MATH2132 Test1 Solutions Summer 2011

Values

10 1. Find limits for the following sequences, if they exist.

(a) $\left\{\left(\frac{n+1}{n}\right)^n\left(\frac{n^2}{2n^2+1}\right)\right\}$ (b) $\left\{\frac{2^n+\mathrm{Cot}^{-1}n}{3(2^n+4)}\right\}$

6 2. Find the limit for the following sequence of functions on the interval $-1 < x \leq 100$, if it exists. Show your reasoning or calculations.

$$\left\{\frac{n^2x^2+5x+n}{3n^2-x^{15}}+x\right\}$$

8 3. Determine whether the following series converge or diverge. Justify your conclusions.

(a) $\sum_{n=1}^{\infty}\frac{n^2}{3n^2+2n+5}$ (b) $\sum_{n=2}^{\infty}\frac{e^n}{3^{2n}}$

6 4. Find the sum of the series $\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{5^n}(x+1)^n$. Include its interval of convergence.

10 5. Find the interval of convergence for the power series

$$\sum_{n=3}^{\infty}\frac{1}{n4^n}(x-2)^{2n}?$$

Justify all results.

Answers

1. a) $e/2$ b) $1/3$

2. $\lim_{n\to\infty} f_n(x) = \frac{x^2}{3} + x$

3. a) series diverges by the n^{th} term test.

   b) Convergent Geometric series with $r = e/9 < 1$.

4. $\frac{x+1}{x+6}$, $-6 < x < 4$

5. $0 < x < 4$

MATH2132 Test1 October 6, 2011 60 minutes

Student Name -

Student Number -

Values

12 1. Determine whether the following series converge or diverge. Justify your answers. If a series converges, find its sum.

(a) $\displaystyle\sum_{n=2}^{\infty} \frac{2^{2n+3}}{5^{n+1}}$ (b) $\displaystyle\sum_{n=1}^{\infty} \frac{n-4}{10n+5}$

13 2. Find the interval of convergence for the power series

$$\sum_{n=1}^{\infty} \frac{2^n}{n+1} x^{2n+2}.$$

11 3. Use Taylor's remainder formula to verify that the Maclaurin series for e^{-2x} converges to e^{-2x} for $x \leq 0$.

6 4. Is it possible for the Maclaurin series for a function $f(x)$ to converge at $x = 5$, but not at $x = 4$? Explain.

8 5. Determine whether the sequence of functions

$$\left\{ \frac{n^2x^2 + 3n^2x + n}{2n^2x + 5nx + 4} \right\}$$

has a limit as $n \to \infty$. If the sequence has a limit, find it; if the sequence does not have a limit, indicate why not. Do this on the following intervals:

(a) $x \geq 1$ (b) $-1 < x < 1$

Answers

1. a) Convergent Geometric Series with $r = 4/5 < 1$, Sum $= \frac{128}{25}$

b) divergent by the n^{th} term test.

2. $-\frac{1}{\sqrt{2}} < x < \frac{1}{\sqrt{2}}$

3. Hint: try to obtain: $\lim_{n \to \infty} |R_n(0, x)| < \lim_{n \to \infty} e^{-2x} \frac{|2x|^{n+1}}{(n+1)!} = 0$ (for all $x \leq 0$)

4. No. $-R < -5 \leq x \leq 5 < R$, and $x = 4$ is inside the interval.

5. a) $\lim_{n \to \infty} f_n(x) = \frac{x^2+3}{2}$, $x \geq 1$

b) $\lim_{n \to \infty} f_n(x)$ does not exist on the interval: $-1 < x < 1$.

MATH2132 Test1 May 22, 2013 60 minutes

Student Name - Student Number -

Values

6 1. Find the limit for the following sequence of functions on the interval $0 \le x \le 5$, if it exists. Show your reasoning and all calculations.

$$\left\{\frac{n^3x^3 + n^2x^2 + 4}{2n^3x^2 + nx + 1}\right\}$$

12 2. Determine whether the following series converge or diverge. If a series converges, find its sum. Justify your conclusions.

(a) $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n 3^n}{2^{2n+1}}$ (b) $\displaystyle\sum_{n=1}^{\infty} \left(1 + \frac{1}{an}\right)^4$ where $0 < a < 5$ is a constant

10 3. Find the open interval of convergence for the power series

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}[1 \cdot 4 \cdot 7 \cdots (3n+1)]}{2^n}(x+1)^{2n+1}.$$

12 4. (a) For the function $f(x) = \dfrac{1}{4+3x}$, calculate $f'(x)$, $f''(x)$, $f'''(x)$, and more derivatives if necessary, in order to find a formula for the n^{th} derivative $f^{(n)}(x)$ of the function.

