MATH 1210 Problem Workshop 9

1. Find basic solutions, with integer coefficients, for the following system of homogeneous equations.

$$x + 5y + 3z - 5w = 0$$
$$2x - y + 3z - 4w = 0$$

Are they unique?

2. (a) Show that solutions of the system of nonhomogeneous equations

$$2x - y + 3z + 5w = 3$$
$$x + 3y - 2z + w = -2$$
$$3x + 2y + z + 6w = 1$$

can be expressed in the form

$$\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = z \begin{bmatrix} -1 \\ 1 \\ 1 \\ 0 \end{bmatrix} + \frac{w}{7} \begin{bmatrix} -16 \\ 3 \\ 0 \\ 7 \end{bmatrix} + \begin{bmatrix} 1 \\ -1 \\ 0 \\ 0 \end{bmatrix}$$

- (b) Are $\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \\ 1 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \begin{bmatrix} -16 \\ 3 \\ 0 \\ 7 \end{bmatrix}$ basic solutions of the system?
- (c) Is $\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 0 \\ 0 \end{bmatrix}$ a solution of the system? Is it a basic solution?

$\underline{\text{Answers}}$

1.
$$\begin{bmatrix} -18 \\ -3 \\ 11 \\ 0 \end{bmatrix}$$
, $\begin{bmatrix} 25 \\ 6 \\ 0 \\ 11 \end{bmatrix}$. No

2. (b) No, No, (c) Yes, No