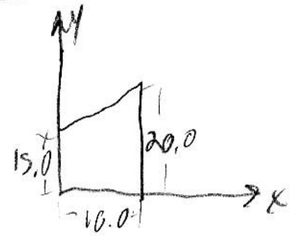
PS #12

(1)



$$I_x = \int_0^{15} 10 y^2 dy + \int_{15}^{20} 10(20-y) y^2 dy$$

$$= \frac{10(15)^3}{3} + 10\left[\frac{20y^3}{3} - \frac{y^4}{4}\right]_{15}^{20}$$

$$= 11250 + \frac{160000}{3} - \frac{160000}{4} + 22500 - \frac{50625}{4}$$

$$\boxed{I_x = 7760.417}$$

$$I_y = \int_0^{10.0} \left(15.0 + \frac{x}{2}\right) x^2 dx$$

$$= 5.0(10.0)^3 + \frac{(10.0)^4}{8} = \boxed{6250 = I_y}$$

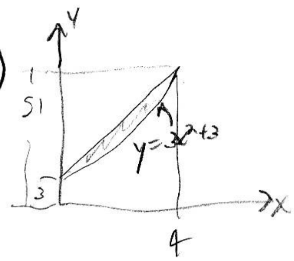
$$J = \int x^2 + y^2 dA$$

$$= \int_0^{10.0} \int_0^{15.0+\frac{x}{2}} (x^2+y^2) dy\, dx = \int_0^{10.0} x^2\left(15+\frac{x}{2}\right) + \frac{\left(15+\frac{x}{2}\right)^3}{3} dx = \int_0^{10} 15x^2 + \frac{x^3}{2} + 3375 + \frac{675x}{2} + \frac{45x^2}{4} + \frac{x^3}{8} dx$$

$$= \left[5x^3 + \frac{x^4}{8} + 3375x + \frac{675x^2}{4} + \frac{15x^3}{4} + \frac{x^4}{32}\right]_0^{10}$$

$$= 5000 + 1250 + 33750 + 16875 + 3750 + 312.5$$

$$\boxed{J = 60937.5}$$

② 

Find limits: $y = 51 = 3x^2+3 \rightarrow 3x^2 = 48$

$x^2 = 16$

$x = 4$

upper edge thus: $51 = 4a + b$

$3 = 0a + b \rightarrow b = 3,\ a = 12$

so: $\underline{y = 12x+3}$

(2 cont.)

$$I_x = \int_0^4 \int_0^{12x+3} y^2\,dy\,dx - \int_0^4 \int_0^{3x^2+3} y^2\,dy\,dx$$

(1st term) (2nd term)

$$= \int_0^4 (1728x^3 + 1296x^2 + 324x + 27)\,dx - \int_0^4 (27x^6 + 81x^4 + 81x^2 + 27)\,dx$$

$$= 140940 - 81620.23$$

$$\boxed{I_x = 59319.77}$$

$$I_y = \int_0^4 \int_0^{12x+3} x^2\,dy\,dx - \int_0^4 \int_0^{3x^2+3} x^2\,dy\,dx$$

$$= \int_0^4 (12x^3 + 3x^2)\,dx - \int_0^4 (3x^4 + 3x^2)\,dx$$

$$= \left[3x^4 + \cancel{x^3} - \frac{3x^5}{5} - \cancel{x^3}\right]_0^4$$

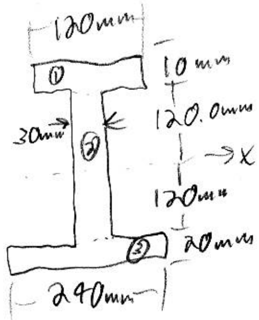
$$= 3(256) - \frac{3(1024)}{5} = \boxed{153.6 = I_y}$$

$$J = \int_0^4 \int_{3x^2+3}^{12x+3} (x^2+y^2)\,dy\,dx = \int_0^4 \int_{3x^2+3}^{12x+3} x^2\,dy\,dx + \int_0^4 \int_{3x^2+3}^{12x+3} y^2\,dy\,dx$$