(b) Use the result in part (a) to find a formula for the Taylor remainder $R_n(1, x)$, simplified as much as possible.

Answers

1. $\displaystyle\lim_{n\to\infty} f_n(x) = \begin{cases} x/2 & 0 < x \le 5 \\ 4 & x = 0 \end{cases}$

2. a) Convergent Geometric Series with $r = -3/4 < 1$, sum $= 9/56$

b) divergent, by the n^{th} term test

3. Series Converges only for $x = -1$

4. a) $f^{(n)}(x) = \dfrac{(-1)^n 3^n n!}{(4+3x)^{n+1}}$ $n \ge 1$

b) $R_n(1, x) = \dfrac{(-1)^{n+1} 3^{n+1}}{(4+3z_n)^{n+2}}(x-1)^{n+1}$, z_n is between 1 and x.

MATH2132 Test1 **September 26, 2013** **60 minutes**

Student Name - **Student Number -**

Values

6 **1.** Find the limit for the following sequence of functions on the interval $0 \le x \le 5$, if it exists. Show your reasoning and all calculations.

$$\left\{ \frac{3^{n+1}x^4 + 2^n x^2 + 11}{3^n x^2 + 5x + 55} \right\}$$

12 **2.** Determine whether the following series converge or diverge. If a series converges, find its sum. Justify your conclusions.

(a) $\displaystyle\sum_{n=2}^{\infty} \frac{17^{n+2}}{4^{2n+3}}$ (b) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \left(\frac{n+2}{14n}\right)^3$

10 **3.** (a) Find all values of the constant a for which the series

$$\sum_{n=2}^{\infty} \frac{a^{2n}+3}{5^{n+1}}$$

converges.

(b) Find the sum of the series for the values of a for which the series converges.

12 **4.** Prove that the Taylor series for $\overset{e^{3-3x}}{\cancel{e^{3x}}}$ about $x = 1$ converges to $\overset{e^{3-3x}}{\cancel{e^{3x}}}$ for all $x \ge 1$.

<u>Answers</u>.

1. $\displaystyle\lim_{n\to\infty} f_n(x) = \begin{cases} 3x^2 & 0 < x \le 5 \\ 1/5 & x = 0 \end{cases}$

2. a) Diverges (Geometric Series with $r = 17/16 > 1$)

   b) Diverges (by the n^{th} term test)

3. a) $-\sqrt{5} < a < \sqrt{5}$ b) $\dfrac{a^4}{25(5-a^2)} + \dfrac{3}{100}$

4. Hint: try to obtain

$$\lim_{n\to\infty} |R_n(1,x)| < \lim_{n\to\infty} \frac{|3(x-1)|^{n+1}}{(n+1)!} = 0 \quad (\text{for all } x \ge 1)$$

MATH2132 Test1 February 4, 2014 70 minutes

1. (a) Determine whether the sequence of functions

$$\{f_n(x)\} = \left\{\frac{3n^2x^2+1}{n^2x^2+2nx+4}\right\}$$

has a limit on the interval $-1 \le x \le 1$. Show your reasoning and all calculations.

(b) Would the series $\sum_{n=1}^{\infty} f_n(x)$ have a sum? Explain.

10 2. Determine whether the following series converge or diverge. If a series converges, find its sum. Justify your conclusions.

(a) $\displaystyle\sum_{n=2}^{\infty}\frac{3^{n+3}}{2^{2n-1}}$ (b) $\displaystyle\sum_{n=1}^{\infty}(-1)^n\left(\frac{n^2+1}{3n^2+4}\right)$

10 3. Find the interval of convergence for the power series

$$\sum_{n=3}^{\infty}\frac{(-1)^n n}{a^n}x^{2n}, \qquad \text{where } a>0 \text{ is a given constant.}$$

8 5. Find the remainder $R_n(1,x)$ when the function $f(x)=\sin 5x$ is expanded with Taylor's remainder formula (about $x=1$). Verify that $\lim_{n\to\infty} R_n(1,x)=0$ for all x.

Answers

1. a) $\lim_{n\to\infty} f_n(x) = \begin{cases} 3 & -1\le x\le 1,\ x\ne 0 \\ 1/4 & x=0 \end{cases}$

b) NO (series diverges by the n^{th} term test)

2. a) Converges (Geometric series with $r = 3/4 < 1$, Sum $= \frac{3^5\cdot 4}{8} = \frac{243}{2}$

3. $-\sqrt{a} < x < \sqrt{a}$

4. $\cos\left(\frac{x}{\sqrt{2}}\right) + \frac{x^2}{4} - 1$