(1)

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Final Exam (Dec. 2005)

1. $\vec{A} = 3\hat{i}+\hat{j}+\hat{k} \qquad \vec{B} = -2\hat{i}+4\hat{j}-\hat{k}$

$$\theta = \cos^{-1}\left[\frac{(3)(-2)+4(1)+(1)(-1)}{\sqrt{3^2+1^2+1^2}\cdot\sqrt{(-2)^2+4^2+(-1)^2}}\right] = \cos^{-1}\left[\frac{-3}{\sqrt{(11)}\cdot\sqrt{21}}\right] = 101.38^\circ \approx \boxed{101.4^\circ}$$

2. $m = 0.150\,kg$

$\vec{r}(t) = 3t\hat{i}+3t^2\hat{j}+3t^3\hat{k}$

$t = 2.00\,s$

$\vec{v}(t) = 3\hat{i}+6t\hat{j}+9t^2\hat{k}$

$\vec{a}(t) = 0\hat{i}+6\hat{j}+18t\hat{k}$

$\vec{v}(2) = 3\hat{i}+12t\hat{j}+36\hat{k}$

$\vec{a}(2) = 0\hat{i}+6\hat{j}+36\hat{k}$

$$P(t) = \vec{F}\cdot\vec{v} = \underbrace{m\frac{d^2\vec{r}}{dt^2}}_{ma}\cdot\underbrace{\frac{d\vec{r}}{dt}}_{v}$$

$$P(2) = m\vec{a}(2)\cdot\vec{v}(2)$$
$$= (0.15)\left[0(3)+12(6)+(36)(36)\right] = 205.2W \approx \boxed{205}$$

3. $t=0$

$\vec{v}_0 = 4.4\hat{j}$

$\vec{a} = 2.1\hat{i}-1.1\hat{j}$

$\vec{v} = \vec{v}_0+\vec{a}t$

$\vec{v} = v_x\hat{i}+v_y\hat{j} = (v_{0x}\hat{i}+v_{0y}\hat{j})+(a_x\hat{i}+a_y\hat{j})t$

$(V_x, V_y) = (V_{0x}+a_xt,\ V_{0y}+a_yt)$

$V_x = V_{0x}+a_xt = 0+2.1t$

$V_y = V_{0y}+a_yt = 4.4-1.1t \rightarrow \max. y = V_y = 0$

$0 = 4.4-1.1t$

$t = 4s \longrightarrow$ $V_x = 2.1(4) = \boxed{8.4\,m/s}$, $V_y = 0$

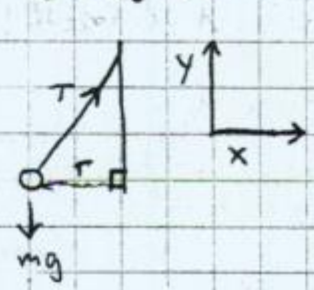
4. $m = 0.12\,kg$

$v = 3\,m/s$

$r = 1.7\,m$

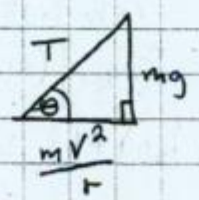
$T = ?$

$F_c = \dfrac{mV^2}{r}$



$\Sigma F_x = T\cos\theta = \dfrac{mv^2}{r}$

$\Sigma F_y = T\sin\theta - mg \rightarrow T\sin\theta = mg$



$$T = \sqrt{\left(\frac{mv^2}{r}\right)^2+(mg)^2} = \sqrt{\frac{m^2v^4}{r^2}+m^2g^2} = \sqrt{m^2\left(\frac{v^4}{r^2}+g^2\right)} = m\sqrt{\frac{v^4}{r^2}+g^2}$$

$$= (0.12)\sqrt{\frac{(3)^4}{(1.7)^2}+(9.8)^2} = 1.3366 \approx \boxed{1.3}$$

(a)

