THE UNIVERSITY OF MANITOBA

6:00 p.m. 11 December 2009 FINAL EXAMINATION

PAPER NO.: 151

DEPARTMENT & COURSE NO.: ENG 1460 TIME: 3 HOURS

EXAMINATION: Introduction to Thermal Sciences EXAMINERS: Dr. S. Ormiston, Dr. J. Bartley

Instructions:

(a) You are permitted to use the course textbook and a calculator.
(b) Clear, systematic solutions are required. **Show all steps in presenting your work.** Marks will not be assigned for solutions that require unreasonable (in the opinion of the instructor) effort to decipher.
(c) Ask for clarification if any problem statement is not clear to you.
(d) Use linear interpolation between table entries as necessary. Use constant specific heat.
(e) Retain all the significant figures in the property values taken from the tables and indicate the units. Use 4 to 5 significant figures in your calculations. Final answers must have 3 to 5 significant figures and units.
(f) There are **five** questions on this exam. The weight of each problem is indicated. The exam will be marked out of **100**.

value **1.**

16

The expansion engine shown in the figure below receives two mass flows of water, $\dot{m}_1 = 5$ [kg/s] at 3 [MPa] and 600 [°C], and $\dot{m}_2 = 2$ [kg/s] at 350 [kPa] and 25 [°C]. The water exits the engine and passes through a nozzle where it exits the nozzle with at temperature of 125 [°C] and quality of 90%. The engine produces shaft power $\dot{W}$ but also loses heat to the surroundings at a rate $|\dot{Q}| = 601.25$ [kW]. The diameter at the exit of the nozzle is $D_3 = 20$ [cm].

4 (a) Calculate the velocity of the flow leaving the nozzle, $\overline{V}_3$, in m/s.

12 (b) Neglecting the kinetic energies of the two inlet streams, determine the power produced by the expansion engine, $\dot{W}$, in kW.

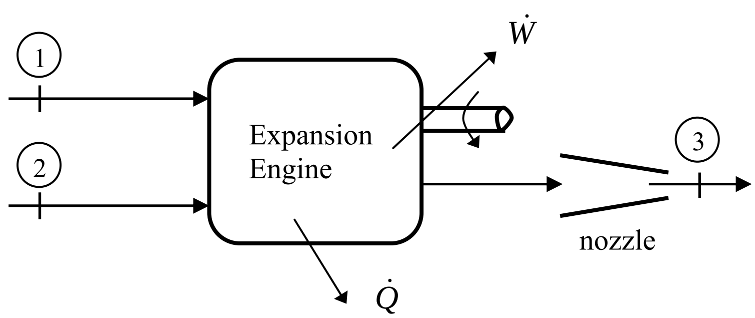


Figure 1: Expansion engine and nozzle for problem 1.

value **2.**
12

A mass $m = 0.146$ [kg] of air at 387 [°C] contained in a frictionless piston-cylinder arrangement undergoes an isothermal (constant temperature) process during which the pressure increases from $P_1 = 100$ [kPa] to $P_2 = 400$ [kPa]. The heat transfer from the air during this process is used as heat input to a Carnot heat engine as shown in Figure 2 below. The Carnot heat engine rejects heat into a thermal reservoir at 24 [°C]. Treat the air as an ideal gas.

3 (a) Determine the thermal efficiency of the Carnot heat engine.

9 (b) Calculate the net work produced by the heat engine, W_{net}, in kJ.

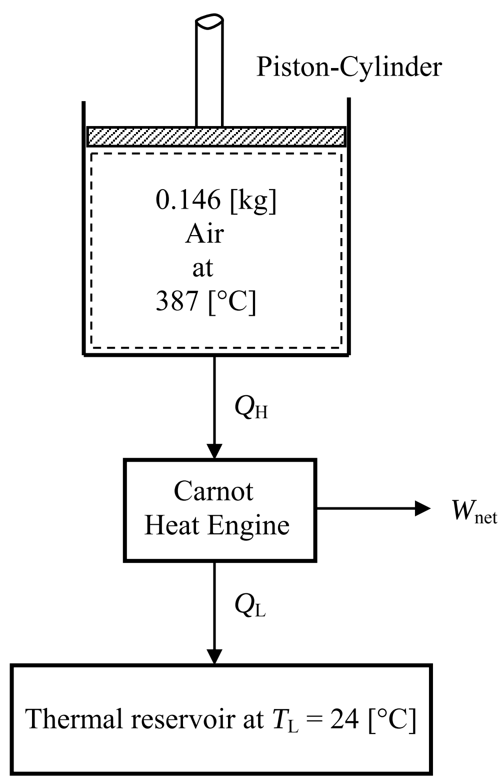


Figure 2: Piston-cylinder and Carnot engine for problem 2.

value 18

3.

An arrangement consisting of a reversible heat engine (HE) operating between the thermal reservoirs of $T_{\mathrm{H,HE}} = 800$ [°C] and $T_{\mathrm{L,HE}} = 200$ [°C] produces exactly enough power, $\dot{W}$, to drive (supply) an irreversible heat pump (HP) whose purpose is to supply enough heat to generate saturated water vapour at 350 [kPa]. A flow of water, $\dot{m}_{\mathrm{w}}$, enters the inlet to the steam generator as a saturated liquid and exits the steam generator as a saturated vapour. It is known that the heat engine, operating in a reversible manner, rejects heat to a thermal reservoir at $T_{\mathrm{L,HE}}$ at a rate of 500 [kW].

