## QUESTION PAPER CODE 65/2/D

# EXPECTED ANSWERS/VALUE POINTS SECTION - A

Marks

1. 
$$\frac{dv}{dr} = -\frac{A}{r^2}, \implies r^2 \frac{d^2v}{dr^2} + 2r \frac{dv}{dr} = 0$$
 \(\frac{1}{2} + \frac{1}{2} m\)

2. I. 
$$F = e^{\int \frac{1}{\sqrt{x}} dx} = e^{2\sqrt{x}}$$

3. 
$$p = \frac{\vec{a} \cdot \vec{b}}{|\vec{b}|} = \frac{8}{7}$$

4. 
$$\begin{vmatrix} 1 & 3 & 1 \\ 2 & -1 & -1 \\ 0 & \lambda & 3 \end{vmatrix} = 0 \implies \lambda = 7$$

$$\frac{1}{2} + \frac{1}{2} m$$

5. 
$$\cos^2 \frac{\pi}{2} + \cos^2 \frac{\pi}{3} + \cos^2 \theta = 1 \implies \theta = \frac{\pi}{6}$$

6. 
$$a_{23} = \frac{|2-3|}{2} = \frac{1}{2}$$

#### **SECTION - B**

7. 
$$\frac{dx}{d\theta} = -a \sin \theta + b \cos \theta$$
 \tag{7} m

$$\frac{dy}{d\theta} = a\cos\theta + b\sin\theta$$

$$\therefore \frac{dy}{dx} = \frac{a \cos \theta + b \sin \theta}{a \sin \theta + b \cos \theta} = -\frac{x}{y}$$

or 
$$y \frac{dy}{dx} + x = 0$$

$$\therefore y \frac{d^2y}{dx^2} + \frac{dy}{dx} \cdot \frac{dy}{dx} + 1 = 0$$

Using (i) we get 
$$y \frac{d^2y}{dx^2} - \frac{x}{y} \frac{dy}{dx} + 1 = 0$$
 1/2 m

$$\therefore y^2 \frac{d^2y}{dx^2} - x \frac{dy}{dx} + y = 0$$

## 8. Let x be the side of an equilateral triangle

$$\therefore \frac{dx}{dt} = 2 \text{ cm/s.}$$

Area (A) = 
$$\frac{\sqrt{3}x^2}{4}$$
 1 m

$$\Rightarrow \frac{dA}{dt} = \frac{\sqrt{3}}{2} \times \frac{dx}{dt}$$
 1 m

$$\Rightarrow \frac{dA}{dt} = \frac{\sqrt{3}}{2} \cdot (20) \cdot (2) = 20\sqrt{3} \text{ cm}^2 | \text{s}$$

9. Writing 
$$x + 3 = -\frac{1}{2}(-4 - 2x) + 1$$

$$\therefore \int (x+3)\sqrt{3-4x-x^2} \ dx = -\frac{1}{2} \int (-4-2x)\sqrt{3-4x-x^2} \ dx + \int \sqrt{7-(x+2)^2} \ dx \quad \frac{1}{2} + \frac{1}{2} m$$

$$= -\frac{1}{3} \left( 3 - 4x - x^2 \right)^{\frac{3}{2}} + \frac{x+2}{2} \sqrt{3 - 4x - x^2} + \frac{7}{2} \sin^{-1} \left( \frac{x+2}{\sqrt{7}} \right) + c$$
 1+1 m

10. HF. M P

A 
$$\begin{pmatrix} 40 & 50 & 20 \\ 25 & 40 & 30 \\ C & 35 & 50 & 40 \end{pmatrix} \begin{pmatrix} 25 \\ 100 \\ 50 \end{pmatrix} = \begin{pmatrix} 7000 \\ 6125 \\ 7875 \end{pmatrix}$$

