ENG 1460 Introduction to Thermal Sciences
Term Test # 2

Textbook and Calculator Only **March 17, 2008**
Section A03 **ANSWER ALL QUESTIONS** **Time: 1¾ hours**

1. Consider the following processes that were undertaken by a system of a control mass. In each process, you are required to determine whether or not the process is polytropic. If the process is polytropic, determine the polytropic exponent *n* (showing your steps clearly), and if the process is not polytropic, explain why:

   (a) An isothermal process (i.e., T = constant) involving an ideal gas. **(2 marks)**
   (b) A process where the pressure and volume are related by the equation $P = 20 + 100V$. **(2 marks)**
   (c) A process where the pressure is proportional to the diameter of the system (i.e., $P \propto D$) and the volume is given by $V = (\pi/6)D^3$. **(2 marks)**

2. A rigid cylindrical tank is divided into two rooms (room A and room B) by an insulated piston, as shown in Fig. 1. The lateral side and the left base of the cylindrical tank are insulted, while the right base receives heat from the surroundings. The insulated piston is free to move and therefore, the pressure in room A at any instant is equal to the pressure in room B. Room A contains 10 kg of R-134a, initially at T_{A1} = 10°C and x_{A1} = 0.8, while room B contains 2.5 kg of CO_2, initially at T_{B1} = 40°C. Heat is now added through the right base of the cylindrical tank and the process continued until the conditions in room A reached T_{A2} = 20°C and x_{A2} = 0.9. Neglect changes in the kinetic and potential energies in rooms A and B.

   (a) Determine the work in [kJ] done by the CO_2 in room B on the R-134a in room A. **(5 marks)**
   (b) Determine the final pressure and the final temperature, P_{B2} and T_{B2}, respectively, of the CO_2 in room B. **(7 marks)**
   (c) Determine the amount of heat transfer in [kJ] that was added through the right base of the cylindrical tank. **(4 marks)**

3. Consider a tube with a variable diameter, as shown in Fig. 2. The tube has an inlet diameter D_1 = 5 cm, and an outlet diameter D_2 = 10 cm. Air (treat as an ideal gas) enters the tube at P_1 = 100 kPa, T_1 = 300 K, and V_1 = 80 m/s, and it leaves at P_2 = 200 kPa. The tube receives heat from the surroundings at a rate of $\dot{Q} = 19\,\text{kW}$. Determine the velocity V_2 and the temperature T_2 at the outlet of the tube. Assume steady-state and steady-flow, neglect the change in potential energy, but take the change in kinetic energy into account. **(13 marks)**

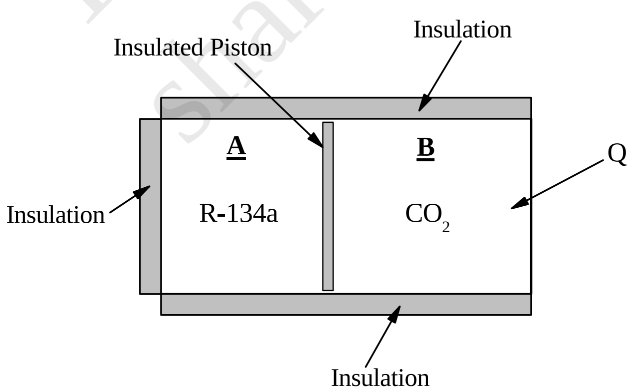


Fig. 1

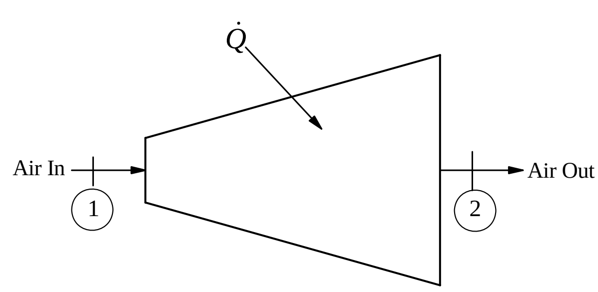


Fig. 2

Solutions March 2008

1. (a) For ideal gases $PV = mRT$

If the process is isothermal, then $T = $ constant

Since $m = $ Const. and $R = $ constant, then

$$PV = C$$

This is a polytropic process with $n = 1$

(b) The relation $P = 20 + 100V$ cannot be expressed in the form $PV^n = C$ with n and C as constant throughout the process.

Therefore, this process is not polytropic.

