Derivative

3.1.1

(a)
$$f'(x)=\lim_{h\to 0}\frac{\sqrt{(x+h)^2+1}-\sqrt{x^2+1}}{h}=$$

$$=\lim_{h\to 0}\frac{\sqrt{(x+h)^2+1}-\sqrt{x^2+1}}{h}\cdot\frac{\sqrt{(x+h)^2+1}+\sqrt{x^2+1}}{\sqrt{(x+h)^2+1}+\sqrt{x^2+1}}=$$

$$=\lim_{h\to 0}\frac{2xh+h^2}{h\left(\sqrt{(x+h)^2+1}+\sqrt{x^2+1}\right)}=\lim_{h\to 0}\frac{2x+h}{\sqrt{(x+h)^2+1}+\sqrt{x^2+1}}=\frac{x}{\sqrt{x^2+1}}$$

(b)
$$f'(x)=\lim_{h\to 0}\frac{\frac{2(x+h)-1}{3-(x+h)}-\frac{2x-1}{3-x}}{h}=\lim_{h\to 0}\frac{(2x+2h-1)\cdot(3-x)-(3-x-h)(2x-1)}{h(3-x-h)\cdot(3-x)}$$

$$=\lim_{h\to 0}\frac{\cancel{6x}+6h\cancel{-3}\cancel{-2x^2}\cancel{-2xh}\cancel{+x}-(\cancel{6x}\cancel{-2x^2}\cancel{-2xh}\cancel{-3}\cancel{+x}+h)}{h(3-x-h)(3-x)}=$$

$$=\lim_{h\to 0}\frac{5\cancel{h}}{\cancel{h}(3-x-h)(3-x)}=\frac{5}{(3-x)^2}$$