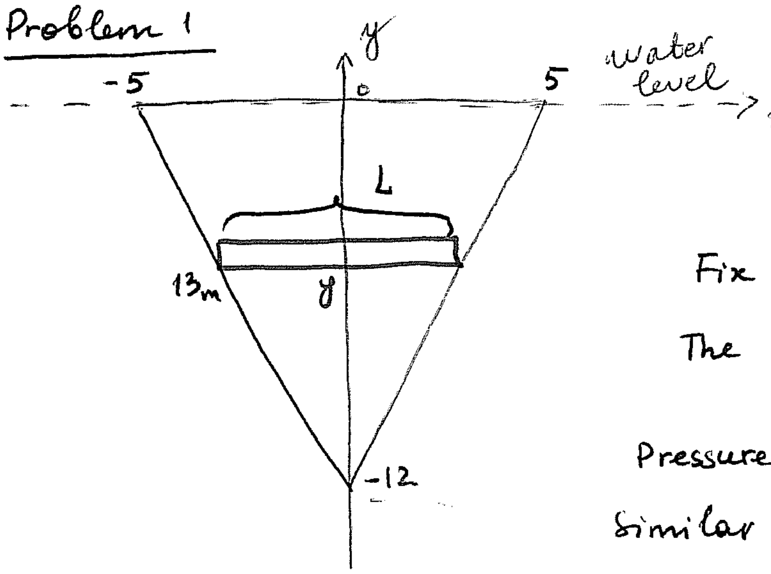
Math 1710: Practice Problems + Tutorial 7. Sketch of the Solutions

Fluid Pressure

Problem 1



The height of the triangle:
$\sqrt{13^2-5^2} = 12$ m.

Fix $y \in (-12, 0)$.

The force acting on a "small" rectangle: (horizontal)

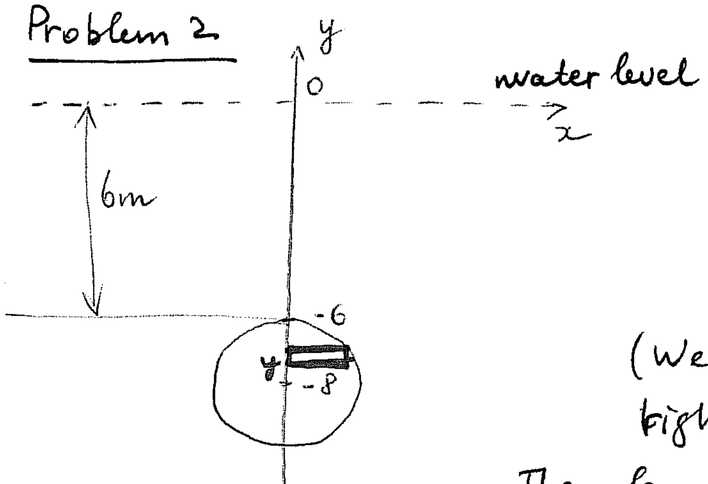
Pressure $\times$ Area $= \rho g \cdot (-y) \times L \times dy$

Similar triangles: $\frac{L}{10} = \frac{y-(-12)}{12}$

$\Rightarrow L = \frac{10}{12}(y+12) = \frac{5}{6}(y+12)$ m.

Total force $= \int_{-12}^{0} \rho g(-y) \cdot \frac{5}{6}(y+12)\, dy$ [N].

Problem 2



Equation of the circle:
$x^2 + (y+8)^2 = 4$
$x = \sqrt{4-(y+8)^2}$ (the right half only)

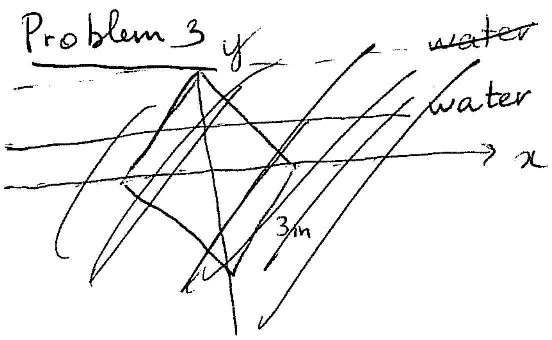
(We can find the total force on the right half only, and then multiply by 2)

The force acting on a "small" horizontal rect.,

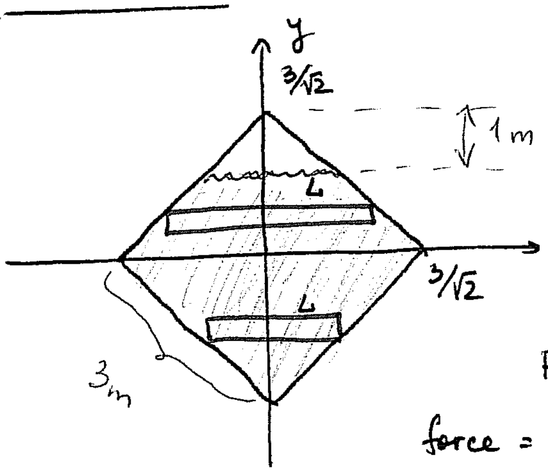
Pressure $\times$ Area $= \rho g(-y)\sqrt{4-(y+8)^2}\, dy$

$\Rightarrow$ Force $= 2 \cdot \int_{-10}^{-6} \rho g(-y)\sqrt{4-(y+8)^2}\, dy.$

Problem 3



Problem 3



The coordinates of the vertices of the square : $\left(\pm \frac{3}{\sqrt{2}}, 0\right), \left(0, \pm\frac{3}{\sqrt{2}}\right)$.

split the area into two regions : $y>0$ and $y<0$

For $y>0$: force on a small rectangle:

force = area x pressure = $L \cdot dy \cdot \rho \cdot g \cdot \left(\frac{3}{\sqrt{2}} - 1 - y\right)$

(the water level is at $y = \frac{3}{\sqrt{2}} - 1$)

Similar triangles: $\frac{L}{3\sqrt{2}} = \frac{\frac{3}{\sqrt{2}} - y}{\frac{3}{\sqrt{2}}} \Rightarrow L = 3\sqrt{2} - 2y$, and so,

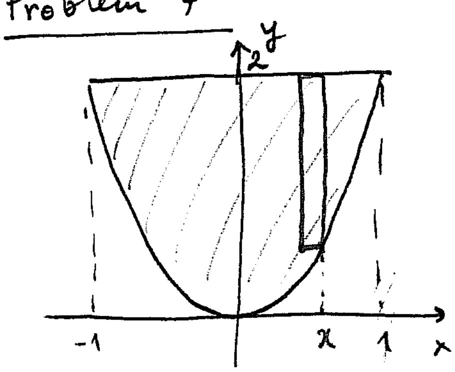
$$I_1 = \int_0^{\frac{3}{\sqrt{2}}-1} (3\sqrt{2} - 2y) \cdot \rho g \left(\frac{3}{\sqrt{2}} - 1 - y\right) dy$$

similarly, for $y<0$,

$$I_2 = \int_{-3/\sqrt{2}}^{0} (3\sqrt{2} + 2y) \rho g \left(\frac{3}{\sqrt{2}} - 1 - y\right) dy$$

Final answer = $I_1 + I_2$.

Problem 4



$y = x^4 + x^2 \Rightarrow y(\pm 1) = 2$.

(cannot use horizontal rectangles, since x cannot be expressed in terms of y)

use vertical rectangles.

