ENG 1440
Introduction to Statics
Distance Education
STUDY PROBLEM SET


J. Frye, Ph. D., P. Eng.
June, 2011

ENG 1440 Introduction to Statics
Distance Education
Study Problem Set

Introduction:

The following Study Problem Set is intended to be used in conjunction with the course material found in the Text/Notes, Introduction to Statics 2011-2012 Edition by M. J. Frye, N. Rattanawangcharoen and A. H. Shah.

There are thirty (30) solved problems contained in the problem set. These problems cover the different concepts and most of the problem types found in each of the chapters of the Text/Notes. Problems have been selected that demonstrate the different concepts that are fundamental to gaining a sound foundation to the understanding of Engineering Statics.

The problem solutions emphasize a step-by-step structured approach that students should find easy to follow. Following this approach and presenting solutions in the manner suggested will quickly lead to a mastery of the concepts while reducing the probability of calculation errors. Professors and Teaching Assistants who grade your work will find your approach to problem solution easy to follow and will have little difficulty in fairly assessing your work. Equally, if not more important is the fact that many of you will be working in engineering offices where your work will be checked by other Professional Engineers.

M. J. Frye, Ph. D., P. Eng.
June, 2011

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CHAPTER 1
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Study Problem # 1
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Right Handed Cartesian Coordinate System

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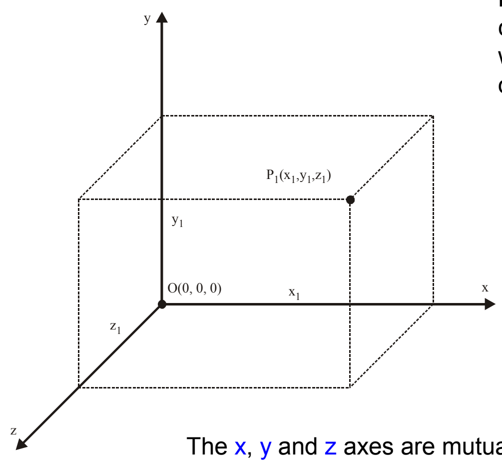
SPACE

In our study of Statics, we define space as the region containing the bodies that are the subject of our study. The physical dimensions of space are described through the use of coordinate axes.

Any set of mutually perpendicular straight lines intersecting at a reference point called the origin, O, can be used as a set of coordinate axes. The position of any point can be described by specifying its coordinates relative to a convenient origin.

We will use a **Right Handed Cartesian Coordinate System** or just a right handed coordinate system.

Right Handed Coordinate System



Point P_1 has coordinates x_1, y_1, z_1 with respect to the origin, O.

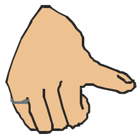
The x, y and z axes are mutually perpendicular to each other and are referred to as the **PRINCIPAL AXES**.

Given the Positive Directions of Any Two (2) of the Principal Axes, What is the Positive Direction of the Third Principal Axis? – We use the <u>Right Hand Rule</u>. To help us do this we use a <u>Right Handed Triad</u>.

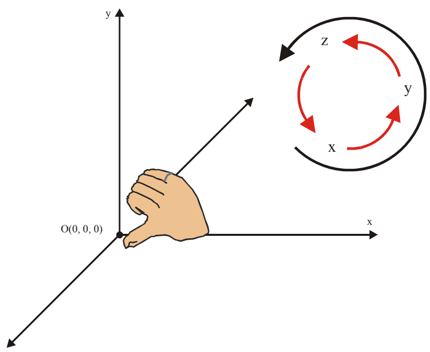
Right Handed Triad

The **Right Handed Triad** is a curved arrow with a counter-clockwise sense. Inside are placed the letters x, y, and z (also in the counter-clockwise direction) to represent the principal coordinate axes.

Interpretation of the Right Handed Triad



Given any two (2) of the positive principal axes, we use the curved fingers and thumb of our right hand to determine the positive sense of the third principal axis.

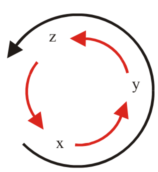


+ve z-axis

Example: given the x-axis and y-axis as shown, does the **positive z-axis** come out of the page or into the page???

Using the Right Handed Triad, we curve the fingers of our right hand so as to rotate the x-axis into the y-axis. The thumb of our right hand points in the positive direction of the z-axis.

Interpretation of the Right Handed Triad



If Given:

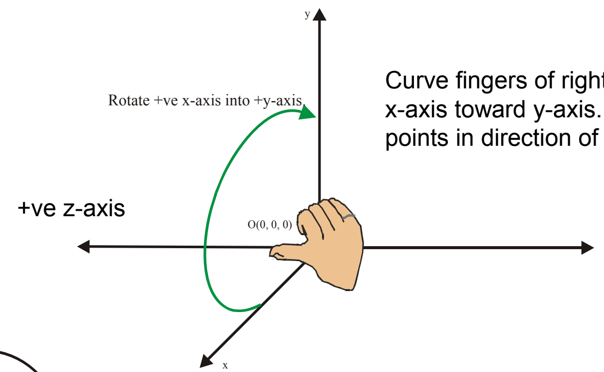
+ve x-axis and +y-axis, we rotate **90°** the x-axis into the y-axis to get **+ve z-axis**

+ve y-axis and +z-axis, we rotate **90°** the y-axis into the z-axis to get **+ve x-axis**

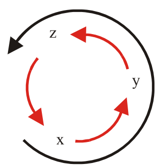
+ve z-axis and +x-axis, we rotate **90°** the z-axis into the x-axis to get **+ve y-axis**

IMPORTANT: To use the Right Handed Triad to apply the Right Hand Rule always proceed **in sequence** in a counter-clockwise direction.

Example: Given +ve x-axis and +y-axis. What is direction of +z-axis?

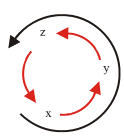


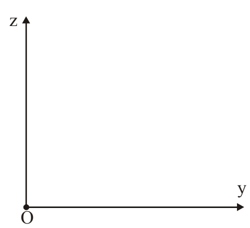
Curve fingers of right hand from x-axis toward y-axis. Thumb points in direction of +ve z-axis.

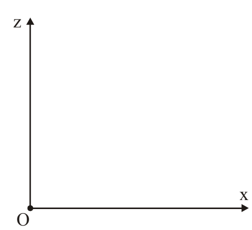


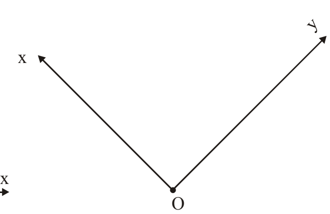
We rotate x-axis into y-axis.

Examples: Determine the direction of the Missing Principal Axis

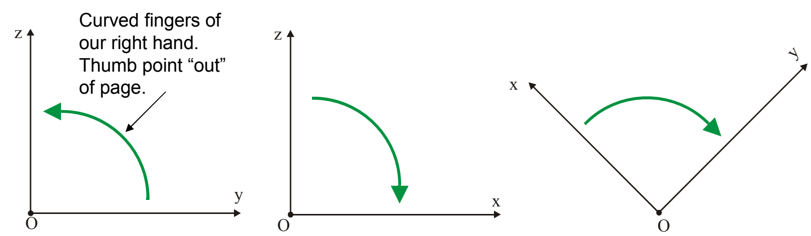




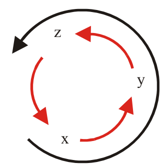




Solution:



+ve x-axis out of page +ve y-axis into page +ve z-axis into page



CHAPTER 2
Distance Education
Study Problem # 2
J. Frye
May, 2011

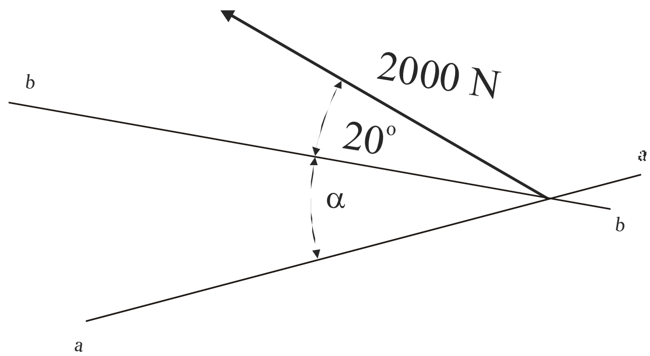
Resultant Force of Two Components

Example: 2.4 - Question 1 Winter Term 2009 – Term Test#1

The force ***F*** of magnitude *2000 N* in the Figure below is to be resolved into two components along line *a-a* and *b-b*. Knowing that the component of F along line *b-b* is to be *3407 N*, determine the angle α and the magnitude of the component along line *a-a*. Use:

a) Graphical method (parallelogram law or triangle rule: (Do not forget to state your scale) (5 marks)

b) Trigonometric method (sine and/or cosine laws) (5 marks)



Recommendation: Make a small Table indicating what you **know** and what you **don't know** and make a "rough" sketch.
In this example we know:

Magnitude and Direction of $\mathbf{F}_{bb}$

Magnitude and Direction of the 2000 N force (the Resultant, **R** of $\mathbf{F}_{aa}$ and $\mathbf{F}_{bb}$)

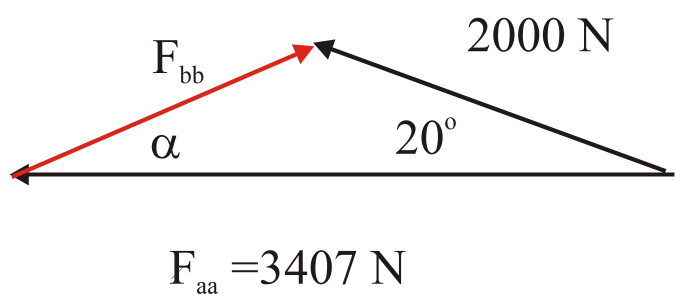
We do not know the magnitude of $\mathbf{F}_{aa}$ (the other component of the 2000 N force**)** nor do we know the direction of its Line of Action (α) relative to $\mathbf{F}_{bb}$.

Force	Magnitude	Direction
$\mathbf{F}_{aa}$	**?**	**?**
$\mathbf{F}_{bb}$	3407	(0°)
R	2000	20°

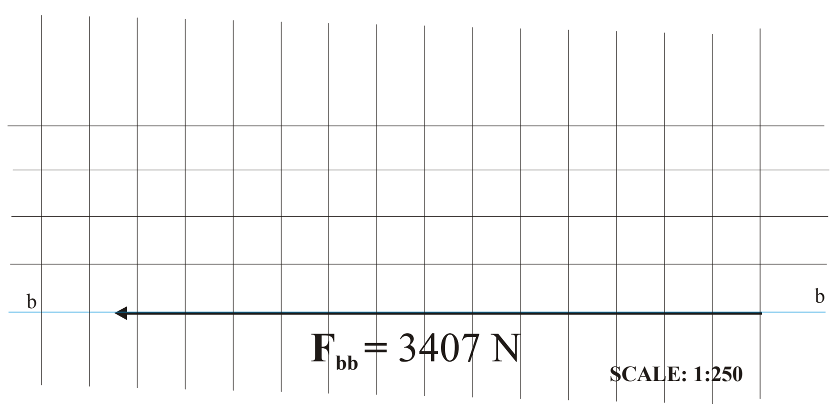
"Rough" sketch" (Start by plotting what you know)

Estimate length of F_{aa}, 2000 N force and angle α = 20^o.
F_{aa} and **F_{bb}** placed **"Tip-to-Tail"** give R = 2000 N.

R = F_{aa} + F_{bb} (Vector Addition)
Our rough sketch will give us a "Ballpark" magnitude of F_{bb} and angle α.

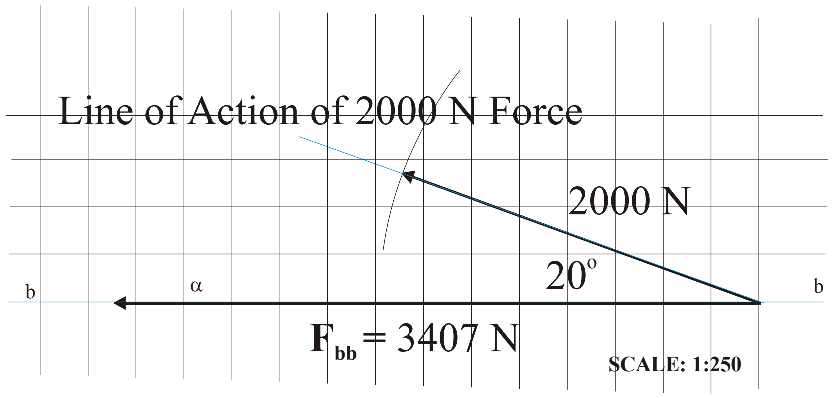


Start by selecting a suitable scale and draw in **F_{bb}** = 3407 N

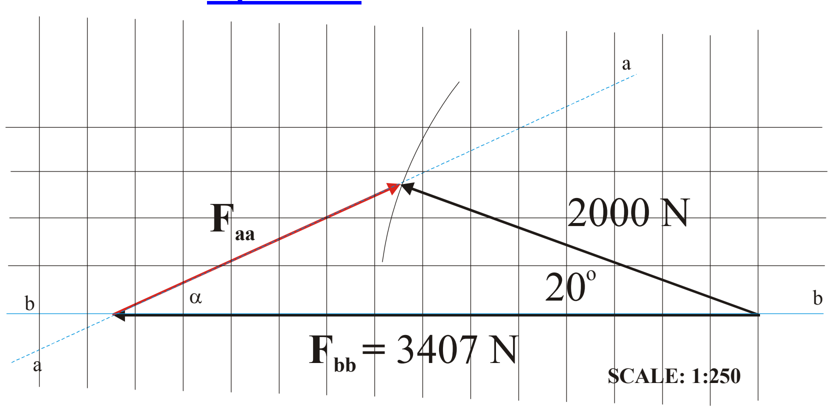


F$_{bb}$ is one of the components of the 2000 N force that has a Line of Action that is 20^o to the Line of action of **F**$_{bb}$

Draw in the Line of Action of the 2000 N force at an angle of 20^o to **F**$_{bb}$. Scale off 2000 N.

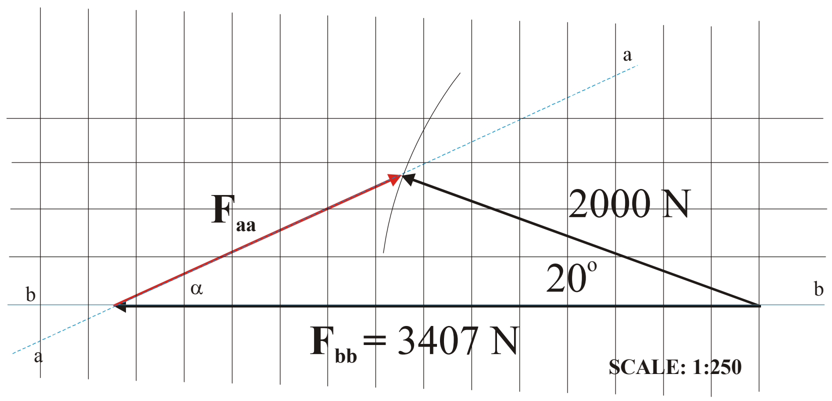


Draw in **F**$_{aa}$ from the "Tip" of **F**$_{bb}$ to the "Tip" of the 2000 N force.
Use your scale to determine the magnitude of **F**$_{aa}$ and use your protractor to measure the angle α.
NOTE: **F**$_{aa}$ andd **F**$_{bb}$ are the **components** of the 2000 N force and are "**Tip-to-Tail**".

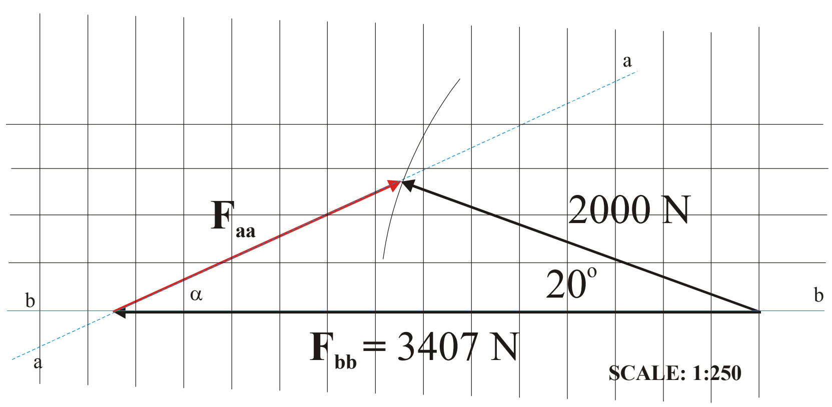


Graphical Solution

(Note – The Scale Drawing can also be used for the TRIG Solution)



Graphical Solution - Results



$$F_{aa} \approx 1667N$$

$$\alpha \approx 25^{o}$$

TRIG Solution

Cosine Rule:

$a^2 = b^2 + c^2 - 2bc\cos A$

$F_{aa}{}^2 = 3407^2 + 2000^2 - 2(3407)(2000)\cos 20^{\circ}$

$F_{aa} = 1673.6\ \text{N}$

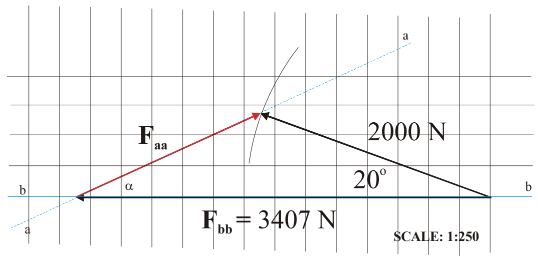
Sine Rule:

$\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

$\dfrac{1673.8}{\sin 20^{\circ}} = \dfrac{2000}{\sin \alpha}$

$\sin \alpha = 0.40868$

$\alpha = 24.12^{\circ}$



CHAPTER 2
Distance Education
Study Problem # 3
J. Frye
May, 2011

Components of a Given Force Along Specified Lines of Action

1. Graphical Solution
2. Trig Solution

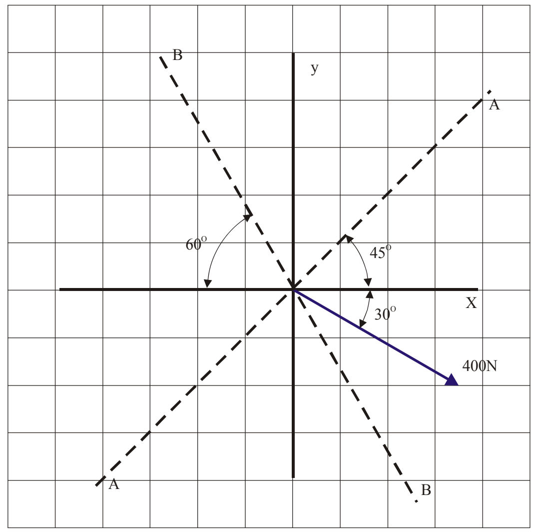
A 400 N force acts at 30° to the horizontal. Determine the components of this force along the Lines A-A and B-B:

a) By a graphical solution and

b) By Trigonometry.

Force	Magnitude	Direction
R	400 N	30°
F_{AA}	?	45°
F_{BB}	?	60°

For Problems involving two (2) components and a resultant, **R** it is recommended that we make a table of information that is given and what we are asked to determine. We then plot the information we are given. In this example we first plot the resultant, **R** and the Lines-of-Action of its components.



SOLUTION

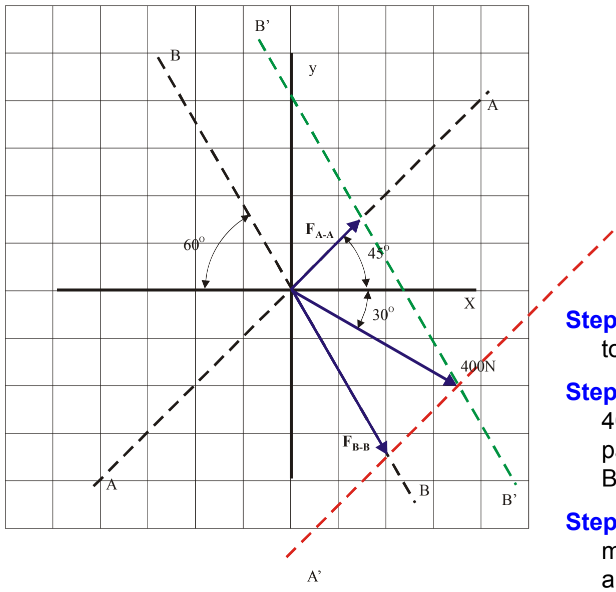
In this problem we are asked to resolve a given force, **F**, into two (2) components, **P** and **Q**, along two given Lines-of-Action.

We apply the **Parallelogram Law for the Addition of Forces**, **F** = **P** + **Q**:

We can determine the magnitude and sense of each of the components of the given force by the Parallelogram Law.

1. Draw the given force, **F**, to be resolved to an appropriate scale.
2. Draw lines through the “Tip” (arrowhead) of **F** parallel to the given Lines-of-Action to construct a parallelogram with **F** as the diagonal.
3. The sides of the parallelogram are the forces, **P** and **Q**. We can determine them graphically or trigonometrically by applying the Law of Sines.

a) Using Parallelogram Rule –Graphical (Approximate) Solution



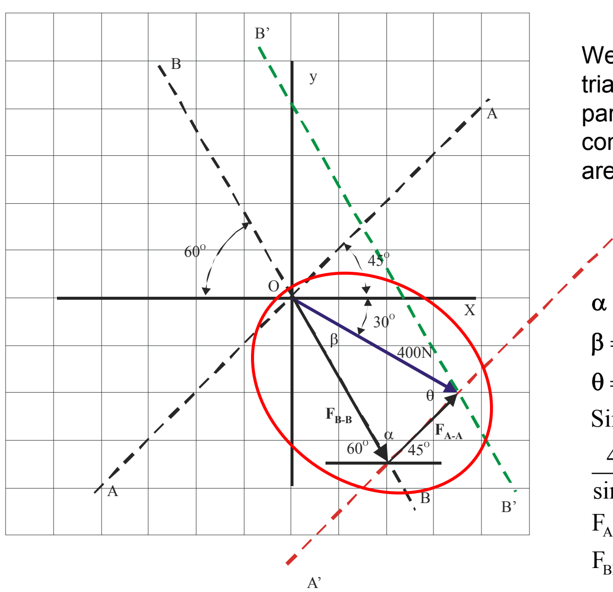
Note: You must state the scale being used when you use a graphical solution..

Step 1 – Draw the 400 N force to a suitable scale at 30°

Step 2 – From the "Tip" of the 400 N force draw Line A'-A' parallel to Line A-A and Line B'-B' parallel to Line B-B

Step 3 – Using the same scale measure components $\mathbf{F_{A\text{-}A}}$ and $\mathbf{F_{B\text{-}B}}$

a) Using Trig (Exact) Solution and Applying the "Triangle Rule"



We use the "lower" triangle of the parallelogram we have constructed. $\mathbf{F_{AA}}$ and $\mathbf{F_{BB}}$ are "Tip" to "Tail".

$$\alpha = 180^\circ - 45^\circ - 60^\circ = 75^\circ$$

$$\beta = 60^\circ - 30^\circ = 30^\circ$$

$$\theta = 180^\circ - 30^\circ - 75^\circ = 75^\circ$$

Sine Rule:

$$\frac{400}{\sin 75^\circ} = \frac{F_{AA}}{\sin 30^\circ} = \frac{F_{BB}}{\sin 75^\circ}$$

$$F_{AA} = 207.06\text{N}$$

$$F_{BB} = 400\text{N}$$

CHAPTER 2
Distance Education
Study Problem # 4
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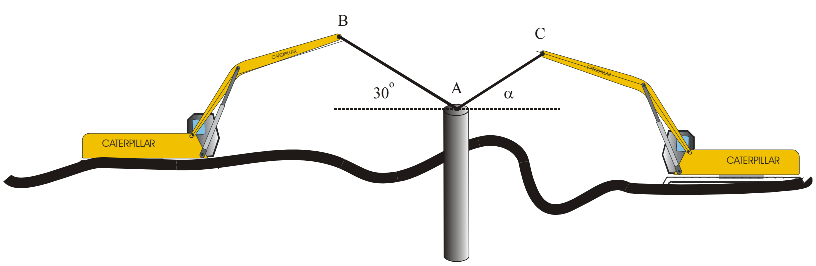
May, 2011

Components of a Resultant Force, R

Given:
1. Magnitude and direction of the Resultant, **R**
2. Magnitude and direction of one of the components of **R**
3. Magnitude and direction of other component of **R** are unknown but given that it is a **MINIMUM**

Example

The two cranes shown in the figure are attempting to pull a pile from the ground. Given that the tension in cable AB attached to the crane on the left is 3.4 kN acting at 30^o. What is the magnitude and direction of the minimum tension in cable AC attached to the crane on the right such that the resultant force, **R** acting on the pile is vertical. What is the magnitude of **R**.



Solution Steps

STEP 1 – It is recommended that you make up a table and indicate the information given in the problem statement,

STEP 2 – Plot the information you are given in the problem statement. (magnitudes and lines-of-action of the Resultant, **R** and any of its components) (Read the question carefully – in this type of component problem it will be stated that either the resultant, **R** or one of the two components is to be the "SMALLEST" or to be a minimum. **Plot to a suitable scale so that we the plot can be used as a graphical solution.**

STEP 3 –In this example you are given the magnitude and direction of $\mathbf{T_{AB}}$ and the line of action of the resultant, **R** of $\mathbf{T_{AB}}$ and $\mathbf{T_{AC}}$. From the plot we have started in **STEP 2**, we examine the plot and place $\mathbf{T_{AC}}$ "Tip-to-Tail" with $\mathbf{T_{AB}}$ such that the length of the $\mathbf{T_{AC}}$ is the shortest ("smallest" or "minimum") distance to the vertical line of action of **R**.

STEP 4 – Measure the length of $\mathbf{T_{AC}}$ and angle α for a graphical solution or use the triangle formed by $\mathbf{T_{AB}}$, $\mathbf{T_{AC}}$ and **R** for a trig solution.

STEP 1 – Table of Given Information

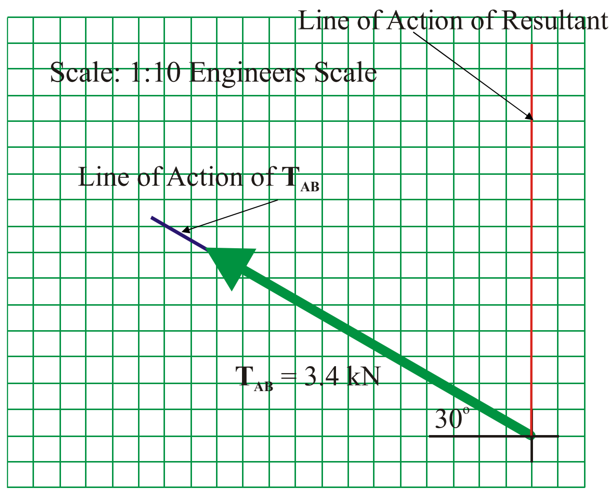
When we fill out the table indicating the given information in the problem statement it may seem that there is not enough information to solve the problem.

Force	Magnitude	Direction
T_{AB}	3.4 kN	30º
T_{AC}	?	?
R	?	Vertical

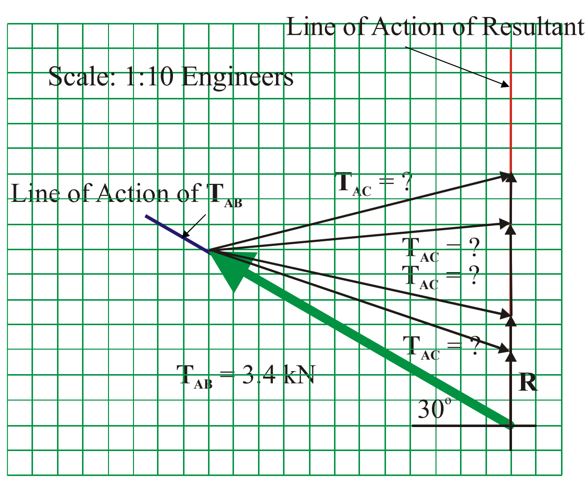
The important piece of information necessary to solve the problem is that the magnitude of T_{AC} is a **MINIMUM**.

Force	Magnitude	Direction
T_{AB}	3.4 kN	30º
T_{AC}	**MINIMUM**	?
R	?	Vertical

STEP 2 – We select a suitable scale and plot the information we are given. In this example we plot $\mathbf{T_{AB}}$ and the Line-of-Action of R.

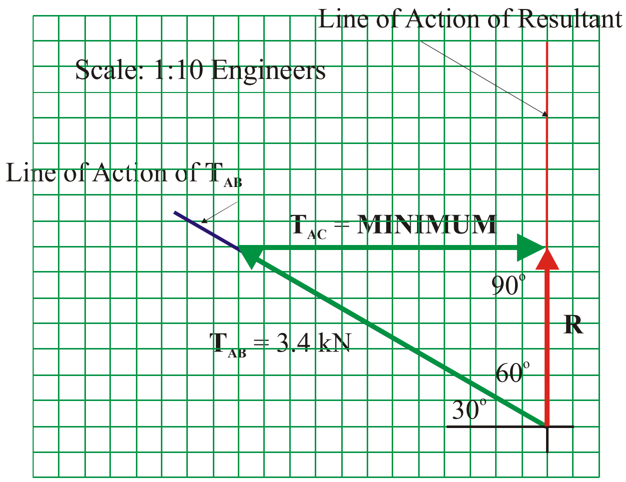


STEP 3 – We examine the plot from STEP 2. We place the "Tail" of $\mathbf{T_{AC}}$ on the "Tip" of $\mathbf{T_{AB}}$. The vertical resultant, **R** is drawn from the "Tail" of TAB to the "Tip" of $\mathbf{T_{AC}}$ such that length of $\mathbf{T_{AC}}$ is "smallest" or a "minimum". In the figure below we have shown a number of possibilities for $\mathbf{T_{AC}}$ that will give a vertical resultant, but only one is a minimum.



We observe that $\mathbf{T_{AC}}$ is a minimum (the length of arrow $\mathbf{T_{AC}}$ is shortest) when $\mathbf{T_{AC}}$ is horizontal.

STEP 4 – We measure $\mathbf{T_{AC}}$ and angle α for a graphical solution and the triangle formed by $\mathbf{T_{AB}}$, $\mathbf{T_{AC}}$ and **R** for the Trig Solution



Trig Solution:

$$\sin 60^{o} = \frac{T_{AC}}{3.4}$$

$$T_{AC} = 2.94\text{kN}$$

$$\cos 60^{o} = \frac{R}{3.4}$$

$$R = 1.7\text{kN}$$

CHAPTER 2
Distance Education
Study Problem # 5
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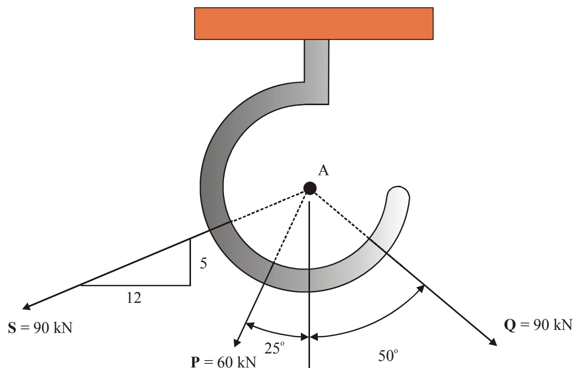
May, 2011

Resultant of Concurrent Forces

Rectangular Components by Inspection

Example: Three (3) forces, **S**, **P** and **Q** are applied to the hook shown in the figure. The lines-of-action of these forces are all CONCURRENT (pass thru the same point). Determine the resultant, **R** of these forces.



SOLUTION STEPS

STEP 1 – Establish a sign convention for positive rectangular force components

STEP 2 – Determine the magnitude of the rectangular (x and y) components using the trig relationships and the algebraic sign by INSPECTION according to the established sign convention.

STEP 3 – Find the algebraic sums ($\mathbf{R_x}$ and $\mathbf{R_y}$) of the x and y components of the concurrent forces.

STEP 4 – Determine the magnitude of the resultant force:

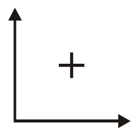
$$R = \sqrt{(R_x)^2 + (R_y)^2}$$

STEP 5 – Show the final answer. Do not forget that the resultant, **R** is a vector and you must specify both magnitude and direction unless you are asked to determine only the magnitude, R of the resultant. Direction may be specified by showing the slope of **R** or by showing the angle **R** makes with either the x-axis or the y-axis.

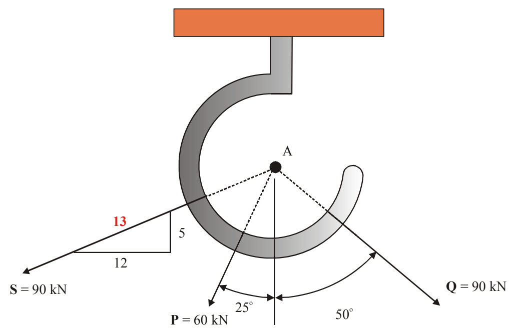
Resolution of Forces into Rectangular Components:

In this problem note that for forces **P** and **Q** the angle that each of these force makes with the vertical is given in degrees, while the slope is specified for force **S**.

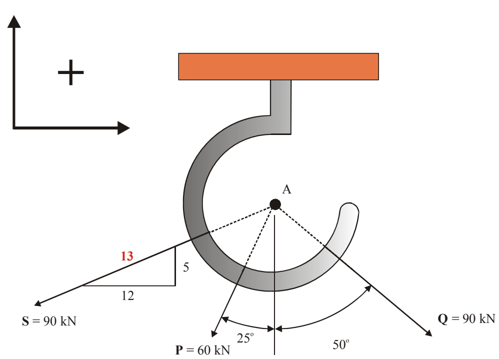
<u>For force **S**, work with the slope triangle to determine the rectangular components of **S.**</u>



Positive Sign Convention



Rectangular Components of force S working with the slope triangle and determining algebraic signs by inspection.



Calculate the hypotenuse of the slope triangle of S and insert it on the figure.

$$\sqrt{5^2 + 12^2} = 13$$

Component Magnitudes:

$$S_x = \frac{12}{13}(90) = 83.08\text{kN}$$

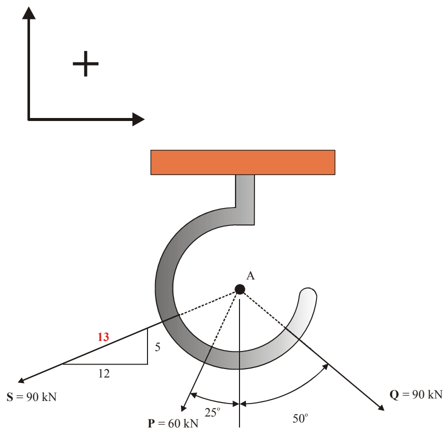
$$S_y = \frac{5}{13}(90) = 34.62\text{kN}$$

Algebraic Signs by Inspection:

$$S_x = -83.08\text{kN}$$

$$S_y = -34.62\text{kN}$$

Resultant Force, R



Rectangular Components of R:

$R_x = \sum F_x$

$-\frac{12}{13}(90) - 60\sin 25^{\circ} + 90\sin 50^{\circ} = -39.49\text{kN}$

$R_y = \sum F_y$

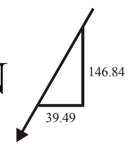
$-\frac{5}{13}(90) - 60\cos 25^{\circ} - 90\cos 50^{\circ} = -146.84\text{kN}$

Magnitude of R:

$R = \sqrt{(-39.49)^2 + (-146.84)^2} = 152.06\text{kN}$

Final Answer:

$\therefore \mathbf{R} = 152.06\text{kN}$



CHAPTER 2
Distance Education
Study Problem # 6
J. Frye
May, 2011

Resultant Force of Two Components

Graphical Solution

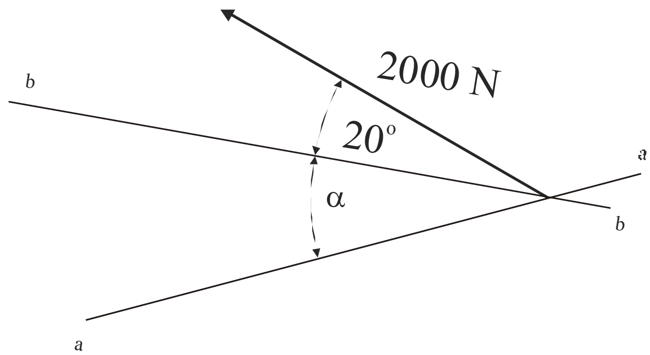
Trig Solution

Example:

Determine the magnitude of the component of the *2000 N* force along the *Line a-a* if the magnitude of the component along the *Line b-b* is *3407 N*. Also determine the angle α. Use

a) Graphical method (parallelogram law or triangle rule: (Do not forget to state the scale that is used!)

b) Trigonometric method (sine and/or cosine laws).



Recommendation: Make a small Table indicating what you **know** and what you **don't know** and make a "rough" sketch.
In this example we know:

Magnitude and Direction of $\mathbf{F_{bb}}$

Magnitude and Direction of the 2000 N force (the Resultant, **R** of $\mathbf{F_{aa}}$ and $\mathbf{F_{bb}}$)

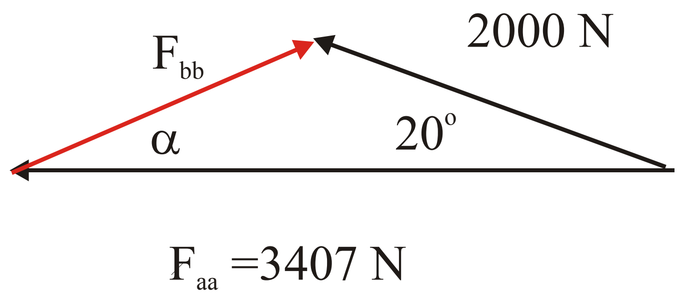
We do not know the magnitude of $\mathbf{F_{aa}}$ (the other component of the 2000 N force**)** nor do we know the direction of its Line of Action relative to $\mathbf{F_{bb}}$.

Force	Magnitude	Direction
$\mathbf{F_{aa}}$	**?**	**?**
$\mathbf{F_{bb}}$	3407	(0°)
R	2000	20°

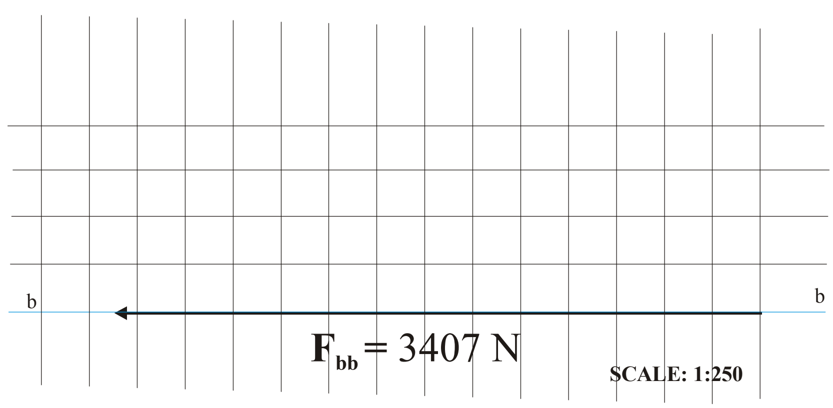
“Rough” sketch”

Estimate length of F_{aa}, 2000 N force and angle α = 20°.
$\mathbf{F_{aa}}$ and $\mathbf{F_{bb}}$ placed “Tip-to-Tail” give R = 2000 N.

$\mathbf{R = F_{aa} + F_{bb}}$ (Vector Addition)
Our rough sketch will give us a “Ballpark” magnitude of Fbb and angle α.

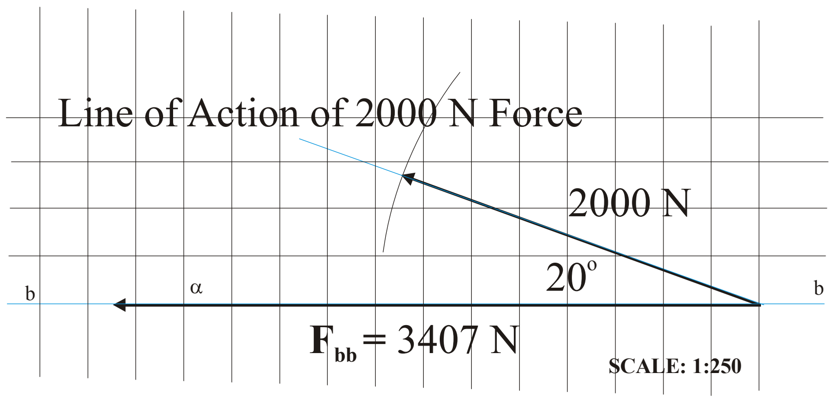


Start by selecting a suitable scale and draw in $\mathbf{F_{bb}}$ = 3407 N

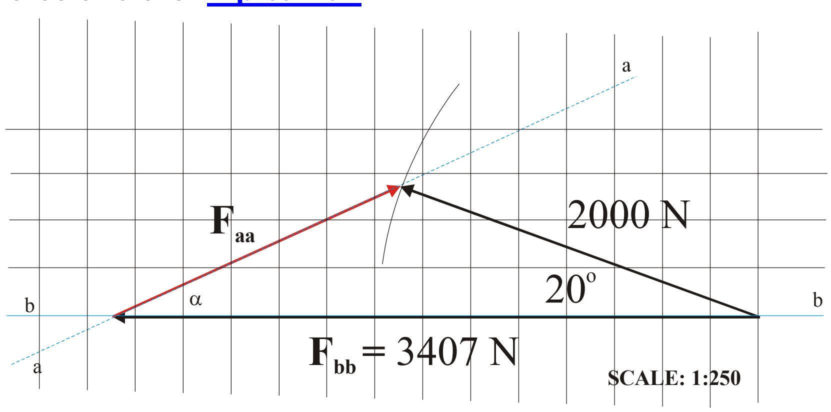


F$_{bb}$ is one of the components of the 2000 N force that has a Line of Action that is 20^o to the Line of action of **F**$_{bb}$

Draw in the Line of Action of the 2000 N force at an angle of 20^o to **F**$_{bb}$. Scale off 2000 N.

