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[5] **1.** Evaluate the following definite integral. Show all your work and simplify your answer. (Note: your answer has to be given in the form $A\sqrt{2}+B$, where A and B are rational numbers.)

$$\int_0^1 x\sqrt{2-x}\,dx$$

$$\int_0^1 x\sqrt{2-x}\,dx = \left\{\begin{array}{l} u=2-x \leftrightarrow x=2-u \\ dx=-du \\ x=0 \rightarrow u=2 \\ x=1 \rightarrow u=1 \end{array}\right\} =$$

$$= \int_2^1 (2-u)\sqrt{u}\,(-1)\,du = \int_1^2 (2-u)\sqrt{u}\,du =$$

$$= \int_1^2 \left(2u^{\frac{1}{2}} - u^{\frac{3}{2}}\right) du = \left(2\cdot\frac{2}{3}u^{\frac{3}{2}} - \frac{2}{5}u^{\frac{5}{2}}\right)\Big|_1^2 =$$

$$= \left(\frac{4}{3}2^{\frac{3}{2}} - \frac{2}{5}\cdot 2^{\frac{5}{2}}\right) - \left(\frac{4}{3} - \frac{2}{5}\right) =$$

$$= \left(\frac{4}{3}\cdot 2\sqrt{2} - \frac{2}{5}\cdot 4\sqrt{2}\right) - \frac{20-6}{15} =$$

$$= \left(\frac{8}{3} - \frac{8}{5}\right)\sqrt{2} - \frac{14}{15} =$$

$$= \frac{16}{15}\sqrt{2} - \frac{14}{15}$$

[5] **2.** Evaluate the following indefinite integral. Show all your work and simplify your answer.

$$\int 3^x \cdot e^x dx$$

$$\int 3^x \cdot e^x dx = \int e^{x \ln 3} \cdot e^x dx = \int e^{x \ln 3 + x} dx =$$

$$= \int e^{x(\ln 3+1)} dx = \left\{ \begin{array}{l} u = (\ln 3+1)x \\ dx = \frac{1}{\ln 3+1} du \end{array} \right\} =$$

$$= \frac{1}{\ln 3+1} \int e^u du = \frac{e^u}{\ln 3+1} + C = \frac{e^{(\ln 3+1)x}}{\ln 3+1} + C =$$

$$= \frac{3^x \cdot e^x}{\ln 3+1} + C.$$

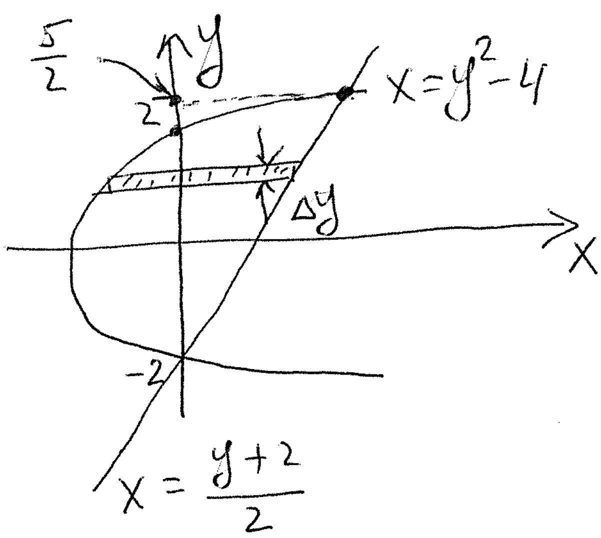
[5] **3.** Set up (but do not evaluate) a definite integral to compute the length of the curve $y = x \ln x + \sin(\cos x)$ from $x = 1$ to $x = 3$.

$$L = \int_1^3 \sqrt{1 + [f'(x)]^2}\, dx, \text{ where } f(x) = x \ln x + \sin(\cos x)$$

$$f'(x) = \ln x + 1 + \cos(\cos x) \cdot (-\sin x)$$

$$\therefore L = \int_1^3 \sqrt{1 + [\ln x + 1 - \sin x \cdot \cos(\cos x)]^2}\, dx$$

[5] **4.** Set up (but do not evaluate) a definite integral (or a sum of integrals) to compute the area of the region bounded by the curves $x = y^2 - 4$ and $y = 2x - 2$.



$$\begin{cases} x = y^2 - 4 \\ x = \frac{y+2}{2} \end{cases} \Rightarrow 2y^2 - 8 = y + 2 \Leftrightarrow$$

$$\Leftrightarrow 2y^2 - y - 10 = 0$$

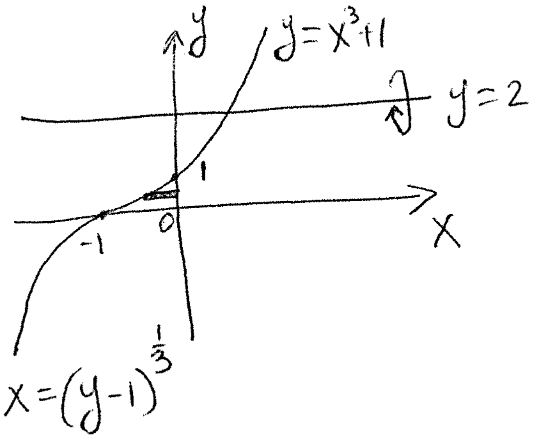
$$\Leftrightarrow y = \frac{1 \pm \sqrt{1 + 4 \cdot 2 \cdot 10}}{4} = \frac{1 \pm 9}{4}$$

$$\therefore y = \frac{5}{2} \text{ or } y = -2.$$

$$A = \int_{-2}^{5/2} \left[y^2 - 4 - \frac{y+2}{2} \right] dy$$

5. Set up (but do not evaluate) definite integrals to determine the volume of the solid of revolution obtained when the region bounded by $y = x^3 + 1$, $x = 0$ and $y = 0$ is revolved about the line $y = 2$, using

