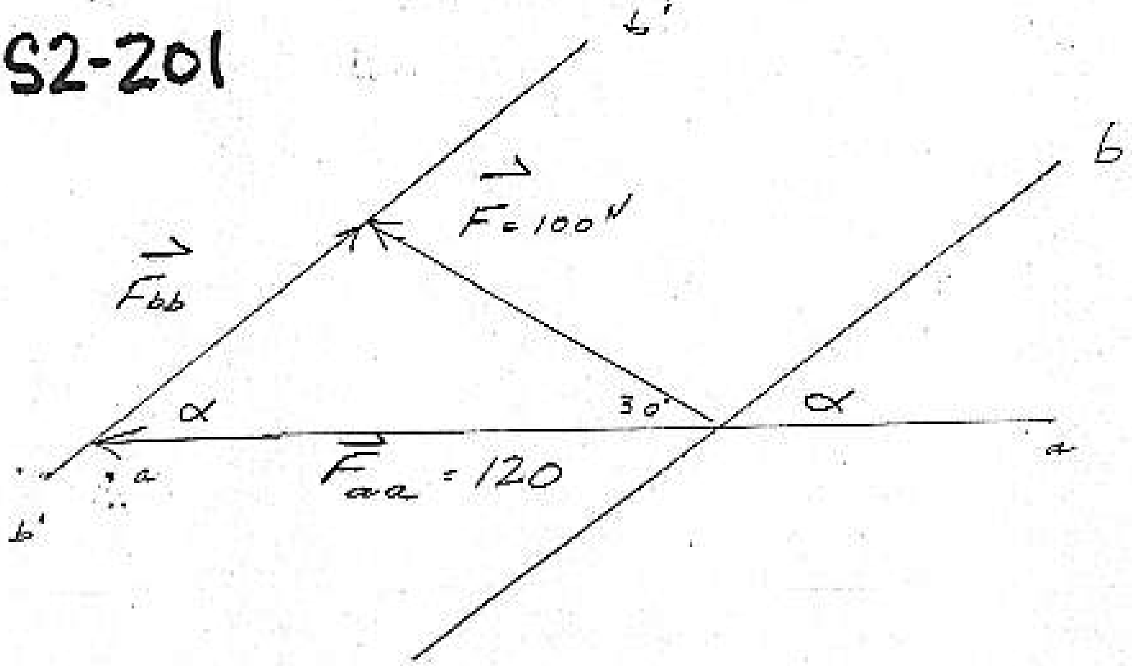
S2-201



$$\vec{F} = \vec{F}_{aa} + \vec{F}_{bb}$$

draw b'-b' parallel to b b
($\vec{F}_{aa}$ & $\vec{F}_{bb}$ placed "tip" to "tail")

Cosine rule:

$$F_{bb}^2 = 100^2 + 120^2 - 2(100)(120)\cos 30°$$

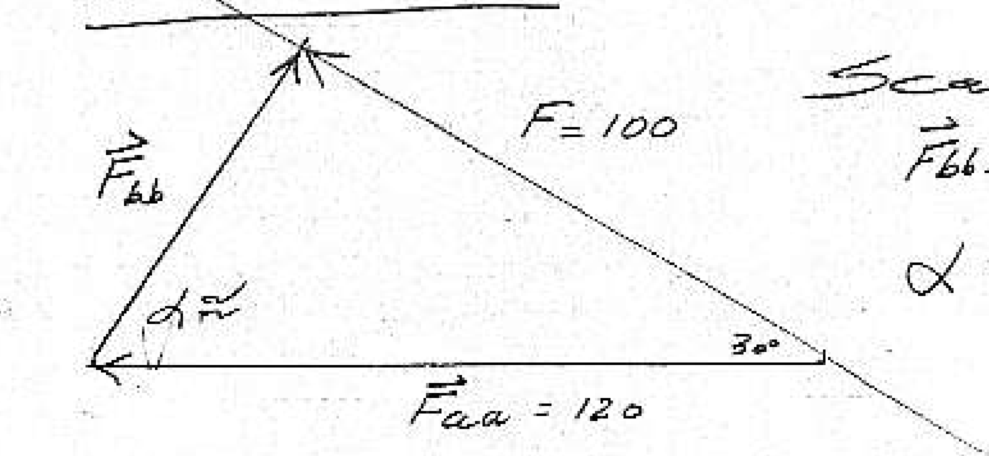
$$F_{bb} = 60.128^N$$

Sine rule

$$\frac{60.128}{\sin 30} = \frac{100}{\sin\alpha} \qquad \sin\alpha = \frac{100\sin 30°}{60.128}$$

$$\alpha = 56.26°$$

Graphical Check

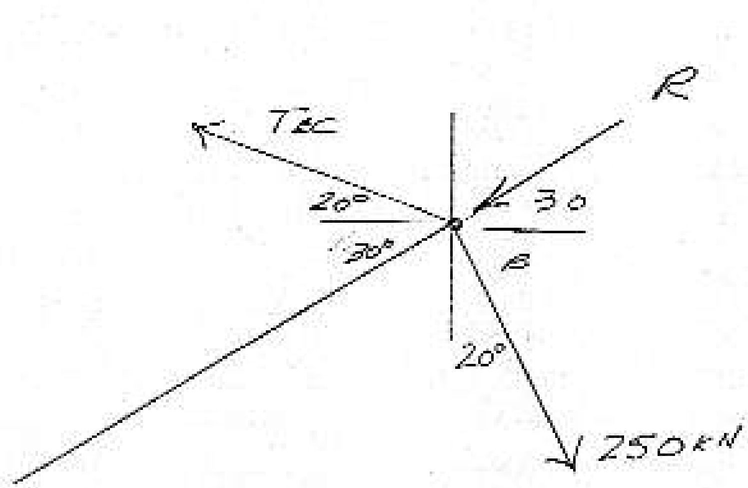


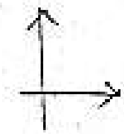
Scale 1:125

$\vec{F}_{bb} \approx 60.2$

$\alpha \approx 56°$

S2-202





Force	Mag	Dir
F_1	250	✓
T_{BC}	?	✓
R	?	✓

$$+R\cos 30° = -T_{BC}\cos 20° + 250\sin 20°$$
$$-R\sin 30° = +T_{BC}\sin 20° - 250\cos 20°$$

$$-0.866R + T_{BC}\cos 20° = 85.51$$
$$-0.5R - T_{BC}\sin 20° = -234.92$$

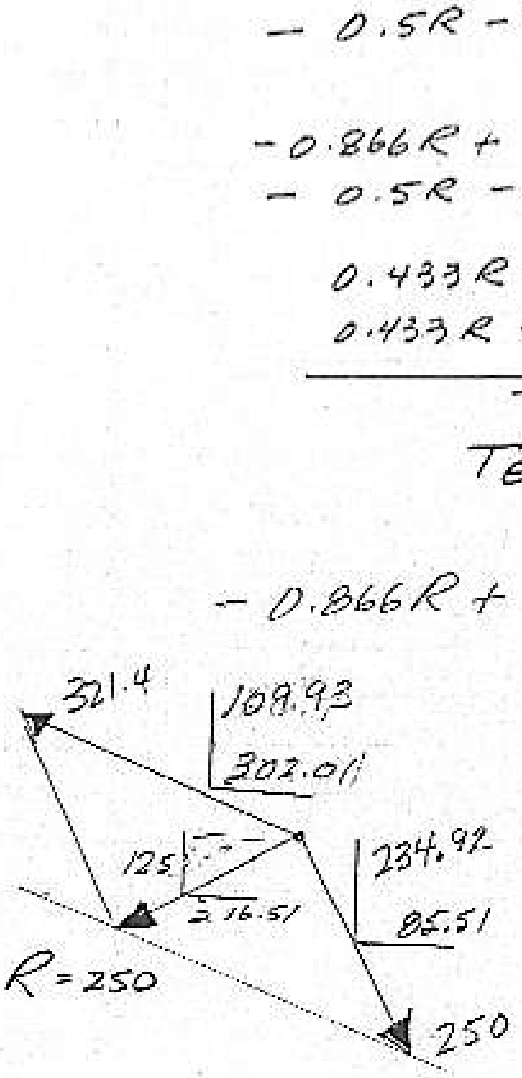
$$-0.866R + 0.94T_{BC} = 85.51$$
$$-0.5R - 0.342T_{BC} = -234.92$$

$$0.433R - 0.47T_{BC} = -42.755$$
$$0.433R + 0.296T_{BC} = 203.44$$
$$-0.766T_{BC} = -246.195$$

$$T_{BC} = \frac{-246.195}{-0.766} = 321.4\text{ kN}$$

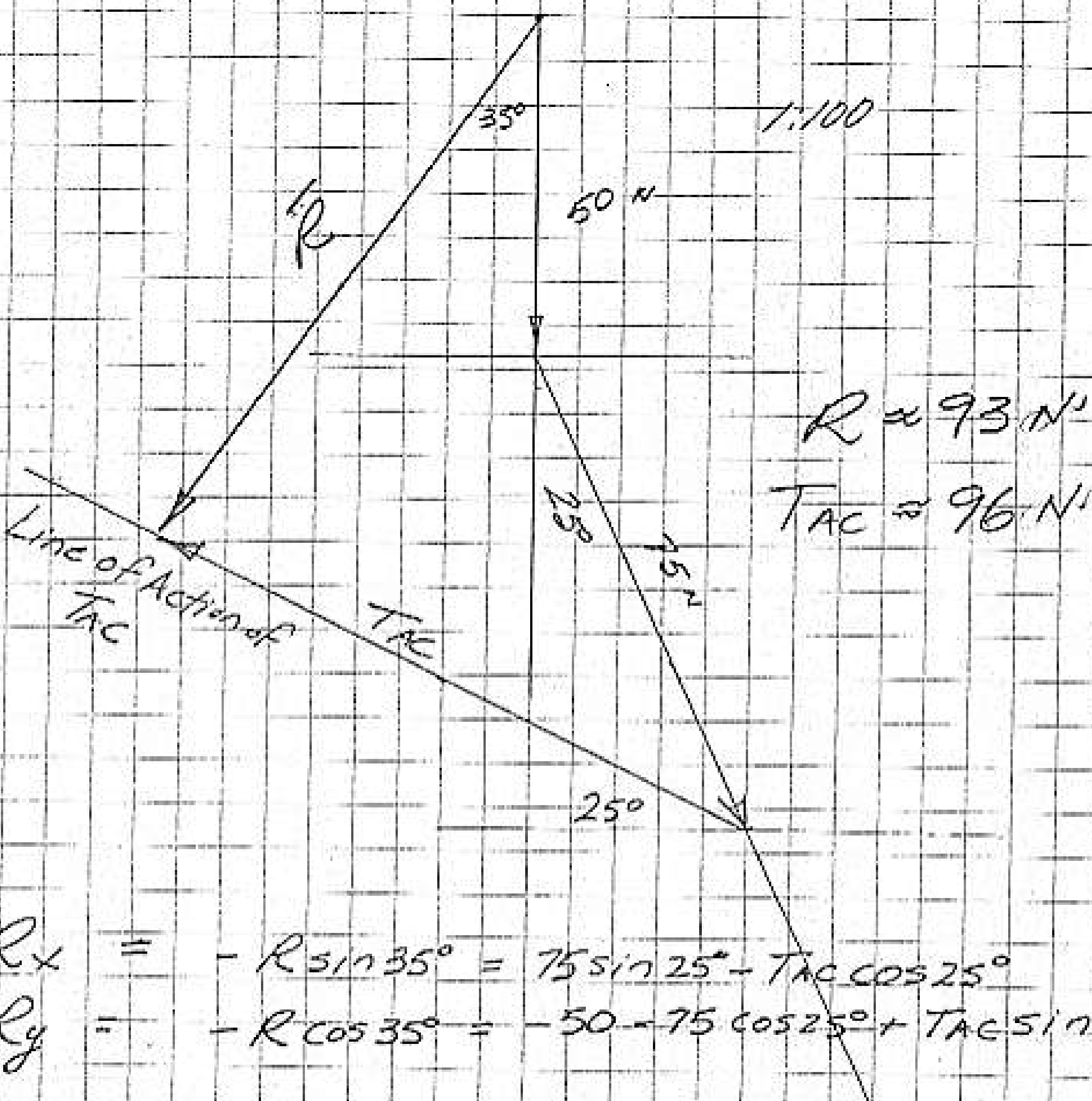
$$-0.866R + 321.4\cos 20° = 85.51$$

$$R = 250\text{ kN}$$



$302.011 - 85.51 = 216.51$ ✓
$234.92 - 109.93 = 125$ ✓

S2-203



$$R_x = -R\sin 35° = 75\sin 25° - T_{AC}\cos 25°$$
$$R_y = -R\cos 35° = -50 - 75\cos 25° + T_{AC}\sin 25°$$

$$-0.5736R + 0.9063T_{AC} = 31.696$$
$$-0.8192R - 0.4226T_{AC} = -117.973$$

$$-0.4699R + 0.7424T_{AC} = 25.965$$
$$-0.4699R - 0.2424T_{AC} = -67.669$$

$$0.9848T_{AC} = 93.634$$

$$T_{AC} = 95.08\text{ N}$$

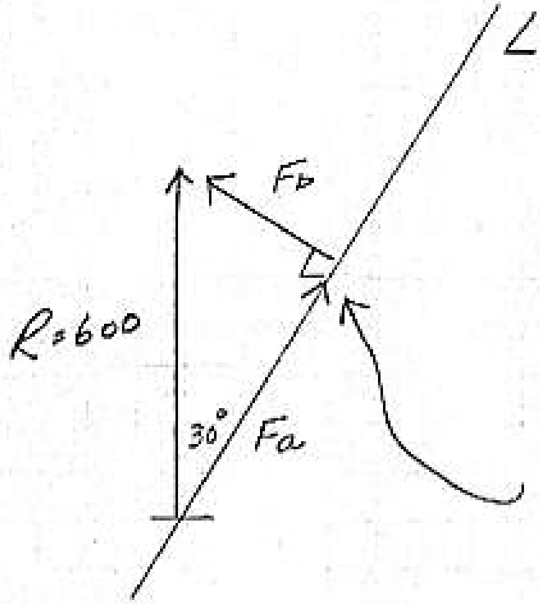
$$-0.5736R = 31.696 - 0.9063(95.08)$$

$$R = 94.97\text{N}$$

S2-204

$$\vec{R} = \vec{F_A} + \vec{F_B}$$

$\vec{R}$ is vertical $\qquad R = 600\text{N}$



We place F_A & F_B "Tip" to "Tail" To get $\vec{R}$

For F_B minimum it MUST be $\perp$ to F_A.

FORCE	MAG	DIR
F_A	?	30°
F_B	? (Min)	[illegible]
R	600	✓

3 unknowns but F_B is MINIMUM

$\therefore \sin 30° = \dfrac{F_B}{600} \qquad F_B = 300\text{ N}$

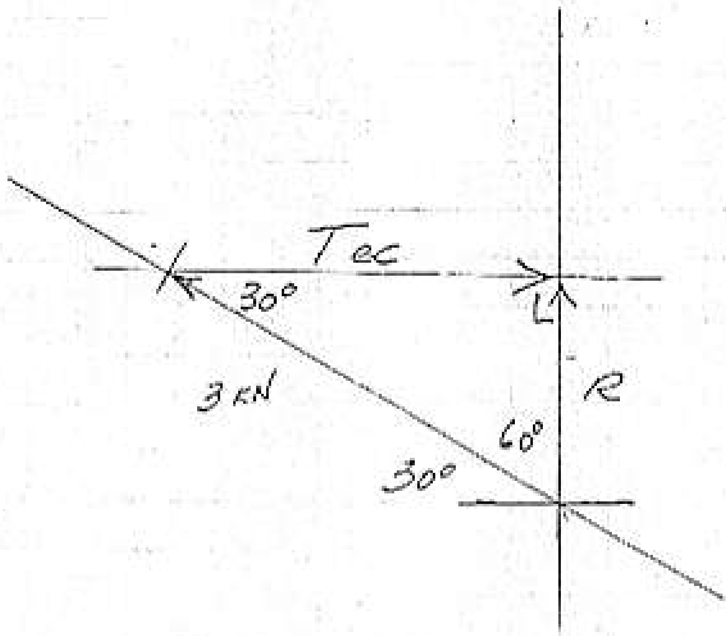
$\cos 30° = \dfrac{F_A}{600} \qquad F_A = 519.62\text{ N}$

S2-205

1/1

FORCE	MAG	DIR
TBA	3	30°
TBC	?	? M.
R	?	vertical

3 unknowns but magnitude of TBC is minimum



Scale 1:50

Tbc = 2.6 kN

R = 1.5 kN

TRIG: $\sin 60° = \frac{T_{BC}}{3}$ $T_{BC} = 2.598$ kN

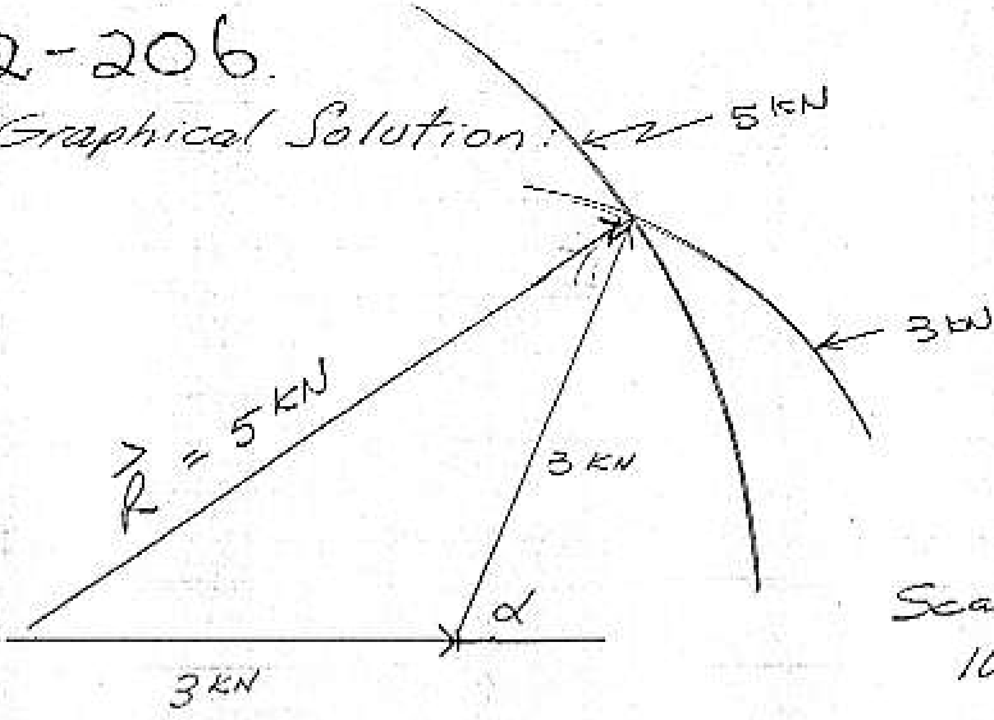
$\cos 60° = \frac{R}{3}$ $R = 1.5$ kN

$\vec{T}_{BC} = 2.598$ kN →

$\vec{R} = 1.5$ kN ↑

S2-206.

a) Graphical Solution:



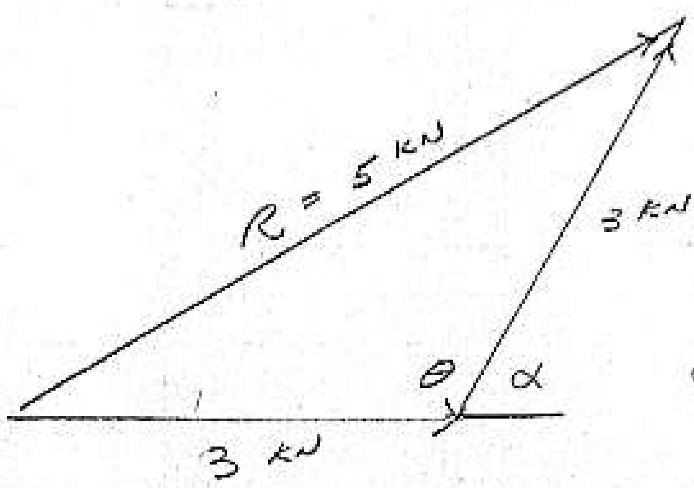
Scale
10 mm = 0.5 kN

$\alpha \approx 68^\circ$

$\therefore \quad 68^\circ \le \alpha \le 90^\circ$

If $\alpha < 68^\circ \quad \vec{R} > 5$ kN

b)



Cosine Rule

$5^2 = 3^2 + 3^2 - 2(3)(3)\cos\theta$

$\cos\theta = -0.38888$

$\theta = 112.89^\circ$

$\alpha = 180^\circ - 112.89^\circ$

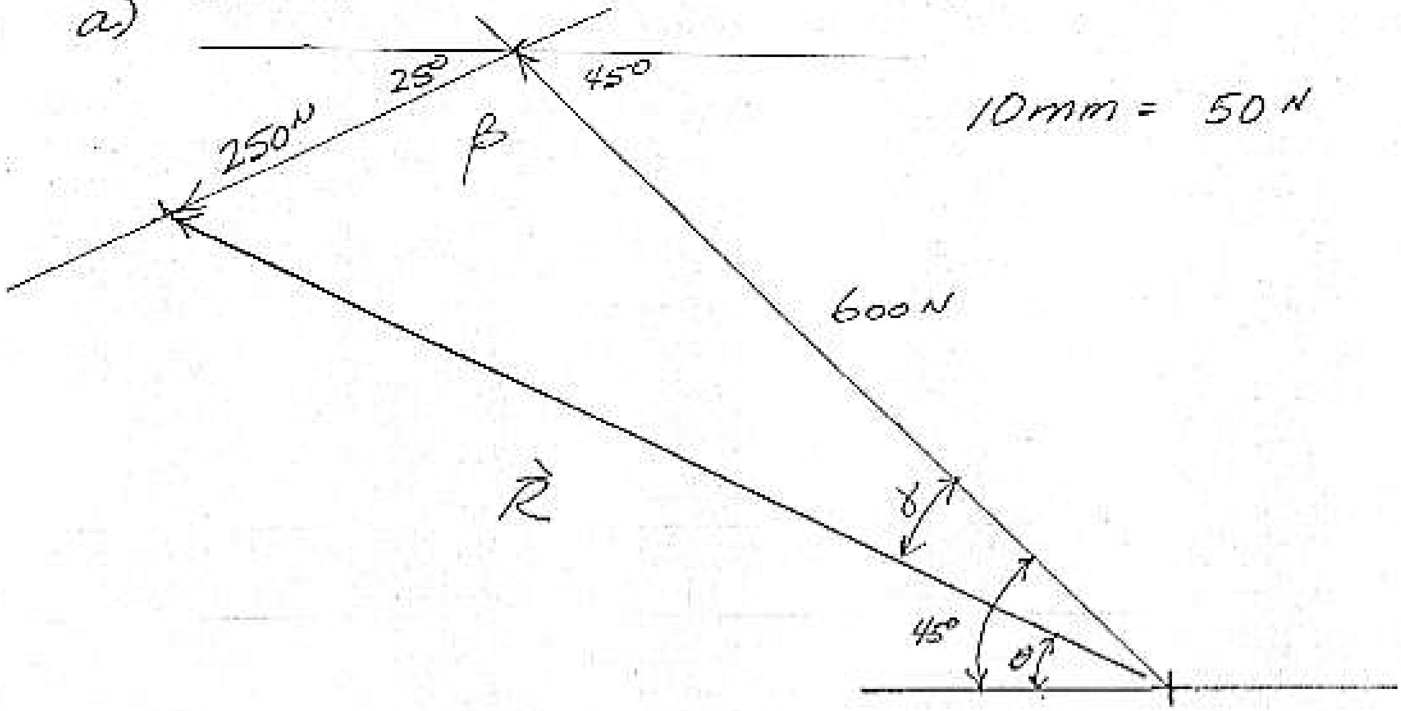
$\alpha = 67.11^\circ$

$67.11^\circ \le \alpha \le 90^\circ$

If $\alpha < 67.11^\circ \quad \vec{R} > 5$ kN

a) S2-207 1/2

$10mm = 50^N$



$R \approx 725^N \quad \theta \approx 25°$

b) TRIG SOLUTION

$\beta = 180° - 25° - 45° = 110°$

$R^2 = 250^2 + 600^2 - 2(250)(600)\cos 110°$

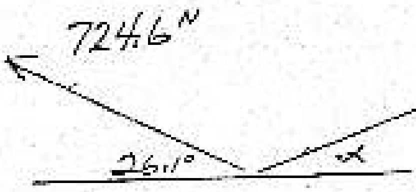
$R = 724.6^N$

$\frac{250}{\sin\gamma} = \frac{724.6}{\sin 110°} \qquad \sin\gamma = \frac{250 \sin 110°}{724.6}$

$\gamma = 18.92°$

$\therefore \theta = 45° - 18.92° = 26.1°$

c) For R Horizontal $\Sigma F_y = 0 \quad \Sigma F_x \neq 0$



$724.6 \sin 26.1 + 350 \sin\alpha = 0$

$\sin\alpha = -0.9108$

$\alpha = -65.62°$

S2-207 2/2

Alternately

$$600\sin 45° - 250\sin 25° \pm 350\sin\alpha = 0$$

$$424.264 - 105.655 \pm 350\sin\alpha = 0$$

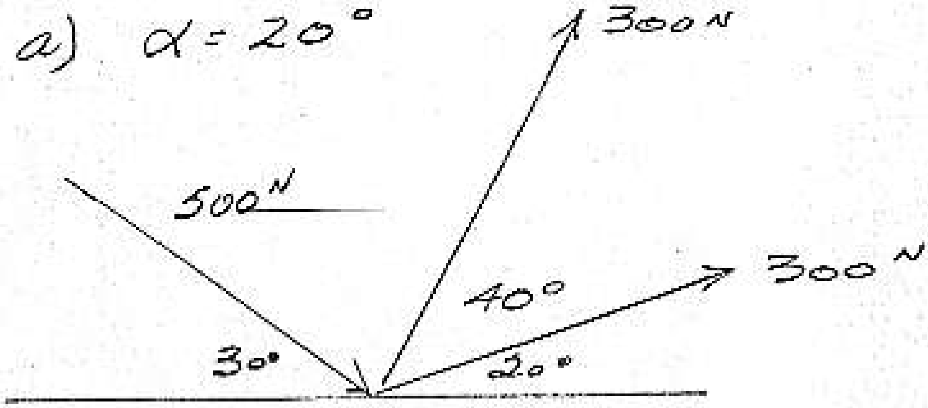
$$318.609 \pm 350\sin\alpha = 0$$

$$\frac{-318.609}{-350} = \sin\alpha = 0.9103$$

$$\alpha = 65.55$$

52-208

a) $\alpha = 20°$



$R_x = \Sigma F_x = 500\cos 30° + 300\cos 20° + 300\cos 60°$

$\vec{R}_x = 864.92\,N \rightarrow$

$R_y = \Sigma F_y = -500\sin 30° + 300\sin 20° + 300\sin 60°$

$\vec{R}_y = 112.41\,N \uparrow$

$R = \sqrt{(864.92)^2 + (112.41)^2} = 872.19\,N$

$\tan\theta = \dfrac{112.41}{864.92}$ $\theta = 7.4°$

$\vec{R} = 872.19°$ 7.4°

b) $\vec{R}$ is in x direction

$\therefore R_y = \Sigma F_y = 0$

$-500\sin 30° + 300\sin\alpha + 300\sin(40+\alpha) = 0$

by calculator $\alpha = 6.3215°$

or Brute Force

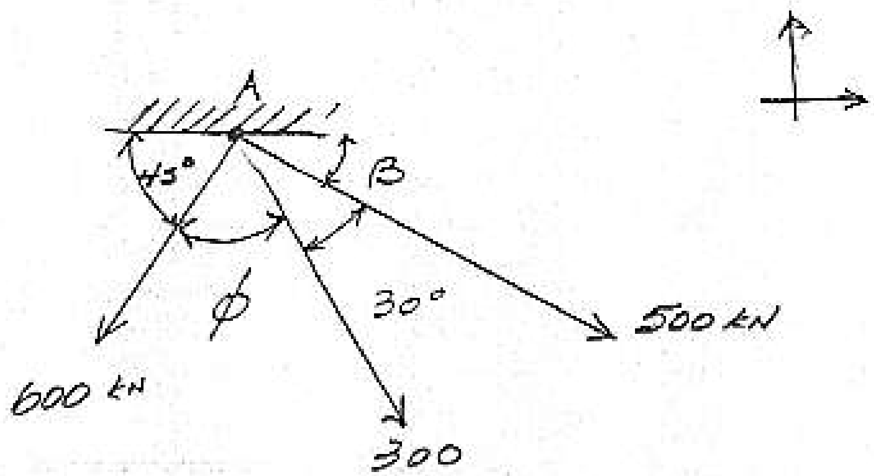
TRY	$\alpha = 60$	$305.2 \neq 0$
	$\alpha = 10$	31.9
	$\alpha = 6$	2.84
	$\alpha = 6.5$	1.57
	$\alpha = 6.3$	0.19
	$\alpha = 6.32$	-0.01 close enough

Alternate Solution:

We find the resultant $\vec{R}_1$ of the two 300 N forces which will act at $\alpha + 20°$. Then find $\vec{R}_2$ = Resultant of $\vec{R}_1$ and the 500 N force such that

$R_{2y} = 0$

S2 - 209



For $\vec{R}$ vertically downward $R_x = \Sigma F_x = 0$

$$\Sigma F_x = -600\cos 45° + 300\cos(\beta + 30°) + 500\cos\beta = 0 \qquad (1)$$

Solution by calculator: $\beta = 45.615°$

$\therefore \phi = 180° - 45° - 30° - 45.615°$

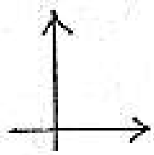
$\phi = 59.385°$

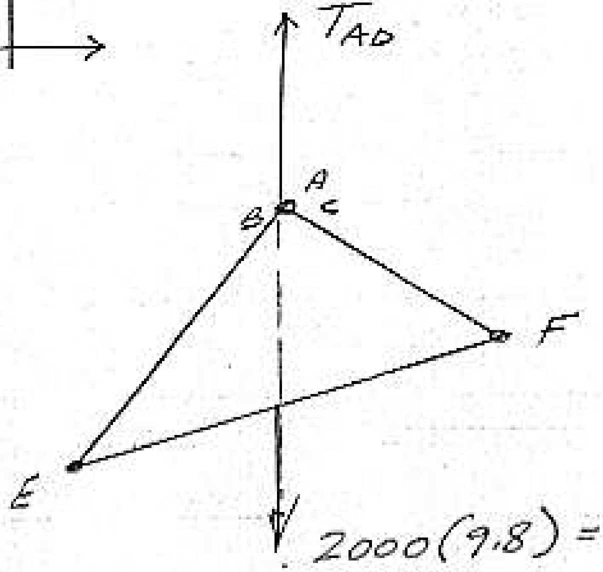
Alternately:
Solve eqn(1) by "brute" force:

guess $\beta = 40° \Rightarrow 61.36$
$\beta = 50° \Rightarrow -50.77$
$\beta = 45° \Rightarrow 6.935$
$\beta = 45.5° \Rightarrow 1.3$
$\beta = 45.6° \Rightarrow 0.17$ close enough.

