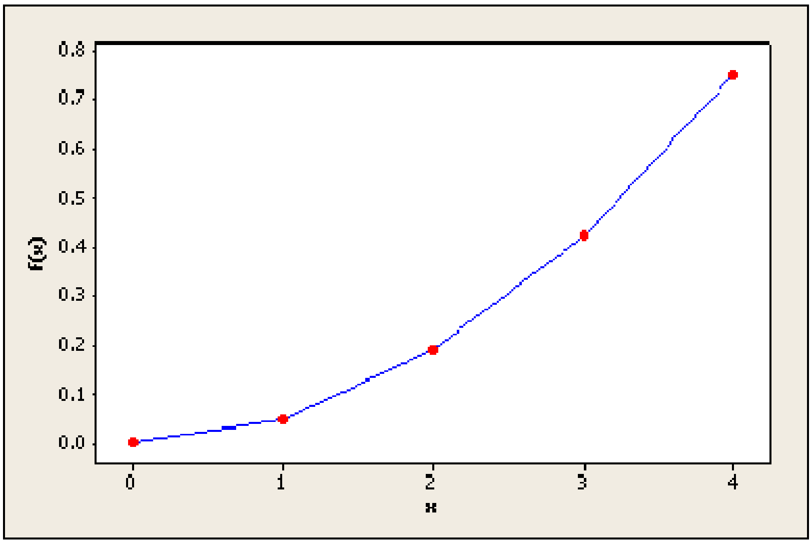
STAT2220: Engineering Statistics

Solution to Assignment 2

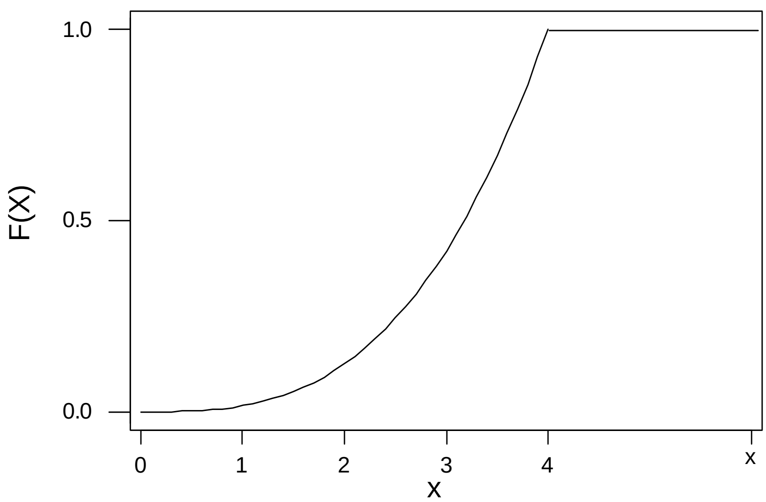
3-20. For 3-19a:

a) $f(x) = \dfrac{3x^2}{64}, \qquad 0 < x < 4$



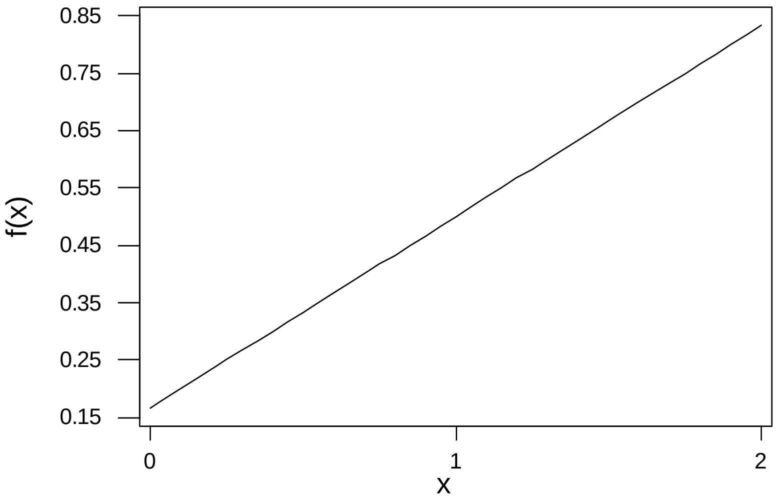
b) F(X) = $\int_0^x f(t)dt = \int_0^x \dfrac{3t^2}{64}dt = \dfrac{x^3}{64}$, 0 < x < 4

c)



