Values

Instructions:

1. You are permitted to use the textbook for the course and a calculator.
2. Clear, systematic solutions are required. **Show your work.** Marks will not be assigned for problems that require unreasonable (in the opinion of the instructor) effort to decipher.
3. Ask for clarification if any problem statement is unclear to you.
4. Use linear interpolation between table entries as necessary. Use constant specific heat.
5. Retain all the significant figures and units of property values from tables. Keep 4 or 5 significant figures in your intermediate results. Final answers must have 3 to 5 significant figures and units.
6. There are **five** questions on this exam. The weight of each problem is indicated. The exam will be marked out of 100.

$\boxed{23}$ **1**. The piston-cylinder arrangement shown in Figure 1 contains R-134a. Two linear springs with different spring constants and different lengths are installed as shown above the frictionless piston. In the initial position, the piston is motionless and floating so that the top of the piston is 20.0 [cm] from the bottom of Spring 1 and 52.0 [cm] from the bottom of Spring 2. In this initial position (State 1) the R-134a is at 0 [°C], occupies a volume of 165 [L], and has a quality of 47.1%.

Energy is added to the system by heat transfer, causing the piston to rise until it just touches Spring 1, but Spring 1 exerts no force on the piston (State 2). Heating continues again and Spring 1 is compressed until the piston just touches Spring 2, but Spring 2 exerts no force on the piston (State 3). Heating continues further while the piston compresses both springs until the pressure is 600 [kPa] (State 4). The frictionless piston has a cross-sectional area of 0.125 [m^2]. The spring constants of the two springs are given in Figure 1.

6 (a) Calculate the quality at State 2, x_2.

5 (b) Determine P_3. (**Hint:** The value of P_3 should correspond to a property table entry.)

3 (c) Calculate the boundary work done by the system during each of the three processes: ${}_1W_2$, ${}_2W_3$, and ${}_3W_4$.

9 (d) Show the three processes on a P-V (pressure–volume) diagram. Label the state pressure and volume values and the relevant constant T lines. Do any necessary additional calculations needed to label the diagram. Mark the areas representing work done.

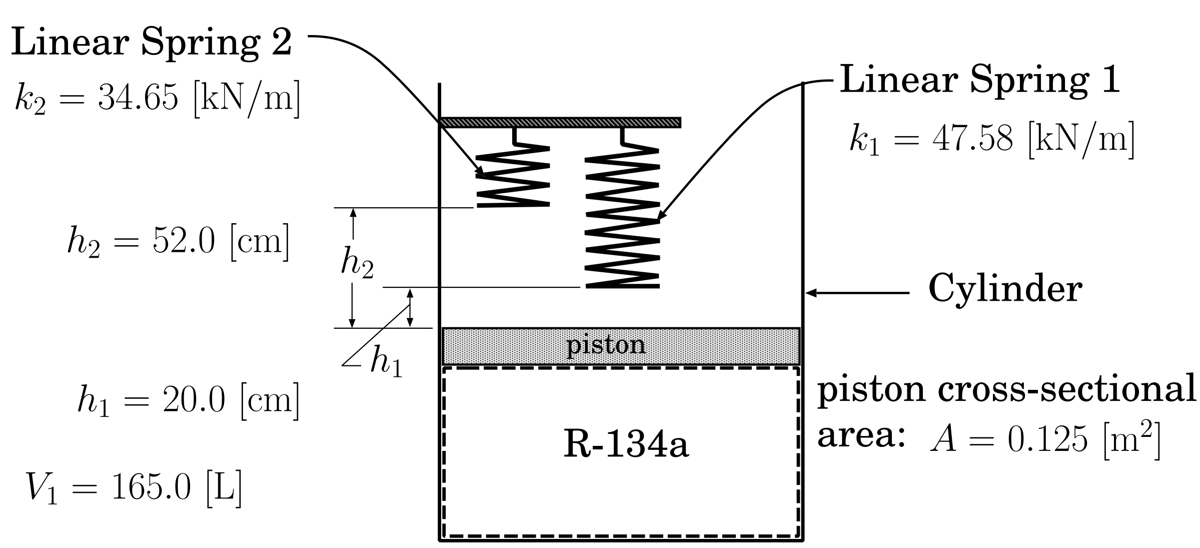

Figure 1: Piston-cylinder arrangement for problem **1**

Values

2. Figure 2 shows a schematic of a power plant that has two steam turbines. The water (the working fluid for the plant) leaves the high pressure turbine (HPT) as saturated vapour at 1000 [kPa]. Part of this flow is diverted to an insulated heat exchanger, which uses a hot air stream to create superheated steam that enters the low pressure turbine (LPT). The air mass flow rate is $\dot{m}_a = 19.5$ [kg/s] and the air temperature drops from 1100 [°C] to 600 [°C] as it goes through one side of the heat exchanger. The remaining portion of the exit flow from the high pressure turbine goes to a mixing chamber. This flow is mixed with the stream coming from the low pressure turbine branch after it has gone through the low pressure turbine, the condenser, and Pump 1. The flow leaves the mixing chamber as saturated liquid and then enters Pump 2. After leaving Pump 2 at a pressure of 8 [MPa], the water enters the steam generator, where it exits at 8 [MPa] and 450 [°C] before flowing into the high pressure turbine. Further details of the plant operating conditions are shown in Figure 2. The turbines and pumps are insulated (*i.e.*, adiabatic). Neglect changes in potential and kinetic energies. $\boxed{34}$

8 (a) Determine the power output of the low pressure turbine, $\dot{W}_{\text{LPT}}$, in [kW].

2.5 (b) Determine the power input to Pump 1, $\dot{W}_{P1}$, in [kW].

7.5 (c) Determine the power input to Pump 2, $\dot{W}_{P2}$, in [kW].

7 (d) Determine the thermal efficiency of the power plant.

9 (e) On a T-v (temperature–specific volume) diagram, draw a process representation for the working fluid in the power plant. On the diagram, clearly indicate the labelled state points, the process paths (use a dashed line if the path is unknown), and the constant pressure lines that pass through the state points. Indicate state T and v values and saturation temperature values for reference as appropriate. Do any additional work necessary to label the diagram.

