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(A1) when displacement is zero, speed is max and hence K is max. K is always positive and oscillates $\boxed{(e)}$

(A2) Frequency rises as train approaches and drops as it passes.

$$f_1 = f_0 \frac{V}{V-V_s} = (440)\frac{340}{340 - \frac{90\times1000}{60\times60}} = 475\ Hz.$$

$$f_2 = f_0 \frac{V}{V+V_s} = (440)\frac{340}{340 + \frac{90\times1000}{60\times60}} = 410\ Hz$$

$\boxed{(a)}$

(A3) For standing waves in a pipe, $f \propto \frac{1}{L}$

Hence $f_1 L_1 = f_2 L_2$

For 3 beats/sec, $\frac{f_2 - f_1}{f_1} = \frac{3}{150} = 2\% = \frac{L_1}{L_2} - 1 = \frac{\Delta L}{L}$ $\boxed{(b)}$

(A4) Period in space is $4\,m = \lambda$

Period in time is $.5\,s = T$

$V = \frac{\lambda}{T} = 8.0\ m/s$ $\boxed{(a)}$

(A5) $y(x,t) = 0.04\sin(\pi x)\cos(2\pi t)$

Particles move vertically in standing wave

$$v_y = \frac{dy}{dt} = -0.04(2\pi)\sin(\pi x)\sin(2\pi t)$$

$$= -0.04(2\pi)\sin(\pi/2)\sin(6\pi) = 0$$ $\boxed{(a)}$

(A6) $\phi = 4.00\times10^{-19}\,J = \frac{4.00\times10^{-19}}{1.60\times10^{-19}}\,eV = 2.5\,eV$

$K = hf - \phi = \frac{hc}{\lambda} - \phi = 0$ when

$\lambda = \frac{hc}{\phi} = \frac{1240\ eV.nm}{2.5\ eV} = 496\ nm$ $\boxed{(c)}$

(A7) $\lambda_{\text{deBroglie}} = \frac{h}{p}$ = same for all $\boxed{(d)}$

(A8) $n_1 \sin\theta_1 = n_2 \sin\theta_2$

when $\theta_2 = \pi/2$, $\sin\theta_1 = \frac{n_2}{n_1} = \frac{1.33}{1.55} = .86$

$\theta_1 = \sin^{-1}(.86) = 59°$ $\boxed{(e)}$

(A9) Light enters at angle $\tan\theta_1 = 4/3$ and in the liquid is moving at angle $\tan\theta_2 = 1/2$ to the normal

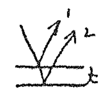
$\sin\theta_1 = 4/5$ and $\sin\theta_2 = \frac{1}{\sqrt5}$

$n_1\sin\theta_1 = n_2\sin\theta_2 = \sin\theta_1$ $(n_1 = 1)$

$n_2 = \frac{\sin\theta_1}{\sin\theta_2} = \frac{4}{\sqrt5} = 1.79$

Speed of light in liquid $= \frac{c}{n_2} = \frac{3.0\times10^8\,m/s}{1.79} = 1.7\times10^8\,m/s$

$\boxed{(a)}$

(A10) 

Both waves have a phase change of π due to reflection. But wave 2 travels further with path difference $2t$

phase difference $\Delta\phi = \frac{2\pi n\, 2t}{\lambda} = \pi$ for minimum

$t = \frac{\lambda}{4n} = \frac{560\,nm}{4(1.38)} = 101\ nm$ $\boxed{(a)}$

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(A11) $d = 0.5\,mm = 0.5\times10^{-3}\,m$
$\lambda = 555\,nm = 555\times10^{-9}\,m$
$D = 6.0\,m$

For bright fringe $d\sin\theta = m\lambda$
Position of screen $y = D\tan\theta \sim D\sin\theta = \frac{Dm\lambda}{d}$

$\therefore \Delta y = \frac{D\lambda}{d} = \frac{(6.0)(555)10^{-6}}{.5} = 6.7\,mm$ $\boxed{(c)}$

(A12) $\lambda = .00150\,nm$

$\lambda' = \lambda + \frac{h}{m_e c}(1-\cos\theta) = \lambda + \frac{hc}{m_e c^2}(1-\cos\theta)$

$= .00150\,nm + \frac{(1240\,eV.nm)}{(511\,keV)}\left(1-\frac{1}{\sqrt{2}}\right)$

