Dec 2007 Final

THE UNIVERSITY OF MANITOBA

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COURSE

YEAR

SECTION

DATE

NAME Solutions originally by Dawit ydawit@yahoo.com

PROBLEM

SHEET OF

A 1. $19.6 = V_{f2}$ $0 = V_{f1}$ $(V_{cm})_f = \frac{m_1 v_{1i} + m_2 v_{2i}}{m_T} = \frac{2 \times 19.6}{3} = 13.06 = \boxed{13.1 \text{ m/s}}$

C 2. $K_2 = \frac{1}{2} m_2 v^2$ $K_1 = \frac{1}{2} m_1 v^2$ $K_p = \frac{1}{2} I \omega^2 = \frac{1}{2} \frac{I}{r^2} v^2$

Compare $m_1, m_2, \frac{I}{r^2}$! $\frac{I}{r^2}$ is biggest at 5 ∴ pulley

B 3. 6m/s, 2kg; 0m/s, 4kg; V_{1f}, 2kg; V_{2f}, 4kg

Rel. Velocity of approach = Velocity of separation

$V_{1i} - V_{2i} = V_{2f} - V_{1f}$ $V_{2f} = V_{1i} + V_{1f}$

$P_i = P_f$

$2(6) = 2V_{1f} + 4V_{2f}$

$12 = 2V_{1f} + 4V_{1i} + 4V_{1f}$ (24)

$-12 = 6V_{1f}$

$\boxed{-2 = V_{1f}}$

C 4. $|A \times B| = |A||B| \sin\theta$

$2|A||B| \sin\theta = |A||B| \sin(\theta + 2\theta)$

$2\sin\theta = \sin(\theta + 2\theta)$

$2\sin\theta = \sin\theta\cos 2\theta + \cos\theta \sin 2\theta$

$(2 - \cos 2\theta)\sin\theta = \sin 2\theta \cos\theta$

$\frac{\sin\theta}{\cos\theta} = \frac{\sin 2\theta}{(2 - \cos 2\theta)}$

$\tan\theta = \frac{\sin 2\theta}{(2 - \cos 2\theta)}$ $\boxed{\theta = 17.9°}$

D 5. Just find centre of mass: $X_{cm} = \frac{m_1 x_1 + m_2 x_2}{m_T} = \frac{60(0) + 140(12)}{140} = \boxed{6.9 \text{ m}}$

E 6. rocket $v = 0.75c$, S′; what is u_x, S

rocket measures comet (u') as $u_x' = 0.5c$

$u_x = \frac{u_x' + V}{1 + \frac{u_x' V}{c^2}} = \frac{0.5c + 0.75c}{1 + \frac{0.5c \cdot 0.75c}{c^2}} = \boxed{\frac{0.5 + 0.75}{1 + (0.5)(0.75)}c}$

$= \boxed{0.9c}$

D 7. $E_i = \frac{1}{2} m v_i^2$ $E_f = \frac{1}{2} k d^2$ $\Delta E = W_{fric} = -F_{fric} d$

$\frac{1}{2} k d^2 - \frac{1}{2} m v_i^2 = -F_f d$

$k d^2 - m v_i^2 + 2\mu_k m g d = 0$

$k d^2 + 2\mu_k m g d - m v_i^2 = 0$

$95 d^2 + 2(0.25)(1.5)(9.8) d - (1.5)(1.8 \text{ m/s})^2 = 0$

$95 d^2 + 7.35 d - 4.86 = 0$

$\boxed{0.191 \text{ m}}$

C 8. $v_i = ?$, $t = 2.5$ s, $v = 2.8$ m/s, 2 m, 18 m, accel.

$\Delta x = v_i t + \frac{1}{2} a t^2$ and $v_i = v_f - at$ $a = \frac{2(v_f t - \Delta x)}{t^2} = 0.32 \text{ m/s}^2$

OR use const. accel eqn $\Delta x = \frac{1}{2}(v_i + v_f) t \rightarrow v_i = \frac{2\Delta x}{t} - v_f$

D 9. $P_i = P_f$ $0 + (-20) + 10(10) = 15 V_f$ $V_f = 0$ ∴ energy lost is E_i

$E_i = \frac{1}{2} m_1 V_1^2 + \frac{1}{2} m_2 V_2^2 = 1500 \text{ J}$

B 10. $\vec{r}_0 = 3\hat{i}$ m

$\vec{L} = \vec{r} \times \vec{p} = \vec{r} \times m\vec{v} = \vec{r} \times m(\vec{v}_0 + \vec{a}t)$

$= (\vec{r}_0 + \vec{v}_0 t + \frac{1}{2}\vec{a}t^2) \times (m\vec{a}t)$

$= (3\hat{i} + \frac{1}{2}(4\hat{i} - 3\hat{j})2^2) \times 2(4\hat{i} - 3\hat{j})2$

$= (8\hat{i} - 6\hat{j}) \times (16\hat{i} - 12\hat{j})$

$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 11 & -6 & 0 \\ 16 & -12 & 0 \end{vmatrix}$ $0 \cdot 0\,\hat{i} - 0\,\hat{j} - 176 - 96\,\hat{k} = \boxed{-36\hat{k}}$

