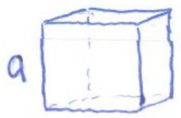
Related rates

4.9.1



If $a=a(t)$ is the length of the edge, then the surface area $A(t)=6(a(t))^2$ and the volume $V(t)=(a(t))^3$.

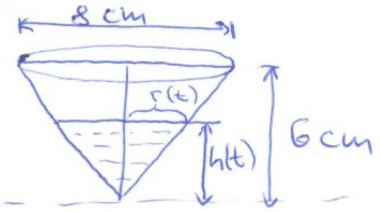
$\frac{dA}{dt}=4\ m^2/s$, $\left.\frac{dV}{dt}\right|_{a=10}$ - ?

$\frac{dA}{dt}=12a(t)\cdot\frac{da}{dt} \Rightarrow \frac{da}{dt}=\frac{\frac{dA}{dt}}{12a}$

$\frac{dV}{dt}=3a^2(t)\frac{da}{dt}=\frac{1}{4}a\cdot\frac{dA}{dt}=a \Rightarrow \left.\frac{dV}{dt}\right|_{a=10}=10\ m^3/s$

Ans the volume is increasing at $10\ m^3/s$.

4.9.2



Let $h(t)$ be the depth of the water in a cup and $r(t)$ be the radius of the top of the water.

$\frac{dV}{dt}=-2\ cm^3/min$, $\left.\frac{dr}{dt}\right|_{h=3}=?$

$V(t)=\frac{1}{3}\pi r^2\cdot h$ $\qquad \frac{r}{4}=\frac{h}{6}$ (by similar triangles)

$\Downarrow$

$h=\frac{3r}{2}$

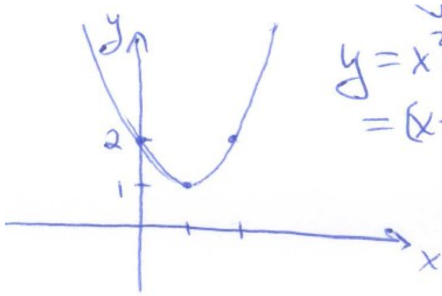
$V(t)=\frac{1}{3}\pi r^2\cdot\frac{3r}{2}=\frac{1}{2}\pi r^3$

$\frac{dV}{dt}=\frac{3}{2}\pi r^2\cdot\frac{dr}{dt}$, when $h=3$, $r=\frac{2h}{3}=2 \Rightarrow$

$\Rightarrow \left.\frac{dr}{dt}\right|_{h=3}=\frac{-2}{\frac{3}{2}\cdot\pi\cdot 2^2}=-\frac{1}{3\pi}\ cm/min$

Ans it's decreasing at $\frac{1}{3\pi}$ cm/min.

4.9.3



$y = x^2 - 2x + 2 =$
$= (x-1)^2 + 1$

Let $D(t)$ be the distance from the particle to a point $(3,0)$.

$$\left.\frac{dx}{dt}\right|_{x=2,\,y=2} = -\frac{1}{2} \ m/s, \quad \left.\frac{dD}{dt}\right|_{x=2,\,y=2} - ?$$

$$D(t) = \sqrt{(y-0)^2 + (x-3)^2} = \sqrt{y^2 + (x-3)^2}$$

$$y(t) = x^2(t) - 2x(t) + 2$$

$$\frac{dD}{dt} = \frac{\cancel{2}y \cdot \frac{dy}{dt} + \cancel{2}(x-3) \cdot \frac{dx}{dt}}{\cancel{2}\sqrt{y^2 + (x-3)^2}}$$

$$\frac{dy}{dt} = 2x \cdot \frac{dx}{dt} - 2\frac{dx}{dt} \Rightarrow \left.\frac{dy}{dt}\right|_{x=2,\,y=2} = 4 \cdot \left(-\frac{1}{2}\right) + 2 \cdot \frac{1}{2} = -1 \ m/s$$

$$\Downarrow$$

$$\left.\frac{dD}{dt}\right|_{x=2,\,y=2} = \frac{2 \cdot (-1) + (-1) \cdot \left(-\frac{1}{2}\right)}{\sqrt{2^2 + (-1)^2}} = -\frac{3}{2\sqrt{5}} \ m/s$$

Ans: it's decreasing at $\frac{3}{2\sqrt{5}}$ m/s.