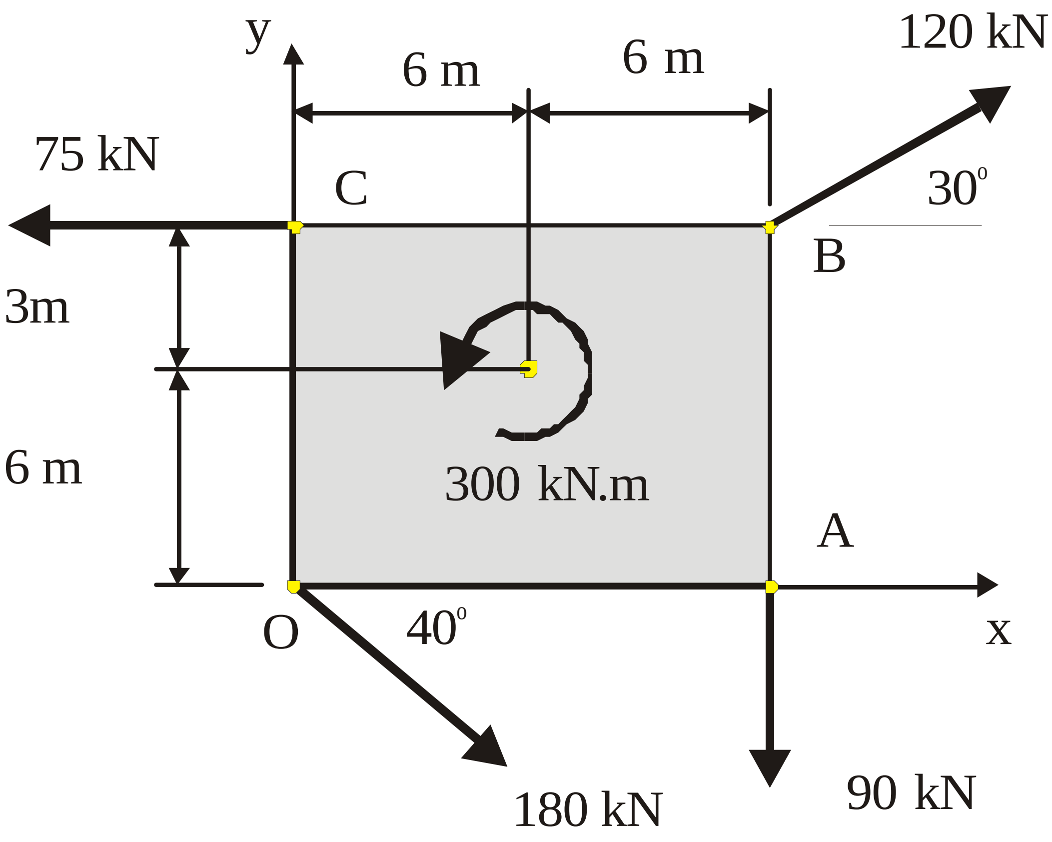
Example 3.7

J. Frye

Example 3.7:

Replace the force system shown by a single force and determine where the line of action of this force intersects the x-axis and the y-axis.



IMPORTANT: In problems where you are asked to replace a system of forces and couples acting on a rigid body by a single force read the question carefully. It will ask you to determine where the line of action of the force **INTERSECTS** two lines. The lines specified will usually (but not always) be the x-axis and the y-axis. Your first step in solution of this type of problem is to find the equivalent force-couple **at this intersection point**.

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Replace the force system shown by a single force and determine where the line of action of this force **intersects the x-axis and the y-axis**.

