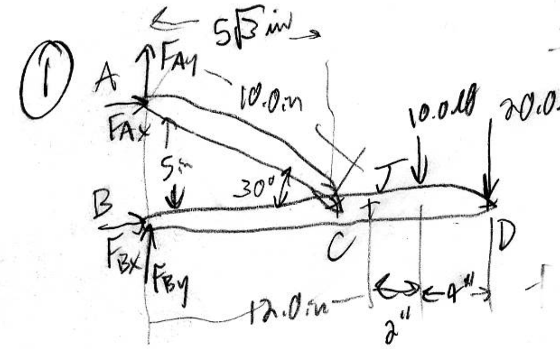
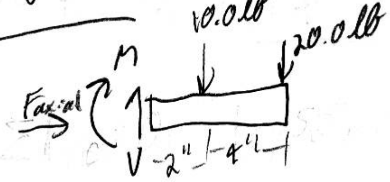
Problem set #8 Solutions

(1)



consider JD:



$$\Sigma F_y = 0 = V - 10\,lb - 20\,lb$$

$$\boxed{V = 30.0\,lb}$$

$$\Sigma F_x = \boxed{0 = F_{axial}}$$

$$\Sigma M_J = 0 = -M - 10.0\,lb(2.00\,in) - 20.0\,lb(6.00\,in)$$

$$\rightarrow \boxed{M = -140\,in \cdot lb}$$

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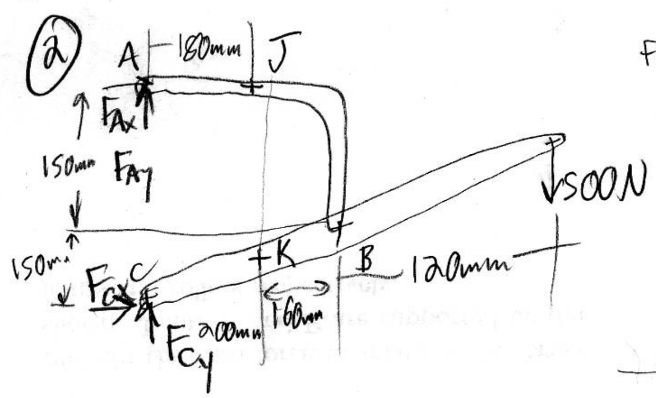
(2)



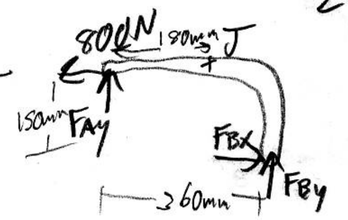
Find internal forces at J, K

body: $\Sigma M_A = 0 = F_{Cx}(300mm) - 500N(480mm)$

$$\rightarrow F_{Cx} = 0 = 800N$$

$$\Sigma F_x = 0 = 800N + F_{Ax} \rightarrow \boxed{F_{Ax} = 800\,N}$$

For AB:



$$\Sigma M_B = 0 = 800N(150mm) - F_{Ay}(360mm)$$

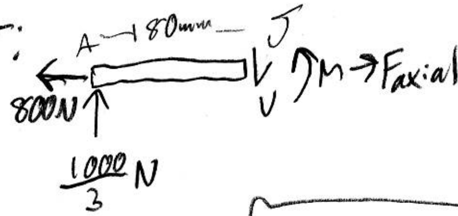
$$\rightarrow F_{Ay} = 800N\left(\frac{150}{360}\right) \quad \boxed{F_{Ay} = \frac{1000}{3}N}$$

$$\Sigma F_y = 0 = F_{Ay} + F_{By} \rightarrow \boxed{F_{By} = -\frac{1000}{3}N}$$

$$\Sigma F_x = 0 = -800N + F_{Bx}$$

$$\rightarrow \boxed{F_{Bx} = 800N}$$

$\boxed{2a}$: internal forces at J:



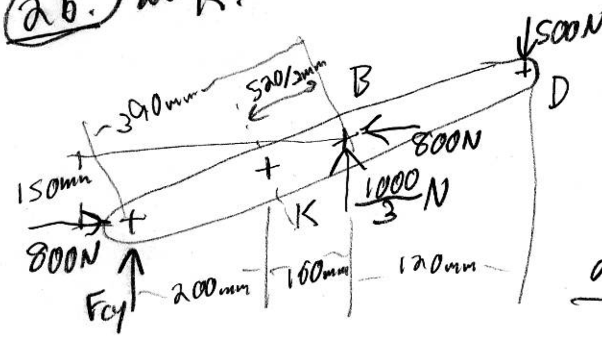
For AJ:

$\Sigma F_x = 0 = -800N + F_{axial}$ $\rightarrow$ $\boxed{F_{axial} = 800N}$

$\Sigma F_y = 0 = \frac{1000}{3}N - V$ $\rightarrow$ $\boxed{V = \frac{1000}{3}N \; (333.\bar{3}N)}$

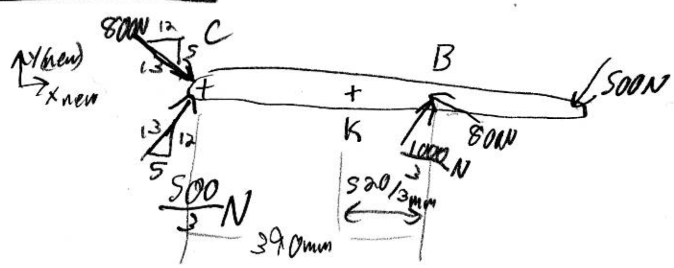
$\Sigma M_J = 0 = M - \frac{1000}{3}N(180\,mm)$ $\rightarrow$ $\boxed{M = 60\,N\cdot m}$

2b: at K:

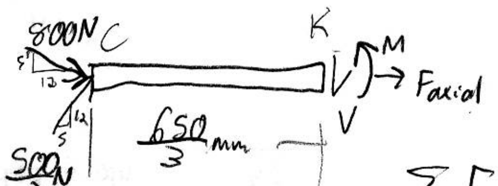


$\Sigma F_y = 0 = F_{cy} + \frac{1000}{3}N - 500N$ $\rightarrow$ $F_{cy} = \frac{500}{3}N$

define new x, y:



so for CK:



$\Sigma F_{y_{new}} = 0 = -V - \left(\frac{5}{13}\right)800N + \left(\frac{12}{13}\right)\frac{500}{3}N$

$\rightarrow$ $\boxed{V = -\frac{2000}{13}N}$ $(-153.85N)$

$\Sigma M_C = 0 = -V\left(\frac{650}{3}mm\right) + M$

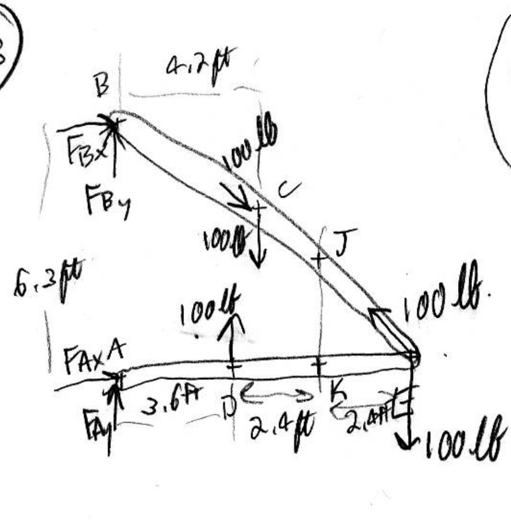
$M = -\left(\frac{2000}{13}N\right)\left(\frac{650}{3}mm\right)$

$M = -\frac{1300}{39}N\cdot m = \boxed{-\frac{100}{3}N\cdot m = M \; (-33.\bar{3}N\cdot m)}$

