Unit 9 Assignment

1. We would like to estimate the true mean amount of money spent on textbooks by U of M students in one semester. Suppose that textbook costs are known to follow a normal distribution with standard deviation \$100. We take a random sample of 25 students and calculate a mean cost of \$375.

   (a) Calculate a 93% confidence interval for the true mean cost of textbooks for one semester.
   (b) Provide an interpretatioin the interval you constructed in (a).
   (c) Calculate a 98% confidence interval for the true mean cost of textbooks for one semester.
   (d) What is the effect of raising the confidence level on the length of the confidence interval?

2. (a) A random variable X follows a normal distribution with standard deviation 5. We take a random sample of 100 individuals from the population and calculate a confidence interval for μ to be (40.973, 43.027). What is the confidence level for this interval?
   (b) A random variable X follows a normal distribution with standard deviation σ. We take a random sample of n individuals from the population and calculate a 90% confidence interval for μ to be (23.7, 26.3). Using the same data, what would be a 99% confidence interval for μ?

3. A Winnipeg researcher would like to estimate the true mean commuting distance between home and the principal place of business for all city workers. Suppose it is known that the population standard deviation of commuting distances is 5.75 km.

   (a) What sample size is required to estimate the true mean commuting distance for all Winnipeg workers to within ± 3 km with 90% confidence?
   (b) What sample size is required to estimate the true mean commuting distance for all Winnipeg workers to within ± 1.5 km with 90% confidence?
   (c) For a given confidence level, how many times larger a sample size is required when we cut the desired margin of error in half?
   (d) Using only the result from (a), what sample size is required to estimate the mean commuting distance for all Winnipeg workers to within ± 1 km with 90% confidence?
   (e) What sample size is required to estimate the true mean commuting distance of all Winnipeg workers to within ± 3 km with 99% confidence?
   (f) What happens to the required sample size when we increase the confidence level?
   (g) The city of Vancouver has three times the population of Winnipeg. Suppose the population standard deviation of commuting distances in Vancouver is the same as in Winnipeg. What sample size would be required to estimate the true mean commuting distance for all Vancouver workers to within ± 3 km with 90% confidence?

4. An experimenter would like to determine whether the true mean voltage of batteries produced on a production line is less than 240. She randomly selects 16 batteries from the production line and measures their voltages. The 16 batteries have an average voltage of 238.3 V. Voltages are known to follow a normal distribution with standard deviation 4.1 V.

   (a) Conduct an appropriate hypothesis test at the 1% level of significance.
   (b) Provide an interpretation of the P-value of the test in (a).

5. We would like to determine if the true mean breaking strength of a certain type of plastic is greater than 7.0. A random sample of 40 specimens is selected, and the sample mean breaking strength is calculated to be 7.1. The population standard deviation of breaking strengths is known to be 0.25.

   (a) Breaking strengths are known to follow some right-skewed distribution. Why is it nevertheless appropriate to use inference methods which rely on the assumption of normality?
   (b) Conduct an appropriate hypothesis test at the 10% level of significance.

6. GPAs of students at a large university are known to follow a normal distribution with standard deviation 0.58. The GPAs for a random sample of 50 students are recorded. The sample has a mean GPA of 3.20 and a standard deviation of 0.65.

   (a) Construct a 95% confidence interval for the true mean GPA of all students at the university.
   (b) Provide an interpretation of the confidence interval in (a).
   (c) Conduct a hypothesis test at the 5% level of significance to determine whether there is significant evidence that the true mean GPA of all students at the university differs from 3.00.
   (d) Provide an interpretation of the P-value of the test in (c).
   (e) Could you have used the interval in (a) to conduct the test in (c)? Explain why or why not. If you could have used the interval, explain what your conclusion would be, and why.

7. In some mining operations, a by-product of the processing is mildly radioactive. Of prime concern is the possibility that release of these by-products into the environment may contaminate the freshwater supply. There are strict regulations for the maximum allowable radioactivity in supplies of drinking water. In order for the water to be considered safe, the population mean radioactivity level must be less than 5 picocuries per litre (pCi/L). A random sample of 30 specimens of water was taken from a city's water supply. The sample mean radioactivity was calculated to be 4.75 pCi/L and the sample standard deviation was calculated to be 0.87 pCi/L. Radioactivity levels are known to follow a normal distribution.

(a) Construct a 95% confidence interval for the true mean radioactivity.
(b) Conduct a hypothesis test at the 5% level of significance to determine whether there is sufficient evidence to declare the city's drinking water safe.
(c) Provide an interpretation of the P-value of the test in (b).
(d) Could you have used the interval in (a) to conduct the test in (b)? Explain why or why not. If you could have used the interval, explain what your conclusion would be, and why.

8. Lumber intended for building houses and other structures must be monitored for strength. We would like to test whether the true mean strength of Southern Pine wood is greater than the minimum specification of 3500 pounds per square inch. A random sample of 25 specimens of Southern Pine is selected. The mean strength is calculated to be 3600 pounds per square inch and the sample standard deviation is calculated to be 225 pounds per square inch. Strengths are known to follow a normal distribution.

(a) If we would like to be very sure that $\mu > 3500$ before we are willing to reject the null hypothesis, should she use a low or a high value of α? Explain.
(b) Conduct an appropriate hypothesis test at the 1% level of significance.
(c) Provide an interpretation of the meaning of the P-value of the test in (b).

9. We would like to determine whether the true mean resistance of a certain type of wire differs from 200 Ohms. A random sample of 22 wires is selected, and their resistances are measured. The sample has a mean resistance of 197 Ohms and a standard deviation of 10 Ohms. Suppose it is known that resistances follow a normal distribution.

(a) Construct a 90% confidence interval for the true mean resistance of all wires of this type.
(b) Provide an interpretation of the interval in (a).
(c) Conduct an appropriate hypothesis test at the 10% level of significance.
(d) Could you have used the interval in (a) to conduct the test in (c)? Explain why or why not. If you could have used the interval, explain what your conclusion would be, and why.