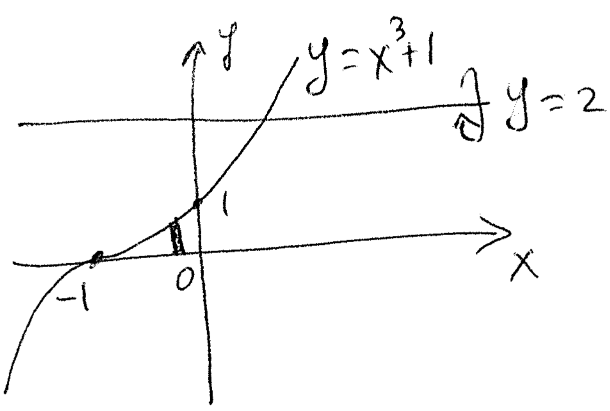
[5] (a) the "cylindrical shells" method



$$V = \int_0^1 \left[0 - (y-1)^{\frac{1}{3}}\right] \cdot 2\pi(2-y)\,dy$$

$$= \int_0^1 (1-y)^{\frac{1}{3}} \cdot 2\pi(2-y)\,dy$$

[5] (b) the "washers" method



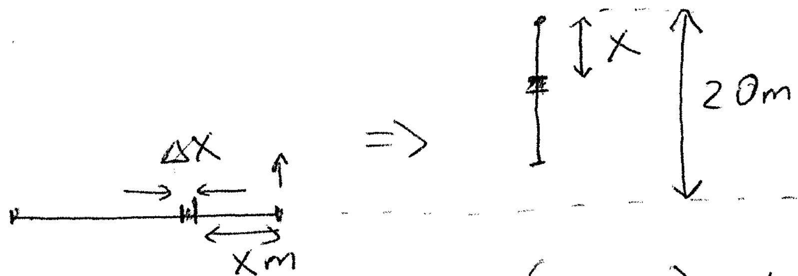
$$V = \int_{-1}^{0} \left(\pi R_x^2 - \pi r_x^2\right) dx =$$

$$= \int_{-1}^{0} \left[\pi \cdot 2^2 - \pi\left(2 - (x^3+1)\right)^2\right] dx =$$

$$= \int_{-1}^{0} \left[4\pi - \pi(1-x^3)^2\right] dx$$

[10] **6.** Determine the work required to lift one end of a chain of length 15 metres and mass 30 kilograms to the top of a 20 metre building (neglect friction).

Approach 1

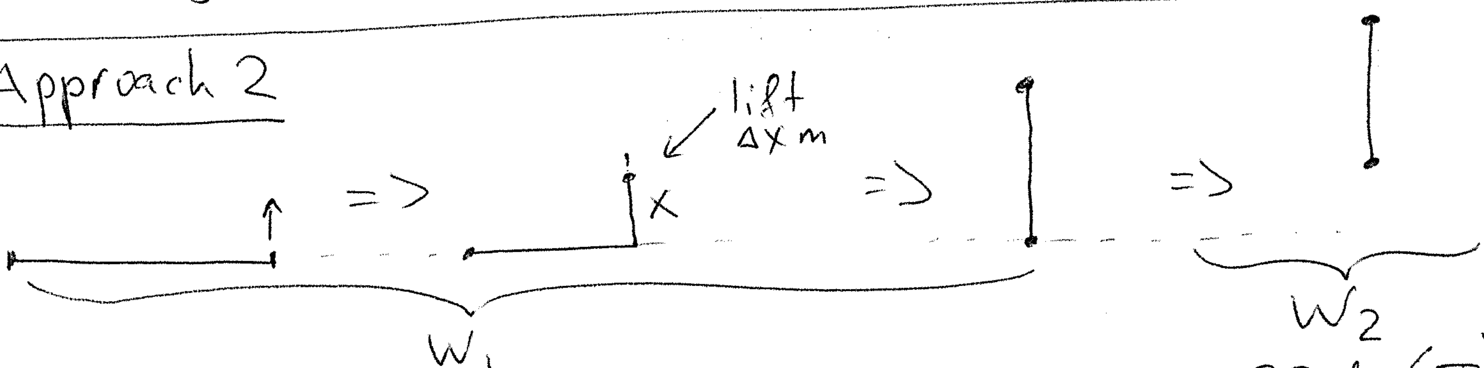


$$W_\Delta = mg\cdot(20-x) = \Delta x\cdot\rho\cdot g(20-x) = \Delta x\cdot\frac{30}{15}\cdot g(20-x)$$

$$\therefore W = \int_0^{15} 2g(20-x)\,dx = 2g\left(20x-\frac{1}{2}x^2\right)\Big|_0^{15} =$$

$$= 2g\left(20\cdot 15-\frac{1}{2}\cdot 15^2\right) = 375g = 375\cdot 9.81\,(J)$$

Approach 2



$$W_2 = (\text{mass of chain})\cdot g\cdot 5 = 30\cdot g\cdot 5 = 150g\ (J)$$

$$W_1 = \int_0^{15} \underbrace{\rho g x}_{\text{mass of hanging part}}\cdot\,dx = \int_0^{15} 2gx\,dx =$$

$$= g\left(x^2\Big|_0^{15}\right) = 225g$$

$$\therefore W = W_1 + W_2 = 225g + 150g = 375g\,(J)$$