S2-210

1/2



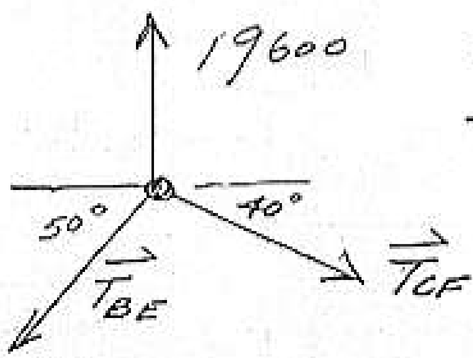


$2000(9.8) = 19600\ N$

$\Sigma F_y = 0 \quad T_{AD} - 19600\ N = 0$

$T_{AD} = 19600\ N$

Draw FBD of ring:



$\Sigma F_x = 0 \rightarrow$

$-T_{BE}\cos 50° + T_{CF}\cos 40° = 0 \quad (1)$

$\Sigma F_y = 0$

$19600 - T_{BE}\sin 50° - T_{CF}\sin 40° = 0 \quad (2)$

Simplify (1) & (2)

$-0.643\,T_{BE} + 0.766\,T_{CF} = 0 \quad (1)$

$-0.766\,T_{BE} - 0.643\,T_{CF} = -19600 \quad (2)$

Multiply (1) by 0.766 and (2) by 0.643

$-0.493\,T_{BE} + 0.587\,T_{CF} = 0$

$-0.493\,T_{BE} - 0.413\,T_{CF} = -12602.8$

subtract:

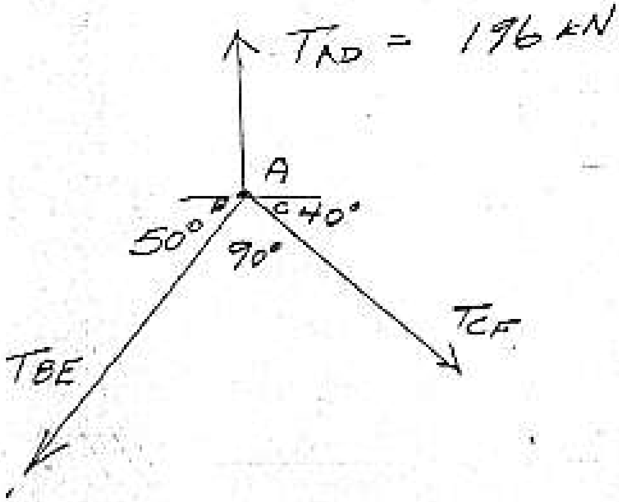
$T_{CF} = 12602.8\ N$

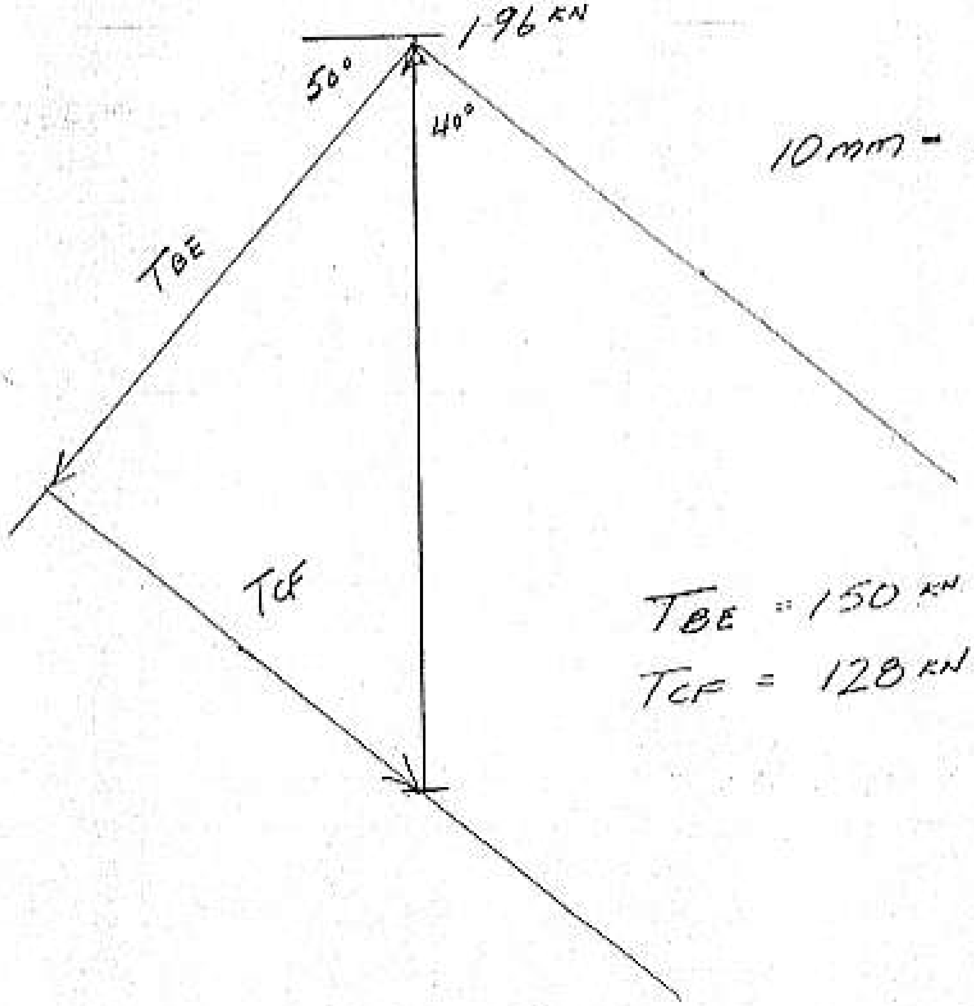
Backsubstitute: $-0.643\,T_{BE} + 0.766(12602.8) = 0$

$T_{BE} = 15020.03\ N$

52-210 GRAPHICAL SOLUTION

2/2



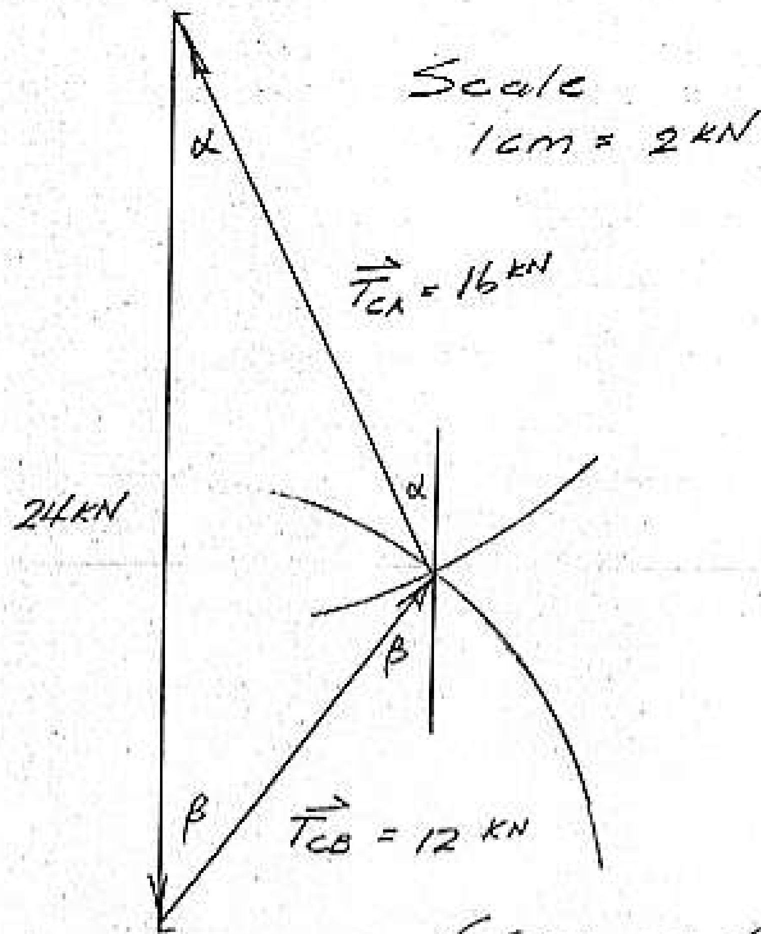


10mm - 20 kN

T_{BE} = 150 kN
T_{CF} = 128 kN

S2-211

a) Graphical Solution



$\alpha \approx 27°$

$\beta \approx 37°$

b) Trig Solution (Cosine Rule)

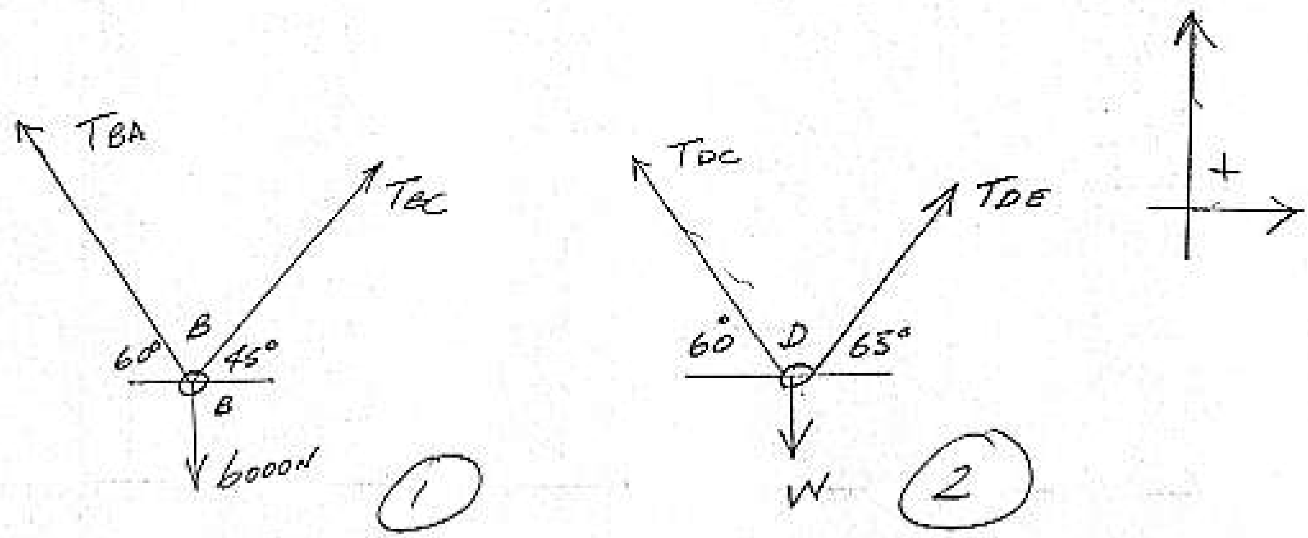
$$12^2 = 24^2 + 16^2 - 2(24)(16)\cos\alpha$$

$$\cos\alpha = 0.89583 \quad \alpha = 26.38°$$

$$16^2 = 24^2 + 12^2 - 2(24)(12)\cos\beta$$

$$\cos\beta = 0.80555 \quad \beta = 36.34°$$

S2-212



From 1

$\Sigma F_x = 0$

$-T_{BA}\cos 60° + T_{BC}\cos 45° = 0$

$$T_{BA} = \frac{T_{BC}\cos 45°}{\cos 60°} = T_{BA} = 1.414\,T_{BC}$$

$\Sigma F_y = 0$

$T_{BA}\sin 60° + T_{BC}\sin 45° - 6000 = 0$

$0.866T_{BA} + 0.707T_{BC} = 6000$

$0.866(1.414\,T_{BC}) + 0.707\,T_{BC} = 6000$

$1.932\,T_{BC} = 6000 \quad \therefore\ T_{BC} = 3106^{N}$

$T_{BA} = 1.414\,T_{BC} = 1.414(3106) = 4391.9^{N}$

But $T_{BC} = T_{DC}$ (same cable) $= 3106^{N}$

From FBD 2 $\quad \Sigma F_x = 0 \quad -3106\cos 60° + T_{DE}\cos 65° = 0$

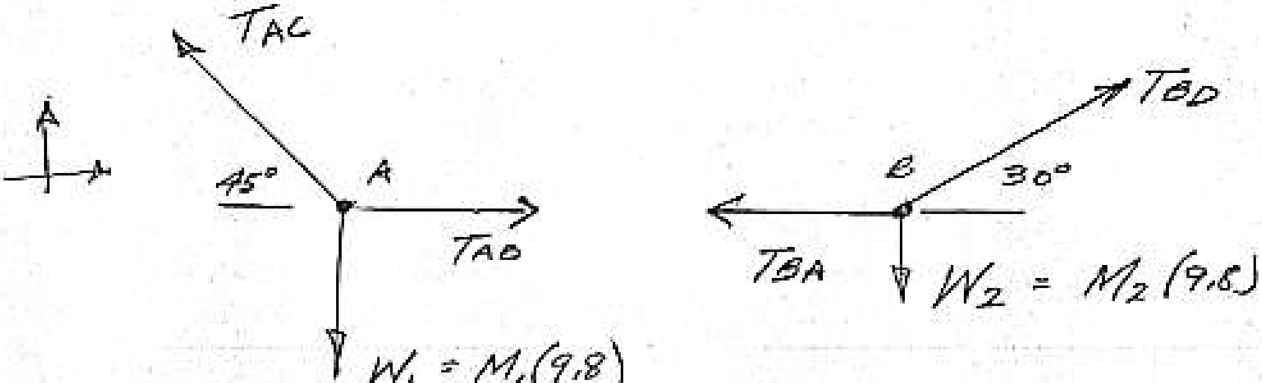
$$T_{DE} = \frac{3106\cos 60°}{\cos 65°} = 3674.71^{N}$$

$\Sigma F_y = 0 \quad 3106\sin 60° + 3674.71\sin 65° - W = 0$

$W = 6020.3^{N}$

S2-213 1/2

We draw FBDs of A & B



Maximum tension in any cable = 100 kN

We assume tenion in one of the cables = 100 kN
& check other cables

Say at B we assume T_{BD} = 100 kN

$\therefore \Sigma F_x = 0 \quad -T_{BA} + 100\cos 30° = 0$

$\therefore T_{BA} = 86.6\ \text{kN} < 100$ ok.

$\Sigma F_y = 0 \quad 100\sin 30° - W_2 = 0$

$\therefore W_2 = 50\ \text{kN}$ ok.

At A:

$\Sigma F_x = 0 \quad -T_{AC}\cos 45° + 86.6 = 0$

$T_{AC} = 122.49\ \text{kN}$ No Good

$\therefore$ Assume T_{AC} = 100 kN & check other cables.

$\Sigma F_x = 0 \quad -100\cos 45° + T_{AB} = 0$

$\therefore T_{AB} = 70.7\ \text{kN}$ ok.

$\Sigma F_y = 0 \quad 100\sin 45 - W_1 = 0 \quad W_1 = 70.7\ \text{kN}$ ok

At B: $\Sigma F_x = 0 \quad -70.7 + T_{BD}\cos 30° = 0$

$T_{BD} = 81.64\ \text{kN} < 100$ ok

S2-213 2/2

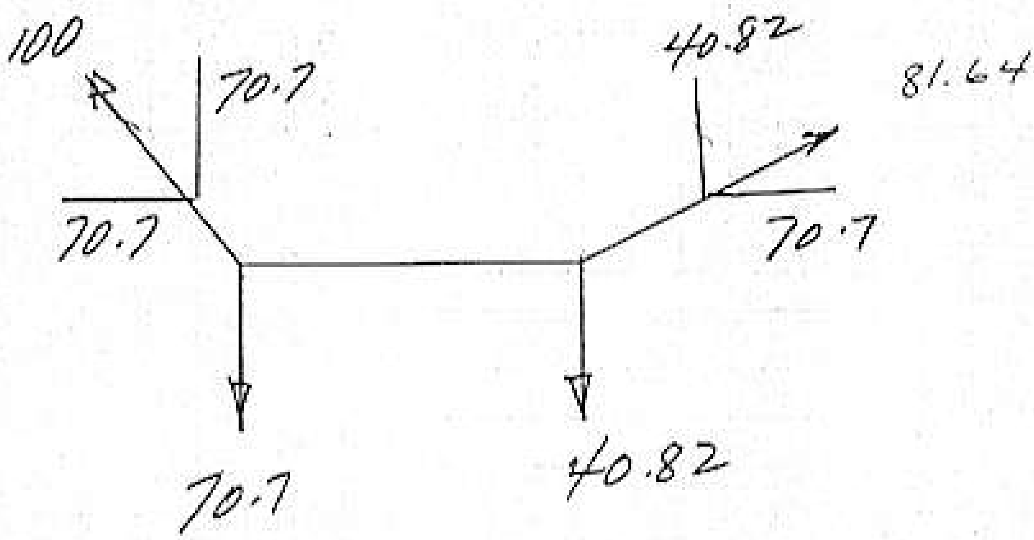
$\sum F_y = 0$

$81.64 \sin 30° - W_2 = 0$

$W_2 = 40.82$

$\therefore$ Combined mass

$$\frac{(W_1 + W_2)}{9.8} = \frac{70.7 + 40.82}{9.8} = \underline{11.38 \text{kg}}$$



S2-214 $\uparrow\!\!\rightarrow$ 1/2

$\Sigma F_x = 0$

$$13 + \frac{2}{\sqrt{5}} F_{BD} + \frac{1}{\sqrt{2}} F_{BC} = 0 \quad (1)$$

$\Sigma F_y = 0$ $\quad 6.5 - 4 + \frac{1}{\sqrt{5}} F_{BD} - \frac{1}{\sqrt{2}} F_{BC} = 0 \ (2)$

$$\frac{2}{\sqrt{5}} F_{BD} + \frac{1}{\sqrt{2}} F_{BC} = -13$$

$$\frac{1}{\sqrt{5}} F_{BD} - \frac{1}{\sqrt{2}} F_{BC} = -2.5$$

$$\frac{3}{\sqrt{5}} F_{BD} = -15.5 \text{ kN}$$

$$F_{BD} = +11.55 \text{ kN}$$

∴ Sense F_{BD} Assumed incorrectly

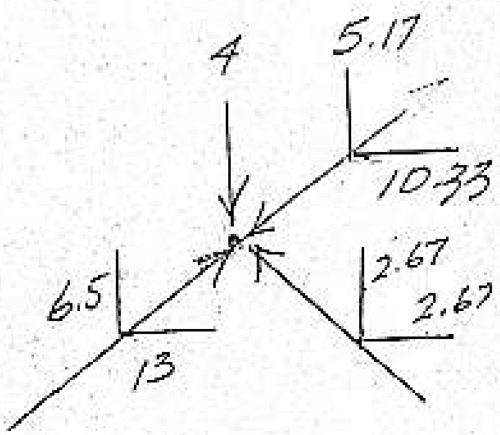
$\vec{F}_{BD} = 11.55 \text{ kN}$

$$\frac{2}{\sqrt{5}}(-11.55) + \frac{1}{\sqrt{2}} F_{BC} = -13$$

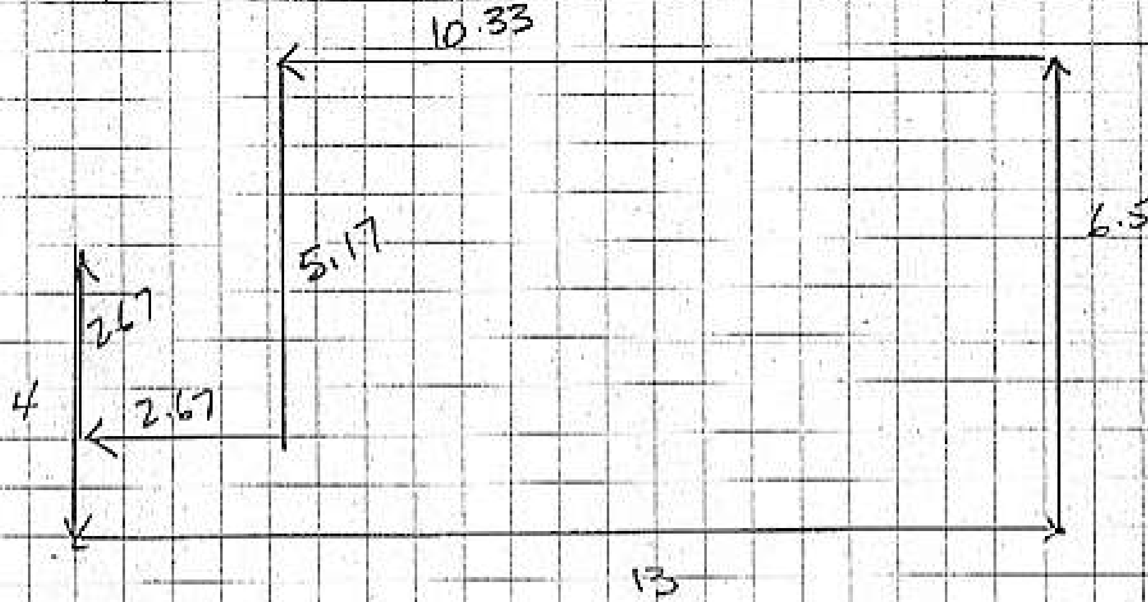
$$F_{BC} = -3.78 \text{ kN}$$

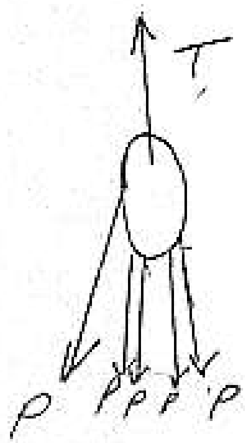
Sense $\vec{F}_{BC}$ assumed incorrectly

$\vec{F}_{BC} = 3.78 \text{ kN}$



52-214 2/2





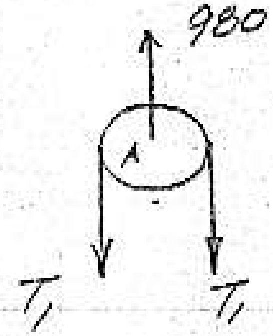
Common cord around pulleys. Tension remains constant

$T = 5P = 5(500)$

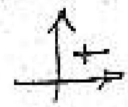
$= 2500\,N$

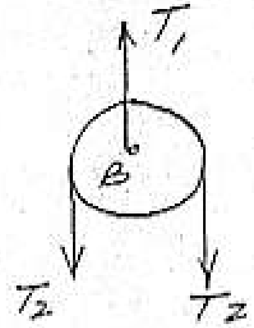
5-2-216

$100\,kg = 100(9.8) = 980\,N$



FBD pulley at A





FBD pulley at B

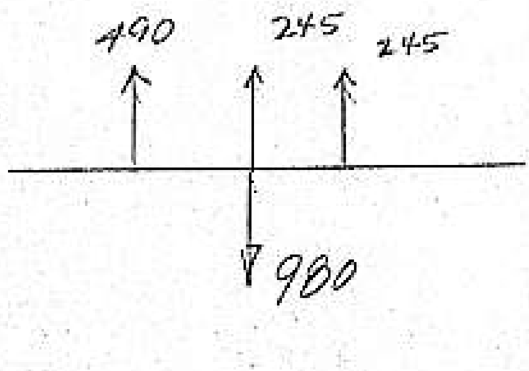
Pulley at A: $\Sigma F_y = 0$

$980 - T_1 - T_1 = 0 \qquad T_1 = 490\,N$

Pulley at B

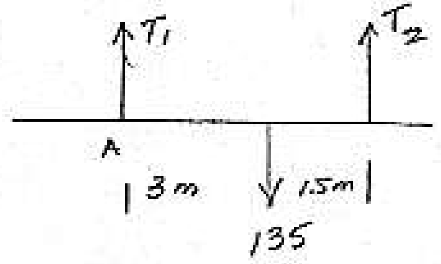
$T_1 - T_2 - T_2 = 0$

$490 - 2T_2 = 0 \qquad T_2 = 245\,N$



52-217

FBD Block



$\Sigma M_A = 0$

$-135(3) + T_2(4.5) = 0$

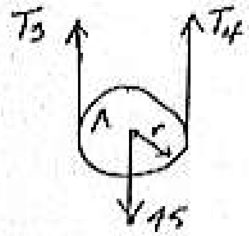
$T_2 = \frac{135(3)}{4.5} = 90\text{ kN}$

$\vec{T_2} = 90\text{ kN} \uparrow$ on the block

$\Sigma F_y = 0 \quad T_1 + 90 - 135 = 0$

$T_1 = 45\text{ kN} \quad \therefore \vec{T_1} = 45\text{ kN} \uparrow$ on the block

FBD Pulley A



$\Sigma M_A = 0 \quad -T_3(r) + T_4(r) = 0$

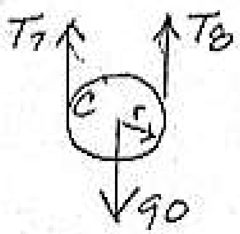
$\therefore T_3 = T_4$

$\Sigma F_y = 0$

$2T_3 - 45 = 0$

$T_3 = T_4 = 22.5\text{ kN}$

FBD Pulley B



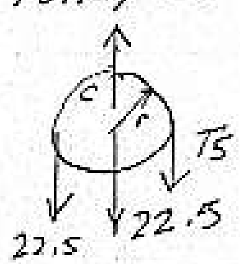
$\Sigma M_C = 0 \quad -T_7(r) + T_8(r) = 0$

$T_7 = T_8$

$2T_7 = 90$

$T_7 = T_8 = 45\text{ kN}$

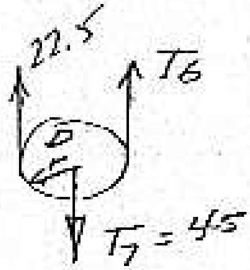
FBD Pulley C



$\Sigma M_C = 0 \quad -22.5(r) + T_5(r) = 0$

$T_5 = 22.5\text{ kN}$

FBD Pulley D



$\Sigma M_D = 0$

$-22.5 + T_6(r) = 0$

$T_6 = 22.5\text{ kN}$

$\Sigma F_y = 0 \quad 22.5 + 22.5 - 45 = 0$

Checks

S2-218

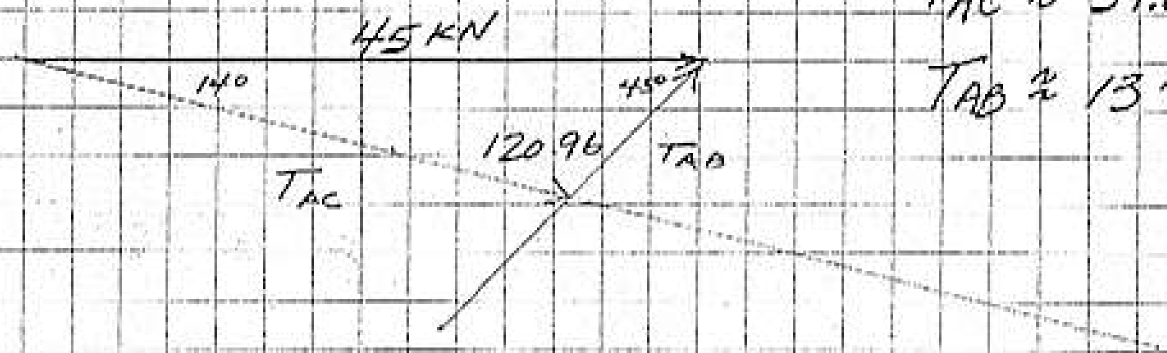
a)

FORCE	MAG	DIR
T_{AB}	?	45°
T_{AC}	?	14°
R	45°	0°

$\tan\theta = \frac{1}{1} = 45°$

$\tan\theta = \frac{1}{4} = 14.04°$

$180 - 14.04 - 45° = 120.96°$



$T_{AC} \approx 37.8$ KN

$T_{AB} \approx 13$ KN

b) $\frac{45}{\sin 120.96} = \frac{T_{AB}}{\sin 14°} = \frac{T_{AC}}{\sin 45°}$

$T_{AB} = \frac{45}{\sin 120.96} \sin 14° = 12.73$ KN

$T_{AC} = \frac{45}{\sin 120.96} \sin 45° = 37.11$ KN

c) $R_x = \Sigma F_x \qquad 45 = T_{AC}\cos 14.04° + T_{AB}\cos 45°$

$R_y = \Sigma F_y = 0 \qquad 0 = -T_{AC}\sin 14.04° + T_{AB}\sin 45°$

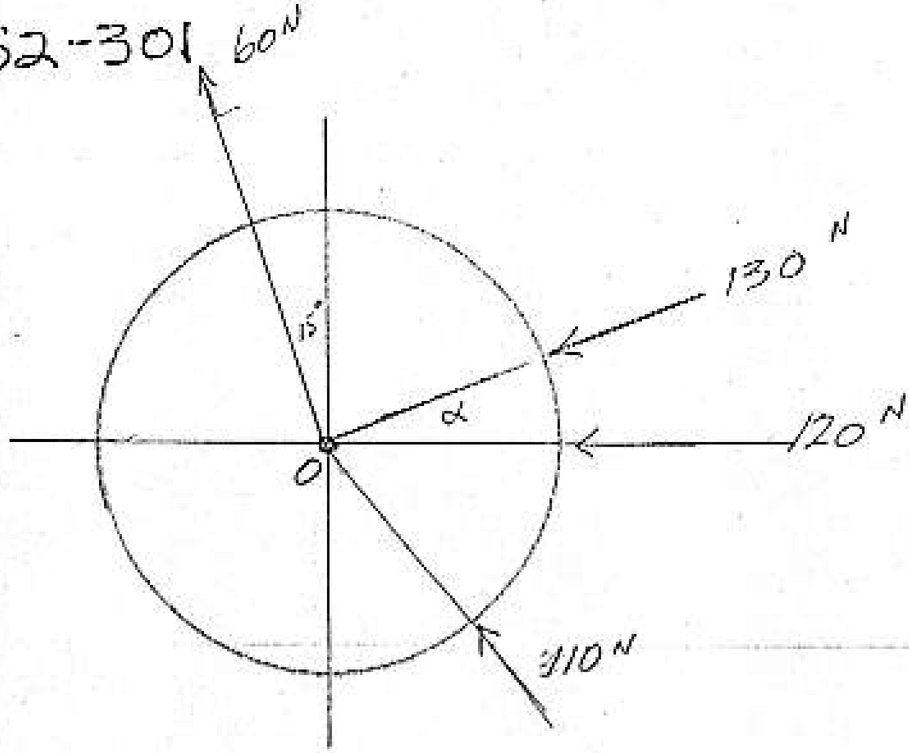
$0.9701\, T_{AC} + 0.707\, T_{AB} = 45$

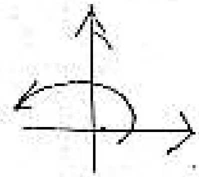
$-0.2426\, T_{AC} + 0.707\, T_{AB} = 0$

$1.2127\, T_{AC} = 45 \qquad T_{AC} = 37.11$ KN

$\frac{+0.2426(37.11)}{0.707} = T_{AB} \qquad T_{AB} = 12.73$ KN

S2-301





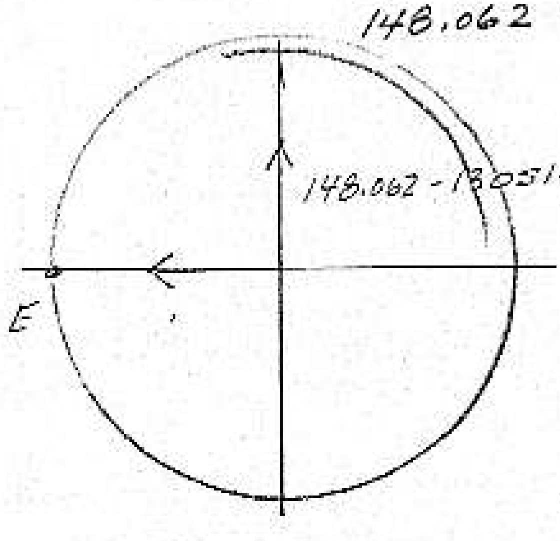
Apply Principle of Transmissibility to apply all forces at O

$$R_x = \Sigma F_x = -60\sin 15° - 130\cos\alpha - 120 - 110\sin 35°$$

$$= -190.625 - 130\cos\alpha$$

$$R_y = \Sigma F_y$$
$$= 60\cos 15° - 130\sin\alpha + 110\cos 35°$$

$$148.062 - 130\sin\alpha$$



$r = 0.75$ m

$\vec{M}_E = 50$ N.m

$$(148.062 - 130\sin\alpha)0.75 = 50$$

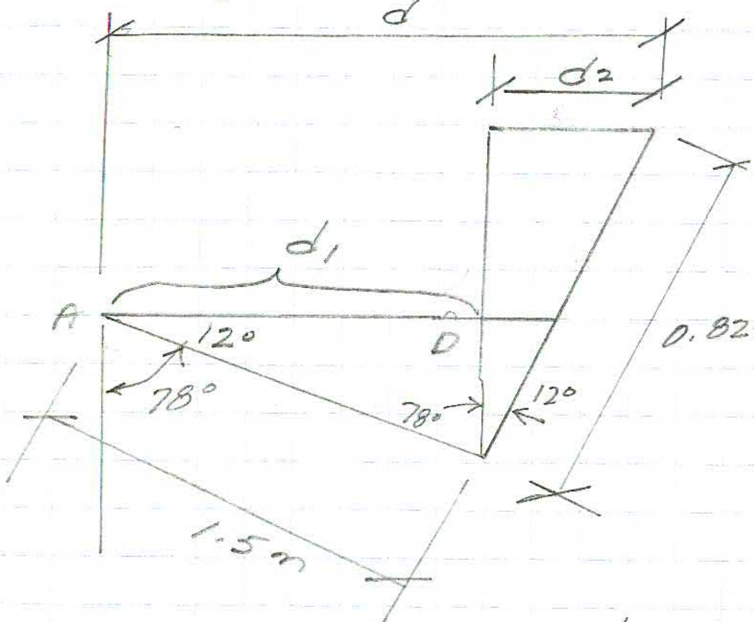
$$111.047 - 97.5\sin\alpha = 50$$

$$\sin\alpha = 0.62612$$

$$\Longrightarrow \alpha = 38.76°$$

52-302

a) From geometry, determine d



$d = d_1 + d_2$

$\cos 12° = \frac{d_1}{1.5}$

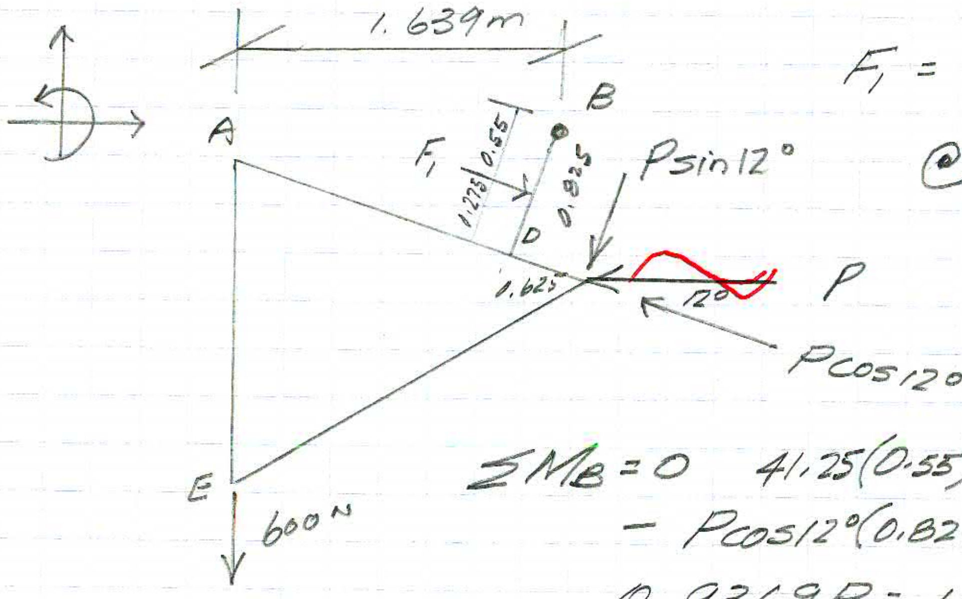
$d_1 = 1.5\cos 12° = 1.467\,m$

$\sin 12° = \frac{d_2}{0.825}$

$d_2 = 0.825\sin 12° = 0.172\,m$

$\therefore\ d = d_1 + d_2 = 1.467 + 0.172 = 1.639\,m$ ◄

b) Given $M_B = 0$ determine magnitude of P



$F_1 = \frac{100(0.825)}{2} = 41.25^N$

@ $\frac{0.825}{3} = 0.275^m$

$\frac{2}{3}(0.825) = 0.55^m$

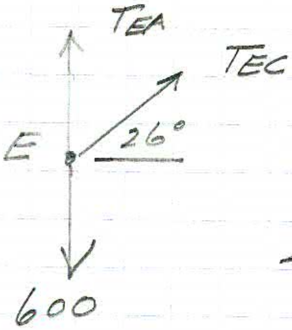
$\Sigma M_B = 0 \quad 41.25(0.55) + 600(1.639)$

