate the planning process to reduce the time needed by the scheduler, decrease the number of rule and regulation violations, improve the schedules' fairness, and eventually simplify the scheduling process so that employees other than the current expert may take on the scheduling task in the future. It currently takes the very experienced (decades of practice) scheduler about seven hours on average to generate the initial schedule. In the expert's opinion, a new scheduler might need two or three days for the described scheduling process.

Improving the quality of the schedules increases physician satisfaction. Therefore, the integration of fairness (for example, evenly distributed standby shifts) as well as time and duty preferences is essential. Additionally, a model should be flexible enough to accommodate changes in the data easily. For example, there are monthly changes in the personnel structure and the qualification of physicians, and sometimes new shifts are defined.

4. Rules, Regulations, and Implementation

It is indispensable to take into account numerous legal restrictions and the collective agreements as well as some in-house rules, which are listed in five blocks below, for a mathematical model to find a feasible, lawful, and useful schedule. The regulations stem from German laws and negotiations with the leading union for German physicians:

Qualification/demand

(A) A physician can be assigned to one shift per day at most.

(B) Any physician may only be assigned to the shifts that they are qualified for.

(C) The demand for physicians for each shift needs to be fulfilled.

(D) Certain minimum qualification requirements need to be met; for example, there may be a minimum amount of senior or chief physicians required to be present.

(E) Some physicians do not work shifts on weekends and/or standby duties.

Quality

(F) Physicians' shift and day-off requests should be considered.

(G) No physician should work more than seven days in a row.

(H) At most five standby shifts should be assigned to a full-time physician each month. No more than two (three) standby duties are assigned to physicians working part time up to (more than) 50% of the full-time equivalent.

(I) There must be at least three days between two standby duties.

Days off

(J) Because the duration of all shift types is designed with full-time employees in mind, every part-time physician is entitled to compensation day assignments (icon A) on weekdays, which are similar to days off. The amount per month depends on the weekly hours of the part-time contract.

(K) Some part-time physicians have guaranteed, some can request, specific "A" days or at least their allotment.

(L) After working a night shift, employees are assigned a day off.

(M) Shifts on weekends and holidays (on a weekday) in the ICU should be compensated, if possible, by connected days off. Shifts in the ICU on Saturday (Sunday) are compensated by Monday (Tuesday) being off, and shifts on Saturday and Sunday by the following Thursday and Fridays being off, respectively. After working a holiday, a day off should be granted relatively soon.

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(N) At least one of the adjacent weekends at the beginning and the end of a granted vacation period (e.g., Monday to Friday) must be off; if possible both should be off.

(O) Every physician should have at least two weekends off per month and not work on two consecutive weekends.

Hours

(P) The time account, that is, the accumulated overtime/undertime of standard time, is only allowed to be greater than 1.5 times the contractual weekly working hours.

(Q) Part-time physicians should work as little overtime (standard time) as possible, because these physicians choose to work a lower workload in terms of their part-time contract.

(R) The legal working time (i.e., the sum of standard, standby, and activated on-call time) of a physician may only exceed 60 hours, or 64 hours if opted out, per week (Monday to Sunday).

(S) The 26-week average of legal working time of a physician is not allowed to be greater than 1.2 times the contractual working week. If a physician opted out, the limit increases to 1.5.

Stints

(T) In the ICU, physicians should work specific stints: from Wednesday to Monday, the early shift; on Tuesday, the late shift; and from Wednesday to Sunday, off.

(U) The shift as internal organization (icon IO) should be assigned to the same physician over four consecutive working days (e.g., Monday to Thursday, Friday and Monday to Wednesday, etc.).

4.1. Model Formulation

As shown in Table A.1 in the appendix, we use a relatively large amount of subsets for physicians, shifts, and days in our model formulation, which we also moved to the appendix because of its size. Subsets for physicians are rather straightforward and we separate them into the different levels of seniority. They are primarily used to model the qualification constraints. In our Excel template, the practitioner would simply need to keep a current list of the anesthesiologists and their experience levels and use the macro to automatically assign the physicians to their respective sets.

Shifts are split into subsets to account for various rules. Some represent a form of absence (S_2), while others denote that a physician is assigned an active duty shift (S_1). They also need to be grouped according to the time when they take place. This may trigger numerous constraints, for example, the ones concerning mandatory days off that should be granted after the assigned shift types collected in the set S_{FZA}. Since the goals include evenly distributing standby shifts among the employees and also avoiding less than three days between assigned standby duties, all shifts that involve being on standby are assigned to the subset S_{BD}.

The days of the one-month planning horizon, plus some days before and after, as explained next, are grouped into numerous subsets. First of all, weekdays and weekend days need to be denoted so that rules regarding limits on worked weekends, among others, can be enforced. The last week of the previous month is added to the model, as well as the days of the following month until a Tuesday is reached (but at least two days). These two sets are important, as they are used to guarantee that the implemented schedule will not cause violations of rules and regulations between two adjacent months. In the worst case, ignoring this possibility could lead to the creation of a schedule that leads to infeasibilities in the next month's schedule. That would be the case, for example, if a month ended on a Thursday and too many physicians were assigned so many shifts at the end of the month, in that first half of the week, that they could not work any other shifts in the second half of the week. This would lead to too many unassigned shifts in the second half of that week, at the beginning of the following month. Without paying attention to this detail, a modeler would always optimize only the current month's schedule performance at the expense of the following month, likely ending up using, on average, worse schedules. Note that assignments on days outside of the scheduled month are not part of the performance measures. Finally, days are also separated into the week of the month they belong to, so that weekly constraints can be properly modeled.

The required parameters and their descriptions as well as the decision variables can be found in Tables A.2 and A.3, respectively. Note that the only *actual* decisions x_{pst} determine which physicians are assigned to which shifts (or absences, formulated as shifts). The remaining decision variables are either auxiliary variables (z_{pw}) or decision expressions (all other variables).

The objective function (A.1) of our model is a weighted sum of 17 terms. The majority of the summands are violations of constraints. To have a feasible model in most cases, we relaxed some hard constraints with very high penalty coefficients α_i ($i = 1, 2$). In general, the weights of the terms must be determined by the user of the model. In our work, the following relationship is proposed: $\alpha_1 \geq \alpha_2 \gg \beta_1 \geq \cdots \geq \beta_4 \gg \gamma_1 \geq \gamma_2 \gg \delta_1 \geq \cdots \geq \delta_7 \gg \omega_1 \geq \omega_2$. Therefore, the objective function's terms are sorted in descending order of importance.

