THE UNIVERSITY OF MANITOBA

December 10, 9:00 A.M. 2003
Paper No.: 239
Department & Course No.: 130.112
Examination: Thermal Sciences

Final Examination
Page No.: Page 1 of 3
Time: 3 hours
Examiner: Dr. H.M. Soliman

Instructions:

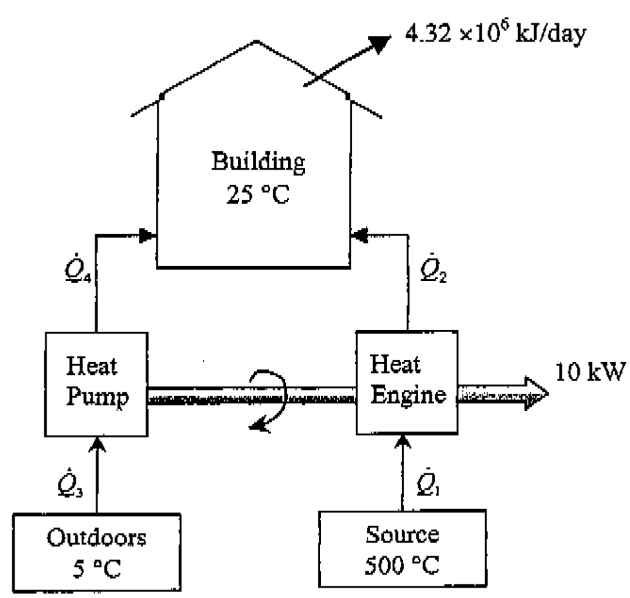
1. This is a three-hour open textbook exam. Students are permitted to use the course textbook, supplementary notes on heat transfer, and a calculator. No other materials (e.g., notes, solved problems, etc.) are allowed.
2. State all assumptions and label your system with dashed lines. Write your solutions clearly and legibly in the booklets provided. Ambiguous solutions, which cannot be interpreted, will be considered incorrect.
3. If interpolation is required in the property tables, then use linear interpolation between table entries. Use **four** significant figures in your calculations and full precision for data from the textbook.
4. Attempt all questions. The values are indicated in the margin.

Value 1. It is required to maintain a building at a steady temperature of 25 °C on a day when the outdoors temperature is 5 °C. The house is estimated to be losing heat at a rate of 4.32×10^6 kJ/day. A heat engine with a thermal efficiency of 37.5% receives heat ($\dot{Q}_1$) from a source at 500 °C and rejects heat ($\dot{Q}_2$) into the building. This heat engine generates enough power to operate a heat pump in addition to 10 kW of excess power (see the figure below). The heat pump rejects heat ($\dot{Q}_4$) into the building. The amounts of heat supplied to the building by the heat engine and the heat pump are equal (i.e., $\dot{Q}_2 = \dot{Q}_4$).

(12) (a) Determine (via appropriate calculations) whether the heat engine and the heat pump are reversible, irreversible, or impossible. Make separate calculations for each device.

(8) (b) Determine the amount of excess power (in kW) that would be developed if both the heat engine and the heat pump are reversible. Assume that the heat losses from the house, as well as all temperatures remain the same as in part (a) and that $\dot{Q}_2 = \dot{Q}_4$ (as in part (a)).

Problem 1



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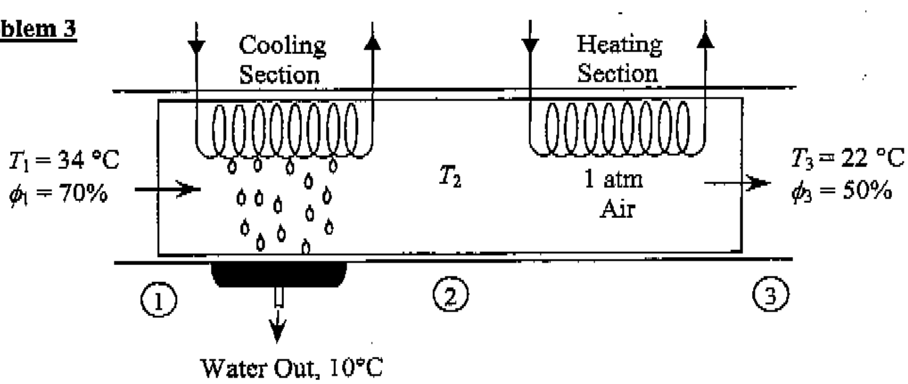
Value 2. A piston-cylinder device contains a mixture of three ideal gases: 4 kg of hydrogen, 6 kg of argon, and 10 kg Nitrogen. Initially the mixture exists at a pressure of 5 MPa and temperature of 50 °C. The mixture now expands slowly in a polytropic process during which $PV^{1.3}$ = constant until the pressure of the mixture drops to 1 MPa. Neglect changes in the kinetic and potential energies.

