$$x[n] = [-1, 3, \underbrace{2, 4}_{\text{kompleksiset}}, 1], \quad n = 0 \ldots N-1$$

missä N on siemenlukumäärä

Ainoa Reaaliluku kompleksisuhdeakselia

Päätelmä $x[k]$ arvoja tarvitsee laskea vain $(N-1)/2$ sekka, sillä loput voidaan päätellä symmetrisyyden perusteella. Tässä esimerkissä

$$X[0], \; X[1], \; X[2]$$

Lisäksi: huomautuksia

$$X[3] \text{ ja } X[4]$$

laskeksen vain vastaksen varmentamiseksi!

$x = [N] = [-1, 3, 2, 4, 1]$ $\quad N = 5$

$$\underline{X}[k] = \sum_{n=0}^{N-1} x[n] e^{-\frac{i2\pi}{N}kn} \qquad k = 0, 1, 2, \ldots, N-1$$

$$\underline{X}[0] = \underbrace{\sum_{n=0}^{N-1} x[n] e^{-\frac{i2\pi}{N}\cdot 0 \cdot n}}_{= 1 \text{, tosto } e^0 = 1} = x[0] + x[1] + x[2] + x[3] + x[4]$$

$$= -1 + 3 + 2 + 4 + 1$$

$$= 9$$

$$\underline{X}[0] = 9$$

$k = 1$

$$\underline{X}[1] = \sum_{n=0}^{N-1} x[n]\, e^{-i\frac{2\pi}{N}\cdot 1\cdot n}$$

$$= x[0]\underbrace{e^{-i\frac{2\pi}{5}\cdot 1\cdot 0}}_{1} + x[1]\underbrace{e^{-i\frac{2\pi}{5}\cdot 1\cdot 1}}_{\substack{= \cos(2\pi/5) - i\sin(2\pi/5) \\ \approx\ 0{,}309 - i\,0{,}951}} \quad *$$

$$+ x[2]\underbrace{e^{-i\frac{2\pi}{5}\cdot 1\cdot 2}}_{= -0{,}809 - i\,0{,}587} \quad * \qquad + x[3]\underbrace{e^{-i\frac{2\pi}{5}\cdot 1\cdot 3}}_{= -0{,}809 + i\,0{,}587} \quad *$$

$$+ x[4]\underbrace{e^{-i\frac{2\pi}{5}\cdot 1\cdot 4}}_{= 0{,}309 + i\,0{,}951} \quad *$$

* hyperbolometry Euler's hacker

$$e^{-i\phi}$$

$x[2] \neq x[3]$ komplekslaşmalıdır

$x[1] \neq x[4]$ komplekslaşmalıdır

$$\underline{X}[1] = -1\cdot 1 + 3(0{,}309 - 0{,}951) + 2(-0{,}809 - i$$

$$+ 3(0{,}309 - i\,0{,}951)$$
$$+ 2(-0{,}809 - i\,0{,}587)$$
$$+ 4(-0{,}809 + i\,0{,}587)$$
$$+ 1(0{,}309 + i\,0{,}951)$$

$$= -4{,}618 - i\,0{,}726$$

$k = 2$

$$\underline{X}[2] = \sum_{n=0}^{N-1} x[n]\, e^{-i\frac{2\pi}{N}\cdot 2\cdot n}$$

$$= x[0]\, \underbrace{e^{-i\frac{2\pi}{5}\cdot 2\cdot 0}}_{=1}$$

$$+ x[1]\, \underbrace{e^{-i\frac{2\pi}{5}\cdot 2\cdot 1}}_{= -0{,}809 - i\,0{,}587}$$ x eulerin kaava

$$+ x[2]\, \underbrace{e^{-i\frac{2\pi}{5}\cdot 2\cdot 2}}_{= 0{,}309 + i\,0{,}951}$$ x eulerin kaava

$$+ x[3]\, \underbrace{e^{-i\frac{2\pi}{5}\cdot 2\cdot 3}}_{0{,}309 - i\,0{,}951}$$

$$+ x[4]\, \underbrace{e^{-i\frac{2\pi}{5}\cdot 2\cdot 4}}_{-0{,}809 + i\,0{,}587}$$ x eulerin kaava

$$\underline{X}[2] = -1(1) + 3(-0{,}809 - i\,0{,}587) + 2(0{,}309 + i\,0{,}951) + 4(0{,}309 - i\,0{,}951) + 1(-0{,}809 + i\,0{,}587)$$

