**MATH 1210 Assignment 1**

Due: 1:30 pm Friday 23 January 2009 (at your instructor's office)

NOTES:

1. **Late assignments will NOT be accepted**.
2. **If your assignment is not accompanied by a signed Faculty of Science "Honesty Declaration", it will NOT be graded**.

***Provide a complete solution to each of the following problems***:

1. Verify that, for all $n \geq 1$,
$$1 + 4 + \cdots + (3n - 2) = \frac{n\,(3n-1)}{2}.$$
[*HINT: You might find it useful to write the left-hand side of this equation using sigma notation.*]

2. Use the Principle of Mathematical Induction to verify that, for $n$ any positive integer, $6^n - 1$ is divisible by 5.

3. Verify that, for all $n \geq 1$, the sum of the squares of the first $2n$ positive integers is given by
$$1^2 + 2^2 + \cdots + (2n)^2 = \frac{n\,(2n+1)\,(4n+1)}{3}.$$
[*HINT: You might find it useful to write the left-hand side of this equation using sigma notation.*]

4. Consider the sequence of real numbers defined by the relations
$$x_1 = 1 \text{ and } x_{n+1} = \sqrt{1 + 2\,x_n} \text{ for } n \geq 1.$$
Use the Principle of Mathematical Induction to show that $x_n < 4$ for all $n \geq 1$.

5. Show that $n! > 3^n$ for $n \geq 7$.

6. Let $p_0 = 1$, $p_1 = cos(\theta)$ (for $\theta$ some fixed constant) and $p_{n+1} = 2\,p_1\,p_n - p_{n-1}$ for $n \geq 1$. Use an extended Principle of Mathematical Induction (see below) to prove that $p_n = cos(n\,\theta)$ for $n \geq 0$.

   [HINT: You may find it useful to employ the compound-angle formula for $cos((n+1)\,\theta)$ and $cos((n-1)\,\theta)$ in order to complete this problem.]

7. Consider the famous Fibonacci sequence $\{x_n\}_{n=1}^{\infty}$, defined by the relations $x_1 = 1$, $x_2 = 1$ and $x_n = x_{n-1} + x_{n-2}$ for $n \geq 3$.

   (a) Compute $x_{20}$,

   (b) Use an extended Principle of Mathematical Induction (see below) in order to show that

$$x_n = \frac{1}{\sqrt{5}}\left[\left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{1-\sqrt{5}}{2}\right)^n\right] \quad \text{for } n \geq 1,$$

   (c) Use a calculator and the formula of part (b) to compute $x_{20}$.

For the last two problems you will need the following result.
***An extended version of the Principle of Mathematical Induction***:

Consider the infinite sequence of statements $\{P_n\}_{n=N}^{\infty}$ for $N$ a fixed integer.

IF

(A) $P_N$ and $P_{N+1}$ are both true,

(B) the truth of both $P_k$ and $P_{k+1}$ implies the truth of $P_{k+2}$ for $k \geq N$,

THEN

(C) we may conclude that $P_n$ is true for all $n \geq N$.