

Term Test 2

DATE: November 12, 2009
COURSE: MATH 2132

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TIME: 70 minutes
EXAMINER: G.I. Moghaddam

[8] 1. Find the sum of the series $\sum_{n=1}^{\infty} \left(\frac{n+2}{n!} \right) x^{n+1}$.

- [9] 2. Use binomial expansion to find the Maclaurin series of the function

$$f(x) = \left(\frac{x^2}{1+x^3} \right)^3.$$

What is the open interval of convergence? Express your answer in sigma notation and simplify as much as possible.

- [9] 3. 50 g of a certain chemical is added to 200 mL of water; this chemical dissolves in water at a rate proportional to the product of the amount of undissolved chemical and the difference between concentrations in a saturated solution and the existing concentration in the solution. A saturated solution contains 25 g of chemical in 100 mL of solution.

- (a) Show that the differential equation that describes the situation is

$$\frac{dx}{dt} = \frac{k}{200} (50 - x)^2, \quad x(0) = 0,$$

where $x(t)$ is the number of grams of dissolved chemical at time t .

- (b) Solve the differential equation in part (a).

- [8] 4. Find in explicit form the solution of the initial value problem

$$\frac{dy}{dx} + \frac{y}{2x} = \frac{1}{x^2 \sqrt{x}} e^{1/x}, \quad y(1) = e.$$

- [8] 5. Find a 2-parameter family of solutions for the differential equation

$$y'' = \frac{(y')^2 - y'}{x}.$$

- [8] 6. Consider the homogeneous linear differential equation

$$y''' - 3y'' - 4y' + 12y = 0.$$

- (a) Write the differential equation in form $\phi(D)y = 0$, where $\phi(D)$ is the differential operator.
- (b) Find the general solution for this homogeneous linear differential equation.