Term Test 2
Nov 7, 2012

SOLUTIONS

QUESTION 1

Distributed loads are applied to the shape shown in Figure 1.
In addition to these distributed loads, a *25 kN* force and a *75 kN.m* clockwise couple moment are applied.

Replace the system of forces and a couple acting on the shape with a single force and state where this force intersects the *x* and *y* axes.

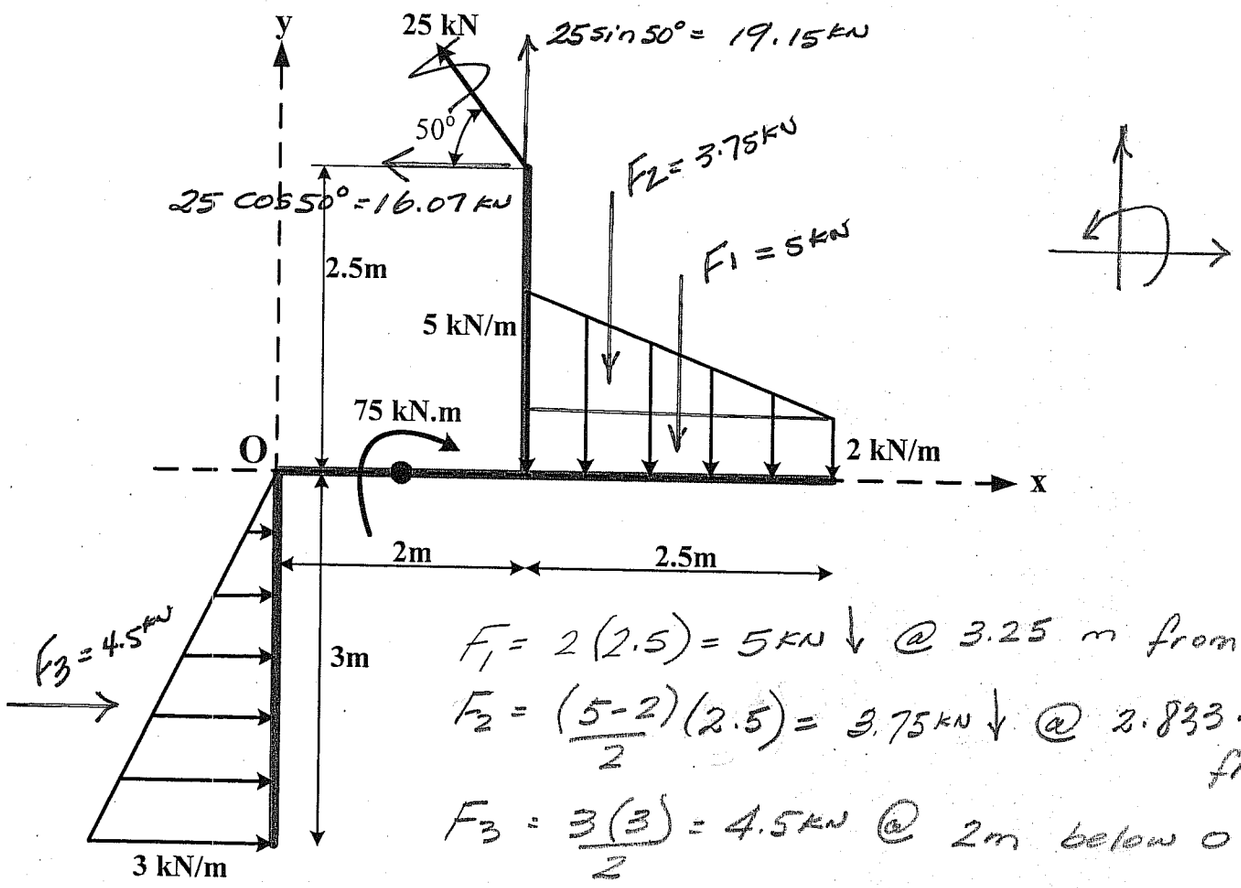


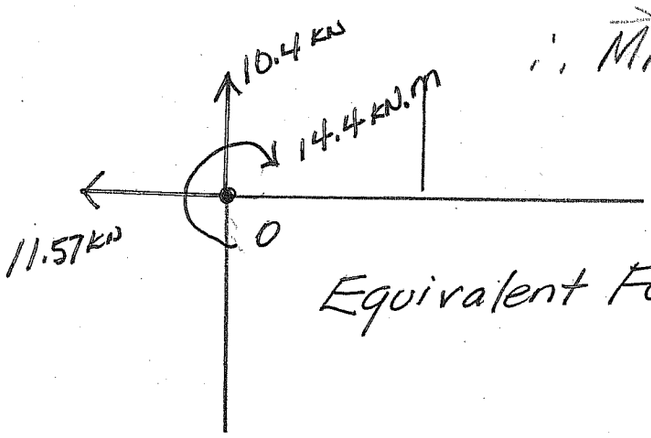
Figure 1

$R_x = \Sigma F_x = 4.5 - 16.07 = -11.57\,kN \therefore \vec{R}_x = 11.57\,kN \leftarrow$

