# THE UNIVERSITY OF MANITOBA

**1:30 pm, December 9, 2008** | **Final Examination (E2-229 EIT Complex)**

**Paper No: 251** | **Page No: Page 1 of 5**

**Dept. and Course No.: ENG 1460** | **Time: 3 Hours**

**Examination.: Intro Thermal Sciences** | **Examiners: Drs. J. Bartley and H. Soliman**

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1. This is a three-hour, open-book examination. Students are permitted to use the course textbook and a non-programmable calculator. No other material is allowed.
2. Answer all **five** questions. The value of each question is indicated beside the problem number.
3. Write your solutions clearly (showing all steps) in the booklets provided. Ambiguous solutions, which cannot be interpreted, will be considered incorrect.
4. If interpolation is required in the property tables, then use **linear** interpolation between table entries. Use **four** significant figures in your calculations and full precision for data taken from the textbook.

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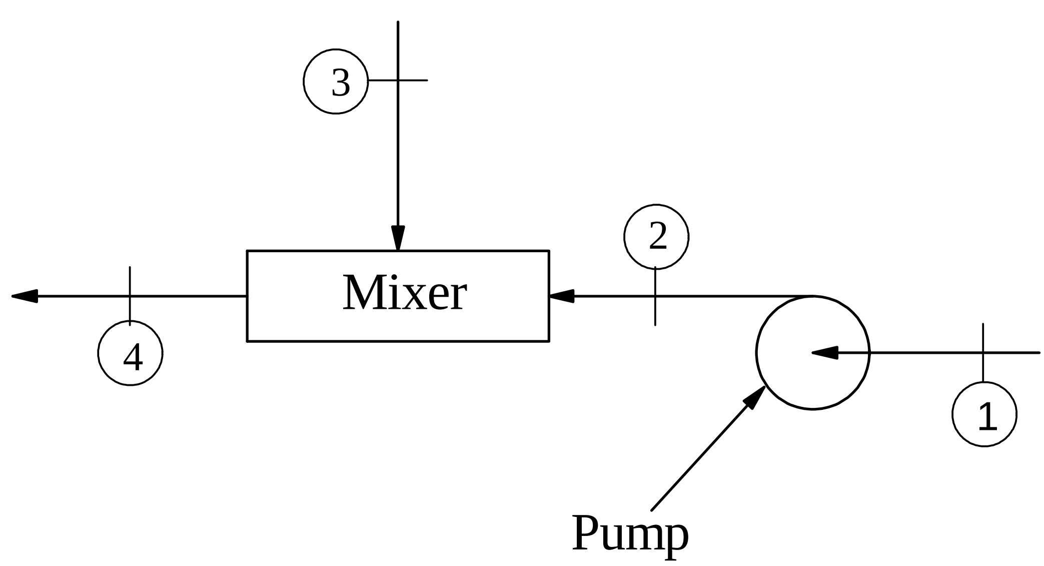
**Problem #1 (15 Points):**

The feed-water heater of a steam power plant consists of a pump and a mixer, as shown in the figure below. Water with a mass flow rate of 20 kg/s enters the pump at $P_1$ = 10 kPa and $T_1$ = 20°C and exits at $P_2$ = 5 MPa and $T_2$ = 20°C. The water then enters the mixer where it is mixed with a supply of steam at $P_3$ = 5 MPa and $T_3$ = 400°C. The flow exiting the mixer is at $P_4$ = 5 MPa and $T_4$ = 160°C. Assume steady-state and steady-flow with no heat transfer between the pump and the surroundings, or between the mixer and the surroundings. Kinetic and potential energy changes are negligible throughout the system.

(a) Determine the power required to operate the pump.
(b) Determine the mass flow rate of steam required in the mixer.
(c) Determine the diameter of the pipe connecting the pump and the mixer if the water velocity in this pipe is 5 m/s.



**Problem #2 (24 Points):**

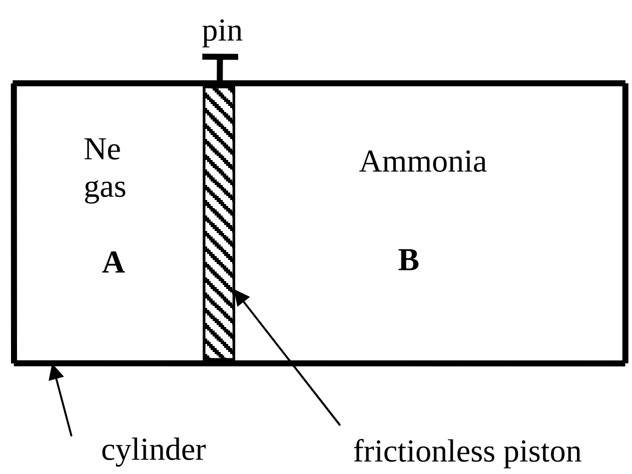
A closed cylinder is divided into two chambers by a moveable piston that is initially locked (held stationary) in place by a pin, as shown in the figure below. Chamber **A** contains $m_A = 1.15$ kg of neon gas at a temperature $T_A = -10\,°C$ (treat the neon as an ideal gas with constant specific heats). Chamber **B** contains ammonia at a saturated vapor state and with a temperature $T_B = -10\,°C$. The neon gas initially occupies a volume of $V_A = 60$ L. The sum of the volumes of chambers **A** and **B** together is 500 L. A process is carried out whereby the pin holding the piston stationary is removed and the piston is allowed to move freely in a frictionless manner. Heat is transferred between the cylinder and the surroundings so that the temperature of the neon gas remains at $-10\,°C$ and the ammonia is a saturated mixture at $-10\,^oC$. It is assumed that any changes in state of the substances occur in a quasi-equilibrium manner. The process ends when the fluids in chambers **A** and **B** come to a state of equilibrium. Assume that the changes in kinetic and potential energies are negligible.

