Related Rate Problems

1. A launch whose deck is 7 ft below the level of a windlass (winch) on a wharf is being pulled toward the wharf by a rope attached to a ring on the deck. If the windlass pulls in the rope at the rate of 15 ft/min, how fast is the launch moving through the water when there are 25 ft of rope out?

2. A light is on the ground 40 ft from a building. A man 6 ft tall walks from the light towards the building at 5 ft/sec. How rapidly is his shadow on the building growing shorter when he is 10 ft from the building?

3. A trough 10 ft long has as its ends isoceles triangles with altitude 2 ft and base 2 ft; their vertices being at the bottom. If water is let into the trough at the rate of 3 ft^3/min, how fast is the water level rising when the water is 1 ft deep?

4. Sand is issuing from a spout at the rate of 3 ft^3/min and falling on a conical pile whose diameter at the base is always 3 times the altitude. At what rate is the altitude increasing when the altitude is 4 ft?

5. A baseball diamond is 90 ft on a side. A man runs from first base to second base at 25 ft/sec. At what rate is his distance from third base decreasing when he is 30 ft from first base? At what rate is his distance from home plate increasing? (at this same instant)

6. A woman starts walking eastward at 5 ft/sec from a point A. Ten minutes later a man starts walking west at the rate of 5 ft/sec from a point B, 3000 ft north of A. How fast are they separating 10 min after the man starts?

7. A point moves along the upper half of the curve $y^2 = 2x + 1$ in such a way that $dx/dt = x^3$. Find dy/dt when $x = 4$.

8. A man 6 ft tall is walking away from a lamppost 24 ft high at the rate of 4 ft/sec. At what rate is the end of his shadow moving away from the lamppost? At what rate is the length of his shadow increasing?

9. A cylindrical can is being heated with its height increasing at the rate of .01 cm/min and its diameter at the rate of .005 cm/min. At what rate is the volume increasing when the can has diameter 15 cm and height 20 cm?

10. An airplane at an altitude of 4400 ft is flying horizontally directly away from an observer. At the instant when the angle of elevation is 45°, the angle is decreasing at the rate of 0.05 radians/sec. How fast is the airplane flying at that instant?

11. A lighthouse is three miles off a straight shore. Its light makes three revolutions per minute. How fast does the light beam move along a sea wall at a point two miles down the coast?

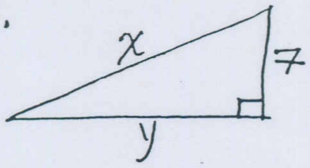
12. A man is walking along a sidewalk at the rate of 5 ft/sec. A searchlight on the ground 30 ft from the walk is kept trained on him. At what rate is the searchlight revolving when the man is 20 ft away from the point on the sidewalk nearest the light?

13. A ladder 10 ft long is leaning against a wall 8 ft high, with its upper end projecting over the wall. If the lower end of the ladder slides away from the wall at the rate of 2 ft/sec, find the rate at which the upper end of the ladder is approaching the ground when the upper end reaches the top of the wall?

14. A bridge is 30 ft above a canal. A motorboat going 10 ft/sec passes under the center of the bridge at the same instant that a man walking 5 ft/sec reaches that point. How rapidly are they separating 3 sec later?

Solutions

1.



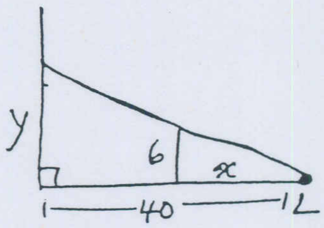
Rate sought: $\left[\frac{dy}{dt}\right]_{x=25}$

Rate given: $\frac{dx}{dt} = -15$

Now $y = \sqrt{x^2-49}$ so $\frac{dy}{dt} = \frac{2x}{2\sqrt{x^2-49}}\left(\frac{dx}{dt}\right)$

Thus $\left[\frac{dy}{dt}\right]_{x=25} = \frac{2(25)}{2\sqrt{25^2-49}}(-15) = -\frac{125}{8}$ ft./sec.

2.



