

Term Test 2

Nov 7, 2012

SOLUTIONS

QUESTION 1

Distributed loads are applied to the shape shown in Figure 1.

In addition to these distributed loads, a 25 kN force and a 75 kN.m clockwise couple moment are applied.

Replace the system of forces and a couple acting on the shape with a single force and state where this force intersects the x and y axes.

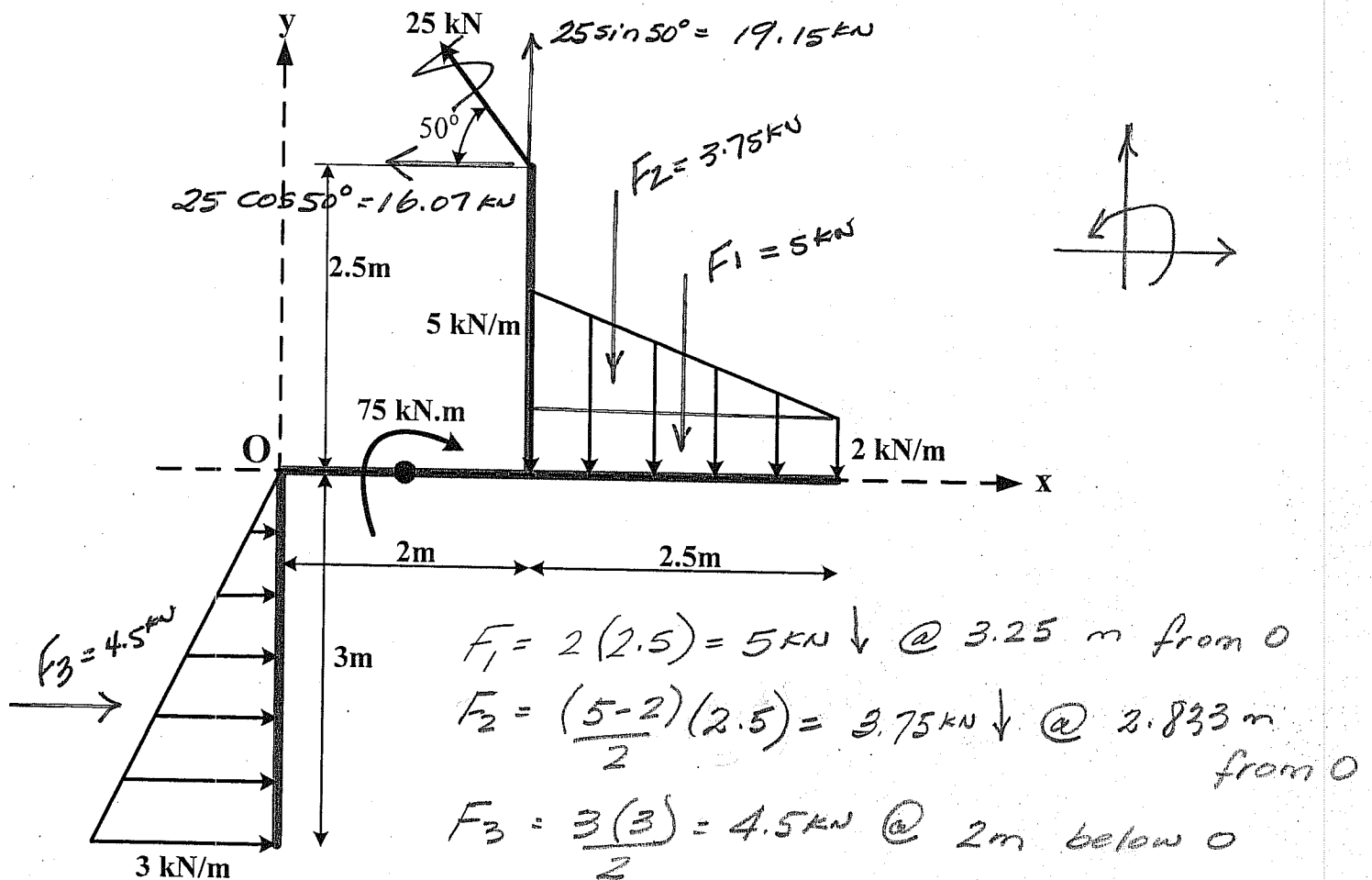


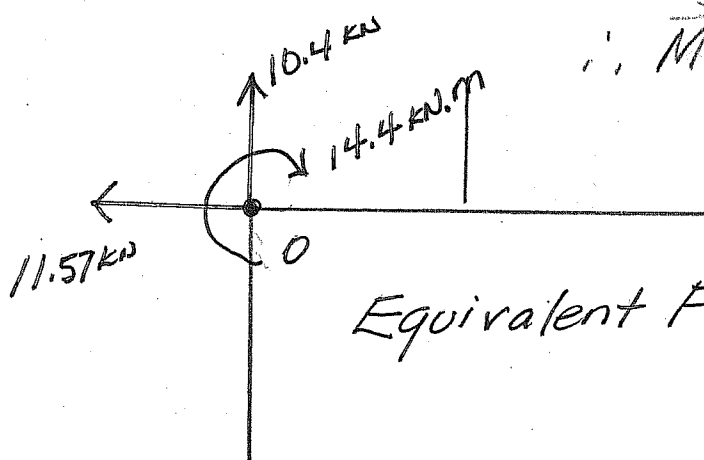
Figure 1

$$R_x = \sum F_x = 4.5 - 16.07 = -11.57 \text{ kN} \therefore \vec{R}_x = 11.57 \text{ kN} \leftarrow$$

