Test 1

DATE: October 4, 2007 TIME: 60 minutes
COURSE: MATH 2132 EXAMINER: G.I. Moghaddam

NAME:_____

STUDENT # : _____

There are 5 questions of total mark 50.

[6] 1. The following sequence of functions is defined on the interval $-1 \leq x \leq 1$.

$$\left\{ \frac{1}{n} + \frac{n^4x^2 + 2nx^4}{n^4x + x^4 + 1} \right\}_{n=1}^{\infty}$$

Determine whether the sequence is convergent or divergent. If it converges, find the limit function.

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[15] 2. Let $f(x) = \ln x$ for $0 < x < 4$ Then:

[5] (a) Find the first 4 terms of the Taylor series of $f(x)$ about 2.

[5] (b) Find the n^{th}-remainder (i.e. $R_n(2, x)$).

[5] (c) Show that $\lim_{n\to\infty} R_n(2, x) = 0$ *only* for the case $2 < x < 4$.

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[8] 3. Find the radius of convergence and the open interval of convergence for the series

$$\sum_{n=0}^{\infty} \frac{(-1)^n\,(n!)^2\,4^n}{(2n+1)!}\,(x-7)^{4n}$$

[6] 4. Find the sum of the series: $-\dfrac{\sqrt{2}}{3}\,x^3 + \dfrac{2}{9}\,x^6 - \dfrac{2\sqrt{2}}{27}\,x^9 + \cdots + \dfrac{(-1)^n\,2^{\frac{n}{2}}}{3^n}\,x^{3n} + \cdots$

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[15] 5. Find the Taylor series about 1 for the function

$$f(x) = \frac{1}{x^2} + \ln x$$

Express your answer in sigma notation and simplify as much as possible. Determine its open interval of convergence.