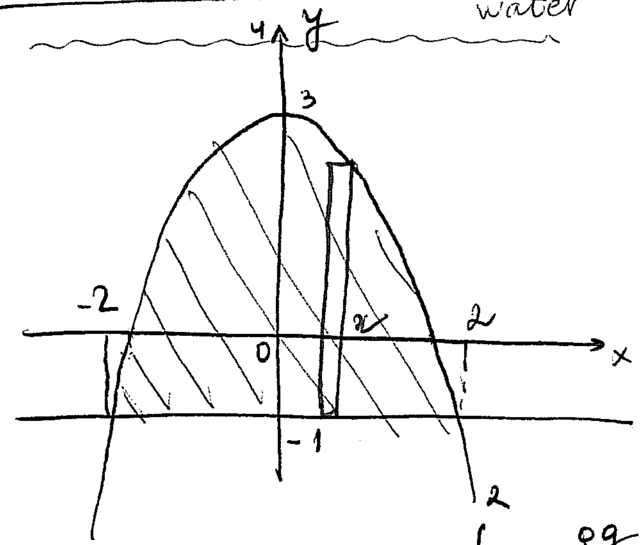
Force acting on a small vertical rectangle (at position x):

$$+\frac{\rho g \times \text{width of rect.}}{2} \times \left[\text{depth}^2_{\text{bottom}} - \text{depth}^2_{\text{top}}\right] \Leftrightarrow$$

$$\Leftrightarrow \frac{\rho g}{2} \left[\left(2 - (x^4 + x^2)\right)^2 - 0^2\right] dx \Rightarrow$$

$$\text{Force} = \int_{-1}^{1} \frac{\rho g}{2} \left[\left(2 - x^4 - x^2\right)^2 - 0^2\right] dx$$

Problem 5



Water level at $y=4$.

Use vertical rectangles

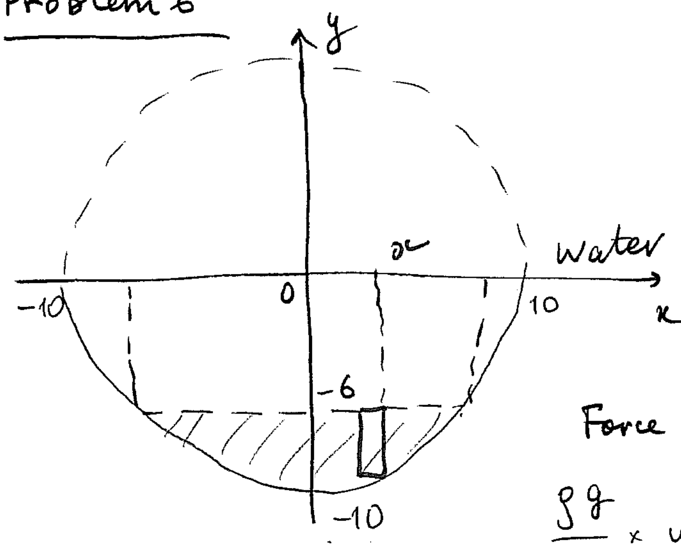
Force on a small rectangle:

$$\frac{\rho g \times \text{width}}{2}\left[\text{depth}_{\text{bottom}}^2 - \text{depth}_{\text{top}}^2\right]$$

$$= \frac{\rho g \cdot dx}{2}\left[\left(4-(-1)\right)^2 - \left(4-(3-x^2)\right)^2\right]$$

$$\Rightarrow \text{Total force} = \int_{-2}^{2} \frac{\rho g}{2}\left[5^2 - (1+x^2)^2\right]dx$$

Problem 6



Equation of the circle:

$x^2+y^2=100$

$y=-\sqrt{100-x^2}$ (the bottom semicircle)

Use vertical rectangles.

Force on a small vertical rectangle:

$$\frac{\rho g}{2} \times \text{width}\left[\text{depth}_{\text{bottom}}^2 - \text{depth}_{\text{top}}^2\right] =$$

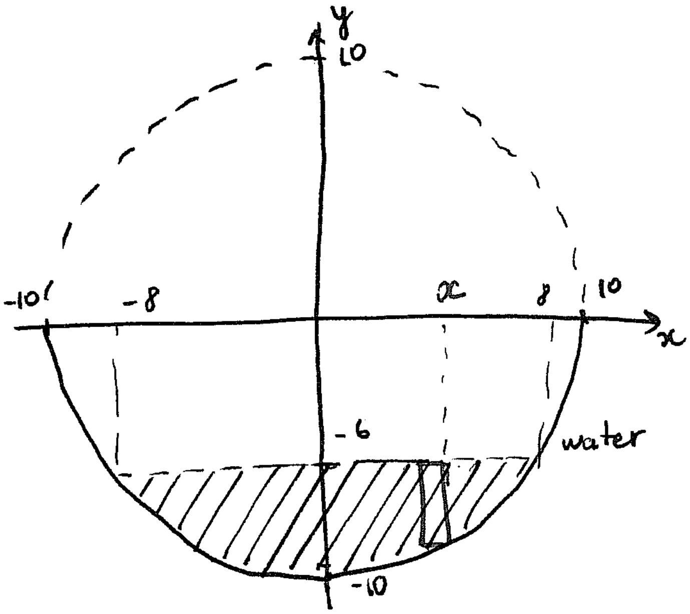
$$= \frac{\rho g}{2} \times dx\left[6^2 - \left(0-(-\sqrt{100-x^2})\right)^2\right]$$

The bounds on x: $y=-6 \Rightarrow x^2+(-6)^2=100$

$x=\pm 8$

$$\text{Total force} = \int_{-8}^{8} \frac{\rho g}{2}\left[36-(100-x^2)\right]dx.$$

Problem 6 (corrected)



Equation of the circle : $x^2 + y^2 = 100$

$y = -\sqrt{100 - x^2}$ (the bottom semicircle)

Use vertical rectangles.

Force on a small vertical rectangle:

$$\frac{\rho g}{2} \times \text{width} \times \left[\text{depth}^2_{\text{bottom}} - \text{depth}^2_{\text{top}} \right] =$$

$$= \frac{\rho g}{2} \times dx \times \left[\left(-6 - (-\sqrt{100 - x^2})\right)^2 - 0^2 \right]$$

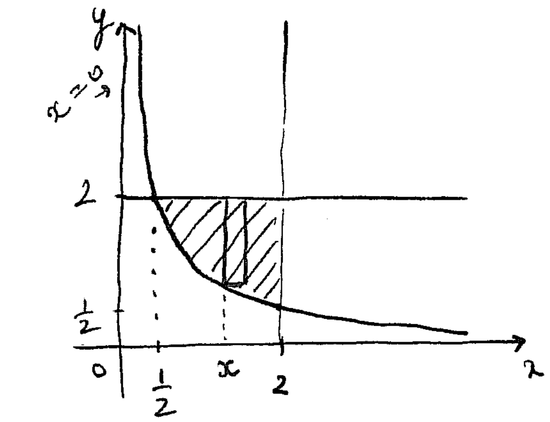
The bounds on x : $y = -6 \Rightarrow$ $x^2 + (-6)^2 = 100$

$x = \pm 8$

$$\text{Total force} = \int_{-8}^{8} \frac{\rho g}{2} \left(\sqrt{100 - x^2} - 6 \right)^2 dx$$

Centroids (sketch of the solutions)

Problem 7 (a) $y=\frac{1}{x}$, $x=2$, $y=2$, line: $x=0$

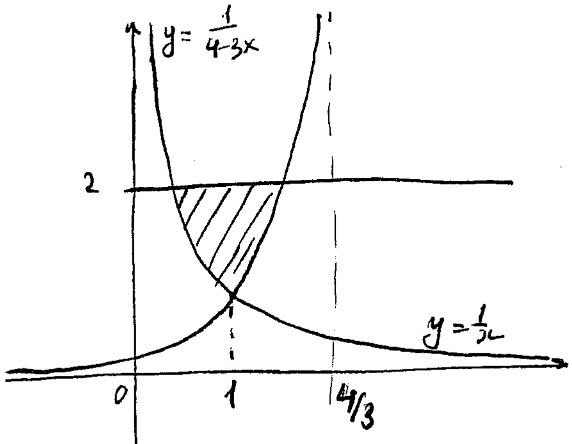


Pts of intersection: $(2,\frac{1}{2})$, $(\frac{1}{2},2)$, $(2,2)$.

