THE UNIVERSITY OF MANITOBA

6:00 p.m. 23 April 2010 FINAL EXAMINATION

SEAT NO.: 1 − 217

DEPARTMENT & COURSE NO.: ENG 1460 TIME: 3 HOURS

EXAMINATION: Introduction to Thermal Sciences EXAMINERS: Drs. D. Kuhn, B.C. Wang, and J. Bartley

Instructions:

(a) You are permitted to use the course textbook, your course notes and a calculator. Copies and solutions to any previous exams are not permitted for reference during this exam.

(b) Clear, systematic solutions are required. **Show all steps in presenting your work.** Marks will not be assigned for solutions that require unreasonable (in the opinion of the instructor) effort to decipher.

(c) Ask for clarification if any problem statement is not clear to you.

(d) Use linear interpolation between table entries as necessary. Use constant specific heat.

(e) Retain all the significant figures in the property values taken from the tables and indicate the units. Use 4 to 5 significant figures in your calculations. Final answers must have 3 to 5 significant figures and units.

(f) There are **five** questions on this exam. The weight of each problem is indicated. The exam will be marked out of **75**.

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value 8 **1.**

A new type of bio-fuel provides 45 000 [kJ/kg] of energy in the form of heat to a heat engine operating between two thermal reservoirs: T_H = 2500 [K], which corresponds to the fuel burning temperature, and a low temperature reservoir of T_L = 360 [K]. The heat engine operates with an actual thermal efficiency that is half that of a Carnot heat engine operating between the two temperatures T_H and T_L; i.e., $\eta = 0.5 \times \eta_{Carnot}$. The heat engine produces $\dot{W} = 14.98$ [kW] of power.

1.5 (a) Calculate the actual thermal efficiency of the heat engine, η.

3 (b) Determine the mass flow rate of fuel that is burned in one hour, in kg/h.

3.5 (c) If the fuel costs \$1.10 per Litre, and the fuel has a density of 750 [kg/m^3], calculate the cost of operating the heat engine per hour, in \$(*dollars*)/h.

Question 2 was removed.

value 15 **3.**

An arrangement consisting of an irreversible heat engine (HE) operating between two thermal reservoirs, $T_{\text{H,HE}} = 800$ [K] and $T_{\text{L,HE}} = 350$ [K], produces exactly enough power, $\dot{W}$, to drive (supply) a reversible (Carnot) heat pump (HP) operating between two thermal environments, $T_{\text{L,HP}} = 250$ [K] and $T_{\text{H,HP}} = 320$ [K]. The purpose of the heat pump is to deliver enough heat to maintain the high-temperature thermal environment at $T_{\text{H,HP}}$. The heat engine operates with a thermal efficiency that is 75% of that of a Carnot heat engine operating between the thermal reservoirs $T_{\text{H,HE}}$ and $T_{\text{L,HE}}$; that is, $\eta = 0.75 \times \eta_{\text{Carnot}}$. The heat engine rejects heat to the low temperature reservoir, $T_{\text{L,HE}}$, at the rate $\dot{Q}_{\text{L,HE}} = 8$ [kW]. The heat pump delivers heat to the high-temperature thermal environment, $T_{\text{H,HP}}$, at a rate that is directly proportional to the temperature difference as $\dot{Q}_{\text{H,HP}} = K\left(T_{\text{H,HP}} - T_{\text{L,HP}}\right)$.

5 (a) Calculate the thermal efficiency of the heat engine, η, and the power produced by the heat engine, $\dot{W}$.

5 (b) Determine the value of the proportionality constant, K, for $\dot{Q}_{\text{H,HP}}$.

5 (c) Considering only the heat pump, and if the value of $\dot{W}$ is increased to $\dot{W} = 7.5$ [kW], determine the minimum value of $T_{\text{L,HP}}$ such that the heat pump is still able to maintain $T_{\text{H,HP}} = 320$ [K].

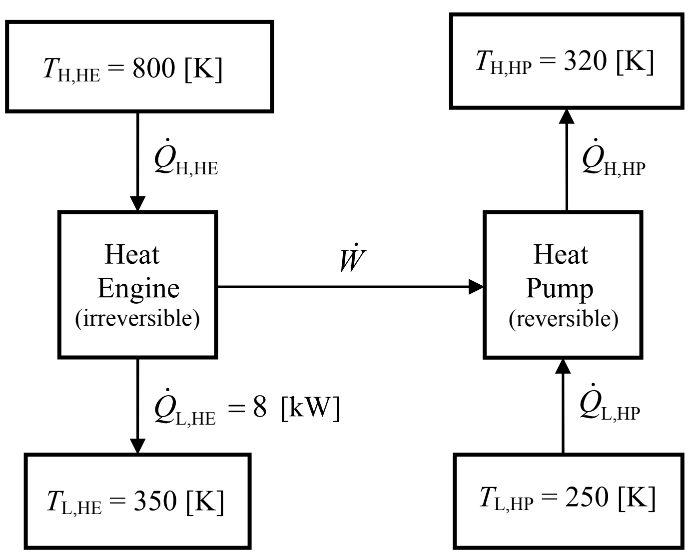


Figure 2: Heat engine and heat pump arrangement for problem 3.

value 16

4.

