MATH REVIEW

1. Solution of Equations
2. Trigonometry
3. Determinants
4. Similar Triangles

1. Solution of Equations

Two Algebraic Equations in **Two** Unknowns

- Substitution Method
- **Elimination Method**

To determine the roots of two equations in two unknowns, either the substitution method or the elimination method may be used. However, for most cases, the Elimination Method is recommended.

Example: Solve the following set of equations by **Elimination Method**

$$3x - 8y = 14 \qquad \textbf{(1)}$$

$$5x + 4y = 6 \qquad \textbf{(2)}$$

Our strategy will be to eliminate either x or y from Equation (2).

If we multiply both sides of Equation (1) by **5** and Equation (2) by **3** we get:

$$5*(3x - 8y) = 5*14 \qquad \textbf{(1)}$$

$$3*(5x + 4y) = 3*6 \qquad \textbf{(2)}$$

$$15x - 40y = 70 \qquad \textbf{(1)'}$$

$$15x + 12y = 18 \qquad \textbf{(2)'}$$

Subtracting Equation (2)' from (1)' we eliminate x:

$$-52y = 52$$

$$y = \frac{52}{-52} = -1$$

We back substitute in (1)':

$$15x + 12(-1) = 18$$

$$15x = 30$$

$$x = \frac{30}{15} = 2$$

$$\therefore x = 2, y = -1$$

We check in (1)

$$15(2) - 40(-1) = 70$$

$$70 = 70$$

Example: Solve the following set of equations by **Substitution Method**

$$3x - 8y = 14 \qquad \textbf{(1)}$$

$$5x + 4y = 6 \qquad \textbf{(2)}$$

Our strategy will be to use one of the two equations to express variable x in terms of y OR variable y in terms of x and then substitute in the other equations to obtain one equation in one unknown.

$$3x = 14 + 8y$$

$$x = \frac{(14 + 8y)}{3}$$

$$\text{From Equation (2):}$$

$$5\left(\frac{14 + 8y)}{3}\right) + 4y = 6$$

$$\left(\frac{70 + 40y}{3}\right) + 4y = 6$$

$$23.33 + 13.33y + 4y = 6$$

$$17.33y = -17.33$$

$$y = -1$$

$$\therefore 5x + 4(-1) = 6$$

$$x = 2$$

Example: Determine F_1 and F_2 given

$$-20\cos 45^{\circ} - F_2\cos 30^{\circ} - F_1\cos 60^{\circ} = 0 \qquad \textbf{(1)}$$

$$20\cos 45^{\circ} + 18 - F_2\sin 30^{\circ} - F_1\sin 60^{\circ} = 0 \qquad \textbf{(2)}$$

VERY IMPORTANT: If the equations involve trig functions etc. **ALWAYS simplify** (convert trig functions to decimals) before starting the solution.

Simplify:

$$-0.866F_2 - 0.5F_1 = 14.142 \qquad \textbf{(1)'}$$

$$-0.5F_2 - 0.866F_1 = -32.142 \qquad \textbf{(2)'}$$

Now use Elimination to determine F_1 and F_2.

$$-20\cos 45^{o} - F_2\cos 30^{o} - F_1\cos 60^{o} = 0 \qquad \mathbf{(1)}$$

$$20\cos 45^{o} + 18 - F_2\sin 30^{o} - F_1\sin 60^{o} = 0 \qquad \mathbf{(2)}$$

DO NOT DO THIS!!!!!!!!!!!!!!!

$$F_2 = \frac{-F_1\cos 60^{o} - 20\cos 45^{o}}{\cos 30^{o}}$$

Substitute in (2):

$$20\cos 45^{o} + 18 - \left(\frac{-F_1\cos 60^{o} - 20\cos 45^{o}}{\cos 30^{o}}\right)\sin 30^{o} - F_1\sin 60^{o} = 0$$

Simplify:

$-0.866F_2 - 0.5F_1 = 14.142$ **(1)**

$-0.5F_2 - 0.866F_1 = -32.142$ **(2)**

Our strategy will be to eliminate either F_1 or F_2 from Equation (2).

If we multiply both sides of Equation (1) by 0.5 and Equation (2) by 0.866 we get:

$0.5*(-0.866F_2 - 0.5F_1) = 0.5*(14.142)$ **(1)**

$0.866*(-0.5F_2 - 0.866F_1) = 0.866*(-32.142)$ **(2)**

$-0.433F_2 - 0.25F_1 = 7.071$ **(1)'**

$-0.433F_2 - 0.75F_1 = -27.835$ **(2)'**

Subtracting Equation (2)' from (1)' we eliminate x:

$0.5F_1 = 34.906$

$$F_1 = \frac{34.906}{0.5} = 69.812$$

We back substitute in (1):

$-0.866F_2 - 0.5(69.812) = 14.142$

$-0.866F_2 = 49.948$

$$F_2 = \frac{49.048}{-0.866} = -56.637$$

$\therefore F_1 = 69.812, F_2 = -56.637$

We check in (1)

$-0.866(-56.637) - 0.5(69.812) = 14.142$

$14.142 = 14.142$

Algebraic Equations involving **Three or More** Unknowns

In most cases, the **ELIMINATION Method** is recommended.

3 Equations in 3 Unknowns

$$2x + y - 2z = 10 \qquad \mathbf{(1)}$$

$$3x + 2y + 2z = 1 \qquad \mathbf{(2)}$$

$$5x + 4y + 3z = 4 \qquad \mathbf{(3)}$$

Our strategy will be to use Equation (1) to eliminate x from Equations (2) and (3);

STEP 1 – Initialize Equation (1)

To make our calculations easier before we start the elimination, we want to divide Equation (1) through by 2 so that the first term is x rather than 2x.

$$\text{Dividing Equation (1) by 2:}$$

$$x + 0.5y - z = 5 \qquad \textbf{(1)}$$

$$3x + 2y + 2z = 1 \qquad \textbf{(2)}$$

$$5x + 4y + 3z = 4 \qquad \textbf{(3)}$$

STEP 2 – Eliminate

To eliminate x from Equations (2) and (3), we:

Subtract 3*(Equation 1) from Equation 2

Subtract 5 *(Equation 1) from Equation 3

$$x + 0.5y - z = 5 \quad \textbf{(1)}$$

$$3x + 2y + 2z = 1 \quad \textbf{(2)}$$

$$5x + 4y + 3z = 4 \quad \textbf{(3)}$$

$$x + 0.5y - z = 5 \quad \textbf{(1)'}$$

$$+0.5y + 5z = -14 \quad \textbf{(2)'}$$

$$+1.5y + 8z = -21 \quad \textbf{(3)'}$$

Elimination of x from Equations (2) and (3)

$$x + 0.5y - z = 5 \qquad \textbf{(1)}$$

$$3x + 2y + 2z = 1 \qquad \textbf{(2)}$$

$$5x + 4y + 3z = 4 \qquad \textbf{(3)}$$

$$x + 0.5y - z = 5 \qquad \textbf{(1)'}$$

$$+0.5y + 5z = -14 \qquad \textbf{(2)'}$$

$$+1.5y + 8z = -21 \qquad \textbf{(3)'}$$

STEP 3 – Initialize Equation (2)'

$$x + 0.5y - z = 5 \qquad \mathbf{(1)'}$$

$$+0.5y + 5z = -14 \qquad \mathbf{(2)'}$$

$$+1.5y + 8z = -21 \qquad \mathbf{(3)'}$$

To initialize Equation 2', we divide Equation (2)' through by 0.5:

$$x + 0.5y - z = 5 \qquad \mathbf{(1)'}$$

$$+y + 10z = -28 \qquad \mathbf{(2)''}$$

$$+1.5y + 8z = -21 \qquad \mathbf{(3)'}$$

STEP 4 – Eliminate y from Equation 3’

To eliminate y from Equations (3)’ we:

Subtract 1.5*[Equation (2)’’] from Equation (3)’:

$$x + 0.5y - z = 5 \qquad \mathbf{(1)'}$$

$$+ y + 10z = -28 \qquad \mathbf{(2)''}$$

$$+1.5y + 8z = -21 \qquad \mathbf{(3)'}$$

$$x + 0.5y - z = 5 \qquad \mathbf{(1)'}$$

$$+ y + 10z = -28 \qquad \mathbf{(2)''}$$

$$-7z = 21 \qquad \mathbf{(3)''}$$

STEP 5 – Solve for z and Back Substitute

$$x + 0.5y - z = 5 \qquad \textbf{(1)'}$$

$$+ y + 10z = -28 \qquad \textbf{(2)''}$$

$$-7z = 21 \qquad \textbf{(3)''}$$

$$\therefore z = -\frac{21}{7} = -3$$

Back substitute in Equation (2)'':

$$y + 10(-3) = -28$$

$$y = 2$$

Back substitute in Equation (1)':

$$x + 0.5(2) - (-3) = 5$$

$$x = 1$$

STEP 7 – CHECK!!!!!!!!

