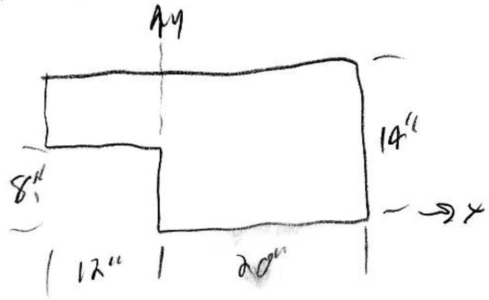
Problem set #4

(1)



rt.: $\bar{x} = 10''$, $\bar{y} = 7''$, area $= 280 in^2$

lt: $\bar{x} = -6''$, $\bar{y} = 11''$, area $= 96 in^2$

$Q_y = \bar{x}A$

so: $$\bar{x} = \frac{10''(280 in^2) + -6''(96 in^2)}{280 in^2 + 96 in^2} = \frac{2224 in^3}{376 in^2}$$

$\bar{x} = 5.915''$

$$\bar{y} = \frac{7''(280 in^2) + 11''(96 in^2)}{376 in^2} = 8.0213''$$

$$\boxed{\begin{array}{l}\bar{x} = 5.915'' \\ \bar{y} = 8.021''\end{array}}$$

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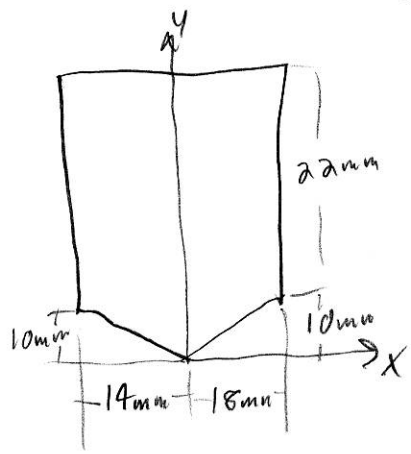
(2)



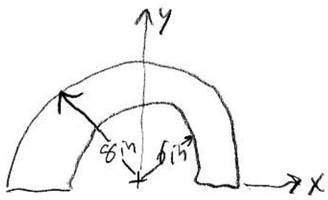
upper: $\bar{x} = 2mm$, $\bar{y} = 21mm$, $A = 704 mm^2$

lower: $$\bar{x} = \frac{6mm(90mm)^2 + \left(-\frac{14}{3}mm\right)70mm^2}{160mm^2} = 1\tfrac{1}{3}mm$$

$\bar{y} = \frac{20}{3}mm$ $\quad\quad A = \frac{1}{2}(10mm)(32mm) = 160mm^2$

$$\bar{x} = \frac{2mm(704mm^2) + \frac{4}{3}mm(160mm^2)}{864mm^2} = 1.8765 mm$$

③ (omit)

④ 

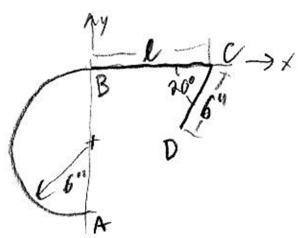
by inspection, symmetric about y axis, so $\boxed{\bar{x}=0}$

out: $\bar{y}=\frac{4r}{3\pi}=\frac{32}{3\pi}\text{ in}$, area $=\frac{\pi r^2}{2}=32\pi\text{ in}^2$

in: $\bar{y}=\frac{24}{3\pi}\text{ in}$, area $=18\pi\text{ in}^2$

$$\bar{y}=\frac{\frac{32}{3\pi}\text{ in}(32\pi\text{ in}^2)-\frac{24}{3\pi}\text{ in}(18\pi\text{ in}^2)}{(32-18)\pi\text{ in}^2}=\frac{\frac{592}{3}\text{ in}^3}{14\pi\text{ in}^2}$$

$$\boxed{\bar{y}=4.487\text{ in}}$$

⑤ 

C : $L=\pi r=6\pi''$, $\bar{x}=-\frac{2(6'')}{\pi}$

$-$: $L=l$, $\bar{x}=\frac{l}{2}$

$/$: $L=6''$, $\bar{x}=(l-3''\cos 20°)$

if horizontal, $\bar{x}$ is at $x=0$ (or B)

$$\bar{x}=0=\frac{6\pi''\left(-\frac{12''}{\pi}\right)+l\left(\frac{l}{2}\right)+(l-3''\cos 20°)(6'')}{6''+l+6\pi''}$$

$$=\frac{-72\text{ in}^2+\frac{l^2}{2}+6l\text{ in}-18\cos 20°\text{ in}^2}{6(1+\pi)\text{ in}+l}=\boxed{8.623\text{ in}=\bar{x}}$$

