Final Exam

December 10, 2009

1. $x(t) = 20t^2 - 10t^3 + 109t$

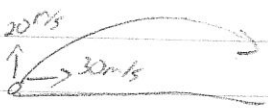
$v(t) = 40t - 30t^2 + 109$

$a(t) = 40 - 60t$

$a(2) = 40 - 60(2)$

$a(2) = -80\ m/s^2$ (c)

2. $10 \sin 45° - 7 \sin 60° = 1.0\ m$ (a)

3. 

in y-direction: $v_f = v_i + at$

$0 = 20 + (-10)t$

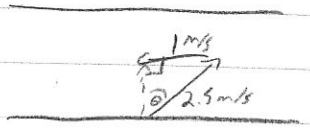
$-20 = -10t$

$t = 2s$

in x-direction: $d = v_i t + \frac{1}{2}at^2$

$d = 30(2) + \frac{1}{2}$

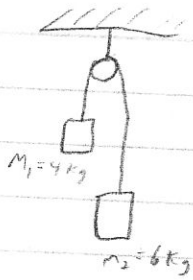
$d = 60\ m$ (b)

4. 

$\theta = \sin^{-1}\left(\frac{1}{2.5}\right)$

$\theta = 23.6°$

5.



$F_{net} = m_t a$

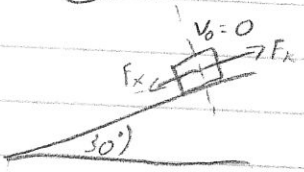
~~$F_{net} = (m_2 + m_1)a$~~

$(m_2 - m_1)g = (m_1 + m_2)a$

$a = \frac{m_2 - m_1}{m_1 + m_2} g = \frac{2}{10} g = \frac{1}{5} g \uparrow$

(d)

6.



$\mu_K = 0.40$

$t = 5s$

$V_f = ?$

$V_f = V_0 + at$

$F_x = mg \sin\theta$

$F_K = \mu_K mg \cos\theta$

$F_{net} = m(g \sin\theta - \mu_K g \cos\theta)$

$= mg(\sin\theta - \mu_K \cos\theta)$

$= m[9.8(\sin 30 - 0.4 \cos 30)]$

$= 1.51m$

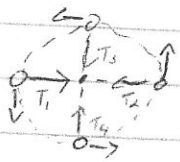
$a = \frac{F_{net}}{m}$ $\quad a = 1.51 m/s^2$

$V_f = 1.51(5)$

$V_f = 7.55 m/s$

(c)

7.



$F_{net} = ma_c$

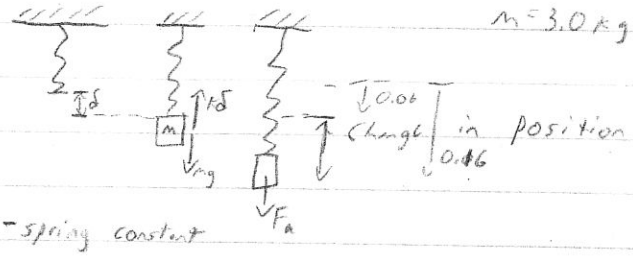
$\uparrow: T_3 + mg = \frac{mv^2}{r}$

$T_3 = m\left(\frac{v^2}{r} - g\right)$

$T_3 = 4\left(\frac{6^2}{2} - 9.8\right)$

$T_3 = 32.8$

(b)

8. 

k - spring constant

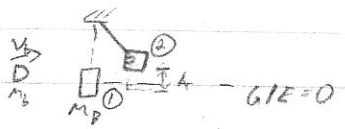
δ = static deflection

$k = \frac{mg}{\delta} = \frac{3(9.8)}{0.06} = 490 \frac{N}{m}$

$W_{spring} = -\frac{1}{2} k (x_f^2 - x_i^2)$ $\quad W_{sp} = -\Delta U_s$

$= -\frac{1}{2}(490)(0.16^2 - 0.06^2)$

$W_{spring} = -5.39\ J$ $\quad -5.4\ J$ $\quad$ (b)

9. 

