DEPARTMENT & COURSE NO: MATH 2130
COURSE: Engineering Mathematical Analysis 1

EXAMINERS: Lui, Williams

PART A Clearly write down your final answers to each question in the box provided. We will mark *only* what is written in the box. Note Q.3 and 6 would have possible partial marks.

[4] 1. Given the parametric equation of a line: $x(t) = 1 + 2t,\ y(t) = -1 + 3t,\ z(t) = t$. Find the length of the line segment between the points defined by $t = 0$ and $t = 2$.

$t=0$: $(1,-1,0)$

$t=2$: $(5,5,2)$

length $= \sqrt{4^2+6^2+2^2}$

$\boxed{\sqrt{56} = 2\sqrt{14}}$

[4] 2. Find all values of the constant α so that the line defined by $\dfrac{x-1}{\alpha} = y = z+1$ is parallel to the plane $4x + 5z + 4 = 0$.

direction of line is $(\alpha,1,1)$ which is perpendicular to the normal $(4,0,5)$ of the plane.

$(\alpha,1,1)\cdot(4,0,5) = 0$

$4\alpha + 5 = 0$

$\boxed{\alpha = -5/4}$

[4] 3. Given the vector equation of a curve $\mathbf{r}(t) = (2, 3t, 5t^2)$. Set up but DO NOT EVALUATE an integral for the length of the curve between the points $(2,0,0)$ and $(2,6,20)$.

$r'(t) = (0,3,10t)$

length $= \int_0^2 |r'(t)|\,dt$

$\boxed{\int_0^2 \sqrt{9+100t^2}\,dt}$

[4] 4. Let $f(x,y) = \begin{cases} \alpha, & (x,y) = (0,0); \\ \dfrac{\sin(3x+6y)}{x+2y}, & (x,y) \neq (0,0). \end{cases}$ Find the value of α so that f is continuous at $(0,0)$.

let $u = x+2y$

$$\lim_{(x,y)\to(0,0)} f(x,y) = \lim_{u\to 0} \frac{\sin 3u}{u}$$

$$= \lim_{u\to 0} \frac{3\cos 3u}{1} \quad \text{(L'Hopital)}$$

$$= 3$$

$$\boxed{\alpha = 3}$$

[4] 5. Evaluate $\lim\limits_{(x,y)\to(0,0)} \dfrac{2x^2+y^2}{x^2+y^2}$.

approach along x-axis $(y=0)$: $\lim\limits_{x\to 0} \dfrac{2x^2}{x^2} = 2$

approach along y-axis $(x=0)$: $\lim\limits_{y\to 0} \dfrac{y^2}{y^2} = 1$.

$$\boxed{\text{D.N.E.}}$$

[4] 6. Let $f(x,y) = \sin(x^2 + e^{3y})$. Find $\dfrac{\partial f}{\partial x}$ and $\dfrac{\partial^2 f}{\partial x \partial y}$.

$$\frac{\partial f}{\partial x} = 2x\cos(x^2+e^{3y})$$

$$\frac{\partial^2 f}{\partial x \partial y} = 2x(-1)\sin(x^2+e^{3y})\, e^{3y}\cdot 3$$

$$\boxed{\frac{\partial f}{\partial x} = 2x\cos(x^2+e^{3y}), \quad \frac{\partial^2 f}{\partial x \partial y} = -6x\sin(x^2+e^{3y})\,e^{3y}}$$

PART B For questions in this part, show all your work.

[6] 7. Let $z = xyt + 3xy^2$, $x = t + e^t$, $y = t\sin t$. Find $\frac{dz}{dt}$. Write your answer as a function of t alone. There is no need to simplify.

$$\frac{dz}{dt} = \frac{\partial z}{\partial x}\frac{dx}{dt} + \frac{\partial z}{\partial y}\frac{dy}{dt} + \frac{\partial z}{\partial t}$$

$$= (yt + 3y^2)(1+e^t) + (xt + 6xy)(\sin t + t\cos t) + xy$$

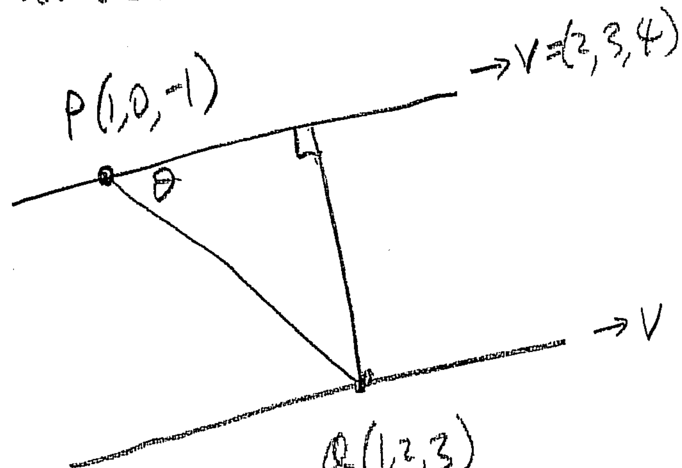
$$= (t^2\sin t + 3t^2\sin^2 t)(1+e^{-t}) +$$