$= .00150\,nm + .00071\,nm$
$= .00221\,nm$

$E = \frac{hc}{\lambda'} = \frac{1240\,eV.nm}{.00221\,nm} = 5.61\times10^{5}\,eV$ $\boxed{(b)}$

(A13) Probability density $p(x) = \frac{1}{(4cm-0)} = .25/cm$

Probability of finding particle in dx is $p(x)dx$

$dx = 3.4cm - 3.0cm = .4cm$

probability $= \frac{.4cm}{4cm} = 0.1$ $\boxed{(c)}$

(A14) Ground state has no nodes inside well
Each higher state has an additional node
Fourth state has 3 nodes
None of these! $\boxed{(e)}$

(A15) $E_n = \frac{n^2h^2}{8mL^2}$

$E_4 - E_3 = \left[(16)-(9)\right]\frac{h^2}{8m_eL^2} = \frac{hc}{\lambda}$

$\lambda = \frac{hc\,8m_eL^2}{7h^2} = \frac{8}{7}\frac{m_ec^2L^2}{hc} = \frac{8}{7}\left(\frac{511\,keV}{1240\,eV.nm}\right)(0.2\,nm$

$= 18.8\,nm$ $\boxed{(a)}$

(A16) $l = 0, 1, 2, \quad n-1$
$m = -l, -l+1, \ldots, l \quad = 2l+1$ values.

For $n=4$, $l = 0, 1, 2, 3$
$l=0$ has 1 value $m=0$
$l=1$ has 3 values $m=-1,0,1$
$l=2$ has 5 values $m=-2,-1,0,1,2$
$l=3$ has 7 values $m=-3,-2,-1,0,1,2,3$

$\therefore$ total number is 16 $\boxed{(d)}$

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(A17) $\Delta t = \gamma \Delta t_0 = \dfrac{1}{\sqrt{1-\left(\frac{2.7\times10^8}{3\times10^8}\right)^2}} 26\,ns = 60\ ns$

$\boxed{(d)}$

(A18) $L = \dfrac{L_0}{\gamma} = L_0\sqrt{1-(.6)^2} = (100\,m)\,0.8 = 80\,m$

$\boxed{(b)}$

(A19) Electron 1 in frame of atom is moving at $V_1 = .6c$
Electron 2 in frame of atom is moving at $V_2 = -.6c$

In frame of electron 2, observer is moving at $V_0 = .6c$
and electron 1 is moving at $V_1 = .6c$ in this frame

$\therefore\ u = -.6c$
$V = +.6c$

$V' = \dfrac{V-u}{1-uV/c^2} = \dfrac{1.2c}{1+(.6)^2} = .88c$ $\boxed{(b)}$

(A20) $K = \dfrac{mc^2}{\sqrt{1-v^2/c^2}} - mc^2 = (511\,keV)\left(\dfrac{1}{\sqrt{1-(.5)^2}} - 1\right)$

$= 79\ keV$ $\boxed{(b)}$

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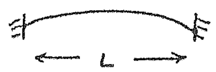
LONG ANSWER #1

a) 

$L = \frac{\lambda}{2}$ fundamental

$\lambda f = v \quad \therefore \quad 2Lf = v$

$2\,(0.22\text{ m})\,(920\text{ Hz}) = \boxed{405\text{ m/s}}$

b) Sound waves have the same frequency but different speed.

$\lambda f = v\,; \quad \lambda = \frac{340\text{ m/s}}{920\text{ Hz}} = \boxed{0.37\text{ m}}$

c) $y(x,t) = 3.0\sin\left(\frac{\pi x}{55}\right)\cos(7360\pi t)$

$\frac{\partial^2 y}{\partial x^2} = -3.0\left(\frac{\pi}{55}\right)^2\sin\left(\frac{\pi x}{55}\right)\cos(7360\pi t)$

$= -\left(\frac{\pi}{55}\right)^2 y(x,t)$

$\frac{\partial^2 y}{\partial t^2} = -(7360\pi)^2\, y(x,t)$

wave equation: $\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$

$-\left(\frac{\pi}{55}\right)^2 = -(7360\pi)^2 = \frac{1}{v^2}$

$v = (7360)(55)\ \frac{\text{mm}}{\text{s}} = 405\text{ m/s}$ ✓

d) which harmonic? $kL = n\pi$ for n^{th} harmonic

$\frac{2\pi}{\lambda}L = n\pi\,;$ standing wave $y(x,t) = A\sin(kx)\cos(\omega t)$