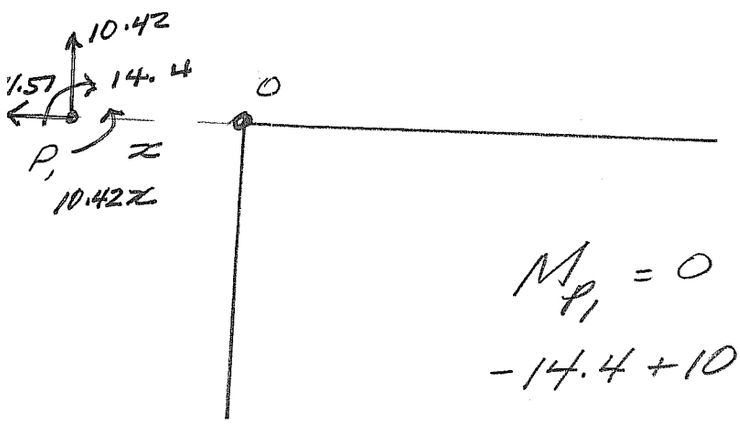
$R_y = \Sigma F_y = 19.15 - 3.75 - 5 = +10.4\,kN \therefore \vec{R}_y = 10.4\,kN \uparrow$

$M_{R_O} = \Sigma M_O = 4.5(2) - 75 + 16.07(2.5) + 19.15(2) - 3.75(2.833) - 5(3.25) = -14.4\,kN.m$

$\therefore \vec{M}_{R_O} = 14.4\,kN.m \circlearrowright$



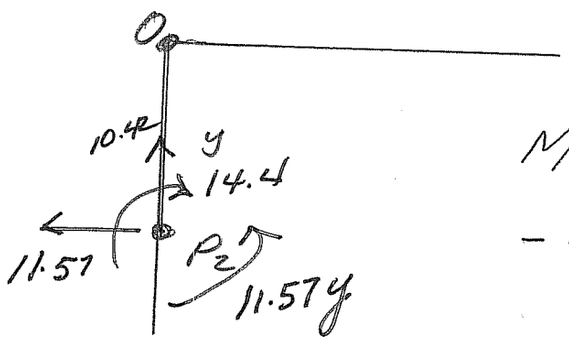
Equivalent Force-Couple at O



$$M_{P_1} = 0$$

$$-14.4 + 10.42x = 0$$

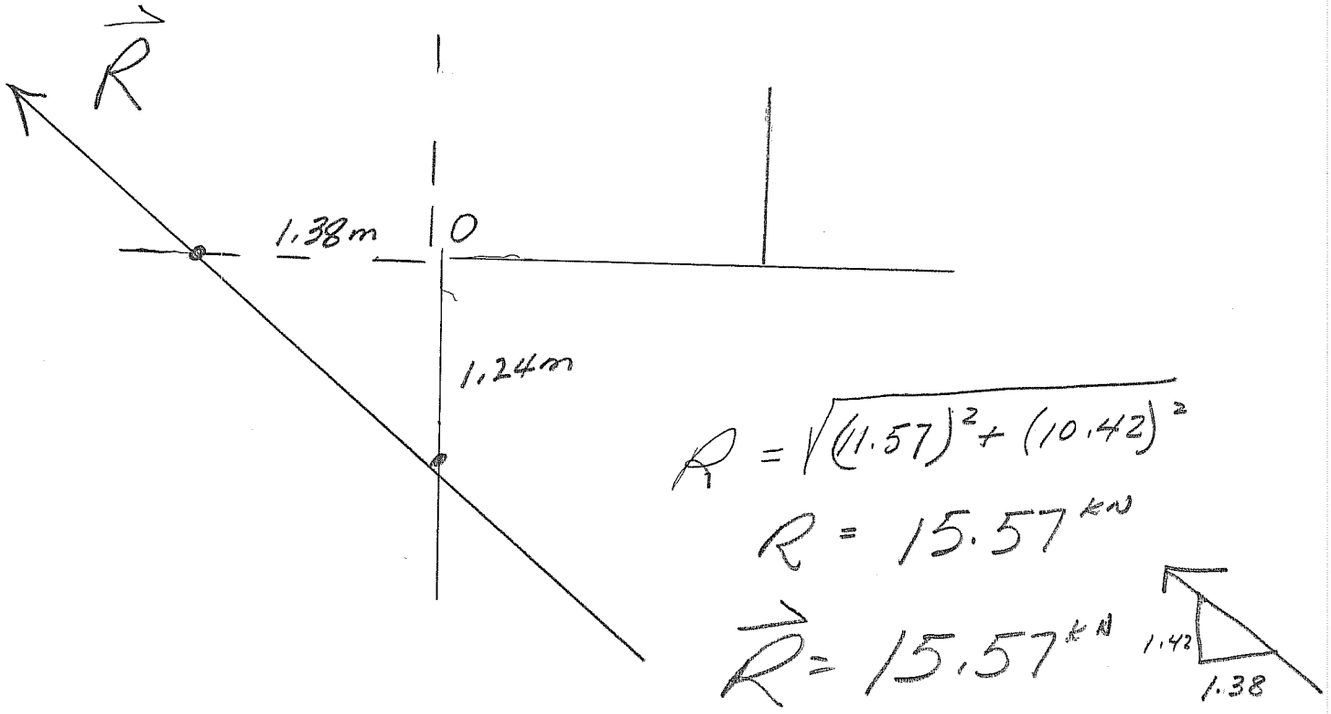
$$x = \frac{14.4}{10.42} = 1.38\text{m left of } O$$



$$M_{P_2} = 0$$

$$-14.4 + 11.57y = 0$$

$$y = \frac{14.4}{11.57} = 1.24\text{m below } O$$



$$R = \sqrt{(11.57)^2 + (10.42)^2}$$

$$R = 15.57\ \text{kN}$$

$$\vec{R} = 15.57\ \text{kN}$$

$$\frac{10.4}{11.57} = 0.899$$

$$\frac{1.24}{1.38} = 0.899$$

QUESTION 2

The *12 m* long beam *JKL* has a cable attached to the center of the beam at *K*. The right half of the beam carries a distributed load that goes from *0 kN/m* at *K* to *4 kN/m* at end *L*. The left half of the beam carries a distributed load that varies from *0 kN/m* at *K* to *w kN/m* at end *J*. Forces of *2 kN*, *4 kN* and *3 kN* are applied to the truss at joints *A*, *C* and *E* respectively.

