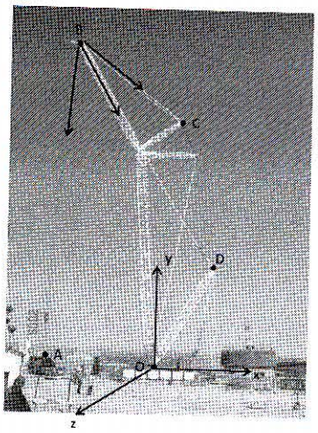
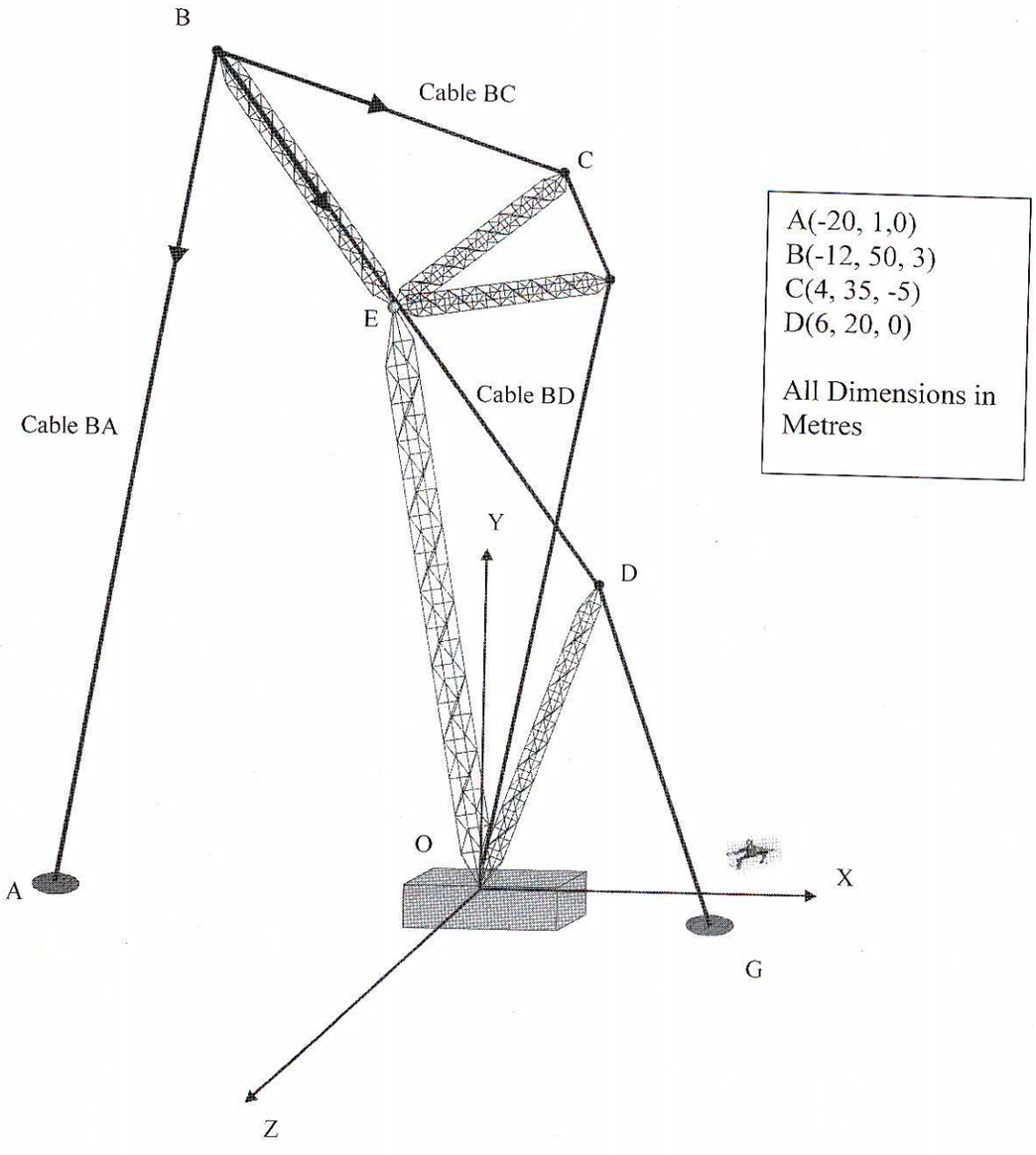
Question 1

