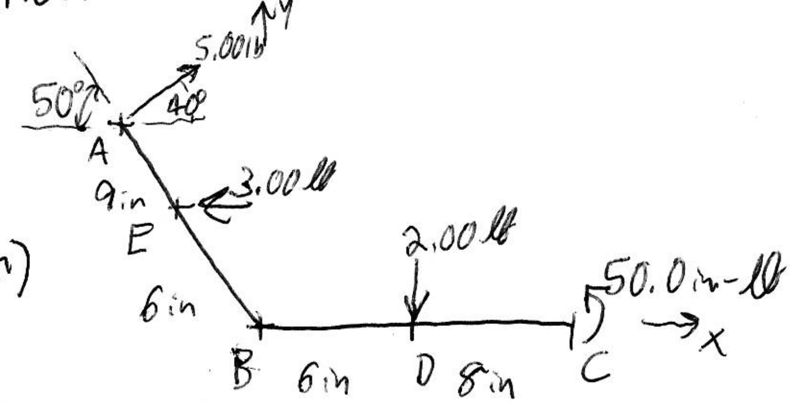
Problem set #3 Solutions



1a:

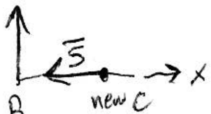
$$\Sigma \bar{F}_{body} = 5.00\,lb(\cos 40°\hat{i} + \sin 40°\hat{j}) - 3.00\,lb\,\hat{i} - 2.00\,lb\,\hat{j}$$

$$\boxed{= 0.83\,lb\,\hat{i} + 1.214\,\hat{j}}$$

(b): at B $\Sigma \bar{F}_{body} = 0.83022\,lb\,\hat{i} + 1.21394\,lb\,\hat{j}$

$$\Sigma \bar{M}_B = \underset{(C)}{50.0\,in\text{-}lb} - \underset{D}{2.00\,lb\,(6\,in)} + \underset{E}{3.00\,lb\,(6\,in)\sin(50°)} - 5.00\,lb\,\sin(50°+40°)$$

$$= -23.211\,in\cdot lb$$



$$\Sigma M_{new} = 0 = \bar{M}_B + \bar{s}\times\bar{F}_{body} = -23.211\,in\cdot lb + \overset{F_{body,y}}{1.21394\,lb}(sx)$$

$$-\frac{23.211\,in\cdot lb}{1.21394\,lb} = sx \rightarrow \boxed{19.1\text{ in to } \cancel{\text{right}}\ \text{left of B}}$$

(C) $\bar{M}_{new} = \bar{M}_B + \bar{s}\times\bar{F}$

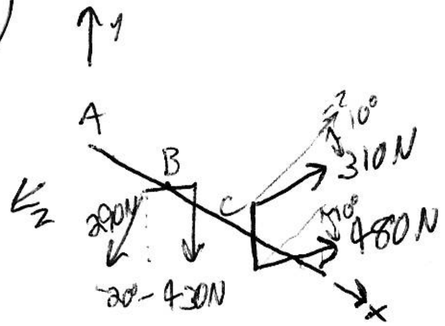
$$= 23.211\,in\cdot lb\,\hat{k} + \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ s\cos(-50°) & s\sin(-50°) & 0 \\ 0.83022\,lb & 1.21394\,lb & 0 \end{vmatrix} = 0$$

$$23.211\,in\,lb\,\hat{k} = s\cos(-50°)(1.21394\,lb) - s\sin(50°)(0.83022\,lb)$$

$$= s(.7803\,lb + .635985\,lb)$$

$\rightarrow s = 16.4$ in, so the point is 16.4 in to the ~~upper left~~ lower right of A

(2)



at B: $\Sigma \bar{F}_B = 430N(-\hat{j}) + 290N\cos 20°(-\hat{j}) + 290N\sin 20° \hat{k}$

$= -702.51N\hat{j} + 99.186N\hat{k}$

$\Sigma \bar{M}_B = [290N(150mm) - 430N\ 150mm]\hat{i}$

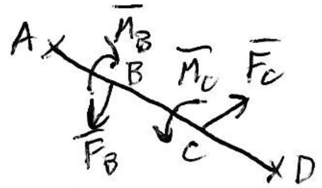
$= -21.0\ N\cdot m\ \hat{i}$

at C: $\Sigma \bar{F}_c = (310N + 480N)[\sin 10°(-\hat{j}) + \cos 10°(-\hat{k})]$

