DATE: March 11, 2010
TIME: 70 minutes
COURSE: MATH 2130
EXAMINER: G.I. Moghaddam

[6] 1. If $z = f(r, s, y)$, $r = g(x, y)$, $s = h(x, y)$, and $y = k(x)$, find a formula for $\dfrac{dz}{dx}$.

[10] 2. Evaluate each of the following limit or explain why it does not exist.

(a) $\displaystyle \lim_{(x,y)\to(0,-2)} \frac{x^3 + (y+2)^5}{4x^3 - (y+2)^5}$

(b) $\displaystyle \lim_{(x,y)\to(-1,0)} \frac{x^4 + y^4 + 2x^2y^2 - 1}{\sin(x^2 + y^2 - 1)}$

[10] 3. Let u, v, and z be functions of x and y. Find $\dfrac{\partial v}{\partial x}$ and $\dfrac{\partial z}{\partial y}$ if

$$\begin{aligned} x^2 + y^2 - z + u^2 - v^3 &= 0 \\ x + 2y + z^2 - u^4 &= 0 \\ v + x^2 &= 0. \end{aligned}$$

Simplify your answer.

[8] 4. Let $f(x, y, z) = x^3y + xy^2 + \frac{1}{2}z^2$ and let $\mathbf{v}$ be a vector such that $\mathbf{v} = (a, 2a, a^2)$. Find the value(s) of a such that the directional derivative of f in the direction $\mathbf{v}$ at the point $(1, -1, 1)$ is -3.

[8] 5. Find parametric equations of the tangent line to the curve

$$x^3 + y = -z \quad , \quad x^3 - y = 3z$$

at the point $(1, -2, 1)$.

[8] 6. Show that the curve

$$\frac{1}{2}x^2 - \frac{1}{2}y^2 + \frac{1}{2}z^2 - 1 = 0 \quad , \quad xy + xz - 1 = 0$$

is tangent to the surface $x^2 - xyz + y + 2z - 3 = 0$ at the point $(1, 0, 1)$.

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Answers to Math 2130 Test 2, March 11, 2010

1) $\dfrac{dz}{dx} = \left.\dfrac{\partial z}{\partial r}\right)_{s,y}\left[\left.\dfrac{\partial r}{\partial x}\right)_y + \left.\dfrac{\partial r}{\partial y}\right)_x \dfrac{dy}{dx}\right] + \left.\dfrac{\partial z}{\partial s}\right)_{r,y}\left[\left.\dfrac{\partial s}{\partial x}\right)_y + \left.\dfrac{\partial s}{\partial y}\right)_x \dfrac{dy}{dx}\right] + \left.\dfrac{\partial z}{\partial y}\right)_{r,s}\dfrac{dy}{dx}$

2) a) let $y+2 = m\,x^{3/5}$ $\Rightarrow$ $\lim\limits_{x\to 0}$ gives an answer $\dfrac{1+m^5}{4-m^5}$

along x axis $\overset{m\to\infty}{\Longrightarrow}$ $\lim = -1$
along $y=-2$ $\underset{m=0}{\Longrightarrow}$ $\lim = 1/4$ $\quad\therefore$ limit Does not exist.

b) 2

3) $\dfrac{\partial v}{\partial x} = -2x$ provided $u\neq 0,\ z\neq u^2$; $\dfrac{\partial z}{\partial y} = \dfrac{2u^2y-1}{z-u^2}$ $\quad u\neq 0,\ z\neq u^2$

4) Since Max rate of Chang is $\pm\left|\nabla f|_p\right| = \pm\sqrt{6}$ and $-3<-\sqrt{6}$.
there is no direction at which the rate of change is equal to -3

5) $x = 1-2t$, $y = -2+12t$, $z = 1-6t$ or $x = 1+2t$, $y = -2-12t$, $z = 1+6t$ or $x = 1+t$, $y = -2-6t$, $z = 1+3t$

6) $\nabla(x^2-xyz+y+2z-3)\Big|_{(1,0,1)} = (2,0,2) = \vec{N}$ (vector normal to the Surface)

$(1,0,1)\times(1,1,1) = (-1,0,1) = \vec{V} \leftarrow$ (vector along the curve at the given point.)

$\vec{N}\cdot\vec{V} = (2,0,2)\cdot(-1,0,1) = 2-2 = 0$

$\therefore$ the Curve is tangent to the Surface @ point $(1,0,1)$