5. $V_0 = 60.0\ m/s$

$H = 150\ m$

$R = ?$

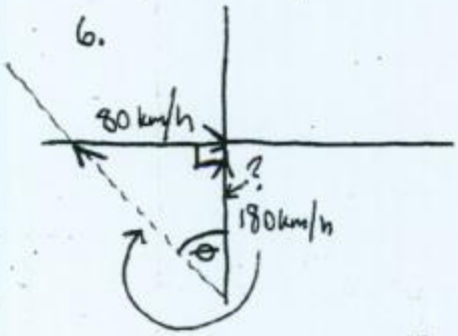
$R = \dfrac{V_0^2 \sin 2\theta}{g}$

$H = \dfrac{V_0^2 \sin^2\theta}{2g}$

$H = \dfrac{V_0^2 \sin^2\theta}{2g} \rightarrow \sin^2\theta = \dfrac{2gH}{V_0^2} \rightarrow \theta = \sin^{-1}\left[\sqrt{\dfrac{2gH}{V_0^2}}\right] = \sqrt{\dfrac{2(9.8,}{60^2}} = \underline{64.6^\circ}$

$R = \dfrac{(60)^2 \sin(2(64.6))}{9.8} = 284.67 \approx \boxed{284}$ ✓

6.



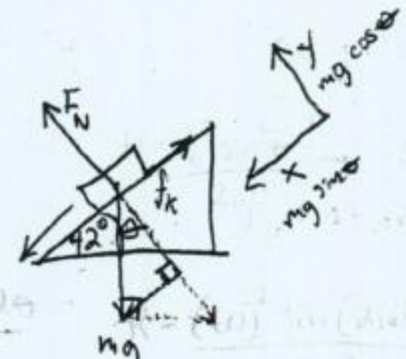
$\theta = \sin^{-1}\left(\dfrac{80}{180}\right) = 26.38^\circ \approx \boxed{26^\circ \text{ W of N.}}$ ✓

7. $m = 45\ kg$

$\mu_k = 0.35$

$a = ?$

$f_k = \mu_k F_N = \mu_k mg$



$\Sigma F_x = \cancel{m}g\sin\theta - \mu_k \cancel{m}g\cos\theta = \cancel{m}a$

$a = g(\sin\theta - \mu_k\cos\theta)$

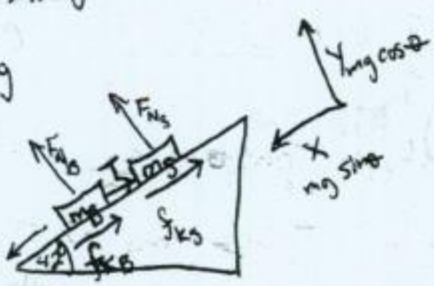
$= (9.8)(\sin 42 - (0.35)\cos 42)$

$= 4.0085 \approx \boxed{4.0\ m/s^2}$ ✓

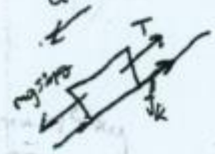
8. $m_B = 25\ kg$

$\mu_{kB} = 0.48$

$T = ?$



$a = 3.67\ m/s^2$



$\Sigma F_x = (m_B + m_S)g\sin\theta - \mu_{k_B} m_B g\cos\theta - \mu_{k_S} m_S g\cos\theta$

$= (m_B + m_S)a$

$a = \dfrac{[(m_B+m_S)\sin\theta - \mu_{k_B} m_B\cos\theta - \mu_{k_S} m_S\cos\theta]g}{(m_B+m_S)}$

$= \dfrac{[(45+25)\sin 42 - (0.35)(45)\cos 42 - (0.48)(25)\cos 42](9.8)}{(45+25)}$

$= \underline{3.67\ m/s^2}$

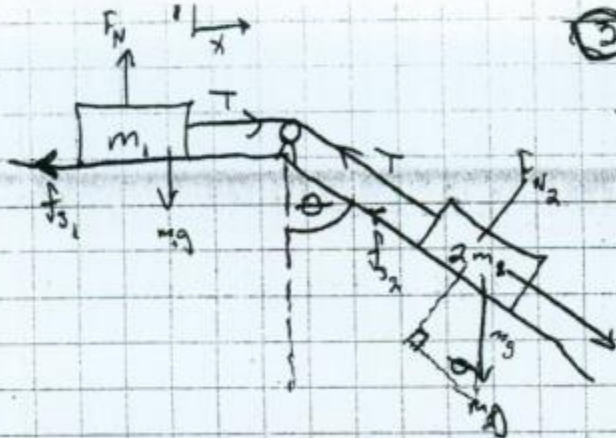
$\Sigma F_x = m_B g\sin\theta - T - \mu_k m_B g\cos\theta = m_B a$

$T = m_B g\sin\theta - \mu_{kB} m_B g\cos\theta - m_B a = m_B(g(\sin\theta - \mu_{k_B}\cos\theta) - a)$

$= 45[9.8(\sin 42 - (0.35)\cos 42) - 3.67]$

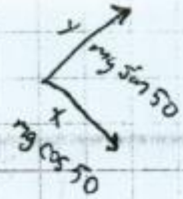
$T = 15.23\ N \approx \boxed{15\ N}$ ✓

9.



