THE UNIVERSITY OF MANITOBA

1:30 pm, December 16, 2005	**Final Examination (Room 229 EIT Complex)**
Paper No: 457	**Page No: Page 1 of 5**
Dept. and Course No.: 130.112	**Time: 3 Hours**
Examination.: Thermal Science	**Examiners: Drs. J. Bartley and H. Soliman**

1. This is a three-hours, open-book examination. Students are permitted to use the course textbook, supplementary notes on heat transfer, and a non-programmable calculator. No other material is allowed.
2. Answer all **five** questions. The value of each question is indicated in the margin.
3. Write your solutions clearly (showing all steps) in the booklets provided. Ambiguous solutions, which cannot be interpreted, will be considered incorrect.
4. If interpolation is required in the property tables, then use **linear** interpolation between table entries. Use **four** significant figures in your calculations and full precision for data taken from the textbook.

Values **Problem #1 (20 Points):**

Consider the piston-cylinder-spring arrangement shown in the figure below. The piston has a surface area $A = 1.2\,\mathrm{m}^2$ and the (linear) spring has a constant $k = 250\,\mathrm{kN/m}$. The cylinder contains a mixture of two gases: 4 kg oxygen (O_2) and 8 kg ethane (C_2H_6). The mixture has an initial pressure and temperature of P_1 = 400 kPa and T_1 = 75°C (State 1). Heat is added and the piston moves freely until it touches the spring (State 2), at which point the volume is V_2 = 3.2 m^3. Heat addition continued until the spring is compressed by a distance of 1 m (State 3). Treat the oxygen-ethane mixture as an ideal gas mixture and neglect changes in the kinetic and potential energies.

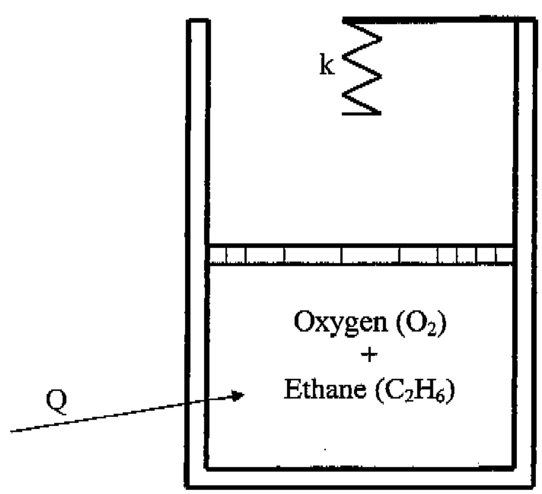
(6) (a) Determine the initial volume occupied by the gas mixture, V_1.

(2) (b) Calculate the temperature of the gas mixture at State 2, and the work done between States 1 and 2, ${}_1W_2$.

(5) (c) Determine the volume V_3, pressure P_3, and temperature T_3. Also, calculate the work done between States 2 and 3, ${}_2W_3$.

(5) (d) Calculate the amounts of heat transfer for the two processes, ${}_1Q_2$ and ${}_2Q_3$.

(2) (e) Sketch a P versus V diagram showing all state points, all temperature lines, the two process lines and label the areas representing work.



