Math 1710 Tutorial 5.

Arc Length. Work

Problem 1. Find the length of the curve in (a)–(c).

(a) $y = \frac{3}{2}\left(\sqrt[3]{x} - \frac{1}{5}\sqrt[3]{x^5}\right), \quad 1 \le x \le 8;$

(b) $y = \frac{1}{2}\big(\ln(\sin x) + \ln(\cos x)\big), \quad \pi/4 \le x \le \pi/3;$

(c) $3x = 2\,(y^2+1)^{3/2}$ from $(\frac{4\sqrt{2}}{3}, 1)$ to $(18, -2\sqrt{2})$;

(d) **Set up** a definite integral for the length of the curve $x^2 - 4y^2 = 1$ from $(\sqrt{5}, 1)$ to $(3, -\sqrt{2})$;

(e) Find the length of the curve $y = \sqrt{2x - x^2} - 1, \quad \frac{1}{2} \le x \le 1.$

Problem 2. A chain is L meters long weighing M kg is lying on the ground. How much work is needed to pull the chain to the top of a bridge that is $L + 5$ meters tall?

Problem 3. A block of ice weighing 500 kg is to be lifted to the top of a 30 m building. In the 20 minutes it will take to do this, the block will lose 12 kg (the block loses weight linearly). How much work is needed to lift the block of ice to the top of the building?

Problem 4. A cylindrical tank (see Fig. 1 for the dimensions of the tank) is full of water. How much work is needed to pump all of the water out of the tank to a point 3 meters above the tank?

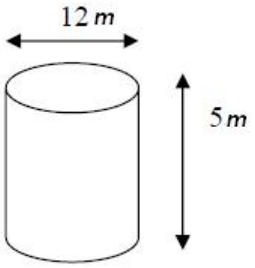

Fig. 1

Problem 5. An inverted conical tank filled with kerosene is buried 4 meters underground (see Fig. 2). The density of kerosene is 817 kg/m^3. The kerosene is pumped out to the surface until the tank is empty. How much work is needed to pump the kerosene to the surface?

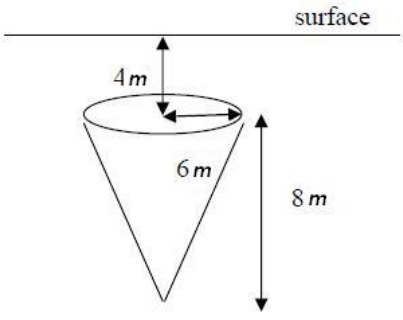

Fig. 2