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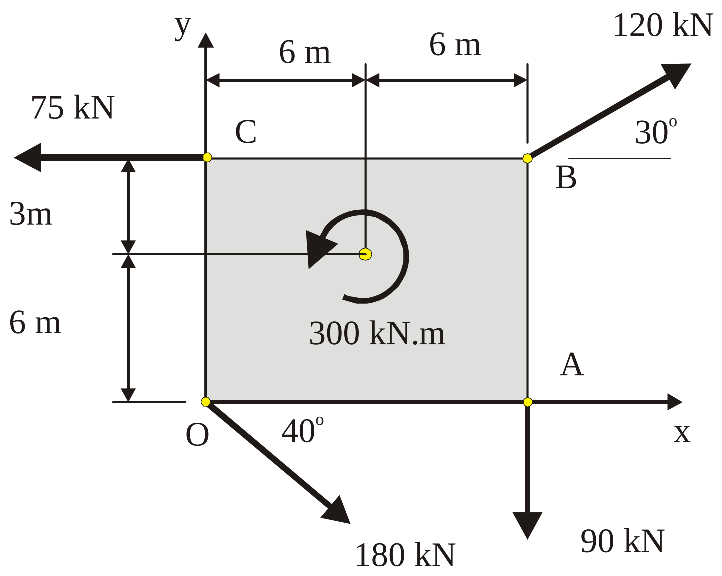
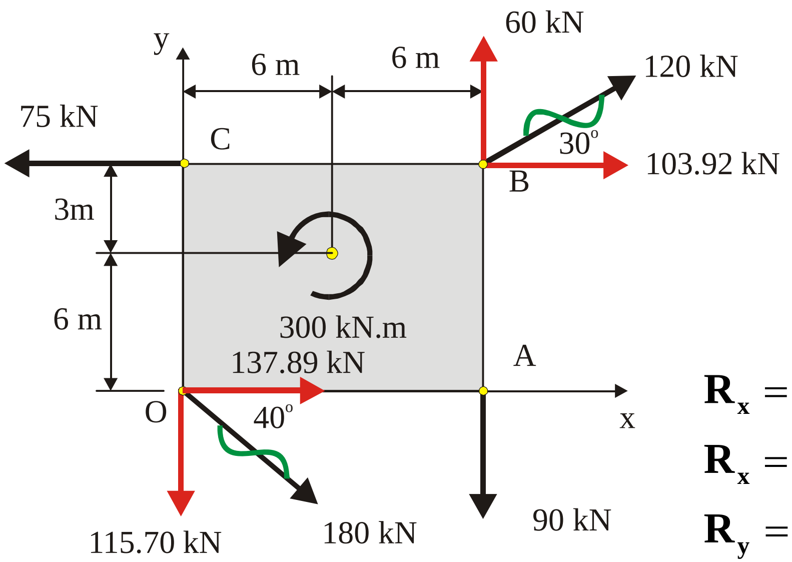


We first find the **equivalent force-couple** at O (intersection point of x-axis and y-axis).

1. Resolve sloping forces into x and y components.
2. Determine the resultants $\mathbf{R_x}$ and $\mathbf{R_y}$
3. Determine the resultant moment, $\mathbf{M_{RO}}$ of all forces and couples applied to the rigid body about the intersection point

Determination of the Equivalent Force-Couple at O



$$\mathbf{R_x} = \sum F_x = -75 + 103.92 + 137.89 = +166.81\ \text{kN}$$

$$\mathbf{R_x} = 166.81\ \text{kN} \rightarrow$$

$$\mathbf{R_y} = \sum F_y = +60 - 90 - 115.70 = -145.70\ \text{kN}$$

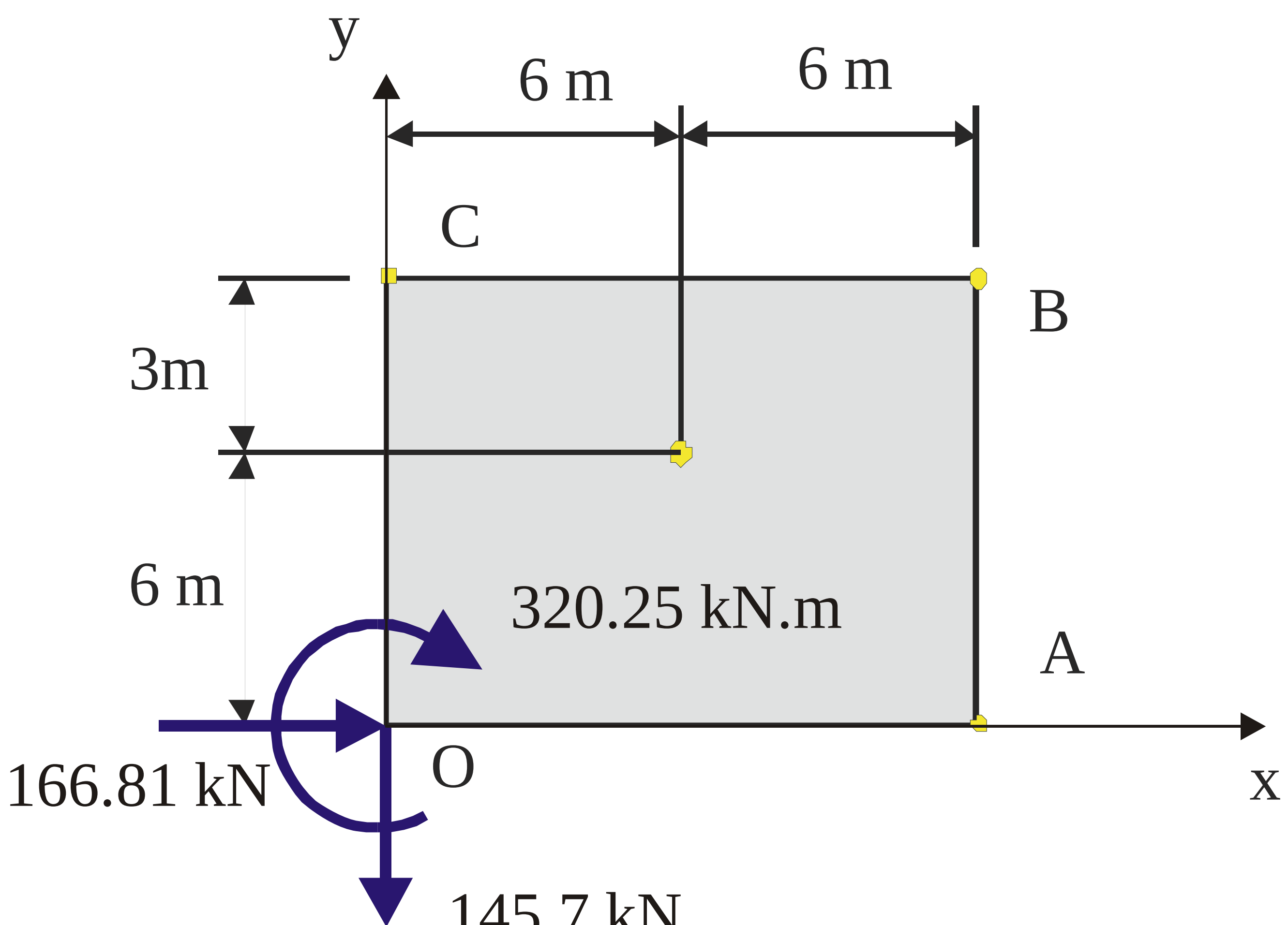
$$\mathbf{R_y} = 145.70\ \text{kN} \downarrow$$