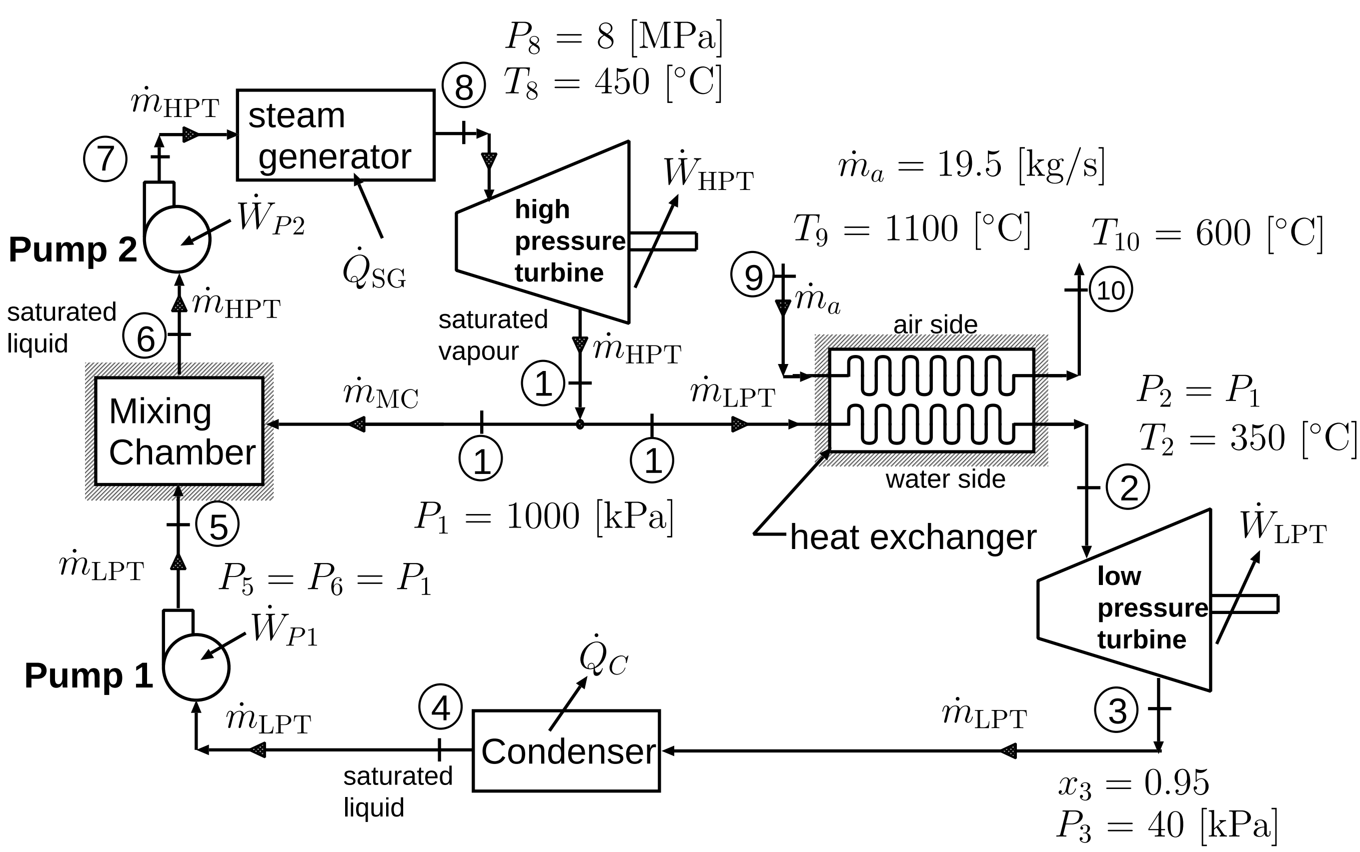

Figure 2: Figure for problem **2**

Values

3. A piston-and-cylinder system consists of a volume containing 1.5 [kg] of water, and a separate region containing air with one side bounded by a movable insulating piston as shown in Figure 3. The partition separating the water and the air is fixed in place and it has negligible mass such that it allows heat transfer from one side to the other. Both the water and air are at an initial temperature of $T_{\mathrm{w},1} = T_{\mathrm{a},1} = 150$ [°C]. A process occurs whereby a block of copper, of mass $m_{\mathrm{Cu}} = 8.5$ [kg], is brought into contact with the cylinder volume containing the water. The copper block is at an initial temperature of $T_{\mathrm{Cu},1} = 600$ [°C] when it is placed in contact with the cylinder as shown in the figure. The entire piston-cylinder and copper-block system is then surrounded with insulation to prevent heat transfer to the surroundings; the air in the cylinder is compressed polytropically with $n = 1.15$. The process occurs until the pressure of the water reaches $P_{\mathrm{w},2} = 1200$ [kPa] and all three masses (copper, water, and air) reach the same final equilibrium temperature, $T_{\mathrm{Cu},2} = T_{\mathrm{w},2} = T_{\mathrm{a},2} = 400$ [°C].

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6 (a) Determine the pressure of the water at the initial state and the mass of liquid water present at the initial state.

13 (b) Determine the mass of air contained in the cylinder.

3 (c) Show the process for the water on a $T - v$ diagram, indicating all relevant information.

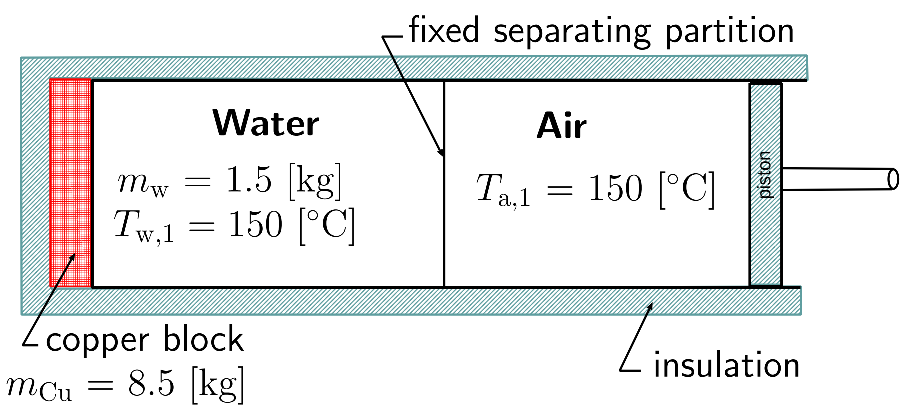

Figure 3: Figure for problem **3**

4. A rigid container that contains 0.1 [m^3] of R-12 at 30 [°C] and 400 [kPa] is placed inside a Carnot refrigerator that maintains the cooled space at −5 [°C]. The container of R-12 is cooled to 0 [°C]. The Carnot refrigerator is situated in a room that has an ambient temperature of 30 [°C].

