Lecture 4

1. A paint manufacturer produces many primers, as shown in the *ch4_P3paintprimer.xlsx* file, and wants to determine how many of each product should be made daily to maximize net profits. Due to contractual agreements, there is a maximum production level of 1,000 units per each primer, and a minimum production level of 500 units. There are 10,000 machine hours available and $100,000 budget for raw materials.
   The net profit for each component is a function of the amount of each primer to be produced. Specifically, the net profit can be calculated as
   $$NP = ((x_1^3 + x_2^3 + \cdots + x_n^3) - 3(x_1^2 + x_2^2 + \cdots + x_n^2) + 2(x_1 + x_2 + \cdots + x_n))/10000$$
   where
   NP is the net profit for all products.
   $x_1, x_2, \ldots\ x_n$, are the amounts of first, second, ... n-th primer to be produced per month.

   a. Formulate an NLP model that represents the above business description.
   b. Use Solver and generate an Answer Report and a Sensitivity Report.
   c. Perform a scenario analysis using the findings in the above reports.

   Answers: see *ch4_P3paintprimer_solution.xlsx*

2. Toy manufacturing company makes to types of toys: toy trucks and toy cars. The manufacturing requirements for each toy production lot are shown in the following table

Raw Materials	Truck Toy	Car Toy	Available
Plastic	6 kg.	8 kg.	72 kg.
Labor hours	10 hrs.	8 hrs.	80 hrs.
Machine time	10 hrs.	4 hrs.	60 hrs.

   The cost of producing T lots of toy trucks can be calculated as $700T + 40T^2 + 1{,}000$. The cost of producing C lots of toy cars is $200C + 20C^2 + 1{,}500$. There is a total budget of $5,000 per week. The profit for either toy is $500 per lot. The operational data can be found in the Excel file named *ch4_P4toys.xlsx*.
   Formulate an NLP model that represents the above business description.

   a. Solve the problems and indicate
   • What is the optimal number of toy truck and toy car lots to be purchased each month
   • What is the value of the objective function (total profit) for the above solution
   • Identify binding and not binding constraints for the optimal solution.
   b. Perform a sensitivity analysis using Solver's output.

Answers: see *ch4_P4toys_solution.xlsx*

3. Nonlinear programming models are based on the assumptions that the objective function and constraints are nonlinear equations.
   a. True
   b. *False

4. Nonlinear programming models have the same structure as the linear programming models. Both models consist of the objective function, a set of constraints, and a set of non-negativity constraints.
   a. *True
   b. False

5. Relationships in nonlinear programming models with two decision variables can be represented by straight lines.
   a. True
   b. *False

6. A local optimum is a point in the feasible region with a better value than any other feasible point in the small neighborhood around it.
   a. *True
   b. False

7. A global optimum is a point in the feasible region with a better value than any other feasible point in the entire area of feasible solutions.
   a. *True
   b. False

8. When constraints are nonlinear, any local optimum is also a global optimum.
   a. True
   b. *False

9. When an objective function is nonlinear, any local optimum is also a global optimum.
   a. True
   b. *False

10. The reduced gradient values in sensitivity analysis for nonlinear programming models are valid only at the point of the optimal solution.
    a. *True
    b. False

11. The Lagrange multiplier values in sensitivity analysis for nonlinear programming models are valid only at the point of the optimal solution.
    a. *True
    b. False

12. When solving linear or nonlinear programming models, a constraint with a zero slack variable is a binding constraint.
    a. *True
    b. False

13. A nonlinear model has at least one nonlinear equation in either the constraint or the objective function.
    a. *True
    b. False

14. Solver's GRG algorithm is best suited for linear programming models.
    a. True
    b. *False

15. Which of the following must be satisfied in a nonlinear programming model?
    a. The objective function must be nonlinear.
    b. The constraints must be nonlinear equations.
    c. *Either a or b
    d. Neither a nor b

16. By definition, any linear equation must be:
    a. *Proportional and additive.
    b. Proportional or additive.
    c. Neither proportional nor additive.
    d. Either non-proportional or non-additive.

17. The additivity assumption may fail under certain conditions such as:
    a. Economies of scale
    b. *Buy one, get a second item of an equal or lower price for free.
    c. Both a and b
    d. Neither a nor b

18. Nonlinear relationships in nonlinear programming models can be represented by:
    a. Straight lines when the model has two decision variables.
    b. Planes when the model has three decision variables.
    c. Both a and b are true.

d. *None of the above

19. Nonlinear relationships in nonlinear programming models can be represented by:
    a. *Curved lines when the model has two decision variables.
    b. Planes when the model has three decision variables.
    c. Both a and b are true.
    d. None of the above

20. The final value in the variable cells of the sensitivity report of nonlinear programming models can indicate:
    a. The initial solution for the decision variables.
    b. *The optimal solution for the decision variables.
    c. A range of optimal solutions for the decision variables.
    d. Any of the above

21. Which of the following distinguishes the sensitivity reports of nonlinear programming models from the sensitivity reports of regular linear programming models?
    a. The "reduced cost" in linear programming models is called the "Lagrange multiplier" in nonlinear programming models.
    b. The "shadow price" in linear programming models is called the "reduced gradient" in nonlinear programming models.
    c. *The "reduced cost" in linear programming models is called the "reduced gradient" in nonlinear programming models.
    d. All of the above

22. The dual values in sensitivity analysis for nonlinear programming models:
    a. *Change when these values move away from the optimal solution.
    b. Remain constant within the range between the upper and lower limits.
    c. Become invalid at the point of the optimal solution.
    d. None of the above statements are true about the dual values for nonlinear programming models.

23. Solutions of nonlinear programming models with Microsoft Excel will often generate "division by zero" errors. To avoid these errors, the decision maker should:
    a. Ignore them and read the solution as provided by Excel.
    b. Accept the errors as part of the solution (i.e., there is no solution to the given problem).
    c. *Add a non-negativity constraint for decision variables instead of checking the non-negativity box in Solver.
    d. All of the above