All questions are of equal value. Answer all 25 questions. No marks are subtracted for wrong answers.

Record all answers on the bubble sheet provided. USE PENCIL ONLY! Black pen will look good but may not be read reliably by the scoring machine. **Mark only <u>one</u> answer for each question!** Select the answer that is closest to yours.

A formula sheet is provided for your use; you may not use your own formula sheet or any other materials or notes. Calculators of any type are allowed, but not devices that store text or that can communicate with other such devices.

Be sure your name and student number are printed on the score sheet and the student number correctly coded in the box at the top right-hand side of the sheet. **<u>DO NOT start your student number with 00.</u>**

1. Over a short interval, starting at time $t = 0$, the coordinate of an automobile in meters is given by $x(t) = 12t - 2t^3$, where t is in seconds. The magnitudes of the initial (at $t = 0$) velocity and acceleration of the auto respectively are:

   a) 0 m/s; 12 m/s^2
   b) 0 m/s; 24 m/s^2
   ✓c) 12 m/s; 0 m/s^2
   d) 12 m/s; 12 m/s^2
   e) 12m/s; 24 m/s^2

1. $v = \frac{dx}{dt} = 12 - 6t^2$
$a = \frac{dv}{dt} = -12t$
$v(0) = 12\ m/s$
$a(0) = 0\ m/s^2$

2. The coordinate of an object is given as a function of time by $x = 4t - 3t^4$, where x is in meters and t is in seconds. Its average acceleration over the interval from $t = 0$ to $t = 2$ s is:

   a) –8 m/s^2 b) 8 m/s^2 ✓c) –48 m/s^2 d) 48 m/s^2 e) –18 m/s^2

$v = \frac{dx}{dt} = 4 - 12t^3$
2. $a = \frac{\Delta v}{\Delta t} = \frac{v(2) - v(0)}{2 - 0} = \frac{-92}{2} = -48\ m/s^2$

3. At a stop light, a truck traveling at 30 m/s passes a car as the car starts from rest. The truck travels at constant velocity and the car accelerates at 3 m/s^2. How much time does the car take to catch up to the truck?

   a) 5 s b) 10 s c) 15 s ✓d) 20 s e) 25 s

3. $x = 30t = \frac{1}{2}at^2 = \frac{3}{2}t^2$
$\Rightarrow \frac{1}{2}t(3t - 60) = 0$
$t = 0,\ 20$

4. At time $t = 0$ a car has a velocity of 16 m/s. It slows down with an acceleration given by $a = -0.50t$, in m/s^2 for t in seconds. At the end of 2.0 s it has traveled:

   a) 0 b) 12 m c) 16 m d) 20 m ✓e) 31 m

4. $v = \int -\frac{1}{2}t\,dt = -\frac{1}{4}t^2 + C$
$v(0) = 16 \rightarrow C = 16$
$x = \int \left(16 - \frac{1}{4}t^2\right)dt = 16t - \frac{1}{12}t^3 + D$
$x(0) = 0 = D$
$x = 16(2) - \frac{8}{12} = 32 - 0.75 = 31.25\,m$

5. Two vectors lie with their tails at the same point. When the angle between them is increased to 90°, the magnitude of their vector product doubles. The original angle between them was about:

   a) 0° b) 15° ✓c) 30° d) 45° e) 90°

5. $|\vec{a}\times\vec{b}| = ab\sin\theta$
$|\vec{a}| = a,\ |\vec{b}| = b$
$2ab\sin\theta = ab$
$\sin\theta = \frac{1}{2} \Rightarrow \theta = 30^\circ$

6. Let $\vec{S} = (1\text{ m})\hat{i} + (2\text{ m})\hat{j} + (2\text{ m})\hat{k}$ and $\vec{T} = (4\text{ m})\hat{k}$. The angle between these two vectors is:

   a) 0° ✓b) 48° c) 58° d) 132° e) 270°

6. $\theta = \cos^{-1}\left(\frac{8}{3\cdot 4}\right) = \cos^{-1}\left(\frac{2}{3}\right) = 48.2^\circ$
$\theta = \cos^{-1}\left[\frac{\vec{S}\cdot\vec{T}}{|\vec{S}||\vec{T}|}\right]$