We check in original Equation (3) :

$$5x + 4y + 3z = 4 \qquad \textbf{(3)}$$

$$5(1) + 4(2) + 3(-3) = 4$$

$$4 = 4$$

CHECKS!!!!

Equations Involving Trig Functions

$$0.5\mathrm{T}_{\mathrm{DA}} \sin\phi = 0 \qquad \textbf{(1)}$$

$$0.866\mathrm{T}_{\mathrm{DA}} - \mathrm{W} = 0 \qquad \textbf{(2)}$$

$$0.5\mathrm{T}_{\mathrm{DA}} \cos\phi = 50 \qquad \textbf{(3)}$$

Example: Equations involving Trig Functions

$$0.5\mathrm{T}_{\mathrm{DA}}\sin\phi = 0 \qquad \textbf{(1)}$$

$$0.866\mathrm{T}_{\mathrm{DA}} - \mathrm{W} = 0 \qquad \textbf{(2)}$$

$$0.5\mathrm{T}_{\mathrm{DA}}\cos\phi = 50 \qquad \textbf{(3)}$$

Divide Equation (1) by Equation (3)

$$\frac{0.5\mathrm{T}_{\mathrm{DA}}\sin\phi}{0.5\mathrm{T}_{\mathrm{DA}}\cos\phi} = \frac{0}{50} = 0$$

$$\therefore \frac{\sin\phi}{\cos\phi} = \tan\phi = 0$$

$$\therefore \phi = 0^{\mathrm{o}}$$

Substitute in Equation (3):

$$0.5\mathrm{T}_{\mathrm{DA}}\cos 0^{\mathrm{o}} = 50$$

$$0.5\mathrm{T}_{\mathrm{DA}}(1) = 50$$

$$\therefore \mathrm{T}_{\mathrm{DA}} = 100$$

Substitute in Equation (2):

$$0.866(100) - \mathrm{W} = 0$$

$$\therefore \mathrm{W} = 86.6$$

Alternate solution:

$$0.5\mathrm{T_{DA}}\sin\phi = 0 \qquad \mathbf{(1)}$$

$$0.866\mathrm{T_{DA}} - \mathrm{W} = 0 \qquad \mathbf{(2)}$$

$$0.5\mathrm{T_{DA}}\cos\phi = 50 \qquad \mathbf{(3)}$$

For a non – trivial solution (T_{DA} not equal 0), from Equation (1) sin$\phi = 0$

$$\sin\phi = 0$$

$$\therefore \phi = 0^{o}$$

From Equation (3):

$$0.5\mathrm{T_{DA}}\cos 0^{o} = 50$$

$$\therefore \mathrm{T_{DA}} = 100$$

From Equation (2):

$$0.866(100) - \mathrm{W} = 0$$

$$\mathrm{W} = 86.6$$

Example: Transcendetal Equation

$$100\text{Sin}\alpha - 400\text{Cos}(30^{o} - \alpha) = 210$$

Example: Transcendental Equations

(Function that is not algebraic)

$$100\text{Sin}\alpha - 400\text{Cos}(30^o - \alpha) = 210$$

Solution by Calculator: $\alpha = -218.276^o$ or $\alpha = 141.724^o$

Solution by "Brute Force" (Guessing):

Guess $\alpha = 30^o \longrightarrow -350 = 210$

Guess $\alpha = 90^o \longrightarrow -100 = 210$

Guess $\alpha = 120^o \longrightarrow 86.6 = 210$

Guess $\alpha = 150^o \longrightarrow 250 = 210$

Guess $\alpha = 140^o \longrightarrow 201.1 = 210$

Guess $\alpha = 141^o \longrightarrow 206.3 = 210$

Guess $\alpha = 142^o \longrightarrow 211.4 = 210$

Guess $\alpha = 141.7^o \longrightarrow 209.9 = 210$ Close Enough!

2. Trigonometry

Trig Relations for the Right Triangle

Sine and Cosine Rules (Laws)

GRAPHICAL SOLUTIONS

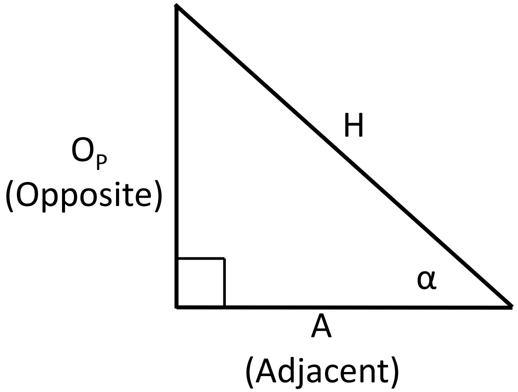
Trig Relations for the Right Triangle

Right Angle Triangle

$$\sin\alpha = \frac{O_p}{H}$$

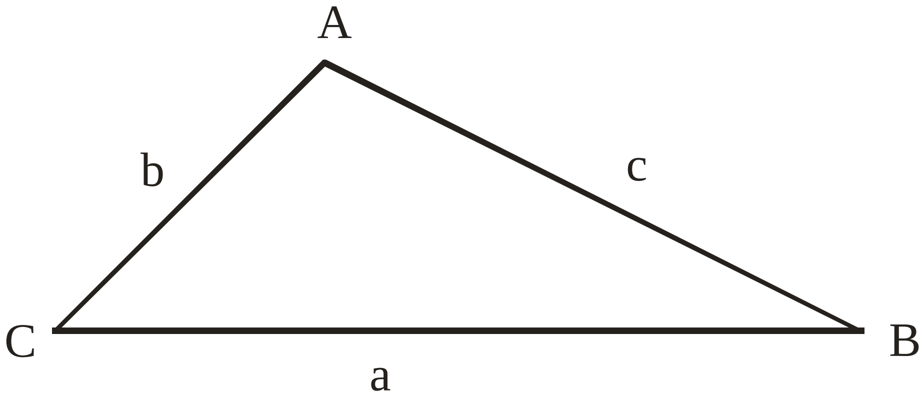
$$\cos\alpha = \frac{A}{H}$$

$$\tan\alpha = \frac{O_p}{A}$$



$$H^2 = A^2 + O^2$$

Cosine Rule:



Sides of the triangle: a, b, and c

Internal Angles: A, B and C

$A + B + C = 180^o$

$$a^2 = b^2 + c^2 - 2bcCosA$$

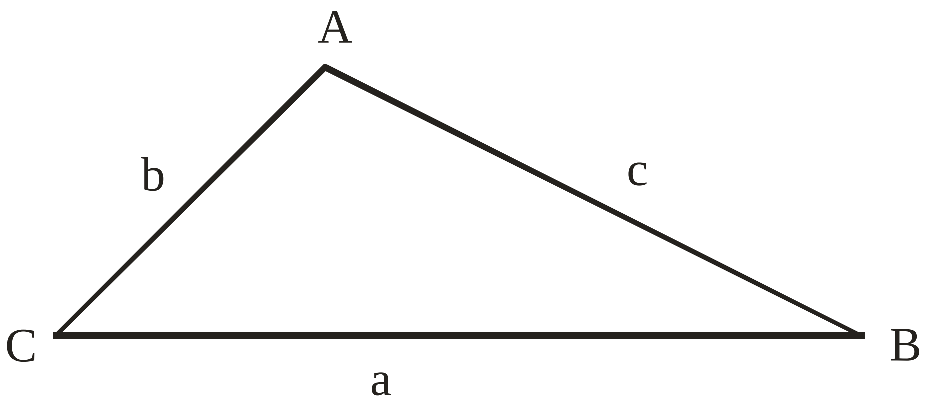
$$b^2 = a^2 + c^2 - 2acCosB$$

$$c^2 = a^2 + b^2 - 2abCosC$$

Given: Any two sides and the included angle, use the **Cosine Rule** to determine the third side. E.g. Given sides b, c and included angle A determine the other two angles and the third side a.