If the actual COP (coefficient of performance) of the heat pump is 65% of the Carnot COP operating between the same two thermal reservoirs, determine the power produced by the heat engine, $\dot{W}$, and the mass flow rate of water through the steam generator, $\dot{m}_{\mathrm{w}}$, in kg/s.

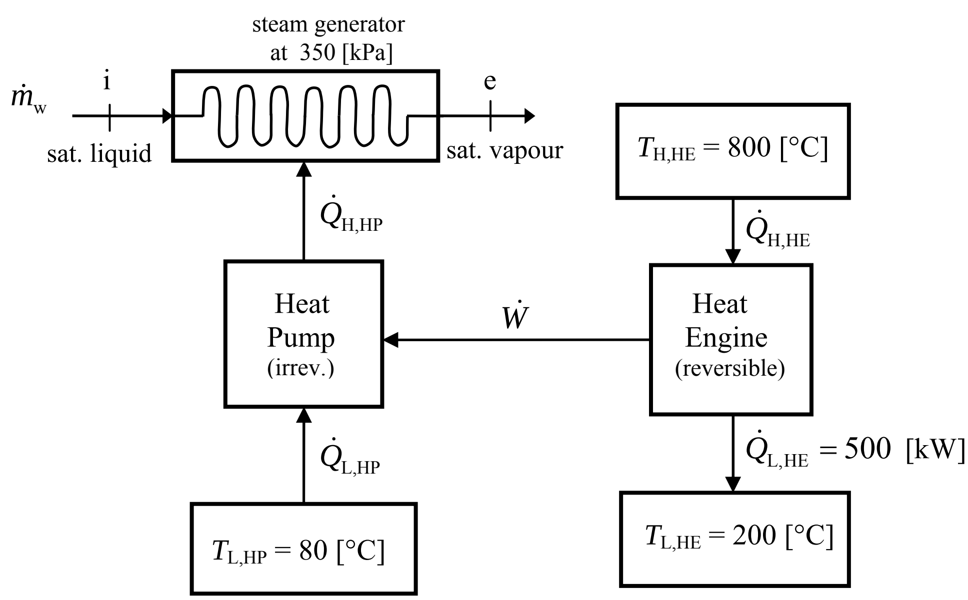


Figure 3: Heat pump and heat engine arrangement for problem 3.

value 21 **4.**

A rigid container is divided by a fixed partition into two compartments: A and B, as shown in Figure 4. Compartment A contains 2.5 [kg] of air initially at $P_{A,1} = 300$ [kPa]. Compartment B contains 1.5 [kg] of water, initially as saturated vapour at $P_{W,1} = 375$ [kPa]. The partition is very thin and highly conductive so that the temperature in Compartment A is always the same as the temperature in Compartment B during quasi-equilibrium conditions. The container is heavily insulated on all but one surface on the water side where energy exchange (by heat transfer) with the surroundings can take place. A quasi-equilibrium process was conducted whereby heat was transferred from the water such that its pressure decreased to $P_{W,2} = 125$ [kPa]. Treat the air as an ideal gas and neglect changes in kinetic and potential energy during the process. Assume that the partition has a negligible volume and stores no energy.

6 (a) Determine the total volume of the container, in m^3.

4 (b) Determine the final pressure of the air in Compartment A, $P_{A,2}$, in kPa.

3 (c) Determine the amount of energy transfer by heat between the compartments (through the partition), in kJ. Is this heat going from the water to the air, or vice versa? Briefly explain how you know the direction.

8 (d) Determine the amount of heat transfer from the water to the surroundings, in kJ.

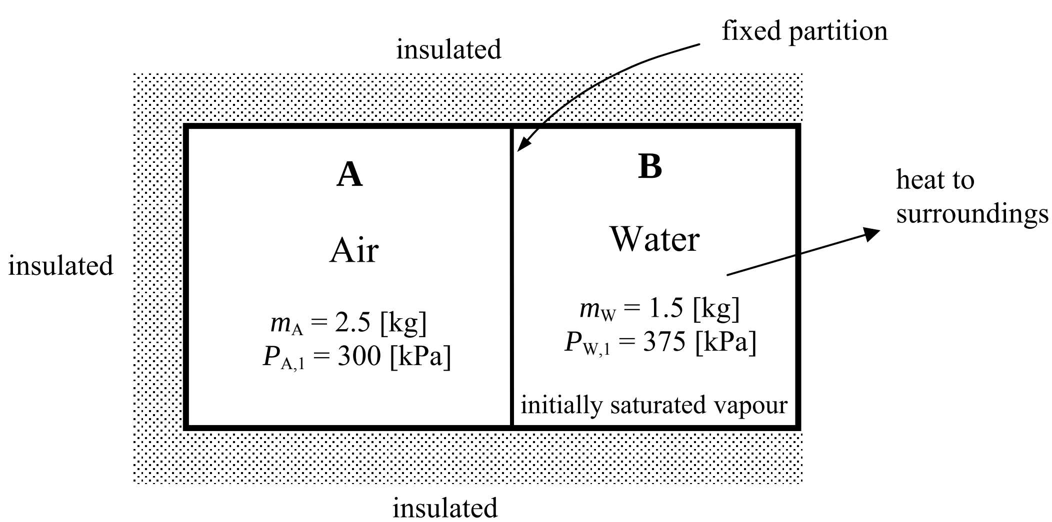


Figure 4: Compartments in the rigid container for problem 4.

value 33 **5.**

A vapour-compression refrigeration cycle is modified to include a well-insulated counterflow heat exchanger as shown in Figure 5 below. Refrigerant 134a leaves the evaporator as saturated vapour at $P_6 = P_5 = 150$ [kPa] and is heated at constant pressure to 10 [°C] before entering the compressor. The refrigerant leaves the compressor at 1200 [kPa] and 90 [°C]. The refrigerant passes through the condenser, exiting at 40 [°C]. After passing through the heat exchanger, the liquid enters the expansion valve (throttling valve) at $P_4 = P_3 = 1200$ [kPa]. The mass flow rate of the R-134a is 6 [kg/min].

