Solutions

Term Test #2

Nov 9, 2010

QUESTION 1

A T-section has two smooth pulleys attached at points *A* and *B*. A single cable passes over the two pulleys as shown in the figure. The tension in the cable is constant at *10 kN*. A distributed load varying from *0 kN/M* to *5 kN/m* is applied as shown. There is also a *20 kN* force and a *30 kN.m* counterclockwise couple applied to the T-section.

Replace the system of forces and couples with a single force and state where this force intersects the *x* and *y* axis. Show your results on the figure provided.

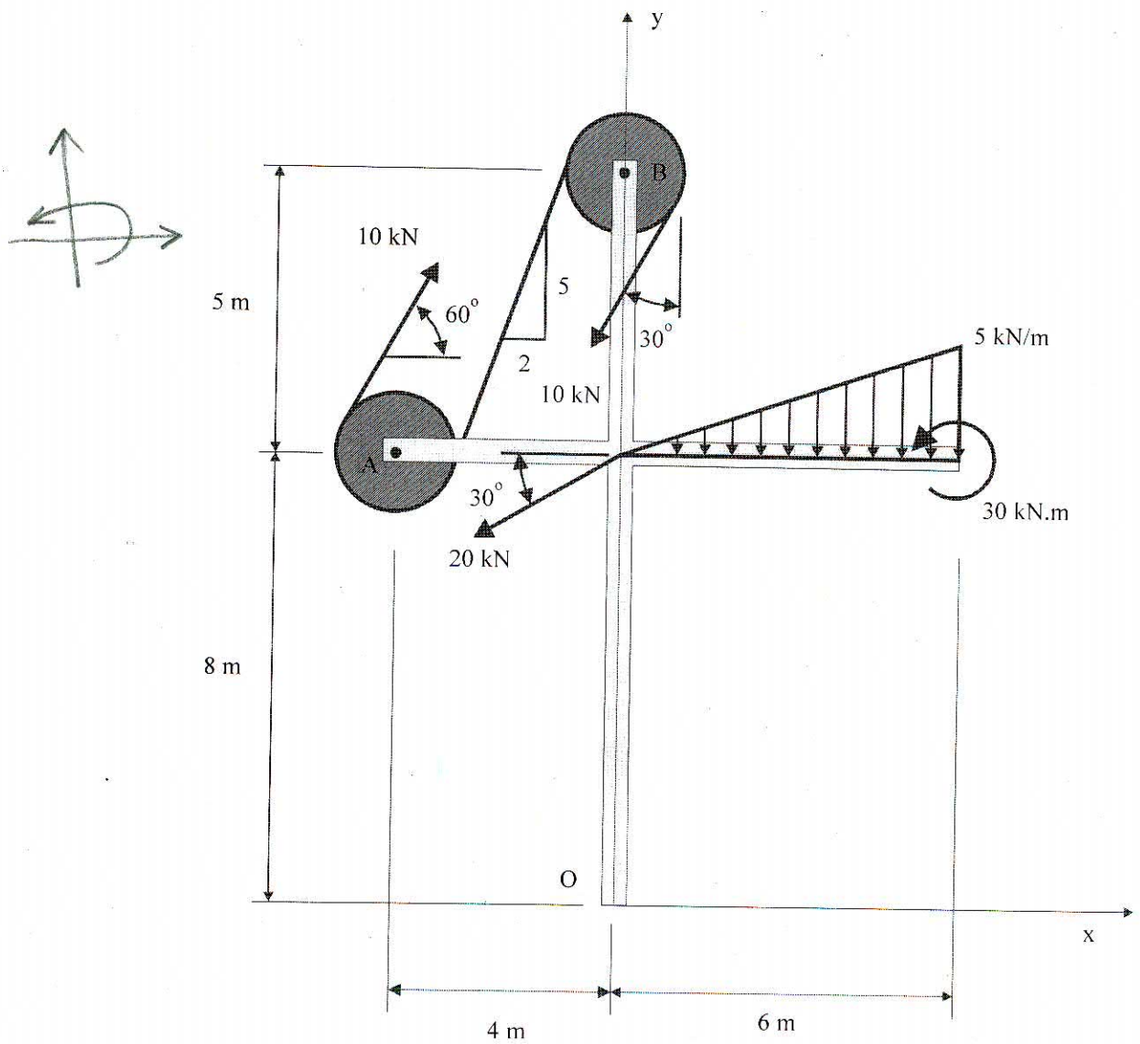
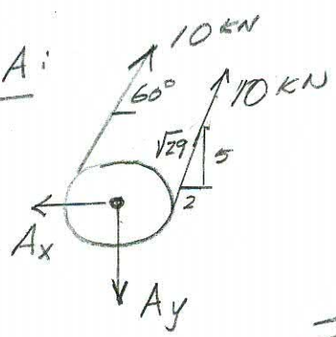


Figure 1

APPLY PULLEY FORCES AT A & B

AT A:



$\Sigma F_x = 0$

$-A_x + 10\cos 60° + \frac{2}{\sqrt{29}}(10) = 0$

$A_x = +8.714\ kN \quad \therefore \vec{A}_x = 8.714\ kN \leftarrow$ on the pulley

