Sort Algorithms

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Git URL:

https://github.com/studentRiphah/Sorting-Algorithms

Sorting

- ***Sorting*** is a process that organizes a collection of data into either ascending or descending order.
- An ***internal sort*** requires that the collection of data fit entirely in the computer's main memory.
- We can use an ***external sort*** when the collection of data cannot fit in the computer's main memory all at once but must reside in secondary storage such as on a disk.
- We will analyze only internal sorting algorithms.

Sorting

- Any significant amount of computer output is generally arranged in some sorted order so that it can be interpreted.
- Sorting also has indirect uses. An initial sort of the data can significantly enhance the performance of an algorithm.
- Majority of programming projects use a sort somewhere, and in many cases, the sorting cost determines the running time.
- A comparison-based sorting algorithm makes ordering decisions only on the basis of comparisons.

Sorting Algorithms

- There are many comparison based sorting algorithms, such as:
  - Bubble Sort
  - Selection Sort
  - Merge Sort
  - Quick Sort

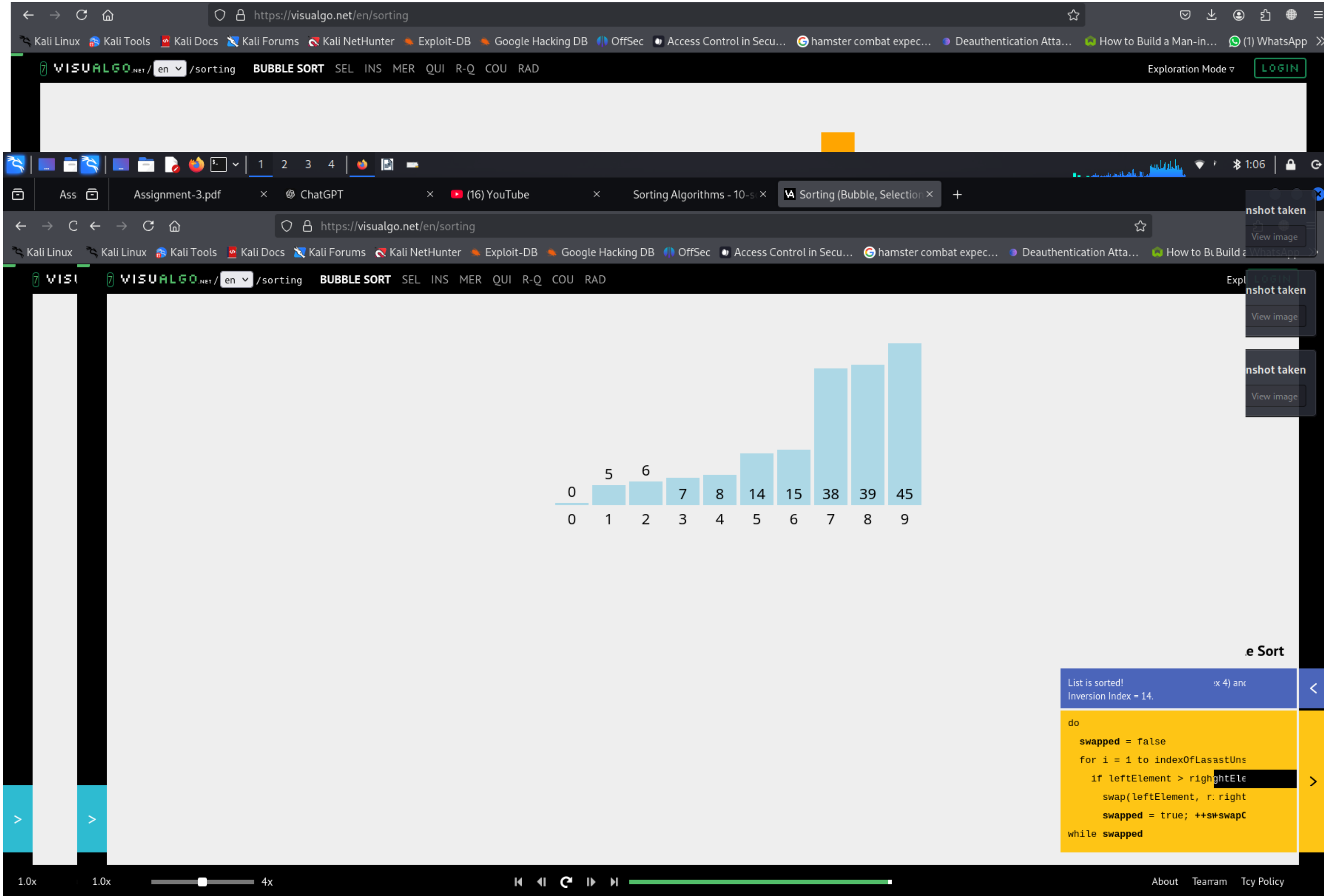
Bubble Sort

- The list is divided into two sublists: sorted and unsorted.
- Starting from the bottom of the list, the smallest element is bubbled up from the unsorted list and moved to the sorted sublist.
- After that, the wall moves one element ahead, increasing the number of sorted elements and decreasing the number of unsorted ones.
- Each time an element moves from the unsorted part to the sorted part one sort pass is completed.
- Given a list of n elements, bubble sort requires up to n-1 passes to sort the data.

Bubble Sort

23	78	45	8	32	56	Original List
8	23	78	45	32	56	After pass 1
8	23	32	78	45	56	After pass 2
8	23	32	45	78	56	After pass 3
8	23	32	45	56	78	After pass 4

Bubble Sort


https://visualgo.net/en/sorting
BUBBLE SORT SEL INS MER QUI R-Q COU RAD
0 5 6 7 8 14 15 38 39 45
0 1 2 3 4 5 6 7 8 9
List is sorted!
Inversion Index = 14.
do
swapped = false
for i = 1 to indexOfLasastUns
if leftElement > righghtEle
swap(leftElement, r. right
swapped = true; ++s+swapC
while swapped

Bubble Sort

```
void swap( int &lhs, int &rhs );

void bubleSort(int a[], int n) {
   bool sorted = false;
   int last = n-1;
   for (int i = 0; (i < last) && !sorted; i++){
      sorted = true;
      for (int j=last; j > i; j--)
```

Bubble Sort

```
        if (a[j-1] > a[j]{
            swap(a[j],a[j-1]);
            sorted = false;
        }
    }
}

    void swap( int &lhs, int &rhs
   ){int tmp = lhs;
   lhs = rhs;
   rhs = tmp;
}
```

Tick Function

```
int calculateTicks(int arr[], int n)
{ int ticks = 0;  for (int i = 0; i < n - 1; i++)
{ bool swapped = false;
for (int j = 0; j < n - i - 1; j++)
{ ticks++;
 if (arr[j] > arr[j + 1])
{ swap(arr[j], arr[j + 1]); ticks++;
```

Bubble Sort

```
 swapped = true; }
}
if (!swapped) break;
 }
return ticks; }
int main()
{ int originalArray[] = {23, 78, 45, 8, 32, 56};
int n = sizeof(originalArray) / sizeof(originalArray[0]);
int arr[n];
```

Tick Function

```
// Worst Case:
  copy(begin(originalArray), end(originalArray), arr);
 sort(arr, arr + n, greater<int>());
 cout << "Ticks (Worst Case): " << calculateTicks(arr, n) << endl;
// Best Case:
 copy(begin(originalArray), end(originalArray), arr);
```

Bubble Sort

```
 sort(arr, arr + n);

/cout << "Ticks (Best Case): " << calculateTicks(arr, n) << endl;
// Average Case:
copy(begin(originalArray), end(originalArray), arr);
 cout << "Ticks (Average Case): " << calculateTicks(arr, n) << endl;
```

Bubble Sort – Analysis

- In general, we compare keys and move items (or exchange items) in a sorting algorithm (which uses key comparisons).

  So, to analyze a sorting algorithm we should count the number of key comparisons and the number of moves.

  - Ignoring other operations does not affect our final result.

Bubble Sort – Analysis

Time Calculation Function for all cases

```
import time
 def bubble_sort(arr):
n = len(arr)
for i in range(n):
    swapped = False
    for j in range(0, n - i - 1):
        if arr[j] > arr[j + 1]:
            arr[j], arr[j + 1] = arr[j + 1], arr[j]
            swapped = True
    if not swapped:
        break
```

Bubble Sort – Analysis

```
def measure_time(arr):
start_time = time.time()
bubble_sort(arr)
return time.time() - start_time
```

Time Calculation Function for all cases

```
best_case = [1, 2, 3, 4, 5]
average_case = [3, 1, 4, 5, 2]
worst_case = [5, 4, 3, 2, 1]

print(f"Best-case time: {measure_time(best_case[:]):.6f} seconds")
print(f"Average-case time: {measure_time(average_case[:]):.6f} seconds")
print(f"Worst-case time: {measure_time(worst_case[:]):.6f} seconds")
```

Bubble Sort – Analysis

- ***Best-case:*** **O(n)**
  - Array is already sorted in ascending order.
  - Outer loop executes 1 time and inner loop n-1 times.
  - The number of moves: 0 O(1)
  - The number of key comparisons: (n-1) O(n)
- ***Worst-case:*** **$O(n^2)$**
  - Array is in reverse order:
  - Outer loop is executed n-1 times and inner loop executes (n-1-i) times,
  - The number of moves: 3*((n-1)+(n-2)+...+3+2+1) = 3 * n*(n-1)/2 $O(n^2)$
  - The number of key comparisons: ((n-1)+(n-2)+...+3+2+1) = n*(n-1)/2 $O(n^2)$
- **Average-case**: **$O(n^2)$**
  - We have to look at all possible initial data organizations.
- **So, Bubble Sort is $O(n^2)$**

Comparison of *N*, *logN* and *N*^{2}

N	O(LogN)	O(N^2)
16	4	256
64	6	4K
256	8	64K
1,024	10	1M
16,384	14	256M
131,072	17	16G
262,144	18	6.87E+10
524,288	19	2.74E+11
1,048,576	20	1.09E+12
1,073,741,824	30	1.15E+18