$\therefore k = \frac{\pi}{55}\text{ mm}^{-1} \rightarrow n = \frac{kL}{\pi} = \frac{220\text{ mm}}{55\text{ mm}} = 4$

$\therefore$ $\boxed{4^{th}\text{ harmonic}}$

e) $\beta = 10\log\left(\frac{I}{I_o}\right) = 60 \quad \therefore I = 10^6 I_o = 10^6(10^{-12}) = 10^{-6}\ \frac{\text{W}}{\text{m}^2}$

But $I = \frac{P}{4\pi r^2}$ and $r = 20$ m $\therefore$ power is $P = 4\pi(20)^2 10^{-6}$

$\rightarrow \boxed{P = 5.0\times10^{-3}\text{ Watts}}$

Solutions Long Answer #2

a) Assumption:
- exact circular orbits
- circumference is an integer multiple of de Broglie wavelength

$2\pi r = n\frac{h}{p} \qquad n = 1, 2, 3 \ldots$

[or $\ell = n\hbar$]

This is not consistant with quantum mechanics, which specifies that the electron is described as a probability wave and does not follow an exact path.

b) Quantum numbers: to solve Schrödinger's eq^n for the wave function in 3d, we have one quantization condition for each space dimension. Bohr's n is associated with r; ℓ with θ (angular momentum) and m with ϕ. Bohr got lucky because the energy only depends on n. States with the same n but different ℓ and m are degenerate.

c)

n	Energy $= -13.6/n^2$ eV
1	-13.6 eV
2	-3.4
3	-1.5
4	-0.85
⋮	⋮
∞	0

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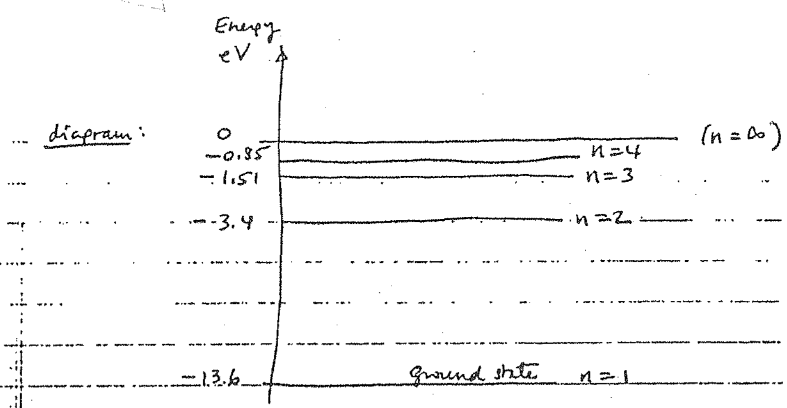
diagram:



$E \geq 0$ means the electron is free; it is not bound to the atom. ∴ there are no restrictions on the allowed values of E in this case

d) $n=3 \to n=2$; $\Delta E = 13.6 \left[\frac{1}{4} - \frac{1}{9}\right]$ eV

$\Delta E = 1.88$ eV photon energy

$\Delta E = \frac{hc}{\lambda}$ $\lambda = \frac{1240 \text{ eV}\cdot\text{nm}}{1.88 \text{ eV}} = 660$ nm

1st order line $\lambda = d \sin\theta$

$d = \frac{1 \text{ cm}}{4000} = 2.5 \times 10^{-6}$ m

$\sin\theta = \frac{\lambda}{d} = \frac{660 \times 10^{-9}}{2.5 \times 10^{-6}} = 0.264$

$\theta = 0.267$ rad $= 15.3°$

lines occur at $m\lambda = d \sin\theta$

$$\sin\theta = \frac{m\lambda}{d} = \begin{cases} 0.264 & (m=1) \\ 0.528 & (m=2) \\ 0.792 & (m=3) \\ \cancel{1.056} & (m=4) \end{cases}$$

$m=4$ line has $\sin\theta > 1$
∴ not observed

∴ we see 3 lines between 0 and 90°
[$m = 1, 2, 3$]