

# STAT 2220 – Sample Final Exam Questions

## Part A

1. We take a sample of cars driving down a highway. Which of the following pairs of variables will likely have a positive correlation?
- (A)  $X$  = age of the car,  $Y$  = value of the car (in \$)
  - (B)  $X$  = amount of gas in car's gas tank,  $Y$  = amount of money it would cost to fill the tank with gas
  - (C)  $X$  = age of the driver,  $Y$  = speed of the vehicle
  - (D)  $X$  = year the car was made,  $Y$  = total distance the car has been driven (in km)
  - (E) none of the above

2. Test scores for two different sections of a course (A01 and A02) are displayed in the back-to-back stemplot shown below:

| <u>A01</u>      |   | <u>A02</u>  |
|-----------------|---|-------------|
| 7 1             | 3 | 5 8         |
| 8 4             | 4 | 4 6 9       |
| 9 7 0           | 5 | 5 8         |
| 5 5 4           | 6 | 0 3 6 7     |
| 9 6 3 2 0       | 7 | 2 4 4 4 5 8 |
| 9 8 7 5 4 1 0 0 | 8 | 2 7 9       |
| 7 7 6 5 3 1     | 9 | 2 3 6 9     |

Which of the following statements is/are **true**?

- (I) The distribution of scores for A01 is skewed to the right.
- (II) The range of scores for A01 is 60.
- (III) The median score for A02 is 73.

(A) I only      (B) II only      (C) III only      (D) I and III only      (E) II and III only

3. The horsepowers of a sample of five automobiles are shown below:

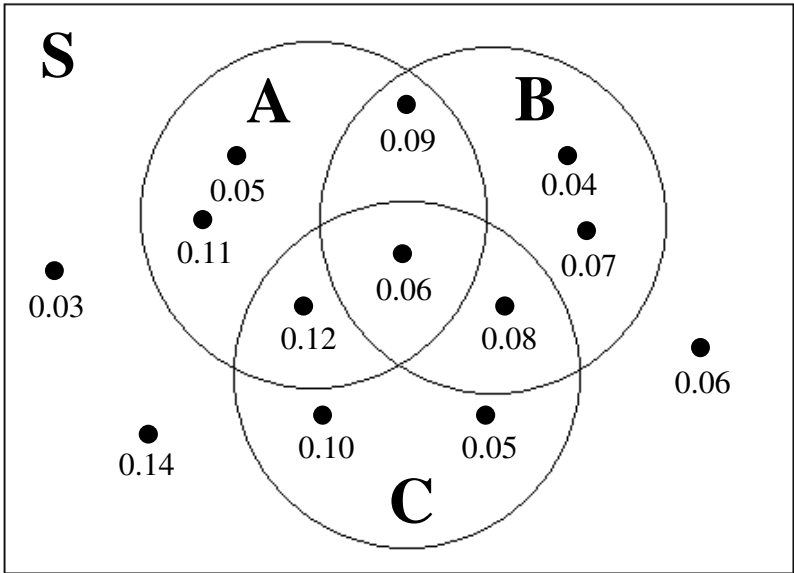
85      110      97      126      102

What is the variance of horsepowers for this sample of automobiles?

(A) 233.5      (B) 13.7      (C) 62.8      (D) 186.8      (E) 15.3

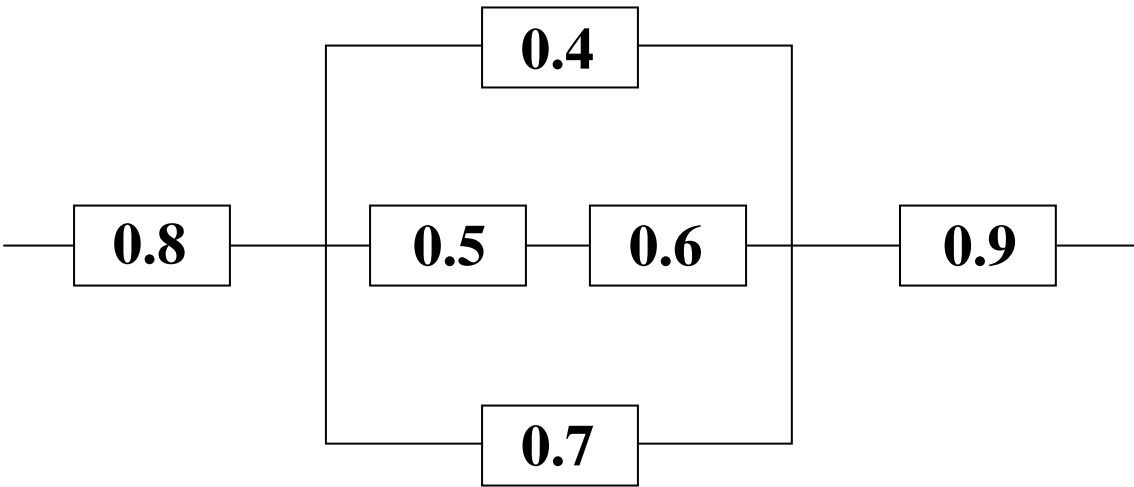
4. An engineer would like to investigate the surface charge on a silicon wafer. Specifically, she would like to determine how the surface charge varies, depending on the cleaning method (rinse dry or spin dry) and the position on the wafer where the charge was measured (left, center or right). She has 30 wafers available for the experiment. What is (are) the factor(s) in this experiment?
- (A) type of wafer
  - (B) surface charge
  - (C) cleaning method and position
  - (D) rinse dry, spin dry, left, right, center
  - (E) rinse dry/left, rinse dry/center, rinse dry/right, spin dry/left, spin dry/center, spin dry/right

