Example 5.2

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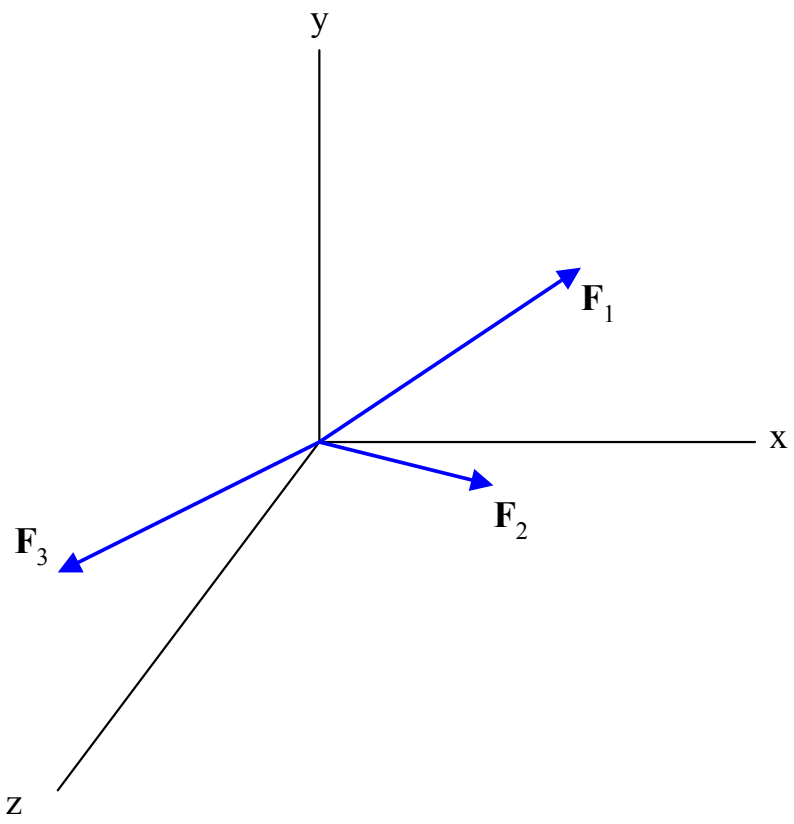
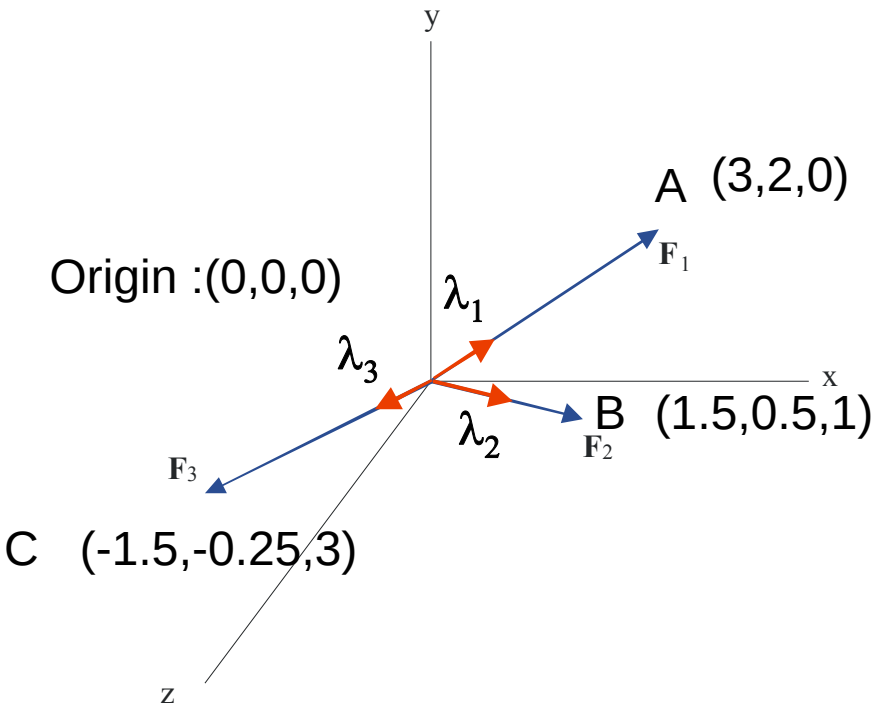
Determine the resultant of the three forces shown if **F**1 passes through the origin and point (3, 2, 0), **F**2 passes through the origin and point (1.5, 0.5, 1), and **F**3 passes through the origin and point (-1.5, -0.25, 3). The magnitudes of the three forces are 1500, 2000, and 1500 N, respectively



