MATH 2132 Problem Workshop 1

1. Determine whether the sequence of constants converge diverge. Justify your answer. Find the sum of any convergent series

   (a) $\sum_{n=1}^{\infty} \frac{n^2+3n+1}{3n^2-4}$

   (b) $\sum_{n=2}^{\infty} \left(-\frac{7}{3}\right)^{n+1}$

   (c) $\sum_{n=2}^{\infty} \frac{3^{n+3}}{4^{2n-5}}$

   (d) $\sum_{n=3}^{\infty} \left(1+\frac{1}{n}\right)^{n}$

   (e) $\sum_{n=1}^{\infty} \frac{2^n+3^n}{4^{3n}}$

   (f) $\sum_{n=1}^{\infty} (-e)^{-n}$

   (g) $\sum_{n=100}^{\infty} \frac{1}{n}\tan^{-1} n$

   (h) $\sum_{n=1}^{\infty} \frac{1}{n^2+n}$ Hint: Find the sequence of partial sums

2. (a) Find the first five Taylor polynomials for the function $f(x) = \cos 2x$ about $x = 0$.

   (b) Show that the Maclaurin series for $\cos 2x$ converges to $\cos 2x$ for all x using the remainder formula.

3. Find the Taylor series about $x = 1$ for the function $f(x) = \frac{1}{(x-2)^2}$. Express your answer in sigma notation, simplified as much as possible.

4. Find the Maclaurin series for the function $f(x) = \frac{1}{(8+3x)^{1/3}}$. Express your answer in sigma notation, simplified as much as possible.

5. Find the open interval of convergence for the power series.

   (a) $\sum_{n=3}^{\infty} \frac{2^n}{n3^{n+1}} x^n$

   (b) $\sum_{n=0}^{\infty} \frac{(-1)^n 3^n}{n!} x^n$

   (c) $\sum_{n=0}^{\infty} \frac{(n+5)^4}{3^n} x^n$

(d) $\sum_{n=1}^{\infty} \frac{1 \cdot 3 \cdot 5 \cdots (2n+1)}{2 \cdot 5 \cdot 8 \cdots (3n+2)} (2x)^n$

6. Find the sum of the series $\sum_{n=1}^{\infty} \frac{(-1)^n}{2^{2n}} (x-1)^n$. What is its interval of convergence.