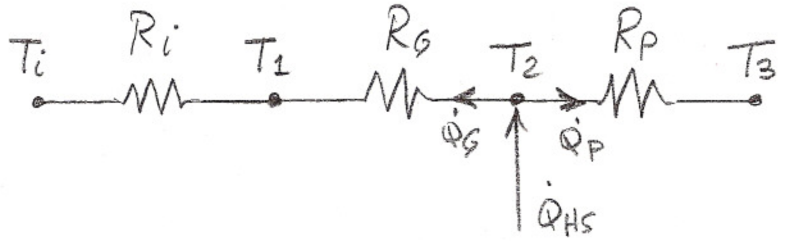
130.112 Thermal Sciences W06
Final Exam Solution

1. (a)



(b)

$$R_i = \frac{1}{h_i A} = \frac{1}{10(0.8)} = 0.125 \left[\frac{K}{W}\right] \leftarrow$$

$$R_G = \frac{L_G}{k_G A} = \frac{0.0144}{1.20\ (0.8)} = 0.0150 \left[\frac{K}{W}\right] \leftarrow$$

$$R_P = \frac{L_P}{k_P A} = \frac{0.0225}{0.750\ (0.8)} = 0.0375 \left[\frac{K}{W}\right] \leftarrow$$

(c) $\dot{Q}_{HS} = 0$ Calculate $\dot{Q}_P$ $\dot{Q}_G = -\dot{Q}_P$

$$\dot{Q}_P = \frac{(T_i - T_3)}{(R_i + R_G + R_P)} = \frac{(25 - 19.5)}{(0.125 + 0.015 + 0.0375)} = \frac{8.5}{0.1775} = 30.986\ (W)$$

$$T_1 = T_i - \dot{Q}_P R_i = 25 - 30.986\ (0.125) = 21.13\ [°C] \leftarrow$$

what is the dew point temperature? $\phi = 0.98$

$$P_g = P_{sat}(25\ °C) = 3.169\ (kPa)$$

$$P_v = \phi P_g = 0.98\ (3.169) = 3.1056\ (kPa)$$

$$T_{dp} = T_{sat}(3.1056\ kPa) = 24.08 + \frac{(3.1056 - 3.0)}{(4.0 - 3.0)}(28.96 - 24.08)$$

$$T_{dp} = 24.60\ [°C]$$

$T_1 < T_{dp}$ $\Rightarrow$ Yes, the mirror will fog up in this case because the surface temperature is below the dew point temperature.

1 (d) $T_1 = 27\ (°C)$, $\dot{Q}_{HS} = ?$

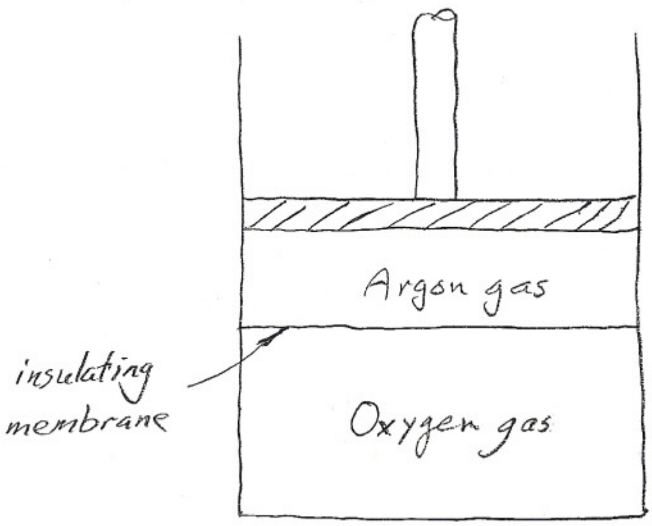
In this case $\dot{Q}_G = \dfrac{(T_1 - T_i)}{R_i} = \dfrac{27.0 - 25.0}{0.125} = 16.0\ [W]$

$$T_2 = T_1 + \dot{Q}_G R_G = 27.0 + 16.0\,(0.0150) = 27.24\ (°C)$$

$$\dot{Q}_P = \frac{(T_2 - T_1)}{R_P} = \frac{(27.24 - 19.5)}{0.0375} = 206.40\ (W)$$

$$\dot{Q}_{HS} = \dot{Q}_G + \dot{Q}_P = 16.0 + 206.40 = 222.4\ [W] \leftarrow$$

#C, April 2006 solution



Initial state:

$T_{Ar,1} = 300\ K$

$V_{Ar,1} = 0.6\ m^3$

$n_{Ar} = 0.09\ kmol$

$T_{O_2,1} = 500\ K$

$V_{O_2,1} = 0.8\ m^3$

$n_{O_2} = 0.18\ kmol$

(a) $PV = n\bar{R}T, \quad \bar{R} = 8.31451\ kJ/kmol\cdot K$

$$\left.\begin{aligned} P_{Ar,1} &= 0.09 \times 8.31451 \times 300/0.6 \\ &= 374.152\ kPa \\ P_{O_2,1} &= 0.18 \times 8.31451 \times 500/0.8 \\ &= 935.382\ kPa \end{aligned}\right\}$$

not required (information only)

$M_{Ar} = 39.948\ kg/kmol, \quad M_{O_2} = 31.999\ kg/kmol$

$m_i = n_i M_i$

$m_{Ar} = 0.09 \times 39.948 = 3.59532\ kg$

$m_{O_2} = 0.18 \times 31.999 = 5.75982\ kg$

(b) membrane ruptures and gases mix

$$\left\{\begin{aligned} n_{tot} &= n_{Ar} + n_{O_2} = 0.09 + 0.18 = 0.27\ kmol \\ m_{tot} &= m_{Ar} + m_{O_2} = 3.59532 + 5.75982 = 9.35514\ kg \end{aligned}\right.$$

(b) continued: $(T_2)_{mix} = 400\ K$

First law: ${}_1Q_2 - \cancel{{}_1W_2} = U_2 - U_1 + \cancel{\Delta KE} + \cancel{\Delta PE}$
(process 1-2)

$U_2 - U_1 = m_{Ar}(u_2 - u_1)_{Ar} + m_{O_2}(u_2 - u_1)_{O_2}$

$= m_{Ar} C_{v0,Ar}(T_2 - T_1)_{Ar} + m_{O_2} C_{v0,O_2}(T_2 - T_1)_{O_2}$

$T_{2,Ar} = T_{2,O_2} = T_{2,mix}$

(Table A.5) $C_{v0,Ar} = 0.312\ kJ/kg\cdot K$, $C_{v0,O_2} = 0.662\ kJ/kg\cdot K$

$\therefore {}_1Q_2 = 3.59532 \times 0.312 \times (400 - 300) +$

$+ 5.75982 \times 0.662 \times (400 - 500)$

${}_1Q_2 = 112.173 + (-381.300) = -269.126\ kJ$

Negative indicates that heat is transferred from the cylinder.

(c) $P_2 V_2 = n_{tot} \bar{R}\, T_2$ (or, $P_2 V_2 = m_{tot} R_{mix} T_2$)

$\therefore P_2 = \dfrac{0.27 \times 8.31451 \times 400}{1.4} = 641.405\ kPa$

(d) $n_{tot} = 0.09 + 0.18 = 0.27\ kmol$

$C_{Ar} = \dfrac{m_{Ar}}{m_{tot}} = \dfrac{3.59532}{9.35514} = 0.384314$

these could be found at the beginning also

$C_{O_2} = \dfrac{m_{O_2}}{m_{tot}} = \dfrac{5.75982}{9.35514} = 0.615685$

$m_{tot} = 3.59532 + 5.75982 = 9.35514\ kg$

$$M_{mix} = \frac{1}{\underbrace{\sum C_i/M_i}_{\left(\frac{C_{Ar}}{M_{Ar}} + \frac{C_{O_2}}{M_{O_2}}\right)}} = \frac{1}{\frac{0.384314}{39.948} + \frac{0.615685}{31.999}} = 34.6486 \text{ kg/kmol}$$

$$R_{mix} = \frac{\bar{R}}{M_{mix}} = \frac{8.31451}{34.6486} = 0.239966 \text{ kJ/kg·K}$$