Three cables are attached at point *B* on the boom of a large construction crane as shown in the figure below. The cable forces acting at point *B* are $\boldsymbol{F}_{BC} = 50\ kN$, $\boldsymbol{F}_{BD} = 100\ kN$ and $\boldsymbol{F}_{BA} = 75\ kN$. Determine:

a) The resultant, $\boldsymbol{R}$, of the three *(3)* forces acting at *B*,
b) The moment of $\boldsymbol{R}$ about the line *AD*,
c) The angle between cables *BD* and *BC*.





a) $\vec{R}$ at $\vec{B} = \vec{F}_{BC} + \vec{F}_{BD} + \vec{F}_{BA}$

$\vec{F}_{BC} = F_{BC}\vec{\lambda}_{BC} = 50\vec{\lambda}_{BC}$

$\vec{\lambda}_{BC} = \frac{\vec{BC}}{BC}$ $\quad \vec{BC} = 16\hat{i} - 15\hat{j} - 8\hat{k}$

$BC = \sqrt{(16)^2 + (-15)^2 + (-8)^2} = \sqrt{545}$

$\vec{F}_{BC} = 50\left(\frac{16\hat{i} - 15\hat{j} - 8\hat{k}}{\sqrt{545}}\right) = 34.27\hat{i} - 32.13\hat{j} - 17.13\hat{k}$

$\vec{F}_{BD} = F_{BD}\vec{\lambda}_{BD} = 100\vec{\lambda}_{BD}$

$\vec{\lambda}_{BD} = \frac{\vec{BD}}{BD}$ $\quad \vec{BD} = 18\hat{i} - 30\hat{j} - 3\hat{k}$

$BD = \sqrt{18^2 + (-30)^2 + (-3)^2} = \sqrt{1233}$

$\vec{F}_{BD} = 100\left(\frac{18\hat{i} - 30\hat{j} - 3}{\sqrt{1233}}\right) = 51.26\hat{i} - 85.44\hat{j} - 8.54\hat{k}$

$\vec{F}_{BA} = F_{BA}\vec{\lambda}_{BA} = 75\vec{\lambda}_{BA}$ $\quad \vec{\lambda}_{BA} = \frac{\vec{BA}}{BA}$

$\vec{BA} = -8\hat{i} - 49\hat{j} - 3\hat{k}$

$BA = \sqrt{(-8)^2 + (-49)^2 + (-3)^2} = \sqrt{2474}$

$\vec{F}_{BA} = \frac{75(-8\hat{i} - 49\hat{j} - 3\hat{k})}{\sqrt{2474}} = -12.06\hat{i} - 73.89\hat{j} - 4.52\hat{k}$

$\vec{R} = (34.27 + 51.26 - 12.06)\hat{i} + (-32.13 - 85.44 - 73.89)\hat{j}$

$+ (-17.13 - 8.54 - 4.52)\hat{k}$

$\vec{R} = (73.47\hat{i} - 191.46\hat{j} - 30.19\hat{k})$ kN

b) $M_{AD} = \vec{\lambda}_{AD} \cdot (\vec{r}_{DB} \times \vec{R}) = \vec{\lambda}_{AD} \cdot (\vec{r}_{AB} \times \vec{R})$

$\vec{\lambda}_{AD} = \frac{\vec{AD}}{AD}$ $\quad \vec{AD} = \frac{\vec{AD}}{AD}$ $\quad \vec{AD} = 26\hat{i} + 19\hat{j} + 0\hat{k}$

$AD = \sqrt{(26)^2 + (19)^2} = \sqrt{1037}$

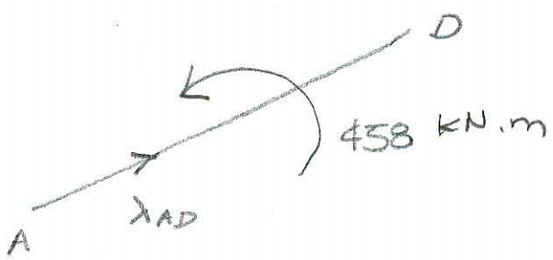
$\vec{\lambda}_{AD} = \frac{26\hat{i} + 19\hat{j}}{\sqrt{1037}} = 0.807\hat{i} + 0.59\hat{j}$