(a) Calculate the initial pressure of the neon gas, $P_{A1}$. Determine the mass of ammonia in chamber **B**, $m_B$, and the initial pressure of the ammonia, $P_{B1}$.

(b) Determine the final pressure in chambers **A** and **B**, and calculate the new volumes of each chamber, $V_{A2}$ and $V_{B2}$.

(c) Considering the entire cylinder, determine the amount of heat transferred to or from the surroundings, ${}_1Q_2$, and any work transferred to or from the surroundings, ${}_1W_2$.

(d) Show the state points and process paths on a *T-V* (temperature–volume) diagram for the neon, and on a *T-v* (temperature–specific volume) diagram for the ammonia. Label all temperatures, pressure lines, volumes and specific volumes.



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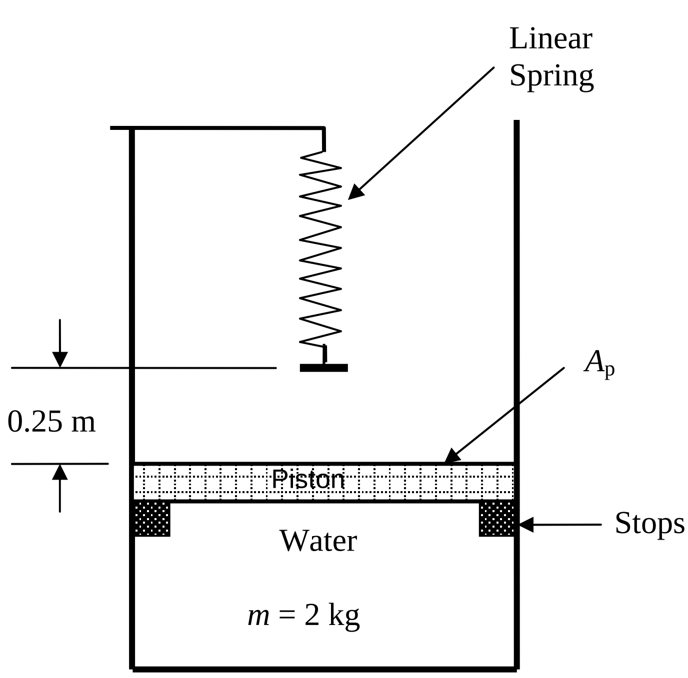
**Problem #3 (26 Points):**

A piston/cylinder system containing a saturated mixture of water is shown in the figure below. Initially, the piston is resting on a set of stops and a mass $m = 2$ kg of water is contained in the cylinder. The initial volume occupied by the water is $V_1 = 450$ L and the initial pressure of the water is $P_1 = 150$ kPa. It is known that a pressure of 300 kPa is required to float the piston; i.e., to begin to raise the piston off the set of stops. A series of processes occur during which heat is added to the water until the spring is compressed a certain distance and the water occupies its final volume. The cross-sectional area of the piston is $A_p = 0.8$ m$^2$ and the spring constant is $k = 689.3$ kN/m. The vertical distance between the piston's initial position at the stops to the location where it first comes into contact with the linear spring is 0.25 m. Heat transfer continues from where the piston first contacts the spring until the volume reaches 1.3 m$^3$ and compresses the spring to the final state. Assume that the changes in kinetic and potential energies are negligible in all processes.

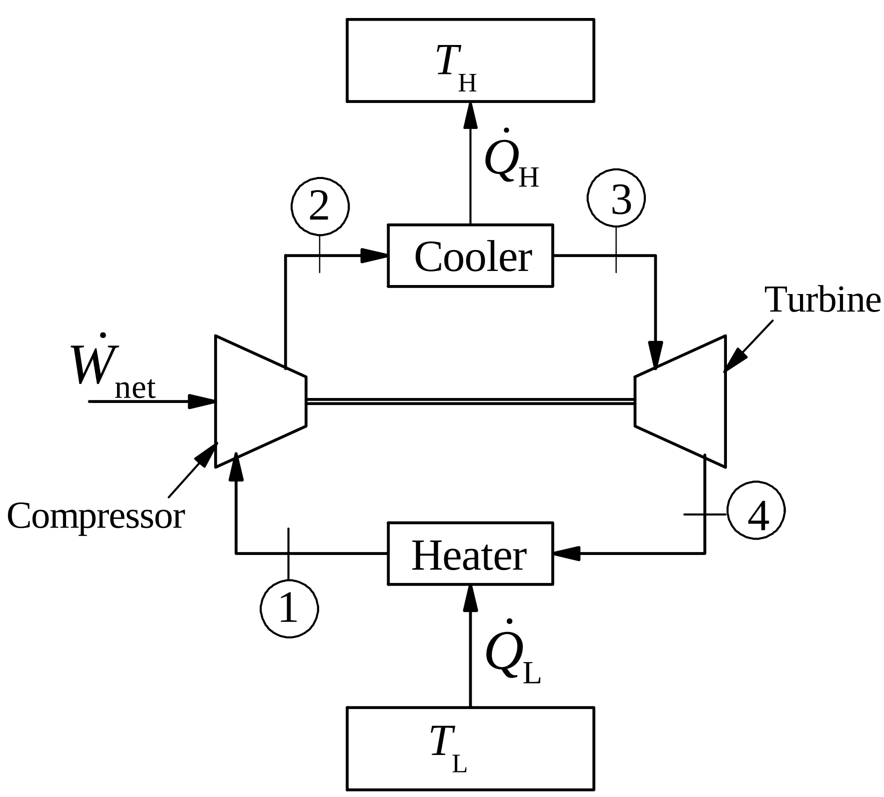
(a) For the initial state of the water, determine the temperature, $T_1$, specific volume, $v_1$, and quality, $x_1$.
(b) Describe the states when the piston first leaves the stops and when the piston first comes in contact with the linear spring, but does not exert any force on it. For each of the two states, calculate the volume, $V$, specific volume, $v$; and state the temperature, $T$, and pressure, $P$.
(c) Describe the final state and determine the final pressure, $P$, and temperature, $T$.
(d) Calculate the work done, $W$, for each process described above.
(e) Determine the overall heat transfer, $Q$, between the initial state and the final state.
(f) On a $P - v$ diagram show all state points and process lines. Label the diagram with the appropriate pressure, specific volume, and temperature lines. Indicate the area of the diagram that represents the total work done.