$- P\cos 12°(0.825) - P\sin 12°(0.625) = 0$

$0.9369P = 1006.09$

$\therefore\ \vec{P} = 1073.85^N \leftarrow$ ◄

c)



$\Sigma F_x = 0$

$T_{EC}\cos 26° = 0 \quad \therefore\ T_{EC} = 0$ ◄

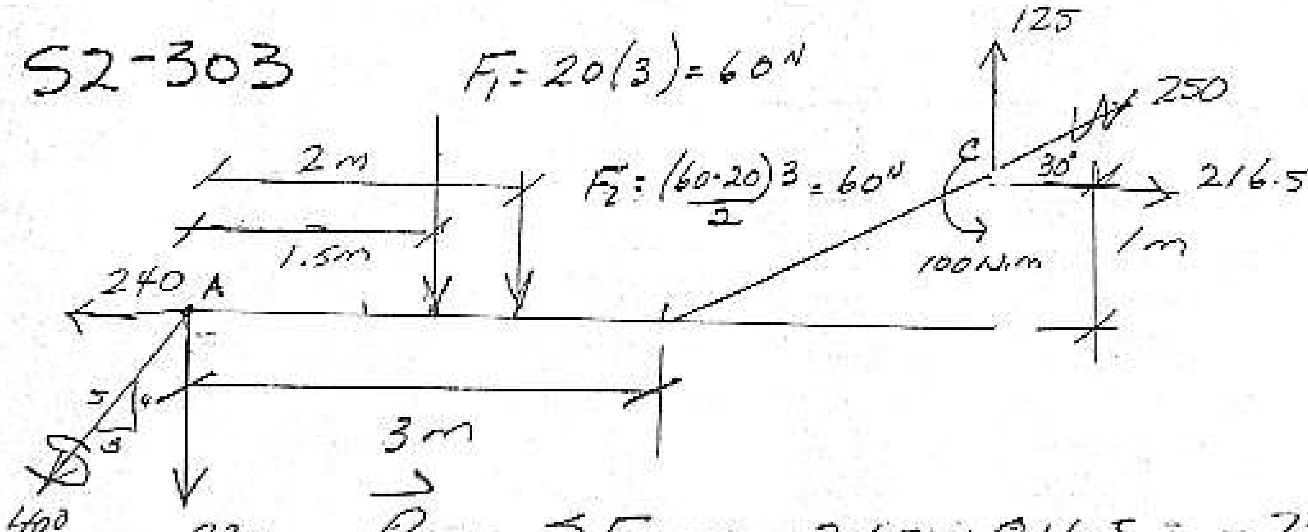
$\Sigma F_y = 0 \quad T_{EA} + T_{EC}\sin 26 - 600 = 0$

$\therefore\ T_E = +600N \quad \vec{T}_{EA} = 600^N \uparrow$ ◄

S2-303

$F_1 = 20(3) = 60^N$

$F_2 = \frac{(60-20)}{2} 3 = 60^N$



$\vec{R}_x = \Sigma F_x = -240 + 216.5 = -23.5^N$

$\vec{R}_x = 23.5^N \leftarrow$

$\vec{R}_y = \Sigma F_y = -320 - 60 - 60 + 125 = -315^N$

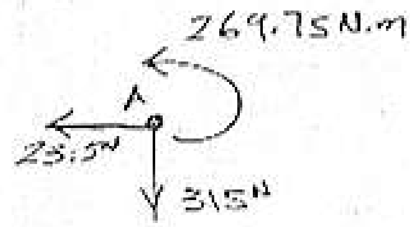
$\vec{R}_y = 315^N \downarrow$

$\vec{M}_{R_A} = -240(0.25) - 60(1.5) - 60(2) + 125(5.25) - 216.5(1) + 100$

$= +269.75 N.m$

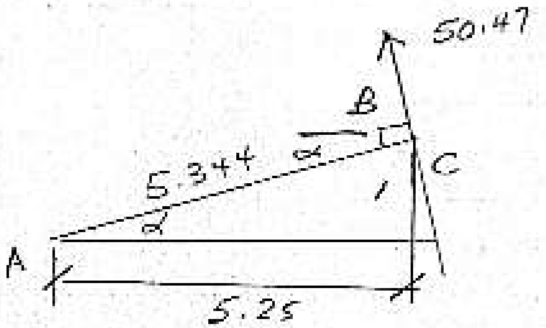
$\vec{M}_{R_A} = 269.75 N.m \circlearrowleft$

a) Equivalent Force-Couple at A



b) $\vec{AC} = \sqrt{5.25^2 + 1^2} = 5.344^m$

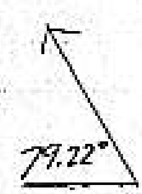
$M = Fd \quad F = \frac{M}{d} = \frac{269.75}{5.344} = 50.47 N$



$\tan \alpha = \frac{1}{5.25}$

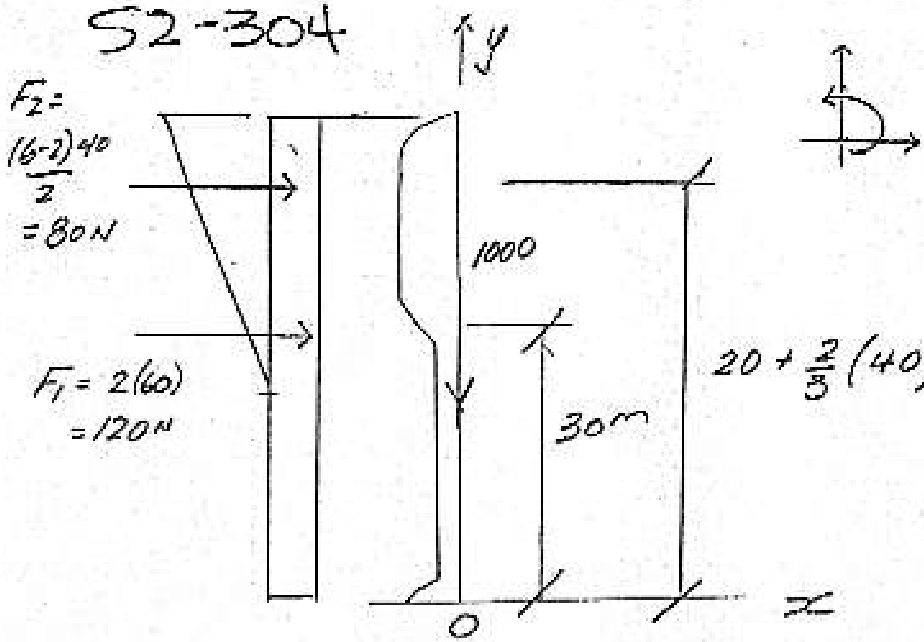
$\alpha = 10.78°$

$\vec{F} = 50.47^N$



$\beta = 90 - 10.78° = 79.22°$

52-304



$20 + \frac{2}{3}(40) = 46.67m$

a)

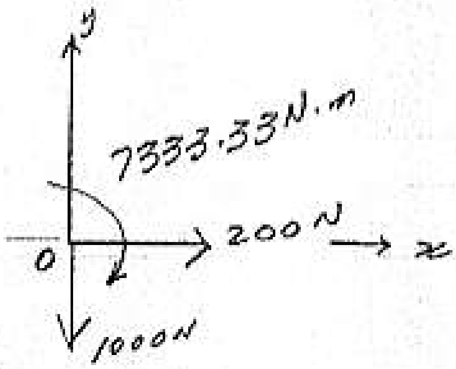
$R_x = \Sigma F_x = 120 + 80 = +200N \quad \vec{R}_x = 200N \rightarrow$

$R_y = \Sigma F_y = -1000N \quad \vec{R}_y = 1000N \downarrow$

$R = \sqrt{1000^2 + 200^2} = 1019.8N$

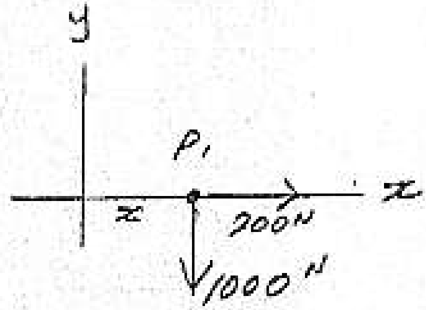
$M_{OR} = -80(46.67) - 120(30) = -7333.33 N.m$

$\therefore \vec{M}_{OR} = 7333.33 N.m$ ↻



equivalent force-couple at O

b)

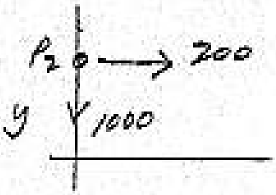


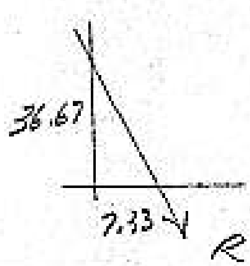
$M_{P_1} = 0 \quad -7333.33 + 1000x = 0$

$x = 7.33m$

$M_{P_2} = 0 \quad -7333.33 + 200y = 0$

$y = 36.67m$



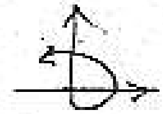


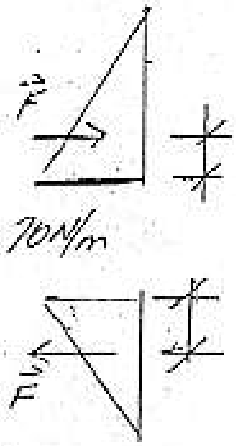
$\frac{7.33}{36.67} = 0.2$

$\frac{200}{1000} = 0.2$

S2-305

a) $\vec{F} = \frac{1}{2}(70)(0.21) = 7.35\ N$





$\frac{0.21}{3} = 0.07m$

$0.07m$

$M = 7.35(0.21 - 0.07 - 0.07)$

$\vec{M} = 0.5145\ N.m$

b) $\vec{R}_x = \Sigma F_x = 7.35 - 150\cos 60° + 100\cos 60° - 7.35$

$R_x = -75 + 50 = -25N$

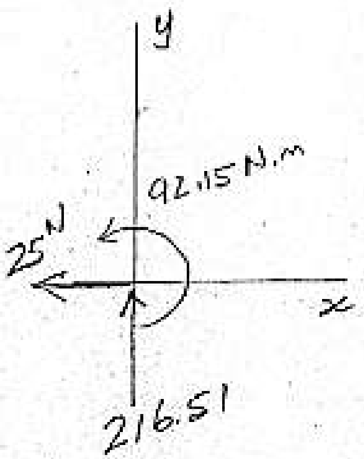
$\therefore \vec{R}_x = 25N \leftarrow$

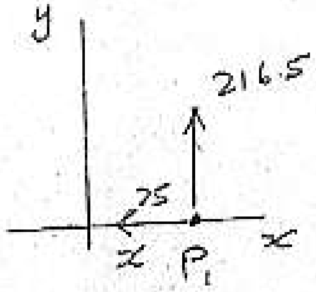
$\vec{R}_y = \Sigma F_y \quad R_y = 150\sin 60 + 100\sin 60°$

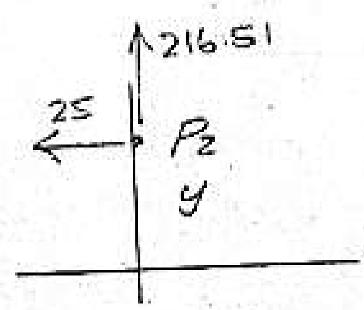
$R_y = +216.51N \quad \therefore \vec{R}_y = 216.51N \uparrow$

$\vec{M}_{R_O} = 0.5145 + 150\cos 60°(0.43)$
$+ 150\sin 60°(0.72) - 100\cos 60°(0.43)$
$+ 100\sin 60°(0.72) - 75$

$M_{R_O} = +92.15\ N.m \quad \vec{M}_{R_O} = 92.15\ N.m$







$M_{P_1} = 0$
$92.15 - 216.51x = 0$

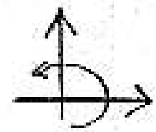
$x = 0.426m$

$M_{P_2} = 0$
$92.15 - 25y = 0$

$y = 3.686\ m$

$R = \sqrt{25^2 + 216.51^2} = 217.95^N$

S2-306



$R_x = \Sigma F_x = +60 - 20 = +40N$

$\therefore \vec{R_x} = 40N \rightarrow$

$R_y = \Sigma F_y = +60 - 90 = -30N$

$\therefore \vec{R_y} = 30N \downarrow$

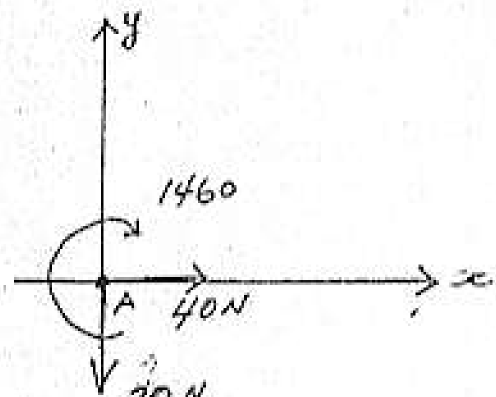
$M_{R_A} = -90(12) - 60(4) - 140 = -1460 \, N.m$

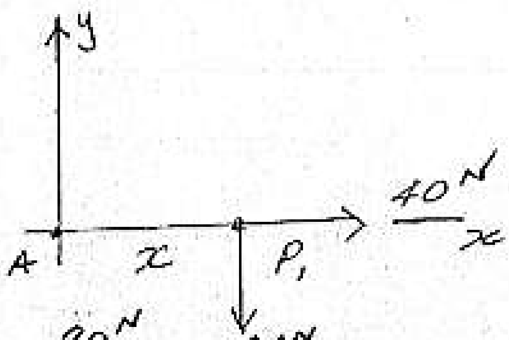
$\therefore \vec{M_{R_A}} = 1460 \, N.m$

$R = \sqrt{(40)^2 + (30)^2}$

$R = 50N$

a)





$M_{P_1} = 0$

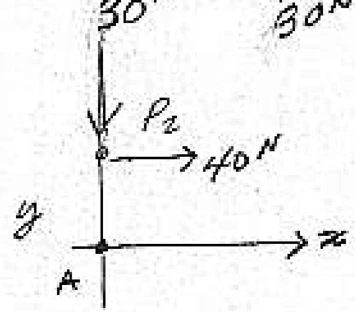
$-1460 + 30x = 0$

$x = 48.66 \, m$

$M_{P_2} = 0$

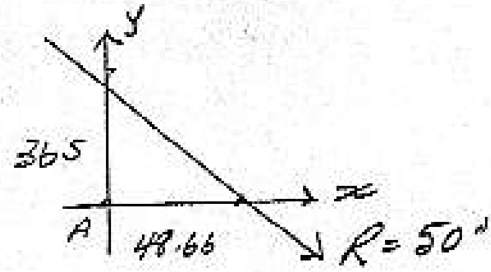
$-1460 + 40y = 0$

$y = 36.5 m$



$\frac{36.5}{48.66} = 0.75$

$\frac{30}{40} = 0.75$

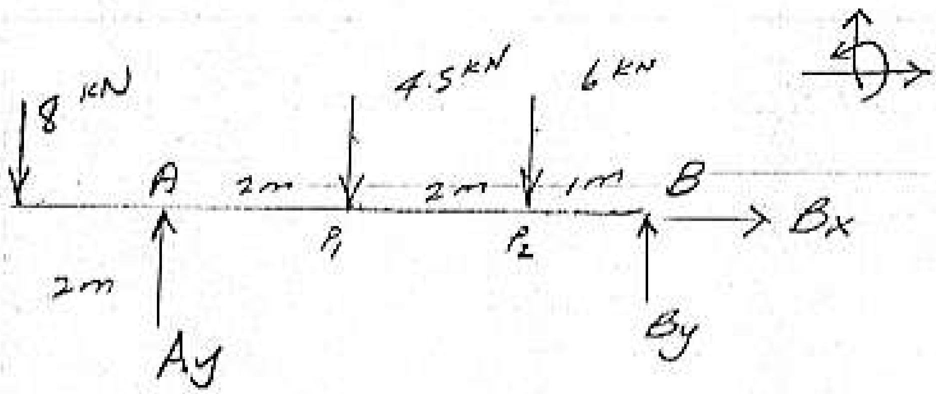


52-307

We replace distributed loads by point loads and draw the FBD

$$F_1 = \frac{3(3)}{2} = 4.5^{kN} \quad @ 2m \text{ from } A$$

$$F_2 = 3(2) = 6\ kN \quad @ 4m \text{ from } A$$



$\Sigma F_x = 0 \quad B_x = 0$

$\Sigma M_B = 0 \quad 8(7) - A_y(5) + 4.5(3) + 6(1) = 0$

$A_y = +15.1 \quad \therefore \vec{A_y} = 15.1^{kN} \uparrow$

$\Sigma F_y = 0 \quad -8 + 15.1 - 4.5 - 6 + B_y = 0$

$B_y = +3.4$

$\therefore \vec{B_y} = 3.4^{kN} \uparrow$

Check $\quad \Sigma M_{P_1} = 0$

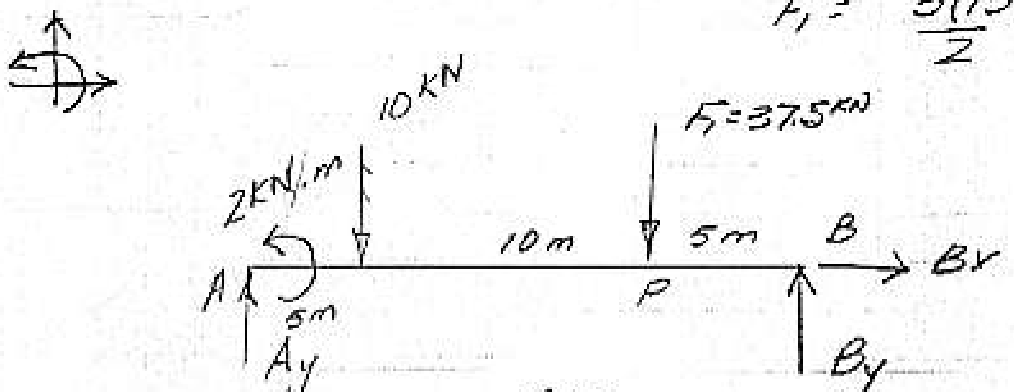
$8(4) - 15.1(2) - 6(2) + 3.4(3) = 0$

$0 = 0 \checkmark$

S2-308

We replace the distributed load by a point load and draw the FBD.

$F_1 = \frac{5(15)}{2} = 37.5^{kN}$



FBD

$\Sigma F_x = 0 \quad B_x = 0$

$\Sigma M_B = 0$

$$2 - A_y(20) + 10(15) + 37.5(5) = 0$$

$$A_y = +16.975 \quad \therefore \vec{A}_y = 16.975^{kN} \uparrow$$

$\Sigma F_y = 0 \quad 16.975 - 10 - 37.5 + B_y = 0$

$$B_y = +30.525$$

$$\therefore \vec{B}_y = 30.525^{kN} \uparrow$$

Check $\Sigma M_P = 0$

$$2 - 16.975(15) + 10(10) + 30.525(5) = 0$$

$$0 = 0 \checkmark$$

52-309

We replace the distributed loads by point loads and draw the FBD.

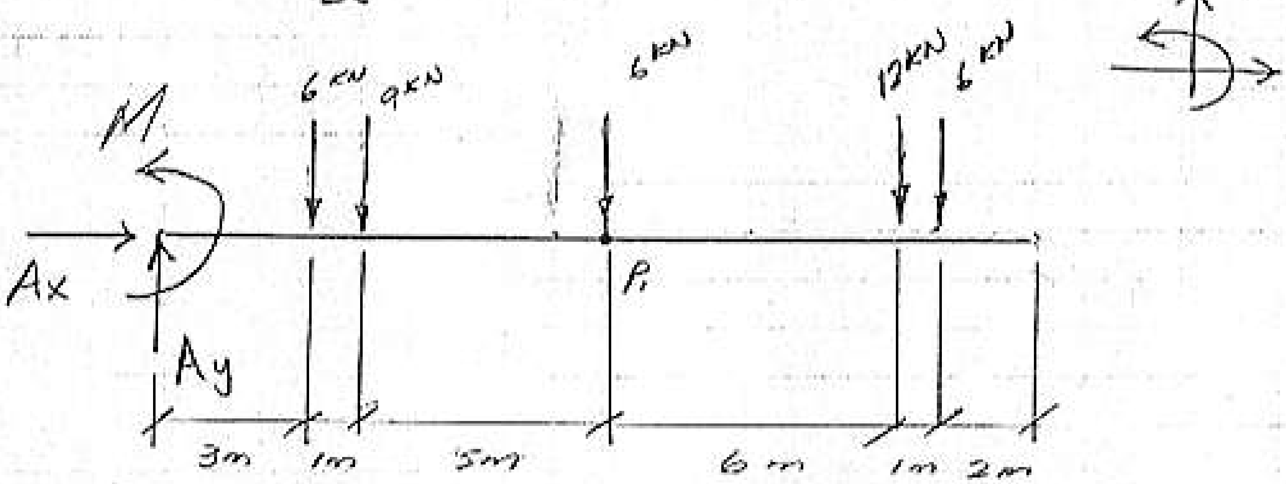
$F_1 = (1)(6) = 6\,kN$ @ 3 m from A

$F_2 = \frac{(4-1)(6)}{2} = 9\,kN$ @ 4 m from A

$F_3 = (1)(6) = 6\,kN$ @ 9 m from A

$F_4 = 2(6) = 12\,kN$ @ 15 m from A

$F_5 = \frac{(4-2)(6)}{2} = 6\,kN$ @ 16 m from A



$\Sigma F_x = 0 \quad A_x = 0$

$\Sigma F_y = 0 \quad A_y - 6 - 9 - 6 - 12 - 6 = 0$

$A_y = +39\,kN \qquad A_y = 39\,kN \uparrow$

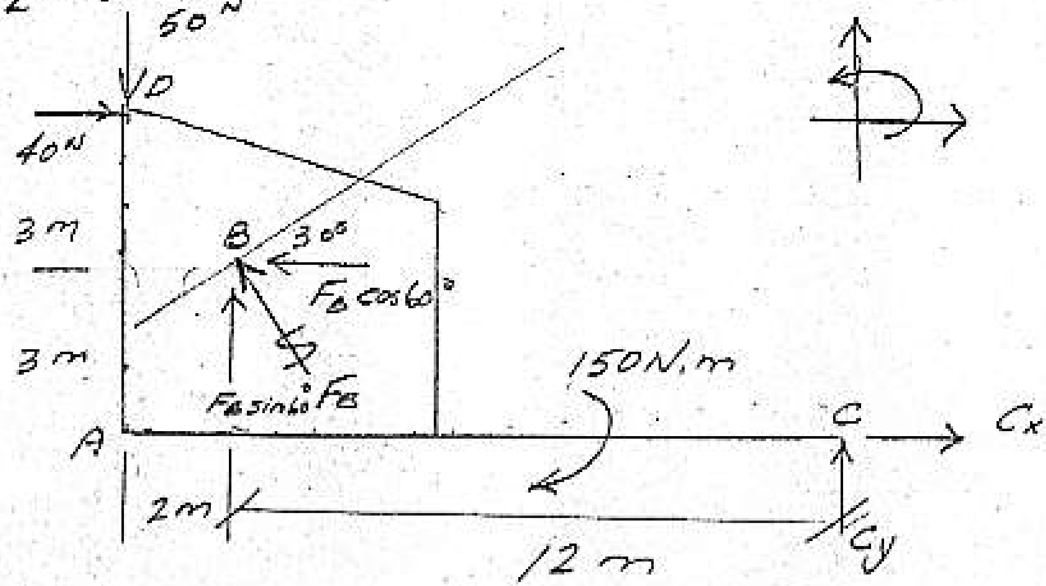
$\Sigma M_A = 0 \quad M - 6(3) - 9(4) - 6(9) - 12(15) - 6(16) = 0$

$M = +384\,kN.m \qquad \vec{M} = 384\,kN.m \circlearrowleft$

check $M_{P_1} = 0 \quad 384 - 39(9) + 6(6) + 9(5) - 12(6) - 6(7) = 0$

$0 = 0$ ✓

S2-310



$$\sum F_x = 0 \rightarrow \quad 40 - F_B \cos 60° + C_x = 0 \quad (1)$$

$$\sum F_y = 0 \uparrow \quad -50 + F_B \sin 60° + C_y = 0 \quad (2)$$

$$\sum M_C = 0 \circlearrowleft \quad 50(14) - 40(6) + F_B \cos 60°(3)$$
$$- F_B \sin 60°(12) - 150 = 0 \quad (3)$$

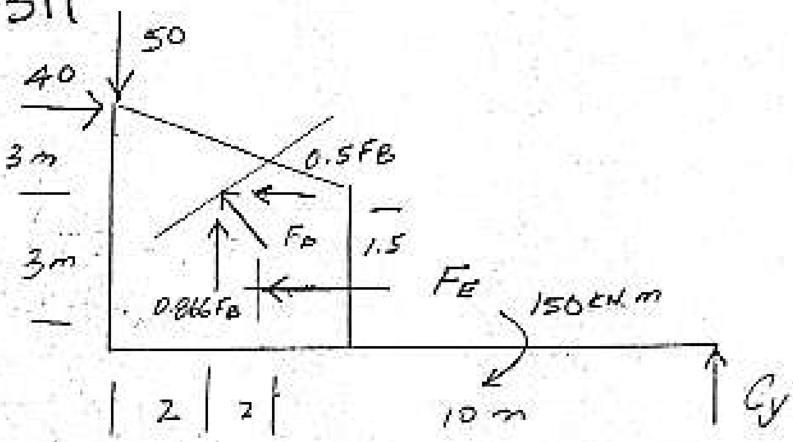
$$310 + 1.5F_B - 10.39F_B = 0$$
$$-8.89F_B = -310$$
$$F_B = +34.86^N$$
$$\therefore \vec{F_B} = 34.86^N \nwarrow 60°$$

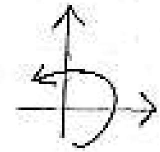
$$C_x = 34.86 \cos 60° - 40$$
$$C_x = -22.57^N \quad \therefore \vec{C_x} = 22.57^N \leftarrow$$
$$C_y = 50 - 34.86 \sin 60° = +19.81^N$$
$$\therefore \vec{C_y} = 19.81^N \uparrow$$

