Math 1710. Homework Problems I (January 4, 2012)

1. $\displaystyle\int \frac{(x+1)(x-3)}{\sqrt[3]{x}}\,dx = \int \frac{x^2-2x-3}{\sqrt[3]{x}}\,dx = \int \frac{x^2}{\sqrt[3]{x}}\,dx - \int \frac{2x}{\sqrt[3]{x}}\,dx - \int \frac{3}{\sqrt[3]{x}}\,dx$

$$= \int x^{5/3}\,dx - 2\int x^{2/3}\,dx - 3\int x^{-1/3}\,dx = \frac{x^{8/3}}{8/3} - 2\frac{x^{5/3}}{5/3} - 3\frac{x^{2/3}}{2/3} + C$$

$$= \boxed{\frac{3x^{8/3}}{8} - \frac{6x^{5/3}}{5} - \frac{9x^{2/3}}{2} + C, \quad C \in \mathbb{R}}$$

2. $\displaystyle\int x^5\sqrt{x^2+1}dx = \int x^4\sqrt{x^2+1}\,x\,dx$

We make substitution $u = x^2+1: \quad du = 2xdx, \quad x^2 = u-1 \Rightarrow x^4 = (u-1)^2$

$$= \int (u-1)^2\sqrt{u}\,\frac{du}{2} = \frac{1}{2}\int (u^2-2u+1)\sqrt{u}\,du$$

$$= \frac{1}{2}\left(\int u^2\sqrt{u}\,du - 2\int u\sqrt{u}\,du + \int \sqrt{u}\,du\right) = \frac{1}{2}\left(\frac{u^{7/2}}{7/2} - 2\frac{u^{5/2}}{5/2} - \frac{u^{3/2}}{3/2}\right) + C$$

$$= \boxed{\frac{(x^2+1)^{7/2}}{7} - \frac{2(x^2+1)^{5/2}}{5} - \frac{(x^2+1)^{3/2}}{3} + C, \quad C \in \mathbb{R}}$$

3. $\displaystyle\int \sin x \tan^2 x\,dx = \int \sin x(\sec^2 x - 1)\,dx = \int \left(\frac{1}{\cos^2 x} - 1\right)\sin x\,dx$

We make substitution $u = \cos x: \quad du = -\sin x dx$

$$\int \left(\frac{1}{u^2} - 1\right)(-du) = \frac{1}{u} + u + C = \boxed{\frac{1}{\cos x} + \cos x + C, \quad C \in \mathbb{R}}$$

4. $\displaystyle\int \frac{dx}{x^2+x} = \int \frac{dx}{x^2(1+\frac{1}{x})}$

We make substitution $u = 1 + \dfrac{1}{x}: \quad du = -\dfrac{dx}{x^2}$

$$= \int \frac{-du}{u} = -\ln|u| + C = \boxed{-\ln\left|1+\frac{1}{x}\right| + C, \quad C \in \mathbb{R}}$$