Math 1210 Assignment # 3 - Oct 29, 2008 Solutions

1a) $\overrightarrow{P_1P_2} = [8,2,-6]$, $\overrightarrow{P_1P_3} = [3,-4,-3]$, $\overrightarrow{P_3P_2} = [5,6,-3]$

$$\|\overrightarrow{P_1P_2}\|^2 = \left(\sqrt{8^2+2^2+(-6)^2}\right)^2 = 64+4+36 = 104$$
$$\|\overrightarrow{P_1P_3}\|^2 + \|\overrightarrow{P_3P_2}\|^2 = \left(\sqrt{3^2+(-4)^2+(-3)^2}\right)^2 + \left(\sqrt{5^2+6^2+(-3)^2}\right)^2$$
$$= (9+16+9)+(25+36+9) = 34+70 = 104$$

hence $\|\overrightarrow{P_1P_2}\|^2 = \|\overrightarrow{P_1P_3}\|^2 + \|\overrightarrow{P_3P_2}\|^2$

b) $\overrightarrow{P_3P_2} = [5,6,-3]$, $\overrightarrow{P_3P_1} = [-3,4,3]$

$$\overrightarrow{P_3P_2} \cdot \overrightarrow{P_3P_1} = (5)(-3)+(6)(4)+(-3)(3) = -15+24-9 = 0$$

so $\cos\theta = 0$, hence θ is $\pi/2$.

2 $\overrightarrow{P_1P_2} = [3,3,1]$ and $\overrightarrow{P_1P_3} = [4,-1,6]$ are two vectors in the plane, so $\vec{n} = \overrightarrow{P_1P_2} \times \overrightarrow{P_1P_3}$ is normal to the plane

$$\vec{n} = \overrightarrow{P_1P_2} \times \overrightarrow{P_1P_3} = \left[(3)(6)-(-1)(1),\ (1)(4)-(3)(6),\ (3)(-1)-(4)(3)\right]$$
$$= [19,-14,-15]$$

using $\vec{n} \cdot \overrightarrow{P_0P_x} = 0$

$$[19,-14,-15] \cdot [x-1,\ y-3,\ z+2] = 0$$
$$19(x-1) - 14(y-3) - 15(z+2) = 0$$
$$19x - 19 - 14y + 42 - 15z - 30 = 0$$
$$19x - 14y - 15z = 7$$

3a) $x = 1 + 3t$ $\quad x = 3 + s \quad 1 + 3t = 3 + s \quad 3t - s = 2$

$y = 2 + 4t \quad y = 2 + 5s \quad 2 + 4t = 2 + 5s \quad 4t - 5s = 0$

$z = -3 + 2t \quad z = 7 - 2s \quad -3 + 2t = 7 - 2s \quad 2t + 2s = 10$

(using Gaussian Elimination:)

$$\left[\begin{array}{cc|c} 3 & -1 & 2 \\ 4 & -5 & 0 \\ 2 & 2 & 10 \end{array}\right] \xrightarrow{R_1 \leftrightarrow R_3} \left[\begin{array}{cc|c} 2 & 2 & 10 \\ 4 & -5 & 0 \\ 3 & -1 & 2 \end{array}\right] \xrightarrow{R_1 \to \frac{1}{2}R_1} \left[\begin{array}{cc|c} 1 & 1 & 5 \\ 4 & -5 & 0 \\ 3 & -1 & 2 \end{array}\right] \begin{array}{l} \\ R_2 \to R_2 - 4R_1 \\ R_3 \to R_3 - 3R_1 \end{array}$$

$$\left[\begin{array}{cc|c} 1 & 1 & 5 \\ 0 & -9 & -20 \\ 0 & -4 & -13 \end{array}\right] \xrightarrow{R_2 \to -\frac{1}{9}R_2} \left[\begin{array}{cc|c} 1 & 1 & 5 \\ 0 & 1 & 20/9 \\ 0 & -4 & -13 \end{array}\right] \begin{array}{l} \\ \\ R_3 \to R_3 + 4R_2 \end{array} \left[\begin{array}{cc|c} 1 & 1 & 5 \\ 0 & 1 & 20/9 \\ 0 & 0 & -37/9 \end{array}\right]$$

This system has no solution.

These lines do not intersect

b) $x = 1 - t \quad x = 4 + 5s \quad 1 - t = 4 + 5s \quad -t - 5s = 3$

$y = 3 + 3t \quad y = 1 - 8s \quad 3 + 3t = 1 - 8s \quad 3t + 8s = -2$

$z = -2 + 6t \quad z = 3 - 7s \quad -2 + 6t = 3 - 7s \quad 6t + 7s = 5$

$$\left[\begin{array}{cc|c} -1 & -5 & 3 \\ 3 & 8 & -2 \\ 6 & 7 & 5 \end{array}\right] \xrightarrow{R_1 \to -R_1} \left[\begin{array}{cc|c} 1 & 5 & -3 \\ 3 & 8 & -2 \\ 6 & 7 & 5 \end{array}\right] \begin{array}{l} \\ R_2 \to R_2 - 3R_1 \\ R_3 \to R_3 - 6R_1 \end{array} \left[\begin{array}{cc|c} 1 & 5 & -3 \\ 0 & -7 & 7 \\ 0 & -23 & 23 \end{array}\right] \xrightarrow{R_2 \to -\frac{1}{7}R_2} \left[\begin{array}{cc|c} 1 & 5 & -3 \\ 0 & 1 & -1 \\ 0 & -23 & 23 \end{array}\right] \begin{array}{l} \\ \\ R_3 \to R_3 + 23R_2 \end{array}$$

$$\left[\begin{array}{cc|c} 1 & 5 & -3 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{array}\right]$$

