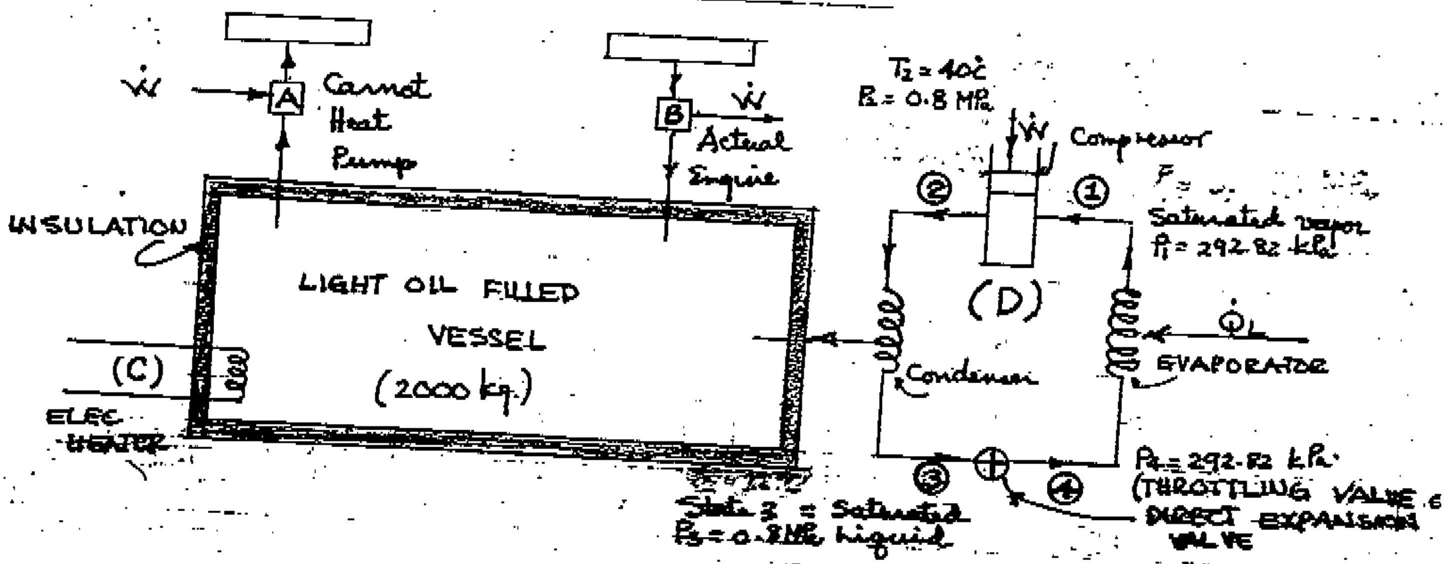
April 2000 + Solutions

textbook "Thermodyanamics" Third or Second Edition by Y.A. Cengel and M.A. Boles.

April. 2000 Final. 130.112 1/4



(24 marks)

1. A Carnot heat pump (A) with a $COP_{H.P.} = 3.0$ extracts heat from the inside of the insulated, oil filled vessel. The power input to this heat pump is 600 kJ/minute.

An actual heat engine (B) with an efficiency $\eta = 30\%$ burns fuel at the rate of 1.636 kg/hr. The fuel has a higher heating value of 44,000 $\frac{kJ}{kg}$. The waste heat is rejected into the vessel as shown.

A 220 volt, 50-amp electric resistance heater (C) supplies heat to the oil.

An actual refrigerator (D), with the properties as shown, uses R-134a, as a refrigerant; has a mass flow rate of $\dot{m} = 9.99 \frac{kg}{min}$ and rejects heat from the condenser into the same insulated tank.

The tank is filled with 2000 kg of light oil (See table A-3 for properties). If all the energy transfers to and from the tank start at the same time, and remain constant, then calculate the length of time (hours) for the temperature in the tank to increase 35°C.

