Statistics 5.2220

Fall 2007

Solution to Assignment 4

4-30. a) $SE\ Mean = \frac{\sigma}{\sqrt{n}} = \frac{2.4}{\sqrt{25}} = 0.48$

$P-value = 1-\Phi(z) = 1-\Phi(3.25) = 1-0.999423 = 0.000577$

```
One-Sample Z

Test of mu = 100 vs > 100
The assumed standard deviation = 2.4

                                95%
                              Lower
 N      Mean  SE Mean         Bound     Z         P
25   101.560     0.48       100.770  3.25  0.000577
```

Yes, the null hypothesis can be rejected at the 0.05 level since the p-value = 0.000577 < 0.05.

b) It is a one-sided test.

c) The null hypothesis will be rejected at the 0.05 level since $\mu = 99$ is not included in the 95% lower bound.

No additional calculations are needed since the lower bound calculation is $\bar{x} - z_\alpha\left(\frac{\sigma}{\sqrt{n}}\right)$ has nothing to do with the change of μ_0 in the hypothesis.

d) 95% two-sided CI on the mean

$$\bar{x} - z_{0.05/2}\left(\frac{\sigma}{\sqrt{n}}\right) \le \mu \le \bar{x} + z_{0.05/2}\left(\frac{\sigma}{\sqrt{n}}\right)$$

$$101.560 - 1.96\left(\frac{2.4}{\sqrt{25}}\right) \le \mu \le 101.560 + 1.96\left(\frac{2.4}{\sqrt{25}}\right)$$

$$101.560 - 0.9408 \le \mu \le 101.560 + .9408$$

$$100.6192 \le \mu \le 102.5008$$

e) $P-value = 2[1-\Phi(|Z|)] = 2[1-(\Phi|3.25|)] = 2[1-0.999423] = 0.001154$

4-34. a) The parameter of interest is the true mean ppm of benzene, μ.

H_0: $\mu = 7980$

H_1: $\mu < 7980$

Test statistic is $z_0 = \dfrac{\overline{x} - \mu}{\sigma / \sqrt{n}}$

$\overline{X} = 7906$, $\sigma = 80$

$$z_0 = \frac{7906 - 7980}{80/\sqrt{10}} = -2.925$$

The p-value = $\Phi(-2.925) \cong 0.00172$. Since the p-value < 0.05, we reject the null hypothesis and conclude manufacturers exit water meets federal regulation.

b) $\beta = \Phi\left(-z_{0.01} + \dfrac{7920 - 7980}{80/\sqrt{10}}\right) = \Phi(-4.70) = 0$

c) Set $\beta = 1 - 0.90 = 0.10$

$$n = \frac{(z_\alpha + z_\beta)^2 \sigma^2}{\delta^2} = \frac{(z_{0.01} + z_{0.10})^2 \sigma^2}{(7920 - 7980)^2} \cong \frac{(2.33 + 1.29)^2 (80)^2}{(-60)^2} = 23.29,$$

$n \cong 24.$

d) For $\alpha = 0.01$, $z_\alpha = z_{0.01} = 2.33$

$$\mu \le \bar{x} + z_{0.01}\left(\frac{\sigma}{\sqrt{n}}\right)$$

$$\mu \le 7906 + 2.33\left(\frac{80}{\sqrt{10}}\right)$$

$$\mu \le 7965$$

e) The confidence interval constructed does not contain the value 7980, thus the manufacturer's exit water meets federal regulation using a 99% level of confidence.

4-44. a) $StDev = SE\,Mean \times \sqrt{n} = 0.4673 \times \sqrt{25} = 2.3365$

$$Upper\ Limit\ 95\%CI = \bar{x} + t_{0.025,24}\left(\frac{s}{\sqrt{n}}\right)$$

$$= 92.5805 + 2.064\left(\frac{2.3365}{\sqrt{25}}\right)$$

$$= 93.5450$$

One-Sample T: X

```
Test of mu = 91 vs not = 91


Variable   N     Mean   StDev  SE Mean        95% CI             T      P
X         25  92.5805  2.3365   0.4673  (91.6160, 93.5450)    3.38  0.002
```

The null hypothesis can be rejected at the 0.05 level since the p-value = 0.002 < 0.05.

b) It is a two-sided test.

c) The null hypothesis $H_0 : \mu = 90$ will be rejected at the 0.05 level since 90 is not included in the 95%CI

d) 99% CI

$$\bar{x} - t_{0.005,24}\left(\frac{s}{\sqrt{n}}\right) \le \mu \le \bar{x} + t_{0.005,24}\left(\frac{s}{\sqrt{n}}\right)$$

