MATH2132 Test1 **February 4, 2014** **70 minutes**

1. (a) Determine whether the sequence of functions

$$\{f_n(x)\} = \left\{ \frac{3n^2x^2 + 1}{n^2x^2 + 2nx + 4} \right\}$$

has a limit on the interval $-1 \le x \le 1$. Show your reasoning and all calculations.

(b) Would the series $\sum_{n=1}^{\infty} f_n(x)$ have a sum? Explain.

10 **2.** Determine whether the following series converge or diverge. If a series converges, find its sum. Justify your conclusions.

(a) $\sum_{n=2}^{\infty} \frac{3^{n+3}}{2^{2n-1}}$ (b) $\sum_{n=1}^{\infty} (-1)^n \left(\frac{n^2+1}{3n^2+4} \right)$

10 **3.** Find the interval of convergence for the power series

$$\sum_{n=3}^{\infty} \frac{(-1)^n n}{a^n} x^{2n}, \qquad \text{where } a > 0 \text{ is a given constant.}$$

8 **5.** Find the remainder $R_n(1, x)$ when the function $f(x) = \sin 5x$ is expanded with Taylor's remainder formula (about $x = 1$). Verify that $\lim_{n\to\infty} R_n(1, x) = 0$ for all x.