Selection Sort

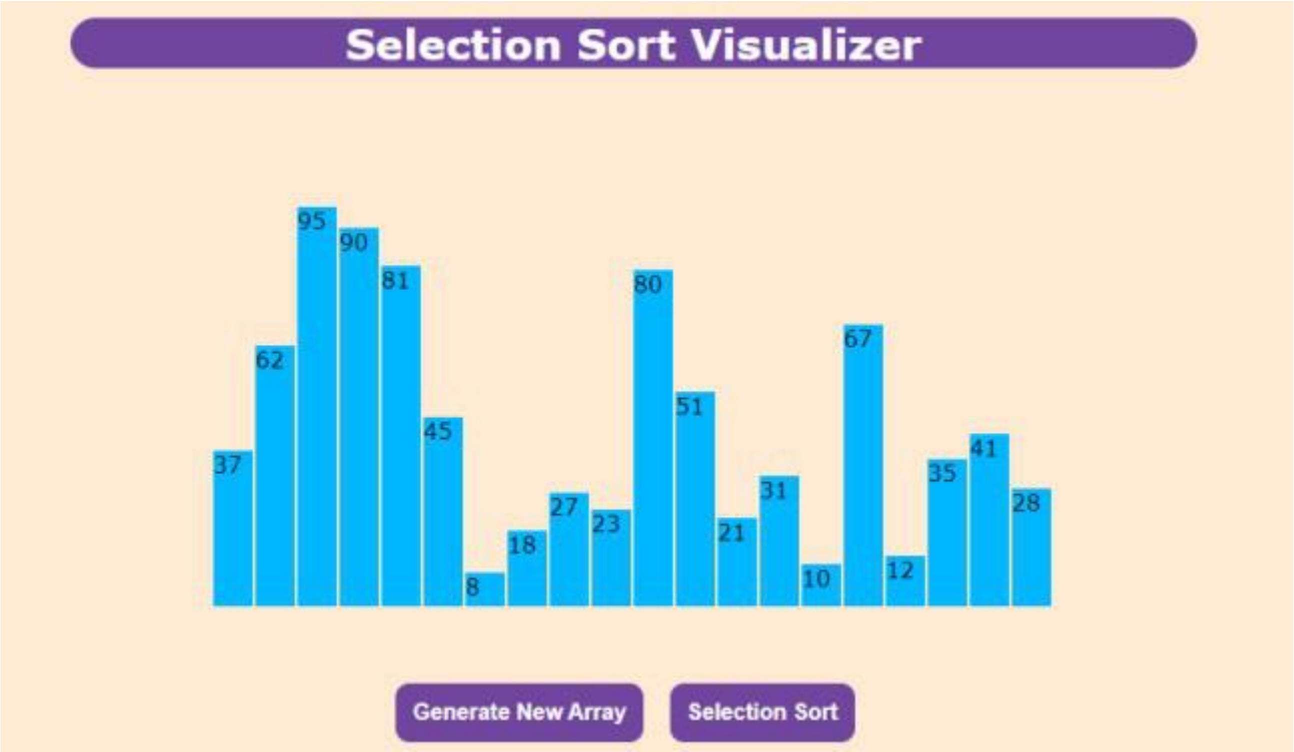
- The list is divided into two sublists, *sorted* and *unsorted*, which are divided by an imaginary wall.
- We find the smallest element from the unsorted sublist and swap it with the element at the beginning of the unsorted data.
- After each selection and swapping, the imaginary wall between the two sublists move one element ahead, increasing the number of sorted elements and decreasing the number of unsorted ones.
- Each time we move one element from the unsorted sublist to the sorted sublist, we say that we have completed a sort pass.
- A list of *n* elements requires *n-1* passes to completely rearrange the data.

Selection Sort

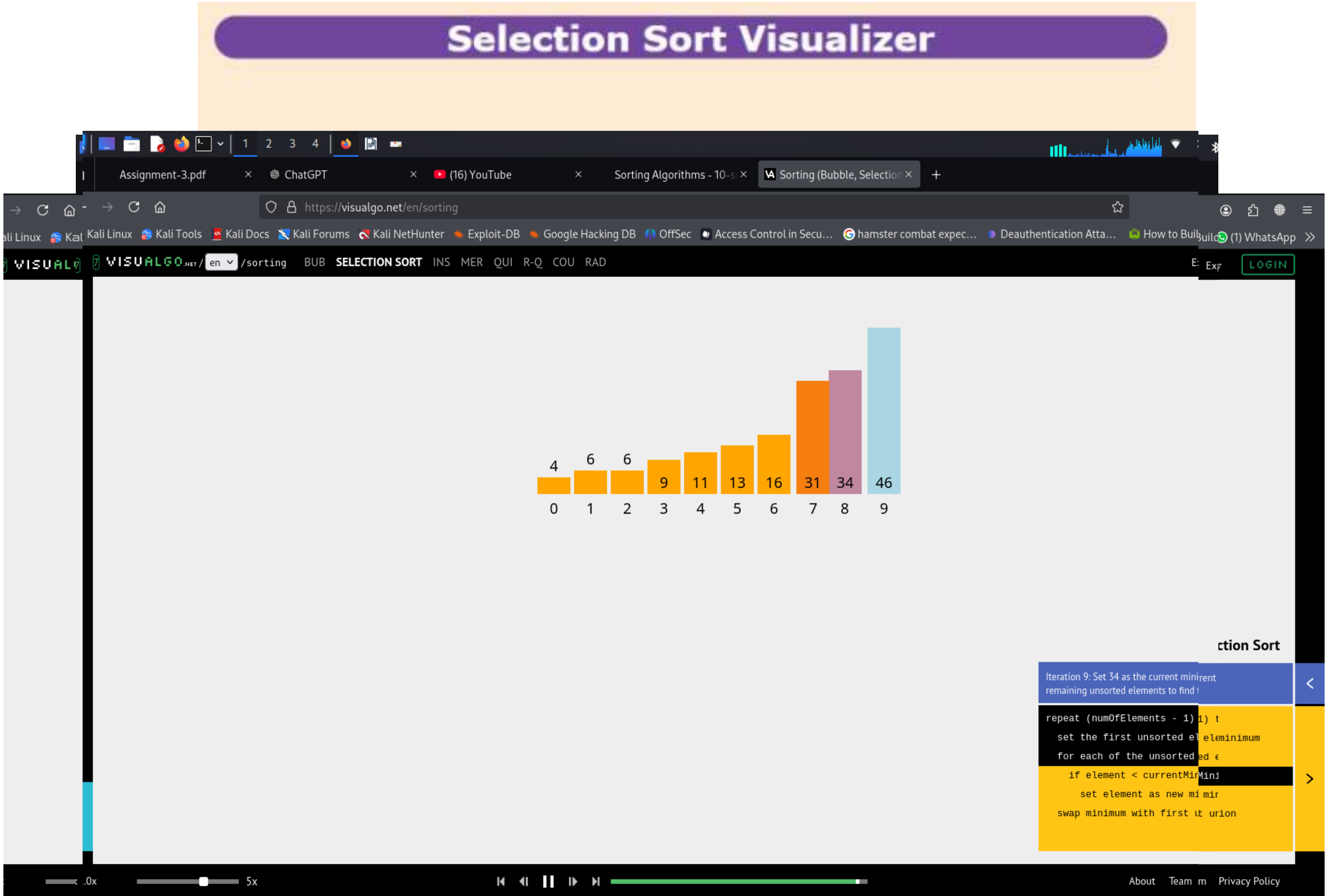
Sorted **Unsorted**

23	78	45	8	32	56	Original List
8	78	45	23	32	56	After pass 1
8	23	45	78	32	56	After pass 2
8	23	32	78	45	56	After pass 3
8	23	32	45	78	56	After pass 4
8	23	32	45	56	78	After pass 5

Selection Sort



Selection Sort



Selection Sort

```
void swap( int &lhs, int &rhs );

void selectionSort( int a[], int n) {
  for (int i = 0; i < n-1; i++) {
    int min = i;
    for (int j = i+1; j < n; j++){
       if (a[j] < a[min]) min = j;
    }
    swap(a[i], a[min]);
  }
}
```

Selection Sort

Tick Function

```
def selection_sort(arr):
    ticks = 0
    n = len(arr)
    for i in range(n):
        min_idx = i
        ticks += 1
        for j in range(i+1, n):
            ticks += 1
            if arr[j] < arr[min_idx]:
                min_idx = j
                ticks += 1
        arr[i], arr[min_idx] = arr[min_idx], arr[i]
        ticks += 1
    return ticks

arr = [64, 25, 12, 22, 11]
ticks = selection_sort(arr)
print(arr)
```

Selection Sort

```
print(ticks)
```

Time calculation for all cases(Best, Worst, Average)

```
import time

def selection_sort(arr):
    for i in range(len(arr)):
        min_idx = i
        for j in range(i+1, len(arr)):
            if arr[j] < arr[min_idx]:
                min_idx = j
        arr[i], arr[min_idx] = arr[min_idx], arr[i]

def time_sort(arr):
    start = time.time()
    selection_sort(arr[:])
    return time.time() - start

arr_best = [1, 2, 3, 4, 5]
arr_average = [64, 25, 12, 22, 11]
arr_worst = [5, 4, 3, 2, 1]

print(f"Best case: {time_sort(arr_best):.6f} seconds")
```

Selection Sort

```
print(f"Average case: {time_sort(arr_average):.6f} seconds")
print(f"Worst case: {time_sort(arr_worst):.6f} seconds")
```

Selection Sort -- Analysis

- In selectionSort function, the outer for loop executes n-1 times.
- We invoke swap function once at each iteration.

  Total Swaps: n-1

  Total Moves: 3*(n-1)   (Each swap has three moves)

Selection Sort – Analysis (cont.)

- The inner for loop executes the size of the unsorted part minus 1 (n-1-i), and in each iteration we make one key comparison.

  of key comparisons = $((n-1)+(n-2)+...+3+2+1) = n*(n-1)/2$

  So, Selection sort is $O(n^2)$

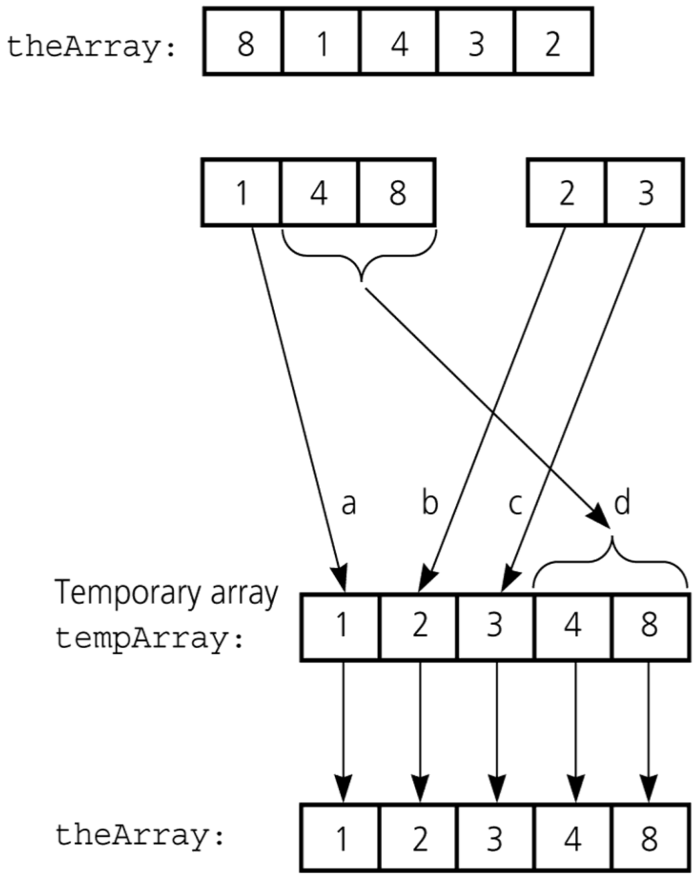
- The best case, the worst case, and the average case of the selection sort algorithm are same. all of them are **$O(n^2)$**
  - This means that the behavior of the selection sort algorithm does not depend on the initial organization of data.
  - Since $O(n^2)$ grows so rapidly, the selection sort algorithm is appropriate only for small n.
  - Although the selection sort algorithm requires $O(n^2)$ key comparisons, it only requires $O(n)$ moves.
  - A selection sort could be a good choice if data moves are costly but key comparisons are not costly (short keys, long records).