(c) If $P \propto D$, then $P = kD$ where k is a constant

If $V = \frac{\pi}{6} D^3$, then $D = \left(\frac{6}{\pi}\right)^{1/3} V^{1/3}$

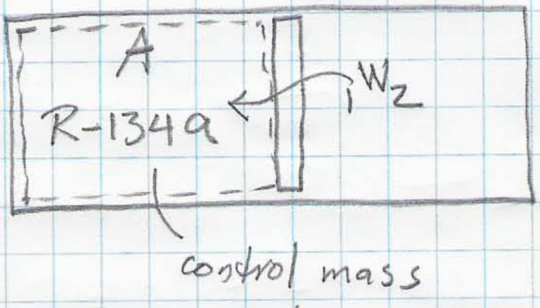
Thus, $P = k\left(\frac{6}{\pi}\right)^{1/3} V^{1/3}$

or $PV^{-1/3} = \left(\frac{6}{\pi}\right)^{1/3} k = C$

Therefore, this is a polytropic process with

$$n = -1/3$$

2. (a) Apply the 1st law of thermodynamics on the R-134a control mass.



$\Delta KE = \Delta PE = 0$

$$\cancel{{}_1Q_2} - {}_1W_2 = m_A \left[u_{A2} - u_{A1} \right]$$

$$u_{A1} = x_{A1} u_g + (1 - x_{A1}) u_f$$
$$= 0.8 \times 383.67 + 0.2 \times 213.25$$
$$= 349.6 \text{ kJ/kg}$$

$$u_{A2} = 0.9 \times 389.19 + 0.1 \times 227.03$$
$$= 373.0 \text{ kJ/kg}$$

$$-{}_1W_2 = 10 (373.0 - 349.6) = 234 \text{ kJ}$$
$${}_1W_2 = -234 \text{ kJ}$$

Work done by the CO_2 on the R-134a is 234 kJ

2. (b) $P_{B1} = P_{A1} = 415.8$ kPa

$P_{B2} = P_{A2} = 572.8$ kPa

$$P_{B1} V_{B1} = m_B R_{CO_2} T_{B1}$$

$$V_{B1} = \frac{2.5 \times 0.1889 \times 313.15}{415.8} = 0.3557 \text{ m}^3$$

$$V_{A1} + V_{B1} = V_{A2} + V_{B2}$$

$$V_{B2} = V_{A1} + V_{B1} - V_{A2}$$

$$v_{A1} = x_{A1} v_g + (1 - x_{A1}) v_f$$

$$= 0.8 \times 0.04945 + 0.2 \times 0.000794$$

$$= 0.03972 \text{ m}^3/\text{kg}$$

$$V_{A1} = m_A v_{A1} = 10 \times 0.03972 = 0.3972 \text{ m}^3$$

$$v_{A2} = 0.9 \times 0.03606 + 0.1 \times 0.000817$$

$$= 0.03254 \text{ m}^3/\text{kg}$$

$$V_{A2} = 0.3254 \text{ m}^3$$

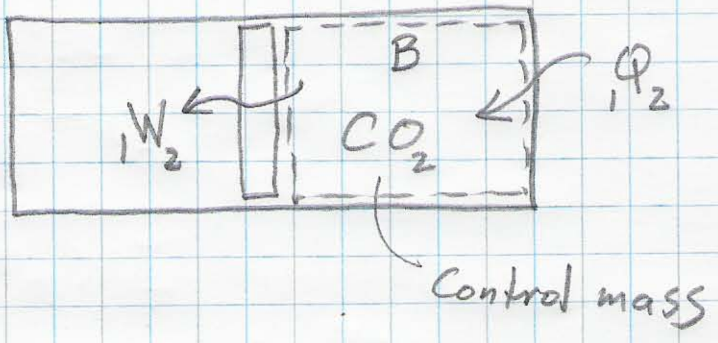
$$V_{B2} = 0.3972 + 0.3557 - 0.3254 = 0.4275 \text{ m}^3$$

$$\frac{P_{B1} V_{B1}}{T_{B1}} = \frac{P_{B2} V_{B2}}{T_{B2}}$$

$$T_{B2} = \frac{313.15 \times 572.8 \times 0.4275}{415.8 \times 0.3557} = 518.5 \text{ K}$$

$$= 245.3^\circ\text{C}$$

(c) Apply the 1^{st} law of thermodynamics on the CO_2 control mass.