THE UNIVERSITY OF MANITOBA

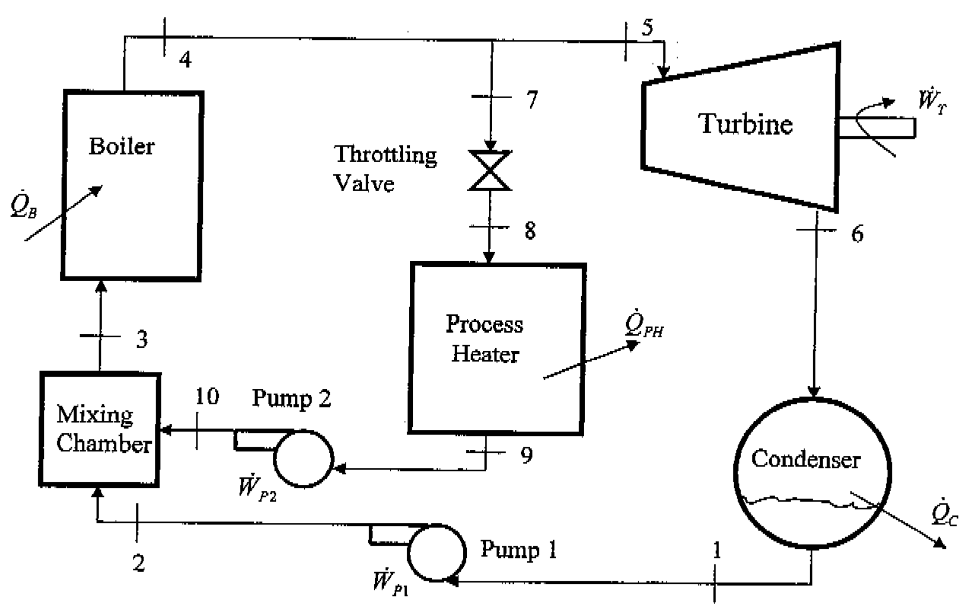
1:30 pm, December 16, 2005	**Final Examination (Room 229 EIT Complex)**
Paper No: 457	**Page No: Page 2 of 5**
Dept. and Course No.: 130.112	**Time: 3 Hours**
Examination.: Thermal Science	**Examiners: Drs. J. Bartley and H. Soliman**

Values **Problem #2 (24 Points):**

A steam cogeneration plant produces power $\dot{W}_T$ from a steam turbine along with steam for a process heater (e.g., industrial process heating or building heating); see the figure below. Steam at $\dot{m}_4 = 20$ kg/s, P_4 = 6 MPa, and T_4 = 600°C is produced by the boiler. Part of this steam ($\dot{m}_7$ = 10 kg/s) is used for heating in the process heater (note that $P_7 = P_4$ and $T_7 = T_4$). This steam flow is first throttled to a pressure P_8 = 500 kPa and then heat is extracted from it in the process heater until the flow becomes saturated liquid at P_9 = 500 kPa. The saturated liquid leaving the process heater is then pumped by pump 2 to boiler pressure P_{10} = 6 MPa. The remaining steam from the boiler ($\dot{m}_5$ = 10 kg/s) expands in the turbine to P_6 = 50 kPa and x_6 = 0.95. The steam exiting the turbine is condensed at constant pressure to saturated liquid at P_1 = 50 kPa and then pumped by pump 1 to boiler pressure P_2 = 6 MPa. Streams 2 and 10 are then combined in a mixing chamber before returning to the boiler. Assume that:

(i) The entire process cycle is at steady-state, steady-flow operating conditions.
(ii) The value of $\dot{Q}$ in all components is zero, except the boiler, the process heater and the condenser.
(iii) The change in internal energy is negligible in both pumps.
(iv) All changes in the potential and kinetic energies are negligible throughout the cycle.

(5) (a) Determine the power output from the turbine.
(5) (b) Calculate the rate of work required by pump 1 and by pump 2.
(5) (c) Determine the rate of heat extracted from the steam in the process heater.
(4) (d) Calculate the rate of heat transfer from the steam in the condenser.
(5) (e) Calculate the enthalpy at state 3, h_3.



THE UNIVERSITY OF MANITOBA

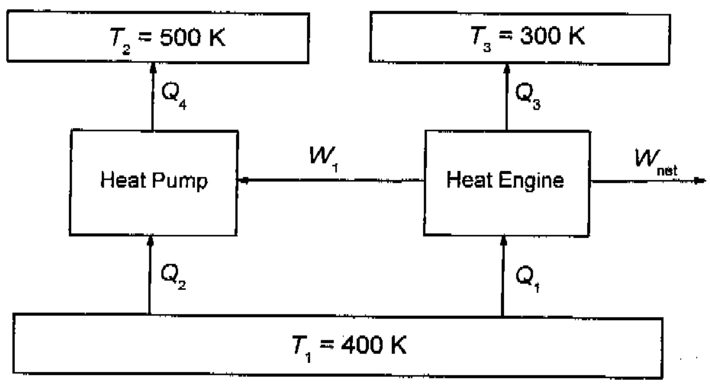
1:30 pm, December 16, 2005	**Final Examination (Room 229 EIT Complex)**
Paper No: 457	**Page No: Page 3 of 5**
Dept. and Course No.: 130.112	**Time: 3 Hours**
Examination.: Thermal Science	**Examiners: Drs. J. Bartley and H. Soliman**

Values **Problem #3 (15 Points):**

A heat engine receives an amount of heat Q_1 from a thermal reservoir at $T_1 = 400$ K and rejects an amount of heat Q_3 into a thermal reservoir at $T_3 = 300$ K. The engine delivers enough work to drive a heat pump (W_1) and in addition, the engine produces net work W_{net}. The heat pump removes an amount of heat Q_2 from the thermal reservoir at T_1 and it delivers an amount of heat Q_4 to a thermal reservoir at $T_2 = 500$ K.

