EXERCISES FOR CHAPTER 7: Differential Equations

1. Prove that the equation $\frac{dy}{dx} = F(\frac{y}{x})$ becomes the separable equation $\frac{dv}{F(v)-v} = \frac{dx}{x}$ after the substitution $y = vx$.

Solution

$y = vx \Rightarrow \frac{dy}{dx} = \frac{dv}{dx}x + v$. Hence $\frac{dy}{dx} = F(\frac{y}{x})$ becomes $\frac{dv}{dx}x + v = F(v)$, i.e. the separable equation

$$\frac{dv}{dx}x = F(v) - v$$

$$\frac{dv}{F(v)-v} = \frac{dx}{x}$$

2. Find the solution to $(x+y)\frac{dy}{dx} = y$ where x and y are restricted to be positive and $y = 1$ when $x = 1$.

Solution

$y = vx \Rightarrow \frac{dy}{dx} = \frac{dv}{dx}x + v$. Hence

$$(x+vx)(\frac{dv}{dx}x + v) = vx$$

$$x(1+v)(\frac{dv}{dx}x + v) = v$$

$$(1+v)(\frac{dv}{dx}x + v) = v$$

$$(1+v)\frac{dv}{dx}x = -v^2$$

$$\frac{(1+v)}{v^2}dv = -\frac{dx}{x}$$

$$\int \frac{(1+v)}{v^2}dv = -\int \frac{dx}{x}$$

$$-\frac{1}{v} + \ln v = -\ln x + C$$

$$-\frac{x}{y} + \ln\frac{y}{x} = -\ln x + C \Rightarrow -1 + 0 = 0 + C \Rightarrow C = -1$$

Hence $-\frac{x}{y} + \ln\frac{y}{x} = -\ln x - 1 \quad \Rightarrow \quad \ln y = \frac{x}{y} - 1$.

3. Find the solution to $x\frac{dy}{dx} = x + y$ given that $y = \ln 2$ when $x = 1$.

Solution

$y = vx \Rightarrow \frac{dy}{dx} = \frac{dv}{dx}x + v$. Hence

$$x(\frac{dv}{dx}x + v) = x + xv$$
$$\frac{dv}{dx}x + v = 1 + v$$
$$x\frac{dv}{dx} = 1$$
$$dv = \frac{dx}{x}$$
$$v = \ln|x| + \ln C$$
$$\frac{y}{x} = \ln(C|x|)$$
$$y = x\ln(C|x|)$$

Then $\ln 2 = \ln(C) \Rightarrow C = 2$. The solution is thus $y = x\ln(2|x|)$.

4. Find the solution to (a) $-y\frac{dy}{dx} = x + 2y$ and (b) $x\frac{dy}{dx} = x + 2y$.

Solution

$y = vx \Rightarrow \frac{dy}{dx} = \frac{dv}{dx}x + v$. Hence,

$$-vx(\frac{dv}{dx}x + v) = x + 2vx$$
$$-v(\frac{dv}{dx}x + v) = 1 + 2v$$
$$-vx\frac{dv}{dx} - v^2 = 1 + 2v$$
$$-vx\frac{dv}{dx} = 1 + 2v + v^2$$
$$-\frac{v}{(1+v)^2}dv = \frac{dx}{x}$$

To do the v integral, write

$$\begin{aligned}\int\frac{v}{(1+v)^2}dv &= \int\frac{1+v-1}{(1+v)^2}dv \\ &= \int\frac{1+v}{(1+v)^2}dv - \int\frac{1}{(1+v)^2}dv \\ &= \ln(1+v) + \frac{1}{1+v}\end{aligned}$$

so that

$$-\ln|(1+v)| - \frac{1}{1+v} = \ln|x| + \ln C$$

$$-\ln\left|(1+\frac{y}{x})\right| - \frac{1}{1+\frac{y}{x}} = \ln C|x|$$

$$-\ln\left|(\frac{y+x}{x})\right| - \frac{x}{x+y} = \ln C|x|$$

or $\ln\left|(\frac{y+x}{x})\right| + \frac{x}{x+y} = -\ln C|x|$.

