Unit 7 Assignment

1. A standard deck of cards consists of 52 cards. There are 13 cards (2, 3, 4, 5, 6, 7, 8, 9, 10, Jack, Queen, King and Ace) in each of four suits (Clubs, Diamonds, Hearts and Spades).

   (a) You randomly select cards from the deck, one at a time **with replacement** (i.e. you replace the card after each draw and shuffle the deck). What is the probability that the first face card appears on the sixth draw? (A face card is a Jack, Queen or King).
   (b) If you randomly select cards **without replacement**, what is the probability that the first face card appears on the sixth draw?
   (c) If you randomly select cards **with replacement**, what is the probability that the fourth Club is chosen on the 15^{th} draw?
   (d) If you randomly select 12 cards **with replacement**, what is the probability that exactly three of them are Diamonds?
   (e) If you randomly select 20 cards **with replacement**, what is the probability that at least two of them are Aces?
   (f) If five people each randomly select 20 cards **with replacement**, what is the probability that three people get at least two Aces?
   (g) If you randomly select 15 cards **without replacement**, what is the probability that 3 of them are Spades?

2. Keno is a casino game in which the player selects eight numbers from the numbers 1 to 80. Twenty numbers are then drawn at random and the player wins a prize if she correctly matches at least three of the numbers. What is the probability that the player wins at least three times in her next ten games?

3. Repeat Question 4(c) from the Unit 5 Assignment (the probability that Roddick wins the match), using the negative binomial distribution.

4. A random variable is said to possess the "lack of memory property" if

$$P(X > s + t \mid X > s) = P(X > t)$$

   Show that a random variable X with a geometric distribution possesses this lack of memory property.

5. The number of crickets in a large field follows a Poisson distribution with at a rate of 2.7 per square meter.

   (a) What is the probability of observing at least three crickets in a randomly selected square meter of the field?
   (b) What is the probability of observing 15 crickets in a five square meter area of the field?
   (c) If we randomly select a circular area of the field, what must the radius of the circle be in order to have a 99% chance of observing at least one cricket?

6. Students in an 8:30 a.m. class fall asleep according to a Poisson process with parameter $\lambda = 0.1$ per minute.

   (a) What is the probability that first student falls asleep after 8:40?
   (b) If no students have fallen asleep by 8:50, what is the probability that the first student falls asleep after 9:00?
   (c) What is the interquartile range of the time between students falling asleep?
   (d) What is the probability that the fourth student falls asleep after 9:00? (Hint: There is an easier way to do this than using the distribution in (d)).

7. Swimming's 4 x 100 medley relay consists of teams of four swimmers, each swimming 100 meters in a different style (backstroke, breaststroke, butterfly and freestyle). From past experience, the four swimmers on Team Canada know that their race times follow normal distributions with the following means and standard deviations (in seconds):

Swimmer	μ	σ
Swimmer 1 (Backstroke)	54.2	0.7
Swimmer 2 (Breaststroke)	60.3	0.9
Swimmer 3 (Butterfly)	52.1	0.6
Swimmer 4 (Freestyle)	47.0	0.5

Times for all swimmers are known to be independent.

(a) In any given race, what is the probability that Swimmer 1 has a time less than 55 seconds?
(b) What is the probability that Swimmer 2 has a time between 60 and 62 seconds?
(c) What is the probability that Swimmer 3 has a time greater than 51 seconds?
(d) What is the 85^{th} percentile of Swimmer 4's times?
(e) In the last competition, Swimmer 1 had a time of 54.7 seconds, Swimmer 2 had a time of 59.5 seconds, Swimmer 3 had a time of 51.6 seconds and Simmer 4 had a time of 46.6 seconds. Which swimmer did the best relative to their past times?
(g) What is the probability that the team's total time is less than three and a half minutes?
(h) One of the teams that Canada is racing against is Team U.S.A. From past experience, the four American swimmers know that their race times follow normal distributions with the following means and standard deviations (in seconds):

Swimmer	μ	σ
Swimmer 1 (Backstroke)	53.6	0.8
Swimmer 2 (Breaststroke)	60.1	1.0
Swimmer 3 (Butterfly)	50.9	0.7
Swimmer 4 (Freestyle)	47.9	0.4

What is the probability that Team Canada beats Team U.S.A. in the race?

8. A coffee machine is regulated so it dispenses an average of μ ounces per cup. Fill volumes are normally distributed with a standard deviation of 0.2 ounces, and are independent for any two cups.

(a) What value should μ be set at so that 6-ounce cups will overflow only 2% of the time?
(b) What proportion of cups will contain less than 5.7 ounces of coffee?
(c) What proportion of cups will contain more than 5.2 ounces of coffee?
(d) What proportion of cups will contain between 5.5 and 5.8 ounces of coffee?
(e) What is the interquartile range of fill volumes?