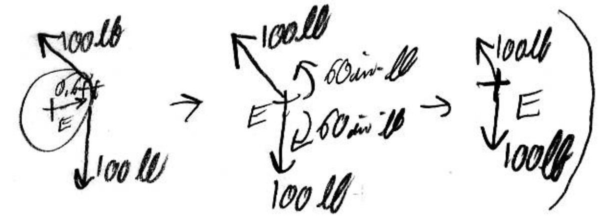
$\Sigma F_{x_{new}} = 0 = F_{axial} + \frac{5}{13}\left(\frac{500}{3}N\right) + \left(\frac{12}{13}\right)800N$

$\boxed{F_{axial} = -\frac{31300}{39}N = -802.56N}$

(3)



(For example at E:



)

For body: $\Sigma M_A = 0 = -F_{Bx}(6.3\,ft) - 100\,lb(8.4\,ft)$

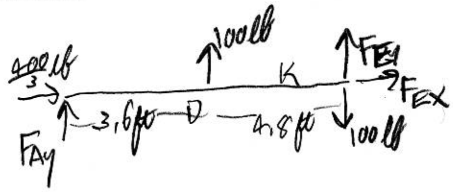
→ $\boxed{F_{Bx} = \frac{-400}{3}\,lb}$

$\Sigma F_x = 0 = F_{Ax} + F_{Bx} + \left(\frac{4}{5}\right)100\,lb - \left(\frac{4}{5}\right)100\,lb$

→ $\boxed{F_{Ax} = \frac{400}{3}\,lb}$

$\Sigma F_y = 0 = F_{Ay} + F_{By} + 100\,lb - 2(100\,lb)$ → $\boxed{F_{By} = \frac{1100}{7}\,lb}$

For AE:

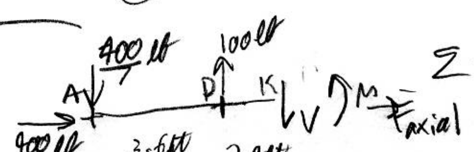


$\Sigma M_E = 0 = -100\,lb(4.8\,ft) - F_{Ay}(8.4\,ft)$

$\boxed{\Sigma F_{Ay} = -\frac{400\,lb}{7}}$

(b)

For ADK: (at K)



$\Sigma F_y = 0 = -\frac{400}{7}\,lb + 100\,lb - V$

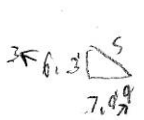
→ $\boxed{V = \frac{300}{7}\,lb}$ (42.857 lb)

$\Sigma M_K = 0 = M - 100\,lb(2.4\,ft) + \frac{400}{7}\,lb(6.0\,ft)$

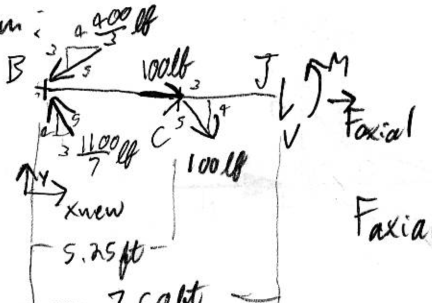
$M = 240\,ft\cdot lb - \frac{2400}{7}\,ft\cdot lb$ → $\boxed{M = -\frac{720}{7}\,ft\cdot lb}$ (-102.857 lb)

$\Sigma F_x = 0 = \frac{400}{3}\,lb + F_{axial}$ → $\boxed{F_{axial} = \frac{-400}{3}\,lb}$ (-133.3 lb)

(a) new coordinate system:



BCJE as 3:4:5 triangle



$\Sigma F_{x_{new}} = F_{axial} + \left(\frac{3}{5}\right)100\,lb + 100\,lb - \left(\frac{4}{5}\right)\frac{400}{3}\,lb - \left(\frac{3}{5}\right)\frac{1100}{7}\,lb$

$F_{axial} = 40.9524\,lb$

3(a) (cont.) $\sum F_{y\,new} = -\left(\frac{3}{5}\right)\frac{400}{3}\,lb + \left(\frac{4}{5}\right)\frac{1100}{7}\,lb - \left(\frac{4}{5}\right)100\,lb - V$

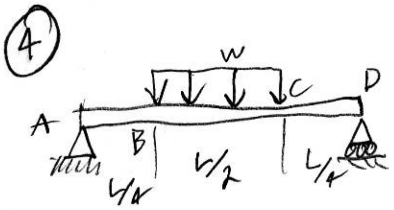
$V = -80\,lb + \frac{880}{7}\,lb - 80\,lb \rightarrow V = -34.286\,lb$

$\sum M_B = 0 = -\left(\frac{4}{5}\right)100\,lb(5.25\,ft) - V(7.50\,ft) + M$

$M = -(34.286\,lb)(7.50\,ft) + 80\,lb(5.25\,ft) \rightarrow M = 162.855\,lb\cdot ft$

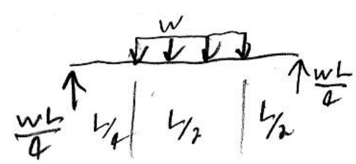
$\boxed{V = -34.3\,lb,\quad M = 163\,ft\cdot lb,\quad F_{axial} = 41.0\,lb}$

4

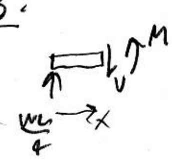


body: $\sum M_A = 0 = -w\left(\frac{L}{2}\right)\left(\frac{L}{2}\right) + F_{Dy}(L) \rightarrow F_{Dy} = \frac{wL}{4}$

$\sum F_y = 0 = \frac{wL}{4} - \frac{wL}{2} + F_{Ay} \rightarrow F_{Ay} = \frac{wL}{4}$



for AB:



$\sum F_y = 0 = -V + \frac{wL}{4} \rightarrow V = \frac{wL}{4}$

$\sum M_0 = 0 = M - V(x) \rightarrow M = \frac{wL}{4}(x)$

for BC:

$\sum F_y = 0 = \frac{wL}{4} - V - w(x - L/4)$

$V = -w(x) + \frac{wL}{2}$

$\sum M_0 = 0 = -V(x) - w(x - L/4)\left(\frac{x - L/4}{2} + \frac{L}{4}\right) + M$

$M = \left[-w(x) + \frac{wL}{2}\right](x) + w\left(x - \frac{L}{4}\right)\left(\frac{x + L/4}{2}\right)$

$M = -w(x^2) + \frac{wL}{2}(x) + \frac{w}{2}(x^2) - \frac{w}{2}\left(\frac{L^2}{16}\right)$

$M = -\frac{wL^2}{32} + \frac{wL}{2}(x) - \frac{w}{2}(x^2)$

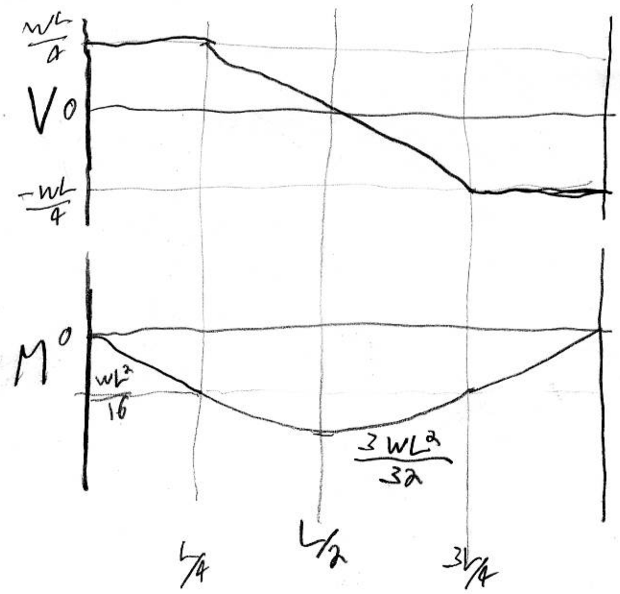
For CD:

$\sum F_y = 0 = V + \frac{wL}{4} \rightarrow V = -\frac{wL}{4}$

$\sum M_x = 0 = \frac{wL}{4}(L - x) - M$

$M = \frac{wL^2}{4} - \frac{wL(x)}{4}$

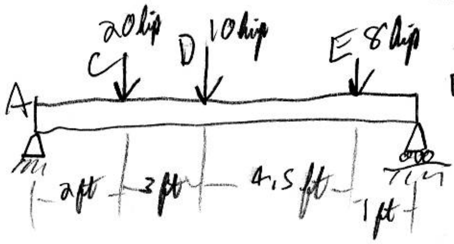
4a



4b: $\boxed{\text{maximum shear} = \frac{wL}{4}}$

maximum moment magnitude = $\boxed{\frac{3wL^3}{32}}$

5



for body: $\Sigma M_A = 0 = -20\,kip(2\,ft) - 10\,kip(3\,ft) - 8\,kip(9.5\,ft) + F_{Fy}(10.5\,ft)$

$F_{Fy} = 15.8$ kip

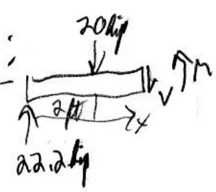
$\Sigma F_y = 0 = -20\,kip - 10\,kip - 8\,kip + 15.8\,kip + F_{Ay}$

$F_{Ay} = 22.2$ kip

AC:

$\Sigma F_y = 0 = 22.2\,kip - V \Rightarrow \underline{V = 22.2\,kip}$ | $\Sigma M = 0 = M - 22.2\,kip(x) \Rightarrow \underline{M = 22.2(x)\,kip}$

CD:

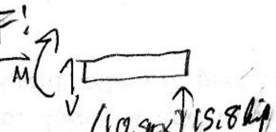


$\Sigma F_y = 0 = 22.2\,kip - 20\,kip - V$ | $\Sigma M = 0 = -20\,kip(2\,ft) + M - 2.2\,kip(x)$

$\underline{V = 2.2\,kip}$

$\underline{M = 40\,kip \cdot ft + 2.2\,kip(x)}$

EF:



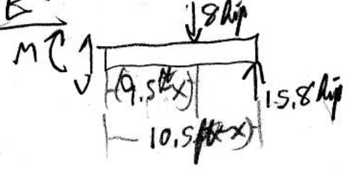
$\Sigma F_y = 0 = V + 15.8\,kip$

$\underline{V = -15.8\,kip}$

$\Sigma M_x = 0 = 15.8\,kip(10.5\,ft - x) - M$

$\underline{M = 165.9\,kip \cdot ft - 15.8\,kip(x)}$

DE:



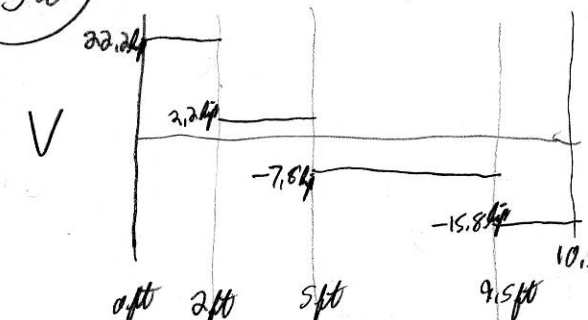
$\Sigma F_y = 0 = -V - 8\,kip + 15.8\,kip \Rightarrow \underline{V = -7.8\,kip}$