(b) $y = vx \Rightarrow \frac{dy}{dx} = \frac{dv}{dx}x + v$. Hence,

$$x(\frac{dv}{dx}x + v) = x + 2vx$$

$$\frac{dv}{dx}x + v = 1 + 2v$$

$$x\frac{dv}{dx} = 1 + v$$

$$\frac{1}{1+v}dv = \frac{dx}{x}$$

$$\ln|(1+v)| = \ln C|x|$$

$$\ln\left|1+\frac{y}{x}\right| = \ln C|x|$$

$$y = x \pm Cx^2$$

5. Find the solution to (a) $(x-y)\frac{dy}{dx} = x + y$ and (b) $x^2\frac{dy}{dx} = x^2 + xy + y^2$.

Solution

(a) $y = vx \Rightarrow \frac{dy}{dx} = \frac{dv}{dx}x + v$. Hence,

$$(x-vx)(\frac{dv}{dx}x+v)=x+vx$$
$$(1-v)(\frac{dv}{dx}x+v)=1+v$$
$$(1-v)x\frac{dv}{dx}=1+v-v(1-v)$$
$$(1-v)x\frac{dv}{dx}=1+v^2$$
$$\frac{(1-v)}{1+v^2}dv=\frac{dx}{x}$$
$$(\frac{1}{1+v^2}-\frac{v}{1+v^2})dv=\frac{dx}{x}$$
$$\arctan v-\frac{1}{2}\ln\left|(1+v^2)\right|=\ln|x|+\ln C$$
$$\arctan\frac{y}{x}-\frac{1}{2}\ln\left|(\frac{x^2+y^2}{x^2})\right|=\ln C|x|$$

This can be simplified to

$$\arctan\frac{y}{x}-\frac{1}{2}\ln(x^2+y^2)+\frac{1}{2}\ln(x^2)=\ln C|x|$$
$$\arctan\frac{y}{x}-\frac{1}{2}\ln(x^2+y^2)+\ln|x|=\ln C|x|$$
$$\arctan\frac{y}{x}-\frac{1}{2}\ln(x^2+y^2)=\ln C$$

(b) $y=vx\Rightarrow\frac{dy}{dx}=\frac{dv}{dx}x+v$. Hence,

$$x^2(\frac{dv}{dx}x+v)=x^2+vx^2+v^2x^2$$
$$(\frac{dv}{dx}x+v)=1+v+v^2$$
$$\frac{dv}{dx}x=1+v^2$$
$$\frac{1}{1+v^2}dv=\frac{dx}{x}$$
$$\arctan v=\ln(C|x|)$$
$$v=\tan(\ln(C|x|))$$
$$y=x\tan(\ln(C|x|))$$

6. Find the solution to $xy\frac{dy}{dx}=x^2-y^2$.

Solution

$y=vx\Rightarrow\frac{dy}{dx}=\frac{dv}{dx}x+v$. Hence,

$$vx^2(\frac{dv}{dx}x+v)=x^2-(vx)^2$$
$$v(\frac{dv}{dx}x+v)=1-v^2$$
$$vx\frac{dv}{dx}=1-2v^2$$
$$\frac{v}{1-2v^2}dv=\frac{dx}{x}$$
$$-\frac{1}{4}\ln(1-2v^2)=\ln|x|+\ln C$$
$$-\frac{1}{4}\ln(1-2\frac{y}{x^2}^2)=\ln(C|x|)$$

Thus, $\ln(1-2\frac{y}{x^2}^2)=-4\ln(C|x|)=\ln\left(\frac{1}{C^4x^4}\right)$, implying $1-2\frac{y}{x^2}^2=\frac{1}{C^4x^4}$ and finally $y=\sqrt{\frac{x^2}{2}-\frac{k}{x^2}}$ where k is a constant.

7. Find the solution to $x(y-x)\frac{dy}{dx}=y(x+y)$ where x and y are positive.

Solution

$y=vx\Rightarrow\frac{dy}{dx}=\frac{dv}{dx}x+v$. Hence,

$$(\frac{y}{x}-1)(\frac{dv}{dx}x+v)=\frac{y}{x}(1+\frac{y}{x})$$
$$(v-1)(\frac{dv}{dx}x+v)=v(1+v)$$
$$\frac{dv}{dx}x=\frac{v(1+v)}{v-1}-v$$
$$\frac{dv}{dx}x=\frac{2v}{v-1}$$
$$\frac{(v-1)dv}{2v}x=\frac{dx}{x}$$
$$\frac{v}{2}-\frac{1}{2}\ln v=\ln Cx$$
$$v-\ln v=\ln C^2x^2$$
$$\frac{y}{x}=\ln C^2x^2\frac{y}{x}$$
$$\frac{y}{x}=\ln(C^2xy)$$

