Problem Set #6 Solutions

①

BC	NM
CD	ML
IJ	
IL	

②

BC	MN
BE	NK
FG	IJ
FE	
DE	

③

DI	FK
EI	GK
AI	CK
BJ	

④
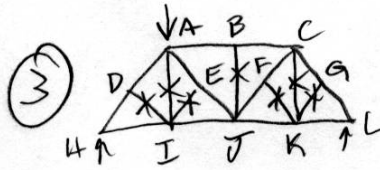
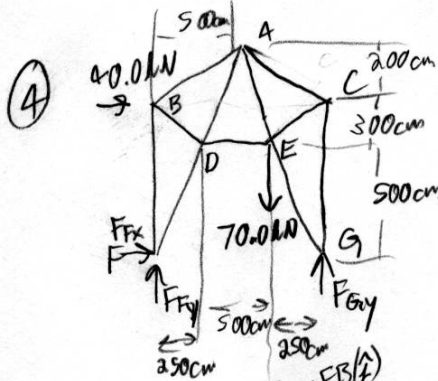$$\text{body}\quad \sum M_F = 0 = -40.0\,kN(8.00m) - 70.0\,kN(7.50m) + F_{Gy}(10.00m)$$

$$\boxed{F_{Gy} = 52.5\,kN}$$

$$\sum F_y = 0 = F_{Gy} + F_{Fy} - 70.0\,kN \rightarrow \boxed{F_{Fy} = 17.5\,kN}$$

$$\sum F_x = 0 = 40.0\,kN + F_{Fx} \rightarrow \boxed{F_{Fx} = -40.0\,kN}$$

at F:

$$\vec{F}_{FB} = FB(\hat{j})$$
$$\vec{F}_{FD} = FD\left(\frac{2.5}{\sqrt{31.25}}\hat{i} + \frac{5.0}{\sqrt{31.25}}\hat{j}\right)$$

$$\sum F_x = 0 = FD\left(\frac{2.5}{\sqrt{31.25}}\right) - 40.0\,kN \rightarrow \boxed{FD = 89.443\,kN}$$
$$\leftarrow \frac{40\sqrt{31.25}}{2.5}\,kN$$

$$\sum F_y = 0 = F_{Fy} + FD\left(\frac{5.0}{\sqrt{31.25}}\right) + FB$$

$$\rightarrow \boxed{FB = -97.5\,kN}$$

at G:

$$\sum F_y = 0 = 52.5\,kN + GC + GE\left(\frac{5}{\sqrt{31.25}}\right) \rightarrow \boxed{GC = -52.5\,kN}$$

$$\sum F_x = 0 = GE\left(\frac{2.5}{\sqrt{31.25}}\right) \rightarrow \boxed{GE = 0}$$

at B:

$$\vec{F}_{BA} = BA\left(\frac{5}{\sqrt{29}}\hat{i} + \frac{2}{\sqrt{29}}\hat{j}\right)$$
$$\vec{F}_{BD} = BD\left(\frac{2.5}{\sqrt{15.25}}\hat{i} - \frac{3}{\sqrt{15.25}}\hat{j}\right)$$

$$\sum F_x = 0 = BA\left(\frac{5}{\sqrt{29}}\right) + BD\left(\frac{2.5}{\sqrt{15.25}}\right) + 40.0\,kN$$

$$\sum F_y = 0 = FB - \left(\frac{3}{\sqrt{15.25}}\right)BD + \left(\frac{2}{\sqrt{29}}\right)BA$$
$$= -97.5\,kN - \frac{3\,BD}{\sqrt{15.25}} + \frac{2\,BA}{\sqrt{29}}$$

$$BA = \left[97.5\,kN + \frac{3\,BD}{\sqrt{15.25}}\right]\frac{\sqrt{29}}{2}$$

$$0 = \frac{5}{2}\left[97.5\,kN + \frac{3}{\sqrt{15.25}}BD\right] + \frac{2.5}{\sqrt{15.25}}BD + 40\,kN$$

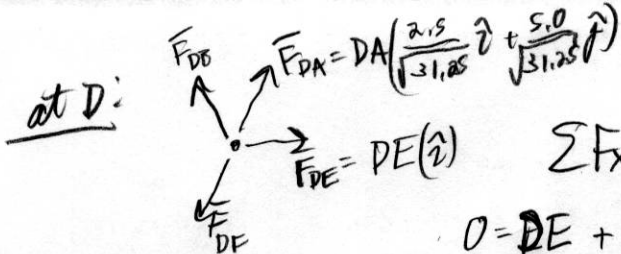$$\frac{10}{\sqrt{15.25}}BD = -283.75\,kN \rightarrow BD = -28.375\sqrt{15.25}\,kN$$
$$= -110.808\,kN \rightarrow \boxed{BA = 33.321\,kN}$$

