16.107 April 1998 Final

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A1 Simple harmonic motion $x = x_m \cos(\omega t + \phi)$

at $t=0$, $x=0$ $\therefore$ $\cos\phi = 0$, $\phi = \frac{\pi}{2}$ or $\frac{3\pi}{2}$

at $t=0$, $v_x < 0$; $v_x = \frac{dx}{dt} = -\omega x_m \sin(\omega t + \phi)$

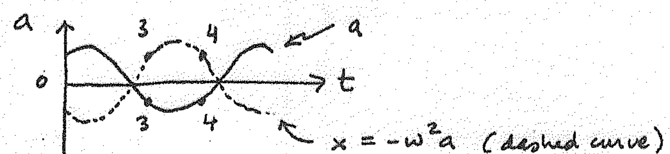
at $t=0$ $v_x = -\omega x_m \sin\phi$ $\therefore \sin\phi > 0$

$\therefore \phi = \pi/2$ (a)

A2 acceleration $a = \frac{d^2x}{dt^2} = -\omega^2 x$

$\therefore a < 0$ when $x > 0$.

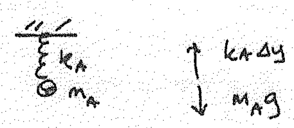
On the graph, this eliminates all but 2 possibilities, 3 and 4. But we know a, so we can sketch x



So as t increases from 3, x increases, getting farther from x=0 (equilibrium). Therefore, point 4 is the only point with $x>0$ and moving towards $x=0$, as required

$\therefore$ point 4 (d)

A3 At equilibrium (static):



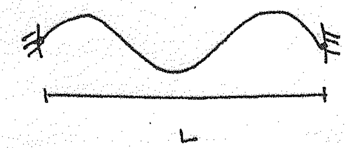
forces balance. $k_A \Delta y = mg$

$k_B \Delta y = mg/2$

$\therefore k_A/k_B = 2$ from this info.

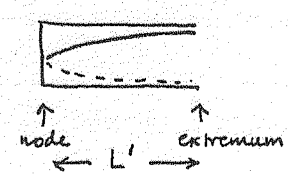
When oscillating, $E = \frac{1}{2} k y_{max}^2$ $\therefore \frac{E_A}{E_B} = \frac{k_A}{k_B} = 2$ (b)

A4 string of length L, 3rd harmonic:



Ends are tied, so $\boxed{L = \frac{3\lambda}{2}}$

open pipe, length L', closed at one end:



$L' = \frac{\lambda}{4}, \frac{\lambda}{4} + \frac{\lambda}{2} \ldots$

$= \frac{n\lambda}{4}$ where $n = 1, 3, 5 \ldots$

but in this problem $L' = \frac{L}{2}$ $\therefore$ $\frac{L}{2} = \frac{n\lambda}{4}$ or $L = \frac{n\lambda}{2}$

compare and deduce that $n=3$ for the pipe (b)

A5 $f = 200$ Hz. Speed of sound $v = 340$ m/s

source moves away at $v_s = 80$ m/s

$\therefore$ frequency <u>drops</u>: $f' = \frac{f v}{v + v_s}$

wavelength ? $f'\lambda' = v$

$\lambda' = \frac{v}{f'} = \frac{v + v_s}{f} = 2.1$ m (d)

A6 $y = A\cos(kx - \omega t)$

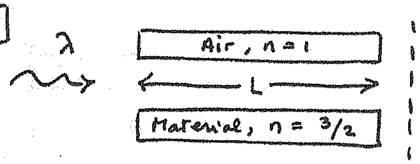
phase difference between 2 points is $\boxed{\Delta\phi = k\,\Delta x}$

frequency $f = 40$ Hz ; $\lambda f = v$

$k = \frac{2\pi}{\lambda} = \frac{2\pi f}{v}$ $\therefore$ $\frac{2\pi f}{v}\Delta x = \Delta\phi = \frac{\pi}{6}$ rad

$\therefore$ $v = 12 f\,\Delta x = 24$ m/s (c)

