Math 2130 - Engineering Mathematical Analysis 1

Tutorial 9 - Questions for §13.5, 13.6, and 13.7 (Polar Coordinates) - Part 2.

13.5.1. A thin plate with constant mass per unit area ρ has edges defined by the curves

$$x = \sqrt{a^2 - y^2}, \quad y = x, \quad y = 0,$$

where $a > 0$ is a constant. Find the first moment of the plate about the x-axis.

13.5.2. A triangular plate has sides of lengths 2, 3 and 3, and constant mass per unit area ρ. Find its moment of inertia about the shorter side.

13.6.1. Evaluate the area of that part of the surface $z = xy$ inside the cylinder $x^2 + y^2 = a^2$, where, $a > 0$ is a constant.

13.6.2. Set up, but do **NOT** evaluate, a double iterated integral for the surface area of the ellipsoid, where $a > 0$, $b > 0$, and $c > 0$ are constants.

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1.$$

13.6.3. Set up, but do **NOT** evaluate, a double iterated integral for the area of the surface $z = \sqrt{1 + x^2 + y^2}$ below $z = 2$.

13.6.4. Set up, but do **NOT** evaluate, a double iterated integral for the area of the surface $z = 2x^2 + y^2$ bounded by $y = 0$, $x = 0$, and $x + y = 1$.

13.7.1. Evaluate the area bounded by $(x^2 + y^2)^3 = 4a^2x^2y^2$, where $a > 0$ is a constant.

13.7.2. Evaluate the double integral of $f(x, y) = xy(x + y)$ over the region in the first quadrant bounded by $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$.

13.7.3. A plate with constant mass per unit area ρ is bounded by the curve $(x^2 + y^2)^2 = 9(x^2 - y^2)$. Evaluate its moment of inertia about the x-axis.

13.7.4. Evaluate the surface area of that part of the sphere $x^2 + y^2 + z^2 = 16$ that lies above the cone $z = \sqrt{x^2 + y^2}$.

Answers:

13.5.1. $\rho a^3(\sqrt{2}-1)/(3\sqrt{2})$.

13.5.2. $8\sqrt{2}\rho/3$.

13.6.1. $2\pi[(1+a^2)^{3/2}-1]/3$.

13.6.2. $8\displaystyle\int_0^a \int_0^{(b/a)\sqrt{a^2-x^2}} \sqrt{1+\left(\frac{-cx}{a^2\sqrt{1-x^2/a^2-y^2/b^2}}\right)^2+\left(\frac{-cy}{b^2\sqrt{1-x^2/a^2-y^2/b^2}}\right)^2}\,dy\,dx.$

13.6.3. $4\displaystyle\int_0^{\pi/2} \int_0^{\sqrt{3}} \sqrt{\frac{1+2r^2}{1+r^2}}\,r\,dr\,d\theta.$

13.6.4. $\displaystyle\int_0^1 \int_0^{1-x} \sqrt{1+16x^2+4y^2}\,dy\,dx.$

13.7.1. $\pi a^2/2$.

13.7.2. $62/15$.

13.7.3. $27(3\pi-8)\rho/16$.

13.7.4. $16\pi(2-\sqrt{2})$.
