THE UNIVERSITY OF MANITOBA

Faculty of Engineering

1440 Introduction to Statics

October 10, 2012 Term Test # 1
Time: 6:00 pm – 7:30 pm

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NAME: ______________________________

STUDENT NMBER: ______________

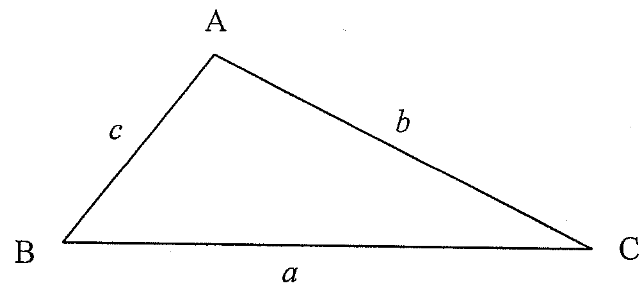
There are three (3) questions.
All questions are of equal value (10 marks each).
CLOSED BOOK: textbooks, notes, problems, etc., are not permitted.
Calculators are permitted.

Wherever necessary a FBD must be drawn.

STRAIGHT EDGE IS REQUIRED.
UNDERLINE YOUR ANSWERS.

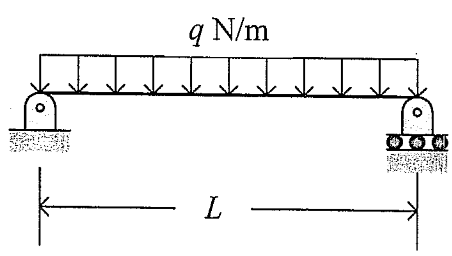
$g = 9.8\ \text{m/sec}^2$

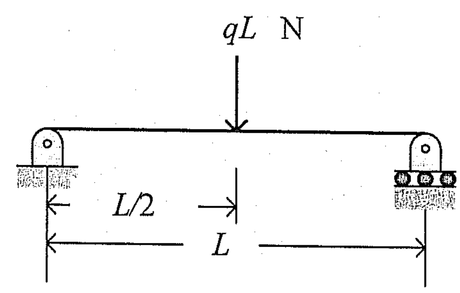
Cosine and Sine Laws:

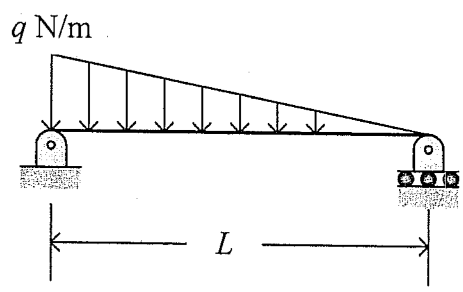


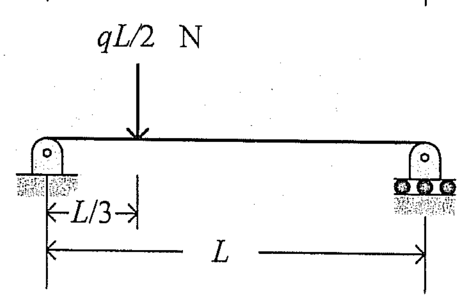
$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$b^2 = a^2 + c^2 - 2ac\cos B$$
$$c^2 = a^2 + b^2 - 2ab\cos C$$
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Resultant forces of distributed loads:









TERM TEST #1

SOLUTIONS

QUESTION 1

A disabled snowmobile is towed by two other snowmobiles by using two cables attached to the disabled snowmobile at A as shown in Figure 1.

The tension force in the cable AB is $(20\boldsymbol{i} + 40\boldsymbol{j})$ kN, and the tension force in cable AC is 50 kN. Cable AC makes an angle of $\alpha = 30^\circ$ with the horizontal x axis. Calculate the magnitude and the direction of the resultant force, $\boldsymbol{R}$ on the disabled snowmobile by:

(a) Using a graphical solution (State the scale you are using),
(b) Using a trig solution (sine and/or cosine rule),
(c) Using a rectangular component solution.