1½ m

Funds collected by school A: Rs. 7000,

For writing one value 1 m

11. Getting 
$$A^2 = \begin{pmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{pmatrix}$$
 1½ m

$$A^{2} - 5A + 4I = \begin{pmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{pmatrix} + \begin{pmatrix} -10 & 0 & -5 \\ -10 & -5 & -15 \\ -5 & 5 & 0 \end{pmatrix} + \begin{pmatrix} 4 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 4 \end{pmatrix}$$

$$1 \text{ m}$$

$$= \begin{pmatrix} -1 & -1 & -3 \\ -1 & -3 & -10 \\ -5 & 4 & 2 \end{pmatrix}$$
 1 m

$$\therefore X = \begin{pmatrix} 1 & 1 & 3 \\ 1 & 3 & 10 \\ 5 & -4 & -2 \end{pmatrix}$$
<sup>1</sup>/<sub>2</sub> m

OR

$$A' = \begin{pmatrix} 1 & 0 & -2 \\ -2 & -1 & 2 \\ 3 & 4 & 1 \end{pmatrix}$$
 1 m

$$|A'| = 1(-9)-2(-5) = -9+10 = 1 \neq 0$$
 ½ m

$$Adj A' = \begin{pmatrix} -9 & -8 & -2 \\ 8 & 7 & 2 \\ -5 & -4 & -1 \end{pmatrix}$$
 2 m

$$\therefore (A')^{-1} = \begin{pmatrix} -9 & -8 & -2 \\ 8 & 7 & 2 \\ -5 & -4 & -1 \end{pmatrix}$$
<sup>1</sup>/<sub>2</sub> m

12. 
$$f(x) = \begin{vmatrix} a & -1 & 0 \\ ax & a & -1 \\ ax^2 & ax & a \end{vmatrix}$$

$$R_2 \rightarrow R_2 - x R_1$$
 and  $R_3 \rightarrow R_3 - x^2 R_1$ 

$$f(x) = \begin{bmatrix} a & -1 & 0 \\ 0 & a+x & -1 \\ 0 & ax+x^2 & a \end{bmatrix}$$
 (For bringing 2 zeroes in any row/column 1+1 m

$$f(x) = a (a^2 + 2ax + x^2) = a (x + a)^2$$

$$f(2x) - f(x) = a [2x + a]^{2} - a (x + a)^{2}$$

$$= a x (3x + 2a)$$
1 m

$$= -\int \frac{dt}{(1-t)(1+t)(1+2t)}$$
 where  $\cos x = t$  \(\frac{1}{2}\) m

$$= \int \left( \frac{-\frac{1}{6}}{1-t} + \frac{\frac{1}{2}}{1+t} - \frac{\frac{4}{3}}{1+2t} \right) dt$$

$$= + \frac{1}{6} \log \left| 1 - t \right| + \frac{1}{2} \log \left| 1 + t \right| - \frac{2}{3} \log \left| 1 + 2t \right| + c$$

$$= \frac{1}{6} \log \left| 1 - \cos x \right| + \frac{1}{2} \log \left| 1 + \cos x \right| - \frac{2}{3} \log \left| 1 + 2 \cos x \right| + c$$

OR

$$\int \frac{x^2 - 3x + 1}{\sqrt{1 - x^2}} dx = \int \frac{2 - 3x - (1 - x^2)}{\sqrt{1 - x^2}} dx$$

$$= 2 \int \frac{1}{\sqrt{1-x^2}} dx - 3 \int \frac{x}{\sqrt{1-x^2}} dx - \int \sqrt{1-x^2} dx$$

$$= 2\sin^{-1}x + 3\sqrt{1-x^2} - \frac{x}{2}\sqrt{1-x^2} - \frac{1}{2}\sin^{-1}x + c \qquad (\frac{1}{2}+1+1)m$$

or = 
$$\frac{3}{2} \sin^{-1}x + \frac{1}{2} (6-x) \sqrt{1-x^2} + c$$

14. 
$$I = \int_{-\pi}^{\pi} (\cos ax - \sin bx)^{2} dx = \int_{-\pi}^{\pi} (\cos^{2}ax + \sin^{2}bx) dx - \int_{-\pi}^{\pi} 2\cos ax \sin bx dx$$
$$= I_{1} - I_{2}$$
<sup>1</sup>/<sub>2</sub> m

$$I_1 = 2 \int_0^{\pi} (\cos^2 ax + \sin^2 bx) dx \quad \text{(being an even fun.)}$$

$$I_2 = 0$$
 (being an odd fun.) 1 m

$$I = I_1 = \int_0^{\pi} (1 + \cos 2ax + 1 - \cos 2bx) dx$$

$$= \left[2x + \frac{\sin 2ax}{2a} - \frac{\sin 2bx}{2b}\right]_0^{\pi}$$

$$= \left[2\pi + \frac{1}{2a} \cdot \sin 2a\pi - \frac{\sin 2b\pi}{2b}\right] \text{ or } 2\pi$$