$kg \cdot \frac{kJ}{kg \cdot K} \cdot \frac{kJ}{K} \div \frac{kJ}{s}$

lines.

2/4

(16 marks) a) A mixture of water vapor and oxygen is in a container and has a total pressure of 100 kPa. The mixture is saturated and the temperature of the mixture is 20°C. Calculate the ratio $\frac{m_{wv}}{m\ \text{oxygen}}$ for this mixture. (Both gases are ideal)

b) An air water vapor mixture is at a pressure of 1 atmosphere; a dry bulb temperature of 29°C and a relative humidity of 60%. Determine the ratio of T wet bulb (°C) $\div\ w\left(\frac{\text{grams of wv}}{\text{kg dry air}}\right)$ for this mixture.

c) An air water vapor mixture is at atmospheric pressure. It is at its dew point temperature of 5°C. It is heated and humidified until its enthalpy is $80\frac{\text{kg}}{\text{kg dry air}}$ and relative humidity is 40%. Determine the change in the humidity ratio. i.e. $w_2 - w_1$ for this process.

d) An air water vapor mixture at a total pressure of 20 kPa, a temperature of 30°C and a relative humidity of 20%. It is cooled at a constant pressure until the temperature is 20°C. Will any water vapor condense? Support your answer with an explanation.

(30 marks) 3. A combined air-turbine cycle and a steam-turbine cycle power plant are partially shown in the figure below. Heat is transferred in the heat exchanger from the air in the air cycle to the water of the steam cycle. (The air turbine produces enough power output to drive the compressor and to have a surplus of net power $(\dot{W}_{net})$. The following data of the two systems is known:

a) <u>Gas-Turbine Cycle</u>:
Air enters the compressor at P_1=100kPa and T_1=25°C. The pressure at the exit of the compressor is P_2=900kPa and the temperature T_2=45°C. The inlet air temperature to the turbine is T_3=500°C. The air-turbine exit pressure is P_4=100kPa. The exhaust air leaves the heat exchanger at a temperature of T_5=45°C and atmospheric pressure, P_5=100kPa. The mass flow rate of the air is 15 kg/s. The total power output $\left(\text{i.e. } \dot{W}_{COMPRESSOR} + \dot{W}_{NET}\right)$ of the air-turbine is 3,450 kW.