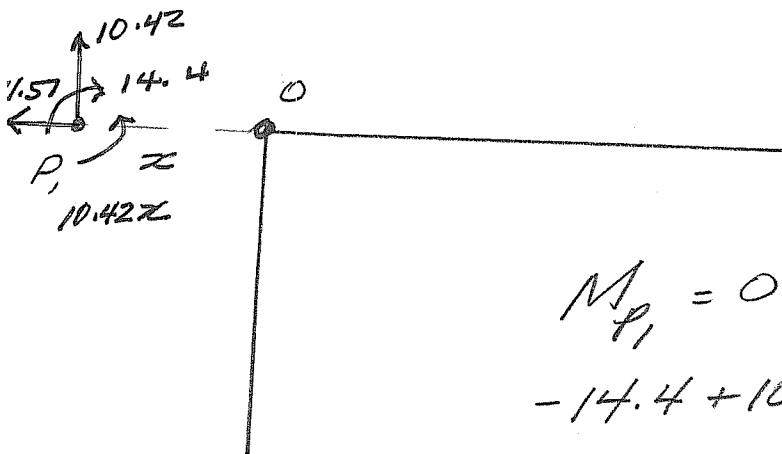
$$R_y = \sum F_y = 19.15 - 3.75 - 5 = +10.4 \text{ kN} \therefore \vec{R}_y = 10.4 \text{ kN} \uparrow$$

$$M_{R_0} = \sum M_0 = 4.5(2) - 75 + 16.07(2.5) + 19.15(2) - 3.75(2.833) - 5(3.25) = -14.4 \text{ kN.m}$$

$$\therefore \vec{M}_{R_0} = 14.4 \text{ kN.m} (\curvearrowright)$$



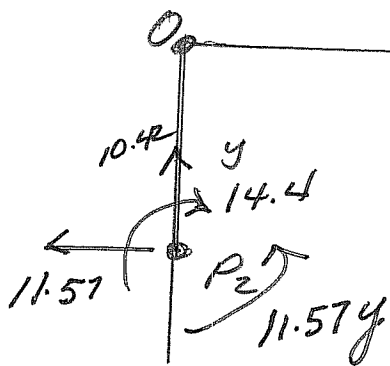
Equivalent Force-Couple at O



$$M_{P_1} = 0$$

$$-14.4 + 10.42x = 0$$

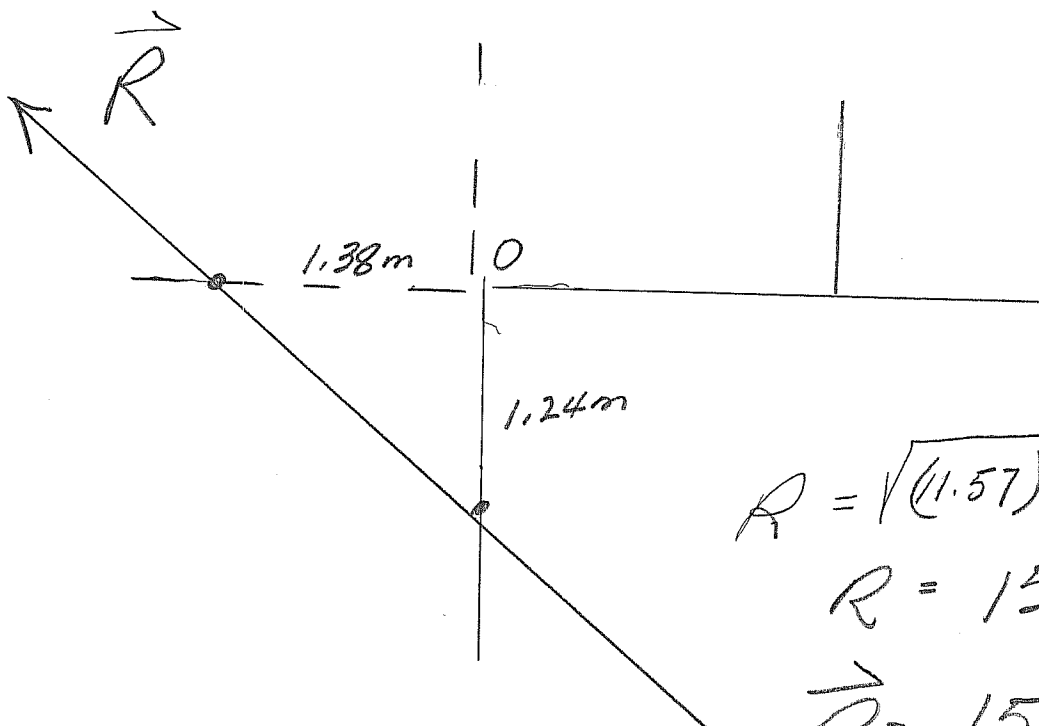
$$x = \frac{14.4}{10.42} = 1.38 \text{ m left of } O$$



$$M_{P_2} = 0$$

$$-14.4 + 11.57y = 0$$

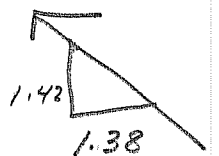
$$y = \frac{14.4}{11.57} = 1.24 \text{ m below } O$$



$$R = \sqrt{(11.57)^2 + (10.42)^2}$$

$$R = 15.57 \text{ kN}$$

$$\vec{R} = 15.57 \text{ kN}$$



$$\frac{10.4}{11.57} = 0.899$$

$$\frac{1.24}{1.38} = 0.899$$

QUESTION 2

The 12 m long beam JKL has a cable attached to the center of the beam at K . The right half of the beam carries a distributed load that goes from 0 kN/m at K to 4 kN/m at end L . The left half of the beam carries a distributed load that varies from 0 kN/m at K to w kN/m at end J . Forces of 2 kN, 4 kN and 3 kN are applied to the truss at joints A , C and E respectively.

To keep the beam in the horizontal equilibrium position shown in the Figure 2, a 48 kN.m clockwise couple moment is applied to the beam. A cable then attaches the beam to a ring at I .

Two cables each at 45° then attach the ring to the truss at joints H and F .

Determine:

- The magnitude of the distributed load, w , applied to the left half of the beam,
- The force in each member of the truss and state whether it is in tension, compression or zero.

PLACE YOUR RESULTS FOR THE TRUSS MEMBER FORCES ON THE FIGURE PROVIDED NEXT PAGE.

