MATH 1210 Assignment 2 Fall 2013 **Due date: October 18 in class**

1. **Let $P(x) = x^4 + x^3 + x^2 + x + 1$; find all values of a for which $a^2P(1) + 5aP(i) = 10P(0)$.**

2. **Find the remainder when the polynomial $P(x) = -2x^5 - 2ix^4 - ix^3 + x^2 + 5$ is divided by $(1+i)x - 1 + i$.**

3. **For each of the following, if it is true prove it and if it is not true give a counter example.**
   - **(a)** **If r_1 is a zero of polynomial $P_1(x)$ and r_2 is a zero of polynomial $P_2(x)$, then $r_1 + r_2$ is a zero of polynomial $P_1(x) + P_2(x)$.**
   - **(b)** **If r is a zero of polynomial $P(x)$, then $2r$ is a zero of polynomial $P(2x)$.**
   - **(c)** **If r is a zero of polynomial $P(x)$ of multiplicity k, then r is a zero of polynomial $(P(x))^n$ of multiplicity nk where $n \geq 1$ is an integer.**

4. **Consider the polynomial $P(x) = 7x^6 - 33x^5 + 53x^4 - 3x^3 - 62x^2 + 54x + 20$.**
   - **(a)** **Use Rational Root Theorem to find all possible rational zeros of $P(x)$.**
   - **(b)** **Use Descartes' Rules of Signs to determine how many positive or negative real zeros $P(x)$ may have.**
   - **(c)** **Use Bounds Theorem to determine how large the absolute value of a root of $P(x)$ may be.**
   - **(d)** **Use your answers in part (c) to improve your list of all possible rational zeros of $P(x)$ in part (a).**
   - **(e)** **If $2+i$ is a complex root of $P(x)$, find an irreducible real quadratic factor of $P(x)$.**

5. **Find all the roots of the polynomial $P(x) = 2x^4 - 3x^3 - 7x^2 - 5x - 3$.**

6. **Given that $2i$ is a root of $P(x) = x^6 - x^5 + x^4 - 16x^2 + 16x - 16$, find all roots of $P(x)$ and write $P(x)$ as a product of real linear and irreducible real quadratic factors.**

7. **Let $P(x) = -2x^{17} - x^{15} + x^2 - 20$; prove each of the following statements or explain why it is not correct.**
   - **(a)** **$P(x)$ has at least one zero in the interval $[11, 20]$.**
   - **(b)** **It is impossible for $P(x)$ to have 2 negative real zeros.**

8. **Let $A = \begin{bmatrix} 1 & 2 & -3 \\ 0 & 4 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 4 & 0 \\ 2 & -1 \end{bmatrix}$, $C = \begin{bmatrix} 1 & 5 \\ -1 & 1 \\ 2 & 6 \end{bmatrix}$ and $D = \begin{bmatrix} 2 & 3 \\ -1 & 1 \end{bmatrix}$. Evaluate each of the following expressions or explain why it is not defined.**
   - **(a)** $(3B + 2D)(6C - A^T)$
   - **(b)** $BD^T + AC$

9. **Let $A = \begin{bmatrix} 1 & a \\ -a & 2 \end{bmatrix}$; find all values of a for which $A^2 - 3A = 3aI_2$ where I_2 is the 2×2 identity matrix.**