$$x[n] = [-1, \underbrace{3, \underbrace{2, 4}_{\text{konjugoituja}}, 1}_{\text{kompleksisia konjugaatteja}}], \quad n = 0 \ldots N-1$$

ainoa reaalinen

missä $N$ on sinusoidinen

Päätelmä $x[k]$ ainoa tarvitsee laskea kaksi $(N-1)/2$ sekka, siksi lopuilla pääsemme symmetrisyyden perusteella. Tässä esimerkissä

$X[0]$, $X[1]$, $X[2]$

Lähde: luentomateriaali

$X[3]$ ja $X[4]$

laskettu vain vastaksen varmentamiseksi!

$x = [N] = [-1, 3, 2, 4, 1]$ $\qquad N = 5$

$$X[k] = \sum_{n=0}^{N-1} x[n] e^{-\frac{i2\pi}{N}kn} \qquad k = 0, 1, 2, \dots, N-1$$

$$X[0] = \underbrace{\sum_{n=0}^{N-1} x[n] e^{-\frac{i2\pi}{N}\cdot 0 \cdot n}}_{= 1 \text{, tosto } e^0 = 1} = x[0] + x[1] + x[2] + x[3] + x[4]$$

$$= -1 + 3 + 2 + 4 + 1$$

$$= 9$$

$$\underline{\underline{X[0] = 9}}$$

$k = 1$

$$\underline{X}[1] = \sum_{n=0}^{N-1} x[n]\, e^{-i\frac{2\pi}{N}\cdot 1\cdot n}$$

$$= x[0]\underbrace{e^{-i\frac{2\pi}{5}\cdot 1\cdot 0}}_{1} + x[1]\underbrace{e^{-i\frac{2\pi}{5}\cdot 1\cdot 1}}_{\substack{= \cos(2\pi/5) - i\sin(2\pi/5) \\ \approx\ 0{,}309 - i\,0{,}951}} \quad *$$

$$+ x[2]\underbrace{e^{-i\frac{2\pi}{5}\cdot 1\cdot 2}}_{= -0{,}809 - i\,0{,}587} \quad * \qquad + x[3]\underbrace{e^{-i\frac{2\pi}{5}\cdot 1\cdot 3}}_{= -0{,}809 + i\,0{,}5} \quad *$$

$$+ x[4]\underbrace{e^{-i\frac{2\pi}{5}\cdot 1\cdot 4}}_{= 0{,}309 + i\,0{,}951} \quad *$$

* trigonometria Euler-i kaava

$$e^{-i\phi}$$

$x[2]$ ja $x[3]$ kompleksikonjugaatteja

$x[1]$ ja $x[4]$ kompleksikonjugaatteja

$$\underline{X}[1] = -1\cdot 1 + 3(0{,}309 - 0{,}951) + 2(-0{,}809 - i$$

$$+ 3(0{,}309 - i\,0{,}951)$$
$$+ 2(-0{,}809 - i\,0{,}587)$$
$$+ 4(-0{,}809 + i\,0{,}587)$$
$$+ 1(0{,}309 + i\,0{,}951)$$

$$= -4{,}618 - i\,0{,}726$$

$k = 2$

$$\underline{X}[2] = \sum_{n=0}^{N-1} x[n]\, e^{-i\frac{2\pi}{N}\cdot 2\cdot n}$$

$$= x[0]\, \underbrace{e^{-i\frac{2\pi}{5}\cdot 2\cdot 0}}_{=1}$$

$$+ x[1]\, \underbrace{e^{-i\frac{2\pi}{5}\cdot 2\cdot 1}}_{= -0{,}809 - i\,0{,}587}$$ x euler-n facta

$$+ x[2]\, \underbrace{e^{-i\frac{2\pi}{5}\cdot 2\cdot 2}}_{= 0{,}309 + i\,0{,}951}$$ x euler-n facta

$$+ x[3]\, \underbrace{e^{-i\frac{2\pi}{5}\cdot 2\cdot 3}}_{0{,}309 - i\,0{,}951}$$

