1:30 p.m. 19 April 2008 FINAL EXAMINATION

PAPER NO.: 425 PAGE NO.: 1 of 4

DEPARTMENT & COURSE NO.: ENG 1460 TIME: 3 HOURS

EXAMINATION: Introduction to Thermal Sciences EXAMINERS: M. Guyot, J. Bartley M. Abou Al-Sood, H. Soliman

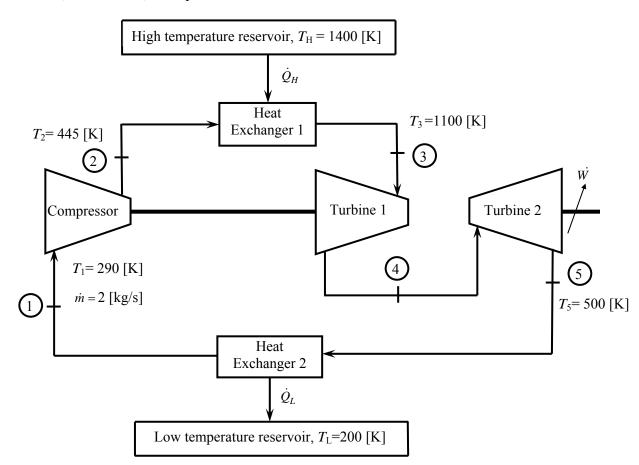
### Instructions:

- (a) You are permitted to use the course textbook and a non-programmable calculator.
- (b) Clear, systematic solutions are required. **Show all the steps in presenting your work.** In general, write the applicable equation to be used, substitute for the quantities appearing in the equation, and calculate a result. Marks will not be assigned for solutions that require unreasonable (in the opinion of the instructor) effort to decipher.
- (c) Ask for clarification if any problem statement is not clear to you.
- (d) Use linear interpolation between table entries as necessary.
- (e) Retain all the significant figures in the property values taken from the tables and indicate the units. Use 4 to 5 significant figures in your calculations. Final answers must have 4 to 5 significant figures with units.
- (f) Unless the state is given in the question, provide the reasoning (justification) that you used in deciding the state of a substance (*i.e.*, compressed liquid, superheated vapour, saturated mixture).

value 15 <u>1.</u>

A gas power cycle uses air as the working fluid and has two turbine sections as shown in the figure below. The power produced by Turbine 1 is used to drive the compressor, and Turbine 2 produces output power  $\dot{W}$ . Air enters the compressor with a temperature of 290 K and a mass flow rate of  $\dot{m}=2$  [kg/s] (State 1); the air exits the compressor at 445K (State 2). Heat is transferred from a high-temperature thermal reservoir,  $T_{\rm H}=1400$  [K], to Heat Exchanger 1 at a rate  $\dot{Q}_{\rm H}$ . The temperature of the air is raised to 1100 [K] (State 3) as it flows through Heat Exchanger 1. The air enters Turbine 1 where it is expanded to State 4. The flow then proceeds to Turbine 2 and exits with a temperature of 500 [K] (State 5). Heat is then removed from the air through Heat Exchanger 2 at a rate  $\dot{Q}_{\rm L}$  which is rejected to the low-temperature thermal reservoir,  $T_{\rm L}=200$  [K].

- 6 (a) Determine the temperature at the inlet to Turbine 2,  $T_4$ .
- 2 (b) Calculate the power output from Turbine 2,  $\dot{W}$ .
- 3 (c) Calculate the rate of heat transfer to the air through Heat Exchanger 1,  $\dot{Q}_{\rm H}$ .
- 4 (d) Calculate the thermal efficiency  $\eta_{th}$  of this gas power cycle and determine if the cycle is reversible, irreversible, or impossible.



1:30 p.m. 19 April 2008 FINAL EXAMINATION

PAPER NO.: 425 PAGE NO.: 2 of 4

DEPARTMENT & COURSE NO.: ENG 1460 TIME: 3 HOURS

EXAMINATION: Introduction to Thermal Sciences EXAMINERS: M. Guyot, J. Bartley M. Abou Al-Sood, H. Soliman

<u>value</u> 2.