$\Delta KE = \Delta PE = 0$

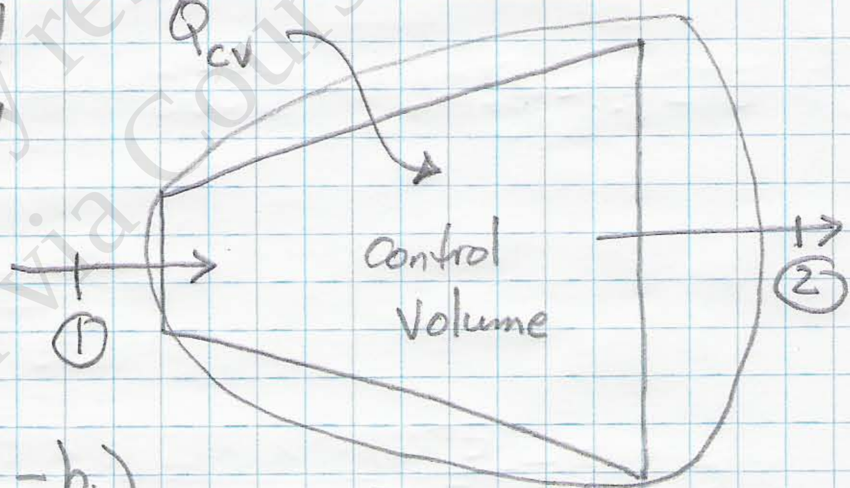
$$ {}_1Q_2 - {}_1W_2 = m_B \left(u_{B2} - u_{B1} \right) $$

$$ = m_B \, C_{v_0} \left(T_{B2} - T_{B1} \right) $$

$$ {}_1Q_2 - 234 = 2.5 \times 0.653 (245.3 - 40) $$

$$ {}_1Q_2 = 569.2 \text{ kJ} $$

3. Consider the Control Volume shown and apply the 1^{st} law of thermodynamics.



$$ \dot{Q}_{cv} - \cancel{\dot{W}_{cv}} = \dot{m} \left[(h_2 - h_1) + \frac{V_2^2 - V_1^2}{2} \right] $$

$$ \dot{Q}_{cv} = \dot{m} \left[c_{p_0} (T_2 - T_1) + \frac{V_2^2 - V_1^2}{2} \right] \quad (1) $$

Also, from mass balance, we have

$$\dot{m} = \frac{A_1 V_1}{v_1} = \frac{A_2 V_2}{v_2}$$

$$v_1 = \frac{R T_1}{P_1} \quad \text{and} \quad v_2 = \frac{R T_2}{P_2}$$

Therefore

$$\frac{A_1 V_1}{\left(\frac{R T_1}{P_1}\right)} = \frac{A_2 V_2}{\left(\frac{R T_2}{P_2}\right)}$$

$$V_2 = V_1 \left(\frac{A_1}{A_2}\right)\left(\frac{T_2}{T_1}\right)\left(\frac{P_1}{P_2}\right)$$

$$A_1 = \frac{\pi}{4} D_1^2 \quad \text{and} \quad A_2 = \frac{\pi}{4} D_2^2$$

$$V_2 = V_1 \left(\frac{D_1^2}{D_2^2}\right)\left(\frac{T_2}{T_1}\right)\left(\frac{P_1}{P_2}\right)$$

$$= 80\left(\frac{25}{100}\right)\left(\frac{T_2}{300}\right)\left(\frac{100}{200}\right)$$

$$V_2 = \frac{T_2}{30} \qquad (2)$$

Equations (1) and (2) give two equations in two unknows. Substituting from (2) into (1), we get

$$19 = \dot{m}\left[1.004\,(T_2 - 300) + \frac{T_2^2}{900 \times 2000} - \frac{80^2}{2000}\right]$$

We need $\dot{m}$

$$v_1 = \frac{0.287 \times 300}{100} = 0.861 \ m^3/kg$$

$$\dot{m} = \frac{\frac{\pi}{4}(0.05)^2(80)}{0.861} = 0.1824$$

$$104.2 = 1.004\,T_2 - 301.2 + 5.556\times10^{-7}\,T_2^2 - 3.2$$

$$5.556\times10^{-7}\,T_2^2 + 1.004\,T_2 - 408.6 = 0$$

$$T_2 = \frac{-1.004 \pm \sqrt{1.008016 + 9.08\times10^{-4}}}{2\times 5.556\times10^{-7}}$$

$$T_2 = 406.9 \ K$$

$$V_2 = \frac{406.9}{30} = 13.56 \ m/s$$