# THE UNIVERSITY OF MANITOBA

**Date : Friday, December 15, 2006**
**Department & Course No : 1440**
**Paper No : 471**
**Examination : Introduction to Statics**

**Place : Frank Kennedy Gold Gym**


**Time : 6:00 p.m.**
**Duration : 2 Hours**
**Examiners : Dr. M. J. Frye, Dr. C. Gheorghiu, Dr. D. Polyzois,**
**Seats: 1-227**

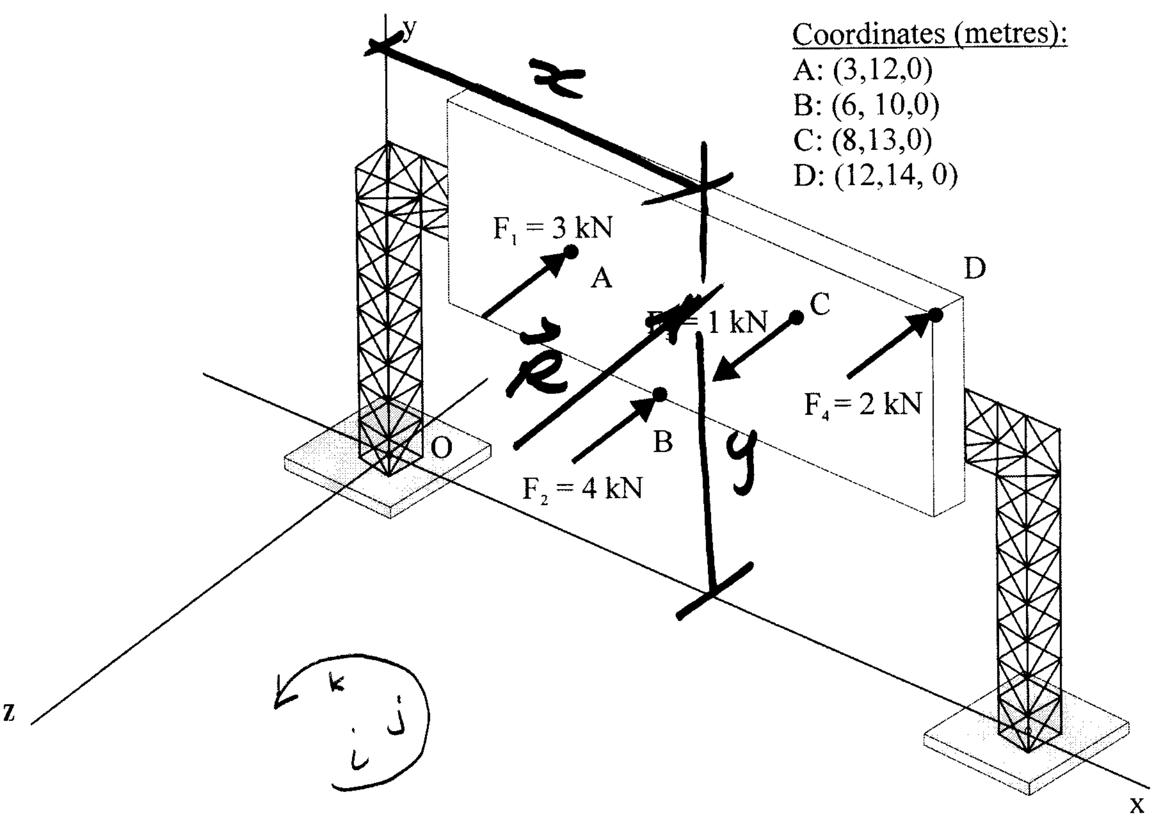
**Question 1**

Four forces are applied to the highway sign at points $A$, $B$ , $C$, and $D$ as shown. (All forces are parallel to the $z$ – axis.) The coordinates of the points with respect to the origin $O$ are also specified.