A7



after length L,

phase diff. $\Delta\phi = k_{mat}L - k_{air}L$

$\boxed{\Delta\phi = \Delta k . L}$

Now $k = \frac{2\pi}{\lambda} = 2\pi f \cdot \frac{n}{c}$ for light

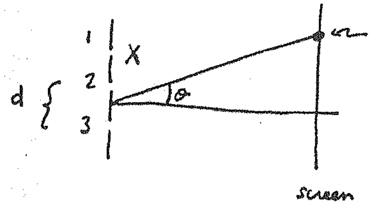
Air: $n = 1$ $k_{air} = 2\pi/\lambda$

Material: $n = \frac{3}{2}$ $k_{mat} = 2\pi f \cdot \frac{n}{c} = 2\pi \frac{c}{\lambda}\frac{n}{c} = \frac{2\pi n}{\lambda} = \frac{3\pi}{\lambda}$

$\therefore \Delta\phi = \left(\frac{3\pi}{\lambda} - \frac{2\pi}{\lambda}\right)L = \frac{\pi L}{\lambda}$ e)

A8 3 slit "spectrometer", but one is closed in each case

(i) close end slit. 1st Max. is at $\lambda = d\sin 60°$

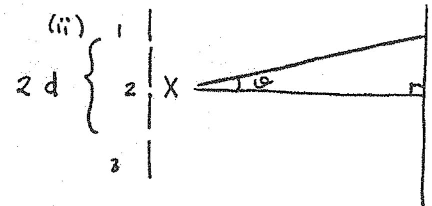


$\theta = 60°$, 1st Max

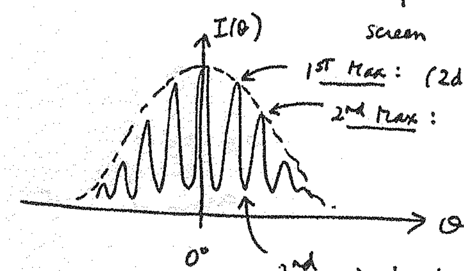
This tells us λ

(A8 cont'd) Now cover the middle slit:

(ii)



2nd order Minimum



1st Max: $(2d)\sin\theta = \lambda$

2nd Max: $(2d)\sin\theta = 2\lambda$

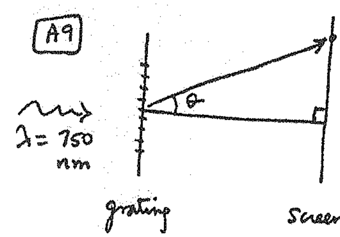
2nd min: halfway in between!

$\therefore$ $(2d)\sin\theta = \frac{3}{2}\lambda$; find this θ.

$2d\sin\theta = \frac{3}{2}(d\sin 60°)$ with $\lambda = d\sin 60°$

$\therefore$ $\sin\theta = \frac{3}{4}\sin 60°$ $\theta = 40.5°$ (b)

A9



nth bright line : $n\lambda = d\sin\theta$

$d = \left(\frac{4000 \text{ lines}}{\text{cm}}\right)^{-1} = 2.5\times10^{-6}$ m

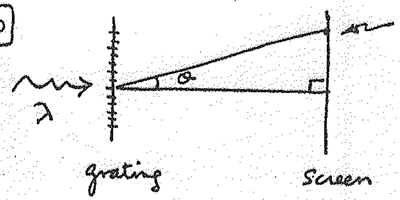
max. n is when $\theta = 90°$, $\sin\theta = 1$

then $n = \frac{d}{\lambda} = 3.3$ @ 90°

But in practise, $\theta < 90°$ and n must be an integer

$\therefore$ $n = 3$ (c)