$\vec{r}_{DB} = -18\hat{i} + 30\hat{j} + 3\hat{k}$

$$M_{AD} = \begin{vmatrix} 0.807 & 0.59 & 0 & 0.807 & 0.59 \\ -18 & 30 & 3 & -18 & 30 \\ 73.47 & -191.46 & -30.19 & 73.47 & -191.46 \end{vmatrix}$$

$$= [-730.9 + 130.0 + 0] - [0 - 463.5 + 320.6]$$

$$= -458 \text{ kN.m}$$



$$\vec{r}_{AB} = 8\hat{i} + 49\hat{j} + 3\hat{k}$$

$$M_{AD} = \begin{vmatrix} 0.807 & 0.59 & 0 & 0.807 & 0.59 \\ 8 & 49 & 3 & 8 & 49 \\ 73.47 & -191.46 & -30.19 & 73.47 & -191.46 \end{vmatrix}$$

$$= [-1193.8 + 130 + 0] - [0 - 463.5 - 142.5]$$

$$= -457.8 \text{ kN.m}$$

c) Angle between BD & BC

$$\vec{\lambda}_{BD} \cdot \vec{\lambda}_{BC} = \cos\theta$$

$$\left(\frac{18\hat{i} - 30\hat{j} - 3\hat{k}}{\sqrt{1233}}\right) \cdot \left(\frac{16\hat{i} - 15\hat{j} - 8\hat{k}}{\sqrt{545}}\right)$$

$$\cos\theta = \frac{(18)(16) + (-30)(-15) + (-3)(-8)}{(\sqrt{1233})(\sqrt{545})}$$

$$\cos\theta = \frac{762}{819.747} = 0.9296$$

$$\theta = 21.63^\circ$$

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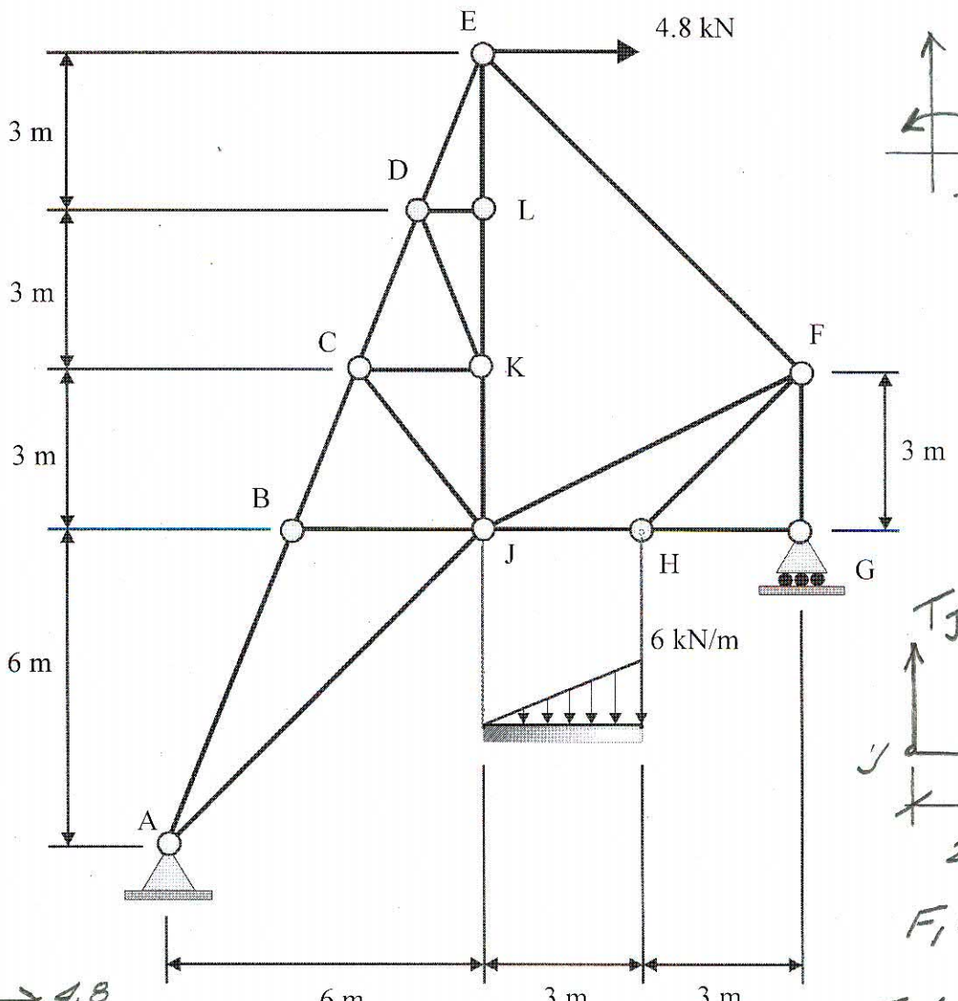
Examination : Introduction to Statics | **Examiners : Dr. M. J. Frye**
Dr. D. Polyzois
Place : Frank Kennedy Brown Gym | **Seats: 1 - 223**

