MATH 1210 ASSIGNMENT #4 SOLUTIONS

① (a) $\vec{v} = (2-1)\hat{i} + (1-0)\hat{j} + (3-(-1))\hat{k} = \hat{i} + \hat{j} + 4\hat{k}$

$\therefore$ $\left.\begin{aligned} x &= 1 + t \\ y &= 0 + t \\ z &= -1 + 4t \end{aligned}\right\}$ are parametric eqns for l_1

(b) $\vec{v} = (-1-1)\hat{i} + (2-0)\hat{j} + (0-(-1)) = -2\hat{i} + 2\hat{j} + \hat{k}$

$l_2 : \begin{cases} x = 2 - 2s \\ y = 1 + 2s \\ z = 3 + s \end{cases}$

(c) for l_1 to intersect l_2 we must have

$\left.\begin{aligned} x &= 1+t = 2-2s \\ y &= t = 1+2s \\ z &= -1+4t = 3+s \end{aligned}\right\} \Longrightarrow t = 1+2s$

$1+t = 2+2s = 2-2s$
$\Longrightarrow s = 0$
$\&\ t = 1$

if $s=0$, the point on l_2 is $(2,1,3)$
while if $t=1$, the point on l_1 is $(2,1,3)$
i.e., P_2 lies on both l_1 & l_2

(d) let $\vec{N} = \overrightarrow{P_2P_1} \times (\overrightarrow{P_1P_3})$
$= -7\hat{i} - 9\hat{j} + 4\hat{k}$
be the normal vector to π_1

Thus, the equation is $-7x - 9y + 4z = D$
but P_2 is on $\Longrightarrow D = -7(2) - 9(1) + 4(3)$
$= -14 - 9 + 12 = -11$

Thus, π_1 has equation : $-7x - 9y + 4z = -11$

or equivalently

$7x + 9y - 4z = 11$

(e) l_3: $x = 2 - 7r$

$y = 1 - 9r$

$z = 3 + 4r$

(f) since π_2 is parallel to π_1, it has equation

$7x + 9y - 4z = D_1$

But $(0,-5,0)$ is on $\longrightarrow$ $7(0) + 9(-5) - 4(0) = D_1 = -45$

$\therefore$ its equation is π_2: $7x + 9y - 4z = -45$

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(2) $$\underset{\sim}{A}\underset{\sim}{B} = \begin{pmatrix} 1 & -2 & 3 \\ -3 & 1 & 4 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 2 & 1 \\ -1 & 2 \end{pmatrix} = \begin{pmatrix} -6 & 4 \\ -5 & 9 \end{pmatrix}$$

$$\underset{\sim}{B}\underset{\sim}{A} = \begin{pmatrix} 1 & -2 & 3 \\ -1 & -3 & 10 \\ -7 & 4 & 5 \end{pmatrix}$$

note: $\underset{\sim}{A}\underset{\sim}{B} \neq \underset{\sim}{B}\underset{\sim}{A}$

$$BI_{(2)} = \underset{\sim}{B} \quad \text{so} \quad (BI_{(2)})\underset{\sim}{A} = \underset{\sim}{B}\underset{\sim}{A} = \begin{pmatrix} 1 & -2 & 3 \\ -1 & -3 & 10 \\ -7 & 4 & 5 \end{pmatrix}$$

$I_{(3)}\underset{\sim}{A}$ does not exist (since $I_{(3)}$ is a 3×3 matrix & $\underset{\sim}{A}$ is a 2×3 matrix).

$$\underset{\sim}{F}^T\underset{\sim}{E} = \begin{pmatrix} -1 & 0 & 2 \\ 2 & 2 & -1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ 2 & 0 & -1 \\ 1 & 1 & -2 \end{pmatrix} = \begin{pmatrix} 1 & 2 & -4 \\ 5 & -1 & 0 \end{pmatrix}$$

