Mean Value Theorem

3.14.1

2. (a) Yes. Polynomials are continuous and differentiable everywhere.
(b) NO. Not defined at $x=\frac{\pi}{2}$, hence, not continuous at $x=\frac{\pi}{2}$.
(c) Yes. $\cos x$ is cont. & diff. everywhere.

a) $f'(x) = 4x - 1$

$$\frac{f(2)-f(-3)}{2-(-3)} = \frac{4-19}{5} = \frac{-14}{5}$$

$$f'(c) = \frac{-14}{5}$$

$$4c - 1 = \frac{-14}{5} \qquad 4c = \frac{-9}{5}$$

$$c = \frac{-9}{20} \in (-3,2)$$

(c) $f'(x) = -\sin x$

$$\frac{f(\frac{5\pi}{2}) - f(-\frac{\pi}{2})}{\frac{5\pi}{2} - (-\frac{\pi}{2})} = \frac{0-0}{3\pi} = 0$$

$$f'(c) = 0 = -\sin c$$

$$\sin c = 0, \qquad c \in \left(-\frac{\pi}{2}, \frac{3\pi}{2}\right)$$

$$c = 0, \quad c = \pi$$

Corrections to the above:

Part (a): 4-19=-15 leading to c=-1/2 as answer.

Part (c): the upper bound of the interval is 5pi/2, not 3pi/2, leading to 2pi as a solution in addition to the two listed solutions.