6. Find a...

- Consider the following

FACULTY OF ENGINEERING

DATE | NAME | PAGE
COURSE | TITLE | OF

3) L_1: $x = 3 + 4t_1$ $\quad$ L_2: $x = -1 + 12t_2$

$y = 4 + t_1$ $\quad$ $y = 7 + 6t_2$

$z = 1$ $\quad$ $z = 5 + 3t_2$

z) $1 = 5 + 3t_2 \Rightarrow t_2 = \frac{-4}{3}$

y) $4 + t_1 = 7 + 6\left(\frac{-4}{3}\right) \Rightarrow t_1 = 3 - 8 = -5$

$\underline{t_1 = -5}$ $\quad$ $\underline{t_2 = \frac{-4}{3}}$

Check: 1+2

$x_1 = 3 + 4(-5) = -17$ $\quad$ $x_2 = -1 + \frac{12(-4)}{3} = -1$

$y_1 = 4 - 5 = -1$ $\quad$ $y_2 = 7 + \frac{6(-4)}{3} = -1$

$z_1 = 1$ $\quad$ $z_2 = 5 + \frac{3(-4)}{3} = 1$

$(x_1, y_1, z_1) = (x_2, y_2, z_2)$

$\therefore$ point of intersection at $(-17, -1, 1)$

FACULTY OF ENGINEERING

DATE | NAME | PAGE

COURSE | TITLE | OF

$$\left[\begin{array}{cccc|c} 1 & -2 & 0 & 3 & 0 \\ 2 & -3 & -1 & -4 & 0 \\ 3 & -5 & -1 & -1 & 0 \end{array}\right] \quad R_2 \Rightarrow R_2 - 2R_1 \quad R_3 \Rightarrow R_3 - 3R_1$$

$$\left[\begin{array}{cccc|c} 1 & -2 & 0 & 3 & 0 \\ 0 & 1 & -1 & -10 & 0 \\ 0 & 1 & -1 & -10 & 0 \end{array}\right] \quad R_3 \Rightarrow R_3 - R_2$$

$$\left[\begin{array}{cccc|c} 1 & -2 & 0 & 3 & 0 \\ 0 & 1 & -1 & -10 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

Put into equations.

1) $x_1 - 2x_2 + 3x_4 = 0$

2) $x_2 - x_3 - 10x_4 = 0$

2) $x_2 = x_3 + 10x_4$

1) $x_1 = 2x_3 + 20x_4 - 3x_4$

$x_1 = 2x_3 + 17x_4$

$$\boxed{\begin{aligned} x_1 &= 2t + 17w \\ x_2 &= t + 10w \\ x_3 &= t \\ x_4 &= w \end{aligned}}$$

By Gaussian Elimination

Infinite amount of solutions

$t, w \in \mathbb{R}$

FACULTY OF ENGINEERING

DATE	NAME	PAGE
COURSE	TITLE	OF

5) a) A determinant is defined by a square matrix, therfore there is no solution as the matrix is 3x4.

b) $\begin{bmatrix} 1 & 0 & -1 & 2 \\ 2 & 1 & -3 & 4 \\ 0 & 2 & -2 & 3 \\ 1 & 1 & -4 & -2 \end{bmatrix} \xrightarrow[R_4 \to R_4 - R_1]{R_2 \to R_2 - 2R_1} \begin{bmatrix} 1 & 0 & -1 & 2 \\ 0 & 1 & -1 & 0 \\ 0 & 2 & -2 & 3 \\ 0 & 1 & -3 & -4 \end{bmatrix} \xrightarrow[R_4 \to R_4 - R_2]{R_3 \to R_3 - 2R_2}$

$\begin{bmatrix} 1 & 0 & -1 & 2 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 3 \\ 0 & 0 & -2 & -4 \end{bmatrix} \xrightarrow{R_3 \leftrightarrow R_4} - \begin{bmatrix} 1 & 0 & -1 & 2 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & -2 & -4 \\ 0 & 0 & 0 & 3 \end{bmatrix}$

Triangular matrix, determinant is the product of the main diagonal entries

$\therefore -\left(1 \times 1 \times (-2) \times 3\right) = -(-6) = \boxed{6}$

c) $\det A^n = (\det A)^n$ Therefore $6^5 = \boxed{7776}$

d) $\det A = 6$, 4x4 matrix therefore

$\det(-A) = (-1)^4 \det A = \boxed{6}$

e) $\det A^T = \det A$ therefore $\det A^T = \boxed{6}$

FACULTY OF ENGINEERING

DATE

NAME

COURSE

TITLE

6)

$$A = \begin{vmatrix} \lambda-4 & 0 & 0 \\ 0 & \lambda & 2 \\ 0 & 3 & \lambda-1 \end{vmatrix} = 0$$

