ENG 1460 Introduction to Thermal Sciences **Fall 2011** **Term Test 2**

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Duration: 110 min

1. You are permitted to use the textbook for the course, a one-page double-sided aide sheet, and a calculator.
2. Ask for clarification if any problem statement is unclear.
3. Clear, systematic solutions are required. **Show your work**. Marks will not be assigned for problems that require unreasonable (in the opinion of the instructor(s)) effort for the marker to decipher.
4. Use linear interpolation in the property tables as necessary.
5. Keep 5 significant figures in intermediate calculations, and use 4 or 5 significant figures in final answers. For temperature, keep two decimal places in your final answer.
6. There are three problems on this test. The weight of each problem is indicated. The test will be marked out of **85**.

1. *(18 marks)*

A spherical balloon contains nitrogen gas. At the initial state (State 1), the pressure of the nitrogen gas is $P_1 = 100$ kPa and the diameter of the balloon is $D_1 = 2$ m. Then the nitrogen gas undergoes a polytropic process to reach the final state (State 2). At State 2, the pressure and temperature of the nitrogen gas are $P_2 = 500$ kPa and $T_2 = 150$ ^oC, respectively; and the final diameter of the balloon is 2/3 of its initial diameter, i.e. $D_2/D_1 = 2/3$. The nitrogen can be treated as an ideal gas with constant specific heats. The formula for calculating the volume of a sphere is $V = \frac{1}{6}\pi D^3$.

(a) Determine the value of the exponent/index of the polytropic process, n. *(4 marks)*
(b) Determine the mass of the nitrogen, m, in kg. *(2 marks)*
(c) Determine the initial temperature of the nitrogen gas, T_1, in ^oC. *(1.5 marks)*
(d) Determine the total work of the process, ${}_1W_2$, in kJ. *(3 marks)*
(e) Determine the total heat transfer of the process, ${}_1Q_2$, in kJ. *(3.5 marks)*
(f) Show both state points and the process on a P–V (pressure–volume) diagram;
Indicate the area that represents the total work of the process on the P-V diagram;
Label the pressure and volume values for both states. *(4 marks)*

2. *(35 marks)*

A frictionless piston-cylinder and linear-spring assembly containing neon gas is shown in the figure. A tungsten plate uniformly covers the bottom of the cylinder, and this tungsten plate is **always** in thermal equilibrium with the neon gas. The mass of the tungsten plate is $m_T = 0.5$ kg. The distance from the top surface of the tungsten plate to the bottom surface of the piston is denoted as y. At the initial state (State 1), the piston is in mechanical equilibrium with its surroundings (the piston floats and is not moving), with $y_1 = 0.5$ m and the temperature of the neon gas being $T_1 = 30.3$ ^oC. Then heat is transferred to the neon gas and tungsten plate from the outside, and the piston begins to move up. At the exact moment when the piston just begins to touch the spring, the state of the neon gas is referred to as State 2. At State 2, $y_2 = 0.8$ m. When the piston just begins to touch the stops, the state of the neon gas is referred to as State 3. At State 3, $y_3 = 1.1$ m and $P_3 = 200$ kPa. The neon gas and tungsten plate continue to receive heat from the outside until the final state (State 4) is reached, when the temperature of the neon gas becomes $T_4 = 1100$ ^oC. The inner cross-sectional area of the cylinder is $A = 0.25$ m^2. The piston mass is $m_p = 509.7$ kg. The local gravitational acceleration is $g = 9.81$ m/s^2 and the local atmospheric pressure is $P_0 = 100$ kPa. The neon gas can be treated as an ideal gas with constant specific heats.

