FALL 2008

FINAL

SOLUTION

THE UNIVERSITY OF MANITOBA

Date : Wednesday, December 10, 2008
Department & Course No : ENG 1440 **Time : 1:30 p.m.**
Paper No : 326/322 Sections 1-2, D01 **Duration : 2 Hours**
Examination : Introduction to Statics **Examiners : Dr. M. J. Frye**
Dr. D. Polyzois

Place : Frank Kennedy Brown Gym **Seats: 1 - 223**

Question 1

A UFO (Unidentified Flying Object) landed in South Winnipeg (near the U of M campus) and was seen taking off carrying three (3) unidentified packages (rumor has it they were Engineering students). The masses of the students are $M_1 = 100\ kg$, $M_2 = 65\ kg$ and $M_3 = 140\ kg$. Their location in the $5\ m$ radius space craft is ~~shown in~~ Figure 1(b) below.

The message back from the space craft is that the students be ~~will returned~~ if you can replace these forces by a single force and correctly locate its point of application with respect to the origin, O in the figure. **(Use g = 9.8 m/sec^2)**

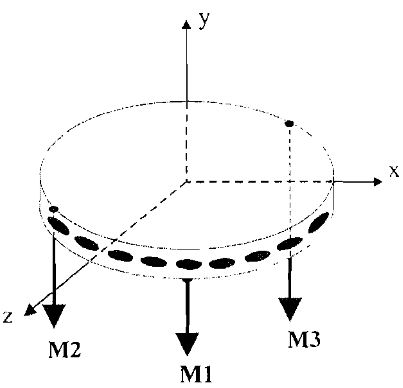


Figure 1(a)

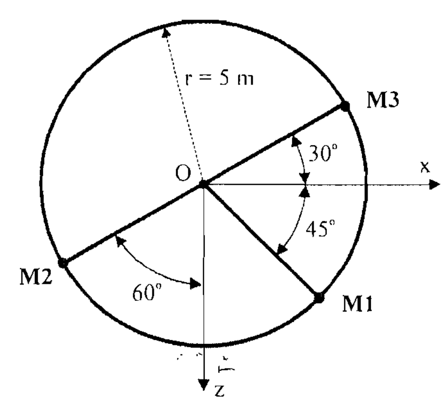


Figure 1(b) - Location of Masses

COORDINATES

M_1: $100 \times 9.8 = 980^N$ M_3: $(4.33, 0, -2.5)$

M_2: $65 \times 9.8 = 637^N$ M_1: $(3.535, 0, 3.535)$

M_3: $140 \times 9.8 = 1372^N$ M_2 $(-4.33, 0, 2.5)$

Force	$\vec{r}$	$\vec{F}$	$\vec{M} = \vec{r} \times \vec{F}$
M_1	$3.535\hat{i} + 3.535\hat{k}$	$-980\hat{j}$	$-3464.3\hat{k} + 3464.3\hat{i}$
M_2	$-4.33\hat{i} + 2.5\hat{k}$	$-637\hat{j}$	$2758.21\hat{k} + 1592.5\hat{i}$
M_3	$4.33\hat{i} - 2.5\hat{k}$	$-1372\hat{j}$	$-5940.76\hat{k} - 3430\hat{i}$
	$\bar{x}\hat{i} + \bar{z}\hat{k}$	$-2989\hat{j}$	$-6646.85\hat{k} + 1626.8\hat{i}$

$$-2989\bar{x}\,\hat{k} + 2989\bar{z}\,\hat{i} = -6646.85\hat{k} + 1626.8\hat{i}$$

