UNIVERSITY OF MANITOBA

DATE: November 12, 2008 — Midterm 2
COURSE: MATH 1010 — PAGE: 1 of 7
EXAMINATION: Applied Finite Mathematics — TIME: 1 hour

Instructions:
1. Answer all questions on the machine-scored answer sheet provided.
USE PENCIL ONLY.
2. Return examination paper with machine-scored answer sheet.
3. Single-line display calculators permitted. No other aids permitted.
4. Fill in the information requested below.
5. The examination invigilators may not interpret or explain questions to you.
6. Fill in your student number on the machine-scored sheet and encode it as well.

FAMILY NAME: (Print in ink) _____

GIVEN NAME(S): (Print in ink) _____

STUDENT NUMBER: _____

SECTION: _____

SIGNATURE: (in ink) _____
(I understand that cheating is a serious offence.)

For the first eight problems,use the following matrices:

$$A = \begin{pmatrix} 3 & -2 & -1 \\ 2 & -1 & 0 \end{pmatrix}, B = \begin{pmatrix} 0 & -2 \\ 1 & 2 \\ -1 & 3 \end{pmatrix}, C = \begin{pmatrix} 2 & -1 \\ 1 & 3 \end{pmatrix}, D = \begin{pmatrix} 1 & 2 & -1 \\ 0 & 1 & -1 \\ -2 & -1 & 1 \end{pmatrix},$$

1. What is the $(2,1)$ entry of $A^T + B$?

   A. 0

   B. -1

   C. 3

   D. $A^T + B$ is not defined

   E. none of the above

2. What is the (2,3) entry of AD?

   A. -1

   B. 2

   C. 3

   D. AD is not defined

   E. none of the above

3. What is the (2,2) entry of $D - 2B^T$?

   A. -1

   B. 0

   C. 3

   D. $D - 2B^T$ is not defined

   E. none of the above

$$A = \begin{pmatrix} 3 & -2 & -1 \\ 2 & -1 & 0 \end{pmatrix}, B = \begin{pmatrix} 0 & -2 \\ 1 & 2 \\ -1 & 3 \end{pmatrix}, C = \begin{pmatrix} 2 & -1 \\ 1 & 3 \end{pmatrix}, D = \begin{pmatrix} 1 & 2 & -1 \\ 0 & 1 & -1 \\ -2 & -1 & 1 \end{pmatrix}$$

4. What is the (3,1) entry of BC?

   A. 1

   B. 10

   C. 0

   D. BC is not defined

   E. none of the above

5. What is the (1,2) entry of C^2?

   A. -5

   B. 1

   C. 5

   D. C^2 is not defined

   E. none of the above

6. What is the (1,1) entry of $C^T A^T$?

   A. 6

   B. 4

   C. 3

   D. $C^T A^T$ is not defined

   E. none of the above

7. What is the (3,2) entry of AD?

   A. -1

   B. -4

   C. 0

   D. AD is not defined

   E. none of the above

8. What are the dimensions of the matrix BCA?

   A. 3×2

   B. 3×3

   C. 2×2

   D. BCA is not defined

   E. none of the above

9. The following matrix is not in row echelon form:

$$\begin{pmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

What is the reason?

A. The first nonzero entry in some row is not a 1
B. There are nonzero entries above a leading 1
C. There are nonzero entries below a leading 1
D. The leading 1's are not in the correct places
E. none of the above

10. What SINGLE elementary row operation would simplify the accompanying augmented matrix to reduced row echelon form?

$$\left(\begin{array}{cccc|c} 1 & 3 & 2 & 0 & 4 \\ 0 & 1 & 1 & 0 & -5 \\ 0 & 0 & 0 & 1 & 17 \end{array}\right)$$

A. Multiply a row by a nonzero constant
B. Add a multiple of a row to another row
C. Switch two rows
D. This cannot be done in a single operation
E. none of the above

11. Which of the following accurately describes the REF and RREF forms of a matrix?

A. The number of leading 1's in the REF form is larger than the number of leading 1's in the RREF form.
B. The number of leading 1's in the REF form is smaller than the number of leading 1's in the RREF form.
C. The number of leading 1's in the REF form is the same as the number of leading 1's in the RREF form.
D. The number of leading 1's in the REF form is unrelated to the number of leading 1's in the RREF form.
E. none of the above

