Tutorial 9: 13.7.3

$$A = 4\int_0^{\pi/4}\int_0^{3\sqrt{\cos 2\theta}} \rho r^3 \sin^2\theta \, dr\, d\theta = \cancel{4}\int_0^{\pi/4} \frac{81\rho}{\cancel{4}} \cos^2 2\theta \sin\theta \, d\theta$$

$$= 81\rho \int_0^{\pi/4} \cos^2 2\theta \cdot \frac{1}{2}\left[1 - \cos 2\theta\right] d\theta$$

$$= \frac{81\rho}{2} \int_0^{\pi/4} \left(\cos^2 2\theta - \cos^3 2\theta\right) d\theta \qquad , \cos^2 2\theta = 1 - \sin^2 2\theta$$

$$= \frac{81\rho}{2}\left[\int_0^{\pi/4} \cos^2\theta \, d\theta - \int_0^{\pi/4} \left(1 - \sin^2 2\theta\right) \cos 2\theta \, d\theta\right]$$

$$= \frac{81\rho}{2}\left[\int_0^{\pi/4} \frac{1}{2}\left(1 + \cos 2\theta\right) d\theta - \int_0^{\pi/4} \cos 2\theta \, d\theta + \int_0^{\pi/4} \sin^2 2\theta \cos 2\theta \, d\theta\right]$$

$$= \frac{81\rho}{2}\left[\frac{1}{2}\left(\theta + \frac{1}{2}\sin 2\theta\right)\Big|_0^{\pi/4} - \frac{1}{2}\sin 2\theta \Big|_0^{\pi/4} + \frac{1}{6}\sin^3 2\theta \Big|_0^{\pi/4}\right]$$

$$= \frac{81\rho}{2}\left[\left(\frac{\pi}{8} + 0\right) - \frac{1}{2}(0 - 0) - \frac{1}{2}(1 - 0) + \frac{1}{6}(1 - 0)\right]$$

$$= \frac{81\rho}{2}\left(\frac{\pi}{8} - \frac{1}{2} + \frac{1}{6}\right) = \frac{27}{16}\rho(3\pi - 8)$$