$$-2989\bar{x} = -6646.85 \qquad \bar{x} = \cancel{2.22\,m}$$

$$2989\bar{z} = 1626.8 \qquad \bar{z} = \cancel{0.544\,m}$$

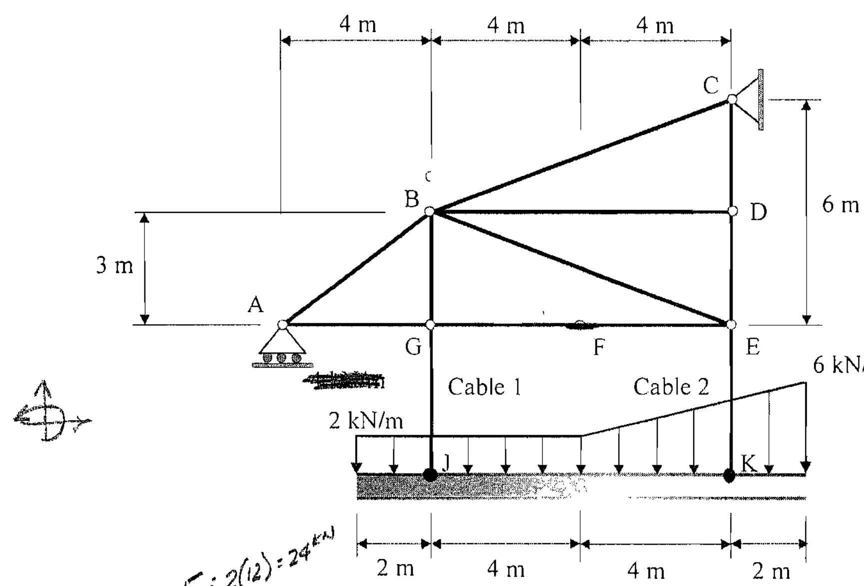
Check $\Sigma M_x = 637(2.5) + 980(3.535) - 1372(2.5) = 1626.8\hat{i}$

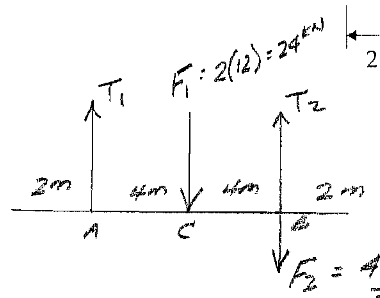
$\Sigma M_z = -980(3.535) + 637(4.33) - 1372(4.33) = 6646.85\hat{k}$

Question 2

The truss shown in the figure below has a beam suspended from two *(2)* cables. The beam supports the distributed load indicated. The truss has a pin support at *C* and a roller support at *A*. Determine:

a) The tension in *Cable 1 (GJ)* and *Cable 2(EK)*,
b) The reactions at *A* and *C*, and
c) The force in each member of the truss and state whether it is in tension or compression. (**Indicate your results on the figure provided on the next page.**)





