University of Manitoba
Dept. of Mechanical and Industrial Engineering

130.112 Thermal Sciences (F00)

$\boxed{\texttt{L01/B01}}$ (Prof. Ormiston)

Term Test # 1 | 12 October 2000 | Time: 90 minutes

---

1. You are permitted to use the textbook for the course and a calculator.
2. Clear, systematic solutions are required. Marks will not be assigned for problems that require unreasonable (in the opinion of the instructor) effort for the marker to decipher.
3. Ask for clarification if any problem statement is unclear.
4. The weight of each problem is indicated. The test will be marked out of **50**.
5. Do not interpolate in the property tables; use the nearest table entry.
6. There are **two** problems on this test.

Values

1. A system containing 0.12 $[kg]$ of air initially (state 1) at a pressure of 187 $[kPa]$ and a temperature equal to 1900 $[°C]$ undergoes two quasi-equilibrium processes, one after the other. In the first process (state 1 to state 2), the air is cooled at constant volume until the temperature reaches 1500 $[°C]$. It is then (state 2 to state 3) cooled at constant pressure until the volume is reduced to half its initial value. Keep 5 significant figures in all your calculations in the parts below.

$\boxed{16}$

   (a) Determine the final temperature, T_3.

   (b) Show the two processes on a P-V diagram. Clearly identify the states and show the process paths with respect to constant temperature lines.

   (c) Calculate the total work done by the system for the two processes (i.e. $W_{12} + W_{23}$).

   (d) Can the second process be described as a polytropic process? Briefly explain your answer.

34 2. A control mass system in the piston-cylinder arrangement shown in Figure 1 initially (at state 1) contains a saturated liquid-vapour mixture of water at 225 $[kPa]$. The frictionless piston is initially resting on the stops and the total volume is 0.22000 $[m^3]$. At the initial state, the volume of vapour is 0.19896 $[m^3]$ and the remainder is liquid. The system undergoes two quasi-equilibrium processes, one after the other. First (state 1 to state 2), heat is transferred to the water until the pressure reaches 400 $[kPa]$. When the pressure reaches 400 $[kPa]$, the piston just starts to move upward. Second (state 2 to state 3), heat transfer continues until the temperature rises to 250 $[°C]$. Keep 5 significant figures in all your calculations in the parts below.

(a) Determine the total mass of water in the system.

(b) Determine the temperature at the end of the first process, T_2.

(c) Calculate the volume of vapour at state 2.

(d) Calculate the total work done by the system for both processes (i.e. $W_{12} + W_{23}$).

(e) Show the processes on a P-v diagram. Clearly identify the states and show the process paths with respect to the saturation lines.

(f) Calculate the change in total internal energy between the initial and final states (i.e. between states 1 and 3).

(g) Assuming changes in potential and kinetic energy of the system are negligible, calculate the total heat transfer to the system for both processes (i.e. $Q_{12} + Q_{23}$).

Parts (f) & (g) do not apply to 130.112 W06 term.

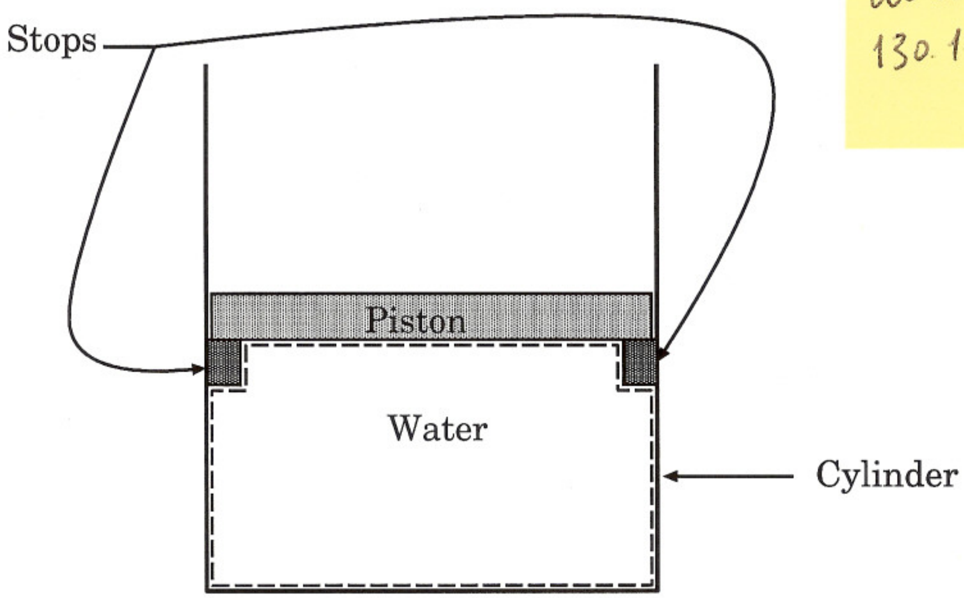
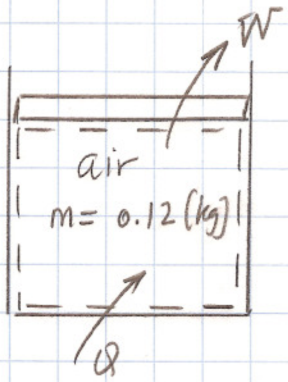


