

University of Manitoba  
Department of Mechanical and Manufacturing Engineering  
**ENG 1460 Introduction to Thermal Sciences (W11)**

A01, A02, A03

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Term Test # 2

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Duration: **100** minutes

1. You are permitted to use the textbook for the course, a one-page double-sided aide sheet, and a calculator.
2. Ask for clarification if any problem statement is unclear.
3. Clear, systematic solutions are required. **Show your work.** Marks will not be assigned for problems that require unreasonable (in the opinion of the instructor(s)) effort for the marker to decipher.
4. Use linear interpolation in the property tables as necessary.
5. Keep 5 significant figures in intermediate calculations, and use 4 or 5 significant figures in final answers. For temperature, keep two decimal places in your final answer.
6. There are **three** problems on this test. The weight of each problem is indicated. The test will be marked out of **70**.

Values

1. A 40 L rigid tank contains refrigerant 134a (R-134a) and is stored in the University Fine Arts Studio. The tank has a pressure relief valve that opens at 1.6 MPa. At room temperature ( $T_1 = 20^\circ\text{C}$ ), the R-134a has a specific volume of  $0.02 \text{ m}^3/\text{kg}$  (State 1). A fire breaks out in the art studio due to careless smoking and the tank heats up until the pressure relief valve just opens (State 2).
 

(a) At State 2, determine the specific internal energy,  $u_2$ , in kJ/kg.

(b) Determine the change in specific internal energy from State 1 (*i.e.*, at the initial room conditions) to State 2, in kJ/kg.

(c) For the process from State 1 to State 2, determine the work done by the R-134a,  ${}_1W_2$ , in kJ, and the heat transfer to the R-134a,  ${}_1Q_2$ , in kJ.
2. A control mass system contains 244.0 L of chlorine gas ( $R = 0.11726 \text{ kJ/kg} \cdot \text{K}$ ). Initially, the chlorine is at a pressure of 1460 kPa and a temperature of 1600 K. The system then undergoes two quasi-equilibrium processes, one after the other. In the first process (State 1 to State 2), the chlorine is compressed in a polytropic process until the volume is 152.5 L. At State 2 the temperature is 800 K and the pressure is 1168 kPa. In the second process (State 2 to State 3), the system is expanded isothermally (*i.e.*, at constant temperature) until the volume reaches the original volume (*i.e.*,  $V_3 = V_1$ ). Treat the chlorine as an ideal gas.
 

(a) Determine the polytropic exponent,  $n$ , for the first process.

(b) Determine the final pressure,  $P_3$ , in kPa.

(c) Can the second process (State 2 to State 3) be described as a polytropic process? If so, determine the polytropic exponent  $n$ .

(d) Calculate the **total** boundary work done by the system,  ${}_1W_3$ , in kJ.

(e) Show the two processes on a  $P$ - $V$  (pressure–volume) diagram. Clearly label the three state points, the two process paths, pressure and volume values, and the relevant constant  $T$  lines. Indicate the area representing the **total** work done,  ${}_1W_3$ .

3. A very long frictionless piston-cylinder and linear-spring assembly containing water is shown in Figure 1. The inner cross-sectional area of the cylinder is  $A = 0.05 \text{ m}^2$ . The distance from the bottom surface of the cylinder to the bottom surface of the piston is denoted as  $y$ . At the initial state (State 1), the piston is detached from spring, the water is a saturated liquid at  $T_1 = 125^\circ\text{C}$ , and the value of  $y$  is  $0.02 \text{ m}$  (*i.e.*,  $y_1 = 0.02 \text{ m}$ ). Then, heat is transferred to the water from the outside. At the exact moment when the piston just begins to touch the spring, the state of the water is referred to as State 2. The quality of the water at State 2 is  $x_2 = 0.15$ . The system continues to receive heat until the final state (State 3) is reached. At State 3, the temperature of the water is  $T_3 = 200^\circ\text{C}$  and the value of  $y$  is twice that at the State 2 (*i.e.*,  $y_3 = 2y_2$ ). The local gravitational acceleration is  $g = 9.81 \text{ m/s}^2$  and the local atmospheric pressure is  $P_0 = 100 \text{ kPa}$ .

