

University of Manitoba  
Department of Mechanical and Manufacturing Engineering  
**ENG 1460 Introduction to Thermal Sciences (F07)**  
**A01 & A02** (Profs. Ormiston and Bartley)

Term Test # 1

10 October 2007

Duration: 90 minutes

1. You are permitted to use the textbook for the course and a calculator.
2. Ask for clarification if any problem statement is unclear.
3. Clear, systematic solutions are required. **Show your work.** Marks will not be assigned for problems that require unreasonable (in the opinion of the instructor) effort for the marker to decipher.
4. Use linear interpolation in the property tables as necessary.
5. Keep 5 significant figures in intermediate calculations, and use 4 or 5 significant figures in final answers.
6. There are **three** problems on this test. The weight of each problem is indicated. The test will be marked out of **70**.

Values

1. A closed system contains 1.25 [kg] of a gas whose gas constant is 0.21882 [kJ/kg·K]. The gas, initially at a pressure of 1440 [kPa] and a temperature equal to 385 [°C] (State 1), undergoes two quasi-equilibrium processes, one after the other.

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In the first process (State 1 to State 2), the gas is heated at constant pressure until the temperature reaches 780 [°C] (at State 2). The second process (State 2 to State 3) is an isothermal expansion at 780 [°C] until the volume reaches 0.275 [m<sup>3</sup>] (at State 3). Assume ideal gas behaviour.

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|-----|---|
| 6   | (a) Determine the system volume at State 2, $V_2$ .   |
| 2.5 | (b) Determine the final system pressure, $P_3$ .  |
| 8   | (c) Draw a representation of both processes on a $T$ - $v$ (temperature-specific volume) diagram. On the diagram, clearly label the three state points, the two process paths, and the constant pressure lines that pass through the three state points. Indicate the values of $T$ <u>and</u> $v$ for the state points on the diagram. |
| 1.5 | (d) Determine the molar mass of the gas.  |

2. A saturated mixture of water is contained in a piston-cylinder assembly as shown in Figure 1. The water has an initial quality of 15% and is at an initial pressure of 400 [kPa]. The diameter of the piston is 20 [cm] and it is initially at a distance of 40 [cm] from the bottom of the cylinder. The cylinder has a set of stops that limit the vertical rise of the piston. Heat is continuously transferred to the cylinder until a final pressure of 1000 [kPa] is reached. The atmospheric pressure above the piston is  $P_{\text{atm}} = 100$  [kPa]. Use  $g = 9.81$  [m/s<sup>2</sup>].

- (a) Calculate the mass of the piston,  $m_p$ , that is necessary for the initial state of equilibrium with  $P_1 = 400$  [kPa].
- (b) Determine the initial specific volume,  $v_1$ , and the temperature,  $T_1$ , of the water. Also determine the mass,  $m$ , of the water.
- (c) It is required that the water be saturated vapour when the piston first reaches the set of stops (*i.e.*, the piston is at the stops but it does not exert any force on them). Calculate the vertical distance  $L_2$  (from the bottom of the cylinder to the bottom of the piston) needed to meet this requirement.
- (d) Describe the final state when  $P = 1000$  [kPa] and determine the final temperature and the final specific volume.
- (e) Show the state points and process path(s) on a  $P$ - $v$  (pressure-specific volume) diagram. On the diagram, clearly label the constant temperature lines that pass through the state points and the values of  $P$  and  $v$  for the state points on the diagram.