STEP 1 – Express ALL forces in Component Form

$$\mathbf{F} = \mathbf{F}\boldsymbol{\lambda}$$

Force and Unit (λ) Vector



(1) $\mathbf{F_1} = F_1\boldsymbol{\lambda}_1$

$$\boldsymbol{\lambda}_1 = \frac{\mathbf{OA}}{OA} \qquad \mathbf{OA} = 3\mathbf{i} + 2\mathbf{j} + 0\mathbf{k} \quad \text{and} \quad OA = \sqrt{3^2 + 2^2} = \sqrt{13}$$

$$\mathbf{F_1} = F_1\left(\frac{3\mathbf{i} + 2\mathbf{j}}{\sqrt{13}}\right) = 1500\left(\frac{3\mathbf{i} + 2\mathbf{j}}{\sqrt{13}}\right)$$

$$\mathbf{F_1} = 1248.08\mathbf{i} + 832.05\mathbf{j}$$

(2) $\mathbf{F_2} = F_2\boldsymbol{\lambda}_2$

$$\boldsymbol{\lambda}_2 = \frac{\mathbf{OB}}{OB} \qquad \mathbf{OB} = 1.5\mathbf{i} + 0.5\mathbf{j} + 1\mathbf{k} \quad \text{and} \quad OB = \sqrt{1.5^2 + 0.5^2 + 1^2} = \sqrt{3.5}$$

$$\mathbf{F_2} = F_2\left(\frac{1.5\mathbf{i} + 0.5\mathbf{j} + 1\mathbf{k}}{\sqrt{3.5}}\right) = 2000\left(\frac{1.5\mathbf{i} + 0.5\mathbf{j} + 1\mathbf{k}}{\sqrt{3.5}}\right)$$

$$\mathbf{F_2} = +1603.57\mathbf{i} + 534.52\mathbf{j} + 1069.04\mathbf{k}$$

(3) $\mathbf{F_3} = F_3\boldsymbol{\lambda}_3$

$$\boldsymbol{\lambda}_3 = \frac{\mathbf{OC}}{OC} \qquad \mathbf{OC} = -1.5\mathbf{i} - 0.25\mathbf{j} + 3\mathbf{k} \quad \text{and} \quad OC = \sqrt{(-1.5)^2 + (-0.25)^2 + 3^2} = \sqrt{11.3125}$$

$$\mathbf{F_3} = F_3\left(\frac{-1.5\mathbf{i} - 0.25\mathbf{j} + 3\mathbf{k}}{\sqrt{11.3125}}\right) = 1500\left(\frac{-1.5\mathbf{i} - 0.25\mathbf{j} + 3\mathbf{k}}{\sqrt{11.3125}}\right)$$

$$\mathbf{F_3} = -668.96\mathbf{i} - 111.49\mathbf{j} + 1337.93\mathbf{k}$$

Collect the x, y and z components

$$\mathbf{R_x} = \sum F_x = \left(+1248.08 + 1603.57 - 668.96\right)\mathbf{i} = 2182.69\mathbf{i}$$

$$\mathbf{R_y} = \sum F_y = \left(+832.05 + 534.52 - 111.49\right)\mathbf{j} = 1255.08\mathbf{j}$$

$$\mathbf{R_z} = \sum F_z = \left(+1069.04 + 1337.93\right)\mathbf{k} = 2406.97\mathbf{k}$$

