THE UNIVERSITY OF MANITOBA

1:30 pm to 4:30 pm, April 25, 2007	Final Examination: Location depends on sections
Paper No: 720, 721, 722	Page No: Page 1 of 5
Dept. and Course No.: Eng 1460	Time: 3 Hours
Examination.: Thermal Science	Examiners: Bibeau, Fraser and Bartley

- Answer **5 out of the 6** questions in this exam. Each problem worth is indicated in the margin.
- Follow the "Problem-Solving Technique" discussed in class.
- The exam is 3 hours long, open book, and use of a calculator is permitted.
- You are allowed 8 crib sheets or the equivalent write up in your text book.
- When you are finished, take time to review your work and double check units and formulas; make sure you have assumptions in every problem solution, have made a sketch, have shown the units and inputs, and have drawn a T-V or P-V diagram, if applicable.
- Ask for clarification in any problem statement is unclear to you.
- Retain all the significant figures of properties taken from tables. Final results should have at least 3 to 5 significant digits.
- Use constant specific heats and state your assumptions.

Values

20

Problem #1: An ideal gas having a C_v of 0.600 kJ/kg K, a specific gas constant of 0.300 kJ/kg K, and at State **1** has an initial pressure of P_1 = 300 kPa and a temperature of T_1 = 1000 K, is contained inside a frictionless piston/cylinder assembly, as shown in the figure, and undergoes a cyclic process as follows:

- Process 1-2: A **constant pressure** heat addition which doubles the specific volume from v_1 to $v_2 = 2\,v_1$,
- Process 2-3: A **constant volume** process which reduces the system pressure from P_2 = 300 kPa to P_3 = 100 kPa
- Process 3-1: A **constant temperature** compression from P_3 = 100 kPa to P_1 = 300 kPa to come back to State **1**.

3 (a) Draw a PV diagram of the cyclic indicating temperature isotherms, pressures, processes and state all assumptions

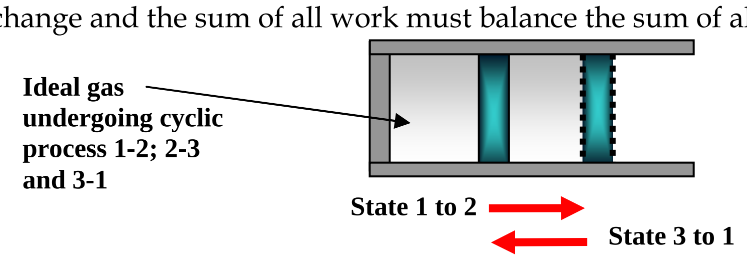
4 (b) Determine v_1, v_2, T_2

3 (c) Calculate the specific work, heat and change in internal energy, $w_{1\text{-}2}$, $q_{1\text{-}2}$ and Δu_{12}

3 (d) Calculate the specific work, heat and change in internal energy, $w_{2\text{-}3}$, $q_{2\text{-}3}$ and Δu_{23}

3 (e) Calculate the specific work, heat and change in internal energy, $w_{3\text{-}1}$, $q_{3\text{-}1}$ and Δu_{31}

4 (f) Check your answer by applying the first law to a cycle which says that the energy does not change and the sum of all work must balance the sum of all heat.



Values

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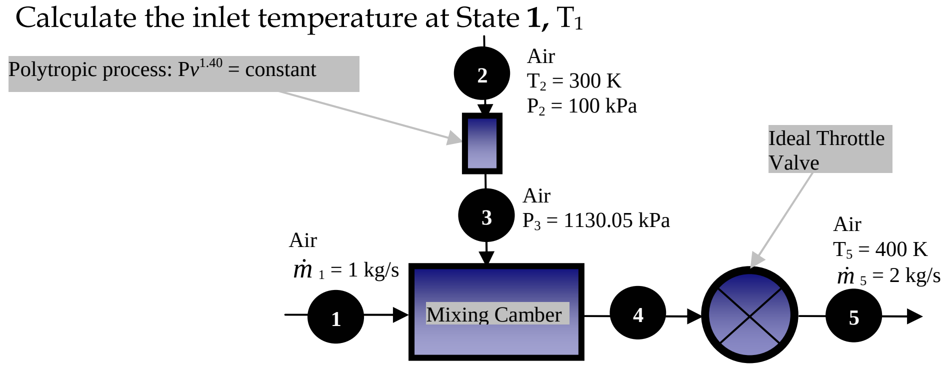
Problem #2: A first stream of air flows at $\dot{m}_1$ = 1 kg/s into a mixing chamber at State **1.** A second stream of air at P_3 = 1130.05 kPa also flows into a mixing chamber at State **3**. This second stream of air was previously compressed following a Polytropic process, $Pv^{1.40}$ = constant, using a steady-state compression device, as shown in the figure. The inlet conditions at State **2** are: P_2 = 100 kPa and T_2 = 300 K. The two air streams entering the mixing chamber are mixed, the air then exits the chamber at State **4,** and then flows through an ideal throttling valve, exiting at State **5**: T_5 = 400 K and $\dot{m}_5$ = 2 kg/s.