Rate sought: $\left[\frac{dy}{dt}\right]_{x=3.0}$

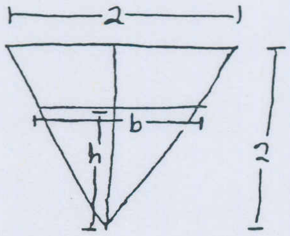
Rate given: $\frac{dx}{dt} = 5$

Now $\frac{y}{6} = \frac{40}{x}$ so $y = \frac{240}{x}$

Thus $\frac{dy}{dt} = -\frac{240}{x^2}\frac{dx}{dt}$. Therefore $\left[\frac{dy}{dt}\right]_{x=30} = -\frac{240}{(30)^2}(5)$

$= -\frac{4}{3}$ ft./sec.

3.



Rate sought: $\left[\frac{dh}{dt}\right]_{h=1}$

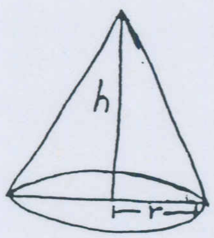
Rate given: $\frac{dV}{dt} = 3$

Now $V = \left(\frac{1}{2}bh\right)10 = 5bh$ and

$\frac{b}{2} = \frac{h}{2}$ so $b = h$. Thus $V = 5h^2$ and $\frac{dV}{dt} = 10h\frac{dh}{dt}$

Therefore $\frac{dh}{dt} = \frac{1}{10h}\frac{dV}{dt}$ so $\left[\frac{dh}{dt}\right]_{h=1} = \frac{1}{10(1)}(3) = \frac{3}{10}$ ft./sec.

4.



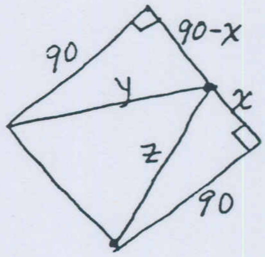
Rate sought: $\left[\frac{dh}{dt}\right]_{h=4}$

Rate given: $\frac{dV}{dt} = 3$

Now $V = \frac{1}{3}\pi r^2 h$ and $2r = 3h$ so

$V = \frac{3\pi}{4}h^3$. Thus $\frac{dV}{dt} = \frac{9\pi}{4}h^2\frac{dh}{dt}$ so $\frac{dh}{dt} = \frac{4}{9\pi h^2}\frac{dV}{dt}$

Therefore $\left[\frac{dh}{dt}\right]_{h=4} = \frac{4}{9\pi(4)^2}(3) = \frac{1}{12\pi}$ ft./min.

5. 

Rates sought: $\left[\frac{dy}{dt}\right]_{x=30}$, $\left[\frac{dz}{dt}\right]_{x=30}$

Rate given: $\frac{dx}{dt} = 25$

$$y = \sqrt{90^2 + (90-x)^2} = \sqrt{16200 - 180x + x^2}$$

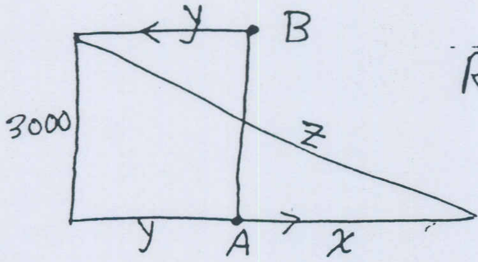
$$z = \sqrt{90^2 + x^2} = \sqrt{8100 + x^2}$$

Thus $\frac{dy}{dt} = \frac{(-180+2x)}{2\sqrt{16200-180x+x^2}} \frac{dx}{dt}$

$$\frac{dz}{dt} = \frac{2x}{2\sqrt{8100+x^2}} \frac{dx}{dt}$$

Therefore $\left[\frac{dy}{dt}\right]_{x=30} = \frac{[-180+2(30)]\ (25)}{2\sqrt{16200-180(30)+30^2}} = -\frac{50}{\sqrt{13}}$ ft./sec

$\left[\frac{dz}{dt}\right]_{x=30} = \frac{2(30)}{2\sqrt{8100+(30)^2}}(25) = \frac{25}{\sqrt{10}}$ ft./sec.

6. 

