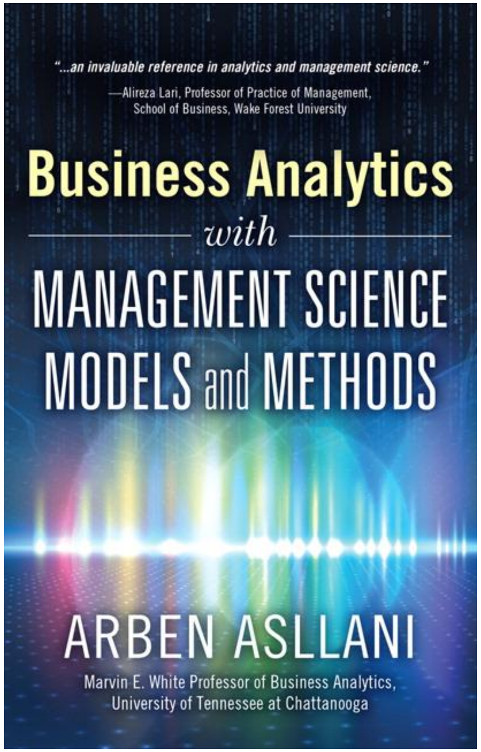
Business Analytics
Prescriptive Models



<u>Based on</u>

Business Analytics
With
Management Science
Models and Methods
by
Arben Asllani

CHAPTER 7

Business Analytics with Shipment Models

Business Analytics with Management Science Models and Methods

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Chapter Outline

- Chapter Objectives
- Prescriptive Analytics in Action: Danaos Corporation
- Introduction
- Transportation Models
- Example: National Xpress Corporation
- Sensitivity Analysis
- Transshipment Method
- Exploring Big Data with Shipment Model
- Wrap Up

Chapter Objectives

- Explain shipment models and their structure as special cases of mathematical programming
- Demonstrate the use of network diagrams for the transportation and transshipment problems
- Learn how to expand transshipment models by adding layers of warehouses or distribution centers
- Learn how to check for the feasibility of the transportation models before even attempting to formulate and solve the model
- Use Excel templates to set up and formulate transportation or transshipment models as linear programming models

Chapter Objectives

- Demonstrate the Solver solutions for the transportation and transshipment problems
- Discuss the benefits of using a network diagram to represent a transportation or transshipment model
- Perform sensitivity analysis and explain the meaning of the shadow price and reduced cost in the solution of transportation models
- Discuss practical recommendation when exploring big data with transportation and transshipment models

Prescriptive Analytics in Action

- Danaos Corporation is a leading international owner of containerships
  - Owns over 59 containerships
  - 345,000 20-foot standard containers
  - Global operations: chartering, crewing, and vessel management
- ORISMA (Operations Research in Ship Management)
  - Combines internal and external sources and information such as financial data, hydrodynamic models and weather forecasting
  - Led to optimum solutions at the operational, tactical and strategic level
  - Average net profit increase of 8%
  - Approximately $1 million per vessel per year
  - The anticipated cost savings for the year 2012 for all ORISMA clients was estimated at $500 million

Introduction

- Optimization of shipment
  - Shipment of goods from plants to warehouses, to distribution centers, or other destinations
  - Include organizational, intra-organizational, and inter-organizational shipments
  - Count for a significant part of costs
- Two types of shipment problems (both are LP models)
  - Transportation model (two layers of shipment: sources-destinations)
  - Transshipment model (more than two layers of shipment)
- Shipment models can be represented with network diagrams
  - Nodes (represent locations: cities, warehouses, machines)
  - Arcs (represent transfer: of goods, materials, people, funds)

General Formulation of the Transportation Model

$$\text{Minimize } Z = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij}$$

subject to:

$$\sum_{i=1}^{m} x_{ij} \geq D_j \quad \text{for all } j = 1,2,3, \ldots, n$$

$$\sum_{j=1}^{n} x_{ij} \leq S_i \quad \text{for all } i = 1,2,3, \ldots, m$$

$$x_{ij} \geq 0$$

x_{ij} = amount of units to transports from source i to destination j (decision variables)

