MATH 1210 Summer 2015 Quiz 3

Surname: ______________________________

Given Name: ______________________________

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[5] 1. Let $A = \begin{bmatrix} 1 & -2 & -1 & 0 \\ 3 & 0 & 1 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 1 & 0 & 0 \\ 0 & 4 & 1 & -1 \end{bmatrix}$. Find a matrix X such that $AB^T - 3X = I$.

Solution: Some simple manipulations leads to

$$X = \frac{1}{3}\big(AB^T - I\big)$$

where B^T is the transpose which is

$$\begin{bmatrix} -1 & 1 & 0 & 0 \\ 0 & 4 & 1 & -1 \end{bmatrix}^T = \begin{bmatrix} -1 & 0 \\ 1 & 4 \\ 0 & 1 \\ 0 & -1 \end{bmatrix}$$

Then we can find

$$AB^T = \begin{bmatrix} 1 & -2 & -1 & 0 \\ 3 & 0 & 1 & 4 \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 1 & 4 \\ 0 & 1 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} -3 & -9 \\ -3 & -3 \end{bmatrix}$$

Hence

$$\begin{aligned} X &= \frac{1}{3}\big(AB^T - I\big) \\ &= \frac{1}{3}\left(\begin{bmatrix} -3 & -9 \\ -3 & -3 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right) \\ &= \frac{1}{3}\begin{bmatrix} -4 & -9 \\ -3 & -4 \end{bmatrix} \\ &= \begin{bmatrix} -4/3 & -3 \\ -1 & -4/3 \end{bmatrix} \end{aligned}$$

2. For the polynomial $P(x) = 8x^3 - 6x^2 + 7x - 3$,

[3] (a) Find the possible rational roots using the rational root theorem.

Solution: If p/q is a rational solution, then p divides 3 and q divides 8. Hence p is one of ±1 or ±3. and q is one of ±1, ±2, ±4 or ±8. Hence the possible rational solutions are

$$\pm1,\ \pm3,\ \frac{1}{2},\pm\frac{3}{2}\,\frac{1}{4},\pm\frac{3}{4}\,\frac{1}{8},\pm\frac{3}{8}$$

[3] (b) Use Descrates' Rules of Signs to find the number of possible positive and negative solutions.

Solution: $P(x)$ has three sign changes, so there are 3 or 1 positive solutions. $P(-x) = -8x^3 - 6x^2 - 7x - 3$ has no sign changes so there are no negative solutions.

[2] (c) Find a bound on the solutions using the Bounds Theorem.

Solution: If x is a solution of the polynomial equation, then

$$|x| < \frac{M}{|a_n|} + 1 = \frac{7}{|8|} + 1 = \frac{15}{8}.$$

[1] (d) Write the list of possible rational solutions using the restrictions from parts (a)-(c)

Solution: We eliminate the negative solutions as well as 3 from the Bounds Theorem.Hence the possible rational solutions are

$$1,\ \frac{1}{2},\ \frac{3}{2}\,\frac{1}{4},\ \frac{3}{4}\,\frac{1}{8},\ \frac{3}{8}.$$

[6] (e) Given that $P(1/2) = 0$, solve for all roots of $P(x)$.

Solution: Since

$$P(1/2) = 0.$$

we know that $2x - 1$ is a factor of $P(x)$. Long division leads to

$$P(x) = (2x-1)(4x^2 - x + 3).$$

The remaining quadratic yields solutions

$$x = \frac{1 \pm \sqrt{(-1)^2 - 4(4)(3)}}{2(4)} = \frac{1 \pm \sqrt{-47}}{8} = \frac{1 \pm i\sqrt{47}}{8}.$$

Therefore the roots are

$$\frac{1}{2}, \frac{1 \pm i\sqrt{47}}{8}$$