MATH 1210 Fall 2013 Assignment 1

Attempt all questions and show all your work. The assignment is due Friday, September 27.

1. Show the following are true by induction:

   (a) $3^3 + 3^5 + \cdots + 3^{2n-1} = \frac{1}{8}\big(3^{2n+1} - 27\big)$ for all $n \geq 2$.

   (b) 6 divides $n(n^2+5)$ for all $n \geq 1$.

   (c) For $n \geq 1$,
   $$\frac{1}{1^2} + \frac{1}{2^2} + \cdots + \frac{1}{n^2} \leq 2 - \frac{1}{n}.$$

2. Show for all $n \geq 1$ that
   $$\sum_{i=n}^{3n} i^2 = \frac{26n^3 + 15n^2 + n}{3}:$$

   (a) by using summation formulas $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$ and/or $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6.}$

   (b) by induction.

3. Possibly using the summation formulas in question 2a, find the value of the following summations.

   (a) $\sum_{i=10}^{40} (2i-19)^2$

   (b) $\sum_{j=-20}^{59} \big((j+21)^2 - 4(j+21)\big)$

4. Simplify $\frac{(2i-5)(3-2i)}{(3+i)^2}$ in Cartesian form.

5. Find all 5^{th} roots of $4-4i$. Leave your answers in Polar form.

6. Find z^{20} if $z = \sqrt{3} - i$. Leave your answers in Cartesian form.

7. Solve the following equations. Leave your answers in Cartesian form.

   (a) $(\overline{3+2i})z = i^6(1+2i)(3-4i)$

   (b) $z^4 - 2z^2 + 4 = 0$

8. Show that
   $$1 + e^{2\pi i/5} + e^{4\pi i/5} + e^{6\pi i/5} + e^{8\pi i/5} = 0.$$
   (Hint: If $z = e^{2\pi i/5}$, what is $z^5 - 1$?)