To keep the beam in the horizontal equilibrium position shown in the Figure 2, a *48 kN.m* clockwise couple moment is applied to the beam. A cable then attaches the beam to a ring at *I*.

Two cables each at *45°* then attach the ring to the truss at joints *H* and *F*.

Determine:

a) The magnitude of the distributed load, *w*, applied to the left half of the beam,
b) The force in each member of the truss and state whether it is in tension, compression or zero.

PLACE YOUR RESULTS FOR THE TRUSS MEMBER FORCES ON THE FIGURE PROVIDED NEXT PAGE.

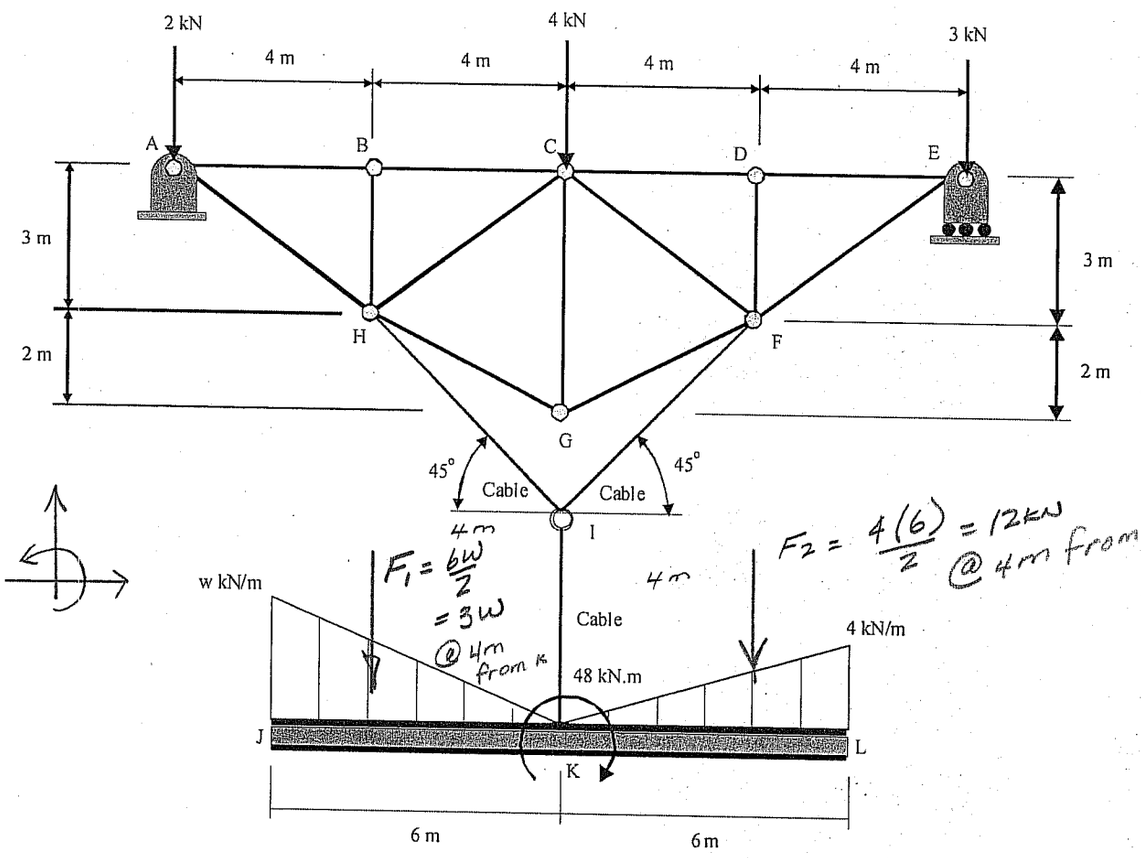
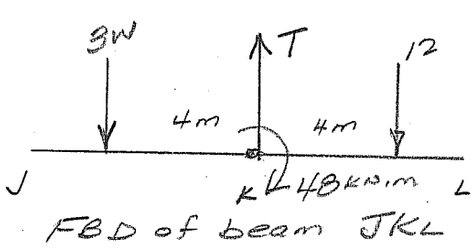


