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[9] 1. (a) The following sequence of functions is defined on the interval $[0, \infty)$

$$\{1 + (x^2 - 2x + 1)\, e^{-nx}\}_{n=1}^{\infty}.$$

Determine whether the sequence is convergent or divergent. If it converges, find the limit function.

(b) Find all values of x for which the sequence $\left\{1 + \frac{|x-2|^n}{n!}\right\}_{n=1}^{\infty}$ converges. (Explain your work.)

[12] 2. Let $f(x) = e^{1-2x}$ for $-\infty < x < \infty$. Then:

[4] (a) Find the first 4 terms of the Taylor series of $f(x)$ about 1.

[4] (b) Find the n^{th}-remainder (i.e. $R_n(1, x)$).

[4] (c) Show that $\lim_{n\to\infty} R_n(1, x) = 0$ *only* for the case $x > 1$.

[8] 3. Let $f(x) = \frac{1+a}{1+ax}$, find the value of a such that the 4^{th} term of Taylor series of $f(x)$ about 1 is $-\frac{1}{27}(x-1)^3$.
(**Hint**: You may use geometric series)

[8] 4. Find the radius of convergence and the open interval of convergence for the series $\sum_{n=0}^{\infty} \frac{(-1)^n}{3^{2n}\, e^{\sqrt{n}}}\,(x+1)^{2n}$

[6] 5. Find the sum and the interval of convergence of the series

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{(2n+1)!\, e^{(4n+2)x}}$$

[12] 6. Let $f(x) = \frac{x - x^2}{(1+x)^2}$ use the binomial expansion to find the Maclaurin series of $f(x)$. Express your answer in sigma notation and simplify as much as possible. Determine its open interval of convergence.