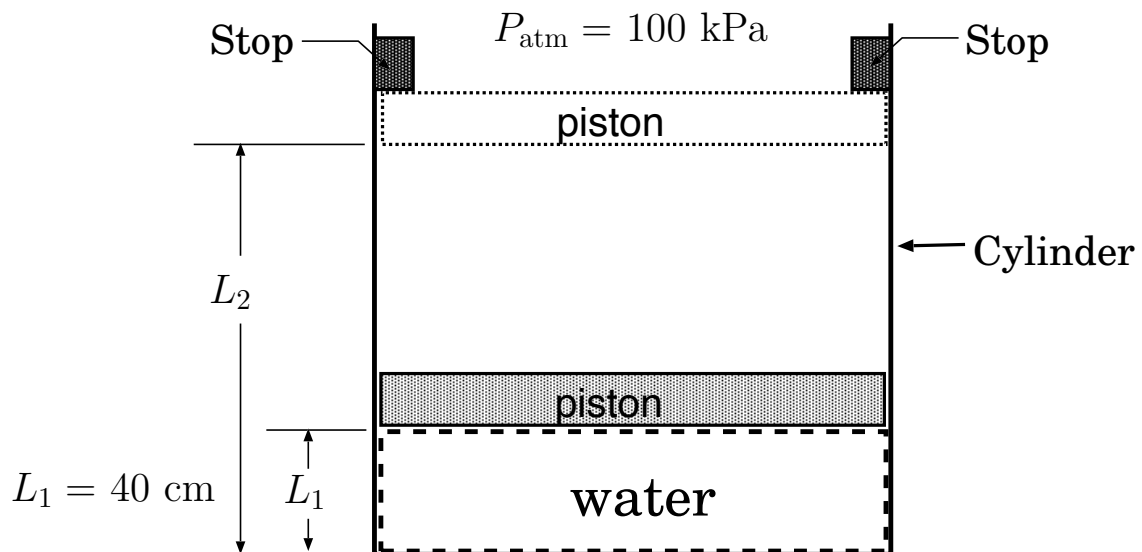


Figure 1: Schematic for Problem 2

3. Two cylindrical enclosed volumes, A and B, are connected by a piston, as shown in Figure 2. Cylindrical volume A contains a gas and the diameter of the piston on the end in contact with the gas is  $D_A = 4.5$  [cm]. The absolute pressure in cylindrical volume A is  $P_A = 398$  [kPa]. Cylindrical volume B contains oil and the diameter of the piston on the end in contact with the oil is  $D_B = 24$  [cm]. The surfaces of the piston in between the two cylindrical volumes are exposed to ambient air at  $P_o = 101.4$  [kPa]. The mass of the piston is  $12.02$  [kg] and gravitational acceleration is  $9.81$  [m/s<sup>2</sup>], acting downward as shown in the figure.

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- (a) Determine the absolute pressure in cylindrical volume B,  $P_B$ .

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- (b) Assuming that a manometer (filled with the same oil) connected to cylindrical volume B provides a good measurement of  $P_B$ , calculate  $h_B$  shown in the figure, in [mm]. Take the density of the oil to be  $910$  [kg/m<sup>3</sup>].