Notes:
1. Neglect changes in kinetic and potential energies for all devices in the cycle.
2. The saturation temperature for 150 [kPa] for R-134a is −17.29 [°C] from Table B.5.2.
3. The following values have been calculated (by interpolation) for you to use in the solution of the problem: h_f @ 150 [kPa] = 177.10 [kJ/kg] and v_f @ 150 [kPa] = 0.000742 [m^3/kg].

5 (a) Calculate the power input to the compressor, $\dot{W}_{\text{comp}}$, in kW.

8 (b) Calculate the refrigeration capacity, $\dot{Q}_{\text{L}}$, in kW.

2 (c) Determine the coefficient of performance (COP) of the cycle.

2 (d) Calculate the quality at the inlet to the evaporator.

12 (e) Draw a process representation of this cycle on a *T–v* (temperature – specific volume) diagram. Label all states and show process paths (use a dashed line for an unknown process path). Label state point temperatures and specific volumes and label constant pressure lines that pass through state points. Show your work for any extra calculations needed for the diagram.

4 (f) What is the COP of the cycle without the heat exchanger? Assume States 2, 3, and 6 are unchanged.

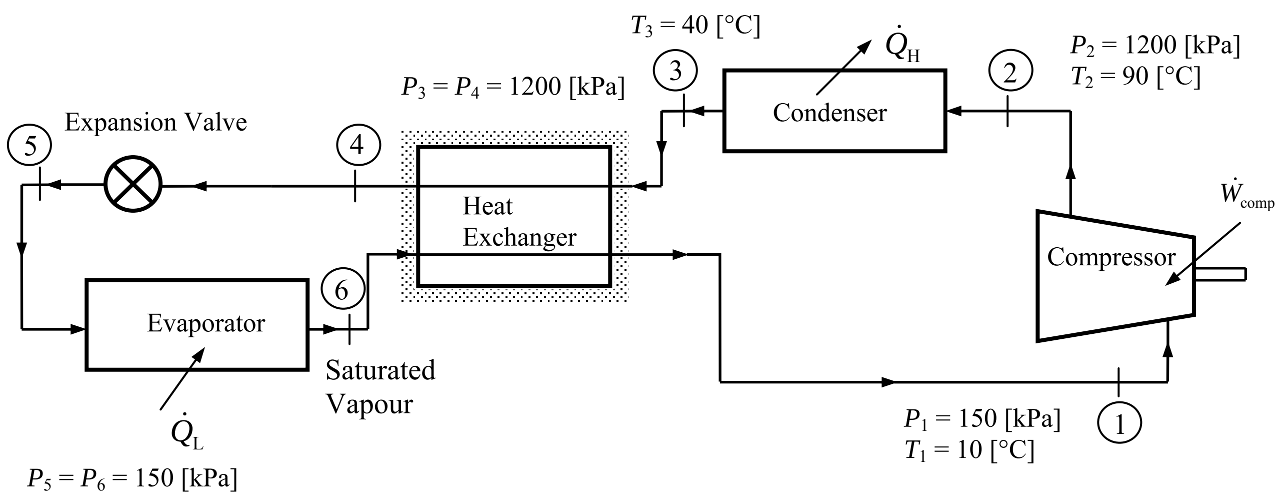
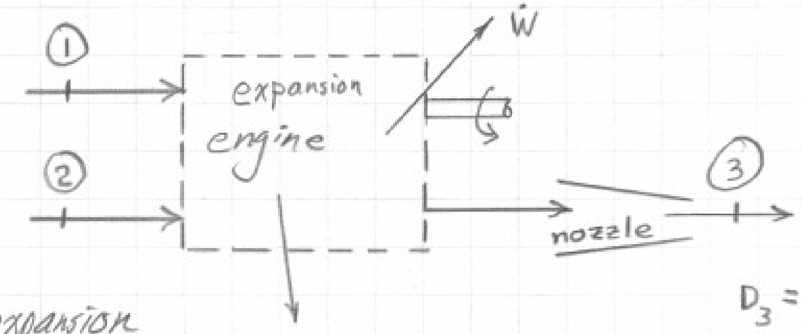


Figure 5: Schematic diagram of refrigeration cycle for problem 5.

#1.