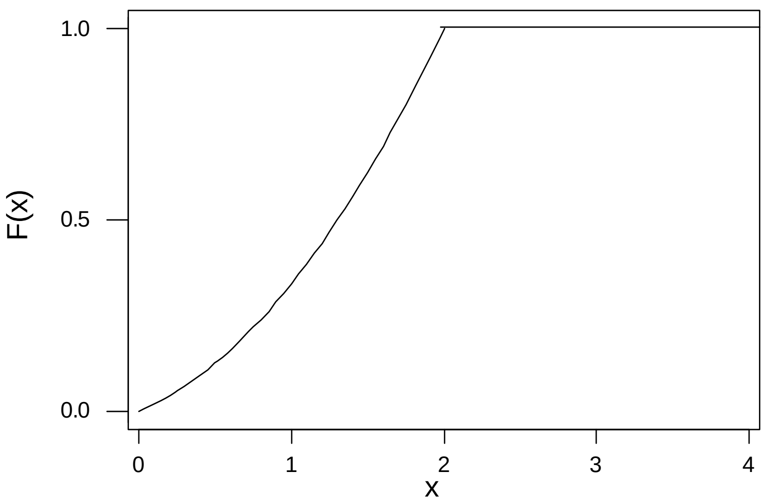
3-19b:

a) $f(x) = \frac{(1+2x)}{6}, \qquad 0 < x < 2$



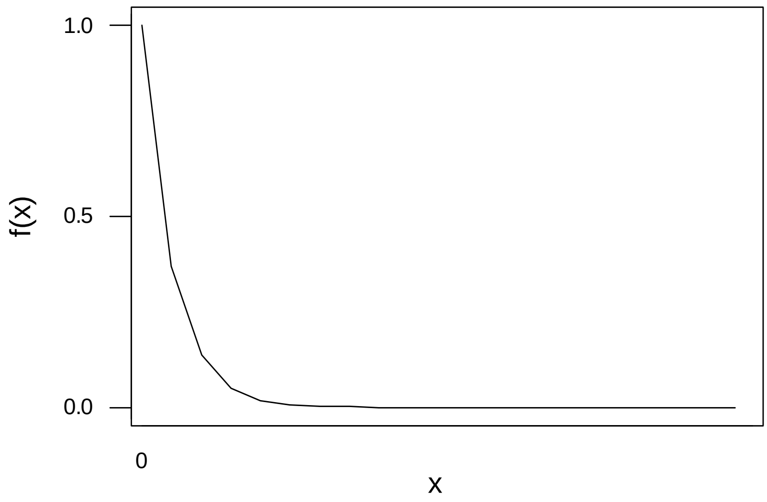
b) $F(X) = \int_0^x \frac{(1+2t)}{6} dt = \frac{1}{6}\left(x + x^2\right), \qquad 0 < x < 2$

c)



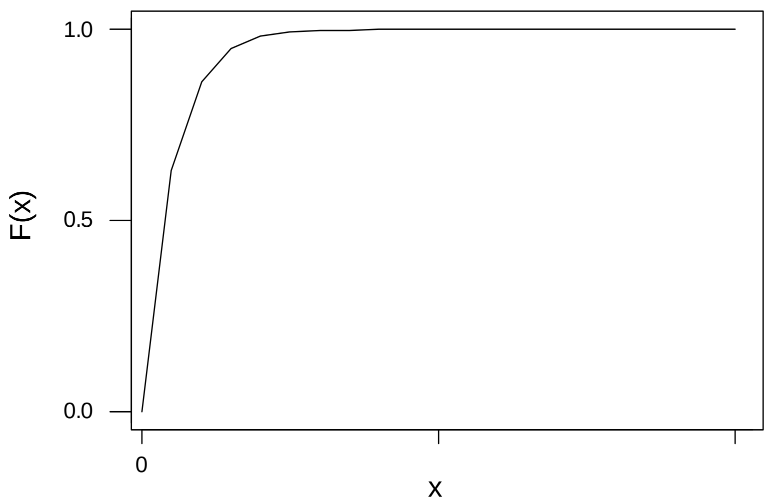
3-19c: $f(x) = e^{-x}$, x >0

a)



