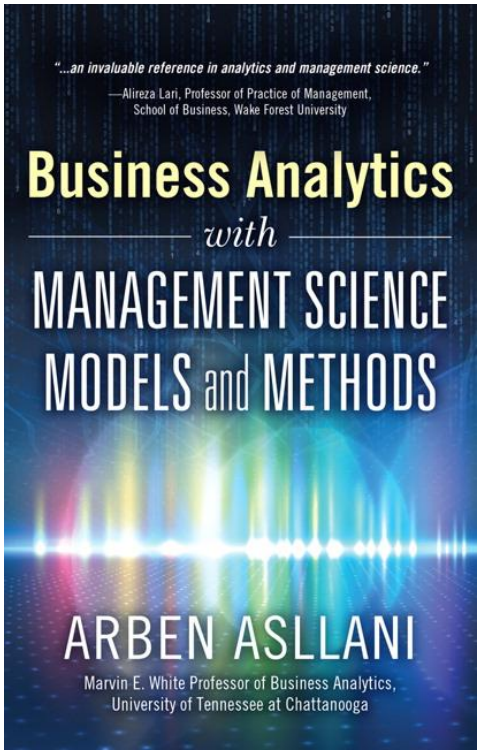
Business Analytics
Prescriptive Models



Based on

Business Analytics With Management Science Models and Methods

by

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Chapter 3

Business Analytics with Linear Programming

Prescriptive Analytics

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Chapter Outline

- Chapter Objectives
- Prescriptive Analytics in Action
- Introduction
- General Formulation of LP Models
- Formulating a Large LP Model
- Example: Primer Manufacturer Inc. Production Mix
- Solving Linear Programming Models with Excel
  - Sensitivity Analysis
- Big Optimization with big data
- Wrap up!

Chapter Objectives

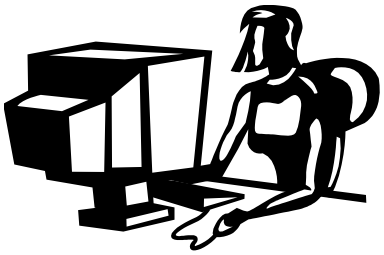
- Provide a general formulation to LP models using mathematical notations
- Demonstrate the use of Excel and Solver for solving LP models with large number of decision variables and constraints
- Discuss the importance of data preparation techniques which can be used to summarize data and refresh data
- Explore the difference between binding and not binding constraints
- Understand the impact of the changes in the right-hand values and contribution coefficients
- Discuss the challenges of implementing linear programming models in real business settings

Nu-kote Minimizes Shipment Cost

- World's the largest independent remanufacturer of Paint Primer
- Located in the US, China, Thailand, and Mexico
- Needed to better plan shipments of finished goods
  - Goal: minimize cost
  - Constraints: distance requirements
- LP models were relatively large:
  - between 5,000 and 9,700 variables
  - 2,500 constraints
- Saved $1,000,000 in its first year from operating the business

Introduction

- Popularity of the Linear Programming
  - LP algorithms are considered to be one of the top ten most important tools of the last century
- LP Facing Challenge
  - Computational difficulty – takes the users a significant time
    - Analytic Solver Platform
    - CMPL- COIN Mathematical Programming Language
    - IBM's CPLEX Optimization Studio
    - GAMS - General Algebraic Modeling System
    - GIPALS - Linear Programming Environment
    - GNU Linear Programming Kit
    - LINDO/LINGO - Linear, Interactive, and Discrete Optimizer
    - Risk Solver Platform
- Spreadsheet Modeling



General Formulation of LP Models

$$Max\ (or\ Min)\ Z = \sum_{j=1}^{n} c_j x_j$$

$$\text{s.t. } \sum_{j=1}^{n} a_{ij} x_j \begin{cases} \leq \\ \geq \\ = \end{cases} b_i \text{ for all } i=1, 2, \ldots m$$

$$x_1, x_2, \ldots x_n \geq 0$$

(s.t. – subject to)

Formulating a Large LP Model

1. Calculate model parameters
2. Define decision variables
3. Formulate the objective function
4. Identify the set of constraints
5. Identify a set of non-negativity constraints

Example (ch3_PaintPrimer.xlsx): **Primer Manufacturer Inc. Production Mix**

- PMI produces and distributes **48** different paint primer every week
- Available machine hours: **8,500 h/week** (cost $10/h)
- Minimum production level for each primer:

  200 gallons/week
- Maximum production level for each primer:

  1200 gallons/week
- Budget for raw material: **$90,000/week**
- Assume we know selling prices (see: ch3_PaintPrimer.xlsx)
- How many gallons of each primer to produce every week?