THE UNIVERSITY OF MANITOBA

YEAR _____ NAME _____

SECTION _____

COURSE _____ DATE _____ PROBLEM _____ SHEET OF _____

11. [A]

$I_i\omega_i = I_f\omega_f$

$600(0.5) = [600 + 20(3^2)]\omega_f$

$\omega_f = 0.615 \text{ rad/s}$

12. [B]

$a_T = 4.4 \text{ m/s}^2$

$a = 6.0 \text{ m/s}^2$

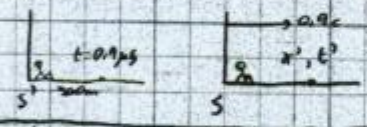
$a_N = \frac{v^2}{r}$

$a^2 = a_T^2 + a_n^2$ $\quad a_n = \sqrt{a^2 - a_T^2} = 4.08 \text{ m/s}^2$

$v = \sqrt{r a_n} = \sqrt{2(4.08)} = 2.85 \text{ m/s}$

a_{normal} is $a_{centrip}$

13. [E]



$\delta = \frac{1}{\sqrt{1-0.9^2}} = 2.294$

$x' = \delta(x - vt) = 589.6$

$t' = \delta\left(t - \frac{vx}{c^2}\right) = -1.38 \mu s$

14. [D]

$W = mgl(1-\cos\theta) + \frac{1}{2}mv^2$

$= mgl(1-\cos\theta) + \frac{1}{2}m(l\sin\theta\sqrt{g\tan\theta})$

$= mgl(1-\cos\theta) + \frac{1}{2}mgl\sin\theta\tan\theta$

$T\sin\theta = \frac{mv^2}{r}$ $\quad T\cos\theta = mg$ $\quad \rightarrow T\tan\theta = \frac{v^2}{r\sin\theta}$ $\quad \rightarrow v^2 = gl\sin\theta\tan\theta$

15. [E]

$E = \gamma E_0$ $\quad KE = E - E_0 = (\gamma - 1)E_0$

$E_0 = mc^2 = (9.11\times10^{-31})(3\times10^8)^2 \text{ J} = 8.199\times10^{-14}$

$\gamma = \sqrt{\frac{a^2c^2}{a^2c^2 - v^2a^2}} = 3.2$

$KE = (\gamma - 1)E_0 = (3.2 - 1)8.199\times10^{-14} = 1.8\times10^{-13}$

16. [C]

$mg = 2(kh\cos30)$

$\Delta x = \frac{mg}{2k\cos30} = 0.0578 \text{ m}$

17. [A] On paper

18. [D]

$ME_i = ME_f$

$\frac{1}{2}k(\Delta d)^2 = mgh \rightarrow h = \frac{k(\Delta d)^2}{2mg} = 0.466 \text{ m}$

$(d + \Delta d)\sin\theta = 0.466$

$d + \Delta d = \frac{0.466}{\sin35} = 0.81 \text{ m}$

19. [A]

$0^2 = 120^2 + 2(-9.8)y_2$ $\quad y_2 = 734.69 \text{ m}$ $\quad > 1094.69 \text{ m} = 1.1 \text{ km}$

$120^2 = 0^2 + 2(120)y_1$ $\quad y_1 = 360 \text{ m}$

20. [E]

max spring energy = ΔPE

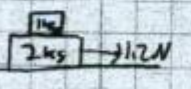
$U_h = 0$

max energy stored = $mg(13.6) = 9329.6 \text{ J}$

21. [B] On paper

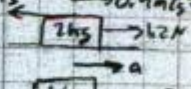
22. [E] On paper

23.



$a = \frac{F}{m_1 + m_2} = 0.4 \text{ m/s}^2$

forces on 2kg



1kg

$f_3 = 1(0.4) = 0.4 \text{ N}$

THE UNIVERSITY OF MANITOBA

YEAR _____ NAME _____

SECTION _____

COURSE _____ DATE _____ PROBLEM _____ SHEET OF _____

24. [A]

$KE = (\gamma - 1) E_0$

$\frac{1}{2} E_0 = (\gamma - 1) E_0$

$\gamma = 2$

$E_0 = mc^2$

$E = \gamma mc^2$

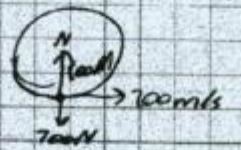
$\gamma mc^2 = KE + mc^2$

$KE = \gamma mc^2 - mc^2$

$= mc^2(\gamma - 1)$ → $KE = (\gamma - 1) E_0$

$\beta = \sqrt{1 - \frac{1}{\gamma^2}} = \sqrt{1 - \frac{1}{2^2}} = 0.866 \Rightarrow v = \beta c = 0.866c$

25.



$N - mg = \frac{mv^2}{r}$ → $N = m\left(g + \frac{v^2}{r}\right)$

$= \frac{700}{9.8}\left(9.8 + \frac{200^2}{800}\right) = 4271 N$ up