Practice Midterm 1B Answers

Multiple-Choice

1. A
2. A
3. E
4. B
5. E
6. C
7. D
8. A
9. A
10. D
11. C
12. A
13. C
14. B
15. B

Long-Answer

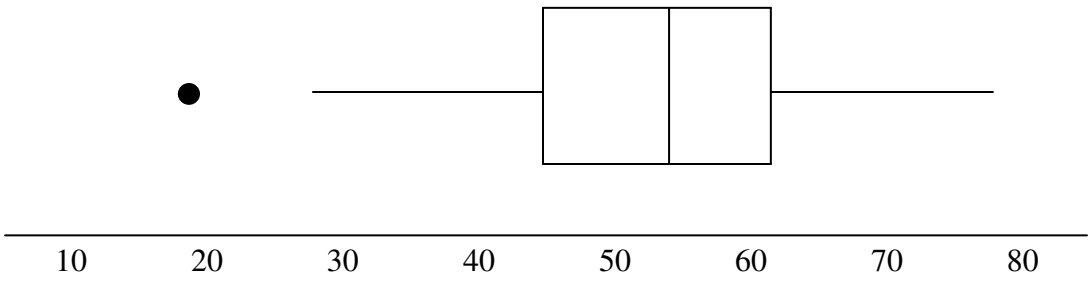
1. (a) An observation will be considered an outlier if it is either less than the lower fence LF or greater than the upper fence UF, where

$$LF = Q1 - 1.5IQR = Q1 - 1.5(Q3 - Q1) = 45 - 1.5(61 - 45) = 45 - 24 = 21$$
$$UF = Q3 + 1.5IQR = Q3 + 1.5(Q3 + Q1) = 61 + 1.5(61 - 45) = 61 + 24 = 85$$

There is one value less than LF ($19 < 21$), so 19 will be labeled as an outlier. The whisker coming out of the left side of the box will extend to the lowest value that is not an outlier (i.e., the "new minimum"), which is 28.

There are no values greater than UF, so there are no outliers on the right. The whisker coming out of the right side of the box will extend to the "new maximum", which is the same as the "old maximum", i.e., 79.

The outlier boxplot is shown below:



If we exclude the outlier from consideration, the distribution is **approximately symmetric**.

(b) The back-to-back stemplot is shown below:

Symphony		Elton John
9	1	7 8
8	2	3 5 9
6 3	3	0 3 7 7
8 5 2	4	1 2 6 9
9 8 5 3 2 2 0	5	0 4 4
8 5 1 1 0	6	2
9 2 0	7	

Since the distribution of ages for the Elton John concert is **approximately symmetric** with **no outliers**, it would be more appropriate to report the **mean and standard deviation**.

2. (a) The explanatory variable X is temperature and the response variable Y is ice cream sales. We know that

$$b_1 = r\frac{s_y}{s_x} \quad \Rightarrow \quad r = b_1\frac{s_x}{s_y} = 22.58\left(\frac{4.86}{123.98}\right) = \mathbf{0.8851}$$

(b) If the temperature increases by 1°C, we predict ice cream sales to increase by $22.58.

(c) The residual for this day is

$$y_i - \hat{y} = 350 - (-231.92 + 22.58(27)) = 350 - 377.74 = \mathbf{-\$27.74}$$

The sign of the residual is negative, meaning that the point falls **below** the least squares regression line (i.e., the sales for this day are below what we would have predicted them to be using the regression line.

(d) The fraction of variation in Y accounted for by its regression on X is, by definition, equal to r^2. Here we have $r^2 = (0.8851)^2 = 0.7834 =$ **78.34%**.

3. (a) The factors in an experiment are the explanatory variables. There are two factors in this experiment – **type of tire and speed**.

