ENG 1460 Introduction to Thermal Sciences **Winter 2010** **Term Test 2**

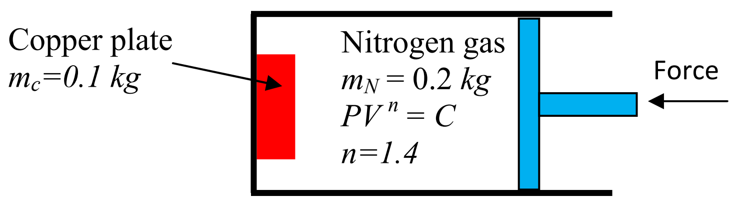
Instructor: Dr. B.-C. Wang

March 12, 2010
Duration: 100 min

(a) You are permitted to use the course textbook and a calculator.
(b) Ask for clarification if a problem statement is not clear.
(c) Use linear interpolation in the property tables as necessary.
(d) **Show your work** (or, **the complete solution steps**) in a clear and logical manner. Solutions that require unreasonable effort (in the opinion of the instructor) for the marker to decipher will not be credited.

1. *(14 marks)*
A piston-cylinder assembly contains nitrogen gas with a mass of m_N = 0.2 *kg*. At the initial state (state 1), the pressure and temperature of the nitrogen are P_1 = 100 *kPa* and T_1 = 100 ^oC, respectively. The nitrogen is compressed in a polytropic process with an exponent n = 1.4. At the final state (state 2), the pressure of the nitrogen gas is P_2 = 500 *kPa*. A copper plate inside the cylinder is in thermal equilibrium with the nitrogen gas at both the initial and the final states of the process. The mass of the copper plate is m_C =0.1 *kg*. The nitrogen can be treated as an ideal gas.

(a) Determine the initial and final volume of the nitrogen gas (i.e., V_1 and V_2). *(4 marks)*
(b) Determine the final temperature of the nitrogen gas (i.e., T_2). *(2 mark)*
(c) Determine the *total work* of the process (i.e., ${}_1W_2$). *(2 marks)*
(d) Determine the *total heat transfer* of the process (i.e., ${}_1Q_2$) between the cylinder and surroundings. *(3 marks)*
(e) Show the state points and the process on a *P–V* (pressure–volume) diagram. *(2 mark)*
(f) Indicate the area that represents the *total work* of the process on the *P-V* (pressure-volume) diagram. *(1 mark)*



2. *(17 marks)*
A rigid tank is divided into two rooms by a membrane. The volume for room A and room B is V_A=1m^3 and V_B=60m^3, respectively. At the initial state (state 1), the water temperature and pressure in room A are T_{A1}=100 ^oC and P_{A1}=500 *kPa*, respectively; and room B contains m_B=200 *kg* water at 800 *kPa*. The membrane raptures and heat transfers between the tank and its surroundings. At the final state (state 2), water inside the tank reaches an equilibrium state, with a final pressure of P_2=600 *kPa*.

(a) Determine the mass and specific internal energy of the water in room A at the initial state (i.e., m_A and u_{A1}). *(3 marks)*
(b) Determine the temperature and specific internal energy of water in room B at the initial state (i.e., T_{B1} and u_{B1}). *(5 marks)*
(c) Determine the temperature and specific internal energy of water at the final state (i.e., T_2 and u_2). *(6 marks)*
(d) Determine the total heat transfer (between the tank and its surroundings) during the process (i.e., ${}_1Q_2$). *(3 marks)*

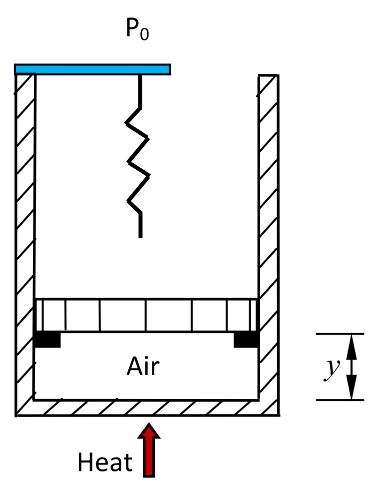
A	**B**
water	water
V_A=1 m^3	V_B =60 m^3
T_{A1}=100 ^oC	m_B=200 kg
P_{A1}=500 kPa	P_{B1}=800 kPa

Hint:
The phase of water is not given, and therefore needs to be determined.

