April 2002 + Solutions

THE UNIVERSITY OF MANITOBA

April 27 1:30 p.m., 2002 — Final Examination

Paper No.: 683 — Page No.: Page 1 of 4

Department & Course No.: 130.112 — Time: 3 Hours

Examination: Thermal Sciences — Examiner(s): Dr. G.F. Naterer, Dr. H.M. Soliman, Dr. J.T. Bartley

Instructions

1. This is a three-hour open textbook exam. Students are permitted to use the course textbook and a calculator. No other materials (i.e. notes, solved problems, etc.) are allowed.
2. State all assumptions and label your system with dashed lines. Write your solutions clearly and legibly in the booklets provided. Ambiguous solutions which cannot be interpreted will be considered incorrect.
3. If interpolation is required in the property tables, then use linear interpolation between table entries. Use **four** significant figures in your calculations and full precision from data in the textbook.
4. Attempt all **five** questions. The values are indicated in the margin.

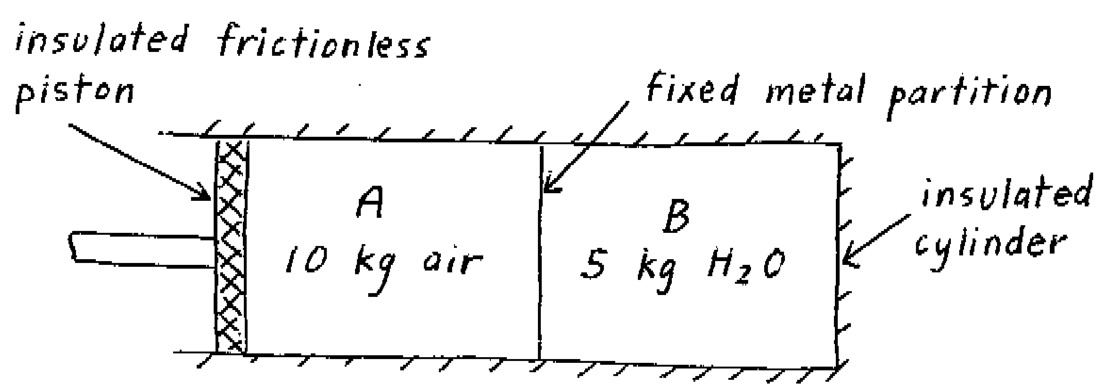
Value

1. An insulated cylinder is constructed so that compartments A and B are separated by a fixed metal partition (see figure). Compartment A is fitted with a frictionless, insulated piston. This compartment (A) contains 10 kg of air (treat as ideal gas) initially at 0.1 MPa and 150 °C, while compartment B initially contains a mixture of 4 kg of saturated liquid water and 1 kg of saturated water vapor at 150 °C. The piston is now moved to the right, thus compressing the air in a quasi-equilibrium process until the air temperature reaches 200 °C. It can be assumed that the temperature difference between A and B is only infinitesimal during the process (i.e. the temperature of both compartments is practically the same throughout the process). Heat transfer takes place across the fixed metal partition, but there is no heat transfer across the piston or cylinder walls (i.e., the heat transfer associated with the process in compartment A is all going to compartment B). Assume that changes in the kinetic and potential energies are negligible.

(10) (a) Determine the heat transfer across the partition in kJ.

(7) (b) Determine the work done by the piston on the air in kJ.

(3) (c) Assuming that the compression process for the air is polytropic, determine the exponent n.