8 (a) Assuming that air is an ideal gas, determine at State **3,** T_3 and the mass flow rate through the compression device, $\dot{m}_3$ (Hint: apply the ideal gas law to substitute for v)

4 (b) Calculate the temperature at State **4**, T_4, and state your assumptions

8 (c) Calculate the inlet temperature at State **1,** T_1

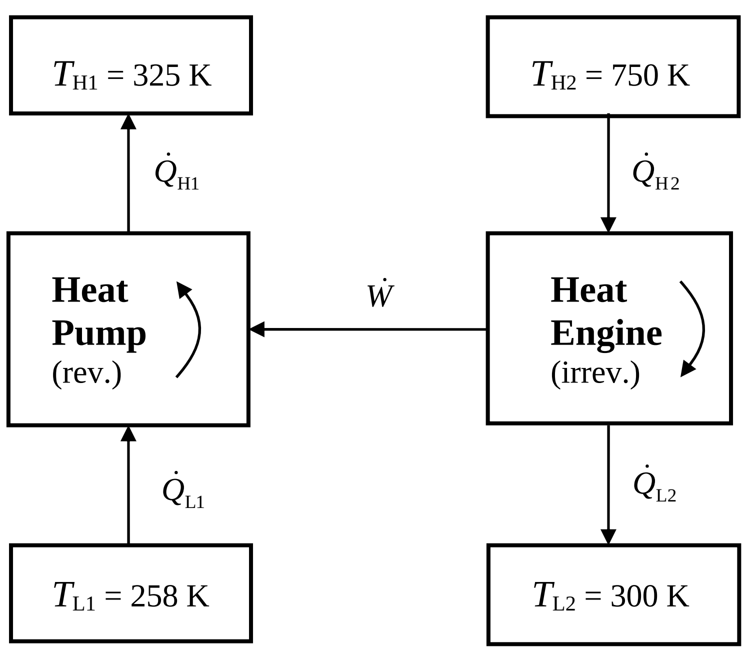


Values **Problem #3:**

20 The figure below shows a reversible Heat Pump that operates between two thermal reservoirs at T_{H1} = 325 K and T_{L1} = 258 K and requires a power input of $\dot{W}$ which is produced by an irreversible Heat Engine. The Heat Engine operates between two thermal reservoirs with the temperatures of T_{H2} = 750 K and T_{L2} = 300 K. Heat is transferred to the Heat Engine from a thermal reservoir at T_{H2} and at a rate $\dot{Q}_{H2}$; and heat is "pumped" by the Heat Pump to a thermal reservoir at T_{H1} at a rate $\dot{Q}_{H1}$. It is known that the sum of these two rates of heat transfer is $\dot{Q}_{H1} + \dot{Q}_{H2} = 350$ kW. Also, the thermal efficiency of the heat engine is 80% of the thermal efficiency of a reversible heat engine operating between the same thermal reservoirs at T_{H2} and T_{L2}, that is $\eta_{th} = 0.8 \cdot \eta_{th,rev}$.

12 (a) Calculate the thermal efficiency of the Heat Engine, η_{th}, and the coefficient of performance of the heat pump, β'_{HP}. Also determine the power, $\dot{W}$, for this system.

8 (b) Determine the rates of heat transfer, $\dot{Q}_{H1}$ and $\dot{Q}_{L1}$, for the Heat Pump, and rates of heat transfer, $\dot{Q}_{H2}$ and $\dot{Q}_{L2}$, for the Heat Engine.



