Practice Midterm 1A Answers

Multiple-Choice

1. C
2. A
3. B
4. E
5. B
6. A
7. C
8. D
9. A
10. D
11. A
12. E
13. D
14. C
15. C

Long-Answer

1. (a) We know that y = a + bx, and so

$$\bar{y} = \frac{\sum_{i=1}^{n} y_i}{n} = \frac{1}{n}\sum_{i=1}^{n}(a + bx_i) = \frac{1}{n}\left(\sum_{i=1}^{n} a + b\sum_{i=1}^{n} x_i\right) = \frac{na}{n} + b\frac{\sum_{i=1}^{n} x_i}{n} = a + b\bar{x}.$$

$$s_y = \sqrt{\frac{\sum_{i=1}^{n}(y_i - \bar{y})^2}{n-1}} = \sqrt{\frac{\sum_{i=1}^{n}(a + bx_i - (a + b\bar{x}))^2}{n-1}} = \sqrt{\frac{\sum_{i=1}^{n}(bx_i - b\bar{x})^2}{n-1}}$$
$$= \sqrt{\frac{b^2\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1}} = |b|\sqrt{\frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1}} = |b|\,s_x$$

(b) Let X be the length of a call and let Y be the charge for the call. Then

$$y = 1.49 + 0.25x$$

This is a linear relationship of the form y = a + a + bx, where a = 1.49 and b = 0.25, so

$$\bar{y} = a + b\bar{x} = 1.49 + 0.25\bar{x} = 1.49 + 0.25(12) = \$4.49$$
$$s_y = |b|\,s_x = 0.25 s_x = 0.25(7) = \$1.75$$

2. (a) The fraction of variation in Y explained by its regression on X is, by definition, equal to r^2. We know that $r^2 = 0.90$, and so

$$r = -\sqrt{r^2} = -\sqrt{0.90} = -0.9487$$

Note that we take the negative square root of r^2, as we see from the slope of the regression line that the relationship between X and Y is negative.

(b) If the temperature increases by 1°C, we predict the reaction time to decrease by 0.5 seconds.

(c) The residual for this trial is

$$y_i - \hat{y} = 54 - (87.2 - 0.5(70)) = 54 - 52.2 = 1.8 \text{ seconds}$$

3. (a) Michelle is basing her conclusions on an **observational study**. A proper experiment was not conducted. (For example, no randomization was used.) In addition, since Fertilizer A was only used in the back yard, which was watered once a week, and Fertilizer B was only used in the front yard, which was watered twice a week, fertilizer and water frequency are **confounded**. It is impossible to separate their effect on the response. (Did the front yard do better because of the fertilizer, because it was watered more, or because of something else? We don't know!) Because we haven't observed all combinations of fertilizer and watering frequency, we were also unable to establish any possible **interaction** that might exist.

(b) (i) The appropriate type of experimental design to use is a **randomized block design**.

(ii) The experimental units are the plots of grass in the yard.

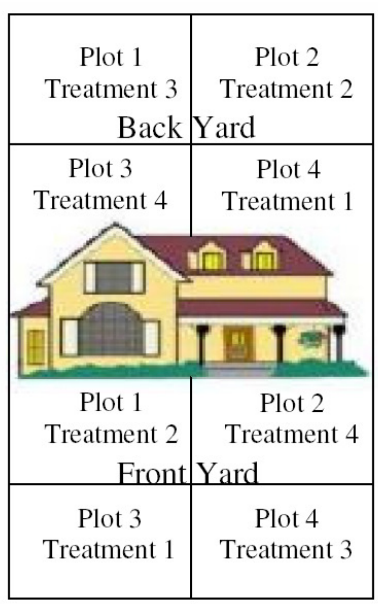
The factors are Fertilizer and Watering Frequency.

Fertilizer has two levels – A and B. Watering frequency has two levels – once or twice a week.

There are four treatments – Fertilizer A/Once a week, Fertilizer A/Twice a week, Fertilizer B/Once a week, Fertilizer B/Twice a week.

The response variable is growth (or greenness) of their lawn.

(c) Since we expect the grass in the front and back yards to behave differently, we will conduct a randomized block design with two blocks – the front yard and the back yard. In the back yard, we will divide the yard into four plots of grass, as shown below. We will then randomly assign each of the four treatments to one plot. We will then do the same in the front yard. For the duration of the summer, the appropriate treatment will be applied to each plot of grass, and at the end of the summer, we will compare the four treatments for the back yard and for the front yard separately.



4. (a) The outcomes in the sample space are shown below (using the first letter in the team's name to indicate that they win the game). The probability of each outcome is also calculated, multiplying respective probabilities for each team winning because of independence.

Outcome	Probability
MTV	$(0.4)(0.2)(0.7) = 0.056$
MTL	$(0.4)(0.2)(0.3) = 0.024$
MDV	$(0.4)(0.8)(0.7) = 0.224$
MDL	$(0.4)(0.8)(0.3) = 0.096$
BTV	$(0.6)(0.2)(0.7) = 0.084$
BTL	$(0.6)(0.2)(0.3) = 0.036$
BDV	$(0.6)(0.8)(0.7) = 0.336$
BDL	$(0.6)(0.8)(0.3) = 0.144$

(b) $P(B \cup L) = P(B) + P(L) - P(B \cap L) = P(B) + P(L) - P(B)P(L) = 0.6 + 0.3 - (0.6)(0.3) = 0.72$

We could also find this probability by adding the probabilities of the corresponding outcomes in the sample space:

$$P(B \cup L) = P(MTL) + P(MDL) + P(BTV) + P(BTL) + P(BDV) + P(BDL)$$
$$= 0.024 + 0.096 + 0.084 + 0.036 + 0.336 + 0.144 = 0.72$$

(c) $P(\text{two Canadian teams win}) = P(MTL) + P(MDV) + P(BTV) = 0.024 + 0.224 + 0.084 = 0.332$