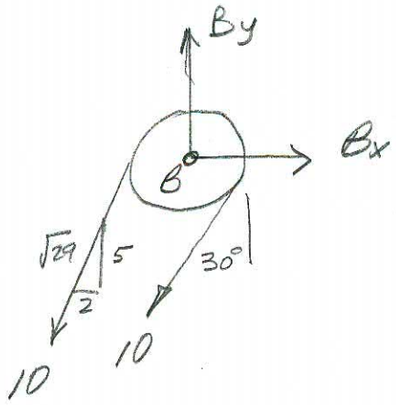
$\Sigma F_y = 0$

$10\sin 60° + \frac{5}{\sqrt{29}}(10) - A_y = 0$

$A_y = +17.945\ kN$

$\therefore \vec{A}_y = 17.945\ kN \downarrow$ on the pulley

At B:



$\Sigma F_x = 0$

$B_x - \frac{2}{\sqrt{29}}(10) - 10\sin 30° = 0$

$B_x = +8.714\ kN$

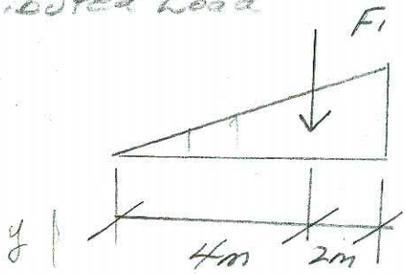
$B_x = 8.714\ kN \rightarrow$ on the pulley