Values **Problem #4:**

20 The figure below is a schematic of a steam power cycle. The thermal efficiency of the cycle is improved by pre-heating the supply water to the boiler with steam that has been partially expanded through the turbine to a lower pressure P_6 = 2 MPa. This pre-heating is accomplished by mixing the extracted steam (State 6) with water from Pump 1 (State 2) in the mixing chamber. The mixing process is assumed to occur adiabatically; *i.e.*, there is no heat transfer with the surroundings. The saturated liquid water from the condenser (State 1) is pumped to the mixing chamber to a pressure P_2 = 2MPa. Water leaves the mixing chamber as a saturated liquid and is then pumped by Pump 2 to a pressure P_4 = 10 MPa for the boiler. Heat is added to the water in the boiler at a rate $\dot{Q}_B$, at constant pressure, until State 5 is reached where P_5 = 10 MPa and T_5 = 800°C. Steam is expanded through the Turbine to produce power $\dot{W}_T$; part of the steam is extracted and fed to the Mixing Chamber at a rate $\dot{m}_6 = 10$ kg/s, and the remainder of the steam expands to State 7 with pressure P_7 = 40 kPa and quality x_7 = 0.95. The condenser removes heat from the steam at a rate $\dot{Q}_C$ at constant pressure. The total mass flow rate of the steam entering the Turbine is $\dot{m}_5 = 50$ kg/s. Assume that the Turbine, Pumps 1 and 2 and the Mixing Chamber are all adiabatic devices (insulated); also, changes is kinetic and potential energies may be neglected. (Note that all energy flow arrows for $\dot{W}$ and $\dot{Q}$ are shown according to the sign convention used in the course.)

7 (a) Determine the power required for Pump 1, $\dot{W}_{P1}$; and by considering the Mixing Chamber, determine the enthalpy and temperature for State 6, h_6 and T_6.

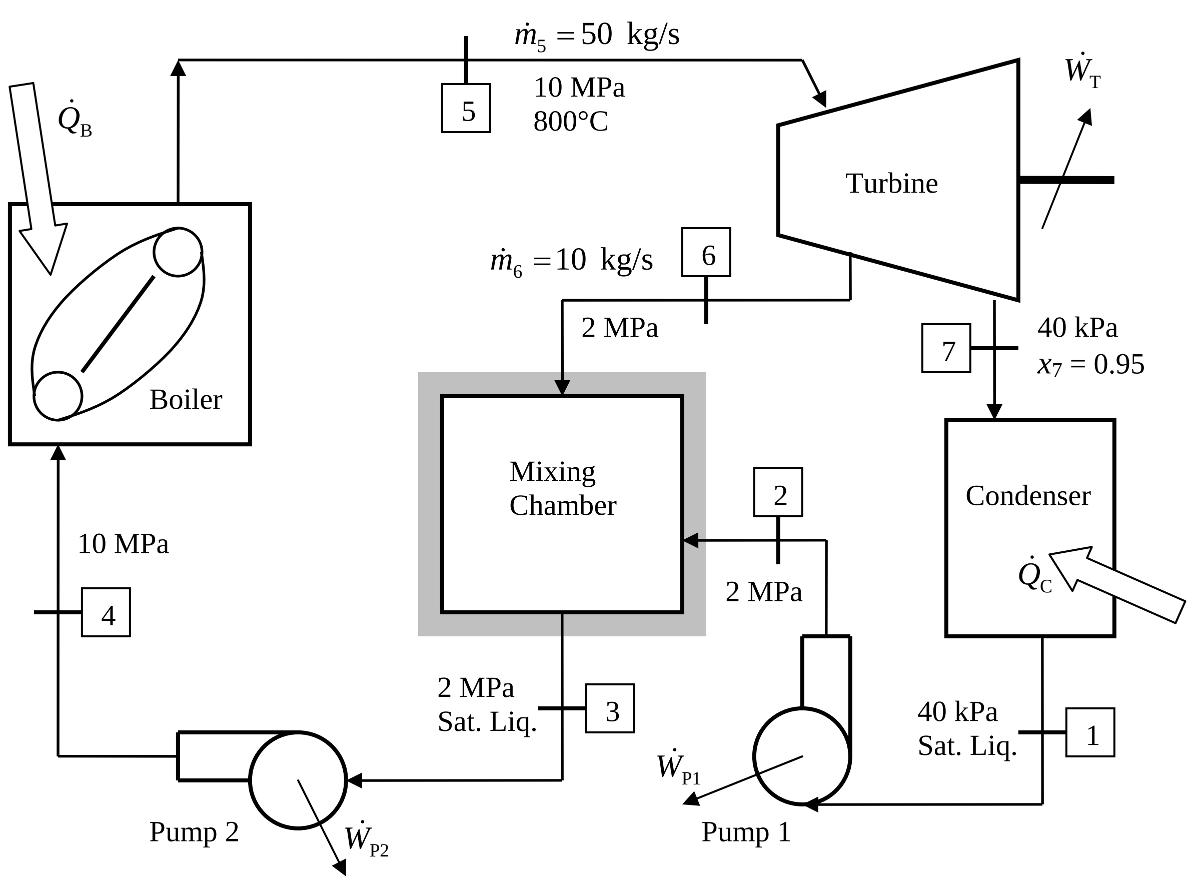
3 (b) Determine the power required for Pump 2, $\dot{W}_{P2}$, and the rate of heat transfer to the Boiler, $\dot{Q}_B$.

