Some spring mass system questions from recent finals*(by Trim)*

11 **4.** (a) A 1 kilogram mass is suspended from a spring with constant 20 newtons per metre. It is set into motion by pulling it 10 centimetres below its equilibrium position and releasing it. During its subsequent motion, it is subjected to a damping force that is equal to 12 times its velocity.

(a) Find the position of the mass as a function of time t.

(b) Determine all times at which the mass passes through its equilibrium position, or prove that it never passes through the equilibrium position.

10 **4.** (a) A 500 gram mass is suspended from a spring with constant 50 newtons per metre. It is set into motion by releasing it from a position 10 centimetres above its equilibrium position. If a damping force proportional to velocity with coefficient $\beta = 2$ acts on the mass, find its position as a function of time.

(b) If your solution is expressed in the form $Ae^{-\beta t}\sin(\omega t + \phi)$, where A and ω are constants, what is A?

15 **6.** (a) A mass of 2 kilograms is suspended from a spring with constant 50 newtons per metre. At time $t = 0$, it is lifted 10 centimetres above its equilibrium position and given velocity 4 metres per second downward. During its subsequent motion, it is also subjected to a damping force that (in newtons) is equal to 20 times its velocity (in metres per second). (b) Determine the maximum distance from its equilibrium position that the mass ever achieves.

c (c) If damping is removed, and an additional force $4\cos\omega t$ acts on the mass, what value of ω causes resonance?

15 **3.** (a) A 1 kilogram mass is suspended from a spring with constant 50 newtons per metre. It is set into motion by striking it at its equilibrium position so as to give it an initial downward speed of 2 metres per second. If a damping force proportional to velocity with coefficient $\beta = 2$ acts on the mass, find its position as a function of time.

(b) When does the mass come to a stop for the first time?

11 **10.** A mass of 100 grams is suspended from a spring with constant 600 newtons per metre. At time $t = 0$, it is 10 cm above its equilibrium position and is given velocity 2 metres per second downward. Find the amplitude and the period of the resulting motion of the mass.

→ Answers on page 2.

13 10. A mass of 2 kilograms resting on a table is attached to a spring with constant 40 Newtons per metre. The other end of the spring is attached to a wall. The mass is set into horizontal motion (directly away from the wall) by pulling it 10 centimetres to the right of its equilibrium position and releasing it. During its subsequent motion, there is friction between the mass and the table with coefficient of kinetic friction equal to $\mu = 0.1$, but motion is free of any damping force proportional to velocity. After 1/2 second, the mass is hit to the left with an impulse force of 2 Newtons.

(a) Set up an initial-value problem for the displacement $x(t)$ of the mass from its equilibrium position.

(b) Show that, as long as the mass moves to the left, the Laplace transform of $x(t)$ is

$$X(s) = \frac{\frac{g}{5s} - 2e^{-s/2} + \frac{s}{5}}{2s^2 + 40},$$

c where $g = 9.81$.

(c) Find the position of the mass as a function of time until it comes to rest for the first time.

Answers (by Dawit)

11 4. a) $1\frac{d^2x}{dt^2} + 12\frac{dx}{dt} + 20x = 0$, $x(0) = -\frac{1}{10}$, $x'(0) = 0$ b) $x(t) = -\frac{1}{8}e^{-2t} + \frac{1}{40}e^{-10t}$ (in m) meter

c) Will never pass (Hint: set $x=0$ in (b) and show no $t>0$ solution can be found)

10 4. a) $\frac{1}{2}\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + 50x = 0$, $x(0) = \frac{1}{10}$, $x'(0) = 0$ b) $A = \sqrt{\left[\left(\frac{1}{10}\right)^2 + \left(\frac{1}{20\sqrt{6}}\right)^2\right]}$ Amplitude of damp free motion

15 6. a) $2\frac{d^2x}{dt^2} + 20\frac{dx}{dt} + 50x = 0$, $x(0) = \frac{1}{10}$, $x'(0) = -4$ b) $-\frac{7}{10}e^{-8/7}$ m c) $\omega = 5$ (in rad per sec)

15 3. a) $x(t) = -\frac{2}{7}e^{-t}\sin 7t$ (in m) b) $\frac{1}{7}\tan^{-1}7$ (in sec)

11 10. Amplitude $= \frac{2}{5\sqrt{15}}$ (in m) $= \frac{2\sqrt{15}}{75}$ (in m), period $= \frac{\sqrt{15}}{150}\pi$ (in s) $= \frac{\pi}{10\sqrt{15}}$ (in s)

13 10 a) $2\frac{d^2x}{dt^2} + 40x = \frac{1}{10}(2g) - 2\delta(t-\frac{1}{2})$, $x(0) = \frac{1}{10}$, $x'(0) = 0$ c) $x(t) = \frac{g}{200}\left[1 - \cos 2\sqrt{5}t\right] + \frac{1}{10}\cos 2\sqrt{5}t - \frac{1}{2\sqrt{5}}\sin 2\sqrt{5}(t-\frac{1}{2})\,h(t-\frac{1}{2})$