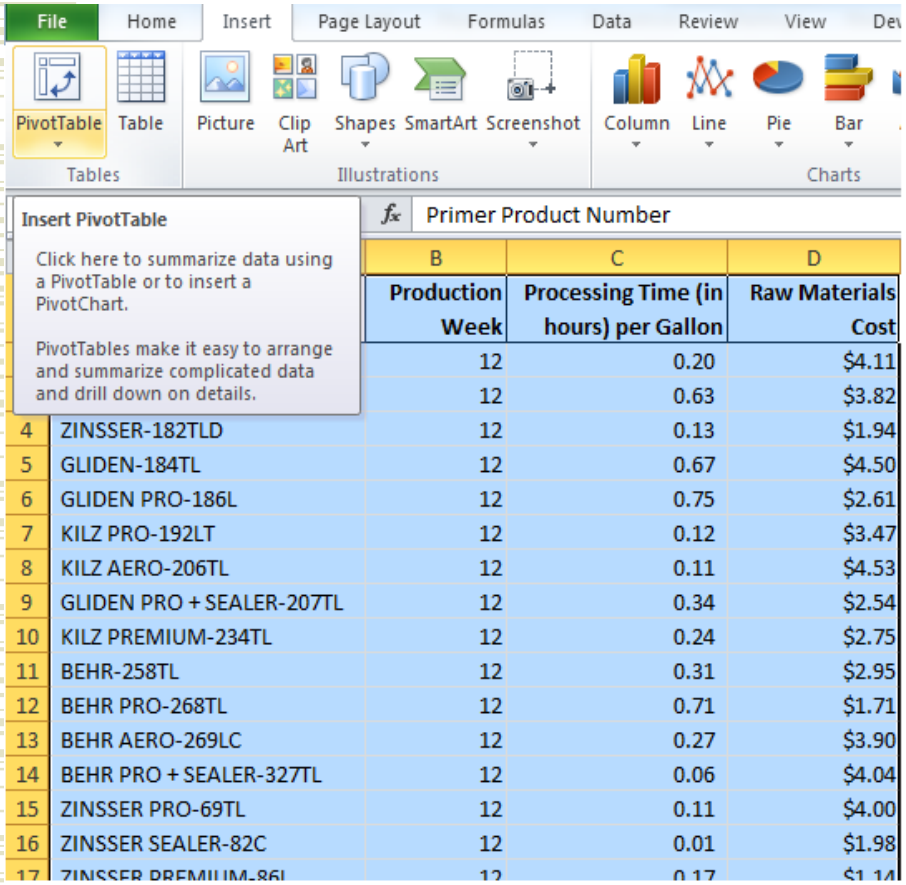
Operational data

	A	B	C	D
1	Primer Product Number	Production Week	Processing Time (in hours) per Gallon	Raw Materials Cost
2	KILZ-163TL	12	0.20	$4.11
3	KILZ2-166TL	12	0.63	$3.82
4	ZINSSER-182TLD	12	0.13	$1.94
5	GLIDEN-184TL	12	0.67	$4.50
6	GLIDEN PRO-186L	12	0.75	$2.61
7	KILZ PRO-192LT	12	0.12	$3.47
8	KILZ AERO-206TL	12	0.11	$4.53
9	GLIDEN PRO + SEALER-207TL	12	0.34	$2.54
10	KILZ PREMIUM-234TL	12	0.24	$2.75
11	BEHR-258TL	12	0.31	$2.95
12	BEHR PRO-268TL	12	0.71	$1.71
13	BEHR AERO-269LC	12	0.27	$3.90
14	BEHR PRO + SEALER-327TL	12	0.06	$4.04
15	ZINSSER PRO-69TL	12	0.11	$4.00
16	ZINSSER SEALER-82C	12	0.01	$1.98
17	ZINSSER PREMIUM-86L	12	0.17	$1.14
18	GILDEN PREMIUM-86LTORG	12	0.58	$3.60
19	KILZ-86TL	12	0.16	$1.22
20	KILZ2-B115	12	0.50	$4.52
21	ZINSSER-B126F	12	0.09	$2.97
22	GLIDEN-B127	12	0.85	$4.65
23	GLIDEN PRO-B128	12	0.66	$3.59
24	KILZ PRO-B129	12	0.14	$2.92

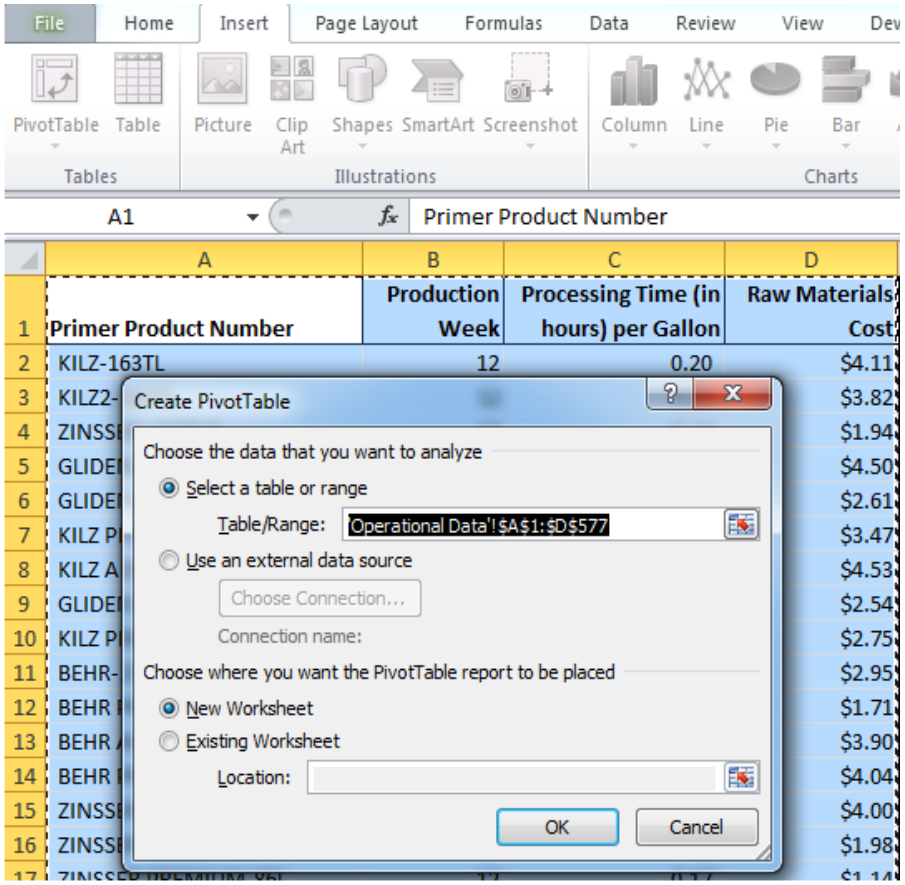
Step 1: Calculate model parameters

- **Pivot Table:**
  - Automatically sort, count, total or average data stored in spreadsheet
  - The "*refresh*" option (Data => Refresh All)
- **The features and capabilities of the Pivot Table:**
  - Select relevant columns for data processing
  - Created *Pivot Table* using INSERT menu as shown in Figure 3-2a
  - In the *Create Pivot Table* window, select to place the pivot table in a *New Worksheet* (Figure 3-2b)
  - Select Rows and Columns of the pivot table accordingly.

Step 1: Calculate model parameters



a. Selecting Columns and Inserting Pivot Table

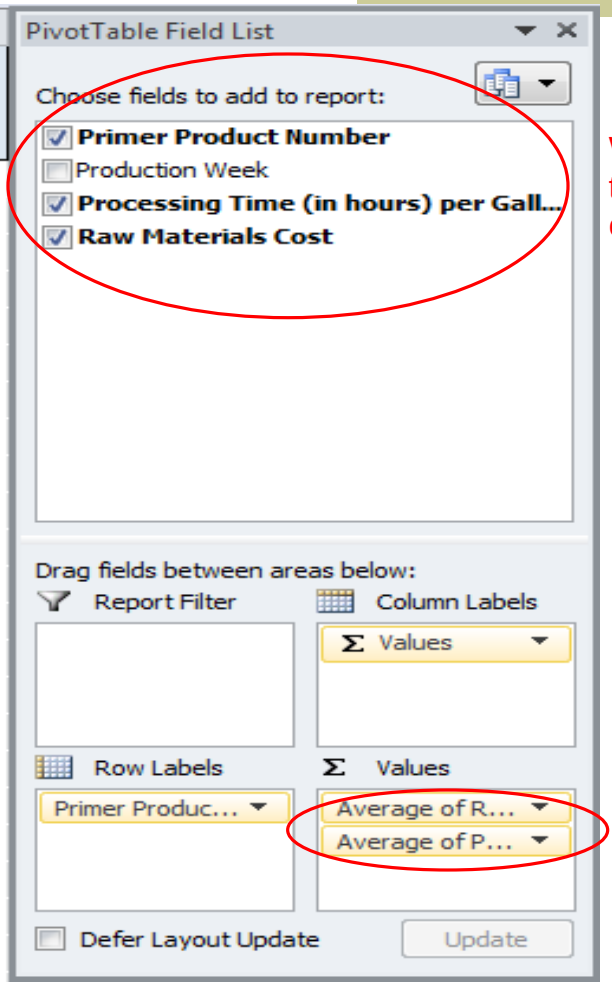


b. Creating Pivot Table in a New Worksheet

Step 1: Calculate model parameters

Result of the pivot table:

	A	B	C
1	Primer Product Number	Average of Raw Materials Cost	Average of Processing Time (in hours) per Gallon
2	BEHR AERO-269LC	$3.17	0.34
3	BEHR AERO-B41	$3.19	0.46
4	BEHR AERO-B82	$4.17	0.42
5	BEHR PRO + SEALER-327TL	$3.84	0.44
6	BEHR PRO + SEALER-B42	$2.35	0.47
7	BEHR PRO + SEALER-B86HD	$4.31	0.32
8	BEHR PRO-268TL	$3.48	0.46
9	BEHR PRO-B40	$4.34	0.39
10	BEHR PRO-B81	$3.40	0.52
11	BEHR-258TL	$4.28	0.31
12	BEHR-B39	$3.74	0.51
13	BEHR-B80N	$5.37	0.35
14	GILDEN PREMIUM-86LTORG	$2.59	0.37
15	GILDEN PREMIUM-B64	$2.84	0.42
16	GLIDEN PRO + SEALER-207TL	$3.95	0.37
17	GLIDEN PRO + SEALER-B29	$2.41	0.35
18	GLIDEN PRO + SEALER-B76	$2.78	0.36
19	GLIDEN PRO-186L	$3.18	0.53
20	GLIDEN PRO-B128	$4.20	0.39
21	GLIDEN PRO-B70	$4.15	0.36
22	GLIDEN-184TL	$4.76	0.46
23	GLIDEN-B127	$4.12	0.40