7. A large cannon is fired from ground level over level ground with the speed of 980 m/s at an angle of 30° above the horizontal. Neglecting air resistance, how long does it take the projectile to strike the ground?

a) 50s b) 100s ✓ c) 150s d) 200s e) 250s

7. $t = \frac{2V_0 \sin\theta}{g} = \frac{2(980)\sin 30}{9.8} = 100s$

8. An airplane makes a gradual 90° turn along a circular path while flying at a constant speed of 200 m/s. The process takes 20.0 seconds to complete. The magnitude of the centripetal acceleration of the plane is:

a) 0 m/s^2 b) 40 m/s^2 c) 20 m/s^2 d) 16 m/s^2 ✓ e) 14 m/s^2

8. $t = \frac{\pi r / 2}{v} \Rightarrow r = \frac{2vt}{\pi}$

$a_c = \frac{v^2}{r}$

$a = \frac{\pi v}{2t} = \frac{\pi(200)}{2(20)} = 5\pi s \approx 16s$

9. A boat is traveling upstream at 14 km/hr with respect to a river that is flowing at 6 km/hr (with respect to the ground). A man runs directly from the back of the boat to the front of the boat at 2 km/hr (with respect to the boat). The speed of the man with respect to the ground is:

a) 10 km/hr ✓ b) 14 km/hr c) 18 km/hr d) 22 km/hr e) 6 km/hr

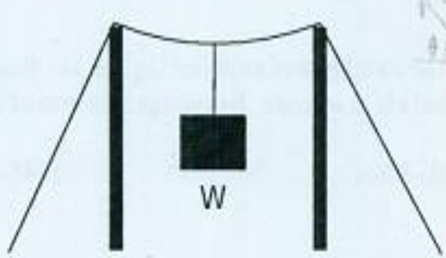
9. down ← → up

+

$\vec{V}_{mg} = \vec{V}_{mB} + \vec{V}_{Br} + \vec{V}_{rg} = 2 + 14 - 6 = 10$ km/h (upstream)

10. A weight W is hanging on a wire which is attached to two poles, see figure. The tension in the wire between the two poles is:

a) approximately zero
b) approximately W
c) approximately $W/2$
d) much more than W ✓
e) much less than W



T T θ θ W

θ is very small

$2T\sin\theta = W$

$T = \frac{W}{2\sin\theta} \gg W$

(for small θ)

11. A 5 kg block is initially at rest on a rough horizontal surface with friction coefficients μ_s=0.5 and μ_k=0.2. Applying a 27 N horizontal force, the magnitude of the friction force is

a) 25 N b) 20 N c) 15 N d) 10 N ✓ e) 5 N

$f_{s\,max} = N\mu_s = 5\times 9.8\times(0.5) = 24.5\text{ N} < F_a$

$\Rightarrow F_f = \mu_k N = 5\times 9.8\times 0.2 = 9.8\text{ N}$

12. A 3 kg box falls from a height of 1 m onto a spring scale with a spring constant k=1.5 · 10^5 N/m. At its greatest compression the magnitude of the force exterted by the spring onto the box is:

a) 30 N b) 60 N c) 1.5 · 10^4 N d) 1.5 · 10^3 N e) 3 · 10^3 N ✓

1m d

$mg(1+d) = \frac{1}{2}kd^2$

$kd^2 - 2mgd - 2mg = 0$

$15000d^2 - 58.8d - 58.8 = 0$

$d = 0.02$m

$F_s = kd = 2.999\times 10^3$ N

13. A force $\vec{F} = (4\text{ N})\hat{\imath} - (2\text{ N})\hat{\jmath} + (8\text{ N})\hat{k}$ acts on a mass of 4 kg as it moves in the x direction at a velocity of $\vec{v} = (2\text{ m/s})\hat{\imath} + (1\text{ m/s})\hat{\jmath} - (0.75\text{ m/s})\hat{k}$. What is the rate at which the force is doing work?

