Math 1710. Homework Problems II (January 6, 2012)

1. $\displaystyle\int_4^{16} \sqrt{x\sqrt{x\sqrt{x}}}\,dx = \int_4^{16} \left(x\left(x\cdot x^{1/2}\right)^{1/2}\right)^{1/2} dx = \int_4^{16} \left(x\cdot x^{3/4}\right)^{1/2} dx = \int_4^{16} x^{7/8}\,dx$

   $\displaystyle= \left.\frac{x^{15/8}}{15/8}\right|_4^{16} = \boxed{\frac{8}{15}\left(16^{15/8} - 4^{15/8}\right)}$

2. $\displaystyle\int_1^4 \frac{\sqrt{1+\sqrt{u}}}{\sqrt{u}}\,du$

   We make substitution $t = 1 + \sqrt{u}$: $\quad dt = \dfrac{1}{2\sqrt{u}}\,du,$

   $u = 1 \to t = 2, \qquad u = 4 \to t = 3.$

   $\displaystyle\int_2^3 2\sqrt{t}\,dt = \left.\frac{2t^{3/2}}{3/2}\right|_2^3 = \left.\frac{4\sqrt{t^3}}{3}\right|_2^3 = \frac{4\sqrt{3^3}}{3} - \frac{4\sqrt{2^3}}{3} = \boxed{\frac{12\sqrt{3} - 8\sqrt{2}}{3}}$

3. $\displaystyle\int_0^{\pi/2} \frac{dx}{1+\tan^2 x} = \int_0^{\pi/2} \frac{dx}{\sec^2 x} = \int_0^{\pi/2} \cos^2 x\,dx = \int_0^{\pi/2} \frac{1+\cos 2x}{2}\,dx = \left.\left[\frac{x}{2} + \frac{\sin 2x}{4}\right]\right|_0^{\pi/2} = \boxed{\frac{\pi}{4}}$

   Comment: The last question becomes **much harder** if you replace power of $\tan x$ with $\sqrt{2}$ instead of 2. Try it!!!