m = number of sources

n = number of destinations

c_{ij} = transportation cost per unit from source i to destination j

D_j = amount of units demanded by destination j

S_i = amount of units available at source i

Network Diagram of Transportation Models

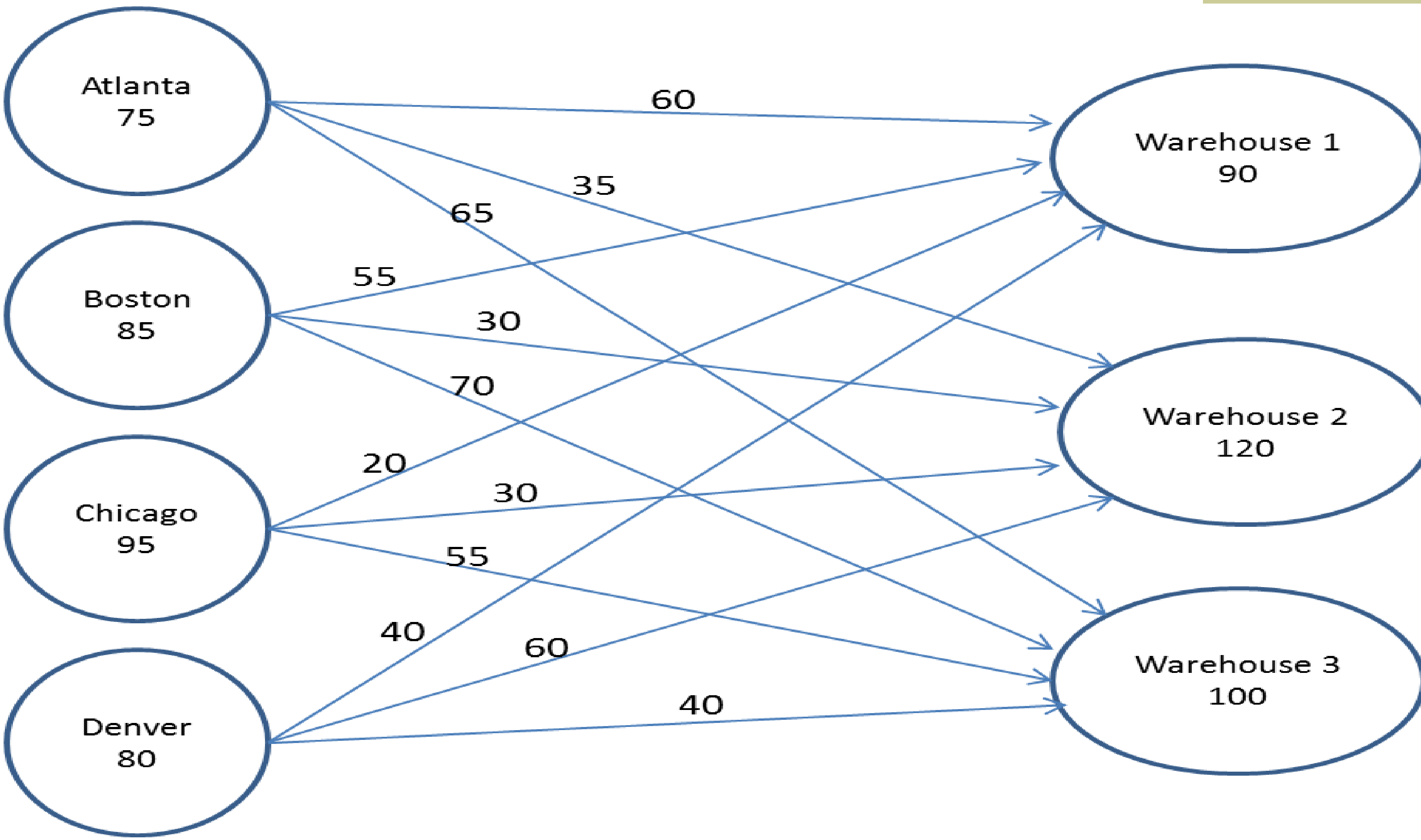
- NXT is planning the next month shipments from its four production plans to its three warehouses
- Goal: To minimize the total shipping cost

Operational Data for the NXT Transportation Model

Plants	Warehouse 1	Warehouse 2	Warehouse 3	**Monthly Production Capacity**
Atlanta	$60	$35	$65	**75**
Boston	$55	$30	$70	**85**
Chicago	$20	$30	$55	**95**
Denver	$40	$60	$40	**80**
Monthly Demand	**90**	**120**	**100**	

