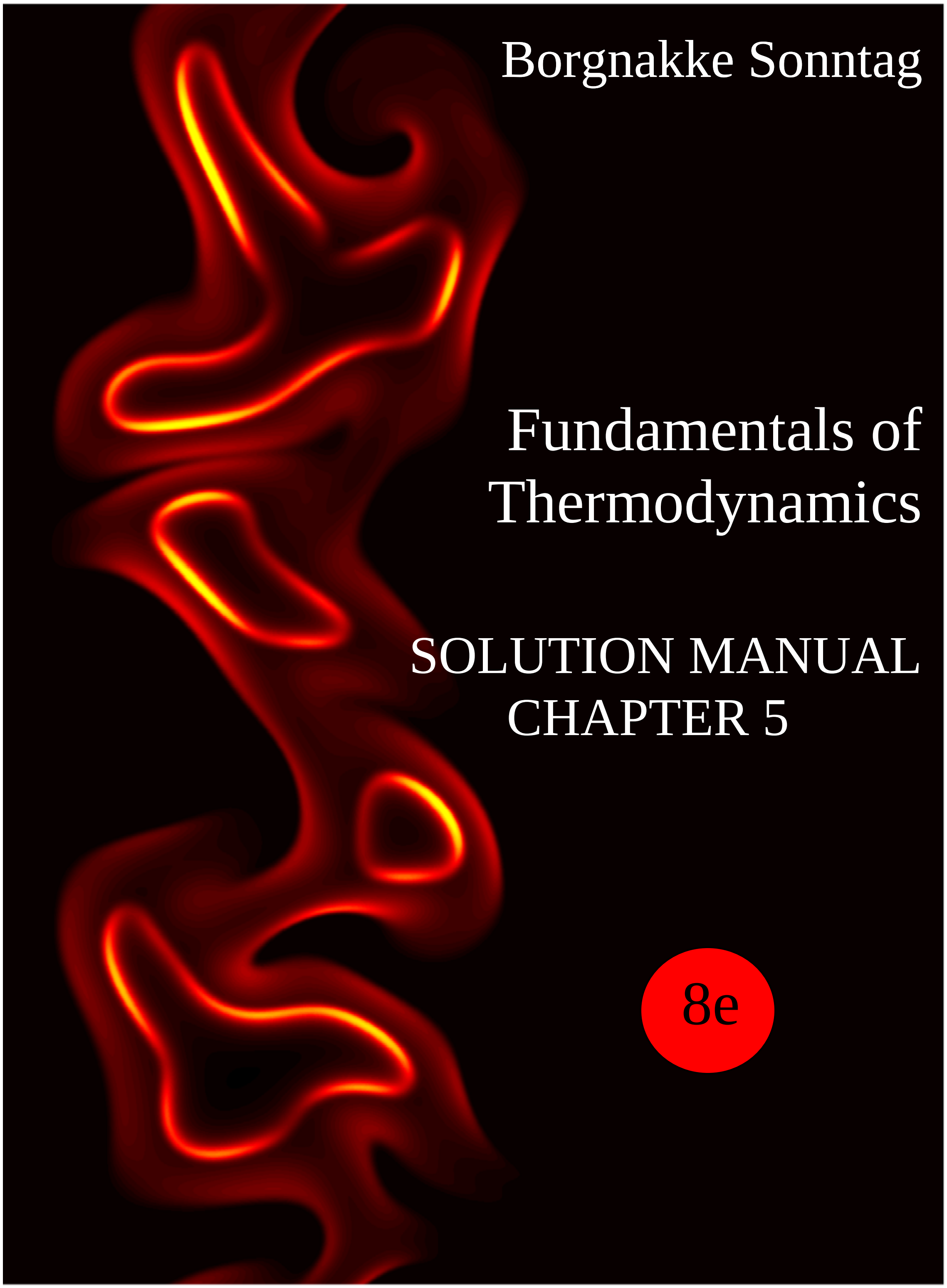

Updated June 2013

CONTENT CHAPTER 5

The clip-art used in the solution manual is from **Microsoft**.

In-Text Concept Questions

5.a

Electrical appliances (TV, stereo) use electric power as input. What happens to the power? Are those heat engines? What does the second law say about those devices?

Most electric appliances such as TV, VCR, stereo and clocks dissipate power in electrical circuits into internal energy (they get warm) some power goes into light and some power into mechanical energy. The light is absorbed by the room walls, furniture etc. and the mechanical energy is dissipated by friction so all the power eventually ends up as internal energy in the room mass of air and other substances.

These are not heat engines, just the opposite happens, namely electrical power is turned into internal energy and redistributed by heat transfer. These are irreversible processes.

5.b

Geothermal underground hot water or steam can be used to generate electric power. Does that violate the second law?

No.
Since the earth is not uniform we consider the hot water or steam supply as coming from one energy source (the high T) and we must reject heat to a low temperature reservoir as the ocean, a lake or the atmosphere which is another energy reservoir.

Iceland uses a significant amount of steam to heat buildings and to generate electricity. Microsoft clipart.

5.c

A windmill produces power on a shaft taking kinetic energy out of the wind. Is it a heat engine? Is it a perpetual machine? Explain.

Since the wind is generated by a complex system driven by solar heat input and radiation out to space it is a kind of heat engine.

Microsoft clipart.

Within our lifetime it looks like it is perpetual. However with a different time scale the climate will change, the sun will grow to engulf the earth as it burns out of fuel. There is a storage effect and a non-uniform distribution of states in the system that drives this.

5.d

heat engines and heat pumps (refrigerators) are energy conversion devices altering amounts of energy transfer between Q and W. Which conversion direction (Q $\rightarrow$ W or W $\rightarrow$ Q) is limited and which is unlimited according to the second law.

The work output of a heat engine is limited (Q to W).
You can transform W to Q unlimited (a heat pump that does not work well or you may think about heat generated by friction).

5.e

Ice cubes in a glass of liquid water will eventually melt and all the water approach room temperature. Is this a reversible process? Why?

There is heat transfer from the warmer ambient to the water as long as there is a temperature difference. Eventually the temperatures approach each other and there is no more heat transfer. This is irreversible, as we cannot make ice-cubes out of the water unless we run a refrigerator and that requires a work from the surroundings, which does not leave the surroundings unchanged.


Microsoft clipart.

5.f

Does a process become more or less reversible with respect to heat transfer if it is fast rather than slow? *Hint*: Recall from Chapter 3 that $\dot{Q} = CA\ \Delta T$.

If the higher heat transfer rate is caused by a larger ΔT then the process is more irreversible so as the process would be slower due to a lower ΔT then it approaches a reversible process. If the rate of heat transfer is altered due to the factor CA with the same ΔT then it is irreversible to the same degree.

5.g

If you generated hydrogen from, say, solar power, which of these would be more efficient: (1) transport it and then burn it in an engine or (2) convert the solar power to electricity and transport that? What else would you need to know in order to give a definite answer?

Case (1):
First there is a certain efficiency when converting solar power to hydrogen. Then the transport and packaging of hydrogen has some energy expenditures associated with it. The hydrogen could be compressed to a high pressure (typically 70 MPa) which is expensive in terms of work input and then stored in a tank. One alternative would be to cool it down to become a liquid to have a much smaller volume but the temperature at which this happens is very low so the cooling and continued cooling under transport requires a significant work input also. Certain materials like metal-hydrides, boron salt slurries and nano-carbon fibers allows storage of hydrogen at more modest pressures and are all currently under investigation as other alternative storage methods. After the hydrogen is transported to an engine then the engine efficiency determines the work output.

Case (2):
If the solar power is located where there is access to electrical transmission lines then it can be used in solar panels, solar heating of water or other substance to run a heat engine cycle like a power plant to generate electricity. All of these processes have a certain efficiency that must be evaluated to estimate the overall efficiency. To make new transmission lines is costly and has an impact on the environment that must be considered.

You also need to look at the time of day/year at which the power is required and when it is available. The end use also presents some limitations like if the power should be used for a car then the energy must be stored temporarily like in a battery.

Concept Problems

5.1

Two heat engines operate between the same two energy reservoirs and both receives the same Q_H. One engine is reversible and the other is not. What can you say about the two Q_L's?

The reversible heat engine can produce more work (has a higher efficiency) than the irreversible heat engine and due to the energy conservation it then gives out a smaller Q_L compared to the irreversible heat engine.

$$W_{rev} = Q_H - Q_{L\ rev} \quad > \quad W_{irrev} = Q_H - Q_{L\ irrev}$$

$$\Rightarrow \quad Q_{L\ rev} < Q_{L\ irrev}$$

5.2

Compare two domestic heat pumps (A and B) running with the same work input. If A is better than B which one heats the house most?

The statement that A is better means it has a higher COP and since

$$Q_{H\,A} = COP_A\,W \quad > \quad Q_{H\,B} = COP_B\,W$$

it can thus provide more heat to the house. The higher heat comes from the higher Q_L it is able to draw in.

5.3

Suppose we forget the model for heat transfer as $\dot{Q} = CA\ \Delta T$, can we draw some information about direction of Q from the second law?

One of the classical statements of the second law is the Clausius statement saying that you cannot have heat transfer from a lower temperature domain to a higher temperature domain without work input.

The opposite, namely a transfer of heat from a high temperature domain towards a lower temperature domain can happen (which is a heat engine with zero efficiency). That is the only direction the heat transfer can have namely from the high T towards the low T environment.

5.4

A combination of two heat engines is shown in Fig. P5.4. Find the overall thermal efficiency as a function of the two individual efficiencies.

The overall efficiency

$$\eta_{TH} = \dot{W}_{net} / \dot{Q}_H = (\dot{W}_1 + \dot{W}_2) / \dot{Q}_H = \eta_1 + \dot{W}_2 / \dot{Q}_H$$

For the second heat engine and the energy Eq. for the first heat engine

$$\dot{W}_2 = \eta_2 \dot{Q}_M = \eta_2 (1 - \eta_1) \dot{Q}_H$$

so the final result is

$$\eta_{TH} = \eta_1 + \eta_2 (1 - \eta_1)$$

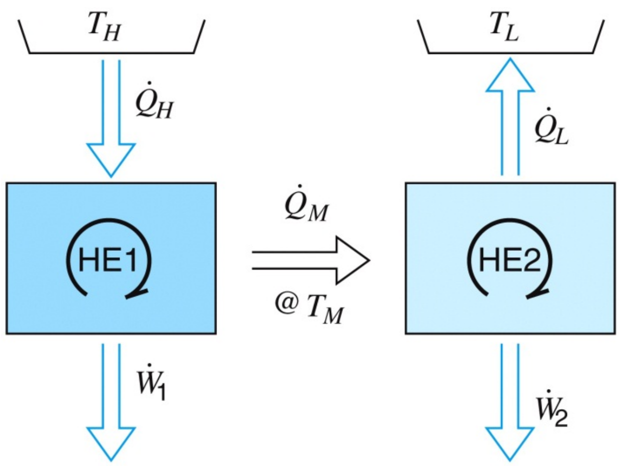

Figure P5.4

5.5

Compare two heat engines receiving the same Q, one at 1200 K and the other at 1800 K; they both reject heat at 500 K. Which one is better?

The maximum efficiency for the engines are given by the Carnot heat engine efficiency as

$$\eta_{TH} = \dot{W}_{net} / \dot{Q}_H = 1 - \frac{T_L}{T_H}$$

Since they have the same low temperature the one with the highest T_H will have a higher efficiency and thus presumably better.

5.6

A car engine takes atmospheric air in at 20^{o}C, no fuel, and exhausts the air at –20^{o}C producing work in the process. What do the first and the second laws say about that?

Energy Eq.: $W = Q_H - Q_L$ = change in energy of air. **OK**

2nd law: Exchange energy with only one reservoir. **NOT OK**.
This is a violation of the statement of Kelvin-Planck.

Remark: You cannot create and maintain your own energy reservoir.

5.7

A combination of two refrigerator cycles is shown in Fig. P5.7. Find the overall COP as a function of COP_1 and COP_2.

The overall COP becomes

$$COP = \beta = \frac{\dot{Q}_L}{\dot{W}_{tot}} = \frac{\dot{Q}_L}{\dot{W}_1}\frac{\dot{W}_1}{\dot{W}_{tot}} = COP_1 \frac{\dot{W}_1}{\dot{W}_{tot}} = COP_1 \frac{1}{1 + \dot{W}_2/\dot{W}_1}$$

where we used $\dot{W}_{tot} = \dot{W}_1 + \dot{W}_2$. Use definition of COP_2 and energy equation for refrigerator 1 to eliminate $\dot{Q}_M$ as

$$COP_2 = \dot{Q}_M / \dot{W}_2 \qquad \text{and} \qquad \dot{Q}_M = \dot{W}_1 + \dot{Q}_L$$

so we have

$$\dot{W}_2 = \dot{Q}_M / COP_2 = (\dot{W}_1 + \dot{Q}_L) / COP_2$$

and then

$$\dot{W}_2 / \dot{W}_1 = (1 + \dot{Q}_L/\dot{W}_1) / COP_2 = (1 + COP_1) / COP_2$$

Finally substitute into the first equation and rearrange a little to get

$$COP = \beta = \frac{COP_1\, COP_2}{COP_1 + COP_2 + 1}$$

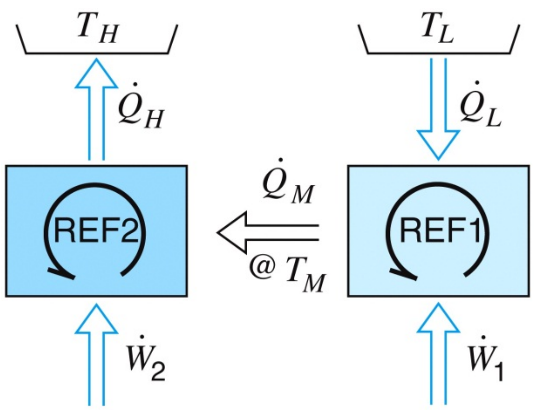

Figure P5.7

5.8

After you have returned from a car trip the car engine has cooled down and is thus back to the state in which it started. What happened to all the energy released in the burning of the gasoline? What happened to all the work the engine gave out?

Solution:
All the energy from the fuel generates heat and work out of the engine. The heat is directly dissipated in the atmosphere and the work is turned into kinetic energy and internal energy by all the frictional forces (wind resistance, rolling resistance, brake action). Eventually the kinetic energy is lost by braking the car so in the end all the energy is absorbed by the environment increasing its internal energy.


Microsoft clipart.

5.9

Does a reversible heat engine burning coal (which, in practice, cannot be done reversibly) have impacts on our world other than depletion of the coal reserve?

Solution:

When you burn coal you form carbon dioxide CO_2 which is a greenhouse gas. It absorbs energy over a wide spectrum of wavelengths and thus traps energy in the atmosphere that otherwise would go out into space.

Coal from various locations also has sulfur and other substances like heavy metals in it. The sulfur generates sulfuric acid (resulting in acid rain) in the atmosphere and can damage the forests.

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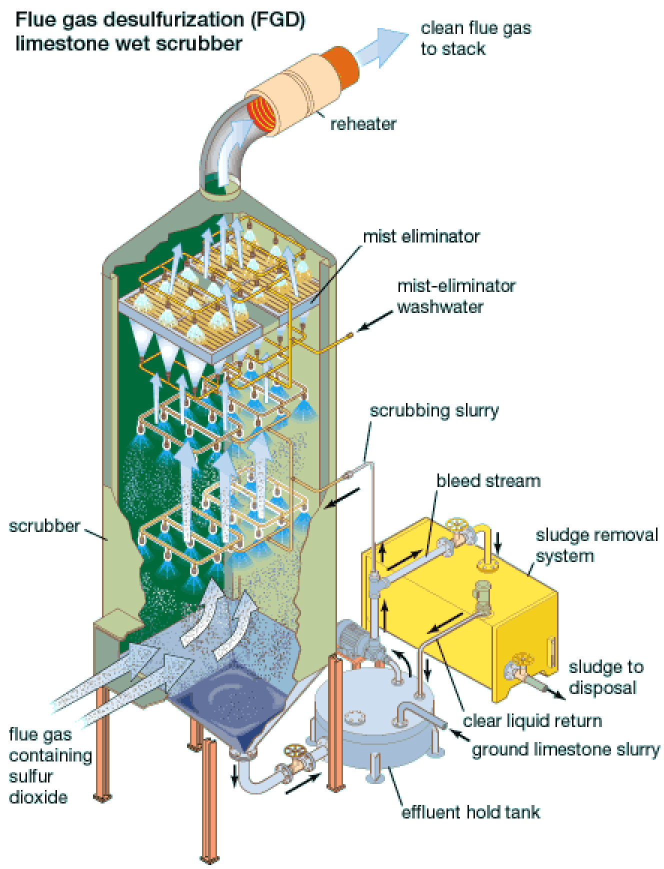
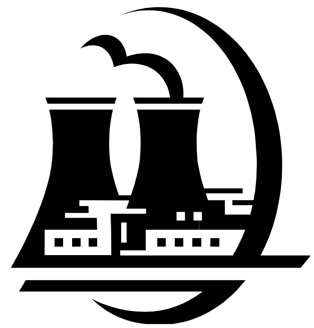


5.10

If the efficiency of a power plant goes up as the low temperature drops, why do power plants not just reject energy at say $-40^{o}C$?

In order to reject heat the ambient must be at the low temperature. Only if we moved the plant to the North Pole would we see such a low T.

Remark: You cannot create and maintain your own energy reservoir.



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5.11

If the efficiency of a power plant goes up as the low temperature drops why not let the heat rejection go to a refrigerator at, say, $-10^{o}C$ instead of ambient $20^{o}C$?

The refrigerator must pump the heat up to $20^{o}C$ to reject it to the ambient. The refrigerator must then have a work input that will exactly offset the increased work output of the power plant, if they are both ideal. As we can not build ideal devices the actual refrigerator will require more work than the power plant will produce extra.

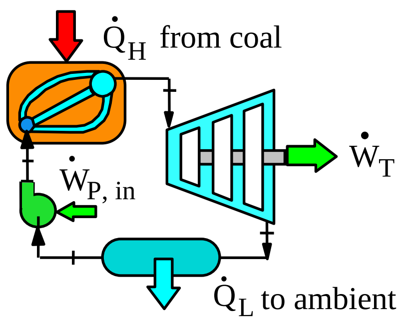
5.12

A coal-fired power plant operates with a high T of 600°C whereas a jet engine has about 1400 K. Does that mean we should replace all power plants with jet engines?

The thermal efficiency is limited by the Carnot heat engine efficiency.

That is, the low temperature is also important. Here the power plant has a much lower T in the condenser than the jet engine has in the exhaust flow so the jet engine does not necessarily have a higher efficiency than the power plant.

Gas-turbines are used in power plants where they can cover peak power demands needed for shorter time periods and their high temperature exhaust can be used to boil additional water for the steam cycle.



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5.13

A heat transfer requires a temperature difference, see chapter 3, to push the $\dot{Q}$. What implications does that have for a real heat engine? A refrigerator?

This means that there are temperature differences between the source of energy and the working substance so T_H is smaller than the source temperature. This lowers the maximum possible efficiency. As heat is rejected the working substance must have a higher temperature T_L than the ambient receiving the $\dot{Q}_L$, which lowers the efficiency further.

For a refrigerator the high temperature must be higher than the ambient to which the $\dot{Q}_H$ is moved. Likewise the low temperature must be lower than the cold space temperature in order to have heat transfer from the cold space to the cycle substance. So the net effect is the cycle temperature difference is larger than the reservoir temperature difference and thus the COP is lower than that estimated from the cold space and ambient temperatures.

Both of these situations and statements are illustrated in Fig.5.27.

5.14

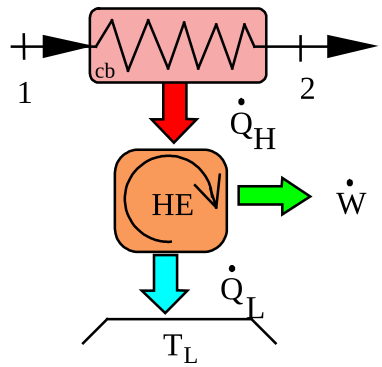
Hot combustion gases (air) at 1500 K are used as the heat source in a heat engine where the gas is cooled to 750 K and the ambient is at 300 K. This is not a constant T source. How does that affect the efficiency?

Solution:

If the efficiency is written as

$$\eta_{TH} = \dot{W}_{net} / \dot{Q}_H = 1 - \frac{T_L}{T_H}$$

then T_H is somewhere between 1500 K and 750 K and it is not a linear average.



After studying chapter 6 and 7 we can solve this problem and find the proper average high temperature based on properties at states 1 and 2.

Heat Engines and Refrigerators

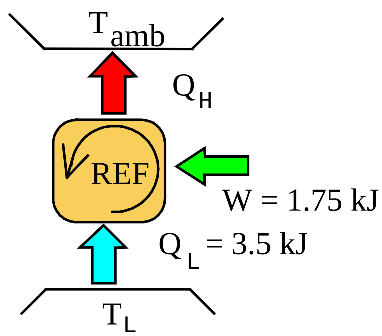
5.15

A window mounted air conditioner removes 3.5 kJ from the inside of a home using 1.75 kJ work input. How much energy is released outside and what is its coefficient of performance?

C.V. A/C unit. The energy Q_H goes into the outside air.

Energy Eq.: $Q_H = W + Q_L = 1.75 + 3.5 = \mathbf{5.25\ kJ}$

COP: $\beta = \dfrac{Q_L}{W} = 3.5 / 1.75 = \mathbf{2}$



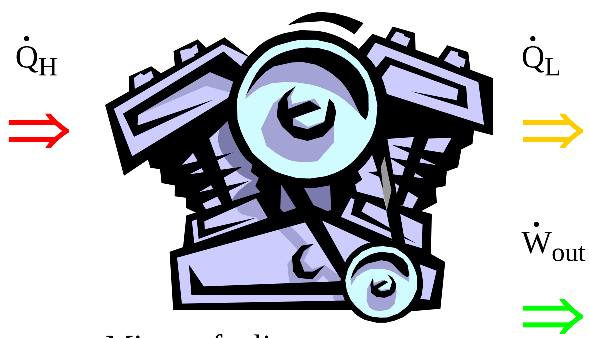
5.16

A lawnmower tractor engine produces 18 hp using 40 kW of heat transfer from burning fuel. Find the thermal efficiency and the rate of heat transfer rejected to the ambient?

Conversion Table A.1: $18 \text{ hp} = 18 \times 0.7457 \text{ kW} = 13.423 \text{ kW}$

Efficiency: $\eta_{TH} = \dot{W}_{out}/\dot{Q}_H = \frac{13.423}{40} = \mathbf{0.33}$

Energy equation: $\dot{Q}_L = \dot{Q}_H - \dot{W}_{out} = 40 – 13.423 = \mathbf{26.6\ kW}$



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5.17

Calculate the thermal efficiency of the steam power plant cycle described in Example 4.9.

Solution:

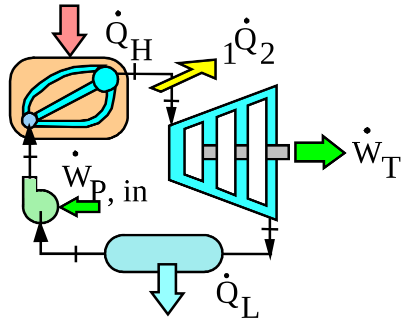
From solution to Example 4.9,

$w_{net} = w_t + w_p = 640.7 - 4$

$= 636.7$ kJ/kg

$q_H = q_b = 2831$ kJ/kg

$\eta_{TH} = w_{net}/q_H = \frac{636.7}{2831} = \mathbf{0.225}$



Notice we cannot write $w_{net} = q_H - q_L$ as there is an extra heat transfer $_1\dot{Q}_2$ as a loss in the line. This needs to be accounted for in the overall energy equation.

5.18

Assume we have a refrigerator operating at steady state using 500 W of electric power with a COP of 2.5. What is the net effect on the kitchen air?

Take a C.V. around the whole kitchen. The only energy term that crosses the control surface is the work input $\dot{W}$ apart from energy exchanged with the kitchen surroundings. That is the kitchen is being heated with a rate of $\dot{W}$.

Remark: The two heat transfer rates are both internal to the kitchen. $\dot{Q}_H$ goes into the kitchen air and $\dot{Q}_L$ actually leaks from the kitchen into the refrigerated space, which is the reason we need to drive it out again.

5.19

A room is heated with a 1500 W electric heater. How much power can be saved if a heat pump with a COP of 2.5 is used instead?

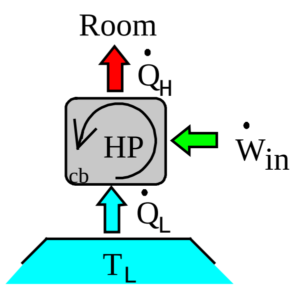
Assume the heat pump has to deliver 1500 W as the $\dot{Q}_H$.

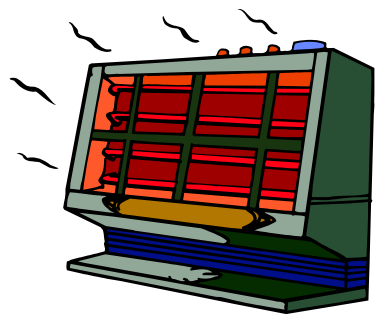
Heat pump: $\beta' = \dot{Q}_H / \dot{W}_{IN}$

$$\dot{W}_{IN} = \dot{Q}_H / \beta' = \frac{1500}{2.5} = 600 \text{ W}$$

So the heat pump requires an input of 600 W thus saving the difference

$$\dot{W}_{saved} = 1500 \text{ W} - 600 \text{ W} = \mathbf{900\ W}$$





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5.20

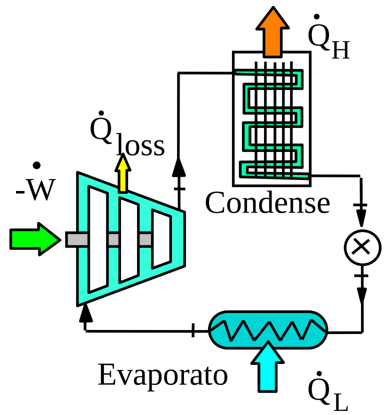
Calculate the coefficient of performance of the R-134a refrigerator given in Example 4.8.

Solution:

From the definition

$$\beta = \dot{Q}_L/\dot{W}_{IN} = \frac{14.54}{5} = 2.91$$

Notice we cannot write $\dot{W}_{IN} = \dot{Q}_H - \dot{Q}_L$ as there is a small $\dot{Q}$ in the compressor. This needs to be accounted for in the overall energy equation.



5.21

Calculate the thermal efficiency of the steam power plant cycle described in Problem 4.118.

