UNIVERSITY OF MANITOBA
DEPARTMENT OF MATHEMATICS
MATH 1710 Applied Calculus II
FIRST MIDTERM EXAMINATION
February 9, 2012 5:30 pm

LAST NAME: (Print in ink) ______________________

FIRST NAME: (Print in ink) Solutions

STUDENT NUMBER: (in ink) ______________________

SIGNATURE: (in ink) ______________________
(I understand that cheating is a serious offense)

Please indicate your instructor and section by checking the appropriate box below:

☐	A01	MWF (8:30am – 9:20am)	O. Maizlish
☐	A02	TR (1pm – 2:15pm)	A. Gumel

Total	/35
1	/12
2	/14
3	/9

DO NOT WRITE IN THIS COLUMN

INSTRUCTIONS TO STUDENTS:

Fill in all the information above. *This is a 1 hour exam.*
You are permitted to bring one information page (21.6cm. by 28.0 cm. or 8.5 in. by 11 in.) which may contain information on one side only, must be hand-written (not mechanically reproduced) and must bear your name and the student identification number. Information pages not meeting these criteria will be confiscated. ***No*** *other aids, calculators, texts, notes, cellphones, pagers or translators are permitted.*

This exam has a title page, 3 pages of questions and 1 blank page at the end for rough work. Please check that you have all pages.

The value of each question is indicated in the left-hand margin beside the statement of the question.. The total value of all questions is 35.

Answer all questions on the exam paper in the space provided. If you need more room, you may continue your work on the ***reverse*** *side of the page, but* ***clearly indicate*** *that your work is continued there.*

Show all your work clearly and justify your answers (unless it is explicitly stated that you do not have to do that). **Unjustified answers will receive LITTLE or NO CREDIT.**

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[12] **1.** Find the following integrals. Show all your work and simplify your answers.

(a) $\displaystyle\int \frac{\cos(\ln x)}{x}\,dx.$

$$\int \frac{\cos(\ln x)}{x}\,dx = \left\{ \begin{array}{l} \text{Let } u = \ln x \\ du = \dfrac{dx}{x} \end{array} \right\} = \int \cos(u)\,du = \sin u + C$$

$$= \sin(\ln x) + C, \quad C \in \mathbb{R}$$

(b) $\displaystyle\int_0^{\sqrt[3]{3}} \frac{x^5}{\sqrt{x^3+1}}\,dx.$

$$\int_0^{\sqrt[3]{3}} \frac{x^5}{\sqrt{x^3+1}}\,dx = \left\{ \begin{array}{ll} \text{Let } u = x^3+1, & x^3 = u-1 \\ du = 3x^2\,dx, & x=0 \Rightarrow u = 1 \\ \dfrac{du}{3} = x^2\,dx, & x = \sqrt[3]{3} \Rightarrow u = 4 \end{array} \right\}$$

$$= \int_1^4 \frac{u-1}{\sqrt{u}}\,\frac{du}{3} = \frac{1}{3}\int_1^4 \left[u^{\frac{1}{2}} - u^{-\frac{1}{2}}\right] du = \frac{1}{3}\left[\frac{2}{3}u^{3/2} - 2u^{\frac{1}{2}}\right]\Bigg|_1^4$$

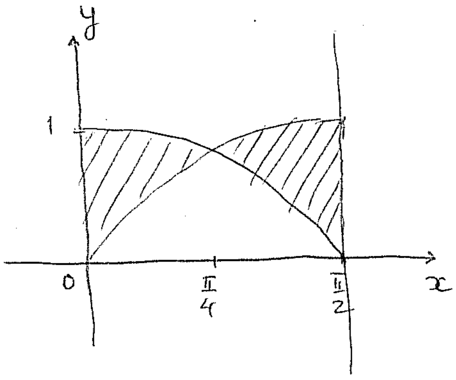
$$= \left[\frac{2}{9}\left(\sqrt{u}\right)^3 - \frac{2}{3}\sqrt{u}\right]\Bigg|_1^4 = \left[\frac{2}{9}\cdot 8 - \frac{2}{3}\cdot 2 - \frac{2}{9} + \frac{2}{3}\right] = \frac{16}{9} - \frac{4}{3} - \frac{2}{9} + \frac{2}{3}$$

$$= \frac{8}{9}$$

[14] **2.** Set up (but do not evaluate) integrals to determine the following physical quantities:

(a) The area of the region enclosed by the curves $y = \sin x$, $y = \cos x$, $x = 0$ and $x = \frac{\pi}{2}$.

