6 **1.** Find the equation of the tangent plane to the surface

$$x^2y^3 + xz^3 + z = 3$$

at the point $(1, 1, 1)$.

8 **2.** Set up, but do **NOT** evaluate, a definite integral for the length of the curve

$$x + y + z = 1, \quad x^2 + y^2 = 4$$

between the points $(2, 0, -1)$ and $(0, 2, -1)$. Simplify the integrand as much as possible, but again, do not evaluate the integral.

6 **3.** The angle γ between two planes is defined as the acute angle between their normal vectors. Show that the cosine of the angle between the planes

$$x - y + z = 4, \quad 2x + y + 3z = 6$$

is $\cos\gamma = \dfrac{2\sqrt{42}}{21}$.

7 **4.** If

$$u = r^3 + rs^4, \quad r = \sqrt{x^2 + y^2 + z^2}, \quad s = \sin(xy), \quad y = z^2,$$

find $\dfrac{\partial u}{\partial z}$. Do not simplify your answer.

8 **5.** Find the distance between the lines

$$x = 1 - t, \quad y = 3 + 3t, \quad z = 3 - t, \qquad \text{and} \qquad x = 2t, \quad y = 2, \quad z = 4 + t.$$

5 **6.** For what value, or values, of the constant b is the function

$$f(x, y) = e^{bx}\cos 5y$$

harmonic in the entire xy-plane?

8 **7.** The equations

$$u^3x^2 + uv^2 - xz = y + v + 1, \qquad v^5y^2 + u^4x + uz = 6v + 3$$

define u and v as functions of x, y, and z. Set up the Jacobians necessary to find $\partial v/\partial y$. Calculate the partial derivatives in the Jacobians, but do **NOT** evaluate the determinants.

12 **8.** Find the maximum value of the function

$$f(x, y) = x^2 - y^2 + x$$

considering only points inside and on the boundary of the area bounded by the curves

$$x = \sqrt{1 - y^2}, \qquad x = 0.$$

8 **9.** Set up, but do **NOT** evaluate, a double iterated integral to find the volume of the solid of revolution obtained by rotating the area bounded by the curves

$$x = \sqrt{6 - y}, \quad y = x, \quad y = 0$$

about the line $2x + y = 10$.

7 **10.** Set up, but do **NOT** evaluate, a double iterated integral in polar coordinates to find the area of that part of the surface

$$z = 4 - x^2 - y^2$$

above the plane $z = 1$.

8 **11.** Set up, but do **NOT** evaluate, a double iterated integral, (or double iterated integrals), for the first moment about the line $y = 2$ of a plate with constant mass per unit area ρ if its edges are the following curves

$$y = x^3 + 1, \quad x + y = 1, \quad y = 3.$$

6 **12.** Set up, but do **NOT** evaluate, a triple iterated integral, (or triple iterated integrals), to evaluate

$$\iiint_V (x^2 + y - z^3)\, dV$$

where V is the volume bounded by the surfaces

$$y = z, \quad y + z = 2, \quad x = 0, \quad x = 2, \quad z = 0.$$

11 **13.** Find the volume bounded by the surfaces

$$z = 2\sqrt{x^2 + y^2}, \quad x^2 + y^2 = 4, \quad z = 0.$$

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[8] 1. Find the distance between the two lines $\frac{x+1}{2} = \frac{y-3}{3} = z+4$ and $x = 1-t, \quad y = -2t, \quad z = -3+2t.$

[6] 2. Find an equation for the tangent plane to the surface $x^3 + 3\,y^2 - 3\,z^2 = 3$ at the point $(3,\, 1,\, 3)$.

[8] 3. Let $u(x,y) = f(x^3+y^2) + g(x^3+y^2)$ such that f and g are differentiable functions. Show that

$$2y\,\frac{\partial u}{\partial x} - 3x^2\,\frac{\partial u}{\partial y} = 0.$$

[8] 4. Given that the equations

$$e^x + \sin y = u^2 - v^2, \quad \text{and} \quad e^y + \sin x + 2u^2 + v^2 = 0$$

define u and v as functions of x and y find $\frac{\partial u}{\partial x}$. Simplify your answer.

