130.135 Engineering Statics December 16, 1999

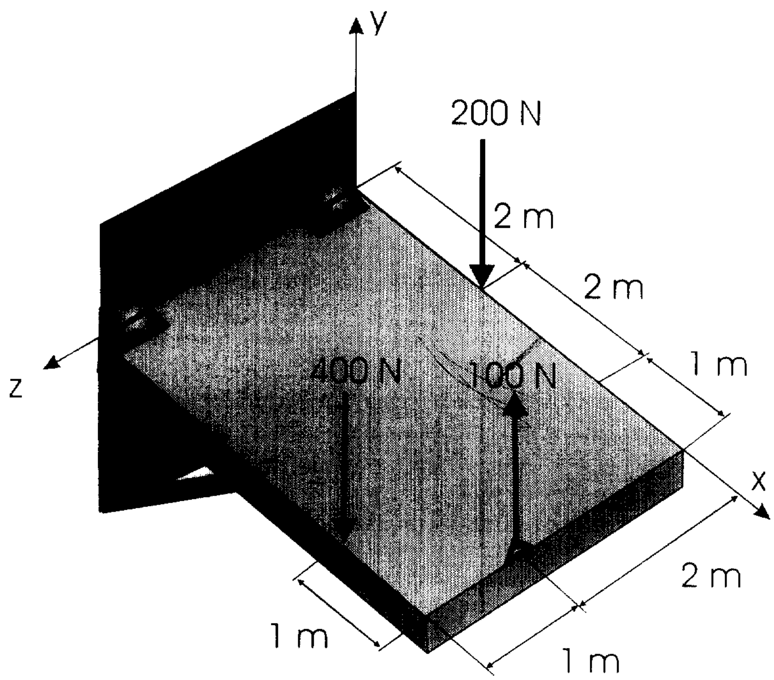
Examination Problems with Solutions

~~and~~

M~~arking Scheme~~

Problem 1.

Find the magnitude and the position of the resultant of the three vertical forces shown!



Solution:

Forces are $F_1 = - 200\,j$ $F_2 = - 400\,j$ $F_3 = 100\,j$
The Resultant $R = F_1 + F_2 + F_3 = - 500\,j$

For position write moments about x and z:

$$\sum M_x: \quad -\bar{x}\,(500) = 0\,(200) + 3\,(400) - 2\,(100)$$

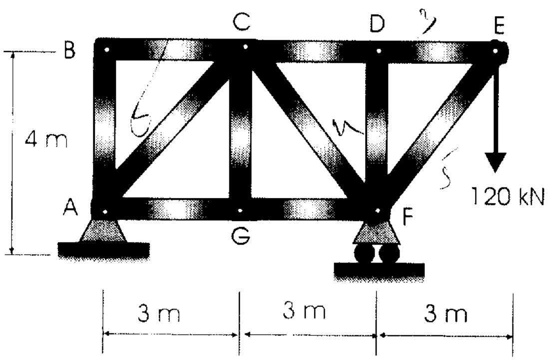
$$\sum M_z: \quad -\bar{z}\,(500) = -2\,(200) - 4\,(400) + 5\,(100)$$

$$\bar{x} = 3\text{ m}$$
$$\bar{z} = 2\text{ m}$$

Problem 2

Using the method of joints find the force in members:

AB, AC, BC, CG, ED, EF



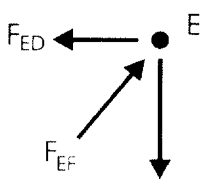
Solution:

AB and BC form a V-joint; Both are zero-force members

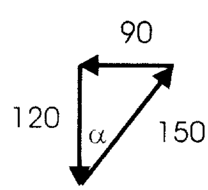
The collinear AG/GF and CG form a T-joint; CG is a zero-force member

Joint E has only two unknowns; they can be determined graphically, or by writing the two force-equilibrium equations for joint E:

free-body



force-polygon



from geometry: $\tan\alpha = 3/4$

$\alpha = 36.9$

$$\sum F_x = -F_{ED} + (3/4)120 = 0$$

$$F_{ED} = 90 \text{ kN}$$

$$\sum F_y = -120 + F_{EF}(\cos\alpha) = 0$$

$$F_{EF} = 150 \text{ kN}$$

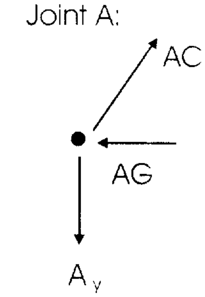
Member ED is in tension and EF is in compression

