Curve Sketching Problems

For the following functions, f, find all local extrema, inflection points, vertical and horizontal asymptotes. Sketch the graph of f. Find the domain and range of f.

1. a) $f(x) = x^3 - 9x^2 + 15x$ $\quad [f'(x) = 3(x-1)(x-5),\ f''(x) = 6(x-3)]$

   b) $f(x) = 5 - 9x + 6x^2 - x^3$ $\quad [f'(x) = -3(x-1)(x-3),\ f''(x) = -6(x-2)]$

   c) $f(x) = x^3 - 3x^2 + 3x$ $\quad [f'(x) = 3(x-1)^2,\ f''(x) = 6(x-1)]$

   d) $f(x) = x^4 - 6x^2$ $\quad [f'(x) = 4x(x^2-3),\ f''(x) = 12(x^2-1)]$

   e) $f(x) = -x^4 - 4x^3 + 16x$ $\quad [f'(x) = -4(x-1)(x+2)^2,\ f''(x) = -12x(x+2)]$

   f) $f(x) = -3x^5 + 10x^3 - 15x$ $\quad [f'(x) = -15(x-1)^2(x+1)^2,\ f''(x) = -60x(x^2-1)]$

   g) $f(x) = (x^2-1)^3$ $\quad \left[f'(x) = 6x(x-1)^2(x+1)^2,\ f''(x) = 30(x^2-1)(x^2-\frac{1}{5})\right]$

2. a) $f(x) = \dfrac{3}{3+x^2}$ $\quad \left[f'(x) = \dfrac{-6x}{(3+x^2)^2},\ f''(x) = \dfrac{18(x+1)(x-1)}{(3+x^2)^3}\right]$

   b) $f(x) = \dfrac{x}{x^2+1}$ $\quad \left[f'(x) = \dfrac{1-x^2}{(x^2+1)^2},\ f''(x) = \dfrac{2x(x^2-3)}{(x^2+1)^3}\right]$

   c) $f(x) = \dfrac{x^2}{x+1}$ $\quad \left[f'(x) = \dfrac{x^2+2x}{(x+1)^2},\ f''(x) = \dfrac{2}{(x+1)^3}\right]$

   d) $f(x) = \dfrac{4(x+1)}{x^2}$ $\quad \left[f'(x) = \dfrac{-4(x+2)}{x^3},\ f''(x) = \dfrac{8(x+3)}{x^4}\right]$

   e) $f(x) = \dfrac{-3(x-1)^2}{(x+1)^2}$ $\quad \left[f'(x) = \dfrac{-12(x-1)}{(x+1)^3},\ f''(x) = \dfrac{24(x-2)}{(x+1)^4}\right]$

   f) $f(x) = \dfrac{x^3}{x^2-4}$ $\quad \left[f'(x) = \dfrac{x^2(x^2-12)}{(x-2)^2(x+2)^2},\ f''(x) = \dfrac{8x(x^2+12)}{(x-2)^3(x+2)^3}\right]$

   g) $f(x) = \dfrac{3x^2}{x^2-9}$ $\quad \left[f'(x) = \dfrac{-54x}{(x^2-9)^2},\ f''(x) = \dfrac{162(x^2+3)}{(x^2-9)^3}\right]$

   h) $f(x) = \dfrac{9(x^2-3)}{x^3}$ $\quad \left[f'(x) = \dfrac{-9(x^2-9)}{x^4},\ f''(x) = \dfrac{18(x^2-18)}{x^5}\right]$

3. a) $f(x) = \dfrac{1}{\sqrt{x^2+1}}$ $\quad \left[f'(x) = \dfrac{-x}{(x^2+1)^{3/2}},\ f''(x) = \dfrac{2x^2-1}{(x^2+1)^{5/2}}\right]$

   b) $f(x) = \dfrac{x^2+3}{\sqrt{x^2+1}}$ $\quad \left[f'(x) = \dfrac{x(x^2-1)}{(x^2+1)^{3/2}},\ f''(x) = \dfrac{5x^2-1}{(x^2+1)^{5/2}}\right]$

   c) $f(x) = x^{1/3}(x+4)$ $\quad \left[f'(x) = \dfrac{4(x+1)}{3x^{2/3}},\ f''(x) = \dfrac{4(x-2)}{9x^{5/3}}\right]$

   d) $f(x) = x^{2/3}(5-x)$ $\quad \left[f'(x) = \dfrac{5(2-x)}{3x^{1/3}},\ f''(x) = \dfrac{-10(x+1)}{9x^{4/3}}\right]$

   e) $f(x) = \sqrt{\dfrac{4-x}{4+x}}$ $\quad \left[f'(x) = \dfrac{-4}{\sqrt{(4-x)(4+x)^3}},\ f''(x) = \dfrac{8(2-x)}{\sqrt{(4-x)^3(4+x)^5}}\right]$