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- 3 (a) Determine the mass of the water,  $m$ , in kg.
- 5 (b) Determine the volume of the water at State 2,  $V_2$ , in  $\text{m}^3$ .
- 5 (c) Determine the pressure of the water at State 3,  $P_3$ , in kPa.
- 6 (d) Determine the **total** work done by the water,  ${}_1W_3$ , in kJ.
- 5 (e) Determine the **total** heat transfer to the water,  ${}_1Q_3$ , in kJ.
- 6 (f) Show all the **state points** and **processes** on a  $P$ - $V$  (pressure-volume, in  $\text{kPa}\text{-}\text{m}^3$ ) diagram. Indicate the area that represents the total work done by the water on the  $P$ - $V$  diagram. Note that all information should be presented on **one single diagram**.
- 5 (g) Determine the spring constant,  $k_s$ , in  $\text{kN/m}$ .

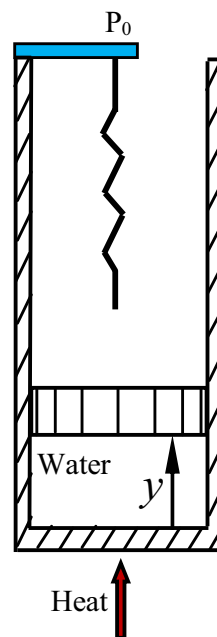


Figure 1: Schematic for Problem 3 (not to scale)

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#1 R134a  $V = 40\text{L} = 0.04\text{ m}^3$   
 $T_1 = 20^\circ\text{C}$   
 $v_1 = 0.02\text{ m}^3/\text{kg}$   
 $P_2 = 1.6\text{ MPa}$

a) Find  $T_2$  AND  $u_2$ 

$v_2 = v_1 = 0.02\text{ m}^3/\text{kg}$  RIGID CONTAINER  
 @  $P_2 = 1.6\text{ MPa}$ ,  $v_g = 0.01146\text{ m}^3/\text{kg}$  TABLE B.5.2  
 $v_2 > v_g$  @  $1.6\text{ MPa}$   $\therefore$  superheated steam

@  $P_2 = 1.6\text{ MPa}$   $v_2 = 0.02\text{ m}^3/\text{kg}$  is between  $T = 150^\circ\text{C}$  &  $160^\circ\text{C}$   
 TABLE B.5.2

$v$ [ $\text{m}^3/\text{kg}$ ]	$u$ [ $\text{kJ/kg}$ ]
0.01949	500.24
0.02	$u_2$
0.02013	510.33

$$\frac{u_2 - 500.24}{510.33 - 500.24} = \frac{0.02 - 0.01949}{0.02013 - 0.01949}$$

$$u_2 = 508.28\text{ kJ/kg}$$

b)  $T_1 = 20^\circ\text{C}$   $v_1 = 0.02\text{ m}^3/\text{kg}$   
 $v_f = 0.000817\text{ m}^3/\text{kg}$ ,  $v_g = 0.03606$  TABLE B.5.1  
 $v_f < v_1 < v_g$   $\therefore$  saturated mixture  
 $v_{fs} = 0.03529\text{ m}^3/\text{kg}$

$$x = \frac{v_1 - v_f}{v_{fg}}$$

$$= \frac{0.02 - 0.000817}{0.03529}$$

$$x = 0.544353$$

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$$\begin{aligned}
 u_1 &= x u_{fg} + u_f \\
 &= 0.544353 (162.16) + 227.03 \quad \text{TABLE B.5.1} \\
 &= 315.30 \text{ kJ/kg} \quad @ T_1 = 20^\circ\text{C}
 \end{aligned}$$

$$\begin{aligned}
 \Delta u &= u_2 - u_1 \\
 &= 508.28 - 315.30 \\
 &= 192.98 \text{ kJ/kg}
 \end{aligned}$$

c)  ${}_1W_2 = 0$  CONSTANT VOLUME PROCESS

$${}_1Q_2 - \cancel{{}_1W_2}^0 = m(u_2 - u_1) + \cancel{\Delta KE}^{0, \text{no motion}} + \cancel{\Delta PE}^{0, \text{no change in centre of gravity}}$$

$$\therefore {}_1Q_2 = m(u_2 - u_1)$$

$$\begin{aligned}
 m &= \frac{V}{v_1} \\
 &= \frac{0.04 \text{ m}^3}{0.02 \text{ m}^3/\text{kg}} \\
 m &= 2.000 \text{ kg}
 \end{aligned}$$

$$\begin{aligned}
 {}_1Q_2 &= m(\Delta u) \\
 &= 2.000 \times 192.98 \\
 &= 385.96 \text{ kJ}
 \end{aligned}$$

