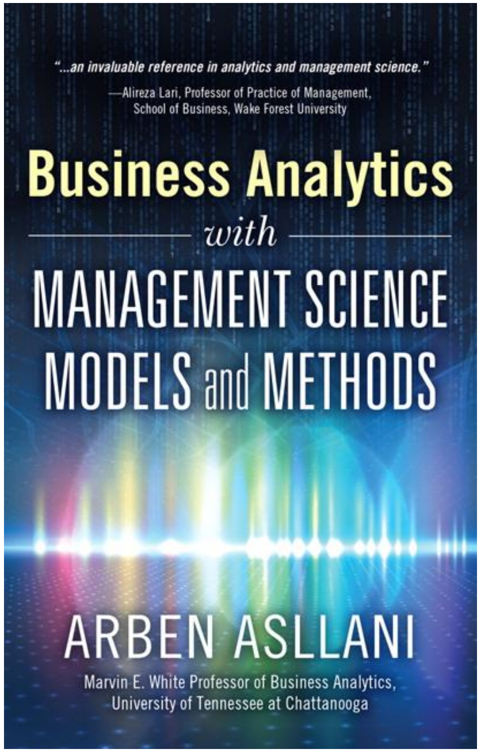
Business Analytics Prescriptive Models



Based on

Business Analytics
With
Management Science
Models and Methods
by
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Chapter 4

Business Analytics with Nonlinear Programming

Business Analytics with Management Science Models and Methods

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Chapter Outline

- Chapter Objectives
- Introduction
- Challenges to NLP Models
  - Local optimum versus global optimum
  - The solution is not always found at an extreme point
  - Multiple feasible areas
- Example1: World Class furniture
- Example2: Optimizing an Investment Portfolio
- Exploring Big Data with Nonlinear Programming
- Wrap up

Chapter Objectives

- Explain what are nonlinear programming models and why they are more difficult to solve than LP models
- Demonstrate how to formulate nonlinear programming models
- Show how to use Solver to reach solution for nonlinear programming models
- Explain how to read the answer report and how to perform sensitivity analysis for nonlinear programming models
- Offer practical recommendations when using nonlinear models in the era of big data