**Problem #4 (25 Points):**

The figure below shows a refrigeration cycle consisting of four components (compressor, cooler, turbine, and heater). The cycle uses air as the working medium with a mass flow rate of $\dot{m} = 2.5\,\text{kg/s}$ in each component. The air enters the compressor at $P_1$ = 100 kPa and $T_1$ = 270 K, and it exits at $P_2$ = 300 kPa and $T_2$ = 350 K. The conditions at the exit of the cooler are: $P_3$ = 300 kPa and $T_3$ = 300 K, and the pressure at the exit of the turbine is $P_4$ = 100 kPa. The compressor consumes all the power generated by the turbine and, in addition, it receives power from an external source in the amount of $\dot{W}_{\text{net}} = 25.1\text{ kW}$. The cooler rejects heat into a high temperature reservoir at $T_H$ and the heater receives heat from a low temperature reservoir at $T_L$. All four components of the cycle operate under steady-state and steady-flow conditions. Treat the air as an ideal gas with constant specific heats. The changes in the kinetic and potential energies are negligible throughout the cycle. Also, the flows in the compressor and the turbine are adiabatic.

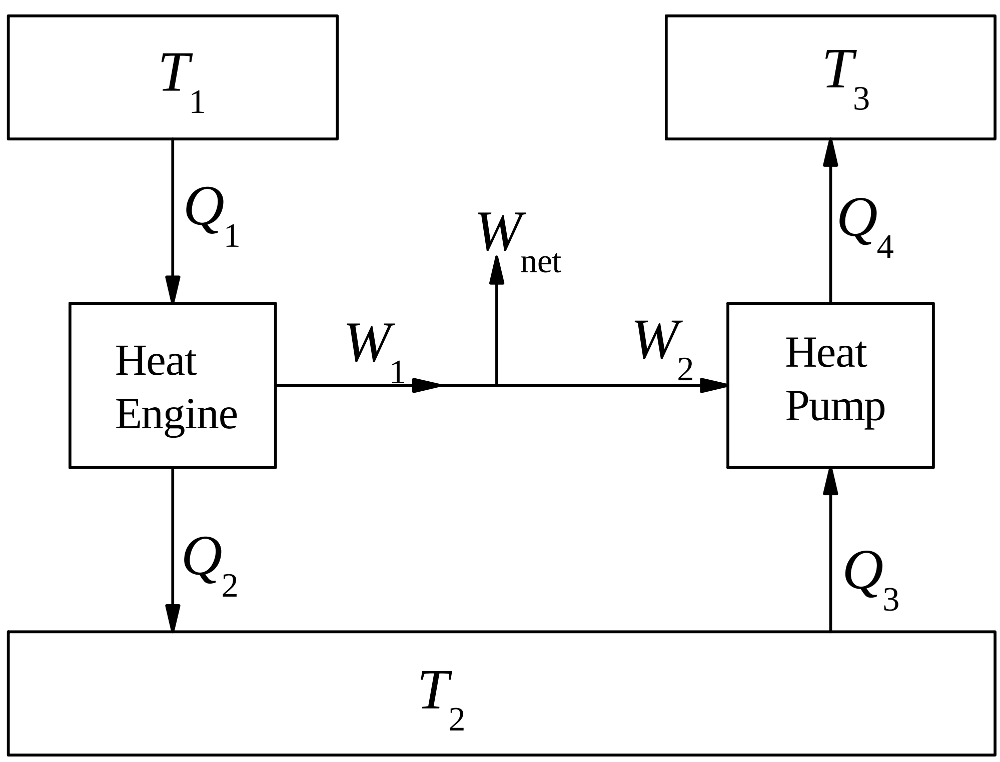
(a) Determine the temperature at the exit of the turbine, $T_4$.
(b) Sketch a $P - v$ diagram of the cycle showing the four processes and the relevant pressure and temperature lines.
(c) Determine the coefficient of performance of this refrigeration cycle.
(d) What is the highest possible value for $T_H$ and what is the lowest possible value for $T_L$? Explain your answers briefly.
(e) Based on the values of $T_H$ and $T_L$ from part (d), determine the highest possible coefficient of performance for a refrigeration cycle operating between these two heat reservoirs. Compare this value with the value obtained in part (c) and determine whether the given cycle is reversible, irreversible, or impossible.



