University of Manitoba
Department of Mechanical and Manufacturing Engineering

**ENG 1460 Introduction to Thermal Sciences** (F06)

A01/B01 (Prof. Ormiston)

Term Test # 2 9 November 2006 Duration: 120 minutes

1. You are permitted to use the textbook for the course and a calculator.
2. Ask for clarification if any problem statement is unclear.
3. Clear, systematic solutions are required. **Show your work.** Marks will not be assigned for problems that require unreasonable (in the opinion of the instructor) effort for the marker to decipher.
4. Use linear interpolation in the property tables as necessary.
5. Keep 5 significant figures in intermediate calculations, and use 4 or 5 significant figures in final answers.
6. There are **two** problems on this test. The weight of each problem is indicated. The test will be marked out of **95**.

Values

50 1. A control mass system contains 1.250 [$kg$] of water. Initially (at state 1), the water is at a pressure of 800 [$kPa$] and a temperature of 900 [°C]. The system then undergoes two quasi-equilibrium processes, one after the other.

In the first process (state 1 to state 2), the water is compressed in a polytropic process until the pressure is 400 [$kPa$] and the volume is 634.5 [$L$]. In the second process (state 2 to state 3), the system volume is held constant and heat is removed until the temperature reaches 130 [°C].

Assume changes in potential and kinetic energy of the system are negligible.

7 (a) Determine the temperature after the first process, $T_2$.

7 (b) Determine the polytropic exponent, $n$, for the first process (state 1 to state 2).

4 (c) Calculate the boundary work done by the system during the first process, ${}_1W_2$.

7 (d) Calculate the heat transfer to the system in the first process, ${}_1Q_2$.

11 (e) Calculate the heat transfer to the system in the second process, ${}_2Q_3$.

14 (f) Show the two processes on a $P$-$v$ (pressure–specific volume) diagram. Clearly label the states and show the process paths with respect to the saturation lines and the critical point. Label the state and relevant saturation pressure values and the relevant constant $T$ lines. Mark the area(s) representing specific work done. Do any additional interpolations that may be needed to find values needed for labelling those $P$ and $T$ values. Labelling $v$ values is optional.

2. The piston-cylinder arrangement shown in Figure 1 contains air. Two linear springs with the same spring constant but different lengths are installed as shown above the piston. In the initial position, the piston is motionless and floating so that the top of the piston is 20.0 $[cm]$ from the bottom of Spring 1 and 60.0 $[cm]$ from the bottom of Spring 2. In this initial position
45 (state 1) the air is at 450 $[kPa]$ and 900 $[K]$.

   Energy is added to the system by heat transfer, causing the piston to rise until it just touches Spring 1, but Spring 1 exerts no force on it (state 2). In this state the air has a volume of 0.080 $[m^3]$. Heating continues again and Spring 1 is compressed until the piston just touches Spring 2, but Spring 2 exerts no force on it (state 3). At this point the pressure is 550 $[kPa]$. Heating continues further while the piston compresses both springs until the pressure is 700 $[kPa]$ (state 4).

   The frictionless piston has a cross-sectional area of 0.0250 $[m^2]$.

7 (a) Calculate the spring constant, $k$, in $[kN/m]$.

(b) Calculate the boundary work done by the system during each of the three processes:
15 ${}_1W_2$, ${}_2W_3$, and ${}_3W_4$.

10 (c) Determine the temperatures $T_2$, $T_3$ and $T_4$.

(d) Show the three processes on a $P$-$V$ (pressure–volume) diagram. Label the state pressure values and the relevant constant $T$ lines. Mark the area(s) representing work done.
13 Labelling $V$ values is optional.