From solution to Problem 4.118,

Turbine $A_5 = (\pi/4)(0.2)^2 = 0.03142 \text{ m}^2$

$$\mathbf{V}_5 = \dot{m}v_5/A_5 = 25 \text{ kg/s} \times 0.06163 \text{ m}^3/\text{kg} / 0.03142 \text{ m}^2 = 49 \text{ m/s}$$

$$h_6 = 191.83 + 0.92 \times 2392.8 = 2393.2 \text{ kJ/kg}$$

$$w_T = 3404 - 2393.2 - (200^2 - 49^2)/(2 \times 1000) = 992 \text{ kJ/kg}$$

$$\dot{W}_T = \dot{m}w_T = 25 \times 992 = 24\ 800 \text{ kW}$$

$$\dot{W}_{NET} = 24800 - 300 = 24\ 500 \text{ kW}$$

From the solution to Problem 4.120

Economizer $A_7 = \pi D_7^2/4 = 0.004\ 418 \text{ m}^2$, $v_7 = 0.001\ 008 \text{ m}^3/\text{kg}$

$$\mathbf{V}_2 = \mathbf{V}_7 = \dot{m}v/A_7 = 25 \times 0.001008 / 0.004418 = 5.7 \text{ m/s},$$

$$\mathbf{V}_3 = (v_3/v_2)\mathbf{V}_2 = (0.001\ 118 / 0.001\ 008)\ 5.7 = 6.3 \text{ m/s} \approx \mathbf{V}_2$$

so kinetic energy change is unimportant

$$q_{ECON} = h_3 - h_2 = 744 - 194 = 550.0 \text{ kJ/kg}$$

$$\dot{Q}_{ECON} = \dot{m}q_{ECON} = 25\ (550.0) = 13\ 750 \text{ kW}$$

Generator $A_4 = \pi D_4^2/4 = 0.031\ 42 \text{ m}^2$, $v_4 = 0.060\ 23 \text{ m}^3/\text{kg}$

$$\mathbf{V}_4 = \dot{m}v_4/A_4 = 25 \times 0.060\ 23/0.031\ 42 = 47.9 \text{ m/s}$$

$$q_{GEN} = 3426 - 744 + (47.9^2 - 6.3^2)/(2\times1000) = 2683 \text{ kJ/kg}$$

$$\dot{Q}_{GEN} = 25 \times (2683) = 67\ 075 \text{ kW}$$

The total added heat transfer is

$$\dot{Q}_H = 13\ 758 + 67\ 075 = 80\ 833 \text{ kW}$$

$$\Rightarrow\ \eta_{TH} = \dot{W}_{NET}/\dot{Q}_H = \frac{24500}{80833} = \mathbf{0.303}$$

5.22

A large coal fired power plant has an efficiency of 45% and produces net 1,500 MW of electricity. Coal releases 25 000 kJ/kg as it burns so how much coal is used per hour?

From the definition of the thermal efficiency and the energy release by the combustion called heating value HV we get

$$\dot{W} = \eta\ \dot{Q}_H = \eta \cdot \dot{m} \cdot HV$$

then

$$\dot{m} = \frac{\dot{W}}{\eta \times HV} = \frac{1500\ \text{MW}}{0.45 \times 25000\ \text{kJ/kg}} = \frac{1500 \times 1000\ \text{kJ/s}}{0.45 \times 25000\ \text{kJ/kg}}$$

$$= 133.33\ \text{kg/s} = \mathbf{480\ 000\ kg/h}$$

5.23

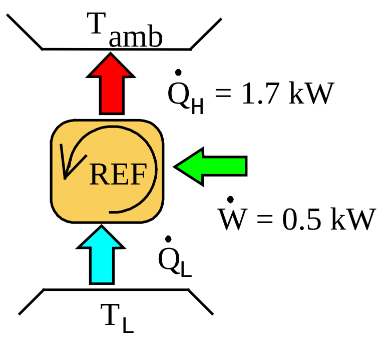
A window air-conditioner discards 1.7 kW to the ambient with a power input of 500 W. Find the rate of cooling and the coefficient of performance.

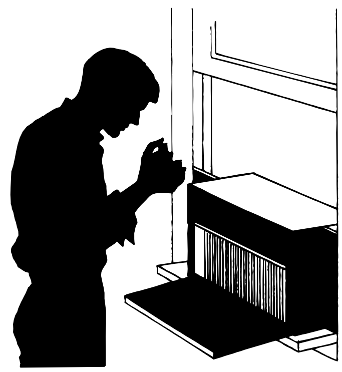
Solution:

In this case $\dot{Q}_H$ = 1.7 kW goes to the ambient so

Energy Eq. : $\dot{Q}_L = \dot{Q}_H - \dot{W} = 1.7 - 0.5 = \mathbf{1.2\ kW}$

$$\beta_{REFRIG} = \frac{\dot{Q}_L}{\dot{W}} = \frac{1.2}{0.5} = \mathbf{2.4}$$





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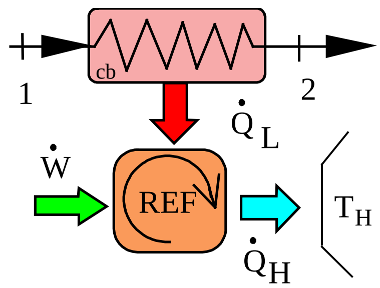
5.24

An industrial machine is being cooled by 0.4 kg/s water at 15^{o}C that is chilled from 35^{o}C by a refrigeration unit with a COP of 3. Find the rate of cooling required and the power input to the unit.



Energy equation for heat exchanger

$$\dot{Q}_L = \dot{m}(h_1 - h_2) = \dot{m}\, C_P\, (T_1 - T_2)$$
$$= 0.4 \text{ kg/s} \times 4.18 \text{ kJ/kg-K} \times (35 - 15) \text{ K}$$
$$= 33.44 \text{ kW}$$

$$\beta = \text{COP} = \dot{Q}_L / \dot{W} \quad \Rightarrow$$

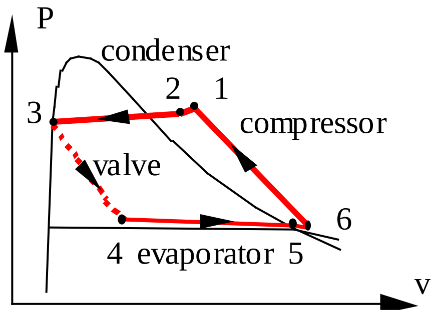
$$\dot{W} = \dot{Q}_L / \beta = 33.44 / 3 = \mathbf{11.15\ kW}$$

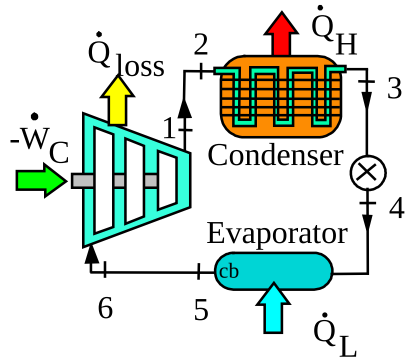
Comment: An outside cooling tower is often used for this, see Chapter 11.

5.25

Calculate the COP of the R-410A heat pump cycle described in Problem 4.123.

The cycle is given by the following states:





Where

$$\dot{Q}_H = \dot{m}\ (h_2 - h_3) = 0.05\ \text{kg/s}\ (367 - 134)\ \text{kJ/k} = 11.65\ \text{kW}$$

The COP is

$$\beta' = \text{COP} = \dot{Q}_H / \dot{W}_{IN} = \frac{11.65\ \text{kW}}{5\ \text{kW}} = \mathbf{2.33}$$

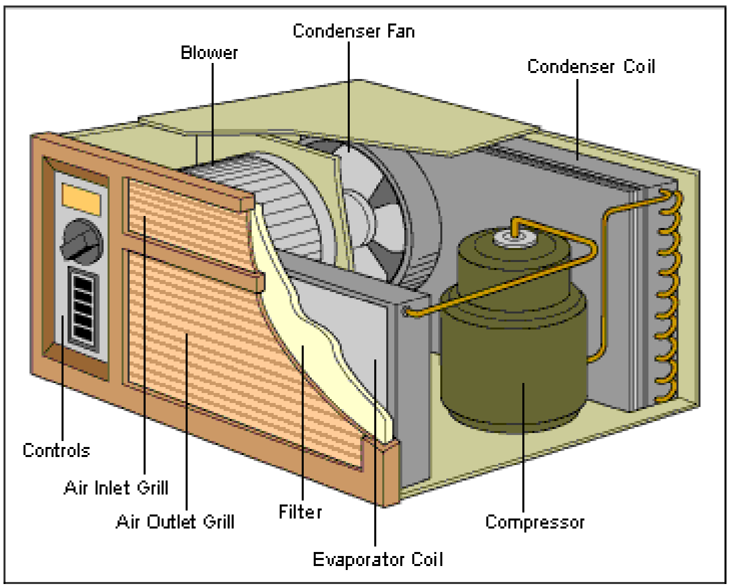
5.26

A window air-conditioner unit is placed on a laboratory bench and tested in cooling mode using 750 W of electric power with a COP of 1.75. What is the cooling power capacity and what is the net effect on the laboratory?

Definition of COP: $\beta = \dot{Q}_L / \dot{W}$

Cooling capacity: $\dot{Q}_L = \beta \, \dot{W} = 1.75 \times 750 =$ **1313 W**

For steady state operation the $\dot{Q}_L$ comes from the laboratory and $\dot{Q}_H$ goes to the laboratory giving a net to the lab of $\dot{W} = \dot{Q}_H - \dot{Q}_L = 750$ W, that is heating it.



5.27

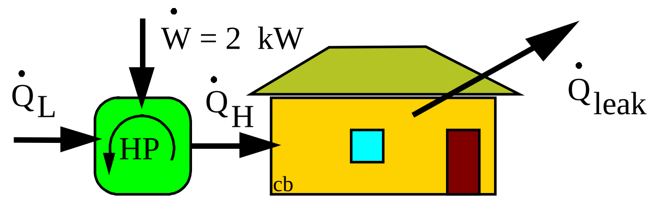
A farmer runs a heat pump with a 2 kW motor. It should keep a chicken hatchery at 30^{o}C, which loses energy at a rate of 10 kW to the colder ambient T_{amb}. What is the minimum coefficient of performance that will be acceptable for the heat pump?

Solution:

Power input: $\dot{W} = 2$ kW

Energy Eq. for hatchery: $\dot{Q}_H = \dot{Q}_{Loss} = 10$ kW

Definition of COP: $\beta = \text{COP} = \frac{\dot{Q}_H}{\dot{W}} = \frac{10}{2} = \mathbf{5}$



5.28

A sports car engine delivers 100 hp to the driveshaft with a thermal efficiency of 25%. The fuel has a heating value of 40 000 kJ/kg. Find the rate of fuel consumption and the combined power rejected through the radiator and exhaust.

Solution:

Heating value (HV): $\dot{Q}_H = \dot{m} \cdot HV$

From the definition of the thermal efficiency

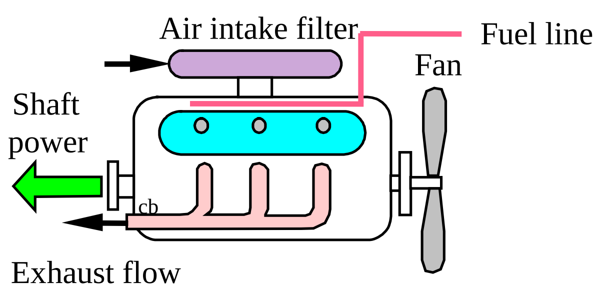
$$\dot{W} = \eta\ \dot{Q}_H = \eta \cdot\ \dot{m} \cdot HV$$

$$\dot{m} = \frac{\dot{W}}{\eta \cdot HV} = \frac{100 \times 0.7355}{0.25 \times 40\ 000} = 0.00736 \text{ kg/s} = \mathbf{7.36\ g/s}$$

Conversion of power from hp to kW in Table A.1.

$$\dot{Q}_L = \dot{Q}_H - \dot{W} = (\dot{W}/\eta - \dot{W}\ ) = (\frac{1}{\eta} - 1\ )\dot{W}$$

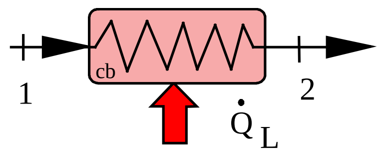
$$= (\frac{1}{0.25} - 1)\ 100 \text{ hp} \times 0.7355 \text{ kW/hp} = \mathbf{221\ kW}$$



5.29

R-410A enters the evaporator (the cold heat exchanger) in an A/C unit at -20^{o}C, x = 28% and leaves at -20^{o}C, x = 1. The COP of the refrigerator is 1.5 and the mass flow rate is 0.003 kg/s. Find the net work input to the cycle.

Energy equation for heat exchanger



$$\dot{Q}_L = \dot{m}(h_2 - h_1) = \dot{m}[h_g - (h_f + x_1\ h_{fg})]$$

$$= \dot{m}[h_{fg} - x_1\ h_{fg}] = \dot{m}\ (1 - x_1)h_{fg}$$

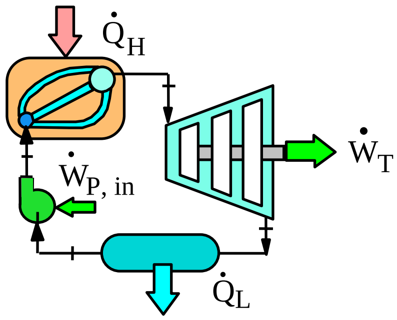
$$= 0.003\ \text{kg/s} \times 0.72 \times 243.65\ \text{kJ/kg} = 0.5263\ \text{kW}$$

$$\beta = \text{COP} = \dot{Q}_L / \dot{W} \quad \Rightarrow \quad \dot{W} = \dot{Q}_L / \beta = 0.5263 / 1.5 = \mathbf{0.35\ kW}$$

5.30

In a Rankine cycle steam power plant 0.9 MW is taken out in the condenser, 0.63 MW is taken out in the turbine, and the pump work is 0.03 MW. Find the plant thermal efficiency. If everything could be reversed find the COP as a refrigerator.

Solution:



CV. Total plant:

Energy Eq.:

$$\dot{Q}_H + \dot{W}_{P,in} = \dot{W}_T + \dot{Q}_L$$

$$\dot{Q}_H = \dot{W}_T + \dot{Q}_L - \dot{W}_{P,in}$$
$$= 0.63 + 0.9 - 0.03 = 1.5 \text{ MW}$$

$$\eta_{TH} = \frac{\dot{W}_T - \dot{W}_{P,in}}{\dot{Q}_H} = \frac{0.63 - 0.03}{1.5} = \mathbf{0.40}$$

$$\beta = \frac{\dot{Q}_L}{\dot{W}_T - \dot{W}_{P,in}} = \frac{0.9}{0.63 - 0.03} = \mathbf{1.5}$$

5.31

An experimental power plant generates 130 MW of electrical power. It uses a supply of 1200 MW from a geothermal source and rejects energy to the atmosphere. Find the power to the air and how much air should be flowed to the cooling tower (kg/s) if its temperature cannot be increased more than 12$^{\text{o}}$C.

Solution:

C.V. Total power plant.

Energy equation gives the amount of heat rejection to the atmosphere as

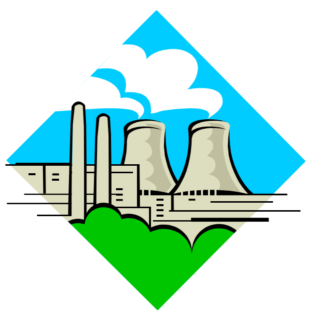
$$\dot{Q}_L = \dot{Q}_H - \dot{W} = 1200 - 130 = \mathbf{1070\ MW}$$

The energy equation for the air flow that absorbs the energy is

$$\dot{Q}_L = \dot{m}_{air}\,\Delta h = \dot{m}_{air}\,C_p\,\Delta T$$

$$\dot{m}_{air} = \frac{\dot{Q}_L}{C_p \Delta T} = \frac{1070 \times 1000}{1.004 \times 12}\,\frac{\text{MW}}{\text{(kJ/kg-K)} \times \text{K}} = \mathbf{88\ 811\ kg/s}$$

This is too large to make, so some cooling by liquid water or evaporative cooling should be used, see chapter 11.



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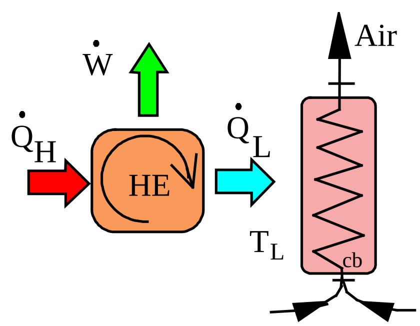
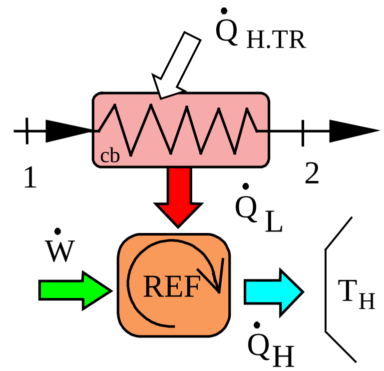


5.32

A water cooler for drinking water should cool 25 L/h water from 18^{o}C to 10^{o}C while the water reservoir also gains 60 W from heat transfer. Assume a small refrigeration unit with a COP of 2.5 does the cooling. Find the total rate of cooling required and the power input to the unit.



The mass flow rate is

$$\dot{m} = \boldsymbol{\rho}\dot{V} = \frac{25 \times 10^{-3}}{0.001002} \frac{1}{3600} \text{ kg/s} = 6.93 \text{ g/s}$$

Energy equation for heat exchanger

$$\dot{Q}_L = \dot{m}(h_1 - h_2) + \dot{Q}_{H\ TR}$$

$$= \dot{m}\ C_P\ (T_1 - T_2) + \dot{Q}_{H\ TR}$$

$$= 6.93 \times 10^{-3} \text{ kg/s} \times 4.18 \text{ kJ/kg-K} \times (18 - 10) \text{ K} + 60 \text{ W}$$
$$= 291.8 \text{ W}$$

$$\beta = \text{COP} = \dot{Q}_L / \dot{W} \quad \Rightarrow \quad \dot{W} = \dot{Q}_L / \beta = 291.8 / 2.5 = \mathbf{116.7\ W}$$

Comment: The unit does not operate continuously so the instantaneous power is higher during the periods it does operate.

5.33

A large stationary diesel engine produces 5 MW with a thermal efficiency of 40%. The exhaust gas, which we assume is air, flows out at 800 K and the intake air is 290 K. How large a mass flow rate is that, assuming this is the only way we reject heat? Can the exhaust flow energy be used?

Heat engine: $\dot{Q}_H = \dot{W}_{out}/\eta_{TH} = \frac{5}{0.4} = 12.5 \text{ MW}$

Energy equation: $\dot{Q}_L = \dot{Q}_H - \dot{W}_{out} = 12.5 – 5 = 7.5 \text{ MW}$

Exhaust flow: $\dot{Q}_L = \dot{m}_{air}(h_{800} - h_{290})$

$$\dot{m}_{air} = \frac{\dot{Q}_L}{h_{800} - h_{290}} = \frac{7.5 \times 1000}{822.2 - 290.43} \frac{\text{kW}}{\text{kJ/kg}} = \mathbf{14.1\ kg/s}$$

The flow of hot gases can be used to heat a building or it can be used to heat water in a steam power plant since that operates at lower temperatures.

5.34

For each of the cases below determine if the heat engine satisfies the first law (energy equation) and if it violates the second law.

a. $\dot{Q}_H = 6$ kW, $\dot{Q}_L = 4$ kW, $\dot{W} = 2$ kW

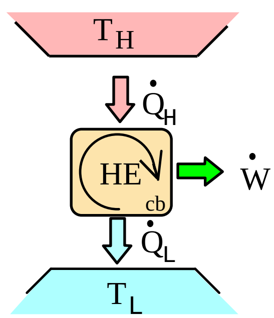
b. $\dot{Q}_H = 6$ kW, $\dot{Q}_L = 0$ kW, $\dot{W} = 6$ kW

c. $\dot{Q}_H = 6$ kW, $\dot{Q}_L = 2$ kW, $\dot{W} = 5$ kW

d. $\dot{Q}_H = 6$ kW, $\dot{Q}_L = 6$ kW, $\dot{W} = 0$ kW

Solution:

	1^{st}. law	2^{nd} law
a	Yes	Yes (possible)
b	Yes	No, impossible Kelvin - Planck
c	No	Yes, but energy not conserved
d	Yes	Yes (Irreversible $\dot{Q}$ over ΔT)



5.35

For each of the cases in problem 5.34 determine if a heat pump satisfies the first law (energy equation) and if it violates the second law.

a. $\dot{Q}_H = 6$ kW, $\dot{Q}_L = 4$ kW, $\dot{W} = 2$ kW

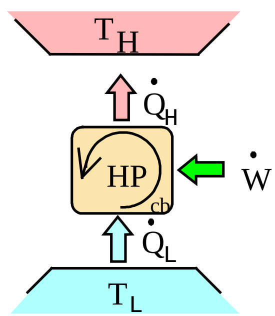
b. $\dot{Q}_H = 6$ kW, $\dot{Q}_L = 0$ kW, $\dot{W} = 6$ kW

c. $\dot{Q}_H = 6$ kW, $\dot{Q}_L = 2$ kW, $\dot{W} = 5$ kW

d. $\dot{Q}_H = 6$ kW, $\dot{Q}_L = 6$ kW, $\dot{W} = 0$ kW

Solution:

	1^{st}. law	2^{nd} law
a	Satisfied	Does not violate
b	Satisfied	Does not violate
c	Violated	Does not violate, but 1^{st} law
d	Satisfied	Does violate, Clausius



5.36

Calculate the amount of work input a refrigerator needs to make ice cubes out of a tray of 0.25 kg liquid water at 10^oC. Assume the refrigerator has β = 3.5 and a motor-compressor of 750 W. How much time does it take if this is the only cooling load?

C.V. Water in tray. We neglect tray mass.

Energy Eq.: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Process : $P = \text{constant} = P_o$

$${}_1W_2 = \int P\, dV = P_o m(v_2 - v_1)$$

$${}_1Q_2 = m(u_2 - u_1) + {}_1W_2 = m(h_2 - h_1)$$

Tbl. B.1.1 : h_1 = 41.99 kJ/kg, Tbl. B.1.5 : h_2 = - 333.6 kJ/kg

$${}_1Q_2 = 0.25(-333.4 - 41.99\ ) = -\ 93.848 \text{ kJ}$$

Consider now refrigerator

$$\beta = Q_L/W$$

$$W = Q_L/\beta = -\ {}_1Q_2/\ \beta = 93.848/3.5 = \mathbf{26.81\ kJ}$$

For the motor to transfer that amount of energy the time is found as

$$W = \int \dot{W}\, dt = \dot{W}\, \Delta t$$

$$\Delta t = W/\dot{W} = (26.81 \times 1000)\text{J} / 750 \text{ W} = \mathbf{35.75\ s}$$

Comment: We neglected a baseload of the refrigerator so not all the 750 W are available to make ice, also our coefficient of performance is very optimistic and finally the heat transfer is a transient process. All this means that it will take much more time to make ice-cubes.

Second Law and Processes

5.37

Prove that a cyclic device that violates the Kelvin–Planck statement of the second law also violates the Clausius statement of the second law.

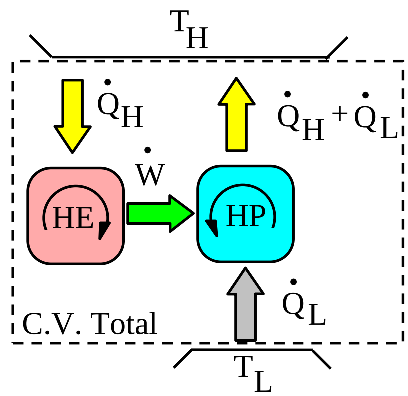
Solution: Proof very similar to the proof in section 5.2.

H.E. violating Kelvin receives Q_H from T_H and produces net $W = Q_H$.

This W input to H.P. receiving Q_L from T_L.

H.P. discharges $Q_H + Q_L$ to T_H .

Net Q to T_H is : $-Q_H + Q_H + Q_L = Q_L$.



H.E. + H.P. together transfers Q_L from T_L to T_H with no W thus violates Clausius.

5.38

Discuss the factors that would make the power plant cycle described in Problem 4.118 an irreversible cycle.

Solution:

General discussion, but here are a few of the most significant factors.

1. Combustion process that generates the hot source of energy.
2. Heat transfer over finite temperature difference in boiler.
3. Flow resistance and friction in turbine results in less work out.
4. Flow friction and heat loss to/from ambient in all pipes.
5. Heat transfer over finite temperature difference in condenser.

5.39

Discuss the factors that would make the heat pump described in Problem 4.123 an irreversible cycle.

Solution:

General discussion but here are a few of the most significant factors.

1. Unwanted heat transfer in the compressor.
2. Pressure loss (back flow leak) in compressor
3. Heat transfer and pressure drop in line 1 => 2.
4. Pressure drop in all lines.
5. Throttle process 3 => 4.

5.40

Assume a cyclic machine that exchanges 6 kW with a 250^{o}C reservoir and has

a. $\dot{Q}_L$ = 0 kW, $\dot{W}$ = 6 kW

b. $\dot{Q}_L$ = 6 kW, $\dot{W}$ = 0 kW

and $\dot{Q}_L$ is exchanged with a 30^{o}C ambient. What can you say about the processes in the two cases a and b if the machine is a heat engine? Repeat the question for the case of a heat pump.

Solution:

Heat engine

a. Since $\dot{Q}_L = 0$ impossible Kelvin – Planck

b. Possible, irreversible, $\eta_{\mathbf{eng}} = 0$

Heat pump

a. Possible, irreversible (like an electric heater)

b. Impossible, $\beta \rightarrow \infty$, Clausius

5.41

Consider a heat engine and heat pump connected as shown in figure P5.41. Assume $T_{H1} = T_{H2} > T_{amb}$ and determine for each of the three cases if the setup satisfy the first law and/or violates the 2nd law.

	$\dot{Q}_{H1}$	$\dot{Q}_{L1}$	$\dot{W}_1$	$\dot{Q}_{H2}$	$\dot{Q}_{L2}$	$\dot{W}_2$
a	6	4	2	3	2	1
b	6	4	2	5	4	1
c	3	2	1	4	3	1

Solution:

	1st. law	2nd law
a	Yes	Yes (possible)
b	Yes	No, combine Kelvin - Planck
c	Yes	No, combination clausius

5.42

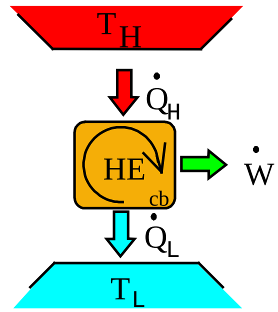
Consider the four cases of a heat engine in problem 5.34 and determine if any of those are perpetual machines of the first or second kind.

a. $\dot{Q}_H = 6$ kW, $\dot{Q}_L = 4$ kW, $\dot{W} = 2$ kW

b. $\dot{Q}_H = 6$ kW, $\dot{Q}_L = 0$ kW, $\dot{W} = 6$ kW

c. $\dot{Q}_H = 6$ kW, $\dot{Q}_L = 2$ kW, $\dot{W} = 5$ kW

d. $\dot{Q}_H = 6$ kW, $\dot{Q}_L = 6$ kW, $\dot{W} = 0$ kW



Solution:

	1^{st}. law	2^{nd} law
a	Yes	Yes (possible)
b	Yes	No, impossible Kelvin - Planck Perpetual machine second kind It violates the 2^{nd} law converts all $\dot{Q}$ to $\dot{W}$
c	No	Yes, but energy not conserved Perpetual machine first kind It generates energy inside
d	Yes	Yes (Irreversible $\dot{Q}$ over ΔT)

5.43

The simple refrigeration cycle is shown in Problem 5.23 and in Fig. 5.6. Mention a few of the processes that are expected to be irreversible.

The throttling process is highly irreversible.