8. Solve the equations (a) $-2x^2\dfrac{dy}{dx}=x^2+y^2$ and (b) $x^2\dfrac{dy}{dx}=3x^2+xy$.

Solution

(a) $y=vx\Rightarrow\dfrac{dy}{dx}=\dfrac{dv}{dx}x+v$. Hence,

$$-2x^2(\frac{dv}{dx}x+v)=x^2+x^2v^2$$
$$-2x^2(\frac{dv}{dx}x+v)=x^2(1+v^2)$$
$$-2(\frac{dv}{dx}x+v)=(1+v^2)$$
$$-2x\frac{dv}{dx}=1+v^2+2v$$
$$\frac{2dv}{(v+1)^2}=-\frac{dx}{x}$$
$$-\frac{2}{v+1}=-\ln|x|-\ln C$$
$$\frac{2}{v+1}=\ln C|x|$$
$$\frac{2x}{y+x}=\ln C|x|$$

The solution is thus $y=\dfrac{2x}{\ln C|x|}-x$.

(b) $y=vx\Rightarrow\dfrac{dy}{dx}=\dfrac{dv}{dx}x+v$. Hence,

$$x^2(\frac{dv}{dx}x+v)=3x^2+x^2v$$
$$\frac{dv}{dx}x=3$$
$$dv=3\frac{dx}{x}$$
$$v=3\ln(C|x|)$$
$$y=3x\ln(C|x|)$$

9. Consider the non-homogeneous equation $\dfrac{dy}{dx}=\dfrac{2x+y-3}{x-3y-5}$. Change variables to $x=X+a$ and $y=Y+b$. (a) Choose a and b so that the equation becomes a homogeneous equation in the variables X and Y. (b) Hence solve the equation.

Solution

$$\frac{dY}{dX}=\frac{2X+Y+2a+b-3}{X-3Y+a-3b-5}$$

$2a+b=3$

$a-3b=5$

Hence $a=2, b=-1$.

$\frac{dY}{dX}=\frac{2X+Y}{X-3Y}$. Write $Y=vX$ so that $\frac{dY}{dX}=v+X\frac{dv}{dX}$. Then,

$$v+X\frac{dv}{dX}=\frac{2+v}{1-3v}\Rightarrow X\frac{dv}{dX}=\frac{2+v}{1-3v}-\frac{v(1-3v)}{1-3v}=\frac{2+3v^2}{1-3v}$$

$$\frac{1-3v}{3v^2+2}dv=-\frac{dX}{X}$$

$$\left(\frac{1}{3v^2+2}-\frac{1}{2}\frac{6v}{3v^2+2}\right)dv=-\frac{dX}{X}$$

$$\frac{1}{\sqrt{2}}\arctan\frac{v\sqrt{3}}{\sqrt{2}}-\frac{1}{2}\ln(3v^2+2)=-\ln X+C$$

$$\frac{1}{\sqrt{2}}\arctan\frac{Y\sqrt{3}}{X\sqrt{2}}-\frac{1}{2}\ln(\frac{3Y^2}{X^2}+2)=-\ln X+C$$

Finally, $\frac{1}{\sqrt{2}}\arctan\frac{(y+1)\sqrt{3}}{(x-2)\sqrt{2}}-\frac{1}{2}\ln(\frac{3(y+1)^2}{(x-2)^2}+2)=-\ln(x-2)+C$ is the required solution.

10. Given the differential equation $x\frac{dy}{dx}+y=\cos x$, realize that $x\frac{dy}{dx}+y=\frac{d}{dx}(xy)$ and hence provide the solution of the equation given that $y=0$ when $x=\pi$.

Solution

$\frac{d}{dx}(xy)=x\frac{dy}{dx}+y$ and so $x\frac{dy}{dx}+y=\cos x$ becomes

$$\frac{d}{dx}(xy)=\cos x$$

$$xy=\int\cos x\,dx$$

$$xy=\sin x+C$$

$$y=\frac{\sin x}{x}+\frac{C}{x}$$

Setting $x=\pi$ we must get $y=0$ i.e. $0=0+\frac{C}{\pi}\Rightarrow C=0$. Hence $y=\frac{\sin x}{x}$.

11. Given the differential equation $\frac{x}{y}\frac{dy}{dx}+\ln y=\sin x$, realize that $\frac{x}{y}\frac{dy}{dx}+\ln y=\frac{d}{dx}(x\ln y)$ and hence provide the solution of the equation given that when $x=\pi/2$, $y=1$.

Solution

$\frac{d}{dx}(x\ln y)=\frac{x}{y}\frac{dy}{dx}+\ln y$ and so $\frac{x}{y}\frac{dy}{dx}+\ln y=e^{-x}$ becomes

$$\frac{d}{dx}(x\ln y)=\sin x$$
$$x\ln y=\int \sin x\,dx=-\cos x+C$$
$$\ln y=-\frac{\cos x}{x}+\frac{C}{x}$$

Then, $\ln 1=-\frac{\cos\pi/2}{\pi/2}+\frac{C}{\pi/2}\Rightarrow 0=0+\frac{C}{\pi/2}\Rightarrow C=0$. Thus $y=e^{-\frac{\cos x}{x}}$.

