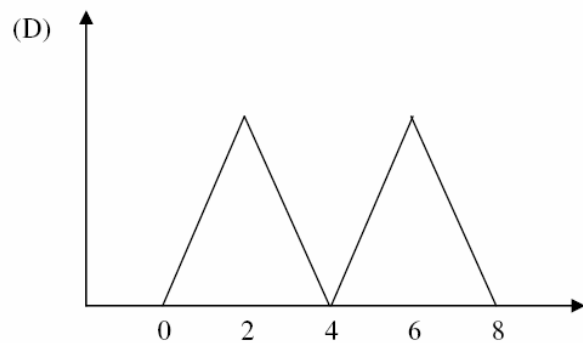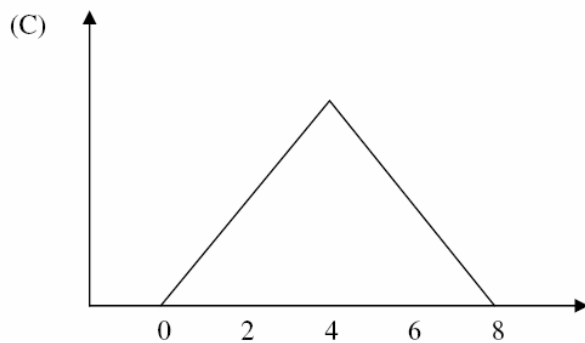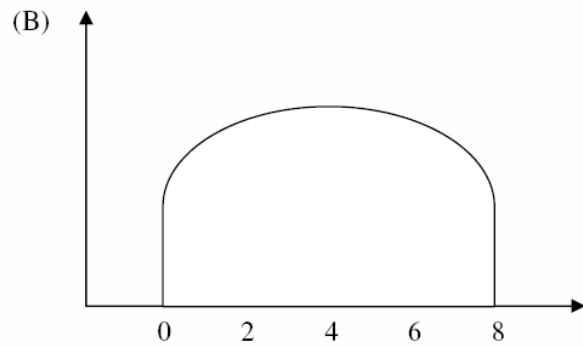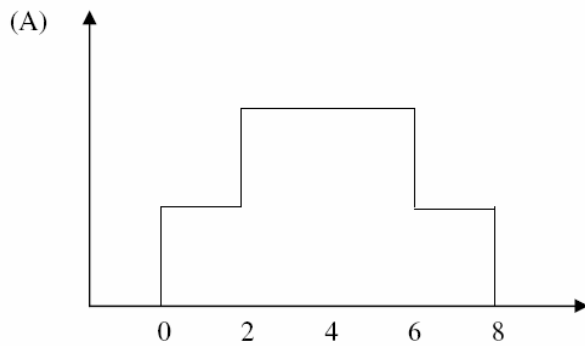# Sample Midterm Test 2A

<u>Part A</u>

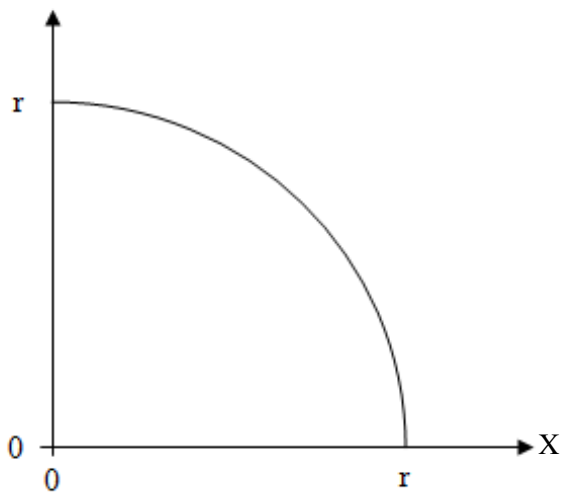1. A random variable X follows a uniform distribution, as shown below:
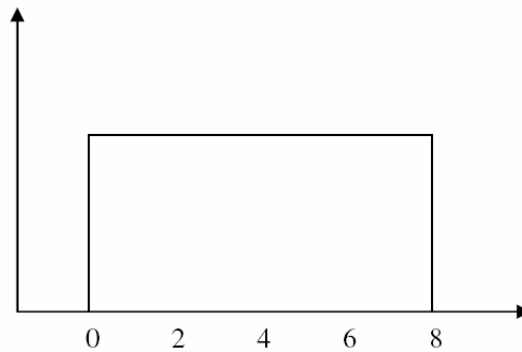
Which of the following distributions has the same interquartile range as the uniform distribution above? (Assume that all curves are valid density curves).

