UNIVERSITY OF MANITOBA

DATE: March 12, 2009 MIDTERM II

COURSE: MATH 2130 TIME: 70 minutes

EXAMINATION: Engineering Mathematical Analysis 1 EXAMINER: M. Davidson

1. For each of the following, find the value of the limit (with appropriate justification) if it exists. If it does not exist, state so and explain why not.

[4] (a) $\lim_{(x,y)\to(0,0)} \dfrac{3x^4y}{x^8+y^2}$

[3] (b) $\lim_{(x,y)\to(0,0)} \dfrac{x^3+xy+4x}{x^2+2x+xy^2}$

[5] (c) $\lim_{(x,y)\to(0,0)} \dfrac{xy^2}{x^2+y^2}$

(Hint, you may assume $\left|\dfrac{y^2}{x^2+y^2}\right| \leq 1$.)

[5] 2. If we have the following functions: $u = f(v,w,x)$, $v = g(x,y,z)$, $w = h(x,y)$, and $z = z(x)$; find an expression for $\left(\dfrac{\partial u}{\partial x}\right)_y$

[8] 3. The following equations define s and t as functions of x and y. Find $\left(\dfrac{\partial t}{\partial y}\right)_x$

$$x^3ys + s^2t + yt^2 = 4$$
$$y\cos(s^2) + e^{xt} = xy$$

[8] 4. Find the directional derivative of $f(x,y,z) = xe^{y^2-z^2}$ at the point $(1,2,-2)$ in the direction of the curve $\vec{r}(t) = t\hat{\imath} + 2\cos(t-1)\hat{\jmath} + (-2e^{t-1})\hat{k}$

[7] 5. Find the line tangent to the curve of intersection of the surfaces $5x^2+3y^2-2z^2 = 0$ and $z = 4x^2 + 5xy + 3y^2$ at the point (-1,1,2).

(HINT: DO NOT try to find an expression for this curve.)

[10] 6. Find **all** the critical points of the function

$$f(x,y) = 4x - 3x^3 - 2xy^2 .$$

Classify **one** of them (as either a relative maximum, relative minimum or a saddle point).

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Answers to Math 2130 Test 2 March 12/2009

1. a) DNE (doesn't exist) b) 2 c) 0

2. $\frac{\partial u}{\partial y}\frac{\partial y}{\partial x} + \frac{\partial u}{\partial v}\frac{\partial v}{\partial z}\frac{dz}{dx} + \frac{\partial u}{\partial w}\frac{\partial w}{\partial x} + \frac{\partial u}{\partial x}\Big)_{v,w}$

3. $-\dfrac{2sy\sin s^2(x^3s+t^2) + (\cos s^2 - x)(x^3y+2st)}{2sy\sin s^2(s^2+2yt) + xe^t(x^3y+2st)}$

4. $-\frac{7}{\sqrt{5}} = -\frac{7\sqrt{5}}{5}$

5. $x = -1+t$ or
   $y = 1+7t$; $x+1 = \frac{y-1}{7} = \frac{z-2}{4}$
   $z = 2+4t$

6. Critical points : $(0,\sqrt{2}), (0,-\sqrt{2}), (-\frac{2}{3},0), (\frac{2}{3},0)$

   ⓐ $(0,-\sqrt{2})$ and $(0,\sqrt{2})$ → Saddle point
   ⓐ $(-\frac{2}{3},0)$ → rel. Min ⓐ $(+\frac{2}{3},0)$ → rel. Max.