Both of the heat transfer processes are externally irreversible, large ΔT between the working substance and the source or sink energy reservoir.

Both of the heat transfer processes are also internally irreversible, smaller ΔT in the substance so there is not a single uniform T across the flow cross sectional area. This is necessary to redistribute the energy uniformly in the working substance.

The compressor has friction and flow losses so not all of the shaft work goes into raising the substance pressure. Such an effect is described by a device efficiency in chapter 7.

Carnot Cycles and Absolute Temperature

5.44

Calculate the thermal efficiency of a Carnot cycle heat engine operating between reservoirs at 300^{o}C and 45^{o}C. Compare the result to that of Example 4.7.

Solution:

$$\eta_{TH} = W_{net} / Q_H = 1 - \frac{T_L}{T_H} = 1 - \frac{45 + 273}{300 + 273} = \mathbf{0.445} \;\; \text{(Carnot)}$$

$$\eta_{EX\ 4.7} = w_{net} / q_H = \frac{640.8 - 4}{2831.1} = 0.225$$

(efficiency about ½ of the Carnot)

5.45

An ideal (Carnot) heat engine has an efficiency of 40%. If the high temperature is raised 15% what is the new efficiency keeping the same low temperature?

Solution:

$$\eta_{TH} = W_{net} / Q_H = 1 - \frac{T_L}{T_H} = 0.4 \qquad \Rightarrow \qquad \frac{T_L}{T_H} = 0.6$$

so if T_H is raised 15% the new ratio becomes

$$\frac{T_L}{T_{H\,new}} = 0.6 / 1.15 = 0.5217 \quad \Rightarrow \qquad \eta_{TH\,new} = 1 - 0.5217 = \mathbf{0.478}$$

5.46

At a few places where the air is very cold in the winter, like –30°C it is possible to find a temperature of 13°C down below ground. What efficiency will a heat engine have operating between these two thermal reservoirs?

Solution:

$$\eta_{TH} = 1 - \frac{T_L}{T_H}$$

The ground becomes the hot source and the atmosphere becomes the cold side of the heat engine

$$\eta_{TH} = 1 - \frac{273 - 30}{273 + 13} = 1 - \frac{243}{286} = \mathbf{0.15}$$

This is low because of the modest temperature difference.

Microsoft clipart.

5.47

Consider the combination of a heat engine and a heat pump, as in Problem 5.41, with a low temperature of 400 K. What should the high temperature be so that the heat engine is reversible? For that temperature what is the COP for a reversible heat pump?

For all three cases of the heat engine the ratio between the heat transfers and the work term is the same as:

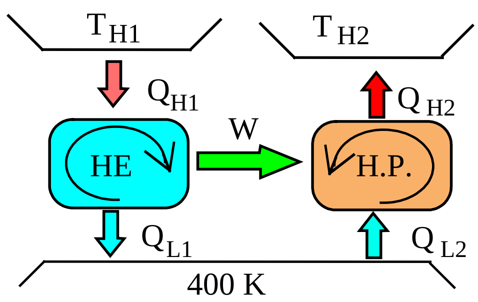
$$\dot{Q}_H : \dot{Q}_L : \dot{W} \quad = \quad 6:4:2 \; = \; 3:2:1$$

For a reversible heat engine we must have the heat transfer ratio equal to the temperature ratio so

$$\frac{\dot{Q}_H}{\dot{Q}_L} = \frac{T_H}{T_L} = \frac{3}{2} = \frac{T_H}{400\text{ K}} \quad => \quad T_H = (3/2)\ 400\text{ K} = \mathbf{600\ K}$$

The COP is

$$COP_{HP} = \dot{Q}_H / \dot{W} = \frac{3}{1} = \mathbf{3} \quad \left(= \frac{T_H}{T_H - T_L} = \frac{600}{600 - 400} \right)$$



5.48

Find the power output and the low T heat rejection rate for a Carnot cycle heat engine that receives 6 kW at 250^{o}C and rejects heat at 30^{o}C as in Problem 5.40.

Solution:

From the definition of the absolute temperature Eq. 5.4

$$\eta_{carnot} = 1 - \frac{T_L}{T_H} = 1 - \frac{303}{523} = 0.42$$

Definition of the heat engine efficiency gives the work as

$$\dot{W} = \eta\ \dot{Q}_H = 0.42 \times 6 = \mathbf{2.52\ kW}$$

Apply the energy equation

$$\dot{Q}_L = \dot{Q}_H - \dot{W} = 6 - 2.52 = \mathbf{3.48\ kW}$$

5.49

A large heat pump should upgrade 4 MW of heat at 65°C to be delivered as heat at 145°C. What is the minimum amount of work (power) input that will drive this?

For the minimum work we assume a Carnot heat pump and $\dot{Q}_L = 4$ MW.

$$\beta_{HP} = \frac{\dot{Q}_H}{\dot{W}_{in}} = \frac{T_H}{T_H - T_L} = \frac{273.15 + 145}{145 - 65} = 5.227$$

$$\beta_{REF} = \beta_{HP} - 1 = \frac{\dot{Q}_L}{\dot{W}_{in}} = 4.227$$

Now we can solve for the work

$$\dot{W}_{in} = \dot{Q}_L/\beta_{REF} = 4/4.227 = \mathbf{0.946\ MW}$$

This is a domestic or small office building size A/C unit, much smaller than the 4 MW in this problem.

5.50

A temperature of about 0.01 K can be achieved by magnetic cooling. In this process a strong magnetic field is imposed on a paramagnetic salt, maintained at 1 K by transfer of energy to liquid helium boiling at low pressure. The salt is then thermally isolated from the helium, the magnetic field is removed, and the salt temperature drops. Assume that 1 mJ is removed at an average temperature of 0.1 K to the helium by a Carnot-cycle heat pump. Find the work input to the heat pump and the coefficient of performance with an ambient at 300 K.

Solution:

$$\beta = \frac{Q_L}{W_{in}} = \frac{T_L}{T_H - T_L} = \frac{0.1}{299.9} = \mathbf{0.00033}$$

$$W_{in} = \frac{Q_L}{\beta} = \frac{1 \times 10^{-3}}{0.00033} = \mathbf{3\ J}$$

Remark: This is an extremely large temperature difference for a heat pump. A real one is built as a refrigerator within a refrigerator etc.

5.51

The lowest temperature that has been achieved is about 1×10^{-6} K. To achieve this an additional stage of cooling is required beyond that described in the previous problem, namely nuclear cooling. This process is similar to magnetic cooling, but it involves the magnetic moment associated with the nucleus rather than that associated with certain ions in the paramagnetic salt. Suppose that 10 μJ is to be removed from a specimen at an average temperature of 10^{-5} K (ten micro-joules is about the potential energy loss of a pin dropping 3 mm). Find the work input to a Carnot heat pump and its coefficient of performance to do this assuming the ambient is at 300 K.

Solution:

The heat removed from the cold space is

$$Q_L = 10\ \mu J = 10 \times 10^{-6}\ J \qquad \text{at}\ \ T_L = 10^{-5}\ K$$

Carnot heat pump satisfies Eq.7.4

$$\Rightarrow Q_H = Q_L \times \frac{T_H}{T_L} = 10 \times 10^{-6}\ J \times \frac{300}{10^{-5}} = 300\ J$$

From the energy equation for the heat pump

$$W_{in} = Q_H - Q_L = 300 - 10 \times 10^{-6} \cong \mathbf{300\ J}$$

$$\beta = \frac{Q_L}{W_{in}} = \frac{10 \times 10^{-6}}{300} = \mathbf{3.33 \times 10^{-8}}$$

5.52

Consider the setup with two stacked (temperature wise) heat engines as in Fig. P5.4. Let T_H = 850 K, T_M = 600 K and T_L = 350 K. Find the two heat engine efficiencies and the combined overall efficiency assuming Carnot cycles.

The individual efficiencies

$$\eta_1 = 1 - \frac{T_M}{T_H} = 1 - \frac{600}{850} = \mathbf{0.294}$$

$$\eta_2 = 1 - \frac{T_L}{T_M} = 1 - \frac{350}{600} = \mathbf{0.417}$$

The overall efficiency

$$\eta_{TH} = \dot{W}_{net} / \dot{Q}_H = (\dot{W}_1 + \dot{W}_2) / \dot{Q}_H = \eta_1 + \dot{W}_2 / \dot{Q}_H$$

For the second heat engine and the energy Eq. for the first heat engine

$$\dot{W}_2 = \eta_2 \, \dot{Q}_M = \eta_2 \, (1 - \eta_1) \, \dot{Q}_H$$

so the final result is

$$\eta_{TH} = \eta_1 + \eta_2 \, (1 - \eta_1) = 0.294 + 0.417 \, (1 - 0.294) = \mathbf{0.588}$$

Comment: It matches a single heat engine $\eta_{TH} = 1 - \frac{T_L}{T_H} = 1 - \frac{350}{850} = 0.588$

5.53

Find the maximum coefficient of performance for the refrigerator in your kitchen, assuming it runs in a Carnot cycle.

Solution:

The refrigerator coefficient of performance is

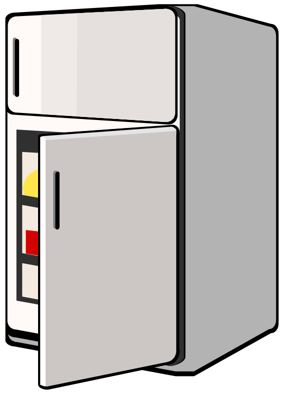
$$\beta = Q_L/W = Q_L/(Q_H - Q_L) = T_L/(T_H - T_L)$$

Assuming $T_L \sim 0°C$, $T_H \sim 35°C$,

$$\beta \le \frac{273.15}{35 - 0} = \mathbf{7.8}$$

Actual working fluid temperatures must be such that

$$T_L < T_{refrigerator} \quad \text{and} \quad T_H > T_{room}$$



A refrigerator does not operate in a Carnot cycle. The actual vapor compression cycle is examined in Chapter 9.

5.54

A car engine burns 5 kg fuel (equivalent to addition of Q_H) at 1500 K and rejects energy to the radiator and the exhaust at an average temperature of 750 K. If the fuel provides 40 000 kJ/kg what is the maximum amount of work the engine can provide?

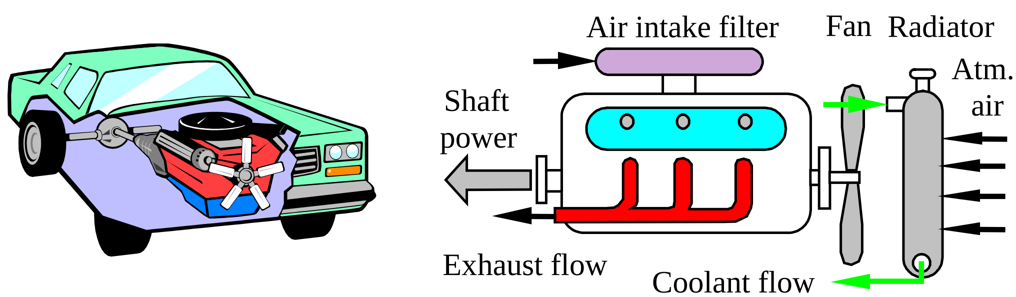
Solution:

A heat engine $Q_H = m\, q_{fuel} = 5 \text{ kg} \times 40\,000 \text{ kJ/kg} = 200\,000 \text{ kJ}$

Assume a Carnot efficiency (maximum theoretical work)

$$\eta = 1 - \frac{T_L}{T_H} = 1 - \frac{750}{1500} = 0.5$$

$$W = \eta\, Q_H = \mathbf{100\,000\ kJ}$$



5.55

An air-conditioner provides 1 kg/s of air at 15°C cooled from outside atmospheric air at 35°C. Estimate the amount of power needed to operate the air-conditioner. Clearly state all assumptions made.

Solution:

Consider the cooling of air which needs a heat transfer as

$$\dot{Q}_{air} = \dot{m}\ \Delta h \cong \dot{m}\ C_p\Delta T = 1\text{ kg/s} \times 1.004\text{ kJ/kg K} \times 20\text{ K} = 20\text{ kW}$$

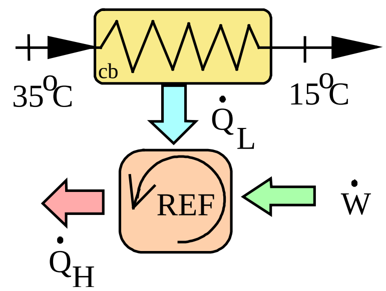
Assume Carnot cycle refrigerator

$$\beta = \frac{\dot{Q}_L}{\dot{W}} = \dot{Q}_L / (\dot{Q}_H - \dot{Q}_L) \cong \frac{T_L}{T_H - T_L} = \frac{273 + 15}{35 - 15} = 14.4$$

$$\dot{W} = \dot{Q}_L / \beta = \frac{20.0}{14.4} = \mathbf{1.39\ kW}$$

This estimate is the theoretical maximum performance. To do the required heat transfer $T_L \cong 5°C$ and $T_H = 45°C$ are more likely; secondly

$$\beta < \beta_{carnot}$$



5.56

A refrigerator should remove 400 kJ from some food. Assume the refrigerator works in a Carnot cycle between –15^{o}C and 45^{o}C with a motor-compressor of 400 W. How much time does it take if this is the only cooling load?

Assume Carnot cycle refrigerator

$$\beta = \frac{\dot{Q}_L}{\dot{W}} = \dot{Q}_L / (\dot{Q}_H - \dot{Q}_L) \cong \frac{T_L}{T_H - T_L} = \frac{273 - 15}{45 - (-15)} = 4.3$$

This gives the relation between the low T heat transfer and the work as

$$\dot{Q}_L = \frac{Q}{t} = \beta\,\dot{W} = 4.3\,\dot{W}$$

$$t = \frac{Q}{\beta\,\dot{W}} = \frac{400 \times 1000}{4.3 \times 400}\,\frac{\text{J}}{\text{W}} = \mathbf{233\ s}$$

5.57

Calculate the amount of work input a refrigerator needs to make ice cubes out of a tray of 0.25 kg liquid water at 10^oC. Assume the refrigerator works in a Carnot cycle between -8^oC and 35^oC with a motor-compressor of 600 W. How much time does it take if this is the only cooling load?

Solution:

C.V. Water in tray. We neglect tray mass.

Energy Eq.: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Process : $P = \text{constant} + P_o$

$${}_1W_2 = \int P\, dV = P_o m(v_2 - v_1)$$

$${}_1Q_2 = m(u_2 - u_1) + {}_1W_2 = m(h_2 - h_1)$$

Tbl. B.1.1 : $h_1 = 41.99$ kJ/kg, Tbl. B.1.5 : $h_2 = - 333.6$ kJ/kg

$${}_1Q_2 = 0.25 \text{ kg } (-333.4 - 41.99) \text{ kJ/kg} = - 93.848 \text{ kJ}$$

Consider now refrigerator

$$\beta = \frac{Q_L}{W} = \frac{Q_L}{Q_H - Q_L} = \frac{T_L}{T_H - T_L} = \frac{273 - 8}{35 - (-8)} = 6.16$$

$$W = \frac{Q_L}{\beta} = - \frac{{}_1Q_2}{\beta} = \frac{93.848}{6.16} = 15.24 \text{ kJ}$$

For the motor to transfer that amount of energy the time is found as

$$W = \int \dot{W}\, dt = \dot{W}\, \Delta t$$

$$\Delta t = \frac{W}{\dot{W}} = \frac{15.24 \times 1000}{600} \frac{J}{W} = \mathbf{25.4\ s}$$

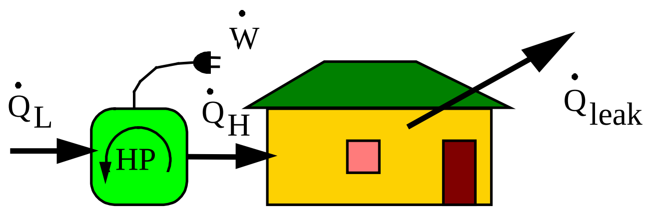
Comment: We neglected a baseload of the refrigerator so not all the 600 W are available to make ice, also our coefficient of performance is very optimistic and finally the heat transfer is a transient process. All this means that it will take much more time to make ice-cubes.

5.58

We propose to heat a house in the winter with a heat pump. The house is to be maintained at 20°C at all times. When the ambient temperature outside drops to −10°C, the rate at which heat is lost from the house is estimated to be 25 kW. What is the minimum electrical power required to drive the heat pump?

Solution:

Minimum power if we assume a Carnot cycle



Energy equation for the house (steady state): $\dot{Q}_H = \dot{Q}_{leak} = 25$ kW

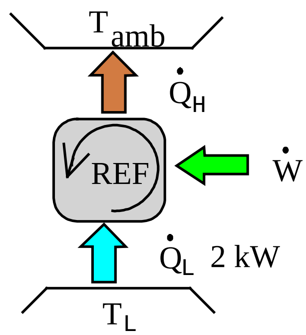
$$\beta' = \frac{\dot{Q}_H}{\dot{W}_{IN}} = \frac{T_H}{T_H - T_L} = \frac{293.2}{20 - (-10)} = 9.773 \quad \Rightarrow \quad \dot{W}_{IN} = \frac{25}{9.773} = \mathbf{2.56\ kW}$$

5.59

A household freezer operates in a room at 20°C. Heat must be transferred from the cold space at a rate of 2 kW to maintain its temperature at –30°C. What is the theoretically smallest (power) motor required to operate this freezer?

Solution:

Assume a Carnot cycle between T_L = -30°C and T_H = 20°C:



$$\beta = \frac{\dot{Q}_L}{\dot{W}_{in}} = \frac{T_L}{T_H - T_L} = \frac{273.15 - 30}{20 - (-30)} = 4.86$$

$\dot{W}_{in} = \dot{Q}_L/\beta = 2/4.86 =$ **0.41 kW**

This is the theoretical minimum power input.
Any actual machine requires a larger input.

5.60

A thermal storage is made with a rock (granite) bed of 2 m^3 which is heated to 400 K using solar energy. A heat engine receives a Q_H from the bed and rejects heat to the ambient at 290 K. The rock bed therefore cools down and as it reaches 290 K the process stops. Find the energy the rock bed can give out. What is the heat engine efficiency at the beginning of the process and what is it at the end of the process?

Solution:

Assume the whole setup is reversible and that the heat engine operates in a Carnot cycle. The total change in the energy of the rock bed is

$u_2 - u_1 = q = C\ \Delta T = 0.89$ kJ/kgK $\times$ (400 - 290) K = 97.9 kJ/kg

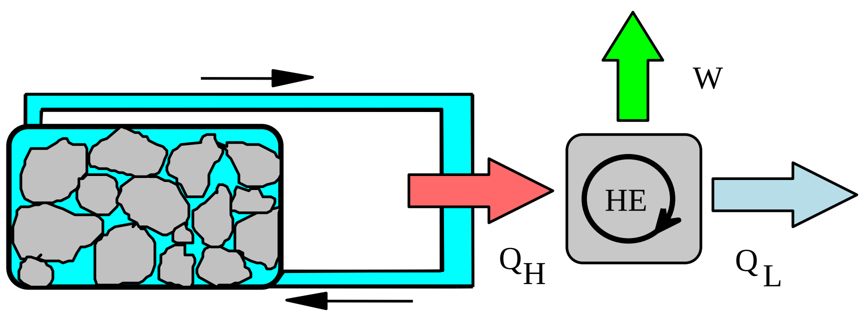
$m = \rho V = 2750$ kg/m$^3 \times 2$ m^3 = 5500 kg ,

$Q = mq = 5500$ kg $\times$ 97.9 kJ/kg = **538 450 kJ**

To get the efficiency use the CARNOT cycle result as

$\eta = 1 - T_o/T_H = 1 - 290/400 =$ **0.275** at the beginning of process

$\eta = 1 - T_o/T_H = 1 - 290/290 =$ **0.0** at the end of process



5.61

It is proposed to build a 1000-MW electric power plant with steam as the working fluid. The condensers are to be cooled with river water (see Fig. P5.61). The maximum steam temperature is 550°C, and the pressure in the condensers will be 10 kPa. Estimate the temperature rise of the river downstream from the power plant.

Solution:

$$\dot{W}_{NET} = 10^6 \text{ kW}, \quad T_H = 550°\text{C} = 823.3 \text{ K}$$

$$P_{COND} = 10 \text{ kPa} \rightarrow T_L = T_G\ (P = 10 \text{ kPa}) = 45.8°\text{C} = 319 \text{ K}$$

$$\eta_{TH\ CARNOT} = \frac{T_H - T_L}{T_H} = \frac{823.2 - 319}{823.2} = 0.6125$$

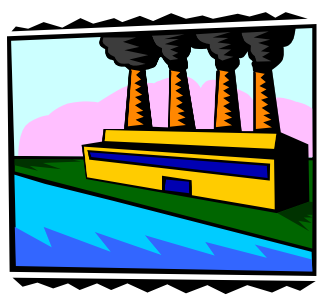
$$\Rightarrow \dot{Q}_{L\ MIN} = 10^6\left(\frac{1 - 0.6125}{0.6125}\right) = 0.6327 \times 10^6 \text{ kW}$$

But $\dot{m}_{H_2O} = \dfrac{60 \times 8 \times 10/60}{0.001} = 80\,000$ kg/s having an energy flow of

$$\dot{Q}_{L\ MIN} = \dot{m}_{H_2O}\,\Delta h = \dot{m}_{H_2O}\, C_{P\ LIQ\ H_2O}\, \Delta T_{H_2O\ MIN}$$

$$\Rightarrow \Delta T_{H_2O\ MIN} = \frac{\dot{Q}_{L\ MIN}}{\dot{m}_{H_2O} C_{P\ LIQ\ H_2O}} = \frac{0.6327 \times 10^6}{80000 \times 4.184} \frac{\text{kW}}{\text{kg/s} \times \text{kJ/kg-K}}$$

$$= \mathbf{1.9°C}$$



5.62

A certain solar-energy collector produces a maximum temperature of 100°C. The energy is used in a cyclic heat engine that operates in a 10°C environment. What is the maximum thermal efficiency? If the collector is redesigned to focus the incoming light, what should the maximum temperature be to produce a 25% improvement in engine efficiency?

Solution:

For T_H = 100°C = 373.2 K & T_L = 283.2 K

$$\eta_{th\ max} = \frac{T_H - T_L}{T_H} = \frac{90}{373.2} = \mathbf{0.241}$$

The improved efficiency is

$$\eta_{th\ max} = 0.241 \times 1.25 = 0.301$$

With the Carnot cycle efficiency

$$\eta_{th\ max} = \frac{T_H - T_L}{T_H} = 1 - \frac{T_L}{T_H} = 0.301$$

Then

$$T_H = T_L / (1 - 0.301) = 405 \text{ K} = \mathbf{132°C}$$

5.63

A constant temperature of –125°C must be obtained in a cryogenic experiment, although it gains 120 W due to heat transfer. What is the smallest motor you would need for a heat pump absorbing heat from the container and rejecting heat to the room at 20°C?

Solution:

We do not know the actual device so find the work for a Carnot cycle

$$\beta_{REF} = \dot{Q}_L / \dot{W} = \frac{T_L}{T_H - T_L} = \frac{148.15}{20 - (-125)} = 1.022$$

$$=> \qquad \dot{W} = \dot{Q}_L / \beta_{REF} = 120 \text{ W}/1.022 = \mathbf{117.4\ W}$$

5.64

Helium has the lowest normal boiling point of any of the elements at 4.2 K. At this temperature the enthalpy of evaporation is 83.3 kJ/kmol. A Carnot refrigeration cycle is analyzed for the production of 1 kmol of liquid helium at 4.2 K from saturated vapor at the same temperature. What is the work input to the refrigerator and the coefficient of performance for the cycle with an ambient at 300 K?

Solution:

For the Carnot cycle the ratio of the heat transfers is the ratio of temperatures

$$Q_L = n\,\bar{h}_{fg} = 1 \text{ kmol} \times 83.3 \text{ kJ/kmol} = 83.3 \text{ kJ}$$

$$Q_H = Q_L \times \frac{T_H}{T_L} = 83.3 \times \frac{300}{4.2} = 5950 \text{ kJ}$$

$$W_{IN} = Q_H - Q_L = 5950 - 83.3 = \mathbf{5886.7\ kJ}$$

$$\beta = \frac{Q_L}{W_{IN}} = \frac{83.3}{5886.7} = \mathbf{0.0142} \qquad \left[\ = \frac{T_L}{T_H - T_L}\ \right]$$

5.65

R-134a fills a 0.1-m^3 capsule at 20°C, 200 kPa. It is placed in a deep freezer, where it is cooled to −10°C. The deep freezer sits in a room with ambient temperature of 20°C and has an inside temperature of −10°C. Find the amount of energy the freezer must remove from the R-134a and the extra amount of work input to the freezer to perform the process.

Solution:

C.V. R-134a out to the -10°C space.

Energy equation: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Process : V = Const $\Rightarrow$ $v_2 = v_1$ $\Rightarrow$ ${}_1W_2 = 0$

Table B.5.2: $v_1 = 0.11436\ m^3/kg$, $u_1 = 395.27$ kJ/kg

$m = V/ v_1 = 0.87443$ kg

State 2: $v_2 = v_1 > v_g = 0.09921\ m^3/kg$ Table B.5.1 => sup vap

(T,v) interpolate between 150 kPa and 200 kPa in B.5.2

$$P_2 = 150 + 50\,\frac{0.11436 - 0.13602}{0.10013 - 0.13602} = 150 + 50\times 0.6035 = 180 \text{ kPa}$$

$$u_2 = 373.44 + 0.6035 \times (372.31 - 373.44) = 372.76 \text{ kJ/kg}$$

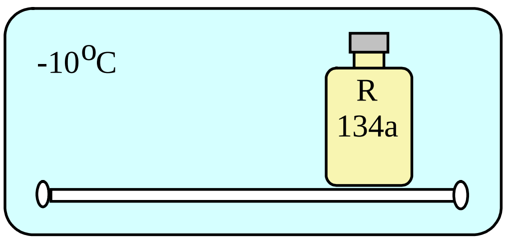
$${}_1Q_2 = m(u_2 - u_1) = 0.87443 \text{ kg} \times (372.76 - 395.27) \text{ kJ/kg}$$

$= \mathbf{-\ 19.68\ kJ}$

Consider the freezer and assume Carnot cycle

$$\beta = \frac{Q_L}{W} = \frac{Q_L}{Q_H - Q_L} = \frac{T_L}{T_H - T_L} = \frac{273 - 10}{20 - (-10)} = 8.767$$

$W_{in} = Q_L / \beta = 19.68 \text{ kJ}/ 8.767 = \mathbf{2.245\ kJ}$



5.66

A heat engine has a solar collector receiving 0.2 kW per square meter inside which a transfer media is heated to 450 K. The collected energy powers a heat engine which rejects heat at 40^{o}C. If the heat engine should deliver 2.5 kW what is the minimum size (area) solar collector?