3 (c) Calculate the power produced by the Turbine, $\dot{W}_T$.

1.5 (d) Determine the rate of heat transfer from the Condenser, $\dot{Q}_C$.

2 (e) Calculate the thermal efficiency η_{th} for the steam power cycle.

3.5 (f) On a *T*-*v* diagram, carefully show and label all state points and process lines (omit showing a process for the mixing chamber). Label the constant pressure lines that pass through the state points and indicate the state temperature values and saturation temperature values as appropriate. Labelling of specific volumes is optional.



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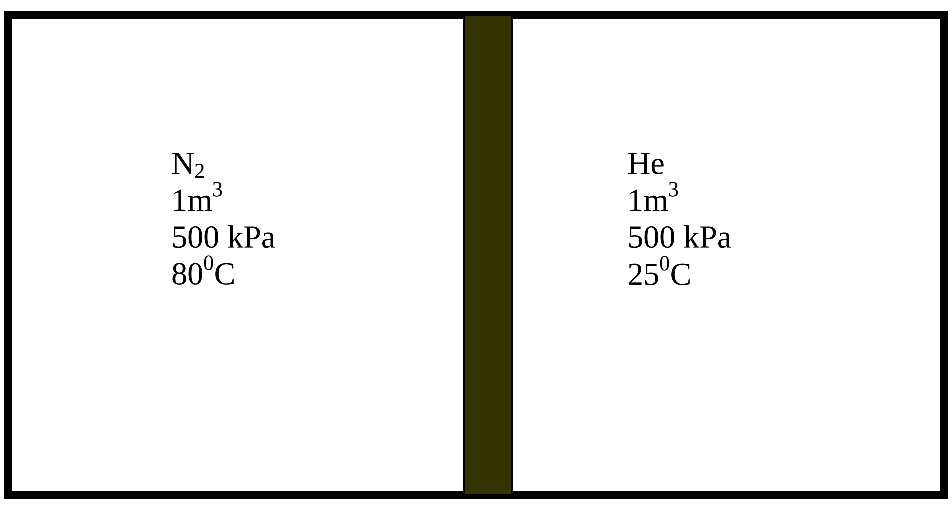
Values **Problem #5:**

20 Consider a well-insulated rigid cylinder as shown below. The cylinder is divided into two compartments by a piston that is free to move but does not allow either gas to get by it (e.g. leak to the other side). The two different gases on either side and their volumes are given in the figure. The piston however allows heat to cross it. Hence, once thermal equilibrium is established, determine the final equilibrium temperature in the cylinder. You can assume constant specific heats (Table A.5 Van Wylen and Sonntag etc.). Would your answer change if the piston was not free to move?



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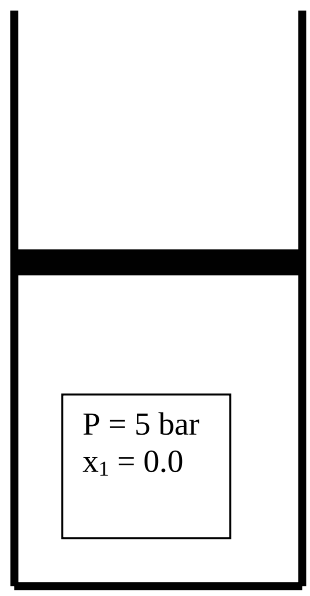
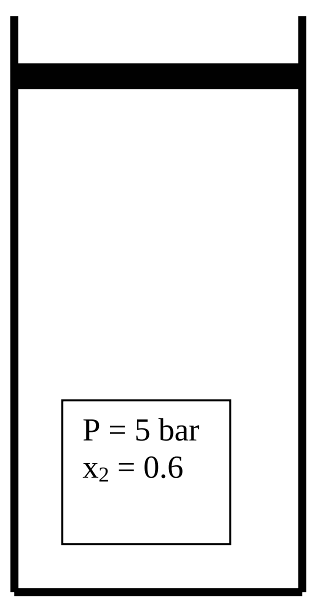
Values **Problem #6**:

20 Three (3) kilograms of saturated liquid water are contained in a constant pressure system at 5 bars (500 kPa) as shown below. Heat or energy is added to the system until the fluid has a quality of 60%. Determine the initial and final pressure or temperature. Also include the final volume and enthalpy changes. Show the equation for calculating the mass of the piston as a function of the cross sectional area of the cylinder. Note that the energy added could be by a stirrer or heat addition.





