P1/1

Test 2

Page: 1

DATE: November 8, 2007
COURSE: MATH 2132

TIME: 60 minutes
EXAMINER: G.I. Moghaddam

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[12] 1. Use bionomial expansion to find the Maclaurin series of the function $f(x) = \frac{1}{\sqrt{2-x}}$. What is the open interval of convergence? Express your answer in sigma notation and simplify as much as possible.

[8] 2. Choose and answer *only* one the following two parts:

(a) Find the sum of the series $\sum_{n=1}^{\infty} \frac{2^{2n-2}}{n} x^{2n}$.

(b) Evaluate the following limit using infinite series.

$$\lim_{x \to 0} \frac{\sqrt[5]{(1-x^2)^3} - 1}{x^2}$$

[12] 3. Find, in explicit form, the solution of the differential equation

$$x^2 \frac{dy}{dx} + 3xy = 2 \ln x, \quad y(1) = \tfrac{1}{2}.$$

[10] 4. Find a 2 -parameter family of solutions of differential equation

$$y'' - 3(y')^2 = 3.$$

[8] 5. Find a general solution for a homogeneous linear differential equation $\Phi(D)y = 0$ whose auxiliary equation is:

$$(m+1)^2(m-\sqrt{2})^4(m^2+m+1)^3 = 0$$

Dawit's Answers (plankton@yahoo.com)

1. $\sum_{n=0}^{\infty} \frac{(2n)!}{2^{3n+\frac{1}{2}}(n!)^2} x^n$, $-2 < x < 2$.

2. a) $-\frac{1}{4}\ln(1-4x^2)$, $-\frac{1}{2} < x < \frac{1}{2}$

b) $-3/5$

3. $y(x) = \frac{1}{2x^3}\left[x^2(\ln x^2 - 1) + 2\right]$

4. $y(x) = \frac{1}{3}\ln\left|\sec(3x+C)\right| + D$

5. $y(x) = (c_1 + c_2 x)e^{-x} + (c_3 + c_4 x + c_5 x^2 + c_6 x^3)e^{\sqrt{2}x} + e^{-\frac{x}{2}}\left[(c_7 + c_8 x + c_9 x^2)\cos\frac{\sqrt{5}}{2}x + (c_{10} + c_{11}x + c_{12}x^2)\sin\frac{\sqrt{5}}{2}x\right]$