Summer2012

MATH2132 Test1

Values

6 **1.** Find the limit for the following sequence of functions on the interval $0 < x \leq 2$, if it exists. Show your reasoning or calculations.

$$\left\{\left(\frac{n^2x^2+x-1}{n^2x+1}\right)\cos\left(\frac{5x}{n}\right)\right\}$$

10 **2.** Determine whether the following series converge or diverge. If a series converges, find its sum. Justify your conclusions.

(a) $\displaystyle\sum_{n=1}^{\infty}(-1)^n \operatorname{Sin}^{-1}\left(\frac{n^2+1}{2n^2}\right)$ (b) $\displaystyle\sum_{n=2}^{\infty}\frac{2^{n+1}+1}{3^{2n}}$

12 **3.** Find the interval of convergence for the power series

$$\sum_{n=3}^{\infty}\frac{(-1)^n n^2 4^n}{2n^2+1}(x-2)^{2n}.$$

Express your answer in one of the forms $a < x < b$, $a \leq x < b$, $a < x \leq b$, or $a \leq x \leq b$. Justify all results.

12 **4.** Use Taylor remainders to verify that the Maclaurin series for $\cos 2x$ converges to $\cos 2x$ for all x.

Answers.

1. $\begin{cases} x & 0 < x \leq 2 \\ -1 & x = 0 \end{cases}$

2. a) diverges (by the n-th term test)

b) Sum of two convergent geometric series

Sum $= 71/504$

3. $3/2 < x < 5/2$

4. hint: try to obtain, $|R_n(0,x)| \leq \dfrac{2^{n+1}|x|^{n+1}}{(n+1)!} = \dfrac{|2x|^{n+1}}{(n+1)!}$