Figure 2

$F_2 = \frac{4(6)}{2} = 12\text{kN}$ @ 4m from K



FBD of beam JKL

$\Sigma F_y = 0 \quad -3w + T - 12 = 0$

$T = 12 + 3w$

$\Sigma M_K = 0$

$3w(4) - 48 - 12(4) = 0$

$w = \frac{96}{12} = 8\ \text{kN/m}$

$\therefore F_1 = 3(8) = 24\text{kN}$

$T = 36\text{kN} \uparrow$ on the beam

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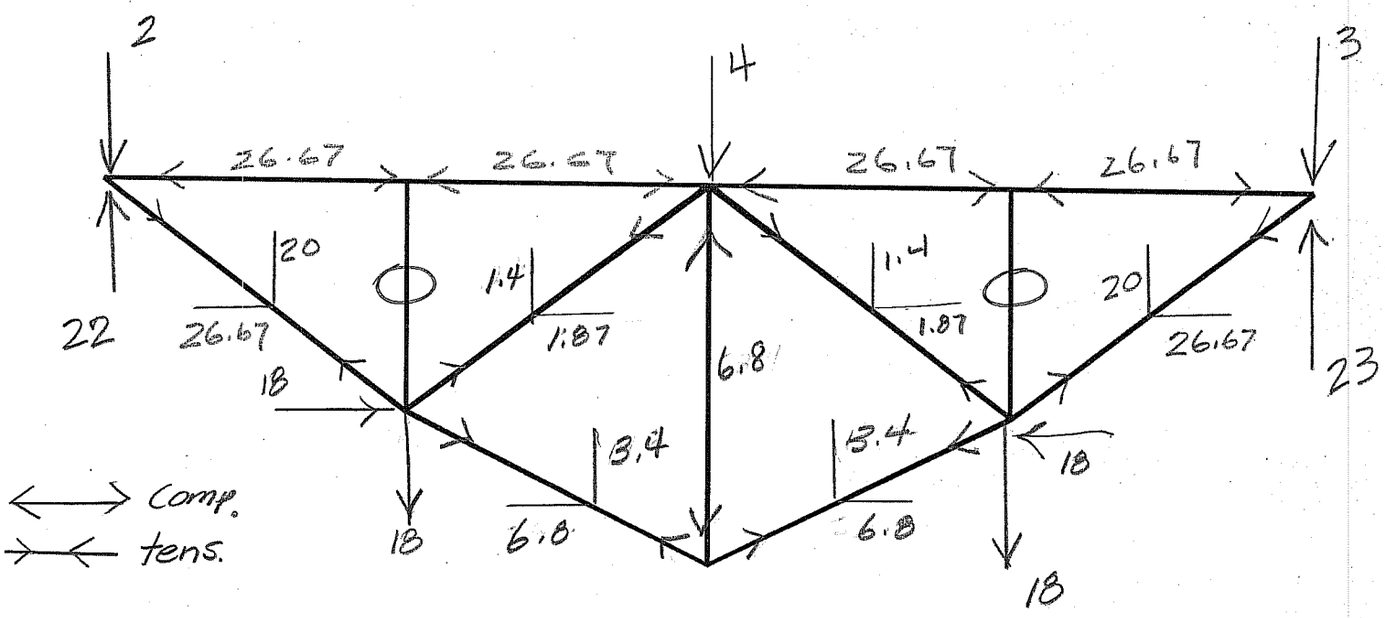
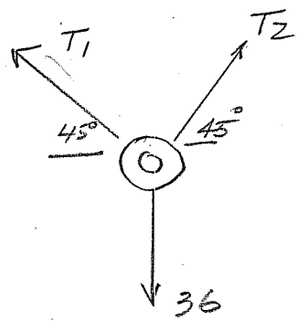


$\frac{20}{3} = \frac{x}{4}$ $x =$

FBD of Ring at I



$\Sigma F_x = 0$

$-T_1 \cos 45 + T_2 \cos 45° = 0$

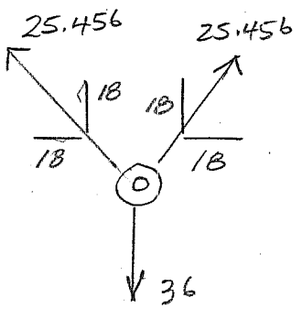
$\therefore T_1 = T_2$

$\Sigma F_y = 0$

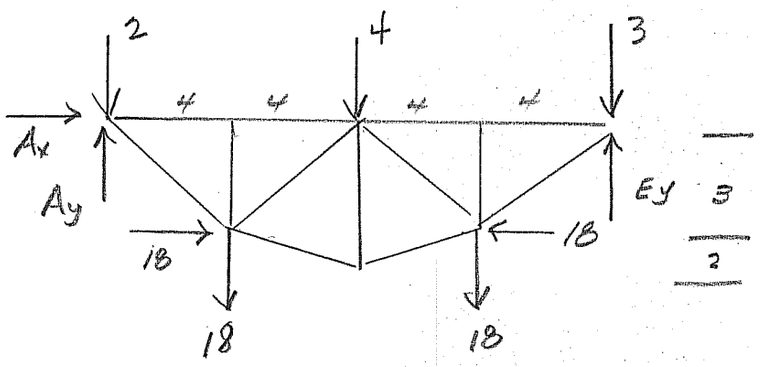
$T_1 \sin 45 + T_2 \sin 45 - 36 = 0$

$2T_1 \sin 45° = 36$

$T_1 = T_2 = \frac{36}{2 \sin 45°} = 25.456\ kN$



FBD of TRUSS



$\Sigma F_x = 0$

$A_x + 18 - 18 = 0 \quad A_x = 0$

$\Sigma M_A = 0 \quad 18(3) - 18(4) - 4(8) - 3(16) - 18(3) - 18(12) + E_y(16) = 0$