$CB + D$ is undefined since CB is a 3×2 matrix & D is a 2×2 matrix

$$AB + 2D = \begin{pmatrix} -6 & 4 \\ -5 & 9 \end{pmatrix} + \begin{pmatrix} -2 & 4 \\ 6 & 4 \end{pmatrix} = \begin{pmatrix} -8 & 8 \\ 1 & 13 \end{pmatrix}$$

see above

$$AB + D^2 = \begin{pmatrix} -6 & 4 \\ -5 & 9 \end{pmatrix} + \begin{pmatrix} -1 & 2 \\ 3 & 2 \end{pmatrix}\begin{pmatrix} -1 & 2 \\ 3 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} -6 & 4 \\ -5 & 9 \end{pmatrix} + \begin{pmatrix} 7 & 2 \\ 3 & 10 \end{pmatrix} = \begin{pmatrix} 1 & 6 \\ -2 & 19 \end{pmatrix}$$

$DA + B$ does not exist since DA is 2×3 & B is 3×2

$$EC = \begin{pmatrix} 3 & -1 & 2 \\ 4 & -2 & 3 \\ 0 & -3 & -1 \end{pmatrix}$$

$$CE = \begin{pmatrix} 3 & 2 & -3 \\ -4 & -1 & 4 \\ 3 & 1 & -2 \end{pmatrix}$$

note: $EC \neq CE$

$$EB + F = \begin{pmatrix} 1 & 0 \\ 3 & -2 \\ 5 & -3 \end{pmatrix} + \begin{pmatrix} -1 & 2 \\ 0 & 2 \\ 2 & -1 \end{pmatrix} = \begin{pmatrix} 0 & 2 \\ 3 & 0 \\ 7 & -4 \end{pmatrix}$$

$FC + D$ is undefined since F is a 3×2 matrix & C is a 3×3 matrix

$$FD - 3B = \begin{pmatrix} 7 & 2 \\ 6 & 4 \\ -5 & 2 \end{pmatrix} + \begin{pmatrix} -3 & 0 \\ -6 & -3 \\ 3 & -6 \end{pmatrix} = \begin{pmatrix} 4 & 2 \\ 0 & 1 \\ -2 & -4 \end{pmatrix}$$

$$\underset{\sim}{E}^T\underset{\sim}{B} + \underset{\sim}{D} = \begin{pmatrix} -3 & 4 \\ 7 & 0 \end{pmatrix} + \begin{pmatrix} -1 & 2 \\ 3 & 2 \end{pmatrix} = \begin{pmatrix} -4 & 6 \\ 10 & 2 \end{pmatrix}$$

$2\underset{\sim}{E} - 3(\underset{\sim}{A}\underset{\sim}{E})$: does not exist since $\underset{\sim}{E}$ is a 3x2 matrix & $\underset{\sim}{A}\underset{\sim}{E}$ is is a 2x3 matrix

$$(2(\underset{\sim}{A}\underset{\sim}{B}))^T = \begin{pmatrix} -12 & 8 \\ -10 & 18 \end{pmatrix}^T = \begin{pmatrix} -12 & -10 \\ 8 & 18 \end{pmatrix}$$

$$\underset{\sim}{B}\underset{\sim}{D} = \begin{pmatrix} -1 & 2 \\ 1 & 6 \\ 7 & 2 \end{pmatrix} \quad \text{so} \quad \underset{\sim}{A}(\underset{\sim}{B}\underset{\sim}{D}) = \begin{pmatrix} 18 & -4 \\ 32 & 8 \end{pmatrix}$$

$$(\underset{\sim}{A}\underset{\sim}{B})\underset{\sim}{D} = \begin{pmatrix} -6 & 4 \\ -5 & 9 \end{pmatrix}\begin{pmatrix} -1 & 2 \\ 3 & 2 \end{pmatrix} = \begin{pmatrix} 18 & -4 \\ 32 & 8 \end{pmatrix}$$