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Table 1. Scheduling Restrictions and Constraint Representation

Block	Restrictions in the problem	Model formulation
Qualification/demand	(A)	(A.2)
	(B)	(A.3)
	(C)	(A.4)
	(D)	(A.5)–(A.8)
	(E)	(A.28), (A.14)
Quality	(F)	(A.10)
	(G)	(A.11)
	(H)	(A.14)
	(I)	(A.15)
Days off	(J)	(A.17)
	(K)	(A.16), (A.9)–(A.10)
	(L)	(A.18)–(A.20)
	(M)	(A.21)–(A.23), (A.44)–(A.45)
	(N)	(A.9)–(A.10)
	(O)	(A.29)–(A.30)
Hours	(P)	(A.32)
	(Q)	(A.1), (A.33)
	(R)	(A.34)
	(S)	(A.35)
Stints	(T)	(A.19), (A.21)–(A.22), (A.36)–(A.37)
	(U)	(A.38)–(A.40)

The term in the objective function with weight α_2 penalizes all vacant shifts except for the replacement shifts denoted AO and TBA, which are replacement shifts, which are penalized less heavily if left unassigned ($\beta_1 < \alpha_2$). Similarly, overtime of part-time physicians is more important and therefore is weighted higher than overtime of full-time physicians (δ_2 versus ω_2). The eleventh term (δ_3) penalizes the difference between the maximum and minimum number of assigned standby duties, thus targeting a fair distribution among physicians. Furthermore, all assignments of set S_2 are minimized because their size is not exactly predefined as S_1. Hence, an assignment of shift type "Off" is always preferred.

Tables 1 and 2 clarify how the previously described rules and regulations are linked to the constraints in the mathematical model. The constraints that are not mentioned, (A.12), (A.13), (A.24)–(A.27), and (A.31), are necessary for determining the variables for distributing the standby duties (bd^{MIN}, bd^{MAX}), working on weekends (z_{pw}), and overtime/undertime (ot_p, ut_p). In total, every verbal constraint above is integrated in the model formulation. Hence, the model formulation is well suited to create feasible schedules at the hospital in Berlin. Furthermore, constraints (A.44) and (A.45) represent a suggestion to integrate the very vaguely worded restriction in (M) about compensation for working on holidays. Therein, working on a holiday in one week is compensated by a day off on a workday in the following week.

4.2. Implementation

The MIP presented in the appendix is implemented within IBM ILOG CPLEX Optimization Studio (Version 12.5.1) and solved with standard settings. An Excel sheet is used to read data and to illustrate the results. To read the data, we used variable range names in Excel accessed by ILOG Studio. Thus, the Excel sheet is expandable in a flexible manner if new physicians or shifts are added. Furthermore, a programmed macro in Visual Basic for Applications (VBA) helps to generate the sets for weeks and days for each month. After solving the MIP, the results are postprocessed in ILOG Studio and then written to Excel with variable ranges of results.

Table 2. Restriction Dimensions and Associated Constraint Types

	Type of constraint	
Block	Soft	Hard
Qualification/demand		(A), (B), (C), (D), (E)
Quality	(F), (G), (H)	(I)
Days off	(K), (M), (N), (O)	(J), (K), (L), (N)
Hours	(Q)	(P), (R), (S)
Stints	(T), (U)	

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Table 3. Exemplary Schedule for Chief Physician (Two Week Excerpt)

February	…	9 Mon	10 Tue	11 Wed	12 Thu	13 Fri	14 Sat	15 Sun	16 Mon	17 Tue	18 Wed	19 Thu	20 Fri	21 Sat	22 Sun	…
Name: ___ Level: Chief physician Working time: 40 h	…	OP	OP	OP	OP	A73	Off	Off	AO	FW	FW	FW	FW	Off	Off	…
Standard time	…	8	8	8	8	9			8	8	8	8	8			…
Standby time	…					8.8										…

Another macro in Excel ensures that the results are illustrated clearly in a readable format similar to the original schedules. All computations were performed on a laptop with a 1.8 GHz Intel Celeron CPU 1000M processor with 4 GB of memory and a Windows 8.1 operating system.

4.3. Model Output

A schedule provides a daily roster icon for each physician. These icons are divided into shifts and absences. An example of a roster for a single physician is given in Table 3. The physician might have the icon OP (which is an eight-hour standard time shift in the operating room) Monday through Thursday and then be assigned to icon A73 (which is a late shift in the ICU, consisting of nine hours standard time and 8.8 hours standby time) on Friday. The A73 icon triggers the following day to be given off (icon Off), and the Sunday might be off as well. In the next week, the doctor might work icon AO on Monday (which is a replacement shift that helps other physicians in the operating room on shortage and accounts for eight hours standard time) and be absent due to education from Tuesday till Friday (icon FW; accounts for eight hours standard time), followed by an off weekend.

5. Results

We worked parallel to the scheduling expert at the hospital in Berlin over a time span of several months. In this testing phase, we generated schedules using the same inputs (workforce composition, individual time accounts, demand, etc.) as the expert. The weights of the violation terms in the objective function were determined in close collaboration with the practice partner involving multiple feedback rounds. Input on the prioritization of objectives from the hospital was implemented in the MIP optimization using ranges for the weights of the different penalty variables. The resulting set of alternative schedules was presented to the expert, who provided feedback on the most sensible selection of penalty parameters. Our results were subsequently compared to the manually created schedules to identify any possible shortcomings in the chosen penalty coefficients until a consensus on the most sensible values was reached. The final version is a set of penalty coefficients that ranks the rules and regulations in lexicographical order. The optimal solution will have the lowest possible sum of violations of the rules associated with the highest coefficient value (here 100,000; see Table 4). When there are multiple such solutions, the optimal solution will be the one that minimizes the sum of the violations with the next lower penalty coefficient. This pattern continues down to the least penalized rules and regulations.

In the following section, we provide an overview of the improvements in the performance measures that the automated schedule generation achieved over the manually created schedules. We refer to the schedules generated by the mathematical model as “model” and the ones manually created as “original.”

