

## MATH 2132 Tutorial 7

In Problems 1–3, find a one-parameter family of solutions of the differential equation. Find any singular solutions.

1.  $(y - 1)\frac{dy}{dx} = yx^2$
  2.  $\frac{y - 1}{y}\frac{dy}{dx} = x^2$
  3.  $x^2\frac{dy}{dx} = y^2 - 1$
4. Find an explicit solution of the initial-value problem

$$\frac{dy}{dx} = \frac{x^4}{y + 1}, \quad y(1) = 2.$$

What is the largest possible domain of the solution?

5. Two substances A and B react to form a third substance C in such a way that 2 grams of A react with 1 gram of B to produce 3 grams of C. The rate at which C is formed is proportional to the product of the amounts of A and B present in the mixture. Find the amount of C present in the mixture as a function of time when the original amounts of A and B brought together at time  $t = 0$  are 20 grams and 5 grams, respectively.
6. Find a general solution of the differential equation

$$x\frac{dy}{dx} = 3y + x^5\sqrt{1 + x^2}.$$

7. Find a one-parameter family of solutions of the differential equation

$$\frac{dy}{dx} = \sin^3 x - y \sin x.$$

Are there any singular solutions?

8. When a substance such as glucose is administered intravenously into the bloodstream, it is used up by the body at a rate proportional to the amount present at that time. If it is added at a variable rate  $R(t)$  units per unit time, and  $A_0$  is the amount present when the intravenous feeding begins, find a formula for the amount present at any time. Simplify your solution when  $R(t)$  is a constant value  $R$ .
9. A tank originally contains 1000 litres of water in which has been dissolved 10 kilograms of sugar. A mixture containing 2 kilograms of sugar per 100 litres of water is added to the tank at 15 millilitres per minute. At the same time, 20 millilitres of well-stirred mixture is removed from the tank each minute. Find the amount of sugar in the tank as a function of time  $t$ . For how long is your solution valid?

**Answers:** 1.  $y - \ln|y| = x^3/3 + C$ ,  $y = 0$     2.  $y - \ln|y| = x^3/3 + C$

3.  $y = (1 + Ce^{-2/x})/(1 - Ce^{-2/x})$ ,  $y = -1$     4.  $y = (-5 + \sqrt{10x^5 + 215})/5$ ,  $x > -(41/2)^{1/5}$

5.  $30(1 - e^{-10kt/3})/(2 - e^{-10kt/3})$  g    6.  $y = Cx^3 + (x^3/3)(1 + x^2)^{3/2}$

7.  $y = -(1 + \cos x)^2 + Ce^{\cos x}$

8.  $A = A_0e^{-kt} + \int_0^t R(u)e^{k(u-t)}du$ ,  $A = A_0e^{-kt} + \frac{R}{k}(1 - e^{-kt})$

9.  $(10^6 - 5t)/50 - 10^{-20}(10^6 - 5t)^4$  g,  $t \leq 200,000$  m