UNIVERSITY OF MANITOBA

DATE: December 13, 2007, 9:00am — FINAL EXAMINATION
PAPER # 384 — PAGE: 1 of 7
DEPARTMENT & COURSE NO: MATH 1510 — TIME: 2 hours
EXAMINATION: Applied Calculus I — EXAMINER: W. Korytowski, T. Kucera

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[9] 1. Evaluate the following limits:

(a) $\lim_{x \to 2} \dfrac{x^2 - 4}{8 - x^3}$

(b) $\lim_{x \to -\infty} \dfrac{\sqrt{3x^2 + 4}}{2x + 5}$

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[15] 2. Find $\frac{dy}{dx}$ in each case (DO NOT SIMPLIFY YOUR ANSWERS):

(a) $y = \dfrac{\sec(x)}{x^4 + 10}$

(b) $y = \mathrm{e}^{-x} \cos\left(\frac{\pi}{4}x\right)$

(c) $y = (x^3 + 3)^{10}$

(d) $y = \ln(3^x + x^2)$

[19] 3. Evaluate the following indefinite and definite integrals:

(a) $\int (5x - 14)^{10}\, dx$

(b) $\int_0^1 \frac{x}{(x^2+4)^2}\, dx$

(c) $\int \frac{1}{x \ln(x)}\, dx$

(d) $\int_0^5 x\sqrt{9-x}\, dx$

[14] 4. A particle moves on the x-axis with acceleration $a(t) = (4 - 6t)\text{m/s}^2$. At time $t = 0\text{s}$ the position is $x = 3\text{m}$ and the velocity is 4m/s.

(a) What is the velocity of the particle at $t = 1\text{s}$?

(b) What is the position of the particle at $t = 1\text{s}$?

(c) Is the particle speeding up or slowing down at $t = 1\text{s}$? (Explain!)

(d) Is the particle speeding up or slowing down at $t = 3\text{s}$? (Explain!)

[8] 5. Find the absolute maximum value and the absolute minimum value of $g(t) = t^2 e^{-t}$ on the interval $[-1, 3]$.

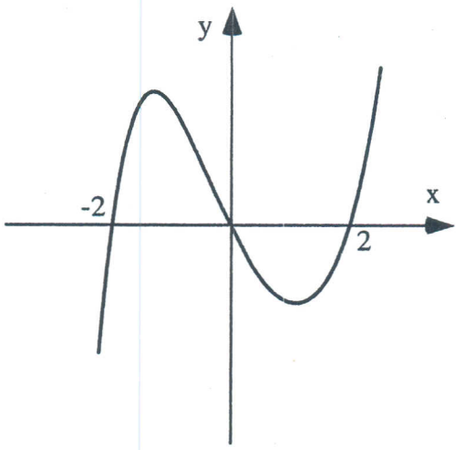
[8] 6. Consider the following word problem:

> "A jeweler is going to cut a piece of gold wire 30cm long into two pieces. One piece will be bent into a square, the other piece will be bent into a circle. Find the length of the piece that will be bent into a square so that the total area enclosed by the square and the circle is maximized."

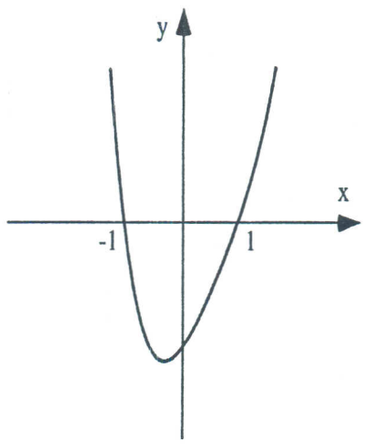
DO NOT SOLVE THIS WORD PROBLEM! Just set up the equivalent mathematical question: draw a neat sketch illustrating the situation described; identify the variables involved; set up the equations described by the problem; and find a function of one variable to be maximized. State any restrictions on the variables involved (that is, determine the domain of the function).

[10] 7. A particle moves on the parabola $y = x^2 - 2x + 2$, x and y measured in metres. When the particle is at the point $(2, 2)$, its x-coordinate is decreasing at $\frac{1}{20}$m/s. How fast is the distance from the particle to the point $(3, 0)$ changing at this time?

[17] 8. Consider the following two sketches and table of information about the function $f(x)$, which is defined and continuous on $(-\infty, \infty)$:



$y = f'(x)$



$y = f''(x)$

x	-3	-2	-1	0	1	2	3
$f(x)$	0	-3	-2	0	$-1/2$	-1	0

This information includes EVERYTHING that is "interesting" about the curve. Please note that there are no "tricks" hidden in minor flaws in the sketches!

(a) On what intervals is f increasing? _____

(b) On what intervals is f decreasing? _____

(c) Find the coordinates of all the local maxima of f. _____

(d) Find the coordinates of all the local minima of f. _____

(e) On what intervals is f concave up? _____

(f) On what intervals is f concave down? _____

(g) Find the coordinates of all the inflection points of f. _____

(h) Give a rough sketch of the graph of $y = f(x)$.