Shipping cost per container, monthly demand and production

Diagram for the NXT Transportation Model

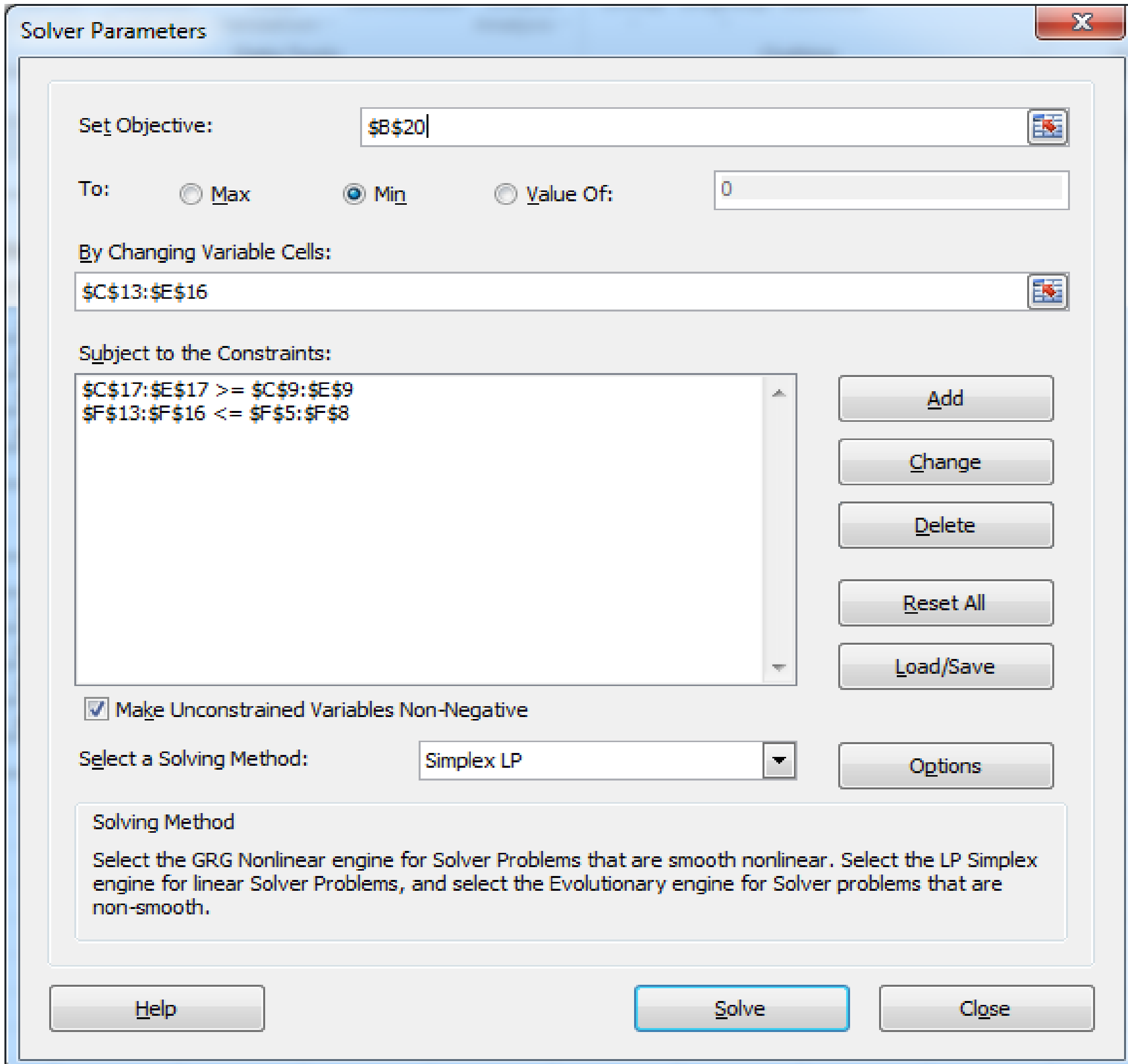


Solving Transportation Model with Solver (ch7_transportation_transshipment.xlsx)

	A	B	C	D	E	F	G	H
1	**NXC Transportation Model**							
2								
3	Parameters							
4		*From/To*	***Warehouse 1***	***Warehouse 2***	***Warehouse 3***	***Monthly Production Capacity***		
5		Atlanta	60	35	65	***75***		
6		Boston	55	30	70	***85***		
7		Chicago	20	30	55	***95***		
8		Denver	40	60	40	***80***		
9		***Monthly Demand***	***90***	***120***	***100***			
10								
11	Decisions							
12		*From/To*	*Warehouse 1*	*Warehouse 2*	*Warehouse 3*	*Delivered*		
13		*Atlanta*	0	0	0	***0***	=SUM(C13:E13)	
14		*Boston*	0	0	0	***0***	=SUM(C14:E14)	
15		*Chicago*	0	0	0	***0***	=SUM(C15:E15)	
16		*Denver*	0	0	0	***0***	=SUM(C16:E16)	
17		*Received*	***0***	***0***	***0***			
18			=SUM(C13:C16)	=SUM(D13:D16)	=SUM(E13:E16)			
19	Objective function							
20		0	=SUMPRODUCT(C5:E8, C13:E16)					

