Instructor: Dr. B.-C. Wang

February 12, 2010
Duration: 100 min

(a) You are permitted to use the course textbook and a calculator.
(b) Ask for clarification if a problem statement is not clear.
(c) Use linear interpolation in the property tables as necessary.
(d) **Show your work** (or, **the complete solution steps**) in a clear and logical manner. Solutions that require unreasonable effort (in the opinion of the instructor) for the marker to decipher will not be credited.

1. *(15 marks)*

As shown in the figure, two cylinders (A and B) are filled with water and connected using a very thin pipeline with a closed valve in the middle. In cylinder A, a piston is floating on the surface of the water. The piston mass is $m_P = 500$ kg, and the friction between the piston and cylinder A can be ignored. The density of water is $\rho = 1000\ kg/m^3$. The local acceleration of gravity is $g = 9.81\ m/s^2$. The local atmospheric pressure is $P_0 = 1$ bar. The initial mass of water contained in cylinder A and cylinder B is: $m_{A1} = 200$ kg and $m_{B1} = 800$ kg, respectively. The internal cross sectional areas of cylinder A and cylinder B are $A_A = 0.4\ m^2$ and $A_B = 0.2\ m^2$, respectively. The height $h_0 = 0.75$ m.

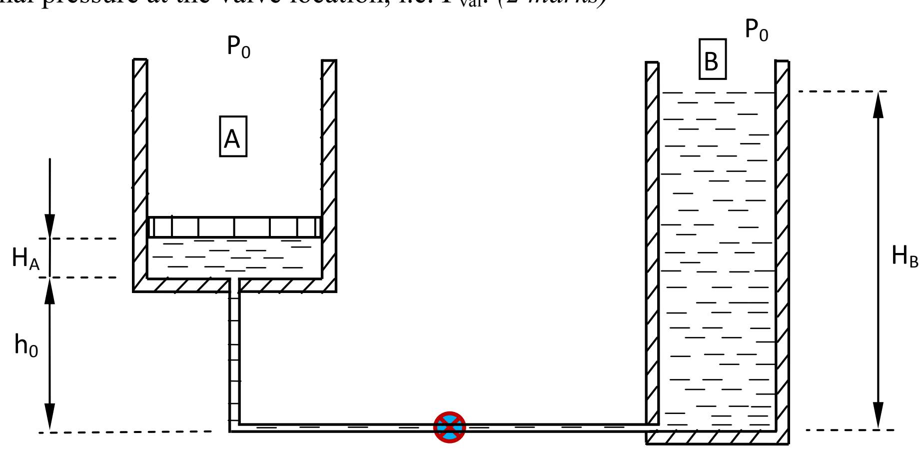
Questions:

At the initial state, the valve is closed,

(a) Find the initial volumes of the water contained in cylinder A and cylinder B, i.e. V_{A1} and V_{B1}. *(1 marks)*
(b) Find the initial heights of the water contained in cylinder A and cylinder B, i.e. H_{A1} and H_{B1}. *(1 mark)*
(c) Find the initial pressure on *each side* of the valve, i.e. $P_{val,\ left}$ and $P_{val,\ right}$. *(4 marks)*
(d) If the valve is opened, will the piston move up or down? Explain. *(2 mark)*

The valve is fully opened and water flows to an equilibrium. At the final state:

(e) Find the final heights of the water contained in cylinder A and cylinder B, i.e. H_{A2} and H_{B2}. *(5 marks)*
(f) Find the final pressure at the valve location, i.e. P_{val}. *(2 marks)*



Note that:

(1) in the above problem statement, subscripts "1" and "2" are used to indicate the initial and the final states, respectively; and
(2) the volume of the thin pipeline can be ignored.

2. *(8 marks)*

Two rigid tanks (A and B) are filled with helium and connected by a very thin pipeline. Helium contained in both tanks can be treated as an ideal gas. The thin pipeline goes through a valve which controls the flow rate between these two tanks. Initially, the valve is closed. The volume for these two rigid tanks is $V_A = 1\ m^3$ for tank A, and $V_B = 2\ m^3$ for tank B, respectively. The initial pressure and temperature of helium are P_{A1}=100 kPa and T_{A1}=100 ^oC in tank A, and P_{B1}=300 kPa and T_{B1}=500 ^oC in tank B, respectively. At the final state, the valve is opened and the helium gas in these two tanks reaches an equilibrium state, with a final temperature T_2 =300 ^oC.

