Math 1710 Tutorial 5. Centroids. Moments of Inertia.

Problem 1. Find the first moment of a plate with constant mass per area ρ that is bounded by the given curves about the specified line:

(a) $y = x^5$, $x = y^5$ (the part of the region in the first quadrant only) about $y = 0$;

(b) $x = y^2 - 2y + 4$, $x - 3y + 2 = 0$ about $x = 1$;

(c) $x = y^2 - 2y + 4$, $x - 3y + 2 = 0$ about $y = -1$;

(d) $y = \sin x^2\, (0 \le x \le \sqrt{\pi})$, $y = 0$, about $x = 0$.

Problem 2. Find the centroid of the region bounded by the curves:

(a) $x = y^2 - 2y + 4$, $x - 3y + 2 = 0$; (can you obtain the answer using 1(b) and 1(c)?)

(b) $y = \sin x\, (0 \le x \le \pi)$, $y = 0$; (use symmetry to find $\bar{x}$)

(c) $y = \sqrt{x}$, $x = y + 2$, $y = 0$;

(d) $y = \sqrt{1 - x^2}\, (0 \le x \le 1)$, $x = 0$, $y = 0$.

Problem 3. The given curves determine a thin plate with constant mass per unit area ρ. Find its moment of inertia about the given line.

(a) $y = \sin x\, (0 \le x \le \pi/2)$, $x = \pi/2$, $y = 0$ about $y = -1$ (set up a definite integral only);

(b) $y = 3e^{-2x}$, $y = 3$, $x = 2$ about $x = -1$ (set up a definite integral only).

Problem 4. Triangular thin plate with constant mass per unit area ρ has vertices $(2, 0)$, $(0, 1)$, $(-1, -1)$. Find its moment of inertia about the line $y = 2$.