Math 1710 Tutorial 1. Solutions

I (a) $\int\sqrt{1-5x}\,dx = \int(1-5x)^{\frac{1}{2}}dx = \dfrac{(1-5x)^{\frac{3}{2}}}{\frac{3}{2}(-5)} + C, = -\dfrac{2(1-5x)^{\frac{3}{2}}}{15} + C, \quad C\in\mathbb{R}$

(or you can make substitution $u=1-5x$)

(b) $\int\dfrac{x}{(x^2+13)^3}dx = \left\{\begin{array}{l} u=(x^2+13) \\ du = 2x\,dx \\ \frac{du}{2} = x\,dx\end{array}\right\} = \int\dfrac{du}{2u^3} = \int 2u^{-3}du = \dfrac{2u^{-2}}{-2} + C \;(=)$

$(=)\; -\dfrac{1}{u^2} + C = -\dfrac{1}{x^2+13} + C, \quad C\in\mathbb{R}$

(c) $\int\tan x\,dx = \int\dfrac{\sin x}{\cos x}dx = \left\{\begin{array}{l} u=\cos x \\ du = -\sin x \\ -du = \sin x\end{array}\right\} = \int\dfrac{-du}{u} = -\ln|u| + C \;(=)$

$(=)\; -\ln|\cos x| + C = \ln|\sec x| + C, \quad C\in\mathbb{R}$

(d) $\int 2^{2x}\cdot e^x dx = \int(2^2)^x\cdot e^x dx = \int 4^x\cdot e^x dx = \int(4e)^x dx = \dfrac{(4e)^x}{\ln(4e)} + C \;(=)$

$(=)\; \dfrac{(4e)^x}{1+2\ln 2} + C, \quad C\in\mathbb{R}$

(e) $\int x^2\,2^{x^3+1}dx = \left\{\begin{array}{l} u = x^3+1 \\ du = 3x^2dx \\ \frac{du}{3} = x^2dx\end{array}\right\} = \int 2^u\dfrac{du}{3} = \dfrac{2^u}{3\ln 2} + C \;(=)$

$(=)\; \dfrac{2^{x^3+1}}{3\ln 2} + C, \quad C\in\mathbb{R}$

(f) $\int\dfrac{e^{2x}+e^x}{e^x-1}dx = \int\dfrac{e^x(e^x+1)}{e^x-1}dx = \left\{\begin{array}{l} u = e^x-1 \\ du = e^x dx \\ e^x+1 = u+2\end{array}\right\} \;(=)$

$(=)\; \int\dfrac{u+2}{u}du = \int\left(1+\dfrac{2}{u}\right)du = u + 2\ln|u| + C = e^x - 1 + 2\ln|e^x-1| + C$

possible to combine -1 and C $(C\in\mathbb{R})$

(h) $\int\left(\dfrac{x}{x^5+2}\right)^4 dx = \int\dfrac{x^4}{(x^5+2)^4}dx = \left\{\begin{array}{l} u = x^5+2 \\ du = 5x^4dx \\ \frac{du}{5} = x^4dx\end{array}\right\} = \int\dfrac{du}{5u^4} \;(=)$

$(=)\; \dfrac{u^{-3}}{-3\cdot 5} + C = -\dfrac{1}{15(x^5+2)^3} + C, \quad C\in\mathbb{R}$

II (a) $\int_{-1}^{2}\sqrt[3]{x}\,dx = \left.\dfrac{3x^{4/3}}{4}\right|_{-1}^{2} = \dfrac{3\sqrt[3]{16}}{4} - \dfrac{3}{4}$ ($\sqrt[3]{x}$ can be extended for negative values of x)

(b) $\int_{-1}^{1}(x^3-2x^2+x-1)\,dx = \left.\left(\dfrac{x^4}{4} - \dfrac{2x^3}{3} + \dfrac{x^2}{2} - x\right)\right|_{-1}^{1} \;(=)$

$$\Leftrightarrow \left(\frac{1}{4} - \frac{2}{3} + \frac{1}{2} - 1\right) - \left(\frac{1}{4} + \frac{2}{3} + \frac{1}{2} + 1\right) = -\frac{4}{3} - 2 = -\frac{10}{3}$$

(Perhaps, mention about integrals of odd/even functions over symmetric interval).