(a) Determine the mass of the neon gas, m_N, in kg. *(5 marks)*
(b) Determine the temperature of the neon gas at State 2, T_2, in ^oC. *(4 marks)*
(c) Determine the spring-constant of the linear spring, k_s, in kN/m. *(4 marks)*
(d) Determine the temperature of the neon gas at State 3, T_3, in ^oC. *(1.5 marks)*
(e) Determine the pressure of the neon gas at State 4, P_4, in kPa. *(2.5 marks)*
(f) Determine the total work done by the neon gas, ${}_1W_4$, in kJ. *(6 marks)*

(g) Determine the total heat received by the neon gas and tungsten plate, ${}_1Q_4$, in kJ. *(5 marks)*

(h) Show all state points and all processes on a P-V (pressure-volume) diagram;
Indicate the area that represents the total work, ${}_1W_4$;
Show isotherms (constant temperature curves/lines) for all 4 states;
Label the pressure, volume and temperature values for all states. *(7 marks)*

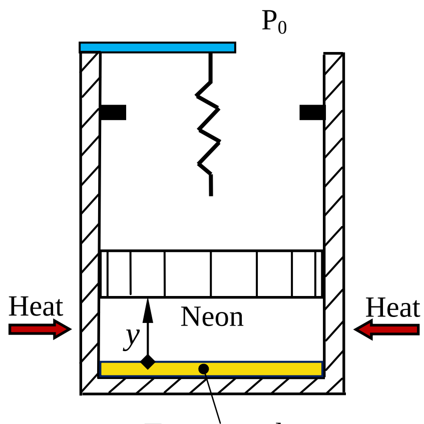


Figure for Problem 2

3. *(32 marks)*

A piston-cylinder assembly (compartment A) and a rigid tank (compartment B) contain water. The piston has negligible mass. The top surface of the piston is exposed to the atmosphere. The local atmospheric pressure is $P_0 = 100$ kPa. A thin pipe of negligible volume connects the two compartments, with a valve in the middle controlling the flow rate. Initially (State 1), the valve is closed, the water inside compartment A has temperature $T_{A,1} = 10$ °C and mass $m_{A,1} = 0.35$ kg, and the water inside compartment B has pressure $P_{B,1} = 50$ kPa, volume $V_{B,1} = 1.65\ m^3$ and mass $m_{B,1} = 0.55$ kg. The valve opens slowly and the system undergoes a quasi-equilibrium process. At the end of this process (State 2), the volume inside compartment A is $V_{A,2} = 0.15\ m^3$.

(a) Determine the initial volume of compartment A, $V_{A,1}$, in m^3. (*5.5 marks*)

(b) Determine the work ${}_1W_2$ done during this process, in kJ. Is this work done *by* the water or *on* the water? (*5 marks*)

(c) Determine the final temperature of the water at State 2, T_2, in °C. *(6.5 marks)*

(d) Determine the heat transfer ${}_1Q_2$ during this process, in kJ. Is this heat transferred *into* the water or *out of* the water? (*15 marks*)

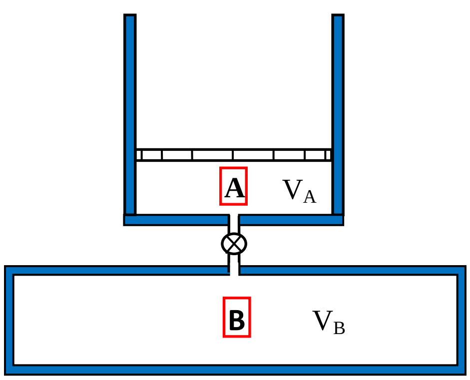


Figure for Problem 3

$\boxed{\text{Problem 1}}$

From Table A 5, for nitrogen, $R = 0.2968 \frac{kJ}{kg \cdot K}$, $C_{V0} = 0.745 \frac{kJ}{kg \cdot K}$

(a) $\left.\begin{aligned} P_1 V_1^n &= P_2 V_2^n \\ V_1 &= \tfrac{1}{6}\pi D_1^3 \\ V_2 &= \tfrac{1}{6}\pi D_2^3 \end{aligned}\right\} \Rightarrow \frac{P_1}{P_2} = \left(\frac{V_2}{V_1}\right)^n = \left(\frac{D_2}{D_1}\right)^{3n}$

