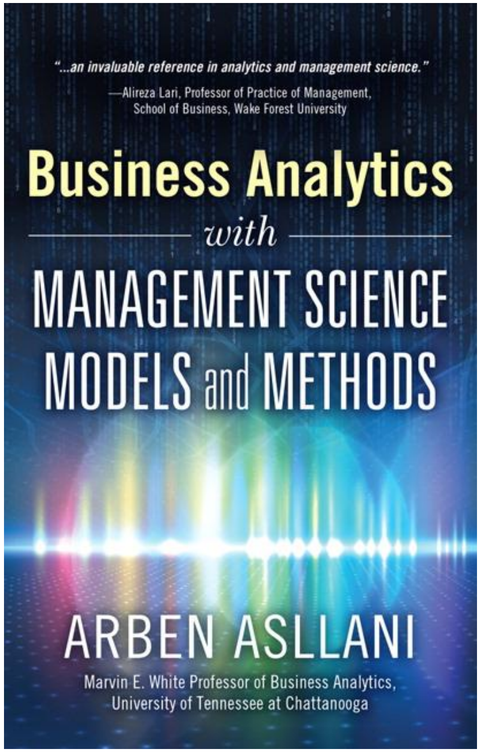
Business Analytics Prescriptive Models



Based on

Business Analytics
With
Management Science
Models and Methods
by
Arben Asllani

CHAPTER 6

Business Analytics with Integer Programming

Business Analytics with Management Science Models and Methods

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Chapter Outline

- Prescriptive Analytics in Action: Zara
- Introduction
- Formulation and Graphical Solution of IP Models
- Types of Integer Programming Models
  - Solving Integer LP Models with Solver
  - Solving Integer GP Models with Solver
- The Assignment Method
  - General Formulation of the Assignment Problem
  - Solving the Assignment Method with Solver
- The Knapsack Problem
  - General Formulation of the Knapsack Problem
- Exploring Big Data with Integer Programming
- Wrap up

Chapter Objectives

- Discuss the need to require integer or binary values for the solution of some programming models
- Offer a graphical explanation of the integer programming models
- Discuss different types of integer programming models and when to choose them
- Demonstrate the process of seeking integer or binary solutions for linear, nonlinear, or goal programming models via Solver
- Discuss the main assumptions of the knapsack and assignment problems
- Describe the challenges of requiring binary, integer, or mixed integer solutions for programming models
- Offer practical recommendations when using integer, binary, or mixed programming models in the era of big data

Prescriptive Analytics in Action

- Zara: one of the largest international fashion companies
  - Vertically integrate its supply chain
  - Replenish inventory directly to every store twice a week
- Challenge:
  - To determine the exact number of each size to ship to each store
  - Decision must be made in a few hours
  - The limitation of the available inventory in the warehouse
- Customer preference data on the PDAs
- Point of Sale (POS) transaction processing system
- Seasonal sale increased over 3-4% and transshipment cost reduced significantly

Introduction

- The assumption of Divisibility
  - Allows decision variables to take integer as well as factional values
- There are business applications where the solutions must be restricted to be an integer
- Integer programming (IP) models
  - Seek optimal solutions
  - All/some of the decision variables are required to be integers
  - Same structure as the LP, NLP, or GP models
    - Objective function and a set of constraints
  - A set of constraints that forces decision variables to be integers

Formulation and Graphical Solution of IP Models

$$\text{Max } Z = 400x_1 + 300x_2 \quad \text{(objective function)}$$

subject to:

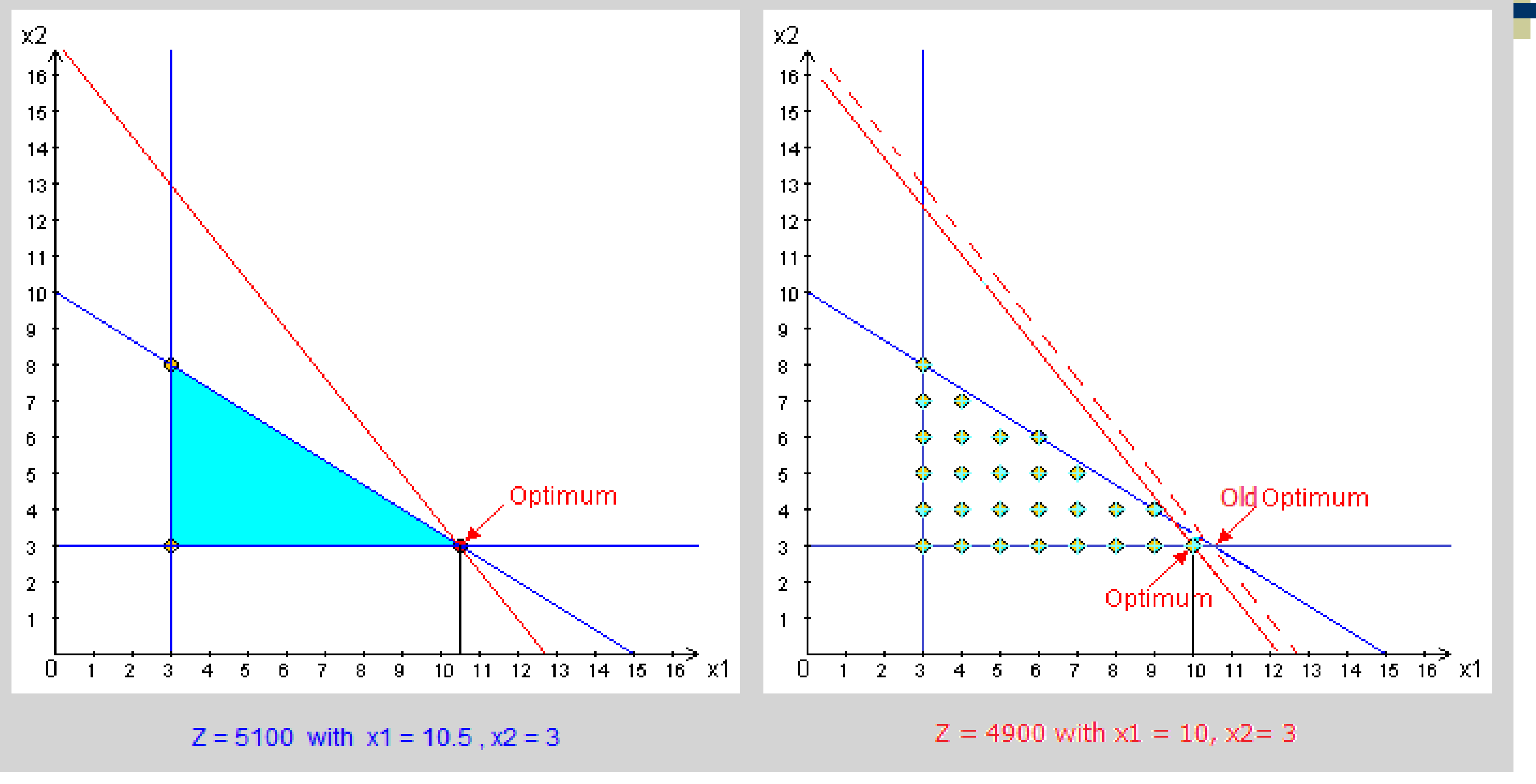
$$10x_1 + 15x_2 \le 150 \quad \text{(constraint 1)}$$

$$x_1 \ge 3 \quad \text{(constraint 2)}$$

$$x_2 \ge 3 \quad \text{(constraint 3)}$$

$$x_1, x_2 \ge 0 \text{ and integer} \quad \text{(non-negativity and integer constraints)}$$

