DATE: February 12, 2013
COURSE: MATH 2132

TIME: 70 minutes
EXAMINER: G.I. Moghaddam & M. Virgilio

[9] 1. Find all values of x for which the sequence of functions

$$\left\{\frac{(x^2-1)n^2+(x^2+5x+6)n}{(x+1)n^2+(x-1)n+1}\right\}_{n=1}^{\infty}$$

is convergent. Find the limit function for those values of x.

[8] 2. Find the radius of convergence and the open interval of convergence for the series

$$\sum_{n=1}^{\infty}\frac{(-1)^n 3^n n^3}{\ln(n+2)}x^{2n}.$$

[12] 3. Let $f(x)=e^{3x}$ for $-\infty<x<\infty$. Then:

[4] (a) Find the first 4 terms of the Taylor series of $f(x)$ about -2.

[4] (b) Find $R_n(-2,x)$ (i.e. the n^{th}-remainder with $c=-2$).

[4] (c) Show that $\lim_{n\to\infty} R_n(-2,x)=0$ *only* for the case $x>-2$.

[6] 4. Find the value of a for which the sum of the series

$$\frac{2a}{\sqrt{3}}x^2-\frac{4a}{3}x^4+\frac{8a}{3\sqrt{3}}x^6-\frac{16a}{9}x^8+\cdots$$

is $\dfrac{20x^2}{\sqrt{3}+2x^2}$.

[15] 5. Find the Taylor series about 2 for the function

$$f(x)=\frac{x-3}{x^2}.$$

Express your answer in sigma notation and simplify as much as possible. Determine the open interval of convergence.

Answers

1. $$\lim_{n\to\infty} f_n(x)=\begin{cases} x-1, & x\neq -1,1\\ -1, & x=-1\\ 0, & x=1\end{cases}$$

2. $-\frac{1}{\sqrt{3}}<x<\frac{1}{\sqrt{3}}$

3. a) $e^{-6}+3e^{-6}(x+2)+\frac{3^2e^{-6}}{2!}(x+2)^2+\frac{3^3e^{-6}}{3!}(x+2)^3$

b) $R_n(-2,x)=3^{n+1}e^{3z_n}\frac{(x+2)^{n+1}}{(n+1)!}$, z_n being between -2 and x.

c) Hint: try to show $\lim_{n\to\infty}|R_n(-2,x)|\leq\lim_{n\to\infty}e^{3x}\frac{|3(x+2)|^{n+1}}{(n+1)!}=0$

4. $a=10$

5. $$\sum_{n=0}^{\infty}\frac{(-1)^{n+1}(3n+1)}{2^{n+2}}(x-2)^n$$

$0<x<4$