Dec 99 + Solutions

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UNIVERSITY OF MANITOBA

ENGINEERING 130.112 - THERMAL SCIENCES

9 December 1999

Final Exam
Dr. G. F. Naterer

Name: ____________
ID#: ____________

- This is a three-hour *open – textbook* exam. Students may use the textbook entitled *Thermodynamics: An Engineering Approach* (by Y. A. Cengel, M. A. Boles) and the *Thermodynamics* excerpt (by J. P. Holman). No other materials (i.e. notes, solved problems, extra pages in textbook, etc.) are allowed.
- State all assumptions and label your system with dashed lines. If iteration is required, then perform one complete iteration, i.e. one initial calculation plus iterative recalculation.
- Write your solutions clearly and legibly in the booklets provided (not on question pages). Ambiguous solutions which cannot be interpreted will be considered incorrect.

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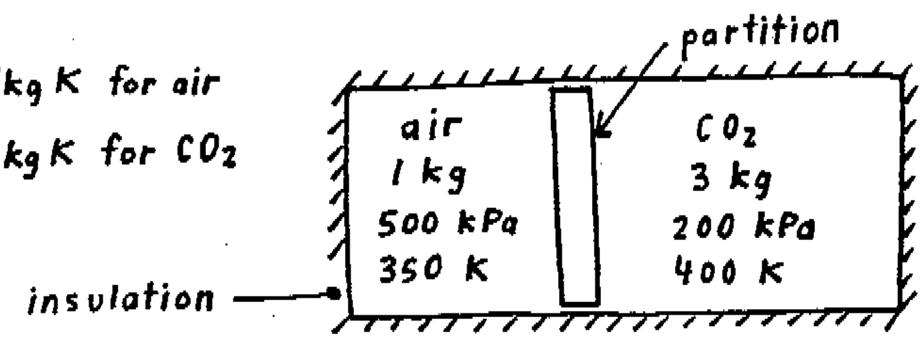
1. (**10 MARKS**) One kilogram of air (initially at 500 kPa, 350 K) and 3 kg of of carbon dioxide, CO_2 (initially at 200 kPa, 450 K) are confined to opposite sides of a rigid, insulated container (see figure). The dividing partition is free to move and heat is transferred by conduction from one gas to the other gas through the partition (without energy storage in the partition itself). Assuming ideal gas behaviour, find the equilibrium temperature and final pressure in the container.

$c_v \sim 726$ J/kg K for air

$c_v \sim 750$ J/kg K for CO_2



2. (**10 MARKS**) A rigid tank having a volume of 0.028 m^3 initially contains a mixture at 21 $°C$ and 100 kPa consisting of 79 % N_2 and 21 % O_2 on a molar basis. Helium at 21 $°C$ is allowed to flow into the tank until the pressure reaches 136 kPa. If the final temperature of the mixture within the tank is 21 $°C$, then find the mass of each component present at the final state.

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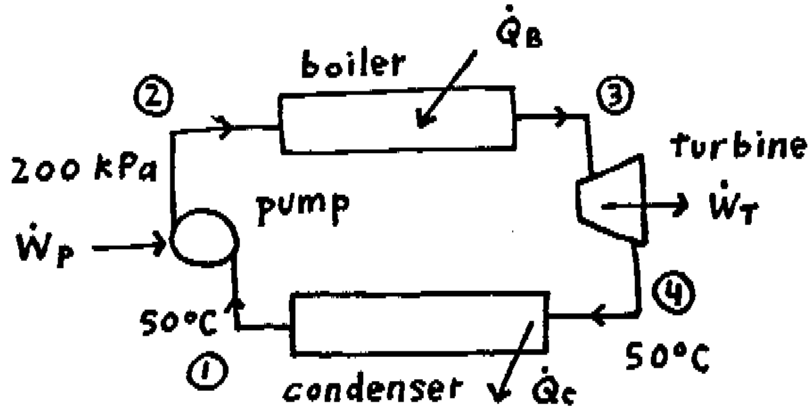
3. **(15 MARKS)** A small solar heat engine for water pumping uses steam as the working fluid (see figure). Water enters the pump as saturated liquid at 50 $°C$ and is pumped up to 200 kPa. The boiler evaporates the water at 0.2 MPa and saturated vapor at this pressure enters the small turbine. The steam leaves the turbine with a quality of x = 0.94 at 50 $°C$ and is subsequently condensed. The flow rate is 140 kg/h and the pump is driven by a 1/2 hp motor (note: 1 hp = 746 W) operating at full load.



