[12] 1. Find each of the following limits or explain why it does not exist.

(a) $\displaystyle\lim_{(x,y)\to(0,-2)} \frac{x^4y + 2x^4}{x^8 + y^2 + 4y + 4}$ $\frac{0}{0}$

(b) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^2y}{\sqrt{x^4 + y^4 + 2x^2y^2}}$

[8] 2. Find a formula for $\dfrac{dz}{dt}$ if $z = f(x,y,s,t)$, $x = g(s,t)$, $y = h(t)$ and $s = k(t)$. $\neq$

[6] 3. Find $f(x,y)$ if $f(0,1) = 5$ and $\nabla f(x,y) = (ye^x, e^x + 1)$.

[12] 4. Let $u(x,y) = f(y^2 - x) + g(y^2 - x)$, where f and g are twice differentiable functions. Show that

$$4y^2 \frac{\partial^2 u}{\partial x^2} - \frac{\partial^2 u}{\partial y^2} - 2\frac{\partial u}{\partial x} = 0.$$

f g

u

[12] 5. If $z = \ln(u^2 + v^2)$, where u and v are functions of x defined by

$$x = u^2 - e^{u^2} + \sqrt{2}, \quad v^2e^x + 2v^3x - \sqrt{2} = 0,$$

find $\dfrac{dz}{dx}$ and simplify your answer..