MATH2132 Test 3 November 24, 2011 60 minutes

Values

14 1. (a) Find a 2-parameter family of solutions of the differential equation

$$xy'' - 3y' = x^5.$$

(b) Can there be any singular solutions to your family of solutions in part (a)? Explain.

14 2. Two substances A and B react to form a third substance C in such a way that 1 gram of A reacts with 1 gram of B to produce 2 grams of C. The rate at which C is formed is proportional to the product of the amounts of A and B present in the mixture. If 10 grams of A and 10 grams of B are originally brought together at time $t = 0$, find the amount of C present in the mixture as a function of time.

7 3. Let $\phi(m) = 0$ be the auxiliary equation associated with the differential equation $\phi(D)y = 0$. It is known that

$$\phi(m) = (m+1)(m-7)^3(m^2 - 4m + 13)^2.$$

What is a general solution of the differential equation?

15 4. Find a general solution of the differential equation

$$y'' - 4y' - 5y = 8xe^x.$$

Dawit's Answers.

1. a) $y = \frac{x^6}{12} + C\frac{x^4}{4} + D$ b) NO. (the DE is linear)

2. $C(t) = \frac{100kt}{5kt+1}$ (in grams)

3. $y(x) = c_1e^{-x} + (c_2 + c_3x + c_4x^2)e^{7x} + e^{2x}\left[(c_5+c_6x)\sin 3x + (c_7+c_8x)\cos 3x\right]$

4. $y(x) = c_1e^{-x} + c_2e^{5x} + \frac{1}{4}e^x(1-4x)$