[9] 5. Evaluate the following double intergal.

$$\int_0^1 \int_0^{\frac{1}{2}(1-y)} e^{x-x^2}\, dx\, dy$$

[12] 6. Find the absolute maximum and the absolute minimum of the function

$$f(x,y) = x^2 + 2xy - y^2$$

on the region bounded by $x = \sqrt{1-y^2}$, $y = x$ and $y = 0$.

[11] 7. Consider a thin plate with mass per unit area $\rho(x,y) = x^2 + y$ such that the edges of the plate are defined by the parabola $y = (x-2)^2$ and the line $y = x$. **Set up but do not evaluate** double intergrals for each of the following.

(a) First moment of the plate about the y–axis.

(b) Moment of inertia of the plate about the line $4x - 3y + 1 = 0$.

(c) Centre of mass of the plate.

[10] 8. Consider the double intergral

$$\iint_R \sqrt{1+\left[\frac{\partial}{\partial x}(y^2-x^2)\right]^2+\left[\frac{\partial}{\partial y}(y^2-x^2)\right]^2}\ dA$$

where R is the region between the two circles $x^2+y^2=1$ and $x^2+y^2=4$.

(a) Simplify the intergral.

(b) Give a (natural) geometrical interpretation of the integral.

(c) Rewrite the integral in terms of polar coordinates and then evaluate the integral.

[6] 9. **Set up but do not evaluate** a set of iterated integrals to evaluate

$$\iiint_V dV$$

where V is a region in $\mathbf{R}^3$ bounded by the planes

$$z=3x, \quad x+z=4, \quad y=0, \quad y=2.$$

[6] 10. (a) Find the spherical coordinates of the point P with cartesian coordinates $(\sqrt{2}, \sqrt{2}, 2\sqrt{3})$.

(b) Find the cylinderical coordinates of the point Q with spherical coordinates $(2\sqrt{2}, \frac{\pi}{6}, \frac{\pi}{4})$.

[8] 11. Find the volume of the region inside the cylinder $x^2+y^2=1$ and between the plane $z=0$ and the paraboloid $z=x^2+y^2$.
(Hint: you may use cylinderical coordinates system.)

[8] 12. A solid half ball (semisphere) V of radius 3 has density ρ depending on the distance $\Re$ from the centre of the base disk. The density is given by $\rho=k(6-\Re)$ where k is a constant. Use **spherical coordinates** system to find the mass of the half ball.

UNIVERSITY OF MANITOBA

DATE: December 11, 2008	FINAL EXAMINATION
PAPER # 402	
EXAMINATION: Engineering Mathematical Analysis 1	TIME: 3 hours
COURSE: MATH 2130	EXAMINER: G.I. Moghaddam

[10] 1. Find the distance from the point of intersection of the two lines

$$x = 1 - t, \quad y = -2t, \quad z = -3 + 2t \quad \text{and} \quad x = 2 + r, \quad y = 4 + 2r, \quad z = -r$$

to the plane $3x + \sqrt{15}\, y + 5z - 8 = 0$.

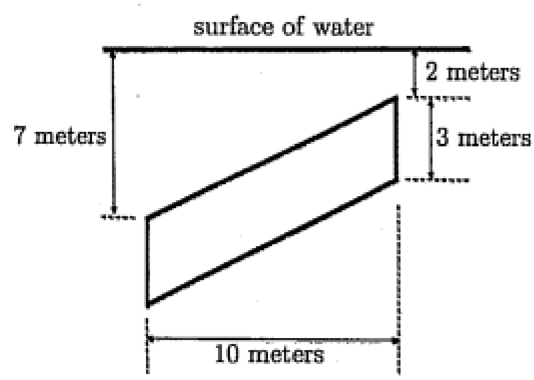
[10] 2. If $u = x^2 + y^3 - z^2$ such that $x^3 - e^t = 1$, $e^y - t^2 = 7$, $z = 2r^2 + 1$, find $\dfrac{\partial u}{\partial t}$. Simplify your answer.

[12] 3. Find the absolute maximum and the absolute minimum of the function

$$f(x, y) = y^2 - x^2 - 2x$$

on and **inside** the circle $x^2 + y^2 = 1$, except where both $x < 0$ and $y < 0$.

[10] 4. Find the force due to water pressure on each side of a vertical plate in the form of a parallelogram in the figure below.



[10] 5. Find the surface area of that part of the sphere $x^2 + y^2 + z^2 = 8$ which is inside of the cylinder $x^2 + y^2 = 2$.

