Dec 2008
Final

THE UNIVERSITY OF MANITOBA

COURSE Dec 2008 final.

YEAR ______ SECTION ______ NAME Solutions by Dawit
ydawit@yahoo.com.

DATE ______ PROBLEM ______ SHEET ______ OF ______

1. B

$\vec{r}(t) = 2.0t^2\hat{i} - 5.0t^2\hat{j} + t^2\hat{k}$

$\vec{v}(t) = 4.0t\,\hat{i} - 10.0t\,\hat{j} + 2t\,\hat{k}$

$\vec{a}(t) = 4.0\hat{i} - 10.0\hat{j} + 2\hat{k}$

$\vec{F} = m\vec{a} = 0.3[4.0\hat{i} - 10.0\hat{j} + 2\hat{k}] = 1.2\hat{i} - 3.0\hat{j} + 0.6\hat{k}$

$P = \vec{F}\cdot\vec{v} = [1.2\hat{i} - 3.0\hat{j} + 0.6\hat{k}]\cdot[8.0\hat{i} - 20.0\hat{j} + 4\hat{k}]$

$= 9.6 + 60 + 2.4 =$

$\boxed{P = 72\ W}$

2. E

$\vec{r}(t) = 0.5t^3\hat{i} - 3.0\hat{j} + (0.5t^2 - t)\hat{k}$

$\vec{v}(t) = 1.5t^2\hat{i} + (1.0t - 1)\hat{k}$

$\vec{L} = \vec{r}\times\vec{p} = \vec{r}\,m\vec{v} = m\,\vec{r}\times\vec{v}$

$\vec{r}(1) = 0.5\hat{i} - 3.0\hat{j} - 0.5\hat{k}$

$\vec{v}(1) = 1.5\hat{i}$

$\vec{r}\times\vec{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0.5 & -3 & -0.5 \\ 1.5 & 0 & 0 \end{vmatrix} = -0.75\hat{j} + 4.5\hat{k}$

$\vec{L} = m\,\vec{r}\times\vec{v} = 0.1[-0.75\hat{j} + 4.5\hat{k}]$

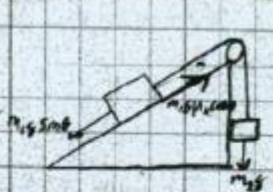
$= -0.075\hat{j} + 0.045\hat{k}$

$|\vec{L}| = \sqrt{0.075^2 + 0.45^2} = \boxed{0.456\ m^2/s}$

3. C

Compare forces due to weight:

$m_1 g\sin\theta = 98.827$ ↓ $m_2 g = 13.72\ N$



$a = \frac{F_{net}}{m_T} = \frac{m_1 g\sin\theta - m_1 g\mu_k\cos\theta - m_2 g}{m_1 + m_2} =$

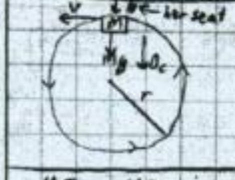
$\boxed{a = 0.86\ m/s^2}$

4. E

$T = \frac{mv^2}{r}$

$\therefore\ \frac{m_1 v^2}{r} = m_2 g \rightarrow m_2 = \frac{m_1 v^2}{rg} = \frac{6\cdot 9}{1.5\cdot 9.8} = \boxed{3.67\ kg}$

5. D



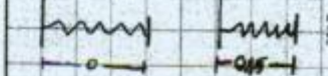
$N + mg = \frac{mv^2}{r}$

$v = \sqrt{\frac{(N + mg)r}{m}}$ = V_{min} happens when $N = 0$ ∴

$V = \sqrt{\frac{\cancel{m}gr}{\cancel{m}}} = \sqrt{gr} = \boxed{31.3\ m/s}$

6. A

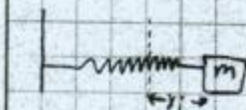
$ME_i = ME_f \rightarrow \frac{1}{2}mV_i^2 + \cancel{\frac{1}{2}kx_i^2}^{\,0} = \frac{1}{2}mV_f^2 + \frac{1}{2}kX_f^2$



$V_F = \sqrt{V_i^2 - \frac{k}{m}x_f^2} = \sqrt{6.0^2 - \frac{2000}{2.0}(-0.15)^2} = \boxed{3.67\ m/s}$

