MATH 2130 Problem Workshop 7

1. Set up but do not evaluate a triple iterated integral for the volume bounded by the surfaces
$$z = 9 - x^2 - y^2, \qquad z = x^2$$

2. Find the volume bounded by the surfaces
$$z = xy, \qquad x^2 + y^2 = 1, \qquad z = 0.$$

3. Find the volume bounded by the surfaces
$$z = 2\sqrt{x^2 + y^2}, \qquad z = 9 - x^2 - y^2$$
(Do not simplify your numerical answer.

4. Set up but do not evaluate a triple iterated integral for the triple integral of the function $f(x, y, z) = x^2$ over the region bounded by the surfaces
$$(x^2 + y^2)^2 = 2xy, \qquad z = \sqrt{1 - x^2 - y^2}, \qquad z = 0.$$

5. Evaluate $\iiint_V x^2 dV$ where V is the region bounded by the xz-plane and the hemispheres $y = \sqrt{9 - x^2 - z^2}$ and $y = \sqrt{16 - x^2 - z^2}$.

6. Find the volume and centroid of the region V that lies above the cone $z = \sqrt{x^2 + y^2}$ and below the sphere $x^2 + y^2 + z^2 = 1$.

Answers:

1. $4\int_0^{3/\sqrt{2}}\int_0^{\sqrt{9-2x^2}}\int_{x^2}^{9-x^2-y^2} dzdydx$
2. $1/2$
3. $2\pi\left[\frac{9(\sqrt{10}-1)^2}{2} - \frac{(\sqrt{10}-1)^4}{4} - \frac{2(\sqrt{10}-1)^3}{3}\right]$
4. $2\int_0^{\pi/2}\int_0^{\sqrt{\sin 2\theta}}\int_0^{\sqrt{1-r^2}} r^3\cos^2\theta dzdrd\theta$
5. $\frac{1562}{15}\pi$
6. $\frac{\pi}{3}(2-\sqrt{2}), \left(0, 0, \frac{3}{8(2-\sqrt{2})}\right)$