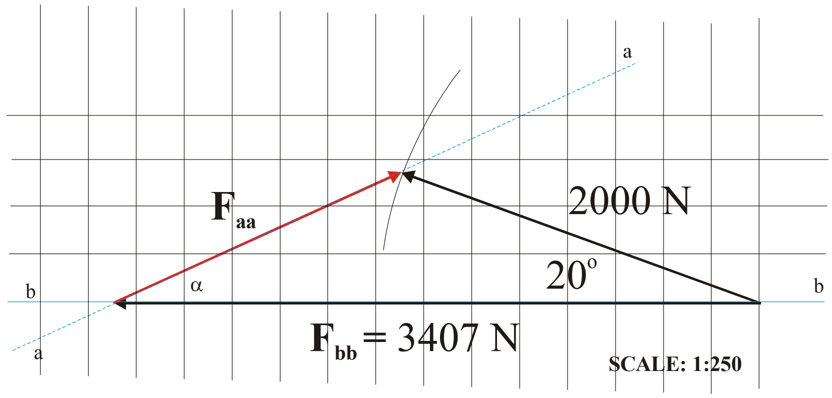


Draw in **F**$_{aa}$ from the "Tip" of **F**$_{bb}$ to the "Tip" of the 2000 N force.
Use your scale to determine the magnitude of **F**$_{aa}$ and use your protractor to measure the angle α.
NOTE: **F**$_{aa}$ andd **F**$_{bb}$ are the **components** of the 2000 N force and are "**Tip-to-Tail**".

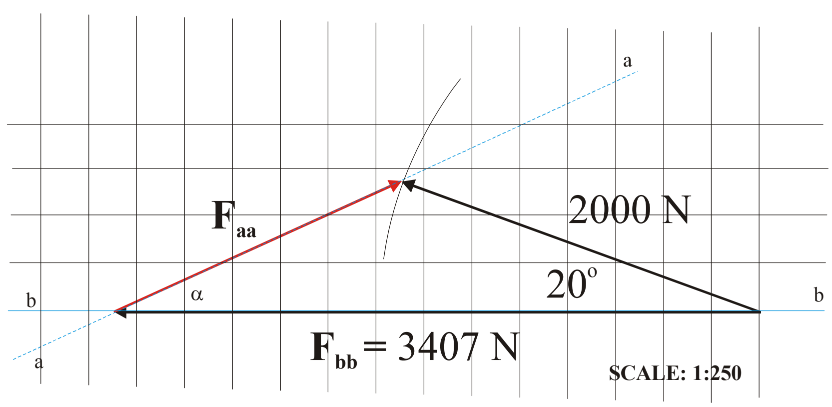


Graphical Solution

(Note – The Scale Drawing can also be used for the TRIG Solution)



Graphical Solution - Results



$$F_{aa} \approx 1667\text{N}$$

$$\alpha \approx 25^o$$

TRIG Solution

Cosine Rule:

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$F_{aa}{}^2 = 3407^2 + 2000^2 - 2(3407)(2000)\cos 20^\circ$$

$$F_{aa} = 1673.6\ \mathrm{N}$$

Sine Rule:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{1673.8}{\sin 20^\circ} = \frac{2000}{\sin\alpha}$$

$$\sin\alpha = 0.40868$$

$$\alpha = 24.12^\circ$$

CHAPTER 3
Distance Education
Study Problem # 7
J. Frye
May, 2011

Distributed Loads (Forces)

Replacing distributed loads acting on a rigid body by concentrated force(s) acting at a point on the rigid body.

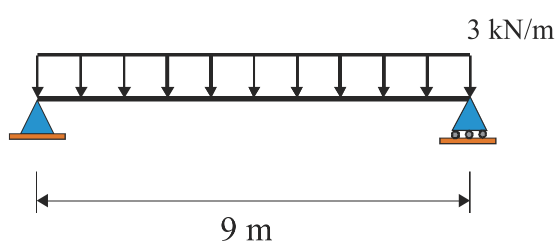
DEFINITION: Forces applied over an area or over a length are referred to as distributed forces or loads. The distribution can be uniform or non-uniform. Examples include snow load and wind load.

It is often convenient to replace a distributed load by a resultant force that is equivalent to the distributed load.

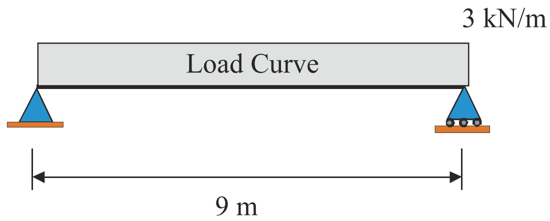
We will look at three basic shapes of distributed loads applied to a beam:

1. **Load Case 1** - Uniformly distributed load (rectangular shape) in which the magnitude of the load does not vary along the application length on the beam.
2. **Load Case 2** - Triangular distributed load (triangular shape) in which the magnitude goes from a given value to zero over the application length on the beam.
3. **Load Case 3** - A combination of Load Case 1 and Load Case 2 (trapezoidal shape).

Load Case 1 – Uniformly (evenly) distributed load



In this example, the load of 3 kN/m is constant (or uniform) over the entire length of the beam (10 m)



We refer to the shaded area in the figure above as the **LOAD CURVE** for the uniformly distributed load.

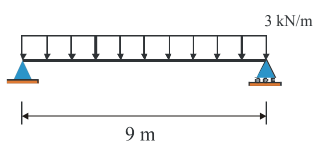
The concentrated force that is equivalent to the rectangular (uniformly) distributed load is the area of the Load Curve. Its point of application is at the center of the rectangle.

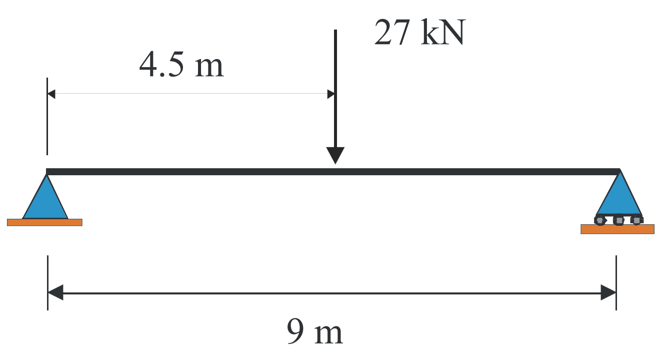
$$F1 = (3kN/m)(10m) = 30kN$$

Point of Application is at :

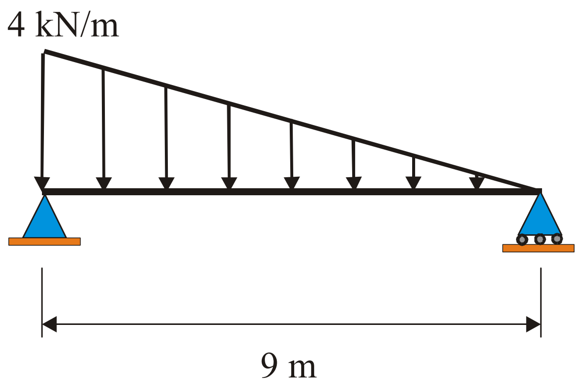
$$10m/2 = 5m$$

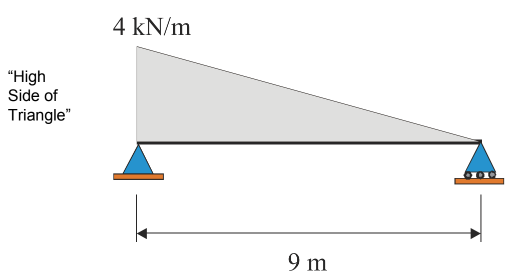
Concentrated Load replacing 3kN/m uniformly distributed load





Load Case 2 – Triangular Distributed Load





We refer to the shaded area in the figure above as the **LOAD CURVE** for the triangular distributed load.

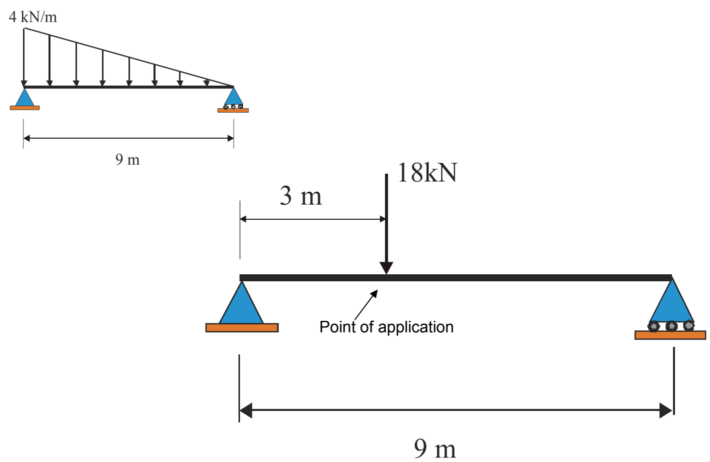
The concentrated force that is equivalent to the triangular distributed load is the area of the Load Curve. Its point of application is at **1/3 the length of the base of the triangle** measured from the right (90°) angle (high side of triangle).

$$F1 = (4kN/m)(9m)/2 = 18kN$$

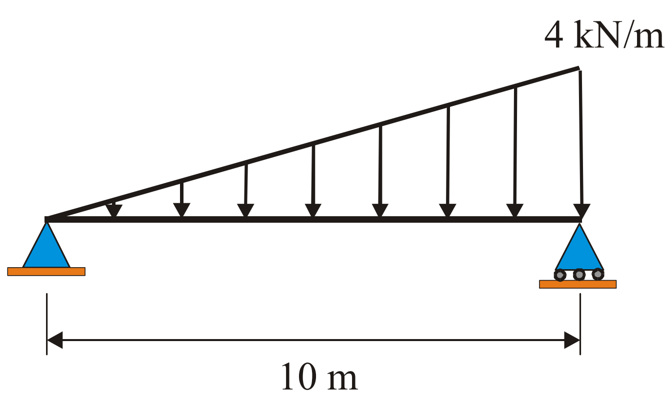
Point of Application is at :

9m/3 = 3 m from the 90° angle (high side of triangle).

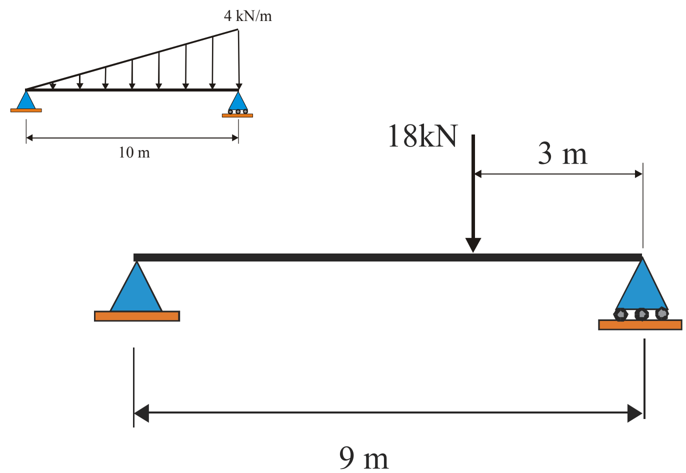
Concentrated load replacing triangular distributed load



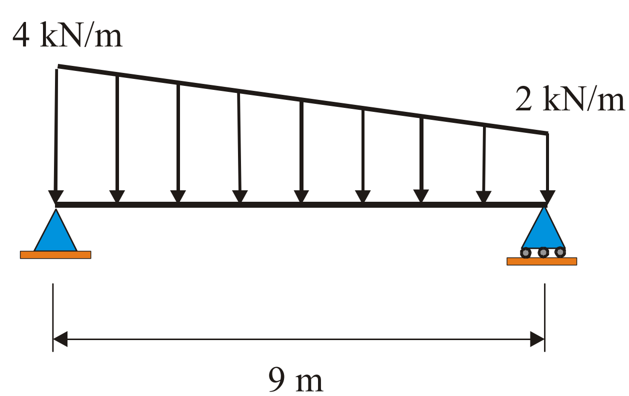
Load Case 2 (reversed)



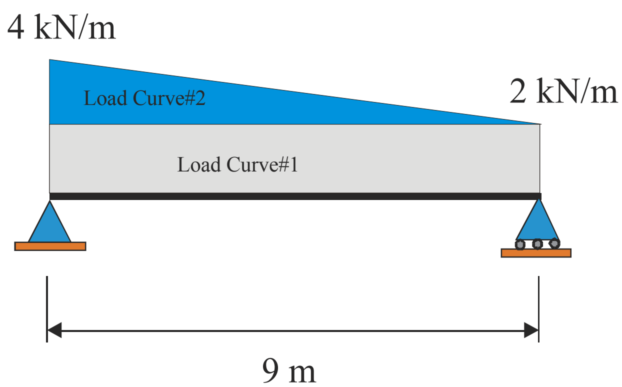
Concentrated load replacing triangular distributed load.
We measure 3 m from the right (90^o) angle

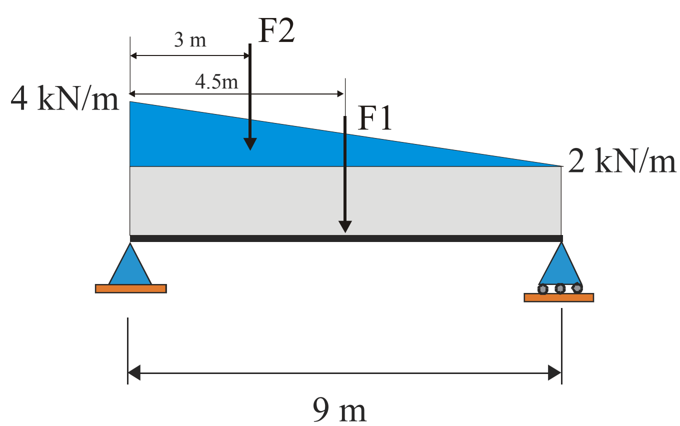


Load Case 3 – Combined Uniformly and Triangular Distributed Load (Trapezoidal Load Curve)



We divide the Load Curve into two load curves – a rectangle and a triangle as shown in the figure.

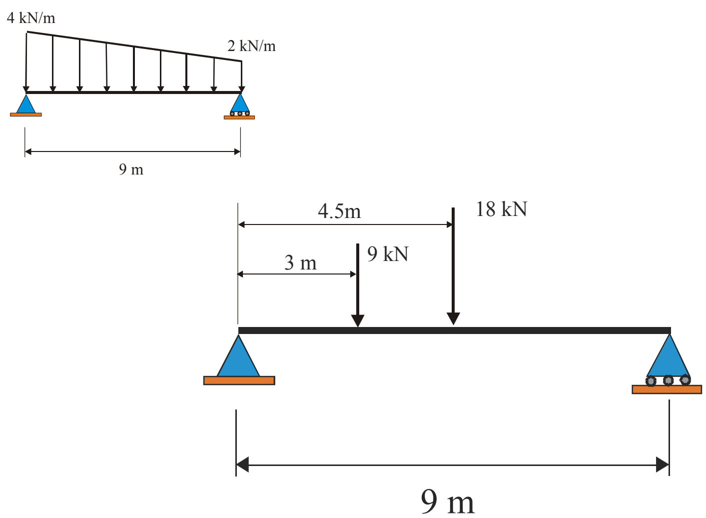




$$F1 = (2kN/m)(9m) = 18kN$$

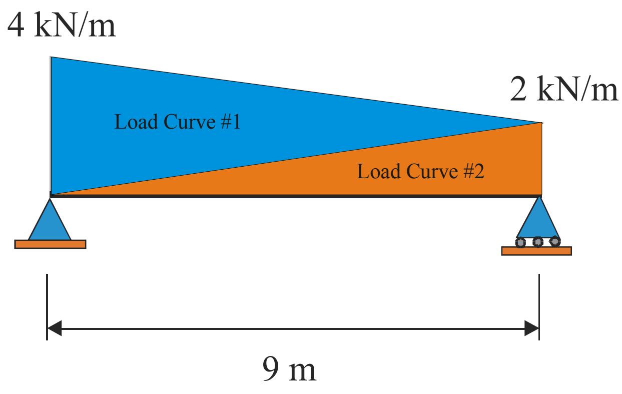
$$F2 = (4kN/m - 2kN/m)(9m)/2 = 9kN$$

Concentrated load replacing rectangular and triangular distributed load

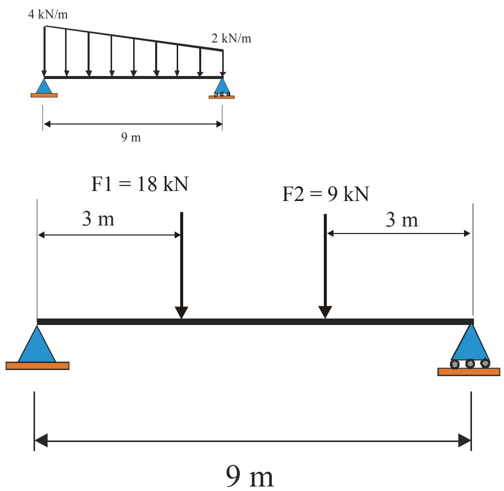


Load Case 3 - Alternate Solution

We divide the trapezoid into two (2) triangles. Concentrated forces equal to the areas of each of the triangles are located 1/3 of the 9 m length measured from the left support for Load Curve #1 and 1/3 of the 9 m length measured from the right angle for Load Curve #2

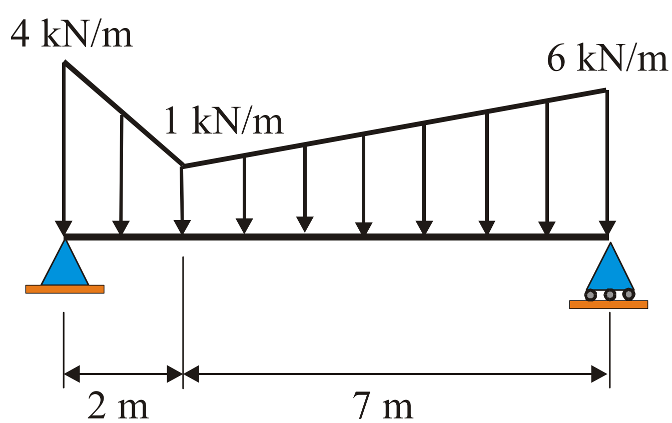


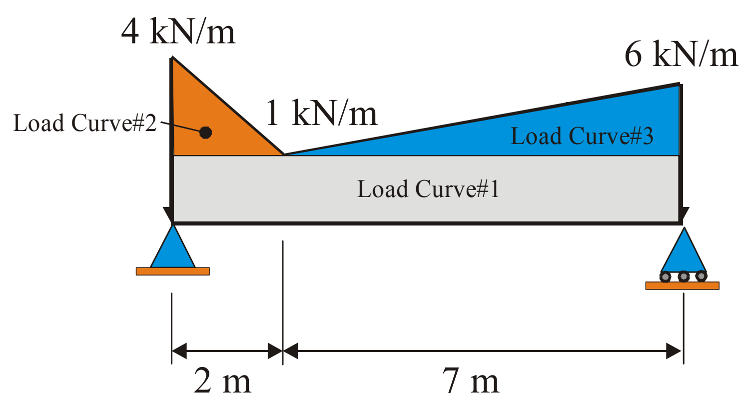
Load Case 3 - Alternate Solution

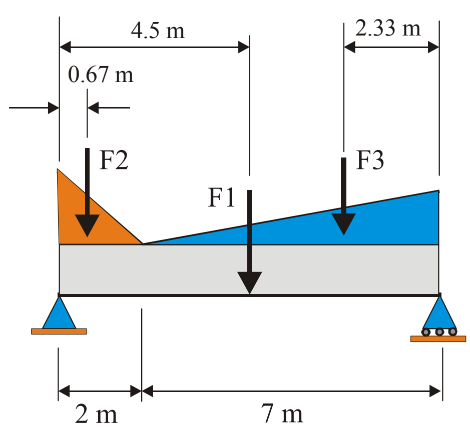


$$F1 = 4(9)/2 = 18\text{kN}$$
$$F2 = 2(9)/2 = 9\text{kN}$$

Load Case 3 – with rectangular and two triangular distributed loads.



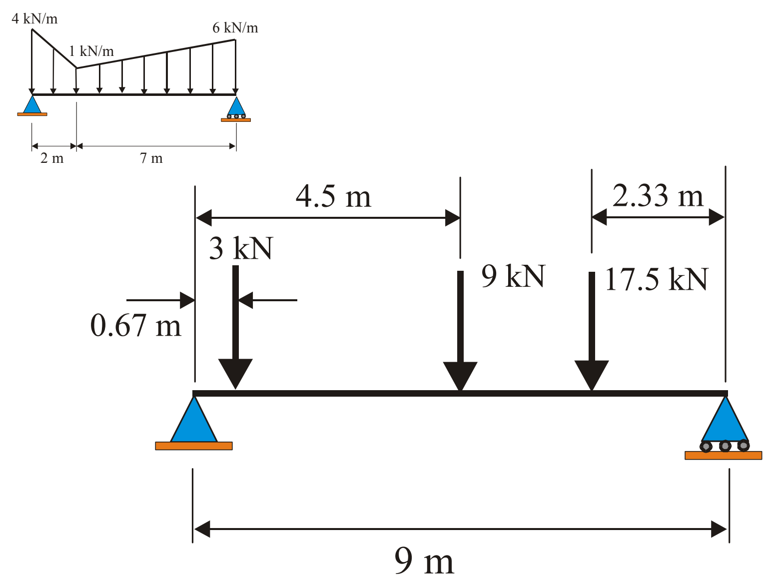




$$F1 = (1kN/m)(9m) = 9kN$$

$$F2 = (4kN/m - 1kN/m)(2m)/2 = 3kN$$

$$F3 = (6kN/m - 1kN/m)(7m)/2 = 17.5kN$$



CHAPTER 3
Distance Education
Study Problem # 8
J. Frye
May, 2011

Couples
Couple-Moments
Equivalent Force-Couples

Definition of a Couple

Two forces **F** and **-F** having
a) same magnitude,
b) parallel (non-concurrent) lines of action, and
c) opposite sense
are said to form a **couple**.

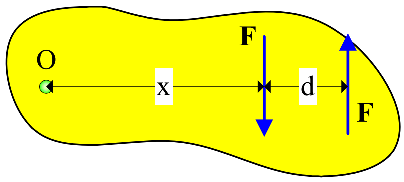
Definition of a Couple-Moment – The moment produced by two equal, opposite, and non-collinear (not on the same straight line) forces is called a **couple-moment**.

Moment of a Couple (Couple-Moment)

The moment of a couple, sometimes called the **couple moment** or just the **couple**, is a moment and is a vector.

It is a **free vector**, i.e. it is the same everywhere in the space.

Moment of a Couple (Couple-Moment)



d = the perpendicular distance between the two forces comprising the couple

$$M_O = -(F)(x) + F(x+d) = Fd$$

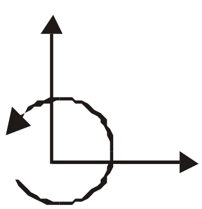
, or

$$\mathbf{M}_o = Fd\mathbf{k}$$

The Couple-Moment is **independent** of the location of the Point O (Moment of the couple does not depend on x). It depends only on F and d. We say it is a **"Free Vector".**

Direction of the Couple (Couple-Moment)

We will call Counter clockwise moments Positive and clockwise moments Negative.



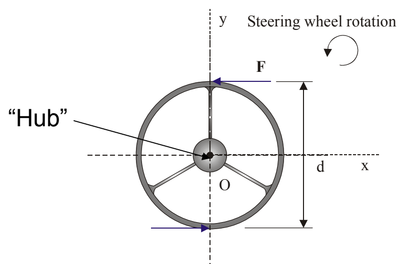
Positive sign convention for forces and couples

Couple-Moment Direction by Inspection

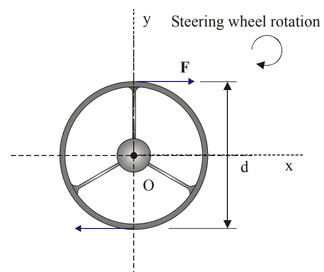
TWO-DIMENSIONAL PROBLEMS – COUNTERCLOCKWISE AND CLOCKWISE MOMENTS BY INSPECTION

In **two-dimensional problems** it is quite natural and **acceptable** to refer to the sense of the moment of a force **F** about a point as **counterclockwise** or **clockwise**.

In **two-dimensional problems** it is easy to determine the sense of the **couple-moment** by **INSPECTION if we consider the direction a steering wheel would rotate about the "hub" when a couple is applied**.



Positive (Counterclockwise) Moment



Negative (Clockwise) Moment

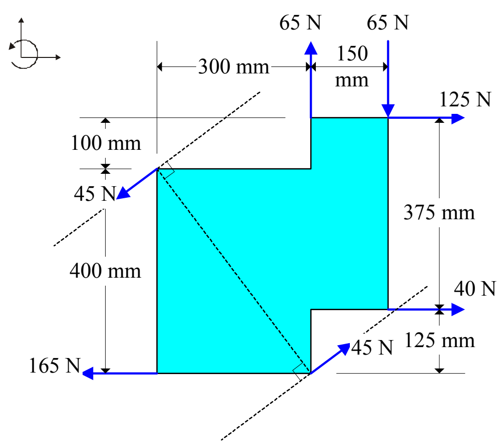
Addition of Couples

Addition of Couples:
The sum of two or more couples is simply the algebraic sum of their moments.

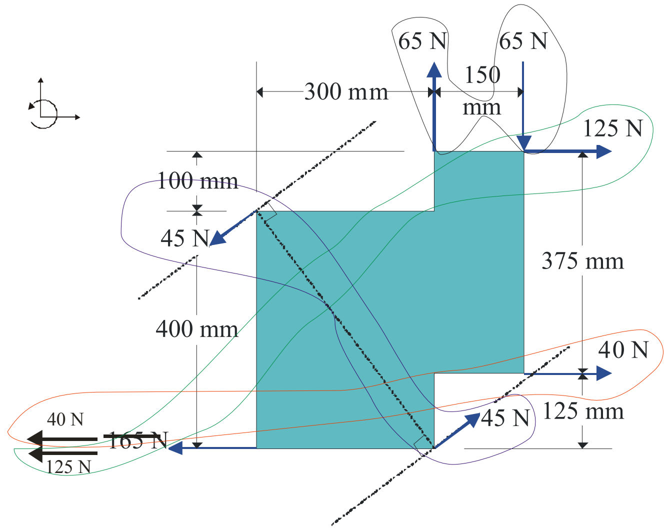
In our sign convention, counter clockwise couples are positive moments and clockwise couples are negative. That is, two or more couples acting on a rigid body may be replaced by a single couple (resultant couple) by algebraically adding the moments of the couples.

Example 3.3 of the Text/Notes Illustrates Addition of Couples

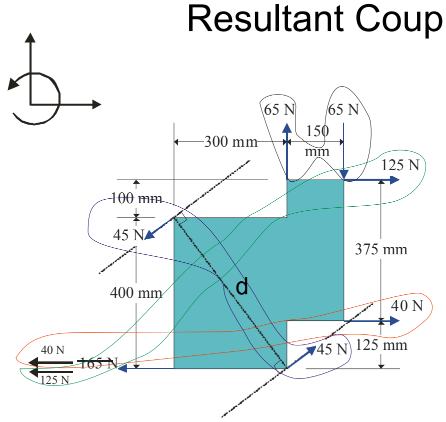
Example 3.3:
Determine the magnitude and the direction of the resultant couple which acts on the system shown.



We break the 165 N force applied at the bottom left corner of the block into a 125 N and 40 N force and identify four (4) couples applied to the block.



Resultant Couple Moment, $\mathbf{M_R}$



$$d = \sqrt{0.4^2 + 0.3^2} = 0.5$$

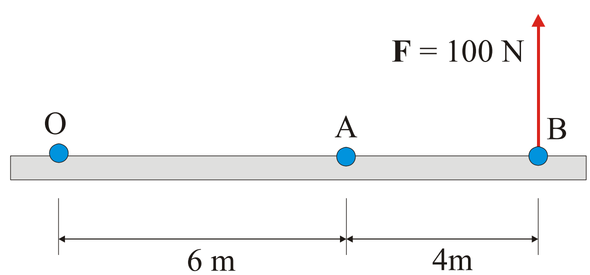
$$M_R = -40(0.125) - 125(0.5) + 45(0.5) - 65(0.15)$$
$$= -54.75\,\text{N.m}$$
$$\mathbf{M_R} = 54.75\,\text{N.m}$$

Resolution of a Given Force Applied at a Point on a Rigid Body to an Equivalent Force-Couple at another Point on the Rigid Body

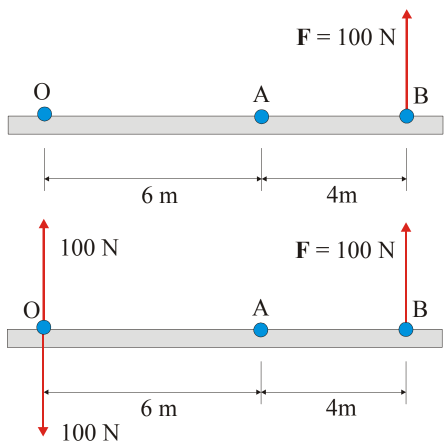
In the following slides, we will demonstrate that any force applied to a rigid body can be replaced by the same force and a couple moment **(EQUIVALENT FORCE-COUPLE)** applied at a different point on the rigid body

Consider the 100 N force acting at point B on the beam shown. If this is the only force acting on the beam, it will cause the beam to move.

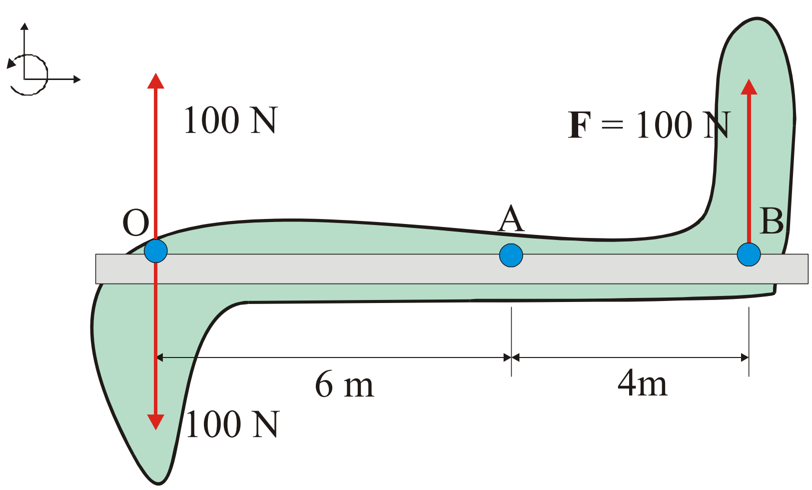
Can the 100 N force be applied at a **different point** on the beam so that its effect on the beam (motion) will be **exactly** the same as if it were applied at point B (its original point of application)??????



We apply equal and opposite 100 N forces at point O

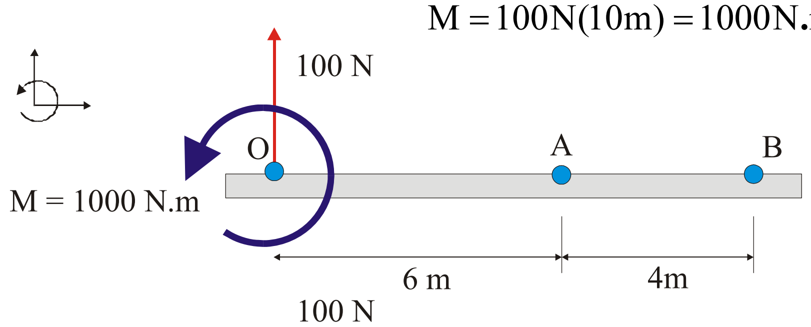


Note that two of the 100 N forces form a couple!!!!



We calculate the couple-moment as 1000 N.m. counter-clockwise and show it as follows:

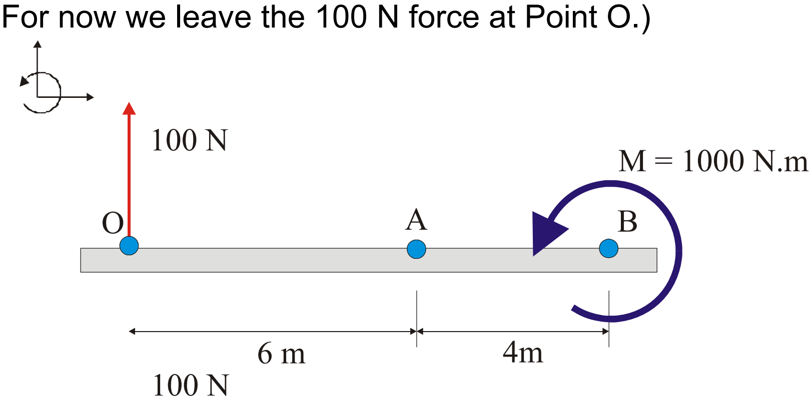
$$M = 100N(10m) = 1000N.m$$



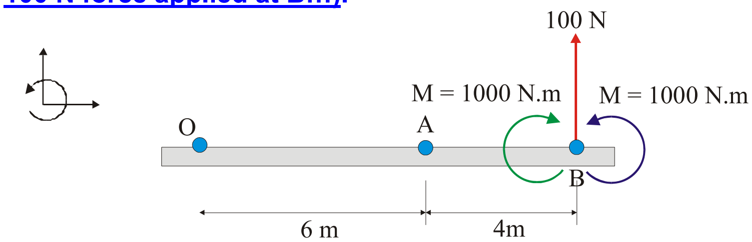
IMPORTANT!!!! We have just demonstrated that we can move any force applied to a rigid body to any other point on the rigid body if we **add a couple-moment equal to the moment that the force has about the point we want to move the force to**.

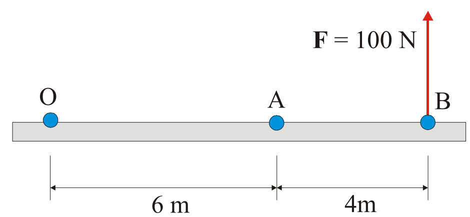
That is, we moved the 100 N force originally applied at Point B to Point O and added in a 1000 N.m counter-clockwise couple-moment since this is the moment that the 100 N force applied at B has about the Point O.

Let us reverse the process and move the 100 N force and 1000 N.m couple-moment back to Point B. The couple is a "free vector" so we can show it acting anywhere on the beam. We move the couple directly to B as shown below. (For now we leave the 100 N force at Point O.)

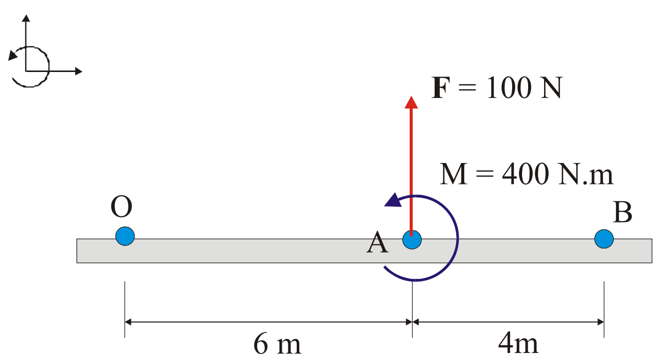


We now move the 100 N force at O back to B. If we do this, we must add into the system a couple-moment equal to the moment that the 100 N force at O has about the point B which is **1000 N.m clockwise** as shown below. The 1000 N.m clockwise couple eliminates the 1000 N.m counter-clockwise couple and we are left with what we started with **(a 100 N force applied at B!!!).**

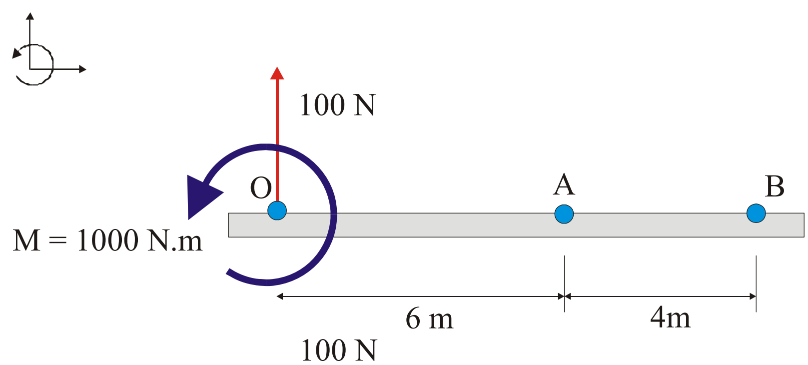




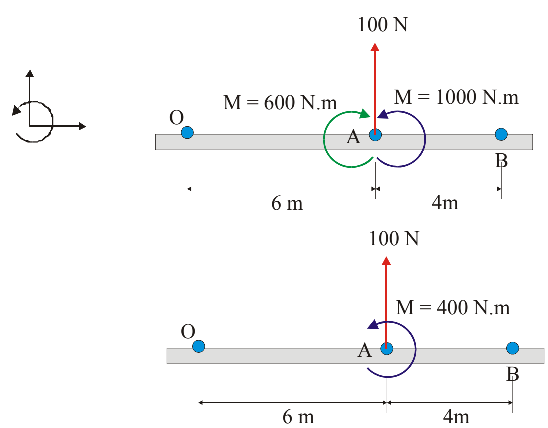
In the following figure, we have moved the 100 N force applied at B to the Point A which is 4 m from B.



We should get the same result if we moved the 100 N force from B to point O and then back to A. The following figure indicates our previous result when we moved the 100 N force from B to O.



When we move the 100 N force from O to A we must add into the system the moment that the 100 N force has about A which is 600 N.m clockwise. The resultant moment at A is therefore 400 N.m counter-clockwise. This is the same result that we got when we moved the 100 N force from B to A.



Equivalent Force-Couple Systems

We can replace any system of forces and couples applied to a rigid body by a Single force and a Couple (Couple-Moment) at any point on the rigid body.

STEP 1 – Express all sloping forces in terms of their x and y components.

STEP 2 – Calculate $\mathbf{R_x}$ and $\mathbf{R_y}$ (resultants of x and y forces respectively).

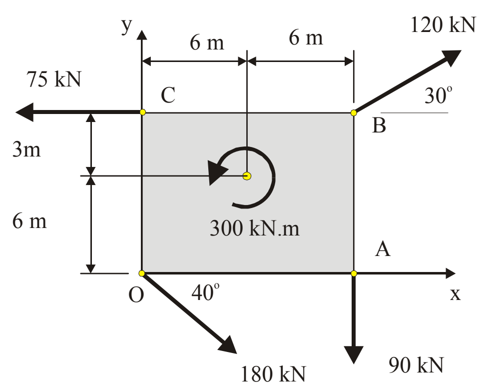
STEP 3 – Calculate $\mathbf{M_R}$ (the resultant moment) of all forces and couples about the point on the rigid body where you wish to determine the equivalent force-couple).

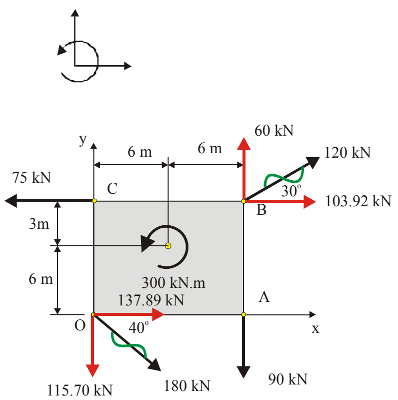
STEP 4 – Draw the rigid body showing **Rx**, **Ry** and $\mathbf{M_R}$ at the point selected for the equivalent force-couple.

We illustrate the steps in the following example:

Example: Replace the forces and couple(s) applied to the rigid body by an equivalent force-couple system at Point C.

In this example, we have 4 forces and a 300 kN.m counter-clockwise couple-moment ("free vector") applied to the rigid body. We want to replace this system of forces and couple by a single force and couple applied at C. that is, we want to move all forces and couples to Pont C. **We refer to this as the equivalent force-couple at C.** We follow the steps just outlined.





$\mathbf{R}_x = \sum F_x = -75 + 103.92 + 137.89 = +166.81\,\text{kN}$

$\mathbf{R}_x = 166.81\text{ kN} \rightarrow$

$\mathbf{R}_y = \sum F_y = +60 - 90 - 115.70 = -145.70\,\text{kN}$

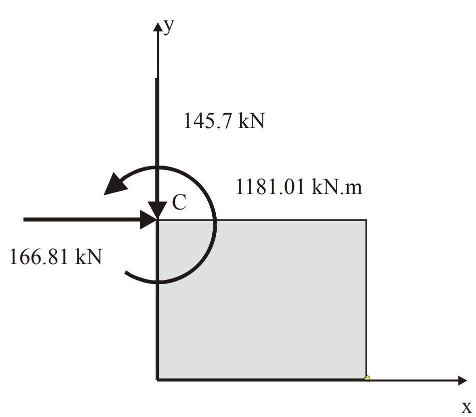
$\mathbf{R}_y = 145.70\text{ kN} \downarrow$

Resultant Moment at C :

$\mathbf{M}_{\mathbf{R}_C} = +60(12) - 90(12) + 137.89(9) + 300 = +1181.01\,\text{kN.M}$

$\mathbf{M}_{\mathbf{R}_C} = 1181.01\,\text{kN.M}$

Equivalent Force-Couple at Point C



CHAPTER 3
Distance Education
Study Problem # 9
J. Frye
May, 2011

Equivalent Force Systems

Comparing Equivalent Force-Couple at the same point for different force systems applied to the same rigid body.

SOLUTION STEPS

STEP 1 – Declare a sign convention for positive forces and moments and establish a point on the rigid body where moments are to be taken about. (The origin, O is usually as good as any choice.)

For each rigid body carry out Step 2 and Step 3:

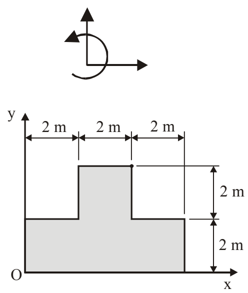
STEP 2 – Determine the resultant of all forces in the x-direction and y-direction, $\mathbf{R_x}$ and $\mathbf{R_y}$. Any sloping forces applied to the rigid body must first be resolved into their rectangular components.

STEP 3 – Determine the resultant moment, $\mathbf{M_{RO}}$ about the common point that you have chosen to take moments about (say the origin O).