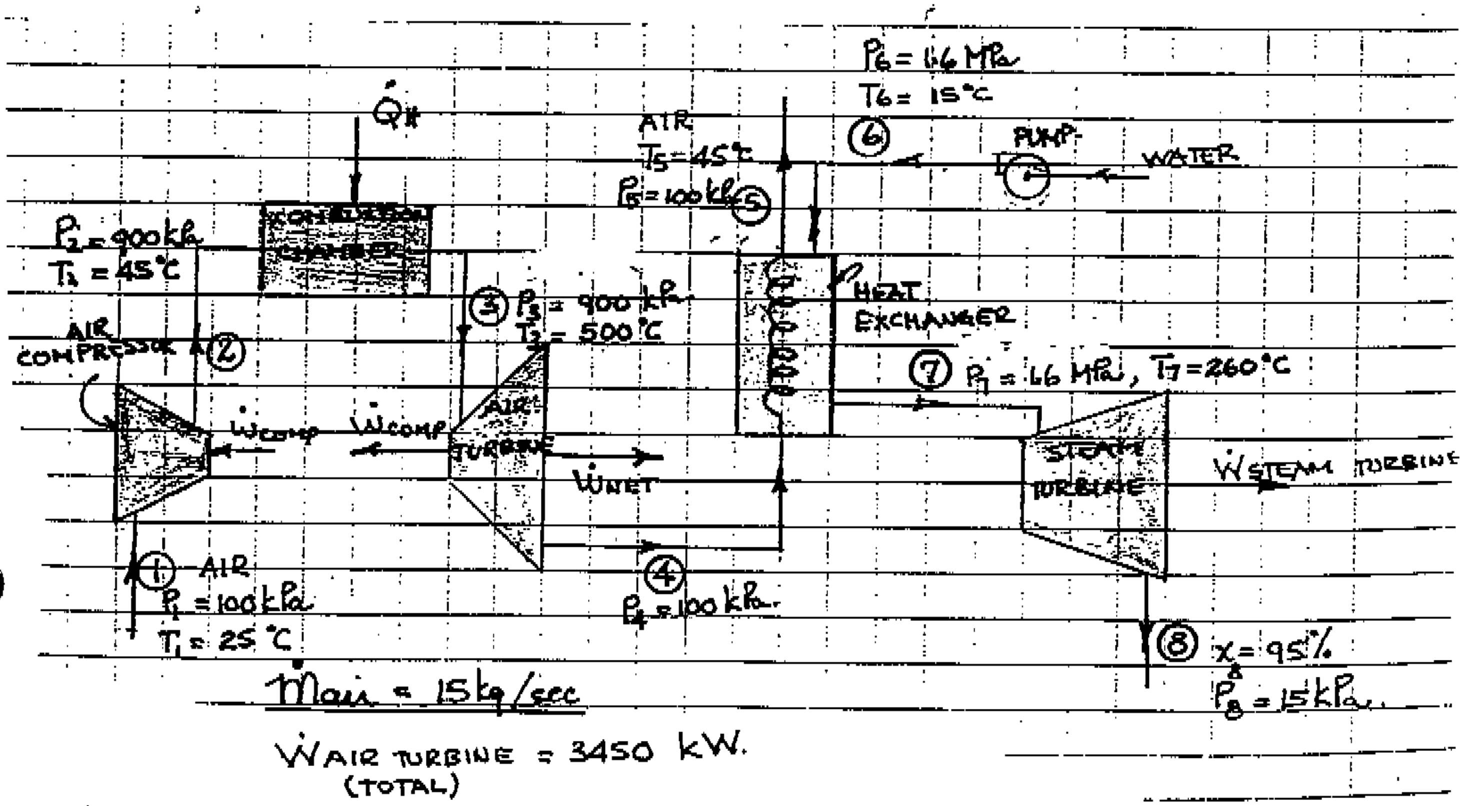
$$\left(\text{Note: for air } C_p = 1.005\frac{\text{kJ}}{\text{kgK}}\right)$$

b) Steam-Turbine Cycle:

The turbine exit state is a saturated mixture with a quality of 95% and a pressure of P_8=15kPa. The turbine inlet temperature T_7=260°C. The temperature at the outlet of the pump is T_6=15°C, and water can be assumed to be a saturated liquid at state 6. The pump outlet pressure is P_6=1.6MPa.

Determine:

1) The amount of heat added $(\dot{Q}_H)$ to the air cycle.
2) The exit air temperature T_4 from the air-turbine.
3) Calculate the rate of heat exchange between the air and the water in the heat exchanger.
4) The net power output of the air tubine.
5) The mass flow rate through the steam-turbine $(\dot{m}_{H20})$.
6) The power output of the steam-turbine $(\dot{W}_T)$
7) The thermal efficiency of the air cycle only.

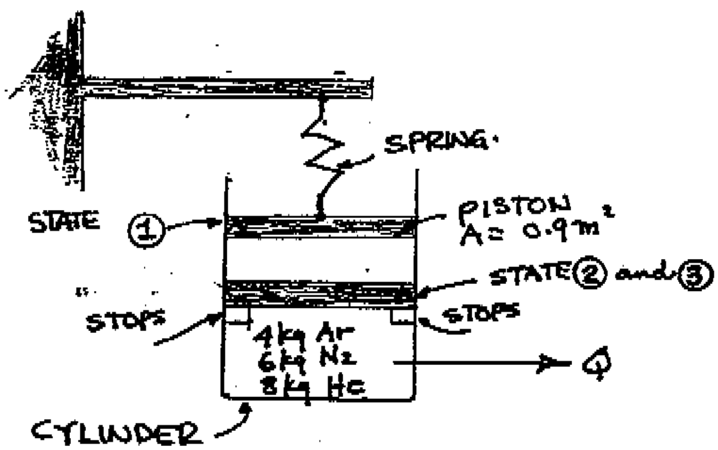


Dept. and Course No.: 130.112 Time: 3 Hours

Examination: Thermal Sciences Examiners: Professors L. Magalhaes, A. Elshaboury and R. Schilling

