UNIVERSITY OF MANITOBA

DATE: April 13 , 2010 FINAL EXAMINATION

PAGE: 1 of 10

EXAMINATION: Engineering Mathematical Analysis 1 TIME: 3 hours

COURSE: MATH 2130 EXAMINER: G.I. Moghaddam

Answers by Dawit
ydawit@yahoo.com

[9] 1. Find the distance between the two lines

$$\frac{x-5}{-4}=\frac{y}{2}=\frac{z-3}{3} \qquad \text{and} \qquad \frac{x+8}{1}=\frac{y-1}{-1}=\frac{z-7}{-1}.$$

1. $\sqrt{6}$

[11] 2. Let u and v be functions of x , y and z . Find $\dfrac{\partial u}{\partial z}$ if

$$x^2+y^5-xz-xu^3+yv^2=0$$
$$x^4+y^3+xz^2-yv^4=0.$$

Simplify your answer as much as possible.

2. $\dfrac{z-v^2}{3u^2v^2}$

[11] 3. Let $f(x,y,z)=2xy+\ln(xy)+z^2$ be a function of x, y and z.

(a) Find the direction in which f increases most rapidly at the point $(2,\frac{1}{2},1)$. What is the rate of change in that direction ?

(b) What is the rate of change of f in a direction perpendicular to the gradient of f ? Why?

3. a) $\langle \frac{3}{13},\frac{12}{13},\frac{4}{13}\rangle$ or any vector which is a multiple of it.
$|\langle \frac{3}{2},6,2\rangle| = \frac{13}{2}$

b) 0 , $D_{\vec{v}}f=\nabla f\cdot\vec{v}=0$
($\vec{v}$ is a vector that lies on the tangent plane at the given point)

[11] 4. Find all critical points for the function

$$f(x,y)=x^2+2y^2-x^2y.$$

Classify each critical point to determine if it is a relative maximum, a relative minimum, or a saddle point. Show your work.

4. $(0,0)$, $(-2,1)$, $(2,1)$
$(0,0)$ yields a rel-min
$(-2,1)$ " " Saddle pt.
$(2,1)$ " " " "

[12] 5. Find the absolute maximum and the absolute minimum of the function

$$f(x,y)=x^2+2y^2$$

on the region bounded by the y−axis and $x+|y|=1$.

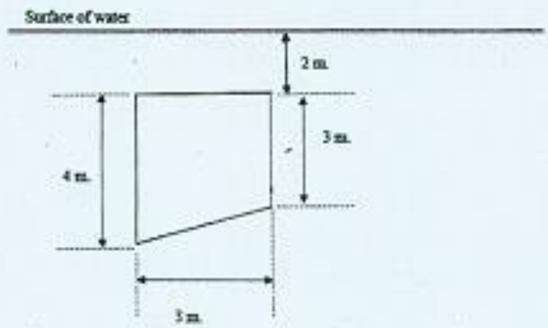
5. Abs-max = 2
Abs-min = 0

[9] 6. Evaluate the double integral

$$\int_0^1\int_{\sqrt{y}}^1 \frac{1}{1+x^3}\,dx\,dy.$$

6. $\frac{1}{3}\ln 2$

[10] 7. Find the force due to water pressure on each side of a vertical plate in the form of a trapezoid in the figure below.



7. $\frac{79}{2}\rho g = 3.871\times 10^5 N$

[10] 8. Find the surface area of that part of $z = \frac{2}{3}(x^{\frac{3}{2}} + y^{\frac{3}{2}})$ in the first octant cut off by the plane $x + y = 1$.

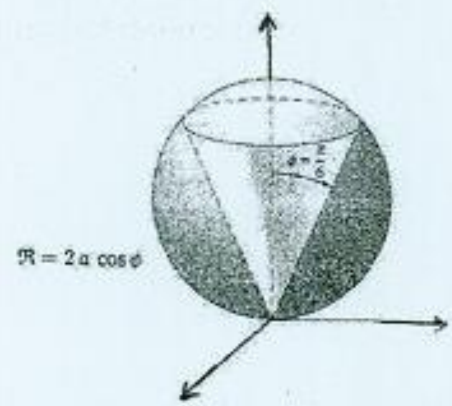
8. $\frac{4}{15}\left(\sqrt{2}-1\right)$

[7] 9. **Set up but do not evaluate** a triple integral to find the mass of a solid that lies within the cylinder $x^2 + y^2 = 1$, below the plane $z = 4$ and above the paraboloid $z = 1 - x^2 - y^2$. The density at any point is proportional to its distance from the z-axis. (Hint: you may use cylindrical coordinate system.)

9. $\int_0^{2\pi}\int_0^1\int_{1-r^2}^4 \rho_0 r^2 \, dz\, dr\, d\theta$

$[\ \rho(x,y,z) = \rho_0\sqrt{x^2+y^2},$
$\rho(r,\theta,z) = \rho_0 r\ ]$

[10] 10. Use **Spherical Coordinate System** to find the volume of the ice-cream cone that is bounded by the cone $\phi = \frac{\pi}{6}$ and the sphere $\mathfrak{R} = 2a\cos\phi$ of radius a.



10. $\frac{7a^3\pi}{12}$