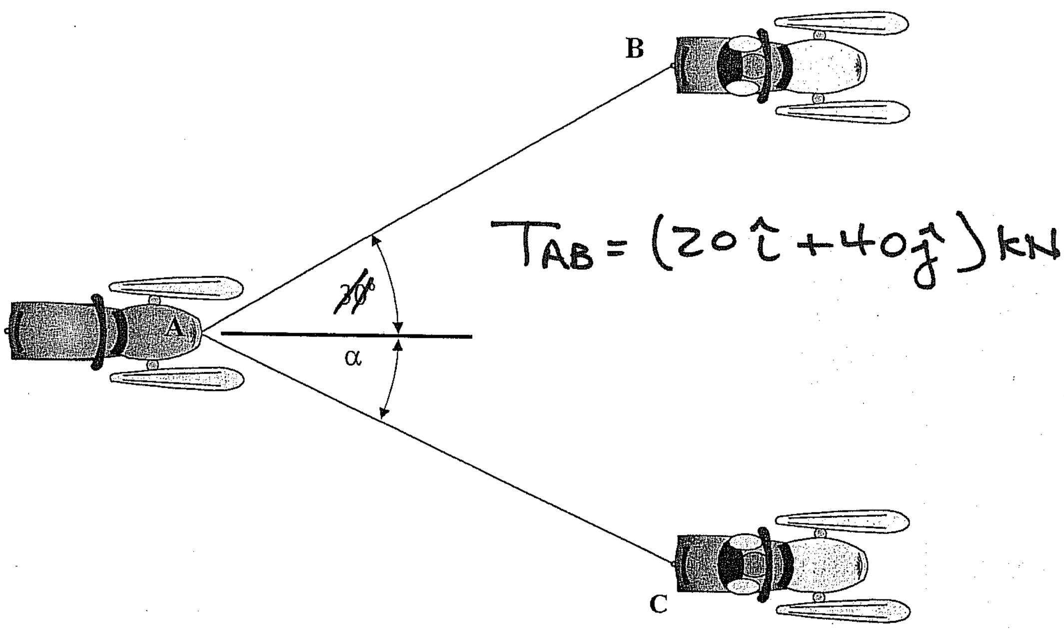
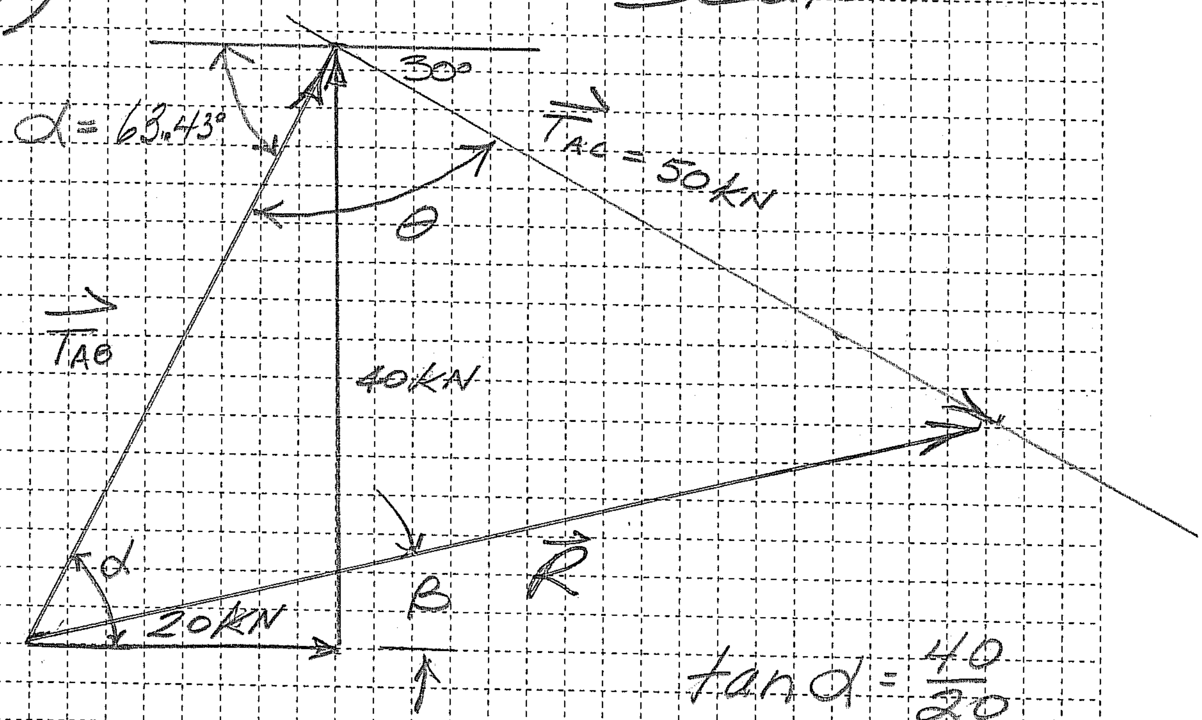