An insulated piston-and-cylinder system consists of two cylindrical regions separated by a thin metal plate that is fixed in place, as shown in the figure below. It is assumed that the plate conducts heat very well such that there is no temperature difference between the two cylindrical regions at any given time (*i.e.*, the temperature among both regions is uniform during any quasi-equilibrium process). One of the cylindrical regions contains 2.5 [kg] of R-134a and the other region contains 2.5 [kg] of oxygen gas,  $O_2$ , bounded on one side by an insulating piston. Both the R-134a and the oxygen gas have an initial temperature of -10 [°C]. The oxygen gas is compressed polytropically until a final temperature of +5 [°C] is reached and the R-134a becomes saturated vapour. Note that heat transfer may take place across the thin metal partition but there is no heat transfer across the cylinder walls or through the moveable piston. Assume that the oxygen gas,  $O_2$ , behaves as an ideal gas.

- 10 (a) Determine all of the relevant properties of the R-134a corresponding to the initial state (State 1) and the final state (State 2).
- 10 (b) Determine the value of the polytropic exponent n, the amount of heat transferred across the thin plate,  ${}_{1}Q_{2}$ , and the work done during the compression process,  ${}_{1}W_{2}$ .

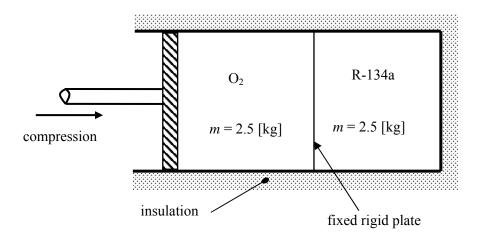


Figure. Schematic diagram for Problem 2.

 $\frac{\text{value}}{Q}$   $\frac{3.}{Q}$ 

A refrigeration/heat-pump unit functions as an air conditioner (providing cooling) in the summer months and as a heat pump (providing heating) during the winter months. It is desired to maintain a home at a temperature of 21 [°C] in both the summer and winter. In the summer, the house gains heat from the outdoors at a rate of 6 [kW], while in the winter, the house loses heat to the surrounding outside air at a rate of 25 [kW]. In addition, heat is generated within the house at a rate  $\dot{Q}_{\rm gen}$  during both the summer and winter months due to the presence of people and the regular use of lighting and appliances. The power input to the refrigeration/heat-pump unit is 5 [kW] for either the cooling mode or the heating mode of operation. If the coefficient of performance for the refrigeration (cooling) mode,  $\beta_{\rm R}$ , is related to the coefficient of performance for the heat pump mode,  $\beta'_{\rm HP}$ , by the equation,  $\beta_{\rm R} = \beta'_{\rm HP} - 1$ , then determine the rate of heat generation in the home,  $\dot{Q}_{\rm gen}$ , in [kW].

1:30 p.m. 19 April 2008 FINAL EXAMINATION

PAPER NO.: 425 PAGE NO.: 3 of 4

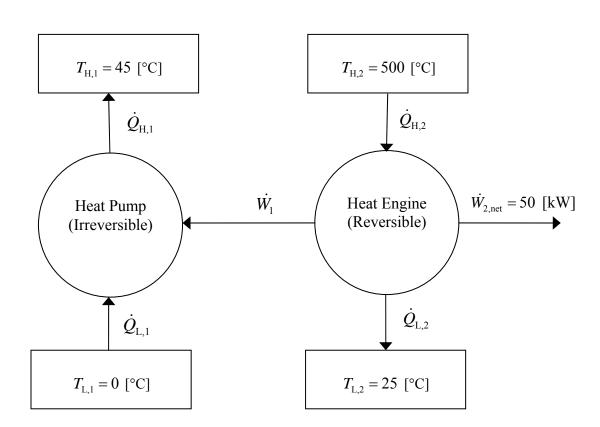
DEPARTMENT & COURSE NO.: ENG 1460 TIME: 3 HOURS

EXAMINATION: Introduction to Thermal Sciences EXAMINERS: M. Guyot, J. Bartley M. Abou Al-Sood, H. Soliman