[12] 6. **Set up but do not evaluate** the *six* triple iterated integrals in **Cartesian Coordinate System** for the triple integral of a function $f(x, y, z)$ over the region V in the first octant bounded by the surfaces

$$y^2 + z^2 = 1, \quad y = x, \quad z = 0, \quad , x = 0.$$

[8] 7. **Set up but do not evaluate** a double integral to find the moment of inertia about the line $3x - 2y = 6$ of a thin plate with constant mass per unit area ρ if its edges are defined by the curves

$$y = x^3, \quad y = \sqrt{2 - x}, \quad \text{and} \quad x = 0.$$

[10] 8. Find the volume of a region V in the first octant bounded by the plane $2x + y + z = 2$ and inside the cylinder $y^2 + z^2 = 1$.

[10] 9. (a) In Cartesian Coordinates System , the equation

$$z^2 - x^2 - y^2 = 1,$$

represents a surface; find the equation for the surface in Spherical Coordinates System.(simplify your answer)

(b) In Cylindrical Coordinates System , the equation

$$r(r\cos^2\theta + \sin\theta + 2rz - 1) + 1 = 0,$$

represents a surface; find the equation for the surface in Cartesian Coordinates System. Express z as a function of x and y.

[8] 10. **Set up but do not evaluate** a triple integral in **Spherical Coordinate System** to evaluate volume of a region V, where V is the region on and above the plane $z = 1$ and bounded by the semi sphere $z = \sqrt{4 - x^2 - y^2}$.

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[3] 1. Complete the following formal definition of the limit of a function of two variables:

$$\lim_{(x,y)\to(a,b)} f(x,y) = L \text{ if ...}$$

[5] 2. Show that the function $f(x,y) = \sin(6x)\sin(2y)$ satisfies the specific instance of the wave equation given by $\dfrac{\partial^2 f}{\partial x^2} = 9\left(\dfrac{\partial^2 f}{\partial y^2}\right)$.

[6] 3. Find an equation of the plane tangent to the surface $xyz^2 = 6x^2 - 3xy + xyz$ at the point $(1,2,1)$.

[7] 4. Evaluate the following double integral:

$$\int_0^4 \int_{\frac{y}{2}}^2 \cos(x^2)\, dx dy.$$

[8] 5. Let $f(u,v) = 3u^2v^3 + e^{uv}$ where u and v are both functions of x and y implicitly defined by the equations:

$$6u^2x^4 = -5vy^3 + 1$$
$$8x^7y = 5u^2v^3 + 13$$

Find $\dfrac{\partial f}{\partial x}$. (You may express your answer in terms of unsimplified determinants, but you must find all partial derivatives for the solution.)

6. Let C be the curve of intersection of the $y + z = 4x^2$ and $4x + 3y - z = 12$.

[4] (a) Find a parametric representation for C in the direction of increasing x.

[3] (b) Set up but **do not evaluate** the integral to find the length of the curve C from the point $(1,3,1)$ to the point $(3,9,27)$.

[12] 7. Find and classify all of the critical points of $f(x,y) = x^3 - 6xy + y^3$.

[10] 8. Use Lagrange multipliers (i.e. the Lagrangian) to find the maximum and minimum value of the function $f(x,y) = xy$ subject to $9x^2 + y^2 = 8$.

[10] 9. A thin ring has inner radius 1m, outer radius 3m, and has density directly proportional to the distance from the centre of the ring. In polar coordinates, set up but **do not evaluate** the double iterated integral to find the *moment of inertia* about a line tangent to the outside of the ring.

[10] 10. Set up but **do not evaluate** the double iterated integral to find the surface area of $z = x^2+3xy+y^2$ on the inside of the cylinders $x = (y-2)^2$ and $x = 12-2(y-2)^2$.

[12] 11. Set up **but do not evaluate** the triple iterated integrals necessary to find the center of mass $(\bar{x}, \bar{y}, \bar{z})$ of the solid bounded by the equations:

$$y = 6 + x - x^2 \ ; \ 2x + 2y + z = 20 \ ; \ y = 0 \ ; \ z = 0.$$

having density directly proportional to height (i.e. $\rho = kz$).

[18] 12. Find all six triple iterated integrals (in Cartesian coordinates) for the volume of the solid that is bounded by the functions:

$$z = 36 - 9y^2 \ ; \ y = 2x \ ; \ x = 0 \ ; \ z = 0.$$

Do not evaluate.

