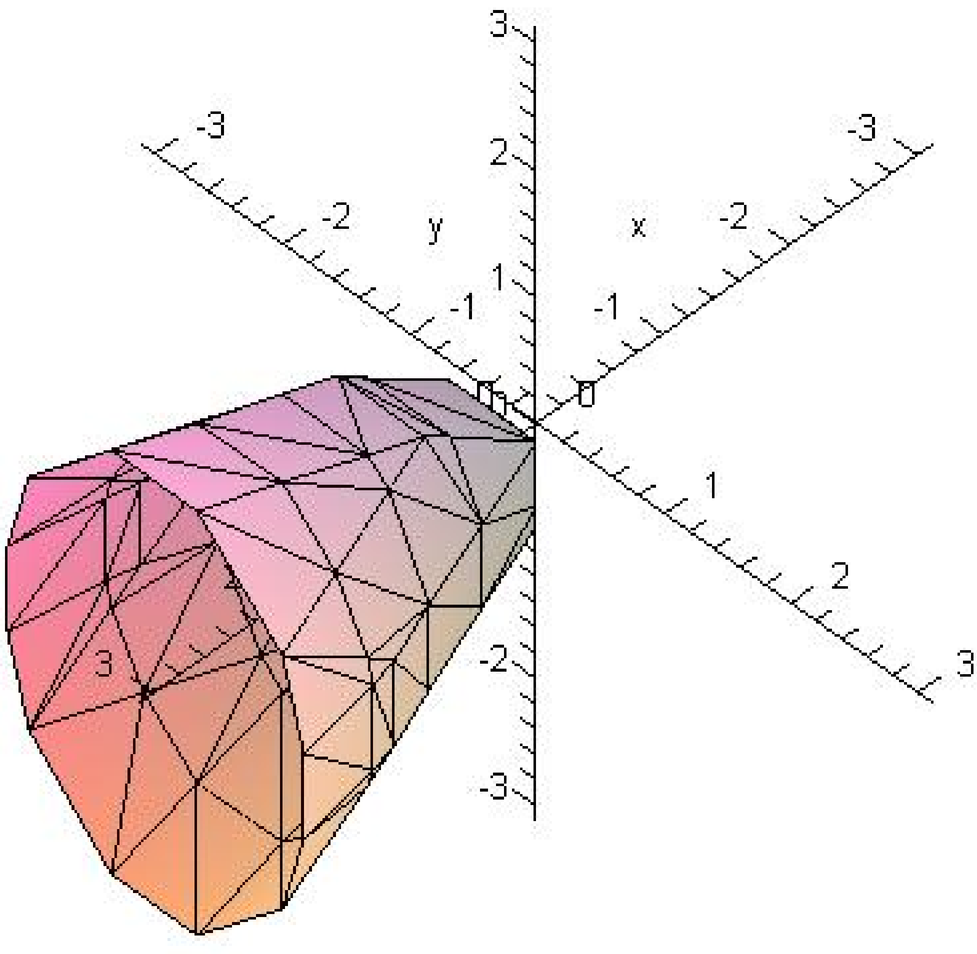
MATH 2130 Problem Workshop 1 Solutions

In questions 1-12, draw the surface defined by the question. In questions 13-16, draw the curve and find the projections in the xy, yz and xz-coordinate planes.

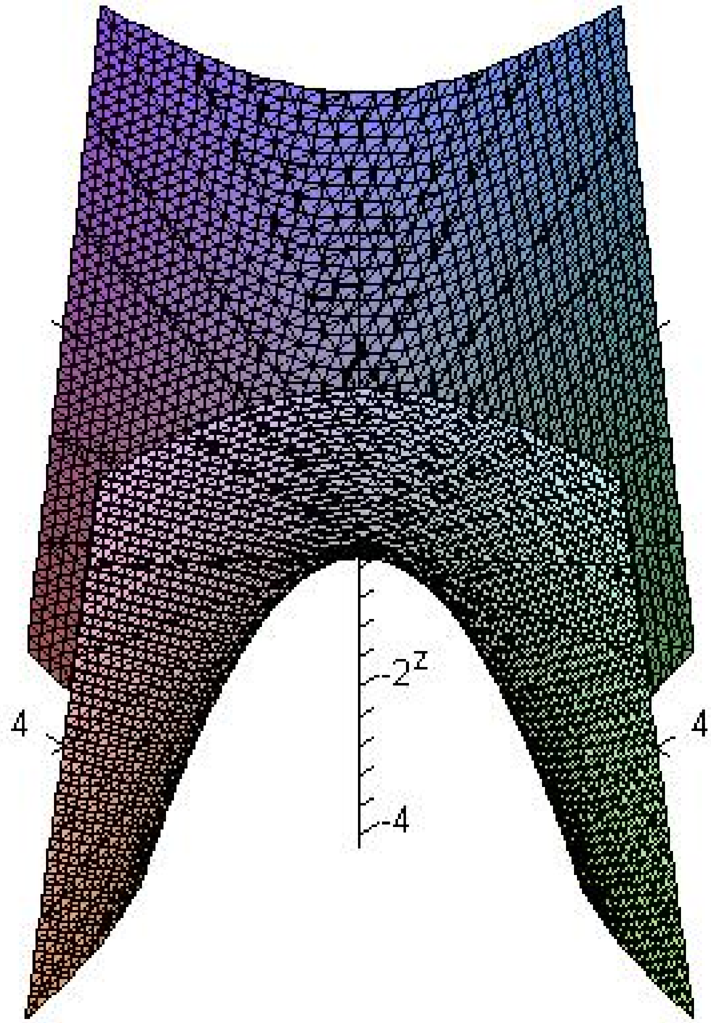
1. $x = 2y^2 + z^2$

   This is an elliptic paraboloid opening in the positive x-direction. Hence the picture looks like



