MATH 1210 Summer 2015 Quiz 4 (20 minutes)

1. For the following system

$$\begin{aligned} 2x + 5y - 4z &= 6 \\ x + 2y - 3z &= -2 \\ 3x + 8y - 5z &= 14 \end{aligned}$$

[1] (a) Put the system into an augmented matrix

Solution:

$$\left[\begin{array}{ccc|c} 2 & 5 & -4 & 6 \\ 1 & 2 & -3 & -2 \\ 3 & 8 & -5 & 14 \end{array}\right]$$

[5] (b) Use elementary row operations to get the matrix into reduced row–echelon form.

Solution:

Using $R_1 \leftrightarrow R_2$ yields

$$\left[\begin{array}{ccc|c} 1 & 2 & -3 & -2 \\ 2 & 5 & -4 & 6 \\ 3 & 8 & -5 & 14 \end{array}\right]$$ Using $R_2 \to R_2 - 2R_1, R_3 \to R_3 - 3R_1$ yields

$$\left[\begin{array}{ccc|c} 1 & 2 & -3 & -2 \\ 0 & 1 & 2 & 10 \\ 0 & 2 & 4 & 20 \end{array}\right]$$ Using $R_1 \to R_1 - 2R_2$ and $R_3 \to R_3 - 2R_2$ yields

$$\left[\begin{array}{ccc|c} 1 & 0 & -7 & -22 \\ 0 & 1 & 2 & 10 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

[3] (c) Solve the system.

Solution:

The column with no leading one is for z so z is arbitrary. Therefore from equation 1, we get

$$x - 7z = -22 \Rightarrow x = -22 + 7z.$$

From equation 2, we get

$$y + 2z = 10 \Rightarrow y = 10 - 2z$$

Hence the solution is

$x = -22 + 7z, \quad y = 10 - 2z$ and z is arbitrary.

[7] 2. Find an equation of the plane through the points $P(3,-1,2)$, $Q(8,2,4)$ and $R(-1,-2,-3)$.

Solution:

We start by finding the vectors $\vec{PQ}$ and $\vec{PR}$ parallel to the plane (note that there are other possibilities as well.)

$$\begin{aligned}\vec{PQ} &= \langle 8-3, 2-(-1), 4-2\rangle = \langle 5,3,2\rangle\\ \vec{PR} &= \langle -1-3, -2-(-1), -3-2\rangle = \langle -4,-1,-5\rangle\end{aligned}$$

Hence a normal vector perpendicular to the plane is

$$\begin{aligned}\mathbf{n} &= \langle 5,2,3\rangle \times \langle -4,-1,-5\rangle\\ &= \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 5 & 3 & 2 \\ -4 & -1 & -5 \end{vmatrix}\\ &= \begin{vmatrix} 3 & 2 \\ -1 & -5 \end{vmatrix}\hat{\mathbf{i}} - \begin{vmatrix} 5 & 2 \\ -4 & -5 \end{vmatrix}\hat{\mathbf{j}} + \begin{vmatrix} 5 & 3 \\ -4 & -1 \end{vmatrix}\hat{\mathbf{k}}\\ &= (-15-(-2))\hat{\mathbf{i}} - (-25-(-8))\hat{\mathbf{j}} + (-5-(-12))\hat{\mathbf{k}}\\ &= -13\hat{\mathbf{i}} + 17\hat{\mathbf{j}} + 7\hat{\mathbf{k}}\end{aligned}$$

Therefore an equation of the plane is

$$-13(x-3)+17(y+1)+7(z-2)=0.$$

Note: Other acceptable answers may include (but are not limited to)

$13(x-3)-17(y+1)-7(z-2)=0.$

$-13(x-8)+17(y-2)+7(z-4)=0.$

$13(x-8)-17(y-2)-7(z-4)=0.$

$-13(x+1)+17(y+2)+7(z+3)=0.$

$13(x+1)-17(y+2)-7(z+3)=0.$

$-13x+17y+7z=-42.$

$13x-17y-7z=42.$

[4] 3. A given system has been reduced to the following augmented matrix.

$$\left[\begin{array}{ccc|c} 1 & 3 & 5 & 13 \\ 0 & 1 & 6 & 15 \\ 0 & 0 & a & b \\ 0 & 0 & 0 & 0 \end{array}\right]$$

For which values of a and b does the system have

(a) No solution

> **Solution:**
> $a = 0 \quad b \neq 0.$

(b) Exactly one solution

> **Solution:**
> $a \neq 0.$

(c) Exactly three solutions

> **Solution:**
> Never.

(d) Infinitely many solutions

> **Solution:**
> $a = 0 \quad b = 0.$