MATH 1210 Assignment 1 Winter 2014 **Due date: January 31**

1. **Use mathematical induction on positive integer n to prove each of the following:**

   (a) $1^2 + 4^2 + 7^2 + \cdots + (3n-2)^2 = \dfrac{1}{2}\, n(6n^2 - 3n - 1)$, **for** $n \geq 1$ **;**

   (b) $3 + 7 + 11 + \ldots + (8n-1) = 2n(4n+1)$, **for** $n \geq 1$ **;**

   (c) $2^{4n} - 3^{2n}$ **is divisible by** 7 **for** $n \geq 1$ **.**

2. **Consider the sum** $(5)^2 + (11)^2 + (17)^2 + \cdots + (18n-1)^2$**:**

   (a) **Write the sum in sigma notation.**

   (b) **Use identities** $\displaystyle\sum_{k=1}^{m} k = \frac{1}{2}\,[m(m+1)]$ **and** $\displaystyle\sum_{k=1}^{m} k^2 = \frac{1}{6}\,[m(m+1)(2m+1)]$ **to prove that**

   $$(5)^2 + (11)^2 + (17)^2 + \cdots + (18n-1)^2 = 3n\,(108n^2 + 36n + 1)\,.$$

3. **Prove that** $\displaystyle\sum_{\ell=1}^{n} \ell(\ell+2) = \frac{1}{6}\,[n(n+1)(2n+7)]$ **by each of the following two methods:**

   (a) **By mathematical induction on positive integer** $n \geq 1$ **.**

   (b) **By using the identities mentioned in part (b) of question** 2 **.**

4. **For each of the following sums, rewrite the sum such that it starts from the given number. Keep your answer in sigma notation but simplify it.**

   (a) $\displaystyle\sum_{j=6}^{25} \left[(3j-15)^3 + j(j-10) + 25\right]$ **starting with** $j = 1$ **.**

   (b) $\displaystyle\sum_{k=-3}^{n-3} \left[(6+2k)^2 + \frac{k+3}{k(k+4)}\right]$ **starting with** $k = 0$ **.**

5. **Find all** 4^{th} **roots of** -17 **in Cartesian form. Simplify as much as possible.**

6. **Let** $z = \dfrac{1}{2}i - \dfrac{\sqrt{3}}{2}$ **, evaluate** $z^{123} + 2i\,\overline{z} + \dfrac{-3+\sqrt{3}i}{1+\sqrt{3}i}$ **. Simplify as much as possible.**

7. **For each of the following statements, if it is true prove it, and if it is false give a counter example.**

   (a) $\overline{z} = \dfrac{|z|^2}{z}\,, \quad (z \neq 0)$ **;**

   (b) $\arg(z) = \arg(\overline{z})$ **;**

   (c) $z\,\overline{(z + z|z|)} \;=\; |z|^2\,(1 \,+\, |z|)$ **;**

   (d) $\dfrac{e^{i\,\theta^2}\,(e^{i\,\theta})^2}{e^{i^3}} \;=\; \cos(\theta+1)^2 \,+\, i\sin(\theta+1)^2$ **.**