MATH 1210 Assignment 2 Fall 2016 **Due date: October21**

1. **Find all 4^{th} roots of -8 in Cartesian form. Simplify as much as possible.**

Solution: Either simply notice that $-8 = 8(-i) = 8e^{i\pi}$, or say that $-8 = -8 + 0i$ so $r = \sqrt{(-8)^2 + 0^2} = 8$ and $\theta = \tan(\theta) = \frac{0}{-8}$ so $\theta = \pi$.

$$-8 = 8e^{i\pi}$$

Therefore all 4^{th} roots of -8 are of the form

$$z_k = \sqrt[4]{8e^{i\pi}} = \sqrt[4]{8}e^{i\frac{\pi+2k\pi}{4}}$$

where $k = -2, -1, 0, 1$.

If $k = -2$ then

$$z_{-2} = \sqrt[4]{8}e^{i\frac{\pi-4\pi}{4}} = \sqrt[4]{8}e^{i\frac{-3\pi}{4}}$$

If $k = -1$ then

$$z_{-1} = \sqrt[4]{8}e^{i\frac{\pi-2\pi}{4}} = \sqrt[4]{8}e^{i\frac{-\pi}{4}}$$

If $k = 0$ then

$$z_0 = \sqrt[4]{8}e^{i\frac{\pi}{4}}$$

If $k = 1$ then

$$z_1 = \sqrt[4]{8}e^{i\frac{\pi+2\pi}{4}} = \sqrt[4]{8}e^{i\frac{3\pi}{4}}$$

Therefore the 4^{th} roots of -8 are $\left\{\sqrt[4]{8}e^{i\frac{-3\pi}{4}}, \sqrt[4]{8}e^{i\frac{-\pi}{4}}, \sqrt[4]{8}e^{i\frac{\pi}{4}}, \sqrt[4]{8}e^{i\frac{3\pi}{4}}\right\}$.

Because

$$\cos\left(\frac{\pi}{4}\right) = \sin\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2} = 2^{\frac{-1}{2}}$$

and

$$\sqrt[4]{8} = 2^{\frac{3}{4}}$$

we have

$$\frac{\sqrt{2}}{2}\sqrt[4]{8} = 2^{\frac{-1}{2}}2^{\frac{3}{4}} = 2^{\frac{1}{4}} = \sqrt[4]{2}.$$

Then the above set of 4^{th} roots can be simplified to

$$\left\{-\sqrt[4]{2} - i\sqrt[4]{2}, \sqrt[4]{2} - i\sqrt[4]{2}, \sqrt[4]{2} + i\sqrt[4]{2}, -\sqrt[4]{2} + i\sqrt[4]{2}\right\}$$

2. **For each of the following statements, if it is true prove it, and if it is false give a counter example.**

   (a) $z = \dfrac{|\sqrt{2}z|^2}{2z}, \quad (z \neq 0)$**;**

   (b) $\overline{z}\,(z + z|z|) = |\overline{z}|^2\,(1 + |\overline{z}|)$**;**

   (c) $\dfrac{e^{4\theta^2 i}\, e^{i^5}}{(e^{\theta i})^4} = \cos(2\theta - 1)^2 + i\sin(2\theta - 1)^2$**.**

Solution:

(a) **It is true, because let** $z = x + yi$ **then**

$$\frac{|\sqrt{2}z|^2}{\overline{2z}} = \frac{(\sqrt{2})^2\,|z|^2}{2\,\overline{z}} = \frac{|z|^2}{\overline{z}} = \frac{x^2+y^2}{x-yi} = \frac{x^2+y^2}{x-yi}\times\frac{x+yi}{x+yi} = \frac{(x^2+y^2)(x+yi)}{x^2+y^2} = x+yi = z.$$

(b) **It is true, because** $|z| = |\overline{z}|$ **and** $\overline{z}z = |z|^2$ **and**

$$\overline{z}\,(z + z|z|) = \overline{z}z\,(1+|z|) = \overline{z}z\,(1+|\overline{z}|) = |z|^2\,(1+|\overline{z}|)\,.$$

(c) **It is true, because** $i^5 = i$ **and** $(e^{\theta i})^4 = e^{4\theta i}$

$$\begin{aligned}\frac{e^{4\theta^2 i}\,e^{i^5}}{(e^{\theta i})^4} &= \frac{e^{4\theta^2 i}\,e^{i}}{e^{4\theta i}}\\ &= e^{4\theta^2 i}\,e^{i}\,e^{-4\theta i} = e^{(4\theta^2-4\theta+1)\,i} = e^{(2\theta-1)^2\,i} = \cos(2\theta-1)^2 + i\sin(2\theta-1)^2\,.\end{aligned}$$

