MATH 1210 Assignment 3

1.a) $2\underset{\sim}{u} + \underset{\sim}{v} = 2(1,1,1) + (-1,2,5)$

$= (2,2,2) + (-1,2,5)$

$= (1,4,7).$

$\underset{\sim}{v} - 3\underset{\sim}{w} = (-1,2,5) - 3(0,1,1)$

$= (-1,2,5) - (0,3,3)$

$= (-1,-1,2).$

$(2\underset{\sim}{u} + \underset{\sim}{v}) \cdot (\underset{\sim}{v} - 3\underset{\sim}{w}) = (1,4,7) \cdot (-1,-1,2)$

$= -1 - 4 + 14 = \underline{9}$

b) $\|\underset{\sim}{u}\| = \|(1,1,1)\| = \sqrt{3}$

$\|\underset{\sim}{v}\| = \|(-1,2,5)\| = \sqrt{1+4+25} = \sqrt{30}$

$\|\underset{\sim}{w}\| = \|(0,1,1)\| = \sqrt{2}.$

$\|\underset{\sim}{u}\| - 2\|\underset{\sim}{v}\| + \|(-3)\underset{\sim}{w}\| = \|\underset{\sim}{u}\| - 2\|\underset{\sim}{v}\| + 3\|\underset{\sim}{w}\|$

$= \sqrt{3} - 2\sqrt{30} + 3\sqrt{2}.$

2. (a) Let $u = (u_1, u_2, u_3)$
$v = (v_1, v_2, v_3)$
$w = (w_1, w_2, w_3)$

$$\Big((u_1, u_2, u_3) + (v_1, v_2, v_3) \Big) + (w_1, w_2, w_3)$$

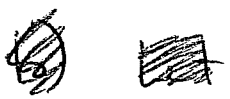
$$= (u_1 + v_1, u_2 + v_2, u_3 + v_3) + (w_1, w_2, w_3)$$

$$= \Big((u_1 + v_1) + w_1, (u_2 + v_2) + w_2, (u_3 + v_3) + w_3 \Big)$$

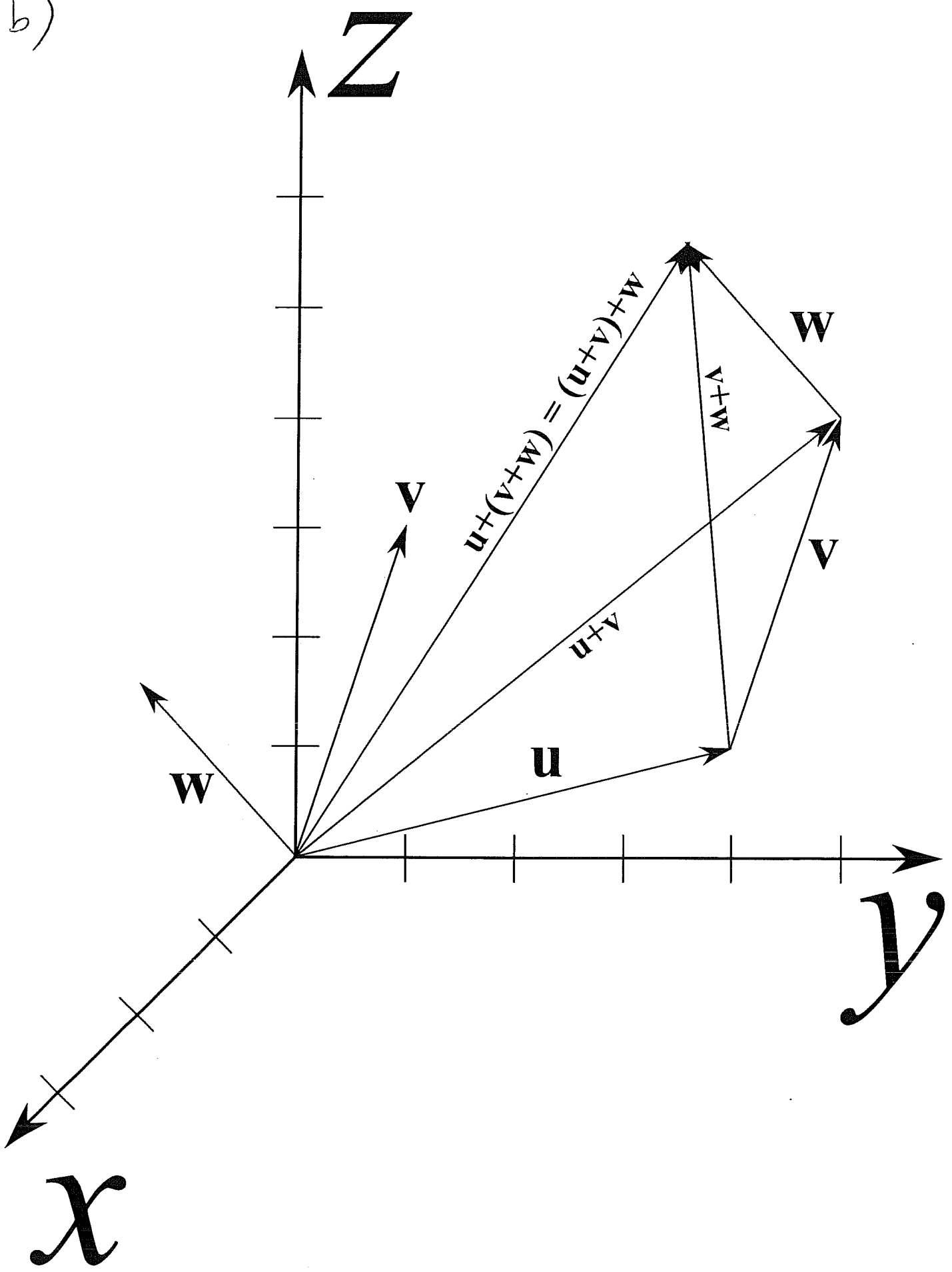
$$= \Big(u_1 + (v_1 + w_1), u_2 + (v_2 + w_2), u_3 + (v_3 + w_3) \Big)$$

