SOLUTIONS

FINAL EXAM

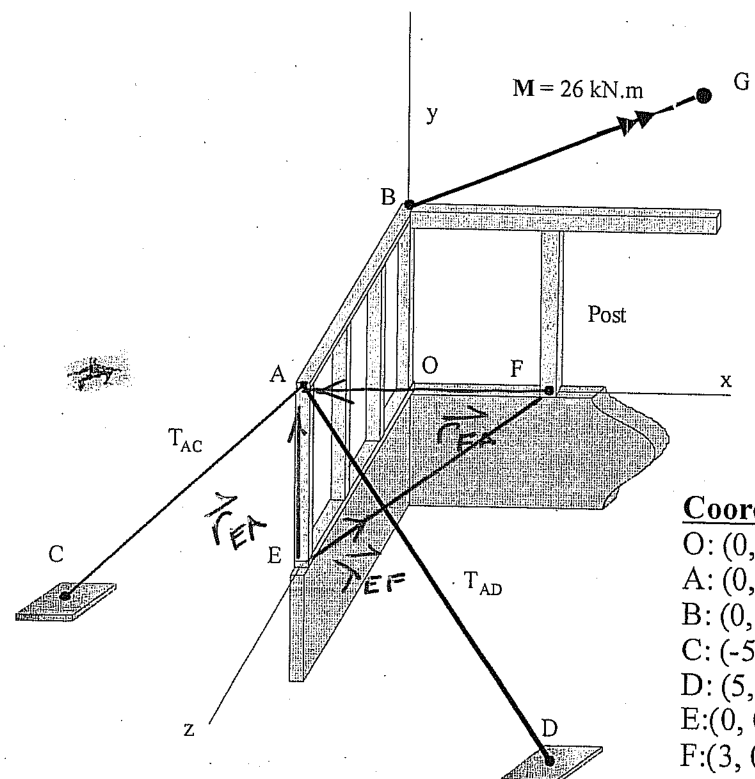
Dec 2012

## Question 1

Two wood framed walls are supported on top of concrete walls as shown in Figure 1. Two cables are attached to the stud wall at *A*. The tension force in cable AC is $T_{AC} = 20\ kN$ and the tension force in cable *AD* is $T_{AD} = 10\ kN$. A couple moment of *26 kN.m* is applied at point *B*.

Determine:

a) The equivalent force-couple at *E*,
b) The angle between cables *AC* and *AD*,
c) The projection of the $T_{AC}$ force applied at *A* onto the line *AD*,
d) The moment of the $T_{AC}$ force applied at *A* about the point *F*.
e) The moment of the $T_{AC}$ force applied at *A* about the line *EF* and
f) The perpendicular distance from point *E* to the line of action $T_{AC}$.




**Coordinates (metres)**
O: (0, 0, 0)
A: (0, 3, 4)
B: (0, 3, 0)
C: (-5, -1, 5)
D: (5, -3, 7)
E:(0, 0, 4)
F:(3, 0, 0)
G:(12, 6, -4))

**Figure 1**

$\vec{T}_{AC} = T_{AC}\vec{\lambda}_{AC} = 20\vec{\lambda}_{AC}$ $\quad \vec{\lambda}_{AC} = \frac{\vec{AC}}{AC}$

$\vec{AC} = -5\hat{i} - 4\hat{j} + 1\hat{k}$ $\quad AC = \sqrt{(-5)^2+(-4)^2+(1)^2} = \sqrt{42}$

$\therefore \vec{T}_{AC} = 20\left(\frac{-5\hat{i} - 4\hat{j} + 1\hat{k}}{\sqrt{42}}\right) = (-15.43\hat{i} - 12.34\hat{j} + 3.09\hat{k})\ kN$

$\vec{T}_{AD} = T_{AD}\vec{\lambda}_{AD} = 10\vec{\lambda}_{AD}$ $\quad \vec{\lambda}_{AD} = \frac{\vec{AD}}{AD}$

$\vec{AD} = 5\hat{i} - 6\hat{j} + 3\hat{k}$ $\quad AD = \sqrt{(5)^2+(-6)^2+(3)^2} = \sqrt{70}$

$\therefore \vec{T}_{AD} = 10\left(\frac{5\hat{i} - 6\hat{j} + 3\hat{k}}{\sqrt{70}}\right) = (5.98\hat{i} - 7.17\hat{j} + 3.59\hat{k})\ kN$

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$$\vec{M} = M\vec{\lambda}_{BG} = 26\,\vec{\lambda}_{BG}$$

$$\vec{\lambda}_{BG} = \frac{\vec{BG}}{BG} \qquad \vec{BG} = 12\hat{i} + 3\hat{j} - 4\hat{k}$$

$$BG = \sqrt{(12)^2 + (3)^2 + (-4)^2} = \sqrt{169} = 13$$

$$\therefore \vec{M} = 26\left(\frac{12\hat{i} + 3\hat{j} - 4\hat{k}}{13}\right) = (24\hat{i} + 6\hat{j} - 8\hat{k})\ \text{kN.m}$$

a) Equivalent Force-Couple at E:

$$R_x = \Sigma F_x = (-15.43 + 5.98)\hat{i} = -9.45\hat{i}\ \text{kN}$$

$$R_y = \Sigma F_y = (-12.34 - 7.17)\hat{j} = -19.51\hat{j}\ \text{kN}$$

$$R_z = \Sigma F_z = (3.09 + 3.59)\hat{k} = 6.68\hat{k}\ \text{kN}$$

$$\vec{R} = (-9.45\hat{i} - 19.51\hat{j} + 6.68\hat{k})\ \text{kN}$$

$$R = \sqrt{(-9.45)^2 + (-19.51)^2 + (6.68)^2} = 22.68\ \text{kN}$$

$$M_{RE} = \vec{r}_{EA} \times \vec{T}_{AC} + \vec{r}_{EA} \times \vec{T}_{AD} + \vec{M}$$

$$\vec{r}_{EA} = (0\hat{i} + 3\hat{j} + 0k)\ \text{m}$$

$$\vec{r}_{EA} \times \vec{T}_{AC} = \left|\begin{array}{ccc|cc} i & j & k & i & j \\ 0 & 3 & 0 & 0 & 3 \\ -15.43 & -12.34 & +3.09 & -15.43 & -12.34 \end{array}\right|$$