**Problem #5 (10 Points):**

The figure below shows a heat engine driving a heat pump and, in addition, producing a net amount of work $W_{net}$. The heat engine receives an amount of heat $Q_1$ = 100 kJ from a heat reservoir at $T_1$ = 600 K, rejects an amount of heat $Q_2$ into a heat reservoir at $T_2$ = 300 K, and produces an amount of work $W_1$. The heat pump removes an amount of heat $Q_3$ = 60 kJ from the reservoir at $T_2$, delivers an amount of heat $Q_4$ to a heat reservoir at $T_3$ = 500 K, and receives an amount of work $W_2$. The work from the engine is sufficient to drive the pump and, in addition, a net amount of work $W_{net}$ = 20 kJ is produced, i.e., $W_1 = W_2 + 20$.

Determine whether the system described above is possible or impossible, supporting your answer with appropriate calculations.



1. (a) For a control volume around the pump

$$\cancel{\dot{Q}_{cv}} - \dot{W}_{cv} = \dot{m}\left[(h_2 - h_1) + \cancel{\frac{V_2^2 - V_1^2}{2}} + \cancel{g(z_2 - z_1)}\right]$$

$$h_1 = u_1 + P_1 v_1 = 83.94 + 10 \times 0.001002$$
$$= 83.95 \text{ kJ/kg}$$

$$h_2 = u_2 + P_2 v_2 = 83.94 + 5000 \times 0.001002$$
$$= 88.95 \text{ kJ/kg}$$

<u>OR</u> $h_2 = 88.64$ kJ/kg from the compressed liquid tables

$$\dot{W}_{pump} = -20(88.95 - 83.95) = -100 \text{ kW}$$

(b) For a control volume around the mixer

$$\cancel{\dot{Q}_{cv}} - \cancel{\dot{W}_{cv}} = \dot{m}_4\left(h_4 + \cancel{\frac{V_4^2}{2}} + \cancel{g z_4}\right)$$
$$- \left[\dot{m}_2\left(h_2 + \cancel{\frac{V_2^2}{2}} + \cancel{g z_2}\right) + \dot{m}_3\left(h_3 + \cancel{\frac{V_3^2}{2}} + \cancel{g z_3}\right)\right]$$

$$\dot{m}_4 = \dot{m}_2 + \dot{m}_3$$

$$(\dot{m}_2 + \dot{m}_3) h_4 = \dot{m}_2 h_2 + \dot{m}_3 h_3$$

$$\dot{m}_3 = \frac{(h_4 - h_2)\dot{m}_2}{h_3 - h_4}$$

$$h_3 = 3195.64 \text{ kJ/kg}$$

$$h_4 = u_4 + P_4 v_4$$

$$= 674.85 + 5000 \times 0.001102 = 680.36 \text{ kJ/kg}$$

<u>OR</u> $h_4 = 678.10$ kJ/kg from the compressed liquid table

$$\dot{m}_3 = 20 \frac{680.36 - 88.95}{3195.64 - 680.36} = 4.703 \text{ kg/s}$$

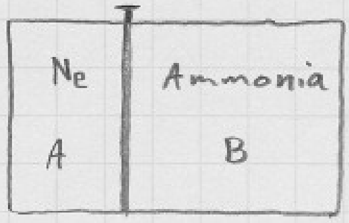
(C) $$A_2 = \frac{\dot{m}_2 v_2}{V_2} = \frac{20 \times 0.001002}{5} = 0.004008 \text{ m}^2$$

$$A_2 = \frac{\pi}{4} D_2^2$$

$$D_2 = \sqrt{\frac{4 A_2}{\pi}} = \sqrt{\frac{4 \times 0.004008}{\pi}} = 0.07144 \text{m}$$

$$= 7.144 \text{ Cm}$$

#2. Final Exam, Dec. 2008




$V_T = 500\ L$

Ne

$T_{A_1} = -10°C$

$V_{A_1} = 60\ L = 0.060\ m^3$

$m_{Ne} = 1.15\ kg$

(a) Table A.5 $R = 0.4120\ kJ/kg \cdot K$

$C_{v_0} = 0.6180\ kJ/kg \cdot K$

$P_A V_A = m_A R_A T_A \quad \therefore P_{A_1} = \dfrac{1.15 \times 0.4120 \times 263.15}{0.060}$

$= 2078.0\ kPa$

Ammonia Table B.2.1 $T_{B_1} = -10°C$, saturated vapour

$v_g = 0.41808\ m^3/kg$, $P_{B_1} = P_{sat,} = 290.9\ kPa$

$u_g = 1309.2\ kJ/kg$

$V_{B_1} = V_T - V_{A_1} = 0.5 - 0.060 = 0.440\ m^3$

$m_B = \dfrac{V_{B_1}}{v_g} = \dfrac{0.440}{0.41808} = 1.0524\ kg$

(b) If the temperature of the Neon and Ammonia both remain at -10°C during the process, and the Ammonia is a saturated mixture, then the final pressure for both Ne and Ammonia are the same and equal to $P_{sat.}$ at -10°C for Ammonia

i.e., $P_{A_2} = P_{B_2} = P_{sat.@-10°C} = 290.9\ kPa$

$V_{A_2} = \dfrac{m_A R_A T_{A_2}}{P_{A_2}} = \dfrac{1.15 \times 0.4120 \times 263.15}{290.9} = 0.4286\ m^3$

(or, $P_{A_2} V_{A_2} = P_{A_1} V_{A_1}$ since temperature remains constant)

$\therefore V_{B_2} = V_T - V_{A_2} = 0.5 - 0.4286 = 0.07140\ m^3$

(c) To apply the First law to this system, we will need $u_{B_1}$ and $u_{B_2}$ for Ammonia.

$$v_{B_2} = \frac{V_{B_2}}{m_B} = \frac{0.07140}{1.0524} = 0.067844 \text{ m}^3/\text{kg}$$