$\Sigma M_A = 0 \quad -24(4) + T_2(8) - 12(8) = 0$

$T_2 = +24^{kN} \quad \vec{T_2} = 24 kN \uparrow$

$\Sigma F_y = 0$

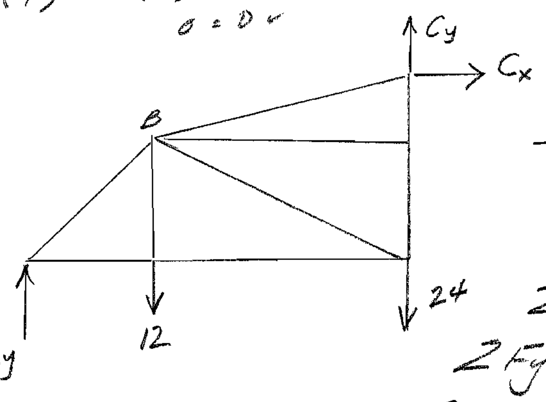
$T_1 - 24 + 24 - 12 = 0$

$T_1 = +12^{kN} \quad T_1 = 12^{kN} \uparrow$

Check $\Sigma M_C = 0$

$-12(4) + 24(4) - 12(4) = 0$

$0 = 0$ ✓



$\Sigma M_C = 0$

$-A_y(12) + 12(8) = 0$

$A_y = +8^{kN} \quad \vec{A_y} = 8^{kN} \uparrow$

$\Sigma F_x = 0 \quad C_x = 0$

$\Sigma F_y = 0 \uparrow$

$8 - 12 - 24 + C_y = 0$

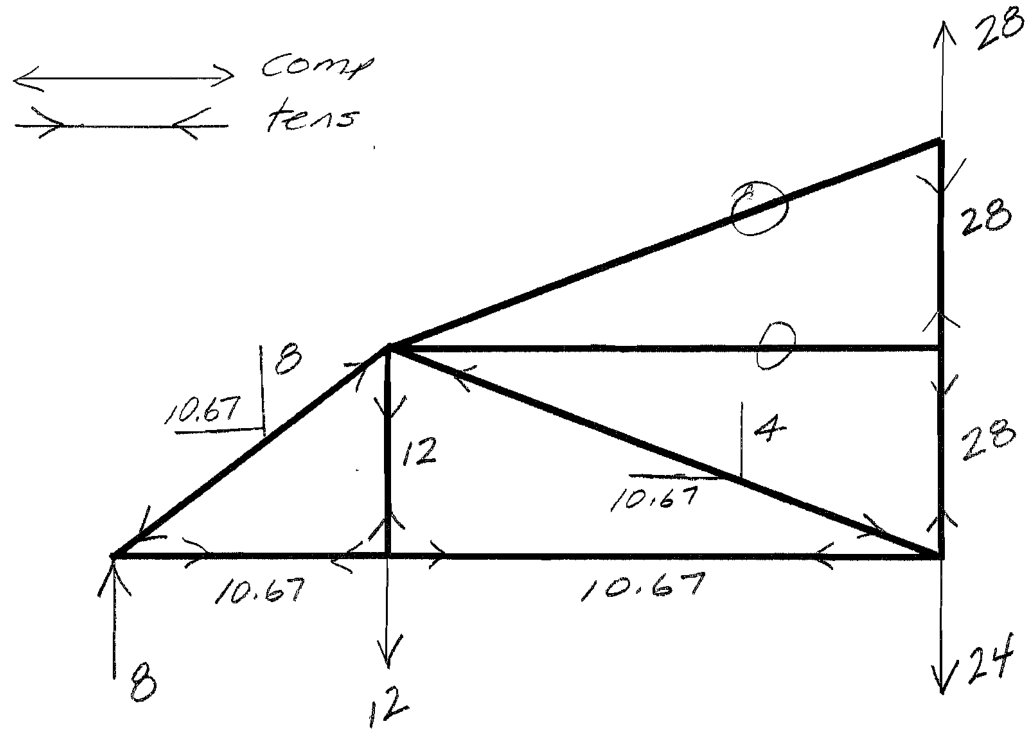
$C_y = +28 \therefore \vec{C_y} = 28 kN \uparrow$

Check $\Sigma M_B = 0$

$-8(4) + 28(8) - 24(8) = 0$

$0 = 0$ ✓

Question 2 (continued)