15. Let  $E_1$ : selecting bag A, and  $E_2$ : selecting bag B.

$$P(E_1) = \frac{1}{3}, P(E_2) = \frac{2}{3}$$

Let A: Getting one Red and one balck ball

$$\therefore P(A|E_1) = \frac{{}^{4}C_1 \cdot {}^{6}C_1}{{}^{10}C_2} = \frac{8}{15}, \ P(A|E_2) = \frac{{}^{7}C_1 \cdot {}^{3}C_1}{{}^{10}C_2} = \frac{7}{15}$$

$$P(A) = P(E_1) \cdot P(A|E_1) + P(E_2) \cdot P(A|E_2)$$

$$= \frac{1}{3} \cdot \frac{8}{15} + \frac{2}{3} \cdot \frac{7}{15} = \frac{22}{45}$$

$$x$$
 : 0 1 2 3 4  $\frac{1}{2}$  m

$$P\left(x\right) \quad : \quad {}^{4}C_{0}\left(\frac{1}{2}\right)^{4} \quad {}^{4}C_{1}\left(\frac{1}{2}\right)^{3}\left(\frac{1}{2}\right) \quad {}^{4}C_{2}\left(\frac{1}{2}\right)^{2}\left(\frac{1}{2}\right)^{2} \quad {}^{4}C_{3}\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)^{3} \quad {}^{4}C_{4}\left(\frac{1}{2}\right)^{4} \quad \quad 1\frac{1}{2}\,m$$

$$= \frac{1}{16} \qquad = \frac{4}{16} \qquad = \frac{6}{16} \qquad = \frac{4}{16} \qquad = \frac{1}{16} \qquad \frac{1}{2} \text{ m}$$

$$x P(x) : 0 \frac{4}{16} \frac{12}{16} \frac{12}{16} \frac{4}{16}$$

$$x^{2}P(x)$$
: 0  $\frac{4}{16}$   $\frac{24}{16}$   $\frac{36}{16}$   $\frac{16}{16}$   $\frac{1}{2}m$ 

Mean = 
$$\sum x P(x) = \frac{32}{16} = 2$$
 ½ m

Variance = 
$$\sum x^2 P(x) - (\sum x P(x))^2 = \frac{80}{16} - (2)^2 = 1$$

16. 
$$\vec{r} \times \vec{i} = (x \hat{i} + y \hat{j} + z \hat{k}) x \hat{i} = -y \hat{k} + z \hat{j}$$

$$\vec{r} \times \vec{j} = \left(x\hat{i} + y\hat{j} + z\hat{k}\right)\hat{j} = x\hat{k} - z\hat{i}$$
1½ m

$$(\overrightarrow{r} \times \widehat{i}), (\overrightarrow{r} \times \overrightarrow{j}) = (o\overrightarrow{i} + z\overrightarrow{j} - y \overrightarrow{k}) \cdot (-z\overrightarrow{i} + o\overrightarrow{j} + x \overrightarrow{k}) = -xy$$

$$(\overrightarrow{r} \times \widehat{i}) \cdot (\overrightarrow{r} \times \overrightarrow{j}) + xy = -xy + xy = 0$$
<sup>1</sup>/<sub>2</sub> m

17. Any point on the line 
$$\frac{x-2}{3} = \frac{y+1}{4} = \frac{z-2}{12}$$
 is  $(3\lambda + 2, 4\lambda - 1, 12\lambda + 2)$ 