<u>Check</u> $\sum M_B = 0 \quad 50(2) - 40(3) - 150$
$$+ 19.81(12) - 22.57(3) = 0$$

$$0.01 = 0 \checkmark$$

52-311





$$\Sigma F_x = 0 \quad 40 - 0.5F_B - F_E = 0 \quad (1)$$

$$0.5F_B + F_E = 40 \qquad F_E = 40 - 0.5F_B$$

$$\Sigma F_y = 0 \quad -50 + 0.866F_B + C_y = 0 \quad (2)$$

$$0.866\,F_B + C_y = 50 \qquad C_y = 50 - 0.866F_B$$

$$\Sigma M_B = 0 \quad 50(2) - 40(3) - F_E(1.5) - 150 + C_y(12) = 0 \quad (3)$$

$$-1.5F_E + 12C_y = 170$$

$$-1.5(40 - 0.5F_B) + 12(50 - 0.866F_B) = 170$$

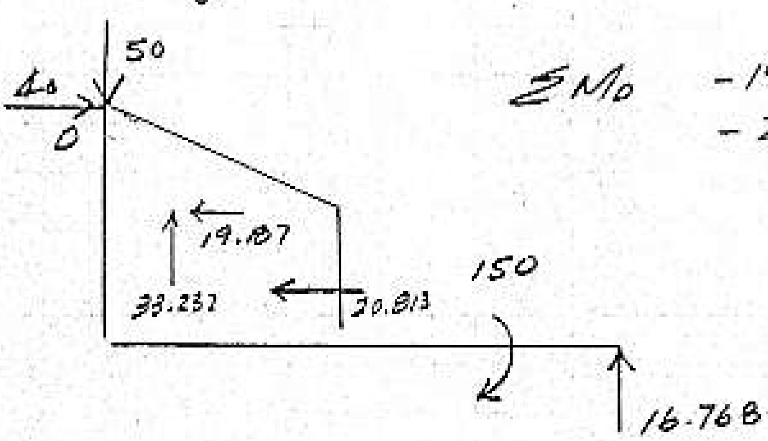
$$-60 + 0.75F_B + 600 - 10.392\,F_B = 170$$

$$-9.642F_B = -370$$

$$F_B = 38.374\text{kN}$$

$$F_E = 40 - 0.5(38.374) = 20.813\text{kN}$$

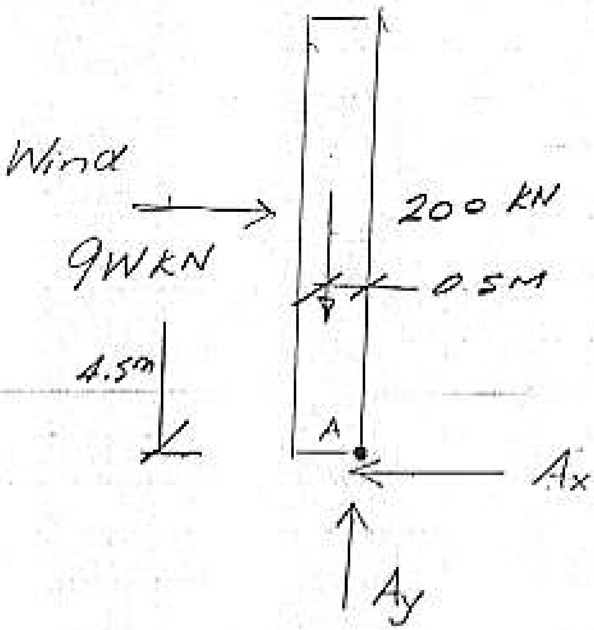
$$C_y = 50 - 0.866(38.374) = 16.768\text{ kN}$$



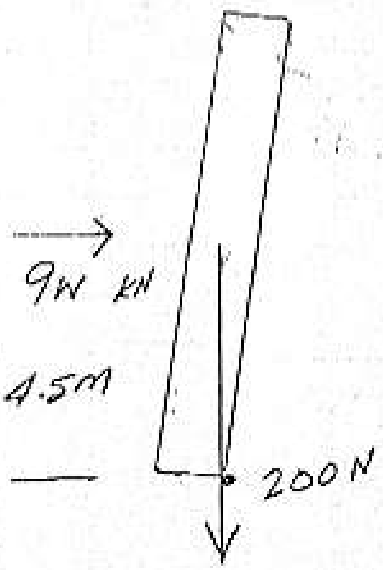
$$\Sigma M_O \quad -19.187(3) + 33.232(2)$$
$$-20.813(4.5) - 150$$
$$+16.768(14) = 0$$

$$0 = 0 \checkmark$$

S2-312



On theverge of overturning,



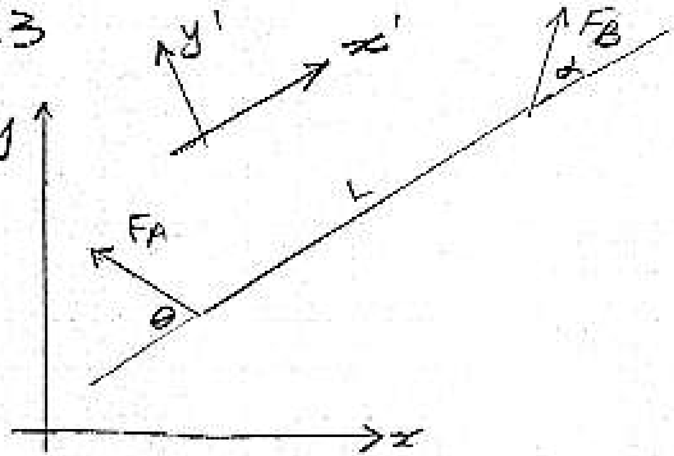
$$-9W(4.5) + 200(0.5) = 0$$

$$W = \frac{200(0.5)}{9(4.5)} = 2.47 \text{ kN/m}$$

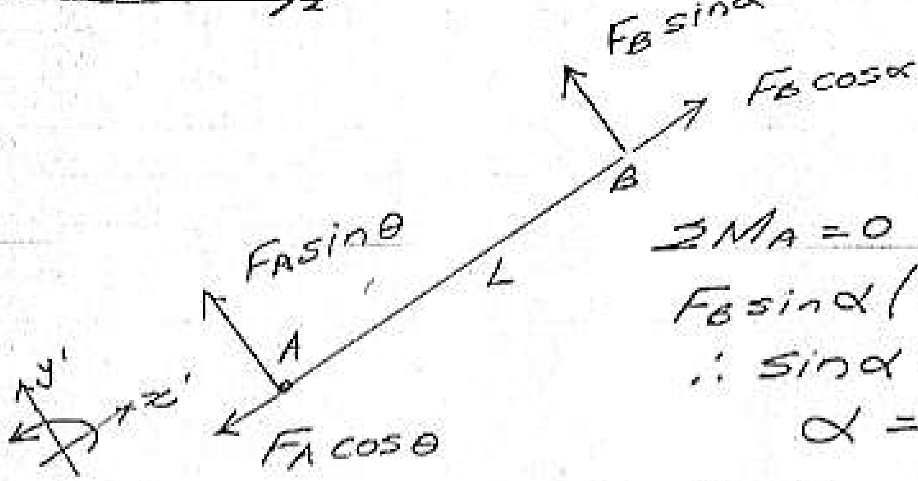
SAY 2.5 kN/m

52-313

1/2



We resolve F_A & F_B into rectangular components in x' & y' directions



$\Sigma M_A = 0$

$F_B \sin\alpha (L) = 0$

$\therefore \sin\alpha = 0$

$\alpha = 0°$

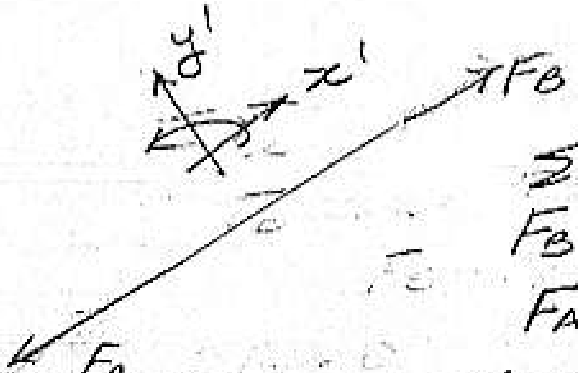
similiarly

$\Sigma M_B = 0$

$-F_A \sin\theta (L) = 0$

$\therefore \sin\theta = 0$

$\theta = 0°$

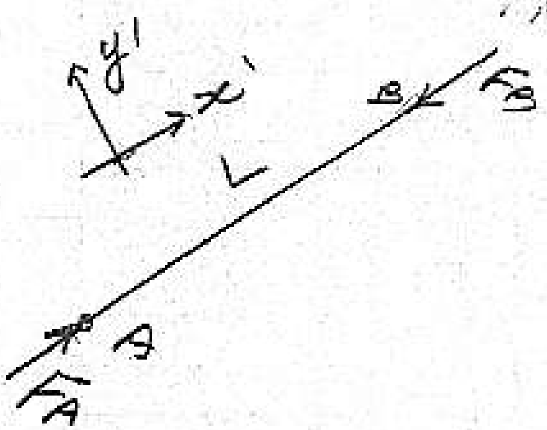


$\Sigma F_{x'} = 0$

$F_B - F_A = 0$

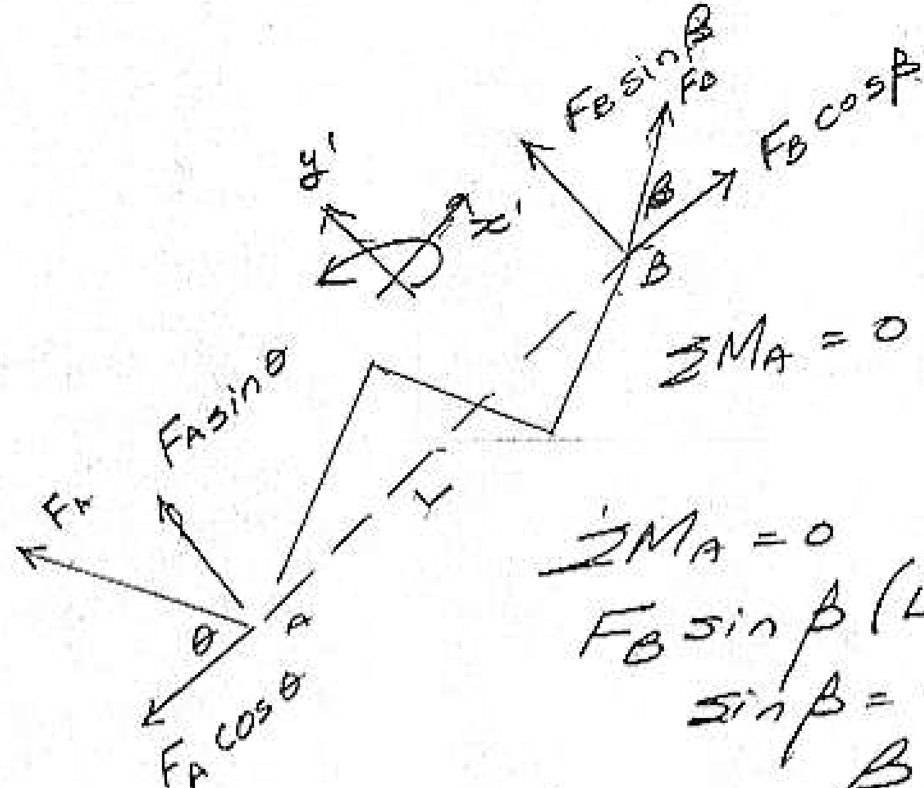
$F_A = F_B$

Magnitude $F_A = F_B$
opposite sense



S2-313 2/2

b)



$\Sigma M_A = 0$

$\Sigma M_A = 0$

$F_B \sin\beta (L) = 0$

$\sin\beta = 0$

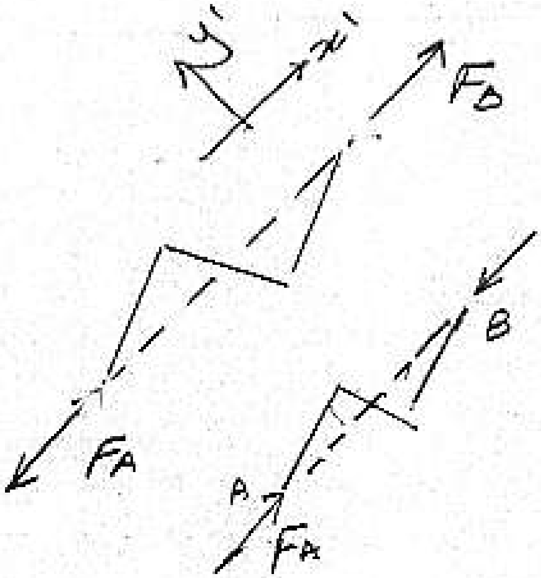
$\beta = 0^\circ$

$\Sigma M_B = 0$

$-F_A \sin\theta (L) = 0$

$\sin\theta = 0$

$\theta = 0^\circ$



$\Sigma F_{x'}$

$F_B \cos 180^\circ - F_A \cos 0^\circ = 0$

$F_A = F_B$

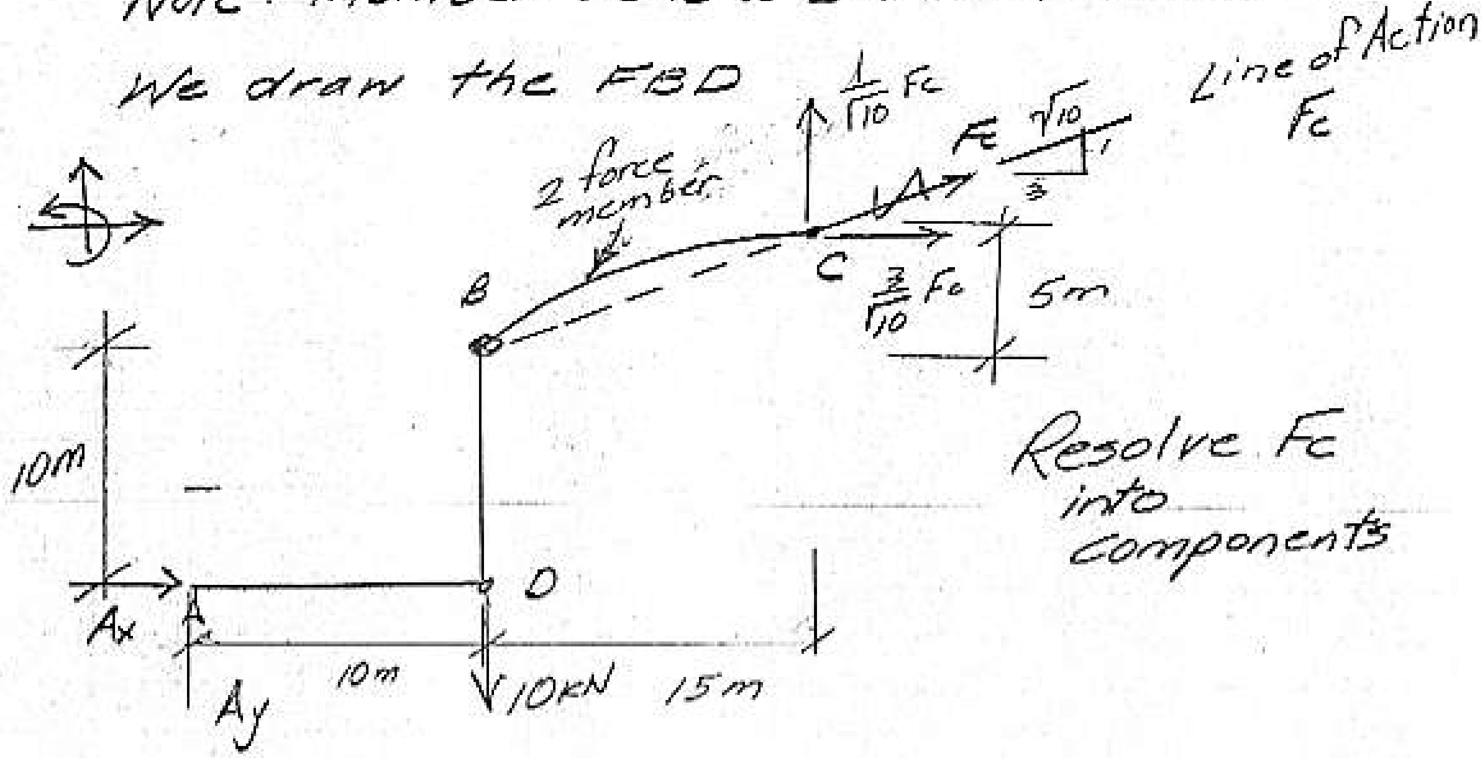
Magnitube $F_A = F_B$

Opposite sense

52-314

Note: Member BC is a 2-force member

We draw the FBD



$$\Sigma M_A = 0 \qquad \frac{1}{\sqrt{10}} F_C (25) - \frac{3}{\sqrt{10}} F_C (15) - 10(10) = 0$$

$$\frac{-20}{\sqrt{10}} F_C = 100 \quad \therefore F_C = -15.811 \text{ kN}$$

$$\therefore \vec{F_C} = 15.811 \text{ kN} \swarrow \; (3, 1)$$

$$\Sigma F_x = 0 \qquad A_x + \frac{3}{\sqrt{10}}(-15.811) = 0$$

$$A_x = +15 \quad \therefore \vec{A_x} = 15 \text{ kN} \rightarrow$$

$$\Sigma F_y = 0 \qquad A_y - 10 + \frac{1}{\sqrt{10}}(-15.811) = 0$$

$$A_y = +15 \text{ kN} \quad \vec{A_y} = 15 \text{ kN} \uparrow$$

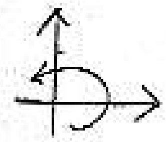
Check $\Sigma M_D = 0$

$$-15(10) - \frac{1}{\sqrt{10}}(15.811)(15) + \frac{3}{\sqrt{10}}(15.811)(15) = 0$$

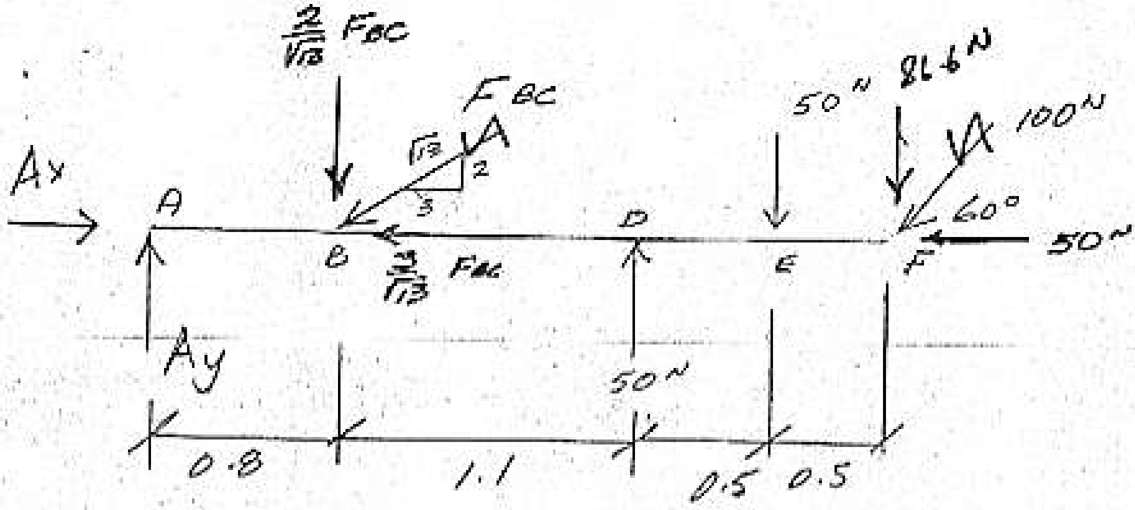
$$0 = 0 \checkmark$$

52-315

Member BC is a 2-force member



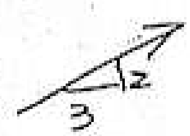
FBD of beam ABDEF



$$\Sigma M_A = 0 \quad -\frac{2}{\sqrt{13}} F_{BC}(0.8) + 50(1.9) - 50(2.4) - 86.6(2.9) = 0$$

$$-0.44376 F_{BC} - 276.14 = 0$$

$$F_{BC} = -622.27\ N$$

$$\therefore \vec{F}_{BC} = 622.27\ N$$ 

on ABDEF

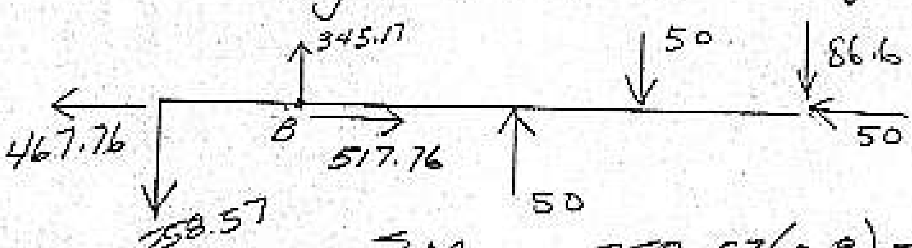
$$\Sigma F_x = 0$$

$$A_x - \frac{3}{\sqrt{13}}(-622.27) - 50 = 0$$

$$A_x = -467.76\ N \quad \therefore \vec{A}_x = 467.76\ N \leftarrow$$

$$\Sigma F_y = 0 \quad A_y - \frac{2}{\sqrt{13}}(-622.27) + 50 - 50 - 86.6 = 0$$

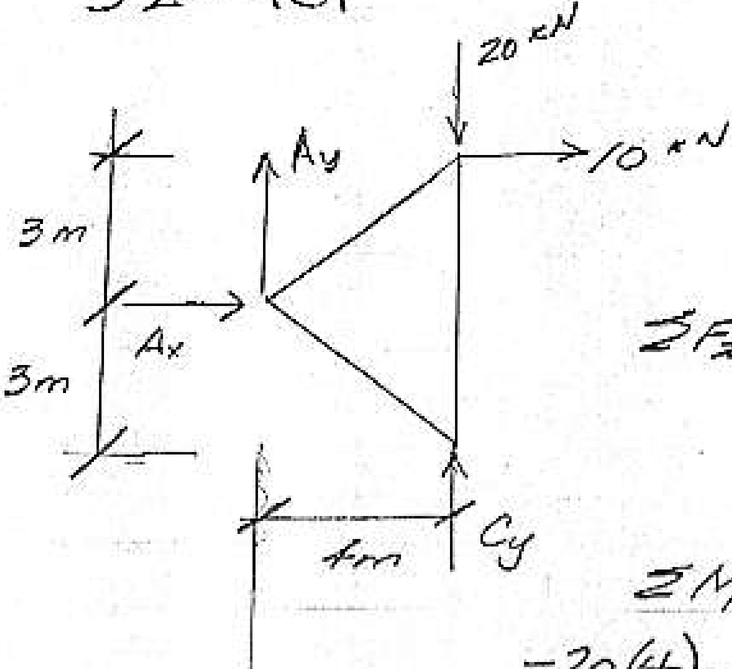
$$A_y + 258.57 = 0 \quad \therefore \vec{A}_y = 258.57 \downarrow$$

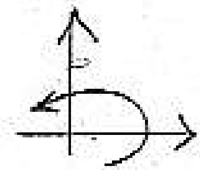


$$\Sigma M_B = 258.57(0.8) - 50(0.5) - 86.6(2.1) = 0$$

$$0.004 = 0 \checkmark$$

52-401





$$\Sigma F_x = 0 \qquad A_x + 10 = 0$$

$$\therefore A_x = -10\ kN$$

$$\therefore \vec{A_x} = 10\ kN \leftarrow$$

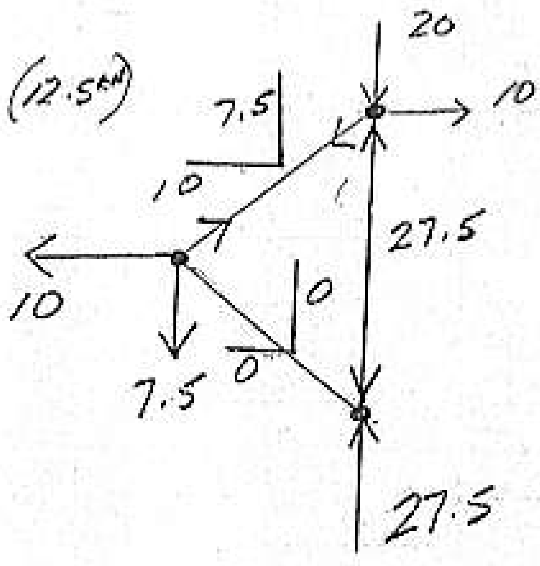
$$\Sigma M_A = 0$$

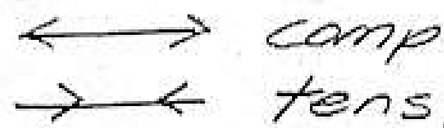
$$-20(4) - 10(3) + C_y(4) = 0$$

$$C_y = +27.5\ kN \quad \therefore \vec{C_y} = 27.5\ kN \uparrow$$

$$\Sigma F_y = 0 \qquad A_y - 20 + 27.5 = 0$$

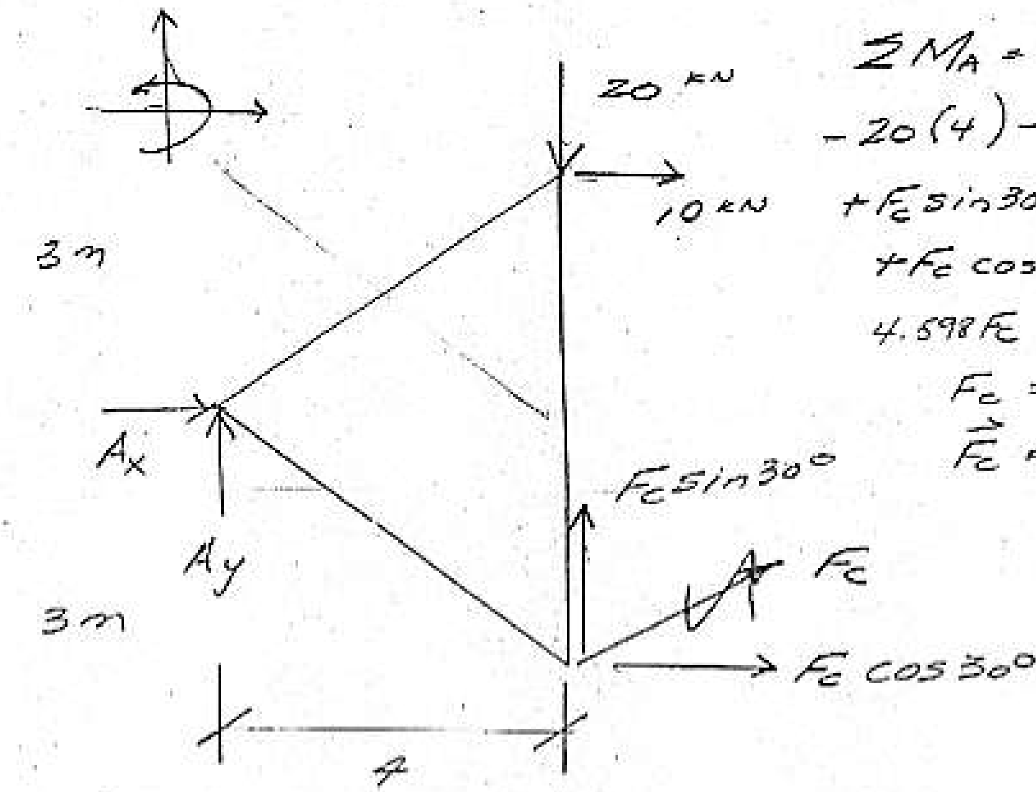
$$A_y = -7.5\ kN, \therefore \quad \vec{A_y} = 7.5\ kN \downarrow$$





SZ-402

CD is a short link. (2 force member)



$\Sigma M_A = 0$

$-20(4) - 10(3)$

$+ F_c \sin 30°(4)$

$+ F_c \cos 30°(3) = 0$

$4.598 F_c = 110$

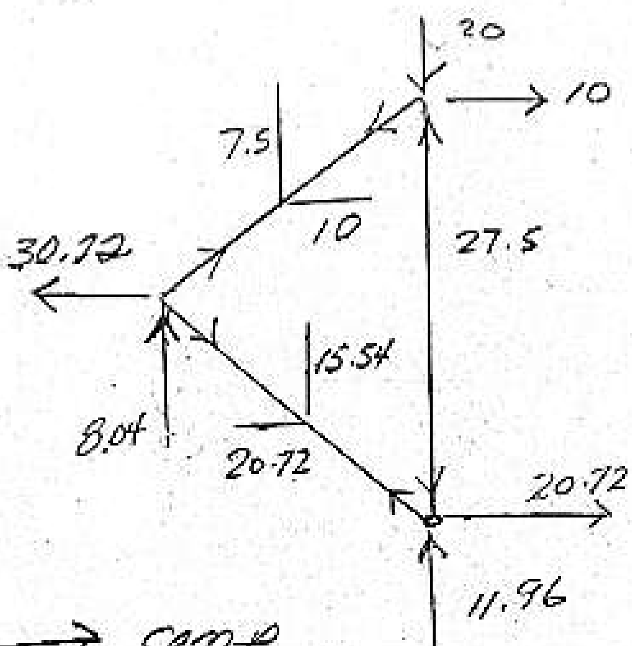
$F_c = +23.92$ kN

$\vec{F_c} = 23.92$ kN $\angle 30$

$\Sigma F_x = 0 \quad A_x + 10 + 23.92 \cos 30° = 0$

$A_x = -10.72$ kN $\therefore \vec{A_x} = 30.72$ kN $\leftarrow$

$\Sigma F_y = 0 \quad A_y - 20 + 23.92 \sin 30° = 8.04$ kN $\therefore \vec{A_y} = 8.04$ kN $\uparrow$



$\frac{20.72}{4} = \frac{y}{3} \quad y = 15.54$

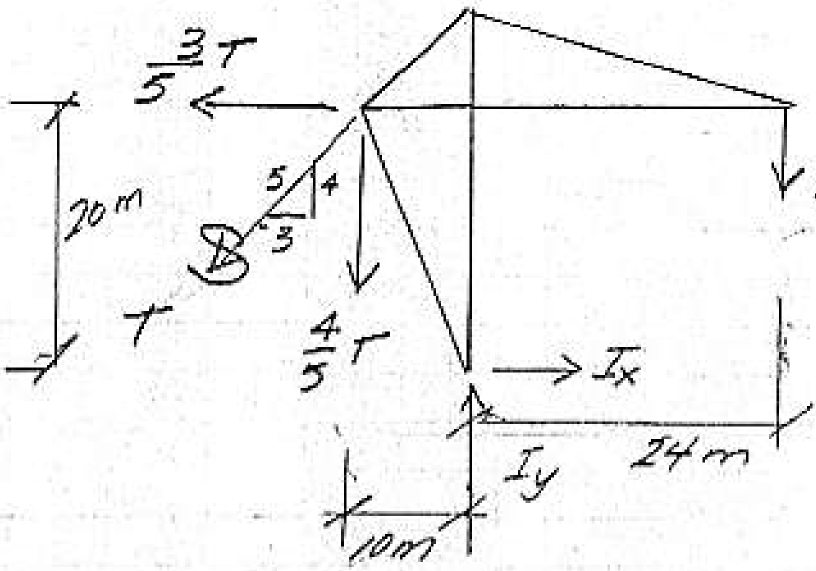
$\frac{7.5}{3} = \frac{x}{4} \quad x = 10$

