MATH 2210 - Limits of Multivariable Functions

Summary: Give a function $f(x,y)$, we want to calculate the limit

$$\lim_{(x,y)\to(0,0)} f(x,y).$$

- We have studied two ways to show that the limit does not exist:
  - Find two lines for which the limits of $f(x,y)$ along those two lines exist and they are different from each other.
  - If all the lines give you the same limit, substitute a function of x for y (or a function of y for x) in order to obtain a limit which is different from the limits along lines.
- To show that the limit does exit, we have also studied two ways:
  - Find a function $g(x,y)$ such that

    $$0 \le |f(x,y)| \le |g(x,y)| \quad \text{for every } x, y,$$

    and show that

    $$\lim_{(x,y)\to(0,0)} g(x,y) = 0.$$

  - Change $f(x,y)$ into polar coordinates, and show that the limit as $r \to 0$ exists no matter what θ is.

Calculate the following limits or show that they do not exist:

$$\lim_{(x,y)\to(0,0)} \frac{x+2y^2}{3x^2+2y^2} \qquad \lim_{(x,y)\to(0,0)} \frac{2x^3y^2}{x^6+y^4}$$

$$\lim_{(x,y)\to(0,0)} \frac{xy}{\sqrt{3x^2+2y^2}} \qquad \lim_{(x,y)\to(0,0)} \frac{x^2y}{3x^2+2y^2}$$

$$\lim_{(x,y)\to(0,0)} \frac{x^2y}{x^4+3y^2} \qquad \lim_{(x,y)\to(0,0)} \frac{3x^3+2\sqrt{y}}{x^2+y^2}$$

$$\lim_{(x,y)\to(0,0)} \frac{-3x^3-y^2}{3x^3+2y^2} \qquad \lim_{(x,y)\to(0,0)} \frac{x^4+y^4}{x^2+3y^2}$$

$$\lim_{(x,y)\to(0,0)} \frac{x^2y^5}{2x^4+3y^{10}} \qquad \lim_{(x,y)\to(0,0)} \frac{x^2y^2}{2x^2+3y^4}$$