If this is the point of intersection with plane x - y + z = 5

then 
$$3\lambda + 2 - 4\lambda + 1 + 12\lambda + 2 - 5 = 0 \implies \lambda = 0$$

$$\therefore$$
 Point of intersection is  $(2, -1, 2)$  1 m

Required distance = 
$$\sqrt{(2+1)^2 + (-1+5)^2 + (2+10)^2} = 13$$

18. Writing 
$$\cot^{-1}(x+1) = \sin^{-1} \frac{1}{\sqrt{1+(x+1)^2}}$$

and 
$$tan^{-1}x = cos^{-1} \frac{1}{\sqrt{1+x^2}}$$
 1½ m

$$\therefore \sin \left( \sin^{-1} \frac{1}{\sqrt{1 + (x+1)^2}} \right) = \cos \left( \cos^{-1} \frac{1}{\sqrt{1 + x^2}} \right)$$
 \(\frac{1}{\sqrt{1 + x^2}} \)

$$1+x^2+2x+1=1+x^2 \implies x=-\frac{1}{2}$$

OR

$$\left(\tan^{-1}x\right)^{2} + \left(\cot^{-1}x\right)^{2} = \frac{5\pi^{2}}{8} \implies \left(\tan^{-1}x\right)^{2} + \left(\frac{\pi}{2} - \tan^{-1}x\right)^{2} = \frac{5\pi^{2}}{8}$$

$$2 \left( \tan^{-1} x \right)^{2} - \pi \tan^{-1} x - \frac{3\pi^{2}}{8} = 0$$

$$\tan^{-1}x = \frac{\pi \pm \sqrt{\pi^2 + 3\pi^2}}{4} = 3\pi/4, -\pi/4$$

$$\Rightarrow x = -1$$

19. Putting 
$$x^2 = \cos \theta$$
, we get  $\frac{1}{2}$  m

$$y = \tan^{-1} \left( \frac{\sqrt{1 + \cos\theta} + \sqrt{1 - \cos\theta}}{\sqrt{1 + \cos\theta} - \sqrt{1 - \cos\theta}} \right)$$
<sup>1</sup>/<sub>2</sub> m

$$= \tan^{-1} \left( \frac{\cos \frac{\theta}{2} + \sin \frac{\theta}{2}}{\cos \frac{\theta}{2} - \sin \frac{\theta}{2}} \right) = \tan^{-1} \left( \frac{1 + \tan \frac{\theta}{2}}{1 - \tan \frac{\theta}{2}} \right)$$

$$1 + \frac{1}{2} \text{ m}$$

$$y = \frac{\pi}{4} + \frac{\theta}{2} = \frac{\pi}{4} + \frac{1}{2} \cos^{-1} x^2$$

$$\frac{dy}{dx} = -\frac{1}{2} \frac{1}{\sqrt{1-x^4}} \cdot 2x = -\frac{x}{\sqrt{1-x^4}}$$

#### **SECTION - C**

20.  $f(x) = \sin x - \cos x, \ 0 < x < 2\pi$ 

$$f'(x)=0 \implies \cos x + \sin x = 0 \text{ or } \tan x = -1,$$
 1 m

$$\therefore x = \frac{3\pi}{4}, \frac{7\pi}{4}$$

$$f''(x) = \cos x - \sin x$$

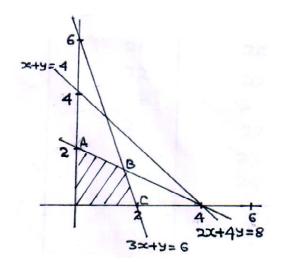
$$f''(3\pi/4) = -\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}$$
 i.e - ve so,  $x = 3\pi/4$  is Local Maxima

and 
$$f''(7\pi/4) = -\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}$$
 i.e + ve so,  $x = 7\pi/4$  is Local Minima

Local Maximum value 
$$=\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} = \sqrt{2}$$

Local Minimum value 
$$= -\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} = \sqrt{2}$$

21. Correct graphs of three lines 
$$1x3 = 3 \text{ m}$$
Correctly shading feasible region 1 m



Vertices are

Z = 2x + 5y is maximum

at 
$$A(0, 2)$$
 and maximum value = 10 1 m

22.  $\forall a, b \in N, (a, b) R (a, b) as ab (b + a) = ba (a + b)$ 

2 m

Let (a, b) R (c, d) for  $(a, b), (c, d) \in N \times N$ 

$$\therefore ad(b+c) = bc(a+d) \dots (ii)$$

Also (c, d) R (a, b) 
$$\therefore$$
 cb (d + a) = da (c + b) (using ii)