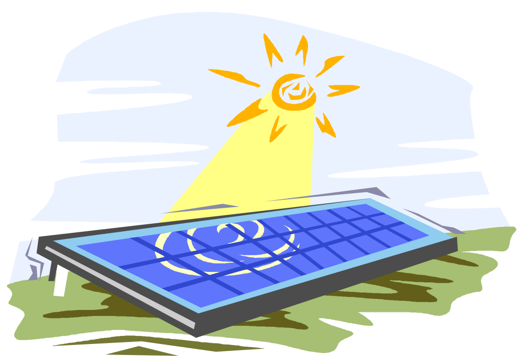
Solution:

$$T_H = 450 \text{ K} \qquad T_L = 40^oC = 313.15 \text{ K}$$

$$\eta_{HE} = 1 - \frac{T_L}{T_H} = 1 - \frac{313.15}{450} = 0.304$$

$$\dot{W} = \eta\, \dot{Q}_H \quad \Rightarrow \quad \dot{Q}_H = \frac{\dot{W}}{\eta} = \frac{2.5}{0.304} \text{ kW} = 8.224 \text{ kW}$$

$$\dot{Q}_H = 0.2 \text{ (kW/m}^2\text{) } A \quad \Rightarrow \quad A = \frac{\dot{Q}_H}{0.2} = \mathbf{41\ m^2}$$



5.67

A heat pump is driven by the work output of a heat engine as shown in figure P5.67. If we assume ideal devices find the ratio of the total power $\dot{Q}_{L1} + \dot{Q}_{H2}$ that heats the house to the power from the hot energy source $\dot{Q}_{H1}$ in terms of the temperatures.

$$\beta_{HP} = \dot{Q}_{H2}/\dot{W} = \dot{Q}_{H2}/(\dot{Q}_{H2} - \dot{Q}_{L2}) = \frac{T_{room}}{T_{room} - T_{amb}}$$

$$\dot{W} = \eta_{HE} \cdot \dot{Q}_{H1} = (1 - \frac{T_{room}}{T_H})\ \dot{Q}_{H1}$$

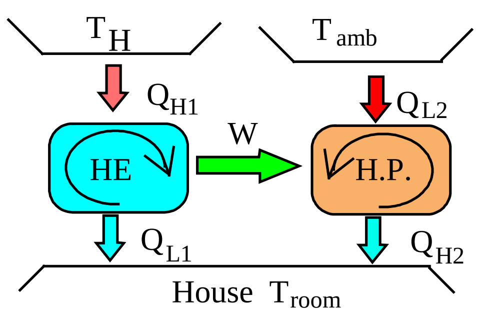
$$\dot{W} = \dot{Q}_{H2}/\beta_{HP} = \frac{T_{room}}{T_{room} - T_{amb}}\ \dot{Q}_{H2}$$

$$\dot{Q}_{L1} = \dot{Q}_{H1} - \dot{W} = [1 - 1 + \frac{T_{room}}{T_H}]\ \dot{Q}_{H1}$$

$$\frac{\dot{Q}_{H2} + \dot{Q}_{L1}}{\dot{Q}_{H1}} = 1 - 1 + \frac{T_{room}}{T_H} + \frac{1 - \frac{T_{room}}{T_H}}{\frac{T_{room} - T_{amb}}{T_{room}}} = \frac{T_{room}}{T_H} + \frac{T_{room} - T^2_{room}/T_H}{T_{room} - T_{amb}}$$

$$= T_{room}\ [\frac{1}{T_H} + \frac{1 - \frac{T_{room}}{T_H}}{T_{room} - T_{amb}}] = \frac{T_{room}}{T_H}[1 + \frac{T_H - T_{room}}{T_{room} - T_{amb}}]$$

$$= \mathbf{\frac{T_{room}}{T_H} \left[\frac{T_H - T_{amb}}{T_{room} - T_{amb}} \right]}$$

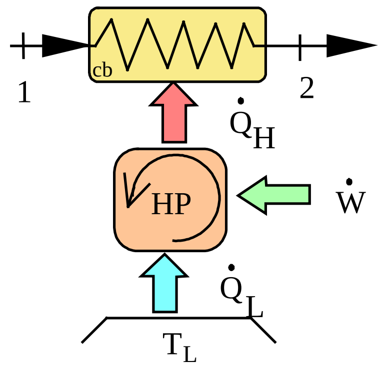


5.68

Sixty kilograms per hour of water runs through a heat exchanger, entering as saturated liquid at 200 kPa and leaving as saturated vapor. The heat is supplied by a Carnot heat pump operating from a low-temperature reservoir at 16°C with a COP of half that of a similar Carnot unit. Find the rate of work into the heat pump.

Solution:

C.V. Heat exchanger



$\dot{m}_1 = \dot{m}_2$; $\qquad \dot{m}_1 h_1 + \dot{Q}_H = \dot{m}_1 h_2$

Table B.1.2: $h_1 = 504.7$ kJ/kg,

$h_2 = 2706.7$ kJ/kg

$T_H = T_{sat}(P) = 120.93 + 273.15$

$= 394.08$ K

$$\dot{Q}_H = \frac{60}{3600}(2706.7 - 504.7) = 36.7 \text{ kW}$$

First find the COP of a Carnot heat pump.

$$\beta' = \dot{Q}_H/\dot{W} = T_H / (T_H - T_L) = 394.08 / (394.08 - 289.15) = 3.76$$

Now we can do the actual one as $\beta'_H = 3.76/2 = 1.88$

$$\dot{W} = \dot{Q}_H/\beta'_H = 36.7 \text{ kW} / 1.88 = \mathbf{19.52\ kW}$$

5.69

A power plant with a thermal efficiency of 40% is located on a river similar to Fig. P5.61. With a total river mass flow rate of 1×10^5 kg/s at 15^{o}C find the maximum power production allowed if the river water should not be heated more than 1 degree.

The maximum heating allowed determines the maximum $\dot{Q}_L$ as

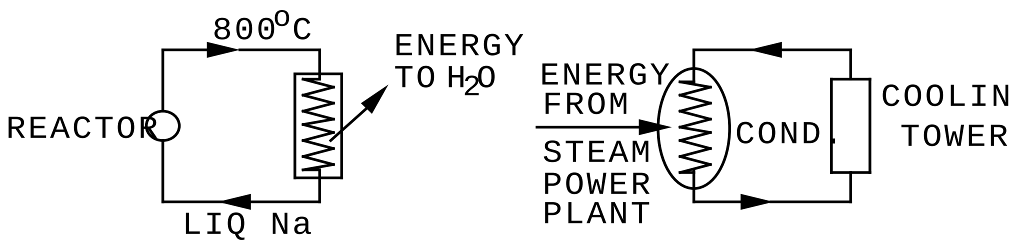
$$\dot{Q}_L = \dot{m}_{H_2O}\ \Delta h = \dot{m}_{H_2O}\ C_{P\ LIQ\ H_2O}\ \Delta T_{H_2O}$$
$$= 1 \times 10^5 \text{ kg/s} \times 4.18 \text{ kJ/kg-K} \times 1 \text{ K} = 418 \text{ MW}$$
$$= \dot{W}_{NET}(1/\eta_{TH\ ac} - 1)$$

$$\dot{W}_{NET} = \dot{Q}_L \ / \ (1/\eta_{TH\ ac} - 1) = \dot{Q}_L \frac{\eta_{TH\ ac}}{1 - \eta_{TH\ ac}}$$
$$= 418 \text{ MW} \times \frac{0.4}{1 - 0.4} = \mathbf{279\ MW}$$

5.70

Liquid sodium leaves a nuclear reactor at 800°C and is used as the energy source in a steam power plant. The condenser cooling water comes from a cooling tower at 15°C. Determine the maximum thermal efficiency of the power plant. Is it misleading to use the temperatures given to calculate this value?

Solution:



T_H = 800°C = 1073.2 K, T_L = 15°C = 288.2 K

$$\eta_{TH\ MAX} = \frac{T_H - T_L}{T_H} = \frac{1073.2 - 288.2}{1073.2} = \mathbf{0.731}$$

It might be misleading to use 800°C as the value for T_H, since there is not a supply of energy available at a constant temperature of 800°C (liquid Na is cooled to a lower temperature in the heat exchanger).

⇒ The Na cannot be used to boil H_2O at 800°C.

Similarly, the H_2O leaves the cooling tower and enters the condenser at 15°C, and leaves the condenser at some higher temperature.

⇒ The water does not provide for condensing steam at a constant temperature of 15°C.

5.71

The management of a large factory cannot decide which of two fuels to purchase. The selected fuel will be used in a heat engine operating between the fuel burning temperature and a low exhaust temperature. of. Fuel A burns at 2200 K and exhausts at 450 K, delivering 30 000 kJ/kg, and costs $1.50/kg. Fuel B burns at 1200 K and exhausts at 350 K, delivering 40 000 kJ/kg and costs $1.30/kg. Which fuel would you buy and why?

Solution:

Fuel A: $\eta_{TH,A} = 1 - \frac{T_L}{T_H} = 1 - \frac{450}{2200} = 0.795$

$W_A = \eta_{TH,A} \times Q_A = 0.795 \times 30\,000 = 23\,850$ kJ/kg

$W_A/\$_A = 23\,850/1.5 = 15\,900$ kJ/\$

Fuel B: $\eta_{TH,B} = 1 - \frac{T_L}{T_H} = 1 - \frac{350}{1200} = 0.708$

$W_B = \eta_{TH,B} \times Q_B = 0.708 \times 40\,000 = 28\,320$ kJ/kg

$W_B/\$_B = 28\,320/1.3 = 21\,785$ kJ/\$

Select fuel B for more work per dollar though it has a lower thermal efficiency.

5.72

A sales person selling refrigerators and deep freezers will guarantee a minimum coefficient of performance of 4.5 year round. How would you evaluate that? Are they all the same?

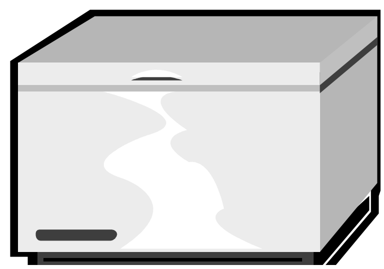
Solution:

Assume a high temperature of 35°C. If a freezer compartment is included T_L ~ -20°C (deep freezer) and fluid temperature is then T_L ~ -30°C

$$\beta_{\text{deep freezer}} \le \frac{T_L}{T_H - T_L} = \frac{273.15 - 30}{35 - (-30)} = 3.74$$

A hot summer day may require a higher T_H to push Q_H out into the room, so even lower β.

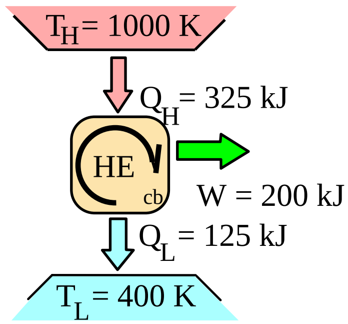
Claim is possible for a refrigerator, but not for a deep freezer.



5.73

A cyclic machine, shown in Fig. P5.73, receives 325 kJ from a 1000 K energy reservoir. It rejects 125 kJ to a 400 K energy reservoir and the cycle produces 200 kJ of work as output. Is this cycle reversible, irreversible, or impossible?

Solution:



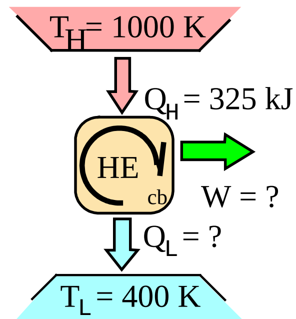
$$\eta_{Carnot} = 1 - \frac{T_L}{T_H} = 1 - \frac{400}{1000} = 0.6$$

$$\eta_{eng} = \frac{W}{Q_H} = \frac{200}{325} = 0.615 > \eta_{Carnot}$$

This is **impossible.**

5.74

Consider the previous problem and assume the temperatures and heat input are as given. If the actual machine has an efficiency that is half that of the corresponding Carnot cycle, find the work out and the rejected heat transfer.



$$\eta_{Carnot} = 1 - \frac{T_L}{T_H} = 1 - \frac{400}{1000} = 0.6$$

$$\eta_{eng} = \eta_{Carnot}/2 = 0.3 = \frac{W}{Q_H}$$

$$W = 0.3\ Q_H = 0.3 \times 325 = \mathbf{97.5\ kJ}$$

$$Q_L = Q_H - W = (325 - 97.5)\ \text{kJ} = \mathbf{227.5\ kJ}$$

5.75

Repeat problem 5.61 using a more realistic thermal efficiency of 45%.

$$\dot{W}_{NET} = 10^6 \text{ kW} = \eta_{TH\ ac}\, \dot{Q}_H, \qquad \eta_{TH\ ac} = 0.45$$

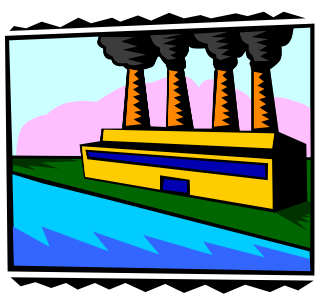
$$\Rightarrow \dot{Q}_L = \dot{Q}_H - \dot{W}_{NET} = \dot{W}_{NET}/\eta_{TH\ ac} - \dot{W}_{NET} = \dot{W}_{NET}(1/\eta_{TH\ ac} - 1)$$

$$= 10^6 \text{ kW} \left(\frac{1 - 0.45}{0.45}\right) = 1.222 \times 10^6 \text{ kW}$$

But $\dot{m}_{H_2O} = \dfrac{60 \times 8 \times 10/60}{0.001} = 80\,000$ kg/s having an energy flow of

$$\dot{Q}_L = \dot{m}_{H_2O}\, \Delta h = \dot{m}_{H_2O}\, C_{P\ LIQ\ H_2O}\, \Delta T_{H_2O}$$

$$\Rightarrow \Delta T_{H_2O} = \frac{\dot{Q}_L}{\dot{m}_{H_2O} C_{P\ LIQ\ H_2O}} = \frac{1.222 \times 10^6}{80\,000 \times 4.18} \frac{\text{kW}}{\text{kg/s} \times \text{kJ/kg-K}} = \mathbf{3.65°C}$$

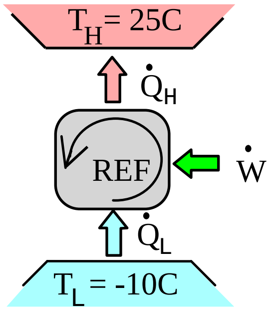


5.76

An inventor has developed a refrigeration unit that maintains the cold space at −10°C, while operating in a 25°C room. A coefficient of performance of 8.5 is claimed. How do you evaluate this?

Solution:

$$\beta_{Carnot} = \frac{Q_L}{W_{in}} = \frac{T_L}{T_H - T_L} = \frac{263.15}{25 - (-10)} = 7.52$$



$8.5 > \beta_{Carnot} \Rightarrow$ **impossible claim**

5.77

A heat pump receives energy from a source at 80°C and delivers energy to a boiler that operates at 350 kPa. The boiler input is saturated liquid water and the exit is saturated vapor both at 350 kPa. The heat pump is driven by a 2.5 MW motor and has a COP that is 60% of a Carnot heat pump COP. What is the maximum mass flow rate of water the system can deliver?

$$T_H = T_{sat} = 138.88^{o}C = 412\text{ K},\quad h_{fg} = 2148.1\text{ kJ/kg}$$

$$\beta_{HP\ Carnot} = \frac{\dot{Q}_H}{\dot{W}_{in}} = \frac{T_H}{T_H - T_L} = \frac{412}{138.88 - 80} = 7$$

$$\beta_{HP\ ac} = 0.6 \times 7 = 4.2 = \dot{Q}_H/\dot{W}_{in}$$

$$\dot{Q}_H = 4.2\ \dot{W}_{in} = 4.2 \times 2.5\text{ MW} = 10.5\text{ MW} = \dot{m}\ h_{fg}$$

$$\dot{m} = \dot{Q}_H / h_{fg} = 10\,500\text{ kW} / 2148.1\text{ kJ/kg} = \mathbf{4.89\ kg/s}$$

5.78

In a remote location, you run a heat engine to provide power to run a refrigerator. The input to the heat engine is 800 K and the low T is 400 K; it has an actual efficiency equal to half of that of the corresponding Carnot unit. The refrigerator has T_L = -10°C and T_H = 35°C, with a cop that is one-third that of the corresponding Carnot unit. Assume a cooling capacity of 2 kW is needed and find the rate of heat input to the heat engine.

Heat engine: $\eta_{Carnot} = 1 - \frac{T_L}{T_H} = 1 - \frac{400}{800} = 0.5; \qquad \eta_{ac} = 0.25$

Refrigerator: $\beta_{ref.\ Carnot} = \frac{T_L}{T_H - T_L} = \frac{273 - 10}{35 - (-10)} = 5.844; \quad \beta_{ref.\ ac} = 1.95$

Cooling capacity: $\dot{Q}_L = 2 \text{ kW} = \beta_{ref.\ ac} \dot{W}; \quad \dot{W} = 2 \text{ kW}/1.95 = 1.026 \text{ kW}$

This work must be provided by the heat engine $\dot{W} = \eta_{ac} \dot{Q}_H$

$$\dot{Q}_H = \dot{W} / \eta_{ac} = 1.026 \text{ kW} / 0.25 = \mathbf{4.1\ kW}$$

5.79

A car engine with a thermal efficiency of 33% drives the air-conditioner unit (a refrigerator) besides powering the car and other auxiliary equipment. On a hot (35^oC) summer day the A/C takes outside air in and cools it to 5^oC sending it into a duct using 2 kW of power input and it is assumed to be half as good as a Carnot refrigeration unit. Find the rate of fuel (kW) being burned extra just to drive the A/C unit and its COP. Find the flow rate of cold air the A/C unit can provide.

$$\dot{W}_{extra} = \eta\ \dot{Q}_{H\ extra} \quad \Rightarrow \quad \dot{Q}_{H\ extra} = \dot{W}_{extra} / \eta = 2\text{ kW} / 0.33 = \mathbf{6\ kW}$$

$$\beta = \frac{Q_L}{W_{IN}} = 0.5\ \beta_{Carnot} = 0.5\ \frac{T_L}{T_H - T_L} = 0.5\ \frac{5 + 273.15}{35 - 5} = 4.636$$

$$\dot{Q}_L = \beta\ \dot{W} = 4.636 \times 2\text{ kW} = 9.272\text{ kW} = \dot{m}_{air}\ C_{P\ air}\ \Delta T_{air}$$

$$\dot{m}_{air} = \dot{Q}_L / [C_{P\ air}\ \Delta T_{air}\ ] = \frac{9.272\text{ kW}}{1.004\text{ kJ/kg-K} \times (35 - 5)\text{ K}} = \mathbf{0.308\ kg/s}$$

5.80

A large heat pump should upgrade 5 MW of heat at 85°C to be delivered as heat at 150°C. Suppose the actual heat pump has a COP of 2.5 how much power is required to drive the unit. For the same COP how high a high temperature would a Carnot heat pump have assuming the same low T?

This is an actual COP for the heat pump as

$$\beta_{HP} = \text{COP} = \dot{Q}_H/\dot{W}_{in} = 2.5 \quad \Rightarrow \quad \dot{Q}_L/\dot{W}_{in} = 1.5$$

$$\dot{W}_{in} = \dot{Q}_L / 1.5 = 5 / 1.5 = \mathbf{3.333\ MW}$$

The Carnot heat pump has a COP given by the temperatures as

$$\beta_{HP} = \dot{Q}_H/\dot{W}_{in} = \frac{T_H}{T_H - T_L} = 2.5 \quad \Rightarrow \quad T_H = 2.5\ T_H - 2.5\ T_L$$

$$\Rightarrow \qquad T_H = \frac{2.5}{1.5}\ T_L = \frac{5}{3}(85 + 273.15) = \mathbf{597\ K}$$

Finite ΔT Heat Transfer

5.81

A refrigerator keeping 5°C inside is located in a 30°C room. It must have a high temperature ΔT above room temperature and a low temperature ΔT below the refrigerated space in the cycle to actually transfer the heat. For a ΔT of 0, 5 and 10°C respectively calculate the COP assuming a Carnot cycle.

Solution:

From the definition of COP and assuming Carnot cycle

$$\beta = \frac{Q_L}{W_{IN}} = \frac{T_L}{T_H - T_L} \quad \text{when T's are absolute temperatures}$$

	ΔT	T_H °C	T_H K	T_L °C	T_L K	β
a	0	30	303	5	278	11.1
b	5	35	308	0	273	7.8
c	10	40	313	-5	268	5.96

Notice how the COP drops significantly with the increase in ΔT.

5.82

The ocean near Havaii has 20^{o}C near the surface and 5^{o}C at some depth. A power plant based on this temperature difference is being planned. How large an efficiency could it have? If the two heat transfer terms (Q_H and Q_L) both require a 2 degree difference to operate what is the maximum efficiency then?

Solution:

T_H = 20°C = 293.2 K; T_L = 5°C = 278.2 K

$$\eta_{TH\ MAX} = \frac{T_H - T_L}{T_H} = \frac{293.2 - 278.2}{293.2} = \mathbf{0.051}$$

$$\eta_{TH\ mod} = \frac{T_{H'} - T_{L'}}{T_{H'}} = \frac{291.2 - 280.2}{291.2} = \mathbf{0.038}$$

This is a very low efficiency so it has to be done on a very large scale to be economically feasible and then it will have some environmental impact.

5.83

A house is cooled by a heat pump driven by an electric motor using the inside as the low-temperature reservoir. The house gains energy in direct proportion to the temperature difference as $\dot{Q}_{gain} = K(T_H - T_L)$. Determine the minimum electric power to drive the heat pump as a function of the two temperatures.

Solution:

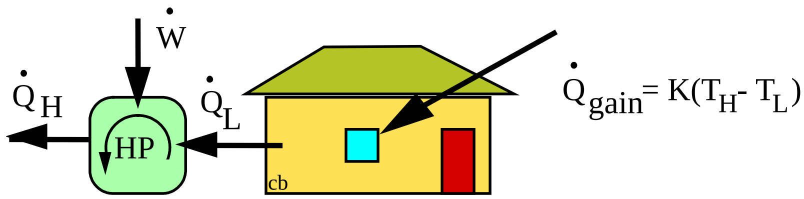
Refrigerator COP: $\beta = \dot{Q}_L/\dot{W}_{in} \le T_L/(T_H - T_L)$;

Heat gain must be removed: $\dot{Q}_L = \dot{Q}_{gain} = K(T_H - T_L)$

Solve for required work and substitute in for β

$$\dot{W}_{in} = \dot{Q}_L/\beta \;\ge\; K(T_H - T_L) \times (T_H - T_L)/T_L$$

$$\dot{W}_{in} \ge K(T_H - T_L)^2/T_L$$



5.84

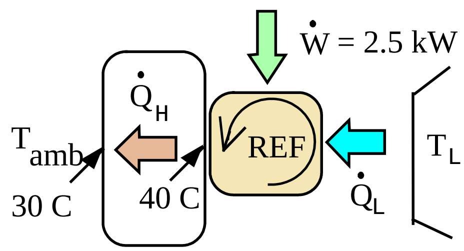
An air conditioner in a very hot region uses a power input of 2.5 kW to cool a 5°C space with the high temperature in the cycle as 40°C. The Q_H is pushed to the ambient air at 30°C in a heat exchanger where the transfer coefficient is 50 W/m^2K. Find the required minimum heat transfer area.

Solution:

$$\dot{W} = 2.5 \text{ kW} = \dot{Q}_H / \beta_{HP}; \qquad \beta_{HP} = T_H/(T_H - T_L) = \frac{273 + 40}{40 - 5} = 8.943$$

$$\dot{Q}_H = \dot{W} \times \beta_{HP} = 2.5 \times 8.943 = 22.36 \text{ kW} = h\, A\, \Delta T$$

$$A = \frac{\dot{Q}_H}{h\, \Delta T} = \frac{22.36 \times 10^3}{50 \times (40 - 30)} = \mathbf{44.72\ m^2}$$



5.85

A small house is kept at 20°C inside loses 12 kW to the outside ambient at 0°C. A heat pump is used to help heat the house together with possible electric heat. The heat pump is driven by a motor of 2.5 kW and it has a COP that is ¼ of a Carnot heat pump unit. Find the actual COP for the heat pump and the amount of electric heat that must be used (if any) to maintain the house temperature.

CV. House Energy: $0 = \dot{Q}_H + \dot{W}_{el.} - \dot{Q}_{Loss}$

Definition of COP: $\beta' = COP_{HP} = \dfrac{\dot{Q}_H}{\dot{W}} = \dfrac{1}{4}\dfrac{T_H}{T_H - T_L} = \dfrac{1}{4}\dfrac{293.15}{20 - 0} = \mathbf{3.664}$

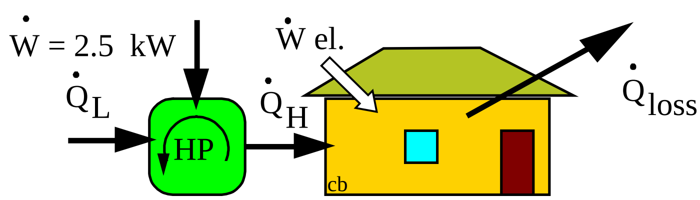
$\dot{Q}_H = COP_{HP}\,\dot{W}_{HP} = 3.664 \times 2.5 \text{ kW} = 9.16 \text{ kW}$

$\dot{W}_{el.} = \dot{Q}_{Loss} - \dot{Q}_H = 12 - 9.16 = \mathbf{2.84\ kW}$

C.V. Total Energy: $\dot{Q}_L = \dot{Q}_H - \dot{W}_{HP} = 9.16 - 2.5 = 6.66 \text{ kW}$

Entropy: $0 = \dot{Q}_L/T_L - \dot{Q}_{loss}/T_L + \dot{S}_{gen}$

$\dot{S}_{gen} = (\dot{Q}_{loss} - \dot{Q}_L)\,/T_L = \dfrac{12 - 6.66}{273.15} = 0.0195 \text{ kW/K}$



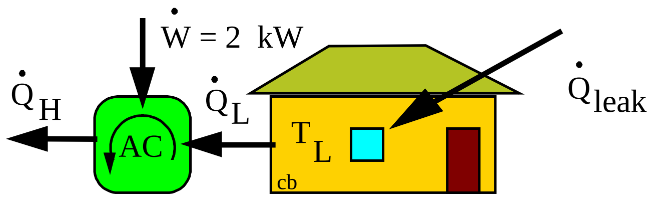
5.86

Consider a room at 20^{o}C that is cooled by an air conditioner with a COP of 3.2 using a power input of 2 kW and the outside is at 35^{o}C. What is the constant in the heat transfer Eq. 5.14 for the heat transfer from the outside into the room?

Definition of the coefficient of performance, Eq.5.2 and Eq.5.14

$$\dot{Q}_L = \beta_{AC}\dot{W} = 3.2 \times 2 \text{ kW} = 6.4 \text{ kW} = \dot{Q}_{\text{leak in}} = CA\ \Delta T$$

$$CA = \dot{Q}_L / \Delta T = \frac{6.4 \text{ kW}}{(35 - 20) \text{ K}} = \mathbf{0.427\ kW/K}$$



Here:

$T_L = T_{house} = 20^oC$

$T_H = T_{amb} = 35^oC$

5.87

A car engine operates with a thermal efficiency of 35%. Assume the air-conditioner has a coefficient of performance of $\beta = 3$ working as a refrigerator cooling the inside using engine shaft work to drive it. How much fuel energy should be spend extra to remove 1 kJ from the inside?

