UNIVERSITY OF MANITOBA

DATE: June 12, 2007
54
COURSE: MATH 1210
EXAMINATION: Classical and Linear Algebra

FINAL EXAMINATION
TITLE PAGE
TIME: 2 hours
EXAMINER: M. Davidson

FAMILY NAME: (Print in ink) ____________________

GIVEN NAME(S): (Print in ink) ____________________

STUDENT NUMBER: ____________

SEAT NUMBER: ____________

SIGNATURE: (in ink) ____________________
(I understand that cheating is a serious offense)

INSTRUCTIONS TO STUDENTS:

This is a 2 hour exam. **Please show your work clearly.**

No texts, notes, or other aids are permitted. There are no calculators, cellphones or electronic translators permitted.

This exam has a title page, 9 pages of questions and also 2 blank pages for rough work. Please check that you have all the pages. You may remove the blank pages if you want, but be careful not to loosen the staple.

The value of each question is indicated in the lefthand margin beside the statement of the question. The total value of all questions is 100 points.

Answer all questions on the exam paper in the space provided beneath the question. If you need more room, you may continue your work on the reverse side of the page, but CLEARLY INDICATE that your work is continued.

Question	Points	Score
1	32	
2	12	
3	10	
4	10	
5	10	
6	12	
7	14	
Total:	100	

1. The following are short answer questions.

[2] (a) What is the Cartesian form of $16e^{\frac{-\pi}{4}i}$?

[4] (b) What does Descartes' rule of signs imply about the polynomial $P(x) = 5x^4 - 4x^3 + 2x^2 + 7x - 13$?

[3] (c) Use the adjoint to find the inverse of the matrix $A = \begin{pmatrix} 3 & 5 \\ 1 & 4 \end{pmatrix}$.

[2] (d) Let T be the transformation from $\mathbb{R}^4$ to $\mathbb{R}^4$ defined by $T(\tilde{x}) = A\tilde{x}$ where $A = \begin{pmatrix} 1 & 2 & -1 & 3 \\ 2 & 4 & 7 & 1 \\ -1 & 7 & 6 & -2 \\ 3 & 1 & -2 & 1 \end{pmatrix}$ How many eigenvalues does T have? How many of them are real?

---

[2] (e) Write $3 - 3i$ in polar form.

[3] (f) Use the remainder theorem to find the remainder when the polynomial $P(x) = 3x^3 + 2x^2 - x + 3$ is divided by $x - 2i$.

[2] (g) Write the following in sigma notation (do not evaluate) :
$1 - 3 + 5 - 7 + 9 - 11 + 13 - 15$

[4] (h) Are the vectors $\{(1, 1, 0), (2, 3, 4), (-1, 2, 6)\}$ linearly dependent or linearly independent. Justify your answer.

---

[3] (i) Given $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$ evaluate the following :

$$\sum_{i=1}^{43}(i+17)$$

[2] (j) Given $A = \begin{pmatrix} 0 & -2 \\ 1 & 2 \end{pmatrix}$ and $B = \begin{pmatrix} 3 & -1 & 5 \\ 1 & 2 & 1 \end{pmatrix}$ then $AB = \begin{pmatrix} -2 & -4 & -2 \\ 5 & 3 & 7 \end{pmatrix}$.
What is $B^T A^T$?

[2] (k) Are the vectors $\{(1,3),(2,-5),(6,7)\}$ linearly dependent or linearly independent. Justify your answer.

[3] (l) If $z = 7 + 7i$, what is z^3? (hint: this may be easier using DeMoivre's theorem.)

---

[12] 2. Use mathematical induction to show that for all $n \geq 1$ that

$$1 + 3 + 5 + \ldots + (4n - 1) = (2n)^2 \,.$$

---

[10] 3. Find all solutions to $w^3 = -4\sqrt{2} + 4\sqrt{2}i$. Give your answers in exponential form.

---

[10] 4. Find all roots of the polynomial $P(x) = x^3 - 5x^2 + 11x - 15$. (hint: Start by considering the rational roots)

[10] 5. Use Cramer's rule to find the solution to the system of equations:

$$\begin{array}{rcrcrl} 2x & + & y & + & 2z = & 1 \\ 3x & - & y & + & 4z = & -8 \\ 5x & + & 4y & + & 3z = & 11 \end{array}$$

---

[12] 6. (a) Find the inverse of the matrix $A = \begin{pmatrix} 1 & 3 & 1 \\ 3 & 10 & 4 \\ 2 & 8 & 5 \end{pmatrix}$.

(b) Use the information from part a to find a solution to :

$$\begin{array}{rcrcrl} x_1 & + & 3x_2 & + & x_3 = & 1 \\ 3x_1 & + & 10x_2 & + & 4x_3 = & -1 \\ 2x_1 & + & 8x_2 & + & 5x_3 = & 3 \end{array}$$

---

[14] 7. Let T be the transformation from $\mathbb{R}^3$ to $\mathbb{R}^3$ defined by $T(\tilde{x}) = A\tilde{x}$ where $A = \begin{pmatrix} -1 & 7 & -7 \\ 0 & 2 & -3 \\ 0 & -4 & 3 \end{pmatrix}$. Find all eigenvalues of T. Find all eigenvectors associated with each eigenvalue of T.