A rigid tank (corresponding to compartment A) is connected to a frictionless piston-cylinder-spring assembly (corresponding to compartment B) through a very thin tube. Compartments A and B contain air, and a valve in the middle of the connecting tube controls the flow between the two compartments. The air can be treated as an ideal gas. The volume of the very thin tube can be ignored. The volume of the rigid tank is $V_A = 1\ \text{m}^3$. The inner cross-sectional area of the cylinder (compartment B) is $A_{cross} = 0.8\ \text{m}^2$. The spring-constant for the linear-spring attached to the piston is $k_S = 40$ kN/m. At the initial state (state 1), the pressure and temperature of the air are $P_{A1} = 500$ kPa and $T_{A1} = 300\,°\text{C}$ in compartment A, and $P_{B1} = 100$ kPa and $T_{B1} = 50\,°\text{C}$ in compartment B, respectively. As shown in Figure 3, the vertical distance from the bottom of the cylinder to the lower surface of the piston is denoted as y. At the initial state, the value of y is 0.5 m. If the piston touches the stops, the value of y is 0.9 m. The valve is opened slowly such that the entire process can be treated as quasi-equilibrium. Heat is dissipated from the system to the surroundings until the final temperature of the air in compartments A and B reaches $30\,°\text{C}$. During the entire process, the pressures and temperatures in compartments A and B may be different until the final state is reached (at the final state, compartments A and B are completely in thermal and mechanical equilibrium).

3 (a) Determine the *total mass* (m) of the air contained in both compartments A and B.
2 (b) Determine the pressure (P_2) in compartment B when the piston *just* touches the stops.
2 (c) Determine the final pressure of the air (P_3).
3 (d) Determine the *total work* ($_1W_3$) done by the air during the process.
3 (e) Determine the *total heat transfer* ($_1Q_3$) during the process.
2 (f) For the thermodynamic process taking place in compartment B, show the state points and process paths on a P–V (pressure – volume) diagram.
1 (g) Indicate the area that represents the total work on the P–V diagram obtained in step (f).

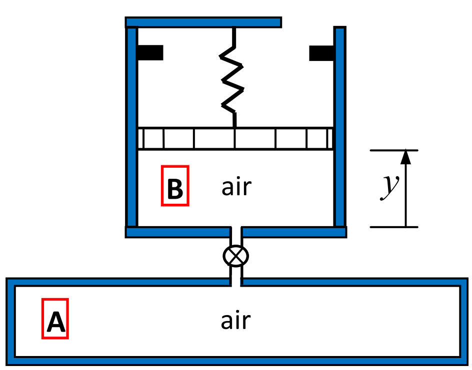


Figure 3: Piston-cylinder, valve, and rigid compartment for problem 4.

value
25 **5.**

A two-stage vapour-compression refrigeration cycle consists of two separate compression stages and a constant-pressure mixing device that links the two refrigeration loops, as shown in Figure 4. The lower (first) refrigeration loop functions using the substance R-410a with a flow rate $\dot{m}_{C1} = 8$ [kg/s] and the upper (second) refrigeration loop also uses R-410a and has the flow rate $\dot{m}_{C2}$. The flows from the lower and upper refrigeration loops are combined in the mixer. All fluid streams directly entering and exiting the mixer (States 2, 3, 6, 7) are at the saturation pressure corresponding to −20 [°C] for R-410a. Both the evaporator and condenser operate at constant pressure. All necessary state information for the two refrigeration loops is shown in Figure 4. The evaporator absorbs heat at a rate $\dot{Q}_L$ and the condenser removes heat from the R-410a at a rate $\dot{Q}_H$. Assume that all devices operate adiabatically except for the evaporator and the condenser.

Notes:
1. Neglect changes in kinetic and potential energies for all devices in the cycle.
2. The following property information, interpolated from Table B.4.2, is provided for you to use in the solution of the problem:

Pressure, P	Temperature, T	Specific volume, v	Enthalpy, h
399.6 [kPa]	0 [°C]	0.07237 [m³/kg]	290.43 [kJ/kg]
2140.2 [kPa]	80 [°C]	0.01620 [m³/kg]	341.24 [kJ/kg]

2 (a) Calculate the power input to compressor 1 of the lower-pressure loop, $\dot{W}_{C1}$, in kW.

3 (b) Calculate the refrigeration capacity, $\dot{Q}_L$, in kW.