(5) (a) Determine the magnitude of $(Q_1 + Q_2)$ given that $Q_4 = 100$ kJ, $W_{net} = 10$ kJ, and both the heat engine and the heat pump are reversible.

(10) (b) Determine the magnitude of $(Q_1 + Q_2)$ given that $Q_4 = 100$ kJ, $W_{net} = 10$ kJ, the heat engine is irreversible with η_{th} equal to 70% of the reversible η_{th}, and the heat pump is irreversible with β' equal to 80% of the reversible β'.



THE UNIVERSITY OF MANITOBA

Values **Problem #4 (16 Points):**

(8) (a) A glass window (surface area, $A = 4$ m^2, and thickness, $L_g = 3$ mm) separates the indoors where the temperature is $T_i = 20$°C and the heat transfer coefficient is $h_i = 10$ W/m^2·K, from the outdoors where the temperature is $T_o = 5$°C and the heat transfer coefficient is $h_o = 100$ W/m^2·K, as shown in Part (a) of the figure below.

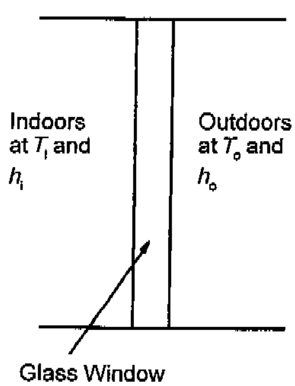
(i) Determine the rate of heat transfer through the window.

(ii) If the indoors pressure is 100 kPa and the relative humidity is 64%, would condensation take place on the inner surface of the window? Support your answer with appropriate calculations. If you use the Psychrometric Chart, make a sketch to show how you obtained the value(s) from the Chart.

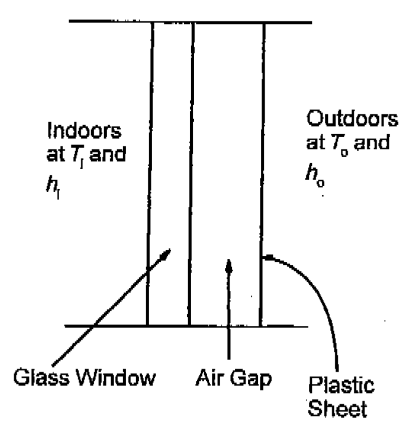
(8) (b) A plastic sheet (surface area, $A = 4$ m^2) was installed so that an air gap was created between the outer surface of the window and the outdoors; see Part (b) of the figure below. The air gap has a thickness $L_a = 1$ cm and heat transfer in this air gap is assumed to be by conduction (i.e., no convection in the air gap). The thickness of the plastic sheet is very small and therefore, its thermal resistance is negligible. Use the same values of L_g, T_i, h_i, T_o, and h_o given in Part (a) of this question.

(i) Determine the rate of heat transfer through the window after installing the plastic sheet.

(ii) If the indoors pressure is 100 kPa and relative humidity is 64%, would condensation take place on the inner surface of the window? Support your answer with appropriate calculations.



Part (a)



Part (b)

THE UNIVERSITY OF MANITOBA

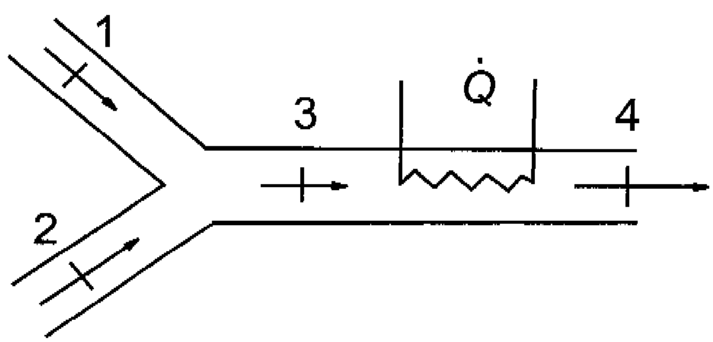
1:30 pm, December 16, 2005	**Final Examination (Room 229 EIT Complex)**
Paper No: 457	**Page No: Page 5 of 5**
Dept. and Course No.: 130.112	**Time: 3 Hours**
Examination.: Thermal Science	**Examiners: Drs. J. Bartley and H. Soliman**