Conservation of momentum

$m_b v_{bi} + m_B \cancel{v_{Bi}}^{0} = (m_b + m_B) V$

$v_{bi} = \frac{m_b + m_B}{m_b} V$ $\quad$ ①

Conservation of energy (mechanical)

$(ME)_1 = (ME)_2$

$\frac{1}{2}\cancel{(m_b + m_B)} V^2 = \cancel{(m_b + m_B)} g h$

$V = \sqrt{2gh}$ $\quad$ ②

①+② $v_{bi} = \frac{m_b + m_B}{m_b}\sqrt{2gh} = \frac{6\times10^{-3} + 20}{6\times10^{-3}}\sqrt{2(9.8)(3\times10^{-3})}$

$v_{bi} = 808\ m/s$ or $8.0 \times 10^2\ m/s$

(b)

10. $m = 0.1\ kg$

$v_i = 2\ m/s$

$k = 10\ N/m$

$\mu_k = 0.4$

Energy Conversion:

$$KE_i = W_f + U_s$$

$$\tfrac{1}{2}mv^2 = 2\mu_k mgx + \tfrac{1}{2}kx^2$$

$$10x^2 + 0.784x - 0.4 = 0$$

$$x = \frac{-0.784 \pm \sqrt{0.784^2 + 4(10)(0.4)}}{20} \qquad 4.076$$

$x = 0.165\ m$ or ~~$-0.24\ m$~~

$$W_f = E_f - E_i = \tfrac{1}{2}kx^2 - \tfrac{1}{2}mv^2$$

$$= \tfrac{1}{2}(10)(0.165)^2 - \tfrac{1}{2}(0.1)(2)^2$$

$W_f = -0.064\ J$ ~~$= -6.5 \times 10^{-2}\ J$~~

(d)

11. $\omega_0 = 20\ \frac{rad}{sec}$

$\omega_f = 10\ \frac{rad}{sec}$

$t = 5\ s$

$$\alpha = \frac{\omega_f - \omega_0}{t} = \frac{10 - 20}{5} = -2\ \frac{rad}{sec^2}$$

$$\theta = \omega_0 t + \tfrac{1}{2}\alpha t^2 = 20(5) - \tfrac{1}{2}(2)(5)^2$$

$$\theta = 75\ rad$$

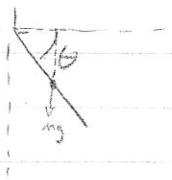
$$\theta = \tfrac{1}{2}(\omega_f + \omega_0)t$$

$$\theta = \tfrac{1}{2}(20 + 10)5$$

$\theta = 75\ rad$

(d)

12



$\tau = mg\frac{L}{2}\cos\theta$

$= 2(9.8)\left(\frac{1.2}{2}\right)\cos 37°$

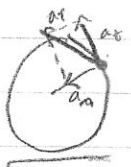
$\tau = 9.392 \frac{N}{m}$

$I = \frac{1}{3}(2)(1.2)^2$

$I = 0.96 \ kg\,m^2$

$\alpha = \frac{\tau}{I} = \frac{9.392}{0.96} = 9.78 \frac{rad}{s^2} \approx 1.8 \frac{rad}{sec^2}$ (a)

13.



$a = \sqrt{a_n^2 + a_t^2}$

$v = r\omega$

$a_R = \frac{v^2}{r} = r\omega^2$

$a_T = \alpha r$

$r = 0.08m$

$\omega_0 = 0$

$t = 2s$

$\omega_f = 4 \frac{rad}{s}$

$\omega = 1.5 \frac{rad}{s}$

$\alpha = \frac{\omega_f - \omega_0}{t} = 2 \frac{rad}{s^2}$

$a_T = 0.08(2) = 0.16 \frac{m}{s^2}$

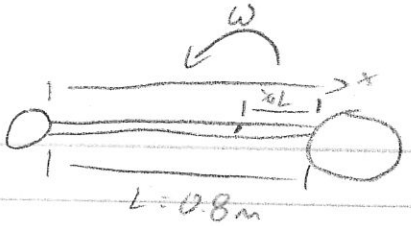
$a_R = 0.08(1.5)^2 = 0.18 \frac{m}{s^2}$

$a = \sqrt{0.16^2 + 0.18^2}$

$a = 0.24 \frac{m}{s^2}$

(a)

14.



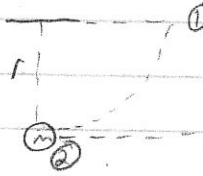
$$x_{cm} = \frac{3M(L)}{(3M+M)} = \frac{3}{4}L$$

$$I_{3M} = 3M\left(\tfrac{3}{4}L\right)^2 = \frac{3}{16}ML^2 =$$
$$I_M = M\left(\tfrac{1}{4}L\right)^2 = \frac{9}{16}ML^2 =$$

$$KE_{Rot} = \frac{1}{2}I_{3M}\omega^2 + \frac{1}{2}I_M\omega^2$$
$$= \frac{1}{2}\omega^2\left[\frac{3}{16}+\frac{9}{16}\right]ML^2$$
$$= \frac{1}{2}\left(\frac{12}{16}\right)ML^2\omega^2$$
$$= \frac{3}{8}ML^2\omega^2 = \frac{3}{8}(2)(0.8)^2(5)^2$$
$$KE_{Rot} = 12\ J$$

