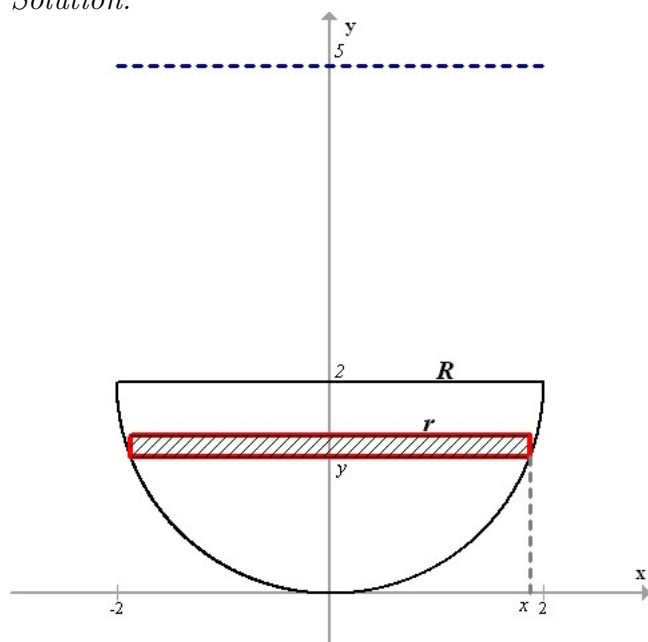


Math 1710. Homework Problems VIII (Feb 1, 2012)

Problem. SET UP (BUT DO NOT EVALUATE) integral(s) to determine the minimum amount of WORK DONE to pump the oil, having a constant density ρ [kilograms per cubic meter] from a hemi-spherical tank (with horizontal planar top) of radius 2 meters, to a height 3 meters above the top of the tank. [You may ignore frictional forces.]

Solution:



$$R = 2m, H = 5m,$$

$$x^2 + (y - 2)^2 = 4,$$

y – level position of the oil.

Mass of a “small” slice of oil (which is a disc):

$$\rho V = \rho \cdot \pi r^2 \cdot dy = \rho \cdot \pi x^2 \cdot dy = \rho \pi (4 - (y - 2)^2) dy = \rho \pi (4y - y^2) dy.$$

Therefore, total work needed to pump oil to a height 3 meters above the tank is

$$W = \int_0^2 \pi \rho (4y - y^2) g (5 - y) dy \quad [J]$$