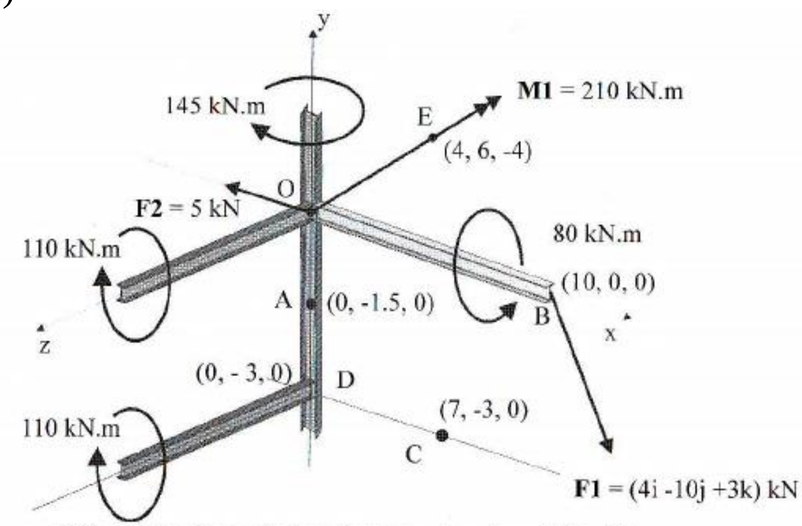
1)



a) Determine the equivalent-force couple acting at Point D.
b) What is the direction of the resultant moment vector at Point D?
c) What is the direction of the resultant force vector at Point D?
d) What is the perpendicular distance from Point D to the line-of-action of F1?
e) What is the moment of F1 about the Line AC?

Using the "Right Hand Rule" and $\vec{M} = \vec{r} \times \vec{F}$

a) Equivalent Force-Couple at D:

$$\vec{M_D} = +80\hat{i} - 145\hat{j} - 110\hat{k} - 110\hat{k} + \vec{r}_{DB} \times \vec{F_1} + \vec{r}_{DO} \times \vec{F_2} + \vec{M_1}$$

$$\vec{M_D} = 80\hat{i} - 145\hat{j} - 220\hat{k} + \vec{r}_{DB} \times \vec{F_1} + \vec{r}_{DO} \times \vec{F_2} + \vec{M_1}$$

$$\vec{r}_{DB} = 10\hat{i} + 3\hat{j} + 0\hat{k} \qquad \vec{F_1} = 4\hat{i} - 10\hat{j} + 3\hat{k}$$

$$\vec{r}_{DO} = 0\hat{i} + 3\hat{j} + 0\hat{k} \qquad \vec{F_2} = -5\hat{i}$$

$$\vec{r}_{DB} \times \vec{F_1} = \begin{vmatrix} i & j & k \\ 10 & 3 & 0 \\ 4 & -10 & 3 \end{vmatrix} \begin{matrix} i & j \\ 10 & 3 \\ 4 & -10 \end{matrix}$$

$$= [9\hat{i} - 100\hat{k}] - [12\hat{k} + 30\hat{j}]$$

$$= (9\hat{i} - 30\hat{j} - 112\hat{k}) \text{ kN.m}$$

$$\vec{r}_{DO} \times \vec{F}_2 = \begin{vmatrix} i & j & k \\ 0 & 3 & 0 \\ -5 & 0 & 0 \end{vmatrix} \begin{matrix} i & j \\ 0 & 3 \\ -5 & 0 \end{matrix}$$

$$= [\,0\,] - [-15\hat{k}] = (15k)\ kN.m$$

$$\vec{M}_1 = M_1 \vec{\lambda}_{OE} = 210\,\lambda_{OE} \qquad \vec{\lambda}_{OE} = \frac{\vec{OE}}{OE}$$

$$\vec{OE} = 4\hat{i} + 6\hat{j} - 4\hat{k} \qquad OE = \sqrt{(4)^2 + (6)^2 + (-4)^2}$$

$$= \sqrt{68}$$

$$\vec{M}_1 = 210\left(\frac{4\hat{i} + 6\hat{j} - 4\hat{k}}{\sqrt{68}}\right) = (101.86\hat{i} + 152.8\hat{j} + 101.86\hat{k})\ kN.m$$

$$\therefore \vec{M}_D = (80\hat{i} - 145\hat{j} - 220\hat{k}) + (9\hat{i} - 30\hat{j} - 112\hat{k}) + (15\hat{k}) + (101.86\hat{i} + 152.8\hat{j} - 101.86\hat{k})$$