$$= [9.27\hat{i}] - [-46.29\hat{k}] = (9.27\hat{i} + 46.29\hat{k})\ \text{kN.m}$$

$$\vec{r}_{EA} \times \vec{T}_{AD} = \left|\begin{array}{ccc|cc} i & j & k & i & j \\ 0 & 3 & 0 & 0 & 3 \\ 5.98 & -7.17 & 3.59 & 5.98 & -7.17 \end{array}\right|$$

$$= [10.77\hat{i}] - [17.94\hat{k}] = (10.77\hat{i} - 17.94\hat{k})\ \text{kN.m}$$

$$\therefore \vec{M}_{RE} = [(9.27\hat{i} + 46.29\hat{k}) + (10.77\hat{i} - 17.94\hat{k}) + (24\hat{i} + 6\hat{j} - 8\hat{k})]\ \text{kN.m}$$

$$\vec{M}_{RE} = (44.04\hat{i} + 6\hat{j} + 20.35\hat{k})\ \text{kN.m}$$

$$R = (-9.45\hat{i} - 19.51\hat{j} + 6.68\hat{k})\ \text{kN}$$

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b) Angle between AC & AD

$$\vec{\lambda}_{AC} \bullet \vec{\lambda}_{AD} = \cos\theta$$

$$\left(\frac{-5\hat{i} - 4\hat{j} + \hat{k}}{\sqrt{42}}\right) \bullet \left(\frac{5\hat{i} - 6\hat{j} + 3\hat{k}}{\sqrt{70}}\right) = \cos\theta$$

$$\frac{(-25) + (24) + (3)}{(\sqrt{42})\sqrt{70}} = \cos\theta$$

$$= 3.6885 \times 10^{-2}$$

$$\Rightarrow \theta = 87.89^\circ$$

c) The projection of TAC onto Line AD

$$\text{Projection} = \vec{T}_{AC} \bullet \vec{\lambda}_{AD} = (T_{AC})(1)\cos\theta$$

$$= 20\cos 87.89^\circ = 0.74\,kN$$

d) Moment of $\vec{T}_{AC}$ about F:

$$\vec{M}_F = \vec{r}_{FA} \times \vec{T}_{AC} = \vec{r}_{FC} \times \vec{T}_{AC}$$

use $\vec{r}_{FA} = -3\hat{i} + 3\hat{j} + 4\hat{k}$

$$\vec{M}_F = \left|\begin{array}{ccc|cc} i & j & k & i & j \\ -3 & 3 & 4 & -3 & 3 \\ -15.43 & -12.34 & 3.09 & -15.43 & -12.34 \end{array}\right|$$

$$= [9.29\hat{i} - 61.72\hat{j} + 37.02\hat{k}] - [-46.29\hat{k} - 49.36\hat{i} - 9.27\hat{j}]$$

$$\vec{M}_F = 58.63\hat{i} - 52.45\hat{j} + 83.31\hat{k}$$

e) Moment of $\vec{T}_{AC}$ about Line EF

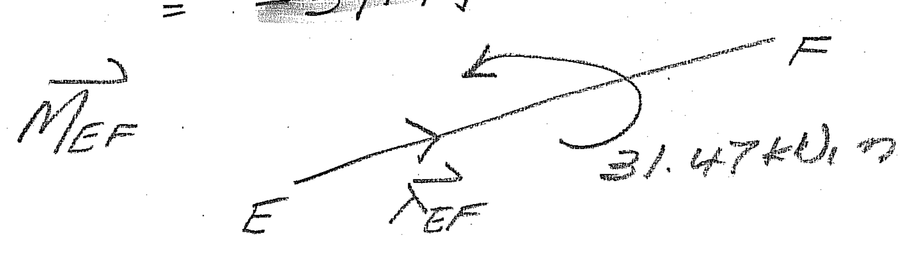
$$M_{EF} = \vec{\lambda}_{EF} \bullet \vec{M}_F \qquad \vec{\lambda}_{EF} = \frac{\vec{EF}}{EF}$$

$$\vec{EF} = 3\hat{i} + 0\hat{j} - 4\hat{k} \qquad EF = \sqrt{3^2 + (-4)^2} = 5$$

$$\vec{\lambda}_{EF} = \frac{3\hat{i} + 0\hat{j} - 4\hat{k}}{5}$$

$$\therefore M_{EF} = \left(\frac{3\hat{i} + 0\hat{j} - 4\hat{k}}{5}\right) \bullet (58.63\hat{i} - 52.45\hat{j} - 83.31\hat{k})$$

$$= 31.47\,kN.m$$



A simple truss has a pin (hinge) support at *A*. *Member* DM is a "*short link*". The beam *JK* is suspended from the truss by cables attached at joints *H* and *B*. The beam *JK* supports a distributed load as shown in Figure 2. Two smooth pulleys each of radius *0.2 m* are attached to the truss at joints *D* and *I*, respectively. A cable passes over the pulleys and is attached to an external support at *L*. A *10 kN* weight is suspended from the other end of the cable.

Determine:
The force in each member of the truss and state whether the member is in tension or compression or a zero force member.

