EXAMINATION: Sequences and Series EXAMINER: Harland

[6] 1. *Find the limit of the sequence*

$$\left\{ n \sin\left(\frac{4}{n}\right) + \frac{n^2+3}{3n-4n^2} \right\}_{n\geq 1}.$$

4. Determine whether the sequence $a_n = \dfrac{\sin(2n)+n}{1+n}$ converges or diverges. If it converges, find the limit.

1. Determine if the sequence $\{a_n\}$ converges of diverges. In the case it converges, find the limit.

   (a) $a_n = \dfrac{1+3^n}{5^{n+1}}$

   (b) $a_n = \sqrt{n+1} - \sqrt{n}$

   (c) $a_n = \dfrac{n!}{4^n}$

2. Determine if the series converges or diverges. If it converges, fin the sum.

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{(-3)^{n+1}}{2(5^{n-1})}$

   (b) $\displaystyle\sum_{n=1}^{\infty} \frac{2^n}{n^2}$

3. *Determine, with justification, whether the following series converge or diverge. If the series converges, find its sum.*

[5] (a) $\displaystyle\sum_{n=1}^{\infty} \frac{(-3)^{n-1}}{4^n}$

[4] (b) $\displaystyle\sum_{n=1}^{\infty} \cos\left(\frac{1}{n^2}\right)$

4. *For the series* $\displaystyle\sum_{n=1}^{\infty} \frac{4}{n^2+4n+3}$:

[6] (a) *Derive a formula for* s_n. *(It is not necessary to prove the formula, but you must show the steps as to how it was created.)*

[2] (b) *Determine the sum of the series.*

1. *For the series:*

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^5}$$

[4] (a) *Show the series is convergent.*

[11] 3. *Determine the radius and interval of convergence for*

$$\sum_{n=1}^{\infty} \frac{1}{n3^n} x^n.$$

[6] 4. *Determine the radius and interval of convergence for*

$$\sum_{n=0}^{\infty} n!(x-3)^n.$$

[6] 5. *Determine a power series representation for* $f(x) = \ln(2-x)$ *centered at* 0.

[4] 7. *Suppose the series* $\sum_{n=0}^{\infty} c_n x^n$ *has a radius of convergence* R. *Determine the radius of convergence of* $\sum_{n=0}^{\infty} c_n x^{3n}$.

3. Represent $f(x) = \ln(5-x)$ as a power series in x and determine the radius of convergence.

4. Find the center, radius of convergence and interval of convergence of the power series:

   (a) $\sum_{n=0}^{\infty} n(x+5)^n$

   (b) $\sum_{n=0}^{\infty} \frac{1}{n+1}(2x-1)^n$

4. Find the radius and interval of convergence of the series

   (a) $\sum_{n=1}^{\infty} \frac{(-2)^n x^n}{\sqrt{n+1}}$

   (b) $\sum_{n=2}^{\infty} \frac{x^n}{(\ln n)^n}$

5. (a) Find a power series representation for $f(x) = \frac{x}{(1-x)^2}$, $-1 < x < 1$.

   (b) Use your answer on part (a) to evaluate $\sum_{n=1}^{\infty} \frac{n}{2^n}$

6. (a) Find the MacLaurin series representation for the function $f(x) = x^3 \cos 2x$ and find its associated radius convergence.

   (b) Find $f^{(}2009)(0)$. You do not need to simplify your answer.

3. Determine whether the series converges or diverges. If it converges, find the exact value of the sum

   (a) $\sum_{n=2}^{\infty} \frac{3}{n(n+1)}$

   (b) $\sum_{n=1}^{\infty} \left(\frac{3^n}{2^{2n}} + (-1)^n \frac{1}{4^n} \right)$