(e) $PV^{0.25} = C$ polytropic process with $n = 0.25$

$$_2W_3 = \frac{P_3V_3 - P_2V_2}{1-n} = \frac{(mR)_{mix}(T_3 - T_2)}{1-n}$$

$$_2W_3 = \frac{(9.35514 \times 0.239966) \times (600 - 400)}{1 - 0.25}$$

$$_2W_3 = 598.644 \text{ kJ}$$

First law for process 2-3:

$$_2Q_3 - {_2W_3} = U_3 - U_2 + \cancel{\Delta KE} + \cancel{\Delta PE}$$

$$_2Q_3 = {_2W_3} + m_{tot}\, C_{v0,mix}\,(T_3 - T_2)$$

Need $C_{v0,mix} = C_{Ar}C_{v0,Ar} + C_{O_2}C_{v0,O_2}$

$$= 0.384314 \times 0.312 + 0.615685 \times 0.662$$

$$= 0.527489 \text{ kJ/kg·K}$$

$$\therefore\ _2Q_3 = 598.644 + 9.35514 \times 0.527489 \times (600 - 400)$$

$$_2Q_3 = 598.644 + 986.946 = 1585.59 \text{ kJ}$$

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(f) $P_2 V_2^{0.25} = P_3 V_3^{0.25}$, $P_3 V_3 = n \bar{R} T_3$

$$P_2 V_2^{0.25} = \frac{n_{tot} \bar{R} T_3}{V_3} V_3^{0.25} = \frac{n_{tot} \bar{R} T_3}{V_3^{0.75}}$$

$$V_3^{0.75} = \frac{n_{tot} \bar{R} T_3}{P_2 V_2^{0.25}}$$

$$\left(V_3^{0.75}\right)^{\frac{1}{0.75}} = V_3 = \left(\frac{n_{tot} \bar{R} T_3}{P_2 V_2^{0.25}}\right)^{\frac{1}{0.75}}$$

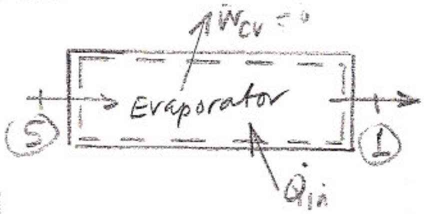
$$V_3 = \left(\frac{0.27 \times 8.31451 \times 600}{641.405 \times (1.4)^{0.25}}\right)^{\frac{1}{0.75}}$$

$$V_3 = 2.4039 \ m^3$$

$$P_3 = \frac{P_2 V_2^{0.25}}{V_3^{0.25}} = 641.405 \times \left(\frac{1.4}{2.4039}\right)^{0.25}$$

$$P_3 = 560.318 \ kPa.$$

3. (a) • First Law for the Evaporator



$$\dot{Q}_{in} - \cancel{\dot{W}_{cv}}^{\,0} = \dot{m}_1 (h_1 - h_5)$$

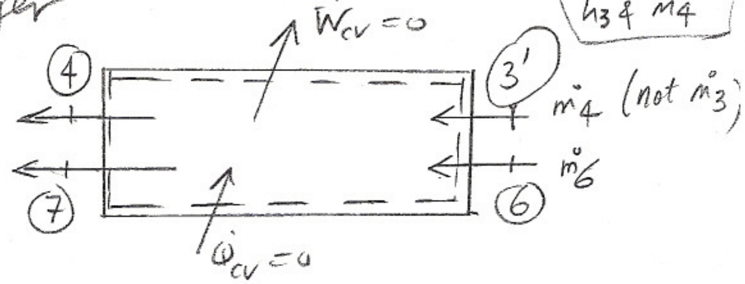
$$\dot{m}_1 = \frac{\dot{Q}_{in}}{(h_1 - h_5)}$$

