16.107 Final Exam April 2000 29/11/00 SAP.

(A1)

vertical SHM $y = y_m \cos(\omega t + \phi)$

$\therefore v_y = \frac{dy}{dt} = -\omega y_m \sin(\omega t + \phi)$

at $t = 0$ $y = 0.43$ cm $= y_m \cos\phi$

$v_y > 0$; $y_m = 0.82$ cm

$\therefore \cos\phi = \frac{0.43}{0.82} \Rightarrow \phi = \pm 58°$ and from v_y, $\sin\phi < 0$

$\therefore \boxed{\phi = -58° + 360° = 302° \quad e)}$

(A2)

initial SHM condition: $E_{Tot} = \frac{1}{2} k y_m^2 = \frac{1}{2} m v_{MAX}^2$

piece falls off at $y = -y_m$ (at rest) $\therefore$ same total energy

$m \rightarrow m' = \frac{3m}{4}$ keeps oscillating

$E_{Tot} = \frac{1}{2} m v_{MAX}^2 = \frac{1}{2} m' (v'_{max})^2$; $\left(\frac{v'}{v}\right)_{max} = \sqrt{\frac{m}{m'}} = \sqrt{\frac{m}{3m/4}}$

or $\boxed{(v'_{max} / v_{max}) = 2/\sqrt{3} \quad a)}$

(A3)

** strings will oscillate at the same frequency **

string A: tension T_A ; $n = 1$ mode of string $\therefore L = \frac{\lambda}{2}$

string B: " T_B ; $n = 3$ " " $L = \frac{3\lambda'}{2}$

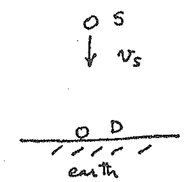
$\lambda f = \sqrt{\frac{T_A}{\mu}}$ and $\lambda' f = \sqrt{\frac{T_B}{\mu}}$ where μ = mass density

from the standing wave condition

$\frac{\lambda}{\lambda'} = \frac{2L}{\frac{2}{3}L} = 3 = \sqrt{\frac{T_A}{T_B}} \rightarrow \boxed{\frac{T_B}{T_A} = \frac{1}{9} \quad a)}$

(A4)

$f_o = 1800$ Hz ; diver is a moving source. detector hears f'
v = speed of sound in air.



$f' = f_o \frac{v}{(v - v_s)} = 2150$ Hz

$\frac{2150}{1800} = \frac{1}{1 - \frac{v_s}{v}} = 1.194 \Rightarrow \frac{v_s}{v} = 0.163$

after reflecting off the earth, sound travels back upwards at frequency $f' = 2150$ Hz, but the diver falls towards it and so hears a higher frequency.

$f'' = f' \frac{(v + v_D)}{v}$ with $\frac{v_D}{v} = 0.163$ as above

so $f'' = 2150(1 + 0.163) = \boxed{2500 \text{ Hz} \quad c)}$

(A5)

maxima seen at $n\lambda = d \sin\theta$

$n = 3$ seen at $\theta = 30°$ for $\lambda = 500$ nm

$\therefore$ slit spacing $d = \frac{3(500 \times 10^{-9})}{\sin 30°} = 3 \times 10^{-6}$ m

Maximum deflection angle is 90° for $\lambda = 700$ nm

$n_{max}(700 \times 10^{-9}) = (3 \times 10^{-6}) \sin(90°) \Rightarrow n_{max} = 4.29$

round down to 4 for complete maxima. Total seen $= \boxed{2n + 1 = 9 \quad d)}$
(4 maxima either side of $n = 0$ central max.)

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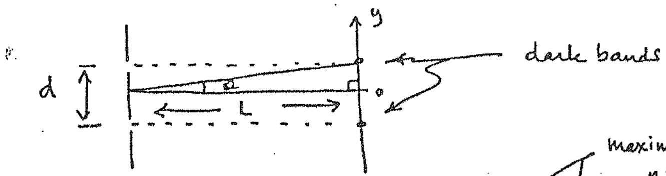
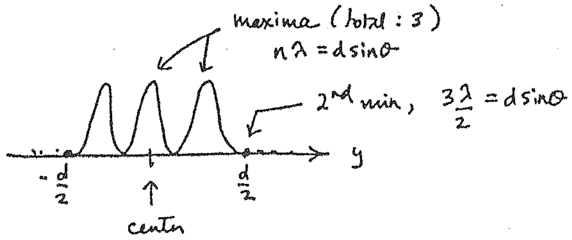
A6 $\lambda = 442$ nm ; $d = 0.400$ nm

screen at distance L.



