UNIVERSITY OF MANITOBA

DATE: August 10, 2013 FINAL EXAMINATION

TITLE PAGE

EXAMINATION: Engineering Mathematical Analysis 1 TIME: 3 hours

COURSE: MATH 2130 EXAMINER: Harland

NAME: ______________________________

STUDENT NUMBER: ______________________________

SIGNATURE: (in ink) ______________________________

(I understand that cheating is a serious offense)

INSTRUCTIONS TO STUDENTS:

This is a 3 hour exam. **Please show your work clearly.**

No texts, notes, or other aids are permitted. There are no calculators, cellphones or electronic translators permitted.

This exam has a title page, 15 pages of questions, a formula sheet and also 1 blank page for rough work. Please check that you have all the pages. You may remove the blank page if you want, but **be careful not to loosen the staple.**

The value of each question is indicated in the left hand margin beside the statement of the question. The total value of all questions is 110 points.

Answer all questions on the exam paper in the space provided beneath the question. If you need more room, you may continue your work on the reverse side of the page, but CLEARLY INDICATE that your work is continued.

Question	Points	Score
1	8	
2	5	
3	8	
4	6	
5	10	
6	6	
7	20	
8	6	
9	5	
10	5	
11	9	
12	7	
13	8	
14	7	
15	0	
Total:	110	

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[8] 1. Find the distance between the lines

$$\frac{x-4}{3} = \frac{y+3}{2} = \frac{z-2}{-1} \text{ and}$$

$$x = 1 + t, \quad y = -1 - 2t, \quad z = 3 - t.$$

You may use without proof that the two lines are skew.

[5] 2. Let $f(t) = t$ and $\mathbf{v}(t) = t^2\hat{\mathbf{i}} + \ln t\hat{\mathbf{j}} - \dfrac{1}{t}\hat{\mathbf{k}}$. Find

$$\int (f\mathbf{v})(t)\,dt.$$

[4] 3. (a) Find and sketch the largest possible domain of $f(x, y) = \sqrt{4 - x^2 - 4y^2}$.

[4] (b) Sketch level curves of $f(x, y) = y - x^2$ corresponding to $k = -2, 0, 1$.

4. Let $z = x^2 + xy + y^2$. Show that z is a solution to:

[3] (a) $xz_x + yz_y = 2z$.

[3] (b) $x^2 z_{xx} + 2xyz_{xy} + y^2 z_{yy} = 2z$.

[5] 5. (a) Find a chain rule for $\left.\dfrac{\partial z}{\partial t}\right)_s$ if $z = f(x, y, s), x = g(r), y = h(r), r = k(s, t)$.

[5] (b) Use your chain rule in part (a) to find $\left.\dfrac{\partial z}{\partial t}\right)_s$ if

$$z = e^{y^2+xs}, \quad x = \frac{\ln r}{r}, \quad y = \sec(r^2), \quad r = \sqrt{s^2+t^2}.$$

[6] 6. Calculate the derivative of $f(x, y, z) = xy + z^2$ at $(0, 10, -2)$ in the direction of increasing t along the line

$$x = -3 + t, \quad y = -2 + 4t, \quad z = 4 - 2t.$$

7. For the function $f(x, y) = x^3 - 3x + y^2 + 2y + 2$.

[4] (a) Find the critical point(s) of f.

[6] (b) Classify the critical points of f.

[10] (c) Find the absolute maximum and minimum of $f(x, y)$ on the triangle bounded by the lines $x = 0$, $\quad y = 0$ and $x - y = 2$.

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[6] 8. Evaluate the double iterated integral

$$\int_{-1}^{0}\int_{-y}^{1} y(x^2+y^2)^{2013}\,dx\,dy.$$

[5] 9. Set up, but do not integrate a multiple integral, or sum of multiple integrals in Cartesian coordinates to find the moment of inertia of a thin plate with constant mass per unit area ρ defined by the region bounded by $x = y^2 - 2$ and $x = y$ rotated about $x + y = 1$.

[5] 10. Set up, but do not integrate a multiple integral, or sum of multiple integrals in polar coordinates to find the volume of the region bounded by the polar curve $r = 2 - \cos\theta$ rotated about the line $x = 3$.

[9] 11. Find the surface area of $y^2 = z + x^2$ inside the cylinder $x^2 + y^2 = 9$.

[7] 12. Set up, but do not integrate multiple integrals, or sum of multiple integrals in Cartesian coordinates to find the z-coordinate of the center of mass of the solid bounded by the surfaces $y = 4 - x^2, \quad z = 0, \quad y = z$ with density equal to the distance to the y-axis.

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[8] 13. Use cylindrical coordinates to find the volume of the solid bounded by the surfaces

$$z = 4 + x^2 + y^2, \quad x^2 + y^2 = 16 \text{ and } z = 0.$$

[7] 14. Set up, but do not integrate multiple integrals, or sum of multiple integrals in spherical coordinates to find the volume of the region bounded by $z = \sqrt{8 - x^2 - y^2}$ and $z = 2$.

15. (Bonus: Max 5 marks) Find

$$I = \int_0^\infty e^{-x^2}\, dx.$$

(Hint: I is also equal to $\displaystyle\int_0^\infty e^{-y^2}\, dy$)