$$= (u_1, u_2, u_3) + (v_1 + w_1, v_2 + w_2, v_3 + w_3)$$

$$= (u_1, u_2, u_3) + \Big((v_1, v_2, v_3) + (w_1, w_2, w_3) \Big).$$



2. b)



3. $3x - 2y + 5z = 30$

x-axis int: $y = z = 0$

$3x = 30$

$x = 10$ (10, 0, 0)

y-axis int: $x = z = 0$

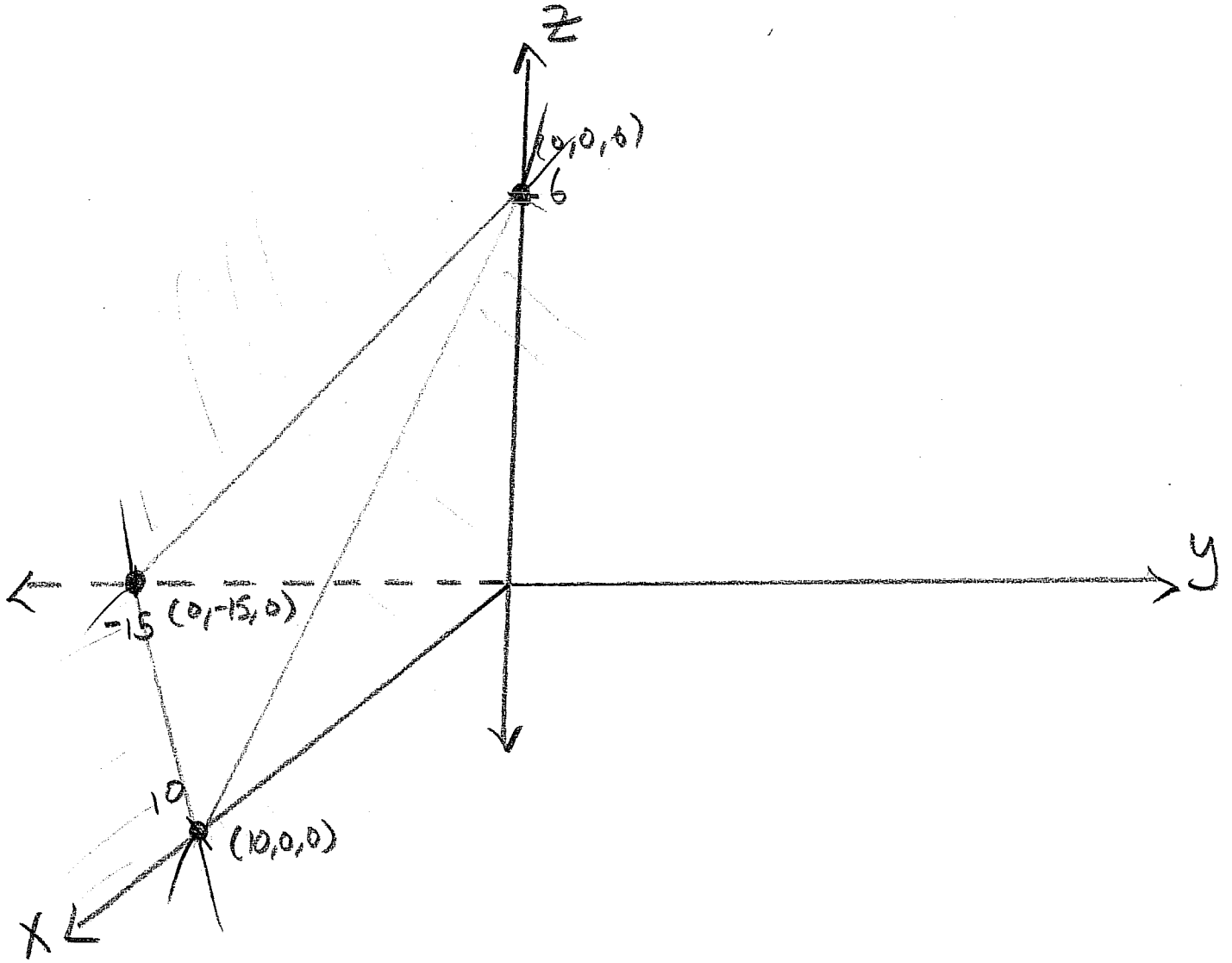
$-2y = 30$

$y = -15$ (0, -15, 0)

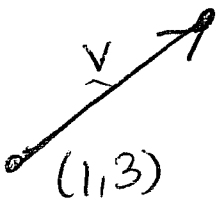
z-axis int: $x = y = 0$

$5z = 30$

$z = 6$ (0, 0, 6)



4. a)



$\underset{\sim}{v} = (5,4) - (1,3)$

