# Math 2132 Summer 2012 Final



10 **1.** Find the interval of convergence for the power series

$$\sum_{n=3}^{\infty} \frac{(-1)^n n}{4^{n+1}} (x-1)^{2n}.$$

14 **2.** Find the Maclaurin series for the function

$$f(x) = \frac{x}{x^2 - x - 2}.$$

Use a method that guarantees that the series converges to $f(x)$. Express your answer in sigma notation, simplified as much as possible. Determine the interval of convergence for the series.

6 **3.** Find a maximum possible error when the function $e^{-3x}$ is approximated by the first three terms in its Maclaurin series on the interval $0 \le x \le 0.2$.

15 **4.** Find a general solution for the differential equation

$$3y''' + 2y'' + 2y' - y = x - e^{-2x}.$$

6 **5.** You are given that the roots of the auxiliary equation associated with the linear, differential equation

$$\phi(D)y = 2xe^{4x} + x^3 - 2 + 3e^{2x}\cos 5x$$

are $m = 0, 2 \pm i, 2 \pm i, \pm 3, 4$. Write down the form of a particular solution of the differential equation as predicted by the method of undetermined coefficients. Do **NOT** find the coefficients, just the form of the particular solution.

6 **6.** When a substance such as glucose is administered intravenously into the bloodstream, it is used up by the body at a rate proportional to the amount present at that time. If it is added at a variable rate $R(t)$, where $t$ is time, and $A_0$ is the amount in the bloodstream when the intravenous feeding begins, set up, but **DO NOT SOLVE**, an initial value problem for the amount of glucose in the bloodstream at any time. Is the differential equation separable?

7 **7.** Find an implicit definition for the solution of the initial value problem

$$y^2 \frac{dy}{dx} = (x+1)(y^3+1), \qquad y(0) = 1.$$

9 **8.** Find the Laplace transform for the function

$$f(t) = \begin{cases} t, & 0 \le t \le 2 \\ 4-t, & 2 < t \le 4 \end{cases} \qquad f(t+4) = f(t).$$

Simplify the transform as much as possible.

9 **9.** Find the inverse Laplace transform for the function

$$F(s) = \frac{e^{-2s}(3s^2+2)}{s^3 - s^2 + 2}.$$

8 **10.** A mass of 1 kilogram is suspended from a spring with constant 400 newtons per metre. At time $t = 0$, it is at its equilibrium position and is given velocity 2 metres per second upward. During its subsequent motion, it is also subjected to a damping force that (in newtons) is equal to 40 times its velocity (in metres per second). Use Laplace transforms to find the position of the mass as a function of time.

10 **11.** Solve the initial value problem

$$y'' - 3y' - 4y = 3\delta(t-2), \qquad y(0) = 0, \quad y'(0) = 1.$$

Dawit's Answers

1. $-1 < x < 3$
2. $\sum_{n=1}^{\infty} \frac{1}{3}\left[(-1)^n - \frac{1}{2^n}\right] x^n, \quad -1 < x < 1$
3. $\frac{9}{2}(0.2)^3$
4. $c_1 e^{x/3} + e^{-x/2}\left[c_2 \sin\frac{\sqrt{3}}{2}x + c_3 \cos\frac{\sqrt{3}}{2}x\right] - (x+2) + \frac{1}{21}e^{-2x}$
5. $(Ax^2+Bx)e^{4x} + Cx^4 + Dx^3 + Ex^2 + Fx + e^{2x}\left[G\cos 5x + H\sin 5x\right]$
6. $\frac{dA}{dt} = R(t) - kA, \quad A(0) = A_0, \quad k>0$, No. DE isn't Separable
7. $\frac{1}{3}\ln|y^3+1| = \frac{x^2}{2} + x + \frac{1}{3}\ln 2$
8. $\frac{1-e^{-2s}}{s^2(1+e^{-2s})}$
9. $\left[e^{-2(t-2)} + 2e^{t-2}(\cos(t-2) + \sin(t-2))\right]h(t-2)$
10. $2te^{-20t}$ m
11. $\frac{1}{5}e^{4t} - \frac{1}{5}e^{-t} + \frac{3}{5}\left(e^{4(t-2)} - e^{-(t-2)}\right)h(t-2)$