$\therefore \frac{100}{500} = \left(\frac{2}{3}\right)^{3n}$

$\therefore \ln\left(\frac{100}{500}\right) = 3n \ln\left(\frac{2}{3}\right)$

$\therefore n = 1.323$

(b) $V_2 = \frac{1}{6}\pi D_2^3 = \frac{1}{6}\pi \times \left(\frac{2}{3}\times 2\right)^3 = 1.2411 \ (m^3)$

$m = \frac{P_2 V_2}{R T_2} = \frac{500 \times 1.2411}{0.2968 \times (273.15+150)} = 4.9411 \ (Kg)$

(c) $V_1 = \frac{1}{6}\pi D_1^2 = \frac{1}{6}\pi \times 2^3 = 4.1888 \ (m^3)$

$T_1 = \frac{P_1 V_1}{m R} = \frac{100 \times 4.1888}{4.9411 \times 0.2968} = 285.63 \ K$

or, $12.48\ (^\circ C)$

[Alternative method

$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{n-1} \Rightarrow T_1 = T_2 \cdot \left(\frac{V_2}{V_1}\right)^{n-1} = T_2 \cdot \left(\frac{D_2}{D_1}\right)^{3(n-1)}$

$\therefore T_1 = (273.15+150) \times \left(\frac{2}{3}\right)^{3\times(1.323-1)}$

$= 285.67 \ K$

or, $12.52\ (^\circ C)$]

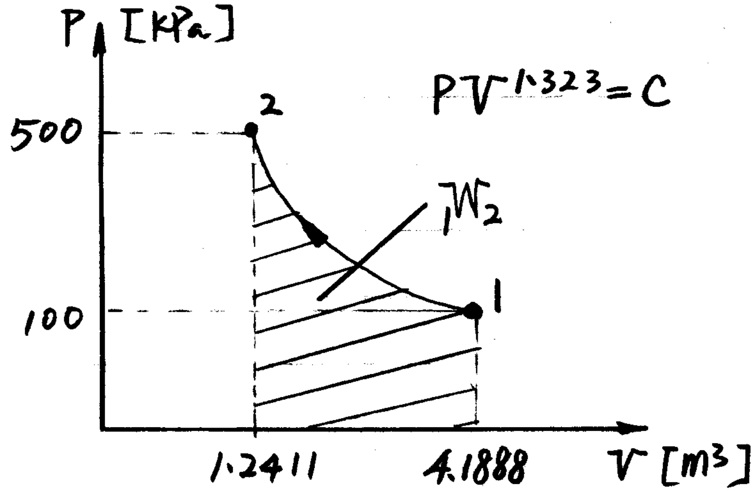
(d) ${}_1W_2 = \frac{P_2 V_2 - P_1 V_1}{1-n} = \frac{500 \times 1.2411 - 100 \times 4.1888}{1-1.323} = -624.37 \ (kJ)$

(e) ${}_1Q_2 = {}_1W_2 + m C_{V0} (T_2 - T_1)$

$= -624.37 + 4.9411 \times 0.745 \times (150 - 12.48)$

$= -118.14 \ (kJ)$

(f)



$\boxed{\text{Problem 2}}$

Table A5: for the neon gas, $R = 0.412 \frac{kJ}{kg \cdot K}$, $C_{V_0} = 0.618 \frac{kJ}{kg \cdot K}$

Table A3: for the tungsten plate, $C_T = 0.13 \frac{kJ}{kg \cdot K}$

(a) $$P_1 = P_0 + \frac{m_p g}{A} = 100 + \frac{509.7 \times 9.81}{0.25} \frac{1}{1000} = 120.0 \text{ (kPa)}$$

$$V_1 = A y_1 = 0.25 \times 0.5 = 0.125 \text{ (m}^3\text{)}$$

$$m_N = \frac{P_1 V_1}{R T_1} = \frac{120.0 \times 0.125}{0.412 \times (30.3 + 273.15)} = 0.12 \text{ (kg)}$$