Resultant Moment at O :

$$\mathbf{M_{RO}} = 75(9) + 60(12) - 103.92(9) - 90(12) + 300 = -320.28\ \text{kN.M}$$

$$\mathbf{M_{RO}} = 320.25\ \text{kN.M}$$

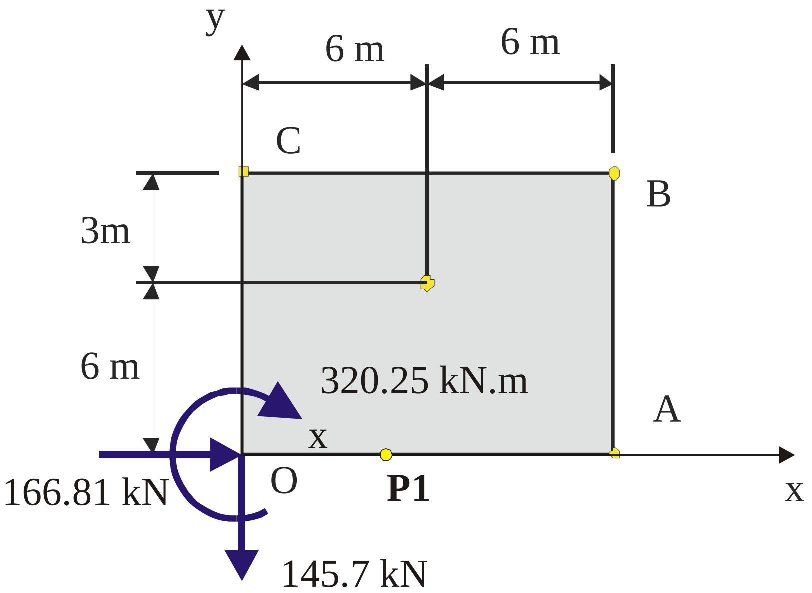
Equivalent Force-Couple at O



We first move the equivalent force-couple system at O to the point **P1** on the x-axis where the 320.25 kN.m clockwise couple will be eliminated or $\mathbf{M_{P1}} = 0$.

