

Instructors: Drs. J. Abelló and B.-C. Wang

October 11, 2011

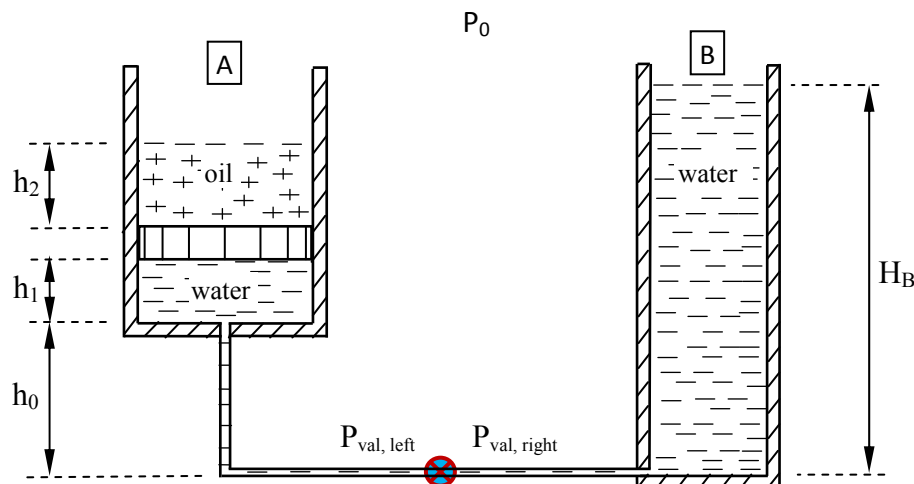
Duration: 100 min

1. You are permitted to use the textbook for the course, a one-page double-sided aide sheet, and a calculator.
2. Ask for clarification if any problem statement is unclear.
3. Clear, systematic solutions are required. **Show your work.** Marks will not be assigned for problems that require unreasonable (in the opinion of the instructor(s)) effort for the marker to decipher.
4. Use linear interpolation in the property tables as necessary.
5. Keep 5 significant figures in intermediate calculations, and use 4 or 5 significant figures in final answers. For temperature, keep two decimal places in your final answer.
6. There are three problems on this test. The weight of each problem is indicated. The test will be marked out of **50**.

1. (9 marks)

A hydraulic system consisting of two cylinders A and B is shown in the figure. The two cylinders are connected by a very thin pipeline with a valve in the middle. The volume of the pipeline can be ignored. Inside cylinder A, there is a piston whose mass is $m_p = 60$ kg. The piston separates oil from water, and oil and water cannot mix by leaking through the seal between the piston and the cylinder. The piston-cylinder assembly can be assumed to be frictionless. At the initial state, the valve is closed, and as shown in the figure, the geometry of the hydraulic system is described by four heights: $h_0 = 0.7$ m, $h_1 = 0.3$ m, $h_2 = 0.5$ m and $H_B = 2$ m. The cross sectional area of the piston is $A_p = 0.05$ m². The local gravitational acceleration is $g = 9.81$ m/s². The local atmospheric pressure is $P_0 = 100$ kPa. The density of water is $\rho_{\text{water}} = 1000$ kg/m³ and the density of oil is $\rho_{\text{oil}} = 900$ kg/m³.

- (a) Determine the *initial* pressure on each side of the valve (when it is closed), i.e. $P_{\text{val, left}}$ and $P_{\text{val, right}}$, in kPa (6marks)
- (b) If the valve is opened, will the piston move up or down? Justify your answer. (3 marks)

**Figure for Problem 1****2. (22 marks)**

A rigid tank is divided into three chambers by two membranes. The volume of chambers A, B and C is 0.1101 m³, 1 m³ and 2 m³, respectively. Initially, chamber A contains water at $T_A = 160$ °C and $P_A = 2000$ kPa, chamber B contains 1 kg water at $P_B = 500$ kPa, and chamber C is empty (vacuum). Both membranes rupture and heat transfer takes place between the rigid tank and its surroundings. Eventually, water inside the entire rigid tank reaches a uniform state, with a final temperature $T_2 = 220$ °C.

