Concavity and points of inflection

4.4.1

(a) $f'(x) = 2x \ln x + x^2 \frac{1}{x} = x(2\ln x + 1) = 2x\left(\ln x + \frac{1}{2}\right)$

domain of $f$: $x > 0$. $\ln x + \frac{1}{2} = 0$ $x = e^{-\frac{1}{2}}$ - critical point.

$f''(x) = 2\left(\left(\ln x + \frac{1}{2}\right) + x \cdot \frac{1}{x}\right) = 2\left(\ln x + \frac{3}{2}\right)$

$f''(e^{-\frac{1}{2}}) = 2\left(\ln(e^{-\frac{1}{2}}) + \frac{3}{2}\right) = 2 > 0$ - $\underline{\text{relative min @ } x = e^{-\frac{1}{2}}}$

(b) $f'(x) = e^{x^3 + x^2 - x + 5}(3x^2 + 2x - 1) = e^{x^3 + x^2 - x + 5}(3x - 1)(x + 1)$

$= 3e^{x^3 + x^2 - x + 5}\left(x - \frac{1}{3}\right)(x + 1)$ $x = \frac{1}{3}$, $x = -1$ - critical points.

$f''(x) = e^{x^3 + x^2 - x + 5}(3x^2 + 2x - 1)^2 + e^{x^3 + x^2 - x + 5}(6x + 2)$

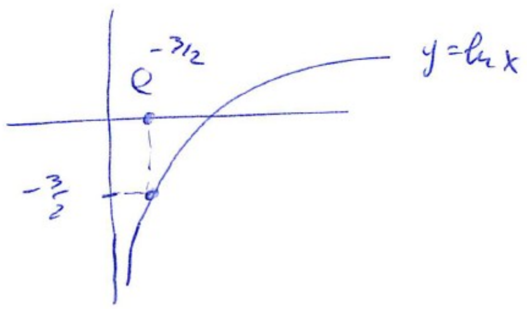
$f''\left(\frac{1}{3}\right) = f\left(\frac{1}{3}\right) \cdot (0)^2 + f\left(\frac{1}{3}\right)(2 + 2) > 0$ - $\underline{\text{rel. min @ } x = \frac{1}{3}}$

$f''(-1) = f(-1) \cdot (0)^2 + f(-1)(-6 + 2) < 0$ - $\underline{\text{rel. max @ } x = -1}$

$f(x) = e^{x^3 + x^2 - x + 5} > 0$ - was used

4.4.2

$\ln x + \frac{3}{2} = 0$ when $x = e^{-3/2}$




As $\ln x$ is increasing,

$\ln x + 3/2 > 0$ for $x \in (0, e^{-3/2})$

$\ln x + 3/2 < 0$ for $x \in (e^{-3/2}, \infty)$

So $f$ is concave up on $(0, e^{-3/2}]$, concave down on $[e^{-3/2}, \infty)$.

$f(e^{-3/2}) = e^{-3}\ln(e^{-3/2}) = -\frac{3}{2}e^{-3}$

$\left(e^{-3/2}, -\frac{3}{2}e^{-3}\right)$ is the only inflection point.