Given: Three sides, use the **Cosine Rule** to determine the internal angles.

Sine Rule



Sides of the triangle: a, b, and c

Internal Angles: A, B and C

A + B + C = 180º

$$\frac{a}{\mathrm{Sin}A} = \frac{b}{\mathrm{Sin}B} = \frac{c}{\mathrm{Sin}C}$$

Given: Any two sides and the angle opposite one of the given sides, use the **Sine Rule** to determine the angle opposite the other given side.

Given: Any one side, an angle opposite that side and one of the other angles, use the **Sine Rule** to determine the side opposite the other given angle.

Summary:

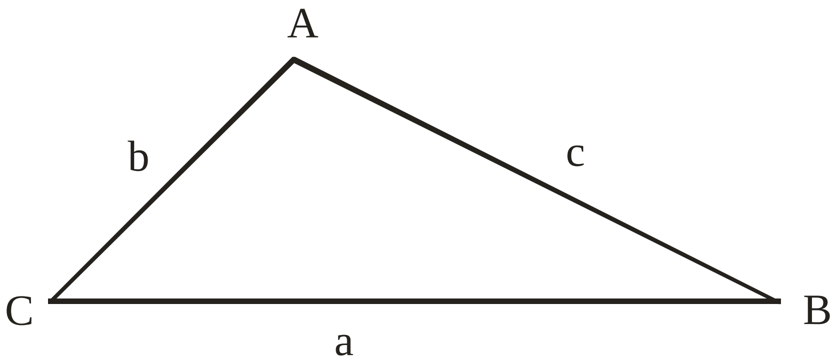
If you are given any three (3) of the sides or internal angles of a triangle, we can determine the other three(3) unknowns using the **Sine** and **Cosine Rules** **AND:**

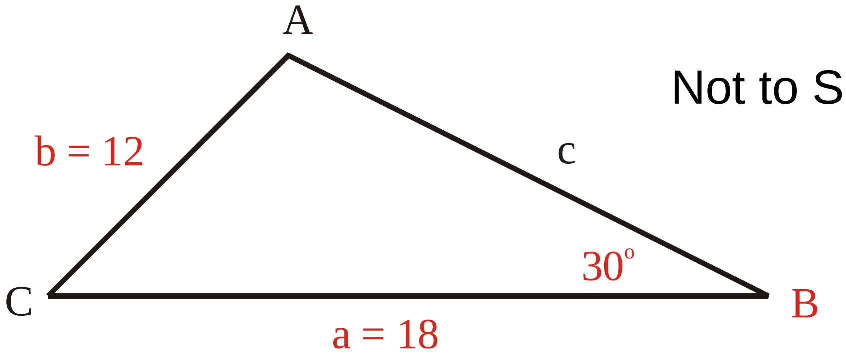
$$A + B + C = 180^{o}$$

Example:

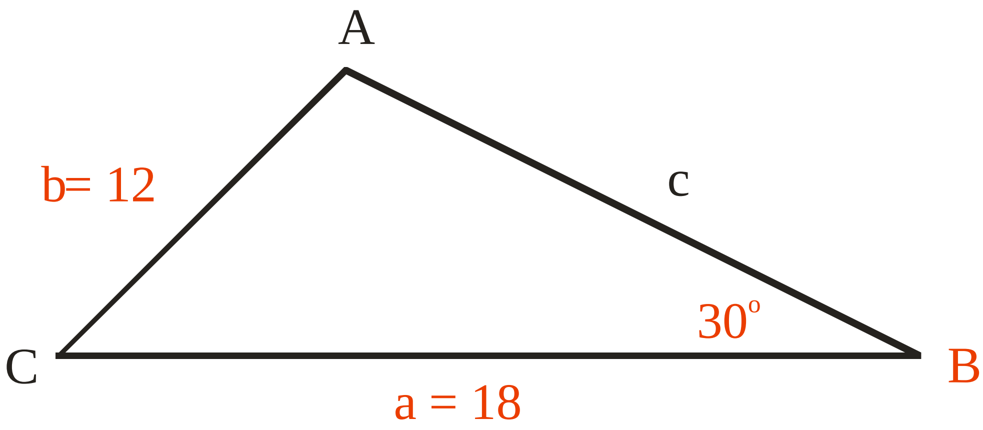
Given:

a = 18
b =12
B = 30º





Not to Scale:



This drawing is not to scale but looks reasonable for the given information?? (But does angle C look like it is 101.41° and does side c look like 23.5 in relation to side a = 18?????

Using the Sine Rule:

$$\frac{a}{\text{SinA}} = \frac{b}{\text{SinB}} = \frac{c}{\text{SinC}}$$

$$\frac{18}{\text{SinA}} = \frac{12}{\text{Sin}30^{o}} = \frac{c}{\text{SinC}}$$

$$\text{SinA} = \frac{18\left(\text{Sin}30^{o}\right)}{12} = 0.75$$

$$A = 48.590^{o}$$

$$\text{But}: A + B + C = 180^{o}$$

$$\therefore C = 180^{o} - 30^{o} - 48.590^{o}$$

$$C = 101.410^{o}$$

$$\frac{12}{\text{Sin}30^{o}} = \frac{c}{\text{Sin}101.410^{o}}$$

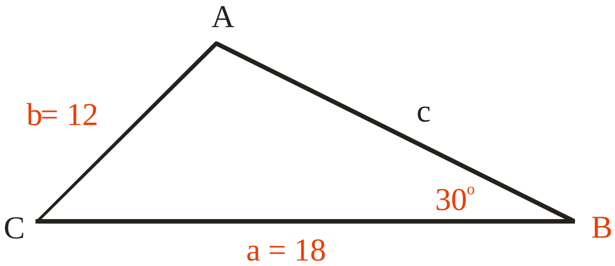
$$c = 23.526$$

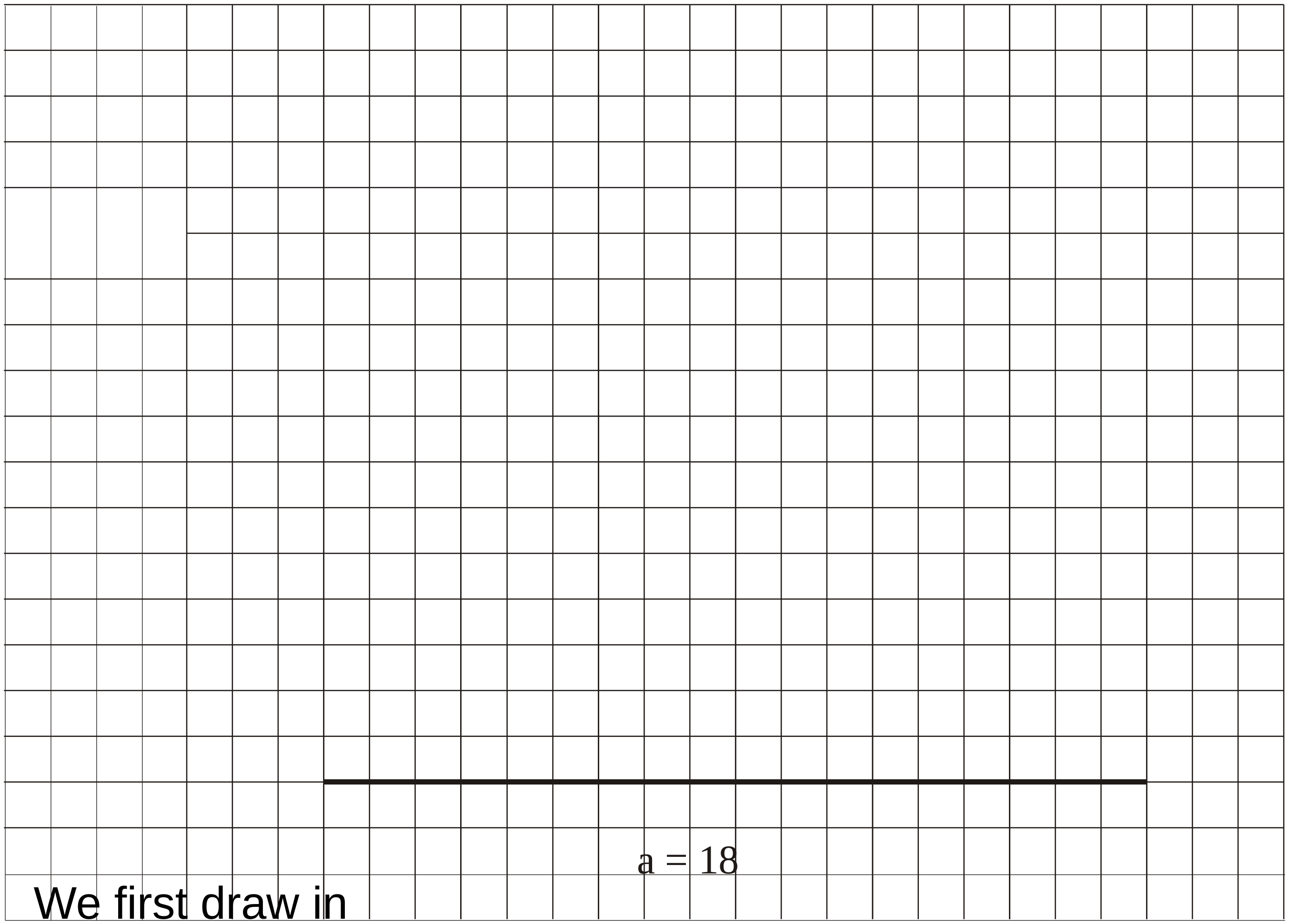
BUT:

Is this the only possible solution???????????

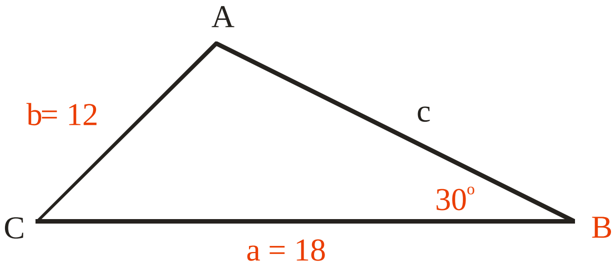
WHY WE SHOULD DRAW THE TRIANGLE TO SCALE!!!!!!!!!!

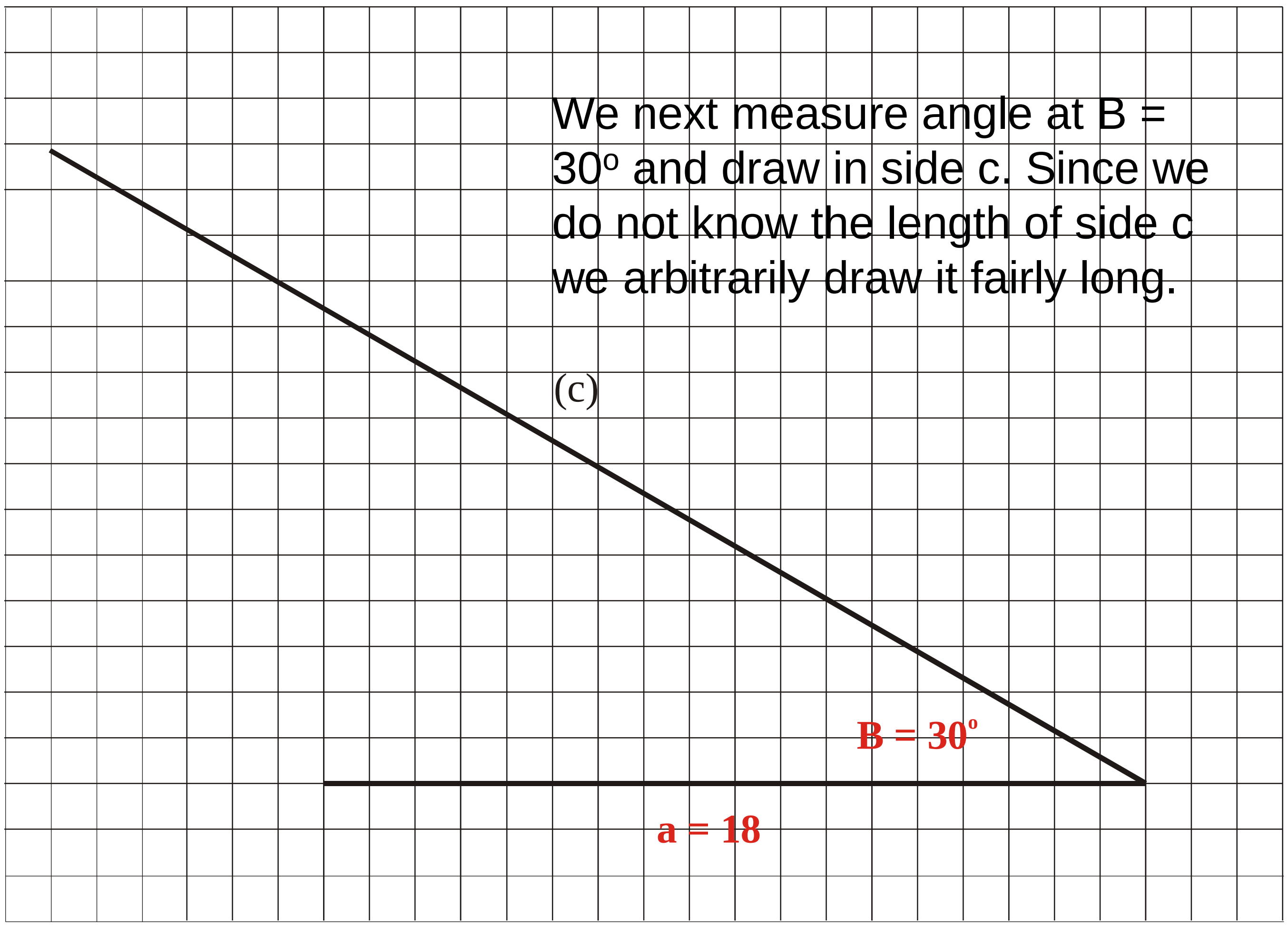
We now draw the triangle to scale.