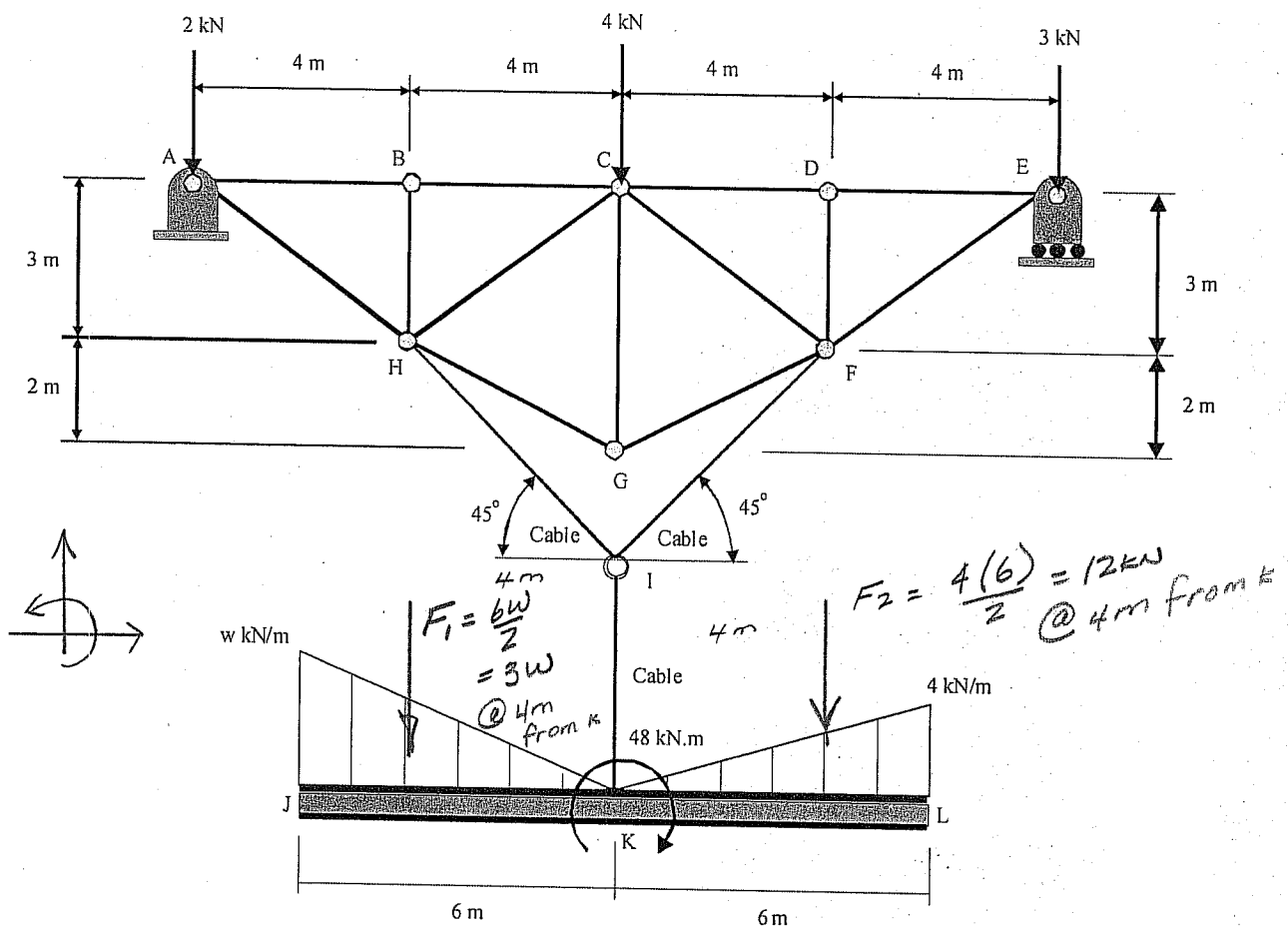
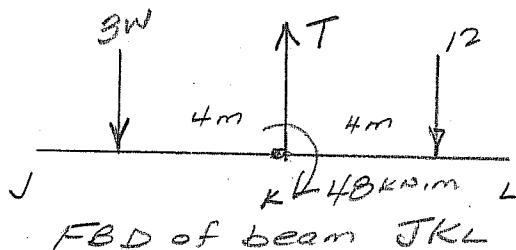