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4 (a) Find the amount of heat the Carnot refrigerator must remove from the R-12 only (neglect cooling of the container walls).

2 (b) For this process, determine the work input to the Carnot refrigerator.

Values

15 **5**. Consider the system of three Carnot (reversible) heat engines as shown in Figure 4. Heat Engine 1 (HE1) operates between the temperature reservoirs $T_H = 1200$ [K] and T_R, where T_R is unknown. Heat Engine 2 (HE2) operates between temperature reservoirs T_R and $T_L = 300$ [K]. Heat Engine 3 (HE3) operates between temperature reservoirs $T_H = 1200$ [K] and $T_L = 300$ [K]. It is known that the ratio of work produced from HE1 to work produced from HE2 is 2 (*i.e.*, $W_1/W_2 = 2$) and that $Q_{H2} = Q_{L1}$.

2 (a) If $Q_{H3} = 100$ [kJ], determine the work produced by HE3, W_3, in [kJ].

9 (b) Determine the temperature of thermal reservoir T_R.

4 (c) If $Q_{H1} = Q_{H3} = 100$ [kJ], determine W_1 and W_2. What can you conclude about the combined work output of HE1 and HE2 compared to the work output of HE3?

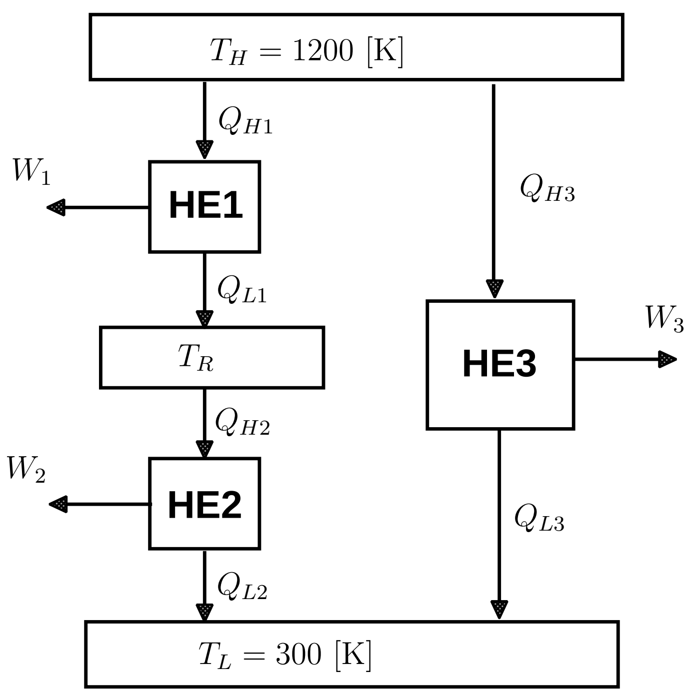


Figure 4: Figure for problem **5**

ENG 1460 Introduction to Thermal Sciences (F07)

Final Exam Solution

1. (a) Process 1 → 2 constant pressure (R-134a)

$P_2 = P_1 = P_{sat}(0\,[°C]) = 294.0\,[kPa]$ (Table B5.1)

State 1: Saturated mixture $x_1 = 0.471$

$\forall_1 = 165.0\,[L] = 0.165\,[m^3]$

$m = \dfrac{\forall_1}{v_1}$ $\quad v_1 = (1-x_1)\,v_f\big/_{0°C} + x_1\,v_g\big/_{0°C}$

$v_1 = (1-0.471)\underbrace{0.000773}_{v_f} + (0.471)\underbrace{0.06919}_{v_g}$ ← Table B5.1

$v_1 = 0.032997\,[m^3/kg]$

$m = \dfrac{0.165}{0.032997} = 5.00\,[kg]$

$\forall_2 = \forall_1 + A_c \cdot h_1 = 0.165 + (0.125)(0.200) = 0.190\,(m^3)$

$v_2 = \dfrac{\forall_2}{m} = \dfrac{0.190}{5.00} = 0.0380\,[m^3/kg]$

$v_f < v_2 < v_g$

⇒ Still as saturated mixture

$x_2 = \dfrac{(v_2 - v_f)}{(v_g - v_f)} = \dfrac{(0.0380 - 0.000773)}{(0.06919 - 0.000773)}$

$x_2 = 0.5441$ ←

(b) $P_3 = P_2 + \dfrac{k_s}{A^2}(\forall_3 - \forall_2)$ $\quad \forall_3 = \forall_2 + A(h_2 - h_1)$

$= 0.190 + 0.125(0.52 - 0.20)$

$P_3 = 294.0 + \dfrac{47.58}{(0.125)^2}(0.230 - 0.190)$ $\quad \forall_3 = 0.230\,(m^3)$

$P_3 = 294.0 + 121.80 = 415.80\,[kPa]$ ←

matches T = 10 °C entry in Table B5.1

1 (c) $_1W_2 = P_1(\forall_2 - \forall_1) = 294.0\,(0.190 - 0.165)$

$_1W_2 = 7.35\,[kJ]$ ←

$_2W_3 = \frac{1}{2}(P_2 + P_3)(\forall_3 - \forall_2) = \frac{1}{2}(294.0 + 415.80)(0.230 - 0.190)$

$_2W_3 = 14.196\,[kJ]$ ←

$_3W_4 = \frac{1}{2}(P_3 + P_4)(\forall_4 - \forall_3)$

Need $\forall_4$: $\qquad \forall_4 = \forall_3 + \frac{A^2}{(k_1 + k_2)}(P_4 - P_3)$

$\forall_4 = 0.230 + \frac{(0.125)^2}{(47.58 + 34.65)}(600 - 415.80)$

$\forall_4 = 0.265\,(m^3)$

$_3W_4 = \frac{1}{2}(415.80 + 600)(0.265 - 0.230)$

$_3W_4 = 17.777\,[kJ]$ ←

(d)