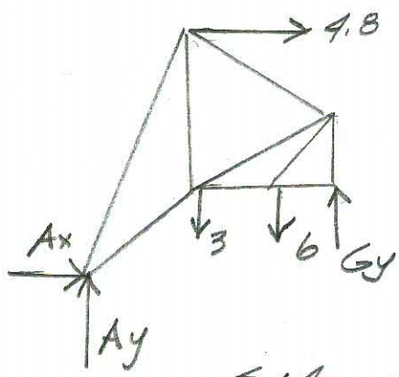
Question 2

The truss shown in the figure below has a pin (hinge) support at *A* and a roller support at *G*. A beam that carries a distributed load is suspended from the truss at joints *J* and *H*. The distributed load varies from *0 kN/m* to *6 kN/m* as shown. A *4.8 kN* load is applied at joint *E*.

Determine:

a) The tension in the two cables supporting the beam and
b) The force in each member of the truss. State whether the member is in tension, compression or a zero *(0)* force member. (**Indicate your results on the figure provided on Page 5. and put your name and student number on this page.**)





$\Sigma F_x = 0 \quad A_x + 4.8 = 0 \quad (1)$

$A_x = -4.8\ kN$

$\therefore \vec{A}_x = 4.8\ kN \leftarrow$

$\Sigma F_y = 0$

$A_y - 3 - 6 + G_y = 0$

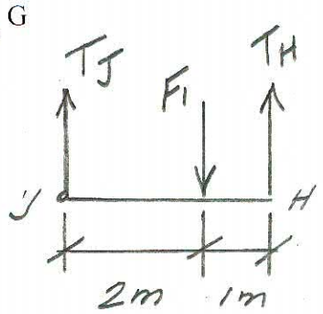
$A_y + G_y = 9 \quad (2)$