 $2 \, \mathrm{m}$ 

Let (a, b) R (c, d) and (c, d) R (e, f), for  $a, b, c, d, e, f \in N$ 

$$\therefore$$
 ad  $(b+c) = bc (a+d)$  and  $cf (d+e) = de (c+f)$ 

1 m

$$\therefore \frac{b+c}{bc} = \frac{a+d}{ad} \text{ and } \frac{d+e}{de} = \frac{c+f}{cf}$$

i.e 
$$\frac{1}{c} + \frac{1}{b} = \frac{1}{d} + \frac{1}{a}$$
 and  $\frac{1}{e} + \frac{1}{d} = \frac{1}{f} + \frac{1}{c}$ 

adding we get 
$$\frac{1}{c} + \frac{1}{b} + \frac{1}{e} + \frac{1}{d} = \frac{1}{d} + \frac{1}{a} + \frac{1}{f} + \frac{1}{c}$$

$$\Rightarrow$$
 af (b + e) = be (a + f)

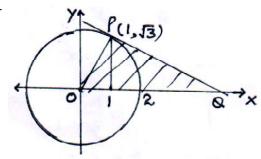
Hence 
$$(a, b) R (e, f)$$
  $\therefore$  R is transitive ......(iv)

 $\frac{1}{2}$  m

 $\frac{1}{2}$  m

23.

Correct Fig. 1 m



Eqn. of normal (OP): 
$$y = \sqrt{3}x$$
  $\frac{1}{2} + \frac{1}{2}m$ 

Eqn. of tangent (PQ) is

$$y - \sqrt{3} = -\frac{1}{\sqrt{3}} (x - 1)$$
 i.e.  $y = \frac{1}{\sqrt{3}} (4 - x)$  1 m  
Coordinates of Q (4, 0)

$$\therefore \text{ Req. area } = \int_{0}^{1} \sqrt{3}x \, dx + \int_{1}^{4} \frac{1}{\sqrt{3}} (4-x) dx$$

$$= \sqrt{3} \frac{x^2}{2} \bigg]_0^1 + \frac{1}{\sqrt{3}} \left[ 4x - \frac{x^2}{2} \right]_1^4$$

$$= \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{3}} \left[ 16 - 8 - 4 + \frac{1}{2} \right] = 2\sqrt{3} \text{ sq. units}$$

OR

$$\int_{1}^{3} \left( e^{2-3x} + x^2 + 1 \right) dx \qquad \text{here } h = \frac{2}{n}$$

$$= \lim_{h \to 0} h \left[ f(1) + f(1+h) + f(1+2h) + \dots + f(1+(n-1)h) \right]$$
1 m

$$= \lim_{h \to 0} h \left[ \! \left( \! e^{-l} + 2 \right) \! + \left( \! e^{-l-3h} + 2 + 2h + h^2 \right) \! + \left( \! e^{-l-6h} + 2 + 4h + 4h^2 \right) \! + \ldots \right]$$

$$+ \left. \left( e^{-l - 3(n - l) \, h} + 2 + 2 \left( n - 1 \right) h + \left( n - 1 \right)^2 h^2 \right) \right] \qquad \quad 1 \, \, m$$

$$= \lim_{h \to 0} h \left[ e^{-l} \left( 1 + e^{-3h} + e^{-6h} + \dots + e^{-3(n-l)h} \right) + 2n + 2h \left( 1 + 2 + \dots + (n-1) \right) + h^2 \left( 1^2 + 2^2 + \dots + (n-l)^2 \right) \right] \ 1\frac{1}{2} \ m$$

$$= \lim_{h \to 0} h \left( e^{-1} \cdot \frac{e^{-3nh} - 1}{e^{-3n} - 1} \cdot h + 2nh + 2 \frac{nh (nh - h)}{2} + \frac{nh (nh - h)(2nh - h)}{6} \right)$$
 1 m

$$= e^{-1} \cdot \frac{\left(e^{-6} - 1\right)}{-3} + 4 + 4 + \frac{8}{3} = - e^{-1} \cdot \frac{\left(e^{-6} - 1\right)}{3} + \frac{32}{3}$$