(a) Determine the power output and thermal efficiency of this power plant.

(b) In an effort to conserve energy in this power cycle, somebody suggests incorporating a refrigeration system that will absorb some of the waste heat (i.e. heat output from condenser) and transfer it to the energy source (boiler) of the heat engine. Could this suggestion improve the thermal efficiency obtained in part (a) ? Explain your response.

(c) Another proposal suggests not rejecting any waste heat in the condenser. Is this proposal feasible for improving the efficiency obtained in part (a) ? Explain your response.

4. **(15 MARKS)** A closed, rigid tank initially contains 0.24 m^3 of moist air at 0.1 MPa in equilibrium with 0.06 m^3 of liquid water at 29 $°C$. The vapor in the moist air and the liquid water can be treated as a saturated mixture at 29 $°C$. The tank contents are then heated to 140 $°C$. Assume ideal gas behaviour in the gas phase.

(a) Calculate the final quality of the mixture (x_2).

(b) Determine the heat transfer to the tank.

(c) Find the specific humidity and pressure at the final state.

5. **(10 MARKS)** Water flows through a tube with a diameter of 2 cm and length of 10 m at at average flow velocity of 8 m/s. The water enters the tube at 20 $°C$ and leaves at 30 $°C$. Determine the average wall temperature necessary to effect the required heat transfer.

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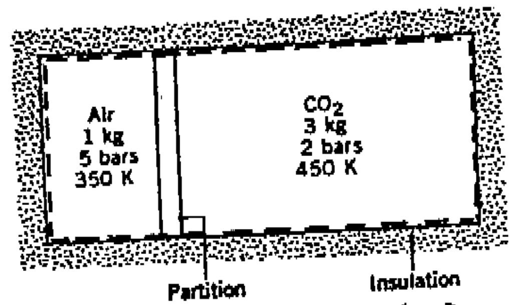
PROBLEM #1

KNOWN: Air and carbon dioxide are confined to opposite sides of a rigid, wall-insulated container. The partition moves and allows conduction from one gas to the other until equilibrium is achieved.

FIND: Determine the final temperature and pressure.

SCHEMATIC & GIVEN DATA:



note: 1 bar = 100 kPa

note: at 400 K, c_v(air) ~ 0.726 kJ/kgK, $c_v(CO_2)$ ~ 0.75 kJ/kg K

ASSUMPTIONS: (1) The contents of the container form a closed system. (2) The air and CO_2 behaves as ideal gases with constant specific heats. (3) The system is isolated, so Q=0 and W=0. (4) There is no energy stored in the partition. (5) Kinetic and potential energy effects are negligible.

ANALYSIS: To determine the final temperature, begin with the energy balance

$$\cancel{\Delta KE}^{0} + \cancel{\Delta PE}^{0} + \Delta U = \cancel{Q}^{0} - \cancel{W}^{0}$$

or, with $\Delta U = m_{air}\Delta u_{air} + m_{co_2}\Delta u_{co_2}$ and using Eq. 3.50

$$m_{air} c_{v,air}(T_2 - T_{1,air}) + m_{co_2} c_{v,co_2}(T_2 - T_{1,co_2}) = 0$$

Solving for T_2

$$T_2 = \frac{m_{air} c_{v,air} T_{1,air} + m_{co_2} c_{v,co_2} T_{1,co_2}}{m_{air} c_{v,air} + m_{co_2} c_{v,co_2}}$$

The specific heats are evaluated using data from Table A-20 at a mean temperature of 400K ; $c_{v,air}$ = 0.726 kJ/kg·K and c_{v,co_2} = 0.750 kJ/kg·K. Thus, the final temperature is

$$T_2 = \frac{(1\ kg)(0.726\ kJ/kg\cdot K)(350\ K) + (3)(.750)(450)}{(1\ kg)(0.726\ kJ/kg\cdot K) + (3)(.750)}$$

5 $$= 425.6\ K \leftarrow T_2$$

Next, to find the final pressure, the total volume is needed. The initial volume of the air is

$$V_{1,air} = \frac{m_{air} R_{air} T_{1,air}}{P_{1,air}}$$

$$= \frac{(1\ kg)\left(\frac{8.314}{28.97}\frac{kJ}{kg\cdot K}\right)(350\ K)}{(5\ bars)}\left(\frac{1\ bar}{10^5\ N/m^2}\right)\left(\frac{10^3\ N\cdot m}{1\ kJ}\right) = 0.201\ m^3$$