$= (4,1)$.

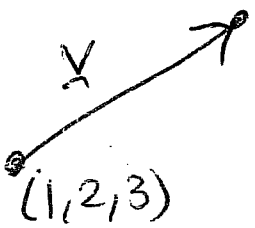
$\underset{\sim}{x} = (1,3) + t(4,1)$

$(x,y) = (1,3) + (4t, t)$

$x = 1 + 4t$

$y = 3 + t$ $\quad t \in \mathbb{R}$

b)



$\underset{\sim}{v} = (5,5,0) - (1,2,3)$

$= (4,3,-3)$.

$\underset{\sim}{x} = (1,2,3) + t(4,3,-3)$

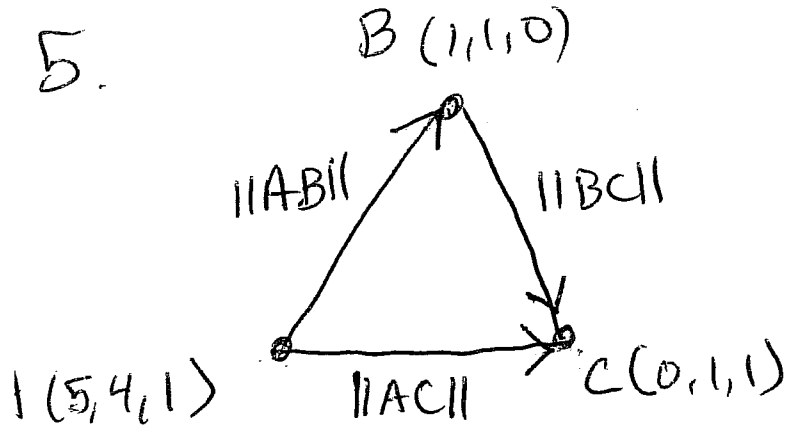
$(x,y,z) = (1,2,3) + (4t, 3t, -3t)$.

$x = 1 + 4t$

$y = 2 + 3t$ $\quad t \in \mathbb{R}$

$z = 3 - 3t$

5.



$AB = B - A = (1,1,0) - (5,4,1)$
$= (-4,-3,-1)$.

