MATH1210 Test #2 — 19 March 2009 — Time : 60 minutes

Instructor: (check one) [ ] Berry (A01) [ ] Borgersen (A02)

NAME: ____________________ **ID#:** ______________

SHOW ALL YOUR WORK!!

[4] 1. Find the angle between the two vectors $\vec{v}_1 = \hat{\imath} + \hat{\jmath}$ and $\vec{v}_2 = -\hat{\jmath} + \hat{k}$.

$\|\vec{v_1}\| = \sqrt{2} \qquad \|\vec{v_2}\| = \sqrt{2}$

$\vec{v_1} \circ \vec{v_2} = 1(0) + (1)(-1) + 0(1) = -1$

$= (\sqrt{2})(\sqrt{2}) \cos\theta \Rightarrow \cos\theta = -1/2$

$\Rightarrow \theta = 2\pi/3$

[5] 2. Given the vectors $\vec{u}_1 = [3,2,-1]$ and $\vec{u}_2 = [2,1,-3]$, find $\|\vec{u}_1 \times \vec{u}_2\|^2$.
(Show your work.)

$$\vec{u_1} \times \vec{u_2} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 3 & 2 & -1 \\ 2 & 1 & -3 \end{vmatrix} = \hat{\imath}(-5) - \hat{\jmath}(-7) + \hat{k}(-1)$$

$$= -5\hat{\imath} + 7\hat{\jmath} - \hat{k}$$

$\|\vec{u_1} \times u_2\|^2 = 25 + 49 + 1 = 75$

ALTERNATIVE : use Lagrange's identity

$$\|\vec{u_1} \times \vec{u_2}\|^2 = \|\vec{u_1}\|^2 \|\vec{u_2}\|^2 - (\vec{u_1} \circ \vec{u_2})^2$$

$$= (9+4+1)(4+1+9) - 11^2 = 75$$

since $\vec{u_1} \cdot \vec{u_2} = 6 + 2 + 3 = 11$

[3] 3. Find parametric equations for the line passing through the point $P_0(4,-1,2)$ in the direction of the vector $\vec{a} = -5\hat{\imath} + 7\hat{\jmath} - 3\hat{k}$.

$x = 4 - 5t$

$y = -1 + 7t$

$z = 2 - 3t$

[5] 4. Determine whether or not the two lines given parametrically by the equations

$$\ell_1 : x = 2 - t,\ y = 4 - 2t,\ z = 3 + t$$

and

$$\ell_2 : x = -1 + s,\ y = s,\ z = 7 - 2s$$

intersect. If they do, find the point of intersection.

For intersection: $2-t=-1+s$, $4-2t=s$, $3+t=7-2s$

$$\left.\begin{array}{l} s+t=3 \\ s+2t=4 \\ 2s+t=4 \end{array}\right\} \Rightarrow \left(\begin{array}{cc|c} 1 & 1 & 3 \\ 1 & 2 & 4 \\ 2 & 1 & 4 \end{array}\right)$$

$$\rightarrow \left(\begin{array}{cc|c} 1 & 1 & 3 \\ 0 & 1 & 1 \\ 0 & -1 & -2 \end{array}\right) \rightarrow \left(\begin{array}{cc|c} 1 & 1 & 3 \\ 0 & 1 & 1 \\ 0 & 0 & -1 \end{array}\right)$$ system is inconsistent

These two lines do not intersect.

[5] 5. Find the equation (in standard form) of the plane, through $P_0(4,-1,2)$, which contains the two vectors $\vec{v}_1 = \hat{i} + \hat{j}$ and $\vec{v}_2 = -\hat{j} + \hat{k}$ (which are assumed to be positioned at P_0).

$$\vec{v_1} \times \vec{v_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 0 \\ 0 & -1 & 1 \end{vmatrix} = \hat{i}(1) - \hat{j}(1) + \hat{k}(-1) = \hat{i} - \hat{j} - \hat{k}$$

plane has equation $x - y - z = D$

$(4,1,-2)$ on $\Rightarrow$ $4+1-2 = D \Rightarrow D = 3$

$$\boxed{x - y - z = 3}$$

[5] 6. Given $A = \begin{pmatrix} 2 & -1 & 2 \\ 1 & 3 & -1 \end{pmatrix}$ and $B = \begin{pmatrix} -1 & 0 \\ 0 & 2 \\ 1 & 4 \end{pmatrix}$, compute $AB - 2I_{(2)}$.

$$\underset{\sim}{A}\underset{\sim}{B} = \begin{pmatrix} 2 & -1 & 2 \\ 1 & 3 & -1 \end{pmatrix} \begin{pmatrix} -1 & 0 \\ 0 & 2 \\ 1 & 4 \end{pmatrix} = \begin{pmatrix} 0 & 6 \\ -2 & 2 \end{pmatrix}$$

$$\underset{\sim}{A}\underset{\sim}{B} - 2\underset{\sim}{I}_{(2)} = \begin{pmatrix} -2 & 6 \\ -2 & 0 \end{pmatrix}$$

[5] 7. Consider the matrix $\begin{pmatrix} 1 & 4 & 3 & -2 & 4 & 7 \\ 0 & 0 & \frac{1}{2} & 1 & \frac{3}{2} & \frac{1}{4} \\ 0 & 0 & 3 & 5 & -7 & 9 \end{pmatrix}$.

Describe and perform the NEXT TWO elementary row operations (ERO) which must be performed in reducing this matrix to row-echelon form (REF).

multiply row 2 by 2: $\begin{pmatrix} 1 & 4 & 3 & -2 & 4 & 7 \\ 0 & 0 & 1 & 2 & 3 & 1/2 \\ 0 & 0 & 3 & 5 & -7 & 9 \end{pmatrix}$

row 3 − 3 · row 2: $\begin{pmatrix} 1 & 4 & 3 & -2 & 4 & 7 \\ 0 & 0 & 1 & 2 & 3 & 1/2 \\ 0 & 0 & 0 & -1 & -16 & 15/2 \end{pmatrix}$

[3] 8. Find all solutions of the homogeneous linear system of equations

$$x - y + z = 0$$
$$y + 2z = 0$$

$\left(\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 \end{array}\right) \longrightarrow \underset{\text{RREF}}{\left(\begin{array}{ccc|c} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 0 \end{array}\right)}$

$\therefore\ z = t,\ y = -2t,\ x = -3t$

Alt: let $z = t$: $y = -2t$, $x = y - z = -2t - t$
$= -3t$.

[5] 9. Consider a linear system of equations $Ax = b$ for which the augmented matrix has the reduced row-echelon form (RREF)

$$\begin{pmatrix} 1 & 2 & 0 & 0 & -3 & 0 & 4 \\ 0 & 0 & 1 & 0 & 7 & 0 & 3 \\ 0 & 0 & 0 & 1 & 2 & 0 & 2 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}.$$

Find all solutions of this system, writng your answer in vector from.

$x_6 = 1$

$x_5 = t$

$x_4 = 2 - 2t$

$x_3 = 3 - 7t$

$x_2 = s$

$x_1 = 4 - 2s + 3t$

$$\underset{\sim}{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{pmatrix} = \begin{pmatrix} 4 - 2s + 3t \\ s \\ 3 - 7t \\ 2 - 2t \\ t \\ 1 \end{pmatrix}$$

Problem	1	2	3	4	5	6	7	8	9	TOTAL
MARK										
Total	4	5	3	5	5	5	5	3	5	40