

[12] 1. Compute each limit, if it exists.

(a) $\lim_{x \rightarrow 1/3} \frac{(2 - 6x)^2}{(3x - 1)(9x^2 - 1)}$

(b) $\lim_{x \rightarrow \infty} \left(\sqrt{x^2 + x} - \sqrt{x^2 - 3x} \right)$

(c) $\lim_{x \rightarrow 0} \frac{x^2 + 2x - \sin 3x}{2x}$

- [8] 2. Find $f'(x)$ for each of the following functions. Do NOT simplify your answers.

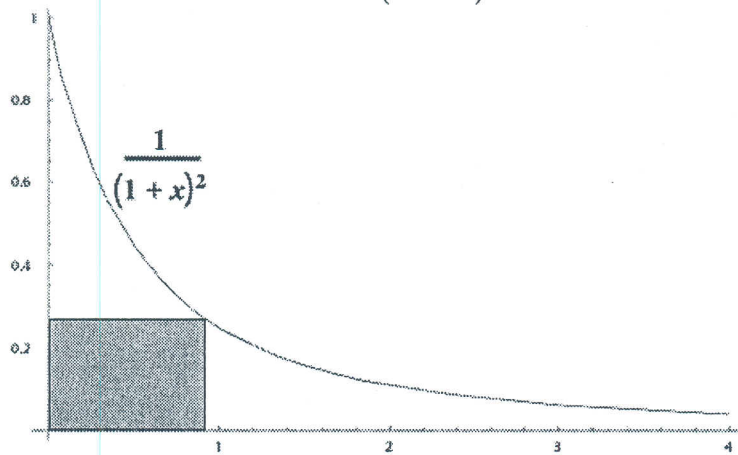
(a) $f(x) = \sqrt{5 - 2\sin^3(7x)}$

(b) $f(x) = \frac{\ln(x^3 + x)}{e^{4\cos(x)}}$

- [10] 3. Find the equation of the tangent line to the curve $3x^5y^5 - y = 2x$ at the point $(1, 1)$.

- [20] 4. (a) Draw the graph of the function $f(x) = x^2e^{-x}$. (You may use the fact that $\lim_{x \rightarrow \infty} x^2e^{-x} = 0$ without proving it.)
- (b) Find all points of inflection on this graph, and determine all intervals where the graph of the function is concave upward.
- (c) Find the absolute maximum and minimum values of the function $f(x) = x^2e^{-x}$ on the interval $-1 \leq x \leq 3$.

- [12] 5. Find the area of the largest rectangle that lies entirely in the first quadrant, has one side on the x -axis, another side on the y -axis, and a vertex on the curve $y = \frac{1}{(1+x)^2}$.



- [10] 6. At noon a ship S_1 is 20 *km* north of ship S_2 . If S_1 sails south at 6 *km/h* and S_2 sails east at 8 *km/h*. How fast are the ships separating at 4 : 00 pm?

[20] 7. Find the following indefinite and definite integrals:

(a) $\int t^2 \cos(t^3) dt$

(b) $\int \frac{\sqrt{x} - x^2}{x} dx$

(c) $\int_{\pi/2}^{\pi} \left(\sin x - \frac{1}{x} \right) dx$. Simplify as much as possible.

(d) $\int_0^1 x^5 \sqrt{x^3 + 1} dx$

- [8] 8. Find $f(x)$ if $f''(x) = 36x^2 + 12x$ with $f'(1) = 19$ and $f(1) = 7$.

- [8] Bonus. Find the points on the curve $y^2 - x^2 + 2x = 10$ closest to the point $(5, 0)$.