[note : $\underset{\sim}{A}(\underset{\sim}{B}\underset{\sim}{D}) = (\underset{\sim}{A}\underset{\sim}{B})\underset{\sim}{D}$]

$$\underset{\sim}{A}^T = \begin{pmatrix} 1 & -3 \\ -2 & 1 \\ 3 & 4 \end{pmatrix} \qquad \underset{\sim}{B}^T = \begin{pmatrix} 1 & 2 & -1 \\ 0 & 1 & 2 \end{pmatrix}$$

$$\underset{\sim}{A}^T\underset{\sim}{B}^T = \begin{pmatrix} 1 & -1 & -7 \\ -2 & -3 & 4 \\ 3 & 10 & 5 \end{pmatrix}$$

$$(\underset{\sim}{A}\underset{\sim}{B})^T = \begin{pmatrix} -6 & -5 \\ 4 & 9 \end{pmatrix}$$

note $(\underset{\sim}{A}\underset{\sim}{B})^T \neq \underset{\sim}{A}^T\underset{\sim}{B}^T$

$$\underset{\sim}{B}^T\underset{\sim}{A}^T = \begin{pmatrix} -6 & -5 \\ 4 & 9 \end{pmatrix}$$

but $(\underset{\sim}{A}\underset{\sim}{B})^T = \underset{\sim}{B}^T\underset{\sim}{A}^T$

③ (a)

```
> with(linalg):
> matrix([[1,2,-3,1],[-1,0,3,4],[0,1,2,-1],[2,3,0,-3]]);
```

$$\begin{bmatrix} 1 & 2 & -3 & 1 \\ -1 & 0 & 3 & 4 \\ 0 & 1 & 2 & -1 \\ 2 & 3 & 0 & -3 \end{bmatrix}$$

```
> addrow(%,1,2,1):addrow(%,1,4,-2);
```

$$\begin{array}{r} \\ [2]+[1] \\ \\ [4]-2[1] \end{array} \begin{bmatrix} 1 & 2 & -3 & 1 \\ 0 & 2 & 0 & 5 \\ 0 & 1 & 2 & -1 \\ 0 & -1 & 6 & -5 \end{bmatrix}$$

```
> swaprow(%,2,3);
```

$$[2]\Leftrightarrow[3] \quad \begin{bmatrix} 1 & 2 & -3 & 1 \\ 0 & 1 & 2 & -1 \\ 0 & 2 & 0 & 5 \\ 0 & -1 & 6 & -5 \end{bmatrix}$$

```
> addrow(%,2,3,-2):addrow(%,2,4,1);
```

$$\begin{array}{r} \\ \\ [3]-2[2] \\ [4]+[2] \end{array} \begin{bmatrix} 1 & 2 & -3 & 1 \\ 0 & 1 & 2 & -1 \\ 0 & 0 & -4 & 7 \\ 0 & 0 & 8 & -6 \end{bmatrix}$$

```
> mulrow(%,3,-1/4);
```

$$\begin{array}{r} \\ \\ -\frac{1}{4}[3] \\ \\ \end{array} \begin{bmatrix} 1 & 2 & -3 & 1 \\ 0 & 1 & 2 & -1 \\ 0 & 0 & 1 & \frac{-7}{4} \\ 0 & 0 & 8 & -6 \end{bmatrix}$$

```
> addrow(%,3,4,-8);
```

$$\begin{array}{r} \\ \\ \\ [4]-8[3] \end{array} \begin{bmatrix} 1 & 2 & -3 & 1 \\ 0 & 1 & 2 & -1 \\ 0 & 0 & 1 & \frac{-7}{4} \\ 0 & 0 & 0 & 8 \end{bmatrix}$$

```
> mulrow(%,4,1/8);
```

$$\begin{array}{r} \\ \\ \\ \frac{1}{8}[4] \end{array} \begin{bmatrix} 1 & 2 & -3 & 1 \\ 0 & 1 & 2 & -1 \\ 0 & 0 & 1 & \frac{-7}{4} \\ 0 & 0 & 0 & 1 \end{bmatrix} \longleftarrow \text{Row-echelon form}$$

```
> 
> 
```

