# THE UNIVERSITY OF MANITOBA

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|---|---|
| 9:00 am, April 18, 2005 | Final Examination (FR Kennedy Gold Gym) |
| Paper No: 250 | Page No: Page 1 of 6 |
| Dept. and Course No.: 130.112 | Time: 3 Hours |
| Examination.: Thermal Science | Examiners: Drs. Bibeau, Naterer, Soliman |

- Answer 5 out of the 6 questions in this exam. Each question is worth 20 points.
- Follow the "Problem-Solving Technique" discussed in class.
- The exam is 3 hours long, open book, and use of a calculator is permitted.
- You are allowed copies of chapter 12 from the heat transfer text book.
- When you are finished, take time to review your work and double check units and formulas; make sure you have made a sketch, have shown the units and inputs, and have drawn a T-V or P-V diagram if applicable.
- Ask for clarification if any problem statement is unclear to you.
- Retain all the significant figures of properties and molecular weights taken from tables. Final results should have at least 3 to 5 significant digits.
- Use constant specific heats.

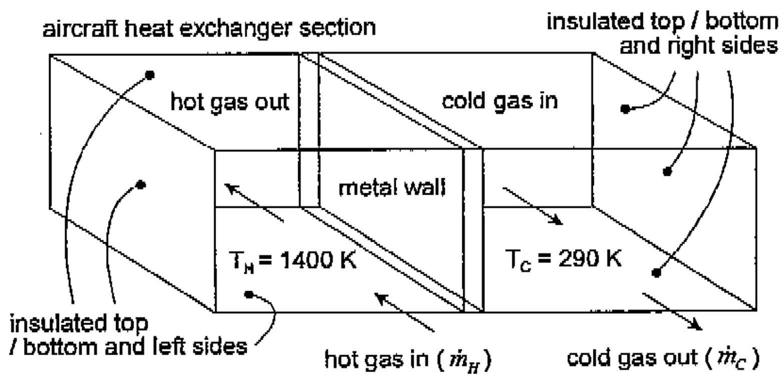
Values **Problem #1 (20 Points):** In the design of an aircraft heat exchanger section, the maximum temperature in the metal wall must not exceed 900 K (see figure). The heat transfer coefficients on the hot side and cold side of the metal wall are 220 W/m$^2$·K and 420 W/m$^2$·K, respectively (including both convection and radiation). All walls are insulated, except the metal wall separating the hot and cold gas sides.

(3) (a) Sketch the thermal circuit for heat flow through the metal wall in this problem. Show the thermal resistances, temperature nodes and heat flows in the diagram.

(7) (b) If the hot gas temperature is $T_H = 1400$ K and the cold gas temperature is $T_C = 290$ K, what is the maximum permissible thermal resistance per unit area (m$^2$·K/W) of the metal wall, in order that the wall temperature does not exceed 900 K?

(4) (c) Under these conditions (part b), what is the temperature difference across the metal wall?

(6) (d) Consider the entire heat exchanger section with the cold gas experiencing a temperature rise of 30 K from the inlet to the outlet. What ratio of mass flow rates between the cold and hot gas sides $(\dot{m}_C/\dot{m}_H)$ is needed, in order to ensure net cooling of the hot gas by 160 K from the inlet to the outlet? Assume constant specific heats for the cold gas ($c_{p,c} = 1$ kJ/kg.K) and hot gas ($c_{p,h} = 1.2$ kJ/kg.K). Neglect changes in the kinetic and potential energies and treat the gases as ideal gases.