$D_3 = 20\ cm$

expansion engine ✓ $\quad \dot{Q} = 601.25\ kW$

state at 1:
$\dot{m}_1 = 5\ kg/s$
$P_1 = 3\ MPa$
$T_1 = 600°C$

state at 2:
$\dot{m}_2 = 2\ kg/s$
$P_2 = 350\ kPa$
$T_2 = 25°C$

state at 3:
$T_3 = 125°C$
$x_3 = 0.90$

(a) conservation of mass:

$$\dot{m}_1 + \dot{m}_2 = \dot{m}_3 \qquad \therefore \dot{m}_3 = 5 + 2 = 7\ kg/s$$

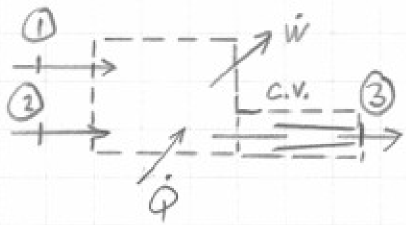
$$\dot{m} = \frac{A \cdot \bar{V}}{v}$$

Table B.1.1 at $T = 125°C$, $v_f = 0.001065\ m^3/kg$, $v_{fg} = 0.76953\ m^3/kg$

$$v_3 = v_f + x_3 \cdot v_{fg}$$
$$= 0.001065 + 0.9 \times 0.76953$$
$$= 0.693642\ m^3/kg$$

$$\bar{V}_3 = \frac{\dot{m}_3 \cdot v_3}{A} = \frac{7 \times 0.693642}{\pi/4\,(0.20)^2} = 154.55\ m/s$$

(b) Let the control volume include the expansion engine and the nozzle:



- assume no changes in potential energies
- neglect kinetic energies at 1 and 2

conservation of energy (First law):

$$\dot{Q} - \dot{W} = \dot{m}_3\left(h_3 + \tfrac{1}{2}\bar{V}_3^2\right) - \left[\dot{m}_1\left(h_1 + \cancel{\tfrac{1}{2}\bar{V}_1^2}\right) + \dot{m}_2\left(h_2 + \cancel{\tfrac{1}{2}\bar{V}_2^2}\right)\right]$$

#1. continued: state 1, at 3 MPa $T_{sat.} = 233.90°C$

$T_1 = 600°C > T_{sat.}$ ∴ superheated vapour

Table B.1.3

$h_1 = 3682.34 \text{ kJ/kg}$

state 2 At 25°C, $P_{sat.} = 3.169 \text{ kPa}$, $P_2 > P_{sat.}$ ∴ compressed liquid

$v_2 \simeq v_{f_{25°C}} = 0.001003 \text{ m}^3/\text{kg}$

$h_{f_{25°C}} = 104.87 \text{ kJ/kg}$

$$h_2 = h_{f_{25°C}} + v_{f_{25°C}}\left(P_2 - P_{sat_{25°C}}\right)$$

$$= 104.87 + 0.001003\,(350 - 3.169)$$

$$= 105.217 \text{ kJ/kg}$$

state 3 state is saturated mixture at 125°C

$h_f = 524.96 \text{ kJ/kg}$, $h_{fg} = 2188.50 \text{ kJ/kg}$

$$h_3 = h_f + x_3 \cdot h_{fg} = 524.96 + 0.90 \times 2188.50 = 2494.61 \text{ kJ/kg}$$

substitute values into first law equation:

$\dot{Q} = 601.25$ kW out from engine

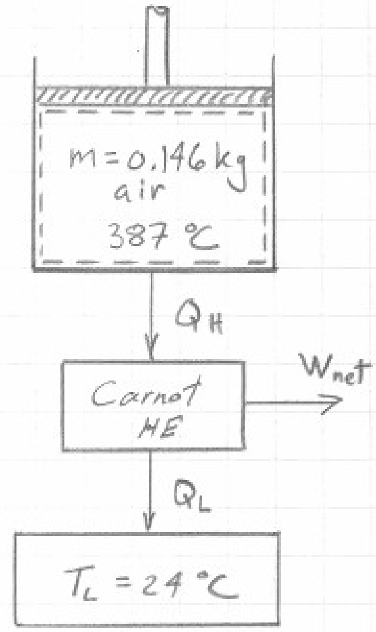
$$\therefore -601.25 - \dot{W} = 7\left(2494.61 + \tfrac{1}{2}(154.55)^2 \times \tfrac{1}{1000}\right) - \left(5 \times 3682.34 + 2 \times 105.217\right)$$

$\dot{W} = 475.01 \text{ kW}$

#2.

air undergoes isothermal process

$P_1 = 100$ kPa to $P_2 = 400$ kPa

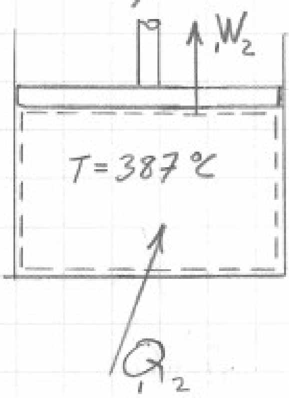


$T_{air} = 387$ °C Carnot HE:

(a) $\eta_{th,carnot} = 1 - \frac{T_L}{T_H} = 1 - \frac{(24+273.15)}{(387+273.15)}$

$\eta_{th,carnot} = 0.5498$

(b) piston-cylinder system:



air is control mass.