<u>value</u> <u>4.</u>

A reversible (Carnot) heat engine operates between two thermal reservoirs at  $T_{\rm H,2} = 500$  [°C] and  $T_{\rm L,2} = 25$  [°C]. The heat engine is used to drive an irreversible heat pump that removes heat from a low temperature reservoir at  $T_{\rm L,1} = 0$  [°C] and rejects heat to a high temperature reservoir at  $T_{\rm H,1} = 45$  [°C]. It is desired to provide input power,  $\dot{W}_1$ , to the heat pump such that the coefficient of performance of the irreversible heat pump is 60% of that for a reversible heat pump; *i.e.*,  $\beta'_{\rm HP} = 0.6 \left(\beta'_{\rm HP,REV}\right)$ . The total power developed by the heat engine is divided into two parts: an amount  $\dot{W}_1$  that is used to drive the heat pump, and  $\dot{W}_{2,\rm net}$  as the remaining power, where  $\dot{W}_{2,\rm net} = 50$  [kW]. Heat is transferred to the heat engine from a high temperature reservoir at the rate of  $\dot{Q}_{\rm H,1}$ , and heat is "pumped" by the heat pump to a high temperature reservoir at the rate of  $\dot{Q}_{\rm H,1}$ . It is known that the sum of these two rates of heat transfer is as follows:  $\dot{Q}_{\rm H,1} + \dot{Q}_{\rm H,2} = 500$  [kW].

- 4 (a) Determine the thermal efficiency,  $\eta_{th}$ , for the Heat Engine and the coefficient of performance,  $\beta'_{HP}$ , for the Heat Pump.
- (b) Determine the power input required for the Heat Pump,  $\dot{W}_1$ , in [kW].
- 4 (c) Determine the rates of heat transfer  $\dot{Q}_{\rm H,1}$  and  $\dot{Q}_{\rm L,1}$  for the Heat Pump, and determine the rates of heat transfer  $\dot{Q}_{\rm H,2}$  and  $\dot{Q}_{\rm L,2}$  for the Heat Engine.



1:30 p.m. 19 April 2008 FINAL EXAMINATION

PAPER NO.: 425 PAGE NO.: 4 of 4

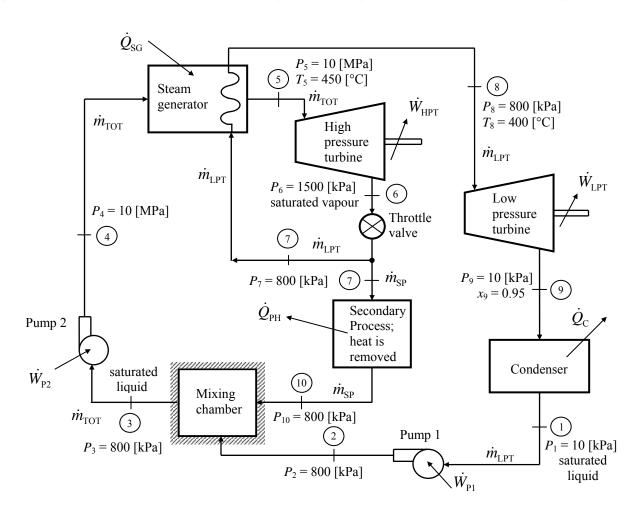
DEPARTMENT & COURSE NO.: ENG 1460 TIME: 3 HOURS

EXAMINATION: Introduction to Thermal Sciences EXAMINERS: M. Guyot, J. Bartley M. Abou Al-Sood, H. Soliman

value 40 <u>5.</u>

The figure below shows a schematic diagram of a steam power plant that uses a portion of the steam generated for a secondary process; e.g., for building heating. Power is produced from two steam turbines: a high-pressure turbine (HPT) and a low-pressure turbine (LPT) that together produce a total of 15 [MW]. Steam enters the high-pressure turbine with mass flow rate  $\dot{m}_{TOT} = 16.72$  [kg/s]. The steam leaving the high-pressure turbine exits as a saturated vapour at 1500 [kPa]. The steam is then throttled across an expansion valve to a pressure of 800 [kPa] at which point part of the steam,  $\dot{m}_{\rm SP}$ , is used to meet the secondary demand for process heating,  $\hat{Q}_{\rm PH}$ . This heat is removed from the steam by some heat exchange process; the flow then enters an insulated mixing chamber at 800 [kPa]. The other part of the steam that was diverted after the throttling valve,  $\dot{m}_{\rm LPT}$ , is reheated in a separate section of the steam generator to a temperature of 400 [°C] (State 8). This steam is then expanded across the lowpressure turbine (LPT) and exits with a pressure of 10 [kPa] and a quality of 95% (State 9). The flow is then condensed to the state of saturated liquid at 10 [kPa] and then pumped by Pump 1 to a pressure of 800 [kPa] to enter the mixing chamber and recombine with the flow from the secondary process. Pump 2 delivers the recombined flow to the steam generator at the required pressure of 10 [MPa]. Further details of the cycle operating states are shown in the figure below. For purposes of analysis, assume that the turbines and mixing chamber are insulated (adiabatic) and that the pumps operate adiabatically. Neglect changes in potential and kinetic energies across all devices.

- 4.5 (a) Calculate the power produced from each turbine,  $\dot{W}_{HPT}$  and  $\dot{W}_{LPT}$ .
- 11 (b) Determine the mass flow rates  $\dot{m}_{LPT}$  and  $\dot{m}_{SP}$  and the power required by each pump.
- 11 (c) Determine the amount of heat removed for the secondary process heating,  $\dot{Q}_{\rm PH}$ .
- 5.5 (d) Determine the rates of heat transfer  $\dot{Q}_{SG}$  and  $\dot{Q}_{C}$ , and calculate the thermal efficiency,  $\eta_{th}$ , of the power plant.
- 8 (e) Show all of the state points on a  $T-\nu$  diagram, including constant-pressure lines and process paths. Label the diagram with relevant temperature and pressure values ( $\nu$  values are optional).



## **Question #1**

(a)

➤ From the first law of thermodynamic for control volume enclosing the compressor

➤ Since turbine 1 is used only for driving compressor, then

$$\dot{W}_{T1} = -\dot{W}_C$$

$$= 311.24 \qquad \text{kW}$$

**(b)** 

From the first law of thermodynamic for control volume enclosing the turbine 1

(c)

From the first law of thermodynamic for control volume enclosing the heat exchanger 1

> Thermal efficiency is defined as

$$\eta_{th} = \frac{\dot{W}_{net}}{\dot{Q}_{add}} = \frac{\dot{W}_{T2}}{\dot{Q}_{H}} \\
= \frac{893.56}{1315.24} \\
= 0.6794$$

➤ An alternative way to calculate Thermal efficiency is

$$\eta_{th} = 1 - \frac{\left|\dot{Q}_{rejected}\right|}{\left|\dot{Q}_{add}\right|} = 1 - \frac{\left|\dot{Q}_{L}\right|}{\left|\dot{Q}_{H}\right|}$$

 $\triangleright$   $\dot{Q}_L$  is calculated form the energy balance of heat exchanger 2

$$\begin{split} \dot{Q}_{CV} - \dot{W}_{CV} &= \dot{m} \bigg[ (h_1 - h_5) + \bigg( \frac{\vec{V}_1^2 - \vec{V}_5^2}{2} \bigg) + g (z_1 - z_5) \bigg] \\ \dot{Q}_L - 0 &= \dot{m} \bigg[ c_{p_o} (T_1 - T_5) + 0 + 0 \bigg] \\ \dot{Q}_L &= \dot{m} c_{p_o} (T_1 - T_5) \\ &= 2 \times 1.004 \times (290 - 500) \\ &= -421.68 \qquad \text{kW} \\ \eta_{th} &= 1 - \frac{\left| \dot{Q}_L \right|}{\left| \dot{Q}_H \right|} \\ &= 1 - \frac{421.68}{1315.24} \\ &= 0.6794 \end{split}$$

(d)

Carnot (reversible) efficiency of a heat engine working between two reservoirs of  $T_H$  and  $T_L$  is defined as

$$\eta_{rev} = 1 - \frac{T_L}{T_H} \\
= 1 - \frac{200}{1400} \\
= 0.8571$$

ightharpoonup Since  $\eta_{th} < \eta_{rev}$ , the cycle is <u>irreversible</u>

2. (a) For the R-134a: State 2 is saturated vapor at 5°C. Therefore.

$$P_2 = 350.9 \text{ kPa}$$
,  $T_2 = 5°C$ ;  $V_2 = 0.05833 \text{ m}^3/kg$ ;

and  $U_2 = 380.85 \text{ kJ/kg}$ .

Since the partition does not move,  $V_1 = V_2 = 0.05833 \text{ m}^3$ .

 $T_1 = -10°C$ , Since  $V_1 < V_2 + 1 \text{ then State 1 is}$ .

Saturated mix ture, and  $P_1 = 201.7 \text{ kPa}$ .

 $V_1 - V_2 = 0.05833 - 0.000755 = 0.5848$ .

 $U_1 = V_1 + U_2 + (1 - V_1)U_2$ .

 $U_2 + (1 - V_1)U_2$ .

 $U_3 + (1 - V_1)U_4$ .

 $U_4 + (1 - V_1)U_4$ .

