DATE: October 4, 2007
COURSE: MATH 2132

TIME: 60 minutes
EXAMINER: G.I. Moghaddam

Answers by Dawit yohannes
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[6] 1. The following sequence of functions is defined on the interval $-1 \leq x \leq 1$.

$$\left\{\frac{1}{n} + \frac{n^4x^2 + 2nx^4}{n^4x + x^4 + 1}\right\}_{n=1}^{\infty}$$

Determine whether the sequence is convergent or divergent. If it converges, find the limit function.

[15] 2. Let $f(x) = \ln x$ for $0 < x < 4$ Then:

[5] (a) Find the first 4 terms of the Taylor series of $f(x)$ about 2.

[5] (b) Find the n^{th}-remainder (i.e. $R_n(2, x)$).

[5] (c) Show that $\lim_{n\to\infty} R_n(2, x) = 0$ <u>only</u> for the case $2 < x < 4$.

[8] 3. Find the radius of convergence and the open interval of convergence for the series

$$\sum_{n=0}^{\infty} \frac{(-1)^n (n!)^2 4^n}{(2n+1)!} (x-7)^{4n}$$

[6] 4. Find the sum of the series: $-\frac{\sqrt{2}}{3}x^3 + \frac{2}{9}x^6 - \frac{2\sqrt{2}}{27}x^9 + \cdots + \frac{(-1)^n 2^{\frac{n}{2}}}{3^n}x^{3n} + \cdots$

[15] 5. Find the Taylor series about 1 for the function

$$f(x) = \frac{1}{x^2} + \ln x$$

Express your answer in sigma notation and simplify as much as possible. Determine its open interval of convergence.

1. x

2 a) $\ln 2 + \frac{1}{2}(x-2) - \frac{1}{8}(x-2)^2 + \frac{1}{24}(x-2)^2$

b) $\frac{(-1)^n (x-2)^{n+1}}{z_n^{n+1}(n+1)}$, $2 < z_n < x < 4$

c) $\lim_{n\to\infty} |R_n(2,x)| \leq \lim_{n\to\infty} \frac{\left|\frac{x-2}{z_n}\right|^{n+1}}{n+1}$

$\leq \lim_{n\to\infty} \frac{1}{n+1} = 0$

3. 1 , $6 < x < 8$

4. $-\frac{\sqrt{2}\,x^3}{3+\sqrt{2}\,x^3}$; $-\sqrt[3]{\frac{3}{\sqrt{2}}} < x < \sqrt[3]{\frac{3}{\sqrt{2}}}$

5. $1 + \sum_{n=1}^{\infty} (-1)^n \left(\frac{n^2+n-1}{n}\right)(x-1)^n$, $0 < x < 2$

MATH2132 Test1 **February 7, 2008** **60 minutes**

Values

4 1. Find the limit of the sequence of functions $\{f_n(x)\}$ on the interval $0 \leq x \leq 5$, if it exists. Justify your answer.

$$f_n(x) = \frac{2n^2x + nx}{n^2+1}$$

5 2. Find the Taylor series about $x = -2$ for the function $f(x) = e^{2x+1}$. Include its interval of convergence.

9 3. Find the open interval of convergence for the power series

$$\sum_{n=3}^{\infty} \frac{(-1)^n 2^n}{n^3}(x+1)^{3n+1}.$$

Express your answer in the form $a < x < b$ for appropriate values of a and b.

10 4. Find the Maclaurin series for the function $f(x) = \frac{x}{(2+x)^2}$. What is the interval of convergence of the series?

12 5. Find the Maclaurin series for the function $f(x) = \frac{1}{\sqrt[3]{8+3x}}$. Find the radius of convergence of the series.

