MATH 1210 Assignment 3

March 07, 2014

Due: March 14, 2014, in class

Question 1. Consider the two points $P(1,2,3)$ and $Q(2,3,1)$, and let $\mathbf{v} = \vec{OQ}$ be the vector with its tail at the origin and with its head at Q.

(a) Find an equation of the line passing through P and Q.

(b) Find an equation of the plane through the origin containing P and Q.

(c) Find an equation of the plane containing P and normal to $\mathbf{v}$.

Question 2. Let ℓ_1 be the line with equation $\langle x, y, z \rangle = \langle 1, 4, 2 \rangle + s \langle 2, 0, 1 \rangle$ and let ℓ_2 be the line with equation $\langle x, y, z \rangle = \langle 2, -3, 1 \rangle + t \langle 1, 1, 3 \rangle$.

(a) Show that the two lines do not intersect.

(b) Find a vector perpendicular to both lines.

(c) Find equations of the two parallel planes Π_1 and Π_2 such that Π_1 contains ℓ_1 and Π_2 contains ℓ_2.

Question 3. Solve the following systems of linear equations by Gaussian elimination.

$$\begin{cases} 2x + 2y + = 3 \\ x + z = 4 \\ 3y + 3z = 9 \end{cases} \qquad \begin{cases} 2X + 2Y = 3 \\ X + Z = 4 \\ 3Y - 3Z = 9 \end{cases}$$

Question 4. Solve the following system of linear equations by Gaussian elimination.

$$\begin{cases} w - 2x - 2y + 7z = -1 \\ -2w + 4x + 3y - 12z = -1 \\ w - 2x + 3z = 5 \end{cases}$$

Question 5. Solve the following system of linear equations by Gauss-Jordan elimination:

$$\begin{cases} 4x + 6y - z = 9 \\ 2x + 3y = 1 \\ 2x + 2y - 3z = -9 \end{cases}$$

Question 6. Solve the following system of linear equations by Gauss-Jordan elimination.

$$\begin{cases} x_1 + 2x_2 + 2x_3 - 5x_4 = -1 \\ 2x_1 + 4x_2 + 5x_3 - 12x_4 + x_5 = 1 \\ -3x_1 - 6x_2 - 9x_3 + 21x_4 - 2x_5 = -1 \end{cases}$$

Question 7. Consider the following matrix A, representing the augmented matrix of a system of linear equations, where c is some unknown constant.

$$A = \left[\begin{array}{ccc|c} 1 & 2 & -1 & c \\ -2 & -3 & 2-c & -3c \\ 3 & 6+c & -c-3 & c \end{array}\right]$$

(a) By a few simple row operations, you can find the first two rows of a row-echelon form of this matrix, and reduce the third row to the point where it has a (quadratic) polynomial in c as its leading entry. Leave it in this form, and use this information to answer the remaining parts of this question:

(b) Find all values of c, if any, such that the system has no solutions.

(c) Find all values of c, if any, such that the system has a unique solution.

(d) Find all the values of c, if any, such that the system has infinitely many solutions.

Question 8. Students in three sections of a course take a midterm exam. An average percentage score is computed for each of the sections. There are 100 students in Section A01, 180 students in Section A02, and 120 students in Section A03. You are given the following pieces of information:

- The average of the three averages is 72%;
- The overall average of the three sections together is 74%;
- The average for section A02 is 20% more than the combined average for sections A01 and A03 together.

Set up a suitable system of linear equations and solve it in order to find the average percentage score for each section.