Math 2130 - Engineering Mathematical Analysis 1

Tutorial § 11.1 – 11.4

In the question numbers below, the first two parts correspond to the section number from the text. For example, question 11.4.2 refers to the material in section 11.4.

Answers only are given at the end of this Tutorial. Detailed solutions will be discussed during each Tutorial (Lab) session.

11.1.1 Calculate the distance between the points: $(2,0,-3)$ and $(-1,-2,4)$.

11.2.1 Sketch the following surfaces and give the name of each surface:

(a) $z = x+1$; (b) $z = y^2+1$;
(c) $x^2 - y + z^2 = 0$; (d) $x^2 - y^2 - z^2 = 0$;
(e) $x^2 - y^2 + z^2 = 4$; (f) $x^2 + y^2 + z^2 - 2x + 4y - 6z + 10 = 0$ (complete the square).

11.2.2 In each of the following, we have a curve which is the intersection of a paraboloid and a plane. Find the equation of the projection of this curve onto the *xy*-plane and describe the resulting projection. If appropriate, complete the square:

(a) $z = x^2 + y^2$ and $z = 4$;
(b) $z = x^2 + y^2$ and $z = 4x - 2y - 4$.

11.3.1 Calculate the *unit* vector which points from the point $(2,1,3)$ to $(3,-1,5)$.

11.4.1 (a) Show that the angle between the vectors $(1, 0)$ and $(1, 1)$ is $\pi/4$.

(b) Show that the same is *not* true for the angle between the vectors (1, 0, 0) and (1, 1, 1). Determine an expression for this angle.

11.4.2 Let $\boldsymbol{u} = (2,1,3)$ and $\boldsymbol{v} = (1,-3,-1)$.
(a) Calculate $\boldsymbol{u}\cdot\boldsymbol{v}$ and $\boldsymbol{u}\times\boldsymbol{v}$; show that $\boldsymbol{u}\times\boldsymbol{v}$ is perpendicular to each of $\boldsymbol{u}$ and $\boldsymbol{v}$.
(b) Can you tell if the angle between $\boldsymbol{u}$ and $\boldsymbol{v}$ is acute or obtuse?
(An angle θ, with $0 \le \theta \le \pi$, is *acute* if $0 \le \theta < \frac{\pi}{2}$, and it is *obtuse* if $\frac{\pi}{2} < \theta \le \pi$).

Answers: 11.1.1: $\sqrt{62}$. 11.2.1 (These will be given on a separate sheet).

11.2.2 (a): $x^2 + y^2 = 4$, the circle, centre at (0, 0), radius 2.

11.2.2 (b): $(x-2)^2 + (y+1)^2 = 1$, the circle, centre at $(2,-1)$, radius 1.

11.3.1: $\frac{1}{3}(1,-2,2)$. 11.4.1 (a): $\cos(\theta) = \frac{1}{\sqrt{2}}$; (b): $\cos(\theta) = \frac{1}{\sqrt{3}} \ne \frac{1}{\sqrt{2}}$; $\theta = C\text{os}^{-1}\left(\frac{1}{\sqrt{3}}\right)$.

11.4.2 (a): $\boldsymbol{u}\cdot\boldsymbol{v} = -4$; $\boldsymbol{u}\times\boldsymbol{v} = (8,5,-7)$; $(\boldsymbol{u}\times\boldsymbol{v}).\boldsymbol{u} = (\boldsymbol{u}\times\boldsymbol{v}).\boldsymbol{v} = 0$.

11.4.2 (b): The angle is obtuse ($\boldsymbol{u}\cdot\boldsymbol{v} < 0$ and thus, $\cos(\theta) < 0$).

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Tutorial § 11.5 – 11.6

Answers only are given at the end of this Tutorial. Detailed solutions will be discussed during the Tutorial (Lab) sessions.

11.5.1 Find the equation of the plane through the points: $(3,0,2)$, $(2,1,3)$ and $(1,3,7)$.

11.5.2 Find parametric and symmetric equations of the line that passes through the point $(3,2,1)$ and is perpendicular to the plane: $2x+z=4$.

11.5.3 Find the equation of the plane that contains the line: $x-3=\dfrac{y-1}{7}=\dfrac{z+3}{2}$, and passes through the point $(2,2,-1)$.

11.6.1 Determine the component of $\boldsymbol{v}=(2,-4,1)$ in the direction of the perpendicular to the plane: $3x-y-2z=1$, which points in the direction of increasing values of z.