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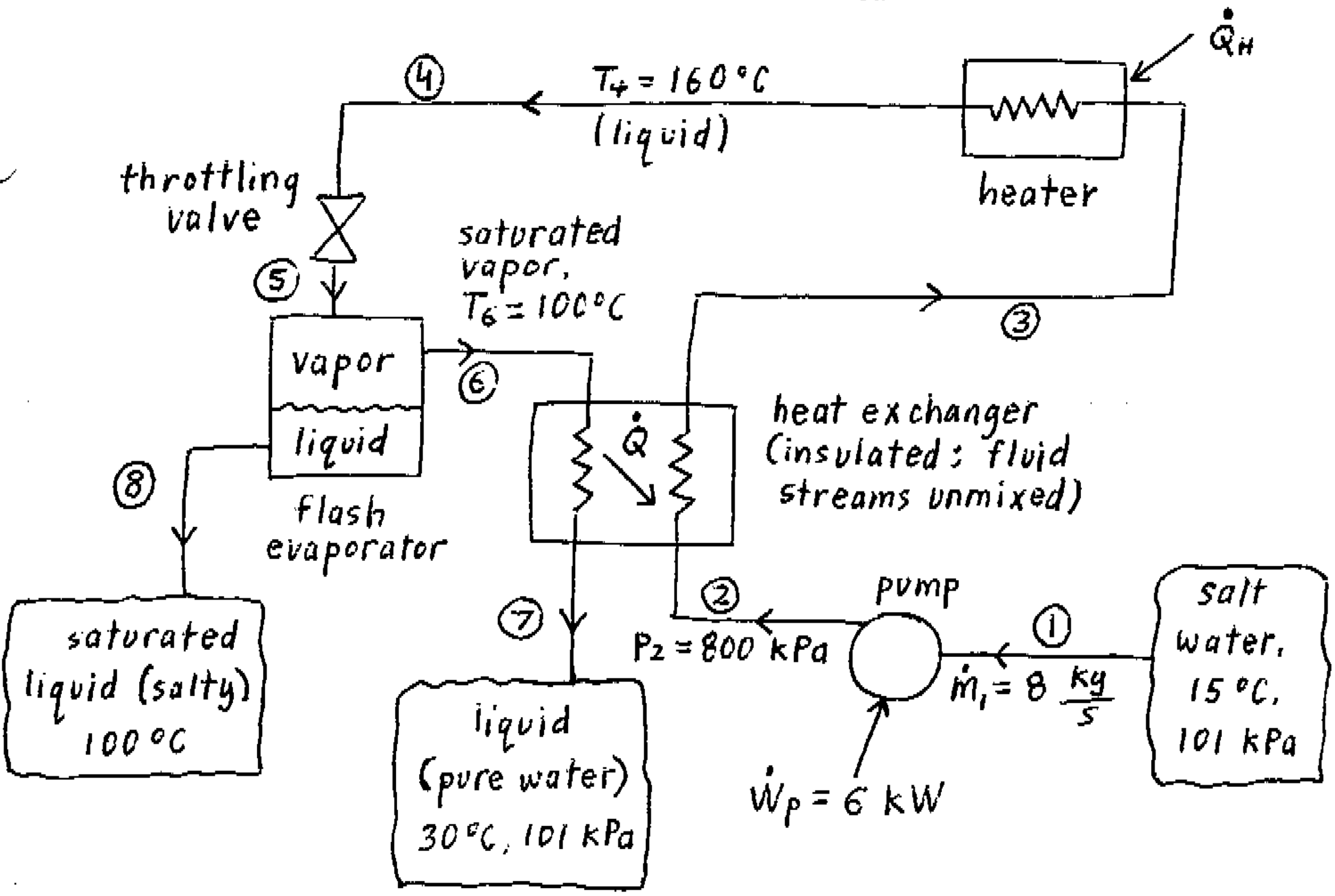
2. A proposed desalination plant involves a scheme for producing fresh water from salt (ocean) water. Salt water enters the plant at 15 °C, 101 kPa and 8 kg/s (see figure). Afterwards, it flows through a pump, two heat exchangers, throttling valve and flash evaporator, which separates salt and fresh water due to the phase change. The power input to the pump is 6 kW. Assume that the pump and flash chamber are externally insulated. Also, assume that pure water properties can be used for salt water. Changes in kinetic and potential energies can be assumed to be negligible throughout the plant.

(8) (a) Show all 8 state points on a $T-v$ diagram with respect to saturation lines. Label all known values of pressure and temperature. Neglect pressure changes through the two heat exchangers.

(6) (b) Find the fluid enthalpy and temperature at state 2.

(8) (c) What mass flow rate of pure water is produced (location 7) ?

(8) (d) Determine the required heat transfer to the heater ($\dot{Q}_H$) ?



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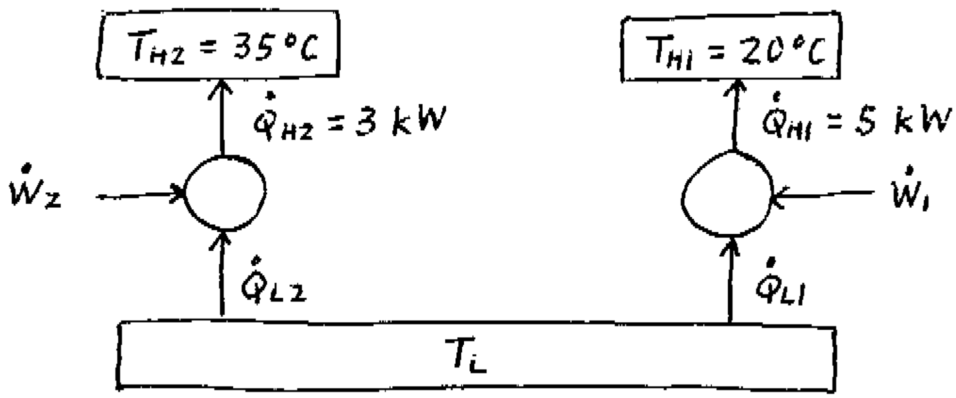
3. Two Carnot refrigerators are used to cool a large room. One refrigerator rejects heat at a rate of $\dot{Q}_{H1} = 5$ kW into a reservoir that has a temperature of $T_{H1} = 20$ °C. The second refrigerator rejects heat at a rate of $\dot{Q}_{H2} = 3$ kW into a reservoir that has a temperature of $T_{H2} = 35$ °C. The total power used by the two refrigerators is $\dot{W}_1 + \dot{W}_2 = 2$ kW.