intensity pattern :



for 2nd min directly behind slits we have $\frac{3\lambda}{2} = d\sin\theta$

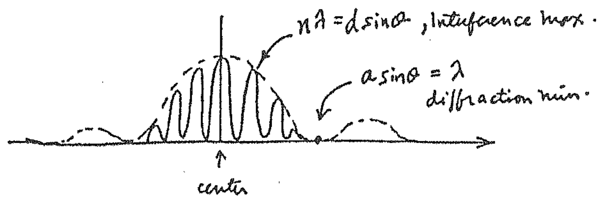
$$\sin\theta = \frac{3\lambda}{2d} = 1.66\times10^{-3} << 1$$

so this is a small angle, $\sin\theta \simeq \theta \simeq \tan\theta$

$$\tan\theta = \frac{d}{2L} = 1.66\times10^{-3} \Rightarrow \boxed{L = 0.12\ \text{m}\quad a)}$$

A7 Intensity pattern :

$\lambda = 500$ nm
$d = 0.2$ mm
$a = 0.1$ mm



out to 1st diffraction min., $\sin\theta_{max} = \frac{\lambda}{a} = 5\times10^{-3}$

this gives a value $n = \frac{d\sin\theta_{max}}{\lambda} = \frac{d}{a} = 2$ in the interference pattern.

So the $n=2$ interference peaks are not visible as they lie under the diffraction minima. $\therefore$ $n=0$ and $n=1$ are visible either side of the central max. Total: $\boxed{3 \text{ are visible } (e)}$

A8

waves are in phase at A and B

phase change $\Phi = k\Delta x = 2\pi\frac{\Delta x}{\lambda}$ after passing distance Δx.

but $\lambda \to \frac{\lambda}{n}$ in a material and so this has to be taken into account.

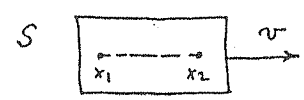
path 2 : $\Phi_2 = 2\pi\left[\frac{(L_1 - L_2)}{\lambda} + \frac{n_2 L_2}{\lambda}\right]$

path 1 : $\Phi_1 = 2\pi\frac{n_1 L_1}{\lambda}$

phase difference after emerging is $(\Phi_2 - \Phi_1) = \boxed{5.24 \text{ rad } (b)}$

A9 meson created at x_1, t_1 ; decays at x_2, t_2

within moving frame :



$$x_2 - x_1 = L = v\Delta t = \Delta x$$

lifetime at rest is $\Delta t' = \gamma\left(\Delta t - \frac{v\Delta x}{c^2}\right)$

(rest frame moves in opposite direction at speed v relative to the first measurement.)

put $\Delta t' = \gamma\left[\frac{L}{v} - \frac{vL}{c^2}\cdot\frac{v}{v}\right]$

$$= \frac{\gamma L}{v}\left[1 - \frac{v^2}{c^2}\right] \qquad \text{But } \gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$$

$$\therefore \quad \Delta t' = \boxed{\frac{L}{v}\sqrt{1 - v^2/c^2}\quad d)}$$

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A10

frame S : $x_A = 100$; $x_B = 0$; $\Delta x = x_A - x_B = +100$ m
$t_A = 0$; $t_B = 9\times10^{-8}$; $\Delta t = t_A - t_B = -9\times10^{-8}$ s

frame S': $(v = 0.95c)$

$x' = \gamma(x - vt)$ $\therefore$ $\Delta x' = \gamma(\Delta x - v\Delta t)$

$\gamma = \dfrac{1}{\sqrt{1-(0.95)^2}} = 3.20$

$\Delta x' = 3.20(100 + 0.95c(9\times10^{-8})) = \boxed{402 \text{ m} \quad e)}$

A11

0.8c ← Q_1 Q_2 → 0.5c

addition of velocities : $u' = \dfrac{u+v}{1+\frac{uv}{c^2}}$

velocity of Q_1 seen by Q_2 is $\dfrac{0.8+0.5}{1+0.8(0.5)} = \boxed{0.93c \quad c)}$

A12

doppler shift, source receding: $f' = f_0\sqrt{\dfrac{1-v/c}{1+v/c}}$

$\lambda f = c$ $\therefore$ $\dfrac{1}{\lambda'} = \dfrac{1}{\lambda_0}\sqrt{\dfrac{1-v/c}{1+v/c}}$

$\lambda' = 500\text{ nm}\sqrt{\dfrac{1.9}{0.1}} = \boxed{2180 \text{ nm} \quad d)}$

A13

relativistic momentum $p = \gamma m v = 2mc$

$\therefore$ $2c = \gamma v$

$\therefore$ $4 = \dfrac{(v/c)^2}{1-(v/c)^2}$; $\boxed{\dfrac{v}{c} = \sqrt{\dfrac{4}{5}} = 0.89 \quad e)}$

