MATH 1210 Assignment 4

Due: 1:30 pm Friday 13 March 2009 (at your instructor's office)

NOTES:

1. **Late assignments will NOT be accepted**.
2. **If your assignment is not accompanied by a signed Faculty of Science "Honesty Declaration", it will NOT be graded**.

Provide a complete solution to each of the following problems:

1. Let $P_1(1,0,-1)$, $P_2(2,1,3)$ and $P_3(-1,2,0)$ be three fixed points in 3-space.
   (a) Find parametric equations for the line l_1 through P_1 in the direction of the vector $\overrightarrow{v} = \overrightarrow{P_2P_1}$.
   (b) Find parametric equations for the line l_2 through P_2 in the direction of the vector $\overrightarrow{v} = \overrightarrow{P_1P_3}$.
   (BE CAREFUL: Do NOT use the same parameter for these two lines, as this would cause some confusion.)
   (c) Find the point of intersection of lines l_1 and l_2.
   (d) Find the equation of a plane Π_1 containing both l_1 and l_2.
   (e) Find parametric equations of the line which is perpendicular to Π_1 and passes through point P_2.
   (COMMENT: To avoid confusion, you should use a different parameter for this line than for the earlier two lines.)
   (f) Find the equation of the plane Π_2 which is parallel to Π_1 and passes through the point $(0,-5,0)$.

2. You are given the following matrices:

$$A = \begin{pmatrix} 1 & -2 & 3 \\ -3 & 1 & 4 \end{pmatrix}, \qquad E = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 0 & -1 \\ 1 & 1 & -2 \end{pmatrix},$$

$$B = \begin{pmatrix} 1 & 0 \\ 2 & 1 \\ -1 & 2 \end{pmatrix}, \qquad F = \begin{pmatrix} -1 & 2 \\ 0 & 2 \\ 2 & -1 \end{pmatrix},$$

$$C = \begin{pmatrix} 3 & -1 & 2 \\ 1 & -2 & -1 \\ 2 & 0 & 1 \end{pmatrix}, \qquad I_{(2)} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix},$$

$$D = \begin{pmatrix} -1 & 2 \\ 3 & 2 \end{pmatrix}, \qquad I_{(3)} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

Compute each of the following matrices, if it exists:

(i) AB

(ii) BA

(iii) $\left(BI_{(2)}\right)A$

(iv) $B\left(I_{(3)}A\right)$

(v) F^TE

(vi) $CB+D$

(vii) $AB+2D$

(viii) $AB+D^2$ (with $D^2=DD$)

(ix)$DA+B$

(x) EC

(xi) CE

(xii) $EB+F$

(xiii) $FC+D$

(xiv) $FD-3B$

(xv) F^TB+D

(xvi) $2F-3(AE)$

(xvii) $(2\,(AB))^T$

(xviii) $A(BD)$

(xix) $(AB)D$

(xx) A^T

(xxi) B^T

(xxii) A^TB^T

(xxiii) $(AB)^T$

(xxiv)B^TA^T

3. Reduce each of the following matrices to **row-echelon form** (R.E.F.):

(a) $\begin{pmatrix} 1 & 2 & -3 & 1 \\ -1 & 0 & 3 & 4 \\ 0 & 1 & 2 & -1 \\ 2 & 3 & 0 & -3 \end{pmatrix}$

(b) $\begin{pmatrix} 1 & 1 & 3 & -3 & 0 \\ 0 & 2 & 1 & -3 & 3 \\ 1 & 0 & 2 & -1 & -1 \end{pmatrix}$

(c) $\begin{pmatrix} 0 & 2 & 3 & -4 & 1 \\ 0 & 0 & 2 & 3 & 4 \\ 2 & 2 & -5 & 2 & 4 \\ 2 & 0 & 6 & 9 & 7 \end{pmatrix}$

NOTE: In part (c) it will be necessary to perform fractional arithmetic.

4. Reduce each of the following matrices to **reduced row-echelon form** (R.R.E.F.):

(a) $\begin{pmatrix} 1 & 3 & 2 & 1 \\ -1 & 2 & 3 & 4 \\ 3 & 0 & 1 & 2 \end{pmatrix}$

(b) $\begin{pmatrix} 1 & 1 & 2 & -1 \\ 1 & 2 & 1 & 0 \\ -1 & -4 & 1 & -2 \\ 1 & -2 & 5 & -4 \end{pmatrix}$

(c) $\begin{pmatrix} 1 & 1 & 3 & -3 & 0 \\ 0 & 2 & 1 & -3 & 3 \\ 1 & 0 & 2 & -1 & -1 \end{pmatrix}$

5. Use **Gaussian elimination** to solve each of the following systems of equations:

   (a) $x + 2y + 3z = 9$
   $2x - y + z = 8$
   $3x - z = 3$

   (b) $a + b + 3c - 3d = 0$
   $2b + c - 3d = 3$
   $a + 2c - d = -1$

   (c) $(1 - i)x + (2 + i)y = 2 + i$
   $2x + (1 - 2i)y = 1 + 3i$
   Comment: in part (c) $i = \sqrt{-1}$.

6. Use **Gaussian-Jordan elimination** to solve each of the following systems of equations:

   (a) $p + q + 3r - 3s = 0$
   $2q + r - 3s = 3$
   $p + 2r - s = -1$

   (b) $2x_5 + 8x_6 = 4$
   $x_4 + 3x_5 + 11x_6 = 9$
   $3x_2 - 12x_3 - 3x_4 - 9x_5 - 24x_6 = -33$
   $-2x_2 + 8x_3 + x_4 + 6x_5 + 17x_6 = 21$
   $x_1 = 2$