Math 1210 Assignment 2 Solutions Oct 10, 2008

1\. a) $\dfrac{(4-3i)^3}{2+i} + \overline{7+2i} = \dfrac{64-144i-108+27i}{2+i} + 7-2i$

$= \dfrac{-44-117i}{2+i} \cdot \dfrac{2-i}{2-i} + 7-2i = \dfrac{-88+44i-234i+117i^2}{4-i^2} + 7-2i$

$= \dfrac{-205-190i}{5} + 7-2i = -41-38i+7-2i = -34+40i$

b) $(2-3i)(4+7i) - \left(\dfrac{2-i}{3+i}\right)^2 = 8+14i-12i-21i^2 - \left(\dfrac{(2-i)(3-i)}{(3+i)(3-i)}\right)^2$

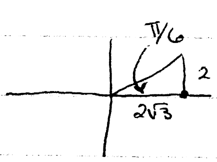
$= 29+2i - \left(\dfrac{6-2i-3i+i^2}{9-i^2}\right)^2 = 29+2i - \left(\dfrac{5-5i}{10}\right)^2$

$= 29+2i - \left(\frac{1}{2}-\frac{1}{2}i\right)^2 = 29+2i - \left(\frac{1}{4}-\frac{1}{4}i-\frac{1}{4}i+\frac{1}{4}i^2\right) = 29+2i-\left(-\frac{1}{2}i\right)$

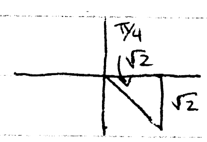
$= 29+\frac{5}{2}i$

2 a) $|2\sqrt{3}+2i| = \sqrt{(2\sqrt{3})^2+(2)^2} = \sqrt{12+4} = \sqrt{16} = 4$

$|\sqrt{2}-\sqrt{2}i| = \sqrt{(\sqrt{2})^2+(\sqrt{2})^2} = \sqrt{2+2} = \sqrt{4} = 2.$



$\arg(2\sqrt{3}+2i) = \pi/6$



$\arg(\sqrt{2}-\sqrt{2}i) = -\pi/4$

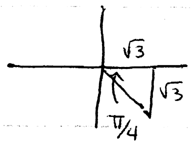
$(2\sqrt{3}+2i)(\sqrt{2}-\sqrt{2}i) = \left(4(\cos\pi/6 + i\sin\pi/6)\right)\left(2(\cos -\pi/4 + i\sin -\pi/4)\right)$

$= 8\left(\cos(\pi/6-\pi/4) + i\sin(\pi/6-\pi/4)\right)$

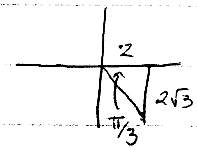
$= 8\left(\cos(-\pi/12) + i\sin(-\pi/12)\right)$

2 b) $|\sqrt{3}-\sqrt{3}i| = \sqrt{(\sqrt{3})^2+(\sqrt{3})^2} = \sqrt{3+3} = \sqrt{6}$

$|2-2\sqrt{3}i| = \sqrt{(2)^2+(-2\sqrt{3})^2} = \sqrt{4+12} = \sqrt{16} = 4$



$\arg(\sqrt{3}-\sqrt{3}i) = -\pi/4$



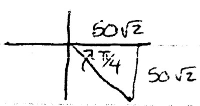
$\arg(2-2\sqrt{3}i) = -\pi/3$

$$\frac{\sqrt{3}-\sqrt{3}i}{2-2\sqrt{3}i} = \frac{\sqrt{6}\,(\cos -\pi/4 + i\sin -\pi/4)}{4\,(\cos -\pi/3 + i\sin -\pi/3)} = \sqrt{6}/4\,(\cos(-\pi/4-(-\pi/3)) + i\sin(-\pi/4-(-\pi/3)))$$

$$= \sqrt{6}/4\,(\cos \pi/12 + i\sin \pi/12)$$

3a $z^4 = 50\sqrt{2} - 50\sqrt{2}i$

$|50\sqrt{2}-50\sqrt{2}i| = \sqrt{(50\sqrt{2})^2+(-50\sqrt{2})^2} = \sqrt{50^2\cdot 2 + 50^2\cdot 2} = \sqrt{50^2\cdot 4} = 100$



$\arg(50\sqrt{2}-50\sqrt{2}i) = -\pi/4$

Suppose $z = re^{\theta i}$ then $z^4 = 50\sqrt{2}-50\sqrt{2}i$ becomes

$$r^4 e^{4\theta i} = 100\, e^{-\frac{\pi}{4}i}$$

Setting module equal

$r^4 = 100$

$r = \sqrt[4]{100} = \sqrt[2]{10}$

(reject neg answers)

Setting angles equal

$4\theta = -\pi/4 + 2n\pi$

$\theta = -\pi/16 + 2n\pi/4 = \dfrac{-\pi + 8n\pi}{16}$

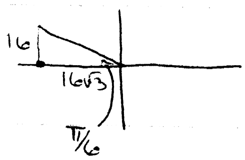
Finding four solutions ($n = 0, 1, 2, 3$)

$\theta_0 = -\pi/16 \quad \theta_1 = 7\pi/16$

$\theta_2 = 15\pi/16 \quad \theta_3 = 23\pi/16 \left(\equiv -\frac{9\pi}{16}\right.$ in principal value)

The solutions are: $\sqrt[2]{10}\, e^{-\frac{\pi}{16}i}$, $\sqrt[2]{10}\, e^{\frac{7\pi}{16}i}$, $\sqrt[2]{10}\, e^{\frac{15\pi}{16}i}$, $\sqrt[2]{10}\, e^{\frac{23\pi}{16}i}$