5.1. How to Compare the Model Output with the Original Schedules

Creating a schedule for a specific month requires the input data resulting from the previous month’s schedule. Using our optimization tool simplifies this task considerably, as all outputs from one month’s schedule are automatically written into the Excel interface to serve as inputs for the following month. To acquire performance measures that result from using the original schedules, we run our optimization model with all decision variables set to the values used in the expert’s schedule, de facto turning them into model parameters. As the original schedules frequently violate hard constraints, that is, constraints that should never be violated in the automated scheduling process, we add variables to constraints (A.7), (A.8), (A.15), (A.32), and (A.34) to count the number of violations in the original schedules. These violations are added to the objective function and given penalty coefficient values (100,000) much larger than the highest penalty coefficient value in the optimization model (10,000) to illustrate that they should be avoided at all costs.

5.2. Long-Term Comparison

By design, there exists a general tendency in our results toward the biggest improvements being achieved by reducing the violations of the more heavily penalized constraints. Actual trade-offs, where reductions of some

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Table 4. Performance Measure Comparison Between Model and Original Schedules

Verbal expression of constraints	Weight	February		March		April		May		June		July		Total		Average per month		Average relative improvement (%)
		Model	Original	Model	Original	Model	Original	Model	Original	Model	Original	Model	Original	Model	Original	Model	Original	
# days/nights without senior physicians	100,000	0	4	0	5	0	4	0	2	0	2	0	5	0	22	0	3.7	100
# violations: "less than 3 days between standby duties"	100,000	0	5	0	3	0	4	0	2	0	2	0	1	0	17	0	2.8	100
# hours above upper bound of time account	100,000	0	3.1	0	0	0	0	0	0	0	8.9	0	0	0	11.9	0	2	100
# hours above upper bound of weekly working time	100,000	0	10	4	25.1	0	3.8	0	0	0	9.8	0	3.8	4	52.4	0.7	8.7	92
# shifts assigned to underqualified physicians	10,000	2	2	1	4	2	6	0	1	0	6	2	4	7	23	1.2	3.8	70
# vacant shifts (w/o replacement shifts)	10,000	0	0	0	5	0	2	0	0	0	1	0	0	0	8	0	1.3	100
# vacant shifts (replacement shifts)	1,000	0	0	1	5	0	0	0	0	0	0	2	0	3	5	0.5	0.8	40
# preferred shift requests denied	1,000	10	13	4	10	5	16	6	12	3	2	2	8	30	61	5	10.2	51
# not-preferred shift requests denied	1,000	0	0	0	0	0	2	0	1	0	0	0	0	0	3	0	0.5	100
# requests in total	—	80	80	65	65	195	195	171	171	185	185	312	312	1,008	1,008	168	168	—
# off days not granted	1,000	0	2	0	2	0	0	0	2	1	2	1	2	2	10	0.3	1.7	80
# standby duties above upper bound	1,000	2	3	1	3	1	2	2	3	1	5	1	5	8	21	1.3	3.5	62
# weekends that should not be worked by a physician	100	2	2	1	2	1	3	0	0	0	2	0	3	4	12	0.7	2	67

Table 4. (Continued)

Verbal expression of constraints	Weight	February		March		April		May		June		July		Total		Average per month		Average relative improvement (%)
		Model	Original	Model	Original	Model	Original	Model	Original	Model	Original	Model	Original	Model	Original	Model	Original	
# not enough weekends off	100	1	6	0	1	1	2	0	2	0	2	0	4	2	17	0.3	2.8	88
# violations of personnel structure	100	0	7	0	9	0	26	0	7	0	7	1	6	1	62	0.2	10.3	98
# consecutive weekends worked	10	6	19	5	19	3	10	4	16	5	13	3	16	26	93	4.3	15.5	72
Range between min. and max. of standby duty assignments	10	1	3	2	4	3	4	2	3	3	4	2	3	13	21	2.2	3.5	38
Min. of standby duty assignments	—	2	1	2	1	1	1	1	1	1	0	2	1	9	5	1.5	0.8	80
Max. of standby duty assignments	—	3	4	4	5	4	5	3	4	4	4	4	4	—	—	3.7	4.3	15
# violations of maximum work stretch	10	1	3	1	3	1	2	0	2	1	6	1	4	5	20	0.8	3.3	75
# compensation days not granted	10	2	0	3	3	0	0	1	1	2	3	3	4	11	11	1.8	1.8	0
# violations of stints in ICU	10	0	0	0	1	1	0	2	2	0	1	1	1	4	5	0.7	0.8	20
# violations of stints at IO	10	10	12	9	13	8	9	7	10	10	11	8	10	52	65	8.7	10.8	20
Hours overtime part time	10	0	12.3	3.2	11.2	0	2	4.8	12.2	4.3	13.7	0	6	12.3	57.4	2.1	9.6	79
# assignments of absences	1	252	252	279	286	286	279	271	276	349	346	348	337	1,785	1,776	297.5	296	−1
Hours overtime full time	0.1	38.7	35.7	19	14	0	11	15	45	73.3	60.3	62.7	28	208.6	194	34.8	32.3	−8
Hours undertime in total	0	68.7	78	90.8	97.8	157.7	202.7	47.2	84.6	64	60.4	116.6	88	545.1	611.6	90.9	101.9	11
Comp. time in seconds	—	84	—	108	—	67	—	89	—	89	—	75	—	—	—	84.8	—	—
Objective value		32,755	2,245,248	416,612	3,417,029	26,646	1,278,650	8,480	429,642	5,608	2,345,968	26,634	1,032,079	475,502	8,073,727	118,875	2,018,432	

violations are achieved by increasing other violations, occur on only three occasions, two in June and one in July (see Table 4).

The degree of improvements from our optimization model over the manually created and eventually implemented schedule is striking. We are able to reduce violations of most rules and regulations considerably. With only a small exception in March, all four of the most important, most heavily penalized constraints (the top four in Table 4) can be satisfied in the optimal solutions, while the schedules developed by the expert violate all of them in three months, and at least half of them in every month. We also decrease the amount of shifts assigned to underqualified physicians by 70%. Shifts that cannot be filled with any physicians are eliminated everywhere except in the acute care department, where three shifts remain vacant—compared to five in the original schedules. These major violations in the original schedules occur despite the expert having decades of experience, despite the employee council checking the schedule against violations, and despite physicians having plenty of time to provide feedback, starting from the time the schedule is posted, two months in advance. But they should come as no surprise, since the scheduling problem is incredibly complex and impossible to solve optimally by hand. This exemplifies how much potential there is for improvement in scheduling quality from mathematical optimization.

