MATH 2130 Summer Evening 2012 Problem Workshop 3

1. If $z = f(u, v, t), u = g(x, y, t), v = h(x, y, t)$ and $y = k(t)$, find the chain rule for $\frac{\partial z}{\partial t}\big)_x$
2. If $f(s)$ and $g(t)$ are differentiable functions, show that $\nabla f(x^2 - y^2) \cdot \nabla g(xy) = 0$.
3. If $z = x^2 + y^2, x = u\cos v$, and $y = u\sin v$, find and simplify $\frac{\partial^2 z}{\partial v^2}\big)_u$.
4. If $f(v)$ is differentiable, show that $u(x, y) = x^3 f(x/y)$ satisifes the equation
$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 3u.$$
5. The equations
$$x^2 + y + 3s^2 + s = 2t - 1, \qquad y^2 - x^4 + 2st + 7 = 6s^2t^2$$
define s and t are functions of x and y. Find $\frac{\partial s}{\partial x}$ when $s = 0$ and $t = 1$. Assume $x > 0$.
6. The equations
$$x = r\sin\phi\cos\theta, \qquad y = r\sin\phi\sin\theta, \qquad z = r\cos\phi$$
define r, ϕ, θ as functions of x, y and z. Find $\frac{\partial \phi}{\partial y}$.
7. Find the rate of change of the function $f(x, y, z) = \sin(xy) - z^3$ at the point $(2, 0, 3)$ in the direction of the upward normal to the surface $xz^2 - x^2z = 6$.
8. Find equations for the tangent line to the following curve at the point $(1, -1, 3)$:
$$xyz + z^3 = 24, \qquad x^3y^2z + y^3 = 4x - 2.$$
9. Find an equation for the tangent plane to the following surface at the point $(2, -1, -1)$:
$$x^2y + y^2z + z^2x + 3 = 0.$$
10. Find all critical points for the function $f(x, y) = x^3y^3 - x^2y^2 + 6$.
11. Find all critical points for the function $f(x, y) = x^3y^2 - xy + 3y$.
12. Find and classify all critical points of the function as giving relative minima, maxima, saddle points or neither.
    (a) $f(x, y) = x^3 + xy + y^3$
    (b) $f(x, y) = x^3 - xy^2 + 3xy$
    (c) $f(x, y) = x^4 - 3x^2y^2 + y^4$

(d) $f(x, y) = y^2 + |x - 1|$

Answers:

1. $\frac{\partial z}{\partial u}\frac{\partial u}{\partial y}\frac{dy}{dt} + \frac{\partial z}{\partial u}\frac{\partial u}{\partial t} + \frac{\partial z}{\partial v}\frac{\partial v}{\partial y}\frac{dy}{dt} + \frac{\partial z}{\partial v}\frac{\partial v}{\partial t} + \frac{\partial z}{\partial t}$
2.
3. 0
4.
5. 16
6. $r^{-1}\cos\phi\sin\theta$
7. $-216/\sqrt{73}$
8. $x = 1 + 81t, y = -1 + 133t, z = 3 - 6t$
9. $x - 2y + z = 3$.
10. All points on the x-axis, y-axis and on the curve $y = 2/(3x)$.
11. $(3, 0), (9, 1/243)$
12. (a) $(0, 0)$ gives a saddle point. $(-1/3, -1/3)$ gives a relative maximum.
    (b) $(0, 0), (0, 3)$ both give saddle points.
    (c) $(0, 0)$ gives a saddle point.
    (d) $(1, 0)$ gives a relative minimum. Points $(1, y)$ for $y \neq 0$ give neither.