Limits at infinity. Asymptotes.

2.3.1

(a) $$\lim_{x\to\infty}\frac{x^2+1}{2x^3+5}=\lim_{x\to\infty}\frac{\frac{1}{x}+\frac{1}{x^3}}{2+\frac{5}{x^3}}=\frac{0+0}{2+0}=0$$

(b) $$\lim_{x\to-\infty}\frac{2+x-x^2}{3+4x^2}=\lim_{x\to-\infty}\frac{\frac{2}{x^2}+\frac{1}{x}-1}{\frac{3}{x^2}+4}=\frac{0+0-1}{0+4}=-\frac{1}{4}$$

(c) $$\lim_{x\to-\infty}\frac{\sqrt{1+2x^2}}{x+2}=\lim_{x\to-\infty}\frac{\sqrt{x^2\left(\frac{1}{x^2}+2\right)}}{x+2}=\lim_{x\to-\infty}\frac{-x\sqrt{\frac{1}{x^2}+2}}{x+2}=$$

$$=\lim_{x\to-\infty}\frac{-\sqrt{\frac{1}{x^2}+2}}{1+\frac{2}{x}}=\frac{-\sqrt{0+2}}{1+0}=-\sqrt{2}$$

(d) $$\lim_{x\to\infty}\left(\sqrt{x^2+4}-\sqrt{x^2-1}\right)=\lim_{x\to\infty}\left(\sqrt{x^2+4}-\sqrt{x^2-1}\right)\frac{\sqrt{x^2+4}+\sqrt{x^2-1}}{\sqrt{x^2+4}+\sqrt{x^2-1}}=$$

$$=\lim_{x\to\infty}\frac{(x^2+4)-(x^2-1)}{\sqrt{x^2+4}+\sqrt{x^2-1}}=\lim_{x\to\infty}\frac{5}{\sqrt{x^2+4}+\sqrt{x^2-1}}=0$$

2.3.2

(a)

$$\lim_{x\to\infty}\frac{1-1/x-6/x^2}{1-9/x^2}=\frac{1}{1}=1$$

(b)

$$\lim_{x\to-\infty}\frac{\sqrt{x^2(4-1/x^2)}}{x+10000}=\lim_{x\to-\infty}\frac{-x\sqrt{4-1/x^2}}{x\left(1+\frac{10000}{x}\right)}=\lim_{x\to-\infty}\frac{-\sqrt{4-1/x^2}}{1+\frac{10000}{x}}=-2$$

2.3.3

$\therefore \lim_{x\to\infty} f(x) = \lim_{x\to\infty} \frac{5x^2+1}{-2+x+x^2} = \lim_{x\to\infty} \frac{5+\frac{1}{x^2}}{\frac{-2}{x^2}+\frac{1}{x}+1} = \frac{5+0}{0+0+1} = 5$

Similarly, $\lim_{x\to-\infty} f(x) = 5$. $\underline{y=5}$ is the only <u>horizontal</u> asymptote

solve denominator $= 0$: $-2+x+x^2=0$ $x=1$ or $x=-2$

$\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} \frac{5x^2+1}{(x-1)(x+2)} = \frac{"6"}{0^+ \cdot 3} = +\infty$ $\underline{x=1}$ is a <u>vertical</u> asympto

$\lim_{x\to -2^+} f(x) = \lim_{x\to -2^+} \frac{5x^2+1}{(x-1)(x+2)} = \frac{"21"}{-3\cdot 0^+} = -\infty$ $\underline{x=-2}$ is a <u>vertial</u> asymptote