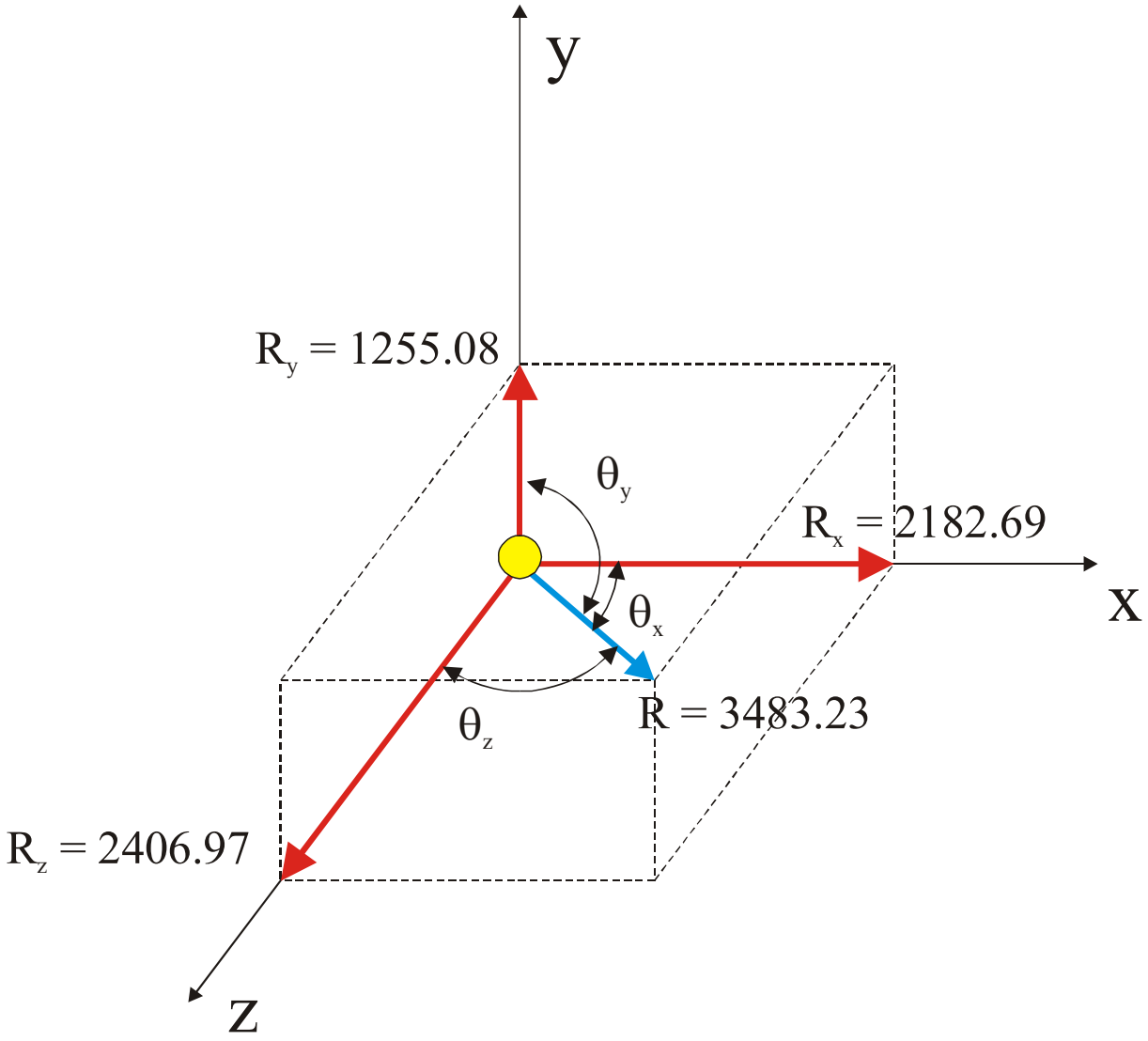
$$\mathbf{R} = \sqrt{2182.69^2 + 1255.08^2 + 2406.97^2} = 3483.23 \text{ N}$$

Direction Cosines

$$\cos\theta_x = \frac{2182.69}{3483.23} \Rightarrow \theta_x = 51.2^{O}$$

$$\cos\theta_y = \frac{1255.08}{3483.23} \Rightarrow \theta_y = 68.88^{O}$$

$$\cos\theta_z = \frac{2406.97}{3483.23} \Rightarrow \theta_z = 46.29^{O}$$


y
$R_y = 1255.08$
θ_y
$R_x = 2182.69$
x
θ_x
θ_z
$R = 3483.23$
$R_z = 2406.97$
z