$= -137.182N\hat{j} - 777.998N\hat{k}$

concentrated B,C so:

at A:



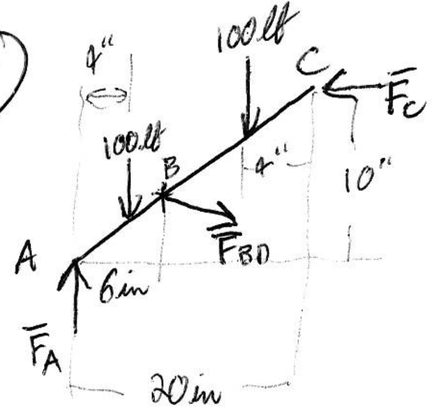
$\bar{F}_A = \bar{F}_B + \bar{F}_C = -702.51N\hat{j} + 99.186N\hat{k} - 137.182N\hat{j} - 777.998N\hat{k}$

$$\boxed{\bar{F}_A = -840\ N\hat{j} - 679\ N\hat{k}}$$

$$\bar{M}_A = \bar{M}_B + \bar{M}_C + \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 450mm & 0 & 0 \\ 0 & -702.51N & 99.186N \end{vmatrix} + \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 900mm & 0 & 0 \\ 0 & -137.182N & -777.998N \end{vmatrix}$$

$$\boxed{\bar{M}_A = 4.5N\cdot m\ \hat{i} + 656\ N\cdot m\ \hat{j} - 440\ N\cdot m\ \hat{k}}$$

(3)



$$\bar{U}_{BD} = \frac{14\,in}{\sqrt{205\,in^2}}\,\hat{i} - \frac{3\,in}{\sqrt{205\,in^2}}\,\hat{j}$$

$$\bar{F}_{BD} = T_{BD}\left(\bar{U}_{BD}\right)$$

$$\sum F_x = 0 = -F_c + \frac{14}{\sqrt{205}}T_{BD} \rightarrow F_c = \frac{14}{\sqrt{205}}T_{BD}$$

$$\sum F_y = 0 = \frac{-3}{\sqrt{205}}T_{BD} + F_A - 100\,lb - 100\,lb$$

$$\rightarrow F_A = 200\,lb + \frac{3}{\sqrt{205}}T_{BD}$$

$$\sum M_B = 0 = -F_8(6\,in) + 100\,lb(2\,in) - 100\,lb(10\,in) + F_c(7\,in)$$

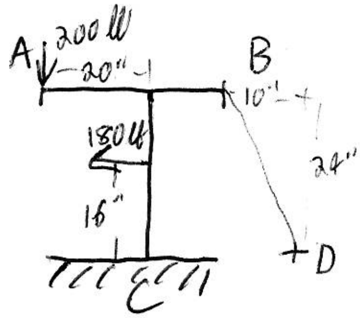
$$= -800\,in\cdot lb - 6\,in\left(200\,lb + \frac{3}{\sqrt{205}}T_{BD}\right) + 7\,in\,\frac{14}{\sqrt{205}}T_{BD}$$

$$= -2000\,in\cdot lb + \frac{80\,in}{\sqrt{205}}T_{BD} \rightarrow \boxed{T_{BD} = 357.946\,lb. \quad (a) \quad (\rightarrow 358\,lb)}$$

$$\sum F_x \rightarrow \boxed{F_c = 350\,lb \quad (c)}$$

$$\sum F_y \rightarrow \boxed{F_A = 275\,lb \quad (b)}$$

(4)



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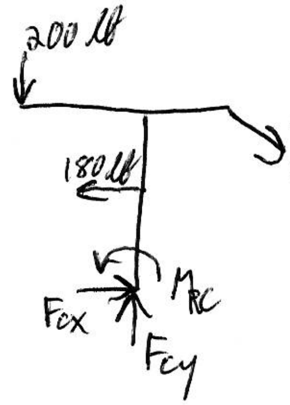


$\bar{F}_{BD} = T_{BD}(\bar{U}_{BD})$

$= 300\,lb(\bar{U}_{BD})$

$\bar{U}_{BD} = \frac{5}{13}\hat{i} - \frac{12}{13}\hat{j}$

$$\Sigma M_C = 0 = 200\,lb(20\,in) + 180\,lb(16\,in) + M_{RC} - \frac{12}{13}(300\,lb)6\,in + \frac{5}{13}300\,lb(24\,in)$$