First moment about $x=0$:

$$\int_{\frac{1}{2}}^{2} x\cdot\rho\cdot\left(2-\frac{1}{x}\right)dx$$

(b) $y=\frac{1}{x}$, $y=\frac{1}{4-3x}$, $y=2$ about $x=0$ (set up only)



Pts of int-n: $\frac{1}{x}=\frac{1}{4-3x} \Rightarrow x=4-3x$
$4x=4 \Rightarrow \underline{x=1}$

$\frac{1}{x}=2 \Rightarrow x=\frac{1}{2}$

$\frac{1}{4-3x}=2 \Rightarrow (4-3x)2=1$
$8-6x=1$
$x=\frac{7}{6}$

(I) Use vertical rectangles $\Rightarrow$ 2 integrals

First moment about $x=0$: $\displaystyle\int_{\frac{1}{2}}^{1} x\rho\left(2-\frac{1}{x}\right)dx+\int_{1}^{7/6} x\rho\left(2-\frac{1}{4-3x}\right)dx$

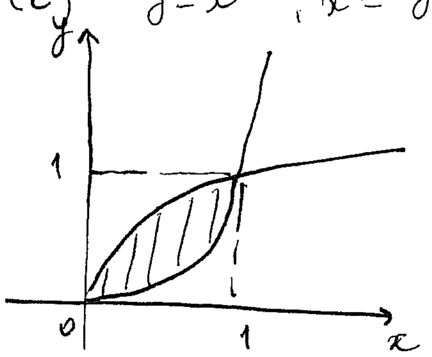
(II) Use horizontal rectangles $\Rightarrow$ 1 integral

First moment about $x=0$: (express functions as $x=f(y)$, $x=g(y)$):

$y=\frac{1}{x} \Rightarrow x=\frac{1}{y}$, $\quad y=\frac{1}{4-3x} \Rightarrow 4-3x=\frac{1}{y} \Rightarrow x=-\frac{\frac{1}{y}-4}{3}=\frac{4y-1}{3y}$

$$\int_{1}^{2} \rho\,\frac{1}{2}\left[\left(\frac{4y-1}{3y}\right)^2-\left(\frac{1}{y}\right)^2\right]dy.$$

(c) $y=x^5$, $x=y^5 \leftarrow y=x^{\frac{1}{5}}$ about $y=0$.



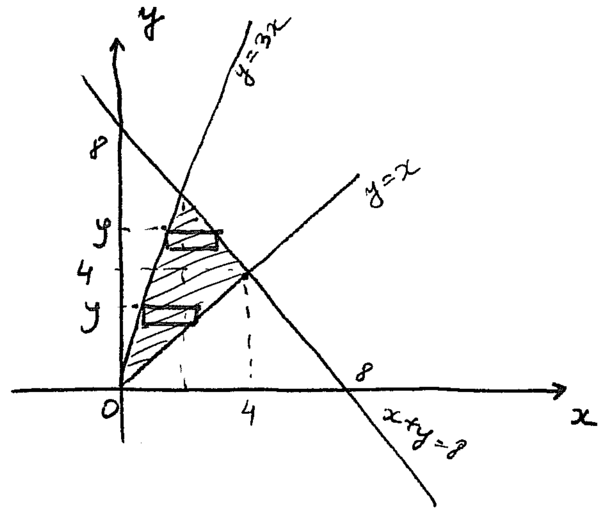
(I) Use vertical rectangles:

First moment about $y=0$: $\displaystyle\int_0^1 \rho\cdot\frac{1}{2}\left[(x^5)^2-(x^{\frac{1}{5}})^2\right]dx$

or (II) Use horizontal rectangles:

First moment about $y=0$: $\displaystyle\int_0^1 y\cdot\rho\cdot\left(y^{\frac{1}{5}}-y^5\right)dy$

(d) $y = x$, $y = 3x$, $x + y = 8$ about $y = 0$.



Pts of intersection:

$(2,6)$, $(4,4)$

In any method we choose, there will be two integrals. So use, horizontal rectangles

Functions in the form $x = f(y)$:

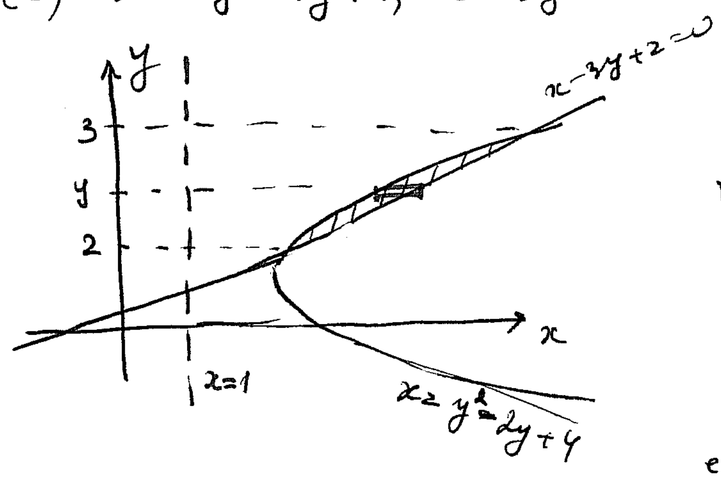
$y = x \Rightarrow x = y$

$y = 3x \Rightarrow x = \frac{y}{3}$

$x + y = 8 \Rightarrow x = 8 - y$.

First moment about $y=0$: $\displaystyle\int_0^4 y\rho\left(y - \frac{y}{3}\right)dy + \int_4^6 y\rho\left(8 - y - \frac{y}{3}\right)dy$.

(e) $x = y^2 - 2y + 4$, $x - 3y + 2 = 0$ about $x = 1$.



Pts of intersection:

$$\begin{cases} x = y^2 - 2y + 4 \\ x = 3y - 2 \end{cases} \Rightarrow y^2 - 2y + 4 = 3y - 2$$

$$y^2 - 5y + 6 = 0$$

$$y = 2, 3$$

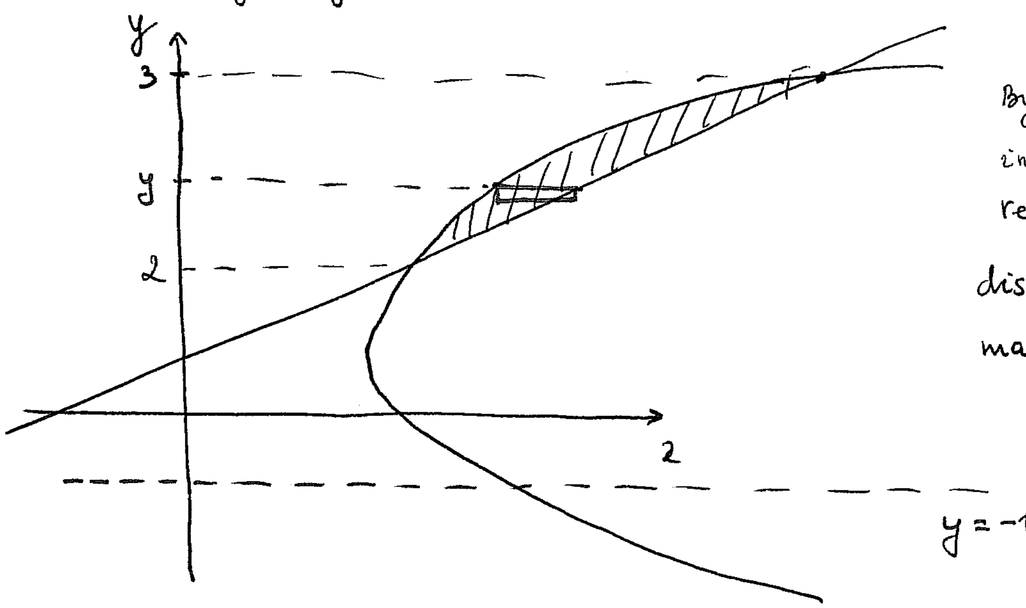
use horizontal rectangles since the function $x = y^2 - 2y + 4$ cannot be easily (nicely) solved for y.