Excel Template and Initial Calculations for the Transportation Mode

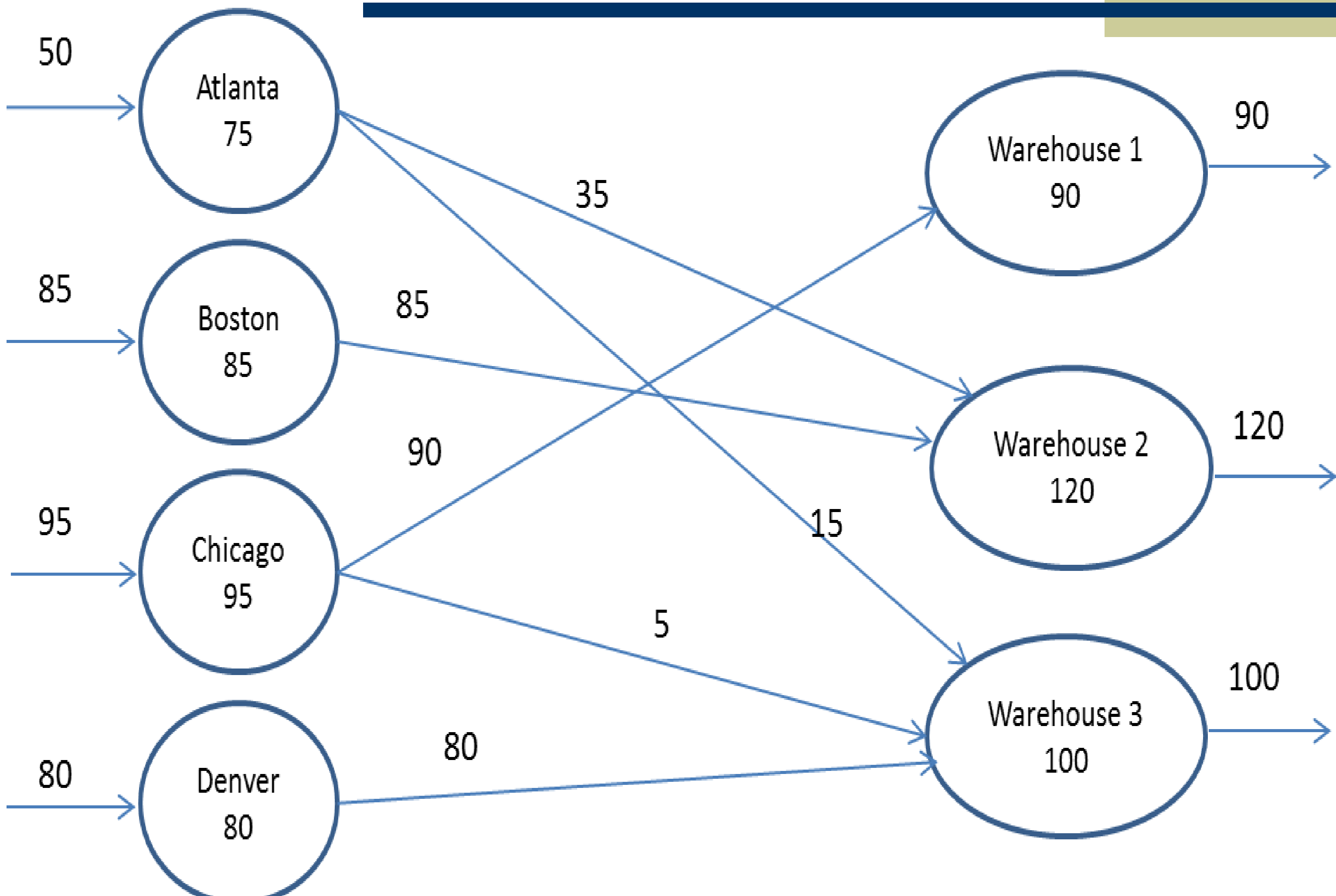
Solver Parameters for NXT Models



Final Solution for the NXT Transportation Model

	A	B	C	D	E	F
1	**NXC Transportation Model**					
2						
3	Parameters					
4		*From/To*	***Warehouse 1***	***Warehouse 2***	***Warehouse 3***	***Monthly Production Capacity***
5		Atlanta	60	35	65	***75***
6		Boston	55	30	70	***85***
7		Chicago	20	30	55	***95***
8		Denver	40	60	40	***80***
9		***Monthly Demand***	***90***	***120***	***100***	
10						
11	Decisions					
12		*From/To*	*Warehouse 1*	*Warehouse 2*	*Warehouse 3*	*Delivered*
13		*Atlanta*	0	35	15	***50***
14		*Boston*	0	85	0	***85***
15		*Chicago*	90	0	5	***95***
16		*Denver*	0	0	80	***80***
17		*Received*	***90***	***120***	***100***	
18						
19	Objective function					
20		10025				

Network Diagram for the Final Solution



Sensitivity Analysis:
Changes in the Right-hand Side Values

- Transportation model has two types of constraints:

1. Demand Constraints
2. Supply Constraints

22	Constraints						
23			Final	Shadow	Constraint	Allowable	Allowable
24	Cell	Name	Value	Price	R.H. Side	Increase	Decrease
25	C17	Received Warehouse 1	90	30	90	5	15
26	D17	Received Warehouse 2	120	35	120	25	35
27	E17	Received Warehouse 3	100	65	100	25	15
28	F13	Atlanta Delivered	50	0	75	1E+30	25
29	F14	Boston Delivered	85	-5	85	35	25
30	F15	Chicago Delivered	95	-10	95	15	5
31	F16	Denver Delivered	80	-25	80	15	25