(E) all of the above

2. A variable X has a density curve as shown below:

What must be the value of r? (Recall the area of a circle is πr^2.)

(A) $\sqrt{\pi}$

(B) $\sqrt{\dfrac{4}{\pi}}$

(C) $\sqrt{\dfrac{1}{4\pi}}$

(D) $\sqrt{\dfrac{1}{\pi}}$

(E) $\sqrt{\dfrac{\pi}{4}}$

3. The c.d.f. for some discrete random variable X is

$$F(x) = \begin{cases} 0, & x < 0 \\ 0.14, & 0 \le x < 1 \\ 0.31, & 1 \le x < 2 \\ 0.53, & 2 \le x < 3 \\ 0.68, & 3 \le x < 4 \\ 0.86, & 4 \le x \le 5 \\ 1, & x > 5 \end{cases}$$

What is the probability $P(2 \le X \le 4)$?

(A) 0.15   (B) 0.35   (C) 0.37   (D) 0.44   (E) 0.55

4. The probability distribution for the number of stars X given by a food critic to restaurants in her weekly newspaper column is shown below:

x	0	1	2	3	4	5
P(X = x)	0.1	0.1	0.1	0.3	0.2	0.2

What is the variance of the number of stars given by the food critic?

(A) 1.2   (B) 1.5   (C) 2.0   (D) 2.4   (E) 2.8

5. You will continuously roll a fair six-sided die until you have rolled the number 5 three times. What is the probability that the third 5 appears on the 20th roll?

(A) 0.0357   (B) 0.0621   (C) 0.1450   (D) 0.2022   (E) 0.2379

6. Lotto Super 7 was a national lottery game in Canada. It was launched in June 1994, and its last draw was on September 18, 2009. Players purchased a ticket and selected seven numbers from 1 to 47. Seven numbers were randomly drawn (without replacement) and players won prizes depending on how many numbers they matched. If you selected your seven numbers, what would be the probability of correctly matching exactly three of the numbers drawn?

(A) 0.0403   (B) 0.0509   (C) 0.0607   (D) 0.0704   (E) 0.0801

7. A basketball player makes 90% of his free-throw shots. Assuming shots are independent, what is the probability that it takes him at least 12 shots from the beginning of the season for him to miss his first shot?

(A) 0.3138   (B) 0.6176   (C) 0.0314   (D) 0.6862   (E) 0.2824

8. Ten people each have a deck of 52 playing cards. Each person randomly selects 15 cards from their deck without replacement. What is the probability that exactly three of the people select exactly five Clubs?

(A) 0.1449   (B) 0.1508   (C) 0.1653   (D) 0.1781   (E) 0.1826

9. Which of the following variables X have a binomial distribution?

    I. Take a random sample of 25 Canadians.
       X = number of people in the sample with blood type A.

    II. There are ten people waiting in line at a bank.
        X = number of people in line who are served in the next 15 minutes.

    III. A boy asks a girl on a date five different times.
         X = number of times the girl agrees to go on a date with the boy.

(A) I only   (B) II only   (C) I and II only   (D) I and III only   (E) I, II and III

10. We have a deck of 30 cards. There are 6 red cards, 15 blue cards and 9 green cards. We randomly select three cards from the deck. After we select a card and record the colour, we replace the card in the deck and thoroughly shuffle them before we select another card.

