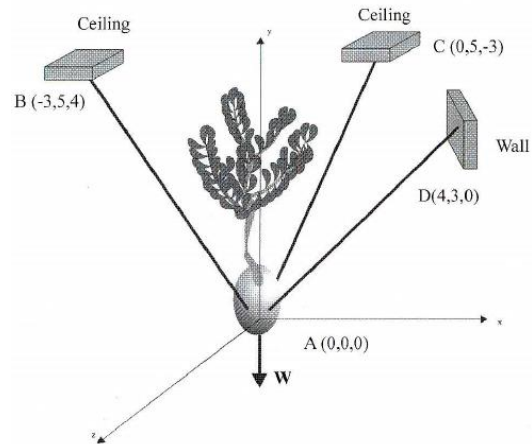
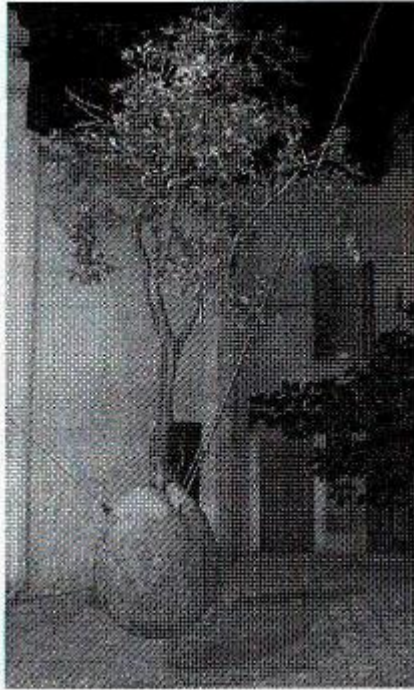


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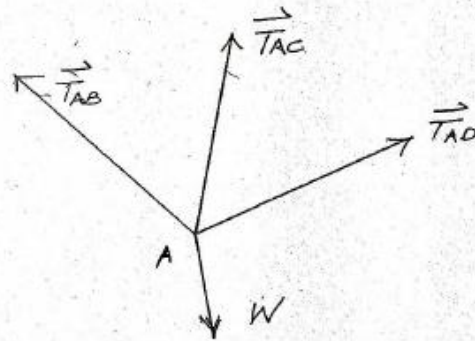
ENG 1440 Assignment #7 Solution

S2-504 A tree is suspended by two cables from the ceiling at B and C and by a third cable attached to the wall at D . If the maximum safe tension force in any one of the three cables is 1.2 kN , Determine the maximum weight of the tree that can be supported.



$$\begin{aligned} A: (0, 0, 0) \\ B: (-3, 5, 4) \\ C: (0, 5, -3) \\ D: (4, 3, 0) \end{aligned}$$

$$-W\hat{j}$$



$$\begin{aligned} \vec{T}_{AD} &= T_{AD} \vec{\lambda}_{AD} \\ \vec{\lambda}_{AD} &= \frac{\vec{AD}}{AD} \end{aligned}$$

$$\begin{aligned} \vec{AD} &= 4\hat{i} + 3\hat{j} \\ AD &= \sqrt{4^2 + 3^2} = 5 \end{aligned}$$

$$\vec{T}_{AD} = T_{AD} \left(\frac{4\hat{i} + 3\hat{j}}{5} \right) = 0.8 T_{AD} \hat{i} + 0.6 T_{AD} \hat{j}$$

$$\begin{aligned} \vec{T}_{AC} &= T_{AC} \vec{\lambda}_{AC} & \vec{\lambda}_{AC} &= \frac{\vec{AC}}{AC} & \vec{AC} &= 5\hat{j} - 3\hat{k} \\ & & & & AC &= \sqrt{5^2 + (-3)^2} \end{aligned}$$

$$\vec{T}_{AC} = T_{AC} \left(\frac{5\hat{j} - 3\hat{k}}{\sqrt{34}} \right)$$

$$\vec{T}_{AC} = 0.857 T_{AC} \hat{j} - 0.514 T_{AC} \hat{k}$$

$$\begin{aligned} \vec{T}_{AB} &= T_{AB} \vec{\lambda}_{AB} & \vec{\lambda}_{AB} &= \frac{\vec{AB}}{AB} & \vec{AB} &= -3\hat{i} + 5\hat{j} + 4\hat{k} \\ & & & & AB &= \sqrt{(-3)^2 + 5^2 + 4^2} = \sqrt{50} \end{aligned}$$