$t + 5s = -3$

$s = -1$

$\therefore t = 2$

$x = 1 - 2 = -1$

$y = 3 + 3(2) = 9$

$z = -2 + 6(2) = 10$

The point of intersection is $(-1, 9, 10)$.

4 $x=3-t,\ y=1+t,\ z=-3+2t$

so $3x-2y+z=10$

$$3(3-t)-2(1+t)+(-3+2t)=10$$
$$9-3t-2-2t-3+2t=10$$
$$-3t=6$$
$$t=-2$$

$x=3-(-2)=5$

$y=1+(-2)=-1$

$z=-3+2(-2)=-7$

The point of intersection is $(5,-1,-7)$.

5 a) $$2\begin{pmatrix}3 & -2\\ 2 & -1\end{pmatrix}+\left(\begin{pmatrix}2 & -1 & 3\\ 1 & 3 & -1\end{pmatrix}\begin{pmatrix}1 & 2\\ 0 & 1\\ -2 & -1\end{pmatrix}\right)^T$$

$$=\begin{pmatrix}6 & -4\\ 4 & -2\end{pmatrix}+\begin{pmatrix}-4 & 0\\ 3 & 6\end{pmatrix}^T=\begin{pmatrix}6 & -4\\ 4 & -2\end{pmatrix}+\begin{pmatrix}-4 & 3\\ 0 & 6\end{pmatrix}=\begin{pmatrix}2 & -1\\ 4 & 4\end{pmatrix}$$

b) $$\begin{pmatrix}4\\ 1\end{pmatrix}\begin{pmatrix}3 & -2 & 1\end{pmatrix}+\begin{pmatrix}1 & 2\\ 0 & 1\\ -2 & -1\end{pmatrix}^T=\begin{pmatrix}12 & -8 & 4\\ 3 & -2 & 1\end{pmatrix}+\begin{pmatrix}1 & 0 & -2\\ 2 & 1 & -1\end{pmatrix}$$

$$=\begin{pmatrix}13 & -8 & 2\\ 5 & -1 & 0\end{pmatrix}$$

c) This is undefined. Since B is 3x3 and D is 3x2, BD is 3x2; since A is 2x2, 3A is 2x2, and a 3x2 matrix cannot be added to a 2x2 matrix (since they are different sizes)

(Q5 cont'd)

d) $\begin{pmatrix} 2 & -1 & 3 \\ 1 & 3 & -1 \end{pmatrix}\begin{pmatrix} 3 & -2 & 1 \end{pmatrix}^T - 2\begin{pmatrix} 4 \\ 1 \end{pmatrix}$

$= \begin{pmatrix} 2 & -1 & 3 \\ 1 & 3 & -1 \end{pmatrix}\begin{pmatrix} 3 \\ -2 \\ 1 \end{pmatrix} - \begin{pmatrix} 8 \\ 2 \end{pmatrix} = \begin{pmatrix} 11 \\ -4 \end{pmatrix} - \begin{pmatrix} 8 \\ 2 \end{pmatrix} = \begin{pmatrix} 3 \\ -6 \end{pmatrix}$

e) $\begin{pmatrix} 1 & 2 \\ 0 & 1 \\ -2 & -1 \end{pmatrix}\begin{pmatrix} 3 & -2 \\ 2 & -1 \end{pmatrix}\begin{pmatrix} 2 & -1 & 3 \\ 1 & 3 & -1 \end{pmatrix} + \begin{pmatrix} 0 & -2 & -1 \\ 1 & 2 & 4 \\ -1 & 3 & 2 \end{pmatrix}$

$= \begin{pmatrix} 7 & -4 \\ 2 & -1 \\ -8 & 5 \end{pmatrix}\begin{pmatrix} 2 & -1 & 3 \\ 1 & 3 & -1 \end{pmatrix} + \begin{pmatrix} 0 & -2 & -1 \\ 1 & 2 & 4 \\ -1 & 3 & 2 \end{pmatrix}$

$= \begin{pmatrix} 10 & -19 & 25 \\ 3 & -5 & 6 \\ -11 & 23 & -29 \end{pmatrix} + \begin{pmatrix} 0 & -2 & -1 \\ 1 & 2 & 4 \\ -1 & 3 & 2 \end{pmatrix} = \begin{pmatrix} 10 & -21 & 24 \\ 4 & -3 & 10 \\ -12 & 26 & -27 \end{pmatrix}$

f) This is undefined. Since E is 1×3, B is 3×3; EB is 1×3. Since F is 2×1, F^T is 1×2 and $-3F^T$ is 1×2. a 1×2 matrix cannot be added to a 1×3 matrix, they are diffuent sizes.