Other notable improvements include the almost complete disappearance of violations of the required workforce qualification composition in all shifts, the decrease of all weekend-related regulation violations, and a 75% reduction of occurrences of physicians working on more consecutive days than allowed per the maximum work stretch rule.

Furthermore, the automated process is able to improve the desirability of the schedule from the physicians' perspective: we increase the amount of granted shift requests by over 50% and eliminate all denials of requests to not work specific shifts. We also grant almost all day-off requests. On top of that, the number of standby duties assigned above the allowed maximum per physician is reduced by 62%. Standby duties are much more evenly distributed among the staff because of increased minimum and decreased maximum numbers of standby shift assignments. Thereby, the perceived unfairness among the workforce is reduced.

Additionally, we manage to eliminate most overtime assigned to part-time physicians, a reduction of 7.5 hours per month, while only assigning 2.5 more hours of overtime to full-time physicians. This trade-off was specifically requested by the collaborating scheduler and is incorporated by choosing the corresponding penalty coefficients. The amount of undertime worked in total also decreased by 11 hours per month, leading to a more balanced workload.

Overall, we conclude that our approach not only achieves much more consistent, fair, preferable, and "more lawful" schedules. Also, it only takes the scheduler about 30 minutes of input preparation and output checking, with an additional mere 85 second solver runtime (on average). This is a huge improvement over the seven-plus hours the scheduling expert at the collaborating hospital needs to create manually generated, inferior schedules.

6. Conclusion

Our work examines the personnel scheduling problem of physicians in an anesthesiology department in a hospital in Berlin, Germany. The manual creation of monthly schedules, conducted by a senior physician, for more than 30 physicians with different contracts (full- and part-time) and heterogeneous experience levels as well as a variety of scheduling rules is extremely complex and time-consuming. Consequently, the schedules frequently violate rules and regulations, are perceived as being unfair as uneven amounts of undesired shifts are assigned among the workforce, and cause uneven workloads by creating frequently allocated overtime and undertime hours for the physicians. Therefore, the focus of this work is to develop a mathematical optimization model that implements all the various laws, regulations, and rules. The model is flexible enough to easily adapt to changes in the workforce and the scheduling horizon, while at the same time providing a user-friendly interface that allows for automated input generation from very basic required manual data entries. Additionally, we consider physicians' work preferences and evenly distributed on-call duty shifts. The conceptualized model is implemented in IBM ILOG CPLEX Optimization Studio and the data/results are processed in Excel using macros. Using real-world data, we show that optimal solutions are obtained with very reasonable computational effort. From a practical point of view, the senior physician receives a high-quality schedule proposal; however, human expertise is still needed, particularly for performing manual fine-tuning and short-term adjustments.

The developed model represents a very realistic, detailed representation of the monthly personnel scheduling problem at the partner hospital in Berlin without making any simplifying assumptions. While the model itself is very large, we accomplish great accessibility and usability for practitioners by embedding it in a sophisticated Excel environment. Thus, the ever-changing surrounding parameters, such as employed physicians, the actual scheduling horizon with monthly variations in length, starting and ending days, and national holidays, can be easily adjusted prior to the schedule generation.

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Appendix. Mathematical Model

Tables A.1–A.3 describe the notation of our MIP, which is as follows:

$$\begin{aligned}\text{Minimize}\ \Bigg\{&\alpha_1\cdot\sum_{p\in P}\sum_{s\in S}v_{ps}^{Quali}+\alpha_2\cdot\sum_{s\in S\setminus\{\mathrm{AO,TBA}\}}\sum_{t\in T}v_{st}+\beta_1\cdot\sum_{s\in\{\mathrm{AO,TBA}\}}\sum_{t\in T}v_{st}+\beta_2\cdot\sum_{p\in P}\sum_{s\in S}\sum_{t\in T}(v_{pst}^{req1}+v_{pst}^{req0})+\beta_3\cdot\sum_{p\in P}\sum_{t\in T}v_{pt}^{Off}\\&+\beta_4\cdot\sum_{p\in P}v_p^{BD}+\gamma_1\cdot\sum_{p\in P}(v_p^{WEyn}+v_p^{WE\min})+\gamma_2\cdot\sum_{s\in S}\sum_{t\in T}(v_{st}^{SP}+v_{st}^{CP})+\delta_1\cdot\sum_{p\in P}\sum_{w\in W}v_{pw}^{WErow}+\delta_2\cdot\sum_{p\in P:\,h_p^{contract}<40}ot_p\\&+\delta_3\cdot(bd^{MAX}-bd^{MIN})+\delta_4\cdot\sum_{p\in P}v_p^{Stretch}+\delta_5\cdot\sum_{p\in P}\sum_{w\in W}v_{pw}^{Ady}+\delta_6\cdot\sum_{p\in P}\sum_{t\in T}v_{pt}^{\mathrm{IOPL}}+\delta_7\cdot\left(\sum_{p\in P}\sum_{t\in T}v_{pt}^{IO}+\sum_{p\in P}\sum_{w\in W}v_{pw}^{IO\,Max}\right)\\&+\omega_1\cdot\sum_{p\in P}\sum_{s\in S_2}\sum_{t\in T}x_{pst}+\omega_2\cdot\sum_{p\in P:\,h_p^{contract}\geq 40}ot_p\Bigg\}\end{aligned}\tag{A.1}$$

