

Math 2130 Summer2014 Test 1 (by N Harland)

[3] 1. (a) For the curve, $x = y^2 + z^2$, identify the type of curve and give a sketch.

[5] (b) Find the projection of the intersection of $z^2 = x^2 + y^2$ and $x^2 + 3y^2 + z^2 = 4$ in the xy -plane.

2. Let l_1 be the line

$$\frac{x-1}{2} = y-2 = \frac{z-1}{3}$$

and l_2 be the line

$$x = 3 - 2t, y = 1 + t, z = 2 - t.$$

[3] (a) Show that the lines are intersecting and find the point of intersection.

[5] (b) Find the equation of the plane containing both lines.

[3] (c) Find the distance from the point $R(1, 2, 3)$ to the plane found in part (b).

3. Let l_1 be the line

$$\frac{x-3}{2} = y+5 = \frac{z-1}{-2}$$

and l_2 has parametric equations

$$x = 1 \quad y = 3 + 2t \quad z = 1 + 2t.$$

[5] (a) Determine whether the lines are parallel, intersecting or skew.

[6] (b) Find the shortest distance between the lines.

[6] 4. Find a parametric representation for the curves of intersection of $x^2 + y^2 = z^2$ and $x^2 + y^2 + z^2 = 8$ directed so that x increases when y is positive. Justify your answer. Assume $z \geq 0$.

5. Let a curve C be defined by a position vector $\mathbf{r}_1(t) = \langle t, 2-t, 3-2t^2 \rangle$ and let a curve D be defined by a position vector $\mathbf{r}_2(s) = \langle s, s^2, s^3 \rangle$.

[4] (a) Find parametric equations for the tangent line to $\mathbf{r}_1(t)$ at the point $(0, 2, 3)$.

[6] (b) Find the point of intersection of the two curves and find the cosine of the angle between the curves at that point.

[4] (c) Set up, but don't evaluate an integral to find the length of the curve C from the point $(0, 2, 3)$ to $(2, 0, -5)$.