MATH 2130 Problem Workshop 2

1. Find the distance from the point $(3, -1, 5)$ to the line $x = 2 + 3t, y = 2t - 1, z = 4 + t$.

2. Find the distance between the lines $y = 2x + 3z - 4, 3x + y - 2z = 6$ and $x = 2 + t, y = 3 - 2t, z = 1 + t$.

3. The following three lines define a triangle

$$\begin{array}{lll} x = -11 + 5s, & y = s, & z = -2 + 2s \\ x = 1 + 2u, & y = 1 - u, & z = -2 - 4u \\ x = -2 + 3t, & y = -1 + 2t, & z = -8 + 6t \end{array}$$

   Find the area of the triangle.

4. Find the centroid of the triangle in question 3. It is the point of intersection of the three medians of the triangle, which occurs on a median which is 2/3 of the way from the vertex to the opposite midpoint.

5. Find $\mathbf{v}'(3)$ if $\mathbf{v}(t) = t^2\hat{\mathbf{i}} + \arcsin(t/4)\hat{\mathbf{j}} + \ln(2t + 1)\hat{\mathbf{k}}$.

6. If $f(t) = t^2 + 1$ and $\mathbf{v}(t) = e^t\hat{\mathbf{i}} + \left[t/(t^2 + 1)^3\right]\hat{\mathbf{j}} - t\sqrt{t^2 + 1}\hat{\mathbf{k}}$, evaluate $\int f(t)\mathbf{v}(t)dt$.

7. Find a parameterization of the following curves.
   (a) $z = 2\sqrt{x^2 + y^2}, x^2 + y^2 = 3 - z$ from $(1, 0, 2)$ to $(-1, 0, 2)$ directed so y is always non-positive.
   (b) First octant part of $x^2 + z^2 = 4, x + y = 1$ directed so that z increases along the curve.
   (c) $z = x^2 + y^2, x^2 + y^2 - 4y = 0$ directed clockwise viewed from above.

8. Find all unit tangent vectors to the curve $x^2 + z^2 = 4, x + y = 1$ at the point $(\sqrt{2}, 1 - \sqrt{2}, \sqrt{2})$.

9. Find the unit tangent vector to the curve $x = t^2, y = 2t^3, z = 3t^2$ at the origin.

10. Find the angle between the tangent vectors to the curves

$$x^2 + y = z + 4, x + 2y = 5 \qquad \text{and} \qquad x + y^2 = 5, 2x + 3y + 4z = 4$$

   at the point of intersection between the curves.

11. Find the length of the curve $x = t + 1, y = 2t^{3/2} - 3, z = 4t - 2$ between the points $(2, -1, 2)$ and $(1, -3, -2)$

12. Set up but do not evaluate a definite integral to find the length of the curve $x^2 + y^2 = z^2 - 4, x + y = 4$ joining the points $(4, 0, 2\sqrt{5})$ and $(2, 2, 2\sqrt{3})$. Simplify the integrand as much as possible.

Answers:

1. $\sqrt{6/7}$
2. $1/\sqrt{14}$
3. $\sqrt{629}/2$
4. $(4/3, 2, 4/3)$
5. $6\hat{\mathbf{i}}+\left(1/\sqrt{7}\right)\hat{\mathbf{j}}+(2/7)\hat{\mathbf{k}}$.
6. $(t^2 - 2t + 3)e^t\hat{\mathbf{i}} + \left[1/2(t^2 + 1)\right]\hat{\mathbf{j}} - \frac{1}{5}(t^2 + 1)^{5/2}\hat{\mathbf{k}} + \mathbf{C}$, where $\mathbf{C}$ is a constant vector.
7. (a) $x = \cos t, y = -\sin t, z = 2, 0 \le t \le \pi$.
   (b) $x = 2\cos t, y = 1 - 2\cos t, z = 2\sin t, \pi/3 \le t \le \pi/2$.
   (c) $x = 2\cos t, y = 2 - 2\sin t, z = 8(1 - \sin t), 0 \le t \le 2\pi$.
8. $\pm\frac{1}{\sqrt{3}}(-\hat{\mathbf{i}}+\hat{\mathbf{j}}+\hat{\mathbf{k}}.)$
9. $\frac{1}{\sqrt{10}}(\hat{\mathbf{i}}+3\hat{\mathbf{k}}.)$
10. $\arccos\left(\frac{-21}{\sqrt{14}\sqrt{297}}\right)$.
11. $\frac{2}{\sqrt{27}}\left(26^{3/2} - 17^{3/2}\right)$.
12. $2\int_2^4 \sqrt{\frac{t^2-4t+7}{t^2-4t+10}}dt$