3. *(19 marks)*

A frictionless piston-cylinder and spring assembly containing air is shown in the figure below. Initially, the piston rests on the stops. Heat is transferred to the air from the outside. As shown in the figure, the distance between the bottom surface of the piston and the bottom surface of the cylinder is denoted as y. The value of y is: $0.2\,m$ and $0.4\,m$ when the piston rests on the stops and just begins to touch the spring, respectively. The piston mass is $m_P = 300kg$. The inner cross-sectional area of the cylinder is $A = 0.05\,m^2$. The local acceleration of gravity is $g = 9.81\,m/s^2$ and the local atmospheric pressure is $P_0 = 100\,kPa$, respectively. The spring-constant of the linear spring is $k_S = 50\,kN/m$. The initial temperature and the initial pressure of the air are $T_1 = 30\,^oC$ and $P_1 = 95\,kPa$, respectively. At the final state, the pressure is $P_4 = 300\,kPa$. The air can be treated as an ideal gas. The volume of the stops can be ignored.

- **(a)** Determine the mass (m) of the air inside the cylinder. *(1 marks)*
- **(b)** Determine the pressure (P_2) at which the piston starts to float (or, just about to leave the stops). *(2 marks)*
- **(c)** Determine the pressure (P_3) and temperature (T_3) when the piston *just* begins to touch the spring. *(2 marks)*
- **(d)** Determine the final volume (V_4). *(2 marks)*
- **(e)** Determine the final temperature (T_4). *(1 marks)*
- **(f)** Determine the *total work* (${}_1W_4$) done during the process. *(3 marks)*
- **(g)** Determine the *total heat transfer* (${}_1Q_4$) during the process. *(3 marks)*
- **(h)** Show the state points and process paths on a *P–V* (pressure–volume) diagram. *(2.5 marks)*
- **(i)** Indicate the area that represents the total work on the *P-V* (pressure–volume) diagram. *(1.5 mark)*



Suggestion & hint:

Subscripts "1", "2", "3",... are used to indicate a succession of state points associated with this multi-step process. For instance, "T_1" and "V_1" represent the initial temperature and initial volume of the air, respectively.

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Prof. B.-C. Wang March 12, 2010

Problem 1

From Table A.5, for nitrogen,

(a) $R = 0.2968 \frac{kJ}{kg \cdot K}$, $C_{v0} = 0.745 \frac{kJ}{kg \cdot K}$

$$V_1 = \frac{mRT_1}{P_1} = \frac{0.2 \times 0.2968 \times (100+273.15)}{100} = 0.2215 (m^3)$$

$$P_1 V_1^n = P_2 V_2^n$$

$$\therefore V_2 = V_1 \cdot \left(\frac{P_1}{P_2}\right)^{\frac{1}{n}} = 0.2215 \times \left(\frac{100}{500}\right)^{\frac{1}{1.4}} = 0.07016 \ (m^3)$$

(b) $P_2 V_2 = mRT_2$

$$\therefore T_2 = \frac{P_2 V_2}{mR} = \frac{500 \times 0.07016}{0.2 \times 0.2968} = 590.97 \ (K)$$

or, $317.82 \ (^\circ C)$

(c)

$${}_1W_2 = \frac{mR(T_2 - T_1)}{1-n} = \frac{0.2 \times 0.2968 \times (317.82 - 100)}{1 - 1.4} = -32.324 \ (kJ)$$

