MATH 1210 Problem Workshop 3

1. Simplify each of the following expressions to Cartesian form:

   (a) $\dfrac{(1+2i^3)^2\overline{(3-i)}}{4+i}$

   (b) $(\sqrt{3}-i)^{14}$

2. Express each of the following in exponential form. Your final answer should have the principal argument.

   (a) $(3+3\sqrt{3}i)^7 e^{5\pi i/6}$

   (b) $\dfrac{(1+i)e^{3\pi i/4}}{3e^{-\pi i/3}}$

3. Find exact values for all solutions of the following equations. Express final answers in Cartesian form.

   (a) $2x^4+3x^2-1=0$

   (b) $z^4=-4i$

4. Find the square roots of $5+12i$ by:

   (a) using the procedure of Exercises 44 in section 2.1

   (b) using the procedure of section 2.2

5. Use Euler's identity ($e^{i\theta}=\cos\theta+i\sin\theta$) and DeMoivre's theorem to prove the triple angle formulae

$$\cos 3\theta = \cos^3\theta - 3\cos\theta\sin^2\theta \quad \text{and} \quad \sin 3\theta = 3\cos^2\theta\sin\theta - \sin^3\theta.$$

6. Use Euler's identity to prove that

$$\cos\theta = \frac{e^{i\theta}+e^{-i\theta}}{2} \quad \text{and} \quad \sin\theta = \frac{e^{i\theta}-e^{-i\theta}}{2i}.$$

7. Find the fifth roots of $-2-2i$. Write answers in Cartesian form.

Answers

1. (a) $-\dfrac{35}{17}-\dfrac{55}{17}i$

   (b) $2^{13}-2^{13}\sqrt{3}i$

2. (a) $6^7 e^{-5\pi i/6}$

   (b) $\dfrac{\sqrt{2}}{3}e^{-2\pi i/3}$

3. (a) $\pm\dfrac{\sqrt{\sqrt{17}-3}}{2}, \pm\dfrac{\sqrt{\sqrt{17}+3}}{2}i$

   (b) $\sqrt{2}\cos\left(\dfrac{-5\pi}{8}\right)+\sqrt{2}\sin\left(\dfrac{-5\pi}{8}\right)i,\ \sqrt{2}\cos\left(\dfrac{-\pi}{8}\right)+\sqrt{2}\sin\left(\dfrac{-\pi}{8}\right)i,$
   $\sqrt{2}\cos\left(\dfrac{3\pi}{8}\right)+\sqrt{2}\sin\left(\dfrac{3\pi}{8}\right)i,\ \sqrt{2}\cos\left(\dfrac{7\pi}{8}\right)+\sqrt{2}\sin\left(\dfrac{7\pi}{8}\right)i$

4. $\pm(3+2i)$

7. $2^{-1/5}+2^{-1/5}i,\quad 2^{3/10}\cos\left(\dfrac{13\pi}{20}\right)+2^{3/10}\sin\left(\dfrac{13\pi}{20}\right),\quad 2^{3/10}\cos\left(\dfrac{21\pi}{20}\right)+2^{3/10}\sin\left(\dfrac{21\pi}{20}\right),$
   $2^{3/10}\cos\left(\dfrac{29\pi}{20}\right)+2^{3/10}\sin\left(\dfrac{29\pi}{20}\right),\quad 2^{3/10}\cos\left(\dfrac{37\pi}{20}\right)+2^{3/10}\sin\left(\dfrac{37\pi}{20}\right)$