5. Suppose it is known that 61% of Winnipeg adults read the *Winnipeg Free Press*, 29% read the *Winnipeg Sun* and 18% read both newspapers. What is the probability that a randomly selected adult in Winnipeg reads only one of the two papers (but not both)?
- (A) 0.54                      (B) 0.50                      (C) 0.45                      (D) 0.72                      (E) 0.47
6. Consider the sample space and events shown in the Venn diagram below:



What is  $P(A \cap B' | C')$ ?

- (A) 0.30                      (B) 0.27                      (C) 0.48                      (D) 0.35                      (E) 0.21
7. Consider the following system, where the values inside the boxes represent the component reliabilities:



What is the reliability of this system?

- (A) 0.6048                      (B) 0.6387                      (C) 0.6195                      (D) 0.6444                      (E) 0.6293

The next **two** questions (**8 and 9**) refer to the following:

The amount of gravel  $X$  (in tons) sold by a particular construction supply company in a given week follows a continuous distribution with p.d.f.

$$f(x) = \begin{cases} \frac{3}{38}\sqrt{x}, & 4 \leq x \leq 9 \\ 0, & \text{otherwise} \end{cases}$$

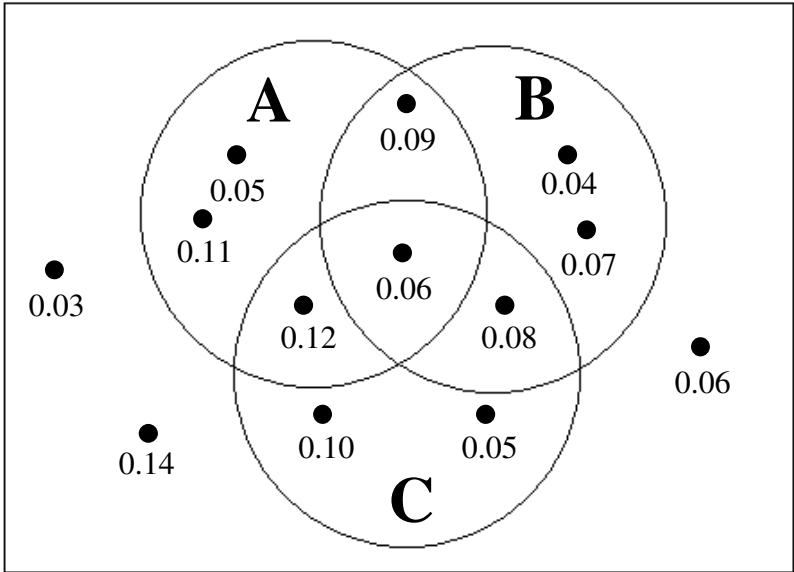
8. What is the expected value of  $X$ ?
- (A) 6.357      (B) 5.942      (C) 7.018      (D) 6.663      (E) 6.854
9. What is the median of the distribution of  $X$ ?
- (A) 6.441      (B) 6.537      (C) 6.675      (D) 6.740      (E) 6.882
10. When an archer shoots her arrow, she hits the bullseye on the target 78% of the time. It is also known that her shots are independent. If she shoots 15 arrows, what is the probability that she hits the bullseye exactly ten times?
- (A) 0.1290      (B) 0.1390      (C) 0.1490      (D) 0.1590      (E) 0.1690
11. The probability that a certain machine will produce a defective item is  $1/4$ . If a random sample of six items is taken from the output of this machine, what is the probability that there will be at least five defectives in the sample?
- (A)  $\frac{1}{4096}$       (B)  $\frac{3}{4096}$       (C)  $\frac{4}{4096}$       (D)  $\frac{18}{4096}$       (E)  $\frac{19}{4096}$

The next **two** questions (**12 and 13**) refer to the following:

Goals are scored in a hockey game according a Poisson process with rate 0.1 per minute.

