P1/1

Test 2

DATE: November 8, 2007
COURSE: MATH 2132

TIME: 60 minutes
EXAMINER: G.I. Moghaddam

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[12] 1. Use bionomial expansion to find the Maclaurin series of the function $f(x) = \frac{1}{\sqrt{2-x}}$. What is the open interval of convergence? Express your answer in sigma notation and simplify as much as possible.

[8] 2. Choose and answer *only* one the following two parts:

(a) Find the sum of the series $\sum_{n=1}^{\infty} \frac{2^{2n-2}}{n} x^{2n}$.

(b) Evaluate the following limit using infinite series.

$$\lim_{x \to 0} \frac{\sqrt[5]{(1-x^2)^3} - 1}{x^2}$$

[12] 3. Find, in explicit form, the solution of the differential equation

$$x^2 \frac{dy}{dx} + 3x\,y = 2\ln x, \qquad y(1) = \tfrac{1}{2}.$$

[10] 4. Find a 2 -parameter family of solutions of differential equation

$$y'' - 3(y')^2 = 3.$$

[8] 5. Find a general solution for a homogeneous linear differential equation $\Phi(D)y = 0$ whose auxiliary equation is:

$$(m+1)^2(m-\sqrt{2})^4(m^2+m+1)^3 = 0$$

Dawit's
Answers (plankon@yahoo.com)

1. $\sum_{n=0}^{\infty} \frac{(2n)!}{2^{3n+1/2}(n!)^2} x^n$, $-2 < x < 2$.

2. a) $-\frac{1}{4}\ln(1-4x^2)$, $-1/2 < x < 1/2$

b) $-3/5$

3. $y(x) = \frac{1}{2x^3}\left[x^2(\ln x^2 - 1) + 2\right]$

4. $y(x) = \frac{1}{3}\ln|\sec(3x+C)| + D$

5. $y(x) = (c_1 + c_2x)e^{-x} + (c_3 + c_4x + c_5x^2 + c_6x^3)e^{\sqrt{2}x} + e^{-x/2}\left[(c_7 + c_8x + c_9x^2)\cos\frac{\sqrt{3}}{2}x + (c_{10} + c_{11}x + c_{12}x^2)\sin\frac{\sqrt{3}}{2}x\right]$

Term Test 2

DATE: March 10, 2009
COURSE: MATH 2132
TIME: 70 minutes
EXAMINER: G.I. Moghaddam

[9] 1. Find the sum of the series $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(n+3)}{2^n}\, x^n.$

[8] 2. (a) Evaluate the following integral using infinite series

$$\int_0^1 x\, e^{-x^4}\, dx.$$

Express your answer in sigma notation.

(b) If you truncate the series in part (a) after the **third** term, what is a maximum possible error? Explain why you can claim that your answer is a maximum error.

[8] 3. Find a 1 -parameter family of solutions for differential equation

$$xy + x - y - 1 - y\frac{dy}{dx} = 0.$$

Is there any singular solution? Explain.

[8] 4. Find a 2 -parameter families of solutions for differential equation

$(y')^{\frac{3}{2}}\, y'' = 4x\,(y')^2.$

[8] 5. Newton's second law of motion says that an object of mass m falling near the surface of the earth is retarded by air resistance proportional to its velocity i.e. $m\frac{dv}{dt} = mg - kv$, where $v = v(t)$ is the velocity of the object at time t and g is the gravitational constant and k is constant of proportionality.
If an object of mass 1 kilogram is dropped (with no initial velocity) from a hovering helicopter, such that the air resistance is proportional to the velocity of the object; then:

(a) Create and solve an initial-value problem to find the velocity of the object as a function of time t.

[9] 6. Find the general solution for the homogeneous linear differential equation :

$$y^{(8)} + 4y^{(6)} + 4y^{(4)} = 0$$

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Answers

1. $\frac{3x^2+8x}{(x+2)^2}$, $|x| < 2$

2. a) $\sum_{n=0}^{\infty} \frac{(-1)^n}{(4n+2)\, n!}$ b) Max error $\le 1/84$

3. $y - \ln|y+1| = \frac{x^2}{2} - x + D$, $y = -1$ is a Singular Solution.