Figure 2



$$\sum F_y = 0 \quad -3w + T - 12 = 0$$

$$T = 12 + 3w$$

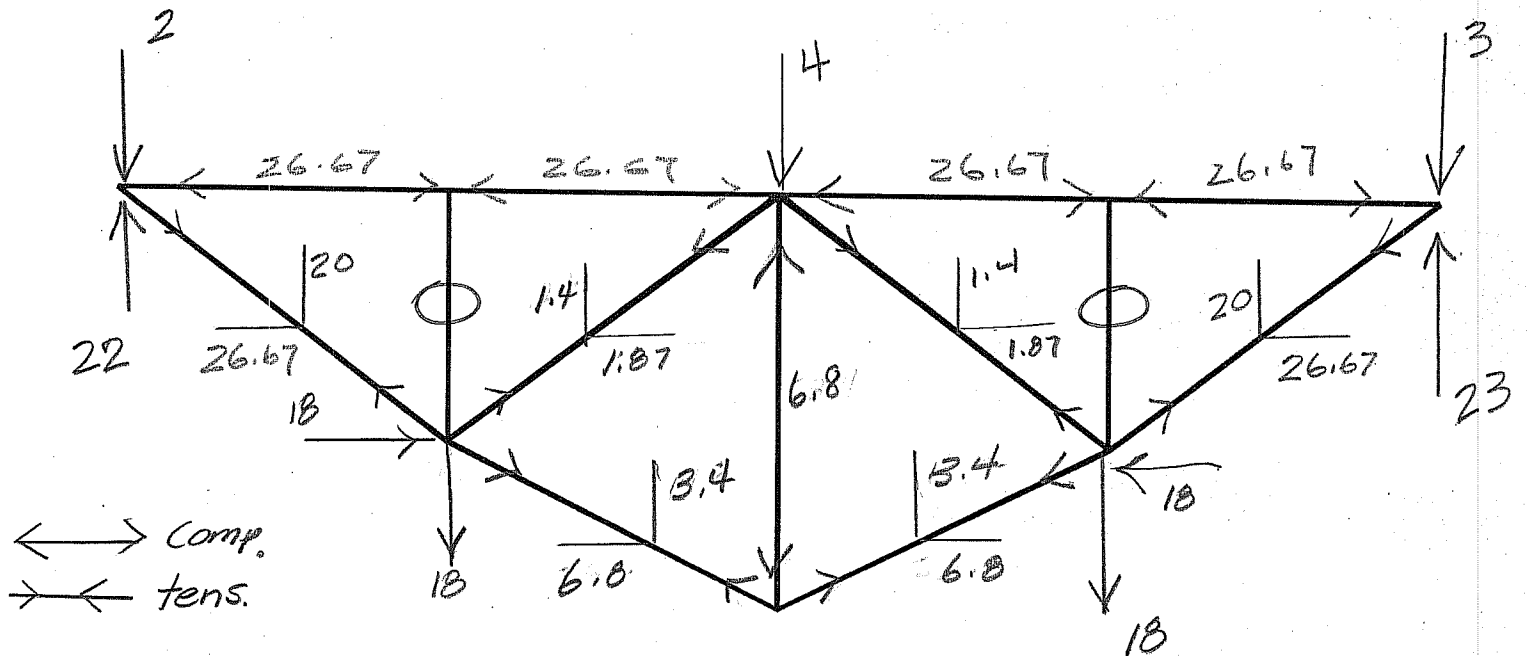
$$\sum M_K = 0$$

$$3w(4) - 48 - 12(4) = 0$$

$$w = \frac{96}{12} = 8 \text{ kN/m}$$

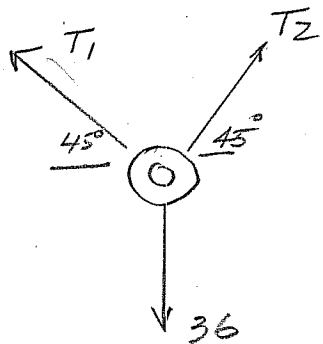
$$\therefore F_1 = 3(8) = 24 \text{ kN}$$

$$T = 36 \text{ kN} \uparrow \text{ on the beam}$$



$$\frac{20}{3} = \frac{x}{4} \quad x =$$

FBD of Ring at I



$$\sum F_x = 0$$

$$-T_1 \cos 45^\circ + T_2 \cos 45^\circ = 0$$

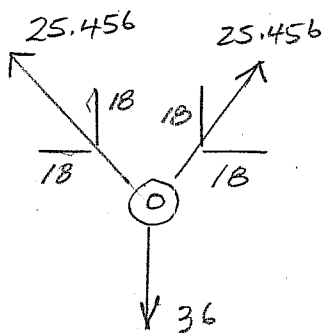
$$\therefore T_1 = T_2$$

$$\sum F_y = 0$$

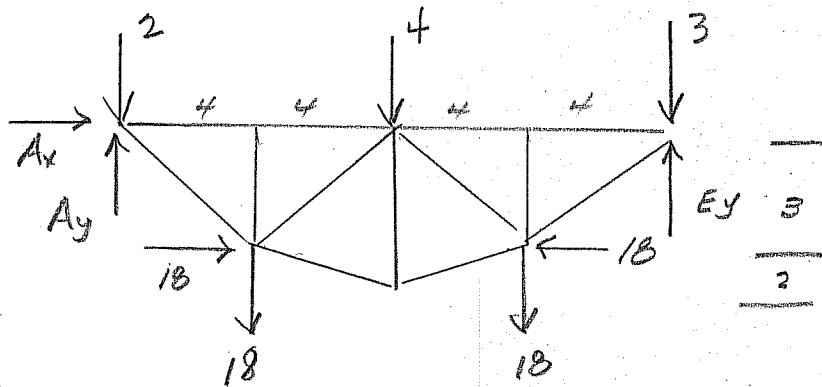
$$T_1 \sin 45^\circ + T_2 \sin 45^\circ - 36 = 0$$

$$2T_1 \sin 45^\circ = 36$$

$$T_1 = T_2 = \frac{36}{2 \sin 45^\circ} = 25.456 \text{ kN}$$



FBD of TRUSS



$$\sum F_x = 0$$

$$A_x + 18 - 18 = 0 \quad A_x = 0$$

$$\sum M_A = 0 \quad 18(3) - 18(4) - 4(8) - 3(16) - 18(3) - 18(12) + E_y(16) = 0$$

$$E_y = +23 \text{ kN} \quad \therefore \vec{E}_y = 23 \text{ kN} \uparrow$$

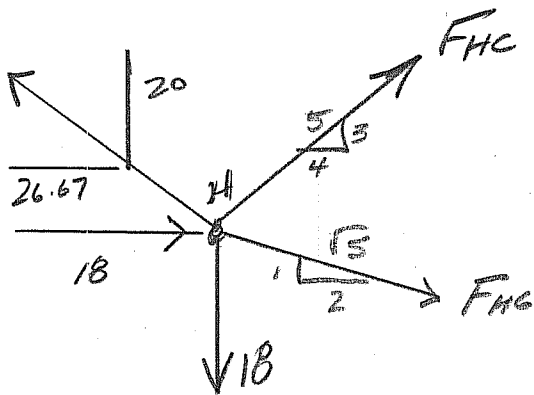
$$\sum F_y = 0$$

$$A_y - 2 - 4 - 3 - 18 - 18 + 23 = 0$$

$$A_y = +22 \text{ kN}$$

$$\vec{A}_y = 22 \text{ kN} \uparrow$$

FBD JOINT H



$$\sum F_x = 0 \quad -26.67 + 18 + \frac{4}{5} F_{HC} + \frac{2}{\sqrt{5}} F_{HG} = 0$$

$$\frac{4}{5} F_{HC} + \frac{2}{\sqrt{5}} F_{HG} = +8.67 \quad (1)$$

$$\sum F_y = 0 \quad 20 - 18 + \frac{3}{5} F_{HC} - \frac{1}{\sqrt{5}} F_{HG} = 0$$

