MATH 1210 Tutorial # 2

1. Prove that

$$\sum_{k=1}^{n} ((2k-1)\sqrt{3})^2 = n(4n^2 - 1)$$

for every positive integer n.

2. Decide whether or not the equalities

(a)
$$\sum_{k=1}^{n} (k+1)^3 = \left(\sum_{k=1}^{n} (k+1)\right)^2$$

and

(b)
$$\sum_{k=0}^{n} (k+1)^3 = \left(\sum_{k=0}^{n} (k+1)\right)^2$$

hold for all positive integers n.

3. Rewrite the sum

$$\sum_{r=12}^{122} \frac{r-6}{r+9}$$

using an index whose initial and terminal values are 1 and 111, respectively (HINT: use a change of variables).

4. Simplify

$$\frac{169}{5+12i} + \left(\overline{(1-2i)^3+4}\right)^2$$

and express in Cartesian form.

5. Given that $i^2 = -1$, show that

$$\sum_{k=0}^{4n} i^k = 1$$

for all integers $n \geq 0$.