$\theta = 50°$

$\mu_s = ?$



$f_{s_1} = \mu_{s_1} m_1 g$

$f_{s_2} = \mu_s m_2 g \sin 50$

$m_2 \cancel{g} \cos 50 = f_{s_1} + f_{s_2}$

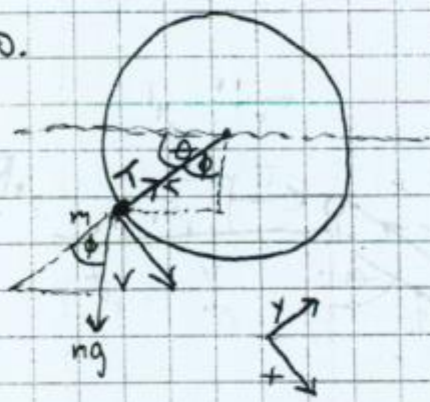
$= \mu_{s_1} m_1 \cancel{g} + \mu_s m_2 \cancel{g} \sin 50$

$m_2 \cos 50 = \mu_s (m_1 + m_2 \sin 50)$

$\mu_s = \dfrac{m_2 \cos 50}{(m_1 + m_2 \sin 50)}$

$= \dfrac{(2)\cos 50}{1 + 2\sin 50} = 0.5077 \approx \boxed{0.51}$ ✓

10.



$m = 1.0\,kg$
$r = 0.30\,m$
$\theta = 30°$
$V = 3.0\,m/s$
$\phi = 60°$

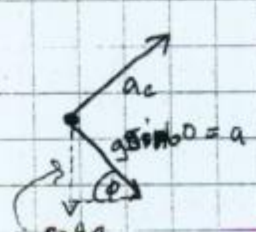
$\sum F_y = T - mg\cos 60 = \dfrac{mV^2}{r}$

$T = mg\cos 60 + \dfrac{mV^2}{r}$

$= m\left(g\cos 60 + \dfrac{V^2}{r}\right)$

$T = \left(9.8\cos 60 + \dfrac{(3.0)^2}{0.3}\right) = 34.9 \approx \boxed{35\,N}$ ✓

11.



$\phi = 60$

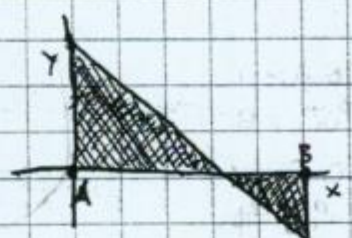
$a = \dfrac{V^2}{r}$

$a_c = \dfrac{V^2}{r} = \dfrac{(3.0)^2}{.3} = 30\,m/s^2$

$a = 9.8(\sin 60) = 8.487\,m/s^2$

$a_r = \sqrt{a_c^2 + a^2} = \sqrt{(30)^2 + (8.487)^2} = 31.177 \approx \boxed{31\,m/s^2}$ ✓

12. $m = 1.5\,kg$
$v_0 = 4.0\,m/s$
$x = 3.0\,m$



W = area under the curve

$W = \Delta K$

$W_{AB} = K_B - K_A$

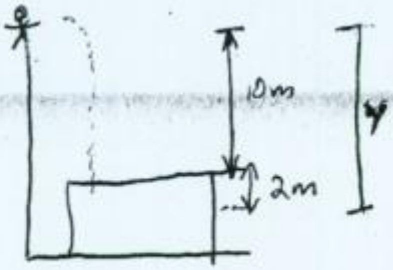
$W_{AB} = \frac{1}{2}(2)(8) - \frac{1}{2}(1)(4) = \underline{6J}$

$K_B = K_A + W_{AB}$

$K_A = \frac{1}{2} m v_0^2 = \frac{1}{2}(1.5)(4.0)^2 = \underline{12J}$

$K_B = K_A + W_{AB} = 12 + 6 = \boxed{18J}$ ✓

13.



$$\Delta U = \Delta SPE$$

$$mgy = \Delta SPE = \tfrac{1}{2}kx^2$$

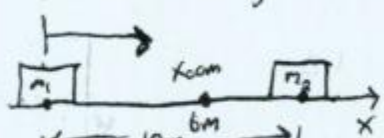
$$(700)(12) = \Delta SPE = \boxed{8400\ \text{J}}\ (\text{N-m})$$