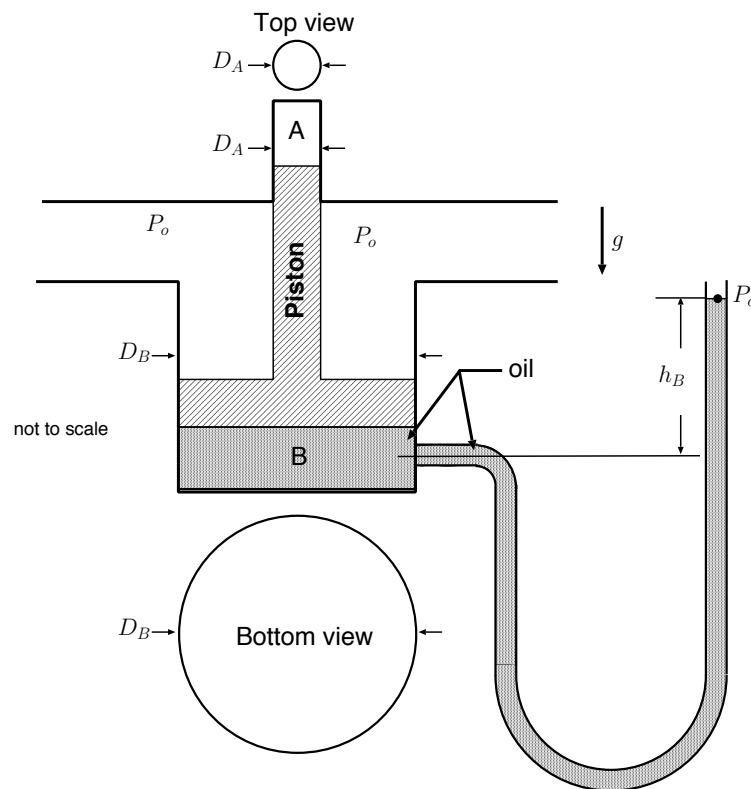


Figure 2: Piston and cylindrical volumes for Problem 3

#1. (a)  $P_1 = 1440 \text{ (kPa)}$   $R = 0.21882 \text{ (kJ/kgK)}$   
 $m = 1.25 \text{ (kg)}$   
 $T_1 = 385 \text{ (}^\circ\text{C)} = 658.15 \text{ (K)}$

$$V_1 = \frac{m R T_1}{P_1} = \frac{(1.25)(0.21882)(658.15)}{1440} = 0.12501 \text{ (m}^3\text{)}$$

$$P_2 = P_1$$

$$\frac{P_2 V_2}{R T_2} = \frac{m R T_2}{m R T_1} \Rightarrow \frac{V_2}{V_1} = \frac{T_2}{T_1}$$

$$V_2 = V_1 \cdot \left( \frac{T_2}{T_1} \right)$$

$$T_2 = 780 \text{ (}^\circ\text{C)} = 1053.15 \text{ (K)}$$

$$V_2 = 0.12501 \left( \frac{1053.15}{658.15} \right) = 0.2000 \text{ (m}^3\text{)} \leftarrow$$

(b)  $T_3 = T_2$   $V_3 = 0.275 \text{ (m}^3\text{)}$

$$\frac{P_3 V_3}{P_2 V_2} = \frac{T_3}{T_2} = 1 \Rightarrow P_3 = \left( \frac{V_2}{V_3} \right) P_2$$

$$P_3 = \frac{0.2000}{0.275} (1440) = 1047.3 \text{ (kPa)} \leftarrow$$

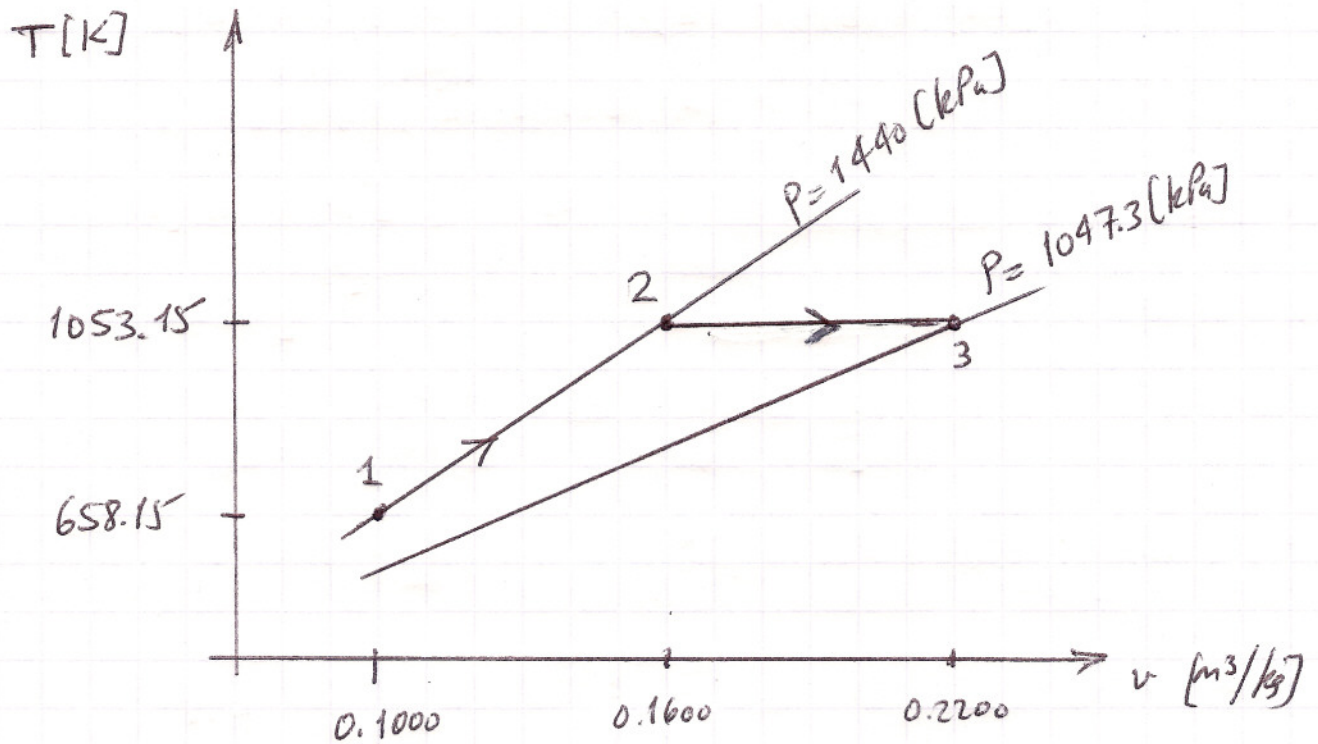
(c) need  $v_1, v_2, v_3$  for the diagram