$BC = C - B = (0,1,1) - (1,1,0)$
$= (-1,0,1)$

$AC = C - A = (0,1,1) - (5,4,1)$
$= (-5,-3,0)$

a) $AB \cdot AC = (-4,-3,-1) \cdot (-5,-3,0)$
$= 20 + 9 + 0 = 29 \neq 0.$

$BA \cdot BC = (-AB) \cdot BC$
$= -(AB \cdot BC)$
$= -((-4,-3,-1) \cdot (-1,0,1))$
$= -(4 + 0 - 1) = -3. \neq 0.$

$CA \cdot CB = (-AC) \cdot (-BC)$
$= (-1)\cdot(-1)\cdot(AC \cdot BC)$
$= (-5,-3,0) \cdot (-1,0,1)$
$= 5 + 0 + 0 = 5. \neq 0.$

None of the dot products are zero, and thus this is not a right angle triangle.

b)

$$\|AB\| = \|(-4,-3,-1)\| = \sqrt{16+9+1} = \sqrt{26}$$

$$\|BC\| = \|(-1,0,1)\| = \sqrt{1+1} = \sqrt{2}$$

$$\|AC\| = \|(-5,-3,0)\| = \sqrt{25+9} = \sqrt{34}.$$

No two sides are the same length. Thus this is NOT an isosceles triangle.

c) Since $\|AB\| \neq \|BC\|$, this is not equilateral either.

d) If there is an obtuse angle, then one of the angles is $> 90^\circ = \frac{\pi}{2}$ rads. Thus, $\cos\theta < 0$, and so the dot product of the two vectors creating the angle, say $\underset{\sim}{u}$ and $\underset{\sim}{v}$, is $\underset{\sim}{u} \cdot \underset{\sim}{v} = \|\underset{\sim}{u}\| \cdot \|\underset{\sim}{v}\| \cdot \cos\theta < 0$ (since $\|\underset{\sim}{u}\| > 0$ and $\|\underset{\sim}{v}\| > 0$)

So, since $BA \cdot BC = -3 < 0$, we know that the angle at B is obtuse.

6. a)

$\underset{\sim}{n} = (5,0,2)$ $\quad$ $P = (2,2,1)$

$\underset{\sim}{n} \cdot \underset{\sim}{X} = \underset{\sim}{n} \cdot P$

$(5,0,2) \cdot (x,y,z) = (5,0,2) \cdot (2,2,1)$

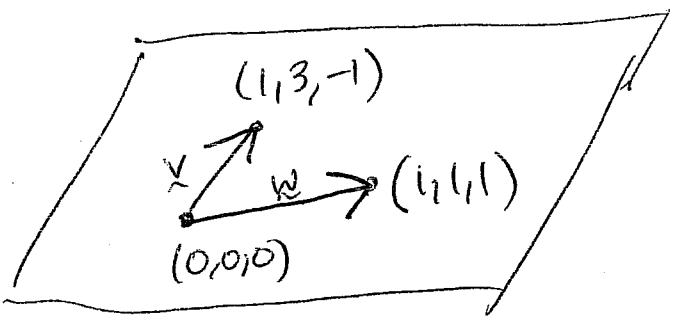
↑ Point-normal form (or anything equivalent)

$5x + 2z = 10 + 2$

$5x + 2z = 12$ ← Standard form.

b)

Q:



$\underset{\sim}{v} = (1,3,-1)$

$\underset{\sim}{w} = (1,1,1)$.

$\underset{\sim}{X} = (0,0,0) + t(1,3,-1) + s(1,1,1)$

$x = t + s$

$y = 3t + s$ $\quad$ $t, s \in \mathbb{R}$.

$z = -t + s$

c) $s+t=x$

$-\quad s+3t=y$

$-2t=x-y$

$t=\frac{x-y}{-2}=\frac{y-x}{2}$

$\therefore s=x-t$

$s=\frac{2x}{2}-\frac{y-x}{2}$

$s=\frac{3x-y}{2}$

$s-t=z$

$\left(\frac{3x-y}{2}\right)-\left(\frac{y-x}{2}\right)=z$

$\frac{3}{2}x-\frac{1}{2}y-\frac{1}{2}y+\frac{1}{2}x=z$

$2x-y-z=0$

↑ Standard Form

d) The line of intersection consists of all points that satisfy both

$2x-y-z=0$

$5x+2z=12.$

e) $z = t$

$$\Rightarrow \begin{aligned} 2x - y &= t \\ 5x &= 12 - 2t \end{aligned}$$

$$\Rightarrow x = \frac{12-2t}{5} = \frac{-2}{5}t + \frac{12}{5}$$

$$\Rightarrow 2\left(\frac{-2}{5}t + \frac{12}{5}\right) - y = t$$

$$y = \frac{-4}{5}t + \frac{24}{5} - \frac{5t}{5} =$$

$$= -\frac{9}{5}t + \frac{24}{5}$$

Thus the line of intersection is:

$$x = -\frac{2}{5}t + \frac{12}{5}$$

$$y = -\frac{9}{5}t + \frac{24}{5} \qquad t \in \mathbb{R}.$$

$$z = t$$