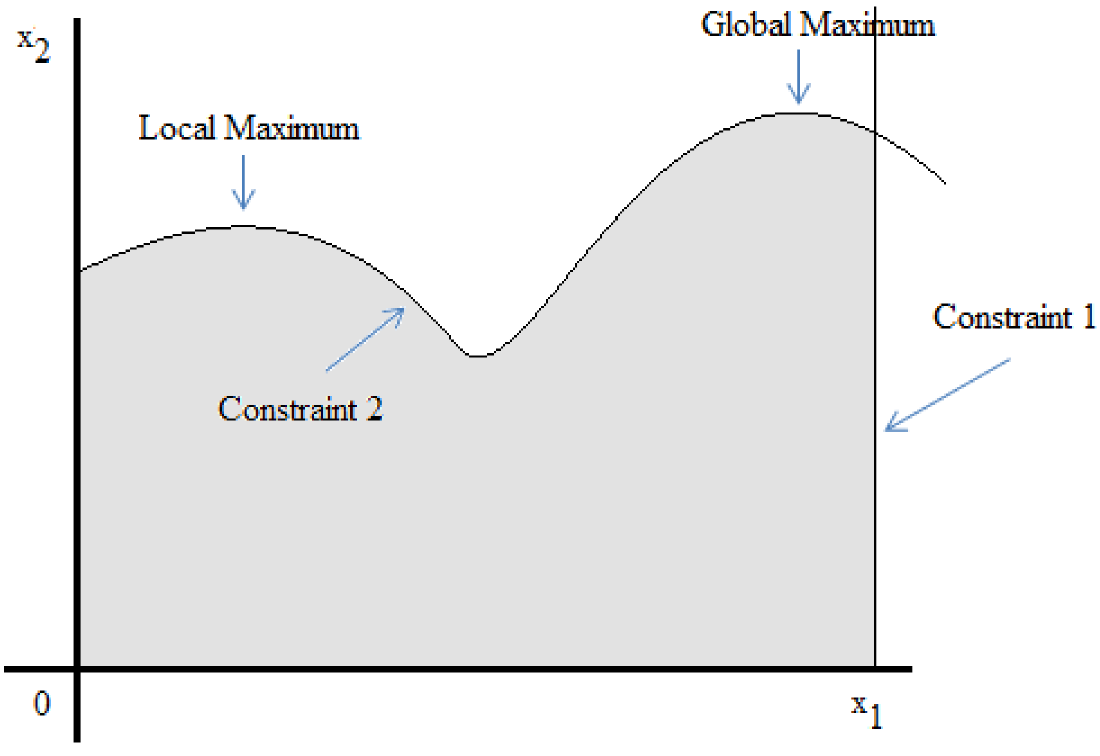
Introduction

- Linear Programming
  - The objective function and constraints are linear equations
  - Both proportional and additive
- Nonlinear Programming (NLP)
  - To deal with not proportional or additive business relationships
  - Same structure: objective function and a set of constraints
  - More challenging to solve
- Necessity of using NLP
  - Difficult to use
  - But more accurate than linear programming

Challenge to NLP Models

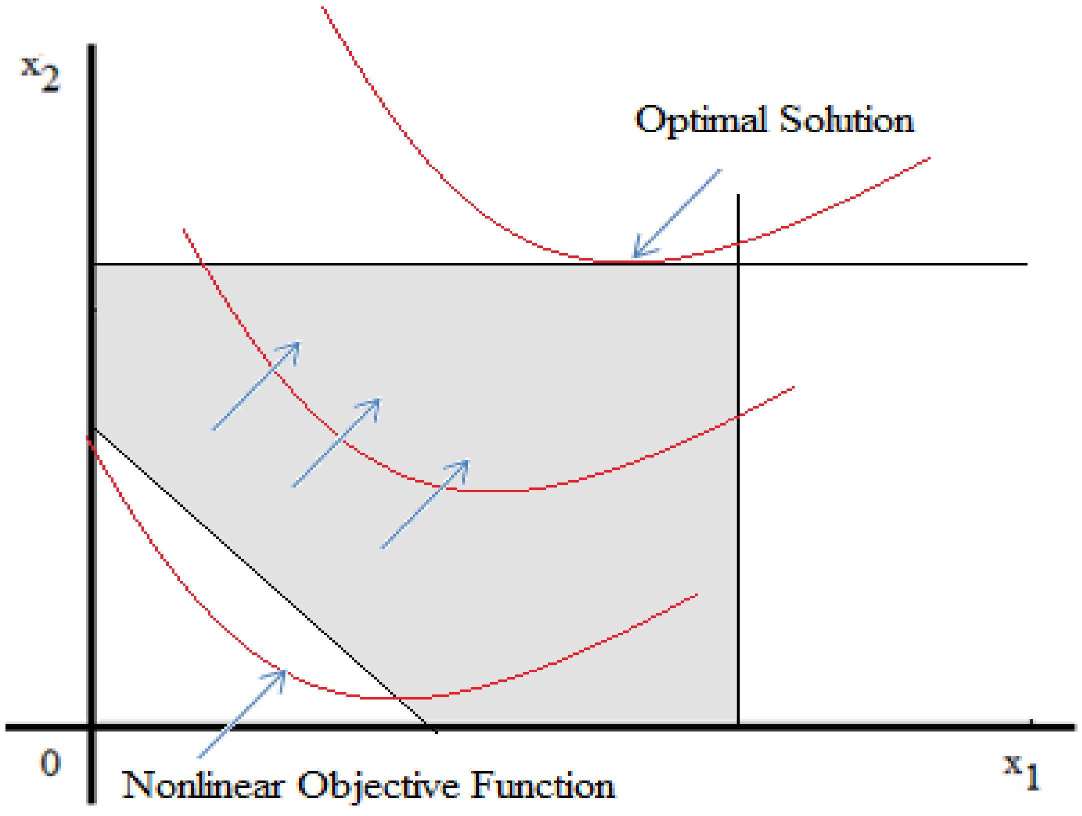
- NLP Models
  - Represented with curved lines or curved surface
  - Complicated to represent relationship with a large number of decision variables

