P 1/2

UNIVERSITY OF MANITOBA

DATE: August 10, 2013 FINAL EXAMINATION

EXAMINATION: Engineering Mathematical Analysis 1 TIME: 3 hours

COURSE: MATH 2130 EXAMINER: Harland

Answers by Dawit
ydawit@yahoo.com

[8] 1. Find the distance between the lines

$$\frac{x-4}{3} = \frac{y+3}{2} = \frac{z-2}{-1} \text{ and}$$

$$x = 1+t, \quad y = -1-2t, \quad z = 3-t.$$

$\longrightarrow$ $\frac{4}{\sqrt{21}} = \frac{4\sqrt{21}}{21}$

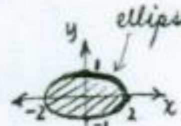
You may use without proof that the two lines are skew.

[5] 2. Let $f(t) = t$ and $\mathbf{v}(t) = t^2\hat{\mathbf{i}} + \ln t\hat{\mathbf{j}} - \frac{1}{t}\hat{\mathbf{k}}$. Find

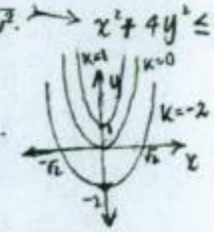
$$\int (f\mathbf{v})(t)\,dt.$$

$\longrightarrow$ $\frac{t^4}{4}\hat{i} + \frac{t^2}{4}(\ln t^2 - 1)\hat{j} - t\hat{k} + \vec{C}$

[4] 3. (a) Find and sketch the largest possible domain of $f(x,y) = \sqrt{4 - x^2 - 4y^2}$. $\longrightarrow$ $x^2 + 4y^2 \le 4$



[4] (b) Sketch level curves of $f(x,y) = y - x^2$ corresponding to $k = -2, 0, 1$. $\longrightarrow$



4. Let $z = x^2 + xy + y^2$. Show that z is a solution to:

[3] (a) $xz_x + yz_y = 2z$. $\longrightarrow$ $LHS = x(2x+y) + y(x+2y) = 2(x^2+xy+y^2) = 2z = RHS$

[3] (b) $x^2 z_{xx} + 2xy z_{xy} + y^2 z_{yy} = 2z$. $\longrightarrow$ $LHS = x^2(2) + 2xy(1) + y^2(2) = 2(x^2+xy+y^2) = 2z = RHS$

[5] 5. (a) Find a chain rule for $\left.\frac{\partial z}{\partial t}\right)_s$ if $z = f(x,y,s), x = g(r), y = h(r), r = k(s,t)$.

$\longrightarrow$ a) $\frac{\partial z}{\partial x}\frac{dx}{dr}\frac{\partial r}{\partial t} + \frac{\partial z}{\partial y}\frac{dy}{dr}\frac{\partial r}{\partial t} = \frac{\partial r}{\partial t}\left[\frac{\partial z}{\partial x}\frac{dx}{dr} + \frac{\partial z}{\partial y}\frac{dy}{dr}\right]$

[5] (b) Use your chain rule in part (a) to find $\left.\frac{\partial z}{\partial t}\right)_s$ if

$$z = e^{y^2+xs}, \quad x = \frac{\ln r}{r}, \quad y = \sec(r^2), \quad r = \sqrt{s^2+t^2}.$$

$\longrightarrow$ b) $\frac{t}{\sqrt{s^2+t^2}}\left\{se^{y^2+xs}\left[\frac{1}{r^2}(1-\ln r)\right] + 2ye^{y^2+xs}(2r\sec r^2 \tan r^2)\right\}$

[6] 6. Calculate the derivative of $f(x,y,z) = xy + z^2$ at $(0,10,-2)$ in the direction of increasing t along the line

$$x = -3+t, \quad y = -2+4t, \quad z = 4-2t.$$

$\longrightarrow$ $\frac{18}{\sqrt{21}} = \frac{6\sqrt{21}}{7}$

7. For the function $f(x,y) = x^3 - 3x + y^2 + 2y + 2$.

[4] (a) Find the critical point(s) of f. $\longrightarrow$ $(-1,-1), (1,-1)$

[6] (b) Classify the critical points of f. $\longrightarrow$ $(-1,-1) \to$ Saddle point
$(1,-1) \to$ relative-min

[10] (c) Find the absolute maximum and minimum of $f(x, y)$ on the triangle bounded by the lines $x = 0$, $y = 0$ and $x - y = 2$. $\longrightarrow$ abs-max $= 4$, abs-min $= -1$

[6] 8. Evaluate the double iterated integral

$$\int_{-1}^{0} \int_{-y}^{1} y(x^2 + y^2)^{2013}\, dx\, dy.$$

$\longrightarrow$ $\dfrac{1-2^{2014}}{(4028)(4029)}$

[5] 9. Set up, but do not integrate a multiple integral, or sum of multiple integrals in Cartesian coordinates to find the moment of inertia of a thin plate with constant mass per unit area ρ defined by the region bounded by $x = y^2 - 2$ and $x = y$ rotated about $x + y = 1$. $\longrightarrow$ $\displaystyle\int_{-1}^{2}\int_{y^2-2}^{y} \rho\left(\frac{x+y-1}{\sqrt{2}}\right)^2 dx\,dy$

[5] 10. Set up, but do not integrate a multiple integral, or sum of multiple integrals in polar coordinates to find the volume of the region bounded by the polar curve $r = 2 - \cos\theta$ rotated about the line $x = 3$. $\longrightarrow$ $\displaystyle\int_{0}^{2\pi}\int_{0}^{2-\cos\theta} 2\pi(3 - r\cos\theta)\, r\, dr\, d\theta$

[9] 11. Find the surface area of $y^2 = z + x^2$ inside the cylinder $x^2 + y^2 = 9$. $\longrightarrow$ $\frac{\pi}{6}(37\sqrt{37} - 1)$

[7] 12. Set up, but do not integrate multiple integrals, or sum of multiple integrals in Cartesian coordinates to find the z-coordinate of the center of mass of the solid bounded by the surfaces $y = 4 - x^2$, $z = 0$, $y = z$ with density equal to the distance to the y-axis. $\longrightarrow$

$$M = \int_{-2}^{2}\int_{0}^{4-x^2}\int_{0}^{y} \sqrt{x^2+z^2}\, dz\, dy\, dx$$

$$\bar{z} = \frac{1}{M}\int_{-2}^{2}\int_{0}^{4-x^2}\int_{0}^{y} z\sqrt{x^2+z^2}\, dz\, dy\, dx$$

[8] 13. Use cylindrical coordinates to find the volume of the solid bounded by the surfaces

$$z = 4 + x^2 + y^2, \quad x^2 + y^2 = 16 \text{ and } z = 0.$$ $\longrightarrow$ 192π

[7] 14. Set up, but do not integrate multiple integrals, or sum of multiple integrals in spherical coordinates to find the volume of the region bounded by $z = \sqrt{8 - x^2 - y^2}$ and $z = 2$. $\longrightarrow$ $\displaystyle\int_{0}^{2\pi}\int_{0}^{\pi/4}\int_{2\sec\phi}^{2\sqrt{2}} R^2 \sin\phi\, dR\, d\phi\, d\theta$

15. (Bonus: Max 5 marks) Find

$$I = \int_0^\infty e^{-x^2}\, dx.$$

(Hint: I is also equal to $\int_0^\infty e^{-y^2}\, dy$) $\longrightarrow$ $\dfrac{\sqrt{\pi}}{2}$