Figure 1

a) Scale 1:50



$\tan\alpha = \frac{40}{20}$

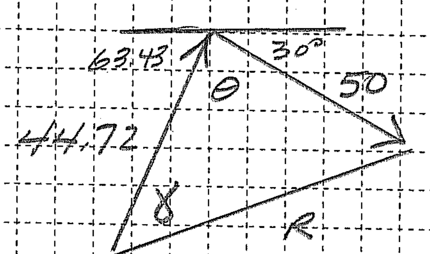
$\alpha = 63.43°$

GRAPHICAL $R \simeq 65.5kN$

$\beta \simeq 13.5°$

$\therefore \vec{R} = 65.5kN \ \angle 13.5°$

b) TRIG SOLN: using



$T_{AB} = \sqrt{20^2 + 40^2}$

$= 44.72 \ kN$

$\theta = 180° - 30° - 63.43° = 86.57°$

Cosine rule:

$$R^2 = 44.72^2 + 50^2 - 2(44.72)(50)\cos 86.57°$$

$$R = 65.05 \ kN$$

Sine rule:

$$\frac{65.05}{\sin 86.57} = \frac{50}{\sin\gamma} \qquad \gamma = 50.10°$$

$$\therefore \beta = 63.43 - 50.10 = 13.32°$$

$$\vec{R} = 65.05 \ kN \ \angle 13.32°$$

c) Rectangular Components

$$R_x = \Sigma F_x = 44.72\cos 63.43° + 50\cos 30° = +63.30 \ kN$$

$$\vec{R}_x = 63.30 \ kN \rightarrow$$

$$R_y = \Sigma F_y = 44.72\sin 63.43 - 50\sin 30° = 15 \ kN$$

$$R = \sqrt{63.30^2 + 15^2} = 65.05 \ kN$$

$$\tan\beta = \frac{15}{63.3}$$

$$\beta = 13.33^\circ$$

$$\vec{R} = 65.05^{kN} \angle 13.33^\circ$$

QUESTION 2

A weight, $W = 60\ N$ is held in the equilibrium position shown in Figure 2 by a system of cables (Cable BE is horizontal.).

Determine the tension in cables AC, AB, BD and BE when $W = 60\ N$.

