Math2130 Test 2 Summer 2015 (by N Harland)

[4] 1. (a) Show $\lim\limits_{(x,y)\to(0,0)} \dfrac{x^4 y}{x^6+y^3}$ does not exist.

[3] (b) Show $\lim\limits_{(x,y)\to(0,0)} \dfrac{|x|\,y^2}{x^2+y^2} = 0.$

[6] 2. Determine parametric equations for the tangent line to

$$2x^2 - y^2 + z^2 = 7,\ xy + xz = 5$$

at the point $P(1,2,3)$.

[2] 3. Let $f(x,y) = y\cos(x^7)$. Calculate $\dfrac{\partial^2}{\partial y^2}\left(\dfrac{\partial^7 f}{\partial x^7}\right)$. Explain your answer.

[4] 4. (a) Find a chain rule for $\left(\dfrac{\partial z}{\partial t}\right)_s$ if

$$z = f(x,y,t), \quad x = g(y,s,t), \quad y = h(t).$$

(Note: Notation will be marked)

[4] (b) Use part (a) to find $\left(\dfrac{\partial z}{\partial t}\right)_s$ if

$$z = e^{x^2+y^2+t^2}, \quad x = \ln(yst), \quad y = \sec t.$$

Do not simplify your answer.

[10] 5. Given

$$\begin{aligned} x^2 - y\cos(uv) + 2z &= 0 \\ x^2 + y^2 - \sin(uv) + 2z^2 - 2 &= 0 \\ xy - \sin v\cos u + 3z - 1 &= 0 \end{aligned}$$

Calculate $\left(\dfrac{\partial y}{\partial v}\right)_u$ when $x=1,\ y=1,\ u=\pi/2,\ v=0,\ z=0.$

[4] 6. (a) Determine all critical points of $f(x,y) = x^6 + y^6 - 6xy + 1$.

[7] (b) Classify the critical points you found in part (a).

[6] 7. Calculate $D_{\mathbf{v}} f$ at the point $P(1,2,3)$ if $f(x,y,z) = \ln(x^2+y^2+z^2)$ along the curve $\mathbf{r}(t) = (t^2,\ t^3+1,\ 3t)$.

[4 (bonus)] 8. Let $\mathbf{v} = \langle \cos\alpha, \sin\alpha\rangle$ and f be a twice differentiable function. Determine and simplify a formula for $D_{\mathbf{v}}\big(D_{\mathbf{v}} f\big)$ in terms of α and the second partial derivatives.