

MATH 2130 Problem Workshop 5

1. Find the maximum and minimum values of the function $f(x, y) = x^2 - y^2$ on the region $x^2 + y^2 \leq 1$.
2. Find the maximum value of the function $f(x, y) = xy(3 - x - 2y)$ on the triangle R bounded by the positive axes and the line $x + y = 1$.
3. Find the maximum value of the function $f(x, y) = x^2 - y^2 + 2x + \frac{9y}{2}$ on the region R bounded by $x = 1 - y^2, x = 0$.
4. Evaluate the double iterated integral

$$\int_{-2}^0 \int_0^{-x} \sqrt{y-x} dy dx.$$

5. Evaluate the double integral of $f(x, y) = x^3 y^3 - 3xy^2 + y$ over the region bounded by the curves $y = -x^2, y = x^2 - 1$.
6. Evaluate the double iterated integral

$$\int_{-2}^0 \int_{-3x}^6 e^{y^2} dy dx.$$

7. Evaluate the double integral

$$\iint_R \frac{1}{y-1} dy dx$$

where R is the region bounded by the curves $y = 2x, y = x, x = 2, x = 3$.

8. Find the volumes of the solids of revolution when the area bounded by the curves

$$y = 2x - x^2, \quad y = x$$

is rotated about the lines (a) $x = 3$ (b) $y = 1$ (c) $x + y = -1$.

9. A triangular plate has sides with lengths 3, 4 and 5 metres. It is submerged vertically in oil with density 950 kilograms per cubic metre. The side of length 3 metres is vertical, the side of length 4 is horizontal and the uppermost vertex is 1 metre below the surface of the oil. Find the force due to oil pressure on each side of the plate.

Answers:

1. $1, -1$
2. $2/3(\sqrt{3})$
3. $65/16$
4. $16(4 - \sqrt{2})/15$
5. $-\sqrt{2}/3$
6. $(e^{36} - 1)/6$
7. $(5/2) \ln 5 - (3/2) \ln 3 - 2 \ln 2$
8. (a) $5\pi/6$, (b) $2\pi/15$ (c) $7\sqrt{2}\pi/20$
9. $1.68 \times 10^5 N$