MATH 2130 Tutorial 8

1. Find all critical points for the function $f(x, y) = x^3y^3 - x^2y^2 + 6$.

   Answer: Every point on x-axis, every point on y-axis, and every point on the curve $y = 2/(3x)$.

2. Find all critical points for the function $f(x, y) = x^3y^2 - xy + 3y$.

   Answer: $(3, 0)$, $(9, 1/243)$

3. You are given that $(0, 0)$ and $(-1/3, -1/3)$ are critical points of the function $f(x, y) = x^3 + xy + y^3$. Classify each critical point as yielding a relative maximum, a relative minimum, or a saddle point.

   Answer: $(0, 0)$ gives a saddle point; $(-1/3, -1/3)$ gives a relative maximum

4. Find all critical points of the function $f(x, y) = x^3 - xy^2 + 3xy$. Classify any one of the critical points (your choice as to which one) as yielding a relative maximum, a relative minimum, or a saddle point.

   Answer:$(0, 0)$ gives a saddle point; $(0, 3)$ also gives a saddle point

5. Find all critical points of the function
   $$f(x, y) = x^4 - 3x^2y^2 + y^4.$$
   Classify each critical point as giving a relative maximum, a relative minimum, or a saddle point.

   Answer: $(0, 0)$ gives a saddle point

6. Find the maximum value of the function $f(x, y) = x^2 - y^2$ on the region $x^2 + y^2 \leq 1$.

   Answer: 1

7. Find the maximum value of the function $f(x, y) = xy(3 - x - 2y)$ on the triangle R bounded by the positive x- and y-axes and the line $x + y = 1$. Assume that $f(x, y)$ has no critical points in the interior of R

   Answer: $2\sqrt{3}/9$

8. Find the maximum value of the function $f(x, y) = x^2 - y^2 + x$ considering only points inside and on the boundary of the region bounded by the curve
   $$x = \sqrt{1 - y^2}, \quad x = 0.$$

   Answer: 2