MATH 1210 A01 Summer 2013 Problem Workshop 2 Solutions

1. (a) $\sum_{n=1}^{100} \frac{n+1}{\sqrt{n}}$

   (b) $\sum_{n=1}^{20} \frac{2}{n\sqrt{n}}$

   (c) $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}n!}{(2n-1)^2}$

2.

$$\begin{aligned}
\sum_{n=16}^{39}(2n^2+3n+4) &= \sum_{n=1}^{39}(2n^2+3n+4) - \sum_{n=1}^{15}(2n^2+3n+4) \\
&= \left(2\sum_{n=1}^{39} n^2 + 3\sum_{n=1}^{39} n + 4\sum_{n=1}^{39} 1\right) - \left(2\sum_{n=1}^{15} n^2 + 3\sum_{n=1}^{15} n + 4\sum_{n=1}^{15} 1\right) \\
&= \left(2\left(\frac{39(40)(79)}{6}\right) + 3\left(\frac{39(40)}{2}\right) + 4(39)\right) \\
&\quad - \left(2\left(\frac{15(16)(31)}{6}\right) + 3\left(\frac{15(16)}{2}\right) + 4(15)\right) \\
&= \Big(2(20540) + 3(780) + 4(39)\Big) - \Big(2(1240) + 3(120) + 4(15)\Big) \\
&= \big(41080 + 2340 + 156\big) - \big(2480 + 360 + 60\big) \\
&= 43576 - 2900 \\
&= 40676
\end{aligned}$$

3. Using a substitution $n = j + 12$ (or $j = n - 12$) we get

$$\sum_{n=1}^{20}((n)^3 + (n)^2 - n + 12 - 12) = \sum_{n=1}^{20}(n^3 + n^2 - n).$$

Hence

$$\begin{aligned}
\sum_{n=1}^{20}(n^3+n^2-n) &= \sum_{n=1}^{20} n^3 + \sum_{n=1}^{20} n^2 - \sum_{n=1}^{20} n \\
&= \frac{(20)^2(21)^2}{4} + \frac{20(21)(41)}{6} - \frac{20(21)}{2} \\
&= 44100 + 2870 - 210 \\
&= 46760
\end{aligned}$$

4. First we need to turn the sum into sigma notation. We notice the first factor goes up by 1 from 1 to 33. Hence if i goes from 1 to 33, then the first factor is just i. The second factor goes down by 1 from 52 to 20. Hence if i goes from 1 to 33, then the second factor is just $53 - i$. Therefore in sigma notation, the sum becomes

$$\begin{aligned}
\sum_{n=1}^{33} i(53-i)^2 &= \sum_{n=1}^{33} (i^3 - 106i^2 + 2809i) \\
&= \sum_{n=1}^{33} i^3 - 106 \sum_{n=1}^{33} i^2 + 2809 \sum_{n=1}^{33} i \\
&= \frac{(33)^2(34)^2}{4} - 106\left(\frac{(33)(34)(67)}{6}\right) + 2809\left(\frac{(33)(34)}{2}\right) \\
&= 314721 - 106(12529) + 2809(561) \\
&= 314721 - 1328074 + 1575849 \\
&= 562496
\end{aligned}$$

5. (a) Part A:

When $n = 1$, the left hand side is

$$\sum_{l=1}^{2} (l+1) = 2 + 3 = 5.$$

The right hand side is

$$\frac{1}{2}(1+1)(3+2) = 5.$$

Therefore the formula is true for $n = 1$.

Part B:

Suppose the formula is true for $n = k$, that is

$$\sum_{l=k}^{2k} (l+1) = \frac{1}{2}(k+1)(3k+2).$$

We need to show that

$$\sum_{l=k+1}^{2(k+1)} (l+1) = \frac{1}{2}(k+1+1)(3(k+1)+2) = \frac{1}{2}(k+2)(3k+5).$$

The left hand side is

$$
\begin{aligned}
LHS &= \sum_{l=k+1}^{2k+2} (l+1) \\
&= \sum_{l=k}^{2k} (l+1) - (k+1) + (2k+2) + (2k+3) \\
&= \sum_{l=k}^{2k} (l+1) + 3k + 4 \\
&= \frac{1}{2}(k+1)(3k+2) + 3k + 4 \\
&= \frac{1}{2}\left(3k^2 + 5k + 2 + 6k + 8\right) \\
&= \frac{3k^2 + 11k + 10}{2} \\
&= \frac{(k+2)(3k+5)}{2}
\end{aligned}
$$

which is equal to the right hand side. Hence the formula is true for $n = k+1$. By the principle of mathematical induction the formula is true for all $n \geq 1$.

(b)

$$
\begin{aligned}
S &= \sum_{l=n}^{2n} (l+1) \\
&= \sum_{l=1}^{2n} (l+1) - \sum_{l=1}^{n-1} (l+1) \\
&= \sum_{l=1}^{2n} l + \sum_{l=1}^{2n} 1 - \sum_{l=1}^{n-1} l - \sum_{l=1}^{n-1} 1 \\
&= \frac{2n(2n+1)}{2} + 2n - \frac{(n-1)n}{2} - (n-1) \\
&= \frac{4n^2 + 2n}{2} + \frac{4n}{2} - \frac{n^2 - n}{2} - \frac{2n-2}{2} \\
&= \frac{4n^2 + 2n + 4n - n^2 + n - 2n + 2}{2} \\
&= \frac{3n^2 + 5n + 2}{2} \\
&= \frac{(3n+2)(n+1)}{2}.
\end{aligned}
$$