MATH1210 Assignment #4

Due: 1:30 pm Monday 13 November 2006 T. G. Berry

NOTES:

(1) ***The assignment is due at the start of our class on Monday 13 November 2006.***

(2) ***Late assignments will NOT be accepted.***

(3) ***If your assignment is not accompanied by a Faculty of Science "Honesty Declaration", it will NOT be graded.***

1. You are given the following matrices:

$$A=\begin{pmatrix}1 & -2 & 3\\ -3 & 1 & 4\end{pmatrix},\ B=\begin{pmatrix}1 & 0\\ 2 & 1\\ -1 & 2\end{pmatrix},\ C=\begin{pmatrix}3 & -1 & 2\\ 1 & -2 & -1\\ 2 & 0 & 1\end{pmatrix},\ D=\begin{pmatrix}-1 & 2\\ 3 & 2\end{pmatrix},$$

$$E=\begin{pmatrix}1 & 0 & 0\\ 2 & 0 & -1\\ 1 & 1 & -2\end{pmatrix},\ F=\begin{pmatrix}-1 & 2\\ 0 & 2\\ 2 & -1\end{pmatrix},\ I_{(2)}=\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix} \text{ and } I_{(3)}=\begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{pmatrix}.$$

Compute each of the following matrices, if it exist:

(i) AB (ii) BA (iii) $(BI_{(2)})A$

(iv) $B(I_{(3)}A)$ (v) F^TE (vi) $CB+D$

(vii) $AB+2D$ (viii) $AB+D^2$ (with $D^2=DD$)

(ix) $DA+B$ (x) EC (xi) CE

(xii) $EB+F$ (xiii) $FC+D$ (xiv) $FD-3B$

(xv) F^TB+D (xvi) $2F-3(AE)$ (xvii) $(2(AB))^T$

(xviii) $A(BD)$ (xix) $(AB)D$ (xx) A^T

(xxi) B^T (xxii) A^TB^T (xxiii) $(AB)^T$

(xxiv) B^TA^T

2. Reduce each of the following matrices to **row-echelon form** (R.E.F.).:

(i) $\begin{pmatrix} 1 & 2 & -3 & 1 \\ -1 & 0 & 3 & 4 \\ 0 & 1 & 2 & -1 \\ 2 & 3 & 0 & -3 \end{pmatrix}$ (ii) $\begin{pmatrix} 1 & 1 & 3 & -3 & 0 \\ 0 & 2 & 1 & -3 & 3 \\ 1 & 0 & 2 & -1 & -1 \end{pmatrix}$

(iii) $\begin{pmatrix} 0 & 2 & 3 & -4 & 1 \\ 0 & 0 & 2 & 3 & 4 \\ 2 & 2 & -5 & 2 & 4 \\ 2 & 0 & 6 & 9 & 7 \end{pmatrix}$ (Note: it will be necessary to do fractional arithmetic)

3. Reduce each of the following matrices to **reduced row-echelon form** (R.R.E.F.).:

(i) $\begin{pmatrix} 1 & 3 & 2 & 1 \\ -1 & 2 & 3 & 4 \\ 3 & 0 & 1 & 2 \end{pmatrix}$ (ii) $\begin{pmatrix} 1 & 1 & 2 & -1 \\ 1 & 2 & 1 & 0 \\ -1 & -4 & 1 & -2 \\ 1 & -2 & 5 & -4 \end{pmatrix}$ (iii) $\begin{pmatrix} 1 & 1 & 3 & -3 & 0 \\ 0 & 2 & 1 & -3 & 3 \\ 1 & 0 & 2 & -1 & -1 \end{pmatrix}$

4. Use **Gaussian elimination** to solve each of the following linear systems of equations:

(i) $\begin{Bmatrix} x+2y+3z=9 \\ 2x-y+z=8 \\ 3x-z=3 \end{Bmatrix}$ (ii) $\begin{Bmatrix} a+b+3c-3d=0 \\ 2b+c-3d=3 \\ a+2c-d=-1 \end{Bmatrix}$

(iii) $\begin{Bmatrix} (1-i)x+(2+i)y=2+i \\ 2x+(1-2i)y=1+3i \end{Bmatrix}$ where $i=\sqrt{-1}$.

5. Use **Gauss-Jordan elimination** to solve each of the following linear systems;

(i) $\begin{Bmatrix} p+q+3r-3s=0 \\ 2q+r-3s=3 \\ p+2r-s=-1 \end{Bmatrix}$ (ii) $\begin{Bmatrix} 2x_5+8x_6=4 \\ x_4+3x_5+11x_6=9 \\ 3x_2-12x_3-3x_4-9x_5-24x_6=-33 \\ -2x_2+8x_3+x_4+6x_5+17x_6=21 \\ x_1=2 \end{Bmatrix}$

6. The following problem investigates the relationship between

(i) the complex function

$$w = e^{i\phi}z\ , \quad \text{(for } \phi \text{ a given real angle)}$$

[in which $w = u + iv = Re^{i\omega}$ and $z = x + iy = re^{i\theta}$ denote complex numbers]

and

(ii) the linear matrix equation

$$w = Az,$$

[in which $w = \begin{pmatrix} u \\ v \end{pmatrix}$, $A = \begin{pmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{pmatrix}$ and $z = \begin{pmatrix} x \\ y \end{pmatrix}$].

(a) By writing w and z in exponential form, explain why we may interpret the function $w = e^{i\phi}z$ as representing a simple rotation of z about the origin through the angle of ϕ.

(b) By writing w and z in Cartesian form show that the function $w = e^{i\phi}z$ may be written as the pair of real linear equations

$$\begin{pmatrix} u = x\cos\phi - y\sin\phi \\ v = x\sin\phi + y\cos\phi \end{pmatrix}.$$

(c) Show that the equations of part (b) may be written in matrix form as

$$w = Az \text{ with } w = \begin{pmatrix} u \\ v \end{pmatrix},\ A = \begin{pmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{pmatrix} \text{ and } z = \begin{pmatrix} x \\ y \end{pmatrix}.$$

Use the above information to describe the effect of the transformation $w = Az$ on an arbitrary vector $z = \begin{pmatrix} x \\ y \end{pmatrix}$.

(d) Suppose that $w = A_1 z$ with $A_1 = \begin{pmatrix} \cos\phi_1 & -\sin\phi_1 \\ \sin\phi_1 & \cos\phi_1 \end{pmatrix}$, and $y = A_2 w$ with $A_2 = \begin{pmatrix} \cos\phi_2 & -\sin\phi_2 \\ \sin\phi_2 & \cos\phi_2 \end{pmatrix}$. Substituting the first equation into the second gives the matrix equation $y = A_2 A_1 z = A_3 z$. Show that $A_3 = \begin{pmatrix} \cos(\phi_1+\phi_2) & -\sin(\phi_1+\phi_2) \\ \sin(\phi_1+\phi_2)_1 & \cos(\phi_1+\phi_2) \end{pmatrix}$

and justify both algebraically and geometrically why the transformation $y = A_2 A_1 z = A_3 z$ may be interpreted as a rotation of the vector z about the origin through an angle $(\phi_1 + \phi_2)$.