ES201 Problem set #2 Solutions

(1a) M_B=800N · 100. mm = 80.0 N·m CCW($\curvearrowright$)

(1b) M_B= 0 = -450 mm (**P**)sin $70° - \alpha$ + 80.0 N·m

$\rightarrow$ P=205.3 N 800N $\rightarrow$ P = 205 N

(1c) The maximum moment applied about B occurs when the angle between AB and $\overline{\mathsf{P}} = 90°$, so in that case, the moment of $\overline{\mathsf{P}}$ about B = P·450 mm = 80.0 N·m:

$\rightarrow$ P=177.$\overline{7}$ N $\rightarrow$ P = 178 N

(2a) $\bar{\mathsf{u}}_{BA} = \frac{-0.1m}{3.61m^2}\imath + \frac{1.8m}{3.61m^2}\jmath + \frac{-0.6m}{3.61m^2}\hat{k} = \frac{-1}{19}\imath + \frac{18}{19}\jmath + \frac{-6}{19}\hat{k}$

so $\overline{\mathsf{F}}_{BA}$ = 228 N · $[\frac{-1}{19}\imath + \frac{18}{19}\jmath + \frac{-6}{19}\hat{k}]$

$\bar{\mathsf{r}}_{CA} = 0.96m\imath - 0.12m\jmath + 0.72m\hat{k}$

$\overline{\mathsf{M}}_C = \bar{\mathsf{r}}_{CA} \times \overline{\mathsf{F}}_{BA} = -146.9\mathsf{N}\cdot\mathsf{m}\imath + 60.5\mathsf{N}\cdot\mathsf{m}\jmath + 205.9\mathsf{N}\cdot\mathsf{m}\hat{k} = = -147\mathsf{N}\cdot\mathsf{m}\imath + 60.5\mathsf{N}\cdot\mathsf{m}\jmath + 206\mathsf{N}\cdot\mathsf{m}\hat{k}$

(2b) $\bar{\mathsf{r}}_{DA} = 0\imath - 0.12m\jmath + 0.72m\hat{k}$;

$\overline{\mathsf{M}}_D = \bar{\mathsf{r}}_{DA} \times \overline{\mathsf{F}}_{BA} = -146.9\mathsf{N}\cdot\mathsf{m}\imath - 8.6\mathsf{N}\cdot\mathsf{m}\jmath - 1.4\mathsf{N}\cdot\mathsf{m}\hat{k}$

(3) $\Sigma\overline{\mathsf{F}}_{body} = 450\ \mathsf{N}\imath - 450\ \mathsf{N}\imath$

Useful shortcut: the sides of a 30°, 60°, 90° triangle scale as 1, $\sqrt{3}$, 2, and the x and y components of vectors relevant to the hexagon scale this way as well.

The moment of $\overline{\mathsf{P}}$ about E is maximum when $\overline{\mathsf{P}} \perp$ the line EB, so the angle is the same as $\angle$FBE = 30° (via similar triangles), so α = 30°.

Then replacing moment and force combination with pure a force only: $\Sigma\overline{\mathsf{M}}_E = 0 = 450\ \mathsf{N}$(600 mm) - P$cos\alpha$(300$\sqrt{3}$ mm) - P$sin\alpha$(300 mm)

0 = 270 N·m - P·0.450 m - P·0.150 m

$\rightarrow$ P=450 N , α=30°

(4) The center of mass (really the location where the concentrated forces due to all the masses) occurs at the point where the moment about the point is zero. Since the desired center of mass is at point C, if I define d as the location of the center of mass to the right of C:

part (4a):

$\Sigma\overline{\mathsf{M}}_C$ = 0 = 38 kg(g)(2.00 m) - (d)(27kg)(g) - 29kg(g)(2.00 m) (where g is the acceleration due to gravity)

solve for d = $\frac{76m\cdot kg - 58m\cdot kg}{27kg} = \frac{2}{3}$ m to the right of C.

part (4b):

$\Sigma\overline{\mathsf{M}}_C$ = 0 = 38 kg(g)(2.00 m) - (d)(24kg)(g) - 29kg(g)(2.00 m)

solve for d = $\frac{76m\cdot kg - 58m\cdot kg}{24kg}$ = 0.75 m to the right of C.

(5a) The center of mass, again, occurs where $\Sigma\overline{\mathsf{M}}_{centerofmass} = 0$. Approaching it differently from problem #4,:

$\Sigma\overline{\mathsf{F}}_{body}$ = -1.8 kg(g)$\jmath$ -1.8 kg(g)$\jmath$ -1.6 kg(g)$\jmath$ = -5.2 kg(g)$\jmath$

$\Sigma\overline{\mathsf{M}}_D$ = -(100 mm)(1.8kg(g) - (1300mm)(1.8kg(g)-(2050mm)(1.6kg)(g) = -6.16 kg(g)m

Now if this force-moment concentration is shifted to the center of mass (center of force) from point D, the moment about that point is zero, so:

$\overline{\mathsf{M}}_{centerofmass} = \overline{\mathsf{M}}_D + \bar{\mathsf{s}} \times \overline{\mathsf{F}}_{body}$ = 0 = -6.16kg(g)·m - r[-5.2kg(g)], where r is the distance from D to the center of mass.

$\rightarrow$ r = 1.185 m from D

(5b) Similarly to how done in 5(a):
$\overline{\mathsf{M}}_{\text{center of mass}} = \overline{\mathsf{M}}_{\mathsf{D}} + \bar{\mathsf{s}} \times \overline{\mathsf{F}}_{\text{body}} = 0 =$ -(.300 m)(1.8kg)(g) - (1.300m)(1.8kg)(g) - (1.300m + d)(1.6kg)(g) + 5.2kg(g)(1.250m), taking d as the distance to the right of point B of point C.

$\rightarrow$ d = 0.9625 m from D $\rightarrow$ d = 963 mm from D