**UNIVERSITY OF MANITOBA**

DATE: November 5, 2014 | TERM TEST 2



EXAMINATION: Engineering Mathematical Analysis 1 | TIME: 70 minutes

COURSE: MATH 2130 | EXAMINER: various

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1. *Evaluate the limit, if it exists. Justify your answer.*

[3] (a) $\displaystyle\lim_{(x,y)\to(0,0)} \left(\frac{x^2-y^2}{x^2+y^2}\right)^2.$

[3] (b) $\displaystyle\lim_{(x,y)\to(1,-4)} \frac{(x-1)^3-2(y+4)^2}{3(x-1)^2+(y+4)^2}.$

2. *Let* $z=\cos(xy^2)$ *where* $x=3u^2+v^3, \quad y=2uv^2$.

[5] (a) *Use the chain rule to determine* $\left.\dfrac{\partial z}{\partial u}\right)_v$.

[5] (b) *Use the chain rule to determine* $\left.\dfrac{\partial^2 z}{\partial u^2}\right)_v$. *Do not simplify your answer.*

[10] 3. *The equations*

$$x^2y^3\cos v+2u^2\sin w=\sqrt{2}$$
$$y^3-\cos v+u\cos^2 v+y^2=0$$
$$yu+x\cos v+\sin w=\frac{\sqrt{2}}{2}-1$$

*define* $u$, $v$, *and* $w$ *as functions of* $x$ *and* $y$.
*Compute* $\left.\dfrac{\partial v}{\partial y}\right)_x$ *when* $x=1, y=-1, u=1, v=\pi/2, w=\pi/4$.

4. *The temperature at a point* $(x,y,z)$ *is given by*

$$T(x,y,z)=2e^{-x^2-2y^2-3z^2}$$

*where* $T$ *is measured in degrees Celsius and* $x,y,z$ *in meters. Further let* $P$ *be the point* $P(2,-1,2)$.

[4] (a) *Compute the directional derivative of the function* $T$ *at the point* $P$ *in the direction toward the point* $Q(2,-3,3)$. *Include units in your answer.*

[2] (b) *In which direction does the temperature increase fastest at* $P$ *?*

[2] (c) *Compute the maximum rate of increase of* $T$ *at* $P$. *Include units in your answer.*

[6] 5. *Determine parametric equations for the tangent line to the curve*

$$x^2z+3y^2x-3z^2y=-11, \qquad x^2+2xy+z^2+3y^2=10$$

*at* $P_0(1,1,-2)$.

[10] 6. *Determine all critical points of* $f(x,y)=-x^3+4xy-2y^2+1$, *and classify each point as yielding a relative maximum, relative minimum, a saddle point, or none of these.*