First moment about $x=1$; (we concentrate the mass in the center of the rectangle: dist from the center $= \frac{f+g}{2} - 1$

$$\int_2^3 \underbrace{\left(\frac{3y - 2 + y^2 - 2y + 4}{2} - 1\right)}_{\text{distance}} \cdot \underbrace{\rho \cdot \left[(3y-2) - (y^2 - 2y + 4)\right] \cdot dy}_{\text{mass}}$$

$$= \int_2^3 \frac{y^2 - y + 2}{2} \cdot \rho \cdot \left(-y^2 + 5y - 6\right) dy.$$

(f) $x = y^2 - 2y + 4$, $x - 3y + 2 = 0$ about $y = -1$.



By the same reasoning as in (e), we use horizontal rectangles:

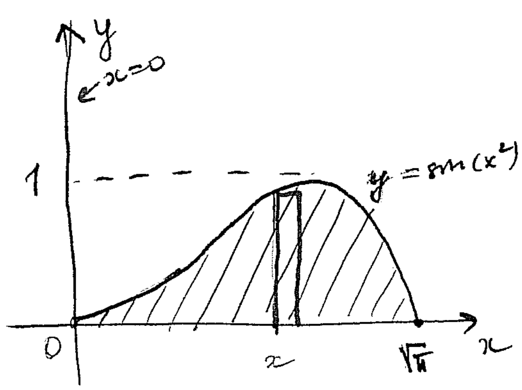
distance $= y - (-1) = y + 1$

mass $= \rho \times$ area

First moment about $y = -1$:

$$\int_2^3 (y+1)\cdot\rho\left[(3y-2) - (y^2 - 2y + 4)\right] dy = \int_2^3 (y+1)\,\rho\,(-y^2 + 5y - 6)\, dy.$$

(g) $y = \sin(x^2)$, $0 \le x \le \sqrt{\pi}$; $y = 0$, about $x = 0$.



– we use vertical rectangles (x cannot be easily and nicely expressed in terms of y).

First moment about $x = 0$: $\displaystyle\int_0^{\sqrt{\pi}} x \cdot \rho \cdot \sin(x^2)\, dx =$

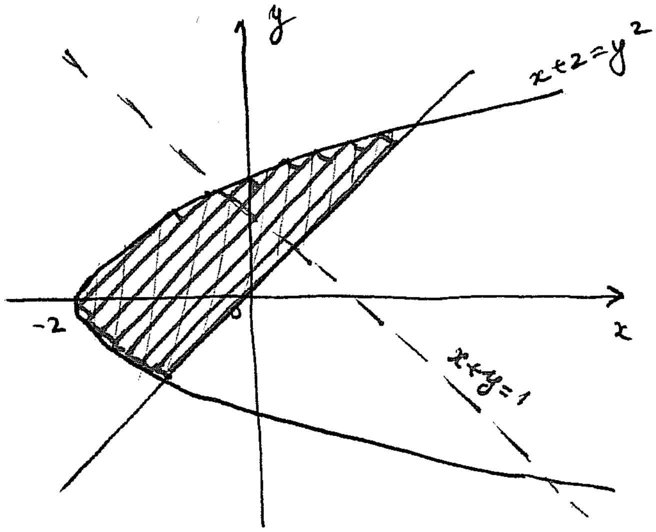
* make substitution $u = x^2 \Rightarrow du = 2x\,dx \Rightarrow x\,dx = \frac{du}{2}$

$x = 0 \Rightarrow u = 0$

$x = \sqrt{\pi} \Rightarrow u = \pi$

$$= \int_0^{\pi} \rho \sin u \,\frac{du}{2}$$

(h)* $x+2=y^2$, $y=x$ about $x+y=1$



Use rectangles shown at the picture. Fix $x=a\in[-2,0]$, and draw the line that is parallel to $y=x$ through this point on x-axis:

$y=x-a$.

Find points of intersection with parabola:

$$\begin{cases} y=x-a \\ x+2=y^2 \end{cases} \Rightarrow y+a=y^2-2 \Rightarrow$$

$$\Rightarrow y^2-y-a-2=0$$

$\Rightarrow$ two roots y_1, y_2.

The length of the rectangle is $\sqrt{2}\,|y_1-y_2|$, $\Rightarrow$

$$\Rightarrow |y_1-y_2| = \sqrt{(y_1-y_2)^2} = \sqrt{y_1^2+y_2^2-2y_1y_2} = \sqrt{(y_1+y_2)^2-4y_1y_2} =$$

$$= \sqrt{1-4(-a-2)} = \sqrt{1+4(a+2)} = \sqrt{4a+9}.$$

The width of the rectangle is $\frac{dx}{\sqrt{2}}$.

The center of the rectange is at $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$,

where x_1, x_2 are roots of $(x-a)^2=x+2 \iff x^2-x(2a+1)+a^2-2=0$

$$\Rightarrow \frac{1}{2}(x_1+x_2) = \frac{2a+1}{2}, \quad \frac{1}{2}(y_1+y_2) = \frac{1}{2}.$$

The distance from the center of rectangle to the axis:

$$\sqrt{2}\left(\frac{1-2a}{2}-\frac{1}{2}\right) = \left(-\frac{2a}{2}\right)\cdot\sqrt{2} = -\sqrt{2}a,$$ since the intersection of $\begin{cases} y=x-a \\ x+y=1 \end{cases}$ has y-coordinate $\frac{1-2a}{2}$

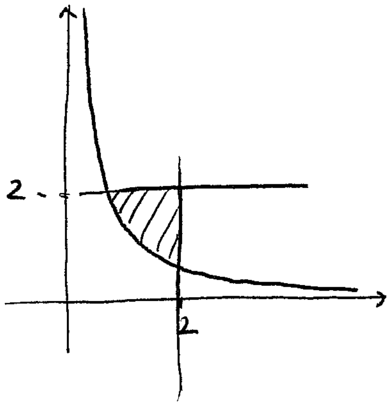
So dist $=-\sqrt{2}a$, mass $=\rho\cdot$ length $\cdot$ width $=\rho\cdot\sqrt{2}\,\sqrt{4a+9}\cdot\frac{dx}{\sqrt{2}}$

$$= \sqrt{4a+9}\,\rho\,dx$$

$$\Rightarrow \text{First moment} = \int_{-2}^{0} -\sqrt{2}x\cdot\sqrt{4x+9}\,\rho\,dx$$

Problem 8

(a) $y = \frac{1}{x}$, $x = 2$, $y = 2$;



From Problem 7 (a), first moment about y-axis is $\int_{1/2}^{2} x \cdot \rho \cdot \left(2 - \frac{1}{x}\right) dx$.