Determine:

(a) the magnitude and direction of the resultant of the resultant of the four forces, and

(b) the point of application of the resultant with respect to the origin $O$.




| Force | $\vec{r}$ , m | $\vec{F}$ (kN) | $\vec{M}_O = \vec{r} \times \vec{F}$ |
|---|---|---|---|
| $F_1$ | $3\hat{i} + 12\hat{j}$ | $-3\hat{k}$ | $9\hat{j} - 36\hat{i}$ |
| $F_2$ | $6\hat{i} + 10\hat{j}$ | $-4\hat{k}$ | $24\hat{j} - 40\hat{i}$ |
| $F_3$ | $8\hat{i} + 13\hat{j}$ | $1\hat{k}$ | $-8\hat{j} + 13\hat{i}$ |
| $F_4$ | $12\hat{i} + 14\hat{j}$ | $-2\hat{k}$ | $24\hat{j} - 28\hat{i}$ |
| | $x\hat{i} + y\hat{j}$ | $-8k$ | $\Sigma\ 49\hat{j} - 91\hat{i}$ |

$M = \sqrt{49^2 + (-91)^2}$
$= 103.35$

$(x\hat{i} + y\hat{j}) \times (-8\hat{k}) = 49\hat{j} - 91\hat{i}$

$8x\hat{j} - 8y\hat{i} = 49\hat{j} - 91\hat{i}$

$8x = 49 \quad x = 6.125\ m \qquad -8y = -91$
$y = 11.375\ m$

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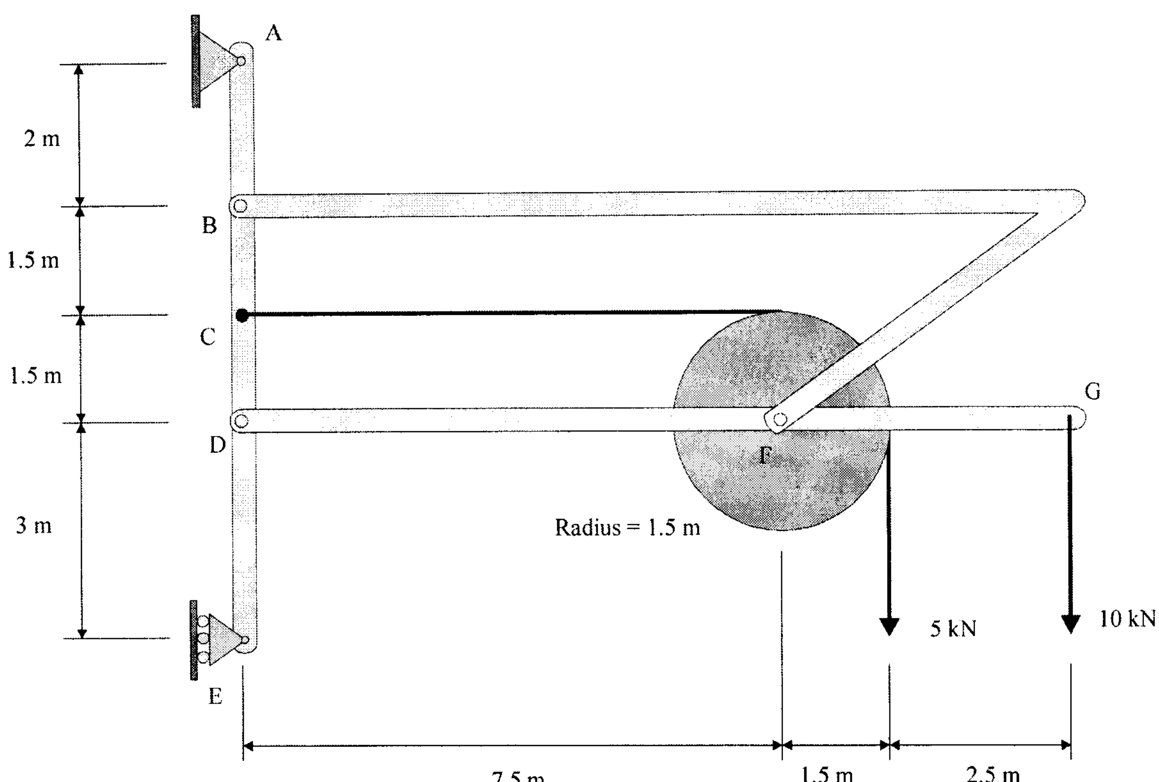
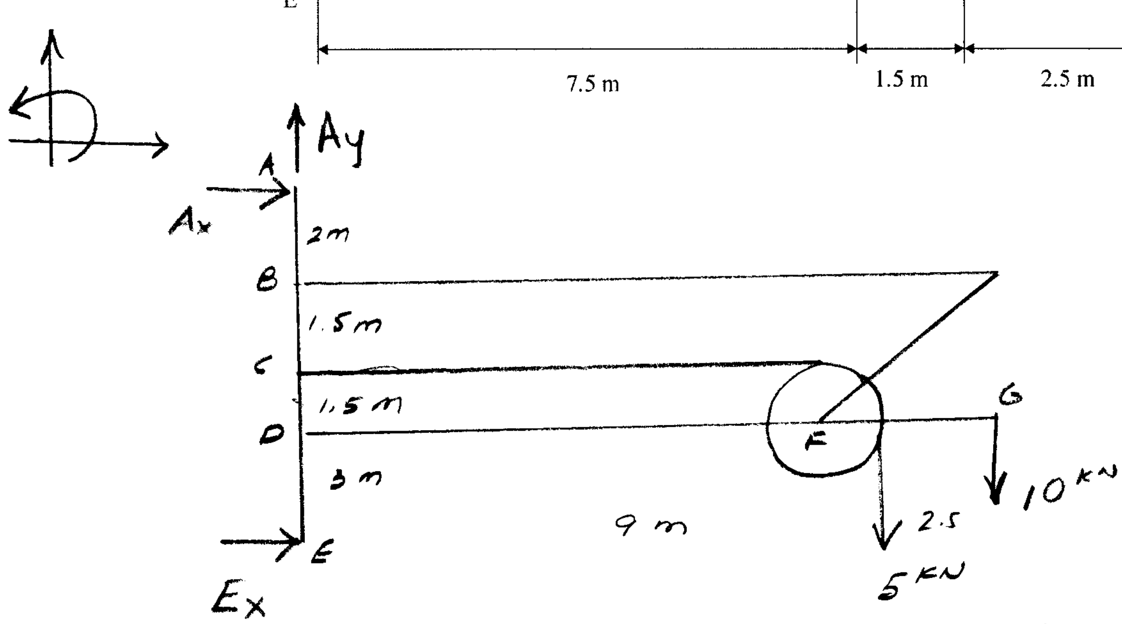
## QUESTION 2