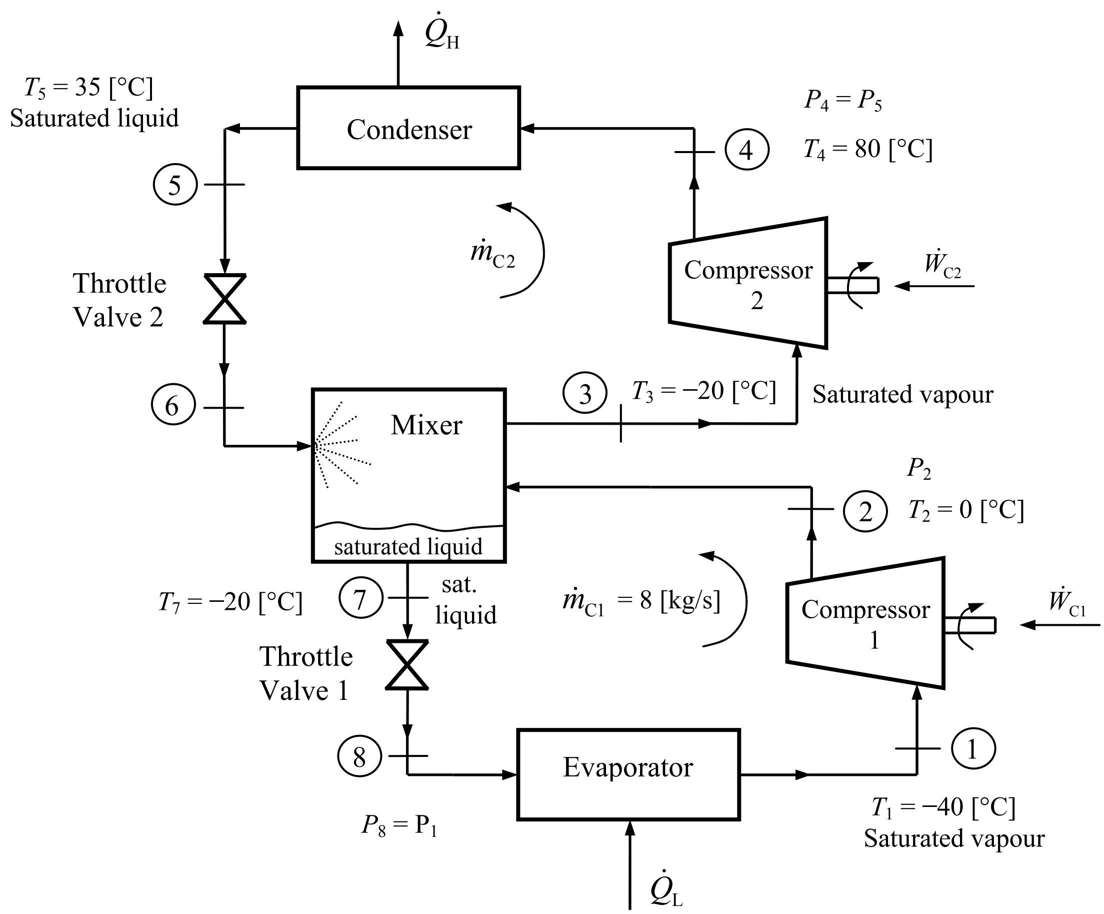
9 (c) Determine the mass flow rate through the secondary loop, $\dot{m}_{C2}$, in kg/s; and calculate the power input to compressor 2, $\dot{W}_{C2}$, in kW.

2 (d) Calculate the rate of heat transfer from the condenser, $\dot{Q}_H$, in kW.

3 (e) Calculate the quality x at each of the exits of throttle valve 1 and throttle valve 2.

1 (f) Determine the overall refrigeration coefficient of performance, β_R, of the combined cycle.

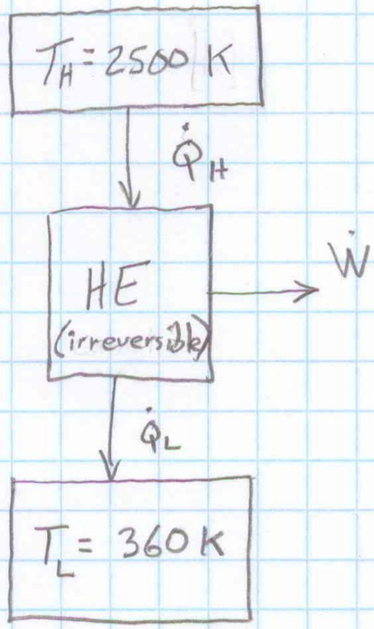
5 (g) Draw a process representation of this dual cycle on a T–v (temperature – specific volume) diagram. Label all states and show process paths (use a dashed line for an unknown process path). Label all state-point temperatures and specific volumes, and label the constant pressure lines that pass through the state points. Show your work for any extra calculations needed for the diagram; e.g., determining values for specific volumes.



$$P_2 = P_3 = P_6 = P_7 = P_{\text{sat. at } -20\ [°C]}$$

Figure 4: Schematic diagram of the dual refrigeration cycle for problem 5.

#1.