First moment about x-axis is (use horiz. rectangles)

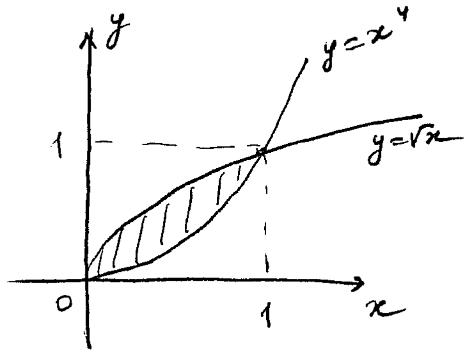
$$\int_{\frac{1}{2}}^{2} y \, \rho \left(2 - \frac{1}{y}\right) dy$$

Mass of the region is $\int_{\frac{1}{2}}^{2} \rho\left(2 - \frac{1}{x}\right) dx$

* May not include ρ in these calculations.

$$\bar{x} = \frac{\int_{1/2}^{2} x \left(2 - \frac{1}{x}\right) dx}{\int_{\frac{1}{2}}^{2}\left(2 - \frac{1}{x}\right) dx}, \quad \bar{y} = \frac{\int_{1/2}^{2} y \left(2 - \frac{1}{y}\right) dy}{\int_{1/2}^{2}\left(2 - \frac{1}{y}\right) dy}.$$

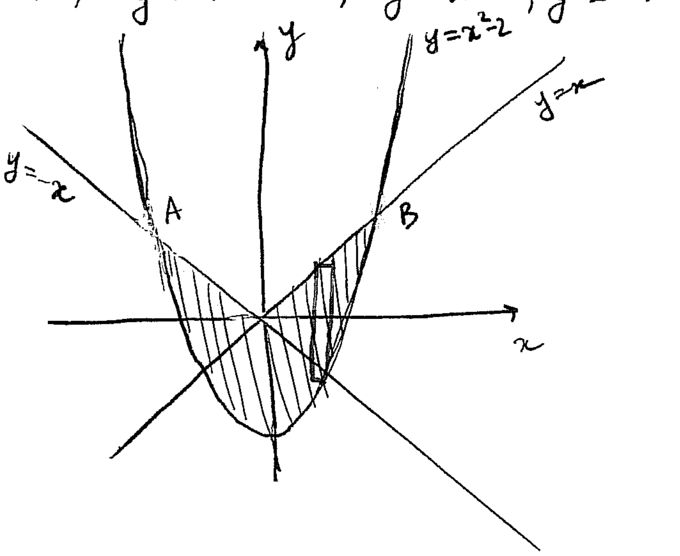
(b) $y = x^4$, $y = \sqrt{x}$



$$\bar{x} = \frac{\int_0^1 x \left(\sqrt{x} - x^4\right) dx}{\int_0^1 \left(\sqrt{x} - x^4\right) dx}$$

$$\bar{y} = \frac{\int_0^1 \frac{(\sqrt{x})^2 - (x^4)^2}{2} dx}{\int_0^1 \left(\sqrt{x} - x^4\right) dx} \quad \text{(or)} \quad \frac{\int_0^1 y \cdot \left(\sqrt[4]{y} - y^2\right) dy}{\int_0^1 \left(\sqrt{x} - x^4\right) dx}$$

(c) $y = x^2 - 2$, $y = x$, $y = -x$

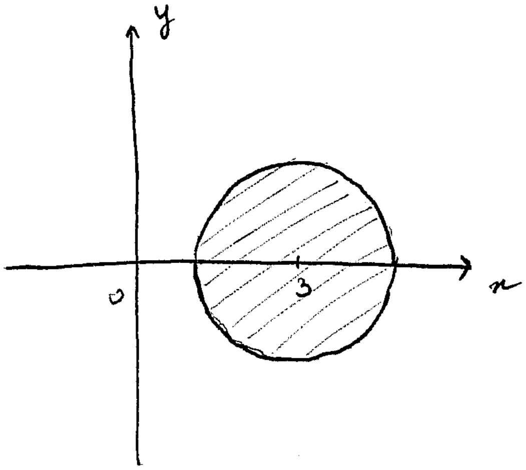


Pts of intersection: $A(-2, 2)$, $B(2, 2)$

Since the region is symmetrical about the y-axis $\Rightarrow$ $\bar{x} = 0$.

$$\bar{y} = \frac{2 \int_0^2 \frac{1}{2}\left[x^2 - (x^2-2)^2\right] dx}{2 \int_0^2 \left[x - (x^2 - 2)\right] dx}$$

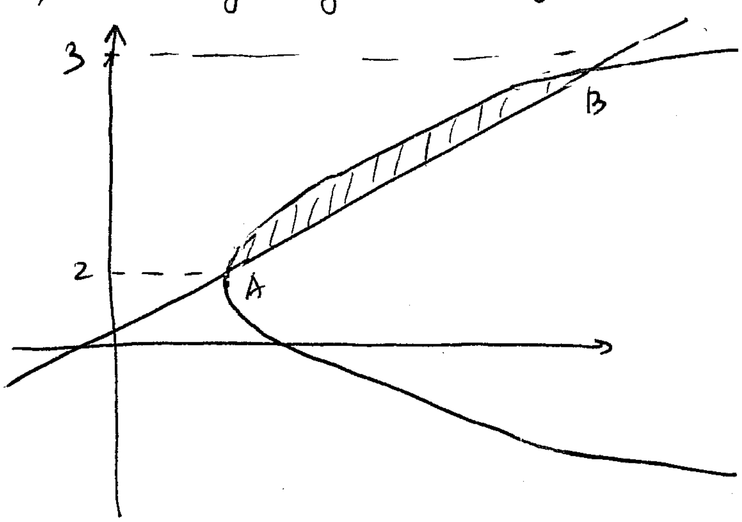
(d) $(x-3)^2 + y^2 = 4$ — circle. Obviously, the centroid of a circle is the center of the circle : $(3,0)$.



$\bar{x} = 3, \quad \bar{y} = 0$

(Could be done analyticaly as well).

(e) $x = y^2 - 2y + 4, \quad x - 3y + 2 = 0.$

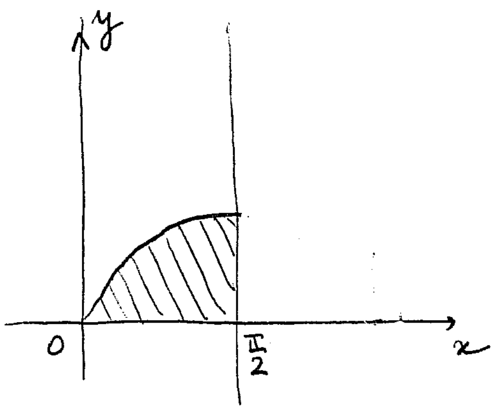


Pts of intersection $A(4,2)$, $B(7,3)$

$$\bar{x} = \frac{\int_2^3 \frac{1}{2}\left[(3y-2)^2 - (y^2-2y+4)^2\right] dy}{\int_2^3 \left[(3y-2) - (y^2-2y+4)\right] dy}$$

$$\bar{y} = \frac{\int_2^3 y \left[(3y-2) - (y^2-2y+4)\right] dy}{\int_2^3 \left[(3y-2) - (y^2-2y+4)\right] dy}$$