$$\dot{m}_1 = \frac{162.9}{(237.73 - 50.49)} = 0.8700 \left[\frac{kg}{s}\right]$$

$$h_1 = h_g\big|_{-30°C} = 237.73 \left[\frac{kJ}{kg}\right]$$

$$h_5 = h_4 = h_f\big|_{\substack{1200\,kPa \\ 5\%}} = 50.49 \left[\frac{kJ}{kg}\right]$$

• First Law for the Heat Exchanger



state 3' has h_3 & $\dot{m}_4$

$$\cancel{\dot{Q}_{cv}}^{\,0} + \sum \dot{m}_i h_i = \cancel{\dot{W}_{cv}}^{\,0} + \sum \dot{m}_e h_e$$

$$\dot{m}_4 h_3 + \dot{m}_6 h_6 = \dot{m}_4 h_4 + \dot{m}_7 h_7$$

Note

$$\dot{m}_6 = \dot{m}_7 \qquad \dot{m}_4 = \dot{m}_1$$

$$\dot{m}_1 h_3 + \dot{m}_7 h_6 = \dot{m}_1 h_4 + \dot{m}_7 h_7$$

$h_6 = h_3$

$$\dot{m}_7 = \frac{\dot{m}_1 (h_3 - h_4)}{(h_7 - h_3)}$$

$$\dot{m}_7 = 0.8700 \frac{(81.57 - 50.49)}{(246.14 - 81.57)}$$

$$\dot{m}_7 = 0.1643 \,[kg/s]$$

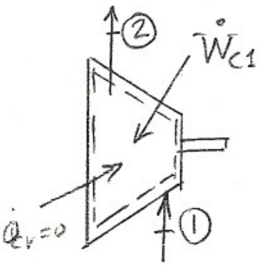
$$h_3 = h_f\big|_{1200\,kPa} = 81.25 + \frac{(1200 - 1191.9)}{(1354.8 - 1191.9)}(87.70 - 81.25)$$

$$h_3 = 81.57 \,[kJ/kg]$$

$$h_4 = 50.49 \,[kJ/kg]$$

$$h_7 = h_g\big|_{-10°C} = 246.14 \left[\frac{kJ}{kg}\right]$$

(b) • First Law for Compressor 1



compressor 1
Note direction of $\dot{W}_{c1}$

$$\dot{W}_{c1} + \dot{m}_1 h_1 = \dot{m}_2 h_2 \qquad \dot{m}_2 = \dot{m}_1$$

$$\dot{W}_{c1} = \dot{m}_1 (h_2 - h_1)$$

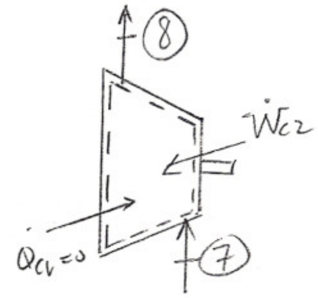
$$\dot{W}_{c1} = 0.8700 (292.42 - 237.73)$$

$$\dot{W}_{c1} = 47.58 \,[kW]$$

$$h_2 = h\big|_{\substack{1200\,kPa \\ 70°C}} = 292.42 \left[\frac{kJ}{kg}\right]$$

3 (b) continued

· First Law for Compressor 2



$$\dot{W}_{c2} = \dot{m}_7 (h_8 - h_7)$$

$$= 0.1643(276.01 - 246.14)$$

$$\dot{W}_{c2} = 4.908 \text{ (kW)}$$

$$h_8 = h\Big|_{\substack{1200\text{ kPa} \\ 50\,°C}} = 276.01 \left(\frac{kJ}{kg}\right)$$

