MATH 1210 (WINTER 2014)
ASSIGNMENT TWO

Due Date: February 14, 2014

Please write clearly. Your submission must be accompanied by the **Honesty Declaration** to be found on the course web-page.

Q1. Consider the polynomial

$$p(x) = x^3 + k\,x + (3 - 2\,i),$$

where k is an unknown complex number. It is given to you that if $p(x)$ is divided by $2\,x - (4 - i)$, then the remainder is $5 + i$. Find the value of k.

Q2. Find all values of k (which may be complex numbers) such that $k\,x - (k + 1)$ is a factor of the polynomial

$$p(x) = x^2 + (2 + 3\,i)\,x - (17 + i)\,.$$

Q3. Let

$$p(x) = x^5 + a\,x^4 + b\,x^3 + c\,x^2 + d\,x + e$$

where a, b, c, d, e are unknown real numbers. It is given that

$$2 + i\,\sqrt{3}, \quad 1 - i\,\sqrt{5}, \quad -3$$

are three of the roots of $p(x)$. Find the remaining two roots, and then find the values of a, b, c, d, e.

Q4. Consider the polynomial

$$p(x) = 3\,x^4 - 2\,x^3 - 22\,x^2 + 23\,x + 10.$$

(a) Use the Rational Root Theorem to list all possible rational roots of $p(x)$.
(b) Use the Bounds Theorem to eliminate some of the possibilities from your list in (a).
(c) Use direct substitution to check whether any of the remaining possibilities are in fact roots.
(d) Use your results from (c) to find all the roots of $p(x)$.

Q5. Consider the polynomial

$$p(x) = x^7 - 3\,x^5 + m\,x - 2,$$

where m is some integer. Find all possible values of m such that $p(x)$ has a rational root.

Q6. Consider the polynomial

$$p(x) = x^4 + (-1 + 2\,i)\,x^3 - 3\,x^2 - (2 + i)\,x + 1 - 3\,i\,.$$

It is given that $x^2 + i$ is a factor of $p(x)$. Use long division followed by the quadratic formula to find all the roots of $p(x)$.

Q7. Consider the polynomial

$$p(x) = 1 + \sum_{k=1}^{13} \frac{(-1)^k}{k^2}\,x^k\,.$$

(a) Show that $p(x)$ must have at least one positive real root.
(b) Show that $p(x)$ has no negative real roots.
(c) Show that if z is any root of $p(x)$, then $|z| < 170$.

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