Math 1710: Tutorial 11 (Parametric curves, polar coordinates)

1. Find the length of the curve.

   (a) $x = 1 + 3t^2, y = 4 + 2t^3, \quad 0 \le t \le 1;$

   (b) $x = e^t + e^{-t}, y = 5 - 2t, \quad 0 \le t \le 3;$

   (c) $x = \dfrac{t}{1+t}, y = \ln(1+t), \quad 0 \le t \le 2.$

2. Find the area bounded by the curve $x = \sin^3 t$, $y = \cos^3 t$, $0 \le t \le 2\pi$.

3. Sketch graphs of the following curves in the *polar coordinate system*. Use the regular definition of polar coordinates, *i.e.*, r is not allowed to be negative.

   (a) $r = 2\sin(\theta)$

   (b) $r = 2\sin(2\theta)$

   (c) $r = 2\sin(3\theta)$

   (d) $r = |2\sin(\theta)|$

   (e) $r = |2\sin(2\theta)|$

   (f) $r = |2\sin(3\theta)|$

4. Consider the curve defined in polar coordinates by $r = 1 + \cos(2\theta)$.

   (a) Sketch of the graph of this curve on the interval $0 \le \theta \le 2\pi$.

   (b) Find the slope of the tangent line to this curve at $\theta = \pi/4$.

5. Show that the curves $x = t^3 - t$, $y = t + t^4$ and $x = t^2 - 2t$, $y = t^2 + 2t$ have a common tangent line at their *only* point of intersection. (You do not have to show that the curves intersect at exactly one point, but you may use that information to find that point).