MATH2132 Test1 February 2, 2010 60 minutes

5 1. Find the limit of the sequence of functions

$$\left\{\frac{n^2x^3+3nx}{2n^2x+1}\operatorname{Tan}^{-1}\left(\frac{nx}{n+3}\right)\right\}$$

on the interval $0 \le x \le 3$, if it exists. Justify your answer.

8 2. Determine whether the following series converge or diverge. Justify you answers. If a series converges, find its sum.

(a) $\sum_{n=1}^{\infty}\frac{(-1)^n n^2}{1+2n^2}$ (b) $\sum_{n=3}^{\infty}\frac{2^n}{3^{n+1}}$

12 3. (a) Find the first four Taylor polynomials $P_0(x)$, $P_1(x)$, $P_2(x)$, and $P_3(x)$ about $x = 0$ for the function $\cos 3x$.

(b) Use Taylor's remainder formula to verify that the Maclaurin series for $\cos 3x$ converges to $\cos 3x$ for all x.

8 4. Find the interval of convergence for the power series

$$\sum_{n=3}^{\infty}\frac{(-1)^{n+1}n^2}{3^n}x^{2n+1}.$$

7 5. Find the open interval of convergence for the power series

$$\sum_{n=2}^{\infty}\frac{2^{n+1}}{n^3+100n^2}(x+2)^n.$$

Answers by Dawit Yohannes (ydawit@yahoo.com)

1) $\frac{x^2}{2}\operatorname{Tan}^{-1}x$, $0 \le x \le 3$

2) a) diverges (by the nth term test)

b) Converges (Geometric series, $r = \frac{2}{3} < 1$) with Sum $= \frac{8}{27}$

3) a. $P_0(x) = 1$, $P_1(x) = 1$, $P_2(x) = 1 - \frac{9x^2}{2}$, $P_3(x) = 1 - \frac{9x^2}{2}$

b) $\lim_{n\to\infty}|R_n(0,x)| \le \lim_{n\to\infty}\frac{|3x|^{n+1}}{(n+1)!} = 0$ (for all x)

4) $-\sqrt{3} < x < \sqrt{3}$

5) $-5/2 < x < -3/2$.