

THE UNIVERSITY OF MANITOBA

DATE: December 17, 2011

FINAL EXAMINATION

DEPARTMENT & NO: MATH2132

TIME: 3 hours

EXAMINATION: Engineering Mathematical Analysis 2 EXAMINER: M. Despic, D. Trim

- 10 1. For what value of the constant a will the radius of convergence of the power series

$$\sum_{n=0}^{\infty} \frac{n^2 + 1}{a^n} (x - 1)^{2n}$$

be equal to 5?

- 13 2. Find the Taylor series about $x = 3$ for the function

$$f(x) = (x - 3) \ln(x + 1).$$

Use a method that guarantees that the series converges to $f(x)$. Express your answer in sigma notation, simplified as much as possible. Determine the open interval of convergence for the series.

- 7 3. Find a maximum possible error for all x in the interval $-1 \leq x \leq 3$ if the series

$$\sum_{n=1}^{\infty} \frac{n(-1)^{n+1}}{4^n} (x - 1)^n$$

is truncated after its 10^{th} term. Justify your answer.

- 6 4. You are given that the roots of the auxiliary equation associated with the linear, differential equation

$$\phi(D)y = 3 - 5x + 4 \cos x + x^3 e^{-2x}$$

are $m = 0, 3 \pm i, 3 \pm i, -2, -2, -2, \pm\sqrt{5}$. Write down the form of a particular solution of the differential equation as predicted by the method of undetermined coefficients. You must use the minimum number of terms possible. Do NOT find the coefficients, just the form of the particular solution.

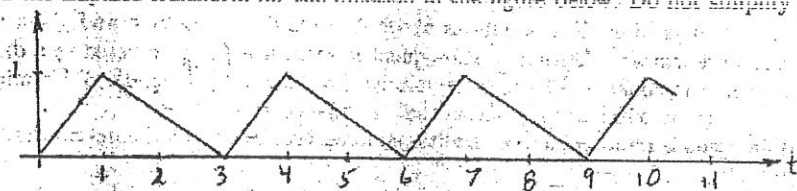
- 9 5. Find a one-parameter family of solution for the differential equation

$$y \frac{dy}{dx} = (y + 2)(\sin 3x - x).$$

Are there any singular solutions of your family?

- 15 6. (a) A mass of 2 kilograms is suspended from a spring with constant 50 newtons per metre. At time $t = 0$, it is lifted 10 centimetres above its equilibrium position and given velocity 4 metres per second downward. During its subsequent motion, it is also subjected to a damping force that (in newtons) is equal to 20 times its velocity (in metres per second). Determine the maximum distance from its equilibrium position that the mass ever achieves.
(b) If damping is removed, and an additional force $4 \cos \omega t$ acts on the mass, what value of ω causes resonance?

13. 7. (a) Find the Laplace transform for the function in the figure below. Do not simplify your answer.



- (b) Find the inverse Laplace transform for

$$F(s) = \frac{se^{-2s}}{s^2 + 4s + 7}$$

11. 8. Find a general solution of the following initial value problem where $f(t)$ is some unspecified function of time,

$$y'' + 4y' - 5y = f(t), \quad y(0) = 1, \quad y'(0) = 0.$$

16. 9. A mass of 1 kilogram is suspended from a spring with constant 6 newtons per metre. At time $t = 0$, it is released from 5 centimetres below its equilibrium position. During its subsequent motion, it is subjected to a constant force of 4 newtons upward, and at time $t = 5$ seconds, it is struck upward with an instantaneous force of 3 newtons. Find its position as a function of time.

Dawit's Answers (plankion@yahoo.com)

1. ± 25

2. $\ln 4(x-3) + \sum_{n=2}^{\infty} \frac{(-1)^n}{4^{n-1}(n-1)} (x-3)^n, \quad -1 < x < 7$

3. $\frac{11}{2^{11}}$

4. $Ax^2 + Bx + C \cos x + D \sin x + (Ex^6 + Fx^5 + Gx^4 + Hx^3)e^{-2x}$

5. $y - 2 \ln|y+2| = -\frac{1}{6}(2 \cos 3x + 3x^2) + C$; yes, $y = -2$ is a singular solution.

6. $y(t) = \frac{1}{10}(1-35t)e^{-5t}$, a) $y_{\max} = -\frac{7}{10}e^{-8/7}$ (in meters) b) 5 rad/s

7. a) $\frac{1}{1-e^{-35}} \frac{e^{-35}-3e^{-5}+2}{2s^2}$ b) $e^{-2(t-2)} \left[\cos \sqrt{3}(t-2) - \frac{2}{\sqrt{3}} \sin \sqrt{3}(t-2) \right] h(t-2)$

8. $y(t) = e^{-5t} + 5e^t + \frac{1}{6} \left[\int_0^t [\bar{e}^{-5(t-u)} + e^{t-u}] f(u) du \right]$

9. $\frac{2}{3} - \frac{37}{60} \cos \sqrt{6}t + \frac{3}{\sqrt{6}} \sin \sqrt{6}(t-5) h(t-5)$ (in meters)