4. $\frac{x^5}{5} + \frac{2}{3}Dx^3 + D^2x + E$

5. a) $g/k\,(1 - e^{-kt})$ b) g/k

6. $c_1 + c_2x + c_3x^2 + c_4x^3 + (c_5 + c_6x)\cos\sqrt{2}x + (c_7 + c_8x)\sin\sqrt{2}x$

Term Test 2

DATE: November 12, 2009
COURSE: MATH 2132

TIME: 70 minutes
EXAMINER: G.I. Moghaddam

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[8] 1. Find the sum of the series $\sum_{n=1}^{\infty}\left(\frac{n+2}{n!}\right)x^{n+1}$.

[9] 2. Use binomial expansion to find the Maclaurin series of the function

$$f(x)=\left(\frac{x^2}{1+x^3}\right)^3.$$

What is the open interval of convergence? Express your answer in sigma notation and simplify as much as possible.

[9] 3. 50 g of a certain chemical is added to 200 mL of water; this chemical dissolves in water at a rate proportional to the product of the amount of undissolved chemical and the difference between concentrations in a saturated solution and the existing concentration in the solution. A saturated solution contains 25 g of chemical in 100 mL of solution.

(a) Show that the differential equation that describes the situation is

$$\frac{dx}{dt}=\frac{k}{200}(50-x)^2, \qquad x(0)=0,$$

where $x(t)$ is the number of grams of dissolved chemical at time t.

(b) Solve the differential equation in part (a).

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[8] 4. Find in explicit form the solution of the initial value problem

$$\frac{dy}{dx} + \frac{y}{2x} = \frac{1}{x^2\sqrt{x}}\, e^{1/x}, \qquad y(1) = e.$$

[8] 5. Find a 2-parameter family of solutions for the differential equation

$$y'' = \frac{(y')^2 - y'}{x}.$$

[8] 6. Consider the homogeneous linear differential equation

$$y''' - 3y'' - 4y' + 12y = 0.$$

(a) Write the differential equation in form $\phi(D)y = 0$, where $\phi(D)$ is the differential operator.

(b) Find the general solution for this homogeneous linear differential equation.

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Answers

1. $x^2e^x + 2xe^x - 2x$
2. $\frac{1}{2}\sum_{n=2}^{\infty}(-1)^n n(n-1)\, x^{3n}$, $|x|<1$
3. b) $x(t) = 50 - \frac{200}{kt+4}$ or, $x(t) = \frac{50kt}{kt+4}$
4. $\frac{1}{\sqrt{x}}\left(2e - e^{1/x}\right)$ or $\frac{e}{\sqrt{x}}\left(2 - e^{\frac{1-x}{x}}\right)$
5. $E - \frac{1}{D}\ln|Dx-1|$ or $F\ln|x+F| + G$
6. $c_1e^{-2x} + c_2e^{2x} + c_3e^{3x}$

MATH 2132 Test 2 March 9, 2010 60 minutes

Student Name - Student Number -

Values

7 1. (a) Find the Taylor series of $\ln x$ about $x = 3$. Express your answer in sigma notation.
(b) What is the open interval of convergence of the series?

9 2. (a) Find the Taylor series about $x = -2$ for $f(x) = \dfrac{1}{1+3x}$. Express your final answer in sigma notation. Use a technique that guarantees that the series converges to the function.
(b) What is the interval of convergence of the series?

8 3. Evaluate

$$\sum_{n=0}^{\infty} \frac{1}{n+1} x^{2n}.$$

Justify all steps in your solution.

8 4. Find, in explicit form $y = f(x)$, a 1-parameter family of solutions for the differential equation

$$x\frac{dy}{dx} = (x+1)y^2.$$

Does the 1-parameter family of solutions have any singular solutions? Explain.

8 5. Find the solution of the initial value problem

$$2\frac{dy}{dx} = y + 2x^2 e^{x/2}, \quad y(0) = 3.$$

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Answers

1. a) $\ln 3 + \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n 3^n} (x-3)^n$ b) $0 < x < 6$.

2. a) $-\sum_{n=0}^{\infty} \frac{3^n}{5^{n+1}} (x+2)^n$ b) $-11/3 < x < -1/3$

3. $S(x) = \begin{cases} -\frac{1}{x^2} \ln(1-x^2) & ; \ -1<x<1,\ x \neq 0 \\ 1 & , \ x = 0 \end{cases}$

4. $y = \dfrac{-1}{x + \ln|x| + C}$, yes $y=0$ is a Singular Solution.

5. $y = \frac{1}{3}(9 + x^3) e^{x/2}$