Test 1

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COURSE: MATH 2132

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[6] 1. The following sequence of functions is defined on the interval $-1 \le x \le 1$.

$$\left\{\frac{1}{n} + \frac{n^4x^2 + 2nx^4}{n^4x + x^4 + 1}\right\}_{n=1}^{\infty}$$

Determine whether the sequence is convergent or divergent. If it converges, find the limit function.

1. x

[15] 2. Let $f(x) = \ln x$ for $0 < x < 4$ Then:

[5] (a) Find the first 4 terms of the Taylor series of $f(x)$ about 2.

[5] (b) Find the n^{th}-remainder (i.e. $R_n(2,x)$).

[5] (c) Show that $\lim\limits_{n\to\infty} R_n(2,x) = 0$ *only* for the case $2 < x < 4$.

2 a) $\ln 2 + \frac{1}{2}(x-2) - \frac{1}{8}(x-2)^2 + \frac{1}{24}(x-2)^2$

b) $\dfrac{(-1)^n (x-2)^{n+1}}{z_n^{n+1}(n+1)}, \quad 2 < z_n < x < 4$

c) $\lim\limits_{n\to\infty} |R_n(2,x)| \le \lim\limits_{n\to\infty} \dfrac{\left|\frac{x-2}{z_n}\right|^{n+1}}{n+1}$

$\le \lim\limits_{n\to\infty} \dfrac{1}{n+1} = 0$

[8] 3. Find the radius of convergence and the open interval of convergence for the series

$$\sum_{n=0}^{\infty} \frac{(-1)^n (n!)^2 4^n}{(2n+1)!}(x-7)^{4n}$$

3. $1, \quad 6 < x < 8$

[6] 4. Find the sum of the series: $-\frac{\sqrt{2}}{3}x^3 + \frac{2}{9}x^6 - \frac{2\sqrt{2}}{27}x^9 + \cdots + \frac{(-1)^n 2^{\frac{n}{2}}}{3^n}x^{3n} + \cdots$

4. $-\dfrac{\sqrt{2}\,x^3}{3+\sqrt{2}\,x^3}; \quad -\sqrt[3]{\frac{3}{\sqrt{2}}} < x < \sqrt[3]{\frac{3}{\sqrt{2}}}$

[15] 5. Find the Taylor series about 1 for the function

$$f(x) = \frac{1}{x^2} + \ln x$$

Express your answer in sigma notation and simplify as much as possible. Determine its open interval of convergence.

5. $1 + \sum\limits_{n=1}^{\infty} (-1)^n \left(\dfrac{n^2+n-1}{n}\right)(x-1)^n$

$, \quad 0 < x < 2$