STAT2220: Engineering Statistics

Solution to Assignment 3 – Part I

3-140. a) $P(X \leq 5, Y \leq 8) = P(X \leq 5)P(Y \leq 8)$
$= (0.6321)(0.6321)$
$= 0.3996$

b) $P(X > 5, Y \leq 6) = P(X > 5)P(Y \leq 6)$
$= (1 - P(X \leq 5))P(Y \leq 6)$
$= (0.3679)(0.5276)$
$= 0.1941$

c) $P(3 < X \leq 7, Y > 7) = P(3 < X \leq 7)P(Y > 7)$
$= (P(Y \leq 7) - P(Y \leq 3))(1 - P(Y \leq 7))$
$= (0.3022)(0.4169)$
$= 0.1260$

d) $P(X > 7, 5 < Y \leq 7) = P(X > 7)P(5 < Y \leq 7)$
$= (1 - P(X \leq 7))(P(Y \leq 7) - P(Y \leq 5))$
$= (0.2466)(0.1184)$
$= 0.0292$

3-154. a) $E(Y) = E(2.5X_1 - 0.5X_2 + 1.5X_3) = 2.5(1.2) - 0.5(0.8) + 1.5(0.5) = 3.35$
b) $V(Y) = V(2.5X_1 - 0.5X_2 + 1.5X_3) = 6.25(1)^2 + 0.25(0.25)^2 + 2.25(2.2)^2 = 17.1556$
c) $E(-3Y) = -3E(Y) = -3(3.35) = -10.05$
d) $V(-3Y) = 9V(Y) = 9(17.1556) = 154.4004$

3-200. Let $\overline{X}$ denote the average time to locate 10 parts. Then, $E(\overline{X}) = 45$ and $\sigma_{\overline{X}} = \frac{30}{\sqrt{10}}$

a) $P(\overline{X} > 60) = P\left(Z > \frac{60-45}{30/\sqrt{10}}\right) = P(Z > 1.58) = 0.0571$

b) Let Y denote the total time to locate 10 parts. Then, $Y > 600$ if and only if $\overline{X} > 60$. Therefore, the answer is the same as part a.

3-216. a) P(system operates) = $P(C_1)P(C_2 \text{ or } C_3)P(C_4) = (0.9)(1 - P(C_2')P(C_3'))(0.95)$

$= (0.9)(1-(0.1)(0.05))(0.95)$

$= 0.8507$

b) Components in parallel do not fail.

$P(C_1' \text{ or } C_4' \text{ or both fail}) = 1 - P(C_1)P(C_4) = 1 - 0.855 = 0.145$

c) P(Both fail) = $P(C_2')P(C_3') = (0.1)(0.05) = 0.005$

d) $0.145(0.995) + 0.9(0.95)(0.1)(0.05) + 0.145(0.005) = 0.1493$

e) Fail due to series and not to parallel components

Fail due to parallel and not series components

Fail due to both series and parallel components

f) P(system fails) = $1 - 0.8507 = 0.1493$

g) Recompute parts a, b, c, and f from 3-202

a) $(0.95)(0.995)(0.95) = 0.8980$

b) $1 - 0.95^2 = 0.0975$

c) 0.005

f) $1 - 0.8980 = 0.1020$

h) Recompute parts a, b, c, and f

a) $0.9(1-0.05^2)(0.95) = 0.8529$

b) 0.1450

c) $0.05^2 = 0.0025$

f) $1 - 0.8529 = 0.1471$

i) Increase reliability of series component since improvement in series reduced the probability of failure from 0.1493 to 0.102, whereas the improvement in parallel component only resulted in a decrease to 0.1471

Statistics 5.2220

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Solution to Assignment 3 - Part II

Q4-20 (a). $H_0 = 175$ vs. $H_1 > 175$

$Z = \frac{\overline{X}-\mu}{\sigma/\sqrt{n}} = \frac{190-175}{20/\sqrt{10}} = 2.372 > 1.645 = Z_{0.05}$

Conclusion: Reject H_0 at $\alpha = 0.05$

Q4-20 (b). Calculate the probability

$$\begin{aligned} \alpha &= P(\bar{X} > 190| = 175) \\ &= p[\frac{\bar{X}-\mu}{\sigma/\sqrt{n}} > \frac{190-175}{20/\sqrt{10}}] \\ &= P(Z > 2.37) \\ &= 1 - 0.99111 \\ &= 0.0089 \end{aligned}$$

Q4-22 (a). n=16

0.05=P($\bar{x} > c$ when $\mu = 175$)=P($Z > \frac{c-175}{20/\sqrt{16}}$)=P($Z > 1.645$)

Thus, $1.645 = \frac{c-175}{20/\sqrt{16}}$, and $c = 183.225$

Q4-22 (b). For $\alpha = 0.05$, fail to reject H_0 if $\bar{X} < 175 + 1.645(\frac{20}{\sqrt{16}}) = 183.23$

β=P($\bar{X} \leq 183.23$ when $\mu = 195$)=P($Z \leq \frac{183.23-195}{20/\sqrt{16}}$)=P($Z \leq -2.35$)=0.009387

Q4-22 (c). For the same level of α, with the increased sample size, β is reduced. With an increased sample size, the power has also increased.