7. B

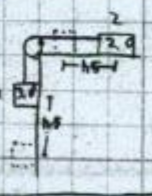
$ME_i = \frac{1}{2}kx_i^2 = 4.48\ J$



$W_{friction} = F_{fric}\,x_i = 30(0.08) = 2.4\ J$

$KE = ME - W_{fric} = 4.48 - 2.4 = \boxed{2.1\ J}$

8. D



gravitational pot. energy

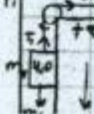
$i: 1: \Delta GPE = -mgy = -3(9.8)(1.5) = -44.1\ J$

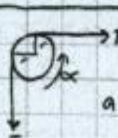
$2: \Delta GPE = 0$ b/c same height

$\Delta KE = \overset{\frac{1}{2}m_{tot}v^2}{\frac{1}{2}(2+3)(3.8)^2} = 36.1\ J$

$W = \Delta PE + \Delta KE = -8\ J$

9. A





$a = \alpha r$

$m_1 g - T_1 = m_1 a$

$T_2 = m_2 a$

$\tau - \tau_1$

$T_1\cancel{r} - T_2\cancel{r} = \frac{I\alpha}{r}$

$T_1 - T_2 = \frac{I}{r}\alpha\frac{a}{r}$

$m_1 g - (m_1 + m_2)a = \frac{I}{r^2}a$

$m_1 g = (m_1 + m_2 + \frac{I}{r^2})a$

$a = \frac{m_1 g}{m_1 + m_2 + \frac{I}{r^2}}$

THE UNIVERSITY OF MANITOBA

YEAR ______ NAME ______

SECTION ______

COURSE ______ DATE ______ PROBLEM ______ SHEET ______ OF ______

$m_1 > m_2$

$a = \frac{F_{net}}{M_T} = \frac{m_1 g - m_2 g}{m_1 + m_2} = \frac{(m_1 - m_2)g}{m_1 + m_2}$

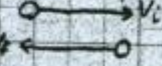
If pulley has mass (given in I):
$I = mr^2 \rightarrow m = \frac{I}{r^2}$ and then
$a = \frac{(m_1 - m_2)g}{m_1 + m_2 + \frac{I}{r^2}}$

[A] 10. $(V_{cm})_x = \frac{m_1 v_{1x} + m_2 v_{2x} \ldots}{m_T}$ $(V_{cm})_y = \ldots$ $V_{cm} = \sqrt{(V_{cm})_x^2 + (V_{cm})_y^2}$

$(V_{cm})_x = \frac{m_1 v_{1x}}{m_T} = 9.7 \text{ m/s}$ $(V_{cm})_y = \frac{m_2 v_{2y}}{m_T} = 13.3 \text{ m/s}$ $V_{cm} = \sqrt{9.7^2 + 13.3^2} = 16.4 \rightarrow \boxed{16 \text{ m/s}}$

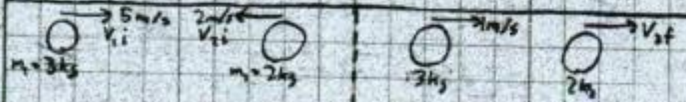
[D] 11. $m_T x_{cm} = \sum m_i x_i \Rightarrow$ origin $x_{cm} = 0$
$m_T y_{cm} = \sum m_i y_i \Rightarrow$ $y_{cm} = 0$
$0 = 40(3) + 50(-2) + 20x_3$ $x_3 = -1$ $\boxed{(-1, 7)}$
$0 = 40(4) + 50(-6) + 20y_3$ $y_3 = 7$

[B] 12. Impulse = $m \Delta v$



Impulse = 4kg(−3 − 5) = −32 N |Impulse| = $\boxed{32 N}$

[E] 13.



