Dec 11, 2010
Final

THE UNIVERSITY OF MANITOBA

COURSE _____ YEAR _____ NAME _____ SECTION _____ DATE _____ PROBLEM _____ SHEET OF _____

1. $\vec{a}\cdot\vec{b} = ab\cos\theta$ $\quad \cos\theta = \frac{\vec{a}\cdot\vec{b}}{ab} = \frac{1-1+1}{\sqrt{3}\sqrt{3}} = \frac{1}{3} = 70.3°$ [C]

2. $P = \frac{dE}{dt} = \vec{F}\cdot\vec{v}$

$\vec{r} \to \vec{F} = m\vec{a} = m\frac{d^2\vec{r}}{dt^2}$, $\vec{v} = \frac{d\vec{r}}{dt}$

$\frac{d\vec{r}}{dt} = 4t^3\hat{i} + 3t^2\hat{j} + 2t\hat{k}$, $\left.\frac{d\vec{r}}{dt}\right|_{t=2} = 32\hat{i} + 12\hat{j} + 4\hat{k}$ m/s

$\frac{d^2\vec{r}}{dt^2} = 12t\hat{i} + 6t\hat{j} + 2\hat{k}$, $\left.\frac{d^2\vec{r}}{dt^2}\right|_{t=2} = 48\hat{i} + 12\hat{j} + 2\hat{k}$ m/s²

$P = 0.1(48\hat{i} + 12\hat{j} + 2\hat{k})\cdot(32\hat{i} + 12\hat{j} + 4\hat{k})$

$= 0.1(2)(4)\left[(24)(8) + 6(3) + 1(1)\right]$

$= 169.6$ [D]

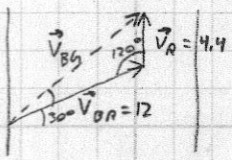
3. $\vec{v}(t) = \vec{v}_0 + \vec{a}t \to \vec{v}(t) = 5.0\hat{j} + (3.1\hat{i} - 2.4\hat{j})t$

$= 3.1\hat{i} + (5.0 - 2.4t)\hat{j}$ [D]

max-y-corr $\Rightarrow 5.0 - 2.4t = 0 \Rightarrow t = \frac{5.0}{2.4} = 2.08$ s

$\vec{v}(2.08) = (3.1)(2.08)\hat{i} = 6.45$

4.

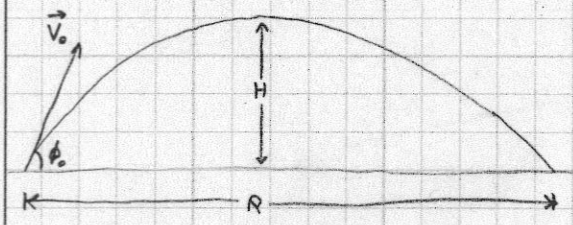


$V_{BG} = \sqrt{12^2 + 4.4^2 - 2(12)(4.4)\cos 120} = 14.4$ km/h

$\frac{\sin\beta}{4.4} = \frac{\sin 120}{14.4} \Rightarrow \beta = \sin^{-1}\left(\frac{4.4}{14.4}\sin 120\right) = 15.3°$

$30° + 15.3° = 45°$ E of N [B]

5.

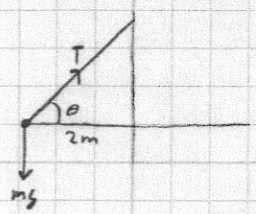


$H = \frac{V_{0y}^2}{2g} = \frac{(V_0\sin\phi)^2}{2g} = \frac{V_0^2\sin^2\phi}{2g}$

$R = \frac{V_0^2\sin 2\phi}{g} = \frac{V_0^2 2\sin\phi_0\cos\phi_0}{g}$ [D]

$\frac{H}{R} = \frac{\tan\phi_0}{4} \Rightarrow \phi_0 = \tan^{-1}\left(\frac{4H}{R}\right) = 64.98°$

6.



$\Sigma F_x = T\cos\theta = \frac{mv^2}{r}$ ← b/c it's rotating

$T\cos\theta = \frac{0.15\cdot 16}{2} = 1.2$

$\Sigma F_y = T\sin\theta - mg = 0$

$T\sin\theta = 0.15\cdot 9.8 = 1.47$

$T^2\cos^2\theta = 1.2^2$

$T^2\sin^2\theta = 1.47^2$ add eqns

$T^2 = 1.2^2 + 1.47^2$

$T = \sqrt{1.2^2 + 1.47^2} = 1.9$ [B]

7.

