MATH 2130 Summer Evening 2013 Problem Workshop 4

1. If $z = f(u, v, t)$, $u = g(x, y, t)$, $v = h(x, y, t)$ and $y = k(t)$, find the chain rule for $\left(\dfrac{\partial z}{\partial t}\right)_x$

2. If $f(s)$ and $g(t)$ are differentiable functions, show that $\nabla f(x^2 - y^2) \cdot \nabla g(xy) = 0$.

3. If $z = x^2 + y^2$, $x = u \cos v$, and $y = u \sin v$, find and simplify $\left(\dfrac{\partial^2 z}{\partial v^2}\right)_u$.

4. If $f(v)$ is differentiable, show that $u(x, y) = x^3 f(x/y)$ satisifes the equation

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 3u.$$

5. The equations

$$x^2 + y + 3s^2 + s = 2t - 1, \qquad y^2 - x^4 + 2st + 7 = 6s^2 t^2$$

define $s$ and $t$ are functions of $x$ and $y$. Find $\dfrac{\partial s}{\partial x}$ when $s = 0$ and $t = 1$. Assume $x > 0$.

6. The equations

$$x = r \sin \phi \cos \theta, \qquad y = r \sin \phi \sin \theta, \qquad z = r \cos \phi$$

define $r, \phi, \theta$ as functions of $x, y$ and $z$. Find $\dfrac{\partial \phi}{\partial y}$.

7. Find the rate of change of the function $f(x, y, z) = \sin(xy) - z^3$ at the point $(2, 0, 3)$ in the direction of the upward normal to the surface $xz^2 - x^2 z = 6$.

8. Find equations for the tangent line to the following curve at the point $(1, -1, 3)$ :

$$xyz + z^3 = 24, \qquad x^3 y^2 z + y^3 = 4x - 2.$$

9. Find an equation for the tangent plane to the following surface at the point $(2, -1, -1)$ :

$$x^2 y + y^2 z + z^2 x + 3 = 0.$$

10. Find all critical points for the function $f(x, y) = x^3 y^3 - x^2 y^2 + 6$.

11. Find all critical points for the function $f(x, y) = x^3 y^2 - xy + 3y$.

12. Find and classify all critical points of the function as giving relative minima, maxima, saddle points or neither.

    (a) $f(x, y) = x^3 + xy + y^3$

(b) $f(x, y) = x^3 - xy^2 + 3xy$

(c) $f(x, y) = x^4 - 3x^2y^2 + y^4$

(d) $f(x, y) = y^2 + |x - 1|$

Answers:

1. $\frac{\partial z}{\partial u}\frac{\partial u}{\partial y}\frac{dy}{dt} + \frac{\partial z}{\partial u}\frac{\partial u}{\partial t} + \frac{\partial z}{\partial v}\frac{\partial v}{\partial y}\frac{dy}{dt} + \frac{\partial z}{\partial v}\frac{\partial v}{\partial t} + \frac{\partial z}{\partial t}$

2.

3. 0

4.

5. 16

6. $r^{-1}\cos\phi\sin\theta$

7. $-216/\sqrt{73}$

8. $x = 1 + 81t, y = -1 + 133t, z = 3 - 6t$

9. $x - 2y + z = 3$.

10. All points on the x-axis, y-axis and on the curve $y = 2/(3x)$.

11. $(3, 0), (9, 1/243)$

12.  (a) $(0, 0)$ gives a saddle point. $(-1/3, -1/3)$ gives a relative maximum.

   (b) $(0, 0), (0, 3)$ both give saddle points.

   (c) $(0, 0)$ gives a saddle point.

   (d) $(1, 0)$ gives a relative minimum. Points $(1, y)$ for $y \neq 0$ give neither.