Local Optimum versus Global Optimum



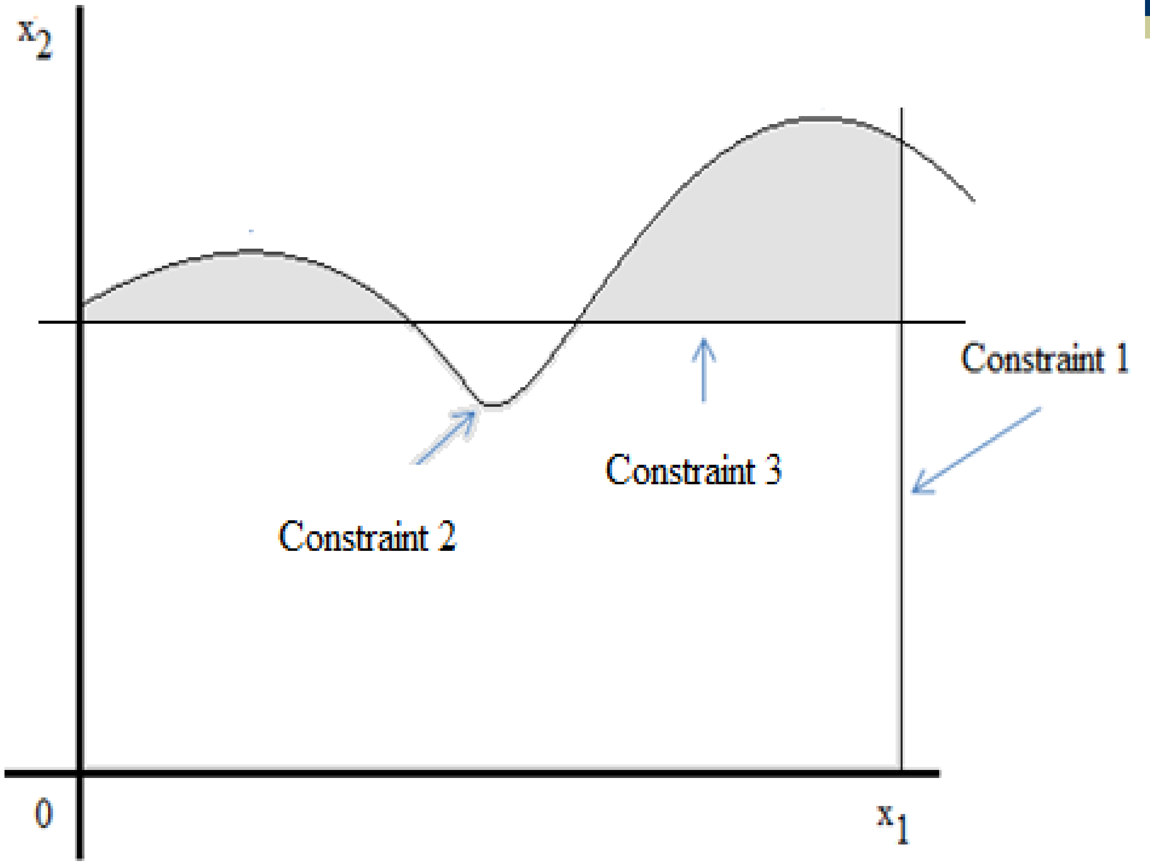
Area of Feasible Solution with Local and Global Maximum

The Solution Is Not Always Found at an Extreme Point



Possible Optimal Solution For NLP Model

Multiple Feasible Areas



Multiple Areas of Feasible Solutions

Challenge to NLP Models

- Three Challenges NLP is Facing:
  - Local Optimum versus Global Optimum
  - The Solution Is Not Always Found at and Extreme Point
  - Multiple Feasible Areas
- Solutions developed to deal with the challenges
  - Advanced heuristics such as genetic algorithms
  - Simulated annealing
  - Generalized reduced gradient (GRG) method
  - Quadratic programming
  - Barrier methods
- However, these algorithms often are not successful

Example1: World Class Furniture

(ch4_Furniture.xlsx)