$\frac{8}{3} = \frac{x}{4}$ $x =$

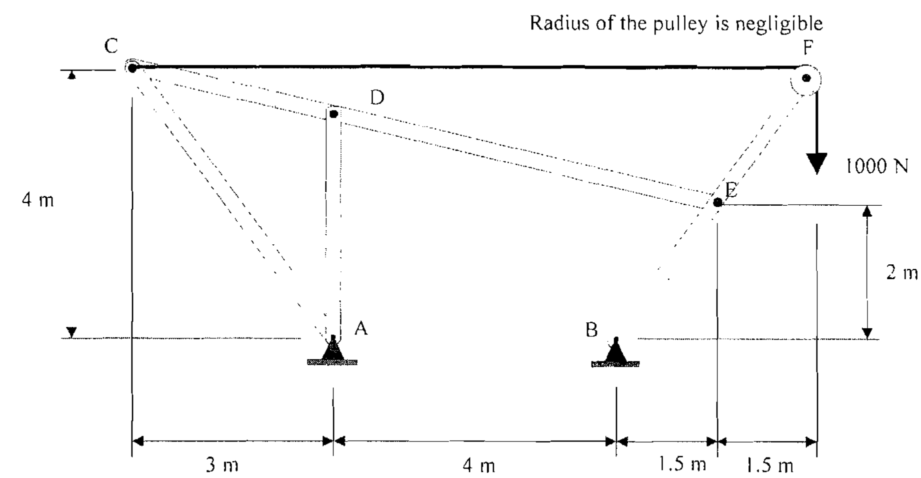
$\frac{10.67}{8} = \frac{y}{3}$

$y =$

Question 3

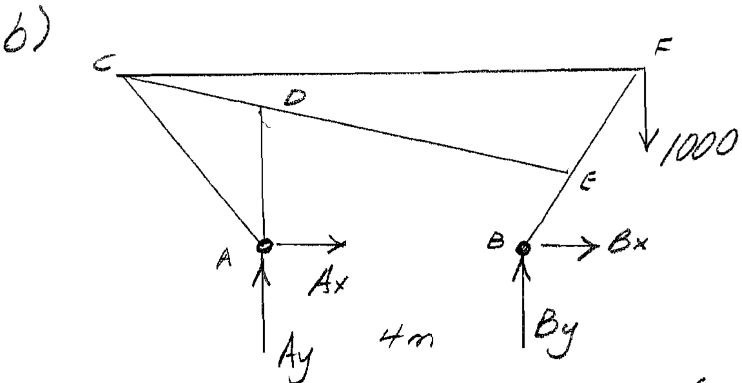
The frame shown in the figure below supports a *1000 N* load suspended from a pulley (neglect the radius of the pulley) and has pin supports at *A* and *B*.

a) Identify any two force members in the frame.
b) Determine the reactions at pin supports *A* and *B*.
c) Determine the forces acting on **ALL** members of the frame.



a) AC & AD are 2 force members.

b)



$\Sigma M_A = 0 \quad B_y(4) - 1000(7) = 0$

$B_y = +1750\text{ N} \quad \vec{B_y} = 1750\text{ N} \uparrow$

$\Sigma F_y = 0 \quad A_y + 1750 - 1000 = 0$

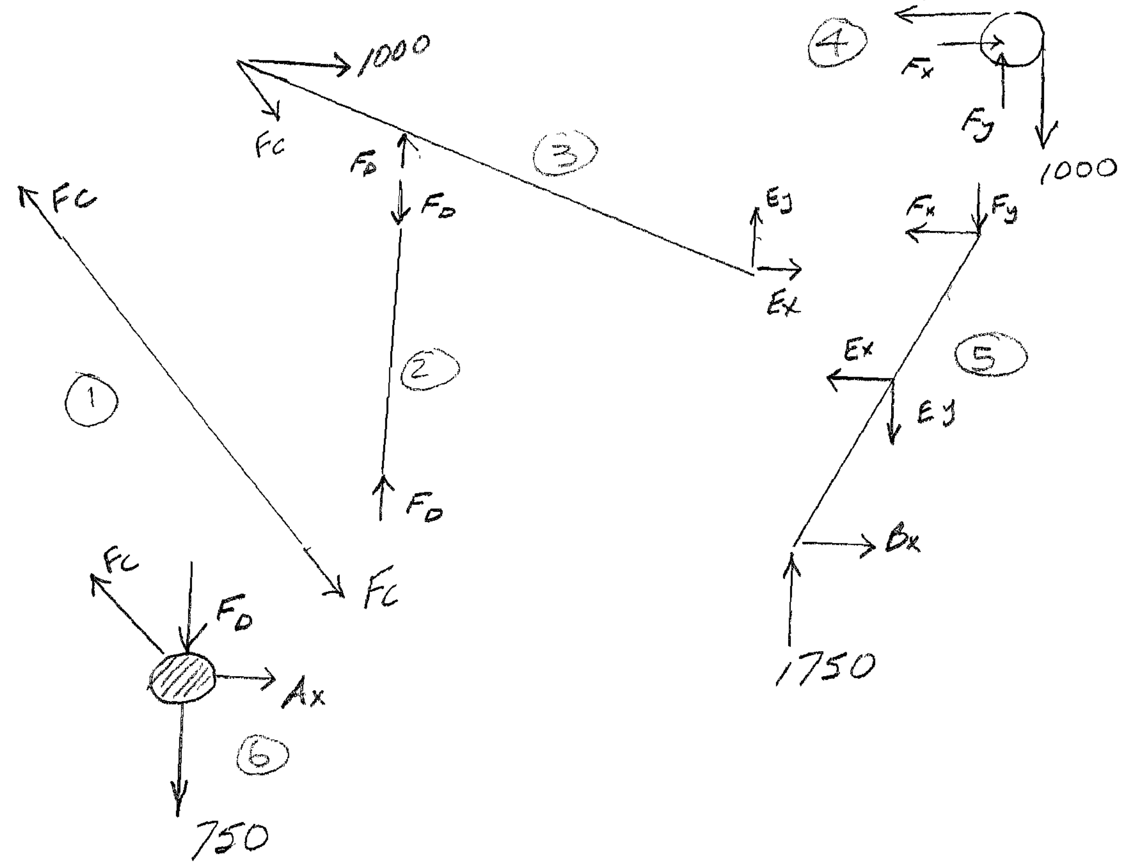
$A_y = -750\text{ N}$

$\therefore \vec{A_y} = 750\text{ N} \downarrow$

$\Sigma F_x = 0$

$A_x + B_x = 0$

Substructure:



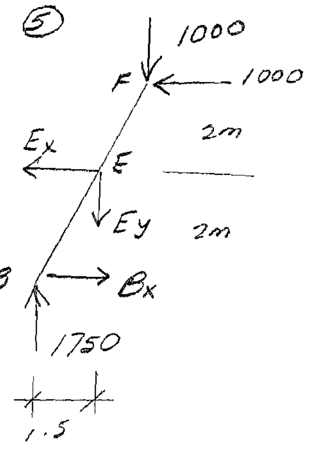
From 4 $\quad \Sigma F_x = 0 \quad F_x - 1000 = 0$

$F_x = +1000 \quad \vec{F_x} = 1000^N \rightarrow$ on the pulley

$\Sigma F_y = 0. \quad F_y - 1000 = 0 \quad F_y = +1000$

$\vec{F_y} = 1000^N \uparrow$ on the pulley

From 5



$\Sigma M_E = 0$

$-1750(1.5) + B_x(2) - 1000(1.5)$

$+1000(2) = 0$

$B_x = +1062.5^N$

$\vec{B_x} = 1062.5^N \longrightarrow$

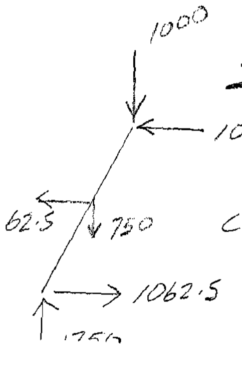
$\Sigma F_x = 0$

$1062.5 - E_x - 1000 = 0$

$E_x = +62.5^N \quad \vec{E_x} = 62.5^N \leftarrow$ on BEF

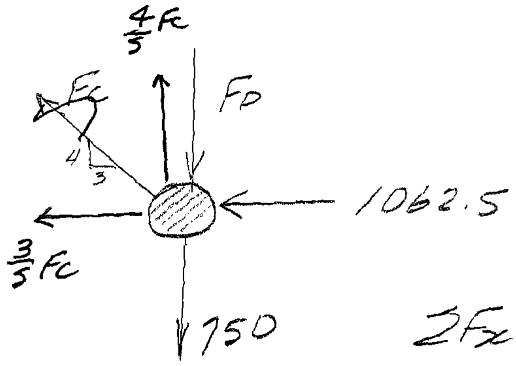
$\Sigma F_y = 0 \quad 1750 - E_y - 1000 = 0$