$\longleftrightarrow$ comp

$\rightarrow\leftarrow$ tens

52-403

We draw the FBD of the truss & solve for equilibrium equations



$\Sigma M_I = 0$

$+\frac{3}{5}T(20) + \frac{4}{5}T(10) - 30(24) = 0$

$20T - 720 = 0$

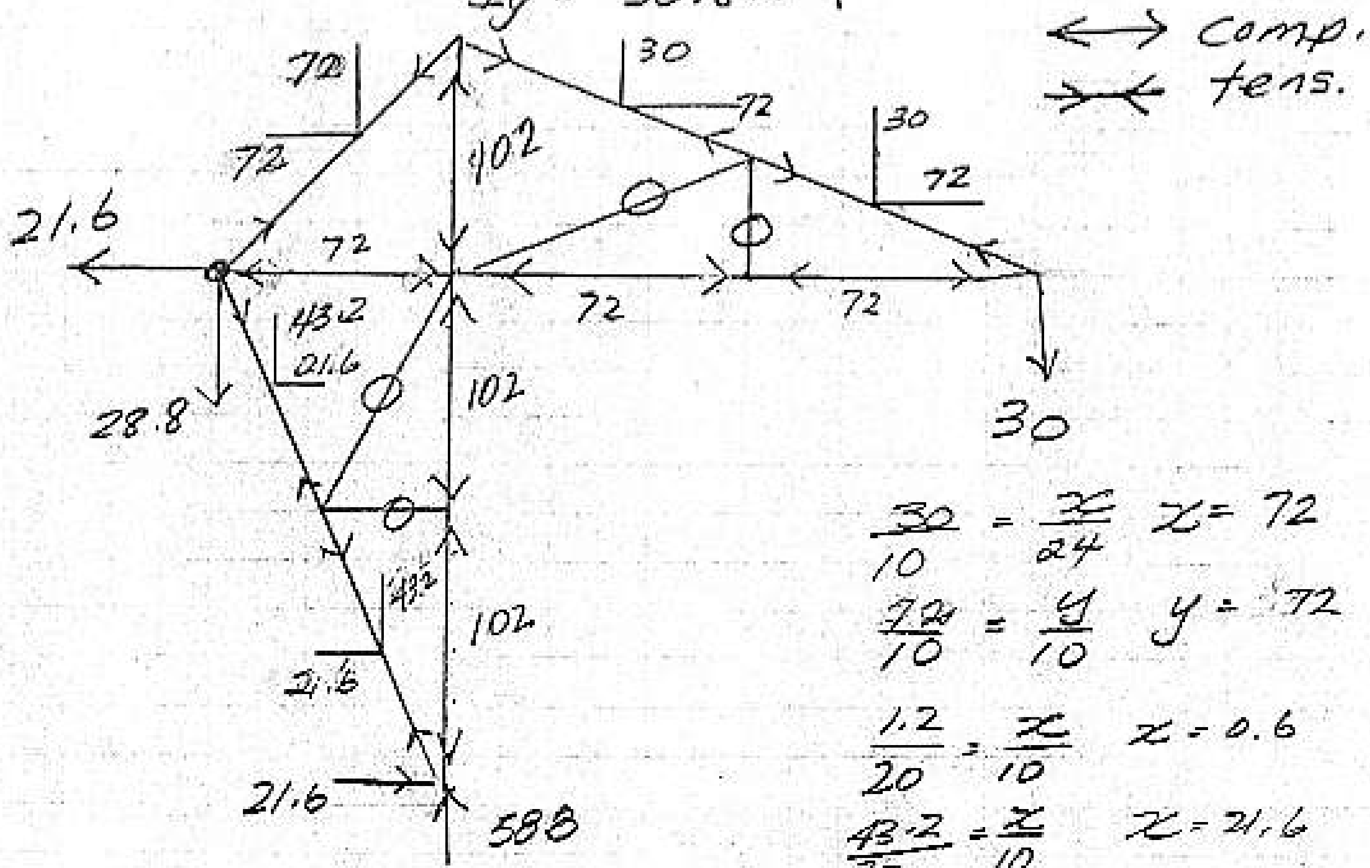
$T = 36$ kN

$\Sigma F_x = 0 \quad -\frac{3}{5}(36) + I_x = 0$

$I_x = +21.6$ kN $\therefore \vec{I_x} = 21.6$ kN $\rightarrow$

$\Sigma F_y = 0 \quad -\frac{4}{5}(36) + I_y - 30 = 0 \quad I_y = +58.8$ kN

$\vec{I_y} = 58.8$ N $\uparrow$



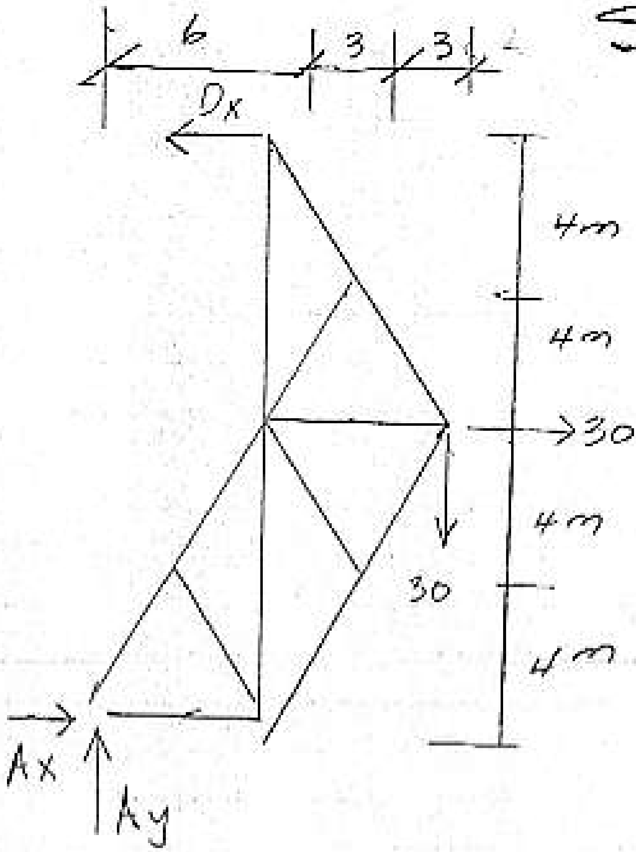
$\frac{30}{10} = \frac{x}{24} \quad x = 72$

$\frac{72}{10} = \frac{y}{10} \quad y = 72$

$\frac{1.2}{20} = \frac{x}{10} \quad x = 0.6$

$\frac{43.2}{20} = \frac{x}{10} \quad x = 21.6$

S2-404



$$\Sigma M_A = 0$$

$$D_x(16) - 30(12) - 30(8) = 0$$

$$D_x = +37.5 \text{ kN}$$

$$\therefore \vec{D}_x = 37.5 \text{ kN} \leftarrow$$

$$\Sigma F_x = 0$$

$$-37.5 + 30 + A_x = 0$$

$$A_x = +7.5 \text{ kN}$$

$$\therefore \vec{A}_x = 7.5 \text{ kN} \rightarrow$$

$$\Sigma F_y = 0 \qquad A_y - 30 = 0 \qquad A_y = +30 \text{ kN} \; \therefore \vec{A}_y = 30 \text{ kN} \uparrow$$

comp

tens

Joint C

$\Sigma F_x = 0$

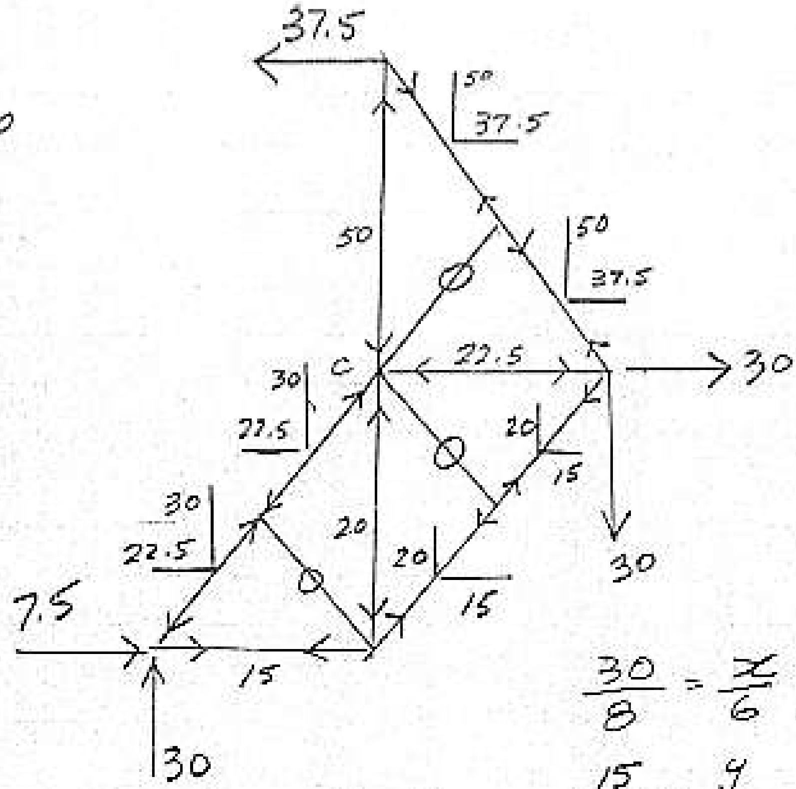
$22.5 - 22.5 = 0$

$0 = 0$ ✓

$\Sigma F_y = 0$

$30 + 20 - 50 = 0$

$0 = 0$ ✓



$$\frac{30}{8} = \frac{x}{6} \qquad x = 22.5$$

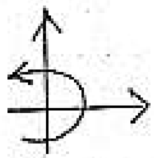
$$\frac{15}{6} = \frac{y}{8} \qquad y = 20$$

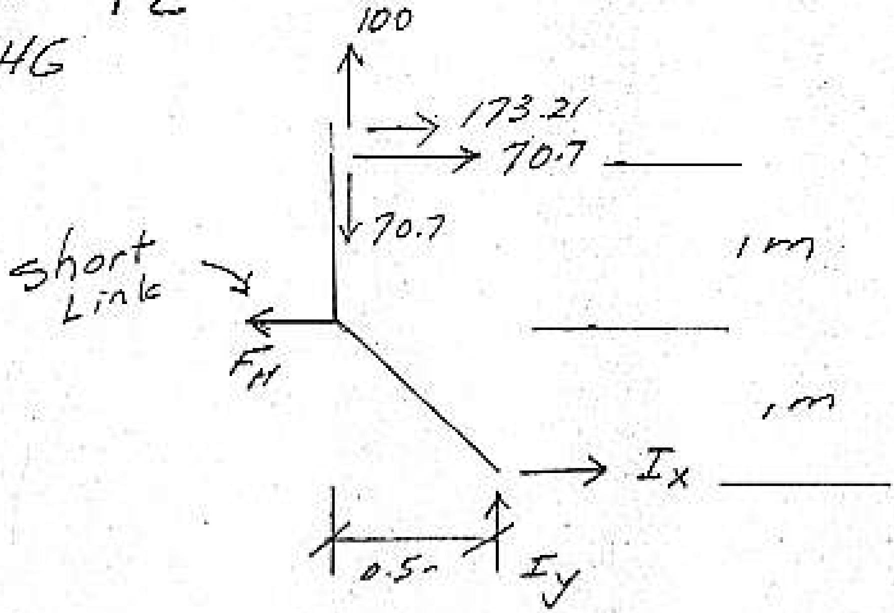
$$\frac{50}{8} = \frac{x}{6}$$

$$x =$$

52-405 1/2

FBD IHG





$\Sigma M_I = 0$ $\quad F_H(1) - 100(0.5) + 70.7(0.5)$
$- 173.21(2) - 70.7(2) = 0$

$F_H = +502.47\,kN \therefore \vec{F_H} = 502.47\,kN \leftarrow$ on IHG

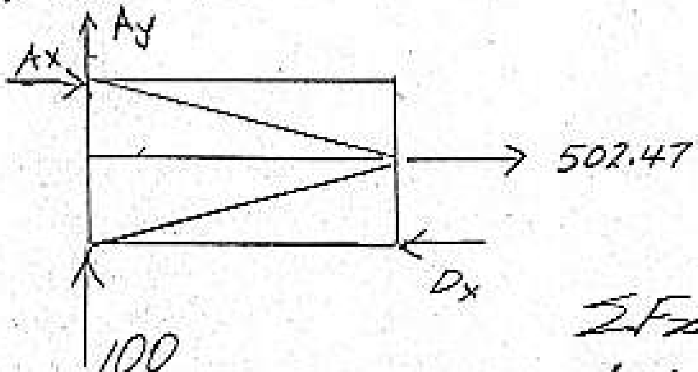
$\Sigma F_x = 0$ $\quad -502.47 + 173.21 + 70.7 + I_x = 0$

$I_x = +258.56\,N \therefore \vec{I_x} = 258.56\,kN \rightarrow$

$\Sigma F_y = 0$ $\quad 100 - 70.7 + I_y = 0 \quad \vec{I_y} = 29.3\,kN \downarrow$

$I_y = -29.3\,N \therefore$

FBD TRUSS



$\Sigma M_K = 0$

$502.47(1) - D_x(2) = 0$

$D_x = +251.24\,kN$

$\vec{D_x} = 251.24\,kN \leftarrow$

$\Sigma F_x = 0$

$A_x + 502.47 - 251.24 = 0$

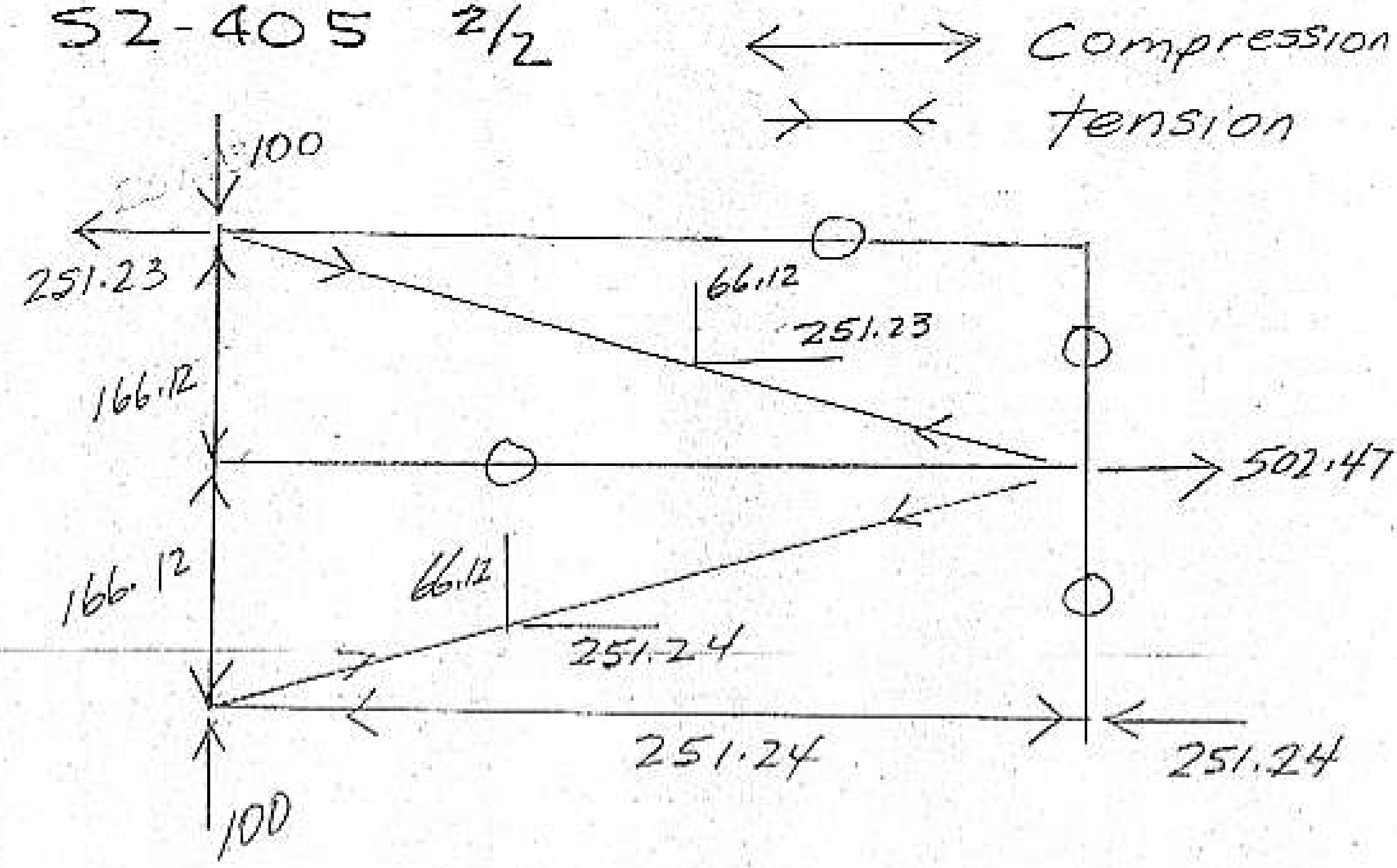
$A_x = -251.23$

$\therefore \vec{A_x} = 251.23\,kN \leftarrow$

$\Sigma F_y = 0 \quad A_y + 100 = 0$

$A_y = -100\,kN \quad \therefore \vec{A_y} = 100\,kN \downarrow$

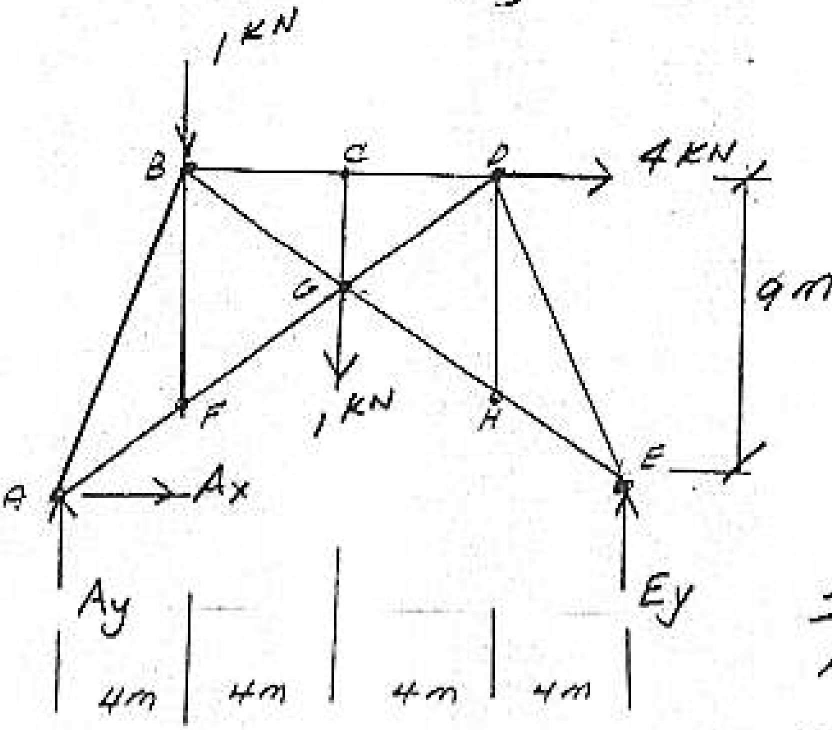
52-405 2/2

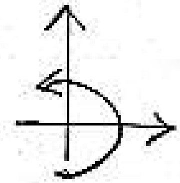


$$\frac{251.24}{3.8} = \frac{y}{1} \qquad y = 66.12$$

$$\frac{251.23}{3.8} = \frac{y}{1} \qquad y = 66.12$$

S2-406 1/3





$\Sigma M_A = 0$

$-1(4) - 1(8) - 4(9) + E_y(16) = 0$

$E_y = +3\,kN$

$\therefore \vec{E}_y = 3\,kN \uparrow$

$\Sigma F_x = 0$

$A_x + 4 = 0 \quad A_x = -4\,kN$

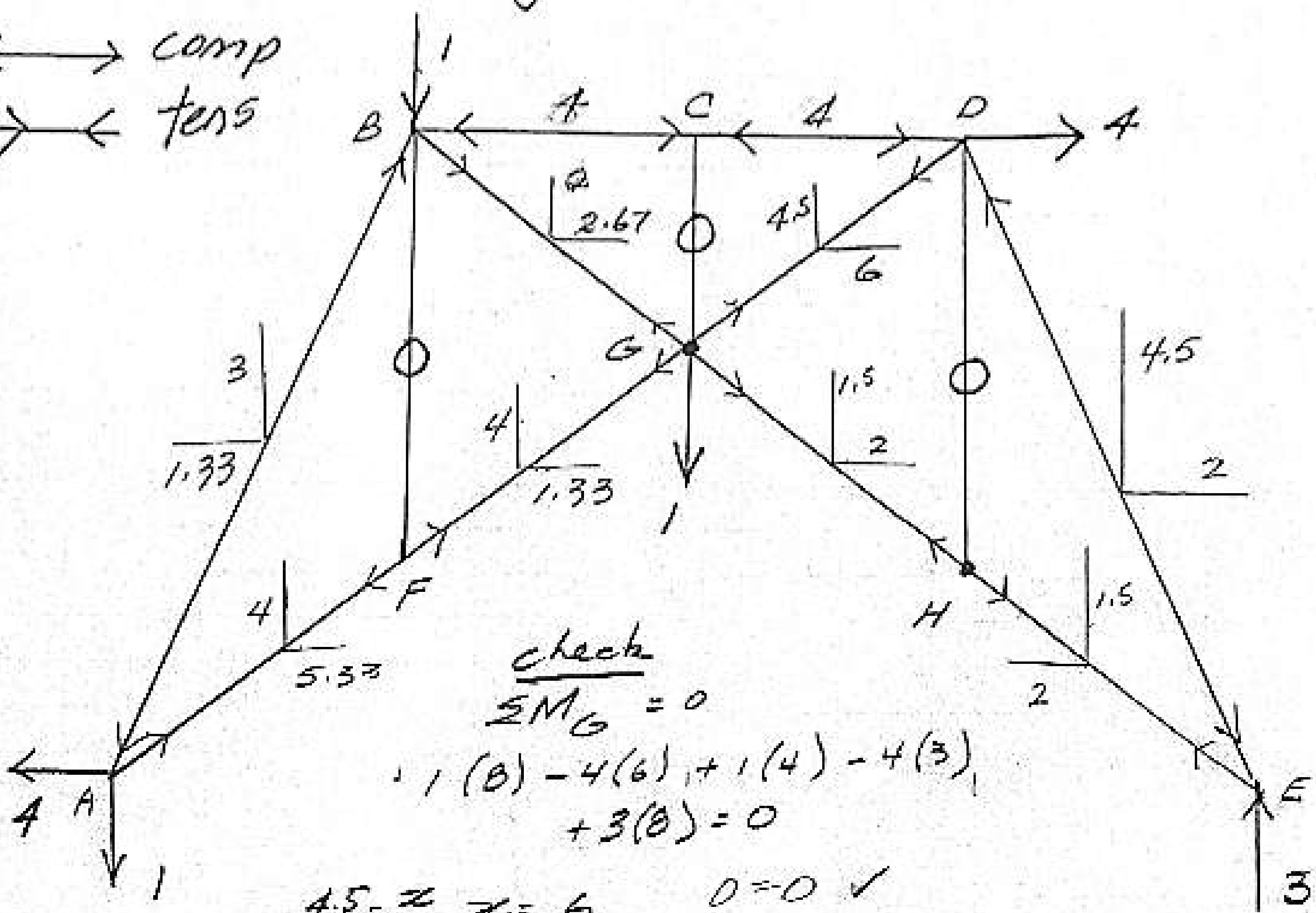
$\therefore \vec{A}_x = 4\,kN \leftarrow$

$\Sigma F_y = 0 \quad A_y - 1 - 1 + 3 = 0 \quad A_y = -1\,kN$

$\therefore \vec{A}_y = 1\,kN \downarrow$

$\longleftrightarrow$ comp

$\rightarrow\!\leftarrow$ tens



check

$\Sigma M_G = 0$

$1(8) - 4(6) + 1(4) - 4(3) + 3(8) = 0$

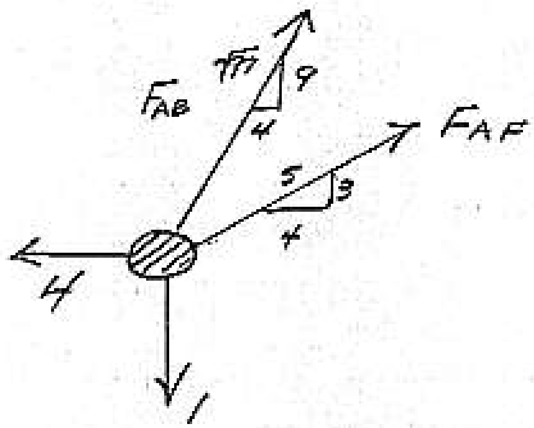
$0 = 0$ ✓

$\frac{4.5}{3} = \frac{x}{4} \quad x = 6$

$\frac{2.67}{4} = \frac{y}{3} \quad y$

S2-406 2/3

FBD JOINT A



$$\Sigma F_x = 0 \quad -4 + \frac{4}{\sqrt{97}} F_{AB} + \frac{4}{5} F_{AF} = 0$$

$$\Sigma F_y = 0 \quad -1 + \frac{9}{\sqrt{97}} F_{AB} + \frac{3}{5} F_{AF} = 0$$

$$0.8 F_{AF} + 0.406 F_{AB} = 4$$
$$0.6 F_{AF} + 0.914 F_{AB} = 1$$

$$0.48 F_{AF} + 0.2436 F_{AB} = 2.4$$
$$0.48 F_{AF} + 0.7312 F_{AB} = 0.8$$

$$-0.4876 F_{AB} = 1.6$$

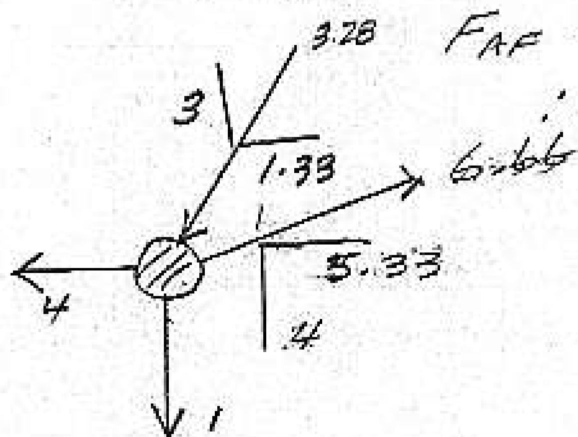
$$F_{AB} = -3.28 \text{ kN}$$

$$\therefore \overrightarrow{F_{AB}} = 3.28 \text{ kN}$$

$$0.8 F_{AF} + 0.406(-3.28) = 4$$

$$F_{AF} = +6.66 \text{ kN}$$

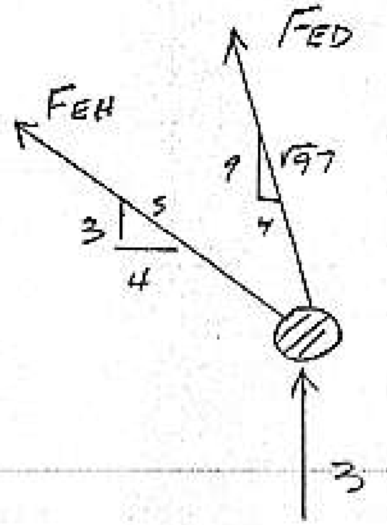
$$\therefore \overrightarrow{F_{AF}} = 6.66 \text{ kN}$$



Place results on truss placeholders

52-406 3/3

Pin at E



$\Sigma F_x = 0$

$-\frac{4}{5} F_{EH} - \frac{4}{\sqrt{97}} F_{ED} = 0$

$\Sigma F_y = 0$

$3 + \frac{3}{5} F_{EH} + \frac{9}{\sqrt{97}} F_{ED} = 0$

$-0.8 F_{EH} = \frac{4}{\sqrt{97}} F_{ED}$

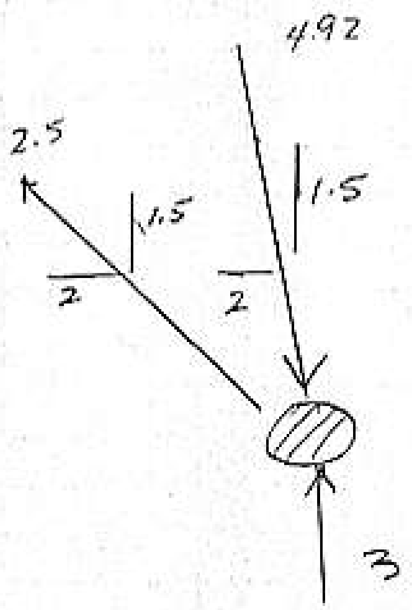
$F_{EH} = -0.507 F_{ED}$

$0.6(-0.507 F_{ED}) + \frac{9}{\sqrt{97}} F_{ED} = -3$

$F_{ED} = -4.92$

$F_{EH} = -0.507(-4.92)$

$F_{EH} = +2.5$

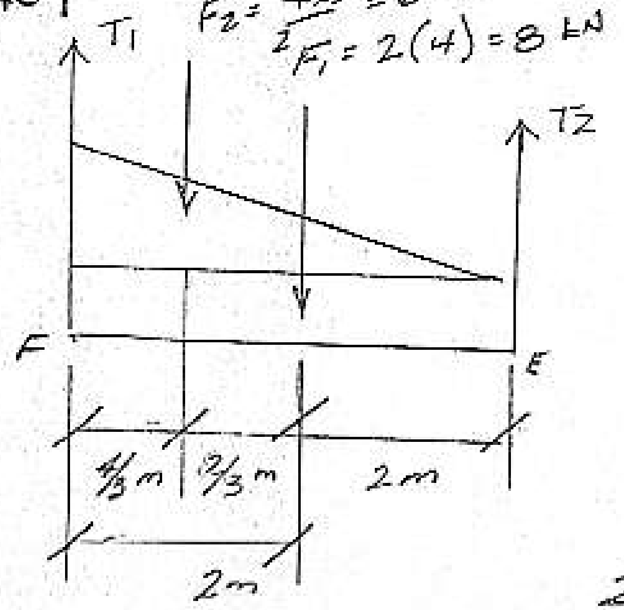


Place results on truss placeholders

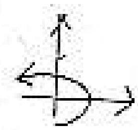
52-407

$F_2 = \frac{4 \times 3}{2} = 6 \text{ kN}$

$F_1 = 2(4) = 8 \text{ kN}$



FBD Beam



$\Sigma M_F = 0$

$-6\left(\frac{4}{3}\right) - 8(2) + T_2(4) = 0$

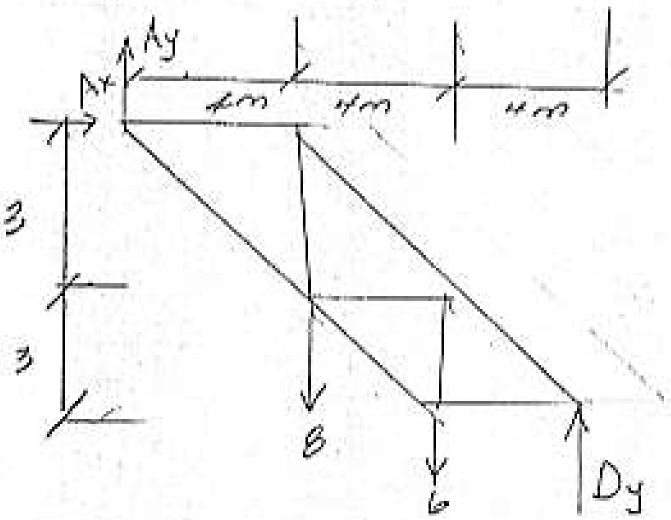
$T_2 = +6 \text{ kN}$

$\vec{T_2} = 6 \text{ kN} \uparrow$ on the beam

$\Sigma F_y = 0$

$T_1 + 6 - 6 - 8 = 0$

$T_1 = +8 \qquad \vec{T_1} = 8 \text{ kN} \uparrow$ on the beam



$\Sigma F_x = 0 \qquad A_x = 0$

$\Sigma M_A = 0$

$-8(4) - 6(8) + D_y(12) = 0$

$D_y = +6.66 \text{ kN}$

$\therefore \vec{D_y} = 6.66 \text{ kN} \uparrow$

$\Sigma F_y = 0$

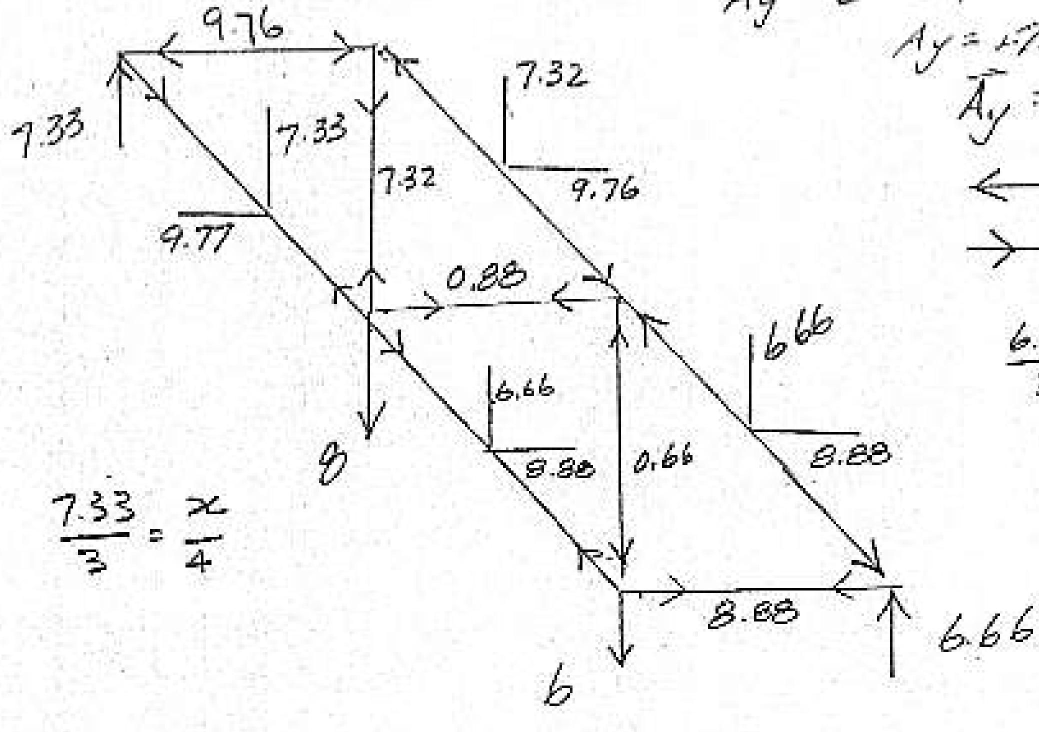
$A_y - 8 - 6 + 6.66 = 0$

$A_y = +7.34 \text{ kN}$

$\vec{A_y} = 7.34 \text{ kN} \uparrow$

$\longleftrightarrow$ comp.