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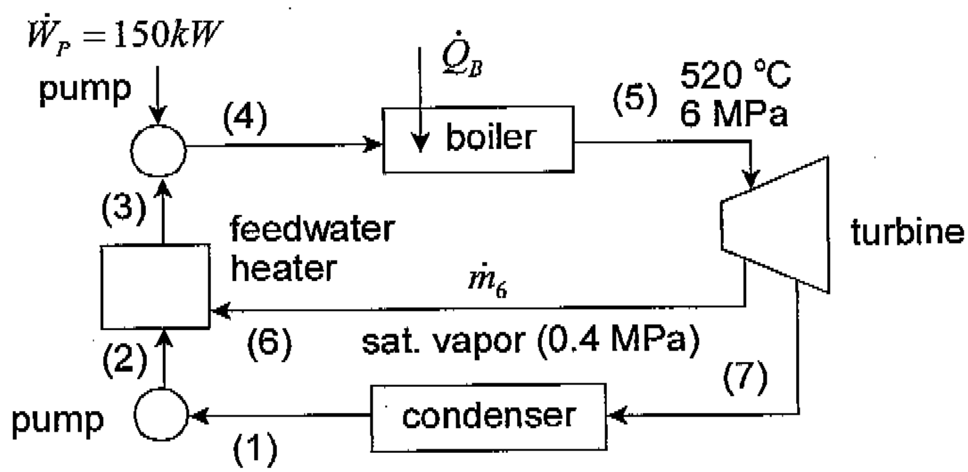
Values **Problem #2 (20 Points):** In a power plant, regeneration extracts steam from the turbine to heat the feedwater (liquid leaving the pump as shown in the figure) to improve the overall system efficiency. The mass flow rate of steam entering the turbine is 20 kg/s and the power input to the pump between states 3 and 4 is 150 kW. Water enters the feedwater heater (State 2) at $P_2 = 0.4$ MPa and $T_2 = 120$ °C. Another incoming stream of saturated vapor at 0.4 MPa enters at State 6 and fluid leaves the feedwater heater as saturated liquid at 0.4 MPa (State 3). Assume that the pumps and feedwater heater are externally insulated (adiabatic) and neglect changes in the kinetic and potential energies. Also, assume steady-state, steady-flow operation

(10) (a) What heat input to the boiler is needed $\left(\dot{Q}_B\right)$, if steam at 6.0 MPa pressure and 520°C temperature enters the turbine?

(10) (b) What mass flow rate enters the feedwater heater at state 6 (denoted by $\dot{m}_6$)?



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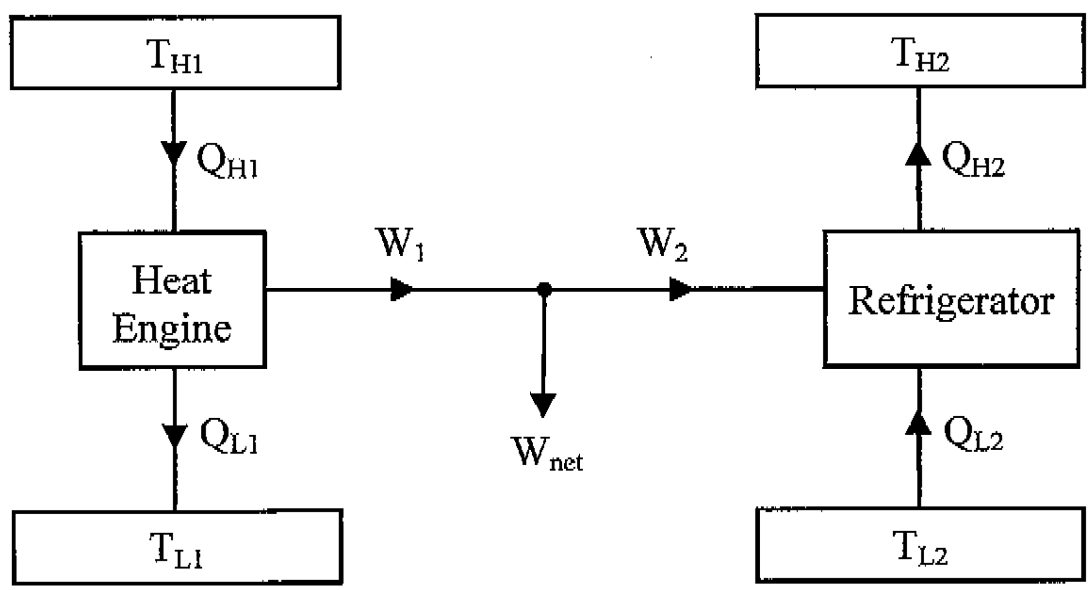
Values **Problem #3 (20 Points):** Consider a heat engine and a refrigerator connected as shown in the figure below. The heat engine operates between two heat reservoirs at $T_{H1}$ = 600 °C and $T_{L1}$ = 50 °C, and it receives an amount of heat $Q_{H1}$ = 1000 kJ from the reservoir at $T_{H1}$. The refrigerator operates between two heat reservoirs at $T_{H2}$ = 20 °C and $T_{L2}$ = –20 °C and it extracts an amount of heat $Q_{L2}$ = 500 kJ from the reservoir at $T_{L2}$. The refrigerator is irreversible and its coefficient of performance ($\beta$) is 60% of the reversible coefficient of performance. The heat engine delivers an amount of work $W_1$ and the refrigerator consumes an amount of work $W_2$. The net amount of work developed from the combined cycles is $W_{net}$, where $W_{net} = W_1 - W_2$.

