Practice Final Exam Answers

Multiple-Choice

1. E
2. C
3. A
4. C
5. A

6. B
7. E
8. D
9. D
10. A

11. E
12. D
13. C
14. A
15. D

16. A
17. E
18. B
19. E
20. A

21. E
22. C
23. E
24. C
25. C

26. C
27. A
28. B
29. D
30. B

31. C
32. D
33. A
34. D
35. D

36. C
37. B
38. C
39. A
40. A

41. C
42. B
43. A
44. A
45. E

46. D
47. B
48. C
49. B
50. E

51. B
52. A
53. A
54. B
55. D

Long-Answer

1. (a) Two events are mutually exclusive if they cannot occur together, i.e. P(A and B) = 0. Two events are independent if one event occurring does not affect the probability of the other event occurring. If two events are independent, P(A and B) = P(A)P(B).

   (b) A discrete random variable is one that can take only a countable number of values (and usually represents a count). A continuous random variable can take any value within a given range of values, and has a distribution that is described by a smooth curve.

2. (a) The sample space is shown below, as well as the probability of each outcome. Since the colour of the coin selected from the first hat is independent of the colour coin selected from the second hat, probabilities of outcomes are obtained by multiplying respective probabilities.

Outcome	Probability
GG	(0.4)(0.3) = 0.12
GS	(0.4)(0.6) = 0.24
GC	(0.4)(0.1) = 0.04
SG	(0.1)(0.3) = 0.03
SS	(0.1)(0.6) = 0.06
SC	(0.1)(0.1) = 0.01
CG	(0.5)(0.3) = 0.15
CS	(0.5)(0.6) = 0.30
CC	(0.5)(0.1) = 0.05

   (b) P(X = 0) = P(SS) + P(SC) + P(CS) + P(CC) = 0.06 + 0.01 + 0.30 + 0.05 = 0.42
   P(X = 1) = P(GS) + P(GC) + P(SG) + P(CG) = 0.24 + 0.04 + 0.03 + 0.15 = 0.46
   P(X = 2) = P(GG) = 0.12

   The p.m.f. of X is shown below:

x	0	1	2
P(X = x)	0.42	0.46	0.12

   (c) $E(X) = \sum x\,p(x) = 0(0.42) + 1(0.46) + 2(0.12) = 0.7$

   $E\left(X^2\right) = \sum x^2\,p(x) = (0)^2(0.42) + (1)^2(0.46) + (2)^2(0.12) = 0.94$

   And so $\mathrm{Var}(X) = E\left(X^2\right) - \left(E(X)\right)^2 = 0.94 - (0.7)^2 = 0.45$

(d) Let X be the number of number of selections required to get the third gold coin.
Then X has a negative binomial distribution with parameters r = 3 and p = 0.4, and so

$$P(X = 15) = \binom{14}{2}(0.4)^3(0.6)^{12} = 0.0127$$

(e) Let X be the number of gold coins selected in five draws without replacement.
Then X has a hypergeometric distribution with parameters N = 10, r = 4 and n = 5, and so

$$P(X = 3) = \frac{\binom{4}{3}\binom{6}{2}}{\binom{10}{5}} = 0.2381$$

(f) $P(\text{both silver} \mid \text{same colour}) = \dfrac{P(\text{both silver} \cap \text{same colour})}{P(\text{same colour})} = \dfrac{P(\text{both silver})}{P(\text{same colour})}$

$$= \frac{P(SS)}{P(GG) + P(SS) + P(CC)} = \frac{0.06}{0.12 + 0.06 + 0.05} = \frac{0.06}{0.23} = 0.2609$$

3. (a) $P(X < 1) = \displaystyle\int_0^1 1.25e^{-1.25x}\,dx = \left[-e^{-1.25x}\right]_0^1 = 1 - e^{-1.25} = 0.7135.$

(b) The Central Limit Theorem states that, regardless of the population distribution of a random variable X, the sampling distribution of the sample mean $\overline{X}$ will be approximately normal with mean μ and standard deviation $\sigma/\sqrt{n}$, provided that the sample size is large (we use $n \geq 30$).

