Winter 2007 Test 1/2 (Question were taken from both tests that cover the material for our test)

1. Set up, but do not evaluate, integral(s) to determine the length of the portion of the curve $x = (y+1)\ln y$ which lies between $(0, 1)$ and $e + 1 = e$.

2. Set up, but do not evaluate integral(s) to determine the minimum amount of work done to pump the oil having a constant density ρ from a hemi-spherical tank (with horizontal planar top) of radius 2 metres, to a height 3 metres above the top of the tank. (ignoring friction)

3. Set up, but do not evaluate integral(s) to determine the total fluid force exerted on one face of a circular plate of radius 3 metres which is immersed vertically into a fluid of density ρ so that the top of the plate is 1 metre above the surface of the fluid.

4. Consider a thin plate of constant mass per unit area ρ which occupies the region in the first quadrant inside the curve $x^2 + 4y^2 = 4$. Using vertical strips, set up but do not evaluate integrals for the following physical quantites

   (a) The mass $M =$

   (b) The first moment of mass about the y-axis. $M_y =$

   (c) The first moment of mass about the x-axis. $M_x =$

   (d) The moment of inertia about the line $x = -3$. $I_{(x=-3)} =$

   (e) The moment of inertia about the line $y = 2$. $I_{(y=2)} =$

5. Evaluate the integrals

   (a) $\displaystyle\int cos^{-1} x\, dx$

   (b) $\displaystyle\int sec^{2014} x \tan^3 x\, dx$

Answers

1. $\displaystyle\int_1^e \sqrt{1 + \left(\ln y + \frac{y+1}{y}\right)^2}\, dy$

2. $\displaystyle\int_{-2}^0 \rho g(4 - z^2)(3 - z)\, dz$

3. $\displaystyle 2\int_{-3}^2 \rho g(\sqrt{9 - x^2})(2 - y)\, dy$

4. (a) $\displaystyle M = \rho \int_0^2 \sqrt{1 - \frac{x^2}{4}}\, dx$

   (b) $\displaystyle M_y = \rho \int_0^2 x\sqrt{1 - \frac{x^2}{4}}\, dx$

   (c) $\displaystyle M_x = \frac{\rho}{2} \int_0^2 \left(1 - \frac{x^2}{4}\right) dx$

   (d) $\displaystyle I_{(x=-3)} = \rho \int_0^2 (x+3)^2\sqrt{1 - \frac{x^2}{4}}\, dx$

   (e) $\displaystyle I_{(y=2)} = \frac{\rho}{3} \int_0^2 \left(3 - \sqrt{1 - \frac{x^2}{4}}\right)^3 dx$

5. (a) $x \cos^{-1} x - (1 - x^2)^{1/2} + C$

   (b) $\displaystyle \frac{\sec^{2016} x}{2016} - \frac{\sec^{2014} x}{2014} + C$