Table B.2.1 at $-10°C$ $\quad v_f = 0.001534 \text{ m}^3/\text{kg}$

$v_g = 0.41808 \text{ m}^3/\text{kg}$

$v_f < v_{B_2} < v_g$
(saturated mixture)

$$x_{B_2} = \frac{v_{B_2} - v_f}{v_g - v_f} = \frac{0.067844 - 0.001534}{0.41808 - 0.001534}$$

$$\therefore x_{B_2} = 0.15919$$

$u_{B_2} = (1-x)u_f + x u_g \quad ; \quad u_f = 133.96 \text{ kJ/kg}$

$u_g = 1309.2 \text{ kJ/kg}$

$\therefore u_{B_2} = (1 - 0.15919) \times 133.96 + 0.15919 \times 1309.2$

$u_{B_2} = 321.04 \text{ kJ/kg}$

First law: (applied to whole cylinder) $\quad {}_1Q_2 - {}_1W_2 = U_2 - U_1 + \cancel{\Delta KE} + \cancel{\Delta PE}$

${}_1W_2 = 0$ for the system of both A & B together

$$\therefore {}_1Q_2 = (m_A u_{A_2} + m_B u_{B_2}) - (m_A u_{A_1} + m_B u_{B_1})$$

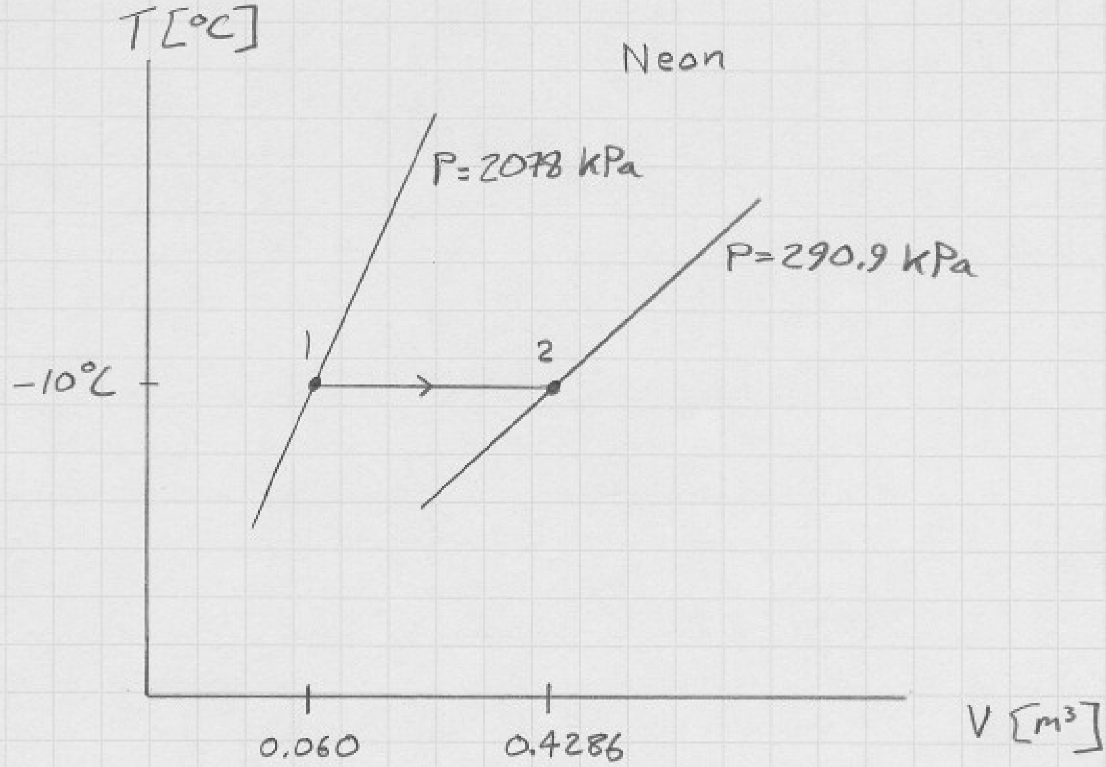
$$= m_A (u_2 - u_1)_A + m_B (u_2 - u_1)_B$$

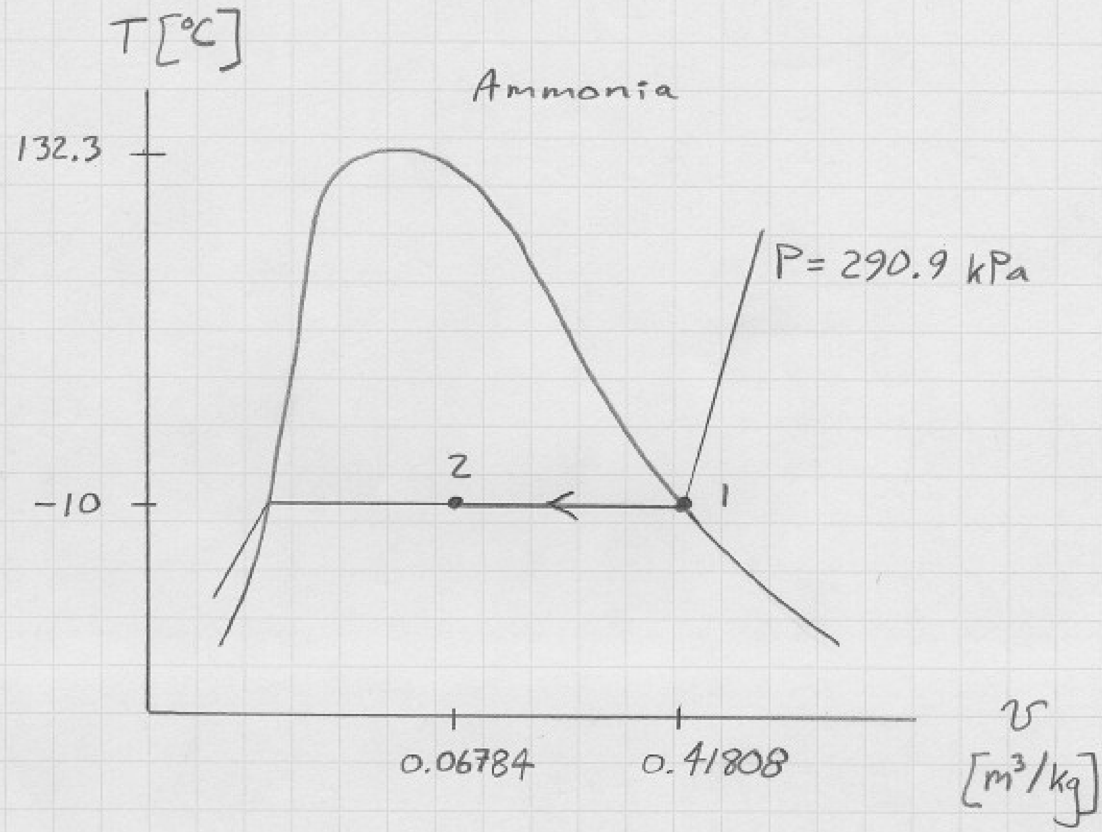
$$= m_A \underset{Ne}{C_{v_0}} (T_2 - T_1)_A + m_B (u_2 - u_1)_B$$