(5) (a) Calculate the initial volume occupied by the mixture (in m^3).
(4) (b) Calculate the final volume (in m^3) and the final temperature (in °C) of the mixture.
(3) (c) Calculate the work done during this process (in kJ).
(4) (d) Determine the heat transfer during this process in (kJ). Assume that the constant values of specific heats at 300 K are applicable.
(4) (e) Show process on a P-V diagram with respect to constant- temperature lines; identify the area representing the work.

3. Atmospheric air enters an air-conditioning system at a rate of 10 m^3/min. The system takes in the air at 1 atm, 34 °C, and 70% relative humidity and delivers it at 1 atm, 22 °C, and 50% relative humidity. The air flows first over cooling coils, where it is cooled and dehumidified, and then over heating coils, where it is heated to the desired temperature (see the figure below). The condensed water is removed from the cooling section at 10 °C. Using the psychrometric chart or the property relations of atmospheric air, determine

(5) (a) The temperature of air as it enters the heating coils, T_2 (in °C).
(8) (b) The rate of heat removal in the cooling section (in kW).
(4) (c) The rate of heat addition in the heating section (in kW).
(3) (d) The volume flow rate of atmospheric air leaving the air-conditioning system (in m^3/min).

Problem 3



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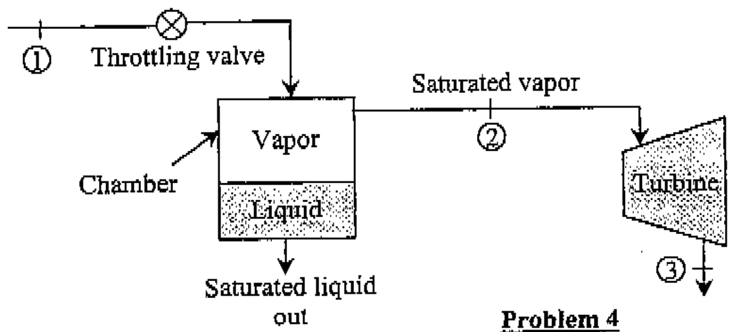
Value 4. A proposal is made to use a geothermal supply of hot water to operate a steam turbine, as shown in the figure below. The high-pressure water at P_1 = 1.5 Mpa and T_1 = 180 °C is throttled into a chamber, thus forming saturated liquid and saturated vapor at a pressure of P_2 = 40 kPa. The liquid is discarded while the vapor feeds the turbine and exits at P_3 = 10 kPa with 90% quality. If the required power output from the turbine is 1 MW, determine the required mass flow of hot geothermal water in kg/h. Neglect the changes in kinetic and potential energies in all components.

(20)



Problem 4

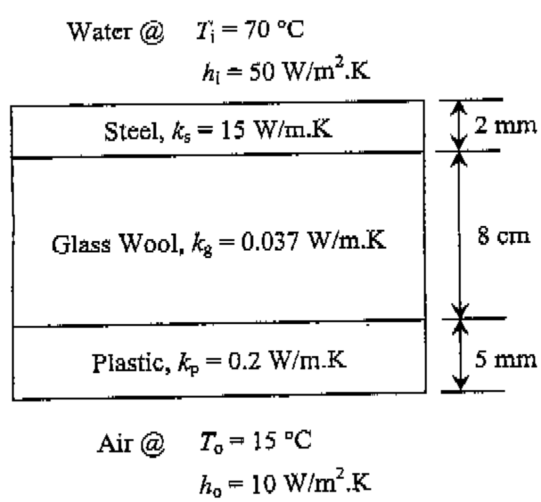
5. Hot water inside a tank is stored at T_i = 70 °C. The bottom of the tank has a surface area of 2 m^2 and it can be treated as a plane composite wall. This composite wall consists of 2 mm of steel (k_s = 15 W/m·K), 8 cm of glass wall insulation (k_g = 0.037 W/m·K), and 5 mm of plastic (k_p = 0.2 W/m·K), as shown in the figure below. The convection heat transfer coefficient between the water and the inner surface is h_i = 50 W/m^2·K, and the convection heat transfer coefficient between the outside air and the outer surface is h_o = 10 W/m^2·K. The outside air is at T_o = 15 °C

(10) (a) Sketch the thermal resistance network for heat transfer through the composite wall representing the bottom of the tank and calculate the numerical value of each resistance in the network.

