8 1. Find the distance between the plane $2x + 4y + z = 6$ and the line

$$x = 2 + t, \quad y = 3 - t, \quad z = 1 + 2t.$$

Ans. $\frac{11}{\sqrt{21}} = \frac{11\sqrt{21}}{21}$

7 ~~10~~ 2. Find the equation of the plane perpendicular to the curve

$$x = 2 + \sin t, \quad y = \cos 2t, \quad z = 2t + 1,$$

at the point $(2, 1, 1)$. Simplify your equation as much as possible.

Ans) $x + 2z = 4$

9 ~~8~~ 3. Find equations for the tangent line to the curve

$$2x^2 + y^2 + xz = 1, \quad 2xyz + z^3 = -12,$$

at the point $(1, 1, -2)$.

Ans) $x = 1 + t$
$y = 1 - t$
$z = -2$

5 4. Are there directions at the point $(1, 2, -1)$ in which the rate of change of the function

$$f(x, y, z) = x^2 + y^2z^3 - 15z$$

is equal to ~~6~~ -6? Justify your answer. If directions do exist, it is not necessary for you to find them.

Ans.) None

6 5. Show that every function of the form $f(y^2 - x^3)$ satisfies the equation

$$2y\frac{\partial f}{\partial x} + 3x^2\frac{\partial f}{\partial y} = 0.$$

Hint: let $u = y^3 - x^3$ and Use a tree diagram.

10 **6.** Show that $(0, 0)$ is a critical point for the function

$$f(x, y) = x^4y^4(x^2 + y^2).$$

Are there any other critical points? If so, what are they? Classify $(0, 0)$ as yielding a relative maximum, a relative minimum, or a saddle point. Hint: Do not use the second derivative test. It will fail.

Ans.) yes there are: every point on the Coordinate axes (x-axis & y-axis)

(0,0) yields relative min

5

13 **7.** Find the maximum value of the function

$$f(x, y) = xy(2x + y - 1)$$

on the region R bounded by the lines

$$x = 0, \qquad y = 0, \qquad 2x + y = 2.$$

Ans.) $\frac{1}{2}$

10 **8.** Evaluate the double iterated integral

$$\int_0^2 \int_{y/2}^1 e^{x^2}\, dx\, dy.$$

Ans) $e-1$

7 **9.** Set up, but do **NOT** evaluate, a double iterated integral for the area of that part of the surface $z = (x - 1)^2 + 3(2 - y)^2$ bounded by the planes $y = 0$, $x = 2y$, and $x + 2y = 4$.

Ans.) $$\int_0^1 \int_{2y}^{4-2y} \sqrt{1 + [2(x-1)]^2 + [-6(2-y)]^2}\, dx\, dy$$

8 **10.** Set up, but do **NOT** evaluate, a double iterated integral for the volume of the solid of revolution if the area bounded by the curves

$$3y = 4 - x^2, \quad x = 3y - 2,$$

is revolved about the line $x + y + 2 = 0$. Simplify the integrand as much as possible.

Ans.) $$\sqrt{2}\pi \int_{-2}^{1} \int_{\frac{x+2}{3}}^{\frac{4-x^2}{3}} (x+y+2)\, dy\, dx$$

7 **11.** Set up, but do **NOT** evaluate, a triple iterated integral for the volume bounded by the surfaces

$$x^2 + y^2 = z^2 - 4, \quad x^2 + 4y^2 = 4.$$

Ans.) $$8 \int_0^1 \int_0^{2\sqrt{1-y^2}} \int_0^{\sqrt{4+x^2+y^2}} dz\, dx\, dy = 8 \int_0^2 \int_0^{\sqrt{1-\frac{x^2}{4}}} \int_0^{\sqrt{4+x^2+y^2}} dz\, dy\, dx$$

8 **12.** Set up, but do **NOT** evaluate, a triple iterated integral in cylindrical coordinates to evaluate the triple integral

$$\iiint_V xyz\, dV$$

over the region in the first octant bounded by the surfaces

$$z = \sqrt{9 - x^2 - y^2}, \quad z = 0, \quad y = \frac{x}{\sqrt{3}}, \quad y = \sqrt{3}x.$$

Ans.) $$\int_{\pi/6}^{\pi/3} \int_0^3 \int_0^{\sqrt{9-r^2}} z r^3 \cos\theta \sin\theta \, dz\, dr\, d\theta$$