MATH 1210 Assignment 1 Fall 2016 Due date: October 3

Attempt all questions and show all your work. Attach to Honesty Declaration Form.

1. **Use mathematical induction on positive integer n to prove each of the following:**
   - **(a)** $1(4) + 2(7) + 3(10) + \cdots + (n-1)(3n-2) = n^2(n-1)$, **for** $n \geq 2$ **;**
   - **(b)** $n! + 2^n < (2n)!$ **for** $n \geq 2$**;**
   - **(c)** $(1^3 + 4^3) + (2^3 + 8^3) + (3^3 + 12^3) + \cdots + [n^3 + (4n)^3] = \dfrac{65}{4} n^2(n+1)^2$**;**
   - **(d)** $(n+1)^3 + 2n + 5$ **is divisible by** 3**, for** $n \geq 0$**.**

2. **Consider the sum** $(3)^2 + (7)^2 + (11)^2 + \cdots + (12n-1)^2$**:**
   - **(a) Write the sum in sigma notation.**
   - **(b) Use identities** $\displaystyle\sum_{k=1}^{m} k = \frac{1}{2}[m(m+1)]$ **and** $\displaystyle\sum_{k=1}^{m} k^2 = \frac{1}{6}[m(m+1)(2m+1)]$ **to prove that**

     $$(3)^2 + (7)^2 + (11)^2 + \cdots + (12n-1)^2 = n\,(144n^2 + 36n - 1)\,.$$

3. **First write the sum** $n + (n+2) + (n+4) + (n+6) + \cdots + (3n)$ **in sigma notation and then use the identity** $\displaystyle\sum_{k=1}^{m} k = \frac{1}{2}[m(m+1)]$ **to find the value of the sigma in terms of** n**.**

4. **Prove that** $\displaystyle\sum_{\ell=1}^{n} \ell(\ell-3) = \frac{1}{3}[n(n+1)(n-4)]$ **by each of the following two methods:**
   - **(a)** **By mathematical induction on positive integer** $n \geq 1$**.**
   - **(b)** **By using the identities mentioned in part (b) of question** 2 **.**

5. **Rewrite the sum** $\displaystyle\sum_{j=-4}^{50} \Big[(4j+20)^3 + j(j+10) + 25\Big]$ **such that it starts with** $j = 1$**. Simplify your answer but keep it in sigma notation.**

6. **Find values of** x **and** y **if**

   $$\sum_{j=4}^{14}[(4j+1)^{10} \;-\; (2j+6)^2] \;=\; \sum_{j=7}^{17}[(4j+x)^{10} \;+\; yj^2]\,.$$

7. **Find all real and complex solutions of the equation**

   $$x^5 + x^4 + 3x^3 - x^2 - x - 3 = 0\,.$$

8. **Find the Cartesian form of each of the following expression. Simplify as much as possible.**
   - **(a)** $\Big(\dfrac{\sqrt{3}}{\sqrt{2}} - \dfrac{1}{\sqrt{2}}\,i\Big)^{12} - \Big(\dfrac{\sqrt{3}}{\sqrt{2}} + \dfrac{1}{\sqrt{2}}\,i\Big)^{14}$**.**
   - **(b)** $\dfrac{(\overline{-1+i})^{13}\,(3+3\sqrt{3}\,i)^6}{6^6\,i^3(1-i)}$**.**