MATH 1210 TUTORIAL #8 - Revised SOLUTIONS

① $\det(A) = \det\begin{pmatrix} 0&1&1&1\\ 1&0&1&1\\ 0&1&-1&0\\ 0&1&0&-1 \end{pmatrix} = (1)(-1)^{2+1}\det\begin{pmatrix} 1&1&1\\ 1&-1&0\\ 1&0&-1 \end{pmatrix}$

$R_3 \to R_3 - R_2$
$R_4 \to R_4 - R_2$

$= -\det\begin{pmatrix} 1&1&1\\ 1&-1&0\\ 2&1&0 \end{pmatrix}$

$R_3 \to R_3 + R_1$

See p. 3 for another faster solution

$= -(1)(-1)^{1+3}\det\begin{pmatrix} 1&-1\\ 2&1 \end{pmatrix} = -((1)(1)-(-1)(2))$

$= -(1+2) = -3$

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②

$$\det\begin{bmatrix} a_1+b_1 & a_1-b_1 & c_1 & d_1\\ a_2+b_2 & a_2-b_2 & c_2 & d_2\\ a_3+b_3 & a_3-b_3 & c_3 & d_3\\ a_4+b_4 & a_4-b_4 & c_4 & d_4 \end{bmatrix} = \det\begin{bmatrix} 2a_1 & a_1-b_1 & c_1 & d_1\\ 2a_2 & a_2-b_2 & c_2 & d_2\\ 2a_3 & a_3-b_3 & c_3 & d_3\\ 2a_4 & a_4-b_4 & c_4 & d_4 \end{bmatrix}$$

$C_1 \to C_1 + C_2$

$$= \det\begin{bmatrix} 2a_1 & -b_1 & c_1 & d_1\\ 2a_2 & -b_2 & c_2 & d_2\\ 2a_3 & -b_3 & c_3 & d_3\\ 2a_4 & -b_4 & c_4 & d_4 \end{bmatrix} = 2\cdot(-1)\det\begin{bmatrix} a_1 & b_1 & c_1 & d_1\\ a_2 & b_2 & c_2 & d_2\\ a_3 & b_3 & c_3 & d_3\\ a_4 & b_4 & c_4 & d_4 \end{bmatrix}$$

$C_2 \to C_2 - \frac{1}{2}C_1$

$= 2(-1)(1) = -2$

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③ $\det\begin{pmatrix} x+2 & -2 & -3\\ 2 & x-3 & -2\\ 4 & -2 & x-5 \end{pmatrix} = \det\begin{pmatrix} x-3 & -2 & -3\\ x-3 & x-3 & -2\\ x-3 & -2 & x-5 \end{pmatrix} =$

$C_1 \to C_1 + C_2 + C_3$

$$= (x-3)\det\begin{pmatrix} 1 & -2 & -3 \\ 1 & x-3 & -2 \\ 1 & -2 & x-5 \end{pmatrix} \underset{\substack{R_2\to R_2-R_1 \\ R_3\to R_3-R_1}}{=} (x-3)\det\begin{pmatrix} 1 & -2 & -3 \\ 0 & x-1 & 1 \\ 0 & 0 & x-2 \end{pmatrix}$$

$$= (x-3)\big((1)(x-1)(x-2)\big) = (x-1)(x-2)(x-3).$$

It follows that the given determinant equals 0 whenever $x=1$, $x=2$, or $x=3$

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④ $$\det\begin{pmatrix} 1+i & 2 & 3 \\ 4 & i & 5 \\ 6 & 0 & i-1 \end{pmatrix} - \det\begin{pmatrix} 1 & 2 & 3 \\ 4 & i & 5 \\ 6 & 0 & i-1 \end{pmatrix}$$

$$= (1+i)(-1)^{1+1}\det\begin{pmatrix} i & 5 \\ 0 & i-1 \end{pmatrix} + 2(-1)^{1+2}\det\begin{pmatrix} 4 & 5 \\ 6 & i-1 \end{pmatrix}$$

$$+ 3(-1)^{1+3}\det\begin{pmatrix} 4 & i \\ 6 & 0 \end{pmatrix} - \Bigg(1(-1)^{1+1}\det\begin{pmatrix} i & 5 \\ 0 & i-1 \end{pmatrix}$$

$$+ 2(-1)^{1+2}\det\begin{pmatrix} 4 & 5 \\ 6 & i-1 \end{pmatrix} + 3(-1)^{1+3}\det\begin{pmatrix} 4 & i \\ 6 & 0 \end{pmatrix}\Bigg)$$

$$= i\det\begin{pmatrix} i & 5 \\ 0 & i-1 \end{pmatrix} = (i)(i)(i-1) = i^2(i-1) = 1-i$$

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⑤ $$\det\begin{bmatrix} 4 & 0 & 0 & 1 & 0 \\ 3 & 3 & 3 & -1 & 0 \\ 1 & 2 & 4 & 2 & 3 \\ 9 & 4 & 6 & 2 & 3 \\ 2 & 2 & 4 & 2 & 3 \end{bmatrix} \underset{R_5\to R_5-R_3}{=} \det\begin{bmatrix} 4 & 0 & 0 & 1 & 0 \\ 3 & 3 & 3 & -1 & 0 \\ 1 & 2 & 4 & 2 & 3 \\ 9 & 4 & 6 & 2 & 3 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$= (1)(-1)^{5+1} \det \begin{bmatrix} 0 & 0 & 1 & 0 \\ 3 & 3 & -1 & 0 \\ 2 & 4 & 2 & 3 \\ 4 & 6 & 2 & 3 \end{bmatrix} = (1)(-1)^{1+3} \det \begin{pmatrix} 3 & 3 & 0 \\ 2 & 4 & 3 \\ 4 & 6 & 3 \end{pmatrix}$$

$$= \det \begin{pmatrix} 3 & 0 & 0 \\ 2 & 2 & 3 \\ 4 & 2 & 3 \end{pmatrix} = 0.$$

identical columns

Alternate solution for #1:

$$\det \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix} = \det \begin{bmatrix} 3 & 3 & 3 & 3 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$$

$R_1 \to R_1 + R_2 + R_3 + R_4$

$$= 3 \det \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix} = 3 \det \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} =$$

$R_2 \to R_2 - R_1$
$R_3 \to R_3 - R_1$
$R_4 \to R_4 - R_1$

matrix is diagonal

$$= 3(1)(-1)(-1)(-1) = -3$$