Logarithmic differentiation

3.12.1

(a) $\ln y = 5x \ln(x^4+1)$

$y'/y = 5\ln(x^4+1) + \frac{5x}{x^4+1}\cdot(4x^3)$

$y' = (x^4+1)^{5x}\cdot\left(5\ln(x^4+1) + \frac{20x^4}{x^4+1}\right)$

(b) $u = x^{\cos 3x}$ $\quad v = 7^{x^2}$ $\quad y = u+v$ $\quad y' = u'+v'$

$\ln u = \cos 3x \cdot \ln x$ $\quad v' = 7^{x^2}\cdot \ln 7 \cdot 2x$

$u'/u = -3\sin 3x \cdot \ln x + \cos 3x \cdot \frac{1}{x}$

$u' = x^{\cos 3x}\left(\frac{\cos 3x}{x} - 3\sin 3x \cdot \ln x\right)$

$y' = x^{\cos 3x}\left(\frac{\cos 3x}{x} - 3\sin 3x \cdot \ln x\right) + 7^{x^2}\cdot 2x \cdot \ln 7$

(c) $\ln y = \ln x \cdot \ln \ln x$

$y'/y = \frac{1}{x}\ln\ln x + \ln x \frac{1}{\ln x}\cdot\frac{1}{x}$

$y' = (\ln x)^{\ln x}\cdot\left(\frac{\ln\ln x}{x} + \frac{1}{x}\right)$

(d) $\ln|y| = \frac{1}{3}\ln|x-\tan x| + 5\ln|1+2x^3| - \frac{1}{2}\ln(1+x^2)$

$y'/y = \frac{1}{3}\frac{1}{x-\tan x}\left(1-\frac{1}{\cos^2 x}\right) + \frac{5}{1+2x^3}\cdot 6x^2 - \frac{1}{2(1+x^2)}\cdot 2x$

$y' = \frac{\sqrt[3]{x-\tan x}\,(1+2x^3)^5}{\sqrt{1+x^2}}\cdot\left(\frac{1}{3}\cdot\frac{1}{x-\tan x}\cdot\left(1-\frac{1}{\cos^2 x}\right) + \frac{30x^2}{1+2x^3} - \frac{x}{1+x^2}\right)$