Interpolate for T_4

$v_4 = \frac{\forall_4}{m} = \frac{0.265}{5.00} = 0.053\ \frac{m^3}{kg}$

$v_g\big|_{600\,kPa} = 0.03442\left(\frac{m^3}{kg}\right)$ Table B.5.2

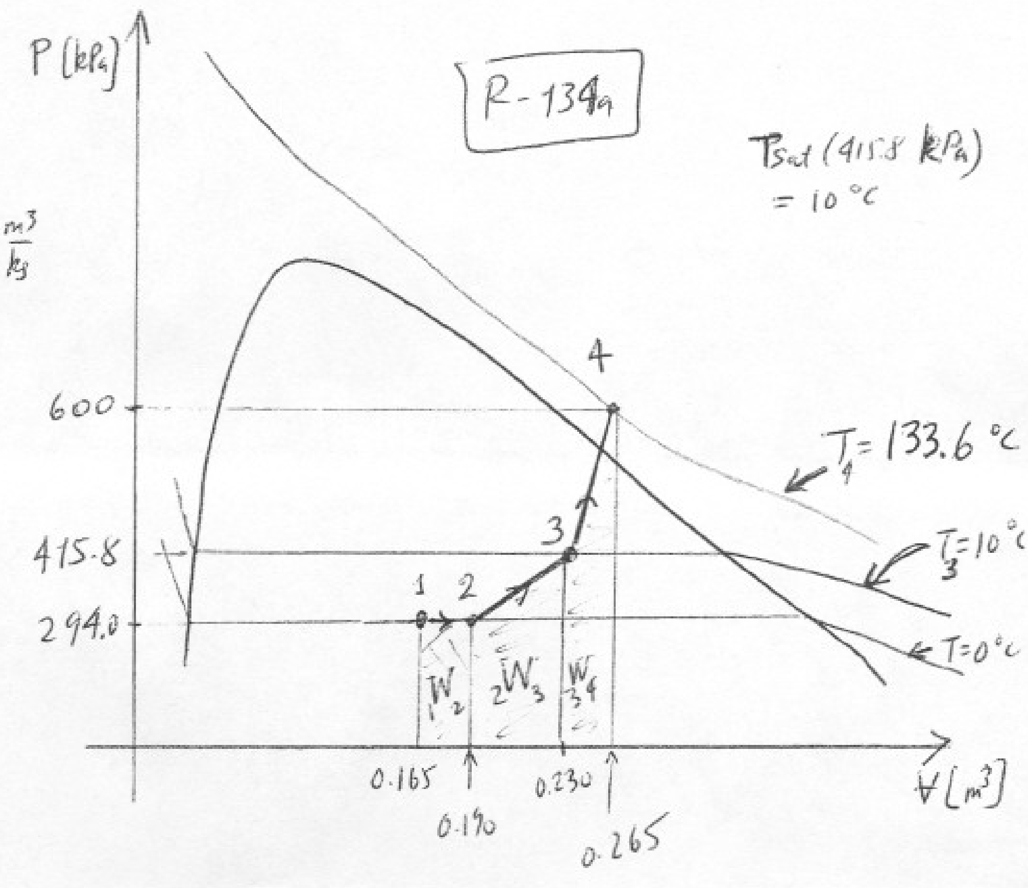
$v_2 > v_g\big|_{P_2}$

⇒ superheated vapour

Table B.5.2 600 (kPa)

T (°C)	v (m^3/kg)
130	0.05246
T_4	0.053
140	0.05396

$T_4 = 133.6\,[°C]$



2. (a) First law for the LP Turbine (neglect $\Delta KE, \Delta PE$) 3/

$${}_{i}\dot{Q}_{3}^{\nearrow 0} - \dot{W}_{LPT} = \dot{m}_{LPT}(h_3 - h_2)$$

→ need $\dot{m}_{LPT}$ ⇒ analyze the heat exchanger

First law for the heat exchanger neglect $\Delta KE, \Delta PE$; $\dot{Q}_{cv}=0, \dot{W}_{cv}=0$)

$$0 = \Sigma \dot{m}_e h_e - \Sigma \dot{m}_i h_i = \dot{m}_2 h_2 + \dot{m}_{10} h_{10} - \dot{m}_1 h_1 - \dot{m}_9 h_9$$

$\dot{m}_1 = \dot{m}_2 = \dot{m}_{LPT}$ $\quad \dot{m}_9 = \dot{m}_{10} = \dot{m}_a$

$$0 = \dot{m}_{LPT}(h_2 - h_1) + \dot{m}_a(h_{10} - h_9)$$

For air as an ideal gas $\quad (h_{10} - h_9) = C_{p0}(T_{10} - T_9)$

$$\dot{m}_{LPT} = -\frac{\dot{m}_a C_{p0}(T_{10} - T_9)}{(h_2 - h_1)} = \frac{\dot{m}_a C_{p0}(T_9 - T_{10})}{(h_2 - h_1)}$$

$C_{p0} = 1.004$ (kJ/kg·K) (Tab6 A.5)

$h_1 = h_g|_{1000\,kPa} = 2778.08$ [kJ/kg]

$h_2 = h(1000\text{ kPa}, 350\,°C)$ $\quad$ $T_{sat}(1000\text{ kPa}) = 179.91$ (°C) (Table B.1.2)

$T_2 > T_{sat}(P_2)$ ⇒ Superheated vapor

$h_2 = 3157.65 \left(\frac{kJ}{kg}\right)$ (Table B.1.3)

$$\dot{m}_{LPT} = \frac{(19.5)(1.004)(1100 - 600)}{(3157.65 - 2778.08)} = 25.79\ [kg/s]$$

$$h_3 = (1 - x_3) h_f|_{P_3} + (x_3) h_g|_{P_3} = (1-0.95)\underbrace{317.55}_{h_f @ 40kPa} + (0.95)\underbrace{2636.74}_{h_g @ 40kPa}$$

(Table B1.2)

