Math 1710: Tutorial 8 (integration: substitutions, integration by parts, trigonometric integrals)

1. In each of the following cases, employ a substitution in order to verify the given integration formulae:

   (a) $\displaystyle\int \csc^n x \cot x \, dx = \frac{-\csc^n x}{n} + c\,, \quad n \neq 0\,,$
   and, when $n = 0$:
   $$\int \cot x \, dx = -\ln|\csc x| + c\,.$$

   (b) $\displaystyle\int \cot^n x \csc^2 x \, dx = \frac{-\cot^{n+1} x}{n+1} + c\,, \quad n \neq -1\,,$
   and, when $n = -1$:
   $$\int \frac{\csc^2 x}{\cot x}\, dx = -\ln|\cot x| + c\,.$$

2. (a) Use the method of integration by parts to derive the following "reduction formula":
   $$\int \cos^n x \, dx = \frac{1}{n}\cos^{(n-1)} x \sin x + \frac{n-1}{n}\int \cos^{(n-2)} x \, dx\,, \quad n > 0\,.$$

   (b) Use the reduction formula of part (a) in order to evaluate $\displaystyle\int \cos^7 x \, dx$.

3. By writing
   $$\tan^n x = \tan^{(n-2)} x \tan^2 x = \tan^{(n-2)} x \left(\sec^2 - 1\right)\,,$$
   derive the "reduction formula"
   $$\int \tan^n x \, dx = \frac{1}{n-1}\tan^{(n-1)} x - \int \tan^{(n-2)} x \, dx\,, \quad n > 1\,,$$
   and thus evaluate $\displaystyle\int \tan^5 x \, dx$.