$$\vec{M}_D = (190.86\hat{i} - 22.2\hat{j} - 418.86\hat{k})\ kN.m$$

$$\vec{R}_D = \vec{F}_1 + \vec{F}_2$$

$$\vec{R}_D = (4\hat{i} - 10\hat{j} + 3\hat{k}) - 5\hat{i}$$

$$\vec{R}_D = (-\hat{i} - 10\hat{j} + 3\hat{k})\ kN$$

$\therefore$ Equivalent Force - Couple at D:

$$\vec{R}_D = (-\hat{i} - 10\hat{j} + 3\hat{k})\ kN$$

$$\vec{M}_D = (190.86\hat{i} - 22.2\hat{j} - 418.86\hat{k})\ kN.m$$

b) Direction of the resultant moment at D:

$$M_D = \sqrt{(190.86)^2 + (-22.2)^2 + (-418.86)^2} = 460.83\ kN.m$$

$$\cos\theta_x = \frac{190.86}{460.83} \Rightarrow \theta_x = 65.65^\circ$$

$$\cos\theta_y = \frac{-22.2}{460.83} \Rightarrow \theta_y = 92.76^\circ$$

$$\cos\theta_z = \frac{-418.86}{460.83} \Rightarrow \theta_z = 155.36^\circ$$

c) Direction of Resultant force at D:

$$R_D = \sqrt{(-1)^2 + (-10)^2 + (3)^2} = \sqrt{110}$$

$$\cos\theta_x = \frac{-1}{\sqrt{110}} \Rightarrow \theta_x = 95.47^\circ$$

$$\cos\theta_y = \frac{-10}{\sqrt{110}} \Rightarrow \theta_y = 162.45^\circ$$

$$\cos\theta_z = \frac{3}{\sqrt{110}} \Rightarrow \theta_z = 73.38^\circ$$

d) $M = F_1 d$

$$\vec{M}_{F_1} = (9\hat{i} - 30\hat{j} - 112\hat{k})\ kN.m$$

$$M_{F_1} = \sqrt{(9)^2 + (-30)^2 + (-112)^2} = 116.3\ kN.m$$

$$\vec{F}_1 = (4\hat{i} - 10\hat{j} + 3\hat{k})\ kN$$

$$F_1 = \sqrt{(4)^2 + (-10)^2 + (3)^2} = \sqrt{125} = 11.18\ kN$$

$$116.3 = 11.18\, d \qquad d = 10.4\ m$$

e) Moment of $\vec{F}_1$ about Line AC

$$M_{AC} = \vec{\lambda}_{AC} \cdot (\vec{r}_{AB} \times \vec{F}_1)$$

$$\vec{\lambda}_{AC} = \frac{\vec{AC}}{AC} \qquad \vec{AC} = 7\hat{i} - 1.5\hat{j} + 0\hat{k}$$

$$AC = \sqrt{(7)^2 + (-1.5)^2} = \sqrt{51.25}$$

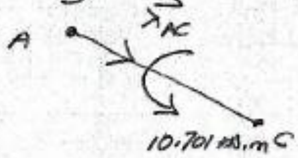
$$\vec{\lambda}_{AC} = \frac{7\hat{i} - 1.5\hat{j} + 0\hat{k}}{\sqrt{51.25}} = 0.978\hat{i} - 0.21\hat{j}$$

$$\vec{r}_{AB} = 10\hat{i} + 1.5\hat{j} + 0\hat{k}$$

$$\vec{F}_1 = 4\hat{i} - 10\hat{j} + 3\hat{k}$$

$$M_{AC} = \left|\begin{array}{ccc|cc} 0.978 & -0.21 & 0 & 0.978 & -0.21 \\ 10 & 1.5 & 0 & 10 & 1.5 \\ 4 & -10 & 3 & 4 & -10 \end{array}\right|$$

