Values

6 1. The only point at which the curve

$$x = y^2 + z, \quad x + y = 1$$

intersects the surface

$$2y + 2z - x^2 y = 0$$

is $(0, 1, -1)$. Find the cosine of the angle between the tangent to the curve and the normal to the surface at this point.

6 2. If $u = f(x, y, z)$, $x = g(y, z)$, and $z = h(t)$, find the chain rule for $\left.\dfrac{\partial u}{\partial t}\right)_y$.

6 3. Find all directions in which the rate of change of the function $f(x, y, z) = x^2yz + xy$ is equal to zero at the point $(1, -1, 2)$. Express your answer as a vector.

11 4. The equations

$$u^2 + v^3 + xu^3 + 2y = 1, \quad u^3 + uy - 3ux - 3vx = 0,$$

define u and v as functions of x and y. Find $\dfrac{\partial v}{\partial y}$ when $x = 0$ and $y = 1$.

11 5. (a) Find all critical points for the function $f(x, y) = 4x^2 - 12xy + 9y^2$.

(b) Verify that the second derivative test fails to classify any of the critical points as yielding relative maxima, relative minima, or saddle points.

(c) Find a classification for each critical point.

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1. $\dfrac{\sqrt{22}}{11}$

2. $\dfrac{\partial u}{\partial x}\dfrac{\partial x}{\partial z}\dfrac{dz}{dt} + \dfrac{\partial u}{\partial z}\dfrac{dz}{dt}$

3. $a\hat{i} + b\hat{j} + (3b - 5a)\hat{k}$ (a and b are arbitrary constants)

4. $2/3$

5. a) all points on the line $y = 3x/2$ are critical

b) Show $f_{xy}^2 - f_{xx}f_{yy} = 0$

c) all points on $y = 3x/2$ yield relative minimum.