Differentiability and continuity. Angle between curves.

3.3.1

$$|1-(1+h)| = |-h| = |h|$$

(a) $$f'_+(1) = \lim_{h\to 0^+} \frac{f(1+h)-f(1)}{h} = \lim_{h\to 0^+} \frac{(1+h)|h| - 0}{h}$$

$$\underset{h>0}{=} \lim_{h\to 0^+} \frac{(1+h)h}{h} = \lim_{h\to 0^+} (1+h) = \underline{1}$$

$$f'_-(1) = \ldots \underset{\substack{h<0 \\ |h|=-h}}{=} \lim_{h\to 0^-} \frac{(1+h)(-h)}{h} = \lim_{h\to 0^-} -(1+h) = \underline{-1}$$

As $f'_+(1) \neq f'_-(1)$, $f'(1)$ $\underline{\text{D.N.E.}}$

(b) Def. of derivative (even of one-sided) at x contains $f(x)$. But f is not defined at $x=2$.

Answer: no, no, no.

(although disc. is removable)

3.3.2

Pts of intersection:

$$2x^2 - x = x^2 + kx - k$$

$$x^2 - (k+1)x + k = 0$$

$$\underline{x = k} \quad \text{or} \quad \underline{x = 1}$$

Derivatives: $(2x^2 - x)' = 4x - 1$

$(x^2 + kx - k)' = 2x + k$

To be at a right angle, we must have

$(4x-1)(2x+k) = -1$ at $x = 1$ or $x = k$.

$x = 1$: $3(2+k) = -1$

$$2 + k = -\frac{1}{3}$$

$$\underline{k} = -\frac{1}{3} - 2 = \underline{-\frac{7}{3}}$$

$x = k$: $(4k-1)(3k) = -1$

$$12k^2 - 3k + 1 = 0$$

$$(-3)^2 - 4 \cdot 12 < 0$$

no real sol-ns.

Ans: $\underline{-\frac{7}{3}}$.