3. Let $P(x) = 8x^4 - 2kx^3 + 2k^2x^2 + \frac{k}{2}$**, where** k **is a complex number. Find all values of** k **such that the remainder of** $P(x)$ **divided by** $2x-1$ **is** $\frac{15}{32} + \frac{1}{32}i$**.**

Solution:

By the Remainder Theorem the remainder of $P(x)$ **divided by** $2x-1$ **is** $P(\frac{1}{2})$**.**

$$\begin{aligned}P\left(\frac{1}{2}\right) &= \frac{8}{2^4} - \frac{2k}{2^3} + \frac{2k^2}{2^2} + \frac{k}{2}\\ &= \frac{1}{2} - \frac{k}{4} + \frac{k^2}{2} + \frac{k}{2}\\ &= \frac{k^2}{2} + \frac{k}{4} + \frac{1}{2}\\ &= \frac{1}{4}\left(2k^2+k+2\right)\end{aligned}$$

Then the remainder of $P(x)$ **divided by** $2x-1$ **is** $\frac{15}{32} + \frac{1}{32}i$ **if and only if**

$$\frac{1}{4}\left(2k^2+k+2\right) = \frac{15}{32} + \frac{1}{32}i$$

which is equivalent to

$$2k^2 + k + 2 - \frac{15}{8} - \frac{i}{8} = 0$$

which is further equivalent to

$$2k^2 + k + \frac{1}{8} - \frac{i}{8} = 0$$

We then apply the quadratic formula to find k

$$
\begin{aligned}
k &= \frac{-1 \pm \sqrt{1^2 - 4(2)(\frac{1}{8} - \frac{1}{8}i)}}{4} \\
&= \frac{-1 \pm \sqrt{1^2 - (1-i)}}{4} \\
&= \frac{-1 \pm \sqrt{i}}{4}.
\end{aligned}
$$

Since $i = e^{i\frac{\pi}{2}}$ we have

$$\sqrt{i} = e^{i\frac{\pi}{4}} = \left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i\right).$$

Therefore k can take the values

$$k = \left(\frac{-1}{4} + \frac{\sqrt{2}}{8}\right) + \frac{\sqrt{2}}{8}i \quad \textbf{or} \quad k = \left(\frac{-1}{4} - \frac{\sqrt{2}}{8}\right) - \frac{\sqrt{2}}{8}i$$

which can reduce to

$$k = \left(\frac{-2+\sqrt{2}}{8}\right) + \frac{\sqrt{2}}{8}i \quad \textbf{or} \quad k = \left(\frac{-2-\sqrt{2}}{8}\right) - \frac{\sqrt{2}}{8}i.$$

4. Let $P(x) = 1 + \sum_{i=1}^{6}(-1)^i x^{2i} + \sum_{i=0}^{5}(-1)^i x^{2i+1}$.

Use Descartes' Rule of Signs to determine

(a) The number of positive real roots.

(b) The number of negative real roots.

(c) The total number of real roots. How many real linear factors does the total number of real roots imply, are their roots positive or negative, how many irreducible quadratics divide $P(x)$ (i.e. what configuration of real linear and irreducible quadratics does each number of total real roots imply)?

Solution:

(a)

$$P(x) = 1 \underbrace{+x - x^2}_{\textbf{sign change 1}} \underbrace{-x^3 + x^4}_{\textbf{sign change 2}} \underbrace{+x^5 - x^6}_{\textbf{sign change 3}} \underbrace{-x^7 + x^8}_{\textbf{sign change 4}} \underbrace{+x^9 - x^{10}}_{\textbf{sign change 5}} \underbrace{-x^{11} + x^{12}}_{\textbf{sign change 6}}$$

$\therefore$ **There are 6 or 4 or 2 or 0 real positive roots of $P(x)$ by Descartes' Rules of Signs.**

(b)

$$P(-x) = \underbrace{1 - x}_{\textbf{sign change 1}} \underbrace{-x^2 + x^3}_{\textbf{sign change 2}} \underbrace{+x^4 - x^5}_{\textbf{sign change 3}} \underbrace{-x^6 + x^7}_{\textbf{sign change 4}} \underbrace{+x^8 - x^9}_{\textbf{sign change 5}} \underbrace{-x^{10} + x^{11}}_{\textbf{sign change 6}} +x^{12}$$