$$(t^2 + te^{-t} + 6(t+e^t)\,t\sin t)(\sin t + t\cos t) +$$

$$(t+e^t)\,t\sin t$$

[8] 8. Find the distance between the line $\frac{x-1}{2} = \frac{y}{3} = \frac{z+1}{4}$ and the line with vector equation $\mathbf{r}(t) = (1,2,3) + t(4,6,8)$.

Note that the lines are parallel.



$$\text{distance} = |PQ|\,|\sin\theta|$$

$$= \left| PQ \times \frac{v}{|v|} \right|$$

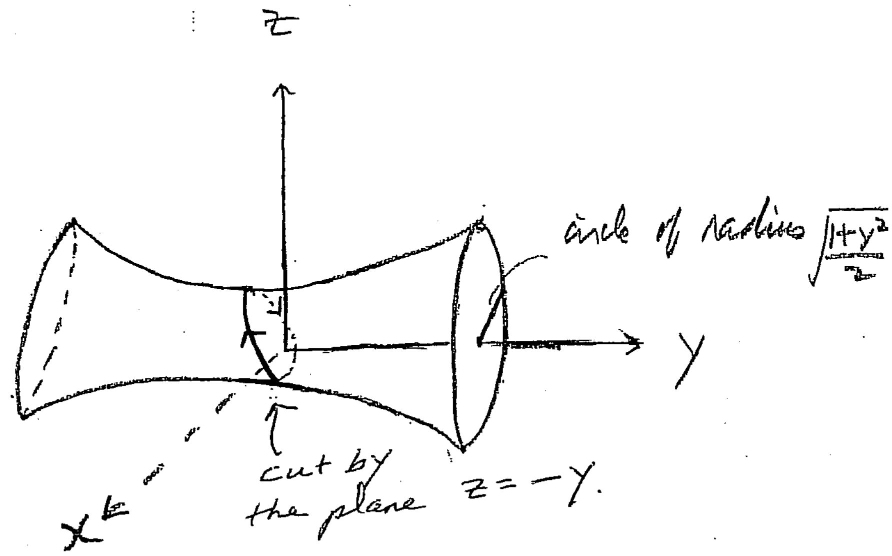
$$= \frac{|(0,2,4)\times(2,3,4)|}{\sqrt{4+9+16}}$$

$$= \frac{|(-4,8,-4)|}{\sqrt{29}}$$

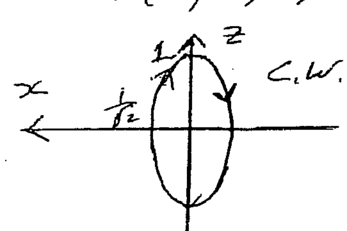
$$= \frac{4\sqrt{6}}{\sqrt{29}}$$

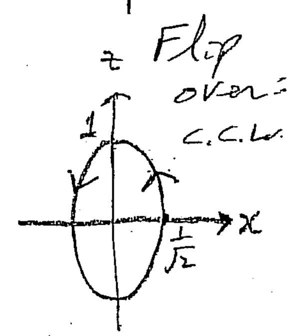
[12] 9. Sketch the surface $y^2 - 2x^2 - 2z^2 + 1 = 0$. Find parametric equations for the curve given by the intersection of the above surface and the plane $y + z = 0$. The curve is oriented in the clockwise direction when viewing from $(0, 10, 0)$.

$$1 + y^2 = 2x^2 + 2z^2$$



Looking from $(0,10,0)$:





$$y = -z \Rightarrow \quad 1 + z^2 = 2x^2 + 2z^2$$

$$1 = 2x^2 + z^2$$

$$1 = \frac{x^2}{\left(\frac{1}{\sqrt{2}}\right)^2} + z^2$$

$\therefore \quad x = \frac{1}{\sqrt{2}}\cos t, \quad z = \sin t, \quad y = -\sin t$

$0 \le t \le 2\pi$

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or $\quad x = -\frac{1}{\sqrt{2}}\sin t, \quad z = \cos t, \quad y = -\cos t$

$0 \le t \le 2\pi$