(13) (a) If $W_{net}$ is equal to 600 kJ, determine (using appropriate First Law and Second Law calculations) whether the engine is reversible, irreversible, or impossible.

(7) (b) For the same given values of $T_{L1}$, $T_{H1}$, $T_{L2}$, $T_{H2}$, $Q_{L2}$, and $Q_{H1}$, determine the maximum possible value of $W_{net}$ that can be developed by any combined heat engine-refrigerator system.



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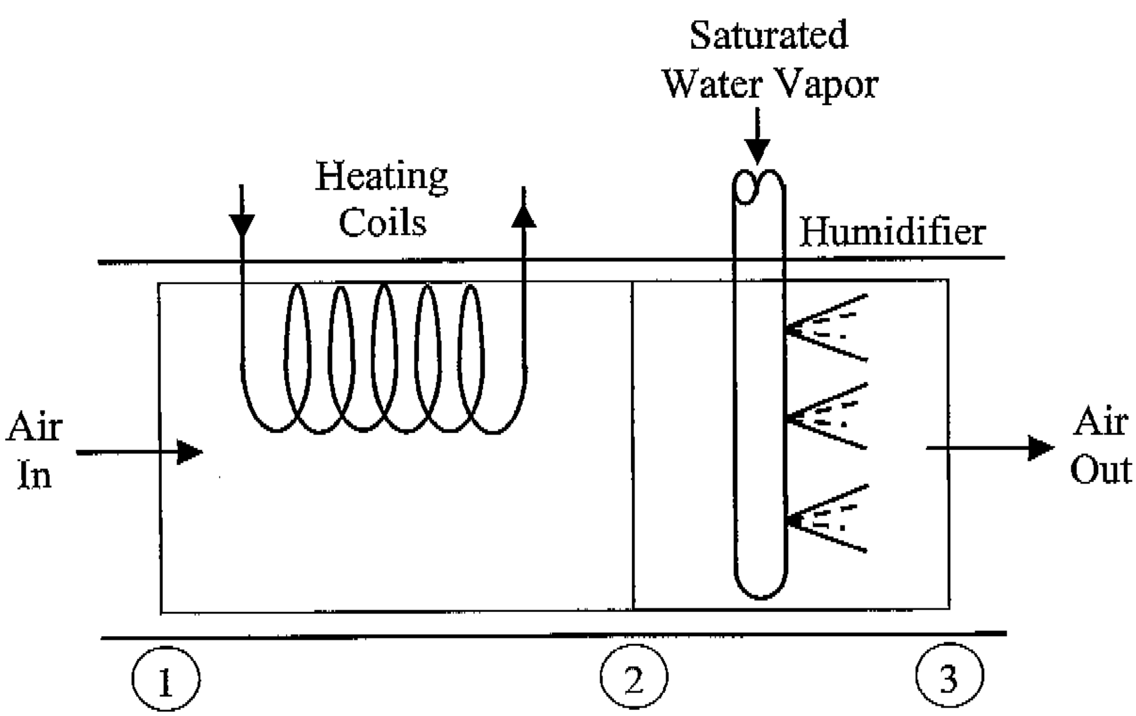
Values **Problem #4 (20 Points):** An air-conditioning system receives outside ambient air at $T_1 = 10\ ^{\circ}C$, $P_1 = 95$ kPa, and $\phi_1 = 70\%$ with a volume flow rate $\dot{V}_1 = 36\ m^3/min$. The air-conditioning system, shown in the figure below, consists of two sections; a heating section (from State 1 to State 2) and a humidifying section (from State 2 to State 3). The air leaves the heating section at $T_2 = 20\ ^{0}C$ and $P_2 = 95$ kPa. Saturated water vapor at 100 °C is added to the air in the humidifying section. The air leaves the air conditioner at $T_3 = 21\ ^{\circ}C$ and $P_3 = 95$ kPa. Assume steady-state, steady-flow operation and neglect changes in the kinetic and potential energies.

