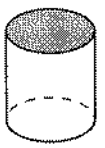
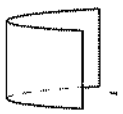
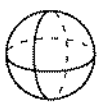
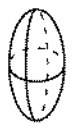
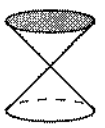
QUADRIC SURFACES

name	equation in standard form	$x =$ const cross–section	$y =$ const cross–section	$z =$ const cross–section	sketch
plane	$ax + by + cz = d$	line	line	line	
elliptic cylinder	$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$	two lines	two lines	ellipse	
parabolic cylinder	$y = ax^2$	one line	two lines	parabola	
sphere	$x^2 + y^2 + z^2 = d^2$	circle	circle	circle	
ellipsoid	$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$	ellipse	ellipse	ellipse	
elliptic paraboloid	$\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{z}{c}$	parabola	parabola	ellipse	
elliptic cone	$\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{z^2}{c^2}$	two lines if $x = 0$ hyperbola if $x \neq 0$	two lines if $y = 0$ hyperbola if $y \neq 0$	ellipse	
hyperboloid of one sheet	$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$	hyperbola	hyperbola	ellipse	
hyperboloid of two sheets	$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = -1$	hyperbola	hyperbola	ellipse	
hyperbolic paraboloid	$\frac{y^2}{b^2} - \frac{x^2}{a^2} = \frac{z}{c}$	parabola	parabola	hyperbola	

IDENTIFYING A QUADRIC SURFACE FROM THE FORM OF ITS EQUATION

Equation	$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$	$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1$	$\frac{z^2}{c^2}-\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$	$z^2-\frac{x^2}{a^2}-\frac{y^2}{b^2}=0$	$z-\frac{x^2}{a^2}-\frac{y^2}{b^2}=0$	$z-\frac{y^2}{b^2}+\frac{x^2}{a^2}=0$
Characteristic	No minus signs	One minus sign	Two minus signs	No linear terms	One linear term; two quadratic terms with the same sign	One linear term; two quadratic terms with opposite signs
Classification	Ellipsoid	Hyperboloid of one sheet	Hyperboloid of two sheets	Elliptic cone	Elliptic paraboloid	Hyperbolic paraboloid

→ $Ax+By+Cz=D$ → plane

→ linear and qudratic terms of the same variable ⇒ Shift

→ product of linear terms ⇒ Rotation about the 3rd variable axis

Ex: $z=xy$ ⇒ ($z=x^2-y^2$ rotated about the z-axis by 45°)

→ Any Surface Area involving the above surface bounded by a Cylinder (of radius a) can be done in the form of

$$8\int_0^{\pi/4}\int_0^{a}\int_0^{r^2\cos\theta\sin\theta}\sqrt{1+r^2}\,r\,dz\,dr\,d\theta$$

→ Any volume integral can also be performed as above

→ given $x^2+y^2=a^2$ ⇒ $y=\pm\sqrt{a^2-x^2}$ $\begin{cases}+\sqrt{a^2-x^2}\\ -\sqrt{a^2-x^2}\end{cases}$

⇒ $x=\pm\sqrt{a^2-y^2}$ $\begin{cases}+\sqrt{a^2-y^2}\\ -\sqrt{a^2-y^2}\end{cases}$