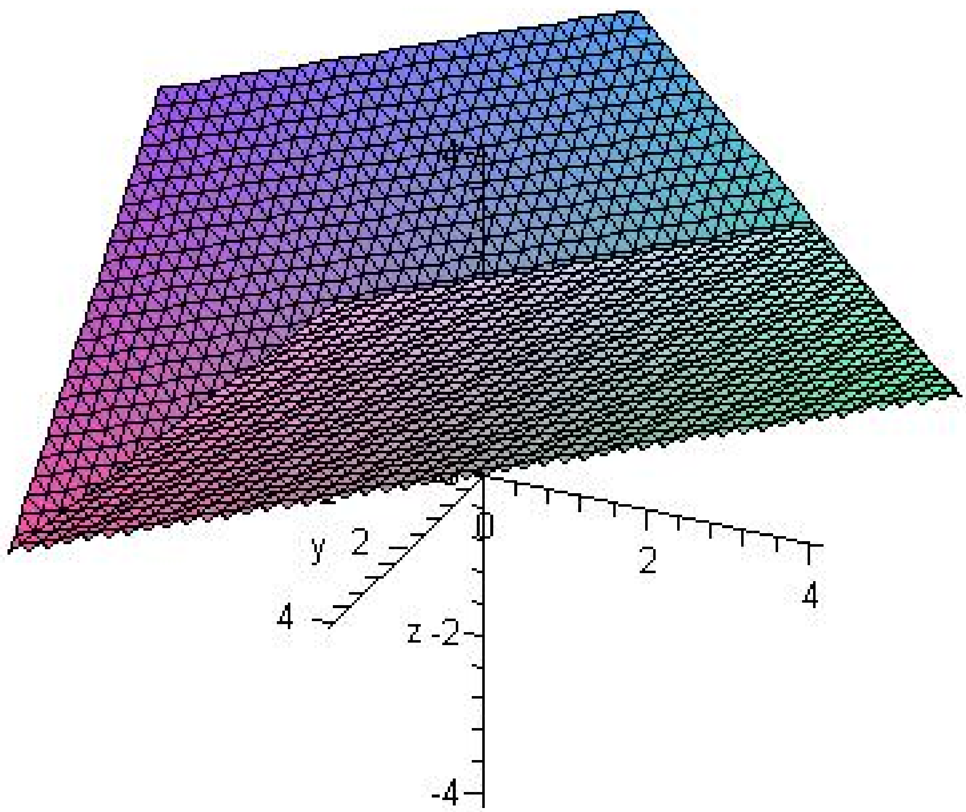
2. $z = 2xy$

   We could draw some level curves to see for fixed values of $z = k$ we have $y = \dfrac{k}{2x}$ which are hyperbolas. For fixed values of x, y we have lines. Hence the graph looks like



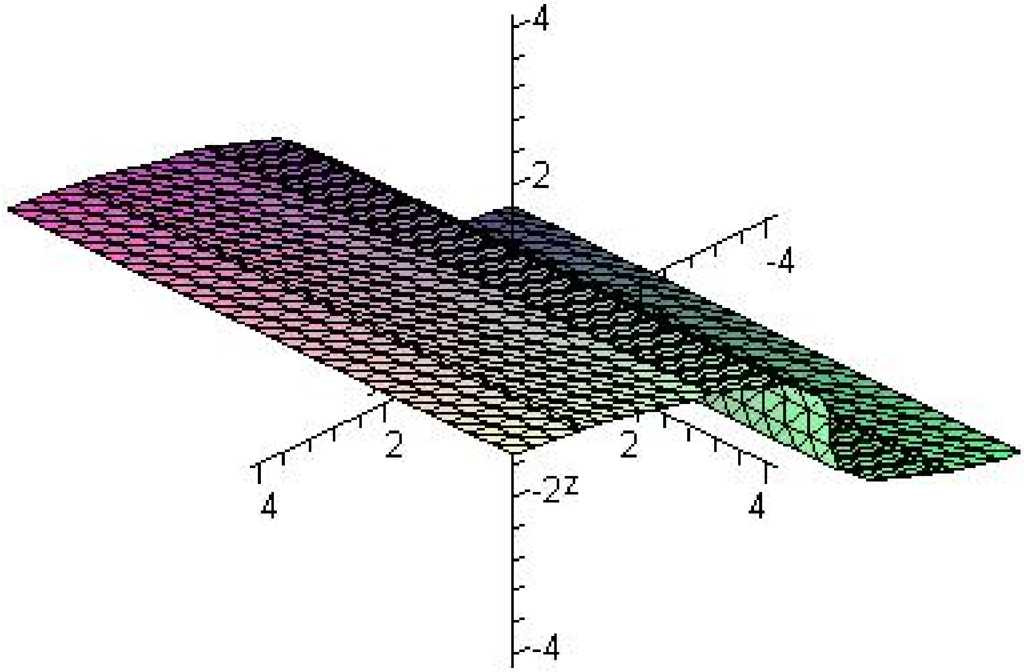
3. $z = |x + y|$

   For fixed values of $z = k$ we have that $x + y = \pm k \Rightarrow y = \pm k - x$. Hence the graph looks like



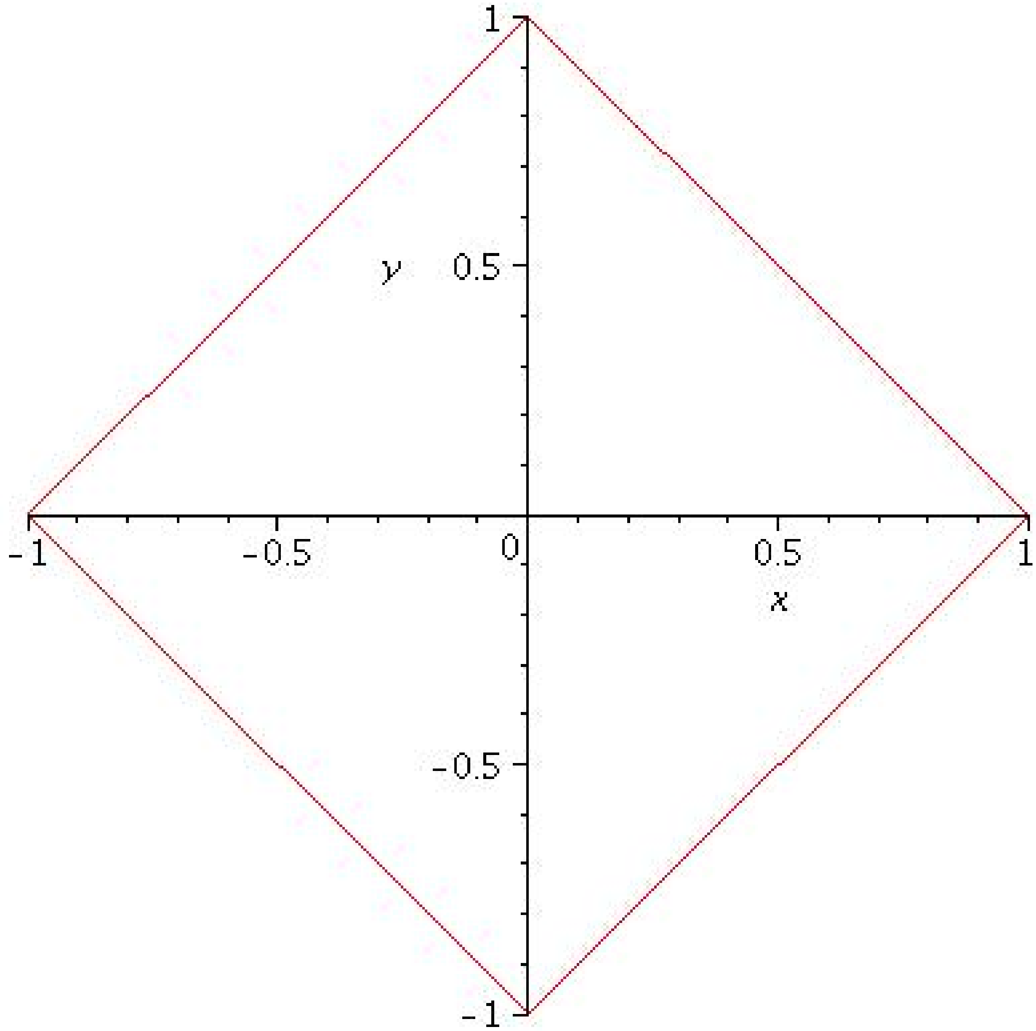
4. $x = z^3 + 1$

   This is a cyclinder going in the y direction since there is no y in the equation. The equation $x = z^3 - 1$ is a cubic, and hecne the picture is

