MATH 1210
Assignment 3

Due November 15, in class.

1. Given the points $A : (2, 3, 1)$, $B : (3, 5, -2)$, and $C : (-2, 9, -1)$, find the angle between $\overrightarrow{AB}$ and $\overrightarrow{AC}$

2. For vectors $\overrightarrow{u}$ and $\overrightarrow{v}$ in 3-space, prove that:

$$|\overrightarrow{u} \times \overrightarrow{v}|^2 = |\overrightarrow{u}|^2 |\overrightarrow{v}|^2 - (\overrightarrow{u} \cdot \overrightarrow{v})^2$$

3. Find the point of intersection of the line $\dfrac{x-1}{-2} = \dfrac{y+4}{3} = z - 2$ and the plane that passes through the point $(1, 1, 1)$ and is parallel to the lines $x = 3 + t\,;\ y = -2 - 2t\,;\ z = 4 - t$ and $x = 4 - t\,;\ y = 1 + 6t\,;\ z = 3 - t$.

4. Find all values of a and b such that the following system of equations:

$$\begin{array}{rcrcrcr} x & - & y & + & 2\,z & = & 4 \\ 3\,x & - & 2\,y & + & 9\,z & = & 14 \\ 2\,x & - & 4\,y & + & a\,z & = & b \end{array}$$

   (a) has no solutions.

   (b) has exactly one solution.

   (c) has exactly three solutions.

   (d) has infinitely many solutions.

5. Solve the following systems of equations using the Gauss Jordan method:

(a)
$$\begin{array}{rcrcrcrcr} x_1 & - & x_2 & + & 2\,x_3 & + & 8\,x_4 & = & 1 \\ -\,x_1 & + & x_2 & - & x_3 & - & 5\,x_4 & = & -2 \\ 3\,x_1 & - & 3\,x_2 & + & 4\,x_3 & + & 18\,x_4 & = & 5 \end{array}$$

(b)
$$\begin{array}{rcrcrcr} x & - & y & - & 2\,z & = & 3 \\ 3\,x & - & 2\,y & - & 4\,z & = & 9 \\ -2\,x & + & 2\,y & + & 5\,z & = & -7 \\ 4\,x & - & 3\,y & - & 6\,z & = & 12 \end{array}$$

(c)
$$\begin{array}{rcrcrcr} x_1 & - & x_2 & + & 2\,x_3 & = & 0 \\ 4\,x_1 & - & x_2 & + & 6\,x_3 & = & 0 \\ -3\,x_1 & - & 3\,x_2 & - & 2\,x_3 & = & 0 \end{array}$$

6. Let $A = \begin{pmatrix} x & 0 & 0 & 0 \\ 0 & x & 1 & 1 \\ 0 & 2 & 1 & 3 \\ 0 & 4 & -1 & 2 \end{pmatrix}$, and $B = \begin{pmatrix} 4 & 1 & 2 \\ 3 & x & -1 \\ 2 & 2 & 5 \end{pmatrix}$.

   Find all values of x such that $\det A = \det B$.

7. Let $A = \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix}$. Suppose $\det A = 4$, find the determinant of the following matrices:

   (a) $B_1 = \begin{pmatrix} a & b & c \\ 2d + a & 2e + b & 2f + c \\ g + 3a & h + 3b & i + 3c \end{pmatrix}$.

   (b) $B_2 = \begin{pmatrix} a + g & b + h & c + i \\ d & e & f \\ a + d + g & b + e + h & c + f + i \end{pmatrix}$.

   (c) $B_3 = \begin{pmatrix} g & h & i \\ d + 2a & e + 2b & f + 2c \\ a + g & b + h & c + i \end{pmatrix}$.

   (d) $B_4 = \begin{pmatrix} 3d & 3e & 3f \\ 2a + 3d & 2b + 3e & 2c + 3f \\ a + \frac{1}{12}g & b + \frac{1}{12}h & c + \frac{1}{12}i \end{pmatrix}$.