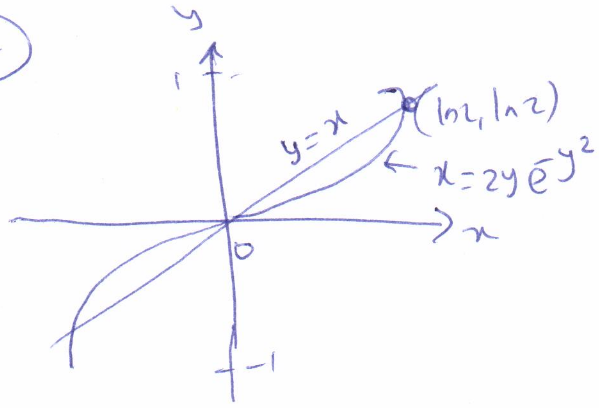
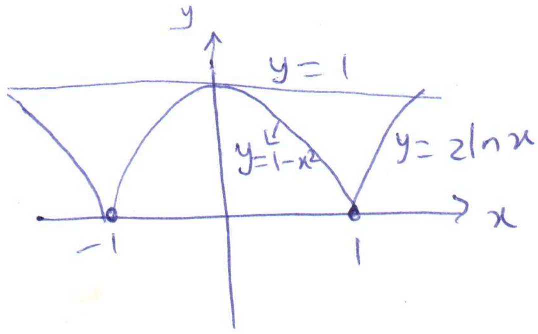
Tutorial 2: Solutions

(1a)



$$A = 2\int_0^{\ln 2} \left(2y e^{-y^2} - y\right) dy$$
$$= 2\left[-e^{-y^2} - \frac{y^2}{2}\right]\Big|_0^{\ln 2}$$
$$= 2\left(-e^{-\ln 2} - \frac{\ln 2}{2}\right) - 2(-1)$$
$$= \underline{1 - \ln 2}$$

(1b)



$\frac{y}{2} = \ln x$
$\Rightarrow x = e^{y/2}$
$y = 1 - x^2 \Rightarrow x = \pm\sqrt{1-y}$

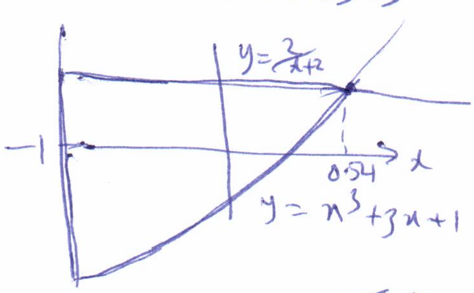
$$A = 2\int_0^1 \left(e^{y/2} - \sqrt{1-y}\right) dy$$
$$= 2\left[2e^{y/2} + \frac{2}{3}(1-y)^{3/2}\right]_0^1$$
$$= 4\sqrt{e} - \frac{16}{3}$$

(1c) The x-coordinate of the point of intersection of the curves is defined by the equation $\frac{2}{x+2} = x^3 + 3x - 1$

$\Rightarrow x^4 + 2x^3 + 3x^2 + 5x - 4 = 0$

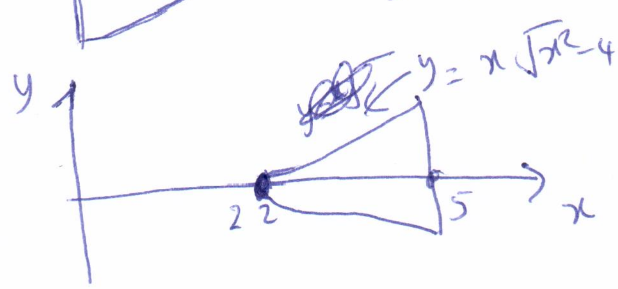
Solve using iterative method for nonlinear eqn., such as the Newton's method $\left(x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}\right)$. One root is

$x_1 = 0.542373$



$$\therefore A = \int_0^{0.542373} \left(\frac{2}{x+2} - x^3 - 3x + 1\right) dx$$
$$= 0.559.$$

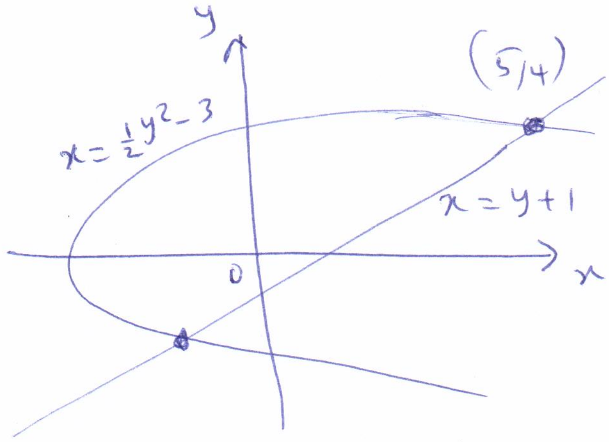
(1d)



$$A = 2\int_2^5 x\sqrt{x^2-4}\, dx$$
$$= 2\left[\frac{1}{3}(x^2-4)^{3/2}\right]_2^5$$
$$= \frac{2}{3}(21)^{3/2} = 14\sqrt{21}$$