A10



$n = 1$, first order line
$\theta = 30°$

$$\lambda = d \sin 30°$$

$$d = \left(\frac{1\text{ cm}}{10^4\text{ lines}}\right) = 10^{-6}\text{ m}$$

$$\therefore \lambda = 10^{-6}\text{ m} \sin 30 = 500\text{ nm} \quad (c)$$

A11 Doppler shift for light ; $\lambda f = c$

$$\therefore \lambda = \lambda_0 \sqrt{\frac{1-\beta}{1+\beta}} \quad ; \quad \beta = v/c$$

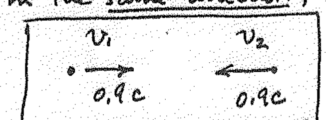
red light $\lambda_0 = 650$ nm ; shifted light appears $\lambda = 525$ nm

$$\left(\frac{\lambda}{\lambda_0}\right)^2 = \frac{1-\beta}{1+\beta} = 0.652 \rightarrow \beta = 0.210$$

or $v = 0.210c$ (c)

A12 Relativistic velocity transformation
for frame and object moving in the same direction,

$$u = \frac{u' + V}{1 + u'V/c^2}$$



u = velocity measured in fixed frame
u' = " " moving frame
V = " of the frame

so, observe v_1' from the viewpoint of v_2

$$v_1 = \frac{v_1' - v_2}{1 + \frac{v_1' v_2}{c^2}}$$

(sign change since v_2 moves towards #1)

So $$0.9c = \frac{v_1' - 0.9c}{1 - 0.9 v_1'/c}$$

(find v_1', the speed of ship 1 relative to ship 2)

$$0.9c - 0.81 v_1' = v_1' - 0.9c$$

$$\therefore \frac{v_1'}{c} = \frac{1.80}{1.81} = 0.995 \quad (b)$$

A13 Electron $m_e c^2 = 0.511 \times 10^6$ eV
Total energy $E_{TOT} = 0.6 \times 10^6$ eV $= \gamma m_e c^2$

$$\therefore \gamma = 1.174 = \frac{1}{\sqrt{1 - v^2/c^2}}$$

where v = electron speed ; $\gamma^2 = \frac{1}{1 - v^2/c^2}$

$$\therefore v/c = 0.524 \quad (d)$$

A14 photon : $\lambda = 550 \times 10^{-9}$ m
energy E, all kinetic $= \frac{hc}{\lambda}$

Conversion factor : $\boxed{hc = 1.242 \times 10^{-6}\text{ eV}\cdot\text{m}}$

$\therefore E = K = 2.26$ eV , small compared to e^- rest mass.
$\therefore$ electron with the same kinetic energy is <u>nonrelativistic</u>

$$K = p^2/2m \quad \text{and} \quad \lambda_{deB} = h/p = \frac{h}{\sqrt{2mK}}$$

$$\therefore \lambda_{de.B.} = \frac{h}{\sqrt{2m \cdot \frac{hc}{\lambda}}} = \sqrt{\frac{hc\lambda}{2(mc^2)}} \quad \text{for the electron}$$

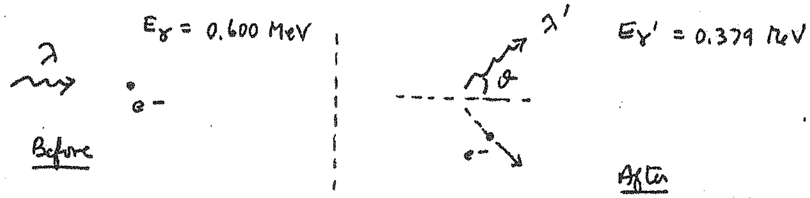
$$\therefore \lambda_{de B.} = \sqrt{\frac{(4.14 \times 10^{-15}\text{ eV.s})(3 \times 10^8\text{ m/s})(550 \times 10^{-9}\text{ m})}{2\,(0.511 \times 10^6\text{ eV})}} = 8.2 \times 10^{-10}\text{ m}$$

$$= 0.82\text{ nm} \quad (b)$$

16.107 Final Examination (April 16, 1998) - Solutions

A15 Compton scattering wavelength shift of photon:

$$\Delta\lambda = \frac{h}{mc}(1-\cos\theta) = \frac{hc}{(mc^2)}(1-\cos\theta)$$



$E = \frac{hc}{\lambda}$ $\therefore \lambda = \frac{hc}{E}$

$\lambda = 2.07 \times 10^{-12}$ m

$\lambda' = hc/E' = 3.28 \times 10^{-12}$ m

$\therefore \Delta\lambda = \lambda' - \lambda = 1.21 \times 10^{-12}$ m

$$(1-\cos\theta) = \frac{(mc^2)(\Delta\lambda)}{hc} = 0.498 \rightarrow \theta = 60^\circ \text{ (e)}$$