Merge Sort

Mergesort algorithm is one of two important divide-and-conquer sorting algorithms (the other one is quicksort).

- It is a recursive algorithm.
  - Divides the list into halves,
  - Sort each halve separately, and
  - Then merge the sorted halves into one sorted array.

Mergesort - Example



Divide the array in half

Sort the halves

Merge the halves:

a. $1 < 2$, so move 1 from left half to `tempArray`

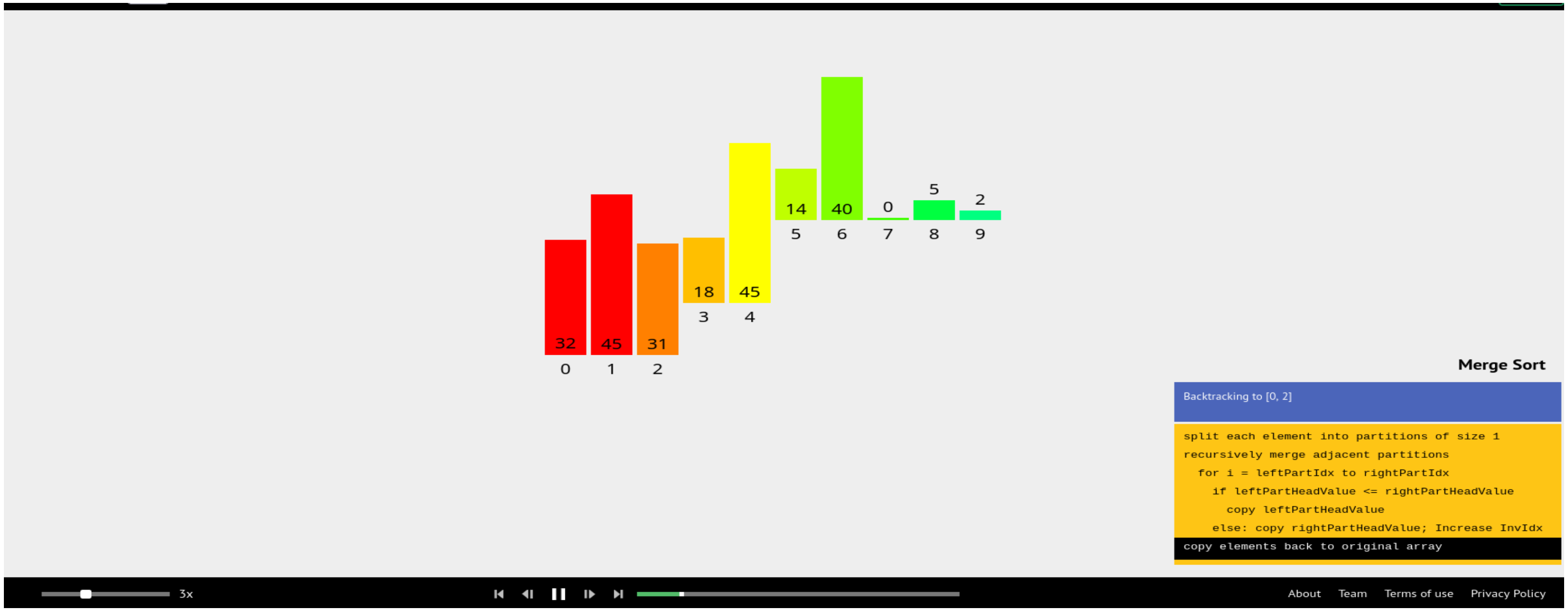
b. $4 > 2$, so move 2 from right half to `tempArray`

c. $4 > 3$, so move 3 from right half to `tempArray`

d. Right half is finished, so move rest of left half to `tempArray`

Copy temporary array back into original array

Merge Sort



32
45
31
18
45
14
40
0
5
2
0
1
2
3
4
5
6
7
8
9
Merge Sort
Backtracking to [0, 2]
split each element into partitions of size 1
recursively merge adjacent partitions
for i = leftPartIdx to rightPartIdx
if leftPartHeadValue <= rightPartHeadValue
copy leftPartHeadValue
else: copy rightPartHeadValue; Increase InvIdx
copy elements back to original array
3x
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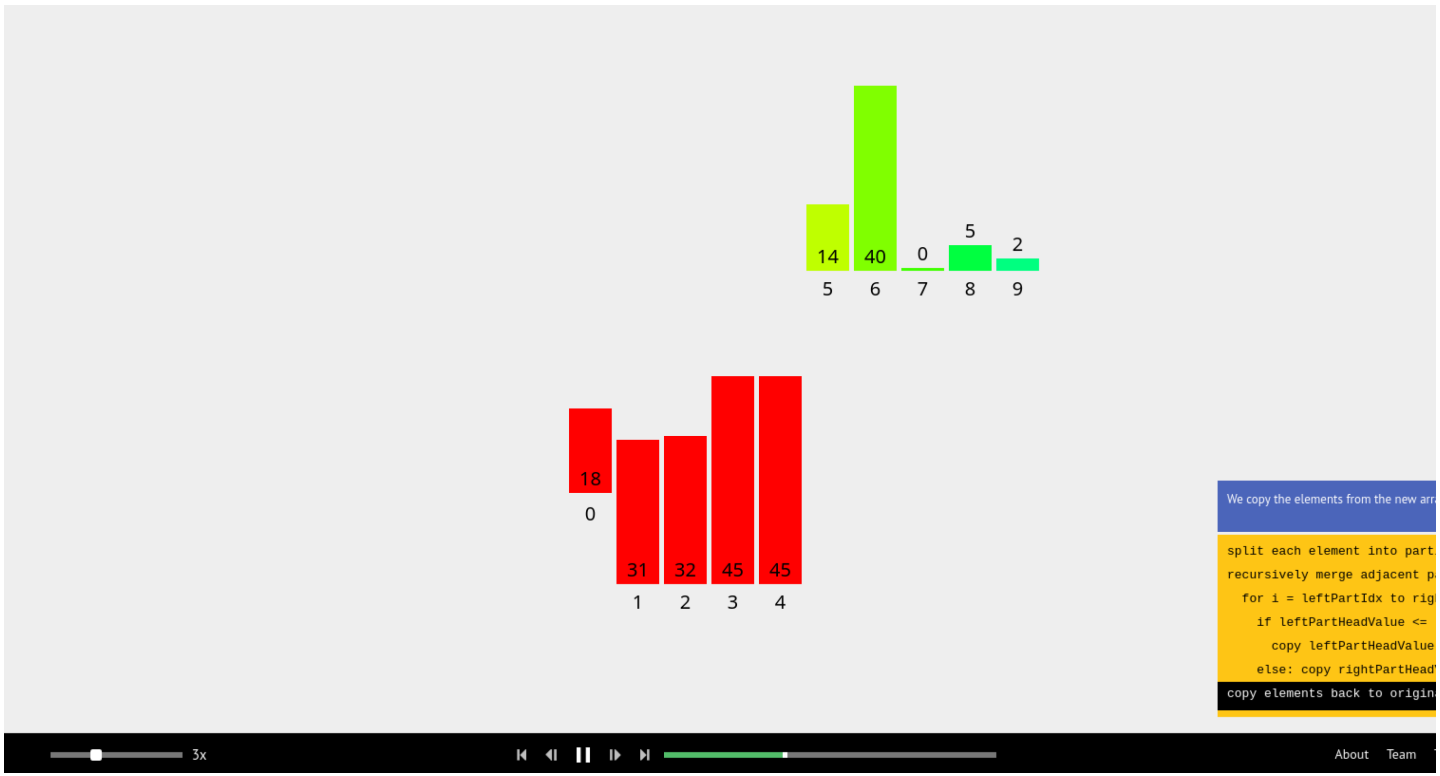

14
40
0
5
2
5
6
7
8
9
18
0
31
32
45
45
1
2
3
4
We copy the elements from the new arr
split each element into part
recursively merge adjacent p
for i = leftPartIdx to rig
if leftPartHeadValue <=
copy leftPartHeadValue
else: copy rightPartHead
copy elements back to origin
3x
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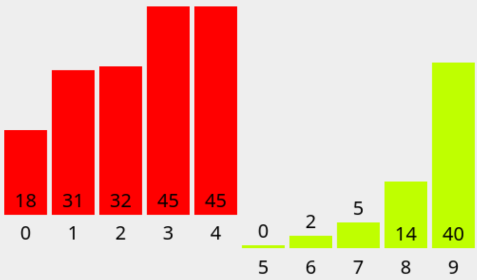

Merge Sort

Backtracking to [0, 9]

```
split each element into partitions of size 1
recursively merge adjacent partitions
  for i = leftPartIdx to rightPartIdx
    if leftPartHeadValue <= rightPartHeadValue
      copy leftPartHeadValue
    else: copy rightPartHeadValue; Increase InvIdx
copy elements back to original array
```

3x

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```
void merge(int theArray[], int first, int mid, int last)
{int tempArray[last+1];  // temporary array
int first1 = first;     // beginning of first subarray
int last1 = mid;        // end of first subarray
int first2 = mid + 1;   // beginning of second subarray
int last2 = last;       // end of second subarray
int index = first1; // next available location in tempArray
for ( ; (first1 <= last1) && (first2 <= last2); ++index) {
   if (theArray[first1] < theArray[first2]) {
      tempArray[index] = theArray[first1];
      ++first1;
   }
   else {
       tempArray[index] = theArray[first2];
       ++first2;
   }
}
```

Merge Sort (cont.)