$$\frac{3}{5} F_{HC} - \frac{1}{\sqrt{5}} F_{HG} = -2 \quad (2)$$

$$\frac{4}{5} F_{HC} + \frac{2}{\sqrt{5}} F_{HG} = 8.67$$

$$\frac{3}{5} F_{HC} - \frac{1}{\sqrt{5}} F_{HG} = -2$$

$$\frac{2 F_{HC} = 4.67}{\therefore F_{HC} = +2.34 \text{ kN}}$$

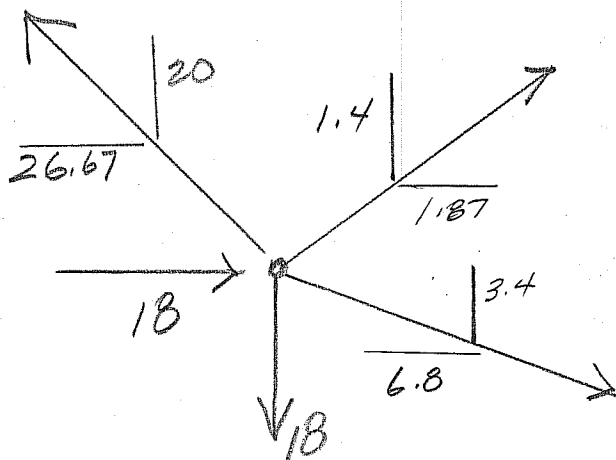
$$\therefore \vec{F}_{HC} = 2.34 \text{ kN} \nearrow_{43}$$

$$\frac{4}{5} (2.34) + \frac{2}{\sqrt{5}} F_{HG} = 8.67$$

$$\frac{2}{\sqrt{5}} F_{HG} = 6.8$$

$$F_{HG} = +7.6$$

$$\vec{F}_{HG} = 7.6 \text{ kN} \searrow_{12}$$



$$\sum F_x = 0$$

$$-26.67 + 18 + 1.87 + 6.8 = 0$$

$$0 = 0$$

$$\sum F_y = 0$$

$$20 - 18 + 1.4 - 3.4 = 0$$

$$0 = 0$$

Determine the forces acting on each member including the pulleys of the frame shown in Figure 3.

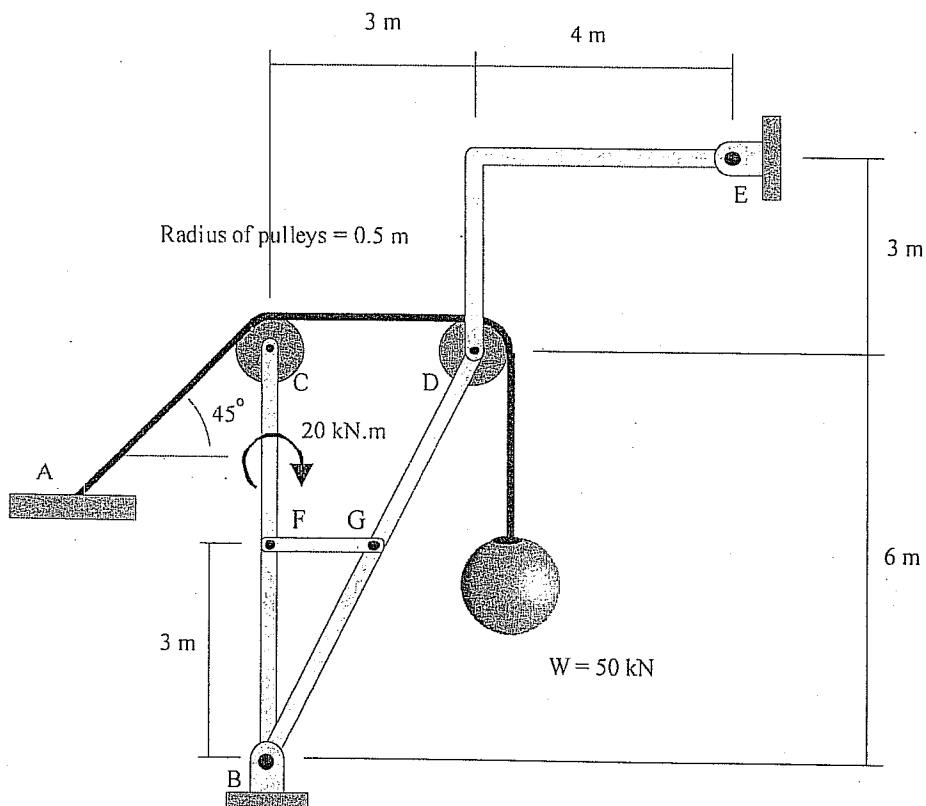
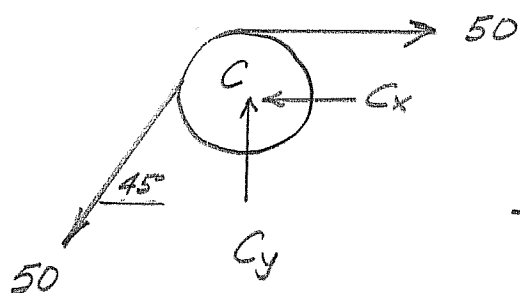
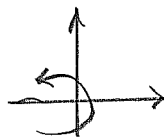


Figure 3

Pulleys:



$$\sum F_x = 0$$

$$-50 \cos 45^\circ - C_x + 50 = 0$$

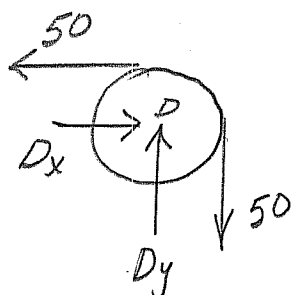
$$C_x = +14.64 \text{ kN}$$

$$\therefore \vec{C}_x = 14.64 \text{ kN} \leftarrow \text{on the pulley}$$

$$\sum F_y = 0$$

$$-50 \sin 45^\circ + C_y = 0 \quad C_y = +35.36 \text{ kN}$$

$$\therefore \vec{C}_y = 35.36 \text{ kN} \uparrow \text{ on the pulley}$$



$$\sum F_x = 0$$

$$-50 + D_x = 0$$

$$D_x = +50 \text{ kN}$$

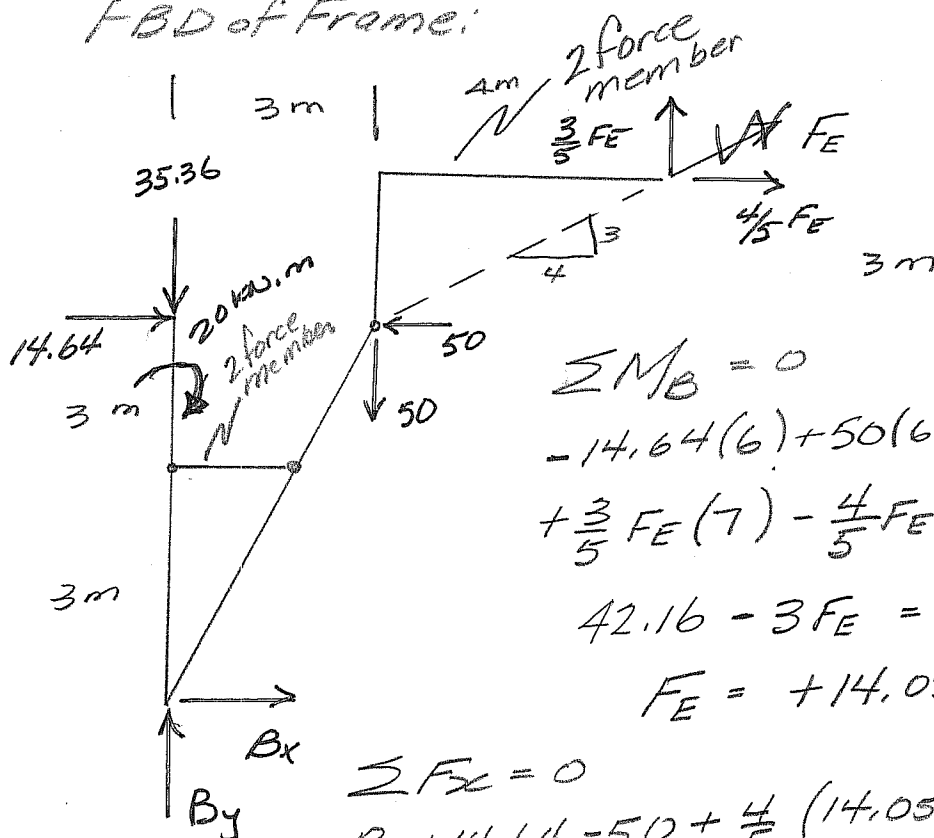
$$\therefore \vec{D}_x = 50 \text{ kN} \rightarrow \text{on the pulley}$$

$$\sum F_y = 0$$

$$D_y - 50 = 0$$

$$D_y = +50 \text{ kN} \therefore \vec{D}_y = 50 \text{ kN} \uparrow \text{ on the pulley}$$

FBD of Frame:



$$\sum M_B = 0$$

$$-14.64(6) + 50(6) - 50(3) - 20$$

$$+ \frac{3}{5} F_E (7) - \frac{4}{5} F_E (9) = 0$$

$$42.16 - 3 F_E = 0$$

$$F_E = +14.05 \text{ kN} \therefore \vec{F}_E = 14.05 \text{ kN} \nearrow$$

$$\sum F_x = 0$$

$$B_x + 14.64 - 50 + \frac{4}{5} (14.05) = 0$$

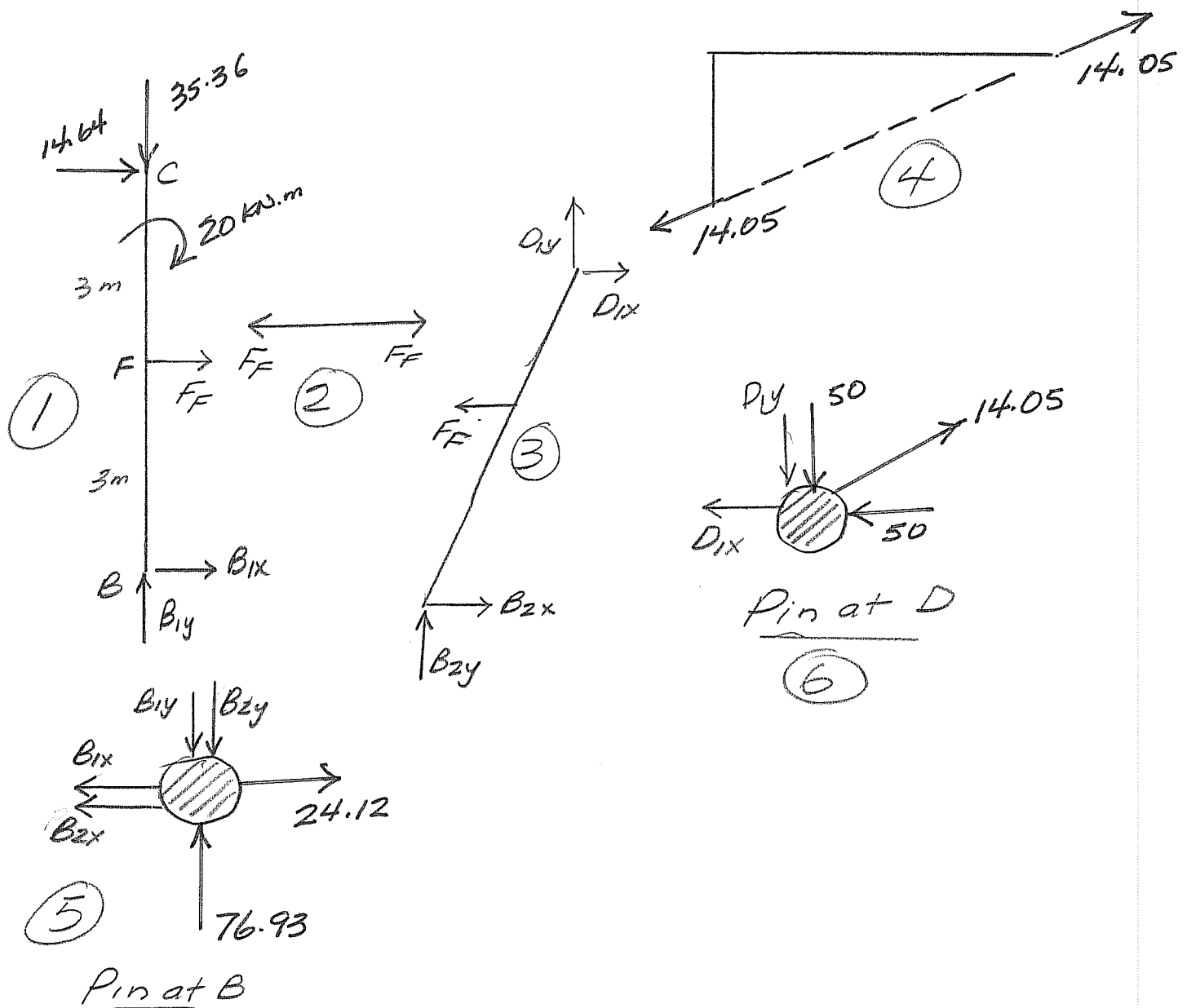
$$B_x = +24.12 \text{ kN} \therefore \vec{B}_x = 24.12 \text{ kN} \rightarrow$$

