University of Manitoba
Department of Mechanical and Manufacturing Engineering

**ENG 1460 Introduction to Thermal Sciences** (F07)

A01 & A02 (Profs. Ormiston and Bartley)

Term Test # 2 7 November 2007 Duration: 120 minutes

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1. You are permitted to use the textbook for the course and a calculator.
2. Ask for clarification if any problem statement is unclear.
3. Clear, systematic solutions are required. **Show your work.** Marks will not be assigned for problems that require unreasonable effort for the marker to decipher.
4. Use linear interpolation in the property tables as necessary.
5. Keep 5 significant figures in intermediate calculations, and use 4 or 5 significant figures in final answers.
6. There are **two** problems on this test. The weight of each problem is indicated. The test will be marked out of **100**.

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Values

56.5 1. A piston-cylinder arrangement containing R-12 (Refrigerant 12) is shown in Figure 1. In this configuration, the piston is in contact with a linear spring which has a spring constant of 0.2374 [kN/m]. Initially (State 1), the control mass (R-12) is at a temperature of −30 [°C], has a quality of 25%, and occupies a volume of 250 [L]. Heat is added to the control mass, and during this process the piston (whose cross-sectional area is 0.0200 [m$^2$]) rises against the spring until it reaches the stops (thus reaching State 2). The volume of the R-12 when the piston hits the stops is 450 [L]. Heating continues until the temperature reaches 30 [°C] (State 3).

18 (a) Determine the pressure, temperature, and specific volume at State 2. (**Hint:** The value of $P_2$ should correspond to a property table entry.)

4 (b) Determine the work done by the system during the process from State 1 to State 2: ${}_1W_2$.

16 (c) Determine the heat transferred to the system during the process from State 1 to State 2: ${}_1Q_2$.

5.5 (d) Determine the final pressure, $P_3$.

2 (e) Determine the total work done for both processes.

11 (f) Show the state points and process paths on a $P$-$v$ (pressure–specific volume) diagram. On the diagram, clearly label the constant temperature lines that pass through the state points and the values of $P$ and $v$ for the state points on the diagram. Indicate the area(s) representing the specific work.

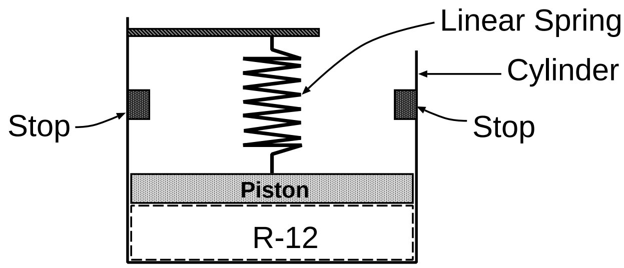



Figure 1: Schematic for Problem 1

2. Consider a control mass consisting of helium gas (He) at an initial pressure, volume, and temperature of $P_1 = 125$ [kPa], $V_1 = 2$ [m$^3$], and $T_1 = 300$ [K] (State 1). A polytropic process occurs whereby heat is transferred to the control mass until the volume doubles (thus reaching State 2); the polytropic exponent for this process is $n = -2/3$. The control mass then expands isothermally (*i.e.*, at constant temperature) until the volume doubles again (thus
43.5 reaching State 3). Take the helium to be an ideal gas.

3.5 (a) Calculate the mass of the helium gas, $m$.

6.5 (b) Determine the pressure $P_2$, and the temperature $T_2$ for the end of the polytropic process (State 2).

4 (c) Determine the pressure $P_3$, and the temperature $T_3$ for the end of the isothermal process (State 3).

18.5 (d) Determine the total work done by the system for both processes and the total heat transferred to the system for both processes.

11 (e) Show the state points and process paths on a $P$–$v$ diagram. Label all pressures, specific volumes and temperatures (with respect to the constant temperature lines). Indicate the area(s) representing the specific work done.