The frame shown in the figure has a pin support at $A$ and a roller support at $E$. A single pin at $F$ connects member $BF$, member $DFG$ and a $1.5\ m$ radius pulley.
Determine:

a) The reactions at supports $A$ and $E$,
b) The force exerted by the pins at $B$ and $D$ on member $ABCDE$.
c) The force exerted by member $BF$, member $DFG$ and the pulley on the pin at $F$.




FBD

$\Sigma M_A = 0 \circlearrowleft \quad E_x(8) - 5(9) - 10(11.5) = 0$

$E_x = +20\ kN \quad \therefore\ \vec{E_x} = 20\ kN \rightarrow$

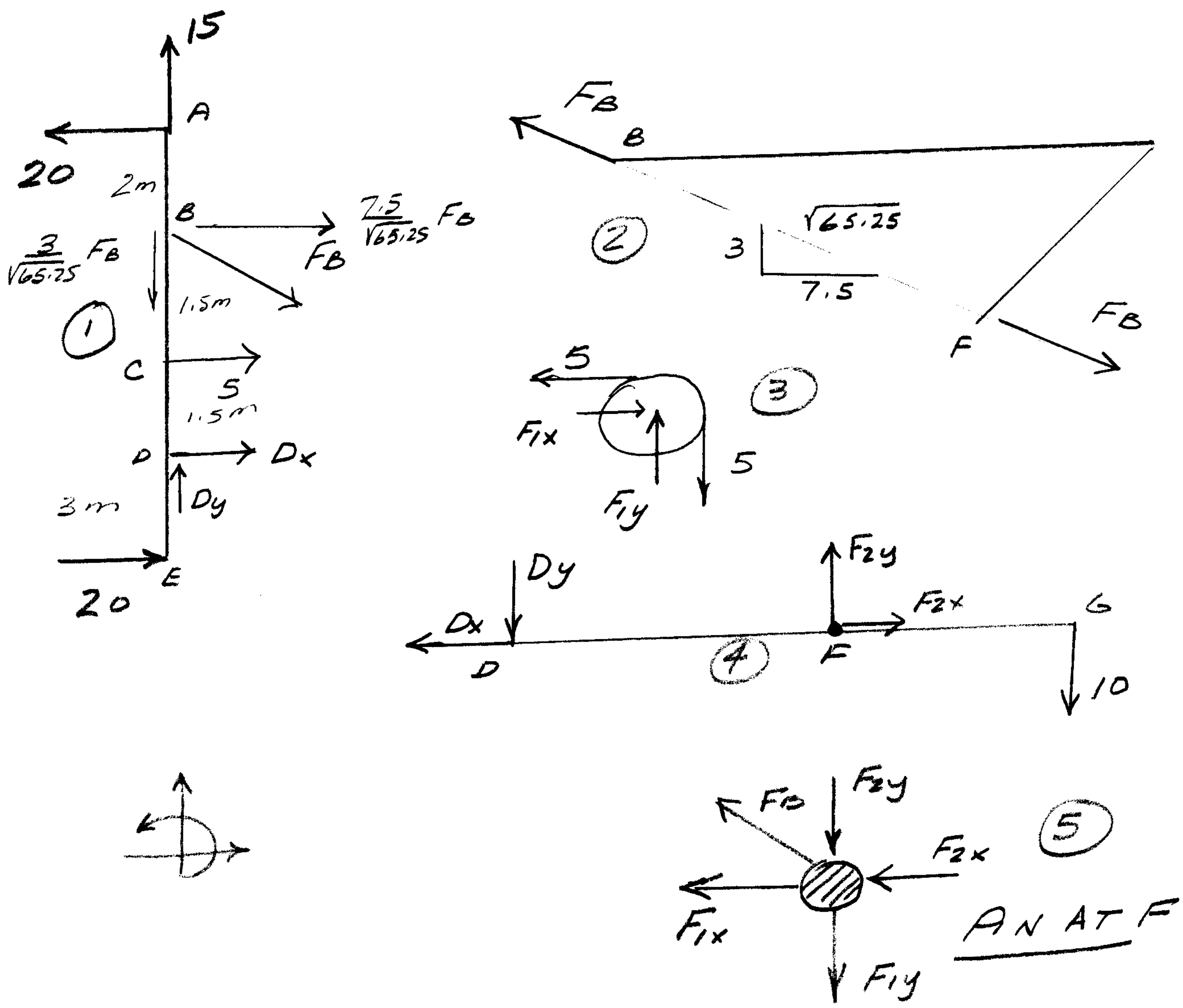
$\Sigma F_x = 0 \rightarrow \quad A_x + 20 = 0 \quad A_x = -20\ kN$

