10   **1.** Find the interval of convergence for the power series

$$\sum_{n=2}^{\infty} \frac{2^n}{n}(x+1)^{2n+3}.$$

12   **2.** Find the Taylor series about $x = -2$ for the function $\dfrac{1}{\sqrt{2x+7}}$. Express your answer in sigma notation simplified as much as possible. Include its open interval of convergence. You must use a method that guarantees that the series converges to the function.

8   **3.** (a) Find a series of constants that represents the value of the integral

$$\int_0^{1/2} \frac{x - \sin x}{x^3} dx.$$

     Express your answer in sigma notation.

     (b) Explain how you would obtain an approximation to the integral correct to 6 decimal places. Justify your reasoning.

10   **4.** Find, in explicit form, a one-parameter family of solutions of the differential equation

$$x\frac{dy}{dx} = \frac{\sqrt{x-2}}{x} - 3y.$$

Is your solution a general solution? Explain.

6   **5.** A tank originally contains 1000 litres of water in which 5 kilograms of sugar has been dissolved. A mixture containing 2 kilograms of sugar for each 100 litres of solution is being added to the tank at a rate of 10 millilitres per second. At the same time, 20 millilitres of well-stirred mixture is being withdrawn from the tank. Set up an initial-value problem for the number of grams of sugar in the tank as a function of time. Make **NO ATTEMPT** to solve the problem, but determine how long the differential equation is in effect.

7 6. You are given that the roots of the auxiliary equation associated with the differential equation

$$\phi(D)y = 3x^2 + 4\cos 2x - 3xe^{3x},$$

are $m = 0, 0, 1 \pm 2i, 1 + 2i, 3, -2$. What is the form of a particular solution to the differential equation as predicted by undetermined coefficients? Your answer should contain the minimum number of terms possible. Make **NO** attempt to find the coefficients.

10 7. Find a general solution for the differential equation

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 4y = \sin 2x - x.$$

10 8. Find Laplace transforms for the following functions. Do **NOT** simplify your answers.

(a) $f(t) = 3t^2 - 1,\ 0 < t < 3,\quad f(t+3) = f(t)$

(b) $f(t) = \begin{cases} 0, & 0 \le t < \pi \\ e^{2t}\sin t, & t > \pi \end{cases}$

14 9. Find inverse Laplace transforms for the following functions:

(a) $F(s) = \dfrac{6s^2 + 11s - 1}{s^3 + 3s^2 + s + 3}$

(b) $F(s) = \dfrac{se^{-s}}{s^2 + 2s + 4}$

13 10. A mass of 2 kilograms resting on a table is attached to a spring with constant 40 Newtons per metre. The other end of the spring is attached to a wall. The mass is set into horizontal motion (directly away from the wall) by pulling it 10 centimetres to the right of its equilibrium position and releasing it. During its subsequent motion, there is friction between the mass and the table with coefficient of kinetic friction equal to $\mu = 0.1$, but motion is free of any damping force proportional to velocity. After $1/2$ second, the mass is hit to the left with an impulse force of 2 Newtons.

(a) Set up an initial-value problem for the displacement $x(t)$ of the mass from its equilibrium position.

(b) Show that, as long as the mass moves to the left, the Laplace transform of $x(t)$ is

$$X(s) = \frac{\dfrac{g}{5s} - 2e^{-s/2} + \dfrac{s}{5}}{2s^2 + 40},$$

where $g = 9.81$.

(c) Find the position of the mass as a function of time until it comes to rest for the first time.