A fuel provides 45000 kJ/kg of heat, from a combustion temperature of $T_H = 2500$ K, to a heat engine.

$$\eta = \frac{1}{2} \times \eta_{carnot} = 0{,}5 \times \frac{T_H - T_L}{T_H}$$

$$= 0{,}5 \times \frac{2500 - 360}{2500}$$

$$= 0.4280$$

$$\dot{W} = 14.98 \text{ kW}$$

$$\dot{Q}_H = 45000 \frac{kJ}{kg} \times \dot{m} \frac{kg}{s} = 45000 \times \dot{m} \text{ KW}$$

$$\eta = \frac{\dot{W}}{\dot{Q}_H} \quad \therefore \quad 45000 \times \dot{m} = \frac{14.98}{0.4280}$$

$$\therefore \underset{(\text{Fuel})}{\dot{m}} = \frac{35.0}{45000} \frac{kg}{s} \quad (7.778 \times 10^{-4} \text{ kg/s})$$

or, per hour, $\dot{m}_{fuel} = \frac{35.0}{45000} \frac{Kg}{s} \times \frac{3600 \text{ s}}{1 \text{ h}} = 2.8 \frac{kg}{h}$

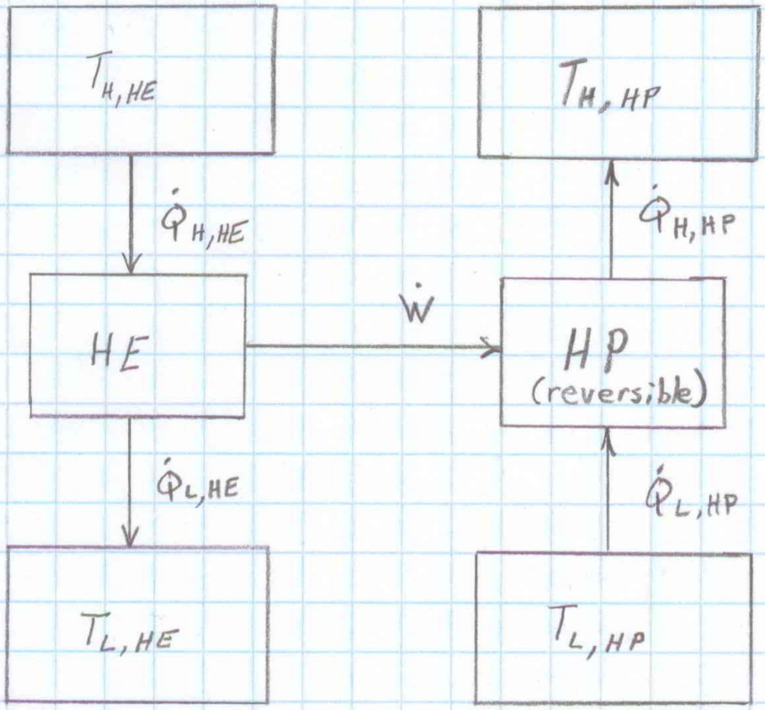
Fuel costs \$1.10/Litre, fuel density $\rho = 750$ kg/m^3

$$\dot{m} = \frac{\dot{V}}{v}, \quad v = \frac{1}{\rho} \quad \therefore \dot{V} = \frac{\dot{m}}{\rho} = \frac{2.8 \text{ kg/h}}{750 \text{ kg/m}^3} = 3.733 \times 10^{-3} \text{ m}^3/\text{h}$$

$$1 \text{ m}^3 \equiv 1000 \text{ L} \quad \therefore \dot{V} = 3.733 \times 10^{-3} \frac{m^3}{h} \times \frac{1000 \text{ L}}{1 \text{ m}^3} = 3.733 \frac{L}{h}$$

Fuel operating cost: $\frac{\$1.10}{L} \times 3.733 \frac{L}{h} = \$4.106/h$

#3.



HE: $T_{H,HE} = 800\ K$

$T_{L,HE} = 350\ K$

$\dot{Q}_{L,HE} = 8\ kW$

HP: $T_{H,HP} = 320\ K$

$T_{L,HP} = 250\ K$

(a)

$$\eta = 0.75 \times \eta_{carnot} = 0.75 \times \frac{T_{H,HE} - T_{L,HE}}{T_{H,HE}} = 0.75 \times \left(\frac{800-350}{800}\right)$$

$$\eta = 0.42187$$

$$\eta = \frac{\dot{W}}{\dot{Q}_{H,HE}} = \frac{\dot{Q}_{H,HE} - \dot{Q}_{L,HE}}{\dot{Q}_{H,HE}} \quad \therefore \quad 0.42187 = \frac{\dot{Q}_{H,HE} - 8}{\dot{Q}_{H,HE}}$$

solve, $\dot{Q}_{H,HE} = 13.838\ kW$

$$\dot{W} = \dot{Q}_{H,HE} - \dot{Q}_{L,HE} = 13.838 - 8.0 = 5.838\ kW$$

(b)

Heat Pump: $\dot{W}$ from HE is entirely used to drive the HP.

$$\underset{(Carnot)}{\beta'_{HP}} = \frac{T_{H,HP}}{T_{H,HP} - T_{L,HP}} = \frac{320}{320-250} = 4.5714$$

(coefficient of performance

Also, $\beta'_{HP} = \dfrac{\dot{Q}_{H,HP}}{\dot{W}}$ and $\dot{Q}_{H,HP} = k(T_{H,HP} - T_{L,HP})$

