

Name: _____

Student Number: _____

Answer all questions and show all your work. No calculators allowed. (Total Marks: 24).
 You have 20 minutes to complete the quiz.

1. Express in the forms required, with all arguments in your answers reduced to numbers in the interval $(-\pi, \pi]$.

- [6] (a) $-6 + i\sqrt{108}$ in polar and exponential forms

$$\begin{aligned} |z| &= \sqrt{6^2 + 108} = \sqrt{144} = 12 \\ \cos \varphi &= -\frac{6}{12} = -\frac{1}{2} \quad \varphi = \frac{2\pi}{3} \\ z &= 12 e^{i\frac{2\pi}{3}} = 12 \left(\cos \frac{2\pi}{3} + i \sin \frac{2\pi}{3} \right) \end{aligned}$$

- [6] (b) $\sqrt{18} \left(\cos \frac{19\pi}{4} + i \sin \frac{19\pi}{4} \right)$ in Cartesian and exponential forms

$$\begin{aligned} &= \sqrt{18} e^{i\frac{19\pi}{4}} = \sqrt{18} = e^{i\left(\frac{3\pi}{4} + 4\pi\right)} = \sqrt{18} e^{\frac{3\pi}{4}i} = \\ &= \left(-\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i \right) \sqrt{18} = -\frac{6}{2} + \frac{6}{2}i = \\ &= -3 + 3i \end{aligned}$$

continued on back...

[12] 2. Use Mathematical Induction to prove that for all $n \geq 1$,

$$n + (n+1) + (n+2) + \dots + (2n) = \frac{3n(n+1)}{2}.$$

Let P_n be the statement for
 $n \geq 1$ $n + (n+1) + (n+2) + \dots + (2n) = \frac{3n(n+1)}{2}$

Assume that for $n=k \geq 1$, P_k is also true
that is

$$k + (k+1) + (k+2) + \dots + 2k = \frac{3k(k+1)}{2}$$

We'll prove for $n=k+1$, P_{k+1} is also true
that is

$$(k+1) + (k+2) + \dots + 2(k+1) = \frac{3(k+1)(k+2)}{2}$$