Practice Midterm 2A Answers

Part A

1. D
2. C
3. D
4. B
5. E
6. D
7. A
8. B
9. A
10. D
11. E
12. B
13. A
14. D
15. A

Part B

1. (a) The complete sample space of outcomes is shown below, together with the probabilities of each outcome. Notice that probabilities can be multiplied, as outcomes of games are independent.

Outcome	Probability
FUJ	$(0.2)(0.3)(0.4) = 0.024$
FUG	$(0.2)(0.3)(0.6) = 0.036$
FSJ	$(0.2)(0.7)(0.4) = 0.056$
FSG	$(0.2)(0.7)(0.6) = 0.084$
CUJ	$(0.8)(0.3)(0.4) = 0.096$
CUG	$(0.8)(0.3)(0.6) = 0.144$
CSJ	$(0.8)(0.7)(0.4) = 0.224$
CSG	$(0.8)(0.7)(0.6) = 0.336$

(b) $P(X = 0) = P(CUJ) = 0.096$
$P(X = 1) = P(FUJ) + P(CUG) + P(CSJ) = 0.024 + 0.144 + 0.224 = 0.392$
$P(X = 2) = P(FUG) + P(FSJ) + P(CSG) = 0.036 + 0.056 + 0.336 = 0.428$
$P(X = 3) = P(FSG) = 0.084$

The p.m.f. of X is shown in tabular form below:

x	0	1	2	3
$P(X = x)$	0.096	0.392	0.428	0.084

(c) $E(X) = \sum p_i x_i = 0.096(0) + 0.392(1) + 0.428(2) + 0.084(3) = 1.5$

$E(X^2) = \sum p_i x_i^2 = 0.096(0^2) + 0.392(1^2) + 0.428(2^2) + 0.084(3^2) = 2.86$

And so $\mathrm{Var}(X) = E(X^2) - (E(X))^2 = 2.86 - (1.5)^2 = 2.86 - 2.25 = 0.61$

2. (a) $$F(x) = \int_{-\infty}^{x} f(y)\,dy = \int_{1}^{x} \frac{3}{7} y^2\,dy = \left[\frac{y^3}{7}\right]_1^x = \frac{x^3 - 1}{7} \quad \text{for } 1 \le x \le 2$$

We also know that F(x) = 0 for x < 1 and F(x) = 1 for x > 2.

(b) The median of the distribution of X is the value x such that

$$F(x) = 0.5 \quad \Rightarrow \quad \frac{x^3 - 1}{7} = 0.5 \quad \Rightarrow \quad x^3 = 7(0.5) + 1 = 4.5 \quad \Rightarrow \quad x = (4.5)^{1/3} = 1.65$$

(c) We can calculate this probability in either of two ways:

(i) Using the p.d.f.:

$$P(1.2 < X < 1.5) = \int_{1.2}^{1.5} f(x)\,dx = \int_{1.2}^{1.5} \frac{3}{7} x^2\,dx = \left[\frac{x^3}{7}\right]_{1.2}^{1.5} = \frac{(1.5)^3 - (1.2)^3}{7} = \frac{3.375 - 1.728}{7} = \frac{1.647}{7} = 0.235$$

(ii) Using the c.d.f.:

$$P(1.2 < X < 1.5) = F(1.5) - F(1.2) = \left(\frac{(1.5)^3 - 1}{7}\right) - \left(\frac{(1.2)^3 - 1}{7}\right) = 0.339 - 0.104 = 0.235$$

(d) $$E(X) = \int_{-\infty}^{\infty} x f(x)\,dx = \int_{1}^{2} \frac{3}{7} x^3\,dx = \left[\frac{3}{28} x^4\right]_1^2 = \frac{3}{28}(2^4 - 1^4) = \frac{3}{28}(15) = \frac{45}{28} = 1.61$$

3. (a) We verify the four conditions of the binomial distribution:

- There is a fixed sample size (n = 10).
- There are only two outcomes of interest for each trial – either the person smokes or the person doesn't smoke.
- Trials are independent – whether one person smokes does not affect whether any other person smokes.
- The probability of selecting a smoker is constant from trial to trial (p = 0.21).

Since the four conditions of the binomial setting are satisfied, it follows that X has a binomial distribution with parameters n = 10 and p = 0.21.

(b) $P(X \geq 3) = 1 - P(X \leq 2) = 1 - [P(X = 0) + P(X = 1) + P(X = 2)]$

$$= 1 - \left[\binom{10}{0}(0.21)^0(0.79)^{10} + \binom{10}{1}(0.21)^1(0.79)^9 + \binom{10}{2}(0.21)^2(0.79)^8\right]$$

$$= 1 - (0.0947 + 0.2517 + 0.3011) = 1 - 0.6475 = 0.3525$$

(c) We know that $E(X) = np = 3.6$ and

$Var(X) = np(1 - p) = 3.6(1 - p) = 2.52$

$$\Rightarrow \quad 1 - p = \frac{2.52}{3.6} = 0.7 \quad \Rightarrow \quad p = 0.3$$

Substituting this back into the equation for the mean of X,

$$E(X) = np = n(0.3) = 3.6 \Rightarrow \quad n = \frac{3.6}{0.3} = 12$$