MATH 2130 – Midterm 1 Solutions

1. Let $\mathbf{r}(t) = (1, \cos(t^3), \sin(t^3))$. Calculate the indefinite integral

$$\int \mathbf{r}(t) \times \mathbf{r}'(t)\, dt.$$

2. Let Π be the plane

$$3x - 2y + 6z - 6 = 0,$$

and let ℓ be the line with parametric representation

$$x = 4 + t, \quad y = 6 + 4t, \quad z = 1 + 2t, \quad t \in \mathbb{R}.$$

Let P be the point $(2, 1, -2)$.

(a) Find the distance between ℓ and Π.

(b) Find the distance between P and Π.

(c) Find the equation of the plane containing P and ℓ.

3. On a large, clearly labeled diagram, sketch the surface

$$2x^2 - 4x - 3y^2 - 6y + z^2 - 6z + 2 = 0$$

Mark at least one important point. Give either the name of the surface, or the names of two different cross sections.

4. Find a parametric representation for the intersection of the surfaces

$$x^3 + y + z = 0, \quad x^2 y - z = 0,$$

having the property that y increases when x is positive.

5. Let $\mathcal{C}$ be the curve with vector representation

$$\mathbf{r}(t) = \left(\frac{2}{\sqrt{3}}t^3 - \frac{1}{\sqrt{3}}\right)\widehat{\mathbf{i}} + \left(\frac{3}{2}t^2\right)\widehat{\mathbf{j}} + \left(2 - \frac{2}{3}t^3\right)\widehat{\mathbf{k}}, \quad t \in \mathbb{R}.$$

(a) Find a unit tangent vector $\widehat{\mathbf{T}}$ to $\mathcal{C}$ at the point $(\frac{1}{\sqrt{3}}, \frac{3}{2}, \frac{4}{3})$, in the direction of decreasing t.

(b) Find the arc length of $\mathcal{C}$ between the points $(-\frac{1}{\sqrt{3}}, 0, 2)$ and $(\frac{1}{\sqrt{3}}, \frac{3}{2}, \frac{4}{3})$.