Combining equations, $k = \dfrac{\beta'_{HP}\ \dot{W}}{T_{H,HP} - T_{L,HP}} = \dfrac{4.5714 \times 5.838}{320-250}$

$$k = 0.38125$$

$\therefore\ \dot{Q}_{H,HP} = 0.38125(320-250) = 26.687\ \text{kW}$

$\dot{Q}_{L,HP} = \dot{Q}_{H,HP} - \dot{W} = 26.687 - 5.838 = 20.849\ \text{kW}$

(c)

If $\dot{W}$ was increased to 7.5 kW:

$$\dot{Q}_{H,HP} = k(T_{H,HP} - T_{L,HP}) = \underset{(rev.)}{\beta'_{HP}} \cdot \dot{W}$$

i.e., $k(T_{H,HP} - T_{L,HP}) = \dfrac{T_{H,HP}}{(T_{H,HP} - T_{L,HP})} \times \dot{W}$

$$(T_{H,HP} - T_{L,HP})^2 = \frac{T_{H,HP} \times \dot{W}}{k}$$

$$(320 - T_{L,HP})^2 = \frac{320 \times 7.5}{0.38125}$$

solve for $T_{L,HP} = 240.65\ K$

4.

From table A.5, for air, $R = 0.287 \frac{kJ}{kg \cdot K}$, $C_{v0} = 0.717 \frac{kJ}{kg \cdot K}$

$$m_{A1} = \frac{P_{A1} V_{A1}}{R T_{A1}} = \frac{500 \times 1}{0.287 \times (273.15 + 300)} = 3.040 \ (kg)$$

$$V_{B1} = A_{cross} \cdot y_1 = 0.8 \times 0.5 = 0.4 \ (m^3)$$

$$m_{B1} = \frac{P_{B1} V_{B1}}{R \cdot T_{B1}} = \frac{100 \times 0.4}{0.287 \times (273.15 + 50)} = 0.431 \ (kg)$$

<u>Total mass</u>

- $m = m_{A1} + m_{B1} = 3.040 + 0.431 = 3.471 \ (kg)$

When the piston just hits the stops,

- $P_2 = P_1 + \frac{k}{A}(y_2 - y_1) = 100 + \frac{40}{0.8}(0.9 - 0.5) = 120 \ (kPa)$

Aussume that the piston contacts the stops at the final state,

$V_3 = V_{A3} + V_{B3} = 1.0 + A_{cross} \times y_2 = 1.0 + 0.8 \times 0.9 = 1.72 \ (m^3)$

- $P_3 = \frac{m R T_3}{V_3} = \frac{3.471 \times 0.287 \times (273.15 + 30)}{1.72} = 175.58 \ (kPa)$

- Because $P_3 > P_2$, the piston contacts the stops at the final state.

Total work

$${}_1W_3 = {}_1W_2 + \cancel{{}_2W_3} = \frac{1}{2}(P_1 + P_2)(V_{B2} - V_{B1}) = \frac{1}{2}(P_1 + P_2)[A_{cross} \cdot (y_2 - y_1)]$$

$$\therefore {}_1W_3 = \frac{1}{2}(100 + 120)[0.8 \times (0.9 - 0.5)] = 35.2 \ (kJ)$$

