THE UNIVERSITY OF MANITOBA

DATE: April 11, 2011 (Morning) FINAL EXAMINATION

DEPARTMENT & COURSE NO: MATH2132 TIME: 3 hours

EXAMINATION: Engineering Mathematical Analysis 2 EXAMINER: D. Trim

4 ~~5~~ 1. Find the radius of convergence for the power series

$$\sum_{n=1}^{\infty} n(-1)^n 2^n (x+3)^{2n+1}$$

ans) $R = 1/\sqrt{2}$

6 ~~5~~ 2. Find a maximum possible error if e^{-2x^2} is approximated by the first three nonzero terms in its Maclaurin series on the interval $-0.05 \le x \le 0.5$.

ans) $\frac{4}{3}\left(\frac{1}{2}\right)^6 = 1/48$

1~~0~~ 3. Find the sum of the series $\sum_{n=2}^{\infty} \frac{n(-1)^n}{3^n} x^{2n}$.

ans) $\frac{x^4(6+x^2)}{3(3+x^2)}$; $-\sqrt{3}<x<\sqrt{3}$

10 4. Find the Taylor series about $x = 2$ for the function $\ln(x+1)$.
Express your answer in sigma notation simplified as much as possible. Include the open interval of convergence expressed in the form a < x < b

ans) $\ln 3 + \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{3^n n}(x-2)^n$

, $-1 < x < 5$

9 5. Find a one-parameter family of solutions of the differential equation

$$\sin 2x\frac{dy}{dx} = 4y\cos 2x + \sin^5 2x\cos 2x.$$

Is it a general solution? Explain.

ans) $y(x) = \frac{1}{6}\sin^5 2x + C\sin^2 2x$,

yes, DE is linear

2 1$\cancel{5}$ 6. Find a general solution for the differential equation

$$2\frac{d^2y}{dx^2} + \frac{dy}{dx} + 3y = xe^x.$$

ans) $y(x) = e^{-x/4}\left(C_1\cos\frac{\sqrt{23}}{4}x + C_2\sin\frac{\sqrt{23}}{4}x\right)$

$+ \frac{1}{36}e^x(6x-5)$

6 7. You are given that the roots of the auxiliary equation associated with the linear, differential equation

$$\phi(D)y = 3x^2 - 2 + 5\sin 2x + xe^{4x}$$

are $m = 0, \pm 2i, \pm 2i, \pm 3, 4 \pm 6i$. Write down the form of a particular solution of the differential equation as predicted by the method of undetermined coefficients. Do **NOT** find the coefficients, just the form of the particular solution.

ans) $y_p(x) = Ax^3 + Bx^2 + Cx + Dx^2\sin 2x + Ex^2\cos 2x + Fxe^{4x}$

$+ Ge^{4x}$

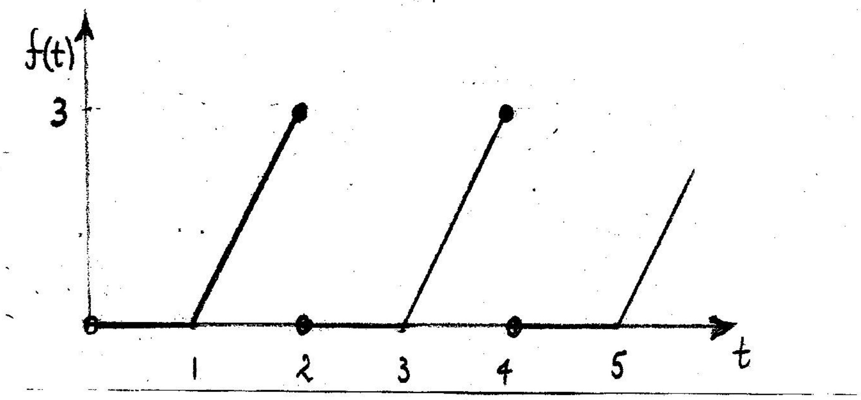
7 8. Find the Laplace transform for the function $t^2e^{5t}\,h(t-3)$.

ans) $e^{3(5-s)}\left[\frac{2}{(s-5)^3} + \frac{6}{(s-5)^2} + \frac{9}{s-5}\right]$

4 **9.** You are given that $f(0) = 2$ and $f'(0) = -4$. Write an expression for the Laplace transform of $e^{-3t} f''(t)$ in terms of the Laplace transform of $f(t)$.

ans) $(s+3)^2 F(s+3) - 2(s+3) + 4$

9 **10.** Find the Laplace transform for the periodic function shown below. Do not simplify your answer.



ans) $\frac{3}{1-e^{-2s}}\left[\frac{e^{-s}}{s^2} - e^{-2s}\left(\frac{1}{s^2}+\frac{1}{s}\right)\right]$

10 **11.** Find the inverse Laplace transform for the function

$$F(s) = \frac{e^{-3s}}{s^3 + 3s^2 + 5s + 3}.$$

ans) $\frac{1}{2} e^{-(t-3)}\left[1 - \cos\sqrt{2}(t-3)\right] h(t-3)$

12 **12.** Solve the following initial value problem

$$\frac{d^2y}{dt^2} + 5\frac{dy}{dt} + 4y = 3\delta(t-2), \qquad y(0) = 2, \quad y'(0) = 1.$$

ans) $y(t) = \left[e^{-(t-2)} - e^{-(t-2)}\right] h(t-2) + 3e^{-t} - e^{-4t}$