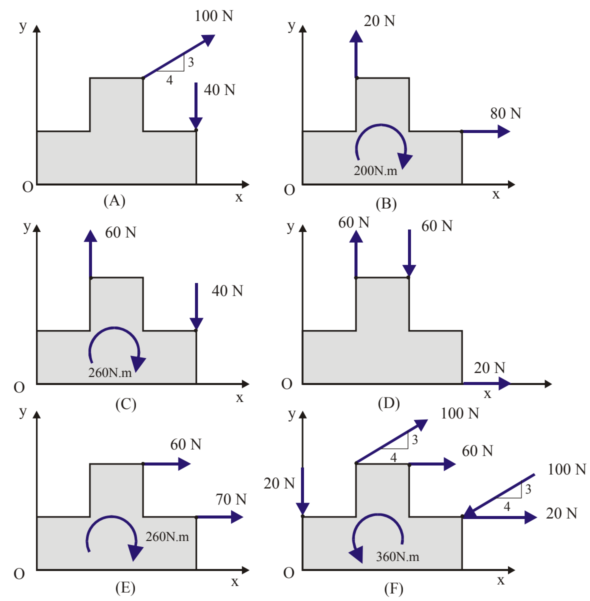
STEP 4 – For each rigid body, compare $\mathbf{R_x}$, $\mathbf{R_y}$ and $\mathbf{M_{RO}}$. Rigid bodies that have the same $\mathbf{R_x}$, $\mathbf{R_y}$ and $\mathbf{M_{RO}}$ are equivalent.

$$R_x = \sum F_x$$
$$R_y = \sum F_y$$
$$M_{RO} = \sum M_O$$

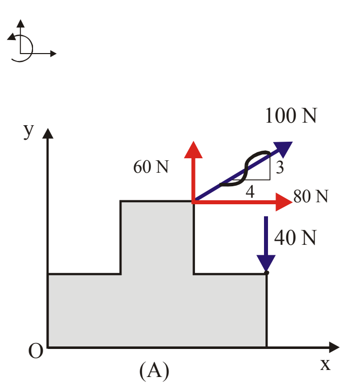
Example – Which of the force systems applied to the rigid body (all blocks are identical) are equivalent?



Block Dimensions

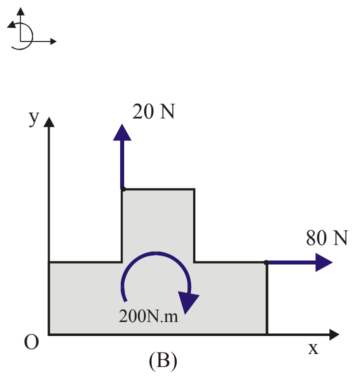


Rigid Body (a)



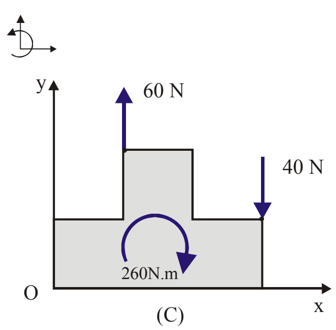
$$R_x = \sum F_x$$
$$R_x = +80\text{N}$$
$$\therefore \mathbf{R_X} = 80\text{N} \rightarrow$$
$$R_y = \sum F_y$$
$$R_y = +60 - 40 = +20\text{N}$$
$$\therefore \mathbf{R_y} = 20\text{N} \uparrow$$
$$M_{RO} = \sum M_O$$
$$M_{RO} = +60(4) - 80(4) - 40(6) = -320\text{N.m}$$
$$\therefore \mathbf{M_{RO}} = 320\text{N.m} \curvearrowright$$

Rigid Body (b)



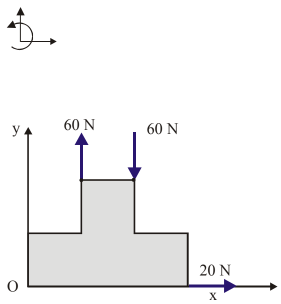
$$R_x = \sum F_x$$
$$R_x = +80\text{N}$$
$$\therefore \mathbf{R_X} = 80\text{N} \rightarrow$$
$$R_y = \sum F_y$$
$$R_y = +20\text{N}$$
$$\therefore \mathbf{R_y} = 20\text{N} \uparrow$$
$$M_{RO} = \sum M_O$$
$$M_{RO} = +20(2) - 80(2) - 200 = -320\text{N.m}$$
$$\therefore \mathbf{M_{RO}} = 320\text{N.m} \curvearrowright$$

Rigid Body (c)



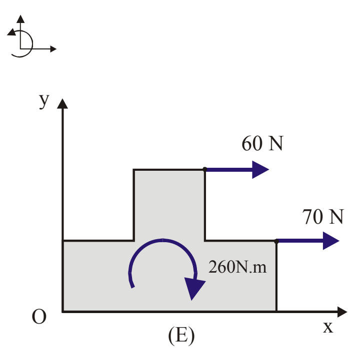
$$R_x = \sum F_x$$
$$R_x = 0$$
$$\mathbf{R_x} = 0$$
$$R_y = \sum F_y$$
$$R_y = +60 - 40 = +20\text{N}$$
$$\therefore \mathbf{R_y} = 20\text{N} \uparrow$$
$$M_{RO} = \sum M_O$$
$$M_{RO} = +60(2) - 40(6) - 260 = -380\text{N.m}$$
$$\therefore \mathbf{M_{RO}} = 380\text{N.m} \curvearrowright$$

Rigid Body (d)



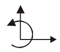
$$R_x = \sum F_x$$
$$R_x = +20$$
$$\mathbf{R_x} = 20\text{N} \rightarrow$$
$$R_y = \sum F_y$$
$$R_y = +60 - 60 = 0$$
$$\therefore \mathbf{R_y} = 0$$
$$M_{RO} = \sum M_O$$
$$M_{RO} = -60(2) = -120\text{N.m}$$
$$\therefore \mathbf{M_{RO}} = 120\text{N.m}$$

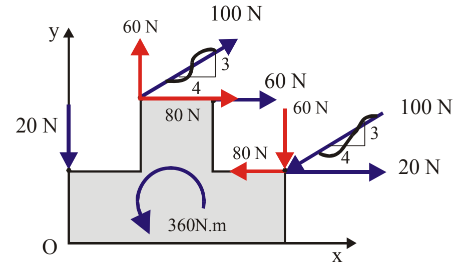
Rigid Body (e)



$$R_x = \sum F_x$$
$$R_x = +60 + 70 = +130\text{N}$$
$$\mathbf{R_x} = 130\text{N} \rightarrow$$
$$R_y = \sum F_y$$
$$R_y = 0$$
$$\therefore \mathbf{R_y} = 0$$
$$M_{RO} = \sum M_O$$
$$M_{RO} = -60(4) - 70(2) - 260 = -640\text{N.m}$$
$$\therefore \mathbf{M_{RO}} = 640\text{N.m}$$

Rigid Body (f)





$$R_x = \sum F_x$$
$$R_x = +80 + 60 - 80 + 20 = +80\text{N}$$
$$\mathbf{R_x} = 80\text{N} \rightarrow$$
$$R_y = \sum F_y$$
$$R_y = -20 + 60 - 60 = -20$$
$$\therefore \mathbf{R_y} = 20 \downarrow$$
$$M_{RO} = \sum M_O$$
$$M_{RO} = +60(2) - 80(4) - 60(4) + 80(2) - 60(6) - 20(2) + 360 = -320\text{N.m}$$
$$\therefore \mathbf{M_{RO}} = 320\text{N.m} \curvearrowright$$

Summary

a) $\mathbf{R_x} = 80\text{N} \rightarrow \quad \mathbf{R_y} = 20\text{N} \uparrow \quad \mathbf{M_{RO}} = 320\text{N.m} \curvearrowright$

b) $\mathbf{R_x} = 80\text{N} \rightarrow \quad \mathbf{R_y} = 20\text{N} \uparrow \quad \mathbf{M_{RO}} = 320\text{N.m} \curvearrowright$

c) $\mathbf{R_x} = 0\text{N} \quad \mathbf{R_y} = 20\text{N} \uparrow \quad \mathbf{M_{RO}} = 380\text{N.m} \curvearrowright$

d) $\mathbf{R_x} = 20\text{N} \rightarrow \quad \mathbf{R_y} = 0\text{N} \quad \mathbf{M_{RO}} = 120\text{N.m} \curvearrowright$

e) $\mathbf{R_x} = 130\text{N} \rightarrow \quad \mathbf{R_y} = 0\text{N} \quad \mathbf{M_{RO}} = 640\text{N.m} \curvearrowright$

f) $\mathbf{R_x} = 80\text{N} \rightarrow \quad \mathbf{R_y} = 20\text{N} \downarrow \quad \mathbf{M_{RO}} = 320\text{N.m} \curvearrowright$

a) and b) are equivalent

CHAPTER 3
Distance Education
Study Problem # 10
J. Frye
May, 2011

Equivalent Force-Couple at a Point

Principle of Transmissibility

The condition of equilibrium or motion of a rigid body remains unchanged if a force acting at a given point of application is replaced by a force of the same magnitude and the same direction but applied at a different point on the rigid body, provided that the two forces have the same line of action.

Solution Steps

STEP 1 – Resolve all forces into their x and y components.

STEP 2 – Look to see if the **PRINCIPLE OF TRANSMISSIBILITY** may be used to simplify the problem. That is, can the forces all be applied at a common application point, P, by sliding them along their lines of action to the common point. (This will not always be the case.)

STEP 3 – Calculate the resultant of the x-force components, $\mathbf{R}_x$, and the y-force components, $\mathbf{R}_y$:

$$\mathbf{R}_x = \sum F_x$$
$$\mathbf{R}_y = \sum F_y$$

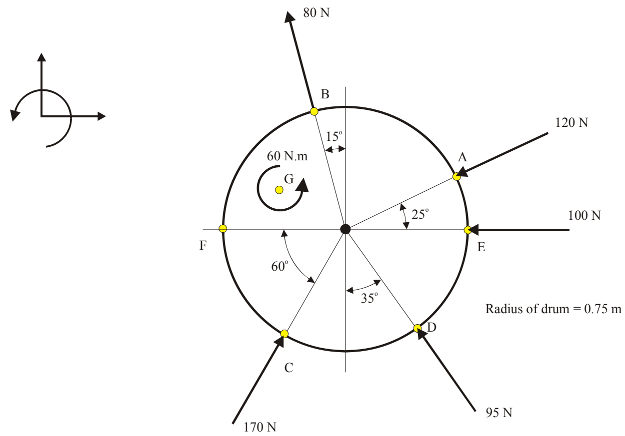
STEP 4 – Calculate the resultant moment, $\mathbf{M}_{PR}$, of all forces and couple moments about the point, P, where you are asked to determine the resultant force-couple at.

$$\mathbf{M}_{RP} = \sum M_P$$

STEP 5 – Redraw the figure showing $\mathbf{R}_x$, $\mathbf{R}_y$ and $\mathbf{M}_{RP}$ all at the point P. This is the equivalent force-couple at point P.

Example Problem

Five forces and a 60 N.m couple moment are applied to a drum. The points of application of these forces is shown. Determine the equivalent force-couple at point F.



Solution

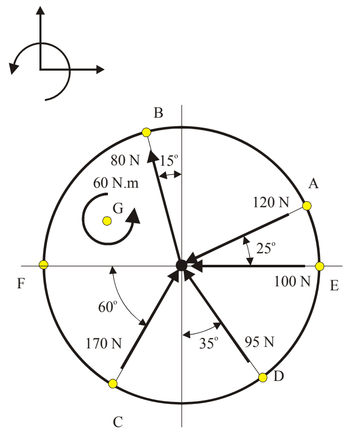
In solving this problem many students would do the following:

1. Resolve all forces into their x and y components,
2. Calculate the resultant of the x and y components, $\mathbf{R_x}$ and $\mathbf{R_y}$,
3. Determine the coordinates of the points of application of each of the five forces,
4. Determine the resultant moment, $\mathbf{M_{RF}}$ at point, F, by adding the applied couple moment to the individual moments of each of the force components about the point F.

While this approach is perfectly correct, it overlooks that the **Principle of Transmissibility** may be used to simplify the calculation of the resultant moment, $\mathbf{M_{RF}}$ at point F.

All of the lines of action of the forces applied at points on the perimeter of the drum intersect at the center of the drum, O. The Principle of Transmissibility says we may slide all of the forces and apply them at this point. This eliminates the need to calculate the coordinates of the points of application of the individual forces!

Applying Principle of Transmissibility to Perimeter Forces



Radius of drum = 0.75 m

$$\mathbf{R_x} = \sum F_x$$

$$R_x = -80\sin 15^o - 120\cos 25^o - 100 - 95\sin 35^o + 170\cos 60^o$$

$$R_x = -198.95\text{N}$$

$$\therefore \mathbf{R_x} = 198.95\text{N} \leftarrow$$

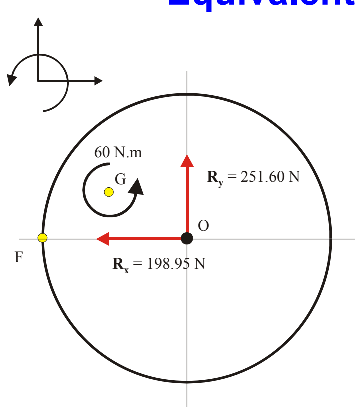
$$\mathbf{R_y} = \sum F_y$$

$$R_y = +80\cos 15^o - 120\sin 25^o + 95\cos 35^o + 170\sin 60^o$$

$$R_y = +251.60\text{N}$$

$$\therefore \mathbf{R_y} = 251.60\text{N} \Uparrow$$

Equivalent Couple-Moment at F:



Radius of drum = 0.75 m

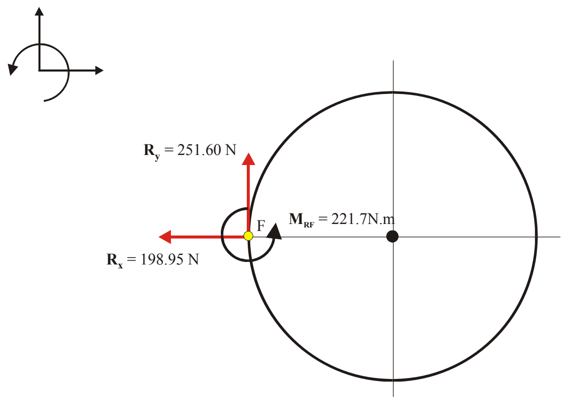
We can now calculate the equivalent couple-moment at point F due to the applied system of forces and couple-moments.

The 60 N.m counter clockwise couple moment is a Free Vector and can moved anywhere on the drum. $\mathbf{R_x}$ has no moment about point F since its Line of Action passes thru F. When we move $\mathbf{R_y}$ to point F we must add into the system the moment that $\mathbf{R_y}$ has about F

$$\mathbf{M_{RF}} = +60\text{N.m} + 215.6\text{N}(0.75\text{m}) = +221.7\text{N.m}$$

$$\therefore \mathbf{M_{RF}} = 221.7\text{N.m} \circlearrowleft$$

Equivalent Force-Couple at F

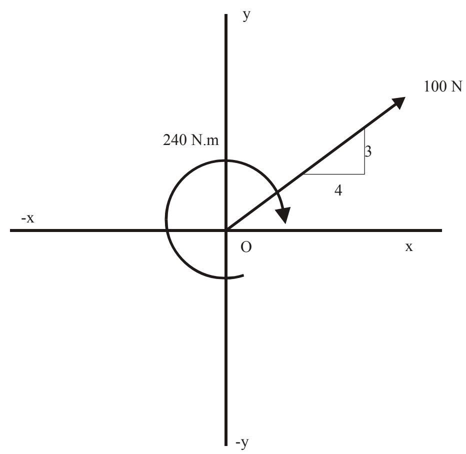


CHAPTER 3
Distance Education
Study Problem # 11
J. Frye
May, 2011

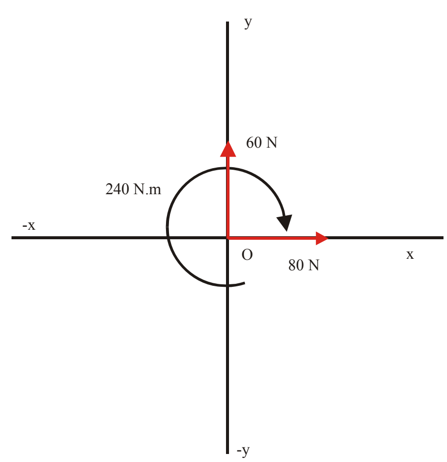
Reduction of a Force-Couple at a Point to a Single Force

Reduction of an Equivalent Force Couple at a point to a Single Force

Let us say that we are given the **100 N** force and the **240 N.m** **clockwise** couple-moment at Point O as shown in the Figure below and we are asked to replace this force-couple by a single force and determine where the line-of-action of this force **intersects** the x-axis and the y-axis. We will illustrate the procedure in steps.



STEP 1 – Express the force in terms of its rectangular components

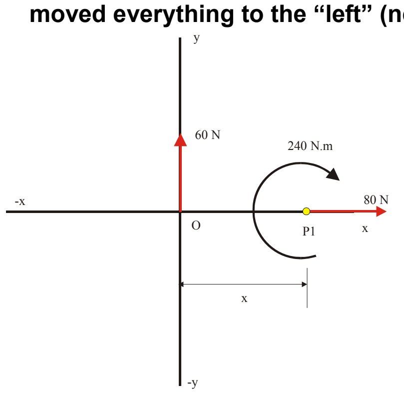


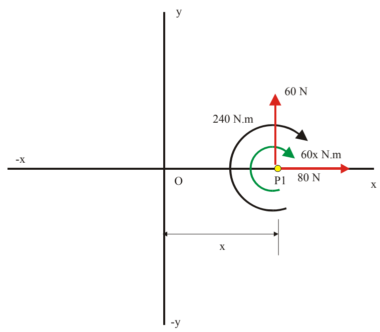
STEP 2 – We will **FIRST** move the system of forces in the x-direction to a new point on the x-axis with the objective of eliminating the 240 N.m clockwise couple-moment.

IMPORTANT:

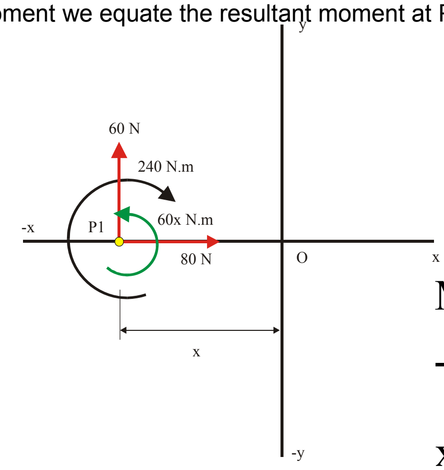
1. The 240 N.m clockwise couple moment is a Free Vector so we can move it anywhere in the x-y plane.
2. The 80 N horizontal force component can move to any “new” point on the x-axis (its Line-of-Action) by the “Principle of Transmissibility.
3. **However**, if we move the 60 N vertical force component we must **add into the system the moment that the 60 N force has about the point we are moving it to**.

Let us suppose that we consider moving the 80 N and 60 N force to the right (along the positive x-axis) to a “new” Point P1 that is a distance x from Point O. If we move the 60 N vertical component to Point P1, we must add into the system the moment that the 60 N force has about P1. That is, 60x N. m **CLOCKWISE as shown below.** **We see that we have not eliminated the 240 N.m clockwise couple. It has in fact increased to a (240 + 60x) N.m clockwise couple at P1. Clearly we should have moved everything to the “left” (negative x-axis).**





Moving the 60 N vertical force component to “new” point P1 introduces a 60x N.m counterclockwise couple as shown below. To determine the distance x that we must move to “eliminate” the 240 N.m clockwise couple-moment we equate the resultant moment at P1 to zero.

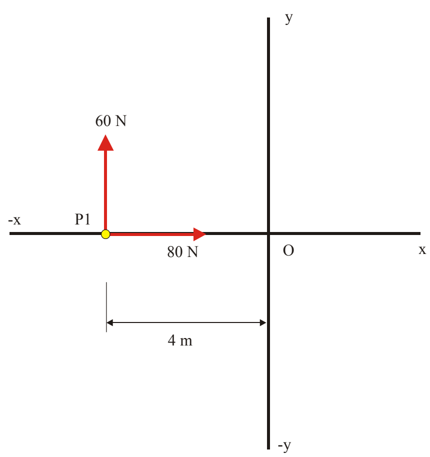


$$\text{Moment at P1} = 0:$$

$$-240\ \text{N.m} + 60\text{x} = 0$$

$$\text{x} = \frac{240}{60} = 4\text{m}$$

At P1 the resultant moment is zero!!!!!!!!

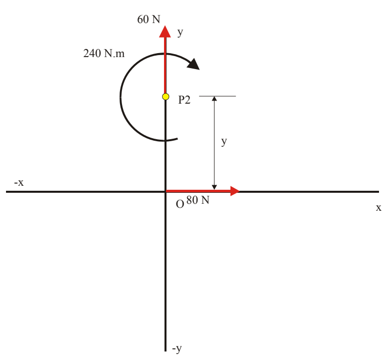


STEP 3 – We will move the system of forces in the y-direction to a new point on the y-axis with the objective of eliminating the 240 N.m clockwise couple-moment.

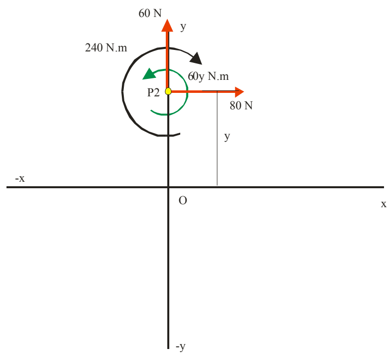
IMPORTANT:

1. The 240 N.m clockwise couple moment is a Free Vector so we can move it anywhere.
2. The 60 N vertical force component can move to any “new” point on the y-axis by the “Principle of Transmissibility.
3. If we move the 80 N horizontal force component we must **add into the system the moment that the 80 N force has about the point we are moving it to**.

As before, we must decide on where point P2 should be chosen so has to eliminate the 240 N.m clockwise couple. This time we note that if we choose P2 at a distance y “above” (along positive y-axis) the 80 N horizontal force component will have a 80y N.m counterclockwise moment about P2 which can be used to “eliminate” the 240 N.m clockwise couple. We therefore choose to move everything in this direction.



We now equate the resultant moment at P2 to zero:

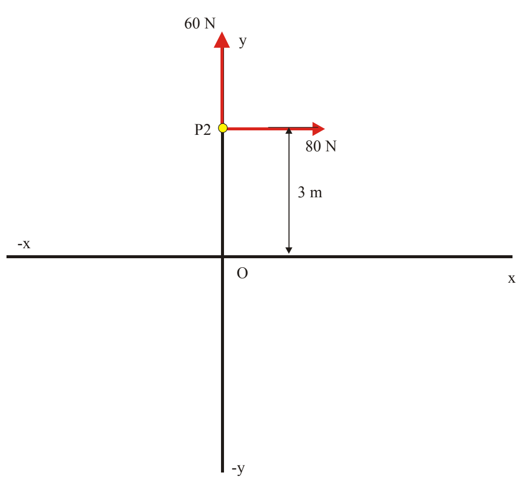


$$\text{Moment at P2} = 0:$$

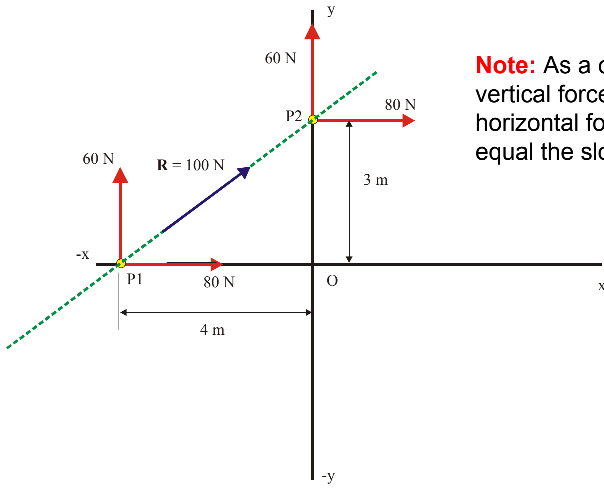
$$-240\ \text{N.m} + 80x = 0$$

$$x = \frac{240}{80} = 3\text{m}$$

At P2 the resultant moment is zero!!!!!!!!



The line connecting points P1 and P2 is the "Line-of-Action" of the single force R = 100 N that replaces the original force-couple system at O. The intersection points of this force on the x-axis and y-axis are x= - 4 m and y = 3 m respectively.

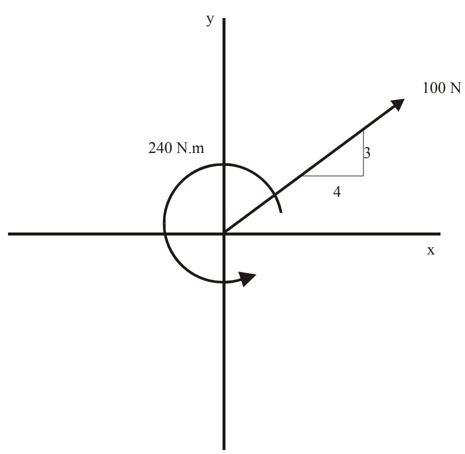


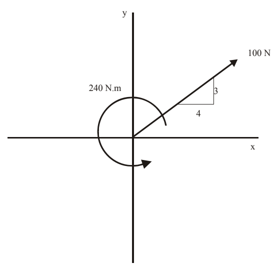
Note: As a check, the ratio of the vertical force component to the horizontal force component will equal the slope of the line.

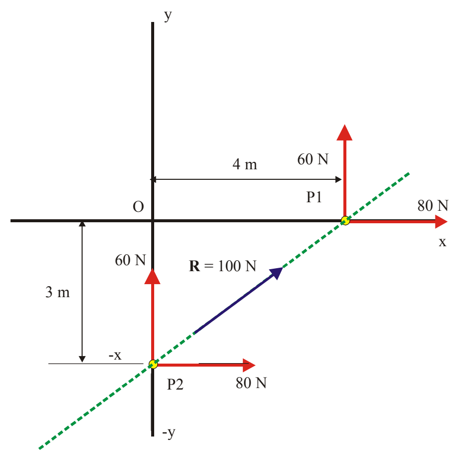
$$\frac{60\text{N}}{80\text{N}} = \frac{3}{4}$$

$$\frac{3\text{m}}{4\text{m}} = \frac{3}{4}$$

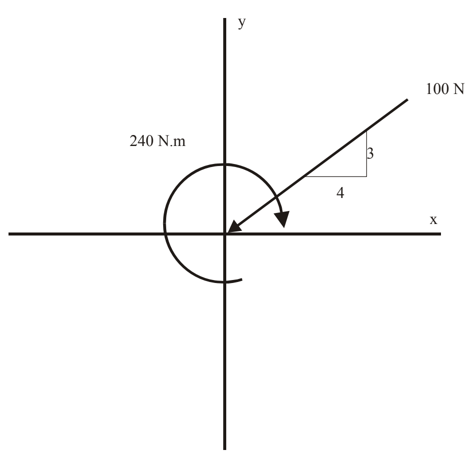
Self-Test – Reduce the following force-couple system to a single force and indicate where its Line-of-Action will intercept the x-axis and y-axis.

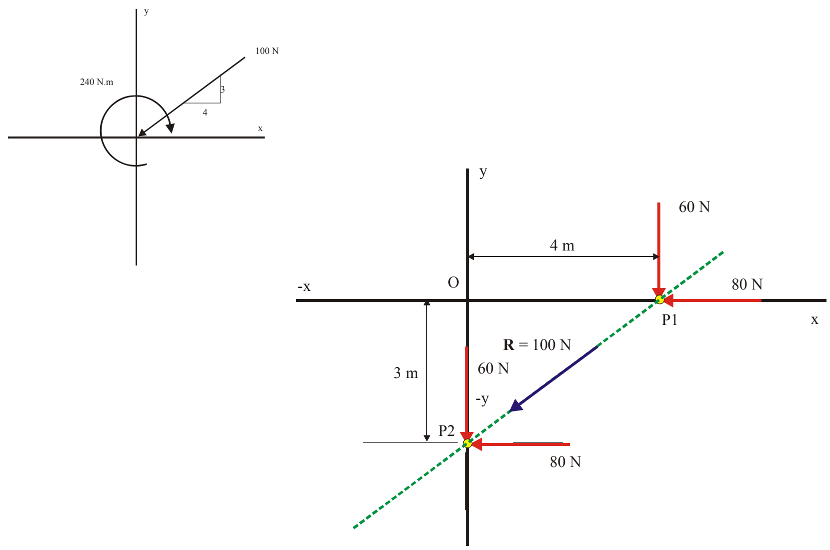






Self-Test – Reduce the following force-couple system to a single force and indicate where its Line-of-Action will intercept the x-axis and y-axis



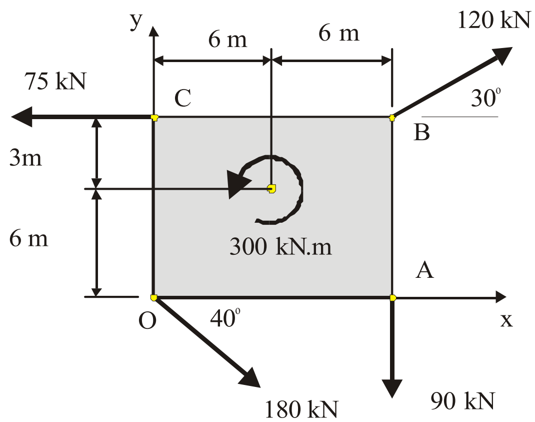


Example 3.7

J. Frye

Example 3.7:

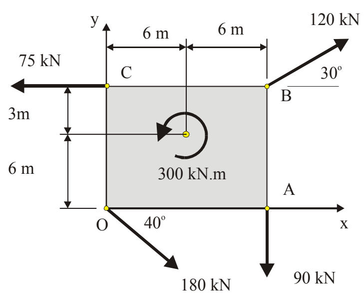
Replace the force system shown by a single force and determine where the line of action of this force intersects the x-axis and the y-axis.



IMPORTANT: In problems where you are asked to replace a system of forces and couples acting on a rigid body by a single force read the question carefully. It will ask you to determine where the line of action of the force **INTERSECTS** two lines. The lines specified will usually (but not always) be the x-axis and the y-axis. Your first step in solution of this type of problem is to find the equivalent force-couple **at this intersection point**.

Example 3.7:
Replace the force system shown by a single force and determine where the line of action of this force **intersects the x-axis and the y-axis**.

Example 3.7



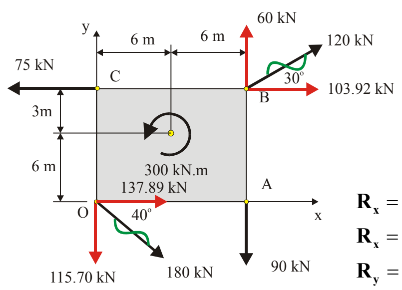
Important: Note that in this example, the 300 kN.m couple-moment is shown acting at (6,6). It is, however, a **"Free Vector"** so it was <u>not necessary</u> to indicate its location on the rigid body.

In Chapter 4, where we will be determining "internal forces" where two or more rigid bodies are connected, there will be examples where the location of the couple-moment will be important.

We first find the **equivalent force-couple** at O (intersection point of x-axis and y-axis).

1. Resolve sloping forces into x and y components.
2. Determine the resultants $\mathbf{R_x}$ and $\mathbf{R_y}$
3. Determine the resultant moment, $\mathbf{M_{RO}}$ of all forces and couples applied to the rigid body about the intersection point

Determination of the Equivalent Force-Couple at O

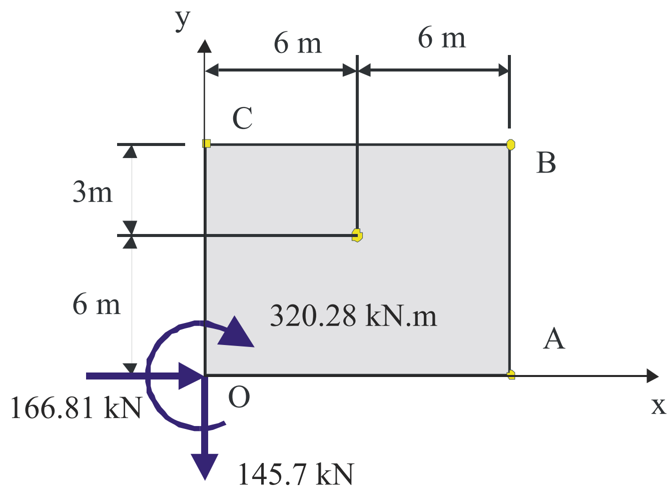


$$\mathbf{R}_x = \sum F_x = -75 + 103.92 + 137.89 = +166.81\,\text{kN}$$
$$\mathbf{R}_x = 166.81\ \text{kN} \rightarrow$$
$$\mathbf{R}_y = \sum F_y = +60 - 90 - 115.70 = -145.70\,\text{kN}$$
$$\mathbf{R}_y = 145.70\,\text{kN} \downarrow$$

Resultant Moment at O :

$$\mathbf{M}_{RO} = 75(9) + 60(12) - 103.92(9) - 90(12) + 300 = -320.28\,\text{kN.M}$$
$$\mathbf{M}_{RO} = 320.28\,\text{kN.M} \circlearrowright$$

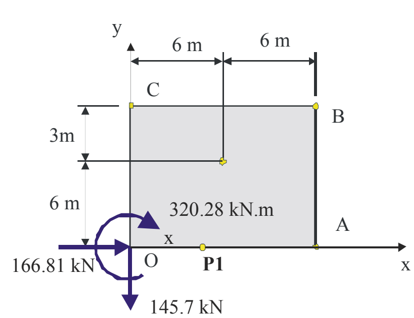
Equivalent Force-Couple at O



We first move the equivalent force-couple system at O to the point **P1** on the x-axis where the 320.25 kN.m clockwise couple will be eliminated or $\mathbf{M}_{P1}$ = 0.

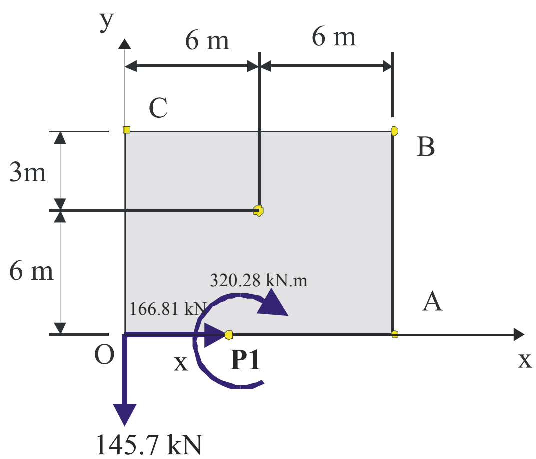
We need to determine the distance x such that $\mathbf{M}_{P1}$ = 0.

To illustrate we will do it in steps.



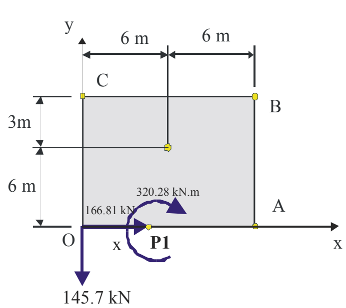
The 320.25 kN.m couple is a “Free” vector so we can move it anywhere. We show it applied at P1.

The 166.81 kN force has a line of action that passes through P1, therefore we apply the Principle of Transmissibility and slide it to P1. We are left with the 145.7 kN force still applied at O.



We know that when we move a force from one point of application to another point we MUST add into the system **the moment that the force has about the point we are moving it to**.

In the figure below we observe that if the 145.7 kN force is moved a distance x to P1 we must add into the system a **COUNTERCLOCKWISE** moment of 145.7x kN.m



We want to eliminate a CLOCKWISE moment of 320.25 kN.m (We have therefore chosen the correct direction to move the 145.7 kN force since it creates a COUNTERCLOCKWISE moment about P1.)

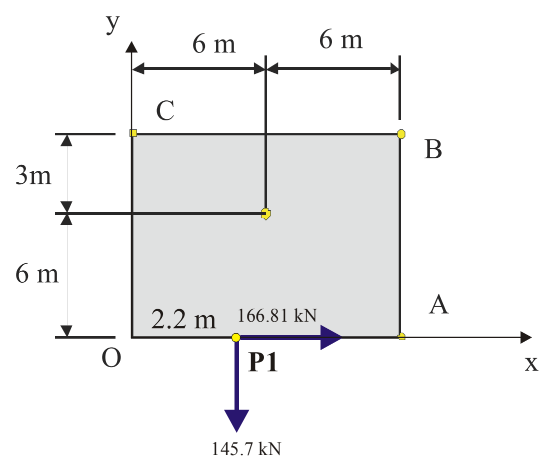
$$\mathbf{M}_{\mathbf{P1}} = 0$$

$$145.7x - 320.28 = 0$$

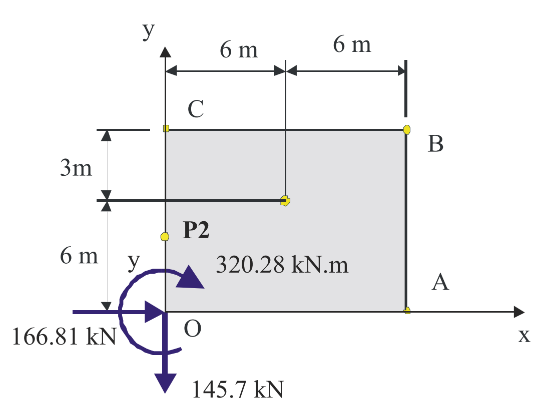
$$\therefore x = 2.2\text{m}$$

Resultant force at P1:

$$R = \sqrt{166.81^2 + 145.7^2} = 221.48\text{kN}$$

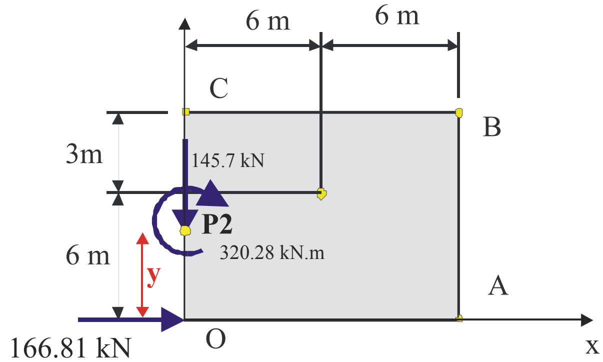


We now move the equivalent force-couple system at O to the point **P2** on the y-axis where the 320.25 kN.m clockwise couple will be eliminated or $\mathbf{M}_{\mathbf{P2}}$ = 0.
We need to determine the distance y such that $\mathbf{M}_{\mathbf{P2}}$ = 0.
To illustrate we will do it in steps.



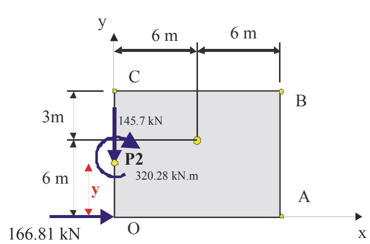
Again, the 320.25 kN.m couple is a “Free” vector so we can move it anywhere. This time, we show it applied at P2.

The 145.7 kN force has a line of action that passes through P2, therefore we apply the Principle of Transmissibility and slide it to P2. This time, we are left with the 166.81 kN force still applied at O.



We know that when we move a force from one point of application to another point we MUST add into the system **the moment that the force has about the point we are moving it to**.

In the figure below we observe that if the 166.81 kN force is move a distance y to P2 we must add into the system a **COUNTERCLOCKWISE** moment of 166.81y kN.m



We want to eliminate a CLOCKWISE moment of 320.25 kN.m (We have therefore chosen the correct direction to move the 166.81 kN force since it creates a COUNTERCLOCKWISE moment about P2.)

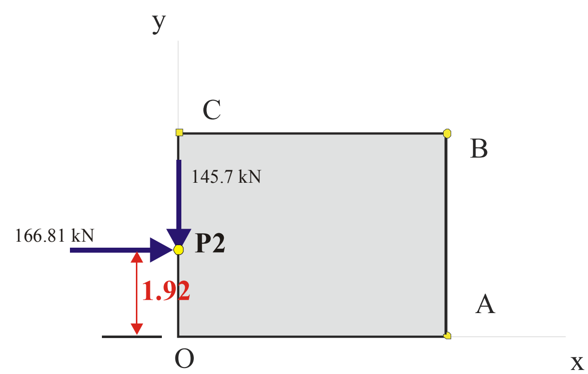
$$\mathbf{M}_{\mathbf{P2}} = 0$$

$$166.81y - 320.28 = 0$$

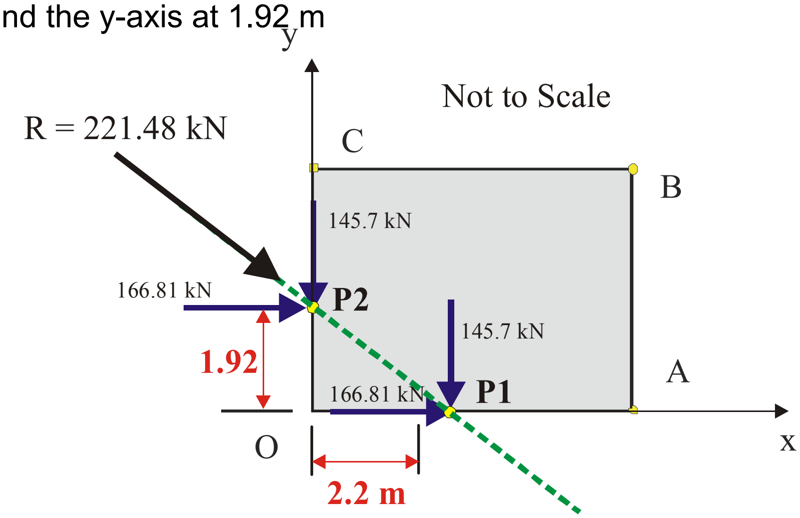
$$\therefore y = 1.92\text{m}$$

Resultant force at P2 :

$$R = \sqrt{166.81^2 + 145.7^2} = 221.48\text{kN}$$



The single force R = 221.48 kN intersects the x-axis at 2.3 m and the y-axis at 1.92 m



As a check, the ratio of the rectangular components of the resultant force will equal the ratio of the intersection points on the x - axis and the y - axis.

$$\frac{1.92}{2.2} = 0.873$$

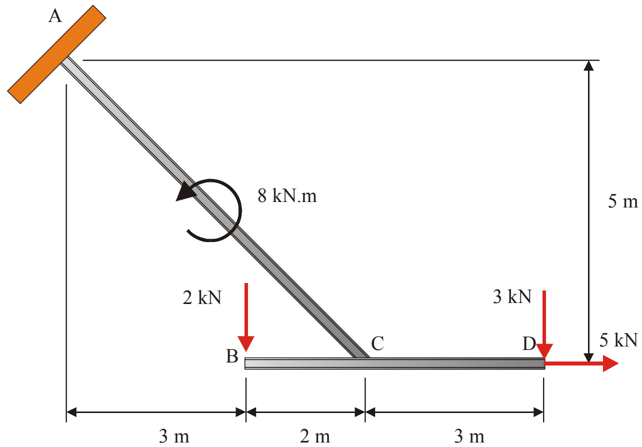
$$\frac{145.7}{166.81} = 0.873$$

CHAPTER 3
Distance Education
Study Problem # 12
J. Frye
May, 2011

External (Support) Reactions

Fixed Support

Example:
Two beams are welded together at C and attached to a sloping ceiling by a “Fixed” (rigid) support at A. A 8 kN.m counter-clockwise couple is applied along with 2 kN, 3 kN and 5 kN loads at B and D as shown. Determine the support reactions at A.