2-1/2

2. Ideal gas: Chlorine

State 1

$$P_1 = 1460 \text{ [kPa]}$$

$$T_1 = 1600 \text{ [K]}$$

$$V_1 = 0.2440 \text{ [m}^3\text{]}$$

State 2

$$P_2 = 1168 \text{ [kPa]}$$

$$T_2 = 800 \text{ [K]}$$

$$V_2 = 0.1525 \text{ [m}^3\text{]}$$

State 3

$$P_3 = ?$$

$$T_3 = T_2$$

$$V_3 = V_1$$

$$(a) \quad n = \frac{\ln\left(\frac{P_1}{P_2}\right)}{\ln\left(\frac{V_2}{V_1}\right)} = \frac{\ln\left(\frac{1460}{1168}\right)}{\ln\left(\frac{0.1525}{0.2440}\right)} = -0.47477$$

Alternative Methods (all based on combining  $PV^n = C$   
and the ideal gas equation  
 $PV = mRT$ )

$$P_1 V_1^n = P_2 V_2^n$$

$$P_1 = \frac{mRT_1}{V_1} \quad P_2 = \frac{mRT_2}{V_2} \quad \Rightarrow \quad T_1 V_1^{1-n} = T_2 V_2^{1-n}$$

$$\Rightarrow n = 1 + \frac{\ln\left(\frac{T_2}{T_1}\right)}{\ln\left(\frac{V_1}{V_2}\right)} = 1 + \frac{\ln\left(\frac{800}{1600}\right)}{\ln\left(\frac{0.2440}{0.1525}\right)} = -0.47477$$

$$P_1 V_1^n = P_2 V_2^n$$

$$V_1 = \frac{mRT_1}{P_1} \quad V_2 = \frac{mRT_2}{P_2} \quad \Rightarrow \quad \left(\frac{T_2}{T_1}\right) = \left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}}$$

$$\Rightarrow n = \frac{1}{\left(1 - \frac{\ln\left(\frac{T_2}{T_1}\right)}{\ln\left(\frac{P_2}{P_1}\right)}\right)} = \frac{1}{\left(1 - \frac{\ln\left(\frac{800}{1600}\right)}{\ln\left(\frac{1168}{1460}\right)}\right)} = -0.47477$$

$2-2/2$ 

$$(b) \quad P_3 = P_2 \left( \frac{V_2}{V_3} \right) = 1168 \left( \frac{0.1525}{0.2440} \right) = 730 \text{ [kPa]}$$

(c) Process 2 to 3 is isothermal (constant T)

Therefore,  $PV = nRT = \text{constant}$

$\therefore PV = C \rightarrow$  this is polytropic with  $n=1$

$$(d) \quad {}_1\bar{W}_3 = {}_1\bar{W}_2 + {}_2\bar{W}_3$$

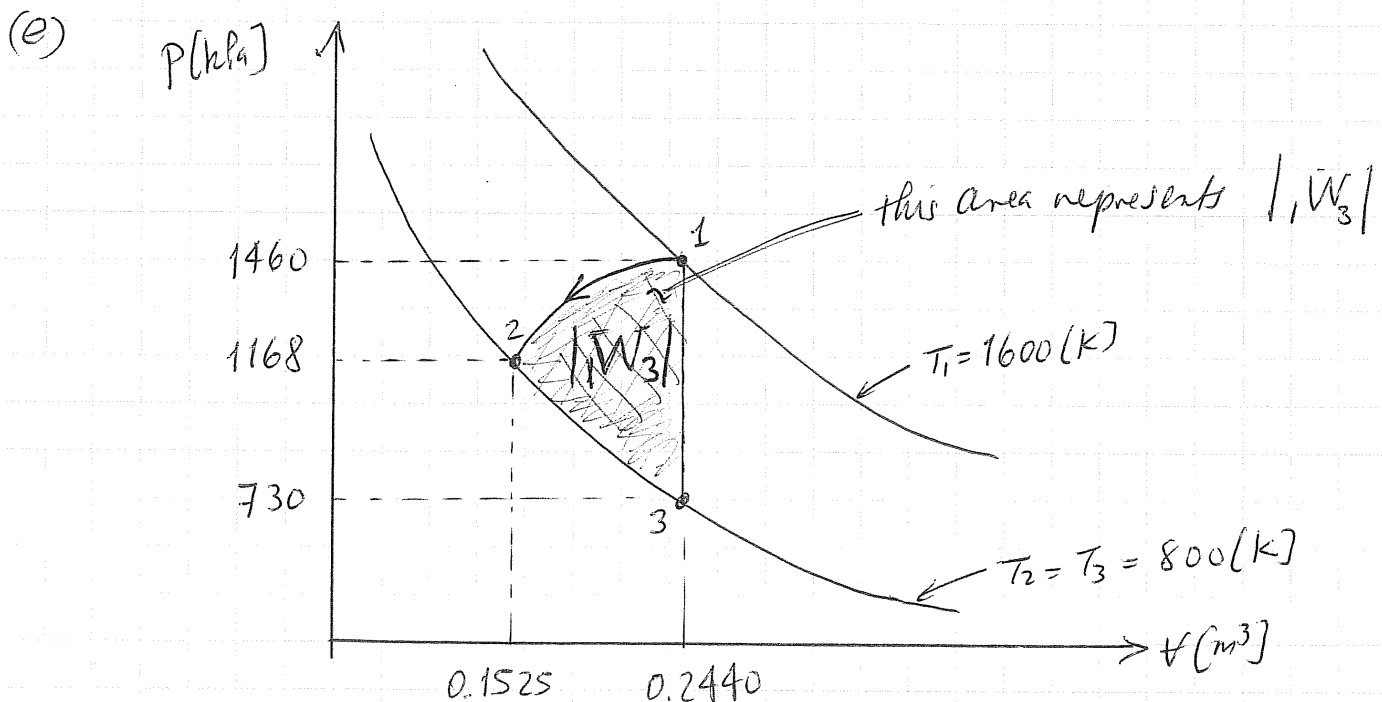
$${}_1\bar{W}_2 = \frac{(P_2 V_2 - P_1 V_1)}{(1-n)} = \frac{(1168)(0.1525) - (1460)(0.2440)}{(1 - -0.47477)}$$