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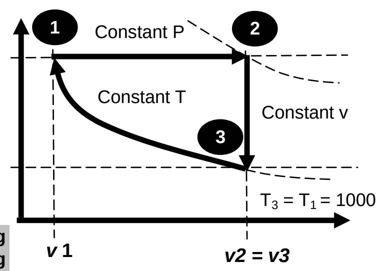
SOLUTIONS

Question 1: Constant Cv, ideal gas, close system

Cv	0.600	kJ/kgK
T1	1000	K
P1	300	kPa
Rg	0.300	kJ/kg K
P3	100	kPa
v2	2	v1



v1	RT1/P1	1.00	m3/kg
v2	=2v1	2.00	m3/kg
v3	=v2	2.00	m3/kg
T2	= T1 P2/P1 V2/V1	2000	K
T3	=T1	1000	K
P2	=P1	300	kPa
w12	=P1*(v2-v1)	300	kJ/kg
u12	=cv(T2-T1)	600	kJ/kg
q12	=du + w12	900	kJ/kg
w23	0 as dv = 0	0	kJ/kg
u23	=cv(T2-T3)	-600	kJ/kg
q23	=du	-600	kJ/kg
w31	R T3 ln(v1/v3)	-207.94	kJ/kg
u31	0 as dT = 0	0.00	kJ/kg
q31	q=w	-207.94	kJ/kg

U sum	0	kJ/kg
w sum	92.06	kJ/kg
q sum	92.06	kJ/kg

Question 2: Assume ideal gas, constant Cv, no enthalpy drop across throttle valve

Cv	0.737	kJ/kgK
Cp	1.032	kJ/kgK
n	1.400	kJ/kgK
m1	1	kg/s
T1	200.00	K
P2	100	kPa
T2	300	K
m2	1	kg/s
P3	1130.05	kPa
T3	600.00	K
m3	1	kg/s
m4	2	kg/s
T4	400	K
m5	2	kg/s
T5	400	K

$(P3/P2)^{n-1}\, T2^n = T3^n$

$dh_{45} = Cp(T5-T4) = 0 \quad T4 = T5$

m1 + m3 = m4 = m5

m1h1 + m3h3 = m4h4

h1 + h3 = 2h4

T1 = 2T4 - T3

The solutions for Questions 3 and 4 were not available for posting. Please contact the instructors directly regarding those problems.

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Question #5 Power Cycle — FE April 2007

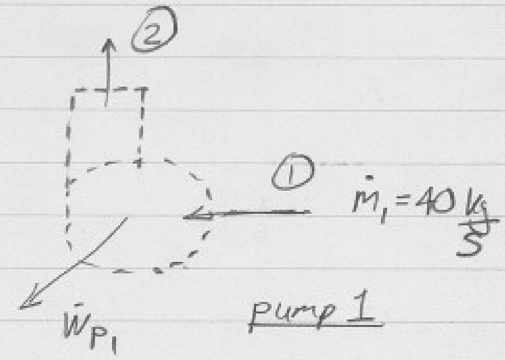
(a) State 1

$P_1 = 40\ kPa$, saturated liquid — Table B.1 (water)

$\therefore T_{sat.} = 75.87°C$

$v_f = 0.001026\ m^3/kg$

$h_f = 317.55\ kJ/kg$



conservation of energy for pump:

$$-\dot{W}_{P_1} = \dot{m}_1 (h_2 - h_1)$$

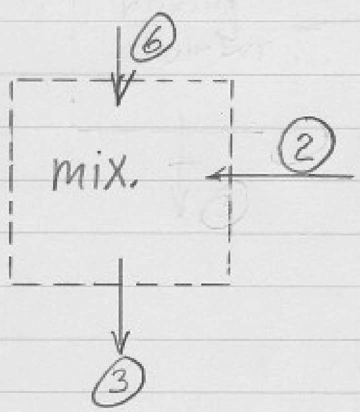
$$\cong \dot{m}_1 v_{f_1} (P_2 - P_1)$$

$$\therefore h_2 - h_1 \cong v_{f_1} (P_2 - P_1)$$

$$h_2 = 0.001026(2000-40) + 317.55 = 319.56\ \frac{kJ}{kg}$$

$$|\dot{W}_{P_1}| = 40 \times 0.001026 \times (2000-40) = 80.43\ kW$$

Mixing Chamber:



conservation of mass: $\dot{m}_6 + \dot{m}_2 = \dot{m}_3$

conservation of energy:

$$\cancel{\dot{Q}} - \cancel{\dot{W}} = \dot{m}_3 h_3 - (\dot{m}_2 h_2 + \dot{m}_6 h_6)$$

