Math 1710 Tutorial 7.
Centroids and Moments of Inertia

Centers of Mass and Centroids (the same set of problems as in the reading weak practice)

Problem 1. Find the first moment of a plate with constant mass per area ρ that is bounded by the given curves about the suggested line:

(a) $y = \dfrac{1}{x}$, $x = 2$, $y = 2$ about $x = 0$;

(b) $y = \dfrac{1}{x}$, $y = \dfrac{1}{4 - 3x}$, $y = 2$ about $x = 0$ **(set up only)**;

(c) $y = x^5$, $x = y^5$ (the part of the region in the first quadrant only) about $y = 0$;

(d) $y = x$, $y = 3x$, $x + y = 8$ about $y = 0$;

(e) $x = y^2 - 2y + 4$, $x - 3y + 2 = 0$ about $x = 1$;

(f) $x = y^2 - 2y + 4$, $x - 3y + 2 = 0$ about $y = -1$;

(g) $y = \sin x^2$ $(0 \le x \le \sqrt{\pi})$, $y = 0$, about $x = 0$;

(h)* $x + 2 = y^2$, $y = x$ about $x + y = 1$.

Problem 2. Find the centroid of the region bounded by the curves:

(a) $y = \dfrac{1}{x}$, $x = 2$, $y = 2$;

(b) $y = x^4$, $y = \sqrt{x}$;

(c) $y = x^2 - 2$, $y = x$, $y = -x$;

(d) $(x - 3)^2 + y^2 = 4$;

(e) $x = y^2 - 2y + 4$, $x - 3y + 2 = 0$;

(f) $y = \sin x$ $(0 \le x \le \pi/2)$, $x = \pi/2$, $y = 0$ **(set up only)**;

(g) $y = \sqrt{x}$, $x = y + 2$, $y = 0$;

(h) $y = \sqrt{1 - x^2}$ $(0 \le x \le 1)$, $x = 0$, $y = 0$.

Moments of Inertia

Problem 3. In this problem, the curves determine a thin plate with constant mass per unit area ρ. Find its moment of inertia about the line.

(a) $y = \sin x$ $(0 \leq x \leq \pi/2)$, $x = \pi/2$, $y = 0$ about $y = -1$ (**set up a definite integral only**);

(b) $y = 3e^{-2x}$, $y = 3$, $x = 2$ about $y = 4$ (**set up a definite integral only**);

(c) $y = 1 - x^2$, $y = 0$ about $x = -4$;

(d) $y = 1 - x^2$, $y = 0$ about $y = 5$;

(e) $x = 4 - y^2$, $x = y^2 - y$ about $x = -2$;

(f) $y = -\sqrt{9 - x^2}$ about $y = 1$ (**set up a definite integral only**);

(g) $y = x^2$, $y = x$ about $x = -1$;

(h) Problems 15-16 from the textbook.