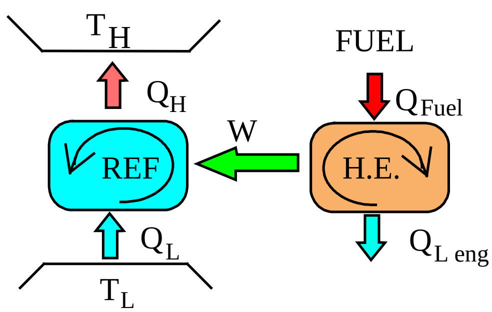
Solution:

Car engine: $W = \eta_{eng}\, Q_{fuel}$

Air conditioner: $\beta = \dfrac{Q_L}{W}$

$$W = \eta_{eng}\, Q_{fuel} = \frac{Q_L}{\beta}$$

$$Q_{fuel} = Q_L / (\eta_{eng}\, \beta) = \frac{1}{0.35 \times 3} = \mathbf{0.952\ kJ}$$



5.88

Arctic explorers are unsure if they can use a 5-kW motor driven heat pump to stay warm. It should keep their shelter at 15°C. The shelter losses energy at a rate of 0.5 kW per degree difference to the colder ambient. The heat pump has a COP that is 50% of a Carnot heat pump. If the ambient temperature can fall to -25°C at night, would you recommend this heat pump to the explorers?

CV Heat pump.
The heat pump should deliver a rate of heating that equals the heat loss to the ambient for steady inside temperature.

$$COP = \beta' = \dot{Q}_H/\dot{W} = 0.5\ \beta'_{Carnot} = \frac{1}{2} \times \frac{T_H}{T_H - T_L} = \frac{1}{2} \times \frac{273 + 15}{15 - (-25)} = 3.6$$

The heat pump can then provide a heating capacity of

$$\dot{Q}_H = \beta'\ \dot{W} = 3.6 \times 5\text{ kW} = 18\text{ kW}$$

The heat loss is

$$\dot{Q}_{leak\ out} = CA\ \Delta T = 0.5\text{ kW/K} \times [15 - (-25)]\text{ K} = 20\text{ kW}$$

The heat pump is not sufficient to cover the loss and **not recommended**.

5.89

Using the given heat pump in the previous problem, how warm would it make the shelter in the arctic night?

The high is now an unknown so both the heat loss and the heat pump performance depends on that. The energy balance around the shelter then gives

$$\dot{Q}_H = \beta' \dot{W} = \dot{Q}_{\text{leak out}} = \text{CA } \Delta T$$

Substitute the expression for β' and CA ΔT to give

$$\frac{1}{2} \times \frac{T_H}{T_H - T_L} \dot{W} = 0.5 \text{ kW/K} \times [T_H - T_L]$$

Multiply with the temperature difference, factor 2 and divide by the work to get

$$T_H = \frac{0.5 \times 2}{5 \text{ K}} [T_H - T_L]^2 = \frac{0.2}{\text{K}} [T_H - T_L]^2$$

Solve this equation like $0.2\, x^2 - x - T_L = 0$, with $x = T_H - T_L$ and $T_L = 248.15$ K

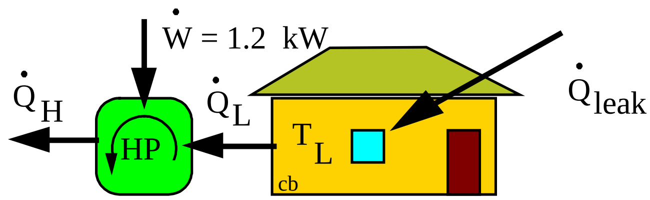
$$x = T_H - T_L = 37.81 \text{ K} \qquad \text{(negative root discarded)}$$

$$T_H = x + T_L = 37.81 - 25 = \mathbf{12.8°C}$$

5.90

An air conditioner cools a house at T_L = 20°C with a maximum of 1.2 kW power input. The house gains 0.6 kW per degree temperature difference to the ambient and the refrigeration COP is $\beta = 0.6\ \beta_{Carnot}$. Find the maximum outside temperature, T_H, for which the air conditioner provides sufficient cooling.

Solution:



Here:

$T_L = T_{house}$

$T_H = T_{amb}$

In this setup the low temperature space is the house and the high temperature space is the ambient. The heat pump must remove the gain or leak heat transfer to keep it at a constant temperature.

$\dot{Q}_{leak} = 0.6\ (T_{amb} - T_{house}) = \dot{Q}_L$ which must be removed by the heat pump.

$$\beta = \dot{Q}_L / \dot{W} = 0.6\ \beta_{carnot} = 0.6\ T_{house} / (T_{amb} - T_{house})$$

Substitute in for $\dot{Q}_L$ and multiply with $(T_{amb} - T_{house})\dot{W}$:

$$0.6\ (T_{amb} - T_{house})^2 = 0.6\ T_{house}\ \dot{W}$$

Since T_{house} = 293.15 K and $\dot{W}$ = 1.2 kW it follows

$$(T_{amb} - T_{house})^2 = T_{house}\ \dot{W} = 293.15 \times 1.2 = 351.78\ K^2$$

Solving $\Rightarrow$ $(T_{amb} - T_{house}) = 18.76$ $\Rightarrow$ T_{amb} = **311.9 K = 38.8 °C**

Comment: We did assume here that $\beta = 0.6\ \beta_{carnot}$, the statement could also have been understood as $\beta' = 0.6\ \beta'_{carnot}$ which would lead to a slightly different result.

5.91

A house is cooled by an electric heat pump using the outside as the high-temperature reservoir. For several different summer outdoor temperatures, estimate the percent savings in electricity if the house is kept at 25°C instead of 20°C. Assume that the house is gaining energy from the outside directly proportional to the temperature difference as in Eq. 5.14.

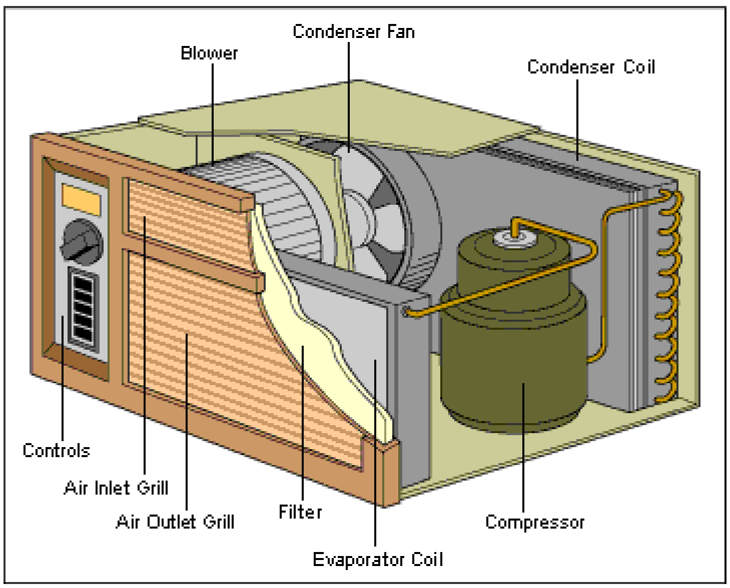
Solution:

Air-conditioner (Refrigerator) $\dot{Q}_{LEAK} \propto (T_H - T_L)$

Max Perf. $$\frac{\dot{Q}_L}{\dot{W}_{IN}} = \frac{T_L}{T_H - T_L} = \frac{K(T_H - T_L)}{\dot{W}_{IN}}, \quad \dot{W}_{IN} = \frac{K(T_H - T_L)^2}{T_L}$$

A: T_{L_A} = 20°C = 293.2 K B: T_{L_B} = 25°C = 298.2 K

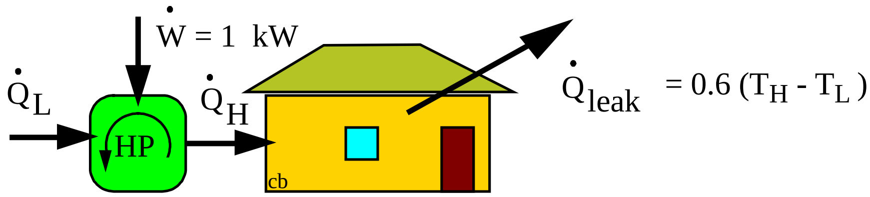
T_H,°C	$\dot{W}_{IN_A}/K$	$\dot{W}_{IN_B}/K$	% saving
45	2.132	1.341	37.1 %
40	1.364	0.755	44.6 %
35	0.767	0.335	56.3 %



5.92

A heat pump has a coefficient of performance that is 50% of the theoretical maximum. It maintains a house at 20°C, which leaks energy of 0.6 kW per degree temperature difference to the ambient. For a maximum of 1.0 kW power input find the minimum outside temperature for which the heat pump is a sufficient heat source.

Solution:



C.V. House. For constant 20°C the heat pump must provide $\dot Q_{leak} = 0.6\ \Delta T$

$$\dot Q_H = \dot Q_{leak} = 0.6\ (T_H - T_L) = \beta'\ \dot W$$

C.V. Heat pump. Definition of the coefficient of performance and the fact that the maximum is for a Carnot heat pump.

$$\beta' = \frac{\dot Q_H}{\dot W} = \frac{\dot Q_H}{\dot Q_H - \dot Q_L} = 0.5\ \beta'_{Carnot} = 0.5 \times \frac{T_H}{T_H - T_L}$$

Substitute into the first equation to get

$$0.6\ (T_H - T_L) = [\ 0.5 \times T_H / (T_H - T_L)\ ]\ 1 \quad =>$$

$$(T_H - T_L)^2 = (0.5 / 0.6)\ T_H \times 1 = 0.5 / 0.6 \times 293.15 = 244.29$$

$$T_H - T_L = 15.63 \quad => \quad T_L = 20 - 15.63 = \mathbf{4.4\ °C}$$

5.93

The room in problem 5.90 has a combined thermal mass of 2000 kg wood, 250 kg steel, and 500 kg drywall (gypsum). Estimate how quickly the room heats up if the air conditioner is turned off on a day when it is 35^{o}C outside.

Without the air-conditioner the house gains heat and the energy equation for the house becomes

$$m\, C \frac{dT}{dt} = \dot{Q}_{in}$$

The gain is due to the temperature difference as

$$\dot{Q}_{in} = 0.6\ (T_H - T_L) = 0.6\ \text{kW/K}\ (35 - 20)\ \text{K} = 9\ \text{kW}$$

The combined (mC) is using an estimate C for gypsum as 1 kJ/kg-K

$$mC = [2000 \times 1.38 + 250 \times 0.46 + 500 \times 1\ ]\ \text{kJ/K} = 3375\ \text{kJ/K}$$

$$\frac{dT}{dt} = \dot{Q}_{in}\ /mC = 9\ \text{kW/}\ 3375\ \text{kJ/K} = \mathbf{0.00267\ K/s}$$

5.94

A window air conditioner cools a room at $T_L = 22^oC$, with a maximum of 1.2 kW power input possible. The room gains 0.6 kW per degree temperature difference to the ambient, and the refrigeration COP is $\beta = 0.6\ \beta_{Carnot}$. Find the actual power required on a day when the temperature is 30^oC outside.

COP refrigerator: $\beta = \dfrac{Q_L}{W_{IN}} = 0.6\ \beta_{Carnot} = 0.6\ \dfrac{T_L}{T_H - T_L} = \dfrac{273.15 + 22}{30 - 22} = 22.1$

Heat gain: $\dot{Q}_L = 0.6\ (T_H - T_L\ ) = 0.6\ \text{kW/K} \times (30 - 22)\ \text{K} = 4.8\ \text{kW}$

$$\dot{W} = \dot{Q}_L/\beta = 4.8\ \text{kW}/22.1 = \mathbf{0.217\ kW}$$

5.95

On a cold (–10^{o}C) winter day a heat pump provides 20 kW to heat a house maintained at 20^{o}C and it has a COP_{HP} of 4. How much power does the heat pump require? The next day a winter storm brings the outside to -15^{o}C, assuming the same COP and the same house heat transfer coefficient for the heat loss to the outside air. How much power does the heat pump require then?

If we look at the heat loss for the house we have

$$\dot{Q}_{loss} = 20 \text{ kW} = CA\ \Delta T \qquad \Rightarrow \qquad CA = \frac{20 \text{ kW}}{20 - (-10) \text{ K}} = 0.667 \text{ kW/K}$$

So now with the new outdoor temperature we get

$$\dot{Q}_{loss} = CA\ \Delta T = 0.667 \text{ kW/K} \times [20 - (-15)] \text{ K} = 23.3 \text{ kW}$$

$$\dot{Q}_{loss} = \dot{Q}_H = COP\ \dot{W} \quad \Rightarrow \quad \dot{W} = \dot{Q}_{loss} / COP = \frac{23.3 \text{ kW}}{4} = \mathbf{5.83\ kW}$$

5.96

In the previous problem, it was assumed that the COP will be the same when the outside temperature drops. Given the temperatures and the actual COP at the -10^{o}C winter day, give an estimate for a more realistic COP for the outside at -15^{o}C case.

As the outside T drops the temperature in the low temperature heat exchanger drops and it will be harder for the heat pump. A reasonable assumption is then that the reduced COP will follow the ideal (Carnot cycle) COP.

At = -10^{o}C: $\beta_{Carnot} = \frac{T_H}{T_H - T_L} = \frac{293.15}{20 - (-10)} = 9.772$; COP = 4

At = -15^{o}C: $\beta_{Carnot} = \frac{T_H}{T_H - T_L} = \frac{293.15}{20 - (-15)} = 8.376$

$$\text{COP} = \frac{8.376}{9.772} \times 4 = \mathbf{3.43}$$

Ideal Gas Carnot Cycles

5.97

Hydrogen gas is used in a Carnot cycle having an efficiency of 60% with a low temperature of 300 K. During the heat rejection the pressure changes from 90 kPa to 120 kPa. Find the high and low temperature heat transfer and the net cycle work per unit mass of hydrogen.

Solution:

As the efficiency is known, the high temperature is found as

$$\eta = 0.6 = 1 - \frac{T_L}{T_H} \qquad => T_H = T_L\,/(1 - 0.6) = 750 \text{ K}$$

Now the volume ratio needed for the heat transfer, $T_3 = T_4 = T_L$, is

$$v_3 / v_4 = (RT_3 / P_3) \,/\, (RT_4 / P_4) = P_4 / P_3 = 120 / 90 = 1.333$$

so from Eq.5.9 we have with R = 4.1243 kJ/kg-K from Table A.5

$$q_L = RT_L \ln (v_3/v_4) = \mathbf{355.95\ kJ/kg}$$

Using the efficiency from Eq.5.5 then

$$q_H = q_L / (1 - 0.6) = \mathbf{889.9\ kJ/kg}$$

The net work equals the net heat transfer

$$w = q_H - q_L = \mathbf{533.9\ kJ/kg}$$

5.98

Carbon dioxide is used in an ideal gas refrigeration cycle, reverse of Fig. 5.24. Heat absorption is at 250 K and heat rejection is at 325 K where the pressure changes from 1200 kPa to 2400 kPa. Find the refrigeration COP and the specific heat transfer at the low temperature.

The analysis is the same as for the heat engine except the signs are opposite so the heat transfers move in the opposite direction.

$$\beta = \dot{Q}_L / \dot{W} = \beta_{carnot} = T_L / (T_H - T_L) = \frac{250}{325 - 250} = \mathbf{3.33}$$

$$q_H = RT_H \ln(v_2/v_1) = RT_H \ln \left(\frac{P_1}{P_2}\right) = 0.1889 \text{ kJ/kg-K} \times 325 \text{ K} \ln\left(\frac{2400}{1200}\right)$$

$$= 42.55 \text{ kJ/kg}$$

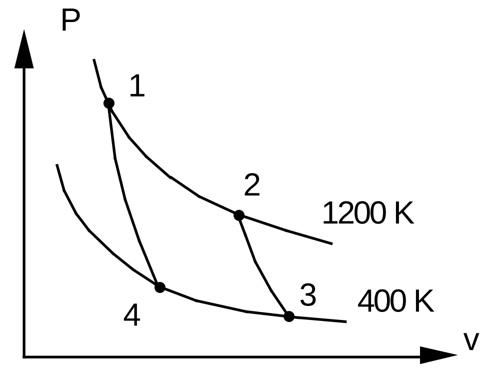
$$q_L = q_H T_L / T_H = 42.55 \text{ kJ/kg} \times 250 / 325 = \mathbf{32.73\ kJ/kg}$$

5.99

An ideal gas Carnot cycle with air in a piston cylinder has a high temperature of 1000 K and a heat rejection at 400 K. During the heat addition the volume triples. Find the two specific heat transfers (q) in the cycle and the overall cycle efficiency.

Solution:

The P-v diagram of the cycle is shown to the right.
From the integration along the process curves done in the main text we have Eq.5.7



$q_H = R\ T_H \ln(v_2/v_1)$

$= 0.287\ \text{kJ/kg} \times 1000 \ln(3)$

$=$ **315.3 kJ/kg**

Since it is a Carnot cycle the knowledge of the temperatures gives the cycle efficiency as

$$\eta_{TH} = 1 - \frac{T_L}{T_H} = 1 - \frac{400}{1000} = \mathbf{0.6}$$

from which we can get the other heat transfer from Eq.5.4

$q_L = q_H\ T_L / T_H = 315.3 \times 400 / 1000 =$ **126.1 kJ/kg**

5.100

Air in a piston/cylinder goes through a Carnot cycle with the P-v diagram shown in Fig. 5.24. The high and low temperatures are 600 K and 300 K respectively. The heat added at the high temperature is 250 kJ/kg and the lowest pressure in the cycle is 75 kPa. Find the specific volume and pressure after heat rejection and the net work per unit mass.

Solution:

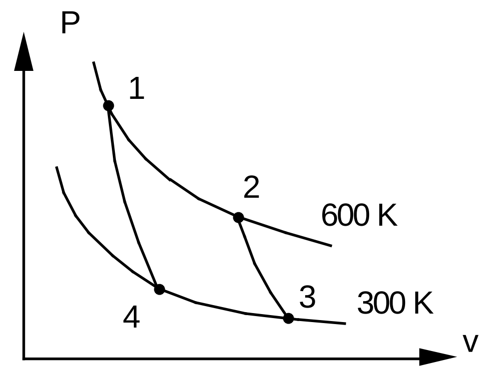
$q_H = 250$ kJ/kg , $T_H = 600$ K, $T_L = 300$ K, $P_3 = 75$ kPa

The states as shown in figure 5.24

1: 600 K , 2: 600 K, 3: 75 kPa, 300 K 4: 300 K

Since this is a Carnot cycle and we know the temperatures the efficiency is

$$\eta = 1 - \frac{T_L}{T_H} = 1 - \frac{300}{600} = 0.5$$



and the net work becomes

$$w_{NET} = \eta q_H = 0.5 \times 250$$
$$= \mathbf{125\ kJ/kg}$$

The heat rejected is

$$q_L = q_H - w_{NET} = 125 \text{ kJ/kg}$$

After heat rejection is state 4. From equation 5.9

$3 \rightarrow 4$ Eq.5.9 : $q_L = RT_L \ln (v_3/v_4)$

$v_3 = RT_3 / P_3 = 0.287$ kJ/kg-K $\times$ 300 K / 75 kPa = 1.148 m^3/kg

$v_4 = v_3 \exp(-q_L/RT_L) = 1.148$ m^3/kg $\exp(-125/0.287 \times 300)$

$= \mathbf{0.2688\ m^3/kg}$

$P_4 = RT_4 / v_4 = 0.287$ kJ/kg-K $\times$ 300 K/ 0.2688 m^3/kg = **320 kPa**

Review Problems

5.101

A 4L jug of milk at 25°C is placed in your refrigerator where it is cooled down to 5°C. The high temperature in the Carnot refrigeration cycle is 45°C, the low temperature is -5°C and the properties of milk are the same as for liquid water. Find the amount of energy that must be removed from the milk and the additional work needed to drive the refrigerator.

Solution:

C.V milk + out to the 5 °C refrigerator space

Energy Eq.: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Process : P = constant = 1 atm => ${}_1W_2 = Pm\,(v_2 - v_1)$

State 1: Table B.1.1, $v_1 \cong v_f = 0.001003\ m^3/kg$, $h_1 \cong h_f = 104.87\ kJ/kg$

$m_2 = m_1 = V_1/v_1 = 0.004 / 0.001003 = 3.988\ kg$

State 2: Table B.1.1, $h_2 \cong h_f = 20.98\ kJ/kg$

${}_1Q_2 = m(u_2 - u_1) + {}_1W_2 = m(u_2 - u_1) + Pm\,(v_2 - v_1) = m(h_2 - h_1)$

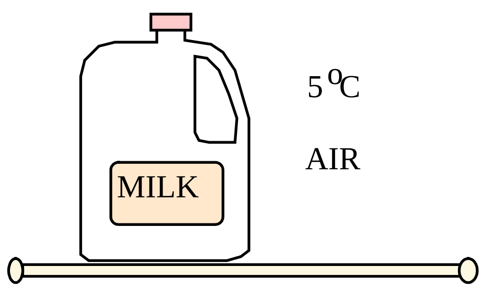
${}_1Q_2 = 3.998\ (20.98 - 104.87) = -3.988 \times 83.89 = \mathbf{-\ 334.55\ kJ}$

C.V. Refrigeration cycle $T_L = -5$ °C ; $T_H = 45$ °C, assume Carnot

Ideal : $\beta = Q_L / W = Q_L / (Q_H - Q_L) = T_L/(T_H - T_L)$

$= 278.15 / [45 - (-5)] = 5.563$

$W = Q_L / \beta = 334.55\ kJ/ 5.563 = \mathbf{60.14\ kJ}$



Remark: If you calculate the work term ${}_1W_2$ you will find that it is very small, the volume does not change (liquid). The heat transfer could then have been done as $m(u_2 - u_1)$ without any change in the numbers.

5.102

Consider the combination of the two heat engines as in Fig. P5.4. How should the intermediate temperature be selected so the two heat engines have the same efficiency assuming Carnot cycle heat engines.

Heat engine 1: $\eta_{TH\,1} = 1 - \frac{T_M}{T_H}$

Heat engine 2: $\eta_{TH\,2} = 1 - \frac{T_L}{T_M}$

$$\eta_{TH\,1} = \eta_{TH\,2} \;\Rightarrow\; 1 - \frac{T_M}{T_H} = 1 - \frac{T_L}{T_M} \;\Rightarrow\; \frac{T_M}{T_H} = \frac{T_L}{T_M}$$

$$\Rightarrow\; T_M = \sqrt{T_L T_H}$$

5.103

Consider a combination of a gas turbine power plant and a steam power plant as shown in Fig. P5.4. The gas turbine operates at higher temperatures (thus called a topping cycle) than the steam power plant (then called a bottom cycle). Assume both cycles have a thermal efficiency of 32%. What is the efficiency of the overall combination assuming Q_L in the gas turbine equals Q_H to the steam power plant?

Let the gas turbine be heat engine number 1 and the steam power plant the heat engine number 2. Then the overall efficiency

$$\eta_{TH} = \dot{W}_{net} / \dot{Q}_H = (\dot{W}_1 + \dot{W}_2) / \dot{Q}_H = \eta_1 + \dot{W}_2 / \dot{Q}_H$$

For the second heat engine and the energy Eq. for the first heat engine

$$\dot{W}_2 = \eta_2 \dot{Q}_M = \eta_2 (1 - \eta_1) \dot{Q}_H$$

so the final result is

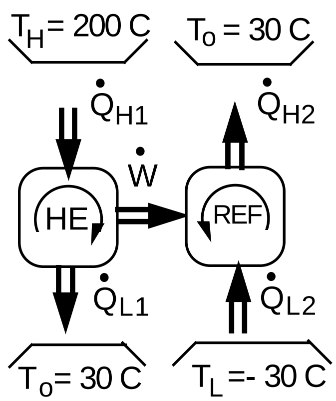
$$\eta_{TH} = \eta_1 + \eta_2 (1 - \eta_1)$$
$$= 0.32 + 0.32(1 - 0.32) = \mathbf{0.538}$$

5.104

We wish to produce refrigeration at −30°C. A reservoir, shown in Fig. P5.104, is available at 200°C and the ambient temperature is 30°C. Thus, work can be done by a cyclic heat engine operating between the 200°C reservoir and the ambient. This work is used to drive the refrigerator. Determine the ratio of the heat transferred from the 200°C reservoir to the heat transferred from the −30°C reservoir, assuming all processes are reversible.

Solution:

Equate the work from the heat engine to the refrigerator.



$$\dot{W} = \dot{Q}_{H1} \left(\frac{T_H - T_0}{T_H} \right)$$

also

$$\dot{W} = \dot{Q}_{L2} \left(\frac{T_0 - T_L}{T_L} \right)$$

$$\dot{Q}_{H1} / \dot{Q}_{L2} = \left(\frac{T_o - T_L}{T_L} \right) \left(\frac{T_H}{T_H - T_o} \right) = \left(\frac{60}{243.2} \right) \left(\frac{473.2}{170} \right) = \mathbf{0.687}$$

5.105

Redo the previous problem, assuming the actual devices both have a performance that is 60% of the theoretical maximum.

For the heat engine this means:

$$\dot{W} = 0.6\ \dot{Q}_{H1} \left(\frac{T_H - T_0}{T_H}\right)$$

For the refrigerator it means:

$$\dot{W} = \dot{Q}_{L2} \left(\frac{T_0 - T_L}{T_L}\right) /0.6$$

The ratio of the two heat transfers becomes

$$\dot{Q}_{H1} / \dot{Q}_{L2} = \left(\frac{T_o - T_L}{T_L}\right) \times \frac{1}{0.6} \times \left(\frac{T_H}{T_H - T_o}\right) \times \frac{1}{0.6}$$

$$= \left(\frac{60}{243.2}\right)\left(\frac{473.2}{170}\right) \frac{1}{0.36} = \mathbf{1.91}$$

As the heat engine delivers less work and the refrigerator requires more work energy from the high T source must increase significantly.

5.106

A house should be heated by a heat pump, $\beta' = 2.2$, and maintained at 20^{o}C at all times. It is estimated that it loses 0.8 kW per degree the ambient is lower than the inside. Assume an outside temperature of -10^{o}C and find the needed power to drive the heat pump?

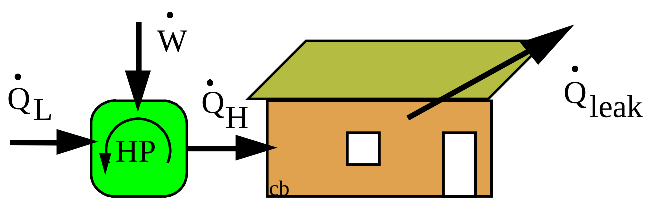
Solution : Ambient $T_L = -10^{o}$C

Heat pump : $\beta' = \dot{Q}_H/\dot{W}$

House : $\dot{Q}_H = \dot{Q}_{leak} = 0.8\ (T_H - T_L)$

$$\dot{W} = \dot{Q}_H/\beta' = \dot{Q}_{leak} / \beta' = 0.8\ (T_H - T_L) / \beta'$$

$$= 0.8[20 - (-10)] /2.2 = \mathbf{10.91\ kW}$$

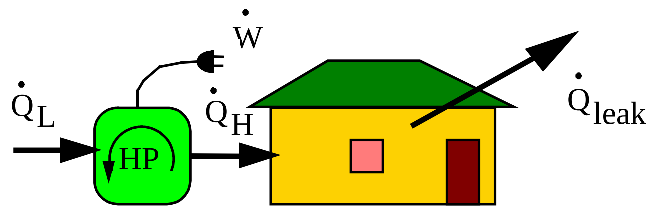


5.107

Give an estimate for the COP in the previous problem and the power needed to drive the heat pump when the outside temperature drops to -15^{o}C.