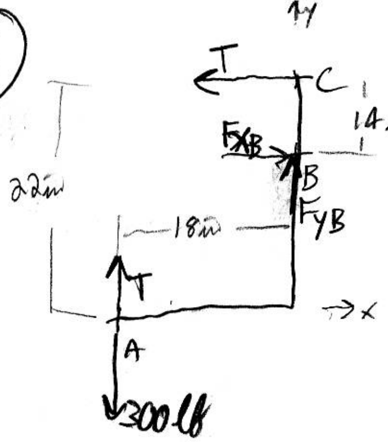
$$\rightarrow \boxed{M_{RC} = -5107.7\,in \cdot lb \rightarrow -5110\,in\,lb}$$

$$\Sigma F_x = 0 = F_{cx} + \frac{5}{13}(300\,lb) - 180\,lb \rightarrow \boxed{F_{cx} = 64.615\,lb \rightarrow 65\,lb}$$

$$\Sigma F_y = 0 = -200\,lb - \frac{12}{13}(300\,lb) + F_{cy}$$

$$\rightarrow F_{cy} = 200\,lb + \frac{12(300\,lb)}{13} = \boxed{476.92\,lb \rightarrow 477\,lb = F_{cy}}$$

(5)



$$\Sigma M_B = 0 = T(14\,in) - T(18\,in) + 300\,lb(18\,in)$$

$$\rightarrow T = \frac{300\,lb(18\,in)}{4\,in} = \boxed{1350\,lb = T}$$

$$\Sigma F_y = 0 = T - 300\,lb + F_{yB}$$

$$0 = 1350\,lb - 300\,lb + F_{yB} \rightarrow \boxed{F_{yB} = -1050\,lb}$$

$\boxed{\text{or } 1050\,lb(-\hat{j})}$

$$\Sigma F_x = 0 = F_{xB} - T$$

$$\rightarrow \boxed{F_{xB} = 1350\,lb} \; \boxed{\text{or } 1350\,lb\,\hat{i}}$$

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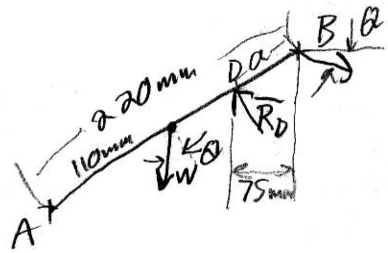
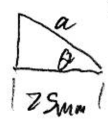
(6) uniform rod → weight centered at midpoint



$$\bar{U}_D = -\sin\theta\,\hat{i} + \cos\theta\,\hat{j} \quad | \quad \bar{R}_D = \bar{U}_D(R_D)$$

$$\bar{U}_B = \cos\theta\,\hat{i} - \sin\theta\,\hat{j} \qquad \bar{F}_B = \bar{U}_B(T_{BC})$$



$$\cos\theta = \frac{75\,mm}{a}$$

$$a = 110\,mm$$

$$\Sigma M_B = 0 = W(110\,mm)(\cos\theta) - R_D\left(\frac{75\,mm}{\cos\theta}\right) \rightarrow R_D = \frac{W(110)\cos\theta^2}{75} = \frac{22}{15}W\cos\theta^2$$

$$\Sigma F_x = 0 = T_{BC}(\cos\theta) - \sin\theta(R_D)$$

$$\frac{T_{BC}}{R_D} = \tan\theta \rightarrow T_{BC} = R_D\tan\theta = \frac{22}{15}W\cos^2\theta\tan\theta$$

$$T_{BC} = \frac{22W}{15}\sin\theta\cos\theta$$

$$\Sigma M_D = 0 = W\cos\theta\left(110\,mm - \underset{\frac{75\,mm}{\cos\theta}}{a}\right) - T_{BC}(a)\sin(180° - 2\theta)$$

$$- \frac{22W}{15}\,\frac{75\,mm}{\cos\theta}$$

$\rightarrow [\sin(-2\theta)] \rightarrow \sin(2\theta) \rightarrow 2\sin\theta\cos\theta$

$$0 = W\cos\theta\,110\,mm - W\,75\,mm - \frac{110\,mm\,W}{\cos\theta}\boxed{\sin(180° - 2\theta)}$$

$$0 = W\cos\theta\,110\,mm - W\,75\,mm - \frac{110\,mm\,W(2\sin\theta)\cos\theta}{\cos\theta}$$