$\Sigma F_y = 0$ $\quad B_y - \frac{5}{\sqrt{29}}(10) - 10\cos 30° = 0$

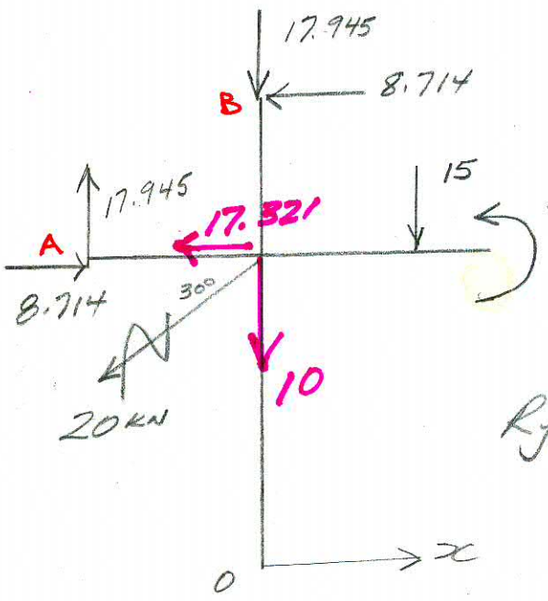
$B_y = +17.945\ kN$

$\therefore \vec{B_y} = 17.945\ kN \uparrow$ on the pulley

Distributed Load



$F_1 = \frac{5 \times 6}{2} = 15\ kN \downarrow$ at $x = 4\ m$



$R_x = \Sigma F_x$

$= 8.714 - 8.714 - 17.321$

$R_x = -17.321\ kN$

$\vec{R_x} = 17.321\ kN \leftarrow$

$R_y = \Sigma F_y$

$-17.945 + 17.945 - 10 - 15$

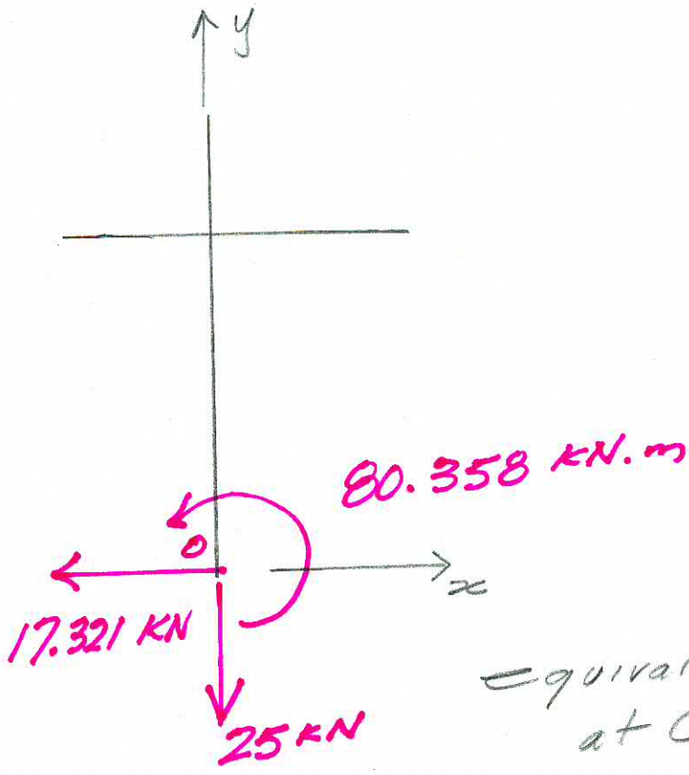
$R_y = -25\ kN$

$\vec{R_y} = 25\ kN \downarrow$

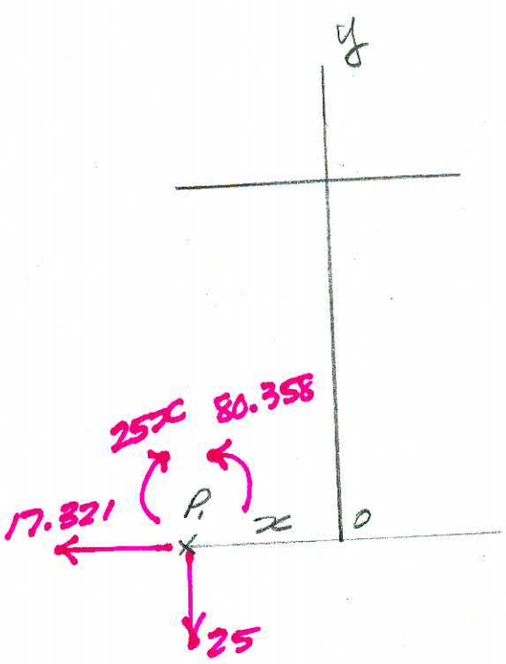
$M_{R_O} = -17.945(4) - 8.714(8) + 17.321(8)$

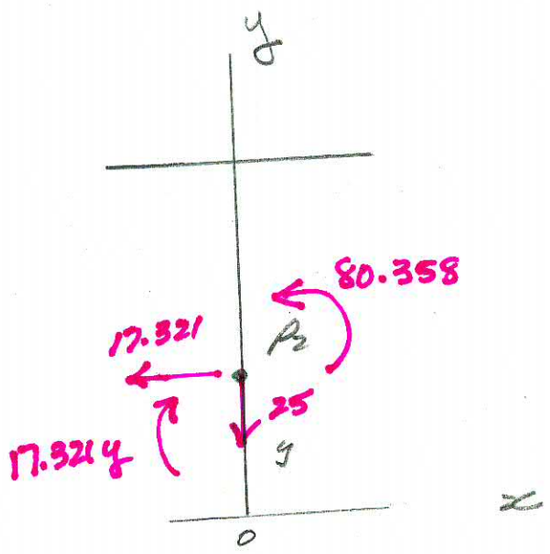
$+ 8.714(13) - 15(4) + 30 = +80.358\ kN.m$

$\vec{M_{R_O}} = 80.358\ kN.m \circlearrowleft$



equivalent force-couple at O





$M_{P_1} = 0 \quad -25x + 80.358 = 0$

$x = 3.214 \text{ m}$

x is left of O

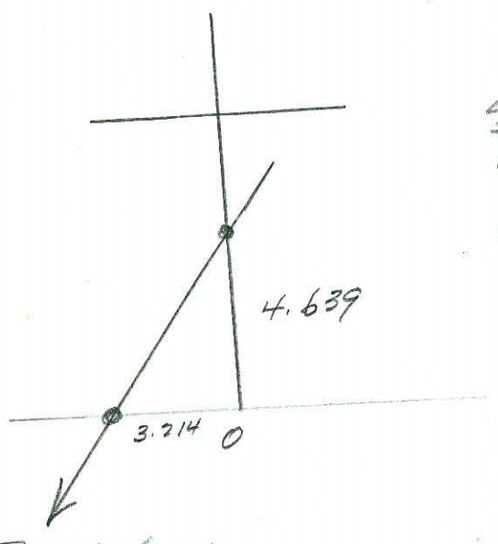
$M_{P_2} = -17.321y + 80.358 = 0$

$y = 4.639 \text{ m}$

y above point O

$R = \sqrt{17.321^2 + 25^2}$

$R = 30.414 \text{ kN}$



$\frac{4.639}{3.214} = 1.443$ ✓

$\frac{25}{17.321} = 1.443$ ✓

QUESTION 2

Determine all forces acting on members *ABC* and *DCE* of the frame shown in the figure and draw final FBDs of the two members.