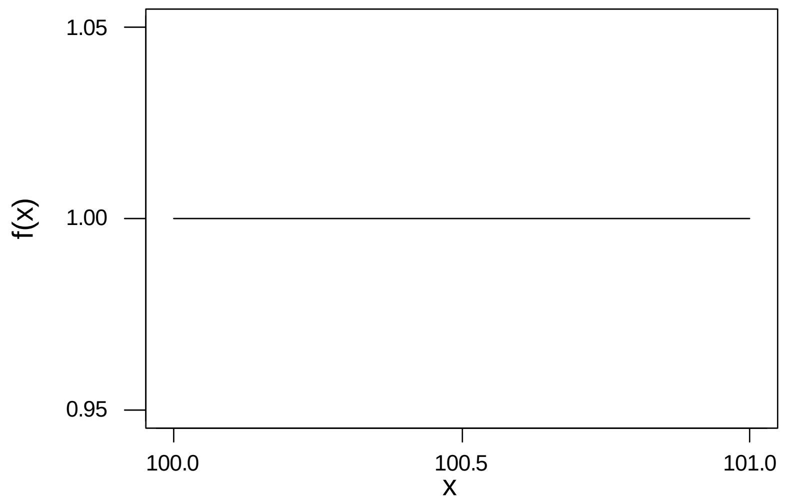
b) $F(x) = 1 - e^{-x}$, $x > 0$

c)



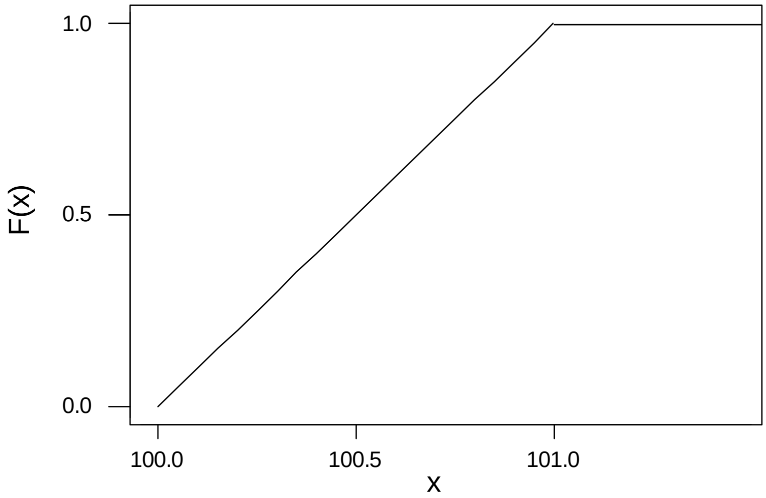
3-19 d: $f(x) = 1$, 100 < x < 101

a)



b) F(X) = x – 100, 100 < x < 101

c)



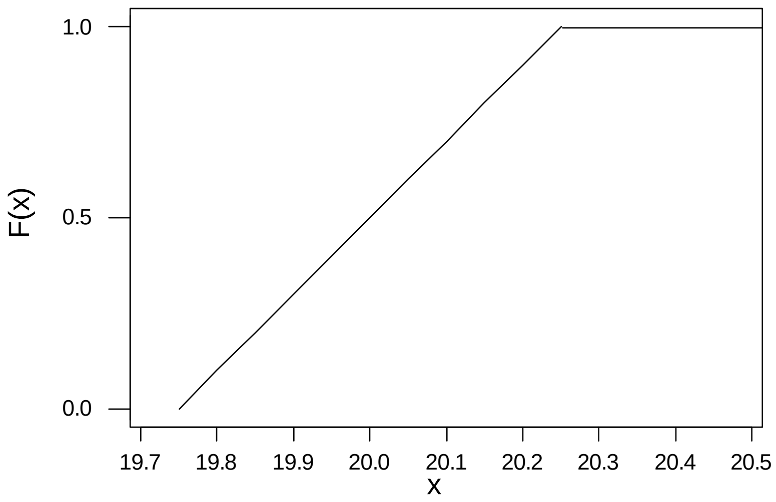
3-24. a) P(19.75 < X < 20) = $\int_{19.75}^{20} 2.0 dx = 0.5$