From analysis of the pump, $h_2 = 319.56\ \frac{kJ}{kg}$

state 3 $P_3 = 2\ MPa$, saturated liquid

$T_{sat.} = 212.42°C$, $v_f = 0.001177\ m^3/kg$

$h_f = 908.77\ kJ/kg$

$\dot{m}_6 = 10\ kg/s$

$\dot{m}_2 = 40\ kg/s$

$\dot{m}_3 = 50\ kg/s$

$\left(\dot{m}_1 = \dot{m}_2 = \dot{m}_5 - \dot{m}_6 = 50 - 10\ kg/s\right)$

$$\dot{m}_6 h_6 = \dot{m}_3 h_3 - \dot{m}_2 h_2$$

$$\therefore h_6 = \frac{50 \times 908.77 - 40 \times 319.56}{10}$$

$$h_6 = 3265.61\ kJ/kg$$

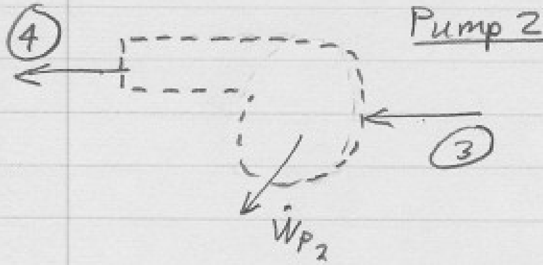
State 6 $h_6 > h_g$ at 2000 kPa $\therefore$ superheated vapor

#5 Solution, continued

Interpolate for T_6:

T	h
400	3247.60
T_6	3265.61
450	3357.48

$\therefore T_6 = 408.19\,°C$



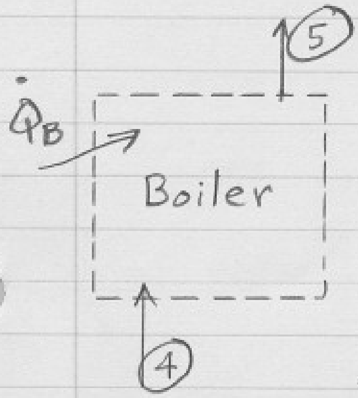
energy equation:

$$-\dot{W}_{P_2} = \dot{m}_3 (h_4 - h_3)$$
$$\cong \dot{m}_3\, h_{f_3} (P_4 - P_3)$$
$$\therefore |\dot{W}_{P_2}| = 50 \times 0.001177 \times (10000 - 2000)$$
$$= 470.80 \text{ kW}$$
$$470.80 = 50 \times (h_4 - 908.77)$$
$$\therefore h_4 = 918.18 \text{ kJ/kg}$$

state 4 is compressed liquid

(could interpolate in B.1.1 on h_F column to Find T_4: i.e., use $h_F = 918.18 \Rightarrow T_4$)

$T_4 \cong 214.46\,°C$



conservation of energy for Boiler,

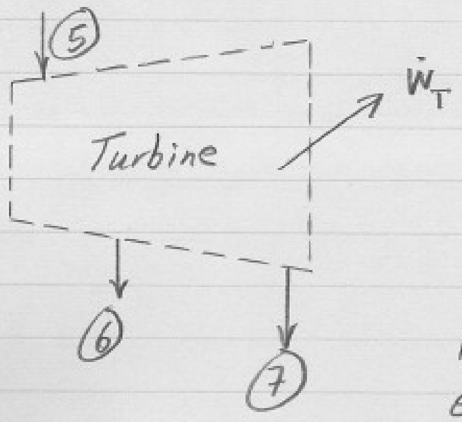
$$\dot{Q}_B = \dot{m}_4 (h_5 - h_4)$$
$$\dot{Q}_B = 50 \times (4114.91 - 918.18)$$
$$\dot{Q}_B = 159\,836 \text{ kW}$$

state 5: $P_5 = 10$ MPa

$T_5 = 800\,°C$

superheated vapor

$\therefore h_5 = 4114.91$ kJ/kg.



state 7: $P_7 = 40$ kPa, $x_7 = 0.95$

$\therefore T_{sat.} = 75.87\,°C$

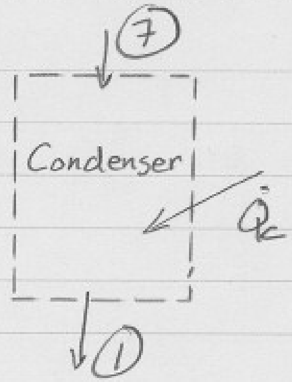
$$h_7 = h_f + x_7 h_{fg} = 317.55 + 0.95 \times 2319.19$$
$$h_7 = 2520.78 \text{ kJ/kg}$$

mass: $\dot{m}_5 = \dot{m}_6 + \dot{m}_7$

energy: $\cancel{\dot{Q}_T} - \dot{W}_T = (\dot{m}_6 h_6 + \dot{m}_7 h_7) - \dot{m}_5 h_5$

$$\therefore -\dot{W}_T = (10 \times 3265.61 + 40 \times 2520.78) - 50 \times 4114.91$$
$$\dot{W}_T = 72\,258.2 \text{ kW}$$