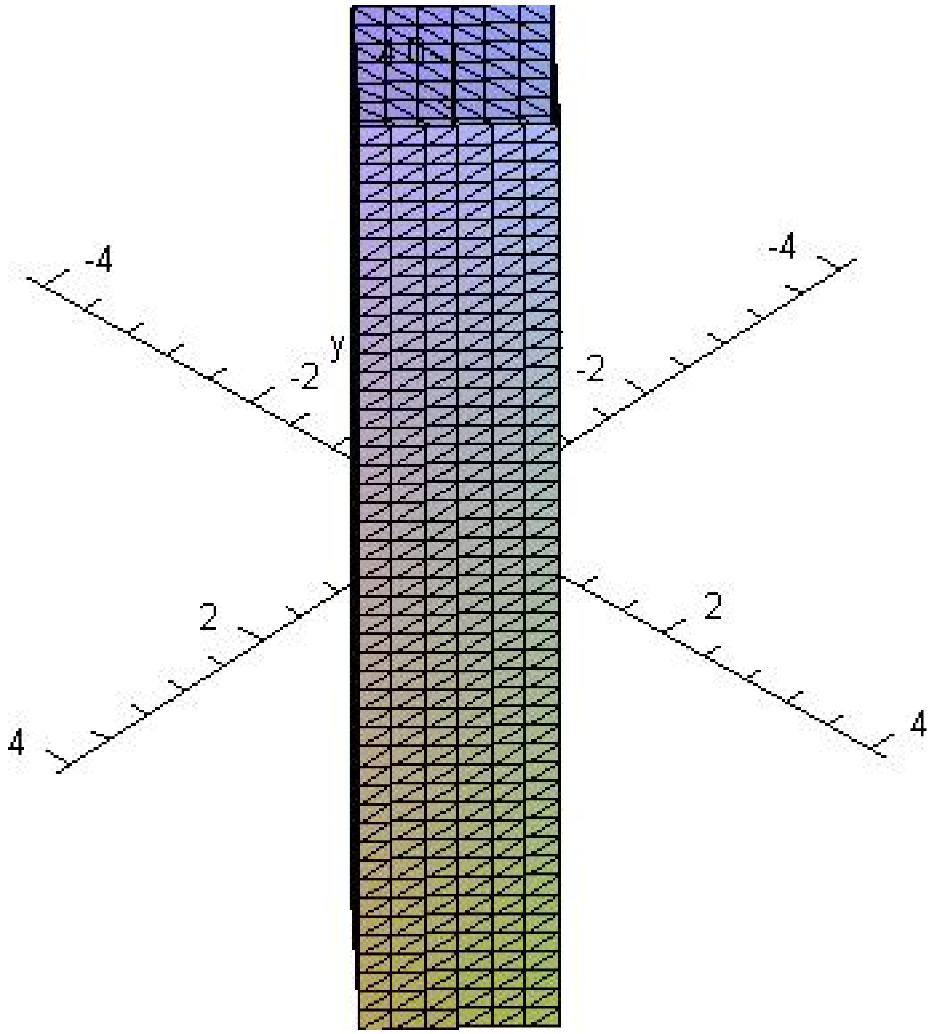


5. $|x| + |y| = 1$

   This is a cylinder going in the z direction. For the part of the curve involving x and y, we have $\pm x \pm y = 1$ leading to the lines $y = \pm x \pm 1$ where $-1 \leq x, y \leq 1$. In two dimensions, this is

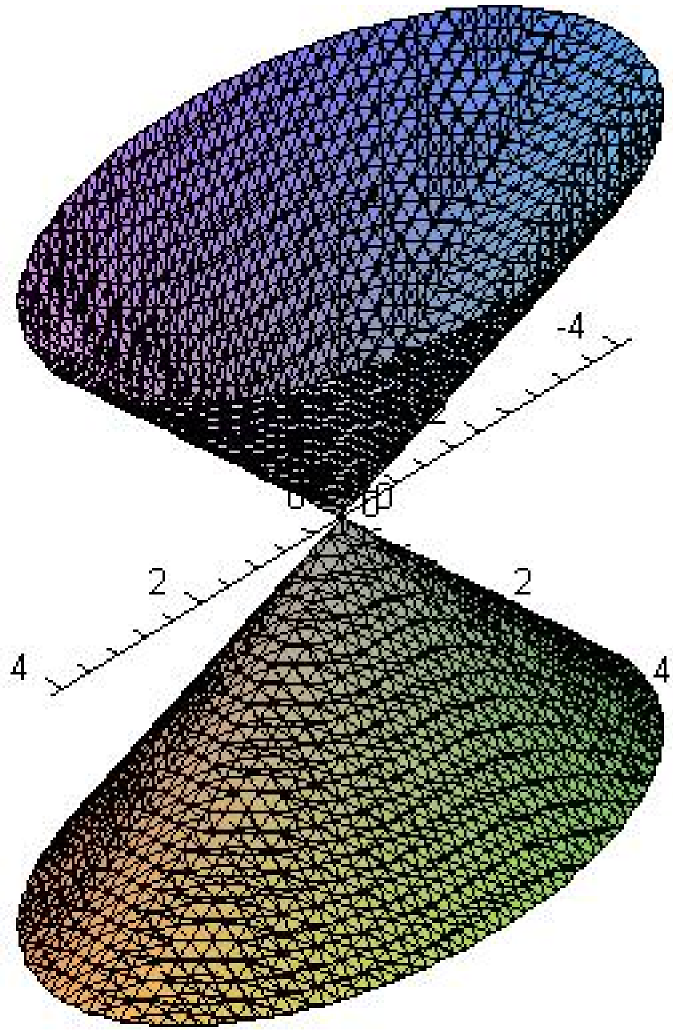


in 3-dimensions, this is



6. $z^2 - x^2 = 3y^2$

   Rearranging the equation leads to $z^2 = x^2 + 3y^2$ which is an elliptic cone going in the z-direction. Hence the graphs look like.

