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[6] 1. Evaluate $\lim_{x\to 2} \dfrac{x^3 - 3x^2 + 4}{x^2 - 4x + 4}$, if it exists. Show all calculations.

[6] 2. Find all intervals on which the function $f(x) = \dfrac{x+3}{x^2+3}$ is increasing.

[6] 3. Let $f(x) = x^5 - x^6$. Determine the intervals on which $f(x)$ is concave up or concave down, and find all inflection points (if any).

[6] 4. Find the maximum value of the function $f(x) = \dfrac{\ln x}{x^3}$ on the interval $1 \leq x \leq 2$.

[6] 5. From a standing start, a cheetah reaches a velocity of 72 km/hr in 2 seconds. If the acceleration of the cheetah is constant, and it runs in a straight line, what distance does it cover in the 2 seconds? You are not permitted to use any physics formulas. You must derive your own.

[5] 6. A particle travels along the curve $y = x^3 - 2x^2 - x + 5$ where x and y are in metres in such a way that its x-coordinate is increasing at a constant rate of 2 metres per second. Find all points on the curve where its y-coordinate is decreasing at 4 metres per second.

[6] 7. Alex can swim 5 km/h and run 12 km/h. He is standing on one bank of a river that is 100 m wide and wants to reach a point located 400 m downstream on the other side as quickly as possible. Alex will swim diagonally across the river and then run along the river bank. Alex wants to determine the best (i.e., the fastest) route to take (assume that the river flow is negligible).

[4] (a) Set up a function that needs to be minimized. State it in terms of only one variable.

[2] (b) Over what domain should this function be minimized?

DO NOT PROCEED ANY FURTHER IN ATTEMPTING TO SOLVE THIS PROBLEM (the set-up is all that is required).

[19] 8. Evaluate the following integrals (simplify your answers):

[3] (a) $\displaystyle\int \left(2\sqrt{x} + \frac{1}{x^3}\right) dx$

[4] (b) $\displaystyle\int x^2(x^3+4)^{77}\, dx$

[4] (c) $\displaystyle\int \frac{\sin(\ln x)}{x}\, dx$

[4] (d) $\displaystyle\int_{-\pi/4}^{\pi/2} \cos 3x \, dx$

[4] (e) $\displaystyle\int_{0}^{\ln 2} e^x (1 + 2e^x)^4 \, dx$

[6] Bonus. Is the function $f(x) = |x| \sin x$ differentiable at $x = 0$? If it is find its derivative at this point. If it is not explain why. (Justify your answer.)