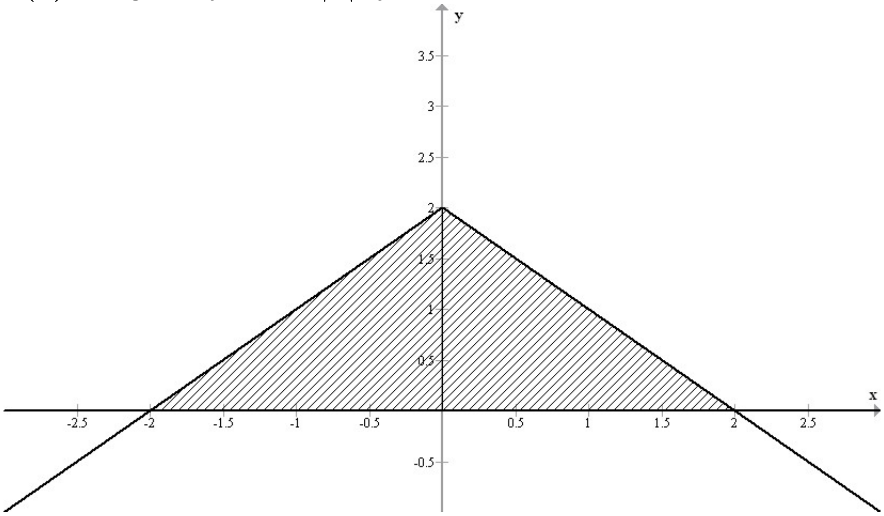
Math 1710. Tutorial 3 (Sketch of the solutions)

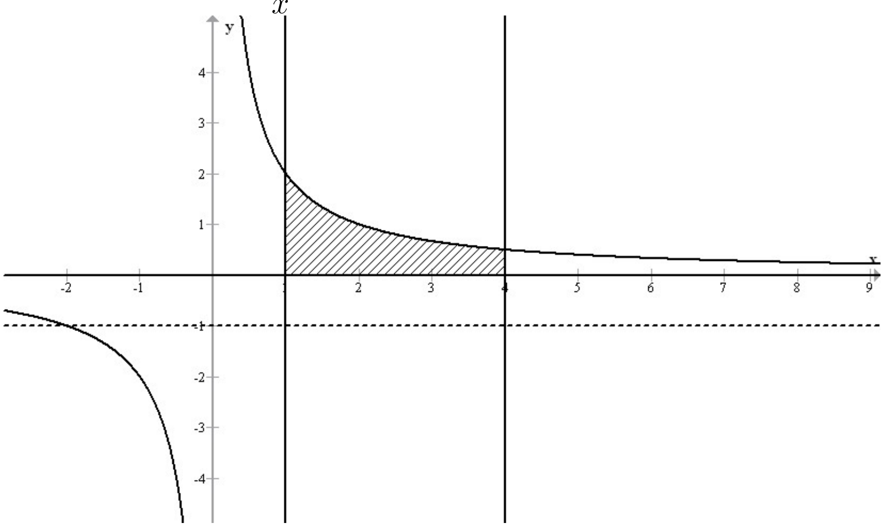
1(a). Region: $y = 2 - |x|$, $y = 0$; rotation about the x-axis.



Use the Washer Method (and symmetry):

$$\textit{outer radius} = 2 - x, \ \textit{inner radius} = 0, \quad 0 \le x \le 2.$$
$$\textit{Volume} = 2\int_0^2 \pi(2-x)^2\, dx.$$