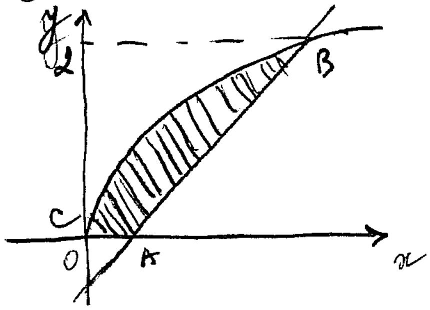
(f)



$$\bar{x} = \frac{\int_0^{\frac{\pi}{2}} x \sin x \, dx}{\int_0^{\pi/2} \sin x \, dx}$$

$$\bar{y} = \frac{\int_0^{\pi/2} \frac{1}{2}\left[\sin^2 x - 0^2\right] dx}{\int_0^{\pi/2} \sin x \, dx}.$$

(g) $y = \sqrt{x}, \quad x = y + 2, \quad y = 0$



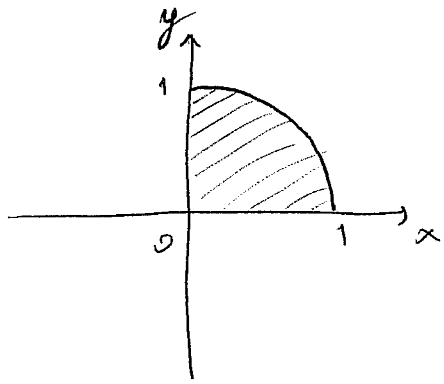
Pts of intersection: $A(2,0)$, $B(4,2)$, $C(0,0)$.

$$\bar{x} = \frac{\int_0^2 \frac{1}{2}\left[(y+2)^2 - (y^2)^2\right) dy}{\int_0^2 \left[(y+2) - (y^2)\right] dy}$$

$$\bar{y} = \frac{\int_0^2 y \left[(y+2) - y^2\right] dy}{\int_0^2 \left[(y+2) - y^2\right] dy}$$

(h) $y = \sqrt{1-x^2}$, $0 \le x \le 1$, $x=0$, $y=0$

$y = \sqrt{1-x^2}$, $0 \le x \le 1$ – quarter of the circle $x^2+y^2=1$ in the first quadrant.



$$\bar{x} = \frac{\int_0^1 x\sqrt{1-x^2}\,dx}{\int_0^1 \sqrt{1-x^2}\,dx}$$

$$\bar{y} = \frac{\int_0^1 \frac{1}{2}\left[(\sqrt{1-x^2})^2 - 0^2\right]dx}{\int_0^1 \sqrt{1-x^2}\,dx}.$$

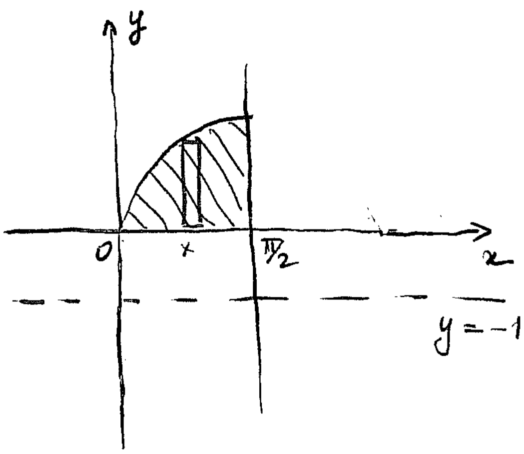
* To solve $\int_0^1 x\sqrt{1-x^2}\,dx$, you can make substitution $u=1-x^2$.

** To solve $\int_0^1 \sqrt{1-x^2}\,dx$, use geometric interpretation of a definite integral.

Moments of Inertia (Tutorial 7)

Problem 3

(a) $y = \sin x$ $\left(0 \le x \le \frac{\pi}{2}\right)$, $x = \frac{\pi}{2}$, $y=0$ about $y=-1$



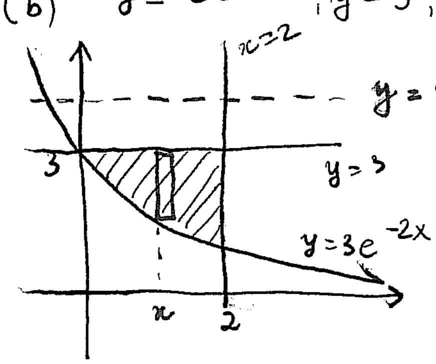
Use vertical rectangles

2nd moment ~~of inertia~~ about $y=1$ is

$$\int_0^{\pi/2} \rho \cdot \frac{1}{3}\left[\left(\sin x - (-1)\right)^3 - \left(0-(-1)\right)^3\right]dx$$

$$= \int_0^{\frac{\pi}{2}} \frac{1}{3}\rho\left[(\sin x+1)^3 - 1^3\right]dx.$$

(b) $y = 3e^{-2x}$, $y=3$, $x=2$ about $y=4$

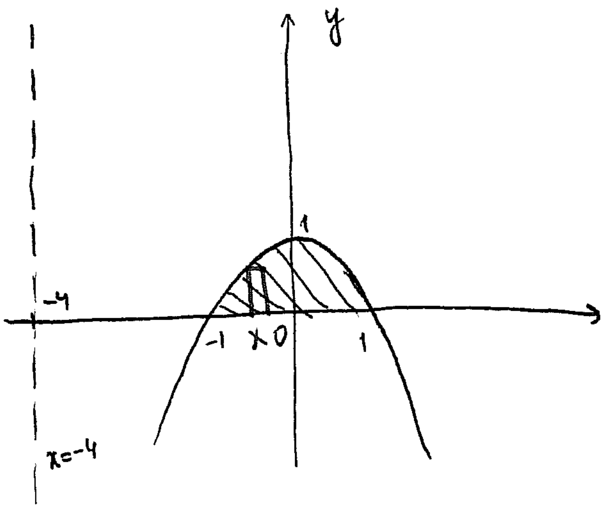


Use vertical rectangles:

2nd moment ~~of inertia~~ about $y=4$ is

$$\int_0^2 \rho \cdot \frac{1}{3}\left[\left(4-3e^{-2x}\right)^3 - (4-3)^3\right]dx$$

(c) $y = 1 - x^2$, $y = 0$ about $x = -4$.



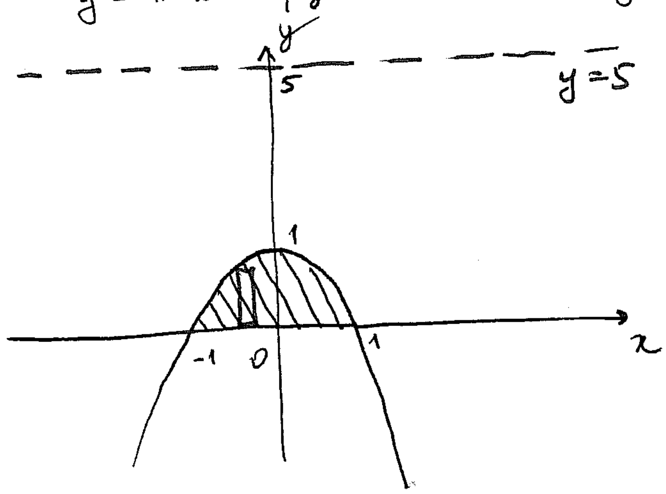
Use vertical rectangles.

2nd moment about $x = -4$ is

$$\int_{-1}^{1} (x-(-4))^2 \rho \left[(1-x^2) - 0\right] dx$$

$$= \int_{-1}^{1} (x+4)^2 \rho \, (1-x^2) \, dx.$$

(d) $y = 1 - x^2$, $y=0$ about $y=5$



Use vertical rectangles.