$$\sum F_y = 0 \quad B_y - 35.36 - 50 + \frac{3}{5} (14.05) = 0$$

$$B_y = +76.93 \text{ kN}$$

$$\vec{B}_y = 76.93 \text{ kN} \uparrow$$

Substructures:



From ① $\sum M_B = 0$
 $-14.64(6) - 20 - F_F(3) = 0$

$$F_F = -35.95 \text{ kN}$$

$$\therefore \vec{F}_F = 35.95 \text{ kN} \leftarrow \text{on BFC}$$

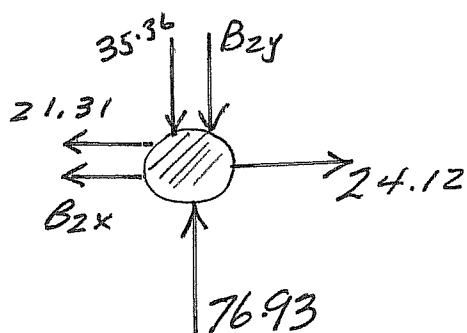
$$\sum F_x = 0 \quad + 14.64 + (-35.95) + B_x = 0$$

$$B_{ix} = +21.31 \text{ kN} \quad \therefore \vec{B}_{ix} = 21.31 \text{ kN} \rightarrow \text{on BFC}$$

$$\sum F_y = 0 \quad -35.36 + B_y = 0$$

$$B_{1y} = +35.36 \text{ kN} \quad \therefore \vec{B}_{1y} = 35.36 \text{ kN} \uparrow \text{ on BFC}$$

Re-draw (5)