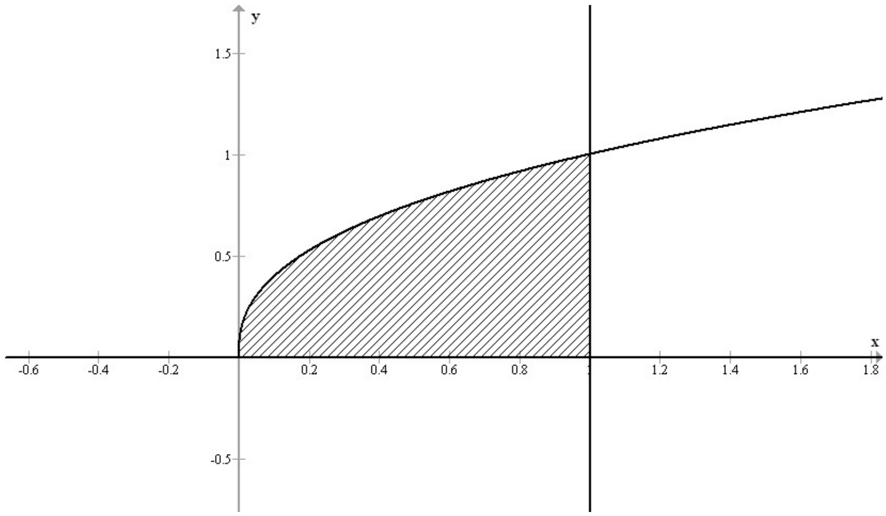
1(b). Region: $y = \dfrac{2}{x}$, $y = 0$, $x = 1$, $x = 4$; rotation about $y = -1$.



Use the Washer Method:

$$\textit{outer radius} = \frac{2}{x} + 1, \ \textit{inner radius} = 1, \quad 1 \le x \le 4.$$
$$\textit{Volume} = \int_1^4 \pi\left(\left(\frac{2}{x}\right)^2 - 1^2\right)\, dx.$$

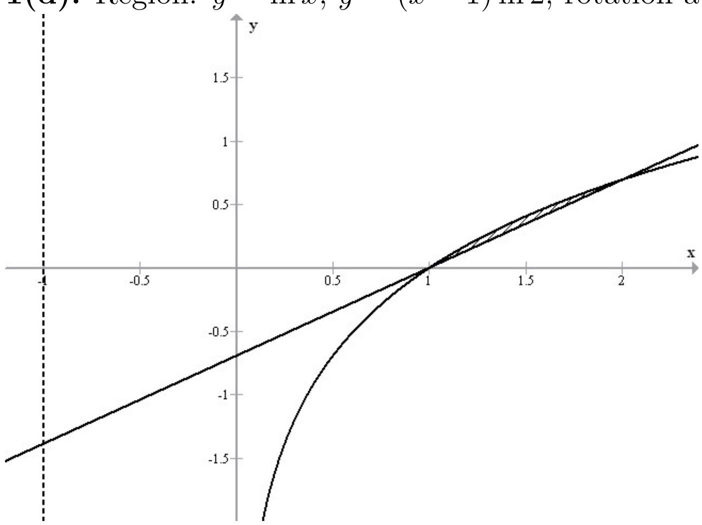
1(c). Region: $y = x^{2/5}\, x = 1$, $y = 0$; rotation about the y-axis.



Use the Washer Method (the axis of rotation is vertical $\Rightarrow$ use integration along the y-axis):

$$y = x^{2/5} \Rightarrow x = y^{5/2}$$
$$\textit{outer radius} = 1,\ \textit{inner radius} = y^{5/2}, \quad 0 \le y \le 1.$$
$$\textit{Volume} = \int_0^1 \pi \left(1^2 - \left(y^{5/2}\right)^2\right)\, dy.$$

1(d). Region: $y = \ln x$, $y = (x-1)\ln 2$; rotation about $x = -1$.



