Math 2130 - Engineering Mathematical Analysis 1

Tutorial Questions for §12.7 - 12.9.

12.7.1. The equations

$$x^2 + y + 3s^2 + s = 2t - 1, \quad y^2 - x^4 + 2st + 7 = 6s^2t^2,$$

define s and t as functions of x and y. Find $\dfrac{\partial s}{\partial x}$ when $s = 0$ and $t = 1$. Assume that $x > 0$.

12.7.2. The equations

$$x^2 - y^2 + u^2 + 2v^2 = 1, \quad x^2 + y^2 = 2 + u^2 + v^2,$$

define u and v as functions of x and y. Find $\dfrac{\partial u}{\partial x}$ and $\dfrac{\partial v}{\partial y}$.

12.8.1. Find the rate of change of the function $f(x, y, z) = \sin(xy) - z^3$ at the point $(2, 0, 3)$ in the direction of the upward normal to the surface $xz^2 - x^2z = 6$.

12.8.2. At the point $(1, 2, -3)$, a vector $\mathbf{v}$ makes an angle of $\pi/3$ radians with the gradient of the function $f(x, y, z) = x^2yz - 3xy^3$. Find the rate of change of $f(x, y, z)$ in direction $\mathbf{v}$.

12.9.1. Find equations for the tangent line to the following curve at the point $(1, -1, 3)$:

$$xyz + z^3 = 24, \quad x^3y^2z + y^3 = 4x - 2.$$

12.9.2. Find an equation for the tangent plane to the following surface at the point $(2, -1, -1)$:

$$x^2y + y^2z + z^2x + 3 = 0.$$

Answers:

12.7.1: 16.

12.7.2: $\dfrac{\partial u}{\partial x} = \dfrac{3x}{u}$ and $\dfrac{\partial v}{\partial y} = 0$.

12.8.1: $-216/\sqrt{73}$.

12.8.2: $\sqrt{2821}/2$.

12.9.1: $x = 1 + 81t$, $y = -1 + 133t$, $z = 3 - 6t$.

12.9.2: $x - 2y + z = 3$.