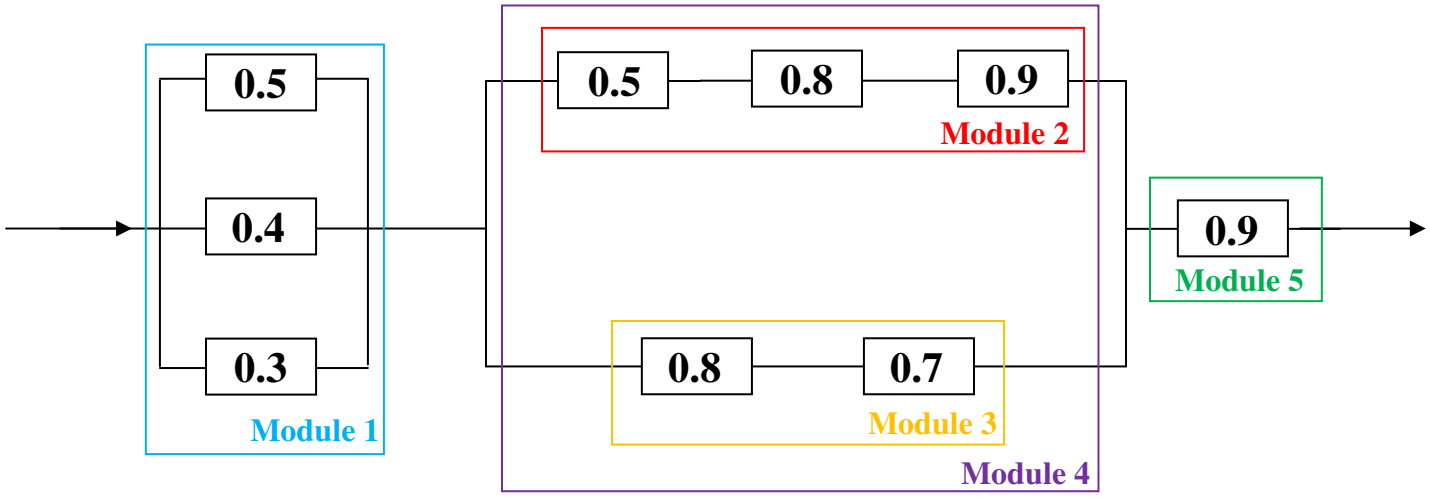
(b) Type of tire has two factor levels – regular or snow tires. Speed has three factor levels – 40 km/h, 60 km/h or 80 km/h. As such, there are 2 x 3 = 6 treatments, as shown below:

regular/40, regular/60, regular/80, snow/40, snow/60, snow/80

(c) The response variable is **stopping distance**.

(d) The blocking variable is **model of car**.

4. (a) We divide the system into modules as shown below:



The reliability of the system is $r_s = P(M_1)P(M_4)P(M_5)$

We calculate

$P(M_1) = 1 - (1 - 0.5)(1 - 0.4)(1 - 0.3) = 1 - (0.5)(0.6)(0.7) = 1 - 0.21 = 0.79$

$P(M_2) = (0.5)(0.8)(0.9) = 0.36$

$P(M_3) = (0.8)(0.7) = 0.56$

$P(M_4) = 1 - (1 - 0.36)(1 - 0.56) = 1 - (0.64)(0.44) = 1 - 0.2816 = 0.7184$

$P(M_5) = 0.9$

And so $r_s = P(M_1)P(M_4)P(M_5) = (0.79)(0.7184)(0.9) =$ **0.5108**

(c) The reliability of the system is $r_s = 1 - (1 - r)^3$. In order for the reliability to be at least 0.95, we must have

$$1-(1-r)^3 \geq 0.95 \quad \Rightarrow \quad -(1-r)^3 \geq -0.05 \quad \Rightarrow \quad (1-r)^3 \leq 0.05$$

$$\Rightarrow \quad 1-r \leq (0.05)^{1/3} \quad \Rightarrow r \geq 1-(0.05)^{1/3} = 1-0.3684 = \mathbf{0.6316}$$