MATH 2130 Problem Workshop 4

1. If $z = f(u, v, t), u = g(x, y, t), v = h(x, y, t)$ and $y = k(t)$, find the chain rule for $\left.\dfrac{\partial z}{\partial t}\right)_x$

2. If $f(s)$ and $g(t)$ are differentiable functions, show that $\nabla f(x^2 - y^2) \cdot \nabla g(xy) = 0$.

3. If $z = x^2 + y^2, x = u\cos v$, and $y = u\sin v$, find and simplify $\left.\dfrac{\partial^2 z}{\partial v^2}\right)_u$.

4. If $f(v)$ is differentiable, show that $u(x, y) = x^3 f(x/y)$ satisifes the equation

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 3u.$$

5. The equations

$$x^2 + y + 3s^2 + s = 2t - 1, \qquad y^2 - x^4 + 2st + 7 = 6s^2t^2$$

define s and t are functions of x and y. Find $\dfrac{\partial s}{\partial x}$ when $s = 0$ and $t = 1$. Assume $x > 0$.

6. The equations

$$x = r\sin\phi\cos\theta, \qquad y = r\sin\phi\sin\theta, \qquad z = r\cos\phi$$

define r, ϕ, θ as functions of x, y and z. Find $\dfrac{\partial \phi}{\partial y}$.

7. Find the rate of change of the function $f(x, y, z) = \sin(xy) - z^3$ at the point $(2, 0, 3)$ in the direction of the upward normal to the surface $xz^2 - x^2z = 6$.

8. Find equations for the tangent line to the following curve at the point $(1, -1, 3)$:

$$xyz + z^3 = 24, \qquad x^3y^2z + y^3 = 4x - 2.$$

9. Find an equation for the tangent plane to the following surface at the point $(2, -1, -1)$:

$$x^2y + y^2z + z^2x + 3 = 0.$$

10. Find all critical points for the function $f(x, y) = x^3y^3 - x^2y^2 + 6$.

11. Find all critical points for the function $f(x, y) = x^3y^2 - xy + 3y$.

12. Find and classify all critical points of the function as giving relative minima, maxima, saddle points or neither.

    (a) $f(x, y) = x^3 + xy + y^3$

(b) $f(x,y) = x^3 - xy^2 + 3xy$

(c) $f(x,y) = x^4 - 3x^2y^2 + y^4$

(d) $f(x,y) = y^2 + |x-1|$

Answers:

1. $\frac{\partial z}{\partial u}\frac{\partial u}{\partial y}\frac{dy}{dt} + \frac{\partial z}{\partial u}\frac{\partial u}{\partial t} + \frac{\partial z}{\partial v}\frac{\partial v}{\partial y}\frac{dy}{dt} + \frac{\partial z}{\partial v}\frac{\partial v}{\partial t} + \frac{\partial z}{\partial t}$
2.
3. 0
4.
5. 16
6. $r^{-1}\cos\phi\sin\theta$
7. $-216/\sqrt{73}$
8. $x = 1 + 81t, y = -1 + 133t, z = 3 - 6t$
9. $x - 2y + z = 3$.
10. All points on the x-axis, y-axis and on the curve $y = 2/(3x)$.
11. $(3,0), (9, 1/243)$
12. (a) $(0,0)$ gives a saddle point. $(-1/3, -1/3)$ gives a relative maximum.
    (b) $(0,0), (0,3)$ both give saddle points.
    (c) $(0,0)$ gives a saddle point.
    (d) $(1,0)$ gives a relative minimum. Points $(1,y)$ for $y \neq 0$ give neither.