$$T_2 = T_1 \quad \therefore (u_2 - u_1)_A = 0$$

$$\therefore {}_1Q_2 = 1.0524 (321.04 - 1309.2) = -1039.9 \text{ kJ}$$

(d)


T [°C]
Neon
P = 2078 kPa
P = 290.9 kPa
1
2
-10°C
0.060
0.4286
V [m³]


T [°C]
Ammonia
132.3
P = 290.9 kPa
2
1
-10
0.06784
0.41808
v [m³/kg]

#3.

(a) State 1 $P_1 = 150$ kPa , $V_1 = 450$ L , $m = 2$ kg

saturated mixture, Table B.1.2 at 150 kPa, $T_{sat,1} = 111.37°C$ $(=T_1)$

$$v_1 = \frac{V_1}{m} = \frac{0.450}{2} = 0.225\ m^3/kg$$

$$x_1 = \frac{v_1 - v_f}{v_g - v_f} = \frac{0.225 - 0.001053}{1.15933 - 0.001053} = 0.19334$$

$v_f = 0.001053\ m^3/kg$
$v_g = 1.15933\ m^3/kg$

State 2 $v_2 = v_1$ , $P_2 = 300$ kPa, saturated mixture

(b) state 3

piston contacts spring after rising 0.25 m off the stops. $V_3 = A_p \cdot L + V_2 = 0.8 \times 0.25 + 0.450 = 0.650\ m^3$

$$\therefore v_3 = \frac{V_3}{m} = \frac{0.650}{2} = 0.325\ m^3/kg$$

$P_3 = 300$ kPa, the pressure needed to float the piston.

Table B.1.2 at 300 kPa, $v_f = 0.001073\ m^3/kg$
$v_g = 0.60582\ m^3/kg$

$v_f < v_3 < v_g$ $\therefore$ saturated mixture.

$T_3 = T_{sat,1} = 133.55\ °C$

(c) state 4 piston compresses the spring to the final volume, $V_4 = 1.3\ m^3$.

$$P_4 = P_3 + \frac{k_s}{A_p^2}(V_4 - V_3) = 300 + \frac{689.3}{(0.8)^2}(1.3 - 0.650)$$

$P_4 = 1000.0$ kPa

$$v_4 = \frac{V_4}{m} = \frac{1.3}{2} = 0.650\ m^3/kg$$

Table B.1.2 at 1000 kPa, $v_g = 0.19444\ m^3/kg$ , $T_{sat,} = 179.91°C$

$v_4 > v_{g,1000\,kPa}$ $\therefore$ state is superheated vapour.

Table B.1.3 at 1000 kPa

| T | v | u |
|---|---|---|
| 1100 | 0.63345 | 4255.09 |
| T | 0.65 | u |
| 1200 | 0.679777 | 4465.58 |

by interpolation,
$T_4 = 1135.7\ °C$

(d) ${}_1W_2 = 0$ , volume remains constant

$${}_2W_3 = P_2 (V_3 - V_2) = 300(0.650 - 0.450) = 60 \text{ kJ}$$

$${}_3W_4 = \frac{P_3 + P_4}{2} (V_4 - V_3) = \frac{300 + 1000}{2} (1.3 - 0.65)$$

$$= 422.5 \text{ kJ}$$

(e) first law, ${}_1Q_4 - {}_1W_4 = U_4 - U_1 + \Delta \cancel{K}E + \Delta \cancel{P}E$