```
    // finish off the first subarray, if necessary
    for (; first1 <= last1; ++first1, ++index)
       tempArray[index] = theArray[first1];

    // finish off the second subarray, if necessary
    for (; first2 <= last2; ++first2, ++index)
       tempArray[index] = theArray[first2];

    // copy the result back into the original array
    for (index = first; index <= last; ++index)
       theArray[index] = tempArray[index];
}
```

Merge Sort

```
void mergesort(int theArray[], int first, int last) {
   if (first < last) {
      int mid = (first + last)/2; // index of midpoint

      // dived into two halves at the middle
      mergesort(theArray, first, mid);
      mergesort(theArray, mid+1, last);

      // merge the two halves
      merge(theArray, first, mid, last);
   }
}
```

Merge Sort Tick Function

```
def merge_sort(arr):
    ticks = 0
    if len(arr) <= 1:
        return arr, ticks

    mid = len(arr) // 2
    left_sorted, left_ticks = merge_sort(arr[:mid])
    right_sorted, right_ticks = merge_sort(arr[mid:])
    ticks += left_ticks + right_ticks

    sorted_arr, merge_ticks = merge(left_sorted, right_sorted)
    ticks += merge_ticks
    return sorted_arr, ticks

def merge(left, right):
    result = []
    i = j = ticks = 0

    while i < len(left) and j < len(right):
        ticks += 2
        if left[i] <= right[j]:
            result.append(left[i])
            i += 1
        else:
            result.append(right[j])
            j += 1
```

```
result.extend(left[i:])
result.extend(right[j:])
ticks += len(left[i:]) + len(right[j:])
return result, ticks

arr = [64, 25, 12, 22, 11]
sorted_arr, ticks = merge_sort(arr)
print("Sorted array:", sorted_arr)
print("Total ticks (operations):", ticks)
```

Merge Sort Time Calculation for (BEST,AVERAGE,WORST) Cases

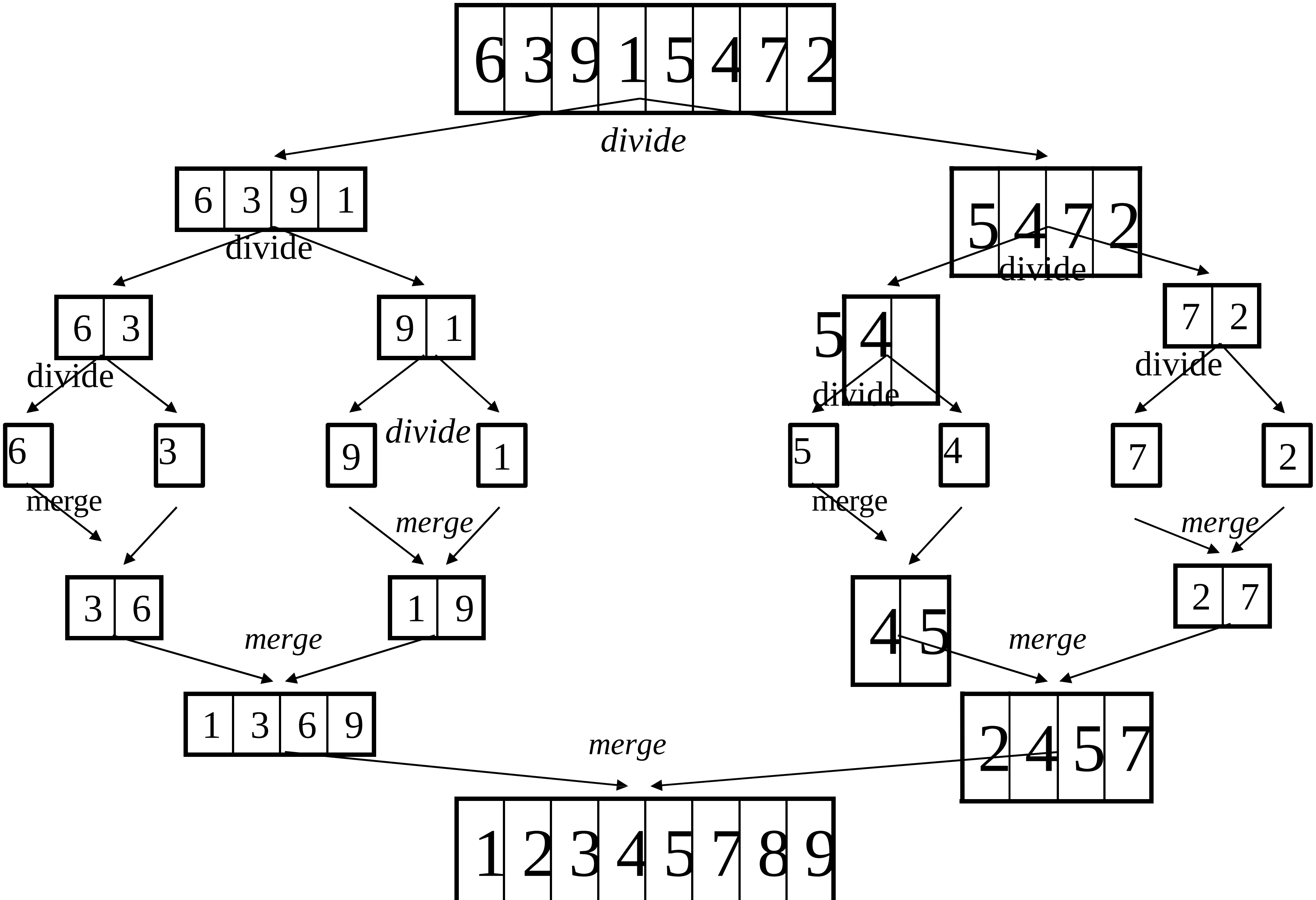
```
import time
import random



def merge_sort(arr):
    if len(arr) > 1:
        mid = len(arr) // 2
        L = arr[:mid]
        R = arr[mid:]
        merge_sort(L)
        merge_sort(R)
        i = j = k = 0
        while i < len(L) and j < len(R):
            if L[i] < R[j]:
                arr[k] = L[i]
                i += 1
            else:
                arr[k] = R[j]
                j += 1
            k += 1
        while i < len(L):
            arr[k] = L[i]
            i += 1
            k += 1
        while j < len(R):
            arr[k] = R[j]
```

```
j += 1
k += 1
def time_merge_sort(arr):
start_time = time.time()
merge_sort(arr)
return time.time() - start_time


def generate_sorted_array(size):
return list(range(size))


def generate_reverse_sorted_array(size):
return list(range(size, 0, -1))


def generate_random_array(size):
return [random.randint(0, 10000) for _ in range(size)]


if __name__ == "__main__":
array_size = 1000
best_case_time = time_merge_sort(generate_sorted_array(array_size))
print(f"Best case (sorted): {best_case_time:.6f} seconds")
```

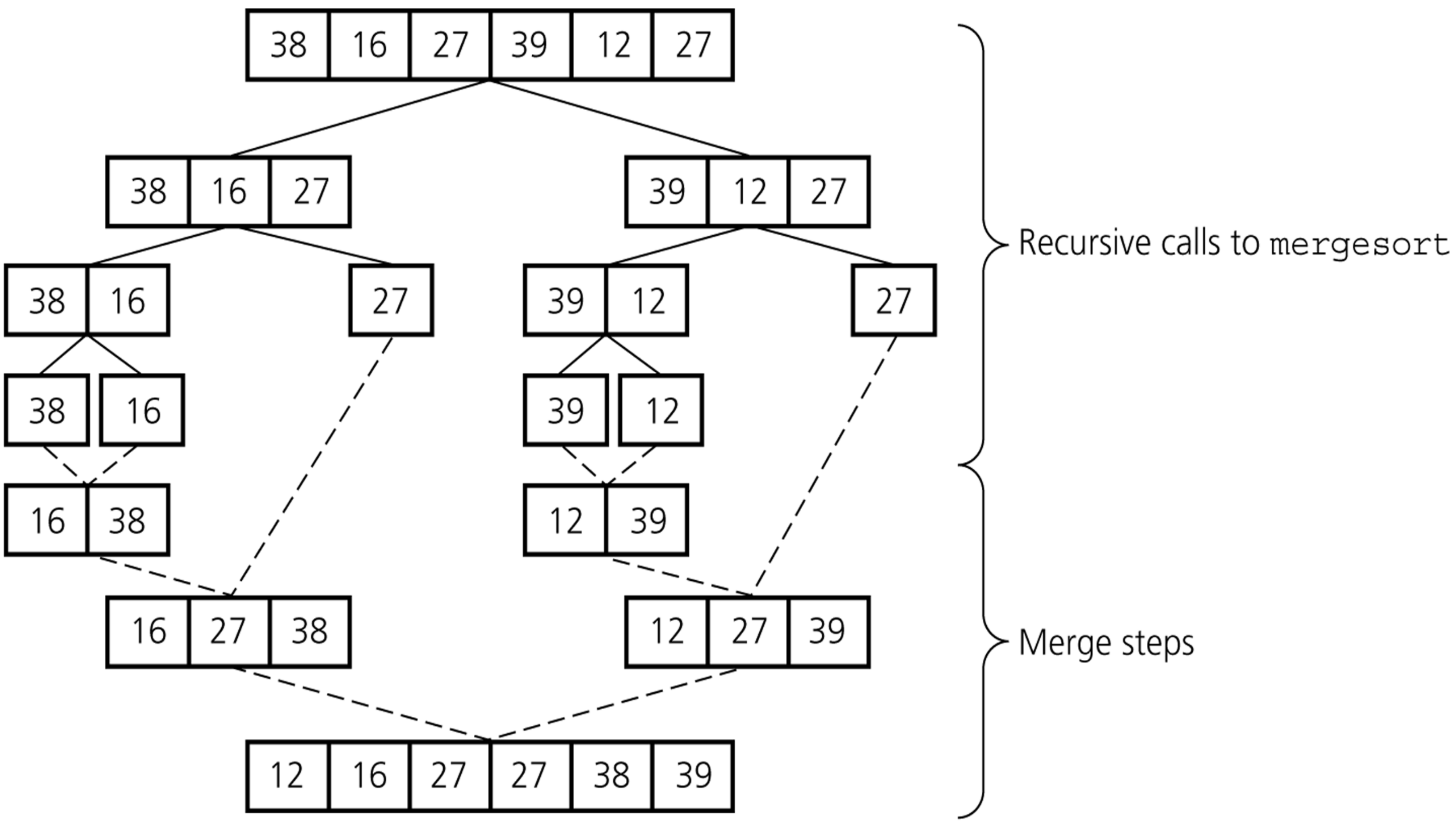
```
worst_case_time = time_merge_sort(generate_reverse_sorted_array(array_size))
print(f"Worst case (reverse sorted): {worst_case_time:.6f} seconds")
```