We need these columns

We need Average not Sum

Step 1: Calculate model parameters

Result of the pivot table:

	A	B	C	D	E	F	G
1	Primer Product Number	j-index	Price per Gallon	Processing time (in hours)	Raw Materials Cost	Contribution Coefficients	
2	BEHR AERO-269LC	1	$10.20	0.34	$3.17	$3.64	<--=C2-(E2+D2*10)
3	BEHR AERO-B41	2	$8.75	0.46	$3.19	$0.97	
4	BEHR AERO-B82	3	$8.46	0.42	$4.17	$0.08	
5	BEHR PRO + SEALER-327TL	4	$8.26	0.44	$3.84	($0.02)	
11	BEHR-258TL	10	$14.42	0.31	$4.28	$7.08	
12	BEHR-B39	11	$7.79	0.51	$3.74	($1.01)	
13	BEHR-B80N	12	$14.99	0.35	$5.37	$6.11	
14	GILDEN PREMIUM-86LTORG	13	$7.53	0.37	$2.59	$1.21	
15	GILDEN PREMIUM-B64	14	$10.42	0.42	$2.84	$3.37	
16	GLIDEN PRO + SEALER-207TL	15	$7.18	0.37	$3.95	($0.47)	
17	GLIDEN PRO + SEALER-B29	16	$12.22	0.35	$2.41	$6.36	
18	GLIDEN PRO + SEALER-B76	17	$9.99	0.36	$2.78	$3.64	
19	GLIDEN PRO-186L	18	$9.67	0.53	$3.18	$1.21	
20	GLIDEN PRO-B128	19	$7.25	0.39	$4.20	($0.84)	
21	GLIDEN PRO-B70	20	$12.94	0.36	$4.15	$5.21	
22	GLIDEN-184TL	21	$10.56	0.46	$4.76	$1.18	
23	GLIDEN-B127	22	$9.01	0.40	$4.12	$0.89	

cc = price – (raw materials cost+ processing times in hours*labor cost per hour)

Step 2: Define Decision Variables

- Decision variables are defined using general notation:
  - **x_j = number of gallons of primer *j* to be produced in the production line during the week where *j*=1, 2, 3, …, 48**
- New worksheet (LP Modeling):
  - The values of decision variables → 1
  - *Maximum Production Level, Minimum Production Level*, *Available Machine Hours*, and *Available Raw Materials*

Step 2: Define Decision Variables

	A	B	C	D	E	F	G	H	I	J	K
1	Primer Product Number	j-index	Decision Variables	Contribution Coefficients	Processing time (in hours)	Raw Materials Cost	Maximum Production Level	Minimum Production Level	Available Machine Hours	Available Raw Materials Budget	
2	BEHR AERO-269LC	1	1	$3.64	0.34	$3.17	1,200	200	8,500	$ 90,000	
3	BEHR AERO-B41	2	1	$0.97	0.46	$3.19					
4	BEHR AERO-B82	3	1	$0.08	0.42	$4.17					
5	BEHR PRO + SEALER-327TL	4	1	($0.02)	0.44	$3.84					
6	BEHR PRO + SEALER-B42	5	1	$5.58	0.47	$2.35		Total Profit	Machine Hours	Raw Materials Purchases	
7	BEHR PRO + SEALER-B86HD	6	1	$6.81	0.32	$4.31		$94.86	19.36	$172.44	
8	BEHR PRO-268TL	7	1	$0.74	0.46	$3.48		=SUMPRODUCT($C$2:$C$49,D2:D49)			
9	BEHR PRO-B40	8	1	($2.12)	0.39	$4.34					
10	BEHR PRO-B81	9	1	$0.43	0.52	$3.40					
11	BEHR-258TL	10	1	$7.08	0.31	$4.28					
12	BEHR-B39	11	1	($1.01)	0.51	$3.74					
13	BEHR-B80N	12	1	$6.11	0.35	$5.37					
14	GILDEN PREMIUM-86LTORG	13	1	$1.21	0.37	$2.59					
15	GILDEN PREMIUM-B64	14	1	$3.37	0.42	$2.84					
16	GLIDEN PRO + SEALER-207TL	15	1	($0.47)	0.37	$3.95					
17	GLIDEN PRO + SEALER-B29	16	1	$6.36	0.35	$2.41					
18	GLIDEN PRO + SEALER-B76	17	1	$3.64	0.36	$2.78					
19	GLIDEN PRO-186L	18	1	$1.21	0.53	$3.18					
20	GLIDEN PRO-B128	19	1	($0.84)	0.39	$4.20					
21	GLIDEN PRO-B70	20	1	$5.21	0.36	$4.15					
22	GLIDEN-184TL	21	1	$1.18	0.46	$4.76					
23	GLIDEN-B127	22	1	$0.89	0.40	$4.12					

Step 3: Formulate the objective function

	A	B	C	D	E	F	G	H	I	J
1	Primer Product Number	j-index	Decision Variables	Contribution Coefficients	Processing time (in hours)	Raw Materials Cost	Maximum Production Level	Minimum Production Level	Available Machine Hours	Available Raw Materials Budget
2	BEHR AERO-269LC	1	1	$3.64	0.34	$3.17	1,200	200	8,500	$ 90,000
3	BEHR AERO-B41	2	1	$0.97	0.46	$3.19				
4	BEHR AERO-B82	3	1	$0.08	0.42	$4.17				
5	BEHR PRO + SEALER-327TL	4	1	($0.02)	0.44	$3.84				
6	BEHR PRO + SEALER-B42	5	1	$5.58	0.47	$2.35		Total Profit	Machine Hours	Raw Materials Purchases
7	BEHR PRO + SEALER-B86HD	6	1	$6.81	0.32	$4.31		$94.86	19.36	$172.44
8	BEHR PRO-268TL	7	1	$0.74	0.46	$3.48		=SUMPRODUCT($C$2:$C$49,D2:D49)		
9	BEHR PRO-B40	8	1	($2.12)	0.39	$4.34				
10	BEHR PRO-B81	9	1	$0.43	0.52	$3.40				
11	BEHR-258TL	10	1	$7.08	0.31	$4.28				
12	BEHR-B39	11	1	($1.01)	0.51	$3.74				
13	BEHR-B80N	12	1	$6.11	0.35	$5.37				
14	GILDEN PREMIUM-86LTORG	13	1	$1.21	0.37	$2.59				
15	GILDEN PRE									
16	GILDEN ■									
17	GILDEN PRO									
18	GILDEN PRO									
19	GILDEN PRO									
20	GILDEN ■RO									
21	GILDEN PRO									
22	GILDEN-184TL	21								
23	GILDEN-B127	22	1	$0.89	0.40	$4.12				

objective function: The sum of the product between the decision variables and the contribution coefficients

SUMPRODUCT() function:

H7 =SUMPRODUCT ($C$2:$C$49, D2:D49)