Questions:

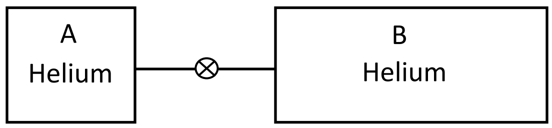
(a) Determine the initial mass of helium contained in tank A and tank B, i.e. m_{A1} and m_{B1}. *(4 marks)*

(b) Determine the final pressure, i.e. P_2. *(4 marks)*

Note that:

(1) in the above problem statement, subscripts "1" and "2" are used to indicate the initial and the final states, respectively; and

(2) the volume of the thin pipeline can be ignored.



3. *(17 marks)*

A frictionless piston-cylinder assembly containing R410a undergoes a cooling process. The mass of R410a is m=100 kg. At the initial state (state 1), the piston pushes against a set of stops. The initial pressure and temperature of R410a are P_1 = 550 kPa and T_1 = 260 ^oC, respectively. It is observed that at the *exact moment* when the piston just begins to drop (state 2), the pressure of R410a is P_2 = 150 kPa. After the piston leaves the stops, the system continues to dissipate heat to the environment until the final state (state 3) is reached, with the final temperature T_3= –55 ^oC.

Questions:

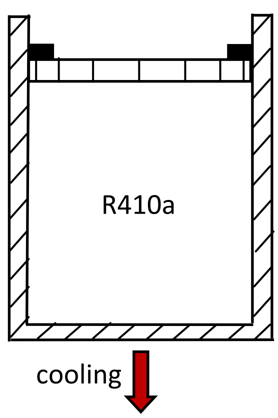
(a) Determine the initial specific volume (v_1). *(3 marks)*

(b) Determine the specific volume (v_2) and temperature (T_2) of state 2. *(5 marks)*

(c) Determine the mass of the liquid (m_{f2}) and the mass of the vapour (m_{g2}) of state 2. *(3 marks)*

(d) Estimate the specific volume (v_3) of state 3. *(2 marks)*

(e) Illustrate all the state points (1, 2, 3) and the entire process (1→2→3) in both P-v and T-v diagrams. *(4 marks)*



Solution ENG 1460 Winter 2010

Term Test 1 Dr. B.-C. Wang

Problem 1

step(a)

$$V_{A1} = m_{A1}/\rho = 200/1000 = 0.2 \ (m^3)$$

$$V_{A2} = m_{A2}/\rho = 800/1000 = 0.8 \ (m^3)$$

step(b)

$$H_{A1} = V_{A1}/A_A = 0.2/0.4 = 0.5 \ (m)$$

$$H_{B1} = V_{B1}/A_B = 0.8/0.2 = 4 \ (m)$$

step(c)

$$P_{Val}^{Left} = P_0 + \frac{m_p g}{A_A} + \rho g (H_{A1} + h_0)$$

$$= 10^5 + \frac{500 \times 9.81}{0.4} + 10^3 \times 9.81 \times (0.5 + 0.75)$$

$$= 124525 \ (Pa)$$

or, 124.525 kPa

$$P_{Val}^{Right} = P_0 + \rho g H_{B1}$$

$$= 10^5 + 10^3 \times 9.81 \times 4$$

$$= 139240 \ (Pa)$$

or, 139.24 kPa

step(d)

Because $P_{Val}^{Left} < P_{Val}^{Right}$,

water will flow from B to A, and piston will move up.

step(e)

From the principle of mass conservation

$$m_{total} = m_{A1} + m_{B1} = m_{A2} + m_{B2}$$

$$\therefore \ 200 + 800 = \rho (A_A H_{A2} + A_B H_{B2})$$

$\therefore\ 1000 = 10^3\,(0.4 H_{A2} + 0.2 H_{B2})$

$\therefore\ 5 = 2H_{A2} + H_{B2}$ ——————— (1)

From the principle of force/pressure balance

$$P^{Left}_{Val,2} = P^{Right}_{Val,2}$$

$$P_0 + \frac{m_p g}{A_A} + \rho g (H_{A2} + h_0) = P_0 + \rho g H_{B2}$$

$$\therefore\ \frac{m_p}{A_A} + \rho (H_{A2} + h_0) = \rho H_{B2}$$

$$\therefore\ \frac{500}{0.4} + 1000 (H_{A2} + 0.75) = 1000 H_{B2}$$

$$2 + H_{A2} = H_{B2} \qquad \text{——————— (2)}$$

From (1) & (2) $\begin{cases} 5 = 2H_{A2} + H_{B2} \\ 2 = -H_{A2} + H_{B2} \end{cases}$

$$\Rightarrow \begin{cases} H_{A2} = 1\ (m) \\ H_{B2} = 3\ (m) \end{cases}$$

Step (f)

