

MATH 1210 Tutorial # 5

Oct. 13 - 19, 2011

1. Given the vectors $\vec{u} = a\hat{i} - 2\hat{j} + \hat{k}$ and $\vec{v} = a\hat{i} + a\hat{j} - 3\hat{k}$. Determine for which values of a these two vectors are

- (a) parallel.
- (b) perpendicular.

2. Consider the vectors $\vec{u} = \hat{i} + \hat{j}$ and $\vec{v} = \hat{j} + \hat{k}$. Find

- (a) $\vec{u} \cdot \vec{v}$.
- (b) $\vec{u} \times \vec{v}$.
- (c) The angle between \vec{u} and \vec{v} .

3. Show that the triangle in \mathbb{E}^3 with vertices $P_1(2, 2, 3), P_2(1, 4, 4), P_3(5, 4, 2)$ is a right triangle and find the length of its hypotenuse.

4. Show that the triangle with vertices $(1, 1, 0), (0, 1, 1), (1, 0, 1)$ is equilateral and find the coordinates of its center.

Hint: The distance of the center C of an equilateral triangle from a vertex V of the triangle equals $\frac{2}{3}|VM|$, where M is the midpoint of the side opposite the vertex V .

5. Show that

$$\vec{u} \times (\vec{v} \times \vec{w}) = (\vec{u} \cdot \vec{w})\vec{v} - (\vec{u} \cdot \vec{v})\vec{w}$$

for any 3 vectors $\vec{u}, \vec{v}, \vec{w} \in \mathbb{E}^3$.

Hint: First verify this for the special cases $\vec{u} = \hat{i}, \vec{u} = \hat{j}$, and $\vec{u} = \hat{k}$. Then use the representation $\vec{u} = u_1\hat{i} + u_2\hat{j} + u_3\hat{k}$.