MATH 2132 Test 2 March 13, 2008 70 minutes

Student Name - Student Number -

Values

7 **1.** Find a numerical expression representing a maximum possible error when the first three nonzero terms in the Maclaurin series are used to approximate the function $f(x) = x^3 \cos x$ for $x = 1/2$. Justify your answer.

6 **2.** A tank originally ($t = 0$) contains 500 L of water in which 3 kg of salt have been dissolved. A mixture containing 2 kg of salt for each 100 L of solution is added at 20 mL/s. At the same time, 10 mL of pure water is added from another source each second. Well-stirred mixture in the tank is removed at 15 mL/s. Set up, but do **NOT** solve, an initial value problem for the number of grams of salt in the tank as a function of time.

10 **3.** Find a 1-parameter family of solutions for the differential equation

$$\frac{dy}{dx} = \frac{3(y^2 - 2)}{4ye^{2x}}.$$

Does the 1-parameter family of solutions have any singular solutions? Explain.

7 **4.** Find a general solution for the differential equation

$$2\frac{d^3y}{dx^3} + \frac{d^2y}{dx^2} + 8\frac{dy}{dx} + y = 0.$$

4 **5.** You are given that the roots of the auxiliary equation $\phi(m) = 0$ associated with the differential equation $\phi(D)y = 0$ are

$$m = -1, 3 \pm \sqrt{7}, 0, 0, 0, 1 \pm 4i, 1 \pm 4i.$$

What is a general solution of the differential equation?

6 **6.** Find the sum of the series

$$\sum_{n=1}^{\infty} \frac{n+1}{3^{n+2}} x^n.$$