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(Fall 07)

Answers to Dec. 10/2007 Math 2130 Final by Dawit yohannes

1) $3x+3y+4z-10=0$

2) $\displaystyle\int_0^{\pi/2}\sqrt{8-8\sin t\cos t}\,dt=\int_0^{\pi/2}2\sqrt{2-\sin 2t}\,dt$

3) $\vec{N}_1=(1,-1,1)$, $\vec{N}_2=(2,1,3)$ $\Rightarrow$ $\displaystyle\cos\gamma=\frac{|\vec{N}_1\cdot\vec{N}_2|}{|\vec{N}_1||\vec{N}_2|}=\frac{4}{\sqrt{42}}=\frac{4\sqrt{42}}{42}=\frac{2\sqrt{42}}{21}$

4) $\displaystyle(3r^2+s^4)\left[\frac{y}{r}(2z)+\frac{z}{r}\right]+(4\hat{r}s^3)(x\cos xy)(2z)$

5) $\dfrac{4\sqrt{46}}{23}$

6.) $b=\pm 5$

7) $-\begin{vmatrix}-1 & 3x^2u^2+v^2\\ 2yv^5 & 4xu^3+z\end{vmatrix}\Bigg/\begin{vmatrix}2uv-1 & 3x^2u^2+v^2\\ 5v^4y^2-6 & 4xu^3+z\end{vmatrix}$.

Note: $\begin{vmatrix}a & b\\ c & d\end{vmatrix}=-\begin{vmatrix}b & a\\ d & c\end{vmatrix}$

$\begin{vmatrix}a & b\\ c & d\end{vmatrix}=+\begin{vmatrix}d & c\\ b & a\end{vmatrix}$

(from properties of determinants)

8) 2

9) $\displaystyle\frac{2\pi}{\sqrt{5}}\int_0^2\int_y^{\sqrt{6-y}}(10-2x-y)\,dx\,dy$

10) $\displaystyle\int_0^{2\pi}\int_0^{\sqrt{3}}\sqrt{1+4r^2}\,r\,dr\,d\theta=4\int_0^{\pi/2}\int_0^{\sqrt{3}}\sqrt{1+4r^2}\,r\,dr\,d\theta.$

11.) $\displaystyle\int_1^3\int_{1-y}^{\sqrt[3]{y-1}}\ell(y-2)\,dx\,dy$

12.) $\displaystyle\int_0^1\int_z^{2-z}\int_0^2(x^2+y-z^3)\,dx\,dy\,dz$

13.) $\displaystyle 4\int_0^{\pi/2}\int_0^2\int_0^{2r}r\,dz\,dr\,d\theta=\frac{32\pi}{3}$

(Winter 08)

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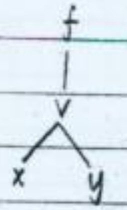
Answers to april 22/2008 Math 2130 Final by Dawit Yohannes

1) $30/\sqrt{90} = \sqrt{10}$

2) $9x+2y-6z-11=0$ or $-9x-2y+6z+11=0$

3)



let $v = x^3+y^2 \Rightarrow u = f(v)+g(v)$

$$\frac{\partial u}{\partial x} = \frac{du}{dv}\frac{\partial v}{\partial x} = (f'(v)+g'(v))(3x^2)$$

$$\frac{\partial u}{\partial y} = \frac{du}{dv}\frac{\partial v}{\partial y} = (f'(v)+g'(v))(2y)$$

$$2y\left[(3x^2)(f'(v)+g'(v))\right] - 3x^2\left[2y(f'(v)+g'(v))\right] = 0$$

4) $\dfrac{e^x - \cos x}{6u}$

5) $e^{1/4}-1$

6) Max $=\sqrt{2}$, min $=0$

7) a) $\displaystyle\int_1^4\int_{(x-2)^2}^{x} (x^2+y)\,x\,dy\,dx$ b) $\displaystyle\int_1^4\int_{(x-2)^2}^{x} (x^2+y)\left(\frac{4x-3y+1}{5}\right)^2 dy\,dx$.

c) $M = \displaystyle\int_1^4\int_{(x-2)^2}^{x}(x^2+y)\,dy\,dx$; $\bar{x} = \dfrac{1}{M}\displaystyle\int_1^4\int_{(x-2)^2}^{x} x(x^2+y)\,dy\,dx$; $\bar{y} = \dfrac{1}{M}\displaystyle\int_1^4\int_{(x-2)^2}^{x} y(x^2+y)\,dy\,dx$

8) a) $\displaystyle\iint_R \sqrt{1+4(x^2+y^2)}\,dA$ b) Surface area of $z=y^2-x^2$ bounded by $x^2+y^2=1$ and $x^2+y^2=4$.

c) $\frac{\pi}{6}\left(17\sqrt{17}-5\sqrt{5}\right)$ 9) $\displaystyle\int_0^3\int_{z/3}^{4-z}\int_0^3 dy\,dx\,dz$ 10) a) $(\rho,\phi,\theta) = (4, \pi/6, \pi/4)$

b) $(r,\theta,z) = (\sqrt{2}, \pi/4, \sqrt{6})$

11) $\pi/2$ 12) $\dfrac{135k\pi}{2}$.