Graphical Solution of Rolls Bakery IP Model



(a) Non-integer solution

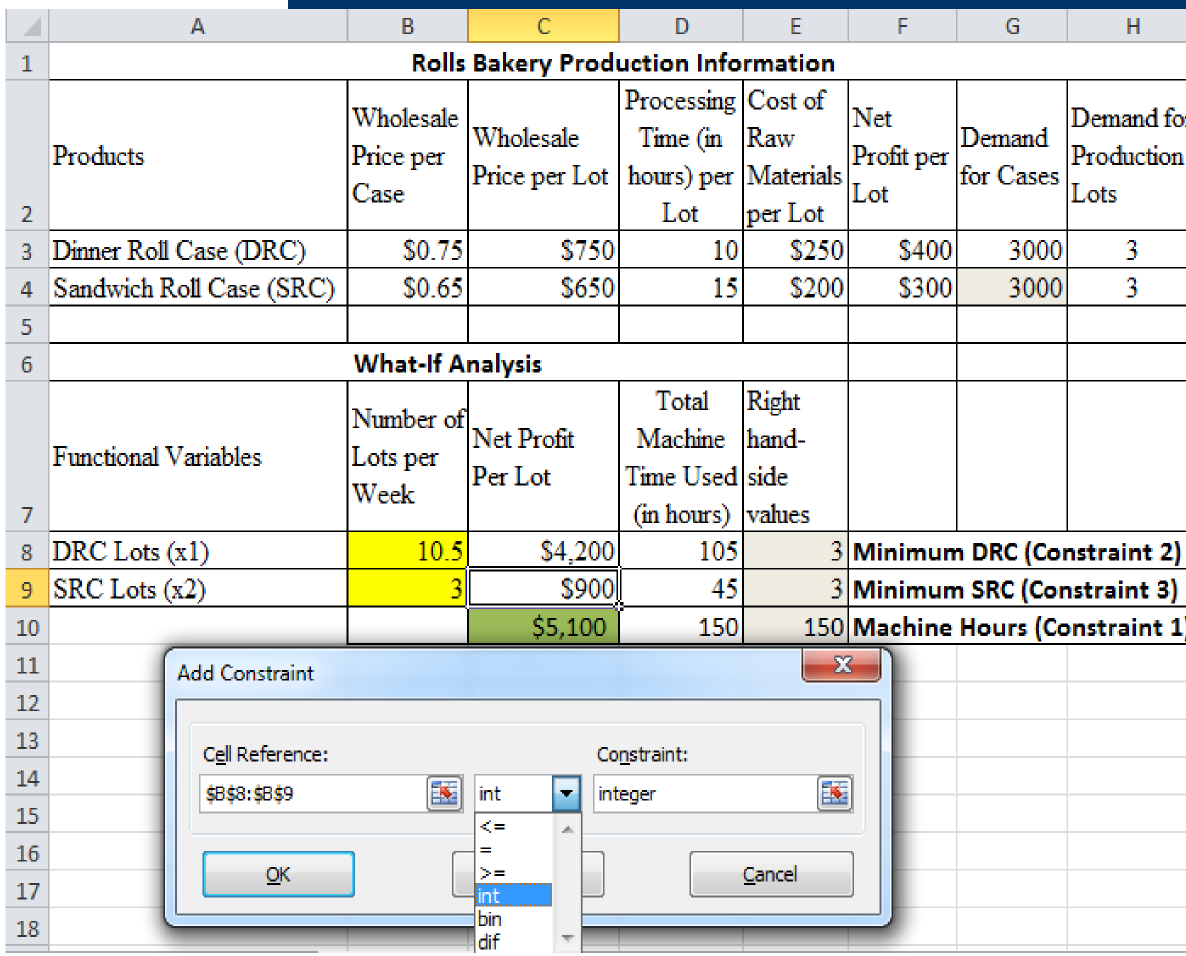
(b) Integer solution

Adding the integer constraints in the regular LP models causes a significant change in the nature of the problem

Types of Integer Programming Models

1. All-integer programming model
2. Mixed-integer programming model
3. Linear IP model
4. Nonlinear IP model
   - Special challenges for solution algorithms
   - Evolutional solving method of Solver
5. Binary integer Programming
   or simply 0-1 programming

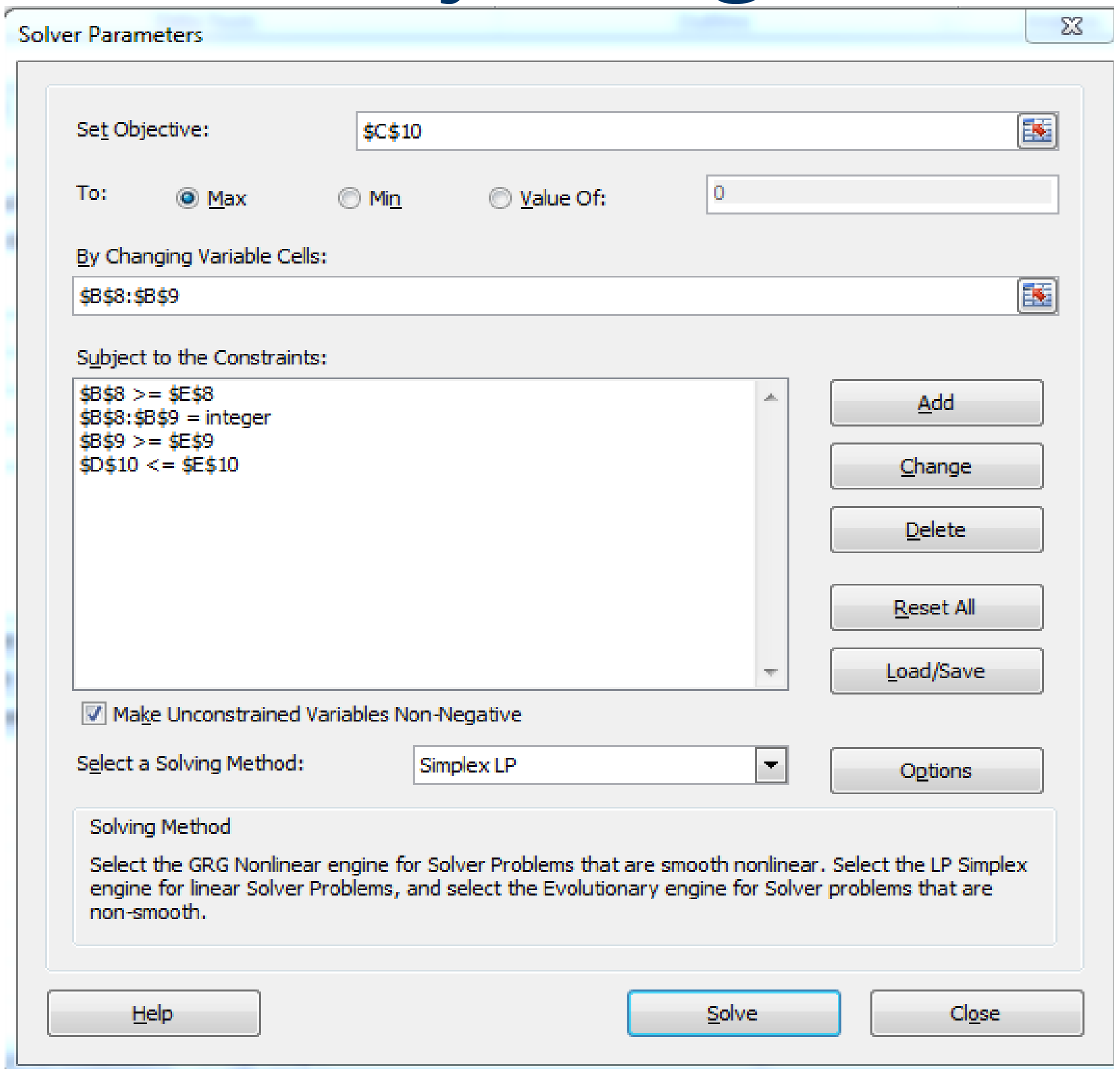
Solving Integer LP Models with Solver (Ch2_RollsBakery.xlsx)



