

[6] 1. Evaluate the limit (or explain why it does not exist).

(a) $\lim_{t \rightarrow 1} \frac{t-1}{\sqrt{t^2+t}-\sqrt{2t}}$

[6]

(c) $\lim_{x \rightarrow \infty} \sqrt{x^2+1000} - x$

[6]

(b) $\lim_{x \rightarrow 2^-} \frac{|x-2|}{x^2-2x}$

2. Find the derivative of the function. Do not simplify your answer.

[6] (a) $f(x) = \frac{(3x+1)^{1/3}}{1+\sqrt{2x-1}}$

[6]

(b) $y = \left(\pi + \frac{1}{x}\right)^{10} (x^2 - 10)^{50}$

[10] 3. Is the function $f(x) = |x-2| + x$ continuous at $x=2$? Is it differentiable at $x=2$? Justify your answer using the definition of continuity and the right and left derivatives.[10] 4. Find the equation of the tangent line to the curve $2x^3 - x^2y^2 + 3y - 4 = 0$ at the point $(1,2)$. What is the equation of the normal line to the curve at that point? (Recall that the normal line to a curve is the line perpendicular to the tangent line.)[10] 5. A ball is thrown upward from ground level with an initial speed of 9.8 m/s so that its height in metres after t seconds is given by $y = 9.8t - 4.9t^2$.

(a) What is the acceleration of the ball at any time?

(b) How high can the ball go?

(c) How fast is it moving when it strikes the ground?

[3] 6. [Bonus question]. Suppose that $f(x)$ is a function that satisfies the equation

$$f(x+y) = f(x) + f(y) + x^2y + xy^2$$

for all real numbers x and y . Suppose also that $\lim_{x \rightarrow 0} \frac{f(x)}{x} = 1$.(a) Find $f(0)$. (Hint: Let $x=y=0$ in the equation.)(b) Find $f'(0)$. (Hint: Use the definition of derivative.)(c) Find $f'(x)$.

[15] 1. Evaluate the limit or explain why it does not exist.

[5] (a) $\lim_{x \rightarrow 3} \frac{|x^2-9|}{x-3}$

[5] (b) $\lim_{x \rightarrow \pi/4} \frac{\tan x - 1}{\cos x - \sin x}$

[5] (c) $\lim_{x \rightarrow \infty} (\sqrt{x^2+x-1} - x)$

[8] 2. Let $f(x) = \frac{1}{x^2+1}$. Using only the definition of the derivative find $f'(1)$.

[8] 3. Find the following derivatives (DO NOT simplify your answer):

[4] (a) $f''(x)$ if $f(x) = \frac{x}{2x+1}$

[4] (b) $g'(t)$ if $g(t) = \frac{t^2}{(1+\sqrt{t})^5}$

[9] 4. At what point(s) on the curve $y = x + \frac{1}{x}$ does the tangent line pass through the point with coordinates $(0,1)$?[10] 5. Find all points (if any) on the curve $x^2 + xy + y^2 = 3$ at which the tangent line has slope 1.[7] Bonus. Let $f(x) = x^{1/3}$. Using only the definition of the derivative find $f'(x)$. (Note that no partial credit will be given if you do not use the definition.)