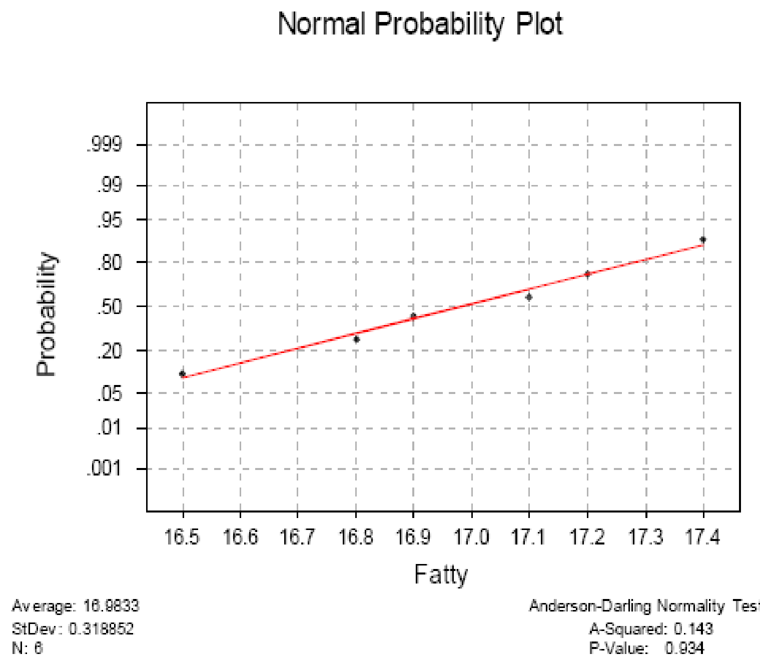
$$92.5805 - 2.797\left(\frac{2.3365}{\sqrt{25}}\right) \le \mu \le 92.5805 + 2.797\left(\frac{2.3365}{\sqrt{25}}\right)$$

$$92.5805 - 1.307 \le \mu \le 92.5805 + 1.307$$

$$91.2735 \le \mu \le 93.8875$$

e) P-value = P(t > 3.38): for degrees of freedom of 24 we obtain 0.001 < P-value < 0.0025

4-50



The normality assumption appears to be satisfied. This is evident by the fact that the data fall along a straight line.

a) The parameter of interest is the true mean level of polyunsaturated fatty acid, μ.

H_0: $\mu = 17$

H_1: $\mu \neq 17$

Test statistic is $t_0 = \dfrac{\overline{x} - \mu}{s / \sqrt{n}}$

$\overline{x} = 16.98 \quad s = 0.319 \quad n = 6$

$$t_0 = \frac{16.98 - 17}{0.319 / \sqrt{6}} = -0.154$$

P-value = 2P(t > 0.154): for degrees of freedom of 5 we obtain 2(0.40) < P-value = 0.80 < P-value. Since the p-value is greater than 0.05, we do not reject the null hypothesis.

b) Using the OC curves on Chart v (a), with $d = \dfrac{0.5}{0.319} = 1.567$, when $\beta \cong 0.1$, n = 7. Therefore, the current sample size of 6 is inadequate.

c) For $\alpha = 0.01$, $t_{\alpha/2,n-1} = t_{0.005,5} = 4.032$

$$\overline{x} - t_{0.005,5}\left(\frac{s}{\sqrt{n}}\right) \le \mu \le \overline{x} + t_{0.005,5}\left(\frac{s}{\sqrt{n}}\right)$$

$$16.98 - 4.032\left(\frac{0.319}{\sqrt{6}}\right) \le \mu \le 16.98 + 4.032\left(\frac{0.319}{\sqrt{6}}\right)$$

$$16.455 \le \mu \le 17.505$$

With 99% confidence, we believe the true mean level of polyunsaturated fatty acid is between 16.455% and 17.505%.

4-60.a) In order to use χ^2 statistic in hypothesis testing and confidence interval construction, we need to assume that the underlying distribution is normal.

1) The parameter of interest is the true variance of the sugar content, σ^2.