Rate sought: $\left[\frac{dz}{dt}\right]_{t=1200}$

Rates given: $\frac{dx}{dt} = 5$, $\frac{dy}{dt} = 5$

Now $z = \sqrt{(3000)^2 + (x+y)^2}$

Thus $\frac{dz}{dt} = \frac{1}{2}\left[3000^2 + (x+y)^2\right]^{-\frac{1}{2}} 2(x+y)\left(\frac{dx}{dt} + \frac{dy}{dt}\right)$

When $t = 1200$, $x = 5(1200) = 6000$ and $y = 5(600) = 3000$

Therefore $\left[\frac{dz}{dt}\right]_{t=1200} = \frac{1}{2}\left[3000^2 + (6000+3000)^2\right]^{-\frac{1}{2}} 2(6000+300)(5+5)$

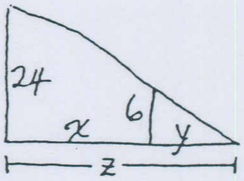
$= \sqrt{90}$ ft./sec

7. Rate sought: $\left[\frac{dy}{dt}\right]_{x=4}$ Rate given: $\frac{dx}{dt} = x^3$

Now $y^2 = 2x+1$ and $y \geq 0$ so $y = \sqrt{2x+1}$.

Thus $\frac{dy}{dt} = \frac{2}{2\sqrt{2x+1}} \frac{dx}{dt} = \frac{1}{\sqrt{2x+1}} x^3$

Therefore $\left[\frac{dy}{dt}\right]_{x=4} = \frac{1}{\sqrt{2(4)+1}} (4)^3 = \frac{64}{3}$

8. 

Rates sought: $\frac{dz}{dt}$ and $\frac{dy}{dt}$
Rate given: $\frac{dx}{dt} = 4$

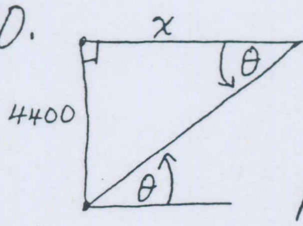
Now $\frac{y}{6} = \frac{z}{24}$ and $z = x+y$. Then $y = \frac{z}{4}$ and $z = x + \frac{z}{4}$ so $z = \frac{4}{3}x$. Thus $\frac{dz}{dt} = \frac{4}{3}\frac{dx}{dt}$ and $\frac{dy}{dt} = \frac{1}{4}\frac{dz}{dt}$. Therefore $\frac{dz}{dt} = \frac{4}{3}(4) = \frac{16}{3}$ ft/sec. and $\frac{dy}{dt} = \frac{1}{4}\left(\frac{16}{3}\right) = \frac{4}{3}$ ft/sec.

9. 

Rate sought: $\left[\frac{dV}{dt}\right]_{\substack{h=20\\D=15}}$
Rates given: $\frac{dh}{dt} = .01$ and $\frac{dD}{dt} = .005$

Now $V = \pi r^2 h$ and $r = \frac{D}{2}$ so $V = \frac{\pi}{4}D^2 h$
Thus $\frac{dV}{dt} = \frac{\pi}{4}\left[D^2\frac{dh}{dt} + 2hD\frac{dD}{dt}\right]$. Therefore

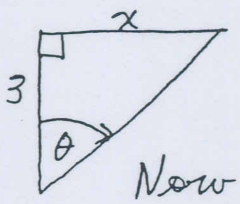
$$\left.\frac{dV}{dt}\right]_{\substack{h=20\\D=15}} = \frac{\pi}{4}\left[(15)^2(.01) + 2(20)(15)(.005)\right] = \frac{5.25}{4}\pi \text{ cm}^3/\text{min}$$

10. 

Rate sought: $\left[\frac{dx}{dt}\right]_{\theta=\frac{\pi}{4}}$
Rate given: $\left[\frac{d\theta}{dt}\right]_{\theta=\frac{\pi}{4}} = -.05$

Now $\frac{x}{4400} = \cot\theta$ so $x = 4400\cot\theta$. Thus $\frac{dx}{dt} = -4400\csc^2\theta\,\frac{d\theta}{dt}$. Therefore

$$\left.\frac{dx}{dt}\right]_{\theta=\frac{\pi}{4}} = -4400\left(\csc\frac{\pi}{4}\right)^2(-.05) = -4400(\sqrt{2})^2(-.05) = 440 \text{ ft./sec.}$$

11. 