$$+ x[4]\, \underbrace{e^{-i\frac{2\pi}{5}\cdot 2\cdot 4}}_{-0{,}809 + i\,0{,}587}$$ x euler-n facta

$$\underline{X}[2] = -1\,(1)$$
$$+ 3\,(-0{,}809 - i\,0{,}587)$$
$$+ 2\,(0{,}309 + i\,0{,}951)$$
$$+ 4\,(0{,}309 - i\,0{,}951)$$
$$+ 1\,(-0{,}809 + i\,0{,}587)$$
$$= -2{,}382 - i\,3{,}076$$

$k = 3$

$$\underline{\bar{X}}[3] \qquad \sum_{n=0}^{N-1} x[n]\, e^{-i\frac{2\pi}{N}3\cdot n}$$

$$x[0]\ \underbrace{e^{-i\frac{2\pi}{5}\cdot 3\cdot 0}}_{1}$$

$$+\ x[1]\ \underbrace{e^{-i\frac{2\pi}{5}\cdot 3\cdot 1}}_{} = -0{,}809 + i\,0{,}587$$

$$+\ x[2]\ \underbrace{e^{-i\frac{2\pi}{5}\cdot 3\cdot 2}}_{} = 0{,}309 - i\,0{,}951$$

$$+\ x[3]\ \underbrace{e^{-i\frac{2\pi}{5}\cdot 3\cdot 3}}_{} = 0{,}309 + i\,0{,}951$$

$$+\ x[4]\ \underbrace{e^{-i\frac{2\pi}{5}\cdot 3\cdot 4}}_{} \quad -0{,}809 - i\,0{,}587$$

$$\begin{aligned}\underline{\bar{X}}[3] = &-1\,(1)\\ &+3\,(-0{,}809 + i\,0{,}587)\\ &+2\,(0{,}309 - i\,0{,}951)\\ &+4\,(0{,}309 + i\,0{,}951)\\ &+1\,(-0{,}809 - i\,0{,}587)\\ &= -2{,}382 + i\,3{,}076\end{aligned}$$

$k = 4$

$$\sum_{n=0}^{N-1} x[n]\, e^{-i\frac{2\pi}{N}4\cdot n}$$

$\underline{X}[4]$

$$x[0]\underbrace{e^{-i\frac{2\pi}{5}\cdot 4\cdot 0}}_{1}$$

$$+ x[1]\underbrace{e^{-i\frac{2\pi}{5}\cdot 4\cdot 1}}_{} = 0{,}309 + i\,0{,}951$$

$$+ x[2]\underbrace{e^{-i\frac{2\pi}{5}\cdot 4\cdot 2}}_{} = -0{,}809 + i\,0{,}587$$

$$+ x[3]\underbrace{e^{-i\frac{2\pi}{5}\cdot 4\cdot 3}}_{} = -0{,}809 - i\,0{,}587$$

$$+ x[4]\underbrace{e^{-\frac{2\pi}{5}\cdot 4\cdot 4}}_{} = 0{,}309 - i\,0{,}951$$

$$\begin{aligned}\underline{X}[4] = & -1(1) \\ & +3(0{,}309 + i\,0{,}951) \\ & +2(-0{,}809 + i\,0{,}587) \\ & +4(-0{,}809 - i\,0{,}587) \\ & +1(0{,}309 - i\,0{,}951) \\ = & -4{,}618 + i\,0{,}726\end{aligned}$$

Vastaus:

$$\underline{\overline{X}}[k] = [\,9,\ -4{,}618 - i\,0{,}726,\ -2{,}382 - i\,3{,}076,\ -2{,}382 + i\,3{,}076,\ -4{,}618 + i\,0{,}726\,]$$

$\underline{\overline{X}}[1]$ ja $\underline{\overline{X}}[4]$ = kompleksikonjugaatteja

$\underline{\overline{X}}[2]$ ja $\underline{\overline{X}}[3]$ = kompleksikonjugaatteja