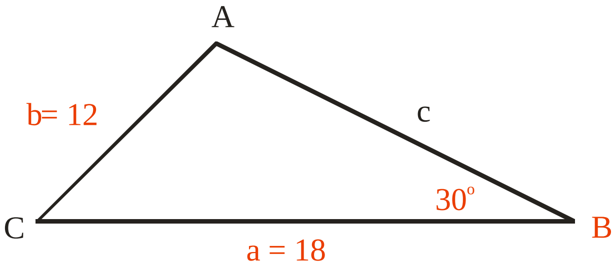




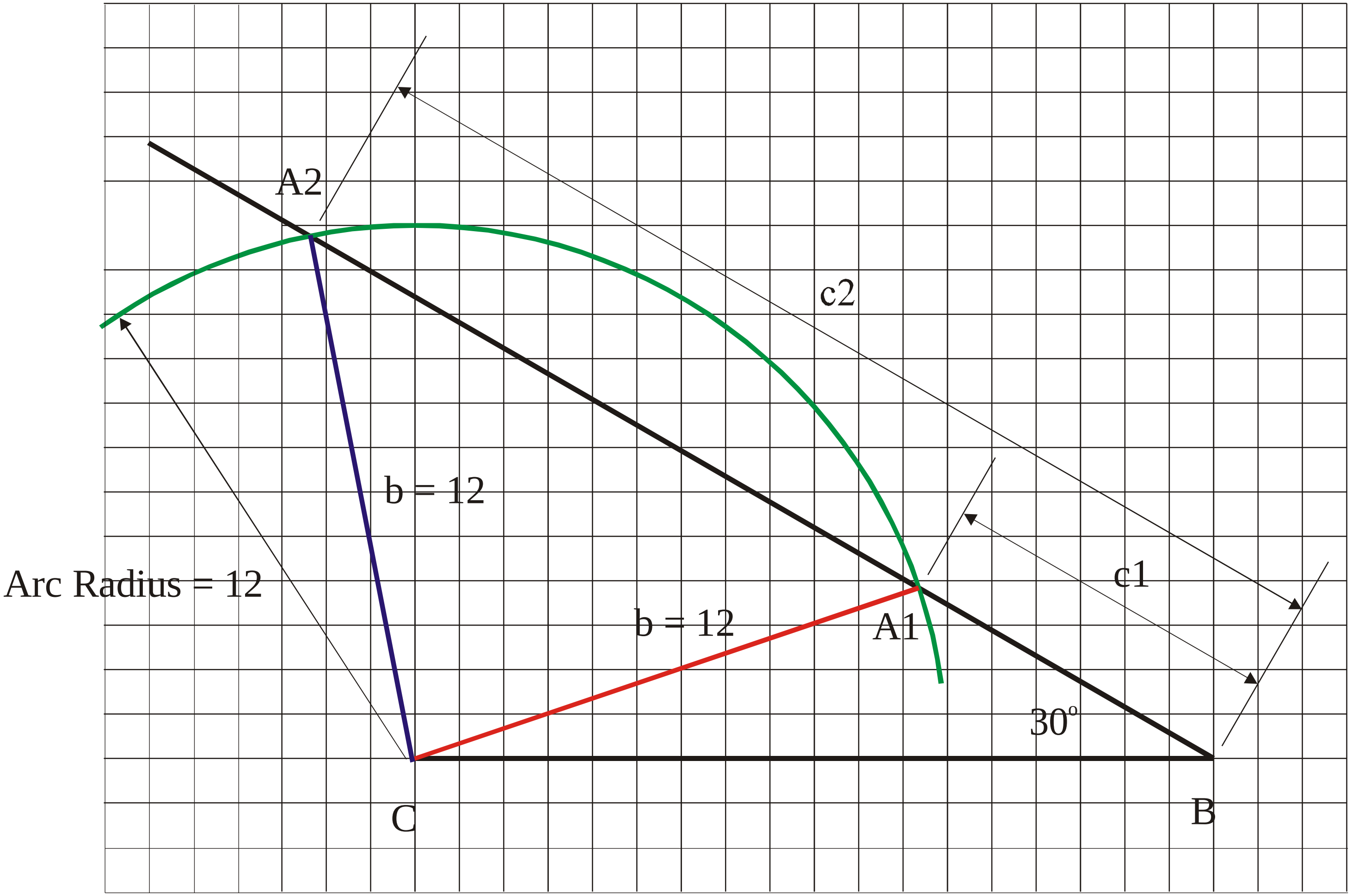
We first draw in side a = 18


A
b= 12
c
30°
C
B
a = 18


We next measure angle at B = 30º and draw in side c. Since we do not know the length of side c we arbitrarily draw it fairly long.
(c)
B = 30º
a = 18

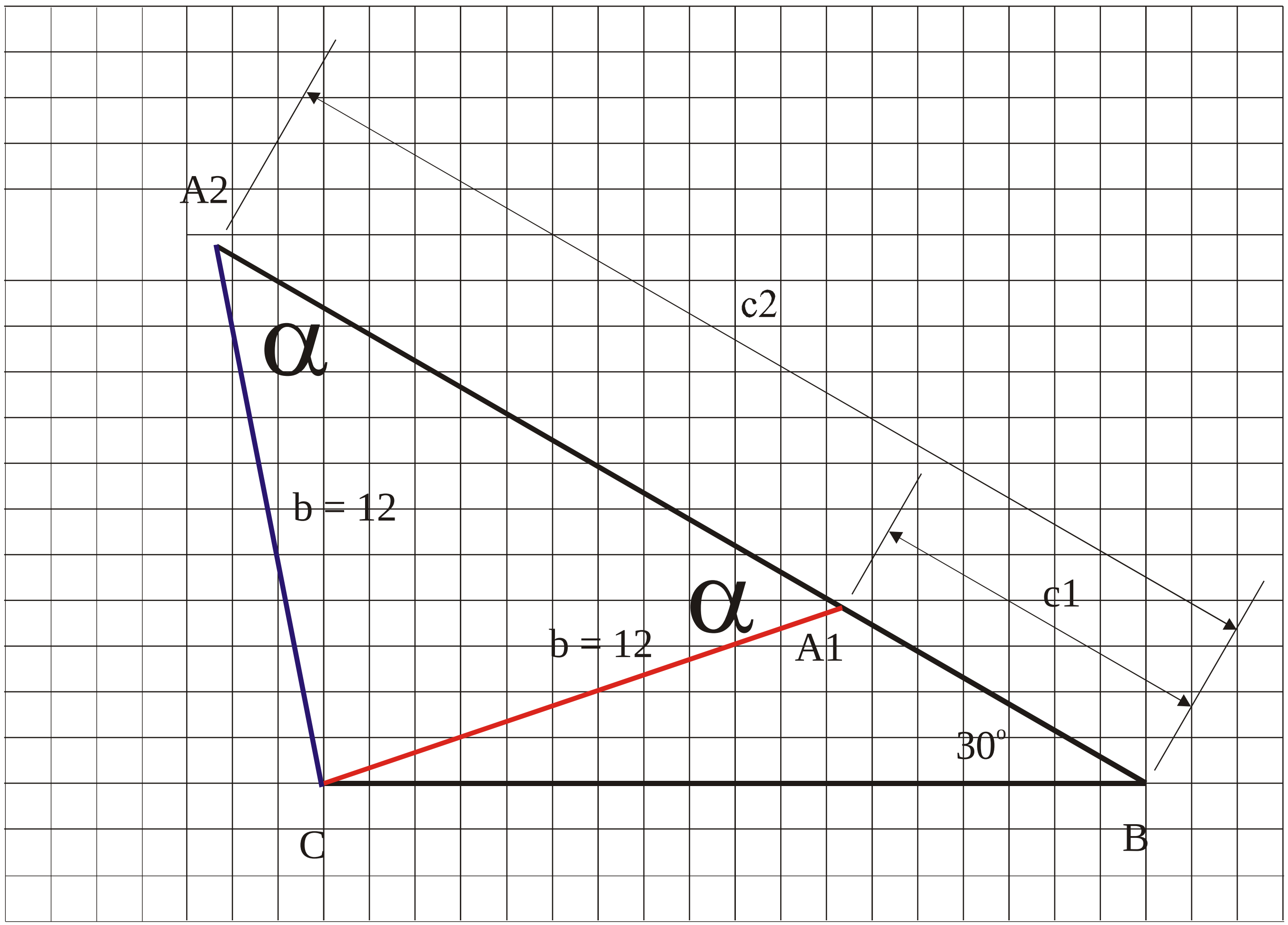


We know that the length of b = 12. Therefore with a compass setting the radius of a circle = 12 and the centre at C we draw the arc of a circle to intersect (c).



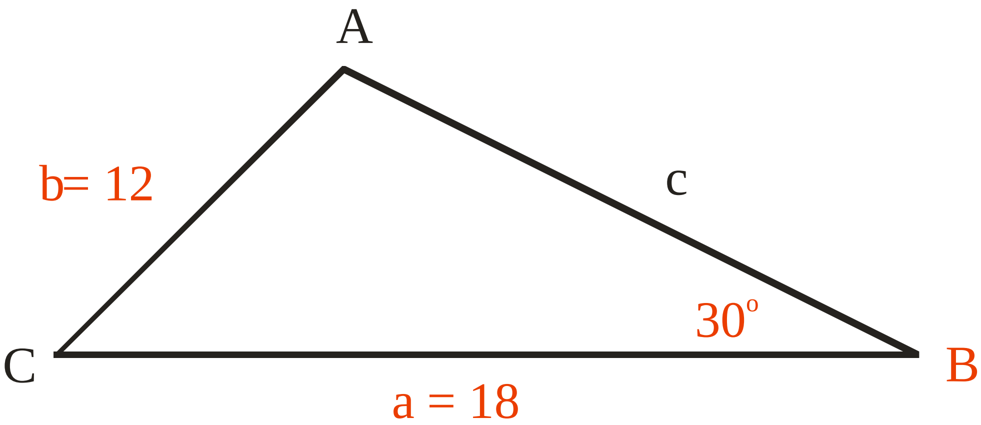
We now see there are two possibilities:

Triangle A1BC and Triangle A2BC.