(30 marks) 4. A piston-cylinder device with a set of stops contains 4 kg of argon (Ar), 6 kg of nitrogen (N_2) and 8 kg of helium (He). A linear spring is attached to the top of the piston without exerting any force on it. The piston cross-sectional area is 0.9 m^2. Initially the pressure and temperature inside the cylinder are 600 kPa and 300 K (state 1). Heat is transferred out of the system such that the piston moves down causing the spring to expand. When the piston just comes to rest on the stops, the pressure is 300 kPa and the volume is ¾ its initial value (state 2). More heat is transferred out of the system until the pressure is 250 kPa (state 3). Assuming ideal gas mixture behaviour and constant specific heat at 300 K, determine:

(a) The initial volume, V_1, in m^3
(b) The spring constant, K, in kN/m
(c) The amount of work done during process 1-2, W_{12}, in kJ
(d) The amount of heat transfer during process 1-2, Q_{12}, in kJ
(e) The amount of heat transfer during process 2-3, Q_{23}, in kJ



130.112 Final Exam April 2000 SOLUTION

1

A) Carnot Ht Pump

$COP_H = 3.0 = \frac{\dot{Q}_H}{\dot{W}}$ $\therefore \dot{Q}_H = 3.0 \times 600 = 1800$ kJ/min

$\dot{W} = |\dot{Q}_H| - |Q_L|$ $\therefore \underline{Q_L} = 1800 - 600 = 1200 \frac{kJ}{min} = \frac{1200}{60} = \underline{20\ kW}$

c) Electric Heater Heat added = 220 v × 50 a = 11,000 watts = 11 kWatts

B) Engine $\eta = \frac{W}{Q_H}$ $\therefore 0.3(Q_H) = W$

$\therefore 0.3(1.636)(44{,}000) = W$

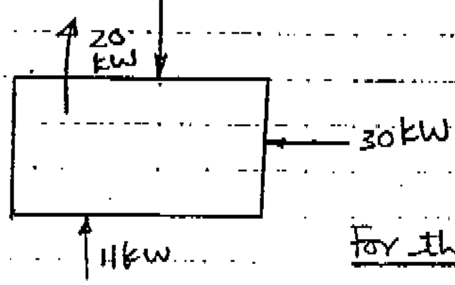
$W = 21595.2\ \frac{kJ}{hr} = \frac{21595.2}{60 \times 60} = \underline{6\ kWatts}$

$\therefore \dot{Q}_L = \dot{Q}_H - \dot{W}$

$= [(1.636 \times 44{,}000) - 21595.2] \div 60 \times 60 = \underline{14\ kW}$

D) Heat Rejected from Condenser

14 kW (See sketch) $\dot{Q}_L = \dot{m}[h_3 - h_2] = (93.42 - 273.66)\frac{9.99}{60} = \underline{30\ kW}$



Net Heat transfer to vessel

$= -20 + 14 + 30 + 11 = \underline{35\ kW}$

For the oil in the tank to increase 35°C

$t\dot{Q} = mc\Delta T$

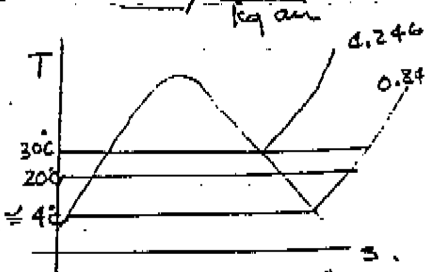
$\underline{t} = \frac{2000(1.8)35}{35 \times 60 \times 60} = \underline{1\ hr}$