(5) (b) Determine the rate of heat loss (in W) from the water through the bottom of the tank.

(5) (c) Determine the temperature at the interface between the plastic and the air (in °C)

Problem 5



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Solutions December 2003

1. (a) The rate of heat loss from the building =

$$\frac{4.32 \times 10^6}{24 \times 3600} = 50 \text{ kW}$$

The reversible efficiency of the heat engine

$$\eta_{Carnot} = 1 - \frac{T_L}{T_H} = 1 - \frac{273.15 + 25}{273.15 + 500} = 0.6144$$

The given efficiency is $\eta_{th} = 0.375$

Therefore the engine is irreversible (since $\eta_{th} < \eta_{Carnot}$)

$$\dot{Q}_2 = 50/2 = 25 \text{ kW}$$

$$\eta_{th} = 1 - \frac{\dot{Q}_2}{\dot{Q}_1} = 0.375 = 1 - \frac{25}{Q_1}$$

$$\dot{Q}_1 = 40 \text{ kW}$$

Total power out from the engine $= 40 - 25$

$= 15$ kW

Power used to run the heat pump =

$= 15 - 10 = 5$ kW

$$\dot{Q}_4 = 50/2 = 25 \text{ kW}$$

$$COP_{HP} = \frac{\dot{Q}_4}{\dot{W}_{HP}} = \frac{25}{5} = 5$$

$$COP_{HP}\Big|_{Carnot} = \frac{T_H}{T_H - T_L} = \frac{298.15}{20} = 14.91$$

Since $COP_{HP} < COP_{HP}\Big|_{Carnot}$, the heat pump is irreversible

(b) With both devices reversible

$$\dot{Q}_2 = 25 \text{ kW}$$

$$\eta_{Carnot} = 0.6144 = 1 - \frac{25}{\dot{Q}_1}$$

$$\dot{Q}_1 = 64.83 \text{ kW}$$

Total power developed by engine = 64.83 − 25

= 39.83 kW

$\dot{Q}_4 = 25$ kW, $\quad COP_{HP} = 14.91 = \frac{\dot{Q}_4}{\dot{W}_{HP}}$

Power required by the heat pump = 1.677 kW

Excess power = 39.83 − 1.677 = 38.15 kW

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2.(a) $m = 4 + 6 + 10 = 20$ kg

$mf_{H_2} = \frac{4}{20} = 0.2$, $mf_{Ar} = \frac{6}{30} = 0.3$

$mf_{N_2} = \frac{10}{20} = 0.5$

$$R = \sum_{i=1}^{k} mf_i R_i = 0.2 \times 4.124 + 0.3 \times 0.2081$$

$$+ 0.5 \times 0.2968 = 1.036 \text{ kJ/kg}\cdot\text{K}$$

$$C_v = \sum_{i=1}^{k} mf_i C_{v_i} = 0.2 \times 10.183 + 0.3 \times 0.3122$$

$$+ 0.5 \times 0.743 = 2.502 \text{ kJ/kg}\cdot\text{K}$$

$$P_1 V_1 = m R T_1$$

$$5000\, V_1 = 20 \times 1.036 \times 323.15$$

$$V_1 = 1.339 \text{ m}^3$$

(b) $$P_1 V_1^{1.3} = P_2 V_2^{1.3}$$

$$\frac{V_2}{V_1} = \left(\frac{P_1}{P_2}\right)^{1/1.3} = \left(\frac{5000}{1000}\right)^{0.7692} = \frac{V_2}{1.339}$$

$$V_2 = 4.618 \text{ m}^3$$

$$P_2 V_2 = m R T_2$$

$$T_2 = \frac{1000 \times 4.618}{20 \times 1.036} = 222.88 \text{ K} = -50.27^\circ\text{C}$$

(c) $$W_b = \frac{P_2 V_2 - P_1 V_1}{1-n} = \frac{(1000)(4.618)-(5000)(1.339)}{1-1.3}$$

