Student Name -   Student Number -

Values

7   1. (a) Find the Taylor series of $\ln x$ about $x = 3$. Express your answer in sigma notation.

      (b) What is the open interval of convergence of the series?

9   2. (a) Find the Taylor series about $x = -2$ for $\quad f(x) = \dfrac{1}{1+3x}$.   Express your final answer in sigma notation. Use a technique that guarantees that the series converges to the function.

      (b) What is the interval of convergence of the series?

8   3. Evaluate

$$\sum_{n=0}^{\infty} \frac{1}{n+1} x^{2n}.$$

Justify all steps in your solution.

8   4. Find, in explicit form $y = f(x)$, a 1-parameter family of solutions for the differential equation

$$x\frac{dy}{dx} = (x+1)y^2.$$

Does the 1-parameter family of solutions have any singular solutions? Explain.

8   5. Find the solution of the initial value problem

$$2\frac{dy}{dx} = y + 2x^2 e^{x/2}, \quad y(0) = 3.$$