MATH 2132 Problem Workshop 4

1. Find a general solution to the following differential equations

   (a) $(y-1)\dfrac{dy}{dx} = yx^2$

   (b) $\dfrac{y-1}{y}\dfrac{dy}{dx} = x^2$

   (c) $x^2\dfrac{dy}{dx} = y^2 - 1$

   (d) $x\dfrac{dy}{dx} = 3y + x^5\sqrt{1+x^2},$ x¿0

2. Find an explicit solution of the initial-value problem and where is the solution valid?

$$\frac{dy}{dx} = \frac{x^4}{y+1}, \qquad y(1) = 2.$$

3. We will be solve the following word problem:

   A tank originally contains 1000 litres of water in which 10 kilograms of sugar has been dissolved (uniformly). A mixture containing 2 kilograms of sugar per 100 litres of water is added to the tank at 15 millilitres per minute. At the same time, 20 millitires of well-stirred mistures is removed from the tank each minute. Find the amount of sugar in the tank as a function of time t. For how long is the solution valid?

   (a) Let $Q(t)$ be the quantity of sugar in the tank. What is $Q(0)$ in kilograms.

   (b) What is the rate which the sugar is entering the tank? Include units.

   (c) What is the rate which the sugar is leaving the tank? Include units.

   (d) Set up an initial value problem which solves the question.

   (e) Solve the differential equation and thus the word problem.

4. Find a general solution of the differential equation $xy'' = x^3 - y'$, $x < 0$

5. Solve the initial value problem

$$y'' = 4yy', \qquad y(0) = 1,\ y'(0) = 0$$

6. Find general solutions for the following differential equations

   (a) $3y''' + 8y'' + 19y' + 10y = 0$

   (b) $6y''' + y'' - y' = 0$