Step 4: Identify the set of constraints

	A	B	C	D	E	F	G	H	I	J
1	Primer Product Number	j-index	Decision Variables	Contribution Coefficients	Processing time (in hours)	Raw Materials Cost	Maximum Production Level	Minimum Production Level	Available Machine Hours	Available Raw Materials Budget
2	BEHR AERO-269LC	1	1	$3.64	0.34	$3.17	1,200	200	8,500	$ 90,000
3	BEHR AERO-B41	2	1	$0.97	0.46	$3.19				
4	BEHR AERO-B82	3	1	$0.08	0.42	$4.17				
5	BEHR PRO + SEALER-327TL	4	1	($0.02)	0.44	$3.84				
6	BEHR PRO + SEALER-B42	5	1	$5.58	0.47	$2.35		Total Profit	Machine Hours	Raw Materials Purchases
7	BEHR PRO + SEALER-B86HD	6	1	$6.81	0.32	$4.31		$94.86	19.36	$172.44
8	BEHR PRO-268TL	7	1	$0.74	0.46	$3.48		=SUMPRODUCT($C$2:$C$49,D2:D49)		
9	BEHR PRO-B40	8	1	($2.12)	0.39	$4.34				
10	BEHR PRO-B81	9	1	$0.43	0.52	$3.40				
11	BEHR-258TL	10	1	$7.08	0.31	$4.28				
12	BEHR-B39	11	1	($1.01)	0.51	$3.74				
13	BEHR-B80N									
14	GILDEN PREMIUM									
15	GILDEN PREMIUM									
16	GLIDEN PRO + SEA									
17	GLIDEN PRO + SEA									
18	GLIDEN PRO + SEA									
19	GLIDEN PRO-186L									
20	GLIDEN PRO-B128									
21	GLIDEN PRO-B70									
22	GLIDEN-184TL	21	1	$1.18	0.46	$4.76				
23	GLIDEN-B127	22	1	$0.89	0.40	$4.12				

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1st set of constraint:

$$0.34x_1 + 0.466x_2 + 0.42x_3 + \cdots + 0.42x_{48} \leq 8500$$

SUMPRODUCT() function:

I7 =SUMPRODUCT ($C$2:$C$49, E2:E49)

Step 4: Identify the set of constraints

	A	B	C	D	E	F	G	H	I	J	K
1	Primer Product Number	j-index	Decision Variables	Contribution Coefficients	Processing time (in hours)	Raw Materials Cost	Maximum Production Level	Minimum Production Level	Available Machine Hours	Available Raw Materials Budget	
2	BEHR AERO-269LC	1	1	$3.64	0.34	$3.17	1,200	200	8,500	$ 90,000	
3	BEHR AERO-B41	2	1	$0.97	0.46	$3.19					
4	BEHR AERO-B82	3	1	$0.08	0.42	$4.17					
5	BEHR PRO + SEALER-327TL	4	1	($0.02)	0.44	$3.84					
6	BEHR PRO + SEALER-B42	5	1	$5.58	0.47	$2.35		Total Profit	Machine Hours	Raw Materials Purchases	
7	BEHR PRO + SEALER-B86HD	6	1	$6.81	0.32	$4.31		$94.86	19.36	$172.44	
8	BEHR PRO-268TL	7	1	$0.74	0.46	$3.48		=SUMPRODUCT($C$2:$C$49,D2:D49)			
9	BEHR PRO-B40	8	1	($2.12)	0.39	$4.34					
10	BEHR PRO-B81	9	1	$0.43	0.52	$3.40					
11	BEHR-258TL	10	1	$7.08	0.31	$4.28					
12	BEHR-B39										
13	BEHR-B80N										
14	GILDEN PREMI										
15	GILDEN PREMI										
16	GLIDEN PRO + S										
17	GLIDEN PRO + S										
18	GLIDEN PRO + S										
19	GLIDEN PRO-18										
20	GLIDEN PRO-B1										
21	GLIDEN PRO-B7										
22	GLIDEN-184TL	21	1	$1.18	0.46	$4.76					
23	GLIDEN-B127	22	1	$0.89	0.40	$4.12					

2nd set of constraint:

$$x\_1 \leq 1200, x\_2 \leq 1200, x\_3 \leq 1200, \ldots, x\_48 \leq 1200$$

Minimum production requirement:

$$x_1 \geq 200, \quad x_2 \geq 200, \quad x_3 \geq 200, \quad \ldots, \quad x_{48} \geq 200$$

Step 4: Identify the set of constraints

	A	B	C	D	E	F	G	H	I	J	K
1	Primer Product Number	j-index	Decision Variables	Contribution Coefficients	Processing time (in hours)	Raw Materials Cost	Maximum Production Level	Minimum Production Level	Available Machine Hours	Available Raw Materials Budget	
2	BEHR AERO-269LC	1	1	$3.64	0.34	$3.17	1,200	200	8,500	$ 90,000	
3	BEHR AERO-B41	2	1	$0.97	0.46	$3.19					
4	BEHR AERO-B82	3	1	$0.08	0.42	$4.17					
5	BEHR PRO + SEALER-327TL	4	1	($0.02)	0.44	$3.84					
6	BEHR PRO + SEALER-B42	5	1	$5.58	0.47	$2.35		Total Profit	Machine Hours	Raw Materials Purchases	
7	BEHR PRO + SEALER-B86HD	6	1	$6.81	0.32	$4.31		$94.86	19.36	$172.44	
8	BEHR PRO-268TL	7	1	$0.74	0.46	$3.48		=SUMPRODUCT($C$2:$C$49,D2:D49)			
9	BEHR PRO-B40	8	1	($2.12)	0.39	$4.34					
10	BEHR PRO-B81	9	1	$0.43	0.52	$3.40					
11	BEHR-258TL	10	1	$7.08	0.31	$4.28					
12	BEHR-B39										
13	BEHR-B80N										
14	GILDEN PREM										
15	GILDEN PREM										
16	GLIDEN PRO +										
17	GLIDEN PRO +										
18	GLIDEN PRO +										
19	GLIDEN PRO-1										
20	GLIDEN PRO-B										
21	GLIDEN PRO-B										
22	GLIDEN-184TL	21	1	$1.18	0.46	$4.76					
23	GLIDEN-B127	22	1	$0.89	0.40	$4.12					

3rd set of constraint:

$$3.17x_1 + 3.19x_2 + 4.17x_3 + \cdots + 5.19x_{48} \leq 9000$$

SUMPRODUCT() function:

$$J7 = SUMPRODUCT\ (\$C\$2{:}\$C\$49, F2{:}F49)$$

Step 5: Identify a set of non-negativity constraints

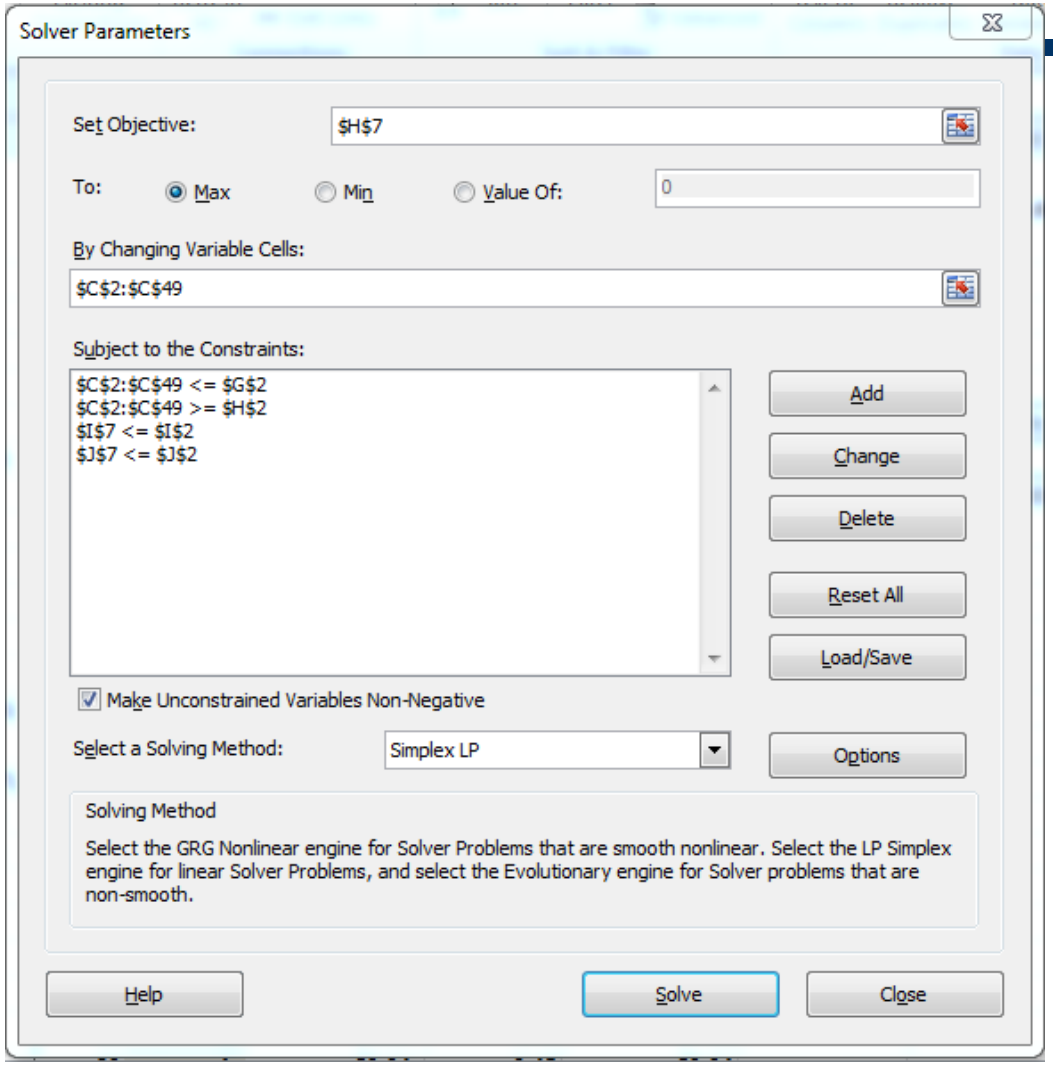
	A	B	C	D	E	F	G	H	I	J	K
1	Primer Product Number	j-index	Decision Variables	Contribution Coefficients	Processing time (in hours)	Raw Materials Cost	Maximum Production Level	Minimum Production Level	Available Machine Hours	Available Raw Materials Budget	
2	BEHR AERO-269LC	1	1	$3.64	0.34	$3.17	1,200	200	8,500	$ 90,000	
3	BEHR AERO-B41	2	1	$0.97	0.46	$3.19					
4	BEHR AERO-B82	3	1	$0.08	0.42	$4.17					
5	BEHR PRO + SEALER-327TL	4	1	($0.02)	0.44	$3.84					
6	BEHR PRO + SEALER-B42	5	1	$5.58	0.47	$2.35		Total Profit	Machine Hours	Raw Materials Purchases	
7	BEHR PRO + SEALER-B86HD	6	1	$6.81	0.32	$4.31		$94.86	19.36	$172.44	
8	BEHR PRO-268TL	7	1	$0.74	0.46	$3.48		=SUMPRODUCT($C$2:$C$49,D2:D49)			
9	BEHR PRO-B40	8	1	($2.12)	0.39	$4.34					
10	BEHR PRO-B81	9	1	$0.43	0.52	$3.40					
11	BEHR-258TL	10	1	$7.08	0.31	$4.28					
12	BEHR-B39	11	1	($1.01)	0.51	$3.74					
13	BEHR-B80N	12	1								
14	GILDEN PREMIUM-86LTORG	13	1								
15	GILDEN PREMIUM-B64	14	1								
16	GLIDEN PRO + SEALER-207TL	15	1	($0.47)	0.37	$3.95					
17	GLIDEN PRO + SEALER-B29	16	1	$6.36	0.35	$2.41					
18	GLIDEN PRO + SEALER-B76	17	1	$3.64	0.36	$2.78					
19	GLIDEN PRO-186L	18	1	$1.21	0.53	$3.18					
20	GLIDEN PRO-B128	19	1	($0.84)	0.39	$4.20					
21	GLIDEN PRO-B70	20	1	$5.21	0.36	$4.15					
22	GLIDEN-184TL	21	1	$1.18	0.46	$4.76					
23	GLIDEN-B127	22	1	$0.89	0.40	$4.12					

$$x_1 \geq 0, x_2 \geq 0, \ldots, x_{48} \geq 0$$

Solving LP Models with Excel

1. Set up constraints and objective functions in *Solver*
2. Generate the solution and *results*
3. Use *sensitivity analysis* to gain greater insight

Step 1: Set up constraints and objective functions in *Solver*



Step 2: General Solution and Results

- Answer Report:

- the value of the objective function, the final values for the decision variables that determine the objective function and the state of each constraint for the given solution.

Step 2: General Solution and Results

Objective Cell (Max)

Cell	Name	Original Value	Final Value
$H$7	BEHR PRO + SEALER-B86HD Total Profit	$94.86	$92,358.64

Variable Cells

Cell	Name	Original Value	Final Value	Integer
$C$2	BEHR AERO-269LC Decision Variables	1	915	Contin
$C$3	BEHR AERO-B41 Decision Variables	1	200	Contin
$C$4	BEHR AERO-B82 Decision Variables	1	200	Contin
$C$5	BEHR PRO + SEALER-327TL Decision Variables	1	200	Contin
$C$6	BEHR PRO + SEALER-B42 Decision Variables	1	1,200	Contin
$C$7	BEHR PRO + SEALER-B86HD Decision Variables	1	1,200	Contin
$C$8	BEHR PRO-268TL Decision Variables	1	200	Contin
$C$9	BEHR PRO-B40 Decision Variables	1	200	Contin
$C$10	BEHR PRO-B81 Decision Variables	1	200	Contin
$C$11	BEHR-258TL Decision Variables	1	1,200	Contin
$C$12	BEHR-B39 Decision Variables	1	200	Contin
$C$13	BEHR-B80N Decision Variables	1	1,200	Contin
$C$14	GILDEN PREMIUM-86LTORG Decision Variables	1	200	Contin
$C$15	GILDEN PREMIUM-B64 Decision Variables	1	200	Contin
$C$16	GLIDEN PRO + SEALER-207TL Decision Variables	1	200	Contin
$C$17	GLIDEN PRO + SEALER-B29 Decision Variables	1	1,200	Contin
$C$18	GLIDEN PRO + SEALER-B76 Decision Variables	1	200	Contin
$C$19	GLIDEN PRO-186L Decision Variables	1	200	Contin

Step 2: General Solution and Results

	A	B	C	D	E	F
14	Objective Cell (Max)					
15		Cell	Name	Original Value	Final Value	
16		$H$7	BEHR PRO + SEALER-B86HD Total Profit	$94.86	$92,358.64	
17						
18						
19	Variable Cells					
20		Cell	Name	Original Value	Final Value	Integer
21		$C$2	BEHR AERO-269LC Decision Variables	1	915	Contin
22		$C$3	BEHR AERO-B41 Decision Variables	1	200	Contin
23		$C$4	BEHR AERO-B82 Decision Variables	1	200	Contin
24		$C$5	BEHR PRO + SEALER-327TL Decision Variables	1	200	Contin
25		$C$6	BEHR PRO + SEALER-B42 Decision Variables	1	1,200	Contin
26		$C$7	BEHR PRO + SEALER-B86HD Decision Variables	1	1,200	Contin
						00 Contin
						00 Contin
						00 Contin
						00 Contin
						00 Contin
						00 Contin
						00 Contin
						00 Contin
						00 Contin
37		$C$18	GLIDEN PRO + SEALER-B76 Decision Variables	1	200	Contin
38		$C$19	GLIDEN PRO-186L Decision Variables	1	200	Contin

PMI can potentially generate revenues equal to $92,358.64 per week. This level of revenue can be achieved when the company produces certain quantities of each type of primer. Specifically, the company must produce 915 gallons of primer BEHR AERO-269LC, 200 gallons of BEHR AERO-B41, 200 gallons of BEHR AERO-B82, 1200 gallons of BEHR PRO + SEALER-B42, and so on.

