Relative maxima and minima

4.3.1 Note: in (b) it should be $\sqrt{2}$ in place of $\sqrt{5}$.

(a) $f'(x) = 5(x-1)^4$

f' is defined everywhere, $f'(x)=0$ $\quad 5(x-1)^4 = 0$ $\quad x=1$

$x=1$ - critical point

$$f' \quad \begin{array}{ccc} + & & + \\ \hline & 1 & \end{array}$$

$x=1$ IS NOT A RELATIVE EXTREMUM

(b) $f'(x) = \dfrac{2x(x-1)-(x^2+1)\cdot 1}{(x-1)^2} = \dfrac{2x^2-2x-x^2-1}{(x-1)^2} = \dfrac{x^2-2x-1}{(x-1)^2}$

$x^2-2x-1=0$

$x_{1,2} = 1 \pm \sqrt{5}$

$f'(x) = \dfrac{(x-(1-\sqrt{5}))(x-(1+\sqrt{5}))}{(x-1)^2}$

$x = 1+\sqrt{5}$, $x = 1-\sqrt{5}$ - critical pts.

$$f' \quad \begin{array}{ccccccc} + & & - & & - & & + \\ \hline & 1-\sqrt{5} & & 1 & & 1+\sqrt{5} & \end{array}$$

$x = 1-\sqrt{5}$ - relative max. $\qquad x = 1+\sqrt{5}$ - relative min.

(c) $f'(x) = \frac{1}{3}(x+2)^{-\frac{2}{3}} = \dfrac{1}{3\sqrt[3]{(x+2)^2}}$ - NOT DEFINED AT $x=-2$, NEVER ZERO

$x=-2$ - critical point

$$f' \quad \begin{array}{ccc} + & & + \\ \hline & -2 & \end{array}$$

$x=-2$ IS NOT A RELATIVE EXTREMUM