Figure 1: Piston-cylinder arrangement for Problem 2.

130.112 Term Test 1 Fall 2000 Solution

1.



Assumptions: 1. Ideal gas
2. Quasi-equilibrium processes

State 1	State 2	State 3
$P_1 = 187$ [kPa]	$P_2 = ?$	$P_3 = P_2$
$T_1 = 1900$ [°C]	$T_2 = 1500$ (°C)	$T_3 = ?$
$V_1 = ?$	$V_2 = V_1$	$V_3 = \frac{1}{2} V_2$

Analysis:

$R_{air} = 0.2870 \left[\frac{kJ}{kg\,K}\right]$ (Table A-1)

(a) $T_3 = ?$

$$PV = mRT$$

$$V_1 = \frac{mRT_1}{P_1} = \frac{0.12\,[kg]\; 0.2870 \left[\frac{kJ}{kg\,K}\right] (1900+273)\,(K)}{187\,(kPa)}$$

$$V_1 = 0.4002\,[m^3]$$

$$V_2 = V_1 = 0.4002\,[m^3]$$

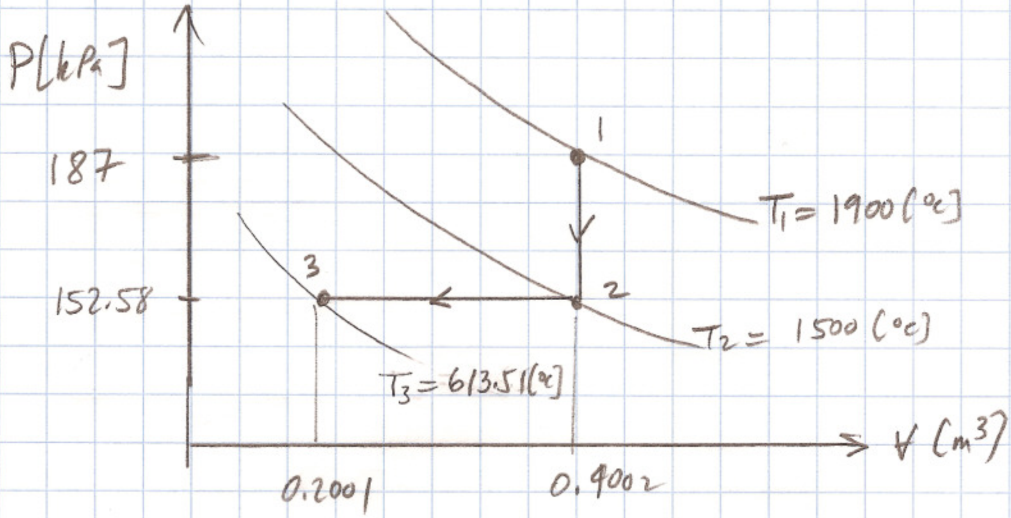
$$P_2 = \frac{mRT_2}{V_2} = \frac{0.12\,(kg)\; 0.2870 \left(\frac{kJ}{kg\,K}\right) (1500+273)}{0.4002}$$

$$P_2 = 152.58\,[kPa] = P_3$$

$$T_3 = \frac{P_3 V_3}{mR} = \frac{152.58\,[kPa]\left(\frac{1}{2}\right)(0.4002)\,[m^3]}{0.12\,[kg]\; 0.2870 \left(\frac{kJ}{kg\,K}\right)}$$

$$T_3 = \underline{886.51\,[K]} = \underline{613.51\,[°C]}$$ ← final temperature

(b)