Solution Steps

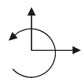
STEP 1 – Identify the type(s) of external supports and **DRAW THE FREE BODY DIAGRAM (FBD)!!!!!!! Label all forces and couple moments in the FBD**

STEP 2 – Indicate the positive sign convention for forces and moments and write the equilibrium equations based on the FBD you have drawn.

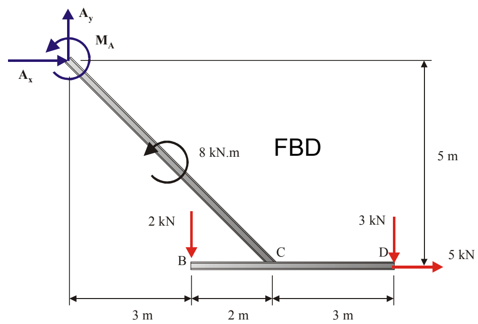
STEP 3 – Solve the equilibrium equations for unknown reactions.

STEP 4 – Check your solution by re-drawing the FBD indicating all reactions with the correct sense and taking moments about a different point.

Solution:



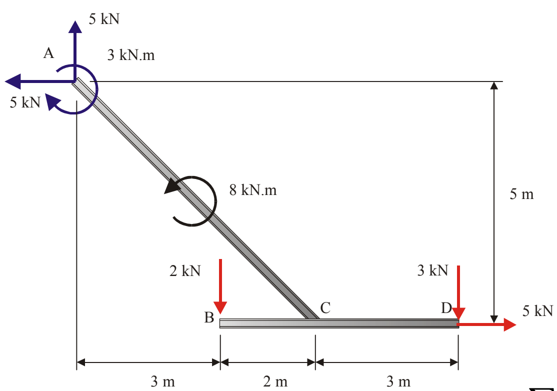
In our FBD for the **"Fixed" support**, we indicate unknown vertical and horizontal force reactions, $\mathbf{A_x}$ and $\mathbf{A_y}$ as well as an unknown couple moment $\mathbf{M_A}$.



Hint: If we assume a positive sense for all unknown reactions, when we solve the equilibrium equations, a negative value means that the reaction has in fact a negative sense.

$$\sum F_x = 0$$
$$A_x + 5 = 0$$
$$A_x = -5\text{kN}$$
$$\therefore \mathbf{A_x} = 5\text{kN} \leftarrow$$
$$\sum F_y = 0$$
$$A_y - 2 - 3 = 0$$
$$A_y = +5\text{kN}$$
$$\therefore \mathbf{A_y} = 5\text{kN} \uparrow$$
$$\sum M_A = 0$$
$$+M_A + 8 - 2(3) - 3(8) + 5(5) = 0$$
$$M_A = -3\text{kN.m}$$
$$\therefore \mathbf{M_A} = 3\text{kN.m} \circlearrowright$$

Check of Results (We take moments about Point C)



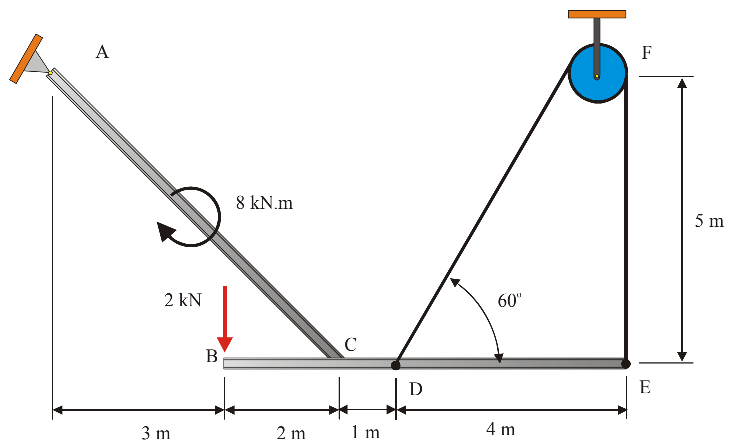
$$\sum M_C = 0$$
$$5(5) - 5(5) - 3 + 8 + 2(2) - 3(3) = 0$$
$$0 = 0 \textbf{ CHECKS!!!!}$$

CHAPTER 3
Distance Education
Study Problem # 13
J. Frye
May, 2011

Pin and Pulley External Supports

Example: Two beams are welded together at C. A 8 kN.m clockwise couple moment and a 2 kN force are applied to the assembly as shown. The external support at A is a “Pin” (Hinge) support. A cable passes over a smooth pulley at F and is attached to the beam at D and E. Determine:

(a) The external reactions at A and
(b) The tension in the cable.



Solution Steps

STEP 1 – Identify the type(s) of external supports and **DRAW THE FREE BODY DIAGRAM (FBD)!!!!!!! Label all forces and couple moments in the FBD.** In this problem, to isolate the beam assembly from the pulley support we "cut" the cable and show a cable tension force, **T** at points D and E. (Always indicate a tension force acting away from the attachment point(s).

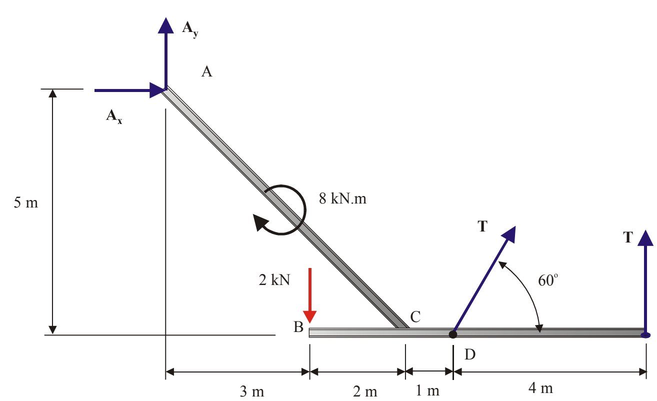
In this example, the "Pin" (hinge) support at A is shown as sloping. IT DOES NOT MATTER – ALWAYS SHOW VERTICAL AND HORIZONTAL REACTIONS FOR A PIN.

STEP 2 – Indicate the positive sign convention for forces and moments and write the equilibrium equations based on the FBD you have drawn.

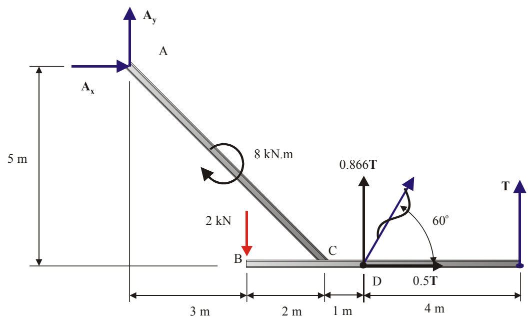
STEP 3 – Solve the equilibrium equations for unknown reactions.

STEP 4 – Check your solution by re-drawing the FBD indicating all reactions with the correct sense and taking moments about a different point.

FBD – We have three (3) unknown reaction components, $\mathbf{A_x}$, $\mathbf{A_y}$ and **T**. Since we have three (3) equilibrium equations, the problem is STATICALLY DETERMINATE and we will be able to solve for ALL unknown force components.



NOTE: Before applying the equilibrium equations, resolve all sloping forces in the FBD into their horizontal and vertical components.



$$\sum F_x = 0$$
$$A_x + 0.5T = 0 \quad \textbf{(1)}$$
$$\sum F_y = 0$$
$$A_y - 2 + 0.866T + T = 0$$
$$A_y + 1.866T = 2 \quad \textbf{(2)}$$
$$\sum M_A = 0$$
$$-8 - 2(3) + 0.866T(6) + 0.5T(5) + T(10) = 0$$
$$17.696T = 14 \quad \textbf{(3)}$$
$$T = 0.791\text{kN}$$

Substitute in (1):

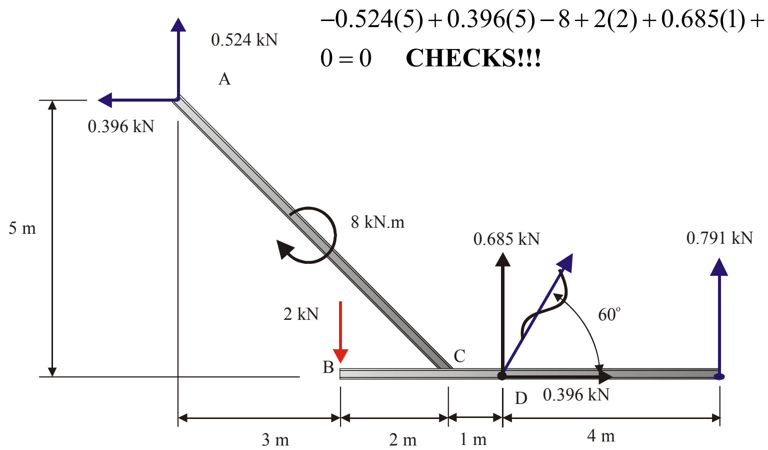
$$A_x + 0.5(0.791) = 0$$
$$A_x = -0.396\text{kN}$$
$$\therefore \mathbf{A_x} = 0.396\text{kN} \leftarrow$$

Substitute in (2):

$$A_y + 1.866(0.791) = 2$$
$$A_y = +0.524\text{kN}$$
$$\therefore \mathbf{A_y} = 0.524\text{kN} \uparrow$$

Check Results by taking moments about point C.

$$\sum M_C = 0$$
$$-0.524(5) + 0.396(5) - 8 + 2(2) + 0.685(1) + 0.791(5) = 0$$
$$0 = 0 \quad \textbf{CHECKS!!!}$$

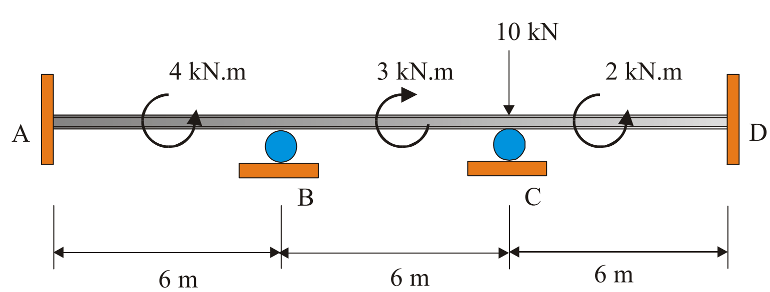


CHAPTER 3
Distance Education
Study Problem # 14
J. Frye
May, 2011

Statically Indeterminate Beam

Number of unknown force and/or couple moments in the Free Body Diagram exceeds the number of equilibrium equations.

Example: The beam shown has “Fixed” (rigid) supports at A and D. It has “Roller” supports at B and C. Draw the Free Body Diagram (FBD) for the beam and write the equilibrium equations for the FBD



Solution Steps

STEP 1 - Identify the external support types and the reaction components associated with each support type.

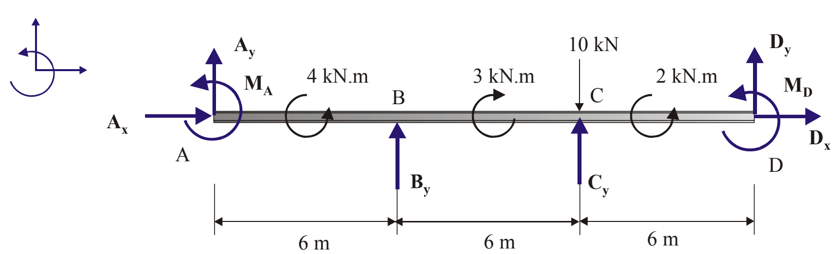
In this example, supports A and D are “Fixed” (rigid) supports so we show horizontal and vertical force components and a couple moment (rotation prevented) at each of these supports. Supports B and C are “Roller” supports so we show only a vertical force component at these supports.

We can assume positive senses for all unknown reaction components.

Indicate a positive sign convention. **MAKE SURE TO LABEL ALL FORCES AND COUPLE MOMENTS IN THE FBD!!!!!!**

STEP 2 - Write the equilibrium equations base on the FBD and solve for any unknowns that you can.

FBD and Equilibrium Equations for the FBD



$$\sum F_x = 0$$
$$A_x + D_x = 0 \quad \textbf{(1)}$$
$$\sum F_y = 0$$
$$A_y + B_y - 10 + C_y + D_y = 0 \quad \textbf{(2)}$$
$$\sum M_A = 0$$
$$+M_A + 4 - 3 + 2 - 10(12) + M_D = 0$$
$$M_A + M_D - 117 = 0 \quad \textbf{(3)}$$

UNKNOWNS:

A_x, D_x, M_A, B_y, C_y, D_x, D_y, M_D

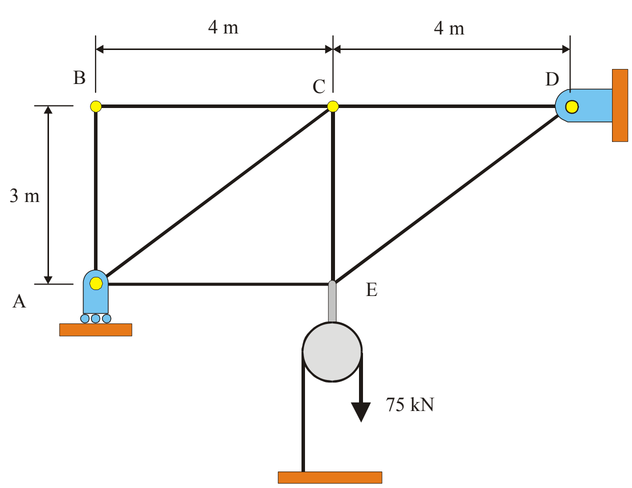
Beam is Statically Indeterminate since there are eight (8) unknowns and only three (3) equilibrium equations. We can write the equilibrium equations but are unable to solve for any of the unknown forces or couple moments. We require five (5) additional equations.

CHAPTER 4
Distance Education
Study Problem # 15
J. Frye
May, 2011

Trusses

Truss with Pulley Load at a Joint

Example - Determine the force in each member of the truss and state whether it is in tension or compression.



Step 1 - We first draw a separate Free Body Diagram of the pulley and determine the force(s) that the pulley applies to Joint E of the truss. (Note: Newton's Third law applies when we remove the pulley from the truss.)

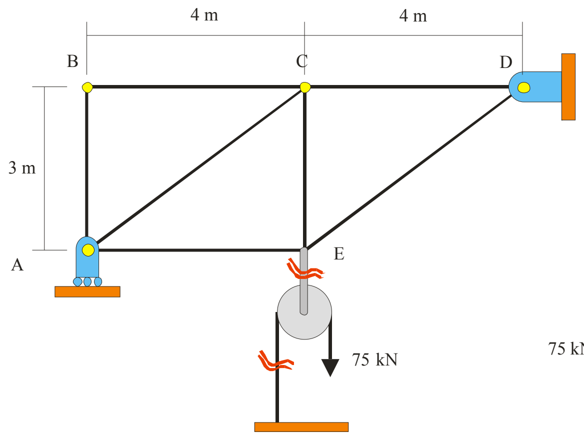
$\sum F_y = 0 \uparrow$

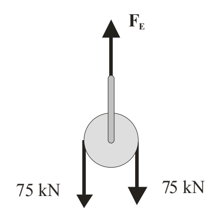
$F_E - 75 - 75 = 0$

$F_E = +150N$

$\therefore \mathbf{F_E} = 150 \uparrow$ on the pulley

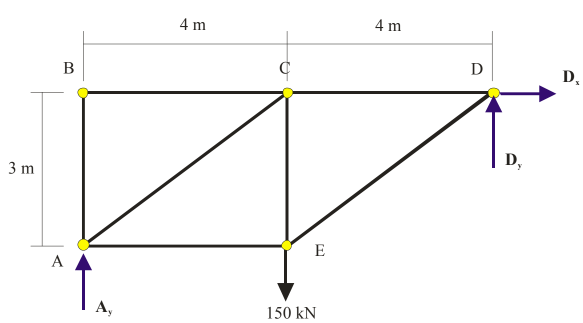
$\therefore \mathbf{F_E} = 150 \downarrow$ on the truss





FBD of Pulley

Step 2 – Draw a separate FBD of the truss and apply the equilibrium equations to determine the support reactions.



$\sum F_x = 0$

$\therefore \mathbf{D_x} = 0$ **(1)**

$\sum F_y = 0$

$A_y + D_y = 0$ **(2)**

$\sum M_D = 0$

$-A_y(8) + 150(4) = 0$ **(3)**

$A_y = +75kN$

$\therefore \mathbf{A_y} = 75kN \uparrow$

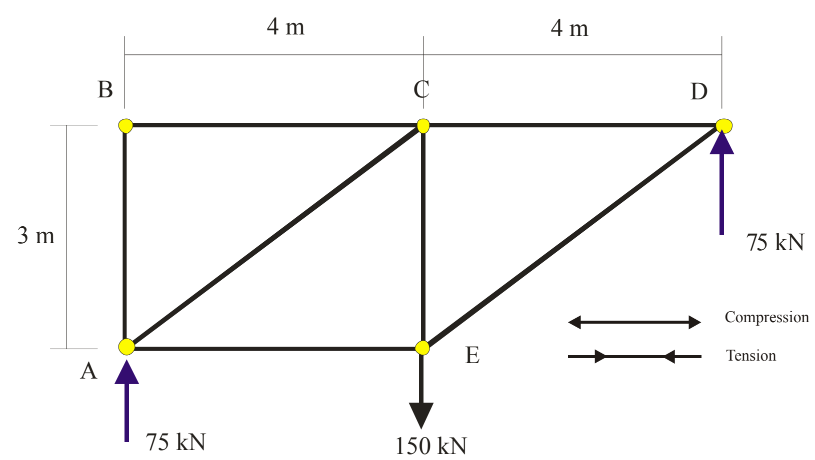
Substitute in (2):

$75 - 150 + D_y = 0$

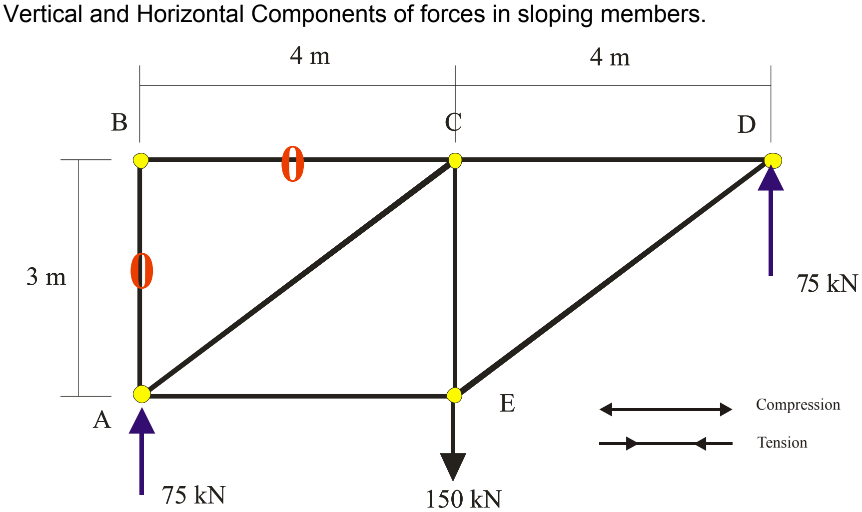
$D_y = +75kN$

$\therefore \mathbf{D_y} = 75kN \uparrow$

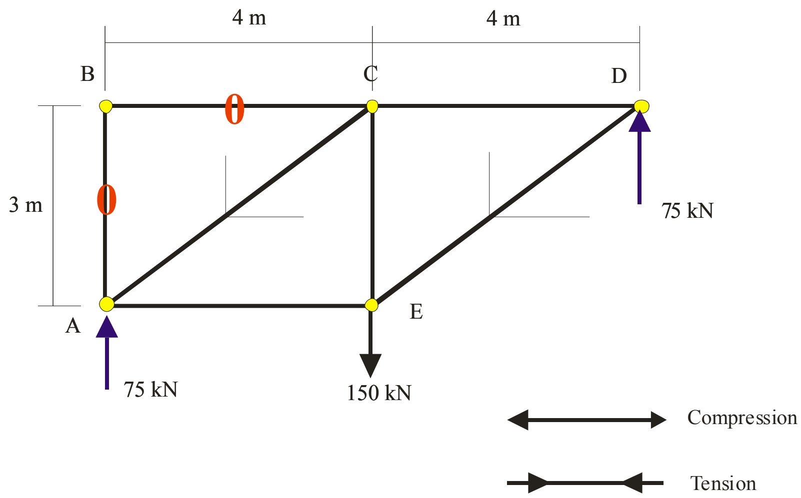
Step 3 –Re-draw a the truss and place a legend for tension and compression adjacent to the figure. Make this drawing fairly large so that it does not become too crowded when you place analysis results directly on this figure.

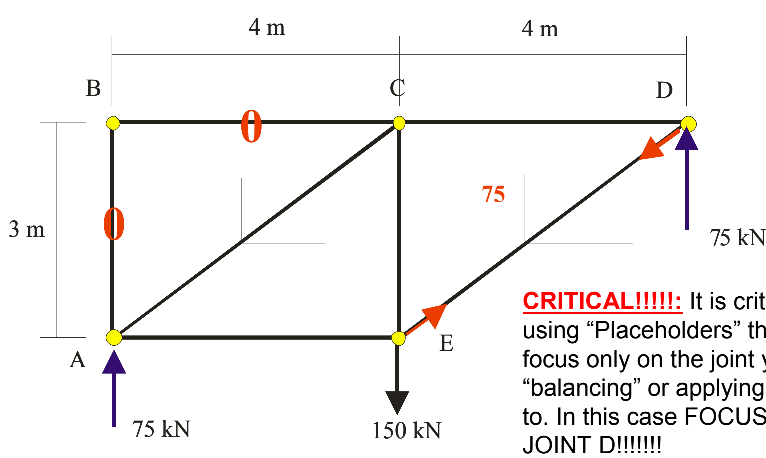


Step 4 – Identify any zero force members (if you can) and begin the analysis. It is recommended that you use the Method of Joints with Placeholders for Vertical and Horizontal Components of forces in sloping members.



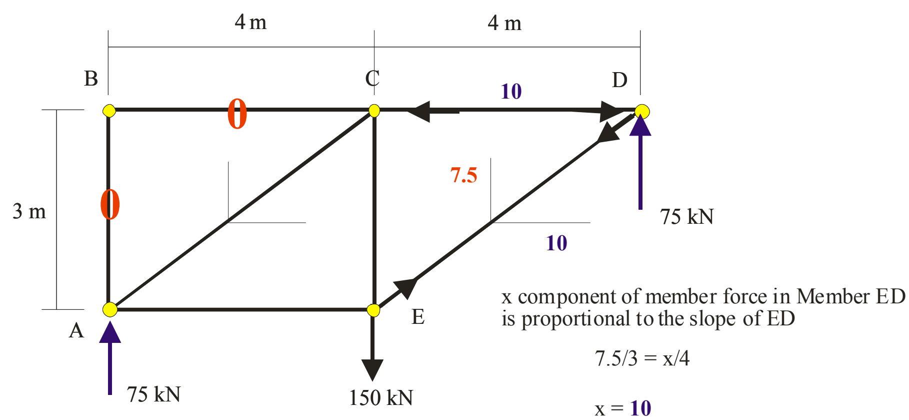
Horizontal and vertical placeholders on all Sloping Members.
Start the analysis at a Joint where there are no more than two (2) unknown member forces and one of the members is either horizontal or vertical. We can choose either Joint A or Joint D.

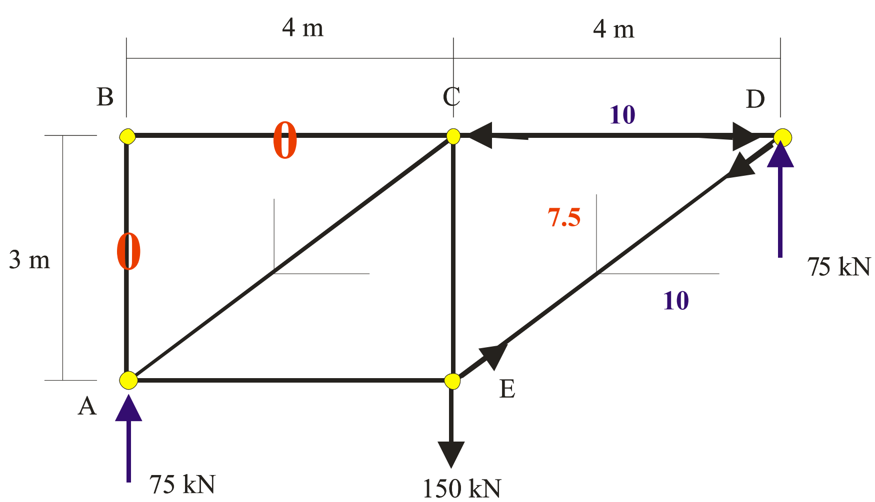




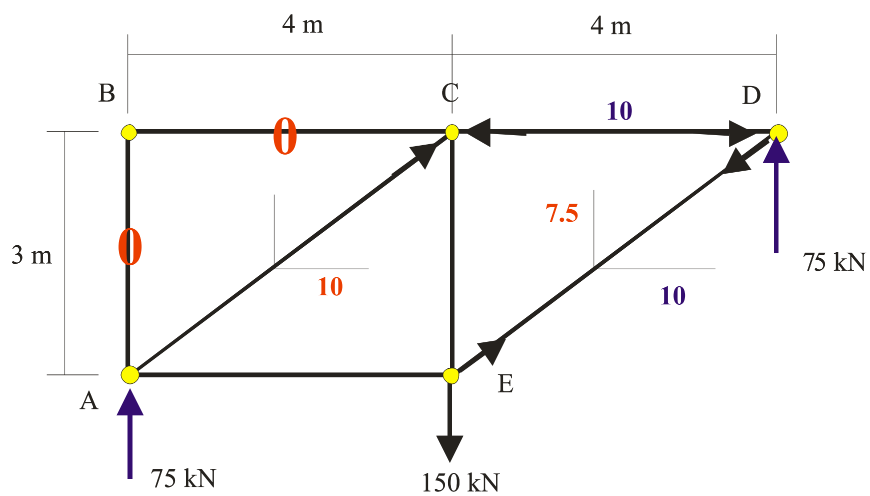
CRITICAL!!!!!: It is critical when using “Placeholders” that you focus only on the joint you are “balancing” or applying equilibrium to. In this case FOCUS ONLY ON JOINT D!!!!!!!

Balance Joint D in the y - direction first:
The vertical component must be 75 kN downward at Joint D to balance the 75 kN vertical reaction at D.
Place an arrow **downward** to the left at Joint D.
At the other end of Member DE place an arrow going **upward** to the right to indicate Member DE is in tension. Place 75 on the vertical placeholder adjacent to Member DE.

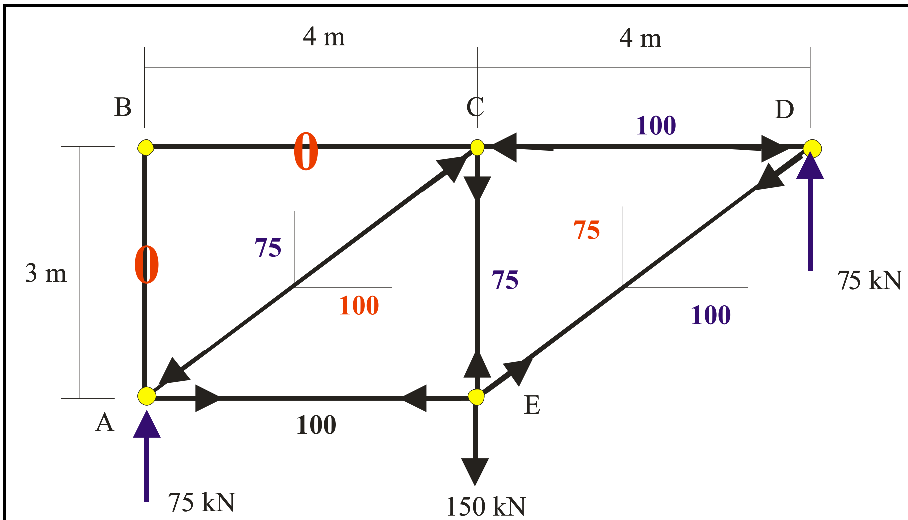




Balance Joint D in the x direction:



Move to Joint C and Balance Joint C in the x direction:



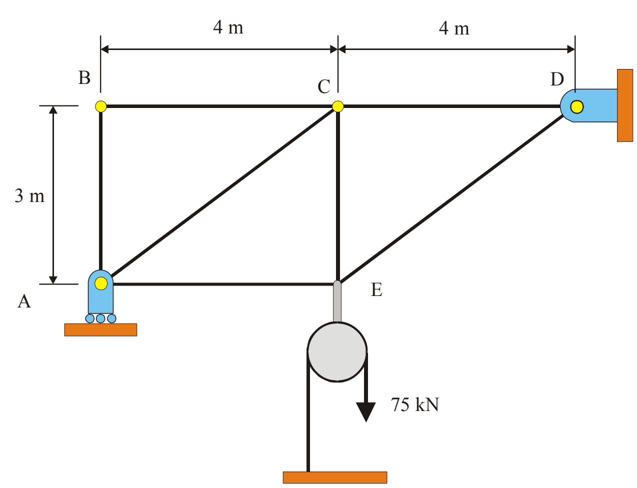
Balance Joint C in the y - direction and move to Joint E.
Balance Joint E in the x - direction to complete the analysis.
Check Equilibrium at Joint A

CHAPTER 4
Distance Education
Study Problem # 15
J. Frye
May, 2011

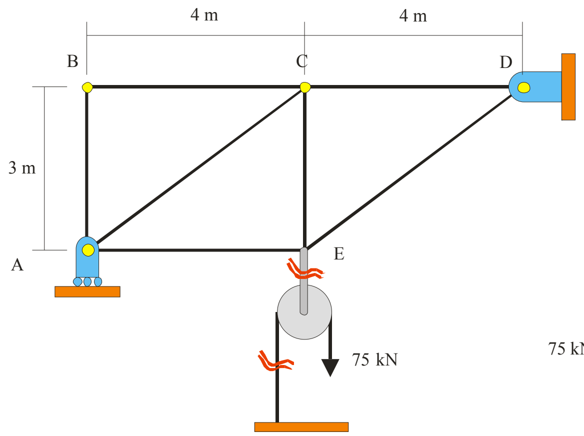
Trusses

Truss with Pulley Load at a Joint

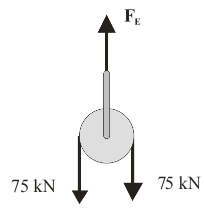
Example - Determine the force in each member of the truss and state whether it is in tension or compression.



Step 1 - We first draw a separate Free Body Diagram of the pulley and determine the force(s) that the pulley applies to Joint E of the truss. (Note: Newton's Third law applies when we remove the pulley from the truss.)

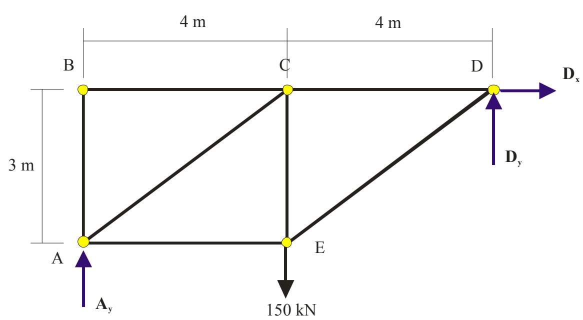


$$\sum F_y = 0 \uparrow$$
$$F_E - 75 - 75 = 0$$
$$F_E = +150N$$
$$\therefore \mathbf{F_E} = 150 \uparrow \text{ on the pulley}$$
$$\therefore \mathbf{F_E} = 150 \downarrow \text{ on the truss}$$



FBD of Pulley

Step 2 – Draw a separate FBD of the truss and apply the equilibrium equations to determine the support reactions.

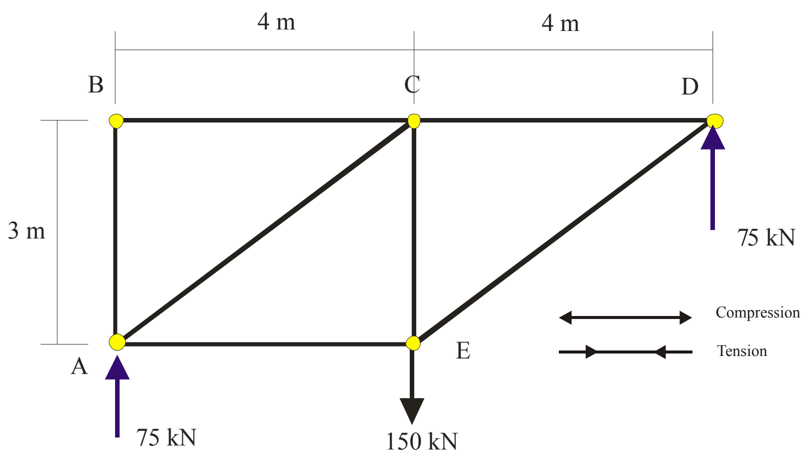


$$\sum F_x = 0$$
$$\therefore \mathbf{D_x} = 0 \quad \mathbf{(1)}$$
$$\sum F_y = 0$$
$$A_y + D_y = 0 \quad \mathbf{(2)}$$
$$\sum M_D = 0$$
$$-A_y(8) + 150(4) = 0 \quad \mathbf{(3)}$$
$$A_y = +75kN$$
$$\therefore \mathbf{A_y} = 75kN \uparrow$$

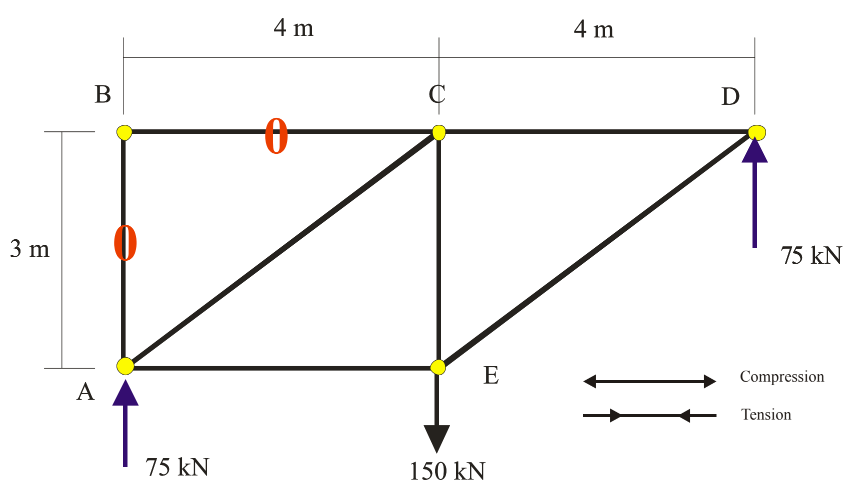
Substitute in (2):

$$75 - 150 + D_y = 0$$
$$D_y = +75kN$$
$$\therefore \mathbf{D_y} = 75kN \uparrow$$

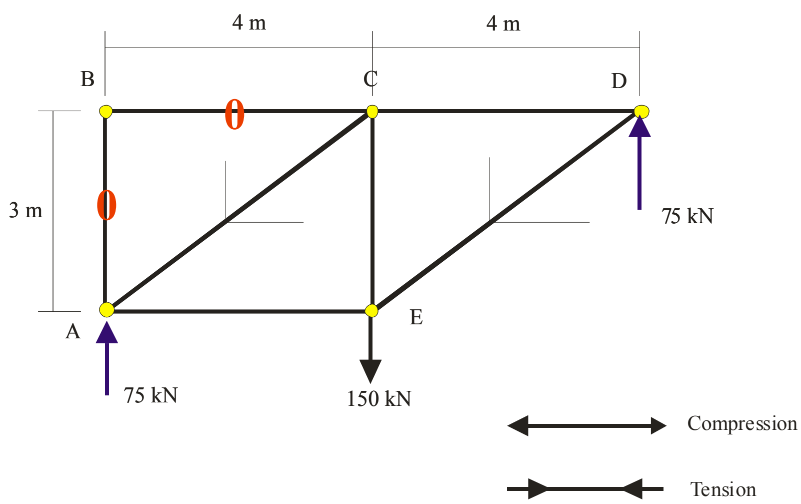
Step 3 –Re-draw a the truss and place a legend for tension and compression adjacent to the figure. Make this drawing fairly large so that it does not become too crowded when you place analysis results directly on this figure.

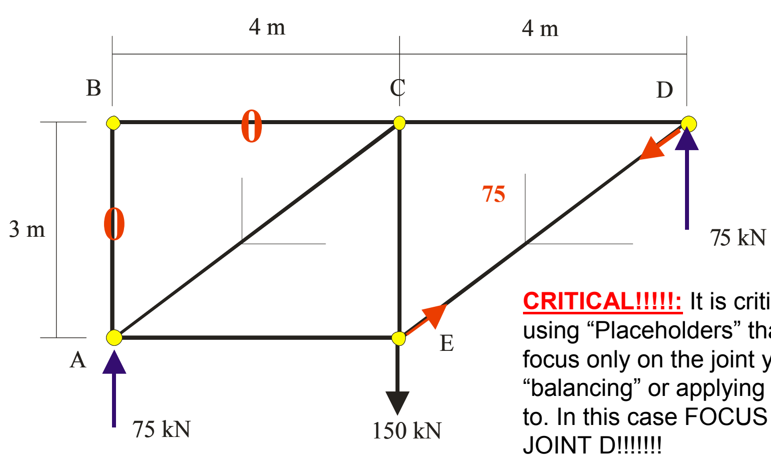


Step 4 – Identify any zero force members (if you can) and begin the analysis. It is recommended that you use the Method of Joints with Placeholders for Vertical and Horizontal Components of forces in sloping members.



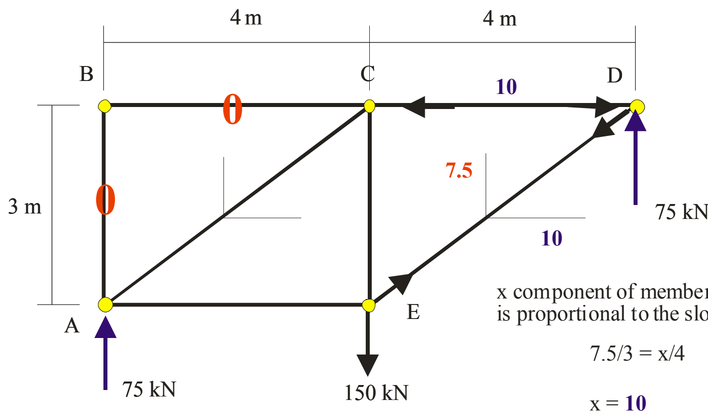
Horizontal and vertical placeholders on all Sloping Members.
Start the analysis at a Joint where there are no more than two (2) unknown member forces and one of the members is either horizontal or vertical. We can choose either Joint A or Joint D.





CRITICAL!!!!!: It is critical when using "Placeholders" that you focus only on the joint you are "balancing" or applying equilibrium to. In this case FOCUS ONLY ON JOINT D!!!!!!!

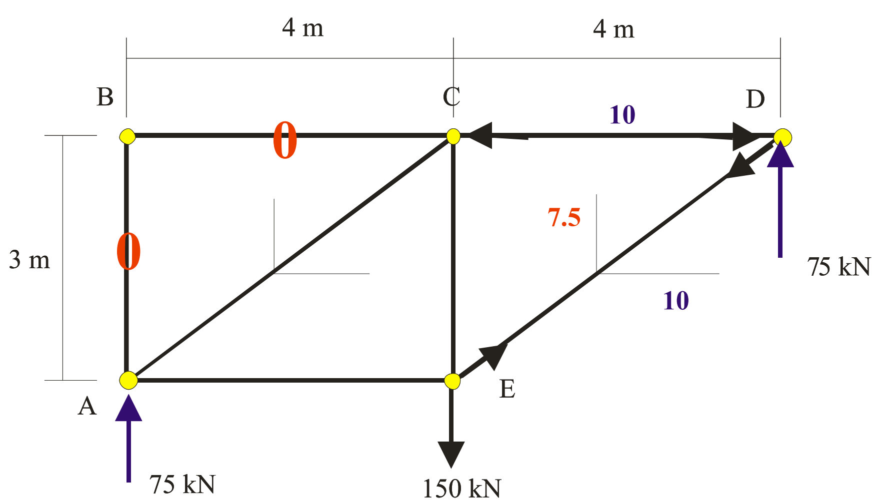
Balance Joint D in the y - direction first:
The vertical component must be 75 kN downward at Joint D to balance the 75 kN vertical reaction at D.
Place an arrow **downward** to the left at Joint D.
At the other end of Member DE place an arrow going **upward** to the right to indicate Member DE is in tension. Place 75 on the vertical placeholder adjacent to Member DE.