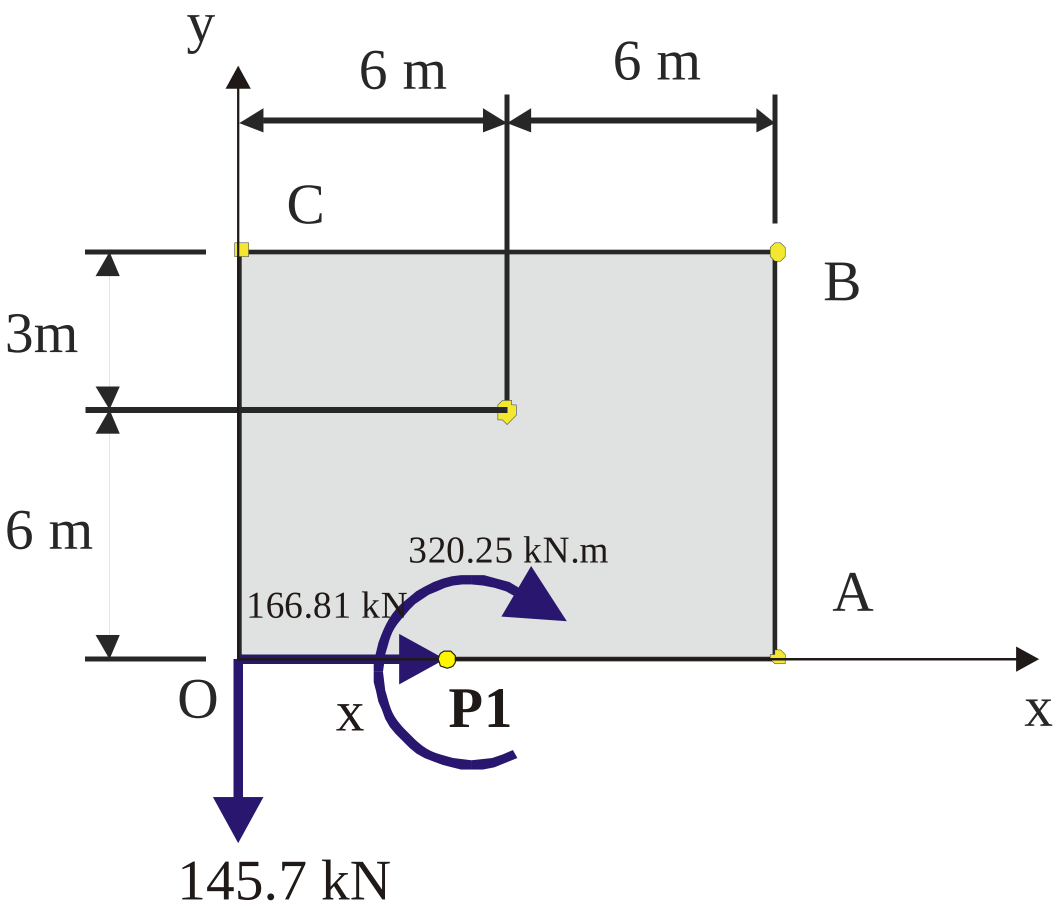
We need to determine the distance x such that $\mathbf{M_{P1}} = 0$.

To illustrate we will do it in steps.



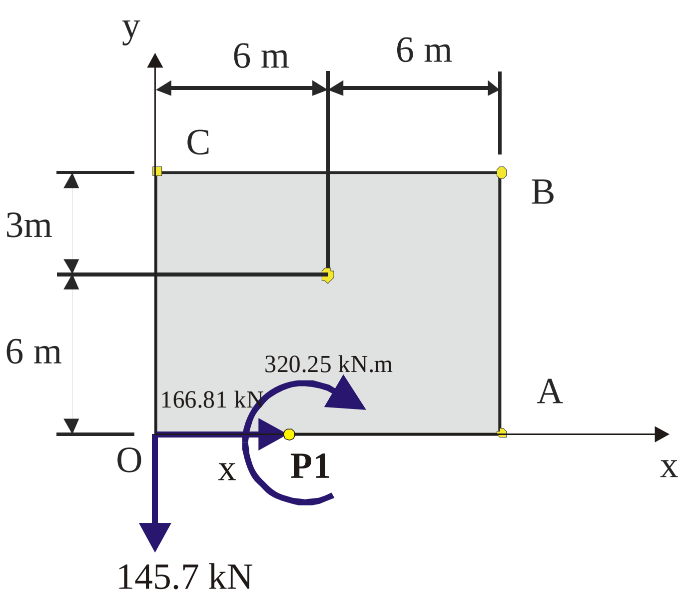
The 320.25 kN.m couple is a “Free” vector so we can move it anywhere. We show it applied at P1.

The 166.81 kN force has a line of action that passes through P1, therefore we apply the Principle of Transmissibility and slide it to P1. We are left with the 145.7 kN force still applied at O.



We know that when we move a force from one point of application to another point we MUST add into the system **the moment that the force has about the point we are moving it to**.

In the figure below we observe that if the 145.7 kN force is moved a distance x to P1 we must add into the system a **COUNTERCLOCKWISE** moment of 145.7x kN.m



We want to eliminate a CLOCKWISE moment of 320.25 kN.m (We have therefore chosen the correct direction to move the 145.7 kN force since it creates a COUNTERCLOCKWISE moment about P1.)