a) 100 W b) 50 W c) 25 W d) 12.5 W e) 0 W ✓

$\vec{P} = \vec{F}\cdot\vec{V} = 4\times 2 - 2\times 1 - 8(0.75) = 0\text{ W}$

14. A mass m, at one end of a string of length L, rotates in a vertical circle just fast enough to prevent the string from going slack at the top of the circle. Assuming mechanical energy is conserved, the tension in the string at the bottom of the circle is:

a) 6 mg ✓ b) mg + 3mg/L c) 5 mg d) $\sqrt{5gL}$ e) 4 mgL

$\frac{mV^2}{L} = mg + T$

$V_{min} = \sqrt{Lg}$

a_c $T = 0$

$\frac{1}{2}mV_{min}^2 + mg(2L) = \frac{1}{2}mV^2$

$\frac{V^2}{L} = 5g$

a_c T V_m mg

$T = mg + \frac{mV^2}{L} = mg + 5mg = 6mg$

15.
$p = dE/dt$
$= 4 + 36t^2$
$P(4) = 4 + 36(4)^2$

15. The energy of a system as function of time is given by $4t + 12t^3$, in Joules. What is the instantaneous power at t=4s?

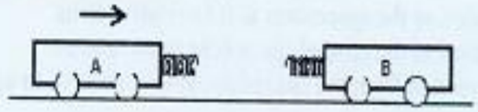
a) 576 W b) 480 W c) 784 W d) 580 W ✓ e) 196 W

16. A 80 kg hockey player sits at rest on a frictionless ice surface. He throws his 1 kg hockey stick at a speed of 20 m/s to his left. As a result the player moves to his right at a speed of

a) 1 m/s b) 0.75 m/s c) 0.5 m/s d) 0.25 m/s ✓ e) 0 m/s

16. $m_p V_p - m_s V_s = 0$
$V_p = \frac{m_s}{m_p} V_s = \frac{1}{80} \times 20\ m/s$
$= 0.25\ m/s$

17. Two carts (A and B), having spring bumpers, collide as shown. Cart A has a mass of 2 kg and is initially moving to the right. Cart B has a mass of 3 kg and is initially stationary. When the separation between the carts is a minimum:



17.
ME = KE + U_{sp} is Conserved
=> $KE_{max} \rightarrow U_{s\,min}$
$U_{max} \rightarrow KE_{min}$

a) the carts have the same kinetic energy
✓ b) the kinetic energy of the system is at a minimum
c) cart B is still at rest
d) cart A has come to rest
e) the carts have the same momentum

18.
$\theta = \frac{1}{2}\alpha t_1^2$
$\alpha = \frac{2\theta_1}{t_1^2}$
$t = \sqrt{\frac{2\theta_3}{\alpha}} = \sqrt{\frac{2\theta_3}{2\theta_1}}\, t_1$
$= \sqrt{3}\, t_1$

18. A wheel starts from rest and rotates with a constant angular acceleration about a fixed axis. It completes the first revolution 6.0 s after it started. How long after it started will the wheel complete the third revolution?

a) 9.9 s b) 7.8 s c) 8.5 s d) 9.2 s e) 10.4 s ✓

19.


19. A thin uniform rod (length L = 1.2 m, mass M = 2.0 kg) is pivoted about a horizontal, frictionless pin through one end of the rod. (The moment of inertia of the rod about this axis is $ML^2/3$.) The rod is released when it makes an angle of 60° with the horizontal. What is the angular acceleration of the rod at the instant it is released?

a) 9.8 rad/s^2 b) 7.4 rad/s^2 c) 8.4 rad/s^2 d) 5.9 rad/s^2 e) 6.1 rad/s^2 ✓

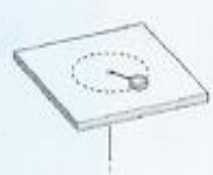
$\alpha = \frac{\tau}{I} = \frac{mg\frac{l}{2}\cos\theta}{\frac{1}{3}ml^2}$
$= \frac{3g\cos\theta}{2l}$
$= \frac{3g}{4l} = 6.125\ rad/s^2$