2) H_0: $\sigma^2 = 18$

3) H_1: $\sigma^2 \neq 18$

4) $\alpha = 0.05$

5) $\chi_0^2 = \dfrac{(n-1)s^2}{\sigma^2}$

6) Reject H_0 if $\chi_0^2 < \chi^2_{1-\alpha/2,n-1}$ where $\chi_{0.975,9} = 2.70$ or $\chi_0^2 > \chi^2_{\alpha,2,n-1}$ where $\chi^2_{0.025,9} = 19.02$

7) $n = 10$, $s^2 = 16$

$$\chi_0^2 = \frac{(n-1)s^2}{\sigma^2} = \frac{9(16)}{18} = 8$$

8) Since $2.70 < 8 < 19.02$ do not reject H_0 and conclude the evidence indicates the true variance of the sugar content is not significantly different from 18 mg^2 at $\alpha = 0.05$.

b) P-value $= 2P(\chi^2 > 8) \cong 1$ for 9 degrees of freedom

c) 95% confidence interval for σ:

First find a confidence interval for σ^2:

For $\alpha = 0.05$ and $n = 10$, $\chi^2_{\alpha/2,n-1} = \chi^2_{0.025,9} = 19.02$ and $\chi^2_{1-\alpha/2,n-1} = \chi^2_{0.975,9} = 2.70$

$$\frac{9(16)}{19.02} \le \sigma^2 \le \frac{9(16)}{2.70}$$

$$7.57 \le \sigma^2 \le 53.33$$

Take the square root of the endpoints of this interval to find the approximate confidence interval for σ:

$2.75 \le \sigma \le 7.30$

With 95% confidence, we believe the true standard deviation of the sugar content is between 2.75 mg and 7.30 mg.

d) Since the hypothesized value lies within this confidence interval, the null hypothesis cannot be rejected.

4-64.

Test and CI for One Proportion

```
Test of p = 0.3 vs p not = 0.3

Sample   X    N  Sample p         95% CI          Z-Value  P-Value
1       95  250  0.380000  (0.319832, 0.440168)     2.76    0.006
```

a) It is a two-sided test

b) Yes, this test was conducted using the normal approximation to the binomial.
Yes, the normal approximation to the binomial is appropriate since $p = 0.3$ is not extremely close to 0 or 1 and the sample size = 250 is relatively large. Also $np = 250(0.3) = 75 > 5$ and $n(1-p) = 250(1-0.3) = 175 > 5$.

c) Yes, the null hypothesis can be rejected at the 0.05 level since the p-value = 0.006 < 0.05.

d) No, the null hypothesis $H_0: p = 0.04$ cannot be rejected at the 0.05 level since 0.4 is included in 95%CI. No additional calculations are necessary.

e) Approximate 90%CI for p: $\hat{p} = \frac{x}{n} = \frac{95}{250} = 0.38$

$$\hat{p} - z_{0.05}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \le p \le \hat{p} + z_{0.05}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

$$0.38 - 1.64\sqrt{\frac{0.38(1-0.38)}{250}} \le p \le 0.38 + 1.64\sqrt{\frac{0.38(1-0.38)}{250}}$$

$$0.38 - 0.0503 \le p \le 0.38 + 0.0503$$

$$0.3297 \le p \le 0.4303$$

4-68. $n = 50, x = 18, \hat{p} = \frac{18}{50} = 0.36, \alpha = 0.05, z_{\alpha/2} = z_{0.025} = 1.96$

a) 1) The parameter of interest is the true proportion of damage, p.

2) $H_0: p = 0.30$

3) $H_1: p \neq 0.30$

4) $\alpha = 0.05$

5) $z_0 = \frac{x - np_0}{\sqrt{np_0(1-p_0)}}$ or $z_0 = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$; Either approach will yield the same conclusion

6) Reject H_0 if $z_0 < -z_{\alpha/2}$ where $-z_{\alpha/2} = -z_{0.025} = -1.96$ or $z_0 > z_{\alpha/2}$ where $z_{\alpha/2} = z_{0.025} = 1.96$

7) $x = 18$ $n = 50$ $\hat{p} = \frac{18}{50} = 0.36$

$$z_0 = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} = \frac{0.36 - 0.30}{\sqrt{\frac{(0.30)(0.70)}{50}}} = 0.926$$

8) Since -1.96 < 0.926 < -1.96, do not reject the null hypothesis and conclude the percentage of helmets damaged is not significantly different from 30%.

b) $0.36 - 1.96\sqrt{\frac{0.36(0.64)}{50}} \le p \le 0.36 + 1.96\sqrt{\frac{0.36(0.64)}{50}}$

$$0.23 \le p \le 0.49$$

c) P-value $= 2 \cdot P(Z > 0.93) = 2 \cdot (0.176185) = 0.35237$

With 95% confidence, we believe the true proportion of helmets damaged lies within 0.23 and 0.49.

d) $n = \left(\frac{z_{\alpha/2}}{E}\right)^2 \hat{p}(1-\hat{p}) = \left(\frac{1.96}{0.02}\right)^2 0.36(1-0.36) = 2212.76$, $n \cong 2213$

e) $n = \left(\frac{z_{\alpha/2}}{E}\right)^2 p(1-p) = \left(\frac{1.96}{0.02}\right)^2 0.5(1-0.5) = 2401$, $n \cong 2401$