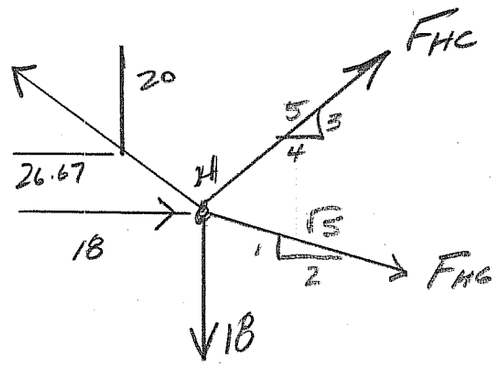
$E_y = +23\ kN \quad \therefore \vec{E_y} = 23\ kN \uparrow$

$\Sigma F_y = 0 \quad A_y - 2 - 4 - 3 - 18 - 18 + 23 = 0$

$A_y = +22\ kN$

$\vec{A_y} = 22\ kN \uparrow$

FBD Joint H



$\Sigma F_x = 0 \quad -26.67 + 18 + \frac{4}{5} F_{HC} + \frac{2}{\sqrt{5}} F_{HG} = 0$

$\frac{4}{5} F_{HC} + \frac{2}{\sqrt{5}} F_{HG} = +8.67 \quad (1)$

$\Sigma F_y = 0 \quad 20 - 18 + \frac{3}{5} F_{HC} - \frac{1}{\sqrt{5}} F_{HG} = 0$

$\frac{3}{5} F_{HC} - \frac{1}{\sqrt{5}} F_{HG} = -2 \quad (2)$

$\frac{4}{5} F_{HC} + \frac{2}{\sqrt{5}} F_{HG} = 8.67$

2*(2) $\frac{6}{5} F_{HC} - \frac{2}{\sqrt{5}} F_{HG} = -4$

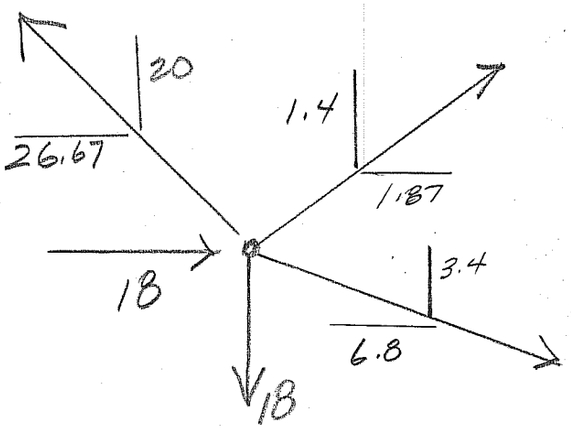
$2 F_{HC} = 4.67 \quad \therefore F_{HC} = +2.34 \text{ kN}$

$\therefore \vec{F}_{HC} = 2.34 \text{ kN}$ (4, 3)

$\frac{4}{5}(2.34) + \frac{2}{\sqrt{5}} F_{HG} = 8.67$

$\frac{2}{\sqrt{5}} F_{HG} = 6.8 \qquad \vec{F}_{HG} = 7.6 \text{ kN}$ (1, 2)

$F_{HG} = +7.6$



$\Sigma F_x = 0$

$-26.67 + 18 + 1.87 + 6.8 = 0$

$0 = 0$

$\Sigma F_y = 0$

$20 - 18 + 1.4 - 3.4$

$0 = 0$

Determine the forces acting on each member including the pulleys of the frame shown in Figure 3.

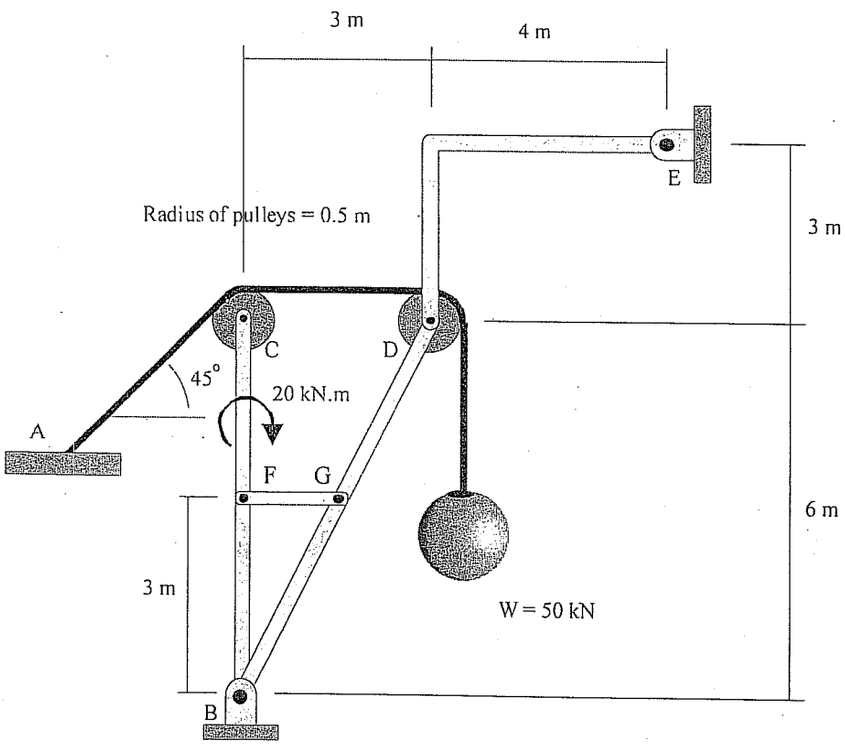
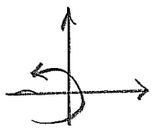
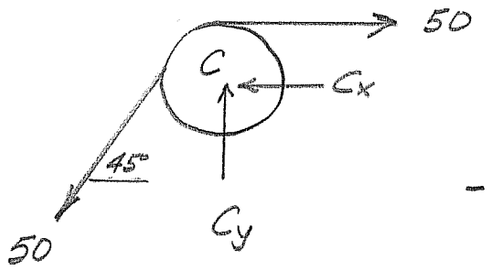


Figure 3

Pulleys:





$\Sigma F_x = 0$

$-50\cos 45° - C_x + 50 = 0$

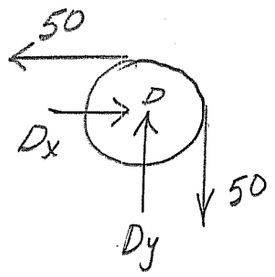
$C_x = +14.64\,kN$

$\therefore \vec{C_x} = 14.64\,kN \leftarrow$ on the pulley

$\Sigma F_y = 0$

$-50\sin 45° + C_y = 0 \quad C_y = +35.36\,kN$

$\therefore \vec{C_y} = 35.36\,kN \uparrow$ on the pulley



$\Sigma F_x = 0$

$-50 + D_x = 0$

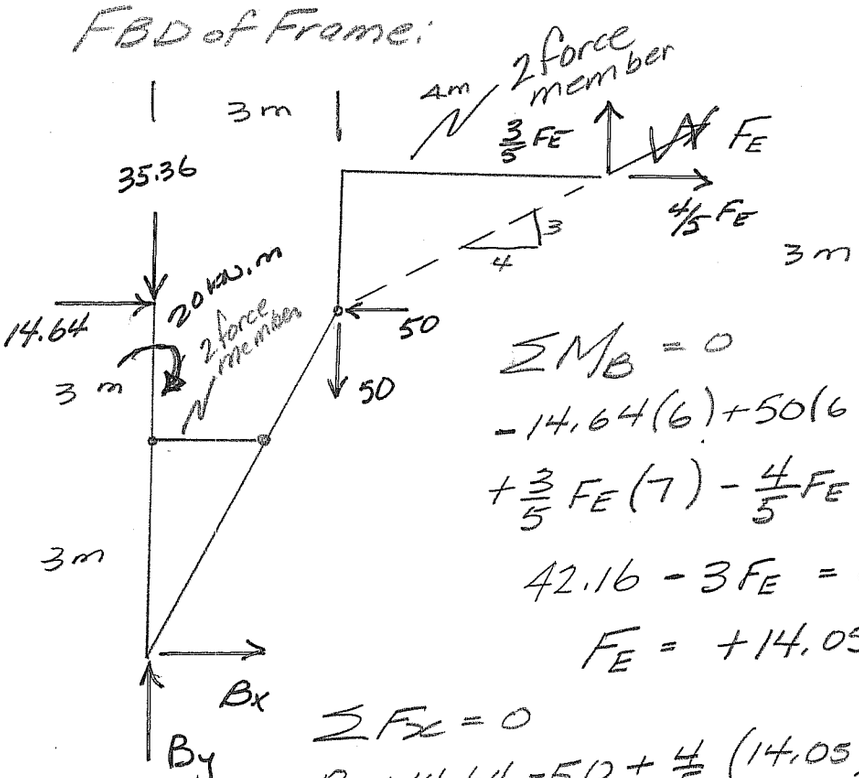
$D_x = +50\,kN \quad \therefore \vec{D_x} = 50\,kN \rightarrow$ on the pulley

$\Sigma F_y = 0$

$D_y - 50 = 0$

$D_y = +50\,kN \therefore \vec{D_y} = 50\,kN \uparrow$ on the pulley

FBD of Frame:



$\Sigma M_B = 0$

$-14.64(6) + 50(6) - 50(3) - 20$

$+\frac{3}{5}F_E(7) - \frac{4}{5}F_E(9) = 0$

$42.16 - 3F_E = 0$

$F_E = +14.05\,kN \therefore \vec{F_E} = 14.05\,kN$ 

$\Sigma F_x = 0$

$B_x + 14.64 - 50 + \frac{4}{5}(14.05) = 0$

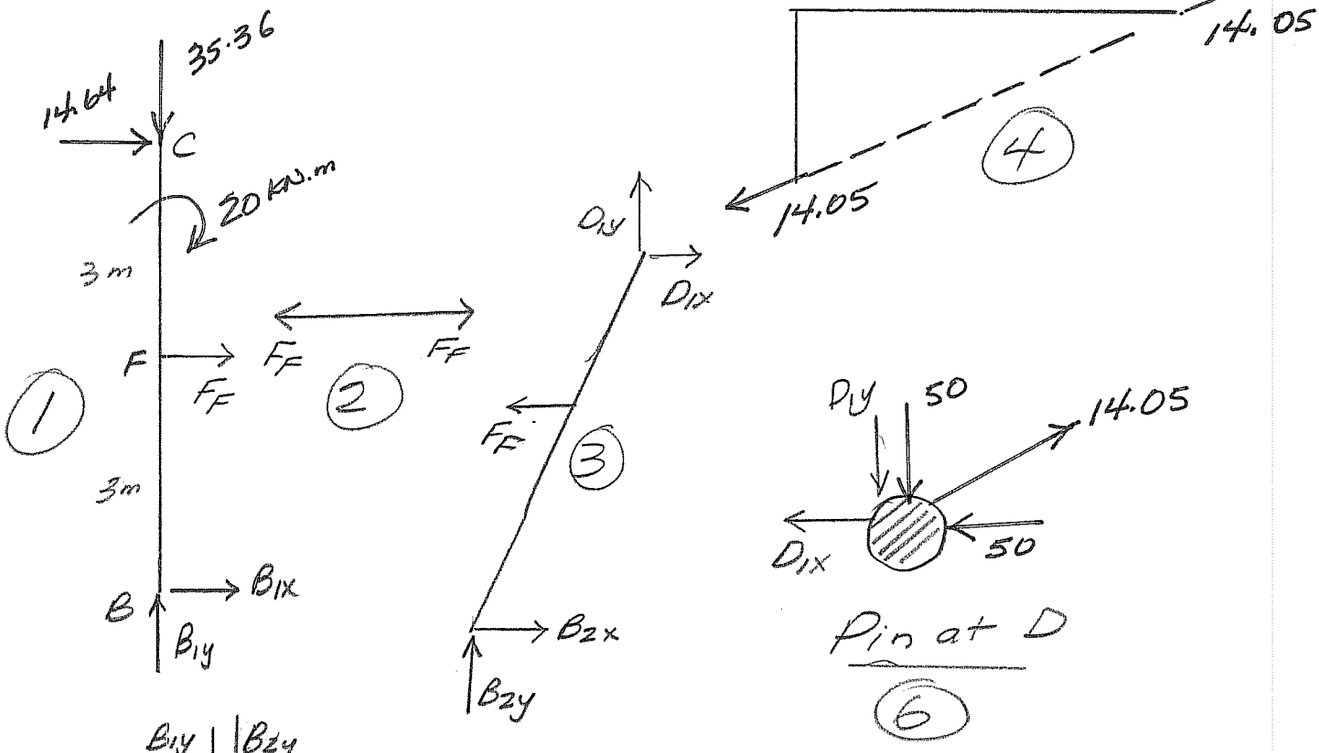
$B_x = +24.12\,kN \therefore \vec{B_x} = 24.12\,kN \rightarrow$

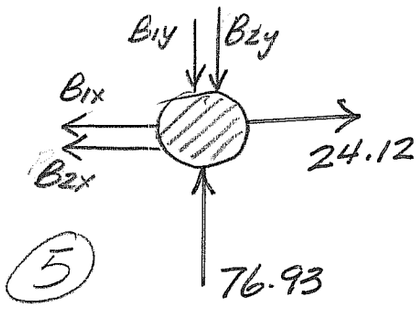
$\Sigma F_y = 0 \quad B_y - 35.36 - 50 + \frac{3}{5}(14.05) = 0$

$B_y = +76.93\,kN$

$\vec{B_y} = 76.93\,kN \uparrow$

Substructures:





From ① $\Sigma M_B = 0$

$-14.64(6) - 20 - F_F(3) = 0$

$F_F = -35.95\,KN$

$\therefore \vec{F_F} = 35.95\,KN \leftarrow$ on BFC

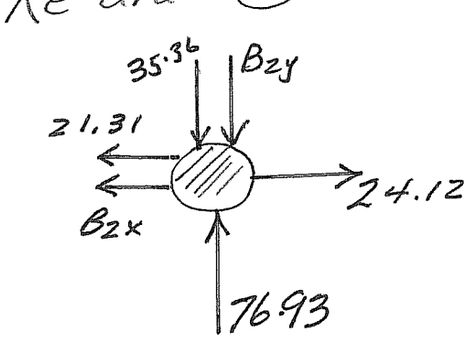
$\Sigma F_x = 0 \quad +14.64 + (-35.95) + B_{1x} = 0$

$B_{1x} = +21.31\,KN \quad \therefore \vec{B}_{1x} = 21.31\,KN \rightarrow$ on BFC

$\Sigma F_y = 0 \quad -35.36 + B_{1y} = 0$

$B_{1y} = +35.36\,KN \quad \therefore \vec{B}_{1y} = 35.36\,KN \uparrow$ on BFC

Re-draw ⑤



$\Sigma F_x = 0$

$-21.31 - B_{2x} + 24.12 = 0$

$B_{2x} = +2.81\,KN$

$\therefore \vec{B}_{2x} = 2.81\,KN \leftarrow$ on pin at B

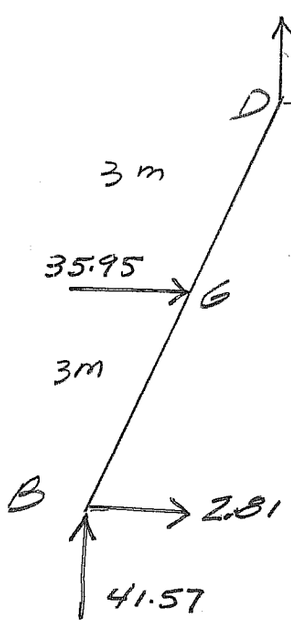
$\Sigma F_y = 0$

$-35.36 - B_{2y} + 76.93 = 0$

$B_{2y} = +41.57\,KN$

$\therefore \vec{B}_{2y} = 41.57\,KN \downarrow$ on pin at B

Re-draw (3)



