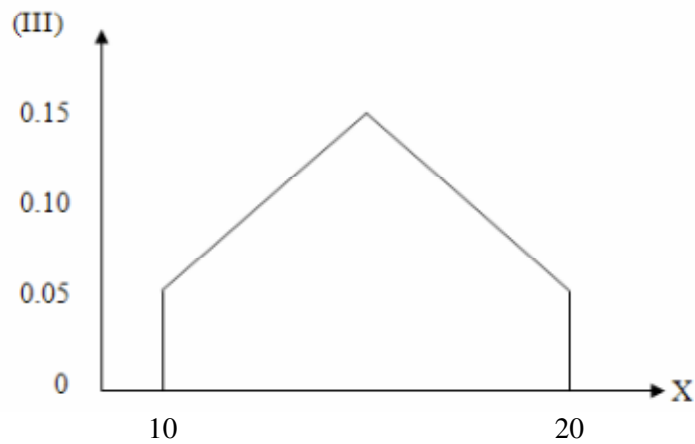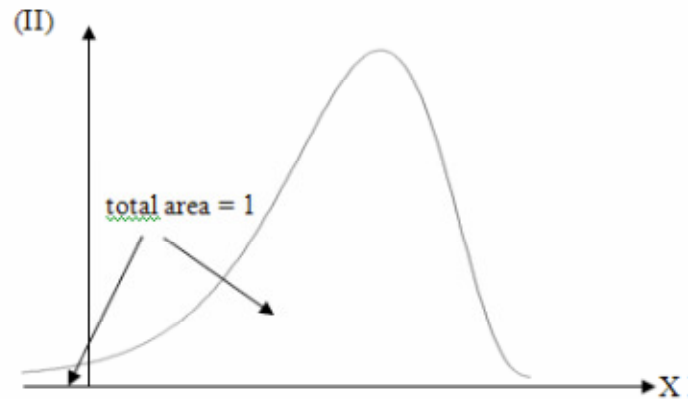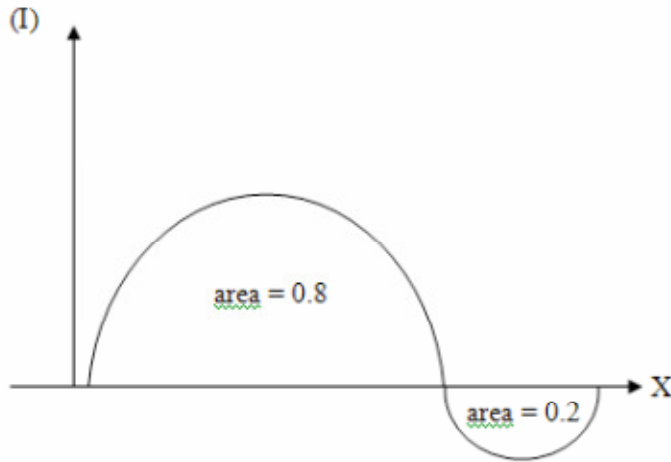# Sample Midterm Test 2A

Part A

1. Which of the following represent a legitimate density curve for some random variable X?

(A) II only   (B) I and II only   (C) I and III only   (D) II and III only   (E) I, II and III

2. A discrete random variable X has probability mass function

$$P(X = x) = \begin{cases} c(0.9)^x, & x = 0, 2, 4, 6, 8, ... \\ 0, & \text{otherwise} \end{cases}$$

What must be the value of the constant c in order for this to be a legitimate p.m.f.?

(A) 0.10   (B) 0.18   (C) 0.19   (D) 0.81   (E) 0.90

3. Consider two independent random variables W and X, the means and variances of which are shown below:

$$E(W) = 4 \qquad Var(W) = 2$$
$$E(X) = 7 \qquad Var(X) = 3$$

Now consider the random variable $Y = 5W - 2X + 6$. What is the variance of Y?

(A) 16   (B) 38   (C) 44   (D) 62   (E) 68

The next **four** questions (**4, 5, 6 and 7**) refer to the following:

A bowl contains 20 fruit-flavoured candies – 6 orange candies, 2 grape, 4 lemon and 8 cherry.

4. If you randomly select candies **with replacement**, what is the probability that the fourth orange candy will be selected on the 11th draw?

(A) 0.2201   (B) 0.0800   (C) 0.1857   (D) 0.3065   (E) 0.1263

5. If you randomly select candies **with replacement**, what is the probability that it takes less than 12 draws to get the first grape candy?