(3) (b)

```
> with(linalg):
> matrix([[1,1,3,-3,0],[0,2,1,-3,3],[1,0,2,-1,-1]]);
```

$$\begin{bmatrix} 1 & 1 & 3 & -3 & 0 \\ 0 & 2 & 1 & -3 & 3 \\ 1 & 0 & 2 & -1 & -1 \end{bmatrix}$$

```
> addrow(%,1,3,-1);
```

$$[3]-[1] \quad \begin{bmatrix} 1 & 1 & 3 & -3 & 0 \\ 0 & 2 & 1 & -3 & 3 \\ 0 & -1 & -1 & 2 & -1 \end{bmatrix}$$

```
> swaprow(%,2,3);
```

$$[2]\leftrightarrow[3] \quad \begin{bmatrix} 1 & 1 & 3 & -3 & 0 \\ 0 & -1 & -1 & 2 & -1 \\ 0 & 2 & 1 & -3 & 3 \end{bmatrix}$$

```
> mulrow(%,2,-1);
```

$$-1[2] \quad \begin{bmatrix} 1 & 1 & 3 & -3 & 0 \\ 0 & 1 & 1 & -2 & 1 \\ 0 & 2 & 1 & -3 & 3 \end{bmatrix}$$

```
> addrow(%,2,3,-2);
```

$$[3]-2[2] \quad \begin{bmatrix} 1 & 1 & 3 & -3 & 0 \\ 0 & 1 & 1 & -2 & 1 \\ 0 & 0 & -1 & 1 & 1 \end{bmatrix}$$

```
> mulrow(%,3,-1);
```

$$-1[3] \quad \begin{bmatrix} 1 & 1 & 3 & -3 & 0 \\ 0 & 1 & 1 & -2 & 1 \\ 0 & 0 & 1 & -1 & -1 \end{bmatrix} \leftarrow \text{Row-echelon form}$$

```
>
>
```

3(c)


```
> with(linalg):
> matrix([[0,2,3,-4,1],[0,0,2,3,4],[2,2,-5,2,4],[2,0,6,9,7]]);
```

$$\begin{bmatrix} 0 & 2 & 3 & -4 & 1 \\ 0 & 0 & 2 & 3 & 4 \\ 2 & 2 & -5 & 2 & 4 \\ 2 & 0 & 6 & 9 & 7 \end{bmatrix}$$

```
> swaprow(%,1,4);;
```

$$[1] \leftrightarrow [4] \quad \begin{bmatrix} 2 & 0 & 6 & 9 & 7 \\ 0 & 0 & 2 & 3 & 4 \\ 2 & 2 & -5 & 2 & 4 \\ 0 & 2 & 3 & -4 & 1 \end{bmatrix}$$

```
> mulrow(%,1,1/2);
```

$$\tfrac{1}{2}[1] \quad \begin{bmatrix} 1 & 0 & 3 & \frac{9}{2} & \frac{7}{2} \\ 0 & 0 & 2 & 3 & 4 \\ 2 & 2 & -5 & 2 & 4 \\ 0 & 2 & 3 & -4 & 1 \end{bmatrix}$$

```
> addrow(%,1,3,-2);
```

$$[3]-2[1] \quad \begin{bmatrix} 1 & 0 & 3 & \frac{9}{2} & \frac{7}{2} \\ 0 & 0 & 2 & 3 & 4 \\ 0 & 2 & -11 & -7 & -3 \\ 0 & 2 & 3 & -4 & 1 \end{bmatrix}$$

```
> swaprow(%,2,4);
```

$$[2] \leftrightarrow [4] \quad \begin{bmatrix} 1 & 0 & 3 & \frac{9}{2} & \frac{7}{2} \\ 0 & 2 & 3 & -4 & 1 \\ 0 & 2 & -11 & -7 & -3 \\ 0 & 0 & 2 & 3 & 4 \end{bmatrix}$$

```
> mulrow(%,2,1/2);
```

$$\tfrac{1}{2}[2] \quad \begin{bmatrix} 1 & 0 & 3 & \frac{9}{2} & \frac{7}{2} \\ 0 & 1 & \frac{3}{2} & -2 & \frac{1}{2} \\ 0 & 2 & -11 & -7 & -3 \\ 0 & 0 & 2 & 3 & 4 \end{bmatrix}$$

```
> addrow(%,2,3,-2);
```

$$[3]-2[2] \quad \begin{bmatrix} 1 & 0 & 3 & \frac{9}{2} & \frac{7}{2} \\ 0 & 1 & \frac{3}{2} & -2 & \frac{1}{2} \\ 0 & 0 & -14 & -3 & -4 \\ 0 & 0 & 2 & 3 & 4 \end{bmatrix}$$