- Stores five different furniture categories
- Economic Order Quantity (EOQ) model
  - Allow to optimally calculate the amount of inventory with the goal of minimizing the total inventory cost
  - Does not consider:
    - Storage capacity (200,000 cubic feet)
    - purchasing budget ($1.5 million)

Warehouse Capacity (cubic feet)	**200,000**				
Average Inventory Budget	**$1,500,000**				
	Tables	**Chairs**	**Beds**	**Sofas**	**Bookcases**
Weekly Demand (units)	**1125**	**2750**	**3075**	**3075**	**750**
Purchase Price per Unit	**$45**	**$85**	**$125**	**$155**	**$125**
Holding Cost (per unit, period)	**$2**	**$3**	**$3**	**$3**	**$4**
Ordering Cost (per order)	**$100**	**$225**	**$135**	**$135**	**$100**
Storage Space Required (cubic feet per u	**84**	**106**	**140**	**70**	**100**

EOQ Model

- Solution for EOQ model (without constraints), for each furniture category:

$$EOQ_j = \sqrt{\frac{2o_jD_j}{h_j}}$$

- But we have to include constraints! So we'll use NLP model

Formulation of NLP Model

- Define decision variables:

We want to „…*calculate the weekly order quantity for each furniture category…*", so:

- $x_1 = number\ of\ tables\ to\ be\ ordered\ every\ week$
- $x_2 = number\ of\ chairs\ to\ be\ ordered\ every\ week$
- $x_3 = number\ of\ beds\ to\ be\ ordered\ every\ week$
- $x_4 = number\ of\ sofas\ to\ be\ ordered\ every\ week$
- $x_5 = number\ of\ bookcases\ to\ be\ ordered\ every\ week$

Formulation of NLP Model

- Formulate the objective function
  - Holding cost: $$H=\sum_{j=1}^{5}h_j\frac{x_j}{2}=2\frac{x_1}{2}+3\frac{x_2}{2}+3\frac{x_3}{2}+3\frac{x_4}{2}+4\frac{x_5}{2}$$
  - Ordering cost:

$$O=\sum_{j=1}^{5}o_j\frac{D_j}{x_j}=100\frac{1125}{x_1}+225\frac{2750}{x_2}+135\frac{3075}{x_3}+135\frac{3075}{x_4}+100\frac{750}{x_5}$$

  - Total cost: $$Z=H+O=\sum_{j=1}^{5}\left(h_j\frac{x_j}{2}+o_j\frac{D_j}{x_j}\right)$$

Formulation of NLP Model

- Step 3: Identify a set of constraints
  1. Warehouse Capacity: $\sum_{j=1}^{5} s_j x_j \leq C$
  2. Purchasing budget: $\sum_{j=1}^{5}(h_j \frac{x_j}{2} + o_j \frac{D}{x_j} + p_j D_j) \leq P$
- Step 4: Identify a set of non-negativity constraints

Minimize $Z = \sum_{j-1}^{5}(h_j \frac{x_j}{2} + o_j \frac{D_j}{x_j})$ (nonlinear objective function)

Subject to:

$\sum_{j=1}^{5} s_j x_j \leq C$ (linear constraint)

$\sum_{j=1}^{5}(h_j \frac{x_j}{2} + o_j \frac{D}{x_j} + p_j D_j) \leq P$ (nonlinear constraint)

$x_j \geq 0$ for all j=1, 2, 3, 4, 5 (non-negativity constraint)