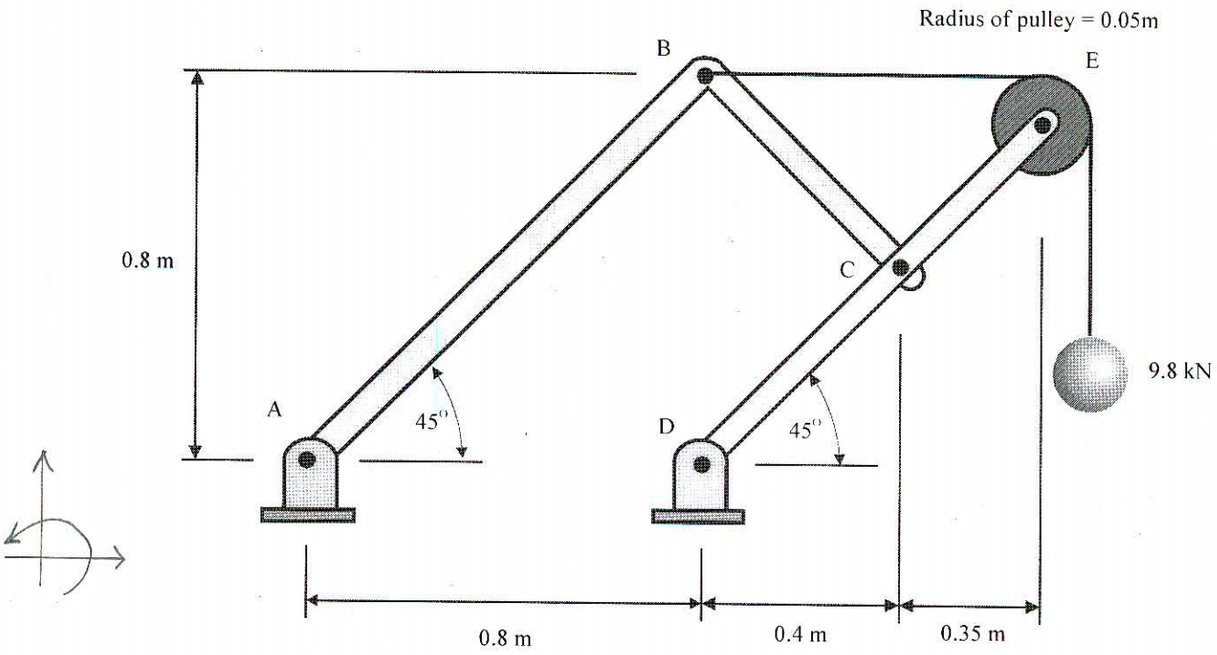
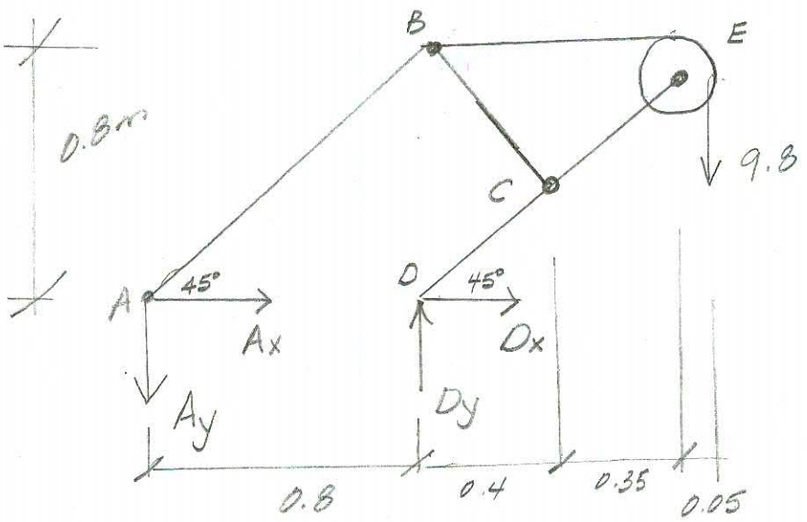


Figure 2



$\Sigma F_x = 0$

$A_x + D_x = 0$ (1)

$\Sigma F_y = 0$

$-A_y + D_y - 9.8 = 0$ (2)

$\Sigma M_A = 0$ (3)

$D_y(0.8) - 9.8(1.6) = 0$

$D_y = +19.6$ kN

$\therefore \vec{D_y} = 19.6$ kN ↑

Substitute in (2)

$-A_y + 19.6 - 9.8 = 0$

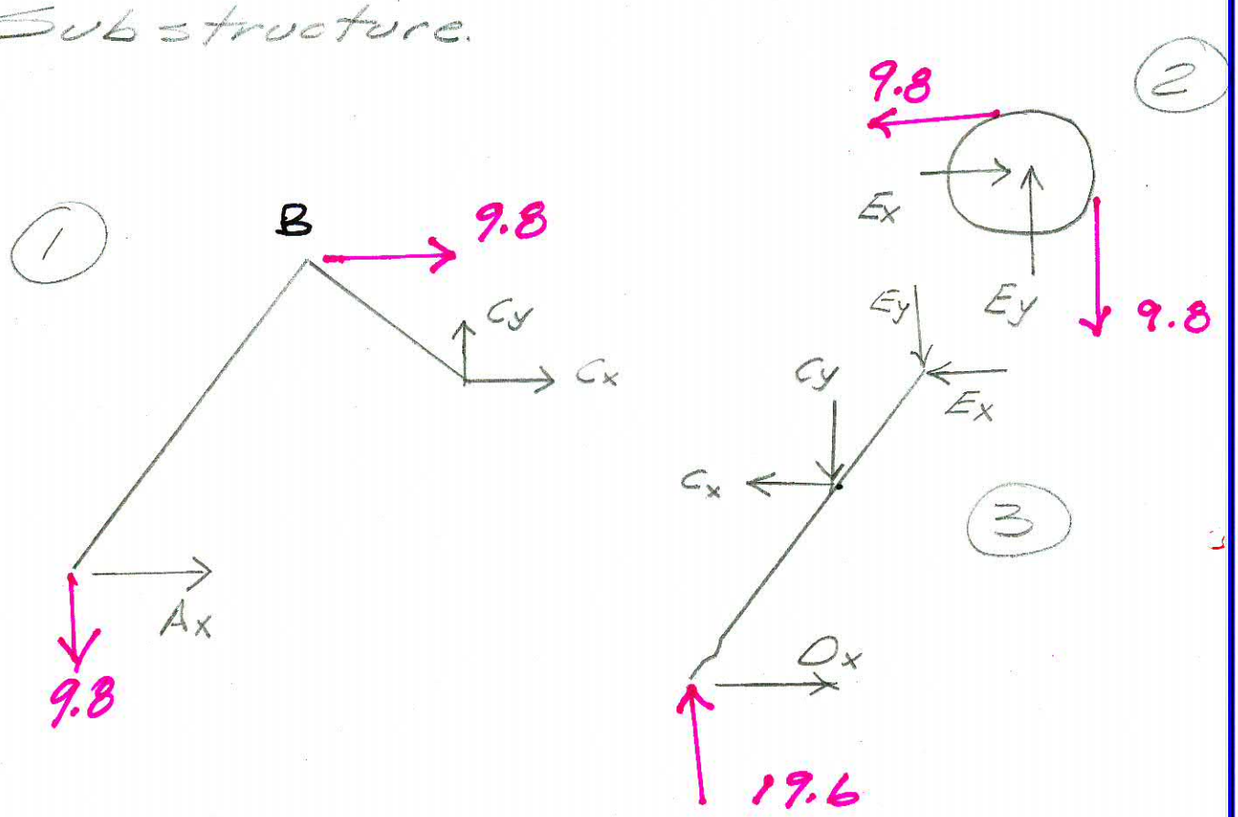
$A_y = +9.8$ kN $\therefore \vec{A_y} = 9.8$ kN ↓

