

MATH 2130 Summer Evening 2012 Problem Workshop 1

- Find the distance from the point $(3, -1, 5)$ to the line $x = 2 + 3t, y = 2t - 1, z = 4 + t$.
- Find the distance between the lines $y = 2x + 3z - 4, 3x + y - 2z = 6$ and $x = 2 + t, y = 3 - 2t, z = 1 + t$.
- The following three lines define a triangle

$$\begin{aligned} x &= -11 + 5s, & y &= s, & z &= -2 + 2s \\ x &= 1 + 2u, & y &= 1 - u, & z &= -2 - 4u \\ x &= -2 + 3t, & y &= -1 + 2t, & z &= -8 + 6t \end{aligned}$$

Find the area of the triangle.

- Find the centroid of the triangle in question 3. It is the point of intersection of the three medians of the triangle, which occurs on a median which is $2/3$ of the way from the vertex to the opposite midpoint.
- Find $\mathbf{v}'(3)$ if $\mathbf{v}(t) = t^2\hat{\mathbf{i}} + \arcsin(t/4)\hat{\mathbf{j}} + \ln(2t + 1)\hat{\mathbf{k}}$.
- If $f(t) = t^2 + 1$ and $\mathbf{v}(t) = e^t\hat{\mathbf{i}} + [t/(t^2 + 1)^3]\hat{\mathbf{j}} - t\sqrt{t^2 + 1}\hat{\mathbf{k}}$, evaluate $\int f(t)\mathbf{v}(t)dt$.
- Find a parameterization of the following curves.
 - $z = 2\sqrt{x^2 + y^2}, x^2 + y^2 = 3 - z$ from $(1, 0, 2)$ to $(-1, 0, 2)$ directed so y is always non-positive.
 - First octant part of $x^2 + z^2 = 4, x + y = 1$ directed so that z increases along the curve.
 - $z = x^2 + y^2, x^2 + y^2 - 4y = 0$ directed clockwise viewed from above.
- Find all unit tangent vectors to the curve $x^2 + z^2 = 4, x + y = 1$ at the point $(\sqrt{2}, 1 - \sqrt{2}, \sqrt{2})$.
- Find the unit tangent vector to the curve $x = t^2, y = 2t^3, z = 3t^2$ at the origin.
- Find the angle between the tangent vectors to the curves

$$x^2 + y = z + 4, x + 2y = 5 \quad \text{and} \quad x + y^2 = 5, 2x + 3y + 4z = 4$$
 at the point of intersection between the curves.
- Find the length of the curve $x = t + 1, y = 2t^{3/2} - 3, z = 4t - 2$ between the points $(2, -1, 2)$ and $(1, -3, -2)$

12. Set up but do not evaluate a definite integral to find the length of the curve $x^2 + y^2 = z^2 + 4$, $x + y = 4$ joining the points $(4, 0, 2/\sqrt{5})$ and $(2, 2, 2\sqrt{3})$. Simplify the integrand as much as possible.

Answers:

1. $\sqrt{6/7}$
2. $1/\sqrt{14}$
3. $\sqrt{629}/2$
4. $(4/3, 2, 4/3)$
5. $6\hat{\mathbf{i}} + (1/\sqrt{7})\hat{\mathbf{j}} + (2/7)\hat{\mathbf{k}}$.
6. $(t^2 - 2t + 3)e^t\hat{\mathbf{i}} + [1/2(t^2 + 1)]\hat{\mathbf{j}} - \frac{1}{5}(t^2 + 1)^{5/2}\hat{\mathbf{k}} + \mathbf{C}$, where \mathbf{C} is a constant vector.
7. (a) $x = \cos t, y = -\sin t, z = 2, 0 \leq t \leq \pi$.
 (b) $x = 2 \cos t, y = 1 - 2 \cos t, z = 2 \sin t, \pi/3 \leq t \leq \pi/2$.
 (c) $x = 2 \cos t, y = 2 - 2 \sin t, z = 8(1 - \sin t), 0 \leq t \leq 2\pi$.
8. $\pm \frac{1}{\sqrt{3}}(-\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}})$
9. $\frac{1}{\sqrt{10}}(\hat{\mathbf{i}} + 3\hat{\mathbf{k}})$
10. $\arccos\left(\frac{-21}{\sqrt{14}\sqrt{297}}\right)$.
11. $\frac{2}{\sqrt{27}}(26^{3/2} - 17^{3/2})$.
12. $2 \int_2^4 \sqrt{\frac{t^2 - 4t + 7}{t^2 - 4t + 10}} dt$