RECALL: From Sine Rule:

$$\frac{a}{\text{Sin}A} = \frac{b}{\text{Sin}B} = \frac{c}{\text{Sin}C}$$



Using the Sine Rule:

$$\frac{18}{\text{Sin}A} = \frac{12}{\text{Sin}30^{o}} = \frac{c}{\text{Sin}C}$$

$$\text{Sin}A = \frac{18\left(\text{Sin}30^{o}\right)}{12} = 0.75$$

$A=48.590^{o}$

$$\text{But}: A + B + C = 180^{o}$$

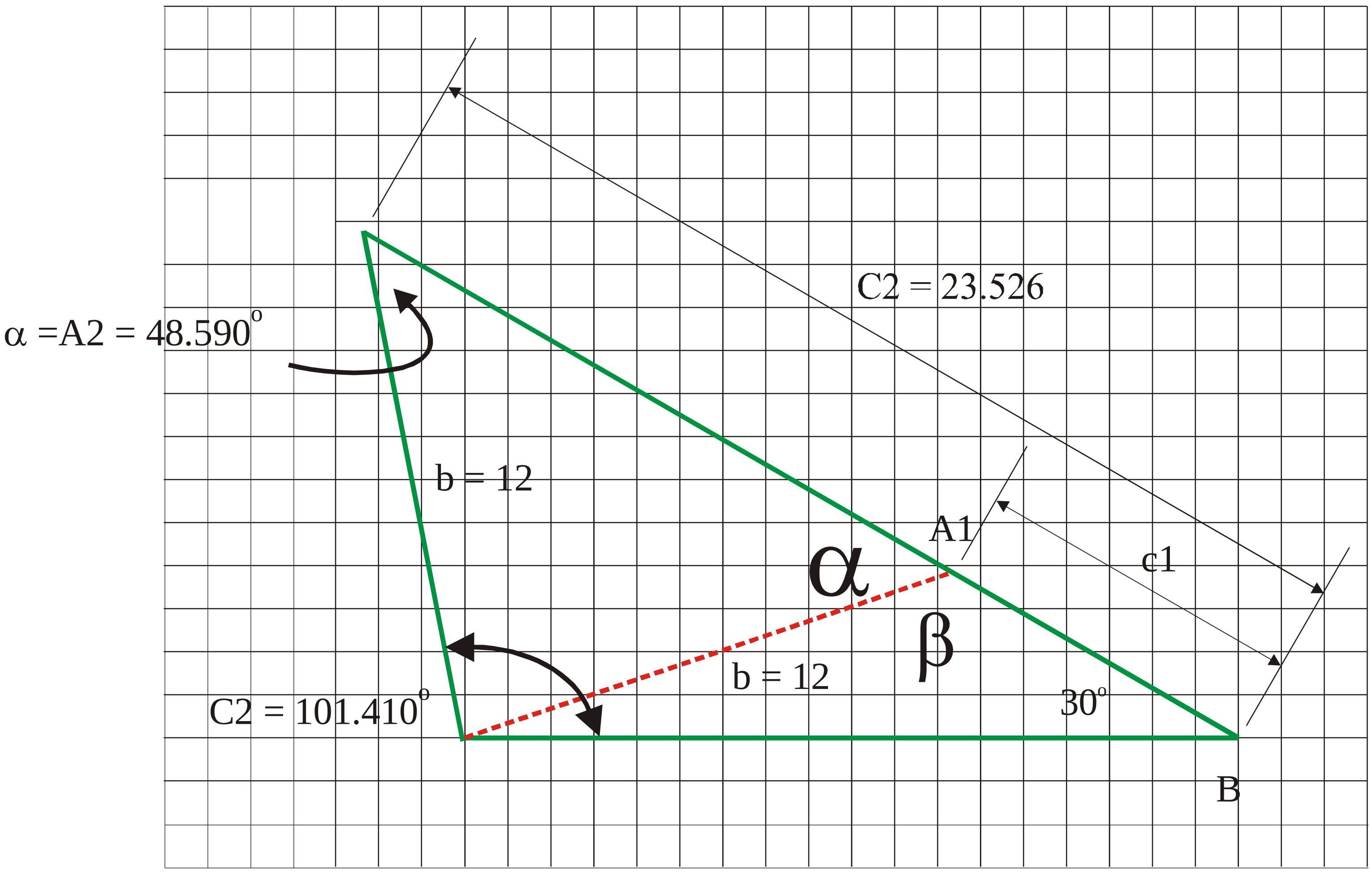
$$\therefore C = 180^{o} - 30^{o} - 48.590^{o}$$

$C=101.410^{o}$

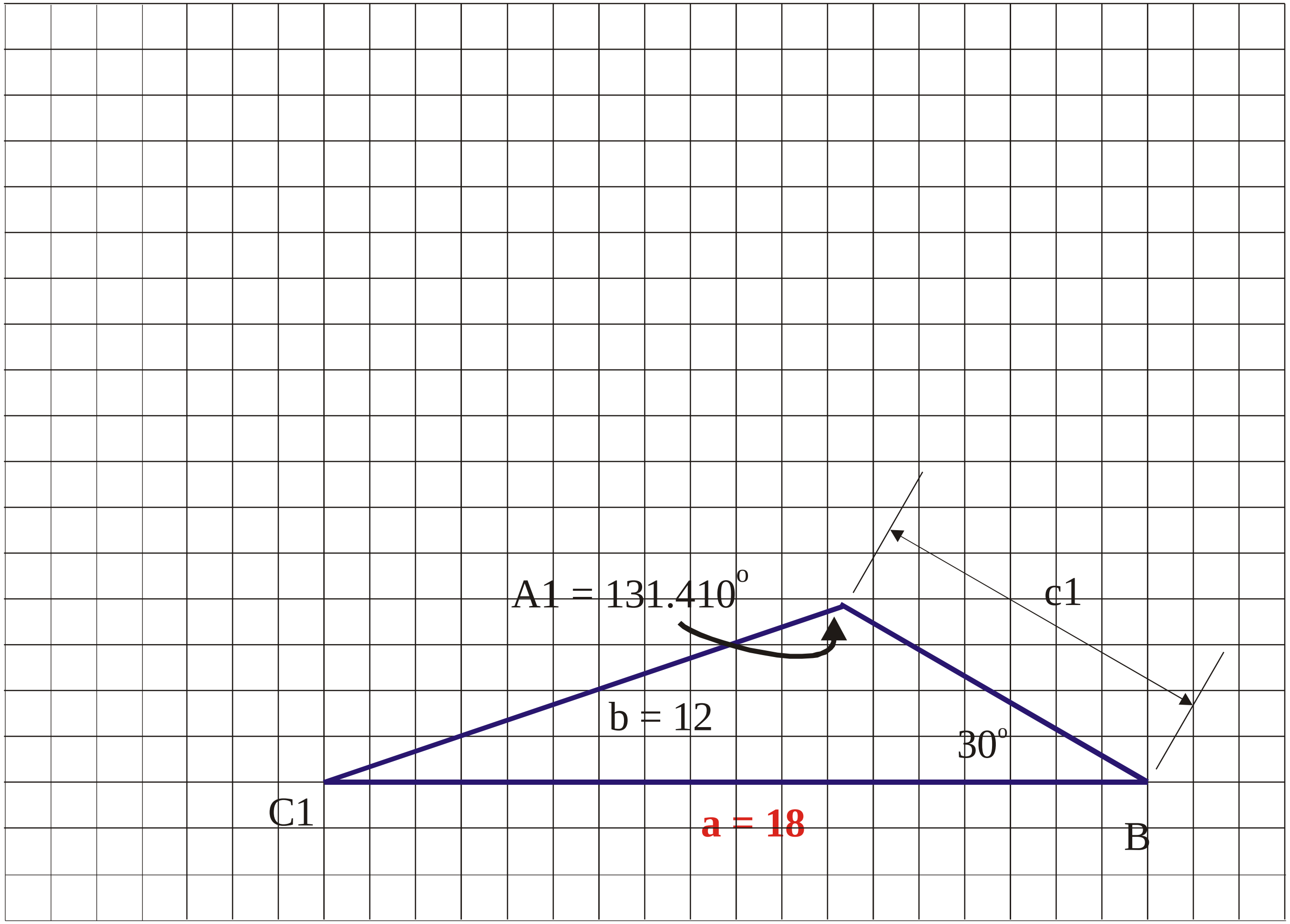
$$\frac{12}{\text{Sin}30^{o}} = \frac{c}{\text{Sin}101.410^{o}}$$

$c=23.526$

The application of the Sine Rule has provided the solution for Triangle A2BC.



