Math 1710: Practice Problems.

Fluid Pressure

Problem 1. A plate is in the shape of an isosceles triangle with equal sides of length $13m$ and the third side of length $10m$. It is suspended vertically in water with its shortest side lying on the surface of the water. Find the force due to the water on one side of the plate.

Problem 2. Set up a definite integral for the hydrostatic force on a flat circular plate of radius $2m$ that is vertically submerged $6m$ in the water ($6m$ from the top of the plate to the water surface).

Problem 3. Find the total fluid force exerted on one face of a flat square plate with side length $3m$ and one diagonal being horizontal, if the plate is immersed vertically into water so that the top of the plate is $1m$ above the surface of the water.

Problem 4. The vertical surface of a dam exposed to the water of a lake has the shape shown on Figure 1. Find the force of the water on the face of the dam.

Problem 5. A flat surface is bounded by the curves

$$y = 3 - x^2, \quad y = -1,$$

where all dimensions are in metres. The plate is submerged in water with its upper most point $1m$ below the surface. Find the force due to water pressure on each side of the plate.

Problem 6. The vertical face of a dam across a river has the shape of a semicircle shown on Figure 2. What is the force that the river exerts on the dam if the water level is $6m$ below the top?
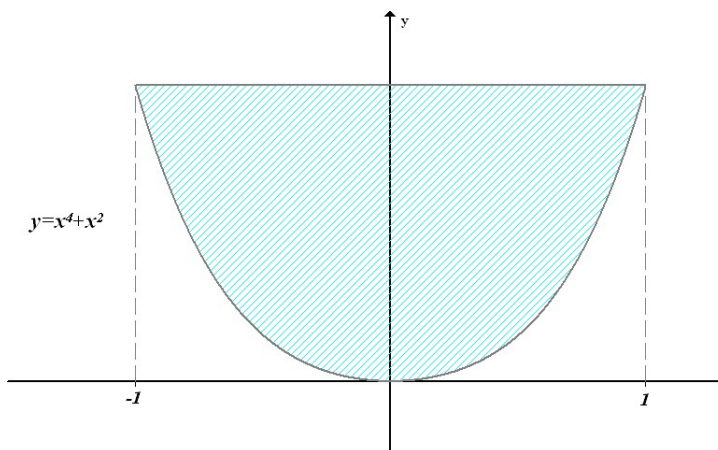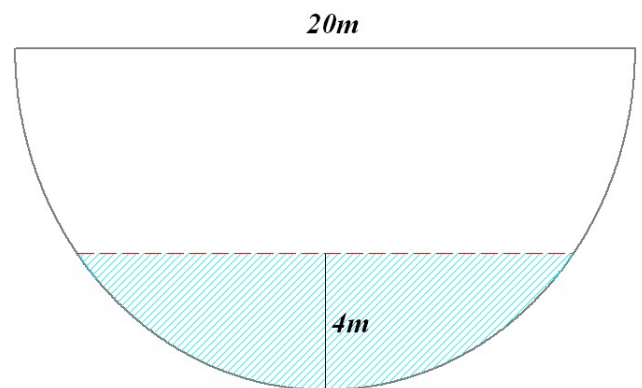

Fig. 1   Fig. 2

Centers of Mass and Centroids

Problem 7. Find the first moment of a plate with constant mass per area ρ that is bounded by the given curves about the suggested line:

(a) $y = \dfrac{1}{x}$, $x = 2$, $y = 2$ about $x = 0$;

(b) $y = \dfrac{1}{x}$, $y = \dfrac{1}{4 - 3x}$, $y = 2$ about $x = 0$ **(set up only)**;

(c) $y = x^5$, $x = y^5$ (the part of the region in the first quadrant only) about $y = 0$;

(d) $y = x$, $y = 3x$, $x + y = 8$ about $y = 0$;

(e) $x = y^2 - 2y + 4$, $x - 3y + 2 = 0$ about $x = 1$;

(f) $x = y^2 - 2y + 4$, $x - 3y + 2 = 0$ about $y = -1$;

(g) $y = \sin x^2$ $(0 \le x \le \sqrt{\pi})$, $y = 0$, about $x = 0$;

(h)* $x + 2 = y^2$, $y = x$ about $x + y = 1$.

Problem 8. Find the centroid of the region bounded by the curves:

(a) $y = \dfrac{1}{x}$, $x = 2$, $y = 2$;

(b) $y = x^4$, $y = \sqrt{x}$;

(c) $y = x^2 - 2$, $y = x$, $y = -x$;

(d) $(x - 3)^2 + y^2 = 4$;

(e) $x = y^2 - 2y + 4$, $x - 3y + 2 = 0$;

(f) $y = \sin x$ $(0 \le x \le \pi/2)$, $x = \pi/2$, $y = 0$ **(set up only)**;

(g) $y = \sqrt{x}$, $x = y + 2$, $y = 0$;

(h) $y = \sqrt{1 - x^2}$ $(0 \le x \le 1)$, $x = 0$, $y = 0$.