$$= m(u_4 - u_1)$$

State 1 , Table B.1.2

at $P = 150$ kPa, $u_f = 466.92$ kJ/kg , $x_1 = 0.19334$

$u_g = 2519.64$ kJ/kg

$$\therefore u_1 = (1 - x_1) u_f + x_1 \cdot u_g$$

$$= (1 - 0.19334) \times 466.92 + 0.19334 \times 2519.64$$

$$= 863.79 \text{ kJ/kg}$$

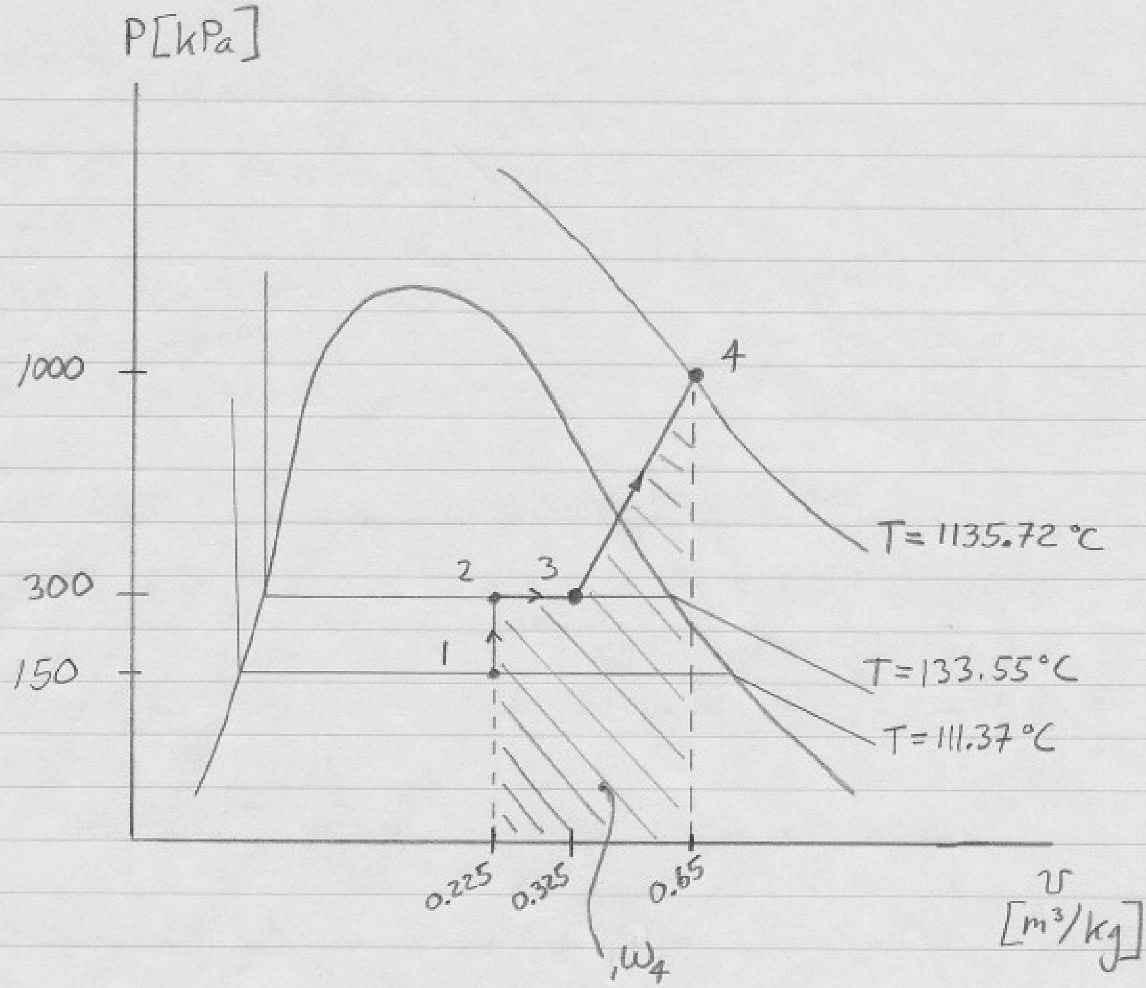
by linear interpolation above, $u_4 = 4330.28$ kJ/kg

$${}_1Q_4 = ({}_1W_2 + {}_2W_3 + {}_3W_4) + m(u_4 - u_1)$$

$$= (0 + 60 + 422.5) + 2 \times (4330.28 - 863.79)$$

$$= 7415.48 \text{ kJ}$$

(f)



P[kPa]
1000
300
150
4
3
2
1
T= 1135.72 °C
T=133.55°C
T=111.37°C
0.225
0.325
0.65
v
[m³/kg]
,W4

4. (a) For the compressor

$$\cancel{\dot{Q}_{cv}} - \dot{W}_{cv} = \dot{m}\left[(h_2 - h_1) + \cancel{\frac{V_2^2 - V_1^2}{2}} + g\cancel{(z_2 - z_1)}\right]$$

$$\dot{W}_{cv} = -\dot{m}\, c_{p_0}(T_2 - T_1)$$

$$= -2.5 \times 1.004\,(350 - 270)$$

$$= -200.8 \text{ kW}$$

The compressor requires 200.8 kW. It gets 25.1 kW from an external source, therefore, the turbine delivers