Use the Washer Method (the axis of rotation is vertical $\Rightarrow$ use integration along the

y-axis):

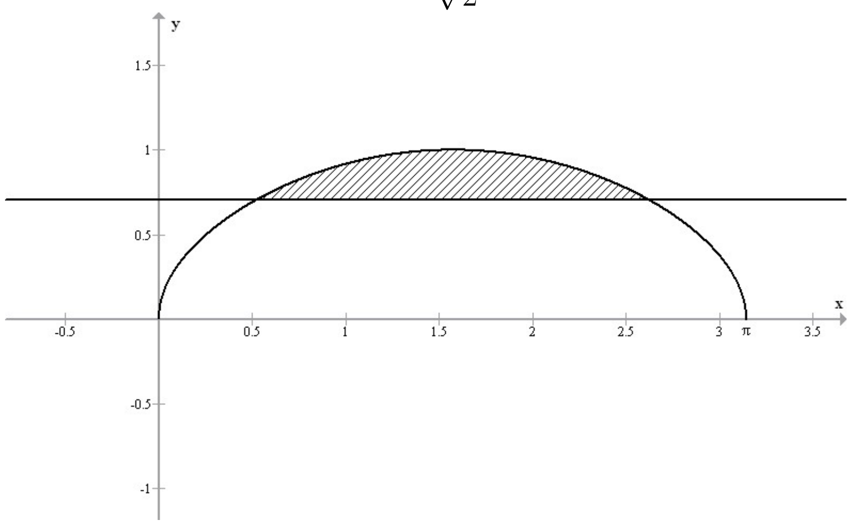
$$y = \ln x \Rightarrow x = e^y, \quad y = (x-1)\ln 2 \Rightarrow x = \frac{y}{\ln 2} + 1;$$

Points of intersection: $x = 1$, $x = 2 \Rightarrow y = 0$, $y = \ln 2$.

outer radius $= \left(\frac{y}{\ln 2} + 1\right) + 1$, *inner radius* $= e^y + 1, \quad 0 \leq y \leq \ln 2$.

$$Volume = \int_0^{\ln 2} \pi \left(\left(\frac{y}{\ln 2} + 2 \right)^2 - (e^y + 1)^2 \right) dy.$$

1(e). Region: $y = \sqrt{\sin x}$, $y = \frac{1}{\sqrt{2}}$, $0 \leq x \leq \pi$; rotation about the x-axis.



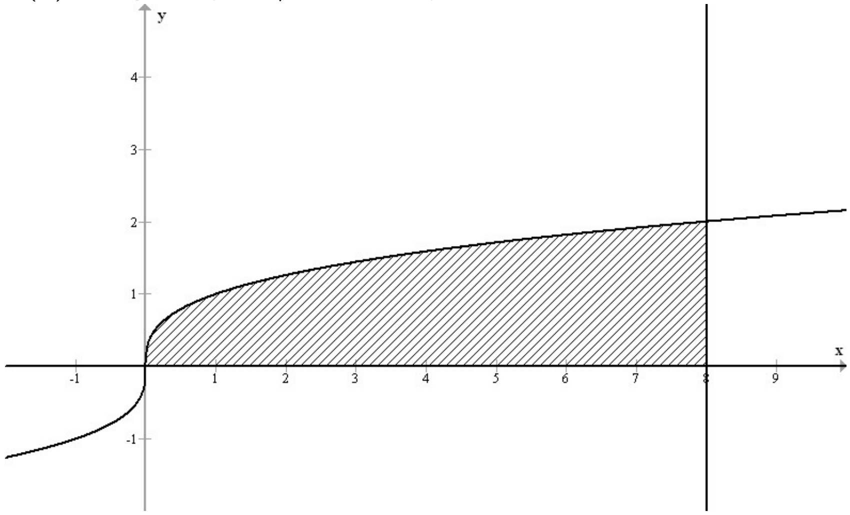
Use the Washer Method:

$$Points\ of\ intersection: \begin{cases} y = \sqrt{\sin x}, \\ y = \frac{1}{\sqrt{2}}, \end{cases} \Rightarrow \sqrt{\sin x} = \frac{1}{\sqrt{2}} \Rightarrow \sin x = \frac{1}{2} \Rightarrow x = \frac{\pi}{6}, \frac{5\pi}{6}.$$

outer radius $= \sqrt{\sin x}$, *inner radius* $= \frac{1}{\sqrt{2}}, \quad \frac{\pi}{6} \leq x \leq \frac{5\pi}{6}$.

$$Volume = \int_{\pi/6}^{5\pi/6} \pi \left(\left(\sqrt{\sin x}\right)^2 - \left(\frac{1}{\sqrt{2}}\right)^2 \right) dx.$$

2(a). Region: $y = \sqrt[3]{x}$, $x = 8$, $y = 0$; rotation about the x-axis.



