W08/1

THE UNIVERSITY OF MANITOBA

DATE: April 17, 2008 (Morning) **FINAL EXAMINATION**

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7 **1.** Find the open interval of convergence for the power series

$$\sum_{n=2}^{\infty} \frac{(-1)^n}{n2^n} x^{2n}.$$

Express your answer in the form $a < x < b$ for appropriate values of a and b.

$\rightarrow$ $-\sqrt{2} < x < \sqrt{2}$

10 **2.** Find the Maclaurin series for the function

$$f(x) = x \ln(1 + 2x).$$

Write your final answer in sigma notation, simplified as much as possible. What is the radius of convergence of the series?

$\rightarrow$ $\sum_{n=2}^{\infty} \frac{(-1)^n 2^{n-1} x^n}{n-1}$

$|x| < 1/2$

8 **3.** Find the Taylor series about $x = 1$ for the function

$$f(x) = \frac{5}{4+x}.$$

Express your answer in sigma notation, simplified as much as possible. What is the interval of convergence of the series?

$\rightarrow$ $\sum_{n=0}^{\infty} \frac{(-1)^n}{5^n} (x-1)^n$

$|x-1| < 5 = R$

$-4 < x \leq 6$

10 **4.** (a) Find a series of constants whose sum is the value of the definite integral

$$\int_0^{1/2} \frac{x - \sin x}{x^3} dx.$$

Write the series in sigma notation.

(b) Explain how you would use the series to find an approximation to the integral with four decimal accuracy. Do **NOT** find the approximation; just explain the steps that you would follow.

$\rightarrow$ Ans: a) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{2^{2n-1}(2n-1)(2n+1)!}$

b) Since the Series is alternating whose absolute terms are decreasing and approach zero, Compute the partial Sums till Consecutive ones round off to the Same four decimal number

Ans: $Ax^3 + Bx^2 + (Cx+D)e^{-3x}$

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5 6. You are given that the roots of the auxiliary equation associated with the linear, differential equation

$$\phi(D)y = 2x - 3xe^{-3x}$$

are $m = \pm 2, 0, 0$. Write down the form of a particular solution of the differential equation as predicted by the method of undetermined coefficients. Do **NOT** find the coefficients, just the form of the particular solution.

8 **5.** Find a one-parameter family of solutions for the differential equation

$$x\frac{dy}{dx} = xe^{x^3} - 2y.$$

Write your solution in explicit form. Can you claim that your solution is a general solution of the differential equation? Explain.

$y = \frac{1}{3x^2}e^{x^3} + \frac{C}{x^2}$

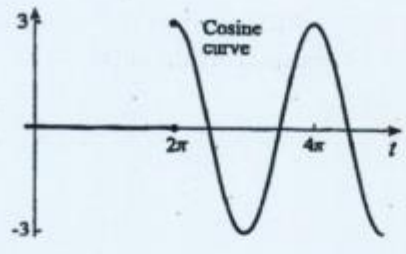
yes, DE is linear

16 **7.** (a) A 200 gram mass hangs motionless on the end of a spring with constant 100 N/m. From this position, the mass is given speed 2 m/s downward at time $t = 0$. During its subsequent motion, the mass is acted upon by a damping force whose magnitude in Newtons is 4 times its velocity in metres per second. In addition, a force $F(t) = 5\sin\omega t$ N acts on the mass. Find the position of the mass as a function of time.

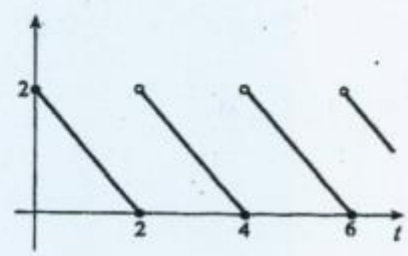
(b) Is it possible to choose ω so that the mass will experience unbounded oscillations? Explain.

12 **8.** Find Laplace transforms for the functions shown below:

(a)



(b)



$-3e^{-2\pi s}\left(\frac{s}{s^2+1}\right)$

$\frac{1}{1-e^{-2s}}\left[\frac{2}{s} - \frac{1}{s^2} + e^{-2s}\left(\frac{1}{s^2} - \frac{4}{s}\right)\right]$

12 **10.** Solve the following initial value problem

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} - 15y = 4\delta(t-2), \qquad y(0) = 1, \quad y'(0) = -1.$$

$y = 4h(t-2)\left[\frac{1}{2}e^{-5(t-2)} + \frac{1}{2}e^{3(t-2)}\right] + \frac{1}{2}e^{-5t} + \frac{1}{2}e^{3t}$