$\therefore$ **There are 6 or 4 or 2 or 0 real negative roots of $P(x)$ by Descartes' Rules of Signs.**

(c) **There are either 0 or 2 or 4 or 6 or 8 or 10 or 12 real roots.**

# +ve roots	# -ve roots	Total # real roots	# irreducible quadratics
0	0	0	6
0	2	2	5
0	4	4	4
0	6	6	3
2	0	2	5
2	2	4	4
2	4	6	3
2	6	8	2
4	0	4	4
4	2	6	3
4	4	8	2
4	6	10	1
6	0	6	3
6	2	8	2
6	4	10	1
6	6	12	0

5. **For each of the following polynomials either use the Rational Root Theorem to make a list of possible rational roots or explain why the Rational Root Theorem cannot be used.**

   (a) $P(x) = 6x^5 + 3x^3 + 2x + 12$.

   (b) $Q(x) = 7x^4 + 10x^2 + 2x$.

   (c) $R(x) = 15x^4 + (8 - 6i)x^3 + 3x + x^2 + 1$.

Solution:

(a) $P(x)$ **has all integer coefficients and a non-zero constant term, therefore the Rational Root Theorem can be used to find possible rational roots of** $P(x)$**. Rational roots of** $P(x)$ **the form** $r = \frac{m}{n}$ **must satisfy the requirements that** m **divides** 12 **and** n **divides** 6**.**

Possible values for m **are** $\{\pm1, \pm2, \pm3, \pm4, \pm6, \pm12\}$.

Possible values for n **are** $\{\pm1, \pm2, \pm3, \pm6\}$

Possible values for $r = \frac{m}{n}$ **are** $\left\{\pm\frac{1}{6}, \pm\frac{1}{3}, \pm\frac{1}{2}, \pm\frac{2}{3}, \pm1, \pm\frac{4}{3}, \pm\frac{3}{2}, \pm2, \pm3, \pm4, \pm6, \pm12\right\}$

(b) $Q(x)$ **does not have a constant term, or alternatively** $a_0 = 0$ **for** $Q(x)$**. The Rational Root Theorem requires the polynomial has a non-zero constant term, which is not true for** $Q(x)$**, therefore the Rational Root Theorem cannot be used.**

(c) $R(x)$ **has a coefficient which is not an integer, namely** $(8 - 6i)$**. The Rational Root Theorem requires all coefficients are integers, which is not true for** $R(x)$**, therefore the Rational Root Theorem cannot be used.**

6. **For the following polynomials use the Bounds Theorem to determine an upper bound for the modulus of their roots.**

   (a) $P(x) = 6x^5 + 3x^3 + 2x + 12$.

   (b) $Q(x) = 10.5x^3 + 12x + 3$.

(c) $R(x) = 15x^4 + (8 - 6i)x^3 + 3x + x^2 + 1$.

Solution:

(a) **For** $P(x)$ **we have** $|a_n| = 6$ **and** $M = \max\{|3|, |2|, |12|\}$. **Therefore** $M = |12| = 12$. **Bounds Theorem states that if** x **is a root of** $P(x)$ **then**

$$|x| < \frac{M}{|a_n|} + 1$$

so in this case

$$|x| < \frac{12}{6} + 1 = 2 + 1 = 3.$$

Therefore 3 **is an upper bound for the modulus of roots of** $P(x)$.

(b) **For** $Q(x)$ **we have** $|a_n| = 10.5$ **and** $M = \max\{|12|, |3|\}$. **Therefore** $M = |12| = 12$. **Bounds Theorem states that if** x **is a root of** $Q(x)$ **then**

$$|x| < \frac{M}{|a_n|} + 1$$

so in this case

$$|x| < \frac{12}{10.5} + 1 = \frac{12}{\frac{21}{2}} + 1 = \frac{24}{21} + \frac{21}{21} = \frac{45}{21} = \frac{15}{7}.$$

Therefore $\frac{15}{7}$ **is an upper bound for the modulus of roots of** $Q(x)$.

(c) **For** $R(x)$ **we have** $|a_n| = 15$ **and** $M = \max\{|(8 - 6i)|, |3|, |1|, |1|\}$.