$\boxed{4(\text{cont.})}$

at D:

$\vec{F}_{DB}$ ↑, $\nearrow \vec{F}_{DA} = DA\left(\frac{2.5}{\sqrt{31.25}}\hat{\imath} + \frac{5.0}{\sqrt{31.25}}\hat{\jmath}\right)$

$\rightarrow \vec{F}_{DE} = DE(\hat{\imath})$

$\sum F_x = 0 = DE + \frac{2.5}{\sqrt{31.25}}DA - \left(\frac{2.5}{\sqrt{31.25}}\right)DF - DB\left(\frac{2.5}{\sqrt{15.25}}\right)$

↓ $\vec{F}_{DF}$

$0 = DE + DA\left(\frac{2.5}{\sqrt{31.25}}\right) - 40 kN + 70.9375 kN$

$DE = -DA\left(\frac{2.5}{\sqrt{31.25}}\right) - 30.9375 kN$

$\sum F_y = 0 = -DF\left(\frac{5}{\sqrt{31.25}}\right) + DA\left(\frac{5}{\sqrt{31.25}}\right) + DB\left(\frac{3}{\sqrt{15.25}}\right)$
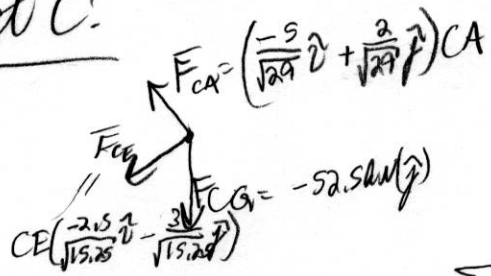
$\boxed{DE = -113.5 kN}$

$= -80 kN + \left(\frac{5}{\sqrt{31.25}}\right)DA - 85.125 kN \rightarrow \boxed{DA = 33.025\sqrt{31.25}\ kN = 184.615 kN}$

at C:

$\vec{F}_{CA} = \left(\frac{-5}{\sqrt{29}}\hat{\imath} + \frac{2}{\sqrt{29}}\hat{\jmath}\right)CA$

$\vec{F}_{CD}$ ←

$CE\left(\frac{-2.5}{\sqrt{15.25}}\hat{\imath} - \frac{3}{\sqrt{15.25}}\hat{\jmath}\right)$ $\vec{F}_{CG} = -52.5 kN(\hat{\jmath})$

$\sum F_x = 0 = CA\left(\frac{-5}{\sqrt{29}}\right) - \left(\frac{2.5}{\sqrt{15.25}}\right)CE$

$CE = -\frac{2\sqrt{15.25}}{\sqrt{29}}CA \rightarrow \boxed{CE = -13.2\sqrt{15.25}\ kN = -51.548 kN}$
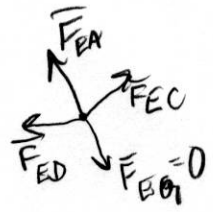
$\sum F_y = 0 = -52.5 kN + \frac{2}{\sqrt{29}}CA - \frac{3}{\sqrt{15.25}}CE$

$52.5 kN = \left(\frac{2}{\sqrt{29}} + \frac{6}{\sqrt{29}}\right)CA$

$\rightarrow \boxed{CA = \frac{52.5\sqrt{29}}{8}\ kN = 35.342 kN}$

$(6.6\sqrt{29}kN)$

at E:

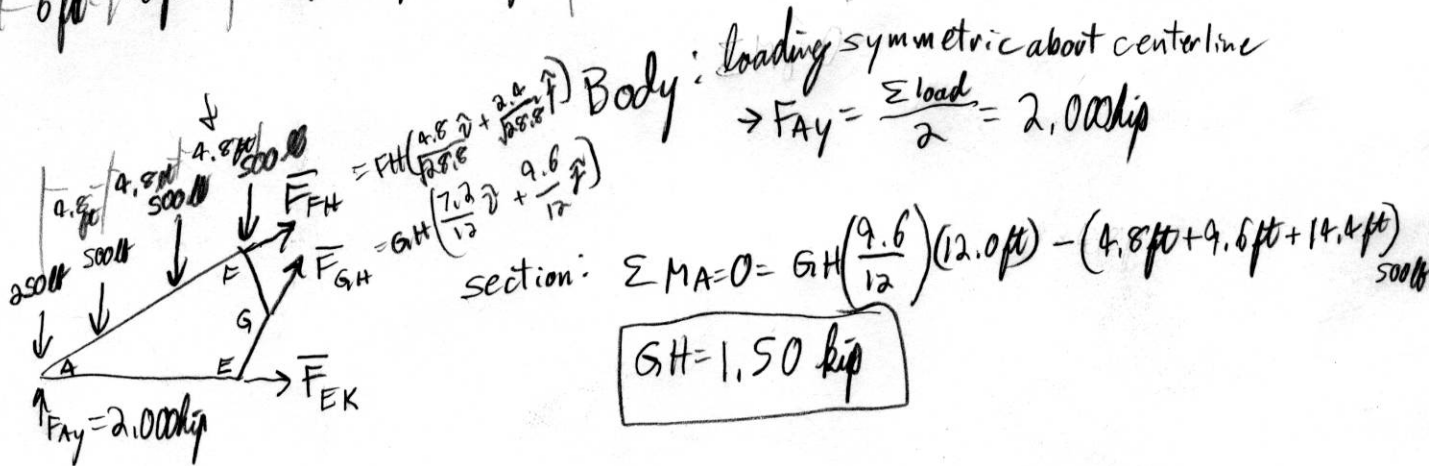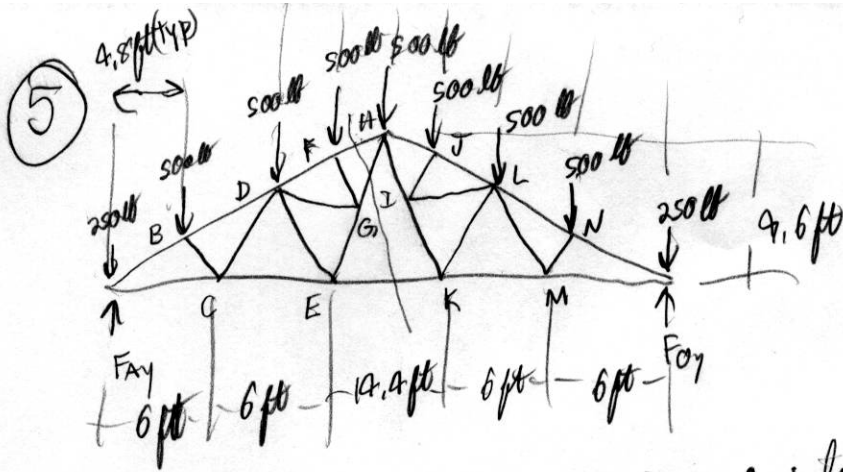$\sum F_x = 0 = \left(-\frac{2.5}{\sqrt{31.25}}\right)EA - DE + CE\left(\frac{2.5}{\sqrt{15.25}}\right) + 0$

$\vec{F}_{EA}$ ↑
$\vec{F}_{EC}$ ↗
$\vec{F}_{ED}$ ← $\vec{F}_{EG} = 0$

$0 = \left(\frac{-2.5}{\sqrt{31.25}}\right)EA + 113.5 kN - 330 kN$

$\boxed{EA = \frac{80.5\sqrt{31.25}}{2.5}\ kN = 32.2\sqrt{31.25}\ kN = 180.0 kN}$

AB: 33.3 kN (T)   BD: 110.8 kN (C)   CG: 52.5 kN (C)

AD: 184.6 kN (T)   CE: 51.5 kN (C)   EG: 0 kN

AC: 35.5 kN (T)   DE: 113.5 kN (C)

AE: 180.0 kN (T)   BF: 97.5 kN (C)

                   DF: 89.4 kN (T)

⑤

4.8 ft (typ)

250 lb   500 lb   500 lb   500 lb 500 lb   500 lb   500 lb   500 lb   250 lb

B   D   A   F   H   I   J   L   N

C   E   G   K   M

F_{AY} F_{OY}

6 ft | 6 ft | 14.4 ft | 6 ft | 6 ft

9.6 ft

Body: loading symmetric about centerline

$$\rightarrow F_{AY} = \frac{\Sigma \text{load}}{2} = 2,000 \text{ lb}$$

$$\overline{F}_{FH} = FH\left(\frac{4.8}{\sqrt{28.8}}\hat{v} + \frac{2.4}{\sqrt{28.8}}\hat{\imath}\right)$$

$$\overline{F}_{GH} = GH\left(\frac{7.2}{12}\hat{v} + \frac{9.6}{12}\hat{\imath}\right)$$