12. Using the method used above solve the equation $xe^{y}\frac{dy}{dx}+e^{y}=2x^{2}$, given that when $x=1$, $y=\ln 2$.

Solution

$\frac{d}{dx}(xe^{y})=xe^{y}\frac{dy}{dx}+e^{y}$ and so $xe^{y}\frac{dy}{dx}+e^{y}=2x^{2}$ becomes

$$\frac{d}{dx}(xe^{y})=2x^{2}$$
$$xe^{y}=\int 2x^{2}\,dx=\frac{2x^{3}}{3}+C$$
$$e^{y}=\frac{2x^{2}}{3}+\frac{C}{x}$$

Then, $2=\frac{2}{3}+C\Rightarrow C=\frac{4}{3}$. Thus $y=\ln\left(\frac{2x^{2}}{3}+\frac{4}{3x}\right)$.

13. (a) Solve the equation $\frac{dy}{dx}+xy=x$ given that when $x=0$, $y=2$. (b) Find the general solution to $x\frac{dy}{dx}-y=\frac{x}{x-1}$.

Solution

(a) The integrating factor is $e^{\int x dx}=e^{x^{2}/2}$. Thus

$$\frac{d}{dx}(ye^{x^{2}/2})=xe^{x^{2}/2}$$
$$ye^{x^{2}/2}=\int xe^{x^{2}/2}\,dx$$
$$ye^{x^{2}/2}=e^{x^{2}/2}+C$$
$$y=1+Ce^{-x^{2}/2}$$

The initial condition requires that $2=1+C\Rightarrow C=1$. Hence $y=1+e^{-x^{2}/2}$.

(b) $x\dfrac{dy}{dx} - y = \dfrac{x}{x-1}$ hence $\dfrac{dy}{dx} - \dfrac{y}{x} = \dfrac{1}{x-1}$. The integrating factor is $e^{-\int \frac{dx}{x}} = e^{-\ln x} = \dfrac{1}{x}$. Hence $\dfrac{d}{dx}(\dfrac{y}{x}) = \dfrac{1}{x(x-1)}$. Using partial fractions $\dfrac{1}{x(x-1)} = \dfrac{1}{x-1} - \dfrac{1}{x}$ and so:

$$\frac{y}{x} = \int\left(\frac{1}{x-1} - \frac{1}{x}\right)dx$$

$$\frac{y}{x} = \ln\left|\frac{x-1}{x}\right| + \ln C = \ln\left|\frac{C(x-1)}{x}\right|$$

$$y = x\ln\left|\frac{C(x-1)}{x}\right|$$

14. Solve the equation $\dfrac{dy}{dx} - xy = e^{x^2/2}$ given that when $x = 0$, $y = 2$.

Solution

The integrating factor is $e^{-\int xdx} = e^{-x^2/2}$. Thus

$$\frac{d}{dx}(ye^{-x^2/2}) = 1$$

$$ye^{-x^2/2} = \int dx$$

$$ye^{-x^2/2} = x + C$$

$$y = (x+C)e^{x^2/2}$$

The initial condition requires that $2 = (0 + C) \Rightarrow C = 2$. Hence $y = (x+2)e^{x^2/2}$.

15. (a) Solve the equation $\dfrac{dy}{dx} + y\sin x = e^{\cos x}$ given that when $x = 0$, $y = 0$. (b) Find the general solution to $\dfrac{dy}{dx} + 2y = e^{-2x}\cos x$.

Solution

(a) The integrating factor is $e^{\int \sin x dx} = e^{-\cos x}$. Thus

$$\frac{d}{dx}(ye^{-\cos x}) = 1$$

$$ye^{-\cos x} = \int dx$$

$$ye^{-\cos x} = x + C$$

$$y = (x+C)e^{\cos x}$$

The initial condition requires that $0 = (0 + C)e \Rightarrow C = 0$. Hence $y = xe^{\cos x}$.

