Term Test 1

DATE: October 8, 2009
COURSE: MATH 2132

TIME: 70 minutes
EXAMINER: G.I. Moghaddam

Answers by Dawit y
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[9] 1. (a) Determine whether the sequence of numbers $\left\{\dfrac{1+\cos\sqrt{n}}{\sqrt{n+1}}\right\}_{n=1}^{\infty}$ is convergent or divergent. If it converges, find the limit.

(b) The sequence of functions $\left\{\dfrac{x^2}{n}+\dfrac{(x-1)n^2-x^2}{(1-x)n^2+8}\right\}_{n=1}^{\infty}$ is defined on the interval $(-\infty,\infty)$. Determine whether the sequence is convergent or divergent. If it converges, find the limit function.

[12] 2. Let $f(x)=\dfrac{4x}{1-4x}$ for $-\dfrac{1}{4}<x\le\dfrac{1}{8}$. It is given that $f^{(n)}(x)=\dfrac{4^n\,n!}{(1-4x)^{n+1}}$ where $n\ge 1$.

(a) Find the first 3 terms of the Maclaurin series of $f(x)$.

(b) Find the n^{th}-remainder (i.e. $R_n(0,x)$).

(c) Show that $\lim_{n\to\infty} R_n(0,x)=0$ *only* for the case $x<0$.

[8] 3. Find the sum and the interval of convergence of the series

$$\sum_{n=1}^{\infty}(-1)^n 2^{4n}\,(x-\frac{1}{2})^n$$

[8] 4. Find the radius of convergence and the open interval of convergence for the series $\displaystyle\sum_{n=0}^{\infty}\frac{(-1)^n n!}{2^n\,(1\cdot 4\cdot 7\cdot 10\cdot\cdots\cdot(3n+1))}\,x^{3n}$.

[13] 5. (a) Find the Maclaurin series of $f(x)=\dfrac{1}{1+2x}$. What is the interval of convergence ?

(b) Find the Maclaurin series of $g(x)=\dfrac{-2(x^2+1)}{(1+2x)^2}$. Express your answer in sigma notation and simplify as much as possible. Determine its open interval of convergence. (Hint : you may use part (a).

1) a. 0 b. $\begin{cases} -1, & -\infty<x<\infty,\ x\neq 1 \\ -\frac{1}{8}, & x=1 \end{cases}$

2) a. $4x+16x^2+64x^3$

b. $R_n(0,x)=\dfrac{4^{n+1}x^{n+1}}{(1-4z_n)^{n+2}}$

(z_n is between 0 and x)

c) $\lim_{n\to\infty}|R_n(0,x)|<\lim_{n\to\infty}|4x|^{n+1}=0$

Since, $-\frac{1}{4}<x<z_n<0$

$0<-4z_n<-4x<1$

$\frac{1}{2}<\frac{1}{1-4z_n}<1$, $|4x|<1$

3) $\dfrac{8-16x}{16x-7}$, $\frac{7}{16}<x<\frac{9}{16}$

4) $\sqrt[3]{6}$, $-\sqrt[3]{6}<x<\sqrt[3]{6}$

5) a. $\displaystyle\sum_{n=0}^{\infty}(-1)^n 2^n x^n$, $-\frac{1}{2}<x<\frac{1}{2}$

b. $\displaystyle -2+8x+\sum_{n=2}^{\infty}(-1)^{n+1}2^{n-1}(5n+3)x^n$

$-\frac{1}{2}<x<\frac{1}{2}$