Values **Problem #5 (25 Points):**

Flow 1 of moist air is at T_1 = 10°C and ϕ_1 = 80%, while flow 2 of moist air is at T_2 = 35°C and dew point temperature $T_{dp,2}$ = 30°C. The mass flow rates of dry air in these two streams are: $\dot{m}_{a,1}$ = 4 kg/s and $\dot{m}_{a,2}$ = 1 kg/s. The two streams are mixed, as shown in the figure, thus producing stream 3. The mixing process is followed by a heater or a cooler in order to produce stream 4, which is desired to be at T_4 = 20°C. The pressure throughout the system is 100 kPa. No liquid water was added or extracted anywhere in the system. Assume steady state and steady flow with no changes in the kinetic and potential energies, and that heat transfer takes place only between states 3 and 4.

Determine whether heating or cooling is required between states 3 and 4 and calculate the rate of heat transfer in kW.

NOTE: Use the formulas in your calculations; do not use the Psychrometric Chart.



THE UNIVERSITY OF MANITOBA

1:30 pm, December 16, 2005 **Final Examination (Room 229 EIT Complex)**

Paper No: 457 **Page No: Page 6 of 5**

Dept. and Course No.: 130.112 **Time: 3 Hours**

Examination.: Thermal Science **Examiners: Drs. J. Bartley and H. Soliman**

SOLUTIONS

Problem #1 Piston-Cylinder and Spring

$P_1 = 400$ kPa, $T_1 = 75$ °C, $k_{spring} = 250$ kN/m,
$m_{O2} = 4$ kg, mC2H6 = 8 kg,
$A_{piston} = 1.2\text{ m}^2$, $V_2 = 3.2\text{ m}^3$,
$M_{O2} = 31.999$ kg/kmol, $M_{C2H6} = 30.070$ kg/kmol

(a)

$$N_{O2} = \frac{m_{O2}}{M_{O2}} = \frac{4}{31.999} = 0.1250\,\text{kmol}$$

$$N_{C2H2} = \frac{m_{C2H2}}{M_{C2H2}} = \frac{8}{30.070} = 0.2660\,\text{kmol}$$

$$N_{mixture} = N_{O2} + N_{C2H6} = 0.1250 + 0.2660 = 0.3910\,\text{kmol}$$

$$M_{mixture} = \frac{m_{total}}{N_{mixture}} = \frac{12}{0.3910} = 30.686\text{ kg/kmol}$$

$$R_{mixture} = \frac{\overline{R}}{M_{mixture}} = \frac{8.31451\text{ kJ/kmol K}}{30.686\text{ kg/kmol}} = 0.27094\text{ kJ/kg K}$$

$$V_1 = \frac{m_{mixture}R_{mixture}T_1}{P_1} = \frac{12\times 0.27094\times(75+273.15)}{400} = 2.8299\text{ m}^3$$

(b)
State 2:
$P_2 = P_1$

$$P_2V_2 = m_{total}R_{mixture}T_2,\quad \therefore\ T_2 = \frac{400\times 3.2}{12\times 0.27094} = 393.67\text{ K}$$

Process 1 to 2 is at constant pressure, so

$${}_1W_2 = P(V_2 - V_1) = 400\times(3.2 - 2.8299) = 148.03\text{ kJ}$$

(c)
State 3:
Spring is compressed by 1 m:

$$\therefore V_3 = V_2 + y\times A = 3.2 + 1\times 1.2 = 4.4\text{ m}^3$$

$$P_3 = P_2 + \frac{k_{spring}}{A^2}(V_3 - V_2) = 400 + \frac{250}{(1.2)^2}(4.4 - 3.2) = 608.33\text{ kPa}$$

$$P_3V_3 = m_{total}R_{mixture}T_3,\quad \therefore\ T_3 = \frac{608.33\times 4.4}{12\times 0.27094} = 823.23\text{ K}$$

$${}_2W_3 = \frac{P_2 + P_3}{2}(V_3 - V_2) = \frac{400 + 608.33}{2}(4.4 - 3.2) = 605\text{ kJ}$$

