Term Test 2

DATE: March 12, 2013
COURSE: MATH 2132

TIME: 70 minutes
EXAMINER: G.I. Moghaddam & M. Virgilio

[8] 1. Use the binomial expansion to find only the first three nonzero terms of the Taylor series about 1 of $f(x) = \sqrt{x} + \frac{1}{\sqrt{x}}$. Simplify your answer.
(You are **not** asked to find all the terms of the Taylor series. No mark will be given for any other method)

[8] 2. Evaluate the following limit using infinite series.

$$\lim_{x\to 0} \frac{1}{x^3}\left[\sqrt[3]{(1+x^3)^2} + x^3 - 1\right]$$

(You are not allowed to use any other method.)

[8] 3. Find the sum and the open interval of convergence of the series $\sum_{n=1}^{\infty} \left(\frac{n+3}{n!}\right) x^{n+2}$.

[11] 4. Find, in **explicit** form , a one parameter family of solutions for the differential equation

$$x\frac{dy}{dx} + (1+x)y = e^{-x}\sin 2x.$$

[15] 5. Find a 2 -parameter family of solutions for the differential equation

$$2\sqrt{x}\,y'' = (y')^2.$$

Is there any singular solution? Explain.

Answers by Dawit (ydawit@yahoo.com)

1. $2 + \frac{1}{4}(x-1)^2 - \frac{1}{4}(x-1)^3$

2. $5/3$

3. $x^2(x+3)e^x - 3x^2$, $-\infty < x < \infty$

4. $\frac{e^{-x}}{2x}(D - \cos 2x)$ $(D = 2C)$

5. $y(x) = 2\ln|\sqrt{x}+C| - 2\sqrt{x} + D$

$y =$ Constant, is a Singular Solution of the above 2-parameter family of Solutions