$\therefore\ \vec{A_x} = 20\ kN \leftarrow$

$\Sigma F_y = 0 \uparrow \quad A_y - 5 - 10 = 0 \quad A_y = +15\ kN$

$\therefore\ \vec{A_y} = 15\ kN \uparrow$

Substructure




From ①

$\sum M_D = 0$ ↺ $\quad 20(5) - \frac{7.5}{\sqrt{65.25}} F_B (3) - 5(1.5) + 20(3) = 0$

$F_B = +54.75 \text{ kN} \quad \therefore \vec{F_B} = 54.75 \text{ kN}$ (3 : 7.5 : $\sqrt{65.25}$) on member ABCDE

$\sum F_x = 0 \rightarrow \qquad F_{B_x} = \frac{7.5}{\sqrt{65.25}}(54.75) = 50.83 \qquad F_{B_y} = \frac{3}{\sqrt{65.25}}(54.75) = 20.33$

$-20 + \frac{7.5}{\sqrt{65.25}}(54.75) + 5 + D_x + 20 = 0$

$D_x = -55.83 \text{ kN} \quad \vec{D_x} = 55.83 \text{ kN} \leftarrow$ on member ABCDE

$\sum F_y = 0 \uparrow \quad 15 - \frac{3}{\sqrt{65.25}}(54.75) + D_y = 0$

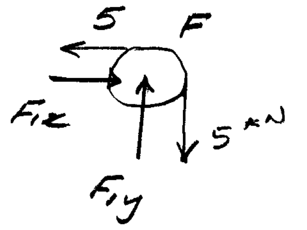
$D_y = +5.33 \text{ kN} \quad \therefore \vec{D_y} = 5.33 \text{ kN} \uparrow$ on member ABCDE

Check $\sum M_A = 0$

$\frac{7.5}{\sqrt{65.25}}(54.75)(2) + 5(3.5) - 55.83(5) + 20(8) = 0$

$0.01 \approx 0$ ✓

From (3)