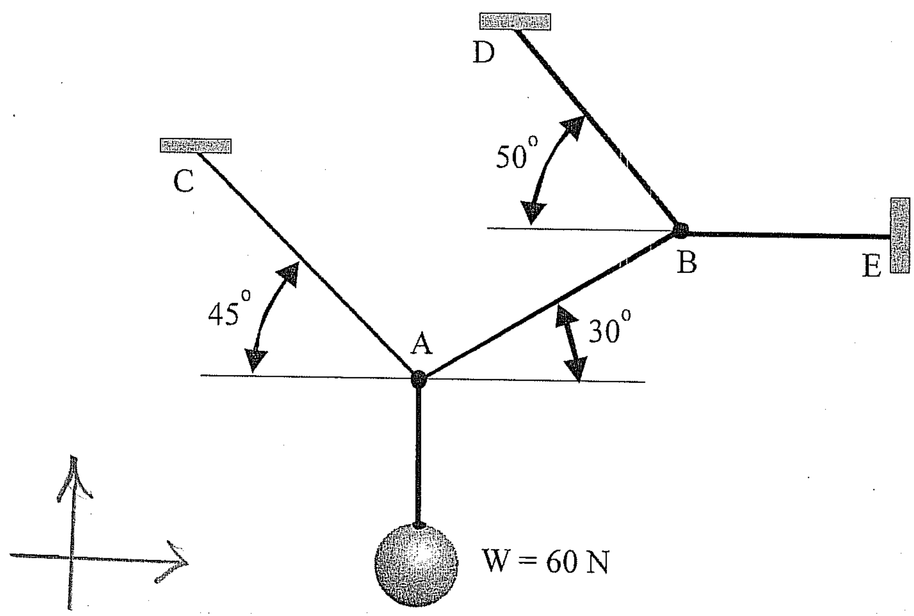
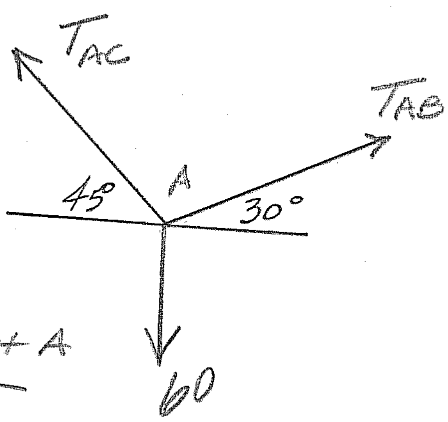
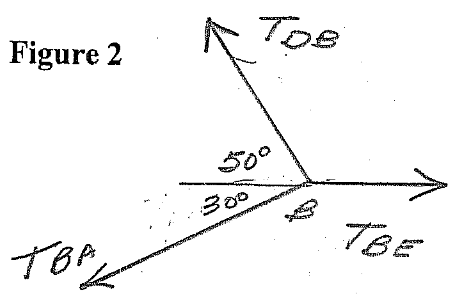


Figure 2



FBD at A



FBD AT B

For A: $\sum F_x = 0$

$-T_{AC}\cos 45° + T_{AB}\cos 30° = 0$ (1)

$-0.707\,T_{AC} + 0.866\,T_{AB} = 0$

$T_{AB} = \dfrac{0.707\,T_{AC}}{0.866} = 0.8164\,T_{AC}$

$\sum F_y = 0$

$T_{AC}\sin 45° + T_{AB}\sin 30° - 60 = 0$ (2)

$0.707\,T_{AC} + 0.5\,T_{AB} = 60$

$0.707\,T_{AC} + 0.5(0.8164\,T_{AC}) = 60$

$1.1152\,T_{AC} = 60$ $\therefore$ $\vec{T}_{AC} = 53.8^{N}$ 45°

$\therefore$ $T_{AB} = 0.8164(53.8) = 43.92^{N}$

$\vec{T}_{AB} = 43.92^{N}$ 30°

FROM FBD B

$\Sigma F_x = 0$

$-43.92\cos 30° - T_{BD}\cos 50° + T_{BE} = 0 \quad (1)$

$-0.6428T_{BD} + T_{BE} = 38.04$

$\Sigma F_y = 0$

$-43.92\sin 30° + T_{BD}\sin 50° = 0 \quad (2)$

$T_{BD} = \dfrac{21.96}{0.766} = 28.67^N$

$\therefore \vec{T_{BD}} = 28.67^N$ 50°

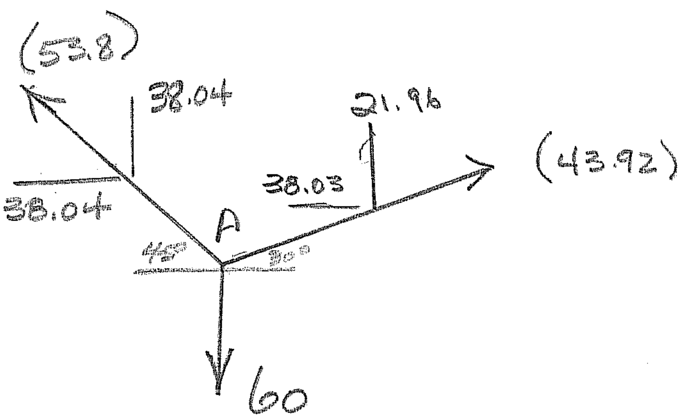
From (1)