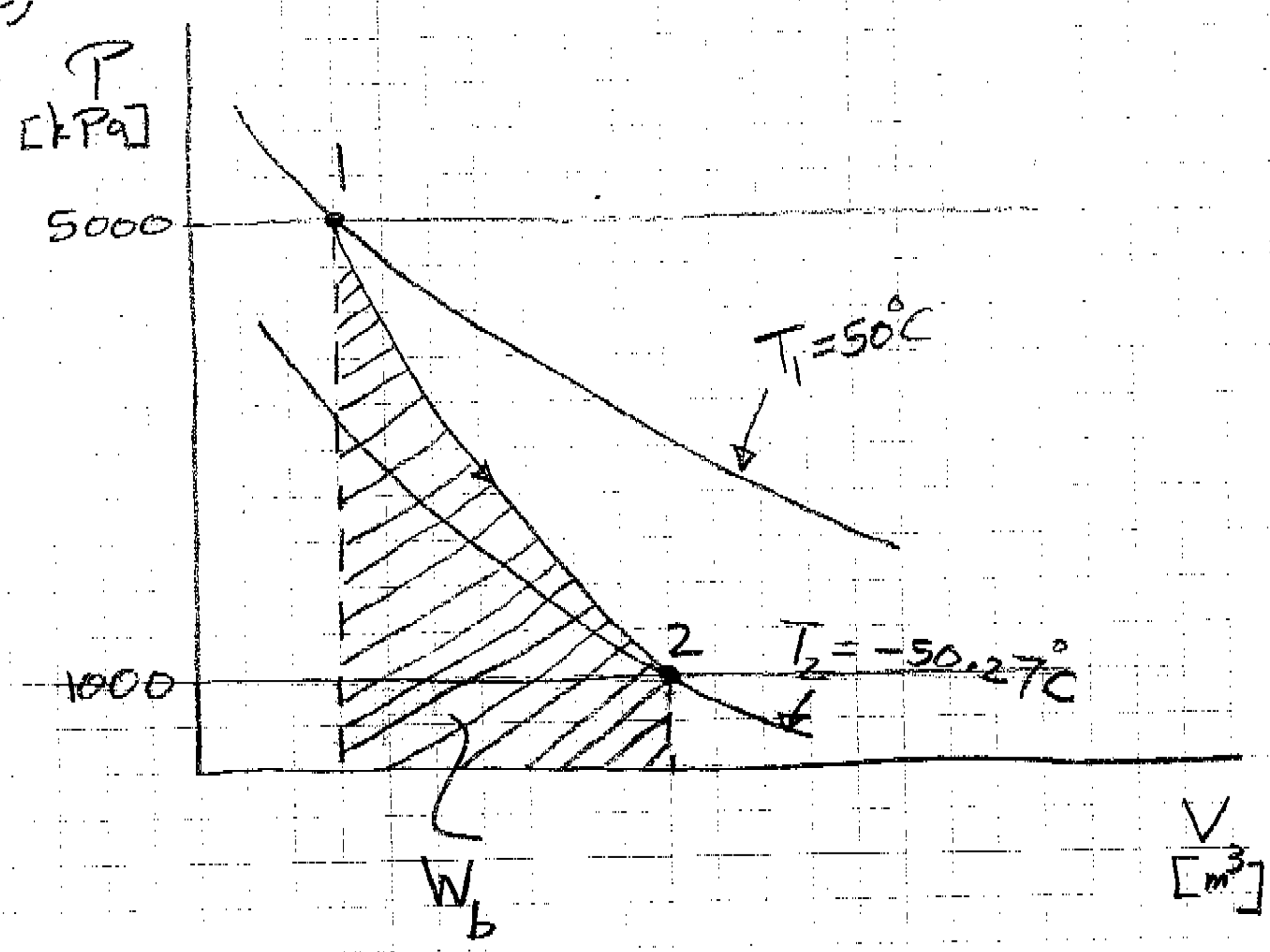
$$= 6923 \text{ kJ}$$

(d) $$Q_{in} - W_{out} = m\left[(u_2-u_1) + \Delta \cancel{KE} + \Delta \cancel{PE}\right]$$
$$= m c_v (T_2 - T_1)$$

$$Q_{in} - 6923 = 20(2.502)(-50.27-50)$$

$$Q_{in} = 1905 \text{ kJ}$$

(e)