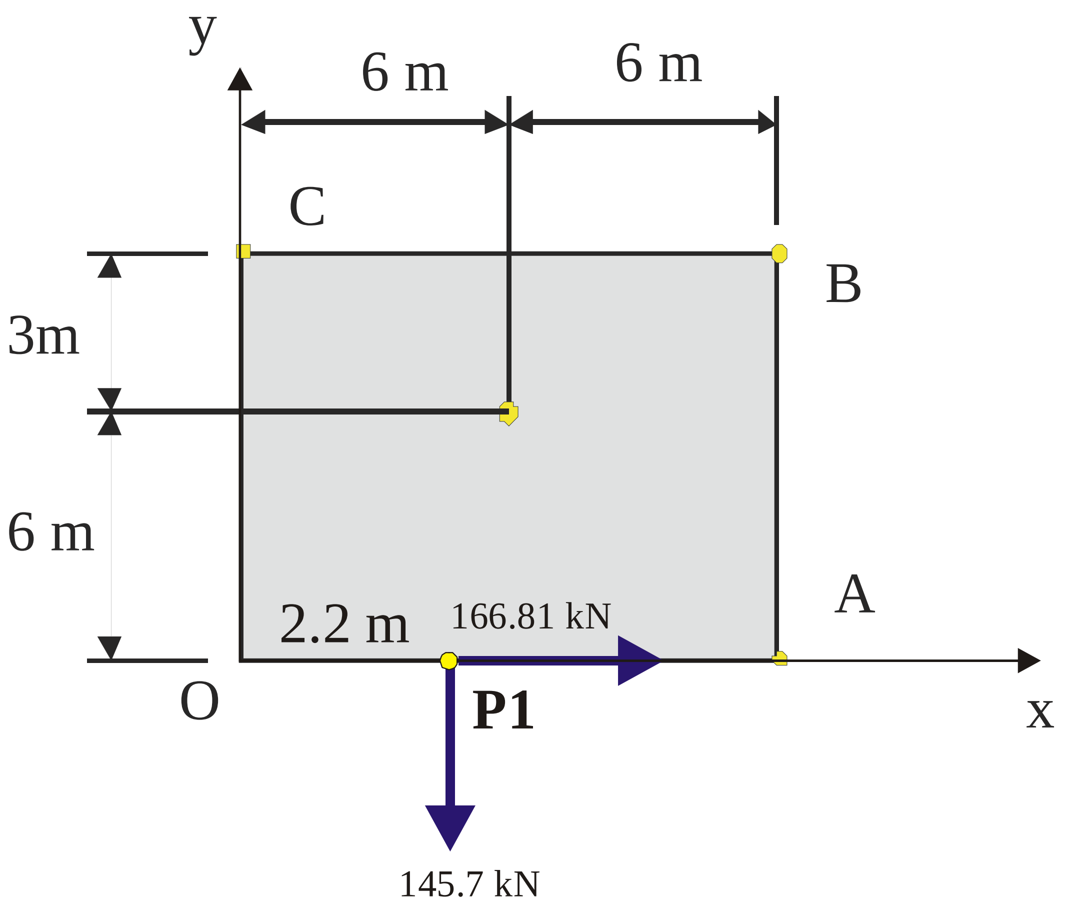
$$\mathbf{M}_{\mathbf{P1}} = 0$$

$$145.7x - 320.25 = 0$$

$$\therefore x = 2.2\text{m}$$

Resultant force at P1:

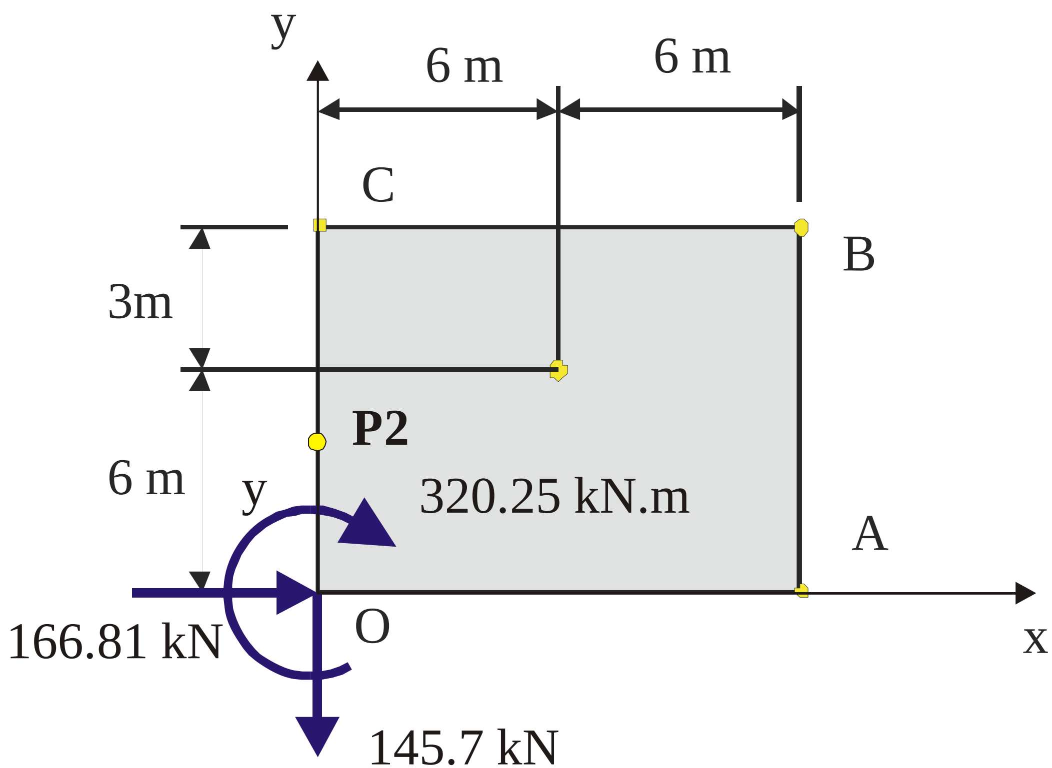
$$R = \sqrt{166.81^2 + 145.7^2} = 221.48\text{kN}$$



We now move the equivalent force-couple system at O to the point **P2** on the y-axis where the 320.25 kN.m clockwise couple will be eliminated or $\mathbf{M_{P2}} = 0$.

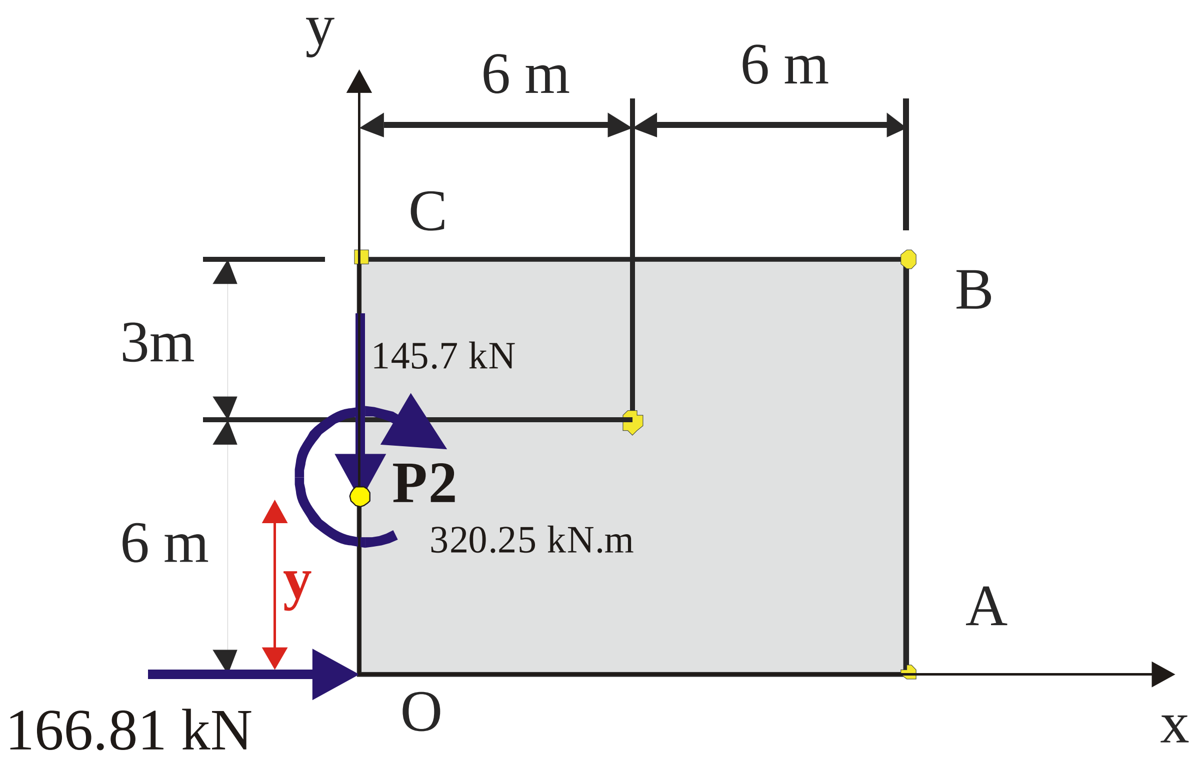
We need to determine the distance y such that $\mathbf{M_{P2}} = 0$.

