UNIVERSITY OF MANITOBA

DATE: June 20, 2009
PAPER # 50
DEPARTMENT & COURSE NO: MATH 2130
EXAMINATION: Engineering Mathematical Analysis 1

FINAL EXAMINATION
TITLE PAGE
TIME: 3 hours
EXAMINER: M. Potter

NAME: (Print in ink) ______________________________

STUDENT NUMBER: ______________

SIGNATURE: (in ink) ______________________________
(I understand that cheating is a serious offense)

INSTRUCTIONS TO STUDENTS:

This is a 3 hour exam.

No texts or notes are permitted. Calculators are **not** permitted. Cell phones, electronic translators, and other electronic devices are **not** permitted.

This exam has a title page and 13 pages of questions, including 2 blank pages for rough work. Please check that you have all the pages. You may remove the blank pages if you want, but be careful not to loosen the staples.

Answer all questions on the exam paper. If you need more room, you may use the back of the exam paper, but if you do, CLEARLY indicate your work is continued. When you are done, hand in this booklet.

Question	Points	Score
1	8	
2	6	
3	7	
4	4	
5	8	
6	10	
7	15	
8	10	
9	10	
10	12	
11	10	
12	14	
13	18	
Total:	132	

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1. Consider the function $f(x,y) = \dfrac{3x^2y + 2xy + 3y + y^3}{x^2y + y^2 + 2y}$.

   (a) [3 points] Give the domain of $f(x,y)$.

   (b) [2 points] Evaluate the limit $\lim\limits_{(x,y)\to(0,0)} f(x,y)$ or justify its nonexistence.

   (c) [3 points] Can the function $f(x,y)$ be extended to include $(0,0)$ in its domain continuously? Justify your answer using the definition of continuity.

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2. [6 points] Give a formula for the plane tangent to the surface $x = y\sin\left(\frac{\pi z}{2}\right)$ at the point $P(-1,-1,1)$.

3. [7 points] Evaluate the double integral

$$\int_0^1 \int_y^1 \sin(x^2)\ \mathrm{d}x\,\mathrm{d}y\ .$$

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4. [4 points] Let $\mathcal{C}$ be the curve given by the function

$$\mathbf{r}(t) = (1 + \cos \pi t)\hat{\mathbf{i}} - (\sin \pi t)\hat{\mathbf{j}} + (\sqrt{4+t})\hat{\mathbf{k}} \quad \text{for } t \geq -4 \, .$$

Set up the integral to find the length of the curve $\mathcal{C}$ from the point $P(2, 0, 2)$ to the point $Q(0, 0, \sqrt{5})$. Simplify the integrand **but DO NOT EVALUATE** the final integral.

5. [8 points] Let $\mathcal{C}$ be the curve of intersection described by $x^2 - y^2 + z^2 = 1$ and $xy + xz = 2$. Find the equation for the *tangent line* to $\mathcal{C}$ at the point $P(1, 1, 1)$ in the direction of decreasing z. Represent the line in parametric form (i.e., as a parametric system). (*Hint*: Do not parameterize $\mathcal{C}$.)

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6. Let $\mathbf{f}_k(x, y) = xy\hat{\mathbf{i}} + (kx^2 + 3y^2)\hat{\mathbf{j}}$.

   (a) [2 points] Show that the only k making $\mathbf{f}_k(x, y)$ consistent with the differential equation $\mathbf{f}_k(x, y) = \nabla F(x, y)$ is $k = \dfrac{1}{2}$.

   (b) [5 points] Setting $k = 1/2$, give an expression for all $F(x, y)$ that solve the differential equation $\mathbf{f}_k(x, y) = \nabla F(x, y)$.

   (c) [3 points] For any solution $F(x, y)$ above, give the direction of the **greatest decrease** in F at the point $P(2, 1)$. (Note: a direction should be a *unit vector*.)

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7. [15 points] Find all critical points of the function $g(x, y) = 6x - 2x^3 - 3xy^2$. Classify the critical points as a local (relative) maximum, minimum, or saddle point. Justify your classification.

8. [10 points] Use Lagrange multipliers to determine the area of the largest possible rectangle bounded by the axes $x = 0$, $y = 0$, and the ellipse $x^2 + \dfrac{y^2}{4} = 1$.

9. [10 points] Set up **but DO NOT EVALUATE** a triple iterated integral in *cylindrical coordinates* that yields the *moment of inertia* around the z-axis, I_z, of a sphere of radius 3 (centered at the origin) with density function

$$\rho(r,\theta,z) = 3 - \sqrt{z^2 + r^2}\ .$$

Express the integrand as a concrete expression in r, θ, and z.

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10. [12 points] Set up **but DO NOT EVALUATE** a set of four triple iterated integrals that yield the *centre of mass* $(\bar{x}, \bar{y}, \bar{z})$ for an object with a linear density function $\rho = kx$ (where $k > 0$) and the boundaries

$$0 \le z \le 1 - y^2 \quad \text{and} \quad 0 \le x \le y + 2\,.$$

Express each integrand as a concrete expression in x, y, and z.

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11. [10 points] Set up **but DO NOT EVALUATE** a double iterated integral that yields the *surface area* of the function $z = x\sin(y+x)$ over the region R bounded between the cylinders

$$x \geq (y-2)^2 \quad \text{and} \quad x \leq 4 - y^2\ .$$

Express the integrand as a concrete expression in x and y.

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12. [14 points] Use *spherical coordinates* to evaluate the volume of the object in the first octant bounded by the surfaces

$$x^2 + y^2 + z^2 = 1 \quad ; \quad y = x \quad ; \quad y = \sqrt{3}x \quad ; \quad z = 0\,.$$

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13. [18 points] Consider the object V in the first octant bounded by $z^2 = x^2 + y^2$ and $z = 3$. Formulate **but DO NOT EVALUATE** *all possible* triple iterated integrals in Cartesian coordinates that give the volume of the object. (Partial marks may be given for sketches.)

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