1210 Tutorial 4 Sols

1. $\overline{-2+3i} = -2-3i$ is also a root

$(x+2-3i)(x+2+3i) = x^2+4x+13$ divides the pol.

$$
\begin{array}{r}
x^2-2x+5 \\
\hline
x^2+4x+13 \;\big)\; x^4+2x^3+10x^2-6x+65 \\
\underline{x^4+4x^3+13x^2} \\
-2x^3-3x^2-6x \\
\underline{-2x^3-8x^2-26x} \\
5x^2+20x+65 \\
\underline{5x^2+20x+65} \\
0
\end{array}
$$

$x^2-2x+5=0$

$x = \frac{2 \pm \sqrt{4-4\cdot 5}}{2} = 1 \pm \sqrt{-4} = 1 \pm 2i$

Ans: $-2-3i,\ 1+2i,\ 1-2i$

2. $4k = P(-2) = -8 + k^2 \cdot 4 + 8 + 1$

$4k^2 - 4k + 1 = 0$

$(2k-1)^2 = 0$

Ans: $k = \frac{1}{2}$.

3. $\begin{cases} P(-1)=0 \\ P(3)=0 \end{cases}$

$$\begin{cases} 1 - k + h + 16 - 24 = 0 \\ 81 + 27k + 9h - 48 - 24 = 0 \end{cases} \quad (\text{div. by } 3)$$

$$\begin{cases} -k + h - 7 = 0 \\ 9k + 3h + 27 - 16 - 8 = 0 \end{cases} \quad (\text{div. by } 3)$$

$$\begin{cases} -k + h - 7 = 0 & (1) \\ 3k + h + 1 = 0 & (2) \end{cases}$$

(2)−(1): $4k - 8 = 0 \quad k = 2$

(1): $h = k + 7 = 9$

Ans: $k = 2, \ h = 9$

4(a) Coefs of $P(x)$: $2, -5, -8, -6, 6, 5$ — 2 sign changes — 2 or 0 positive sols.

Coefs of $P(-x)$: $-2, -5, 8, -6, -6, 5$ — 3 sign changes — 3 or 1 negative sols.

(b) If x is a solution, then

$$|x| < \frac{\max\{|-5|, |-8|, |-6|, |6|, |5|\}}{|2|} + 1 = \frac{8}{2} + 1 = 5$$

$$|x| < 5$$

(c) $\pm \frac{p}{q}$ where p divides 5, so $p = 1$ or $p = 5$

q divides 2, so $q = 1$ or $q = 2$

$$\pm 1, \pm 5, \pm \frac{1}{2}, \pm \frac{5}{2}$$

By (5), we can eliminate ± 5.

~~By (6), we can eliminate ...~~

(a) does not provide any extra info

Ans: $\pm 1, \pm \frac{1}{2}, \pm \frac{5}{2}$.

5. By rational roots, if x is an integer sol., then x divides 14, so it may be

$$\pm 1, \pm 2, \pm 7, \pm 14.$$

$x=1$ doesn't satisfy: $1-7+17-14 \neq 0$

$x=-1$ doesn't satisfy: $-1-7-17-14 \neq 0$

$x=2$ does: $8-28+34-14=0$

$$\begin{array}{r} x^2-5x+7 \\ x-2 \overline{)\, x^3-7x^2+17x-14} \\ \underline{x^3-2x^2} \\ -5x^2+17x \\ \underline{-5x^2+10x} \\ 7x-14 \\ \underline{7x-14} \\ 0 \end{array}$$

$x^2-5x+7=0$

$$x=\frac{5\pm\sqrt{25-28}}{2}=\frac{5\pm i\sqrt{3}}{2}$$

Ans: $2, \frac{5\pm i\sqrt{3}}{2}$.