To illustrate we will do it in steps.



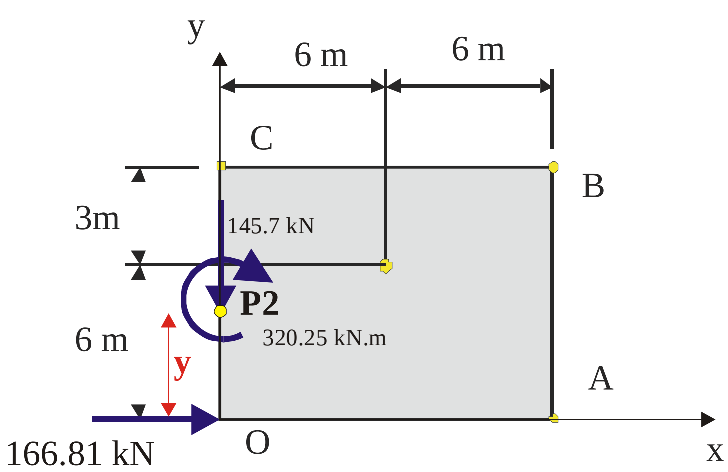
Again, the 320.25 kN.m couple is a “Free” vector so we can move it anywhere. This time, we show it applied at P2.

The 145.7 kN force has a line of action that passes through P2, therefore we apply the Principle of Transmissibility and slide it to P2. This time, we are left with the 166.81 kN force still applied at O.



We know that when we move a force from one point of application to another point we MUST add into the system **the moment that the force has about the point we are moving it to**.

In the figure below we observe that if the 166.81 kN force is move a distance y to P2 we must add into the system a **COUNTERCLOCKWISE** moment of 166.81y kN.m



We want to eliminate a CLOCKWISE moment of 320.25 kN.m (We have therefore chosen the correct direction to move the 166.81 kN force since it creates a COUNTERCLOCKWISE moment about P2.)

