Student Name -   Student Number -

Values

7   1. (a) Find the Taylor series of $\ln x$ about $x = 3$. Express your answer in sigma notation.

(b) What is the open interval of convergence of the series?

9   2. (a) Find the Taylor series about $x = -2$ for $f(x) = \dfrac{1}{1 + 3x}$.   Express your final answer in sigma notation. Use a technique that guarantees that the series converges to the function.

(b) What is the interval of convergence of the series?

8   3. Evaluate

$$\sum_{n=0}^{\infty} \frac{1}{n+1} x^{2n}.$$

Justify all steps in your solution.

8   4. Find, in explicit form $y = f(x)$, a 1-parameter family of solutions for the differential equation

$$x\frac{dy}{dx} = (x+1)y^2.$$

Does the 1-parameter family of solutions have any singular solutions? Explain.

8   5. Find the solution of the initial value problem

$$2\frac{dy}{dx} = y + 2x^2 e^{x/2}, \quad y(0) = 3.$$

Dawit's (plankion@yahoo.com)
Answers

1. a) $\ln 3 + \displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n\, 3^n}(x-3)^n$ b) $0 < x < 6$.

2. a) $-\displaystyle\sum_{n=0}^{\infty} \frac{3^n}{5^{n+1}}(x+2)^n$ b) $-\frac{11}{3} < x < -\frac{1}{3}$

3. $S(x) = \begin{cases} -\frac{1}{x^2}\ln(1-x^2) \, ; & -1 < x < 1, \ x \neq 0 \\ 1 \, , & x = 0 \end{cases}$

4. $y = \dfrac{-1}{x + \ln|x| + C}$,   yes $y = 0$ is a Singular Solution.

5. $y = \frac{1}{3}(9 + x^3) e^{x/2}$