$E_y = +750^N \quad \vec{E_y} = 750^N \downarrow$ on BEF



check $\quad 62.5(2) + 1000(4) - 1000(3) - 750(1.5) = 0$

$0 = 0$ ✓



$\sum F_x = 0 \; \rightarrow$

$-\frac{3}{5}F_C - 1062.5 = 0$

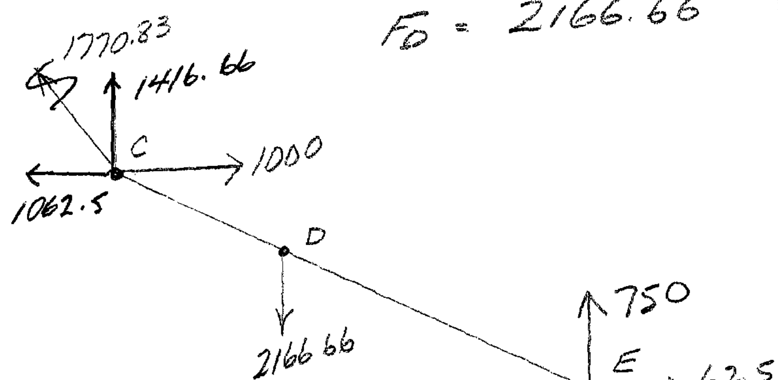
$F_C = -1770.83^N$

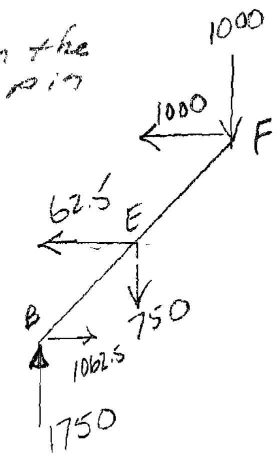
$\vec{F_C} = 1770.83$ on the pin

$\sum F_y = 0 \quad \frac{4}{5}(-1770.83) - F_D - 750 = 0$

$F_D = -2166.66^N$

$F_D = 2166.66^N \uparrow$ on the pin



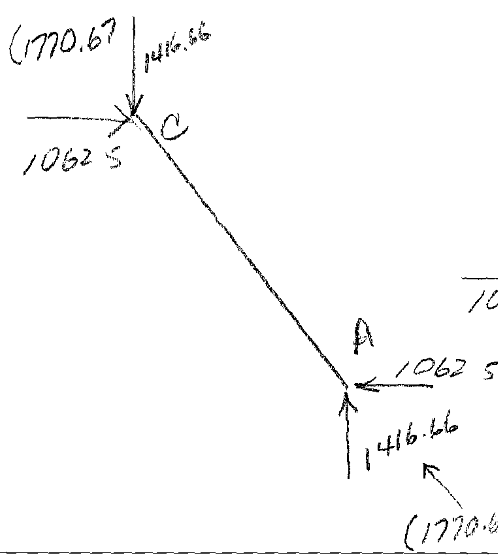


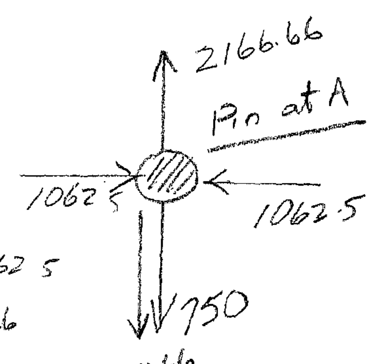
Check $\sum M_E = 0$

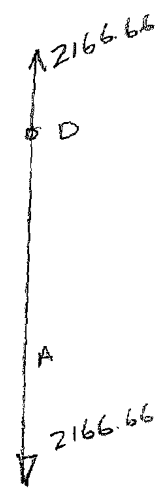
$2166.66(5.5) + 1062.5(2)$

$-1000(2) - 1416.66(8.5) = 0$

$0.02 = 0$ ✓





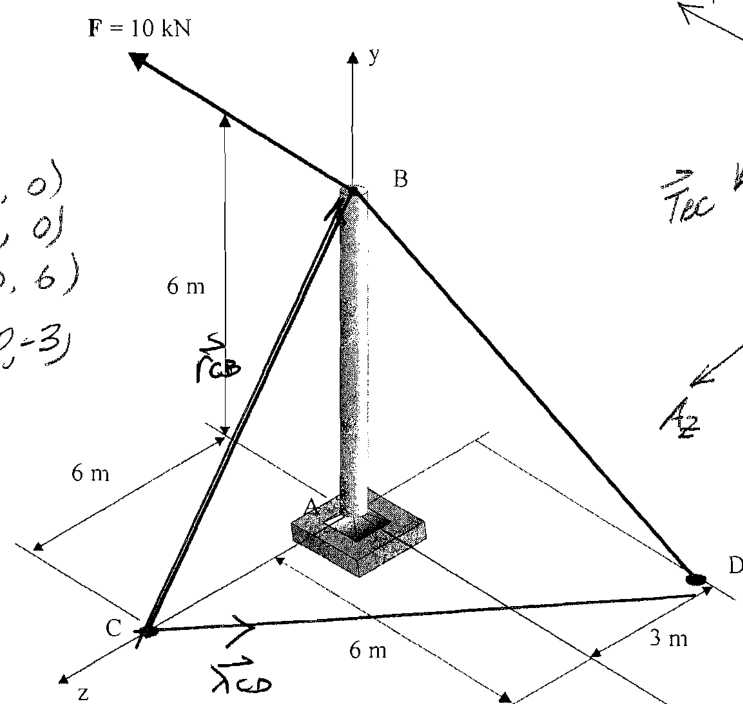


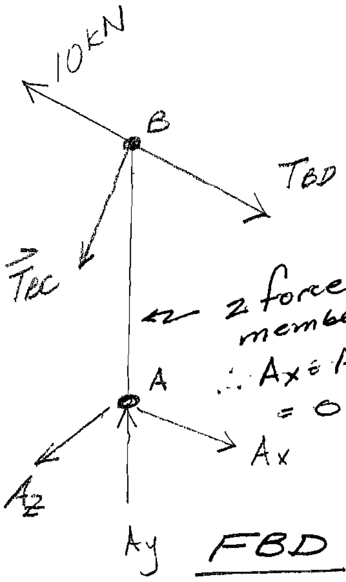
Question 4

A *6 m* long pole has a ball-and socket joint at *A* and is supported by two *(2)* cables, *BD* and *BC* as shown in the figure below. A *10 kN* force acting in the *x-y* plane and parallel to the *x* axis is applied to the pole at *B*. Determine:

a) The tensions in the two cables,
b) The reactions at the ball-and-socket joint at *A*,
c) The angle between cables *BC* and *BD* and
d) The moment of the *10 kN* force about the line *CD*.

A : (0, 0, 0)
B : (0, 6, 0)
C : (0, 0, 6)
D : (6, 0, -3)





$$\vec{T}_{BC} = T_{BC}\vec{\lambda}_{BC} \qquad \vec{\lambda}_{BC} = \frac{\vec{BC}}{BC} \qquad \vec{BC} = 0\hat{i} - 6\hat{j} + 6\hat{k}$$

$$BC = \sqrt{(-6)^2 + (6)^2} = \sqrt{72}$$