(c) $$\int_0^{\pi} \sin x \, dx = (-\cos x)\Big|_0^{\pi} = -\cos\pi - (-\cos 0) = -(-1) - (-1) = 2$$

(d) $$\int_0^{\ln 3} e^x \, dx = e^x \Big|_0^{\ln 3} = e^{\ln 3} - e^0 = 3 - 1 = 2$$

(e) $$\int_{-4}^{-2} \left(\frac{1}{x} + \frac{1}{x^2}\right) dx = \left(\ln|x| - \frac{1}{x}\right)\Big|_{-4}^{-2} = \left(\ln 2 + \frac{1}{2}\right) - \left(\ln 4 + \frac{1}{4}\right) \Leftrightarrow$$

$$\Leftrightarrow \ln\left(\frac{1}{2}\right) + \frac{1}{4} = -\ln 2 + \frac{1}{4}$$

(f) $$\int_1^2 \frac{(\sqrt{x} + \sqrt[3]{x^2})^2}{x} dx = \int_1^2 \frac{(x^{1/2} + x^{2/3})^2}{x} dx = \int_1^2 \frac{x + 2x^{\frac{7}{6}} + x^{4/3}}{x} dx \Leftrightarrow$$

$$\Leftrightarrow \int_1^2 \left(1 + 2x^{\frac{1}{6}} + x^{\frac{1}{3}}\right) dx = \left(x + 2 \cdot \frac{6x^{\frac{7}{6}}}{7} + \frac{3x^{\frac{4}{3}}}{4}\right)\Big|_1^2 \Leftrightarrow$$

$$\Leftrightarrow \left(2 + \frac{12}{7}\sqrt[6]{2^7} + \frac{3}{4}\sqrt[3]{2^4}\right) - \left(1 + \frac{12}{7} + \frac{3}{4}\right) = \frac{24}{7}\sqrt[6]{2} + \frac{3}{2}\sqrt[3]{2} - \frac{41}{28}$$

(g) $$\int_0^2 |1-x| \, dx = \int_0^1 |1-x| \, dx + \int_1^2 |1-x| \, dx = \left\{\begin{array}{l} |1-x| = 1-x, \text{ if } 1-x \geq 0 \Leftrightarrow x \leq 1 \\ |1-x| = -(1-x), \text{ if } 1-x < 0 \Leftrightarrow x > 1 \end{array}\right.$$

$$\Leftrightarrow \int_0^1 (1-x) \, dx + \int_1^2 (x-1) \, dx = \left(x - \frac{x^2}{2}\right)\Big|_0^1 + \left(\frac{x^2}{2} - x\right)\Big|_1^2 \Leftrightarrow$$

$$\Leftrightarrow \left(1 - \frac{1}{2}\right) - (0 - 0) + (2 - 2) - \left(\frac{1}{2} - 1\right) = 1$$

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iii. (a) $$\int_1^2 \frac{dx}{2x-1} = \left\{\begin{array}{ll} u = 2x-1 & x=1 \Rightarrow u=1 \\ du = 2dx & x=2 \Rightarrow u=3 \\ dx = \frac{du}{2}, & \end{array}\right\} = \int_1^3 \frac{du}{2u} = \frac{1}{2}\ln|u|\Big|_1^3 \Leftrightarrow$$

$$\Leftrightarrow \frac{1}{2}\ln 3 - \frac{1}{2}\ln 1 = \frac{\ln 3}{2}$$

(b) $$\int_e^{e^2} \frac{\ln x}{x(\ln^2 x + 1)} dx = \left\{\begin{array}{ll} u = \ln^2 x + 1 & x = e \Rightarrow u = 2 \\ du = 2\ln x \cdot \frac{1}{x} dx & x = e^2 \Rightarrow u = 5 \\ \frac{du}{2} = \frac{\ln x}{x} dx, & \end{array}\right\} = \int_2^5 \frac{du}{2u} \Leftrightarrow$$

$$\Leftrightarrow \frac{1}{2}\ln|u|\Big|_2^5 = \frac{1}{2}\ln 5 - \frac{1}{2}\ln 2 = \frac{1}{2}\ln\frac{5}{2}.$$

(c)
$$\int_0^{\sqrt{\pi}} x\sin(x^2)\,dx = \left\{\begin{array}{ll} u = x^2 & x=0 \Rightarrow u=0 \\ du = 2x\,dx & x=\sqrt{\pi} \Rightarrow u=\pi \\ x\,dx = \frac{du}{2}, & \end{array}\right\} = \int_0^{\pi} \sin u \,\frac{du}{2} =$$

$$= -\frac{1}{2}\cos u \Big|_0^{\pi} = -\frac{1}{2}\cos\pi - \left(-\frac{1}{2}\cos 0\right) = \frac{1}{2} + \frac{1}{2} = 1.$$