$\Sigma F_x = 0 \rightarrow$

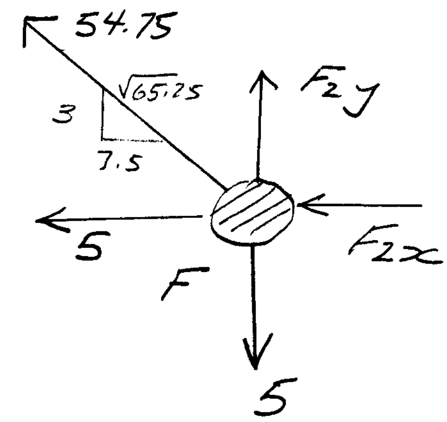
$F_{1x} - 5 = 0$

$F_{1x} = +5 \therefore \vec{F}_{1x} = 5^{kN}$ on the pulley

$\Sigma F_y = 0 \uparrow \quad F_{1y} - 5 = 0 \quad F_{1y} = +5$

$\vec{F}_{1y} = 5\,kN \uparrow$ on the pulley

From (5)




$$\Sigma F_x = 0 \rightarrow \quad -5 - \frac{7.5}{\sqrt{65.25}}(54.75) - F_{2x} = 0$$

$$F_{2x} = -55.83^{kN}$$

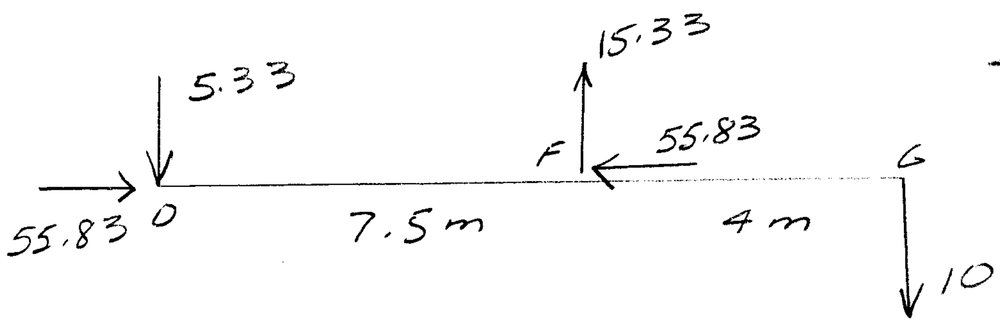
$$\vec{F}_{2x} = 55.83 \rightarrow \text{ on the pin at } F$$

$$\Sigma F_y = 0 \uparrow \quad \frac{3}{\sqrt{65.25}}(54.75) - 5 + F_{2y} = 0$$

$$F_{2y} = -15.33^{kN}$$

$$\therefore \vec{F}_{2y} = 15.33^{kN} \downarrow \text{ on the pin at } F$$

Check member DFG




$\Sigma F_x = 0$ ✓

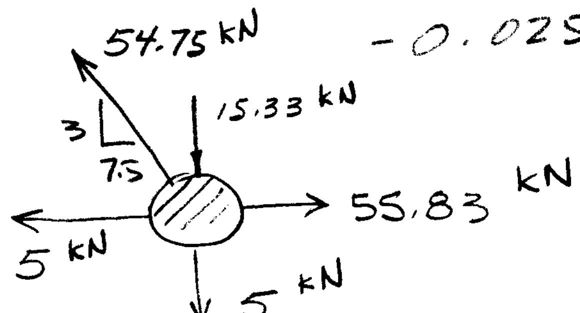
$\Sigma F_y = 0$ ✓

$$\Sigma M_D = 0$$

$$15.33(7.5) - 10(11.5) = 0$$

$$-0.025 = 0 \quad ✓$$

Pin at F



## QUESTION 3

A simple truss shown in Figure A below is supported by a pin support at *A* and by a roller support at *D*. A beam carrying a distributed load is suspended from joints *F* and *E* of the truss.

Determine:

a) The force exerted by the cables attached to the truss at *F* and at *E*,
b) The reactions at supports *A* and *D*, and
c) The force in each member of the truss and state whether it is in tension or compression. Place your results on Figure B.