2. a) $\frac{m_{wv}}{m_{O_2}} = \frac{P_{wv} V_{wv}}{R_{wv} T_{wv}} \frac{R_{O_2} T_{O_2}}{P_{O_2} V_{O_2}} = \frac{0.2598}{0.4615} \frac{P_{wv}}{P_{O_2}} = \frac{0.2598\ (2.339)}{0.4615\ (100 - 2.339)}$

$= \underline{0.01348}$ kg wv / kg O_2

b) From Psy. Chart $T_{wb} = 23$°C $\omega = 15.25$ $\frac{T_{wb}}{\omega} = \underline{1.5°C / \frac{g\ wv}{kg\ air}}$

c) $\omega_1 = 5.5$ $\omega_2 = 16.5$ $\underline{\Delta\omega = 11\ g/kg\ air}$



d) $\frac{P_{wv}}{P_{MAX}} = 0.2$ $\therefore P_{wv} = 0.2(4.246) = 0.8492$ kPa

$T_{DEW\ POINT}$ for 0.8492 kPa ≅ 4°C

Since 20°C > Tdewpoint ... condensation

Problem 3 COMBINED AIR-TURBINE CYCLE and a STEAM-TURBINE CYCLE 2/7

1. Properties

AIR-TURBINE CYCLE ($\Delta h = C_{p_0}(\Delta T)$)

State (1) $P_1 = 100\ KPa$; $T_1 = 25°C$

State (2) $P_2 = 400\ KPa$; $T_2 = 45°C$

State (3) $P_3 = 400\ KPa$; $T_3 = 500°C$

State (4) $P_4 = 100\ KPa$; $T_4 = ?$ $\boxed{= 271.31°C}$

State (5) $P_5 = 100\ KPa$; $T_5 = 45°C$

$_3\dot{W}_4 = +3450\ KW$; $\dot{m}_{air} = 15\ Kg/s$

STEAM TURBINE CYCLE h - from Table

State (6) $P_6 = 1600\ KPa$; $T_6 = 15°C$ (Comp. Liq

Tab A4 $h_6 = h_f(15°C) = 62.99\ KJ/Kg$

State (7) $P_7 = 1600\ KPa$; $T_7 = 260°C$ (S.H.V)

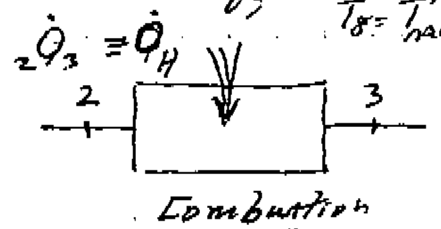
Tab A6 / Interpolation $h_7 = 2,942.3\ KJ/K$

State (8) $P_8 = 15\ KPa$; $x_8 = 0.95$ (Mixture

Tab A5 $h_8 = \underset{h_f}{225.94} + 0.95 * \underset{h_{fg}}{2373} = 2,480.4\ KJ/Kg$

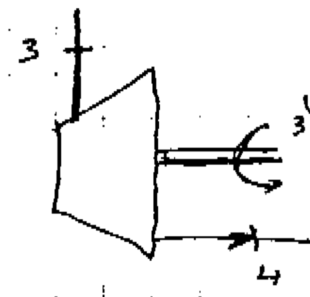
$T_8 = T_{sat} = 53.97$

2. Combustion Chamber



$$_2\dot{Q}_3 - \cancel{_2\dot{W}_3}^{\,0} = \dot{m}_{air}(h_3 - h_2)$$
$$= \dot{m}_{air}\, C_p (T_3 - T_2)$$
$$= 15 \times 1.005 \times (500 - 45) = 6859\ KW$$

Air - Turbine (exit temperature T_4)