A16 Barrier tunnelling: $T = e^{-2kL}$ = transmission probability

where $k = \sqrt{\frac{8\pi^2 m (E_{pot} - E)}{h^2}}$; L = barrier width = 0.75 nm

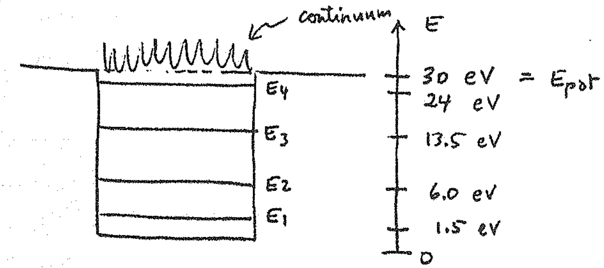
$(E_{pot} - E) = 0.20$ eV $\rightarrow k = 2.29 \times 10^9\ \text{m}^{-1}$

$T = e^{-2kL} = 3.23 \times 10^{-2}$.

Number of electrons that get through the barrier is

$NT = 10^7 \times (3.23 \times 10^{-2}) = 3.23 \times 10^5$ (c)

A17



Energy to free a particle in the lowest state?

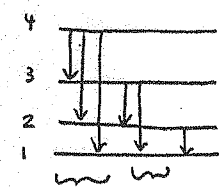
must put in $(E_{pot} - E_1) = 30 - 1.5 = 28.5$ eV (c)

A18 Longest wavelength photon that can be absorbed corresponds to the smallest transition energy:

$(E_2 - E_1) = 4.5$ eV

then $\lambda = \frac{hc}{E} = \frac{1.242 \times 10^{-6}\ \text{eV m}}{4.5\ \text{eV}} = 0.28\,\mu\text{m}$ (a)

A19 $n = 4 \rightarrow n = 1$ de-excitation



3 + 2 + 1 = 6 lines total

$\boxed{A20}$ $n = 2$ wavefunction $\psi_2 = \sqrt{\frac{2}{L}} \sin\left(\frac{2\pi x}{L}\right)$

$\psi^* \psi \, dx$ = probability of being within dx of x

$$\therefore \quad P = \int_{L/4}^{3L/4} \psi^* \psi \, dx = \frac{2}{L} \int_{L/4}^{3L/4} \sin^2\left(\frac{2\pi x}{L}\right) dx$$

let $\alpha = \frac{2\pi x}{L}$ $\quad d\alpha = \frac{2\pi}{L} dx$

α ranges from $\pi/2 \rightarrow 3\pi/2$

$$\therefore \quad P = \frac{2}{L} \cdot \frac{L}{2\pi} \cdot \underbrace{\int_{\pi/2}^{3\pi/2} \sin^2\alpha \, d\alpha}_{\text{integral is } \pi/2}$$

$$\therefore \quad P = \frac{2}{L} \cdot \frac{L}{2\pi} \cdot \frac{\pi}{2} = \frac{1}{2} \quad (b)$$

Part B

B1

a) The special theory of relativity is based on two postulates:
   1. The laws of physics are the same for observers in all inertial reference frames.
   2. The speed of light in vacuum has the same value, c, in all directions and in all inertial reference frames.

b) Since the Lorentz transformation for each event is

$x_1' = \gamma (x_1 - v t_1)$, $y_1' = y_1$, $z_1' = z_1$, $t_1' = \gamma(t_1 - v x_1/c^2)$