$\Sigma M_A = 0 \quad -4.8(15) - 3(6) - 6(9) + G_y(12) = 0 \quad (3)$

$G_y = +12 \quad \therefore \vec{G}_y = 12\ kN \uparrow$

From (2) $\quad A_y + 12 = 9 \quad A_y = -3\ kN$

$\therefore = 3\ kN \downarrow$



$F_1 = \frac{6 \times 3}{2} = 9\ kN$

$\Sigma M_J = 0$

$-9(2) + T_H(3) = 0$

$T_H = +6\ kN$

$\vec{T}_H = 6\ kN \uparrow$ on the beam

$\Sigma F_y = 0$

$T_J - 9 + 6 = 0$

$T_J = +3\ kN$

$\vec{T}_J = 3\ kN \uparrow$ on the beam

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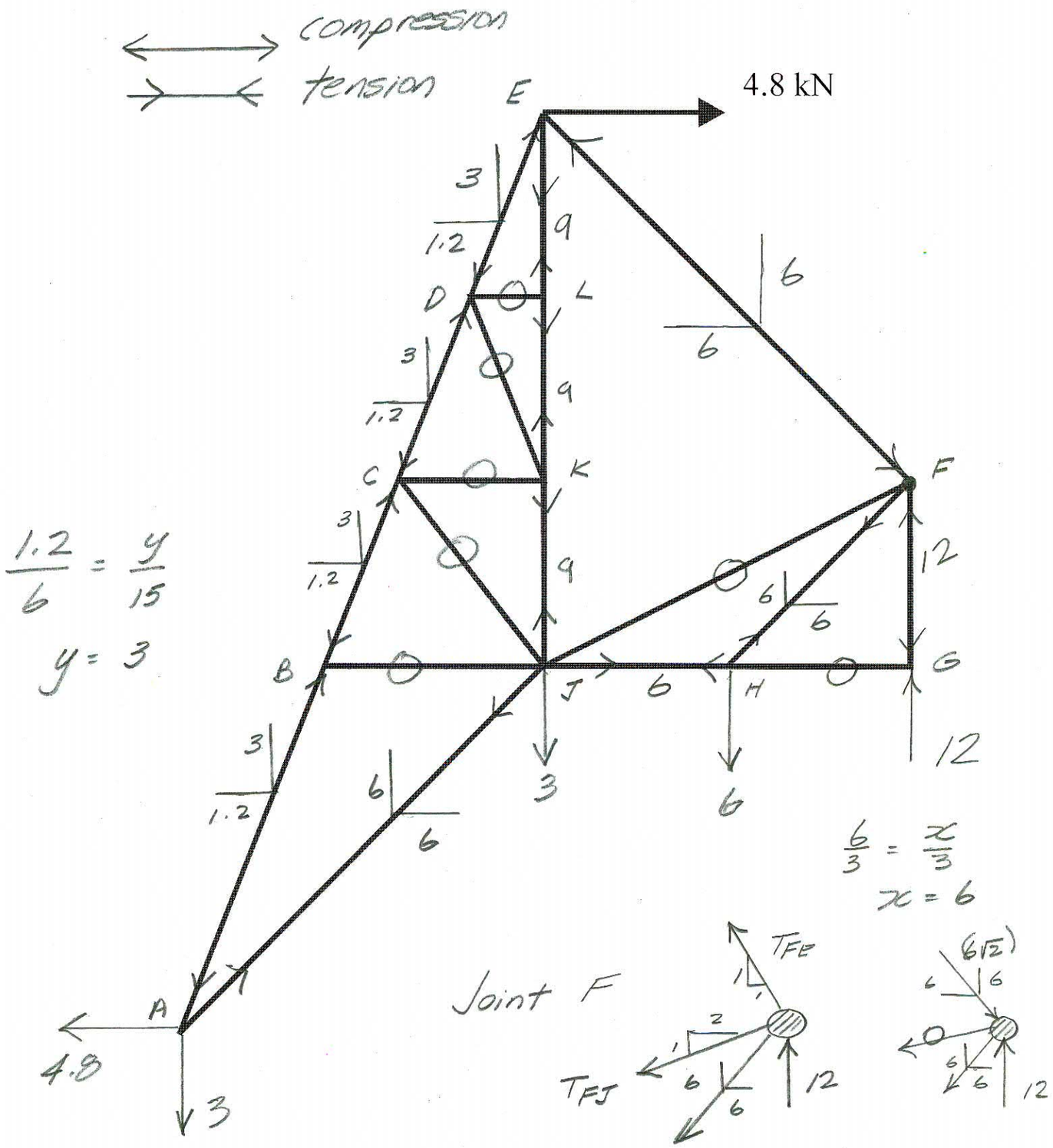
Question 2 – Results Page

NAME: _____ STUDENT NO. _____



$\frac{1.2}{6} = \frac{y}{15}$

$y = 3$

$\frac{6}{3} = \frac{x}{3}$

$x = 6$

$-\frac{1}{\sqrt{2}} T_{FE} = 6$

$T_{FE} = -6\sqrt{2}$

$\Sigma F_x = 0 \quad -6 - \frac{2}{\sqrt{5}} T_{FJ} - \frac{1}{\sqrt{2}} T_{FE} = 0$

$\Sigma F_y = 0 \quad 12 - 6 - \frac{1}{\sqrt{5}} T_{FJ} + \frac{1}{\sqrt{2}} T_{FE} = 0$