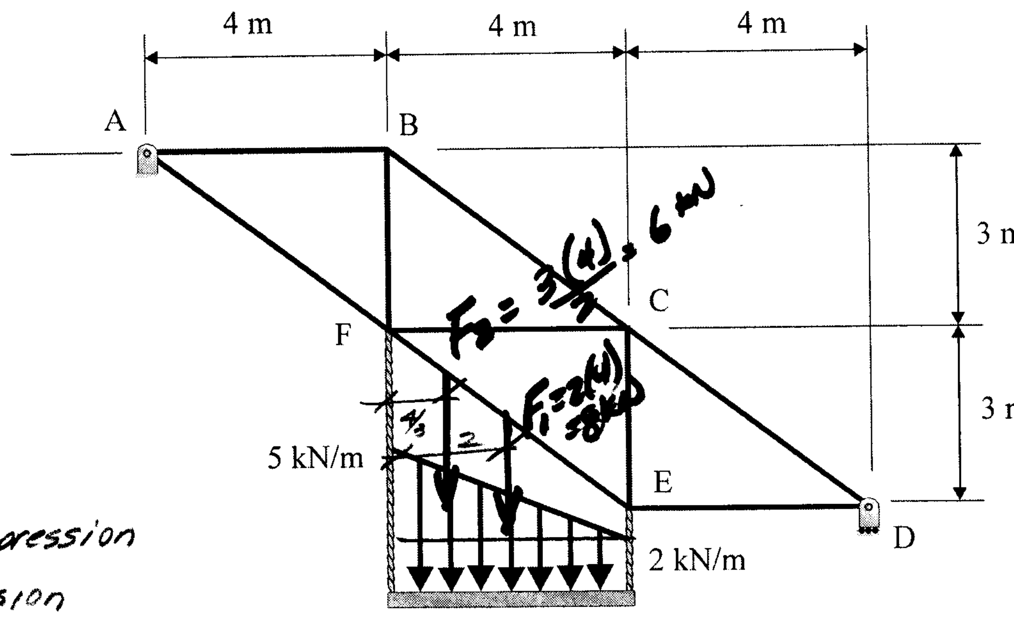



Figure A

compression

tension

$\Sigma F_x = 0 \quad A_x = 0$

$\Sigma M_A = 0 \quad -6(\frac{16}{3}) - 8(6) + D_y(12) = 0$

$D_y = +6.66 \quad \vec{D_y} = 6.66$ kN ↑

$\Sigma F_y = 0 \quad A_y - 6 - 8 + 6.66 = 0$

$A_y = +7.33$ kN

$\vec{A_y} = 7.33$ kN ↑

6   8 kN

4/3   2

$T_1$   $T_2$

G   4

$\Sigma M_G = -6(\frac{4}{3}) - 8(2) + T_2(4) = 0$

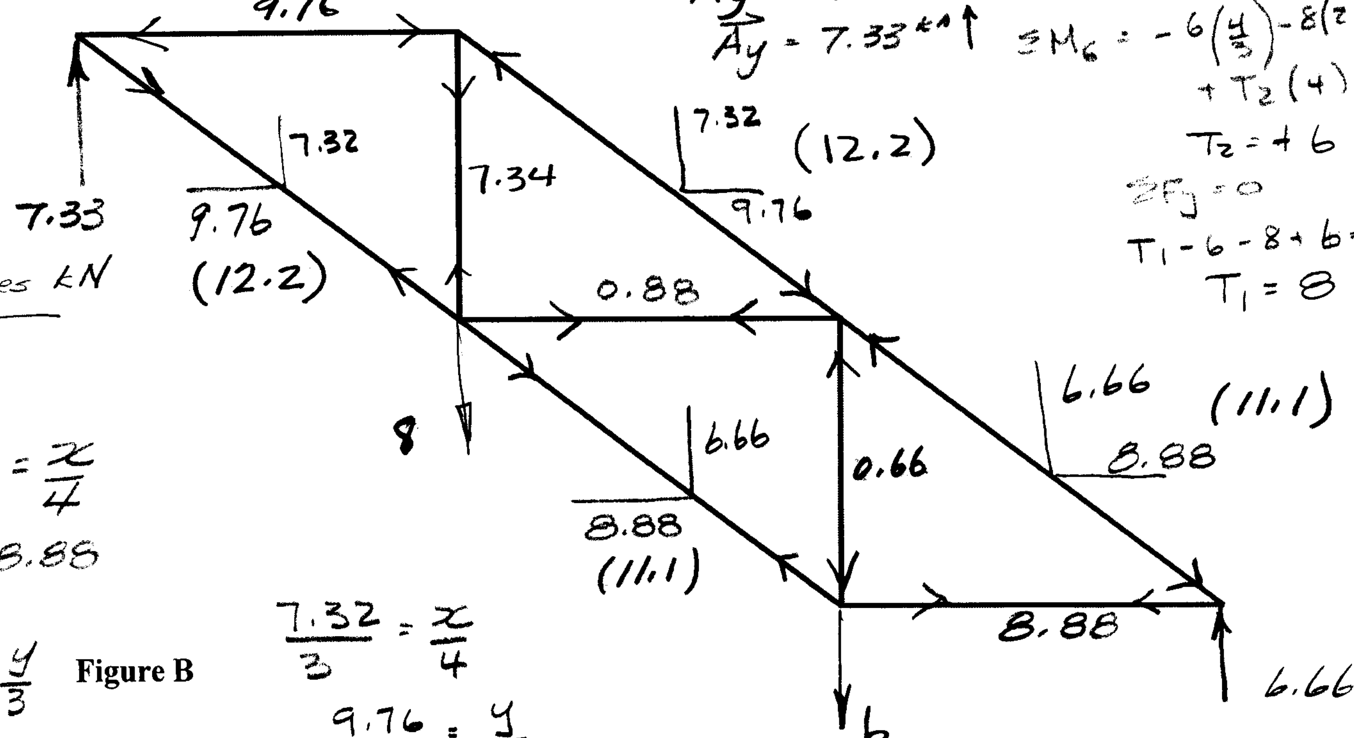
$T_2 = +6$

$\Sigma F_y = 0$

$T_1 - 6 - 8 + 6 = 0$

$T_1 = 8$

All forces kN




$\frac{6.66}{3} = \frac{x}{4}$

$x = 8.88$

$\frac{8.88}{4} = \frac{y}{3}$