$200.8 - 25.1 = 175.7$ kW

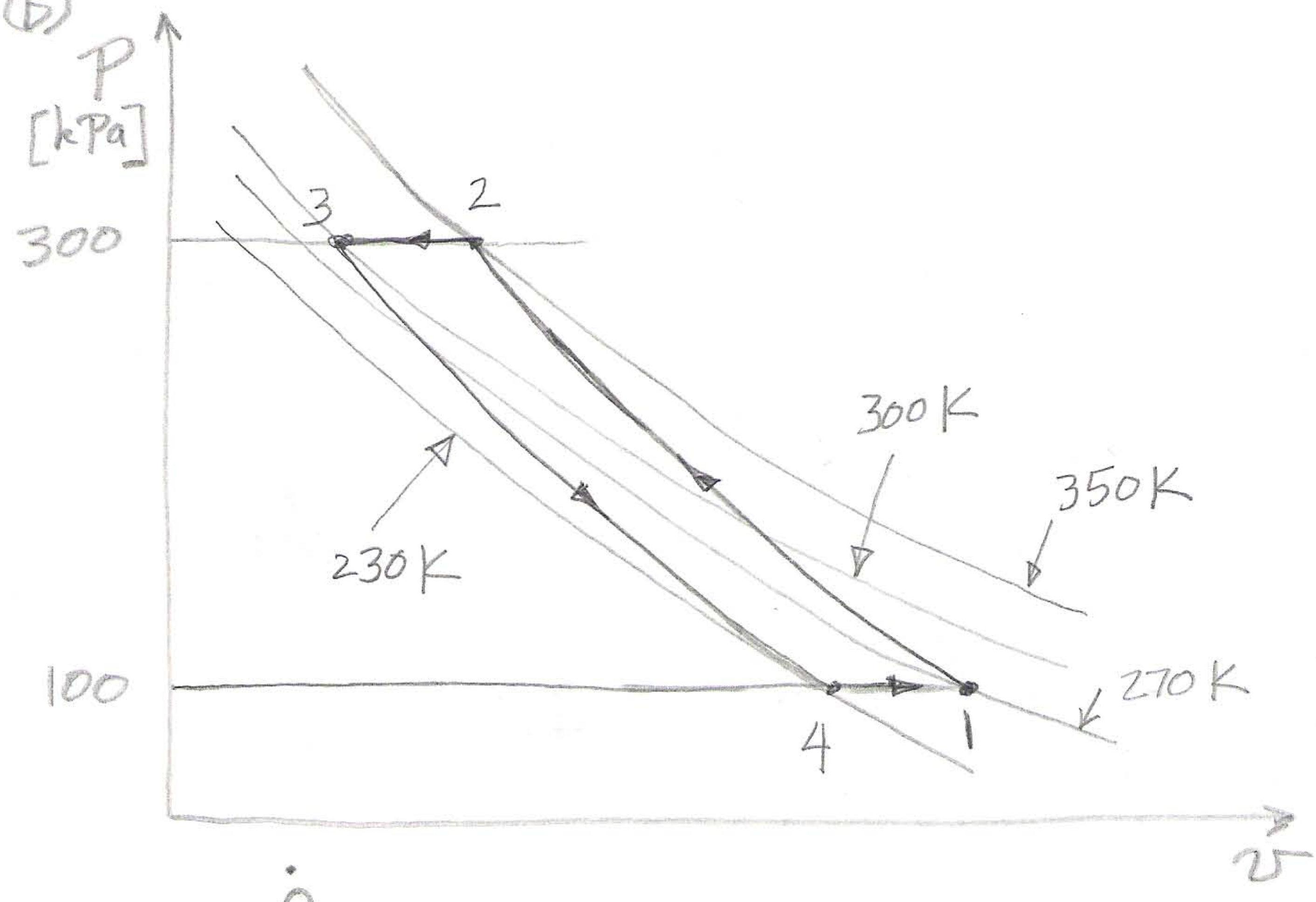
For the turbine

$$\cancel{\dot{Q}_{cv}}^{0} - \dot{W}_{cv} = \dot{m}\left[(h_4 - h_3) + \cancel{\frac{V_4^2 - V_3^2}{2}}^{0} + \cancel{g(z_4 - z_3)}^{0}\right]$$

$$-175.7 = 2.5 \times 1.004\,(T_4 - 300)$$

$$T_4 = 230\ \text{K}$$

(b)




(c) $\beta = \dfrac{\dot{Q}_L}{\dot{W}_{net}}$

For the heaters

$$\dot{Q}_{cv} - \cancel{\dot{W}_{cv}}^{0} = \dot{m}\left[(h_1 - h_4) + \cancel{\frac{V_1^2 - V_4^2}{2}}^{0} + \cancel{g(z_1 - z_4)}^{0}\right]$$

$\dot{Q}_L = \dot{Q}_{cv} = 2.5 \times 1.004\,(270 - 230) = 100.4$ kW

$$\beta = \frac{100.4}{25.1} = 4$$

(d) $T_H$ cannot exceed $T_3$ since the cooler is sending heat to the reservoir at $T_H$. Thus,

$$T_{H,max} = 300 \text{ K}$$

$T_L$ cannot be lower than $T_1$ since the heater receives heat from the reservoir at $T_L$. Thus

$$T_{L,min} = 270 \text{ K}$$

(e) Based on the numbers in (d),

$$\beta_{max} = \frac{270}{300 - 270} = 9$$

Comparing with the $\beta = 4$ in (c), the given cycle is irreversible.

5. To determine whether the system is possible or impossible, we assume that the engine and the pump are both reversible and calculate $W_{net}$ under this assumption. If $W_{net}$ come out higher than 20 kJ, then the system is possible, otherwise, it is impossible.

For a reversible heat engine,

$$\frac{Q_1}{Q_2} = \frac{T_1}{T_2} \longrightarrow \frac{100}{Q_2} = \frac{600}{300}$$

$$Q_2 = 50 \text{ kJ} \rightarrow W_1 = Q_1 - Q_2 = 50 \text{ kJ}$$

For a reversible heat pump,

$$\frac{Q_4}{Q_3} = \frac{T_3}{T_2} \longrightarrow \frac{Q_4}{60} = \frac{500}{300}$$

$$Q_4 = 100 \text{ kJ} \longrightarrow W_2 = Q_4 - Q_3 = 40 \text{ kJ}$$

$$W_{net} = W_1 - W_2 = 50 - 40 = 10 \text{ kJ}$$

Since the claimed $W_{net}$ is higher than the reversible $W_{net}$, the system is impossible