(d)

$$C_{vo-mix} = C_{O2}C_{vo\,O2} + C_{C2H6}C_{vo\,C2H6}$$

From Table A5, $C_{vo\,O2} = 0.662$ kJ/kg K, $C_{vo\,C2H6} = 1.490$ kJ/kg K

$$C_{O2} = \frac{m_{O2}}{m_{total}} = \frac{4}{12} = 0.3333$$

$$C_{C2H6} = \frac{m_{C2H6}}{m_{total}} = \frac{8}{12} = 0.6666$$

SOLUTIONS

$C_{vo-mix} = 0.3333 \times 0.662 + 0.6666 \times 1.490 = 1.2139$ kJ/kg K

First Law:

$$ {}_1Q_2 - {}_1W_2 = m_{total}(u_2 - u_1) $$
$$ = m_{total} C_{vo-mix}(T_2 - T_1) $$

${}_1Q_2 = 148.032 + 12 \times 1.2139 \times (393.67 - 348.15)$

${}_1Q_2 = 811.161$ kJ

Similarly,

${}_2Q_3 = 605 + 12 \times 1.2139 \times (823.23 - 393.67)$

${}_2Q_3 = 6862.77$ kJ

Problem #2 Steam Power Cycle

(a)

Boiler: $T_4 > T_{sat}$ at P = 6 MPa, therefore state is superheated steam

Table B.1.3: $h_4 = 3658.40$ kJ/kg

Turbine: $h_5 = h_4$, $P_5 = P_4 = 6$ MPa, $\dot{m}_5 = 10$ kg/s

$x_6 = 0.95$, $P_6 = 50$ kPa , $T_6 = T_{sat} = 81.33$ °C

$h_6 = h_f + x_6 \cdot h_{fg} = 340.47 + 0.95 \times 2305.40$ kJ/kg (Table B.1.2)

h6 = 2530.60 kJ/kg

First law: $\dot{Q}_T - \dot{W}_T = \dot{m}_6 h_6 - \dot{m}_5 h_5$

$\dot{Q}_T = 0$ (adiabatic) , $\therefore \dot{W}_T = \dot{m}_5 h_5 - \dot{m}_6 h_6 = 10 \times (3658.40 - 2530.60)$

$\dot{W}_T = 11278$ kW

(b)

Pumps: Pump 1

First law: $-\dot{W}_{P1} = \dot{m}(h_2 - h_1)$ (adiabatic)

$(h_2 - h_1) \cong v(P_2 - P_1)$, where $u_2 - u_1$ is neglected and v is essentially a constant

$v = v_1 = v_{f\,@\,50kPa} = 0.001030\,\text{m}^3/\text{kg}$

$\therefore -\dot{W}_{P1} = 10 \times 0.001030 \times (6000 - 50)$

$\dot{W}_{P1} = 61.285$ kW (power input)

Also, $h_2 = \dfrac{-\dot{W}_{P1}}{\dot{m}_1} + h_1 = \dfrac{61.285}{10} + 340.47$, $h_2 = 346.59$ kJ/kg

(where $h_1 = h_{f\,@\,50kPa} = 340.47$ kJ/kg)

Pump 2

similarly, $v = v_9 = v_{f\,@\,500kPa} = 0.001093\,\text{m}^3/\text{kg}$, and

$$ -\dot{W}_{P2} \cong \dot{m}\, v_9\, (P_{10} - P_9) = 10 \times 0.001093 \times (6000 - 500) $$

$\dot{W}_{P2} = 60.115$ kW (power input)

Also, since $h_{10} = \dfrac{-\dot{W}_{P2}}{\dot{m}} + h_9$, then

THE UNIVERSITY OF MANITOBA

1:30 pm, December 16, 2005 | **Final Examination (Room 229 EIT Complex)**

Paper No: 457 | **Page No: Page 8 of 5**

Dept. and Course No.: 130.112 | **Time: 3 Hours**

Examination.: Thermal Science | **Examiners: Drs. J. Bartley and H. Soliman**

SOLUTIONS

$$h_{10} = \frac{-60.115}{10} + 640.21 = 646.22 \text{ kJ/kg}$$
(where $h_9 = h_{f\,@\,500\text{kPa}} = 640.21$ kJ/kg)