$_1Q_2 - {_1W_2} = m(u_2 - u_1) + \cancel{\Delta KE} + \cancel{\Delta PE}$

For an ideal gas undergoing an isothermal process,

$$_1W_2 = mRT \ln \frac{V_2}{V_1}$$

$R_{air} = 0.287$ kJ/kg·K , $C_{v_0} = 0.717$ kJ/kg·K

$$V_1 = \frac{mRT}{P_1} = \frac{0.146 \times 0.287 \times (387+273.15)}{100} = \frac{27.661}{100} = 0.27661 \text{ m}^3$$

$$V_2 = \frac{mRT}{P_2} = \frac{27.661}{400} = 0.069154 \text{ m}^3$$

$$\therefore {_1W_2} = 0.146 \times 0.287 \times (387+273.15) \ln\left(\frac{0.069154}{0.27661}\right) = -38.346 \text{ kJ}$$

$$\text{or, } {_1W_2} = mRT \ln\left(\frac{mRT/P_2}{mRT/P_1}\right) = mRT \ln\left(\frac{P_1}{P_2}\right)$$

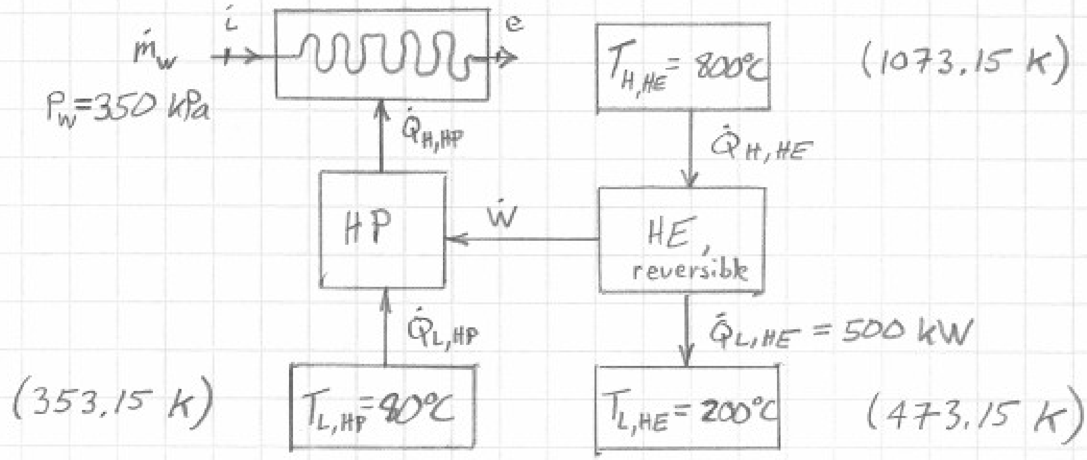
$Q_H = -{_1Q_2} = 38.346$ kJ

$_1Q_2 = {_1W_2} + m C_{v_0} \cancel{(T_2 - T_1)}$ $\therefore {_1Q_2} = {_1W_2} = -38.346$ kJ

$T_2 = T_1$

$W_{net} = \eta_{th} \cdot Q_H = 0.5498 \times 38.346 = 21.08$ kJ

#3.



HE, reversible

$$\therefore \eta_{th,rev} = 1 - \frac{T_{L,HE}}{T_{H,HE}} = 1 - \frac{200+273.15}{800+273.15} = 0.5591$$

$\eta_{th} = \frac{\dot{W}}{\dot{Q}_{H,HE}}$ Apply the first law to the HE, $\dot{Q}_{H,HE} = \dot{W} + \dot{Q}_{L,HE}$

$$\therefore \eta_{th} = \frac{\dot{Q}_{H,HE} - \dot{Q}_{L,HE}}{\dot{Q}_{H,HE}}$$

$$0.5591 = 1 - \frac{500}{\dot{Q}_{H,HE}} \qquad \therefore \dot{Q}_{H,HE} = 1134.04 \text{ kW}$$

$$\dot{W} = \dot{Q}_{H,HE} - \dot{Q}_{L,HE} = 1134.04 - 500 = 634.04 \text{ kW}$$

HP The steam generator acts as the high-T reservoir for the HP.

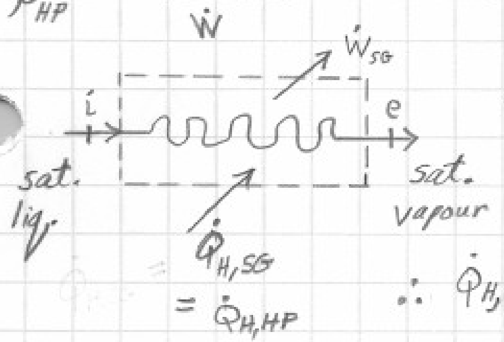
$T_{sat.} = 138.88°C$ for 350 kPa water

$\therefore T_{H,HP} = 138.88°C$ (412.03 K)

$$\beta'_{HP} = 0.65\beta'_{HP,rev} = 0.65 \times \frac{1}{1 - \frac{T_{L,HP}}{T_{H,HP}}} = 0.65 \times \frac{1}{1 - \frac{80+273.15}{138.88+273.15}}$$

$$\beta'_{HP} = 0.65 \times 6.997 = 4.548$$

$$\beta'_{HP} = \frac{\dot{Q}_{H,HP}}{\dot{W}} \qquad \therefore \dot{Q}_{H,HP} = 634.04 \times 4.548 = 2883.6 \text{ kW}$$



First law applied to steam generator: $\dot{W}_{SG} = 0$, $\Delta KE, \Delta PE$ neglect

at 350 kPa, $h_f = 584.31$ kJ/kg

$h_g = 2732.40$ kJ/kg

$$\therefore \dot{Q}_{H,SG} - \cancel{\dot{W}_{SG}}^{0} = \dot{m}_w(h_e - h_i), \quad \dot{m}_w = 2883.6/(2732.40 - 584.31)$$

$$\therefore \dot{m}_w = 1.342 \text{ kg/s}$$

4. (a) ▸ Compartment B $\quad P_{W,1} = 375\ (kPa)$ (Table B.1.2)

