Term Test 1

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TIME: 70 minutes
COURSE: MATH 2130
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Answers by Dawit

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[7] 1. Find standard equation of the plane containing the two points $P(2,3,2)$ and $Q(-1,4,1)$ which is perpendicular to the plane $2x + y + 2z = 6$.

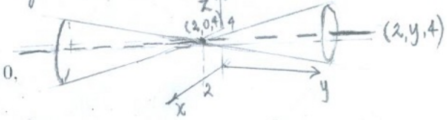
1. $3x + 4y - 5z = 8$

[7] 2. Identify and sketch the surface with the equation

$$x^2 - y^2 + z^2 - 4x - 8z + 20 = 0.$$

Mark the important points.

2. elliptic Cone (with axis $(2, y, 4)$)
$y^2 = (x-2)^2 + (z-4)^2$; Center $(2,0,4)$



[10] 3. Consider the point $P(3,3,1)$, the plane $\Pi: \quad 2x - y + 2z - 11 = 0$, and the line

$$\ell: \quad x = 1 + t \quad , \quad y = 4 - 2t \quad , \quad z = 2 - 2t\,.$$

If d_1 is the distance from the point P to the plane Π, and d_2 is the distance from the point P to the line ℓ, show that $d_1 = d_2^2$.

3. Show that $d_1 = 2$
$d_2 = \sqrt{2}$

[7] 4. Find a parametric representation for the curve

$$x - y = 4z\,, \qquad xy + 4z^2 = 0$$

directed so that z decreases along the curve.

4. $x = -2t$
$y = 2t$, $t \in \mathbb{R}$ (parameter)
$z = -t$

[7] 5. Find a tangent vector of length $5\sqrt{3}$ to the curve C with the vector representation

$$\mathbf{r}(t) = [\,e^{\pi - t}\ln(t - \pi + 1)\,]\,\hat{\mathbf{i}} + [\,e^{\pi - t}\cos t\,]\,\hat{\mathbf{j}} + [\,e^{\pi - t}\sin t\,]\,\hat{\mathbf{k}}$$

at the point $(0, -1, 0)$.

5. $\vec{T} = \langle -5, 5, -5 \rangle$

[12] 6. Given the curve C with vector representation

$$C: \quad \mathbf{r}(t) = 2t\,\hat{\mathbf{i}} + t^2\,\hat{\mathbf{j}} + (\frac{1}{3}t^3)\,\hat{\mathbf{k}}$$

[6] (a) Find the arc length of the curve C from the point $(2, 1, \frac{1}{3})$ to the point $(6, 9, 9)$.

a) $38/3$

[6] (b) First form $(3\,\mathbf{r}(\mathbf{t}) + \mathbf{r}''(\mathbf{t})) \times \mathbf{r}''(\mathbf{t})$ and then show that it is perpendicular to $\mathbf{r}(\mathbf{t})$ for all values of t.

b) $3\vec{r} \times \vec{r}'' = \langle 4t^3, -12t^2, 12t \rangle$
for perpendicularity
Show that $\vec{r} \cdot [3\vec{r} \times \vec{r}''] = 0$, i.e
$\langle 2t, t^2, t^3/3 \rangle \cdot \langle 4t^3, -12t^2, 12t \rangle = 0$