(c)
Throttle Valve:

$\dot{m}_7 = \dot{m}_8 = 10$ kg/s

$h_7 = h_8 = h_4 = 3658.40$ kJ/kg (from First Law considerations)

Process Heater:

$h_9 = h_{f\,@\,500kPa} = 640.21$ kJ/kg

$T_9 = T_{sat} = 151.86\,°C$

First Law: $\dot{Q}_{PH} - \dot{W} = \dot{m}\left(h_9 - h_8\right)$ where $\dot{W} = 0$

$\dot{Q}_{PH} = 10\times(640.21 - 3658.40) = -30181.9$ kW

(d)
Condenser:

$P_6 = 50$ kPa , $\dot{m}_6 = 10$ kg/s

First Law: $\dot{Q}_C - \dot{W} = \dot{m}\left(h_1 - h_6\right)$ where $\dot{W} = 0$

$h_1 = h_{f\,@\,50kPa} = 340.47$ kJ/kg

$T_1 = T_{sat} = 81.33\,°C$

$\dot{Q}_C = 10\times(340.47 - 2530.6) = -21901.3$ kW

(e)
Mixing Chamber:

Assume mixing of fluid streams from 2 and 10 occurs adiabatically.

First Law: $\dot{Q} - \dot{W} = \dot{m}_3 h_3 - \left(\dot{m}_2 h_2 + \dot{m}_{10} h_{10}\right)$, where $\dot{Q} = 0$ and $\dot{W} = 0$

$$\therefore h_3 = \frac{\dot{m}_2 h_2 + \dot{m}_{10} h_{10}}{\dot{m}_3} = \frac{10\times 346.59 + 10\times 646.22}{20}$$

$h_3 = 496.4$ kJ/kg

Problem #3

(a) $Q_2/Q_4 = T_1/T_2 = Q_2/100 = 400/500$

$Q_2 = 80$ kJ

$W_1 = Q_4 - Q_2 = 100 - 80 = 20$ kJ

$$W_{tot} = W_1 + W_{net} = 20 + 10 = 30 = \eta_{th}\times Q_1 = \left(1 - \frac{T_3}{T_1}\right)\times Q_1 = \left(1 - \frac{300}{400}\right)\times Q_1$$

$Q_1 = 120$ kJ and $Q_3 = 120 - 30 = 90$ kJ

$Q_1 + Q_2 = 120 + 80 = 200$ kJ

(b) $$\eta_{th}\big|_{rev} = \left(1 - \frac{T_3}{T_1}\right) = \left(1 - \frac{300}{400}\right) = 0.25$$

THE UNIVERSITY OF MANITOBA

1:30 pm, December 16, 2005 | Final Examination (Room 229 EIT Complex)
Paper No: 457 | Page No: Page 9 of 5
Dept. and Course No.: 130.112 | Time: 3 Hours
Examination.: Thermal Science | Examiners: Drs. J. Bartley and H. Soliman

SOLUTIONS

$$\eta_{th}\big|_{irr} = 0.7\times0.25 = 0.175$$

$$\beta'\big|_{rev} = \frac{T_2}{T_2 - T_1} = \frac{500}{500-400} = 5$$

$$\beta'\big|_{irr} = 0.8\times5 = 4$$

$$\beta'\big|_{irr} = \frac{Q_4}{W_1} = 4 = \frac{100}{W_1}$$

$W_1 = 25$ kJ

$Q_2 = 100 - 25 = 75$ kJ

$W_{tot} = 25 + 10 = 35$ kJ

$$\eta_{th}\big|_{irr} = \frac{W_{tot}}{Q_1} = 0.175 = \frac{35}{Q_1}$$

$Q_1 = 200$ kJ and $Q_3 = 200 - 35 = 165$ kJ

$Q_1 + Q_2 = 200 + 75 = 275$ kJ

Problem #4

(a)-(i) $k_g = 1.4$ W/m·K

$$\dot{Q} = \frac{T_i - T_o}{\frac{1}{h_i A} + \frac{L_g}{k_g A} + \frac{1}{h_o A}} = \frac{20-5}{\frac{1}{10\times4} + \frac{0.003}{1.4\times4} + \frac{1}{100\times4}} = 535.0 \text{ W}$$

