[14] **1.** In each of the following cases, determine whether or the given limit exists. If not, explain why the limit does not exist. In particular if the trend is to ∞ or $-\infty$, indicate so.

(a) $\lim_{x\to 2} \frac{x^2-x-2}{x^2-4}$

$$= \lim_{x\to 2} \frac{(x-2)(x+1)}{(x-2)(x+2)} = \lim_{x\to 2} \frac{x+1}{x+2} = \frac{3}{4}$$

(b) $\lim_{x\to 1^-} \frac{2x^2-1}{|x-1|}$

$$= \lim_{x\to 1^-} \frac{2x^2-1}{1-x} = \infty \quad \left\{ \frac{1}{0^+} \right.$$

(c) $\lim_{x\to -\infty} \frac{\sqrt{x^2+2}}{3x-1}$

$$= \lim_{x\to -\infty} \frac{\sqrt{x^2}\cdot\sqrt{1+\frac{2}{x^2}}}{3x-1} = \lim_{x\to -\infty} \frac{-\sqrt{1+\frac{2}{x^2}}}{3-\frac{1}{x}}$$

$(|x| = -x \text{ as } x\to -\infty)$

$$= -\frac{1}{3}$$

[8] **2.** Use **limits** to determine the value of a and the value of b so that the function defined by

$$f(x) = \begin{cases} ax & \text{if } x < -1 \\ 2 & \text{if } x = -1 \\ x^2 + b & \text{if } x > -1 \end{cases}$$

is continuous for all real numbers.

f is cont. for $x<-1$ and for $x>-1$ as polynomial.

f is cont. at $x=-1$ if and only if

$$\lim_{x\to -1^-} f(x) = \lim_{x\to -1^+} f(x) = f(-1)$$

$$\lim_{x\to -1^-} f(x) = \lim_{x\to -1^-} (ax) = -a$$

$$\lim_{x\to -1^+} f(x) = \lim_{x\to -1^+} (x^2+b) = 1+b$$

$f(-1) = 2$ $\qquad -a = 1+b = 2$

Ans: $a=-2 \quad b=1$

[8] **3.** Let $f(x) = \sqrt{2-x}$. For any $x < 2$ find $f'(x)$ using **only the definition** of the derivative.

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{\sqrt{2-(x+h)} - \sqrt{2-x}}{h}$$

$$= \lim_{h\to 0} \frac{2-(x+h)-(2-x)}{h\left(\sqrt{2-(x+h)}+\sqrt{2-x}\right)} = \lim_{h\to 0} \frac{-h}{h\left(\sqrt{2-x-h}+\sqrt{2-x}\right)}$$

$$= \lim_{h\to 0} \frac{-1}{\sqrt{2-x-h}+\sqrt{2-x}} = \frac{-1}{2\sqrt{2-x}}$$

[20] **4.** Find $\frac{dy}{dx}$. DO NOT SIMPLIFY YOUR ANSWER.

(a) $y = 5x^3 + \sqrt[3]{x^4} + \frac{1}{x} + \pi^2$

$$\frac{dy}{dx} = 15x^2 + \frac{4}{3}x^{\frac{1}{3}} - \frac{1}{x^2} + 0$$

(b) $y = x\sqrt{3 - x^2}$

$$\frac{dy}{dx} = \sqrt{3-x^2} + x\,\frac{1}{2\sqrt{3-x^2}} \cdot (-2x)$$

(c) $y = \frac{x^7 - 3x^4 + 10}{2 - 5x^2}$

$$\frac{dy}{dx} = \frac{(7x^6 - 12x^3)(2-5x^2) - (x^7 - 3x^4 + 10)(-10x)}{(2-5x^2)^2}$$

(d) $y = ((x-1)^{25} - x^2)^{10}$

$$\frac{dy}{dx} = 10\left((x-1)^{25} - x^2\right)^9 \cdot \left(25(x-1)^{24} - 2x\right)$$

[8] **5.** Find $f'''(x)$ if $f(x) = (2010 - x)^{2010}$. YOU DO NOT NEED TO SIMPLIFY YOUR ANSWER.

$$f'(x) = 2010\,(2010 - x)^{2009} \cdot (-1)$$

$$f''(x) = 2010 \cdot 2009 \cdot (2010 - x)^{2008} \cdot (-1)^2$$

$$f'''(x) = 2010 \cdot 2009 \cdot 2008 \cdot (2010 - x)^{2007} \cdot (-1)^3$$

[12] **6.** A ball is thrown upward from the ground level so that its height in meters after t seconds is given by $y = 10t - 5t^2$.

(a) What is the acceleration of the ball at any time?

$v(t) = 10 - 10t$ [m./s.]

$\underline{a(t) = -10}$ [m./s.²]

(b) How high will the ball go?

$v(t) = 0 \qquad 10 - 10t = 0 \qquad t = 1$ [s.]

$y(1) = 10 \cdot 1 - 5 \cdot 1^2 = \underline{5}$ [m.]

(c) How fast is it moving when it strikes the ground?

$y = 0 \qquad 10t - 5t^2 = 0$

$t = 0$ or $t = 2$

initial

$v(2) = 10 - 10 \cdot 2 = -10$ [m./s.]

$s(2) = |v(2)| = 10$ [m./s.]