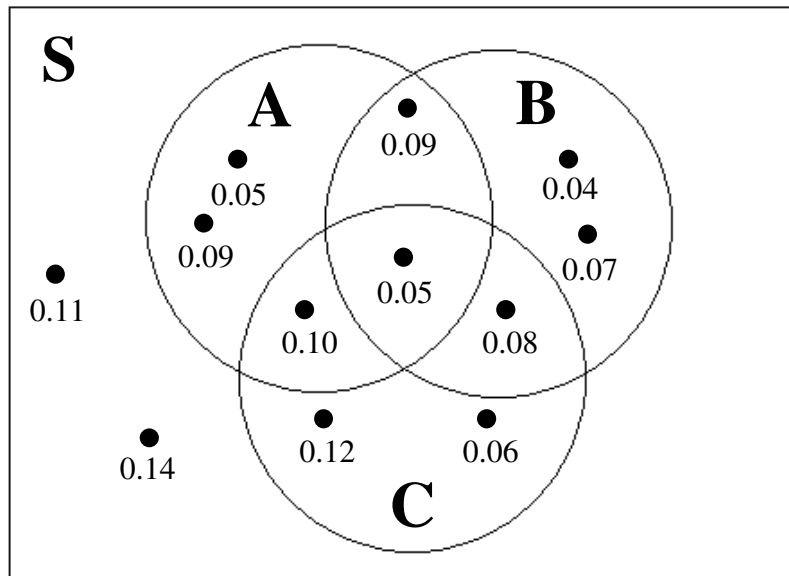


Unit 5 Assignment

1. Consider the sample space and events shown in the Venn diagram below. Calculate the probabilities of the events:

- | | | | |
|--------------------|----------------------------|------------------------------|------------------------------|
| (a) $P(C)$ | (e) $P(B' \cup C)$ | (i) $P(A C)$ | (m) $P(A \cap B A \cup B)$ |
| (b) $P(A \cap B)$ | (f) $P((A \cup B) \cap C)$ | (j) $P(B A \cup C)$ | (n) $P(A' \cap C B')$ |
| (c) $P(B \cup C)$ | (g) $P(A \cup (B \cap C))$ | (k) $P(B A \cap C)$ | |
| (d) $P(A' \cap C)$ | (h) $P((A' \cup B)')$ | (l) $P(A A \cup B \cup C)$ | |



2. Suppose your friend has a fair coin and you have an unfair coin that lands on Tails with probability 0.6. You and your friend will play a game of chance with the coins, and the loser will have to pay the winner one dollar. You will take turns tossing your coins. You let your friend go first. As soon as someone gets a Tail, the game is over and that person wins.
- (a) Your friend complains that you have a better chance than she does, because your coin has a greater chance of landing on Tails. Calculate the probability that your friend wins the game to show her that she actually has the advantage.
- (b) What would have to be the probability of flipping Tails on your coin in order for this to be a fair game?

3. A hockey player compiles the following information:

- He scores a goal in 53% of his games.
- He gets a penalty in 40% of his games.
- In 74% of his games, he scores a goal or his team wins.
- In 20% of his games, he scores a goal and gets a penalty.
- In 39% of his games, he scores a goal and his team wins.
- In 24% of his games, he gets a penalty and his team wins.
- In 12% of his games, he scores a goal, gets a penalty and his team wins.

- (a) In any given game, what is the probability that he scores a goal or gets a penalty?
- (b) What percentage of their games does his team win?
- (c) What is the probability that the player's team wins if he doesn't score a goal?
- (d) Are any of the three events (goal, penalty, win) pairwise independent?
- (e) Show that, for any events A, B and C,

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C).$$

- (f) What is the probability that the player does not score a goal, does not get a penalty, and his team loses?

4. Roger Federer is playing Andy Roddick in a best-of-five match in the final of a Grand Slam tennis tournament. (The first player to win three sets is the winner). Being the higher ranked player, suppose that Federer has a 60% chance of winning any given set and that the outcome of any set is independent of any other.

- (a) The outcome of interest is the winner of each set that is played. List the complete sample space of possible outcomes.
- (b) What is the probability that the match lasts five sets?
- (c) What is the probability that Roddick wins the match?

5. (a) Three components with reliabilities r_1 , r_2 and r_3 are placed in series. A fourth component, with reliability r_4 , can be placed in parallel with any one of these three components. Show that, if $r_1 < r_2 < r_3$, then the reliability will be maximized if the fourth component is placed in parallel with the first component, regardless of the value of r_4 .
- (b) Calculate the reliability of the following system, where the values inside the boxes represent the component reliabilities:

