

### Math 1710 Tutorial 3. Volumes. Length of curves. Work.

**Problem 1.** Find the volume of the solid obtained by rotating the region bounded by the given curves about the specified line. In each case, you may use either the “washer” method or the “cylindrical shells” method, but should identify the “preferred” method.

- (a)  $y = 2 - |x + 1|$ ,  $y = 0$ ; about the  $x$ -axis.
- (b)  $x + 4y = y^2 + 4$ ,  $y = x$ ; about  $y = -1$ .
- (c)  $y = \ln x$ ,  $y = (x - 1) \ln 2$ ; about  $x = -1$  (the  $x$ -coordinates of the points of intersection of the two curves are  $x = 1$  and  $x = 2$ ; bonus problem: prove that there are no other intersection points).

**Problem 2.** Set up, but do NOT evaluate, a definite integral to find the length of the portion of the curve:

- (a)  $y^2 - x^2 = 4$  between the points  $(-1, -\sqrt{5})$  and  $(2, -2\sqrt{2})$ ;
- (b)  $x^3 + 3y^3 = 4$  between the lines  $y = 1$  and  $y = 2$ .

**Problem 3.** Find the length of the portion of the curve:

- (a)  $y = x^2 - \frac{1}{8} \ln x$  between the lines  $x = 1$  and  $x = e$ ;
- (b)  $27(y + 5)^2 = (x - 1)^3$  where  $-7 \leq y \leq -6$ .

**Problem 4.** A spring has a natural length of 20 *cm*. If a 25 *N* force is required to keep it stretched to a length of 30 *cm*, how much work is required to stretch it from 20 *cm* to 25 *cm*?

**Problem 5.** A circular swimming pool has a diameter of 10 *m*, the sides are 2 *m* high, and the depth of the water is 1.5 *m*. Find the work required to pump all of the water out over the side.

**Problem 6.** Find the work done to lift one end of a chain of mass 20 *kg* and length 10 *m*, lying initially on the ground, vertically upward to a height of 15 *m* above ground level (you may ignore friction).