(b) The integrating factor is $e^{\int 2dx} = e^{2x}$. Hence,

$$\frac{d}{dx}(ye^{2x}) = \cos x$$
$$ye^{2x} = \sin x + C$$
$$y = e^{-2x}(\sin x + C)$$

16. Solve the equation $\frac{dy}{dx} + \frac{y}{x} = x$ given that when $x = 1$, $y = \frac{2}{3}$.

The integrating factor is $e^{\int \frac{1}{x}dx} = e^{\ln x} = x$. Thus

$$\frac{d}{dx}(yx) = x^2$$
$$yx = \int x^2\, dx$$
$$yx = \frac{x^3}{3} + C$$
$$y = \frac{x^2}{3} + \frac{C}{x}$$

The initial condition requires that $\frac{2}{3} = \frac{1}{3} + \frac{C}{1} \Rightarrow C = \frac{1}{3}$. Hence $y = \frac{1}{3}(x^2 + \frac{1}{x})$.

17. Solve the equation $\frac{dy}{dx} + \frac{xy}{1-x^2} = 1$.

Solution

The integrating factor is $e^{\int \frac{x}{1-x^2}dx} = e^{-\frac{1}{2}\ln(1-x^2)} = \frac{1}{\sqrt{1-x^2}}$. Thus

$$\frac{d}{dx}(\frac{y}{\sqrt{1-x^2}}) = \frac{1}{\sqrt{1-x^2}}$$
$$\frac{y}{\sqrt{1-x^2}} = \int \frac{1}{\sqrt{1-x^2}}dx$$
$$= \arcsin x + C$$
$$y = \sqrt{1-x^2}(\arcsin x + C\ )$$

18. Solve the equation $\frac{dy}{dx} - \frac{xy}{1-x^2} = 1$ given that when $x = 0$, $y = 1$.

Solution

The integrating factor is $e^{-\int \frac{x}{1-x^2}dx} = e^{\frac{1}{2}\ln(1-x^2)} = \sqrt{1-x^2}$. Thus

$$\frac{d}{dx}(y\sqrt{1-x^2}) = \sqrt{1-x^2}$$
$$y\sqrt{1-x^2} = \int \sqrt{1-x^2}\, dx$$

To do the integral use $x = \sin u \Rightarrow dx = \cos u\, du$ so that

$$\begin{aligned}\int \sqrt{1-x^2}\, dx &= \int \sqrt{1-\sin^2 u}\cos u\, du \\ &= \int \cos^2 u\, du \\ &= \int \frac{\cos 2u + 1}{2} du \\ &= \frac{\sin 2u}{4} + \frac{u}{2} \\ &= \frac{\sin u \cos u}{2} + \frac{u}{2} \\ &= \frac{x\sqrt{1-x^2}}{2} + \frac{\arcsin x}{2} + C\end{aligned}$$

Hence

$$y\sqrt{1-x^2} = \frac{x\sqrt{1-x^2}}{2} + \frac{\arcsin x}{2} + C$$
$$y = \frac{x}{2} + \frac{\arcsin x}{2\sqrt{1-x^2}} + \frac{C}{\sqrt{1-x^2}}$$

The initial condition requires that $1 = C$. Hence $y = \frac{x}{2} + \frac{\arcsin x}{2\sqrt{1-x^2}} + \frac{1}{\sqrt{1-x^2}}$.

19. Verify the claims made at the end of section 7.4.

Solution

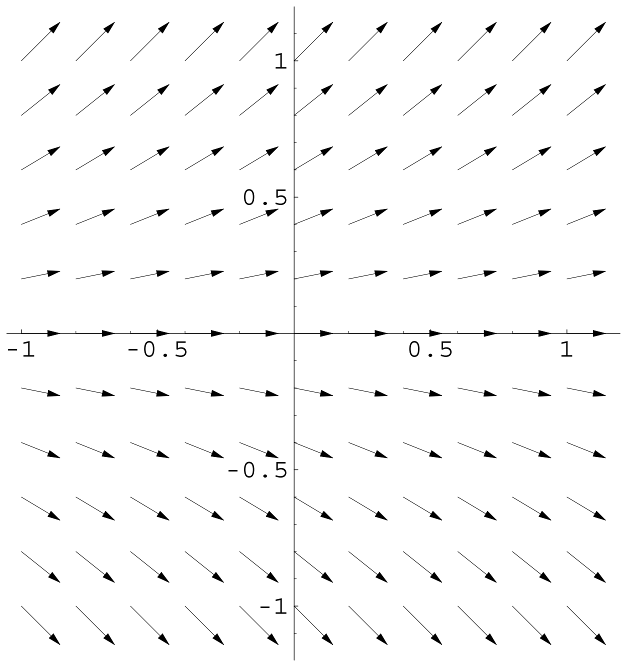
$\frac{dy}{dx} = 1 - y \Rightarrow \frac{dy}{1-y} = dx \Rightarrow -\ln|1-y| = x + c \Rightarrow |1-y| = e^{-x-c}$. As x increases, $e^{-x-c} \to 0$ and so $y \to 1$.