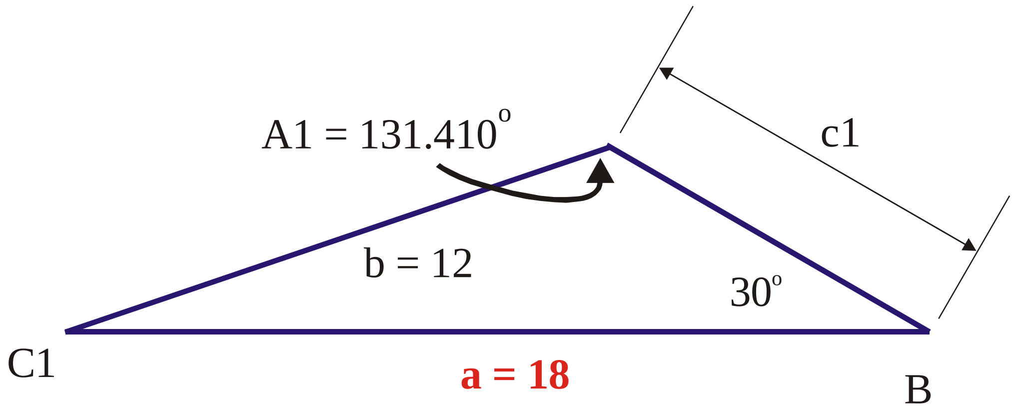
$\alpha + \beta = 180^{o}$ Therefore: $\beta = 180^{o} - 48.590^{o} = 131.410^{o}$



$$C1 + 30^o + 131.410^o = 180^o \quad \text{Therefore: } C1 = 18.590^o$$

Applying the Sine Rule to Triangle A1BC1:

$$\frac{a}{\mathrm{Sin}A} = \frac{b}{\mathrm{Sin}B} = \frac{c}{\mathrm{Sin}C}$$



$$\frac{18}{\mathrm{Sin}131.410^{\circ}} = \frac{12}{\mathrm{Sin}30^{\circ}} = \frac{c1}{\mathrm{Sin}18.590^{\circ}}$$

c1=7.651

Summary of Results:

a	b	c	A^o	B^o	C^o
18	12	23.526	48.590^o	30^o	101.410^o
18	12	7.651	131.410^o	30^o	18.590

Making a scaled drawing (**GRAPHICAL SOLUTION**) using an appropriate scale and using a compass and protractor to measure angles should assure you that you have not overlooked a possible solution. It is also a check of your answers obtained using the Sine and Cosine rules!!!

3. DETERMINANTS
(We will need in Chapter 6)

In linear algebra, the determinant is the value of a square matrix computed by a specific arithmetic expression.

Determinant of ORDER n

A determinant of order n consists of n^2 numbers called **elements** arranged in n rows and n columns and enclosed by vertical lines.

n = 1,2,3 ………

Examples:

First order determinant $\begin{vmatrix} 2 \end{vmatrix}$

Second order determinant $\begin{vmatrix} 3 & 2 \\ 4 & 1 \end{vmatrix}$

Third order determinant $\begin{vmatrix} -3 & 4 & 0 \\ 2 & 6 & 1 \\ 2 & 3 & -4 \end{vmatrix}$

Definition – Minor of a given element

The **minor** of a given element is the determinant of the elements that remain after DELETING the row and column in which the element stands.

$$\text{Example:}$$

$$\begin{vmatrix} -3 & 4 & 0 \\ 2 & 6 & 1 \\ 2 & 3 & -4 \end{vmatrix} \qquad \text{The minor of element -3 is } \begin{vmatrix} 6 & 1 \\ 3 & -4 \end{vmatrix}$$

$$\begin{vmatrix} -3 & 4 & 0 \\ 2 & 6 & 1 \\ 2 & 3 & -4 \end{vmatrix} \qquad \text{The minor of element 4 is } \begin{vmatrix} 2 & 1 \\ 2 & -4 \end{vmatrix}$$

Minors of Elements of Row 1

$$\begin{vmatrix} -3 & 4 & 0 \\ 2 & 6 & 1 \\ 2 & 3 & -4 \end{vmatrix}$$ Determinant of order 3

$$\begin{vmatrix} -3 & 4 & 0 \\ 2 & 6 & 1 \\ 2 & 3 & -4 \end{vmatrix}$$ Minor of element -3 $$\begin{vmatrix} 6 & 1 \\ 3 & -4 \end{vmatrix}$$

$$\begin{vmatrix} -3 & 4 & 0 \\ 2 & 6 & 1 \\ 2 & 3 & -4 \end{vmatrix}$$ Minor of element 4 $$\begin{vmatrix} 2 & 1 \\ 2 & -4 \end{vmatrix}$$

$$\begin{vmatrix} -3 & 4 & 0 \\ 2 & 6 & 1 \\ 2 & 3 & -4 \end{vmatrix}$$ Minor of element 0 $$\begin{vmatrix} 2 & 6 \\ 2 & 3 \end{vmatrix}$$

Value of a Determinant (Expanding a Determinant)

ORDER 1 – (Single Element) – The value of a determinant of order one is the single element of the determinant.

ORDER n (n > 1) – The value of a determinant of order n where n > 1 may be expressed as the SUM of n Products formed by multiplying EACH ELEMENT of ANY CHOSEN row or column by its MINOR and PREFIXING THE APPROPRIATE ALGEBRAIC SIGN.

Proper Algebraic Sign Associated with Each Product

$$(-1)^{i+j}$$

Where :

i = the number of the row for the element

j = the number of the column for the element

$\therefore$ i + j = Even Number, Sign is +

i + j = Odd Number, Sign is -

Expanding by First Row :

$$(-1)^{1+1} = +1$$

$$(-1)^{1+2} = -1$$

$$(-1)^{1+3} = +1$$

Example: Expanding a 2 x 2 Determinant by FIRST ROW

$-1^{(1+1)} = +1$ $\quad$ $-1^{(1+2)} = -1$

$$\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = +a_{11}(\text{minor of } a_{11}) - a_{12}(\text{minor of } a_{12})$$

$$= [(a_{11})(a_{22})] - [(a_{12})(a_{21})]$$

Example:

$$\begin{vmatrix} 3 & 2 \\ 2 & -4 \end{vmatrix} = +[(3)(-4)] - [(2)(2)] = -12 - 4 = -16$$

Example: Expanding a 3 x 3 Determinant by FIRST ROW

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

$$= +a_{11}\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - a_{12}\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + a_{13}\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

$$= a_{11}(a_{22}a_{33} - a_{32}a_{23}) - a_{12}(a_{21}a_{33} - a_{31}a_{23}) + a_{13}(a_{21}a_{32} - a_{31}a_{22})$$

3 x 3 Determinants

Determining the Value of a **3 x 3** Determinant by the

Basket Weave Method

This will be used in Chapter 6

(Moment about a Point in 3-Dimensions)

This is an alternative to expansion of a determinant by method of minors and works for a 3 x 3 determinant. (You will find that you will make fewer algebraic sign errors if you use this method.)

Let us consider the 3 x 3 determinant. It has three rows and three columns for a total of 9 elements. We wish to “expand” or determine the value of this determinant using what is sometimes referred to as the “Basket Weave” method.

