Continuity

2.4.1

point	cont.	right-cont.	left-cont.	discont.	type of discont.
$x=-2$	NO	NO	YES	YES	JUMP
$x=1$	NO	NO	NO	YES	REMOVABLE
$x=4$	NO	YES	NO	YES	INFINITE

2.4.2

(a)

IV.(a) $f(0) = -0^2+1 = 1$

$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} \cos x = \left(\begin{array}{c}\text{as } \cos x \\ \text{is continuous} \\ \text{we can substitute}\end{array}\right) = \cos 0 = 1$

$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} (-x^2+1) = -0^2+1 = 1$

As all three numbers are defined and equal, we get:

Answer: YES, f is continuous at 0.

(b) For any k both $kx+2$ for $x>-1$ and k^2x^2 for $x<-1$ are continuous as polynomials. So, $f(x)$ is continuous for all x if and only if it is continuous at $x=-1$.

$f(-1) = -k+2$ $\lim_{x\to -1^+} f(x) = \lim_{x\to -1^+} (kx+2) = -k+2$

$\lim_{x\to -1^-} f(x) = \lim_{x\to -1^-} k^2x^2 = k^2$

k^2 must be equal to $-k+2$

$k^2 = -k+2$ $k^2+k-2=0$ $k=1$ or $k=-2$

Answer: $k=1$, $k=-2$.