```
average_case_time = time_merge_sort(generate_random_array(array_size))
print(f"Average case (random): {average_case_time:.6f} seconds")
```

Merge Sort - Example



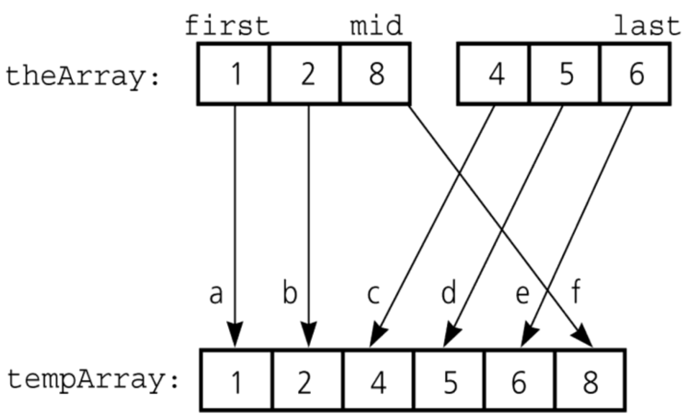
Mergesort – Example2



Mergesort – Analysis of Merge

A worst-case instance of the merge step in *mergesort*

Some elements in the first array are smaller and some elements are larger than all the elements in the second array

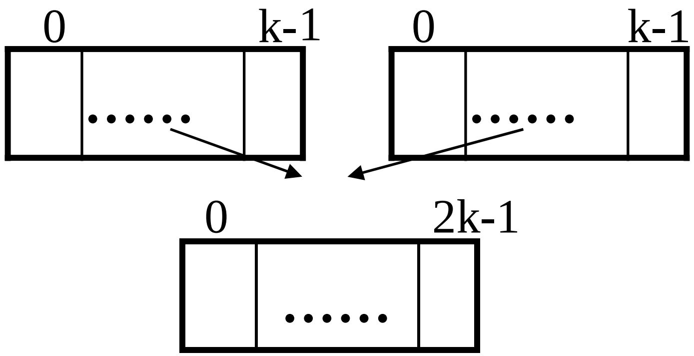


Merge the halves:

a. 1 < 4, so move 1 from `theArray[first..mid]` to `tempArray`
b. 2 < 4, so move 2 from `theArray[first..mid]` to `tempArray`
c. 8 > 4, so move 4 from `theArray[mid+1..last]` to `tempArray`
d. 8 > 5, so move 5 from `theArray[mid+1..last]` to `tempArray`
e. 8 > 6, so move 6 from `theArray[mid+1..last]` to `tempArray`
f. `theArray[mid+1..last]` is finished, so move 8 to `tempArray`

Mergesort – Analysis of Merge (cont.)

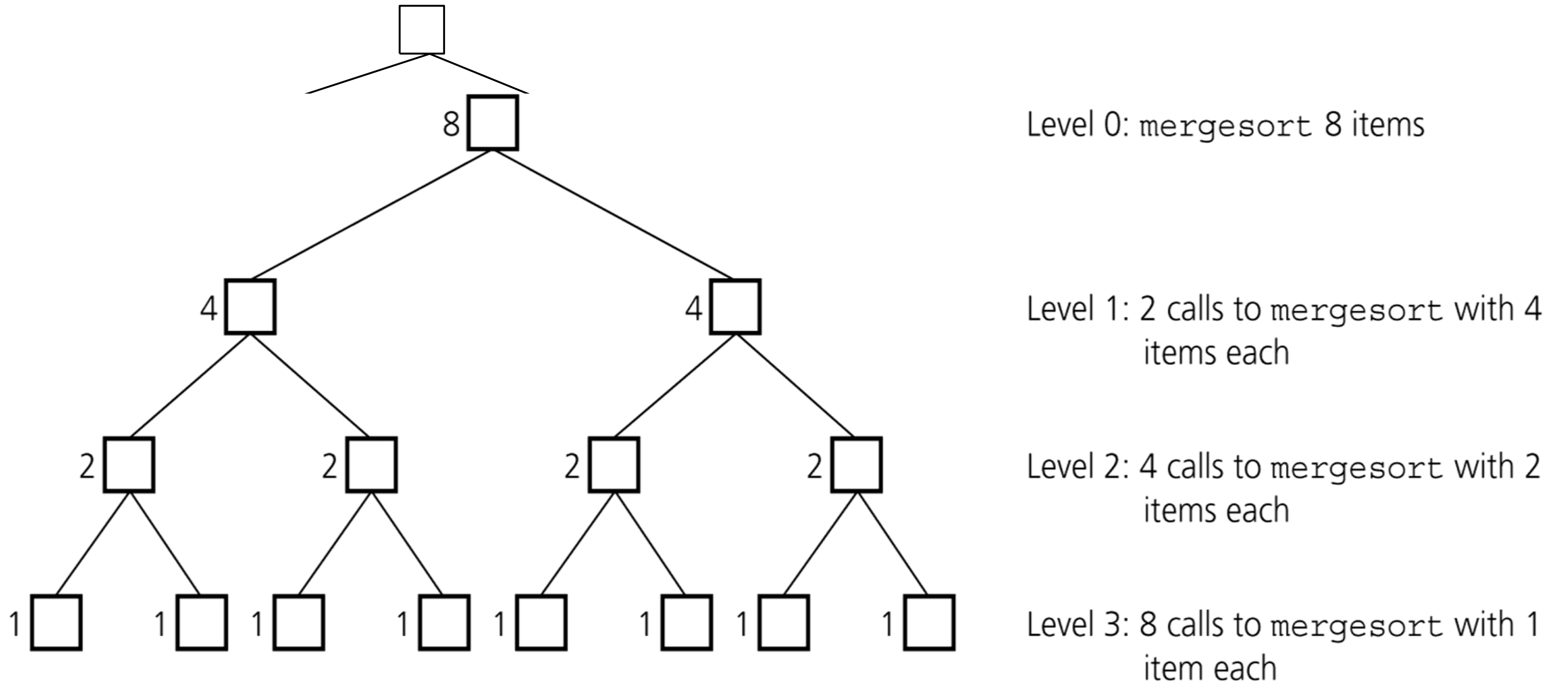
Merging two sorted arrays of size ***k***



- ***Best-case:***
  - All the elements in the first array are smaller (or larger) than all the elements in the second array.
  - The number of moves: 2k + 2k
  - The number of key comparisons: k
- ***Worst-case:***
  - The number of moves: 2k + 2k
  - The number of key comparisons: 2k-1

Mergesort - Analysis

Levels of recursive calls to *mergesort*, given an array of eight items



Mergesort - Analysis

- ***Worst-case –***

The number of key comparisons:

$= 2^0*(2*2^{m-1}-1) + 2^1*(2*2^{m-2}-1) + ... + 2^{m-1}*(2*2^0-1)$

$= (2^m - 2^0) + (2^m - 2^1) + ... + (2^m – 2^{m-1})$ (m terms)

$= m2^m – (2^0 + 2^1 + ... + 2^{m-1})$

$= m*2^m – \sum_{i=0}^{m-1} 2^i$

$= m*2^m – 2^m – 1$

Using m = log n

= n * log₂n – n – 1

O (n * log₂n)

Mergesort – Analysis

- Mergesort is extremely efficient algorithm with respect to time.
  - Both worst case and average cases are **O (n * $\log_2$n)**

- But, mergesort requires an extra array whose size equals to the size of the original array.

Quicksort

- Like mergesort, Quicksort is also based on the *divide-and-conquer* paradigm.
- But it uses this technique in a somewhat opposite manner, as all the hard work is done *before* the recursive calls.
- It works as follows:
  1. First, it partitions an array into two parts with respect to a pivot,
  2. Then, it sorts the parts independently,
  3. Finally, it combines the sorted subsequences by a simple concatenation.

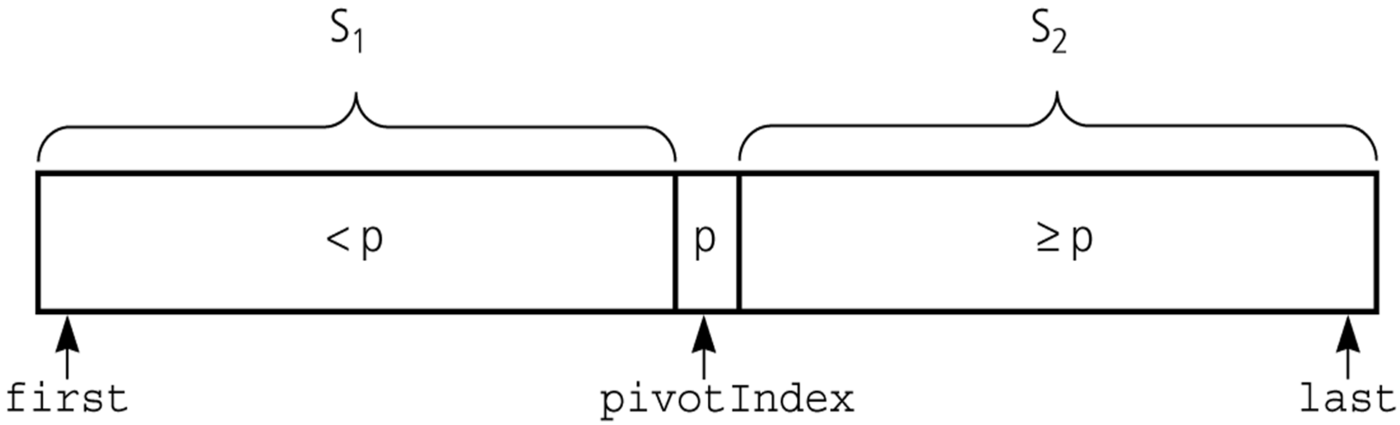
Quicksort (cont.)

The quick-sort algorithm consists of the following three steps:

1. ***Divide***: Partition the list.
   - To partition the list, we first choose some element from the list for which we hope about half the elements will come before and half after. Call this element the ***pivot***.
   - Then we partition the elements so that all those with values less than the pivot come in one sublist and all those with greater values come in another.
2. ***Recursion***: Recursively sort the sublists separately.
3. ***Conquer***: Put the sorted sublists together.

Quick Sort Partition

- Partitioning places the pivot in its correct place position within the array.