(b) $P_2 = P_1 = 120 \text{ KPa}$ (isobaric process)

$$V_2 = A y_2 = 0.25 \times 0.8 = 0.2 \text{ (m}^3\text{)}$$

$$T_2 = \frac{P_2 V_2}{m R} = \frac{120 \times 0.2}{0.12 \times 0.412} = 485.44 \text{ (K)}$$

or, 212.29 (°C)

(C) $$V_3 = A y_3 = 0.25 \times 1.1 = 0.275 \text{ (m}^3\text{)}$$

$$P_3 = P_2 + \frac{k_s}{A^2}(V_3 - V_2)$$

$$200 = 120 + \frac{k_s}{0.25^2}(0.275 - 0.2)$$

$$\therefore K_s = 66.667 \text{ (KN/m)}$$

[Alternative method $$P_3 = P_2 + \frac{k_s}{A}(y_3 - y_2)$$

$$200 = 120 + \frac{k_s}{0.25}(1.1 - 0.8)$$

$$\therefore k_s = 66.667 \text{ kN/m}$$]

(d) $$T_3 = \frac{P_3 V_3}{m R} = \frac{200 \times 0.275}{0.12 \times 0.412} = 1112.46 \text{ (K)}$$

or, 839.31 (°C)

(e) $V_4 = V_3 = 0.275\ m^3$ (isochoric process)

$$P_4 = \frac{mRT_4}{V_4} = \frac{0.12 \times 0.412 \times (1100+273.15)}{0.275} = 246.87\ (kPa)$$

(f) $_1W_4 = {_1W_2} + {_2W_3} + \cancel{_3W_4}^{\,0}$

$_1W_2 = P_1(V_2 - V_1) = 120.0 \times (0.2 - 0.125) = 9\ (kJ)$

$_2W_3 = \frac{1}{2}(P_2 + P_3)\cdot(V_3 - V_2)$

$= \frac{1}{2}(120.0 + 200) \times (0.275 - 0.2)$

$= 12\ (kJ)$

$\therefore\ _1W_4 = {_1W_2} + {_2W_3} = 9 + 12 = 21\ (kJ)$

(g) $_1Q_4 = {_1W_4} + \Delta U$

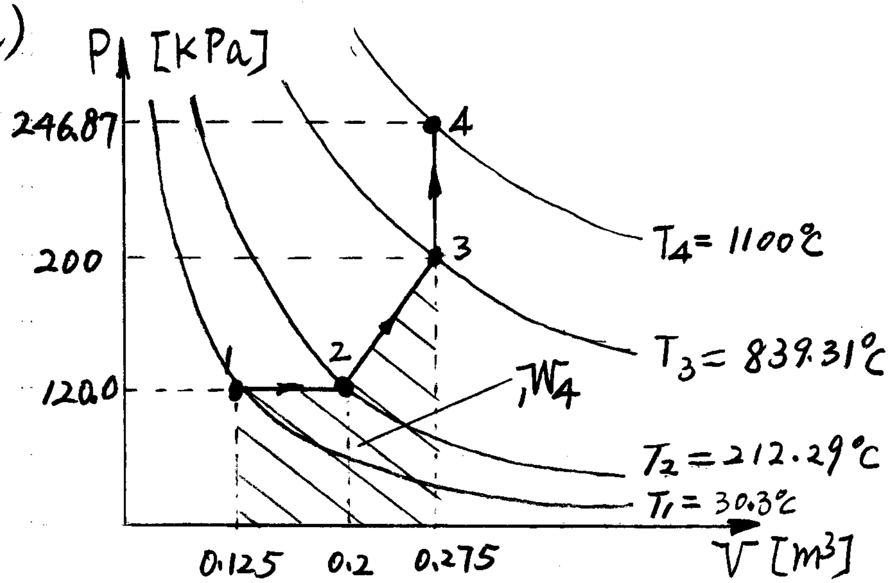
$= {_1W_4} + [m_N C_{V0}(T_4 - T_1) + m_T C_T (T_4 - T_1)]$

