Term Test 2

DATE: March 10, 2011
TIME: 70 minutes
COURSE: MATH 2130
EXAMINER: G.I. Moghaddam

NAME: ______

STUDENT # : ______

Q1	Q2	Q3	Q4	Q5	Total
10	10	9	8	13	50
1	2	6.5	7.5	6.5	23.5

[10] 1. Evaluate each of these limits or explain why it does not exist.

(a) $\lim_{(x,y)\to(1,-1)} \frac{(x^2-y)(1+y)}{x^4-y^2} = \lim_{(x,y)\to(1,-1)} \frac{(x^2-y)(1+y)}{(x^2-y)(x^2+y)}$

$= \lim_{(x,y)\to(1,-1)} \frac{1+y}{x^2+y}$ ✓ $\quad$ let $y = mx$ no!

1

$\lim_{x\to 1} \frac{1+mx}{x^2+mx} = \frac{1+m(1)}{(1)^2+m(1)} = \frac{1+m}{1+m} = 1$

$y=-1 \Rightarrow 0 \ldots 0$

$x=1 \Rightarrow \frac{(1-y)(1+y)}{1-y^2} = 1$

(b) $\lim_{(x,y)\to(0,-1)} \frac{\sin(\sqrt{x^2+y^2}-1)}{x^4+y^4+2x^2y^2-1} = \lim_{(x,y)\to(0,-1)} \frac{\sin(\sqrt{x^2+y^2}-1)}{(x^2+y^2)^2-1}$

let $x = my$ $\quad \lim_{(x,y)\to(0,-1)} \frac{\sin(\sqrt{m^2y^2+y^2}-1)}{(m^2y^2+y^2)^2-1}$

depends on m, $\therefore$ does not exist.

0

$\frac{\sin(\sqrt{x^2+y^2}-1)}{(x^2+y^2)^2-1}$

$\frac{\sin\sqrt{x^2+y^2}-1}{(x^2+y^2+1)(x^2+y^2-1)}$

$\frac{\sin\sqrt{x^2+y^2}-1}{(x^2+y^2+1)(\sqrt{x^2+y^2}+1)(\sqrt{x^2+y^2}-1)}$ ______ $= \frac{1}{4}$

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[10] 2. Let $u(x, y) = f(x^2 + y) + g(x^2 + y)$ where f and g are twice differentiable functions. Show that

$$\frac{\partial^2 u}{\partial x^2} - 4x^2 \frac{\partial^2 u}{\partial y^2} - 2\frac{\partial u}{\partial y} = 0$$

let $x^2+y=V$

$U(x,y)$

No!

?

?

$$\frac{\partial u}{\partial x} = \frac{\partial u}{\partial f}\frac{\partial f}{\partial x} + \frac{\partial u}{\partial g}\frac{\partial g}{\partial x} = \frac{\partial u}{\partial f}(2x) + \frac{\partial u}{\partial g}(2x)$$

$$\frac{\partial u}{\partial y} = \frac{\partial u}{\partial f}\frac{\partial f}{\partial y} + \frac{\partial u}{\partial g}\frac{\partial g}{\partial y} = \frac{\partial u}{\partial f}(1) + \frac{\partial u}{\partial g}(1)$$

2

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial}{\partial f}\left(\frac{\partial u}{\partial f}\right)(2) \cancel{+ \frac{\partial}{\partial f}\left(\frac{\partial u}{\partial g}\right)(2)}$$

$$\frac{\partial^2 u}{\partial y^2} = \frac{\partial}{\partial f}\left(\frac{\partial u}{\partial f}\right) + \frac{\partial}{\partial f}\left(\frac{\partial u}{\partial g}\right)$$

$$\frac{\partial^2 u}{\partial f^2}(2) + \frac{\partial^2 u}{\partial g^2}(2) - 4x^2\left(\frac{\partial^2 u}{\partial f^2} + \frac{\partial^2 u}{\partial g^2}\right) - 2\left(\frac{\partial u}{\partial f} + \frac{\partial u}{\partial g}\right) = 0$$

no !!!!

