MATH 1210 Assignment 5

Due: 1:30 pm Friday 3 April 2009 (at your instructor's office)

NOTES:

1. **Late assignments will NOT be accepted**.
2. **If your assignment is not accompanied by a signed Faculty of Science "Honesty Declaration", it will NOT be graded**.

Provide a complete solution to each of the following problems:

1. Evaluate each of the following determinants in two distinct ways:

   (i) by using a cofactor expansion along some (appropriately chosen) row or column,

   (ii) by using elementary row operations, together with properties of determinants, in order to reduce the determinant to a form which is easily evaluated.

   (a) $\begin{vmatrix} 1 & 3 & -1 \\ 2 & 0 & 5 \\ 1 & 4 & 3 \end{vmatrix}$,

   (b) $\begin{vmatrix} 1 & 0 & 2 & 1 \\ 2 & -1 & 1 & 0 \\ 1 & 0 & 0 & 3 \\ -1 & 0 & 2 & 1 \end{vmatrix}$,

   (c) $\begin{vmatrix} 1 & -1 & 0 & 2 \\ -1 & 1 & 2 & 3 \\ 2 & -2 & 3 & 4 \\ 6 & -6 & 5 & 1 \end{vmatrix}$,

   (d) $\begin{vmatrix} x & x^2 & x^3 & x^4 \\ 1 & 2x & 3x^2 & 4x^3 \\ 0 & 2 & 6x & 12x^2 \\ 0 & 0 & 6 & 24x \end{vmatrix}$.

2. Use Cramer's Rule to determine whether or not each of the following systems of equations possesses a unique solution. *If it does possess a unique solution*, find it using Cramer's Rule. *If it does not possess a unique solution*, find all solutions (possibly "none") by reducing the augmented matrix to reduced row-echelon form (RREF).

   (a) $a + 3b + c = -2$
   $2a + 5b + c = -5$
   $a + 2b + 3c = 6$

   (b) $5x_1 + 6x_2 + 4x_3 = 3$
   $7x_1 + 8x_2 + 6x_3 = 1$
   $6x_1 + 7x_2 + 5x_3 = 0$

   (c) $3x_1 - 3x_2 + 3x_3 = 0$
   $2x_1 - x_2 + 4x_3 = 0$
   $3x_1 - 5x_2 - x_3 = 0$

3. For each of the following sets of vectors , *determine whether the given set is linear dependent or linearly independent.*

   Moreover, *if the set is linearly dependent*:

   (i) provide a relation which explicitly displays how one (or more) of the given vectors may be written as a linear combination of the others,

   (ii) find a subset (of the given set of vectors) containing the largest number of linearly independent vectors. [HINT: If the given set of vectors is linearly dependent, delete one (or more) of the vectors until you find a subset containing the largest number of linearly independent vectors.]

   (a) $\overrightarrow{v_1} = \hat{\imath} + 2\hat{\jmath}$, $\overrightarrow{v_2} = 3\hat{\imath} + 4\hat{\jmath}$

   (b) $\overrightarrow{u_1} = \hat{\imath} + 2\hat{\jmath}$, $\overrightarrow{u_2} = 3\hat{\imath} + 4\hat{\jmath}$, $\overrightarrow{u_3} = 5\hat{\imath} + 6\hat{\jmath}$

   (c) $\overrightarrow{v_1} = \hat{\imath} + 2\hat{\jmath} + 3\hat{k}$, $\overrightarrow{v_2} = 4\hat{\imath} + 5\hat{\jmath} + 6\hat{k}$, $\overrightarrow{v_3} = 7\hat{\imath} + 8\hat{\jmath}$

   (d) $\overrightarrow{w_1} = \hat{\jmath} + \hat{k}$, $\overrightarrow{w_2} = \hat{\jmath} + \hat{k}$, $\overrightarrow{w_3} = \hat{\imath} + \hat{\jmath} + \hat{k}$

   (e) $\overrightarrow{r_1} = \hat{\imath} + \hat{\jmath} + \hat{k}$, $\overrightarrow{r_2} = 2\hat{\imath} + 2\hat{\jmath} + 2\hat{k}$, $\overrightarrow{r_3} = 5\hat{k}$, $\overrightarrow{r_4} = \hat{\imath} + 2\hat{\jmath} + 3\hat{k}$,

   (f) $\overrightarrow{a} = \begin{pmatrix} 2 & 0 & 1 \end{pmatrix}$, $\overrightarrow{b} = \begin{pmatrix} 1 & 1 & -1 \end{pmatrix}$, $\overrightarrow{c} = \begin{pmatrix} 0 & 0 & 0 \end{pmatrix}$, $\overrightarrow{d} = \begin{pmatrix} 5 & 3 & 2 \end{pmatrix}$,

   (g) $\overrightarrow{v_1} = \begin{pmatrix} 2 \\ 1 \\ 3 \\ 1 \end{pmatrix}$, $\overrightarrow{v_2} = \begin{pmatrix} 6 \\ 3 \\ 9 \\ 3 \end{pmatrix}$, $\overrightarrow{v_3} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix}$, $\overrightarrow{v_4} = \begin{pmatrix} 5 \\ 2 \\ 6 \\ 3 \end{pmatrix}$

4. Determine whether or not each of the following matrices has an inverse. If so, find the inverse by two methods, namely:

   (i) using the adjoint method

   (ii) using the "direct" method:

   (a) $\begin{pmatrix} 1 & 6 & 4 \\ 2 & 4 & -1 \\ -1 & 2 & 5 \end{pmatrix}$,

   (b) $\begin{pmatrix} 1 & 2 & 3 \\ 2 & 5 & 3 \\ 1 & 0 & 8 \end{pmatrix}$,

   (c) $\begin{pmatrix} 2 & 0 & 0 & 0 \\ 1 & 2 & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 1 & 2 \end{pmatrix}$,

   (d) $\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & cos\theta \end{pmatrix}$.

5. Determine all values of λ for which the homogeneous linear system of equations

$$\begin{aligned}(5-\lambda)x_1 + 4x_2 + 2x_3 &= 0\\ 4x_1 + (5-\lambda)x_2 + 2x_3 &= 0\\ 2x_1 + 2x_2 + (2-\lambda)x_3 &= 0\end{aligned}$$

possesses **non-trivial solutions**. In addition, for each admissible value of λ, find all non-trivial solutions.

Comment: The values λ determined in the above are known as ***eigenvalues*** of the matrix $\begin{pmatrix} 5 & 4 & 2 \\ 4 & 5 & 2 \\ 2 & 2 & 2 \end{pmatrix}$. For each eigenvalue λ, the corresponding vector(s) $x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$ are known as ***eigenvectors*** of $\begin{pmatrix} 5 & 4 & 2 \\ 4 & 5 & 2 \\ 2 & 2 & 2 \end{pmatrix}$ corresponding to the eigenvalue λ.