MATH 1710: Tutorial 11 (Trigonometric Substitutions and Completing Squares)

1. Evaluate the following indefinite integrals:

(i)
$$\int \frac{1}{\sqrt{x^2 - 5}} dx$$
, (ii) $\int x\sqrt{5x^2 + 3} dx$, (iii) $\int \frac{x^2}{(2 - 9x^2)^{3/2}} dx$,

(iv)
$$\int \frac{x^2}{\sqrt{x^2 - 5}} dx.$$

- 2. Find the length of the portion of the parabola $y=x^2$ from (0,0) to (1,1).
- 3. The parabola $x=y^2$ divides the circle $x^2+y^2=4$ into two parts. Find the second moment of area of the smaller part about the x-axis.
- 4. Evaluate the indefinite integrals:

(i)
$$\int \frac{\sqrt{x^2 + 2x - 3}}{x + 1} dx$$
, (ii) $\int \sqrt{-y^2 + 6y} dy$, $\int \frac{2x - 3}{x^2 + 6x + 13} dx$.

5. One of the gates in a dam is circular with radius 1 metre. If the gate is closed and the surface of the water is 3 metres above the top of the gate, find the force due to water pressure on the gate.