(6) (a) What is the total heat rate (in kW) extracted from the large room by the two refrigerators combined (i.e. $\dot{Q}_{L1} + \dot{Q}_{L2}$) ?

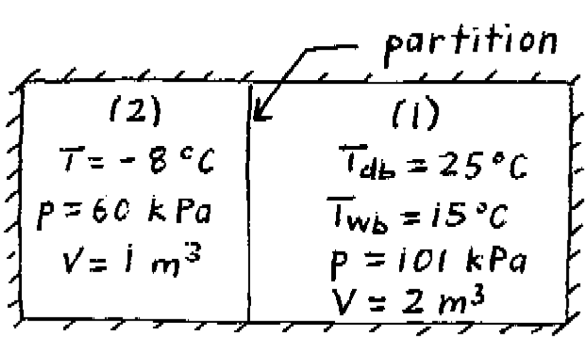
(9) (b) What is the temperature of the cooled room (T_L) and the power input to each individual refrigerator ?



4. An insulated rigid tank is divided into two compartments by a partition. One compartment (1) contains an air- water-vapor mixture in a 2 m^3 volume at a pressure of 101 kPa that has a dry-bulb temperature of $T_{db} = 25$°C and a wet-bulb temperauture of $T_{wb} = 15$ °C. The other compartment (2) contains dry air (no water-vapor) in a 1 m^3 volume at a temperature of −8°C and pressure of 60 kPa. The partition is removed and the air-vapor mixture in compartment (1) mixes with the dry air in compartment (2) until equilibrium is established.

(20)

Find the final temperature and pressure of the new mixture. Assume constant specific heats for air and water vapor that are evaluated at 300 K.



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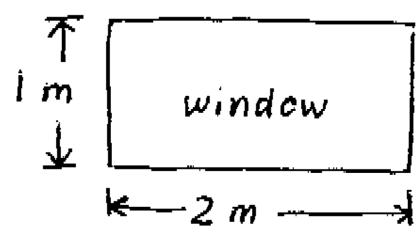
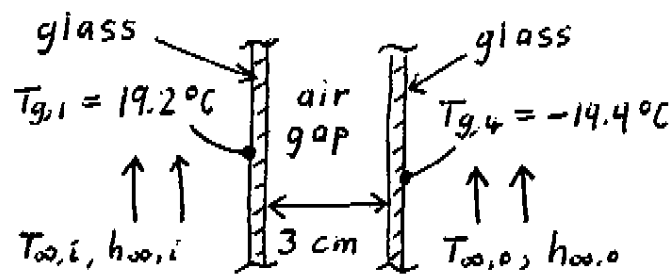
5. A window 1 m high by 2 m wide consists of two panes of glass separated by a distance of 3 cm thereby creating an air gap between the panes. The thickness of each of the glass panes is 6 mm and they each have a thermal conductivity of 0.75 W/m·K. The window is in a room where the ambient indoor air temperature is $T_{\infty,i} = 22°C$ and the outside air is at a temperature of $T_{\infty,o} = -15$ °C. The surface temperatures across the double-pane window are $T_{g,1} = 19.2°C$ and $T_{g,4} = -14.4$ °C, as shown in the diagram below. Assume that the air in the space between the panes of glass is motionless (no convection heat transfer occurs there) and the thermal conductivity of the still air is 0.025 W/m·K.

(6) (a) Draw a thermal resistance circuit representing the heat transfer in the window and calculate the total thermal resistance. Calculate the rate of heat transfer (in Watts) through the window.

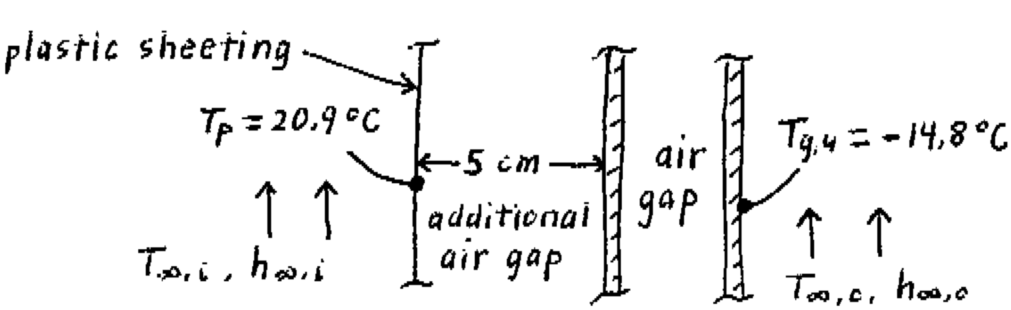




cross-section of window

(4) (b) Calculate the convection heat transfer coefficient of the indoor air at the inside surface of the window, $h_{\infty,i}$, and the convection heat transfer coefficient of the outside air at the outside surface of the window, $h_{\infty,o}$.