continued

```
> swaprow(%,3,4);
```

$$[3] \leftrightarrow [4] \quad \begin{bmatrix} 1 & 0 & 3 & \frac{9}{2} & \frac{7}{2} \\ 0 & 1 & \frac{3}{2} & -2 & \frac{1}{2} \\ 0 & 0 & 2 & 3 & 4 \\ 0 & 0 & -14 & -3 & -4 \end{bmatrix}$$

```
> mulrow(%,3,1/2);
```

$$\tfrac{1}{2}[3] \quad \begin{bmatrix} 1 & 0 & 3 & \frac{9}{2} & \frac{7}{2} \\ 0 & 1 & \frac{3}{2} & -2 & \frac{1}{2} \\ 0 & 0 & 1 & \frac{3}{2} & 2 \\ 0 & 0 & -14 & -3 & -4 \end{bmatrix}$$

```
> addrow(%,3,4,14);
```

$$[4]+14[3] \quad \begin{bmatrix} 1 & 0 & 3 & \frac{9}{2} & \frac{7}{2} \\ 0 & 1 & \frac{3}{2} & -2 & \frac{1}{2} \\ 0 & 0 & 1 & \frac{3}{2} & 2 \\ 0 & 0 & 0 & 18 & 24 \end{bmatrix}$$

```
> mulrow(%,4,1/18);
```

$$\tfrac{1}{8}[4] \quad \begin{bmatrix} 1 & 0 & 3 & \frac{9}{2} & \frac{7}{2} \\ 0 & 1 & \frac{3}{2} & -2 & \frac{1}{2} \\ 0 & 0 & 1 & \frac{3}{2} & 2 \\ 0 & 0 & 0 & 1 & \frac{4}{3} \end{bmatrix} \leftarrow \text{row-echelon form}$$

```
>
>
```

4 (a)

```
> with(linalg):
> A:=matrix([[1,3,2,1],[-1,2,3,4],[3,0,1,2]]);
```

$$A := \begin{bmatrix} 1 & 3 & 2 & 1 \\ -1 & 2 & 3 & 4 \\ 3 & 0 & 1 & 2 \end{bmatrix}$$

```
> addrow(%,1,2,1):addrow(%,1,3,-3);
```

$$[2]+[1] \quad \begin{bmatrix} 1 & 3 & 2 & 1 \\ 0 & 5 & 5 & 5 \\ 0 & -9 & -5 & -1 \end{bmatrix}$$

```
> mulrow(%,2,1/5);
```

$$\tfrac{1}{5}[2] \quad \begin{bmatrix} 1 & 3 & 2 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & -9 & -5 & -1 \end{bmatrix}$$

```
> addrow(%,2,1,-3):addrow(%,2,3,9);
```

$$\begin{array}{r} [1]-3[2] \\ \\ [3]+9[2] \end{array} \begin{bmatrix} 1 & 0 & -1 & -2 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 4 & 8 \end{bmatrix}$$

```
> mulrow(%,3,1/4);
```

$$\begin{array}{r} \\ \\ \tfrac{1}{4}[3] \end{array} \begin{bmatrix} 1 & 0 & -1 & -2 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

```
> addrow(%,3,1,1):addrow(%,3,2,-1);
```

$$\begin{array}{r} [1]+[3] \\ [2]-[3] \\ \\ \end{array} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 2 \end{bmatrix} \longleftarrow \text{reduced row-echelon form}$$

```
>
```