Saturated vapour

$$v_{W,1} = v_g\Big|_{375\,(kPa)} = 0.49137\ \left(\frac{m^3}{kg}\right)$$

$$T_{W,1} = T_{sat}\Big|_{375\,(kPa)} = 141.32\ (°C)$$

$$\forall_W = m_W \cdot v_{W,1}$$

$$\forall_W = 1.5\,(0.49137) = 0.73706\ (m^3)$$

▸ Compartment A $\quad P_{A,1} = 300\ (kPa)$

$$T_{A,1} = T_{W,1} = 141.32\ °C = 414.47\ (K)$$

$$R = 0.287\ (kJ/kg\ K)\ \text{(Tab6 A.5)}$$

$$\forall_A = \frac{m_A R T_{A,1}}{P_{A,1}}$$

$$\forall_A = \frac{2.5(0.287)\,414.47}{300} = 0.99127\ (m^3)$$

$$\forall_{tot} = \forall_W + \forall_A = 0.73706 + 0.99127 = 1.7283\ (m^3) \leftarrow$$

(b) $P_{W,2} = 125\ [kPa] \qquad v_{W,2} = v_{W,1} = 0.49137\ (m^3/kg)$

For $P = 125\ [kPa]$ $\quad v_f\Big|_{125\,(kPa)} = 0.001048\ (m^3/kg)$

$$v_g\Big|_{125\,(kPa)} = 1.37490\ (m^3/kg)$$

$$v_f\Big|_{P_{W,2}} < v_{W,2} < v_g\Big|_{P_{W,2}} \Rightarrow \text{Saturated mixture}$$

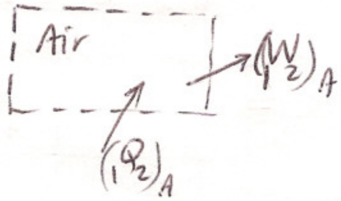
$$\therefore T_{W,2} = T_{sat}\Big|_{125\,(kPa)} = 105.99\ [°C]$$

$$T_{A,2} = T_{W,2} = 105.99\ (°C) = 379.14\ (K)$$

$$P_{A,2} = \frac{m_A R T_{A,2}}{\forall_A} = \frac{(2.5)\,0.287\,(379.14)}{0.99127} = 274.43\ [kPa] \leftarrow$$

4(c) <u>Air First Law</u>

$$({}_1Q_2)_A - ({}_1W_2)_A = m_A C_{v_0} (T_{A,2} - T_{A,1})$$



Neglect ΔKE, ΔPE

$({}_1W_2)_A = 0$ (No volume change)

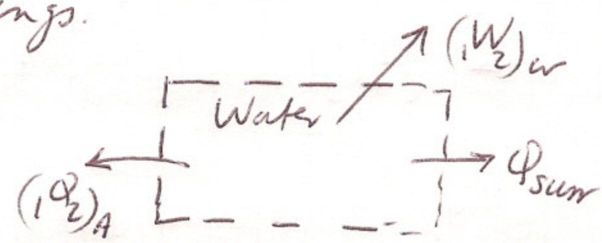
$C_{v_0} = 0.717$ (kJ/kg K) (Table A5)

$$({}_1Q_2)_A = 2.5(0.717)(379.14 - 414.47) = -63.33 \text{ (kJ)}$$

This is heat transfer from the air to the water.
The direction is known because of the negative sign on $({}_1Q_2)_A$.

(d) <u>Water First Law</u> (neglect ΔKE, ΔPE and consider Q transfers at the partition and to the surroundings.

$$-({}_1Q_2)_A - Q_{surr} - \cancel{({}_1W_2)_W}^{\,0} = m_W (u_2 - u_1)$$



$u_1 = u_g|_{375\text{ kPa}} = 2551.31$ (kJ/kg)

$u_2 = (1-x_2)\, u_f|_{125\text{ (kPa)}} + x_2\, u_g|_{125\text{ (kPa)}}$

$$x_2 = \frac{(v_{w,2} - v_f)}{(v_g - v_f)}$$

$$x_2 = \frac{(0.49137 - 0.001048)}{(1.37490 - 0.001048)}$$

$$x_2 = 0.3569$$

$u_2 = (1-0.3569)\,444.16 + (0.3569)\,2513.48$

$u_2 = 1182.7$ (kJ/kg)

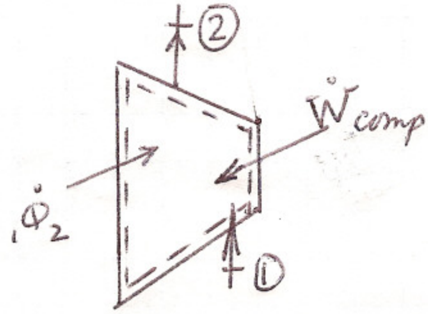
$$Q_{surr} = -({}_1Q_2)_A - m(u_2 - u_1)$$

$$= -(-63.33) - 1.5(1182.7 - 2551.31)$$

$$Q_{surr} = 2116.25 \text{ (kJ)} \leftarrow$$

5. (a) Assume ${}_1\dot{Q}_2 = 0$

(Neglect Δke, Δpe for all devices)



First Law for the Compressor (Using the notation from the test paper).

