Thermal Sciences 130.112 **Section L02** **Test 2**

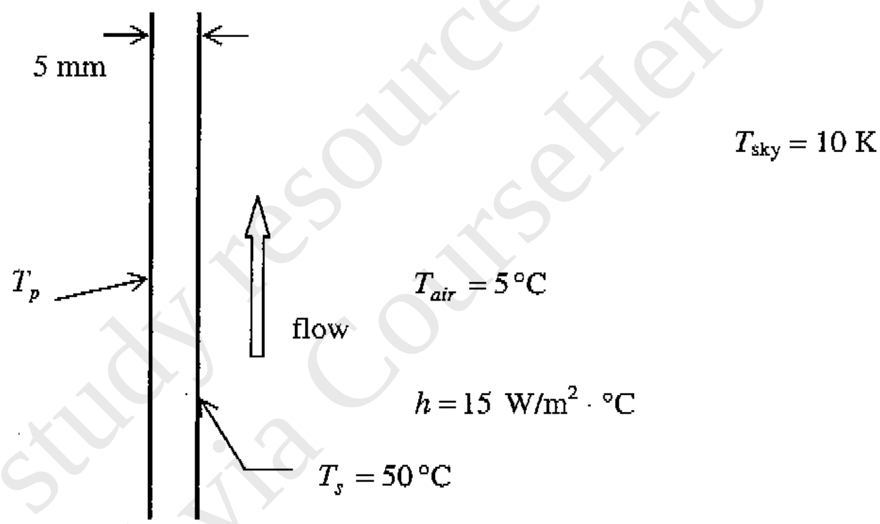
Instructor: Dr. J. Bartley November 14, 2005

Use of text and calculator are permitted. Time permitted: 2 hr

Question 1. (15 marks)

A plaster wall of thickness 5 mm is exposed to air at 5°C on one side (facing the sky). The exposed surface of the plaster wall has an emissivity of $\varepsilon = 0.7$ and the wall is maintained at a uniform surface temperature of 50°C. The exposed surface of the wall exchanges radiation with the cold sky at 10 K.

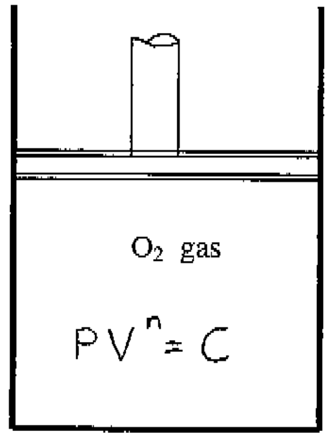
(a) If the convection heat transfer coefficient between the wall surface and the surrounding air is $h = 15\ \text{W/m}^2\cdot°\text{C}$, and the surface area of the wall is $A = 10\ \text{m}^2$, determine the total rate of heat transfer, $\dot{Q}$ [W], from the plate surface.
(b) Determine the temperature of the surface on the other side, T_p, assuming the other surface also has a uniform temperature. The thickness of the wall is 5 mm.
(c) Sketch a thermal circuit diagram for this scenario and label with known quantities.



Question 2. (20 marks)

A piston-cylinder device contains 5 kg of oxygen (O_2) gas having an initial pressure of 500 kPa and occupying an initial volume of 0.9 m^3. The oxygen gas is compressed polytropically according to the relation $PV^n = C$ until the volume of the gas is 0.1 m^3 and its temperature is 1800 K. Changes in kinetic and potential energy can be neglected. (See figure on the following page.)

(a) Determine the initial temperature of the gas.
(b) Determine the final pressure of the gas.
(c) Determine the polytropic exponent n for the process.
(d) Calculate the work done during the compression process.
(e) Calculate the value of ${}_1Q_2$ and indicate whether heat is transferred to the system or from the system.