Expand along 1st row

$$(\lambda-4)\left[\lambda(\lambda-1)-6\right] = 0$$

$$(\lambda-4)(\lambda^2-\lambda-6) = 0$$

$$(\lambda-4)(\lambda-3)(\lambda+2) = 0$$

$$\boxed{\lambda = 4, 3, -2}$$

FACULTY OF ENGINEERING

DATE	NAME	PAGE
COURSE	TITLE	OF

7)

$$2x - y + 2z = 2$$
$$x - y - z = 1$$
$$4x + 2y - z = 0$$

a) Rank of Augmented Matrix = 3
(Three leading zeros in Matrix)

b) Rank of Coeffecient Matrix = 3
(Three leading zeros in determinant).

c) $$\left[\begin{array}{ccc|c} 1 & -1 & -1 & 1 \\ 2 & -1 & 2 & 2 \\ 4 & 2 & -1 & 0 \end{array}\right] \xrightarrow[R_3 \to R_3 - 4R_1]{R_2 \to R_2 - 2R_1} \left[\begin{array}{ccc|c} 1 & -1 & -1 & 1 \\ 0 & 1 & 4 & 0 \\ 0 & 6 & 3 & -4 \end{array}\right] \xrightarrow[R_1 \to R_1 + R_2]{R_3 \to R_3 - 6R_2}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 3 & 1 \\ 0 & 1 & 4 & 0 \\ 0 & 0 & -21 & -4 \end{array}\right] \xrightarrow{R_3 \to \frac{R_3}{-21}} \left[\begin{array}{ccc|c} 1 & 0 & 3 & 1 \\ 0 & 1 & 4 & 0 \\ 0 & 0 & 1 & \frac{4}{21} \end{array}\right] \xrightarrow[R_1 \to R_1 - 3R_3]{R_2 \to R_2 - 4R_3}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & \frac{9}{21} \\ 0 & 1 & 0 & \frac{-16}{21} \\ 0 & 0 & 1 & \frac{4}{21} \end{array}\right] \quad \therefore \quad \boxed{x = \frac{9}{21} \quad y = \frac{-16}{21} \quad z = \frac{4}{21}}$$

FACULTY OF ENGINEERING

DATE	NAME
COURSE	TITLE

1)

$$2x_1 + 2x_2 - x_3 + x_5 = 0$$
$$-x_1 - x_2 + 2x_3 - 3x_4 + x_5 = 0$$
$$x_1 + x_2 - 2x_3 - x_5 = 0$$
$$x_3 + x_4 + x_5 = 0$$

$$\left[\begin{array}{ccccc|c} 1 & 1 & -2 & 0 & -1 & 0 \\ -1 & -1 & 2 & -3 & 1 & 0 \\ 2 & 2 & -1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{array}\right] \quad \begin{array}{l} R_2 \to R_2 + R_1 \\ R_3 \to R_3 - 2R_1 \end{array}$$

$$\left[\begin{array}{ccccc|c} 1 & 1 & -2 & 0 & -1 & 0 \\ 0 & 0 & 0 & -3 & 0 & 0 \\ 0 & 0 & 3 & 0 & 3 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{array}\right] \quad R_2 \Leftrightarrow R_4$$

$$\left[\begin{array}{ccccc|c} 1 & 1 & -2 & 0 & -1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 3 & 0 & 3 & 0 \\ 0 & 0 & 0 & -3 & 0 & 0 \end{array}\right] \quad \begin{array}{l} R_3 \to R_3 - 3R_2 \\ R_1 \to R_1 + 2R_2 \end{array}$$

$$\left[\begin{array}{ccccc|c} 1 & 1 & 0 & 2 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & -3 & 0 & 0 \\ 0 & 0 & 0 & -3 & 0 & 0 \end{array}\right] \quad R_4 \to R_4 - R_3$$

$$\left[\begin{array}{ccccc|c} 1 & 1 & 0 & 2 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & -3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right] \quad R_3 \to \frac{R_3}{-3}$$

$$\left[\begin{array}{ccccc|c} 1 & 1 & 0 & 2 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right] \quad \begin{array}{l} R_1 \to R_1 - 2R_3 \\ R_2 \to R_2 - R_3 \end{array}$$

Cont. $\longrightarrow$

FACULTY OF ENGINEERING

DATE

NAME

COURSE

TITLE

$$\left[\begin{array}{ccccc|c} 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

x_2 & x_5 are parameters.

$x_1 = -x_2 - x_5$

$x_3 = -x_5$

$x_4 = 0$,

b)

$$\begin{aligned} x_1 &= -x_2 - x_5 \\ x_2 &= x_2 + 0x_5 \\ x_3 &= 0x_2 - x_5 \\ x_4 &= 0x_2 + 0x_5 \\ x_5 &= 0x_2 + x_5 \end{aligned}$$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix} = x_2 \begin{pmatrix} -1 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} + x_5 \begin{pmatrix} -1 \\ 0 \\ -1 \\ 0 \\ 1 \end{pmatrix}$$

c) yes, if $x_2 = x_5 = 0$