12. What is the probability that it takes between 10 and 15 minutes for the first goal to be scored?
- (A) 0.1117      (B) 0.1227      (C) 0.1337      (D) 0.1447      (E) 0.1557
13. What is the probability that a total of seven goals are scored in the 60 minute game?
- (A) 0.1177      (B) 0.1277      (C) 0.1377      (D) 0.1477      (E) 0.1577

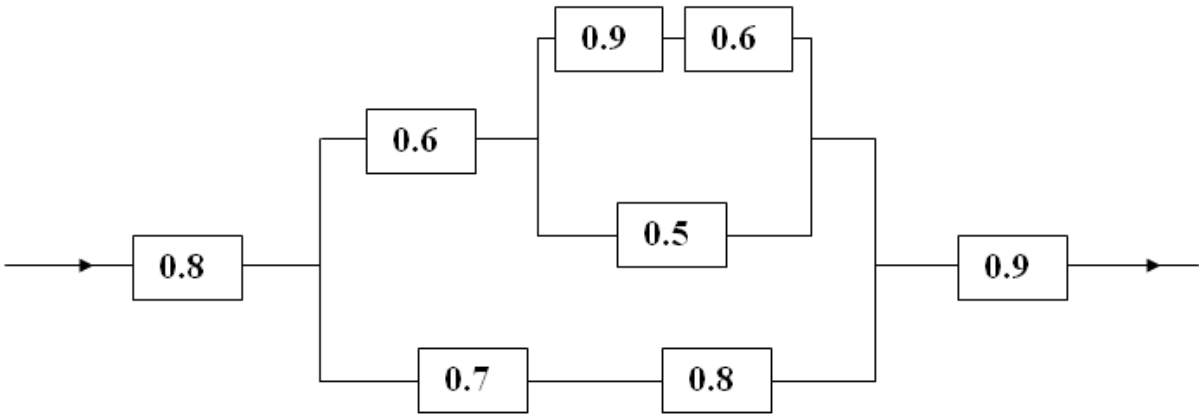
14. Consider the sample space and events shown in the Venn diagram below.



What is  $P(A'|B \cup C)$ ?

- (A) 0.5574      (B) 0.5774      (C) 0.5974      (D) 0.6174      (E) 0.6374

15. Consider the following system, where the value in each box represents the reliability of that component:



What is the reliability of the system?

- (A) 0.5196      (B) 0.5296      (C) 0.5396      (D) 0.5496      (E) 0.5596

16. The p.d.f. for some continuous random variable X is

$$f(x) = \begin{cases} \frac{3}{2}(1 - x^2), & 0 \leq x \leq 1 \\ 0, & \text{otherwise} \end{cases}$$

What is the expected value of X?

- (A)  $\frac{3}{8}$       (B)  $\frac{2}{3}$       (C)  $\frac{1}{4}$       (D)  $\frac{5}{8}$       (E)  $\frac{1}{3}$

The next **two** questions (**17 and 18**) refer to the following:

A machine automatically fills bottles with dish soap. The amount of soap per bottle follows a normal distribution with mean 828 ml and standard deviation 4 ml.

17. What is the probability that a random sample of 10 bottles of dish soap contains a mean amount greater than 830 ml?
- (A) 0.3085      (B) 0.1539      (C) 0.9429      (D) 0.2296      (E) 0.0571
18. What amount should be placed on the label of the bottles so that only 4% of bottles contain less than the amount on the label?
- (A) 820 ml      (B) 821 ml      (C) 822 ml      (D) 835 ml      (E) 836 ml

The next two questions (**19 and 20**) refer to the following:

19. The time  $X$  taken by a cashier in a grocery store express lane follows a normal distribution with mean 90 seconds and standard deviation 20 seconds.
- What is the first quartile ( $Q_1$ ) of the distribution of  $X$ ?
- (A) 73.8 seconds  
(B) 85.0 seconds  
(C) 69.4 seconds  
(D) 81.2 seconds  
(E) 76.6 seconds
20. What is the probability that the average service time for the next three customers is between 80 and 100 seconds? (Assume the next three customers can be considered a simple random sample.)
- (A) 0.6156  
(B) 0.4893  
(C) 0.7212  
(D) 0.5559  
(E) impossible to calculate with the information given

The next two questions (**21 and 22**) refer to the following:

The amount  $X$  spent (in \$) by customers in the grocery store express lane follow some right-skewed distribution with mean \$24 and standard deviation \$15.