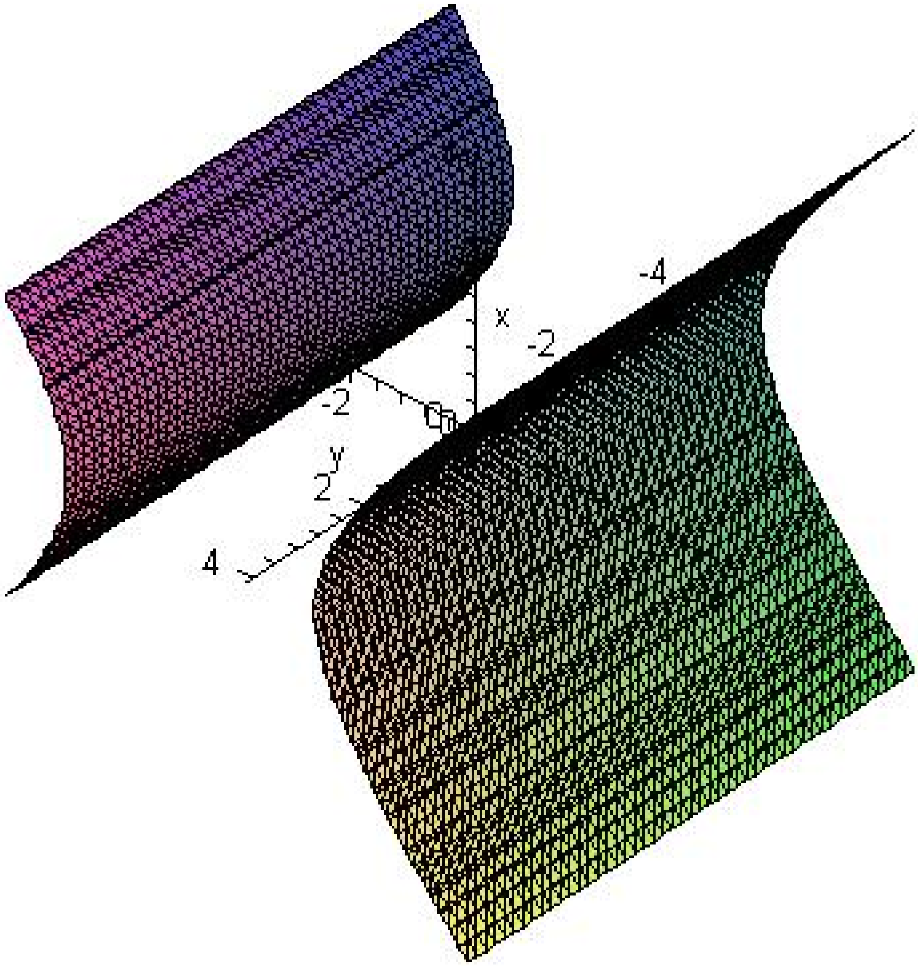


7. $y^2 = z^2 - 2y + 3$

   There is no x in the equation, hence it's a cylinder in the x direction. Rearranging the equation leads to

   $$z^2 - y^2 - 2y + 3 = 0 \Rightarrow z^2 - (y+1)^2 + 4 = 0 \Rightarrow \frac{(y+1)^2}{4} - \frac{z^2}{4} = 1$$