20. Construct the slope field diagram for the equation $\frac{dy}{dx} = y$ for $x, y \in [-1,1]$ and confirm that it gives the curves $y = ce^x$ as solutions.

Solution

The slope field is shown below. The solutions are:

$\int \frac{dy}{y} = \int dx \Rightarrow \ln y = x + k \Rightarrow y = e^{x+k} = ce^x$ as required. The solutions with $c > 0$ are in the upper half-plane, those with $c < 0$ in the lower half-plane and that with $c = 0$ is along the x axis.

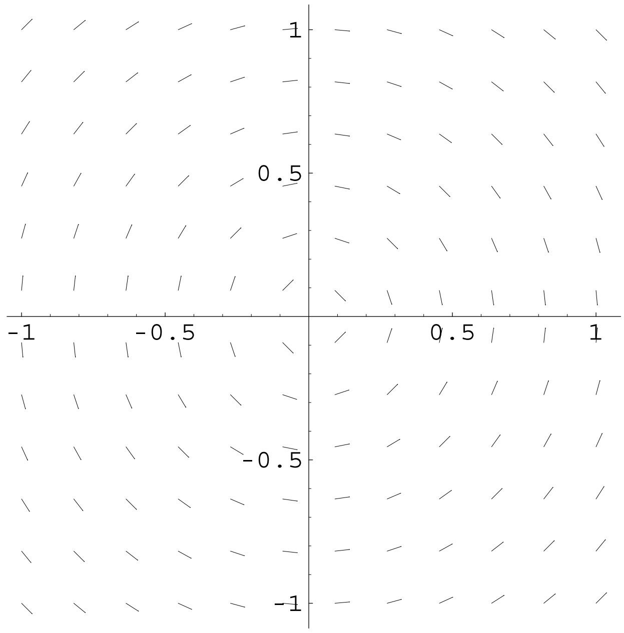


21. Construct the slope field diagram for the equation $\frac{dy}{dx} = -\frac{x}{y}$ for $x, y \in [-1,1]$ and confirm that it gives the circles $y^2 + x^2 = c$ as solutions.

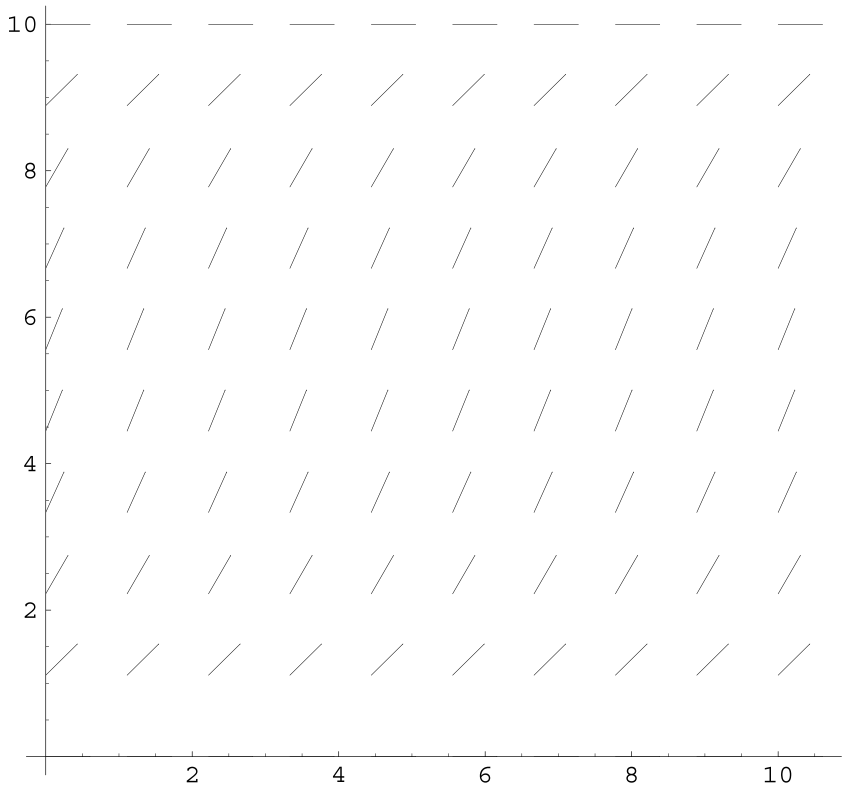
Solution

The differential equation $\frac{dy}{dx} = -\frac{x}{y}$ is separable and becomes

$\int y dy = -\int x dx \Rightarrow \frac{y^2}{2} = -\frac{x^2}{2} + c \Rightarrow y^2 + x^2 = const$, as required. The slope field diagram is shown below indicating that the solutions in the range $x, y \in [-1,1]$ are indeed parts of a circle centered at the origin.