$\Sigma F_x = 0$

$35.95 + D_{1x} + 2.81 = 0$

$D_{1x} = -38.76\ kN$

$\therefore \vec{D}_{1x} = 38.76\ kN \leftarrow$ on BGD

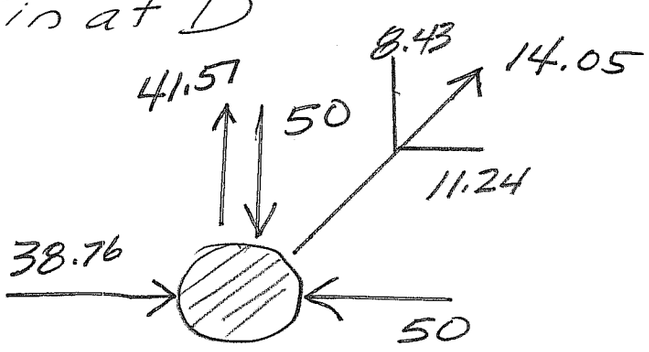
$\Sigma F_y = 0$

$D_{1y} + 41.57 = 0$

$D_{1y} = -41.57\ kN$

$\therefore \vec{D}_{1y} = 41.57\ kN \downarrow$ on BGD

Pin at D



$\Sigma F_x = 0$

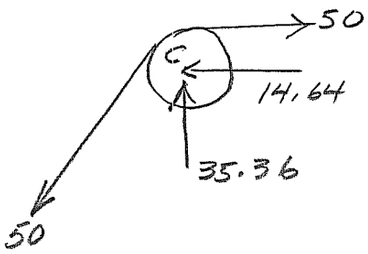
$38.76 + 11.24 - 50 = 0$

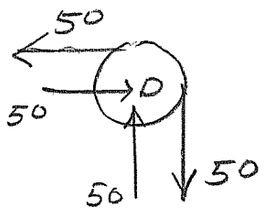
$0 = 0$ ✓

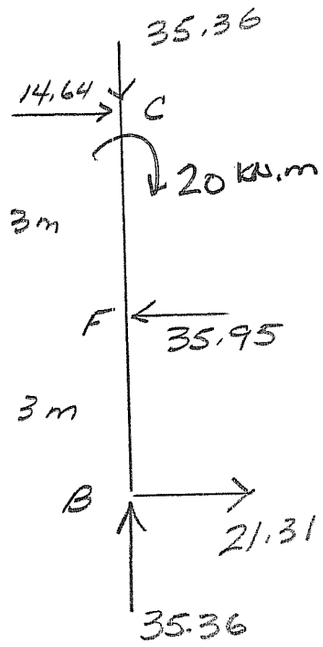
$\Sigma F_y = 0$

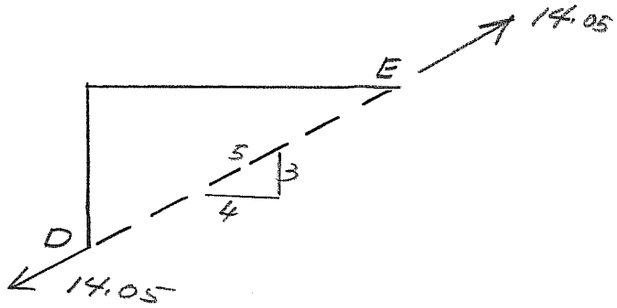
$41.57 + 8.43 - 50 = 0$

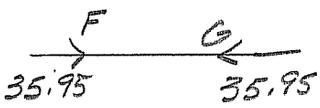
$0 = 0$ ✓

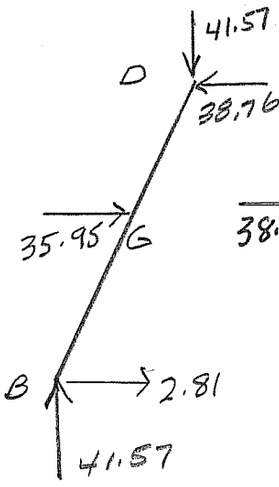

C
50
14.64
35.36
50

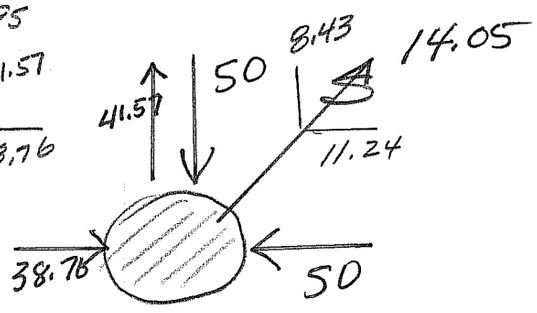

50
D
50
50
50

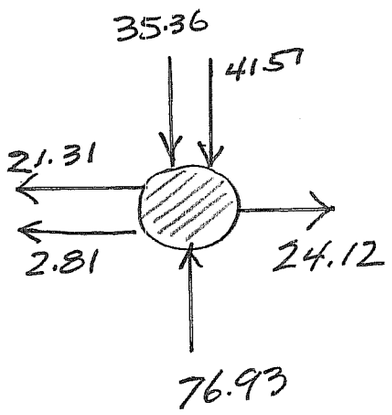

35.36
14.64
C
20 kN.m
3m
F
35.95
3m
B
21.31
35.36


E
14.05
5
3
4
D
14.05


F
G
35.95
35.95


41.57
D
38.76
35.95
G
B
2.81
41.57


8.43
14.05
41.57
50
11.24
38.76
50


35.36
41.57
21.31
2.81
24.12
76.93

Summary