Figure B

$\frac{7.32}{3} = \frac{x}{4}$

$\frac{9.76}{4} = \frac{y}{3}$

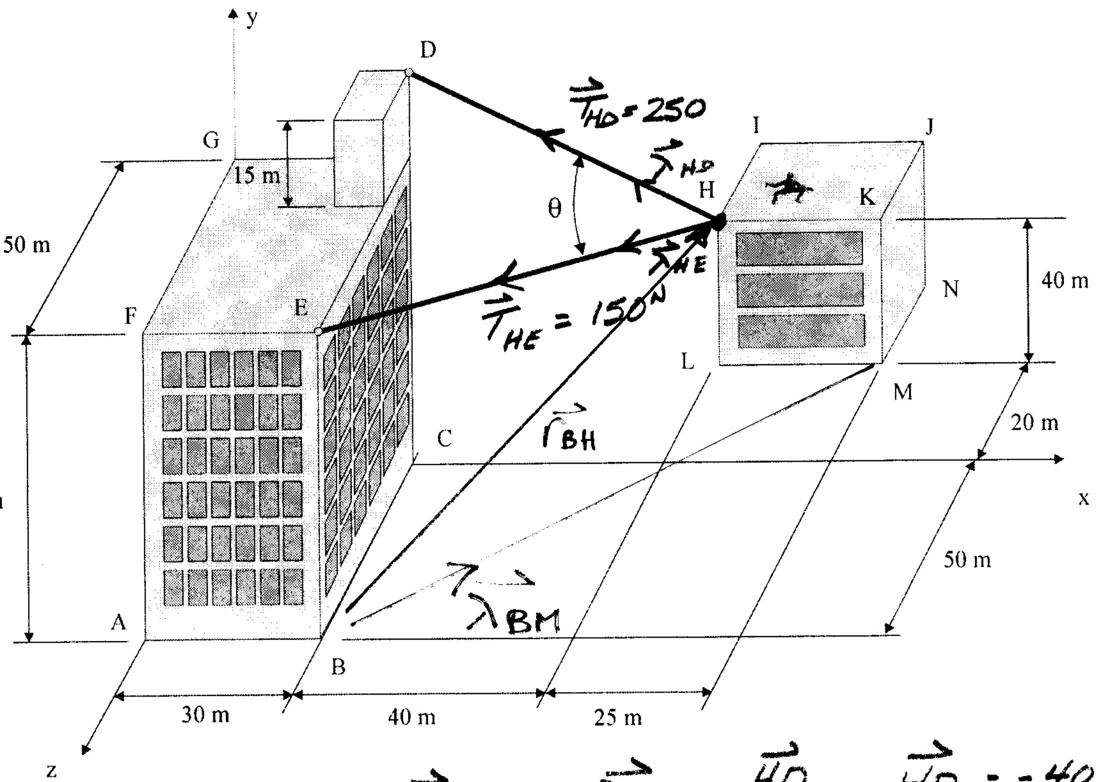
## QUESTION 4

Spiderman casts two web lines *HE* and *HD* across two buildings as shown in the figure below. In doing so, he applied a force of *150 N* on line *HE* and *250 N* on line *HD* pulling from point *H*.

Determine:

a) The resultant, ***R***, of the two forces acting at the point *H*,
b) The angle, $\theta$, between the two web lines at *H*,
c) The moment of the resultant, ***R***, about the point *B*, and
d) The moment of the resultant, ***R***, about the line *BM*.