Let X be the number of green cards that are selected. The distribution of X is:

(A) binomial with parameters n = 3 and p = 0.3.
(B) binomial with parameters n = 9 and p = 0.3.
(C) binomial with parameters n = 3 and p = 0.1.
(D) binomial with parameters n = 9 and p = 0.1.
(E) not binomial.

The next two questions (**11 and 12**) refer to the following:

Accidents occur at a busy intersection according to a Poisson process with a rate of 0.05 per day.

11. What is the probability that there are three accidents at the intersection during the month of April (30 days)?

(A) 0.1148   (B) 0.1255   (C) 0.1367   (D) 0.1473   (E) 0.1580

12. What is the probability that the time between successive accidents is more than a week (7 days)?

(A) 0.7047   (B) 0.6458   (C) 0.6874   (D) 0.7459   (E) 0.6124

13. You walk to the bus stop not knowing when the next bus will come. The time you have to wait follows a uniform distribution from 0 to 7 minutes. What is the probability that you will have to wait either less than one minute or more than five minutes for the bus to come?

(A) 0.5714   (B) 0.1892   (C) 0.4286   (D) 0.2857   (E) 0.3640

14. Which of the following normal variables has the density curve with the highest peak?

(A) $X \sim N(1, 3)$
(B) $X \sim N(2, 1)$
(C) $X \sim N(3, 4)$
(D) $X \sim N(4, 2)$
(E) All four curves have the same height.

15. Weights of chocolate bars produced by a candy company are known to follow a normal distribution with mean 52.82 grams and standard deviation 1.5 grams. What weight should the company put on the label so that only 3% of chocolate bars are underweight?

(A) 50 grams   (B) 51 grams   (C) 52 grams   (D) 53 grams   (E) 54 grams

Part B

1. We have two boxes, Box 1 and Box 2, each containing a large number of balls. In each box, 20% of the balls are numbered 0, 50% are numbered 1 and 30% are numbered 2. We will randomly select one numbered ball from each of the two boxes.

    (a) The outcome of interest is the number on the ball selected from each of the two boxes. List the complete sample space of outcomes and calculate the probability of each outcome.
    (b) Let X be the product of the numbers on the two selected balls. Find the p.m.f. of X.
    (c) Find the c.d.f. of X.

2. A random variable X has p.d.f.

$$f(x) = \begin{cases} \dfrac{1}{10\sqrt{x}}, & 25 \le x \le 100 \\ 0, & \text{otherwise} \end{cases}$$

    (a) Find the c.d.f. of X.
    (b) Find the third quartile of the distribution of X.
    (c) Find $P(36 < X < 81)$.
    (d) Find the expected value of X.

3. Do **only one** of the following questions:

    A random variable is said to possess the "lack of memory property" if

    $$P(X > s + t \mid X > s) = P(X > t)$$

    (a) Show that a random variable X with a geometric distribution possesses this lack of memory property.

    (b) Show that a random variable X with an exponential distribution possesses this lack of memory property.

4. A swimmer is competing in two events at a swim meet. From past experience, he knows that his times in the 100-meter freestyle event follow a normal distribution with mean 53 seconds and standard deviation 0.7 seconds. His times in the 100-meter buttery follow a normal distribution with mean 55 seconds and standard deviation 1.0 seconds.

(a) Use the scale provided below and draw the two density curves.

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_____
50              52              54              56              58              60
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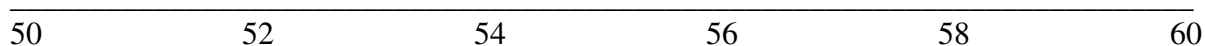
(b) In what proportion of his freestyle races are the swimmer's times between 52.5 and 54.0 seconds?

(c) Assuming his times for the two races are independent, what is the probability that the swimmer has a faster time in the butterfly than in the freestyle?

(d) At this swim meet, the swimmer finished the freestyle race in 52.3 seconds and the backstroke race in 53.0 seconds. In which race did the swimmer do better relative to his previous times?