MATH 2130 Summer 2014 Test 2

1. (a) Show $\lim\limits_{(x,y)\to(0,0)} \dfrac{x^2y}{x^4+y^2}$ does not exist.

   (b) Show $\lim\limits_{(x,y)\to(0,0)} \dfrac{x^2y^4}{x^4+y^4} = 0.$

2. Laplace's equation for a function $f(x,y)$ is

$$\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = 0.$$

   Show $f(x,y) = \ln(x^2+y^2)$ satisfies Laplace's equation.

3. Determine parametric equation for the tangent line to

$$x^2 - y^2 + 2z^2 = 2,\ xy + xz = 2$$

   at the point $P(1,1,1)$.

4. (a) Find a chain rule for $\left.\dfrac{\partial z}{\partial t}\right)_s$ if

$$z = f(x,y,t), \quad x = g(y,s,t), \quad y = h(t).$$

   (b) Use part (a) to find $\left.\dfrac{\partial z}{\partial t}\right)_s$ if

$$z = e^{x^2+y^2+t^2}, \quad x = \tan(yst), \quad y = \ln t.$$

5. Given

$$\begin{aligned} x^2 - y\cos(uv) + 2z &= 0 \\ x^2 + y^2 - \sin(uv) + 2z^2 - 2 &= 0 \\ xy - \sin v \cos u + z &= 0 \end{aligned}$$

   Find $\left.\dfrac{\partial x}{\partial u}\right)_v$ when $x = 1,\ y = 1,\ v = \pi/2,\ u = 0,\ z = 0.$

6. (a) Determine all critical points of $f(x,y) = x^4 + y^4 - 4xy + 1.$

   (b) Classify the critical points you found in part (a).

7. Let $\mathbf{v} = \langle \cos\alpha, \sin\alpha \rangle$ and f be a twice differentiable function.

   (a) Determine and simplify a formula for $D_{\mathbf{v}}f$ in terms of α and the partial derivatives.

   (b) Determine and simplify a formula for $D_{\mathbf{v}}\big(D_{\mathbf{v}}f\big)$ in terms of α and the second partial derivatives. (this was a bonus)

Answers

1.

2.

3. $x = 1 - 3t,\ y = 1 + 3t,\ z = 1 + 3t$

4. (a) $\left.\dfrac{\partial z}{\partial t}\right)_s = \left.\dfrac{\partial z}{\partial x}\right)_{y,t} \left.\dfrac{\partial x}{\partial y}\right)_{s,t} \dfrac{dy}{dt} + \left.\dfrac{\partial z}{\partial x}\right)_{y,t} \left.\dfrac{\partial x}{\partial t}\right)_{s,y} + \left.\dfrac{\partial z}{\partial y}\right)_{x,t} \dfrac{dy}{dt} + \left.\dfrac{\partial z}{\partial t}\right)_{x,y}$

   (b) $\left.\dfrac{\partial z}{\partial t}\right)_s = 2xe^{x^2+y^2+t^2} st \sec^2(yst) \left(\dfrac{1}{t}\right) + 2xe^{x^2+y^2+t^2}(ys)\sec^2(yst) + 2ye^{x^2+y^2+t^2}\left(\dfrac{1}{t}\right) + 2te^{x^2+y^2+t^2}$

5. $\dfrac{\pi}{4}$

6. (a) $(0,0)$, $(1,1)$, $(-1,-1)$

   (b) $(0,0)$ yields a saddle point. $(1,1)$ and $(-1,-1)$ yield relative minima.

7. (a) $D_{\mathbf{v}} f = f_x \cos\alpha + f_y \sin\alpha$

   (b) $D_{\mathbf{v}}\big(D_{\mathbf{v}} f\big) = f_{xx}\cos^2\alpha + 2f_{xy}\cos\alpha\sin\alpha + f_{yy}\sin^2\alpha$