MATH 1210 A01 Summer 2013 Problem Workshop 6 Solutions

1. (a) $\begin{bmatrix} 8 & -2 & 8 \\ -4 & 4 & -6 \\ 0 & 1 & 3 \end{bmatrix}$

   (b) Not possible as there are 3 columns in the first matrix which is not equal to the 1 row of the second matrix

   (c) [10]

   (d) $\begin{bmatrix} 3 & 4 & 5 \\ -6 & -8 & -10 \\ 9 & 12 & 15 \end{bmatrix}$

   (e) Not possible as there is 1 column in the first matrix which is not equal to the 3 rows of the second matrix

2.

$$\begin{aligned} 3(X+I) - 2AC^T &= B^2 \\ 3(X+I) &= B^2 + 2AC^T \\ X + I &= \frac{1}{3}B^2 + \frac{2}{3}AC^T \\ X &= \frac{1}{3}B^2 + \frac{2}{3}AC^T - I \end{aligned}$$

Now

$$B^2 = \begin{bmatrix} 1 & -2 & 3 \\ 4 & 2 & 1 \\ 0 & 5 & 2 \end{bmatrix} \begin{bmatrix} 1 & -2 & 3 \\ 4 & 2 & 1 \\ 0 & 5 & 2 \end{bmatrix} = \begin{bmatrix} -7 & 9 & 7 \\ 12 & 1 & 16 \\ 20 & 20 & 9 \end{bmatrix}$$

$$AC^T = \begin{bmatrix} 2 & 1 \\ -2 & 3 \\ 5 & 2 \end{bmatrix} \begin{bmatrix} 4 & 2 & 0 \\ 5 & 6 & 7 \end{bmatrix} = \begin{bmatrix} 13 & 10 & 7 \\ 7 & 14 & 21 \\ 30 & 22 & 14 \end{bmatrix}$$

Therefore

$$\begin{aligned} X &= \frac{1}{3}B^2 + \frac{2}{3}AC^T - I \\ &= \frac{1}{3}\begin{bmatrix} -7 & 9 & 7 \\ 12 & 1 & 16 \\ 20 & 20 & 9 \end{bmatrix} + \frac{2}{3}\begin{bmatrix} 13 & 10 & 7 \\ 7 & 14 & 21 \\ 30 & 22 & 14 \end{bmatrix} - I \\ &= \begin{bmatrix} 16/3 & 29/3 & 7 \\ 26/3 & 26/3 & 58/3 \\ 80/3 & 64/3 & 34/3 \end{bmatrix} \end{aligned}$$

3. This is untrue since

$$(A-B)(A+B) = A^2 - BA + AB - B^2$$

which is only equal to $A^2 - B^2$ if $AB = BA$ which is not ture in general. Many counterexamples could also verify that this is not always true.

4.

$$\begin{aligned}
\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} &= \begin{bmatrix} 1^2-1 & 1^2-2 & 1^2-3 \\ 2^2-1 & 2^2-2 & 2^2-3 \\ 3^2-1 & 3^2-2 & 3^2-3 \end{bmatrix} \\
&= \begin{bmatrix} 0 & -1 & -2 \\ 3 & 2 & 1 \\ 8 & 7 & 6 \end{bmatrix}
\end{aligned}$$