4 (b)

```
> with(linalg):
> A:=matrix([[1,1,2,-1],[1,2,1,0],[-1,-4,1,-2],[1,-2,5,-4]]);
```

$$A := \begin{bmatrix} 1 & 1 & 2 & -1 \\ 1 & 2 & 1 & 0 \\ -1 & -4 & 1 & -2 \\ 1 & -2 & 5 & -4 \end{bmatrix}$$

```
> addrow(%,1,2,-1):addrow(%,1,3,1):addrow(%,1,4,-1);
```

$$\begin{array}{r} \\ [2]-[1] \\ [3]+[1] \\ [4]-[1] \end{array} \begin{bmatrix} 1 & 1 & 2 & -1 \\ 0 & 1 & -1 & 1 \\ 0 & -3 & 3 & -3 \\ 0 & -3 & 3 & -3 \end{bmatrix}$$

```
> addrow(%,2,1,-1):addrow(%,2,3,3):addrow(%,2,4,3);;
```

$$\begin{array}{r} [1]-[2] \\ \\ [3]+3[2] \\ [4]+3[2] \end{array} \begin{bmatrix} 1 & 0 & 3 & -2 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \longleftarrow \text{reduced row-echelon form}$$

```
>
```

4 (c)

```
> with(linalg):
> A:=matrix([[1,1,3,-3,0],[0,2,1,-3,3],[1,0,2,-1,-1]]);
```

$$A := \begin{bmatrix} 1 & 1 & 3 & -3 & 0 \\ 0 & 2 & 1 & -3 & 3 \\ 1 & 0 & 2 & -1 & -1 \end{bmatrix}$$

```
> addrow(%,1,3,-1);
```

$$\begin{array}{r} \\ \\ [3]-[1] \end{array} \begin{bmatrix} 1 & 1 & 3 & -3 & 0 \\ 0 & 2 & 1 & -3 & 3 \\ 0 & -1 & -1 & 2 & -1 \end{bmatrix}$$

```
> swaprow(%,2,3);
```

$$[2] \leftrightarrow [3] \quad \begin{bmatrix} 1 & 1 & 3 & -3 & 0 \\ 0 & -1 & -1 & 2 & -1 \\ 0 & 2 & 1 & -3 & 3 \end{bmatrix}$$

```
> mulrow(%,2,-1);
```

$$-1[2] \quad \begin{bmatrix} 1 & 1 & 3 & -3 & 0 \\ 0 & 1 & 1 & -2 & 1 \\ 0 & 2 & 1 & -3 & 3 \end{bmatrix}$$

```
> addrow(%,2,1,-1):addrow(%,2,3,-2);
```

$$\begin{array}{r} [1]-[2] \\ \\ [3]-2[2] \end{array} \begin{bmatrix} 1 & 0 & 2 & -1 & -1 \\ 0 & 1 & 1 & -2 & 1 \\ 0 & 0 & -1 & 1 & 1 \end{bmatrix}$$

```
> mulrow(%,3,-1);
```

$$\begin{array}{r} \\ \\ -1[3] \end{array} \begin{bmatrix} 1 & 0 & 2 & -1 & -1 \\ 0 & 1 & 1 & -2 & 1 \\ 0 & 0 & 1 & -1 & -1 \end{bmatrix}$$

```
> addrow(%,3,1,-2):addrow(%,3,2,-1);
```

$$\begin{array}{r} [1]-2[3] \\ [2]-[3] \\ \\ \end{array} \begin{bmatrix} 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & -1 & 2 \\ 0 & 0 & 1 & -1 & -1 \end{bmatrix} \longleftarrow \text{reduced row-echel form}$$

```
>
```

5 (a)

```
> with(linalg):
Warning, the protected names norm and trace have been redefined and unprotected
> matrix([[1,2,3,9],[2,-1,1,8],[3,0,-1,3]]);
```

$$\begin{bmatrix} 1 & 2 & 3 & 9 \\ 2 & -1 & 1 & 8 \\ 3 & 0 & -1 & 3 \end{bmatrix}$$

```
> addrow(%,1,2,-2):addrow(%,1,3,-3);
```

$$\begin{array}{r} \\ [2]-2[1] \\ [3]-3[1] \end{array} \begin{bmatrix} 1 & 2 & 3 & 9 \\ 0 & -5 & -5 & -10 \\ 0 & -6 & -10 & -24 \end{bmatrix}$$

```
> mulrow(%,2,-1/5);
```

$$\begin{array}{r} \\ -\frac{1}{5}[2] \\ \\ \end{array} \begin{bmatrix} 1 & 2 & 3 & 9 \\ 0 & 1 & 1 & 2 \\ 0 & -6 & -10 & -24 \end{bmatrix}$$

```
> addrow(%,2,3,6);
```