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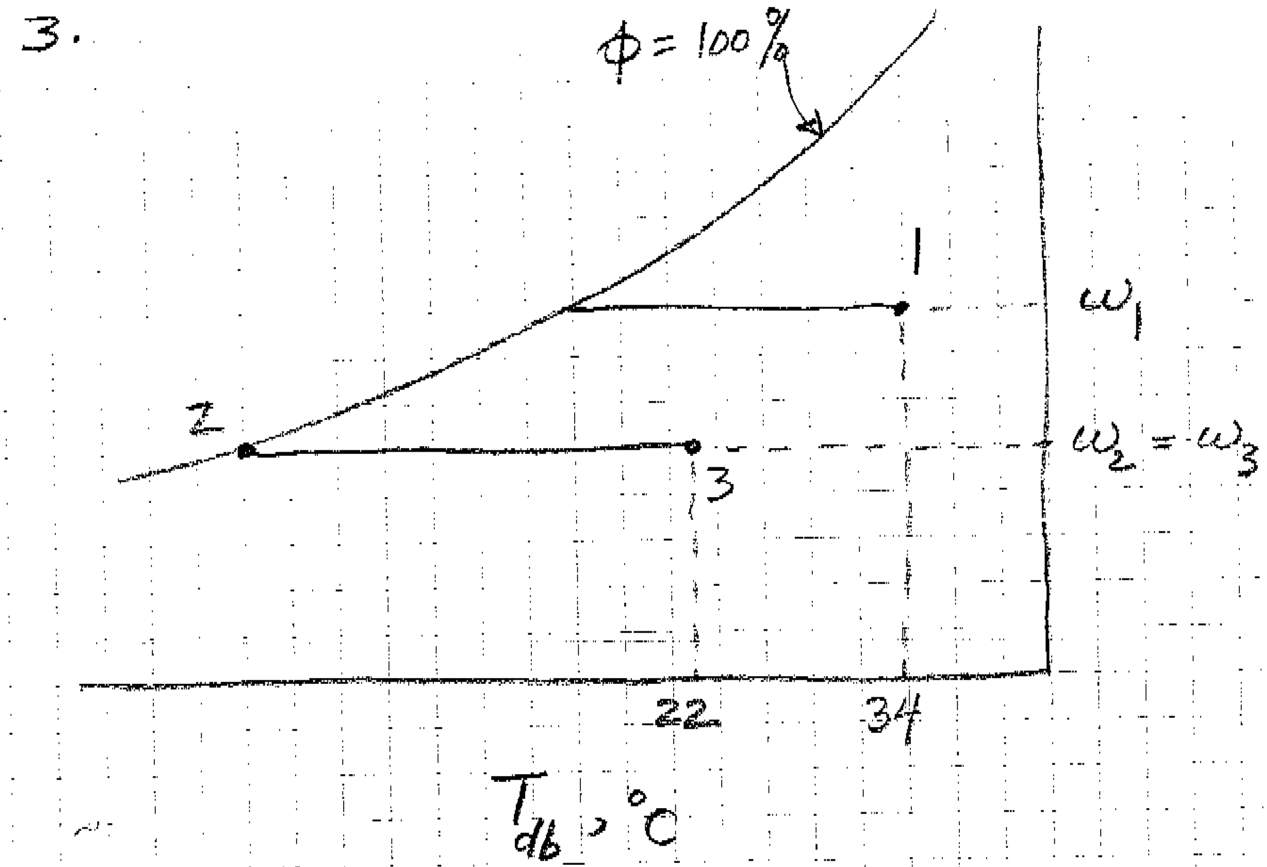
3.



(a) Using the psychrometric chart, $\omega_1 = 0.0237 \frac{kg\ H_2O}{kg\ dry\ air}$

$$\omega_2 = \omega_3 = 0.00825 \frac{kg\ H_2O}{kg\ dry\ air}$$

$$T_2 \doteq 11\,°C$$

(b) $v_1 = 0.903\ m^3/kg\ dry\ air$

$$\dot{m}_a = \frac{\dot{V}_1}{v_1} = \frac{10}{60 \times 0.903} = 0.1846\ kg/s$$

$$\dot{m}_{\substack{water\\out}} = \dot{m}_a(\omega_1 - \omega_2) = 0.1846(0.0237 - 0.00825)$$
$$= 0.002852\ kg/s$$