[12] 13. Use spherical coordinates to evaluate the integral $\iiint_V x^2 + y^2 + z^2 \, dV$ over the solid that is bounded below by the cone $z^2 = x^2 + y^2$ and above by the sphere $x^2 + y^2 + z^2 = 4$.

10 **1.** Consider the curve C of intersection of the surfaces

$$z = 6 - \sqrt{x^2 + (y-1)^2}, \qquad y + z = 5.$$

(a) Find the equation of the projection C_{xy} of C onto the xy-plane, simplified as much as possible.

(b) Use part (a) to find parametric equations for C directed so that x decreases along the curve.

(c) Set up, but do **NOT** evaluate, a definite integral for the length of that part of C that joins the points $(0, 0, 5)$ and $(\sqrt{20}, 5, 0)$.

10 **2.** Find the distance between the lines

$$x = t, \quad y = 3t - 1, \quad z = 1 + 2t, \qquad \text{and} \qquad x = 2u + 1, \quad y = 1 - u, \quad z = 4 + 2u.$$

10 **3.** Find the rate of change of the function

$$f(x, y, z) = \cos(\pi xy) + x \ln(z^2 + 1),$$

with respect to length s along the curve

$$y = -3x, \qquad z = x^2 - y^2 + 9,$$

directed so that x increases, at the point $(-1, 3, 1)$.

8 **4.** Show that the function $G(x, y) = f(3x - 2y^2) + xy$ satisfies the equation

$$4y\frac{\partial G}{\partial x} + 3\frac{\partial G}{\partial y} = 3x + 4y^2.$$

7 **5.** The function

$$f(x, y) = x^4 - 3x^2y^2 + y^4 + x^2 + y^2$$

is known to have five critical points $(0, 0)$, $(1, \pm 1)$, and $(-1, \pm 1)$. It is **NOT** necessary for you to show this. Classify the two critical points $(0, 0)$ and $(1, 1)$ as yielding relative maxima, relative minima, or saddle points.

5 **6.** The function

$$f(x, y) = x^4 - 3x^2y^2 + y^4$$

is known to have critical point $(0, 0)$, and the second derivative test fails to determine whether this critical point gives a relative maximum, a relative minimum, or a saddle point. Use whatever method you can devise to perform this classification.

10 **7.** Find the maximum value of the function

$$f(x, y) = xy(1 - x - y)$$

on the region R bounded by the lines

$$x = 0, \qquad y = 0, \qquad x + y = 1.$$

6 **8.** Evaluate the double iterated integral

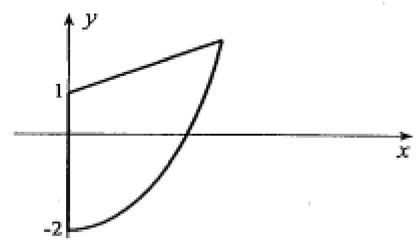
$$\int_0^1 \int_x^1 \sin(\pi y^2)\, dy\, dx.$$

8 **9.** Find the area of that part of the "saddle" $z = x^2 - y^2$ cut out by the cylinder $x^2 + y^2 = a^2$, where $a > 0$ is a constant.

8 **10.** The region bounded by the curves

$$x = \sqrt{y+2}, \quad x = 2y - 2, \quad x = 0$$

is shown to the right.



(a) Set up, but do **NOT** evaluate, double iterated integral(s) for the volume of the solid of revolution obtained by rotating the region about the line $x = -4$.

(b) If the region represents a thin plate with mass per unit area $\rho(x, y) = x^2 + y^2$, set up, but do **NOT** evaluate double iterated integral(s) for the moment of inertia of the plate about the edge $x = 2y - 2$. You may use the formula

$$\frac{|Ax_0 + By_0 + C|}{\sqrt{A^2 + B^2}}$$

for the distance from a point (x_0, y_0) to a line $Ax + By + C = 0$.

8 **11.** Set up, but do **NOT** evaluate, triple iterated integrals to determine the volume of the solid lying below the sphere $x^2 + y^2 + z^2 = 4$ and above the surface $z = \sqrt{x^2 + y^2}$ using:

(a) cylindrical coordinates; and

(b) spherical coordinates.