2nd moment about $y=5$ is

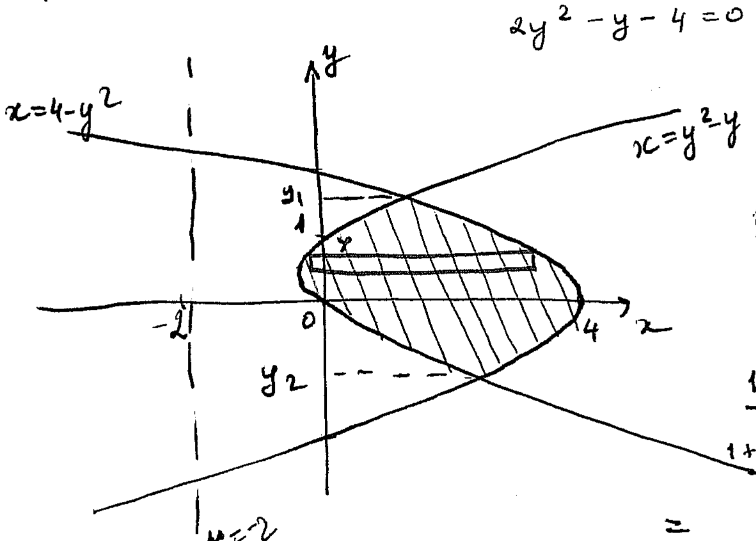
$$\int_{-1}^{1} \rho \cdot \frac{1}{3} \left[(5-0)^3 - (5-(1-x^2))^3 \right] dx$$

$$= \int_{-1}^{1} \frac{1}{3} \rho \left[5^3 - (4+x^2)^3 \right] dx.$$

(e) $x = 4 - y^2$, $x = y^2 - y$, about $x = -2$.

pts of intersection : $4 - y^2 = y^2 - y$

$2y^2 - y - 4 = 0 \Rightarrow y_1 = \dfrac{1+\sqrt{33}}{4}$, $y_2 = \dfrac{1-\sqrt{33}}{4}$



Use horizontal rectangles;

2nd moment about $y=5$ is

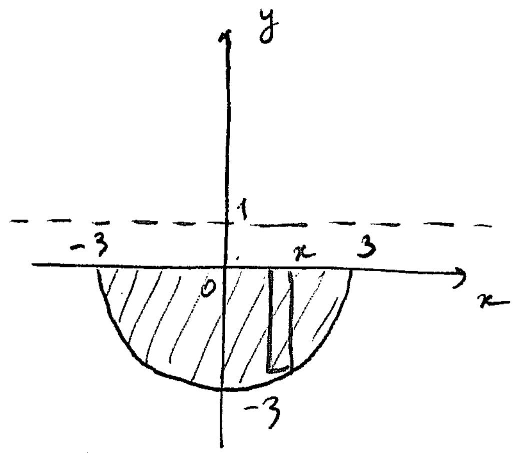
$$\int_{\frac{1-\sqrt{33}}{4}}^{\frac{1+\sqrt{33}}{4}} \rho \cdot \frac{1}{3} \left[\left(4-y^2-(-2)\right)^3 - \left(y^2-y-(-2)\right)^3 \right] dy$$

$$= \int_{\frac{1-\sqrt{33}}{4}}^{\frac{1+\sqrt{33}}{4}} \frac{1}{3} \rho \left[(6-y^2)^3 - (y^2-y+2)^3 \right] dy.$$

* The question has a typo, and the second ~~fn~~ was ~~meant to~~ be

$x = y^2 - 2y$.

(f) $y = -\sqrt{9-x^2}$ about $y=1$.



$y=-\sqrt{9-x^2}$ is the bottom semicircle of the circle $x^2+y^2=9$.

Use vertical rectangles;

2nd moment about $y=1$ is

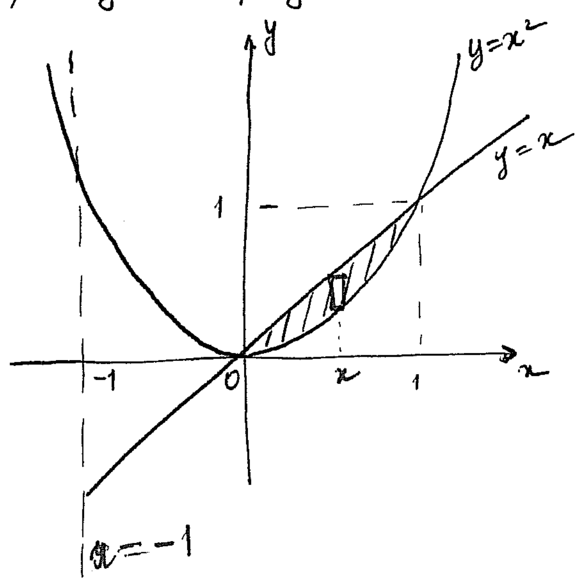
$$\int_{-3}^{3} \rho \cdot \frac{1}{3}\left[\left(1-(-\sqrt{9-x^2})\right)^3-(1-0)^3\right]dx$$

$$= \int_{-3}^{3} \frac{1}{3}\rho\left[\left(1+\sqrt{9-x^2}\right)^3-1^3\right]dx$$

* Could use horizontal rectangles:

$$\int_{-3}^{0} (1-y)^2 \rho\, 2\sqrt{9-y^2}\, dy$$

(g) $y=x^2$, $y=x$ about $x=-1$.

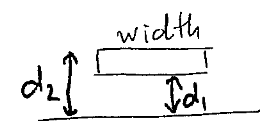


Use vertical rectangles;

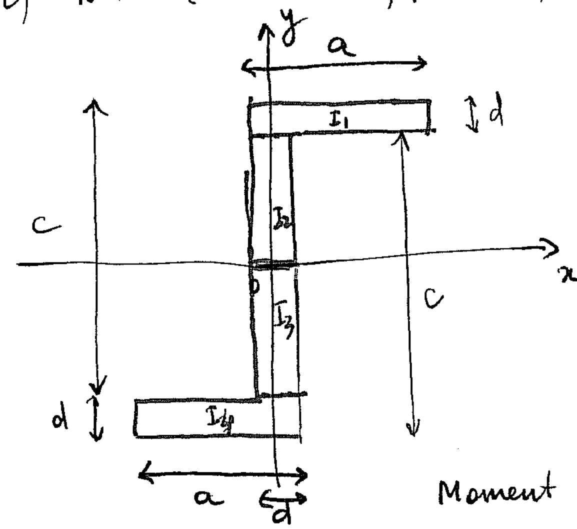
2nd moment about $x=-1$:

$$\int_0^1 (x-(-1))^2 \rho\left[x-x^2\right]dx$$

$$= \int_0^1 (x+1)^2 \rho\left[x-x^2\right]dx.$$



(h) № 15 (textbook, p. 469)



Use formula for moment of inertia of a rectangle: $\frac{1}{3}\text{width}\cdot(d_2^3-d_1^3)$

(a) about x-axis: (split into ~~3~~ 4 rectangles) and add in the end moment of inertia for

I_1: width $= a$, $d_2 = \frac{c+d}{2}$, $d_1 = \frac{c-d}{2}$

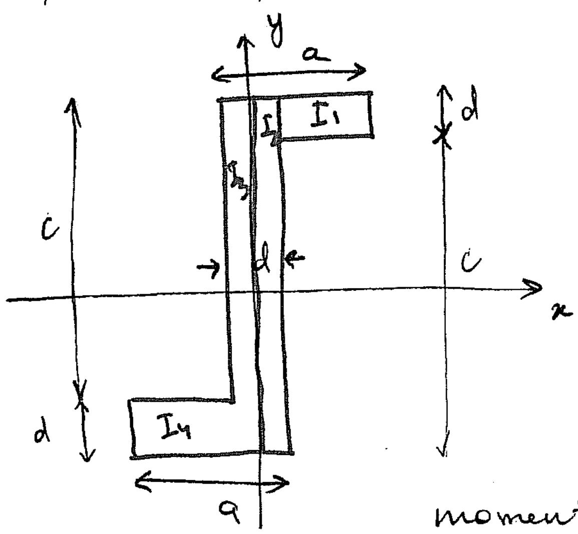
m. of. in. $= \frac{1}{3}a\left[\left(\frac{c+d}{2}\right)^3-\left(\frac{c-d}{2}\right)^3\right]$

I_2: width $= d$, $d_2 = \frac{c-d}{2}$, $d_1 = 0$

m. of in $= \frac{1}{3}d\left[\left(\frac{c-d}{2}\right)^3-0^3\right]$

Moment of inertia of I_1 = m. of in of I_4; m. of. in of I_2 = m. of in of I_3.