Step 2: General Solution and Results

also indicates the status of each constraint

71	Constraints					
72	Cell	Name	Cell Value	Formula	Status	Slack
73	$I$7	Machine Hours	8500	$I$7<=$I$2	Binding	0
74	$J$7	Raw Materials Purchases	79218.13	$J$7<=$J$2	Not Binding	10781.87295
75	$C$2	BEHR AERO-269LC Decision Variables	915	$C$2<=$G$2	Not Binding	285.4679803
76	$C$3	BEHR AERO-B41 Decision Variables	200	$C$3<=$G$2	Not Binding	1000
77	$C$4	BEHR AERO-B82 Decision Variables	200	$C$4<=$G$2	Not Binding	1000
78	$C$5	BEHR PRO + SEALER-327TL Decision Variables	200	$C$5<=$G$2	Not Binding	1000
79	$C$6	BEHR PRO + SEALER-B42 Decision Variables	1,200	$C$6<=$G$2	Binding	0
80	$C$7	BEHR PRO + SEALER-B86HD Decision Variables	1,200	$C$7<=$G$2	Binding	0
81	$C$8	BEHR PRO-268TL Decision Variables	200	$C$8<=$G$2	Not Binding	1000
82	$C$9	BEHR PRO-B40 Decision Variables	200	$C$9<=$G$2	Not Binding	1000
83	$C$10	BEHR PRO-B81 Decision Variables	200	$C$10<=$G$2	Not Binding	1000
84	$C$11	BEHR-258TL Decision Variables	1,200	$C$11<=$G$2	Binding	0
85	$C$12	BEHR-B39 Decision Variables	200	$C$12<=$G$2	Not Binding	1000
86	$C$13	BEHR-B80N Decision Variables	1,200	$C$13<=$G$2	Binding	0
87	$C$14	GILDEN PREMIUM-86LTORG Decision Variables	200	$C$14<=$G$2	Not Binding	1000
88	$C$15	GILDEN PREMIUM-B64 Decision Variables	200	$C$15<=$G$2	Not Binding	1000
89	$C$16	GLIDEN PRO + SEALER-207TL Decision Variables	200	$C$16<=$G$2	Not Binding	1000
90	$C$17	GLIDEN PRO + SEALER-B29 Decision Variables	1,200	$C$17<=$G$2	Binding	0
91	$C$18	GLIDEN PRO + SEALER-B76 Decision Variables	200	$C$18<=$G$2	Not Binding	1000
92	$C$19	GLIDEN PRO-186L Decision Variables	200	$C$19<=$G$2	Not Binding	1000
93	$C$20	GLIDEN PRO-B128 Decision Variables	200	$C$20<=$G$2	Not Binding	1000
94	$C$21	GLIDEN PRO-B70 Decision Variables	1,200	$C$21<=$G$2	Binding	0

Step 2: General Solution and Results

also indicates the status of each constraint

71	Constraints					
72	Cell	Name	Cell Value	Formula	Status	Slack
73	$I$7	Machine Hours	8500	$I$7<=$I$2	Binding	0
74	$J$7	Raw Materials Purchases	79218.13	$J$7<=$J$2	Not Binding	10781.87295
75	$C$2	BEHR AERO-269LC Decision Variables	915	$C$2<=$G$2	Not Binding	285.4679803
76	$C$3	BEHR AERO-B41 Decision Variables	200	$C$3<=$G$2	Not Binding	1000
77	$C$4	BEHR AERO-B82 Decision Variables	200	$C$4<=$G$2	Not Binding	1000
78	$C$5	BEHR PRO + SEALER-327TL Decision Variables	200	$C$5<=$G$2	Not Binding	1000
79	$C$6	BEHR PRO + SEALER-B42 Decision Variables	1,200	$C$6<=$G$2	Binding	0
80	$C$7	BEHR PRO + SEALER-B86HD Decision Variables	1,200	$C$7<=$G$2	Binding	0
81	$C$8	BEHR PRO-268TL Decision Variables	200	$C$8<=$G$2	Not Binding	1000
					Not Binding	1000
					Not Binding	1000
					Binding	0
					Not Binding	1000
					Binding	0
					Not Binding	1000
					Not Binding	1000
					Not Binding	1000
					Binding	0
					Not Binding	1000
					Not Binding	1000
93	$C$20	GLIDEN PRO-B128 Decision Variables	200	$C$20<=$G$2	Not Binding	1000
94	$C$21	GLIDEN PRO-B70 Decision Variables	1,200	$C$21<=$G$2	Binding	0

For example, the *Machine Hours* is indicated as a *binding* constraint. That means that producing the suggested quantities for each primer will fully utilize the available 8500 hours of machine time.

On the contrary, the *Raw Materials Purchases* is not a binding constraint. Out of $90,000 available to purchase raw materials, the optimal solution requires that PMI purchases only $79218.13, which leaves a slack of unused $10,781.87.

Step 2: General Solution and Results

Decision makers can use the analysis of constraints to make better decisions. Normally, not a binding constraint indicates a partial usage of the resource. Adding more resources in this constraint does not change the value of the objective function. That is a waste of resources if PMI decides to allocate more money to the *Raw Materials Purchases* budget, since the existing budget is note fully utilized yet.

On the other hand, a binding constraint indicates a full usage of the resource or a full satisfaction of the constraint. If the factory needs to make more profit, more machine hours must be added through buying new machines or extended the hours of the existing machines (adding another shift for example).

Also, since the company does not want to produce more than the weekly demand, binding constraints (see rows 79 and 80) also mean that the factory has produced the maximum possible allowance for these two primers.

Step 3: Use sensitivity analysis to gain more insights (*Sensitivity Report*)

- Sensitivity Analysis:
  - An important tool in gaining additional insights about the model output

1. Changes in the Right-hand Side Values

58 Constraints

59			Final	Shadow	Constraint	Allowable	Allowable
60	Cell	Name	Value	Price	R.H. Side	Increase	Decrease
61	$I$7	Machine Hours	8500	10.76402065	8500	96.58333333	241.75
62	$J$7	Raw Materials Purchases	79218.13	0	90000	1E+30	10781.87295

Two Constraints

How the value of the objective function changes when one additional unit of the constraint is acquired

To determine the range of right hand side values where the shadow price impact remains true.

Step 3: Use sensitivity analysis to gain more insights (*Sensitivity Report*)

- Sensitivity Analysis:
  - An important tool in gaining additional insights about the model output

1. Changes in the Right-hand Side Values

58 Constraints

59			Final	Shadow	Constraint	Allowable	Allowable
60	Cell	Name	Value	Price	R.H. Side	Increase	Decrease
61	$I$7	Machine Hours	8500	10.76402065	8500	96.58333333	241.75
62	$J$7	Raw Materials Purchases	79218.13	0	90000	1E+30	10781.87295

Suppose that instead of having 8,500 machine hours, the right-hand side value is increased to 8,501 machine hours. How would that affect the objective function? Instead of rerunning the solution, the decision maker can simply look at the shadow price and know that the objective function will increase by exactly $10,76. Alternatively, when the available machine hours are reduced by one hour, the objective function will be reduced by $10,76.