From FBD entire frame Eqn (1)

$A_x + D_x = 0$ $A_x + 19.6 = 0$ $A_x = -19.6$ kN

$\therefore \vec{A_x} = 19.6$ kN ←

Sub-structure.



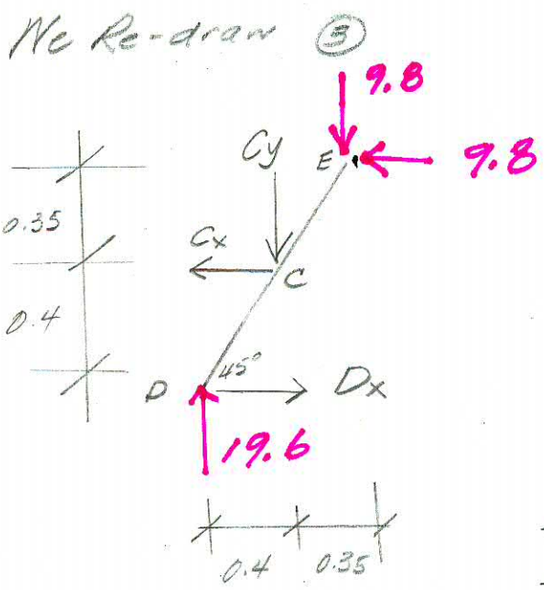
From (2) $\Sigma F_x = 0$ $E_x - 9.8 = 0$

$E_x = +9.8\ kN$ ∴ $\vec{E_x} = 9.8\ kN \rightarrow$ on the pulley

$\Sigma F_y = 0$ $E_y - 9.8 = 0$ $E_y = +9.8\ kN$

∴ $\vec{E_y} = 9.8\ kN \uparrow$ on the pulley

We Re-draw (3)



$\Sigma F_x = 0$

$D_x - C_x - 9.8 = 0$ (1)

$\Sigma F_y = 0$

$19.6 - C_y - 9.8 = 0$ (2)

$C_y = +9.8\ kN$

∴ $\vec{C_y} = 9.8\ kN \downarrow$ on DCE

$\Sigma M_C = 0$

$-19.6(0.4) + D_x(0.4)$

$-9.8(0.35) + 9.8(0.35) = 0$ (3)

$D_x = +19.6\ kN$

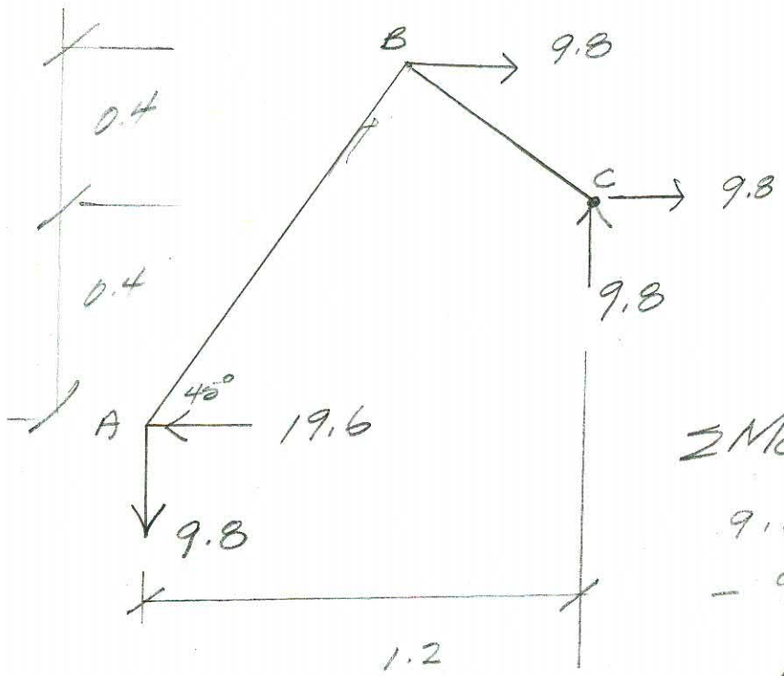
$\vec{D_x} = 19.6\ kN \rightarrow$

Substitute in (1)