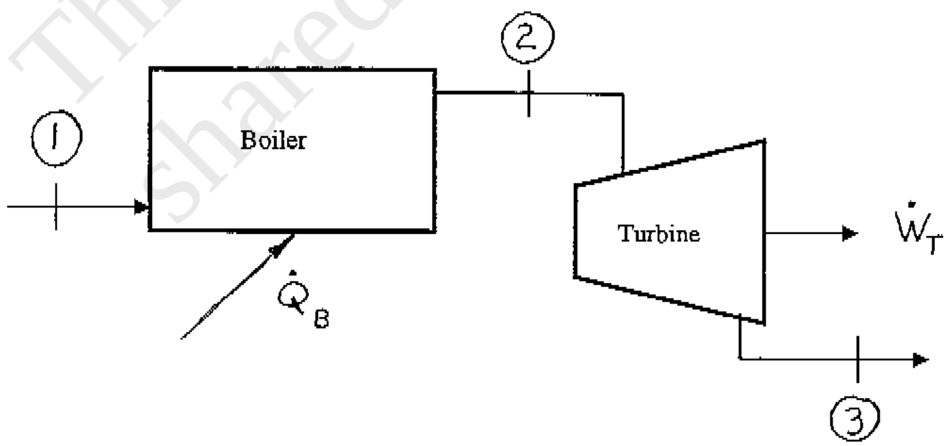


Question 3. (20 marks)

Consider the system of components in the figure below consisting of a Boiler and a Turbine. Water at 85°C and 800 kPa is delivered to the boiler (1) at a mass flow rate of 5 kg/s where 15,000 kW of heat are added to produce superheated vapor (2). The superheated steam is then delivered to a turbine where it expands and produces work. The steam leaves the turbine (3) with a steam quality $x = 0.90$ and a with pressure of 150 kPa. Some heat is lost from the turbine during the expansion process at a rate of 500 kW. Assume that the boiling process occurs at constant pressure, i.e., $P_1 = P_2$. Changes in kinetic and potential energies can be neglected.

(a) Considering the boiler, calculate the enthalpy of the steam, h, leaving the boiler at (2).
(b) Determine the temperature of the steam leaving the boiler.
(c) Considering the turbine, determine the enthalpy and temperature of the steam at (3).
(d) Calculate the power produced by the turbine, $\dot{W}_T$.
(e) By considering an energy balance for both the boiler and turbine together (one control volume), show that the First Law of Thermodynamics is satisfied, *i.e.*, that energy is conserved.

Bonus: Calculate the thermal efficiency η_{th} for the total process.

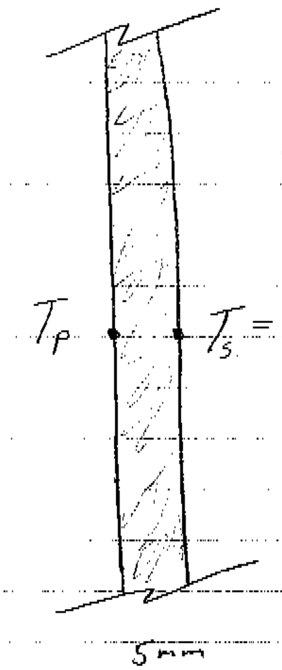


Section L02 — Test #2 Solution

1

Question 1

$T_{sky} = 10\,K$



Flow
$h = 15\ W/m^2\cdot°C$

$T_{air} = 5°C$

$A_{wall} = 10\ m^2$
$\varepsilon = 0.7$

$k = 0.79\ W/m\cdot K$
(plaster)

(a)
$$\dot{Q}_{conv} = hA(T_s - T_{air}) = 15 \times 10 \times (50-5) = 6750\ W$$

$$\dot{Q}_{rad} = \varepsilon \sigma A (T_s^4 - T_{sky}^4)$$ (net radiation from surface)

$$= 0.7 \times \underbrace{5.67\times10^{-8}}_{[W/m^2\cdot K^4]} \times 10 \times [(50+273.15)^4 - 10^4]$$

$$= 4328.1\ W$$

$$\therefore \quad \dot{Q} = \dot{Q}_{conv} + \dot{Q}_{rad}$$
$$= 6750 + 4328.1$$
$$= 11,078.1 \text{ W}$$ rate of heat transfer from plate surface.

(b) To find T_p:

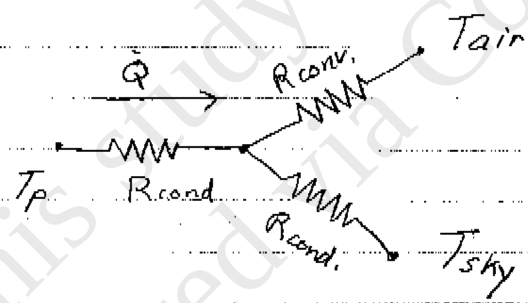
$$\dot{Q}_{cond} = -kA\frac{(T_s - T_p)}{L}$$