$= I_x$ (see above) $+ I_y$ (see above)

$$\boxed{J = 59473.37}$$

3



	$\bar{y}$	$\bar{I}_x$	A
1)	125 mm	$\frac{1}{12}120\,mm(10\,mm)^3$	$1200\,mm^2$
2)	0 mm	$\frac{1}{12}(30\,mm)(240\,mm)^3$	$7200\,mm^2$
3)	−130 mm	$\frac{1}{12}(240\,mm)(20\,mm)^3$	$4800\,mm^2$

$$I_x = \overbrace{10^4\,mm^4 + (125\,mm)^2(1200\,mm^2)}^{1} + \overbrace{600\,mm^2(240\,mm)^2}^{2} + \overbrace{160000\,mm^4 + (130\,mm)^2\,4800\,mm^2}$$

$$= 10^4\,mm^4 + 18750\,000\,mm^4 + 34560\,000\,mm^4 + 160\,000\,mm^4 + 81120\,000\,mm^4$$

$$\boxed{\begin{aligned} I_x &= 134\,440\,000\,mm^4 \\ &= 13\,444\,cm^4 \\ &= 0.000134\,m^4 \end{aligned}}$$

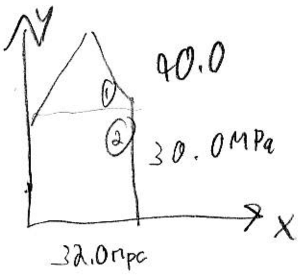
radius of gyration $= \sqrt{\dfrac{I_x}{A}} = \sqrt{\dfrac{134\,440\,000\,mm^4}{1200\,mm^2 + 7200\,mm^2 + 4800\,mm^2}}$

$= 100.92\,mm$

4

Find centroid

body is symmetric in x, by inspection $\boxed{\text{so } \bar{x} = 16.0\,\text{Mpc}}$



	$\bar{y}$	A
1	$\left(30.0 + \frac{10.0}{3}\right)\text{Mpc}$	$\frac{1}{2}(10.0\,\text{Mpc})(32.0\,\text{Mpc}) = 160\,\text{Mpc}^2$
2	$15.0\,\text{MPc}$	$(30.0\,\text{Mpc})(32.0\,\text{Mpc}) = 960\,\text{Mpc}^2$

$$\bar{y} = \frac{33\tfrac{1}{3}\,\text{Mpc}(160\,\text{Mpc}^2) + 15.0\,\text{Mpc}(960\,\text{Mpc}^2)}{160\,\text{Mpc}^2 + 960\,\text{Mpc}^2}$$

$$\boxed{\bar{y} = 17.61905\,\text{Mpc}}$$

new set of areas:

$$\bar{I}_y = \underbrace{\frac{1}{12}(10\,\text{MPc})(16.0\,\text{MPc})^3}_{\text{part 1}} + \underbrace{\frac{1}{12}(10.0\,\text{MPc})(16.0\,\text{Mpc})^3}_{\text{part 2}} + \underbrace{\frac{1}{12}(30.0\,\text{MPc})(32.0\,\text{Mpc})^3}_{\text{part 3}}$$

$$\boxed{\bar{I}_y = 88746\tfrac{2}{3}\,\text{Mpc}^4}$$

$$\bar{I}_x = \underbrace{\frac{1}{12}(32.0\,\text{Mpc})(30.0\,\text{Mpc})^3 + [(17.61905 - 15)\,\text{Mpc}]^2\, 960\,\text{Mpc}^2}_{\text{part 3}} + \frac{1}{36}\, 32.0\,\text{MPc}\left[\left(\frac{10}{3}\right)\text{MPc}\right]^3 + \left(\frac{100}{3}\,\text{Mpc} - 17.61905\,\text{Mpc}\right)^2 (160\,\text{MPc})^2$$

parts 1+2

$$\boxed{\bar{I}_x = 118128.16\,\text{Mpc}^4}$$