Rules for differentiation. Tangent and normal lines.

3.2.1

I. (a) $y'(x) = 5x^4 - 14x + 10$

(b) $f'(t) = (t^{\frac{1}{4}} + t^{\frac{1}{3}})' = \frac{1}{4}t^{-\frac{3}{4}} + \frac{1}{3}t^{-\frac{2}{3}}$

(c) $f'(x) = \left(\frac{5}{2}x^{3/2} - \frac{3}{2}x^{-1} + \frac{1}{2}x^{-3/2}\right)'$

$= \frac{15}{4}x^{1/2} + \frac{3}{2}x^{-2} - \frac{3}{4}x^{-5/2}$

3.2.2

II The line $4x - y = 1$ has the slope 4, so we want to find points on the curve $y = x^3 - \frac{1}{x}$, where the slope of the tangent line is 4, i.e. the points corresponding to x, such that $y'(x) = 4$.

$$y'(x) = (x^3 - x^{-1})' = 3x^2 + x^{-2} = \underline{3x^2 + \frac{1}{x^2} = 4} \quad | \cdot x^2$$

$$3x^4 + 1 = 4x^2 \quad \text{or} \quad \underline{3x^4 - 4x^2 + 1 = 0} \leftarrow \text{this is a biquadratic equation}$$

Solving for x^2, we get $x^2 = 1$ or $x^2 = \frac{1}{3}$

$$\Downarrow$$

$$x = \pm 1 \quad \text{or} \quad x = \pm \frac{1}{\sqrt{3}}$$

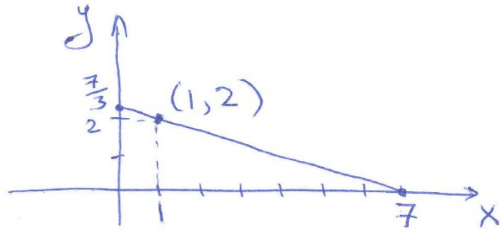
Thus, we have four points: $(1,0)$, $(-1,0)$, $\left(\frac{1}{\sqrt{3}}, -\frac{8}{3\sqrt{3}}\right)$ and $\left(-\frac{1}{\sqrt{3}}, \frac{8}{3\sqrt{3}}\right)$ $\left(y\left(\frac{1}{\sqrt{3}}\right) = \frac{1}{3\sqrt{3}} - \sqrt{3} = \frac{1-9}{3\sqrt{3}} = -\frac{8}{3\sqrt{3}},\ y\left(-\frac{1}{\sqrt{3}}\right) = \right.$

$\left. = -y\left(\frac{1}{\sqrt{3}}\right) = \frac{8}{3\sqrt{3}}, \text{ since } y \text{ is } \underline{\text{odd}} \right)$.

3.2.3

III $y' = 2x+1$, so at $(1,2)$ slope of the tangent line

$m_t = 2\cdot 1+1 = 3 \Rightarrow m_n = -\frac{1}{3}$ - slope of the normal line.



Equation of the normal line at $(1,2)$:

$$y-2 = -\frac{1}{3}(x-1)$$

$$\text{y-intercept} = 2+\frac{1}{3} = \frac{7}{3}$$

$$\text{x-intercept} = (-2)(-3)+1 = 7$$

Area of the triangle $= \frac{1}{2}(\text{base})(\text{haight}) = \frac{1}{2}\cdot\frac{7}{3}\cdot 7 = \frac{49}{6}$