P1/2

THE UNIVERSITY OF MANITOBA

DATE: December 14, 2012 FINAL EXAMINATION

DEPARTMENT & NO: MATH2130 TIME: 3 hours

EXAMINATION: Engineering Mathematical Analysis 1 EXAMINER: M. Davidson, D. Trim

8 1. Find parametric equations for the tangent line to the curve

$$x^2y + z^3 + xz = 9, \quad xy + y^4z = 1,$$

at the point $(1, -1, 2)$.

6 2. Set up, but do **NOT** evaluate a definite integral for the length of the curve

$$z = x^2 + y^2, \quad 2x - 4y + z = 4.$$

You need not simplify the integral.

5 3. Find the equation of the plane that passes through the point $(4, -3, 5)$ and is perpendicular to the line

$$\frac{2x-1}{10} = \frac{y+5}{-7} = \frac{1-z}{3}.$$

Simplify the equation as much as possible.

9 4. The equations

$$x = u^2 + v^3, \quad y = 3uv + u^2v^2$$

explicitly define x and y as functions of u and v. They also implicitly define u and v as functions of x and y. Show that

$$\left.\frac{\partial u}{\partial x}\right)_y \neq \frac{1}{\left.\frac{\partial x}{\partial u}\right)_v}.$$

8 5. Show that for any differentiable function f whatsoever, the function $u(x,y) = f(x^2 - 2y^2) + x^4$ satisfies the equation

$$2y\frac{\partial u}{\partial x} + x\frac{\partial u}{\partial y} = 8x^3y.$$

14 6. Find the maximum and minimum values of the function

$$f(x,y) = xy(1 - 2x - 2y)$$

on the region consisting of the triangle enclosed by the lines

$$x + y = 1, \quad x = 0, \quad y = 0$$

and its edges.

6 7. Evaluate the double iterated integral

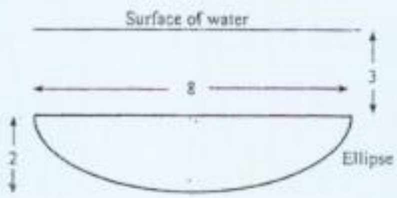
$$\int_0^4 \int_{\sqrt{x}}^2 e^{y^3}\, dy\, dx.$$

7 8. Set up, but do **NOT** evaluate a double iterated integral for the volume of the solid of revolution when the area bounded by the curves

$$y = x^2 - x - 6, \quad y = 0,$$

is rotated about the line $x + y = 3$. Simplify the integrand as much as possible.

8 **9.** Pictured below is a semi-elliptic plate submerged vertically in water. All dimensions are in metres. Set up, but do **NOT** evaluate, a double iterated integral for the force on each side of the plate due to the water.



8 **10.** Set up, but do **NOT** evaluate, a double iterated integral for the surface area of that portion of the surface $ze^{2x+3y} = 1$ that is enclosed by the surfaces $x = 1 - y^2$ and $x = y^2 - 1$.

12 **11.** Evaluate the triple integral of the function $f(x, y, z) = 2(x^2 + y^2)$ over the volume bounded by the surfaces

$$z = x^2 + y^2, \quad z = 4.$$

9 **12.** Set up, but do **NOT** evaluate, a triple iterated integral in spherical coordinates for the triple integral

$$\iiint_V (x^2 + y^2)\, dV$$

where V is the region bounded by the surfaces

$$z^2 = 3x^2 + 3y^2, \quad z = \sqrt{9 - x^2 - y^2}.$$

Answers by Dawit (plankion@yahoo.com)

1. $x = 1 + 92t$, $y = -1 - 13t$, $z = 2 + t$

2. $$\int_0^{2\pi} \left[(-3\sin t)^2 + (3\cos t)^2 + (12\cos t + 6\sin t)^2 \right]^{1/2} dt$$

3. $5x - 7y - 3z - 26 = 0$

4. $$\left.\frac{\partial u}{\partial x}\right)_y = \frac{3u + 2u^2 v}{4u^3 v + 6u^2 - 9v^3 - 6uv^4} \; ; \; \frac{1}{\left.\frac{\partial x}{\partial u}\right)_v} = \frac{1}{2u}$$

6. Max $= 1/108$, Min $= -1/4$

7. $\frac{1}{3}(e^8 - 1)$

8. $$\int_{-2}^{3} \int_{x^2 - x - 6}^{0} 2\pi \left[\frac{3 - x - y}{\sqrt{2}} \right] dy\, dx$$

9. $$2 \int_0^4 \int_{-\frac{1}{2}\sqrt{16 - x^2}}^{0} \rho g (3 - y)\, dy\, dx \quad \text{(in N)}$$

10. $$\int_{-1}^{1} \int_{y^2 - 1}^{1 - y^2} \left[1 + 13 e^{-(4x + 6y)} \right] dx\, dy$$

11. $\frac{64\pi}{3}$

12. $$\int_0^{2\pi} \int_0^{\pi/6} \int_0^3 R^4 \sin^3\phi \, dR\, d\phi\, d\theta$$