

Values

[20] 1. Find $\frac{dy}{dx}$ if: (DO NOT SIMPLIFY YOUR ANSWER.)

(a) $y = (x + \sin(\pi x))^3$

4

$$y' = 3(x + \sin(\pi x))^2 \cdot (1 + (\cos(\pi x) \cdot \pi))$$

6

(b) $y = (5^{x^2+x}) \log_3(x)$

$$y' = \underbrace{5^{x^2+x}}_1 \cdot \underbrace{\ln 5}_{1} \cdot \underbrace{(2x+1)}_1 \cdot \underbrace{\log_3 x}_1 + 5^{x^2+x} \cdot \underbrace{\frac{1}{x \ln 3}}_2$$

4

(c) $y = \sec(e^{2x})$

$$y' = \sec(e^{2x}) \cdot \tan(e^{2x}) \cdot e^{2x} \cdot 2$$

6

(d) $y = \tan(x^{\ln x})$

let $z = x^{\ln x}$

$\ln z = \ln x - \ln x = (\ln x)^2$

$\frac{z'}{z} = 2 \ln x \cdot \frac{1}{x} = \frac{2 \ln x}{x}$

$z' = x^{\ln x} \cdot \frac{2 \ln x}{x}$

5 pts

$$y' = \sec^2(x^{\ln x}) \cdot x^{\ln x} \frac{2 \ln x}{x}$$

Values

- [7] 2. Find an equation of the tangent line to the curve with equation $y^4 + xy = x^3 - x + 2$ at the point $(1,1)$.

$$4y^3 \cdot y' + y + x y' = 3x^2 - 1 \quad - 5 \text{ pts}$$

$$\text{at } x=y=1:$$

$$4y' + 1 + y' = 3 - 1$$

$$y' = \frac{1}{5} \quad - 1 \text{ pts}$$

$$y - 1 = \frac{1}{5} (x - 1) \quad - 1 \text{ pt.}$$

3. Find the absolute minimum and absolute maximum values of $f(x) = x^3 + 3x^2 - 9x + 2$ on the interval $[0, 2]$.

[10]

$$f'(x) = 3x^2 + 6x - 9 \quad (1)$$

$$f'(x) = 0 \quad x^2 + 2x - 3 = 0 = (x+3)(x-1) \quad (2)$$

$$(1) \quad \cancel{x=3} \quad x=1 \quad - \text{crit. point} \quad (1)$$

outside

$$f(0) = 2 \quad (1)$$

$$f(1) = 1 + 3 - 9 + 2 = -3 \quad (1)$$

$$f(2) = 8 + 3 \cdot 4 - 9 \cdot 2 + 2 = 4 \quad (1)$$

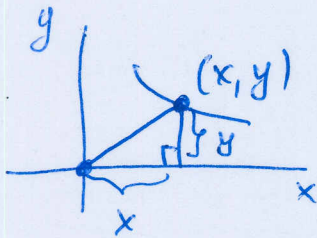
Abs. min value is -3 , at $x=1$ (1)

Abs. max value is 4 , at $x=2$ (1)

Values

4. (a) The hypotenuse of a right triangle has one end at the origin and one end on the curve with equation $y = x^2 e^{-3x}$, with $x \geq 0$. One of the other two sides is on the x -axis, the other side is parallel to the y -axis. Find the maximum area of such a triangle, if all lengths are measured in metres.

[9]



$$A = \frac{1}{2} x \cdot y = \frac{1}{2} x^3 \cdot e^{-3x}, \quad x \geq 0$$

$$\begin{aligned} A'(x) &= \frac{1}{2} \cdot (3x^2 \cdot e^{-3x} + x^3 \cdot e^{-3x} \cdot (-3)) \\ &= \frac{3}{2} x^2 e^{-3x} (1 - x) \end{aligned}$$

$$A' = 0 \text{ at } x = 0 \text{ and at } x = 1$$

$x < 1$	$x > 1$
A' +	A' -
A ↗	A ↘

Abs. max. is at $x = 1$

$$A(1) = \frac{1}{2} e^{-3} \text{ [m}^2\text{]}$$

$$\underline{\text{Ans: } \frac{1}{2} e^{-3} \text{ [m}^2\text{]}}$$

- (b) Find the rate of change of the area of the triangle in (a) when $x = 2$ m and

$$\frac{dx}{dt} = 1 \text{ m/s}$$

[9]

$$\left. \frac{dA}{dt} \right|_{\substack{x=2 \\ \frac{dx}{dt}=1}} = ?$$

$$\frac{dA}{dt} = \frac{d\left(\frac{1}{2} x^3 e^{-3x}\right)}{dt} = \frac{3}{2} x^2 e^{-3x} (1 - x) \cdot \frac{dx}{dt}$$

$$\left. \frac{dA}{dt} \right|_{\substack{x=2 \\ \frac{dx}{dt}=1}} = \frac{3}{2} \cdot 2^2 \cdot e^{-6} \cdot (-1) \cdot 1 = -6e^{-6}$$

$$\underline{\text{Ans: decreasing at } 6e^{-6} \text{ m}^2\text{/s}}$$

Values

5. If $f(x) = \frac{(x-2)^3}{x^2}$ then $f'(x) = \frac{(x-2)^2(x+4)}{x^3}$ and $f''(x) = \frac{24(x-2)}{x^4}$.

(a) Compile the following information about f and its graph. (Give answers only; answer "NONE" if the function does not display a feature listed).

