Math2130 Test 2 Summer 2015 (by N Harland)

Answers by Dawit (ydawit@yahoo.com)

[4] 1. (a) Show $\lim_{(x,y)\to(0,0)} \frac{x^4y}{x^6+y^3}$ does not exist.

$\gg$ 1. a) Hint: let $y = mx^2$

[3] (b) Show $\lim_{(x,y)\to(0,0)} \frac{|x|\,y^2}{x^2+y^2} = 0$.

b) Hint: $0 \le \frac{y^2|x|}{x^2+y^2} \le |x|\frac{y^2}{y^2} \le |x|$

[6] 2. Determine parametric equations for the tangent line to

$$2x^2 - y^2 + z^2 = 7,\ xy + xz = 5$$

at the point $P(1,2,3)$.

$\gg$ 2. $x = 1-5t,\ y = 2+13t,\ z = 3+12t$.

[2] 3. Let $f(x,y) = y\cos(x^7)$. Calculate $\frac{\partial^2}{\partial y^2}\left(\frac{\partial^7 f}{\partial x^7}\right)$. Explain your answer.

$\gg$ 3. 0 (Hint $\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x}$)

[4] 4. (a) Find a chain rule for $\left.\frac{\partial z}{\partial t}\right)_s$ if

$$z = f(x,y,t), \quad x = g(y,s,t), \quad y = h(t).$$

(Note: Notation will be marked)

$\gg$ 4. a) $\left.\frac{\partial z}{\partial x}\right)_{y,t}\left.\frac{\partial x}{\partial y}\right)_{s,t}\frac{dy}{dt} + \left.\frac{\partial z}{\partial x}\right)_{y,t}\left.\frac{\partial x}{\partial t}\right)_{s,y} + \left.\frac{\partial z}{\partial y}\right)_{x,t}\frac{dy}{dt} + \left.\frac{\partial z}{\partial t}\right)_{x,y}$

[4] (b) Use part (a) to find $\left.\frac{\partial z}{\partial t}\right)_s$ if

$$z = e^{x^2+y^2+t^2}, \quad x = \ln(yst), \quad y = \sec t.$$

Do not simplify your answer.

b) $\left(2xe^{x^2+y^2+t^2}\right)\left(\frac{st}{yst}\right)(\sec t \tan t) + \left(2xe^{x^2+y^2+t^2}\right)\left(\frac{ys}{yst}\right) + \left(2ye^{x^2+y^2+t^2}\right)(\sec t \tan t) + \left(2te^{x^2+y^2+t^2}\right)$

[10] 5. Given

$$\begin{aligned} x^2 - y\cos(uv) + 2z &= 0 \\ x^2 + y^2 - \sin(uv) + 2z^2 - 2 &= 0 \\ xy - \sin v \cos u + 3z - 1 &= 0 \end{aligned}$$

Calculate $\left.\frac{\partial y}{\partial v}\right)_u$ when $x = 1$, $y = 1$, $u = \pi/2$, $v = 0$, $z = 0$.

$\gg$ 5. $\pi/9$

[4] 6. (a) Determine all critical points of $f(x,y) = x^6 + y^6 - 6xy + 1$.

[7] (b) Classify the critical points you found in part (a).

$\gg$ 6. a) $(0,0)$; $(-1,-1)$, $(1,1)$

b) $(0,0)$ yields saddle point. $(-1,-1)$ and $(1,1)$ yield Rel-min

[6] 7. Calculate $D_{\mathbf{v}}f$ at the point $P(1,2,3)$ if $f(x,y,z) = \ln(x^2+y^2+z^2)$ along the curve $\mathbf{r}(t) = (t^2, t^3+1, 3t)$.

$\gg$ 7. $\frac{17}{7\sqrt{22}}$

[4 (bonus)] 8. Let $\mathbf{v} = (\cos\alpha, \sin\alpha)$ and f be a twice differentiable function. Determine and simplify a formula for $D_{\mathbf{v}}(D_{\mathbf{v}}f)$ in terms of α and the second partial derivatives.

8. $f_{xx}\cos^2\alpha + 2f_{xy}\sin\alpha\cos\alpha + f_{yy}\sin^2\alpha$

Tree diagram for #4.