Math 2130 Summer 2012 Test 2 (by N Harland)

1. Given that
$$xyu + v = 2 \text{ and } y^2 + u^2 - u^2v = y + 43$$
define u and v as functions of x and y, find $\left.\frac{\partial u}{\partial y}\right)_x$ when $x = 1, y = 2, u = 3$ and $v = -4$. [5]

2. Find the equation of the tangent line (in either parametric, vector or symmetric form) to the curve of intersection of
$$yz + \sin(xyz) = -4 \text{ and } x^2 + y^2 + z^2 = 8$$
at the point $(0, 2, -2)$. [7]

3. For the function $f(x, y) = x^2y + xy^2 + 3y$

   (a) Find the critical point(s) of f. [4]

   (b) Classify the critical point(s) found in (a) as either relative minimum, relative maximum, saddle point, or neither. [5]

   (c) Find the absolute maximum and minimum of f on the region bounded by $y = x^2$ and $y = 4$. [7]

4. Find
$$\iint_R (1 - x)dA$$
where R is the region bounded by the lines $x+y = 1, x+y = -1, x-y = 1, x-y = -1$. [8]

5. The one-dimensional wave equation is
$$\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}$$
where c is any positive constant. Let $f(u)$ and $g(v)$ be twice differentiable functions. Show that
$$y(x, t) = f(x + ct) + g(x - ct)$$
satisfies the wave equation. [4]