MATH 2130 Test 2 **2012** (Winter) **60 minutes**

Values

6 **1.** Determine whether the limit

$$\lim_{(x,y)\to(0,1)} \frac{x^2+y^2-2y+1}{3x^2-4y^2+8y-4}$$

exists. If the limit exists, find its value; if the limit does not exist, give reasons for its nonexistence.

5 **2.** Are there any values of n for which the function $f(x,y)=(2x+3y)^n$ is harmonic in the xy-plane?

9 **3.** The function $f(x,y,z)=x^2e^{-z}+y$ is defined at every point on the curve

$$x^2+y^2=4, \quad z=x,$$

directed counterclockwise as viewed from the origin. Find the rate of change of the function with respect to distance along the curve at the point $(2,0,2)$.

10 **4.** The equations

$$x^2-y^2=2uv, \quad u^2+v^2=2xy$$

define x and y as functions of u and v. Find $\dfrac{\partial x}{\partial v}$, simplified as much as possible.

10 **5.** If $f(x)$ is a differentiable function, verify that the function $u(x,y)=y\,f(3x^2-4y)$ satisfies the equation

$$\frac{2y}{x}\frac{\partial u}{\partial x}+3y\frac{\partial u}{\partial y}=3u.$$

Answers by Dawit.

1. Limit does not exist. (path or mode of approach dependent)
(Hint: let $y-1=mx$ and show the limit depends on m or
Show that: along the y-axis ($x=0$), the limit approaches $-1/4$ and
along the line $y=1$, > > > $1/3$

2. $n=0$ or $n=1$

3. ± 1

4. $\dfrac{ux+vy}{x^2+y^2}$

5. Hint: let $t=3x^2-4y \Rightarrow u=y\,f(t)$

(do the chain-rule by following the tree-digram)