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(Fall 08)

Answers to Dec 11/2008 Math 2130 Final by Dawit yohannes

1.) the question has no solution because the two lines don't intersect. (They are skew-lines)

2) $\frac{2e^t}{3x} + \frac{6ty^2}{e^y}$

3) $Max = 3/2$; $Min = -3$

4) $180\rho g$

5) $2[8\sqrt{2}(\sqrt{8}-\sqrt{6})\pi] = 16(4-2\sqrt{3})\pi = 32(2-\sqrt{3})\pi$

6) $\int_0^1 \int_0^{\sqrt{1-z^2}} \int_0^y f(x,y,z)\,dx\,dy\,dz$; $\int_0^1 \int_0^{\sqrt{1-y^2}} \int_0^y f(x,y,z)\,dx\,d\underset{z}{\cancel{y}}\,d\overset{y}{\cancel{z}}$

$\int_0^1 \int_0^{\sqrt{1-x^2}} \int_x^{\sqrt{1-z^2}} f(x,y,z)\,dy\,dz\,dx$; $\int_0^1 \int_0^{\sqrt{1-z^2}} \int_x^{\sqrt{1-z^2}} f(x,y,z)\,dy\,dx\,dz$

$\int_0^1 \int_0^y \int_x^{\sqrt{1-y^2}} f(x,y,z)\,dz\,dx\,dy$; $\int_0^1 \int_x^1 \int_0^{\sqrt{1-y^2}} f(x,y,z)\,dz\,dy\,dx$

7) $\int_0^1 \int_{x^3}^{\sqrt{2-x}} \rho\left(\frac{3x-2y-6}{\sqrt{13}}\right)^2 dy\,dx$ 8) $\frac{3\pi-2}{6}$

9) a) $R = \frac{1}{\sqrt{\cos 2\phi}} = \sqrt{\sec 2\phi}$ b) $z = \frac{\sqrt{x^2+y^2}-x^2-y-1}{2(x^2+y^2)}$

10) $\int_0^{2\pi} \int_0^{\pi/3} \int_{\sec\phi}^2 R^2 \sin\phi\,dR\,d\phi\,d\theta = 4\int_0^{\pi/2} \int_0^{\pi/3} \int_{\sec\phi}^2 R^2 \sin\phi\,dR\,d\phi\,d\theta$

(Winter 09)

Answers to April 21/2009 Math 2130 Final

by Dawit Yohannes (plankion@yahoo.com)

1) Given any $\epsilon > 0$, we can find $\delta > 0$ such that
$|f(x,y) - L| < \epsilon$ whenever $0 < \left|\sqrt{(x-a)^2 - (y-b)^2}\right| < \delta$.
provided (x, y) is in the domain of $f(x,y)$

2) Show that $\frac{\partial^2 f}{\partial x^2} = -36\sin 6x \sin 2y$ and $\frac{\partial^2 f}{\partial y^2} = -4\sin 6x \sin 2y$

3) $6x - 3y - 2z + 2 = 0$ 4) $\sin 4$

5) $$-(6uv^3 + ve^{uv})\frac{\begin{vmatrix} 24u^2x^3 & 5y^3 \\ 56x^6y & -15u^2v^2 \end{vmatrix}}{\begin{vmatrix} 12ux^4 & 5y^3 \\ -10uv^3 & -15u^2v^2 \end{vmatrix}} - (9u^2v^2 + ue^{uv})\frac{\begin{vmatrix} 12ux^4 & 24u^2x^3 \\ -10uv^3 & 56x^6y \end{vmatrix}}{\begin{vmatrix} 12ux^4 & 5y^3 \\ -10uv^3 & -15u^2v^2 \end{vmatrix}}$$

6) a) $x = t,\ y = t^2 - t + 3,\ z = 3t^2 + t - 3$ b) $\int_1^3 \sqrt{3 + 8t + 40t^2}\, dt$

7) $(0,0)$ yields a saddle point; $(2,2)$ yields a relative minimum.