$P_i = P_f$
$3(5) + 2(-2) = 3(1) + 2V_{2f} \rightarrow V_f = 4 \text{ m/s}$
$E_i = \frac{1}{2}(3)5^2 + \frac{1}{2}(2)2^2 = 41.5 \text{ J}$
$E_f = \frac{1}{2}(3)1^2 + \frac{1}{2}(2)4^2 = 17.5 \text{ J}$
$E_f - E_i = \boxed{-24}$

[C] 14. $\alpha = \pi \text{ rad/s}^2$ $\omega_i = ?$
$\theta = \pi$
$\omega_f = 2\pi \text{ rad/s}$
$\omega_i = \sqrt{\omega_f^2 - 2\alpha\theta} = \sqrt{4\pi^2 - 2\pi^2} = \sqrt{2\pi^2} = \boxed{\sqrt{2}\pi}$

[C] 16. $\alpha = 2 \text{ rad/s}^2$ $W = \Delta KE = \frac{1}{2} I (\omega_f^2 - \cancel{\omega_i^2}^{0})$
$\omega_i = 0 \text{ rad/s}$ $W = 3(100) = \boxed{300 \text{ J}}$
$t = 5 \text{ s}$
$\omega_f = \omega_i + \alpha t = 10 \text{ rad/s}$

[C] 18. $p = mv$ $I_i \omega_i = I_f \omega_f$
$l = I\omega$ $I\omega = (I + 2I)\omega_f$ $\omega_f = \frac{\cancel{I}}{3\cancel{I}} \omega_o = \boxed{\frac{1}{3}\omega_o}$

[E] 19.

$L_i = L_f$ $\xrightarrow{}$ I of boy
$mvr = (mr^2 + I)\omega$ $\omega = \frac{mrv}{mr^2 + I}$

[D] 20.

$l = l_{pulley} + l_{m_1} + l_{m_2}$ $v = r\omega$
$= I\omega + m_1 v r + m_2 v r$ $\omega = \frac{v}{r}$
$= \frac{Iv}{r} + m_1 v r + m_2 v r$
$= \left[\frac{I}{r} + (m_1 + m_2) r\right] v$

E

THE UNIVERSITY OF MANITOBA

YEAR ______ NAME ______

SECTION ______

COURSE ______

DATE ______ PROBLEM ______ SHEET ______ OF ______

21.

consern of p along both axes

x: $p_{ix} = p_{fx}$

x: $2(3) = 2(1.5)\cos 30 + 1V_{2f}\cos\theta$

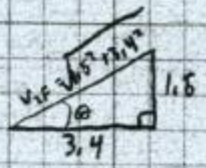
y: $p_{iy} = p_{fy}$

$0 = 2(1.5)\sin 30 - 1V_{2f}\sin\theta$

$\boxed{V_{2f}\cos\theta = 3.4}$

$V_{2f}\sin\theta = 1.5$

B



3.72

$\theta = \tan^{-1}\left(\frac{1.5}{3.4}\right) = 23.8°$

22.

$u_x = \frac{u_x' + V}{1 + \frac{u_x' V}{c^2}}$

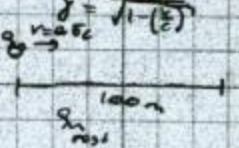
$u_x' = \frac{u_x - V}{1 - \frac{u_x V}{c^2}}$

$V = -0.8c$

E

$= \frac{0.7c - (-0.8c)}{1 - \frac{0.7c(-0.8c)}{c^2}} = \frac{(0.7 + 0.8)c}{1 + (0.7)(0.8)} = \boxed{0.96c}$

23. $L_{\substack{\text{proper}\\\text{length}}} = \gamma L_{\substack{\text{contracted}\\\text{length}}}$



$\gamma = \frac{1}{\sqrt{1-\left(\frac{v}{c}\right)^2}}$ let $\frac{v}{c} = \beta$ $V = \beta c$

$\beta = 0.5$ $\therefore$ $\gamma = \frac{1}{\sqrt{1-\beta^2}} = 1.156$

$L = \frac{L_0}{\gamma} = \frac{100}{1.155} = \boxed{86.6}$

24. $\Delta t = \frac{L_0}{\beta c} = \frac{100}{0.5 \times (3 \times 10^8)} = 6.67 \times 10^{-7}\,\text{s}$ $\boxed{667\,\text{ns}}$

25. $\Delta t = \frac{86.6}{0.5 \times (3 \times 10^8)} = 5.77 \times 10^{-7}\,\text{s}$ $\boxed{577\,\text{ns}}$ OR $\frac{667}{\gamma} = \boxed{587\,\text{ns}}$