$$11,078.1 = -0.79 \times 10 \times \frac{(50 - T_p)}{0.005}$$

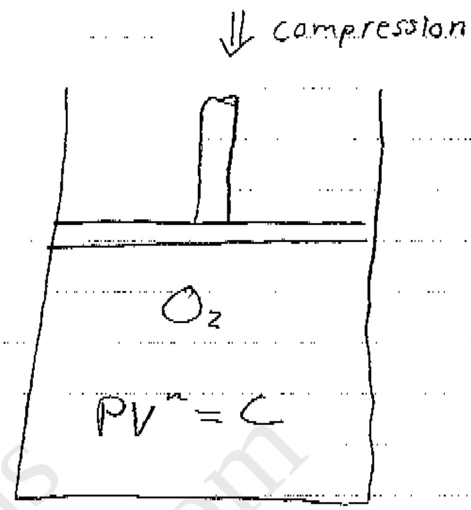
$$\therefore \quad -T_p = \frac{11,078.1 \times 0.005}{-0.79 \times 10} - 50$$

$$T_p = 57.0\ °C$$

c)



Question 2



$m_{O_2} = 5$ kg

$P_1 = 500$ kPa , $V_1 = 0.9\ m^3$

compression until $V_2 = 0.1\ m^3$
(polytropically) $T_2 = 1800$ K

(a) $P_1 V_1 = m R T_1$

$R_{O_2} = 0.2598$ kJ/kg·K

$\therefore T_1 = P_1 V_1 / mR$
$= 500 \times 0.9 / (5 \times 0.2598)$
$= 346.4$ K

(b) $P_2 V_2 = m R T_2$

$$P_2 = \frac{5 \times 0.2598 \times 1800}{0.1}$$

$P_2 = 23,382.0$ kPa (23.382 MPa)

(c) $P_1 V_1^n = C = P_2 V_2^n$

$$\frac{P_1}{P_2} = \left(\frac{V_2}{V_1}\right)^n \qquad \log_e\left(\frac{P_1}{P_2}\right) = n \log_e\left(\frac{V_2}{V_1}\right)$$

$$\therefore n = \log_e\left(\frac{P_1}{P_2}\right) \Big/ \log_e\left(\frac{V_2}{V_1}\right)$$

$$n = \log_e\left(\frac{500}{23382}\right) \Big/ \log_e\left(\frac{0.1}{0.9}\right)$$

$n = 1.75$

(d) Work done:

$$ {}_1W_2 = \frac{P_2 V_2 - P_1 V_1}{1-n} $$

$$ {}_1W_2 = \frac{23382 \times 0.1 - 500 \times 0.9}{1 - 1.75} $$

$$ {}_1W_2 = -2517.6 \text{ kJ} $$

(e) First Law Analysis: Assume a constant specific heat.

$$ {}_1Q_2 - {}_1W_2 = m(u_2 - u_1) \quad , \quad C_{v0} = 0.662 \ \frac{kJ}{kg \cdot K} $$

Table A.5

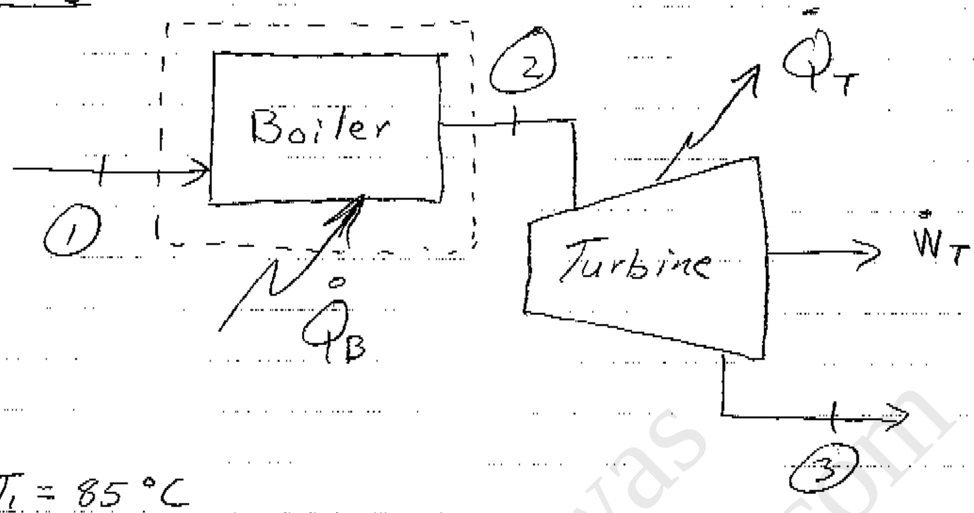
$$ \begin{aligned} (u_2 - u_1) &= C_{v0}(T_2 - T_1) \\ &= 0.662 \times (1800 - 346.4) \\ &= 962.2832 \text{ kJ/kg} \end{aligned} $$

$$ \therefore {}_1Q_2 = 5 \times 962.2832 + (-2517.6) $$

$$ {}_1Q_2 = +2293.81 \text{ kJ} $$

Heat ${}_1Q_2$ is transferred to the system (O_2 gas)