- (a) Determine the initial mass of the water in chamber A, m_A , in kg (3.5 marks)

- (b) Determine the initial temperature of the water in chamber B, T_B , in $^{\circ}\text{C}$ (5 marks)
- (c) Determine the final pressure of the water in the entire rigid tank, P_2 , in kPa (5.5 marks)
- (d) Determine the quality of the water at the final state, x , if applicable. (2 marks)
- (e) Show the three state points (the initial states of the water in chamber A and chamber B, and the final state of the water) on a T - v (temperature–specific volume) diagram. Also show the isobars (i.e., constant pressure curves/lines) corresponding to P_A , P_B and P_2 on the T - v diagram. Clearly label the temperatures, pressures and specific volumes for which you have values. You do not need to show any process paths on the diagram. (6 marks)



Figure for Problem 2

3. (19 marks)

Air of mass $m = 0.065$ kg is contained inside a vertical, frictionless piston-cylinder assembly. The top surface of the piston is open to the atmosphere. The atmospheric pressure is $P_0 = 100$ kPa. The piston has mass $m_p = 200$ kg and cross-sectional area $A_p = 0.2$ m². The distance from the bottom surface of the cylinder to the bottom surface of the piston is denoted as h . The air is initially at state 1. In state 1 the temperature is $T_1 = -23$ $^{\circ}\text{C}$, the piston rests on the stops, and the value of h is $h_1 = 0.25$ m. Heat is then added to the air. The air reaches state 2 when the piston *just* starts to float (just starts to leave the stops). Heat continues being added as the piston rises. The air reaches state 3 when its temperature reaches $T_3 = 330$ $^{\circ}\text{C}$. Treat air as an ideal gas with the gas constant $R = 0.287$ kJ/(kg·K). The local acceleration of gravity is 9.81 m/s².

- (a) Find the pressure at state 1, P_1 , in kPa. (3 marks)
- (b) Find the temperature at state 2, T_2 , in $^{\circ}\text{C}$. (5 marks)
- (c) Find the distance between the bottom surface of the piston and the bottom surface of the cylinder at state 3, h_3 , in meters. (5 marks)
- (d) Draw *either* a P - V (pressure–volume) *or* a P - v (pressure–specific volume) diagram of the 1-2-3 process. Clearly label the states and the process paths. Draw the isotherms (constant temperature lines/curves) corresponding to T_1 , T_2 and T_3 . Clearly label the temperatures and pressures for which you have values. (6 marks)

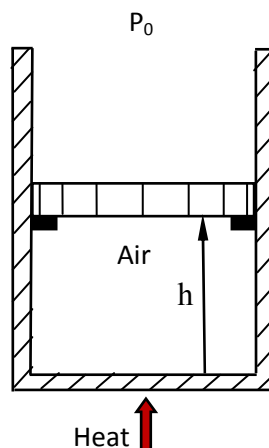


Figure for Problem 3

Problem 1

$$\begin{aligned}P_{\text{valve}}^{\text{Left}} &= P_0 + \rho_w g (h_0 + h_1) + \rho_{\text{oil}} g h_2 + \frac{m_p g}{A_p} \\&= 1 \times 10^5 + 1000 \times 9.81 (0.7 + 0.3) + 900 \times 9.81 \times 0.5 + \frac{60 \times 9.81}{0.05} \\&= 125996.5 \text{ (Pa)} \\&\text{OR, } 125.997 \text{ (kPa)}\end{aligned}$$

$$\begin{aligned}P_{\text{valve}}^{\text{Right}} &= P_0 + \rho_w g H_B \\&= 1 \times 10^5 + 9.81 \times 2 \\&= 119620 \text{ (Pa)} \\&\text{OR, } 119.62 \text{ (kPa)}\end{aligned}$$

B/C $P_{\text{valve}}^{\text{Left}} > P_{\text{valve}}^{\text{Right}}$, the piston will move down
and water will flow from left to right.

