UNIVERSITY OF MANITOBA

DATE: December 10, 2007 — FINAL EXAMINATION
PAPER # 197 — TITLE PAGE
EXAMINATION: Engineering Mathematical Analysis 2 — TIME: 3 hours
COURSE: MATH 2132 — EXAMINER: G.I. Moghaddam

NAME: (Print in ink) ____________________

STUDENT NUMBER: __________

SEAT NUMBER: __________

SIGNATURE: (in ink) ____________________

(I understand that cheating is a serious offense)

INSTRUCTIONS TO STUDENTS:

This is a 3 hour exam. **Please show your work clearly.**

No texts, notes, or other aids are permitted. There are no calculators, cellphones or electronic translators permitted.

This exam has a title page, 11 pages of questions and also 2 blank pages for rough work together with a *formulas sheet*. Please check that you have all the pages. You may remove the blank pages if you want, but be careful not to loosen the staple.

The value of each question is indicated in the left hand margin beside the statement of the question. The total value of all questions is 100 points.

Answer all questions on the exam paper in the space provided beneath the question. If you need more room, you may continue your work on the reverse side of the page, but CLEARLY INDICATE that your work is continued.

Question	Points	Score
1	7	
2	10	
3	7	
4	10	
5	9	
6	10	
7	9	
8	8	
9	10	
10	10	
11	10	
Total:	100	

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[7] 1. Find the radius of convergence and the open interval of convergence for the series

$$\sum_{n=0}^{\infty} \frac{(-1)^n \sqrt{(2n)!}}{2^n \, n!} (x-3)^{4n}.$$

[10] 2. Let $f(x) = \dfrac{4-4x}{4x^2-8x-5}$; given the partial decomposition

$$\frac{4-4x}{4x^2-8x-5} = \frac{1}{5-2x} - \frac{1}{1+2x},$$

find the Taylor series of $f(x)$ about 1. Express your answer in sigma notation and simplify as much as possible. Determine the open interval of convergence.

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[7] 3. Find the value of x for which the ***fourth*** term of the binomial expansion of

$f(x) = \dfrac{1}{(1+\frac{1}{8}x)^3}$ is equal to $\dfrac{-5}{32}$. Show your work.

[10] 4. (a) Evaluate the following integral using infinite series

$$\int_2^3 \frac{1}{1+(x-2)^5}\,dx\,.$$

Express your answer in sigma notation.

(b) If you truncate the series in part (a) after **fifth** term, what is a maximum possible error? Explain why you can claim that your answer is a maximum error.

[9] 5. Find, in explicit form, a general solution for the differential equation

$$\frac{dy}{dx} + \frac{y}{x \ln x} = x^5.$$

[10] 6. When two substances A and B are brought together at time t, they react to form a third substance C in such a way that 2 grams of A reacts with 3 grams of B to produce 5 grams of C. The rate at which C is formed is proportional to the product of the amounts of A and B still present in the mixture. The initial amounts of A and B are 40 and 60 grams, respectively.

(a) Show that the initial-value problem for the amount $x(t)$ of C at any given time t is :

$$\frac{dx}{dt} = k\,(100 - x)^2, \quad x(0) = 0$$

where $k > 0$ is a constant.

(b) Solve the differential equation for $x(t)$.

(c) What is your prediction in long run, i.e. $\lim_{t\to\infty} x(t)$?

[9] 7. Find a 2-parameter family of solutions of differential equation

$$y'' - y' = (y')^2.$$

(Hint: $\dfrac{1}{v(v+1)} = \dfrac{1}{v} - \dfrac{1}{v+1}$)

[8] 8. Given that $m^3(m-2)(m^4-1)=0$ is the auxiliary equation associated with the linear differential equation

$$y^{(8)} - 2y^{(7)} - y^{(4)} + 2y^{(3)} = x^2e^{2x} + x\sin x + 1,$$

what is the **form** of a particular solution $y_p(x)$?

DO NOT EVALUATE THE COEFFICIENTS IN $y_p(x)$.

[10] 9. Find Laplace transform of the function

$$f(t) = \begin{cases} e^t & \text{if} \quad t < 2 \\ t & \text{if} \quad 2 < t < 3 \\ (t-3)^4 & \text{if} \quad t > 3 \ . \end{cases}$$

[10] 10. Find $\mathscr{L}^{-1}\left\{\dfrac{4s^2+19s+25}{(s+1)(s^2+6s+10)}\right\}$.

[10] 11. Use Laplace transforms to solve the initial-value problem

$$\frac{d^2y}{dt^2} + y = \delta(t - 2\pi) + \delta(t - 4\pi)\,, \quad y(0) = 1, \quad y^{'}(0) = 0.$$