MATH 2132 Problem Workshop 5

1. Find general solutions for the following differential equations

   (a) $3y''' + 8y'' + 19y' + 10y = 0$

   (b) $6y'''' + y'' - y = 0$

   (c) $y'' + 3y' + ay = 0$, where $a > 3$ is a constant.

   (d) $y'' + 2y' - 3y = 4e^{5x} - x$

   (e) $y''' + 3y'' - 4y' = 2e^{x} + \cos 4x$

   (f) $y'' + 6y' - 2y = 3x + \sin x$

   (g) $4y''' - 3y'' + 7y' + 2y = e^{-x/4}$

2. The roots of an auxiliary equation $\phi(m) = 0$ associated with the differential equation $\phi(D)y = 0$ are

   $$2 \pm \sqrt{3}i,\ 2 \pm \sqrt{3}i,\ \pm 4,\ \pm 4,\ \pm 4,\ 2,\ -1 \pm \sqrt{6}$$

   What is the general solution of the differential equation?

3. The roots of an auxiliary equation $\phi(m) = 0$ associated with the differential equation

   $$\phi(D)y = 13x^2e^{3x} + e^{x}\cos 2x + 4x^3 - 5$$

   are

   $$3,\ 3,\ 3,\ -1 \pm 2i,\ 1 \pm 2i,\ 3 \pm \sqrt{6},\ 0,\ 0,\ -14$$

   What is the form of the particular solutions as predicting by the method of undtermined coefficients?

4. (a) A 100 gram mass is suspended from a spring with constant 60 N/m. It is lifted 15 cm. above its equilibrium position and given velocity 2 metres per second upward. During its motion, it is acted on by air resistance that is equal, in Newtons, to 5 times the velocity of the mass. Find the position of the mass as a function of time.

   (b) What is the maximum displacement from equilibrium experienced by the mass?

   (c) When, if at all, does the mass pass through its equilibrium position?

5. Recall Newton's Law is that the rate that the temperature changes is proportional to $T_a - T$ where T is the temperature of the object and T_a is the ambient temperature.

   (a) Suppose a potato with initial temperature $20°C$ is placed in an oven with temperature $200°C$. Find the temperature of the potato as a function of time.

   (b) Suppose a potato with initial temperature $20°C$ is placed in an oven with temperature starting at $20°C$ but rises (linearly) to $200°C$ in 5 minutes. Find the temperature of the potato as a function of time.