In Exercises 1-3, for the given transformation $T : \mathbb{R}^3 \to \mathbb{R}^3$, do the following:

(a) Find the image of $\vec{v} = (3, -1, 2)$ under T.
(b) Is T a linear transformation? If so, find the matrix associated with T.
(c) Find all vectors $\vec{v}$ whose image under T is $\vec{v'} = (1, 3, 2)$.
(d) Find the inverse transformation, T^{-1}, or give a reason why T^{-1} does not exist.
(e) Is T^{-1} a linear transformation? If so, find the matrix associated with T^{-1}.

1. $T(v_1, v_2, v_3) = (v_1', v_2', v_3')$, where

$$\begin{aligned} v_1' &= v_3 \\ v_2' &= v_1 + v_2 \\ v_3' &= v_1 v_2 \end{aligned}$$

2. $T : \mathbb{R}^3 \to \mathbb{R}^3$, $T(v_1, v_2, v_3) = (v_1', v_2', v_3')$, where

$$\begin{aligned} v_1' &= v_1 + 2v_2 - v_3 \\ v_2' &= v_2 + v_3 \\ v_3' &= 2 - v_3 \end{aligned}$$

3. $T : \mathbb{R}^3 \to \mathbb{R}^3$, $T(v_1, v_2, v_3) = (v_1', v_2', v_3')$, where

$$\begin{aligned} v_1' &= v_1 - 5v_3 \\ v_2' &= v_2 + 3v_3 \\ v_3' &= 2v_1 + 3v_2 \end{aligned}$$

4. Find all eigenvalues and corresponding eigenvectors for the matrix

$$A = \begin{pmatrix} 7 & -5 & 7 & -1 & 4 \\ 0 & 2 & 5 & -4 & 7 \\ 0 & 0 & 7 & 1 & 17 \\ 0 & 0 & 0 & 7 & 10 \\ 0 & 0 & 0 & 0 & 2 \end{pmatrix}.$$

5. Let A and E be square matrices of the same size. Suppose E is invertible and let $B = E^{-1}AE$. Show that if $(\lambda, \vec{v})$ is an eigenpair of the matrix A, then $(\lambda, E^{-1}\vec{v})$ is an eigenpair of the matrix B.