

## MATH 1210 Problem Workshop 12

1. For the matrix  $E = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 1 & 3 \\ 2 & 1 & 2 \end{bmatrix}$ , find the cofactor matrix and the adjoint of  $E$  and use them to find the inverse.
2. (a) Show  $|A^{-1}| = \frac{1}{|A|}$ .  
(b) Show for any constant  $c$  and any  $n \times n$  matrix  $A$ , that  $|cA| = c^n |A|$ .  
(c) Use parts (a) and (b) to show that  $|adj(A)| = |A|^{n-1}$ .
3. Let  $A, B, C$  be  $3 \times 3$  matrices such that  $|A| = 2$  and  $|B| = 3$ . Find  $|5AB^{-2}|$ .
4. Solve the following system of equations using inverses

$$2x + 3y - 4z = 12$$

$$x + y + 2z = 1$$

$$3x + 4y = 11$$

5. Solve the following system of equations in terms of  $a, b$  and  $c$  using inverses.

$$2x + 3y - 4z = a$$

$$x + y + 2z = b$$

$$3x + 4y = c$$

6. Use both definitions of a transformation being linear to check if the following are linear:
  - (a)  $T(\mathbf{v}) = k\mathbf{v}$ , where  $k$  is a nonzero constant.
  - (b)  $T(\mathbf{v}) = \mathbf{v} + \mathbf{c}$  where  $\mathbf{c}$  is a nonzero vector.
  - (c)  $T(\mathbf{v})$  is the unit vector in the direction of  $\mathbf{v}$ .

Answers

1.  $C = \begin{bmatrix} -1 & 4 & -1 \\ 0 & -4 & 2 \\ 1 & 2 & -1 \end{bmatrix}$  and  $\text{adj}(E) = \begin{bmatrix} -1 & 0 & 1 \\ 4 & -4 & 2 \\ -1 & 2 & -1 \end{bmatrix}$ .

2.

3.  $250/9$ .

4.  $x = 1, \quad y = 2, \quad z = -1$ .

5.  $x = 4a + 8b - 5c, \quad y = -3a - 6b + 4c, \quad z = \frac{-a - b + c}{2}$

6. (a) Yes

(b) No

(c) No