**PLACE YOUR RESULTS FOR THE TRUSS MEMBER FORCES ON THE FIGURE PROVIDED.**

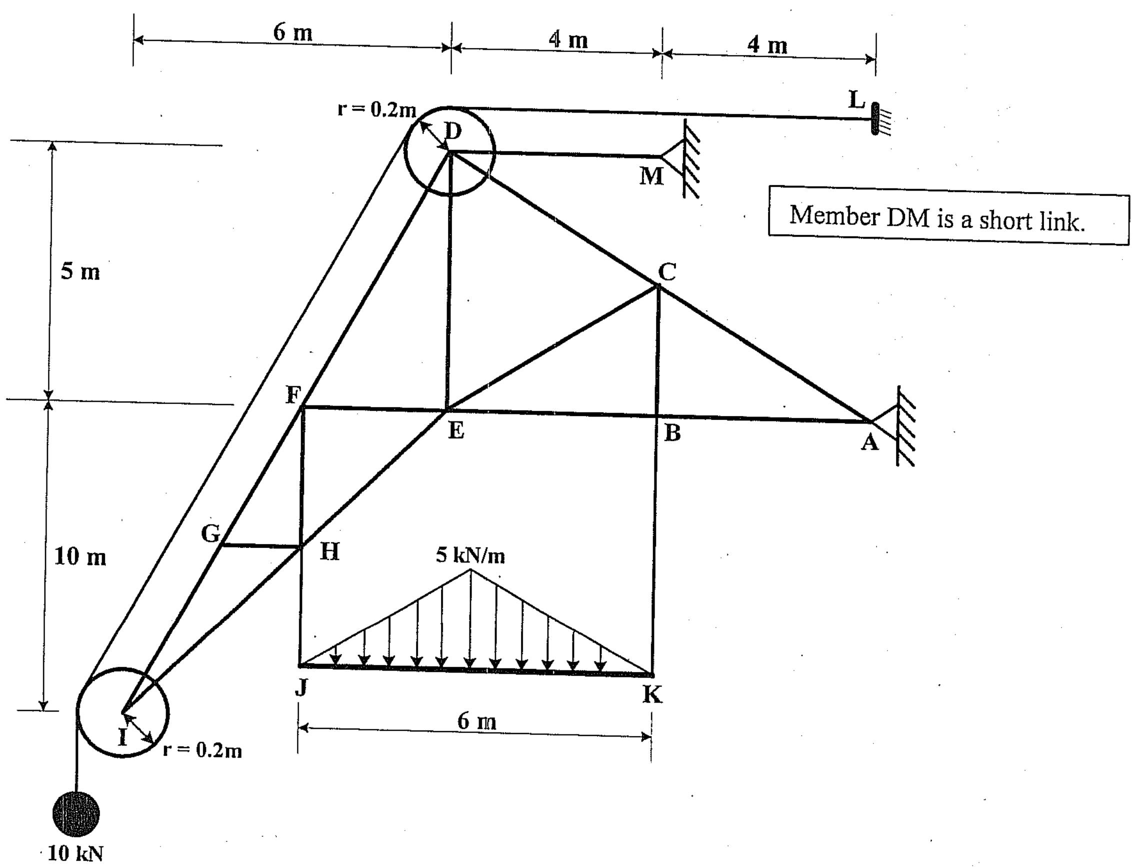
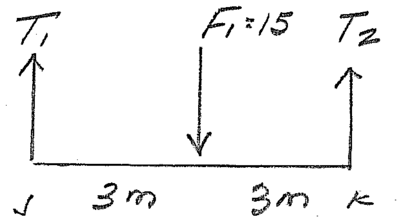



**Figure 2**

Distributed Load:




$F_1 = \frac{5(6)}{2} = 15\text{ kN}$

$\Sigma M_J = 0 \quad -15(3) + T_2(6) = 0$

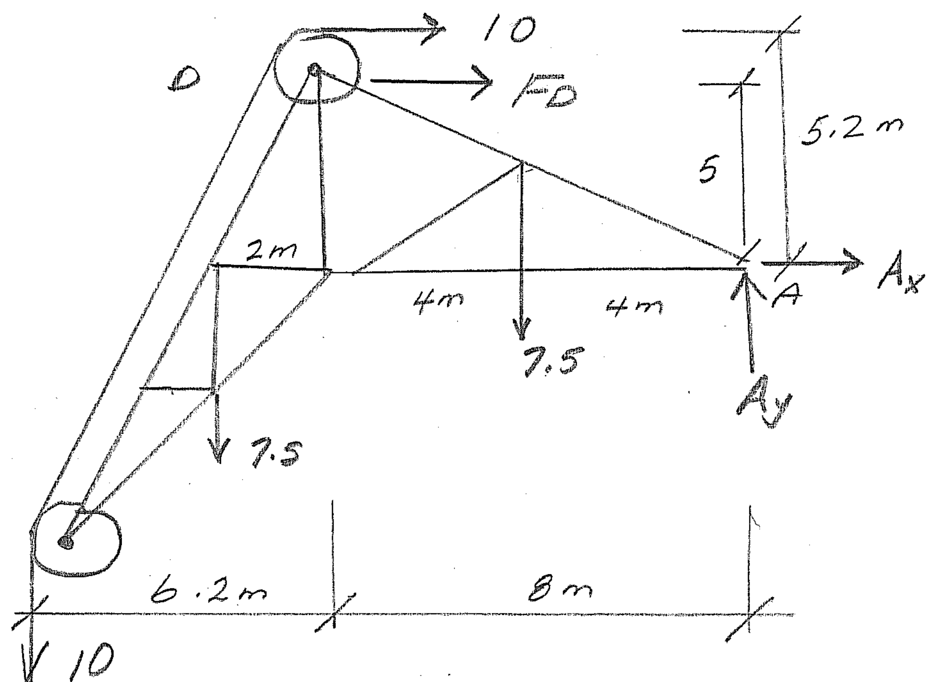
$T_2 = +7.5\text{ kN}$

$\therefore \vec{T_2} = 7.5\text{ kN} \uparrow$ on the beam

$\Sigma F_y = 0 \quad T_1 - 15 - 7.5 = 0$

$T_1 = +7.5\text{ kN} \quad \therefore \vec{T_1} = 7.5\text{ kN} \uparrow$ on the beam

FBD of entire truss (pulleys left attached)




$\Sigma M_A = 0 \quad 10(14.2) + 7.5(10) + 7.5(4) - 10(5.2) - F_D(5) = 0$

$F_D = +39\text{ kN} \quad \therefore \vec{F_D} = 39\text{ kN} \rightarrow$

$\Sigma F_y = 0 \quad -10 - 7.5 - 7.5 + A_y = 0 \quad A_y = +25\text{ kN}$

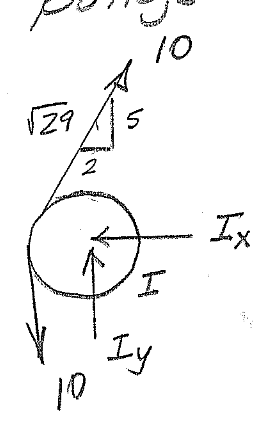
$\therefore \vec{A_y} = 25\text{ kN} \uparrow$