(c) coefficient of performance

$$\beta = \frac{\dot{Q}_{in}}{\dot{W}_{c1} + \dot{W}_{c2}} = \frac{162.9}{47.58 + 4.908} = \underline{3.104}$$

(d) Maximum possible coefficient of performance

$$\beta_{rev} = \frac{T_L}{T_H - T_L}$$

In this case $T_L = -30°C = 243.15\,K$

$T_H = T_{condenser} = T_{sat}(1200 \text{ kPa})$

$$T_{sat}(1200 \text{ kPa}) = 30 + \frac{(1200 - 1191.9)(35-30)}{(1354.9 - 1191.9)} = 30.25\,°C$$

$T_H = 303.40\,K$

(or: $T_{sat} = 30.26$ from Table B.4.2)

$$\beta_{rev} = \frac{243.15}{(303.40 - 243.15)}$$

$$\beta_{rev} = 4.036$$

(e) For the diagram (next page)

$P_{sat}(-10\,°C) = 354.3$ (kPa)

$P_{sat}(-30\,°C) = 163.5$ (kPa)

$T_{sat}(1200 \text{ kPa}) = 30.26\,°C$ (already calculated)

$$x_5 = \frac{(50.49 - 10.73)}{(237.73 - 10.73)} = 0.1752$$

$h_6 = h_3$

$$x_6 = \frac{(81.57 - 33.01)}{(246.14 - 33.01)} = 0.2278$$

$$v_5 = 0.1752(0.000725) + (1-0.1752)\,0.13584$$

$$v_5 = 0.11217 \text{ (m}^3\text{/kg)}$$

$$v_6 = 0.2278(0.000759) + (1-0.2278)\,0.06534$$

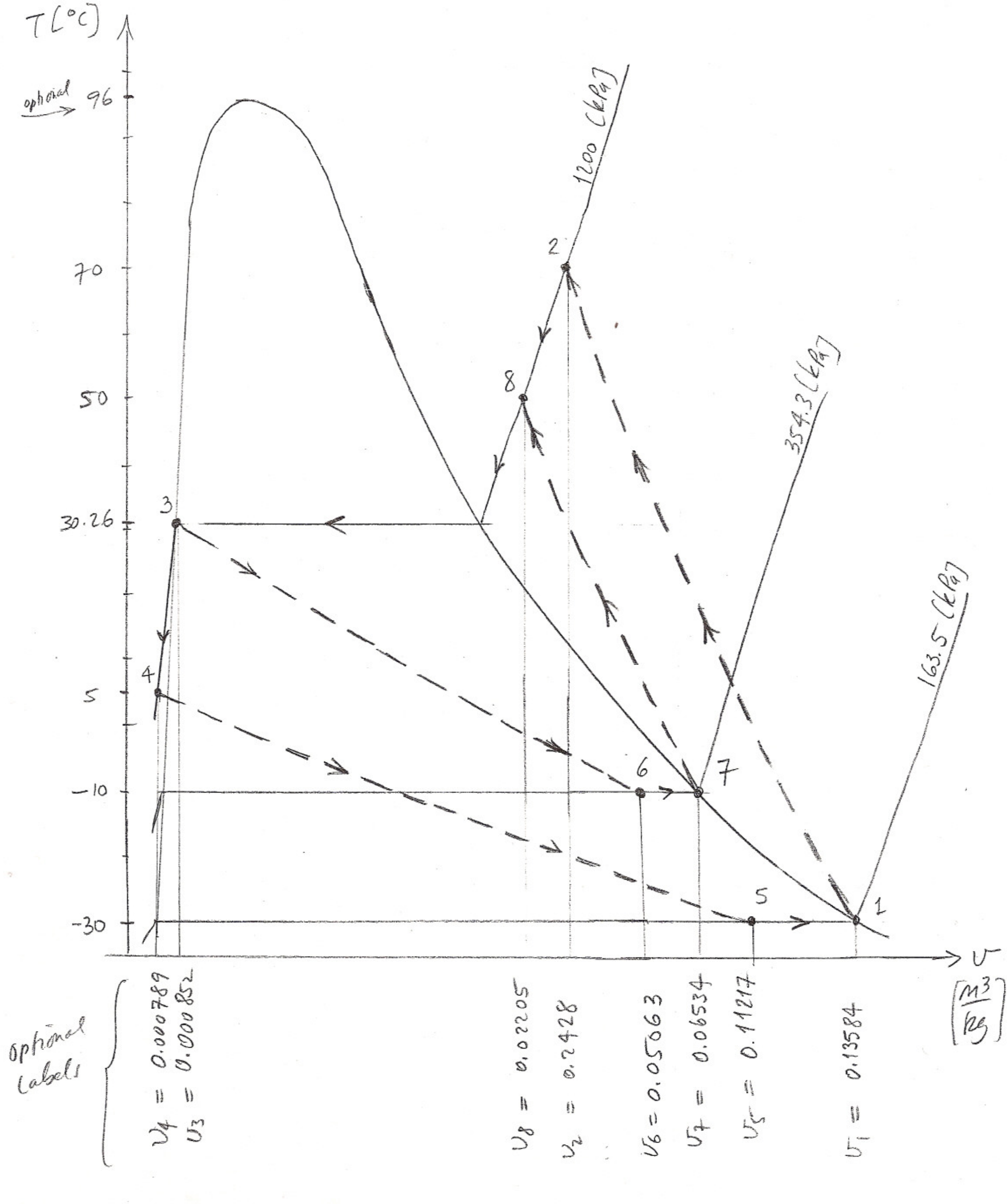
$$v_6 = 0.05063 \text{ (m}^3\text{/kg)}$$