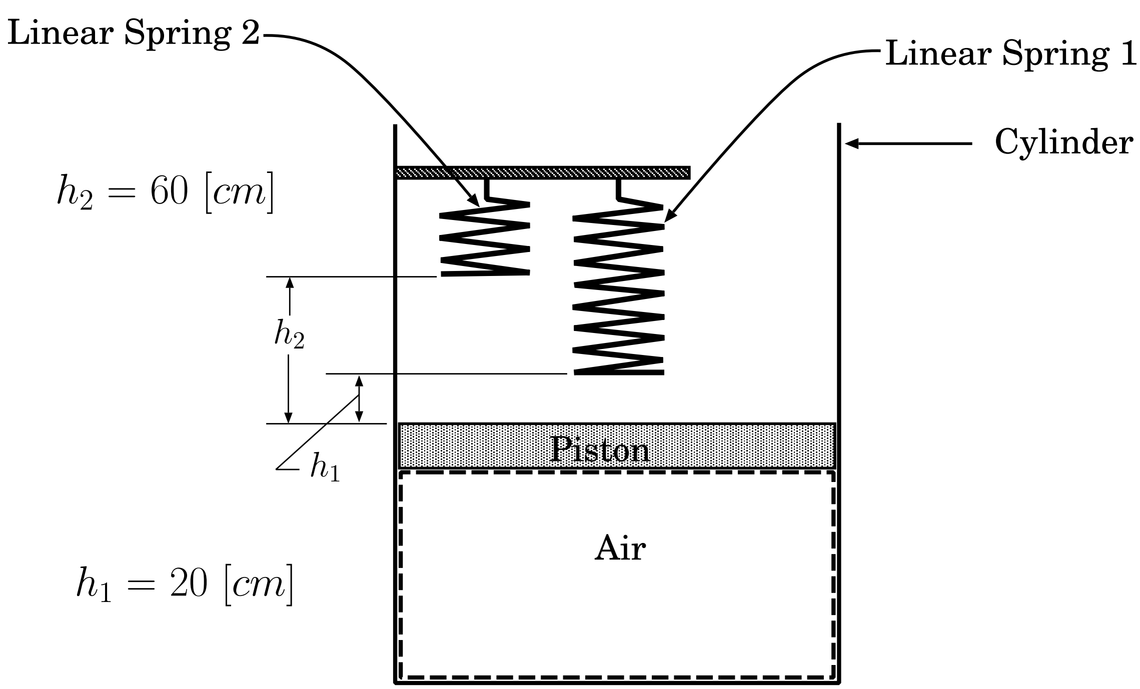



Figure 1: Piston-cylinder arrangement for problem 2

1. (a) $m = 1.250\,[kg]$ (Water)

State 1
$P_1 = 800\,[kPa]$
$T_1 = 900\,[°C]$

State 2
$P_2 = 400\,[kPa]$

$\forall_2 = 634.5\,[L] = 0.6345\,[m^3]$

Polytropic $PV^n = C$ for process $1 \to 2$

$T_{sat}(800\,[kPa]) = 170.43\,[°C]$ $T_1 > T_{sat}(P_1) \Rightarrow$ superheated vapour

$$v_2 = \frac{\forall_2}{m} = \frac{0.6345}{1.250} = 0.5076\,(m^3/kg)$$

$v_g\big|_{400\,(kPa)} = 0.46246\,(m^3/kg)$ (Table B.1.2) $v_2 > v_g\big|_{P_2} \Rightarrow$ superheated vapour

Table B.1.3 $P = 400\,[kPa]$

| $T\,[°C]$ | $v\,(m^3/kg)$ |
|---|---|
| 150 | 0.47084 |
| $T_2$ | 0.5076 |
| 200 | 0.53422 |

$$\frac{T_2 - 150}{200 - 150} = \frac{(0.5076 - 0.47084)}{(0.53422 - 0.47084)} = 0.5800$$

$T_2 = 150 + 0.5800\,(50)$
$T_2 = 179.00\,[°C]$ ←

(b) $P_1 \forall_1^n = P_2 \forall_2^n$

$$\frac{P_1}{P_2} = \left(\frac{\forall_2}{\forall_1}\right)^n$$

$$n = \frac{\ln\left(\frac{P_1}{P_2}\right)}{\ln\left(\frac{\forall_2}{\forall_1}\right)}$$

$\forall_1 = m v_1$

$v_1 = v\,(800\,[kPa], 900\,[°C])$

$v_1 = 0.67610\,(m^3/kg)$ (Table B.1.3)

$\forall_1 = 1.250\,(0.67610) = 0.84513\,(m^3)$

$$n = \frac{\ln\left(\frac{800}{400}\right)}{\ln\left(\frac{0.6345}{0.84513}\right)} = -2.418$$

1(c) $${}_1W_2 = \frac{P_2 V_2 - P_1 V_1}{(1-n)} = \frac{400(0.6345) - 800(0.84513)}{1-(-2.418)}$$

$${}_1W_2 = -123.55\ [kJ]$$

(d) $${}_1Q_2 - {}_1W_2 = m(u_2 - u_1)$$

$${}_1Q_2 = {}_1W_2 + m(u_2 - u_1)$$

$u_1 = u(800[kPa], 900[°C])$

$u_1 = 3852.77\ \left(\frac{kJ}{kg}\right)$

$${}_1Q_2 = -123.55 + 1.250(2612.24 - 3852.77)$$

$$= -123.55 - 1550.66$$

$${}_1Q_2 = -1674.21\ (kJ)$$

$P = 400\ (kPa)$ (Table B.1.3)

| $T$ (°C) | $u$ (kJ/kg) |
|---|---|
| 150 | 2654.48 |
| 179 | $u_2$ |
| 200 | 2646.83 |