   which is a hyperbola opening in the y direction. Hence the graph looks like

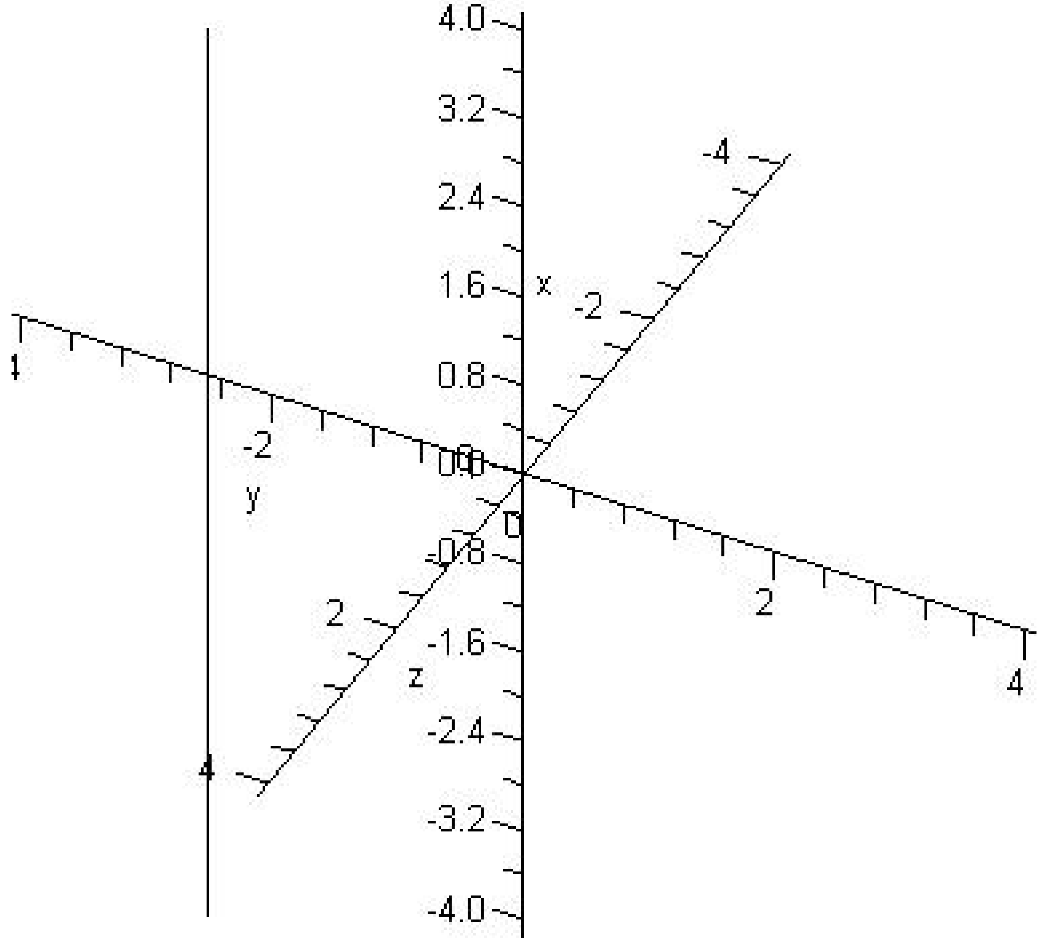


8. $x^2 + y^2 = 2x - 4y - 5$

   This is a cylinder in the z direction. Rearranging the equation leads to

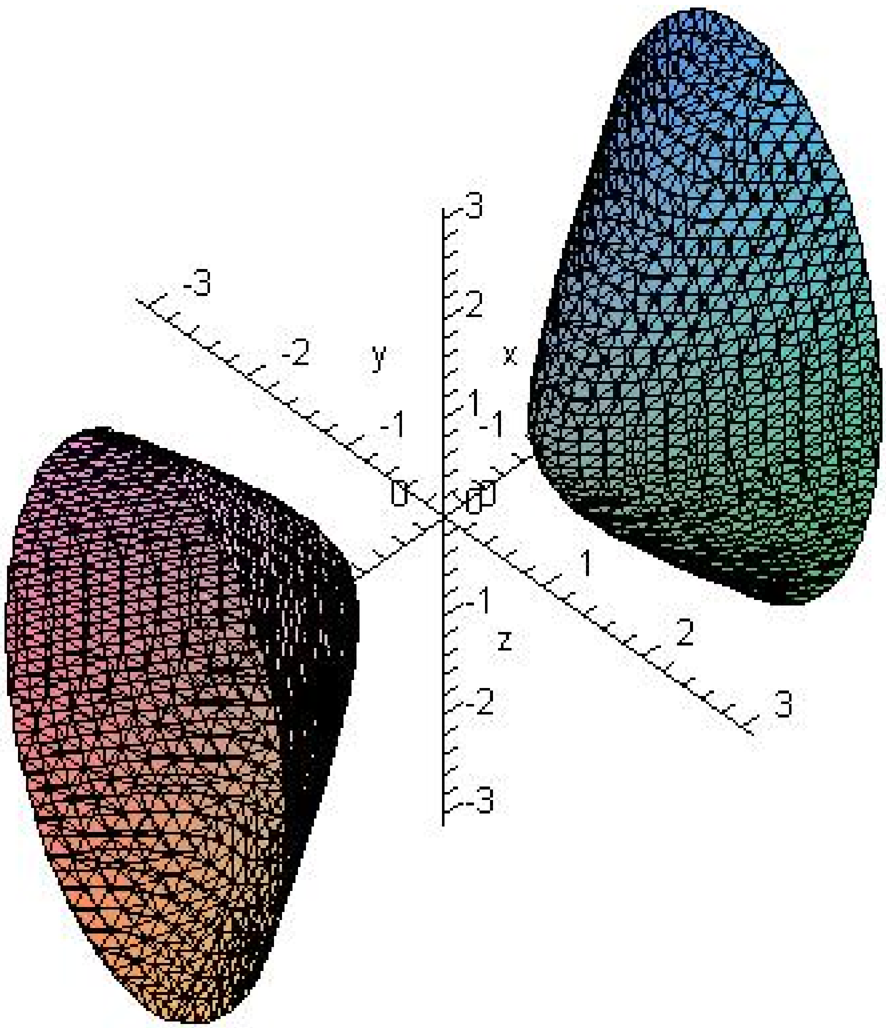
   $$x^2 - 2x + y^2 + 4y = -5 \Rightarrow (x-1)^2 - 1 + (y+2)^2 - 4 = -5 \Rightarrow (x-1)^2 + (y+2)^2 = 0.$$

   Hence the graph is just the points $(1, -2, z)$ for any z. Hence the graph looks like



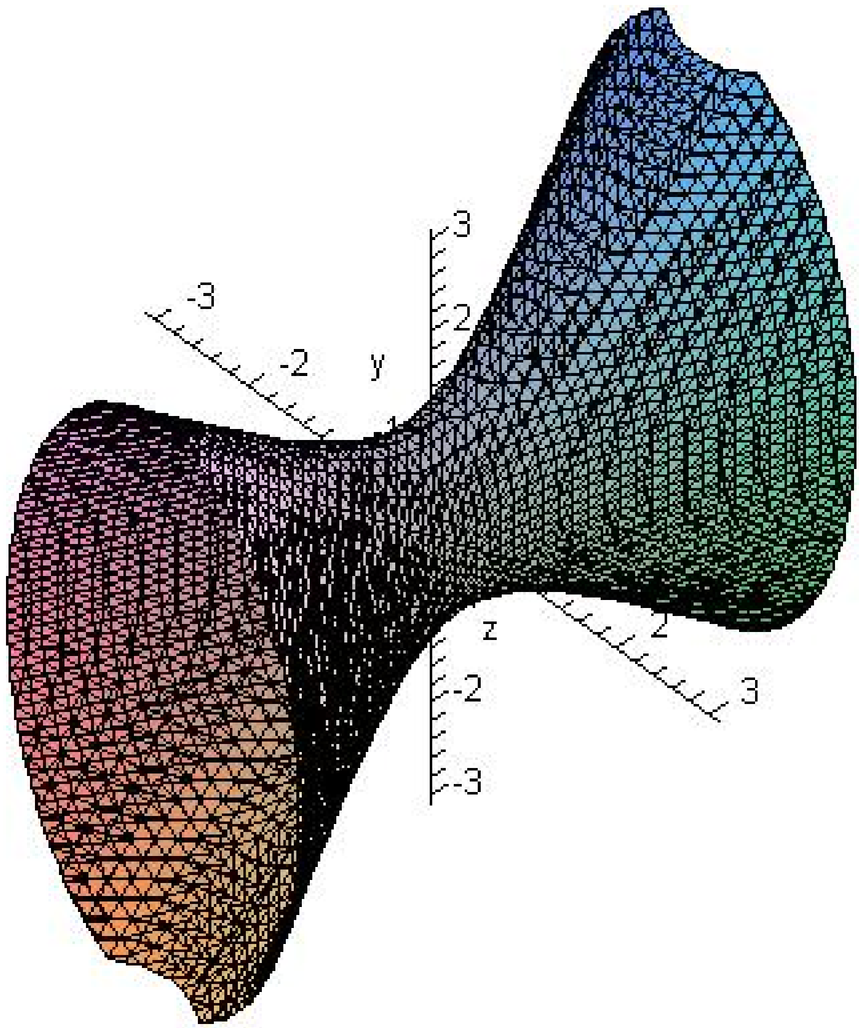
9. $4y^2 + z^2 = x^2 - 1$

   Simplifying leads to $x^2 - 4y^2 - z^2 = 1$ which is an elliptic hyperboloid in two sheets. (Opening in the x direction) Hence the graph is



