Summer 2013 Term Test 1

1. (a) For the curve, $z^2 = x^2 + y^2$, identify the type of curve and give a sketch.

   (b) Find the projection of the intersection of $z^2 = x^2 + y^2$ and $x^2 + y^2 - 2y = 0$ in the yz-plane.

2. Let l_1 be the line

$$\frac{x-5}{2} = 3 - y = \frac{z+3}{3}$$

   and l_2 be the line of intersection of the planes

$$x + y = 3 \quad \text{and} \quad 2x - y + z = 0.$$

   (a) Find parametric equations for the line l_2.

   (b) Determine whether l_1 and l_2 are intersecting, parallel or skew.

3. Let l_1 be the line

$$\frac{x-3}{2} = \frac{y+5}{-4} = \frac{z-6}{-2}$$

   and l_2 has parametric equations

$$x = 1 - t \quad y = 3 + 2t \quad x = 1 + t.$$

   Find the distance between the lines l_1 and l_2.

4. Find a *vector representation* for the curves of intersection of $x = \sqrt{9 - y^2 - z^2}$ and $y^2 - 2y + z^2 = 3$ directed so that y increases when z is positive.

5. Let a curve C be defined by a position vector $\mathbf{r}(t) = \langle 3t, -2\sin t, 2\cos t \rangle$.

   (a) Find the tangent vector.

   (b) Find the unit tangent vector at the point $(0, 0, 2)$.

   (c) Find the length of the curve C from the point $(0, 0, 2)$ to $(9\pi, 0, -2)$.

6. Find the following limits, or show why they do not exist.

   (a) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{-3xy}{2x^2 + y^2}$

   (b) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{e^{x^2+y^2} - 1}{2x^3 + 2xy^2 - 3x^2 - 3y^2}$

Solutions

1. (a) Cone

   (b) $y = \dfrac{z^2}{2}, x = 0, 0 \leq y \leq 2$

2. (a) Some common answers.

   $$x = t, y = 3 - t, z = 3 - 3t \text{ or } x = 3 - t, y = t, z = -6 + 3t \text{ or}$$

   $$x = 3 + t, y = -t, z = -6 - 3t \text{ or } x = 1 + t, y = 2 - t, z = -3t$$

   (b) Skew.

3. $\dfrac{\sqrt{389}}{\sqrt{6}}$

4. A couple common answers

   $\mathbf{r}(t) = \langle \sqrt{4 - 4\cos t}, 1 + 2\cos t, -2\sin t \rangle$ or $\mathbf{r}(t) = \langle \sqrt{4 - 4\sin t}, 1 + 2\sin t, 2\cos t \rangle$

5. (a) $\mathbf{T}(t) = \langle 3, -2\cos t, -2\sin t \rangle$

   (b) $\dfrac{1}{\sqrt{13}} \langle 3, -2, 0 \rangle$

   (c) $3\sqrt{13}\pi$

6. (a) DNE

   (b) $-\dfrac{1}{3}$