Table A.1. Supersets and Subsets

Superset	Subset
P	$:= \{p\colon p$ is a physician in the Department of Anesthesiology$\}$
	$P_{AP} := \{p\in P\colon p$ has qualification assistant$\}$
	$P_{SP} := \{p\in P\colon p$ has qualification senior$\}$
	$P_{CP} := \{p\in P\colon p$ has qualification chief$\}$
	$P_{HP} := \{p\in P\colon p$ has qualification head$\}$
S	$:= \{s\colon s$ is a roster icon in the Department of Anesthesiology$\}$ = {OP, IO, AO, PM, ZD, Z4, Z4F, S1, BD1, BD3, RB3, RB4, IOP, A73, V93, A93, A91, IOPL, TBA, XBD, FW, UR, KR, A, UR0, HO, Off}
	$S_0 := \{$Off$\}$
	$S_1 := \{s\in S\colon s$ is a shift$\}$ = {OP, IO, AO, PM, ZD, Z4, Z4F, S1, BD1, BD3, RB3, RB4, IOP, A73, V93, A93, A91, IOPL, TBA}
	$S_2 := \{s\in S\colon s$ is an absence, $s\notin S_0\}$ = {XBD, FW, UR, KR, A, UR0, HO}
	$S_{FRWE} := \{s\in S_1\colon s$ takes place on Friday and ends after 8 P.M.$\}$ = {Z4F, S1, A73, V93, A91}
	$S_{BD} := \{s\in S_1\colon s$ contains standby time$\}$ = {S1, BD1, BD3, A73, V93, A91}
	$S_{FZA} := \{s\in S_1\colon s$ triggers a following day off$\}$ = {S1, BD1, BD3, A73, V93, A91}
	$S_{RD} := \{s\in S_1\colon s$ takes place on holidays or weekends and will be compensated$\}$ = {A73, IOP, IOPL}
J	:= {Mon, Tue, Wed, Thu, Fri, Sat, Sun}
W	$:= \{w\colon w\in Z, 0\leq w\leq l^{End}\} = \{0,\dots,l^{End}\}$
	$W^{Full} := \{w\in W\colon 0< w\leq l^{WE}\} = \{1,\dots,l^{WE}\}$
T	$:= \{-6,-5,\dots,0,1,2,\dots,m,m+1,m+2,\dots,m+m^{FM}\}$
	$T^{VM} := \{t\in T\colon t\leq 0\} = \{-6,-5,\dots,0\}$
	$T^{AM} := \{t\in T\colon 0<t\leq m\} = \{1,2,\dots,m\}$
	$T^{FM} := \{t\in T\colon t>m\} = \{m+1,m+2,\dots,m+m^{FM}\}$
	$T_j^{Wkdy} := \{t\in T\colon t$ is weekday $j\in J\}$
	$T^{HD} := \{t\in T\colon t$ is a legal holiday, $t\in T_j^{Wkdy}\mid j\in\{$Mon,…,Fri$\}\}$
	$T^{RD} := \{t\in T\colon t\in T^{HD}\cup T_j^{Wkdy}\mid j\in\{Sat, Sun\}\}$
	$T^{WD} := \{t\in T\colon t\in T\setminus T^{RD}\}$
	$T_w^{Week} := \{t\in T\colon t$ is a day in week $w\in W\}$
	$T_0^{Week} := \{t\in T\colon t\leq\tau\mid\tau\in T_{Sun}^{Wkdy}\cap T^{VM}\}$
	$T_w^{Week} := \{t\in T\colon \tau\leq t\leq\tau+6\mid\tau\in T_{Mon}^{Wkdy}$ and $-12+7\cdot w\leq\tau\leq -6+7\cdot w\mid w\in W$ and $w>0\}$
I	$:= \{(p,t,s)\in P\times T\times S\colon (p,t,s)=$ physician p must have icon s on day t in the schedule$\}$
R	$:= \{(p,t,s)\in P\times T\times S\colon (p,t,s)=$ request s on day t of physician $p\}$

Notes. The subset T^{VM} includes the last seven days of the previous month, T^{AM} m days of the actual month, and T^{FM} the first m^{FM} days of the following month. The subsets T^{VM} and T^{FM} are important to ensure a consistent planning of the monthly transition to avoid unnecessary violations and infeasibilities. Basically, their specific sizes are dependent on several aspects (e.g., stints, weekend shifts, and work stretch). The term T^{VM} is already determined by the previous schedule, and T^{FM} is kind of an artificial set that is not taken into account in the schedule of the real following month. In total, there are 28 different sets T possible because each month can start with one of seven weekdays and can have 28, 29, 30, or 31 days.

Table A.2. Parameters

Parameter	
$\alpha_i, \beta_{i'}, \gamma_i, \delta_{i''}, \omega_i$	Weights of violations in the objective function with $i = 1, 2$, $i' = 1, \ldots, 4$, and $i'' = 1, \ldots, 7$
c_{st}	Number of shifts $s \in S$ to staff on day $t \in T$
$c_{st}^{SP}(c_{st}^{CP})$	Minimum number of physicians with qualification senior (chief) or higher with icon $s \in S$ on day $t \in T$
f_p^0	Number shifts worked on holidays in week 0 by physician $p \in P$
h_p^{26Wacc}	Sum of the working time of physician $p \in P$ of the last $L_p^{26W} - \lvert W^{Full} \rvert$ previous weeks starting in week 0 (26-week account)
h_s^{stdrd}	Standard time of icon $s \in S$ (time account)
h_s^{legal}	Legal working time (including standby time) of icon $s \in S$
$h_p^{contract}$	Contractual working hours per weeks of physician $p \in P$ (standard time)
$h_p^{timeacc}$	Time account (standard time) of physician $p \in P$ of the previous month
k_p^0	Equal to 1 if physician $p \in P$ works on the last full weekend of the previous month (in week 0), 0 otherwise
l^{End}	Last week contained in set T
l^{WE}	Week with last full weekend in the actual month
m	Last day in the actual month
m^{FM}	Number of days of the following month with $8 \geq m^{FM} \geq 2$ and the "last" day of T is a Tuesday ($\{m + m^{FM}\} \in T_{Tue}^{Wkdy}$)
n_p^{BD}	Equal to 1 if physician $p \in P$ works standby duties, 0 otherwise
n_p^{WE}	Equal to 1 if physician $p \in P$ works on weekends, 0 otherwise
q_{ps}	Equal to 1 if physician $p \in P$ is able to perform icon $s \in S$, 0 otherwise
r_{pst}	Equal to 1 (0) if physician $p \in P$ requests (not) to be assigned icon $s \in S$ on day $t \in T$
A_{pw}^{MIN}	Minimum number of compensation days for physician $p \in P$ in week $w \in W$
BD_p^{MAX}	Maximum number of standby duties, which are performed by physician $p \in P$ in the actual month
BD^{Days}	Minimum number of days between two standby duties ($BD \geq 1$)
F^{MIN}	Minimum number of physicians with senior or higher qualification that must be present at night
H_p^{26Wacc}	Multiple of contractual hours per week of physician $p \in P$ (upper bound of 26-week account)
H_p^{MAX}	Maximum legal working hours per week of physician $p \in P$
$H^{timeacc}$	Multiple of contractual hours per week (upper bound time account)
K^{MIN}	Minimum number of weekends off for each physician in the actual month
L_p^{26W}	Number of weeks for calculating the average working time $p \in P$ ($\lvert W^{Full} \rvert \leq L_p^{26W} \leq 26$)
M^{MAX}	Maximum number of days that can be worked in a row by a physician ($M^{MAX} \leq \lvert T^{VM} \rvert$)
OT_p^{MAX}	Maximum amount of overtime (standard time) in the actual month by physician $p \in P$