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$$\dot{Q}_{in} - \cancel{\dot{W}_{out}} = \dot{m}_a (h_2 - h_1) + \dot{m}_{\substack{water\\out}} h_f$$

$h_1 = 95$ kJ/kg dry air

$h_2 = 32$ kJ/kg dry air

$h_f = 42.01$ kJ/kg H_2O

$$\dot{Q}_{in} = 0.1846(32-95) + 0.002852 \times 42.01$$
$$= -11.51 \text{ kW}$$

rate of heat removal = 11.51 kW

(c) $\dot{Q}_{in} - \cancel{\dot{W}_{out}} = \dot{m}_a (h_3 - h_2)$

$h_3 = 43.3$ kJ/kg dry air

$$\dot{Q}_{in} = 0.1846(43.3-32) = 2.086 \text{ kW}$$

(d) $v_3 = 0.847$ m^3/kg dry air

$$\dot{V}_3 = \dot{m}_a v_3 = 0.1846 \times 0.847 \times 60$$
$$= 9.381 \ m^3/min$$

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4. Control volume around the turbine

$$\cancel{\dot{Q}} - \dot{W} = \dot{m}_T \left[(h_3 - h_2) + \Delta \cancel{KE} + \Delta \cancel{PE} \right]$$

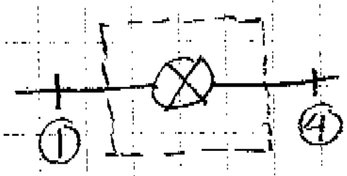
$$-1000 = \dot{m}_T \,(h_3 - h_2)$$

$h_2 = 2636.8$ kJ/kg

$h_3 = 0.9\,(2584.7) + 0.1\,(191.83) = 2345.4$ kJ/kg

$$\dot{m}_T = \frac{1000}{2636.8 - 2345.4} = 3.432 \text{ kg/s}$$

Control volume around throttling valve



$h_1 = h_4$

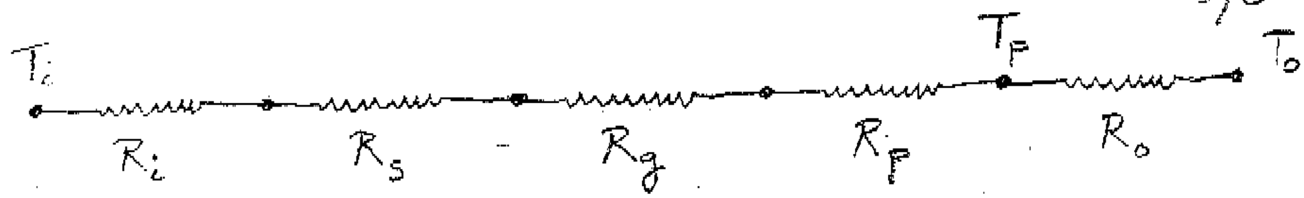
$h_1 = 763.22$ kJ/kg

$h_4 = x_4\,(2636.8) + (1 - x_4)(317.58) = 763.22$

$$x_4 = \frac{763.22 - 317.58}{2636.8 - 317.58} = 0.1922$$

$$\dot{m}_1 = \frac{\dot{m}_T}{x_4} = \frac{3.432}{0.1922} = 17.86 \text{ kg/s}$$

5 (a)



$$R_i = \frac{1}{h_i A} = \frac{1}{50 \times 2} = 0.01 \text{ K/W}$$

$$R_s = \frac{L_s}{k_s A} = \frac{0.002}{15 \times 2} = 6.667 \times 10^{-5} \text{ K/W}$$

$$R_g = \frac{L_g}{k_g A} = \frac{0.08}{0.037 \times 2} = 1.081 \text{ K/W}$$

$$R_p = \frac{L_p}{k_p A} = \frac{0.005}{0.2 \times 2} = 0.0125 \text{ K/W}$$

$$R_o = 1/(h_o A) = 1/(10 \times 2) = 0.05 \text{ K/W}$$

(b)
$$\dot{Q} = \frac{T_i - T_o}{R_i + R_s + R_g + R_p + R_o}$$

$$= \frac{70 - 15}{1.1536} = 47.68 \text{ W}$$

(c)
$$T_p = T_o + R_o \dot{Q}$$

$$= 15 + 0.05 \times 47.68$$

$$T_p = 17.38 \, ^\circ C$$