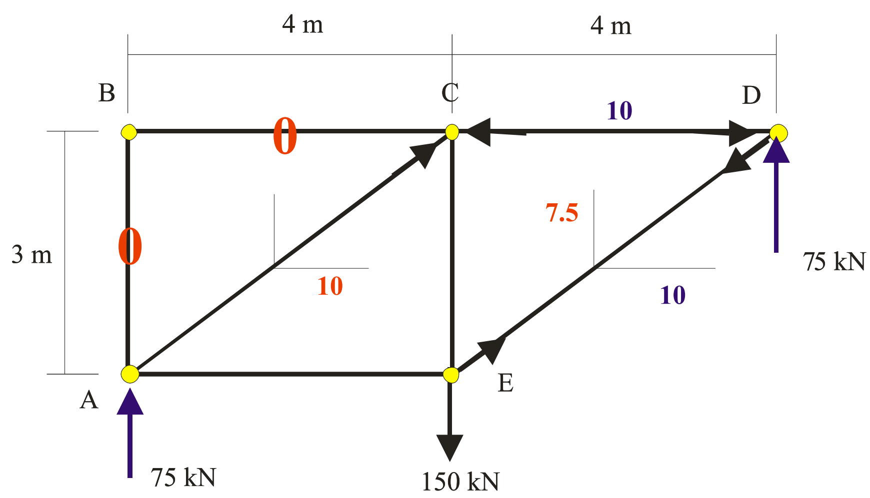
x component of member force in Member ED is proportional to the slope of ED

$7.5/3 = x/4$

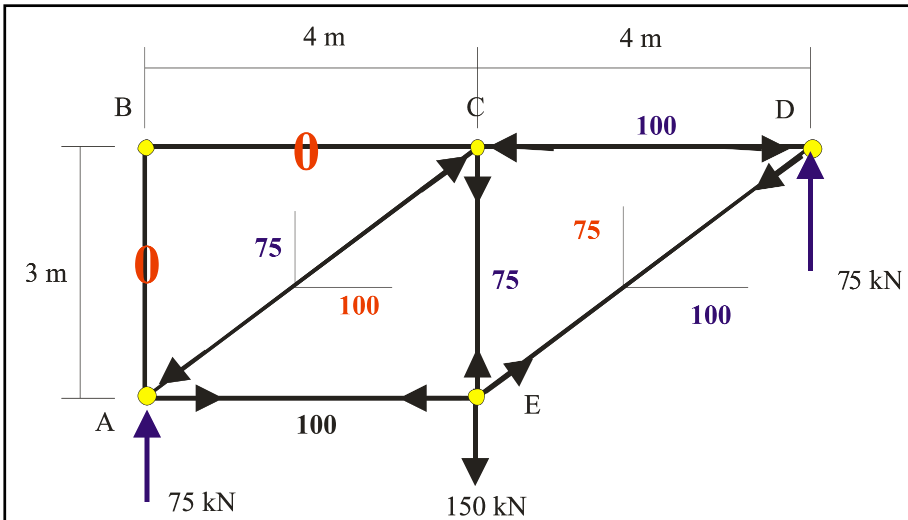
$x = \mathbf{10}$



Balance Joint D in the x direction:



Move to Joint C and Balance Joint C in the x direction:

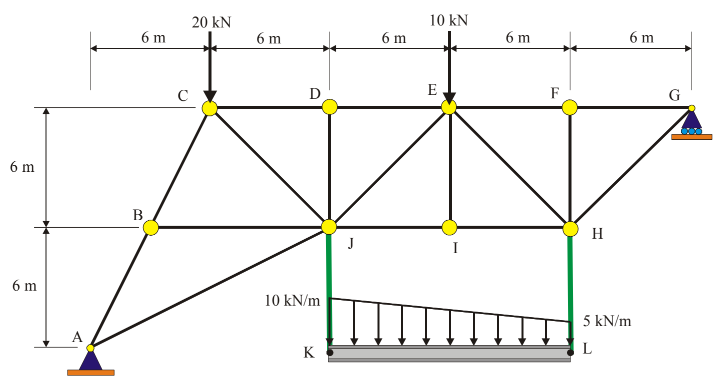


Balance Joint C in the y - direction and move to Joint E.
Balance Joint E in the x - direction to complete the analysis.
Check Equilibrium at Joint A

CHAPTER 4
Distance Education
Study Problem # 16
J. Frye
May, 2011

Truss Analysis

Suspended Load



Analysis Steps – “Method of Joints” using “Placeholders” for Member Horizontal and Vertical Force Components

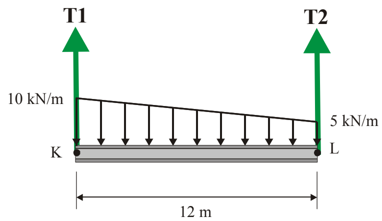
1. Determine the load applied at Joints J and H due to the distributed load on the suspended beam. You must draw a FBD of the suspended beam and apply the equilibrium equations to the beam to determine the tensions in the cables KJ and LH. Newton’s Third Law applies and these tensions are applied opposite on Joints J and H.
2. Draw a FBD of the entire truss and determine the external reactions at the pin Joint A and the roller support Joint G.
3. Draw a separate drawing of the truss (fairly large) since you will be putting member forces directly on this figure. Put the horizontal and vertical “Placeholders” on sloping members.
4. Place all externally applied loads and external reactions on the figure. Any forces that are at a slope must be first resolved into their rectangular components.

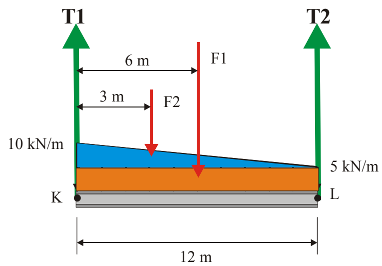
We are now ready to begin the analysis.

TRUSS ANALYSIS

1. Select a joint where there are only two unknown member forces and one of the two members is either vertical or horizontal and begin the analysis. If you cannot find a joint where one of the two members is either vertical or horizontal OR you get to a point where you have two sloping members, draw a FBD of that joint and apply the equilibrium equations to determine the member forces. Then resolve these member forces into there components. Put the components on the placeholders and continue the analysis.
2. When you arrive at the last joint of the truss, horizontal and vertical forces acting on this joint must “balance” or that joint is not in equilibrium indicating a mistake has been made.

Cable Tensions Due to Distributed Load on the Beam





F1 due to 5 kN/m uniformly distributed load :

$F1 = (5kN/m)(12m) = 60kN$

F2 due to 10kN/m to 5 kN/m triangular distributed load :

$F2 = \dfrac{(10kN/m - 5kN/m)(12m)}{2} = 30kN$

$\sum M_k = 0$

$-30(3) - 60(6) + T2(12) = 0$

$T2 = +37.5kN$

$\therefore \mathbf{T2} = 37.5kN \uparrow$ on the beam

$\sum F_y = 0$

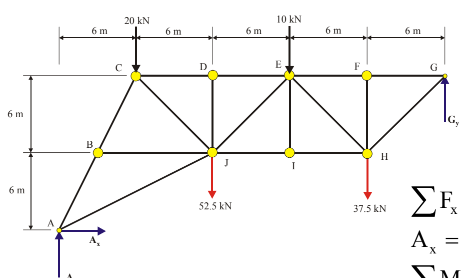
$T1 - 30 - 60 + 37.5 = 0$

$T1 = +52.5kN$

$\therefore \mathbf{T1} = 52.5kN \uparrow$ on the beam

We apply T1 and T2 opposite on truss.

FBD of Entire Truss



$\sum F_x = 0$

$A_x = 0$

$\sum M_A = 0$

$-20(6) - 52.5(12) - 10(18) - 37.5(24) + G_y(30) = 0$

$G_y = +61kN$

$\therefore \mathbf{G_y} = 61kN \uparrow$

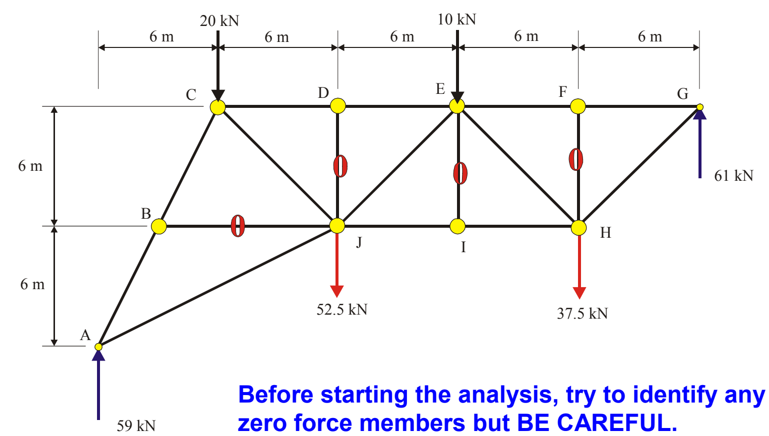
$\sum F_y = 0$

$A_y - 20 - 52.5 - 10 - 37.5 + 61 = 0$

$A_y = +59kN$

$\therefore \mathbf{A_y} = 59kN \uparrow$

Identify any Zero Force Members

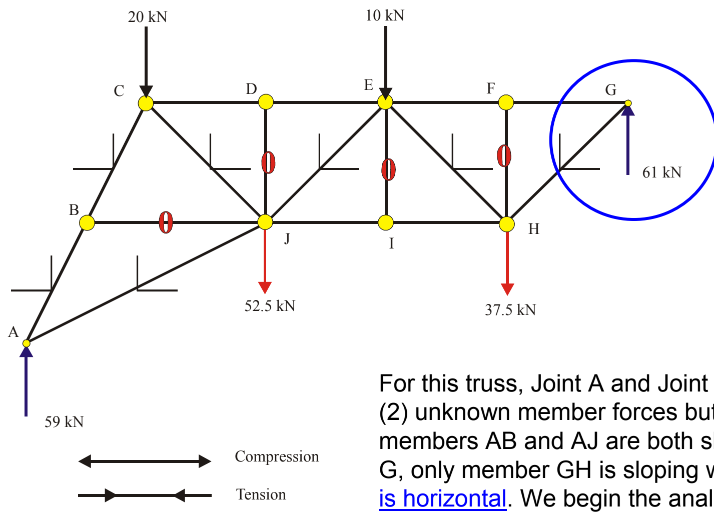


Before starting the analysis, try to identify any zero force members but BE CAREFUL.

Starting the Analysis

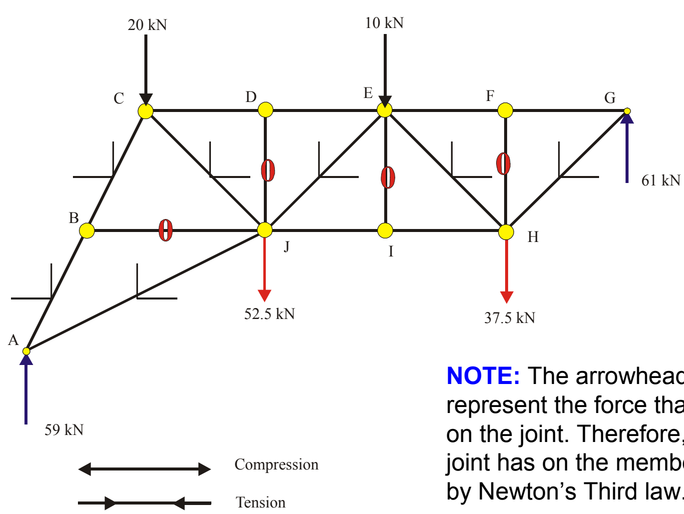
1. Select a joint where there are only two (2) unknown member forces. If you can start at a Joint where one of the members is either Horizontal or Vertical. Apply equilibrium to this Joint noting that the **horizontal and vertical force components are proportional to the member slope**. You will be able to work directly on the component placeholders that you have put on the figure.
2. Proceed to another Joint where there are only two (2) unknown member forces and one of the members is either Horizontal or Vertical. Again apply equilibrium to this joint placing the member force components on the placeholders.
3. Continue until you have completed the truss analysis or you get two a Joint where both member forces are sloping. In this case, you MUST draw a FBD of the Joint and apply the equilibrium equations to determine the member forces. Resolve these member forces into their components and place the on the member placeholders. Continue the analysis until all joints are completed.

Start of Analysis



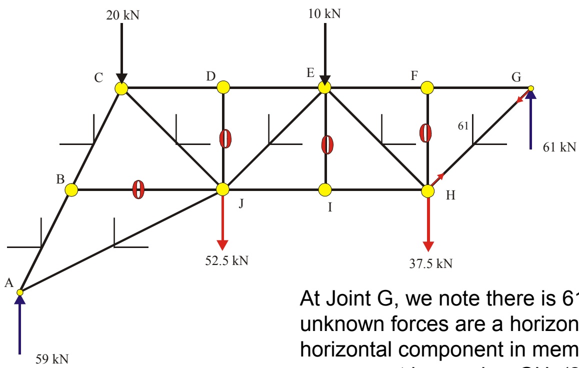
For this truss, Joint A and Joint G each have two (2) unknown member forces but for Joint A, members AB and AJ are both sloping. For Joint G, only member GH is sloping while member GF is horizontal. We begin the analysis at Joint G.

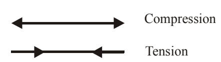
Place the "Compression – Tension" Legend on the Figure



NOTE: The arrowheads of the legend represent the force that the member has on the joint. Therefore, the force that the joint has on the member will be opposite by Newton's Third law.

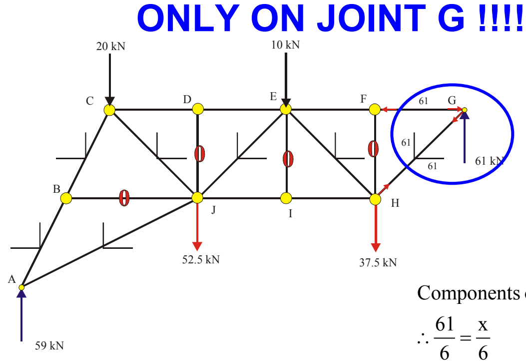
Equilibrium of Joint G – FOCUS ONLY ON JOINT G, DO NOT LOOK AT JOINTS F OR H !!!!!!!!!!!!!!!!!!!!

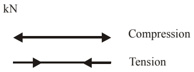




At Joint G, we note there is 61 kN acting vertically. The unknown forces are a horizontal force in member GF, an horizontal component in member GH and an unknown vertical component in member GH. (2 unknown horizontal and only 1 unknown vertical). We therefore apply equilibrium in the vertical direction FIRST. For equilibrium of Joint G in the vertical direction, the vertical component of the member force in GH must be 61 kN acting DOWN. We place 61 on the vertical placeholder and an arrow sloping DOWN at Joint G. We go to the opposite end of member GH and place an arrow sloping UP. Indicating member GH is in Tension

Complete Equilibrium of Joint G- (FOCUS ONLY ON JOINT G !!!!!!!!!!!!!!!!!!!!





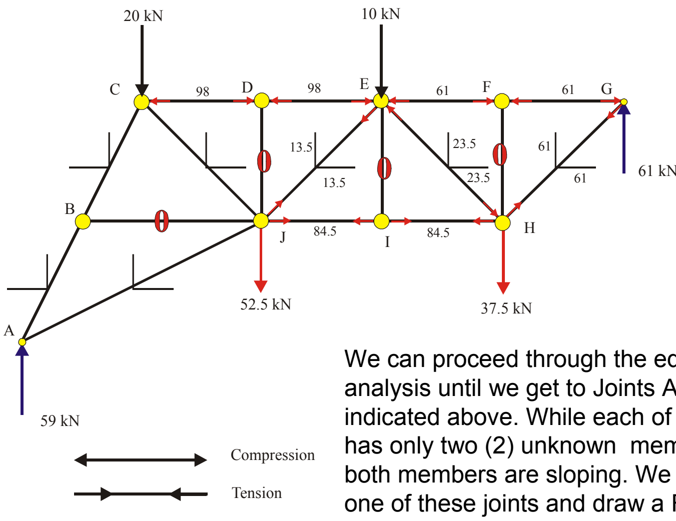
Components of Member GH are Proportional to the Slope :

$$\therefore \frac{61}{6} = \frac{x}{6}$$

x = horizontal component = 61kN

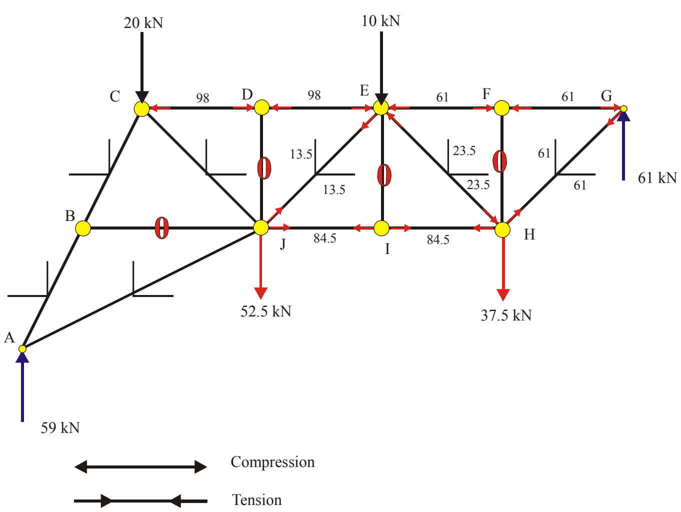
We place 61 kN on the horizontal placeholder for member GH. We now apply equilibrium in the x-direction at Joint G noting that at Joint G we have 61 kN acting to the Left. For equilibrium the member force in GF must be 61 kN acting to the Right. We place 61 kN on member GF and an arrow to the Right at Joint G. We go to the opposite end of member GF and place an arrow opposite. Indicating member GF is in COMPRESSION. We are finished with Joint G and can now proceed to the next Joint.

Joint Equilibrium Analysis Continued
Remember to FOCUS only on one Joint at a time!!!!!!!!!!!

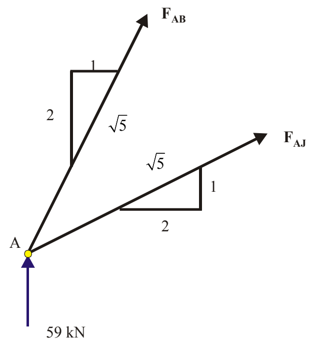


We can proceed through the equilibrium analysis until we get to Joints A, C and J as indicated above. While each of these Joints has only two (2) unknown member forces, both members are sloping. We MUST choose one of these joints and draw a FBD and then apply the equilibrium equations to determine the member forces. We chose Joint A.

Begin the analysis at a Joint where there are only 2 unknown member forces



Equilibrium of Joint A



FBD of Joint A

$$\sum F_x = 0$$

$$\frac{1}{\sqrt{5}}F_{AB} + \frac{2}{\sqrt{5}}F_{AJ} = 0 \quad (1)$$

$$\sum F_y = 0$$

$$\frac{2}{\sqrt{5}}F_{AB} + \frac{1}{\sqrt{5}}F_{AJ} + 59 = 0 \quad (2)$$

Multiply Equation (2) by 2 and subtract from Equation (1):

$$-\frac{3}{\sqrt{5}}F_{AB} = 118$$

$$F_{AB} = -87.95$$

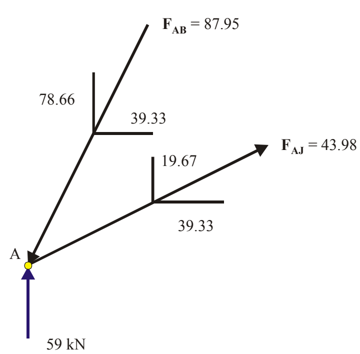
Substitute in Equation (1):

$$-\frac{1}{\sqrt{5}}(-87.95) + \frac{2}{\sqrt{5}}F_{AJ} = 0$$

$$F_{AJ} = +43.98$$

We assume the senses of $\mathbf{F_{AB}}$ and $\mathbf{F_{AJ}}$. From our equilibrium analysis, our assumption of the sense of $\mathbf{F_{AB}}$ was incorrect and our assumption of the sense of $\mathbf{F_{AJ}}$ was correct.

Member Forces F_{AB} and F_{AJ} resolved into components and put on placeholders



$\mathbf{F_{AB}}$:

$$\frac{1}{\sqrt{5}}(87.95) = 39.33$$

$$\frac{2}{\sqrt{5}}(87.95) = 78.66$$

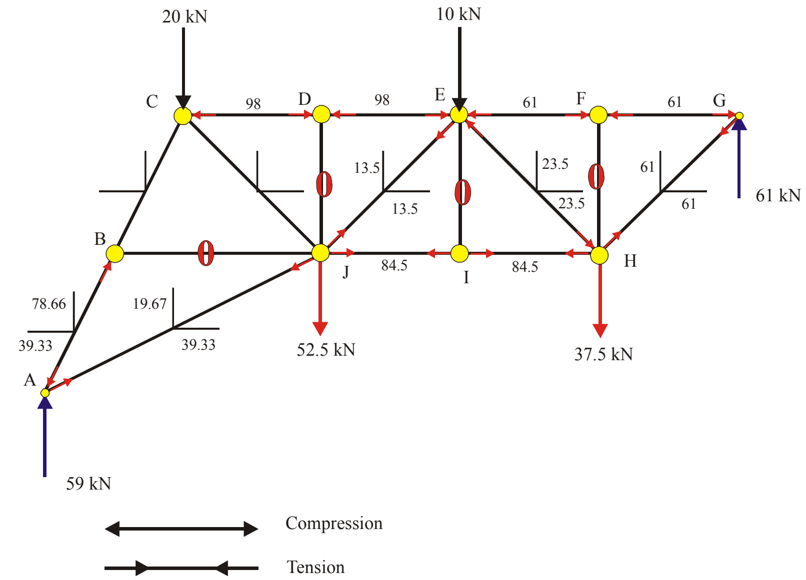
$\mathbf{F_{AJ}}$:

$$\frac{1}{\sqrt{5}}(43.98) = 19.67$$

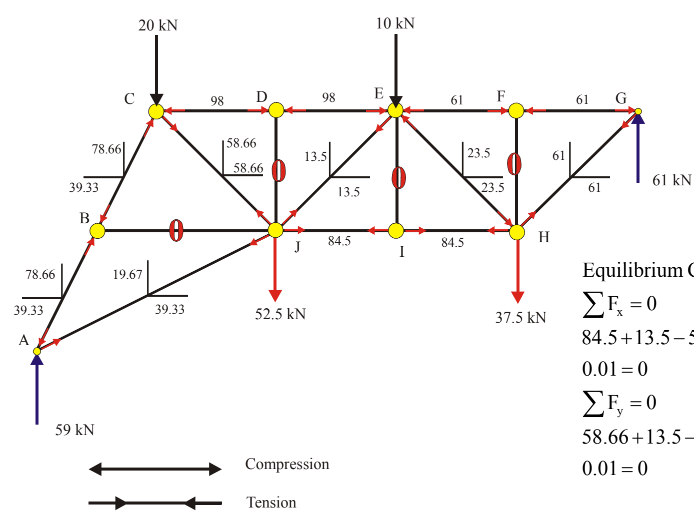
$$\frac{2}{\sqrt{5}}(43.98) = 39.33$$

NOTE: We have changed the Sense of FAB. We can now place the results of Joint A on the truss and finish the analysis.

Joint A Results on the Truss Placeholders



Completed Analysis



Equilibrium Check At Joint J :

$$\sum F_x = 0$$

$$84.5 + 13.5 - 58.66 - 39.33 = 0$$

$$0.01 = 0$$

$$\sum F_y = 0$$

$$58.66 + 13.5 - 19.67 - 52.5 = 0$$

$$0.01 = 0$$

CHECKS!!!!

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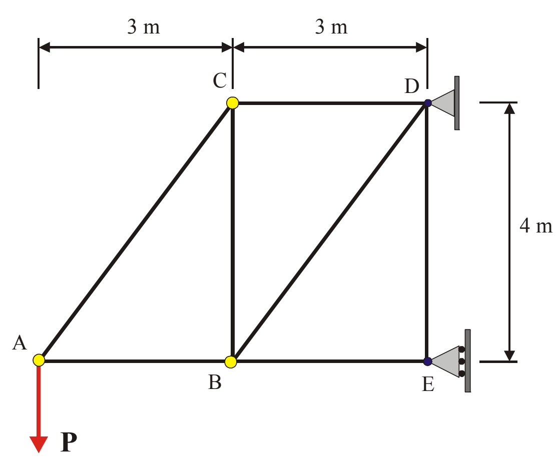
Distance Education
Study Problem # 17
J. Frye
May, 2011

Truss Design for Load P

EXAMPLE: The truss shown in the figure is to be designed to carry a load P applied to Joint A. The truss has a pin (hinge) support at D and a roller support at E.

If the maximum tension force in any member cannot exceed 5 kN and the maximum compressive force in any member cannot exceed 3.9 kN what is the maximum force P that can be applied at Joint A.

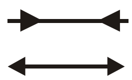


Solution Steps
Method of Joints Using Placeholders for Horizontal and Vertical Components of Member Forces in Sloping Members

STEP 1 – Draw a Free-Body-Diagram (FBD) of the entire truss and determine the external support reactions at D and E in terms of the load **P** applied to Joint A.

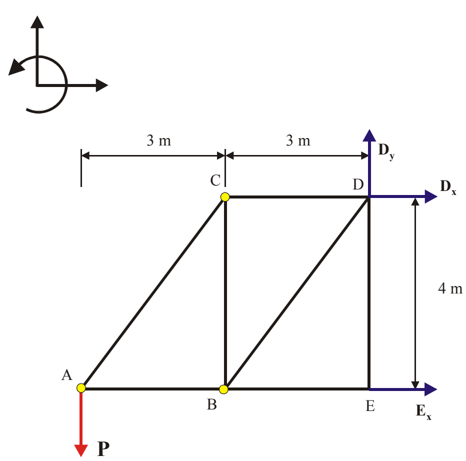
STEP 2 – Re-draw the truss (make a large enough drawing so that it does not become too crowded when you place member forces on this drawing)

Place a legend adjacent to the drawing indicating Tension and Compression



STEP 3 – Analyze the truss (determine all member forces in terms of load **P**. From the analysis determine which member carries the maximum tensile force and which member carries the maximum compression force. Determine the maximum magnitude of **P** such that the maximum tension and compression in any member will not be exceeded. <u>Member DE can be identified as a zero (0) force member!</u>

STEP 1 – Draw the FBD and Determine the External Reactions in Terms of Load P



FBD of Truss

$\sum F_x = 0$

$D_x + E_x = 0 \quad$ **(1)**

$\sum F_y = 0$

$-P + D_y = 0 \quad$ **(2)**

$D_y = +P$

$\therefore \mathbf{D_y} = P \uparrow$

$\sum M_D = 0$

$6P + E_x(4) = 0 \quad$ **(3)**

$E_x = -1.5P$

$\therefore \mathbf{E_x} = 1.5P \leftarrow$

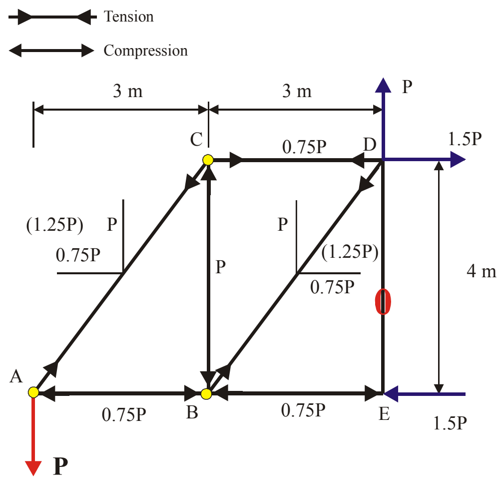
Substitute in (2)

$D_x + (-1.5P) = 0$

$D_x = +1.5P$

$\therefore \mathbf{D_x} = 1.5P \rightarrow$

STEP 2 – Truss Analysis



Starting at Joint A:

$$\frac{P}{4} = \frac{x}{3}$$

x component for member AC

$x = 0.75P$

STEP 3 – Maximum Tension and Compression Member Forces

MEMBER	FORCE	TENSION/COMPRESSION
AB	0.75P	Compression
AC	1.25P	**Tension**
CD	0.75P	Tension
CB	P	**Compression**
BD	1.25P	**Tension**
BE	0.75P	Compression
DE	0	(Zero)

Maximum Tension Member Force = 1.25P in members AC and BD

Maximum Compression Member Force = P in member CB

Maximum Load P

Let us assume tension = 5 kN governs the design :
Member Force in tension members AC and BD :
$1.25P = 5$ (tension)
$P = 4$ kN
Maximum member force in **compression** member CB = 3.9 kN :
$4 > 3.9$
$\therefore$ Compression governs and maximum P = 3.9 kN

MEMBER	FORCE	Force (P = 3.9 kN)	TENSION/COMPRESSION
AB	0.75P	2.925	Compression
AC	1.25P	4.875	Tension
CD	0.75P	2.925	Tension
CB	P	3.9	Compression
BD	1.25P	4.875	Tension
BE	0.75P	2.925	Compression
DE	0		(Zero)

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Distance Education
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May, 2011

Frames – Load Applied at a Joint

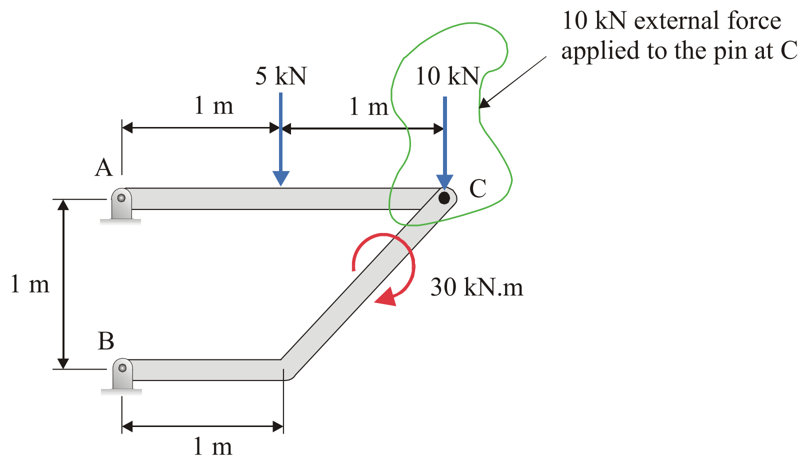
What to Do With the Load applied at a Joint when Sub-structuring?

If a load is applied at a joint where **TWO** members are pinned together, assign it to either one of the members attached at the joint but **NOT BOTH**. This is the same as applying the load to the pin and leaving the pin attached to that member.

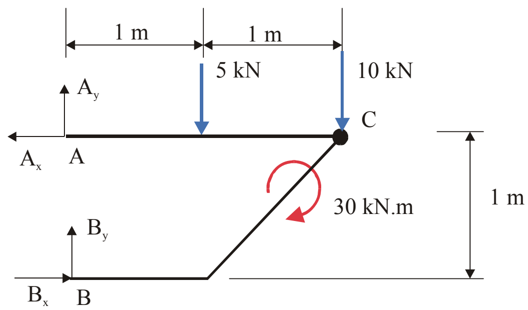
Alternately, you could draw a **SEPARATE** Free Body Diagram of the pin and apply the load to the pin along with the member reactions applied to the pin.

If there are **THREE** or more members (could include a pulley) attached at a pin, draw a separate Free Body Diagram of the pin (even if there is no load appiied at the pin).

10 kN load applied at Joint C and 30 kN.m clockwise couple-moment shown applied to member BC



Draw FBD of entire frame and apply equilibrium equations



Free-Body Diagram of Frame ABC

$$\sum M_A = 0 \circlearrowleft$$

$$B_x(1) - 5(1) - 10(2) - 30 = 0$$

$$B_x = +55 \text{ kN} \quad \therefore \ \mathbf{B_x = 55 \text{ kN}} \rightarrow$$

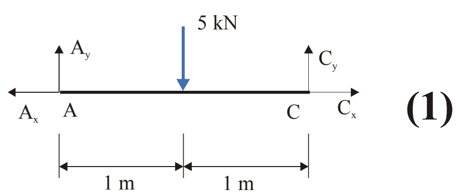
$$\sum F_x = 0 \rightarrow$$

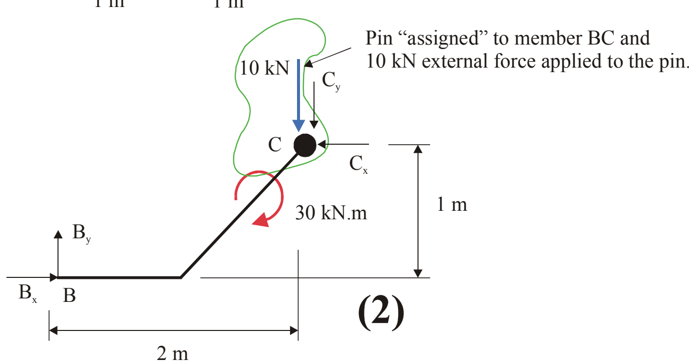
$$-A_x + 55 = 0 \qquad A_x = +55 \text{ kN}$$

(Sense of A_x in FBD assumed correctly.)

$$\therefore \mathbf{A_x} = 55 \text{ kN} \ \leftarrow$$

Substructure an **<u>leave the pin attached to Member BC</u>** and apply the 10 kN to the pin. Leave the 30 kN.m couple-moment on member BC.





From FBD (1):

$$\sum M_A = 0 \circlearrowleft$$

$$-5(1) + C_y(2) = 0$$

$$C_y = +2.5 \text{ kN} \qquad \therefore\ \mathbf{C_y} = 2.5 \text{ kN} \uparrow \text{ on AC}$$

$$\sum F_x = 0 \rightarrow$$

$$-55 + C_x = 0 \qquad C_x = +55 \text{ kN}$$

$$\therefore \mathbf{C_x} = 55 \text{ kN} \rightarrow \text{on AC}$$

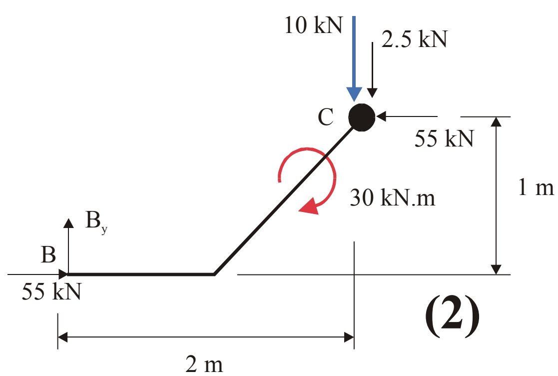
$$\sum F_y = 0 \uparrow$$

$$A_y - 5 + 2.5 = 0 \qquad A_y = +2.5 \text{ kN}$$

$$\therefore\ \mathbf{A_y} = 2.5 \text{ kN} \uparrow$$

From FBD (2):

(We re-draw FBD of BC now indicating all known forces.)



$$\sum F_y = 0 \uparrow$$

$$B_y - 10 - 2.5 = 0 \qquad B_y = +12.5 \text{ kN}$$

$$\therefore\ \mathbf{B_y} = 12.5 \text{ kN} \uparrow$$

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Distance Education
Study Problem # 19
J. Frye
May, 2011

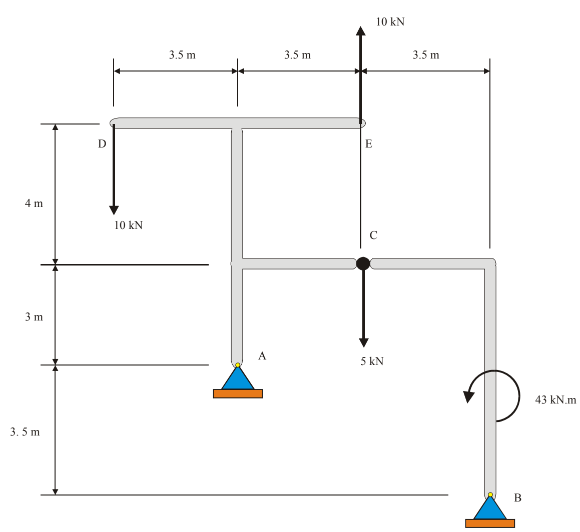
Frame - Example

1. Two pin staggered support
2. Load applied at pin

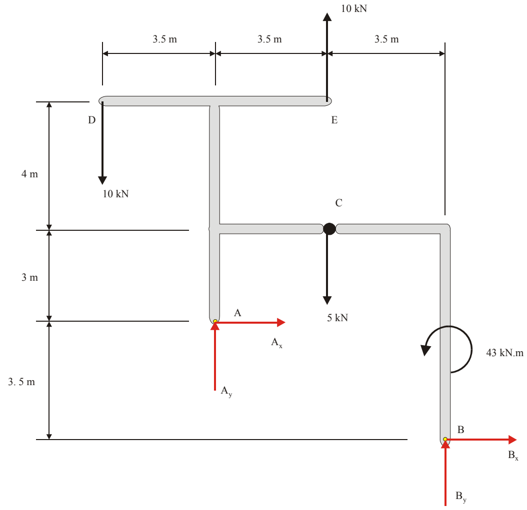
For the frame shown, determine:

a) the reactions at pin supports A and B and

b) All forces acting on the pin at C.



STEP 1- Draw FBD of entire structure and write equilibrium equations.

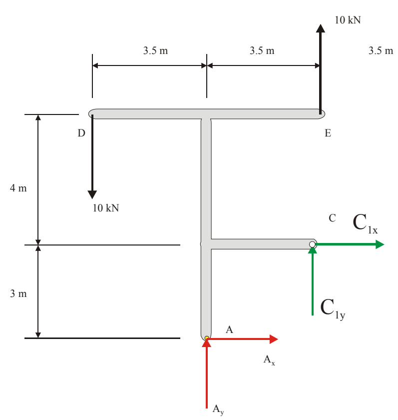


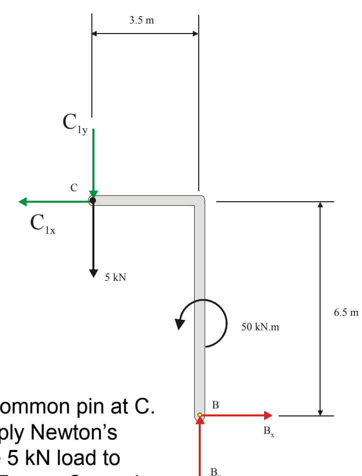
$$\sum F_x = 0$$
$$A_x + B_x = 0$$
$$\sum F_y = 0$$
$$-10 + 10 - 5 + A_y + B_y = 0$$
$$A_y + B_y - 5 = 0$$
$$\sum M_B = 0$$
$$+10(7) - A_y(7) - A_x(3.5) + 5(3.5) + 43 = 0$$
$$7A_y - 3.5A_x + 130.5 = 0$$

4 Unknowns – $\mathbf{A_x}$, $\mathbf{A_y}$, $\mathbf{B_x}$ and $\mathbf{B_y}$

Because the pin supports at A and B are "staggered", we are unable to solve for any of the unknowns. We must substructure.

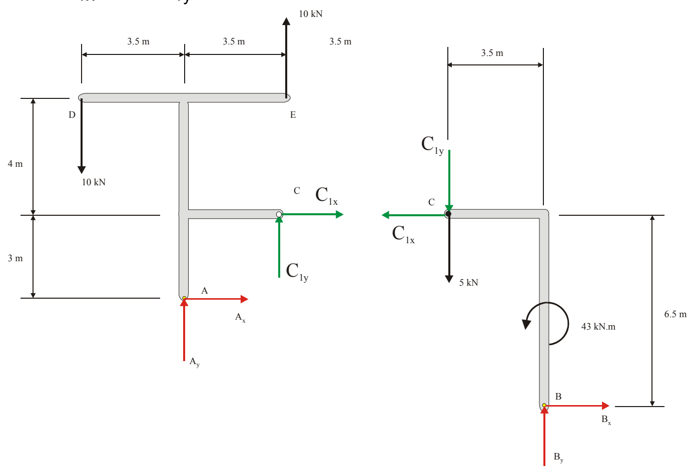
STEP 2 - Substructure

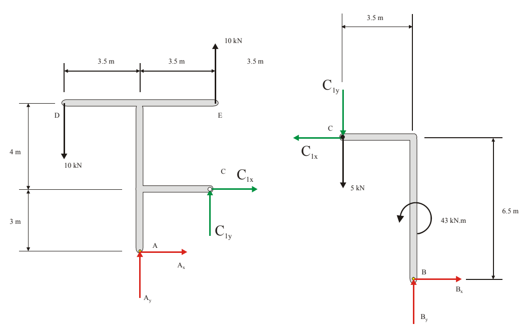




The Members ADEC and CB are connected by a common pin at C. We leave the pin "attached" to member CE and apply Newton's Third Law "member-to-member". We can apply the 5 kN load to either member. We assign the pin to member CB. Forces C_{1x} and C_{1y} are the forces that the pin has on member ADEC.

Substructures – We have a FBD of ADEC and a FBD of CB. We now have 6 unknowns, but we also have 6 equilibrium equations (3 from each FBD). Rather than trying to solve 6 equations in 6 unknowns we note that if we take moments about Point A in the FBD of ADEC and we take moments about Point B in FBD of CB, we get two equations in two unknowns (C_{1x} and C_{1y}).





From FBD of ADEC:

$\sum M_A = 0$

$+10(7) - C_{1x}(3) + C_{1y}(3.5) = 0$

$-3C_{1x} + 3.5C_{1y} = -70 \quad (1)$

From FBD of CB:

$\sum M_B = 0$

$+5(3.5) + 43 + C_{1x}(6.5) + C_{1y}(3.5) = 0$

$6.5C_{1x} + 3.5C_{1y} = -60.5 \quad (2)$

Subtracting (2) from (1)

$-9.5C_{1x} = -9.5$

$C_{1x} = +1\text{kN}$

$\therefore \mathbf{C_{1x}} = 1\text{kN} \rightarrow$ on member ADEC

$\therefore \mathbf{C_{1x}} = 1\text{kN} \leftarrow$ on pn attached to member CB

Substitute in (1):

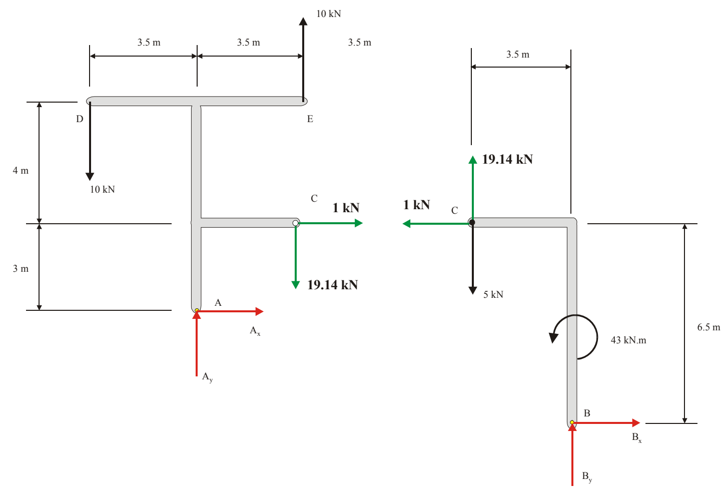
$-3(1) + 3.5C_{1y} = -70$

$C_{1y} = -19.14\text{kN}$

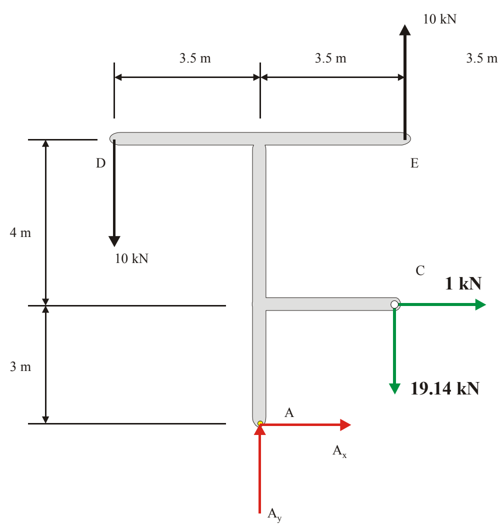
$\therefore \mathbf{C_{1y}} = 19.14\text{kN} \downarrow$ on member ADEC

$\therefore \mathbf{C_{1y}} = 19.14\text{kN} \uparrow$ on pn attached to member CB

We re-draw ADEC and CB showing $\mathbf{C_{1x}}$ and $\mathbf{C_{1y}}$ with the correct sense and apply equilibrium equations to determine $\mathbf{A_x}$, $\mathbf{A_y}$ and $\mathbf{B_x}$, $\mathbf{B_y}$.