$$\vec{T}_{BC} = T_{BC}\frac{(-6\hat{j} + 6\hat{k})}{\sqrt{72}} \qquad \vec{T}_{BC} = -\frac{6}{\sqrt{72}}T_{BC}\hat{j} + \frac{6}{\sqrt{72}}T_{BC}\hat{k}$$

$$\vec{T}_{BD} = T_{BD}\vec{\lambda}_{BD}$$

$$\vec{\lambda}_{BD} = \frac{\vec{BD}}{BD} \qquad \vec{BD} = 6\hat{i} - 6\hat{j} - 3\hat{k}$$

$$BD = \sqrt{6^2 + (-6)^2 + (-3)^2} = 9$$

$$\vec{T}_{BD} = T_{BD}\frac{(6\hat{i} - 6\hat{j} - 3\hat{k})}{9}$$

$$= \frac{2}{3}T_{BD}\hat{i} - \frac{2}{3}T_{BD}\hat{j} - \frac{1}{3}T_{BD}\hat{k}$$

$$\Sigma F_x = 0$$

$$-10 + A_x + \frac{2}{3} T_{BD} = 0 \qquad (1)$$

$$\Sigma F_y = 0$$

$$A_y - \frac{6}{\sqrt{72}} T_{BC} - \frac{2}{3} T_{BD} = 0 \qquad (2)$$

$$\Sigma F_z = 0$$

$$A_z + \frac{6}{\sqrt{72}} T_{BC} - \frac{T_{BD}}{3} = 0 \qquad (3)$$

$$\Sigma M_A = 0 \qquad \vec{M}_A = \vec{r}_{AB} \times (-10\hat{i}) + \vec{r}_{AB} \times \vec{T}_{BC} + \vec{r}_{AB} \times \vec{T}_{BD}$$

$$\vec{r}_{AB} = 6j$$

$$\vec{M}_A = \begin{vmatrix} i & j & k \\ 0 & 6 & 0 \\ -10 & 0 & 0 \end{vmatrix} + T_{BC} \begin{vmatrix} i & j & k \\ 0 & 6 & 0 \\ 0 & -\frac{6}{\sqrt{72}} & \frac{6}{\sqrt{72}} \end{vmatrix}$$

$$+ T_{BD} \begin{vmatrix} i & j & k \\ 0 & 6 & 0 \\ \frac{2}{3} & -\frac{2}{3} & -\frac{1}{3} \end{vmatrix}$$

$$= \left| \begin{matrix} i & j & k \\ 0 & 6 & 0 \\ -10 & 0 & 0 \end{matrix} \middle| \begin{matrix} i & j \\ 0 & 6 \\ -10 & 0 \end{matrix} \right| + T_{BC} \left| \begin{matrix} i & j & k \\ 0 & 6 & 0 \\ 0 & -\frac{6}{\sqrt{72}} & \frac{6}{\sqrt{72}} \end{matrix} \middle| \begin{matrix} i & j \\ 0 & 6 \\ 0 & -\frac{6}{\sqrt{72}} \end{matrix} \right|$$

