MATH 2130 Tutorial 10

1. Find the area of that part of the surface $z = xy$ inside the cylinder $x^2 + y^2 = a^2$.

2. Set up, but do **NOT** evaluate, a double iterated integral for the surface area of the ellipsoid

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1.$$

3. Set up, but do **NOT** evaluate, a double iterated integral for the area of the surface $z = \sqrt{1 + x^2 + y^2}$ below $z = 2$.

4. Set up, but do **NOT** evaluate, a double iterated integral for the area of the surface $z = 2x^2 + y^2$ bounded by $y = 0$, $x = 0$, and $x + y = 1$.

5. Find the area bounded by $(x^2 + y^2)^3 = 4a^2x^2y^2$.

6. Find the double integral of $f(x, y) = xy(x + y)$ over the region in the first quadrant bounded by $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$.

7. A plate with constant mass per unit area ρ is bounded by the curve $(x^2 + y^2)^2 = 9(x^2 - y^2)$. Find its moment of inertia about the x-axis.

Answers

1. $2\pi[(1 + a^2)^{3/2} - 1]/3$

2. $8\displaystyle\int_0^a \int_0^{(b/a)\sqrt{a^2-x^2}} \sqrt{1 + \left(\frac{-cx}{a^2\sqrt{1 - x^2/a^2 - y^2/b^2}}\right)^2 + \left(\frac{-cy}{b^2\sqrt{1 - x^2/a^2 - y^2/b^2}}\right)^2}\, dy\, dx$

3.(a) $4\displaystyle\int_0^{\pi/2} \int_0^{\sqrt{3}} \sqrt{\frac{1 + 2r^2}{1 + r^2}}\, r\, dr\, d\theta$ **4.** $\displaystyle\int_0^1 \int_0^{1-x} \sqrt{1 + 16x^2 + 4y^2}\, dy\, dx$

5. $\pi a^2/2$ **6.** $62/15$ **7.** $27(3\pi - 8)\rho/16$