$\frac{\partial u}{\partial f}$ and $\frac{\partial u}{\partial g}$ do not make sense!

$\frac{\partial u}{\partial v} \cdot \frac{\partial v}{\partial y}$

$$\frac{\partial u}{\partial y} = (f' + g')(1)$$

$$\frac{\partial^2 u}{\partial y^2} = (f'' + g'')$$

$$\frac{\partial u}{\partial x} = \frac{du}{dv}\frac{\partial v}{\partial x} = [f'(v) + g'(v)]\,2x$$

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x}\right) = \left.\frac{\partial^2 u}{\partial x^2}\right)_v + \left.\frac{\partial}{\partial v}\left(\frac{\partial u}{\partial x}\right)\right|_x \frac{\partial v}{\partial x} = 2(f' + g') + 2x(f'' + g'')(2x)$$

$$\cancel{2(f' + g')} + \cancel{4x^2(f'' + g'')} - \cancel{4x^2(f'' + g'')} - \cancel{2(f' + g')} = 0$$

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[9] 3. Find $\dfrac{\partial z}{\partial x}$ if

$$xy + xz + yz = 1 \quad , \quad 2xy - 2y^2 - \frac{1}{2}x^2 = 0\,.$$

Simplify your answer.

$F(x,y) = xy + xz + yz - 1 = 0 \quad , \quad G(x,y) = 2xy - 2y^2 - \frac{1}{2}x^2 = 0$

$$\frac{\partial z}{\partial x} = \frac{-\frac{\partial(F,G)}{\partial(z,y)}}{\frac{\partial(F,G)}{\partial(x,y)}} = \frac{-\begin{vmatrix} F_z & F_y \\ G_z & G_y \end{vmatrix}}{\begin{vmatrix} F_x & F_y \\ G_x & G_y \end{vmatrix}} = \frac{-\begin{vmatrix} x+y & x+z \\ 0 & 2x-4y \end{vmatrix}}{\begin{vmatrix} y+z & x+z \\ 2y-x & 2x-4y \end{vmatrix}}$$

no.

no.

$$= \frac{-[(x+y)(2x-4y)-0]}{(y+z)(2x-4y)-(2y-x)(x+z)} = \frac{-(2x^2-4yx+2yx-4y^2)}{(2xy-4y^2+2xz-4yz)-(2yx+2yz-x^2-xz)}$$

$$= \frac{-2x^2+2yx+4y^2}{-4y^2+3xz-6yz+x^2} = \frac{-2(x^2-yx-2y^2)}{-4y^2+x^2+3xz-6yz} = \frac{-2[(x+y)(x-2y)]}{-4y^2+x^2)+3(xz-2yz)}$$

$$= \frac{-2[(x+y)(\cancel{x-2y})]}{(x+2y)(\cancel{x-2y})+3(xz-2yz)} = \frac{-2x-2y}{x+2y+3(xz-2yz)}$$

no!

it is $-\dfrac{x+2y+3z}{2(x+y)}$

6.5

$y = \dfrac{1-xz}{x+z}$

[8] 4. Let $f(x,y,z) = x^2 + y + z$ and $g(x,y,z) = xz + \frac{1}{4}z^2$. Find the directional derivative of $f+g$, at the origin, along the curve

$$C \ : \quad x = t^2, \quad y = 2t, \quad z = -2t\,.$$

$P(0,0,0)$ $\quad H = f + g = x^2 + y + z + xz + \frac{1}{4}z^2$

$\nabla H(x,y,z) = (2x + z, 1, 1 + x + \frac{1}{2}z)$

$\nabla H(0,0,0) = (0, 1, 1)$

$\vec{r}(t) = (t^2, 2t, -2t)$

$\vec{r}'(t) = (2t, 2, -2)$

$\vec{r}'(0) = (0, 2, -2)$

$\hat{T} = \frac{1}{\cancel{2}}(0, 2, -2)$ $\quad 2\sqrt{2}$

$D_{\hat{T}}H = \nabla H \cdot \hat{T}$

$= (0,1,1) \cdot \frac{1}{2}(0,2,-2)$

$= \frac{1}{\cancel{2}}(2-2) = \frac{0}{2} = 0.$

$2\sqrt{2}$

7.5

[13] 5. Let S_1 be the surface $z = x^3 - y^2$ and also let S_2 be the surface $x + y = xz$.