Notes. The parameters were determined together with the scheduler in the hospital in Berlin, as was the hierarchy of the objectives. The specific weights of the objective function resulted from a lengthy trial-and-error process that involved multiple feedback loops.

subject to
$$\sum_{s \in S} x_{pst} = 1 \qquad \forall\, p \in P,\ t \in T; \quad \text{(A.2)}$$
$$x_{pst} \leq q_{ps} + v_{ps}^{Quali} \qquad \forall\, p \in P,\ s \in S,\ t \in T \backslash T^{VM}; \quad \text{(A.3)}$$
$$\sum_{p \in P} x_{pst} = c_{st} - v_{st} \qquad \forall\, s \in S_1,\ t \in T \backslash T^{VM}; \quad \text{(A.4)}$$
$$\sum_{p \in P \backslash P_{AP}} x_{pst} \geq c_{st}^{SP} - v_{st}^{SP} \qquad \forall\, s \in S,\ t \in T \backslash T^{VM}; \quad \text{(A.5)}$$
$$\sum_{p \in P_{CP} \cup P_{HP}} x_{pst} \geq c_{st}^{CP} - v_{st}^{CP} \qquad \forall\, s \in S,\ t \in T \backslash T^{VM}; \quad \text{(A.6)}$$
$$\sum_{p \in P \backslash P_{AP}} \sum_{s \in \{\text{A73, S1}\}} x_{pst} \geq F^{MIN} \qquad \forall\, t \in T^{WD} \backslash T^{VM}; \quad \text{(A.7)}$$
$$\sum_{p \in P \backslash P_{AP}} \left(\frac{1}{2} \cdot \sum_{s \in S_{RD}} x_{pst} + \sum_{s' \in \{\text{BD1, BD3}\}} x_{ps't} \right) \geq F^{MIN} \qquad \forall\, t \in T^{RD} \backslash T^{VM}; \quad \text{(A.8)}$$
$$x_{pst} = 1 \qquad \forall\, (p, t, s) \in I; \quad \text{(A.9)}$$
$$x_{pst} = r_{pst} - v_{pst}^{req1} + v_{pst}^{req0} \qquad \forall\, (p, t, s) \in R; \quad \text{(A.10)}$$
$$\sum_{s \in S_1} \sum_{\tau = t - M^{MAX}}^{t} x_{ps\tau} \leq M^{MAX} + v_p^{Stretch} \qquad \forall\, p \in P,\ t \in T \backslash T^{VM}; \quad \text{(A.11)}$$
$$\sum_{s \in S_{BD}} \sum_{t \in T^{AM}} x_{pst} \geq bd^{MIN} \cdot n_p^{BD} \qquad \forall\, p \in P; \quad \text{(A.12)}$$
$$\sum_{s \in S_{BD}} \sum_{t \in T^{AM}} x_{pst} \leq bd^{MAX} \qquad \forall\, p \in P; \quad \text{(A.13)}$$

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Table A.3. Decision Variables

Decision variable	
x_{pst}	Equal to 1 if physician $p \in P$ is assigned to shift $s \in S$ on day $t \in T$, 0 otherwise
bd^{MIN}	Minimum of assigned standby duties among all physicians who work standby duties at all
bd^{MAX}	Maximum of assigned standby duties among all physicians
z_{pw}	Equal to 1 if physician $p \in P$ works at weekend in week $w \in W$, 0 otherwise
ot_p	Overtime (standard time) of physician $p \in P$ in the actual month
ut_p	Undertime (standard time) of physician $p \in P$ in the actual month
v_{ps}^{Quali}	Equal to 1 if physician $p \in P$ is assigned to icon $s \in S$, although he or she is not allowed to perform it ($q_{ps} = 0$), 0 otherwise
v_{st}	Number of understaffed physicians in icon $s \in S$ on day $t \in T$
$v_{st}^{SP}(v_{st}^{CP})$	Number of understaffed physicians with qualification senior (chief) or higher in icon $s \in S$ on day $t \in T$
$v_{pst}^{req1}(v_{pst}^{req0})$	Equal to 1 if the request of physician $p \in P$ to (not) be assigned icon $s \in S$ on day $t \in T$ in the schedule is denied, 0 otherwise
$v_p^{Stretch}$	Equal to 1 if physician $p \in P$ exceeds the maximum number M^{MAX} of workdays in a row, 0 otherwise
v_p^{BD}	Violation of the maximum number BD_p^{MAX} of assigned standby duties to physician $p \in P$
v_{pw}^{Ady}	Violation of the minimum number A_{pw}^{MIN} of compensation days of physician $p \in P$ in week $w \in W$
v_{pt}^{Off}	Equal to 1 if day $t \in T$ is not off for physician $p \in P$ (shifts in the ICU), 0 otherwise
v_p^{WEyn}	Physician $p \in P$ has to work on weekends, although he or she is not allowed to ($n_p^{WE} = 0$)
v_p^{WEmin}	Violation of the minimum number K^{MIN} of weekends off for physician $p \in P$
v_{pw}^{WErow}	Equal to 1 if physician $p \in P$ works consecutive weekends $w \in W \backslash \{0\}$ and $w-1$, 0 otherwise
v_{pt}^{IOPL}	Equal to 1 if a stint in the ICU is violated in the schedule of physician $p \in P$ on day $t \in T$, 0 otherwise
v_{pt}^{IO}	Equal to 1 if the stint of icon IO is violated in the schedule of physician $p \in P$ on day $t \in T$, 0 otherwise
v_{pw}^{IOMax}	The upper bound of four IO shifts in a row is exceeded by v_{pw}^{IOMax} in the schedule of physician $p \in P$ within the weeks $w \in W \backslash \{0\}$ and $w-1$

Notes. The decision variables x_{pst} represent the primary decision variables that assign each physician shifts or absences in the planning horizon. Furthermore, variables for distributing standby duties, weekend assignments, and overtime/undertime are defined. Moreover, the "v" variables denote violations of constraints or deviations of parameter values.