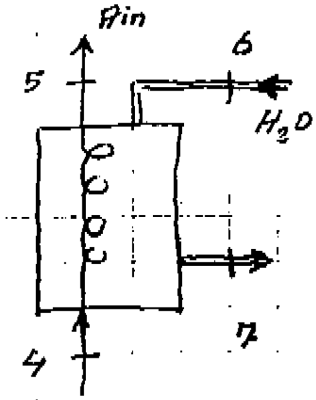


$$ {}_3\dot{Q}_4^{\nearrow 0} - {}_3\dot{W}_4 = \dot{m}_{air}(h_4 - h_3) = \dot{m}\, C_p (T_4 - T_3) $$

$$ 0 - (+3450) = 15 \times 1.005\,(T_4 - 500) $$

$$ T_4 = 271.14\ °C $$
$$ (545.29\ K) $$

4. Rate of Heat Exchange $({}_4\dot{Q}_5)$



$$ {}_4\dot{Q}_5 - {}_4\dot{W}_5^{\nearrow 0} = \dot{m}(h_5 - h_4) = \dot{m}\, C_p (T_5 - T_4) $$

$$ {}_4\dot{Q}_5 - 0 = 15 \times 1.005\,(45 - 271.31) $$

$$ {}_4\dot{Q}_5 = -3{,}411.6\ KJ/s $$

5. Net Power Output of the Air Turbine

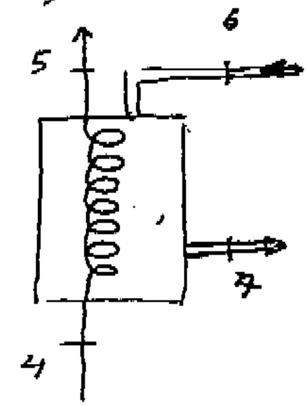
$$ \dot{W}_{net} = \dot{W}_{total} - \dot{W}_{comp} $$

$$ \dot{W}_{total} = 3{,}450\ KJ/s $$

$$ \dot{W}_{comp} = \dot{m}_{air} \times C_p \times (T_2 - T_1) = -301.5\ KJ/s $$

$$ \dot{W}_{net} = 3{,}450 - 301.5 = 3{,}148.5\ KJ/s $$

6. Mass flow rate of steam $(\dot{m}_w)$

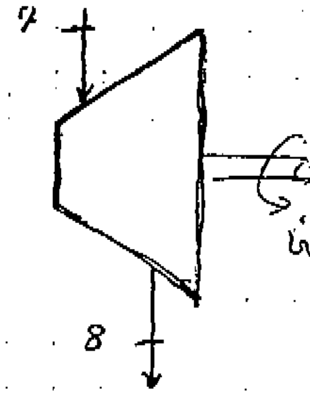


$$ \dot{m}_w = \dot{m}_6 = \dot{m}_7 = \dot{m}_8 $$

$$ {}_6\dot{Q}_7 - {}_6\dot{W}_7^{\nearrow 0} = \dot{m}_w (h_7 - h_6) $$

$$ {}_6\dot{Q}_7 = -{}_4\dot{Q}_5 = +3{,}411.6\ KJ/s $$

$$ 3411.6 = \dot{m}_w\,(2{,}942.3 - 62.99) \Rightarrow \dot{m}_w = 1.185\ Kg/s $$



Power output of steam Turbine

$${}_7\dot{\cancel{Q}}_8^{\,0} - {}_7\dot{W}_8 = \dot{m}_w (h_8 - h_7)$$

$$0 - {}_7\dot{W}_8 = 1.185\,(2480.4 - 2942.3)$$

$${}_7\dot{W}_8 = 547.4 \text{ KJ/s}$$

8. Thermal Efficiency of air cycle.

$$\eta_{th} = 1 - \frac{\dot{Q}_L}{\dot{Q}_H} = \frac{\dot{W}_{net}}{\dot{Q}_H} =$$