(a) Find the equation of the tangent plane to the surface S_1 at the point $(2, 2, 4)$.

$S_1 = x^3 - y^2 - z$ $P(2,2,4)$

$\nabla S_1(x,y,z) = (3x^2, -2y, -1)$

$\nabla S_1(2,2,4) = (12, -4, -1)$

eq of plane: $12(x-2) - 4(y-2) - (z-4) = 0$

$12x - 24 - 4y + 8 - z + 4 = 0$

$12x - 4y - z = 12$

$x - \frac{y}{3} - \frac{z}{12} = 1$ 4

(b) Find a tangent vector for the curve of intersection of the two surfaces S_1 and S_2 at the point $(1, -1, 0)$.

let $x = t$, $z = \frac{t+y}{t} = t^3 + t^4 - t - 1$ at $P(1,-1,0)$, $t = 1$

$y = \frac{t^4 - t - 1}{t}$

$\frac{t+y}{t} = t^3 - y^2$

$t + y = t^4 - ty^2$

$y + ty^2 = t^4 - t$

$y(1+ty) = t^4 - t$

$y = \frac{t^4 - t - 1}{t}$

$\vec{r}(t) = (t, t^4 - t, t^3)$

$\vec{r}'(t) = (1, 4t^3 - 1, 3t^2)$

$\vec{r}'(1) = (1, 3, 3)$

$\vec{T}(1) = (1, 3, 3)$ 0

$z = \frac{t + \left(\frac{t^4 - t - 1}{t}\right)}{t} = t^2 + \left(\frac{t^4 - t - 1}{t}\right) = t^3 + t^4 - t - 1$

(c) Find the point(s) on the surface S_2 at which the tangent plane is parallel to the plane $2x + y - 3z = 0$.

$S_2 = x + y - xz$ $\vec{r} = (2, 1, -3)$

$\nabla S_2(x,y,z) = (1-z, 1, -x)$ no! $(1-z, 1, -x) = \lambda(2,1,-3)$

$1 - z = 2 \Rightarrow -z = 1 \therefore z = 1$!!!?

$1 = 1$

$-x = -3 \Rightarrow x = 3 \therefore$

$\therefore$ parallel at $(3, \cancel{1}, 1)$.

2.5

Normals of both planes are parallel

$\vec{N}_1 = k(2, 1, -3)$

$k \neq 0$

$z = \frac{x+y}{x} = x^3 - y^2$

$F = z + y^2 - x^3 = 0$

$G = x + y - xz = 0$

$\nabla F\big|_{P_0} = (-3x^2, 2y + 1)\big|_{(1,-1,0)} = (-3, -2, 1)$

$= (3, +2, -1)$

$\nabla G\big|_{P_0} = (1-z, 1, -x)\big|_{(1,-1,0)}$

$= (1, 1, -1)$

$\nabla F \times \nabla G = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 2 & -1 \\ 1 & 1 & -1 \end{vmatrix}$

$= (-1, 2, 1)$

$x = t$

$t + y = tz$

$z = \frac{t+y}{t}$

$\frac{t+y}{t} = z$

$\frac{t+y}{t} = t^3 - y^2$

$t^4 - ty^2 = t + y$

$ty^2 + y + t - t^4 = 0$

$y = \frac{-1 \pm \sqrt{1 - 4t(t - t^4)}}{2t}$

$= \frac{-1 - \sqrt{1 - 4t^2 - 4t^5}}{2t}$

$\langle 1-z, 1, -x \rangle = \langle 2, 1, -3 \rangle$

(a, b, c)

$x = 3$

$z = -1$

$y = x(z-1) = 3(-2) = -6$

$1 - a = 2$

$-c = -3$

b → from Surface eqn

$(3, -6, -1)$