(h) N15 (b)



About y-axis:

moment of inertia for

I_1: width $= d$, $d_2 = a - \frac{d}{2}$, $d_1 = \frac{d}{2}$

m. of in. $= \frac{1}{3} d \left[(a - \frac{d}{2})^3 - \left(\frac{d}{2}\right)^3 \right]$

I_2: width $= c + d$, $d_2 = \frac{d}{2}$, $d_1 = 0$

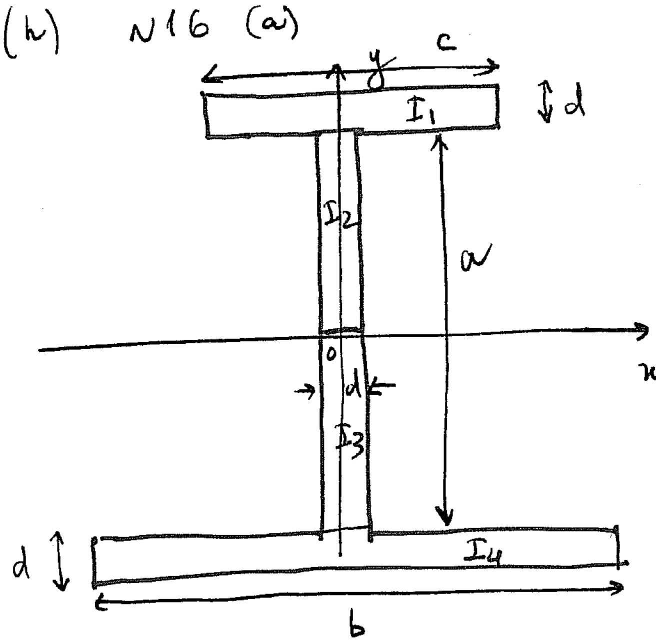
m. of in $= \frac{1}{3}(c+d) \cdot \left[\left(\frac{d}{2}\right)^3 - 0^3 \right]$.

moment of inertia of I_3 = moment of inertia of I_2

moment of inertia of I_4 = moment of inertia of I_1

$\Rightarrow$ Total moment of inertia about y-axis $= 2 \cdot \left[\frac{d}{3} \left((a - \frac{d}{2})^3 - \left(\frac{d}{2}\right)^3 \right) + \frac{c+d}{3} \cdot \left(\frac{d}{2}\right)^3 \right]$

(h) N16 (a)



About x-axis.

moment of inertia for:

I_1: width $= c$, $d_2 = \frac{a}{2} + d$, $d_1 = \frac{a}{2}$

$\frac{1}{3} c \left[\left(\frac{a}{2} + d\right)^3 - \left(\frac{a}{2}\right)^3 \right]$

I_2: width $= d$, $d_2 = \frac{a}{2}$, $d_1 = 0$

$\frac{1}{3} d \left[\left(\frac{a}{2}\right)^3 - 0^3 \right]$

I_3 and I_2 have equal moments of inertia

I_4: width $= b$, $d_2 = \frac{a}{2} + d$, $d_1 = \frac{a}{2}$

$\frac{1}{3} b \left[\left(\frac{a}{2} + d\right)^3 - \left(\frac{a}{2}\right)^3 \right]$

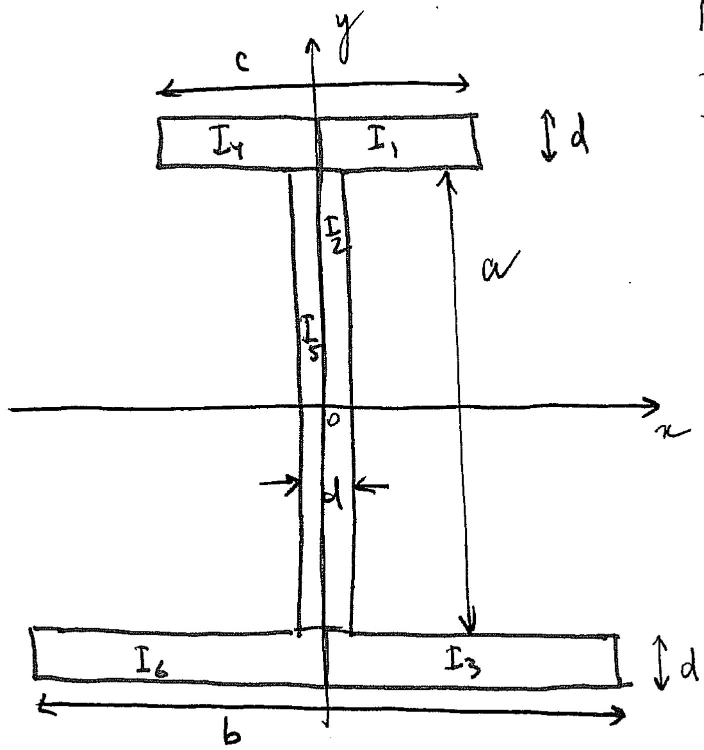
Total moment of inertia $=$

$$\frac{1}{3} c \left[\left(\frac{a}{2} + d\right)^3 - \left(\frac{a}{2}\right)^3 \right] + 2 \cdot \frac{1}{3} d \left(\frac{a}{2}\right)^3 + \frac{1}{3} b \left[\left(\frac{a}{2} + d\right)^3 - \left(\frac{a}{2}\right)^3 \right]$$

$$= \frac{1}{3} \left[\left(\frac{a}{2} + d\right)^3 - \left(\frac{a}{2}\right)^3 \right] (b + c) + \frac{2}{3} d \left(\frac{a}{2}\right)^3$$

(h) N16 (b)

About y-axis.



Moment of inertia for:

I_1, width $= d$, $d_2 = \frac{c}{2}$, $d_1 = 0$

$\frac{1}{3} d \left[\left(\frac{c}{2}\right)^3 - 0^3 \right]$

I_2, width $= a$, $d_2 = \frac{d}{2}$, $d_1 = 0$

$\frac{1}{3} a \left[\left(\frac{d}{2}\right)^3 - 0^3 \right]$

I_3, width $= d$, $d_2 = \frac{b}{2}$, $d_1 = 0$

$\frac{1}{3} d \left[\left(\frac{b}{2}\right)^3 - 0^3 \right]$.

Moments of inertia of I_4, I_5, I_6 are equal to moments of inertia of I_1, I_2, I_3, respectively.

Total moment of inertia =

$$2 \cdot \left(\frac{1}{3} d \cdot \left(\frac{c}{2}\right)^3 + \frac{1}{3} a \left(\frac{d}{2}\right)^3 + \frac{1}{3} d \left(\frac{b}{2}\right)^3 \right)$$

$$= \frac{1}{12} \left(c^3 d + a d^3 + b^3 d \right)$$