$$= \frac{3149}{6859} = 0.459 \Rightarrow 46\%$$

PROBLEM 7. $\frac{5}{7}$

Total mass, $m_m = m_{Ar} + m_{N_2} + m_{He}$
$= 4 + 6 + 8 = 18$ kg

Table A-1: $M_{Ar} = 39.948$ kg/kmol
$M_{N_2} = 28.013$ kg/kmol
$M_{He} = 4.003$ kg/kmol

$$N_{Ar} = \frac{m_{Ar}}{M_{Ar}} = \frac{4}{39.948} = 0.1001 \text{ kmol}$$

$$N_{N_2} = \frac{m_{N_2}}{M_{N_2}} = \frac{6}{28.013} = 0.2142 \text{ kmol}$$

$$N_{He} = \frac{m_{He}}{M_{He}} = \frac{8}{4.003} = 1.9985 \text{ kmol}$$

$$N_m = N_{Ar} + N_{N_2} + N_{He} = 2.3128 \text{ kmol}$$

Initially:

$$P_1 V_1 = N_m R_u T_1$$

$$V_1 = \frac{N_m R_u T_1}{P_1} = \frac{2.3128 * 8.314 * 300}{600} = 9.6143 \text{ m}^3$$

b) $$P_2 = P_1 - \frac{F_s}{A} = P_1 - \frac{K \Delta x}{A}$$

but $\Delta V = (\Delta x) A = 0.25 V_1 = 0.25 * 9.6143 = 2.4036 \text{ m}^3$

$$\Delta x = \frac{\Delta V}{A} \qquad \Delta x = 2.6707 \text{ m}$$

$$P_2 = P_1 - \frac{K \Delta V}{A^2}$$

$$300 = 600 - \frac{K * 2.4036}{(0.9)^2}$$

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$$K = \frac{(600-300)(0.9)^2}{2.4036} = 101.1 \quad kN/m$$

c) $$W_{12} = \frac{P_1 + P_2}{2}(V_1 - V_2)$$

$$= \frac{600+300}{2}(\Delta V)$$

$$= 450 * 2.4036$$

$W_{12} = 1081.62$ kJ done on the system

d) $$Q_{12} - W_{12} = \Delta U_{12} = m_m C_{v,m} (T_2 - T_1)$$

At state 2:

$$P_2 V_2 = N_m R_u T_2$$

$$T_2 = \frac{P_2 V_2}{N_m R_u} = \frac{300 * 0.75 * 9.6143}{2.3128 * 8.314}$$

$$= 112.5 \quad K$$

$$C_{v,m} = \sum_{i=1}^{3} m_{fi} C_{v_i}$$

$$= \frac{4}{18} * 0.3122 + \frac{6}{18} * 0.743 + \frac{8}{18} * 3.1156$$

$$= 1.7018 \quad kJ/kg\,k$$

Table A-2

$C_{v_{He}} = 3.1156$ kJ/kg k

$C_{v_{N_2}} = 0.743$ kJ/kg k

$C_{v_{Ar}} = 0.3122$ kJ/kg k

$$Q_{12} = m_m C_{v,m} (T_2 - T_1) + W_{12}$$

$$= 18 * 1.7018 (112.5 - 300) + (-1081.62)$$

$$= 6825.195 \quad kJ$$

(e) $$Q_{23} - W_{23} = \Delta U_{23} = m_m C_{v,m} (T_3 - T_2)$$

$W_{23} = 0.0$ Const. volume process

$$\frac{P_3}{P_2} = \frac{T_3}{T_2}$$

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$$T_3 = \left(\frac{P_3}{P_2}\right) T_2$$

$$= \left(\frac{250}{300}\right) 112.5$$

$$= 93.75 \quad K$$

$$Q_{23} = m_m C_{v,m} (T_3 - T_2)$$

$$= 18 \times 1.7018 (93.75 - 112.5)$$

$$= -574.3575 \quad kJ$$