Problem 2

(a) Room A

Table B.1.1: At $T_A = 160^\circ\text{C}$, $P_{\text{sat}} = 617.8 \text{ kPa}$

$$\because P_A = 2000 \text{ kPa} > P_{\text{sat}}$$

\therefore Compressed Liquid

Alternative method

Table B.1.2: At $P_A = 2000 \text{ kPa}$, $T_{\text{sat}} = 212.42^\circ\text{C}$

$$\because T_A = 160^\circ\text{C} < T_{\text{sat}}$$

\therefore Compressed Liquid

Table B.1.4: At $P_A = 2000 \text{ kPa}$ & $T_A = 160^\circ\text{C}$

$$V_A = 0.001101 \text{ (m}^3/\text{kg)}$$

$$m_A = \frac{V_A}{v_A} = \frac{0.1101}{0.001101} = 100 \text{ (kg)}$$

(b) Room B

$$V_B = \frac{V_B}{m_B} = \frac{1}{1} = 1 \text{ (m}^3/\text{kg)}$$

Table B.1.2: At $P_B = 500 \text{ kPa}$, $v_g = 0.37489 \text{ (m}^3/\text{kg)}$

$$\because V_B > v_g$$

\therefore Superheated Vapour

Table B.1.3

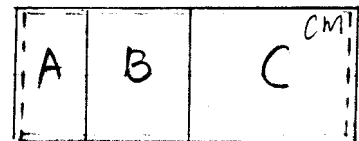
$T [^\circ\text{C}]$	$v \text{ [m}^3/\text{kg}]$
800	0.98959
$T_B = ?$	1
900	1.08217

Interpolation:

$$T_B = 800 + \frac{1 - 0.98959}{1.08217 - 0.98959} (900 - 800)$$

$$= 811.24 (^\circ\text{C})$$

(c) Final state



$$\text{CM: } A + B + C$$

$$m_{\text{tot}} = m_A + m_B + m_C = 100 + 1 = 101 \text{ (kg)}$$

$$V_{\text{tot}} = V_A + V_B + V_C = 0.1101 + 1 + 2 = 3.1101 \text{ (m}^3)$$

$$V_2 = \frac{V_{tot}}{m_{tot}} = \frac{3.1101}{101} = 0.03079 \left(\frac{m^3}{kg} \right)$$

Table B-11: At $T_2 = 220^\circ C$, $V_f = 0.001190 \frac{m^3}{kg}$, $V_g = 0.08619 \frac{m^3}{kg}$
 $V_{fg} = 0.08500$