**NOTES: (1) The humidifying section is adiabatic, i. e., well insulated on the outside. (2) Use the formulas in your calculations; do not use the Psychrometric Chart.**

(6) (a) Determine the mass flow rate of dry air in [kg/s].
(7) (b) Determine the rate of heat addition in the heating section in [kW].
(7) (c) Determine the mass flow rate of the steam in the humidifying section in [kg/h].



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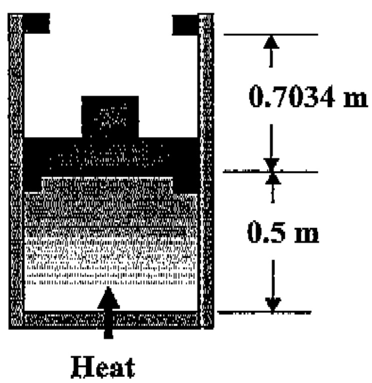
Values **Problem #5 (20 Points):** A piston-cylinder device as shown in the figure below contains 2 kg of saturated water vapour at 400 kPa (state 1). The system pressure is not enough to overcome the weight of the piston and the external pressure at this point and the piston rests on the first set of stops. Heat is added to the water and the pressure increases up to 500 kPa when the water pressure can now overcome the weight of the piston and the external pressure (state 2). Heat is further added to the water raising the piston until it contacts the second set of stops (state 3). The system continues to be heated until the water reaches a pressure of 600 kPa (state 4). The location of the first set of stops is at 0.5 m from the bottom of the cylinder and the location of the second set of stops is at 0.7034 m above the first set, as shown in the figure. Neglect changes in the kinetic and potential energies.

(1) (a) State all assumptions.

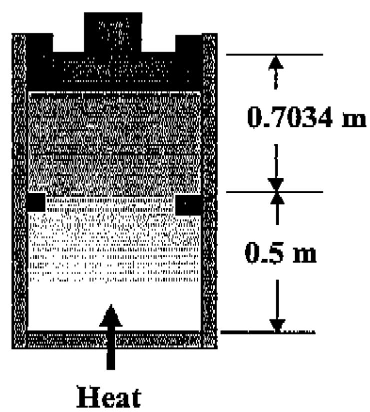
(4) (b) Sketch the P-V diagram (V is the total volume in m³) showing all 4 temperature lines, 4 state points and 3 process lines.

(6) (c) What is the work done by the system between states 1 to 4?

(9) (d) What is the total heat transferred to the system between states 1 to 4?



(State 1 and 2)



(State 3 and 4)

