Math 2130 - Engineering Mathematical Analysis 1

Tutorial § 11.6, 11.10, 11.11

11.6.5 Determine the area of the triangle that passes through the points: $(3,0,2)$, $(2,1,3)$ and $(1,3,7)$.
Hint: See tutorial question 11.5.1.

11.10.1 Find parametric equations of the ellipse: $\frac{(x-x_0)^2}{a^2}+\frac{(y-y_0)^2}{b^2}=1$, traversed (i.e., travelled or gone over) once, counter-clockwise; a and b are positive constants. Note: (x_0, y_0) is the centre of the ellipse and a and b are the semi-axes.
Hint: Use $\cos^2 t+\sin^2 t=1$. Note: Unlike the case of a circle (when $a=b$), if $a \neq b$, then this "t" will *not* be the angle between the line joining a point on the ellipse and the horizontal line through the centre. Can you prove this?

In questions 11.10.2-11.10.4 below, find parametric equations of the given curve and also name the surfaces which intersect in the curve and describe the shape of the curve:

11.10.2 Find parametric equations of the curve: $x+2y+3z=6$, $y-2z=3$, directed so that z increases along the curve.

11.10.3 Find parametric equations of the curve: $z=x^2+y^2$, $z=\sqrt{6-x^2-y^2}$, directed so that y decreases when x is positive.

11.10.4 Find parametric equations of the curve: $z=4x^2+y^2$, $z=5+8x$, so that the curve is traversed once, counter-clockwise, when viewed from above.

11.11.1 For each of 11.10.1-11.10.4 above, determine a *unit* tangent vector in the direction of the curve.

11.11.2 For the curve in 11.10.2 above, calculate the length of that portion where: $-14 \le x \le 7$.

11.11.3 For 11.10.3 above, calculate the length of the curve.

11.11.4 For 11.10.4 above, set up an integral for the length of the curve, and *simplify*, but do *not evaluate* this integral.

11.11.5 Calculate the length of the curve: $y=x^{(2/3)}$, $z=2x^{(2/3)}$, for the portion where: $0 \le x \le 1$.
Use the parametrization where $x=t^3$.

Answers: 11.6.5: Area = $\dfrac{\sqrt{14}}{2}$.

11.10.1: $x = x_0 + a\cos t,\ \ y = y_0 + b\sin t;\ \ 0 \le t \le 2\pi$. Consider the line: $t = \frac{\pi}{4}$.

11.10.2: $x = -7t,\ \ y = 2t + 3,\ \ z = t;$ all real t. Two planes; the curve is a line.

11.10.3: Two standard forms are:

$x = \sqrt{2}\cos(t),\ \ y = -\sqrt{2}\sin(t),\ \ z = 2;\ \ 0 \le t \le 2\pi$.

or, $x = \sqrt{2}\sin(t),\ \ y = \sqrt{2}\cos(t),\ \ z = 2;\ \ 0 \le t \le 2\pi$.

A circular paraboloid and a sphere; the curve is a circle.

11.10.4: $x = 1 + \dfrac{3}{2}\cos t,\ \ y = 3\sin t,\ \ z = 13 + 12\cos t;\ \ 0 \le t \le 2\pi$.

An elliptic paraboloid and a plane; the curve is an ellipse.

11.11.1: (1) $\hat{T} = \dfrac{(-a\sin t,\ b\cos t)}{\sqrt{a^2\sin^2 t + b^2\cos^2 t}}$; (2) $\hat{T} = \dfrac{(-7,2,1)}{\sqrt{54}} = \dfrac{(-7,2,1)}{3\sqrt{6}}$;

(3) $\hat{T} = (-\sin(t), -\cos(t), 0)$, or, $\hat{T} = (\cos(t), -\sin(t), 0)$ (depending on your parametrization);

(4) $\hat{T} = \dfrac{(-\sin t, 2\cos t, -8\sin t)}{\sqrt{4\cos^2 t + 65\sin^2 t}} = \dfrac{(-\sin t, 2\cos t, -8\sin t)}{\sqrt{4 + 61\sin^2 t}}$.

11.11.2: (Re 11.10.2): L = $9\sqrt{6}$.

11.11.3: (Re 11.10.3): L = $\left(2\sqrt{2}\right)\pi$.

11.11.4: (Re 11.10.4): L = $\displaystyle\int_0^{2\pi} \frac{3}{2}\sqrt{4\cos^2 t + 65\sin^2 t}\ dt = \int_0^{2\pi} \frac{3}{2}\sqrt{4 + 61\sin^2 t}\ dt$.

Note: This is called an *elliptic integral* and *cannot* be calculated exactly.

11.11.5: L = $\displaystyle\int_0^1 \sqrt{9t^4 + 20t^2}\ dt = \int_0^1 t\sqrt{9t^2 + 20}\ dt = \frac{1}{27}\left[29^{3/2} - 20^{3/2}\right] = \frac{1}{27}\left[29^{3/2} - 40\sqrt{5}\right]$.