$19.6 - C_x - 9.8 = 0$

$C_x = +9.8\ kN$ $\vec{C_x} = 9.8\ kN \leftarrow$ on DCE

We redraw all members & check



$\Sigma F_x = 0$

$9.8 + 9.8 - 19.6 = 0$

$0 = 0$ ✓

$\Sigma F_y = 0$

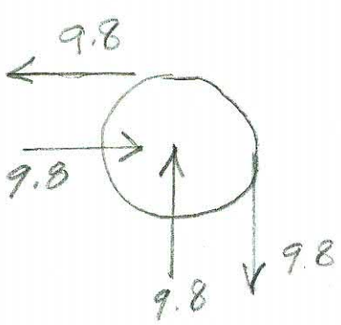
$9.8 - 9.8 = 0$

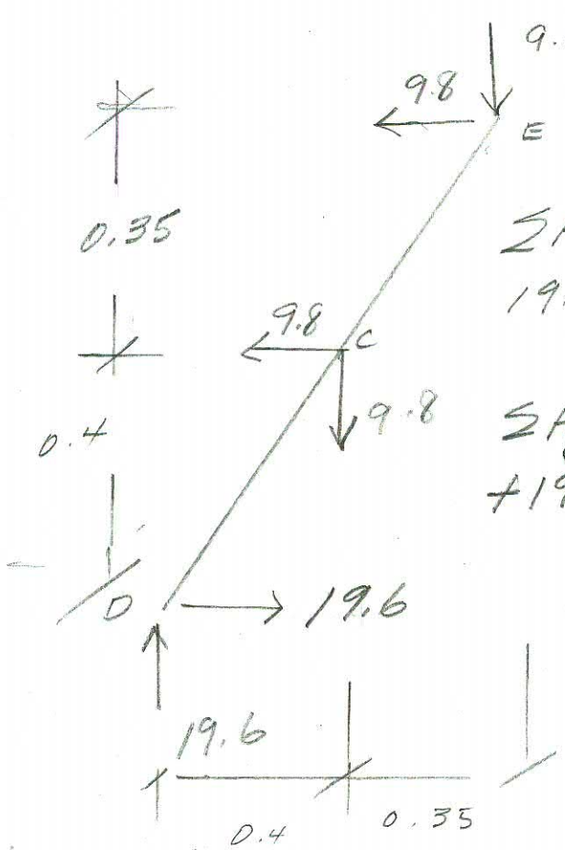
$0 = 0$ ✓

$\Sigma M_C = 0$

$9.8(1.2) - 19.6(0.4) - 9.8(0.4) = 0$

$0 = 0$ ✓





$\Sigma F_x = 0$

$19.6 - 9.8 - 9.8 = 0$

$0 = 0$ ✓

$\Sigma F_y = 0$

$+19.6 - 9.8 - 9.8 = 0$

$0 = 0$ ✓

$\Sigma M_D = 0$

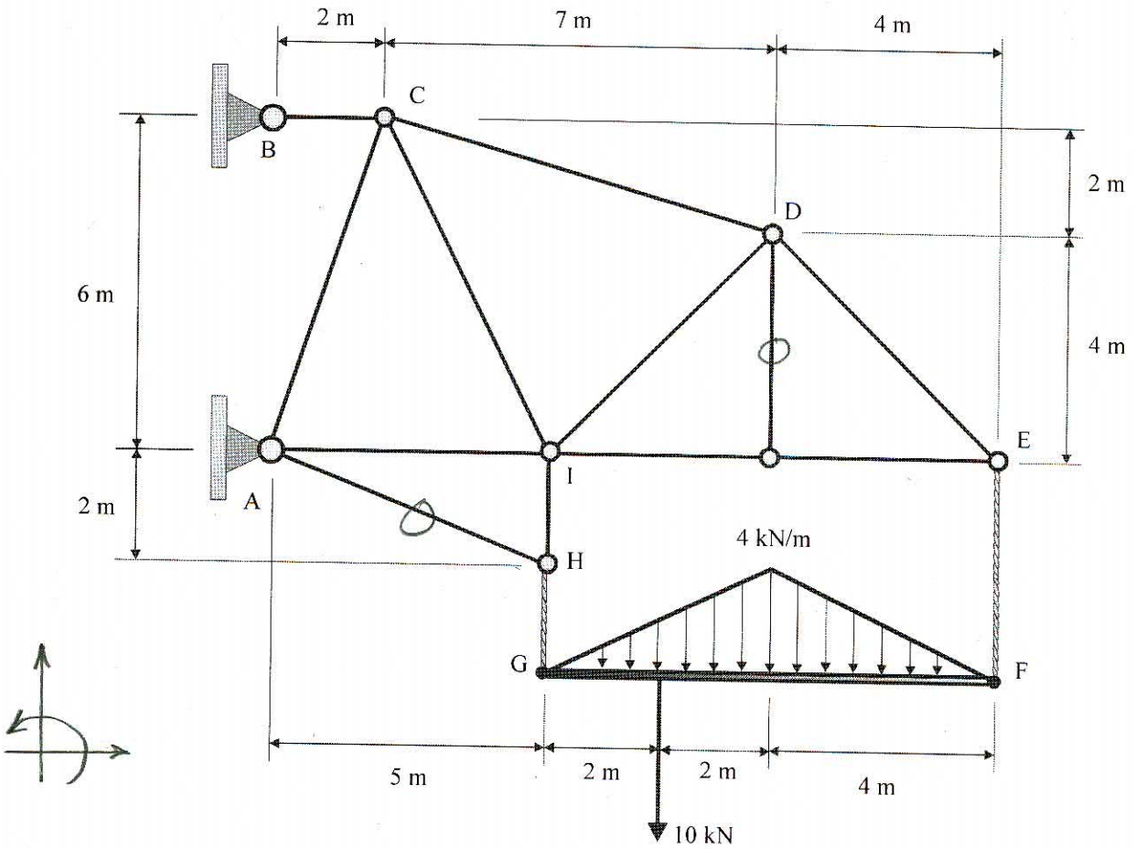
$9.8(0.4) - 9.8(0.4) + 9.8(0.75) - 9.8(0.75) = 0$