21. What is the probability that the average amount spent by the next three customers is more than \$20? (Assume the next three customers can be considered a simple random sample.)
- (A) 0.4619  
(B) 0.6772  
(C) 0.8186  
(D) 0.7673  
(E) impossible to calculate with the information given
22. What is the probability that the next 40 customers spend less than \$1,000 in total? (Assume the next 40 customers can be considered a simple random sample).
- (A) 0.5199      (B) 0.6064      (C) 0.6628      (D) 0.5784      (E) 0.6331

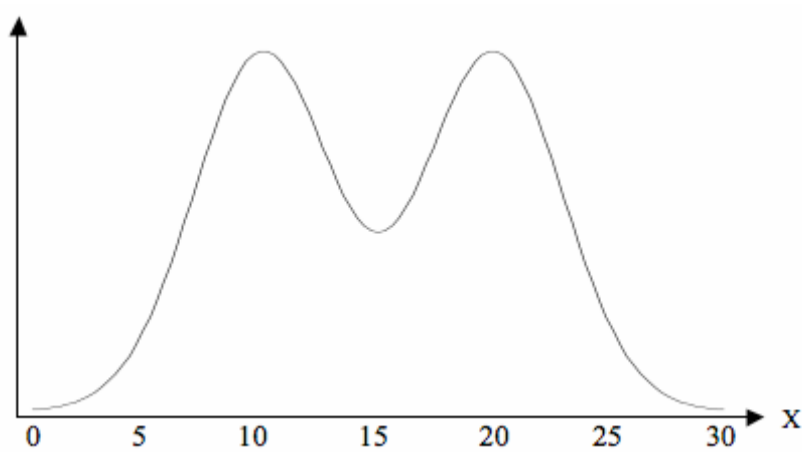
23. An economist calculates that, in order to estimate the true mean amount spent by Canadians on Christmas presents to within \$60 with 95% confidence, she requires a sample of 90 Canadians. What sample size would be required to estimate the true mean amount spent by Canadians on Christmas presents to within \$20 with 95% confidence?

(A) 10                      (B) 30                      (C) 156                      (D) 270                      (E) 810

24. Weights of Granny Smith apples sold at a supermarket follow a normal distribution with mean 140 grams and standard deviation 20 grams. If you select a random sample of ten apples, what is the probability that exactly four of them weigh less than 125 grams?

(A) 0.0985                      (B) 0.1085                      (C) 0.1185                      (D) 0.1285                      (E) 0.1385

25. A bimodal probability distribution is one with two distinct peaks. A random variable  $X$  follows a bimodal distribution with mean 15 and standard deviation 4, as shown below:



We will take a random sample of 10,000 observations from the population above and calculate the sample mean  $\bar{x}$ . The sampling distribution of  $\bar{X}$  is:

- (A) approximately normal with mean close to 15 and standard deviation 0.0004.  
(B) bimodal with mean close to 15 and standard deviation 0.04.  
(C) approximately normal with mean close to 15 and standard deviation 0.04.  
(D) bimodal with mean close to 15 and standard deviation 4.  
(E) approximately normal with mean close to 15 and standard deviation 4.
26. A man accused of committing a crime is taking a polygraph (lie detector) test. The polygraph is essentially testing the hypotheses

$H_0$ : The man is telling the truth. vs.  $H_a$ : The man is lying.

Suppose we use a 5% level of significance. Based on the man's responses to the questions asked, the polygraph determines a P-value of 0.03. We conclude that:

- (A) There is insufficient evidence that the man is telling the truth.  
(B) There is sufficient evidence that the man is telling the truth.  
(C) There is sufficient evidence that the man is lying.  
(D) The probability that the man is lying is 0.03.  
(E) The probability that the man is telling the truth is 0.03.