$$v_1 = \frac{V_1}{m} = \frac{0.12501}{1.25} = 0.1000 \text{ (m}^3\text{/kg)}$$

$$v_2 = \frac{V_2}{m} = \frac{0.2000}{1.25} = 0.1600 \text{ (m}^3\text{/kg)}$$

$$v_3 = \frac{V_3}{m} = \frac{0.275}{1.25} = 0.2200 \text{ (m}^3\text{/kg)}$$

1 (c) continued



$$(d) \quad R = \frac{\bar{R}}{M} \Rightarrow M = \frac{\bar{R}}{R} = \frac{8.3145}{0.21882}$$

$$M = 37.997 \left( \frac{\text{kg}}{\text{kmol}} \right)$$



Question #2

(b)

Initial state

$x_1 = 0.15$

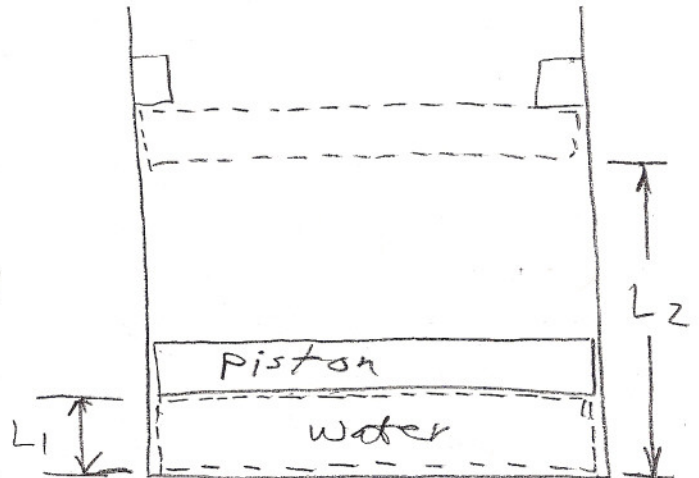
$P_1 = 400 \text{ [kPa]}$

$A_P = \pi/4 (0.20)^2 = \pi \times 10^{-2} \text{ [m}^2\text{]}$

$L_1 = 40 \text{ [cm]} = 0.40 \text{ [m]}$

$V_1 = L_1 A_P = 0.4 \times \pi \times 10^{-2} \text{ [m}^3\text{]}$

$V_1 = 1.25663 \times 10^{-2} \text{ [m}^3\text{]}$



state 1 is saturated mixture because quality is given.  
Table B.1.2, at 400 [kPa],  $v_f = 0.001084 \text{ [m}^3/\text{kg}]$

