MATH 1210 Assignment 3

Due: 1:30 pm Friday 27 February 2009 (at your instructor's office)

NOTES:

1. **Late assignments will NOT be accepted**.
2. **If your assignment is not accompanied by a signed Faculty of Science "Honesty Declaration", it will NOT be graded**.

Provide a complete solution to each of the following problems:

1. In each of the following cases, solve the given polynomial equation by factoring the polynomial completely into real linear and/or irreducible real quadratic Factors::
   (a) $x^2 - 7x + 1 = 0$
   (b) $x^2 - 2x + 9 = 0$
   (c) $x^3 + 5x^2 + 6x = 0$
   (d) $(x-1)^2 (x^4 - 16) = 0$
   (e) $x^4 (x^2 + 4)(x^3 - 3x^2 + 3x - 1) = 0$
   (f) $x^6 + 5x^4 + 4x^2 = 0$
   (g) $p_1(x) = x^4 + x^3 - x^2 + 2x - 6 = 0$ given that $p_1(\sqrt{2}i) = 0$
   (h) $p_2(x) = x^3 + 9x^2 + 16x + 14 = 0$ given that $p_2(-1+i) = 0$
   (i) $p_3(x) = x^5 - 4x^4 + 5x^3 + x^2 - 4x + 5 = 0$ given that $p_3(2+i) = 0$

2. Let $P(x) = x^6 + 12x^5 + 42x^4 + 43x^3 - 30x^2 - 60x - 8$.
   (a) What does *Descarte's Rule of Signs* tell you about the **maximum number of positive real zeros** of $P(x)$?
   (b) List all the **possible rational real zeros** of $P(x)$, and use the result of part (a) to help you **find all of its positive real zeros**.
   (c) By division, remove the factor(s) from $P(x)$ which correspond to the positive real zeros of $P(x)$ identified in part(b).
   (d) Find all the negative real zeros of $P(x)$, one at a time, in each case removing the corresponding factor from $P(x)$, until $P(x)$ is written as a product of only **real** factors.
   (e) List all six roots of $P(x) = 0$, indicating clearly the multiplicity of each.

3. Consider the polynomial ***with complex coefficients*** given by

$$P(x) = ix^4 + 3x^3 + 8ix + 24.$$

   (a) Show that $P\left(-\frac{3}{i}\right) = 0$, and determine the corresponding linear factor of $P(x)$.

   (b) By division, write $P(x)$ as the product of the linear factor of part (a) and a cubic factor.

   (c) Use the information of part (b) to find all roots of $P(x) = 0$.
   (HINT: You will have to remember how to find the cube roots of a complex number in order to complete this part of the problem.)

   (d) Plot the four roots of $P(x) = 0$ in the complex plane.

   (NOTE CAREFULLY THAT **since this equation does NOT have real coefficients, its complex roots do NOT always occur in complex conjugate pairs**.)

4. Let l_1 and l_2 be two **non-parallel**, **non-intersecting** lines in space given parametrically by the equations

$$l_1 : \; x = 1 + s, y = 1 - s, z = 4s \text{ (with parameter } s)$$

and

$$l_2 : \; x = 2 - 3t, y = 6 + 2t, z = 1 - t \text{ (with parameter } t).$$

To **find the shortest distance between these two lines**, we must find that point on each line which is a close as possible to the other line. **We may do this as follows**:

   (a) Find any vector $\overrightarrow{v}_1$ along the line l_1.

   (b) Find any vector $\overrightarrow{v}_2$ along the line l_2.

   (c) Let P_1 with coordinates $(1+s, 1-s, 4s)$ be an **arbitrary** point on l_1 , and P_2 with coordinates $(2-3t, 6+2t, 1-t)$ be an **arbitrary** point on l_2, and construct the vector $\overrightarrow{u}$ from P_1 to P_2.

   (d) In order to guarantee thatt each of these two points is a close as possible to the other line, we now simply **require that the vector $\overrightarrow{u}$ be perpendicular to both** $\overrightarrow{v}_1$ and $\overrightarrow{v}_2$. Use this idea to determine the coordinates of the two fixed points P_1 and P_2 which satisfy these conditions.

   (e) Finally, the desired **minimum distance between the two given lines** is simply the length of the line segment P_1P_2 , in which P_1 and P_2 are the fixed points determine in part (d). Find the minimum distance between the two given lines.

5. Let Π be the plane given by the equation $2x - y + 3z = 4$.
   Let l be the line given by the parametric equations $x = 2+t, y = 3+5t, z = 4+t$.

   (a) Show that l and Π never intersect. (We therefore say that **the line and the plane are parallel**.)

   (b) To find **the shortest distance between the line and the plane**, we may proceed as follows:

      i. Let P be a fixed point on l.

      ii. Let $\overrightarrow{N}$ be any vector perpendicular to Π.

      iii. Find the equation of the line l_1 which passes through P in the direction of the vector $\overrightarrow{N}$.

      iv. Find the point of intersection Q of the line l_1 and the plane Π.

      v. Calculate the distance between P and Q, which clearly represents the distance between the line l and the plane Π.