MATH 1210 Fall 2013 Assignment 4

Attempt all questions and show all your work. The assignment is due Friday, November 29.

1. Let A, B, C and D be 4×4 invertible matrices such that $|A| = 4$, $|B| = -2$, $|C| = 3$. Determine the following if it is possible to do so from the given information. Otherwise state that they cannot be determined.

   (a) $\left|A^2 B^T C^{-1}\right|$

   (b) $\left|AB + C^T A\right|$

   (c) $\left|ADBC^T D^{-1}\right|$

2. Let $A = \begin{bmatrix} 1 & 1 & -2 \\ 0 & 1 & -2 \\ 1 & -2 & 3 \end{bmatrix}$, and $B = \begin{bmatrix} 1 \\ 3 \\ 4 \end{bmatrix}$.

   (a) Use row reduction to find A^{-1} if it exists.

   (b) Solve the system $AX = B$.

   (c) Solve the system $A^T X = B$.

   (d) Solve the system $A^{-1} X = B$.

3. Let $A = \begin{bmatrix} 1 & 1 & -2 & 2 \\ 0 & 1 & -2 & 2 \\ 1 & -2 & 3 & 2 \\ 0 & 0 & 1 & 2 \end{bmatrix}$

   (a) Find $|A|$

   (b) Use Cramer's Rule to solve for x_3 only in $AX = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$

   (c) If $adj(A) = \begin{bmatrix} -8 & 8 & 0 & 0 \\ -6 & 10 & 6 & -10 \\ a & 6 & 2 & -6 \\ b & -3 & -1 & -1 \end{bmatrix}$, find a and b.

   (d) Find A^{-1}.

4. For the matrix $A = \begin{bmatrix} x & 2 & -2 \\ 0 & 1 & -2 \\ 1 & -2 & x \end{bmatrix}$, find x such that A is not invertible.

5. Determine whether the following vectors are linearly independent or dependent.

$$\mathbf{u_1} = \langle 1, -2, 1, 4 \rangle,\ \mathbf{u_2} = \langle 2, 6, -3, 3 \rangle,\ \mathbf{u_3} = \langle 1, 2, -1, 2 \rangle.$$

   If they are linearly dependent, write one vector as a linear combination of the others.

6. Determine whether each of the following sets of vectors are linearly independent or dependent. Explain your answers.

   (a) $\{\langle 1, 2, 3 \rangle,\ \langle 2, 3, 4 \rangle,\ \langle 1, -1, 5 \rangle\}$

   (b) $\{\langle 1, 2, 3, 4, 5, 6 \rangle,\ \langle -2, -4, -6, -8, -10, -6 \rangle\}$

   (c) $\{\langle 1, 2, 3 \rangle,\ \langle 2, 3, 4 \rangle,\ \langle 1, -1, 5 \rangle,\ \langle 4, -10, 3 \rangle\}$

   (d) $\{\langle 1, -4, 3, 5 \rangle,\ \langle 2, 3, 4, -1 \rangle,\ \langle -2, 8, -6, -10 \rangle\}$

7. Show that every vector with 3 components can be written as a linear combination of $\langle 1, 2, 3 \rangle$, $\langle 2, 3, 4 \rangle$ and $\langle 1, -1, 5 \rangle$.