# MATH 2130 – Tutorial Problems, Thu Feb 15

## Gradients and Directional Derivatives

**Example**. Let $f(x,y) = 2x\sin(2\pi xy)$. At the point $(x,y) = (1,1)$, find a direction in which the rate of change of $f$ is $4\pi$. At this point, is there a direction in which the rate of change of $f$ is $8\pi$?

**Example**. Let $f(x,y,z) = xyz$. Let $\mathcal{C}$ be the curve with vector representation

$$\mathbf{r}(t) = (t^2+1)\hat{\mathbf{i}} + \cos(\pi t)\hat{\mathbf{j}} + (t^3 - 2t^2)\hat{\mathbf{k}}, \quad t \in \mathbb{R}.$$

Find the rate of change of $f$ in the direction of $\mathcal{C}$ at the point $(x,y,z) = (5,1,0)$.

## Tangent Lines and Planes

**Example**. Let $\mathcal{D}$ be the curve formed by the intersection of the surfaces $z = \sqrt{y^2 - x^2}$ and $x^2 + 3y^2 + z^2 = 4$ in 3D space. Find the unit tangent vector to $\mathcal{D}$ at the point $P = \left(\frac{\sqrt{3}}{2}, -1, \frac{1}{2}\right)$ that points in the direction of increasing $x$.

**Example**. Consider the surface $x^2 + y^2 + (z-5)^2 = 1$ in 3D space. Find all points $(x,y,z)$ on this surface such that the line through $(x,y,z)$ and the origin is tangent to the surface.

**Example**. Consider the surface

$$\frac{x^2}{4} + \frac{y^2}{9} + z^2 = 1$$

in 3D space. Find all points $(x,y,z)$ on this surface such that the tangent plane to the surface is parallel to the plane $x + y + z = 0$.