$H(70, 40, -20)$
$D(30, 115, 0)$
$E(30, 100, 50)$
$B(30, 0, 50)$
$M(95, 0, -20)$




a) $\vec{T}_{HD} = T_{HD}\vec{\lambda}_{HD} = 250\,\vec{\lambda}_{HD}$ $\quad \vec{\lambda}_{HD} = \dfrac{\vec{HD}}{HD}$, $\quad \vec{HD} = -40\hat{i} + 75\hat{j} + 20\hat{k}$

$$HD = \sqrt{(-40)^2 + (75)^2 + (20)^2} = 87.32 \qquad \vec{\lambda}_{HD} = \frac{-40\hat{i} + 75\hat{j} + 20\hat{k}}{87.32}$$

$$\vec{T}_{HD} = 250\left(\frac{-40\hat{i} + 75\hat{j} + 20\hat{k}}{87.32}\right) = (-114.52\hat{i} + 214.73\hat{j} + 57.26\hat{k})$$

$\vec{T}_{HE} = T_{HE}\vec{\lambda}_{HE} = 150\,\vec{\lambda}_{HE}$ $\quad \vec{\lambda}_{HE} = \dfrac{\vec{HE}}{HE}$, $\quad \vec{HE} = -40\hat{i} + 60\hat{j} + 70\hat{k}$

$$HE = \sqrt{(-40)^2 + (60)^2 + (70)^2} = 100.5$$

$$\vec{\lambda}_{HE} = \frac{-40\hat{i} + 60\hat{j} + 70\hat{k}}{100.5}$$

$$\vec{T}_{HE} = 150\left(\frac{-40\hat{i} + 60\hat{j} + 70\hat{k}}{100.5}\right) = -59.7\hat{i} + 89.55\hat{j} + 104.48\hat{k}$$

$$\vec{R} = \vec{T}_{HD} + \vec{T}_{HE} = (-114.52\hat{i} + 214.73\hat{j} + 57.26\hat{k}) + (-59.7\hat{i} + 89.55\hat{j} + 104.48\hat{k})$$

$$\vec{R} = (-174.22\hat{i} + 304.28\hat{j} + 161.74\hat{k})\ N$$

b) $\vec{\lambda}_{HD} \bullet \vec{\lambda}_{HE} = \cos\theta$

$$\left(\frac{-40\hat{i} + 75\hat{j} + 20\hat{k}}{87.32}\right) \bullet \left(\frac{-40\hat{i} + 60\hat{j} + 70\hat{k}}{100.5}\right)$$

$$= \frac{1}{(87.32)(100.5)}\left[(-40)(-40) + (75)(60) + (20)(70)\right]$$

$$= 0.85464 \implies \theta = 31.28^\circ$$

c) $\vec{M}_B = \vec{r}_{BH} \times \vec{R}$

$$\vec{r}_{BH} = 40\hat{i} + 40\hat{j} - 70\hat{k}$$

$$\vec{M}_B = \left|\begin{array}{ccc|cc} i & j & k & i & j \\ 40 & 40 & -70 & 40 & 40 \\ -174.22 & 304.28 & 161.74 & -174.22 & 304.28 \end{array}\right|$$

$$= \left[6469.6\hat{i} + 12195.4\hat{j} + 12171.2\hat{k}\right] - \left[-6968.8k - 21299.6\,\hat{i} + 6469.6\hat{j}\right]$$

$$\vec{M}_B = \left(27769.2\hat{i} + 5701.6\hat{j} + 19140\hat{k}\right) \text{N.m}$$

d) $M_{BM} = \vec{\lambda}_{BM} \bullet \vec{M}_B$

$$\vec{\lambda}_{BM} = \frac{\vec{BM}}{BM} \qquad \vec{BM} = 65\hat{i} + 0\hat{j} - 70\hat{k}$$

$$BM = \sqrt{(65)^2 + (-70)^2} = \sqrt{9125} = 95.52$$

$$\vec{\lambda}_{BM} = \frac{65\hat{i} + 0\hat{j} - 70\hat{k}}{\sqrt{9125}}$$

$$\vec{\lambda}_{BM} \bullet \vec{M}_B = \left(\frac{65\hat{i} + 0\hat{j} - 70\hat{k}}{\sqrt{9125}}\right)\left(27769.2\hat{i} + 5701.6\hat{j} + 19140\hat{k}\right)$$

$$= \frac{1}{\sqrt{9125}}\left[(65)(27769.2) + (-70)(19140)\right]$$

$$= +4869.9 \text{ N.m}$$