$0 = 0$ ✓

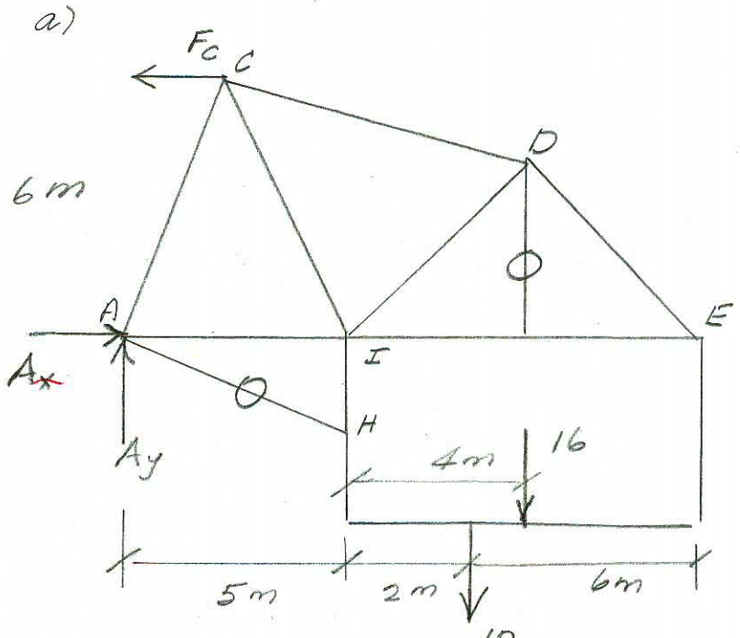
QUESTION 3

A simple truss has a hinge (pin) support at *A*. Member *BC* is a "short link". The beam *GF* is suspended from the truss by cables attached at points *H* and *E*. The beam supports the distributed load shown and a point load of *10 kN*. Determine:

a) The reactions at supports *A* and *B*,
b) The tensions in cables *HG* and *EF*, and
c) The force in each member of the truss and state whether the member is in tension, compression or a zero force member. **PLACE YOUR RESULTS ON THE FIGURE PROVIDED.**



a)



$\Sigma M_A = 0$

$F_C(6) - 10(7) - 16(9) = 0$ $\quad F_C = +35.67\,kN$

$\vec{F_C} = 35.67\,kN \leftarrow$

$\Sigma F_x = 0 \quad -35.67 + A_x = 0$ $\quad \vec{A_x} = 35.67\,kN \rightarrow$

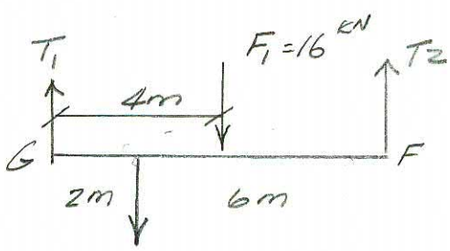
$A_x = +35.67\,kN$

$\Sigma F_y = 0 \quad A_y - 10 - 16 = 0$ $\quad A_y = +26\,kN$

$\vec{A_y} = 26\,kN \uparrow$

b)

