Term Test 2

DATE: March 10, 2009
COURSE: MATH 2132

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TIME: 70 minutes
EXAMINER: G.I. Moghaddam

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[9] 1. Find the sum of the series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(n+3)}{2^n} x^n$.

[8] 2. (a) Evaluate the following integral using infinite series

$$\int_0^1 x\, e^{-x^4}\, dx$$

Express your answer in sigma notation.

(b) If you truncate the series in part (a) after the **third** term, what is a maximum possible error? Explain why you can claim that your answer is a maximum error.

[8] 3. Find a 1 -parameter family of solutions for differential equation

$$xy + x - y - 1 - y\frac{dy}{dx} = 0.$$

Is there any singular solution? Explain.

[8] 4. Find a 2 -parameter families of solutions for differential equation

$$(y')^{\frac{3}{2}} y'' = 4x(y')^2.$$

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[8] 5. Newton's second law of motion says that an object of mass m falling near the surface of the earth is retarded by air resistance proportional to its velocity i.e. $m\frac{dv}{dt} = mg - kv$, where $v = v(t)$ is the velocity of the object at time t and g is the gravitational constant and k is constant of proportionality.
If an object of mass 1 kilogram is dropped (with no initial velocity) from a hovering helicopter, such that the air resistance is proportional to the velocity of the object; then:

(a) Create and solve an initial-value problem to find the velocity of the object as a function of time t.

[9] 6. Find the general solution for the homogeneous linear differential equation :

$$y^{(8)} + 4y^{(6)} + 4y^{(4)} = 0$$

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Answers

1. $\frac{3x^2+8x}{(x+2)^2}$, $|x| < 2$

2. a) $\sum_{n=0}^{\infty} \frac{(-1)^n}{(4n+2)\,n!}$ b) Max error $\leq 1/84$

3. $y - \ln|y+1| = \frac{x^2}{2} - x + D$, $y = -1$ is a Singular Solution.

4. $\frac{x^5}{5} + \frac{2}{3}Dx^3 + D^2x + E$

5. a) $g/k\,(1 - e^{-kt})$ b) g/k

6. $c_1 + c_2x + c_3x^2 + c_4x^3 + (c_5 + c_6x)\cos\sqrt{2}x + (c_7 + c_8x)\sin\sqrt{2}x$