MATH 1210 Assignment 3 (Section A02) Fall 2016 **Due date: December 12**

Attempt all questions and show all your work. Attach to Honesty Declaration Form.

1. **For each of the following linear system of equations, first find reduced row echelon form of the augmented matrix and then find all solutions of the system.**

$$(i)\quad \begin{aligned} x \qquad\quad +z-w &= 1\\ -x+y \quad -z+w &= 0\\ 2y \quad +z+w &= 1\\ 3y \quad +z+w &= 2 \end{aligned} \qquad\qquad (ii)\quad \begin{aligned} 3x+5y&=1\\ 2x+y&=3\\ 4x+2y&=6\\ 5x+6y&=4 \end{aligned}$$

$$(iii)\quad \begin{aligned} x_1 \qquad\quad -x_3-x_4 &= 5\\ -2x_1+x_2 \quad +3x_3+4x_4 &= -2\\ 3x_1+x_2 \quad -2x_3-x_4 &= 9 \end{aligned} \qquad\qquad (iv)\quad \begin{aligned} \frac{1}{x}+\frac{1}{y}+\frac{1}{z}&=4\\ \frac{2}{x}-\frac{3}{y}-\frac{1}{z}&=1\\ -\frac{1}{x}+\frac{2}{y}+\frac{1}{z}&=0 \end{aligned}$$

2. **Find all *basic* solutions of the homogeneous system**

$$\begin{aligned} 9x_1+9x_2+4x_3-4x_4+9x_5-10x_6 &= 0\\ 2x_1+2x_2+x_3-x_4+2x_5-3x_6 &= 0\\ 11x_1+11x_2+3x_3+4x_4+4x_5+22x_6 &= 0 \end{aligned}$$

3. **For any real number n let $A=\begin{pmatrix} n+1 & n+2 & n+3\\ n+4 & n+5 & n+6\\ n+7 & n+8 & n+9 \end{pmatrix}$. Use properties of determinant to find $|A|$. (Explain)**

4. **Let $A=\begin{pmatrix} 1&1&1&0\\ 1&1&0&1\\ 1&0&1&1\\ 0&1&1&1 \end{pmatrix}$. Find $det(-10A)$.**

5. **Use Cramer's rule to solve each of the following linear system of equations.**

(a)
$$\begin{aligned} 3x_1+4x_2 \quad +4x_3 &= 11\\ 4x_1-4x_2 \quad +6x_3 &= 11\\ 6x_1-6x_2 \qquad &= 3 \end{aligned}$$

(b)
$$\begin{aligned} 3y+z-u &= 4\\ y+z &= 0\\ 2y-z+4u &= 6\\ x+5z &= 1 \end{aligned}$$
(For y *only*, so *do not* find x, z and u.)

6. **Determine whether each set of vectors is linearly dependent or linearly independent.**
   (a) $\{\mathbf{u}_1 =< 4,5 >, \mathbf{u}_2 =< 3,10 >, \mathbf{u}_3 =< -11,20 >\}$
   (b) $\{\mathbf{u}_1 =< 4,3,6 >, \mathbf{u}_2 =< 5,7,1 >, \mathbf{u}_3 =< -1,-4,5 >\}$
   (c) $\{\mathbf{u}_1 =< 1,2,-2,4 >, \mathbf{u}_2 =< 1,3,-3,4 >, \mathbf{u}_3 =< 1,2,-1,4 >\}$

7. **Let $\mathbf{u}_1 =< 1,2,1,3 >$, $\mathbf{u}_2 =< 1,3,1,3 >$, $\mathbf{u}_3 =< 4,0,0,0 >$ and $\mathbf{u}_4 =< 6,-9,-2,-6 >$. Show that they are linearly dependent and then write $\mathbf{u}_4$ as a linear combination of $\mathbf{u}_1$, $\mathbf{u}_2$ and $\mathbf{u}_3$.**