MATH 1210 Summer 2015 Quiz 2

[7] 1. Find $(-1+i\sqrt{3})^{10}$. Put your answer in Cartesian form.

Solution: First convert to exponential. The modulus of $-1+i\sqrt{3}$ is

$$\sqrt{(-1)^2+(\sqrt{3})^2}=\sqrt{1+3}=\sqrt{4}=2.$$

To find the argument we need to solve

$$\tan\theta=\frac{y}{x}=\frac{\sqrt{3}}{-1}=-\sqrt{3}$$

which has a solution of $\theta=\dfrac{2\pi}{3}$.

Hence

$$-1+i\sqrt{3}=2e^{2\pi i/3}.$$

This means

$$(-1+i\sqrt{3})^{10}=\left(2e^{2\pi i/3}\right)^{10}=2^{10}e^{20\pi i/3}=1024e^{2\pi i/3}.$$

Converting back to Cartesian leads to

$$\begin{aligned}(-1+i\sqrt{3})^{10}&=1024e^{2\pi i/3}\\&=1024(\cos 2\pi/3+i\sin 2\pi/3)\\&=1024\left(-\frac{1}{2}+i\frac{\sqrt{3}}{2}\right)\\&=-512+512i\sqrt{3}.\end{aligned}$$

[4] 2. Use the Remainder Theorem to find the reminder when $P(x)=2x^2+4$ is divided by $ix+(2i+1)$.

Solution: The remainder theorem says that the remainder is $P\left(-\dfrac{2i+1}{i}\right)$. Using that

$$-\frac{2i+1}{i}=-2-\frac{1}{i}=-2+i$$

we get the remainder is

$$\begin{aligned} P(-2+i) &= 2(-2+i)^2+4 \\ &= 2(4-4i+i^2)+4 \\ &= 2(4-4i-1)+4 \\ &= 2(3-4i)+4 \\ &= 6-8i+4 \\ &= 10-8i \end{aligned}$$

3. Let $P(x)$ be the polynomial $10x^3 - 11x^2 + 34x + 16$.

[3] (a) Find the possible rational roots of $P(x)$ using the rational root theorem.

Solution: Any rational root is of the form p/q where p divides 16 and q divides 10. Hence we get possibilities of

$$\pm 1,\ \pm 2,\ \pm 4,\ \pm 8,\ \pm 16,\ \pm\frac{1}{2},\ \pm\frac{1}{5},\ \pm\frac{2}{5},\ \pm\frac{4}{5},\ \pm\frac{8}{5},\ \pm\frac{16}{5},\ \pm\frac{1}{10}.$$

[6] (b) Given that $P(-2/5) = 0$, solve for all roots of $P(x)$.

Solution:
Since

$$P\left(-\frac{2}{5}\right) = 0.$$

we know that $5x+2$ is a factor of $P(x)$. Long division leads to

$$P(x) = (5x+2)(2x^2-3x+8).$$

The remaining quadratic yields solutions

$$x = \frac{3 \pm \sqrt{(-3)^2 - 4(2)(8)}}{2} = \frac{3 \pm \sqrt{-55}}{2} = \frac{3 \pm i\sqrt{55}}{2}.$$

Therefore the roots are

$$-\frac{2}{5},\ \frac{3 \pm i\sqrt{55}}{2}.$$