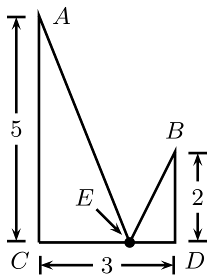
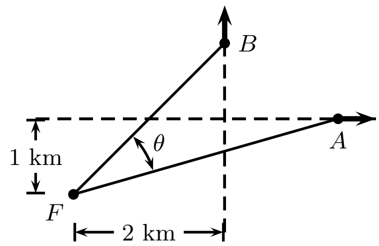
Math 1710 Tutorial 7. Applications of Inverse Trigonometric Functions.

1. Find the position of the point E on the line segment CD in the following figure in order that the angle AEB is as large as possible.

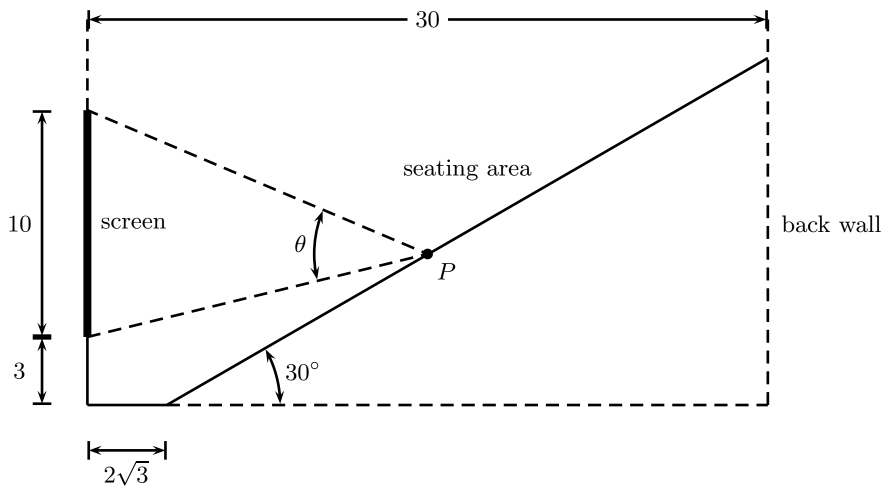


2. A fisherman, sitting in a stationary boat F (see the following diagram), observes two other boats travelling on the lake in which he is fishing. Boat A is travelling due east at a constant speed of 50 kilometres per hour along a straight line one kilometre north of the fisherman. Boat B is travelling due north at a constant speed of 40 kilometers per hour along a line two kilometres east of the fisherman. If θ denotes the angle between the two lines of sight of the fisherman, find the rate at which θ is changing at the instant when boat A is $\sqrt{10}$ kilometres from the fisherman and boat B is $2\sqrt{2}$ kilometres from the fisherman.



Problems continue on the next page.

3. The accompanying diagram shows the side-view cross-section of a movie theatre. On the vertical front wall is mounted a movie screen, which is 10 metres tall, with its bottom edge 3 metres above the floor. The sloped "seating area" of the theatre is inclined at an angle of 30 degrees to the horizontal, and the front edge of this seating area is $2\sqrt{3}$ metres from the front wall. The distance from the front wall to the back wall of the theatre is 30 metres. *The problem is to determine where a patron should be seated in order to get the "best view" of the screen.*



Introduce a rectangular Cartesian coordinate system, with origin at the base of the front wall, and suppose that a patron is positioned at point $P(x, y)$ within the seating area of the theatre, as illustrated in the diagram.

(a) Show that the patron's viewing angle of the screen is given by the relation

$$\theta = \mathrm{Tan}^{-1}\left(\frac{15}{x} - \frac{1}{\sqrt{3}}\right) + \mathrm{Tan}^{-1}\left(\frac{1}{\sqrt{3}} - \frac{5}{x}\right) \quad \text{for} \quad 2\sqrt{3} \le x \le 30.$$

(b) Show that

$$\frac{d\theta}{dx} = \frac{750 - \frac{40}{3}x^2}{\left(x^2 + \left(15 - \frac{x}{\sqrt{3}}\right)^2\right)\left(x^2 + \left(\frac{x}{\sqrt{3}} - 5\right)^2\right)}$$

and deduce that the only critical point of this function occurs when $x = 15/2$.

(c) Show that the maximum viewing angle occurs when $x = 15/2$.

(d) Find the equation of the circle which passes through the point at the top edge of the screen, the point at the bottom edge of the screen and the point at which the patron should sit in order to get the best view of the screen.

(e) Show that the circle obtained in problem 3d is tangent to the "seating area" at the point at which the patron should sit in order to get the best view of the screen.