MATH 1210 A01 Summer 2013 Problem Workshop 13

1. If $\mathbf{v} = T\mathbf{u}$ is a linear transformation from $\mathbb{R}^3$ to $\mathbb{R}^3$ defined by

$$\begin{aligned} v_1 &= 3u_1 - 2u_2 \\ v_2 &= 4u_1 + 3u_2 + u_3 \\ v_3 &= -u_1 + 2u_2 + 3u_3 : \end{aligned}$$

   (a) find $T\langle 2, -1, 3\rangle$.

   (b) find $\mathbf{u}$ if $\mathbf{v} = \langle 1, 1, -1\rangle$

   (c) find all vectors such that $T(\mathbf{v}) = 2\mathbf{v}$.

2. You are told that the characteristic equation for a matrix is

$$6\lambda^4 + 11\lambda^3 - 4\lambda^2 + 11\lambda - 10 = 0.$$

   What are the eigenvalues for the matrix?

3. What are the eigenvalues and eigenvectors for the identity matrix?

4. Find all eigenvalues and all corresponding eigenvectors for each of the following matrices.

   (a) $\begin{bmatrix} 5 & 4 & 2 \\ 4 & 5 & 2 \\ 2 & 2 & 2 \end{bmatrix}$

   (b) $\begin{bmatrix} 1 & 2 & -1 \\ 1 & 0 & 1 \\ 4 & -4 & 5 \end{bmatrix}$

5. Prove that if zero is an eigenvalue of a matrix, then the matrix cannot have an inverse.

<u>Answers</u>

1. (a) $\langle 8, 8, 5\rangle$

   (b) $\dfrac{1}{47}\langle 15, -1, -10\rangle$

   (c) $\langle 0, 0, 0\rangle$

2. $-2/5, 2/3, \pm i$

3. The only eigenvalue is one, and every vector is an eigenvector.

4. (a)

$$\lambda = 1, \quad \mathbf{v} = v_2 \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} + v_3 \begin{bmatrix} -1/2 \\ 0 \\ 1 \end{bmatrix}$$

$$\lambda = 10, \quad \mathbf{v} = v_3 \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix}$$

   Note that $\begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix}$ is perpendicular to $\begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} -1/2 \\ 0 \\ 1 \end{bmatrix}$ which happens since A is symmetric.

   (b)

$$\lambda = 1, \quad \mathbf{v} = v_3 \begin{bmatrix} -1/2 \\ 1/2 \\ 1 \end{bmatrix}$$

$$\lambda = 2, \quad \mathbf{v} = v_3 \begin{bmatrix} -1/2 \\ 1/4 \\ 1 \end{bmatrix}$$

$$\lambda = 3, \quad \mathbf{v} = v_3 \begin{bmatrix} -1/4 \\ 1/4 \\ 1 \end{bmatrix}$$

5.