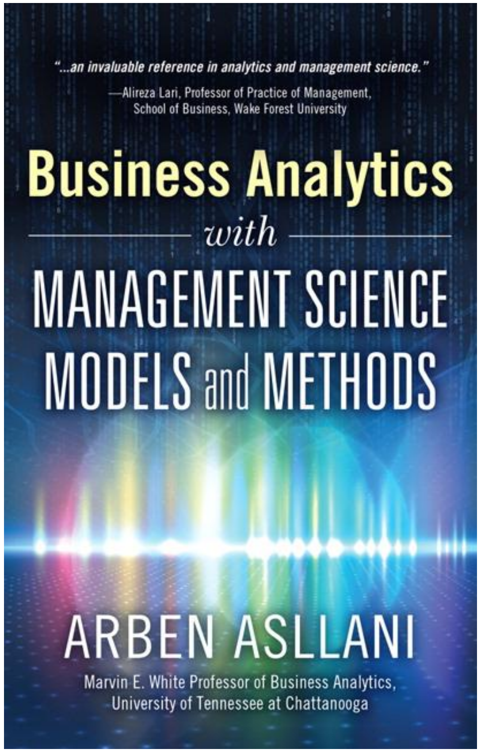
Business Analytics
Prescriptive Models



<u>Based on</u>

Business Analytics
With
Management Science
Models and Methods
by
Arben Asllani

Chapter 5

Business Analytics with Goal Programming

Business Analytics with Management Science Models and Methods

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Chapter Outline

- Chapter Objectives
- Prescriptive Analytics in Action
- Introduction
- GP Formulation
  - Example 1: Rolls bakery
  - Example 2: World Class Furniture
- Exploring Big Data with Goal Programming
- Wrap up

Chapter Objectives

- Discuss the importance of using goal programming models in business applications
- Demonstrate the process of formulating linear and nonlinear goal programming models
- Demonstrate the use of Solver for solving linear and nonlinear goal programming models
- Discuss the concept of aspiration levels and goal priorities
- Distinguish between functional variables and deviational variables in goal programming models
- Distinguish between systems constraints and goal programming constraints
- Offer practical recommendations for implementing goal programming models in business settings

Prescriptive Analytics in Action

- Airbus is the world's leading aircraft manufacturer
- Goals of the company:
  - Improve products design
  - Reduce product development time
  - Reduce cost
- Constraints:
  - Regulatory environments
  - Fuel efficiency
  - Customer expectations
- Use of optimization software called MACROS
  - Enabled engineers to find better design choices for the aircraft with optimum performance relative to their respective seat and range capabilities

Introduction

- Difference of Goal Programming (GP)
  - Seek to achieve not one but multiple goals
  - A set of linear or nonlinear constraints
- History of Goal Programming
  - First introduced by *Charnes*, *Cooper* and *Ferguson* in 1955
  - First described as a decision analysis tool by *Lee* in 1972
  - Later expanded by *Ignizio* in 1974 and *Romero* in 1991
  - *Schniederjans* offered an up-to-date overview in 1995
  - *Jones* and *Tamiz* provided a bibliography of GP applications in 2010
- The value of the objective function in one model becomes a new constraint until all optimization goals are incorporated

GP Formulation

- Components of GP formulation:
  - A minimization objective function
  - A set of goal programming constraints
  - An optional set of system constraints
  - Non-negativity constraints for functional variables and deviational variables

Example1: (ch5_Bakery.xlsx) Rolls Bakery Revisited

- The decision maker wants to determine how many dinner roll cases (DRC) and sandwich roll cases (SRC) to produce in order to maximize the net profit.
- 150 machine hours
- Each product is produced in lots of 1000 cases.
- Products have a different wholesale price, processing time, cost of raw materials, and weekly market demand.

Table 5.1 Production Requirements for One Lot of the Two Products

	(1)	(2) = (1)*1000	(3)	(4)	(5) = (2)-(3)*10-(4)	(6)	(7) = (6)/1000
Product	Wholesale Price per Case	Wholesale Price per Lot	Processing time (in hours) per Lot	Cost of Raw Materials per Lot	Net Profit per Lot	Demand for Cases	Demand for Production Lots
DRC	$0.75	$750	10	$250	$400	3000	3
SRC	$0.65	$650	15	$200	$300	4000	4

Example1: Rolls Bakery Revisited

- Recall from chapter 2 that the LP formulation of the above problem is:

$$\text{Max } Z = 400x_1 + 300x_2$$

Subject to:

- $10x_1 + 15x_2 \leq 150$ (machine hours) (5.1)
- $x_1 \geq 3$ (demand for product 1) (5.2)
- $x_2 \geq 4$ (demand for product 2) (5.3)
- $x_1, x_2 \geq 0$ (Non-negativity constraints) (5.4)

- Solution: The company run nine lots of DRC and four lots of SRC

Example1: Rolls Bakery Revisited

- Revisit the same problem with a new set of goals:
  - *Priority 1:* Company should not produce more than two lots over the weekly demand for each product
  - *Priority 2:* Company should meet the weekly demand for both products
  - *Priority 3:* Company should utilize available machine hours
  - *Priority 4:* Company should make the maximum possible net profit

- Helpful definition:
  - Aspiration Level: indicates the desired or acceptable level of objective
  - Goal deviation: the difference between aspiration level and the actual accomplishment for each goal
  - Goal priority: the order of importance for achieving each goal

Sometimes reflect potential penalties for not achieving the goal

GP Formulation Steps:

- New set of decision variables – deviational variables
  - Represent underachievement or overachievement of a given goal
  - Can be added to represent goals that are not currently represented by the existing constraints in the LP model
  - Decision maker needs to incorporate the deviational variables into a GP objective function and into the newly created or modified constraints
- Step-by-step methodology for GP formulation.
  - Step 1: Formulate the problem as a simple LP model
  - Step 2: Define deviational variables for each goal
  - Step 3: Write GP and system constraints
  - Step 4: Add non-negativity constraints for functional and deviational variables
  - Step 5: Determine the variables to be minimized in the objective function
  - Step 6: Write the objective function with priorities

Step 1: Formulate the problem as a simple LP model

1, the LP model is extended to include two more constraints:

Max $Z = 400x_1 + 300x_2$

Subject to:

- $10x_1 + 15x_2 \leq 150$ (machine hours) (1)
- $x_1 \geq 3$ (demand for product 1) (2)
- $x_2 \geq 4$ (demand for product 2) (3)
- $x_1 \leq 5$ (newly added constraint) (4)
- $x_2 \leq 6$ (newly added constraint) (5)
- $x_1, x_2 \geq 0$ (non-negativity constraints) (6)

Step 2: Define deviational variables for each goal

Goal 1: *Company should not produce more than two lots over the weekly demand for each product*

- s_4^+ = can be defined as the number of lots from DRCs to be over produced when the aspiration level is 5 lots
- s_4^- = can be defined as the number of lots from DRCs to be under produced when the aspiration level is 5 lots
- s_5^+ = can be defined as the number of lots from SRCs to be over produced when the aspiration level is 6lots
- s_5^- = can be defined as the number of lots from SRCs to be under produced when the aspiration level is 6lots

Step 2: Define deviational variables for each goal

Goal 2: Company should meet the weekly demand for both products

- s_2^+ = can be defined as the number of lots from DRCs to be over produced when the aspiration level is 3lots
- s_2^- = can be defined as the number of lots from DRCs to be under produced when the aspiration level is 3lots
- s_3^+ = can be defined as the number of lots from SRCs to be over produced when the aspiration level is 4lots
- s_3^- = can be defined as the number of lots from SRCs to be under produced when the aspiration level is 4lots

Step 2: Define deviational variables for each goal

Goal 3: Company should utilize available machine hours

- s_1^+ = can be defined as the number of machine hours be over utilized when the aspiration level is 150hours
- s_1^- = can be defined as the number of machine hours be underutilized when the aspiration level is 150hours

Goal 4: Company should make the maximum

- s_6^+ = can be defined as the amount of net profit to be overachieved when the aspiration level is \$4,800
- s_6^- = can be defined as the amount of net profit to be under achieved when the aspiration level is \$4,800

Step 3:
Write GP and system constraints

- $10x_1 + 15x_2 + s_1^- - s_1^+ = 150$ (machine hours) (1)
- $x_1 + s_2^- - s_2^+ = 3$ (minimum demand for DRC) (2)
- $x_2 + s_3^- - s_3^+ = 4$ (minimum demand for SRC) (3)
- $x_1 + s_4^- - s_4^+ = 5$ (maximum demand for DRC) (4)
- $x_2 + s_5^- - s_5^+ = 6$ (maximum demand for SRC) (5)
- $400x_1 + 300x_2 + s_6^- - s_6^+ = 4800$ (net profit constraint) (6)

Step 4: Add non-negativity constraints for functional and deviational variables

Similar to LP models, the decision maker must enforce that all variables are zero or positive:

- $x_1, x_2, s_1^-, s_1^+, s_2^-, s_2^+, s_3^-, s_3^+, s_4^-, s_4^+, s_5^-, s_5^+, s_6^-, s_6^+ \geq 0$

Step 5: Determine the variables to be minimized in the objective function

To determine the variables to be minimized, for each constraint we choose variable that allows L.H.side to be possibly close to R.H.side (to assure possibly best utilization of x variables)

It depends on the direction of inequality:

- for ($\leq$) constraint we'll choose (s^{+}) variable
- for ($\geq$) constraint we'll choose (s^{-}) variable

Consider constraint 1 from Step 3:

$$10x_1 + 15x_2 + s_1^- - s_1^+ = 150 \qquad \text{(Constraint 1 was } \leq\text{)}$$

The actual number of machine hours utilized need to be less than 150 hours, so we minimize the positive deviational variable s_1^{+}.

That means that the number of machine hours utilized will not exceed 150 hours

Step 5: Determine the variables to be minimized in the objective function

Applying the same reasoning to all constraints, deviational variables to be included in the objective function of the GP model are:

- Goal 1: s_4^+, s_5^+ (Constraint 4,5: $\leq$)
- Goal 2: s_2^-, s_3^- (Constraint 2,3: $\geq$)
- Goal 3: s_1^+ (Constraint 1: $\leq$)
- Goal 4: s_6^- (Constraint 6: $\geq$)

Step 6: Write the objective function with priorities

Priority values are usually defined by the decision maker using his or her own experience or preferences in specific business settings.

In our example:

- the penalty for not achieving the net profit by $1 is obviously $1,
- the penalty for overutilizing 150 machine hours is $2 per each additional hour in the form of overtime pay,
- the penalty for not meeting or exceeding the demand for each unit is $30 as unsold products can be sold at an average loss of $30 per unit.

Step 6: Write the objective function with priorities

In the GP formulation, the ratio between priorities is important, not the absolute differences between them.

The set of priorities P1 = 300, P2 = 300, P3 = 20, and P4 = 10 is the same as the set P1 = 30, P2 = 30, P3 = 2, and P4 = 1 because the ratio is P1/P2 = 1, P1/P3 = 15, P2/P3 = 15, and P3/P4 = 2 for both sets of priorities.

Priority values (defined by decision maker)

- *Priority 1:* P1 = 300
- *Priority 2:* P2 = 300
- *Priority 3:* P3 = 20
- *Priority 4:* P4 = 10

Step 6: Write the objective function with priorities

- Both goal 1 and goal 2 have the highest priority for the decision maker (P1=P2=300).
- Goal 3 has the second highest priority (P3=20) and goal 4 has the third priority (P4=10). As such, the objective function of the GP model can be written as:

$$\textit{Minimize } Z = 300s_4^+ + 300s_5^+ + 300s_2^- + 300s_3^- + 20s_1^+ + 10s_6^-$$

- Since the optimization algorithm will seek to minimize the value of Z, the first deviational variables to reduced or even become zero are those that are associated with the largest values of contribution coefficients.

Putting it Together

GP Formulation for Rolls Bakery

$$Minimize\ Z = 300s_4^+ + 300s_5^+ + 300s_2^- + 300s_3^- + 20s_1^+ + 10s_6^-$$

Subject to:

- $10x_1 + 15x_2 + s_1^- - s_1^+ = 150$ (machine hours) (1)
- $x_1 + s_2^- - s_2^+ = 3$ (minimum demand for DRC) (2)
- $x_2 + s_3^- - s_3^+ = 4$ (minimum demand for SRC) (3)
- $x_1 + s_4^- - s_4^+ = 5$ (maximum demand for DRC) (4)
- $x_2 + s_5^- - s_5^+ = 6$ (maximum demand for SRC) (5)
- $400x_1 + 300x_2 + s_6^- - s_6^+ = 4800$ (net profit constraint) (6)

And

- $x_1, x_2, s_1^-, s_1^+, s_2^-, s_2^+, s_3^-, s_3^+, s_4^-, s_4^+, s_5^-, s_5^+, s_6^-, s_6^+ \geq 0$

