MATH 2130 Problem Workshop 6

1. A thin plate with constant mass per unit area ρ has edges defined by the curves

$$x = \sqrt{a^2 - y^2}, y = x, y = 0,$$

where $a > 0$ is a constant. (a) Find the mass the plate , (b) Find the first moment of the plate about the x-axis, (c) Find $\overline{y}$.

2. A triangular plate has sides of length 2, 3 and 3. and constant mass per unit area ρ. Find its moment of inertia about the shorter side.

3. Find the area of the part of the surface $z = xy$ inside the cylinder $x^2 + y^2 = a^2$ where $a > 0$ is a constant.

4. Set up but do not evaluate a double iterated integral for the area of the surface $z = 2x^2 + y^2$ bounded by $y = 0, x = 0$ and $x + y = 1$.

5. Find the area bounded by $(x^2 + y^2)^3 = 4a^2x^2y^2$ where $a > 0$ is a constant.

6. Find the double integral of $f(x, y) = xy(x + y)$ over the region in the first quadrant bounded by $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$.

7. Evaluate the triple integral of the function $f(x, y, z) = x$ over the volume bounded by the surfaces

$$2x + 3y + z = 6, x = 0, y = 0, z = 0.$$

8. Find the volume in the first octant bounded by the surfaces

$$4x + 4y + z = 16, \qquad z = 0, \qquad y = x/2, \qquad y = 2x.$$

Answers:

1. (a) $\pi a^2\rho/8$ (b) $\rho a^3(\sqrt{2}-1)/(3\sqrt{2})$, (c) $8a(\sqrt{2}-1)/(3\sqrt{2}\pi)$
2. $8\sqrt{2}\rho/3$
3. $2\pi[(1+a^2)^{3/2}-1]/3$
4. $\int_0^1\int_0^{1-x}\sqrt{1+16x^2+4y^2}dydx$.
5. $\pi a^2/2$
6. $62/15$
7. $9/2$
8. $128/9$