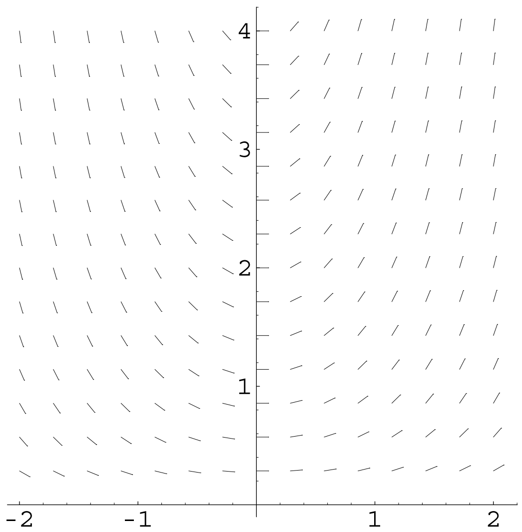
22. Describe qualitatively the solutions to the differential equation whose slope field is given below.



Solution

The solutions appear to grow from the initial value of *y* to a limiting value of 10. If the initial value is 0 the value of *y* stays 0. If it is initially 10 it stays at 10.

23. Shown below are 4 slope field diagrams. Match each one with one of the equations in the list: (A) $\dfrac{dy}{dx}=-1$, (B) $\dfrac{dy}{dx}=x^2$, (C) $\dfrac{dy}{dx}=x-y$ and (D) $\dfrac{dy}{dx}=xy$.



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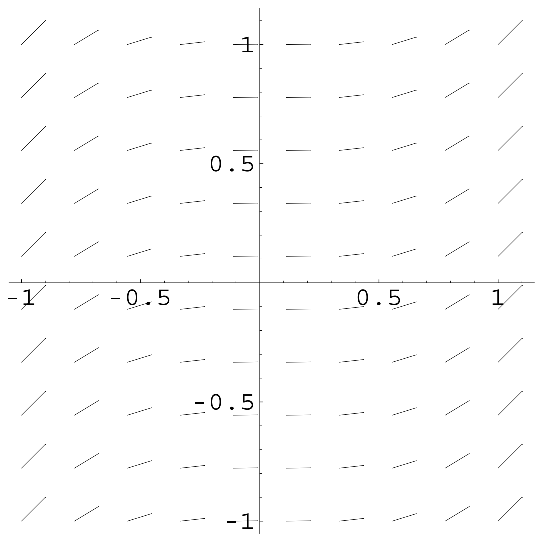

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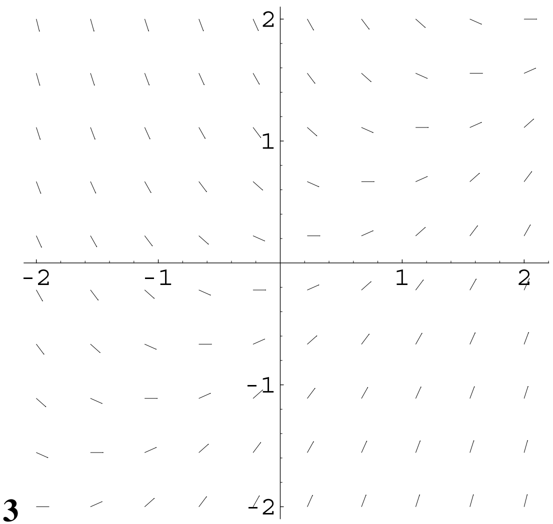

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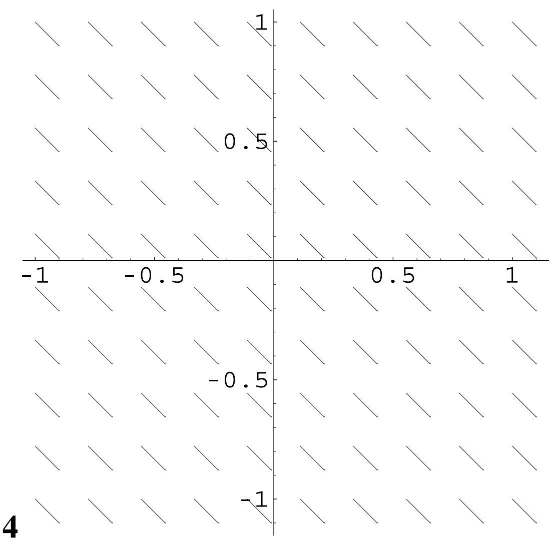

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Solution

Slope field number 4 has a constant negative slope and so $A \to 4$.
Slope field number 2 has always a positive slope and so $B \to 2$.
Equation (C) has zero slope when $y = x$ and this matches slope field 3, $C \to 3$.
Equation D should have a positive slope everywhere in thr first quadrant and negative in the second so $D \to 1$.