(d)
$$\int_0^4 \frac{dx}{1+\sqrt{x}} = \left\{\begin{array}{ll} u = 1+\sqrt{x} & x=0 \Rightarrow u=1 \\ du = \frac{1}{2\sqrt{x}}\,dx & x=4 \Rightarrow u=3 \\ 2\sqrt{x}\,du = dx & \\ 2(u-1)\,du = dx, & \end{array}\right\} = \int_1^3 \frac{2(u-1)}{u}\,du =$$

$$= \int_1^3 \left(2 - \frac{2}{u}\right)du = \left(2u - 2\ln|u|\right)\Big|_1^3 = (6-2\ln 3) - (2-2\ln 1) =$$

$$= 4 - 2\ln 3.$$

(e)
$$\int_2^3 x(x-2)^6\,dx = \left\{\begin{array}{l} u = x-2 \\ du = dx \\ x=2 \Rightarrow u=0 \\ x=3 \Rightarrow u=1 \\ x = u+2 \end{array}\right\} = \int_0^1 (u+2)u^6\,du =$$

$$= \int_0^1 \left(u^7 + 2u^6\right)du = \left(\frac{u^8}{8} + \frac{2u^7}{7}\right)\Big|_0^1 = \left(\frac{1}{8}+\frac{2}{7}\right) - 0 = \frac{23}{56}.$$

(f)
$$\int_{-1}^3 \frac{x^2+1}{\sqrt{x^3+3x+8}}\,dx = \left\{\begin{array}{ll} u = x^3+3x+8 & x=-1 \Rightarrow u=4 \\ du = (3x^2+3)\,dx & x=3 \Rightarrow u=44 \\ \frac{du}{3} = (x^2+1)\,dx, & \end{array}\right\} = \int_4^{44} \frac{du}{3\sqrt{u}}$$

$$= \frac{2}{3}\sqrt{u}\,\Big|_4^{44} = \frac{2}{3}\sqrt{44} - \frac{2}{3}\sqrt{4} = \frac{4}{3}\sqrt{11} - \frac{4}{3}.$$

(g)
$$\int_0^3 \frac{x\,dx}{\sqrt{x+1}} = \left\{\begin{array}{ll} u = x+1 & x=0 \Rightarrow u=1 \\ du = dx & x=3 \Rightarrow u=4 \\ x = u-1, & \end{array}\right\} = \int_1^4 \frac{u-1}{\sqrt{u}}\,du = \int_1^4 \sqrt{u} - \frac{1}{\sqrt{u}}\,du =$$

$$= \left(\frac{2}{3}u^{\frac{3}{2}} - 2u^{\frac{1}{2}}\right)\Big|_1^4 = \left(\frac{2}{3}4^{\frac{3}{2}} - 2\cdot 4^{\frac{1}{2}}\right) - \left(\frac{2}{3} - 2\right) =$$

$$= \frac{16}{3} - 4 - \frac{2}{3} + 2 = \frac{14}{3} - 2 = \frac{8}{3}.$$

(h)
$$\int_0^1 x^3(x^2+1)^{\frac{1}{3}}\,dx = \int_0^1 x^2(x^2+1)^{\frac{1}{3}}\,x\,dx = \left\{\begin{array}{ll} x^2+1 = u, & x^2 = u-1 \\ du = 2x\,dx, & x=0 \Rightarrow u=1 \\ x\,dx = \frac{du}{2}, & x=1 \Rightarrow u=2 \end{array}\right\} =$$

$$= \int_1^2 (u-1)u^{\frac{1}{3}}\frac{du}{2} = \frac{1}{2}\int_1^2 \left(u^{\frac{4}{3}} - u^{\frac{1}{3}}\right)du = \frac{1}{2}\left(\frac{3u^{\frac{7}{3}}}{7} - \frac{3u^{\frac{4}{3}}}{4}\right)\Big|_1^2 =$$

$$= \frac{1}{2}\left[\left(\frac{3\cdot 4\cdot\sqrt[3]{2}}{7} - \frac{3\cdot 2\cdot\sqrt[3]{2}}{4}\right) - \left(\frac{3}{7} - \frac{3}{4}\right)\right] = \frac{6}{7}\sqrt[3]{2} - \frac{3}{4}\sqrt[3]{2} + \frac{9}{28} = \frac{3}{28}\sqrt[3]{2} + \frac{9}{28}$$