1 (a) not defined $A_{2\times 3} \neq C_{2\times 2}$

(b) defined

$$B^T = \begin{pmatrix} 0 & 1 & -1 \\ -2 & 2 & 3 \end{pmatrix} \qquad A - B^T = \begin{pmatrix} 3 & -3 & 0 \\ 4 & -3 & -3 \end{pmatrix}$$

$$(A - B^T)F = \begin{pmatrix} 4 \cdot 3 - 3 \\ 4 \cdot 4 - 3 + 9 \end{pmatrix} = \begin{pmatrix} 9 \\ 22 \end{pmatrix}$$

(c) defined $ED = \begin{pmatrix} 3-2 & 6+2-1 & -3-2+1 \end{pmatrix} = \begin{pmatrix} 1 & 7 & -4 \end{pmatrix}$

(d) ~~not defined~~ $\cancel{F}_{3\times 1}$ $\cancel{E}$

defined

$$FE = \begin{pmatrix} 12 & 8 & 4 \\ 3 & 2 & 1 \\ -9 & -6 & -3 \end{pmatrix}$$

(e) defined $F^T E^T = \begin{pmatrix} 4 & 1 & -3 \end{pmatrix} \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix} = (12 + 2 - 3) = (11)$

(f) defined

$$C^2 = \begin{pmatrix} 4-1 & -2-3 \\ 2+3 & -1+9 \end{pmatrix} = \begin{pmatrix} 3 & -5 \\ 5 & 8 \end{pmatrix}$$

2. (a) Not in R.E.F — 3rd row's leftmost non zero entry is not 1.

$R_3 \to \frac{1}{2} R_3$

(b)

$$\xrightarrow{R_3 \to \frac{1}{2}R_3} \begin{pmatrix} 1 & 1 & 0 & 2 & 3 & 4 \\ 0 & 1 & 3 & 2 & 1 & -1 \\ 0 & 0 & 0 & 1 & -1 & 4 \end{pmatrix} \xrightarrow[R_1 \to R_1 - 2R_3]{R_2 \to R_2 - 2R_3} \begin{pmatrix} 1 & 1 & 0 & 0 & 5 & -4 \\ 0 & 1 & 3 & 0 & 3 & -9 \\ 0 & 0 & 0 & 1 & -1 & 4 \end{pmatrix} \xrightarrow{R_1 \to R_1 - R_2}$$

$$\begin{pmatrix} 1 & 0 & -3 & 0 & 2 & 5 \\ 0 & 1 & 3 & 0 & 3 & -9 \\ 0 & 0 & 0 & 1 & -1 & 4 \end{pmatrix} \qquad \text{R.R.E.F.}$$

3. $\begin{pmatrix} 1 & 1 & -2 & | & -2 \\ 0 & 1 & 2 & | & 3 \\ 1 & 2 & 0 & | & 1 \end{pmatrix}$ $\xrightarrow{R_3 \to R_3 - R_1}$ $\begin{pmatrix} 1 & 1 & -2 & | & -2 \\ 0 & 1 & 2 & | & 3 \\ 0 & 1 & 2 & | & 3 \end{pmatrix}$ $\xrightarrow{R_3 \to R_3 - R_2}$

$\begin{pmatrix} 1 & 1 & -2 & | & -2 \\ 0 & 1 & 2 & | & 3 \\ 0 & 0 & 0 & | & 0 \end{pmatrix}$

z - free variable

$$y + 2z = 3$$
$$y = -2z + 3$$

$$x + y - 2z = -2$$
$$x = -y + 2z - 2 = 2z - 3 + 2z - 2$$
$$= 4z - 5$$

<u>Ans</u> : $x = 4z - 5$ $y = -2z + 3$

4. $\begin{pmatrix} 1 & 3 & 3 & | & 0 \\ 2 & 7 & 2 & | & 6 \\ -1 & 1 & -4 & | & 9 \end{pmatrix}$ $\xrightarrow[R_3 \to R_3 + R_1]{R_2 \to R_2 - 2R_1}$ $\begin{pmatrix} 1 & 3 & 3 & | & 0 \\ 0 & 1 & -4 & | & 6 \\ 0 & 4 & -1 & | & 9 \end{pmatrix}$ $\xrightarrow{R_3 \to R_3 - 4R_2}$