$\Sigma F_x = 0 \quad 10 + 39 + A_x = 0$

$A_x = -49\text{ kN} \quad \therefore \vec{A_x} = 49\text{ kN} \leftarrow$

Check $\Sigma M_G = 0 \quad 10(6.2) + 7.5(2) - 10(0.2) - 7.5(4) + 25(8) - 49(5) = 0$ ✓

FBD pulleys:




$\Sigma F_x = 0$

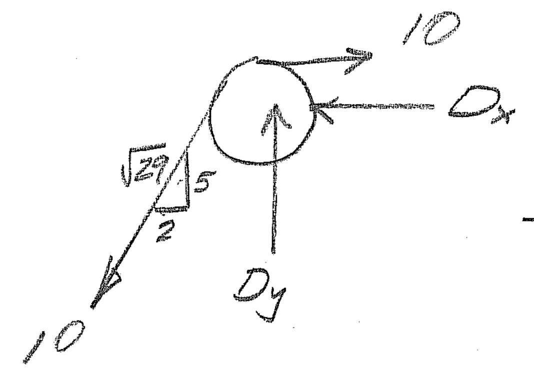
$-I_x + \frac{2}{\sqrt{29}}(10) = 0 \quad I_x = +3.71\text{ kN}$

$\vec{I_x} = 3.71\text{ kN} \leftarrow$ on the pulley at I

$\Sigma F_y = 0 \quad I_y - 10 + \frac{5}{\sqrt{29}}(10) = 0$

$I_y = +0.72\text{ kN} \quad \therefore \vec{I_y} = 0.72\text{ kN} \uparrow$ on the pulley

Pulley at D:




$\Sigma F_x = 0$

$-\frac{2}{\sqrt{29}}(10) - D_x + 10 = 0$

$D_x = +6.29$ kN

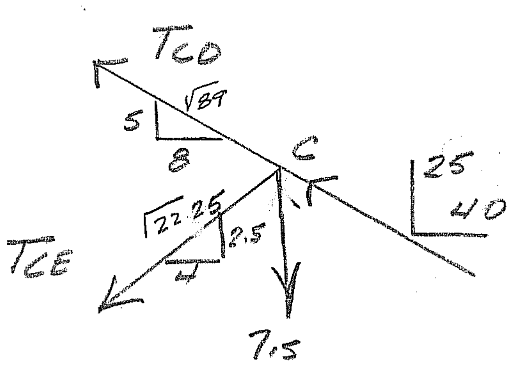
$\therefore \vec{D_x} = 6.29$ kN $\leftarrow$ on the pulley

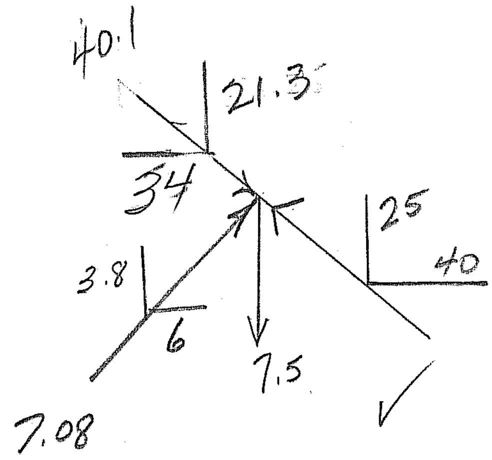
$\Sigma F_y = -\frac{5}{\sqrt{29}}(10) + D_y = 0$

$D_y = +9.28$ kN

$\therefore \vec{D_y} = 9.28$ kN $\uparrow$ on the pulley

## JOINT C




$\Sigma F_x = 0 \qquad -\frac{8}{\sqrt{89}} T_{CD} - \frac{4}{\sqrt{22.25}} T_{CE} - 40 = 0 \quad (1)$

$\Sigma F_y = 0 \qquad \frac{5}{\sqrt{89}} T_{CD} - \frac{2.5}{\sqrt{22.25}} T_{CE} - 7.5 + 25 = 0 \quad (2)$

$-\frac{40}{\sqrt{89}} T_{CD} - \frac{20}{\sqrt{22.25}} T_{CE} = 200$

$\frac{40}{\sqrt{89}} T_{CD} - \frac{20}{\sqrt{22.25}} T_{CE} = -140$

$-\frac{40}{\sqrt{22.25}} T_{CE} = 60$

$T_{CE} = -\frac{60\sqrt{22.25}}{40} = -7.08$ kN

$\vec{T_{CE}} = 7.08$ (2.5, 4)

$-\frac{40}{\sqrt{89}} T_{CD} - \frac{20}{\sqrt{22.25}}(-7.08) = 200$

$-\frac{40}{\sqrt{89}} T_{CD} = 200 - \frac{20(7.08)}{\sqrt{22.25}} \qquad T_{CD} = -40.1 \qquad \vec{T_{CD}} = 40.1$