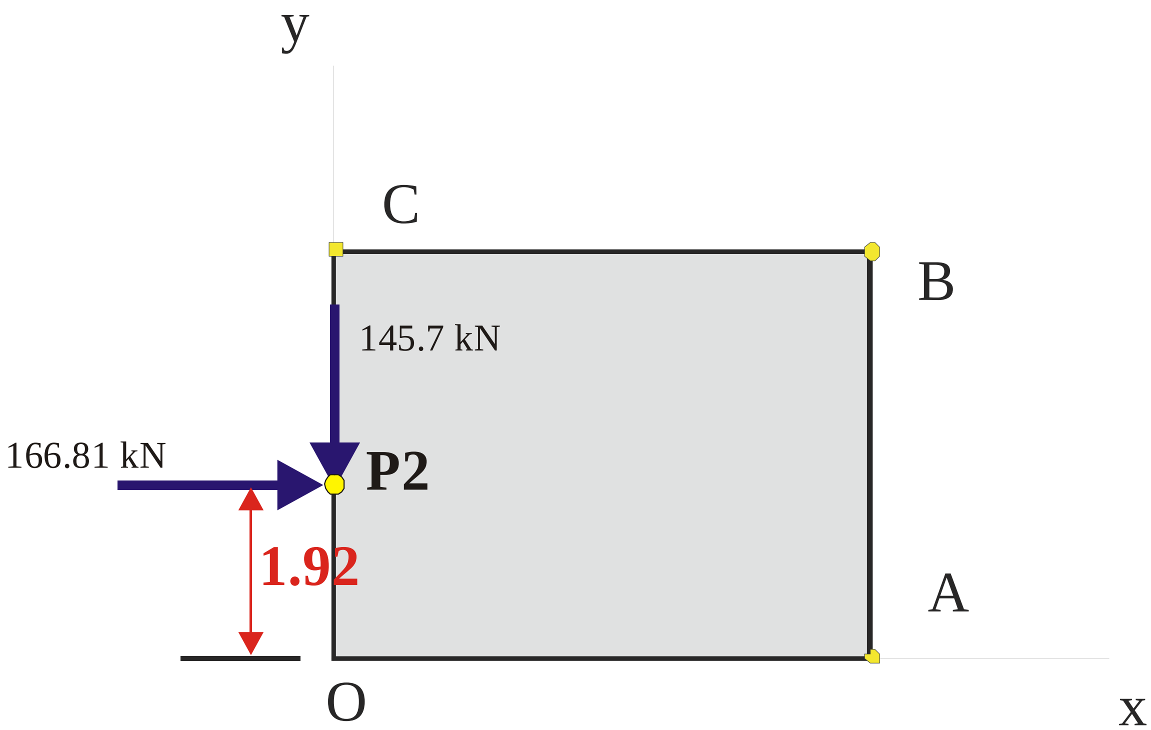
$$\mathbf{M}_{\mathbf{P2}} = 0$$

$$166.81y - 320.25 = 0$$

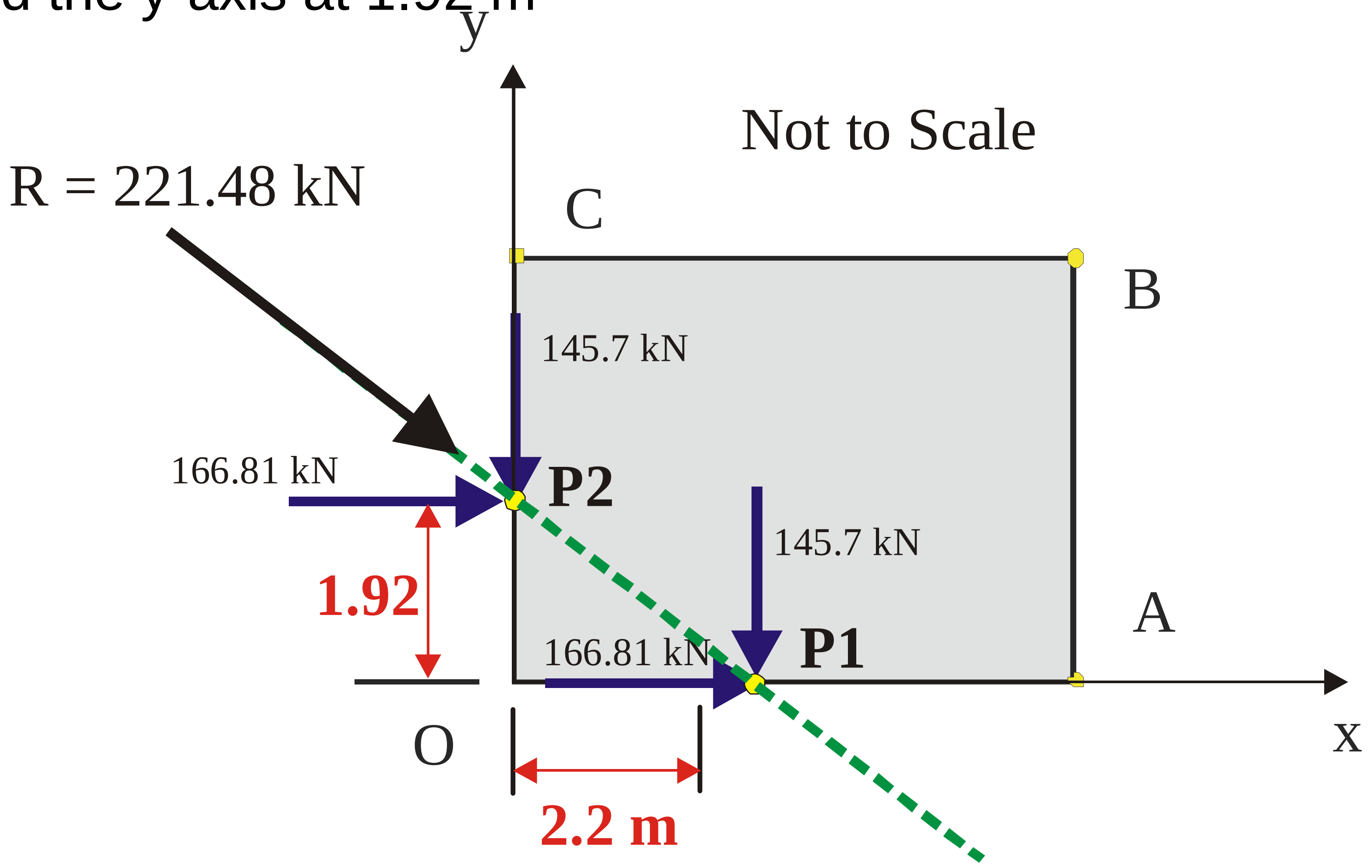
$$\therefore y = 1.92\text{m}$$

Resultant force at P2 :

$$R = \sqrt{166.81^2 + 145.7^2} = 221.48\text{kN}$$



The single force R = 221.48 kN intersects the x-axis at 2.3 m and the y-axis at 1.92 m



As a check, the ratio of the rectangular components of the resultant force will equal the ratio of the intersection points on the x-axis and the y-axis.

$$\frac{1.92}{2.2} = 0.873$$

$$\frac{145.7}{166.81} = 0.873$$