	A	B	C	D	E	F	G	H
1	**Rolls Bakery Production Information**							
2	Products	Wholesale Price per Case	Wholesale Price per Lot	Processing Time (in hours) per Lot	Cost of Raw Materials per Lot	Net Profit per Lot	Demand for Cases	Demand for Production Lots
3	Dinner Roll Case (DRC)	$0.75	$750	10	$250	$400	3000	3
4	Sandwich Roll Case (SRC)	$0.65	$650	15	$200	$300	3000	3
5								
6	**What-If Analysis**							
7	Functional Variables	Number of Lots per Week	Net Profit Per Lot	Total Machine Time Used (in hours)	Right hand-side values			
8	DRC Lots (x1)	10.5	$4,200	105	3	**Minimum DRC (Constraint 2)**		
9	SRC Lots (x2)	3	$900	45	3	**Minimum SRC (Constraint 3)**		
10			$5,100	150	150	**Machine Hours (Constraint 1)**		

Adding integer constraints to the Rolls Bakery Problem

Solver Parameters for the Rolls Bakery Integer LP Model



Final Solution Values for the Integer LP Model

	A	B	C	D	E	F	G	H
2	Products	Wholesale Price per Case	Wholesale Price per Lot	Processing Time (in hours) per Lot	Cost of Raw Materials per Lot	Net Profit per Lot	Demand for Cases	Demand for Production Lots
3	Dinner Roll Case (DRC)	$0.75	$750	10	$250	$400	3000	3
4	Sandwich Roll Case (SRC)	$0.65	$650	15	$200	$300	3000	3
5								
6	**What-If Analysis**							
7	Functional Variables	Number of Lots per Week	Net Profit Per Lot	Total Machine Time Used (in hours)	Right hand-side values			
8	DRC Lots (x1)	10	$4,000	100	3	**Minimum DRC (Constraint 2)**		
9	SRC Lots (x2)	3	$900	45	3	**Minimum SRC (Constraint 3)**		
10			$4,900	145	150	**Machine Hours (Constraint 1)**		

1. The value of the objective function will not be better than the objective function of the non-integer model
2. The values of decision variables will no longer have fractions

Solving Nonlinear IP Model with Solver (ch4_Furniture.xlsx)

$$\text{Minimize } Z = \sum_{j=1}^{5}\left(h_j \frac{x_j}{2} + o_j \frac{D_j}{x_j}\right) \qquad \text{(nolinear objective function)}$$

subject to:

$$\sum_{j=1}^{5} s_j x_j \le C \qquad \text{(linear constraint)}$$

$$\sum_{j=1}^{5}\left(h_j \frac{x_j}{2} + o_j \frac{D}{x_j} + p_j D_j\right) \le P \qquad \text{(nonlinear constraint)}$$

$x_j \ge 0$ and integers for all $j = 1, 2, 3, 4, 5$ (non-negativity and integer constraints)

Non-integer Solutions for the WCF Inventory Problem

	A	B	C	D	E	F	G
3	Warehouse Capacity (cubic feet)	200,000					
4	Inventory Budget	$1,500,000					
5							
6		Tables	Chairs	Beds	Sofas	Bookcases	
7	Weekly Demand (units)	1125	2750	3075	3075	750	
8	Purchase Price per Unit	$45	$85	$125	$155	$125	
9	Holding Cost (per unit, period)	$2	$3	$3	$3	$4	
10	Ordering Cost (per order)	$100	$225	$135	$135	$100	
11	Storage Space Required (cubic feet per unit)	84	106	140	70	100	
12							
13	Calculations and Results						
14		Tables	Chairs	Beds	Sofas	Bookcases	Totals
15	Economic Order Quantity (EOQ)	335	642	526	526	194	
16	Optimized Order Quantity	288.69	574.93	457.21	469.20	179.53	
17	Average Inventory	144	287	229	235	90	
18	Average Number of Orders per Week	3.90	4.78	6.73	6.55	4.18	
19	Total Supply Available	1125	2750	3075	3075	750	
20	Maximum Cubic Foot Storage Required	24250	60942	64010	32844	17953	200000
21	Ordering Cost per Week	$390	$1,076	$908	$885	$418	$3,676
22	Holding Cost per Week	$289	$862	$686	$704	$359	$2,900
23	Inventory Operating Cost per Week	$678	$1,939	$1,594	$1,589	$777	$6,576
24	Total Inventory Value	$51,303	$235,689	$385,969	$478,214	$94,527	$1,245,701