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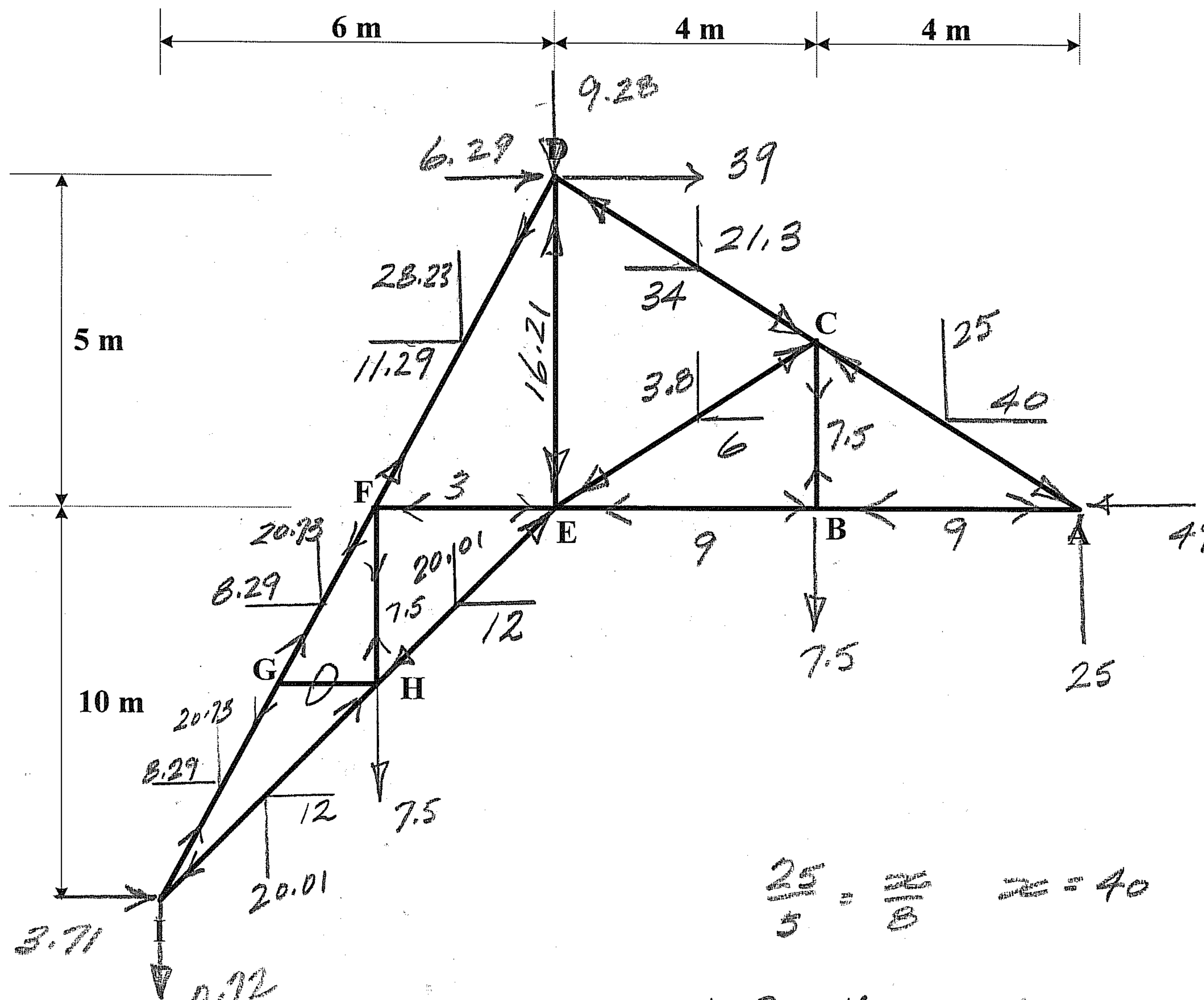
comp

tension




$\frac{25}{5} = \frac{x}{8}$ $x = 40$

$\frac{11.29}{6} = \frac{y}{15}$ $y = 28.23$

$\frac{20.01}{10} = \frac{x}{6}$ $x = 12$

$\frac{8.29}{6} = \frac{y}{15}$ $y = 20.73$

THE UNIVERSITY OF MANITOBA

Date : Friday, December 7, 2012
Department & Course No : ENG 1440
Sections A01-A02-A03
Examination : Introduction to Statics


Time : 6:00 a.m.
Duration : 2 Hours
Examiners : Dr. M. Bassuoni,
Dr. R. Chitikireddy, Dr. M. J. Frye

Place : Frank Kennedy Brown Gym
Seats: 1 - 243

**Question 3**

The frame shown in Figure 3 has external supports at *A* and *C*. A smooth pulley with a radius of *0.75 m* is attached to the frame at *E*. The cable running over the pulley is attached to the wall at *G* and supports a *50 kN* weight. Forces of *60 kN*, *25 kN* and *15 kN* and a clockwise couple moment of *30 kN.m* are applied to the frame as shown. Determine:

a) The external reactions at *A* and *C*, and
b) The forces acting on each member of the frame including the pulley.

**Indicate your final results on separate Free Body Diagrams of each member of the frame including any pins included in the substructures.**