#5 Solution, continued



energy: $\dot{Q}_c = \dot{m}_1 (h_1 - h_7)$, $\dot{m}_1 = \dot{m}_7 = 40 \frac{kg}{s}$

$= 40 \times (317.55 - 2520.78)$

$\therefore \dot{Q}_c = -88129.2$ kW

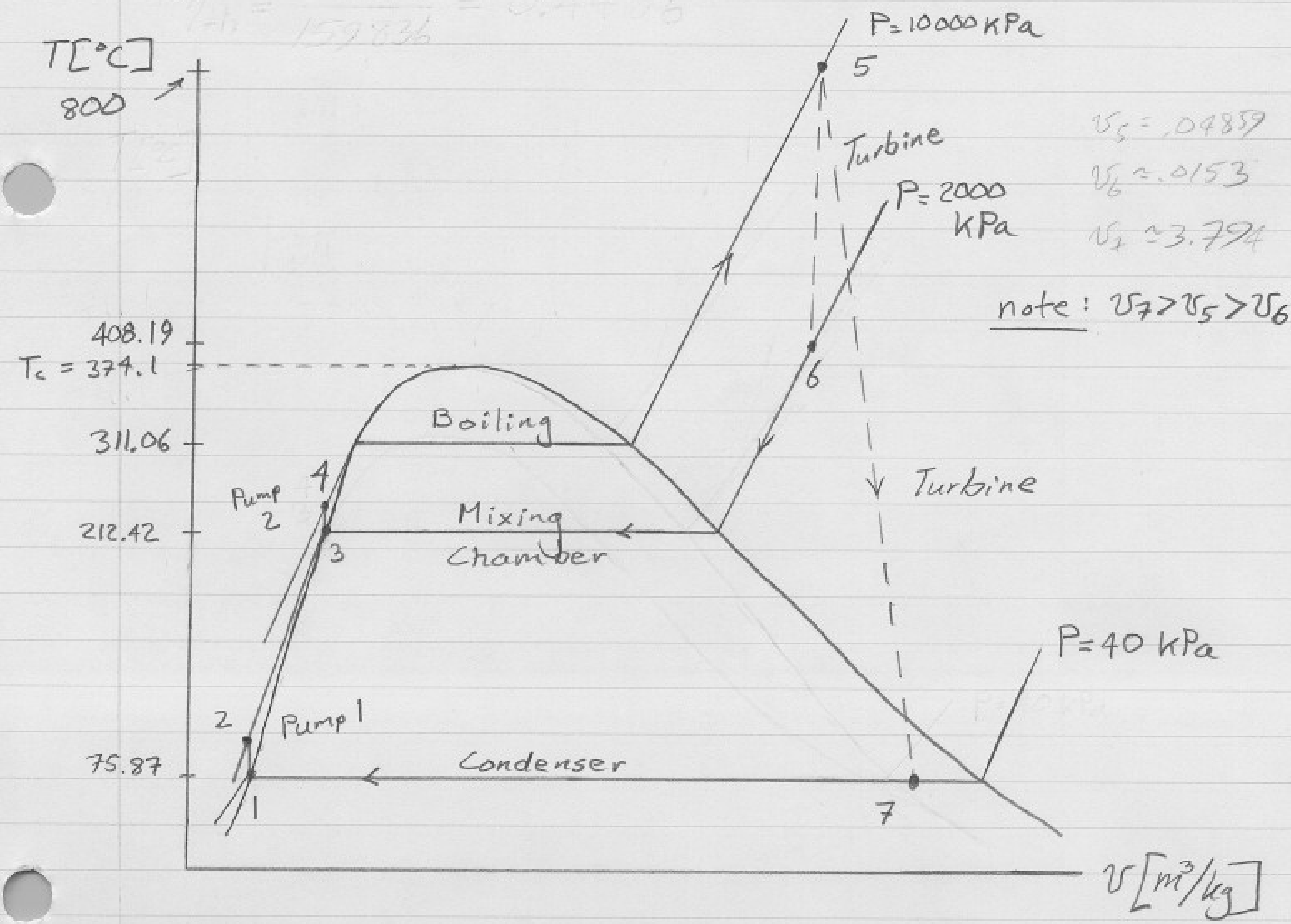
↑ indicates heat removed.