```
Partitions theArray[first..last] such that:
S1 = theArray[first..pivotIndex-1] < pivot
      theArray[pivotIndex] == pivot
S2 = theArray[pivotIndex+1..last] >= pivot
```

Quick Sort Partition

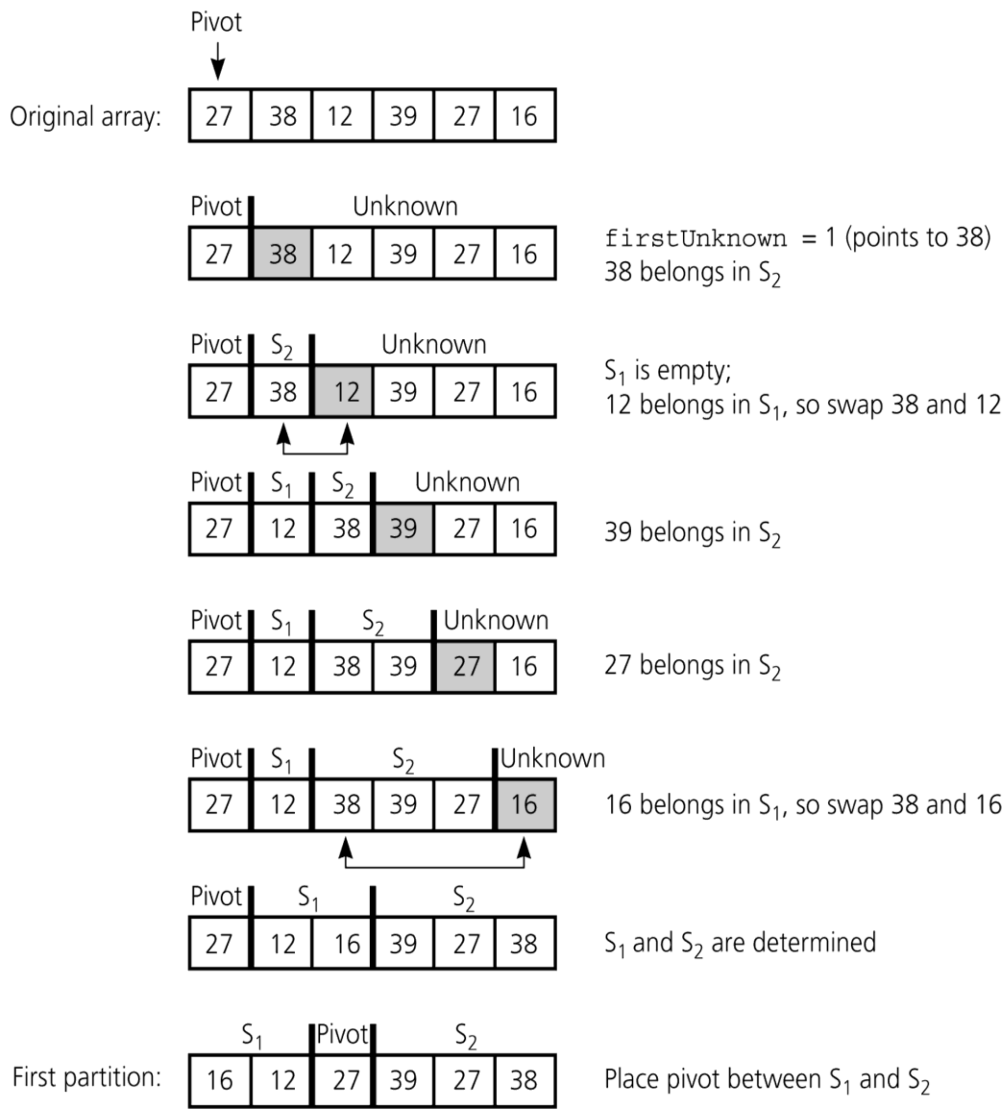
- Generates two smaller sorting problems.
  - Sort the left section of the array
  - Sort the right section of the array
  - Two smaller sorting problems are solved recursively to solve bigger sorting problem.

Quick Sort Partition: Choosing Pivot

- Which array item should be selected as pivot?
  - Somehow we have to select a pivot, and we hope that we will get a good partitioning.
  - If the items in the array arranged randomly, we choose a pivot randomly.
  - We can choose the first or last element as a pivot (it may not give a good partitioning).
  - We can use different techniques to select the pivot.
- Put this pivot into the first location of the array before partitioning

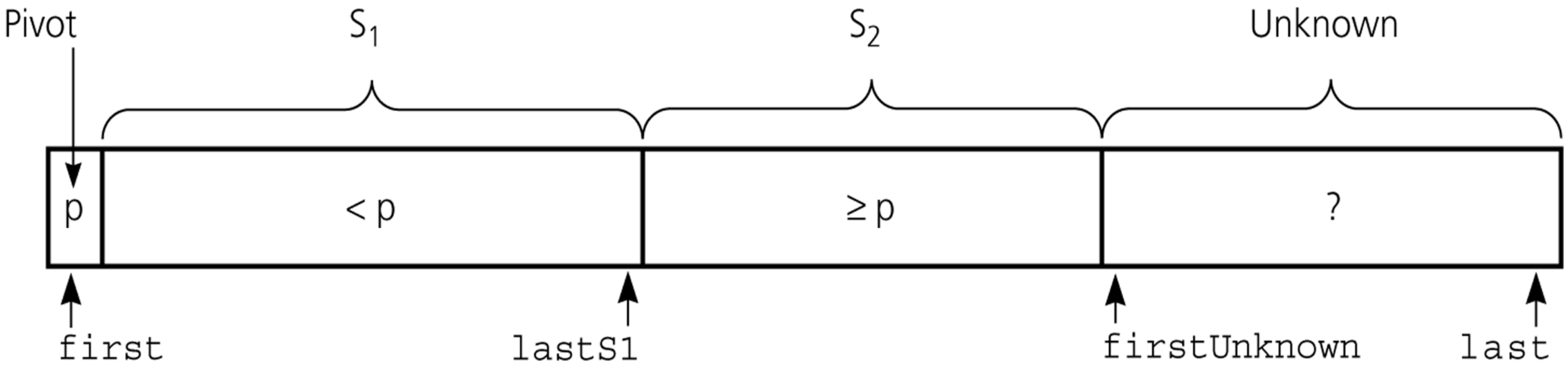
Partition (cont.)

Developing the first partition of an array when the pivot is the first item



Partition (cont.)

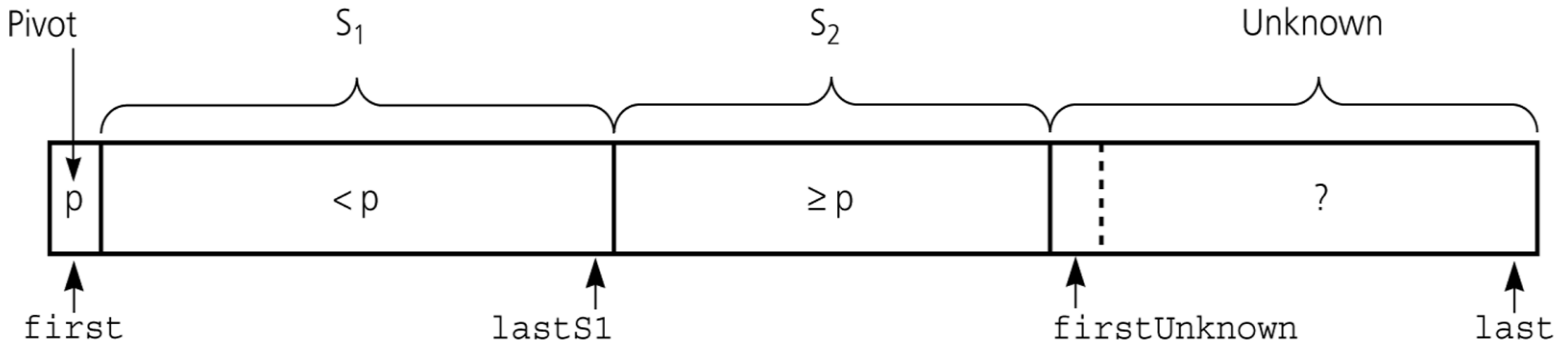
Invariant for the partition algorithm