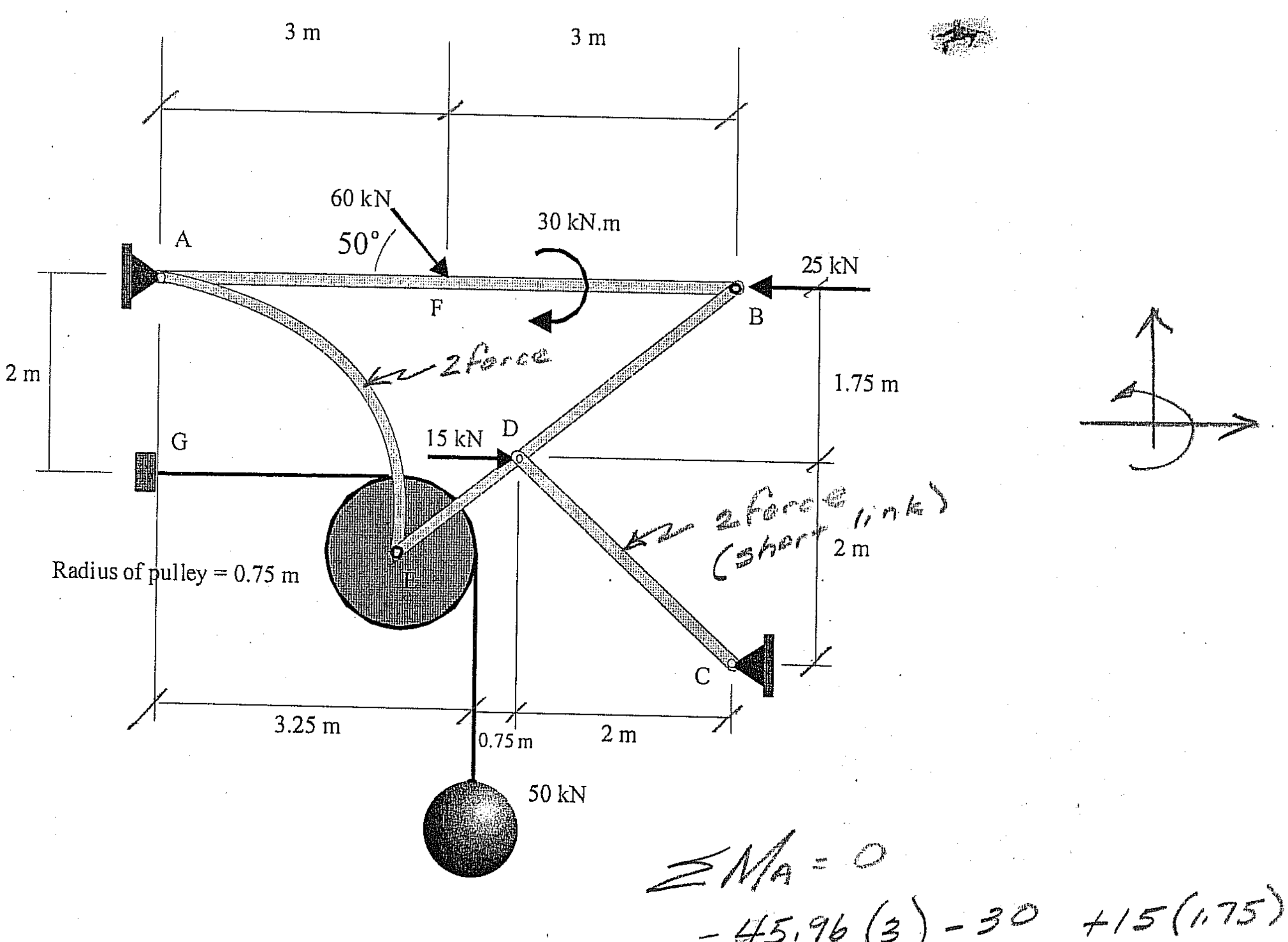
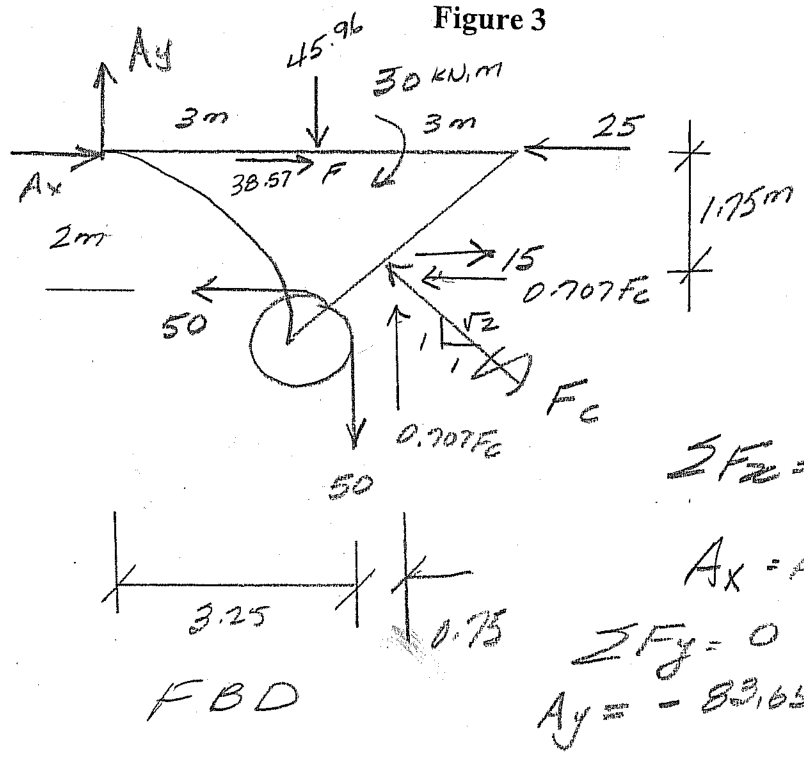



