Math 1710: Tutorial 10 (Partial fractions, Parametric curves)

1. Write the general FORM of the partial fraction decomposition of the following rational functions (DO NOT find the coefficients):

   (a) $\dfrac{x^3+x^2-15x+13}{x(x+1)^3(x^2+x+2)^2(2x+1)^4}$

   (b) $\dfrac{x^2-7x+1}{(x^2-1)(x^3+x)(x^2+x-2)}$

2. Evaluate the following indefinite integrals:

   (a) $\displaystyle\int \frac{x^3+3x^2+2x+1}{x^2+2x+1}\,dx$

   (b) $\displaystyle\int \frac{x+1}{x^3-1}\,dx$

   (c) $\displaystyle\int \frac{x^3}{x^4-1}\,dx$

   (d) $\displaystyle\int \frac{1}{x^4-1}\,dx$

   (e) $\displaystyle\int \frac{-3x^4+x^3-7x^2-x-2}{(x^2+1)^2(x+1)}\,dx$

   (f) $\displaystyle\int \frac{x^4+4x^3+8x^2+9x+3}{(x^2+2x+2)^2(x-1)}\,dx$

3. Try to find the coefficients a and b so that the following decomposition is valid:

$$\frac{x^2+x+2}{(x+1)(x+2)} = \frac{a}{x+1} + \frac{b}{x+2}\,.$$

   Explain why this is not a valid partial fraction decomposition of the given rational function (and so there are no a and b such that this equation is satisfied for all x different from -1 and -2).

4. Eliminate the parameter in parametric equations to find a Cartesian equation of a curve, and then sketch the curves with an arrow indicating the direction in which the curve is traced as parameter increases.

(a) $x = t^2 - 2, y = 5 - 2t, \quad -3 \leq t \leq 4;$

(b) $x = \sin t, y = \csc t, \quad 0 < t < \pi/2;$

(c) $x = \ln t, y = \sqrt{t}, \quad t \geq 1;$

(d) $x = \sec\theta, y = \tan\theta, \quad -\pi/2 < \theta < \pi/2.$

5. Find $\dfrac{dy}{dx}$ and $\dfrac{d^2y}{dx^2}$. For which values of t is the curve concave upward?

(a) $x = 4 + t^2, y = t^2 + t^3;$

(b) $x = 2\sin t, y = 3\cos t.$