Integer Solutions for the WCF Inventory Problem

	A	B	C	D	E	F	G
3	Warehouse Capacity (cubic feet)	200,000					
4	Inventory Budget	$1,500,000					
5							
6		Tables	Chairs	Beds	Sofas	Bookcases	
7	Weekly Demand (units)	1125	2750	3075	3075	750	
8	Purchase Price per Unit	$45	$85	$125	$155	$125	
9	Holding Cost (per unit, period)	$2	$3	$3	$3	$4	
10	Ordering Cost (per order)	$100	$225	$135	$135	$100	
11	Storage Space Required (cubic feet per unit)	84	106	140	70	100	
12							
13	Calculations and Results						
14		Tables	Chairs	Beds	Sofas	Bookcases	Totals
15	Economic Order Quantity (EOQ)	335	642	526	526	194	
16	Optimized Order Quantity	291.00	570.00	452.00	486.00	178.00	
17	Average Inventory	146	285	226	243	89	
18	Average Number of Orders per Week	3.87	4.82	6.80	6.33	4.21	
19	Total Supply Available	1125	2750	3075	3075	750	
20	Maximum Cubic Foot Storage Required	24444	60420	63280	34020	17800	199964
21	Ordering Cost per Week	$387	$1,086	$918	$854	$421	$3,666
22	Holding Cost per Week	$291	$855	$678	$729	$356	$2,909
23	Inventory Operating Cost per Week	$678	$1,941	$1,596	$1,583	$777	$6,575
24	Total Inventory Value	$51,303	$235,691	$385,971	$478,208	$94,527	$1,245,700

Change in the Decision Variables as Indicated in the Answer Report

	Variable Cells			
20	Cell	Name	Original Value	Final Value
21	$B$16	Optimized Order Quantity Tables	288.69	291.00
22	$C$16	Optimized Order Quantity Chairs	574.93	570.00
23	$D$16	Optimized Order Quantity Beds	457.21	452.00
24	$E$16	Optimized Order Quantity Sofas	469.20	486.00
25	$F$16	Optimized Order Quantity Bookcases	179.53	178.00

- A better solution is found now, when the initial values of decision variables have a good starting point.

Solving Integer GP Models with Solver (ch6_BakeryGoalP.xlsx)

Minimize $Z = 300s_4^+ + 300s_5^+ + 300s_2^- + 300s_3^- + 20s_1^+ + 10s_6^-$ (GP objective function)

subject to:

$10x_1 + 15x_2 + s_1^- - s_1^+ = 150$	(machine hours)
$x_1 + s_2^- - s_2^+ = 3$	(minimum demand for DRC)
$x_2 + s_3^- - s_3^+ = 4$	(minimum demand for SRC)
$x_1 + s_4^- - s_4^+ = 5$	(maximum demand for DRC)
$x_2 + s_5^- - s_5^+ = 6$	(maximum demand for SRC)
$400x_1 + 300x_2 + s_6^- - s_6^+ = 4800$	(net profit constraint)

and

$x_1, x_2, s_1^-, s_1^+, s_2^-, s_2^+, s_3^-, s_3^+, s_4^-, s_4^+, s_5^-, s_5^+, s_6^-, s_6^+ \geq 0$ and x_1, x_2 are integers

Linear GP Solution to Rolls Bakery Problem

1		Products	Wholesale Price per Case	Wholesale Price per Lot	Processing Time (in hours) per Lot	Cost of Raw Materials per Lot	Net Profit per Lot	Demand for Cases	Demand for Production Lots
2		Dinner Roll Case (DRC)	$0,75	$750	10	$250	$400	3000	3
3		Sandwich Roll Case (SRC)	$0,65	$650	15	$200	$300	4000	4
4									
5		Functional Variables	Number of Lots per Week	Net Profit Per Lot	Total Machine Time Used (in hours)	Achievement Level for Minimum	Aspiration Level for	Achivement Level for Maximum	Aspiration Level for
6		DRC Lots (x1)	7,5	$3 000	75	3	3	5	**5**
7		SRC Lots (x2)	6	$1 800	90	4	4	6	**6**
8		Machine Hours			165	150	150		
9		Profit		$4 800		4800	4800		
10									
11	**Priority**	**Deviational Variables**	**Values**	**Priority Value**			**Priorities**		
12	No Priority	s1negative	0	0	1		P1	300	
13	P3	s1positive	15	20	-1		P2	300	
14	P2	s2negative	0	300	1		P3	20	
15	No Priority	s2positive	4,5	0	-1		P4	10	
16	P2	s3negative	0	300	1		No Priority	0	
17	No Priority	s3positive	2	0	-1				
18	No Priority	s4negative	0	0	1				
19	P1	s4positive	2,5	300	-1				
20	No Priority	s5negative	0	0	1				
21	P1	s5positive	0	300	-1				
22	P4	s6negative	0	10	1				
23	No Priority	s6positive	0	0	-1				
24		Objective Function	1050						