$$\sum F_x = 0$$

$$-21.31 - B_{2x} + 24.12 = 0$$

$$B_{2x} = +2.81 \text{ kN}$$

$$\therefore \vec{B}_{2x} = 2.81 \text{ kN} \leftarrow \text{on pin at B}$$

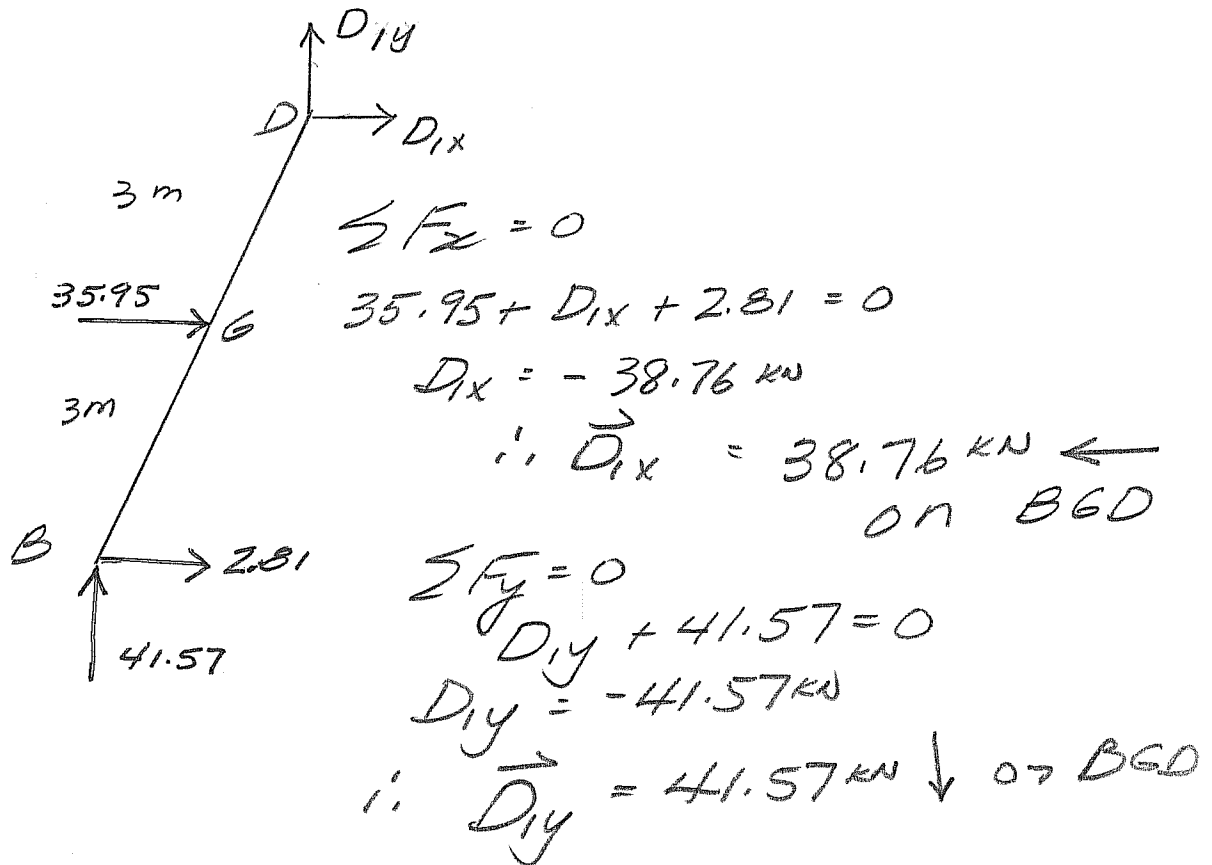
$$\sum F_y = 0$$

$$-35.36 - B_{zy} + 76.93 = 0$$

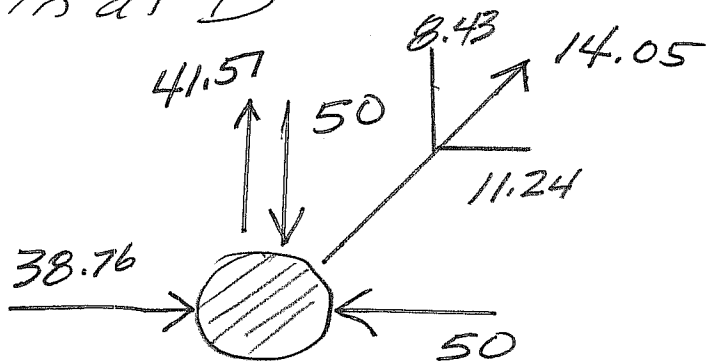
$$B_{2y} = +41.57 \text{ kN}$$

$\therefore B_{2y} = +41.57 \text{ kN}$
 $\therefore B_{2y} = 41.57 \text{ kN} \downarrow$ on pin at B

Re-draw (3)



Pin at D



$$\sum F_x = 0$$

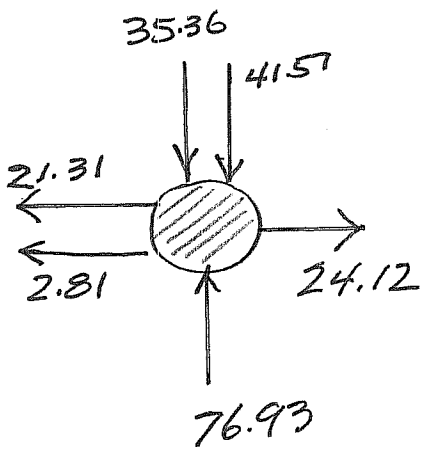
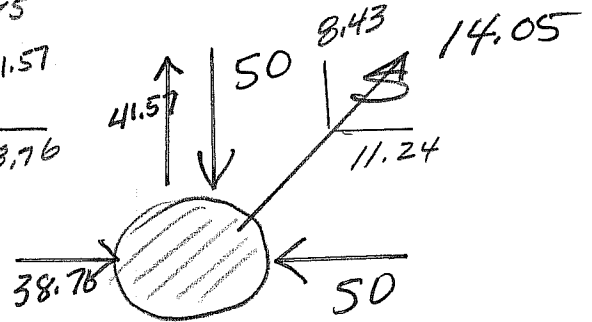
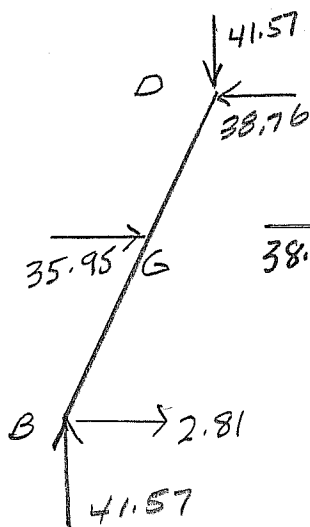
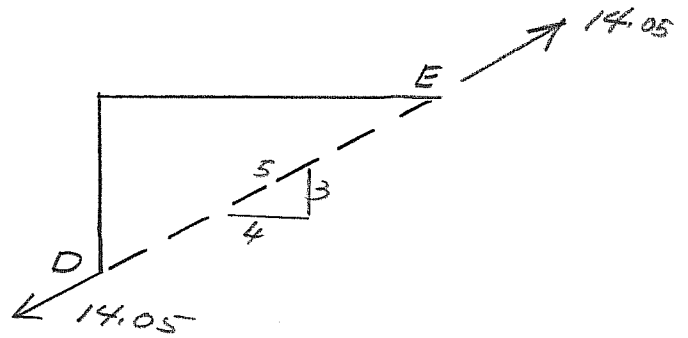
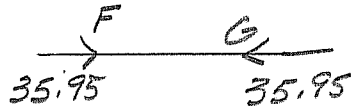
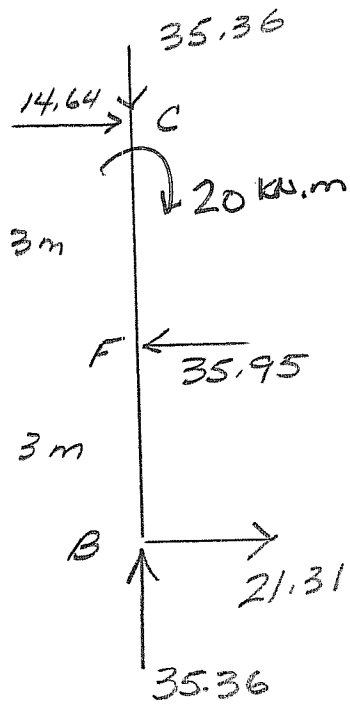
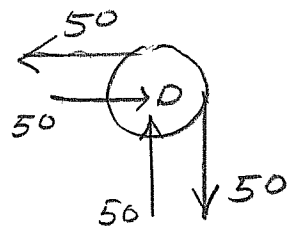
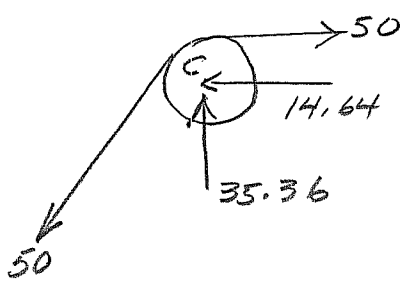
$$38.76 + 11.24 - 50 = 0$$

$$0 = 0 \checkmark$$

$$\sum F_y = 0$$

$$41.57 + 8.43 - 50 = 0$$

$$0 = 0 \checkmark$$



Summary