b) P(19.9 < X < 20.1) = $\int_{19.9}^{20.1} 2.0dx = 0.4$

c) E(X) = $\int_{19.75}^{20.25} 2.0xdx = 20$

V(X) = $\int_{19.75}^{20.25} 2.0x^2dx - [E(X)]^2 = 400.02 - (20)^2 = 0.02$

d) F(x) = $\int_{19.75}^{x} 2.0dx = 2x - 39.5$, 19.75 < x < 20.25



3-26. a) $P(X \le 2.0080) = F(2.0080) = 200(2.0080) - 401 = 0.6$

b) $P(X > 2.0055) = 1 - P(X \le 2.0055) = 1 - F(2.0055) = 1 - 0.1 = 0.9$

c) $P(2.0090 < X < 2.0100) = F(2.0100) - (2.0090) = 1.0 - 0.8 = 0.2$

3-36. a) P(-z < Z < z) = P(Z < z) – P(Z < -z) = 1 – 2P(Z<-z) = 0.95

So P(Z<-z) = 0.5(1 – 0.95) = 0.025

z = 1.96

b) P(-z < Z < z) = P(Z < z) – P(Z < -z) = 1 – 2P(Z<-z) = 0.99

So P(Z<-z) = 0.5(1 – 0.99) = 0.005

z = 2.58

c) P(-z < Z < z) = P(Z < z) – P(Z < -z) = 1 – 2P(Z<-z) = 0.68

So P(Z<-z) = 0.5(1 – 0.68) = 0.16

z = 1

d) P(-z < Z < z) = P(Z < z) – P(Z < -z) = 1 – 2P(Z<-z) = 0.9973

So P(Z<-z) = 0.5(1 – 0.9973) = 0.00135

z = 3

3-48. a) $P(X > 1140) = P(Z > \frac{1140-1000}{60}) = P(Z > 2.33) = P(Z < -2.33) = 0.009903$

b) $P(X < 900) = P(Z < \frac{900-1000}{60}) = P(Z < -1.67) = 0.04746$

3-80. The function is a probability mass function. All probabilities are nonnegative and sum to 1.

a) $P(X \le 1) = P(X=0) + P(X=1) = 0.025 + 0.041 = 0.066$

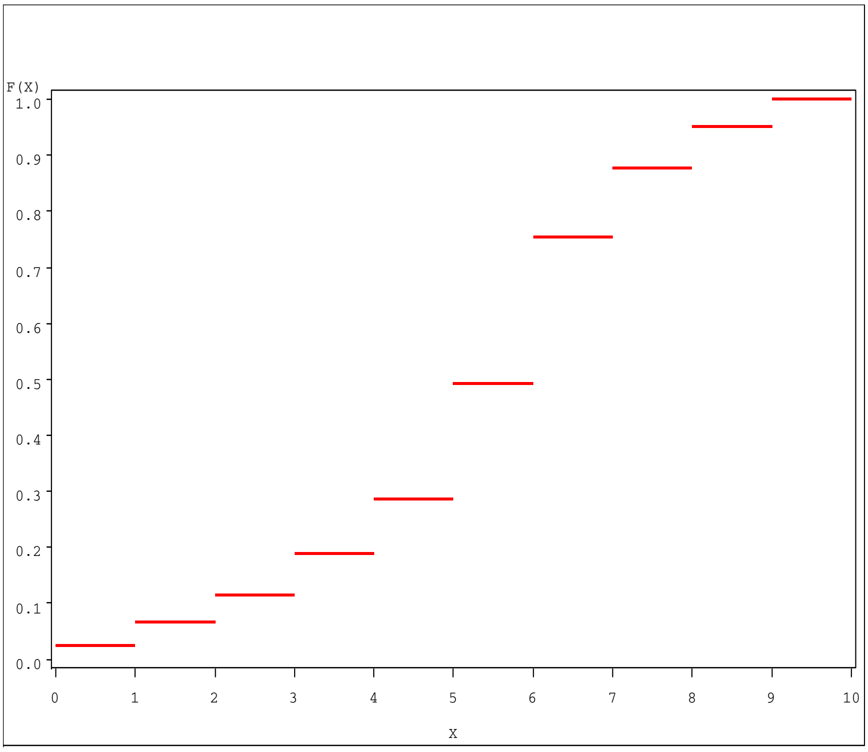
b) $P(2<X<7.2) = P(X=3) + P(X=4) + P(X=5) + P(X=6) + P(X=7) = 0.762$

c) $P(X\ge 6) = P(X=6) + P(X=7) + P(X=8) + P(X=9) = 0.508$

d) $E(X) = 0(0.025) + 1(0.041) + 2(0.049) + 3(0.074) + 4(0.098) + 5(0.205) +6(0.262)$
$+ 7(0.123) + 8(0.074) + 9(0.049)$
$= 5.244$