Net Power output: $\dot{W}_T - (|\dot{W}_{P_1}| + |\dot{W}_{P_2}|)$

$\dot{W}_{T,net} = 72258.2 - (80.43 + 470.80) = 71706.9$ kW

Cycle Thermal Efficiency: $\eta_{th} = \frac{\dot{W}_{T,net}}{\dot{Q}_B} = \frac{71706.9}{159836}$

→ $\eta_{th} = 0.4486$



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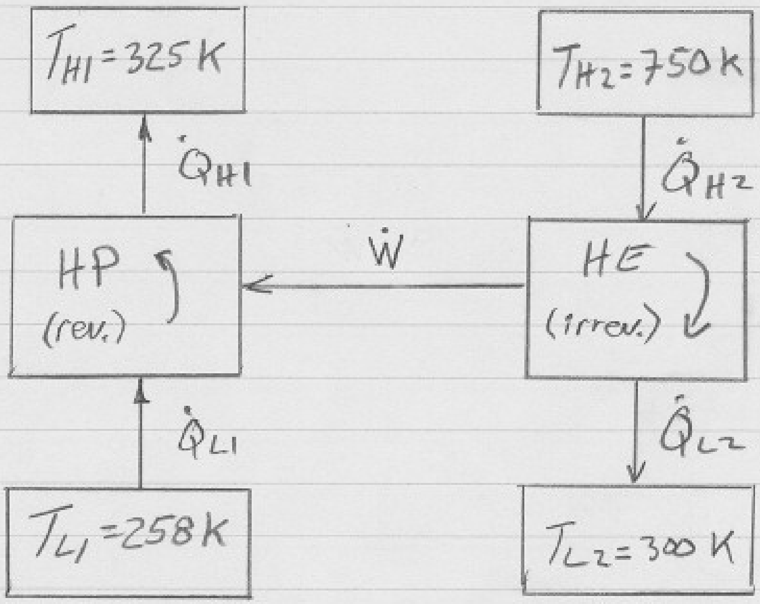
Question # 6.



given: $\dot{Q}_{H_1} + \dot{Q}_{H_2} = 350\ kW$ Eq. (1)

$\eta_{th} = 0.8\,\eta_{th,rev}$ Eq. (2)

(a)

HE $\eta_{th,rev} = \frac{T_{H_2} - T_{L_2}}{T_{H_2}} = \frac{750-300}{750} = 0.6$

$\eta_{th} = \frac{\dot{W}}{\dot{Q}_{H2}}$ Using Eq. (2), $\dot{W} = 0.8 \times 0.6 \times \dot{Q}_{H_2}$

$\eta_{th} = 0.8 \times 0.6 = 0.48$ or, $\dot{Q}_{H_2} = \frac{\dot{W}}{0.48}$ Eq. (3)

HP

$\beta'_{HP\,(rev)} = \frac{T_{H1}}{T_{H1} - T_{L1}} = \frac{325}{325-258} = 4.8507$

$\beta'_{HP} = \frac{\dot{Q}_{H1}}{\dot{W}}$ $\therefore\ \dot{Q}_{H1} = 4.8507 \times \dot{W}$ Eq. (4)

Substitute (3) and (4) into (1)

#6, solution continued

$$4.8507\,\dot{W} + \frac{\dot{W}}{0.48} = 350$$

Solve for $\dot{W}$, $\dot{W} = 50.47$ kW

(b) ∴ using Eq. (4), $\dot{Q}_{H1} = 4.8507 \times 50.47 = 244.81$ kW

using Eq. (3), $\dot{Q}_{H2} = \dfrac{50.47}{0.48} = 105.14$ kW

(c) $\beta'_{HP} = \dfrac{T_{H1}}{T_{H1}-T_{L1}} = \dfrac{325}{325-258} = 4.8507$

$\beta'_{HP} = \dfrac{\dot{Q}_{H1}}{\dot{Q}_{H1}-\dot{Q}_{L1}}$ ∴ $\dot{Q}_{L1} = -\dfrac{\dot{Q}_{H1}}{\beta'_{HP}} + \dot{Q}_{H1}$

$$\dot{Q}_{L1} = -\frac{244.81}{4.8507} + 244.81$$

$$\dot{Q}_{L1} = 194.34 \text{ kW}$$

$$\eta_{th} = \frac{\dot{Q}_{H2}-\dot{Q}_{L2}}{\dot{Q}_{H2}} = 1 - \frac{\dot{Q}_{L2}}{\dot{Q}_{H2}}$$

$$\therefore \dot{Q}_{L2} = -(\eta_{th} - 1)\,\dot{Q}_{H2}$$

$$= (1-0.48) \times 105.14$$

$$\dot{Q}_{L2} = 54.67 \text{ kW}$$