$V_f = V_0 + at = 5 + 2.5(5) = 17.5$ m/s

V_0

$d = 0 + 17.5(6) + \frac{1}{2}(-a)(6)^2 = 103.2$

$\frac{103.2}{6} = 17.2$ m/s [C]

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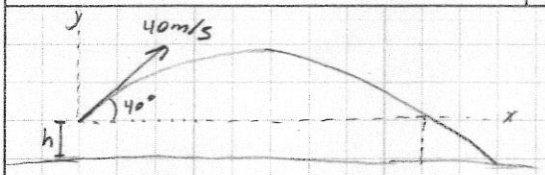
8. 

$t = 5.35s$ $\quad y_0 = 0 \quad y = -h$

$V_{oy} = 40\sin 40 = 25.71$ $\quad a_y = -9.80 m/s$

$V_{ox} = 40\cos 40 = 30.64$ $\quad a_x = 0 m/s$

D

$y = y_0 + V_{oy}t + \frac{1}{2}a_y t^2$

$-h = 0 + 25.71(5.35) + 0.5(-9.8)5.35^2 = -2.7$ $\quad h = 2.7$

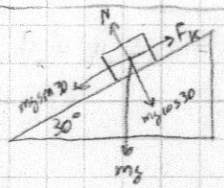
9. 

$T\sin\theta = ma$

$T\cos\theta = mg$

$\tan\theta = \frac{a}{g}$ $\quad \theta = \tan^{-1}\left(\frac{a}{g}\right) = \tan^{-1}\left(\frac{3}{9.8}\right) = 17°$

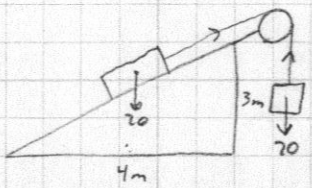
A

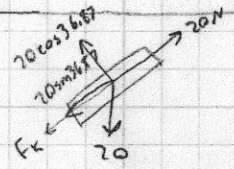
10. 

const. $v \Rightarrow F_k = mg\sin\theta \longrightarrow \mu_k mg\cos\theta = mg\sin\theta$

$\mu_k = \tan\theta = 0.577$

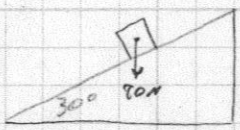
B

11. 



$F_k = 20 - 20\sin 36.87 = 8N$

B

12. 

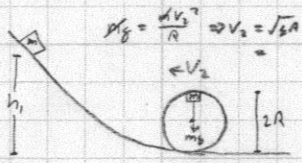
$V_f^2 - V_i^2 = -15 m/s$

C

$F_d - mg(5\sin 30) = \frac{1}{2}m(V_f^2 - V_i^2)$

$F_d - 200 = -61$

$W = 139 N$

13. 

$mg = \frac{mV_2^2}{R} \Rightarrow V_2 = \sqrt{gR}$

cons of ME: $mgh_1 = mg(2R) + \frac{1}{2}mV_2^2$

$mgh_1 = 2mgR + \frac{mgR}{2}$

$h_1 = \frac{5}{2}R$ $\quad y = \frac{5}{2}R - 2R = \frac{R}{2}$

B

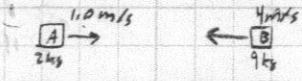
14. $700 \times 12 = 8400$ **E**

15. $m_b V_b + 0 = (m_b + m_B)V$ $\quad V = \frac{m_b}{m_b + m_B}V_b = 0.3996 m/s$

B

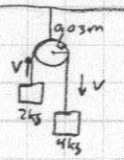
16. $\frac{1}{2}m_A v_A^2 + \frac{1}{2}m_0 V_0^2 = 3750$

C

17. 

$V_{cm} = \frac{1}{(2+9)}\left((2\cdot 1) + (9\cdot -4)\right) = \frac{1}{11}(-34) = \sim 3.1 m/s$ to left

B

18. 