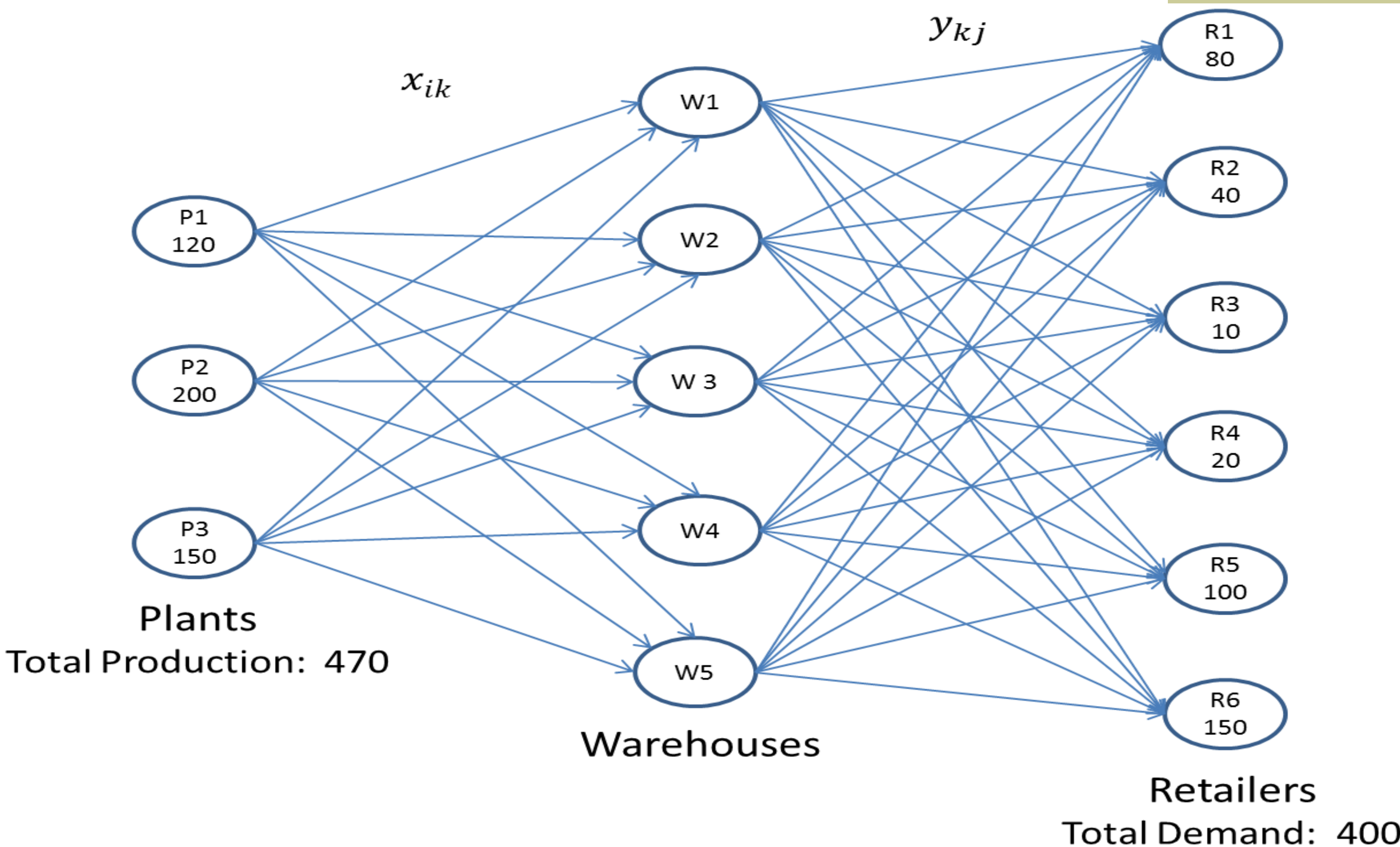
Sensitivity Analysis: Changes in the Decision Variables

6	Variable Cells						
7			Final	Reduced	Objective	Allowable	Allowable
8	Cell	Name	Value	Cost	Coefficient	Increase	Decrease
9	C13	Atlanta Warehouse 1	0	30	60	1E+30	30
10	D13	Atlanta Warehouse 2	35	0	35	5	5
11	E13	Atlanta Warehouse 3	15	0	65	10	5
12	C14	Boston Warehouse 1	0	30	55	1E+30	30
13	D14	Boston Warehouse 2	85	0	30	5	1E+30
14	E14	Boston Warehouse 3	0	10	70	1E+30	10
15	C15	Chicago Warehouse 1	90	0	20	30	30
16	D15	Chicago Warehouse 2	0	5	30	1E+30	5
17	E15	Chicago Warehouse 3	5	0	55	5	30
18	C16	Denver Warehouse 1	0	35	40	1E+30	35
19	D16	Denver Warehouse 2	0	50	60	1E+30	50
20	E16	Denver Warehouse 3	80	0	40	25	1E+30