**BONUS (5 marks)**

The first process described above could be that for a gas in a spherical balloon where the volume and diameter are related as $V = (\pi/6)\, D^3$. Given that the process is polytropic with $n = -2/3$, determine algebraically the equation for the pressure, $P$, as a function of the diameter of the sphere, $D$, for such a process.

1. (a) $T_1 = -30\ [°C]$ 1-1/3

$x_1 = 0.25$ (saturated mixture)

$\therefore P_1 = P_{sat}(T_1) = P_{sat}(-30\ [°C]) = 100.4\ [kPa]$ (Table B.3.1)

$v_1 = (1 - x_1)\, v_f + x_1 v_g$

$v_f\big|_{-30°C} = 0.000672\ \left[\frac{m^3}{kg}\right]$

$v_g\big|_{-30°C} = 0.15937\ \left[\frac{m^3}{kg}\right]$ Table B.3.1

$v_1 = (1-0.25)\, 0.000672 + (0.25)\, 0.15937$

$v_1 = 0.040347\ [m^3/kg]$

At State 2 : The piston is at the stops

$V_2 = 0.450\ (m^3)$

$P_2 = P_1 + \frac{k}{A^2}(V_2 - V_1)$ (linear spring)

$P_2 = 100.4 + \frac{0.2374}{(0.0200)^2}(0.450 - 0.250)$ | $V_1 = 0.250\ (m^3)$

$P_2 = 219.1\ [kPa]$ ←

Check $v_2$ $\quad v_2 = \frac{V_2}{m}$ $\quad m = \frac{V_1}{v_1} = \frac{0.250}{0.040347} = 6.1962\ (kg)$

$v_2 = \frac{0.450}{6.1962} = 0.07263\ \left[\frac{m^3}{kg}\right]$

$v_g\big|_{219.1\ kPa} = 0.07665\ (m^3/kg)$ ← (Table B.3.1) → $v_f\big|_{219.1\ kPa} = 0.000700 \left(\frac{m^3}{kg}\right)$

$v_f\big|_{P_2} < v_2 < v_g\big|_{P_2}$ ⇒ still a saturated mixture

$\therefore T_2 = T_{sat}(P_2) = T_{sat}(219.1\ [kPa]) = -10\ [°C]$ ←

1 (b) $\quad {}_1W_2 = \frac{1}{2}(P_1 + P_2)(V_2 - V_1)$ $\quad$ 1-2/3

$$= \frac{1}{2}(100.4 + 219.1)(0.450 - 0.250)$$

$${}_1W_2 = 31.95 \text{ (kJ)} \leftarrow$$

(c) $\quad {}_1Q_2 - {}_1W_2 = m(u_2 - u_1) + \cancel{\Delta KE}^{\,0} + \cancel{\Delta PE}^{\,0}$

$$u_1 = (1 - x_1)\,u_f\Big|_{-30°C} + (x_1)\,u_g\Big|_{-30°C}$$

$$u_f\Big|_{-30°C} = 8.79 \left(\frac{kJ}{kg}\right), \quad u_g\Big|_{-30°C} = 158.19 \left(\frac{kJ}{kg}\right) \quad \text{Table B.3.1}$$

$$u_1 = (1 - 0.25)\,8.79 + (0.25)\,158.19$$

$$u_1 = 46.14 \left(\frac{kJ}{kg}\right)$$

$$u_2 = (1 - x_2)\,u_f\Big|_{-10°C} + (x_2)\,u_g\Big|_{-10°C}$$

$$x_2 = \frac{v_2 - v_f}{(v_g - v_f)} = \frac{(0.07263 - 0.000700)}{(0.07665 - 0.000700)}$$