$h_3 = 2520.78$ (kJ/kg)

$$\dot{W}_{LPT} = -\dot{m}_{LPT}(h_3 - h_2) = -25.79(2520.78 - 3157.65)$$

$$\dot{W}_{LPT} = 16{,}425\ (kW) = 16.425\ (MW)$$

2 (b) First Law for Pump 1 (neglect ΔKE, ΔPE; $\dot{Q}_{cv}=0$)

$$+\dot{W}_{P1} = \dot{m}_{LPT}\, v_f\big|_{T_4} (P_5 - P_4)$$

$$v_f\big|_{T_4} = v_4 = v_f\big|_{40\,kPa} = 0.001026\ (m^3/kg) \qquad \text{(Table B.1.2)}$$

$$\dot{W}_{P1} = 25.79\,(0.001026)\,(1000-40) = 25.40\ [kW] \leftarrow$$

(c) First law for Pump 2 (neglect ΔKE, ΔPE; $\dot{Q}_{cv}=0$)

$$+\dot{W}_{P2} = \dot{m}_{HPT}\, v_f\big|_{T_6} (P_7 - P_6)$$

$$v_f\big|_{T_6} = v_6 = v_f\big|_{1000\,kPa} = 0.001127\ (m^3/kg) \qquad \text{(Table B.1.2)}$$

- need $\dot{m}_{HPT}$ ⇒ analyze the mixing chamber

First law for the mixing chamber (neglect ΔKE, ΔPE; $\dot{Q}_{cv}=0$, $\dot{W}_{cv}=0$)

$$0 = \Sigma \dot{m}_e h_e - \dot{m}_i h_i = \dot{m}_6 h_6 - \dot{m}_5 h_5 - \dot{m}_1 h_1$$

$$\dot{m}_6 = \dot{m}_{HPT} \qquad \dot{m}_5 = \dot{m}_{LPT} \qquad \dot{m}_1 = \dot{m}_{mc}$$

$$0 = \dot{m}_{HPT} h_6 - \dot{m}_{LPT} h_5 - \dot{m}_{mc} h_1$$

} $\dot{m}_{LPT}$ known; h_1, h_6 known; h_5 found from Pump 1 analysis

Also need mass conservation:

$$\dot{m}_{HPT} = \dot{m}_{LPT} + \dot{m}_{mc}$$

$$h_5 = h_4 + v_4 (P_5 - P_4) \qquad h_4 = h_f\big|_{40\,kPa} = 317.55\ (kJ/kg) \text{ (from before)}$$

$$h_5 = 317.55 + 0.001026\,(1000-40)$$

$$h_5 = 318.53\ [kJ/kg]$$

Substitute for $\dot{m}_{mc}$ from mass conservation into mixing chamber first law

$$0 = \dot{m}_{HPT} h_6 - \dot{m}_{LPT} h_5 - (\dot{m}_{HPT} - \dot{m}_{LPT}) h_1$$

2(c) continued...

$$0 = \dot{m}_{HPT} h_6 - \dot{m}_{LPT} h_5 - \dot{m}_{HPT} h_1 + \dot{m}_{LPT} h_1$$

$$\dot{m}_{HPT} = \dot{m}_{LPT} \frac{(h_5 - h_1)}{(h_6 - h_1)} = \dot{m}_{LPT} \frac{(h_1 - h_5)}{(h_1 - h_6)}$$

$$h_6 = h_f\Big|_{1000\,kPa} = 762.79 \; (kJ/kg) \qquad \text{(Table B.1.2)}$$

$$\dot{m}_{HPT} = 25.79 \frac{(2778.08 - 318.53)}{(2778.08 - 762.79)} = 31.48 \; (kg/s)$$

$$P_7 = P_8 = 8000 (kPa)$$

$$\dot{W}_{P2} = 31.48 (0.001127)(8000 - 1000) = 248.35 \; (kW) \leftarrow$$

(d) $$\eta_{th} = \frac{\text{Net Power Output}}{\text{Energy input}} = \frac{(\dot{W}_{LPT} + \dot{W}_{HPT} - \dot{W}_{P_1} - \dot{W}_{P2})}{(\dot{Q}_{SG} + \dot{Q}_{HE})}$$

where $\dot{Q}_{HE}$ is the energy added to the water from the heat exchanger.

$\Rightarrow$ need $\dot{Q}_{SG}$, $\dot{W}_{HPT}$, $\dot{Q}_{HE}$

First law for the steam generator (neglect ΔKE, ΔPE, $\therefore \dot{W}_{cv} = 0$)

$$\dot{Q}_{SG} = \dot{m}_{HPT} (h_8 - h_7) \qquad h_7 = h_6 + v_6 (P_7 - P_6)$$

$$h_7 = 762.79 + 0.001127)(8000 - 1000)$$

$$\dot{Q}_{SG} = 31.48 (3271.99 - 770.68) \qquad h_7 = 770.68 \; [kJ/kg]$$

$$\dot{Q}_{SG} = 78{,}741 \; \boxed{kW} = 78.741 \; (MW) \qquad h_8 = h(8000\,kPa, 450\,°C)$$

$T_{sat}(8000\,kPa) = 295.06\;(°C)$

$T_8 > T_{sat}(P_8) \Rightarrow$ superheated vapor

$$h_8 = 3271.99 \; (kJ/kg) \; \text{(Table B.1.3)}$$

2(d) continued ...

First law for the HP Turbine (neglect ΔKE, ΔPE; $\dot{Q}_{cv} = 0$)

$$-\dot{W}_{HPT} = \dot{m}_{HPT}(h_1 - h_8)$$

$$\dot{W}_{HPT} = -31.48(2778.08 - 3271.99) = 15{,}548 \text{ [kW]}$$

$$= 15.548 \text{ [MW]}$$

Heat Exchanger: First law for the Water Side

(neglect ΔPE, ΔKE; $\dot{W}_{cv} = 0$)

$$\dot{Q}_{HE} = \dot{m}_{LPT}(h_2 - h_1)$$

$$\dot{Q}_{HE} = 25.79(3157.65 - 2778.08)$$

$$\dot{Q}_{HE} = 9789 \text{ [kW]}$$

$$\eta_{th} = \frac{(16425 + 15548 - 25.40 - 248.35)}{(78741 + 9789)} = 0.3581$$

(e) New values for the diagram

$v_1 = 0.19444$ [m³/kg]

