Term Test 2

DATE: March 10, 2011
COURSE: MATH 2130

TIME: 70 minutes
EXAMINER: G.I. Moghaddam

Answers

[10] 1. Evaluate each of these limits or explain why it does not exist.

(a) $\displaystyle\lim_{(x,y)\to(1,-1)} \frac{(x^2-y)(1+y)}{x^4-y^2}$

→ 1 a) limit does not exist

(b) $\displaystyle\lim_{(x,y)\to(0,-1)} \frac{\sin(\sqrt{x^2+y^2}-1)}{x^4+y^4+2x^2y^2-1}$

b) 1/4

[10] 2. Let $u(x,y) = f(x^2+y) + g(x^2+y)$ where f and g are twice differentiable functions. Show that

$$\frac{\partial^2 u}{\partial x^2} - 4x^2\frac{\partial^2 u}{\partial y^2} - 2\frac{\partial u}{\partial y} = 0$$

→ 2. Hint: let $v = x^2+y$ to write $u = f(v)+g(v)$. then use the tree-diagram

$$\begin{tikzcd} & u \arrow[d, dash] & \\ & v \arrow[dl, dash] \arrow[dr, dash] & \\ x & & y \end{tikzcd}$$

[9] 3. Find $\dfrac{dz}{dx}$ if

$$xy + xz + yz = 1 \quad , \quad 2xy - 2y^2 - \frac{1}{2}x^2 = 0 .$$

Simplify your answer.

→ 3. $-\dfrac{(x+2y+3z)}{2(x+y)}$

[8] 4. Let $f(x,y,z) = x^2+y+z$ and $g(x,y,z) = xz + \frac{1}{4}z^2$. Find the directional derivative of $f+g$, at the origin, along the curve

$$C \; : \quad x = t^2, \quad y = 2t, \quad z = -2t .$$

→ 4. 0

[13] 5. Let S_1 be the surface $z = x^3 - y^2$ and also let S_2 be the surface $x+y = xz$.

(a) Find the equation of the tangent plane to the surface S_1 at the point $(2,2,4)$.

→ 5 a) $12x - 4y - z = 12$

(b) Find a tangent vector for the curve of intersection of the two surfaces S_1 and S_2 at the point $(1,-1,0)$.

b) $-1\hat{i} + 2\hat{j} + \hat{k}$

(c) Find the point(s) on the surface S_2 at which the tangent plane is parallel to the plane $2x+y-3z=0$.

c) $(3\lambda, -6\lambda^2, 1-2\lambda)$
λ is arbitrary Constant, $\lambda \neq 0$