(Value of $v_f$, $v_g$ for $-10°C$ were given on the previous page)

$$x_2 = 0.9471$$

$$u_f\Big|_{-10°C} = 26.72 \left(\frac{kJ}{kg}\right), \quad u_g\Big|_{-10°C} = 166.39 \left(\frac{kJ}{kg}\right) \quad \text{Table B.3.1}$$

$$u_2 = (1 - 0.9471)\,26.72 + (0.9471)\,166.39$$

$$u_2 = 159.00 \left(\frac{kJ}{kg}\right)$$

$${}_1Q_2 = {}_1W_2 + m(u_2 - u_1) = 31.95 + 6.1962(159.00 - 46.14)$$

$${}_1Q_2 = 31.95 + 699.30 = 731.25 \text{ (kJ)} \leftarrow$$

1 (d) State 3 is defined by 1-3/3

$T_3 = 30\ (°C)$

$v_3 = v_2 = 0.07263\ (m^3/kg)$

$v_g\big|_{30°C} = 0.02351\ (m^3/kg)$ (Table B.3.1)

$v_3 > v_g\big|_{T_3} \Rightarrow$ Superheated vapour

Interpolate in Table B.3.2 at $T = 30\ (°C)$ between $P = 200\ (kPa)$ and $P = 400\ (kPa)$ entries.

| P (kPa) | $v\ (m^3/kg)$ |
|---|---|
| 200 | 0.10023 |
| $P_3$ | 0.07263 |
| 400 | 0.04797 |

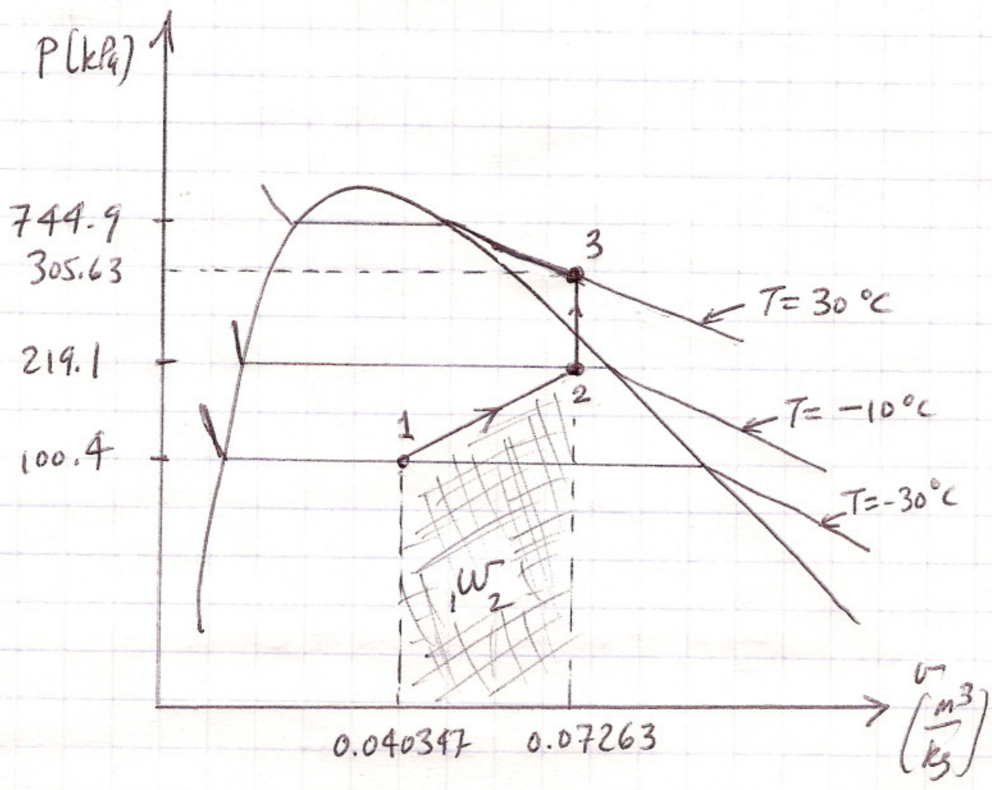
$$P_3 = 200 + (400 - 200)\frac{(0.07263 - 0.10023)}{(0.04797 - 0.10023)}$$

