Math 1710 Tutorial 8.

Inverse Trigonometric Functions and their Derivatives

Problem 1. Simplify the following expressions provided the expression is well-defined:

(a) $\cos^{-1}\left(\cos\left(\frac{\pi}{11}\right)\right)$;

(b) $\cos^{-1}\left(\cos\left(\frac{43\pi}{11}\right)\right)$;

(c) $\sin^{-1}\left(\tan\left(\frac{\pi}{3}\right)\right)$;

(d) $\cos\left(\cot^{-1}\left(-\frac{12}{5}\right)\right)$;

(e) $\tan\left(\sin^{-1}\left(-\frac{15}{17}\right)\right)$;

(f) $\cos\left(2\sin^{-1}\left(\frac{3}{5}\right)\right)$;

(g) $\sec\left(\csc^{-1}\left(-\frac{25}{7}\right)\right)$;

(h) $\cot\left(\sin^{-1}\left(\frac{1}{2}\csc\left(-\frac{11\pi}{4}\right)\right)\right)$.

Problem 2. Using derivatives of inverse trigonometric functions show that for any x,

$$\tan^{-1}(x) + \cot^{-1}(x) = \frac{\pi}{2}.$$

Problem 3. Find $\frac{dy}{dx}$ (simplify the expressions as much as possible):

(a) $y = \sin^{-1}\left(\frac{x^2}{x^2+1}\right)$;

(b) $y = x\tan^{-1}(e^x)$;

(c) $y = 4^{3\cos^{-1}(x)}$;

(d) $y = \sin(\cos^{-1}(x^2))$;

(e) $y = x^{\cot^{-1}(x)}$;

(f) $y = \frac{[\sin^{-1}(x)]^{10}}{x^3}$;

(g) $y = \ln(\sec^{-1}(3x))$.

Problem 4. Find the equation of the tangent line to the curve given by

$$2y\cot^{-1}(x) = \pi(1 + xy)$$

at the point corresponding to $x = 1$.