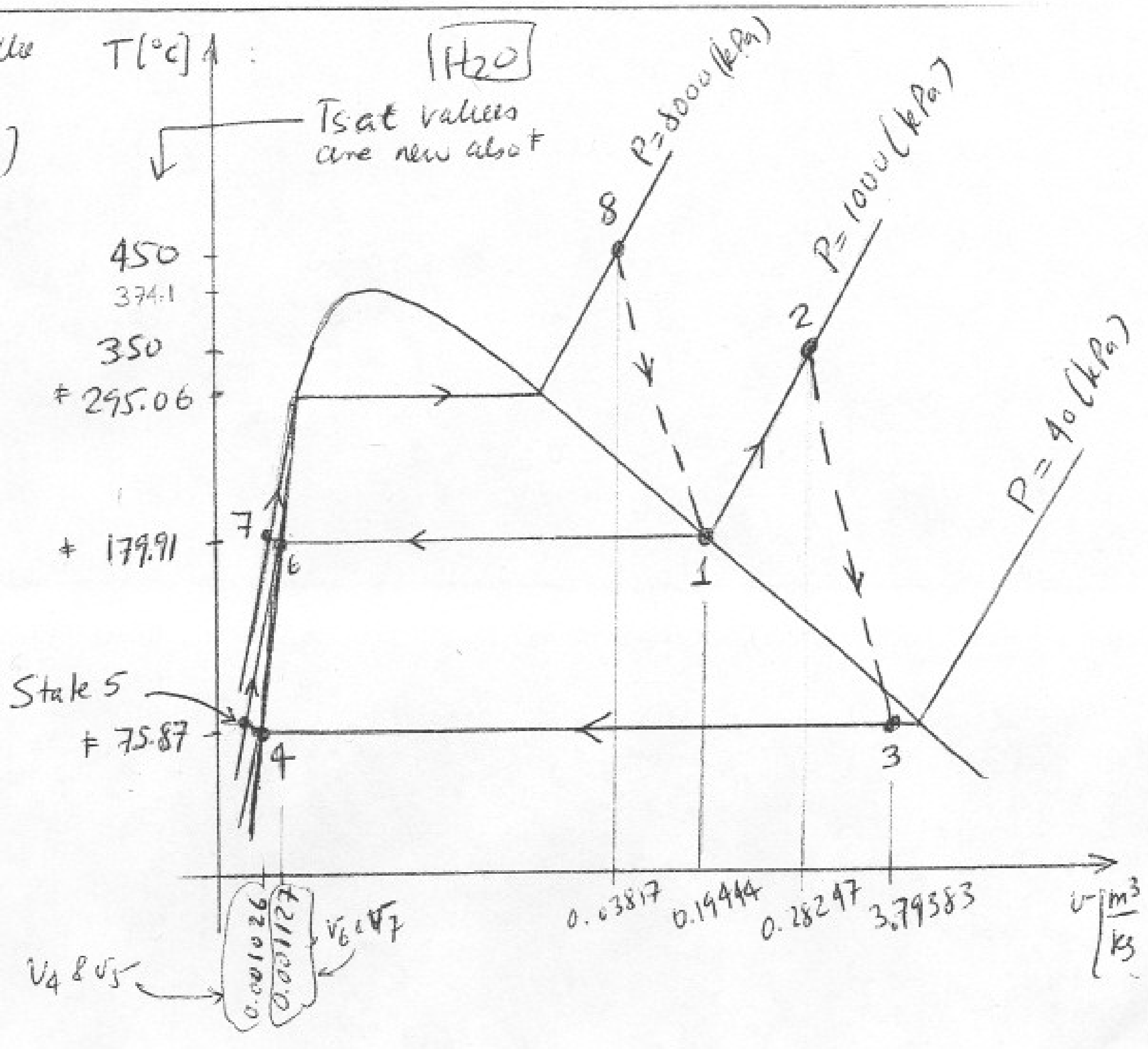
$v_2 = 0.28247 \left(\frac{m^3}{kg}\right)$
Table B.1.3

$v_3 = (1-0.95)\,0.001026 + (0.95)\,3.99395 = 3.79383 \left(\frac{m^3}{kg}\right)$

$v_5 \approx v_4$

$v_7 \approx v_6$

$v_8 = 0.03817 \left[\frac{m^3}{kg}\right]$



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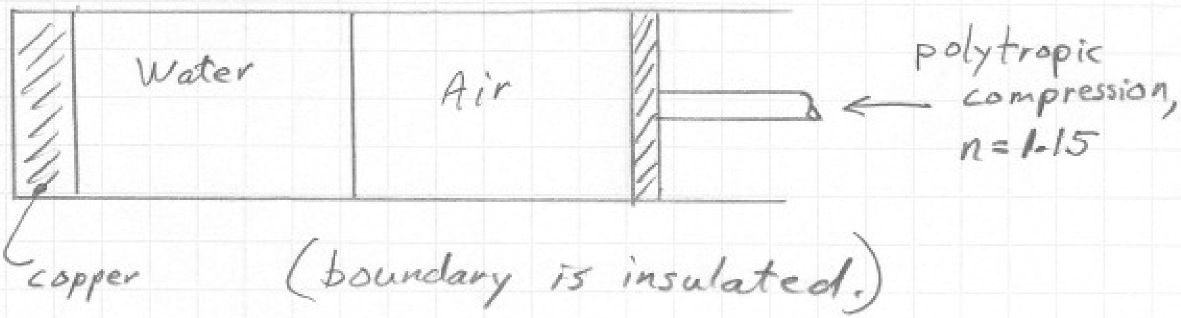
#3.