Enforcing Integer Solution to Rolls Bakery Problem

	A	B	C	D	E	F	G	H	I
1		Products	Wholesale Price per Case	Wholesale Price per Lot	Processing Time (in hours) per Lot	Cost of Raw Materials per Lot	Net Profit per Lot	Demand for Cases	Demand for Production Lots
2		Dinner Roll Case (DRC)	$0,75	$750	10	$250	$400	3000	3
3		Sandwich Roll Case (SRC)	$0,65	$650	15	$200	$300	4000	4
4									
5		Functional Variables	Number of Lots per Week	Net Profit Per Lot	Total Machine Time Used (in hours)	Achievement Level for Minimum	Aspiration Level for	Achivement Level for Maximum	Aspiration Level for
6		DRC Lots (x1)	9	$3 600	90	3	3	5	**5**
7		SRC Lots (x2)	4	$1 200	60	4	4	6	**6**
8		Machine Hours			150	150	150		
9		Profit		$4 800		4800	4800		
10									
11	**Priority**	**Deviational Variables**	**Values**	**Priority Value**				**Priorities**	
12	No Priority	s1negative	0	0	1		P1	300	
13	P3	s1positive	0	20	-1		P2	300	
14	P2	s2negative	0	300	1		P3	20	
15	No Priority	s2positive	6	0	-1		P4	10	
16	P2	s3negative	0	300	1		No Priority	0	
17	No Priority	s3positive	0	0	-1				
18	No Priority	s4negative	0	0	1				
19	P1	s4positive	4	300	-1				
20	No Priority	s5negative	2	0	1				
21	P1	s5positive	0	300	-1				
22	P4	s6negative	0	10	1				
23	No Priority	s6positive	0	0	-1				
24		Objective Function	1200						

Change Constraint

Cell Reference: $C$6:$C$7

=

Constraint: integer

OK Add Cancel

Answer Report for the Integer GP Solution

14	Objective Cell (Min)				
15		Cell	Name	Original Value	Final Value
16		$C$24	Objective Function Values	1050	1200
17					
18					

19	Variable Cells					
20		Cell	Name	Original Value	Final Value	Integer
21		$C$6	DRC Lots (x1) Number of Lots per Week	7.5	9	Integer
22		$C$7	SRC Lots (x2) Number of Lots per Week	6	4	Integer
23		$C$12	s1negative Values	0	0	Contin
24		$C$13	s1positive Values	15	0	Contin
25		$C$14	s2negative Values	0	0	Contin
26		$C$15	s2positive Values	4.5	6	Contin
27		$C$16	s3negative Values	0	0	Contin
28		$C$17	s3positive Values	2	0	Contin
29		$C$18	s4negative Values	0	0	Contin
30		$C$19	s4positive Values	2.5	4	Contin
31		$C$20	s5negative Values	0	2	Contin
32		$C$21	s5positive Values	0	0	Contin
33		$C$22	s6negative Values	0	0	Contin
34		$C$23	s6positive Values	0	0	Contin

The Assignment Method

- The Assignment Method
  - A popular IP model that refers to assigning resources to a specific task
  - Only one resource can be assigned in a task
  - Only one task can be assigned to each resource
  - Goal: maximize the revenue or minimize the cost
  - Examples of business problems
- Decision variables are binary (0/1 type)

General Formulation of the Assignment Problem

$$x_{ij} = \begin{cases} 1 \text{ if item } i \text{ is assigned to project } j \\ 0 \text{ otherwise} \end{cases}$$

Then the problem can be formulated as:

$$\text{Max (or Min) } Z = \sum_{i=1}^{m} \sum_{j=1}^{n} v_{ij}\, x_{ij}$$

Subject to:

$$\sum_{i=1}^{m} x_{ij} = 1 \quad \text{for all } j = 1, 2, 3, \dots, n$$

$$\sum_{j=1}^{n} x_{ij} \le 1 \quad \text{for all } i = 1, 2, 3, \dots, m$$

x_{ij} are binary

Solving the Assignment Method with Solver (ch6_Assignment.xlsx)