(a)-(ii) $T_{s,i} = T_i - \dot{Q}\left(\frac{1}{h_i A}\right) = 20 - \frac{535}{10\times4} = 6.63\,^\circ\text{C}$

$P_{g,i} = 2.339$ kPa $\quad P_{v,i} = \phi_i P_{g,i} = 0.64\times2.339 = 1.497$ kPa $\quad T_{dp,i} \cong 13\,^\circ\text{C}$

$T_{s,i} < T_{dp,i}$, therefore, condensation will occur.

(b)-(i) $k_a = 0.026$ W/m·K

$$\dot{Q} = \frac{T_i - T_o}{\frac{1}{h_i A} + \frac{L_g}{k_g A} + \frac{L_a}{k_a A} + \frac{1}{h_o A}} = \frac{20-5}{\frac{1}{10\times4} + \frac{0.003}{1.4\times4} + \frac{0.01}{0.026\times4} + \frac{1}{100\times4}} = 120.8 \text{ W}$$

(b)-(ii) $T_{s,i} = T_i - \dot{Q}\left(\frac{1}{h_i A}\right) = 20 - \frac{113.4}{10\times4} = 16.98\ ^\circ\text{C}$

This surface temperature is higher than the dew point. Therefore, no condensation will take place.

THE UNIVERSITY OF MANITOBA

1:30 pm, December 16, 2005	**Final Examination (Room 229 EIT Complex)**
Paper No: 457	**Page No: Page 10 of 5**
Dept. and Course No.: 130.112	**Time: 3 Hours**
Examination.: Thermal Science	**Examiners: Drs. J. Bartley and H. Soliman**

SOLUTIONS

Problem #5

$P_{g,1} = 1.2276$ kPa $\qquad P_{v,1} = \phi_1 \times P_{g,1} = 0.8 \times 1.2276 = 0.9821$ kPa

$$\omega_1 = \frac{0.622 P_{v,1}}{P - P_{v,1}} = \frac{0.622 \times 0.9821}{100 - 0.9821} = 0.006169 \text{ kg H}_2\text{O/kg dry air}$$

$P_{v,2} = 4.246$ kPa

$$\omega_2 = \frac{0.622 P_{v,2}}{P - P_{v,2}} = \frac{0.622 \times 4.246}{100 - 4.246} = 0.02758 \text{ kg H}_2\text{O/kg dry air}$$

$$\dot{m}_{a,3} = \dot{m}_{a,4} = \dot{m}_{a,1} + \dot{m}_{a,2} = 4 + 1 = 5 \text{ kg/s}$$

$$\dot{m}_{v,3} = \dot{m}_{v,4} = \dot{m}_{a,1}\omega_1 + \dot{m}_{a,2}\omega_2 = 4 \times 0.006169 + 1 \times 0.02758 = 0.05226 \text{ kg H}_2\text{O/kg dry air}$$

$$\omega_3 = \omega_4 = \dot{m}_{v,3}/\dot{m}_{a,3} = 0.05226/5 = 0.01045 \text{ kg H}_2\text{O/kg dry air}$$

$$h_1 = C_{p,a}T_1 + \omega_1 h_{v,1} = 1.004 \times 10 + 0.006169 \times 2519.74 = 25.58 \text{ kJ/kg dry air}$$

$$h_2 = C_{p,a}T_2 + \omega_2 h_{v,2} = 1.004 \times 35 + 0.02758 \times 2565.28 = 105.89 \text{ kJ/kg dry air}$$

$$\dot{m}_{a,1}h_1 + \dot{m}_{a,2}h_2 = \dot{m}_{a,3}h_3$$

$$h_3 = \frac{4 \times 25.58 + 1 \times 105.89}{5} = 41.64 \text{ kJ/kg dry air}$$

$$h_4 = C_{p,a}T_4 + \omega_4 h_{v,4} = 1.004 \times 20 + 0.01045 \times 2538.06 = 46.60 \text{ kJ/kg dry air}$$

Since $h_4 > h_3$, heating is required

$$\dot{Q} = \dot{m}_{a,3}(h_4 - h_3) = 5(46.60 - 41.64) = 24.8 \text{ kW}$$