CALCULUS 1510 – MIDTERM OCT. 2005

[12] 1. Evaluate the limit or show that it does not exist.

[4] (a) $\lim_{x\to 2}\dfrac{\sqrt{x+2}-2}{x-2}$ [4] (b) $\lim_{x\to\infty}(x-\sqrt{1+x^2})$ [4] (c) $\lim_{x\to 2}\dfrac{x-2}{|x-2|}$

[7] 2. Let $f(x)=3x^2+x$. Find $f'(2)$ using **ONLY** the definition of the derivative.

[13] 3. Compute the following derivatives. Do NOT simplify your answer after differentiating.

[4] (a) $f'(x)$ if $f(x)=\sin(\sqrt{1+x^2})$ [4] (b) $g''(x)$ if $g(x)=\dfrac{x}{1-x}$ [5] (c) $h'(x)$ if $h(x)=x^{(x-1)}$.

[8] 4. Let $f(x)=e^{\cos x}$

[4] (a) Find $f''(0)$. [4] (b) Find the equation of the tangent line of the curve $f(x)=e^{\cos x}$ at the point when $x=\dfrac{\pi}{2}$.

[10] 5. The equation $y^5-y\cos x=0$ defines y as a function on x.

[6] (a) Evaluate $\dfrac{dy}{dx}$ at the point $(0,1)$.

[4] (b) Find an equation of the tangent line to the curve $y^5-y\cos x=0$ at the point $(0,1)$.

[4] 6. [bonus] Suppose $f(x)$ and $g(x)$ are continuous at $x=a$. Show that the function $f(x)g(x)$ is also continuous at $x=a$.

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[11] 1. Evaluate the limit or explain why it does not exist.

(a) $\lim_{x\to 2}\dfrac{\sqrt{x}-\sqrt{2}}{2x^2-8}$ (b) $\lim_{x\to 0}\dfrac{(x^2-1)\sin^2(\pi x)}{x^2}$ (c) $\lim_{x\to 0}\dfrac{(2+x)^3-8}{x}$

[16] 2. Find $f'(x)$ for each of the following functions. DO NOT simplify your answers.

(a) $f(x)=x(\ln x)^2$ (b) $f(x)=\dfrac{\tan x-e^{-x}}{e^{3x}}$ (c) $f(x)=x^3+2^{\cot x}$

[3] 3. Express the function below in terms of the Heaviside function, h. DO NOT simplify your answer. No justification is required.

$$f(x)=\begin{cases}0 & x<-1\\ x-2 & -1<x<3\\ x-4 & 3<x<5\\ 0 & x>5\end{cases}$$

[6] 4. Use the definition of a derivative to find the value of $f'(1)$ if $f(x)=\dfrac{2}{\sqrt{2-x}}$

[3] 5. Determine whether the function F is continuous at $x=3$. Justify your answer using limits.

$$F(x)=\begin{cases}1-x^2 & x<3\\ -8 & x=3\\ 2-3x & x>3\end{cases}$$

[6] 6. Find an equation of the normal line to the curve $ye^{xy}=2$ at the point with coordinates $(\ln 2,1)$.