(d) ${}_1Q_2 = {}_1W_2 + \Delta U + \cancel{\Delta KE}^{0} + \cancel{\Delta PE}^{0}$

$$\therefore {}_1Q_2 = {}_1W_2 + (m_{copper} \cdot \Delta U_{copper} + m_{Nitrogen} \cdot \Delta U_{Nitrogen})$$

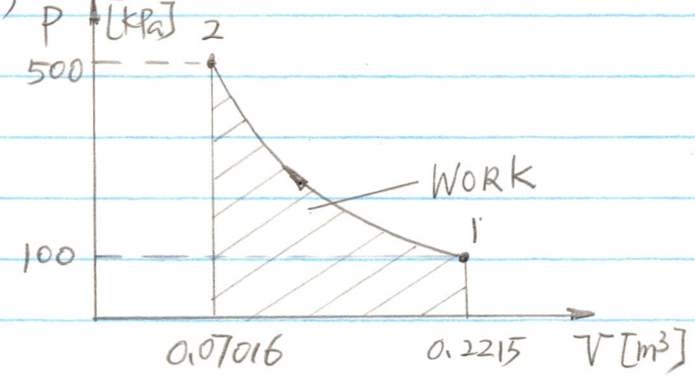
$$= {}_1W_2 + (m_{copper} \cdot C_{copper} + m_{Nitrogen} C_{v0,nitrogen})(T_2 - T_1)$$

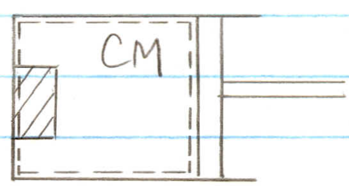
$$= -32.324 + (0.1 \times 0.42 + 0.2 \times 0.745) \cdot (317.82 - 100)$$

$$= 9.280 \ (kJ)$$

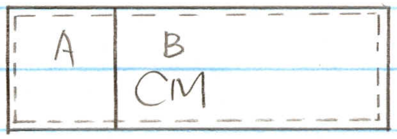
(From table A.3, the specific heat for copper is $C_{copper} = 0.42 \frac{kJ}{kg \cdot K}$)

(e)&(f)





Problem 2



$CM = A + B$

(a) From Table B.1.1

at $T = 100°C$, $P_{sat} = 101.3$ kPa

$P_{A1} = 500$ kPa $> P_{sat}$

$\therefore$ Compressed Liquid

From Table B.1.4

at $P_{A1} = 500$ kPa & $T_{A1} = 100°C$

$v_{A1} = 0.001043$ m³/kg

$u_{A1} = 418.80$ KJ/kg

$$m_A = \frac{V_A}{v_{A1}} = \frac{1}{0.001043} = 958.773 \text{ (kg)}$$

(b) Room B:

$$v_{B1} = \frac{V_B}{m_B} = \frac{60}{200} = 0.3 \ (m^3/kg)$$

From Table B.1.2

at $P_{B1} = 800$ kPa, $v_g = 0.24043$ m³/kg

$v_{B1} > v_g$ $\therefore$ Superheated Vapour.

From table B.1.3

At $P = 800$ kPa

V[m³/kg]	T[°C]	U[KJ/kg]
0.29314	250	2715.46
0.3	T_{B1}=?	U_{B1}=?
0.32411	300	2797.14

$$T_{B1} = 250 + \frac{0.3 - 0.29314}{0.32411 - 0.29314}(300 - 250) = 261.075 (°C)$$

or, 261.08 °C

$$U_{B1} = 2715.46 + \frac{0.3 - 0.29314}{0.32411 - 0.29314}(2797.14 - 2715.46) = 2733.55 \left(\frac{KJ}{kg}\right)$$

(c) $V_2 = V_{tot} = V_A + V_B = 1 + 60 = 61\ (m^3)$

$m_2 = m_{tot} = m_A + m_B = 958.773 + 200 = 1158.773\ (kg)$

$v_2 = \frac{V_2}{m_2} = \frac{61}{1158.773} = 0.052642\left(\frac{m^3}{kg}\right)$

Table B.1.2.