The Transshipment Method

- Company ships products from its three plants to its six retail stores
  - Products are first distributed from three plants to the company's five warehouses
  - At warehouses products are stored according to potential retail destinations
  - Upon demand products are shipped to each of the six retail stores
  - Company produces 470 units per month (120 at plant 1,200 at plant 2, 150 at plant 3)
  - Monthly demand from the retail stores is 400 units (respectively 80, 40, 10, 20, 100, and 150)
- Minimize the shipment cost

Network Diagram for the Transshipment Problem



Shipment Costs Per Unite

From Plants to Warehouses

From/To	Warehouse 1	Warehouse 2	Warehouse 3	Warehouse 4	Warehouse 5	Monthly Production
Plant 1	60	35	65	40	40	120
Plant 2	55	30	70	120	50	200
Plant 3	20	30	55	60	20	150
Total						**470**

From Warehouses to Retail Stores

To/From	Warehouse 1	Warehouse 2	Warehouse 3	Warehouse 4	Warehouse 5	Monthly Demand
Retail store 1	30	70	70	10	10	80
Retail store 2	20	80	10	60	30	40
Retail store 3	80	40	30	100	30	10
Retail store 4	90	30	20	20	90	20
Retail store 5	10	40	70	60	50	100
Retail store 6	100	20	90	30	60	150
Total						**400**

General Formulation of the Transshipment Model

Minimize $Z = \sum_{i=1}^{m} \sum_{k=1}^{l} c_{ik}\, x_{ik} + \sum_{k=1}^{l} \sum_{j=1}^{n} c_{kj}\, y_{kj}$

Subject to:

- $\sum_{k=1}^{l} y_{kj} \geq D_j$ for all $j = 1, 2, 3, \ldots, n$
- $\sum_{k=1}^{l} x_{ik} \leq S_i$ for all $i = 1, 2, 3, \ldots, m$
- $\sum_{i=1}^{m} \sum_{k=1}^{l} x_{ik} - \sum_{k=1}^{l} \sum_{j=1}^{n} y_{kj} = 0$
- $x_{ik} \geq 0$ and $y_{kj} \geq 0$

Solving Transportation Model with Solver(ch7_transportation_transshipment.xlsx)

Parameters: from Plants to Warehouses

From/To	Warehouse 1	Warehouse 2	Warehouse 3	Warehouse 4	Warehouse 5	Monthly Production
Plant 1	60	35	65	40	40	120
Plant 2	55	30	70	120	50	200
Plant 3	20	30	55	60	20	150
Total						470

Decision variables: Xik

From/To	Warehouse 1	Warehouse 2	Warehouse 3	Warehouse 4	Warehouse 5	Monthly Production	Unused Capacity
Plant 1	0	0	0	100	0	**100**	20
Plant 2	0	150	0	0	0	**150**	50
Plant 3	140	0	0	0	10	**150**	0
Total	**140**	**150**	**0**	**100**	**10**		

Parameters: from Warehouses to Retail Stores

To/From	Warehouse 1	Warehouse 2	Warehouse 3	Warehouse 4	Warehouse 5	Monthly Demand
Retail store 1	30	70	70	10	10	80
Retail store 2	20	80	10	60	30	40
Retail store 3	80	40	30	100	30	10
Retail store 4	90	30	20	20	90	20
Retail store 5	10	40	70	60	50	100
Retail store 6	100	20	90	30	60	150
Total						400