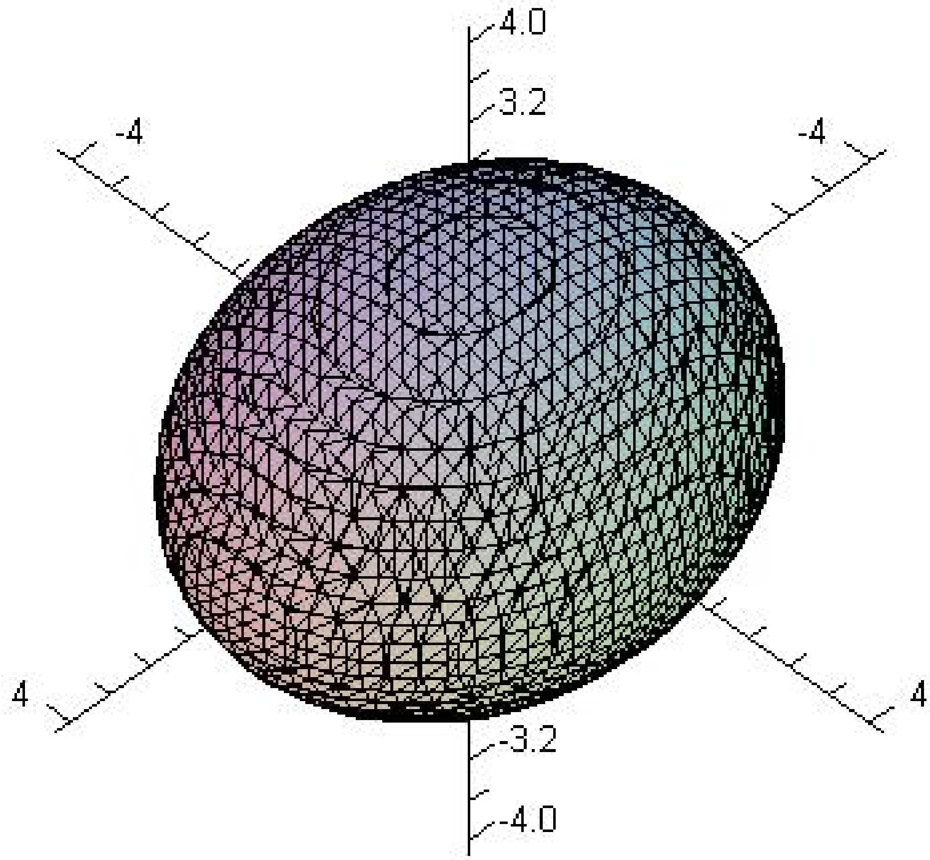
10. $4y^2 + z^2 = x^2 + 1$

Simplifying leads to $4y^2 + z^2 - x^2 = 1$ which is an elliptic hyperboloid in one sheet. (Opening in the x direction) Hence the graph is



11. $2x^2 + 3y^2 + 4z^2 = 12$

    This can be rearranged to yield $\dfrac{x^2}{6} + \dfrac{y^2}{4} + \dfrac{z^2}{3} = 1$ which is an ellipsoid. The graph is