$$\vec{T}_{AB} = T_{AB} \left(\frac{-3\hat{i} + 5\hat{j} + 4\hat{k}}{\sqrt{50}} \right)$$

$$\vec{T}_{AB} = 0.424 T_{AB} \hat{i} + 0.707 T_{AB} \hat{j} + 0.566 T_{AB} \hat{k}$$

$$\begin{aligned} \sum F_x = 0 & \quad 0.8 T_{AD} + 0.424 T_{AB} = 0 \\ T_{AD} &= \frac{0.424}{0.8} T_{AB} = 0.53 T_{AB} \end{aligned}$$

$$\sum F_y = 0 \quad 0.6 T_{AD} + 0.857 T_{AC} + 0.707 T_{AB} - W = 0$$

$$\begin{aligned} \sum F_z = 0 & \quad -0.514 T_{AC} + 0.566 T_{AB} = 0 \\ T_{AC} &= \frac{0.566}{0.514} T_{AB} = 1.101 T_{AB} \end{aligned}$$

$$\text{Let } T_{AC} = 1.2 \text{ kN Governs}$$

$$\therefore T_{AB} = \frac{1.2}{1.101} = 1.09 \text{ kN} < 1.2$$

$$T_{AD} = 0.53 T_{AB} = 0.53(1.09) = 0.578 \text{ kN} < 1.2$$

From ②

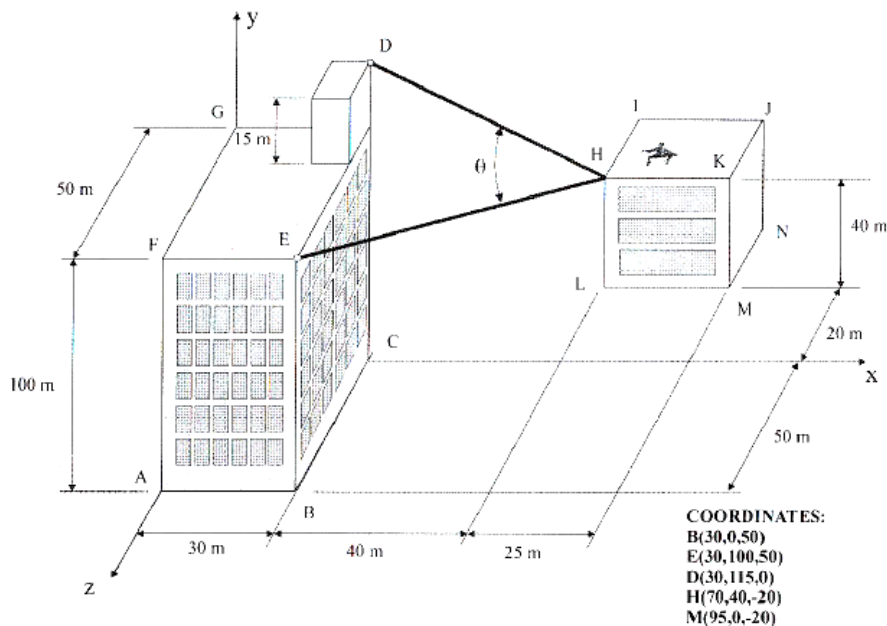
$$0.6(0.578) + 0.857(1.2) + 0.707(1.09) = W$$

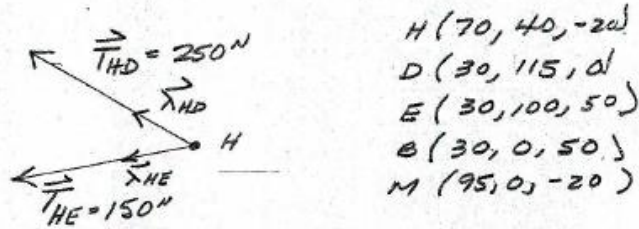
$$W = 2.15 \text{ kN}$$

S2-602 Spiderman casts two web lines HE and HD across two buildings as shown in the figure below. In doing so, he applied a force of 150 N on line HE and 250 N on line HD pulling from point H .

