Math 1210: Review by Dawit Yohannes

* lines and planes

→ line is defined by a point on the line, P_0, and a vector, $\vec{v}$, parallel to the line. $P_0(x_0, y_0, z_0)$, $\vec{v} = \langle a, b, c \rangle$, t = parameter

$$l: \begin{cases} x = x_0 + at \\ y = y_0 + bt \\ z = z_0 + ct \end{cases}$$ parametric equation

$$l: \left\{ \frac{x - x_0}{a} = \frac{y - y_0}{b} = \frac{z - z_0}{c} \right.$$ Symmetric equation

$$l: \left\{ \langle x, y, z \rangle = \langle x_0, y_0, z_0 \rangle + t \langle a, b, c \rangle \right.$$ Vector equation

→ plane is defined by a point (P_0) on the plane and a Normal Vector ($\vec{N}$) to the plane $P_0(x_0, y_0, z_0)$, $\vec{N} = \langle A, B, C \rangle$

$$A(x - x_0) + B(y - y_0) + C(z - z_0) = 0 \quad \text{or} \quad Ax + By + Cz + D = 0$$

→ Two lines are parallel: $\vec{V}_1 = k\vec{V}_2$ ($k \in \mathbb{R}$)

" " " perpendicular: $\vec{V}_1 \cdot \vec{V}_2 = 0$

→ Two planes are parallel: $\vec{N}_1 = k\vec{N}_2$

" " " perpendicular: $\vec{N}_1 \cdot \vec{N}_2 = 0$

→ line and plane:
- $\vec{V} = k\vec{N}$ → perpendicular
- $\vec{V} \cdot \vec{N} = 0$ → parallel

* Acute angles

- line and plane: $\theta = \cos^{-1}\left[\frac{|\vec{N} \cdot \vec{V}|}{\|\vec{N}\| \|\vec{V}\|}\right]$
- line and line: $\theta = \cos^{-1}\left[\frac{|\vec{V}_1 \cdot \vec{V}_2|}{\|\vec{V}_1\| \|\vec{V}_2\|}\right]$

Square matrix

* Eigen Values (EVL) and Eigen Vectors (EVC) of a matrix A.

: The product of the EVL's of A is equal to the det(A)

: The Sum of the EVL's of A is equal to the Trace of A   sum of the principal diagonal elements. $a_{11}+a_{22}+\cdots$

: If one of the EVL's is zero, then $\det(A)=0$

: If λ is an EVL of A then
- λ^{-1} is EVL of A^{-1}
- λ^{n} is EVL of A^{n}
- $k\lambda$ is EVL of (kA)

* Determinants: let $A_{n\times n}$, $B_{n\times n}$

→ : $\det A^{T}=\det A$, $\det A^{-1}=\frac{1}{\det A}$, $\det(kA)=\overset{k\in\mathbb{R}}{k}{}^{n}\det A$

→ $\det(AB)=(\det A)(\det B)$

* Systems of equations (SE): $Ax=b$ (with n unknowns)

let the rank of A be r_1, and the rank of the augmented Matrix be r_2

→
- $r_1\neq r_2 \Rightarrow$ SE is inconsistent (no solution)
- $r_1=r_2 \Rightarrow$ SE is Consistent (has Solution)
  - $r_1=r_2=n \Rightarrow$ unique Solution
  - $r_1=r_2<n \Rightarrow$ infinite Solutions.

* Linear dependence (LD) and independence (LI)

let m = number of Vectors n = number of Components

if
- $m > n$ => linear dependence
- $m = n$, => $\det(\ ) \begin{cases} = 0 \rightarrow \text{linear dependence} \\ \neq 0 \rightarrow \text{linear independence} \end{cases}$
- $m < n$ => use RREF to solve a homogeneous System of equations With Vectors forming Columns. $\longrightarrow$ $\begin{cases} \text{trivial solution} \Rightarrow LI \\ \text{infinite solutions} \Rightarrow LD \end{cases}$

Complex numbers: let $z = a + bi$, $r = \sqrt{a^2+b^2}$, $\theta = \tan^{-1}\left(\frac{b}{a}\right)$ (depends on quadrant)

$$\succ\ :\ z^{1/n} \Rightarrow z_k = r^{1/n} e^{i\left(\frac{\theta + 2k\pi}{n}\right)}, \quad k = 0, 1, 2, \ldots, (n-1)$$

$$= r^{1/n}\left(\cos\left[\frac{\theta+2k\pi}{n}\right] + i\sin\left[\frac{\theta+2k\pi}{n}\right]\right)$$

$$\succ\ :\ z^n = r^n(\cos n\theta + i\sin n\theta)$$

* Summation Notation

$$\rightarrow \sum_{i=k}^{n}(\quad) = \sum_{i=1}^{n}(\quad) - \sum_{i=1}^{k-1}(\quad);$$

Example
$$\sum_{i=5}^{10}(i+1) = \sum_{i=1}^{10}(i+1) - \sum_{i=1}^{4}(i+1)$$

$$\rightarrow \sum_{i=k}^{n} f(i) = \sum_{i=1} f(i+(k-1));$$

Example
$$\sum_{i=5}^{10}(i+1) = \sum_{i=1}^{6}(i+4+1) = \sum_{i=1}^{6}(i+5)$$