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FINAL EXAMINATION

Answers

[8] 1. Find the radius of convergence and open interval of convergence of

$$\sum_{n=3}^{\infty} \frac{3^{2n+5}(2n)!}{2^{4n+3}(n!)((n+1)!)} x^{2n+5}$$

$\rightarrow R = 2/3$

$-2/3 < x < 2/3$

[10] 2. Find the Taylor series of $f(x) = \dfrac{7x-5}{6x^2-7x-3}$ about the point $x = 2$. You may use the fact that

$$\frac{7x-5}{6x^2-7x-3} = \frac{1}{2x-3} + \frac{2}{3x+1}.$$

$\rightarrow \sum_{n=0}^{\infty} (-1)^n \left[2^n + \frac{2 \cdot 3^n}{7^{n+1}}\right](x-2)^n$

$3/2 < x < 5/2$

State your answer in sigma notation, simplifying as much as possible. Find the open interval of convergence.

[6] 3. Use the binomial theorem to find the Maclaurin series of the function

$$f(x) = (1+4x)^{-\frac{3}{5}}$$

$\rightarrow 1 + \sum_{n=1}^{\infty} \frac{(-1)^n 4^n [3 \cdot 5 \cdot 8 \cdots (5n-2)]}{5^n n!} x^n \quad , |x| < 1/4$

[8] 4. Find the sum of the power series

$$\sum_{n=0}^{\infty} \frac{3^{2n}(2n+1)}{2^{4n}} x^{2n+2}$$

$\rightarrow \frac{16x^2(16+9x^2)}{(16-9x^2)^2}, \quad |x| < 4/3$

[9] 5. Solve the following differential equation:

$$\frac{dy}{dx} + 3x^2 y = 2x^2 + x^5$$

$\rightarrow \frac{1}{3}(x^3+1) + Ce^{-x^3}$

[7] 6. Find a two parameter family of solutions to the differential equation:

$$y'y'' = 1$$

$\rightarrow E \pm \frac{1}{3}(2x+D)^{3/2}$

[12] 7. Find a general solution of:

$$y'' - y' - 6y = e^{3x}x^2$$

. ~~Use~~ the ~~operator method to~~ find a particular solution.

$\rightarrow y_p(x) = \left(\frac{1}{15}x^3 - \frac{1}{25}x^2 + \frac{2}{125}x\right)e^{3x}$

[12] 8. A 500 gram mass hangs on a spring with constant $8\frac{N}{m}$. The mass is given a speed of $2\frac{m}{s}$ upwards. The mass is acted upon by a damping force whose magnitude in Newtons is 5 times the instantanious velocity. In addition, a force $F(t) = e^{-3t}$ acts on the mass. Find the position of the mass as a funtion of time.

(Recall the formula: $M\dfrac{d^2x}{dt^2} + \beta\dfrac{dx}{dt} + kx = F(t)$.) $\rightarrowtail -\frac{2}{5}e^{-3t} - \frac{4}{15}e^{-8t} + \frac{2}{3}e^{-2t}$

[10] 9. Find the Laplace transform of the following periodic function:

$$f(x) = \begin{cases} \sin(t) & 0 < t < 3 \\ t^2 & 3 < t < 6 \end{cases} \qquad f(t+6) = f(t)$$

$\rightarrowtail \dfrac{1}{1-e^{-6s}}\left[\dfrac{1}{s+1} + e^{-3s}\left(\dfrac{2}{s^3} + \dfrac{6}{s^2} + \dfrac{9}{s} - \dfrac{(\sin 3)s}{s^2+1} - \dfrac{\cos 3}{s^2+1}\right) + e^{-6s}\left(\dfrac{2}{s^3} + \dfrac{12}{s^2} + \dfrac{36}{s}\right)\right]$

[8] 10. Use convolutions to find the inverse Laplace transform of $\dfrac{2}{s^2(s^2+4)}$

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$\rightarrowtail \frac{1}{2}t - \frac{1}{4}\sin 2t$

[10] 11. Solve the following initial value problem:

$$y'' + 2y' + 10y = 3\,\delta(t-4) \qquad y(0) = 1 \quad y'(0) = 2$$

$\rightarrowtail e^{-(t-4)}\sin 3(t-4)\,h(t-4) + e^{-t}(\cos 3t + \sin 3t)$