

## MATH 2130 Problem Workshop 3

1. For the following limits, determine whether or not the limit exists (with justification) and if the limit exists, determine its value.

(a)  $\lim_{(x,y) \rightarrow (0,0)} \frac{x^2 - xy + y^2}{x^2 + 2y^2}$

(b)  $\lim_{(x,y) \rightarrow (0,0)} \frac{x^3 y}{x^6 + y^3}$

(c)  $\lim_{(x,y) \rightarrow (2,-3)} \frac{x^2 - 4x - y^2 - 6y - 5}{x^2 - 4x + y^2 + 6y + 13}$

(d)  $\lim_{(x,y) \rightarrow (3,2)} \frac{\sin(2x - 3y)}{2x - 3y}$

(e)  $\lim_{(x,y) \rightarrow (0,0)} \frac{x^4 y^4}{x^4 + y^4}$

(f)  $\lim_{(x,y) \rightarrow (0,1)} \arctan \left| \frac{y}{x} \right|$

(g)  $\lim_{(x,y) \rightarrow (0,1)} \arctan \left( \frac{y}{x} \right)$

(h)  $\lim_{(x,y) \rightarrow (0,1)} \arctan \left( \frac{x}{y} \right)$

2. Show that the function  $f(x, y) = 3x^2 + y^2 \cos \left( \frac{2x}{y} \right)$  satisfies the equation

$$x \frac{\partial f}{\partial x} + y \frac{\partial f}{\partial y} = 2f(x, y).$$

Answers:

1. For the following limits, determine whether or not the limit exists (with justification) and if the limit exists, determine its value.

- (a) DNE
- (b) DNE
- (c) DNE
- (d) 1
- (e) 0
- (f)  $\pi/2$
- (g) DNE
- (h) 0