$\rightarrow \leftarrow$ tens



$\frac{6.66}{3} = \frac{x}{4}$

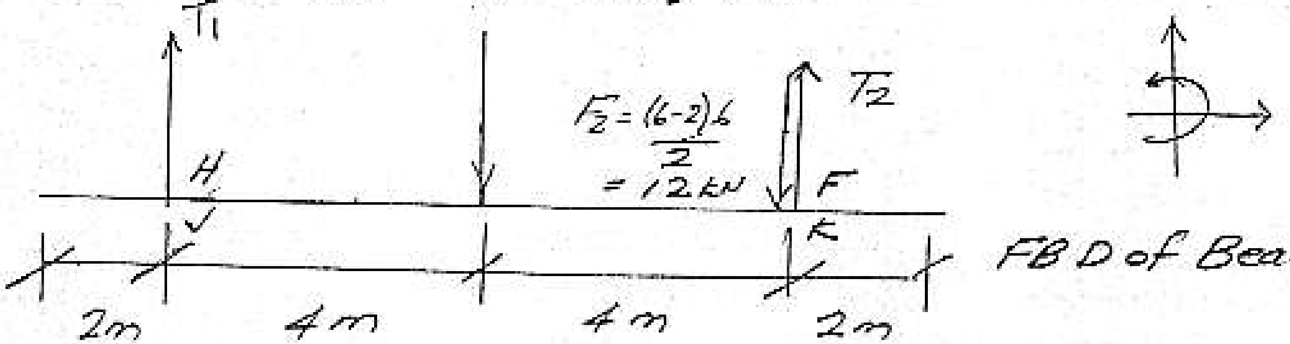
$x = 8.88$

$\frac{7.33}{3} = \frac{x}{4}$

$\frac{7.32}{3} = \frac{x}{4}$

S2 - 408 1/2

$F_1 = 2(12) = 24\,kN$



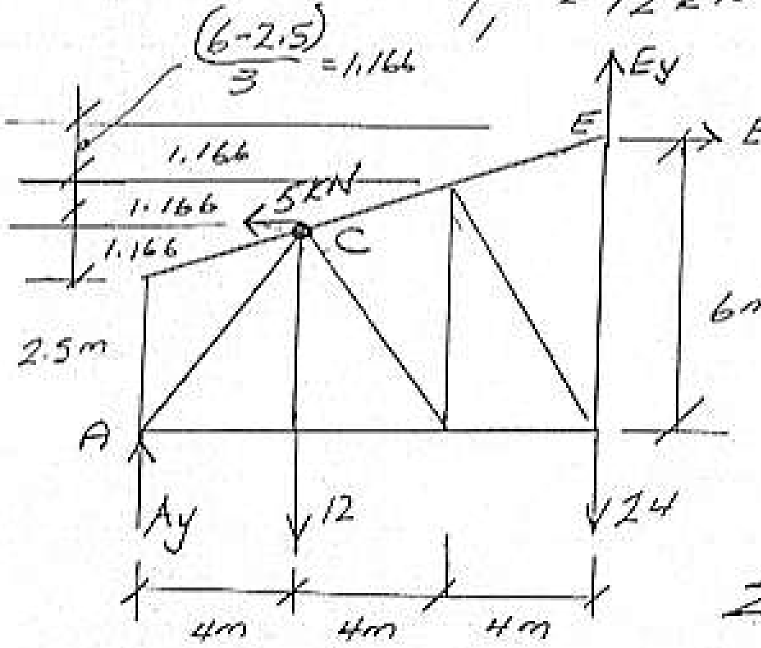
FBD of Beam

$\Sigma M_J = 0 \quad -24(4) - 12(8) + T_2(8) = 0$

$\vec{T_2} = 24\,kN \uparrow$ on the beam

$\Sigma F_y = 0 \quad T_1 - 24 - 12 + 24 = 0$

$\vec{T_1} = 12\,kN \uparrow$ on the beam



$\Sigma M_E = 0$

$-A_y(12) + 12(8) - 5(2.333)$

$\vec{A_y} = 7.03\,kN \uparrow$

$\Sigma F_x = 0$

$-5 + E_x = 0$

$\vec{E_x} = 5\,kN \rightarrow$

$\Sigma F_y = 0$

$7.03 - 12 - 24 + E_y = 0$

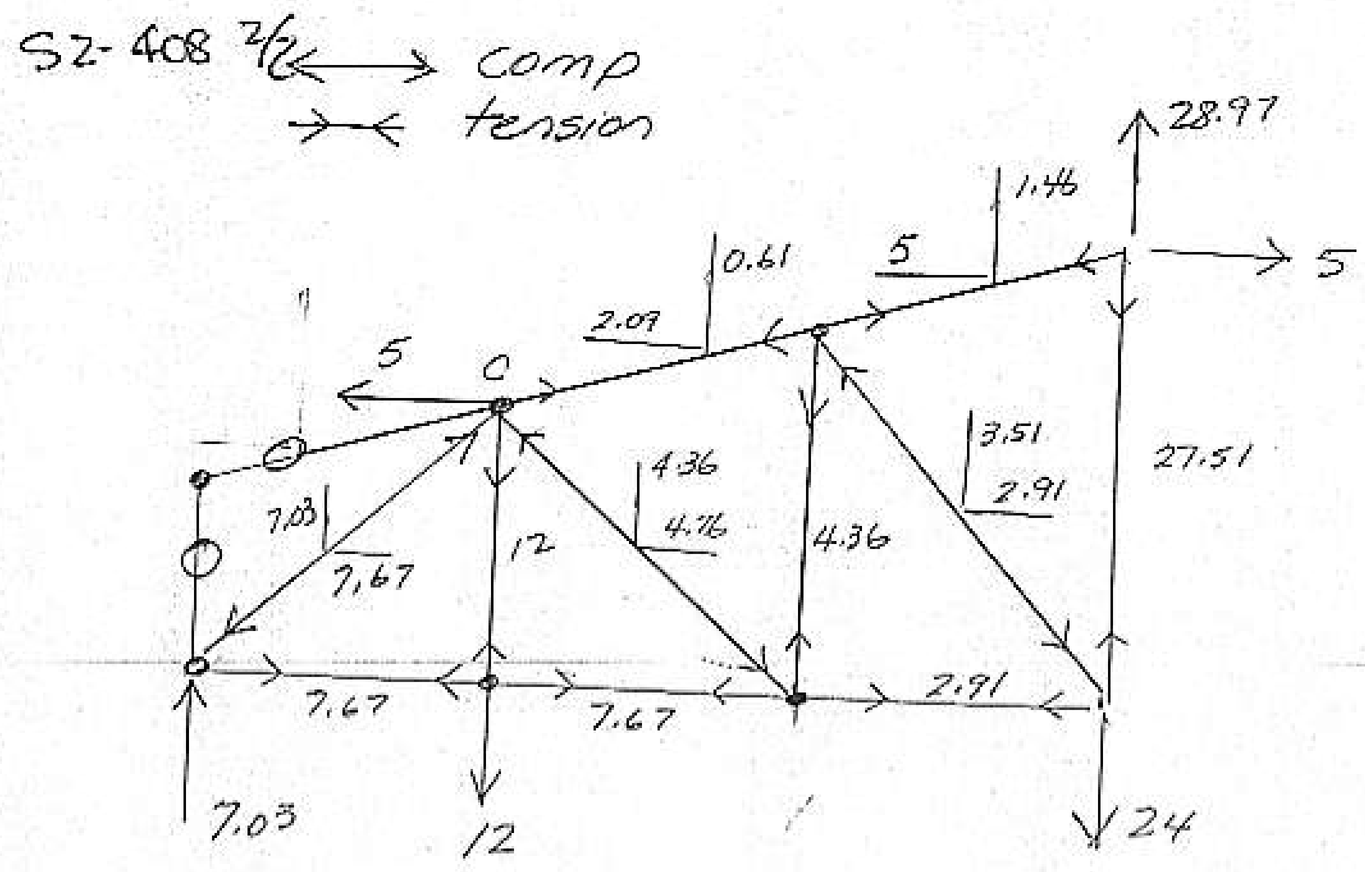
$\vec{E_y} = 28.97\,kN \uparrow$

Check $\Sigma M_C = 0$

$-7.03(4) + 28.97(8) - 5(2.333) - 24(8) = 0$

$-0.025 = 0$ OK ✓

S2-408 2/2



Joint C Check

$\Sigma F_x = 0$

$-5 - 4.76 + 7.67 + 2.09 = 0$

$0 = 0$ ✓

$\Sigma F_y = 0$

$7.03 + 0.61 + 4.36 - 12 = 0$

$0 = 0$ ✓

$\frac{5}{4} = \frac{y}{1.166} \quad y = 3.51$

$\frac{3.51}{4.833} = \frac{x}{4} \quad x = 2.91$

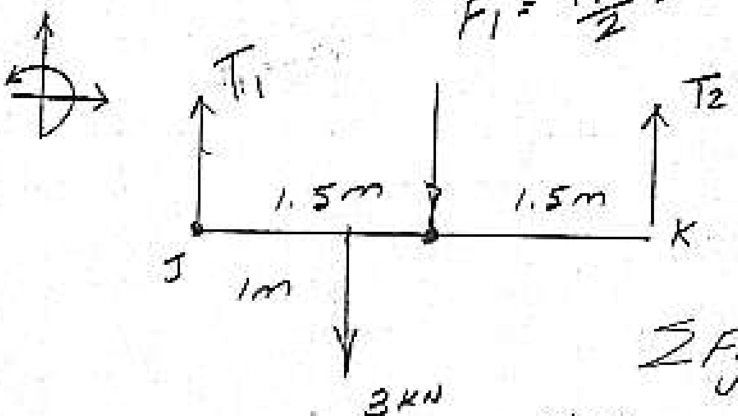
$\frac{2.09}{4} = \frac{y}{1.166} \quad y = 0.61$

$\frac{4.36}{3.666} = \frac{x}{4} \quad x = 4.76$

$\frac{7.67}{4} = \frac{y}{3.666} \quad y$

52-409 1/3

$F_1 = \frac{4(3)}{2} = 6\,kN$



$\Sigma M_J = 0$

$-3(1) - 6(1.5) + T_2(3) = 0$

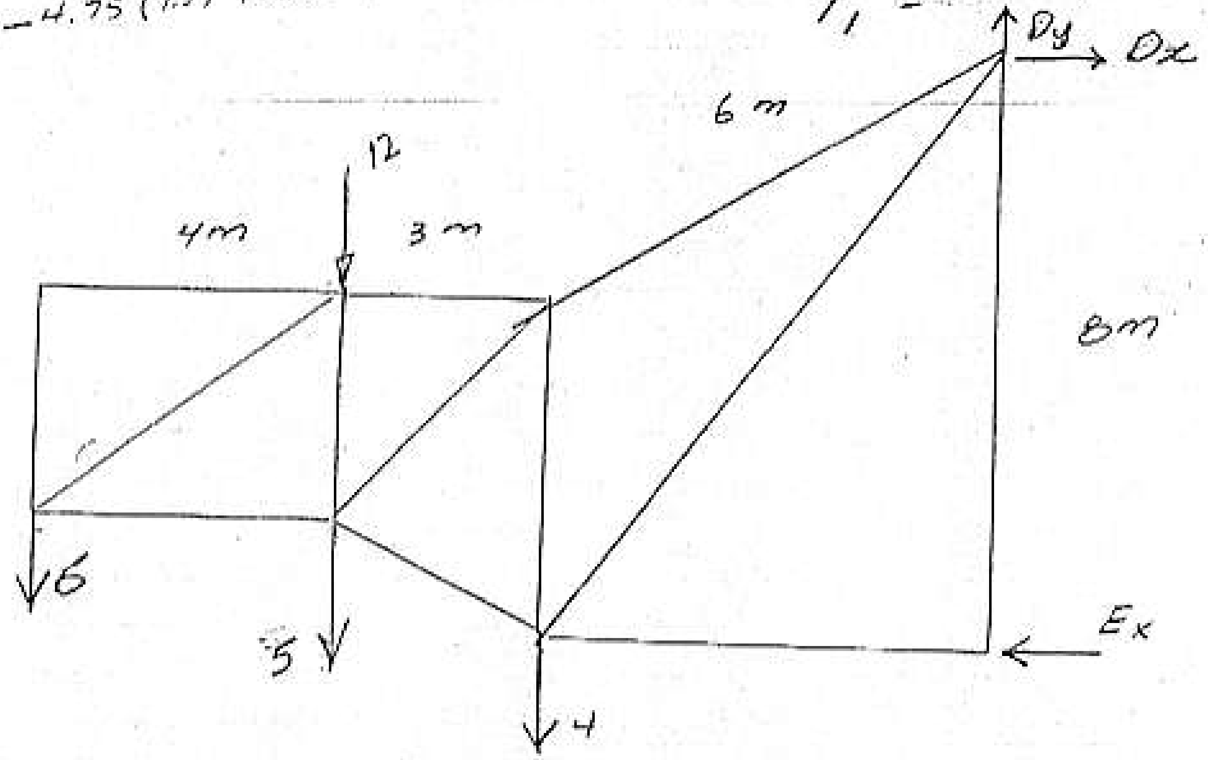
$\vec{T_2} = 4\,kN$ ↑ on Beam JK

$\Sigma F_y = 0$

$T_1 - 3 - 6 + 4.5 = 0$

$\vec{T_1} = 4.5\,kN$ ↑ on Beam JK

$-4.75(1.5) + 3(0.5) + 4.25(2) = 0$



$\Sigma M_D = 0 \quad +6(13) + 12(9) + 5(9) + 4(6) - E_x(8) = 0$

$E_x = +31.875\,kN \quad \vec{E_x} = 31.875\,kN \longleftarrow$

$\Sigma F_x = 0 \quad D_x + 31.875\,kN = 0$

$\vec{D_x} = 31.875\,kN \longrightarrow$

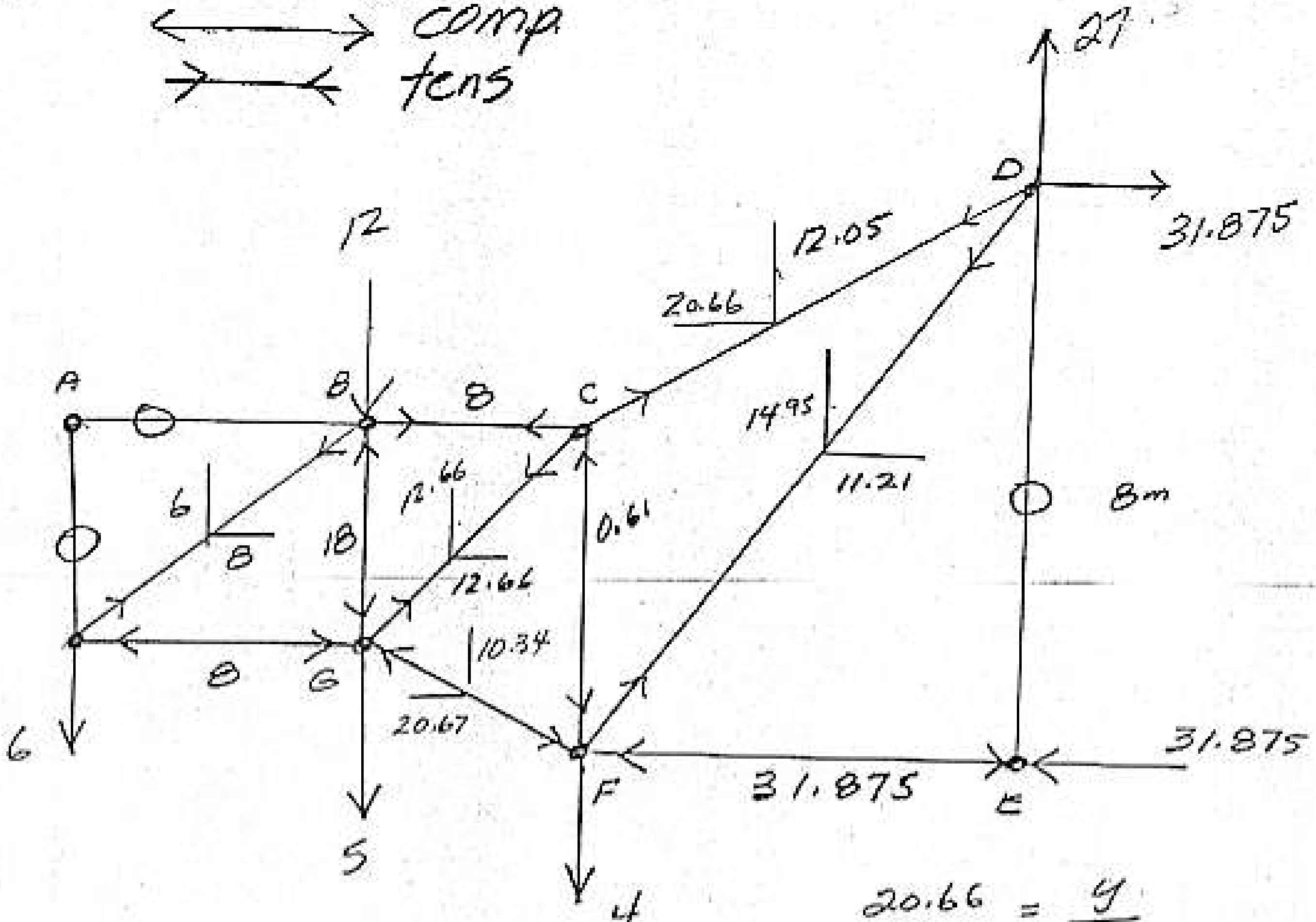
$\Sigma F_y = 0 \quad -6 - 12 - 5 - 4 + D_y = 0$

$\vec{D_y} = 27\,kN \uparrow$

52-409 2/3

comp

tens



$$\frac{20.66}{6} = \frac{y}{3.5}$$

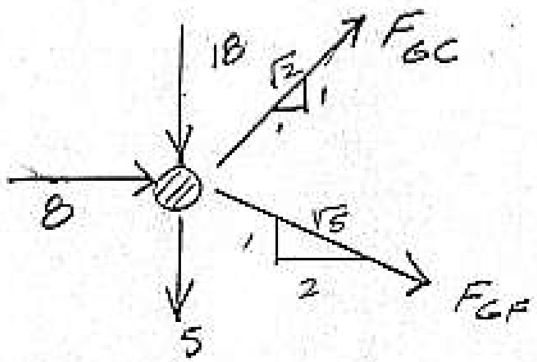
$$y = 12.05$$

$$\frac{6}{3} = \frac{x}{4} \quad x = 8$$

$$\frac{14.95}{8} = \frac{x}{6}$$

$$x =$$

We draw FBD of pin at G



$$\Sigma F_x = 0$$

$$8 + \frac{1}{\sqrt{2}} F_{GC} + \frac{2}{\sqrt{5}} F_{GF} = 0$$

$$\Sigma F_y = 0$$

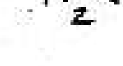
$$-18 - 5 + \frac{1}{\sqrt{2}} F_{GC} - \frac{1}{\sqrt{5}} F_{GF} = 0$$

$$\frac{1}{\sqrt{2}} F_{GC} + \frac{2}{\sqrt{5}} F_{GF} = -8$$

$$\frac{1}{\sqrt{2}} F_{GC} - \frac{1}{\sqrt{5}} F_{GF} = +23$$

$$\frac{3}{\sqrt{5}} F_{GF} = -31$$

$$F_{GF} = -23.11 \quad \therefore F_{GF} = 23.11$$

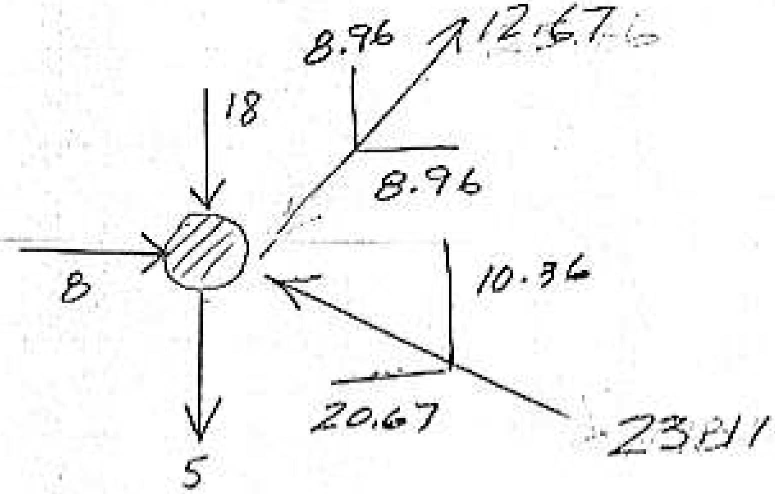


52-409 3/3

$$8 + \frac{1}{\sqrt{2}} F_{GC} + \frac{2}{\sqrt{5}}(-23.11) = 0$$

$$F_{GG} = +12.676 kN$$

$$\therefore F_{GG} = 12.676 k$$



Joint F

ΣF_x $20.67 + 11.21 - 31.875$

$0.005 = 0$ ✓

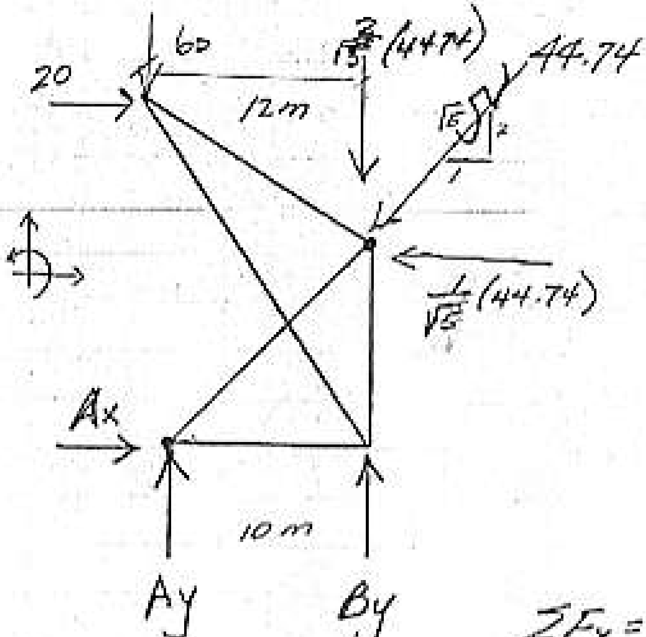
ΣF_y $-10.34 - 0.61 - 4 + 14.95 = 0$

$0 = 0$ ✓

52-410 1/2

The beam supporting the satellite dish is in turn supported by the truss at points E and C. The reactions are provided. (Newton's 3rd Law applies)

We draw an FBD of the truss and solve for the reactions at A & B.



$\Sigma M_A = 0$

$60(2) - 20(16) - \frac{2}{\sqrt{5}}(44.74)(10)$

$+ \frac{1}{\sqrt{5}}(44.74)(10) + B_y(10) = 0$

$B_y = +40$ kN

$\vec{B_y} = 40\text{kN} \uparrow$

$\Sigma F_y = 0 \quad -60 - \frac{2}{\sqrt{5}}(44.74) + 40 + A_y = 0$

$A_y = +60\text{kN}$

$\vec{A_y} = 60\text{kN} \uparrow$

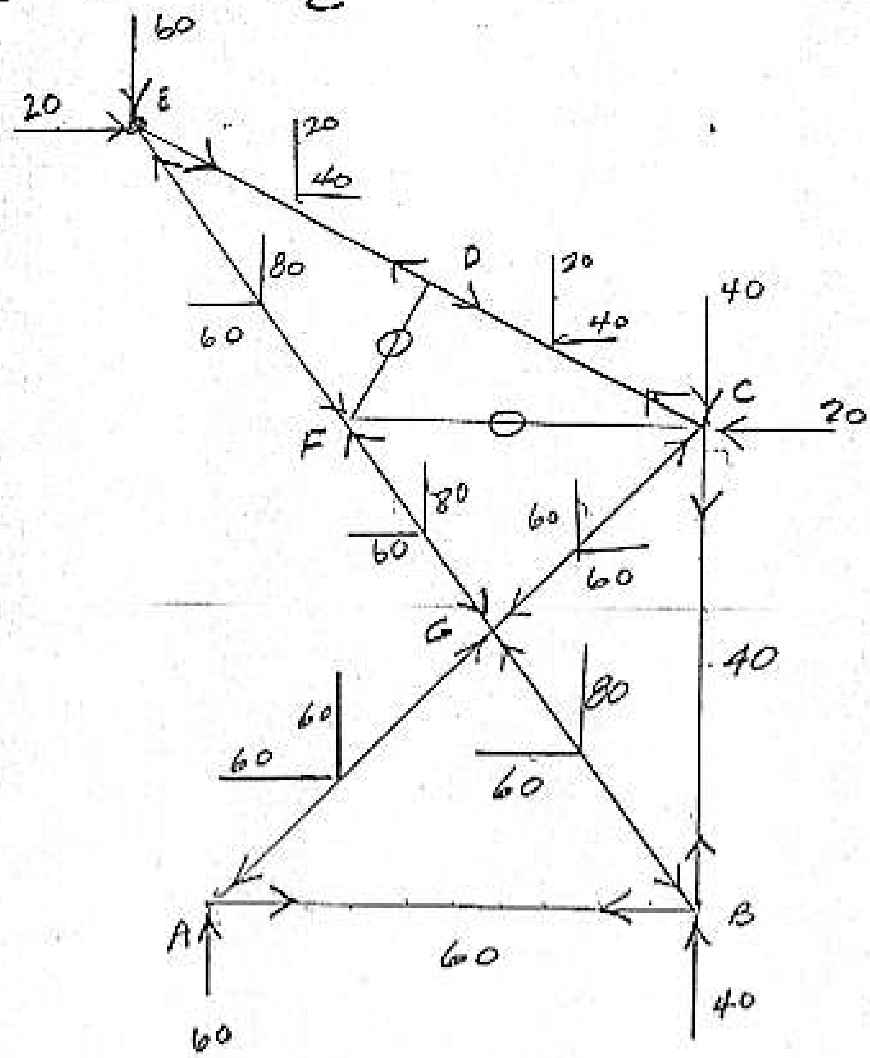
$\Sigma F_x = 0$

$20 + A_x - \frac{1}{\sqrt{5}}(44.74) = 0$

$A_x = 0$

We redraw the truss and use Method of Joints (with Placeholders) to determine all member forces

52-410 2/2



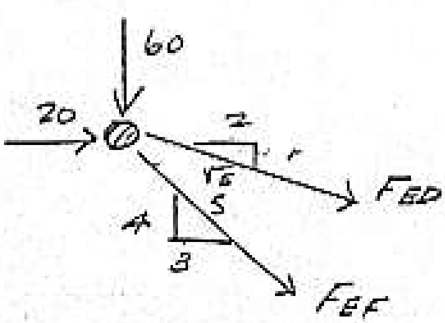
$\longleftrightarrow$ comp

$\rightarrow\!\!-\!\!\leftarrow$ tens

$$\frac{60}{12} = \frac{y}{16}$$

$$y = 80$$

We draw a FBD of Joint E



$$\Sigma F_x = 0 \quad 20 + \frac{2}{\sqrt{5}} F_{ED} + \frac{3}{5} F_{EF} = 0$$

$$\Sigma F_y = 0 \quad -60 - \frac{1}{\sqrt{5}} F_{ED} - \frac{4}{5} F_{EF} = 0$$

$$\frac{2}{\sqrt{5}} F_{ED} + \frac{3}{5} F_{EF} = -20$$

$$-\frac{2}{\sqrt{5}} F_{ED} - \frac{8}{5} F_{EF} = 120$$

$$-F_{EF} = 100$$

$$\therefore \vec{F}_{EF} = 100 \text{ lb}$$

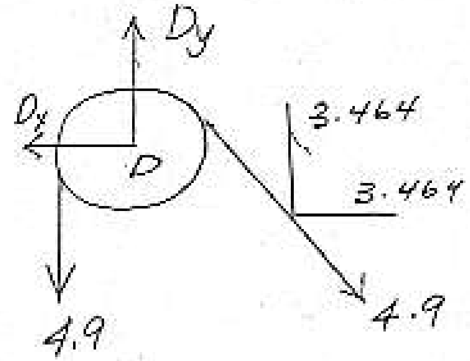
$$\frac{2}{\sqrt{5}} F_{ED} + \frac{3}{5}(-100) = -20$$

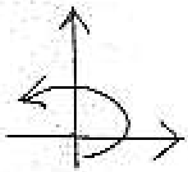
$$F_{ED} = + \frac{40\sqrt{5}}{2}$$

$$\vec{F}_{ED} = \frac{40\sqrt{5}}{2}$$

S2-4.11 1/2

a)