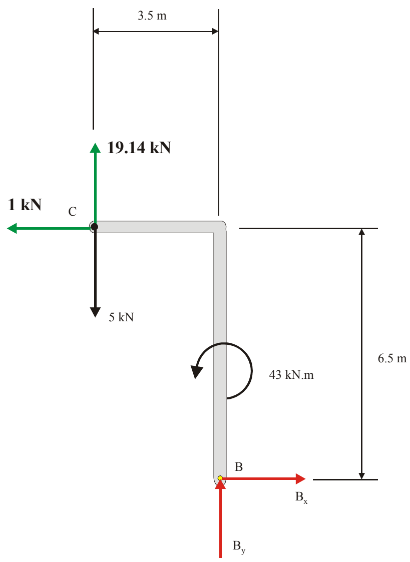


Equilibrium of ADEC



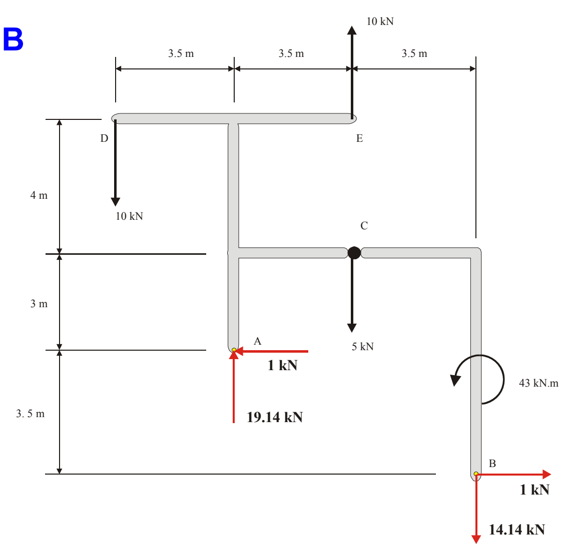
$$\sum F_x = 0$$
$$A_x + 1 = 0$$
$$A_x = -1\text{kN}$$
$$\therefore \mathbf{A_x} = 1\text{kN} \leftarrow$$
$$\sum F_y = 0$$
$$A_y - 10 + 10 - 19.14 = 0$$
$$A_y = +19.14\text{kN}$$
$$\therefore \mathbf{A_y} = 19.14\text{kN} \uparrow$$

Equilibrium of BC



$$\sum F_x = 0$$
$$-1 + B_x = 0$$
$$B_x = +1\text{kN}$$
$$\therefore \mathbf{B_x} = 1\text{kN} \rightarrow$$
$$\sum F_y = 0$$
$$-5 + 19.14 + B_y = 0$$
$$B_y = -14.14\text{kN}$$
$$\therefore \mathbf{B_y} = 14.14\text{kN} \downarrow$$

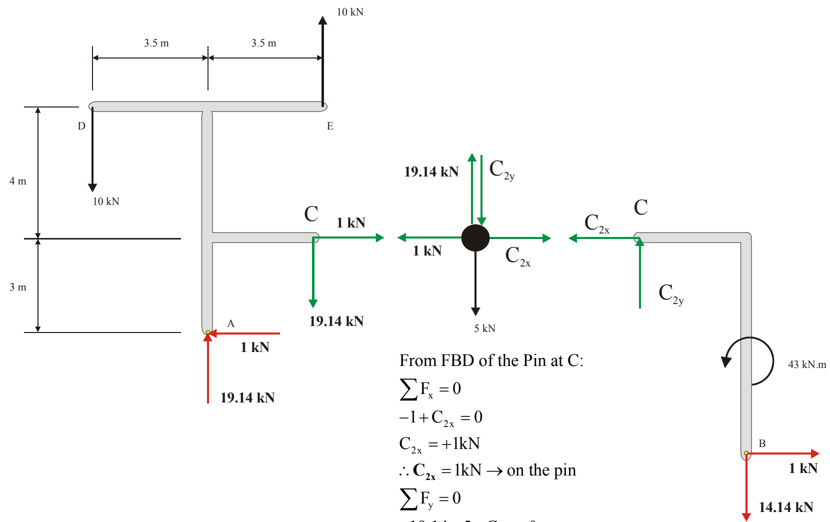
Reactions at A and B



Check:

$$\sum M_C = 0$$
$$+10(7) - 19.14(3.5) - 1(3) + 43 + 1(6.5) - 14.14(3.5) = 0$$
$$0.02 = 0$$

FBDs of Members ADEC, BC and Pin at C
C_{2x} and C_{2y} are forces that Member BC has on the pin



From FBD of the Pin at C:

$\sum F_x = 0$

$-1 + C_{2x} = 0$

$C_{2x} = +1\text{kN}$

$\therefore \mathbf{C_{2x}} = 1\text{kN} \rightarrow$ on the pin

$\sum F_y = 0$

$+19.14 - 5 - C_{2y} = 0$

$C_{2y} = +14.14\text{kN}$

$\therefore \mathbf{C_{2y}} = 14.14\text{kN} \downarrow$ on the pin

CHAPTER 4
Distance Education
Study Problem # 20
J. Frye
May, 2011

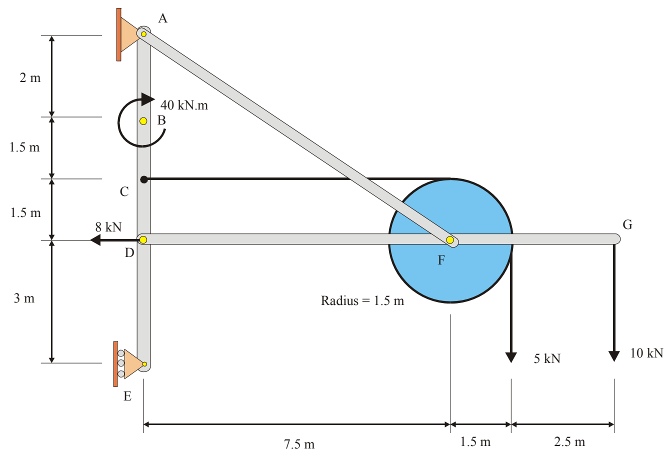
Frame Problem

1. Pulley with cable attached back to frame
2. Two (2) frame members attached to an external support
3. Three (3) members attached by one pin
4. Two Force Member
5. 8 kN force applied at a pin

A *5 kN* weight is suspended from a cable that passes over a frictionless pulley of radius *1.5 m* and is attached back to a frame at *C*. A *40 kN.m* clockwise couple-moment acts on the frame at B and a *8 kN* and *10 kN* loads are applied at *D* and *G*. The frame has a pin (hinge) support at *A* and a roller support at *E*.

a) Determine the reactions at supports *A* and *E*.

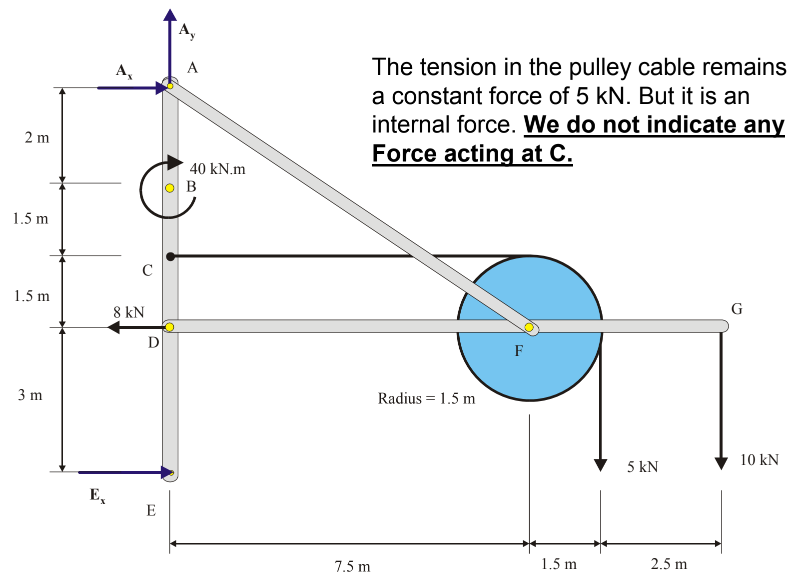
b) Determine all forces exerted on members *ABCDE*, *AF, DFG* and the pulley.



SOLUTION

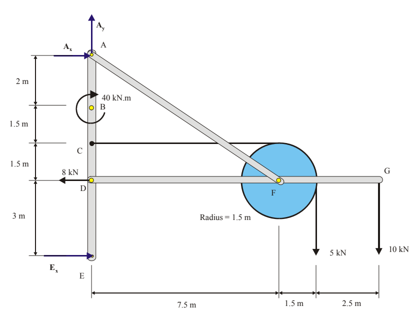
1. **Solving for External Reactions** - Draw a FBD of the entire frame. Label all external reactions. In this problem, there is a pulley attached to the frame. You must decide whether or not you will leave the pulley attached to the frame or if you will detach the pulley and determine the pin reactions on the pulley. If you do this, you will apply Newton's Third Law and apply the pin reactions on the pulley opposite on the frame before applying the equilibrium equations.

FBD – Pulley left attached to the frame



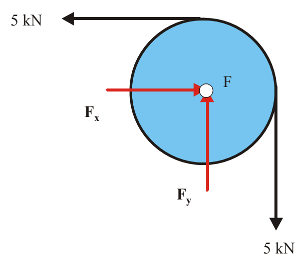
The tension in the pulley cable remains a constant force of 5 kN. But it is an internal force. **We do not indicate any Force acting at C.**

Equilibrium of Entire Frame - External Reactions
Pulley Left Attached to Frame



$$\sum M_A = 0$$
$$-40 - 8(5) - 5(9) - 10(11.5) + \mathbf{E_X}(8) = 0$$
$$\mathbf{E_x} = +30\text{kN}$$
$$\therefore \mathbf{E_x} = 30\text{kN} \rightarrow$$
$$\sum F_x = 0$$
$$A_x - 8 + 30 = 0$$
$$A_x = -22\text{kN}$$
$$\therefore \mathbf{A_x} = 22\text{kN} \leftarrow$$
$$\sum F_y = 0$$
$$A_y - 5 - 10 = 0$$
$$A_y = +15\text{kN}$$
$$\therefore \mathbf{A_y} = 15\text{kN} \uparrow$$

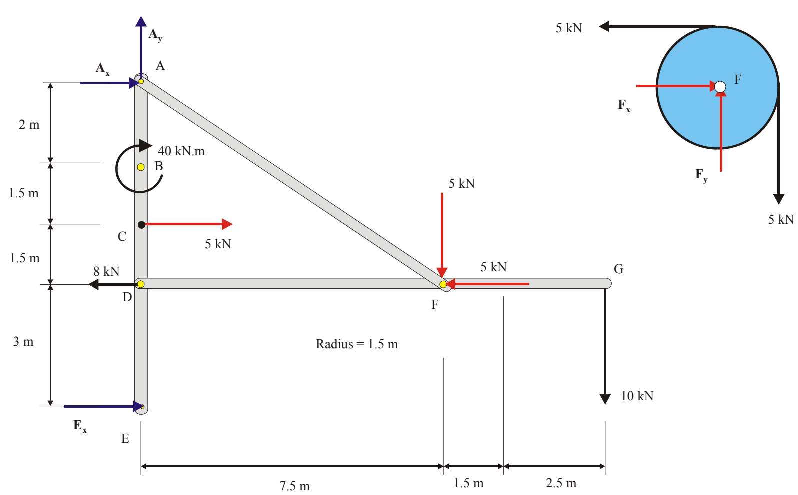
FBD – Pulley first detached from the frame



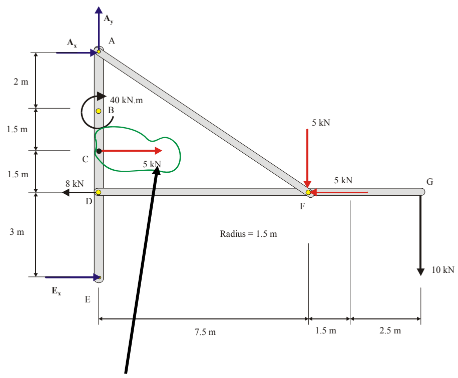
$$\sum F_x = 0$$
$$\mathbf{F_x} - 5 = 0$$
$$\mathbf{F_x} = +5\text{kN}$$
$$\therefore \mathbf{F_x} = 5\text{kN} \rightarrow \text{(on the pulley)}$$
$$\sum F_y = 0$$
$$\mathbf{F_y} - 5 = 0$$
$$\mathbf{F_y} = +5\text{kN}$$
$$\therefore \mathbf{F_y} = 5\text{kN} \uparrow \text{(on the pulley)}$$

Note: $\mathbf{F_x}$ and $\mathbf{F_y}$ are the reactions that the pin at F has on the pulley. WE MUST SAY THIS because Newton's Third Law applies to the pin which we are leaving attached to the frame. They will be shown with the opposite sense on the frame.

FBD of Frame with Pulley Detached



Equilibrium of Entire Frame - External Reactions
Pulley Detached from Frame



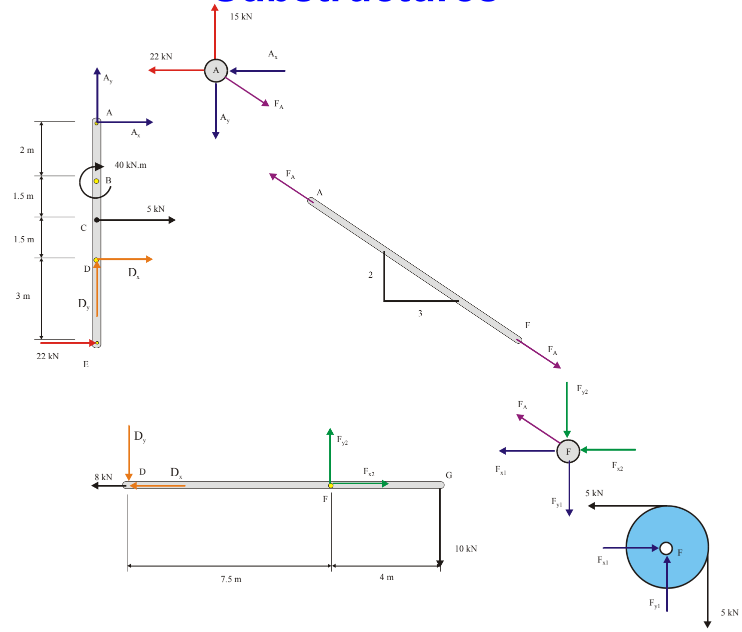
$$\sum M_A = 0$$
$$-40 - 8(5) + 5(3.5) - 5(7.5) - 5(5) - 5(9) - 10(11.5) + \mathbf{E_x}(8) = 0$$
$$\mathbf{E_x} = +30\text{kN}$$
$$\therefore \mathbf{E_x} = 30\text{kN} \rightarrow$$
$$\sum F_x = 0$$
$$A_x - 8 + 5 - 5 + 30 = 0$$
$$A_x = -22\text{kN}$$
$$\therefore \mathbf{A_x} = 22\text{kN} \leftarrow$$
$$\sum F_y = 0$$
$$A_y - 5 - 10 = 0$$
$$A_y = +15\text{kN}$$
$$\therefore \mathbf{A_y} = 15\text{kN} \uparrow$$

When the pulley is detached from the frame, remember to apply the 5 kN force due to the cable attached at point C.

Substructuring

1. Before substructuring take a minute to look for **TWO FORCE** members.
2. Look for external supports where a pin attaches **two or more** members at the support.
3. Look for internal pins where the pin attaches **more than two members**.
4. Look for any locations where an external force is applied at a pin connecting two or more members

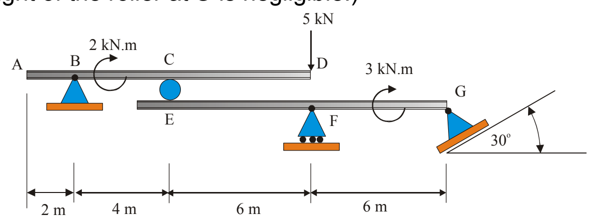
Substructures



CHAPTER 4
Distance Education
Study Problem # 21
J. Frye
May, 2011

STATICALLY DETERMINATE BEAM ARRANGEMENTS

For the beam arrangement shown, determine the reactions at *B, C, F* and *G*. (The height of the roller at *C* is negligible.)



STEP 1 – Identify the external supports and draw a FBD of the entire structure.

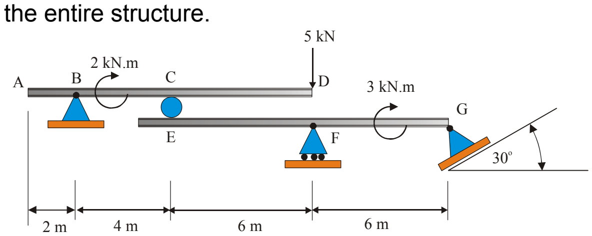
STEP 2 – Write the equilibrium equations for the entire structure and solve for any external reactions that you can.

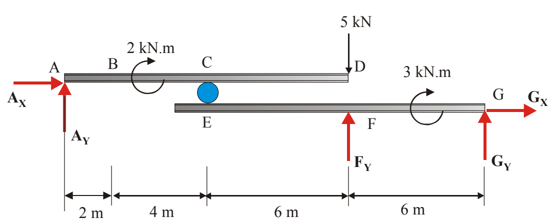
STEP 3 – Substructure – draw FBDs of each member making up the structure. **(MAKE SURE YOU APPLY NEWTON'S THIRD LAW FOR INTERNAL FORCES BETWEEN MEMBERS IN THES FBDs!!!!!!!!) DO NOT PLACE RESULTS ON THESE FBDs!!!!!!!!!!!!!**

STEP 4 – Write and solve the equilibrium equations for each substructure FBD.

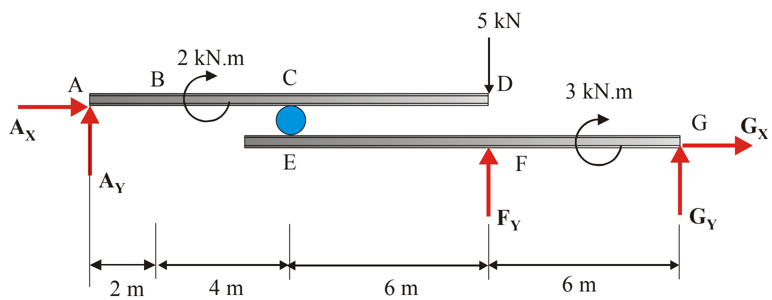
STEP 5 – Check your results by taking moments about a different point.

STEP 1 – We identify A, F and G as **external supports**. A and G are both "Pin" supports and F is a "Roller" support. The beam ABCD is supported at C by a "Roller" support. This is an **internal support** where we will apply Newton's Third Law when we substructure. We draw the FBD of the entire structure.





FBD



In drawing the FBD, there are some points to note:

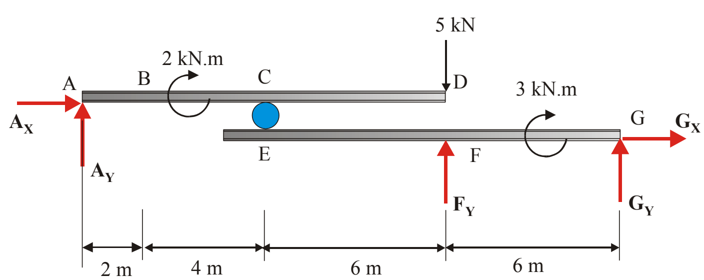
• All unknown forces in the FBD must be properly labelled to match the unknowns when we write the equilibrium equations.

• If we assume a positive sense for the unknowns in the FBD, a negative answer when we solve an equilibrium equation means our assumption of sense was incorrect and the correct sense for equilibrium is negative.

• The fact that the "Pin" support at G is shown at 30º is irrelevant. Show a horizontal and vertical reaction in the FBD at support G.

•DO NOT SHOW ANY FORCES ACTING AT C ON THIS FBD - they are internal forces at this point.

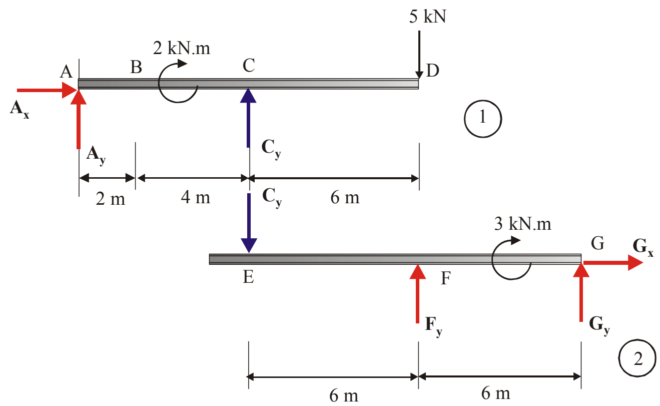
STEP 2 – Equilibrium Equations of Entire Structure



$$\sum F_x = 0 \rightarrow$$
$$A_x + G_x = 0 \quad (1)$$
$$\sum F_y = 0 \uparrow$$
$$A_y - 5 + F_y + G_y = 0 \quad (2)$$
$$\sum M_A = 0$$
$$-2 - 5(12) + F_y(12) - 3 + G_y(18) = 0 \quad (3)$$

We note there are 5 unknowns (A_x, A_y, F_y, G_x and G_y) and only 3 equilibrium equations – we are not able to solve for any of the unknowns so we must substructure. We may choose to use these equations at a later stage.

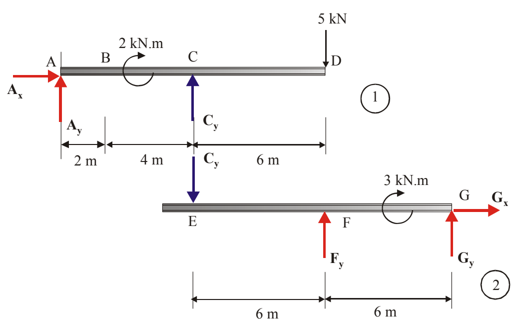
STEP 3 - Substructures



We have 2 substructures – Beam ABCD and Beam EFG.

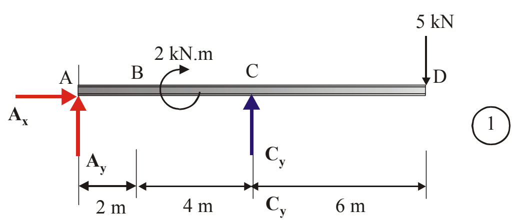
We have applied Newton's Third law at C. Since there is a "Roller" support at C, we show a single vertical unknown force $\mathbf{C_y}$ at this point.

STEP 4 – Write and solve the equilibrium equations for the substructures



Examine the substructure FBDs. We note that FBD #1 of Beam ABCD has only 3 unknown forces while FBD #2 of Beam EFG has 4 unknown forces. Force $\mathbf{C_y}$ is common to both FBDs.

Our solution strategy will be to solve for all unknowns in FBD #1. We can then re-draw FBD#2 indicating the magnitude and correct sense of $\mathbf{C_y}$. We can now solve for the remainder of the unknowns.



$\sum F_x = 0 \rightarrow$

$A_x = 0 \quad (1)$

$\sum F_y = 0 \uparrow$

$A_y + C_y - 5 = 0 \quad (2)$

$\sum M_A = 0$

$-2 + C_y(6) - 5(12) = 0 \quad (3)$

$C_y = +10.33\ \text{kN}$

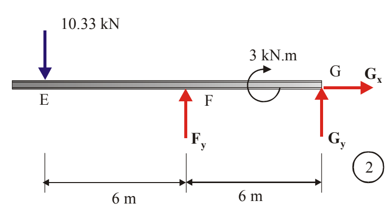
$\therefore \mathbf{C_y} = 10.33\ \text{kN} \uparrow$ on beam ABCD

From Equation 2 :

$A_y + 10.33 - 5 = 0$

$A_y = -5.33\ \text{kN}$

$\therefore \mathbf{A_y} = 5.33\ \text{kN} \downarrow$



$\sum F_x = 0 \rightarrow$

$G_x = 0 \quad (1)$

$\sum F_y = 0 \uparrow$

$-10.33 + F_y + G_y = 0 \quad (2)$

$\sum M_F = 0$

$10.33(6) - 3 + G_y(6) = 0 \quad (3)$

$G_y = -9.83\ \text{kN}$

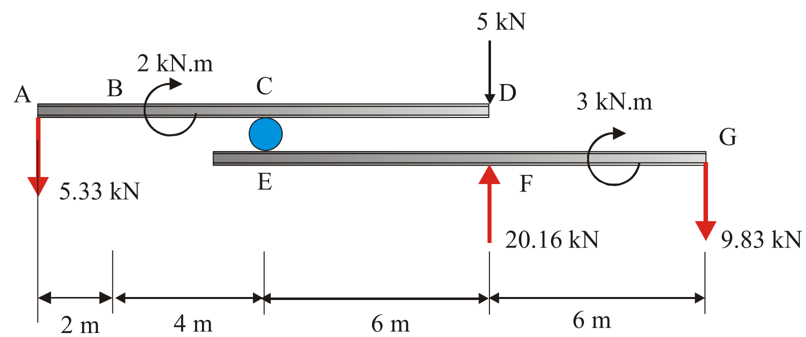
$\therefore \mathbf{G_y} = 9.83\ \text{kN} \downarrow$

From Equation 2 :

$-10.33 + F_y + (-9.83) = 0 \quad (2)$

$F_y = +20.16\ \text{kN}$

$\therefore \mathbf{F_y} = 20.16\ \text{kN} \uparrow$



CHECK

$\sum F_y = 0 \uparrow$

$-5.33 - 5 + 20.16 - 9.83 = 0$

$0 = 0$

$\sum M_F = 0$

$5.33(12) - 2 - 3 - 9.83(6) = 0 \quad (3)$

$-0.02 = 0$

CHECKS!!

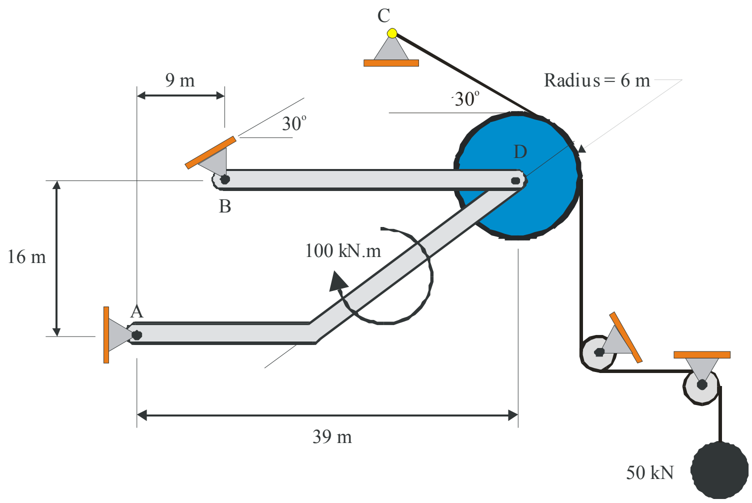
CHAPTER 4
Distance Education
Study Problem # 22
J. Frye
May, 2011

Frame Example

Frame with pulley attached.

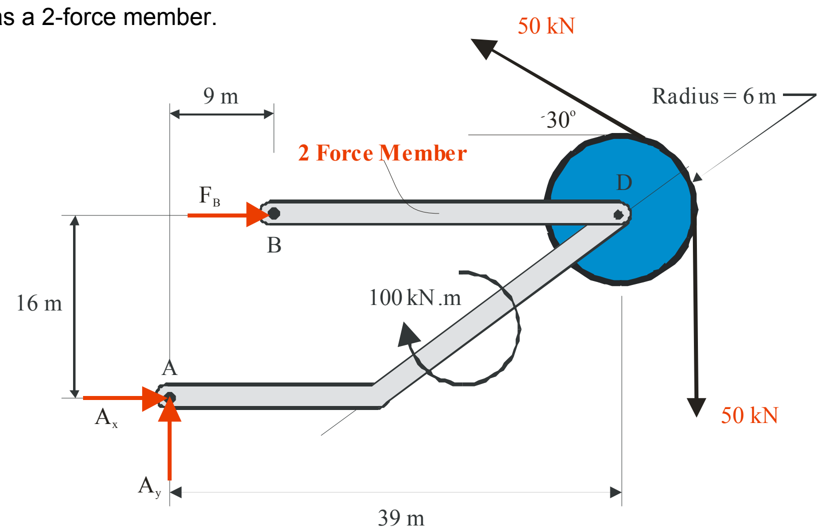
2-Force Member

Determine the reactions at *A* and *B* for the frame shown and the forces exerted by the Pin at *D* on members AD, BD and on the Pulley for the frame shown.



FBD for the entire frame.

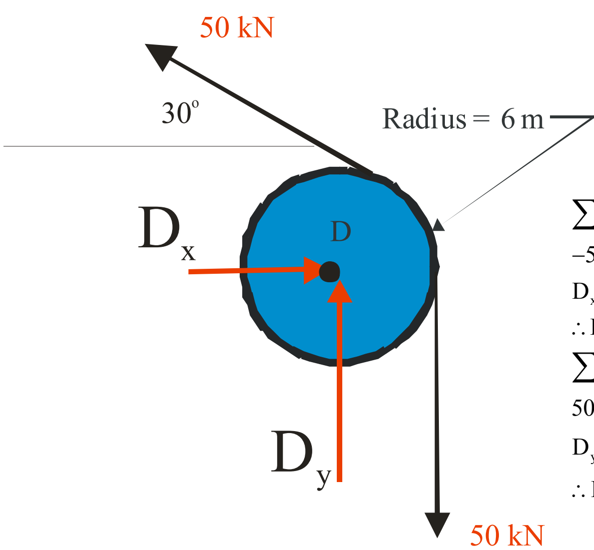
Note: We have identified BD as a 2-force member.



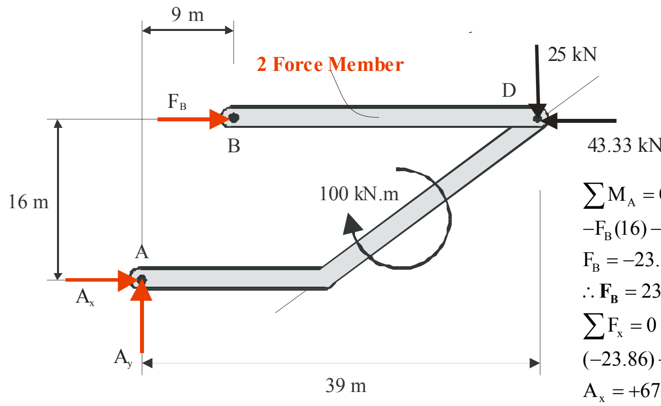
We only have 3 unknown reactions and therefore can solve for all unknowns with the equations of equilibrium applied to the entire frame.

However, because the radius of the pulley is 6 m and the cable is at an angle to the pulley, it is difficult to calculate the perpendicular distance of each of the components of the 50 kN force from Point A in taking moment about A.

We therefore <u>first draw a separate FBD of the pulley</u> and calculate the reactions of the pin on the pulley at D. We then apply these reactions at Point D on the frame.



$$\sum F_x = 0$$
$$-50\cos 30^\circ + D_x = 0$$
$$D_x = +43.3\ \text{kN}$$
$$\therefore \mathbf{D_x} = 43.3\ \text{kN} \rightarrow \text{on the pulley}$$
$$\sum F_y = 0$$
$$50\sin 30^\circ + D_y - 50 = 0$$
$$D_y = +25\ \text{kN}$$
$$\therefore \mathbf{D_y} = 25\ \text{kN} \uparrow \text{on the pulley}$$



$\sum M_A = 0A$

$-F_B(16) - 25(39) + 43.33(16) - 100 = 0$

$F_B = -23.86\,\text{kN}$

$\therefore \mathbf{F_B} = 23.86\,\text{kN} \leftarrow$

$\sum F_x = 0$

$(-23.86) - 43.33 + A_x = 0$

$A_x = +67.19\,\text{kN}$

$\therefore \mathbf{A_x} = 67.19\,\text{kN} \rightarrow$

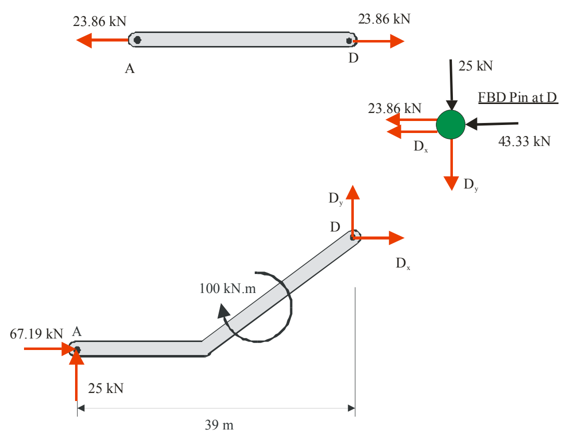
$\sum F_y = 0$

$A_y - 25 = 0$

$A_y = +25\,\text{kN}$

$\therefore \mathbf{A_y} = 25\,\text{kN} \uparrow$

Apply the 25 kN and 43.33 kN directly
on the FBD of the Pin at D



For the Pin at D

$\sum F_x = 0$

$-23.86 - 43.33 - D_x = 0$

$D_x = -67.19\,\text{kN}$

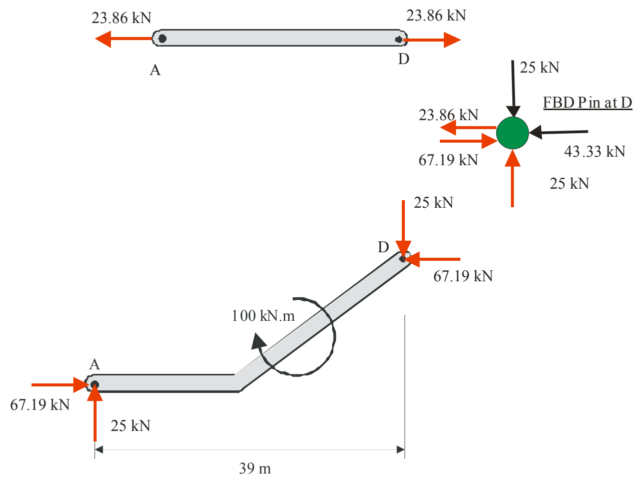
$\therefore \mathbf{D_x} = 67.19\,\text{kN} \rightarrow$ on the Pin at D

$\sum F_y = 0$

$-25 - D_y = 0$

$D_y = -25\,\text{kN}$

$\therefore \mathbf{D_y} = 25\,\text{kN} \uparrow$ on the Pin at D

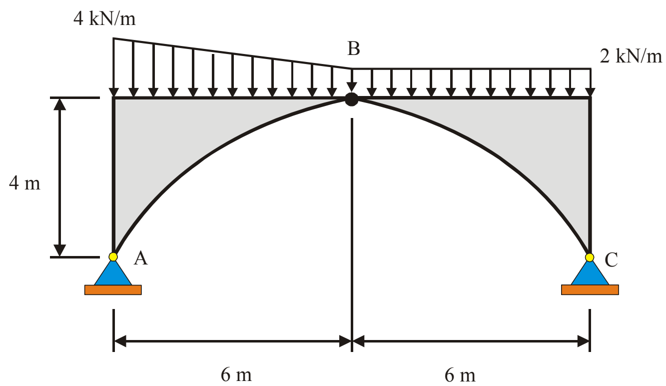


CHAPTER 4
Distance Education
Study Problem # 23
J. Frye
May, 2011

3-Hinged Arch

1. Distributed Loads
2. Pinned supports at same level

Determine the reactions at the pin supports at A and C for the given loading of the 3-hinged arch.



Solution Steps

STEP 1 – Convert the distributed loads to point loads (forces).

STEP 2 – **DRAW THE Free Body Diagram (FBD)** of the entire structure. Make sure you label all forces in the FBD as you will use these labels in the equilibrium equations. Senses of unknown forces are assumed.

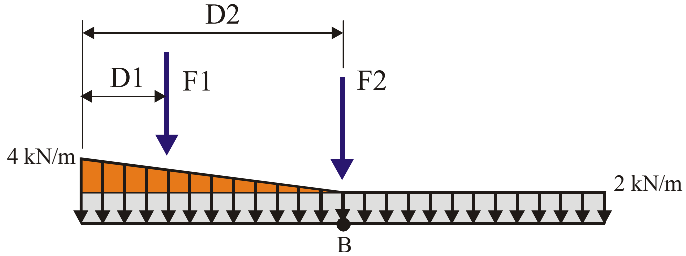
STEP 3 – Write the equilibrium equations for the FBD and solve for any of the unknowns that you can. Remember that a negative value of an unknown force simply means that you have assumed an **incorrect** sense in the FBD (it does not mean that the force is necessarily in the negative direction unless of course you assumed it in a positive direction to begin with).

STEP 4 – Substructure – draw a FBD for each of the members AB and AC making up the structure applying Newton's Third Law at the pin at C. DO NOT TOUCH THESE FBDs AGAIN (That is, do not put results on these FBDs.)

STEP 5 – Write the equilibrium equations for the substructure FBDs and solve for the remaining unknown forces.

STEP 6 – Re-draw the substructures and place all results on these FBDs. Check by taking moments about a different point.

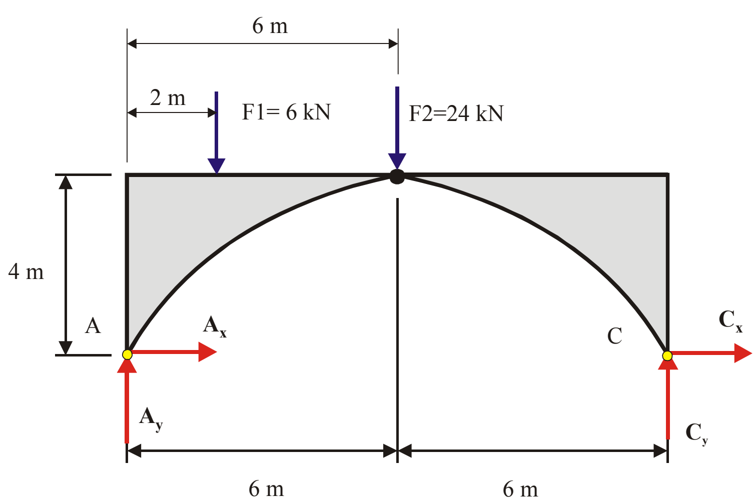
Distributed Load



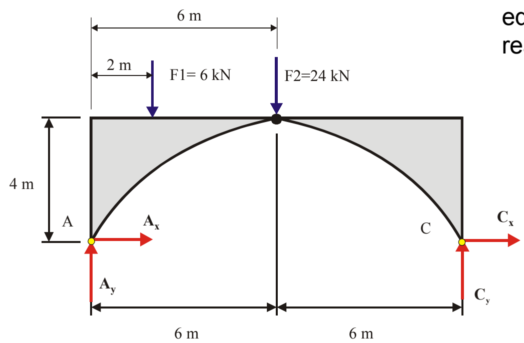
Note: We have a triangle load curve and a rectangular load curve.

$$F1 = (\frac{4kN/m - 2kN/m)}{2}(6m) = 6kN$$
$$D1 = 6m/3 = 2m$$
$$F2 = (2kN/m)(12m) = 24kN$$
$$D2 = 12m/2 = 6m$$

FBD of Entire Structure



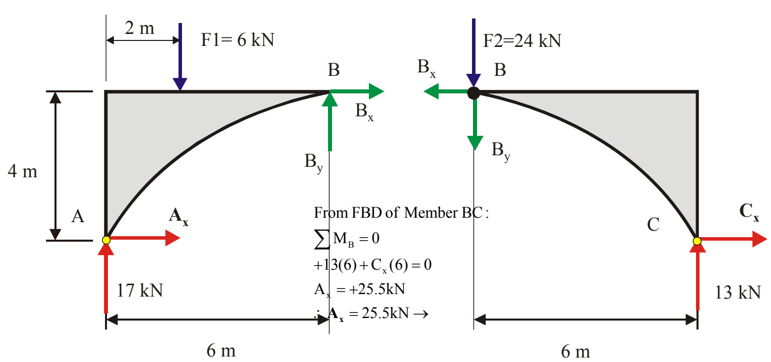
Write the Equilibrium Equations and Solve for any unknowns that you can.