Determine:

- The resultant, \mathbf{R} , of the two forces acting at the point H ,
- The angle, θ , between the two web lines at H ,
- The moment of the resultant, \mathbf{R} , about the point B , and
- The moment of the resultant, \mathbf{R} , about the line BM .





$$\begin{aligned}
 a) \quad \vec{T}_{HD} &= T_{HD} \vec{\lambda}_{HD} = 250 \vec{\lambda}_{HD} \\
 \vec{\lambda}_{HD} &= \frac{\vec{HD}}{HD} \quad HD = -40\hat{i} + 75\hat{j} + 20\hat{k} \\
 HD &= \sqrt{(-40)^2 + (75)^2 + (20)^2} = 87.32 \\
 \vec{\lambda}_{HD} &= \frac{-40\hat{i} + 75\hat{j} + 20\hat{k}}{87.32} \\
 \vec{T}_{HD} &= 250 \left(\frac{-40\hat{i} + 75\hat{j} + 20\hat{k}}{87.32} \right) = (-114.52\hat{i} + 214.73\hat{j} + 57.26\hat{k}) \text{ N} \\
 \vec{T}_{HE} &= T_{HE} \vec{\lambda}_{HE} = 150 \vec{\lambda}_{HE} \quad \vec{\lambda}_{HE} = \frac{\vec{HE}}{HE}
 \end{aligned}$$

$$\begin{aligned}
 \vec{HE} &= -40\hat{i} + 60\hat{j} + 70\hat{k} \\
 HE &= \sqrt{(-40)^2 + (60)^2 + (70)^2} = 100.5
 \end{aligned}$$

$$\begin{aligned}
 \vec{\lambda}_{HE} &= \frac{-40\hat{i} + 60\hat{j} + 70\hat{k}}{100.5} = -59.70\hat{i} + 59.50\hat{j} + 104.48\hat{k} \\
 \vec{T}_{HE} &= 150 \left(\frac{-40\hat{i} + 60\hat{j} + 70\hat{k}}{100.5} \right) = (-59.70\hat{i} + 89.55\hat{j} + 104.48\hat{k}) \text{ N} \\
 \vec{R} &= \vec{T}_{HD} + \vec{T}_{HE} = (-174.22\hat{i} + 304.28\hat{j} + 161.74\hat{k})
 \end{aligned}$$

$$\begin{aligned}
 b) \quad \cos \theta &= \vec{\lambda}_{HD} \cdot \vec{\lambda}_{HE} \\
 \cos \theta &= \frac{1}{(87.32)(100.5)} [(-40)(-40) + (75)(60) + (20)(70)] \\
 \cos \theta &= 0.85464 \Rightarrow \theta = 31.28^\circ
 \end{aligned}$$

$$\begin{aligned}
 c) \quad \vec{M}_B &= \vec{r}_{BH} \times \vec{R} \quad \vec{r}_{BH} = 40\hat{i} + 40\hat{j} - 70\hat{k} \\
 \vec{M}_B &= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 40 & 40 & -70 \\ -174.22 & 304.28 & 161.74 \end{vmatrix} = \begin{vmatrix} \hat{i} & \hat{j} \\ 40 & 40 \\ -174.22 & 304.28 \end{vmatrix} = [6469.6\hat{i} + 12195.4\hat{j} + 12171.2\hat{k}] \\
 \vec{M}_B &= (27769.2\hat{i} + 5701.6\hat{j} + 19140\hat{k}) \text{ N}\cdot\text{m}
 \end{aligned}$$

$$d) M_{BM} = \vec{\lambda}_{BM} \cdot \vec{M}_B$$

$$\vec{\lambda}_{BM} = \frac{\vec{BM}}{BM} \quad BM = 65\hat{i} + 0\hat{j} - 70\hat{k}$$

$$BM = \sqrt{(65)^2 + (-70)^2} = \sqrt{9125}$$

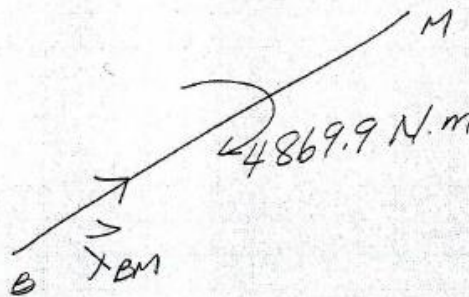
$$= 95.52$$

$$\vec{\lambda}_{BM} = \frac{65\hat{i} + 0\hat{j} - 70\hat{k}}{\sqrt{9125}}$$

$$\vec{\lambda}_{BM} \cdot \vec{M}_B = \left(\frac{65\hat{i} + 0\hat{j} - 70\hat{k}}{\sqrt{9125}} \right) \cdot (27769.2\hat{i} + 5701.6\hat{j} + 19140\hat{k})$$

