THE UNIVERSITY OF MANITOBA

November 10, 2010 — TRITERM EXAMINATION

DEPARTMENT & COURSE NO: MATH 1510 — PAGE NO: 1 of 5

EXAMINATION: Applied Calculus — EXAMINERS: Prymak, Korytowski

Values

[20] 1. Find $\frac{dy}{dx}$ if: (DO NOT SIMPLIFY YOUR ANSWER.)

(a) $y = \left(x + \sin(\pi x)\right)^3$ — 4

$$y' = 3\left(x + \sin(\pi x)\right)^2 \cdot \left(1 + \cos(\pi x) \cdot \pi\right)$$

(b) $y = \left(5^{x^2+x}\right)\log_3(x)$ — 6

$$y' = \underbrace{5^{x^2+x}}_{1} \cdot \underbrace{\ln 5 \cdot (2x+1)}_{1} \cdot \underbrace{\log_3 x}_{1} + \underbrace{5^{x^2+x}}_{1} \cdot \underbrace{\frac{1}{x \ln 3}}_{2}$$

(c) $y = \sec\left(e^{2x}\right)$ — 4

$$y' = \sec(e^{2x}) \cdot \tan(e^{2x}) \cdot e^{2x} \cdot 2$$

(d) $y = \tan\left(x^{\ln x}\right)$ — 6

Let $z = x^{\ln x}$

$\ln z = \ln x \cdot \ln x = (\ln x)^2$

$\frac{z'}{z} = 2 \ln x \cdot \frac{1}{x} = \frac{2 \ln x}{x}$

$z' = x^{\ln x} \cdot \frac{2 \ln x}{x}$ } 5 pts

$$y' = \sec^2\left(x^{\ln x}\right) \cdot x^{\ln x} \frac{2 \ln x}{x}$$

Values

[7] 2. Find an equation of the tangent line to the curve with equation $y^4 + xy = x^3 - x + 2$ at the point $(1,1)$.

$4y^3 \cdot y' + y + x y' = 3x^2 - 1$ - 5 pts

at $x=y=1$:

$4y' + 1 + y' = 3 - 1$

$y' = \frac{1}{5}$ - 1 pts

$y - 1 = \frac{1}{5}(x-1)$ - 1 pt.

3. . Find the absolute minimum and absolute maximum values of $f(x) = x^3 + 3x^2 - 9x + 2$ on the interval $[0,2]$.

[10]

$f'(x) = 3x^2 + 6x - 9$ (1)

$f'(x) = 0$ $x^2 + 2x - 3 = 0 = (x+3)(x-1)$ (2)

(1) ~~$x = -3$~~ outside $x = 1$ - crit. point (1)

$f(0) = 2$ (1)

$f(1) = 1 + 3 - 9 + 2 = -3$ (1)

$f(2) = 8 + 3 \cdot 4 - 9 \cdot 2 + 2 = 4$ (1)

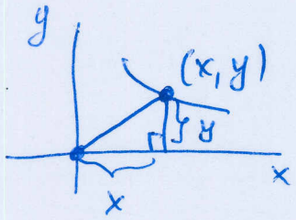
Abs. min value is -3, at $x=1$ (1)

Abs. max value is 4, at $x=2$ (1)

Values

4. (a) The hypotenuse of a right triangle has one end at the origin and one end on the curve with equation $y = x^2 e^{-3x}$, with $x \geq 0$. One of the other two sides is on the x-axis, the other side is parallel to the y-axis. Find the maximum area of such a triangle, if all lengths are measured in metres.

[9]



$A = \frac{1}{2} x \cdot y = \frac{1}{2} x^3 \cdot e^{-3x}$, $x \geq 0$

$A'(x) = \frac{1}{2} \cdot (3x^2 \cdot e^{-3x} + x^3 \cdot e^{-3x} \cdot (-3))$

$= \frac{3}{2} x^2 e^{-3x} (1 - x)$

$A' = 0$ at $x = 0$ and at $x = 1$

	$x < 1$	$x > 1$
A'	+	−
A	↗	↘

Abs. max. is at $x = 1$

$A(1) = \frac{1}{2} e^{-3}$ [m^2]

Ans: $\frac{1}{2} e^{-3}$ [m^2].

(b) Find the rate of change of the area of the triangle in (a) when $x = 2$ m and $\frac{dx}{dt} = 1$ m/s

[9]

$\left.\frac{dA}{dt}\right|_{x=2} = ?$

$\frac{dx}{dt} = 1$

$\frac{dA}{dt} = \frac{d\left(\frac{1}{2} x^3 e^{-3x}\right)}{dt} = \frac{3}{2} x^2 e^{-3x} (1 - x) \cdot \frac{dx}{dt}$

$\left.\frac{dA}{dt}\right|_{x=2} = \frac{3}{2} \cdot 2^2 e^{-6} \cdot (-1) \cdot 1 = -6 e^{-6}$

$\frac{dx}{dt} = 1$

Ans: decreasing at $6e^{-6}$ m^2/s.

