Math 1210 Assignment #4 Nov 10, 2008

1a) $\left[\begin{array}{ccc|c} 1 & -1 & 1 & 1 \\ 4 & 2 & 1 & -2 \\ 9 & 3 & 1 & 1 \end{array}\right] \begin{array}{l} R_2 \to R_2 - 4R_1 \\ R_3 \to R_3 - 9R_1 \end{array} \left[\begin{array}{ccc|c} 1 & -1 & 1 & 1 \\ 0 & 6 & -3 & -6 \\ 0 & 12 & -8 & -8 \end{array}\right] R_2 \to \tfrac{1}{6} R_2$

$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 1 \\ 0 & 1 & -\frac{1}{2} & -1 \\ 0 & 12 & -8 & -8 \end{array}\right] R_3 \to R_3 - 12R_2 \quad \left[\begin{array}{ccc|c} 1 & -1 & 1 & 1 \\ 0 & 1 & -\frac{1}{2} & -1 \\ 0 & 0 & -2 & 4 \end{array}\right] R_3 \to -\tfrac{1}{2} R_3$

$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 1 \\ 0 & 1 & -\frac{1}{2} & -1 \\ 0 & 0 & 1 & -2 \end{array}\right]$

$x - x_2 + x_3 = 1 \qquad x_1 - (-2) + (-2) = 1$

$x_2 - \tfrac{1}{2} x_3 = -1 \qquad x_2 - \tfrac{1}{2}(-2) = -1$

$x_3 = -2$

soln

$\boxed{\begin{array}{l} x_1 = 1 \\ x_2 = -2 \\ x_3 = -2 \end{array}}$

b) $\left[\begin{array}{cccc|c} 3 & -1 & 1 & 2 & -2 \\ 1 & 2 & -1 & 1 & 1 \\ -1 & -3 & 2 & -4 & -6 \end{array}\right] R_1 \leftrightarrow R_2 \left[\begin{array}{cccc|c} 1 & 2 & -1 & 1 & 1 \\ 3 & -1 & 1 & 2 & -2 \\ -1 & -3 & 2 & -4 & -6 \end{array}\right] \begin{array}{l} R_2 \to R_2 - 3R_1 \\ R_3 \to R_3 + R_1 \end{array}$

$\left[\begin{array}{cccc|c} 1 & 2 & -1 & 1 & 1 \\ 0 & -7 & 4 & -1 & -5 \\ 0 & -1 & 1 & -3 & -5 \end{array}\right] R_2 \leftrightarrow R_3 \left[\begin{array}{cccc|c} 1 & 2 & -1 & 1 & 1 \\ 0 & -1 & 1 & -3 & -5 \\ 0 & -7 & 4 & -1 & -5 \end{array}\right] R_2 \to -R_2$

$\left[\begin{array}{cccc|c} 1 & 2 & -1 & 1 & 1 \\ 0 & 1 & -1 & 3 & 5 \\ 0 & -7 & 4 & -1 & -5 \end{array}\right] R_3 \to R_3 + 7R_2 \quad \left[\begin{array}{cccc|c} 1 & 2 & -1 & 1 & 1 \\ 0 & 1 & -1 & 3 & 5 \\ 0 & 0 & -3 & 20 & 30 \end{array}\right] R_3 \to -\tfrac{1}{3} R_3$

$\left[\begin{array}{cccc|c} 1 & 2 & -1 & 1 & 1 \\ 0 & 1 & -1 & 3 & 5 \\ 0 & 0 & 1 & -\frac{20}{3} & -10 \end{array}\right]$

$x_1 + 2x_2 - x_3 + x_4 = 1$

$x_2 - x_3 + 3x_4 = 5$

$x_3 - \tfrac{20}{3} x_4 = -10$

Let $x_4 = t$ $\quad x_3 = -10 + \frac{20}{3}t$

$$x_2 - (-10 + \tfrac{20}{3}t) + 3t = 5 \qquad x_2 = -5 + \tfrac{11}{3}t$$

$$x_1 + 2(-5 + \tfrac{11}{3}t) - (-10 + \tfrac{20}{3}t) + t = 1 \qquad x_1 = 1 - \tfrac{5}{3}t$$

Soln: $x_1 = 1 - \frac{5}{3}t$

$x_2 = -5 + 11/3\, t$

$x_3 = -10 + 20/3\, t$ $\quad t \in \mathbb{R}$

$x_4 = t$

c) $$\left[\begin{array}{ccc|c} 1 & 0 & 2 & 1 \\ 2 & 1 & 5 & 2 \\ 1 & -1 & 1 & 4 \end{array}\right] \begin{array}{l} R_2 \to R_2 - 2R_1 \\ R_3 \to R_3 - R_1 \end{array} \left[\begin{array}{ccc|c} 1 & 0 & 2 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & -1 & -1 & 3 \end{array}\right] R_3 \to R_3 + R_2$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 2 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 3 \end{array}\right] \leftarrow \text{contradiction}$$

This system has no solutions.