Step 3: Use sensitivity analysis to gain more insights (*Sensitivity Report*)

- Sensitivity Analysis:
  - An important tool in gaining additional insights about the model output

1. Changes in the Right-hand Side Values

58 Constraints

59			Final	Shadow	Constraint	Allowable	Allowable
60	Cell	Name	Value	Price	R.H. Side	Increase	Decrease
61	$I$7	Machine Hours	8500	10.76402065	8500	96.58333333	241.75
62	$J$7	Raw Materials Purchases	79218.13	0	90000	+30	10781

The allowable increase and allowable decrease values can be used to determine the range of right-hand side values where the shadow price impact remains true.

That is, the right-hand side value for the machine hours can increase up to 96.58 units to a total of 8,596.58 hours and the objective function (total profit) can improve by $10.76 per each unit.

Step 3: Use sensitivity analysis to gain more insights (*Sensitivity Report*)

1. Changes in the Right-hand Side Values

58 Constraints

59			Final	Shadow	Constraint	Allowable	Allowable
60	Cell	Name	Value	Price	R.H. Side	Increase	Decrease
61	$I$7	Machine Hours	8500	10.76402065	8500	96.58333333	241.75
62	$J$7	Raw Materials Purchases	79218.13	0	90000	E+30	10781

This means that the new profit can be calculated as long as the total machine hours range from the lower limit 8,258.25 (8,500.- 241.75) to the upper limit 8,596.58. So, for example, if the machine hours increase from 8,500 to 8,580 (two additional machines x 40 per week per machine), there will be a total profit increase of $860.80 (80 x $10.76) for an overall total profit of $92,358.64 + $860.80 = $93,219.

On the other side, if machine hours are reduced by 100 hours, the total profit will be reduced by $1,076 dollars.

Step 3: Use sensitivity analysis to gain more insights (*Sensitivity Report*)

1. Changes in the Right-hand Side Values

58 Constraints

59			Final	Shadow	Constraint	Allowable	Allowable
60	Cell	Name	Value	Price	R.H. Side	Increase	Decrease
61	$I$7	Machine Hours	8500	10.76402065	8500	96.58333333	241.75
62	$J$7	Raw Materials Purchases	79218.13	0	90000	1E+30	10781.87295

Shadow prices have the same meaning for not binding constraints. In the case of the Raw Materials Purchases constraint, the shadow price is zero, the allowable increase is 1E +30 (infinity), and the allowable decrease is 10,781.87. That means, the right-hand side value of the purchasing budget can increase beyond $90,000 with no impact in the objective function. After all, the existing budget is not completely used in the optimal solution.

On the other side, the budget can decrease down to approximately $79,218 and the solution will still remain the same. The decision maker needs to rerun the model when the right-hand side values of the constraints change beyond the lower and upper limits.

Step 3: Use sensitivity analysis to gain more insights (*Limits Report*)

Objective Cell	Name	Value
$H$7	Total Profit	$92,358.64

Variable Cell	Name	Value	Lower Limit	Objective Result	Upper Limit	Objective Result
$C$2	BEHR AERO-269LC Decision Variables	915	200	89,756	915	92,359
$C$3	BEHR AERO-B41 Decision Variables	200	200	92,359	200	92,359
$C$4	BEHR AERO-B82 Decision Variables	200	200	92,359	200	92,359
$C$5	BEHR PRO + SEALER-327TL Decision Variables	200	200	92,359	200	92,359
$C$6	BEHR PRO + SEALER-B42 Decision Variables	1,200	200	86,778	1,200	92,359
$C$7	BEHR PRO + SEALER-B86HD Decision Variables	1,200	200	85,544	1,200	92,359
$C$8	BEHR PRO-268TL Decision Variables	200	200	92,359	200	92,359
$C$9	BEHR PRO-B40 Decision Variables	200	200	92,359	200	92,359
$C$10	BEHR PRO-B81 Decision Variables	200	200	92,359	200	92,359
$C$11	BEHR-258TL Decision Variables	1,200	200	85,282	1,200	92,359
$C$12	BEHR-B39 Decision Variables	200	200	92,359	200	92,359
$C$13	BEHR-B80N Decision Variables	1,200	200	86,244	1,200	92,359
$C$14	GILDEN PREMIUM-86LTORG Decision Variables	200	200	92,359	200	92,359
$C$15	GILDEN PREMIUM-B64 Decision Variables	200	200	92,359	200	92,359
$C$16	GLIDEN PRO + SEALER-207TL Decision Variables	200	200	92,359	200	92,359

Step 3: Use sensitivity analysis to gain more insights (*Limits Report*)

Objective Cell	Name	Value
$H$7	Total Profit	$92,358.64

Variable Cell	Name	Value	Lower Limit	Objective Result	Upper Limit	Objective Result
$C$2	BEHR AERO-269LC Decision Variables	915	200	89,756	915	92,359
$C$3	BEHR AERO-B41 Decision Variables	200	200	92,359	200	92,359
$C$4	BEHR AERO-B82 Decision Variables	200	200	92,359	200	92,359
$C$5	BEHR PRO + SEALER-327TL Decision Variables	200	200	92,359	200	92,359
$C$14	GILDEN PREMIUM-86LTORG Decision Variables	200	200	92,359	200	92,359
$C$15	GILDEN PREMIUM-B64 Decision Variables	200	200	92,359	200	92,359
$C$16	GLIDEN PRO + SEALER-207TL Decision Variables	200	200	92,359	200	92,359

Decision variables also have upper bounds and lower bounds and can be seen as constraints. For example, in the case of the PMI model, the requirement was not to produce more than 1,200 gallons of each primer and to produce at least 200 gallons for each primer. Even when there is no simple lower bound, decision variables must always comply with the non-negativity constraint, so the lower bound is zero.

Step 3: Use sensitivity analysis to gain more insights (*Limits Report*)

Objective Cell	Name	Value
$H$7	Total Profit	$92,358.64

Variable Cell	Name	Value	Lower Limit	Objective Result	Upper Limit	Objective Result
$C$2	BEHR AERO-269LC Decision Variables	915	200	89,756	915	92,359
$C$3	BEHR AERO-B41 Decision Variables	200	200	92,359	200	92,359
$C$4	BEHR AERO-B82 Decision Variables	200	200	92,359	200	92,359
$C$5	BEHR PRO + SEALER-327TL Decision Variables	200	200	92,359	200	92,359
$C$15	GILDEN PREMIUM-B64 Decision Variables	200	200	92,359	200	92,359
$C$16	GLIDEN PRO + SEALER-207TL Decision Variables	200	200	92,359	200	92,359

Note the optimal value of the first decision variable 914.5 (rounded to 915). The constraint related to this decision variable can be considered as not binding constraint because the right-hand side value of this constraint is the lower bound of 200. So, there is a slack of 715. If the value of the first decision variable (BEHR AERO-269LC) is reduced to 200, then the objective function will be reduced to the Objective Result, 89,756.(92,358 - 715 x 3.64) where 3.64 is the contribution coefficient of this variable.