Solution:

Minimum power if we assume a Carnot cycle



We assume the heat transfer coefficient stays the same

$$\dot{Q}_H = \dot{Q}_{leak} = 25 \text{ kW} = \text{CA } \Delta T = \text{CA } [20 - (-10)] \quad \Rightarrow \quad \text{CA} = \frac{5}{6} \text{ kW/K}$$

$$\dot{Q}_{leak\ new} = \text{CA } \Delta T = \frac{5}{6} [20 - (-15)] = 29.167 \text{ kW}$$

$$\beta' = \frac{\dot{Q}_H}{\dot{W}_{IN}} = \frac{T_H}{T_H - T_L} = \frac{293.15}{35} = 8.3757 \quad \Rightarrow \quad \dot{W}_{IN} = \frac{29.167}{8.3757} = \mathbf{3.48\ kW}$$

Comment. Leak heat transfer increases and COP is lower when T outside drops.

5.108

A farmer runs a heat pump with a motor of 2 kW. It should keep a chicken hatchery at 30°C which loses energy at a rate of 0.5 kW per degree difference to the colder ambient. The heat pump has a coefficient of performance that is 50% of a Carnot heat pump. What is the minimum ambient temperature for which the heat pump is sufficient?

Solution:

C.V. Hatchery, steady state.

To have steady state at 30°C for the hatchery

Energy Eq.: $\dot{Q}_H = \dot{Q}_{Loss} = \beta_{AC}\dot{W}$

Process Eq.: $\dot{Q}_{Loss} = 0.5\ (T_H - T_{amb})$; $\beta_{AC} = ½\ \beta_{CARNOT}$

COP for the reference Carnot heat pump

$$\beta_{CARNOT} = \frac{\dot{Q}_H}{\dot{W}} = \frac{\dot{Q}_H}{\dot{Q}_H - \dot{Q}_L} = \frac{T_H}{T_H - T_L} = \frac{T_H}{T_H - T_{amb}}$$

Substitute the process equations and this β_{CARNOT} into the energy Eq.

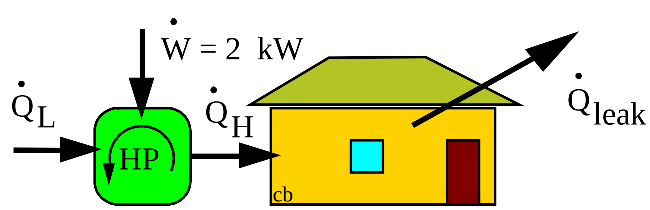
$$0.5\ \text{kW/K}\ (T_H - T_{amb}) = ½\ \frac{T_H}{T_H - T_{amb}}\ \dot{W}$$

$$(T_H - T_{amb})^2 = ½\ T_H\dot{W}/0.5\ \text{kW/K} = T_H\dot{W} = (273 + 30)\ \text{K} \times 2\ \text{K} = 606\ \text{K}^2$$

$$T_H - T_{amb} = 24.62\ \text{K}$$

$$T_{amb} = 30 - 24.62 = \mathbf{5.38^oC}$$

Comment: That of course is not a very low temperature and the size of the system is not adequate for most locations.



5.109

An air-conditioner with a power input of 1.2 kW is working as a refrigerator (β = 3) or as a heat pump (β' = 4). It maintains an office at 20°C year round which exchanges 0.5 kW per degree temperature difference with the atmosphere. Find the maximum and minimum outside temperature for which this unit is sufficient.

Solution:

Analyze the unit in heat pump mode

Replacement heat transfer equals the loss: $\dot{Q} = 0.5 \text{ kW/K } (T_H - T_{amb})$

$$\dot{W} = \frac{\dot{Q}_H}{\beta'} = 0.5 \text{ kW/K} \frac{T_H - T_{amb}}{4}$$

$$T_H - T_{amb} = 4 \frac{\dot{W}}{0.5} = 9.6 \text{ K}$$

Heat pump mode: Minimum $T_{amb} = 20 - 9.6 =$ **10.4 °C**

The unit as a refrigerator must cool with rate: $\dot{Q} = 0.5 (T_{amb} - T_{house})$

$$\dot{W} = \frac{\dot{Q}_L}{\beta}$$ **Error! Bookmark not defined.** $= 0.5 \text{ kW/K } (T_{amb} - T_{house}) / 3$

$$T_{amb} - T_{house} = 3 \frac{\dot{W}}{0.5} = 7.2 \text{ K}$$

Refrigerator mode: Maximum $T_{amb} = 20 + 7.2 =$ **27.2 °C**

5.110

An air-conditioner on a hot summer day removes 8 kW of energy from a house at 21^{o}C and pushes energy to the outside which is at at 31^{o}C. The house has 15 000 kg mass with an average specific heat of 0.95 kJ/kgK. In order to do this the cold side of the air-conditioner is at 5^{o}C and the hot side is at 40^{o}C. The air-conditioner (refrigerator) has a COP that is 60% of a corresponding Carnot refrigerator. Find the actual COP of the air-conditioner and the power required to run it.

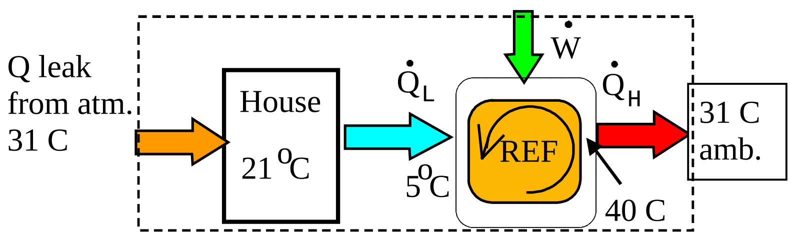
A steady state refrigerator definition of COP

$$COP = \beta_{REF} = \dot{Q}_L / \dot{W} = \dot{Q}_L / (\dot{Q}_H - \dot{Q}_L) = 0.6\ \beta_{Carnot}$$

Carnot: $\beta_{Carnot} = \dfrac{T_L}{T_H - T_L} = \dfrac{5 + 273.15}{40 - 5} = 7.95$

$$\beta_{REF} = 0.6 \times 7.95 = \mathbf{4.77}$$

$\Rightarrow$ $\dot{W} = \dot{Q}_L / \beta_{REF} = 8\text{ kW}/ 4.77 = \mathbf{1.68\ kW}$



Any heat transfer must go from a higher to a lower T domain.

5.111

The air-conditioner in the previous problem is turned off. How quickly does the house heat up in degrees per second ($^oC/s$)?

Once the A/C unit is turned off we do not cool the house so heat leaks in from the atm. at a rate of 8 kW (that is what we had to remove to keep steady state).

Energy Eq.: $\dot{E}_{CV} = \dot{Q}_{leak} = 8 \text{ kW} = m_{house}\, C_P \,\frac{dT}{dt}$

$$\frac{dT}{dt} = \dot{Q}_{leak} / m_{house}\, C_P$$

$$= \frac{8 \text{ kW}}{15\,000 \times 0.95 \text{ kJ/K}} = \mathbf{0.56 \times 10^{-3}\ K/s}$$

5.112

Air in a rigid 1 m^3 box is at 300 K, 200 kPa. It is heated to 600 K by heat transfer from a reversible heat pump that receives energy from the ambient at 300 K besides the work input. Use constant specific heat at 300 K. Since the coefficient of performance changes write $dQ = m_{air} C_v dT$ and find dW. Integrate dW with temperature to find the required heat pump work.

Solution:

COP: $$\beta' = \frac{Q_H}{W} = \frac{Q_H}{Q_H - Q_L} \cong \frac{T_H}{T_H - T_L}$$

$$m_{air} = P_1V_1 / RT_1 = 200 \times 1 / 0.287 \times 300 = 2.322 \text{ kg}$$

$$dQ_H = m_{air} C_v \, dT_H = \beta' \, dW \cong \frac{T_H}{T_H - T_L} \, dW$$

$$\Rightarrow \quad dW = m_{air} C_v \left[\frac{T_H}{T_H - T_L} \right] dT_H$$

$$_1W_2 = \int m_{air} C_v \left(1 - \frac{T_L}{T} \right) dT = m_{air} C_v \int \left(1 - \frac{T_L}{T} \right) dT$$

$$= m_{air} C_v \left[T_2 - T_1 - T_L \ln \frac{T_2}{T_1} \right]$$

$$= 2.322 \text{ kg} \times 0.717 \text{ kJ/kg-K} \left[600 - 300 - 300 \ln \frac{600}{300} \right] \text{K} = \mathbf{153.1 \ kJ}$$

5.113

A Carnot heat engine, shown in Fig. P5.113, receives energy from a reservoir at T_{res} through a heat exchanger where the heat transferred is proportional to the temperature difference as $\dot{Q}_H = K(T_{res} - T_H)$. It rejects heat at a given low temperature T_L. To design the heat engine for maximum work output show that the high temperature, T_H, in the cycle should be selected as $T_H = \sqrt{T_{res}T_L}$

Solution:

$$W = \eta_{TH}Q_H = \frac{T_H - T_L}{T_H} \times K(T_{res} - T_H) \;; \quad \text{maximize } W(T_H) \;\Rightarrow\; \frac{\delta W}{\delta T_H} = 0$$

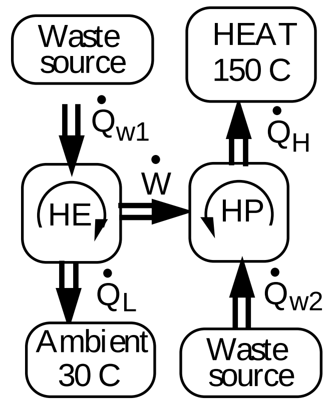
$$\frac{\delta W}{\delta T_H} = K(T_{res} - T_H)T_L T_H^{-2} - K(1 - T_L/T_H) = 0$$

$$\Rightarrow \;\mathbf{T_H = \sqrt{T_{res}T_L}}$$

5.114

A combination of a heat engine driving a heat pump (see Fig. P5.114) takes waste energy at 50°C as a source Q_{w1} to the heat engine rejecting heat at 30°C. The remainder Q_{w2} goes into the heat pump that delivers a Q_H at 150°C. If the total waste energy is 5 MW find the rate of energy delivered at the high temperature.

Solution:



Waste supply: $\dot{Q}_{w1} + \dot{Q}_{w2} = 5$ MW

Heat Engine:

$$\dot{W} = \eta\, \dot{Q}_{w1} = (1 - T_{L1} / T_{H1})\, \dot{Q}_{w1}$$

Heat pump:

$$\dot{W} = \dot{Q}_H / \beta_{HP} = \dot{Q}_{W2} / \beta'$$

$$= \dot{Q}_{w2} / [T_{H1} / (T_H - T_{H1})]$$

Equate the two work terms:

$$(1 - T_{L1} / T_{H1})\, \dot{Q}_{w1} = \dot{Q}_{w2} \times (T_H - T_{H1}) / T_{H1}$$

Substitute $\dot{Q}_{w1} = 5 \text{ MW} - \dot{Q}_{w2}$

$$(1 - 303.15/323.15)(5 - \dot{Q}_{w2}) = \dot{Q}_{w2} \times (150 - 50) / 323.15$$

$$20 (5 - \dot{Q}_{w2}) = \dot{Q}_{w2} \times 100 \quad => \quad \dot{Q}_{w2} = 0.8333 \text{ MW}$$

$$\dot{Q}_{w1} = 5 - 0.8333 = 4.1667 \text{ MW}$$

$$\dot{W} = \eta\, \dot{Q}_{w1} = 0.06189 \times 4.1667 = 0.258 \text{ MW}$$

$$\dot{Q}_H = \dot{Q}_{w2} + \dot{W} = \mathbf{1.09\ MW}$$

(For the heat pump $\beta' = 423.15 / 100 = 4.23$)

5.115

A furnace, shown in Fig. P5.115, can deliver heat, Q_{H1} at T_{H1} and it is proposed to use this to drive a heat engine with a rejection at T_{atm} instead of direct room heating. The heat engine drives a heat pump that delivers Q_{H2} at T_{room} using the atmosphere as the cold reservoir. Find the ratio Q_{H2}/Q_{H1} as a function of the temperatures. Is this a better set-up than direct room heating from the furnace?

Solution:

C.V.: Heat Eng.: $\dot{W}_{HE} = \eta\dot{Q}_{H1}$ where $\eta = 1 - T_{atm}/T_{H1}$

C.V.: Heat Pump: $\dot{W}_{HP} = \dot{Q}_{H2}/\beta'$ where $\beta' = T_{rm}/(T_{rm} - T_{atm})$

Work from heat engine goes into heat pump so we have

$$\dot{Q}_{H2} = \beta' \dot{W}_{HP} = \beta' \eta \dot{Q}_{H1}$$

and we may substitute T's for β', η. If furnace is used directly $\dot{Q}_{H2} = \dot{Q}_{H1}$, so if $\beta'\eta > 1$ this proposed setup is better. Is it? For $T_{H1} > T_{atm}$ formula shows that it is good for Carnot cycles. In actual devices it depends whether $\beta'\eta > 1$ is obtained.

5.116

Consider the rock bed thermal storage in Problem 5.60. Use the specific heat so you can write dQ_H in terms of dT_{rock} and find the expression for dW out of the heat engine. Integrate this expression over temperature and find the total heat engine work output.

Solution:

The rock provides the heat Q_H

$$dQ_H = -dU_{rock} = -mC\ \ dT_{rock}$$

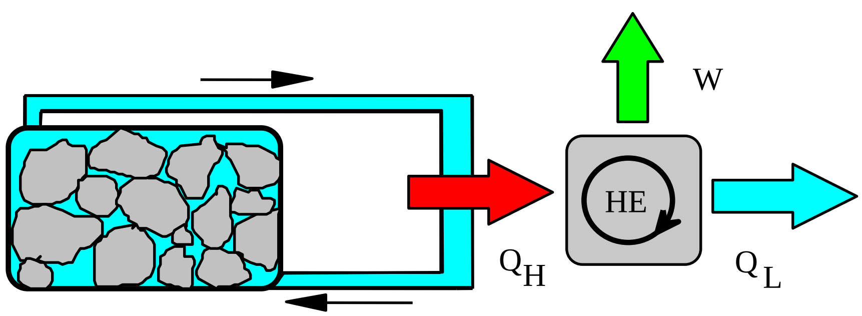
$$dW = \eta dQ_H = -\left(1 - \frac{T_o}{T_{rock}}\right) mC\ dT_{rock}$$

$$m = \rho V = 2750\ kg/m^3 \times 2\ m^3 = 5500\ kg$$

$$_1W_2 = \int -\left(1 - \frac{T_o}{T_{rock}}\right) mC\ dT_{rock} = -\ mC\ [T_2 - T_1 - T_o \ln\frac{T_2}{T_1}]$$

$$= -\ 5500\ kg \times 0.89\ kJ/kg\text{-}K\ [\ 290 - 400 - 290 \ln\frac{290}{400}\ ]\ K$$

$$= \mathbf{81\ 945\ kJ}$$



5.117

Consider a Carnot cycle heat engine operating in outer space. Heat can be rejected from this engine only by thermal radiation, which is proportional to the radiator area and the fourth power of absolute temperature, $\dot{Q}_{rad} \sim KAT^4$. Show that for a given engine work output and given T_H, the radiator area will be minimum when the ratio $T_L/T_H = 3/4$.

Solution:

$$\dot{W}_{NET} = \dot{Q}_H \left(\frac{T_H - T_L}{T_H}\right) = \dot{Q}_L \left(\frac{T_H - T_L}{T_L}\right); \qquad \text{also} \quad \dot{Q}_L = KAT_L^4$$

$$\frac{\dot{W}_{NET}}{KT_H^4} = \frac{AT_L^4}{T_H^4}\left(\frac{T_H}{T_L} - 1\right) = A\left[\left(\frac{T_L}{T_H}\right)^3 - \left(\frac{T_L}{T_H}\right)^4\right] = \text{const}$$

Differentiating,

$$dA\left[\left(\frac{T_L}{T_H}\right)^3 - \left(\frac{T_L}{T_H}\right)^4\right] + A\left[3\left(\frac{T_L}{T_H}\right)^2 - 4\left(\frac{T_L}{T_H}\right)^3\right] d\left(\frac{T_L}{T_H}\right) = 0$$

$$\frac{dA}{d(T_L/T_H)} = - A\left[3\left(\frac{T_L}{T_H}\right)^2 - 4\left(\frac{T_L}{T_H}\right)^3\right] \Big/ \left[\left(\frac{T_L}{T_H}\right)^3 - \left(\frac{T_L}{T_H}\right)^4\right] = 0$$

$$\frac{T_L}{T_H} = \frac{3}{4} \qquad \text{for min. A}$$

Check that it is minimum and not maximum with the 2nd derivative > 0.

5.118

A Carnot heat engine operating between a high T_H and low T_L energy reservoirs has an efficiency given by the temperatures. Compare this to two combined heat engines one operating between T_H and an intermediate temperature T_M giving out work W_A and the other operating between T_M and T_L giving out W_B. The combination must have the same efficiency as the single heat engine so the heat transfer ratio $Q_H/Q_L = \psi(T_H,T_L) = [Q_H/Q_M]\ [Q_M/Q_L]$. The last two heat transfer ratios can be expressed by the same function $\psi()$ involving also the temperature T_M. Use this to show a condition the function $\psi()$ must satisfy.

The overall heat engine is a Carnot heat engine so

$$\dot{Q}_H / \dot{Q}_L = \frac{T_H}{T_L} = \psi(T_H,T_L)$$

The individual heat engines

$$\dot{Q}_H / \dot{Q}_M = \psi(T_H,T_M) \quad \text{and} \quad \dot{Q}_M / \dot{Q}_L = \psi(T_M,T_L)$$

Since an identity is

$$\dot{Q}_H / \dot{Q}_L = [\dot{Q}_H / \dot{Q}_M]\ [\dot{Q}_M / \dot{Q}_L] = \psi(T_H,T_L)$$

it follows that we have

$$\psi(T_H,T_L) = \psi(T_H,T_M) \times \psi(T_M,T_L)$$

Notice here that the product of the two functions must cancel the intermediate temperature T_M, this shows a condition the function $\psi()$ must satisfy. The Kelvin and Rankine temperature scales are determined by the choice of the function

$$\psi(T_H,T_L) = T_H / T_L = \dot{Q}_H / \dot{Q}_L$$

satisfying the above restriction.

5.119

On a cold (-10^{o}C) winter day a heat pump provides 20 kW to heat a house maintained at 20^{o}C and it has a COP_{HP} of 4 using the maximum power available. The next day a winter storm brings the outside to -15^{o}C, assuming that the COP_{HP} changes by the same percentage as a Carnot unit and that the house loses heat to the outside air. How cold is the house then?

If we look at the heat loss for the house we have

$$\dot{Q}_{loss} = 20 \text{ kW} = CA\,\Delta T \quad \Rightarrow \quad CA = \frac{20 \text{ kW}}{20 - (-10) \text{ K}} = 0.667 \text{ kW/K}$$

$$\dot{Q}_{loss} = \dot{Q}_H = COP\ \dot{W} \quad \Rightarrow \quad \dot{W} = \dot{Q}_{loss}/COP = \frac{20 \text{ kW}}{4} = 5 \text{ kW}$$

$$\text{At } = -10^oC: \qquad \beta'_{Carnot\ 1} = \frac{T_H}{T_H - T_{L1}} = \frac{293.15}{20 - (-10)} = 9.772$$

With a changed COP we get

$$COP_{HP} = 4\ \beta'_{Carnot\ 2}/\beta'_{Carnot\ 1} = (4/9.772)\ \beta'_{Carnot\ 2}$$

At the new unknown temperature of the house, T_H, we have

$$\beta'_{Carnot\ 2} = \frac{T_H}{T_H - T_{L2}}\ ; \qquad \dot{Q}_{loss} = CA\,(T_H - T_{L2})$$

The energy equation for the house becomes $\dot{Q}_{loss} = \dot{Q}_H$ and substitution gives

$$0.667 \text{ kW/K}\,(T_H - T_{L2}) = COP_{HP}\ \dot{W} = (4/9.772)\,\frac{T_H}{T_H - T_{L2}}\,5 \text{ kW}$$

or $$(T_H - T_{L2})^2 = \frac{4 \times 5 \text{ kW}}{9.772 \times 0.667 \text{ kW/K}}\,T_H = 3.07 \text{ K} \times T_H$$

Solve with $x = T_H - T_{L2}$, $\ x^2 - 3.07\,x - 3.07(273.15 - 15) = 0$,

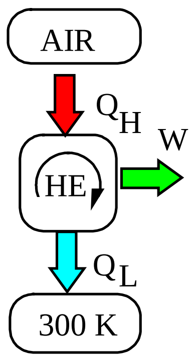
$$x = 3.07/2 + [\,(3.07/2)^2 + 792.52\,]^{1/2} = 29.729$$

$$T_H = x + T_{L2} = 29.729 - 15 = \mathbf{14.7^oC}$$

5.120

A 10-m^3 tank of air at 500 kPa, 600 K acts as the high-temperature reservoir for a Carnot heat engine that rejects heat at 300 K. A temperature difference of 25°C between the air tank and the Carnot cycle high temperature is needed to transfer the heat. The heat engine runs until the air temperature has dropped to 400 K and then stops. Assume constant specific heat capacities for air and find how much work is given out by the heat engine.

Solution:



$$T_H = T_{air} - 25°C, \quad T_L = 300 \text{ K}$$

$$m_{air} = \frac{P_1V}{RT_1} = \frac{500 \times 10}{0.287 \times 600} = 29.04 \text{ kg}$$

$$dW = \eta dQ_H = \left(1 - \frac{T_L}{T_{air} - 25}\right) dQ_H$$

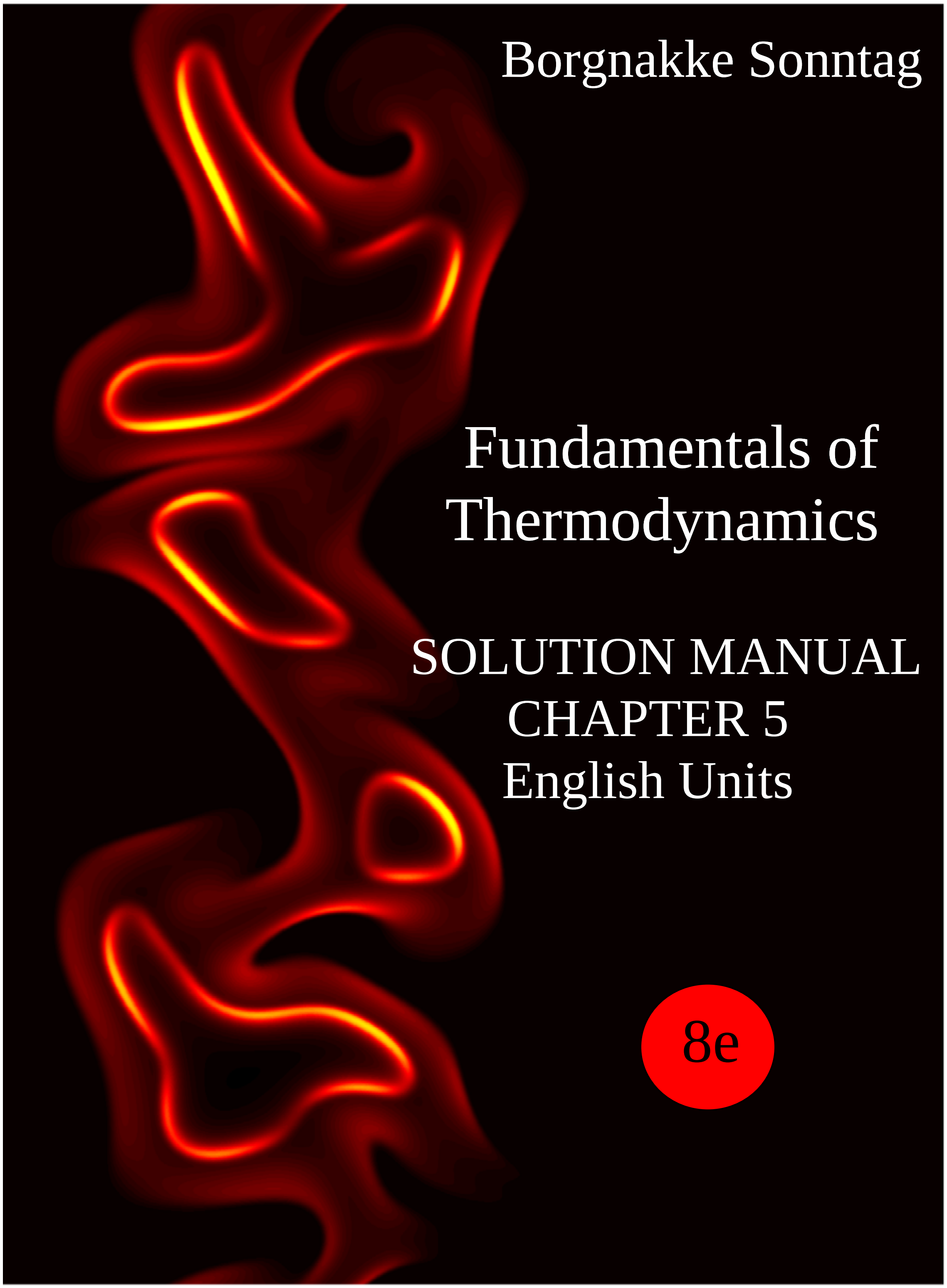
$$dQ_H = -m_{air}du = -m_{air}C_v dT_{air}$$

$$W = \int dW = -m_{air}C_v \int \left[1 - \frac{T_L}{T_a - 25}\right] dT_a$$

$$= -m_{air}C_v \left[T_{a2} - T_{a1} - T_L \ln \frac{T_{a2} - 25}{T_{a1} - 25}\right]$$

$$= -29.04 \text{ kg} \times 0.717 \text{ kJ/kg-K} \times \left[400 - 600 - 300 \ln \frac{375}{575}\right] \text{K}$$

$$= \mathbf{1494.3\ kJ}$$



UPDATED JULY 2013

CHAPTER 5

SUBSECTION	PROB NO.
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Finite ΔT Heat Transfer	146-152
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Heat Engines and Refrigerators

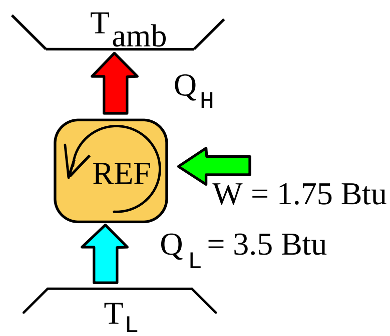
5.121E

A window mounted air-conditioner removes 3.5 Btu from the inside of a home using 1.75 Btu work input. How much energy is released outside and what is its coefficient of performance?

C.V. Refrigerator. The energy Q_H goes into the kitchen air.

Energy Eq.: $Q_H = W + Q_L = 1.75 + 3.5 = \mathbf{5.25\ btu}$

COP: $\beta = \dfrac{Q_L}{W} = 3.5 / 1.75 = \mathbf{2}$



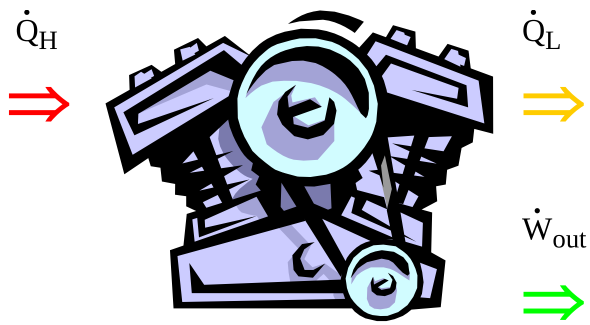
5.122E

A lawnmower tractor engine produces 18 hp using 40 Btu/s of heat transfer from burning fuel. Find the thermal efficiency and the rate of heat transfer rejected to the ambient?