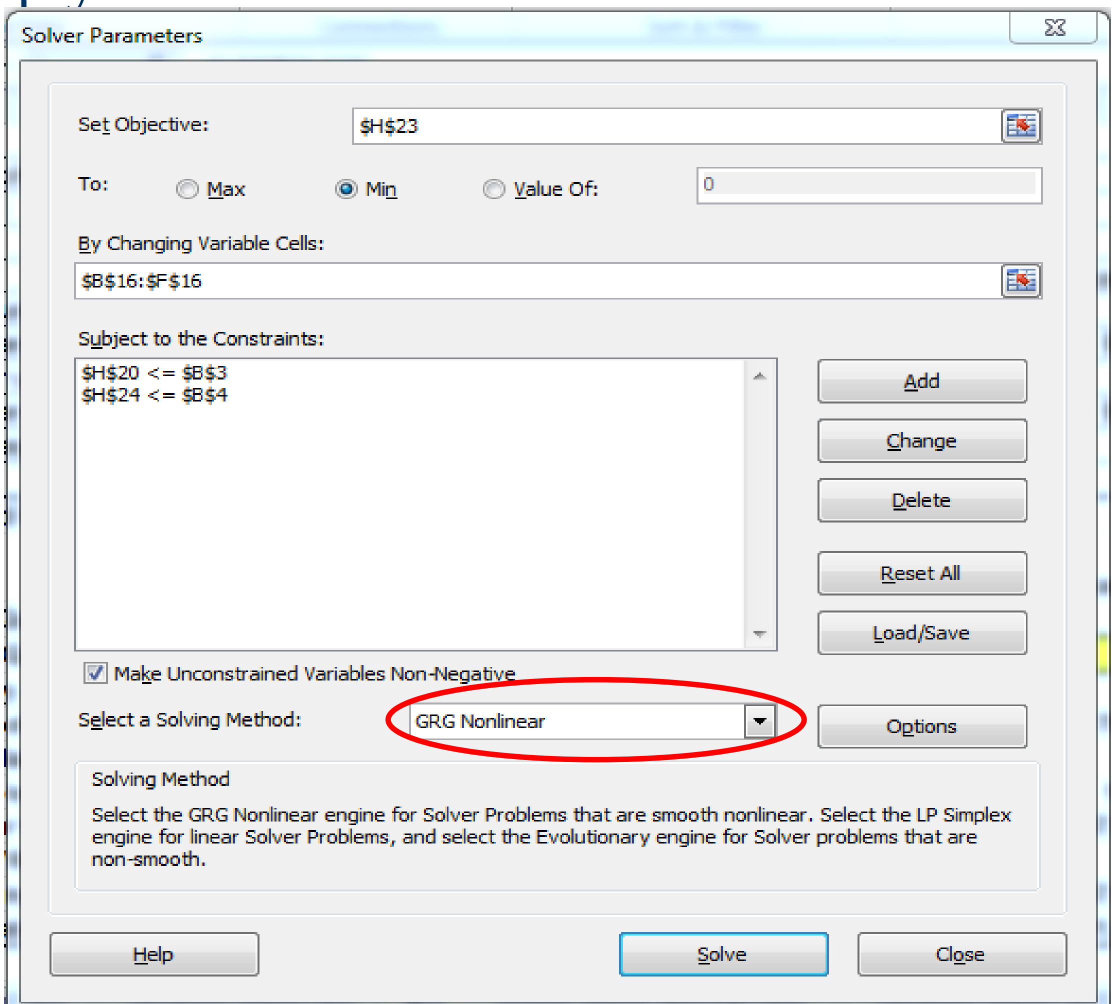
Solving NLP Model with Solver

- Step 1: Create an Excel Template

	A	B	C	D	E	F	G
1	Operational Data for the Inventory Management System at FWC						
2							
3	Warehouse Capacity (cubic feet)	200,000					
4	Average Inventory Budget	$1,500,000					
5							
6		Tables	Chairs	Beds	Sofas	Bookcases	
7	Weekly Demand (units)	1125	2750	3075	3075	750	
8	Purchase Price per Unit	$45	$85	$125	$155	$125	
9	Holding Cost (per unit, period)	$2	$3	$3	$3	$4	
10	Ordering Cost (per order)	$100	$225	$135	$135	$100	
11	Storage Space Required (cubic feet per	84	106	140	70	100	
12							
13	Calculations and Results						
14		Tables	Chairs	Beds	Sofas	Bookcases	Totals
15	Economic Order Quantity (EOQ)	335	642	526	526	194	
16	Optimized Order Quantity	289	575	457	469	180	
17	Average Inventory	144	287	229	235	90	
18	Average Number of Orders per period	3.90	4.78	6.73	6.55	4.18	
19	Total Supply Available	1125	2750	3075	3075	750	
20	Maximum Cubic Foot Storage Required	24250	60942	64010	32844	17953	200,000
21	Ordering Cost per period	$390	$1,076	$908	$885	$418	$3,676
22	Holding Cost per period	$289	$862	$686	$704	$359	$2,900
23	Inventory Operating Cost per period	$678	$1,939	$1,594	$1,589	$777	$6,576
24	Average Inventory Value	$51,303	$235,689	$385,969	$478,214	$94,527	$1,245,701