$-\frac{2}{\sqrt{5}} T_{FJ} - \frac{1}{\sqrt{2}} T_{FE} = 6$

$-\frac{1}{\sqrt{5}} T_{FJ} + \frac{1}{\sqrt{2}} T_{FE} = -6$

$-\frac{3}{\sqrt{5}} T_{FJ} = 0$

$\therefore T_{FJ} = 0$

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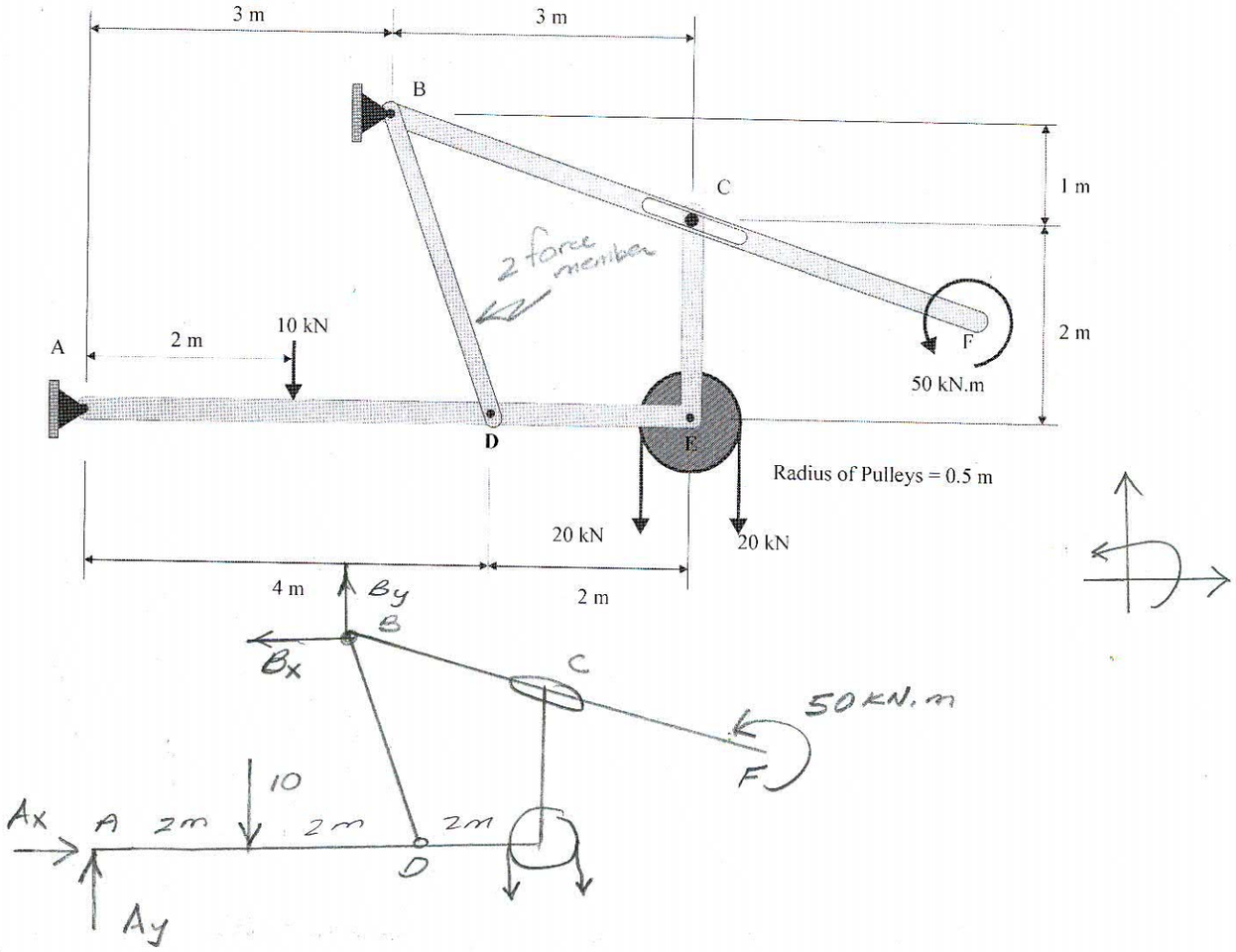
Question~~4~~ 3

The frame shown has three members (*Bent Member ADEC, Member BD and Member BCF*). A smooth pulley having a radius of *0.5 m* is attached to the bent member at *E*. Member *BCF* has a slot at C and a *50 kN.m* counterclockwise couple moment applied at *F*. The frame has pin supports at *A* and *B*.

Determine the forces exerted by the pins on each member of the frame and on the pulleys.



$\Sigma F_x = 0$ $\quad A_x - B_x = 0$ (1)

$\Sigma F_y = 0$ $\quad A_y - 10 + B_y - 20 - 20 = 0$ (2)

$A_y + B_y - 50 = 0$

$A_y + B_y = 50$ (2)

