UNIVERSITY OF MANITOBA

Date: August 6, 2011 **Final Examination**
Department: Mathematics **Title page**
Course: MATH 2130 **Time:** 3 hours
Examination: Engineering Mathematical Analysis 1 **Examiner:** T. Ngatched

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INSTRUCTIONS:

1. No aid permitted.
2. Attempt all questions.
3. Show all your work clearly for full credit.
4. If insufficient space is provided for a solution to a problem, continue your work on the back of the previous page, but **clearly indicate** that your work is continued.
5. Check that your examination booklet contains a title page, pages numbered from 1 to 13, and two blank pages (which may be removed).
6. Fill in the information requested below.

Student Name (Print): _____

Student Signature: _____

Student Number: _____

Seat Number: _____

Question	Points	Score	Question	Points	Score
1	7		8	8	
2	7		9	8	
3	7		10	9	
4	8		11	6	
5	8		12	9	
6	5		13	9	
7	9				

Total: /100

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1. (7 points) Show that the line $x + y + z = 6$, $2x - y + z = 3$ and the plane $2x - 7y - z = 5$ are parallel, and find the distance between them.

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2. (7 points) Evaluate

$$\lim_{(x,y)\to(-1,1)} \frac{x^2 - y^2}{x^2 - xy - 2y^2},$$

if it exists. If the limit does not exist, explain why not.

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3. (7 points) Use Chain Rule to express $\frac{\partial z}{\partial r}$ and $\frac{\partial z}{\partial \theta}$ as functions of r and θ if $z = x^2 + y^2 + u^2$, $x = r\cos\theta$, $y = r\sin\theta$, $u = r$.

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4. (8 points) The equations

$$u^3x^2 + uv^2 - xz = y + v + 1 \quad \text{and} \quad v^5y^2 + u^4x + uz = 6v + 3$$

define u and v as differentiable functions of x, y, and z. Set up the Jacobians necessary to find $\partial v/\partial y$. Calculate the partial derivatives in the Jacobians, but do **NOT** evaluate the determinants.

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5. (8 points) Find symmetric equations and parametric equations of the line that is tangent to the curve of intersection of the plane $2x + 3y + 4z = 1$ and the surface $xyz = 6$ at the point $(3, -1, -2)$.

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6. (5 points) Find an equation of the tangent plane to the surface $z^2 = x^2 + y^2$ at the point $(3, 4, -5)$.

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7. (9 points) Find and classify critical points of the function $f(x, y) = 10 - x^3 - y^3 - 3x^2 + 3y^2$, as yielding relative maxima, relative minima, saddle points, or none of these.

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8. (8 points) The temperature distribution in degrees Celsius at each point (x, y, z) on the surface $x^2 + y^2 + z^2 = 1$ is given by $T(x, y, z) = zx + yz$. Find the hottest spot.

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9. (8 points) Use the method of Lagrange multipliers to find the dimensions of the rectangle with the maximum perimeter that can be inscribed with sides parallel to the coordinate axes in the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

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10. (9 points) Evaluate the following integral by first converting to polar coordinates.

$$\int_0^1 \int_0^{\sqrt{1-y^2}} \frac{1}{1+x^2+y^2} dx dy$$

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11. (6 points) a) Skecth the region bounded by the curves

$$x = \sqrt{y+2}, \quad x = 2y - 2, \quad x = 0$$

b) Set up, but do **NOT** evaluate, double iterated integral(s) for the volume of the solid of revolution obtained by rotating the region about the line $x = -4$.

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12. (9 points) Find the volume of a region V in the first octant bounded by the plane $2x + y + z = 2$ and inside the cylinder $y^2 + z^2 = 1$.

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13. (9 points) The region in first octant that is below the planes $2x + 3y + 3z = 6$ and $x + 2z = 2$ is filled with a substance whose density is given by $\rho(x, y, z) = 16 - z$. Set up, but do **NOT** evaluate, the triple iterated integrals necessary to find the total mass, M, and the centre of mass of the substance in this region.

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