$$\sum_{s \in S_{BD}} \sum_{t \in T^{AM}} x_{pst} \leq BD_p^{MAX} \cdot n_p^{BD} + v_p^{BD} \qquad \forall p \in P; \quad (A.14)$$

$$1 - \sum_{s \in S_{BD}} x_{pst} \geq \sum_{s' \in S_{BD}} x_{ps'(t-\tau)} \qquad \forall p \in P,\ t \in T \backslash T^{VM},\ \tau \in \{1, \ldots, BD^{Days}\}; \quad (A.15)$$

$$\sum_{t \in T^{WD} \cap T_w^{Week}} x_{pst} \geq A_{pw}^{MIN} - v_{pw}^{Ady} \qquad \forall p \in P,\ s \in \{A\},\ w \in W \backslash \{0\}; \quad (A.16)$$

$$\sum_{t \in T^{WD} \cap T^{AM}} x_{pst} \geq \left\lfloor \left(1 - \frac{h_p^{contract}}{40}\right) \cdot |T^{WD} \cap T^{AM}| \right\rfloor \qquad \forall p \in P,\ s \in \{A\}; \quad (A.17)$$

$$\sum_{s \in S_{FZA}} x_{ps(t-1)} \leq x_{ps't} \qquad \forall p \in P,\ s' \in \{\text{Off}\},\ t \in T^{RD} \backslash \{-6\}; \quad (A.18)$$

$$\sum_{s \in S_{FZA}} x_{ps(t-1)} = x_{ps't} \qquad \forall p \in P,\ s' \in \{\text{XBD}\},\ t \in T^{WD} \backslash \{-6\}; \quad (A.19)$$

$$x_{pst} = 0 \qquad \forall p \in P,\ s \in \{\text{XBD}\},\ t \in T^{RD}; \quad (A.20)$$

$$x_{pst} \geq x_{ps'(t-\tau)} - v_{pt}^{Off} \qquad \forall p \in P,\ s \in \{\text{Off}\},\ s' \in \{\text{IOP}\},\ t \in T_{Fri}^{Wkdy} \backslash T^{VM},\ \tau \in \{5, 6\}; \quad (A.21)$$

$$x_{pst} \geq x_{ps'(t-5)} + x_{ps'(t-4)} - 1 - v_{pt}^{Off} \qquad \forall p \in P,\ s \in \{\text{Off}\},\ s' \in \{\text{IOP}\},\ t \in T_{Thu}^{\text{Wkdy}} \backslash T^{VM}; \quad (A.22)$$

$$x_{pst} \geq x_{ps'(t-2)} - v_{pt}^{Off} \qquad \forall p \in P,\ s \in \{\text{Off}\},\ s' \in \{\text{A73}\},\ t \in (T_{Mon}^{Wkdy} \cup T_{Tue}^{Wkdy}) \backslash T^{VM}; \quad (A.23)$$

$$z_{pw} \geq \sum_{s \in S_1} x_{pst} \qquad \forall p \in P,\ w \in W \backslash \{0\},\ t \in T_w^{Week} \cap (T_{Sat}^{Wkdy} \cup T_{Sun}^{Wkdy}); \quad (A.24)$$

$$z_{pw} \geq \sum_{s \in S_{FRWE}} x_{pst} \qquad \forall p \in P,\ w \in W \backslash \{0\},\ t \in T_w^{Week} \cap T_{Fri}^{Wkdy}; \quad (A.25)$$

$$z_{pw} \leq \sum_{s \in S_1} \sum_{t \in T_w^{Week} \cap (T_{Sat}^{Wkdy} \cup T_{Sun}^{Wkdy})} x_{pst} + \sum_{s' \in S_{FRWE}} \sum_{t \in T_w^{Week} \cap T_{Fri}^{Wkdy}} x_{ps't} \qquad \forall p \in P,\ w \in W \backslash \{0\}; \quad (A.26)$$

$$z_{pw} = k_p^0 \qquad \forall p \in P,\ w \in \{0\}; \quad (A.27)$$

$$\sum_{w \in W \backslash \{0\}} z_{pw} \leq l^{End} \cdot n_p^{WE} + v_p^{WEyn} \qquad \forall p \in P; \quad (A.28)$$

$$\sum_{w \in W^{Full}} z_{pw} \leq |W^{Full}| - K^{MIN} + v_p^{WEmin} \qquad \forall p \in P; \quad (A.29)$$

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$$z_{pw} + z_{p(w-1)} \leq 1 + v_{pw}^{WErow} \qquad \forall p \in P,\ w \in W \backslash \{0\}; \qquad (A.30)$$

$$\sum_{s \in S} \sum_{t \in T^{AM}} h_s^{stdrd} \cdot x_{pst} = \frac{h_p^{contract}}{5} \cdot |T^{WD} \cap T^{AM}| - ut_p + ot_p \qquad \forall p \in P; \qquad (A.31)$$

$$h_p^{timeacc} - ut_p + ot_p \leq H^{timeacc} \cdot h_p^{contract} \qquad \forall p \in P; \qquad (A.32)$$

$$ot_p \leq OT_p^{MAX} \qquad \forall p \in P; \qquad (A.33)$$

$$\sum_{s \in S} \sum_{t \in T_w^{Week}} h_s^{legal} \cdot x_{pst} \leq H_p^{MAX} \qquad \forall p \in P,\ w \in W \backslash \{0\}; \qquad (A.34)$$

$$\frac{1}{L_p^{26W}} \cdot \left(h_p^{26Wacc} + \sum_{w \in W^{Full}} \sum_{s \in S} \sum_{t \in T_w^{Week}} h_s^{legal} \cdot x_{pst} \right) \leq H_p^{26Wacc} \cdot h_p^{contract} \qquad \forall p \in P; \qquad (A.35)$$

$$x_{pst} \leq x_{ps'(t+\tau)} \qquad \forall p \in P,\ s \in \{\text{IOPL}\},\ s' \in \{\text{IOP}\},\ t \in T,\ \tau \in \{1, \dots, 5\}; \qquad (A.36)$$

$$x_{pst} \leq x_{ps'(t+6)} + v_{p(t+6)}^{\text{IOPL}} \qquad \forall p \in P,\ s \in \{\text{IOPL}\},\ s' \in \{\text{A73}\},\ t \in T; \qquad (A.37)$$

$$x_{pst} \leq x_{ps(t+1)} + v_{pt}^{IO} \qquad \forall p \in P,\ s \in \{\text{IO}\},\ t \in T_{Mon}^{Wkdy} \cup T_{Tue}^{Wkdy} \cup T_{Wed}^{Wkdy} \cup T_{Thu}^{Wkdy}; \qquad (A.38)$$

