MATH 2130 Test 1 October 10, 2012 60 minutes

10 1. Show that the lines

$$\begin{aligned} x &= 1+t, \\ y &= 2+3t, \\ z &= 4-2t; \end{aligned} \qquad \text{and} \qquad \begin{aligned} y &= 3x, \\ 2x+z &= 9 \end{aligned}$$

determine a plane, and find its equation simplified as much as possible.

6 2. Find the distance between the line $x = 3+2t$, $y = -1+t$, $z = 5+4t$ and the plane $6x-8y-z = 7$.

5 3. Find a vector of length 3 tangent to the curve

$$x = t^3+t, \quad y = 2t-t^2, \quad z = t+1,$$

at the point $(2, 1, 2)$.

8 4. Find parametric equations for the curve

$$z = 4x^2+y^2, \quad 8x+4y+z = 8,$$

directed clockwise as viewed from a point far up the z-axis.

6 5. Show that the following limit does not exist,

$$\lim_{(x,y)\to(0,0)} \frac{xy^3}{x^2+y^6}.$$

5 6. Show that the function $f(x,y) = x^2+y^2e^{y/x}$ satisfies the equation

$$x\frac{\partial f}{\partial x}+y\frac{\partial f}{\partial y} = 2f(x,y).$$

Answers by Dawit (plaukion@yahoo.com)

1. lines are parallel and distinct, they determine a plane.: $11x-3y+z-9=0$

2. $\frac{14}{\sqrt{101}}$ or $\frac{14\sqrt{101}}{101}$

3. $\frac{12}{\sqrt{17}}\hat{i}+\frac{3}{\sqrt{17}}K$ or $\frac{3}{\sqrt{17}}(4\hat{i}+\hat{k})$

4. $x = -1+2\cos t$
$y = -2-4\sin t$ $\qquad 0 \le t \le 2\pi$
$z = 24-16\cos t+16\sin t$

5. limit does not exist (hint: let $x = my^3$)