$-0.6428(28.67) + T_{BE} = 38.04$

$\therefore T_{BE} = 56.47^N$

$\therefore \vec{T_{BE}} = 56.47^N \rightarrow$

Check



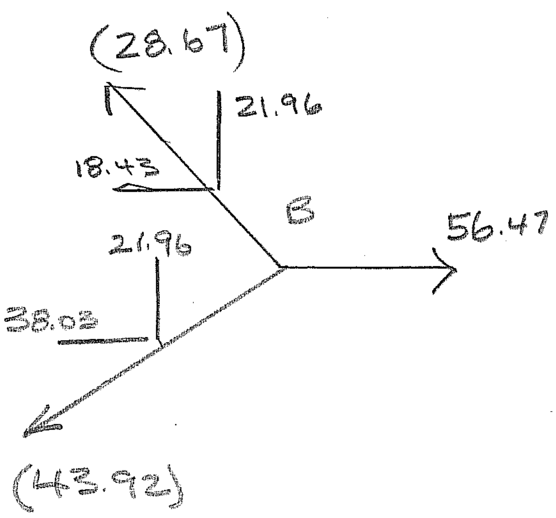
$\Sigma F_x = -38.04 + 38.03 = -0.01$

OK

$\Sigma F_y = 0$

$38.04 + 21.96 - 60 = 0$

$0 = 0$ ✓



$\Sigma F_x =$

$-38.03 - 18.43 + 56.47 = +0.01$

OK

$\Sigma F_y =$

$21.96 - 21.96 = 0$ ✓

Two distributed loads and a clockwise couple-moment of *150 N.m* are applied on the frame ***ABC*** as shown in Figure 3. Determine:

(a) The equivalent force-couple at point ***B***, and
(b) The magnitude and direction of the minimum force applied at point ***A*** that will produce the same moment about point ***B*** as in Part (a).