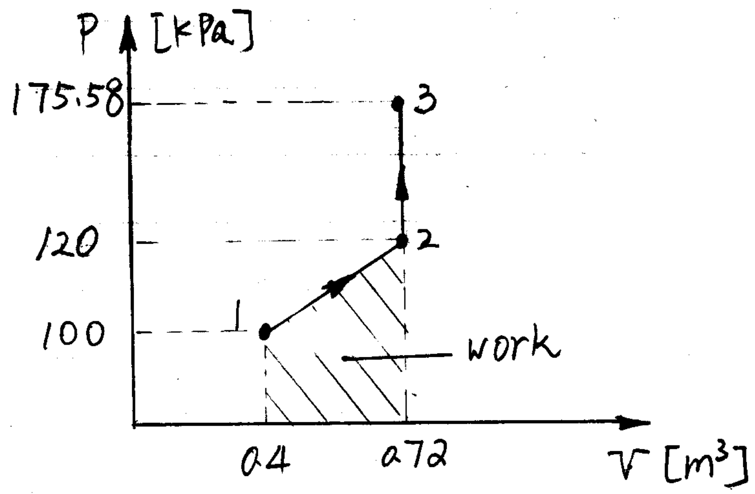
Total heat transfer

$${}_1Q_3 = {}_1W_3 + \Delta U$$

$$= {}_1W_3 + C_{v0}[m_{A1}(T_3 - T_{A1}) + m_{B1}(T_3 - T_{B1})]$$

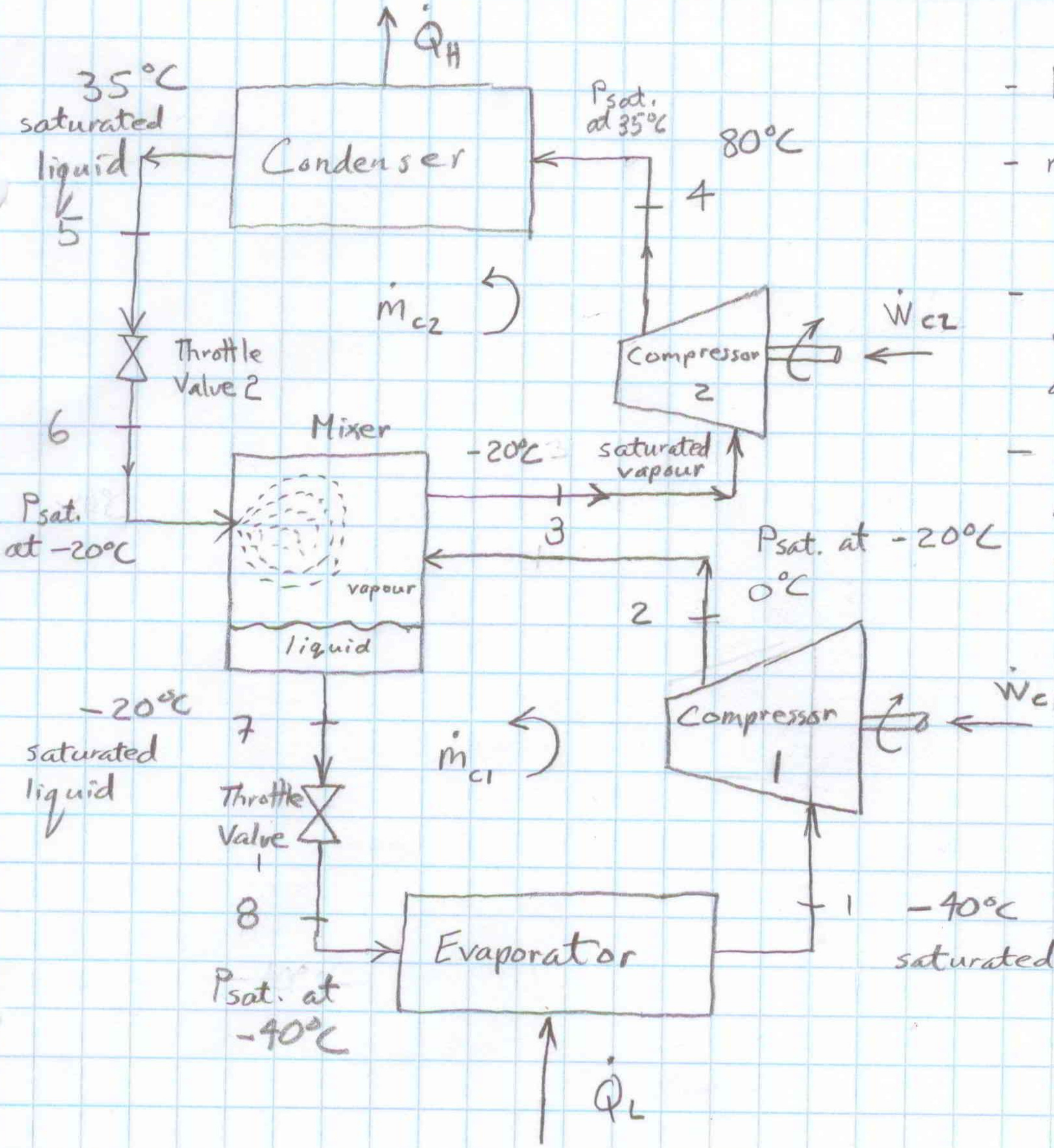
$$= 35.2 + 0.717[3.040 \times (30 - 300) + 0.431 \times (30 - 50)]$$

- $= -559.49 \ (kJ)$



P [kPa]
175.58
120
100
3
2
1
work
0.4
0.72
V [m³]

5.



- R-410a Fluid
- mixing pressure is $P_{sat.}$ at -20°C
- all devices adiabatic except evaporator and condenser
- neglecting all kinetic and potential energy changes.

(a)

State 1 $\quad T_1 = -40°C$, saturated vapour

Table B.4.1, $\quad P_{sat.} = 175.0$ kPa, $\quad v_g = 0.14291\ m^3/kg$

$h_g = 262.83$ kJ/kg

State 2 $\quad T_2 = 0°C$, $P_2 = P_{sat.,\,T=-20°C} = 399.6$ kPa

Table B.4.2, by interpolation, $v_2 = 0.07237\ m^3/kg$, $h_2 = 290.43$ kJ/kg (values provided)

$\dot{m}_{c1} = 8$ kg/s $\quad$ First law on compressor 1,

$$\cancel{\dot{Q}_{cv}} - \dot{W}_{cv} = \dot{m}_{c1}\left[(h_2 - h_1) + \cancel{\Delta ke} + \cancel{\Delta pe}\right]$$

$$-\dot{W}_{cv} = 8(290.43 - 262.83)$$

$\longrightarrow \quad \dot{W}_{cv} = -220.80$ kW, $\quad \dot{W}_{c1} = -\dot{W}_{cv} = 220.80$ kW

