Absolute maxima and minima

4.7.1

$f'(x) = 6x^2 + 6x - 36 \qquad f'(x) = 0$

$x^2 + x - 6 = 0$

$x_1 = 2 \qquad x_2 = -3$

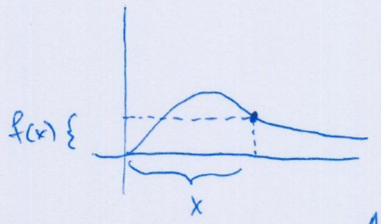
$x = 2$ - critial value inside $[0, 3]$

$f(0) = 0 \leftarrow$ max

$f(2) = 2 \cdot 8 + 3 \cdot 4 - 36 \cdot 2 = -44 \leftarrow$ min

$f(3) = 2 \cdot 27 + 27 - 4 \cdot 27 = -27$

4.7.2



$A(x) = x \cdot f(x) = \frac{x}{1+x^2}, \qquad x \geq 0$

$A'(x) = \frac{(1+x^2) - x \cdot 2x}{(1+x^2)^2} = \frac{1-x^2}{(1+x^2)^2}$

$A' = 0: \quad 1 - x^2 = 0 \quad x_1 = 1, \; x_2 = -1$ - outside $[0, \infty)$

$x = 1$ - critical point

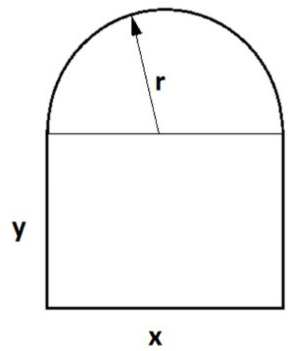
$A(0) = 0$

$A(1) = \frac{1}{2} \leftarrow$ max.

$\lim_{x \to \infty} A(x) = \lim_{x \to \infty} \frac{x}{1+x^2} = \lim_{x \to \infty} \frac{\frac{2}{x}}{1 + \frac{1}{x^2}} = \frac{0}{1+0} = 0$

Answer: $\frac{1}{2}$ cm^2.

4.7.3



$$x + 2y + \pi r = 10$$

$$r = \frac{x}{2} \quad \Rightarrow \quad \left(1 + \frac{\pi}{2}\right)x + 2y = 10 \quad \Rightarrow \quad y = 5 - \left(\frac{2+\pi}{4}\right)x$$

We want to maximize the area of the window.

Area of the window= area of the rectangle + area of the semicircle

$$A = xy + \frac{1}{2}\pi r^2 = 5x - \left(\frac{2+\pi}{4}\right)x^2 + \frac{\pi}{8}x^2$$

$$x \geq 0$$

$$y \geq 0 \quad \Rightarrow \quad x \leq \frac{20}{2+\pi}$$

Therefore,

one must maximize

$$A(x) = 5x - \left(\frac{4+\pi}{8}\right)x^2$$

the interval: $x \in \left[0, \frac{20}{2+\pi}\right]$.

4.7.4

1. Equation of line AB: $2y + 4x = 8$

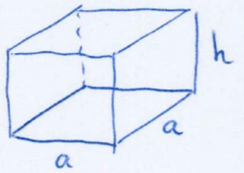
Let $P(x,y)$. Area:

$$A = x \cdot y$$

Find y from $2y+4x=8$: $y = \frac{8-4x}{2} = 4-2x$

$$\underline{A(x)} = x(4-2x) = \underline{2x(2-x)} \qquad \underline{x \in [0,2]}$$

4.7.5



a - side of the base

h - height

volume $V = 1250 = ha^2$

cost $C = 30 \cdot A_{base} + 15 \cdot A_{top} + 20 \cdot A_{sides}$

$$= 30 \cdot a^2 + 15 a^2 + 20 \cdot 4ah$$

$$= 45a^2 + 80\,ah$$

$1250 = ha^2$ $\qquad h = \frac{1250}{a^2}$

To satisfy $ha^2 = 1250$, a may be any positive.

$$\underline{C = 45a^2 + \frac{80 \cdot 1250}{a}} \qquad \underline{\text{interval: } a \in (0, \infty)}$$