Term Test 2

DATE: March 10, 2011
TIME: 70 minutes
COURSE: MATH 2130
EXAMINER: G.I. Moghaddam

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[10] 1. Evaluate each of these limits or explain why it does not exist.

(a) $\displaystyle\lim_{(x,y)\to(1,-1)} \frac{(x^2-y)(1+y)}{x^4-y^2}$

(b) $\displaystyle\lim_{(x,y)\to(0,-1)} \frac{\sin(\sqrt{x^2+y^2}-1)}{x^4+y^4+2x^2y^2-1}$

[10] 2. Let $u(x,y) = f(x^2+y) + g(x^2+y)$ where f and g are twice differentiable functions. Show that

$$\frac{\partial^2 u}{\partial x^2} - 4x^2\frac{\partial^2 u}{\partial y^2} - 2\frac{\partial u}{\partial y} = 0$$

[9] 3. Find $\dfrac{dz}{dx}$ if

$$xy + xz + yz = 1 \quad , \quad 2xy - 2y^2 - \frac{1}{2}x^2 = 0\,.$$

Simplify your answer.

[8] 4. Let $f(x,y,z) = x^2 + y + z$ and $g(x,y,z) = xz + \dfrac{1}{4}z^2$. Find the directional derivative of $f+g$, at the origin, along the curve

$$C \;:\quad x = t^2\,, \quad y = 2t\,, \quad z = -2t\,.$$

[13] 5. Let S_1 be the surface $z = x^3 - y^2$ and also let S_2 be the surface $x + y = xz$.

(a) Find the equation of the tangent plane to the surface S_1 at the point $(2,2,4)$.

(b) Find a tangent vector for the curve of intersection of the two surfaces S_1 and S_2 at the point $(1,-1,0)$.

(c) Find the point(s) on the surface S_2 at which the tangent plane is parallel to the plane $2x + y - 3z = 0$.