

**TKMY3 Tilastollisen päättelyn perusteet**

**Kaavakokoelma**

### Todennäköisyyslaskentaa

$$(1) n! = n \cdot (n-1) \cdot (n-2) \cdots 1$$

$$(2) \binom{n}{k} = \frac{n!}{k!(n-k)!}$$

$$(3) \frac{n!}{(n-k)!} = n \cdot (n-1) \cdots (n-k+1)$$

$$(4) P(\emptyset) = 0$$

$$(5) P(A^*) = 1 - P(A)$$

$$(6) P(A \cup B) = P(A) + P(B)$$

$$(7) P\left(\bigcup_{i=1}^n A_i\right) = \sum_{i=1}^n P(A_i)$$

$$(8) P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$(9) P(A \cap B) = P(A) \cdot P(B)$$

$$(10) P\left(\bigcap_{i=1}^n A_i\right) = \prod_{i=1}^n P(A_i)$$

$$(11) P(A \cap B) = P(A) \cdot P(B | A)$$

### Diskreetit satunnaismuuttujat

$$(12) F(x) = p(X \leq x) = \sum_{x_i \leq x} p_i$$

$$(13) EX = \sum P(x) \cdot x$$

$$(14) D^2 X = \sum P(x) \cdot (x - EX)^2$$

$$(15) P(X = k) = \binom{n}{k} \cdot p^k \cdot (1-p)^{n-k}, k = 0, \dots, n$$

$$(16) EX = np$$

$$(17) D^2 X = n \cdot p \cdot (1-p)$$

$$(18) P(X = k) = \frac{\binom{K}{k} \binom{N-K}{n-k}}{\binom{N}{n}}, k = 0, \dots, n$$

$$(19) EX = \frac{n \cdot K}{N}$$

$$(20) D^2 X = \frac{n \cdot K}{N} \left(1 - \frac{K}{N}\right) \frac{N-n}{N-1}$$

$$(21) P(X = k) = \frac{\lambda^k e^{-\lambda}}{k!}, k = 0, 1, \dots$$

$$(22) EX = \lambda$$

$$(23) D^2 X = \lambda$$

### Jatkuvat satunnaismuuttujat

$$(24) Z = \frac{X - \mu}{\sigma}$$

$$(25) \Phi(-z) = 1 - \Phi(z)$$

$$(26) F_p^{(v_1, v_2)} = \frac{1}{F_{1-p}^{(v_2, v_1)}}$$

$$z_p = -z_{1-p}$$

$$t_p = -t_{1-p}$$

## Luottamusvälejä

**μ:lle**

$$(27) \quad V_{100(1-\alpha)} = [\bar{x} - t_{\alpha/2}^{(n-1)} \frac{s}{\sqrt{n}}, \bar{x} + t_{\alpha/2}^{(n-1)} \frac{s}{\sqrt{n}}]$$

$$(28) \quad V_{100(1-\alpha)} = [\bar{x} - z_{\alpha/2} \frac{s}{\sqrt{n}}, \bar{x} + z_{\alpha/2} \frac{s}{\sqrt{n}}]$$

**μ<sub>1</sub>-μ<sub>2</sub>:lle**

*σ<sub>1</sub> ja σ<sub>2</sub> tuntemattomia, yhtä suuria*

$$(29) \quad V_{100(1-\alpha)} = [\bar{X}_1 - \bar{X}_2 - t_{\alpha/2}^{(n_1+n_2-2)} s \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}, \bar{X}_1 - \bar{X}_2 + t_{\alpha/2}^{(n_1+n_2-2)} s \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}]$$

$$(30) \quad V_{100(1-\alpha)} = [\bar{X}_1 - \bar{X}_2 - z_{\alpha/2} s \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}, \bar{X}_1 - \bar{X}_2 + z_{\alpha/2} s \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}]$$

$$(31) \quad s = \sqrt{\frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1 + n_2 - 2}}$$

*σ<sub>1</sub> ja σ<sub>2</sub> tuntemattomia, eri suuria*

$$(32) \quad V_{100(1-\alpha)} = [\bar{X}_1 - \bar{X}_2 - t_{\alpha/2}^{(\nu)} \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}, \bar{X}_1 - \bar{X}_2 + t_{\alpha/2}^{(\nu)} \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}]$$

$$(33) \quad \frac{1}{v} = \frac{c^2}{n_1 - 1} + \frac{(1-c)^2}{n_2 - 1}$$

$$(34) \quad c = \frac{\frac{s_1^2}{n_1}}{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

$$(35) \quad V_{100(1-\alpha)} = [\bar{X}_1 - \bar{X}_2 - z_{\alpha/2} \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}, \bar{X}_1 - \bar{X}_2 + z_{\alpha/2} \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}]$$

**μ<sub>D</sub>:lle**

$$(36) \quad V_{100(1-\alpha)} = [\bar{D} - t_{\alpha/2}^{(n-1)} \frac{s_D}{\sqrt{n}}, \bar{D} + t_{\alpha/2}^{(n-1)} \frac{s_D}{\sqrt{n}}]$$

$$(37) \quad V_{100(1-\alpha)} = [\bar{D} - z_{\alpha/2} \frac{s_D}{\sqrt{n}}, \bar{D} + z_{\alpha/2} \frac{s_D}{\sqrt{n}}]$$