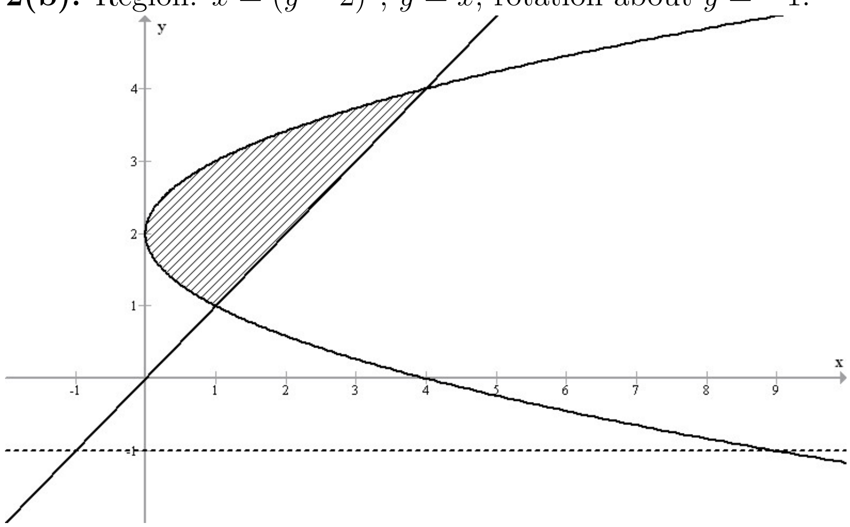
It might be easier to use the Washer method, but just for illustration we will use Cylindrical Shells method:

$$y = \sqrt[3]{x} \Rightarrow x = y^3, \quad 0 \le y \le 2.$$

$$\textit{radius of cylindrical shell} = y,\ \textit{height of cylindrical shell} = 8 - y^3, \quad 0 \le y \le 2.$$

$$\textit{Volume} = \int_0^2 2\pi y \left(8 - y^3\right)\, dy.$$

2(b). Region: $x = (y-2)^2$, $y = x$; rotation about $y = -1$.



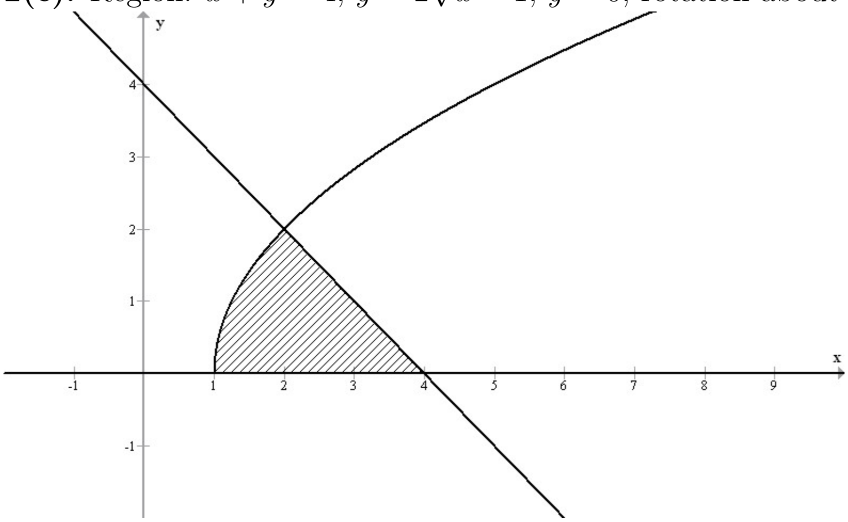
Use Cylindrical Shells method:

$$\textit{Points of intersection:} \begin{cases} x = (y-2)^2, \\ y = x, \end{cases} \Rightarrow y = (y-2)^2 \Rightarrow y^2 - 5y + 4 = 0 \Rightarrow y = 1,\ 4.$$

$$\textit{radius of cylindrical shell} = y + 1,\ \textit{height of cylindrical shell} = y - (y-2)^2, \quad 1 \le y \le 4.$$

$$\textit{Volume} = \int_1^4 2\pi(y+1)\left(y - (y-2)^2\right)\, dy.$$

2(c). Region: $x+y=4$, $y=2\sqrt{x-1}$, $y=0$; rotation about the x-axis.



Use Cylindrical Shells method:

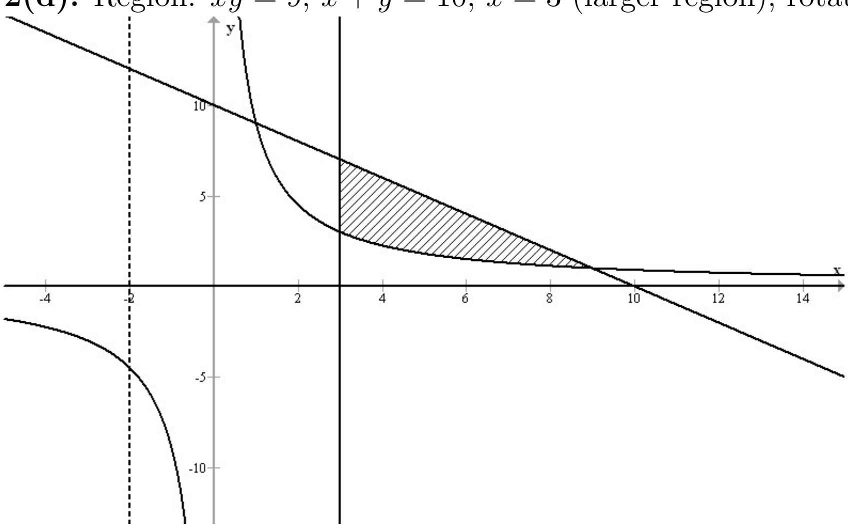
$y=2\sqrt{x-1} \Rightarrow x=\dfrac{y^2}{4}+1.$

Points of intersection: $\begin{cases} x=\dfrac{y^2}{4}+1, \\ x+y=4, \end{cases} \Rightarrow \dfrac{y^2}{4}+1=4-y \Rightarrow y^2+4y-12=0 \Rightarrow y=2,\ (-6\leq 0).$

radius of cylindrical shell $=y$, *height of cylindrical shell* $=(4-y)-\left(\dfrac{y^2}{4}+1\right), \quad 0\leq y\leq 2.$

Volume $=\displaystyle\int_0^2 2\pi y\left((4-y)-\left(\frac{y^2}{4}+1\right)\right)\,dy.$

2(d). Region: $xy=9$, $x+y=10$, $x=3$ (larger region); rotation about $x=-2$.



Use Cylindrical Shells method:

$$y = \frac{9}{x}, \quad y = 10 - x.$$

$$\textit{Points of intersection: } \begin{cases} xy = 9, \\ x + y = 10, \end{cases} \quad \Rightarrow x = 1,\ 9(\text{we are interested in } x = 9)$$

$$\textit{radius of cylindrical shell} = x + 2,\ \textit{height of cylindrical shell} = (10 - x) - \frac{9}{x}, \quad 3 \le x \le 9.$$

$$\textit{Volume} = \int_3^9 2\pi(x+2)\left((10-x) - \frac{9}{x}\right)\, dx.$$