(Same interpolation factor as for $T_2$)

$u_2 = 2654.48 + 0.5800(2646.83 - 2654.48)$

$u_2 = 2612.24\ (kJ/kg)$

(e) $${}_2Q_3 = {}_2W_3 + m(u_3 - u_2)$$

$${}_2W_3 = 0 \qquad (V_3 = V_2)$$

State 3 $\quad T_3 = 130[°C] \quad v_3 = v_2 = 0.5076\ (m^3/kg)$

$v_f|_{130°C} = 0.001070\ (m^3/kg) \qquad v_g)_{130°C} = 0.66850\ (m^3/kg)$

$v_f < v_3 < v_g \rightarrow$ Saturated mixture

$$x_3 = \frac{(v_3 - v_f)}{(v_g - v_f)} = \frac{(0.5076 - 0.001070)}{(0.66850 - 0.001070)} = 0.75893$$

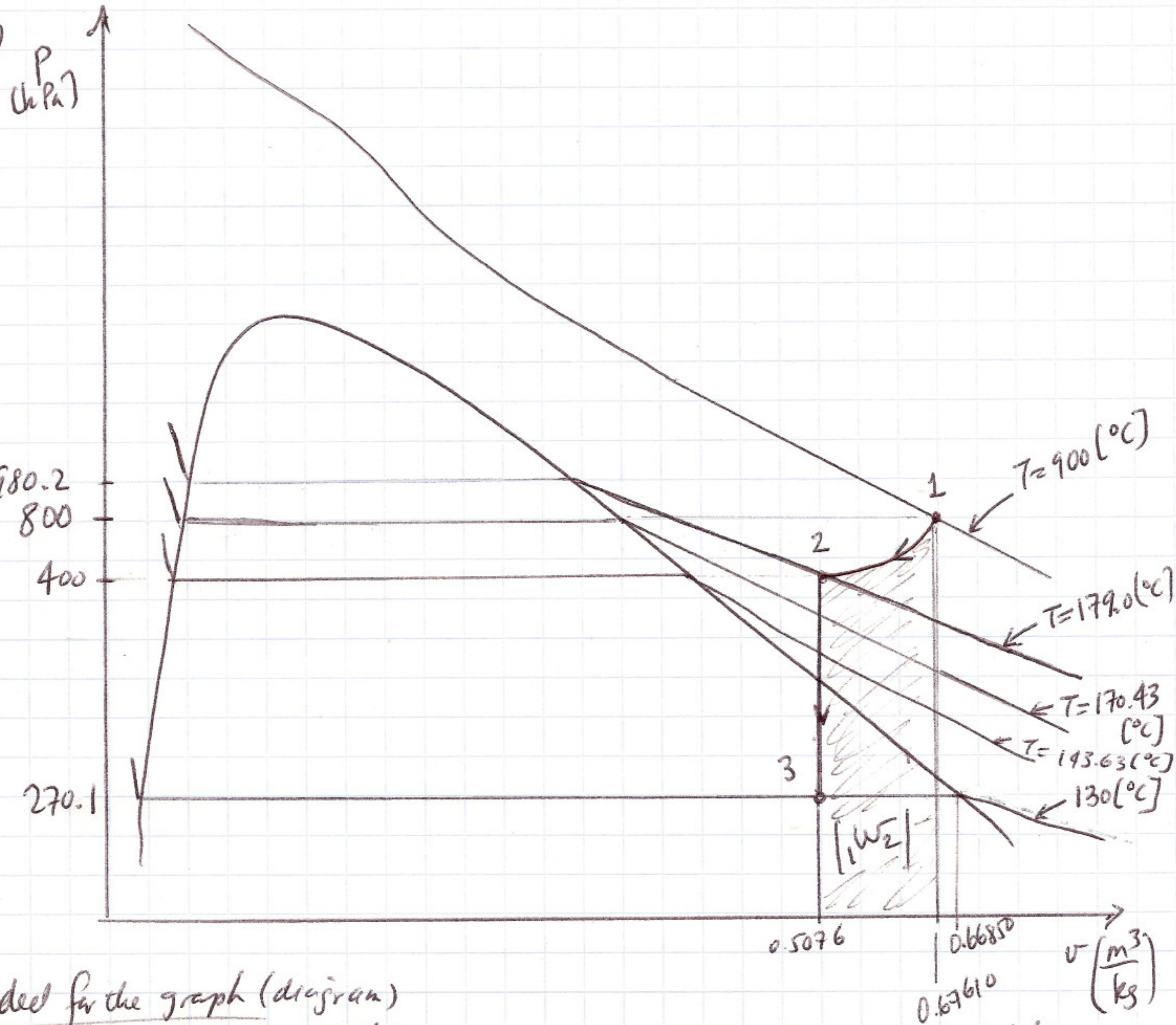
$u_3 = (1-x_3)u_f + x_3 u_g$

$u_f|_{130°C} = 546.00\ [kJ/kg] \qquad u_g)_{130°C} = 2539.90\ (kJ/kg)$

$$u_3 = (1-0.75893)546.00 + (0.75893)2539.90 = 2059.23\left(\frac{kJ}{kg}\right)$$

$${}_2Q_3 = 0 + 1.250(2059.23 - 2612.24) = -691.26\ [kJ]$$

1 (f)




Needed for the graph (diagram)

$P_{sat}(130\,[°C]) = 270.1\ (kPa)$

$T_{sat}(400\,(kPa)) = 143.63\ (°C)$

$T_{sat}(800\,[kPa]) = 170.43\ (°C)$

$$P_{sat}(179\,[°C]) = 892.0 + \frac{(179-175)}{(180-175)}\left(1002.2 - 892.0\right) = 980.2\ (kPa)$$

2. (a) $P_2 = P_1 = 450\ [kPa]$

$P_3 = 550\ [kPa]$

$\forall_2 = 0.080\ [m^3]$

$\forall_3 = \forall_2 + (h_2 - h_1)A = 0.080 + (0.60 - 0.20)\,0.025$

$\forall_3 = \underline{0.090}\ (m^3)$

$P_3 = P_2 + \dfrac{k}{A^2}(\forall_3 - \forall_2)$

$k = \dfrac{(P_3 - P_2)A^2}{(\forall_3 - \forall_2)} = \dfrac{(550 - 450)(0.025)^2}{(0.090 - 0.080)} = 6.25\left[\dfrac{kN}{m}\right]$

(b) • Process 1 → 2 Constant P

${}_1W_2 = P_1(\forall_2 - \forall_1)$ $\qquad (\forall_2 - \forall_1) = h_1(A) = 0.20\,(0.025)$

