UNIVERSITY OF MANITOBA

DATE: April 13 , 2010 FINAL EXAMINATION

EXAMINATION: Engineering Mathematical Analysis 1 TIME: 3 hours

COURSE: MATH 2130 EXAMINER: G.I. Moghaddam

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[9] 1. Find the distance between the two lines

$$\frac{x-5}{-4}=\frac{y}{2}=\frac{z-3}{3} \qquad \text{and} \qquad \frac{x+8}{1}=\frac{y-1}{-1}=\frac{z-7}{-1}.$$

[11] 2. Let u and v be functions of x , y and z . Find $\dfrac{\partial u}{\partial z}$ if

$$x^2+y^5-xz-xu^3+y\,v^2=0$$
$$x^4+y^3+xz^2-y\,v^4=0\,.$$

Simplify your answer as much as possible.

[11] 3. Let $f(x,y,z)=2xy+\ln(xy)\,+z^2$ be a function of x, y and z .

(a) Find the direction in which f increases most rapidly at the point $(2,\frac{1}{2},1)$. What is the rate of change in that direction ?

(b) What is the rate of change of f in a direction perpendicular to the gradient of f ? Why?

[11] 4. Find **all** critical points for the function

$$f(x,y)=x^2+2y^2-x^2y\,.$$

Classify each critical point to determine if it is a relative maximum, a relative minimum, or a saddle point. Show your work.

[12] 5. Find the absolute maximum and the absolute minimum of the function

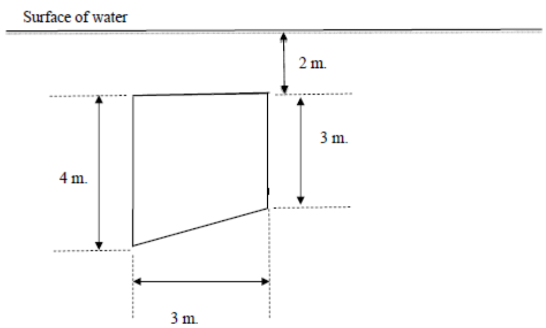
$$f(x,y)=x^2+2y^2$$

on the region bounded by the $y-$axis and $x+\,|y|\,=1\,.$

[9] 6. Evaluate the double integral

$$\int_0^1\int_{\sqrt{y}}^1 \frac{1}{1+x^3}\,dx\,dy\,.$$

[10] 7. Find the force due to water pressure on each side of a vertical plate in the form of a trapezoid in the figure below.



[10] 8. Find the surface area of that part of $z = \frac{2}{3}(x^{\frac{3}{2}} + y^{\frac{3}{2}})$ in the first octant cut off by the plane $x + y = 1$.

[7] 9. **Set up but do not evaluate** a triple integral to find the **mass** of a solid that lies within the cylinder $x^2 + y^2 = 1$, below the plane $z = 4$ and above the paraboloid $z = 1 - x^2 - y^2$. The density at any point is proportional to its distance from the $z-$axis. (Hint: you may use cylindrical coordinate system.)

[10] 10. Use **Spherical Coordinate System** to find the volume of the ice-cream cone that is bounded by the cone $\phi = \frac{\pi}{6}$ and the sphere $\Re = 2a\cos\phi$ of radius a.