Answers by Dawit y. (ydawit@yahoo.Com)

1) $2x$ 2) $\sum_{n=0}^{\infty} \frac{e^{-3}2^n}{n!}(x+2)^n$, $-\infty < x < \infty$, 3) $-(1+\sqrt[3]{1/2}) < x < \sqrt[3]{1/2} - 1$

4) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1} n}{2^{n+1}} x^n$, $-2 < x < 2$, 5) $\frac{1}{2} + \sum_{n=1}^{\infty} \frac{(-1)^n [1\cdot 4 \cdot 7 \cdots (3n-2)]}{2^{3n+1}\, n!} x^n$

, $-8/3 < x < 8/3$

Term Test 1

DATE: February 10, 2009
COURSE: MATH 2132

TIME: 70 minutes
EXAMINER: G.I. Moghaddam

[9] 1. (a) The following sequence of functions is defined on the interval $[0, \infty)$

$$\left\{1 + (x^2 - 2x + 1)\, e^{-nx}\right\}_{n=1}^{\infty}.$$

Determine whether the sequence is convergent or divergent. If it converges, find the limit function.

(b) Find all values of x for which the sequence $\left\{1 + \frac{|x-2|^n}{n!}\right\}_{n=1}^{\infty}$ converges. (Explain your work.)

[12] 2. Let $f(x) = e^{1-2x}$ for $-\infty < x < \infty$. Then:

[4] (a) Find the first 4 terms of the Taylor series of $f(x)$ about 1.

[4] (b) Find the n^{th}-remainder (i.e. $R_n(1, x)$).

[4] (c) Show that $\lim_{n\to\infty} R_n(1, x) = 0$ *only* for the case $x > 1$.

[8] 3. Let $f(x) = \dfrac{1+a}{1+ax}$, find the value of a such that the 4^{th} term of Taylor series of $f(x)$ about 1 is $-\frac{1}{27}(x-1)^3$.
(**Hint**: You may use geometric series)

[8] 4. Find the radius of convergence and the open interval of convergence for the series $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{3^{2n}\, e\sqrt{n}}\,(x+1)^{2n}$

[6] 5. Find the sum and the interval of convergence of the series

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{(2n+1)!\, e^{(4n+2)x}}$$

[12] 6. Let $f(x) = \dfrac{x - x^2}{(1+x)^2}$ use the binomial expansion to find the Maclaurin series of $f(x)$. Express your answer in sigma notation and simplify as much as possible. Determine its open interval of convergence.

Answers by Dawit Y (ydawit@yahoo.ca)

1) a) $\begin{cases} 1, & 0 < x < \infty \\ 2, & x = 0 \end{cases}$

b) $1, \quad -\infty < x < \infty$

2) a) $e^{-1} - 2e^{-1}(x-1) + 2e^{-1}(x-1)^2 - \frac{4}{3}e^{-1}(x-1)^3$

b) $(-1)^{n+1} 2^{n+1} e^{1-2z_n} \dfrac{(x-1)^{n+1}}{(n+1)!}$

z_n is between 1 and x.

c) $\displaystyle\lim_{n\to\infty} |R_n(1,x)| < \lim_{n\to\infty} \frac{e^{-1}|2(x-1)|^{n+1}}{(n+1)!}$
$= 0$.

3) $\frac{1}{2}$

4) $3, \quad -4 < x < 2$

5) $\sin e^{-2x} - e^{-2x}, \quad -\infty < x < \infty$

6) $\displaystyle\sum_{n=1}^{\infty} (-1)^{n+1}(2n-1)\, x^n, \quad -1 < x < 1$

Term Test 1

DATE: October 8, 2009
COURSE: MATH 2132

TIME: 70 minutes
EXAMINER: G.I. Moghaddam

[9] 1. (a) Determine whether the sequence of numbers $\left\{\frac{1+\cos\sqrt{n}}{\sqrt{n}+1}\right\}_{n=1}^{\infty}$ is convergent or divergent. If it converges, find the limit.

(b) The sequence of functions $\left\{\frac{x^2}{n}+\frac{(x-1)n^2-x^2}{(1-x)n^2+8}\right\}_{n=1}^{\infty}$ is defined on the interval $(-\infty,\infty)$. Determine whether the sequence is convergent or divergent. If it converges, find the limit function.

[12] 2. Let $f(x)=\frac{4x}{1-4x}$ for $-\frac{1}{4}<x\leq\frac{1}{8}$. It is given that $f^{(n)}(x)=\frac{4^n\, n!}{(1-4x)^{n+1}}$ where $n\geq 1$.

(a) Find the first 3 terms of the Maclaurin series of $f(x)$.