Question 3



4 (a) $T_1 = 85\,°C$

$P_1 = 800$ kPa

$\dot{m} = 5$ kg/s

$\dot{Q}_B = 15{,}000$ KW   state 2: superheated vapor

first law for the Boiler:

1 $\dot{Q}_B - \cancel{\dot{W}_B} = \dot{m}(h_2 - h_1) + \cancel{\Delta ke} + \cancel{\Delta pe}$

1 $h_1 \cong h_{f@85°C} = 355.88$ kJ/kg (compressed liquid)

(Table B.1.1)

1 $\therefore 15000 = 5(h_2 - 355.88)$

1 $\therefore h_2 = 3355.88$ kJ/kg

4 (b) at 2, the steam is superheated vapor.

assumed that $P_1 = P_2 = 800$ kPa

1 $h_2 = 3355.88$ kJ/kg

From Table B.1.3 For h_2, the temperature is between 400° and 500°C at $P_2 = 800$ kPa

2 Interpolate: $\frac{T_2 - 400}{500 - 400} = \frac{3355.88 - 3267.07}{3480.60 - 3267.07}$

1 $\longrightarrow$ $T_2 = 441.6\ °C$

/5 (c) First law for the turbine:

$$\dot{Q}_T - \dot{W}_T = \dot{m}(h_3 - h_2) + \cancel{\Delta ke} + \cancel{\Delta pe}$$

given that the turbine loses 500 kW of heat (i.e., this particular turbine is not adiabatic)

At 3, $P_3 = 150\ kPa$, $x = 0.90$

1 $\rightarrow$ Table B.1.2: at P = 150 kPa, $T_3 = T_{sat.} = 111.37\ °C$

2 $h_f = 467.08\ kJ/kg$, $h_{fg} = 2226.46\ \frac{kJ}{kg}$

$\therefore h_3 = 467.08 + 0.90 \times 2226.46$

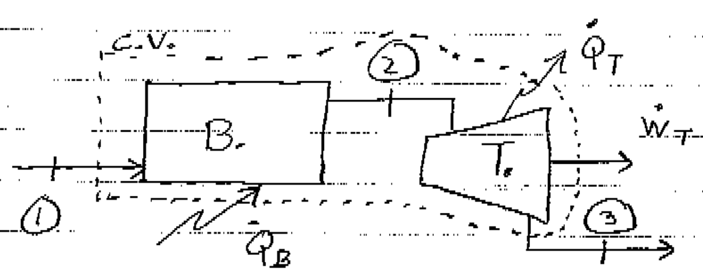
2 $\longrightarrow$ $h_3 = 2470.894\ kJ/kg$

/3 1(d) $\therefore$ First law: $\dot{Q}_T - \dot{W}_T = \dot{m}(h_3 - h_2)$

1 $-500 - \dot{W}_T = 5(2470.894 - 3355.88)$

1 $\longrightarrow$ $\therefore \dot{W}_T = 3924.93\ kW$

/4 (e) energy balance around boiler / turbine system:



(e) continued: first law for system:

2 $\dot{Q}_{net} - \dot{W}_{net} = \dot{m}(h_3 - h_1) + \cancel{\Delta KE} + \cancel{\Delta PE}$

$L.H.S. = \dot{Q}_{net} - \dot{W}_{net}$

$= (+15000 - 500) - (3924.93)$

1 $= 10575.07 \text{ kW}$

$R.H.S. = \dot{m}(h_3 - h_1)$

$= 5 \times (2470.894 - 355.88)$

1 $= 10575.07 \text{ kW}$

$L.H.S = R.H.S. \therefore$ energy is conserved

Bonus

+1 $\eta_{th} = \frac{\dot{W}_{T.}}{\dot{Q}_B} = \frac{3924.93}{15000} = 0.2616$