Math 2132 Winter 2010 Final Exam

10 **1.** Find the interval of convergence for the power series

$$\sum_{n=3}^{\infty} \frac{n4^n}{n+1}(x-2)^{3n+2}.$$

Express your answer in one of the forms $a < x < b$, $a \leq x < b$, $a < x \leq b$, or $a \leq x \leq b$ for appropriate values of a and b.

12 **2.** Find the Maclaurin series for the function

$$f(x) = \sqrt[3]{4-3x^2}.$$

Write your final answer in sigma notation. What is the radius of convergence of the series?

6 **3.** What is a maximum possible error if the first three nonzero terms of the Maclaurin series for $\cos 2x$ are used to approximate the function on the interval $-1/2 \leq x \leq 1/2$?

6 **4.** Find a power series that represents the indefinite integral

$$\int \frac{1-\cos x}{x^2}dx.$$

What is its radius of convergence?

7 **5.** Find a general solution for the differential equation

$$x\frac{dy}{dx} - 2y = \frac{x^4}{\sqrt{3+2x^2}}.$$

6 **6.** You are given that the roots of the auxiliary equation associated with the linear, differential equation

$$\phi(D)y = x^3e^{3x} + 2x^2 - 3\cos 5x$$

are $m = 3, 3, 0, -1$. Write down the form of a particular solution of the differential equation as predicted by the method of undetermined coefficients. Do **NOT** find the coefficients, just the form of the particular solution.

12 **7.** Find a general solution for the differential equation

$$2y''' - y'' + 8y' - 4y = 3\sin x.$$

11 **8.** (a) A 1 kilogram mass hangs motionless on the end of a spring with constant 100 N/m. From this position, the mass is given speed 1 m/s upward at time $t = 0$. During its subsequent motion, the mass is acted upon by a damping force whose magnitude in Newtons is 10 times its velocity in metres per second. Find the position of the mass as a function of time.
(b) Find the first time that the mass comes to an instantaneous stop.

8 **9.** Find the Laplace transform for the function

$$f(t) = 2t - t^2, \quad 0 \le t \le 2, \qquad f(t+2) = f(t).$$

10 **10.** Find the inverse Laplace transform for the function

$$F(s) = \frac{s+1}{s^3 + 2s^2 + 10s}.$$

12 **11.** Solve the following initial value problem

$$\frac{d^2y}{dt^2} + 3y = 4h(t-2), \qquad y(0) = 1, \quad y'(0) = 2.$$