MATH 1710 Review sheet. Questions were taken from Term Test 1/2 in the Winter 2009 term as well as some made up questions on L'Hopital's Rule. There is no guarantee the term test this term will in any way resemble this or any other past test.

1. Evaluate the integral $\displaystyle\int_0^{\sqrt{3}} \frac{x^3}{\sqrt{x^2+1}}\,dx$

2. Set up but don't integrate (although (a) and (c) can be done using methods done so far. (b) will be able to be done using more advanced methods from Chapter 8)

   (a) Determine the area of the region enclosed by the curves $x = 0$, $y = e^x$ and $y = e^{2x} - 2$

   (b) Determine the length of the curve given by $x = \ln(y+1)$ which lies between the vertical lines $x = 0$ and $x = \ln 4$.

   (c) Determine the volume of the solid created by revolving the region enclosed by $x = \sqrt{2-y}$, $y = 0$ and $x = 1$ about the line $x = -1$ using the

      i. washer method

      ii. cylindrical shell method

3. Determine the exact value of $\tan\left(\sec^{-1}\left(-\frac{11}{8}\right)\right)$

4. If $y = \sec^{-1}\sqrt{x^2+1}$, find $\dfrac{dy}{dx}$ simplifying as much as possible.

5. Consider the curve given by the equation

$$x^2 + x\tan^{-1}(e^y - 1) = y - \ln 2.$$

   (a) Find the point on the curve corresponding to $x = 0$.

   (b) Find the equation of the tangent line at that point.

6. Find the following limits using L'Hopital's Rule. Make sure to justify that you can use L'Hopital's Rule.

   (a) $\displaystyle\lim_{x\to 0} \frac{\sin x - x + \frac{1}{6}x^3}{x^5}$

   (b) $\displaystyle\lim_{x\to\infty} (xe^{1/x} - x)$

   (c) $\displaystyle\lim_{x\to 0} (1+2x)^{1/x}$

<u>ANSWERS:</u>

1. $\dfrac{4}{3}$

2. (a) $\displaystyle\int_0^{\ln 2} (e^x - e^{2x} + 2)\,dx = 2\ln 2 - \frac{1}{2}$

   (b) $\displaystyle\int_0^{\ln 4} \sqrt{1+e^{2x}}\,dx$ or $\displaystyle\int_0^{3} \sqrt{1+\frac{1}{(y+1)^2}}\,dy$ both which equal

   $\displaystyle -\sqrt{2} - \frac{1}{2}\ln(\sqrt{2}-1) + \frac{1}{2}\ln(\sqrt{2}+1) + \sqrt{17} + \frac{1}{2}\ln(-1+\sqrt{17}) - \frac{1}{2}\ln(1+\sqrt{17}).$

   (c) i. $\displaystyle\pi\int_0^1 \left((\sqrt{2-y}+1)^2 - 2^2\right)dy = \pi\left(\frac{8}{3}\sqrt{2} - \frac{17}{6}\right).$

   ii. $\displaystyle 2\pi\int_1^{\sqrt{2}} (x+1)(2-x^2)\,dx = \pi\left(\frac{8}{3}\sqrt{2} - \frac{17}{6}\right).$

3. $\dfrac{\sqrt{57}}{8}$

4. $\dfrac{x}{(x^2+1)\,|x|}$

5. (a) $(0, \ln 2)$

   (b) $y - \ln 2 = \dfrac{\pi}{4}x$

6. (a) $1/120$

   (b) 1

   (c) e^2