

Derivatives of exponential and logarithmic functions

3.11.1

(a)

$$\frac{dy}{dx} = \frac{e^{x^2+1} \cdot (2x) \cdot \ln(1-x) - e^{x^2+1} \cdot \frac{1}{1-x} \cdot (-1)}{(\ln(1-x))^2}$$

(b)

$$\frac{dy}{dx} = \cos(\sin(\sin 4^x)) \cdot \cos(\sin 4^x) \cdot \cos 4^x \cdot 4^x \cdot \ln 4$$

(c)

$$y = \log_5 \frac{\cot x}{2x-1} = \log_5 \left| \frac{\cot x}{2x-1} \right| = \log_5 |\cot x| - \log_5 |2x-1|$$

$$\frac{dy}{dx} = \frac{1}{\cot x \cdot \ln 5} \cdot (-\csc^2 x) - \frac{1}{(2x-1) \cdot \ln 5} \cdot 2$$

3.11.2

$$\begin{aligned} \frac{dy}{dx} &= e^x + xe^x = e^x(1+x) \\ \frac{d^2y}{dx^2} &= e^x(1+x) + e^x(1) = e^x(2+x) \\ \frac{d^3y}{dx^3} &= e^x(2+x) + e^x(1) = e^x(3+x) \\ &\vdots \\ \frac{d^ny}{dx^n} &= e^x(n+x) \end{aligned} \quad \Rightarrow \quad \text{Inductive reasoning}$$