27. City engineers in Hamilton, Ontario would like to estimate the true mean commuting distance of all workers in the city between home and their principal place of business. They calculate that, in order to estimate this mean to within  $\pm 1$  kilometer with 99% confidence, they require a sample of 200 people. What sample size would be required to estimate the true mean commuting distance for all workers in Hamilton to within  $\pm 2.5$  kilometers with 99% confidence?
- (A) 32                      (B) 80                      (C) 127                      (D) 500                      (E) 1250
28. We would like to determine whether the true mean systolic blood pressure  $\mu$  of healthy adults differs from 120. We obtain a sample of healthy adults and conduct an appropriate hypothesis test, which results in a P-value of 0.021. Which of the following statements is true?
- (I) A 96% confidence interval for  $\mu$  **would** contain the value 120.  
 (II) A 98% confidence interval for  $\mu$  **would** contain the value 120.  
 (III) A 99% confidence interval for  $\mu$  **would not** contain the value 120.
- (I) I only                  (B) II only                  (C) III only                  (D) I and II only                  (E) II and III only
29. We take a simple random sample of 16 adults and ask them how long they sleep on a typical night. The sample mean is calculated to be 7.2 hours and the sample standard deviation is 1.8 hours. We would like to conduct a test of significance to determine whether there is evidence that the true mean time adults sleep at night differs from 8 hours. The P-value for the appropriate hypothesis test is:
- (A) between 0.01 and 0.02.  
 (B) between 0.02 and 0.04.  
 (C) between 0.025 and 0.05.  
 (D) between 0.05 and 0.10.  
 (E) between 0.10 and 0.20.
30. A random variable  $X$  follows a normal distribution with known standard deviation  $\sigma$ . We would like to construct a confidence interval for the true mean  $\mu$  of the distribution of  $X$ . For which of the following combinations of sample size and confidence level would the confidence interval be the narrowest?
- (A) 96% confidence level with  $n = 25$   
 (B) 96% confidence level with  $n = 100$   
 (C) 98% confidence level with  $n = 25$   
 (D) 98% confidence level with  $n = 100$   
 (E) depends on the value of  $\sigma$ .
31. We would like to conduct a hypothesis test to examine whether there is evidence that the true mean amount spent on textbooks by a U of M student in one semester is greater than \$400. A random sample of 30 students is selected. The mean amount spent on textbooks for one semester for this sample is calculated to be \$450 and the sample standard deviation is calculated to be \$160. At the 5% level of significance, we conclude that we should:
- (A) reject  $H_0$ , since the P-value is between 0.01 and 0.02.  
 (B) fail to reject  $H_0$ , since the P-value is between 0.025 and 0.05.  
 (C) reject  $H_0$ , since the P-value is between 0.025 and 0.05.  
 (D) fail to reject  $H_0$ , since the P-value is between 0.05 and 0.10.  
 (E) reject  $H_0$ , since the P-value is between 0.05 and 0.10.

32. A study was conducted to examine the lifetimes of a certain brand of 60-watt light bulbs. The lifetimes (in hours) for a sample of 10 light bulbs are shown below. The study lasted 1,000 hours, and the lifetime of a light bulb that was still burning after 1,000 hours is denoted as 1,000<sup>+</sup>.

582    1,000+    391    752    872    494    1,000+    737    273    558

What is the interquartile range of lifetimes for this sample of light bulbs?

- (A) 302 hours  
(B) 346 hours  
(C) 358 hours  
(D) 378 hours  
(E) impossible to determine without the exact lifetimes of all light bulbs in the sample
33. We would like to make a histogram (with vertical bars) of the final exam scores for all students in all sections of a course who received a midterm score of 80% or higher. The horizontal and vertical axes represent, respectively:
- (A) final exam score and frequency.  
(B) midterm exam score and frequency.  
(C) final exam score and midterm exam score.  
(D) midterm exam score and final exam score.  
(E) section and final exam score.

The next **two** questions (**3 and 4**) refer to the following:

A coffee shop manager would like to determine how the outside temperature (in °C) affects the sales of hot chocolate (in \$). She records both variables for a sample of ten days. The least squares regression line is calculated to be  $\hat{y} = 45 - 1.42x$ . It is also reported that 41% of the variation in Y can be accounted for by its regression on X.

34. What is the correct interpretation of the slope of the least squares regression line?
- (A) When the temperature increases by  $1.42^{\circ}\text{C}$ , the predicted sales of hot chocolate decrease by \$1.  
(B) When the sales of hot chocolate increase by \$1, the predicted temperature decreases by  $1.42^{\circ}\text{C}$ .  
(C) When the temperature increases by  $1^{\circ}\text{C}$ , the predicted sales of hot chocolate increase by \$1.42.  
(D) When the temperature increases by  $1^{\circ}\text{C}$ , the predicted sales of hot chocolate decrease by \$1.42.  
(E) When the sales of hot chocolate increase by \$1.42, the predicted temperature decreases by  $1^{\circ}\text{C}$ .
35. What is the value of the correlation between hot chocolate sales and temperature?
- (A)  $-0.17$                       (B)  $0.64$                       (C)  $-0.41$                       (D)  $-0.64$                       (E)  $0.41$



The next **two** questions (**36 and 37**) refer to the following:

A dog food manufacturer would like to examine the effect of recipe and portion size on the health of young puppies. Two hundred puppies (100 German shepherds and 100 Labrador retrievers) are available for the experiment. Each dog will be randomly assigned to be fed a diet of one of two recipes (A or B) in portion sizes of either two cups or three cups a day. After three months, each puppy will be examined by a veterinarian to assess their health.

