MATH 2132 Problem Workshop 1

1. Evaluate the limit if it converges. If the limit tends to ∞, $-\infty$ indicate it as such.

   (a) $\displaystyle\lim_{n\to\infty} \frac{n+2}{3n^2+5}$

   (b) $\displaystyle\lim_{n\to\infty} (-1)^n \frac{n+2}{3n^2+5}$

   (c) $\displaystyle\lim_{n\to\infty} \frac{n^2+2}{3n^2+5}$

   (d) $\displaystyle\lim_{n\to\infty} (-1)^n \frac{n^2+2}{3n^2+5}$

   (e) $\displaystyle\lim_{n\to\infty} \frac{n^3+2}{3n^2+5}$

   (f) $\displaystyle\lim_{n\to\infty} (-1)^n \frac{n^3+2}{3n^2+5}$

   (g) $\displaystyle\lim_{n\to\infty} \left(\sqrt{n^2+3n-4} - \sqrt{n^2+6n+5}\right)$

   (h) $\displaystyle\lim_{n\to\infty} \left(1+\frac{3}{n}\right)^{2n}$

   (i) $\displaystyle\lim_{n\to\infty} \left(\frac{3n+2}{2-n}\right) \cot^{-1}\left(\frac{3-\sqrt{3}n^3}{2+3n+n^3}\right)$

   (j) $\displaystyle\lim_{n\to\infty} \left(\frac{3}{n}\right)^{2n}$

   (k) $\displaystyle\lim_{n\to\infty} \frac{\sin n}{n}$

   (l) $\displaystyle\lim_{n\to\infty} \left(\tan^{-1}(1/n)\right)^{1/n}$

2. Find the general term of the sequence $8, \frac{11}{7}, \frac{14}{25}, \frac{17}{79}, \ldots$.

3. Find the general term of the sequence $1, -\frac{6}{5}, \frac{12}{10}, -\frac{20}{17}, \frac{30}{26}, \ldots$.

4. It can be proven that if $\lim_{n\to\infty} c_n = C$ and $\lim_{n\to\infty} d_n = d$, then $\lim_{n\to\infty} c_n d_n = CD$. Use this to prove the following result. Suppose that $\lim_{n\to\infty} c_n = C \neq 0$ and $c_n \neq 0$ for all n. Suppose further that $\lim_{n\to\infty} d_n$ does not exist. Show $\lim_{n\to\infty} c_n d_n$ does not exist.

5. Determine to which function, if it exists, the sequence of functions $\{f_n(x)\}$ converges for x in the given interval.

   (a) $f_n(x) = \dfrac{n^2x^2 + 3nx}{2n^2x + 5}$, $(-\infty,\, \infty)$

   (b) $f_n(x) = \dfrac{\sin nx}{nx}$, $(0\,\infty)$

   (c) $f_n(x) = \dfrac{n\,\sin(x/n)}{x}$, $(0,\, \infty)$

   (d) $f_n(x) = \left(\ln(x^{n+1})\right)^{1/n}$, $(1,\, \infty)$