February 9, 2010 MATH 1710 - Test 1 60 minutes

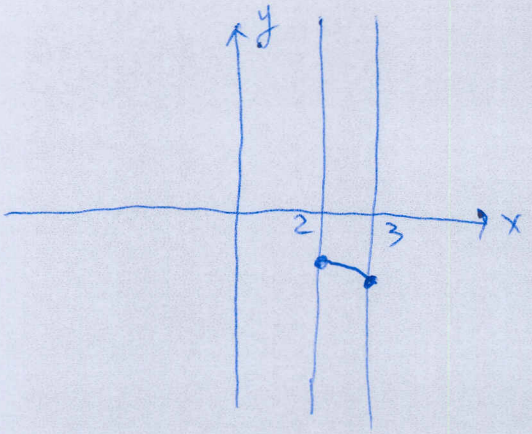
Student Name - ______________________ Student # - ____________

Circle your Instructor's Name: T. Berry D. Trim

Values

9 1. Find the area bounded by the curves

$$y = -\frac{x}{\sqrt{x-1}}, \quad y=0, \quad x=2, \quad x=3.$$



if $y = -\frac{x}{\sqrt{x-1}}$, then when $2 \le x \le 3$ we have $y<0$

so $y = -\frac{x}{\sqrt{x-1}}$ is below $y=0$

$$\int_2^3 \left(0 - \left(-\frac{x}{\sqrt{x-1}}\right)\right) dx = \int_2^3 \frac{x}{\sqrt{x-1}}\, dx$$

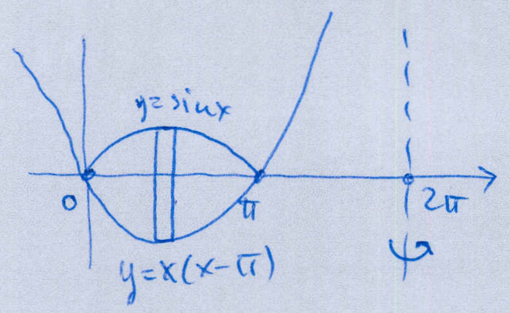
$$= \left| \begin{array}{ll} x-1=t & x=3: t=2 \\ dx = dt & x=2: t=1 \\ x=t+1 & \end{array} \right| = \int_1^2 \frac{t+1}{\sqrt{t}}\, dt = \int_1^2 \left(t^{\frac{1}{2}} + t^{-\frac{1}{2}}\right) dt$$

$$= \left(\frac{2}{3} t^{3/2} + 2t^{\frac{1}{2}}\right)\Big|_1^2 = \frac{4\sqrt{2}}{3} + 2\sqrt{2} - \frac{2}{3} - 2 = \frac{10\sqrt{2}}{3} - \frac{8}{3}$$

6 2. Set up, but do NOT evaluate, a definite integral to find the volume of the solid of revolution when the area bounded by the curves

$$y = \sin x, \qquad y = x^2 - \pi x, \quad 0 \le x \le \pi,$$

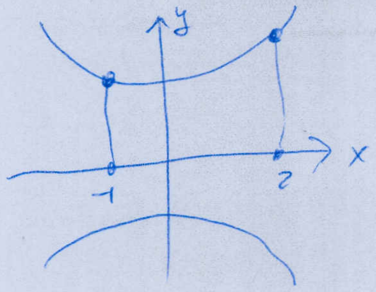
is rotated around the line $x = 2\pi$.



shells

$$\int_0^{\pi} 2\pi \underbrace{(2\pi - x)}_{\text{radius}} \left(\sin x - (x^2 - \pi x)\right) dx$$

5 3. Set up, but do NOT evaluate, a definite integral to find the length of the curve $y^2 - x^2 = 4$ between the points $(-1, \sqrt{5})$ and $(2, 2\sqrt{2})$.



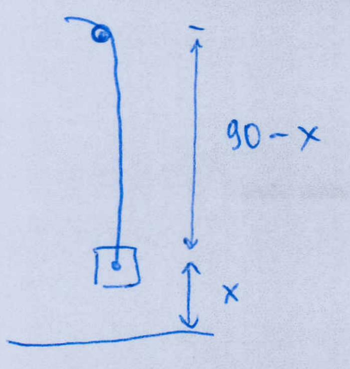
both points are on $y>0$ branch:

$$y = \sqrt{4+x^2}$$

$$\frac{dy}{dx} = \frac{1}{2\sqrt{4+x^2}} \cdot 2x = \frac{x}{\sqrt{4+x^2}}$$

$$\underline{\int_{-1}^{2} \sqrt{1 + \frac{x^2}{4+x^2}}\, dx}$$

7 4. An elevator with mass 5000 kg is sitting on the first floor of a building. The elevator is lifted by a cable with mass 5 kilograms per metre of length. The length of cable from elevator to pulley at the top of the elevator shaft is 90 metres. Set up, but do NOT evaluate, a definite integral to find the work done to lift the elevator and cable a total distance of 30 metres from its present position.



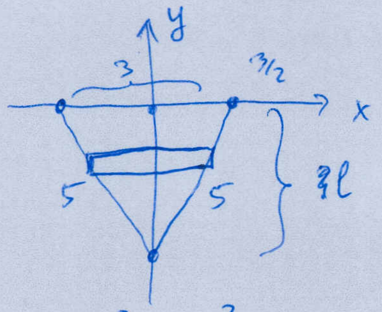
let x be the height (in meters) of the elevator above the original position $(x=0)$

$$m_{ele}(x) = 5000 \quad kg$$

$$m_{cable}(x) = \underbrace{(90-x)}_{\text{length}} \cdot 5 \quad kg$$

$$\underline{\int_{0}^{30} (5000 + (90-x)\cdot 5)\, g\, dx}$$

7 5. A plate is in the shape of an isosceles triangle with equal sides of length 5 metres and the third side of length 3 metres. It is suspended vertically in water with its shortest side in the surface of the water. Set up, but do NOT evaluate, a definite integral to find the force due to the water on one side of the plate.



Put the origin in the midpoint of the shortest side, $y=0$ - surface.

equation of ~~the~~ right side: $y = \frac{3}{2\ell}x - \ell$,

where ℓ is the height of the triangle

$\ell^2 + \left(\frac{3}{2}\right)^2 = 5^2$

$\ell = \sqrt{25 - \frac{9}{4}} = \frac{\sqrt{91}}{2}$

$y + \ell = \frac{3}{2\ell}x$, $x = \frac{2\ell}{3}y + \frac{2\ell^2}{3}$

$x = \sqrt{91}\,y + \frac{91}{6}$

$$\int_{-\frac{\sqrt{91}}{2}}^{0} 1000 \cdot g \cdot (-y) \cdot 2\left(\sqrt{91}\,y + \frac{91}{6}\right) dy$$

6 6. A plate with constant mass per unit area ρ is bounded by the curves

$$x = y^2 - 4, \qquad x + 2y = 4.$$

Set up, but do NOT evaluate, a definite integral to find the first moment of the plate about the y-axis.

This topic is not covered in our test 1.