MATH2132 Test1 **May 2014** **60 minutes**

8 **1.** Determine whether the sequence of functions

$$\{f_n(x)\} = \left\{ \left(\frac{n}{n+1} \right) x + \left(\frac{n+1}{n} \right)^n x^n \right\}$$

has a limit on the interval $0 \leq x \leq 1$. Show your reasoning and all calculations.

10 **2.** Determine whether the series in parts (a) and (b) converge or diverge. If a series converges, find its sum simplified as much as possible. Justify your conclusions. You do not have to do part (c), but if you do, there is a 5 mark bonus.

(a) $\displaystyle\sum_{n=3}^{\infty} \frac{2^{2n}}{5^{n+2}}$ (b) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{n}{n+4} \right)$ (c) $\displaystyle\sum_{n=1}^{\infty} \frac{2n+1}{n^2+n}$

12 **3.** Find the interval of convergence for the power series

$$\sum_{n=4}^{\infty} \frac{n^a}{n+1}(x+2)^n, \qquad \text{where } a \geq 2 \text{ is an integer.}$$

10 **4.** Find the remainder $R_n(2,x)$ when the function $f(x) = e^{4x}$ is expanded with Taylor's remainder formula (about $x = 2$). Verify that $\lim_{n\to\infty} R_n(2,x) = 0$ for all $x \geq 2$.