

Implicit differentiation

3.8.1

$$\frac{d}{dx}(\sqrt{x+y} + 2xy) = \frac{d}{dx}(5x)$$

$$\frac{1}{2\sqrt{x+y}} \cdot (1+y') + 2y + 2xy' = 5$$

$$\frac{dy}{dx} = y' = \frac{5 - 2y - \frac{1}{2\sqrt{x+y}}}{\frac{1}{2\sqrt{x+y}} + 2x} = \frac{2\sqrt{x+y}(5 - 2y) - 1}{1 + 4x\sqrt{x+y}}$$

3.8.2

$$3x^2 + 4xy + 2x^2y' - 3y^2y' = 0$$

$$y' = \frac{3x^2 + 4xy}{3y^2 - 2x^2}$$

$$y'' = \frac{(6x + 4y + 4xy')(3y^2 - 2x^2) - (6yy' - 4x)(3x^2 + 4xy)}{(3y^2 - 2x^2)^2}$$

It is easier to sub. values into y' first:

$$y' \Big|_{(x,y)=(1,-1)} = \frac{3 - 4}{3 - 2} = \frac{-1}{1} = -1$$

$$y'' \Big|_{(x,y)=(1,-1)} = \frac{(6 - 4 - 4) \cdot 1 - (6 - 4)(-1)}{(1)^2}$$

$$= -2 + 2 = 0$$