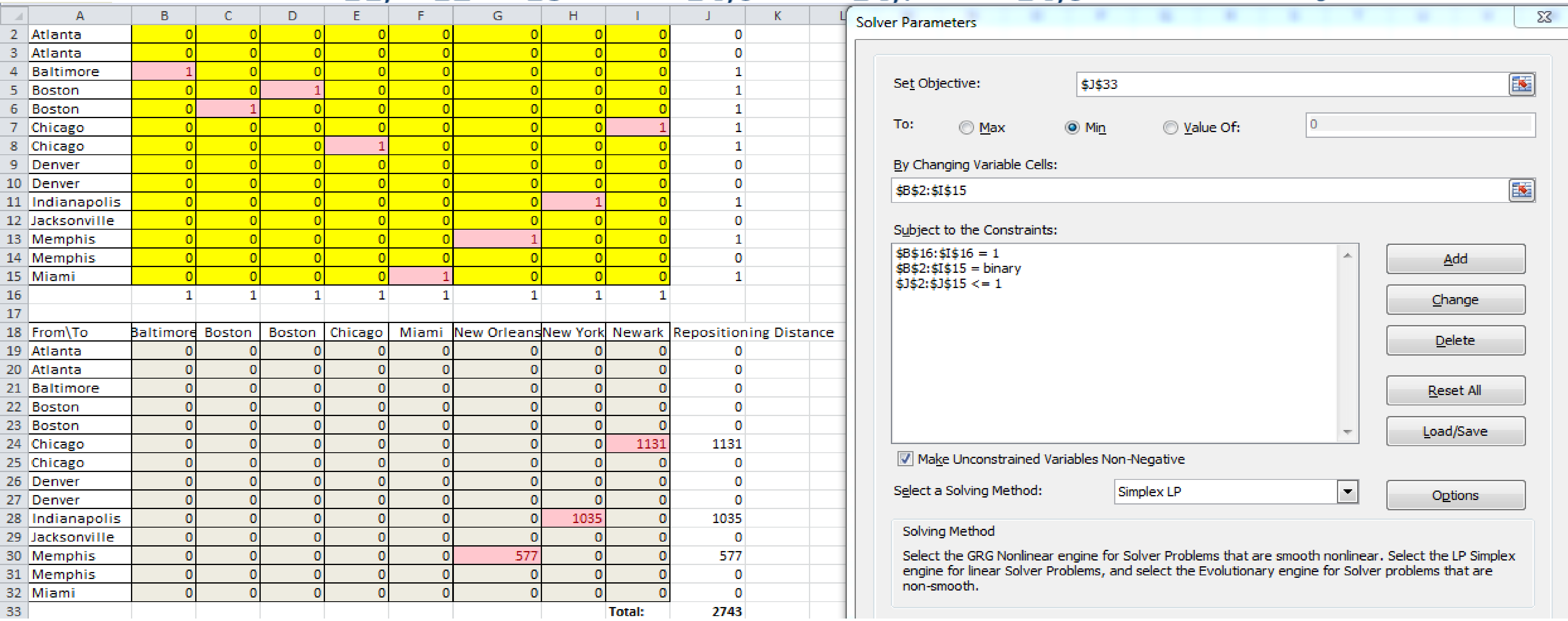
- A dispatcher at a trucking company has 14 trucks
  - Wants each truck to travel to the other cities where eight loads are waiting to be picked up
  - Some of the cities are repeated
  - Each truck can transport only one load at a time
  - Not all trucks will be assigned
- Where should the dispatcher send each truck in order to minimize the total transportation distance?

From\To	Baltimore	Boston	Boston	Chicago	Miami	New Orleans	New York	Newark
Atlanta	927	1505	1505	944	974	682	1200	1190
Atlanta	927	1505	1505	944	974	682	1200	1190
Baltimore	0	578	578	973	1539	1607	272	262
Boston	578	0	0	1367	2022	2184	306	315
Boston	578	0	0	1367	2022	2184	306	315
Chicago	973	1367	1367	0	1912	1340	1145	1131
Chicago	973	1367	1367	0	1912	1340	1145	1131
Denver	2422	2839	2839	1474	2773	1737	2617	2602
Denver	2422	2839	2839	1474	2773	1737	2617	2602
Indianapolis	819	1295	1295	263	1651	1147	1035	1021
Jacksonville	1096	1636	1636	1387	526	810	1344	1338
Memphis	1273	1824	1824	773	1404	577	1533	1520
Memphis	1273	1824	1824	773	1404	577	1533	1520

Solving the Assignment Method with Solver

- Finally, all decision variables must be either 0s or 1s:

$$x_{11}, x_{12}, x_{13}, \ldots, x_{14,6} + x_{14,7} + x_{14,8} \text{ are binary}$$



	A	B	C	D	E	F	G	H	I	J	K
2	Atlanta	0	0	0	0	0	0	0	0	0	
3	Atlanta	0	0	0	0	0	0	0	0	0	
4	Baltimore	1	0	0	0	0	0	0	0	1	
5	Boston	0	0	1	0	0	0	0	0	1	
6	Boston	0	1	0	0	0	0	0	0	1	
7	Chicago	0	0	0	0	0	0	0	1	1	
8	Chicago	0	0	0	1	0	0	0	0	1	
9	Denver	0	0	0	0	0	0	0	0	0	
10	Denver	0	0	0	0	0	0	0	0	0	
11	Indianapolis	0	0	0	0	0	0	1	0	1	
12	Jacksonville	0	0	0	0	0	0	0	0	0	
13	Memphis	0	0	0	0	0	1	0	0	1	
14	Memphis	0	0	0	0	0	0	0	0	0	
15	Miami	0	0	0	0	1	0	0	0	1	
16		1	1	1	1	1	1	1	1		
17											
18	From\To	Baltimore	Boston	Boston	Chicago	Miami	New Orleans	New York	Newark	Repositioning Distance	
19	Atlanta	0	0	0	0	0	0	0	0	0	
20	Atlanta	0	0	0	0	0	0	0	0	0	
21	Baltimore	0	0	0	0	0	0	0	0	0	
22	Boston	0	0	0	0	0	0	0	0	0	
23	Boston	0	0	0	0	0	0	0	0	0	
24	Chicago	0	0	0	0	0	0	0	1131	1131	
25	Chicago	0	0	0	0	0	0	0	0	0	
26	Denver	0	0	0	0	0	0	0	0	0	
27	Denver	0	0	0	0	0	0	0	0	0	
28	Indianapolis	0	0	0	0	0	0	1035	0	1035	
29	Jacksonville	0	0	0	0	0	0	0	0	0	
30	Memphis	0	0	0	0	0	577	0	0	577	
31	Memphis	0	0	0	0	0	0	0	0	0	
32	Miami	0	0	0	0	0	0	0	0	0	
33									Total:	2743	