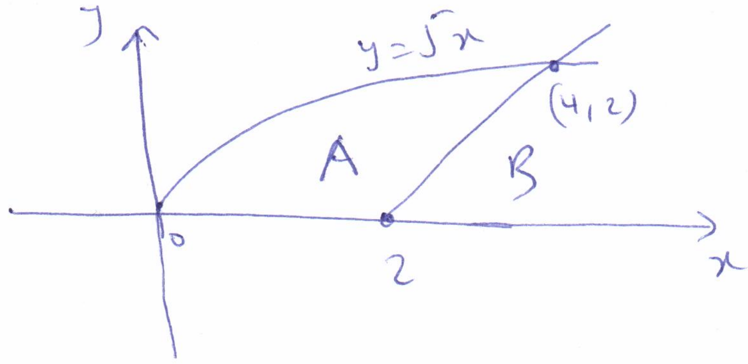
(A1)

(2)



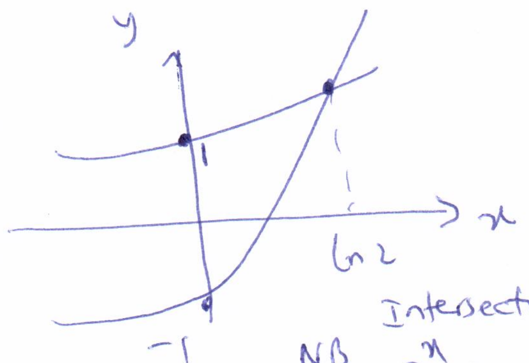
$$A = \int_{-2}^{4}\left[(y+1) - \left(\tfrac{1}{2}y^2 - 3\right)\right] dy$$
$$= \int_{-2}^{4}\left(-\tfrac{1}{2}y^2 + y + 4\right) dy$$
$$= -\tfrac{1}{2}\left(\tfrac{y^3}{3}\right) + \tfrac{y^2}{2} + 4y \Big|_{-2}^{4} = 18$$

(3)



$$A = \underbrace{\int_0^2 \sqrt{x}\,dx}_{\text{Area of region A}} + \underbrace{\int_2^4 (\sqrt{x} - x + 2)\,dx}_{\text{Area of region B}}$$
$$= \left[\tfrac{2}{3}x^{3/2}\right]_0^2 + \left[\tfrac{2}{3}x^{3/2} - \tfrac{x^2}{2} + 2x\right]_2^4$$
$$= 10/3$$

(4)



$$A = \int_0^{\ln 2}\left[e^x - (e^{2x} - 2)\right] dx = \ldots$$

Intersection pts.

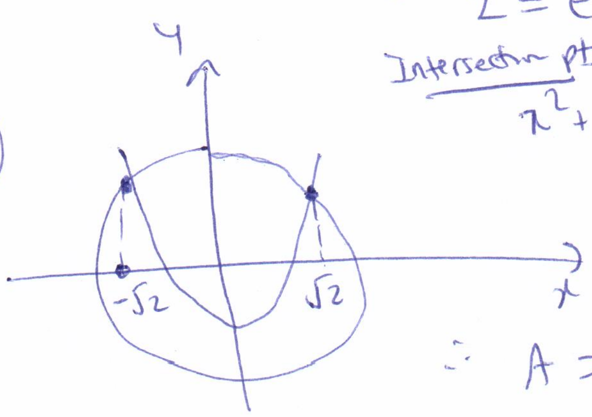
NB $e^x = e^{2x} - 2$. Let $u = e^x$

$\Rightarrow u^2 - u - 2 = 0$

$(u-2)(u+1) = 0 \Rightarrow u = 2$ ✓ or $u = -1$ ✗

$\therefore 2 = e^x \Rightarrow x = \ln 2$

(5)



Intersection pts

$x^2 + (x^2-1)^2 = 3 \Rightarrow x^4 - x^2 - 2 = 0$

$x^2 = 2$ or $x^2 = -1$ ✗

$\therefore x = \pm\sqrt{2}$

$y^2 = 3 - x^2$

$y = \sqrt{3 - x^2}$

$$\therefore A = \int_{-\sqrt{2}}^{\sqrt{2}}\left[\sqrt{3-x^2} - (x^2 - 1)\right] dx$$
$$=$$