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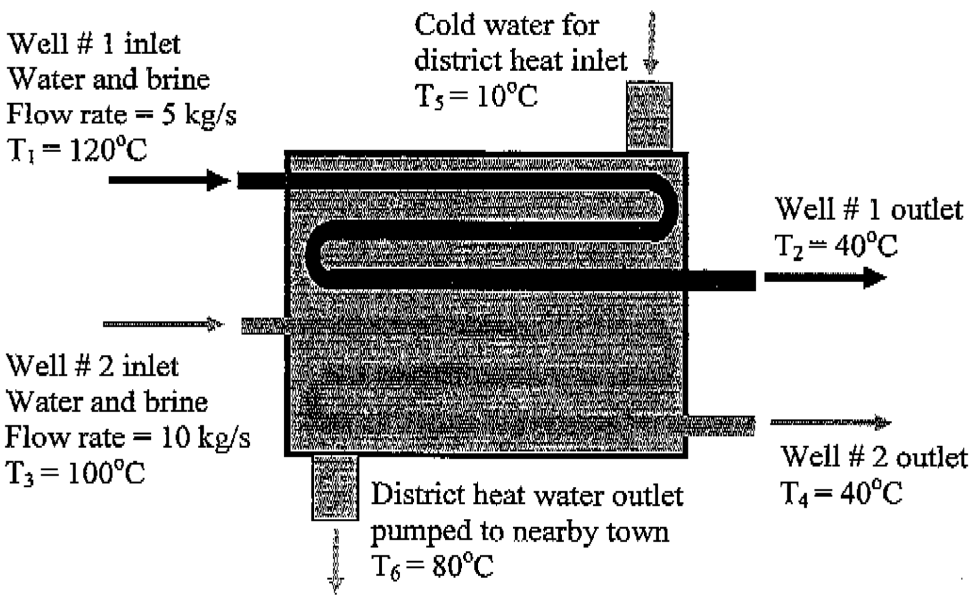
**Problem #6 (20 Points):** Two geothermal wells provide a hot liquid brine solution that is fed to a heat exchanger at the conditions shown in the attached figure. The two compressed-liquid brine solutions flow in separate pipes within the heat exchanger. The specific heat of the geothermal brine solution from both wells is 5.853 kJ/kg·K. Geothermal heat is transferred to cold water entering the heat exchanger at 10°C ($T_5$) and exiting the heat exchanger at 80°C ($T_6$) to provide district heating to the nearby town. Assume steady-state, steady flow operation and neglect changes in the kinetic and potential energies. The heat exchanger is well insulated on the outside.

(2) (a) State all assumptions.

(12) (b) What is the flow rate of the cold water required?

(6) (c) If **only** the temperature of Well #1 dropped by 20°C ($T_1$) while $T_2$, $T_3$, $T_4$, $T_5$ and all flow rates remained the same, what would be the new water temperature that would go to heat the nearby town ($T_6$)?



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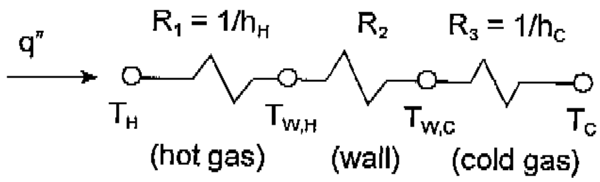
## SOLUTIONS

**Problem 1:**

1. (a) The thermal circuit (starting from the hot gas side) consists of convection / radiation up the wall, conduction through the wall and convection to the cold gas. The convection / radiation portion can be shown as two individual resistances in parallel between $T_W$ and $T_{W,H}$.




1. (b) In this problem, steady state conditions and constant specific heats are assumed. Equating the heat flux on the hot gas side with the total flux through the wall (see thermal circuit),

$$q^{"} = \frac{T_H - T_{W,H}}{1/h_H} = \frac{T_H - T_C}{1/h_H + R_2 + 1/h_C} \quad so \quad \frac{1400-900}{1/220} = \frac{1400-290}{1/220 + R_2 + 1/420}$$

Solving this equation gives $R_2 = 0.0032$ m$^2$K/W with a heat flux of q" = 110 kW/m$^2$.

1. (c) Considering heat conduction through the wall (see thermal circuit),

$$q^{"} = \frac{T_{W,H} - T_{W,C}}{R_2} \quad so \quad 110{,}000 = \frac{900 - T_{W,C}}{0.0032}$$

Solving this equation gives $T_{W,C} = 548$ K so the temperature difference across the wall is 900 – 548 = 352 K.

1. (d) Consider two control volumes encompassing the cold and hot gas sides. Assume SS-SF with negligible changes in kinetic and potential energies, with zero work. Applying the First Law to each CV and equating the heat flow leaving the hot side with heat entering the cold side,

$$\dot{Q} = \dot{m}_C c_{p,c}(T_{C,out} - T_{C,in}) = \dot{m}_H c_{p,h}(T_{H,in} - T_{H,out}) \quad so \quad \frac{\dot{m}_C}{\dot{m}_H} = \frac{1.2(160)}{1(30)} = 6.4$$

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**Problem 2:**

2. (a) Assumptions: SS-SF, negligible changes in KE and PE, adiabatic pump, feedwater heater

State 3: Saturated liquid at 0.4 MPa, so $h_3 = h_f = 604.7$ kJ/kg (Table B.1.2)

First Law (Pump): $h_4 = h_3 + \dot{W}_P / \dot{m}_3 = 604.7 + 150 / 20 = 612.2$ kJ/kg

State 5: $P_5 = 6$ MPa, $T_5 = 520$ °C, so $h_5 = 3{,}469.5$ kJ/kg (interpolation; Table B.1.3)

First Law (Boiler): $\dot{Q}_B = \dot{m}_5(h_5 - h_4) = 20\ (3{,}469.5 - 612.2) = 57.15$ MW (heat input)

2. (b) Define $y = \dot{m}_6 / \dot{m}_3$.