$$= -2{,}382 - i\,3{,}076$$

$k = 3$

$$\underline{X}[3] \qquad \sum_{n=0}^{N-1} x[n]\, e^{-i\frac{2\pi}{N}3\cdot n}$$

$$x[0]\ \underbrace{e^{-i\frac{2\pi}{5}\cdot 3\cdot 0}}_{1}$$

$$+\ x[1]\ \underbrace{e^{-i\frac{2\pi}{5}\cdot 3\cdot 1}}_{} = -0{,}809 + i\,0{,}587$$

$$+\ x[2]\ \underbrace{e^{-i\frac{2\pi}{5}\cdot 3\cdot 2}}_{} = 0{,}309 - i\,0{,}951$$

$$+\ x[3]\ \underbrace{e^{-i\frac{2\pi}{5}\cdot 3\cdot 3}}_{} = 0{,}309 + i\,0{,}951$$

$$+\ x[4]\ \underbrace{e^{-i\frac{2\pi}{5}\cdot 3\cdot 4}}_{} \quad -0{,}809 - i\,0{,}587$$

$$\begin{aligned}\underline{X}[3] = &-1\,(1)\\ &+3\,(-0{,}809 + i\,0{,}587)\\ &+2\,(0{,}309 - i\,0{,}951)\\ &+4\,(0{,}309 + i\,0{,}951)\\ &+1\,(-0{,}809 - i\,0{,}587)\\ &= -2{,}382 + i\,3{,}076\end{aligned}$$

$$\sum_{n=0}^{N-1} x[n]\, e^{-j2\pi \frac{k}{N} n n}$$

$$x[n] = \{1, -1, 1, -1\}$$

$$X[k] = \sum_{n=0}^{3} x[n]\, e^{-j2\pi \frac{k}{4} n n}$$

$$X[0] = x[0]\, e^{-j2\pi \frac{0}{4} \cdot 0 \cdot 0} + x[1]\, e^{-j2\pi \frac{0}{4} \cdot 1 \cdot 0} + x[2]\, e^{-j2\pi \frac{0}{4} \cdot 2 \cdot 0} + x[3]\, e^{-j2\pi \frac{0}{4} \cdot 3 \cdot 0}$$

$$= 1 - 1 + 1 - 1 = 0$$

$$X[1] = x[0]\, e^{-j2\pi \frac{1}{4} \cdot 0 \cdot 1} + x[1]\, e^{-j2\pi \frac{1}{4} \cdot 1 \cdot 1} + x[2]\, e^{-j2\pi \frac{1}{4} \cdot 2 \cdot 1} + x[3]\, e^{-j2\pi \frac{1}{4} \cdot 3 \cdot 1}$$

$$= 0.309 - j0.951$$

$$X[2] = x[0]\, e^{-j2\pi \frac{2}{4} \cdot 0 \cdot 2} + x[1]\, e^{-j2\pi \frac{2}{4} \cdot 1 \cdot 2} + x[2]\, e^{-j2\pi \frac{2}{4} \cdot 2 \cdot 2} + x[3]\, e^{-j2\pi \frac{2}{4} \cdot 3 \cdot 2}$$

$$= -0.809 - j0.587$$

$$X[3] = x[0]\, e^{-j2\pi \frac{3}{4} \cdot 0 \cdot 3} + x[1]\, e^{-j2\pi \frac{3}{4} \cdot 1 \cdot 3} + x[2]\, e^{-j2\pi \frac{3}{4} \cdot 2 \cdot 3} + x[3]\, e^{-j2\pi \frac{3}{4} \cdot 3 \cdot 3}$$

$$= -0.809 + j0.587$$

$$= -0.309 + j0.951$$

$$= -1 + j0 + j0 + 1 + j0 + j0 + 1 - j0$$

$$= -1 + j0.618 + j0.726$$

Vastaus:

$$\underline{X}[k] = [\,9,\ -4{,}618 - i\,0{,}726,\ -2{,}382 - i\,3{,}076,\ -2{,}382 + i\,3{,}076,\ -4{,}618 + i\,0{,}726\,]$$

$\underline{X}[1]$ ja $\underline{X}[4]$ = kompleksikonjugaatteja

$\underline{X}[2]$ ja $\underline{X}[3]$ = kompleksikonjugaatteja