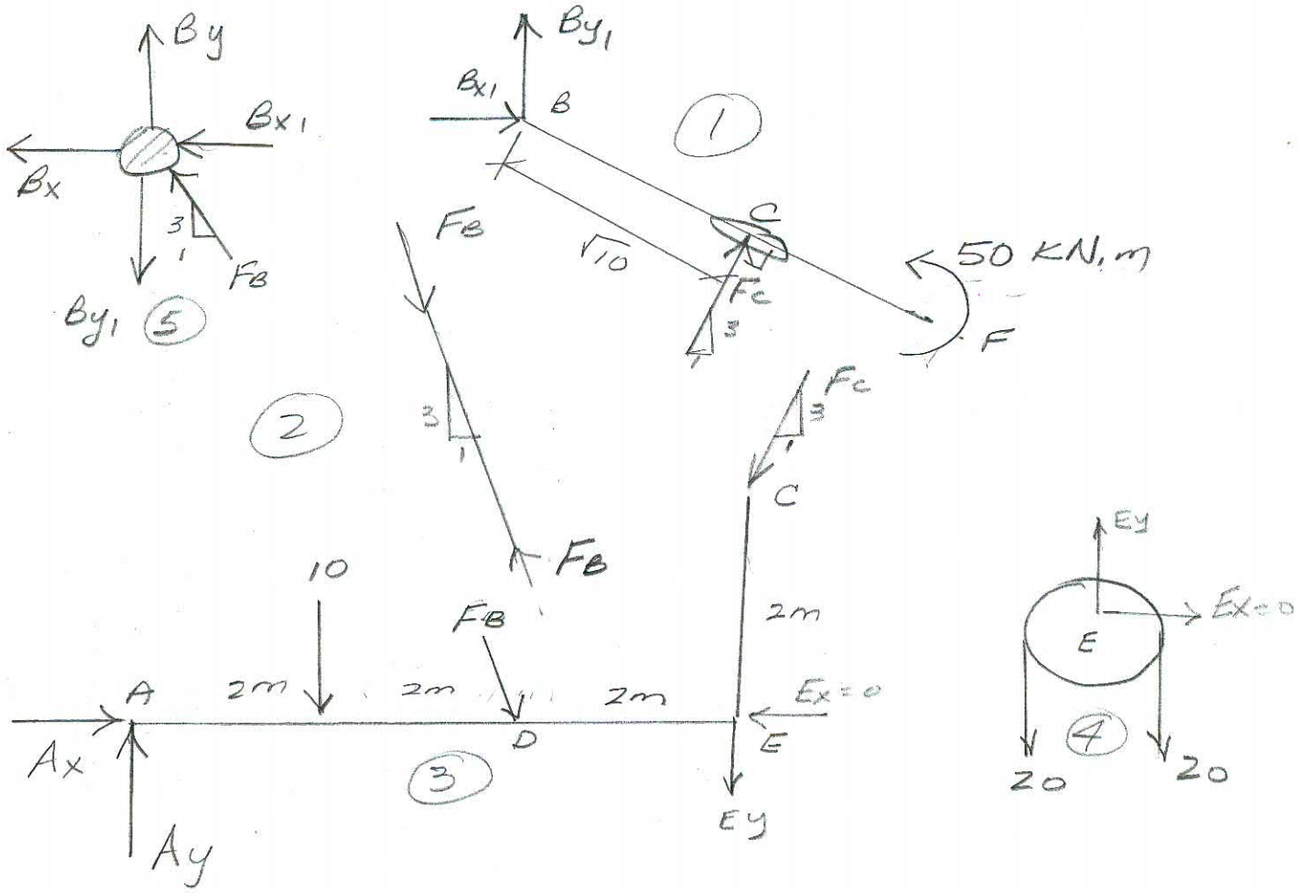
$\Sigma M_A = 0$

$-10(2) + B_x(3) + B_y(3) - 20(5.5) - 20(6.5) + 50 = 0$

$3B_x + 3B_y - 210 = 0$

$3B_x + 3B_y = 210$ (3)

Substructure:



From (4) $\Sigma F_x = 0$ $E_x = 0$

$\Sigma F_y = 0$ $E_y - 20 - 20 = 0$

$E_y = +40\ kN$ $\therefore \vec{E}_y = 40\ kN \uparrow$ on the pulley

From (1)