$$\begin{array}{r} \\ \\ [3]+6[2] \end{array} \begin{bmatrix} 1 & 2 & 3 & 9 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & -4 & -12 \end{bmatrix}$$

```
> mulrow(%,3,-1/4);
```

$$\begin{array}{r} \\ \\ -\frac{1}{4}[3] \end{array} \begin{bmatrix} 1 & 2 & 3 & 9 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 1 & 3 \end{bmatrix}$$

```
>
>
```

equivalent system: $z = 3$

$y = 2 - z = -1$

$x = 9 - 3(3) - 2(-1)$

$= 2$

i.e, $x = 2$, $y = -1$, $z = 3$

(5) (b) augmented matrix is $\begin{pmatrix} 1 & 1 & 3 & -3 & 0 \\ 0 & 2 & 1 & -3 & 3 \\ 1 & 0 & 2 & -1 & -1 \end{pmatrix}$

R.E.F is $\begin{pmatrix} 1 & 1 & 3 & -3 & 0 \\ 0 & 1 & 1 & -2 & 1 \\ 0 & 0 & 1 & -1 & -1 \end{pmatrix}$ see (2) (ii)

equivalent system: $a + b + 3c - 3d = 0$
$b + c - 2d = 1$
$c - d = -1$

Let $d = t$ ("free" variable / parameter)
$c = -1 + t$
$b = -c + 2d + 1 = 1 - (-1+t) + 2t = 2 + t$
$a = -b - 3c + 3d = -2 - t - 3(-1+t) + 3t$
$= 1 - t$

ie, $a = 1-t$, $b = 2+t$, $c = -1+t$; $d = t$
[infinite # of solutions, expressible in terms of one parameter].

(5) (c) augmented matrix is $\begin{pmatrix} 1-i & 2+i & 2+i \\ 2 & 1-2i & 1+3i \end{pmatrix}$

$\frac{1}{1-i}[1]$ $\begin{pmatrix} 1 & (1+3i)/2 & (1+3i)/2 \\ 2 & (1-2i) & (1+3i) \end{pmatrix}$

$[2]-2[1]$ $\begin{pmatrix} 1 & (1+3i)/2 & (1+3i)/2 \\ 0 & -5i & 0 \end{pmatrix}$

$-\frac{1}{5i}[2]$ $\begin{pmatrix} 1 & (1+3i)/2 & (1+3i)/2 \\ 0 & 1 & 0 \end{pmatrix}$

which is in R.E.F.

$\therefore$ $y = 0$ + $x = (1+3i)/2$ – unique solution!

6 (a)

augmented matrix is $\begin{pmatrix} 1 & 1 & 3 & -3 & 0 \\ 0 & 2 & 1 & -3 & 3 \\ 1 & 0 & 2 & -1 & -1 \end{pmatrix}$

rref is $\begin{pmatrix} 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & -1 & 2 \\ 0 & 0 & 1 & -1 & -1 \end{pmatrix}$

equivalent system:
$$p + s = 1$$
$$q - s = 2$$
$$r - s = -1$$

let $s = t$ ("free" variable / parameter)

so $r = -1 + t$
$q = 2 - t$
$p = 1 - t$

i.e, $p = 1 - t$, $q = 2 - t$, $r = -1 + t$, $s = t$

[Compare with the answer in 4 (ii)]

6 (b)

```
> with(linalg):
Warning, the protected names norm and trace have been redefined and unprotected
> matrix([[0,0,0,0,2,8,4],[0,0,0,1,3,11,9],[0,3,-12,-3,-9,-24,-33],[
  0,-2,8,1,6,17,21],[1,0,0,0,0,0,2]]);
```

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 2 & 8 & 4 \\ 0 & 0 & 0 & 1 & 3 & 11 & 9 \\ 0 & 3 & -12 & -3 & -9 & -24 & -33 \\ 0 & -2 & 8 & 1 & 6 & 17 & 21 \\ 1 & 0 & 0 & 0 & 0 & 0 & 2 \end{bmatrix}$$

```
> swaprow(%,1,5);;
```

