

MATH 2130 Tutorial 7

1. For what value, or values, of b is the function $f(x, y) = e^{bx} \cos 5y$ harmonic in the entire xy -plane?

Answer: $b = \pm 5$

2. If $f(x, y) = x^3 y \sin(y^2/x^2)$, find and simplify

$$x \frac{\partial f}{\partial x} + y \frac{\partial f}{\partial y}.$$

Answer: $4x^3 y \sin(y^2/x^2)$

3. You are told that $z = f(u, v, t)$, $u = g(x, y, t)$, $v = h(x, y, t)$, and $y = k(t)$. What is the chain rule for $\frac{\partial z}{\partial t}$?

Answer: $\frac{\partial z}{\partial u} \frac{\partial u}{\partial y} \frac{dy}{dt} + \frac{\partial z}{\partial u} \frac{\partial u}{\partial t} + \frac{\partial z}{\partial v} \frac{\partial v}{\partial y} \frac{dy}{dt} + \frac{\partial z}{\partial v} \frac{\partial v}{\partial t} + \frac{\partial z}{\partial t}$

4. Find the rate of change of the function $f(x, y, z) = \sin(xy) - z^3$ at the point $(2, 0, 3)$ in the direction of the upward normal to the surface $xz^2 - x^2z = 6$.

Answer: $-216/\sqrt{73}$

5. Find equations for the tangent line to the curve

$$xyz + z^3 = 24, \quad x^3 y^2 z + y^3 = 4x - 2$$

at the point $(1, -1, 3)$.

Answer: $x = 1 + 81t, y = -1 + 133t, z = 3 - 6t$

6. At the point $(1, 2, -3)$, a vector \mathbf{v} makes an angle of $\pi/3$ radians with the gradient of the function $f(x, y, z) = x^2 yz - 3xy^3$. Find the rate of change of $f(x, y, z)$ in direction \mathbf{v} .

Answer: $\sqrt{2821}/2$

7. The equations

$$x^2 + y + 3s^2 + s = 2t - 1, \quad y^2 - x^4 + 2st + 7 = 6s^2 t^2,$$

define s and t as functions of x and y . Find $\partial s / \partial x$ when $s = 0$ and $t = 1$. Assume that $x > 0$.

Answer: 16