

Higher order derivatives

3.5.1

$$(a) \quad g'(x) = \frac{1 \cdot (x+1) - x}{(x+1)^2} = \frac{1}{(x+1)^2}$$

$$g''(x) = -2(x+1)^{-3}$$

$$g''(1) = -2 \cdot 2^{-3} = -2^{-2} = -\frac{1}{4}$$

$$(b) \quad y''' = ? \quad y' = -\frac{9}{5} x^{-\frac{14}{5}} \quad y'' = \frac{9 \cdot 14}{5^2} x^{-\frac{19}{5}}$$
$$y''' = -\frac{9 \cdot 14 \cdot 19}{5^3} x^{-\frac{24}{5}}$$

(c) $f(x)$ is a polynomial of degree 2016 with the coefficient in front of x^{2016} equal to $(-1)^{2016}=1$. Differentiation of a polynomial reduces its degree by one. In particular, the derivative of order 2016 of a polynomial of degree 2015 is zero. Moreover, the derivative of the order 2016 of $f(x)$ is equal to the derivative of the order 2016 of the only term of degree 2016, which is x^{2016} (all other terms have smaller degree and will lead to zero derivative of order 2016). Now $(x^{2016})' = 2016x^{2015}$, $(x^{2016})'' = 2016 \cdot 2015 \cdot x^{2014}$, ..., $(x^{2016})^{(2015)} = 2016 \cdot 2015 \cdot \dots \cdot 2x$, and $(x^{2016})^{(2016)} = 2016!$.