Note: Because pin supports A and C are on the same level horizontally, we will be able to solve the equilibrium equations for the vertical reactions at A and C, Ay and Cy

$$\sum F_x = 0$$
$$A_x + C_x = 0 \quad \textbf{(1)}$$
$$\sum F_y = 0$$
$$-6 - 24 + A_y + C_y = 0$$
$$A_y + C_y = 30 \quad \textbf{(2)}$$
$$\sum M_A = 0$$
$$-6(2) - 24(6) + C_y(12) = 0$$
$$C_y = +13\text{kN}$$
$$\therefore \mathbf{C_y} = 13\text{kN} \uparrow$$
Fron (2)
$$A_y + 13 = 30$$
$$A_y = +17\text{kN}$$
$$\therefore \mathbf{A_y} = 17\text{kN} \uparrow$$

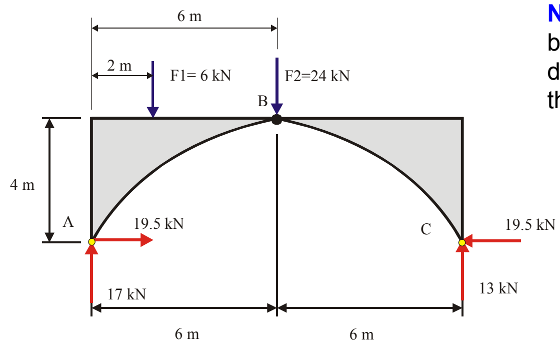
Substructures



From FBD of Member AC :
$$\sum M_B = 0$$
$$-17(6) + A_x(4) + 6(4) = 0$$
$$A_x = +19.5\text{kN}$$
$$\therefore \mathbf{A_x} = 19.5\text{kN} \rightarrow$$

From FBD of Member BC :
$$\sum M_B = 0$$
$$13(6) + C_x(4) = 0$$
$$C_x = -19.5\text{kN}$$
$$\therefore \mathbf{C_x} = 19.5\text{kN} \leftarrow$$

Final Results and Check



Note: We check our results by taking moments about a different point. In this case the moment about Point B.

$$\sum M_B$$

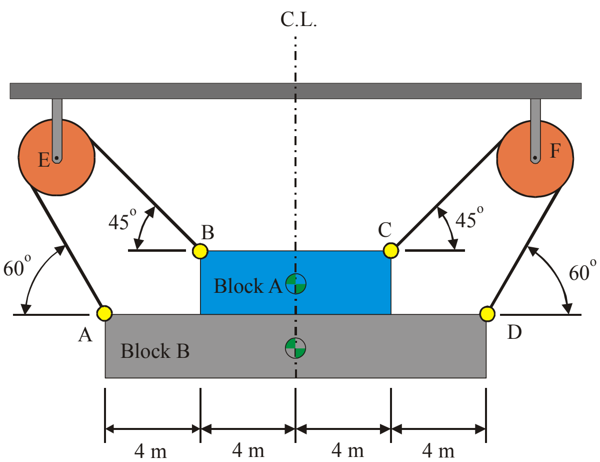
$$-17(6)+19.5(4)+6(4)+13(6)-19.5(4)=0$$

$$0=0 \quad \text{Checks!!!}$$

CHAPTER 4
Distance Education
Study Problem # 24
J. Frye
May, 2011

Rigid Body Equilibrium (Symmetrical System)

EXAMPLE: The system of smooth pulleys and homogeneous blocks shown in the figure is symmetrical about the Center Line (C.L.). Block A weighs 10 kN and Block B weighs kN. Determine the normal force between Block A and Block B. Assume that the weights of the block act at the Centers of Gravity of the blocks .



SOLUTION STEPS

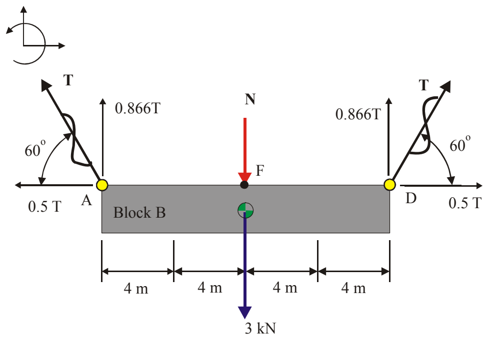
STEP 1 – Draw the Free-Body-Diagrams (FBDs) of each block. (The normal force between the two blocks is internal when the blocks remain in contact. When we draw the FBD of each block we apply Newton's Third law.

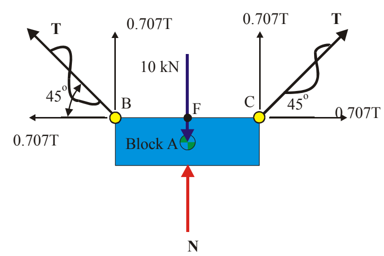
STEP 2 – Declare a positive sign convention for forces and moments. (LABEL ALL FORCES IN THE FBDs!!!!) Write the equilibrium eqautions for each FBD.

STEP 3 – Solve the equilibrium equations for unknown forces.

STEP 4 – Re-draw the FBDs showing all force with the correct senses. Check your solution by taking moments about a different point.

Solution: Because of symmetry, the tension in the two cables is the same. We label the tension force, **T**. There are therefore two (2) unknown forces, **T** and **N**. To obtain two independent equations, we apply equilibrium in the vertical (y-direction in the FBD of each block.





$$\sum F_y = 0$$

$$0.866T - N - 3 + 0.866T = 0$$

$$1.732T - N = 3 \quad \textbf{(1)}$$

$$\sum F_y = 0$$

$$0.707T - 10 + N + 0.707T = 0$$

$$1.414T + N = 10 \quad \textbf{(2)}$$

Solving for T and N and Checking Our Solution:

Solving Equations (1) and (2):

$$1.732T - N = 3 \quad \textbf{(1)}$$

$$1.414T + N = 10 \quad \textbf{(2)}$$

Adding (1) and (2):

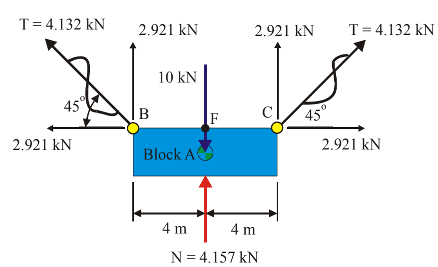
$$3.146T = 13$$

$$\therefore T = 4.132\text{kN}$$

Substitute in (1):

$$1.732(4.132) - N = 3$$

$$\therefore N = 4.157\text{kN}$$



$$\textbf{CHECK - } \sum M_B = 0$$

$$-10(4) + 4.157(4) + 2.921(8) = 0$$

$$-.004 = 0 \quad \textbf{CHECKS!!!}$$

CHAPTER 5
Distance Education
Study Problem # 25
J. Frye
May, 2011

Rectangular Components of a Vector Given Two (2) Points on the Line-of-Action of the Vector (We need for Force and Moment Vectors)

$$\mathbf{F} = F\lambda$$

$$\mathbf{M} = M\lambda$$

Any vector, **V** can be resolved into its **rectangular components** given two (2) points on the line-of-action of the vector using the equation:

$$\mathbf{V} = V\lambda$$

Where:

V is the vector in component form,

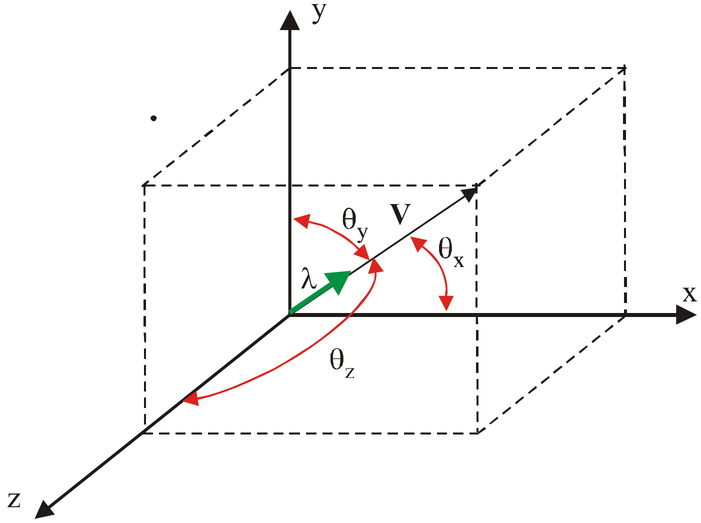
V is the magnitude of the vector and

λ (lamda) is the vector of the direction cosines of **V** directed along the line-of-action of **V.**

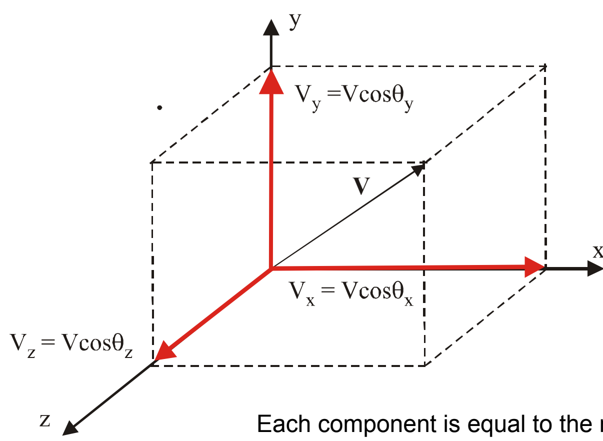
Therefore, if we know the magnitude of vector, **V**, and if we can calculate its direction cosines (**λ,** lamda vector) , we can express the vector in component form.

λ(lamda) vector along Line-of-Action of Vector **V**

The rectangular components of λ are $\cos\theta_x$, $\cos\theta_y$ and $\cos\theta_z$ (called the direction cosines)



Rectangular Components of Vector, V



Each component is equal to the magnitude of the vector times its direction cosine.

The λ Vector

The λ (lamda) vector is the vector of direction cosines:

$$\lambda = \cos\theta_x \mathbf{i} + \cos\theta_y \mathbf{j} + \cos\theta_z \mathbf{k}$$

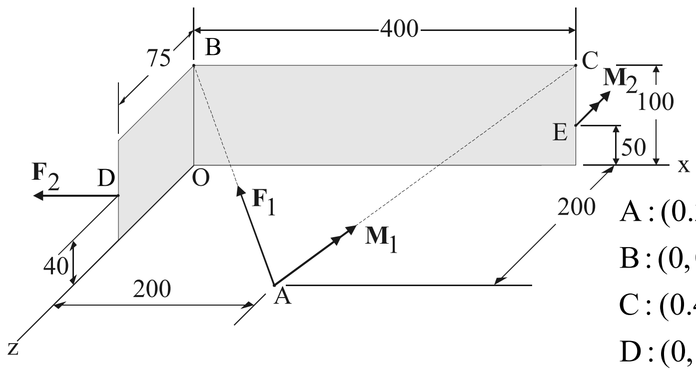
Magnitude of $\lambda = 1$

Example

For the solution of Example 6.7 of the **Introduction to Statics Text/Notes** (Frye, Rattanawangchareon and Shah) it is necessary to express a force and a moment vector in rectangular component form.

Example 6.7:

A force of $\mathbf{F}_1$ in magnitude 480 N and a moment $\mathbf{M}_1$ of magnitude 36 N.m are applied at A on the precast panel. Furthermore, a force $\mathbf{F}_2 = (-100\mathbf{i}+400\mathbf{j}+250\mathbf{k})$ N and a moment $\mathbf{M}_2 = (10\mathbf{i}+15\mathbf{j}-35\mathbf{k})$ N.m are applied at D and E, respectively. **Replace these force-couple systems by an equivalent force-couple system <u>at D</u>**. (All dimensions are in mm.)



$A:(0.2,0,0.2)$

$B:(0,0.1,0)$

$C:(0.4,0.1,0)$

$D:(0,0.04,0.075)$

In the problem statement of Example 6.7, we are given:

a) The magnitude of force vector $\mathbf{F}_1$ and moment vector $\mathbf{M}_1$. That is:

   i. F_1 = 480 N and

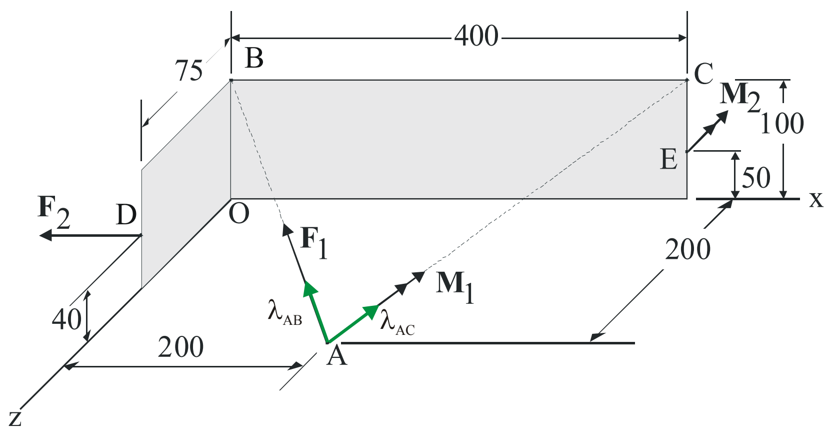
   ii. M_1 = 30 N.m

b) We are also given the coordinates of two points on the lines-of-action of force vector $\mathbf{F}_1$ and moment vector $\mathbf{M}_1$. That is:

   i. For $\mathbf{F}_1$ we are given coordinates of points A and B and

   ii. For $\mathbf{M}_1$ we are given coordinates of points A and C.

   ($\mathbf{F}_2$ and $\mathbf{M}_2$ are already given in component form.)



Line AB is the Line-of-Action of $\mathbf{F_1}$

Line AC is the Line=of-Action of $\mathbf{M_1}$

Put F_1 and M_1 into Component Form

$$\mathbf{F}_1 = F_1\boldsymbol{\lambda}_{AB} = 480\boldsymbol{\lambda}_{AB}$$

$$\boldsymbol{\lambda}_{AB} = \frac{\mathbf{AB}}{AB}$$

$$\mathbf{AB} = -0.2\mathbf{i} + 0.1\mathbf{j} - 0.2\mathbf{k} \qquad AB = \sqrt{\left(-0.2^2\right) + 0.1^2 + \left(-0.2^2\right)} = 0.3$$

$$\boldsymbol{\lambda}_{AB} = \frac{-0.2\mathbf{i} + 0.1\mathbf{j} - 0.2\mathbf{k}}{0.3}$$

$$\mathbf{F}_1 = 480\left(\frac{-0.2\mathbf{i} + 0.1\mathbf{j} - 0.2\mathbf{k}}{0.3}\right) = -320\mathbf{i} + 160\mathbf{j} - 320\mathbf{k}\ \text{N}$$

$$\mathbf{M}_1 = M_1\boldsymbol{\lambda}_{AC} = 36\boldsymbol{\lambda}_{AC}$$

$$\boldsymbol{\lambda}_{AC} = \frac{\mathbf{AC}}{AC}$$

$$\mathbf{AC} = 0.2\mathbf{i} + 0.1\mathbf{j} - 0.2\mathbf{k} \qquad AC = \sqrt{0.2^2 + 0.1^2 + \left(-0.2^2\right)} = 0.3$$

$$\boldsymbol{\lambda}_{AC} = \frac{0.2\mathbf{i} + 0.1\mathbf{j} - 0.2\mathbf{k}}{0.3}$$

$$\mathbf{M}_1 = 36\left(\frac{0.2\mathbf{i} + 0.1\mathbf{j} - 0.2\mathbf{k}}{0.3}\right) = 24\mathbf{i} + 12\mathbf{j} - 24\mathbf{k}\ \text{N.m}$$

$\mathbf{F_1}$ and $\mathbf{M_1}$ in Rectangular Component Form

$$\mathbf{F_1} = -320\mathbf{i} + 160\mathbf{j} - 320\mathbf{k}\ \mathrm{N}$$

$$\mathbf{M_1} = 24\mathbf{i} + 12\mathbf{j} - 24\mathbf{k}\ \mathrm{N.m}$$

What if we were asked to find the directions of $\mathbf{F_1}$ and $\mathbf{M_1}$?

ANSWER:

$\boldsymbol{\lambda}_{AB}$ is the vector of the Direction Cosines of F_1

$$\boldsymbol{\lambda}_{AB} = \frac{-0.2\mathbf{i} + 0.1\mathbf{j} - 0.2\mathbf{k}}{0.3}$$

$$\therefore \cos\theta_x = \frac{-0.2}{0.3} \Rightarrow \theta_x = 131.81^O$$

$$\therefore \cos\theta_y = \frac{0.1}{0.3} \Rightarrow \theta_y = 70.53^O$$

$$\therefore \cos\theta_z = \frac{-0.2}{0.3} \Rightarrow \theta_y = 131.81^O$$

$\boldsymbol{\lambda}_{AC}$ is the vector of the Direction Cosines of M_1

$$\boldsymbol{\lambda}_{AC} = \frac{0.2\mathbf{i} + 0.1\mathbf{j} - 0.2\mathbf{k}}{0.3}$$

$$\therefore \cos\theta_x = \frac{0.2}{0.3} \Rightarrow \theta_x = 48.19^O$$

$$\therefore \cos\theta_y = \frac{0.1}{0.3} \Rightarrow \theta_y = 70.53^O$$

$$\therefore \cos\theta_z = \frac{-0.2}{0.3} \Rightarrow \theta_y = 131.81^O$$

CHAPTER 5
Distance Education
Study Problem # 26
J. Frye
May, 2011

Particle Equilibrium – 3 Dimensions

Solution Steps

1. **STEP 1** – Draw the Free Body Diagram of the particle (point) that is in equilibrium.
2. **STEP 2** – Determine the coordinates of all points you will need to express forces in terms of their rectangular components.
3. **STEP 3** – Express all forces in terms of their rectangular components. ($\mathbf{F} = F\boldsymbol{\lambda}$)
4. **STEP 4** – Write the equilibrium equations based on the FBD.
5. **STEP 5** – Solve the equilibrium equations for unknown quantities.
6. **STEP 6** – CHECK YOUR WORK

THING TO REMEMBER

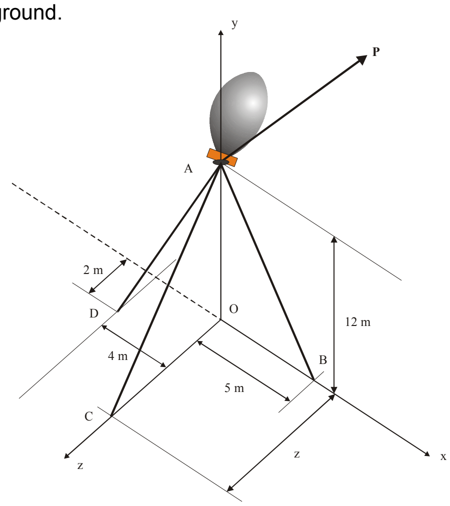
1. You **MUST** draw a FBD and it must be properly labelled!!!!!!!!!!!!!!!
2. You may assume senses of unknown forces, however, always show tension in cables going away from the point.
3. You have 3 equilibrium equations so you can solve for 3 unknowns. The unknowns will typically be forces, dimensions or a combination of the two.
4. A negative answer for a force means you have assumed an incorrect sense for a force in your FBD. When back substituting in an equilibrium equation to solve for another unknown, you must use this negative value

Example – 3D Equilibrium

A giant balloon is held by three cables: *AB*, *AC*, and *AD* attached to the balloon at *A*. A wind force of $\mathbf{P} = -122\mathbf{i} + 985\mathbf{j} - 122\mathbf{k}$ *kN* is also applied to the balloon at *A*.

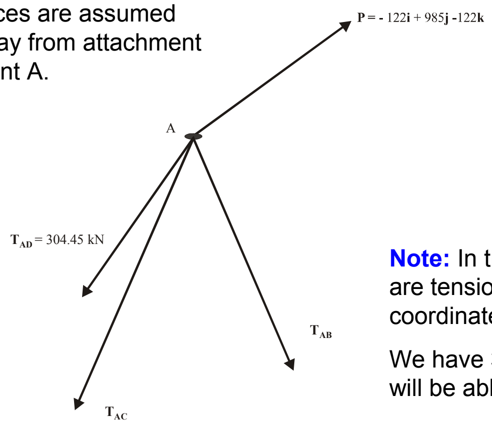
If the tension in cable *AD* is *304.45 kN*, determine:

a) The tension in cables *AC* and *AD* and
b) The *z* coordinate of point *C* on the ground.



Note: In the FBD, the senses of the tension forces are assumed away from attachment Point A.

SOLUTION



FBD of Particle A

COORDINATES:

A(0,12,0)

B(5, 0, 0)

C(0, 0, z)

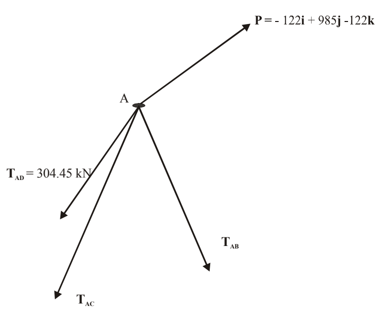
D(-4, 0, 2)

Note: In this problem, the 3 unknowns are tension forces $\mathbf{T_{AC}}$, $\mathbf{T_{AB}}$ and the z coordinate of Point C.

We have 3 equilibrium equations so we will be able to solve for ALL unknowns.

IMPORTANT: Label all forces in the FBD. Labels in the equilibrium equations MUST match the labels used in the FBD.

SOLUTION – Express all forces in terms of Rectangular Components



$$\mathbf{F} = F\boldsymbol{\lambda}$$

$$\mathbf{T_{AD}} = T_{AD}\boldsymbol{\lambda}_{\mathbf{AD}} = 304.45\boldsymbol{\lambda}_{\mathbf{AD}}$$

$$\boldsymbol{\lambda}_{\mathbf{AD}} = \frac{\mathbf{AD}}{AD} \qquad \mathbf{AD} = -4\mathbf{i} - 12\mathbf{j} + 2\mathbf{k}$$

$$AD = \sqrt{(-4)^2 + (-12)^2 + (2)^2} = \sqrt{164}$$

$$\mathbf{T_{AD}} = 304.45\left(\frac{-4\mathbf{i} - 12\mathbf{j} + 2\mathbf{k}}{\sqrt{164}}\right) = (-95.09\mathbf{i} - 285.28\mathbf{j} + 47.55\mathbf{k})\,\text{kN}$$

$$\mathbf{T_{AC}} = T_{AC}\boldsymbol{\lambda}_{\mathbf{AC}}$$

$$\boldsymbol{\lambda}_{\mathbf{AC}} = \frac{\mathbf{AC}}{AC} \qquad \mathbf{AC} = 0\mathbf{i} - 12\mathbf{j} + z\mathbf{k}$$

$$AC = \sqrt{(-12)^2 + (z)^2} = \sqrt{144 + z^2}$$

$$\mathbf{T_{AD}} = T_{AC}\left(\frac{-12\mathbf{j} + z\mathbf{k}}{\sqrt{144 + z^2}}\right) = \left(\frac{-12T_{AC}\mathbf{j}}{\sqrt{144 + z^2}} + \frac{z\mathbf{k}}{\sqrt{144 + z^2}}\right)\text{kN}$$

$$\mathbf{T_{AB}} = T_{AB}\boldsymbol{\lambda}_{\mathbf{AB}}$$

$$\boldsymbol{\lambda}_{\mathbf{AB}} = \frac{\mathbf{AB}}{AB} \qquad \mathbf{AB} = 5\mathbf{i} - 12\mathbf{j} + 0\mathbf{k}$$

$$AB = \sqrt{(5)^2 + (-12)^2} = 13$$

$$\mathbf{T_{AB}} = T_{AB}\left(\frac{5\mathbf{i} - 12\mathbf{j}}{13}\right) = \left(\frac{5T_{AB}\mathbf{i}}{13} - \frac{12T_{AB}\mathbf{j}}{13}\right)\text{kN}$$

SOLUTION – Write and Solve the Equilibrium Equations

$$\sum F_x = 0$$

$$-12z - 95.09 + \frac{5}{13} T_{AB} = 0$$

$$T_{AB} = 564.43\text{kN}$$

$$\sum F_y = 0$$

$$985 - 285.28 - \frac{12T_{AC}}{\sqrt{144+z^2}} - \frac{12}{13}(564.43) = 0$$

$$\frac{12T_{AC}}{\sqrt{144+z^2}} = 178.81$$

$$\sum F_z = 0$$

$$-122 + 47.55 + \frac{T_{AC}z}{\sqrt{144+z^2}} = 0$$

$$-122 + 47.55 + \frac{178.71z}{12} = 0$$

$$z = 5\text{m}$$

$$\therefore T_{AC} = 178.71\left(\frac{\sqrt{144+5^2}}{12}\right) = 193.6\text{kN}$$

CHAPTER 6
Distance Education
Study Problem # 27
J. Frye
May, 2011

Moment of a Force About a Point

Moment of a Force about a Point and about a Line

Moment of a Force about a Point :

$$\mathbf{M} = \mathbf{r} \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$

Moment of a Force about a Line :

$$M_{OL} = \boldsymbol{\lambda}_{OL} \bullet \mathbf{M} = \boldsymbol{\lambda}_{OL} \bullet (\mathbf{r} \times \mathbf{F})$$

Moment of a Force about a Point

Moment of a force about a point is the vector (cross) product of a position vector, **r**, and the force vector, **F**. It is determined from the expansion of the determinant shown.

Moment of a Force about a Point :

$$\mathbf{M} = \mathbf{r} \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$

IMPORTANT!!!!!!!!!

When taking the moment of a force, F about a point, there a number of things to remember:

1. The position vector, **r**, **MUST** be taken or drawn **FROM** the point you want to take moments about to **ANY** point on the Line-of-Action of the force, **F.**
2. The position vector, **r**, is the second row of the determinant and the force vector, **F**, is the third row.

EXAMPLE: If the magnitude of $\mathbf{F}_{AC}$ is 520 N, determine:

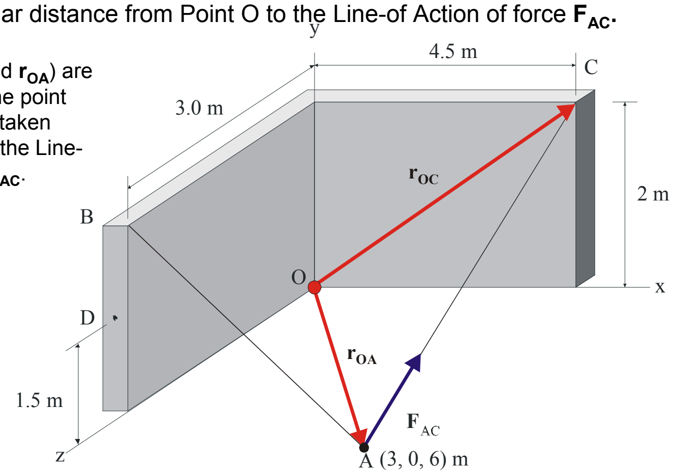
a) The moment of $\mathbf{F}_{AC}$ about the origin O,

b) The moment of $\mathbf{F}_{AC}$ about the Point D,

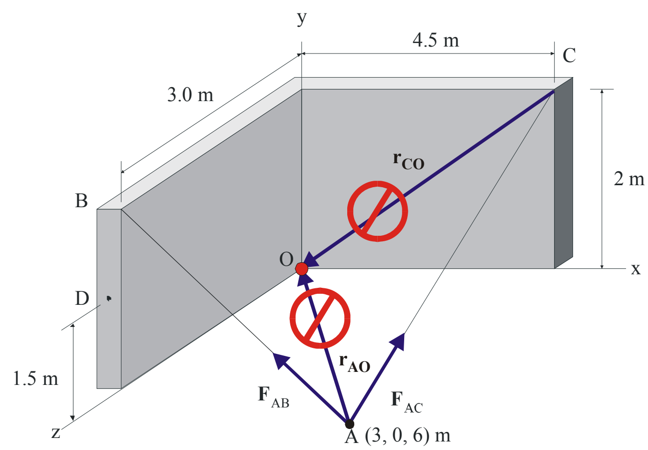
c) The moment of $\mathbf{F}_{AC}$ about the Point C, and

d) The perpendicular distance from Point O to the Line-of Action of force $\mathbf{F}_{AC}$.

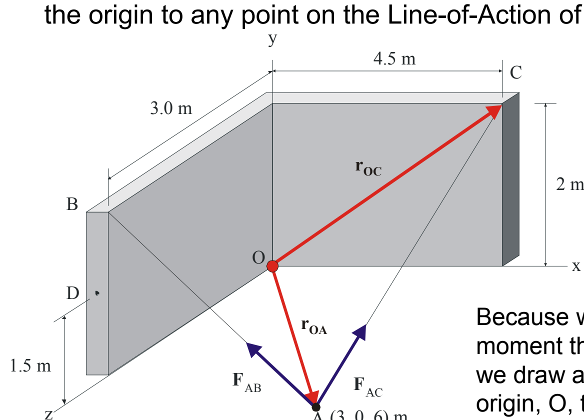
Position vectors ($\mathbf{r}_{OC}$ and $\mathbf{r}_{OA}$) are drawn **From** Point O (the point where moment is to be taken about) to **ANY** point on the Line-of-Action of the force, $\mathbf{F}_{AC}$.



Position vectors drawn incorrectly to take moment of F_{AC} about Point O!!!



Part (a): Step 1 -Determine and record the coordinates of all points that you will require in your calculations of the position vector(s), **r** and the force vector(s) **F**. Draw a position vector, **r**, directed from the origin to any point on the Line-of-Action of the force, **F**.



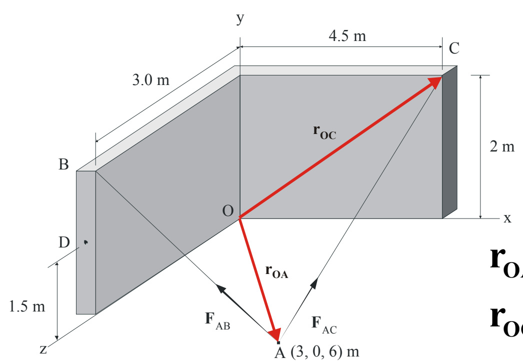
$$O:(0,0,0)$$
$$A:(3,0,6)$$
$$B:(0,2,3)$$
$$C:(4.5,2,0)$$

Because we are asked to find the moment that $\mathbf{F}_{AC}$ has about the origin, we draw a position vector **FROM** the origin, O, to **ANY** point on the Line-of-Action of $\mathbf{F}_{AC}$. In this case, we may use either $\mathbf{r}_{OA}$ or $\mathbf{r}_{OC}$ as our position vector since both Points A and C lie on the Line-of-Action of $\mathbf{F}_{AC}$.

Step 2 – Express the position vector, **r** in component form.

$\mathbf{r}_{OA}$ =(coordinates of Point A)-(coordinates of Point O)

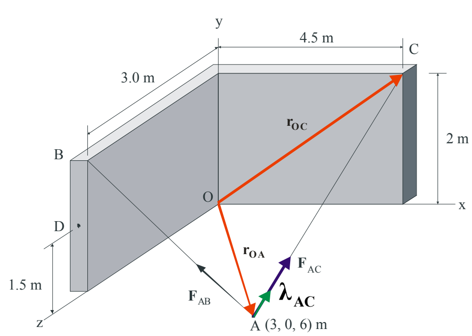
$\mathbf{r}_{OC}$ =(coordinates of Point C)-(coordinates of Point O)



$$\mathbf{r_{OA}} = 3\mathbf{i} + 0\mathbf{j} + 6\mathbf{k}$$

$$\mathbf{r_{OC}} = 4.5\mathbf{i} + 2\mathbf{j} + 0\mathbf{k}$$

Step 3 – Express the force vector F in component form.



$$\mathbf{F_{AC}} = F_{AC}\boldsymbol{\lambda}_{\mathbf{AC}} = 520\boldsymbol{\lambda}_{\mathbf{AC}}$$

$$\boldsymbol{\lambda}_{\mathbf{AC}} = \frac{\mathbf{AC}}{\mathrm{AC}}$$

$$\mathbf{AC} = 1.5\mathbf{i} + 2\mathbf{j} - 6\mathbf{k}$$

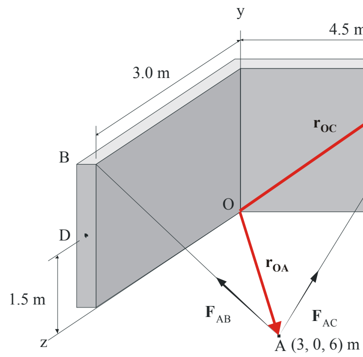
$$\mathrm{AC} = \sqrt{(1.5)^2 + (2)^2 + (-6)^2} = 6.5$$

$$\therefore \boldsymbol{\lambda}_{\mathbf{AC}} = \frac{1.5\mathbf{i} + 2\mathbf{j} - 6\mathbf{k}}{6.5}$$

$$\mathbf{F_{AC}} = 520\boldsymbol{\lambda}_{\mathbf{AC}} = 520\left(\frac{1.5\mathbf{i} + 2\mathbf{j} - 6\mathbf{k}}{6.5}\right)$$

$$\mathbf{F_{AC}} = 120\mathbf{i} + 160\mathbf{j} - 480\mathbf{k} \quad \mathrm{N.}$$

Step 4 – **Moment about a Point** is given by evaluating the cross product of the position vector **r** and the force vector **F**. **M** = **r** x **F**.



$$\mathbf{r_{OA}} = 3\mathbf{i} + 0\mathbf{j} + 6\mathbf{k}$$

$$\mathbf{r_{OC}} = 4.5\mathbf{i} + 2\mathbf{j} + 0\mathbf{k}$$

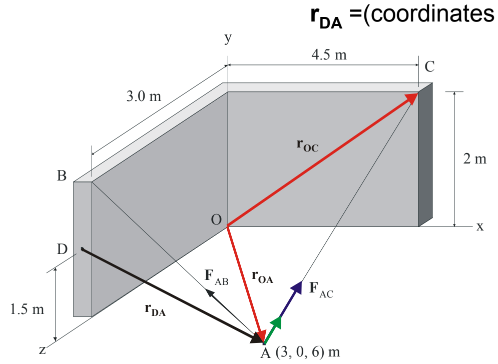
$$\mathbf{F_{AC}} = 120\mathbf{i} + 160\mathbf{j} - 480\mathbf{k} \quad \text{N.}$$

$$\mathbf{M_O} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & 0 & 6 \\ 120 & 160 & -480 \end{vmatrix} = \left(-960\mathbf{i} + 2160\mathbf{j} + 480\mathbf{k}\right)\text{N.m}$$

$$\mathbf{M_O} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 4.5 & 2 & 0 \\ 120 & 160 & -480 \end{vmatrix} = \left(-960\mathbf{i} + 2160\mathbf{j} + 480\mathbf{k}\right)\text{N.m}$$

Part (b): In (b) part of the question we are asked for the moment of force $\mathbf{F_{AC}}$ about Point D. $\mathbf{M_D} = \mathbf{r_{DA}}$ x $\mathbf{F_{AC}}$. We draw the position vector $\mathbf{r_{DA}}$ from point D to any point on the Line-of-Action of $\mathbf{F_{AC}}$. (We also could have used $\mathbf{r_{DC}}$.)

$\mathbf{r_{DA}}$ =(coordinates of Point A)-(coordinates of Point D)



$$\mathbf{r_{DA}} = 3\mathbf{i} - 1.5\mathbf{j} + 3\mathbf{k}$$

$$\mathbf{M_O} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & -1.5 & 3 \\ 120 & 160 & -480 \end{vmatrix} = \left(240\mathbf{i} + 1800\mathbf{j} + 660\mathbf{k}\right)\text{N.m}$$

Part (c): In part (c) of this question we are asked for the moment of $\mathbf{F}_{AC}$ about the point C.

Since the line-of-action of $\mathbf{F}_{AC}$ passes through the point C, the moment about C is zero. Recall that the magnitude of the moment of a force about a point is given by the scalar equation:

$$M = Fd$$

Where F is the magnitude of the force and d is the perpendicular distance from the point to the line-of-action of the force.

Since the line-of-action of $\mathbf{F}_{AC}$ passes through the point C, d = 0 and

$$M = 0$$

PART (d): In Part (d) we are asked to find the perpendicular distance, d, from point O to the line AC.

Recall:

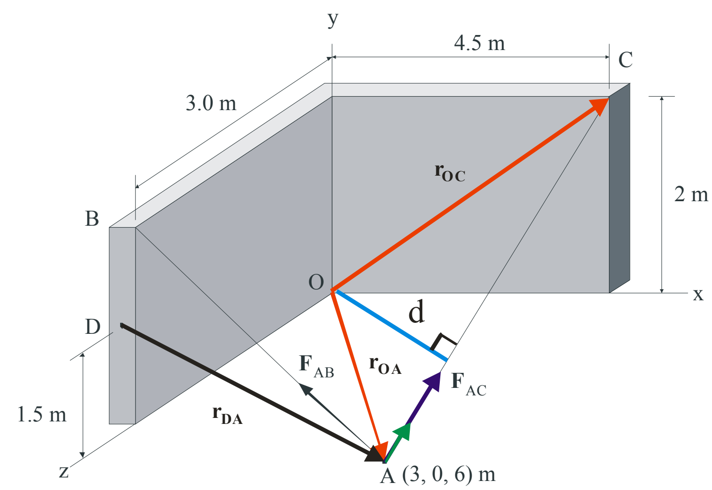
$$\mathbf{M_O} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & 0 & 6 \\ 120 & 160 & -480 \end{vmatrix} = \left(-960\mathbf{i} + 2160\mathbf{j} + 480\mathbf{k}\right) \text{N.m}$$

$$M_O = \sqrt{(-960)^2 + (2160)^2 + (480)^2} = 2411.97\text{N.m}$$

$$M = Fd$$

$$\therefore 2411.97 = 520d$$

$$d = 4.64\text{m}$$



CHAPTER 6
Distance Education
Study Problem # 28
J. Frye
May, 2011

Scalar (Dot) Product of Two Vectors

1. Angle Between Two Vectors
2. Projection of a Vector onto a Line

The scalar product of two vectors **P** and **Q** is defined by the product of the magnitudes P and Q and the cosine of the angle θ formed by vectors **P** and **Q.** The scalar product is most often referred to as the **dot product**.

$$\mathbf{P} \bullet \mathbf{Q} = PQ\cos\theta$$

The **scalar product** is a number (scalar) and it is **commutative**. That is:

$$\mathbf{P} \bullet \mathbf{Q} = \mathbf{Q} \bullet \mathbf{P}$$

Dot Product of Unit Vectors

$$\mathbf{i} \bullet \mathbf{i} = 1$$
$$\mathbf{j} \bullet \mathbf{j} = 1$$, since $(1)(1)\cos 0^{o} = 1$
$$\mathbf{k} \bullet \mathbf{k} = 1$$

$$\mathbf{i} \bullet \mathbf{j} = \mathbf{j} \bullet \mathbf{i} = 0$$
$$\mathbf{j} \bullet \mathbf{k} = \mathbf{k} \bullet \mathbf{j} = 0$$, since $(1)(1)\cos 90^{o} = 0$
$$\mathbf{i} \bullet \mathbf{k} = \mathbf{k} \bullet \mathbf{i} = 0$$

Given $\mathbf{P} = P_x\mathbf{i} + P_y\mathbf{j} + P_z\mathbf{k}$ and $\mathbf{Q} = Q_x\mathbf{i} + Q_y\mathbf{j} + Q_z\mathbf{k}$, then

$$\mathbf{P} \bullet \mathbf{Q} = (P_x\mathbf{i} + P_y\mathbf{j} + P_z\mathbf{k}) \bullet (Q_x\mathbf{i} + Q_y\mathbf{j} + Q_z\mathbf{k})$$

$$\mathbf{P} \bullet \mathbf{Q} = P_xQ_x\,(\mathbf{i} \bullet \mathbf{i}) + \cancel{P_xQ_y\,(\mathbf{i} \bullet \mathbf{j})} + \cancel{P_xQ_z(\mathbf{i} \bullet \mathbf{k})}$$
$$+ \cancel{P_yQ_x\,(\mathbf{j} \bullet \mathbf{i})} + P_yQ_y\,(\mathbf{j} \bullet \mathbf{j}) + \cancel{P_yQ_z(\mathbf{j} \bullet \mathbf{k})}$$
$$+ \cancel{P_zQ_x\,(\mathbf{k} \bullet \mathbf{i})} + \cancel{P_zQ_y\,(\mathbf{k} \bullet \mathbf{j})} + P_zQ_z(\mathbf{k} \bullet \mathbf{k})$$

$$\mathbf{P} \bullet \mathbf{Q} = P_xQ_x + P_yQ_y + P_zQ_z$$

$$\mathbf{P} = 3\mathbf{i} + 6\mathbf{j} - 2\mathbf{k}$$
$$\mathbf{Q} = 2\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}$$
$$\mathbf{P} \bullet \mathbf{Q} = (3)(2) + (6)(-2) + (-2)(3) = -12$$

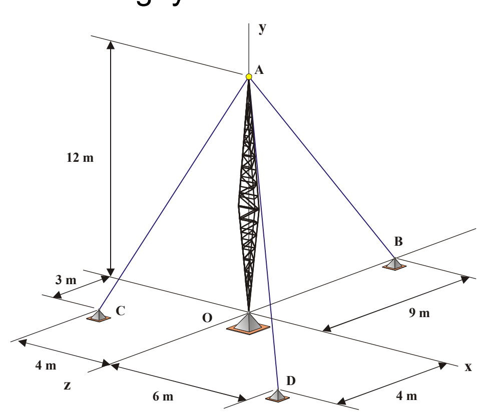
$$\mathbf{P} \cdot \mathbf{Q} = P_xQ_x + P_yQ_y + P_zQ_z$$

$$\mathbf{P} \bullet \mathbf{Q} = PQ\cos\theta$$

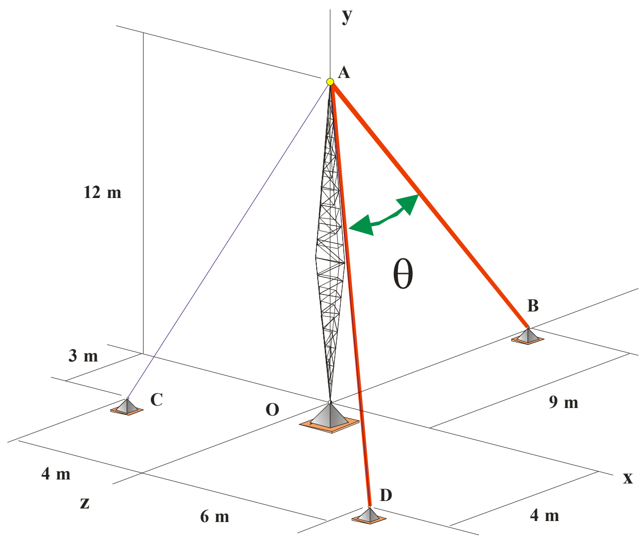
Angle Between Two Vectors

$$\cos\theta = \frac{P_xQ_x + P_yQ_y + P_zQ_z}{PQ} = \boldsymbol{\lambda_P} \bullet \boldsymbol{\lambda_Q}$$

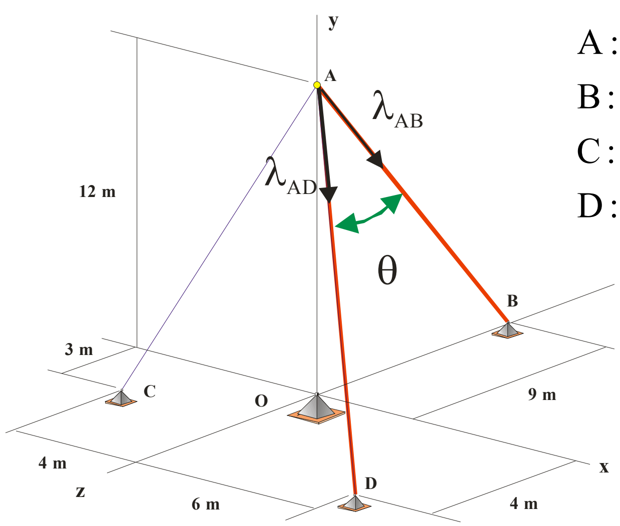
Example 6.4:
For the transmission tower shown, determine:
a) the angle between guy wires AB and AD, and
b) the component of the 1200 N tension exerted by guy AB at point A on guy AD.