Pulley at D

$500(9.8) =$
$4900 N$

$\Sigma F_x = 0 \quad -D_x + 3.464 = 0$

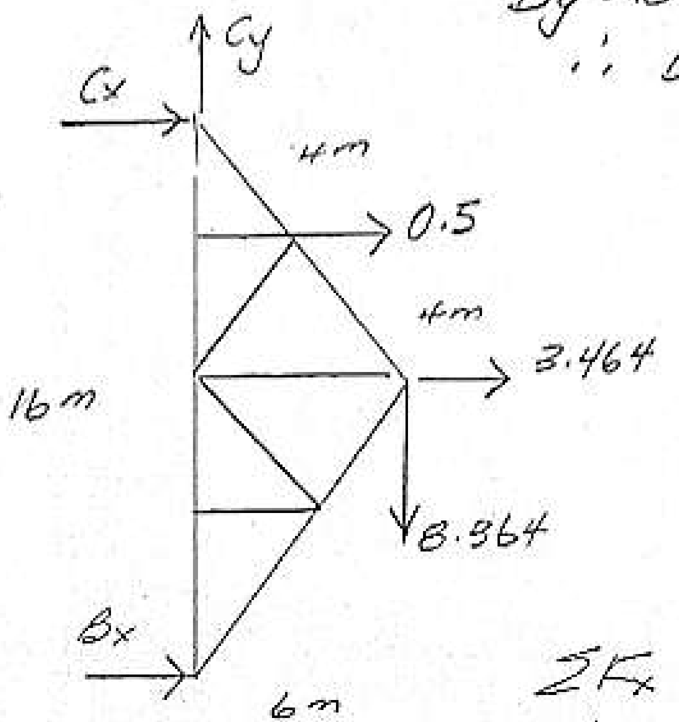
$D_x = +3.464$

$\vec{D_x} = 3.464 \text{ kN} \leftarrow$ on the pulley

$\Sigma F_y = 0 \quad D_y - 3.464 - 4.9$

$D_y = 8.364$

$\therefore \vec{D_y} = 8.364 \text{ kN} \uparrow$ on the pulley



$\Sigma M_C = 0$

$B_x(16) + 0.5(4)$
$+ 8.364(8) - 3.464(6) = 0$

$B_x = +1.28 \text{ kN}$

$\vec{B_x} = 1.28 \text{ kN} \rightarrow$ on the truss

$\Sigma F_x = 0$

$C_x + 0.5 + 3.464 + 1.28 = 0$

$C_x = -5.24 \text{ kN}$

$\therefore C_x = 5.24 \text{ kN} \leftarrow$ on the truss

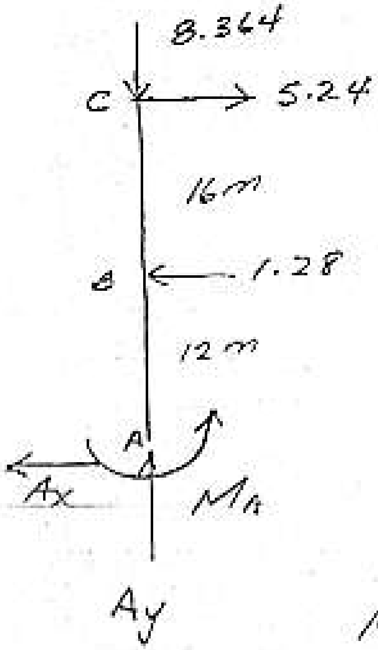
$\Sigma F_y = 0 \quad C_y - 8.364 = 0$

$C_y = +8.364$

$\therefore \vec{C_y} = 8.364 \text{ kN} \uparrow$ on the

52-411 2/2

b) Analysis of Pole ABC



$\Sigma F_x = 0$

$5.24 - 1.28 - A_x = 0$

$A_x = +3.96 \text{ kN}$

$\therefore \vec{A}_x = 3.96 \text{ kN} \leftarrow$

$\Sigma F_y = 0$

$-8.364 + A_y = 0$

$A_y = +8.364 \text{ kN}$

$\therefore \vec{A}_y = 8.364 \text{ kN} \uparrow$

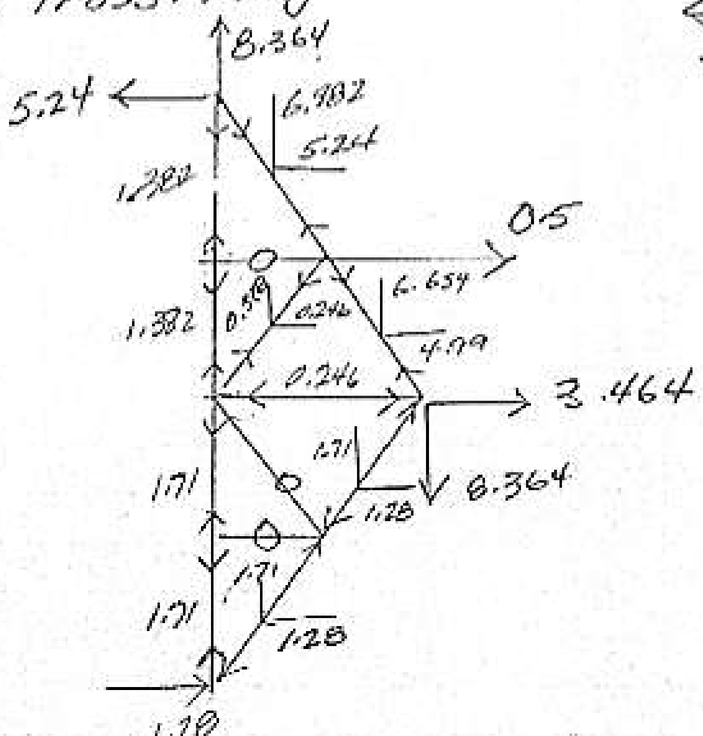
$\Sigma M_A = 0$

$M_A - 5.24(28) + 1.28(12) = 0$

$M_A = +131.36 \text{ kN.m}$

$\therefore \vec{M}_A = 131.36 \text{ kN.m}$

c) Truss Analysis



$\longleftrightarrow$ comp

$\rightarrow\!\!-\!\!\leftarrow$ tens

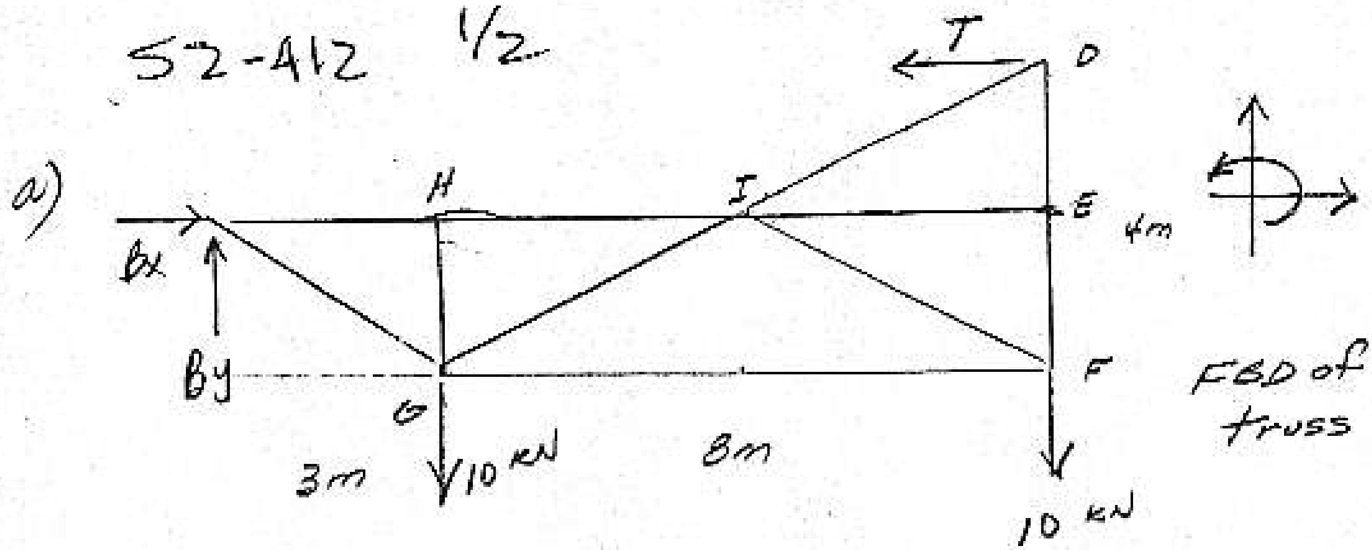
$\frac{1.28}{6} = \frac{y}{8}$ $y = 1.71$

$\frac{6.654}{8} = \frac{x}{6}$ $x = 4.99$

$\frac{0.248}{3} = \frac{y}{4}$ $y = 0.328$

52-A12 1/2

a)



$$\Sigma M_B = 0 \quad -10(3) + T(2) - 10(11) = 0$$

$$T_2 = +70 \text{ kN} \quad \therefore \vec{T_2} = 70 \text{ kN} \leftarrow$$

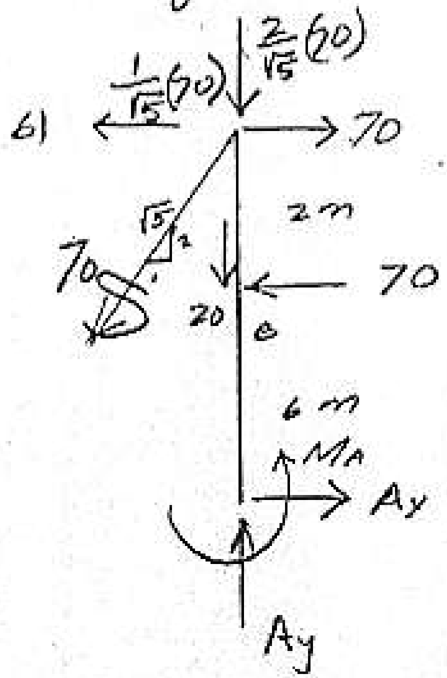
$$\Sigma F_x = 0 \quad B_x - 70 = 0 \quad B_x = +70 \text{ kN}$$

$$\vec{B_x} = 70 \text{ kN} \rightarrow$$

$$\Sigma F_y = 0 \quad B_y - 10 - 10 = 0 \quad \vec{B_y} = 20 \text{ kN} \uparrow$$

$$B_y = +20 \text{ kN}$$

b)



$$\Sigma M_A = 0$$

$$\frac{1}{\sqrt5}(70)(8) - 70(8) + 70(6) + M_A = 0$$

$$M_A = -110.44 \text{ kN}\cdot\text{m}$$

$$\overrightarrow{110.44} \text{ kN}\cdot\text{m} \circlearrowright$$

$$\Sigma F_x = 0$$

$$-\frac{1}{\sqrt5}(70) + 70 - 70 + A_x = 0$$

$$A_x = +31.3 \text{ kN}$$

$$\vec{A_x} = 31.3 \text{ kN} \rightarrow$$

$$\Sigma F_y = 0 \quad -\frac{2}{\sqrt5}(70) - 20 + A_y = 0 \quad A_y = +82.61 \text{ k}$$

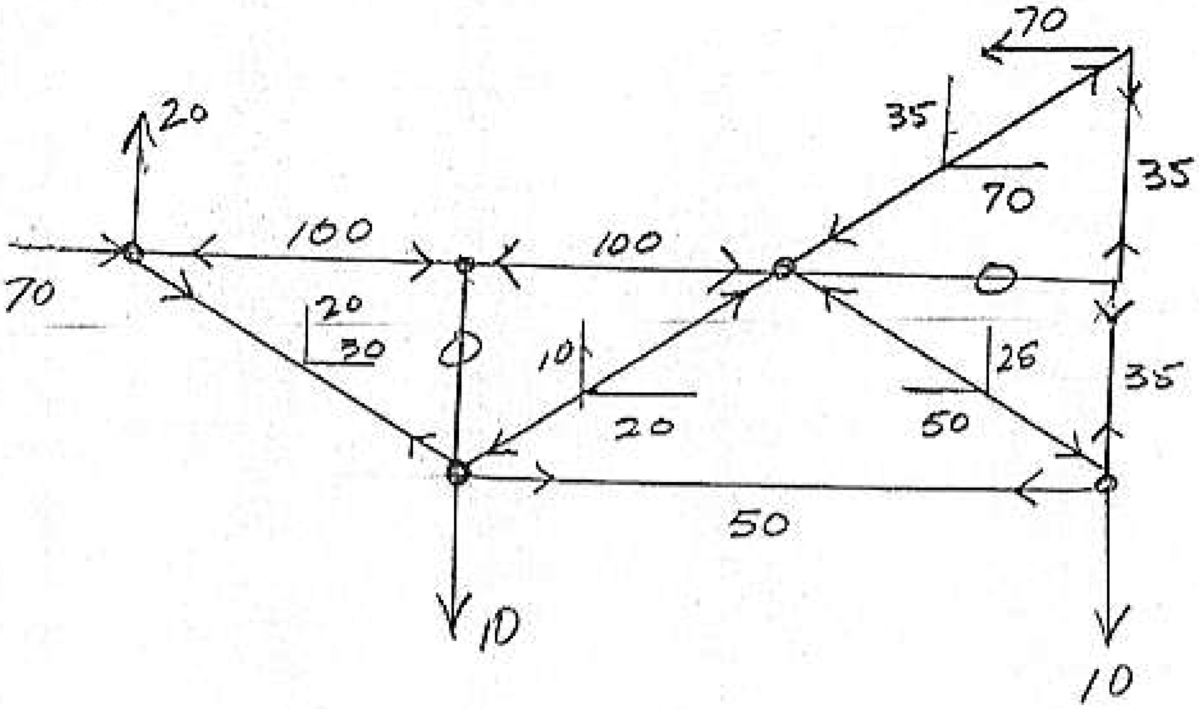
$$\vec{A_y} = 82.61 \text{ kN} \uparrow$$

52-412 2/2

c) Truss

comp

ten



$$\frac{20}{2} = \frac{x}{3} \quad x = 30$$

$$\frac{10}{2} = \frac{x}{4} \quad x = 20$$

$$\frac{50}{4} = \frac{y}{2}$$

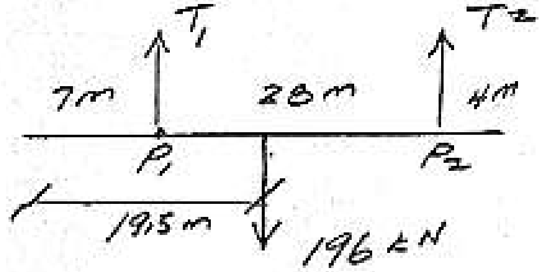
$$y = 25$$

$$\frac{35}{2} = \frac{x}{4}$$

$$x = 70$$

52-413 1/2

We first check cable anchors at P_1 & P_2



$\Sigma M_{P_1} = 0 \quad T_2(20) - 196(12.5) = 0$

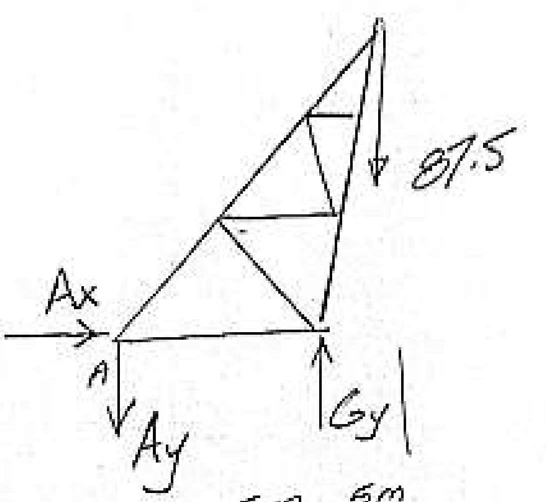
$T_2 = +87.5$ kN < 115 kN ok

$\Sigma F_y = 0 \quad T_1 + 87.5 - 196 = 0$

$T_1 = 108.5 < 115$ kN

$T_1 > T_2 \therefore T_1$ governs

We analyze truss on the left



$\Sigma F_x = 0 \quad A_x = 0$

$\Sigma M_A = 0 \quad -87.5(20) + G_y(15) = 0$

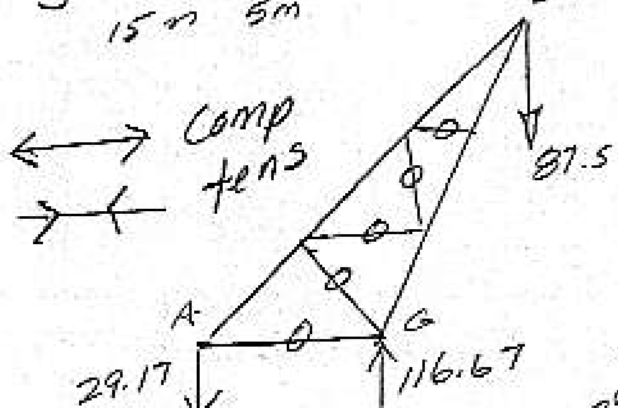
$G_y = +116.67$ kN

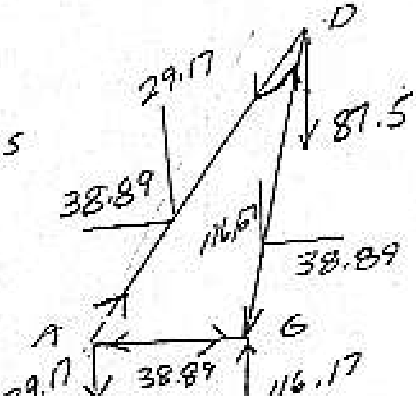
$\therefore G_y = 116.67$ kN ↑

$\Sigma F_y = 0 \quad -A_y - 87.5 + 116.67 = 0$

$\therefore A_y = +29.17$

$\therefore A_y = 29.17$ kN ↓





$\frac{29.17}{15} = \frac{z}{20}$

$z = 38.89$

$\frac{38.89}{5} = \frac{y}{15}$

$y = 116.67$

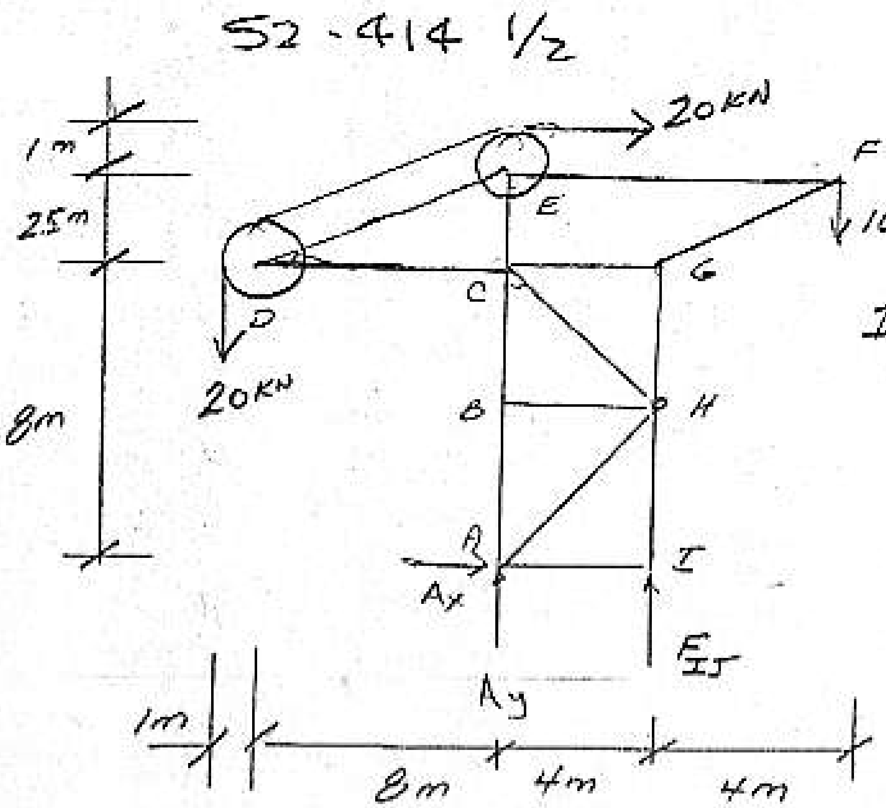
52-413 2/2

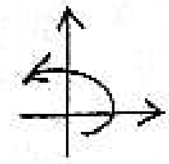
$AD = \sqrt{38.89^2 + 29.17^2} = 48.61$ kN Tension
< 75

$GD = \sqrt{116.67^2 + 38.89^2} = 122.98$ kN COMP.
< 150

OK TO LIFT GIRDER

52-414 1/2





IJ is a "short link"

$$\Sigma M_A = 0 \quad -20(11.5) - 10(8) + 20(9) + F_{IJ}(4) = 0$$

$$F_{IJ} = +32.5\,\text{kN} \quad \therefore \vec{F}_{IJ} = 32.5\,\text{kN} \uparrow$$

$$\Sigma F_x = 0 \quad A_x + 20 = 0 \quad A_x = -20\,\text{kN}$$

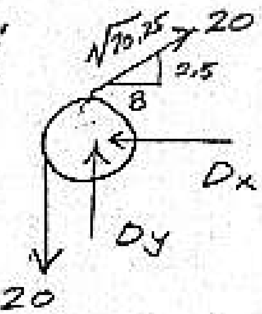
$$\therefore \vec{A}_x = 20\,\text{kN} \leftarrow$$

$$\Sigma F_y = 0 \quad -20 - 10 + 32.5 + A_y = 0$$

$$A_y = -2.5\,\text{kN} \quad \therefore \vec{A}_y = 2.5\,\text{kN} \downarrow$$

Pulley forces on truss

At D:



$$\Sigma F_x = 0$$

$$-D_x + \frac{8}{\sqrt{70.25}}(20) = 0$$

$$D_x = +19.09\,\text{kN}$$

$$\therefore \vec{D}_x = 19.09\,\text{kN} \rightarrow$$ on the truss

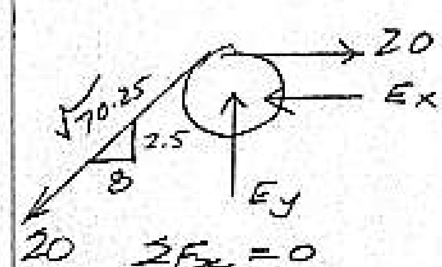
$$\Sigma F_y = 0$$

$$-20 + D_y + \frac{2.5}{\sqrt{70.25}}(20) = 0$$

$$D_y = +14.035\,\text{kN}$$

$$\therefore \vec{D}_y = 14.035\,\text{kN} \downarrow$$ on truss

At E:



$$\Sigma F_x = 0$$

$$20 - \frac{8}{\sqrt{70.25}}(20) - E_x = 0$$

$$E_x = +0.91$$

$$\therefore E_x = 0.91\,\text{kN} \rightarrow$$ on the truss

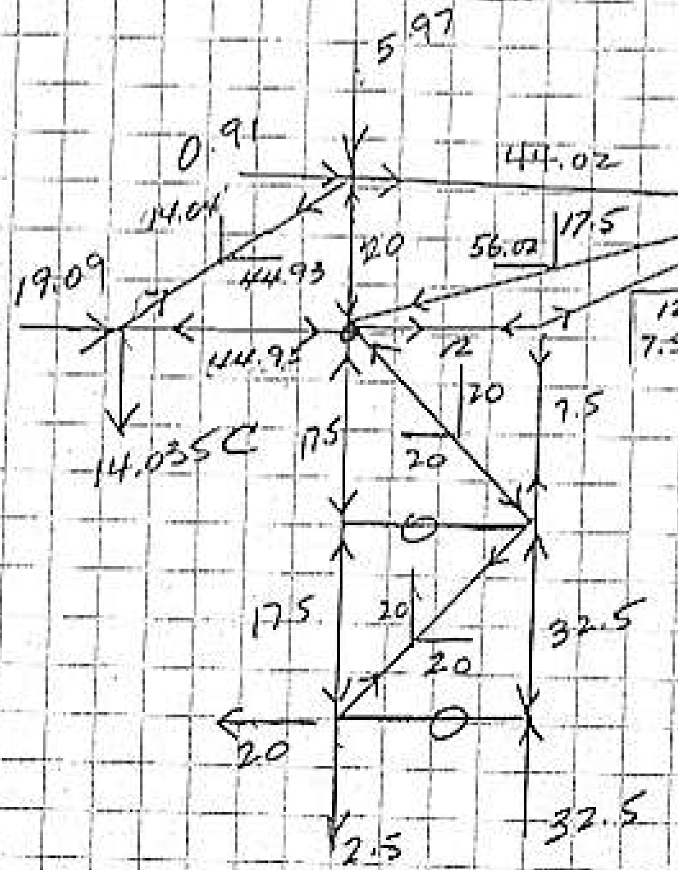
$$\Sigma F_y = 0 \quad E_y - \frac{2.5}{\sqrt{70.25}}(20) = 0$$

$$E_y = +5.97 \quad \ldots \quad 5.97\,\text{kN} \downarrow$$

S2-414 2/2

$\frac{14.04}{2.5} = \frac{x}{8}$ $x = 44.93$

$\frac{7.5}{2.5} = \frac{x}{4}$ $x = 12$



$\frac{56.02}{8} = \frac{y}{2.5}$

AT C:

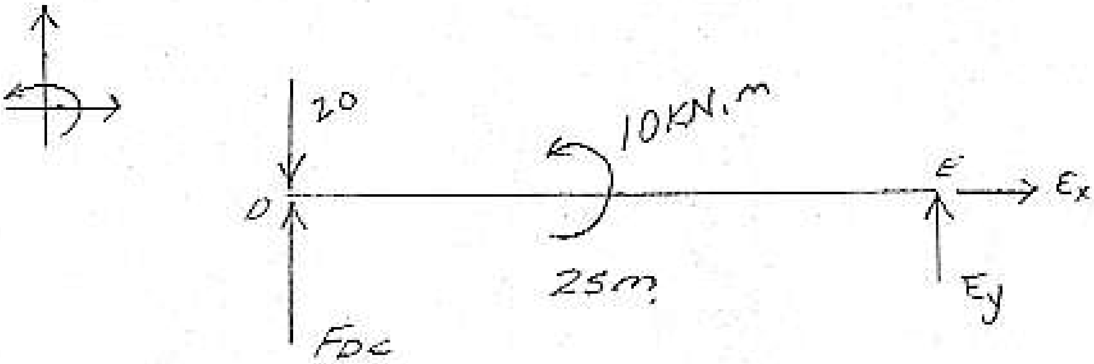
ΣF_x

$64.02 + 12$
$- 56.02 - 20 = 0$
$0 = 0$ ✓

$\Sigma F_y = 0$

$-20 - 17.5 + 17.5 + 20 = 0$ ✓

S2-415



$\Sigma M_E = 0$

$-F_{DC}(25) + 20(25) + 10 = 0$

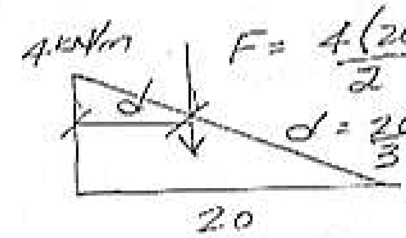
$F_{DC} = +20.4$ kN $\therefore$ $\vec{F}_{DC} = 20.4$ kN $\uparrow$ on beam DE

$\Sigma F_y = 0$

$-20 + 20.4 + E_y = 0$

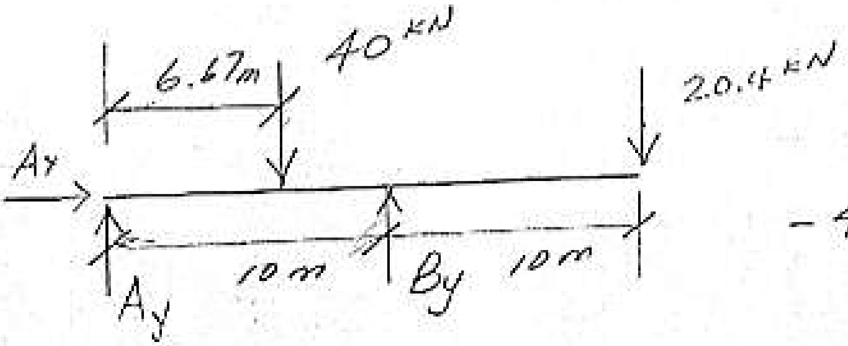
$E_y = -0.4$ $\therefore$ $\vec{E}_y = 0.4$ kN $\downarrow$

$\Sigma F_x = 0$ $\therefore$ $E_x = 0$



$F = \frac{4(20)}{2} = 40$ kN

$d = \frac{20}{3} = 6.67$ m



$\Sigma M_A = 0$

$-40(6.67) + B_y(10) - 20.4(20) = 0$

$B_y = +67.47$ kN

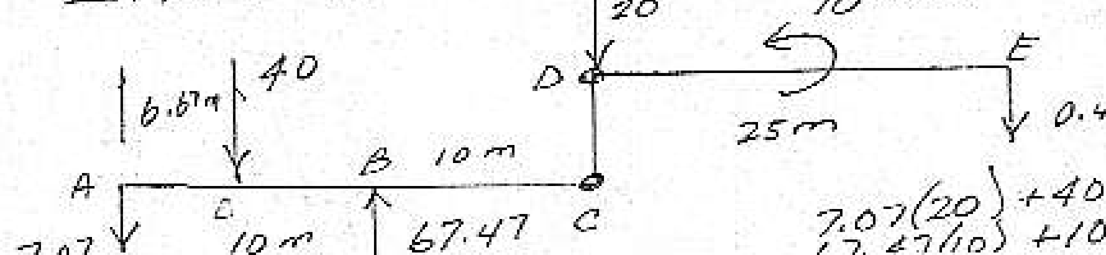
$\therefore$ $\vec{B}_y = 67.47$ kN $\uparrow$

$\Sigma F_y = 0$

$A_y - 40 + 67.47 - 20.4 = 0$

$A_y = -7.07$ kN $\therefore$ $\vec{A}_y = 7.07$ kN $\downarrow$

$\Sigma F_x = 0$ $\therefore$ $A_x = 0$



$\Sigma M_C = 0$

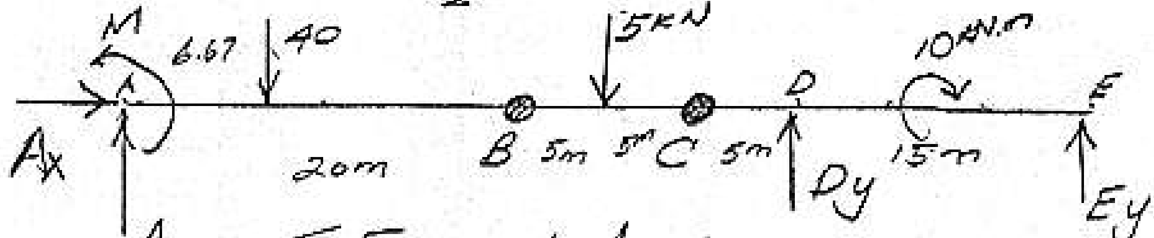
$7.07(20) + 40(13.33) + 10 - 0.4(25) = 0$

$-67.47(10)$

S2-416 1/2

We replace distributed loads by point loads and draw the FBD

$F_1 = \frac{4(20)}{2} = 40\,kN$ @ 6.67m from A

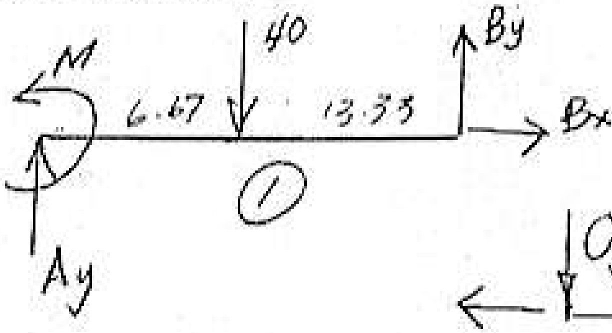


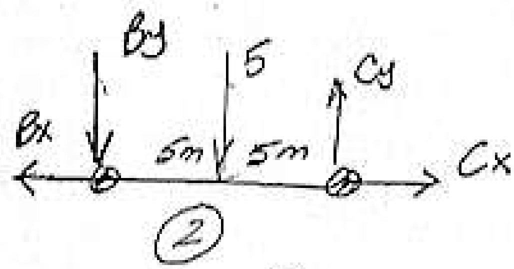
$\Sigma F_x = 0 \therefore A_x = 0$

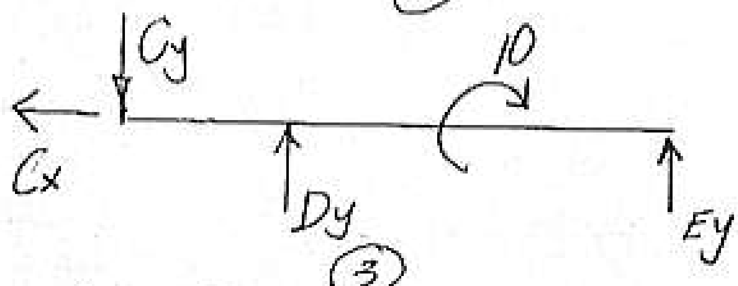
$\Sigma F_y = 0 \quad A_y - 40 - 5 + D_y + E_y = 0$

