MATH 2130 Summer Evening 2012 Problem Workshop 5

1. Find the volumes of the solids of revolution when the area bounded by the curves

$$y = 2x - x^2, \qquad y = x$$

is rotated about the lines $(a) x = 3 \qquad (b) y = 1 \qquad (c) x + y = -1.$

2. A triangular plate has sides with lengeths 3,4 and 5 metres. It is submerged vertically in oil with density 950 kilogrms per cubic metre. The side of length 3 metres is vertical, the side of length 4 is horizontal and the uppermost vertex is 1 metre below the surface of the oil. Find the force due to oil pressure on each side of the plate.

3. A thin plate with constant mass per unit area ρ has edges defined by the curves

$$x = \sqrt{a^2 - y^2}, y = x, y = 0,$$

where $a > 0$ is a constant. (a) Find the mass the plate , (b) Find the first moment of the plate about the x-axis, (c) Find $\overline{y}$.

4. A triangular plate has sides of length 2, 3 and 3. and constant mass per unit area ρ. Find its moment of inertia about the shorter side.

5. Find the area of the part of the surface $z = xy$ inside the cylinder $x^2 + y^2 = a^2$ where $a > 0$ is a constant.

6. Set up but do not evaluate a double iterated integral for the area of the surface $z = 2x^2 + y^2$ bounded by $y = 0, x = 0$ and $x + y = 1$.

7. Find the area bounded by $(x^2 + y^2)^3 = 4a^2x^2y^2$ where $a > 0$ is a constant.

8. Find the double integral of $f(x, y) = xy(x + y)$ over the region in the first quadrant bounded by $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$.

9. Evaluate the triple integral of the function $f(x, y, z) = x$ over the volume bounded by the surfaces

$$2x + 3y + z = 6, x = 0, y = 0, z = 0.$$

10. Find the volume in the first octant bounded by the surfaces

$$4x + 4y + z = 16, \qquad z = 0, \qquad y = x/2, \qquad y = 2x.$$

Answers:

1. (a) $5\pi/6$, (b) $2\pi/15$ (c) $7\sqrt{2}\pi/20$
2. $1.68 \times 10^5 N$
3. (a) $\pi a^2\rho/8$ (b) $\rho a^3(\sqrt{2}-1)/(3\sqrt{2})$, (c) $8a(\sqrt{2}-1)/(3\sqrt{2}\pi)$
4. $8\sqrt{2}\rho/3$
5. $2\pi[(1+a^2)^{3/2}-1]/3$
6. $\int_0^1 \int_0^{1-x} \sqrt{1+16x^2+4y^2}dydx$.
7. $\pi a^2/2$
8. $62/15$
9. $9/2$
10. $128/9$