MATH 1210 Problem Workshop 8 Solutions

1. We start by putting the system into an augmented matrix.

$$\left[\begin{array}{cccc|c} 2 & 3 & -4 & 1 & 16 \\ 0 & 1 & 2 & -3 & -12 \\ 3 & -1 & 0 & 2 & 9 \\ 2 & 1 & 1 & 0 & 3 \end{array}\right]$$ Using $R_3 \to R_3 - R_1$ and $R_1 \leftrightarrow R_3$ yields

$$\left[\begin{array}{cccc|c} 1 & -4 & 4 & 1 & -7 \\ 0 & 1 & 2 & -3 & -12 \\ 2 & 3 & -4 & 1 & 16 \\ 2 & 1 & 1 & 0 & 3 \end{array}\right]$$ Using $R_3 \to R_3 - 2R_1$ and $R_4 \to R_4 - 2R_1$ yields

$$\left[\begin{array}{cccc|c} 1 & -4 & 4 & 1 & -7 \\ 0 & 1 & 2 & -3 & -12 \\ 0 & 11 & -12 & -1 & 30 \\ 0 & 9 & -7 & -2 & 17 \end{array}\right]$$ Using $R_1 \to R_1 + 4R_2, R_3 \to R_3 - 11R_2$ and $R_4 \to R_4 - 9R_2$ yields

$$\left[\begin{array}{cccc|c} 1 & 0 & 12 & -11 & -55 \\ 0 & 1 & 2 & -3 & -12 \\ 0 & 0 & -34 & 32 & 162 \\ 0 & 0 & -25 & 25 & 125 \end{array}\right]$$ Using $R_4 \to -\frac{1}{25}R_4$ and $R_3 \leftrightarrow R_4$ yields

$$\left[\begin{array}{cccc|c} 1 & 0 & 12 & -11 & -55 \\ 0 & 1 & 2 & -3 & -12 \\ 0 & 0 & 1 & -1 & -5 \\ 0 & 0 & -34 & 32 & 162 \end{array}\right]$$ Using $R_1 \to R_1 - 12R_3, R_2 \to R_2 - 2R_3$ and $R_4 \to R_4 + 32R_3$ yields

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 1 & 5 \\ 0 & 1 & 0 & -1 & -2 \\ 0 & 0 & 1 & -1 & -5 \\ 0 & 0 & 0 & -2 & -8 \end{array}\right]$$ Using $R_4 \to -\frac{1}{2}R_4$ yields

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 1 & 5 \\ 0 & 1 & 0 & -1 & -2 \\ 0 & 0 & 1 & -1 & -5 \\ 0 & 0 & 0 & 1 & 4 \end{array}\right]$$ Using $R_1 \to R_1 - R_4, R_2 \to R_2 + R_4$ and $R_3 \to R_3 + R_4$ yields

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 2 \\ 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 & 4 \end{array}\right]$$ Hence the solution is $x = 1, y = 2, z = -1, w = 4$.

2. We start by putting the system into an augmented matrix.

$$\left[\begin{array}{cccc|c} 2 & 3 & -4 & 1 & 3 \\ 1 & -2 & 1 & 0 & 6 \\ 3 & 1 & 0 & 1 & 4 \\ 6 & 2 & -3 & 2 & 13 \end{array}\right]$$ Using $R_1 \leftrightarrow R_2$ yields

$$\left[\begin{array}{cccc|c} 1 & -2 & 1 & 0 & 6 \\ 2 & 3 & -4 & 1 & 3 \\ 3 & 1 & 0 & 1 & 4 \\ 6 & 2 & -3 & 2 & 13 \end{array}\right]$$ Using $R_2 \to R_2 - 2R_1, R_3 \to R_3 - 3R_1$ and $R_4 \to R_4 - 6R_1$ yields

$$\left[\begin{array}{cccc|c} 1 & -2 & 1 & 0 & 6 \\ 0 & 7 & -6 & 1 & -9 \\ 0 & 7 & -3 & 1 & -14 \\ 0 & 14 & -9 & 2 & -23 \end{array}\right]$$ Using $R_4 \to \frac{1}{7}R_3$ and $R_2 \leftrightarrow R_3$ yields

$$\left[\begin{array}{cccc|c} 1 & -2 & 1 & 0 & 6 \\ 0 & 1 & -3/7 & 1/7 & -2 \\ 0 & 7 & -6 & 1 & -9 \\ 0 & 14 & -9 & 2 & -23 \end{array}\right]$$ Using $R_1 \to R_1 + 2R_2, R_3 \to R_3 - 7R_2$ and $R_4 \to R_4 - 14R_2$ yields

$$\left[\begin{array}{cccc|c} 1 & 0 & 1/7 & 2/7 & 2 \\ 0 & 1 & -3/7 & 1/7 & -2 \\ 0 & 0 & -3 & 0 & 5 \\ 0 & 0 & -3 & 0 & 5 \end{array}\right]$$ Using $R_4 \to R_4 - R_3$ and $R_3 \to -\frac{1}{3}R_4$ yields

$$\left[\begin{array}{cccc|c} 1 & 0 & 1/7 & 2/7 & 2 \\ 0 & 1 & -3/7 & 1/7 & -2 \\ 0 & 0 & 1 & 0 & -5/3 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$ Using $R_1 \to R_1 - R_3/7$ and $R_2 \to R_2 + 3R_3/7$ yields

