MATH 2130 Problem Workshop 12

1. Find the area bounded by $(x^2 + y^2)^3 = 4a^2x^2y^2$ where $a > 0$ is a constant.

2. Find the double integral of $f(x, y) = xy(x + y)$ over the region in the first quadrant bounded by $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$.

3. Evaluate the triple integral of the function $f(x, y, z) = x$ over the volume bounded by the surfaces
$$2x + 3y + z = 6, x = 0, y = 0, z = 0.$$

4. Find the volume in the first octant bounded by the surfaces
$$4x + 4y + z = 16, \qquad z = 0, \qquad y = x/2, \qquad y = 2x.$$

5. Set up, but do not integrate, a triple iterated integral for the volume in the first octant bounded by the surfaces
$$z = 2x + y, \qquad 9x^2 + 4y^2 = 1, \qquad x = 0, \qquad y = 0, \qquad z = 0.$$

6. Set up but do not evaluate a triple iterated integral for the volume bounded by the surfaces
$$z = 9 - x^2 - y^2, \qquad z = x^2$$

7. Find the volume bounded by the surfaces
$$z = xy, \qquad x^2 + y^2 = 1, \qquad z = 0.$$

8. Find the volume bounded by the surfaces
$$z = 2\sqrt{x^2 + y^2}, \qquad z = 9 - x^2 - y^2$$
(Do not simplify your numerical answer.)

9. Set up but do not evaluate a triple iterated integral for the triple integral of the function $f(x, y, z) = x^2$ over the region bounded by the surfaces
$$(x^2 + y^2)^2 = 2xy, \qquad z = \sqrt{1 - x^2 - y^2}, \qquad z = 0.$$

10. Evaluate $\iiint_V x^2 dV$ where V is the region bounded by the xz-plane and the hemispheres $y = \sqrt{9 - x^2 - z^2}$ and $y = \sqrt{16 - x^2 - z^2}$.

11. Find the volume and centroid of the region V that lies above the cone $z = \sqrt{x^2 + y^2}$ and below the sphere $x^2 + y^2 + z^2 = 1$.

Answers:

1. $\pi a^2/2$

2. $62/15$

3. $9/2$

4. $128/9$

5. $\int_0^{1/3} \int_0^{(1/2)\sqrt{1-9x^2}} \int_0^{2x+y} \, dz \, dy \, dx$

6. $4 \int_0^{3/\sqrt{2}} \int_0^{\sqrt{9-2x^2}} \int_{x^2}^{9-x^2-y^2} dzdydx$

7. $1/2$

8. $2\pi \left[\frac{9(\sqrt{10}-1)^2}{2} - \frac{(\sqrt{10}-1)^4}{4} - \frac{2(\sqrt{10}-1)^3}{3} \right]$

9. $2 \int_0^{\pi/2} \int_0^{\sqrt{\sin 2\theta}} \int_0^{\sqrt{1-r^2}} r^3 \cos^2 \theta dzdrd\theta$

10. $\frac{1562}{15}\pi$

11. $\frac{\pi}{3}(2-\sqrt{2}), \big(0, 0, \frac{3}{8(2-\sqrt{2})}\big)$