2 a) $$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 1 \\ 2 & 3 & 2 & 0 \\ 1 & 1 & 1 & 2 \end{array}\right] \begin{array}{l} R_2 \to R_2 - 2R_1 \\ R_3 \to R_3 - R_1 \end{array} \left[\begin{array}{ccc|c} 1 & 2 & 1 & 1 \\ 0 & -1 & 0 & -2 \\ 0 & -1 & 0 & 1 \end{array}\right] R_3 \to R_3 - R_2$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 1 \\ 0 & -1 & 0 & -2 \\ 0 & 0 & 0 & 3 \end{array}\right] \leftarrow \text{contradiction}$$

This system has no solutions.

b) $\begin{bmatrix} 3 & -3 & 1 & 3 & -3 \\ 1 & 1 & -1 & -2 & 3 \\ 4 & -2 & 0 & 1 & 0 \end{bmatrix} R_1 \leftrightarrow R_2 \begin{bmatrix} 1 & 1 & -1 & -2 & 3 \\ 3 & -3 & 1 & 3 & -3 \\ 4 & -2 & 0 & 1 & 0 \end{bmatrix} \begin{matrix} \\ R_2 \to R_2 - 3R_1 \\ R_3 \to R_3 - 4R_1 \end{matrix}$

$\begin{bmatrix} 1 & 1 & -1 & -2 & 3 \\ 0 & -6 & 4 & 9 & -12 \\ 0 & -6 & 4 & 9 & -12 \end{bmatrix} R_3 \to R_3 - R_2 \quad \begin{bmatrix} 1 & 1 & -1 & -2 & 3 \\ 0 & -6 & 4 & 9 & -12 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} R_2 \to \frac{-1}{6} R_2$

$\begin{bmatrix} 1 & 1 & -1 & -2 & 3 \\ 0 & 1 & -2/3 & -3/2 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} R_1 \to R_1 - R_2 \begin{bmatrix} 1 & 0 & -1/3 & -1/2 & 1 \\ 0 & 1 & -2/3 & -3/2 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

Soln: $x_1 = 1 + \frac{1}{3}s + \frac{1}{2}t$

$x_2 = 2 + \frac{2}{3}s + \frac{3}{2}t$

$x_3 = s \qquad\qquad s, t \in \mathbb{R}$

$x_4 = t$

c) $\begin{bmatrix} 4 & -3 & -4 & -2 \\ -4 & 2 & 1 & -4 \\ -1 & -3 & 1 & -4 \end{bmatrix} R_1 \leftrightarrow R_3 \begin{bmatrix} -1 & -3 & 1 & -4 \\ -4 & 2 & 1 & -4 \\ 4 & -3 & -4 & -2 \end{bmatrix} R_1 \to -R_1$

$\begin{bmatrix} 1 & 3 & -1 & 4 \\ -4 & 2 & 1 & -4 \\ 4 & -3 & -4 & -2 \end{bmatrix} \begin{matrix} \\ R_2 \to R_2 + 4R_1 \\ R_3 \to R_3 - 4R_1 \end{matrix} \begin{bmatrix} 1 & 3 & -1 & 4 \\ 0 & 14 & -3 & 12 \\ 0 & -15 & 0 & -18 \end{bmatrix} R_2 \to R_2 + R_3$

$\begin{bmatrix} 1 & 3 & -1 & 4 \\ 0 & -1 & -3 & -6 \\ 0 & -15 & 0 & -18 \end{bmatrix} R_2 \to -R_2 \begin{bmatrix} 1 & 3 & -1 & 4 \\ 0 & 1 & 3 & 6 \\ 0 & -15 & 0 & 18 \end{bmatrix} \begin{matrix} R_1 \to R_1 - 3R_2 \\ \\ R_3 \to R_3 + 15R_2 \end{matrix}$

$$\begin{bmatrix} 1 & 0 & -10 & -14 \\ 0 & 1 & 3 & 6 \\ 0 & 0 & 45 & 72 \end{bmatrix} R_3 \to \tfrac{1}{45} R_3 \quad \begin{bmatrix} 1 & 0 & -10 & -14 \\ 0 & 1 & 3 & 6 \\ 0 & 0 & 1 & 8/5 \end{bmatrix} \begin{matrix} R_1 \to R_1 + 10R_3 \\ R_2 \to R_2 - 3R_3 \end{matrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 6/5 \\ 0 & 0 & 1 & 8/5 \end{bmatrix}$$

soln: $x = 2$
$y = 6/5$
$z = 8/5$

3. $$\left[\begin{array}{ccc|c} 1 & 2 & -1 & k \\ 2 & 3 & -2 & 2 \\ -1 & -1 & 1 & 3 \end{array}\right] \begin{matrix} R_2 \to R_2 - 2R_1 \\ R_3 \to R_3 + R_1 \end{matrix} \quad \begin{bmatrix} 1 & 2 & -1 & k \\ 0 & -1 & 0 & -2k+2 \\ 0 & 1 & 0 & k+3 \end{bmatrix} R_3 \to R_3 + R_2$$

$$\left[\begin{array}{ccc|c} 1 & 2 & -1 & k \\ 0 & -1 & 0 & -2k+2 \\ 0 & 0 & 0 & -k+5 \end{array}\right]$$

This will have a solution if and only if
$-k+5=0$
$k=5$

So $k=5$ is the only value that gives a consistent system.