(b) state 7 $T_7 = -20°C$, saturated liquid, $P = P_{sat.} = 399.6$ kPa

$v_7 = v_f = 0.000803\ m^3/kg$, $h_7 = h_f = 28.24$ kJ/kg

state 8 $P_8 = P_{sat.}$ at $-40°C$ = 175.0 kPa

$h_8 = h_7$ from first law applied to throttle valve
$= 28.24$ kJ/kg

At $T = -40°C$ $h_f < h_8 < h_g$ ∴ saturated mixture & $T_8 = -40°C$

$$x_8 = \frac{h_8 - h_f}{h_{fg}} = \frac{28.24 - 0}{262.83} = 0.107445$$

$$v_8 = v_f + x \cdot v_{fg} = 0.000762 + 0.107445 \times 0.14215 = 0.016035\ m^3/kg$$

First law on Evaporator: $\dot{Q}_{cv} - \cancel{\dot{W}_{cv}} = \dot{m}_{c1}\left[(h_1 - h_8) + \cancel{\Delta ke} + \cancel{\Delta pe}\right]$

$\dot{Q}_{cv} = 8 \times (262.83 - 28.24) = 1876.72$ kW

→ $\dot{Q}_L = \dot{Q}_{cv} = 1876.72$ kW

Top Cycle

(c) State 3 $T_3 = -20°C$, saturated vapour ∴ $v_3 = v_g = 0.06480\ m^3/kg$
$h_3 = h_g = 271.89$ kJ/kg $P_{sat.} = 399.6$ kPa

State 4 $T4 = 80°C$, P is P_{sat} at 35°C = 2140.2 kPa ($s_3 = 1.0779$ kJ/kg·K)

$T_4 > T_{sat.} = 35°C$

Interpolation: at 80°C (data provided in exam question)

P [kPa]	v [m³/kg]	h [kJ/kg·K]
2000	0.01717	343.22
2140.2	v	h
3000	0.01025	329.12

$v_4 = 0.01620\ m^3/kg$

$h_4 = 341.24$ kJ/kg

($S_4 = 1.1428$ kJ/kg·K)

state 5 $T_5 = 35°C$, saturated liquid

∴ $v_5 = 0.000995\ m^3/kg$

$h_5 = 114.95$ kJ/kg, $P_5 = P_{sat.} = 2140.2$ kPa state 6 $\quad P_6 = P_{sat.\ at\ -20°C} = 399.6\ kPa$

$h_6 = h_5$ due to throttle valve. At $-20°C$, $h_f < h_6 < h_g$

$\therefore$ saturated mixture, and $T_6 = -20°C$

$$x_6 = \frac{h_6 - h_f}{h_{fg}} = \frac{114.95 - 28.24}{243.65}$$

$x_6 = 0.35587$, $\quad v_6 = v_f + x \cdot v_{fg} = 0.000803 + 0.35587 \times 0.06400$

$v_6 = 0.023578\ m^3/kg$

Mixer $\quad$ first law: (general)

$$\cancel{\dot{Q}_{cv}} - \cancel{\dot{W}_{cv}} = \left[\dot{m}_7(h_7 + \tfrac{1}{2}\bar{V}_7^2 + gZ_7) + \dot{m}_3(h_3 + \tfrac{1}{2}\bar{V}_3^2 + gZ_3)\right] - \left[\dot{m}_6(h_6 + \tfrac{1}{2}\bar{V}_6^2 + gZ_6) + \dot{m}_2(h_2 + \tfrac{1}{2}\bar{V}_2^2 + gZ_2)\right]$$

- neglect all Δke, Δpe

$\dot{m}_7 = \dot{m}_8$, $\dot{m}_8 = \dot{m}_1$, $\dot{m}_1 = \dot{m}_2$ $\therefore$ $\dot{m}_7 = \dot{m}_2$

$\dot{m}_3 = \dot{m}_4$, $\dot{m}_4 = \dot{m}_5$, $\dot{m}_5 = \dot{m}_6$ $\therefore$ $\dot{m}_3 = \dot{m}_6$

$$\therefore 0 = \dot{m}_7(h_7 - h_2) + \dot{m}_3(h_3 - h_6)$$

$$\rightarrow \dot{m}_3 = \dot{m}_7 \frac{(h_2 - h_7)}{(h_3 - h_6)} = 8 \times \frac{290.43 - 28.24}{271.89 - 114.95} = 13.365\ \frac{kg}{s}$$

First Law on Compressor 2

$$\cancel{\dot{Q}_{cv}} - \dot{W}_{cv} = \dot{m}_3\left[(h_4 - h_3) + \cancel{\Delta ke} + \cancel{\Delta pe}\right]$$

$\rightarrow -\dot{W}_{cv} = 13.365\,(341.24 - 271.89) = 926.86\ kW$, $\quad \dot{W}_{c2} = -\dot{W}_{cv} = 926.86\ kW$

(d) First Law on Condenser

$\dot{Q}_{cv} - \cancel{\dot{W}_{cv}} = \dot{m}_4(h_5 - h_4)$, $\Delta ke = \Delta pe = 0$