250 lb   500 lb   500 lb

$F_{AY} = 2,000 \text{ lb}$

Section:

$$\Sigma M_A = 0 = GH\left(\frac{9.6}{12}\right)(12.0 \text{ ft}) - (4.8 \text{ ft} + 9.6 \text{ ft} + 14.4 \text{ ft}) 500 \text{ lb}$$
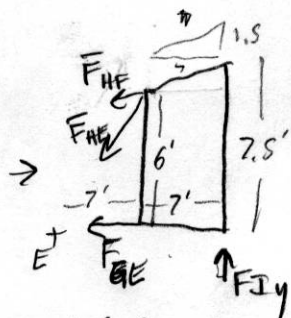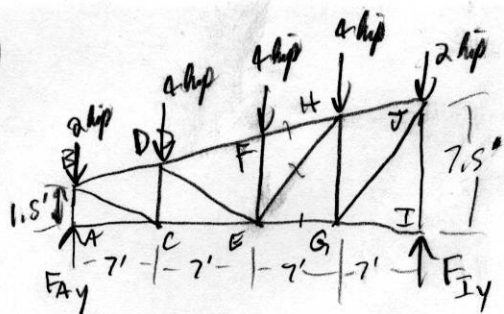
$$\boxed{GH = 1.50 \text{ kip}}$$

$$\Sigma F_y = 0 = -1750 \text{ lb} + 2000 \text{ lb} + GH\left(\frac{9.6}{12}\right) + FH\left(\frac{2.4}{\sqrt{28.8}}\right)$$

$$\rightarrow \boxed{FH = -3.242 \text{ kip}}$$

$$\Sigma F_x = 0 = EK + FH\left(\frac{4.8}{\sqrt{28.8}}\right) + GH\left(\frac{7.2}{12}\right)$$

$$= EK - 2.900 \text{ kip} + 0.900 \text{ kip} \rightarrow \boxed{EK = 2.00 \text{ kip}}$$

⑥

for full body: $\sum M_A = 0 = F_{Iy}(28 ft) - 4 kip(7' + 14' + 21') - 2 kip(28 ft)$

$$\boxed{F_{Iy} = 8.00 kip}$$

$\bar{F}_{HE} = (-.9778\hat{\imath} - .20953\hat{\jmath}) HF$

$\bar{F}_{HE} = \left(\frac{-7}{\sqrt{85}}\hat{\imath} - \frac{6}{\sqrt{85}}\hat{\jmath}\right) HE$

for right section: $\sum M_E = 0 = F_{Iy}(14') + HF(6')(.9778) - HF(7')(.20953)$

$0 = 112.0 kip \cdot ft + 4.4000 HF \cdot ft$

$$\boxed{HF = 25.454 kip}$$

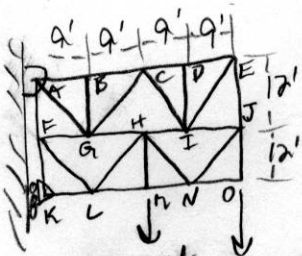$\sum M_G = 0 = HF(.9778)(6.00 ft) + HE\left(\frac{7}{\sqrt{85}}\right)(6.00 ft) + F_{Iy}(7.00 ft)$

$$\boxed{\rightarrow HE = -45.073 kip}$$

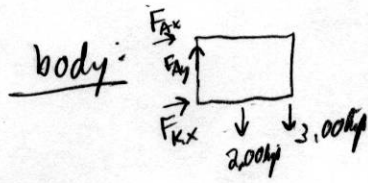$\sum F_x = 0 = GE - (.9778) HF - \frac{7}{\sqrt{85}} HE$

$GE = -24.889 kip + 34.222 kip = \boxed{9.333 kip = GE}$

$$\boxed{\begin{array}{l} HE = -45.1 kip \\ HF = 25.5 kip \\ GE = 9.33 kip \end{array}}$$

body:

$$\sum M_A = 0 = F_{Kx}(24.0\,ft) - 2.00\,kip(18.0\,ft) - 3.00\,kip(36.0\,ft)$$

$$\Rightarrow \boxed{F_{Kx} = 6.00\ kip}$$

for F J O K:
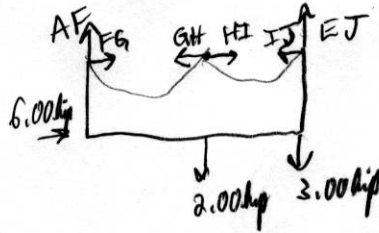
$$\sum M_F = 0 = EJ(36.0\,ft) + 6.00\,kip(12.0\,ft) - 2.00\,kip(18.0\,ft) - 3.00\,kip(36.0\,ft)$$

$$\boxed{EJ = 2.00\ kip}$$

$$\sum F_y = 0 = AF + EJ - 2.00\,kip - 3.00\,kip$$

$$\boxed{AF = 3.00\ kip}$$