$$\cancel{{}_1\dot{Q}_2}^{\,0} + \dot{W}_{comp} = \dot{m}(h_2 - h_1)$$

$$\dot{W}_{comp} = \dot{m}(h_2 - h_1)$$

$$\dot{m} = 6\ \frac{kg}{min} \times \frac{1\ min}{60\ s} = 0.1\ \left(\frac{kg}{s}\right)$$

$$\dot{W}_{comp} = 0.1\,(470.55 - 410.60)$$

$$\dot{W}_{comp} = 5.995\ [kW]$$

State 1

$P_1 = 150\ [kPa]$

$T_1 = 10\ [°C]$

$T_{sat}(150\ [kPa]) = -17.29\ [°C]$ (Table B.5.2)

$T_1 > T_{sat}(P_1)$ ⇒ superheated vapour

$h_1 = 410.60\ \left[\frac{kJ}{kg}\right]$ (Table B.5.2)

State 2

$P_2 = 1200\ (kPa]$

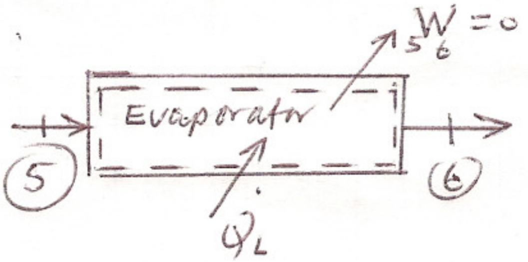
$T_2 = 90\ [°C]$

$T_{sat}(1200\ [kPa]) = 46.31\ [°C]$

$T_2 > T_{sat}(P_2)$ ⇒ superheated vapour

$h_2 = 470.55\ \left[\frac{kJ}{kg}\right]$

(b)



First Law for the Evaporator

$$\dot{Q}_L = \dot{m}\,(h_6 - h_5)$$

Must analyse the heat exchanger to obtain h_4.

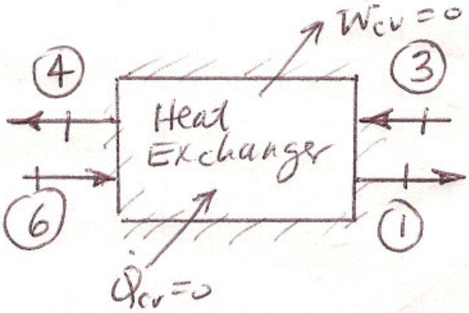
State 6:
saturated vapour

$h_6 = h_g\big|_{150(kPa)} = 387.77\ \left[\frac{kJ}{kg}\right]$ (Table B.5.2)

State 5

$h_5 = h_4$

5 (b) continued



all streams have the same mass flow rate

First law for the Heat Exchanger

$$\dot{m}_1 = \dot{m}_3 = \dot{m}_4 = \dot{m}_6 = \dot{m}$$

$$\sum \dot{m}_i h_i = \sum \dot{m}_e h_e$$

$$\dot{m}_6 h_6 + \dot{m}_3 h_3 = \dot{m}_1 h_1 + \dot{m}_4 h_4$$

$$\Rightarrow \quad h_4 = h_3 + h_6 - h_1$$

State 3

$P_3 = 1200\,(kPa)$

$T_3 = 40\,[°C]$

$T_3 < T_{sat}(P_3)$

$\Rightarrow$ compressed liquid

$$h_3 = h_f|_{T_3} + v_f|_{T_3}(P_3 - P_{sat}|_{T_3})$$

$$h_3 = 256.54 + 0.000873(1200 - 1017.0)$$

$$h_3 = 256.70\,(kJ/kg)$$

$$h_4 = 256.70 + 387.77 - 410.60$$

$$h_4 = 233.87\ (kJ/kg)$$

$$\therefore h_5 = 233.87\,(kJ/kg)$$

$$\dot{Q}_L = 0.1(387.77 - 233.87)$$

$$\dot{Q}_L = 15.39\ [kW]$$

Alternative: $\dot{Q}_H = \dot{m}(h_2 - h_3) = 0.1(470.55 - 256.70)$

$\dot{Q}_H = 21.385\ (kw)$

then $\dot{Q}_L = \dot{Q}_H - \dot{W}_{comp}$
$= 21.385 - 5.995$
$= 15.395\ (kw)$.