	A	B	C	D	E	F	G	H	I
1		Products	Wholesale Price per Case	Wholesale Price per Lot	Processing Time (in hours) per Lot	Cost of Raw Materials per Lot	Net Profit per Lot	Demand for Cases	Demand for Production Lots
2		Dinner Roll Case (DRC)	$0.75	$750	10	$250	$400	3000	3
3		Sandwich Roll Case (SRC)	$0.65	$650	15	$200	$300	4000	4
4									
5		Functional Variables	Number of Lots per Week	Net Profit Per Lot	Total Machine Time Used (in hours)	Achievement Level for Minimum	Aspiration Level for Minimum	Achivement Level for Maximum	Aspiration Level for Maximum
6		DRC Lots (x1)	7.5	$3,000	75	3	3	5	5
7		SRC Lots (x2)	6	$1,800	90	4	4	6	6
8		Machine Hours			165	150	150		
9		Profit		$4,800		4800	4800		
10									
11	Priority	Deviational Variables	Values	Priority Value			Priorities		
12	No Priority	s1negative	0	0	1		P1	300	
13	P3	s1positive	15	20	-1		P2	300	
14	P2	s2negative	0	300	1		P3	20	
15	No Priority	s2positive	4.5	0	-1		P4	10	
16	P2	s3negative	0	300	1		No Priority	0	
17	No Priority	s3positive	2	0	-1				
18	No Priority	s4negative	0	0	1				
19	P1	s4positive	2.5	300	-1				
20	No Priority	s5negative	0	0	1				
21	P1	s5positive	0	300	-1				
22	P4	s6negative	0	10	1				
23	No Priority	s6positive	0	0	-1				
24		Objective Function	1050						

	A	B	C	D	E	F	G	H	I
1		Products	Wholesale Price per	Wholesale Price per Lot	Processing Time (in hours)	Cost of Raw Materials per Lot	Net Profit per Lot	Demand for Cases	Demand for Production
2		Dinner Roll Case (DRC)	0.75	=C2*1000	10	250	=D2-E2*10-F2	3000	=H2/1000
3		Sandwich Roll Case (SRC)	0.65	=C3*1000	15	200	=D3-E3*10-F3	4000	=H3/1000
4									
5		Functional Variables	Number of Lots per	Net Profit Per Lot	Total Machine Time Used (in	Achievement Level for Minimum	Aspiration Level for Minimum	Achivement Level for Maximum	Aspiration Level for
6		DRC Lots (x1)	7.5	=C6*G2	=C6*E2	=C6+SUMPRODUCT(C14:C15,E14:E15)	=I2	=C6+SUMPRODUCT(C18:C19,E18:E19)	5
7		SRC Lots (x2)	6	=C7*G3	=C7*E3	=C7+SUMPRODUCT(C16:C17,E16:E17)	=I3	=C7+SUMPRODUCT(C20:C21,E20:E21)	6
8		Machine Hours			=E6+E7	=E8+SUMPRODUCT(C12:C13, E12:E13)	150		
9		Profit		=D6+D7		=D9+SUMPRODUCT(C22:C23,E22:E23)	4800		
10									
11	Priority	Deviational Variables	Values	Priority Value			Priorities		
12	No Priority	s1negative	0	=VLOOKUP(A12, $G$12:$H$16, 2, FALSE)	1		P1	300	
13	P3	s1positive	15	=VLOOKUP(A13, $G$12:$H$16, 2, FALSE)	-1		P2	300	
14	P2	s2negative	0	=VLOOKUP(A14, $G$12:$H$16, 2, FALSE)	1		P3	20	
15	No Priority	s2positive	4.5	=VLOOKUP(A15, $G$12:$H$16, 2, FALSE)	-1		P4	10	
16	P2	s3negative	0	=VLOOKUP(A16, $G$12:$H$16, 2, FALSE)	1		No Priority	0	
17	No Priority	s3positive	2	=VLOOKUP(A17, $G$12:$H$16, 2, FALSE)	-1				
18	No Priority	s4negative	0	=VLOOKUP(A18, $G$12:$H$16, 2, FALSE)	1				
19	P1	s4positive	2.5	=VLOOKUP(A19, $G$12:$H$16, 2, FALSE)	-1				
20	No Priority	s5negative	0	=VLOOKUP(A20, $G$12:$H$16, 2, FALSE)	1				
21	P1	s5positive	0	=VLOOKUP(A21, $G$12:$H$16, 2, FALSE)	-1				
22	P4	s6negative	0	=VLOOKUP(A22, $G$12:$H$16, 2, FALSE)	1				
23	No Priority	s6positive	0	=VLOOKUP(A23, $G$12:$H$16, 2, FALSE)	-1				
24		Objective Function	=SUMPRODUCT						

Model setup and solution for GP model

Solving GP Models with Solver
(ch5_Furniture.xlsx)

The values in the yellow cells indicate both functional variables (Cells C8 and C9) and deviation variables (Cells C12 through C23).

