Math 2130 Summer 2012 Test 2 (by N Harland)

1. Given that
$$xyu + v = 2 \text{ and } y^2 + u^2 - u^2 v = y + 43$$
define u and v as functions of x and y, find $\left.\frac{\partial u}{\partial y}\right)_x$ when $x = 1, y = 2, u = 3$ and $v = -4$. [5]

2. Find the equation of the tangent line (in either parametric, vector or symmetric form) to the curve of intersection of
$$yz + \sin(xyz) = -4 \text{ and } x^2 + y^2 + z^2 = 8$$
at the point $(0, 2, -2)$. [7]

3. For the function $f(x, y) = x^2 y + xy^2 + 3y$
   (a) Find the critical point(s) of f. [4]

   (b) Classify the critical point(s) found in (a) as either relative minimum, relative maximum, saddle point, or neither. [5]

   (c) Find the absolute maximum and minimum of f on the region bounded by $y = x^2$ and $y = 4$. [7]

4. Find
$$\iint_R (1 - x)\,dA$$
where R is the region bounded by the lines $x + y = 1, x + y = -1, x - y = 1, x - y = -1$. [8]

5. The one-dimensional wave equation is
$$\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}$$
where c is any positive constant. Let $f(u)$ and $g(v)$ be twice differentiable functions. Show that
$$y(x, t) = f(x + ct) + g(x - ct)$$
satisfies the wave equation. [4]

Dawit's Answers

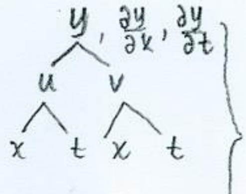
1. $-5/8$

2. $x = 0,\ y = 2 + t,\ z = -2 + t$
or
$x = 0,\ y = 2 - t,\ z = -2 - t$

3. a) $(1, -2), (-1, 2)$
b) $(1, -2)$ yields Saddle point.
$(-1, 2)$ 〃 〃 〃.
c) Abs. Max $= 60$, Abs. min $= -4$

4. 2

5. Hint: let $u = x + ct$
$v = x - ct$ $\Longrightarrow$



$\therefore\ y = f(u) + g(v)$
$u = h(x, t),\ v = J(x, t)$

evaluate: $\frac{\partial y}{\partial x}, \frac{\partial y}{\partial t}, \frac{\partial^2 y}{\partial x^2}, \frac{\partial^2 y}{\partial t^2}$
by employing the tree-diagram