PAPER NO 276 PAGE 1
DEPARTMENT & COURSE NO: 136.151 TIME: 2 hours

EXAMINATION: APPLIED CALCULUS I EXAMINER: Various

FINAL EXAMINATION

[12] 1. Compute each limit, if it exists.

(a)
$$\lim_{x \to 1/3} \frac{(2-6x)^2}{(3x-1)(9x^2-1)}$$

(b)
$$\lim_{x \to \infty} \left(\sqrt{x^2 + x} - \sqrt{x^2 - 3x} \right)$$

(c)
$$\lim_{x \to 0} \frac{x^2 + 2x - \sin 3x}{2x}$$

FINAL EXAMINATION

PAPER NO 276

PAGE 2
TIME: 2 hours

DEPARTMENT & COURSE NO: 136.151

EXAMINATION: APPLIED CALCULUS I EXAMINER: Various

[8] 2. Find f'(x) for each of the following functions. Do NOT simplify your answers.

(a)
$$f(x) = \sqrt{5 - 2\sin^3(7x)}$$

(b)
$$f(x) = \frac{\ln(x^3 + x)}{e^{4\cos(x)}}$$

[10] 3. Find the equation of the tangent line to the curve $3x^5y^5 - y = 2x$ at the point (1,1).

PAPER NO 276

DEPARTMENT & COURSE NO: 136.151

EXAMINATION: APPLIED CALCULUS I EXAMINER: Various

FINAL EXAMINATION

PAGE 3

TIME: 2 hours

4. (a) Draw the graph of the function $f(x) = x^2 e^{-x}$. (You may use the fact that $\lim_{x\to\infty} x^2 e^{-x} = 0$ without proving it.) [20]

- (b) Find all points of inflection on this graph, and determine all intervals where the graph of the function is concave upward.
- (c) Find the absolute maximum and minimum values of the function $f(x) = x^2 e^{-x}$ on the interval $-1 \le x \le 3$.

PAPER NO 276

DEPARTMENT & COURSE NO: 136.151

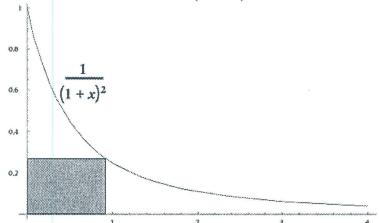
EXAMINATION: APPLIED CALCULUS I EXAMINER: Various

FINAL EXAMINATION

PAGE 4

TIME: 2 hours

5. Find the area of the largest rectangle that lies entirely in the first [12]quadrant, has one side on the x-axis, another side on the y-axis, and a vertex on the curve $y = \frac{1}{(1+x)^2}$.



PAPER NO 276

DEPARTMENT & COURSE NO: 136.151

EXAMINATION: APPLIED CALCULUS I EXAMINER: Various

FINAL EXAMINATION

PAGE 5

TIME: 2 hours

[10] 6. At noon a ship S_1 is 20 km north of ship S_2 . If S_1 sails south at 6 km/h and S_2 sails east at 8 km/h. How fast are the ships separating at 4:00 pm?

FINAL EXAMINATION PAGE 6

PAPER NO 276

TIME: 2 hours

DEPARTMENT & COURSE NO: 136.151 EXAMINATION: APPLIED CALCULUS I EXAMINER: Various

7. Find the following indefinite and definite integrals: [20]

(a)
$$\int t^2 \cos(t^3) dt$$

(b)
$$\int \frac{\sqrt{x} - x^2}{x} dx$$

(c)
$$\int_{\pi/2}^{\pi} \left(\sin x - \frac{1}{x} \right) dx$$
. Simplify as much as possible.

(d)
$$\int_0^1 x^5 \sqrt{x^3 + 1} \, dx$$

FINAL EXAMINATION

PAPER NO 276

PAGE 7 TIME: 2 hours

DEPARTMENT & COURSE NO: 136.151

EXAMINATION: APPLIED CALCULUS I EXAMINER: Various

[8] 8. Find f(x) if $f''(x) = 36x^2 + 12x$ with f'(1) = 19 and f(1) = 7.

[8] Bonus. Find the points on the curve $y^2 - x^2 + 2x = 10$ closest to the point (5,0).