Solving NLP Model with Solver

- Step2: Apply Solver



Solving NLP Model with Solver

- Step3: Interpret Solver Solution
  - Objective Cell
  - Variable Cells
  - Constraints

Objective Cell (Min)

Cell	Original Value	Final Value
$H$23	$163,725	$6,576

Variable Cells

Cell	Name	Original Value	Final Value	Integer
$B$16	Optimized Order Quantity Tables	10	289	Contin
$C$16	Optimized Order Quantity Chairs	10	575	Contin
$D$16	Optimized Order Quantity Beds	10	457	Contin
$E$16	Optimized Order Quantity Sofas	10	469	Contin
$F$16	Optimized Order Quantity Bookcases	10	180	Contin

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
$H$20		200000	$H$20<=$B$3	Not Binding	0.00011507
$H$24		$1,245,701	$H$24<=$B$4	Not Binding	254298.8745

Sensitivity Analysis for NLP Model

Variable Cells

Cell	Name	Final Value	Reduced Gradient
$B$16	Optimized Order Quantity Tables	288.694089	0
$C$16	Optimized Order Quantity Chairs	574.9289799	0
$D$16	Optimized Order Quantity Beds	457.2136075	0
$E$16	Optimized Order Quantity Sofas	469.2041824	0
$F$16	Optimized Order Quantity Bookcases	179.5302673	0

Constraints

Cell	Name	Final Value	Lagrange Multiplier
$G$20	Maximum Cubic Foot Storage Required Totals	199999.9999	-0.003470152
$G$24	Average Inventory Value Totals	$1,245,701.13	0

- Reduced Cost →Reduced gradient
- Shadow Price →Lagrange multiplier
- Valid only at the point of the optimal solution

Example2:
Optimizing an Investment Portfolio

- Trade-off between return on investment and risk is an important aspect in financial planning
- Smart Investment Services (SIS) designs investment products
- Prepare a portfolio as a mix of eight mutual funds that maximizes average return rate and keeps moderate risk level (variance ≤ 15)

Mutal Funds	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6	Year 7	Year 8	Year 9	Year 10
Large-cap U.S. Growth	17%	9%	7%	-2%	-2%	8%	8%	16%	2%	8%
Mid-cap U.S. Growth	11%	13%	-2%	10%	4%	14%	4%	-4%	-9%	-4%
Small-cap U.S. Growth	8%	-13%	-3%	4%	18%	10%	13%	11%	10%	1%
Large-cap U.S. Value	13%	3%	-1%	7%	10%	13%	-6%	6%	8%	17%
Mid-cap U.S. Value	6%	-3%	-7%	18%	1%	5%	12%	12%	-5%	8%
Small-cap U.S. Value	13%	17%	12%	7%	14%	2%	2%	-2%	6%	2%
International Stock	6%	4%	9%	9%	3%	5%	5%	7%	10%	11%
Specialty Funds	8%	1%	6%	8%	8%	8%	-6%	-5%	6%	8%

Investment Portfolio Problem Formulation

(ch4_Portfolio.xlsx)

1. Define decision variables

$$x_j = propotrion\ of\ portfolio\ invested\ in\ j-th\ fund$$
$$(j = 1, \dots, 8)$$