$= {_1W_4} + (m_N C_{V0} + m_T C_T)(T_4 - T_1)$

$= 21 + (0.12 \times 0.618 + 0.5 \times 0.13)(1100 - 30.3)$

$= 169.86\ (kJ)$

(h)



Problem 3

(a) $P_{A1} = P_0 + \cancel{\frac{mg}{A}}^{\,0} \approx P_0 = 100\,kPa$

Table B.1.2: At $P_{A1} = 100\,kPa$, $T_{sat} = 99.62°C$

$T_{A1} = 10°C < T_{sat}$

$\therefore$ Compress Liquid

using the method of approximation:

At $T_{A1} = 10°C$, $u_{A1} \approx u_f|_{10°C} = 41.99\ kJ/kg.K$

$v_{A1} \approx v_f|_{10°C} = 0.001000\ m^3/kg$

$$V_{A1} = m_{A1} v_{A1} = 0.35 \times 0.00100 = 0.00035\,(m^3)$$

(b) $P_2 = P_{A2} = P_{B2} = P_{A1} = 100\,kPa$ (isobaric process)

${}_1W_2 = P_{A1}(V_{A2} - V_{A1}) = 100 \times (0.15 - 0.00035) = 14.965\ (kJ)$

B/C ${}_1W_2 > 0$, work done by the system

(c) $V_2 = V_{A2} + V_B = 0.15 + 1.65 = 1.80\ (m^3)$

$m_2 = m_{A1} + m_{B1} = 0.35 + 0.55 = 0.9\ (kg)$

$v_2 = \frac{V_2}{m_2} = \frac{1.80}{0.9} = 2\ m^3/kg$

Table B.1.2, At $P_2 = 100\,kPa$,

$v_f = 0.001043\ m^3/kg$, $v_g = 1.69400\ m^3/kg$

$v_2 > v_g$

$\therefore$ Superheated Vapour

Table B.1.3

T[°C]	v [m³/kg]	u [kJ/kg.K]
150	1.93636	2582.75
T_2	2	u_2
200	2.17226	2658.05

Interpolations

$T_2 = 150 + \frac{2 - 1.93636}{2.17226 - 1.93636}(200 - 150)$

$= 163.49\ [°C]$

$u_2 = 2582.75 + \frac{2 - 1.93636}{2.17226 - 1.93636}(2658.05 - 2582.75)$

$= 2603.06\ (kJ/kg.K)$

(d) Initial state of B,

$$v_{B1} = \frac{V_B}{m_{B1}} = \frac{1.65}{0.55} = 3.00 \ (m^3/kg)$$

Table B.1.2 At $P_{B1} = 50$ KPa,

$v_f = 0.001030 \ m^3/kg$, $v_{fg} = 3.23931 \ m^3/kg$, $v_g = 3.24034 \ m^3/kg$

$u_f = 340.42 \ KJ/kg.K$, $u_{fg} = 2143.43 \ KJ/kg.K$, $u_g = 2483.85 \ KJ/kg.K$

$v_f < v_{B1} < v_g$

∴ Saturated mixture

$$x_{B1} = \frac{v_{B1} - v_f}{v_{fg}} = \frac{3.00 - 0.001030}{3.23931} = 0.92581$$

$$u_{B1} = u_f + x_{B1} u_{fg} = 340.42 + 0.92581 \times 2143.43 = 2324.83 \ KJ/kg.K$$

$$_1Q_2 = {}_1W_2 + \Delta U$$
$$= {}_1W_2 + [m_2 u_2 - (m_{A1} u_{A1} + m_{B1} u_{B1})]$$
$$= 14.965 + [0.9 \times 2603.06 - (0.35 \times 41.99 + 0.55 \times 2324.83)]$$
$$= 1064.37 \ (KJ)$$

Heat is absorbed into the system, b/c $_1Q_2 > 0$.