Math 253 Homework assignment 8

1. Find the mass and centre of mass of the lamina that occupies the region bounded by $y = e^x$, $y = 0$, $x = 0$ and $x = 1$ and having density $\rho(x, y) = y$.

2. Find the moments of inertia I_x, I_y and I_0 for the lamina of problem 1.

3. A square lamina of constant density ρ occupies a square with vertices $(0, 0)$, $(a, 0)$, (a, a) and $(0, a)$. Find the moments of inertia I_x, I_y and the radii of gyration $\bar{\bar{x}}$ and $\bar{\bar{y}}$.

4. Use integration to verify the following formula for the area of a triangle in $\mathbb{R}^3$ with vertices at $(a, 0, 0)$, $(0, b, 0)$ and $(0, 0, c)$, with a, b, c positive numbers:

$$\text{Area} = \frac{1}{2}\sqrt{a^2b^2 + b^2c^2 + a^2c^2}.$$

5. Find the area of the part of the cylinder $y^2 + z^2 = 9$ that lies above the rectangle with vertices $(0, 0)$, $(4, 0)$, $(4, 2)$ and $(0, 2)$.

6. Find the area of that part of the hyperbolic paraboloid $z = x^2 - y^2$ that lies inside the cylinder $x^2 + y^2 = a^2$.

7. Find the area of the part of the sphere $x^2 + y^2 + z^2 = a^2$ that lies inside the cylinder $x^2 + y^2 = ax$.