MATH 2130 Problem Workshop 3

1. The following three lines define a triangle

$$x = 4 + s, \qquad y = -1 - s, \qquad z = -s$$
$$x = 3 - u, \qquad y = 6 - 2u, \qquad z = 1 + u$$
$$x = -1 + t, \qquad y = -2 + t, \qquad z = 5 - t$$

   Find the area of the triangle.

2. The vertices of the triangle in question 1 are three vertices of a parallelogram. What are the possiblities for the fourth vertex?

3. Find the centroid of the triangle in question 3. It is the point of intersection of the three medians of the triangle, which occurs on a median which is 2/3 of the way from the vertex to the opposite midpoint.

4. Find $\mathbf{v}'(3)$ if $\mathbf{v}(t) = t^2\hat{\mathbf{i}} + \arcsin(t/4)\hat{\mathbf{j}} + \ln(2t+1)\hat{\mathbf{k}}$.

5. If $f(t) = t^2 + 1$ and $\mathbf{v}(t) = e^t\hat{\mathbf{i}} + \left[t/(t^2+1)^3\right]\hat{\mathbf{j}} - t\sqrt{t^2+1}\hat{\mathbf{k}}$, evaluate $\displaystyle\int f(t)\mathbf{v}(t)dt$.

6. Find a parameterization of the following curves.
   (a) $z = 2\sqrt{x^2+y^2}, x^2 + y^2 = 3 - z$ from $(1, 0, 2)$ to $(-1, 0, 2)$ directed so y is always non-positive.
   (b) First octant part of $x^2 + z^2 = 4, x + y = 1$ directed so that z increases along the curve.
   (c) $z = x^2 + y^2, x^2 + y^2 - 4y = 0$ directed clockwise viewed from above.

Answers:

1. $3\sqrt{2}$

2. $(0, -3, 4), (4, 5, 0), (-2, -1, 6)$

3. $(2/3, 1/3, 10/3)$

4. $6\hat{\mathbf{i}} + \dfrac{1}{\sqrt{7}}\hat{\mathbf{j}} + \dfrac{2}{7}\hat{\mathbf{k}}$.

5. $(t^2 - 2t + 3)e^t\hat{\mathbf{i}} - \left[1/2(t^2+1)\right]\hat{\mathbf{j}} - \frac{1}{5}(t^2+1)^{5/2}\hat{\mathbf{k}} + \mathbf{C}$, where $\mathbf{C}$ is a constant vector.

6. (a) $x = \cos t, y = -\sin t, z = 2, 0 \le t \le \pi$.
   (b) $x = 2\cos t, y = 1 - 2\cos t, z = 2\sin t, \pi/3 \le t \le \pi/2$.
   (c) $x = 2\cos t, y = 2 - 2\sin t, z = 8(1 - \sin t), 0 \le t \le 2\pi$.