24. Use Euler's method to find an approximate value for $f(1)$ given that the function $f(x)$ satisfies the equation $\dfrac{df}{dx}=\sin(x^2)$ and $f(0)=0$.

Solution

x_n	y_n
0	0
0.1	0
0.2	0.0009999
0.3	0.004997
0.4	0.013987
0.5	0.029919
0.6	0.054659
0.7	0.089886
0.8	0.136949
0.9	0.196669
1.0	0.269097

The exact answer is 0.3103.

25. Use Euler's method to find an approximate value for $f(1)$ given that the function $f(x)$ satisfies the equation $\frac{df}{dx} = e^{-x^2}$ and $f(0) = 1$.

Solution

x_n	y_n
0	1
0.1	1.1
0.2	1.199
0.3	1.29508
0.4	1.38648
0.5	1.47169
0.6	1.54957
0.7	1.61934
0.8	1.6806
0.9	1.73333
1.	1.77782

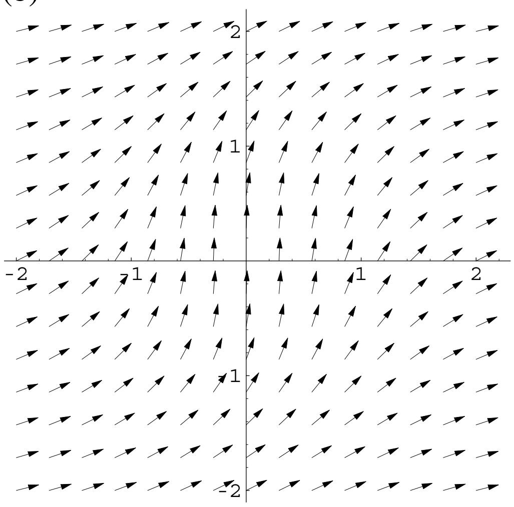
The exact answer is 1.7468.

26. (a) Use Euler's method to find an approximate value for $f(2)$ given that the function $f(x)$ satisfies the equation $\frac{df}{dx} = \frac{1}{x^2 + f^2}$ and $f(1) = 0$, using a step size of 0.2. (b) Sketch the slope field for the equation in (a) for x and y in the interval $[-2, 2]$. (c) Join segments on the slope field diagram starting at the point (1, 0) to predict the value of $f(2)$.

Solution

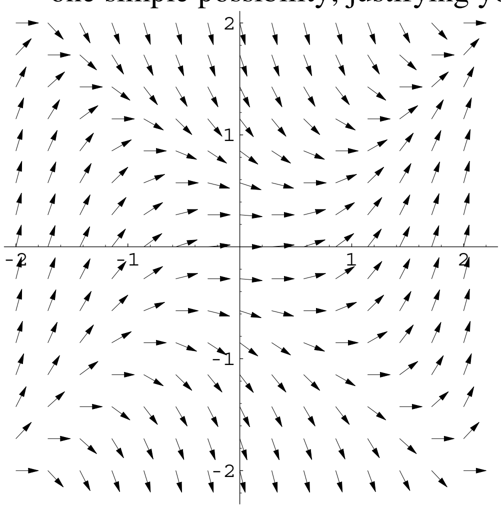
x_n	y_n
1	0
1.2	0.2
1.4	0.335135
1.6	0.431646
1.8	0.504470
2.0	0.561703

(b)



(c) Following the arrows starting at (1, 0) we end up at $x = 2$ with approximately $f(2) \approx 1.15$, substantially above what the Euler method gave in (a).

27. Make the *simplest* guess for the form of $f(x, y)$ in $\dfrac{df}{dx} = f(x, y)$ that gives rise to the following slope field diagram. (There is obviously no unique answer – just find any one simple possibility, justifying your choice.)



The arrows are horizontal (indicating that $f(x, y) = 0$) whenever $y = \pm x$. Thus $f(x, y)$ has at least $y - x$ and $y + x$ as factors i.e. $y^2 - x^2$.