MATH 1210 Summer 2015 Quiz 5

Surname: ______________________________

Given Name: ______________________________

Student ID: ______________________________

1. Determine whether the following vectors are linearly independent or linearly dependent. Justify your answers.

[2] (a) $\langle 1,2,3\rangle, \langle 2,2,-3\rangle, \langle 1,3,9\rangle, \langle -1,5,-2\rangle$

Solution:
Since there are more vectors then components, the vectors are linearly dependent

[4] (b) $\langle 1,2,3\rangle, \langle 4,5,6\rangle, \langle 7,8,9\rangle$

Solution: We must determine whether

$$c_1\langle 1,2,3\rangle + c_2\langle 4,5,6\rangle + c_3\langle 7,8,9\rangle = 0,0,0\rangle$$

has just the trivial solution, or more than the trivial solution. These lead to the equations

$$\begin{aligned} c_1 + 4c_2 + 7c_3 &= 0 \\ 2c_1 + 5c_2 + 8c_3 &= 0 \\ 3c_1 + 6c_2 + 9c_3 &= 0 \end{aligned}$$

Since the coefficient matrix is square, we can check the determinant.

$$\begin{aligned} \begin{vmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{vmatrix} &= 1\begin{vmatrix} 5 & 8 \\ 6 & 9 \end{vmatrix} - 4\begin{vmatrix} 2 & 8 \\ 3 & 9 \end{vmatrix} + 7\begin{vmatrix} 2 & 5 \\ 3 & 6 \end{vmatrix} \\ &= 1(45-48) - 4(18-24) + 7(12-15) \\ &= 1(-3) - 4(-6) + 7(-3) \\ &= -3 + 24 - 21 \\ &= 0 \end{aligned}$$

Since $|A| = 0$ there is more than just the trivial solution and thus the vectors are linearly dependent.

[5] (c) $\langle 1,0,3,0\rangle, \langle 0,2,0,0\rangle, \langle 1,3,0,1\rangle$

Solution:
No theorems can simplify this, so we go to determining whether

$$c_1\langle 1,0,3,0\rangle + c_2\langle 0,2,0,0\rangle + c_3\langle 1,3,0,1\rangle = 0,0,0,0\rangle$$

has just the trivial solution, or more than the trivial solution. These lead to the equations

$$\begin{aligned} c_1 + c_3 &= 0 \\ 2c_2 + 3c_3 &= 0 \\ 3c_1 &= 0 \\ c_3 &= 0 \end{aligned}$$

An augmented matrix can be set up to solve this, or it can be determined that from the last two equations, $c_1 = 0$ and $c_3 = 0$ which imply from the second equation that $c_2 = 0$ as well.

Hence there is only the trivial solution and so the vectors are linearly independent.

[5] 2. Given that $\begin{vmatrix} a & b & c \\ 1 & 2 & 3 \\ d & e & f \end{vmatrix} = 4$, find $\begin{vmatrix} a-2 & b-4 & c-6 \\ 2d & 2e & 2f \\ -3 & -6 & -9 \end{vmatrix}$

Solution:

$$
\begin{aligned}
\begin{vmatrix} a-2 & b-4 & c-6 \\ 2d & 2e & 2f \\ -3 & -6 & -9 \end{vmatrix} &=_{R_2 \to R_2/2} 2 \begin{vmatrix} a-2 & b-4 & c-6 \\ d & e & f \\ -3 & -6 & -9 \end{vmatrix} \\
&=_{R_3 \to R_2/(-3)} 2(-3) \begin{vmatrix} a-2 & b-4 & c-6 \\ d & e & f \\ 1 & 2 & 3 \end{vmatrix} \\
&=_{R_3 \leftrightarrow R_2} 2(-3)(-1) \begin{vmatrix} a-2 & b-4 & c-6 \\ 1 & 2 & 3 \\ d & e & f \end{vmatrix} \\
&=_{R_1 \to R_1 + 2R_2} 2(-3)(-1) \begin{vmatrix} a & b & c \\ 1 & 2 & 3 \\ d & e & f \end{vmatrix} \\
&= 2(-3)(-1)(4) \\
&= 24.
\end{aligned}
$$

[4] 3. A given system (with variables x_1, x_2, x_3, x_4, x_5) has the following matrix in reduced row-echelon form. Find basic solutions for the system.

$$
\left[\begin{array}{ccccc|c} 1 & 3 & 0 & 2 & 0 & 0 \\ 0 & 0 & 1 & -3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]
$$

Solution:

As equations, the solutions would satify

$$
\begin{aligned}
x_1 + 3x_2 + 2x_4 &= 0 \\
x_3 - 3x_4 &= 0 \\
x_5 &= 0
\end{aligned}
$$

and therefore

$$
\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} -3x_2 - 2x_4 \\ x_2 \\ 3x_4 \\ x_4 \\ 0 \end{bmatrix} = \begin{bmatrix} -3x_2 \\ x_2 \\ 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} -2x_4 \\ 0 \\ 3x_4 \\ x_4 \\ 0 \end{bmatrix} = x_2 \begin{bmatrix} -3 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} -2 \\ 0 \\ 3 \\ 1 \\ 0 \end{bmatrix}
$$

Hence the basic solutions are

$$\begin{bmatrix} -3 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \text{ and } \begin{bmatrix} -2 \\ 0 \\ 3 \\ 1 \\ 0 \end{bmatrix}.$$