Pts. of intersection: $\begin{cases} y = \sin x \\ y = \cos x \end{cases} \Rightarrow \sin x = \cos x$, $x \in [0, \frac{\pi}{2}]$ $\Rightarrow \tan x = 1$, $x = \frac{\pi}{4}$

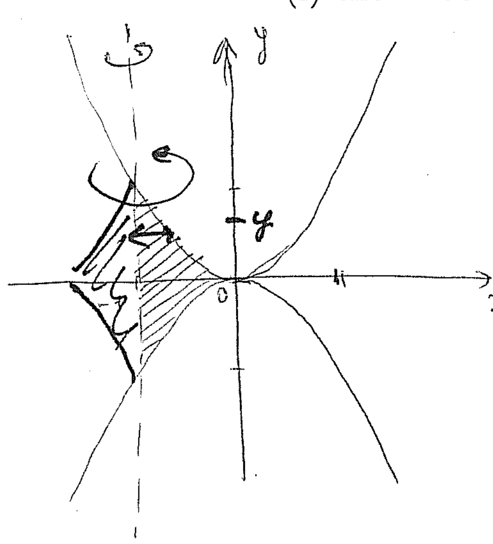


$$\text{Area} = 2\int_0^{\pi/4} [\cos x - \sin x]\,dx,$$

or

$$\int_0^{\pi/4} [\cos x - \sin x]\,dx + \int_{\pi/4}^{\pi/2} [\sin x - \cos x]\,dx$$

(b) The volume of the solid of revolution obtained by rotating the region bounded by the curves $y = x^2$, $y = -x^2$, $x = -1$ about $x = -1$ using (i) the "washers" method and (ii) the cylindrical shells method.



(i) "Washers": $V = 2\int_0^1 \pi(-\sqrt{y}+1)^2\,dy = 2\int_0^1 \pi(1-\sqrt{y})^2\,dy$

$y = x^2$, $x = -\sqrt{y}$

$r_{outer} = -\sqrt{y} - (-1) = -\sqrt{y} + 1$

$r_{inner} = 0$

(ii) "Shells":

$$V = 2\int_{-1}^{0} 2\pi\,(x-(-1))\cdot y\,dx = 2\cdot 2\pi\int_{-1}^{0}(x+1)x^2\,dx$$

(c) The length of the portion of the curve given by $y = \dfrac{e^x + e^{-x}}{2}$ for $0 \le x \le 1$.

$$L = \int_0^1 \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\,dx = \int_0^1 \sqrt{1 + \left(\frac{e^x - e^{-x}}{2}\right)^2}\,dx$$

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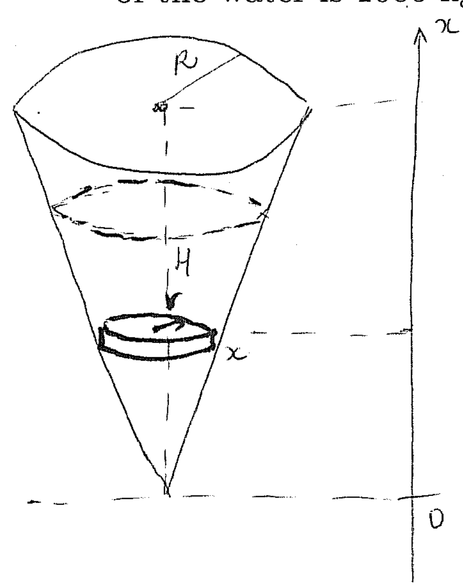
[9] **3.** A tank in the shape of an inverted (circular) cone has a height of 16 meters and a base radius of 4 meters and is filled with water to a depth of 12 meters. Determine the amount of work needed to empty the tank by pumping all of the water to the top of the tank. Assume that the density of the water is 1000 kg/m^3.

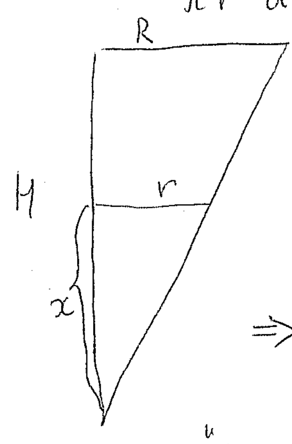


$H = 16m$

$R = 4m$

Volume of a "small" slice:

$\pi r^2 dx$



similar triangles:

$\frac{r}{R} = \frac{x}{H} \Rightarrow$

$\Rightarrow r = \frac{4x}{16} = \frac{x}{4}$

$\Rightarrow$ volume $= \pi \left(\frac{x}{4}\right)^2 dx$

mass of a "small" slice:

$m = V \cdot \rho = 1000\,\pi \left(\frac{x}{4}\right)^2 dx$

$F(x) = mg = 10000\,\pi \left(\frac{x}{4}\right)^2 dx$

A slice positioned at x has to be pumped up by $(16-x)$ m:

$$W = \int_0^{12} 10000\,\pi \left(\frac{x}{4}\right)^2 (16-x)\,dx = 10000\,\pi \cdot \frac{1}{16} \int_0^{12} \left[16x^2 - x^3\right] dx$$

$$= 625\pi \left[\frac{16x^3}{3} - \frac{x^4}{4}\right]\Bigg|_0^{12} = 625\pi \left[\frac{16 \cdot 12^3}{3} - \frac{12^4}{4}\right]$$

$$= 625\pi \left[9216 - 5184\right] = 2\,520\,000 \; [J]$$