$x_2' = \gamma (x_2 - v t_2)$, $y_2' = y_2$, $z_2' = z_2$, $t_2' = \gamma(t_2 - v x_2/c^2)$

Then

$\Delta x' = x_2' - x_1' = \gamma(\Delta x - v\,\Delta t)$
$\Delta y' = \Delta y = 0$
$\Delta z' = \Delta z = 0$
$\Delta t' = t_2' - t_1' = \gamma(\Delta t - v\,\Delta x/c^2)$

c) Simultaneous in frame S' means $\Delta t' = 0 = \gamma(\Delta t - v\,\Delta x/c^2)$

hence $\Delta t = v\,\Delta x/c^2$

$c\Delta t/\Delta x = v/c \le 1$

$\Delta x \ge c\,\Delta t$ in frame S

d) Events at the same place in S means $\Delta x = \Delta y = \Delta z = 0$

hence $\Delta t' = \gamma\Delta t \ge \Delta t$ since $\gamma \ge 1$

Δt is the proper time since the events occur at the same place.

e) Events that are simultaneous in S means $\Delta t = 0$.

hence $\Delta x' = \gamma\Delta x \ge \Delta x$ since $\gamma \ge 1$

$\Delta x'$ is the proper length if both x_1' and x_2' are at rest in frame S'

B2. (a) This type of intensity pattern is given on page 940 and in your double slit experiment.

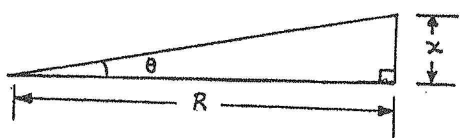
For the *second* minimum, one divides the aperture, a, by 4 to derive the angle for the minimum (see text, p. 932, fig. 37-5).

$$a \sin\theta_{2m} = 4\left(\frac{\lambda}{2}\right) = 2\lambda$$

$$\sin\theta_{2m} = \frac{2\lambda}{a} = \frac{2\times632.8\times10^{-9}}{8\times10^{-6}} = 2\times7.91\times10^{-2} = 1.582\times10^{-1}$$

$$\sin\theta \approx \theta \approx \boxed{\tan\theta = 1.602} \qquad \boxed{\theta \approx 9.06°}$$

$$x_{2d} = R(2\times7.91\times10^{-2}) = \boxed{48.07 \text{ cm}}$$



(b) $$d \sin\theta = n\lambda$$

$$\sin\theta = \left(\frac{\lambda}{d}\right) = 1.26\times10^{-2} = 0.725°$$

$$x_{int} = R(1.26\times10^{-2}) = \boxed{3.78 \text{ cm}}$$

(c) Distance centre to 1st **diffraction** minimum ~ 24.0 cm

Distance centre to 1st **interference** minimum ~ 3.78 (and same distance to the next.

∴ 6 fringes on each side plus the central max.

Central + 6 fringes on either side

$$\boxed{\sim 13}$$

(d) $$d \sin\theta = n\lambda \qquad d = \frac{1\times10^{-3}}{600} = 1.67\times10^{-6} \text{ m.}$$

$$\sin\theta_1 = \frac{632.8\times10^{-9}}{1.667\times10^{-6}} = 3.80\times10^{-1}$$

$$\theta_1 = 22.314° \qquad x_1 = (\tan\theta)R = \boxed{1.23 \text{ m}}$$

$$\sin\theta_2 = .7594$$

$$\theta_2 = 49.4°$$

$$x_2 = R\tan\theta_2 = \boxed{3.50 \text{ m}}$$

$$\boxed{\Delta x = 2.27 \text{ m}}$$

(e) Width of grating = Nd. See derivation in the text (p. 943, fig. 37-21).

$$\frac{Nd}{2}\sin\theta_{hw} = \frac{\lambda}{2}$$

$$\sin\theta_{hw} = \frac{\lambda}{Nd}$$

$$\boxed{\theta_{hw} = \frac{\lambda}{Nd}}$$