Step 3: Use sensitivity analysis to gain more insights (*Limits Report*)

Objective Cell	Name	Value
$H$7	Total Profit	$92,358.64

Variable Cell	Name	Value	Lower Limit	Objective Result	Upper Limit	Objective Result
$C$2	BEHR AERO-269LC Decision Variables	915	200	89,756	915	92,359
$C$3	BEHR AERO-B41 Decision Variables	200	200	92,359	200	92,359
$C$4	BEHR AERO-B82 Decision Variables	200	200	92,359	200	92,359
$C$5	BEHR PRO + SEALER-327TL Decision Variables	200	200	92,359	200	92,359
$C$6	BEHR PRO + SEALER-B42 Decision Variables	1,200	200	86,778	1,200	92,359
$C$7	BEHR PRO + SEALER-B86HD Decision Variables	1,200	200	85,544	1,200	92,359
$C$8	BEHR PRO-268TL Decision Variables	200	200	92,359	200	92,359
$C$15	GILDEN PREMIUM-B64 Decision Variables	200	200	92,359	200	92,359
$C$16	GLIDEN PRO + SEALER-207TL Decision Variables	200	200	92,359	200	92,359

The optimal value of the second decision variable (BEHR AERO-B41) is 200. As such, the constraint, related to the decision variable can be considered as a binding constraint. So there is no slack in this constraint-there is no room for this variable to change without impacting the objective function.

Step 3: Use sensitivity analysis to gain more insights (*Sensitivity Report*)

2. Changes in the Contribution Coefficients

Sensitivity Report for Decision Variables

A partial list of decision variables, their final value, reduced cost, objective coefficient, allowable increase, and allowable decrease

Step 3: Use sensitivity analysis to gain more insights (*Sensitivity Report*)

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$C$2	BEHR AERO-269LC Decision Variables	914.5320197	0	3.641826988	0.375508065	0.091206755
$C$3	BEHR AERO-B41 Decision Variables	200	-3.961359718	0.972149748	3.961359718	1E+30
$C$4	BEHR AERO-B82 Decision Variables	200	-4.456816186	0.077527515	4.456816186	1E+30
$C$5	BEHR PRO + SEALER-327TL Decision Variables	200	-4.79188979	-0.019840634	4.79188979	1E+30
$C$6	BEHR PRO + SEALER-B42 Decision Variables	1200	0.521641746	5.580731453	1E+30	0.521641746
$C$7	BEHR PRO + SEALER-B86HD Decision Variables	1200	3.423839446	6.814505952	1E+30	3.423839446
$C$8	BEHR PRO-268TL Decision Variables	200	-4.234185686	0.744173866	4.234185686	1E+30
$C$9	BEHR PRO-B40 Decision Variables	200	-6.287629182	-2.116571179	6.287629182	1E+30
$C$10	BEHR PRO-B81 Decision Variables	200	-5.154322127	0.425028579	5.154322127	1E+30
$C$11	BEHR-258TL Decision Variables	1200	3.775623714	7.076590047	1E+30	3.775623714
$C$12	BEHR-B39 Decision Variables	200	-6.457221537	-1.01242109	6.457221537	1E+30
$C$13	BEHR-B80N Decision Variables	1200	2.338322825	6.114700071	1E+30	2.338322825
$C$14	GILDEN PREMIUM-86LTORG Decision Variables	200	-2.808911413	1.209656297	2.808911413	1E+30
$C$15	GILDEN PREMIUM-B64 Decision Variables	200	-1.156812025	3.373046666	1.156812025	1E+30
$C$16	GLIDEN PRO + SEALER-207TL Decision Variables	200	-4.450201762	-0.46751412	4.450201762	1E+30
$C$17	GLIDEN PRO + SEALER-B29 Decision Variables	1200	2.648373076	6.361960202	1E+30	2.648373076
$C$18	GLIDEN PRO + SEALER-B76 Decision Variables	200	-0.20901431	3.639123073	0.20901431	1E+30
$C$19	GLIDEN PRO-186L Decision Variables	200	-4.4639646	1.214056294	4.4639646	1E+30
$C$20	GLIDEN PRO-B128 Decision Variables	200	-5.031237287	-0.84223925	5.031237287	1E+30

Step 3: Use sensitivity analysis to gain more insights (*Sensitivity Report*)

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$C$2	BEHR AERO-269LC Decision Variables	914.5320197	0	3.641826988	0.375508065	0.091206755
$C$3	BEHR AERO-B41 Decision Variables	200	-3.961359718	0.972149748	3.961359718	1E+30
$C$4	BEHR AERO-B82 Decision Variables	200	-4.456816186	0.077527515	4.456816186	1E+30

The reduced cost for a decision variable, by definition, is the shadow price of the constraint for that variable. As such, the reduced cost indicates how much the objective function changes if the constraint of the variable changes by one unit.

The optimal solution indicates that the company must make 915 gallons of primer BEHR AERO-269LC. The reduced cost for this variable is 0, contribution coefficient is 3.641826988, allowable increase is 0.375508065, and allowable decrease is 0.091206755. That means that PMI will continue to make 915 gallons of this primer, with no impact in the value of the objective function (because the reduced cost is zero) as long as the contribution coefficient for this variable remains within the lower limit of 3.550620232 (3.641826988 - 0.091206755) and upper limit of 4.017335053 (3.641826988 + 0.375508065).

Step 3: Use sensitivity analysis to gain more insights (*Sensitivity Report*)

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$C$2	BEHR AERO-269LC Decision Variables	914.5320197	0	3.641826988	0.375508065	0.091206755
$C$3	BEHR AERO-B41 Decision Variables	200	-3.961359718	0.972149748	3.961359718	1E+30
$C$4	BEHR AERO-B82 Decision Variables	200	-4.456816186	0.077527515	4.456816186	1E+30
$C$5	BEHR PRO + SEALER-327TL Decision Variables	200	-4.79188979	-0.019840634	4.79188979	1E+30
$C$6	BEHR PRO + SEALER-B42 Decision Variables	1200	0.521641746	5.580731453	1E+30	0.521641746
$C$7	BEHR PRO + SEALER-B86HD Decision Variables	1200	3.423839446	6.814505952	1E+30	3.423839446
$C$8	BEHR PRO-268TL Decision Variables	200	-4.234185686	0.744173866	4.234185686	1E+30
$C$9	BEHR PRO-B40 Decision Variables	200	-6.287629182	2.116571179	6.287629182	1E+30
$C$1						1E+30
$C$1						775623714
$C$1						1E+30
$C$1						338322825
$C$1						1E+30
$C$1						1E+30
$C$1						1E+30
$C$1						648373076
$C$1						1E+30
$C$19	GLIDEN PRO-186L Decision Variables	200	-4.4639646	1.214056294	4.4639646	1E+30
$C$20	GLIDEN PRO-B128 Decision Variables	200	-5.031237287	-0.84223925	5.031237287	1E+30

Now, consider the second decision variable, BEHR AERO-B41. Because $x_2 \geq 200$ is a constraint in the LP formulation, then this constraint has no slack; it is a binding constraint and has a shadow price (in this case, a reduced cost) of —3.9613 (to round for four digits).

Assume for a moment that the constraint is changed to $x_2 \geq 201$. In this case, the value in the objective function will be decreased by 3.9613 (increased by the reduced cost). Of course this is expected because the ideal solution was not to produce 200 units of this primer. Anything other than this amount will provide a less-than-optimal value for the objective function.

Big Optimizations with Big data

- . “For decision-making factories today, the key raw material is information. The contemporary transformation, occurring in an environment of big data, is called big optimization.”
- Examples of Amazon and Google
- Process-driven models require the implementation of three systems:
  - **Magnetism**
  - **Agility** **(MAD approach)**
  - **Depth**

Wrap up!

Create Excel Template

- Use Pivot Table to Calculate Model Parameters
- Formulate the Model
  - Define decision variables
  - Formulate the objective function
  - Identify the set of constraints
  - Identify a set of non-negativity constraints
- Use Excel Formulas to Represent Resource Usage and Objective Function

Use Solver to Generate Results

- Invoke Solver Parameter Window
- Set Objective Function Cell and Constraints
- Generate and Analyze Results
  - Perform sensitivity analysis for contribution coefficients
  - Perform sensitivity analysis for constraints

End of The Lecture

Thank You