14 $KE = E_{sp} + |W_f| \Rightarrow \frac{1}{2} m v_i^2 = \frac{1}{2} k x^2 + \mu_k m g x \Leftarrow \begin{cases} m = 20\ \text{kg},\ \mu_k = 0.3 \\ v_i = 1.3\ \text{m/s},\ k = 120\ \text{N/m} \end{cases}$

$60x^2 + 5.88x - 1.69 = 0 \Rightarrow x = 0.125\ \text{m}$ (the -ve answer is rejected)

15. $m_1 = 40.0\text{kg}$
$m_2 = 60.0\text{kg}$
$x = 10.0\text{m}$
$x_1 = ?$

$$x_{com} = \frac{m_1 x_1 + m_2 x_2}{(m_1 + m_2)} = \frac{1}{M} \sum_{i=1}^{n} m_i x_i$$



$$x_{com} = \frac{40(0) + 60.0(10)}{(40+60)} = \frac{600}{100} = \boxed{6\text{m}}$$

16.

$\vec{P}_i = \vec{P}_f$

$m_A = 2.0\text{kg}$
$V_A = 50\text{m/s}$
$m_B = 4.0\text{kg}$
$V_B = -25\text{m/s}$

$$\vec{P}_{1i} + \vec{P}_{2i} = \vec{P}_{1f} + \vec{P}_{2f}$$

$$m_A V_{Ai} + m_B V_{Bi} = (m_A + m_B) v_f$$

$$(2)(50) + (4)(-25) = 6 v_f$$

$$0 = 6 v_f \rightarrow v_f = 0$$

$$\frac{1}{2} m_A v_A^2 + \frac{1}{2} m_B v_B^2 = \frac{1}{2}(2)(50)^2 + \frac{1}{2}(4)(-25)^2 = \boxed{3750\ \text{J}}$$

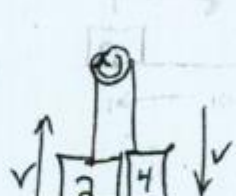
17. $\boxed{V_{4f} \approx -V_{1f}}$

18. $\omega = 20\ \text{rad/s}$
$t = 9.0\text{s}$
$\theta = 450\ \text{rad}$
$\alpha = ?$

$$\theta - \theta_0 = \omega_0 t + \tfrac{1}{2}\alpha t^2$$

$$\frac{(\theta - \omega_0 t)2}{t^2} = \alpha = \frac{(450 - 20(9))2}{9^2} = 6.666 \approx \boxed{6.7\ \text{rad/s}^2}$$

20. $r = 0.03\text{m}$
$I = 4.5\times10^{-3}\ \text{kg}$
$m_1 = 2.0\ \text{kg}$
$m_2 = 4.0\text{kg}$
$v = 2.0\ \text{m/s}$



$$v = \omega r \rightarrow \omega = \frac{v}{r}$$

$$K_T = K_1 + K_2 + K_3$$

$$= \tfrac{1}{2}m_1v_1^2 + \tfrac{1}{2}m_2v_2^2 + \tfrac{1}{2}I\omega^2 = \tfrac{1}{2}m_1v_1^2 + \tfrac{1}{2}m_2v_2^2 + \tfrac{1}{2}I\left(\frac{v}{r}\right)^2$$

$$= \tfrac{1}{2}v^2\left[m_1 + m_2 + \frac{I}{r^2}\right] = \tfrac{1}{2}(2)^2\left[2 + 4 + \frac{4.5\times10^{-3}}{(0.03)^2}\right]$$

$$= 2\left[6 + \frac{4.5\times10^{-3}}{(0.03)^2}\right] = \boxed{22\text{J}}$$

$L_i = L_f$

21. $I_0\omega_0 = I_f\omega_f \rightarrow \frac{I_0}{I_f}\omega_0 = \frac{\omega_0(1I)}{(1I+2I)} = \frac{\omega_0 \cancel{I}}{3\cancel{I}} = \boxed{\frac{\omega_0}{3}}$ ✓

22. $m = 2.0\,kg$
$x = 3\,m$
$\vec{a} = 4.0\hat{i} - 3.0\hat{j}\ m/s^2$
$t = 2.0\,s$
$v_0 = 0\,m/s$

