

MATH 2130 Problem Workshop 4

1. Find all unit vectors tangent to the curve $x^2 + z^2 = 4, x + y = 1$ at the point $(\sqrt{2}, 1 - \sqrt{2}, \sqrt{2})$.
2. Find all unit vector tangent to the curve $x = t^2, y = 2t^3, z = 3t^2$ at the origin.
3. Find the angle between the tangent vectors to the curves

$$x^2 + y = z + 4, x + 2y = 5 \quad \text{and} \quad x + y^2 = 5, 2x + 3y + 4z = 4$$

at the point of intersection between the curves.

4. Find the length of the curve $x = t + 1, y = 2t^{3/2} - 3, z = 4t - 2$ between the points $(2, -1, 2)$ and $(1, -3, -2)$
5. Show that it is impossible for the length of a curve joining the points $(1, -2, 3)$ and $(0, 4, 10)$ to be equal to 9.
6. Set up but do not evaluate a definite integral to find the length of the curve $x^2 + y^2 = z^2 - 4, x + y = 4$ joining the points $(4, 0, 2\sqrt{5})$ and $(2, 2, 2\sqrt{3})$. Simplify the integrand as much as possible.

Answers:

1. $\pm \frac{1}{\sqrt{3}}(-\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}})$
2. $\pm \frac{1}{\sqrt{10}}(\hat{\mathbf{i}} + 3\hat{\mathbf{k}})$
3. $\arccos\left(\frac{21}{\sqrt{14}\sqrt{297}}\right)$.
4. $\frac{2}{27}(26^{3/2} - 17^{3/2})$.
6. $2 \int_2^4 \sqrt{\frac{t^2 - 4t + 7}{t^2 - 4t + 10}} dt$