(b) Find the n^{th}-remainder (i.e. $R_n(0,x)$).

(c) Show that $\lim_{n\to\infty} R_n(0,x)=0$ *only* for the case $x<0$.

[8] 3. Find the sum and the interval of convergence of the series

$$\sum_{n=1}^{\infty}(-1)^n 2^{4n}\left(x-\frac{1}{2}\right)^n$$

[8] 4. Find the radius of convergence and the open interval of convergence for the series $\sum_{n=0}^{\infty}\frac{(-1)^n n!}{2^n(1\cdot 4\cdot 7\cdot 10\cdot\cdots\cdot(3n+1))}x^{3n}$.

[13] 5. (a) Find the Maclaurin series of $f(x)=\frac{1}{1+2x}$. What is the interval of convergence ?

(b) Find the Maclaurin series of $g(x)=\frac{-2(x^2+1)}{(1+2x)^2}$. Express your answer in sigma notation and simplify as much as possible. Determine its open interval of convergence. (Hint : you may use part (a).

Answers by Dawit y
(ydawit@yahoo.com)

1) a. 0 b. $\begin{cases} -1, & -\infty<x<\infty,\ x\neq 1 \\ -\frac{1}{8}, & x=1 \end{cases}$

2) a. $4x+16x^2+64x^3$

b. $R_n(0,x)=\frac{4^{n+1}x^{n+1}}{(1-4z_n)^{n+2}}$

(z_n is between 0 and x)

c) $\lim_{n\to\infty}|R_n(0,x)|<\lim_{n\to\infty}|4x|^{n+1}=0$

Since, $-\frac{1}{4}<x<z_n<0$

$0<-4z_n<-4x<1$

$\frac{1}{2}<\frac{1}{1-4z_n}<1$, $|4x|<1$

3) $\frac{8-16x}{16x-7}$, $\frac{7}{16}<x<\frac{9}{16}$

4) $\sqrt[3]{6}$, $-\sqrt[3]{6}<x<\sqrt[3]{6}$

5) a. $\sum_{n=0}^{\infty}(-1)^n 2^n x^n$, $-\frac{1}{2}<x<\frac{1}{2}$

b. $-2+8x+\sum_{n=2}^{\infty}(-1)^{n+1}2^{n-1}(5n+3)x^n$

$-\frac{1}{2}<x<\frac{1}{2}$

MATH2132 Test1 February 2, 2010 60 minutes

5 1. Find the limit of the sequence of functions

$$\left\{\frac{n^2x^3+3nx}{2n^2x+1}\mathrm{Tan}^{-1}\left(\frac{nx}{n+3}\right)\right\}$$

on the interval $0 \le x \le 3$, if it exists. Justify your answer.

8 2. Determine whether the following series converge or diverge. Justify you answers. If a series converges, find its sum.

(a) $\sum_{n=1}^{\infty} \frac{(-1)^n n^2}{1+2n^2}$ (b) $\sum_{n=3}^{\infty} \frac{2^n}{3^{n+1}}$

12 3. (a) Find the first four Taylor polynomials $P_0(x)$, $P_1(x)$, $P_2(x)$, and $P_3(x)$ about $x = 0$ for the function $\cos 3x$.

(b) Use Taylor's remainder formula to verify that the Maclaurin series for $\cos 3x$ converges to $\cos 3x$ for all x.

8 4. Find the interval of convergence for the power series

$$\sum_{n=3}^{\infty} \frac{(-1)^{n+1}n^2}{3^n}x^{2n+1}.$$

7 5. Find the open interval of convergence for the power series

$$\sum_{n=2}^{\infty} \frac{2^{n+1}}{n^3+100n^2}(x+2)^n.$$

Answers by Dawit yohannes (ydawit@yahoo.Com)

1). $\frac{x^2}{2}\mathrm{Tan}^{-1}x$, $0 \le x \le 3$

2) a) diverges (by the n^{th} term test)

b) Converges (Geometric series, $r = 2/3 < 1$) with Sum $= 8/27$

3) a. $P_0(x) = 1$, $P_1(x) = 1$, $P_2(x) = 1 - \frac{9x^2}{2}$, $P_3(x) = 1 - \frac{9x^2}{2}$

b) $\lim_{n\to\infty} |R_n(0,x)| \le \lim_{n\to\infty} \frac{|3x|^{n+1}}{(n+1)!} = 0$ (for all x)