36. What is (are) the factor(s) in this experiment?
- (A) health of the puppies
  - (B) breed of dog
  - (C) recipe and portion size
  - (D) Recipe A, Recipe B, two cups a day, three cups a day
  - (E) two cups of Recipe A, three cups of Recipe A, two cups of Recipe B, three cups of Recipe B
37. Which of the following statements is false?
- (A) The response variable in this experiment is the health of the puppies.
  - (B) It would be appropriate to use a matched pairs design for this experiment, where each pair consists of one German shepherd and one Labrador retriever.
  - (C) If the experimenters believe the two breeds of dog will respond differently to the treatments, then a randomized block design would be appropriate.
  - (D) If all 100 German shepherds were given Recipe A and all 100 Labrador retrievers were given Recipe B, then recipe would be confounded with dog breed.
  - (E) If we want each treatment to be assigned to the same number of dogs, then 50 dogs will receive each treatment.
38. Which of the following normally distributed variables has the density curve with the highest peak?
- (A)  $X_1 \sim N(53, 18)$
  - (B)  $X_2 \sim N(57, 14)$
  - (C)  $X_3 \sim N(55, 12)$
  - (D)  $X_4 \sim N(51, 16)$
  - (E) All four curves have the same height.
39. We would like to estimate the true mean amount (in \$) students at a large university spend on textbooks in one semester. We record the textbook costs for a simple random sample of 30 students and we calculate a 95% confidence interval for  $\mu$  to be (355, 400), i.e., the length of the interval is 45. Suppose we had instead selected a simple random sample of 90 students and calculated a 95% confidence interval for  $\mu$ . What would be the length of this interval?
- (A) 25.98                      (B) 5.00                      (C) 15.00                      (D) 12.99                      (E) 77.94
40. A major car manufacturer wants to test a new engine to determine if it meets new air pollution standards. Safety regulations require that the mean emission of all engines of this type must be no greater than 20 parts per million (ppm) of carbon. If it is higher than that, they will have to redesign parts of the engine. A random sample of 16 engines is tested and the emission level of each is determined. The sample mean is calculated to be 20.4 ppm and the sample standard deviation is 1.2 ppm. It is known that emission levels are normally distributed with standard deviation 1.6 ppm. We would like to test whether the true mean emission level for the new engine is greater than 20 ppm. The test statistic for the appropriate test of significance is:
- (A)  $z = 1.00$                       (B)  $t = 1.00$                       (C)  $z = 1.33$                       (D)  $t = 1.33$                       (E)  $z = 0.25$

41. Do snow tires help vehicles stop more quickly in winter driving conditions? A sample of 30 vehicles is outfitted with snow tires. The vehicles travel 90 km/h in winter driving conditions and apply the brakes. The sample mean stopping distance for these vehicles is 162 meters and the sample standard deviation is 35 meters. We would like to test whether the true mean stopping distance is greater than 150 meters. At the 10% level of significance, we should:
- (A) reject  $H_0$  since the P-value is between 0.02 and 0.025.
  - (B) fail to reject  $H_0$  since the P-value is between 0.025 and 0.05.
  - (C) reject  $H_0$  since the P-value is between 0.025 and 0.05.
  - (D) fail to reject  $H_0$  since the P-value is between 0.05 and 0.10.
  - (E) reject  $H_0$  since the P-value is between 0.05 and 0.10.

42. The number of trees in a forest follows a Poisson distribution with a rate of 0.25 per meter squared. If we randomly select a circular area of forest, where the radius of the circle is 3 meters, what is the probability there are exactly five trees in the circle? (Area of a circle =  $\pi r^2$ )

(A) 0.0736      (B) 0.1252      (C) 0.1867      (D) 0.2425      (E) 0.2739

The next **two** questions (**43 and 44**) refer to the following:

The contents of bottles of a certain brand of beer are known to follow a normal distribution with mean 341 ml and standard deviation 3 ml.

43. If you buy a six-pack of beer, what is the probability that the average contents of the bottles is between 339 and 342 ml? (Assume that the six bottles can be considered as a simple random sample.)

(A) 0.7423      (B) 0.6802      (C) 0.8029      (D) 0.7817      (E) 0.6381

44. If you buy a case of 24 bottles of beer, what is the probability that the bottles contain more than 8.2 liters of beer in total? (Assume that the 24 bottles can be considered as a simple random sample.)