Figure 3




$\sum M_A = 0$

$-45.96(3) - 30 + 15(1.75) - 0.707F_C(1.75) + 0.707F_C(4) - 50(3.25) - 50(2) = 0$

$1.59F_C = 404.13$

$\therefore F_C = +254.05\ kN$

$\therefore \vec{F_C} = 254.05\ kN$

$\sum F_x = 0 \quad A_x + 38.57 - 25 + 15 - 0.707(254.05) - 50 = 0$

$A_x = +201.04 \quad \therefore \vec{A_x} = 201.04\ kN \rightarrow$

$\sum F_y = 0 \quad A_y - 45.96 + 0.707(254.05) - 50 = 0$

$A_y = -83.65\ kN \quad \therefore \vec{A_y} = 83.65\ kN \downarrow$

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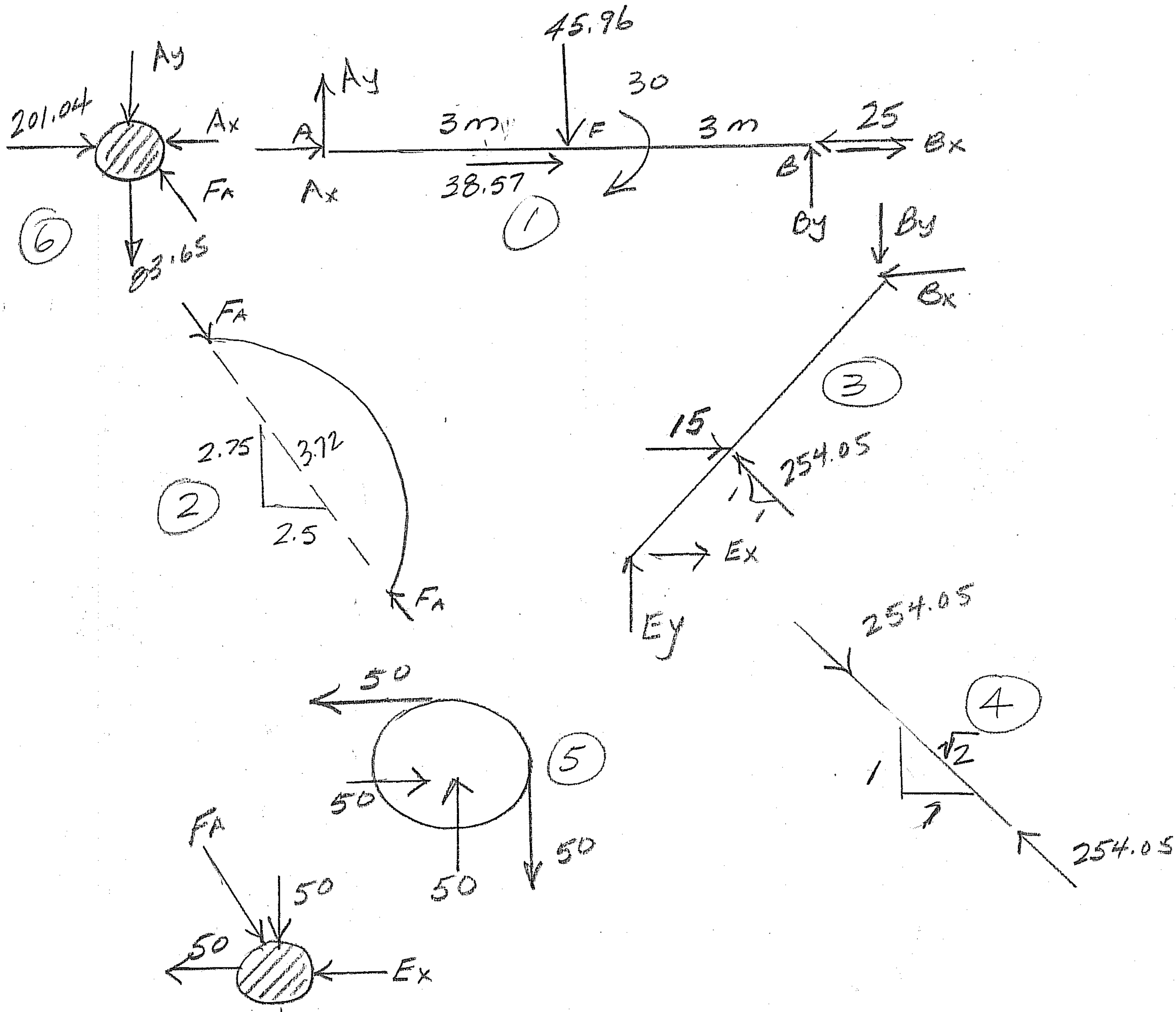
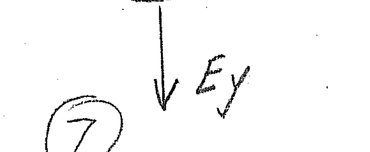
Check Reactions:

$$\Sigma M_F = 0 \quad 83.65(3) - 30 - 50(2) - 50(0.25) + 15(1.75) - 254.05(0.707)(1.75) + 254.05(0.707)(1)$$

$$0 = 0 \checkmark$$

Substructures:




From ①

$$\Sigma M_A = 0 \quad -45.96(3) - 30 + B_y(6) = 0$$

$$B_y = +27.98\text{ kN} \quad \therefore \vec{B_y} = 27.98\text{ kN} \uparrow \text{ on AFB}$$

$$\Sigma F_y = 0 \quad A_y - 45.96 + 27.98 = 0$$

$$A_y = +17.98\text{ kN}$$

$$\therefore \vec{A_y} = 17.98\text{ kN} \uparrow \text{ on AFB}$$

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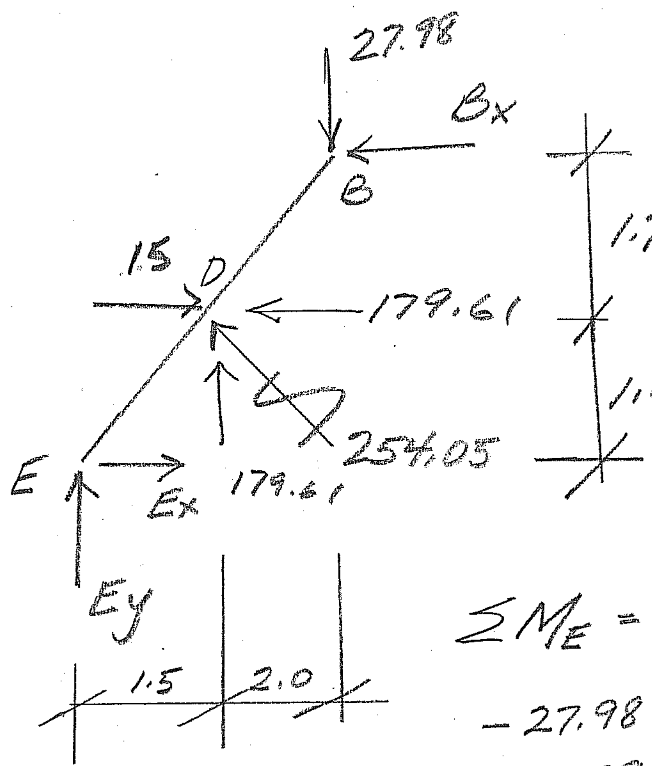
FROM ⑤ Re-draw




$$\Sigma M_E = 0$$

$$-27.98(3.5) + B_x(2.75) - 15(1) + 179.61(1) + 179.61(1.5) = 0$$

$$B_x = -122.22$$

$$\therefore \vec{B_x} = 122.22\text{ kN} \rightarrow \text{ on EDB}$$

$$\Sigma F_x = 0 \quad E_x + 15 - 179.61 - (-122.22) = 0$$

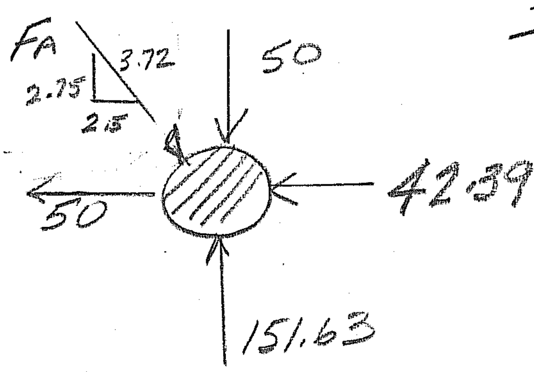
$$E_x = 42.39\text{ kN}$$

$$\therefore \vec{E_x} = 42.39\text{ kN} \rightarrow \text{ on EDB}$$

$$\Sigma F_y = 0 \quad E_y + 179.61 - 27.98 = 0$$

$$E_y = -151.63\text{ kN} \quad \therefore \vec{E_y} = 151.63\text{ kN} \downarrow \text{ on EDB}$$

