Infinite limits

2.2.1

a) $\lim_{x \to -2\pi^-} x \csc x = \lim_{x \to -2\pi^-} \frac{-2\pi}{\sin x} = \frac{-2\pi}{0^-} = +\infty$

b) $x \to \frac{1}{2}^- \Rightarrow 2x-1 < 0$

$\lim_{x \to \frac{1}{2}^-} \frac{2x-1}{|x^2(2x-1)|} = \lim_{x \to \frac{1}{2}^-} \frac{-1}{x^2} = -4$

c) $\lim_{x \to 0^+} \left(\frac{1}{x} - \frac{1}{|x|}\right) = \lim_{x \to 0^+} (0) = 0$

$\lim_{x \to 0^-} \left(\frac{1}{x} - \frac{1}{|x|}\right) = \lim_{x \to 0^-} \left(\frac{2}{x}\right) = -\infty$

(d)

No limit and no trend since $\lim_{x \to 4^+} \frac{x^2-25}{x-4} = -\infty$

while $\lim_{x \to 4^-} \frac{x^2-25}{x-4} = \infty$

2.2.2

a) $$\lim_{x \to +\infty} \left\{ \sqrt{x^2+ax} - \sqrt{x^2+bx} \right\} \frac{\sqrt{x^2+ax} - \sqrt{x^2+bx}}{\sqrt{x^2+ax} - \sqrt{x^2+bx}}$$

$$= \lim_{x \to +\infty} \frac{x^2+ax-x^2-bx}{\sqrt{x^2+ax}+\sqrt{x^2+bx}} = \lim_{x \to +\infty} \frac{a-b}{\frac{1}{x}\left\{\sqrt{x^2+ax}+\sqrt{x^2+bx}\right\}}$$

$$= \lim_{x \to +\infty} \frac{a-b}{\sqrt{1+\frac{a}{x}}+\sqrt{1+\frac{b}{x}}} = \frac{a-b}{2}$$

b) $x \to -\infty \implies x = -\sqrt{x^2}$

.... only the last step is different ...

$$= \lim_{x \to -\infty} \frac{a-b}{-\sqrt{1+\frac{a}{x}} - \sqrt{1+\frac{b}{x}}} = \frac{b-a}{2}$$