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 2/7 & 47/21 \\ 0 & 1 & 0 & 1/7 & -19/7 \\ 0 & 0 & 1 & 0 & 5/3 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

The column with no leading one is for w so w is arbitrary. Hence let $w = t$. Therefore from equation 1, we get

$$x + \frac{2}{7}w = \frac{47}{21} \Rightarrow x = \frac{47}{21} - \frac{2}{7}t.$$

From equation 2, we get

$$y + \frac{1}{7}w = -\frac{19}{7} \Rightarrow y = -\frac{19}{7} - \frac{1}{7}t$$

From equation 3, we get

$$x = -\frac{5}{3}.$$

Hence the solution is
$x = \frac{47}{21} - \frac{2}{7}t, y = -\frac{19}{7} - \frac{1}{7}t, z = -\frac{5}{3}, w = t$

3. We start by putting the system into an augmented matrix.
$\left[\begin{array}{cccc|c} 1 & 5 & 3 & -2 & 6 \\ 2 & -1 & 1 & 0 & -1 \\ 1 & 2 & 0 & -4 & 6 \\ 3 & 7 & 7 & 0 & 3 \end{array}\right]$ Using $R_2 \to R_2 - 2R_1, R_3 \to R_3 - R_1$ and $R_4 \to R_4 - 3R_1$
yields
$\left[\begin{array}{cccc|c} 1 & 5 & 3 & -2 & 6 \\ 0 & -11 & -5 & 4 & -13 \\ 0 & -3 & -3 & -2 & 0 \\ 0 & -8 & -2 & 6 & 15 \end{array}\right]$ Using $R_2 \to R_2 - 4R_3$ yields
$\left[\begin{array}{cccc|c} 1 & 5 & 3 & -2 & 6 \\ 0 & 1 & 7 & 12 & -13 \\ 0 & -3 & -3 & -2 & 0 \\ 0 & -8 & -2 & 6 & 15 \end{array}\right]$ Using $R_1 \to R_1 - 5R_2, R_3 \to R_3 + 3R_2$ and $R_4 \to R_4 + 8R_2$
yields
$\left[\begin{array}{cccc|c} 1 & 0 & -32 & -62 & 71 \\ 0 & 1 & 7 & 12 & -13 \\ 0 & 0 & 18 & 34 & -39 \\ 0 & 0 & 54 & 102 & -89 \end{array}\right]$ Using $R_4 \to R_4 - 3R_3$yields
$\left[\begin{array}{cccc|c} 1 & 0 & -32 & -62 & 71 \\ 0 & 1 & 7 & 12 & -13 \\ 0 & 0 & 18 & 34 & -39 \\ 0 & 0 & 0 & 0 & 28 \end{array}\right]$
Since the last row says $0 = 28$, there is no solution.

4. Plugging the four points in for x and y yields

$$\begin{aligned} 64a + 16b + 4c + d &= -14 \\ 27a + 9b + 3c + d &= -11 \\ a + b + c + d &= 7 \\ 0a + 0b + 0c + d &= 10 \end{aligned}$$

Putting the system into an augmented matrix.
$\left[\begin{array}{cccc|c} 64 & 16 & 4 & 1 & -14 \\ 27 & 9 & 3 & 1 & -11 \\ 1 & 1 & 1 & 1 & 7 \\ 0 & 0 & 0 & 1 & 10 \end{array}\right]$ Using $R_1 \leftrightarrow R_3$ yields

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 7 \\ 27 & 9 & 3 & 1 & -11 \\ 64 & 16 & 4 & 1 & -14 \\ 0 & 0 & 0 & 1 & 10 \end{array}\right]$$ Using $R_1 \to R_1 - R_4, R_2 \to R_2 - R_4$ and $R_3 \to R_3 - R_4$ yields

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 0 & -3 \\ 27 & 9 & 3 & 0 & -21 \\ 64 & 16 & 4 & 0 & -24 \\ 0 & 0 & 0 & 1 & 10 \end{array}\right]$$ Using $R_2 \to R_2 - 27R_2$ and $R_3 \to R_3 - 64R_4$ yields

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 0 & -3 \\ 0 & -18 & -24 & 0 & 60 \\ 0 & -48 & -60 & 0 & 168 \\ 0 & 0 & 0 & 1 & 10 \end{array}\right]$$ Simplifying thing using $R_2 \to -\frac{1}{6}R_2$ and $R_3 \to -\frac{1}{12}R_3$ yields

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 0 & -3 \\ 0 & 3 & 4 & 0 & -10 \\ 0 & 4 & 5 & 0 & -14 \\ 0 & 0 & 0 & 1 & 10 \end{array}\right]$$ Using $R_3 \to R_3 - R_2$ and $R_2 \leftrightarrow R_3$ yields

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 0 & -3 \\ 0 & 1 & 1 & 0 & -4 \\ 0 & 3 & 4 & 0 & -10 \\ 0 & 0 & 0 & 1 & 10 \end{array}\right]$$ Using $R_1 \to R_1 - R_2$ and $R_3 \to R_3 - 3R_2$ yields

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & -4 \\ 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 & 10 \end{array}\right]$$ Using $R_2 \to R_2 - R_3$ yields

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & -6 \\ 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 & 10 \end{array}\right].$$

Hence $a = 1, b = -6, c = 2$ and $d = 10$ leading to the polynomial

$$y = x^3 - 6x^2 + 2x + 10.$$