$${}_1\bar{W}_2 = -120.78 \text{ [kJ]}$$

$${}_2\bar{W}_3 = P_2 V_2 \ln \left( \frac{V_3}{V_2} \right) = (1168)(0.1525) \ln \left( \frac{0.2440}{0.1525} \right)$$

$${}_2\bar{W}_3 = 83.72 \text{ [kJ]}$$

$${}_1\bar{W}_3 = -120.78 + 83.72 = -37.06 \text{ [kJ]} \leftarrow$$



(a) Table B.1.1Saturated liquid, at  $T_1 = 125^\circ\text{C}$ ,

$$P_1 = P_{\text{sat}} = 232.1 \text{ kPa}, V_1 = V_f = 0.001065 \text{ m}^3/\text{kg}, u_1 = u_f = 524.72 \text{ kJ/kg}$$

$$V_1 = A \cdot y_1 = 0.05 \times 0.02 = 0.001 \text{ (m}^3\text{)}$$

$$m = V_1 / V_1 = 0.001 / 0.001065 = 0.9390 \text{ (kg)}$$

(b) Constant Pressure process,  $x_2 = 0.15$  (saturated mixture)

$$\therefore P_2 = P_1 = 232.1 \text{ kPa}, T_2 = T_1 = 125^\circ\text{C}, \text{ From Table B.1.1}$$

$$V_f = 0.001065 \text{ m}^3/\text{kg}, V_{fg} = 0.76953 \text{ m}^3/\text{kg}$$

$$u_f = 524.72 \text{ kJ/kg}, u_{fg} = 2009.91 \text{ kJ/kg}$$

$$V_2 = V_f + x_2 V_{fg} = 0.001065 + 0.15 \times 0.76953 = 0.116495 \text{ (m}^3/\text{kg)}$$

$$u_2 = u_f + x_2 u_{fg} = 524.72 + 0.15 \times 2009.91 = 826.2065 \text{ (kJ/kg)}$$

$$V_2 = m V_2 = 0.9390 \times 0.116495 = 0.1094 \text{ (m}^3\text{)}$$

(c) B/C  $y_3 = 2y_2$ ,  $V_3 = 2V_2 = 2 \times 0.116495 = 0.232989 \text{ (m}^3/\text{kg)}$ .

$$T_3 = 200^\circ\text{C} \text{ (Given)}$$

$$\text{From Table B.1.1, at } T_3 = 200^\circ\text{C}, V_g = 0.12736 \text{ m}^3/\text{kg}$$

$$V_3 > V_g$$

 $\therefore$  Superheated Vapour at state 3

Use Table B.1.3, Interpolation with respect to  $V$ ,  
between 800 kPa & 1000 kPa

