11 **1.** Find the interval of convergence for the power series

$$\sum_{n=3}^{\infty} \frac{(1+1/n)^3}{2^{n+2}}(x-2)^{2n}.$$

10 **2.** Find the Taylor series about $x = 4$ for the function

$$f(x) = \frac{x}{x+3}.$$

Use a method that guarantees that the series converges to $f(x)$. Express your answer in sigma notation, simplified as much as possible. Determine the interval of convergence for the series.

15 **3.** Find a general solution for the differential equation

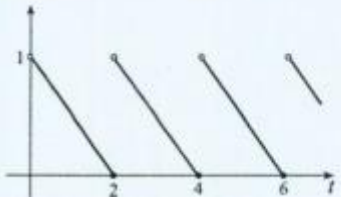
$$2y''' + 3y'' - 5y' - 3y = x + 3e^{2x}.$$

6 **4.** You are given that the roots of the auxiliary equation associated with the linear, differential equation

$$\phi(D)y = x^2 + 4 + xe^{4x} - 2\cos 3x$$

are $m = 0,\ 2\pm 3i,\ 2\pm 3i,\ \pm\sqrt{3},\ 4,\ 4$. Write down the form of a particular solution of the differential equation as predicted by the method of undetermined coefficients. Do **NOT** find the coefficients, just the form of the particular solution.

8 **5.** Find the Laplace transform for the function shown below. You need **NOT** simplify your answer, but you must use a method that does not involve integration by parts.



$f(t) = -\frac{1}{2}(t-2),\ 0 < t \leq 2.$

5 **6.** Find the Laplace transform of the function $f(t) = e^{2t}\sin t\, h(t-2\pi)$.

8 **7.** Find inverse Laplace transforms for the functions:

(a) $F(s) = \dfrac{e^{-3s}}{s^3 - 3s^2 + 3s - 1}$ (b) $F(s) = \dfrac{s}{s^2 + 2s + 4}$.

15 **8.** Solve the initial value problem

$$y'' + 4y = 3\delta(t-2) + h(t-1), \qquad y(0) = 1, \quad y'(0) = 0.$$

11 **9.** A tank contains 10 kilograms of sugar disolved in 500 litres of water. Solution with 3 kilograms of sugar per 1000 litres of water is added to the tank at the rate of 100 millilitres per second. Well-stirred mixture is removed from the tank at 200 millilitres per second. Set up, but DO NOT SOLVE, an initial-value problem for the number of grams of sugar in the tank as a function of time. For how long is your model valid?

11 **10.** A mass of 100 grams is suspended from a spring with constant 600 newtons per metre. At time $t = 0$, it is 10 cm above its equilibrium position and is given velocity 2 metres per second downward. Find the amplitude and the period of the resulting motion of the mass.