$v_7 = 0.07763$ (m³/kg) $v_8 = 0.02205$ (m³/kg)

$v_1 = 0.13584$ (m³/kg) $v_2 = 0.02928$ (m³/kg)

$v_6 < v_5$

3 (P) continued



4. $\eta_{th,rev} = \left(1 - \frac{T_{L_1}}{T_{H_1}}\right) = \frac{\dot{W}}{\dot{Q}_{W1}}$ $\quad T_{H_1} = T_W = 60\,[°C] = 333.15\,(K)$

$T_{L_1} = 20\,[°C] = 293.15\,(K)$

$$\eta_{th,rev} = \left(1 - \frac{293.15}{333.15}\right) = 0.1201$$

$$\dot{W} = 0.1201\,\dot{Q}_{W1}$$

$$\beta' = \frac{T_{H2}}{(T_{H2} - T_{L2})} = \frac{\dot{Q}_{H2}}{\dot{W}}$$ $\quad T_{H_2} = 145\,[°C] = 418.15\,(K)$

$T_{L_2} = T_W = 333.15\,(K)$

$$\beta' = \frac{418.15}{(418.15 - 333.15)} = 4.9194$$

$$\dot{Q}_{H_2} = 4.9194\,\dot{W}$$

$$\dot{Q}_{W2} = \dot{Q}_{H_2} - \dot{W} = 4.9194\,\dot{W} - \dot{W} = 3.9194\,\dot{W}$$

$$\dot{Q}_{W2} = 3.9194\,(0.1201\,\dot{Q}_{W1}) = 0.4707\,\dot{Q}_{W1}$$

$$\dot{Q}_{W1} + \dot{Q}_{W2} = 5.00$$

$$\dot{Q}_{W1} + 0.4707\,\dot{Q}_{W1} = 5.00$$

$$\dot{Q}_{W1} = \frac{5.00}{1.4707} = 3.400\,[MW]$$

$$\dot{Q}_{W_2} = 5.00 - 3.400 = 1.600\,(MW)$$

$$\dot{W} = 0.1201\,(3.400) = 0.4083\,(MW)$$

$$\dot{Q}_{H2} = \dot{Q}_{W_2} + \dot{W} = 1.600 + 0.4083 = \underline{\underline{2.0083\,[MW]}}$$

#5., April 2006 solution   Assume air flows at $P = 100$ kPa.