 $U_5 - 1V_2 = m(U_2 - U_1)$ .

 $U_5 - 1V_2 = m(U_2 - U_1)$ .

For the R-134a:

 $U_5 - 1V_2 = m(U_2 - U_1)$ .

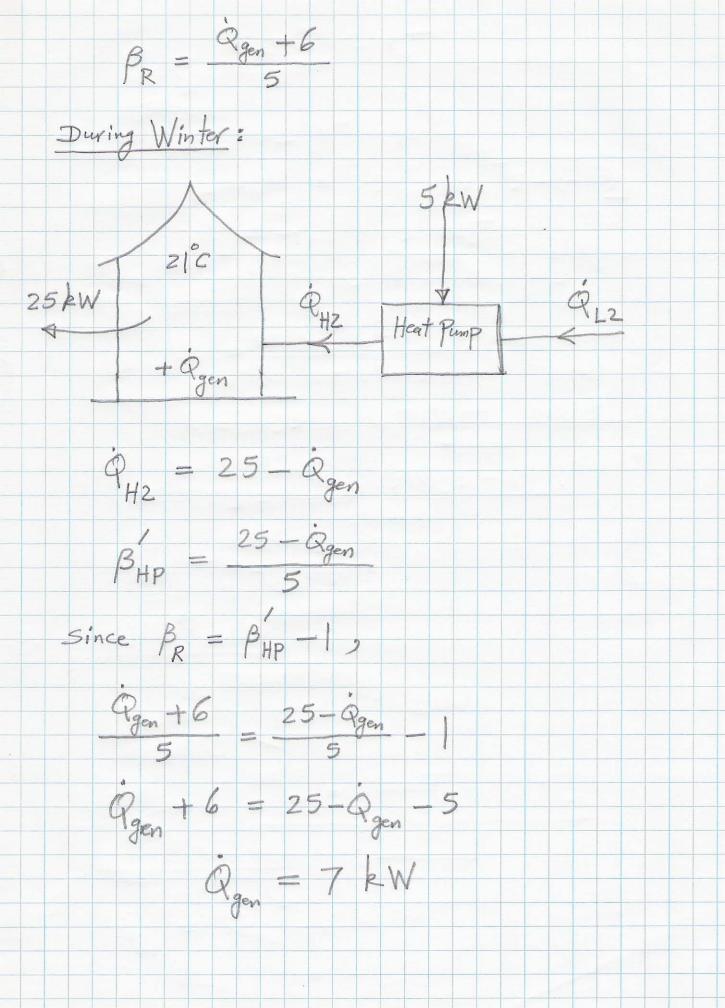
 $U_5 - 1V_2 = m(U_2 - U_1) = mC_3$ .

$$\begin{array}{c} Q_{2} & = -214.2 \text{ kJ} \\ 192 & \text{paygen} \end{array}$$

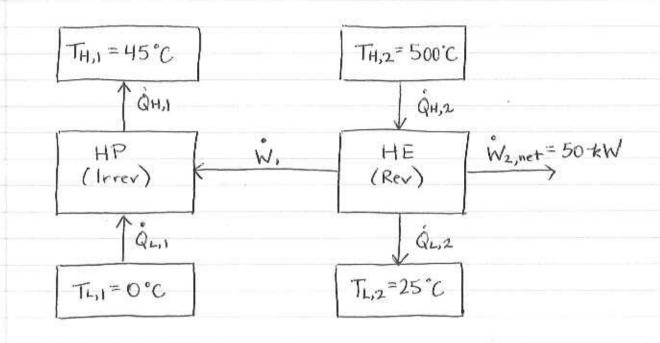
$$\begin{array}{c} C_{V_{0}} = 0.662 \text{ kJ/kg K} \\ -214.2 - |W_{2}| = 2.5 (0.662) [5 - (-10)] \\ |W_{2}| = -239.03 \text{ kJ} \end{array}$$

$$\begin{array}{c} |W_{2}| = -239.03 \text{ kJ} \\ |W_{2}| = -239.03 \text{ kJ/kg} \end{array}$$

$$\begin{array}{c} |W_{2}| = mR (T_{2} - T_{1}) \\ |V_{2}| = 1-n \\ |V_{2}| = 1-n \\ |V_{3}| = 1.0408 \\ |V_{3}| = 1.0408 \\ |V_{3}| = 1.0408 \\ |V_{4}| = 1.0408 \\ |V_{5}| = 1.0408 \\ |V$$



