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[6] 1. Evaluate $\displaystyle\lim_{x\to 2}\frac{x^3-3x^2+4}{x^2-4x+4}$, if it exists. Show all calculations.

$$\lim_{x\to 2}\frac{x^3-3x^2+4}{x^2-4x+4}=\lim_{x\to 2}\frac{(x-2)(x^2-x-2)}{(x-2)^2}=\lim_{x\to 2}\frac{(x-2)(x+1)}{x-2}$$

$$=\lim_{x\to 2}(x+1)=\underline{3}$$

$$\begin{array}{l|l} x^3-3x^2+4 & x-2 \\ \hline x^3-2x^2 & x^2-x-2 \\ \quad -x^2 & \\ \quad -x^2+2x & \\ \qquad -2x+4 & \\ \qquad -2x+4 & \\ \qquad\quad 0 & \end{array}$$

[6] 2. Find all intervals on which the function $f(x)=\dfrac{x+3}{x^2+3}$ is increasing.

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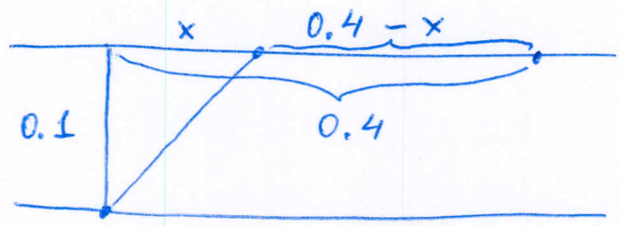
[6] 7. Alex can swim 5 km/h and run 12 km/h. He is standing on one bank of a river that is 100 m wide and wants to reach a point located 400 m downstream on the other side as quickly as possible. Alex will swim diagonally across the river and then run along the river bank. Alex wants to determine the best (i.e., the fastest) route to take (assume that the river flow is negligible).

[4] (a) Set up a function that needs to be minimized. State it in terms of only one variable.

[2] (b) Over what domain should this function be minimized?

DO NOT PROCEED ANY FURTHER IN ATTEMPTING TO SOLVE THIS PROBLEM (the set-up is all that is required).

100 m = 0.1 km
400 m = 0.4 km



Let x (in km) be the distance from the point opposite to Alex on the other side of the river to the point he will swim to.

$\sqrt{0.1^2 + x^2}$ – distance to swim

$0.4 - x$ – distance to run

(a) $$f(x) = \frac{\sqrt{0.1^2 + x^2}}{5} + \frac{0.4 - x}{12}$$

(b) $0 \le x \le 0.4$

[19] 8. Evaluate the following integrals (simplify your answers):

[3] (a) $\int \left(2\sqrt{x} + \frac{1}{x^3}\right) dx$ $= \int (2x^{\frac{1}{2}} + x^{-3})\,dx$

$= 2\cdot\frac{2}{3}x^{3/2} - \frac{1}{2}x^{-2} + C = \frac{4}{3}x^{3/2} - \frac{1}{2}x^{-2} + C, \quad C \in \mathbb{R}$

[4] (b) $\int x^2(x^3+4)^{77}\, dx$ $= \left\{ \begin{array}{l} x^3+4 = u \\ 3x^2\,dx = du \end{array} \right\} = \int u^{77}\frac{du}{3}$

$= \frac{1}{3\cdot 78}u^{78} + C = \frac{1}{234}(x^3+4)^{78} + C, \quad C \in \mathbb{R}$

[4] (c) $\int \frac{\sin(\ln x)}{x}\, dx$ $= \left\{ \begin{array}{l} \ln x = u \\ \frac{1}{x}\,dx = du \end{array} \right\} = \int \sin u\, du$

$= -\cos u + C = -\cos(\ln x) + C, \quad C \in \mathbb{R}$

[4] (d) $\int_{-\pi/4}^{\pi/2} \cos 3x \, dx = \left.\frac{\sin 3x}{3}\right|_{-\frac{\pi}{4}}^{\frac{\pi}{2}} = \frac{1}{3}\left(\sin\frac{3\pi}{2} - \sin\left(-\frac{3\pi}{4}\right)\right)$

$= \frac{1}{3}\left(-1 + \frac{\sqrt{2}}{2}\right) = \frac{\sqrt{2}-2}{6}$

[4] (e) $\int_0^{\ln 2} e^x(1+2e^x)^4 \, dx = \left\{ \begin{array}{l} 1+2e^x = u \\ 2e^x dx = du \\ x=0: \; u=3 \\ x=\ln 2: \; u=1+2\cdot 2=5 \end{array} \right\} = \int_3^5 u^4 \frac{du}{2}$

$= \left.\frac{1}{10}u^5\right|_3^5 = \frac{1}{10}(5^5 - 3^5) = \frac{2882}{10} = \frac{1441}{5}$

[6] Bonus. Is the function $f(x) = |x| \sin x$ differentiable at $x = 0$? If it is find its derivative at this point. If it is not explain why. (Justify your answer.)

$f'(0) = \lim_{x\to 0} \frac{f(x)-f(0)}{x-0} = \lim_{x\to 0} \frac{|x| \sin x}{x}$

This limit exists, because:

$\lim_{x\to 0^+} \frac{|x| \sin x}{x} = \lim_{x\to 0^+} \frac{x \sin x}{x} = \lim_{x\to 0^+} \sin x = \sin 0 = 0$

and

$\lim_{x\to 0^-} \frac{|x| \sin x}{x} = \lim_{x\to 0^-} \frac{-x \sin x}{x} = \lim_{x\to 0^-}(-\sin x) = -\sin 0 = 0.$

So, the function is differentiable at 0 and its derivative is zero at this point.