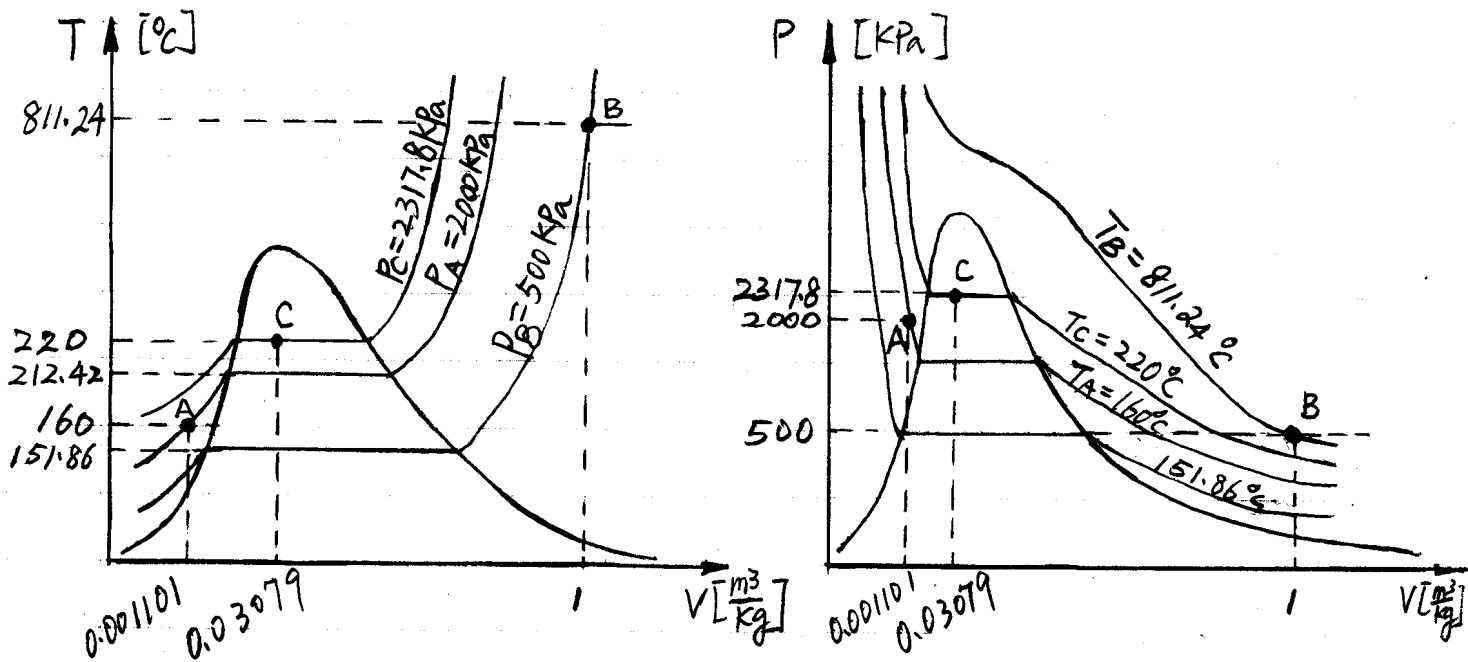
$$\therefore V_f < V_2 < V_g$$

\therefore Saturated mixture

$$P_2 = P_{sat}|_{T_2=220^\circ C} = 2317.8 \text{ kPa}$$

(d) Quality: $x = \frac{V - V_f}{V_{fg}} = \frac{0.03079 - 0.001190}{0.08500} = 0.3483$

(e)



Note that: Only the T-v diagram is required.

QUESTION 3

$$a) V_1 = A_p \cdot h_1 = 0.2 \text{ m}^2 \cdot 0.25 \text{ m} = 0.050 \text{ m}^2$$

$$P_1 V_1 = m R T_1 \rightarrow P_1 = \frac{R T_1 m}{V_1} \quad \text{with } T_1 = (273.15 - 23) \text{ K} = 250.15 \text{ K}$$

$$P_1 = \frac{287 \text{ J/kg} \cdot \text{K} \cdot 250.15 \text{ K} \cdot 0.065 \text{ kg}}{0.050 \text{ m}^2}$$

$$= 93331 \text{ Pa} = \boxed{93.33 \text{ kPa}}$$

$$b) V_2 = V_1 \quad \text{AND} \quad m_2 = m_1 = m \quad (\text{CLOSED SYSTEM}).$$

$$P_2 = P_0 + \frac{m p g}{A_p} = 10^5 \text{ Pa} + \frac{200 \text{ kg} \cdot 9.81 \text{ m/s}^2}{0.2 \text{ m}^2} = 109810 \text{ Pa}$$

$$= 109.81 \text{ kPa}$$

$$P_2 V_2 = m R T_2$$

$$T_2 = \frac{P_2 V_2}{m R} = \frac{P_2 V_1}{m R} = \frac{109.81 \cdot 10^3 \text{ Pa} \cdot 0.050 \text{ m}^2}{0.065 \text{ kg} \cdot 287 \text{ J/kg} \cdot \text{K}} = 294.32 \text{ K}$$

$$T_2 = (294.32 - 273.15) ^\circ \text{C}$$

$$= \boxed{21.17 ^\circ \text{C}}$$

$$c) P_3 = P_2 \quad \text{AND} \quad m_3 = m_2 = m_1 \quad T_3 = (330 + 273.15) \text{ K} = 603.15 \text{ K}$$

$$P_3 V_3 = m R T_3 \rightarrow V_3 = \frac{m R T_3}{P_2} = \frac{0.065 \text{ kg} \cdot 287 \text{ J/kg} \cdot \text{K} \cdot 603.15 \text{ K}}{109.81 \cdot 10^3 \text{ Pa}}$$

$$= 0.10247 \text{ m}^3$$

2
CONTINUES

$$V_3 = A_p \cdot h_3 \rightarrow h_3 = \frac{V_3}{A_p} = \frac{0.10247 \text{ m}^3}{0.2 \text{ m}^2} = \boxed{0.5123 \text{ m}}$$

d)

$P \text{ (kPa)}$

(NOT TO SCALE)

