Example 5.3

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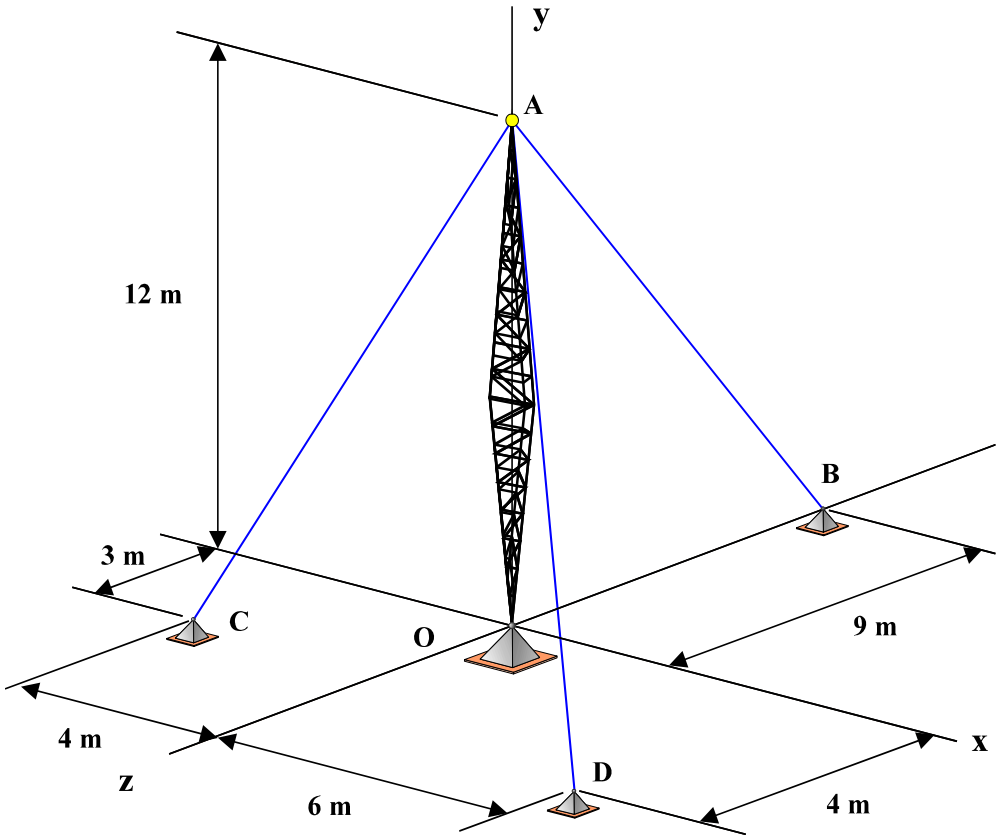
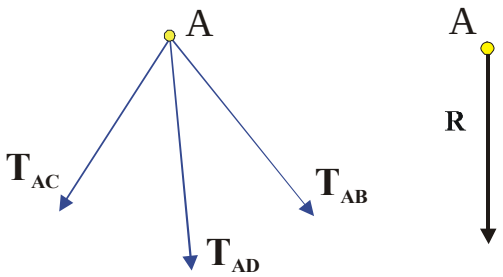
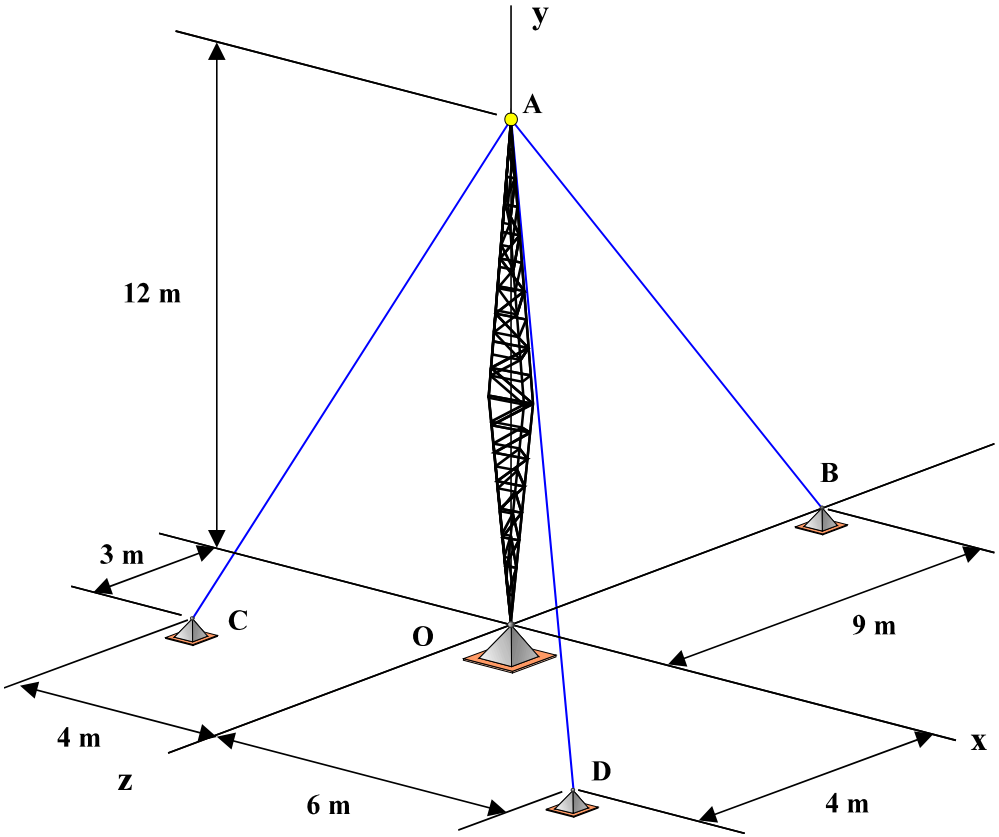
Example 5.3: (Question 1, December, 1983 Exam)
Knowing that the tension in cable AD is 2520 N, determine the required value of the tension in each of the cables AB and AC so that the resultant of the three forces applied by the cables at A is vertical.