$\dot{Q}_{cv} = 13.365\,(114.95 - 341.24) = -3024.13\ kW$

$\dot{Q}_H = -\dot{Q}_{cv} = 3024.3\ kW$

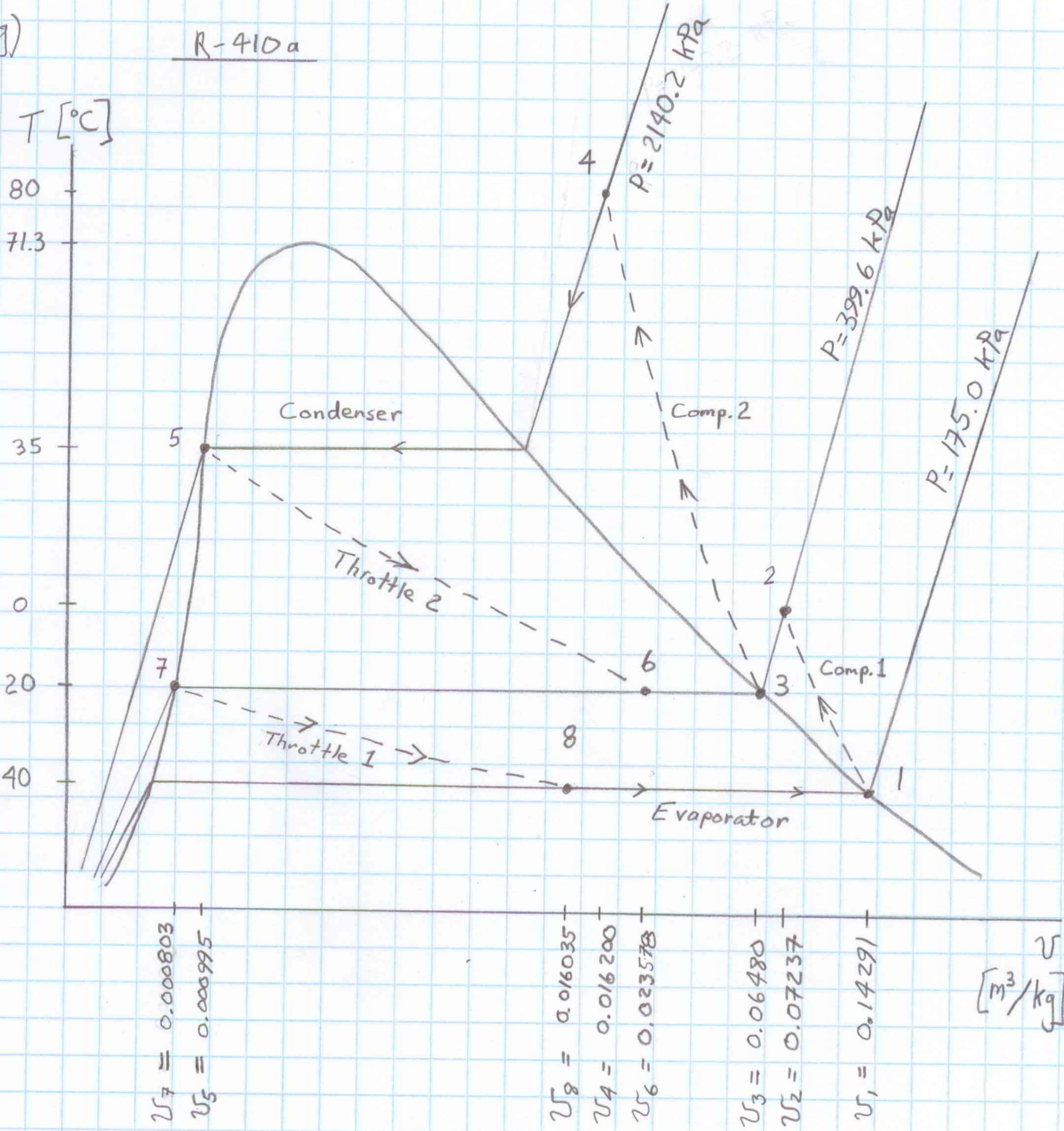
(e) calculated earlier

(f). Overall COP_R : $\beta_R = \dfrac{\dot{Q}_L}{\dot{W}_{c1} + \dot{W}_{c2}} = \dfrac{1876.72}{220.80 + 926.86} = 1.635$

$\dfrac{\dot{m}_{c2}}{\dot{m}_{c1}} = \dfrac{13.365}{8} = 1.67$, $\quad$ verify $\dot{Q}_{net} - \dot{W}_{net} = 0$ for entire dual cycle.

(g)

R-410a



T [°C]
80
71.3
35
0
-20
-40
P = 2140.2 kPa
P = 399.6 kPa
P = 175.0 kPa
4
Comp. 2
Condenser
5
Throttle 2
2
Comp. 1
6
7
3
Throttle 1
8
1
Evaporator
v [m³/kg]
v7 = 0.000803
v5 = 0.000995
v8 = 0.016035
v4 = 0.016200
v6 = 0.023578
v3 = 0.06480
v2 = 0.07237
v1 = 0.14291