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12. You are given that the reduced row echelon form for the augmented matrix associated with the system of equations
$$\begin{array}{rcrcrcrcrcl} x_1 & + & x_2 & + & 5x_3 & - & x_4 & & & = & 3 \\ 2x_1 & + & x_2 & + & 7x_3 & + & x_4 & + & x_5 & = & 7 \\ x_1 & & & + & 2x_3 & + & x_4 & & & = & 3 \end{array}$$
is $\left(\begin{array}{ccccc|c} 1 & 0 & 2 & 0 & -1 & 2 \\ 0 & 1 & 3 & 0 & 2 & 2 \\ 0 & 0 & 0 & 1 & 1 & 1 \end{array}\right)$. Therefore the solution of the system is:

A. $x_1 = 2,\ x_2 = 2,\ x_3 = 0,\ x_4 = 1,\ x_5 = 0$

B. $x_1 = 2+2x_3-x_5,\ x_2 = 2+3x_3+2x_5,\ x_4 = 1+x_5,\ x_3, x_5$ are arbitrary

C. $x_1 = 2,\ x_2 = 2,\ x_4 = 1,\ x_3, x_5$ are arbitrary

D. $x_1 = 2-2x_3+x_5,\ x_2 = 2-3x_3-2x_5,\ x_4 = 1-x_5,\ x_3, x_5$ **are arbitrary**

E. none of the above

13. If $\left(\begin{array}{cccc|c} 1 & 2 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 2 \\ 0 & 0 & 2 & 3 & 3 \\ 0 & 0 & 2 & 3 & 3 \end{array}\right)$ is a partially reduced matrix from the augmented matrix for a system of linear equations, then:

A. The original system has 4 unknowns and 3 equations

B. The original system has 5 unknowns and 4 equations

C. The original system has 3 unknowns and 4 equations

D. It is impossible to determine the number of equations and unknowns of the original system

E. none of the above

14. Which of these is the reduced row echelon form of the augmented matrix $\left(\begin{array}{cccc|c} 1 & 2 & 0 & 1 & 1 \\ 0 & 0 & 1 & -1 & 2 \\ 0 & 0 & 0 & 1 & -1 \end{array}\right)$?

A. $\left(\begin{array}{cccc|c} 1 & 2 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 3 \\ 0 & 0 & 0 & 1 & -1 \end{array}\right)$

B. $\left(\begin{array}{cccc|c} 1 & 2 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & -1 \end{array}\right)$

C. $\left(\begin{array}{cccc|c} 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & -1 & 2 \\ 0 & 0 & 1 & 1 & -1 \end{array}\right)$

D. $\left(\begin{array}{cccc|c} 1 & 2 & 0 & 0 & 2 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & -1 \end{array}\right)$

E. none of the above

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15. When the augmented matrix for a system of 4 equations in 4 unknowns is partially simplified, the following matrix is obtained: $\left(\begin{array}{cccc|c} 1 & 2 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 2 \\ 0 & 0 & 2 & 3 & 3 \\ 0 & 0 & 2 & 3 & 3 \end{array}\right)$. What do you conclude about the number of solutions of the system.

    A. There is exactly one solution

    B. There is no solution

    C. There are infinitely many solutions

    D. There is not enough information to conclude how many solutions the system has

    E. none of the above

16. If the matrix $\left(\begin{array}{cccc|c} 1 & 2 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 2 \\ 0 & 0 & 2 & 3 & 3 \\ 0 & 0 & 2 & 3 & 0 \end{array}\right)$ is simplified to row echelon form, how many leading ones will it have?

    A. 1

    B. 2

    C. 3

    D. 4

    E. none of the above

17. Which of the following operations will reduce the following to REF: $\left(\begin{array}{cccc|c} 1 & 2 & 0 & -1 & -1 \\ 0 & 1 & 0 & 2 & 7 \\ 0 & -2 & 1 & 2 & 3 \end{array}\right)$?