(c) Total Work done

$$W_{12} = \int_1^2 P\,dV = 0 \qquad dV = 0$$

$$W_{23} = \int_2^3 P\,dV = P_2(V_3 - V_2) = 152.58\,[kPa](0.2001 - 0.4002)\,[m^3]$$

$$W_{23} = \underline{-30.53\ (kJ]}$$

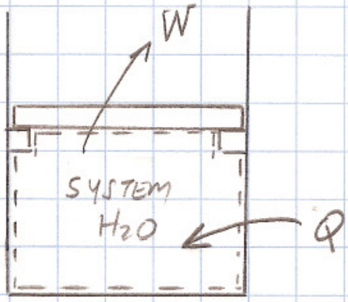
$$W_{TOT} = W_{12} + W_{23} = 0 - 30.53 = -30.53\,(kJ]$$

(d) The second process is constant pressure

$$P = C$$

YES, this can be expressed as a polytropic process with $n = 0$ $\quad P(V^0) = C$

2



State 1

$P_1 = 225$ [kPa]

$V_1 = 0.22000$ [m^3]

$V_{g1} = 0.19896$ [m^3]

$T_1 = T_{sat}(P_1) = 124.00$ [°C]

saturated-mixture

State 2

$P_2 = 400$ [kPa]

$T_2 = ?$

$m_{g_2} = ?$

State 3

$P_3 = 400$ [kPa]

$T_3 = 250$ [°C]

Analysis

(a) $m = ?$

find m from state 1

$V_{g1} = 0.19896$

$V_{f_1} = V_1 - V_{g1} = 0.22000 - 0.19896 = 0.02104$ [m^3]

$v_f(P_1) = 0.001064$ $\left(\frac{m^3}{kg}\right)$ $\quad v_g(P_1) = 0.7933$ $\left[\frac{m^3}{kg}\right]$ (Table A-5)

$$m_{f_1} = \frac{V_{f_1}}{v_{f_1}} = \frac{0.02104}{0.001064} = 19.774 \text{ [kg]}$$

$$m_{g_1} = \frac{V_{g_1}}{v_{g_1}} = \frac{0.19896}{0.7933} = 0.2508 \text{ [kg]}$$

$$M_1 = m = 19.774 + 0.2508 = 19.995 \text{ [kg]} \longleftarrow (a)$$

(b) Process $1 \rightarrow 2$ is CONSTANT VOLUME (piston doesn't move)

$$v_2 = v_1 = \frac{V_1}{m} = \frac{0.22000}{19.995} = 0.01100 \left[\frac{m^3}{kg}\right]$$

Check v_f, v_g at $P_2 = 400$ [kPa]

$v_f(P_2) = 0.001084$ $\left(\frac{m^3}{kg}\right)$

$v_g(P_2) = 0.4625$ [m^3/kg]

$v_f(P_2) < v_2 < v_g(P_2)$

$\rightarrow$ Saturated mixture at state 2

$$\Rightarrow T_2 = T_{sat}(400 \text{ [kPa]}) = 143.63 \text{ [°C]} \longleftarrow (b)$$

2 (c) $V_{g_2} = m_{g_2} v_g(P_2)$ $\qquad x_2 = \frac{m_{g_2}}{m}$

$m_{g_2} = m \cdot x_2$

Find x_2 from v_2: $x_2 = \dfrac{v_2 - v_f(P_2)}{v_g(P_2) - v_f(P_2)}$

$$x_2 = \frac{0.01100 - 0.001084}{0.4625 - 0.001084} = 0.021490$$

$$m_{g_2} = 19.995\,[kg]\;0.021490 = 0.42969\,[kg]$$

$$V_{g_2} = 0.42969\,[kg] \cdot 0.4625\left[\frac{m^3}{kg}\right] = 0.19873\,[m^3] \leftarrow (c)$$

$m_{g_2} > m_{g_1}$ → more vapour by mass at state 2

but vapour volume decreased because saturated vapour at P_2 is more dense than at P_1.