$$v_{fg} = 0.46138 \text{ [m}^3/\text{kg}]$$

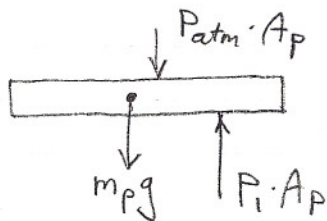
$$T_1 = T_{\text{sat.}} = 143.63 \text{ [}^\circ\text{C}]$$

$$v_1 = v_f + x \cdot v_{fg}$$

$$v_1 = 0.001084 + 0.15 \times 0.46138 = 0.070291 \text{ [m}^3/\text{kg}]$$

$$m_{\text{water}} = \frac{V_1}{v_1} = \frac{1.25663 \times 10^{-2}}{0.070291} = 0.178776 \text{ [kg]}$$

(a)



$$\text{Force balance: } P_i A_P = P_{\text{atm}} A_P + m_P g$$

$$\therefore m_P = \frac{(P_i - P_{\text{atm}}) A_P}{g}$$

$$m_P = \frac{(400 - 100) \times 10^3 \times (\pi \times 10^{-2})}{9.81}$$

$$m_P = 960.73 \text{ [kg]}$$

(c) When the piston just reaches the stops, the state of the water is to be saturated vapour,  $\therefore v_2 = v_g$  at  $P = 400 \text{ [kPa]}$  (pressure is constant while moving to the stops)  
 $\therefore P_2 = 400 \text{ [kPa]}, v_2 = 0.46246 \text{ [m}^3/\text{kg}]$   
 (Table B.1.2)

Also,  $T$  remains constant between states 1 and 2.  
 i.e.,  $T_2 = T_1 = 143.63 \text{ [}^\circ\text{C}]$

$$V_2 = v_2 \cdot m = 0.46246 \cdot 0.178776 = 0.082676 \text{ [m}^3\text{]}$$

$$L_2 = V_2 / A_p = 0.082676 / (\pi \times 10^{-2}) = 2.6316 \text{ [m]} \text{ or } 263.16 \text{ [cm]}$$

(d) Final state, state 3,  $P = 1000 \text{ [kPa]}$

$$v_3 = v_2 = 0.46246 \text{ [m}^3/\text{kg}]$$

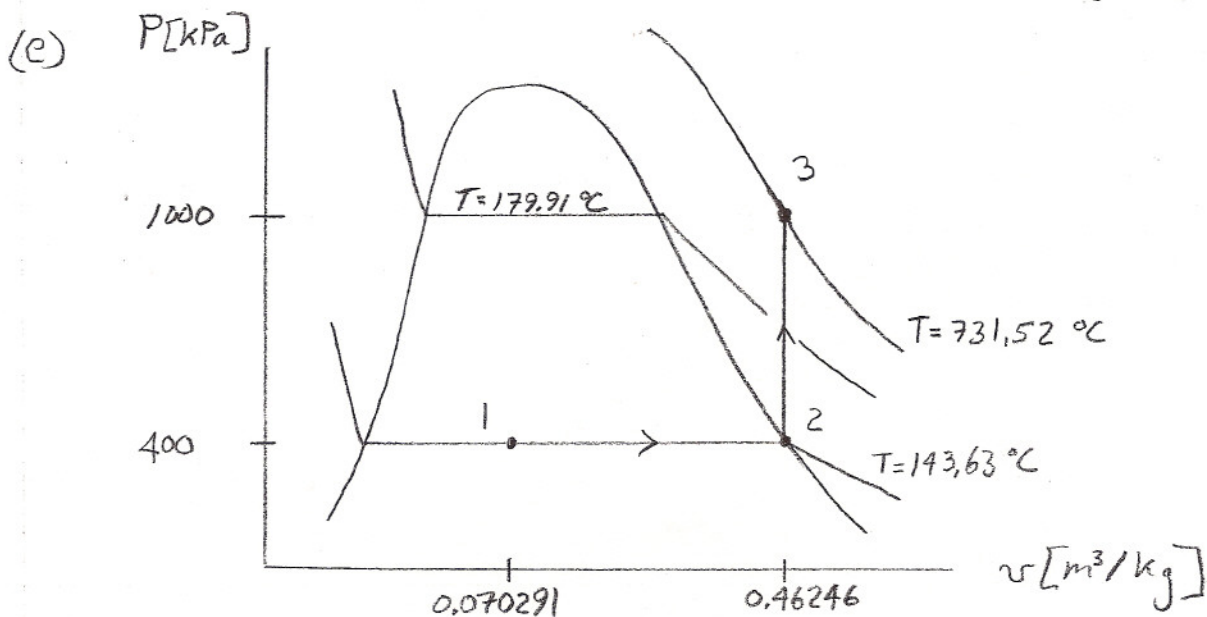
Table B.1.2, at  $1000 \text{ [kPa]}, v_g = 0.19444 \text{ [m}^3/\text{kg}], T_{\text{sat}} = 179.91 \text{ [}^\circ\text{C}]$   
 $v_3 > v_g \therefore$  state 3 is superheated vapor.

