Math 2130 Summer2014 Test 1

Answers by Dawit Y.

[3] 1. (a) *For the curve, $x = y^2 + z^2$, identify the type of curve and give a sketch.*

[5] (b) *Find the projection of the intersection of $z^2 = x^2 + y^2$ and $x^2 + 3y^2 + z^2 = 4$ in the xy-plane.*

2. *Let l_1 be the line*

$$\frac{x-1}{2} = y - 2 = \frac{z-1}{3}$$

and l_2 be the line

$$x = 3 - 2t,\ y = 1 + t,\ z = 2 - t.$$

[3] (a) *Show that the lines are intersecting and find the point of intersection.*

[5] (b) *Find the equation of the plane containing both lines.*

[3] (c) *Find the distance from the point $R(1, 2, 3)$ to the plane found in part (b).*

3. *Let l_1 be the line*

$$\frac{x-3}{2} = y + 5 = \frac{z-1}{-2}$$

and l_2 has parametric equations

$$x = 1 \quad y = 3 + 2t \quad x = 1 + 2t.$$

[5] (a) *Determine whether the lines are parallel, intersecting or skew.*

[6] (b) *Find the shortest distance between the lines.*

[6] 4. *Find a parametric representation for the curves of intersection of $x^2 + y^2 = z^2$ and $x^2 + y^2 + z^2 = 8$ directed so that x increases when y is positive. Justify your answer. Assume $z \geq 0$.*

5. *Let a curve C be defined by a position vector $\mathbf{r_1}(t) = \langle t, 2 - t, 3 - 2t^2 \rangle$ and let a curve D be defined by a position vector $\mathbf{r_2}(s) = \langle s, s^2, s^3 \rangle$.*

[4] (a) *Find parametric equations for the tangent line to $\mathbf{r_1}(t)$ at the point $(0, 2, 3)$.*

[6] (b) *Find the point of intersection of the two curves and find the cosine of the angle between the curves at that point.*

[4] (c) *Set up, but don't evaluate an integral to find the length of the curve C from the point $(0, 2, 3)$ to $(2, 0, -5)$.*

1a) Elliptic paraboloid with Main axis on the x-axis

b) $2x^2 + 4y^2 = 4$, $z = 0$ (Ellipse)

2. a) Hint l_1: $x = 1 + 2s$, $y = 2 + s$, $z = 1 + 3s$ $\quad l_2$: $x = 3 - 2t$, $y = 1 + t$, $z = 2 - t$

Solving for s or t: (Simultaneously) $s = 0$, $t = 1$

$P(1, 2, 1)$ intersection pt.

b) $x + y - z - 2 = 0$

3. Hint: follow the steps above but my favorite is the Vector Method: let $l_1 : P_1, \vec{V_1}$ (P is point on the line) $l_2 : P_2, \vec{V_2}$

faster check $\vec{V_1} = k\vec{V_2}$ (for parallel)

i) $\vec{V_1} \times \vec{V_2} = \begin{cases} \vec{0} \rightarrow \text{parallel} \\ \text{not } \vec{0} \rightarrow \text{non parallel} \end{cases}$

ii) $\vec{V_1} \times \vec{V_2} \neq \vec{0}$ $\begin{cases} \text{may or} \\ \text{May not intersect} \end{cases}$

iii) $\overrightarrow{P_1P_2} \cdot (\vec{V_1} \times \vec{V_2}) \begin{cases} = 0 \rightarrow \text{intersect} \\ \neq 0 \text{ will not intersect (skew-lines)} \end{cases}$

$$d = \frac{|\overrightarrow{P_1P_2} \cdot (\vec{V_1} \times \vec{V_2})|}{|\vec{V_1} \times \vec{V_2}|} \text{ (for skew lines)}$$

3a) Skew b) $\frac{22\sqrt{17}}{17}$

4. $x = 2\cos t$
$y = -2\sin t$
$z = 2$
$0 \leq t < 2\pi$

5. a) $x = s$, $y = 2 - s$, $z = 3$

b) $(1, 1, 1)$, $\cos\theta = -\frac{13}{\sqrt{252}}$

c) $\int_0^2 \sqrt{2 + 16t^2}\, dt$