HOMEWORK ASSIGNMENT #1, Math 253

1. Sketch the curve $r = 1 + \cos\theta, 0 \leq \theta \leq 2\pi$, and find the area it encloses.

2. Find the dot product $\vec{a} \cdot \vec{b}$ in the following cases:

   (a) $\vec{a} =< 1, 0, -2 >, \vec{b} =< 2, 0, 1 >$. Are these vectors orthogonal?

   (b) $\vec{a} =< x_2y_3 - x_3y_2, x_3y_1 - x_1y_3, x_1y_2 - x_2y_1 >, \vec{b} =< x_1, x_2, x_3 >$, where the x_i, y_i are any real numbers. Are these vectors orthogonal?

   (c) $\vec{a}$ is a unit vector having the same direction as $\vec{i} + \vec{j}$ and $\vec{b}$ is a vector of magnitude 2 in the direction of $\vec{i} + \vec{j} - \vec{k}$.

3. Use cross products to find the following areas:

   (a) the area of the triangle through the points $P = (1, 1, 0), Q = (1, 0, 1), R = (0, 1, 1)$.

   (b) the area of the parallelogram spanned by the vectors $\vec{u} =< 1, 2, 0 >, \vec{v} =< a, b, c >$.

   (c) the areas of all 4 faces of the tetrahedron whose vertices are $(0, 0, 0), (a, 0, 0), (0, b, 0)$ and $(0, 0, c)$, where a, b, c are positive numbers.

4. Suppose $\vec{a}$ is a vector in 3-space. Show that $\left(\frac{\vec{a} \cdot \vec{i}}{|\vec{a}|}\right)^2 + \left(\frac{\vec{a} \cdot \vec{j}}{|\vec{a}|}\right)^2 + \left(\frac{\vec{a} \cdot \vec{k}}{|\vec{a}|}\right)^2 = 1.$

   Remark: The direction cosines of the vector $\vec{a}$ are by definition

   $$\cos\alpha = \frac{\vec{a} \cdot \vec{i}}{|\vec{a}|}, \cos\beta = \frac{\vec{a} \cdot \vec{j}}{|\vec{a}|}, \cos\gamma = \frac{\vec{a} \cdot \vec{k}}{|\vec{a}|}.$$

   The angles α, β, γ are the angles $\vec{a}$ makes with the positive directions of the x, y, z axes respectively.

5. (a) Find all vectors of length 2 that make equal angles with the positive directions of the 3 co-ordinate axes.

   (b) Find all unit vectors $\vec{v} = v_1\vec{i} + v_2\vec{j} + v_3\vec{k}$ making respective angles of $\pi/3, \pi/4$ with the positive directions of the x, y axes.

   (c) Find the angles of the triangle whose vertices are $(1, 0, 0), (0, 2, 0), (0, 0, 3)$.

   (d) Find the angle(s) between a diagonal of a cube and one of its edges.

6. A straight river $400m$ wide flows due west at a constant speed of $3km/hr$. If you can row your boat at $5km/hr$ in still water, what direction should you row in if you wish to go from a point A on the south shore to the point B directly opposite on the north shore? How long will the trip take?

7. Find equations of the planes satisfying the following conditions:

   (a) Passing through the point $(0, 2, -3)$ and normal to the vector $4\vec{i} - \vec{j} - 2\vec{k}$.

   (b) Passing through the point $(1, 2, 3)$ and parallel to the plane $3x + y - 2z = 15$.

   (c) Passing through the 3 points $(\lambda, 0, 0), (0, \mu, 0), (0, 0, \nu)$, where λ, μ, ν are non-zero real numbers.

   (d) Passing through the point $(-2, 0, -1)$ and containing the line which is the intersection of the 2 planes $2x + 3y - z = 0$ and $x - 4y + 2z = -5$.

8. Let $v_1 = (0, -1, 0), v_2 = (0, 1, 0), v_3, v_4$ be the 4 vertices of a regular tetrahedron. Suppose $v_3 = (x, 0, 0)$ for some positive x and v_4 has a positive z component. Find v_3 and v_4.