**UNIVERSITY OF MANITOBA**

DATE: April 20, 2013 FINAL EXAMINATION



COURSE: MATH 2130 TIME: 3 hours

EXAMINATION: Engineering Mathematical Analysis 1 EXAMINER: M. Davidson

---

[6] 1. Find the distance between the line $\dfrac{x-4}{3} = \dfrac{y-3}{2} = \dfrac{z-4}{-2}$ and the line $x = 1+t\,,\ y = -4+t\,,\ z = 2$.

[8] 2. Let $C$ be the curve of intersection of the surfaces $z+x = 3y^2$ and $x+3y-2z = 9$.

(a) Find a parametric representation for $C$ in the direction of decreasing $y$.

(b) Set up but **do not evaluate** a definite integral to find the length of the curve $C$ from the point $(4,1,-1)$ to the point $(24,-3,3)$.

[8] 3. Find equations, in parametric form, of the line tangent to the curve $x^2yz + 2x + y = z^3 + 7\ ,\ 3x^2y + 2xyz = -y$ at the point $(1,3,-2)$.

[9] 4. For each of the following, either evaluate the limit, or show that the limit does not exist.

(a) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{2x^2+y^2}{x^2+y^2}$,

(b) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{2x^3}{x^2+y^2}$. (Hint: $\left|\dfrac{x^2}{x^2+y^2}\right| \le 1$.)

[6] 5. Show that for any differentiable function $f$, the function $u(x,y) = f(x^2-y^3)+x^2y$ satisfies the equation $3y^2\dfrac{\partial u}{\partial x} + 2x\dfrac{\partial u}{\partial y} = 6xy^3 + 2x^3$.

[14] 6. Find the absolute maximum and absolute minimum of the function

$$f(x,y) = x + y - xy^2\,,$$

over the triangular region with corners $(0,0), (0,2), (6,2)$.

[6] 7. Evaluate the following double iterated integral:

$$\int_0^4 \int_{\frac{x}{2}}^2 e^{y^2}\,dy dx.$$

[9] 8. Find the volume of the solid obtained by revolving an annulus (the area between concentric circles; a washer) having inner radius of 1m and outer radius of 3m, about a line that is 5m from the center of these circles.

[6] 9. A thin quadrilateral plate has corners $(1,1),(4,1),(4,2)$ and $(2,2)$ and density given by $\rho(x,y) = x+y$. Find the moment of inertia of this plate about the x-axis ($I_x$). (A numerical answer need not be simplified.)

[6] 10. Set up, but **do not evaluate**, a double iterated integral to find the surface area of the portion of $z = 2xy$ that lies below $z = 4 - x^2 - y^2$ in the first octant.

[12] 11. Set up all six triple iterated integrals (in Cartesian coordinates, each having a different order of integration) for the volume of the solid in the first octant that is bounded by the surfaces:

$$y^2 + z^2 = 9 \;\; ; \;\; y = 3x \;\; ; \;\; x = 0 \;\; ; \;\; z = 0.$$

**Do not evaluate.**

[10] 12. Set up, but **do not evaluate**, a triple iterated integral to evaluate

$$\iiint_V (x^2 + y^2)\, dV$$

where $V$ is the region bounded by

$$\sqrt{3}z = \sqrt{x^2 + y^2} \quad \text{and} \quad x^2 + y^2 + z^2 = 4$$

using

(a) Cylindrical coordinates.

(b) Spherical coordinates.