(5) (c) If a sheet of thin clear plastic is fastened onto the inside window frame creating an additional air gap that is 5 cm thick, what is the percentage change in the rate of heat transfer through the window? Assume the plastic sheet has negligible thermal resistance. The new temperatures across the window are $T_p = 20.9$ °C and $T_{g,4} = -14.8$ °C, as shown in the figure below.



Solutions April 2002

(1)

a) For the Water

(1) $x_1 = \frac{1}{5} = 0.2$

(2) $u_1 = (0.2)(2559.5) + 0.8(631.68) = 1017.2$ kJ/kg
$v_1 = (0.2)(0.3928) + 0.8(0.001091) = 0.07943$ m³/kg

(3) $v_2 = v_1$

(1) $0.07943 = x_2(0.12736) + (1-x_2)(0.001157)$
$x_2 = 0.6211$

(1) $u_2 = 0.6211(2595.3) + 0.3789(850.65) = 1934.3$ kJ/kg

(2) ${}_1Q_2 - \cancel{{}_1W_2} = m(u_2 - u_1) + \Delta \cancel{KE} + \Delta \cancel{PE}$
${}_1Q_2 = 5(1934.3 - 1017.2) = 4585.5$ kJ

The heat transfer across the metal partition is 4585.5 kJ (from the air to the water)

(b) For the air

${}_1Q_2 - {}_1W_2 = m(u_2 - u_1) + \Delta \cancel{KE} + \Delta \cancel{PE}$

(2) $= m c_{v_0}(T_2 - T_1)$

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Q.2 (Cont.) 2/2

3

$_1Q_2 = -4585.5$ kJ from part (a)

1

$$-4585.5 - {}_1W_2 = 10(0.7165)(200-150)$$
$${}_1W_2 = -4943.75 \text{ kJ}$$

Work done by piston on air is 4943.75 kJ

(c) Since polytropic, system, ideal gas, and quasiequil^m process

4

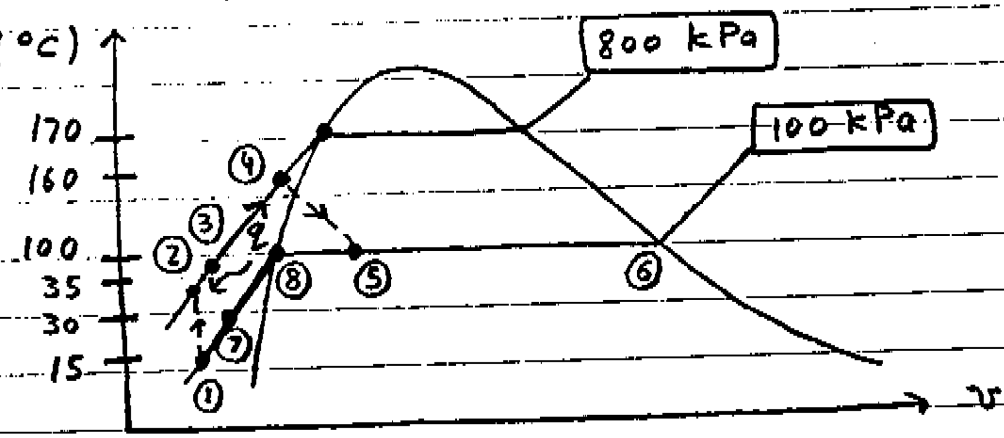
$${}_1W_2 = \frac{mR(T_2-T_1)}{1-n}$$
$$-4943.75 = \frac{(10)(0.287)(200-150)}{1-n}$$
$$n = 1.029$$

4

$$\frac{P_2}{P_1} = \left(\frac{T_2}{T_1}\right)^{\frac{n}{n-1}} = \left(\frac{473.15}{423.15}\right)^{\frac{1.029}{0.029}} = 52.61$$
$$P_2 = 5.261 \text{ MPa}$$

Solution. Assumptions: (i) SSSF, (ii) insulated flash chamber and heat exchanger, (iii) adiabatic pump, (iv) salt water properties same as pure water properties

(a)