Conversion Table A.1: 18 hp = 18 × (2544.4/3600) Btu/s = 12.722 Btu/s

Efficiency: $\eta_{TH} = \dot{W}_{out}/\dot{Q}_H = \dfrac{12.722}{40} = \mathbf{0.318}$

Energy equation: $\dot{Q}_L = \dot{Q}_H - \dot{W}_{out} = 40 - 12.72 = \mathbf{27.3\ Btu/s}$



5.123E

Calculate the thermal efficiency of the steam power plant cycle described in Problem 4.198.

Solution:

From solution to problems 4.198, 199

$$\dot{W}_{NET} = 33\,000 - 400 = 32\,600 \text{ hp} = 8.3 \times 10^7 \text{ Btu/h}$$

$$\dot{Q}_{H,tot} = \dot{Q}_{econ} + \dot{Q}_{gen}$$

$$= 4.75 \times 10^7 + 2.291 \times 10^8 = 2.766 \times 10^8 \text{ Btu/h;}$$

$$\eta = \frac{\dot{W}}{\dot{Q}_H} = \frac{8.3 \times 10^7}{2.766 \times 10^8} = \mathbf{0.30}$$

5.124E

A large coal fired power plant has an efficiency of 45% and produces net 1500 MW of electricity. Coal releases 12 500 Btu/lbm as it burns so how much coal is used per hour?

From the definition of the thermal efficiency and the energy release by the combustion called heating value HV we get

$$\dot{W} = \eta\ \dot{Q}_H = \eta \cdot \dot{m} \cdot HV$$

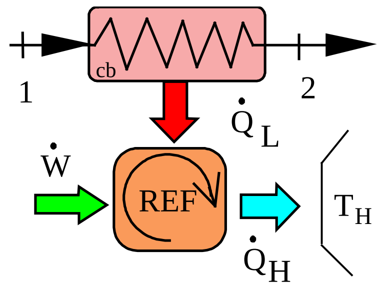
then

$$\dot{m} = \frac{\dot{W}}{\eta \times HV} = \frac{1500\text{ MW}}{0.45 \times 12\,500\text{ btu/lbm}} = \frac{1500 \times (1000/1.055)\text{ Btu/s}}{0.45 \times 12\,500\text{ Btu/lbm}}$$

$$= 252.765\text{ lbm/s} = \mathbf{909\,950\ lbm/h}$$

5.125E

An industrial machine is being cooled by 0.8 lbm/s water at 60 F which is chilled from 95 F by a refrigeration unit with a COP of 3. Find the rate of cooling required and the power input to the unit.



Energy equation for heat exchanger

$$\dot{Q}_L = \dot{m}(h_1 - h_2) = \dot{m}\, C_P\, (T_1 - T_2)$$

$$= 0.8 \text{ lbm/s} \times 1 \text{ Btu/lbm-R} \times (95 - 60) \text{ R}$$

$$= 28 \text{ Btu/s}$$

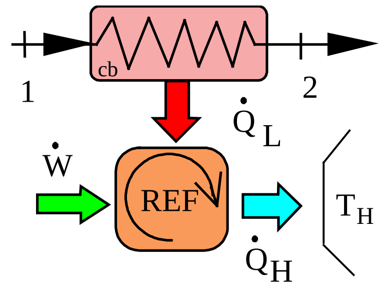
$$\beta = \text{COP} = \dot{Q}_L / \dot{W} \quad \Rightarrow$$

$$\dot{W} = \dot{Q}_L / \beta = 28 / 3 = \mathbf{9.33\ Btu/s}$$

Comment: An outside cooling tower is often used for this, see Chapter 11.

5.126E

A water cooler for drinking water should cool 10 gal/h water from 65 F to 50 F using a small refrigeration unit with a COP of 2.5. Find the rate of cooling required and the power input to the unit.



The mass flow rate is

$$\dot{m} = \boldsymbol{\rho}\dot{V} = \frac{10 \times 231/12^3}{0.01603} \frac{1}{3600} \text{ lbm/s} = 0.0232 \text{ lbm/s}$$

Energy equation for heat exchanger

$$\dot{Q}_L = \dot{m}(h_1 - h_2) = \dot{m}\, C_P\, (T_1 - T_2)$$

$$= 0.0232 \times 1.0 \times (65 - 50) = 0.348 \text{ Btu/s}$$

$$\beta = \text{COP} = \dot{Q}_L / \dot{W} \quad \Rightarrow \quad \dot{W} = \dot{Q}_L / \beta = 0.348 \text{ (Btu/s)} / 2.5 = \mathbf{0.139\ Btu/s}$$

Comment: The unit does not operate continuously.

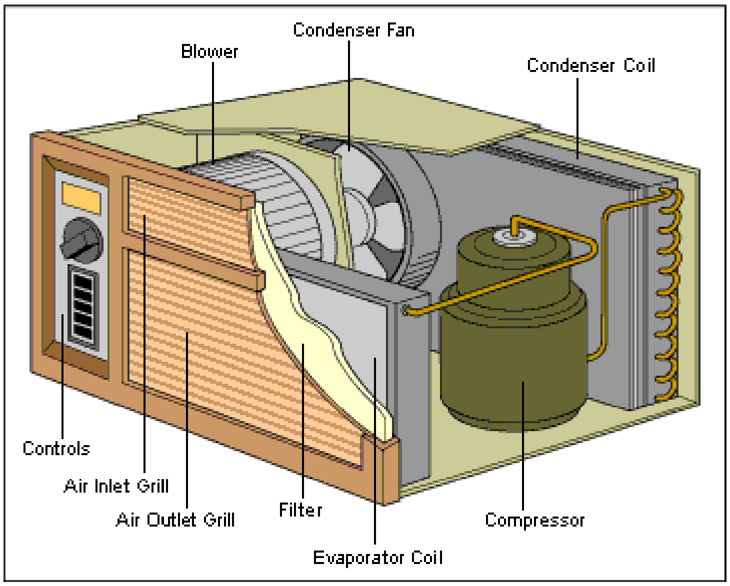
5.127E

A window air-conditioner unit is placed on a laboratory bench and tested in cooling mode using 0.75 Btu/s of electric power with a COP of 1.75. What is the cooling power capacity and what is the net effect on the laboratory?

Definition of COP: $\beta = \dot{Q}_L / \dot{W}$

Cooling capacity: $\dot{Q}_L = \beta \dot{W} = 1.75 \times 0.75 =$ **1.313 Btu/s**

For steady state operation the $\dot{Q}_L$ comes from the laboratory and $\dot{Q}_H$ goes to the laboratory giving a net to the lab of $\dot{W} = \dot{Q}_H - \dot{Q}_L = 0.75$ Btu/s, that is heating it.



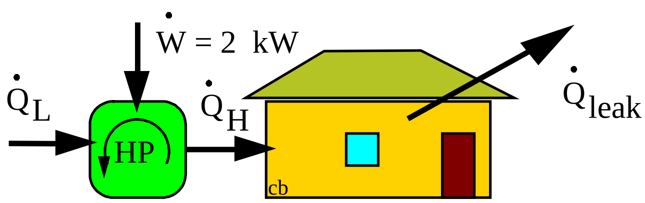
5.128E

A farmer runs a heat pump with a 2 kW motor. It should keep a chicken hatchery at 90 F, which loses energy at a rate of 10 Btu/s to the colder ambient T_{amb}. What is the minimum coefficient of performance that will be acceptable for the heat pump?

Power input: $\dot{W} = 2 \text{ kW} = 2 \times 2544.4 / 3600 = 1.414 \text{ Btu/s}$

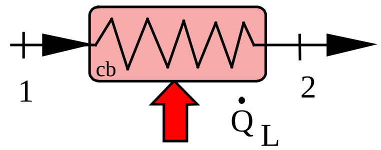
Energy Eq. for hatchery: $\dot{Q}_H = \dot{Q}_{Loss} = 10 \text{ Btu/s}$

Definition of COP: $\beta = \text{COP} = \dfrac{\dot{Q}_H}{\dot{W}} = \dfrac{10}{1.414} = \mathbf{7.07}$



5.129E

R-410A enters the evaporator (the cold heat exchanger) in an A/C unit at 0 F, x = 28% and leaves at 0 F, x = 1. The COP of the refrigerator is 1.5 and the mass flow rate is 0.006 lbm/s. Find the net work input to the cycle.



Energy equation for heat exchanger

$$\dot{Q}_L = \dot{m}(h_2 - h_1) = \dot{m}[h_g - (h_f + x_1 h_{fg})]$$

$$= \dot{m}[h_{fg} - x_1 h_{fg}] = \dot{m}\,(1 - x_1)h_{fg}$$

$$= 0.006 \text{ lbm/s} \times 0.72 \times 103.76 \text{ Btu/lbm} = 0.448 \text{ Btu/s}$$

$$\beta = \text{COP} = \dot{Q}_L / \dot{W} \quad \Rightarrow \quad \dot{W} = \dot{Q}_L / \beta = 0.448 / 1.5 = \mathbf{0.3\ Btu/s}$$

5.130E

A large stationary diesel engine produces 2 000 hp with a thermal efficiency of 40%. The exhaust gas, which we assume is air, flows out at 1400 R and the intake is 520 R. How large a mass flow rate is that if that accounts for half the $\dot{Q}_L$? Can the exhaust flow energy be used?

Power $\quad$ 2 000 hp = 2 000 × 2544.4 / 3600 = 1413.6 Btu/s

Heat engine: $\quad \dot{Q}_H = \dot{W}_{out}/\eta_{TH} = \frac{14\,136}{0.4} = 3534$ Btu/s

Energy equation: $\quad \dot{Q}_L = \dot{Q}_H - \dot{W}_{out} = 3534 – 1413.6 = 2120.4$ Btu/s

Exhaust flow: $\quad \frac{1}{2}\dot{Q}_L = \dot{m}_{air}(h_{1400} - h_{520})$

$$\dot{m}_{air} = \frac{1}{2}\frac{\dot{Q}_L}{h_{1400} - h_{520}} = \frac{1}{2}\frac{2120.4}{343.02 - 124.38} = \mathbf{4.85\ lbm/s}$$

The flow of hot gases can be used to heat a building or it can be used to heat water in a steam power plant since it operates at lower temperatures.

5.131E

Calculate the amount of work input a refrigerator needs to make ice cubes out of a tray of 0.5 lbm liquid water at 50 F. Assume the refrigerator has $\beta = 3.5$ and a motor-compressor of 750 W. How much time does it take if this is the only cooling load?

Solution:

C.V. Water in tray. We neglect tray mass.

Energy Eq.: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Process : $P = \text{constant} = P_o$

$${}_1W_2 = \int P\, dV = P_o m(v_2 - v_1)$$

$${}_1Q_2 = m(u_2 - u_1) + {}_1W_2 = m(h_2 - h_1)$$

Tbl. F.7.1 : $h_1 = 18.05$ Btu/lbm, Tbl. F.7.4 : $h_2 = -143.34$ Btu/lbm

$${}_1Q_2 = 0.5(-143.34 - 18.05) = -80.695 \text{ Btu}$$

Consider now refrigerator

$$\beta = Q_L/W$$

$$W = Q_L/\beta = -{}_1Q_2/\beta = 80.695/3.5 = \mathbf{23.06\ Btu}$$

For the motor to transfer that amount of energy the time is found as

$$W = \int \dot{W}\, dt = \dot{W}\, \Delta t$$

$$\Delta t = W/\dot{W} = (23.06 \times 1055)/750 = \mathbf{32.4\ s}$$

Comment: We neglected a baseload of the refrigerator so not all the 750 W are available to make ice, also our coefficient of performance is very optimistic and finally the heat transfer is a transient process. All this means that it will take much more time to make ice-cubes.

Carnot Cycles and Absolute T

7.132E

Calculate the thermal efficiency of a Carnot-cycle heat engine operating between reservoirs at 920 F and 110 F. Compare the result with that of Problem 5.123.

Solution:

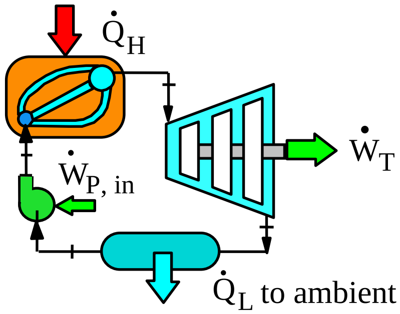
$T_H = 920 \text{ F}, \quad T_L = 110 \text{ F}$

$$\eta_{Carnot} = 1 - \frac{T_L}{T_H} = 1 - \frac{110 + 459.67}{920 + 459.67} = \mathbf{0.587} \quad \text{(about twice 5.123: 0.3)}$$

5.133E

A steam power plant has 1200 F in the boiler, 630 Btu/s work out of turbine, 900 Btu/s is taken out at 100 F in the condenser and the pump work is 30 Btu/s. Find the plant thermal efficiency. Assume the same pump work and heat transfer to the boiler, how much is the turbine power if the plant is running in a Carnot cycle?

Solution:



CV. Total plant:

Energy Eq.:

$$\dot{Q}_H + \dot{W}_{P,in} = \dot{W}_T + \dot{Q}_L$$

$$\dot{Q}_H = 630 + 900 - 30 = 1500 \text{ Btu/s}$$

$$\eta_{TH} = \frac{\dot{W}_T - \dot{W}_{P,in}}{\dot{Q}_H}$$ **Error! Bookmark not defined.** $= \frac{600}{1500} = \mathbf{0.40}$

$$\eta_{carnot} = \dot{W}_{net}/\,\dot{Q}_H = 1 - T_L/T_H = 1 - \frac{100 + 459.67}{1200 + 459.67} = 0.663$$

$$\dot{W}_T - \dot{W}_{P,in} = \eta_{carnot}\dot{Q}_H = 0.663 \times 1500 \text{ Btu/s} = 995 \text{ Btu/s}$$

$$=> \qquad \dot{W}_T = 995 + 30 = \mathbf{1025}\ \frac{\mathbf{Btu}}{\mathbf{s}}$$

5.134E

A large heat pump should upgrade 4000 Btu/s of heat at 175 F to be delivered as heat at 280 F. What is the minimum amount of work (power) input that will drive this?

For the minimum work we assume a Carnot heat pump and $\dot{Q}_L$ = 4000 Btu/s.

$$\beta_{HP} = \frac{\dot{Q}_H}{\dot{W}_{in}} = \frac{T_H}{T_H - T_L} = \frac{459.7 + 280}{280 - 175} = 7.04$$

$$\beta_{REF} = \beta_{HP} - 1 = \frac{\dot{Q}_L}{\dot{W}_{in}} = 6.04$$

Now we can solve for the work

$$\dot{W}_{in} = \dot{Q}_L/\beta_{REF} = 4000/6.04 = \mathbf{662\ Btu/s}$$

This is a domestic or small office building size A/C unit, much smaller than the 4000 Btu/s in this problem.

5.135E

A car engine burns 10 lbm of fuel (equivalent to addition of Q_H) at 2600 R and rejects energy to the radiator and the exhaust at an average temperature of 1300 R. If the fuel provides 17 200 Btu/lbm what is the maximum amount of work the engine can provide?

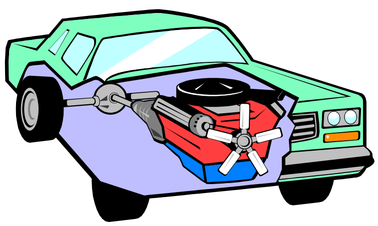
Solution:

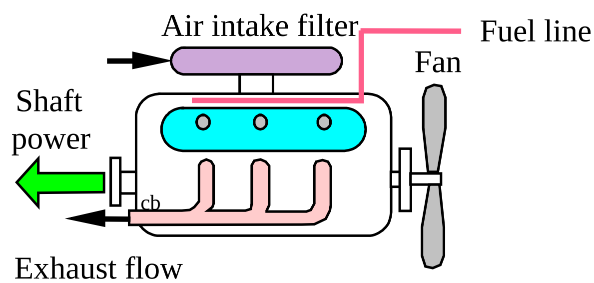
A heat engine $Q_H = m\ q_{fuel} = 10 \times 17200 = 170\ 200$ Btu

Assume a Carnot efficiency (maximum theoretical work)

$$\eta = 1 - \frac{T_L}{T_H} = 1 - \frac{1300}{2600} = 0.5$$

$$W = \eta\ Q_H = 0.5 \times 170\ 200 = \mathbf{85\ 100\ Btu}$$





5.136E

Consider the combination of a heat engine and a heat pump as given in problem 5.41 with a low temperature of 720 R. What should the high temperature be so the heat engine is reversible? For that temperature what is the COP for a reversible heat pump?

For all three cases of the heat engine the ratio between the heat transfers and the work term is the same as:

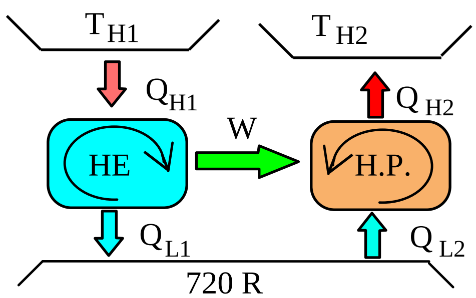
$$\dot{Q}_H : \dot{Q}_L : \dot{W} \quad = \quad 6:4:2 \;\; = \;\; 3:2:1$$

For a reversible heat engine we must have the heat transfer ratio equal to the temperature ratio so

$$\frac{\dot{Q}_H}{\dot{Q}_L} = \frac{T_H}{T_L} = \frac{3}{2} = \frac{T_H}{720\text{ R}} \quad \Rightarrow \quad T_H = (3/2)\ 720\text{ R} = \mathbf{1080\ R}$$

The COP is

$$COP_{HP} = \dot{Q}_H / \dot{W} = \frac{3}{1} = \mathbf{3} \quad \left(= \frac{T_H}{T_H - T_L} = \frac{1080}{1080 - 720} \right)$$



5.137E

An air-conditioner provides 1 lbm/s of air at 60 F cooled from outside atmospheric air at 95 F. Estimate the amount of power needed to operate the air-conditioner. Clearly state all assumptions made.

Solution:

Consider the cooling of air which needs a heat transfer as

$$\dot{Q}_{air} = \dot{m}\,\Delta h \cong \dot{m}\,C_p\,\Delta T = 1 \text{ lbm/s} \times 0.24 \text{ Btu/lbm-R} \times (95 - 60) \text{ R}$$
$$= 8.4 \text{ Btu/s}$$

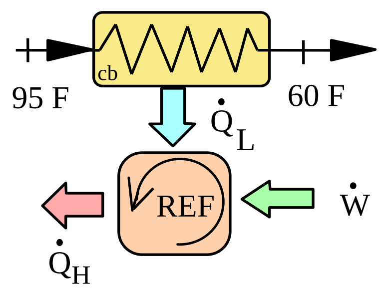
Assume Carnot cycle refrigerator

$$\beta = \frac{\dot{Q}_L}{\dot{W}} = \dot{Q}_L / (\dot{Q}_H - \dot{Q}_L\,) \cong \frac{T_L}{T_H - T_L} = \frac{60 + 459.67}{95 - 60} = 14.8$$

$$\dot{W} \;=\; \dot{Q}_L / \beta = \frac{8.4}{14.8} = \mathbf{0.57\ Btu/s}$$

This estimate is the theoretical maximum performance. To do the required heat transfer $T_L \cong 40$ F and $T_H = 110$ F are more likely; secondly

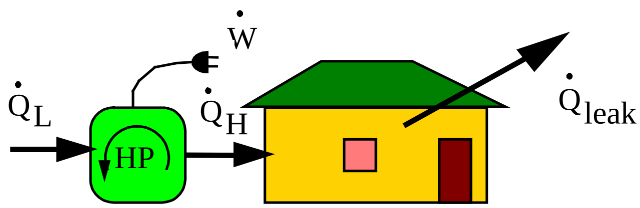
$$\beta < \beta_{carnot}$$



5.138E

We propose to heat a house in the winter with a heat pump. The house is to be maintained at 68 F at all times. When the ambient temperature outside drops to 15 F, the rate at which heat is lost from the house is estimated to be 80 000 Btu/h. What is the minimum electrical power required to drive the heat pump?

Solution:



Minimum power if we assume a Carnot cycle

$\dot{Q}_H = \dot{Q}_{leak} = 80\ 000$ Btu/h

$$\beta' = \frac{\dot{Q}_H}{\dot{W}_{IN}} = \frac{T_H}{T_H - T_L} = \frac{527.7}{68 - 15} = 9.957$$

$$\Rightarrow \dot{W}_{IN} = 80\ 000 / 9.957 = 8035 \text{ Btu/h} = \mathbf{2.355\ kW}$$

5.139E

Consider the setup with two stacked (temperature wise) heat engines as in Fig. P5.4. Let T_H = 1500 R, T_M = 1000 R and T_L = 650 R. Find the two heat engine efficiencies and the combined overall efficiency assuming Carnot cycles.

The individual efficiencies

$$\eta_1 = 1 - \frac{T_M}{T_H} = 1 - \frac{1000}{1500} = \mathbf{0.333}$$

$$\eta_2 = 1 - \frac{T_L}{T_M} = 1 - \frac{650}{1000} = \mathbf{0.35}$$

The overall efficiency

$$\eta_{TH} = \dot{W}_{net} / \dot{Q}_H = (\dot{W}_1 + \dot{W}_2) / \dot{Q}_H = \eta_1 + \dot{W}_2 / \dot{Q}_H$$

For the second heat engine and the energy Eq. for the first heat engine

$$\dot{W}_2 = \eta_2 \, \dot{Q}_M = \eta_2 \, (1 - \eta_1) \, \dot{Q}_H$$

so the final result is

$$\eta_{TH} = \eta_1 + \eta_2 \, (1 - \eta_1) = 0.333 + 0.35 \, (1 - 0.333) = \mathbf{0.566}$$

Comment: It matches a single heat engine $\eta_{TH} = 1 - \frac{T_L}{T_H} = 1 - \frac{650}{1500} = 0.567$

5.140E

A thermal storage is made with a rock (granite) bed of 70 ft^3 which is heated to 720 R using solar energy. A heat engine receives a Q_H from the bed and rejects heat to the ambient at 520 R. The rock bed therefore cools down and as it reaches 520 R the process stops. Find the energy the rock bed can give out. What is the heat engine efficiency at the beginning of the process and what is it at the end of the process?

Solution:

Assume the whole setup is reversible and that the heat engine operates in a Carnot cycle. The total change in the energy of the rock bed is

$$u_2 - u_1 = q = C\,\Delta T = 0.21 \text{ Btu/lbm-R } (720 - 520) \text{ R} = 42 \text{ Btu/lbm}$$

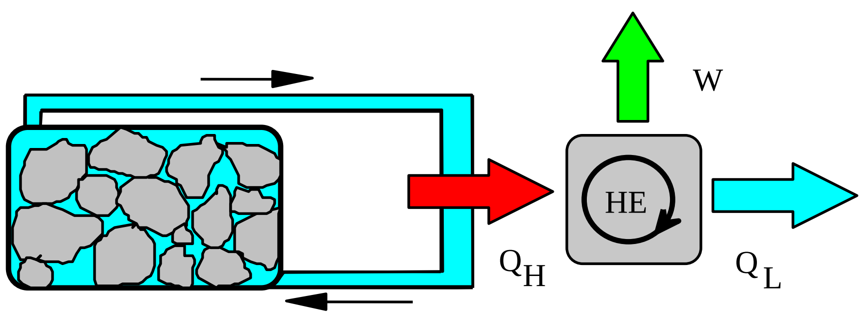
$$m = \rho V = 172 \text{ lbm/ft}^3 \times 70 \text{ ft}^3 = 12040 \text{ lbm;}$$

$$Q = mq = 12040 \text{ lbm} \times 42 \text{ Btu/lbm} = \mathbf{505\ 680\ Btu}$$

To get the efficiency assume a Carnot cycle device

$\eta = 1 - T_o / T_H = 1 - 520/720 = \mathbf{0.28}$ at the beginning of process

$\eta = 1 - T_o / T_H = 1 - 520/520 = \mathbf{0}$ at the end of process



5.141E

A heat engine has a solar collector receiving 600 Btu/h per square foot inside which a transfer media is heated to 800 R. The collected energy powers a heat engine which rejects heat at 100 F. If the heat engine should deliver 8500 Btu/h what is the minimum size (area) solar collector?

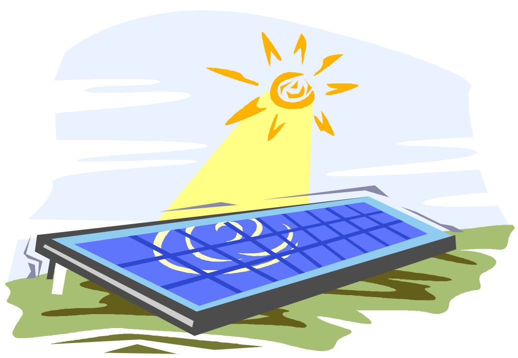
Solution:

$T_H = 800 \text{ R} \qquad T_L = 100 + 459.67 = 560 \text{ R}$

$$\eta_{HE} = 1 - \frac{T_L}{T_H} = 1 - \frac{560}{800} = 0.30$$

$$\dot{W} = \eta \, \dot{Q}_H \quad => \quad \dot{Q}_H = \frac{\dot{W}}{\eta} = \frac{8500}{0.30} = 28\,333 \text{ Btu/h}$$

$$\dot{Q}_H = 600 \text{ A} \quad => \quad A = \frac{\dot{Q}_H}{600} = \mathbf{47 \ ft^2}$$



5.142E

Six-hundred pound-mass per hour of water runs through a heat exchanger, entering as saturated liquid at 250 F and leaving as saturated vapor. The heat is supplied by a Carnot heat pump operating from a low-temperature reservoir at 60 F with a COP of half of the similar carnot unit. Find the rate of work into the heat pump.

Solution:

C.V. Heat exchanger

$\dot{m}_1 = \dot{m}_2$; $\dot{m}_1 h_1 + \dot{Q}_H = \dot{m}_1 h_2$

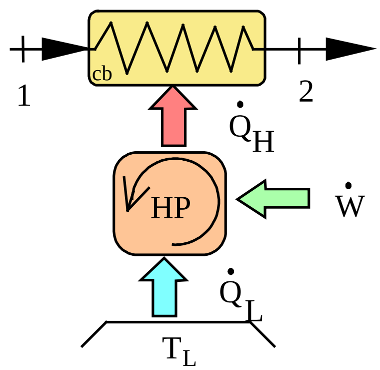


Table F.7.1: h_1 = 218.58 Btu/lbm

h_2 = 1164.19 Btu/lbm

$$\dot{Q}_H = \frac{600}{3600}(1164.19 - 218.58) = 157.6 \text{ Btu/s}$$

For the Carnot heat pump, T_H = 250 F = 710 R.

$$\beta = \dot{Q}_H/\dot{W} = \frac{T_H}{T_H - T_L} = \frac{710}{190} = 3.737$$

$$\beta_{ac} = 3.737/2 = 1.87$$

$$\dot{W} = \dot{Q}_H/\beta_{ac} = 157.6/1.87 = \mathbf{84.3\ Btu/s}$$

5.143E

A power plant with a thermal efficiency of 40% is located on a river similar to Fig. P5.61. With a total river mass flow rate of 2×10^5 lbm/s at 60 F find the maximum power production allowed if the river water should not be heated more than 2 F.