$$= [4.401] - [-6.3] = 10.701\ kN.m$$

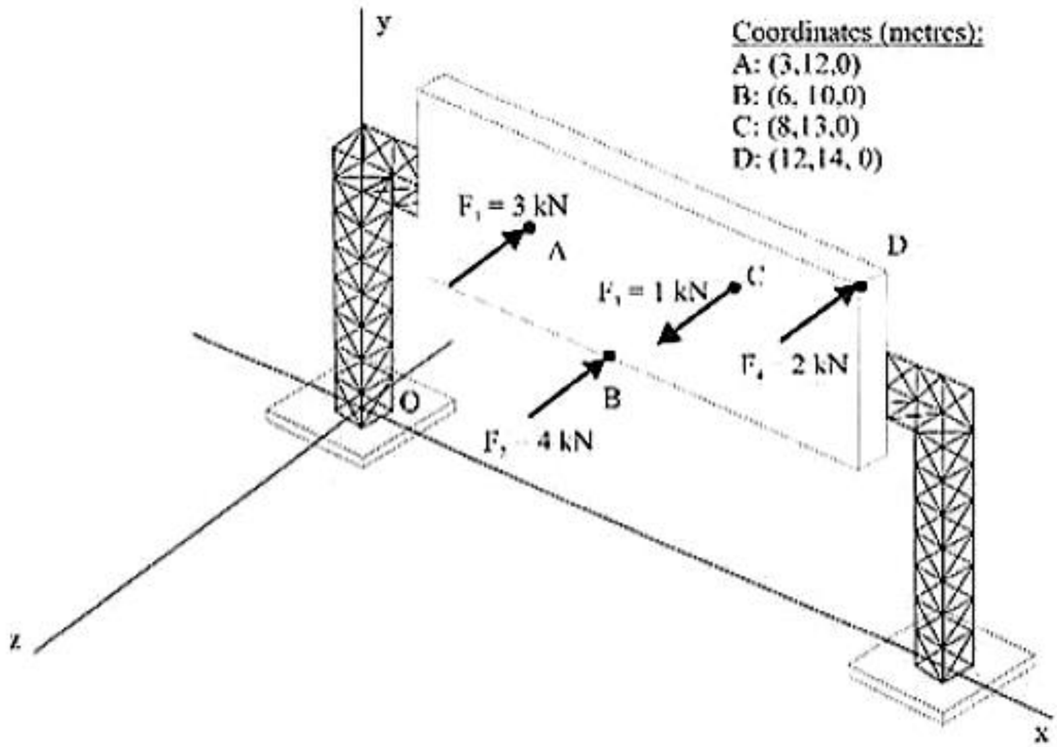


S2-610 Four forces are applied to the highway sign at points A, B, C, and D as shown. (All forces are parallel to the z – axis.) The coordinates of the points with respect to the origin O are also specified.

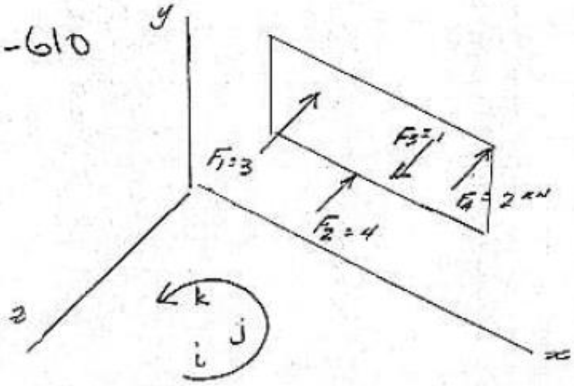
Determine:

(a) the magnitude and direction of the resultant of the resultant of the four forces, and

(b) the point of application of the resultant with respect to the origin O.



S2-610



Force	$\vec{r}$	$\vec{F}$	$\vec{M}_o = \vec{r} \times \vec{F}$
$\vec{F}_1$	$3\hat{i}+12\hat{j}$	$-3\hat{k}$	$9\hat{j}-36\hat{i}$
$\vec{F}_2$	$6\hat{i}+10\hat{j}$	$-4\hat{k}$	$24\hat{j}-40\hat{i}$
$\vec{F}_3$	$8\hat{i}+13\hat{j}$	$+1\hat{k}$	$-8\hat{j}+13\hat{i}$
$\vec{F}_4$	$12\hat{i}+14\hat{j}$	$-2\hat{k}$	$24\hat{j}-28\hat{i}$
$\vec{R}$	$x\hat{i}+y\hat{j}$	$-8\hat{k}$	$\Sigma\ 49\hat{j}-91\hat{i}$

