UNIVERSITY OF MANITOBA

DATE: February 7, 2013 — MIDTERM I

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COURSE: MATH 2130 — TIME: 70 minutes

EXAMINATION: Engineering Mathematical Analysis 1 EXAMINER: M. Davidson

[4] 1. (a) Identify and sketch the surface $z^2 = 6 - x^2 + y^2$.

[3] (b) Find the projection of $z^2 = 6 - x^2 + y^2$, $z + y = 3$ onto the xy-plane.

[8] 2. Find the distance between the line

$$x = 5 + t\,,\ y = -1 - t\,,\ z = 8 + 3t$$

and the line

$$\frac{x-4}{-2} = \frac{y-1}{2} = \frac{z-2}{-6}.$$

[8] 3. Find a vector representation of the curve of intersection of $z = x^2 + 2y^2$ and $4x + 4y + z = 10$ oriented so it is counterclockwise when viewed from far out on the positive z axis.

[6] 4. Find a unit tangent vector to the curve $3x + y = 7$, $z + x^2 + 2y = 9$ at the point $(2, 1, 3)$.

[5] 5. Evaluate the following limit, or show that it does not exist:

$$\lim_{(x,y)\to(0,0)} \frac{2x^2y^8}{x^4 + y^{16}}.$$

[6] 6. Set up but *do not evaluate* an integral for the length of the curve

$$\vec{r}(t) = \langle 2\sin(3t), 2\cos(3t), \tan(t)\rangle$$

from the point $(0, 2, 0)$ to the point $(\sqrt{2}, -\sqrt{2}, 1)$. Simplify the integrand.

Answers by Dawit (plankion@yahoo.com)

1.a) Hyperboloid of one sheet with y-axis as its main axis

b) $6y - x^2 = 3$, $z = 0$ 2. $\sqrt{\frac{10}{11}}$

3. $(-2 + 4\cos t)\,\hat{i} + (-1 + 2\sqrt{2}\sin t)\,\hat{j} + (22 - 16\cos t - 8\sqrt{2}\sin t)\,\hat{k}$

4. $\frac{1}{\sqrt{14}}\,\hat{i} - \frac{3}{\sqrt{14}}\,\hat{j} + \frac{2}{\sqrt{14}}\,\hat{k}$ 5. limit does not exist. (hint: let $x = my^4$)

5. $\int_0^{\pi/4} \sqrt{36 + \sec^4 t}\ dt$