F07/1

UNIVERSITY OF MANITOBA

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PAPER # 197 TITLE PAGE

EXAMINATION: Engineering Mathematical Analysis 2 TIME: 3 hours

COURSE: MATH 2132 EXAMINER: G.I. Moghaddam

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[7] 1. Find the radius of convergence and the open interval of convergence for the series

$$\sum_{n=0}^{\infty} \frac{(-1)^n \sqrt{(2n)!}}{2^n\, n!}(x-3)^{4n}.$$

$\rightarrowtail R=1, \quad 2<x<4$

[10] 2. Let $f(x) = \dfrac{4-4x}{4x^2-8x-5}$; given the partial decomposition

$$\frac{4-4x}{4x^2-8x-5} = \frac{1}{5-2x} - \frac{1}{1+2x},$$

$\rightarrow f(x) = \sum_{n=0}^{\infty} \left(\frac{2}{3}\right)^{2n+2}(x-1)^{2n+1}$

find the Taylor series of $f(x)$ about 1. Express your answer in sigma notation and simplify as much as possible. Determine the open interval of convergence.

$-\frac{1}{2} < x < \frac{5}{2}$

[7] 3. Find the value of x for which the ***fourth*** term of the binomial expansion of

$f(x) = \dfrac{1}{(1+\frac{1}{8}x)^3}$ is equal to $\dfrac{-5}{32}$. Show your work.

$\rightarrowtail x=2$

[10] 4. (a) Evaluate the following integral using infinite series

$$\int_2^3 \frac{1}{1+(x-2)^5}\,dx.$$

$\rightarrowtail$ a) $\sum_{n=0}^{\infty} (-1)^n \frac{1}{5n+1}$

Express your answer in sigma notation.

b) $\leftarrow$ Max-error $\le \frac{1}{26}$

(b) If you truncate the series in part (a) after **fifth** term, what is a maximum possible error? Explain why you can claim that your answer is a maximum error.

$a_n = \frac{1}{5n+1} \Longrightarrow$
i) series is alternatin
ii) $a_{n+1} < a_n$
iii) $\lim_{n\to\infty} a_n = 0$

[9] 5. Find, in explicit form, a general solution for the differential equation

$$\frac{dy}{dx} + \frac{y}{x\ln x} = x^5.$$

$\rightarrowtail y(x) = \frac{x^6}{6}\left(1 - \frac{1}{6\ln x}\right) + \frac{c}{\ln x}$

[10] 6. When two substances A and B are brought together at time t, they react to form a third substance C in such a way that 2 gram of A reacts with 3 grams of B to produce 5 grams of C. The rate at which C is formed is proportional to the product of the amounts of A and B still present in the mixture. The initial amounts of A and B are 40 and 60 grams, respectively.

(a) Show that the initial-value problem for the amount $x(t)$ of C at any given time t is :

$$\frac{dx}{dt} = k(100 - x)^2, \quad x(0) = 0$$

where $k > 0$ is a constant.

(b) Solve the differential equation for $x(t)$.

$\rightarrowtail \quad x(t) = 100\left(1 - \frac{1}{100kt-1}\right)$

(c) What is your prediction in long run, i.e. $\lim_{t\to+\infty} x(t)$?

$\rightarrowtail$ 100 grams.

[9] 7. Find a 2-parameter family of solutions of differential equation

$$y'' - y' = (y')^2.$$

(Hint: $\frac{1}{v(v+1)} = \frac{1}{v} - \frac{1}{v+1}$)

$\rightarrowtail \quad y(x) = E - \ln|1 - De^{x}|$

[8] 8. Given that $m^3(m-2)(m^4-1) = 0$ is the auxiliary equation associated with the linear differential equation

$$y^{(8)} - 2y^{(7)} - y^{(4)} + 2y^{(3)} = x^2e^{2x} + x\sin x + 1,$$

what is the form of a particular solution $y_p(x)$?

DO NOT EVALUATE THE COEFFICIENTS IN $y_p(x)$.

$\rightarrowtail \quad y_p(x) = (Ax^3 + Bx^2 + C)e^{2x} + (Dx^2 + Ex)\cos x + (Fx^2 + Gx)\sin x + Hx^3$

[10] 9. Find Laplace transform of the function

$$f(t) = \begin{cases} e^t & \text{if } t < 2 \\ t & \text{if } 2 < t < 3 \\ (t-3)^4 & \text{if } t > 3\,. \end{cases}$$

$\rightarrowtail \quad F(s) = \frac{1}{s-1} + e^{-2s}\left[\frac{1}{s^2} + \frac{2}{s} - \frac{e^2}{s-1}\right] + e^{-3s}\left[\frac{24}{s^5} - \frac{1}{s^2} - \frac{3}{s}\right]$

[10] 10. Find $\mathscr{L}^{-1}\left\{\frac{4s^2 + 19s + 25}{(s+1)(s^2 + 6s + 10)}\right\}$.

$\rightarrowtail \quad 2e^{-t} + e^{-3t}(2\cos t - \sin t)$

[10] 11. Use Laplace transforms to solve the initial-value problem

$$\frac{d^2y}{dt^2} + y = \delta(t - 2\pi) + \delta(t - 4\pi), \quad y(0) = 1, \quad y'(0) = 0.$$

$\rightarrowtail \quad y(t) = \sin t\,[h(t-2\pi) + h(t-4\pi)] + \cos t$