

- 5 1. Find the limit of the sequence of functions

$$\left\{ \frac{n^2 x^3 + 3nx}{2n^2 x + 1} \tan^{-1} \left( \frac{nx}{n+3} \right) \right\}$$

on the interval  $0 \leq x \leq 3$ , if it exists. Justify your answer.

- 8 2. Determine whether the following series converge or diverge. Justify your answers. If a series converges, find its sum.

(a)  $\sum_{n=1}^{\infty} \frac{(-1)^n n^2}{1 + 2n^2}$

(b)  $\sum_{n=3}^{\infty} \frac{2^n}{3^{n+1}}$

- 12 3. (a) Find the first four Taylor polynomials  $P_0(x)$ ,  $P_1(x)$ ,  $P_2(x)$ , and  $P_3(x)$  about  $x = 0$  for the function  $\cos 3x$ .  
(b) Use Taylor's remainder formula to verify that the Maclaurin series for  $\cos 3x$  converges to  $\cos 3x$  for all  $x$ .

- 8 4. Find the interval of convergence for the power series

$$\sum_{n=3}^{\infty} \frac{(-1)^{n+1} n^2}{3^n} x^{2n+1}.$$

- 7 5. Find the open interval of convergence for the power series

$$\sum_{n=2}^{\infty} \frac{2^{n+1}}{n^3 + 100n^2} (x+2)^n.$$