Question #4



Given: 
$$6H,1 + 6H,2 = 500 \text{ kW}$$
 Eqn (1)  
 $W_{2,net} = 50 \text{ kW}$   
 $BHP = 0.6BHP, REV$ 

a) 
$$\mathcal{N}_{Hh} = 1 - \frac{T_{L,2}}{T_{H,2}} = 1 - \frac{25 + 273.15}{500 + 273.15} = 0.6144$$
  
 $\mathcal{B}_{HP}^{\prime} = 0.6 \left( \frac{T_{H,1}}{T_{H,1} - T_{L,1}} \right) = 0.6 \left( \frac{45 + 273.15}{(45 + 273.15) - 273.15} \right) = 4.242$ 

b) 
$$\mathcal{H}_{HP} = \frac{W_1 + W_2, net}{Q_{H,2}} \quad E_{gn}(2)$$

$$\mathcal{B}_{HP} = \frac{Q_{H,1}}{Q_{H,1}} \quad or \quad Q_{H,1} = \mathcal{B}_{HP} \dot{W}_1 \quad E_{gn}(3)$$

$$W_1$$

#4 solution continued Combine Egn (3) and Egn (1) BHOW, + QH,2= 500 KW or QH,2 = 500 - BHOW, Egn (4) Combine Egn (4) and Egn (2) Mth = W, + W2, net M+ (500 - B+P W) = W+ Wanet W1(1+ BHD Mth) = 5007/4 - Wz, net WI = 500 Mth - Wz,net W, = 500/0.6144) -50 1 + (4.242)(0.6144) W.= 71.32 KW QH,1 = W, BHP = (71.32 kW)(4.242) QH,1 = 302,54 KW QH,2 = 500 KW - QH,1 = 500 kW - 302.54 kW QH2 = 197.46 KW

c)

#4 solution continued

Energy Balance on HP:

QL,1 = QH,1 - W,

= 302.54 kW -71.32 kW

QL,1 = 231. 22 KW

Energy Balance on HE:

QL,2 = QH,2-W, -Wz,net

= 197.46 kW-71.32 kW-50 kW

Q12= 76.14 KW

# ENG 1460 Winter 2008 Final Exam Solution #5. - refer to cycle diagram on test paper. (a) Neglecting AKE, APE in all devices (given) Assuming steady state, steady flow conditions. given: WHPT + WLPT = 15 000 KW (15 MW) high-pressure turbine: ate 5 P= 10 MPa, Ts=450°C A+ 10 MPa, Tsat. = 311.06°C (Table B.1.2) Ts > Tsat.: superheated vapour Table B.1.3, V= 0.02975 m3/kg high-pressure turbine: State 5 P= 10 MPa, T= 450°C Table B.1.3, V= 0.02975 m3/kg h= 3240.83 kJ/kg 5 tate 6 P6 = 1500 kPa, saturated vapour Tsat. = 198.32°C, Vg = 0.13177 m3/kg hg = 2792, 15 kJ/kg First law: SHAT WHAT MING (h6-hs) WHAT = MTOT (hs-h6) = 16.72 (3240,83-2792.15) WHAT = 7501.92 KW WLPT = 15000 - WHPT = 15000 - 7501.92 = 7498.08 KW (b) low-pressure turbine LPT WLPT state 8 P8 = 800 kPa, T8 = 400°C At 800 kPa, Tsat. = 170.43 °C (Table B.1.2) T8 > Tsat. : superheated vapour Table B.1.3, v = 0.38426 m3/kg state 9 hg = 3267,07 kJ/kg Pg = 10 kPa, X = 0,95 is saturated mix ture Table B.1.2, Tsat, = 45.81°C, V\_f=0,001010 m3/kg Vfg=14.67254 m3/kg

state 9: hf = 191.81 KJ/kg, hfg = 2392.82 KJ/kg Q-W=m(h9-h8): WLPT=mLPT(h8-h9) Vg = Vf + x Vfg = 0.001010 + 0.95 (14.67254) = 13.93992 m3/kg hg=hf+xhfg=191.81+0.95(2392,82)=2464.98 kJ/kg 1. 7498.08 = MLPT (3267.07-2464.98) 1. MLPT = 9.348/ Kg/5 conservation of mass: MLPT + MSP = MTOT msp = mrot - mlpt = 16,72-9.3481 = 7.3719 kg/s Pumps: Api - Wpi = mlpr (hz-hi) Apz - Wpz = mTOT (h4 - h3) State | P, = 10 kPa, saturated liquid