$\begin{pmatrix} 1 & 3 & 3 & | & 0 \\ 0 & 1 & -4 & | & 6 \\ 0 & 0 & 15 & | & -15 \end{pmatrix}$ $\xrightarrow{R_3 \to \frac{1}{15} R_3}$ $\begin{pmatrix} 1 & 3 & 3 & | & 0 \\ 0 & 1 & -4 & | & 6 \\ 0 & 0 & 1 & | & -1 \end{pmatrix}$ $\xrightarrow[R_2 \to R_2 + 4R_3]{R_1 \to R_1 - 3R_3}$

$\begin{pmatrix} 1 & 3 & 0 & | & 3 \\ 0 & 1 & 0 & | & 2 \\ 0 & 0 & 1 & | & -1 \end{pmatrix}$ $\xrightarrow{R_1 \to R_1 - 3R_2}$ $\begin{pmatrix} 1 & 0 & 0 & | & -3 \\ 0 & 1 & 0 & | & 2 \\ 0 & 0 & 1 & | & -1 \end{pmatrix}$

<u>Ans</u> : $x = -3$ $y = 2$ $z = -1$

5.
$$2x \quad -3z \quad = 2$$
$$-x +2y \quad +4t = 1$$
$$x +2y + \cancel{4}z \quad = 0$$

$$\begin{pmatrix} 2 & 0 & -3 & 0 & | & 2 \\ -1 & 2 & 0 & 4 & | & 1 \\ 1 & 2 & 1 & 0 & | & 0 \end{pmatrix} \xrightarrow{R_1 \leftrightarrow R_3} \begin{pmatrix} 1 & 2 & 1 & 0 & | & 0 \\ -1 & 2 & 0 & 4 & | & 1 \\ 2 & 0 & -3 & 0 & | & 2 \end{pmatrix}$$

$$\xrightarrow[R_3 \to R_3 - 2R_1]{R_2 \to R_2 + R_1} \begin{pmatrix} 1 & 2 & 1 & 0 & | & 0 \\ 0 & 4 & 1 & 4 & | & 1 \\ 0 & -4 & -5 & 0 & | & 2 \end{pmatrix} \xrightarrow{R_3 \to R_3 + R_2} \begin{pmatrix} 1 & 2 & 1 & 0 & | & 0 \\ 0 & 4 & 1 & 4 & | & 1 \\ 0 & 0 & -4 & 4 & | & 3 \end{pmatrix}$$

$$\xrightarrow[R_3 \to \frac{1}{4} R_3]{R_2 \to \frac{1}{4} R_2} \begin{pmatrix} 1 & 2 & 1 & 0 & | & 0 \\ 0 & 1 & \frac{1}{4} & 1 & | & \frac{1}{4} \\ 0 & 0 & 1 & -1 & | & -\frac{3}{4} \end{pmatrix} \xrightarrow[R_1 \to R_1 - R_3]{R_2 \to R_2 - \frac{1}{4} R_3}$$

$$\begin{pmatrix} 1 & 2 & 0 & \cancel{} 1 & | & \frac{3}{4} \\ 0 & 1 & 0 & \frac{5}{4} & | & \frac{7}{16} \\ 0 & 0 & 1 & -1 & | & -\frac{3}{4} \end{pmatrix} \xrightarrow{R_1 \to R_1 - 2R_2} \begin{pmatrix} 1 & 0 & 0 & \cancel{1} -\frac{3}{2} & | & -\frac{1}{8} \\ 0 & 1 & 0 & \frac{5}{4} & | & \frac{7}{16} \\ 0 & 0 & 1 & -1 & | & -\frac{3}{4} \end{pmatrix}$$

Ans : $x = \frac{3}{2}t - \frac{1}{8} \qquad y = -\frac{5}{4}t + \frac{7}{16} \qquad z = t - \frac{3}{4}.$