DATE: October 8, 2009
COURSE: MATH 2132

TIME: 70 minutes
EXAMINER: G.I. Moghaddam

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[9] 1. (a) Determine whether the sequence of numbers $\left\{\dfrac{1+\cos\sqrt{n}}{\sqrt{n+1}}\right\}_{n=1}^{\infty}$ is convergent or divergent. If it converges, find the limit.

(b) The sequence of functions $\left\{\dfrac{x^2}{n}+\dfrac{(x-1)n^2-x^2}{(1-x)n^2+8}\right\}_{n=1}^{\infty}$ is defined on the interval $(-\infty,\infty)$. Determine whether the sequence is convergent or divergent. If it converges, find the limit function.

[12] 2. Let $f(x)=\dfrac{4x}{1-4x}$ for $-\dfrac{1}{4}<x\leq\dfrac{1}{8}$. It is given that $f^{(n)}(x)=\dfrac{4^n\,n!}{(1-4x)^{n+1}}$ where $n\geq 1$.

(a) Find the first 3 terms of the Maclaurin series of $f(x)$.

(b) Find the n^{th}-remainder (i.e. $R_n(0,x)$).

(c) Show that $\lim\limits_{n\to+\infty} R_n(0,x)=0$ *only* for the case $x<0$.

[8] 3. Find the sum and the interval of convergence of the series

$$\sum_{n=1}^{\infty}(-1)^n\,2^{4n}\,(x-\frac{1}{2})^n$$

[8] 4. Find the radius of convergence and the open interval of convergence for the series $\displaystyle\sum_{n=0}^{\infty}\frac{(-1)^n\,n!}{2^n\,(1\cdot 4\cdot 7\cdot 10\cdot\cdots\cdot(3n+1))}\,x^{3n}$.

[13] 5. (a) Find the Maclaurin series of $f(x)=\dfrac{1}{1+2x}$. What is the interval of convergence ?

(b) Find the Maclaurin series of $g(x)=\dfrac{-2(x^2+1)}{(1+2x)^2}$. Express your answer in sigma notation and simplify as much as possible. Determine its open interval of convergence. (**Hint** : you may use part (a).