Table B. 1, 2, V\_F = 0.001010 m3/kg, h\_F = 191.81 k5/kg Tsat. = 45.81°C state 2 assume  $T_z = T_1$ ,  $P_z = 800 \text{ kPa}$   $\left(U_2 = U_f = U_1\right)$ Referring to state 1,  $P_z > P_{sat}$ . :- compressed liquid for  $T_z$   $V_z = V_f = V_1$ state 3  $P_3$  = 800 kPa, saturated liquid  $V_f = 0.001115 \text{ m}^3/\text{kg}$ ,  $h_f = 721.10 \text{ kJ/kg}$ Tsat. = 170.43°C state 4 assume T4 = T3, P4 = 10 MPa (U4 = Uf = U3)

State 4 assume  $T_4 = T_3$ ,  $P_4 = 10$  MPa  $(U_4 = U_f = U_3)$ Referring to state 3,  $P_4 > P_{sat}$ . :- compressed liquid for  $T_3$   $V_4 = V_F = V_3$ 

```
Pump 1: h_z - h_1 \simeq V_{f_1} (P_z - P_1) since state 2 is compressed liquid

: -\dot{W}_{P_1} = \dot{m}_{LPT} V_{f_1} (P_z - P_1)

= 9.3481 x 0.00/0/0 x (800-10)

= 7.4588 KW i.e., |\dot{W}_{P_1}| = 7.4588 KW \dot{W}_{P_1}
     Pump 2: h_4 - h_3 \approx V_{F_3} (P_4 - P_3)

:. -\dot{W}_{P_2} = \dot{m}_{707} V_{F_3} (P_4 - P_3)

= 16.72 \times 0.001115 \times (10000 - 8000)

= 171.51 \text{ KW} i.e., |\dot{W}_{P_2}| = 171.51 \text{ KW}
(C) Secondary process
      state 7 P7 = 800 kPa, h7 = h6, throttle value
            i. ht= 2792.15 hJ/kg Table B.1.2. hz>hg at 800 kPa
i. state is superheated vapour
                          Table B.1.3
               170.43
                             V
                                          2792,15
                                                          .. Ty = 180.13 ℃
                           0.26080
                                         2839.25
                                                         & V7 = 0.24711 m3/kg
       state 10 - consider the mixing chamber
     mixing in the second servation: m_{TOT} = m_{SP} + m_{LPT}

mixing the second servation: m_{TOT} = m_{SP} + m_{LPT}
             :. h,0 = m TOT h3 - m LPT h2
MSP
        or, using mass cons., hio = mLPT (h3-hz) + h3
```

```
state 10:
                         hz required. State 2 15 a compressed liquid,
                                     hz=h,+VF, (P2-P1)
          hz= 191.81+0.001010 (800-10) = 192.60 KJ/kg
       1. h10 = 16.72 × 721.10 - 9.3481 × 192.60
        hio = 1391-27 KJ/kg , Pio = 800 kPa
                      From Table B.1.2 hf < h10 < hg : saturated 
hf = 721.10 KJ/kg, hfg = 2048.04 KJ/kg mixture
                      \chi_{10} = \frac{h_{10} - h_{f}}{h_{fg}} = \frac{1391.27 - 721.10}{2048.04} = 0.32722

(not required)
                      Vio = Vf + Xio Vfg = 0.001115 + 0.32722 x 0.23931
Vio = 0.079422 m3/kg (not required)
        process PPH
                               OPH - WPH = MSP (10-17)
                             \hat{Q}_{PH} = 7.3719 (1391.27 - 2792.15)
= -10327.1 KW
 (d) steam generator as = WsG = (m TOT hs + mLPT ha)e
i.e., q = m_{TOT} (h_5 - h_4) + \tilde{m}_{LPT} (h_8 - h_7) - (m_{TOT} h_4 + \tilde{m}_{LPT} h_7)_{i}
            ha required. For a compressed liquid,
                          h42h3+Vf3(P4-P3)
                   h4=721.10+0.001115 (10000-800) = 731.35 KJ/kg
      1.\tilde{Q}_{SG} = 16.72(3240.83 - 731.35) + 9.3481(3267.07 - 2792.15)
= 41958.50 + 4439.60
\tilde{Q}_{SG} = 46398.1 \text{ kW}
```