At  $T_3 = 200^\circ\text{C}$ 

$V [\text{m}^3/\text{kg}]$	$P [\text{kPa}]$	$u [\text{kJ/kg}]$
0.26080	800	2630.61
0.23298	$P_3$	$u_3$
0.20596	1000	2621.90

$$P_3 = 800 + \frac{0.23298 - 0.26080}{0.20596 - 0.26080} (1000 - 800)$$

$$= 901.426 \text{ (kPa)}$$

$$u_3 = 2630.61 + \frac{0.23298 - 0.26080}{0.20596 - 0.26080} (2621.90 - 2630.61)$$

$$= 2626.19 \text{ (kJ/kg)}$$

$$(d) \quad {}_1W_3 = {}_1W_2 + {}_2W_3$$

Process 1  $\rightarrow$  2: Constant Pressure

$${}_1W_2 = P_1 (V_2 - V_1) = 232.1 \times (0.1094 - 0.00) = 25.156 \text{ (kJ)}$$

Process 2  $\rightarrow$  3: Linear-Spring (Constant slope on P-V diagrams)

$$\begin{aligned} {}_2W_3 &= \frac{1}{2} (P_2 + P_3) (V_3 - V_2) \\ &= \frac{1}{2} (232.1 + 901.42) \times (0.21877 - 0.1094) \\ &= 61.995 \text{ (kJ)} \end{aligned}$$

$$\therefore {}_1W_3 = {}_1W_2 + {}_2W_3 = 25.156 + 61.995 = 87.151 \text{ (kJ)}$$

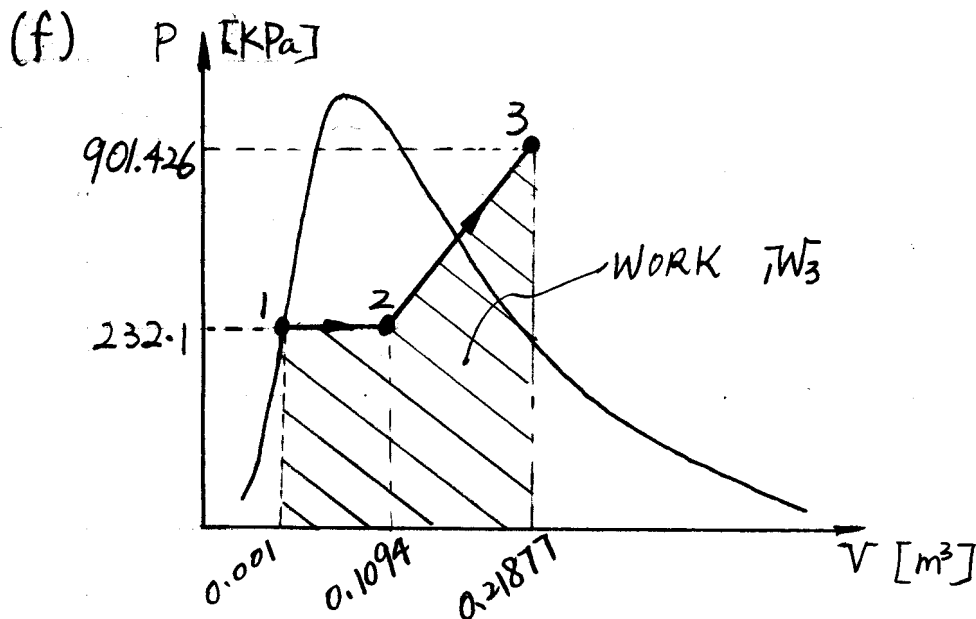
$$(e) \quad Q_3 = \Delta U_3 + {}_1W_3$$

$$= m(u_3 - u_1) + {}_1W_3$$

$$= 0.9390 \times (2626.19 - 524.72) + 87.151$$

$$= 2060.365 \text{ (kJ)}$$





(g)  $V_3 = m V_3 = 0.9390 \times 0.232989 = 0.21877 \text{ (m}^3\text{)}$

$$P_3 = P_2 + \frac{K_S}{A^2} (V_3 - V_2)$$

$$\therefore K_S = (P_3 - P_2) \cdot A^2 / (V_3 - V_2)$$

$$= (901.426 - 232.1) \times 0.05^2 / (0.21877 - 0.1094)$$

$$= 15.298 \text{ (kN/m)}$$

Second Method:

$$y_2 = V_2 / A = 0.1094 / 0.05 = 2.188 \text{ (m)}$$

$$y_3 = 2 y_2 = 2 \times 2.188 = 4.375 \text{ (m)}$$

$$P_3 = P_2 + \frac{K_S}{A} (y_3 - y_2)$$

$$\therefore K_S = (P_3 - P_2) A / (y_3 - y_2)$$

$$= (901.426 - 232.1) \times 0.05 / (4.375 - 2.188)$$

$$= 15.298 \text{ (kN/m)}$$