Values

5. If $f(x)=\frac{(x-2)^3}{x^2}$ then $f'(x)=\frac{(x-2)^2(x+4)}{x^3}$ and $f''(x)=\frac{24(x-2)}{x^4}$.

(a) Compile the following information about f and its graph. (Give answers only; answer "NONE" if the function does not display a feature listed).

domain? $x\neq 0$ $(-\infty,0)\cup(0,\infty)$

y-intercept? NONE

x-intercepts? $x=2$

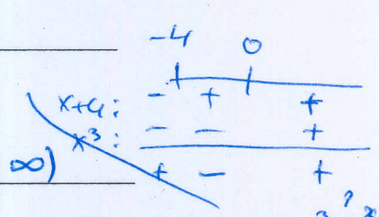
odd, even, or neither? Neither

equation(s) of vertical asymptotes? $x=0$

equation(s) of horizontal asymptotes? NONE

critical number(s) of the function f? $x=2,\ x=-4$

[20]



interval(s) where the function is increasing? $(-\infty,-4),(0,\infty)$

interval(s) where the function is decreasing? $(-4,0)$

x and y coordinates of the point where any local maxima occur? $(-4,-\frac{27}{2})$

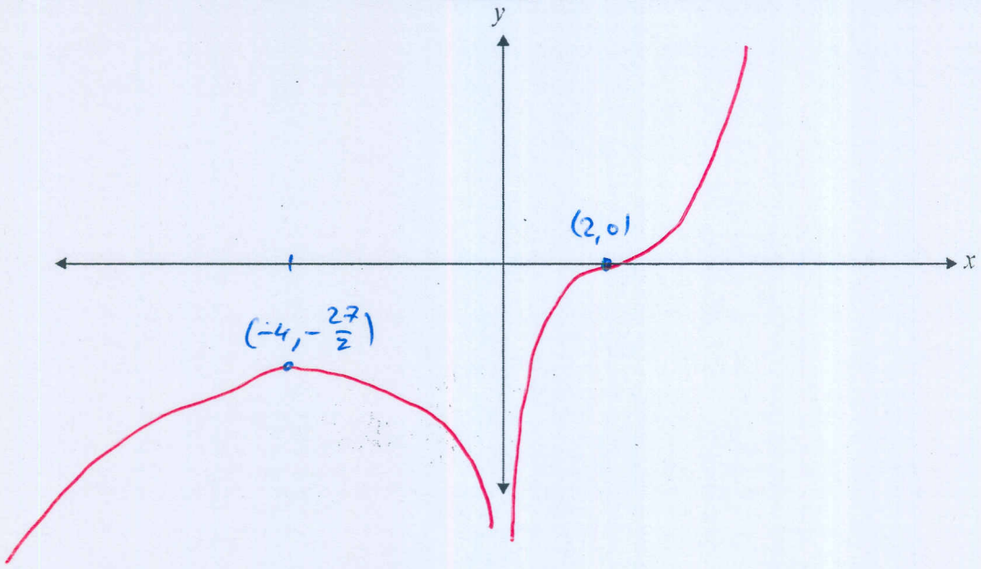
x and y coordinates of the point where any local minima occur? NONE

interval(s) where the function is concave up? $(2,\infty)$

interval(s) where the function is concave down? $(-\infty,0),(0,2)$

x and y coordinates of any inflection points $(2,0)$

(b) Sketch the graph of $y = f(x)$ on the given axes, reflecting all relevant information from (a) and labelling any important features on the graph.



BONUS [Maximum 3 marks.]
Find the values of a, b, and c if the parabola with equation $y = ax^2 + bx + c$ passes through the point with coordinates $(1,4)$ and whose tangent lines at $x = -1$ has slope 6, and whose tangent line at $x = 5$ has slope -2.

$y' = 2ax + b$

$$\begin{cases} 4 = a + b + c \\ 6 = -2a + b \\ -2 = 10a + b \end{cases}$$

$8 = -12a$

$a = -\frac{8}{12} = -\frac{2}{3}$ (1)

$b = 2a + 6 = 6 - \frac{4}{3} = \frac{14}{3}$ (1)

$c = 4 - a - b = 4 + \frac{2}{3} - \frac{14}{3} = \frac{12 + 2 - 14}{3} = 0$ (1)