Velocity and acceleration

4.8.1

(a) $V(t) = x'(t) = 3t^2 - 12t + 9 = 3(t^2 - 4t + 3) = 3(t-1)(t-3)$

crit. points of v: $\quad V'(t) = 6t - 12 = 0$

$\qquad\qquad\qquad\qquad\qquad t = 2 \quad$ does belong to $[0,3]$

$V(2) = 3 \cdot 1 \cdot (-1) = -3$

end points:

$V(0) = 3 \cdot (-1) \cdot (-3) = 9 \qquad$ Ans: max. vel. is 9

$V(3) = 3 \cdot 2 \cdot 0 = 0 \qquad\qquad$ min. vel. is -3

(b) $\quad S(t) = |V(t)| = 3|(t-1)(t-3)|$

Sign of v:

$$\begin{array}{ccccc} & + & - & + & \\ \hline & \overset{\displaystyle |}{1} & & \overset{\displaystyle |}{3} & \end{array}$$

$S(t) = 3(t-1)(t-3), \quad t \in [0,1]$

$S(t) = -3(t-1)(t-3), \quad t \in [1,3]$.

On $[0,1]$: $\quad (S(t))' = (V(t))'$ which vanishes at $t=2 \notin [0,1]$.

endpoints only: $\quad S(0) = V(0) = 9, \quad S(1) = V(1) = 0$.

On $[1,3]$ $\quad (S(t))' = -(V(t))'$ which vanishes where $(V(t))'$

$\qquad\qquad\qquad\qquad\qquad\qquad$ does, i.e. at $t=2$.

$S(1) = 0$
$S(3) = -V(3) = 0 \qquad\qquad\quad S(2) = -3(\cdot 1)(-1) = 3$.

$\qquad\qquad\qquad\qquad$ Ans: max. speed 9

$\qquad\qquad\qquad\qquad\qquad$ min. speed 0

(c)　$a(t) = v'(t) = 6t - 12$

$a'(t) = 6 \neq 0$, no C.P.

$a(0) = -12$

$a(3) = 6$

Ans : max. accel. 6

min. accel. -12