$$+ T_{BD} \left| \begin{matrix} i & j & k \\ 0 & 6 & 0 \\ \frac{2}{3} & -\frac{2}{3} & -\frac{1}{3} \end{matrix} \middle| \begin{matrix} i & j \\ 0 & 6 \\ \frac{2}{3} & -\frac{2}{3} \end{matrix} \right|$$

$$= [0 \quad 0 \quad 0] - [-60\hat{k}] + T_{BC}\left\{ \left[\frac{36}{\sqrt{72}} \hat{i} \right] - [0] \right\}$$

$$+ T_{BD}\left\{ [-2\hat{i}] - [4k] \right\}$$

$$\vec{M}_A = 60\hat{k} + \frac{36}{\sqrt{72}} T_{BC} \hat{i} - 2 T_{BD} \hat{i} - 4 T_{BD} \hat{k}$$

$\Sigma M_x = 0$

$\frac{36}{\sqrt{72}} T_{BC} - 2 T_{BD} = 0$

$\Sigma M_z = 0$

$60 - 4 T_{BD} = 0$

$T_{BD} = \frac{60}{4} = 15\ kN$

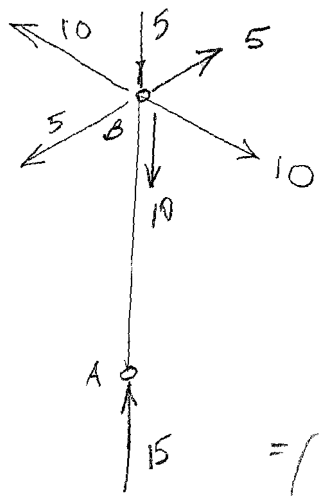
$\frac{36}{\sqrt{72}} T_{BC} - 2(15) = 0$

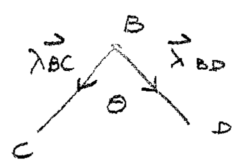
$T_{BC} = \frac{30\sqrt{72}}{36} = 7.07\ kN$

$A_x = 10 - \frac{2}{3}(15) = 0$

$A_y = \frac{6}{\sqrt{72}}(7.07) + \frac{2}{3}(15) = 15$

$A_z = -\frac{6}{\sqrt{72}}(7.07) + \frac{15}{3} = 0$





$\cos\theta = \vec\lambda_{BC} \cdot \vec\lambda_{BD}$

$= \left(\frac{-6\hat j + 6\hat k}{\sqrt{72}}\right) \cdot \left(\frac{6\hat i - 6\hat j - 3\hat k}{9}\right)$

$(-0.707\hat j + 0.707\hat k)\cdot$

$= \left(\frac{-6}{\sqrt{72}}\right)\left(-\frac{6}{9}\right) + \left(\frac{6}{\sqrt{72}}\right)\left(-\frac{3}{9}\right)$

$\theta = 76.37°$

$M_{CD} = \vec\lambda_{CD} \cdot (\vec r_{CB} \times -10\hat i)$

$\vec\lambda_{CD} = \frac{\vec{CD}}{CD}$

$\vec{CD} = 6\hat i + 0\hat j - 9\hat k$

$CD = \sqrt{6^2 + (-9)^2} = \sqrt{117}$

$\vec\lambda_{CD} = \frac{6\hat i - 9\hat k}{\sqrt{117}}$

$\vec r_{CB} = 0\hat i + 6\hat j - 6\hat k$

$= \begin{vmatrix} 0.555 & 0 & -0.832 \\ 0 & 6 & -6 \\ -10 & 0 & 0 \end{vmatrix} =$

$$M_{CD} = \frac{1}{\sqrt{117}} \begin{vmatrix} 6 & 0 & -9 \\ 0 & 6 & -6 \\ -10 & 0 & 0 \end{vmatrix}\begin{matrix} 6 & 0 \\ 0 & 6 \\ -10 & 0 \end{matrix}$$

$$= \frac{1}{\sqrt{117}}\left\{[0] - [540]\right\}$$

$$= -49.92\,kN\,m$$