$P_3 = 305.63\ (kPa)$ ←

(e) ${}_1W_3 = {}_1W_2 + {}_2W_3$ ${}_2W_3 = 0$ because $V_3 = V_2$

${}_1W_3 = 31.95\ (kJ)$

(f) $P_{sat}(30\ (°C)) = 744.9\ (kPa)$



**# 2.** c.m. He gas, consider as ideal gas

$P_1 = 125$ kPa
$V_1 = 2\ m^3$
$T_1 = 300$ K

- polytropic process occurs with $n = -2/3$ until volume doubles
- heat is transferred to the c.m.

(a) $$m_{He} = \frac{P_1 V_1}{R T_1} = \frac{125 \times 2}{2.0771 \times 300} = 0.4012\ [kg]$$ , Table A.5 $R_{He} = 2.0771\ [kJ/kg \cdot K]$

(b) $P_1 V_1^n = P_2 V_2^n$ , $P_2 = \frac{P_1 V_1^n}{V_2^n}$ , $V_2 = 2 \times V_1 = 4\ [m^3]$

$$\therefore P_2 = 125 \left(\frac{2}{4}\right)^{-2/3} \qquad P_2 = 198.425\ [kPa]$$

$$T_2 = \frac{P_2 V_2}{m R} = \frac{198.425 \times 4}{0.4012 \times 2.0771} = 952.44\ [K]$$

(c) - an isothermal process occurs from states 2 to 3 until volume doubles again.

i.e., $T_3 = T_2 = 952.44\ [K]$ , $V_3 = 2 V_2 = 2 \times 4 = 8\ [m^3]$

$$\therefore P_3 = \frac{m R T_3}{V_3} = \frac{0.4012 \times 2.0771 \times 952.44}{8} = 99.21\ [kPa]$$

(d) $${}_1W_2 = \frac{P_2 V_2 - P_1 V_1}{1-n} = \frac{m R (T_2 - T_1)}{1-n} \quad \text{for ideal gas}$$

$${}_1W_2 = \frac{0.4012 \times 2.0771 \times (952.44 - 300)}{1 - (-2/3)} = 326.219\ [kJ]$$

$${}_2W_3 = (m R T_2) \ln \frac{V_3}{V_2} = (0.4012 \times 2.0771 \times 952.44) \ln \left(\frac{8}{4}\right)$$

$${}_2W_3 = 550.15\ [kJ]$$

$${}_1Q_2 - {}_1W_2 = m(u_2 - u_1) + \cancel{\Delta KE} + \cancel{\Delta PE}$$

$${}_1Q_2 = {}_1W_2 + m\,C_{v0}\,(T_2 - T_1) \qquad , \; C_{v0} = 3.116\ [kJ/kg\cdot K]$$