$$x_{pst} \leq x_{ps(t+3)} + v_{pt}^{IO} \qquad \forall p \in P,\ s \in \{\text{IO}\},\ t \in T_{Fri}^{Wkdy}; \qquad (A.39)$$

$$\sum_{t \in T_w^{Week}} x_{pst} + \sum_{\tau \in T_{w-1}^{Week}} x_{ps\tau} \leq 4 + v_{pw}^{IOMax} \qquad \forall p \in P,\ s \in \{\text{IO}\},\ w \in W \backslash \{0\}; \qquad (A.40)$$

$$x_{pst}, z_{pw}, v_{ps}^{Quali}, v_{pst}^{req1}, v_{pst}^{req0}, v_p^{Stretch}, v_{pt}^{Off}, v_{pw}^{WErow}, v_{pt}^{\text{IOPL}}, v_{pt}^{IO} \in \{0, 1\} \qquad \forall p \in P,\ s \in S,\ t \in T,\ w \in W; \qquad (A.41)$$

$$ot_p, ut_p \geq 0 \qquad \forall p \in P; \qquad (A.42)$$

$$bd^{MIN}, bd^{MAX}, v_{st}, v_{st}^{SP}, v_{st}^{CP}, v_p^{BD}, v_{pw}^{Ady}, v_p^{WEyn}, v_p^{WEmin}, v_{pw}^{IOMax} \geq 0 \text{ and integer} \qquad \forall p \in P,\ s \in S,\ t \in T,\ w \in W; \qquad (A.43)$$

$$\sum_{s' \in S_{RD}} \sum_{t' \in T_{w-1}^{Week} \cap T^{HD}} x_{ps't'} \leq \sum_{t \in T_w^{Week} \cap T^{WD}} x_{pst} \qquad \forall p \in P,\ s \in \{\text{HO}\},\ w \in \{2, \dots, l^{End} - 1\}; \qquad (A.44)$$

$$f_p^0 \leq \sum_{t \in T_w^{Week} \cap T^{WD}} x_{pst} \qquad \forall p \in P,\ s \in \{\text{HO}\},\ w \in \{1\}. \qquad (A.45)$$

References

Bard JF, Purnomo HW (2007) Cyclic preference scheduling of nurses using a Lagrangian-based heuristic. *J. Scheduling* 10(1):5–23.
Bard JF, Shu Z, Leykum L (2013) Monthly clinic assignments for internal medicine housestaff. *IIE Trans. Healthcare Systems Engrg.* 3(4): 207–239.
Beaulieu H, Ferland JA, Gendron B, Michelon P (2000) A mathematical programming approach for scheduling physicians in the emergency room. *Health Care Management Sci.* 3(3):193–200.
Benazzouz T, Echchatbi A, Bellabdaoui A (2015) A literature review on the nurses' planning problems. *Internat. J. Math. Comput. Sci.* 1(5):268–274.
Blum K, Löffert S, Offermanns M, Steffen P (2013) Krankenhaus Barometer—Umfrage 2013. Technical report, Deutsches Krankenhaus Institute, Düsseldorf, Germany.
Bruni R, Detti P (2014) A flexible discrete optimization approach to the physician scheduling problem. *Oper. Res. Health Care* 3(4):191–199.
Brunner JO, Edenharter GM (2011) Long term staff scheduling of physicians with different experience levels in hospitals using column generation. *Health Care Management Sci.* 14(2):189–202.
Brunner JO, Bard JF, Kolisch R (2009) Flexible shift scheduling of physicians. *Health Care Management Sci.* 12(3):285–305.
Brunner JO, Bard JF, Kolisch R (2011) Midterm scheduling of physicians with flexible shifts using branch and price. *IIE Trans.* 43(2):84–109.
Burke EK, De Causmaecker P, Berghe GV, Van Landeghem H (2004) The state of the art of nurse rostering. *J. Scheduling* 7(6):441–499.
Carter MW, Lapierre SD (2001) Scheduling emergency room physicians. *Health Care Management Sci.* 4(4):347–360.
Cervesato E, Righini G (2014) Optimization of work shifts in a cardiological hospital ward: A case study. Working paper, University of Milan, Crema, Italy.
Cheang B, Li H, Lim A, Rodrigues B (2003) Nurse rostering problems—A bibliographic survey. *Eur. J. Oper. Res.* 151(3):447–460.
Erhard M, Schoenfelder J, Fügener A, Brunner JO (2018) State of the art in physician scheduling. *Eur. J. Oper. Res.* 265(1):1–18.
Ferrand Y, Magazine M, Rao US, Glass TF (2011) Building cyclic schedules for emergency department physicians. *Interfaces* 41(6):521–533.
Fügener A, Brunner JO, Podtschaske A (2015) Duty and workstation rostering considering preferences and fairness: A case study at a department of anaesthesiology. *Internat. J. Production Res.* 53(24):7465–7487.
Gendreau M, Ferland J, Gendron B, Hail N, Jaumard B, Lapierre S, Pesant G, Soriano P (2007) Physician scheduling in emergency rooms. Burke EK, Rudová H, eds. *Practice and Theory of Automated Timetabling* VI (Springer, Berlin), 53–66.
Knauth P (1993) The design of shift systems. *Ergonomics* 36(1–3):15–28.
Ovchinnikov A, Milner J (2008) Spreadsheet model helps to assign medical residents at the University of Vermont's College of Medicine. *Interfaces* 38(4):311–323.
Rais A, Viana A (2011) Operations research in healthcare: A survey. *Internat. Trans. Oper. Res.* 18(1):1–31.
Santos M, Eriksson H (2014) Insights into physician scheduling: A case study of public hospital departments in Sweden. *Internat. J. Health Care Quality Assurance* 27(2):76–90.
Schall MW, Duffy T, Krishnamurthy A, Levesque O, Mehta P, Murray M, Parlier R, Petzel R, Sanderson J (2004) Improving patient access to the Veterans Health Administration's primary care and specialty clinics. *Joint Commission J. Quality Patient Safety* 30(8):415–423.
Stolletz R, Brunner JO (2012) Fair optimization of fortnightly physician schedules with flexible shifts. *Eur. J. Oper. Res.* 219(3):622–629.
Van den Bergh J, Beliën J, De Bruecker P, Demeulemeester E, De Boeck L (2013) Personnel scheduling: A literature review. *Eur. J. Oper. Res.* 226(3):367–385.

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