20.
$a = \sqrt{a_t^2 + a_r^2}$
$a_t = R\alpha$
$a_r = \frac{V^2}{R}$
$a_r = \sqrt{a_2^2 - R^2\alpha^2}$
$V = \sqrt{R a_r} \approx 1.56\ m/s$

20. A wheel rotates about a fixed axis with a constant angular acceleration of 4.0 rad/s^2. The diameter of the wheel is 1.0 m. What is the linear speed of a point on the rim of this wheel at an instant when that point has a total linear acceleration with a magnitude of 5.0 m/s^2?

✓ a) 1.5 m/s b) 1.7 m/s c) 2.0 m/s d) 2.3 m/s e) 3.0 m/s

21. A puck on a frictionless air hockey table has a mass of 5.0 kg and is attached to a cord passing through a hole in the surface as in the figure. The puck is revolving at a distance 2.0 m from the hole with an angular velocity of 4.0 rad/s. The angular momentum of the puck (in kg · m^2/s) is



✓ a) 80 b) 20 c) 30 d) 60 e) 50

21.
$L = I\omega$
$= mR^2\omega$
$= 5(2^2)4\ kgm^2/s$

22.

$L_i = L_f$

$I\omega_i = (I + mR^2)\omega_f$

$\omega_f = \frac{I}{I+mR^2}\omega_i$

$= \frac{250}{350}(9)\ \text{rev/min}$

$= 6.43\ \text{rev/min}$

22. A merry-go-round of radius $R = 2.0$ m has a moment of inertia $I = 250\ \text{kg} \cdot \text{m}^2$, and is rotating at 9.0 revolutions per minute. A child whose mass is 25 kg jumps onto the edge of the merry-go-round. The new angular speed (in revolutions per minute) of the merry-go-round at the moment when the child jumps onto it is approximately

a) 10.0 b) 9.0 c) 8.5 d) 7.1 e) 6.4 ✓

23. Spaceship A, traveling past us at 0.7c, sends a message capsule to spaceship B, which is in front of A and is traveling in the same direction as A at 0.8c relative to us. The capsule travels at 0.9c relative to us. A clock that measures the proper time between the sending and receiving of the capsule travels:

23. Concept question

a) in the same direction as the spaceships at 0.7c relative to us
b) in the opposite direction from the spaceships at 0.7c relative to us
c) in the same direction as the spaceships at 0.8c relative to us
✓ d) in the same direction as the spaceships at 0.9c relative to us
e) in the opposite direction from the spaceships at 0.9c relative to us

24

$\gamma = \frac{6}{3.6} = 5/3$

$\beta = \sqrt{1 - (1/\gamma)^2} =$

$= 4/5 \approx 0.8$

$v = 0.8c$

24. A certain automobile is 6 m long if at rest. If it is measured to be 3.6 m long by an observer, its speed as measured by the observer is:

a) 0.1c b) 0.3c c) 0.6c d) 0.8c ✓ e) 1.0c

25. Star S1 is moving away from us at a speed of 0.8c. Star S2 is moving away from us in the opposite direction at a speed of 0.2c. The speed of S1 as measured by an observer on S2 is:

a) 0.71c b) 0.86c ✓ c) 0.93c d) 1.0c e) 1.2c

25.

0.8c S1 S'

0.2c S2

S (earth)

$u' = \frac{u - v}{1 - \frac{uv}{c^2}} = \frac{0.2 - (-0.8)}{1 - (0.2)(0.8)}c \approx 0.86c$

THE END

or

$u = \frac{u' + v}{1 + \frac{u'v}{c^2}} = \frac{0.2 + 0.8}{1 + (0.2)(0.8)}c = 0.86c$

0.2c

S1 S

0.8c

S' (earth)

S1 at rest but earth moving with respect to S1.