**Math 2130 - Engineering Mathematical Analysis 1**

***Tutorial 8 - Questions relating to Symmetry,***
***§13.7 (Polar Coordinates) - Part 1, and §13.4 (Fluid Pressure).***

***For questions S.1 to S.4, let $R$ be the region in the $xy$-plane bounded by the lines*** $y = x + 2$, $y = -x - 2$ ***and the portion of the parabola*** $x = 4 - y^2$ ***with*** $x \geq 0$.

***For convenience, let $R_1$, $R_2$, $R_3$, and $R_4$ be the portions of the region $R$ in each of quadrants 1, 2, 3, and 4, respectively.***

***Use symmetry to simplify the double integrals in S.1 to S.3 as much as possible. Do not evaluate the non-zero integrals, but write them as simplified double iterated integrals:***

**S.1.** $\displaystyle\iint_R x\,dA.$ **S.2.** $\displaystyle\iint_R y\,dA.$ **S.3.** $\displaystyle\iint_R \frac{y^2 sin(y)}{1+\sqrt{x^4+y^4}} + x^2\,dA.$

**S.4.** What conditions on the function $f(x, y)$ will guarantee that $\displaystyle\iint_R f\,dA = 2\iint_{R_1 \bigcup R_4} f\,dA,$

where $R_1 \bigcup R_4$ is the part of $R$ with $x \geq 0$ (i.e., the union of $R_1$ and $R_4$),

or that $\displaystyle\iint_R f\,dA = 4\iint_{R_1} f\,dA,$ where $R_1$ is the first quadrant part of $R$?

**S.5.** Use symmetry to simplify the following double integral as much as possible, and then evaluate it:

$$\iint_{x^2+y^2 \leq 9} (xy^2 + x^2y + 2)\,dA.$$

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**13.7.1.** Evaluate the double integral: $\displaystyle\iint_{x^2+y^2 \leq 4} e^{x^2+y^2}\,dA.$

**13.7.2.** Evaluate the double integral: $\displaystyle\iint_R \sqrt{1 + 2x^2 + 2y^2}\,dA,$ where $R$ is the region bounded by

$y = \sqrt{9 - x^2}$, $y = \sqrt{16 - x^2}$, $y = x$, and $x = 0$.

**13.7.3.** Evaluate the double iterated integral: $\displaystyle\int_0^2 \int_0^{\sqrt{4-x^2}} \sqrt{x^2+y^2}\;dy\,dx,$ by changing to polar coordinates.

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**13.4.1.** A triangular plate has sides with lengths 3, 4 and 5 metres. It is submerged vertically in oil with density 950 kilograms per cubic metre. The side of length 3 metres is vertical, the side of length 4 metres is horizontal, and the uppermost vertex is 1 metre below the surface of the oil. Find the force due to oil pressure on each side of the plate. Take $g = 9.81\ m/s^2$.

**13.4.2.** An elliptic plate has major axis of length $2a$ metres and minor axis of length $2b$ metres. Its major axis is horizontal and its minor axes is vertical. It is slowly being lowered into a tank of water. At the instant when only $b/2$ metres of the plate sticks out of the water, set up, but do **NOT** evaluate, a double iterated integral for the force due to the water on each side of the plate. Take $g = 9.81\ m/s^2$. The density of water is $\rho = 1000\ kg/m^3$.

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**Answers:**

**S.1.** $\displaystyle\iint_R x\,dA = 2\iint_{R_1\bigcup R_2} x\,dA = 2\int_0^2\int_{y-2}^{4-y^2} x\,dx\,dy.$

**S.2.** $\displaystyle\iint_R y\,dA = 0.$

**S.3.** $\displaystyle\iint_R \frac{y^2 sin(y)}{1+\sqrt{x^4+y^4}} + x^2\,dA = \iint_R x^2\,dA = 2\iint_{R_1\bigcup R_2} x^2\,dA = 2\int_0^2\int_{y-2}^{4-y^2} x^2\,dx\,dy.$

**S.4.** None (in both cases)! The region is not symmetric about the $y$-axis.

**S.5.** $\displaystyle\iint_{x^2+y^2\le 9} (xy^2+x^2y+2)\,dA = \iint_{x^2+y^2\le 9} 2\,dA = 2\times(\text{Area of region}) = 18\,\pi.$

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**13.7.1.** $\pi\,(e^4-1)$.

**13.7.2.** $\dfrac{\pi}{24}\,[33^{3/2} - 19^{3/2}]$.

**13.7.3.** $\dfrac{4\,\pi}{3}$.

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**13.4.1.** $1.68\times 10^5$ N.

**13.4.2.** $\displaystyle\int_{-b}^{b/2}\int_{-(a/b)\sqrt{b^2-y^2}}^{(a/b)\sqrt{b^2-y^2}} 9810\left(\frac{b}{2}-y\right)dx\,dy\ \text{ N}.$

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