8) Maximum value $= 4/3$, Minimum value $= -4/3$

9) $\int_0^{2\pi}\int_1^3 kr^2(r\cos\theta + 3)^2\, dr\, d\theta$ (about a vertical tangent)

10) $\int_0^4 \int_{(y-2)^2}^{12-2(y-2)^2} \sqrt{1 + 24xy + 13(x^2+y^2)}\, dx\, dy$

11) $M = \int_{-2}^{3}\int_0^{6+x-x^2}\int_0^{20-2(x+y)} kz\, dz\, dy\, dx$; $\bar{x} = \frac{1}{M}\int_{-2}^{3}\int_0^{6+x-x^2}\int_0^{20-2(x+y)} kxz\, dz\, dy\, dx$; $\bar{y} = \frac{1}{M}\int_{-2}^{3}\int_0^{6+x-x^2}\int_0^{20-2(x+y)} kyz\, dz\, dy\, dx$

$\bar{z} = \frac{1}{M}\int_{-2}^{3}\int_0^{6+x-x^2}\int_0^{20-2(x+y)} kz^2\, dz\, dy\, dx$.

12) $\int_0^1\int_{2x}^{2}\int_0^{36-9y^2} dz\, dy\, dx$, $\int_0^2\int_0^{y/2}\int_0^{36-9y^2} dz\, dx\, dy$,

$\int_0^{36}\int_0^{\frac{1}{3}\sqrt{36-z}}\int_0^{y/2} dx\, dy\, dz$, $\int_0^2\int_0^{36-9y^2}\int_0^{y/2} dx\, dz\, dy$,

$\int_0^1\int_0^{36(1-x^2)}\int_{2x}^{\frac{1}{3}\sqrt{36-z}} dy\, dz\, dx$, $\int_0^{36}\int_0^{\frac{1}{6}\sqrt{36-z}}\int_0^{\frac{1}{3}\sqrt{36-z}} dy\, dx\, dz$

13) $\frac{32}{5}(2-\sqrt{2})\pi$

(Fall 09)

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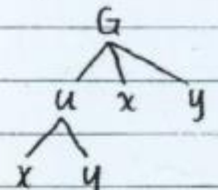
Answers to Dec. 18/2009 Math 2130 Final by Dawit Yohannes

1) a) $y = x^2/4$ b) $x = -t$, $y = t^2/4$, $z = 5 - t^2/4$ c) $\int_{-\sqrt{20}}^{0} \sqrt{1 + t^2/2}\, dt$

2.) $9/\sqrt{117} = \frac{3\sqrt{13}}{13}$ 3) $\frac{1}{\sqrt{266}}(\ln 2 - 16)$

4.) let $u = 3x - 2y^2 \Rightarrow G(x,y) = f(u) + xy$, with $u = u(x,y)$



$$\frac{\partial G}{\partial x} = \left.\frac{\partial G}{\partial u}\right)_{x,y} \frac{\partial u}{\partial x} + \left.\frac{\partial G}{\partial x}\right)_{u,y} = f'(u)\cdot 3 + y$$

$$\frac{\partial G}{\partial y} = \left.\frac{\partial G}{\partial u}\right)_{x,y} \frac{\partial u}{\partial y} + \left.\frac{\partial G}{\partial y}\right)_{x,u} = f'(u)(-4y) + x$$

$$LHS = 4y\left[\cancel{f'(u)\cdot 3} + y\right] + 3\left[\cancel{f'(u)\cdot(-4y)} + x\right] = 4y^2 + 3x = RHS$$

5.) (0,0) yields rel-min; (1,1) yields Saddle point

6) (0,0) yields a Saddle point

7) Max-value $= 1/27$ 8) $1/\pi$

9.) $\frac{\pi}{6}\left[(1+4a^2)^{3/2} - 1\right]$

10.) a) $\int_0^2 \int_{x^2-2}^{\frac{x+2}{2}} 2\pi(x+4)\, dy\, dx$ b) $\int_0^2 \int_{x^2-2}^{\frac{x+2}{2}} (x^2+y^2)\left(\frac{x-2y+2}{\sqrt{5}}\right)^2 dy\, dx$

11) a) $4\int_0^{\pi/2} \int_0^{\sqrt{2}} \int_r^{\sqrt{4-r^2}} r\, dz\, dr\, d\theta$ b) $4\int_0^{\pi/2} \int_0^{\pi/4} \int_0^{2} R^2 \sin\phi\, dR\, d\phi\, d\theta$.