(a) stream 1 $T_1 = 40°C$, $\phi_1 = 60\%$, $\dot{m}_{a_1} = 0.2$ kg/s

$$P_{g@40°C} = 7.384 \text{ kPa} \quad (\text{Table B.1.1})$$

$$\omega_1 = \frac{0.622\,\phi P_g}{P - \phi P_g} = \frac{0.622 \times 0.6 \times 7.384}{100 - 0.6 \times 7.384}$$

$$\omega_1 = 0.0288345 \ \frac{\text{kg water}}{\text{kg dry air}}$$

or, $\phi = \dfrac{P_{v_1}}{P_{g_1}}$ $\therefore$ $P_{v_1} = 0.6 \times 7.384 = 4.4304$ kPa

$$P_{a_1} = P - P_{v_1} = 100 - 4.4304 = 95.5696 \text{ kPa}$$

$$\omega_1 = 0.622 \frac{P_{v_1}}{P_{a_1}} = 0.622 \times \frac{4.4304}{95.5696} = 0.0288345 \ \frac{\text{kg water}}{\text{kg dry air}}$$

(b) state 4

$T_4 = 15°C$, $\phi_4 = 100\%$

$$P_{g@15°C} = 1.705 \text{ kPa}$$

$$\omega_4 = \frac{0.622 \times \phi_4 \times P_{g,4}}{P - \phi_4 P_{g,4}} = \frac{0.622 \times 1.0 \times 1.705}{100 - 1.0 \times 1.705}$$

$$\omega_4 = 0.0107890 \text{ kg water/kg dry air}$$

or, $P_{v_4} = 1 \times 1.705 = 1.705$ kPa

$$P_{a_4} = 100 - 1.705 = 98.295 \text{ kPa}$$

$$\omega_4 = 0.622 \frac{P_{v4}}{P_{a4}} = 0.622 \times \frac{1.705}{98.295} = 0.010789 \ \frac{\text{kg water}}{\text{kg dry air}}$$

conservation of mass for dry air: $\dot{m}_{a_1} + \dot{m}_{a_2} = \dot{m}_{a_3} = \dot{m}_{a4}$

$$\therefore \dot{m}_{a4} = 0.2 + 0.3 = 0.5 \text{ kg/s}$$

(c) conservation of mass for water:

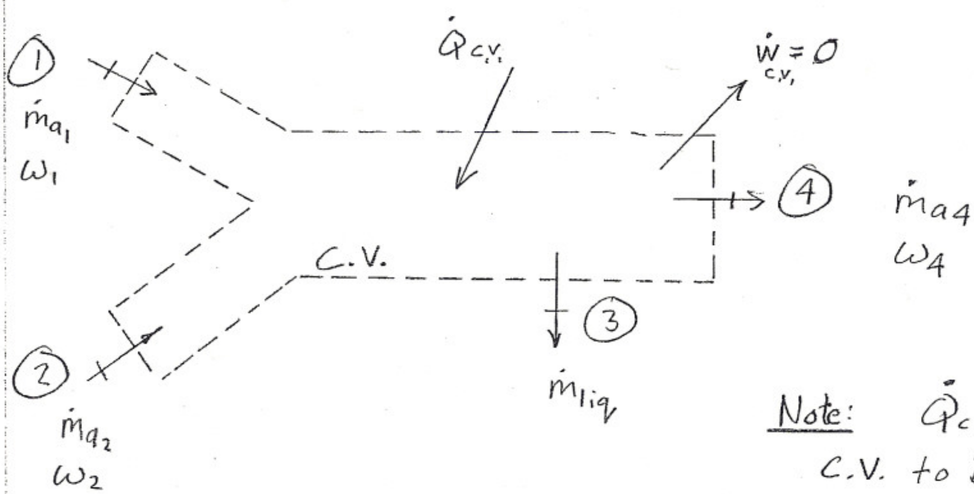
$$\omega_1 \dot{m}_{a_1} + \omega_2 \dot{m}_{a_2} - \dot{m}_{liq} = \omega_4 \dot{m}_{a4}$$

