

Math 1710: Tutorial 10 (Partial fractions, Parametric curves) - answers

1. (a) $\frac{a}{x} + \frac{b}{x+1} + \frac{c}{(x+1)^2} + \frac{d}{(x+1)^3} + \frac{ex+f}{x^2+x+2} + \frac{gx+h}{(x^2+x+2)^2} + \frac{i}{2x+1} + \frac{j}{(2x+1)^2} + \frac{k}{(2x+1)^3} + \frac{l}{(2x+1)^4}$

(b) $\frac{a}{x} + \frac{b}{x-1} + \frac{c}{(x-1)^2} + \frac{d}{x+1} + \frac{e}{x+2} + \frac{fx+g}{x^2+1}$

2. (a) $x^2/2 + x - \ln|x+1| - 1/(x+1) + C$

(b) $(2/3)\ln|x-1| - (1/3)\ln(x^2+x+1) + C$

(c) $(1/4)\ln|x^4-1| + C$

(d) $(1/4)\ln|(x-1)/(x+1)| - (1/2)\tan^{-1}x + C$

(e) $-3\ln|x+1| + \tan^{-1}x + 1/(x^2+1) + C$

(f) $(1/2)\tan^{-1}(x+1) + \frac{x+1}{2(x^2+2x+2)} + \ln|x-1| + C$

3. Not a proper fraction

4. (a) Part of the parabola $x = \frac{y^2}{4} - \frac{10y}{4} + \frac{17}{4}$ for $-\frac{3}{2} \leq y \leq 11$, arrow down.

(b) Part of the hyperbola $y = \frac{1}{x}$ for $0 < x < 1$, arrow down.

(c) Part of the exponential function $y = e^{x/2}$ for $x \geq 0$, arrow up.

(d) Right branch ($x > 0$) of the hyperbola $x^2 - y^2 = 1$, or $x = \sqrt{1+y^2}$ for all y , arrow up.

5. (a) $\frac{dy}{dx} = 1 + \frac{3}{2}t, \quad \frac{d^2y}{dx^2} = \frac{3}{4t}, \quad t \in (0, \infty);$

(b) $\frac{dy}{dx} = -(3/2)\tan t, \quad \frac{d^2y}{dx^2} = -\frac{3}{4\cos^3 t}, \quad t \in (\pi/2 + 2\pi n, 3\pi/2 + 2\pi n),$
 $n = 0, \pm 1, \pm 2, \dots$