Math 2130 - Engineering Mathematical Analysis 1

Tutorial 7 - Questions for §13.1 - 13.3.

13.1.1. Evaluate the double iterated integral $\displaystyle\int_{-2}^{0}\int_{0}^{-x}\sqrt{y-x}\,dy\,dx$.

13.2.1. Evaluate the double iterated integral $\displaystyle\int_{-1}^{0}\int_{-2}^{2x}x\sqrt{x^2+y^2}\,dy\,dx$.

13.2.2. Consider the triangle in the xy-plane with vertices at $(4,0)$, $(4,2)$ and $(0,0)$. Set up and evaluate double iterated integrals that calculate the area of this triangle. Do this using vertical strips and using horizontal strips.

13.2.3. Consider the cylindrical solid whose base is the triangle in the xy-plane given in question 13.2.2 above and whose top is given by the plane $z = 3x + 2y + 1$. Set up and evaluate double iterated integrals that calculate the volume of this solid. Do this using vertical strips and using horizontal strips in the base.

13.3.1. Set up and evaluate double iterated integrals that calculate the volumes of the solids of revolution when the area bounded by the curves

$$y = 2x - x^2, \quad y = x$$

is rotated around the lines: (a) $x = 3$; (b) $y = 1$.

Answers:

13.1.1. $16(4-\sqrt{2})/15$.

13.2.1. $(8-5\sqrt{5})/6$.

13.2.2. Area $= \displaystyle\int_0^4 \int_0^{x/2} dy\,dx = \int_0^2 \int_{2y}^4 dx\,dy = 4.$

13.2.3. Volume $= \displaystyle\int_0^4 \int_0^{x/2} 3x+2y+1\ dy\,dx = \int_0^2 \int_{2y}^4 3x+2y+1\ dx\,dy = \frac{124}{3}.$

13.3.1. (a) Volume $= 2\pi \displaystyle\int_0^1 \int_x^{2x-x^2} 3-x\ dy\,dx = \frac{5\pi}{6}$;

(b) Volume $= 2\pi \displaystyle\int_0^1 \int_x^{2x-x^2} 1-y\ dy\,dx = \frac{2\pi}{15}.$