

MATH 2130 – Midterm 2 Solutions

1. Let $f(x, y, z) = ze^{x^2-y^2}$, and let \mathcal{C} be the curve with vector representation

$$\mathbf{r}(t) = (t^2)\hat{\mathbf{i}} + (t^2 - 2t - 2)\hat{\mathbf{j}} + 2\cos(t + 1)\hat{\mathbf{k}}, \quad t \in \mathbb{R}.$$

Find the rate of change of f in the direction of \mathcal{C} at the point $(1, 1, 2)$.

2. Evaluate each of the following limits, or show that it does not exist.

(a)
$$\lim_{(x,y) \rightarrow (2,1)} \frac{x^2 - xy - 2y^2}{\sqrt{4x + y} - \sqrt{3x + 3y}}$$

(b)
$$\lim_{(x,y) \rightarrow (1,-3)} \frac{x^2 - y^2 - 2x - 6y - 8}{4x^2 + y^2 - 8x + 6y + 13}$$

3. Let $u = s + t^2 + te^s$, where s and t are defined as functions of x and y by

$$F(s, t, x, y) = xs - yt = 1,$$

$$G(s, t, x, y) = xt + ys = 2.$$

Find $\frac{\partial u}{\partial x}$.

4. Let S_1 be the surface $y = 2x^2 - xz$, and let S_2 be the surface $x^2 + 2yz + yz^2 = 4$.

(a) Find the equation of the tangent plane to S_1 at the point $(1, 1, 1)$.

- (b) Let \mathcal{C} be the curve formed by the intersection of S_1 and S_2 . Find a vector representation for the tangent line to \mathcal{C} at $(1, 1, 1)$.

5. Let $f(x, y) = 5x^2 - 4y + 10xy + 2y^3$. Find all critical points of f . Choose **one** critical point and determine if it is a relative maximum, relative minimum, saddle point, or none of these.