8

(b) 1st Law (pump): $\dot{W}_P = \dot{m}_1 (h_2 - h_1)$

$\Rightarrow h_2 \sim h_1 + \frac{\dot{W}_P}{\dot{m}_1} = 62.99 + \frac{6}{8} = 63.74 \text{ kJ/kg}$

Interpolating (Table A-4),

$T_2 \sim 15 + \left(\frac{63.74 - 62.99}{83.96 - 62.99}\right)(20 - 15) = 15.18\ °C$

6

Note: $\Delta h = \Delta(u + pv) = \Delta u + \bar{v}\,\Delta p + p\,\cancel{\Delta v}^{\sim 0}$

Between inlet and outlet pressures, $\bar{v} \sim \frac{0.001043 + 0.001115}{2} = 0.001079 \frac{m^3}{kg}$

$\Rightarrow \dot{m}\,\bar{v}\,\Delta p \sim 6.0$ kW, so most pumping power goes into raising fluid pressure, rather than a temperature change

(c) Cons. Mass : $\dot{m}_1 = \dot{m}_6 + \dot{m}_8 = \dot{m}_7 + \dot{m}_8$ (entire system) (1)

1st Law (flash chamber + valve) : $\dot{m}_4 h_4 = \dot{m}_6 h_6 + \dot{m}_8 h_8$

$\Rightarrow \dot{m}_1 h_4 = \dot{m}_7 h_6 + (\dot{m}_1 - \dot{m}_7) h_8$ using (1)

8) $\Rightarrow \dot{m}_7 = \dot{m}_1 \left(\frac{h_4 - h_8}{h_6 - h_8}\right) = 8\left(\frac{675.55 - 419}{2676 - 419}\right) = 0.91$ kg/s

(d) 1st Law (entire plant) : $\dot{m}_1 h_1 + \dot{W}_p + \dot{Q}_H = \dot{m}_7 h_7 + \dot{m}_8 h_8$

$\Rightarrow \dot{Q}_H = \dot{m}_7 h_7 + (\dot{m}_1 - \dot{m}_7) h_8 - \dot{W}_p - \dot{m}_1 h_1$

$= \dot{m}_1 h_8 + \dot{m}_7 (h_7 - h_8) - \dot{m}_1 h_1 - \dot{W}_p$

8 $= 8(419) + 0.91(125.8 - 419) - 8(63) - 6 = 2,575.2$ kW

Alternative Soln: $\dot{m}_1 (h_3 - h_2) = \dot{m}_7 (h_6 - h_7)$ (1st Law, heat exchanger)

1st Law (heater) : $\dot{Q}_H = \dot{m}_1 (h_4 - h_3) = \dot{m}_1 \left(h_4 - \frac{\dot{m}_7}{\dot{m}_1}(h_6 - h_7) - h_2\right)$

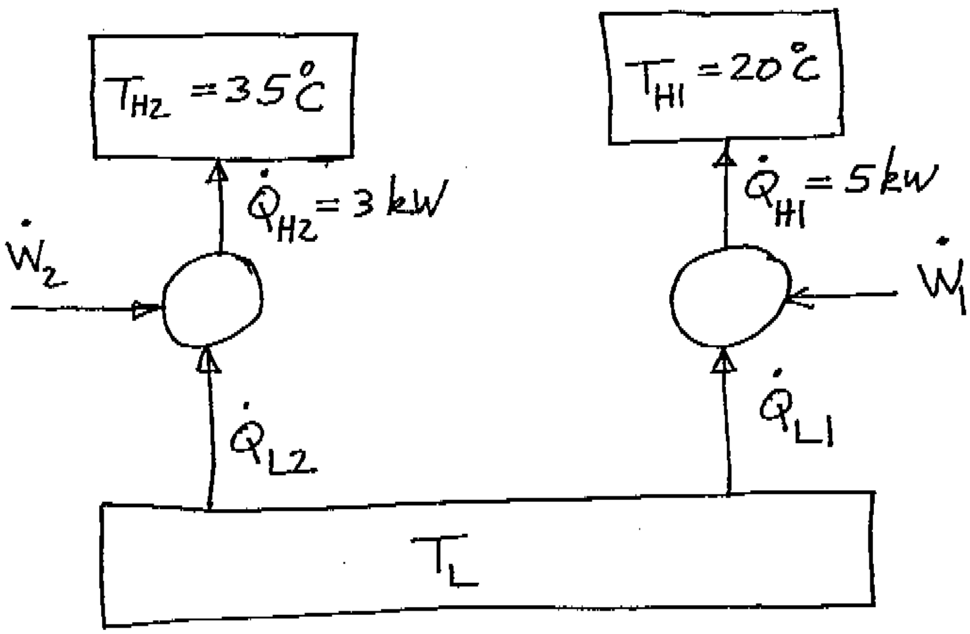
$\Rightarrow \dot{Q}_H = 8\left(675.55 - \frac{0.91}{8}(2676 - 125.8) - 63.74\right) \sim 2574$ kW ~ ✓