Distributed Load



$F_1 = \frac{4 \times 8}{2} = 16\,kN$ @ 4m by symmetry

$\Sigma M_G = 0$

$-10(2) - 16(4) + T_2(8) = 0$

$T_2 = +10.5\,kN$

$\therefore \vec{T_2} = 10.5\,kN \uparrow$ on the beam

$\Sigma F_y = 0$

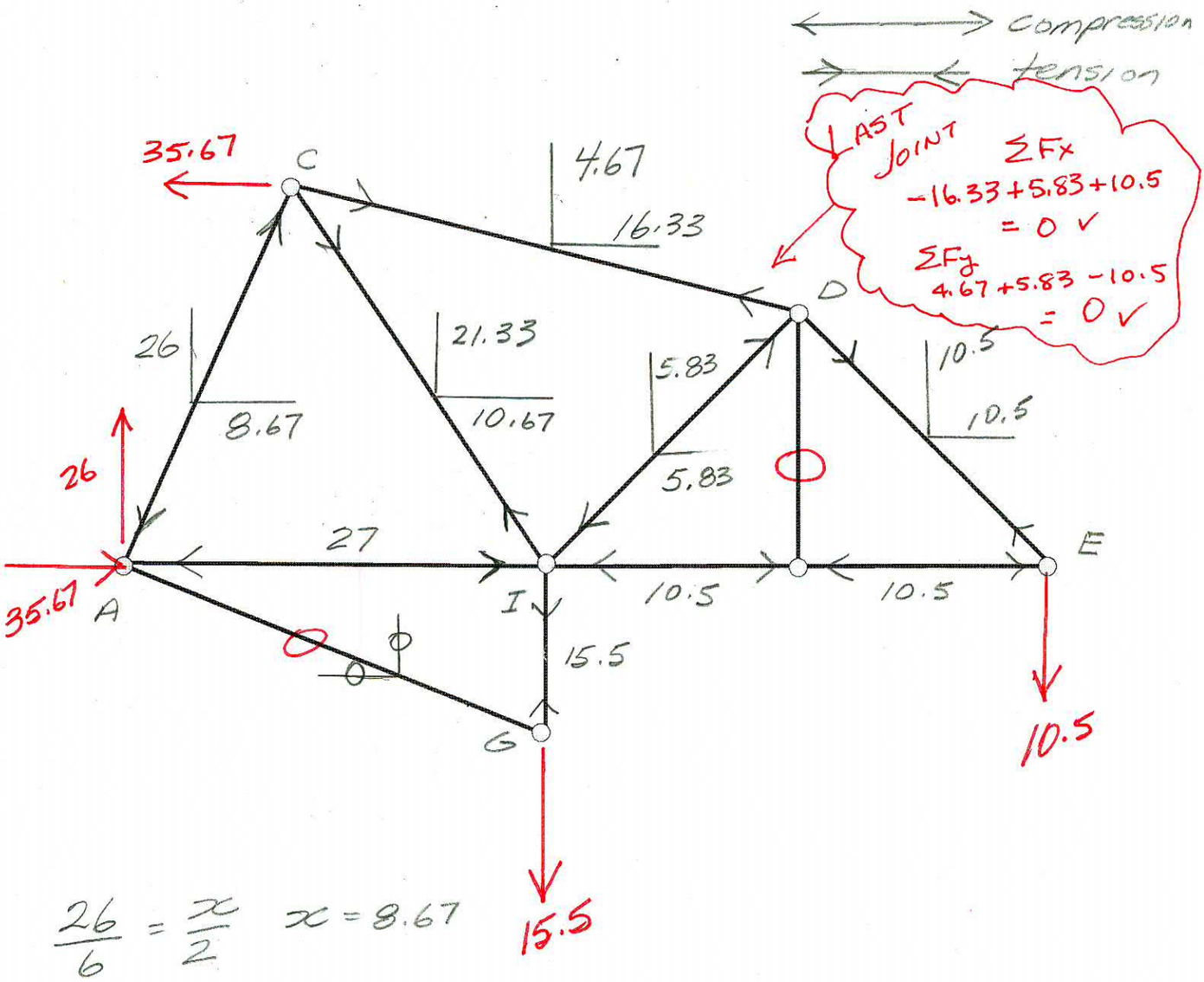
$T_1 - 10 - 16 + 10.5 = 0$

$T_1 = +15.5\,kN$

$\therefore \vec{T_1} = 15.5\,kN \uparrow$ on the beam

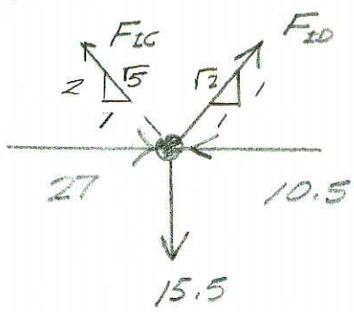
Name : _____ (PLEASE PRINT)

Student Number : _____



$\frac{26}{6} = \frac{x}{2}$ $x = 8.67$

JOINT I



$\Sigma F_x = 0$

$27 - \frac{1}{\sqrt{5}} F_{IC} + \frac{1}{\sqrt{2}} F_{ID} - 10.5 = 0$

$-\frac{1}{\sqrt{5}} F_{IC} + \frac{1}{\sqrt{2}} F_{ID} = -16.5 \quad (1)$

$\Sigma F_y = 0$

$\frac{2}{\sqrt{5}} F_{IC} + \frac{1}{\sqrt{2}} F_{ID} - 15.5 = 0$

$\frac{2}{\sqrt{5}} F_{IC} + \frac{1}{\sqrt{2}} F_{ID} = 15.5 \quad (2)$

Solving (1) & (2)

$-\frac{1}{\sqrt{5}} F_{IC} + \frac{1}{\sqrt{2}} F_{ID} = -16.5$

$\frac{2}{\sqrt{5}} F_{IC} + \frac{1}{\sqrt{2}} F_{ID} = 15.5$

$-\frac{3}{\sqrt{5}} F_{IC} = -32 \quad \therefore F_{IC} = +23.85 \quad \vec{F}_{IC} = 23.85\,kN$

$-\frac{1}{\sqrt{5}}(23.85) + \frac{1}{\sqrt{2}} F_{ID} = -16.5 \quad \frac{1}{\sqrt{2}} F_{ID} = -5.834$

$F_{ID} = -8.25\,kN \quad \vec{F}_{ID} = 8.25\,kN$

Re-draw Joint I & Resolve

F_{IC} & F_{ID} into components:

put on Placeholders