A14

$E_{total} = K + E_{pot}$

after photon absorbed, at $r = 1.00$ nm, $E_{pot} = -1.4$ eV
and $K = 25.0$ eV $\therefore$ $E_{total} = 23.6$ eV

before : $E = -13.6$ eV ; $-13.6 + hf = 23.6$ eV

$\therefore$ $\boxed{hf = 37.2 \text{ eV photon energy} \quad c)}$

A15

diffraction grating has $600 \frac{\text{lines}}{\text{mm}}$ $\therefore$ $d = \dfrac{10^{-3}\text{m}}{600} = 1.67\times10^{-6}$ m

2^{nd} order diffraction at $m=2$; $2\lambda = d\sin\theta$

$\therefore$ $\sin\theta = \dfrac{2\lambda}{d}$; need to know λ

hydrogen spectrum: $E_n = -\dfrac{13.6}{n^2}$; transition $\Delta E = -13.6\left[\frac{1}{9}-\frac{1}{4}\right]$

$\therefore$ $\Delta E = +1.89$ eV $= hf = \dfrac{hc}{\lambda}$ for photon emitted.

$hc = 1240$ eV·nm

$\therefore$ $\lambda = \dfrac{1240}{1.89} = 656$ nm and $\sin\theta = \dfrac{2\lambda}{d}$; $\boxed{\theta = 52° \quad d)}$

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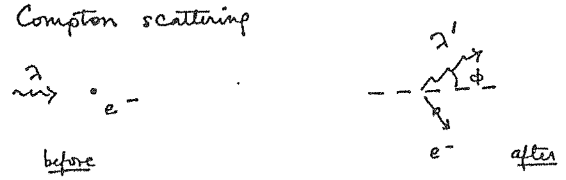
A16 Compton scattering



$\lambda' = \lambda + \Delta\lambda$ with $\Delta\lambda = \frac{h}{mc}(1-\cos\phi) = \lambda' - \lambda$

Note $E = \frac{hc}{\lambda}$ and $\lambda' > \lambda$ $\therefore$ energy is reduced

$\therefore$ max $\Delta\lambda$ corresponds to minimum energy $\leftrightarrow \phi = 180°$

then $\Delta\lambda = \frac{2h}{mc}$; so $\lambda' = \lambda + \Delta\lambda$

$E' = \frac{hc}{(\lambda+\Delta\lambda)}$

$$E' = \frac{hc}{\left(\frac{hc}{E} + \frac{2hc}{mc^2}\right)} = \frac{1}{\frac{1}{E} + \frac{2}{mc^2}}$$

original energy $E = 1.00$ MeV ; $mc^2 = 0.511$ MeV

$$\therefore E' = \frac{1}{1 + \frac{2}{0.511}} = \boxed{0.20 \text{ MeV} \quad a)}$$

A17 $\psi(x) = \psi_0 e^{i(kx-\omega t)}$

probability density $|\psi^2| = \psi_0^2$ has units m^{-1}

$\therefore \boxed{[\psi_0] = m^{-1/2} \quad b)}$

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18 momentum $p = \hbar k$; kinetic energy $K = p^2/2m$

$\therefore \boxed{K = \frac{\hbar^2 k^2}{2m} \quad e)}$

A19 Wave function outside the well is $\psi = \psi_0 e^{-kx}$

Solution to the Schrödinger wave equation :

$$\frac{d^2\psi}{dx^2} + \frac{8\pi^2 m}{h^2}(E - E_{pot})\psi = 0$$

$E = 24.5$ eV , $E_{pot} = 30$ eV $\therefore \frac{8\pi^2 m}{h^2}(E - E_{pot}) = -144 \text{ nm}^{-2}$

$\frac{d^2\psi}{dx^2} = k^2\psi$ with $k^2 = 144 \text{ nm}^{-2}$

$\left[\psi = \psi_0 e^{-kx} \; ; \; \frac{d\psi}{dx} = -k\psi_0 e^{-kx} \; ; \; \frac{d^2\psi}{dx^2} = k^2\psi_0 e^{-kx} \checkmark\right]$

$\therefore k = 12 \text{ nm}^{-1} = \sqrt{\frac{8\pi^2 m (E_{pot} - E)}{h^2}}$ as above

Probability density $|\psi^2| \sim \psi_0^2 e^{-2kx}$

falls off by $\frac{1}{e}$ a distance x into the classically forbidden region given by $2kx = 1$ $\therefore x = \frac{1}{2k}$

$$\therefore x = \frac{1}{24 \text{ nm}^{-1}} = \boxed{42 \text{ pm} \; e)}$$

A20 The quantized energy levels shown in the diagram are evenly spaced. None of the systems listed has evenly spaced energy levels

$\therefore$ answer is $\boxed{e) \text{ none of these}}$

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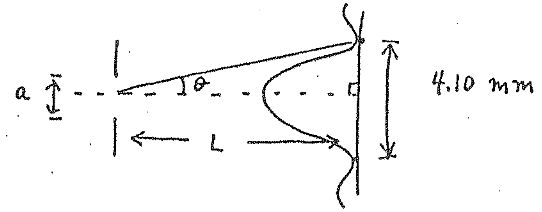
Long Answer