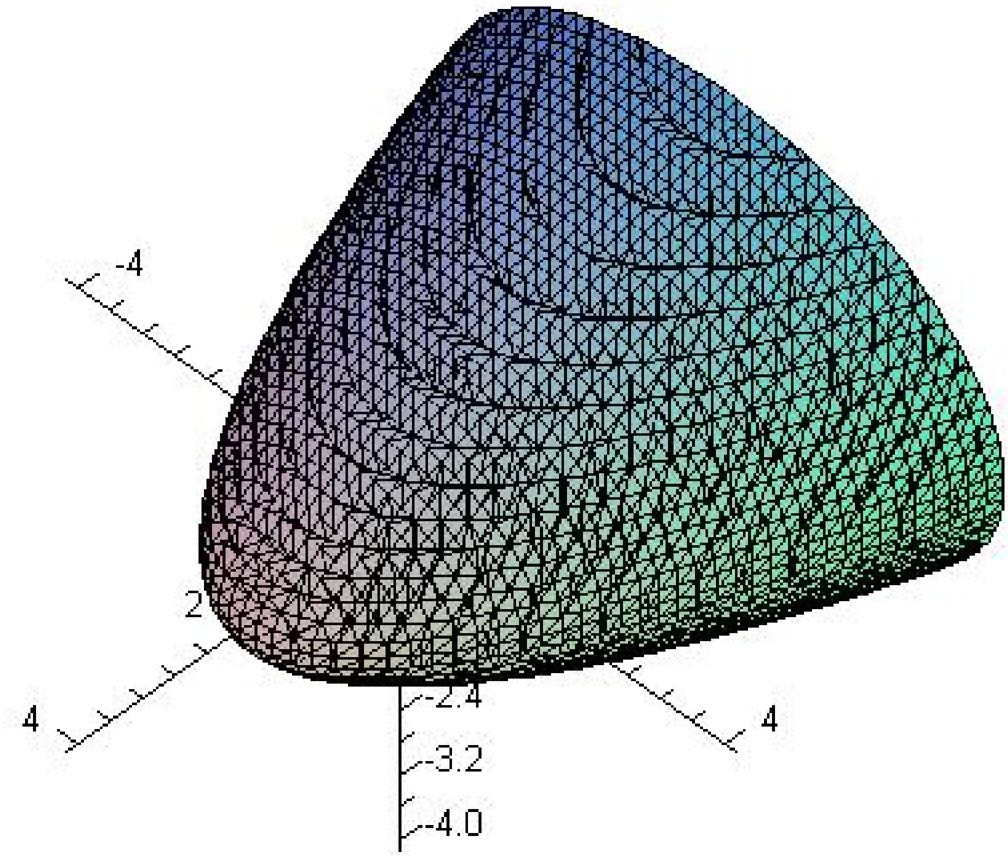


12. $y^2 + 2z^2 = 4 - 2x$

    Rearranging yields

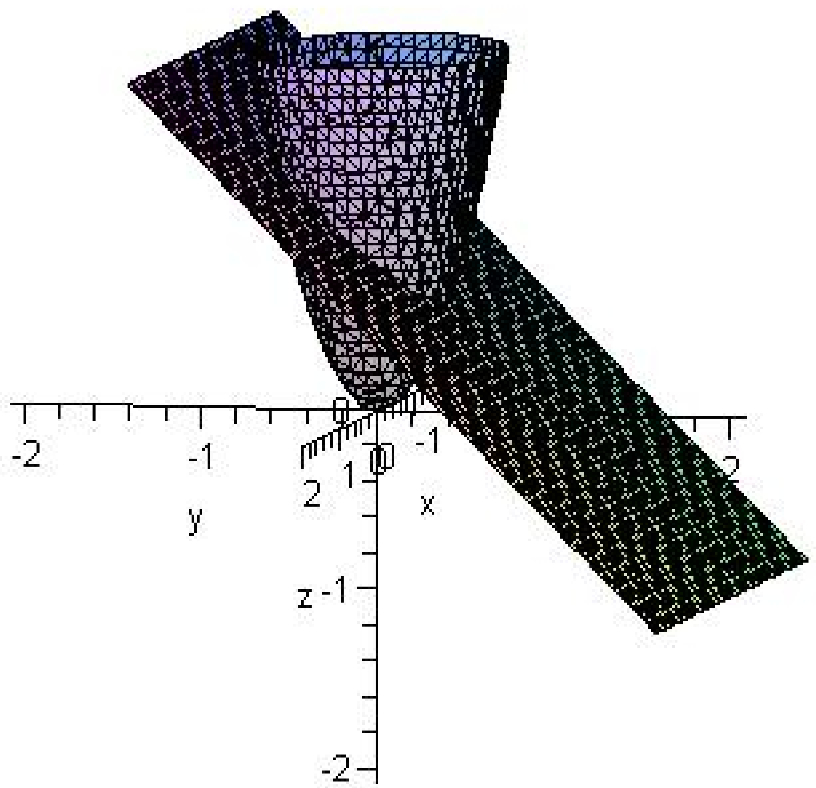
$$x = 2 - \frac{y^2}{2} - z^2$$

which is an elliptic paraboloid opening in the negative x direction. A graph is



13. *(The intersection of)* $z = 2x^2 + 4y^2, \quad y + z = 1$.

This is an intersection of an elliptic paraboloid opening in the z direction and a plane. A picture looks like



For the projection in the xy plane, we need to find values of x, y which satisfy both equations. Hence setting the z values equal to each other yields

$$1 - y = 2x^2 + 4y^2 \Rightarrow 2x^2 + 4y^2 + y = 1.$$

Hence the projection is

$$2x^2 + 4y^2 + y = 1,' quad z = 0.$$

For the projection in the yz plane, we need to find values of y, z which satisfy both equations. Since the second equation only has y, z anything on the line $y + z = 1$ will satisfy both equations provided that they are part of the domain of the first function. Since x^2 cannot be negative, the first equation forces

$$z \geq 4y^2 \Rightarrow 1 - y \geq 4y^2 \Rightarrow 4y^2 + y - 1 \leq 0.$$

Using the quadratic formula yields $(-1-\sqrt{17})/8 \le y \le (-1+\sqrt{17})/8$ and hence the projection is

$$y+z=1, \quad x=0, \qquad (-1-\sqrt{17})/8 \le y \le (-1+\sqrt{17})/8$$

For the projection in the xz plane, we need to find values of x, z which satisfy both equations. Hence setting the y values equal to each other yields

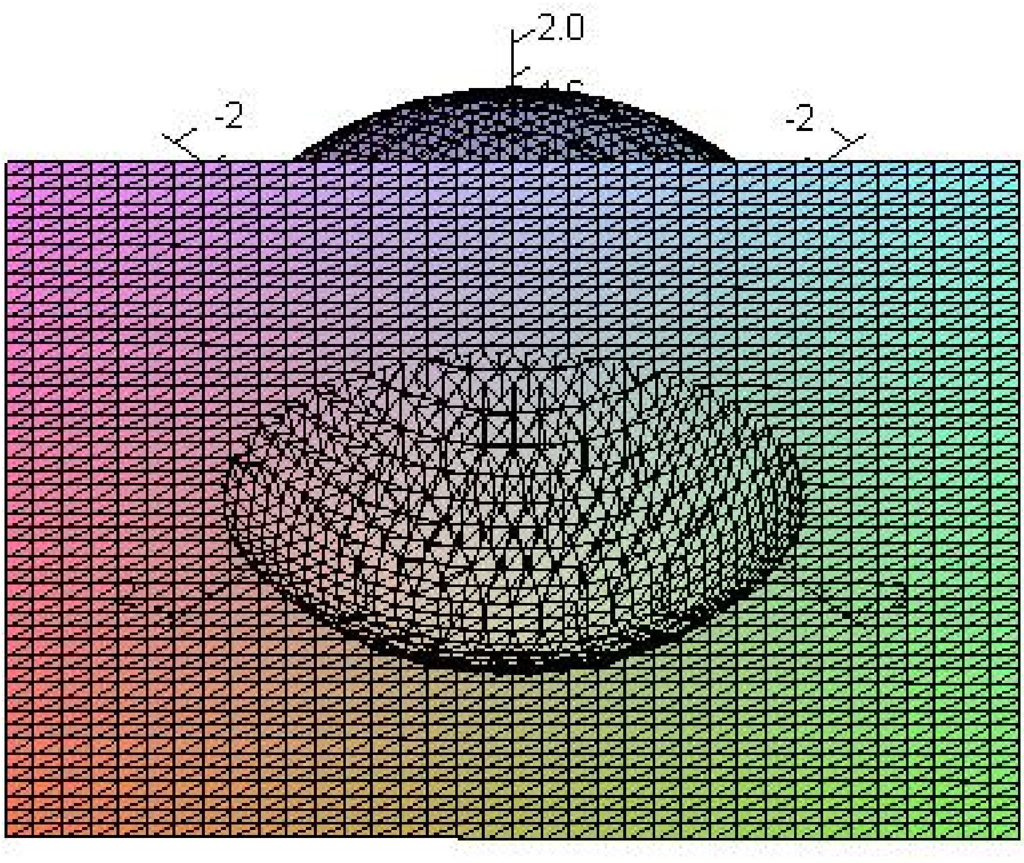
$$z = 2x^2 + 4(1-z)^2 \Rightarrow 2x^2 + 4z^2 - 9z + 4 = 0$$

Hence the projection is

$$4z^2 - 9z + 2x^2 + 4 = 0, \quad y = 0.$$

14. *(The intersection of)* $x^2 + y^2 + 2z^2 = 2, \quad x + y = 1.$

This is an intersection of an ellipsoid and a plane. A picture looks like



For the projection in the xy plane, we need to find values of x, y which satisfy both equations. Since the second equation only has x, y anything on the line $x + y = 1$ will satisfy both equations provided that they are part of the domain of the first function. Since $z^2 \geq 0$ we know that

$$x^2 + y^2 \leq 2 \Rightarrow x^2 + (1-x)^2 \leq 2 \Rightarrow 2x^2 - 2x - 1 \leq 0$$

Using the quadratic formula yields $(1 - \sqrt{3})/2 \leq x \leq (1 + \sqrt{3})/2$ and hence the projection is

$$x + y = 1, \quad z = 0, \qquad (1 - \sqrt{3})/2 \leq x \leq (1 + \sqrt{3})/2$$

For the projection in the yz plane, we need to find values of y, z which satisfy both equations. Setting the x values equal to each other yields

$$(1-y)^2 + y^2 + 2z^2 = 2 \Rightarrow 2y^2 - 2y + 2z^2 = 1.$$

Hence the projection is

$$2y^2 - 2y + 2z^2 = 1, \quad x = 0.$$

For the projection in the xz plane, we need to find values of x, z which satisfy both equations. Hence setting the y values equal to each other yields

