Math 2130 Fall 2013 Test 2 (N Harland)

1. *Evaluate the limit, if it exists. Justify your answers.*

   (a) $\lim_{(x,y)\to(0,0)} \dfrac{x^2-2xy+5y^2}{3x^2+4y^2}$. **3 Marks**

   (b) $\lim_{(x,y)\to(-1,3)} \dfrac{(x+1)^4-2(y-3)^2}{4(x+1)^2+(y-3)^2}$. **3 Marks**

2. (a) *Find a chain rule for* $\left.\dfrac{\partial x}{\partial y}\right)_z$ *if* $x=f(r,s,y)$, $r=g(y)$, $s=h(y,z)$. **5 Marks**

   (b) *Use part (a) to find* $\left.\dfrac{\partial x}{\partial y}\right)_z$ *if*

   $$x=e^{rs}+\tan(sy),\ r=\ln(y^2+2),\ s=y^2z.$$

   3 Marks

3. *The equations*

   $$xyv+uw=0,\quad y^2-v^2+v^2u=y,\quad yu+xv+w-1=0,$$

   define u, v, *and* w *as functions of* x *and* y. *Find* $\left.\dfrac{\partial v}{\partial y}\right)_x$ *when* $x=1, y=2, u=-1$, $v=1, w=2$. **12 Marks**

4. *Find the directional derivative of the function* $f(x,y,z)=x^2e^{y^3+z^3}$ *at the point* $(2,1,-1)$ *along the curve* $\mathbf{r}(t)=(2t^2)\mathbf{i}+(t^3+2)\mathbf{j}+(t^2-2)\mathbf{k}$ *in the direction of increasing* t. **8 Marks**

5. *Find parametric equations for the tangent line to the curve of intersection of the two surfaces*

   $$x^2y+y^2z+z^2x=5,\qquad x^2+xy+yz+z^2=2$$

   at $P_0(1,-1,2)$. **6 Marks**

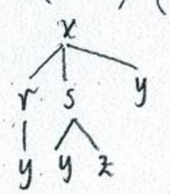
6. *Find all critical points of* $f(x,y)=x^3+3xy-y^3$, *and classify each point as yielding a relative extremum, a saddle point, or none of these.* **10 Marks**

Dawit's Answers

1. a) DNE (path dependent)
Hint: let $y=mx$
limit $= \dfrac{5m^2-2m+1}{4m^2+3}$
along y-axis ($x=0$, $m\to\infty$)
limit $=5/4$
along x-axis ($m=0$, $y=0$)
limit $=1/3$

b) DNE (path dependent)
Hint: along the $x=-1$ line
limit $=-1$
along $y=3$ line
limit $=0$

2. a) $\dfrac{\partial x}{\partial r}\dfrac{dr}{dy}+\dfrac{\partial x}{\partial s}\dfrac{\partial s}{\partial y}+\left.\dfrac{\partial x}{\partial y}\right)_{r,s}$



b) $(se^{rs})\left(\dfrac{2y}{y^2+2}\right)+(re^{rs}+y\sec^2(sy))(2yz)$
$+\ s\sec^2(sy)$

3. $12/19$

4. $4/\sqrt{29}$

5. $x=1-24t$, $y=-1-t$, $z=2+9t$ or $x=1+24t$, $y=-1+t$, $z=2-9t$

6. $(0,0)$, $(1,-1)$
$(0,0)$ yields a Saddle point
$(1,-1)$ " " relative minimum