UNIVERSITY OF MANITOBA

DATE: December 11, 2008 FINAL EXAMINATION
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COURSE: MATH 2132 TIME: 3 hours
EXAMINATION: Engineering Mathematical Analysis 2 EXAMINER: M. Davidson

[8] 1. Find the radius of convergence and open interval of convergence of

$$\sum_{n=3}^{\infty} \frac{3^{2n+5}\,(2n)!}{2^{4n+3}\,(n!)((n+1)!)}\,x^{2n+5}$$

[10] 2. Find the Taylor series of $f(x) = \dfrac{7x-5}{6x^2-7x-3}$ about the point $x = 2$. You may use the fact that

$$\frac{7x-5}{6x^2-7x-3} = \frac{1}{2x-3} + \frac{2}{3x+1}.$$

State your answer in sigma notation, simplifying as much as possible. Find the open interval of convergence.

[6] 3. Use the binomial theorem to find the Maclaurin series of the function

$$f(x) = (1+4x)^{-\frac{3}{5}}$$

[8] 4. Find the sum of the power series

$$\sum_{n=0}^{\infty} \frac{3^{2n}\,(2n+1)}{2^{4n}}\,x^{2n+2}$$

[9] 5. Solve the following differential equation:

$$\frac{dy}{dx} + 3x^2y = 2x^2 + x^5$$

[7] 6. Find a two parameter family of solutions to the differential equation:

$$y'y'' = 1$$

[12] 7. Find a general solution of:

$$y'' - y' - 6y = e^{3x}x^2$$

. Use the operator method to find a particular solution.

[12] 8. A 500 gram mass hangs on a spring with constant $8\frac{N}{m}$. The mass is given a speed of $2\frac{m}{s}$ upwards. The mass is acted upon by a damping force whose magnitude in Newtons is 5 times the instantanious velocity. In addition, a force $F(t) = e^{-3t}$ acts on the mass. Find the position of the mass as a funtion of time.

(Recall the formula: $M\frac{d^2x}{dt^2} + \beta\frac{dx}{dt} + kx = F(t)$.)

[10] 9. Find the Laplace transform of the following periodic function:

$$f(x) = \begin{cases} \sin(t) & 0 < t < 3 \\ t^2 & 3 < t < 6 \end{cases} \qquad f(t+6) = f(t)$$

[8] 10. Use convolutions to find the inverse Laplace transform of $\dfrac{2}{s^2(s^2+4)}$

[10] 11. Solve the following initial value problem:

$$y'' + 2y' + 10y = 3\,\delta(t-4) \qquad y(0) = 1 \quad y'(0) = 2$$