

MATH 2130 Summer Evening 2013 Problem Workshop 3

1. For the following limits, determine whether or not the limit exists (with justification) and if the limit exists, determine its value.

(a) $\lim_{(x,y) \rightarrow (0,0)} \frac{x^2 - xy + y^2}{x^2 + 2y^2}$

(b) $\lim_{(x,y) \rightarrow (0,0)} \frac{x^3 y}{x^6 + y^3}$

(c) $\lim_{(x,y) \rightarrow (2,-3)} \frac{x^2 - 4x - y^2 - 6y - 5}{x^2 - 4x + y^2 + 6y + 13}$

(d) $\lim_{(x,y) \rightarrow (3,2)} \frac{\sin(2x-3y)}{2x-3y}$ (where the function is 1 along the path $2x - 3y = 0$.)

(e) $\lim_{(x,y) \rightarrow (0,0)} \frac{x^4 y^4}{x^4 + y^4}$

(f) $\lim_{(x,y) \rightarrow (0,1)} \arctan \left| \frac{y}{x} \right|$

(g) $\lim_{(x,y) \rightarrow (0,1)} \arctan \left(\frac{y}{x} \right)$

(h) $\lim_{(x,y) \rightarrow (0,1)} \arctan \left(\frac{x}{y} \right)$

2. Show that the function $f(x, y) = 3x^2 + y^2 \cos \left(\frac{2x}{y} \right)$ satisfies the equation

$$x \frac{\partial f}{\partial x} + y \frac{\partial f}{\partial y} = 2f(x, y).$$

Answers:

1. For the following limits, determine whether or not the limit exists (with justification) and if the limit exists, determine its value.

(a) DNE

(b) DNE

(c) DNE

(d) 1

(e) 0

(f) $\pi/2$

(g) DNE

(h) 0