Cells A12 through A23 indicate whether a priority is assigned or not to the deviational variables in Cells B12 through B23.

Priority values in Cells D12 through D23 are calculated with a VLOOKUP function, which automatically assigns the priority values indicated in the area G12:H16. For example, the value in Cell D12 is automatically calculated with VLOOKUP (A12, $G$12:$H$16, 2, FALSE).

The aspiration levels are indicated in cells G6: G9 and I6:I7. These cells hold the right-hand side values of the GP constraints.

Solving GP Models with Solver

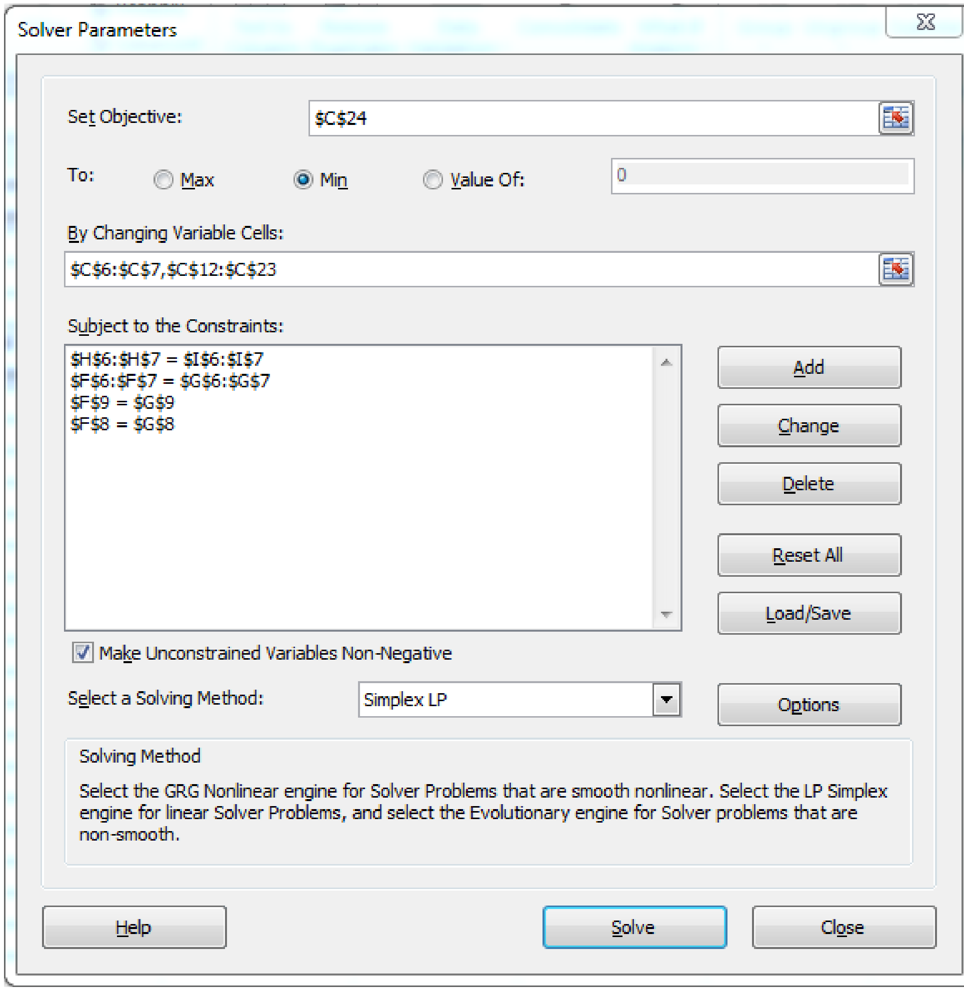
The achievement levels in the adjacent cells (F6:F9 and H6:H7) represent the *left-hand side* values of the same constraints and are calculated as a combination of original LP constraints and their respective negative or positive deviational variables.

For example, the achievement level for the machine hours is placed in cell F8 and is calculated as E8+SUMPRODUCT(C12:C13, E12:E13). In this case, E8 is the actual usage of machine hours (165) and the SUMPRODUCT formula calculated the sum of product of deviational variables and their respective priority coefficients ("+1" for addition and "–1" for subtraction). IN this case, the SUMPRODUCT value is -15.

Solving GP Models with Solver

The objective function, placed in cell C24, is calculated as the sum of the product between deviational variables and their respective weight (priorities) using the following formula: SUMPRODUCT(C12:C23,D12:D23).

Solving GP Models with Solver



Solver setup for GP model

Final Solution

- The bakery must produce 7.5 lots of DRCs and 6 lots of SRCs.
- The values of the deviational variables indicate whether the decision maker has reached the stated goals.
- Since the value of s_1^+ resulted in 15, that shows that the optimal production of rolls required an additional 15 hours to produce for a total of 150+15 = 165.
- Similarly, since s4positive(s_4^+) = 2.5 that shows that the goal of not exceeding five production lots for DRC cases is not achieved.

Example2: World Class Furniture

- Nonlinear programming models can also be transformed into GP models.
- The inventory management example from Furniture World Corporation (from lecture 4):
  - To Calculate the weekly order quantity for each furniture category.
  - Economic Order Quantity (EOQ) model
  - Storage capacity of 200,000 cubic feet
  - Purchasing budget of $1.5 million.

Warehouse Capacity (cubic feet)	**200,000**				
Average Inventory Budget	**$1,500,000**				
	Tables	**Chairs**	**Beds**	**Sofas**	**Bookcases**
Weekly Demand (units)	**1125**	**2750**	**3075**	**3075**	**750**
Purchase Price per Unit	**$45**	**$85**	**$125**	**$155**	**$125**
Holding Cost (per unit, period)	**$2**	**$3**	**$3**	**$3**	**$4**
Ordering Cost (per order)	**$100**	**$225**	**$135**	**$135**	**$100**
Storage Space Required (cubic feet per u	**84**	**106**	**140**	**70**	**100**

Operational data about the inventory management for these five products

NLP Formulation

- Recall the NLP formulation of the problem as follows:
    - Minimize $Z = \sum_{j-1}^{5}(h_j \frac{x_j}{2} + o_j \frac{D_j}{x_j})$
- (Nonlinear objective function seeking to minimize the overall inventory holding and ordering cost)
- Subject to:
    - $\sum_{j=1}^{5} s_j x_j \leq C$ (Linear constraint limiting the storage use to the maximum warehouse capacity C)
    - $\sum_{j=1}^{5}(h_j \frac{x_j}{2} + o_j \frac{D}{x_j} + p_j D_j) \leq P$ (Nonlinear constraint limiting the total storage and purchasing cost to budget P)
    - $x_j \geq 0$ for all j=1, 2, 3, 4, 5 (Non-negativity constraint)

NLP Formulation

The optimal solution

- The warehouse must order 289 tables, 575 chairs, 457 beds, 469 sofas, and 180 bookcases.
- This solution reduced the total inventory cost to $6,576.
- The solution suggested that the warehouse storage capacity is a binding constraint and that total inventory and purchasing cost constraints is not a binding constraint and has a slack of $254,298

Priority and GP Formulation

New requirements:

- Goal 1: maintain a 1 to 4 ration between tables and chairs. This goal is extremely important and is given very high priority (P1=1000).
- Goal 2: avoid overutilization of warehouse capacity (P2=50).
- Goal 3: avoid spending more than $7,000 in holding and ordering cost (P3=1)

Deviational Variables:

- s_1^+ = the degree that the ratio between tables and chairs is over the aspiration levelof0.25
- s_1^- = the degree that the ratio between tables and chairs is under the aspiration levelof0.25
- s_2^+ = the amount of storage capacity over utilized when the aspiration level is 200,000 cubic feet
- s_2^- = the amount of storage capacity underutilized when the aspiration level is 200,000 cubic feet
- s_3^+ = the amount of inventory cost over spent when the aspiration level is \$7,000
- s_3^- = the amount of inventory cost under spent when the aspiration level is \$7,000

x_1

GP Formulation

- The ratio constraint: $x_1/x_2 = 0.25$
- the overall nonlinear GP model :

Minimize $Z = 1000(s_1^- + s_1^+) + 50s_2^+ + s_3^+$

Subject to:

$x_1/x_2 + s_1^- - s_1^+ = 0.25$ (Priority 1: ratio constraint)

$\sum_{j=1}^{5} s_j x_j + s_2^- - s_2^+ = 200{,}000$ (Priority 2: capacity)

$\sum_{j-1}^{5} \left(h_j \frac{x_j}{2} + o_j \frac{D_j}{x_j} \right) + s_3^- - s_3^+ = 7000$ (Priority 3: inventory operating cost)