Solver formulation and solution for the Repositioning problem

Solving the Assignment Method with Solver

1. The Baltimore load should be picked up by a truck in Baltimore.
2. The Boston load should be picked up by a truck in Boston.
3. The other Boston load should be picked up by a truck in Boston.
4. The Chicago load should be picked up by a truck in Chicago.
5. The Miami load should be picked up by a truck in Miami.
6. The New Orleans load should be picked up by a truck in Memphis.
7. The New York load should be picked up by a truck in Indianapolis.
8. The Newark load should be picked up by a truck in Chicago.

- Two trucks in Atlanta, two trucks in Denver, one truck in Jacksonville, and one truck in Memphis are not assigned to pick-up a load. The minimum total repositioning distance is 2,743 miles.

The Knapsack Problem

- A famous IP model that refers to a hiker deciding to select the most valuable items to carry in a hiking venture considering a weight limit
- Examples of business problems:
  - Which items to ship considering limited container space
  - Which investments to choose considering different expected returns on investments and limited budget

General Formulation of The Knapsack Problem

Maximize $Z = \sum_{j=1}^{n} r_j x_j$

subject to:

$$\sum_{j=1}^{n} c_j x_j \le C$$

$x_j \ge 0$ and integer

where:

c_j = the cost or weight of each type-j item, for j = 1, 2, ..., n

r_j = the reward associated with each type-j item, for j = 1, 2, 3, ..., n

C = the total budget or capacity of the knapsack

Exploring Big Data with IP

- Linear IP Models
  - A finite number of possible solutions
  - Can be found relatively fast with Solver
- Nonlinear IP Models
  - Require a more complicated algorithm to reach an optimal solution
  - The likelihood that the solution is a local optimum is high
- Adding an integer or binary constraint will result in a value of the objective function that is equal or worse than noninteger model

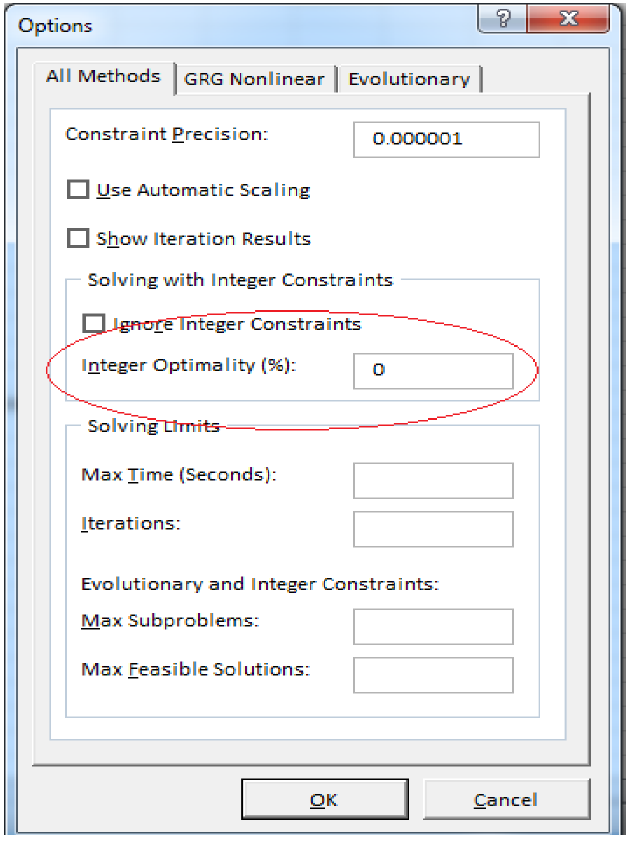
Exploring Big Data with IP

- IP model may provide computational difficulties especially when the number of decision variables is high
  - Typical for assignment problem when number of resources or tasks increases (number of variables is $m \times n$)
- To deal the complexity
  - New software program, such as *MATLAB, XPRESS , CPLEX , and Gurobi* , have added integer solvers into their optimization suites

Exploring Big Data with IP

- Searching for solution with given tolerance:
  - Within 5% of the optimal solution for example

Setting the tolerance level for integer constraints



Wrap up

- Various types of IP models:
  - Linear
  - Nonlinear
  - Linear Goal
  - Nonlinear Goal
- A general formulation of two common IP models:
  - The assignment problem
  - The knapsack problem
- How to deal the model complexity

End of The Lecture

Thank You