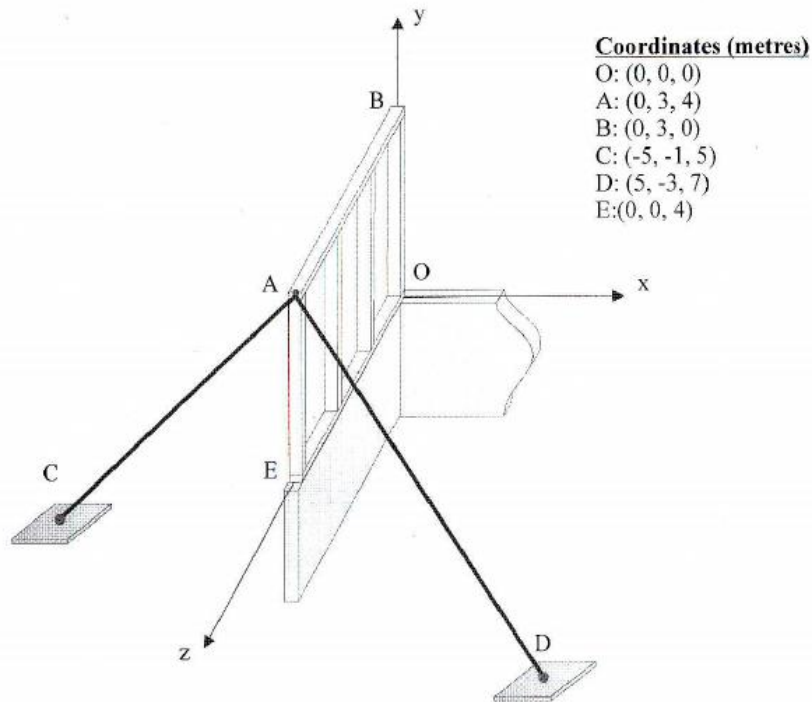
$$= \frac{1}{\sqrt{9125}} [(65)(27769.2) + (-70)(19140)]$$

$$= +4869.9 \text{ N.m}$$



S2-605 A $3\text{ m} \times 4\text{ m}$ wood stud wall sits on top of a concrete foundation wall as shown in the figure. The stud wall is bolted to the foundation wall and is supported by two cables AC and AD attached to the wall at point A . The tension in cable AC is 1.2 kN . Determine:

- The angle between cable AC and cable AD ,
- The moment of the 1.2 kN force applied at A by cable AC about the line EO ,
- The moment of the 1.2 kN force applied at A about the line CD and



a)

$$\vec{AC} = -5\vec{i} - 4\vec{j} + \vec{k}$$

$$AC = 6.48 \text{ m}$$

$$\vec{AD} = 5\vec{i} - 6\vec{j} + 3\vec{k}$$

$$AD = 8.37 \text{ m}$$

$$\vec{AC} \cdot \vec{AD} = (AC)(AD) \cos \theta$$

$$= (6.48)(8.37) \cos \theta$$

$$= 54.24 \cos \theta$$

$$\text{Also } \vec{AC} \cdot \vec{AD} = -25 + 24 + 3 = 2$$

$$\therefore 2 = 54.24 \cos \theta$$

$$\theta = 87.89^\circ$$

b) Find M_{EO}

$$M_{EO} = \vec{M}_E \cdot \vec{r}_{EO}$$

$$\vec{M}_E = \vec{r}_{EA} \times \vec{F}_{AC}$$

$$\vec{F}_{AC} = F_{AC} \vec{r}_{AC}$$

$$\vec{r}_{AC} = \frac{-5\vec{i} - 4\vec{j} + \vec{k}}{6.48}$$

$$= 1.2 \vec{r}_{AC}$$

$$= (-0.9260\vec{i}$$

$$- 0.7408\vec{j}$$

$$+ 0.1852\vec{k}) \text{ kN}$$

$$= -0.7716\vec{i} - 0.6173\vec{j} + 0.1543\vec{k}$$

$$\vec{r}_{EA} = -3\vec{j}$$

$$\bar{M}_E = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 0 & -3 & 0 \\ -0.9260 & -0.7408 & 0.1852 \end{vmatrix} \begin{vmatrix} \bar{i} & \bar{j} \\ 0 & -3 \end{vmatrix}$$

$$= (-0.556 \bar{i}) - (2.778 \bar{j})$$

$$\bar{r}_{EO} = -\bar{k}$$

$$M_{EO} = (+0.556 \bar{i} + 2.778 \bar{j}) \cdot (-\bar{k})$$

$$= -2.778 \text{ kN-m}$$

③ $M_{CD} = 0$ since line CD intersects the line of action of the force F_{AC}