MATH1210 Assignment #5

Due: 1:30 pm Friday 1 December 2006 T. G. Berry

NOTES:

(1) ***The assignment is due at the start of our class on Friday 1 December 2006.***

(2) ***Late assignments will NOT be accepted.***

(3) ***If your assignment is not accompanied by a Faculty of Science "Honesty Declaration", it will NOT be graded.***

1. Evaluate each of the following determinants in two distinct ways:

(i) by using a cofactor expansion along some (appropriately chosen) row or column,

(ii) by using elementary row operations, together with the properties of determinants, to reduce it to a form in which the evaluation is simple.

In each case show all your work, and indicate the operations being performed.

(a) $\begin{vmatrix} 1 & 3 & -1 \\ 2 & 0 & 5 \\ 1 & 4 & 3 \end{vmatrix}$

(b) $\begin{vmatrix} 1 & 0 & 2 & 1 \\ 2 & -1 & 1 & 0 \\ 1 & 0 & 0 & 3 \\ -1 & 0 & 2 & 1 \end{vmatrix}$

(c) $\begin{vmatrix} 1 & -1 & 0 & 2 \\ -1 & 1 & 2 & 3 \\ 2 & -2 & 3 & 4 \\ 6 & -6 & 5 & 1 \end{vmatrix}$

(d) $\begin{vmatrix} x & x^2 & x^3 & x^4 \\ 1 & 2x & 3x^2 & 4x^3 \\ 0 & 2 & 6x & 12x^2 \\ 0 & 0 & 6 & 24x \end{vmatrix}$

2. Use Cramer's Rule to determine whether or not each of the following systems of linear equations possesses a unique solution. ***If it does possess a unique solution***, find it using Cramer's Rule. ***If it does not possess an unique solution***, find all solutions (possibly "none"), by reducing the augmented matrix of the system to reduced row-echelon form (RREF).

(a) $\begin{cases} a+3b+c=-2 \\ 2a+5b+c=-5 \\ a+2b+3c=6 \end{cases}$ (b) $\begin{cases} 5x_1+6x_2+4x_3=3 \\ 7x_1+8x_2+6x_3=1 \\ 6x_1+7x_2+5x_3=0 \end{cases}$

(c) $\begin{cases} 3x_1-3x_2+3x_3=0 \\ 2x_1-x_2+4x_3=0 \\ 3x_1-5x_2-x_3=0 \end{cases}$

3. For each of the following sets of vectors, ***determine if the set is linearly dependent or linearly independent,*** and moreover, ***if the set is linearly dependent***:

(i) provide a relation which explicitly displays how one (or more) of these vectors may be written as a linear combination of the others,

(ii) find a subset (of the given set of vectors) in which the vectors are linearly independent.

(a) $\vec{v}_1=\hat{i}+2\hat{j}$, $\vec{v}_2=3\hat{i}+4\hat{j}$,

(b) $\vec{u}_1=\hat{i}+2\hat{j}$, $\vec{u}_2=3\hat{i}+4\hat{j}$, $\vec{u}_3=5\hat{i}+6\hat{j}$,

(c) $\vec{v}_1=\hat{i}+2\hat{j}+3\hat{k}$, $\vec{v}_2=4\hat{i}+5\hat{j}+6\hat{k}$, $\vec{v}_3=7\hat{i}+8\hat{j}$,

(d) $\vec{w}_1=\hat{j}+\hat{k}$, $\vec{w}_1=\hat{j}+\hat{k}$, $\vec{w}_3=\hat{i}+\hat{j}+\hat{k}$,

(e) $\vec{r}_1=\hat{i}+\hat{j}+\hat{k}$, $\vec{r}_2=2\hat{i}+2\hat{j}+2\hat{k}$, $\vec{r}_3=5\hat{k}$, $\vec{r}_4=\hat{i}+2\hat{j}+3\hat{k}$,

(f) $\vec{a}=(2,0,1)$, $\vec{b}=(1,1,-1)$, $\vec{c}=(0,0,0)$, $\vec{d}=(5,3,2)$

(g) $\vec{v}_1=\begin{pmatrix}2\\1\\3\\1\end{pmatrix}$, $\vec{v}_2=\begin{pmatrix}6\\3\\9\\3\end{pmatrix}$, $\vec{v}_3=\begin{pmatrix}1\\0\\0\\1\end{pmatrix}$, $\vec{v}_4=\begin{pmatrix}5\\2\\6\\3\end{pmatrix}$

4. Determine all values of λ for which the homogeneous linear system of equations

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\begin{aligned}
(5-\lambda)x_1 + 4x_2 + 2x_3 &= 0 \\
4x_1 + (5-\lambda)x_2 + 2x_3 &= 0 \\
2x_1 + 2x_2 + (2-\lambda)x_3 &= 0
\end{aligned}
$$

possesses ***non-trivial solutions***. In addition, for each admissible value of λ, find all non-trivial solutions.

Comment: The values λ determined in the above are known as ***eigenvalues*** of the matrix $\begin{pmatrix} 5 & 4 & 2 \\ 4 & 5 & 2 \\ 2 & 2 & 2 \end{pmatrix}$. For each eignevalue λ, the corresponding vector(s) $x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$ are known as ***eigenvector(s)*** of $\begin{pmatrix} 5 & 4 & 2 \\ 4 & 5 & 2 \\ 2 & 2 & 2 \end{pmatrix}$ corresponding to eigenvalue λ.