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PROBLEM #1 (Cont'd)

Similarly for the carbon dioxide

$$V_{1,CO_2} = \frac{m_{CO_2} R_{CO_2} T_{1,CO_2}}{P_{1,CO_2}} = \frac{(3)\left(\frac{8.314}{44.01}\right)(450)}{(2)}\left(\frac{10^3}{10^5}\right)$$

$$= 1.275 \text{ m}^3$$

Thus

$$V_{tot} = 0.201 \text{ m}^3 + 1.275 \text{ m}^3 = 1.476 \text{ m}^3$$

Since the container is rigid, V_{tot} is constant. At equilibrium

$$V_{tot} = V_{2,air} + V_{2,CO_2}$$

$$= \frac{m_{air} R_{air} T_2}{P_2} + \frac{m_{CO_2} R_{CO_2} T_2}{P_2}$$

$$= \frac{T_2}{P_2}\left(m_{air} R_{air} + m_{CO_2} R_{CO_2}\right)$$

Solving for P_2

$$P_2 = \frac{T_2}{V_{tot}}\left(m_{air} R_{air} + m_{CO_2} R_{CO_2}\right)$$

$$= \frac{(425.6 \text{ K})}{(1.476 \text{ m}^3)}\left[(1 \text{ kg})\left(\frac{8.314}{28.97} \frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right) + (3)\left(\frac{8.314}{44.01}\right)\right]\left(\frac{10^3 \text{N}\cdot\text{m}}{1 \text{ kJ}}\right)\left(\frac{1 \text{ bar}}{10^5 \text{ N/m}^2}\right)$$

5 $= 2.462$ bars $\leftarrow P_2$

COMMENT: The assumption of constant specific heats facilitates the determination of T_2. The assumption is reasonable for the relatively small temperature range involved in this problem.

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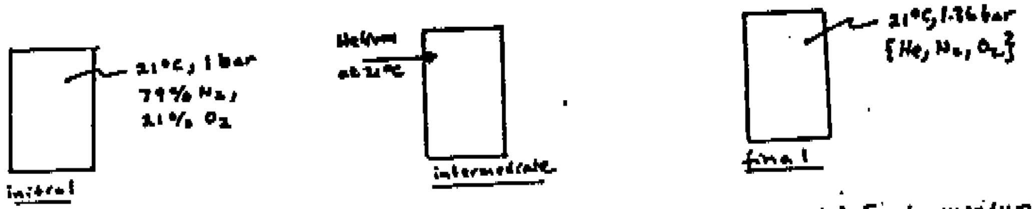
(#2)

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The overall mixture acts as an ideal gas. (2) Each mixture component behaves as if it were an ideal gas occupying the entire volume at the mixture temperature.

ANALYSIS: Using the ideal gas equation of state, $V = n\bar{R}T/p$. Since the total volume is constant: $V_1 = V_2$ (1)

$$n_1 \frac{\bar{R}T_1}{p_1} = n_2 \frac{\bar{R}T_2}{p_2}$$

where 1 denotes the initial total number of moles present, temperature, and pressure, and 2 denotes the final total number of moles present, temperature, and pressure. Since $T_1 = T_2$, Eq. (1) gives (2)

$$n_2 = \frac{p_2}{p_1} n_1$$

Initially

$$n_1 = \frac{p_1 V}{\bar{R}T_1} = \frac{(10^5 N/m^2)(0.028 m^3)}{(8314 \frac{N\cdot m}{kmol\cdot K})(294 K)}$$

$$= 0.00115 \text{ kmol (mixture)}$$

The amounts of N_2 and O_2 present are

$$n_{O_2} = y_{O_2} n_1 = (0.21)(0.00115) = 0.00024 \text{ kmol}$$

$$n_{N_2} = y_{N_2} n_1 = (0.79)(0.00115) = 0.00091 \text{ kmol}$$

(5)

and with Eq(2) the total amount of final mixture is

$$n_2 = \left(\frac{1.36 bar}{1 bar}\right)(0.00115 \text{ kmol}) = 0.00156 \text{ kmol (mixture)}$$

