MATH1210 Assignment #1

Due: 1:30 pm Friday 22 September 2006 T. G. Berry

NOTES:

(1) The assignment is due at the start of our class on Friday 22 September 2006.
(2) Late assignments will NOT be accepted.
(3) If your assignment is not accompanied by a Faculty of Science "Honesty Declaration", it will NOT be graded.

1. Use the Principle of Mathematical Induction (PMI) in order to verify that for n any positive integer the quantity n^3+6n^2+2n is divisible by 3 .

2. (a) For n any positive integer, rewrite the sum $\sum_{i=1}^{2n}(i+1)$ out explicitly in the form "$(term1)+(term2)+\cdots+(last\ term)$" , and describe precisely in words the "meaning" of this sum.
[Note: You might find it useful to consider the special cases $n=1,2,3,4,5$ before considering the general case of any positive integer n .]

(b) Use PMI in order to verify that

for any positive integer n : $\sum_{i=1}^{2n}(i+1)=n(2n+3)$.

(c) Use the result that $\sum_{j=1}^{n} j=\frac{n(n+1)}{2}$ for any positive integer n (which we proved in class) in order to verify the result of part (b).
[Note: This can be done in more than one manner, so you might consider alternate procedures for accomplishing this objective.]

3. (a) Rewrite the sum $\frac{1}{1(4)}+\frac{1}{4(7)}+\cdots+\frac{1}{(3n-2)(3n+1)}$ for n any positive integer in sigma notation.

(b) Conjecture a simple formula for the sum appearing in part (a).

(c) Use PMI in order to verify the conjecture you made in part (b).

4. Use PMI to prove that $(x-y)$ is always a factor of $x^n - y^n$ for n any positive integer.

5. Consider the infinite sequence of numbers $\{x_1, x_2, x_3, x_4, x_5, x_6, \cdots\}$ defined by the relations

$$x_1 = 2 \text{ and } x_{n+1} = \frac{1}{2}\left(x_n + \frac{2}{x_n}\right) \text{ for } n \geq 1 .$$

(a) Evaluate x_1, x_2, x_3, x_4 , expressing your answers both in rational and floating-point form.

(b) Use PMI in order to prove that for $n \geq 1$ $1 \leq x_n \leq 2$.