Summary:

$h_1 = 63$ kJ/kg	$\dot{m}_1 = 8$ kg/s
$h_2 = 63.74$ kJ/kg	$\dot{m}_2 = 8$ kg/s
$h_4 = 675.55$ kJ/kg	$\dot{m}_4 = 8$ kg/s
$h_5 \sim h_4 = 675.55$ kJ/kg	$\dot{m}_5 = 8$ kg/s
$h_6 = 2676$ kJ/kg	$\dot{m}_6 = 0.91$ kg/s
$h_7 = 125.8$ kJ/kg	$\dot{m}_7 = 0.91$ kg/s
$h_8 = 419$ kJ/kg	$\dot{m}_8 = 7.09$ kg/s

$\dot{Q}_H \sim 2575$ kW

(2)



(a)

$$\dot{Q}_{H1} - \dot{Q}_{L1} = \dot{W}_1 \qquad \text{——— (1)}$$

$$\dot{Q}_{H2} - \dot{Q}_{L2} = \dot{W}_2 \qquad \text{——— (2)}$$

Adding (1) and (2)

$$(\dot{Q}_{H1} + \dot{Q}_{H2}) - (\dot{Q}_{L1} + \dot{Q}_{L2}) = \dot{W}_1 + \dot{W}_2$$

$$\dot{Q}_{L1} + \dot{Q}_{L2} = -(\dot{W}_1 + \dot{W}_2) + \dot{Q}_{H1} + \dot{Q}_{H2}$$

$$= -2 + 5 + 3$$

$$\dot{Q}_{L1} + \dot{Q}_{L2} = 6\ kW \qquad \text{——— (3)}$$

(b)

$$\frac{\dot{Q}_{L1}}{\dot{Q}_{H1}} = \frac{T_L}{T_{H1}} \qquad \frac{\dot{Q}_{L1}}{5} = \frac{T_L}{293.15}$$

$$\dot{Q}_{L1} = \frac{5}{293.15} T_L = 0.01706\, T_L \quad \text{——} (4)$$

$$\frac{\dot{Q}_{L2}}{\dot{Q}_{H2}} = \frac{T_L}{T_{H2}} \qquad \frac{\dot{Q}_{L2}}{3} = \frac{T_L}{308.15}$$

$$\dot{Q}_{L2} = \frac{3}{308.15} T_L = 0.009736\, T_L \quad \text{——} (5)$$

Substitute (4) and (5) into (3)

$$0.01706\, T_L + 0.009736\, T_L = 6$$

$$0.026796\, T_L = 6$$

$$T_L = 223.91 \text{ K}$$

$$T_L = -49.24\ ^{\circ}\text{C} \quad \text{——} (6)$$

Substitute (6) into (4) $\quad \dot{Q}_{L1} = 3.82$ kW

Substitute (6) into (5) $\quad \dot{Q}_{L2} = 2.18$ kW

Substitute into (1) $\longrightarrow \quad \dot{W}_1 = 1.18$ kW

Substitute into (2) $\longrightarrow \quad \dot{W}_2 = 0.82$ kW

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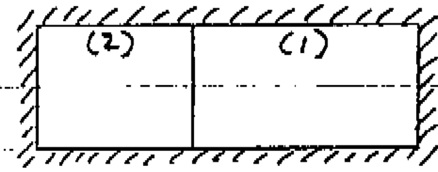
Question 4



given: tank is insulated

$V_1 = 2\ m^3$

(1) contains air-water-vap. mixture

$(T_{db})_1 = 25°C$

$(T_{wb})_1 = 15°C$

$P_1 = 101\ kPa$

$V_2 = 1\ m^3$

(2) contains dry air

$T_2 = -8°C$

$P_2 = 60\ kPa$

Find: mixture T and P after the partition is removed and equilibrium is established.

compartment 1

$$\omega_2 = \frac{0.622\ P_{g_2}}{P_2 - P_{g_2}}$$, subscript 2 means fully saturated

$$P_{g_2} = P_{sat@T_{wb}} = 1.7051\ kPa \quad \text{(Table A-4)}$$

$$\therefore \omega_2 = \frac{0.622 \times 1.7051}{101 - 1.7051} = 1.068 \times 10^{-2}\ \frac{kg\ H_2O}{kg\ dry\ air}$$

$$\omega_1 = \frac{C_p(T_2 - T_1) + \omega_2 h_{fg_2}}{h_{g_1} - h_{f_2}}$$

$h_{fg_2} = h_{fg@T_{wb}} = 2465.9\ kJ/kg$

$h_{g_1} = h_{g@T_{db}} = 2547.2\ kJ/kg$

(Table A-4) $h_{f_2} = h_{f@T_{wb}} = 62.99\ kJ/kg$

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$(C_p)_{air} = 1.005 \ kJ/kg \cdot K$

$$\therefore \omega_1 = \frac{1.005(15-25) + 1.068\times10^{-2} \times 2465.9}{2547.2 - 62.99}$$