(d) $W_{TOT} = W_{12} + W_{23} = 0 + \int_2^3 P\,dV = 0 + P_2(V_3 - V_2)$

W_{12} is zero because $dV = 0$ (no volume change

$W_{TOT} = P_2(V_3 - V_2) = P_2 m(v_3 - v_2)$ $\qquad V_1 = V_2 = 0.22000\,[m^3]$

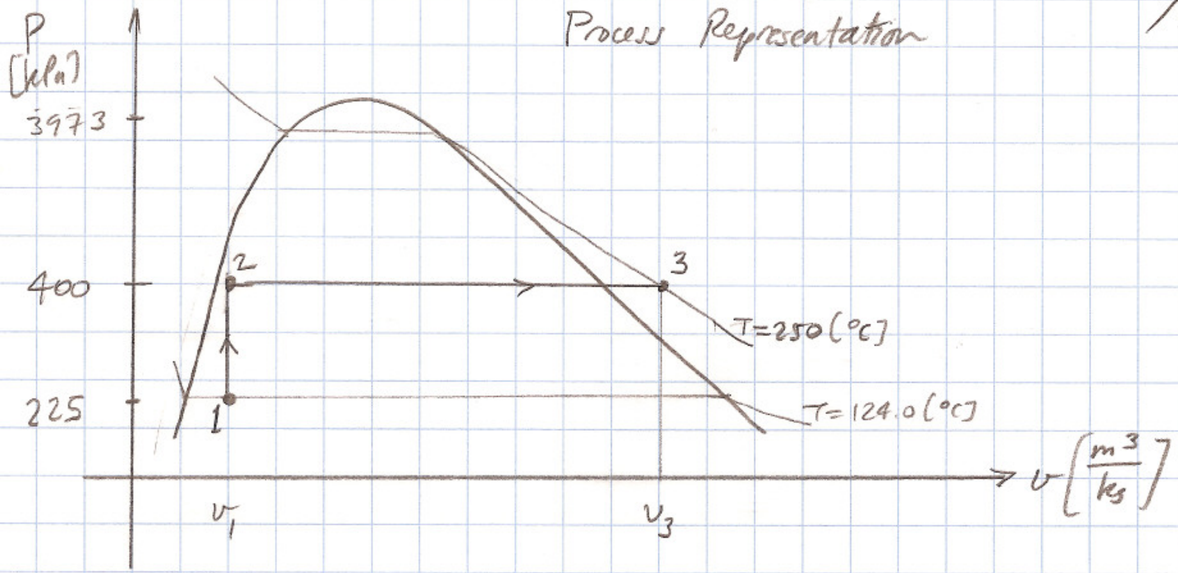
$V_3 = m\,v_3$ $\qquad v_3 = v(400\,[kPa], 250\,[°C])$

$v_3 = 0.5951\left[\frac{m^3}{kg}\right]$ (Table A-6) superheated vapour

$$W_{TOT} = 400\,[kPa] \cdot 19.995\,[kg](0.5951 - 0.01100)$$

$$W_{TOT} = 4671.6\,[kJ] \leftarrow (d)$$

2(e) Process Representation



(f) $U_3 - U_2 = m(u_3 - u_1)$

$u_1 = (1 - x_1)\, u_f(P_1) + x_1\, u_g(P_1)$ $\qquad u_f(P_1) = 520.47 \left[\frac{kJ}{kg}\right]$

$x_1 = \frac{m_{g1}}{m} = \frac{0.2508}{19.995} = 0.012543$ $\qquad u_g(P_1) = 2533.6 \left[\frac{kJ}{kg}\right]$

$u_1 = (1 - 0.012543)\, 520.47 + 0.012543(2533.6)$

$u_1 = 545.72 \left[\frac{kJ}{kg}\right]$

$u_3 = u(400\,[kPa],\ 250\,[°C]) = 2726.1 \left[\frac{kJ}{kg}\right]$

$(U_3 - U_1) = 19.995\,(2726.1 - 545.72) = 43{,}597\ [kJ] \leftarrow$ (f)

(g) $Q_{12} - W_{12} = (U_2 - U_1)$
$Q_{23} - W_{23} = (U_3 - U_2)$) add these to get $Q_{TOT} = Q_{12} + Q_{23}$

$(Q_{12} + Q_{23}) - (W_{12} + W_{23}) = (U_3 - U_1)$ $\qquad W_{TOT} = W_{12} + W_{23}$

$Q_{TOT} = (U_3 - U_1) + W_{TOT}$

$Q_{TOT} = 43{,}597 + 4671.6 = 48{,}269\ [kJ] \leftarrow$ (g)