$K_4 = \frac{1}{2}mv^2 = \frac{1}{2}4v^2 = 2v^2$

$K_2 = \frac{1}{2}2v^2 = v^2$

C

$\omega = \frac{v}{0.03} = \frac{100}{3}v$

$K_{pulley} = \frac{1}{2}I\omega^2 = \frac{1}{2}(4.5\times10^{-3})\left(\frac{100}{3}\right)^2 v^2 = 2.5v^2$

19. $I = 12 kg\,m^2$

$\tau = I\alpha$

$\alpha = \frac{I(\omega^2 - \omega_0^2)}{2\theta} = 12\frac{(6^2 - 5^2)}{20\pi} = 2.2 Nm$

$\theta = 10\pi$

D

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20. $W = 12$ rad/s, $m = 2$ kg, $r = 0.5$ m

$p = mv \leftrightarrow L = I\omega$ (L: angular momentum) $\longrightarrow L = 0.5 \cdot 12 = 6\ \text{kg m}^2/\text{s}$ **[A]**

$I = mr^2 = 0.5$

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21. $\vec{L} = \vec{r} \times \vec{p} = m\vec{r} \times \vec{v}$

$\vec{v} = \vec{a}t = (4.0\hat{i} - 3.0\hat{j})(2) = 8.0\hat{i} - 6.0\hat{j}$ m/s

$\vec{r} = \vec{r}_0 + \vec{v}_0 t + \frac{1}{2}\vec{a}t^2 = 3\hat{i} + 2(4\hat{i} - 3\hat{j}) = 11\hat{i} - 6\hat{j}$ m

$$\vec{L} = 2.0 \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 11 & -6 & 0 \\ 8 & -6 & 0 \end{vmatrix} = 2\left[0\hat{i} - 0\hat{j} + (-66+48)\hat{k}\right] \text{kg m}^2/\text{s} = 2(-18)\hat{k}\ \text{kg m}^2/\text{s} = -36\hat{k}\ \text{kg m}/\text{s}^2$$

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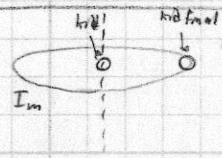
22. $r = 3.0$ m, $I = 600$ kg m^2, $\omega_0 = 0.80$ rad/s



cons. of ang. mom. $L_i = L_f$

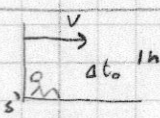
$I_m \omega_i = (I_m + I_k)\omega_f$

$\omega_f = \frac{I_m}{I_m + I_k}\omega_i$ → $I_k = mr^2 = 20(3)^2 = 180$

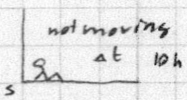
$= \frac{600}{600+180} 0.8 = \frac{600}{780}(0.8) = 0.62$ rad/s

[A]

23.



$\Delta t = \gamma \Delta t_0$ → $\gamma = \frac{1}{\sqrt{1-\beta^2}}$ → $\beta = \frac{v}{c}$



$\frac{\Delta t_0}{\Delta t} = \frac{1}{10} \rightarrow \gamma = \frac{\Delta t}{\Delta t_0} = 10$

$10 = \frac{1}{\sqrt{1-\beta^2}} \Rightarrow \sqrt{1-\beta^2} = \frac{1}{10}$

$\beta^2 = 1 - (\frac{1}{10})^2$

$\beta = \sqrt{1 - \frac{1}{100}} = \sqrt{0.99} = 0.995$

$v = \beta c = 0.995c$

γ is the Lorentz factor ≥ 1

β is the speed factor < 1

[E]

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24. $\Delta L_0 = 6$m, $\Delta L = \frac{4}{5}6$m

$\gamma = \frac{6}{\frac{4}{5}6\text{m}} = \frac{5}{4} = 1.25$

$\gamma = \frac{1}{\sqrt{1-\beta^2}} \Rightarrow \sqrt{1-\beta^2} = \frac{1}{\gamma}$

$\beta = \sqrt{1 - (\frac{1}{\gamma})^2} = \sqrt{1 - (\frac{1}{1.25})^2} = 0.6$

$v = \beta c = \boxed{0.6c}$ **[C]**

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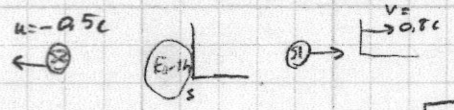
25. $u = -0.5c$



$u = \frac{u' + v}{1 + \frac{u'v}{c^2}} \longrightarrow u' = \frac{u - v}{1 - \frac{uv}{c^2}} = \frac{-0.5c - 0.8c}{1 - \frac{(-0.5c)(0.8c)}{c^2}}$

$= \frac{-1.3c}{1 + 0.4} = \frac{-1.3}{1.4}c = \boxed{-0.9}$

[C]