EXAMPLE:

$$\begin{vmatrix} 1 & 3 & -2 \\ 7 & 4 & 5 \\ -4 & -1 & 2 \end{vmatrix}$$

We first re-write the determinant repeating columns 1 and 2 in columns immediately to the right of column 3 and draw a closing vertical line.

$$\left|\begin{array}{rrr|rr} 1 & 3 & -2 & 1 & 3 \\ 7 & 4 & 5 & 7 & 4 \\ -4 & -1 & 2 & -4 & -1 \end{array}\right|$$

We now create two (2) square brackets with a MINUS sign between them. Always do this first!!!!!!!!

$$[\qquad\qquad\qquad] - [\qquad\qquad\qquad]$$

We Multiply "Down" the three diagonals as shown and place the results inside the first of the two square brackets.

$$\begin{vmatrix} 1 & 3 & -2 \\ 7 & 4 & 5 \\ -4 & -1 & 2 \end{vmatrix} \begin{matrix} 1 & 3 \\ 7 & 4 \\ -4 & -1 \end{matrix}\Bigg|$$

$$\left[(8) \quad + (-60) \quad + (14)\right] - \left[\qquad\qquad \right]$$

We Next Multiply "Up" the three diagonals as shown and place the results inside the second of the two square brackets.

$$\begin{vmatrix} 1 & 3 & -2 \\ 7 & 4 & 5 \\ -4 & -1 & 2 \end{vmatrix}\begin{matrix} 1 & 3 \\ 7 & 4 \\ -4 & -1 \end{matrix}\Bigg|$$

$$=\left[(8) \quad +(-60) \quad +(14)\right]-\left[(32) \quad +(-5) \quad +(42)\right]$$

We Simplify: $$=\left[-38\right]-\left[69\right]=-107$$

Example: Determine the Value of the Determinant

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & -2 & 4 \\ 4 & 3 & -5 \end{vmatrix}$$

$$\left|\begin{array}{ccc|cc} \mathbf{i} & \mathbf{j} & \mathbf{k} & \mathbf{i} & \mathbf{j} \\ 3 & -2 & 4 & 3 & -2 \\ 4 & 3 & -5 & 4 & 3 \end{array}\right|$$

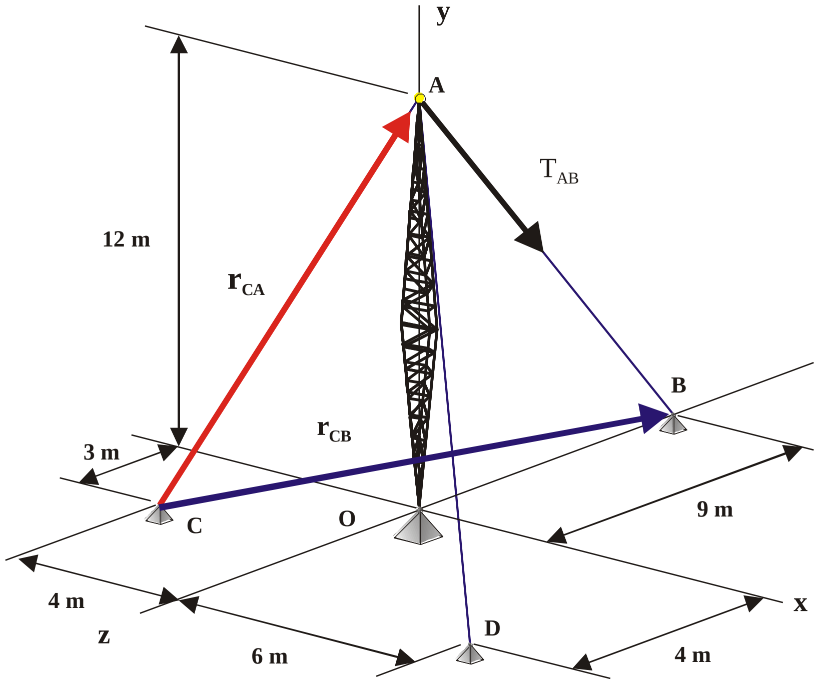
$$= \left[(10\mathbf{i}) + (16\mathbf{j}) + (9\mathbf{k})\right] - \left[(-8\mathbf{k}) + (12\mathbf{i}) + (-15\mathbf{j})\right]$$

$$= -2\mathbf{i} + 31\mathbf{j} + 17\mathbf{k}$$

But WHY do we need to know how to expand a 3 x 3 Determinant???

In Chapter 6 we will cover the topic of Moment of a Force about a point using Vector Algebra for 3 dimensional problems.

3 x 3 Determinant for Moment of a Force about a Point



$$\mathbf{M} = \mathbf{r} \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$

Vector or “Cross” Product

$$\mathbf{AB} = (\mathrm{B} - \mathrm{A}) = (0, -12, -9)$$

$$\mathrm{AB} = \sqrt{0^2 + 12^2 + 9^2} = 15\ \mathrm{m}$$

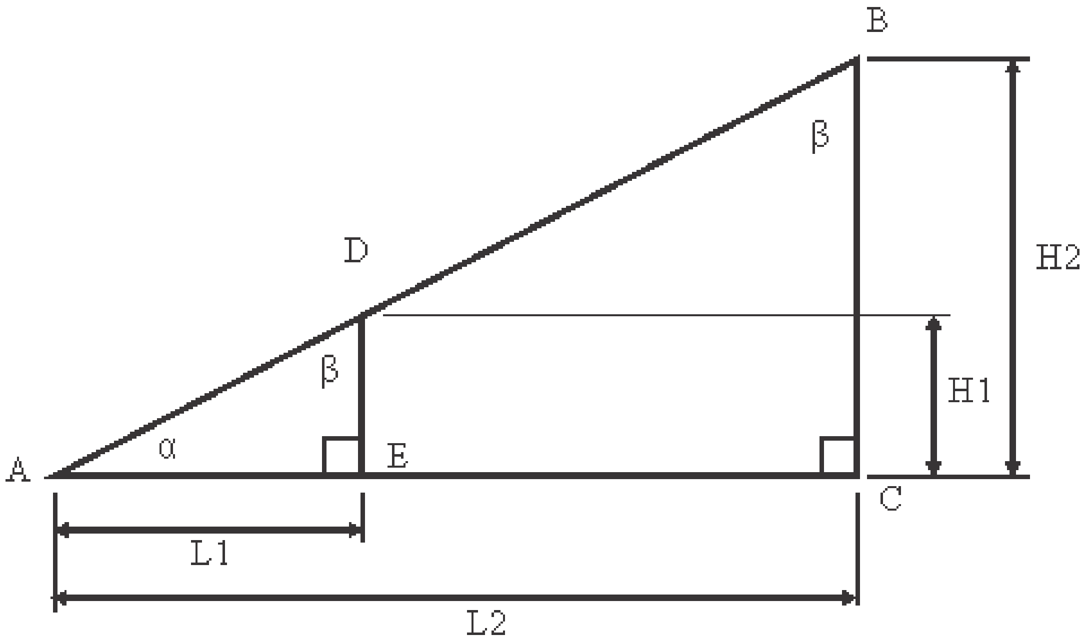
$$\boldsymbol{\lambda}_{\mathrm{AB}} = \left(0, -\frac{12}{15}, -\frac{9}{15}\right) = \left(0, -\frac{4}{5}, -\frac{3}{5}\right)$$

$$\mathbf{T}_{\mathrm{AB}} = \mathrm{T}_{\mathrm{AB}}\boldsymbol{\lambda}_{\mathrm{AB}} = 1200\boldsymbol{\lambda}_{\mathrm{AB}} = (0, -960, -720)\ \mathrm{N}$$

(a) $\mathbf{r} = \mathbf{CA} = (\mathrm{A} - \mathrm{C}) = (4,\ 12,\ -3)$

$$\mathbf{M}_{\mathrm{C}} = \mathbf{CA} \times \mathbf{T}_{\mathrm{AB}} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 4 & 12 & -3 \\ 0 & -960 & -720 \end{vmatrix}$$

Similar Triangles



$$\frac{L1}{H1} = \frac{L2}{H2} \text{ or}$$

$$\frac{L1}{L2} = \frac{H1}{H2}$$

Triangles ABC and ADE are similar in that the internal angles of triangle ABC are the same as the internal angles of triangle ADE.