Table B.1.3

$T$	$v$
700	0.44779
$T$	0.46246
800	0.49432

$$\frac{T_3 - 700}{800 - 700} = \frac{0.46246 - 0.44779}{0.49432 - 0.44779}$$

$$T_3 = 731.52 \text{ [}^\circ\text{C]} > T_{c, \text{water}}$$



#3. (a) Do a force balance on the piston.

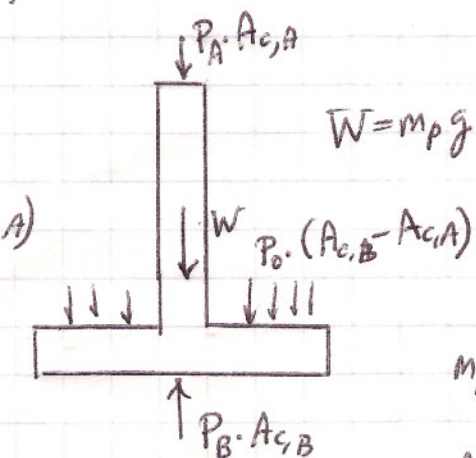
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$$\sum F \uparrow = \sum F \downarrow$$

$$P_B \cdot A_{c,B} = P_A \cdot A_{c,A} + m_p g + P_0 (A_{c,B} - A_{c,A})$$

$$\therefore P_B = P_A \left( \frac{A_{c,A}}{A_{c,B}} \right) + \frac{m_p g}{A_{c,B}} + P_0 \left( 1 - \frac{A_{c,A}}{A_{c,B}} \right)$$

$$P_B = 398000 \cdot \left( \frac{4.5}{24} \right)^2 + \frac{12.02 (9.81)}{\frac{\pi}{4} (0.24)^2} + 101400 \left( 1 - \left( \frac{4.5}{24} \right)^2 \right)$$



$m_p$  = mass of the piston

$A_c$  = cross-sectional area

$$A_{c,A} = \frac{\pi}{4} D_A^2$$

$$A_{c,B} = \frac{\pi}{4} D_B^2$$

$$\frac{A_{c,A}}{A_{c,B}} = \left( \frac{D_A}{D_B} \right)^2$$

$$P_B = 13992.2 + 2606.5 + 97835.2$$

$$P_B = 114433.9 [Pa] = 114.434 [kPa] \leftarrow$$

(b) For the manometer filled with oil with density  $\rho_{oil}$ .

$$(P_B - P_0) = \rho_{oil} g h_B$$

$$h_B = \frac{(P_B - P_0)}{\rho_{oil} g} = \frac{(114434 - 101400)}{(910)(9.81)} = \frac{13034 [Pa]}{(910)(9.81)}$$

$$h_B = 1.4600 [m] = 1460 [mm] \leftarrow$$