$\omega_1 = 6.556 \times 10^{-3}$ kg H_2O/kg dry air

$$\phi = \frac{\omega P}{(0.622+\omega)P_g}, \quad P_g = P_{sat@T_{db}} = 3.169 \ kPa$$

$$\therefore \phi = \frac{6.556\times10^{-3} \times 101}{(0.622 + 6.556\times10^{-3}) \times 3.169} = 0.3324$$

or, could use psychrometric chart

$$\phi = \frac{P_v}{P_g} \quad \therefore P_v = 0.3324 \times 3.169 \ kPa$$

$P_v = 1.053 \ kPa$

$\therefore P_a = 101 - 1.053 = 99.95 \ kPa$

mixture analysis of compartment 1

dry air: $P_a V_1 = N_{a_1} R_u T_1$

$$\therefore N_{a_1} = \frac{99.95 \times 2}{8.314 \times (25+273)} = 8.068\times10^{-2} \ kmol$$

$R_u = 8.314 \ \frac{kJ}{kmol \cdot K}$

(Table A-1)

$M_a = 28.97 \ \frac{kg}{kmol}$

$m_{a_1} = N_{a_1} M_a$

$= 8.068\times10^{-2} \times 28.97$

$\therefore m_{a_1} = 2.337 \ kg$

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water vapor: $P_{v_1} V_1 = N_{v_1} R_u T_1$

$\therefore N_{v_1} = \dfrac{1.053 \times 2}{8.314 \times (25+273)} = 8.500 \times 10^{-4}$ kmol

$m_{v_1} = N_{v_1} M_v$, $M_v = 18.015$ kg/kmol

$= 8.500 \times 10^{-4} \times 18.015$ (Table A-1)

$= 0.01531$ kg

Compartment 2

(dry air only) $N_{a_2} = \dfrac{P_{a_2} V_2}{R_u T_2}$

$N_{a_2} = \dfrac{60 \times 1}{8.314 \times (-8+273)} = 2.723 \times 10^{-2}$ kmol

$m_{a_2} = N_{a_2} M_a$

$= 2.723 \times 10^{-2} \times 28.97$

$\therefore m_{a_2} = 0.7889$ kg

Partition is removed and gases mix:

$\therefore$ for total volume $1+2 = 3\ m^3$,

$m_a = m_{a_1} + m_{a_2} = 2.337 + 0.7889 = 3.126$ kg

$m_v = m_{v_1} = 0.01531$ kg

$m_m = m_a + m_v = 3.126 + 0.01531 = 3.141$ kg

$N_m = (8.068 \times 10^{-2} + 2.723 \times 10^{-2})_{dry\ air} + (8.5 \times 10^{-4})_{vapor}$

$N_m = 0.1088$ kmol

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$$M_m = \frac{m_m}{N_m} = \frac{3.141\ \text{kg}}{0.1088\ \text{kmol}} = 28.87\ \text{kg/kmol}$$

$$R_m = \frac{R_u}{M_m} = \frac{8.314\ \text{kJ/kmol}\cdot\text{K}}{28.87\ \text{kg/kmol}} = 0.2880\ \frac{\text{kJ}}{\text{kg}\cdot\text{K}}$$

Energy Balance between states before and after the partition is removed:

$$\cancel{E_{in}} - \cancel{E_{out}} = \Delta E_{system}$$

$$0 = \Delta U_{system}$$

$$\therefore \Delta U_{a_1} + \Delta U_{v_1} + \Delta U_{a_2} = 0$$

For ideal gases, we have

$$m_{a_1} C_{v_a}(T_m - T_1) + m_{v_1} C_{v_{H_2O}}(T_m - T_1) + m_{a_2} C_{v_a}(T_m - T_2) = 0$$

Assume constant specific heat values at 300 K

$C_{v_a} = 0.7180$ kJ/kg.K (Table A-2a)

$C_{v_{H_2O}} = 1.4108$ kJ/kg.K

$$(m_{a_1} C_{v_a} + m_{v_1} C_{v_{H_2O}} + m_{a_2} C_{v_a}) T_m =$$
$$= (m_{a_1} C_{v_a} + m_{v_1} C_{v_{H_2O}}) T_1 + m_{a_2} C_{v_a} T_2$$