$V(X) = 0^2 (0.025) + 1^2 (0.041) + 2^2 (0.049) + 3^2 (0.074) + 4^2 (0.098) + 5^2 (0.205)$
$+ 6^2 (0.262) + 7^2 (0.123) + 8^2\ 0.074) + 9^2 (0.049) - (5.244)^2$
$= 4.260$

e) Graph of F(x)



3-96. E(X) = 20 (0.01) = 0.2

V(X) = 20 (0.01) (0.99) = 0.198

$\mu_X + 3\sigma_X = 0.2 + 3\sqrt{0.198} = 1.53$

a) $P(X > 1.53) = P(X \geq 2) = 1 - P(X \leq 1)$

$$= 1 - \left[\binom{20}{0} 0.01^0 0.99^{20} + \binom{20}{1} 0.01^1 0.99^{19}\right]$$
$$= 0.0169$$

b) X is binomial with n = 20 and p = 0.04

$P(X > 1) = 1 - P(X \leq 1)$

$$= 1 - \left[\binom{20}{0} 0.04^0 (0.96)^{20} + \binom{20}{1} 0.04^1 (0.96)^{19}\right]$$
$$= 0.1897$$

c) Let Y denote the number of times X exceeds 1 in the next five samples. Then, Y is binomial with n = 5 and p = 0.1897 from part b.

$$P(Y \geq 1) = 1 - P(Y = 0) = 1 - \left[\binom{5}{0} 0.1897^0 (0.8103)^5\right] = 0.6507$$

The probability is 0.6507 that at least one sample from the next five will contain more than one defective.

3-108. a) Let X denote the number of tremors in a 12 month period. Then, X is a Poisson random variable with $\lambda = 6$. $P(W = 10) = \dfrac{e^{-6} 6^{10}}{10!} = 0.0413$

b) $\lambda = 2(6) = 12$ for a two year period. Let Y denote the number of tremors in a two year period. $P(Y = 18) = \dfrac{e^{-12} 12^{18}}{18!} = 0.0255$

c) $\lambda = (1/12)6 = 0.5$ for a one month period. Let W denote the number of tremors in a one-month period. $P(W = 0) = \dfrac{e^{-0.5} 0.5^0}{0!} = 0.6065$

d) $\lambda = (1/2)6 = 3$ for a six month period. Let V denote the number of tremors in a six-month period.

$$P(V > 5) = 1 - P(V \leq 5)$$
$$= 1 - \left[\frac{e^{-3}3^0}{0!} + \frac{e^{-3}3^1}{1!} + \frac{e^{-3}3^2}{2!} + \frac{e^{-3}3^3}{3!} + \frac{e^{-3}3^4}{4!} + \frac{e^{-3}3^5}{5!}\right]$$
$$= 1 - 0.9161$$
$$= 0.0839$$

3-112. a) $\lambda = 3.2$, $P(X = 5) = 0.1140$

b) $\lambda_W = 6.4$, $P(W = 8) = 0.1160$

c) $\lambda_Y = 9.6$, $P(Y = 0) = 0.0001$

3-124. Let X denote the time until a failure occurs. Then, X is an exponential random variable and $\lambda = 1/4$.

a) $P(X < 1) = \int_0^1 \frac{1}{4} e^{-x/4} dx = 1 - e^{-x/4}\Big|_0^1 = 0.2212$

b) $P(X < 2) = \int_0^2 \frac{1}{4} e^{-x/4} dx = 1 - e^{-x/4}\Big|_0^2 = 0.3935$

c) $P(X < 4) = \int_0^4 \frac{1}{4} e^{-x/4} dx = 1 - e^{-x/4}\Big|_0^4 = 0.6321$

d) $P(X < b) = 0.03$

$$\int_0^b \frac{1}{4} e^{-x/4} dx = -e^{-x/4}\Big|_0^b = 1 - e^{-b/4} = 0.03$$
$$e^{-b/4} = 0.97$$
$$-\frac{b}{4} = \ln(0.97)$$
$$b = 0.122$$

Where b = 0.122 years is equivalent to 1.5 months.

e). The mean time to failure in this case is $E(X) = 1/\lambda$. Solve the following for λ.

$$\int_0^1 \frac{1}{\lambda} e^{-x/\lambda} dx = -e^{-x/\lambda}\Big|_0^1 = 1 - e^{-1/\lambda} = 0.03$$
$$e^{-1/\lambda} = 0.97$$
$$-\frac{1}{\lambda} = \ln(0.97)$$
$$\frac{1}{\lambda} = 0.03$$
$$\lambda = 32.83$$