Then, since $n_2 = n_{O_2} + n_{N_2} + n_{He}$

$$n_{He} = 0.00156 - 0.00024 - 0.00091$$

$$= 0.00041 \text{ kmol}$$

With molecular weights from Table A-1

$$\begin{cases} m_{O_2} = n_{O_2} M_{O_2} = (0.00024)(32) = & 0.00768 \text{ kg} \\ m_{N_2} = n_{N_2} M_{N_2} = (0.00091)(28.01) = & 0.02549 \text{ kg} \\ m_{He} = n_{He} M_{He} = (0.00041)(4.003) = & 0.00164 \text{ kg} \end{cases}$$

(5)

(#5)

$$\bar{T}_b = 25°C = 77°F \quad \rho = 996 \quad \mu = 8.96 \times 10^{-4} \quad k = 0.611$$

$$Pr = 6.13 \quad c_p = 4180 \quad Re = \frac{(996)(8)(0.02)}{8.96 \times 10^{-4}} = 1.78 \times 10^5$$

(5)

$$h = (0.023)\frac{0.611}{0.02}(1.78 \times 10^5)^{0.8}(6.13)^{0.4} = 23{,}003 \ W/m^2\cdot°C$$

$$q = hA(\bar{T}_w - \bar{T}_b) = \dot{m} c_p \Delta T_b$$

$$(23003)(\pi)(0.02)(10)(\bar{T}_w - 25) = (996)(8)\pi\left(\frac{0.02}{2}\right)^2(4180)(10)$$

(5)

$$\bar{T}_w = 32.2°C$$

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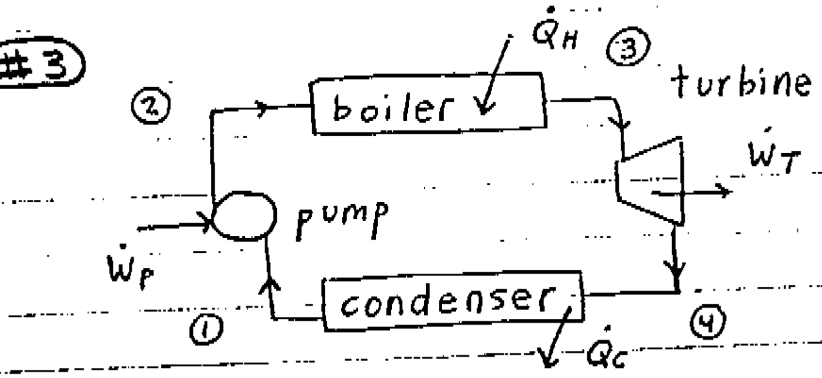
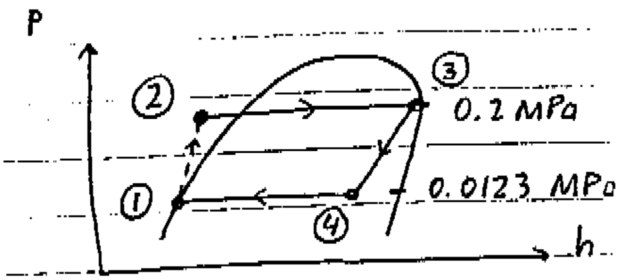
#3





Assumptions: (i) SSSF (ii) adiabatic pump, turbine

(a) $h_1 = 209.3$ kJ/kg $h_{g4} = 2592.1$ kJ/kg

$h_3 = 2706$ kJ/kg $h_4 = 0.94(2592) + 0.06(209)$

$h_{f4} = 209.3$ kJ/kg $= 2449$ kJ/kg

Turbine: $\dot{W}_T = \dot{m}(h_3 - h_4) = 140(2706 - 2449) = 35980$ kJ/h

③ Pump: $\dot{W}_P = 0.5\ hp = 1342$ kJ/h $= \dot{m}(h_2 - h_1)$

$\Rightarrow h_2 = 209.3 + 9.4 = 218.9$ kJ/kg

③ Boiler: $\dot{Q}_B = \dot{m}(h_3 - h_2) = 140(2706 - 218.9) = 348,190$ kJ/hr

③ $\Rightarrow \eta = \frac{\dot{W}_{net}}{\dot{Q}} = \frac{\dot{W}_T - \dot{W}_P}{\dot{Q}} = \frac{34,638}{348,190} \sim 0.10$

③ (b) No. The work input to the refrigerator would be greater than the additional work produced (i.e. lower resulting η). At best (reversible processes), the extra work output would equal new work input.

