Math 2130 - Engineering Mathematical Analysis 1

Tutorial 6 - Questions for §12.10 and 12.11.

12.10.1. Find all critical points for the function $f(x,y) = x^3y^3 - x^2y^2 + 6$.

12.10.2. Find all critical points for the function $f(x,y) = x^3y^2 - xy + 3y$.

12.10.3. You are given that $(0,0)$ and $(-1/3, -1/3)$ are critical points of the function

$$f(x,y) = x^3 + xy + y^3.$$

Classify each critical point as yielding a relative maximum, a relative minimum, or a saddle point.

12.10.4. Find all critical points of the function $f(x,y) = x^3 - xy^2 + 3xy$. Classify each critical point as yielding a relative maximum, a relative minimum, or a saddle point.

12.10.5. Find all critical points of the function

$$f(x,y) = x^4 - 3x^2y^2 + y^4.$$

Classify each critical point as giving a relative maximum, a relative minimum, or a saddle point.

12.11.1. Find the maximum value of the function $f(x,y) = x^2 - y^2$ on the region $x^2 + y^2 \leq 1$.

12.11.2. Find the maximum value of the function $f(x,y) = xy(3 - x - 2y)$ on the triangle R bounded by the positive x- and y-axes and the line $x + y = 1$. Assume that $f(x,y)$ has no critical points in the interior of R.

12.11.3. Find the maximum value of the function $f(x,y) = x^2 - y^2 + x$ considering only points inside and on the boundary of the region bounded by the curves:

$$x = \sqrt{1 - y^2}, \quad x = 0.$$

Answers:

12.10.1: Every point on x-axis, every point on y-axis, and every point on the curve $y = 2/(3x)$.

12.10.2: $(3, 0)$, $(9, 1/243)$.

12.10.3: $(0, 0)$ gives a saddle point; $(-1/3, -1/3)$ gives a relative maximum.

12.10.4: $(0, 0)$ gives a saddle point; $(0, 3)$ also gives a saddle point.

12.10.5: $(0, 0)$ gives a saddle point.

12.11.1: 1.

12.11.2: $2\sqrt{3}/9$.

12.11.3: 2.