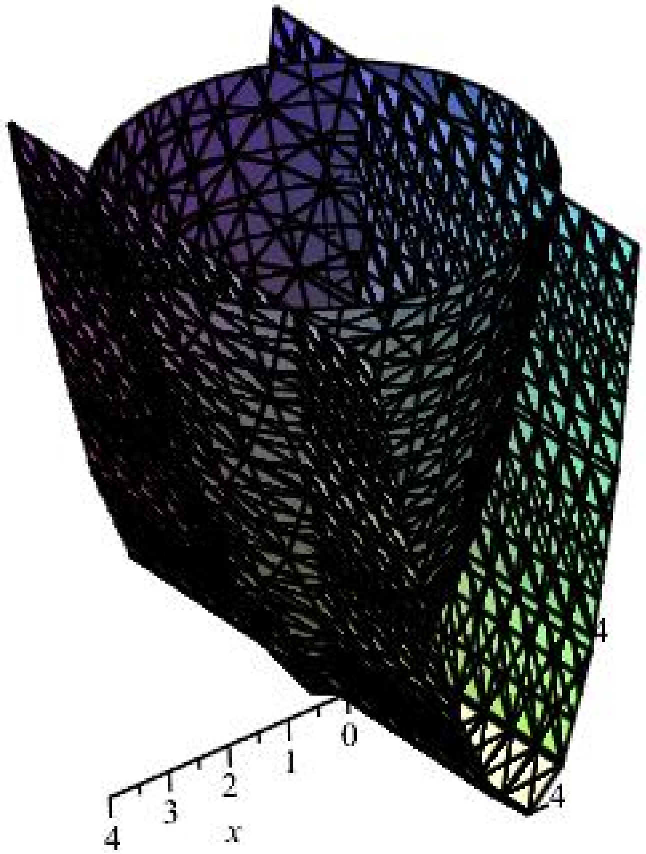
$$x^2 + (1-x)^2 + 2z^2 = 2 \Rightarrow 2x^2 - 2x + 2z^2 = 1.$$

Hence the projection is

$$2x^2 - 2x + 2z^2 = 1, \quad y = 0.$$

15. *(The intersection of)* $z = x^2 + y^2, \quad z = 2x^2.$

This is an intersection of an elliptic paraboloid and a parabolic cylinder. A picture looks like



For the projection in the xy plane, we need to find values of x, y which satisfy both equations. Setting the z values equal to each other yields

$$2x^2 = x^2 + y^2 \Rightarrow x^2 = y^2 \Rightarrow y = \pm x.$$

Hence the projection is

$$y = \pm x, \quad z = 0.$$

For the projection in the yz plane, we need to find values of y, z which satisfy both equations. Hence setting the x values equal to each other yields

$$z = \frac{z}{2} + y^2 \Rightarrow z = 2y^2.$$

Hence the projection is

$$z = 2y^2, \quad x = 0.$$

For the projection in the xz plane, we need to find values of x, z which satisfy both equations. Since the second equation only has x, z anything on the parabola $z = 2x^2$ will satisfy both equations provided that they are part of the domain of the first function. Since $y^2 \geq 0$ we know that

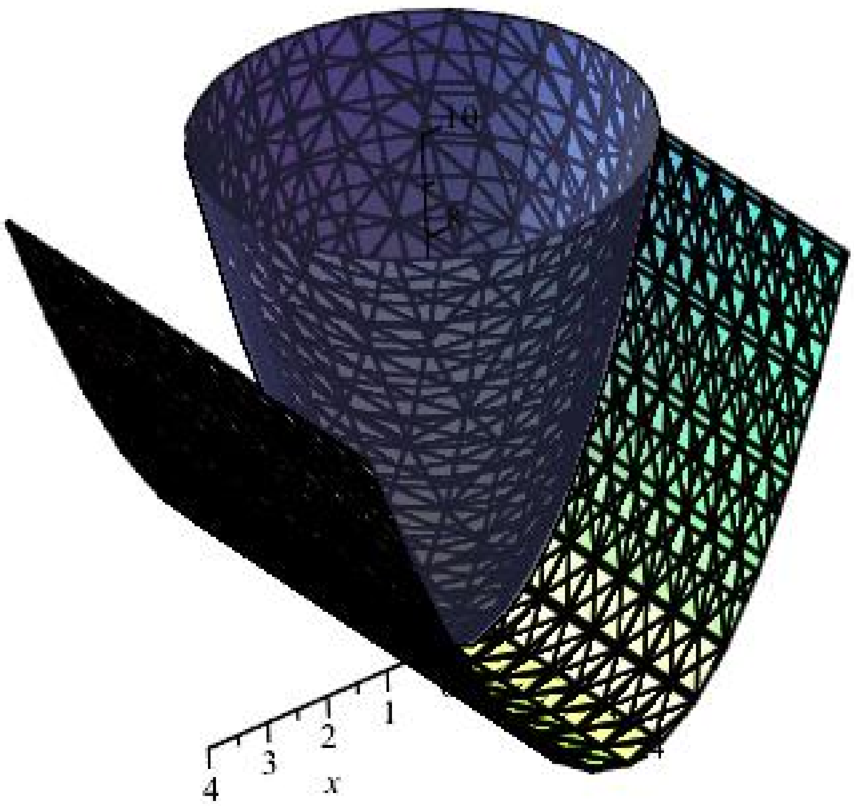
$$z \geq x^2 \Rightarrow 2x^2 \geq x^2$$

which is always true. Hence the projection is

$$z = 2x^2, \quad y = 0.$$

16. *(The intersection of)* $z = x^2 + y^2, \quad 2z = x^2.$

This is an intersection of an elliptic paraboloid and a parabolic cylinder. A picture looks like



For the projection in the xy plane, we need to find values of x, y which satisfy both equations. Setting the z values equal to each other yields

$$2x^2 + 2y^2 = x^2 \Rightarrow x^2 + 2y^2 = 0 \Rightarrow x = y = 0.$$

Hence the projection is the point

$$(0, 0, 0)$$

For the projection in the yz plane, we need to find values of y, z which satisfy both equations. Hence setting the x values equal to each other yields

$$z = 2z + y^2 \Rightarrow z = -2y^2.$$

However since $z \geq 0$ the only possible point is when $y = z = 0$. Hence the projection is the point

$$(0,0,0).$$

For the projection in the xz plane, we need to find values of x, z which satisfy both equations. Since the second equation only has x, z anything on the parabola $2z = x^2$ will satisfy both equations provided that they are part of the domain of the first function. Since $y^2 \geq 0$ we know that

$$z \geq x^2 \Rightarrow \frac{x^2}{2} \geq x^2 \Rightarrow \frac{x^2}{2} \leq 0$$

which can only happen is $x = 0$. Hence the projection is the point

$$(0,0,0).$$