State 2: $P_2 = 0.4$ MPa, $T_2 = 120$ °C (subcooled), so $h_2 = h_f(120) = 503.7$ kJ/kg (Table B.1.1)

State 6: Saturated vapor at 0.4 MPa, so $h_6 = h_g = 2{,}738.5$ kJ/kg (Table B.1.2)

Cons. Mass (Feedwater Heater): $\dot{m}_6 + \dot{m}_2 = \dot{m}_3 \quad (\div \dot{m}_3) \quad \dot{m}_2 / \dot{m}_3 = 1 - y$

First Law (Feedwater Heater): $\dot{m}_6 h_6 + \dot{m}_2 h_2 = \dot{m}_3 h_3 \quad (\div \dot{m}_3) \quad y h_6 + (1-y) h_2 = h_3$

Solving, $y = (h_3 - h_2)/(h_6 - h_2) = (604.7 - 503.7)/(2738.5 - 503.7) = 0.045$.

Thus, $\dot{m}_6 = y \dot{m}_3 = 0.045(20) = 0.9$ kg/s.

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**Problem 3:** [redacted]

(a) $\beta_{\text{rev}} = \dfrac{T_{L2}}{T_{H2} - T_{L2}} = \dfrac{253}{20-(-20)} = 6.325$

$\beta = 0.6\,\beta_{\text{rev}} = 0.6 \times 6.325 = 3.795$

$\beta = \dfrac{Q_{L2}}{W_2}$

$W_2 = \dfrac{500}{3.795} = 131.75 \text{ kJ}$

$W_1 = W_2 + W_{\text{net}}$

$W_1 = 131.75 + 600 = 731.75 \text{ kJ}$

$\eta_{th} = \dfrac{W_1}{Q_{H1}} = \dfrac{731.75}{1000} = 0.73175$

$\eta_{\text{th,rev}} = 1 - \dfrac{T_{L1}}{T_{H1}} = 1 - \dfrac{323}{873} = 0.63$

Since $\eta_{\text{th}} > \eta_{\text{th,rev}}$, the engine is impossible.

(b) The maximum possible $W_{\text{net}}$ would be achieved when both the heat engine and the refrigerator are reversible. In this case,

$W_{2,\min} = \dfrac{Q_{L2}}{\beta_{\text{rev}}} = \dfrac{500}{6.325} = 79.05 \text{ kJ}$

$W_{1,\max} = \eta_{\text{th,rev}} \times Q_{H1} = 0.63 \times 1000 = 630 \text{ kJ}$

$W_{\text{net,max}} = W_{1,\max} - W_{2,\min} = 630 - 79.05 = 550.95 \text{ kJ}$

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**Problem 4:** [redacted]

(a) $P_{g1} = 1.2276$ (at 10 °C)

$$P_{v1} = \phi_1 P_{g1} = 0.7 \times 1.2276 = 0.8593 \text{ kPa}$$

$$P_{a1} = P_1 - P_{v1} = 95 - 0.8593 = 94.14 \text{ kPa}$$

$$v_1 = \frac{R_a T_1}{P_{a1}} = \frac{0.287 \times 283}{94.14} = 0.8628 \text{ m}^3\text{/kg dry air}$$

$$\dot{m}_a = \dot{V}_1 / v_1 = \frac{36}{0.8628 \times 60} = 0.6954 \text{ kg dry air/second}$$

(b) $\omega_1 = \dfrac{0.622\, P_{v1}}{P_{a1}} = \dfrac{0.622 \times 0.8593}{94.14} = 0.005678 \ \dfrac{\text{kg water}}{\text{kg dry air}}$