11.6.2 (a) Show that the line: $\dfrac{x-2}{2}=\dfrac{y+2}{3}=\dfrac{z-1}{4}$ is parallel to the plane: $x-2y+z=3$.
(b) Find the distance between the above line and plane.

11.6.3 (a) Find the distance from the point P $(2,1,-4)$ to the line: $\{\, x=3+2t,\;\; y=2+t,\;\; z=-1-3t \,\}$.
(b) Find the point Q in the above line such that PQ is perpendicular to the line.

11.6.4 Determine the distance between the skew lines: $\{\, x=3+2t,\;\; y=-1-t,\;\; z=2-3t \,\}$ and $\{\, x=2-s,\;\; y=1+2s,\;\; z=4+s \,\}$.

Answers: 11.5.1: $2x+3y-z=4$.

11.5.2: $\{\, x=3+2t,\;\; y=2,\;\; z=1+t \,\}$; $\left\{\dfrac{x-3}{2}=z-1,\;\; y=2\right\}$.

11.5.3: $3x-y+2z=2$.

11.6.1: $\dfrac{-8}{\sqrt{14}}$.

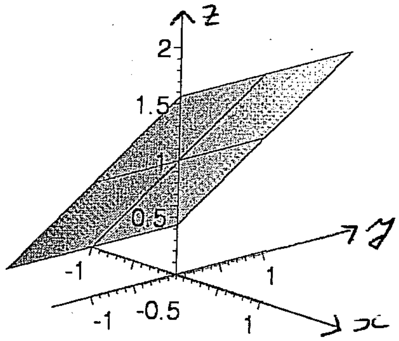
11.6.2: (a): Hint: Show that a direction vector for the line is perpendicular to the normal to the plane.
(b): $\dfrac{4}{\sqrt{6}}$.

11.6.3: (a): $\dfrac{\sqrt{413}}{7}$. (b): $\left(\dfrac{27}{7},\dfrac{17}{7},\dfrac{-16}{7}\right)$.

11.6.4: $\dfrac{3}{\sqrt{35}}$.

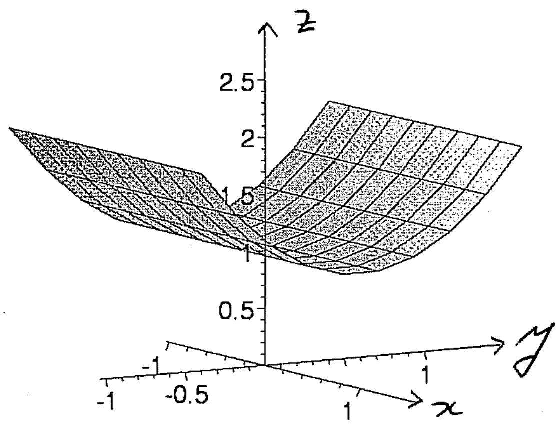
Sketches for question 11.2.1

(a)



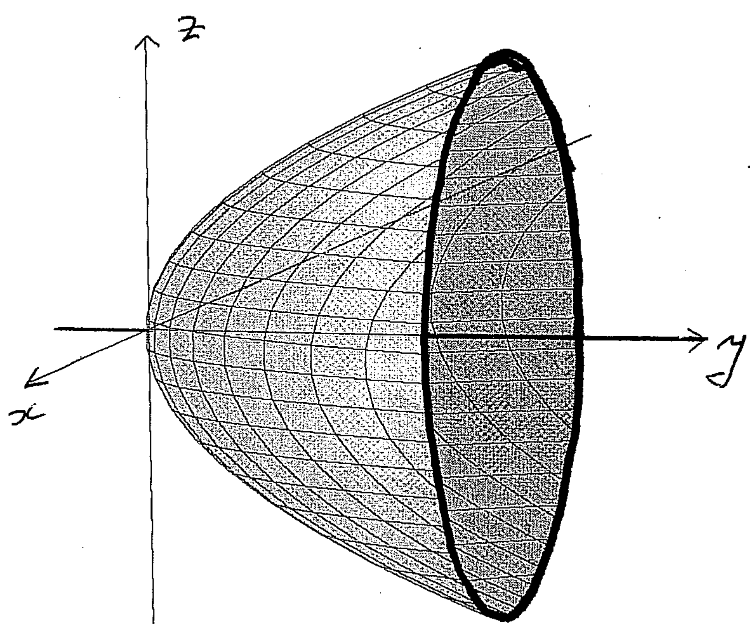
$z = x + 1$. Plane.