To find AC need the reactions at A; then write equilibrium at A:

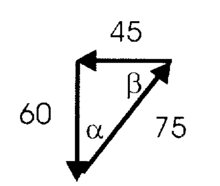
$\sum M_F = -3*120 - 6A_y = 0 \qquad A_y = -60$ (points down)

A_x must be zero There is no other external force in x-direction..

free-body of Joint A:



force-polygon



$\beta = 53.1$

$$\sum F_y = -60 + F_{AC}\sin\beta = 0$$

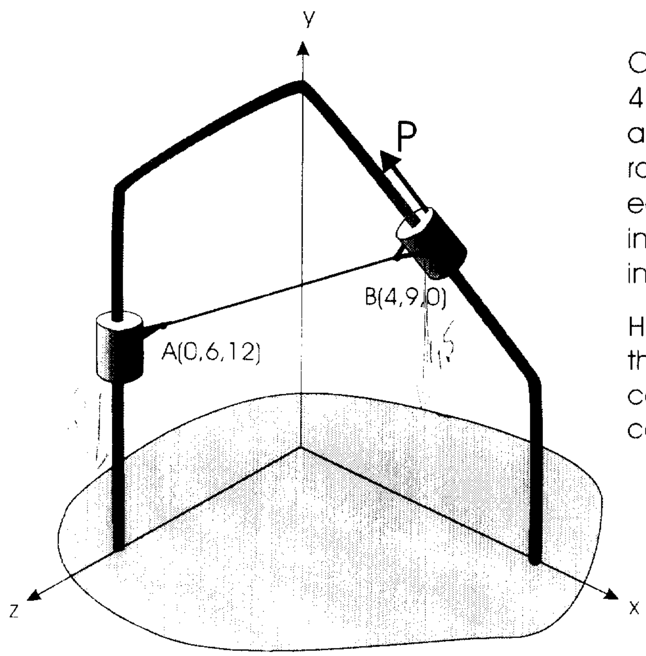
$$F_{AC} = 75$$

$$\sum F_x = -F_{AG} + F_{AC}\cos\beta = 0$$

$$F_{AG} = 45$$

Member AC is in tension and AG is in compression.

Problem 3



Collars A and B, each weighing 4.5 kN are connected by wire AB and may slide freely on the smooth rod. The collars are brought to equilibrium by applying the force P in the position shown. Find the tension in the wire!

Hint: assume that at both A and B, all the forces, including the reactions, are concurrent at the center of gravity of the collar..

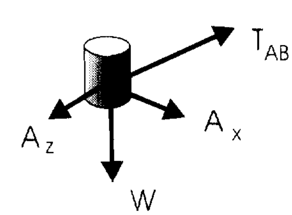
Solution: Collar A has three unknowns: the two reactions A_x and A_y and T_{AB} . Collar B has four. Do collar A!

Write $\vec{T}_{AB}$! in vector form: $\vec{AB}=4i + 3j - 12k \quad AB=13$

$\vec{T}_{AB} = (T_{AB}/13)\ (4i + 3j - 12k)$

The reaction: $\vec{A} = A_x i + A_z k$

The weight: $\vec{W} = -4.5\,j$



For the equilibrium of the collar write:

$$\Sigma\vec{F}=0 = \vec{T}_{AB} + \vec{A} + \vec{W}$$

Substitute and collect the factors of i,j and k:

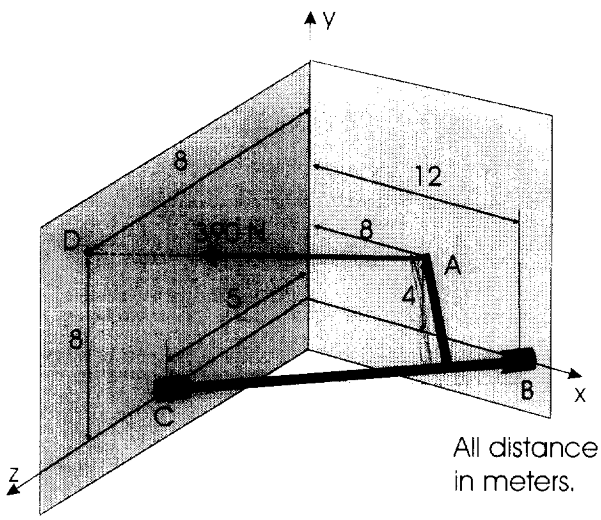
$$((4/13)T_{AB} + A_x)\ i + ((3/13)T_{AB} - 4.5)\ j + ((-12/13)T_{AB} + A_z)\ k = 0$$

Set the factor of j to nil:

$$(3/13)\ T_{AB} - 4.5 = 0 \qquad T_{AB} = 19.5 \text{ kN}$$

Problem 4

A T-shaped lever is supported by bearings at B and C. The lever is found to have an impending rotation about the axis when a force F of 390 N is applied as shown. Determine the moment of F about axis BC!



All distance measures are in meters.

Solution:

Need unit vector along CB, vector equation of the force and the vector equation for the moment arm.

Unit Vector for CB:

$\vec{CB} = 12i - 5k \quad CB = 13 \quad \vec{CB}_o = (12/13)i - (5/13)k$

Force Vector:

$\vec{AD} = -8i + 4j + 8k \quad AD = 12 \quad \vec{AD}_o = (-2/3)i + (1/3)j + (2/3)k$

$\vec{F} = -260i + 130j + 260k$

Moment Arm:

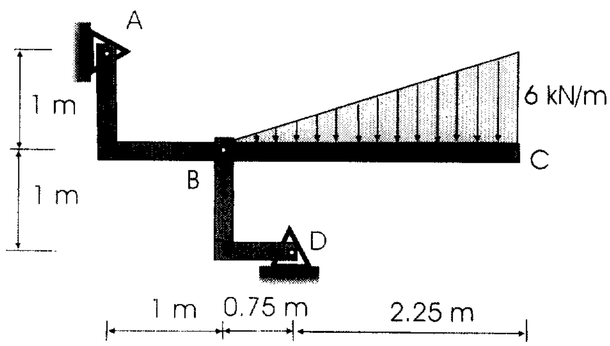
$\vec{BA} = -4i + 4j$

Moment about CB

$$M_{CB} = \begin{vmatrix} 12/13 & 0 & -5/13 \\ -4 & 4 & 0 \\ -260 & 130 & 260 \end{vmatrix} = -760$$

$\left(\frac{12}{13} i - \frac{5}{13} k\right) \cdot \left(-4 i + 4 j\right) \times \left(-260 i + 130 j + 260 k\right)$

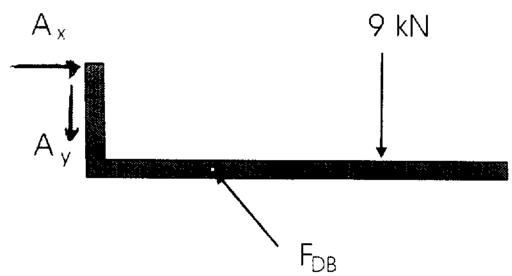
Problem 5



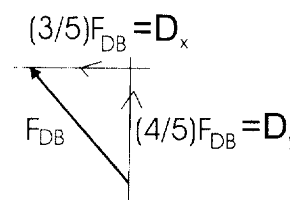
Find the reactions at A and D!

Solution:

Free-body of ABC



Note that BD is a two force body; reaction at D follows DB direction! DB has slope of 4 on 3



$\Sigma M_A = 0 = (1)(4/5)\, F_{DB} - (1)(3/5)F_{DB} - (3)(9)$ $\qquad F_{DB} = 135$ kN

$D_x = -81$

$D_y = 108$

$\Sigma F_x = A_x - (3/5)(135) = 0$ $\qquad A_x = 81$ kN

$\Sigma F_y = A_y - (4/5)(135) + 9 = 0$ $\qquad A_y = 99$ kN