THE UNIVERSITY OF MANITOBA

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DEPARTMENT & COURSE NO: MATH2130 TIME: 3 hours

EXAMINATION: Engineering Mathematical Analysis 1 EXAMINER: D. Trim

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8 1. Find the distance between the plane $3x - 5y + 4z = 10$ and the line

$$x = 2 - t, \quad y = 3 + t, \quad z = -4 + 2t.$$

ans. $7/\sqrt{2}$

10 2. Find equations for the tangent line to the curve

$$xyz - x^2 + y^2 = -3, \qquad z = x^2 + 2y^2,$$

at the point $(1, -1, 3)$.

ans. $x = 1 - 5t$
$y = -1 - 7t$
$z = 3 + 18t$

10 3. Find the rate of change of the function $f(x, y, z) = xy(x + z)$ at the point $(1, 3, -2)$ in the direction normal to the surface $x^2y - 2xy^2z = 39$ at $(1, 3, -2)$.

ans. $-79/\sqrt{2713}$

10 4. The equations

$$xu^2 + v^3 = xy - 8, \qquad 2uv = x^2 + y^2 + v^3$$

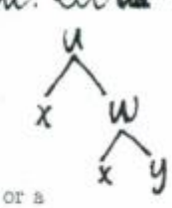
define u and v as functions of x and y. Find $\dfrac{\partial u}{\partial x}$ when $x = 0$ and $y = 2$.

ans. $-5/24$

10 5. If f is a differentiable function, show that every function of the form $u(x, y) = x\,f(6x - 2y^2)$ satisfies the equation

$$2xy\frac{\partial u}{\partial x} + 3x\frac{\partial u}{\partial y} = 2yu.$$

hint: let $w = 6x - 2y^2$



12 6. Find all critical points for the function

$$f(x, y) = x^2y^3 - xy - 4y.$$

Classify any one of the critical points as yielding a relative maximum, a relative minimum, or a saddle point.

ans. $(-4, 0), (8, 1/4), (8, -1/4)$: all yield SP.

12 7. Evaluate

$$\iint_R (xy^4 + y)\, dA$$

where R is the region bounded by the curves

$$y = 1 - x^2, \qquad y = 0.$$

ans. $8/15$

8 8. Set up, but do **NOT** evaluate, a double iterated integral for the volume of the solid of revolution when the area bounded by the curves

$$y = x, \qquad y = 0, \qquad y = (2 - x)^{1/4}$$

is rotated about the line $x + y = -2$. Simplify the integrand as much as possible.

ans. $\sqrt{2}\,\pi \int_0^1 \int_y^{2-y^4} (x + y + 2)\, dx\, dy$

8 9. Set up, but do **NOT** evaluate, a **SINGLE** double iterated integral for the area of that part of the surface $z = 2x^2 + y^2$ bounded by the planes $y = x$, $y = -x$, and the cylinder $x = \sqrt{4 - y^2}$.

ans. $2\int_0^{\sqrt{2}} \int_y^{\sqrt{4-y^2}} \sqrt{1 + 16x^2 + 4y^2}\, dx\, dy$

12 10. Find the volume bounded by the surfaces

$$z = 4 - \sqrt{x^2 + y^2}, \qquad 2z = x^2 + y^2$$

ans. $\dfrac{20\pi}{3}$