$\Sigma M_B = 0$ $F_C\sqrt{10} + 50 = 0$

$F_C = -\frac{50}{\sqrt{10}} = -15.81\ kN$

$\therefore \vec{F}_C = 15.81\ kN$  on BCF

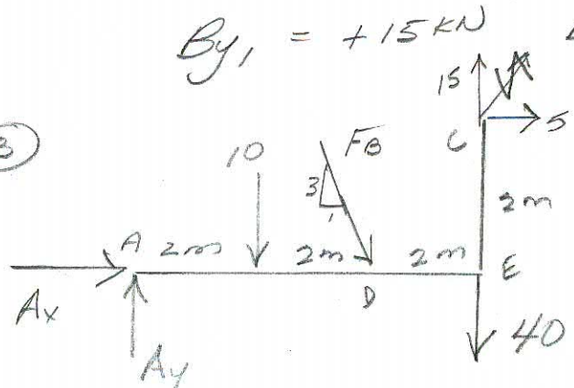
$\Sigma F_x = 0$ $B_{x1} + \frac{1}{\sqrt{10}}(-15.81) = 0$

$B_{x1} = +5\ kN$ $\therefore \vec{B}_{x1} = 5\ kN \rightarrow$ on BCF

$\Sigma F_y = 0$ $B_{y1} + \frac{3}{\sqrt{10}}(-15.81) = 0$

$B_{y1} = +15\ kN$ $\vec{B}_{y1} = 15\ kN \uparrow$ on BCF

Re-draw (3)



$\Sigma M_A = 0 \quad -10(2) - \frac{3}{\sqrt{10}} F_B(4) - 5(2) + 15(6) - 40(6) = 0$

$-\frac{12}{\sqrt{10}} F_B = +180$

$\therefore F_B = -47.43\ kN$

$\therefore \vec{F_B} = 47.43\ kN$  on ADEC

$\Sigma F_x = 0 \quad A_x + \frac{1}{\sqrt{10}}(-47.43) + 5 = 0$

$A_x = +10\ kN \quad \therefore \vec{A_x} = 10\ kN \rightarrow$

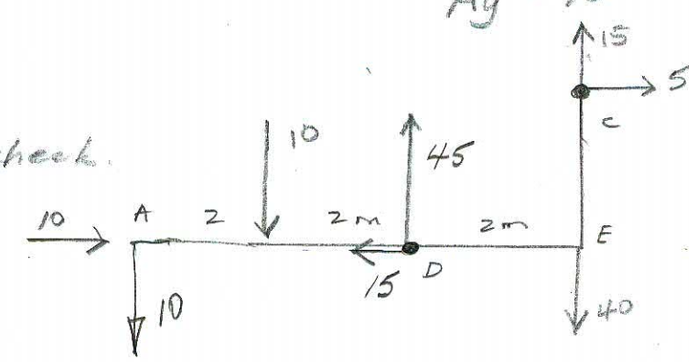
$\Sigma F_y = 0 \quad A_y - 10 - \frac{3}{\sqrt{10}}(-47.43) + 15 - 40 = 0$

$A_y + 10 = 0$

$A_y = -10\ kN$

$A_y = 10\ kN \downarrow$

Check.



$\Sigma F_x = 0$

$10 + 5 - 15 = 0$ ✓

$0 = 0$

$\Sigma F_y = 0$

$-10 - 10 + 45 + 15 - 40 = 0$

$0 = 0$

$\Sigma M_D = 0 \quad 10(4) + 10(2) - 5(2) - 25(2) = 0$

$0 = 0$ ✓

From Original Equilibrium eqns'

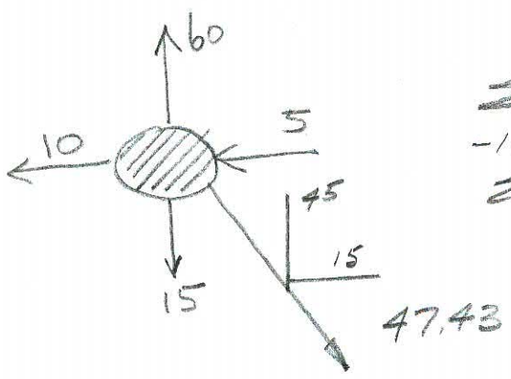
$A_x - B_x = 0$

$10 - B_x = 0 \quad B_x = 10 \quad \therefore \vec{B_x} = 10\ kN \leftarrow$

$\Sigma F_y = 0 \quad -10 - 10 + B_y - 20 - 20 = 0$

$B_y = +60 \quad \therefore \vec{B_y} = 60\ kN \uparrow$

Pin at B.



$\Sigma F_x = 0$ ✓

$-10 - 5 + 15 = 0$

$\Sigma F_y = 0$ ✓

$60 - 15 - 45 = 0$