The maximum heating allowed determines the maximum $\dot{Q}_L$ as

$$\dot{Q}_L = \dot{m}_{H_2O}\, \Delta h = \dot{m}_{H_2O}\, C_{P\ LIQ\ H_2O}\, \Delta T_{H_2O}$$

$$= 2 \times 10^5 \text{ lbm/s} \times 1.0 \text{ Btu/lbm-R} \times 2 \text{ R} = 4 \times 10^5 \text{ Btu/s}$$

$$= \dot{W}_{NET}(1/\eta_{TH\ ac} - 1)$$

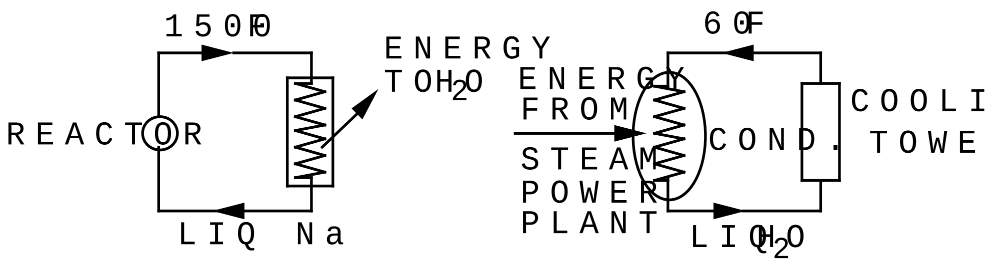
$$\dot{W}_{NET} = \dot{Q}_L / (1/\eta_{TH\ ac} - 1) = \dot{Q}_L \frac{\eta_{TH\ ac}}{1 - \eta_{TH\ ac}}$$

$$= 4 \times 10^5 \text{ Btu/s} \times \frac{0.4}{1 - 0.4} = \mathbf{2.67 \times 10^5\ Btu/s}$$

5.144E

A nuclear reactor provides a flow of liquid sodium at 1500 F, which is used as the energy source in a steam power plant. The condenser cooling water comes from a cooling tower at 60 F. Determine the maximum thermal efficiency of the power plant. Is it misleading to use the temperatures given to calculate this value?

Solution:



T_H = 1500 F = 1960 R, T_L = 60 F = 520 R

$$\eta_{TH\ MAX} = \frac{T_H - T_L}{T_H} = \frac{1960 - 520}{19860} = 0.735$$

It might be misleading to use 1500 F as the value for T_H, since there is not a supply of energy available at a constant temperature of 1500 F (liquid Na is cooled to a lower temperature in the heat exchanger).

⇒ The Na cannot be used to boil H_2O at 1500 F.

Similarly, the H_2O leaves the cooling tower and enters the condenser at 60 F, and leaves the condenser at some higher temperature.

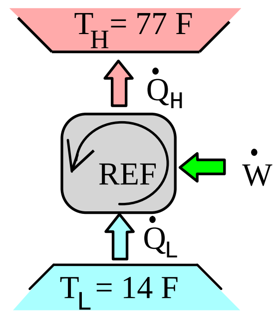
⇒ The water does not provide for condensing steam at a constant temperature of 60 F.

5.145E

An inventor has developed a refrigeration unit that maintains the cold space at 14 F, while operating in a 77 F room. A coefficient of performance of 8.5 is claimed. How do you evaluate this?

Solution:

Assume Carnot cycle then



$$\beta_{Carnot} = \frac{Q_L}{W_{in}} = \frac{T_L}{T_H - T_L} = \frac{14 + 459.67}{77 - 14} = 7.5$$

$8.5 > \beta_{Carnot} \Rightarrow$ **impossible claim**

Finite ΔT Heat Transfer

5.146E

A car engine operates with a thermal efficiency of 35%. Assume the air-conditioner has a coefficient of performance that is one third of the theoretical maximum and it is mechanically pulled by the engine. How much fuel energy should you spend extra to remove 1 Btu at 60 F when the ambient is at 95 F?

Solution:

Air conditioner

$$\beta = \frac{Q_L}{W} = \frac{T_L}{T_H - T_L} = \frac{60 + 459.67}{95 - 60} = 14.8$$

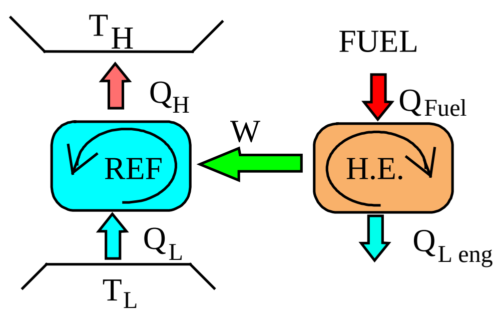
$$\beta_{actual} = \beta / 3 = 4.93$$

$$W = Q_L / \beta = 1 / 4.93 = 0.203 \text{ Btu}$$

Work from engine

$$W = \eta_{eng}\, Q_{fuel} = 0.203 \text{ Btu}$$

$$Q_{fuel} = W / \eta_{eng} = \frac{0.203}{0.35} = \mathbf{0.58\ Btu}$$



5.147E

In a remote location you run a heat engine to provide the power to run a refrigerator. The input to the heat engine is at 1450 R and the low T is 700 R, it has an actual efficiency equal to ½ of the corresponding Carnot unit. The refrigerator has a T_L = 15 F and T_H = 95 F with a COP that is 1/3 of the corresponding Carnot unit. Assume a cooling capacity of 7000 Btu/h is needed and find the rate of heat input to the heat engine.

Heat engine: $\eta_{Carnot} = 1 - \frac{T_L}{T_H} = 1 - \frac{700}{1450} = 0.517; \quad \eta_{ac} = 0.259$

Refrigerator: $\beta_{ref.\ Carnot} = \frac{T_L}{T_H - T_L} = \frac{459.7 + 15}{95 - 15} = 5.934; \quad \beta_{ref.\ ac} = 1.978$

Cooling capacity: $\dot{Q}_L = 7000 \text{ Btu/h} = \beta_{ref.\ ac}\ \dot{W}$;

$$\dot{W} = 7000 \text{ Btu/h} / 1.978 = 3538.9 \text{ Btu/h}$$

This work must be provided by the heat engine $\dot{W} = \eta_{ac}\ \dot{Q}_H$

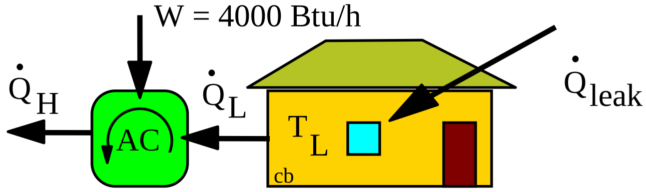
$$\dot{Q}_H = \dot{W} / \eta_{ac} = 3538.9 \text{ (Btu/h)} / 0.259 = 13\ 664 \text{ Btu/h}$$

$$= \mathbf{3.796\ Btu/s}$$

5.148E

A heat pump cools a house at 70 F with a maximum of 4000 Btu/h power input. The house gains 2000 Btu/h per degree temperature difference to the ambient and the heat pump coefficient of performance is 60% of the theoretical maximum. Find the maximum outside temperature for which the heat pump provides sufficient cooling.

Solution:



Here:

$T_L = T_{house}$

$T_H = T_{amb}$

In this setup the low temperature space is the house and the high temperature space is the ambient. The heat pump must remove the gain or leak heat transfer to keep it at a constant temperature.

$$\dot{Q}_{leak} = 2000\ (T_{amb} - T_{house}) = \dot{Q}_L$$

which must be removed by the heat pump.

$\beta' = \dot{Q}_H / \dot{W} = 1 + \dot{Q}_L / \dot{W} = 0.6\ \beta'_{carnot} = 0.6\ T_{amb} / (T_{amb} - T_{house})$

Substitute in for $\dot{Q}_L$ and multiply with $(T_{amb} - T_{house})$:

$$(T_{amb} - T_{house}) + 2000\ (T_{amb} - T_{house})^2 / \dot{W} = 0.6\ T_{amb}$$

Since $T_{house} = 529.7$ R and $\dot{W} = 4000$ Btu/h it follows

$$T_{amb}^2 - 1058.6\ T_{amb} + 279522.7 = 0$$

Solving => T_{amb} = **554.5 R = 94.8 F**

Comment: We did assume here that $\beta' = 0.6\ \beta'_{carnot}$, the statement could also have been understood as $\beta = 0.6\ \beta_{carnot}$ which would lead to a slightly different result.

5.149E

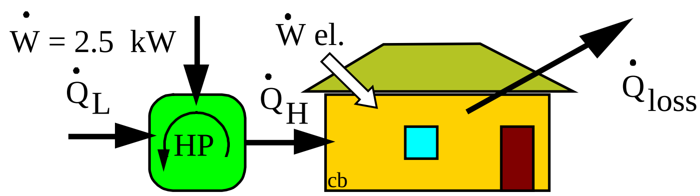
A small house is kept at 77 F inside looses 12 Btu/s to the outside ambient at 32 F. A heat pump is used to help heat the house together with possible electric heat. The heat pump is driven by a motor of 2.5 kW and it has a COP that is ¼ of a Carnot heat pump unit. Find the actual COP for the heat pump and the amount of electric heat that must be used (if any) to maintain the house temperature.

CV. House Energy: $0 = \dot{Q}_H + \dot{W}_{el.} - \dot{Q}_{Loss}$

Definition of COP: $\beta' = COP_{HP} = \frac{\dot{Q}_H}{\dot{W}} = \frac{1}{4}\frac{T_H}{T_H - T_L} = \frac{1}{4}\frac{536.7}{77 - 32} = \mathbf{2.982}$

$$\dot{Q}_H = COP_{HP}\,\dot{W}_{HP} = 2.982 \times (2.5/1.055)\ \text{Btu/s} = 7.066\ \text{Btu/s}$$

$$\dot{W}_{el.} = \dot{Q}_{Loss} - \dot{Q}_H = 12 - 7.066 = \mathbf{4.934\ Btu/s}$$



5.150E

A house is cooled by an electric heat pump using the outside as the high-temperature reservoir. For several different summer outdoor temperatures estimate the percent savings in electricity if the house is kept at 77 F instead of 68 F. Assume that the house is gaining energy from the outside directly proportional to the temperature difference.

Solution:

Air-conditioner (Refrigerator) $\dot{Q}_{LEAK} \propto (T_H - T_L)$

$$\text{Max Perf.} \quad \frac{\dot{Q}_L}{\dot{W}_{in}} = \frac{T_L}{T_H - T_L} = \frac{K(T_H - T_L)}{\dot{W}_{in}}, \quad \dot{W}_{in} = \frac{K(T_H - T_L)^2}{T_L}$$

A: T_{L_A} = 68 F = 527.7 R B: T_{L_B} = 77 F = 536.7 R

T_H, F	$\dot{W}_{IN_A}/K$	$\dot{W}_{IN_B}/K$	% saving
115	4.186	2.691	35.7 %
105	2.594	1.461	43.7 %
95	1.381	0.604	56.3 %

5.151E

Arctic explorers are unsure if they can use a 5 kW motor driven heat pump to stay warm. It should keep their shelter at 60 F which loses energy at a rate of 0.3 Btu/s per degree difference to the colder ambient. The heat pump has a COP that is 50% of a Carnot heat pump. If the ambient temperature can fall to -10 F at night, would you recommend this heat pump to the explorers?

CV Heat pump.

The heat pump should deliver a rate of heating that equals the heat loss to the ambient for steady inside temperature.

$$COP = \beta' = \dot{Q}_H/\dot{W} = 0.5\ \beta'_{Carnot} = \frac{1}{2} \times \frac{T_H}{T_H - T_L} = \frac{1}{2} \times \frac{459.7 + 60}{60 - (-10)} = 3.7$$

The heat pump can then provide a heating capacity of

$$\dot{Q}_H = \beta'\ \dot{W} = 3.7 \times 5\text{ kW} = 18.5\text{ kW} = 17.53\text{ Btu/s}$$

The heat loss is

$$\dot{Q}_{leak\ out} = CA\ \Delta T = 0.3\text{ Btu/s-R} \times [60 - (-10)]\text{ R} = 21\text{ Btu/s}$$

The heat pump is not sufficient to cover the loss and **not recommended**.

5.152E

Using the given heat pump in the previous problem how warm could it make the shelter in the arctic night?

The high is now an unknown so both the heat loss and the heat pump performance depends on that. The energy balance around the shelter then gives

$$\dot{Q}_H = \beta' \dot{W} = \dot{Q}_{\text{leak out}} = CA\,\Delta T$$

Substitute the expression for β' and CA ΔT to give

$$\frac{1}{2} \times \frac{T_H}{T_H - T_L} \dot{W} = 0.3 \text{ Btu/s-R} \times [T_H - T_L]$$

Multiply with the temperature difference, factor 2 and divide by the work,
5 kW = (5/1.055) Btu/s, so we get

$$T_H = \frac{0.3 \times 2}{5/1.055 \text{ R}} [T_H - T_L]^2 = \frac{0.1137}{\text{R}} [T_H - T_L]^2$$

Solve this equation like $0.1137\, x^2 - x - T_L = 0$, with $x = T_H - T_L$ and
$T_L = 459.7 - 10 = 449.7$ R

$$x = T_H - T_L = 67.44 \text{ R} \qquad \text{(negative root discarded)}$$

$$T_H = x + T_L = 67.44 - 10 = \mathbf{57.4\ F}$$

Ideal Gas Garnot Cycle

5.153E

Carbon dioxide is used in an ideal gas refrigeration cycle, reverse of Fig. 5.24. Heat absorption is at 450 R and heat rejection is at 585 R where the pressure changes from 180 psia to 360 psia. Find the refrigeration COP and the specific heat transfer at the low temperature.

The analysis is the same as for the heat engine except the signs are opposite so the heat transfers move in the opposite direction.

$$\beta = \dot{Q}_L / \dot{W} = \beta_{carnot} = T_L / (T_H - T_L) = \frac{450}{585 - 450} = \mathbf{3.33}$$

$$q_H = RT_H \ln(v_2/v_1) = RT_H \ln\left(\frac{P_1}{P_2}\right) = 35.1 \text{ ft-lbf/lbm-R} \times 585 \text{ R} \times \ln\left(\frac{360}{180}\right)$$

$$= 14\ 232.7 \text{ ft-lbf/lbm} = 18.29 \text{ Btu/lbm}$$

$$q_L = q_H\, T_L / T_H = 18.29 \times 450 / 585 = \mathbf{14.07\ Btu/lbm}$$

5.154E

Air in a piston/cylinder goes through a Carnot cycle with the P-v diagram shown in Fig. 5.24. The high and low temperatures are 1200 R and 600 R respectively. The heat added at the high temperature is 100 Btu/lbm and the lowest pressure in the cycle is 10 lbf/in.^{2}. Find the specific volume and pressure at all 4 states in the cycle assuming constant specific heats at 80 F.

Solution:

q_H = 100 Btu/lbm, T_H = 1200 R, T_L = 600 R, P_3 = 10 lbf/in.^{2}

C_v = 0.171 Btu/lbm R ; R = 53.34 ft-lbf/lbm-R

The states as shown in figure 5.24

1: 1200 R , 2: 1200 R, 3: 10 psi, 600 R 4: 600 R

As we know state 3 we can work backwards towards state 1

$$v_3 = RT_3 / P_3 = 53.34 \times 600 /(10 \times 144) = 22.225 \text{ ft}^3\text{/lbm}$$

Process 2→3 from Eq.5.8 & C_v = constant

$$= > C_v \ln (T_L / T_H) + R \ln (v_3/v_2) = 0$$

$$= > \ln (v_3/v_2) = - (C_v / R) \ln (T_L / T_H)$$

$$= - (0.171/53.34) \ln (600/1200) = 1.7288$$

$$= > v_2 = v_3 / \exp (1.7288) = 22.225/5.6339 = 3.9449 \text{ ft}^3\text{/lbm}$$

Process 1→2 and Eq.5.7: $q_H = RT_H \ln (v_2 / v_1)$

$$\ln (v_2 / v_1) = q_H /RT_H = 100 \times 778/(53.34 \times 1200) = 1.21547$$

$$v_1 = v_2 / \exp (1.21547) = 1.1699 \text{ ft}^3\text{/lbm}$$

$$v_4 = v_1 \times v_3 / v_2 = 1.1699 \times 22.225/3.9449 = 6.591 \text{ ft}^3\text{/lbm}$$

$$P_1 = RT_1 / v_1 = 53.34 \times 1200/(1.1699\times144) = 379.9 \text{ psia}$$

$$P_2 = RT_2 / v_2 = 53.34 \times 1200/(3.9449 \times 144) = 112.7 \text{ psia}$$

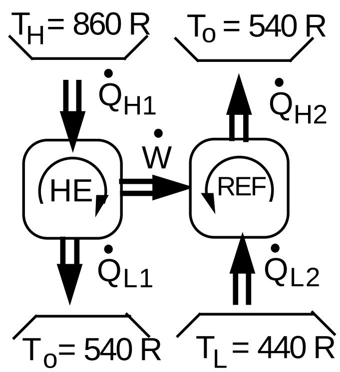
$$P_4 = RT_4 / v_4 = 53.34 \times 600/(6.591 \times 144) = 33.7 \text{ psia}$$

Review Problems

5.155E

We wish to produce refrigeration at –20 F. A reservoir is available at 400 F and the ambient temperature is 80 F, as shown in Fig. P5.104. Thus, work can be done by a cyclic heat engine operating between the 400 F reservoir and the ambient. This work is used to drive the refrigerator. Determine the ratio of the heat transferred from the 400 F reservoir to the heat transferred from the –20 F reservoir, assuming all processes are reversible.

Solution: Equate the work from the heat engine to the refrigerator.



$$\dot{W} = \dot{Q}_{H1}\left(\frac{T_H - T_0}{T_H}\right)$$

also

$$\dot{W} = \dot{Q}_{L2}\left(\frac{T_0 - T_L}{T_L}\right)$$

$$\dot{Q}_{H1} / \dot{Q}_{L2} = \left(\frac{T_O - T_L}{T_L}\right)\left(\frac{T_H}{T_H - T_O}\right) = \frac{100}{440} \times \frac{860}{320} = \mathbf{0.611}$$

5.156E

An air-conditioner on a hot summer day removes 8 Btu/s of energy from a house at 70 F and pushes energy to the outside which is at 88 F. The house has 30 000 lbm mass with an average specific heat of 0.23 Btu/lbm-R. In order to do this the cold side of the air-conditioner is at 40 F and the hot side is 100 F. The air conditioner (refrigerator) has a COP that is 60% of a corresponding Carnot refrigerator. Find the actual air-conditioner COP and the required power to run it.

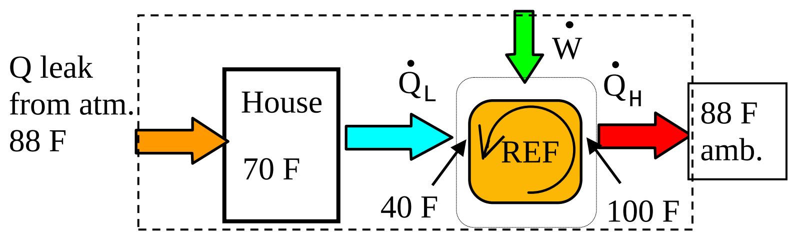
A steady state refrigerator definition of COP

$$COP = \beta_{REF} = \dot{Q}_L / \dot{W} = \dot{Q}_L / (\dot{Q}_H - \dot{Q}_L) = 0.6\ \beta_{Carnot}$$

Carnot: $$\beta_{Carnot} = \frac{T_L}{T_H - T_L} = \frac{40 + 459.7}{100 - 40} = 8.328$$

$$\beta_{REF} = 0.6 \times 8.328 = \mathbf{5}$$

$$\Rightarrow \quad \dot{W} = \dot{Q}_L / \beta_{REF} = 8 \text{ Btu/s} / 5 = \mathbf{1.6\ Btu/s}$$



Any heat transfer must go from a higher to a lower T domain.

5.157E

The air conditioner in the previous problem is turned off. How fast does the house heat up in degrees per second (F/s)?

Once the A/C unit is turned off we do not cool the house so heat leaks in from the atm. at a rate of 8 Btu/s (that is what we had to remove to keep steady state).

Energy Eq.: $\dot{E}_{CV} = \dot{Q}_{leak} = 8 \text{ Btu/s} = m_{house}\, C_P \frac{dT}{dt}$

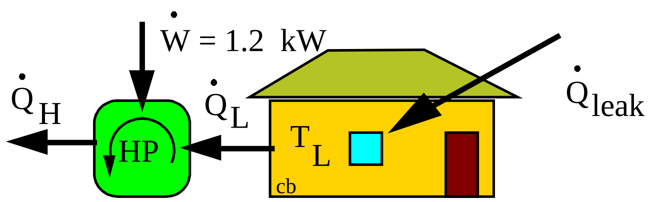
$$\frac{dT}{dt} = \dot{Q}_{leak} / m_{house}\, C_P$$

$$= \frac{8 \text{ Btu/s}}{30\,000 \times 0.23 \text{ Btu/R}} = \mathbf{1.16 \times 10^{-3} \text{ R/s}}$$

5.158E

A window air conditioner cools a room at T_L = 68 F with a maximum of 1.2 kW power input. The room gains 0.33 Btu/s per degree temperature difference to the ambient and the refrigeration COP is $\beta = 0.6\ \beta_{Carnot}$. Find the maximum outside temperature, T_H, for which the air conditioner provides sufficient cooling.

Solution:



Here:

$T_L = T_{house}$

$T_H = T_{amb}$

In this setup the low temperature space is the house and the high temperature space is the ambient. The heat pump must remove the gain or leak heat transfer to keep it at a constant temperature.

$$\dot{Q}_{leak} = 0.33\ (T_{amb} - T_{house}) = \dot{Q}_L$$

which must be removed by the heat pump.

$$\beta = \dot{Q}_L / \dot{W} = 0.6\ \beta_{carnot} = 0.6\ T_{house} / (T_{amb} - T_{house})$$

Substitute in for $\dot{Q}_L$ and multiply with $(T_{amb} - T_{house})\dot{W}$:

$$0.33\ (T_{amb} - T_{house})^2 = 0.6\ T_{house}\ \dot{W}$$

Since T_{house} = 459.7 + 68 = 527.7 R and $\dot{W}$ = 1.2 kW = 1.1374 Btu/s, it follows

$$(T_{amb} - T_{house})^2 = (0.6/0.33) \times 527.7 \times 1.1374 = 1091.28\ R^2$$

Solving $\Rightarrow$ $(T_{amb} - T_{house}) = 33.03$ $\Rightarrow$ T_{amb} = **560.73 R = 101 F**

Comment: We did assume here that $\beta = 0.6\ \beta_{carnot}$, the statement could also have been understood as $\beta' = 0.6\ \beta'_{carnot}$ which would lead to a slightly different result.

5.159E

The room in problem 5.158E has a combined thermal mass of 4 000 lbm wood, 500 lbm steel and 1000 lbm plaster board. Estimate how fast the room heats up if the air-conditioner is turned off on a day it is 95 F outside.

Without the air-conditioner the house gains heat and the energy equation for the house becomes

$$m\,C\,\frac{dT}{dt} = \dot{Q}_{in}$$

The gain is due to the temperature difference as

$$\dot{Q}_{in} = 0.33\,(T_H - T_L) = 0.33 \text{ Btu/s-R } (95 – 68) \text{ R} = 8.91 \text{ Btu/s}$$

The combined (mC) is using an estimate C for gypsum as 0.24 Btu/lbm-R

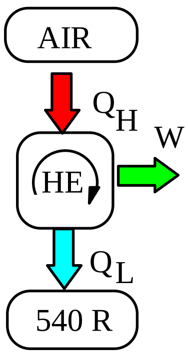
$$mC = [4000 \times 0.33 + 500 \times 0.11 + 1000 \times 0.24\,] \text{ Btu/R} = 1615 \text{ Btu/R}$$

$$\frac{dT}{dt} = \dot{Q}_{in}\,/mC = 8.91 \text{ (Btu/s)} / 1615 \text{ Btu/R} = \mathbf{0.0055\ R/s}$$

5.160E

A 350-ft^3 tank of air at 80 lbf/in.^{2}, 1080 R acts as the high-temperature reservoir for a Carnot heat engine that rejects heat at 540 R. A temperature difference of 45 F between the air tank and the Carnot cycle high temperature is needed to transfer the heat. The heat engine runs until the air temperature has dropped to 700 R and then stops. Assume constant specific heat capacities for air and find how much work is given out by the heat engine.

Solution:



$$T_H = T_{air} - 45\,, \qquad T_L = 540 \text{ R}$$

$$m_{air} = \frac{P_1V}{RT_1} = \frac{80 \times 350 \times 144}{53.34 \times 1080} = 69.991 \text{ lbm}$$

$$dW = \eta dQ_H = \left(1 - \frac{T_L}{T_{air} - 45}\right) dQ_H$$

$$dQ_H = -m_{air}du = -m_{air}C_v dT_{air}$$

$$W = \int dW = -m_{air}\, C_v \int \left[1 - \frac{T_L}{T_a - 45}\right] dT_a$$

$$= -m_{air}\, C_v \left[T_{a2} - T_{a1} - T_L \ln \frac{T_{a2} - 45}{T_{a1} - 45}\right]$$

$$= -69.991 \text{ lbm} \times 0.171 \text{ Btu/lbm-R} \times \left[700 - 1080 - 540 \ln \frac{655}{1035}\right] \text{R}$$

= **1591 Btu**

5.161E

Air in a rigid 40 ft^3 box is at 540 R, 30 lbf/in.^{2}. It is heated to 1100 R by heat transfer from a reversible heat pump that receives energy from the ambient at 540 R besides the work input. Use constant specific heat at 540 R. Since the coefficient of performance changes write $dQ = m_{air} C_v\, dT$ and find dW. Integrate dW with temperature to find the required heat pump work.

Solution:

$$\text{COP:} \quad \beta' = \frac{Q_H}{W} = \frac{Q_H}{Q_H - Q_L} \cong \frac{T_H}{T_H - T_L}$$

$$m_{air} = P_1V_1 / RT_1 = (30 \times 40 \times 144) / (540 \times 53.34) = 6.0 \text{ lbm}$$

$$dQ_H = m_{air}\, C_v\, dT_H = \beta'\, dW \cong \frac{T_H}{T_H - T_L}\, dW$$

$$=> \qquad dW = m_{air}\, C_v \left[\frac{T_H}{T_H - T_L}\right] dT_H$$

$${}_1W_2 = \int m_{air}\, C_v \left(1 - \frac{T_L}{T}\right) dT = m_{air}\, C_v \int \left(1 - \frac{T_L}{T}\right) dT$$

$$= m_{air}\, C_v \left[T_2 - T_1 - T_L \ln \frac{T_2}{T_1}\right]$$

$$= 6.0 \text{ lbm} \times 0.171 \text{ Btu/lbm-R} \times \left[1100 - 540 - 540 \ln\left(\frac{1100}{540}\right)\right] \text{R}$$

$$= \mathbf{180.4\ Btu}$$