At $P_2 = 600\ kPa$, $T_{sat} = 158.85\ °C$

$v_f = 0.001101\ \frac{m^3}{kg}$, $v_{fg} = 0.31457\ \frac{m^3}{kg}$, $v_g = 0.31567\ \frac{m^3}{kg}$

$u_f = 669.88\ \frac{kJ}{kg}$, $u_{fg} = 1897.52\ \frac{kJ}{kg}$, $u_g = 2567.40\ \frac{kJ}{kg}$

$v_f < v_2 < v_g$ ∴ Saturated mixture

$x = \frac{v_2 - v_f}{v_{fg}} = \frac{0.052642 - 0.001101}{0.31457} = 0.16385$

$u_2 = u_f + x\,u_{fg} = 669.88 + 0.16385 \times 1897.52 = 980.78\ \left(\frac{kJ}{kg}\right)$

$T_2 = T_{sat} = 158.85\ (°C)$

(d)

$_1Q_2 = \cancel{_1W_2}^{0} + \Delta U + \cancel{\Delta KE}^{0} + \cancel{\Delta PE}^{0}$

$\therefore\ _1Q_2 = \Delta U = U_2 - U_1$

$_1Q_2 = m_2 u_2 - (m_{A1} u_{A1} + m_{B1} u_{B1})$

$= 1158.773 \times 980.78 - (958.773 \times 418.8 + 200 \times 2733.55)$

$= 188257.25\ (kJ)$

or, $188.257\ MJ$

Problem 3

(a) $V_1 = Ay_1 = 0.05 \times 0.2 = 0.01\ (m^3)$

From table A.5, for air:

$R = 0.287\ kJ/kg.K\ ,\quad C_{V0} = 0.717\ kJ/kg.K\ ,$

$$m = \frac{P_1 V_1}{R T_1} = \frac{95 \times 0.01}{0.287 \times (273.15 + 30)} = 0.01092\ (kg)$$

(b) $$P_2 = P_0 + \frac{m_p g}{A} = 100 \times 10^3 + \frac{300 \times 9.81}{0.05} = 158860\ (Pa)$$

or, $158.86\ (kPa)$

(c) $P_3 = P_2 = 158.86\ kPa$

$V_3 = Ay_3 = 0.05 \times 0.4 = 0.02\ (m^3)$

$$T_3 = \frac{P_3 V_3}{mR} = \frac{158.86 \times 0.02}{0.01092 \times 0.287} = 1013.77\ (K)$$

or, $740.62\ (°C)$

(d) For a linear spring,

$$P_4 = P_3 + \frac{k_s}{A^2}(V_4 - V_3)$$

$$300 = 158.86 + \frac{50}{0.05^2}(V_4 - 0.02)$$

$\Rightarrow V_4 = 0.02706\ (m^3)$

(e) $$T_4 = \frac{P_4 V_4}{mR} = \frac{300 \times 0.02706}{0.01092 \times 0.287} = 2589.98\ (K)$$

or, $2316.83\ (°C)$

(f) ${}_1W_4 = \cancel{{}_1W_2} + {}_2W_3 + {}_3W_4$

${}_2W_3 = P_2(V_3 - V_2) = 158.86 \times (0.02 - 0.01) = 1.5886\ (kJ)$

${}_3W_4 = \frac{1}{2}(P_3 + P_4)(V_4 - V_3) = \frac{1}{2}(158.86 + 300)(0.02706 - 0.02) = 1.6198\ (kJ)$

$\therefore {}_1W_4 = 1.5886 + 1.6198 = 3.208\ (kJ)$

(g) ${}_1Q_4 = {}_1W_4 + \Delta U + \cancel{\Delta KE} + \cancel{\Delta PE}$

$\therefore {}_1Q_4 = {}_1W_4 + m C_{V0}(T_4 - T_1)$

$\therefore {}_1Q_4 = 3.208 + 0.01092 \times 0.717 \times (2316.83 - 30)$

$\therefore {}_1Q_4 = 21.11$ (kJ)

(h) & (i)