$\Sigma M_x = 0 = -M - 8\,kip(9.5\,ft - x) + 15.8\,kip(10.5\,ft - x)$

$M = -76\,kip \cdot ft + 8\,kip(x) + 165.9\,kip \cdot ft - 15.8\,kip(x)$

$\underline{M = 89.9\,kip \cdot ft - 7.8\,kip(x)}$

5a

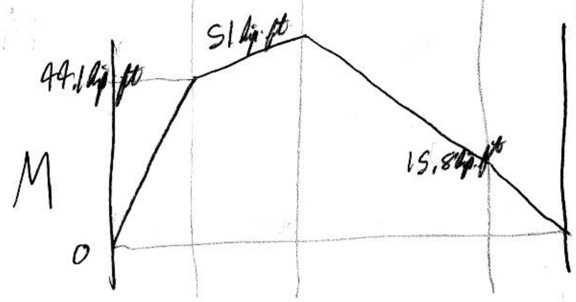


AC: $V = 22.2$ kip (0-2ft) 0ft $M=0$; 2ft $M = 44.4$ kip·ft linear

CD: $V = 2.2$ kip (2-5ft) 2ft $M =$ " ; 5ft $M = 51$ kip ft linear, rounding error

DE: $V = -7.8$ kip (5-9.5ft) 5ft $M = 50.9$ kip·ft ; 9.5ft $M = 15.8$ kip·ft linear

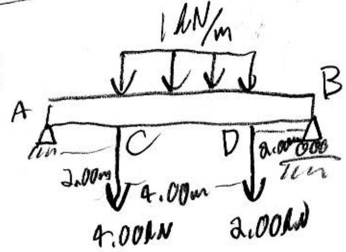
EF: $V = -15.8$ kip (9.5-10.5ft) 9.5ft $M = 15.8$ kip·ft ; 10.5ft $M = 0$ linear



5b $|V|_{max} = 22.2$ kip

$|M|_{max} = 51$ kip·ft.

6



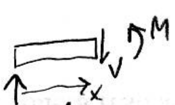
for body: $\Sigma M_A = 0 = -4kN(2m) - 4kN(4m) - 2kN(6m) + F_{By}(8m)$

$\underline{F_{By} = 4.50kN}$

$\Sigma F_y = 0 = -4.00kN - 2.00kN - 4.00kN + 4.50kN + F_{Ay}$

$\underline{F_{Ay} = 5.50kN}$

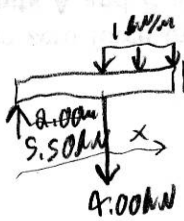
AC:



$\Sigma F_y = 0 = 5.50kN - V$ $\quad \Sigma M = 0 = M - V(x)$

$\underline{V = 5.50kN}$ $\quad \underline{M = 5.50kN(x)}$

CD:



$\Sigma F_y = 0 = 5.50kN - 4.00kN - 1.00kN/m(x - 2.00m) - V$

$\underline{V = 3.50kN - 1.00kN/m(x)}$

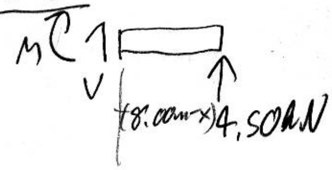
$\Sigma M = 0 = -4.00kN(2.00m) - 1.00kN/m(x - 2.00m)\left[\left(\frac{x - 2.00m}{2}\right) + 2.00m\right] - V(x) + M$ $\quad \frac{x+2.00m}{2}$

$M = 8.00kN\cdot m - 2.00kN\cdot m + \frac{1.00kN/m(x^2)}{2} + 3.50kN(x) - 1.00kN/m(x^2)$

$\underline{M = 6.00kN\cdot m + 3.50kN(x) - 1.00kN/m\left(\frac{x^2}{2}\right)}$

6 (cont.)

