MATH 1210 A01 Summer 2013 Problem Workshop 9 Solutions

1. We start by putting the system into an augmented matrix.

$$\left[\begin{array}{cccc|c} 1 & 5 & 3 & -5 & 0 \\ 2 & -1 & 3 & -4 & 0 \end{array}\right] \text{Using } R_2 \to R_2 - 2R_1 \text{ yields}$$

$$\left[\begin{array}{cccc|c} 1 & 5 & 3 & -5 & 0 \\ 0 & -11 & -3 & 6 & 0 \end{array}\right] \text{Using } R_2 \to -R_2/11 \text{ yields}$$

$$\left[\begin{array}{cccc|c} 1 & 5 & 3 & -5 & 0 \\ 0 & 1 & 3/11 & -6/11 & 0 \end{array}\right] \text{Using } R_1 \to R_1 - 5R_2 \text{ yields}$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 18/11 & -25/11 & 0 \\ 0 & 1 & 3/11 & -6/11 & 0 \end{array}\right]$$

Hence the solution is $x = -\frac{18}{11}z + \frac{25}{11}w$,$y = -\frac{3}{11}z + \frac{6}{11}w$ where z and w are arbitrary. Rewriting this in matrix notation yields

$$\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \frac{z}{11}\begin{bmatrix} -18 \\ -3 \\ 11 \\ 0 \end{bmatrix} + \frac{w}{11}\begin{bmatrix} 25 \\ 6 \\ 0 \\ 11 \end{bmatrix}.$$

Hence two basic solutions could be

$$\begin{bmatrix} -18 \\ -3 \\ 11 \\ 0 \end{bmatrix}, \begin{bmatrix} 25 \\ 6 \\ 0 \\ 11 \end{bmatrix}.$$

This is not unique. For example we could have rewitten the solutions as

$$\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \frac{z}{22}\begin{bmatrix} -36 \\ -6 \\ 22 \\ 0 \end{bmatrix} + \frac{w}{11}\begin{bmatrix} 25 \\ 6 \\ 0 \\ 11 \end{bmatrix}$$

and gotten the basic solutions

$$\begin{bmatrix} -36 \\ -6 \\ 22 \\ 0 \end{bmatrix}, \begin{bmatrix} 25 \\ 6 \\ 0 \\ 11 \end{bmatrix}.$$

2. (a) We start by putting the system into an augmented matrix.

$$\left[\begin{array}{cccc|c} 2 & -1 & 3 & 5 & 3 \\ 1 & 3 & -2 & 1 & -2 \\ 3 & 2 & 1 & 6 & 1 \end{array}\right] \text{ Using } R_1 \leftrightarrow R_2 \text{ yields}$$

$$\left[\begin{array}{cccc|c} 1 & 3 & -2 & 1 & -2 \\ 2 & -1 & 3 & 5 & 3 \\ 3 & 2 & 1 & 6 & 1 \end{array}\right] \text{ Using } R_2 \to R_2 - 2R_1 \text{ and } R_3 \to R_2 - 3R_1 \text{ yields}$$

$$\left[\begin{array}{cccc|c} 1 & 3 & -2 & 1 & -2 \\ 0 & -7 & 7 & 3 & 7 \\ 0 & -7 & 7 & 3 & 7 \end{array}\right] \text{ Using } R_3 \to R_3 - R_2 \text{ yields}$$

$$\left[\begin{array}{cccc|c} 1 & 3 & -2 & 1 & -2 \\ 0 & -7 & 7 & 3 & 7 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right] \text{ Using } R_2 \to -R_2/7 \text{ yields}$$

$$\left[\begin{array}{cccc|c} 1 & 3 & -2 & 1 & -2 \\ 0 & 1 & -1 & -3/7 & -1 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right] \text{ Using } R_1 \to R_1 - 3R_2 \text{ yields}$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 1 & 16/7 & 1 \\ 0 & 1 & -1 & -3/7 & -1 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

Hence the solution is $x = -z - \frac{16}{7}w + 1$,$y = z + \frac{3}{7}w - 1$ where z and w are arbitrary. Hence we get our solution

$$\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \begin{bmatrix} -z - \frac{16}{7}w + 1 \\ z + \frac{3}{7}w - 1 \\ z \\ w \end{bmatrix} = z \begin{bmatrix} -1 \\ 1 \\ 1 \\ 0 \end{bmatrix} + \frac{w}{7} \begin{bmatrix} -16 \\ 3 \\ 0 \\ 7 \end{bmatrix} + \begin{bmatrix} 1 \\ -1 \\ 0 \\ 0 \end{bmatrix}$$

$\begin{bmatrix} -1 \\ 1 \\ 1 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} -16 \\ 3 \\ 0 \\ 7 \end{bmatrix}$ are not basic solutions since they are not solutions at all. They would make the left hand side of each equation equal to 0 instead of 3,-2 and 1 respectively.

$\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 0 \\ 0 \end{bmatrix}$ is a solution of the system. We can either check this directly, or note that since every solution is of the form $x = -z - \frac{16}{7}w + 1$,$y = z + \frac{3}{7}w - 1$

where z and w are arbitrary, that we can let both w and z be zero to get the solution

$$x = 1, \quad y = -1, \quad z = 0, \quad w = 0.$$

It is not a basic solution since if it was, any multiple of it would have to be a solution. However if we take the values

$$x = 2, \quad y = -2, \quad z = 0, \quad w = 0$$

we would not satisfy the equations. For example in the first equation

$$2x - y + 3z + 5w = 2(2) - (-2) + 3(0) + 5(w) = 4 + 2 + 0 + 0 = 6 \neq 3.$$