MATH1210 Assignment #3

Due: 1:30 pm Friday 27 October 2006 T. G. Berry

NOTES: ***(1)*** ***The assignment is due at the start of our class on Friday 27 October 2006.***

(2) ***Late assignments will NOT be accepted.***

(3) ***If your assignment is not accompanied by a Faculty of Science "Honesty Declaration", it will NOT be graded.***

1. Solve each of the following real polynomial equations, by factoring the polynomial appearing on the left-hand side completely into real linear and/or irreducible real quadratic factors

(a) $x^2 - 7x + 11 = 0$,

(b) $x^2 - 2x + 9 = 0$,

(c) $x^3 + 5x^2 + 6x = 0$,

(d) $(x-1)^2(x^4 - 16) = 0$,

(e) $x^4(x^2+4)(x^3 - 3x^2 + 3x - 1) = 0$,

(f) $x^6 + 5x^4 + 4x^2 = 0$,

(g) $p_1(x) = x^4 + x^3 - x^2 + 2x - 6 = 0$ given that $p_1(\sqrt{2}\,i) = 0$,

(h) $p_2(x) = x^3 + 9x^2 + 16x + 14 = 0$ given that $p_2(-1+i) = 0$,

(i) $p_3(x) = x^5 - 4x^4 + 5x^3 + x^2 - 4x + 5 = 0$ given that $p_3(2+i) = 0$.

2. Let $P(x) = x^6 + 12x^5 + 42x^4 + 43x^3 - 30x^2 - 60x - 8$.

(a) What does Descartes' Rule of Signs tell you about the maximum possible number of positive real zeros of $P(x)$?

(b) List all the possible rational zeros of $P(x)$, and use the result of part (a) to help you find all of its positive real zeros.

(c) By long division, remove the factor(s) from $P(x)$ which correspond to the positive real zeros of $P(x)$ identified in part (b).

(d) Find all the negative real zeros of $P(x)$, one at a time, in each case removing the corresponding factor from $P(x)$, until $P(x)$ is written as a product of only real linear factors.

(e) List all six roots of $P(x) = 0$, indicating clearly the multiplicity of each.

3. Consider the polynomial ***with complex coefficients*** given by

$$P(x) = ix^4 + 3x^3 + 8ix + 24 .$$

(a) Show that $P\left(-\frac{3}{i}\right) = 0$, and determine the corresponding linear factor of $P(x)$.

(b) By long division, write $P(x)$ as a product of the linear factor of part (a) and a cubic factor.

(c) Use the information of part (b) to find all roots of $P(x) = 0$.

[HINT: You will have to remember how to calculate the cube roots of a complex number in order to complete this problem.]

(d) Plot the four roots of $P(x) = 0$ in the complex plane.

Note: since the equation does not have real coefficients, its complex roots do not always occur in complex conjugate pairs.

You are not responsible for the following two problems as they are not officially part of the course. However, these problems may help you understand three-dimensional geometry, which in turn may help you understand some of the linear algebra which we will discuss in upcoming weeks.

4. Let ℓ_1 and ℓ_2 be two non-parallel, non -intersecting lines in space , given parametrically by the equations

$$\ell_1 : x = 1 + s, y = 1 - s, z = 4s \text{ (with parameter } s\text{)}$$

and

$$\ell_1 : x = 2 - 3t, y = 6 + 2t, z = 1 - t \text{ (with parameter } t\text{)}.$$

To find the shortest distance between these two lines, we must determine the point on each line which is closest to the other line. We may do this as follows:

(a) Find a vector $\vec{v}_1$ along the line ℓ_1.

(b) Find a vector $\vec{v}_2$ along the line ℓ_2.

(c) Let P_1 with coordinates $(1+s, 1-s, 4s)$ be an arbitrary point on ℓ_1, and P_2 with coordinates $(2-3t, 6+2t, 1-t)$ be an arbitrary point on ℓ_2, and construct the vector $\vec{u}$ from P_1 to P_2.

(d) In order to guarantee that each of the above two points are as close as possible to the other line, we now demand that $\vec{u}$ be perpendicular to both $\vec{v}_1$ and $\vec{v}_2$.

Use this idea to find the coordinates of the two fixed points P_1 and P_2 which satisfy these conditions.

(e) Finally, the desired minumum distance between the given two lines is simply the length of the line segment P_1P_2, in which P_1 and P_2 are the fixed points determined in part (d).

Hence find the shortest distance between the given two lines.

5. Let Π be the plane given by the equation $2x - y + 3z = 4$.

Let ℓ be the line given by the parametric equations $x = 2 + t, y = 3 + 5t, z = 4 + t$.

(a) Show that ℓ and Π never intersect.

(We therefore say that the line and the plane are parallel.)

(b) To find the shortest distance between the line and the plane, we may proceed as follows:

(i) Let P be a fixed point on ℓ .

(ii) Let $\vec{N}$ be any vector perpendicular to Π .

(iii) Find the equation of the line ℓ_1 which passes through P in the direction of $\vec{N}$.

(iv) Find the point of intersection Q of ℓ_1 and Π .

(v) Calculate the distance between P and Q , which clearly represents the distance between the given line and the given plane.