$$= 326.219 + 0.4012 \times 3.116 \times (952.44 - 300)$$

$$= 1141.85\ [kJ]$$

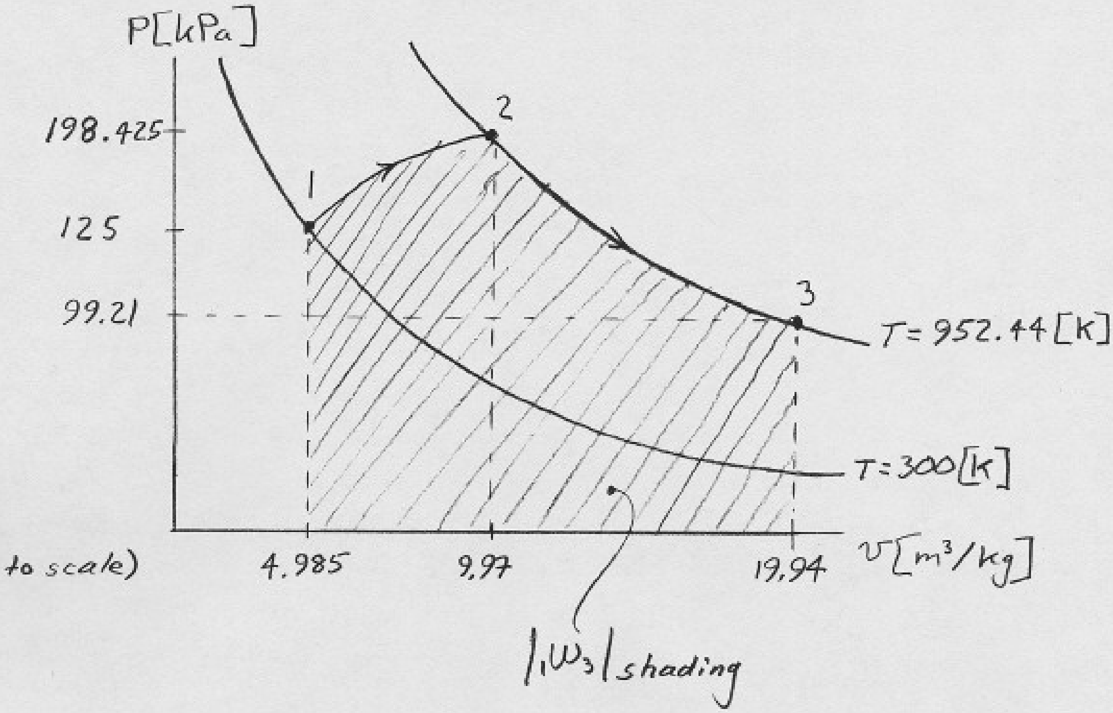
$${}_2Q_3 - {}_2W_3 = m(\underbrace{\cancel{u_3 - u_2}}_{0\ \text{isothermal}}) + \cancel{\Delta KE} + \cancel{\Delta PE}$$

$$\therefore\ {}_2Q_3 = {}_2W_3 = 550.15\ [kJ]$$

$${}_1Q_3 = {}_1Q_2 + {}_2Q_3 = 1141.85 + 550.15 = 1692\ [kJ]$$

$${}_1W_3 = 326.219 + 550.15 = 876.369\ [kJ]$$

(e)




(not to scale)

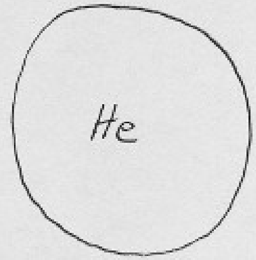
$$v_1 = \frac{V_1}{m} = \frac{2}{0.4012} = 4.985\ [m^3/kg]$$

$$v_2 = \frac{V_2}{m} = \frac{4}{0.4012} = 9.97\ [m^3/kg]$$

$$v_3 = 2\,v_2 = 2 \times 9.97 = 19.94\ [m^3/kg]$$

6 / 6

## Bonus




← sphere, $V = \frac{\pi}{6} D^3$

process equation,

$$PV^n = C \quad , \quad n = -\tfrac{2}{3}$$

$$\therefore P\left(\frac{\pi}{6} D^3\right)^{-2/3} = C$$

$$P\left(\pi/6\right)^{-2/3} D^{-2} = C \qquad \text{or} \qquad P = \underbrace{\left(\frac{\pi}{6}\right)^{2/3} \cdot C}_{\text{new constant, } K} \cdot D^2$$

$$\therefore P = kD^2$$

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For the conditions given the constant could be evaluated as

$$P_1 V_1^{-2/3} = C \quad \therefore \quad 125 \times 2^{-2/3} = 78.745 = C$$

$$\therefore k = \left(\pi/6\right)^{2/3} \times 78.745 = 51.155$$

i.e., $P = 51.155\, D^2$