Math 2130 Winter 2015 Test 2

[12] 1. Find each of the following limits or explain why it does not exist.

(a) $\displaystyle\lim_{(x,y)\to(0,-2)} \frac{x^4y+2x^4}{x^8+y^2+4y+4}$

(b) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^2y}{\sqrt{x^4+y^4+2x^2y^2}}$

[8] 2. Find a formula for $\dfrac{dz}{dt}$ if $z=f(x,y,s,t)$, $x=g(s,t)$, $y=h(t)$ and $s=k(t)$.

[6] 3. Find $f(x,y)$ if $f(0,1)=5$ and $\nabla f(x,y)=(ye^x\,,\,e^x+1)$.

[12] 4. Let $u(x,y)=f(y^2-x)+g(y^2-x)$, where f and g are twice differentiable functions. Show that

$$4y^2\frac{\partial^2 u}{\partial x^2}-\frac{\partial^2 u}{\partial y^2}-2\frac{\partial u}{\partial x}=0.$$

[12] 5. If $z=\ln(u^2+v^2)$, where u and v are functions of x defined by

$$x=u^2-e^{u^2}+\sqrt{2},\quad v^2e^x+2v^3x-\sqrt{2}=0,$$

find $\dfrac{dz}{dx}$ and simplify your answer..

Answers by Dawit

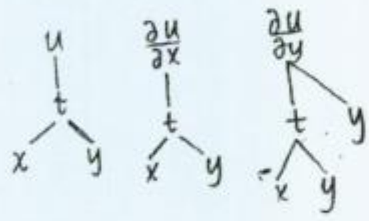
1.a) Does not exist (path dependent) ← Hint: factorization followed by $y+2=mx$ Substitution

b) 0 (Hint: factorization then Simplification then squeeze Theorem.)

2. $\dfrac{\partial z}{\partial x}\dfrac{\partial x}{\partial s}\dfrac{ds}{dt}+\dfrac{\partial z}{\partial x}\dfrac{\partial x}{\partial t}+\dfrac{\partial z}{\partial y}\dfrac{dy}{dt}+\dfrac{\partial z}{\partial s}\dfrac{ds}{dt}+\dfrac{\partial z}{\partial t}\Big|_{x,y,s}$

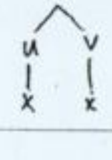
3. ye^x+y+3 or $y(e^x+1)+3$

4. Hint: let $t=y^2-x \Rightarrow u=f(t)+g(t)$ and do the chain-rule based on the tree digram.



5. $\dfrac{1}{u^2+v^2}\left[\dfrac{1}{1-e^{u^2}}-\dfrac{v^2e^x+2v^3}{e^x+3xv}\right]$

$\left\{ \text{Hint: let } \begin{array}{l} F=x-u^2+e^{u^2}-\sqrt{2}=0 \\ G=v^2e^x+2v^3x-\sqrt{2}=0 \end{array}\right\}$



$\Rightarrow \dfrac{dz}{dx}=\dfrac{\partial z}{\partial u}\dfrac{du}{dx}+\dfrac{\partial z}{\partial v}\dfrac{dv}{dx} = \dfrac{\partial z}{\partial u}\left(-\dfrac{F_x}{F_v}\right)+\dfrac{\partial z}{\partial v}\left(-\dfrac{G_x}{G_v}\right)$