Consider

$$|(8 - 6i)| = \sqrt{8^2 + 6^2} = \sqrt{100} = 10.$$

Therefore $M = |(8 - 6i)| = 10$. **Bounds Theorem states that if** x **is a root of** $R(x)$ **then**

$$|x| < \frac{M}{|a_n|} + 1$$

so in this case

$$|x| < \frac{10}{15} + 1 = \frac{10}{15} + \frac{15}{15} = \frac{25}{15} = \frac{5}{3}.$$

Therefore $\frac{5}{3}$ **is an upper bound for the modulus of roots of** $P(x)$.

7. Let $P(x) = x^5 + 6x^4 + 8x^3 - 4x^2 - 9x - 2$

(a) **Using Descartes' Rules of Signs determine the number of real positive roots of** $P(x)$ **and the number of real negative roots of** $P(x)$.

(b) **Use the Rational Root Theorem to determine all possible rational roots of** $P(x)$.

(c) **Evaluate $P(x)$ at possible rational roots and use the Factor Theorem to find one or more linear factor(s) which divide $P(x)$.**

(d) **Show that $P(x)$ has no roots in the interval $[2,5]$.**

(e) **Find all roots of $P(x)$.**

Solution:

(a)

$$P(x) = x^5 + 6x^4 \underbrace{+8x^3 - 4x^2}_{\textbf{sign change 1}} -9x - 2$$

There is one sign change between consecutive coefficients of $P(x)$ therefore there is one real positive root by Descartes' Rules of Signs.

$$P(-x) = \underbrace{-x^5 + 6}_{\textbf{sc 1}}\underbrace{x^4 - 8x^3}_{\textbf{sc 2}}\underbrace{-4x^2 + 9}_{\textbf{sc 3}}\underbrace{x - 2}_{\textbf{sc 4}}$$

There are 4 sign changes between consecutive coefficients of $P(-x)$ therefore there are either 4 or 2 or 0 negative real roots by Descartes' Rules of Signs.

(b) **By the rational root theorem if $r = \dfrac{m}{n}$ is a root of $P(x)$ then m divides 2 and m divides 1.**

Possible m are $\{\pm1, \pm2\}$, possible n are $\{\pm1\}$ therefore possible $r = \dfrac{m}{n}$ are $\{\pm1, \pm2\}$.

(c)

$$\begin{aligned}
P(1) &= 1^5 + 6(1)^4 + 8(1)^3 - 4(1)^2 - 9(1) - 2 = 1 + 6 + 8 - 4 - 9 - 2 = 0.\\
P(-1) &= (-1)^5 + 6(-1)^4 + 8(-1)^3 - 4(-1)^2 - 9(-1) - 2 = -1 + 6 - 8 - 4 + 9 - 2 = 0.\\
P(2) &= (2)^5 + 6(2)^4 + 8(2)^3 - 4(2)^2 - 9(2) - 2 = 32 + 96 + 64 - 16 - 18 - 2 = 156 \neq 0\\
P(-2) &= (-2)^5 + 6(-2)^4 + 8(-2)^3 - 4(-2)^2 - 9(-2) - 2 = -32 + 96 - 64 - 16 + 18 - 2 = 0
\end{aligned}$$

Therefore $x = 1$, $x = -1$, and $x = -2$ are roots of x so $(x-1)$, $(x-(-1))$, and $(x-(-2))$ are linear factors of $P(x)$ by the Factor Theorem.

(d) **Descartes' Rules of Signs tells us that $P(x)$ has only one real positive root and we found that $x = 1$ is a real positive root, therefore there can be no other real positive root, therefore there is no real positive root in the interval $[2,5]$.**

(e) **Given $(x-1)$, $(x-(-1))$, and $(x-(-2))$ are all linear factors of $P(x)$ their product $x^3 + 2x^2 - x - 2$ is also a factor of $P(x)$.**

$$\begin{array}{r|l}
 & x^2 + 4x + 1 \\
\hline
x^3 + 2x^2 - x - 2 & x^5 + 6x^4 + 8x^3 - 4x^2 - 9x - 2 \\
 & -x^5 - 2x^4 + x^3 + 2x^2 \\
\hline
 & 4x^4 + 9x^3 - 2x^2 - 9x \\
 & -4x^4 - 8x^3 + 4x^2 + 8x \\
\hline
 & x^3 + 2x^2 - x - 2 \\
 & -x^3 - 2x^2 + x + 2 \\
\hline
 & 0
\end{array}$$

Therefore $x^2 + 4x + 1$ is a factor of $P(x)$, whose roots can be found using the quadratic formula. The roots of $x^2 + 4x + 1$ are of the form

$$\frac{-4 \pm \sqrt{16-4}}{2}$$
$$= -2 \pm \sqrt{3}.$$

Therefore the roots of $P(x)$ are $\{-2-\sqrt{3}, -2, -1, -2+\sqrt{3}, 1\}$.