$(\forall_2 - \forall_1) = 0.005\ (m^3)$

${}_1W_2 = 450\,(0.005)$

${}_1W_2 = 2.25\ [kJ]$

$\forall_1 = \forall_2 - h_1 A$
$= 0.080 - 0.005$
$\forall_1 = 0.075\ (m^3)$

• Process 2 → 3 linear spring
<u>Spring 1 only</u>

${}_2W_3 = 0.5\,(P_2 + P_3)(\forall_3 - \forall_2) = 0.5\,(450 + 550)(0.090 - 0.080)$

${}_2W_3 = 5.00\ [kJ]$

• Process 3 → 4 two linear spring with the same k
→ same as one linear spring with
<u>twice</u> the spring constant

${}_3W_4 = 0.5\,(P_3 + P_4)(\forall_4 - \forall_3)$ $\qquad P_4 = 700\ (kPa)$

need $\forall_4$. $\qquad \forall_4 = \forall_3 + (P_4 - P_3)\dfrac{A^2}{2k}$ ← $\boxed{\text{NOTE } \underline{\underline{2k}}}$

$\forall_4 = 0.090 + \dfrac{(700 - 550)(0.025)^2}{2\,(6.25)} = 0.0975\ (m^3)$

2(b) continued

$${}_3W_4 = 0.5(550+700)(0.0975-0.090) = 4.6875\ [kJ]$$

(c) • $T_2$:

$$P_2\forall_2 = mRT_2 \qquad P_2 = P_1$$
$$P_1\forall_1 = mRT_1$$

$$\frac{\forall_2}{\forall_1} = \frac{T_2}{T_1} \qquad T_2 = T_1\left(\frac{\forall_2}{\forall_1}\right)$$

$$T_2 = 900\left(\frac{0.080}{0.075}\right) = 960\ [K]$$

• $T_3$:

(Assuming m is known)

$$T_3 = \frac{P_3\forall_3}{mR}$$

$$T_3 = \frac{(550)(0.090)}{(0.13066)(0.287)} = 1320\ (K]$$

also by $T_3 = T_2\left(\frac{P_3\forall_3}{P_2\forall_2}\right)$

Alternative is to calculate m from State 1 and then use $P\forall = mRT$ at all three states

$$m = \frac{P_1\forall_1}{RT_1} = \frac{450(0.075)}{0.287\ 900}$$

$$\boxed{m = 0.13066\ [kg]}$$

($R_{air} = 0.287\ \left(\frac{kJ}{kg\,K}\right)$ was used in this case)

