UNIVERSITY OF MANITOBA

DATE: April 11, 2011 | FINAL EXAMINATION
COURSE: MATH 1210 | TITLE PAGE
EXAMINATION: Techniques of Classical & Linear Algebra | TIME: 2 hours

FAMILY NAME: (Print in ink, capitals) ______________________

GIVEN NAME(S): (Print in ink, capitals) ______________________

STUDENT NUMBER: ____________

SEAT NUMBER: ____________

SIGNATURE: (in ink) ______________________
(I understand that cheating is a serious offence. I have read the instructions below twice.)

❑ A01 R. Thomas

❑ A02 T. Mohammed

INSTRUCTIONS TO STUDENTS:

This is a 2 hour exam. **Please show your work clearly.**

No calculators, texts, notes, or other aids are permitted. No cellphones or electronic translators, or other electronic devices able to receive or transmit a signal are permitted.

This exam has a title page and 7 pages of questions. Please check that you have all the pages.

The value of each question is indicated in the lefthand margin beside the statement of the question. The total value of all questions is 80 points.

Answer all questions on the exam paper in the space provided beneath the question. If you need more room, you may continue your work on the reverse side of the previous page, but CLEARLY INDICATE that your work is continued.

Question	Points	Score
1	5	
2	7	
3	6	
4	5	
5	5	
6	7	
7	5	
8	5	
9	4	
10	8	
11	7	
12	6	
13	10	
Total:	80	

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[5] 1. Given matrices Y and S, let matrices C, Z, and R be defined as follows:

$$C = Y^\top Y, \quad Z = YS, \quad R = Z^\top Z.$$

Show that $R = S^\top CS$.

[7] 2. Prove by mathematical induction (no other method) that $\sum_{j=1}^{n}(6j-1) = n(3n+2)$ for all $n = 1, 2, \ldots$.

[6] 3. Given a mapping $T : \mathbb{R}^3 \to \mathbb{R}^3$ defined by $T(x, y, z) = (x + 3y - 4z, -2x + y, 6y + 2z)$,

(a) find the value of $T(1, 2, 3)$, and

(b) find a matrix representation for T using the standard basis $\hat{\mathbf{i}}$, $\hat{\mathbf{j}}$, $\hat{\mathbf{k}}$ for $\mathbb{R}^3$.

[5] 4. Find all solutions of the system

$$\begin{aligned} 2x_1 + 5x_2 - x_3 &= 14, \\ x_1 + 2x_2 + x_3 &= 5, \\ x_1 + 3x_2 - 2x_3 &= 9. \end{aligned}$$

[5] 5. Has the equation $AX = B$ any solutions when $A = \begin{bmatrix} 1 & 2 \\ 3 & 3 \\ 2 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} -1 \\ -6 \\ -5 \end{bmatrix}$? If so, find them all.

[7] 6. Are the vectors $\{(1, 4, -2, 6), (2, 1, 3, -4), (-4, 5, -13, 24)\}$ linearly dependent or independent? Justify your answer, stating the linear relationship if one exists.

[5] 7. If $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 4 \\ 1 & 2 & 4 \end{bmatrix}$ and $(AB)^{-1} = \begin{bmatrix} 3 & 3 & 0 \\ 0 & 4 & 2 \\ -1 & 0 & 1 \end{bmatrix}$, then what is B^{-1}?

[5] 8. Find the inverse of the matrix $\begin{bmatrix} 1 & a & b \\ 0 & 1 & c \\ 0 & 0 & 1 \end{bmatrix}$, where a, b, and c are any real numbers.

[4] 9. Let A be any $n \times n$ invertible matrix. Show that $\det(\text{adj}A) = [\det(A)]^{n-1}$ by using the fact that $\text{adj}A = [\det(A)]A^{-1}$ or otherwise.

[8] 10. Solve the equation $ix^3 - \sqrt{2}(1+i) = 0$, giving any complex solutions in exponential form.

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[7] 11. Let L be the line in space through the two points $(5, 2, 5)$ and $(-1, 4, 3)$.

(a) Find parametric equations for L.

(b) Find an equation for the plane P perpendicular to L and passing through the point $(0, 6, 2)$.

[6] 12. Find the eigenvalues only of the matrix $\begin{bmatrix} 3 & -2 & -2 \\ -2 & 3 & 2 \\ 3 & -3 & -2 \end{bmatrix}$.

[10] 13. Verify that 1 and -3 are eigenvalues of the matrix $\begin{bmatrix} 5 & 8 & 16 \\ 4 & 1 & 8 \\ -4 & -4 & -11 \end{bmatrix}$ and systematically determine the eigenvectors corresponding to each eigenvalue.