**π:lle**

$$(38) \quad V_{100(1-\alpha)} = \left[ p - z_{\alpha/2} \sqrt{\frac{p(1-p)}{n}}, p + z_{\alpha/2} \sqrt{\frac{p(1-p)}{n}} \right]$$

**$\pi_1$ -  $\pi_2$ :lle**

$$(39) \quad V_{100(1-\alpha)} = \left[ p_1 - p_2 \pm z_{\alpha/2} \sqrt{\frac{p_1(1-p_1)}{n_1} + \frac{p_2(1-p_2)}{n_2}} \right]$$

**$\pi_D$ :lle**

$$(40) \quad V_{100(1-\alpha)} = \left[ (b-c) \Big/ n \pm z_{\alpha/2} \sqrt{\frac{(b+c) - (b-c) \Big/ n}{n(n-1)}} \right]$$

**Tilastollisia testejä**

**$\mu$ :lle**

$$(41) \quad t_{hav} = \frac{\bar{x} - \mu_0}{s \Big/ \sqrt{n}} \sim t^{(n-1)}$$

$$(42) \quad z_{hav} = \frac{\bar{x} - \mu_0}{s \Big/ \sqrt{n}} \sim N(0,1)$$

**$\mu_1$ - $\mu_2$ :lle**

$\sigma_1$  ja  $\sigma_2$  tuntemattomia, yhtä suuria

$$(43) \quad t_{hav} = \frac{\bar{X}_1 - \bar{X}_2}{s \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} \sim t^{(n_1+n_2-2)}$$

$$(44) \quad z_{hav} = \frac{\bar{X}_1 - \bar{X}_2}{s \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} \sim N(0,1)$$

$$(45) \quad s = \sqrt{\frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1 + n_2 - 2}}$$

$\sigma_1$  ja  $\sigma_2$  tuntemattomia, eri suuria

$$(46) \quad t_{hav} = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} \sim t^{(v)}$$

$$(47) \quad z_{hav} = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} \sim N(0,1)$$

$$(48) \quad \frac{1}{v} = \frac{c^2}{n_1 - 1} + \frac{(1-c)^2}{n_2 - 1}$$

(49)

$$c = \frac{\frac{s_1^2}{n_1}}{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

**μ<sub>D</sub>:lle**

(50)

$$t_{hav} = \frac{\overline{D}}{\frac{s_D}{\sqrt{n}}} \sim \mathfrak{t}^{(n-1)}$$

(51)

$$z_{hav} = \frac{\overline{D}}{\frac{s_D}{\sqrt{n}}} \sim \text{N}(0,1)$$

(52)

$$\overline{D} = \frac{1}{n} \sum_{i=1}^n D_i$$

(53)

$$s_D = \sqrt{\frac{1}{n-1} \sum_{i=1}^n (D_i - \overline{D})^2}$$

**σ<sup>2</sup><sub>1</sub>/σ<sup>2</sup><sub>2</sub>:lle**

(54)

$$F_{\text{hav}} = \frac{s_1^2}{s_2^2} \sim F^{(n_1-1,n_2-1)}$$

**ρ:lle**

(55)

$$t_{hav} = \frac{r\sqrt{n-2}}{\sqrt{1-r^2}} \sim \mathfrak{t}^{(n-2)}$$

**β<sub>i</sub>:lle**

(56)

$$t_{hav} = \frac{b_1 - \beta_0}{s_{b_i}} \sim \mathfrak{t}^{(n-k-1)}$$

**Epäparametrisiä testejä**

**π:lle**

(57)

$$z_{hav} = \frac{p - \pi_0}{\sqrt{\frac{\pi_0(1-\pi_0)}{n}}} \sim \text{N}(0,1)$$

**π<sub>1</sub>- π<sub>2</sub>:lle**

(58)

$$z_{hav} = \frac{p_1 - p_2}{\sqrt{p(1-p)\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} \sim \text{N}(0,1)$$

(59)

$$p = \frac{n_1 p_1 + n_2 p_2}{n_1 + n_2}$$

$\pi_D$ -lle

(60)

$$z_{hav} = \frac{\frac{(b-c)/n}{(b+c) - \frac{(b-c)^2}{n}}}{\sqrt{\frac{n(n-1)}}} \sim N(0,1)$$

$\chi^2$ -yhteensopivuustesti

(61)

$$\chi^2_{hav} = \sum_{i=1}^k \frac{(f_i - e_i)^2}{e_i} \sim \chi^2_{(k-1)}$$

$\chi^2$ -riippumattomuustesti

(62)

$$\chi^2_{hav} = \sum_{i=1}^r \sum_{j=1}^s \frac{(f_{ij} - e_{ij})^2}{e_{ij}} \sim \chi^2_{((r-1)(s-1))}$$

(63)

$$e_{ij} = \frac{f_{i.} f_{.j}}{n}$$