**MATH 2130 Sample Test 2** **60 minutes**

**Values**

**6** **1.** (a) Considering all directions that one could move away from the point $(0,1,4)$, in which one does the function

$$f(x,y,z) = z\ln(2x+y) - xyz^2$$

decrease most rapidly?

(b) Is it possible to find directions in which the function in part (a) changes at the rate of $-10$? Explain.

**Answer**: (a) $(8,-4,0)$ (b) No

**7** **2.** It is known that the curve

$$x = 4t - 2t^2 - 1, \quad y = 2t^3 - 1, \quad z = 3t^2 - 5$$

and surface $x^2y + xz + 1 = 0$ intersect at the point $(1,1,-2)$. You need **NOT** show this. Prove that the curve is perpendicular to the surface at the point.

**7** **3.** The equations

$$u^3x^2 + uv^2 - xz = y + v + 1, \quad v^5y^2 + u^4x + uz = 6v + 3$$

define $u$ and $v$ as functions of $x$, $y$, and $z$. Set up the Jacobians necessary to find $\partial v/\partial y$. Calculate the partial derivatives in the Jacobians, but do **NOT** evaluate the determinants.

**Answer**: $-\dfrac{\begin{vmatrix} 3u^2x^2+v^2 & -1 \\ 4u^3x+z & 2v^5y \end{vmatrix}}{\begin{vmatrix} 3u^2x^2+v^2 & 2uv-1 \\ 4u^3x+z & 5v^4y^2-6 \end{vmatrix}}$

**8** **4.** If $u = r^3 + rs^4$, $r = \sqrt{x^2+y^2+z^2}$, $s = \sin(xy)$, and $y = z^2$, find $\left.\dfrac{\partial u}{\partial z}\right)_x$. Do **NOT** simplify your answer.

**Answer**: $\dfrac{2yz(3r^2+s^4)}{\sqrt{x^2+y^2+z^2}} + \dfrac{(3r^2+s^4)z}{\sqrt{x^2+y^2+z^2}} + 4rs^3[x\cos(xy)](2z)$

8 **5.** The function

$$f(x, y) = x^4 - 3x^2y^2 + y^4 + x^2 + y^2$$

is known to have five critical point $(0, 0)$, $(1, \pm 1)$, and $(-1, \pm 1)$. It is **NOT** necessary for you to show this. Classify the two critical points $(0, 0)$ and $(1, 1)$ as yielding relative maxima, relative minima, or saddle points.

**Answer**: $(0, 0)$ gives a relative minimum and $(1, 1)$ gives a saddle point

4 **6.** The function

$$f(x, y) = x^4 - 3x^2y^2 + y^4$$

is known to have critical point $(0, 0)$, and the second derivative test fails to determine whether this critical point gives a relative maximum, a relative minimum, or a saddle point. Use whatever method you can devise to perform this classification.

**Answer**: Show that the cross-section of the surface with the $yz$-plane gives a curve that is concave upward, whereas intersection with the plane $y = x$ gives a curve that is concave downward.