Decision Variables: Ykj

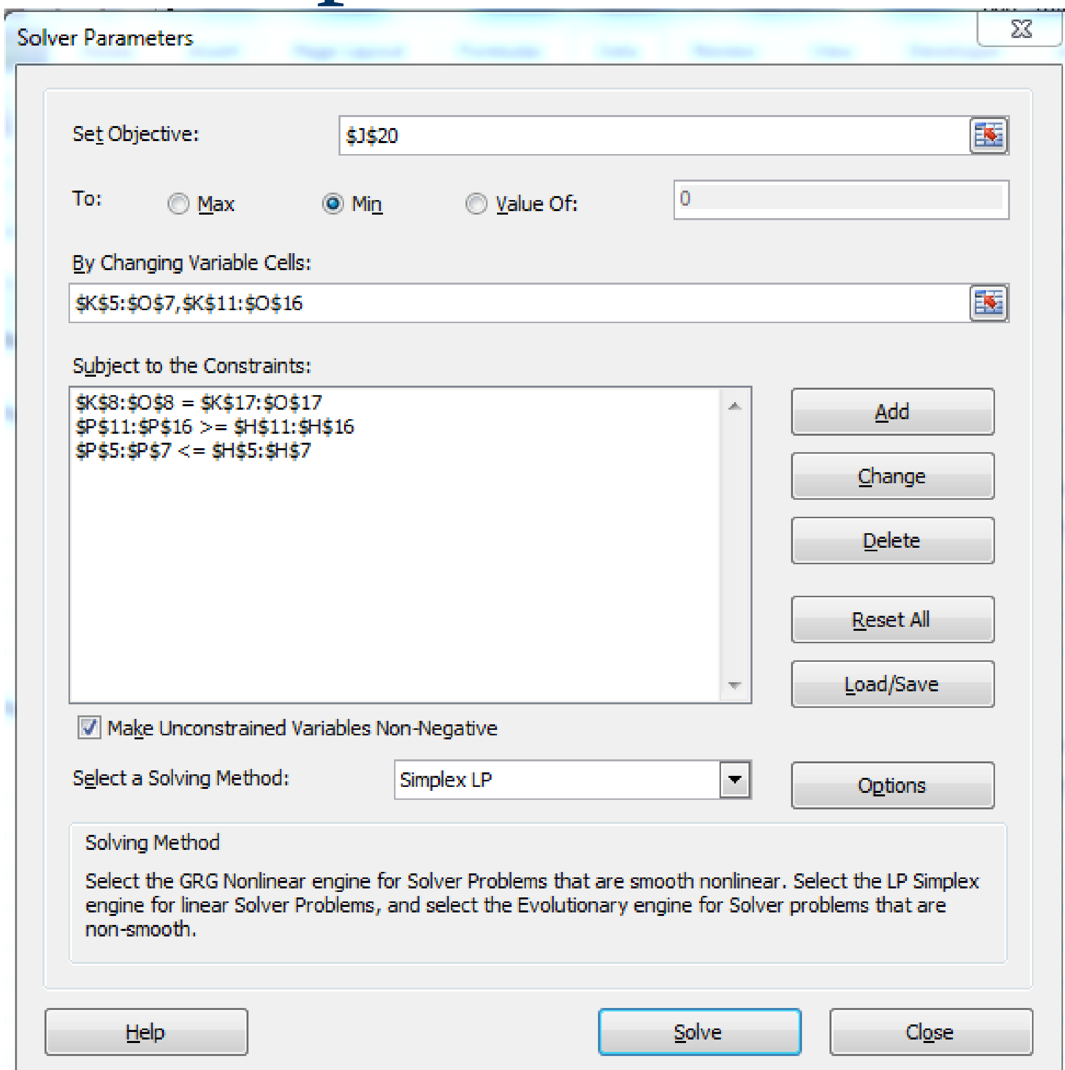
To/From	Warehouse 1	Warehouse 2	Warehouse 3	Warehouse 4	Warehouse 5	Monthly Demand	Unmet Demand
Retail store 1	0	0	0	80	0	**80**	0
Retail store 2	40	0	0	0	0	**40**	0
Retail store 3	0	0	0	0	10	**10**	0
Retail store 4	0	0	0	20	0	**20**	0
Retail store 5	100	0	0	0	0	**100**	0
Retail store 6	0	150	0	0	0	**150**	0
Total	**140**	**150**	**0**	**100**	**10**		

Objective function

17800 =SUMPRODUCT(C5:G7, K5:O7)+SUMPRODUCT(C11:G16, K11:O16)