8. Let $P(x) = x^6 - 6x^5 + \frac{17}{2}x^4 - 7x^3 + \frac{21}{2}x^2 + 3x$

(a) **Using Descartes' Rules of Signs determine the number of real positive roots of $P(x)$ and the number of real negative roots of $P(x)$. What is the minimum number of real roots? What is the maximum?**

(b) **Use the Rational Root Theorem to determine all possible rational roots of $P(x)$. (Hint: If $P(x) = Q(x) \cdot R(x)$ where $Q(x)$ and $R(x)$ are polynomials then rational roots of $Q(x)$ and $R(x)$ are rational roots of $P(x)$.)**

(c) **Use the Bounds Theorem to determine an upper bound for the modulus of roots of $P(x)$. Does this eliminate any possible rational roots? Which ones?**

(d) **Evaluate $P(x)$ at possible rational roots and use the Factor Theorem to find one or more linear factor(s) which divide $P(x)$.**

(e) **Given that $(\sqrt{2}x - i\sqrt{3})$ divides $P(x)$ find all roots of $P(x)$.**

Solution:

(a)

$$P(x) = \underbrace{x^6 - 6}_{\textbf{sc 1}}\underbrace{x^5 + \frac{17}{2}}_{\textbf{sc 2}}\underbrace{x^4 - 7}_{\textbf{sc 3}}\underbrace{x^3 + \frac{21}{2}x^2}_{\textbf{sc 4}} + 3x$$

There are 4 sign changes between consecutive coefficients of $P(x)$ therefore there are 4 or 2 or 0 real positive roots of $P(x)$ by Descartes' Rules of Signs.

$$P(-x) = P(x) = x^6 + 6x^5 + \frac{17}{2}x^4 + 7x^3 + \underbrace{\frac{21}{2}x^2 - 3x}_{\textbf{sign change 1}}$$

There is one sign change between consecutive coefficients of $P(-x)$ therefore there is one real negative root of $P(x)$ by Descartes' Rules of Signs.

Because the constant term for this polynomial is 0 we know $x = 0$ is a real root and there is exactly one negative real root, there are therefore a minimum of 2 real roots (when there are 0 positive real roots) and a maximum of 6 (when there are 4 positive real roots).

(b)

$$P(x) = \frac{x}{2}\left(2x^5 - 12x^4 + 17x^3 - 14x^2 + 21x + 6\right)$$

So all rational roots of $Q(x) = \frac{x}{2}$ and $R(x) = 2x^5 - 12x^4 + 17x^3 - 14x^2 + 21x + 6$ are rational roots of $P(x)$.

$Q(x)$ has one rational root at $x = 0$.

If $r = \frac{m}{n}$ is a rational root of $R(x)$ then m divides 6 and n divides 2.

Possible values for m are $\{\pm 1, \pm 2, \pm 3, \pm 6\}$.
Possible values for n are $\{\pm 1, \pm 2\}$.
Possible values for $r = \frac{m}{n}$ are $\left\{\pm\dfrac{1}{2}, \pm 1, \pm\dfrac{3}{2}, \pm 2, \pm 3, \pm 6\right\}$

Therefore possible rational roots of $P(x)$ are $\left\{0, \pm\dfrac{1}{2}, \pm 1, \pm\dfrac{3}{2}, \pm 2, \pm 3, \pm 4, \pm 6\right\}$

(c) For $P(x)$ we have $a_n = 1$ and $M = \max\{|-6|, |\frac{17}{2}|, |-7|, |\frac{21}{2}|, |-3|\} = \frac{21}{2}$.
Bounds Theorem states that if x is a root of $P(x)$ then

$$|x| < \frac{M}{a_n} + 1$$

So in this case

$$|x| < \frac{21}{2} + 1 = \frac{23}{2}.$$

As $\frac{23}{2} > 6$ no possible rational roots are eliminated.