(A) 0.1379      (B) 0.4207      (C) 0.2578      (D) 0.8621      (E) 0.3594

45. The Central Limit Theorem states that:

- (A) when  $n$  gets large, the sample mean  $\bar{X}$  gets closer and closer to the population mean  $\mu$ .
- (B) when  $n$  gets large, the sample mean  $\bar{X}$  becomes an unbiased estimator of the population mean  $\mu$ .
- (C) when  $n$  gets large, the standard deviation of the sample mean  $\bar{X}$  gets closer and closer to  $\sigma/\sqrt{n}$ .
- (D) if a random variable  $X$  follows a normal distribution, then when  $n$  gets large, the sampling distribution of  $\bar{X}$  is exactly normal.
- (E) regardless of the population distribution of a random variable  $X$ , when  $n$  gets large, the sampling distribution of  $\bar{X}$  is approximately normal.

The next **two** questions (**46 and 47**) refer to the following:

Bags of sugar are filled automatically by a machine in a factory. Weights of sugar in the bags are known to follow a normal distribution with mean 505 grams and standard deviation 4 grams.

46. What is the probability that a randomly selected bag contains between 500 and 508 grams of sugar?

(A) 0.6547      (B) 0.6032      (C) 0.6954      (D) 0.6678      (E) 0.6322

47. What weight should be placed on the label of the bags so that only 4% of bags are underweight?

(A) 497 grams      (B) 498 grams      (C) 499 grams      (D) 500 grams      (E) 501 grams

48. Suppose that a certain mechanical component produced by a company has a width that is normally distributed with a mean of 499.6 mm and a standard deviation of 0.15 mm. In an assembly procedure, two of these components need to be fitted side-by-side into a slot in another part. The widths of the slots are normally distributed with a mean of 1000 mm and a standard deviation of 0.25 mm. What is the probability that two randomly selected components can fit into a randomly selected slot?
- (A) 0.9834            (B) 0.9875            (C) 0.9927            (D) 0.9979            (E) 0.9996
49. Speeds of vehicles on a highway follow some right-skewed distribution with mean 105 km/h and standard deviation 19 km/h. What is the probability that a random sample of 40 cars on this highway have a mean speed less than 100 km/h?
- (A) 0.0228  
(B) 0.0485  
(C) 0.0307  
(D) 0.0132  
(E) impossible to calculate without the exact form of the population distribution.

The next **two** questions (**50 and 51**) refer to the following:

We take a simple random sample of 16 adults and ask them how long they sleep on a typical night. The sample mean is calculated to be 7.2 hours. Suppose it is known that the number of hours adults sleep at night follows a normal distribution with standard deviation 1.8 hours.

50. An 88% confidence interval for the true mean time adults sleep at night is:
- (A) (6.80, 7.60)  
(B) (6.30, 8.10)  
(C) (6.60, 7.80)  
(D) (6.70, 7.70)  
(E) (6.50, 7.90)
51. We would like to conduct a test of significance to determine whether there is evidence that the true mean time adults sleep at night differs from 8 hours. The P-value for the appropriate hypothesis test of  $H_0: \mu = 8$  vs.  $H_a: \mu \neq 8$  is:
- (A) 0.0129            (B) 0.0750            (C) 0.0918            (D) 0.0375            (E) 0.0516
52. Percentage grades in a Physics course follow a normal distribution with mean 67.5 and standard deviation 9.0. The professor decides to assign a grade of A+ to the top 10% of students, a grade of A to the next 15% of students, and a grade of B+ to the next 12% of students. What is the minimum grade required to receive a grade of B+?
- (A) 70.5            (B) 72            (C) 73.5            (D) 75            (E) 76.5

The next **three** questions (**53, 54 and 55**) refer to the following:

A 4x100 meter relay team consists of four runners. During the race, each runner runs 100 meters, one after the other, and the team that finishes first wins. From past experience, the four runners on a team know that their 100-meter race times follow normal distributions with the following means and standard deviations (in seconds):

|          | $\mu$ | $\sigma$ |
|----------|-------|----------|
| Runner 1 | 10.32 | 0.15     |
| Runner 2 | 10.54 | 0.12     |
| Runner 3 | 10.48 | 0.17     |
| Runner 4 | 10.22 | 0.08     |

It is known that the race times of the four runners are independent.