(c)

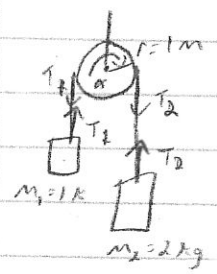
15. $L = I\omega = (mr^2)\left(\frac{v}{r}\right) = mvr = mr\sqrt{2gr}$
$= 1.6(2)\sqrt{2(9.8)(2)}$
$L = 20\ kg\cdot\frac{m^2}{s}$



$(ME)_1 = (ME)_2$
$mgr = \frac{1}{2}mv^2$
$v = \sqrt{2gr}$

(d)

16.



$\alpha r = a \qquad \tau = I\alpha$

m_1: $T_1 - m_1g = m_1a$
m_2: $m_2g - T_2 = m_2a$
pulley: $(T_2 - T_1)r = I\alpha = I\frac{a}{r}$
$T_2 - T_1 = I\frac{a}{r^2}$

$T_1 = m_1(g+a)$
$T_2 = m_2(g-a)$
$T_2 - T_1 = (m_2 - m_1)g - (m_1+m_2)a = \frac{I}{r^2}a$

$a = \frac{(m_2-m_1)g}{(m_1+m_2+\frac{I}{r^2})} = \frac{1(9.8)}{(3+5)} = 1.225\ m/s^2$
$= \left(\frac{1}{8}\right)g\ m/s^2$

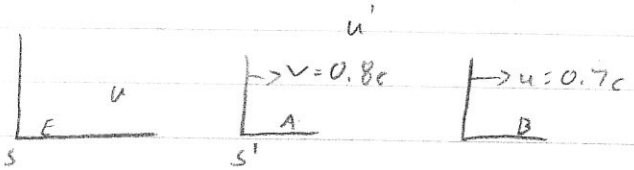
(c)

17. conservation of angular momentum

$$I_{mg}\omega_i = (I_{mg} + I_{cm})\omega_f$$

$$\omega_f = \frac{I_{mg}}{I_{mg} + I_{cm}}\omega_i$$

22.



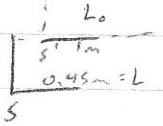
$$u = \frac{u' + v}{\left(1 + \frac{u'v}{c^2}\right)} \iff u' = \frac{u - v}{1 - \frac{uv}{c^2}}$$

$$u' = \frac{0.7c - 0.8c}{1 - (0.7)(0.8)} = \frac{(0.7 - 0.8)c}{1 - (0.7)(0.8)} = -0.227c\,\hat{i}$$

$(-0.23c)\,\hat{i}$

(d)

23.



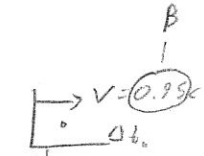
$$L_0 = \gamma L \qquad \gamma = \frac{1}{0.45} = 2.22$$

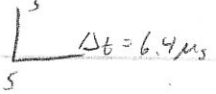
$$\gamma = \frac{1}{\sqrt{1 - \beta^2}} \rightarrow \beta = \sqrt{1 - \left(\frac{1}{\gamma}\right)^2} = \sqrt{1 - \left(\frac{1}{2.22}\right)^2} = 0.893.$$

$$v = \beta c = 0.89c$$

(b)

$\frac{v}{c} = \beta$



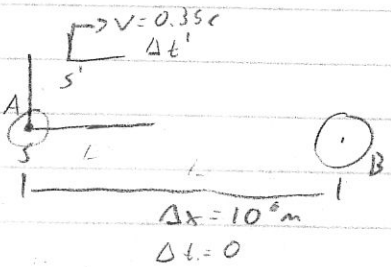
24. 

$\gamma = \frac{1}{\sqrt{1-\beta^2}} = \frac{1}{\sqrt{1-0.95^2}} = 3.203$

$\Delta t = \gamma \Delta t_0$

$\Delta t_0 = \frac{\Delta t}{\gamma} = \frac{6.4}{3.203}\ \mu s = 1.998\ \mu s \approx 2.0\ \mu s$

(d)

25. 

$\Delta t' = \gamma\left(\Delta t^{\nearrow 0} - \frac{v\Delta x}{c^2}\right)$

$= 1.068\left(-\frac{(0.35)(10^6)}{2.998\times 10^8}\right)$

$\Delta t' = 1.25\ ms$

$\gamma = \frac{1}{\sqrt{1-0.35^2}} = 1.068$

(d)