UNIVERSITY OF MANITOBA
DEPARTMENT OF MATHEMATICS

MATH 1510 Applied Calculus I

FIRST TERM EXAMINATION - VERSION A

October 13, 2016 1:00 pm

LAST NAME: ______________________________

FIRST NAME: ______________________________

STUDENT NUMBER: ______________________________

SIGNATURE: ______________________________
(I understand that cheating is a serious offense)

DO NOT WRITE IN THIS TABLE

Question	Points	Score
1	8	
2	13	
3	9	
4	9	
5	7	
Total:	46	

INSTRUCTIONS TO STUDENTS:

Fill in clearly *all the information above.*

This is a 50 minute exam.

No *calculators, texts, notes, cellphones or other aids are permitted.*

Show your work clearly *for full marks.*

This exam has a title page, 5 pages of questions and 1 blank page at the end for rough work. Please check that you have all pages.

The value of each question is indicated in the left-hand margin beside the statement of the question. The total value of all questions is 46.

Answer all questions on the exam paper in the space provided. If you need more room, you may continue your work on the ***reverse*** *side of the page, but* ***clearly indicate*** *that your work is continued there.*

1. In each of the following cases, compute the limit. If the limit does not exist, determine with proof whether the trend is ∞, $-\infty$ or neither.

[4] (a) $\lim_{x\to 2} \dfrac{\sqrt{2x}-2}{x-2}$

Solution:

$$\begin{aligned}\lim_{x\to 2} \frac{\sqrt{2x}-2}{x-2} &= \lim_{x\to 2}\left(\frac{\sqrt{2x}-2}{x-2}\cdot\frac{\sqrt{2x}+2}{\sqrt{2x}+2}\right)\\ &= \lim_{x\to 2}\frac{2x-4}{(x-2)(\sqrt{2x}+2)} = \lim_{x\to 2}\frac{2(x-2)}{(x-2)(\sqrt{2x}+2)}\\ &= \lim_{x\to 2}\frac{2}{\sqrt{2x}+2}\\ &= \frac{2}{2+2} = \frac{1}{2}.\end{aligned}$$

[4] (b) $\lim_{x\to -\infty} \dfrac{\sqrt{x^6-x+1}}{2x^3-1}$

Solution:

$$\begin{aligned}\lim_{x\to -\infty} \frac{\sqrt{x^6-x+1}}{2x^3-1} &= \lim_{x\to -\infty}\frac{\sqrt{x^6\left(1-\frac{1}{x^5}+\frac{1}{x^6}\right)}}{2x^3-1} = \lim_{x\to -\infty}\frac{|x|^3\cdot\sqrt{1-\frac{1}{x^5}+\frac{1}{x^6}}}{2x^3-1}\\ &\underset{|x|=-x \text{ since } x<0}{=} \lim_{x\to -\infty}\frac{-x^3\cdot\sqrt{1-\frac{1}{x^5}+\frac{1}{x^6}}}{2x^3-1}\\ &= \lim_{x\to -\infty}\frac{-\sqrt{1-\frac{1}{x^5}+\frac{1}{x^6}}}{2-\frac{1}{x^3}}\\ &= \frac{-\sqrt{1-0+0}}{2-0} = -\frac{1}{2}.\end{aligned}$$

[4] **2.** (a) For $f(x) = (1+2x-3x^2)(4-5x)$, find $f'(x)$ using the product rule.
DO NOT SIMPLIFY YOUR ANSWER.

Solution:

$$f'(x) = (2-6x)(4-5x) + (1+2x-3x^2)(-5)$$

[4] (b) For $y = \dfrac{x^{2\pi} + x\sqrt{x}}{\sqrt{x}} + e^e$, find $\dfrac{dy}{dx}$. DO NOT SIMPLIFY YOUR ANSWER.

Solution:

Since $y = x^{2\pi - 1/2} + x + e^e$, $\dfrac{dy}{dx} = (2\pi - 1/2)x^{2\pi - 3/2} + 1 + 0.$

[5] (c) For $z(t) = \dfrac{t^{2016}}{1+t}$, find $z'(1)$.

Solution:

$$z'(t) = \frac{2016t^{2015}(1+t) - t^{2016}\cdot 1}{(1+t)^2}$$

so $z'(1) = \frac{4031}{4}$.

---

[9] **3.** Find all vertical asymptotes of the graph of the function $f(x) = \dfrac{x^2+|x|-2}{x^2+x}$.
Justify your answer using limits.

Solution:

Zeros of the denominator: $x^2 + x = 0$ implies that $x = 0$ or $x = -1$. There is no vertical asymptotes at other points as f is continuous on its domain which is $\{x : x \neq 0, -1\}$. At $x = 0$:

$$\lim_{x\to 0^+} \frac{x^2+|x|-2}{x^2+x} = \lim_{x\to 0^+} \frac{x^2+|x|-2}{x(x+1)} = -\infty \qquad \left[\frac{-2}{0^+\cdot 1}\right]$$

Therefore, the graph of f does have a vertical asymptote $x = 0$.

$$\lim_{x\to -1} \frac{x^2+|x|-2}{x^2+x} \underset{x<0}{=} \lim_{x\to -1} \frac{x^2-x-2}{x^2+x} = \lim_{x\to -1} \frac{(x+1)(x-2)}{x(x+1)}$$
$$= \lim_{x\to -1} \frac{x-2}{x} = \frac{-1-2}{-1} = 3.$$

Because $\lim\limits_{x\to -1} f(x)$ is finite, the line $x = -1$ is not a vertical asymptote of the graph of $f(x)$.

---

4. A particle is moving along the x axis, and its displacement in meters after t seconds is given by $x(t) = t^3 - 9t^2 + 15t$, $t \geq 0$.

[4] (a) Find all the values of $t \geq 0$ when the particle is instantaneously at rest.

Solution:
We need $t \geq 0$ where $v(t) = x'(t) = 0$. We have $v(t) = 3t^2 - 18t + 15 = 3(t^2 - 6t + 5) = 3(t-1)(t-5)$.
Answer: $t = 1\ s$, $t = 5\ s$.

[5] (b) Find all the values of $t \geq 0$ when the particle is slowing down.

Solution:
We need $t \geq 0$ where $v(t) \cdot a(t) < 0$. $a(t) = v'(t) = 6t - 18 = 6(t-3)$ and $a(t)v(t) = 18(t-1)(t-3)(t-5)$. This product will be negative when either one or three factors are negative. We conclude that the desired values of t are $t \in [0, 1) \cup (3, 5)$.

[7] **5.** Use limits to find the values of a and b such that the function

$$f(x) = \begin{cases} 1 + ax & x < -1 \\ 3 & x = -1 \\ (a+b)x^2 + 1 & x > -1 \end{cases}$$

is continuous for all real numbers x.

Solution:

The function f is continuous for $x < -1$ and $x > -1$ since in each of these cases it is a polynomial, and polynomials are continuous everywhere.

For the function f to be continuous at $x = -1$ we must have that

$$\lim_{x\to-1^-} f(x) = f(-1) = \lim_{x\to-1^+} f(x).$$

$\lim_{x\to-1^-} f(x) = \lim_{x\to-1^-} (1 + ax) = 1 - a$, $\lim_{x\to-1^+} f(x) = \lim_{x\to-1^+} (a+b)x^2 + 1 = a + b + 1$, and $f(-1) = 3$.

Hence we should have that $1 - a = 3 = a + b + 1$. It follows that $a = 1 - 3 = -2$, and $b = 3 - a - 1 = 4$. So, $a = -2$ and $b = 4$.

This page is intentionally left blank. Use it for rough work.