Substitute in values:

$$(2.337 \times 0.7180 + 0.01531 \times 1.4108 + 0.7889 \times 0.7180)\, T_m =$$

$$= (2.337 \times 0.7180 + 0.01531 \times 1.4108) \times (25 + 273) +$$
$$+ 0.7889 \times 0.7180 \times (-8 + 273)$$

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Solving for T_m, we get $T_m = 289.75\ K$

$\longrightarrow$ or, $T_m = 16.8\ ^\circ C$

$$P_m = \frac{m_m R_m T_m}{V_m}$$

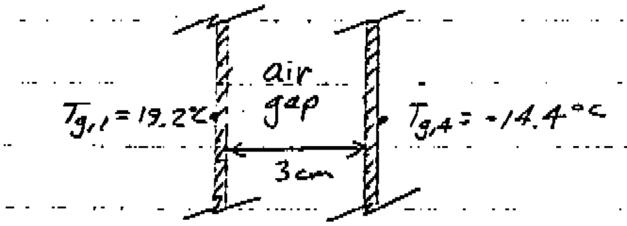
$$\therefore P_m = \frac{3.141 \times 0.2880 \times (289.8)}{(1+2)}$$

$\longrightarrow P_m = 87.39\ kPa$

Question 5

$T_{\infty,i}$ $T_{\infty,o}$

$h_{\infty,i}$ $h_{\infty,o}$



given:

$T_{g,1}$ $T_{\infty,i} = 22°C$ $k_{glass} = 0.75$ W/m·K

$T_{g,4}$ $T_{\infty,o} = -15°C$ $k_{air} = 0.025$ W/m·K

$\Delta x_{glass} = 6$ mm Window dimensions: 1m high × 2m wi

assumptions:

- air in gap between windows is motionless (no convection)
- uniform temperatures over surfaces of glass panes

a) thermal resistance circuit:

$$R_{gP1} = \frac{(\Delta x)_{glass}}{k_{glass} A} = \frac{0.006 \text{ m}}{0.75 \text{ W/m·K} \times (1 \times 2) \text{ m}^2} = 0.004 \frac{K}{W}$$

$$R_{air} = \frac{(\Delta x)_{air}}{k_{air} A} = \frac{0.030 \text{ m}}{0.025 \text{ W/m·K} \times (1 \times 2) \text{ m}^2} = 0.6 \frac{K}{W}$$

$$R_{gP2} = R_{gP1}$$

$$\therefore \sum R_t = 2 \times 0.004 + 0.6 = 0.608 \text{ K/W}$$

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$$\dot{Q} = \frac{T_{g,1} - T_{g,4}}{\sum R_t} = \frac{19.2 - -14.4}{0.608} = \underline{55.26\ W}$$

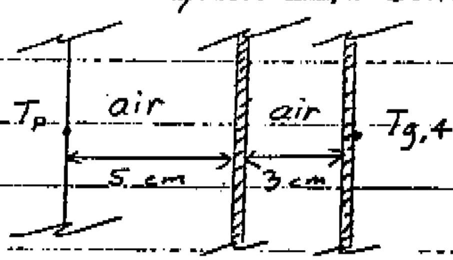
b) $\dot{Q} = \dot{Q}_{conv,i} = h_{\infty,i} A (T_{\infty,i} - T_{gp1})$

$$\therefore h_{\infty,i} = \frac{55.26}{(1\times2)(22-19.2)} = \underline{9.868\ W/m^2\cdot K}$$

$$\dot{Q} = \dot{Q}_{conv,o} = h_{\infty,o} A (T_{gp4} - T_{\infty,o})$$

$$\therefore h_{\infty,o} = \frac{55.26}{(1\times2)(-14.4 - -15)} = \underline{46.05\ W/m^2\cdot K}$$

c) With layer of plastic on inside window frame:



Assume temperature of plastic is uniform and the same on either side ($R_{t,plastic} \approx 0$)

$T_p = 20.9\,°C$, $T_{g,4} = -14.8\,°C$ (changes due to new thermal resistance,

$$R_{\substack{additional\\ air\ gap}} = \frac{(\Delta x)_{air}}{k_{air}\ A} = \frac{0.050\ m}{0.025\ W/m\cdot K \times (1\times2)\,m^2} = 1\ \frac{K}{W}$$

$$\therefore \sum R_t = \underbrace{0.608}_{\text{from part (a)}} + 1 = 1.608\ K/W$$

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$$\dot{Q} = \frac{T_P - T_{g,4}}{\sum R_t} = \frac{20.9 - -14.8}{1.608} = \underline{22.20\ W}$$

$$\%\text{ change in } \dot{Q} \text{ is } = \left|\frac{55.26 - 22.20}{55.26}\right| \times 100\%$$

% change is $\underline{59.83\ \%}$ reduction in $\dot{Q}$ across window due to additional air gap