$\vec{r} = \vec{r}_0 + \cancel{\vec{v}_0 t}^{\,0} + \frac{1}{2}\vec{a}t^2$

$(x,y) = (3,0) + \frac{1}{2}(4,-3)(2)^2 = (3,0) + \frac{1}{\cancel{2}}(4,-3)\cancel{4}^{\,2}$

$\vec{r} = (3,0) + (8,-6) = (11,-6)$

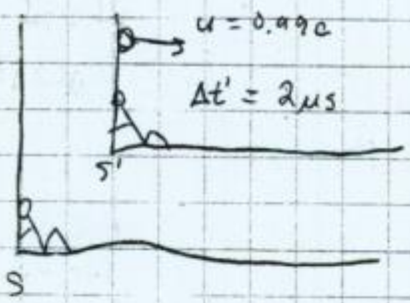
$\vec{r} = 11\hat{i} - 6\hat{j}$

$\vec{v} = v_0 + at$
$= (0,0) + (4,-3)2$
$\vec{v} = 8\hat{i} - 6\hat{j}$

$(\vec{p} = m\vec{v})$

$\vec{L} = \vec{r} \times \vec{p}$
$= \vec{r} \times m\vec{v}$
$= m(\vec{r} \times \vec{v}) = 2\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 11 & -6 & 0 \\ 8 & -6 & 0 \end{vmatrix} = 2[-6(11) + 8(-6)]\hat{k} = \boxed{-36\hat{k}\,(kg \cdot m^2/s)}$

23.

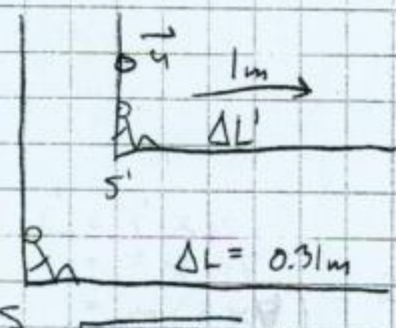


$\beta = \frac{u}{c} = \frac{0.99\cancel{c}}{\cancel{c}} = 0.99$

$\gamma = \frac{1}{\sqrt{1-\beta^2}} = \frac{1}{\sqrt{1-0.99^2}} = 7.0888$

$\Delta t = \gamma\Delta t' = 7.0888(2\mu s) = \boxed{14\,\mu s}$ ✓

24.



$\gamma = \frac{\Delta L'}{\Delta L} = \frac{1}{0.31} = 3.2258$

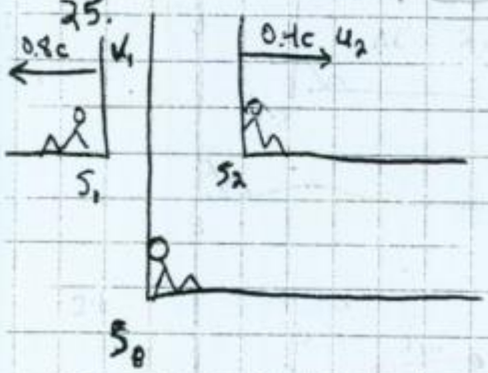
$\gamma = \frac{1}{\sqrt{1-\beta^2}} \rightarrow \gamma^2 = \frac{1}{1-\beta^2} \rightarrow (1-\beta^2)\gamma^2 = 1$

~~$\gamma^2 - \gamma^2\beta^2 = 1 \rightarrow (1-\beta^2)\gamma^2 = 1 \rightarrow 1-\beta^2 = \frac{1}{\gamma^2}$~~

$-\gamma^2\beta^2 = 1-\gamma^2 \rightarrow -\beta^2 = \frac{1-\gamma^2}{\gamma^2} \rightarrow \beta^2 = \frac{\gamma^2-1}{\gamma^2} = \beta = \sqrt{\frac{\gamma^2-1}{\gamma^2}}$

$\sqrt{\frac{(3.2258)^2-1}{3.2258^2}} = \boxed{0.95c}$

25.