(b)



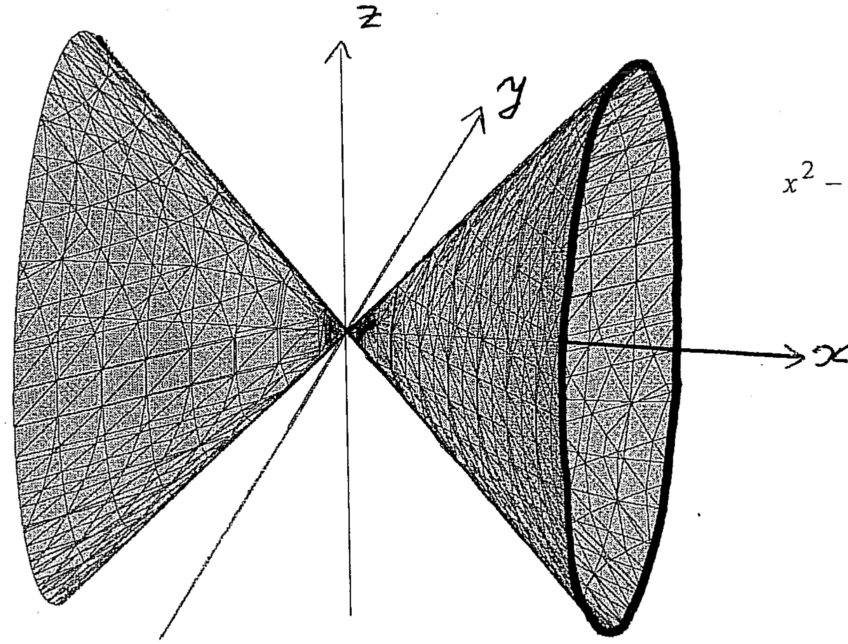
$z = y^2 + 1$. Parabolic cylinder.

(c)



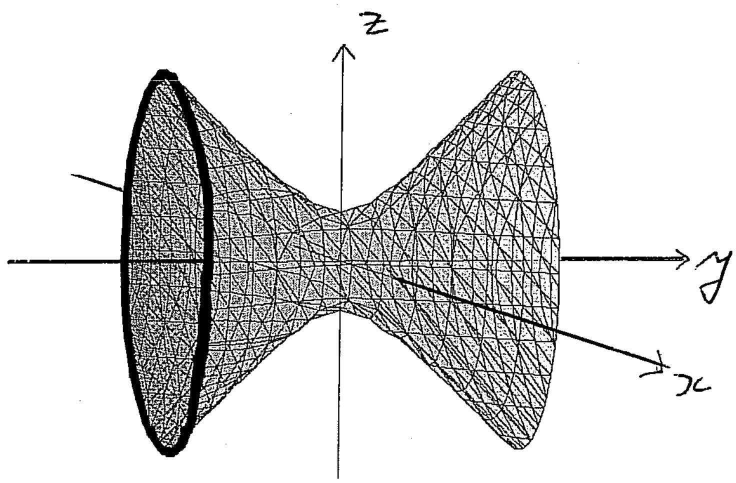
$x^2 - y + z^2 = 0$. Circular paraboloid.

(d)



$x^2 - y^2 - z^2 = 0$. Circular cone.

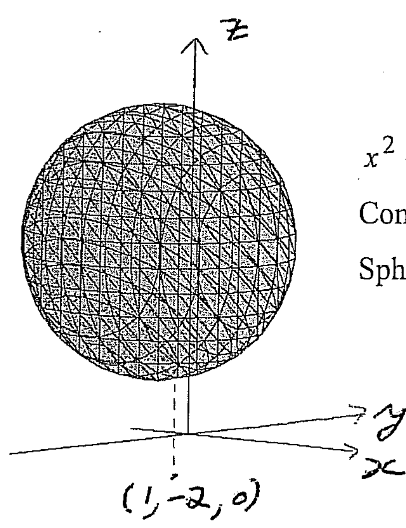
(e)



$x^2 - y^2 + z^2 = 4$.

Circular hyperboloid of one sheet.

(f)



$x^2 + y^2 + z^2 - 2x + 4y - 6z + 10 = 0$.

Completing the square: $(x-1)^2 + (y+2)^2 + (z-3)^2 = 4$

Sphere, with centre at $(1,-2,3)$, radius is 2.