③ (c) No. The engine would violate the Kelvin-Planck statement of the Second Law of Thermodynamics.

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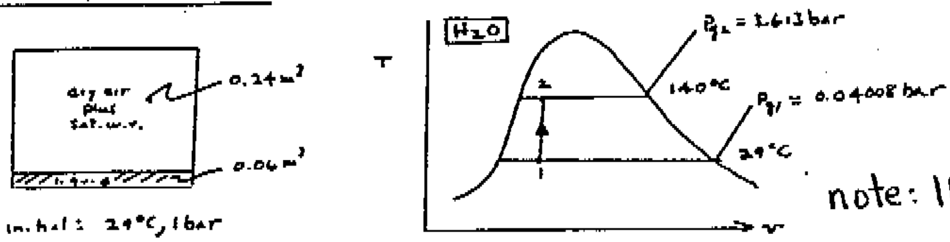
#4

KNOWN: A closed, rigid tank initially containing moist air in equilibrium with liquid water is heated.

FIND: Determine (a) the final pressure and (b) the heat transfer.

SCHEMATIC & GIVEN DATA:



note: 1bar = 100 kPa

ASSUMPTIONS: (1) The system consists of the tank contents. (2) The gas phase adheres to ideal gas principles. (3) Kinetic and potential energy effects are absent and $W = 0$.

ANALYSIS: Reducing an energy balance, $Q = \Delta U$. This requires the final state to be fixed. Since volume remains unchanged, the water present undergoes a constant specific volume process. Calculating the initial amounts of liquid and vapor are

$$m_{vap} = \frac{V_{vap}}{v_g(29°C)} = \frac{0.24\ m^3}{34.733\ m^3/kg} = 0.0069\ kg$$

$$m_{liq} = \frac{V_{liq}}{v_f(29°C)} = \frac{0.06}{1.004\times10^{-3}} = 59.761\ kg$$

$\Big\}\ m_{water} = 59.768\ kg.$

The final specific volume is then $v = (0.30\ m^3/59.768\ kg) = 5.019\times10^{-3}\ m^3/kg$. Accordingly, at the final state there is a two-phase liquid-vapor mixture with quality

$$x_2 = \frac{(5.019\times10^{-3}) - (1.0797\times10^{-3})}{0.5089 - (1.0797\times10^{-3})} = 0.0078 \quad \left(= \frac{m_{v2}}{m_{water}}\right)$$

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To evaluate ΔU, then,

$$U_1 = m_{liq}\,u_f(29°C) + m_{vap}\,u_g(29°C) + m_a u_a(29°C) \qquad (1)$$

$$U_2 = m_{water}\,[u_f(140°C) + x_2 u_{fg}(140°C)] + m_a u_a(140°C) \qquad (2)$$

This requires the mass of dry air. Since $P_1 = P_{a1} + P_{v1} = P_{a1} + P_{g1}$, $P_{a1} = (1\ bar) - (0.04008\ bar) = 0.9599$ bar. And

$$m_a = \frac{P_{a1} V_{gas}}{\bar{R}/M_a\, T_1} = \frac{(0.9599\times10^5\ N/m^2)(0.24\ m^3)}{\left(\frac{8314}{29}\frac{N\cdot m}{kg\cdot K}\right)(302\ K)} = 0.266\ kg$$

Returning to Eqs (1), (2), and combining with $Q = \Delta U$ and data from Tables A-2, A-4,

$$Q = m_{water}[u_f(140°C) + x_2 u_{fg}(140°C)] - [m_{liq} u_f(29°C) + m_{vap} u_g(29°C)] + m_a\underbrace{[u_a(140°C) - u_a(29°C)]}_{c_v \Delta T}$$

$$= 59.768[588.74 + 0.0078(1961.26)] - [59.761(121.6) + 0.0069(2415.2)] + 0.266\times0.718(140-29)$$

$$= 36{,}102 - 7284 + 21 = 28{,}839\ kJ \longleftarrow Q$$

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At the final state, $\omega_2 = m_{v2}/m_a = [(0.0078)(59.768)]/0.266 = 1.7523$. Then, an rearrangement of Eq. 13.8

$$P_2 = P_{v2}\left[1 + \frac{0.622}{\omega_2}\right] = P_{g2}\left[1 + \frac{0.622}{\omega_2}\right] = 3.613\left[1 + \frac{0.622}{1.7523}\right] = 4.895\ bar \longleftarrow P_2$$

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