Summary 2013 Term Test 2

[7] 1. Given that

$$x^2y + u^2v = 11 \text{ and } xy = u^2 + v^2$$

define u and v as functions of x and y, find $\left.\frac{\partial v}{\partial x}\right)_y$ when $x = 2, y = 5, u = 3$ and $v = -1$.

[6] 2. Find the equation of the tangent line (in either parametric, vector or symmetric form) to the curve

$$yz + \sin(xy) = 0, \quad x^2 + y^2 - z^2 = 3$$

at the point $(2, 0, -1)$.

3. For the function $f(x, y) = 2x^3 + xy^2 + 5x^2 + y^2$

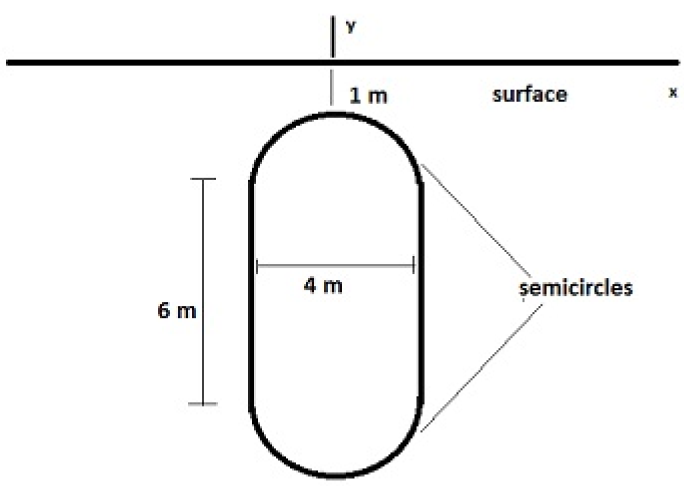
[5] (a) Find the critical point(s) of f. **Show all work.**

[6] (b) Classify any two of the critical points found in (a) as either relative minimum, relative maximum, saddle point, or neither.

[7] 4. Given that the function $f(x, y) = 2x^3 + y^4$ has one critical point $(0, 0)$, find the absolute maximum and minimum of f on the region $\{(x, y)|x^2 + y^2 \leq 1\}$.

[7] 5. Find $\iint_R x \sin y \, dA$ where R is the region bounded by the curves $y = 0, \quad y = x^2$ and $x = 1$.

[6] 6. Set up, but do not integrate a multiple integral, or sum of multiple integrals to find the force due to water pressure on each side of the flat vertical plate in the following figure:



[6] 7. Set up, but do not integrate a multiple integral, or sum of multiple integrals to find the volume of revolution of the region bounded by $y = x^2, \quad y = x+2$ rotated about the line $y = 2x + 5$. (Your final answer must be in terms of x, y and must not include absolute values.)

Solutions

1. $-\dfrac{15}{7}$

2. $x = 2 + 2t, \quad y = 0, \quad z = -1 - 4t.$

3. (a) $(0,0), (-5/3, 0), (-1, 2), (-1, -2)$.

   (b) $(0,0)$ and $(-5/3, 0)$ are relative maximums. $(-1, 2)$ and $(-1, -2)$ are saddle points.

4. Minimum of -2 at $(-1, 0)$. Maximum of 2 at $(1, 0)$.

5. $\dfrac{1 - \sin 1}{2}$

6. $$\int_{-2}^{2} \int_{-9-\sqrt{4-x^2}}^{-3+\sqrt{4-x^2}} (9.81)(1000)(-y)\, dy\, dx$$

7. $$\int_{-1}^{2} \int_{x^2}^{x+2} 2\pi \left(\frac{2x - y + 5}{\sqrt{5}} \right) dy\, dx$$