

MATH 2132 Problem Workshop 3

1. Find the open interval of convergence for the power series:

(a) $\sum_{n=3}^{\infty} \frac{n3^n}{n^2 + 1} x^{2n+3}$

(b) $\sum_{n=0}^{\infty} (-1)^{n+1} \sqrt{\frac{2n+3}{n+6}} \ln(n+6)(x+2)^n$

(c) $\sum_{n=2}^{\infty} \frac{n!}{(3n)!} (3x-1)^n$

2. Find the interval of convergence for the power series

(a) $\sum_{n=1}^{\infty} \frac{(3n)4^n}{n+1} (x+1)^n$

(b) $\sum_{n=1}^{\infty} \frac{(-1)^n 3^n}{5^n} (2x-3)^n$

3. (a) Find the Maclaurin series for the function $f(x) = x/(4x+1)$. Express your answer in sigma notation, simplifying as much as possible. What is the interval of convergence of the series.
- (b) Repeat part (a), but find the Taylor series about $x = 1$.
4. Find the Maclaurin series of $\sin^2 2x$. What is its interval of convergence?
5. Find the Taylor series about $x = 5$ for $\ln(3+x)$. What is its open interval of convergence?
6. Find the Maclaurin series of $f(x) = 1/(4+3x)^2$. What is its interval of convergence?
7. Find the Maclaurin series for the function $\tan^{-1}(2x^2)$. Express your answer in sigma notation, simplifying as much as possible. What is the open interval of convergence of the series.
8. Find the Taylor series for the function $1/\sqrt{10-3x}$ about $x = 2$. Express your answer in sigma notation, simplifying as much as possible. What is the radius of convergence of the series.
9. Find the Maclaurin series for the function $(x^2+2)/(x+3)^2$. Express your answer in sigma notation, simplifying as much as possible. What is the open interval of convergence of the series.
10. Find the sum of the series. Include the open interval of convergence.

(a) $\sum_{n=0}^{\infty} \frac{1}{n+2} x^n$

(b) $\sum_{n=2}^{\infty} \frac{(-1)^n n}{(2n)!} x^{2n}$

(c) $\sum_{n=1}^{\infty} \frac{(-1)^n 2^{2n}}{n} x^{2n+1}$

(d) $\sum_{n=1}^{\infty} (2n+1)(x-1)^n$

11. Use series to evaluate $\lim_{x \rightarrow 0} \frac{\sqrt{3+4x^3} - \sqrt{3}}{x^3}$.