$\sum_{j=1}^{5} \left(h_j \frac{x_j}{2} + o_j \frac{D}{x_j} + p_j D_j \right) \leq 1{,}500{,}000$ (system constraint: budget)

and

$x_j, s_1^+, s_1^-, s_2^+, s_2^-, s_3^+, s_3^- \geq 0$ for all j=1, 2, 3, 4, 5

(non-negativity requirements)

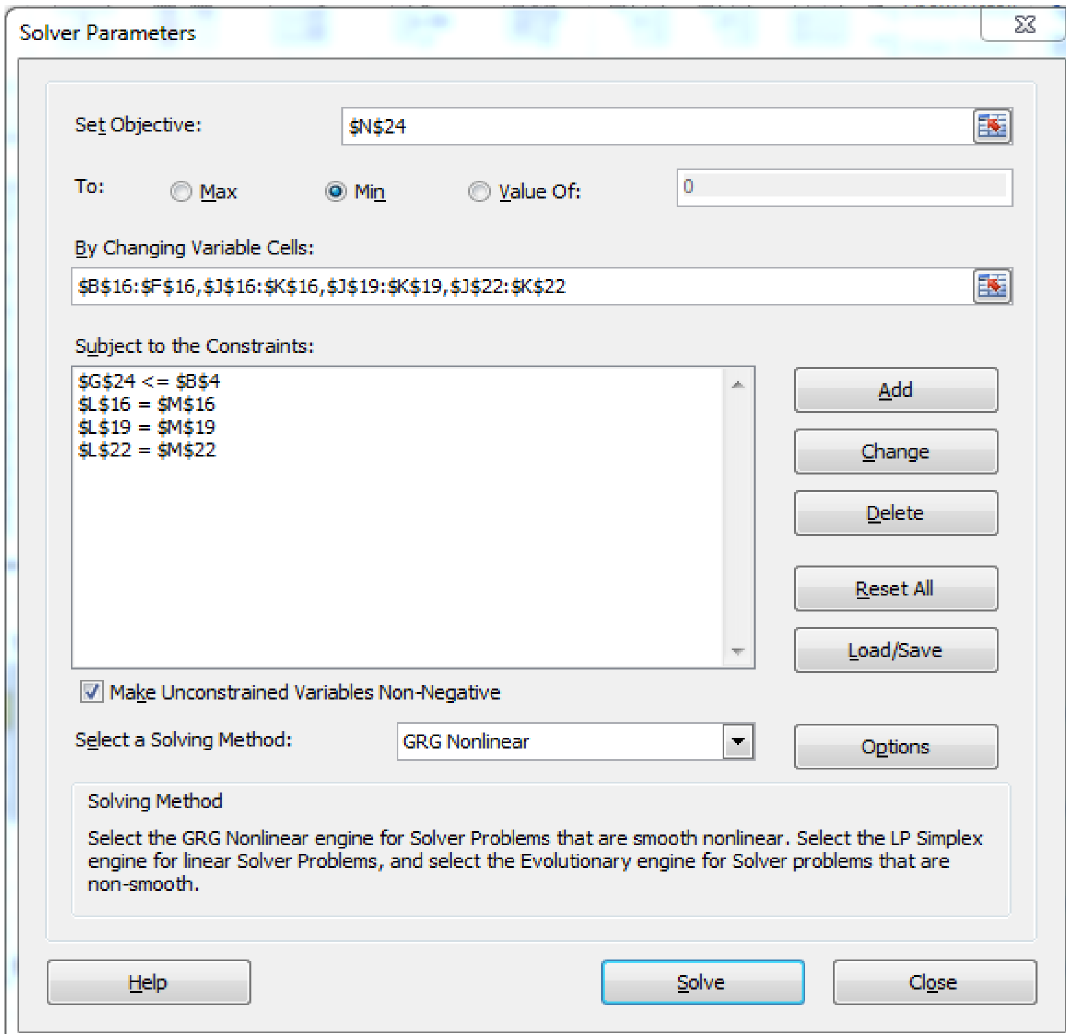
Solving NGP Models with Solver (ch5_Furniture.xlsx)

- GP model is expressed as a minimization NLP model with 11 decision variables and four constraints.