```
S1: theArray[first+1..lastS1] < pivot
S2: theArray[lastS1+1..firstUnknown-1] >= pivot
```

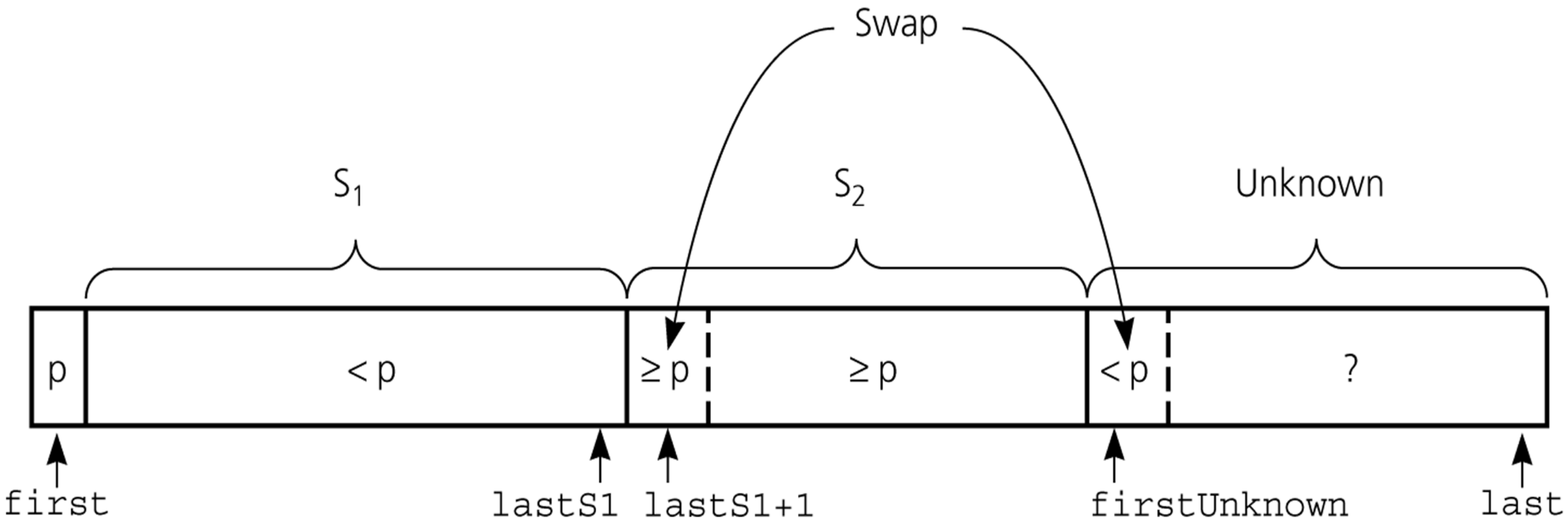
Partition (cont.)

When ***theArray[firstUnknown]*** >= ***pivot***
Move ***theArray[firstUnknown]*** into ***S_2*** by incrementing **firstUnknown**.



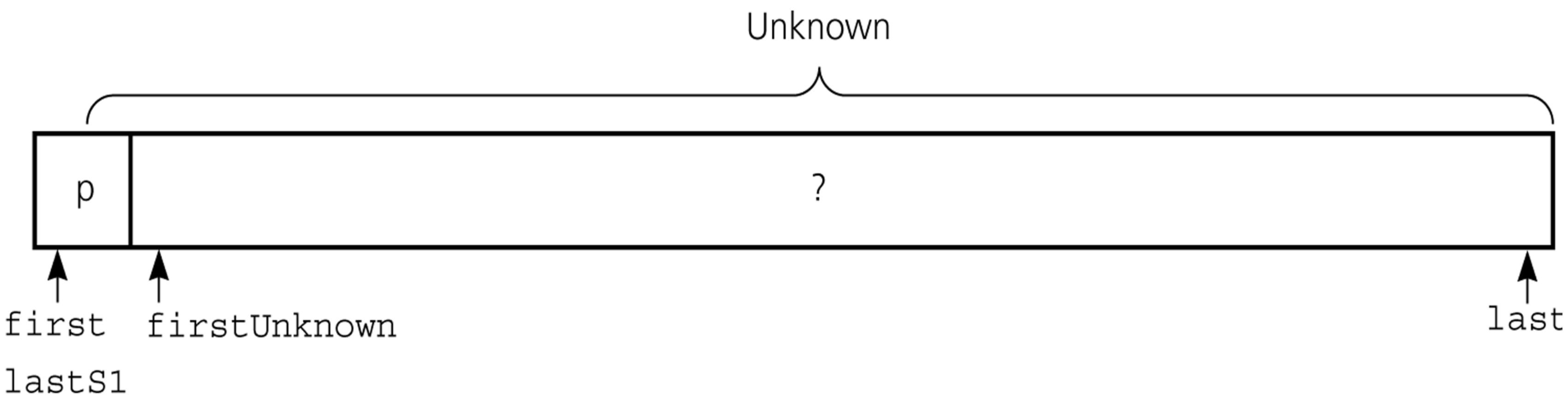
Partition (cont.)

When ***theArray[firstUnknown]*** < ***pivot***
Move ***theArray[firstUnknown]*** into $\mathbf{S_1}$ by swapping ***theArray[firstUnknown]*** with ***theArray[lastS1+1]*** and incrementing both ***lastS1*** and ***firstUnknown***.



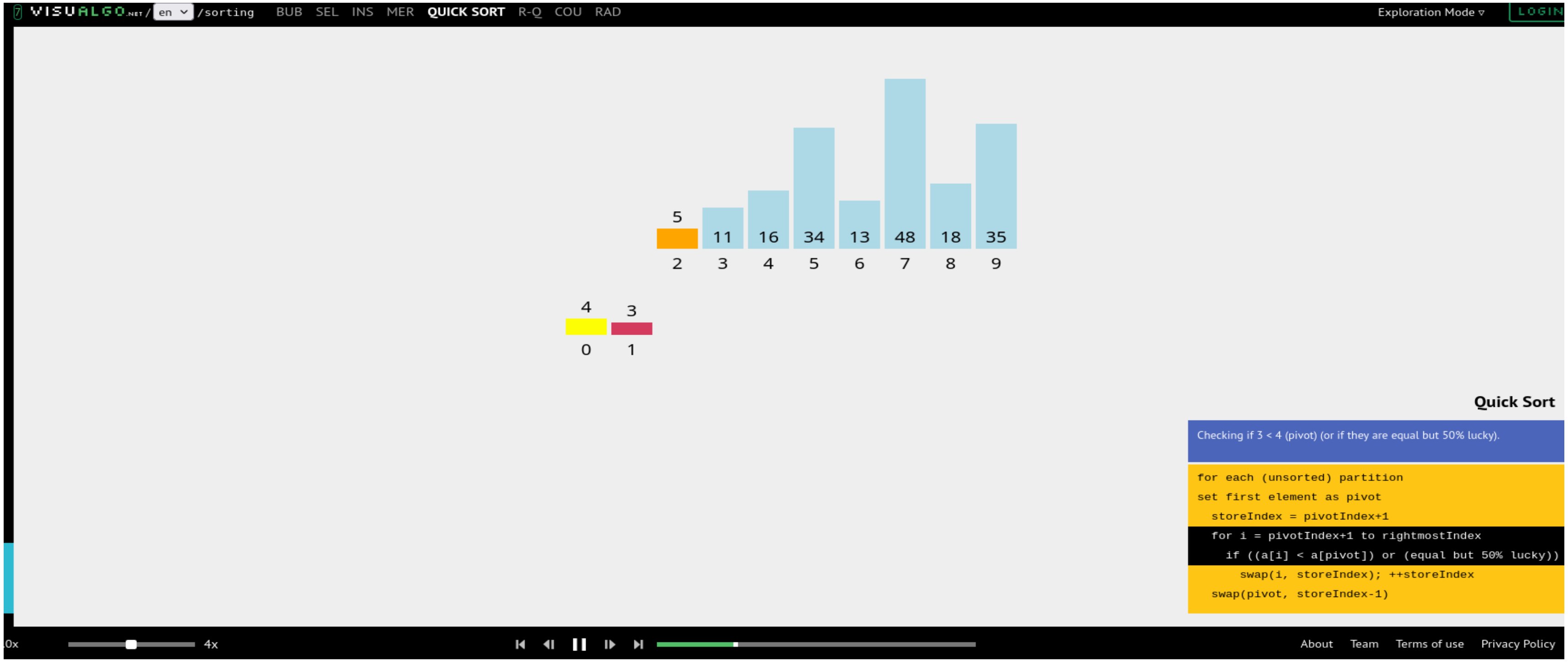
Partition (cont.)

Initial state of the array



lastS1 = first
firstUnknown = first + 1
S1: theArray[first+1..lastS1]: **Empty**
S2: theArray[lastS1+1..firstUnknown-1]: **Empty**

Quick Sort



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Exploration Mode
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5
11
16
34
13
48
18
35
2 3 4 5 6 7 8 9
4
3
0 1
Quick Sort
Checking if 3 < 4 (pivot) (or if they are equal but 50% lucky).
for each (unsorted) partition
set first element as pivot
storeIndex = pivotIndex+1
for i = pivotIndex+1 to rightmostIndex
if ((a[i] < a[pivot]) or (equal but 50% lucky))
swap(i, storeIndex); ++storeIndex
swap(pivot, storeIndex-1)
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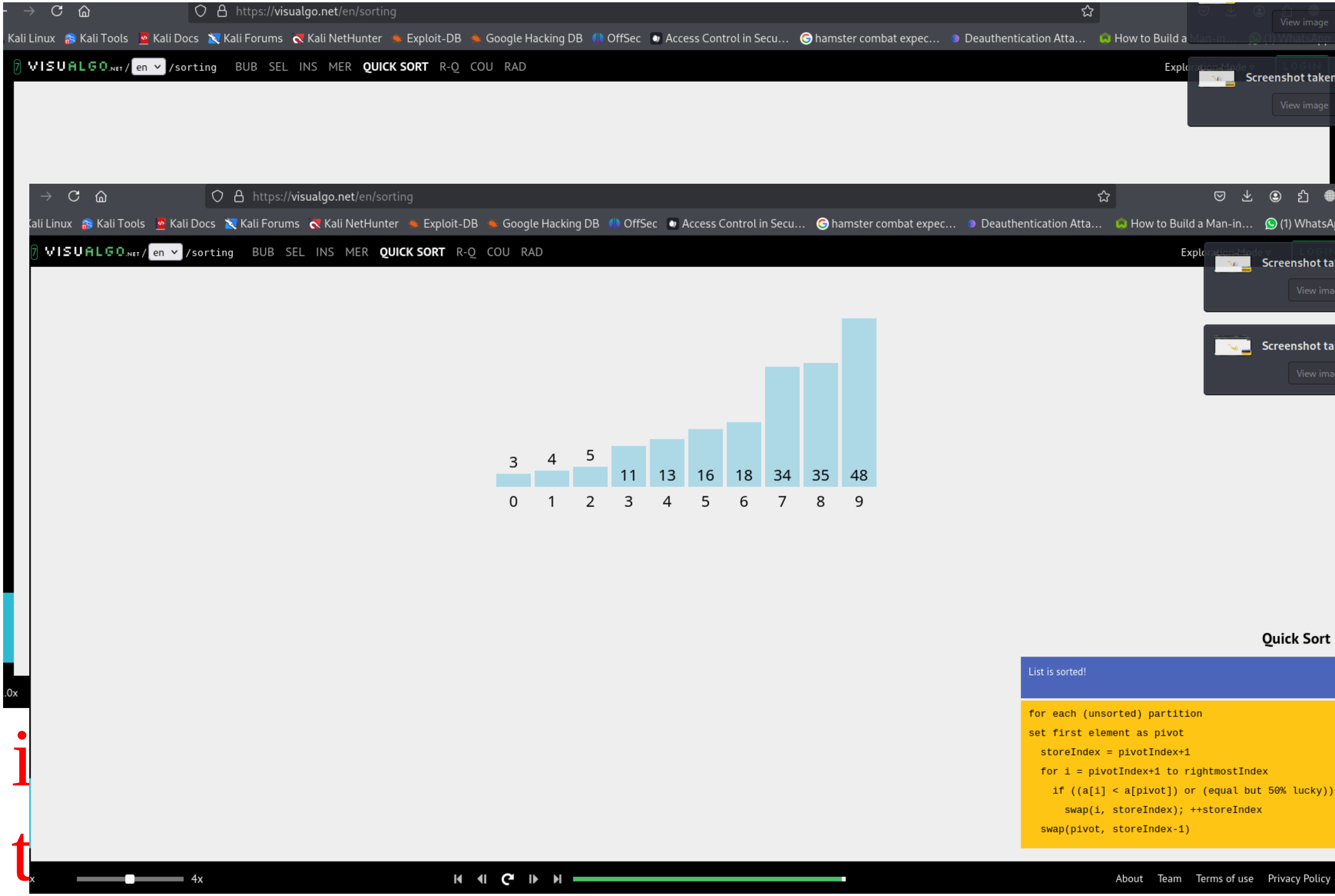
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3 4 5 11 13 16 18 34 35 48
0 1 2 3 4 5 6 7 8 9
Quick Sort
List is sorted!
for each (unsorted) partition
set first element as pivot
storeIndex = pivotIndex+1
for i = pivotIndex+1 to rightmostIndex
if ((a[i] < a[pivot]) or (equal but 50% lucky))
swap(i, storeIndex); ++storeIndex
swap(pivot, storeIndex-1)
4x
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ion Function

```
void swap( int &lhs, int &rhs );

void partition(int theArray[], int first, int last,
            int &pivotIndex) {

   // Choose and place pivot in theArray[first]
   choosePivot(theArray, first, last);

   // Initialize
   int pivot = theArray[first];
   int lastS1 = first;
   int firstUnknown = first + 1;
```

Partition Function (cont.)