or, $\dot{m}_{liq} = \omega_1 \dot{m}_{a_1} + \omega_2 \dot{m}_{a_2} - \omega_4 \dot{m}_{a4}$, given: $\omega_2 = 0.018 \frac{\text{kg } H_2O}{\text{kg d.a.}}$

$$= 0.0288345 \times 0.2 + 0.018 \times 0.3 - 0.0107890 \times 0.5$$

$$\dot{m}_{liq} = 5.7724 \times 10^{-3} \frac{\text{kg water}}{\text{s}}$$

(d)



Note: $\dot{Q}_{c.v.}$ assumed into C.V. to be consistent with the First law.

First law (conservation of energy):

$$\dot{Q}_{c.v.} = \left(\dot{m}_{a4} h_4 + \dot{m}_{liq} h_{liq}\right) - \left(\dot{m}_{a_1} h_1 + \dot{m}_{a_2} h_2\right)$$

we know $\dot{m}_{a4} = \dot{m}_{a_1} + \dot{m}_{a_2}$ from mass conservation

$$\dot{Q}_{c.v.} = (\dot{m}_{a_1} + \dot{m}_{a_2}) h_4 - \dot{m}_{a_1} h_1 - \dot{m}_{a_2} h_2 + \dot{m}_{liq} h_{f@T_4}$$

$$\dot{Q}_{c.v.} = \dot{m}_{a_1}(h_4 - h_1) + \dot{m}_{a_2}(h_4 - h_2) + \dot{m}_{liq} h_{f@T_4}$$

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$h \simeq C_{P_0,air} T + \omega h_g$ for an air-water-vapour mixture

$$\therefore \dot{Q}_{c.v.} = \dot{m}_{a_1} C_{P_0,air} (T_4 - T_1) + \dot{m}_{a_1} (\omega_4 h_{g_4} - \omega_1 h_{g_1}) +$$

$$+ \dot{m}_{a_2} C_{P_0,air} (T_4 - T_2) + \dot{m}_{a_2} (\omega_4 h_{g_4} - \omega_2 h_{g_2}) +$$

$$+ \dot{m}_{liq} h_{f@15°C}$$

From Table B.1.1, $h_{g_1} = h_{g@40°C} = 2574.26$ kJ/kg

$h_{g_2} = h_{g@25°C} = 2547.17$ kJ/kg

$h_{g_4} = h_{g@15°C} = 2528.91$ kJ/kg

$C_{P_0,air} = 1.004$ kJ/kg·K (Table A.5)

$$\therefore \dot{Q}_{c.v.} = 0.2 \times 1.004 \times (15 - 40) + 0.2 \times (0.0107890 \times 2528.91 -$$

$$- 0.0288345 \times 2574.26) + 0.3 \times 1.004 \times (15 - 25) +$$

$$+ 0.3 \times (0.0107890 \times 2528.91 - 0.018 \times 2547.17) +$$

$$+ 5.7724 \times 10^{-3} \times 62.98$$

$\dot{Q}_{c.v.} = -22.6264$ KW

or, $\dot{Q}_{cooling} = 22.626$ KW

(i.e., $\dot{Q}_{c.v.} = \cancel{\dot{Q}_{in}} - \dot{Q}_{cooling}$)