$$(x\hat{i}+y\hat{j})\times(-8\hat{k}) = 49\hat{j}-91\hat{i}$$

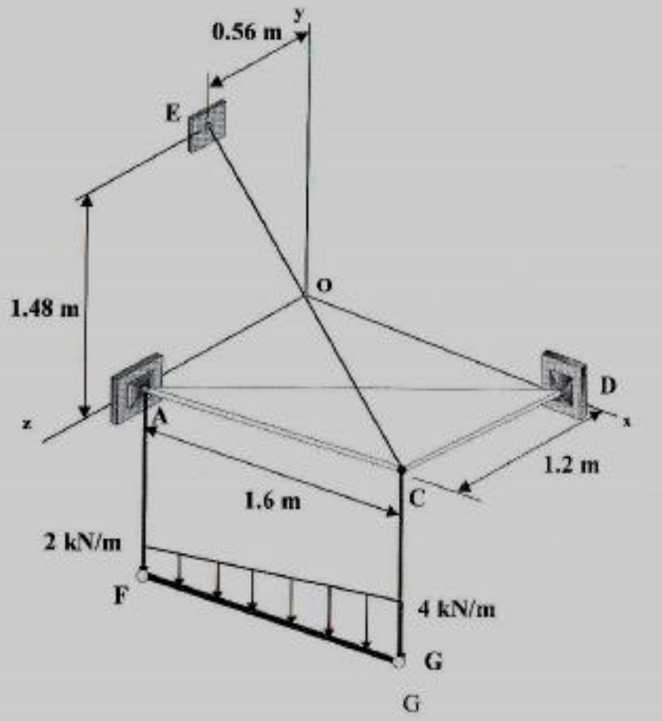
$$8x\hat{j}-8y\hat{i} = 49\hat{j}-91\hat{i}$$

$$\therefore\ 8x = 49 \qquad x = 6.125\,m$$

$$-8y = -91 \qquad y = 11.375\,m$$

S2-611 A triangular plate is supported by ball-and-socket joints at *A* and *D* and by a cable attached to the plate at *C*. The beam, *FG*, is suspended from the plate by cables *AF* and *CG* attached to the plate at A and at C as shown. The beam supports a distributed load that varies from *2 kN/m* to *4 kN/m* as shown in the figure. You may neglect the we ght of the beam. Determine:

a) the tension in the cables supporting the beam, and
b) the tension in the cable *CE*.



$$\vec{\lambda}_{AD} = \frac{\vec{AD}}{AD} \qquad \vec{AD} = 1.6\hat{i} + 0\hat{j} - 1.2\hat{k}$$

$$AD = \sqrt{(1.6)^2 + (-1.2)^2} = 2$$

$$\vec{\lambda}_{AD} = \frac{1.6\hat{i} + 0\hat{j} - 1.2\hat{k}}{2} = 0.8\hat{i} + 0\hat{j} - 0.6\hat{k}$$

$$\vec{r}_{AC} = 1.6\hat{i} + 0\hat{j} + 0\hat{k}$$

$$\vec{T}_{CE} = T_{CE}\vec{\lambda}_{CE} \qquad \vec{\lambda}_{CE} = \frac{\vec{CE}}{CE}$$

$$\vec{CE} = -1.6\hat{i} + 1.48\hat{j} - 0.64\hat{k}$$

$$CE = \sqrt{(-1.6)^2 + (1.48)^2 + (-0.64)^2} = \sqrt{5.16}$$

$$\vec{T}_{CE} = T_{CE}\left(\frac{-1.6\hat{i} + 1.48\hat{j} - 0.64\hat{k}}{\sqrt{5.16}}\right.$$

$$M_{AD} = 0$$

$$\frac{T_{CE}}{\sqrt{5.16}}\begin{vmatrix} 0.8 & 0 & -0.6 \\ 1.6 & 0 & 0 \\ -1.6 & 1.48 & -0.64 \end{vmatrix} + \begin{vmatrix} 0.8 & 0 & -0.6 \\ 1.6 & 0 & 0 \\ 0 & -2.67 & 0 \end{vmatrix}$$

$$\frac{T_{CE}}{\sqrt{5.16}}\left|\begin{array}{ccc|cc} 0.8 & 0 & -0.6 & 0.8 & 0 \\ 1.6 & 0 & 0 & 1.6 & 0 \\ -1.6 & 1.48 & -0.64 & -1.6 & 1.48 \end{array}\right|$$

$$+ \left|\begin{array}{ccc|cc} 0.8 & 0 & -0.6 & 0.8 & 0 \\ 1.6 & 0 & 0 & 1.6 & 0 \\ 0 & -2.67 & 0 & 0 & -2.67 \end{array}\right|$$

$$\frac{T_{CE}}{\sqrt{5.16}}\left\{[-1.4208] - [0]\right\} + \left\{[2.5632] - [0]\right\}$$

$$T_{CE} = \frac{2.5632\sqrt{5.16}}{1.4208} = 4.09\ kN$$