Excel Template and Initial Solution for the Transshipment Model

Solver Parameters for the Transshipment Model



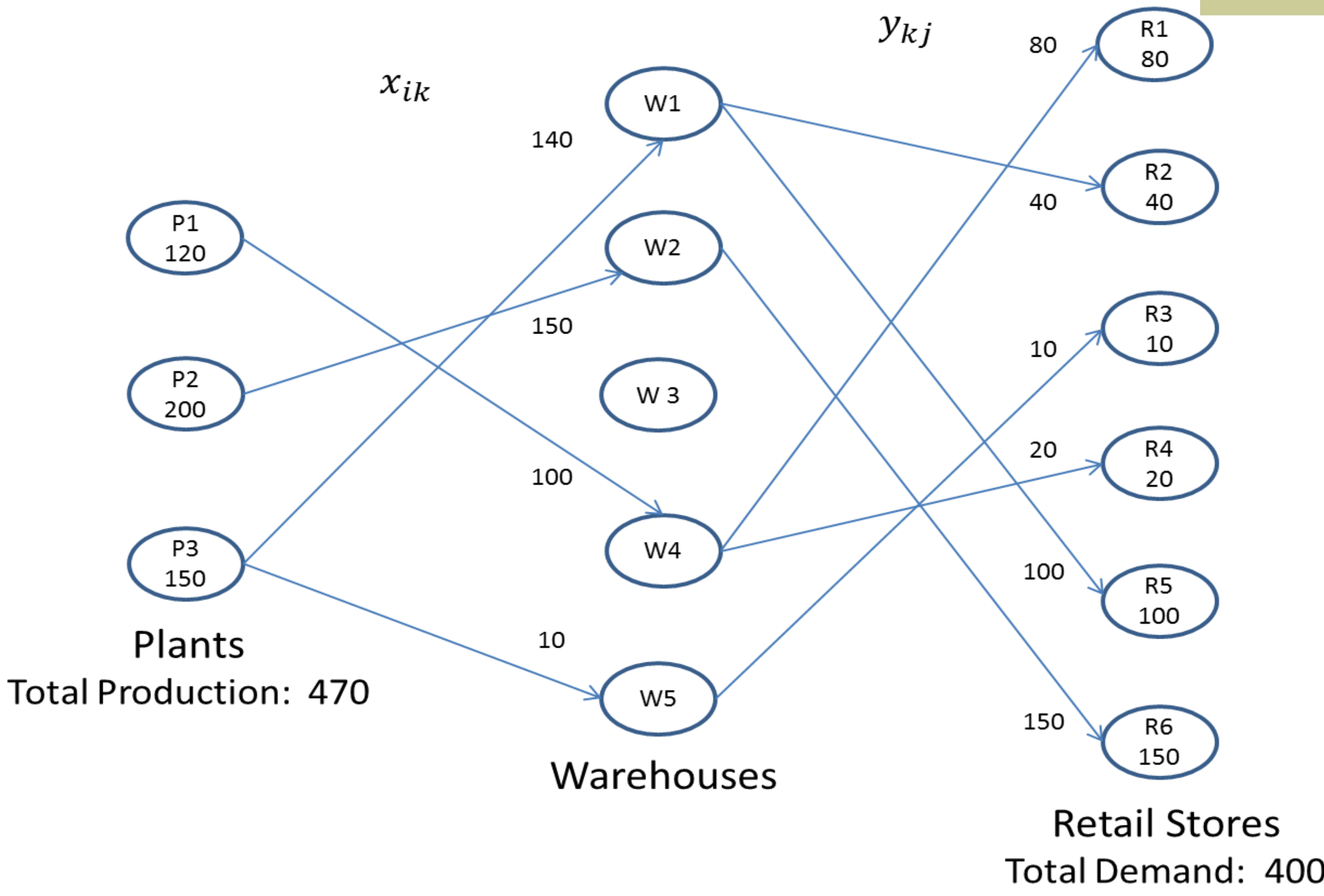
Final Solution for the NXC Transshipment Model

Decision variables:	*Xik*					
From/To	*Warehouse 1*	*Warehouse 2*	*Warehouse 3*	*Warehouse 4*	*Warehouse 5*	*Monthly Production*
Plant 1	0	0	0	100	0	**100**
Plant 2	0	150	0	0	0	**150**
Plant 3	140	0	0	0	10	**150**
Total	**140**	**150**	**0**	**100**	**10**	

Decision Variables:	*Ykj*					
To/From	*Warehouse 1*	*Warehouse 2*	*Warehouse 3*	*Warehouse 4*	*Warehouse 5*	*Monthly Demand*
Retail store 1	0	0	0	80	0	**80**
Retail store 2	40	0	0	0	0	**40**
Retail store 3	0	0	0	0	10	**10**
Retail store 4	0	0	0	20	0	**20**
Retail store 5	100	0	0	0	0	**100**
Retail store 6	0	150	0	0	0	**150**
Total	**140**	**150**	**0**	**100**	**10**	

Objective function	
17800	=SUMPRODUCT(C5:G7, K5:O7)+SUMPRODUCT(C11:G16, K11:O16)

Network Diagram for the Final Solution of the Transshipment Model



Exploring Big Data with Shipment Models

- The availability of big data in the transportation industry
- Radio Frequency Identification (RFID) technology gives decision makers instant access to
  - Delivery times
  - Resource utilizations
  - Geographical coverages
  - Delivery statuses
- The implementation of advanced technologies in transportation and transshipment models allows decision makers to retrieve massive real-time information.

Wrap Up

- Two types of shipment problems:
  - Transportation
  - Transshipment
- Can be represented with network diagrams
- Can be formulated as a special case of LP models
- Requirement:
  - the amount of goods available generated by sources be greater than or equal to the demand for these goods
  - (specific situation) the amount of goods available generated by sources is exactly the same as the demand for these goods
  - then the logistics must assign shipments utilizing all products available and not exceeding market demand

Wrap Up

- In Transshipment Model:
  - Two sets of decision variables:
    - Number of units to be shipped from each source to each intermediary destination (warehouse, distribution center)
    - Number of units to be shipped from each intermediary destination to the final destination.
  - An equality constraint: balances the number of units entering an intermediary destination with the number of units leaving the same location
  - Value of the objective function: the sum of two or more scalar products of the costs of shipment with their respective decision variables.
- The solver methodology and sensitivity analysis are similar to the ones used in LP models.

End of The Lecture

Thank You