MATH 1210 A01 Summer 2013 Problem Workshop 12

1. For the matrix $E = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 1 & 3 \\ 2 & 1 & 2 \end{bmatrix}$, find the cofactor matrix and the adjoint of E and use them to find the inverse.

2. (a) Show $|A^{-1}| = \dfrac{1}{|A|}$.

   (b) Show for any constant c and any $n \times n$ matrix A, that $|cA| = c^n |A|$.

   (c) Use parts (a) and (b) to show that $|adj(A)| = |A|^{n-1}$.

3. Let A, B, C be 3×3 matrices such that $|A| = 2$ and $|B| = 3$. Find $|5AB^{-2}|$.

4. Solve the following system of equations using inverses

$$2x + 3y - 4z = 12$$
$$x + y + 2z = 1$$
$$3x + 4y = 11$$

5. Solve the following system of equations in terms of a, b and c using inverses.

$$2x + 3y - 4z = a$$
$$x + y + 2z = b$$
$$3x + 4y = c$$

6. Use both definitions of a transformation being linear to check if the following are linear:

   (a) $T(\mathbf{v}) = k\mathbf{v}$, where k is a nonzero constant.

   (b) $T(\mathbf{v}) = \mathbf{v} + \mathbf{c}$ where $\mathbf{c}$ is a nonzero vector.

   (c) $T(\mathbf{v})$ is the unit vector in the direction of $\mathbf{v}$.

Answers

1. $C = \begin{bmatrix} -1 & 4 & -1 \\ 0 & -4 & 2 \\ 1 & 2 & -1 \end{bmatrix}$ and $adj(E) = \begin{bmatrix} -1 & 0 & 1 \\ 4 & -4 & 2 \\ -1 & 2 & -1 \end{bmatrix}$.

2.

3. $250/9$.

4. $x = 1$, $y = 2$, $z = -1$.

5. $x = 4a + 8b - 5c$, $y = -3a - 6b + 4c$, $z = \dfrac{-a - b + c}{2}$

6. (a) Yes

   (b) No

   (c) No