3	Warehouse Capacity (cubic feet)	200,000												
4	Average Inventory Budget	$1,500,000												
5														
6		Table	Chairs	Beds	Sofas	Bookcases								
7	Weekly Demand (units)	1125	2750	3075	3075	750								
8	Purchase Price per Unit	$45	$85	$125	$155	$125								
9	Holding Cost (per unit, period)	$2	$3	$3	$3	$4								
10	Ordering Cost (per order)	$100	$225	$135	$135	$100								
11	Storage Space Required (cubic feet per u	84	106	140	70	100								
12														
13	Calculations and Results													
14		Table	Chairs	Beds	Sofas	Bookcases	Totals	Table to						
15	Economic Order Quantity (EOQ)	335	642	526	526	194		Chair Ratio	s_negative	s_positive	LHV	RHV		
16	Optimized Order Quantity	133	533	403	403	155		0.25	0	0	0.25	0.25		0
17	Average Inventory	67	266	202	202	78		P1:	1000	1000				
18	Average Number of Orders per period	8.45	5.16	7.62	7.62	4.83			s_negative	s_positive				
19	Total Supply Available	1125	2750	3075	3075	750			32074	0	200,000	200,000		0
20	Maximum Cubic Foot Storage Required	11189	56479	56479	28239	15540	167,926	P2:	0	50				
21	Ordering Cost per period	$845	$1,161	$1,029	$1,029	$483	$4,547		s_negative	s_positive				
22	Holding Cost per period	$133	$799	$605	$605	$311	$2,453	P3:	0	0	$7,000	7000		0
23	Inventory Operating Cost per period	$978	$1,961	$1,634	$1,634	$793	$7,000			10				
24	Average Inventory Value	$51,603	$235,711	$386,009	$478,259	$94,543	$1,246,125					Objective function		0

Setup and Solution for GP Model

Solver Parameters for the Furniture GP Model



Final Solution for the Furniture Goal Programming Model

3	Warehouse Capacity (cubic feet)	200,000												
4	Average Inventory Budget	$1,500,000												
5														
6		Table	Chairs	Beds	Sofas	Bookcases								
7	Weekly Demand (units)	1125	2750	3075	3075	750								
8	Purchase Price per Unit	$45	$85	$125	$155	$125								
9	Holding Cost (per unit, period)	$2	$3	$3	$3	$4								
10	Ordering Cost (per order)	$100	$225	$135	$135	$100								
11	Storage Space Required (cubic feet per	84	106	140	70	100								
12														
13	Calculations and Results													
14		Table	Chairs	Beds	Sofas	Bookcases	Totals	Table to						
15	Economic Order Quantity (EOQ)	335	642	526	526	194		Chair Ratio	s_negative	s_positive	LHV	RHV		
16	Optimized Order Quantity	133	533	403	403	155		0.25	0	0	0.25	0.25		0
17	Average Inventory	67	266	202	202	78		P1:	1000	1000				
18	Average Number of Orders per period	8.45	5.16	7.62	7.62	4.83			s_negative	s_positive				
19	Total Supply Available	1125	2750	3075	3075	750			32074	0	200,000	200,000		0
20	Maximum Cubic Foot Storage Required	11189	56479	56479	28239	15540	167,926	P2:	0	50				
21	Ordering Cost per period	$845	$1,161	$1,029	$1,029	$483	$4,547		s_negative	s_positive				
22	Holding Cost per period	$133	$799	$605	$605	$311	$2,453	P3:	0	0	$7,000	7000		0
23	Inventory Operating Cost per period	$978	$1,961	$1,634	$1,634	$793	$7,000			10				
24	Average Inventory Value	$51,603	$235,711	$386,009	$478,259	$94,543	$1,246,125						Objective function	0

- Can order 133 tables and 533 chairs, with an almost 1 to 4 ratio.
- Also can order 403 beds, 403 sofas, and 155 bookcases at a time.
- Allow an additional space of 32074 cubic feet
- Inventory operating cost of $7,000
- Average inventory value budget of $1.5 million

Exploring Big Data with GP

- GP could be the favorite tool for data analyst
  - Organizations try to meet multiple objectives under fierce competition
  - Variety of big data allows the decision maker to analyze business problems from many dimensions and multiple goals

- GP can be formulated and solved as a series of connected programming models or a single programming model
  - Designed as a LP model, with the first goal as objective function
  - Once a solution is achieved, the objective function is transformed into a constraint

Wrap up

Step 1:
Formulate the problem as a simple LP or NLP model

Step 2:
Define deviational variables for each goal

Step 3:
Write GP and system constraints

Step 4:
Add non-negativity constraints for functional and deviational variables

Step 5:
Determine the variables to be minimized in the objective function

Step 6:
Write the objective function with priorities.

End of The Lecture

Thank You