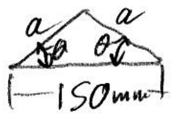
$$\frac{W\ 75\,mm}{W\ 110\,mm} = \cos\theta - 2\sin\theta = \frac{15}{22}$$

θ	$\cos\theta - 2\sin\theta$
10°	
15°	

$$\boxed{\text{(a)} \rightarrow \theta \approx 8.812^\circ}$$

solved numerically

(b) ~~180mm~~



$$\frac{75\,mm}{a} = \cos\theta \rightarrow \boxed{a = \frac{75\,mm}{\cos\theta} = 75.9\,mm}$$

(7)

$\bar{U}_{FH} = \frac{-3.10m}{3.8305m}\hat{i} - \frac{2.25m}{3.8305m}\hat{k}$

$= -.809300\hat{i} - .587395\hat{k}$

$\bar{U}_{CI} = \frac{-1.50m}{\sqrt{3.46m^2}}\hat{i} + \frac{1.10m}{\sqrt{3.46m^2}}\hat{j}$

$= -.806405\hat{i} + .591364\hat{j}$

$\bar{U}_{EG} = \frac{-3.10m}{4.41956m}\hat{i} + \frac{3.15m}{4.41956m}\hat{k}$

$= -.701427\hat{i} + .712741\hat{k}$

$\bar{F}_{FH} = T_1(\bar{U}_{FH})$

$\bar{F}_{CI} = T_1(\bar{U}_{CI})$

$\bar{F}_{EG} = T_2(\bar{U}_{EG})$

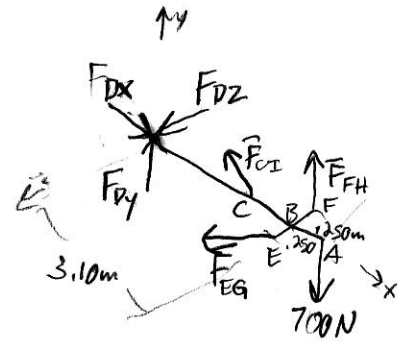
$\bar{r}_{DC} = 1.50m\hat{i}$

$\bar{r}_{DA} = 3.50m\hat{i}$

$\bar{r}_{DF} = 3.10m\hat{i} - 0.250m\hat{k}$

$\bar{r}_{DE} = 3.10m\hat{i} + 0.250m\hat{k}$

FBD:



$\Sigma M_{Dz} = 0 = r_{DCx}F_{CIy} - r_{DCy}F_{CIx}$

$-700N(3.50m)$

$0 = 1.50m\, T_1(.591364) - 2450 N\cdot m$

$\rightarrow \boxed{T_1 = 2761.98N}$

$\Sigma F_z = 0 = T_2(.712741) + F_{Dz} - T_1(.587395)$

$\Sigma M_{Dy} = 0 = r_{DFz}F_{FHx} - r_{DFx}F_{FHz} + r_{DEz}F_{EGx} - r_{DEx}F_{E}$

$0 = (-0.250m)T_1(-.809300) - (3.10m)T_1(-.587395)$

$+(0.250m)T_2(-.701427) - (3.10m)T_2(.712741)$

$0 = 2.0232495T_1 - 2.384854T_2$

$T_2 = \frac{5588.17N}{2.384854} = \boxed{2343.2N = T_2}$

$F_{Dz} = -1622.37N + 1670.09N = \boxed{47.7N = F_D}$

$\Sigma F_y = 0 = F_{Dy} + T_1(.591364) - 700N$

$\rightarrow \boxed{F_{Dy} = -933.3N}$

$\Sigma F_x = 0 = F_{Dx} - T_1[(.806405) + (.809300)]$

$-T_2(.701427)$

$0 = F_{Dx} - 4462.54N - 1643.58N$

$\boxed{F_{Dx} = 6106.1N}$

Final:

$T_1 = 2760\ N$

$T_2 = 2340\ N$

$F_{Dx} = 6110\ N$

$F_{Dy} = -933\ N$

$F_{Dz} = 48\ N$