3 b) $|-16\sqrt{3}+16i| = \sqrt{(-16\sqrt{3})^2+(16)^2} = \sqrt{16^2\cdot 3+16^2} = \sqrt{16^2\cdot 4} = 32$



$\arg(-16\sqrt{3}+16i) = 5\pi/6$

Suppose $z = re^{\theta i}$ the $z^5 = -16\sqrt{3}+16i$ becomes

$$r^5 e^{5\theta i} = 32 e^{5\pi/6\, i}$$

$r^5 = 32$ $\qquad 5\theta = 5\pi/6 + 2n\pi$

$r = 2$ $\qquad \theta = \pi/6 + 2n\pi/5 = \frac{5\pi + 12n\pi}{30}$

$\theta_0 = \pi/6 \quad \theta_1 = 17\pi/30 \quad \theta_2 = 29\pi/30$

$\theta_3 = \frac{41\pi}{30} \left(\equiv -\frac{19\pi}{30}\right) \quad \theta_4 = 53\pi/30 \left(\equiv -\frac{7\pi}{30}\right)$

The solutions are: $2e^{\pi/6 i}, 2e^{17\pi/30 i}, 2e^{29\pi/30}, 2e^{41\pi/30}, 2e^{53\pi/30}$.

4 (omitted) $P(x) = 2x^4 - 6x^3 + 7x^2 - 2x - 2$

If $1-i$ is a root of $P(x)$, then so is $1+i$. Hence $(x-(1-i))$ and $(x-(1+i))$ are both factors, so (x^2-2x+2) is a factor of $P(x)$.

$$\begin{array}{r|l} & 2x^2 - 2x - 1 \\ \hline x^2-2x+2 & 2x^4 - 6x^3 + 7x^2 - 2x - 2 \\ & \underline{2x^4 - 4x^3 + 4x^2} \\ & -2x^3 + 3x^2 - 2x \\ & \underline{-2x^3 + 4x^2 - 4x} \\ & -x^2 + 2x - 2 \\ & \underline{-x^2 + 2x - 2} \\ & 0 \end{array}$$

The roots of $2x^2-2x-1$ are:

$$x = \frac{2 \pm \sqrt{4-4(2)(-1)}}{4} = \frac{2 \pm \sqrt{12}}{4} = \frac{1 \pm \sqrt{3}}{2}$$

So $P(x) = 2(x-(1-i))(x-(1+i))\left(x - \left(\frac{1+\sqrt{3}}{2}\right)\right)\left(x - \left(\frac{1-\sqrt{3}}{2}\right)\right)$

5 a) $P(x) = 6x^4 - 13x^3 - 3x^2 + 12x - 4$ There are 3 sign switches

$P(x)$ has 3 or 1 positive real root.

$P(-x) = 6x^4 + 13x^3 - 3x^2 - 12x - 4$ There is 1 sign switch

$P(x)$ has 1 negative real root.

b) If $P(\alpha) = 0$ then $|\alpha| < \frac{13}{6} + 1$; $|\alpha| < \frac{19}{6}$

c) Divisors of p $\{\pm 1, \pm 2, \pm 4\}$ Divisors of q $\{\pm 1, \pm 2, \pm 3, \pm 6\}$

fractions p/q $\{\pm 1, \pm 2, \underset{(6)}{\pm 4}, \pm \frac{1}{2}, \underset{1}{\cancel{\pm \frac{2}{2}}}, \underset{2}{\cancel{\pm \frac{4}{2}}}, \pm \frac{1}{3}, \pm \frac{2}{3}, \pm \frac{4}{3}, \pm \frac{1}{6}, \underset{1/3}{\cancel{\pm \frac{2}{6}}}, \underset{2/3}{\cancel{\pm \frac{4}{6}}}\}$

Possible rational Roots $\{\pm 1, \pm 2, \pm \frac{1}{2}, \pm \frac{1}{3}, \pm \frac{2}{3}, \pm \frac{4}{3}, \pm \frac{1}{6}\}$

d) (Testing Possible rational roots):

$P(1) = 2$ $P(-1) = 0$ so $(x+1)$ is a factor

$$
\begin{array}{r l}
 & 6x^3 - 19x^2 + 16x - 4 \\
x+1 \,\big) & 6x^4 - 13x^3 - 3x^2 + 12x - 4 \\
 & \underline{6x^4 + 6x^3} \\
 & -19x^3 - 3x^2 \\
 & \underline{-19x^3 - 19x^2} \\
 & 16x^2 + 12x \\
 & \underline{16x^2 + 16x} \\
 & -4x - 4
\end{array}
$$

So $P(x) = (x+1)(6x^3 - 19x^2 + 16x - 4)$

Set $Q(x) = 6x^3 - 19x^2 + 16x - 4$ and continue testing (note: $P(x)$ has only the one neg. root)

$Q(2) = 0$

$$
\begin{array}{r l}
 & 6x^2 - 7x + 2 \\
x-2 \,\big) & 6x^3 - 19x^2 + 16x - 4 \\
 & \underline{6x^3 - 12x^2} \\
 & -7x^2 + 16x \\
 & \underline{-7x^2 + 14x} \\
 & 2x - 4
\end{array}
$$

So $P(x) = (x+1)(x-2)(6x^2 - 7x + 2)$

The roots of $6x^2 - 7x + 2$ are

$$x = \frac{7 \pm \sqrt{49 - 4(2)(6)}}{12} = \frac{7 \pm 1}{12} = \frac{1}{2}, \frac{2}{3}$$

The roots of $P(x)$ are -1, 2, $\frac{1}{2}$ and $\frac{2}{3}$.