$V_x' = \frac{V_x - u}{1 - \frac{uV_x}{c^2}} = \frac{-0.8c - 0.4}{1 + \frac{0.4\cancel{c}(+0.8\cancel{c})}{\cancel{c^2}}} = \frac{-0.8c - 0.4c}{1 + 0.32} = \frac{-1.2c}{1.32} = -0.909$

$\approx \boxed{0.91c}$ ✓

$L_i = L_f$

21. $I_0\omega_0 = I_f\omega_f \rightarrow \frac{I_0}{I_f}\omega_0 = \frac{\omega_0(1I)}{(1I+2I)} = \frac{\omega_0 \cancel{I}}{3\cancel{I}} = \boxed{\frac{\omega_0}{3}}$

22. $m = 2.0\,kg$

$x = 3\,m$

$\vec{a} = 4.0\hat{i} - 3.0\hat{j}\ m/s^2$

$t = 2.0\,s$

$v_0 = 0\,m/s$

$\vec{r} = \vec{r}_0 + \cancelto{0}{\vec{v}_0 t} + \frac{1}{2}\vec{a}t^2$

$(x,y) = (3,0) + \frac{1}{2}(4,-3)(2)^2 = (3,0) + \frac{1}{\cancel{2}}(4,-3)\cancelto{2}{4}$

$\vec{r} = (3,0) + (8,-6) = (11,-6)$

$\vec{r} = 11\hat{i} - 6\hat{j}$

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$= (0,0) + (4,-3)2$

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$(\vec{p} = m\vec{v})$

$\vec{L} = \vec{r} \times \vec{p}$

$= \vec{r} \times m\vec{v}$

$= m(\vec{r} \times \vec{v}) = 2 \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 11 & -6 & 0 \\ 8 & -6 & 0 \end{vmatrix} = 2[-6(11) + 8(-6)]\hat{k} = \boxed{-36\hat{k}\,(kg \cdot m^2/s)}$

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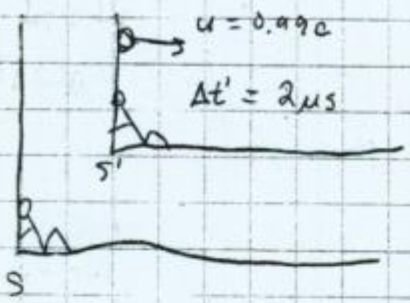

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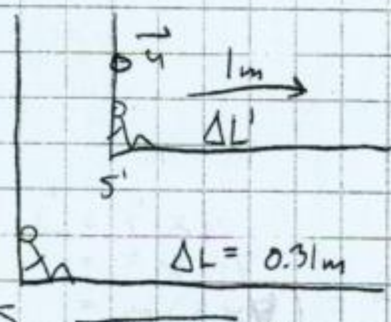
24.



$\gamma = \frac{\Delta L'}{\Delta L} = \frac{1}{0.31} = 3.2258$

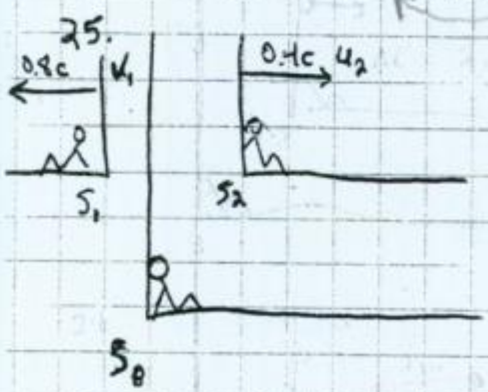
$\gamma = \frac{1}{\sqrt{1-\beta^2}} \rightarrow \gamma^2 = \frac{1}{1-\beta^2} \rightarrow (1-\beta^2)\gamma^2 = 1$

$\gamma^2 - \gamma^2\beta^2 \cancel{= 1 \rightarrow (1-\beta^2)\gamma^2 = 1 \rightarrow 1-\beta = \frac{1}{\gamma^2}}$

$-\gamma^2\beta^2 = 1 - \gamma^2 \rightarrow -\beta^2 = \frac{1-\gamma^2}{\gamma^2} \rightarrow \beta^2 = \frac{\gamma^2 - 1}{\gamma^2} = \beta = \sqrt{\frac{\gamma^2-1}{\gamma^2}}$

$\sqrt{\frac{(3.2258)^2 - 1}{3.2258^2}} = \boxed{0.95c}$

25.



$V_x' = \frac{V_x - u}{1 - \frac{uV_x}{c^2}} = \frac{-0.8c - 0.4}{1 + \frac{0.4\cancel{c}(+0.8\cancel{c})}{\cancel{c^2}}} = \frac{-0.8c - 0.4c}{1 + 0.32} = \frac{-1.2c}{1.32} = -0.909$

$\approx \boxed{0.91c}$