MATH 2130 – Tutorial Problems, Thu Feb 8

Multivariable Limits

Example. Evaluate the limit

$$\lim_{(x,y)\to(1,2)} \frac{x^2-2x+2y^2-8y+9}{3x^2-6x-y^2+4y-1},$$

or show that it does not exist.

Example. Evaluate the limit

$$\lim_{(x,y)\to(1,-1)} \frac{x^2-y^2-2x-2y}{\sqrt{x-2y+1}-\sqrt{2x-y+1}},$$

or show that it does not exist.

Example. (a) Evaluate

$$\lim_{(x,y)\to(0,0)} \frac{\sin(x+2y)}{x+2y}.$$

(b) Consider the limit

$$\lim_{(x,y)\to(0,0)} \frac{\cos(x+2y)-1}{x+y}.$$

Verify that all paths of the form $y=mx$ yield the same limit. Do you think that this limit exists?

Chain Rules

Example. Let $z=f(x,y,s,t)$, where $x=g(s,t)$, $y=h(s,t)$, $s=k(v)$ and $t=m(v,w)$. Find $\left.\dfrac{\partial z}{\partial v}\right)_w$.

Example. Let $z=x^2y^2+yt^3$, $x=t^2+s^3$, $y=1+st+s^2t^2$. Find $\left.\dfrac{\partial^2 z}{\partial t^2}\right)_s$.

Implicit Differentiation

Example. Let the equations

$$\begin{aligned} F(x,y,z,s,t) =& x\sin(ys)+z\cos(yt)=0,\\ G(x,y,z,s,t) =& x^2+y^2+z^2-s^2-t^2=0,\\ H(x,y,z,s,t) =& ye^{s+t}+xz^3=0 \end{aligned}$$

define x,y,z implicitly as functions of s,t. Find $\frac{\partial x}{\partial s}$ and $\frac{\partial x}{\partial t}$ at the point $(x,y,z,s,t)=(1,0,0,0,1)$.

Example. Let $z=\ln(x^2+y^2+1)$, where x,y are defined implicitly as functions of t by

$$\begin{aligned} F(x,y,t) =& x^3-yt^3=0,\\ G(x,y,t) =& xe^{yt-1}-t=0. \end{aligned}$$

Find $\frac{dz}{dt}$ when $(x,y,t)=(1,1,1)$.