(c) $$COP = \beta = \frac{\dot{Q}_L}{\dot{W}_{comp}} = \frac{15.39}{5.995}$$

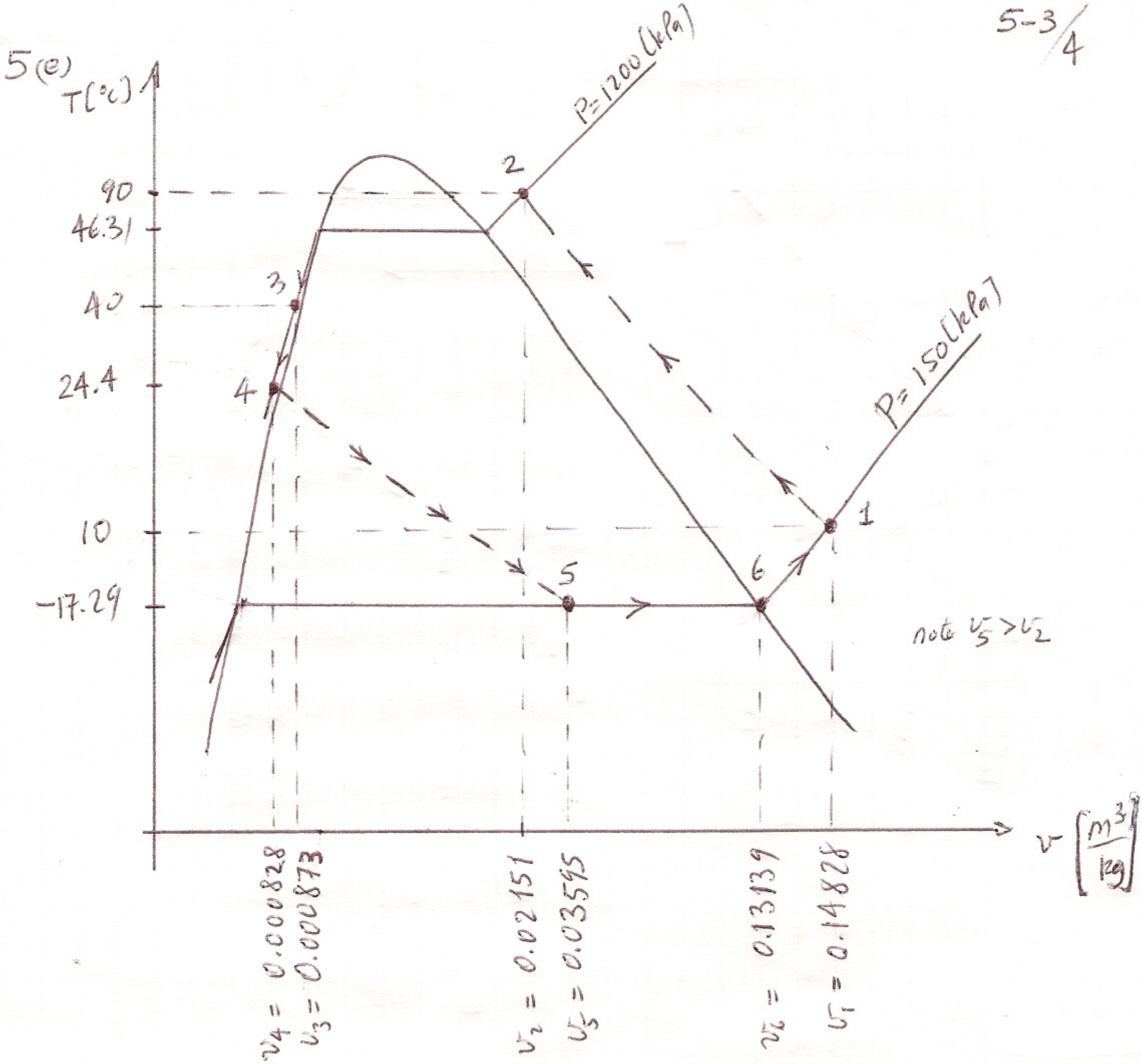
$$COP = 2.567$$

(d) $$x_5 = \frac{(h_5 - h_f|_{150kPa})}{(h_g|_{150kPa} - h_f|_{150kPa})} = \frac{(233.87 - 177.10)}{(387.77 - 177.10)}$$

given on test paper

$$x_5 = 0.2695$$

5(e)



v_1, v_2, v_6 from Table B.5.2 ; v_3 from Table B.5.1

given on test paper

$v_5 = (1-x_5)\, v_f|_{150\,kPa} + x_5\, v_g|_{150\,kPa} = (1-0.2695)\, 0.000742 + (0.2695)\, 0.13139$

$v_5 = 0.03595 \; [m^3/kg]$

State 4 $\quad h_4 = 233.87$ (kJ/kg). As a first approximation $h_4 = h_f|_{T_4}$

Interpolate in Table B.5.1 $\quad \frac{T_4 - 20}{25-20} = \frac{(233.87 - 227.49)}{(234.59 - 227.49)}$

$\Rightarrow T_4 = 24.4\ [°C]$

$\Rightarrow v_4 = 0.000828\ (m^3/kg)$ by interpolation

5(e) Aside If one wanted to use the more complete compressed liquid calculation of h to estimate T_4, one could do the following. 5-4/4

Noting that $h_4 = 233.87 \left(\frac{kJ}{kg}\right)$ is near 25°C

use $v_f \approx v_f|_{25°C} = 0.000829$ (m³/kg)

$P_{sat} \approx P_{sat}|_{25°C} = 666.3$ (kPa)

then write $h_4 \approx h_f|_{T4} + v_f(P_4 - P_{sat})$

$$h_f|_{T4} \approx h_4 - v_f(P_4 - P_{sat}) = 233.87 - 0.000829(1200 - 666.3)$$

$$h_f|_{T4} \approx 233.43 \text{ (kJ/kg)}$$

Then use this to interpolate for T_4. This yields $T_4 \approx 24.2$(°C). Further refinement could be obtained by interpolating for v_f & P_{sat} at this new T_4.

not required

5(f) If the heat exchanger did not exist in this cycle and states 2, 3, and 6 were not changed, then

$$\dot{W}_{comp} = \dot{m}(h_2 - h_6) = 0.1(470.55 - 387.77)$$

$$\dot{W}_{comp} = 8.278 \text{ (kW)}$$

$$\dot{Q}_L = \dot{m}(h_6 - h_3) = 0.1(387.77 - 256.70)$$

$$\dot{Q}_L = 13.107 \text{ (kW)}$$

$$COP\Big|_{\substack{\text{no}\\\text{heat}\\\text{exchanger}}} = \frac{13.107}{8.278} = 1.583$$

which is lower than the actual COP calculated in part (c).