53. What is the probability that Runner 3 takes between 10.3 and 10.6 seconds to run 100 meters?  
(A) 0.6165      (B) 0.7244      (C) 0.6540      (D) 0.7504      (E) 0.6994
54. What is the probability that the team’s total time is less than 41 seconds?  
(A) 0.0034      (B) 0.0188      (C) 0.0336      (D) 0.0427      (E) 0.0582
55. What is the probability that Runner 4 has a faster time than Runner 1?  
(A) 0.5882      (B) 0.6179      (C) 0.6808      (D) 0.7224      (E) 0.7995

## **Part B**

1.
  - (a) Briefly explain the difference between two events being mutually exclusive (disjoint) and two events being independent.
  - (b) Briefly explain the difference between a discrete random variable and a continuous random variable.
  
2. Hat # 1 contains four gold coins, one silver coin and five copper coins. Hat # 2 contains three gold coins, six silver coins and one copper coin.
  - (a) You will randomly select one coin from each of the two hats. The outcome of interest is the colour of each of the selected coins. Give the complete sample space of possible outcomes, and calculate the probability of each outcome.
  - (b) Let  $X$  be the number of gold coins that are chosen. Find the probability mass function of  $X$ .
  - (c) Find the expected value and variance of  $X$ .
  - (d) If you randomly select coins from Hat # 1 with replacement, what is the probability that you will select your third gold coin on the 15<sup>th</sup> draw?
  - (e) If you randomly select five coins from Hat # 1 without replacement, what is the probability that exactly three of them will be gold?
  
3. Suppose it is known that service times (in minutes) at a Tim Hortons drive-through follow an exponential distribution with parameter  $\lambda = 1.25$ .
  - (a) What is the probability that it takes less than one minute to serve the next customer?
  - (b) Explain the Central Limit Theorem.
  - (c) Assuming there is always a car waiting in line, what is the approximate probability that the total service time for the next 40 customers is less than 30 minutes? (Recall that the mean and standard deviation of an exponential random variable are both  $1/\lambda$ , so the mean service time in this drive through is  $1/1.25 = 0.8$  minutes and the standard deviation is also 0.8 minutes.)
  
4. We would like to estimate the true mean size  $\mu$  (in square feet) of all two-bedroom apartments in Winnipeg. A random sample of 30 two-bedroom apartments in the city is selected, and the mean size of these apartments is calculated to be 1000 square feet. Suppose it is known that sizes of two-bedroom apartments in the city follow a normal distribution with standard deviation 200 square feet.
  - (a) Calculate a 93% confidence interval for the true mean size of all two-bedroom apartments in Winnipeg.
  - (b) Interpret the meaning of the interval you calculated in (a) to someone with little or no background in statistics.

5. Many consumers pay careful attention to the nutritional contents on packaged foods when making purchases. It is therefore important that the information on the package is accurate. A random sample of eight frozen dinners of a certain type was selected from production and the calorie content of each one was determined. The frozen dinners in the sample had a mean calorie content of 247 and a standard deviation of 10. Assume that calorie contents are normally distributed.
- Calculate a 95% confidence interval for the true mean number of calories for this type of frozen dinner.
  - Interpret the confidence interval you calculated in (a) to someone with little or no background in statistics.
  - Conduct an appropriate hypothesis test at the 5% level of significance, to determine whether there is evidence that the true mean calorie content for this type of frozen dinner differs from the amount on the label, which is 240 calories.
  - Interpret the P-value you calculated in (d) to someone with little or no background in statistics.
  - Could you have used the confidence interval in (a) to conduct the hypothesis test in (c)? Why or why not? If you could have used the interval, what would your conclusion be, and why?

6. When the NDP formed the provincial government in 1999, the mean wait time for a particular type of surgery was 60 days. Public health officials take a sample of 30 individuals who have had the surgery in 2011 and record the number of days the patients had to wait prior to having surgery. The data are as follows:

|    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |
|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|
| 8  | 13 | 17 | 22 | 25 | 30 | 36 | 39 | 40 | 40 | 42 | 51 | 53 | 56 | 57 |
| 60 | 64 | 64 | 66 | 68 | 68 | 71 | 73 | 75 | 75 | 76 | 79 | 80 | 86 | 88 |

The sample mean is calculated to be  $\bar{x} = 54.07$ . Suppose the population standard deviation of wait times for this type of surgery is known to be 20 days.

- Create a stemplot for this data set. What is the shape of the distribution of wait times?
- It is clear from the stemplot in (a) that wait times do not follow a normal distribution. Why is it nevertheless appropriate to use inference methods which rely on the assumption of normality?
- Conduct an appropriate hypothesis test at the 1% level of significance to determine whether the true mean wait time has decreased since the NDP formed government.
- Interpret the P-value of the test in (c).