B/C the pressure from the left & right sides of the valve are balanced, we can optionally use the right side to calculate the pressure at the valve location

$$P_{val,2} = P_0 + \rho g H_{B2}$$
$$= 10^5 + 10^3 \times 9.81 \times 3$$
$$= 129430\ (Pa)$$

or, 129.43 KPa

Problem 2

step (a)

From Table A.5

$R = 2.0771 \ kJ/kg \cdot K$ for Helium

For an ideal gas: $PV = mRT$

$$\therefore m_{A1} = \frac{P_{A1} V_A}{R T_{A1}} = \frac{100 \times 1}{2.0771 \times (273.15 + 100)} = 0.1290 \ kg$$

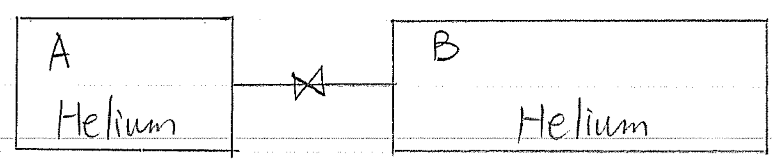
$$m_{B1} = \frac{P_{B1} V_B}{R T_{B1}} = \frac{300 \times 2}{2.0771 \times (273.15 + 500)} = 0.3736 \ (kg)$$

step (b)

$$V = V_A + V_B = 1 + 2 = 3 \ (m^3)$$

$$m = m_{A1} + m_{B1} = 0.1290 + 0.3736 = 0.5026 \ (kg)$$

$$P = \frac{m R T_2}{V} = \frac{0.5026 \times 2.0771 \times (273.15 + 300)}{3} = 199.447 \ (kPa)$$



Problem 3

Step (a) From Table B.4.1,

T[°C]	P[kPa]
−15	480.4
Tsat = ?	550
−10	573.1

$$T_{sat} = -15 + \frac{550-480.4}{573.1-480.4}(-10+15)$$
$$= -11.26\ (°C)$$

$T = 260°C > T_{sat}$

∴ Superheated vapour at state 1.

From Table B.4.2, at $T_1 = 260°C$

P[kPa]	V[m³/kg]
500	0.12129
550	V₁ = ?
600	0.10093

Interpolation

$$V_1 = 0.12129 + \frac{550-500}{600-500}(0.10093-0.12129)$$
$$= 0.11111\ (m^3/kg)$$

Step (b)

Constant-v-process from state 1 to state 2

∴ $V_2 = V_1 = 0.11111\ m^3/kg$

From Table B.4.1 (Sat. Table)

P[kPa]	T[°C]	Vf[m³/kg]	Vg[m³/kg]
138.8	−45	0.000752	0.17804
150	T₂ = ?	Vf₂ = ?	Vg₂ = ?
175.0	−40	0.000762	0.14291

$$V_{f2} = 0.000752 + \frac{150-138.8}{175-138.8}(0.000762-0.000752) = 0.00075509\ (m^3/kg)$$

$$V_{g2} = 0.17804 + \frac{150-138.8}{175-138.8}(0.14291-0.17804) = 0.16717\ (m^3/kg)$$

B/c $V_{f2} \le V_2 < V_{g2}$

∴ Saturated mixture (state 2)

$$T_2 = -45 + \frac{150-138.8}{175-138.8}(-40+45) = -43.45\ (°C)$$

step(c)

Quality: $X_2 = \dfrac{V_2 - V_f}{V_g - V_f} = \dfrac{0.11111 - 0.00075509}{0.16717 - 0.00075509} = 0.66313$

$m_{g2} = m \cdot X_2 = 100 \times 0.66313 = 66.313$ (Kg)

$m_{f2} = m - m_{g2} = 33.687$ (Kg)

step(d)

Constant-P-process from state 2 to state 3

$\therefore P_3 = P_2 = 150$ KPa

From step(b) / Table B.4.1

$T_{sat} = -43.45°C$

$T < T_{sat}$

$\therefore$ Compressed liquid at state 3.

use the method of approximation:

$V_3 \approx V_f|_{-55°C} = 0.000735$ (m^3/Kg)

step (e)