MATH 2130 Problem Workshop 8

1. Find the rate of change of the function $f(x, y, z) = \sin(xy) - z^3$ at the point $(2, 0, 3)$ in the direction of the upward normal to the surface $xz^2 - x^2z = 6$.

2. Find equations for the tangent line to the following curve at the point $(1, -1, 3)$:

$$xyz + z^3 = 24, \qquad x^3y^2z + y^3 = 4x - 2.$$

3. Find an equation for the tangent plane to the following surface at the point $(2, -1, -1)$:

$$x^2y + y^2z + z^2x + 3 = 0.$$

4. Find the acute angle between the normal to the surface $x + z = 3$ and the tangent line to the curve $xy^3z + z^3 = 6, \quad xy + yz = -3$ at their point of intersection.

5. Find all critical points for the function $f(x, y) = x^3y^3 - x^2y^2 + 6$.

6. Find all critical points for the function $f(x, y) = x^3y^2 - xy + 3y$.

7. Find and classify all critical points of the function as giving relative minima, maxima, saddle points or neither.

   (a) $f(x, y) = x^3 + xy + y^3$
   (b) $f(x, y) = x^3 - xy^2 + 3xy$
   (c) $f(x, y) = x^4 - 3x^2y^2 + y^4$
   (d) $f(x, y) = y^2 + |x - 1|$

Answers:

1. $-\dfrac{216}{\sqrt{73}}$
2. $x = 1 + 81t, y = -1 + 133t, z = 3 - 6t$
3. $x - 2y + z = 3.$
4. $\arccos\left(\dfrac{3\sqrt{5}}{10}\right)$
5. All points on the x-axis, y-axis and on the curve $y = \dfrac{2}{3x}$.
6. $(3, 0), (9, 1/243)$
7. (a) $(0, 0)$ gives a saddle point. $(-1/3, -1/3)$ gives a relative maximum.
   (b) $(0, 0), (0, 3)$ both give saddle points.
   (c) $(0, 0)$ gives a saddle point.
   (d) $(1, 0)$ gives a relative minimum. Points $(1, y)$ for $y \neq 0$ give neither.