B1) Interference and Diffraction

a) one slit : diffraction pattern



$L = 2.06$ m , $a = 0.550$ mm

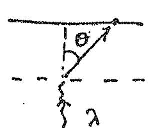
1st diffraction minima are at $\lambda = a \sin\theta$

small angle: $\sin\theta \sim \tan\theta = \frac{2.05 \text{ mm}}{L} = 1 \times 10^{-3}$

$\therefore$ wavelength $\lambda = (0.55 \times 10^{-3} \text{m})(1 \times 10^{-3}) = \boxed{550 \text{ nm}}$

b) diffraction grating 400 lines/mm

$\therefore d = \frac{1}{400}$ mm $= 0.0025$ mm $= 2.5 \times 10^{-6}$ m



bright spots when $m\lambda = d\sin\theta$

max. $\theta = 90°$, $\sin\theta = 1$

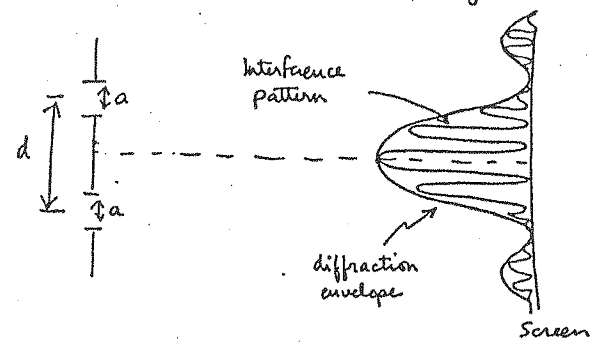
$\therefore m_{max}\lambda = d$

$$m_{max} = \frac{d}{\lambda} = \frac{2.5 \times 10^{-6} \text{ m}}{[(400 - 700) \times 10^{-9} \text{ m}]}$$

range of m_{max} is 3.6 – 6.5 . To see full order for the whole spectrum we are limited to $m = 0, 1, 2, 3$.
This gives a total of 7 bright bands on the screen for the full visible spectrum.

c) diffraction grating

Consider just 2 slits , spacing d and width a :



diffraction envelope has minima when $m\lambda = a\sin\theta$

or $\sin\theta = \frac{m\lambda}{a}$

interference pattern has maxima when $n\lambda = d\sin\theta$

or $\sin\theta = \frac{n\lambda}{d}$ for $n = 0, 1, 2, 3 \ldots$

Now the special case in the problem has $d = 2a$

$\therefore$ diffraction minima occur when $\sin\theta = 2m\frac{\lambda}{d}$

for $m = 1, 2, 3$

Interference maxima are blocked if $\frac{n\lambda}{d} = \frac{m\lambda}{a} = 2m\frac{\lambda}{d}$

that is, for $n = 2m$!

Orders of interference (max) : $n = 0 \;,\; 1 \;,\; 2 \;,\; 3 \;,\; 4 \ldots$

Diffraction min : $m = \text{---} \;,\; \text{----} \;,\; 1 \;,\; \ldots \;,\; 2 \ldots$

so $n = 2, 4, 6, 8 \ldots$ etc are all eliminated. Multiple slits in a grating just narrows the bands but does not change the spacing.

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(B2) <u>Relativity</u>

(A) $\longrightarrow$ $0.65c$ (EARTH) $0.50\,c$ $\longleftarrow$ (B)

each ship is 25 m long in its rest frame.

a.) length of B as measured in earth frame :

B moves at $v/c = 0.5$ $\quad \therefore \gamma = \dfrac{1}{\sqrt{1-(0.5)^2}} = 1.155$

moving ship appears shorter

$$L = \frac{L_0}{\gamma} = 21.6 \text{ meters}$$

b) speed of B relative to A :

velocities add according to $\quad u = \dfrac{u'+v}{1+\frac{u'v}{c^2}}$

$$\therefore \quad u = \frac{0.50 + 0.65}{1 + (0.50)(0.65)} = 0.87\ c$$

c) two events in B : $\quad x_1 = 0 \quad x_2 = 0$
$\quad t_1 = 0 \quad t_2 = 5.0$ sec

$\Delta x = 0 \qquad \Delta t = 5.0 \text{ sec} = (t_2 - t_1)$

$\Delta t' = \gamma\left(\Delta t - \dfrac{v\Delta x}{c^2}\right) = \gamma\,\Delta t$ seen in A

where γ describes relative motion of B and A

$$\therefore \gamma = \frac{1}{\sqrt{1-(0.87)^2}} = 2.02$$

$\Delta t' = 2.02\,(5.0) = 10$ sec according to ship A

the end.