(d)

$$P(0) = 0$$
$$P\left(\frac{1}{2}\right) = \frac{231}{64} \neq 0$$
$$P\left(\frac{-1}{2}\right) = \frac{175}{64} \neq 0$$
$$P(1) = 10 \neq 0$$
$$P(-1) = 30 \neq 0$$
$$P\left(\frac{3}{2}\right) = \frac{855}{64} \neq 0$$
$$P\left(\frac{-3}{2}\right) = \frac{9135}{64} \neq 0$$
$$P(2) = 0$$
$$P(-2) = 484 \neq 0$$
$$P(3) = -126 \neq 0$$
$$P(-3) = 3150 \neq 0$$
$$P(6) = 9900 \neq 0$$
$$P(-6) = 106200 \neq 0$$

Then $(x-2)$ and x are linear factors which divide $P(x)$.

(e) Given $(\sqrt{2}x + i\sqrt{3})$ divides $P(x)$, and all coefficients of $P(x)$ are real, by Theorem 3.5 of the textbook we have $(\sqrt{(2)}x - i\sqrt{3})$ divides $P(x)$. Then

$$S(x) = (x)(x-2)(\sqrt{2}x + i\sqrt{3})(\sqrt{2}x - i\sqrt{2}) = 2x^4 - 4x^3 + 3x^2 - 6x$$

divides $P(x)$. Polynomial long division gives us

$$
\begin{array}{r}
\frac{1}{2}x^2 - 2x - \frac{1}{2} \\
2x^4 - 4x^3 + 3x^2 - 6x \overline{\big)\ x^6 - 6x^5 + \frac{17}{2}x^4 - 7x^3 + \frac{21}{2}x^2 + 3x} \\
\underline{-x^6 + 2x^5 - \frac{3}{2}x^4 + 3x^3} \\
-4x^5 + 7x^4 - 4x^3 + \frac{21}{2}x^2 \\
\underline{4x^5 - 8x^4 + 6x^3 - 12x^2} \\
-x^4 + 2x^3 - \frac{3}{2}x^2 + 3x \\
\underline{x^4 - 2x^3 + \frac{3}{2}x^2 - 3x} \\
0
\end{array}
$$

The roots of $\frac{x^2}{2} - 2x - \frac{1}{2}$ can be found using the quadratic formula and are of the form

$$\frac{2 \pm \sqrt{4 - 4(\frac{1}{2})(\frac{-1}{2})}}{2 \cdot \frac{1}{2}} = 2 \pm \sqrt{5}.$$

Therefore the roots of $P(x)$ are $\left\{2 - \sqrt{5}, 0, 2, 2 + \sqrt{5}, i\sqrt{\frac{3}{2}}, -i\sqrt{\frac{3}{2}}\right\}$

9. Consider the following matrices

$$A = \begin{bmatrix} 1 & 2 & 0 \\ 3 & 5 & 13 \end{bmatrix}, \qquad B = \begin{bmatrix} 3 & 5 \\ 1 & 9 \\ -1 & 13 \end{bmatrix}, \qquad C = \begin{bmatrix} 5 & 23 \\ 1 & k \end{bmatrix}.$$

(i) Find all values of k such that $AB = C$.

(ii) Compute each of the following, or explain why it is undefined

(a) $(B^T + A)C$.

(b) $A + 3B^T$.

(d) BA.

(e) $A^T B$.

Solution:

(i)

$$AB = \begin{bmatrix} 5 & 23 \\ 1 & 229 \end{bmatrix}$$

Therefore $C = AB$ only when $k = 229$.

(ii) (a) $(B^T + A)$ **is well defined and 2×3, however C is 2×2 and therefore $(B^T + A)C$ is not well defined as the number of columns of $(B^T + A)$ is not equal to the number of rows of C.**

(b)

$$A + 3B^T = \begin{bmatrix} 1 & 2 & 0 \\ 3 & 5 & 13 \end{bmatrix} + 3\begin{bmatrix} 3 & 1 & -1 \\ 5 & 9 & 13 \end{bmatrix} = \begin{bmatrix} 10 & 5 & -3 \\ 18 & 32 & 52 \end{bmatrix}$$

(c)

$$BA = \begin{bmatrix} 18 & 31 & 65 \\ 28 & 47 & 117 \\ 38 & 63 & 169 \end{bmatrix}$$

(d) A^T **is** 3×2 **and** B **is** 3×2**, therefore the number of columns of** A^T **is not equal to the number of rows of** B **and so the multiplication is not well defined.**