• $T_4$:

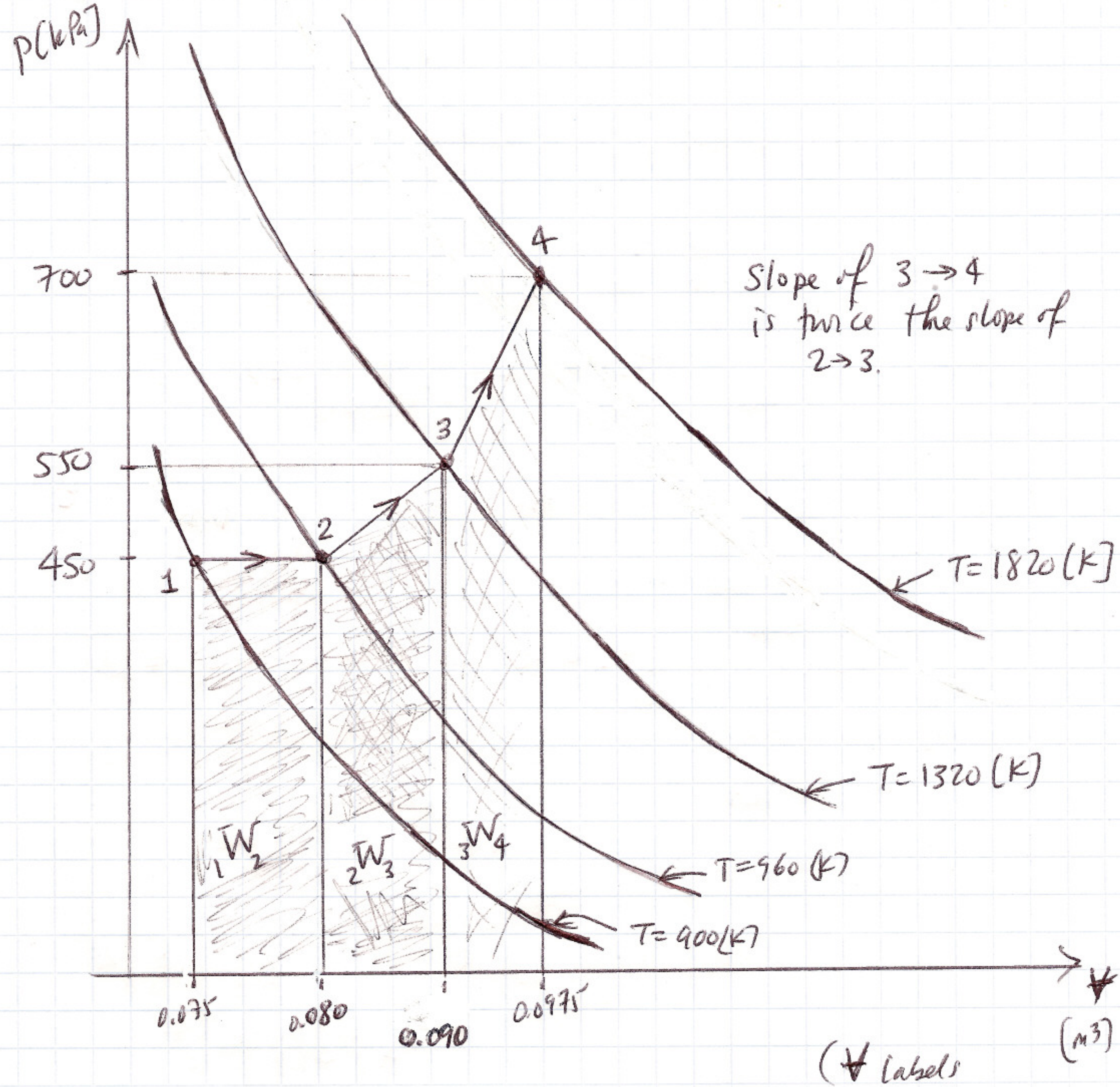
$$T_4 = \frac{P_4\forall_4}{mR} = \frac{(700)(0.0975)}{0.13066\ 0.287}$$

$$T_4 = 1820\ [K]$$

Also by: $T_4 = T_3\left(\frac{P_4\forall_4}{P_3\forall_3}\right)$

2 (d)






Slope of 3 → 4 is twice the slope of 2 → 3.

(∀ labels optional)