STAT 2220 - Fall 2007

Solution to Assignment 5

5-4. $\bar{x}_1 = 30.61 \quad \bar{x}_2 = 30.34$

$\sigma_1 = 0.10 \quad \sigma_2 = 0.15$

$n_1 = 12 \quad n_2 = 10$

a) 90% two-sided confidence interval:

$$\left(\bar{x}_1 - \bar{x}_2\right) - z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} \le \mu_1 - \mu_2 \le \left(\bar{x}_1 - \bar{x}_2\right) + z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

$$(30.61 - 30.34) - 1.645\sqrt{\frac{(0.10)^2}{12} + \frac{(0.15)^2}{10}} \le \mu_1 - \mu_2 \le (30.61 - 30.34) + 1.645\sqrt{\frac{(0.10)^2}{12} + \frac{(0.15)^2}{10}}$$

$$0.179 \le \mu_1 - \mu_2 \le 0.361$$

We are 90% confident that the mean fill volume for machine 1 exceeds that of machine 2 by between 0.179 and 0.361 fl. oz.

b) 95% two-sided confidence interval:

$$\left(\bar{x}_1 - \bar{x}_2\right) - z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} \le \mu_1 - \mu_2 \le \left(\bar{x}_1 - \bar{x}_2\right) + z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

$$(30.61 - 30.34) - 1.96\sqrt{\frac{(0.10)^2}{12} + \frac{(0.15)^2}{10}} \le \mu_1 - \mu_2 \le (30.61 - 30.34) + 1.96\sqrt{\frac{(0.10)^2}{12} + \frac{(0.15)^2}{10}}$$

$$0.161 \le \mu_1 - \mu_2 \le 0.379$$

We are 95% confident that the mean fill volume for machine 1 exceeds that of machine 2 by between 0.161 and 0.379 fl. oz.

Comparison of parts a and b:

As the level of confidence increases, the interval width also increases (with all other values held constant).

c) 95% upper-sided confidence interval:

$$\mu_1 - \mu_2 \le \left(\bar{x}_1 - \bar{x}_2\right) + z_{\alpha}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

$$\mu_1 - \mu_2 \le (30.61 - 30.34) + 1.645\sqrt{\frac{(0.10)^2}{12} + \frac{(0.15)^2}{10}}$$

$$\mu_1 - \mu_2 \le 0.361$$

With 95% confidence, we believe the fill volume for machine 1 exceeds the fill volume of machine 2 by no more than 0.361 fl. oz.

5-24. a) 1) The parameter of interest is the difference in mean melting point, $\mu_1 - \mu_2$

2) H_0: $\mu_1 - \mu_2 = 0$ or $\mu_1 = \mu_2$

3) H_1: $\mu_1 - \mu_2 \neq 0$ or $\mu_1 \neq \mu_2$

4) $\alpha = 0.05$

5) The test statistic is

$$t_0 = \frac{(\bar{x}_1 - \bar{x}_2) - \Delta_0}{s_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

6) Reject the null hypothesis if $t_0 < -t_{\alpha/2, n_1+n_2-2}$ where $-t_{0.025,40} = -2.021$ or $t_0 >$ $t_{\alpha/2, n_1+n_2-2}$ where $t_{0.025,40} = 2.021$

7) $\bar{x}_1 = 420.48$ $\quad \bar{x}_2 = 425$, $\quad \Delta_0 = 0$

$s_1 = 2.34$ $\quad s_2 = 2.5$

$n_1 = 21$ $\quad n_2 = 21$

$$s_p = \sqrt{\frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2}} = \sqrt{\frac{20(2.34)^2 + 20(2.5)^2}{40}} = 2.42$$

$$t_0 = \frac{(420.48 - 425) - 0}{2.42\sqrt{\frac{1}{20} + \frac{1}{20}}} = -5.99$$

8) Since $-5.99 < -2.021$ reject the null hypothesis and conclude that the data do not support the claim that both alloys have the same melting point at $\alpha = 0.05$

b) P-value $= 2P\left(t < -5.99\right)$ P-value < 0.0010

5-38. a) The parameter of interest is the difference in blood cholesterol level, μ_d where d_i = Before – After.

H_0: $\mu_d = 0$ versus H_1: $\mu_d > 0$

The test statistic is

$$t_0 = \frac{\bar{d}}{s_d / \sqrt{n}}$$

$\bar{d} = 26.867$

$s_d = 19.04$

$n = 15$

$$t_0 = \frac{26.867}{19.04 / \sqrt{15}} = 5.465$$

P-value = $P(T > 5.465) < 0.0005$.

Since p-value < 0.05, we reject the null and conclude the data support the claim that the low- fat diet along with an aerobic exercise program are of value in reducing mean blood cholesterol levels.

b) 95% confidence interval:

$$\bar{d} - t_{\alpha/2,n-1}\left(\frac{s_d}{\sqrt{n}}\right) \le \mu_d \le \bar{d} + t_{\alpha/2,n-1}\left(\frac{s_d}{\sqrt{n}}\right)$$

$$26.867 - 2.145\left(\frac{19.04}{\sqrt{15}}\right) \le \mu_d \le 26.867 + 2.145\left(\frac{19.04}{\sqrt{15}}\right)$$

$$16.322 \le \mu_d \le 37.412$$

6-2. a) The regression equation is
Usage = - 6.34 + 9.21 Temp

Predictor	Coef	StDev	T	P
Constant	-6.336	1.668	-3.80	0.003
Temp	9.20836	0.03377	272.64	0.000

S = 1.943 R-Sq = 100.0% R-Sq(adj) = 100.0%

Analysis of Variance

Source	DF	SS	MS	F	P
Regression	1	280583	280583	74334.36	0.000
Residual Error	10	38	4		
Total	11	280621			

$$\hat{y} = -6.34 + 9.21x$$

b)
-1.25010
-0.19519
-0.30208
-1.61751
0.49740
2.07214
1.71688
-0.02329
-2.55294
-1.15260
-1.25734
4.06464

c) SSE = 38 $\hat{\sigma}^2 = 4$

d) se($\hat{\beta}_0$) = 1.668, se($\hat{\beta}_1$) = 0.03377

g) See the output given in part a.

- Based on the t-tests, we conclude that the slope and intercept are nonzero.
- Based on P-values, both intercept has p-value = 0.003 and slope has p-values = 0.000 which are less than $\alpha = 0.05$.

We can conclude that the intercept and slope are significant (nonzero).

i) β_0: $-6.34 \pm 2.228(1.668)$; -10.06, - 2.62
β_1: $9.21 \pm 2.228(0.03377)$; 9.13, 9.29

6-8. a) 472.499

b) (471.183, 473.816)

c) (467.975, 477.024)

d) The prediction interval is wider than the confidence interval.