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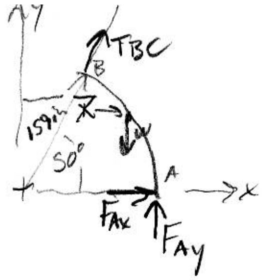


weight = 167 lb

$T_{BC}, F_{Ay}, F_{Ax} = ?$

$\bar{F}_B = \bar{U}_{BC}(T_{BC}) = T_{BC}\left[(\cos 50°)\hat{i} + \sin 50° \hat{j}\right]$

$\bar{r} = \dfrac{r \sin \alpha}{\alpha} = \dfrac{159\text{ in} \sin 25°}{\pi(25/180)} = 154.0026\text{ in}$

$\bar{x} = \cos(25°)(\bar{r}) = 139.574\text{ in}$

$\Sigma M_{center} = 0 = -167\text{ lb}(\bar{x}) + F_{Ay}(159\text{ in})$

$F_{Ay} = \dfrac{167\text{ lb}(139.574\text{ in})}{159\text{ in}} = \boxed{146.596\text{ lb} = F_{Ay}}$

$\Sigma F_y = 0 = F_{Ay} - 167\text{ lb} + \sin 50°\, T_{BC}$

$\rightarrow \boxed{T_{BC} = 26.635\text{ lb}}$

$\Sigma F_x = 0 = \cos(50°)(T_{BC}) + F_{Ax}$

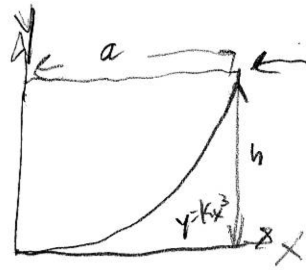
$\rightarrow \boxed{F_{Ax} = -17.12\text{ lb}}$

final: $F_{Ax} = 17\text{ lb } (-x)$

$F_{Ay} = 147\text{ lb } (+y)$

$T_{BC} = 27\text{ lb}$

(7)



$h = k a^3$

$\rightarrow k = \frac{h}{a^3} \rightarrow y = \frac{h}{a^3}(x^3)$

$$Q_x = \int_0^a \bar{y}\, dA = \int_0^a \underbrace{\frac{\left(h + \frac{h}{a^3}x^3\right)}{2}}_{\bar{y}} \underbrace{\left(h - \frac{h}{a^3}x^3\right) dx}_{dA}$$

$$= \int_0^a \left[\frac{h^2}{2} - \frac{h^2}{a^6}x^6\right] dx = \left[\frac{h^2 x}{2} - \frac{h^2}{7a^6}x^7\right]_0^a$$

$$\text{area} = \int_0^a \left(h - \frac{h}{a^3}x^3\right) dx$$

$$\boxed{A = ha - \frac{ha}{4} = \frac{3ha}{4}}$$

$$Q_x = \frac{h^2 a}{2} - \frac{h^2 a}{7} = \frac{5h^2 a}{14}$$

$$\bar{y} = \frac{Q_x}{A} = \frac{\frac{5h^2a}{14}}{\frac{3ha}{4}} = \frac{20h^2a}{42ha} = \boxed{\frac{10h}{21} = \bar{y}}$$

$$Q_y = \iint x\, dy\, dx = \int_0^a \int_0^{\left(\frac{h}{a^3}x^3\right)} x\, dy\, dx$$

$$= \int_0^a \frac{h x^4}{a^3} dx = \left[\frac{ha^5}{5a^3}\right] = \frac{ha^2}{5}$$

$$\bar{x} = \frac{Q_y}{A} = \frac{\frac{ha^2}{5}}{3ha/4} = \frac{4ha^2}{15ha} = \boxed{\frac{4}{15}a} \boxed{= \bar{x}}$$

(8) extra credit