4) $-\sqrt{3} < x < \sqrt{3}$

5) $-5/2 < x < -3/2$

MATH2132 Test1 February 1, 2011 60 minutes

Answers

Values

2 1. The limit of the sequence $\left\{\frac{(-1)^n n^2+3n}{2n^2+5}\right\}$ is

(a) $1/2$ (b) $\pm 1/2$ (c) ∞ (d) $-\infty$ (e) None of these

ans. e

2 2. The limit of the sequence $\left\{\frac{2n^2+3}{5-3n^2}\mathrm{Sin}^{-1}\left(\frac{n+2}{2n-3}\right)\right\}$ is

(a) -1 (b) $\pi/10$ (c) $-\pi/9$ (d) $\pi/6$ (e) None of these

ans. c

2 3. The sum of the series $\sum_{n=1}^{\infty}\left(-\frac{3}{4}\right)^{n+1}$ is

(a) $9/28$ (b) $9/4$ (c) $-3/7$ (d) -3 (e) None of these

ans. a

2 4. The sum of the series $\sum_{n=1}^{\infty} n\left(\frac{7}{4}\right)^n$ is

(a) $-7/3$ (b) $7/3$ (c) ∞ (d) $-\infty$ (e) None of these

ans. c

2 5. The limit of the sequence of functions $\left\{\left(1+\frac{x}{2n}\right)^n\right\}$ on the interval $0 \le x < 1$ is

(a) 1 (b) $e^{x/2}$ (c) $x/2$ (d) Does not exist (e) None of these

ans. b

10 6. Prove that the Maclaurin series for e^{3x} converges to e^{3x} for all x.

hint: $R_n(0,x) = 3^{n+1}e^{3z_n}\frac{x^{n+1}}{(n+1)!}$

8 7. What is the interval of convergence for the power series

$$\sum_{n=1}^{\infty}\frac{n+1}{n4^n}x^n?$$

Justify all results.

ans. $-4<x<4$

12 8. Find the Taylor series about $x=4$ for the function

$$f(x)=\frac{1}{(x-2)^2}.$$

Express your answer in sigma notation simplified as much as possible. You must use a technique that guarantees that the Taylor series converges to the function. What is the radius of convergence of the series?

ans. $\sum_{k=0}^{\infty}\frac{(-1)^n(n+1)}{2^{n+2}}(x-4)^n$

$R=2$

Summer2012

MATH2132 Test1

Values

6 **1. Find the limit for the following sequence of functions on the interval $0 < x \le 2$, if it exists. Show your reasoning or calculations.**

$$\left\{ \left(\frac{n^2x^2 + x - 1}{n^2x + 1} \right) \cos\left(\frac{5x}{n}\right) \right\}$$

10 2. Determine whether the following series converge or diverge. If a series converges, find its sum. Justify your conclusions.

(a) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \operatorname{Sin}^{-1}\left(\frac{n^2+1}{2n^2}\right)$ (b) $\displaystyle\sum_{n=2}^{\infty} \frac{2^{n+1}+1}{3^{2n}}$

12 3. Find the interval of convergence for the power series

$$\sum_{n=3}^{\infty} \frac{(-1)^n n^2 4^n}{2n^2+1}(x-2)^{2n}.$$

Express your answer in one of the forms $a < x < b$, $a \le x < b$, $a < x \le b$, or $a \le x \le b$. Justify all results.

12 4. Use Taylor remainders to verify that the Maclaurin series for $\cos 2x$ converges to $\cos 2x$ for all x.

Answers.

1. $\begin{cases} x & 0 < x \le 2 \\ -1 & x = 0 \end{cases}$

2. a) diverges (by the nth term test)

b) Sum of two convergent geometric series

Sum = $71/504$

3. $3/2 < x < 5/2$

4. hint: try to obtain, $|R_n(0,x)| \le \dfrac{2^{n+1}|x|^{n+1}}{(n+1)!} = \dfrac{|2x|^{n+1}}{(n+1)!}$