$[1] \leftrightarrow [5]$ $$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 1 & 3 & 11 & 9 \\ 0 & 3 & -12 & -3 & -9 & -24 & -33 \\ 0 & -2 & 8 & 1 & 6 & 17 & 21 \\ 0 & 0 & 0 & 0 & 2 & 8 & 4 \end{bmatrix}$$

```
> swaprow(%,2,3);
```

$[2] \leftrightarrow [3]$ $$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 2 \\ 0 & 3 & -12 & -3 & -9 & -24 & -33 \\ 0 & 0 & 0 & 1 & 3 & 11 & 9 \\ 0 & -2 & 8 & 1 & 6 & 17 & 21 \\ 0 & 0 & 0 & 0 & 2 & 8 & 4 \end{bmatrix}$$

```
> mulrow(%,2,1/3);
```

$$\begin{array}{r} \\ \frac{1}{3}[2] \\ \\ \\ \\ \end{array}\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 2 \\ 0 & 1 & -4 & -1 & -3 & -8 & -11 \\ 0 & 0 & 0 & 1 & 3 & 11 & 9 \\ 0 & -2 & 8 & 1 & 6 & 17 & 21 \\ 0 & 0 & 0 & 0 & 2 & 8 & 4 \end{bmatrix}$$

```
> addrow(%,2,4,2);
```

$$\begin{array}{r} \\ \\ \\ [4]+2[2] \\ \\ \end{array}\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 2 \\ 0 & 1 & -4 & -1 & -3 & -8 & -11 \\ 0 & 0 & 0 & 1 & 3 & 11 & 9 \\ 0 & 0 & 0 & -1 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 2 & 8 & 4 \end{bmatrix}$$

```
> addrow(%,3,2,1):addrow(%,3,4,1);
```

$$\begin{array}{r} \\ [2]+[3] \\ \\ [4]+[3] \\ \\ \end{array}\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 2 \\ 0 & 1 & -4 & 0 & 0 & 3 & -2 \\ 0 & 0 & 0 & 1 & 3 & 11 & 9 \\ 0 & 0 & 0 & 0 & 3 & 12 & 8 \\ 0 & 0 & 0 & 0 & 2 & 8 & 4 \end{bmatrix}$$

```
> addrow(%,5,4,-1);
```

$$\begin{array}{r} \\ \\ \\ [4]-[5] \\ \\ \end{array}\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 2 \\ 0 & 1 & -4 & 0 & 0 & 3 & -2 \\ 0 & 0 & 0 & 1 & 3 & 11 & 9 \\ 0 & 0 & 0 & 0 & 1 & 4 & 4 \\ 0 & 0 & 0 & 0 & 2 & 8 & 4 \end{bmatrix}$$

```
> addrow(%,4,3,-3):addrow(%,4,5,-2);;
```

$$\begin{array}{r} \\ \\ [3]-3[4] \\ \\ [5]-2[4] \end{array}\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 2 \\ 0 & 1 & -4 & 0 & 0 & 3 & -2 \\ 0 & 0 & 0 & 1 & 0 & -1 & -3 \\ 0 & 0 & 0 & 0 & 1 & 4 & 4 \\ 0 & 0 & 0 & 0 & 0 & 0 & -4 \end{bmatrix}$$

```
> mulrow(%,5,-1/4);
```

$$\begin{array}{r} \\ \\ \\ \\ -\frac{1}{4}[5] \end{array}\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 2 \\ 0 & 1 & -4 & 0 & 0 & 3 & -2 \\ 0 & 0 & 0 & 1 & 0 & -1 & -3 \\ 0 & 0 & 0 & 0 & 1 & 4 & 4 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

```
> addrow(%,5,1,-2):addrow(%,5,2,2):addrow(%,5,3,3):addrow(%,5,4,-4);
;
```

$$\begin{array}{r} [1]-2[5] \\ [2]+2[5] \\ [3]+3[5] \\ [4]-4[5] \\ \\ \end{array}\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & -4 & 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

```
>
>
```

note: since the last row corresponds to the equation $0=1$, this system is inconsistent + has no solutions.

X