## 24. Given differential equation can be written as

$$\frac{dx}{dy} + \frac{1}{1+y^2} \cdot x = \frac{\tan^{-1}y}{1+y^2}$$
 1 m

$$\therefore$$
 Integrating factor is  $e^{\tan^{-1}y}$  1 m

$$\therefore \text{ Solution is : } x \cdot e^{\tan^{-1}y} = \int \frac{\tan^{-1}y \cdot e^{\tan^{-1}y}}{1+y^2} dy$$

$$\Rightarrow x \cdot e^{\tan^{-1}y} = \int t e^{t} dt \text{ where } \tan^{-1}y = t$$

$$= t e^{t} - e^{t} + c = e^{\tan^{-1}y} (\tan^{-1}y - 1) + c$$

or 
$$x = tan^{-1}y - 1 + c e^{-tan^{-1}y}$$

OR

Given differential equation is  $\frac{dy}{dx} = \frac{\frac{y}{x}}{1 + \left(\frac{y}{x}\right)^2}$ 

Putting 
$$\frac{y}{x} = v$$
 to get  $v + x \frac{dv}{dx} = \frac{v}{1 + v^2}$ 

$$\therefore x \frac{dv}{dx} = \frac{v}{1 + v^2} - v = \frac{-v^3}{1 + v^2}$$
 1½ m

$$\Rightarrow \int \frac{v^2 + 1}{v^3} dv = -\int \frac{dx}{x}$$
 ½ m

$$\Rightarrow \log |v| - \frac{1}{2v^2} = -\log |x| + c$$

$$\therefore \log y - \frac{x^2}{2y^2} = c$$

$$x = 0, y = 1 \implies c = 0 : \log y - \frac{x^2}{2y^2} = 0$$
 1/2 m

25. Any point on line 
$$\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4}$$
 is  $(2\lambda + 1, 3\lambda - 1, 4\lambda + 1)$ 

$$\therefore \frac{2\lambda+1-3}{1} = \frac{3\lambda-1-k}{2} = \frac{4\lambda+1}{1} \implies \lambda = -\frac{3}{2}, \text{ hence } k = \frac{9}{2}$$

Eqn. of plane containing three lines is

$$\begin{vmatrix} x-1 & y+1 & z-1 \\ 2 & 3 & 4 \\ 1 & 2 & 1 \end{vmatrix} = 0$$
1 m

$$\Rightarrow -5(x-1)+2(y+1)+1(z-1) = 0$$
 1 m

i.e. 
$$5x-2y-z-6=0$$
 ½ m

26. 
$$P(\overline{A} \cap B) = \frac{2}{15} \Rightarrow P(\overline{A}) \cdot P(B) = \frac{2}{15}$$

$$P(A \cap \overline{B}) = \frac{1}{6} \Rightarrow P(A) \cdot P(\overline{B}) = \frac{1}{6}$$

$$\therefore (1-P(A))P(B) = \frac{2}{15} \text{ or } P(B)-P(A)\cdot P(B) = \frac{2}{15} \dots (i)$$
 1 m

$$P(A)(1-P(B)) = \frac{1}{6} \text{ or } P(A)-P(A)\cdot P(B) = \frac{1}{6} \dots (ii)$$

From (i) and (ii) 
$$P(A) - P(B) = \frac{1}{6} - \frac{2}{15} = \frac{1}{30}$$
 1/2 m

Let P (A) = x, P (B) = y : 
$$x = \left(\frac{1}{30} + y\right)$$

(i) 
$$\Rightarrow y - \left(\frac{1}{30} + y\right) y = \frac{2}{15} \therefore 30y^2 - 29y + 4 = 0$$
 \(\frac{1}{30} + y\)

Solving to get  $y = \frac{1}{6}$  or  $y = \frac{4}{5}$ 

$$x = \frac{1}{5}$$
 or  $x = \frac{5}{6}$ 

Hence 
$$P(A) = \frac{1}{5}$$
,  $P(B) = \frac{1}{6}$  OR  $P(A) = \frac{5}{6}$ ,  $P(B) = \frac{4}{5}$