Investment Portfolio Problem Formulation

2. Formulate objective function

(to maximize the average return rate over 10 years)

return from fund *j* in year *i*: r_{ij}

average return in year *i*: $R_i = \sum_{j=1}^{8} r_{ij} x_j$

average return in 10 years: $\bar{R} = \frac{1}{10} \sum_{i=1}^{10} R_i$

$$Z = \bar{R} = \frac{1}{10} \sum_{i=1}^{10} \sum_{j=1}^{8} r_{ij} x_j$$

Investment Portfolio Problem Formulation

- Step 3: Identify a set of constraints
  1. that the sum of the proportions must equal to one
  2. The variance of the return: $Var = \frac{1}{10}\sum_{i=1}^{10}(R_i - \overline{R})^2$
  3. The risk constraint: $\frac{1}{10}\sum_{i=1}^{10}(R_i - \overline{R})^2 \leq 15$
- Step 4: Identify a set of non-negativity constraints

$$\text{Maximize } Z = \frac{R_1+R_2+\cdots+R_{10}}{10} \text{ (linear objective function)}$$

Subject to:

$$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 = 1 \quad \text{(linear constraint)}$$

$$\frac{1}{10}\sum_{i=1}^{10}(R_i - \overline{R})^2 \leq 15 \text{ (nonlinear constraint)}$$

$$\sum_{j=1}^{8} r_{ij}x_j = R_i \text{ for all } i\text{=1,2,…10 (linear constraint)}$$

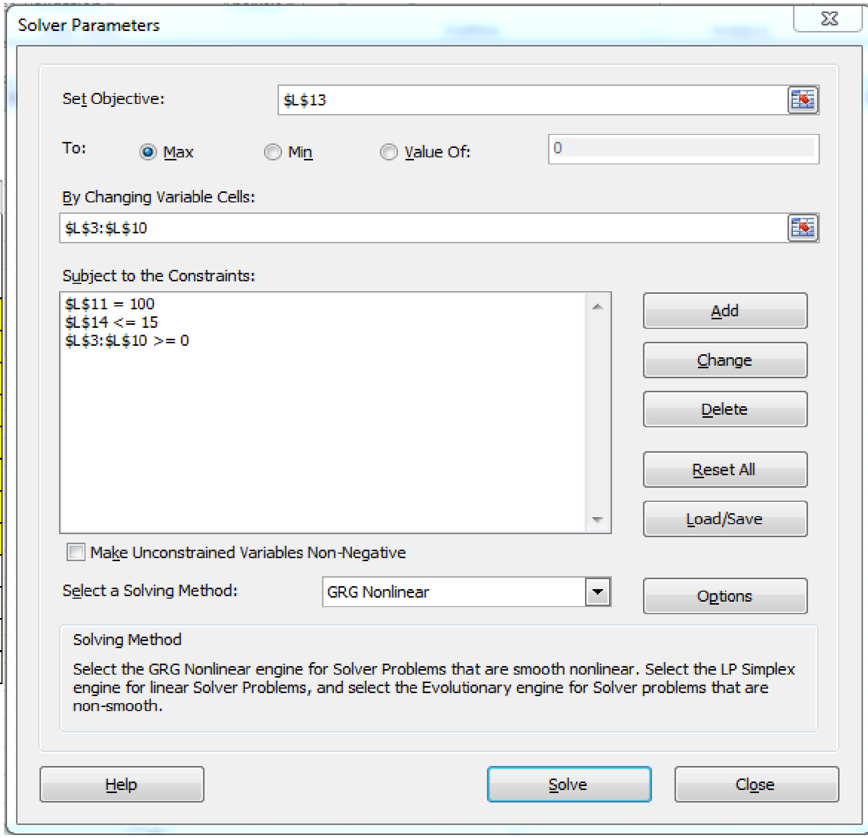
$x_i \geq 0$ for all i=1, 2, ... 10 (non-negativity constraint)