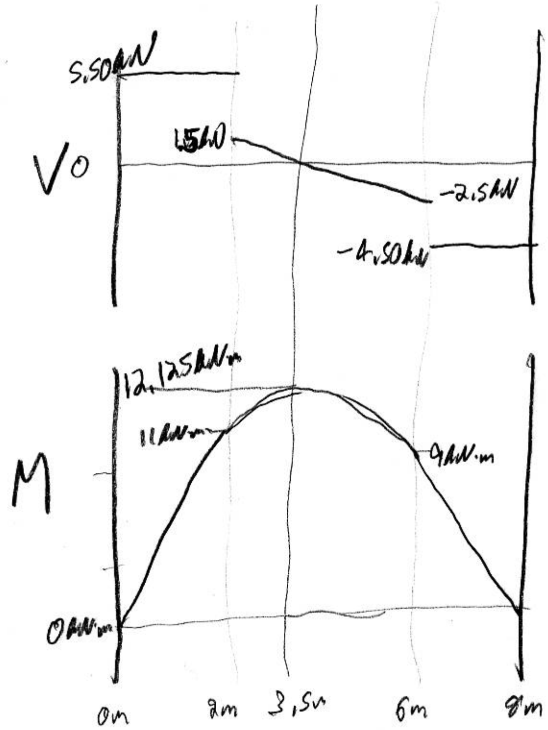
DB:



$\Sigma F_y = 0 = V + 4.50\,kN \rightarrow \underline{V = -4.50\,kN}$

$\Sigma M_x = 0 = -M + 4.50\,kN(8.00\,m - x)$

$\underline{M = 36.0\,kN\cdot m - 4.50\,kN(x)}$



AC: $V = 5.50\,kN$ — 0m $M = 0$; 2m $M = 11.0\,kN\cdot m$ — linear

CD: $V = 3.50\,kN - 1.00\,kN(x)$

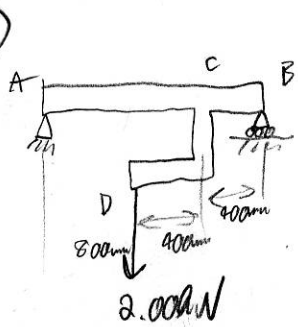
	V	M
2m	1.50kN	11.0kN·m
3.5m:		12.125kN·m
6m	−2.50kN	9.00kN·m
	linear	parabolic; max at x=3.5m

DB: $V = -4.50\,kN$

	M
6m	9.00kN·m
8m	0

linear

7



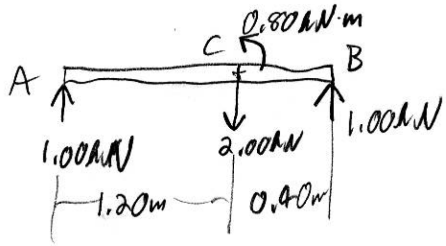
body: $\Sigma M_A = 0 = -800mm(2.00kN) + F_{By}(1.60m)$

$\underline{F_{By} = 1.00kN}$

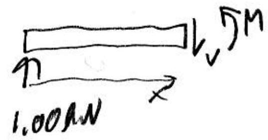
$\Sigma F_y = 0 = F_{Ay} - 2.00kN + 1.00kN$

$\underline{F_{Ay} = 1.00kN}$

transfer load at D to C:



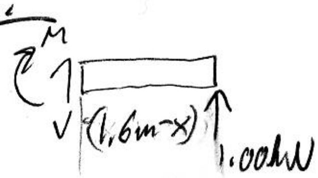
For AC:



$\Sigma F_y = 0 = 1.00kN - V \rightarrow \underline{V = 1.00kN}$

$\Sigma M_{x=0} = 0 = -V(x) + M \rightarrow \underline{M = 1.00kN(x)}$

For CB:



$\Sigma F_y = 0 = V + 1.00kN \rightarrow \underline{V = -1.00kN}$

$\Sigma M_x = 0 = -M + 1.00kN(1.60m - x)$

$\underline{M = 1.60kN \cdot m - 1.00kN(x)}$

AC: V = 1.00kN 0m : $\frac{M}{0}$ linear
1.2m : 1.20kN·m

CB: V = −1.00kN 1.2m 0.40kN·m linear
1.6m 0