Math 2130 Fall 2013 Test 2 (N Harland)

1. *Evaluate the limit, if it exists. Justify your answers.*

   (a) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^2-2xy+5y^2}{3x^2+4y^2}$. **3 Marks**

   (b) $\displaystyle\lim_{(x,y)\to(-1,3)} \frac{(x+1)^4-2(y-3)^2}{4(x+1)^2+(y-3)^2}$.

   3 Marks

2. (a) *Find a chain rule for* $\left.\dfrac{\partial x}{\partial y}\right)_z$ *if* $x=f(r,s,y),\ r=g(y),\ s=h(y,z)$.

   5 Marks

   (b) *Use part (a) to find* $\left.\dfrac{\partial x}{\partial y}\right)_z$ *if*

   $$x=e^{rs}+\tan(sy),\ r=\ln(y^2+2),\ s=y^2z.$$

   3 Marks

3. *The equations*

   $$xyv+uw=0,\quad y^2-v^2+v^2u=y,\quad yu+xv+w-1=0,$$

   define u, v, *and* w *as functions of* x *and* y. *Find* $\left.\dfrac{\partial v}{\partial y}\right)_x$ *when* $x=1, y=2, u=$
   -1,
   $v=1, w=2$. **12 Marks**

4. *Find the directional derivative of the function* $f(x,y,z)=x^2e^{y^3+z^3}$ *at the point* $(2,1,-1)$ *along the curve* $\mathbf{r}(t)=(2t^2)\mathbf{i}+(t^3+2)\mathbf{j}+(t^2-2)\mathbf{k}$ *in the direction of increasing* t.

   8 Marks

5. *Find parametric equations for the tangent line to the curve of intersection of the two surfaces*

   $$x^2y+y^2z+z^2x=5,\qquad x^2+xy+yz+z^2=2$$

   at $P_0(1,-1,2)$. **6 Marks**

6. *Find all critical points of* $f(x,y)=x^3+3xy-y^3$, *and classify each point as yielding a relative extremum, a saddle point, or none of these.* **10 Marks**