(a)

$m_w = 1.5$ kg $\quad T_{w,1} = 150°C$, $\quad T_{w,2} = 400°C$

$P_{w,2} = 1200$ kPa

For state 2, Table B.1.2 at $P = 1200$ kPa, $T_{sat.} = 187.99°C$

Since $T_2 > T_{sat.}$, state is superheated vapour,

Table B.1.3 $\quad v_2 = 0.25480 \ m^3/kg$, $u_2 = 2954.90$ kJ/kg

Volume and mass of water is constant, $\therefore v_1 = v_2$

Table B.1.1, at 150°C $\quad v_f < v_1 < v_g \quad \therefore$ state 1 is saturated mixture

$v_f = 0.001090 \ m^3/kg$

$v_{fg} = 0.39169 \ m^3/kg$ $\qquad P_{sat\,(150°C)} = 475.9$ kPa

$\therefore P_{w,1} = 475.9$ kPa

$$\longrightarrow \; x_1 = \frac{v_1 - v_f}{v_{fg}} = \frac{0.25480 - 0.001090}{0.39169}$$

$x_1 = 0.647731$, $\quad m_f = (1 - x_1)\, m_w = (1 - 0.647731) \times 1.5$

$\longrightarrow \quad m_f = 0.52840$ kg

$m_g = x_1 \cdot m = 0.97159$ kg

$u_1 = u_f + x_1 u_{fg} = 631.66 + 0.647731 \times 1927.87 = 1880.40 \ \frac{kJ}{kg}$

(b) Apply First law to the entire system:

$$\cancel{Q}^{\,0} - W = \Delta U_{air} + \Delta U_{water} + \Delta U_{copper}$$

insulated

$$\Delta U_{air} = m_{air}\, C_{v0,air} \left(T_2 - T_1\right)_{air}$$

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$$\Delta U_{water} = m_w(u_2-u_1) \quad ; \quad \Delta U_{Cu} = m_{Cu} C_{v_{Cu}} (T_2-T_1)_{Cu}$$

Polytropic work done on air: $_1W_2 = m_{air} \dfrac{R_{air}(T_2-T_1)_{air}}{1-n}$

$C_v = C_p$ for copper, Table A.3 $C_{p_{Cu}} = 0.42$ kJ/kg·K

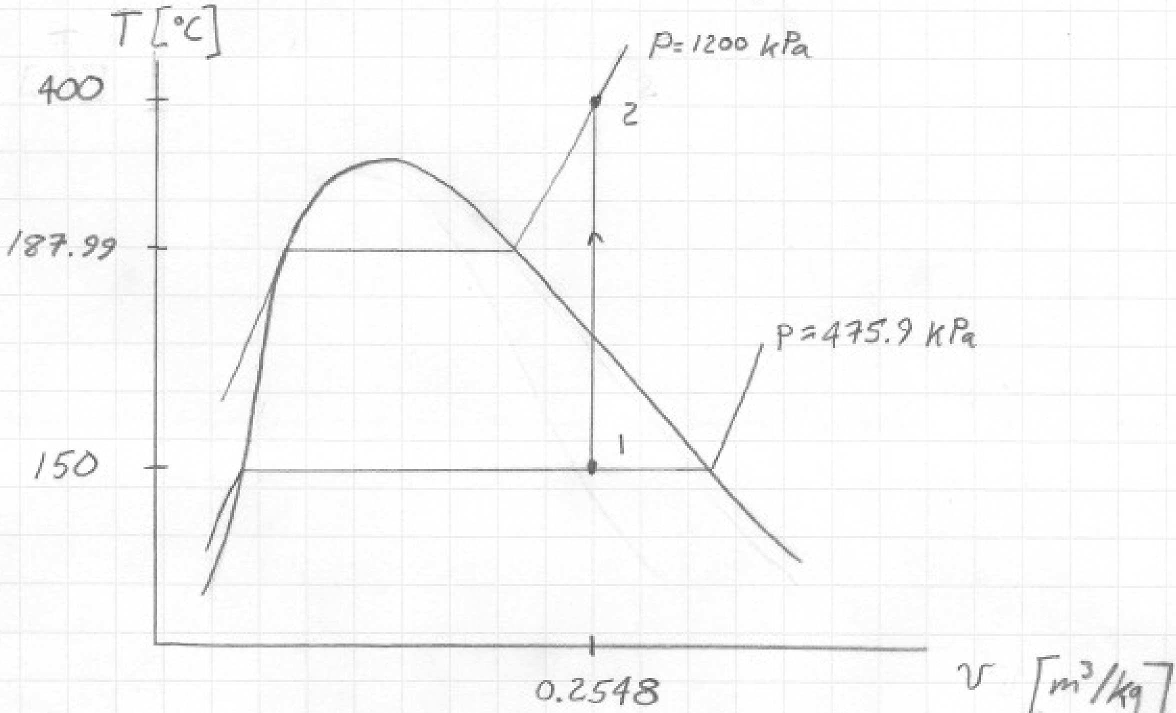
$C_{v_0,air} = 0.717$ kJ/kg·K , $R_{air} = 0.287$ kJ/kg·K

$$\therefore -m_{air}\frac{R_{air}(T_2-T_1)_{air}}{1-n} = m_{air} C_{v_0,air}(T_2-T_1)_{air} + m_w(u_2-u_1) + m_{Cu} C_{p_{Cu}} (T_2-T_1)_{Cu}$$

$$-m_{air}\frac{(0.287)(400-150)}{1-1.15} = m_{air}\, 0.717(400-150) + 1.5(2954.90-1880.40) + 8.5 \times 0.42 \times (400-600)$$

Solve for m_a: $m_a = 3.00$ kg

(c)



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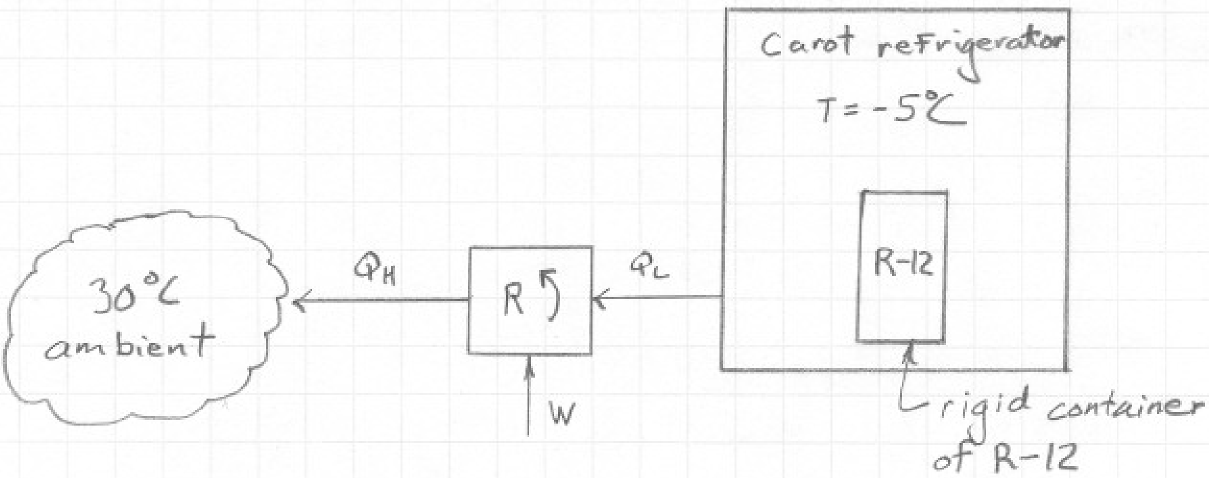
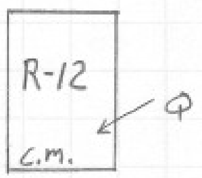
#4.



