Unit 6 Assignment

1. There are three games scheduled in the National Hockey League one night. The games are shown below, together with the probabilities that each team will win their respective game, as determined by odds-makers. Note that tied games are not possible.

   Game 1: **Calgary Flames** (0.6) vs. Minnesota Wild (0.4)
   Game 2: **Edmonton Oilers** (0.2) vs. Pittsburgh Penguins (0.8)
   Game 3: **Ottawa Senators** (0.3) vs. New Jersey Devils (0.7)

   (a) The outcome of interest is the set of winners of the three games. List the complete sample space of possible outcomes, and calculate the probability of each outcome.
   (b) Let X be the number of Canadian teams that win their game. (Canadian teams are shown in **bold**). Find the p.m.f. of X.
   (c) Find the c.d.f. of X.
   (d) Find the expected value and variance of X.

2. You roll two fair six-sided dice – one red and one blue. Let X_R be the value showing on the red die and let X_B be the value showing on the blue die. Define $X = |X_R - X_B|$ to be the absolute difference of the values showing on the two dice.

   (a) Find the p.m.f. of X and draw its graph.
   (b) Find the c.d.f. of X and draw its graph.
   (c) Find the expected value and variance of X.

3. A continuous random variable X has p.d.f.

$$f(x) = \begin{cases} \dfrac{c}{x^3}, & x \geq 1 \\ 0, & \text{otherwise} \end{cases}$$

   (a) Find the value of the constant c.
   (b) Find the c.d.f. of X.
   (c) Find P(X > 1.5), first using the p.d.f and then using the c.d.f.
   (d) Find the median of the distribution of X.
   (e) Find E(X).

4. Low-grade steel ball bearings are being manufactured in a factory. The c.d.f. of the radius X of the ball bearings (in mm) is

$$F(x) = \begin{cases} 0, & x < 14 \\ \dfrac{x^2 - 196}{29}, & 14 \le x \le 15 \\ 1, & x > 15 \end{cases}$$

(a) Find the p.d.f. of X.
(b) Find $P(14.3 \le X \le 14.6)$, first using the p.d.f. and then using the c.d.f.
(c) Find the median of the distribution of X.
(d) Find E(X).
(e) Find Var(X).
(f) Find the probability that the mass of a ball bearing is less than 100 grams. (The density of low-grade steel is 0.00785 g/mm^3.)

5. Consider two independent random variables X and Y, the means and variances of which are shown below:

E(W) = 10	Var(W) = 6
E(X) = 15	Var(X) = 8

(a) Find the mean and variance of $Y = 0.5W + 0.5X$.
(b) Find the mean and variance of $Y = 7W - 3X$.
(c) Find the value of the constant δ (where $0 \le \delta \le 1$) such that the variance of $Y = \delta W + (1 - \delta)X$ is minimized.