

MATH 2130 Problem Workshop 11

1. A triangular plate has sides with lengths 3, 4 and 5 metres. It is submerged vertically in oil with density 950 kilograms per cubic metre. The side of length 3 metres is vertical, the side of length 4 is horizontal and the uppermost vertex is 1 metre below the surface of the oil. Find the force due to oil pressure on each side of the plate.
2. An elliptic plate has major axis of length $2a$ metres and minor axis of length $2b$. Its major axis is horizontal and its minor axis is vertical. It is slowly being lowered into a tank of water. At the instant when only $b/2$ metres of the plate sticks out of the water, set up, but do not integrate a double iterated integral for the force due to water pressure on each side of the plate.
3. A thin plate with constant mass per unit area ρ has edges defined by the curves

$$x = \sqrt{a^2 - y^2}, \quad y = x, \quad y = 0,$$

where $a > 0$ is a constant. (a) Find the mass the plate, (b) Find the first moment of the plate about the x -axis, (c) Find \bar{y} .

4. A triangular plate has sides of length 2, 3 and 3. and constant mass per unit area ρ . Find its moment of inertia about the shorter side.
5. Find the area of the part of the surface $z = xy$ inside the cylinder $x^2 + y^2 = a^2$ where $a > 0$ is a constant.
6. Set up, but do not evaluate, a double iterated integral for the surface area of the ellipsoid

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1.$$

7. Set up but do not evaluate a double iterated integral for the area of the surface $z = 2x^2 + y^2$ bounded by $y = 0$, $x = 0$ and $x + y = 1$.

Answers:

1. $1.68 \times 10^5 \text{ N}$

2. $\int_{-b}^{b/2} \int_{-(a/b)\sqrt{b^2-y^2}}^{(a/b)\sqrt{b^2-y^2}} 9810 \left(\frac{b}{2} - y \right) dx dy \text{ N}$

3. (a) $\frac{\pi a^2 \rho}{8}$ (b) $\frac{\rho a^3 (\sqrt{2} - 1)}{3\sqrt{2}}$, (c) $\frac{8a(\sqrt{2} - 1)}{3\sqrt{2}\pi}$

4. $\frac{8\sqrt{2}\rho}{3}$

5. $\frac{2\pi[(1+a^2)^{3/2} - 1]}{3}$

6. $8 \int_0^a \int_0^{(b/a)\sqrt{a^2-x^2}} \sqrt{1 + \left(\frac{-cx}{a^2\sqrt{1-x^2/a^2-y^2/b^2}} \right)^2 + \left(\frac{-cy}{b^2\sqrt{1-x^2/a^2-y^2/b^2}} \right)^2} dy dx.$

7. $\int_0^1 \int_0^{1-x} \sqrt{1+16x^2+4y^2} dy dx.$