R-12 container, $V = 0.1\ m^3$, $T_1 = 30°C$, $P_1 = 400\ kPa$

(a)



considering R-12 control mass

$${}_1Q_2 - \cancel{W}^{0} = m(u_2 - u_1)$$

$T_1 > T_{sat.}$ at $P \simeq 400\ kPa$ ∴ state is superheated vapour.

∴ Table B.3.2, $v_1 = 0.04797\ m^3/kg$

$u_1 = 186.39\ kJ/kg$

$$\therefore m_{R-12} = \frac{V}{v} = \frac{0.1}{0.04797} = 2.08463\ kg$$

state 2, $T = 0°C$, $v_2 = v_1 = 0.04797\ m^3/kg$

From Table B.3.1, at 0°C, $v_f < v_2 < v_g$

$P_{sat.} = 308.6\ kPa$

$$x_2 = \frac{v - v_f}{v_{fg}} = \frac{0.04797 - 0.000716}{0.05467}$$

$$x_2 = 0.86434$$

$u_2 = u_f + x_2 u_{fg}$ ∴ $u_2 = 35.83 + 0.86434 \times 134.61$

$u_2 = 152.18\ kJ/kg$

$$\therefore {}_1Q_2 = 2.08463 \times (152.18 - 186.39)$$

${}_1Q_2 = -71.315\ kJ$ i.e., the refrigerator must remove 71.315 kJ of heat.

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#4.

(b) Carnot refrigerator

$$\beta_{R\,(carnot)} = \frac{T_L}{T_H - T_L} = \frac{268.15}{303.15 - 268.15} = 7.6614$$

$$\beta_R = \frac{Q_L}{W} \quad , \quad Q_L = |Q_2| = 71.315 \text{ kJ}$$

$$\therefore W = \frac{71.315}{7.6614} \doteq 9.308 \text{ kJ}$$

This is the additional amount of work the Carnot refrigerator must do to remove the 71.315 kJ of heat from the container of R-12.

\# 5. $Q_{H_3} = 100\ kJ$

(a) $W_3 = Q_{H_3} - Q_{L_3}$ $\qquad \eta_{HE_3} = 1 - \dfrac{T_L}{T_H} = 1 - \dfrac{300}{1200} = 0.75$

(All H.E. are reversible in this problem.)

$$\eta_{HE_3} = \frac{W_3}{Q_{H_3}} \qquad \therefore\ W_3 = 0.75 \times 100 = 75\ kJ$$

(b) $\dfrac{W_1}{W_2} = 2$ $\qquad W_1 = Q_{H_1} - Q_{L_1} \qquad W_2 = Q_{H_2} - Q_{L_2}$

$$\eta_{HE_1} = 1 - \frac{T_R}{T_H} = \frac{W_1}{Q_{H_1}} \qquad \therefore\ W_1 = Q_{H_1}\left(1 - \frac{T_R}{T_H}\right)$$

$$\eta_{HE_2} = 1 - \frac{T_L}{T_R} = \frac{W_2}{Q_{H_2}} \qquad \therefore\ W_2 = Q_{H_2}\left(1 - \frac{T_L}{T_R}\right)$$

$$\frac{W_1}{W_2} = \frac{Q_{H_1}}{Q_{H_2}} \cdot \frac{\left(\frac{T_H - T_R}{T_H}\right)}{\left(\frac{T_R - T_L}{T_R}\right)} = 2 \quad , \quad Q_{H_2} = Q_{L_1}$$

$$\frac{Q_{H_1}}{Q_{L_1}} \frac{(T_H - T_R)/T_H}{(T_R - T_L)/T_R} = 2 \quad , \quad \frac{Q_{H_1}}{Q_{L_1}} = \frac{T_H}{T_R}$$

$$\therefore\ \frac{\cancel{T_H}}{\cancel{T_R}} \cdot \frac{(T_H - T_R)}{\cancel{T_H}} \cdot \frac{\cancel{T_R}}{(T_R - T_L)} = 2$$

$$\frac{T_H - T_R}{T_R - T_L} = 2 \qquad \therefore\ 2(T_R - T_L) = T_H - T_R$$

$$3T_R = T_H + 2T_L$$

$$T_R = \frac{T_H + 2T_L}{3}$$

$$T_R = \frac{1200 + 2 \times 300}{3} = 600\ K$$

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#5

(c) Let $Q_{H_1} = Q_{H_3} = 100 \text{ kJ}$

$$W_1 = Q_{H_1}\left(1 - \frac{T_R}{T_H}\right) = 100 \times \left(1 - \frac{600}{1200}\right) = 50 \text{ kJ}$$

$W_1 = Q_{H_1} - Q_{L_1}$ $\quad \therefore 50 = 100 - Q_{L_1}$ $\quad \therefore Q_{L_1} = 50 \text{ kJ}$

$Q_{H_2} = Q_{L_1} = 50 \text{ kJ}$

$$W_2 = Q_{H_2}\left(1 - \frac{T_L}{T_R}\right) = 50 \times \left(1 - \frac{300}{600}\right) = 25 \text{ kJ}$$

$\therefore W_1 = 50 \text{ kJ}, \quad W_2 = 25 \text{ kJ}, \quad W_3 = 75 \text{ kJ}$

The combined work output of HE1 and HE2 $W_1 + W_2$ is the same as the work ouput of HE3. i.e., $W_1 + W_2 = W_3$

Also, $\eta_{HE1} = 1 - \frac{T_R}{T_H} = 1 - \frac{600}{1200} = 0.5$

$\eta_{HE2} = 1 - \frac{T_L}{T_R} = 1 - \frac{300}{600} = 0.5$