Solving the Portfolio Problem

	A	B	C	D	E	F	G	H	I	J	K	L
2	Mutal Funds	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6	Year 7	Year 8	Year 9	Year 10	Decision Variables
3	Large-cap U.S. Growth	17%	9%	7%	-2%	-2%	8%	8%	16%	2%	8%	12.50
4	Mid-cap U.S. Growth	11%	13%	-2%	10%	4%	14%	4%	-4%	-9%	-4%	12.50
5	Small-cap U.S. Growth	8%	-13%	-3%	4%	18%	10%	13%	11%	10%	1%	12.50
6	Large-cap U.S. Value	13%	3%	-1%	7%	10%	13%	-6%	6%	8%	17%	12.50
7	Mid-cap U.S. Value	6%	-3%	-7%	18%	1%	5%	12%	12%	-5%	8%	12.50
8	Small-cap U.S. Value	13%	17%	12%	7%	14%	2%	2%	-2%	6%	2%	12.50
9	International Stock	6%	4%	9%	9%	3%	5%	5%	7%	10%	11%	12.50
10	Specialty Funds	8%	1%	6%	8%	8%	8%	-6%	-5%	6%	8%	12.50
11												100
12												Average
13	Rate of Return	10.25	3.88	2.63	7.63	7.00	8.13	4.00	5.13	3.50	6.38	5.85
14	Variance	19.36	3.90	10.40	3.15	1.32	5.18	3.42	0.53	5.52	0.28	5.31

=SUMPRODUCT(B3:B10,$L$3:$L$10)

=POWER((B13-$L$13),2)



The what-if template for this investment problem

Solving the Portfolio Problem

Objective Cell (Max)

Cell	Name	Original Value	Final Value
$L$13	Rate of Return Average	5.85	7.22

Variable Cells

Cell	Name	Original Value	Final Value	Integer
$L$3	Large-cap U.S. Growth Decision Variables	12.50	32.03	Contin
$L$4	Mid-cap U.S. Growth Decision Variables	12.50	-	Contin
$L$5	Small-cap U.S. Growth Decision Variables	12.50	-	Contin
$L$6	Large-cap U.S. Value Decision Variables	12.50	5.85	Contin
$L$7	Mid-cap U.S. Value Decision Variables	12.50	-	Contin
$L$8	Small-cap U.S. Value Decision Variables	12.50	62.12	Contin
$L$9	International Stock Decision Variables	12.50	-	Contin
$L$10	Specialty Funds Decision Variables	12.50	-	Contin

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
$L$11	Decision Variables	100	$L$11=100	Binding	0
$L$14	Variance Average	15.00	$L$14<=15	Binding	0
$L$3	Large-cap U.S. Growth Decision Variables	32.03	$L$3>=0	Not Binding	32.03
$L$4	Mid-cap U.S. Growth Decision Variables	-	$L$4>=0	Binding	-
$L$5	Small-cap U.S. Growth Decision Variables	-	$L$5>=0	Binding	-
$L$6	Large-cap U.S. Value Decision Variables	5.85	$L$6>=0	Not Binding	5.85
$L$7	Mid-cap U.S. Value Decision Variables	-	$L$7>=0	Binding	-
$L$8	Small-cap U.S. Value Decision Variables	62.12	$L$8>=0	Not Binding	62.12
$L$9	International Stock Decision Variables	-	$L$9>=0	Binding	-
$L$10	Specialty Funds Decision Variables	-	$L$10>=0	Binding	-

Exploring Big data with NLP

- Volume
  - The availability of more data allows organization to explore, formulate and solve previously unsolvable problem
- Variety and Velocity
  - Offer significant challenges for optimization models
- Advanced software programs
  - Used to navigate trillions of permutations, variables and constraints
  - Such as Solver

Wrap up

- The NLP formulation shares the same with LP model
- GRG algorithm is best suited for NLP models
- A risk that the algorithm will result in a local optimum
- Provide a good starting point in the trial template
- Add a non-negativity constraint for decision variables
- Pay close attention when selecting Solver parameters

Wrap up

Consider Potential Challenges of NLP Models

- Choose between LP models and NLP models
- Nonlinear models are more difficult to solve
- Nonlinear models offer better representation of nonlinear business relationships

Formulate NLP Models

- Step 1: Define decision variables
- Step 2: Formulate the objective function
- Step 3: Identify constraints
- Step 4: Identify non-negativity constraints

Solve NLP Models with Solver

- Step 1: Create an Excel template
- Step 2: Apply Solver
- Step 3: Interpret Solver solution

Perform Sensitivity Analysis for NLP Models

- Reduced cost vs. Reduced gradient
- Shadow price vs. Lagrange multiplier
- Upper & Lower limits vs. Valid only at point of optimal solution

End of The Lecture

Thank You