domain? $x \neq 0$ $(-\infty, 0) \cup (0, \infty)$

y-intercept? NONE

x-intercepts? $x=2$

odd, even, or neither? neither

equation(s) of vertical asymptotes? $x=0$

equation(s) of horizontal asymptotes? NONE

critical number(s) of the function f ? $x=2, x=-4$

[20]

interval(s) where the function is increasing? $(-\infty, -4), (0, \infty)$

interval(s) where the function is decreasing? $(-4, 0)$

x and y coordinates of the point where any local maxima occur? $(-4, -\frac{27}{2})$

x and y coordinates of the point where any local minima occur? NONE

interval(s) where the function is concave up? $(2, \infty)$

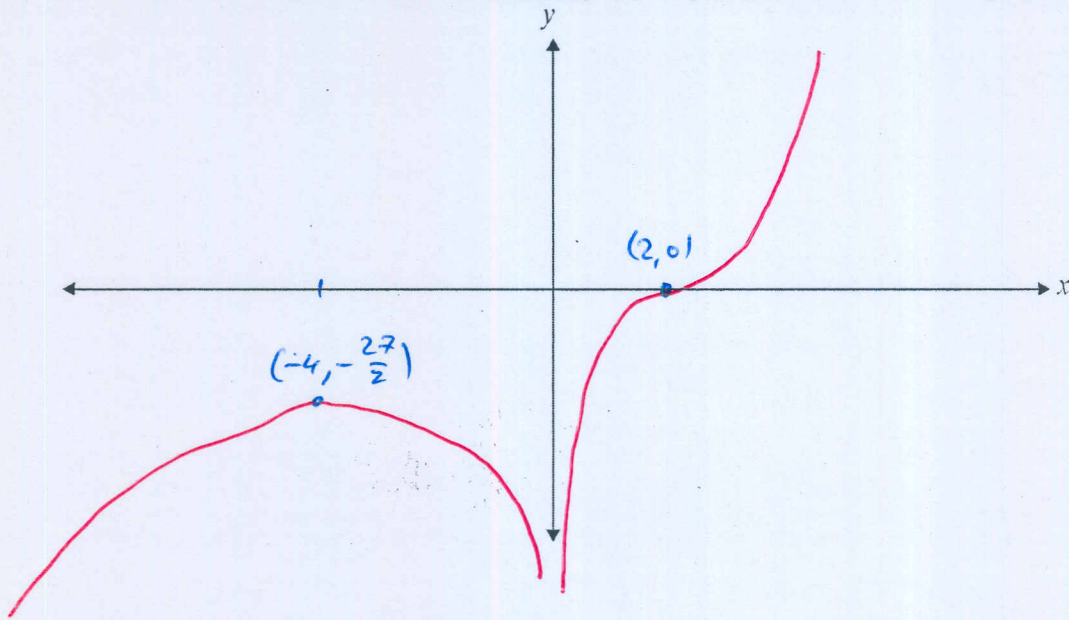
interval(s) where the function is concave down? $(-\infty, 0), (0, 2)$

x and y coordinates of any inflection points $(2, 0)$

	-4	0	
$x+4$	-	+	+
x^3	-	-	+
	+	-	+

$3^3 - 2^3 = 27 - 8 = 19$
 $19 \cdot 2 = 38$
 $38 \cdot 2 = 76$
 $76 \cdot 2 = 152$
 $152 \cdot 2 = 304$
 $304 \cdot 2 = 608$
 $608 \cdot 2 = 1216$
 $1216 \cdot 2 = 2432$
 $2432 \cdot 2 = 4864$
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 5553

- (b) Sketch the graph of $y = f(x)$ on the given axes, reflecting all relevant information from (a) and labelling any important features on the graph.



BONUS [Maximum 3 marks.]

Find the values of a , b , and c if the parabola with equation $y = ax^2 + bx + c$ passes through the point with coordinates $(1, 4)$ and whose tangent lines at $x = -1$ has slope 6, and whose tangent line at $x = 5$ has slope -2 .

$$\begin{cases} 4 = a + b + c \\ 6 = -2a + b \\ -2 = 10a + b \end{cases}$$

$$y' = 2ax + b$$

$$8 = -12a$$

$$a = -\frac{8}{12} = -\frac{2}{3} \quad (1)$$

$$b = 2a + 6 = 6 - \frac{4}{3} = \frac{14}{3} \quad (1)$$

$$c = 4 - a - b = 4 + \frac{2}{3} - \frac{14}{3} = \frac{12 + 2 - 14}{3} = 0 \quad (1)$$