(c) Let X_T be the total service time for the next 40 customers. By the Central Limit Theorem,

$$P(X_T < 30) = P\left(\overline{X} < \frac{30}{40}\right) = P(\overline{X} < 0.75) \approx P\left(Z < \frac{0.75 - 0.8}{0.8/\sqrt{40}}\right) = P(Z < -0.40) = 0.3446.$$

We could have equivalently calculated this probability using the distribution of the sample total.

4. (a) We must find the critical value z* for a 93% confidence interval, i.e., we must find the value z* such that

$$P(-z^* < Z < z^*) = 0.9300 \;\Rightarrow\; P(Z < z^*) = 0.9300 + 0.0350 = 0.9650 \;\Rightarrow\; z^* = 1.81.$$

A 93% confidence interval for μ is therefore

$$\bar{x} \pm z^* \frac{\sigma}{\sqrt{n}} = 1000 \pm 1.81\left(\frac{200}{\sqrt{30}}\right) = 1000 \pm 66 = (934,\ 1066)$$

(b) If we took repeated samples of 30 two-bedroom apartments in Winnipeg and calculated the interval in a similar manner, then 93% of all such intervals would contain the true mean size of all two-bedroom apartments in Winnipeg.

5. (a) A 95% confidence interval for μ is

$$\bar{x} \pm t^* \frac{\sigma}{\sqrt{n}} = 247 \pm 2.365\left(\frac{10}{\sqrt{8}}\right) = 247 \pm 8.36 = (238.64,\ 255.36)$$

where $t^* = 2.365$ is the upper 0.025 critical value from the t distribution with 7 d.f.

(b) If we repeatedly selected samples of eight frozen dinners and calculated the interval in a similar manner, 95% of such intervals would contain the true mean calorie content of all frozen dinners of this type.

(c) Let $\alpha = 0.05$.

We are testing the hypotheses

H_0: The true mean calorie content for this type of frozen dinner is equal to the amount stated on the label.
H_a: The true mean calorie content for this type of frozen dinner differs from the amount stated on the label.

Equivalently, H_0: $\mu = 240$ vs. H_a: $\mu \neq 240$.

We will reject H_0 if the P-value $\leq \alpha = 0.05$.

The test statistic is

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{247 - 240}{10/\sqrt{8}} = 1.98.$$

The P-value is $2P(T(7) \geq 1.98)$.

We see from the t table that $P(T(7) \geq 1.895) = 0.05$ and $P(T(7) \geq 2.365) = 0.025$.

Since the value of our test statistic is between these two values, our P-value is between 2(0.025) and 2(0.05), i.e., between 0.05 and 0.10.

Since the P-value > $\alpha = 0.05$, we fail to reject H_0. At the 5% level of significance, we have insufficient evidence that the true mean calorie content differs from 240.

(d) If the true mean calorie content of all frozen dinners of this type was 240, the probability of getting a sample mean at least as extreme as 247 would be between 0.05 and 0.10.

(e) Since this is a two-sided test, and since the confidence level (95%) and the level of significance (5%) add up to 100%, we could have used the interval in (a) to conduct the test in (c). Since $\mu_0 = 240$ is contained in the 95% confidence interval for μ, we fail to reject H_0 at the 5% level of significance.

6. (a) The stemplot for this data set is shown below:

0	8
1	3 7
2	2 5
3	0 6 9
4	0 0 2
5	1 3 6 7
6	0 4 4 6 8 8
7	1 3 5 5 6 9
8	0 6 8

The distribution of wait times is skewed to the left.

(b) Although the population distribution of wait times is not normal, our inference methods require that the distribution of the sample mean $\overline{X}$ be normal. Since our sample size is high, the Central Limit Theorem tells us that $\overline{X}$ will have an approximate normal distribution.

(c) Let $\alpha = 0.01$.

We are testing the hypotheses

H_0: The true mean wait time for this type of surgery is the same as it was in 1999.
H_a: The true mean wait time for this type of surgery has decreased since 1999.

Equivalently, $H_0 : \mu = 60$ vs. $H_a : \mu < 60$.

We will reject H_0 if the P-value $\leq \alpha = 0.01$.

The test statistic is

$$z = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}} = \frac{54.07 - 60}{20 / \sqrt{30}} = -1.62$$

The P-value is $P(Z \leq -1.62) = 0.0526$.

Since the P-value $> \alpha = 0.01$, we fail to reject H_0. At the 1% level of significance, we have insufficient evidence that the true mean wait time for this type of surgery has decreased since 1999.