```
 // Move one item at a time until unknown region is empty
for (; firstUnknown <= last; ++firstUnknown) {
   if (theArray[firstUnknown] < pivot) {  // Belongs to S1
       ++lastS1; // Expands S1 by incrementing lastS1
       // Swap firstUnknown with lastS1
       swap(theArray[firstUnknown], theArray[lastS1]);
   }
    // else belongs to S2, ++firstUnknown in the loop
    // places it to S2
}
// Place pivot in proper position and mark its location
swap(theArray[first], theArray[lastS1]);
pivotIndex = lastS1;
}
```

Quick Sort

```
void quicksort(int theArray[], int first, int last) {
    int pivotIndex;
   if (first < last) {
      // create the partition: S1, pivot, S2
      partition(theArray, first, last, pivotIndex);
      // sort regions S1 and S2
      quicksort(theArray, first, pivotIndex-1);
      quicksort(theArray, pivotIndex+1, last);
   }
}
```

Quick Sort

Tick Function Quick Sort

```
def quick_sort(arr, low, high):
    ticks = 0
    if low < high:
        pivot_index, part_ticks = partition(arr, low, high)
        ticks += part_ticks
        ticks += quick_sort(arr, low, pivot_index - 1)
        ticks += quick_sort(arr, pivot_index + 1, high)
    return ticks

def partition(arr, low, high):
    pivot = arr[high]
    i = low - 1
    ticks = 0
```

Quick Sort

```
  for j in range(low, high):
      ticks += 1
      if arr[j] < pivot:
          i += 1
          arr[i], arr[j] = arr[j], arr[i]

 ticks += 1

  arr[i + 1], arr[high] = arr[high], arr[i + 1]
  ticks += 1
  return i + 1, ticks


arr = [64, 25, 12, 22, 11]
ticks = quick_sort(arr, 0, len(arr) - 1)
print("Sorted array:", arr)
print("Total ticks (operations):", ticks)
```

Quick Sort

Quick Sort Time Calculation for (BEST,AVERAGE,WORST) Cases

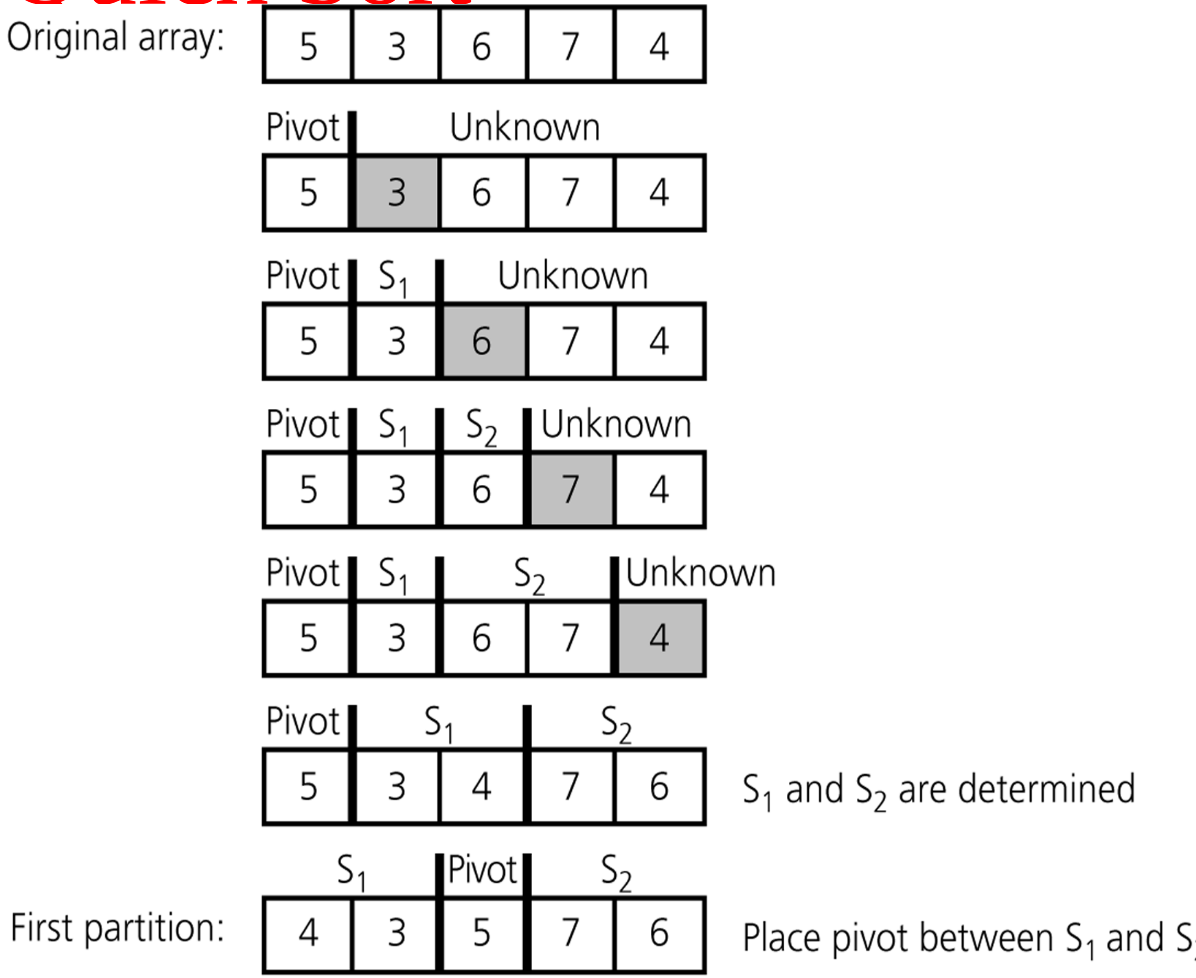
```
import time
import random

def quick_sort(arr):
    if len(arr) <= 1:
        return arr
    pivot = arr[len(arr) // 2]
    left = [x for x in arr if x < pivot]
    middle = [x for x in arr if x == pivot]
    right = [x for x in arr if x > pivot]
    return quick_sort(left) + middle + quick_sort(right)

def time_quick_sort(arr):
```

Quick Sort

```
    start_time = time.time()
    quick_sort(arr)
    return time.time() - start_time

def generate_sorted_array(size):
    return list(range(size))

def generate_reverse_sorted_array(size):
    return list(range(size, 0, -1))

def generate_random_array(size):
    return [random.randint(0, 10000) for _ in range(size)]

if __name__ == "__main__":
    array_size = 1000

    best_case_time = time_quick_sort(generate_sorted_array(array_size))
    print(f"Best case (sorted): {best_case_time:.6f} seconds")

    worst_case_time = time_quick_sort(generate_reverse_sorted_array(array_size))
    print(f"Worst case (reverse sorted): {worst_case_time:.6f} seconds")

    average_case_time = time_quick_sort(generate_random_array(array_size))
    print(f"Average case (random): {average_case_time:.6f} seconds")
```

Quick Sort

An average-case partitioning with quicksort

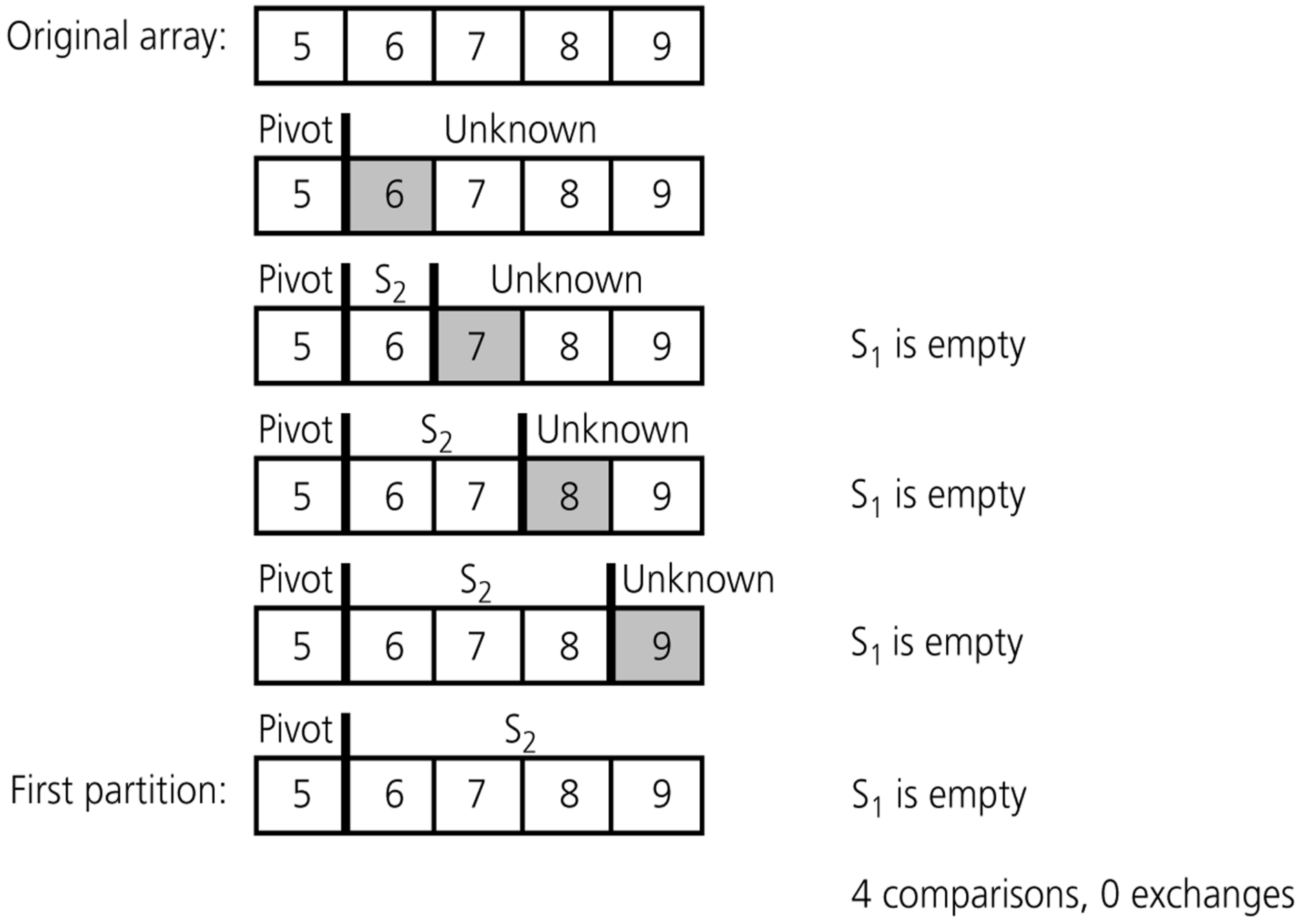


Quicksort – Analysis

- Quicksort is **$O(n*\log_2 n)$** in the best case and average case.
- Quicksort is slow when the array is sorted and we choose the first element as the pivot.
- Although the worst case behavior is not so good, its average case behavior is much better than its worst case.
  - So, Quicksort is one of best sorting algorithms using key comparisons.

Quicksort – Analysis

A worst-case partitioning with quicksort



Quicksort – Analysis

Worst Case: (assume that we are selecting the first element as pivot)

- The pivot divides the list of size n into two sublists of sizes 0 and n-1.
- The number of key comparisons

  $= n-1 + n-2 + ... + 1$

  $= n(n-1)/2$

  $= \mathbf{n^2/2 - n/2}$ **$O(n^2)$**
- The number of swaps =

  $= (n-1 + n-2 + ... + 1) + (n-1)$

  $= (n-1) + n(n-1)/2$

  $= \mathbf{n^2/2 + n/2 - 1}$ **$O(n^2)$**

- So, Quicksort is **$O(n^2)$** in worst case

swaps inside of the for loop

swaps outside ofthe for loop

Comparison of Sorting Algorithms

	Worst case	Average case
Selection sort	n^2	n^2
Bubble sort	n^2	n^2
Insertion sort	n^2	n^2
Mergesort	$n * \log n$	$n * \log n$
Quicksort	n^2	$n * \log n$
Radix sort	n	n
Treesort	n^2	$n * \log n$
Heapsort	$n * \log n$	$n * \log n$