Determine Angle, θ



We Need to Determine λ_{AB} and λ_{AD}



$A:(0,12,0)$

$B:(0,0,-9)$

$C:(-4,0,3)$

$D:(6,0,4)$

$$(1)\quad \boldsymbol{\lambda}_{AD} = \frac{\mathbf{AD}}{AD} \qquad \mathbf{AD} = 6\mathbf{i} - 12\mathbf{j} + 4\mathbf{k} \quad \text{and} \quad AD = \sqrt{6^2 + \left(-12^2\right) + 4^2} = 14$$

$$\boldsymbol{\lambda}_{AD} = \frac{6\mathbf{i} - 12\mathbf{j} + 4\mathbf{k}}{14}$$

$$(2)\quad \boldsymbol{\lambda}_{AB} = \frac{\mathbf{AB}}{AB} \qquad \mathbf{AB} = 0\mathbf{i} - 12\mathbf{j} - 9\mathbf{k} \quad \text{and} \quad AB = \sqrt{\left(-12^2\right) + \left(-9^2\right)} = 15$$

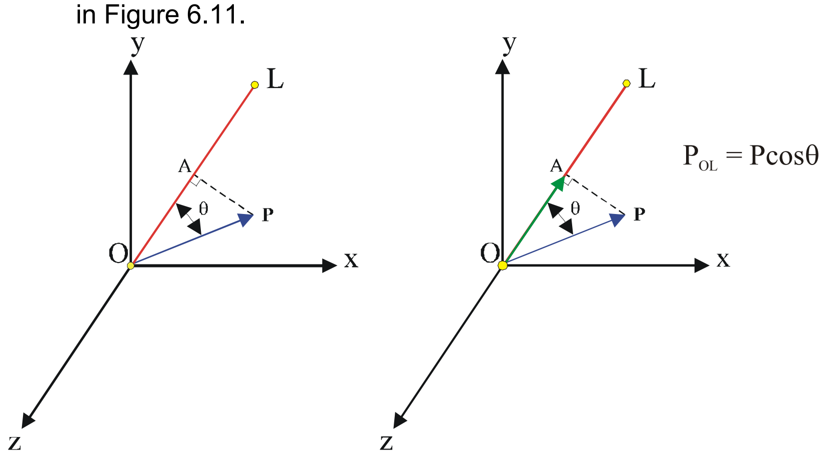
$$\boldsymbol{\lambda}_{AB} = \frac{-12\mathbf{j} - 9\mathbf{k}}{15}$$

$$\boldsymbol{\lambda}_{AD} \bullet \boldsymbol{\lambda}_{AB} = \left(\frac{6\mathbf{i} - 12\mathbf{j} + 4\mathbf{k}}{14}\right) \bullet \left(\frac{-12\mathbf{j} - 9\mathbf{k}}{15}\right) = \cos\theta$$

$$\cos\theta = \left(\frac{6}{14}\right)(0) + \left(\frac{-12}{14}\right)\left(\frac{-12}{15}\right) + \left(\frac{4}{14}\right)\left(\frac{-9}{15}\right) = 0.51428 \Rightarrow \theta = 59.05^{\circ}$$

Projection of a Vector onto a Given Line

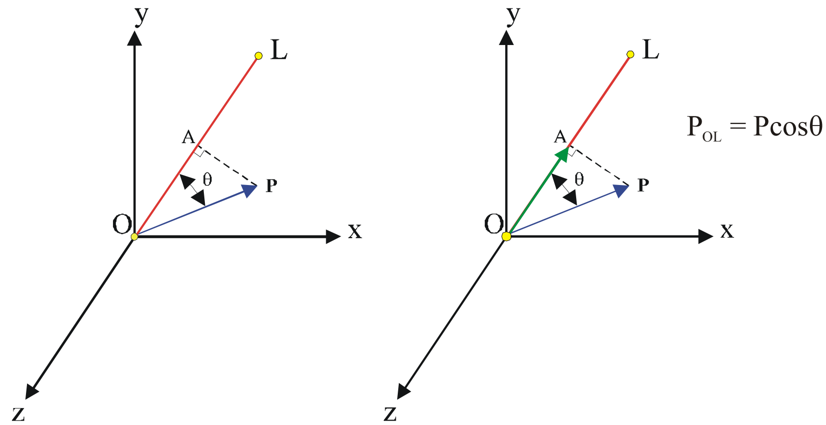
Consider the vector **P** and the line or axis OL as shown in Figure 6.11.



$$P_{OL} = P\cos\theta$$

The projection of **P** on the line OL is defined as the scalar:

$$P_{OL} = P\cos\theta$$ = The length of OA.



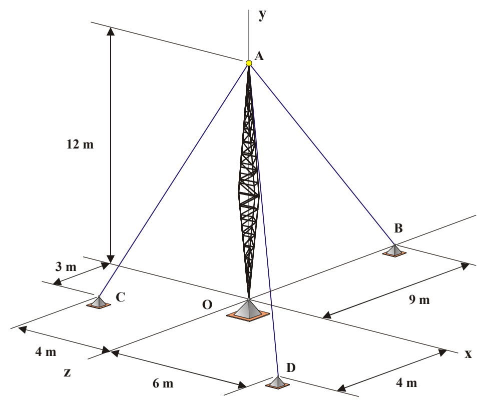
The component of vector **P** on line OL can be determined from the **Dot Product** of the unit vector λ_{OL} in the direction of the line OL and the vector **P**.

$$P\cos\theta = \frac{P_x Q_x + P_y Q_y + P_z Q_z}{Q}$$

$$= P_x\left(\frac{Q_x}{Q}\right) + P_y\left(\frac{Q_y}{Q}\right) + P_z\left(\frac{Q_z}{Q}\right)$$

$$= \mathbf{P} \bullet \boldsymbol{\lambda}_{\mathbf{Q}} = \mathbf{P} \bullet \boldsymbol{\lambda}_{\mathbf{OL}}$$

Example 6.4:
For the transmission tower shown, determine:
a) the angle between guy wires AB and AD, and
b) the component of the 1200 N tension exerted by guy AB at point A on guy AD.



b) Projection of the 1200 N tension onto guy AD

$$\mathbf{T_{AB}} = T_{AB}\boldsymbol{\lambda}_{\mathbf{AB}} = 1200\boldsymbol{\lambda}_{\mathbf{AB}} = 1200\left(\frac{-12\mathbf{j}-9\mathbf{k}}{15}\right) = -960\mathbf{j}-720\mathbf{k}\ \mathrm{N}$$

$$\textbf{Projection onto Line AD} = T_{AB} \bullet \boldsymbol{\lambda}_{\mathbf{AD}}$$

$$\boldsymbol{\lambda}_{\mathbf{AD}} = \frac{6\mathbf{i}-12\mathbf{j}+4\mathbf{k}}{14}$$

$$T_{AB} \bullet \boldsymbol{\lambda}_{\mathbf{AD}} = (-960\mathbf{j}-720) \bullet \left(\frac{6\mathbf{i}-12\mathbf{j}+4\mathbf{k}}{14}\right) = 617.14\ \mathrm{N}$$

Also:

$$1200\cos 59.05^{\circ} = 617.14\ \mathrm{N}$$

CHAPTER 6
Distance Education
Study Problem # 29
J. Frye
May, 2011

Mixed Triple Product

Moment of a Force about a Line

6.5 Mixed Triple Product of Three Vectors

We define the mixed triple product of three vectors **S**, **P** and **Q** as:

$$\mathbf{S} \bullet (\mathbf{P} \times \mathbf{Q})$$

which can be expressed in a determinant form as:

$$\mathbf{S} \bullet (\mathbf{P} \times \mathbf{Q}) = \begin{vmatrix} S_x & S_y & S_z \\ P_x & P_y & P_z \\ Q_x & Q_y & Q_z \end{vmatrix}$$

Thus, we can write:

$$\boldsymbol{\lambda} \bullet (\mathbf{r} \times \mathbf{F}) = \begin{vmatrix} \lambda_x & \lambda_y & \lambda_z \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$

6.6 Moment of a Force about a Line

The moment of a force about a line, OL, is found by taking the **dot product** of a unit vector λ_{OL} **directed along the line we wish to take moments about** and **moment, M_O, of the force about** ANY point on the line.

$$M_{OL} = \boldsymbol{\lambda}_{OL} \bullet \mathbf{M_o} = \boldsymbol{\lambda}_{OL} \bullet (\mathbf{r} \times \mathbf{F})$$

$$M_{OL} = \begin{vmatrix} \lambda_x & \lambda_y & \lambda_z \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$

$$M_{OL} = \begin{vmatrix} \lambda_x & \lambda_y & \lambda_z \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$

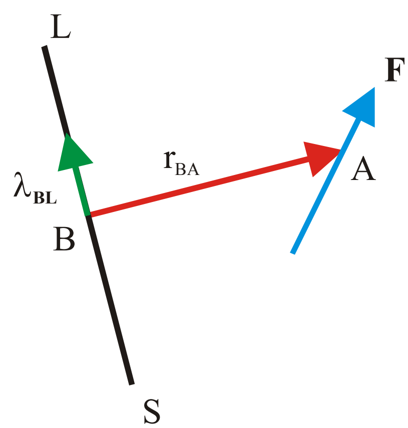
where,

λ_x, λ_y and λ_z the rectangular components of the unit $\boldsymbol{\lambda}_{OL}$ **vector directed along the line OL,**

r_x, r_y and r_z **the rectangular components of the position vector, r** drawn from **ANY** point on the line OL to the point of application of the force **F** (or **ANY** point on the line of action of force **F**), and

F_x, F_y and F_z **are the rectangular components of the force F**.

Summary – Moment of a Force about a Line SL (from S to L)
STEPS INVOLVED:



Step 1
Select a convenient point, say B, on SL. (Convenient means that the coordinates of B are known.)

Step 2
Select a convenient point, say A, on **F.** That is, on the line of action of **F**.

Step 3
Find $\mathbf{r} = \mathbf{BA} = d_x\mathbf{i} + d_y\mathbf{j} + d_z\mathbf{k}$

Step 4
Compute $\mathbf{M}_B = \mathbf{r} \times \mathbf{F} =$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ d_x & d_y & d_z \\ F_x & F_y & F_z \end{vmatrix} = M_{Bx}\mathbf{i} + M_{By}\mathbf{j} + M_{Bz}\mathbf{k}$$

Step 5

Compute the unit vector, λ_{BL}, **in the SL direction (from S to L)**

$$\lambda_{BL} = \lambda_x\mathbf{i} + \lambda_y\mathbf{j} + \lambda_z\mathbf{k}$$

Step 6

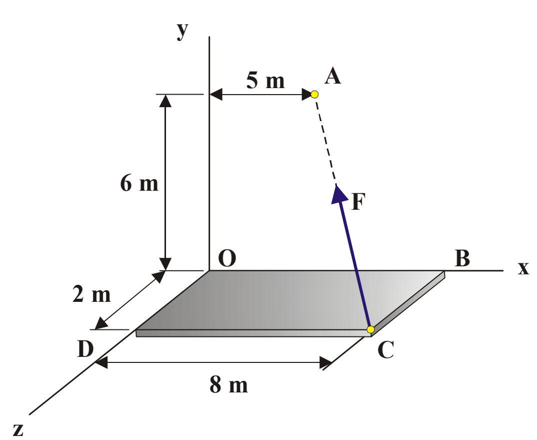
Then $M_{BL} = \lambda_{BL} \cdot \mathbf{M}_B = \lambda_x \mathbf{M}_{Bx} + \lambda_y \mathbf{M}_{By} + \lambda_z \mathbf{M}_{Bz}$ **or alternately,** evaluate the determinant.

$$M_{BL} = \begin{vmatrix} \lambda_x & \lambda_y & \lambda_z \\ d_x & d_y & d_z \\ F_x & F_y & F_z \end{vmatrix}$$

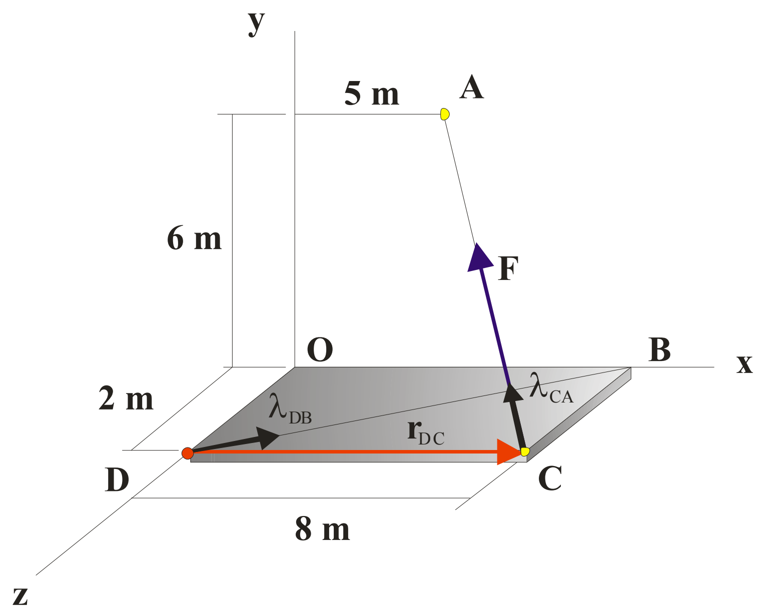
Sample Problem 6.3:

The flat plate OBCD has a force **F** of 280 N applied at point C as shown. Determine

(a) The moment of **F** about the line DB, and
(b) The moment of **F** about the line OA.



$$O:(0,0,0)$$
$$A:(5,6,0)$$
$$B:(8,0,0)$$
$$C:(8,0,2)$$
$$D:(0,0,2)$$



$$\mathbf{CA} = (A - C) = (-3, 6, -2)$$

$$CA = \sqrt{3^2 + 6^2 + 2^2} = \sqrt{49} = 7$$

$$\lambda_{CA} = \left(-\frac{3}{7}, \frac{6}{7}, -\frac{2}{7}\right)$$

We express $\mathbf{T}_{CA}$ in component form:

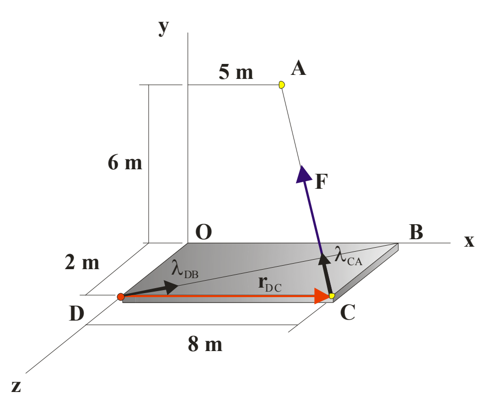
$$\mathbf{T}_{CA} = 280\boldsymbol{\lambda}_{\mathbf{CA}} = (-120, 240, -80) = -120\mathbf{i} + 240\mathbf{j} - 80\mathbf{k}$$

Moment of $\mathbf{T}_{CA}$ about the line DB:

$$\mathbf{M}_{DB} = \boldsymbol{\lambda}_{\mathbf{DB}} \bullet \mathbf{M}_{D} = \boldsymbol{\lambda}_{\mathbf{DB}} \bullet \mathrm{r}_{\mathbf{DC}} \times \mathbf{T}_{CA}$$

In this problem, convenient **r**'s are **DC**, **DA**, **BC**, or **BA**.

DC and **BC** are <u>**better choices**</u> than **DA** and **BA** because they have only one component. We will choose $\mathbf{r}_{DC}$ = **DC** = (C-D) = (8, 0, 0)

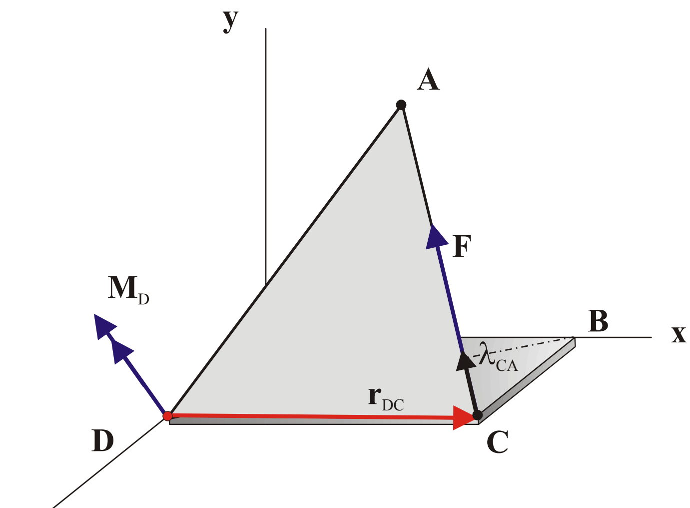


$$\mathbf{M}_D = \mathbf{r}_{\mathbf{DC}} \times \mathbf{T}_{CA} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 8 & 0 & 0 \\ -120 & 240 & -80 \end{vmatrix}$$

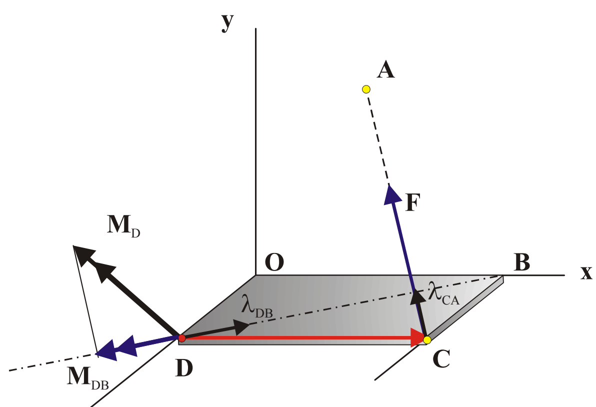
$$= \mathbf{i}\left[(0)(-80)-(240)(0)\right] - \mathbf{j}\left[(8)(-80)-(0)(-120)\right] + \mathbf{k}\left[(8)(240)-(0)(-120)\right]$$

$$= 0\mathbf{i} - \mathbf{j}\left[-640-0\right] + \mathbf{k}\left[1920-0\right]$$

$$= 0\mathbf{i} + 640\mathbf{j} + 1920\mathbf{k} \text{ N.m}$$



The moment vector $\mathbf{M}_D$ is perpendicular to the plane containing $\mathbf{r}_{DC}$ and $\mathbf{F}$.



The moment $\mathbf{M}_{DB}$ is the projection of the moment vector $\mathbf{M}_D$ onto the line DB and is a measure of the tendency of the force $\mathbf{F}$ to cause a rigid body rotation about DB.

We now need λ_{DB} to calculate M_{DB}:

$$\mathbf{DB} = (B - D) = (8, 0, -2);$$

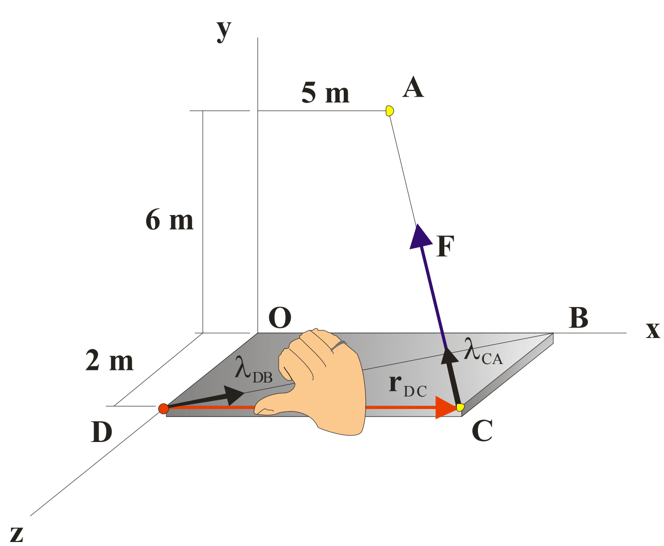
$$DB = \sqrt{8^2 + 0^2 + 2^2} = \sqrt{68} = 8.246 \text{ (Note: must use } \mathbf{DB} \text{ \& not } \mathbf{BD}\text{)}$$

$$\lambda_{DB} = \left(\frac{8}{8.246}, 0, -\frac{2}{8.246} \right)$$

$$= (0.97014, 0, -0.2425)$$

$$\therefore M_{DB} = \lambda_{DB} \bullet \mathbf{M}_D = (0.97014\mathbf{i} + 0\mathbf{j} - 0.2425\mathbf{k}) \bullet (0\mathbf{i} + 640\mathbf{j} + 1920\mathbf{k})$$

$$= 0 + 0 - (0.2425)(1920) = -465.67 \text{ N.m}$$



We note that in our solution M_{DB} = -465.67 N.m.

In this solution we directed the unit vector λ_{DB} from D to B.

We can determine the direction of rotation about the line DB by using the "Right Hand Rule" and noting that our answer is **negative**. The negative answer tells us to point the thumb of our right hand opposite to the sense of λ_{DB}.

The fingers of our right hand will curl around the line DB indicating the direction of the rotation as shown in the figure.

The moment of **F** about the line OA.
Let us choose **r** = **AA** = (0,0,0).

$$d = |\mathbf{r}| = 0$$

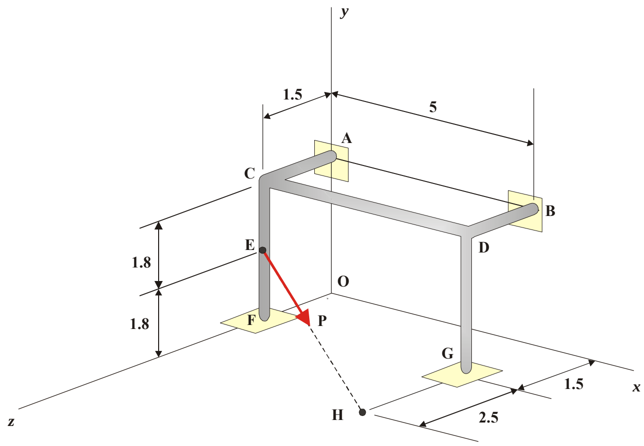
$$\therefore \mathbf{M}_A = \mathbf{AA} \times \mathbf{T}_{CA} = \text{zero vector}$$

$$M_{OA} = 0.$$

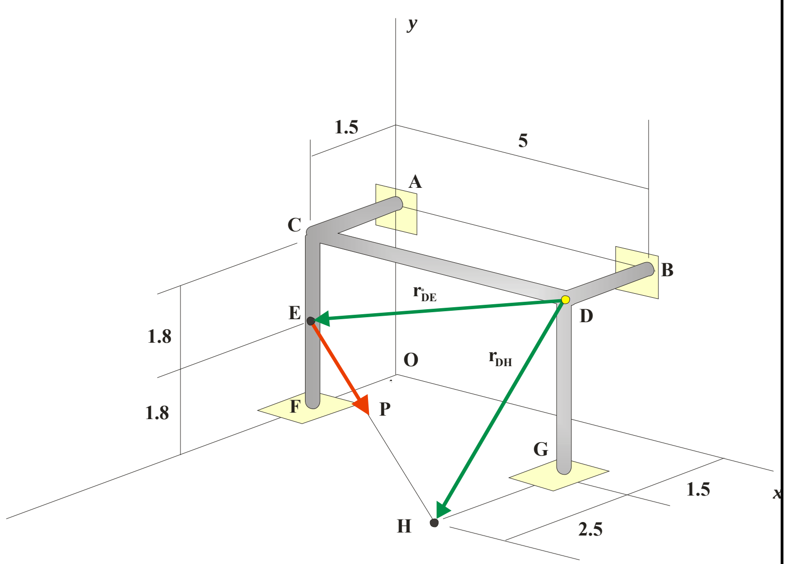
If the line of action of the force passes through **any point** on the line we want to take moments about, the moment = 0.

Example 6.5:
A force ***P*** of magnitude *2600 N* acts on the frame shown at point *E*.
Determine:

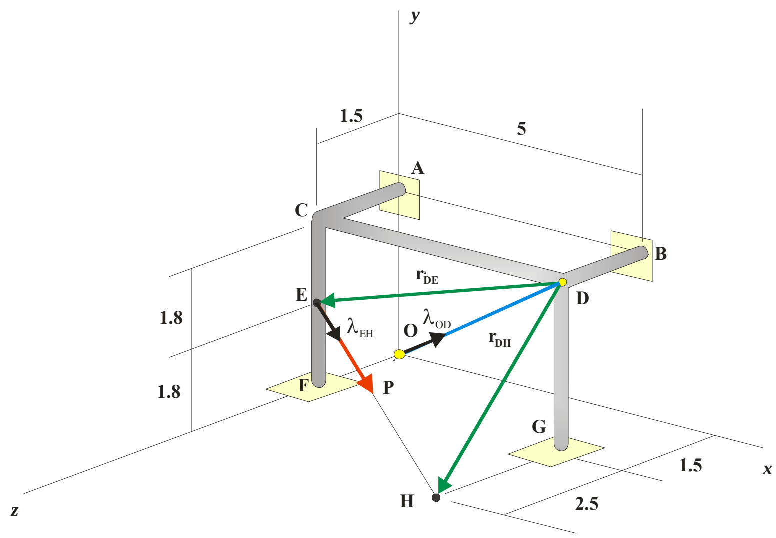
a) The moment of ***P*** about the point *D*.
b) The moment of ***P*** about the line joining points *O* and *D*.
c) The perpendicular distance from the line of action of ***P*** to the point *D*.



Choose an **r** vector ($\mathbf{r_{DE}}$ or $\mathbf{r_{DH}}$)



Draw in Unit Vectors



Moment of force **P** about Point D

$$\mathbf{M_D} = \mathbf{r_{DH}} \times \mathbf{P}$$

$$\mathbf{r_{DH}} = 0\mathbf{i} - 3.6\mathbf{j} + 2.5\mathbf{k}$$

$$\mathbf{P} = \mathrm{P}\boldsymbol{\lambda}_{\mathbf{EH}} = 2600\boldsymbol{\lambda}_{\mathbf{EH}} \qquad \boldsymbol{\lambda}_{\mathbf{EH}} = \frac{\mathbf{EH}}{\mathrm{EH}} \qquad \mathbf{EH} = 5\mathbf{i} - 1.8\mathbf{j} + 2.5\mathbf{k}$$

$$\mathrm{EH} = \sqrt{5^2 + \left(-1.8^2\right) + 2.5^2} = 5.873$$

$$\therefore \mathbf{P} = 2600\left(\frac{5\mathbf{i} - 1.8\mathbf{j} + 2.5\mathbf{k}}{5.873}\right) = 2213.52\mathbf{i} - 796.87\mathbf{j} + 1106.76\mathbf{k}$$

$$\mathbf{M_D} = \mathbf{r_{DH}} \times \mathbf{P} = \left|\begin{array}{ccc|cc} \mathbf{i} & \mathbf{j} & \mathbf{k} & \mathbf{i} & \mathbf{j} \\ 0 & -3.6 & 2.5 & 0 & -3.6 \\ 2213.52 & -796.87 & 1106.76 & 2213.52 & -796.87 \end{array}\right|$$

$$\mathbf{M_D} = \left[-3984.34\mathbf{i} + 5533.8\mathbf{j}\right] - \left[-7968.67\mathbf{k} - 1992.18\mathbf{i}\right]$$

$$\mathbf{M_D} = -1992.16\mathbf{i} + 5533.8\mathbf{j} - 7968.67\mathbf{k} \text{ N.m}$$

Moment of **P** about the Line OD

$$M_{OD} = \boldsymbol{\lambda}_{\mathbf{OD}} \bullet \mathbf{M_D}$$

$$\boldsymbol{\lambda}_{\mathbf{OD}} = \frac{\mathbf{OD}}{OD} \qquad \mathbf{OD} = 5\mathbf{i} + 3.6\mathbf{j} + 1.5\mathbf{k}$$

$$OD = \sqrt{5^2 + 3.6^2 + 1.5^2} = \sqrt{40.21}$$

$$\boldsymbol{\lambda}_{\mathbf{OD}} = \frac{5\mathbf{i} + 3.6\mathbf{j} + 1.5\mathbf{k}}{\sqrt{40.21}}$$

$$M_{OD} = \left(\frac{5\mathbf{i} + 3.6\mathbf{j} + 1.5\mathbf{k}}{\sqrt{40.21}}\right) \bullet \left(-1992.16\mathbf{i} + 5533.8\mathbf{j} - 7968.67\mathbf{k}\right)$$

$$M_{OD} = \left(\frac{5}{\sqrt{40.21}}\right)(-1992.16) + \left(\frac{3.6}{\sqrt{40.21}}\right)(+5533.8) + \left(\frac{1.5}{\sqrt{40.21}}\right)(-7968.67) = 3455.83 \text{ N.m}$$

Alternately :

$$M_{OD} = \boldsymbol{\lambda}_{\mathbf{OD}} \bullet \mathbf{M_D} = \frac{1}{\sqrt{40.21}} \left| \begin{array}{ccc|cc} 5 & 3.6 & 1.5 & 5 & 3.6 \\ 0 & -3.6 & 2.5 & 0 & -3.6 \\ 2213.52 & -796.87 & 1106.76 & 2213.52 & -796.87 \end{array} \right| = 3455.83 \text{ N.m}$$

c) The perpendicular distance from the line of action of ***P*** to the point *D*.

$$M = Fd$$

$$M = \sqrt{(-1992.16)^2 + 5533.8^2 + 7968.67^2} = 9904.11 \text{ N.m}$$

and

$$F = 2600\text{N}$$

$$\therefore d = \frac{9904.11 \text{ N.m}}{2600 \text{ N}} = 3.81 \text{ m}$$

CHAPTER 6
Distance Education
Study Problem # 30
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May, 2011

Reduction of a System of Parallel Forces to a Single Force

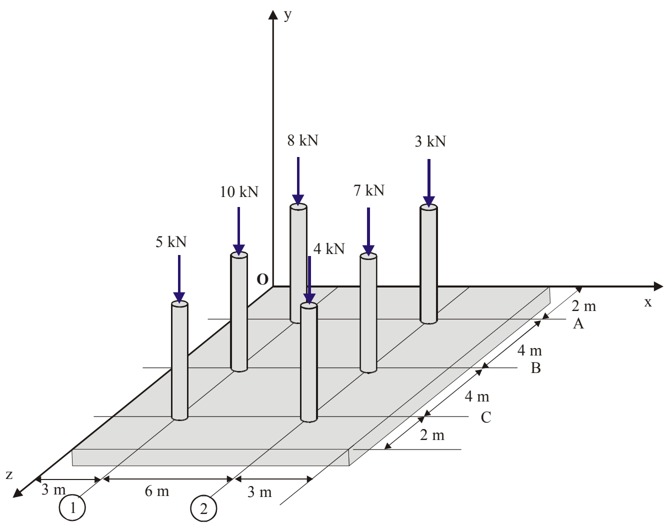
Problem Type 1 – Given a system of parallel forces, determine the magnitude and location (e.g. x and z coordinates with respect to an origin, O) of the resultant force, **R**.

Problem Type 2 – Given the location of the resultant force, **R** and the magnitude of an unknown force, determine the location of the unknown force.

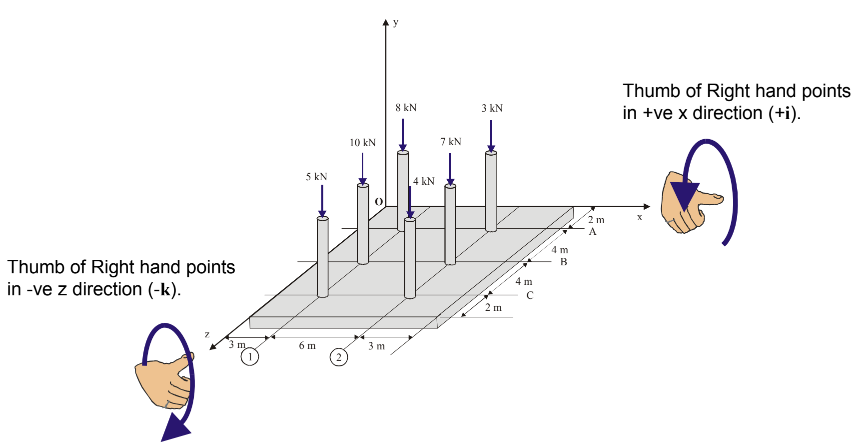
Problem Type 1

1. To establish the location of **R**, we will equate the summation of the moments that each of the forces has about a common point to the moment that the resultant, **R** of the parallel forces has about the same point.
2. To calculate the moment that all forces including the resultant, R have about the common point, we will set up the calculations in tabular form and will show how to use the right handed triad to determine algebraic signs for the moments.

Example Problem Type 1: Six (6) column loads act on a "Raft" foundation. Determine the location with respect to the origin, O, of a single resultant force, **R**, that will have the same effect on the foundation as the six column loads.



Observations: We see that the all six parallel forces want to rotate the raft foundation about the x-axis and also about the z-axis. By the Right Hand Rule if we curl our fingers in the direction of rotation about the x-axis and then about the z-axis our thumb points in the direction of the moment that the forces have about the respective axes. (Positive **i** about the x-axis and negative **k** about the z-axis.)



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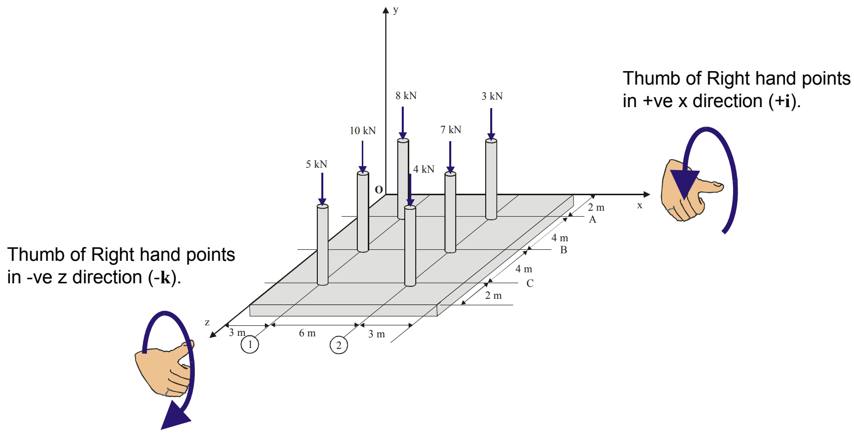
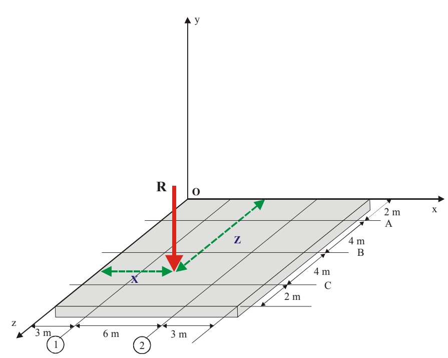


Location of Resultant, **R** on the Raft Foundation

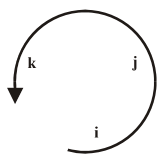
We denote the location of **R** with respect to the origin, O as:

$$x\mathbf{i} + z\mathbf{k}$$



Resultant Moment, M_R of six (6) column forces about the Origin, O

$$\mathbf{M} = \mathbf{r} \times \mathbf{F}$$



Right handed Triad

$$i \times j = k$$
$$j \times k = i$$
$$k \times i = j$$
$$k \times j = -i$$
$$j \times i = -k$$
$$i \times k = -j$$

Force	Position, **r**	Magnitude, **F** (kN)	Magnitude Moment **M = r x F** (kN.m)
A-1	3**i** + 2**k**	-8**j**	-24**k** +16**i**
B-1	3**i** + 6**k**	-10**j**	-30**k** +60**i**
C-1	3**i** + 10**k**	-5**j**	-15**k** +50**i**
A-2	9**i** + 2**k**	-3**j**	-27**k** +6**i**
B-2	9**i** + 6**k**	-7**j**	-63**k** +42**i**
C-2	9**i** + 10**k**	-4**j**	-36**k** +40**i**
	x**i** +z**k**	**R** = -37**j**	$\mathbf{M}_R$ = -195**k** + 214**i**

We now equate the moment of the Resultant, **R**, about the origin, O, to the summation of the moments of the individual forces about O.

$$(x\mathbf{i} + z\mathbf{k}) \times (-37\mathbf{j}) = -205\mathbf{k} + 214\mathbf{i}$$

$$-37x\mathbf{k} + 37z\mathbf{i} = -205\mathbf{k} + 214\mathbf{i}$$

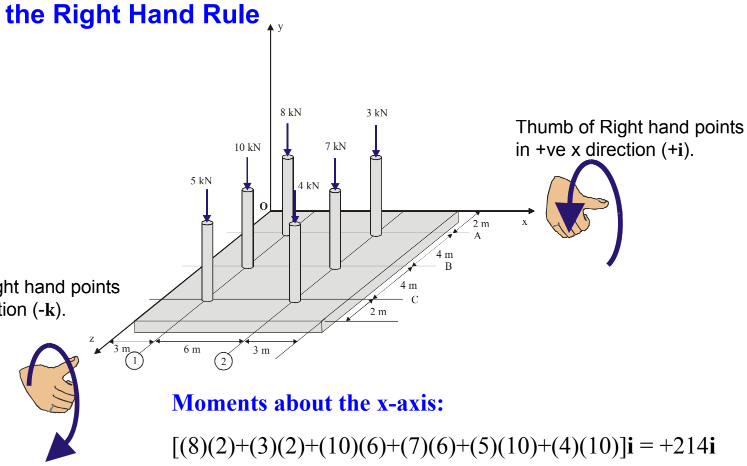
$$\therefore x = \frac{-195}{-37} = 5.27\text{m}$$

$$z = \frac{214}{37} = 5.78\text{m}$$

Alternate Calculation to Tabular Form: Using the Right Hand Rule

Thumb of Right hand points in +ve x direction (+**i**).

Thumb of Right hand points in -ve z direction (-**k**).



Moments about the x-axis:

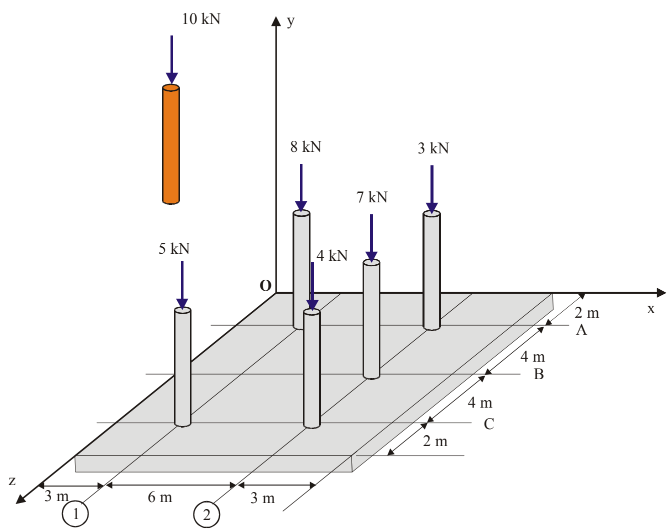
$[(8)(2)+(3)(2)+(10)(6)+(7)(6)+(5)(10)+(4)(10)]\mathbf{i} = +214\mathbf{i}$

Moments about the z-axis:

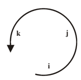
$[-(8)(3)-(3)(9)-(10)(3)-(7)(9)-(5)(3)-(4)(9)]\mathbf{k} = -195\mathbf{k}$

$$x = \frac{-195}{-37} = 5.27\text{m} \qquad z = \frac{214}{37} = 5.78\text{m}$$

Example Problem Type 2: Five (5) column loads act on a "Raft" foundation. A sixth column load of 10 kN acting downward is to be applied such that the location of the Resultant, **R** of the six column loads will be located at x = 5.27 m and z = 5.78 m. Determine the location of the sixth column.



Resultant Moment, M_R of six (6) column forces about the Origin, O when R is located at x = 5.27 m and z = 5.78 m



Right handed Triad

$i \times j = k$
$j \times k = i$
$k \times i = j$
$k \times j = -i$
$j \times i = -k$
$i \times k = -j$

Force	Position, **r**	Magnitude, **F** (kN)	Magnitude Moment **M** = **r x F** (kN.m)
A-1	3**i** + 2**k**	-8**j**	-24**k** +16**i**
C-1	3**i** + 10**k**	-5**j**	-25**k** +50**i**
A-2	9**i** + 2**k**	-3**j**	-27**k** +6**i**
B-2	9**i** + 6**k**	-7**j**	-63**k** +42**i**
C-2	9**i** + 10**k**	-4**j**	-36**k** +40**i**
6th Column	x**i** + z**k**	-10**j**	-10x**k** + 10z**i**
		R = -37**j**	$\mathbf{M_R}$ = (-175-10x)**k** + (154 + 10z)**i**

$$(5.27\mathbf{i} + 5.78\mathbf{k}) \times (-37\mathbf{j}) = (-175 - 10x)\mathbf{k} + (154 + 10z)\mathbf{i}$$
$$-195\mathbf{k} + 213.86\mathbf{i} = (-175 - 10x)\mathbf{k} + (154 + 10z)\mathbf{i}$$
$$\therefore -195 = -175 - 10x$$
$$x = \frac{20}{10} = 2\text{m}$$
$$+213.86 = 154 + 10z$$
$$z = \frac{59.86}{10} = 6\text{m}$$