MATH 2132 Tutorial 6

1. Use (a) differentiation and (b) binomial expansion to find the Taylor series about $x = 1$ for $1/(2x-3)^3$. What is the interval of convergence of the series?

2. Find the Taylor series for $(15-2x)^{3/2}$ about $x = -2$. Express your answer in sigma notation, simplified as much as possible. What is the open interval of convergence?

3. Use series to evaluate

$$\lim_{x\to 0} \frac{\sqrt{3+4x^3}-\sqrt{3}}{x^3}.$$

4. Find a four-decimal approximation for

$$\int_0^1 \cos(x^2/4)\, dx.$$

Verify that your answer has the required accuracy.

5. Find a polynomial that approximates the antiderivative

$$\int (1+x^4)^{-1/4} dx$$

on the interval $0 < x < 1/2$ with error less than 10^{-6}. Justify your answer.

6. Find a maximum possible error in using

$$\ln 7 + \frac{2}{7}(x-1) - \frac{2^2}{2\cdot 7^2}(x-1)^2 + \frac{2^3}{3\cdot 7^3}(x-1)^3$$

to approximate the function $\ln(2x+5)$ on the interval $-0.1 \le x \le 0$. Justify your conclusions.

Answers:

1. $\displaystyle\sum_{n=0}^{\infty} -2^{n-1}(n+2)(n+1)(x-1)^n,\ \frac{1}{2} < x < \frac{3}{2}$

2. $\displaystyle\sqrt{19}\left\{19 - 3(x+2) + \sum_{n=2}^{\infty} \frac{3(2n-4)!}{2^{n-2}n!(n-2)!19^{n-1}}(x+2)^n\right\},\ -\frac{23}{2} < x < \frac{15}{2}$

3. $2/\sqrt{3}$

4. 0.9938

5. $\displaystyle x - \frac{x^5}{20} + \frac{5x^9}{288}$

6. 0.011