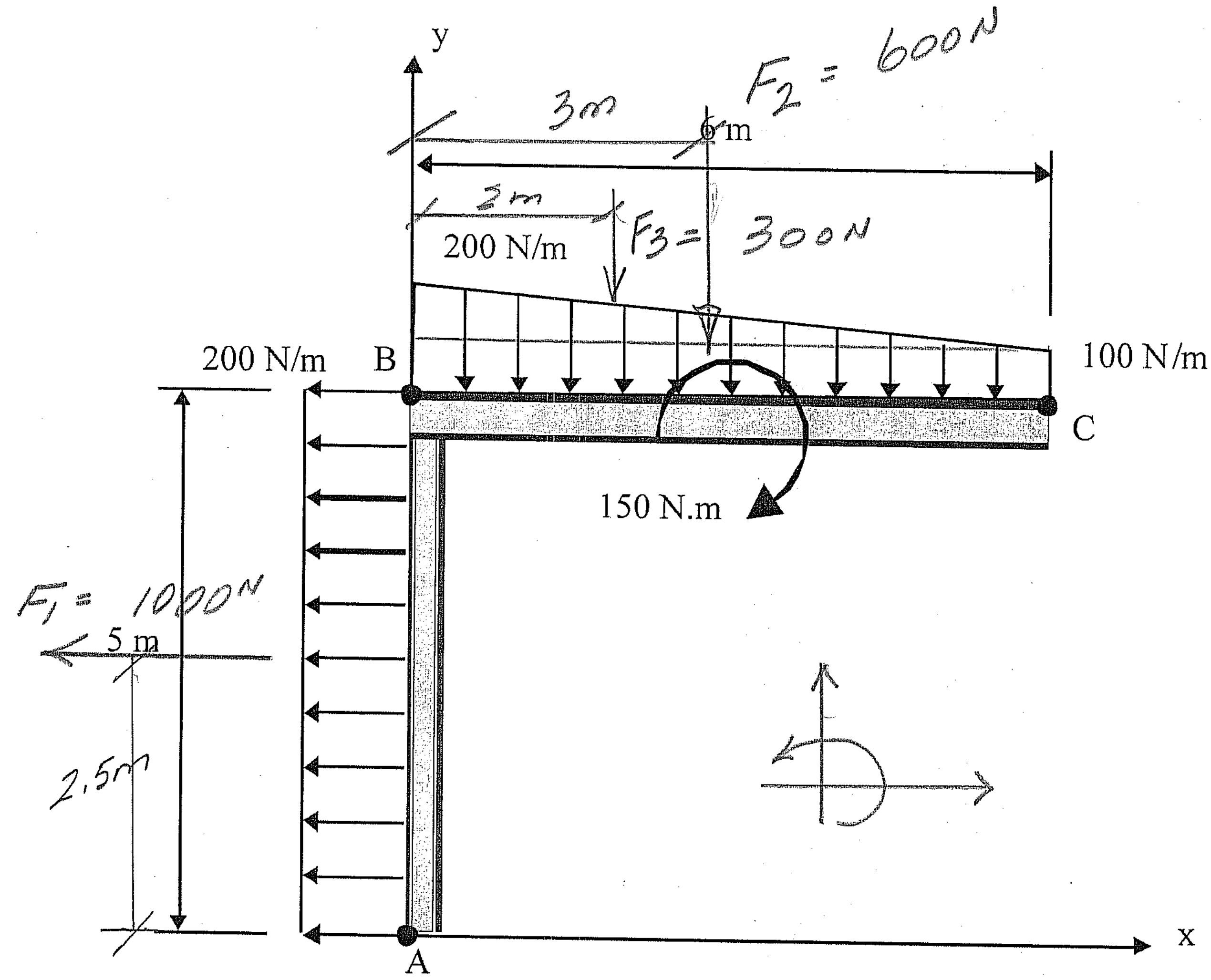


Figure 3

$\vec{F_1} = 200N/m\,(5m) = 1000N \leftarrow$ @ 2.5m from A

$\vec{F_2} = 100N/m\,(6m) = 600N \downarrow$ @ 3m from A

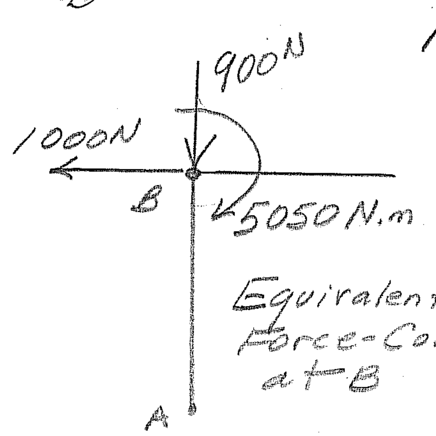
$\vec{F_3} = \dfrac{(200-100)(6)}{2} = 300N\downarrow$ @ $\frac{6}{3} = 2m$ from A

$R_x = \Sigma F_x = -1000N \;\therefore\; \vec{R_x} = 1000N \leftarrow$

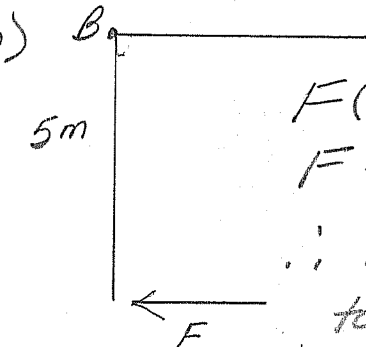
$R_y = \Sigma F_y = -600 - 300 = -900N \;\therefore\; \vec{R_y} = 900N \downarrow$

$M_{R_B} = -150 - 1000(2.5) - 300(2) - 600(3) = -5050\ N.m$

$\vec{M_{R_B}} = 5050\ N.m$ ↻



b)



$F(5) = 5050\ N.m$

$F = 1010N$

$\therefore\; \vec{F} = 1010N \leftarrow$

to produce 5050 N.m clockwise moment at B