

MATH 2130 Tutorial 9

1. Evaluate the double iterated integral $\int_{-2}^0 \int_0^{-x} \sqrt{y-x} \, dy \, dx$.
2. Evaluate the double iterated integral $\int_{-1}^0 \int_{-2}^{2x} x\sqrt{x^2+y^2} \, dy \, dx$.
3. Find the volumes of the solids of revolution when the area bounded by the curves

$$y = 2x - x^2, \quad y = x$$

is rotated around the lines: (a) $x = 3$ (b) $y = 1$ (c) $x + y = -1$.

4. A triangular plate has sides with lengths 3, 4 and 5 metres. It is submerged vertically in oil with density 950 kilograms per cubic metre. The side of length 3 metres is vertical, the side of length 4 metres is horizontal, and the uppermost vertex is 1 metre below the surface of the oil. Find the force due to oil pressure on each side of the plate.
5. An elliptic plate has major axis of length $2a$ metres and minor axis of length $2b$ metres. Its major axis is horizontal and its minor axis is vertical. It is slowly being lowered into a tank of water. At the instant when only $b/2$ metres of the plate sticks out of the water, set up, but do **NOT** evaluate, a double iterated integral for the force due to the water on each side of the plate.
6. A thin plate with constant mass per unit area ρ has edges defined by the curves

$$x = \sqrt{a^2 - y^2}, \quad y = x, \quad y = 0,$$

where $a > 0$ is a constant. Find the first moment of the plate about the x -axis.

7. A triangular plate has sides of lengths 2, 3 and 3, and constant mass per unit area ρ . Find its moment of inertia about the shorter side.

Answers

1. $16(4 - \sqrt{2})/15$
2. $(8 - 5\sqrt{5})/6$
3. (a) $5\pi/6$ (b) $2\pi/15$ (c) $7\sqrt{2}\pi/20$
4. 1.68×10^5 N
5. $\int_{-b}^{b/2} \int_{-(a/b)\sqrt{b^2-y^2}}^{(a/b)\sqrt{b^2-y^2}} 9810 \left(\frac{b}{2} - y \right) dx \, dy$ N
6. $\rho a^3(\sqrt{2} - 1)/(3\sqrt{2})$
7. $8\sqrt{2}\rho/3$