Re-draw ⑦




$$\Sigma F_x = 0$$

$$-50 + \frac{2.5}{3.72} F_A - 42.39 = 0$$

$$F_A = +137.48\text{ kN}$$

$$\vec{F_A} = 137.48\text{ kN}$$ (2.75, 2.5, 3.72)

on pin at E

$$\Sigma F_y = 0 \quad -\frac{2.75}{3.72}(137.48) - 50 + 151.63 = 0$$

$$0 = 0 \checkmark$$

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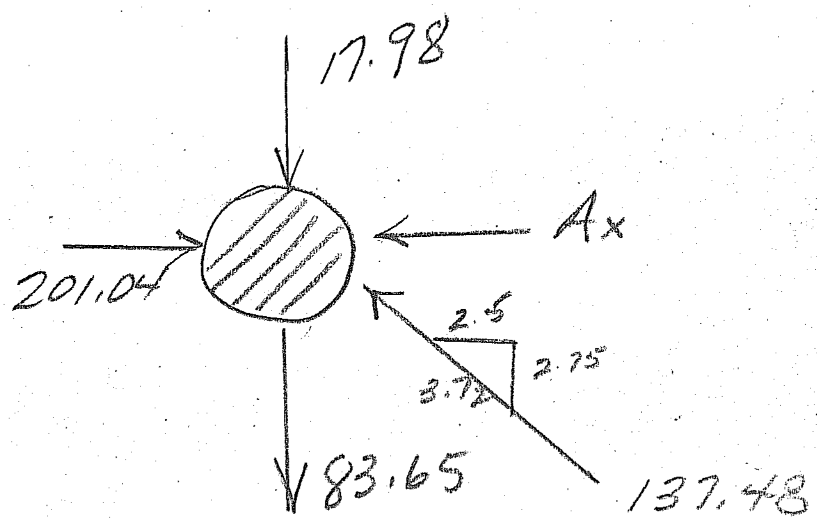
Re-draw 6



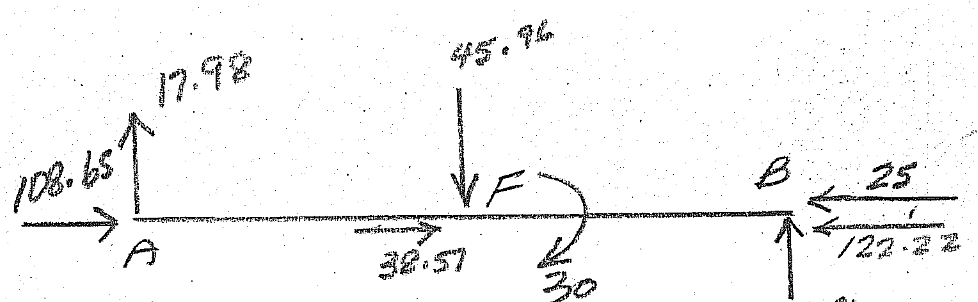

$$\sum F_x = 0 \quad 201.04 - A_x - \frac{2.5}{3.72}(137.48) = 0$$

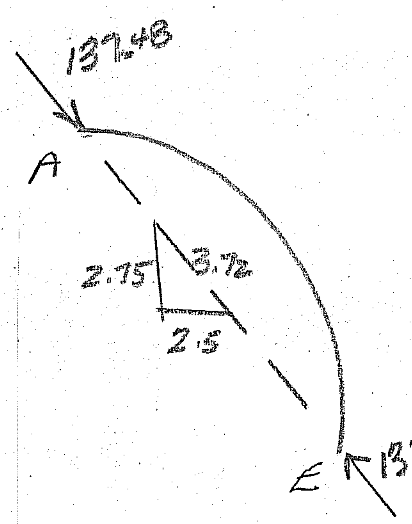
$$A_x = +108.65 \text{ kN}$$

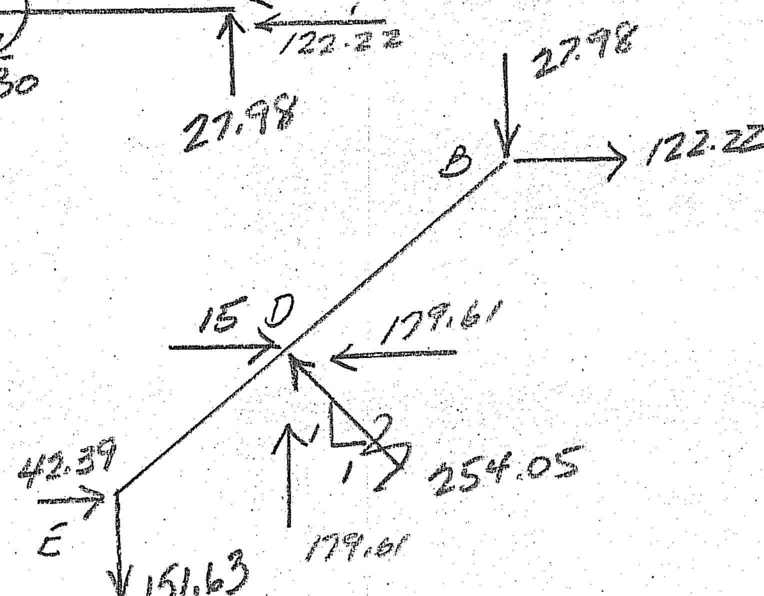
$$\vec{A_x} = 108.65 \text{ kN} \leftarrow \text{ on pin at } A$$

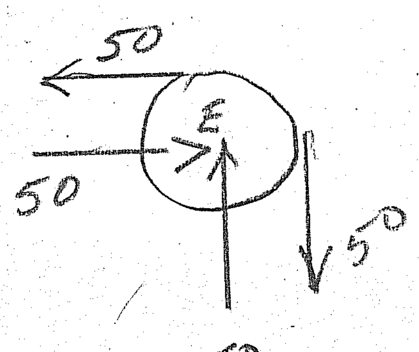
$$\sum F_y = 0 \quad -17.98 - 83.65 + \frac{2.75}{3.72}(137.48) = 0$$

$$0 = 0 \checkmark$$




External Reactions!

$\vec{A_x} = 201.04 \text{ kN} \rightarrow$

$\vec{A_y} = 83.65 \text{ kN} \downarrow$

$\vec{F_C} = 254.05 \text{ kN}$