UNIVERSITY OF MANITOBA

DATE: October 8, 2014 — TERM TEST 1

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EXAMINATION: Engineering Mathematical Analysis 1 — TIME: 70 minutes

COURSE: MATH 2130 — EXAMINER: various

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[4] 1. (a) *For the curve,* $z^2 - 2z = x^2 + y^2 - 1$, *identify the type of curve and give a sketch.*

[4] (b) *Determine the projection of* $x^2 = z^2 - 4$, $x^2 + y^2 - z^2 - 2z = 0$ *in the* yz*-plane.*

2. *Let* l_1 *be the line*

$$x - 5 = \frac{y+3}{-2} = \frac{z-4}{3}$$

and l_2 *be the line*

$$x = 7 + 2t,\ y = -5 - 3t,\ z = 8 + 5t.$$

[3] (a) *Show that the lines are intersecting and determine the point of intersection.*

[3] (b) *Determine the cosine of the smallest angle between the lines.*

[5] (c) Determine an equation of the plane containing both lines.

3. *Let* l_1 *be the line with symmetric equations*

$$\frac{x-3}{2} = y + 5 = \frac{z-1}{-2}$$

and l_2 *be the line with parametric equations*

$$x = 1 - 4t \quad y = 3 - 2t \quad x = 1 + 4t.$$

[2] (a) *Determine whether the lines are parallel, intersecting or skew.*

[6] (b) Determine the shortest distance between the lines.

[5] 4. *Let* $f(t) = t$ *and* $\mathbf{v}(t) = \left\langle t,\ \dfrac{1}{t^2+1},\ e^t \right\rangle$. *Evaluate*

$$\int (f\mathbf{v})(t)\, dt$$

[6] 5. *Determine a parametric representation for the curves of intersection of* $z = \sqrt{9 - x^2 - y^2}$ *and* $x^2 - 2x + y^2 = 0$ *directed so that* x *decreases when* y *is positive. Justify your answer.*

6. *Let a curve* C *be defined by a position vector* $\mathbf{r}(t) = \langle t,\ t^{3/2},\ 4t^{3/2} \rangle$.

[4] (a) *Determine parametric equations for the tangent line to* $\mathbf{r}(t)$ *at the point* $(4, 8, 32)$.

[2] (b) Determine the unit tangent vector to $\mathbf{r}(t)$ at the point $(4, 8, 32)$.

[6] (c) *Determine the length of the curve* C *from the point* $(0, 0, 0)$ *to* $(4, 8, 32)$.