Determine Coordinates of 2-points on each force vector



$$O:[0,0,0]$$

$$A:[0,12,0]$$

$$B:[0,0,-9]$$

$$C:[-4,0,3]$$

$$D:[6,0,4]$$

STEP 1 – Express ALL forces in Component Form

$$\mathbf{F} = \mathbf{F}\boldsymbol{\lambda}$$

Resultant

$$\mathbf{R} = \mathbf{T}_{\mathbf{AC}} + \mathbf{T}_{\mathbf{AD}} + \mathbf{T}_{\mathbf{AD}}$$

(1) $\mathbf{T_{AC}} = T_{AC}\boldsymbol{\lambda}_{\mathbf{AC}}$

$$\boldsymbol{\lambda}_{\mathbf{AC}} = \frac{\mathbf{AC}}{AC} \qquad \mathbf{AC} = -4\mathbf{i} - 12\mathbf{j} + 3\mathbf{k} \quad \text{and} \quad AC = \sqrt{\left(-4\right)^2 + \left(-12^2\right) + 3^2} = 13$$

$$\mathbf{T_{AC}} = T_{AC}\left(\frac{-4\mathbf{i} - 12\mathbf{j} + 3\mathbf{k}}{13}\right)$$

$$\mathbf{T_{AC}} = -\frac{4T_{AC}}{13}\mathbf{i} - \frac{12T_{AC}}{13}\mathbf{j} + \frac{3T_{AC}}{13}\mathbf{k}$$

(2) $\mathbf{T_{AD}} = T_{AD}\boldsymbol{\lambda}_{\mathbf{AD}} = 2520\boldsymbol{\lambda}_{\mathbf{AD}}$

$$\boldsymbol{\lambda}_{\mathbf{AD}} = \frac{\mathbf{AD}}{AD} \qquad \mathbf{AD} = 6\mathbf{i} - 12\mathbf{j} + 4\mathbf{k} \quad \text{and} \quad AD = \sqrt{6^2 + \left(-12^2\right) + 4^2} = 14$$

$$\mathbf{T_{AD}} = 2520\left(\frac{6\mathbf{i} - 12\mathbf{j} + 4\mathbf{k}}{14}\right)$$

$$\mathbf{T_{AD}} = 1080\mathbf{i} - 2160\mathbf{j} + 720\mathbf{k}$$

(3) $\mathbf{T_{AB}} = T_{AB}\boldsymbol{\lambda}_{\mathbf{AB}}$

$$\boldsymbol{\lambda}_{\mathbf{AB}} = \frac{\mathbf{AB}}{AB} \qquad \mathbf{AB} = 0\mathbf{i} - 12\mathbf{j} - 9\mathbf{k} \quad \text{and} \quad AB = \sqrt{\left(-12^2\right) + 9^2} = 15$$

$$\mathbf{T_{AB}} = T_{AB}\left(\frac{-12\mathbf{j} - 9\mathbf{k}}{15}\right)$$

$$\mathbf{T_{AB}} = -\frac{12T_{AB}}{15}\mathbf{j} - \frac{9T_{AB}}{15}\mathbf{k}$$

$$\mathbf{R} = \left(-\frac{4T_{AC}}{13} + 1080 \right)\mathbf{i} + \left(\frac{12T_{AC}}{13} - 2160 - \frac{12T_{AB}}{15} \right)\mathbf{j} + \left(\frac{3T_{AC}}{13} + 720 - \frac{9T_{AB}}{15} \right)\mathbf{k}$$

If the Resultant, **R**, is vertical then the x and z components of $\mathbf{R} = 0$.

$$\mathbf{R}_x = 0$$

$$-\frac{4T_{AC}}{13} + 1080 = 0$$

$$T_{AC} = 3510 \text{ N}$$

$$\mathbf{R}_z = 0$$

$$\frac{3T_{AC}}{13} + 720 - \frac{9T_{AB}}{15} = 0$$

$$\frac{3(3510)}{13} + 720 - \frac{9T_{AB}}{15} = 0$$

$$T_{AB} = 2550 \text{ N}$$