Math 2130 Summer2012 Test 1

1. Let l_1 be the line with symmetric equations

$$\frac{x-1}{2} = \frac{y+3}{3} = 4 - z$$

and l_2 be the line of intersection of the planes

$$x + y + 5z = 2 \text{ and } y + 3z = 1.$$

(a) Find parametric equations for the line l_2. [4]

(b) Determine whether l_1 and l_2 are intersecting, parallel or skew. [2]

(c) Find the distance between lines l_1 and l_2. [6]

2. Find a vector representation for the curves of intersection of $z = \sqrt{4 - x^2 - y^2}$ and $x^2 + y^2 - 2y = 0$ directed so that z increases when x is positive. [6]

3. Let a curve C be defined by a position vector $\mathbf{r}(t) = \langle 2\sin t, 2\cos t, 3t \rangle$.

(a) Find the tangent vector. [3]

(b) Find the unit tangent vector. [2]

(c) Find the length of the curve C from the point $(0, 2, 0)$ to $(0, 2, 6\pi)$. [4]

4. Find the following limits, or show why they do not exist.

(a) $\lim\limits_{(x,y)\to(0,0)} \frac{2xy}{x^2+3y^2}$ [4]

(b) $\lim\limits_{(x,y)\to(0,0)} \frac{\sin(x^2+y^2)}{x^3+xy^2+2x^2+2y^2}$ [3]

5. Show that the function $f(x, y) = \frac{x^3 y}{x-y}$ satisfies $x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} = 3f(x, y)$. [6]