Rate sought: $\left[\frac{dx}{dt}\right]_{x=2}$
Rate given: $\frac{d\theta}{dt} = 6\pi$

Now $\frac{x}{3} = \tan\theta$ so $x = 3\tan\theta$. Thus $\frac{dx}{dt} = 3\sec^2\theta\,\frac{d\theta}{dt}$. When $x=2$, $\tan\theta = \frac{2}{3}$ so $\sec^2\theta = \tan^2\theta + 1 = \left(\frac{2}{3}\right)^2 + 1 = \frac{13}{9}$. Therefore

$$\left[\frac{dx}{dt}\right]_{x=2} = 3\left(\frac{13}{9}\right)(6\pi) = 26\pi \text{ mi./min.}$$

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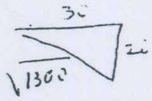
$\frac{x}{30} = \tan\theta$

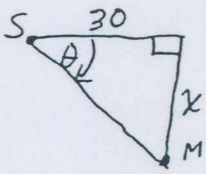
$\frac{1}{30}\frac{dx}{dt} = \sec^2\theta \frac{d\theta}{dt}$

when $x = 20$



$\therefore \frac{1}{30}(5) = \frac{1300}{900}\frac{d\theta}{dt}$

$\therefore \frac{d\theta}{dt} = \frac{3}{26}$

12. 

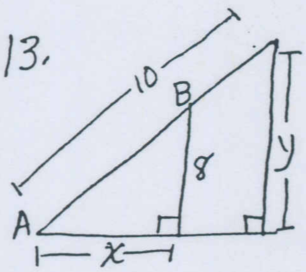
Rate sought: $\left[\frac{d\theta}{dt}\right]_{x=20}$

Rate given: $\frac{dx}{dt} = 5$

Now $\frac{x}{30} = \tan\theta$ so $\theta = \arctan\left(\frac{x}{30}\right)$. Thus

$\frac{d\theta}{dt} = \frac{1}{1+\left(\frac{x}{30}\right)^2}\left(\frac{1}{30}\right)\frac{dx}{dt}$. Therefore

$\left.\frac{d\theta}{dt}\right]_{x=20} = \frac{1}{1+\left(\frac{20}{30}\right)^2}\left(\frac{1}{30}\right)(5) = \frac{3}{26}$ rad/sec (or $\frac{3}{52\pi}$ rev./sec.)

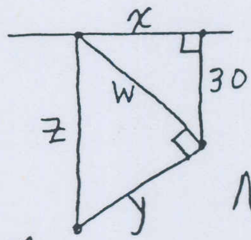
13. 

Rate sought: $\left[\frac{dy}{dt}\right]_{x=6}$ Rate given: $\frac{dx}{dt} = 2$

Now $\frac{y}{8} = \frac{10}{AB}$ and $\overline{AB} = \sqrt{x^2+8^2}$ so

$y = 80\,(x^2+64)^{-\frac{1}{2}}$. Thus $\frac{dy}{dt} = 80\left(-\frac{1}{2}\right)(x^2+64)^{-\frac{3}{2}}\, 2x\,\frac{dx}{dt}$

Therefore $\left[\frac{dy}{dt}\right]_{x=6} = 80\left(-\frac{1}{2}\right)(6^2+64)^{-\frac{3}{2}}\,2(6)(2) = -\frac{24}{25}$ ft./sec.

14. 

Rate sought: $\left[\frac{dz}{dt}\right]_{t=3}$

Rates given: $\frac{dy}{dt} = 10$ and $\frac{dx}{dt} = 5$

Now $z = \sqrt{w^2+y^2}$ and $w^2 = x^2+30^2$

Thus $z = \sqrt{x^2+y^2+900}$ and $\frac{dz}{dt} = \frac{2x\frac{dx}{dt}+2y\frac{dy}{dt}}{2\sqrt{x^2+y^2+900}}$

Now $[x]_{t=3} = 15$ and $[y]_{t=3} = 30$

Therefore $\left[\frac{dz}{dt}\right]_{t=3} = \frac{2(15)(5)+2(30)(10)}{2\sqrt{15^2+30^2+900}} = \frac{25}{3}$ ft./sec.