(A) 0.2824   (B) 0.3138   (C) 0.7176   (D) 0.0314   (E) 0.6862

6. If you randomly select nine candies **with replacement**, what is the probability that less than two of them will be lemon?

(A) 0.4041   (B) 0.3746   (C) 0.3558   (D) 0.4362   (E) 0.3020

7.   If you randomly select seven candies **without replacement**, what is the probability that exactly three of them will be cherry?

(A) 0.3576   (B) 0.3030   (C) 0.4187   (D) 0.2634   (E) 0.2903

The next **two** questions (**8 and 9**) refer to the following:

Customers enter a clothing store according to a Poisson process with a rate of 0.17 per minute. The store opens at 9:00 a.m.

8.   What is the probability that exactly seven customers have entered the store by 9:30 a.m.?

(A) 0.0986   (B) 0.1086   (C) 0.1186   (D) 0.1286   (E) 0.1386

9.   If no customers have entered the store by 9:10 a.m., what is the probability that the first customer enters before 9:15 a.m.?

(A) 0.5726   (B) 0.5926   (C) 0.6126   (D) 0.6326   (E) 0.6526

10. A random variable X has a Poisson distribution with parameter λ. If P(X = 1) = P(X = 3), what is the value of λ?

(A) $\sqrt{2}$ (B) $\sqrt{3}$ (C) $\sqrt{5}$ (D) $\sqrt{6}$ (E) $\sqrt{8}$

The next **two** questions (**11 and 12**) refer to the following:

Percentage grades in a large geography class follow a normal distribution with mean 67.5 and standard deviation 12.5.

11. What proportion of students in the class receive percentage grades between 60 and 70?

(A) 0.2650   (B) 0.2750   (C) 0.2850   (D) 0.2950   (E) 0.3050

12. The professor decides to assign a grade of A+ to the students with the top 8% of the grades, and a grade of A to the next best 12%. What is the minimum percentage a student needs to earn a grade of A?

(A) 77   (B) 78   (C) 79   (D) 80   (E) 81

13. The yearly rainfall in Vancouver, B.C. follows a normal distribution with standard deviation 172 mm. In 20% of years, the city gets over 1200 mm of rain. What is the mean annual rainfall in Vancouver?

   (A) 1055.5 mm   (B) 1165.6 mm   (C) 982.6 mm   (D) 1344.5 mm   (E) 1234.4 mm

14. A random variable W follows a normal distribution with mean 18 and standard deviation 6. A random variable X follows a normal distribution with mean 25 and standard deviation 9. It is known that W and X are independent. Consider the random variable $Y = 3W - 2X$. What is $P(Y > 10)$?

   (A) 0.2357   (B) 0.5948   (C) 0.3632   (D) 0.4052   (E) 0.7643

15. For a certain brand of potato chips, the weight of chips per bag follows a normal distribution with mean 55 grams and standard deviation 2 grams. If you randomly select seven bags, what is the probability that exactly three of them contain more than 56 grams?

   (A) 0.2350   (B) 0.2088   (C) 0.3741   (D) 0.3088   (E) 0.4217

Part B

1. The following three games are scheduled to be played at the World Curling Championship one morning. The values in parentheses are the probabilities of each team winning their respective game.

    Game 1: Finland (0.2) vs. Canada (0.8)
    Game 2: USA (0.3) vs. Switzerland (0.7)
    Game 3: Japan (0.4) vs. Germany (0.6)

    (a) The outcome of interest is the set of winners for each of the three games. List the complete sample space of outcomes and calculate the probability of each outcome.
    (b) Let X be the number of European teams that win their respective games. Find the p.m.f. of X.
    (c) Find the expected value and variance of X.

2. A continuous random variable X has p.d.f.

$$f(x) = \begin{cases} \dfrac{3}{7}x^2, & 1 \le x \le 2 \\ 0, & \text{otherwise} \end{cases}$$

    (a) Find the c.d.f. of X.
    (b) Find the median of the distribution of X.
    (c) Find the probability $P(1.2 < X < 1.5)$.
    (d) Find the expected value of X.

3. (a) Suppose it is known that 21% of all adults in Winnipeg are smokers. We take a simple random sample of ten adults in Winnipeg. Let X be the number of smokers in the sample. Verify the four conditions of the binomial setting to show that X has a binomial distribution.
    (b) What is the probability that at least three of the ten adults in the sample are smokers?
    (c) A random variable X has a binomial distribution with mean 3.6 and variance 2.52. What are the values of the parameters n and p?