    A. $R_3 \to R_3 + 2R_2$

    B. $R_2 \to 2R_2 + R_3$

    C. $R_3 \to R_3 + R_1$

    D. $R_3 \to R_3 - R_3$

    E. None of the above

18. If the inverse of the matrix $A = \begin{pmatrix} 1 & 1 & -5 \\ 2 & -1 & 2 \\ 2 & 1 & -7 \end{pmatrix}$ is $A^{-1} = \frac{1}{3}\begin{pmatrix} 5 & 2 & -3 \\ 18 & 3 & -12 \\ 4 & 1 & -3 \end{pmatrix}$ then the solution to the system of equations $\begin{array}{rcrcrcr} x & + & y & - & 5z & = & 3 \\ 2x & - & y & + & 2z & = & 6 \\ 2x & + & y & - & 7z & = & -1 \end{array}$ for z is

    A. -10

    B. 21

    C. -6

    D. 7

    E. none of the above

19. Which of the following is the inverse of $A = \begin{pmatrix} 1 & 2 & -1 \\ 0 & 1 & -1 \\ -2 & -1 & 0 \end{pmatrix}$?

A. $\begin{pmatrix} -1 & 1 & -1 \\ 2 & -2 & 1 \\ 2 & -3 & 1 \end{pmatrix}$

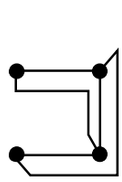
B. $\begin{pmatrix} 1 & \frac{1}{2} & -1 \\ 0 & 1 & -1 \\ -\frac{1}{2} & -1 & 0 \end{pmatrix}$

C. $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

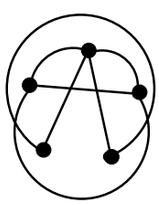
D. $\begin{pmatrix} 1 & 2 & 2 \\ 1 & -2 & 1 \\ -1 & 1 & 1 \end{pmatrix}$

E. none of the above

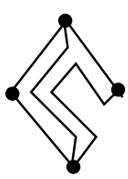
20. Which of the following graphs are simple



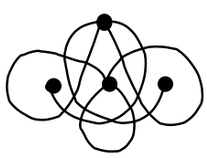
Graph I



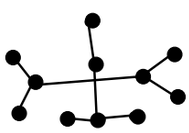
Graph II



Graph III



Graph IV



Graph V

A. All Graphs except Graph IV are simple

B. Only Graph I, Graph III, and Graph V are simple

C. Only Graph I and Graph III are simple

D. Only Graph I, Graph II, and Graph III are simple

E. none of the above

21. If the degree set of a graph is $\{1, 1, 1, 2, 2, 3, 4, 4\}$ then how many edges does it have?

A. 36

B. 9

C. 18

D. This is not a valid degree set

E. none of the above

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22. A graph is known to have 16 edges. If 3 nodes have degree 4, and ever other node has degree 5, what is the total number of nodes?

    A. 4

    B. 7

    C. 10

    D. This is impossible

    E. none of the above

23. Which of the following is impossible for the degree set of a simple graph?

    A. $\{1, 1, 2, 2\}$

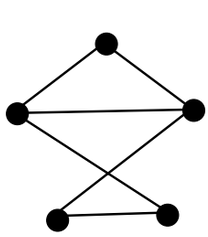
    B. $\{0, 1, 2, 3\}$

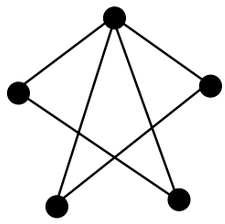
    C. $\{0, 0, 0, 0\}$

    D. $\{1, 2, 2, 3\}$

    E. All of these graphs are possible

24. The following graphs





    A. are not equivalent because they do not have the same number of nodes.

    B. are not equivalent because they do not have the same number of edges.

    C. are not equivalent because, although they the same number of degrees and nodes, they have different degree sets.

    D. are not equivalent for some other reason.

    E. are equivalent.

25. How many nonequivalent graphs are there on 4 nodes where at least one of the nodes is isolated?

    A. 4

    B. 3

    C. 2

    D. 1

    E. none of the above