$h_1 = c_{po}\, T_1 + \omega_1\, h_{g1} = 1.004 \times 10 + 0.005678 \times 2519.74 = 24.347$ kJ/kg dry air

$\omega_2 = \omega_1 = 0.005678 \ \dfrac{\text{kg water}}{\text{kg dry air}}$

$h_2 = c_{po}\, T_2 + \omega_2\, h_{g2} = 1.004 \times 20 + 0.005678 \times 2538.06 = 34.491$ kJ/kg dry air

Control volume around the heating section

$\dot{Q}_{cv} - \dot{W}_{cv} = \dot{m}_a (h_2 - h_1)$

$\dot{Q}_{cv} = 0.6954\ (34.491 - 24.347) = 7.054$ kW

(c) Control volume around the humidifier

$\dot{Q}_{cv} - \dot{W}_{cv} = \dot{m}_a\, h_3 - (\dot{m}_a\, h_2 + \dot{m}_s\, h_s) = 0$

$\dot{m}_s = \dot{m}_a\, (\omega_3 - \omega_2)$

$h_3 = h_2 + (\omega_3 - \omega_2)\, h_s$

$1.004 \times 21 + \omega_3 \times 2539.88 = 34.491 + (\omega_3 - 0.005678) \times 2676.05$

$\omega_3 = 0.01313$ kg water/kg dry air

$\dot{m}_s = 0.6954 \times (0.01313 - 0.005678) \times 3600 = 18.66$ kg/h

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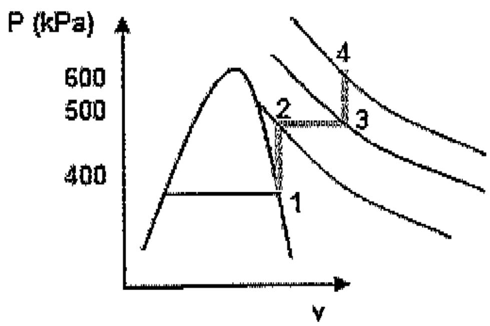
| Problem 5 | state 1 | state 2 | state 3 | state 4 |
|---|---|---|---|---|
| Pressure (kPa) | 400 | 500 | 500 | 600 |
| Temperature C | 143.63 | | | 1200 |
| v ($m^3$/kg) | 0.46246 | 0.46246 | 1.11302 | 1.11302 |
| m (kg) | 2 | 2 | 2 | 2 |
| u (kJ/kg) | 2553.55 | | | 4466.52 |

| | | | |
|---|---|---|---|
| l1 | 0.5 | | |
| l2 | 0.703369 | factor | 2.40673788 |

a) close system, no PE, no KE, contsant v, contant p, constant v

b)




c) $W_{1\text{-}4} = Wb_{2\text{-}3} =$ 650.56 kJ

d) $U_{1\text{-}4} = m(u_4 - u_1) =$ 3825.94 kJ

$Q = W_{1\text{-}4} + U_{1\text{-}4} =$ 4476.5 kJ

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| Problem 6 | Well 1 | Well 2 | Cold water |
|---|---|---|---|
| Flow rate (kg/s) | 5 | 10 | find |
| Temperature inlet C | 120 | 100 | 10 |
| Temperature outlet C | 40 | 40 | 80 |
| Cp (kJ/kg K) | 5.853 | 5.853 | 4.18 |

a) No Pe, No Ke, Steady-state, 3 inlets, 3 outlets, no mixing, constant Cp

b) First law control volume

$m_{w1}(h_1 - h_2) + m_{w2}(h_3 - h_4) = m_{cw}(h_6 - h_5)$ [W]

$m_{w1}\,Cp_b(T_1 - T_2) + m_{w2}\,Cp_b(T_3 - T_4) = m_{cw}\,Cp_w(T_6 - T_5)$ [W]

$2341.2 + 3511.8 = m_{cw}\,292.6$

$m_{cw} =$ **20.00 kg/s**

c)

Well 1 $T_1 =$ 100 C

$m_{w1}\,Cp_b(T_1 - T_2) + m_{w2}\,Cp_b(T_3 - T_4) = m_{cw}\,Cp_w(T_6 - T_5)$ [W]

$1755.9 + 3511.8 = 83.61\,(T_6 - T_5)$

$T_6 =$ 73.00 C