

## MATH 1210 Problem Workshop 5

1. For each of the following, find:

- the number of possible positive and negative real roots.
- the bounds on the modulus of the possible roots.
- the remaining possible rational roots.
- the roots of the polynomial.

(a)  $P(x) = x^3 + \frac{3}{2}x^2 + \frac{9}{2}x + 2$

(b)  $P(x) = 2x^3 - x^2 + 3x - 9$

(c)  $P(x) = 45x^3 + 54x^2 + 19x + 2$

(d)  $P(x) = 15x^3 - 13x^2 + 2x + 30$

(e)  $P(x) = 6x^5 - 13x^4 - 15x^3 + 23x^2 - 6x$

2. Determine with justification, whether or not there are any roots of  $P(x) = x^7 + 4x^5 - 3x^4 - 5x^2 + 7x - 9$  in the interval  $[-5, -2]$ . (Note: the question did not say rational roots)

## Answers

1. (a)
  - 0 positive, either 1 or 3 negative
  - $|x| < 11/2$
  - $x = -1, -2, -4, -1/2$
  - $x = -\frac{1}{2}, \frac{-1 \pm i\sqrt{15}}{2}$
- (b)
  - 3 or 1 positive, 0 negative
  - $|x| < 11/2$
  - $x = 1, 3, 1/2, 3/2, 9/2$
  - $x = \frac{3}{2}, \frac{-1 \pm i\sqrt{11}}{2}$
- (c)
  - 0 positive, either 1 or 3 negative
  - $|x| < 11/5$
  - $x = -1, -1/3, -1/5, -1/9, -1/15, -1/45, -2, -2/3, -2/5, -2/9, -2/15, -2/45$
  - $x = -2/3, -1/3, -1/5$
- (d)
  - 2 or 0 positive, 1 negative
  - $|x| < 3$
  - $x = \pm 1, \pm 1/3, \pm 1/5, \pm 1/15, \pm 2, \pm 2/3, \pm 2/5, \pm 2/15, \pm 3/5, \pm 5/3, \pm 6/5$
  - $x = -1, \frac{14 \pm i\sqrt{254}}{15}$
- (e)
  - 3 or 1 positive, 1 negative
  - $|x| < 29/6$
  - $x = \pm 1, \pm 1/2, \pm 1/3, \pm 1/6, \pm 2, \pm 2/3, \pm 3, \pm 3/2$  (other than 0 which is obviously a solution)
  - $x = 0, -\frac{3}{2}, \frac{2}{3}, \frac{3 \pm \sqrt{5}}{2}$