12   6. Find the Taylor series about $x = 2$ for $(2x - 7)^{1/3}$. Express your answer in sigma notation, simplified as much as possible. What is the radius of convergence of the series?

10   7. (a) Find the Maclaurin series for the function

$$F(x) = \int_0^x \frac{1 - \cos t}{t}\, dt, \qquad F(0) = 0.$$

Express your answer in sigma notation.

(b) Explain how you could find a value for $F(2)$ that is correct to 3 decimal places.

8   8. Find a 1-parameter family of solutions of the differential equation

$$(1 + x^2)\frac{dy}{dx} = y^2 - y.$$

Identify any singular solutions.

Dawit's
Answers

6. $-\sqrt[3]{3} + \dfrac{2(x-2)}{3^{5/3}} + \displaystyle\sum_{n=2}^{\infty} \dfrac{2^n\left[2\cdot 5\cdot 8\cdots(3n-4)\right]}{3^{2n-1/3}\, n!}(x-2)^n$, $|x-2| < \frac{3}{2}$

    $R = \frac{3}{2}$

7. a) $\displaystyle\sum_{n=1}^{\infty} \dfrac{(-1)^{n+1}}{(2n)(2n)!}\, x^{2n}$ b) $F(2) = \dfrac{2^2}{2\cdot 2!} - \dfrac{2^4}{4\cdot 4!} + \dfrac{2^6}{6\cdot 6!} - \cdots$

is an alternating series and absolute values of terms are decreasing and approach zero, calculate the partial sums until two successive partial sums round to the same 3 decimal places.

8. $\ln|y-1| - \ln|y| = \operatorname{Tan}^{-1}(x) + C$, $y=0$ and $y=1$ are singular solutions.

Values

15   1. (a) Find the Taylor series about $x = 2$ for the function $\dfrac{1}{(3+2x)^{1/3}}$. Express your answer in sigma notation simplified as much as possible. You must use a technique that ensures that the series converges to the function.

   (b) What is the radius of convergence of the series?

13   2. (a) Find the Maclaurin series for the function $f(x) = \dfrac{x^4}{(4-3x)^2}$. Express your final answer in sigma notation simplified as much as possible.

   (b) What is the interval of convergence of the series?

12   3. Evaluate

$$\sum_{n=2}^{\infty} \frac{2^n}{(n+1)!}x^n.$$

Justify all steps in your solution.

10   4. Find, in explicit form $y = f(x)$, the solution of the initial value problem

$$x^2 y\frac{dy}{dx} + x^2 = 1, \qquad y(1) = 1.$$

Dawit's
Answers

1, a) $\dfrac{1}{\sqrt[3]{7}} + \displaystyle\sum_{n=1}^{\infty} \dfrac{(-1)^n \, 2^n \, [1\cdot4\cdot7\cdots(3n-2)]}{3^n \, 7^{n+1/3} \, n!}(x-2)^n$,   b) $|x-2| < \frac{1}{2}$

$R = \frac{1}{2}$

2. a) $\displaystyle\sum_{n=4}^{\infty} \dfrac{3^{n-4}(n-3)}{4^{n-2}} x^n$,   b) $|x| < \frac{4}{3}$ or $-\frac{4}{3} < x < \frac{4}{3}$

3. $S(x) = \begin{cases} \frac{1}{2x}(e^{2x}-1) - (x+1) , & x \neq 0 \\ 0 , & x = 0 \end{cases}$

4. $y(x) = \sqrt{5 - \frac{2}{x}(1+x^2)}$