Unit 8 Assignment Solutions

1. (a) Since weights follow a normal distribution, we can calculate this probability:

$$P(X < 3.5) = P\left(Z < \frac{5.1 - 5.0}{0.2}\right) = P(Z < 0.50) = 0.6915.$$

(b) $$P(4.97 < \overline{X} < 5.04) = P\left(\frac{4.97 - 5.00}{0.2/\sqrt{50}} < Z < \frac{5.04 - 5.00}{0.2/\sqrt{50}}\right) = P(-1.06 < Z < 1.41)$$

$$= P(Z < 1.41) - P(Z < -1.06) = 0.9207 - 0.1446 = 0.7761.$$

(c) $$P(X_T > 502) = P\left(\overline{X} > \frac{502}{100}\right) = P(\overline{X} > 5.02) = P\left(Z > \frac{5.02 - 5.00}{0.2/\sqrt{100}}\right)$$

$$= P(Z > 1.00) = 1 - P(Z < 1.00) = 1 - 0.8413 = 0.1587.$$

(d) Since weight X follows a normal distribution, the distribution of $\overline{X}$ is normal for any sample size n. Therefore, the probabilities calculated in (b) and (c) are exact.

2. (a) Since the amount spent by customers does not follow a normal distribution (and since we don't know the true distribution of amounts spent), we cannot calculate this probability.

(b) $P(17.00 < \overline{X} < 22.00) \approx P\left(\frac{17.00-18.50}{14.25/\sqrt{35}} < Z < \frac{22.00-18.50}{14.25/\sqrt{35}}\right) = P(-0.62 < Z < 1.45)$

$= P(Z < 1.45) - P(Z < -0.62) = 0.9265 - 0.2676 = 0.6589.$

(c) $P(X_T > 1000) = P\left(\overline{X} > \frac{1000}{50}\right) = P(\overline{X} > 20.00) = P\left(Z > \frac{20.00-18.50}{14.25/\sqrt{50}}\right)$

$= P(Z > 0.74) = 1 - P(Z < 0.74) = 1 - 0.7704 = 0.2296.$

(d) The population distribution is not normal. However, the Central Limit Theorem tells us that, regardless of the form of the population distribution, the sampling distribution of $\overline{X}$ will be approximately normal when the sample size is large (we use $n \geq 30$). As such, the probabilities in (b) and (c) are approximate.

3. (a) Since times follow an exponential distribution, we can calculate this probability. Note that we convert 1 and 2 minutes to 60 and 120 seconds, respectively.

$$P(60 < X < 120) = \int_{60}^{120} 0.0125e^{-0.0125x}dx = \left[-e^{-0.0125x}\right]_{60}^{120} = e^{-0.0125(60)} - e^{-0.0125(120)}$$

$= e^{-0.75} - e^{-1.50} = 0.4724 - 0.2231 = 0.2493.$

(b) $P(\overline{X} > 90) \approx P\left(Z > \frac{90-80}{80/\sqrt{30}}\right) = P(Z > 0.68) = 1 - P(Z < 0.68) = 1 - 0.7517 = 0.2483.$

(c) One hour = 3600 seconds, so

$$P(X_T < 3600) = P\left(\overline{X} < \frac{3600}{50}\right) = P(\overline{X} < 72) \approx P\left(Z < \frac{72-80}{80/\sqrt{50}}\right) = P(Z < -0.71) = 0.2389.$$

(d) The population distribution is not normal. However, the Central Limit Theorem tells us that, regardless of the form of the population distribution, the sampling distribution of $\overline{X}$ will be approximately normal when the sample size is large (we use $n \geq 30$). As such, the probabilities in (b) and (c) are approximate.