$\Sigma M_A = 0 \quad M - 40(6.67) - 5(25) + D_y(35) - 10 + E_y(50) = 0$

We substructure:



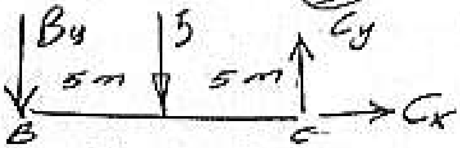




From ①

$\Sigma F_x = 0 \quad B_x = 0$

Redraw ②



$\Sigma F_x = 0 \quad C_x = 0$

$\Sigma M_B = 0 \quad -5(5) + C_y(10) = 0$

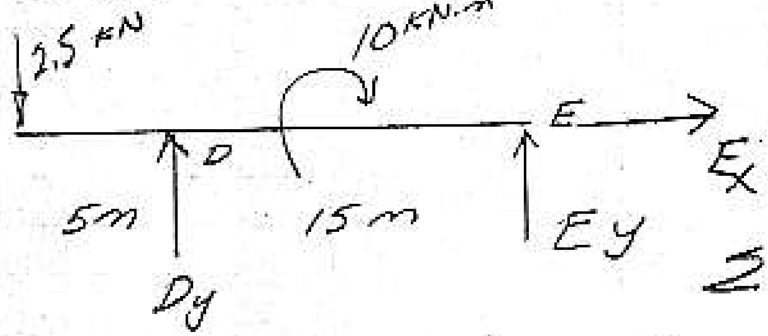
$C_y = +2.5\,kN \quad \vec{C_y} = 2.5\,kN \uparrow$ on BC

$\Sigma F_y = 0 \quad -B_y - 5 + 2.5 = 0$

$B_y = -2.5 \quad \therefore \vec{B_y} = 2.5\,kN \uparrow$ on BC

SZ-416 2/2

We re-draw (3)



$\Sigma F_x = 0 \quad E_x = 0$

$\Sigma M_E = 0 \quad 2.5(20) - D_y(15) - 10 = 0$

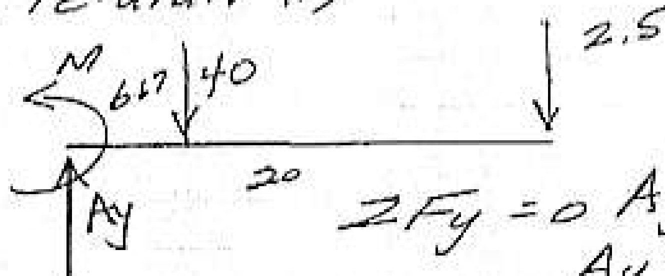
$D_y = +2.66 \text{ kN}$

$\therefore \vec{D_y} = 2.66 \text{ kN} \uparrow$

$\Sigma F_y = 0 \quad -2.5 + 2.66 + E_y = 0$

$E_y = -0.167 \text{ kN} \quad \therefore \vec{E_y} = 0.167 \text{ kN} \downarrow$

We re-draw (1)



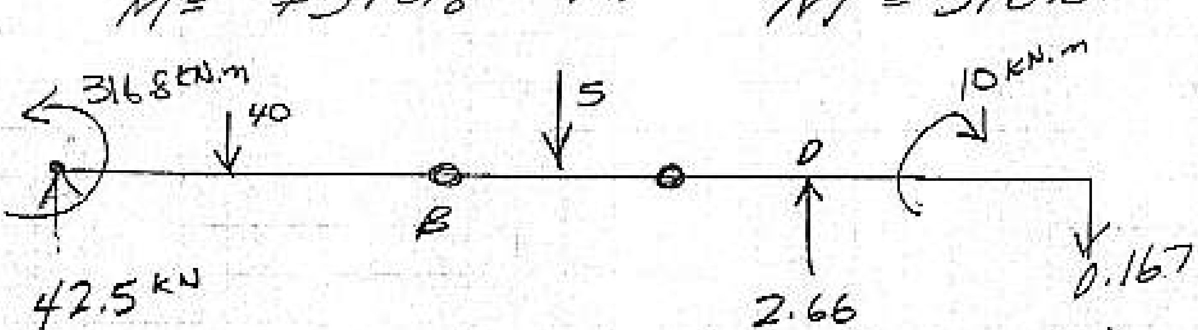
$\Sigma F_y = 0 \quad A_y - 40 - 2.5 = 0$

$A_y = +42.5$

$\therefore \vec{A_y} = 42.5 \text{ kN} \uparrow$

$\Sigma M_A = 0 \quad M - 40(6.67) - 2.5(20) = 0$

$M = +316.8 \text{ kN.m} \quad \vec{M} = 316.8 \text{ kN.m} \circlearrowleft$

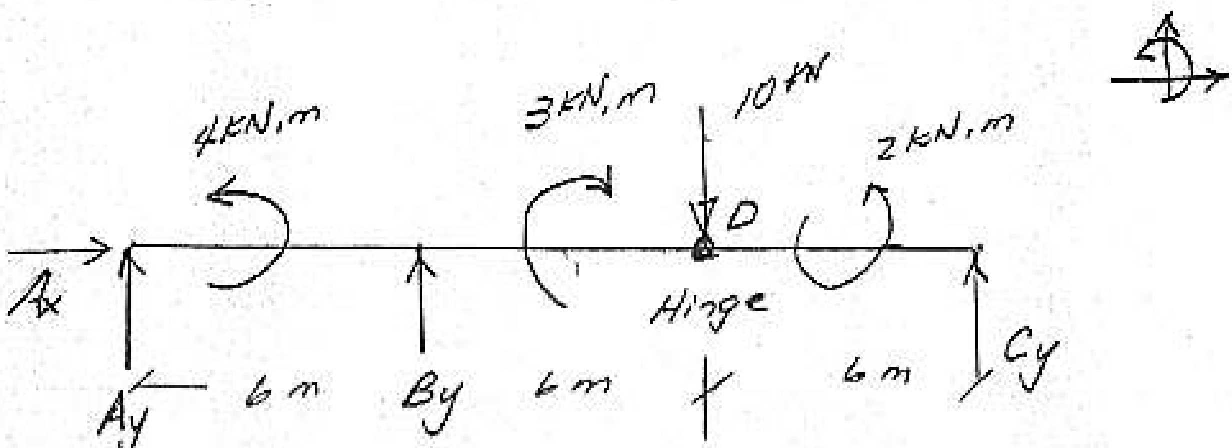


$\Sigma M_B = 316.8 - 42.5(20) + 40(13.33) - 5(5) + 2.66(15)$
$- 10 - 0.167(30) = 0$

$-0.1 = 0 \quad \underline{OK}$

S2-417 1/2

We draw the FBD of the entire structure

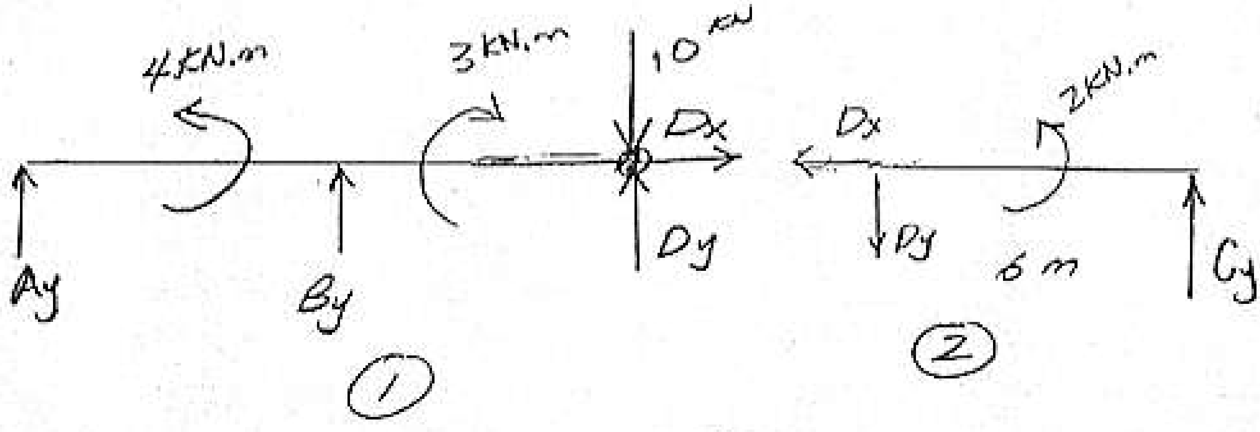


$\Sigma F_x = 0 \quad A_x = 0$

$\Sigma F_y = 0 \quad A_y + B_y - 10 + C_y = 0$

$\Sigma M_A = 0 \quad 4 + B_y(6) - 3 - 10(12) + 2 + C_y(18) = 0$

4 unknowns ∴ Substructure



From ② $\Sigma F_x = 0 \quad D_x = 0$

$\Sigma M_D = 0 \quad 2 + C_y(6) = 0 \quad C_y = -0.333\ kN$

$\therefore \vec{C_y} = 0.333\ kN \downarrow$

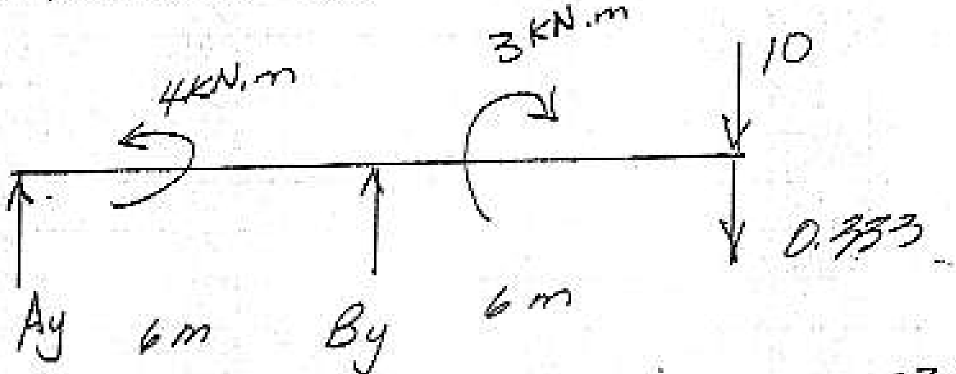
$\Sigma F_y = 0 \quad -D_y + (-0.333) = 0$

$D_y = -0.333 \therefore$

$\vec{D_y} = 0.333\ kN \uparrow$

on DC

We re-draw ①   52-417 2/2



$$\Sigma M_A = 0 \quad 4 + B_y(6) - 3 - 10.333(12) = 0$$

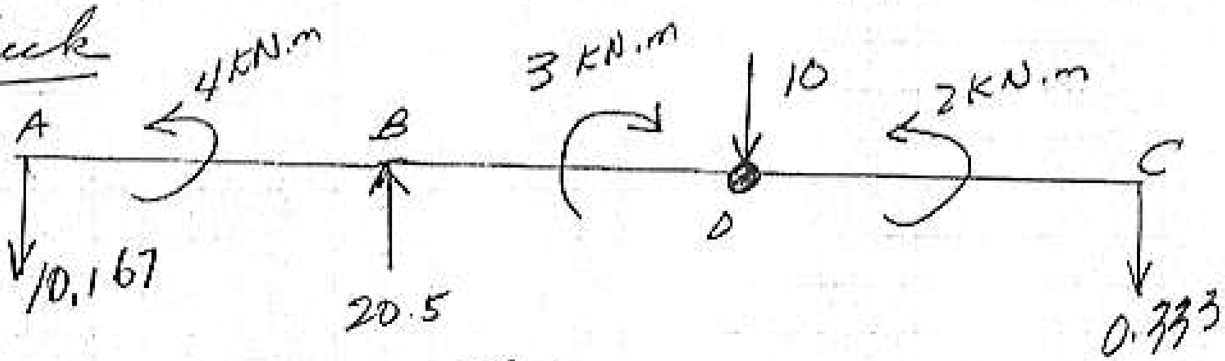
$$\underline{B_y} = +20.5^{kN}$$

$$B_y = 20.5^{kN} \uparrow$$

$$\Sigma F_y = 0 \quad A_y + 20.5 - 10 - 0.333 = 0$$

$$A_y = -10.167^{kN} \quad \therefore \vec{A_y} = 10.167^{kN} \downarrow$$

Check



$$\Sigma F_x = 0$$

$$\Sigma F_y = -10.167 + 20.5 - 10 - 0.333 = 0$$

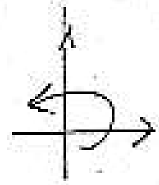
$$0 = 0$$

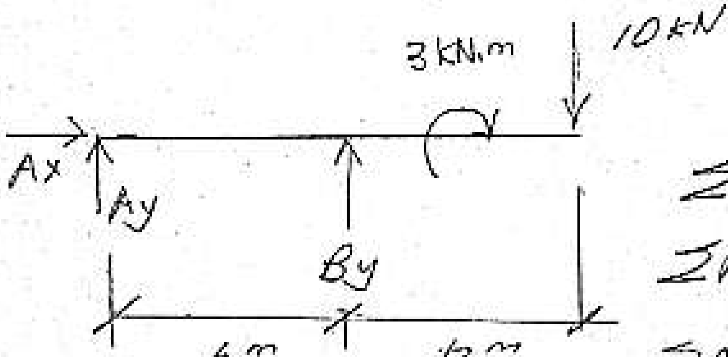
$$\Sigma M_D = 0$$

$$10.167(12) + 4 - 20.5(6) - 3 + 2 - 0.333(6) = 0$$

$$0 = 0 \checkmark$$

S2-418





$\Sigma F_x = 0 \quad A_x = 0 \quad (1)$

$\Sigma F_y = 0 \quad A_y + B_y - 10 = 0 \quad (2)$

$\Sigma M_A = 0$

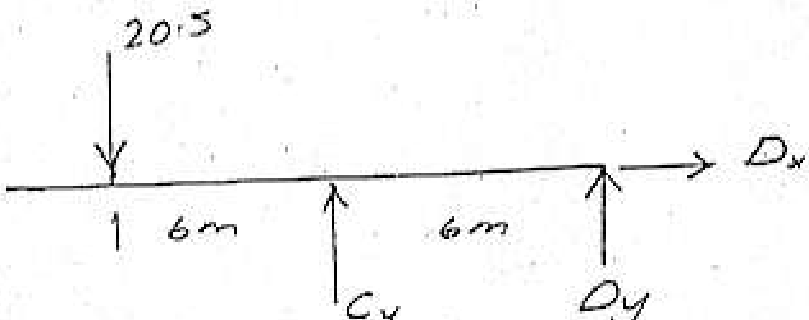
$B_y(6) - 3 - 10(12) = 0$

$B_y = +20.5\,kN \therefore \vec{B_y} = 20.5\,kN \uparrow$ on beam AB

From (2)

$A_y + 20.5 - 10 = 0$

$A_y = -10.5 \therefore \vec{A_y} = 10.5\,kN \downarrow$



$\Sigma F_x = 0 \quad D_x = 0$

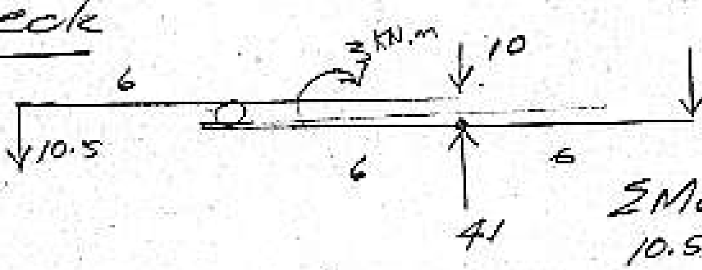
$\Sigma M_C = 0 \quad 20.5(6) + D_y(6) = 0$

$D_y = -20.5\,kN \therefore \vec{D_y} = 20.5\,kN \downarrow$

$\Sigma F_y = 0 \quad -20.5 + C_y + D_y = 0$

$C_y = +41\,kN \therefore \vec{C_y} = 41\,kN \uparrow$

check



$\Sigma F_y = 0$

$-10.5 - 10 - 20.5 + 41 = 0$

$0 = 0$

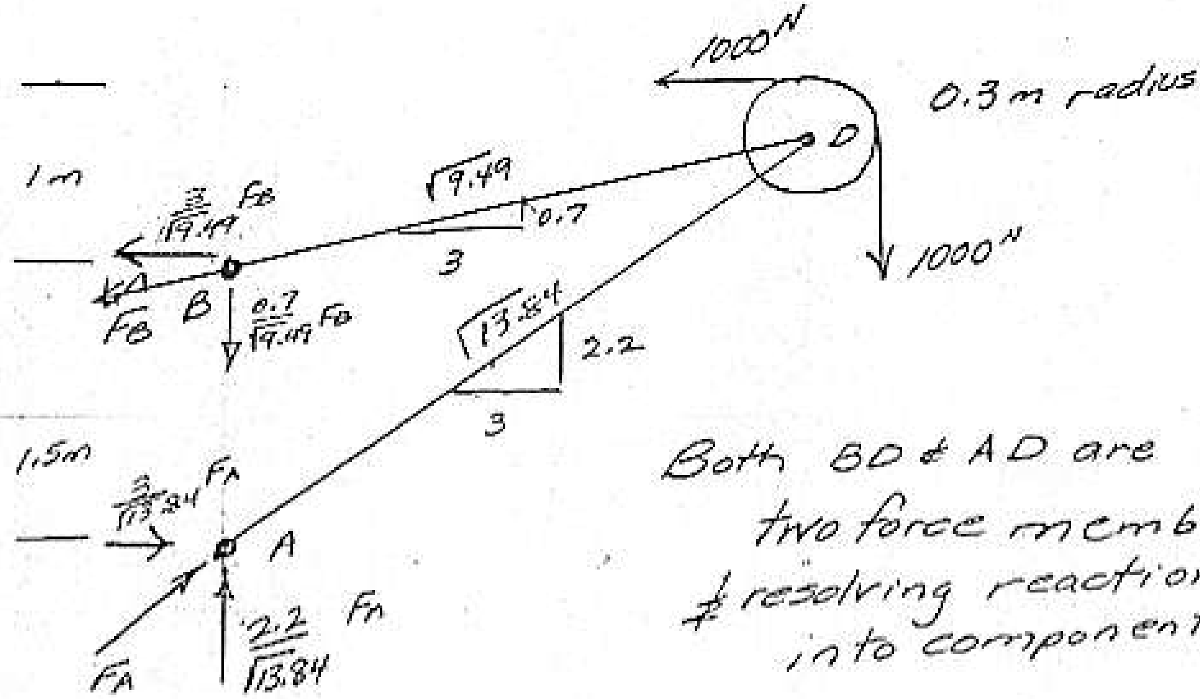
$\Sigma M_C = 0$

$10.5(12) - 3 + 20.5(6)$

$123 - 123 = 0$

$0 = 0$ ✓

S2-419



Both BD & AD are
two force members
& resolving reactions
into components

$$\Sigma M_A = 0 \quad \frac{3}{\sqrt{9.49}} F_B (1.5) + 1000(2.5) - 1000(3.3) = 0$$

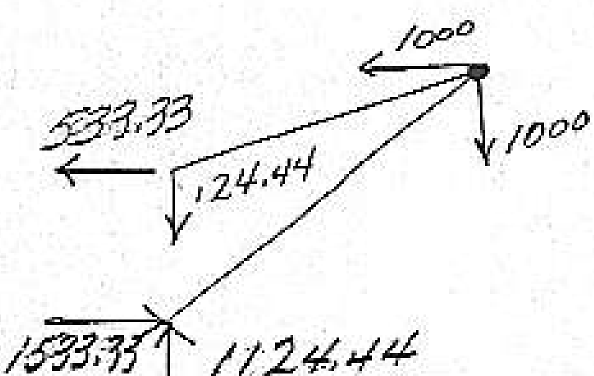
$F_B = +547.66$ N

∴ BD in tension
821.49 N

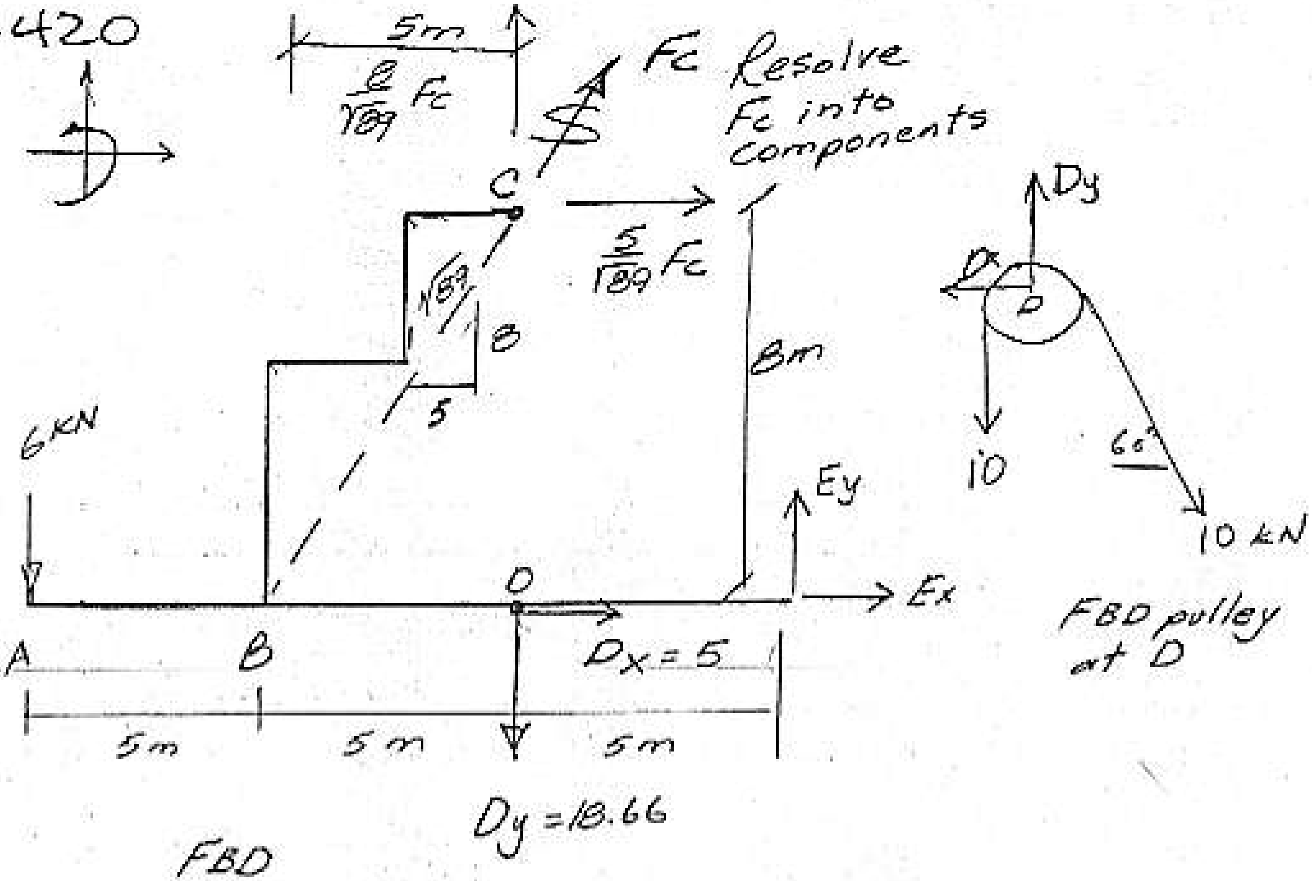
$$\Sigma F_x = 0 \quad -\frac{3}{\sqrt{9.49}}(547.66) - 1000 + \frac{3}{\sqrt{13.84}} F_A = 0$$

$F_A = +1901.44$ N

∴ AD in compression 1901.44 N



S2-420



For pulley at D: $\Sigma F_x = 0 \rightarrow \quad -D_x + 10\cos 60° = 0$

$$D_x = +5 \text{ kN}$$

$\vec{D}_x = 5 \text{ kN} \leftarrow$ on the pulley

$\therefore \vec{D}_x = 5 \text{ kN} \longrightarrow$ on the frame

$\Sigma F_y = 0 \uparrow$

$-10 + D_y - 10\sin 60° = 0 \quad D_y = 18.66 \text{ kN}$

$\therefore \vec{D}_x = 18.66 \text{ kN} \uparrow$ on the pulley

$\vec{D}_x = 18.66 \text{ kN} \downarrow$ on the frame

$\therefore$ From FBD of frame:

$\Sigma M_E = 0 \circlearrowleft \quad 6(15) - \frac{8}{\sqrt{89}}F_C(5) - \frac{5}{\sqrt{89}}F_C(8) + 18.66(5) = 0$

$-8.48 F_C = -183.3 \quad \therefore F_C = +21.62 \text{ kN}$

$\therefore \vec{F}_C = 21.62$ (slope 8:5)

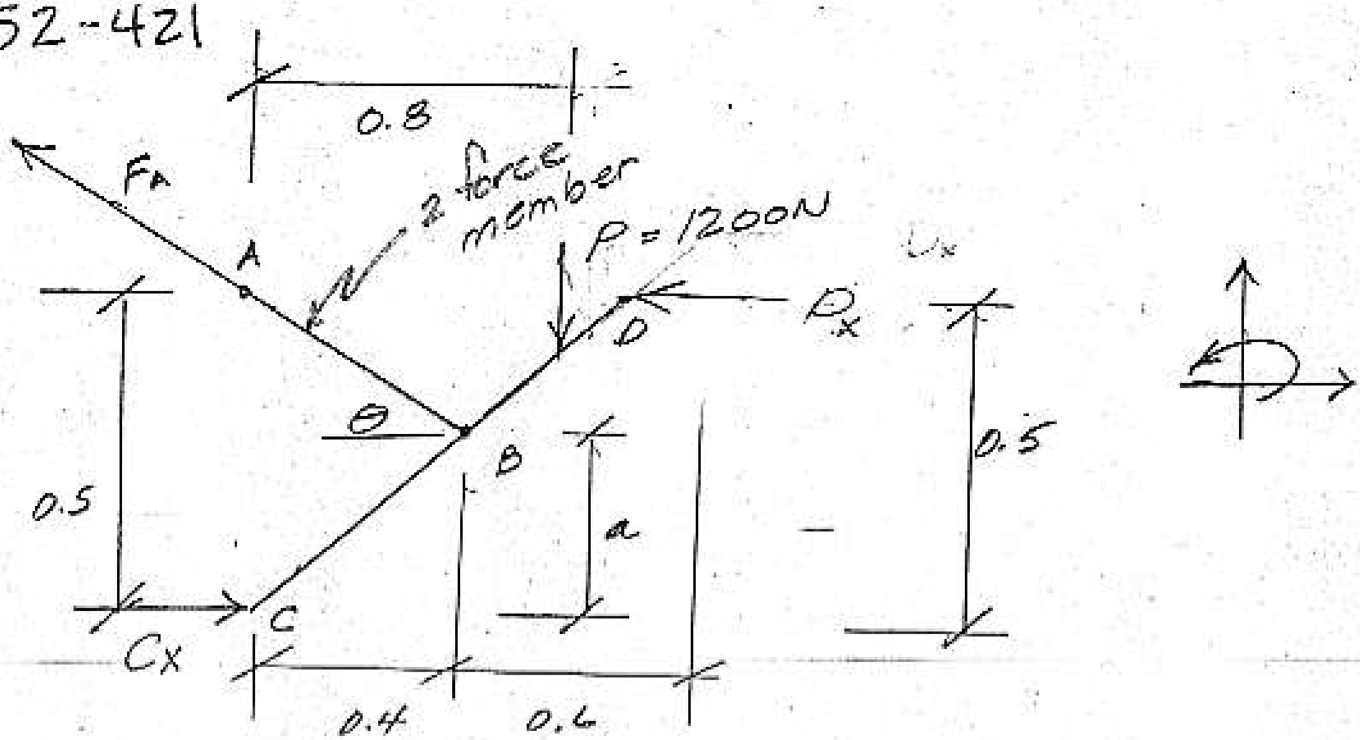
$\Sigma F_x = 0 \rightarrow \quad \frac{5}{\sqrt{89}}(21.62) + 5 + E_x = 0 \quad E_x = -16.46 \text{ kN}$

$\therefore \vec{E}_x = 16.46 \text{ kN} \longleftarrow$

$\Sigma F_y = 0 \uparrow \quad -6 + \frac{8}{\sqrt{89}}(21.62) - 18.66 + E_y = 0$

$E_y = +6.33 \text{ kN} \quad \therefore \vec{E}_y = 6.33 \text{ kN} \uparrow$

52-421



Geometry: Similar triangles $\frac{a}{0.4} = \frac{0.5}{1.0}$ $a = 0.2\ m$

$\tan\theta = \frac{3}{4}$ $\cos\theta = \frac{4}{5}$ $\sin\theta = \frac{3}{5}$

$\Sigma F_y = 0$

$\frac{3}{5} F_A - 1200 = 0$ $F_A = +2000N$

$F_A = 2000N$ (3, 4)

$\Sigma M_A = 0$ $C_x(0.5) - 1200(0.8) = 0$

$C_x = +1920N$ $\therefore$ $\vec{C}_x = 1920N \rightarrow$

$\Sigma F_x = 0$ $C_x - F_{BA}\left(\frac{4}{5}\right) - D_x = 0$

$1920 - 2000\left(\frac{4}{5}\right) - D_x = 0$

$D_x = +320N$

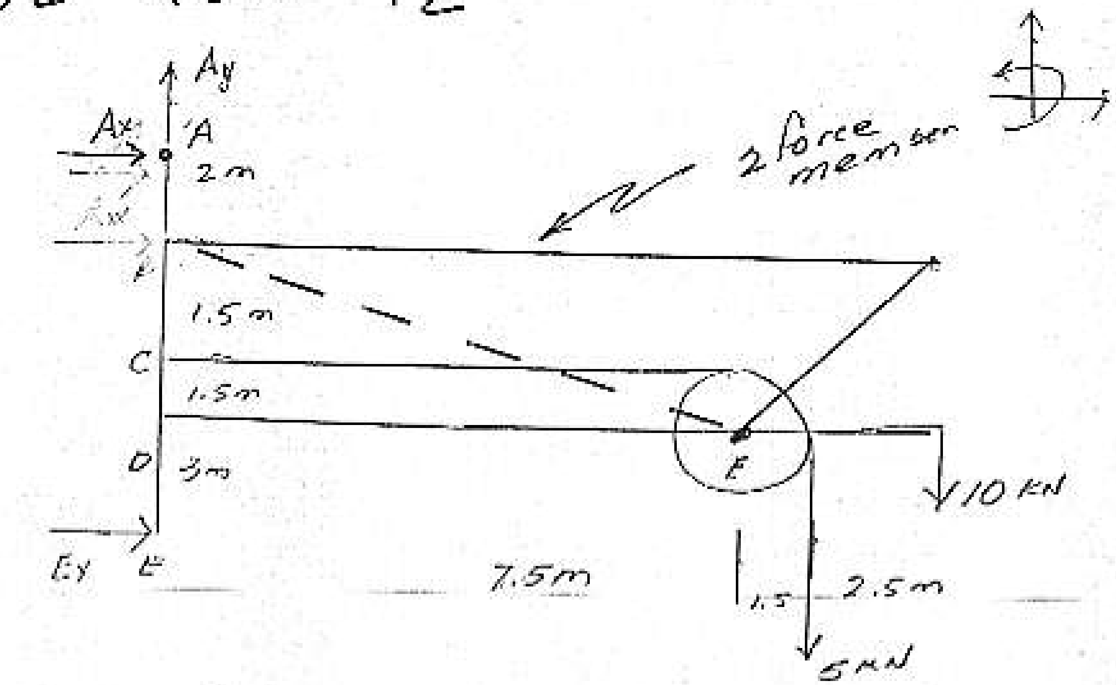
$\therefore$ $\vec{D}_x = 320N \leftarrow$

Check $\Sigma M_B = 0$

$1920(0.2) + 320(0.3) - 1200(0.4) = 0$

$0 = 0$ ✓

52-422 1/2



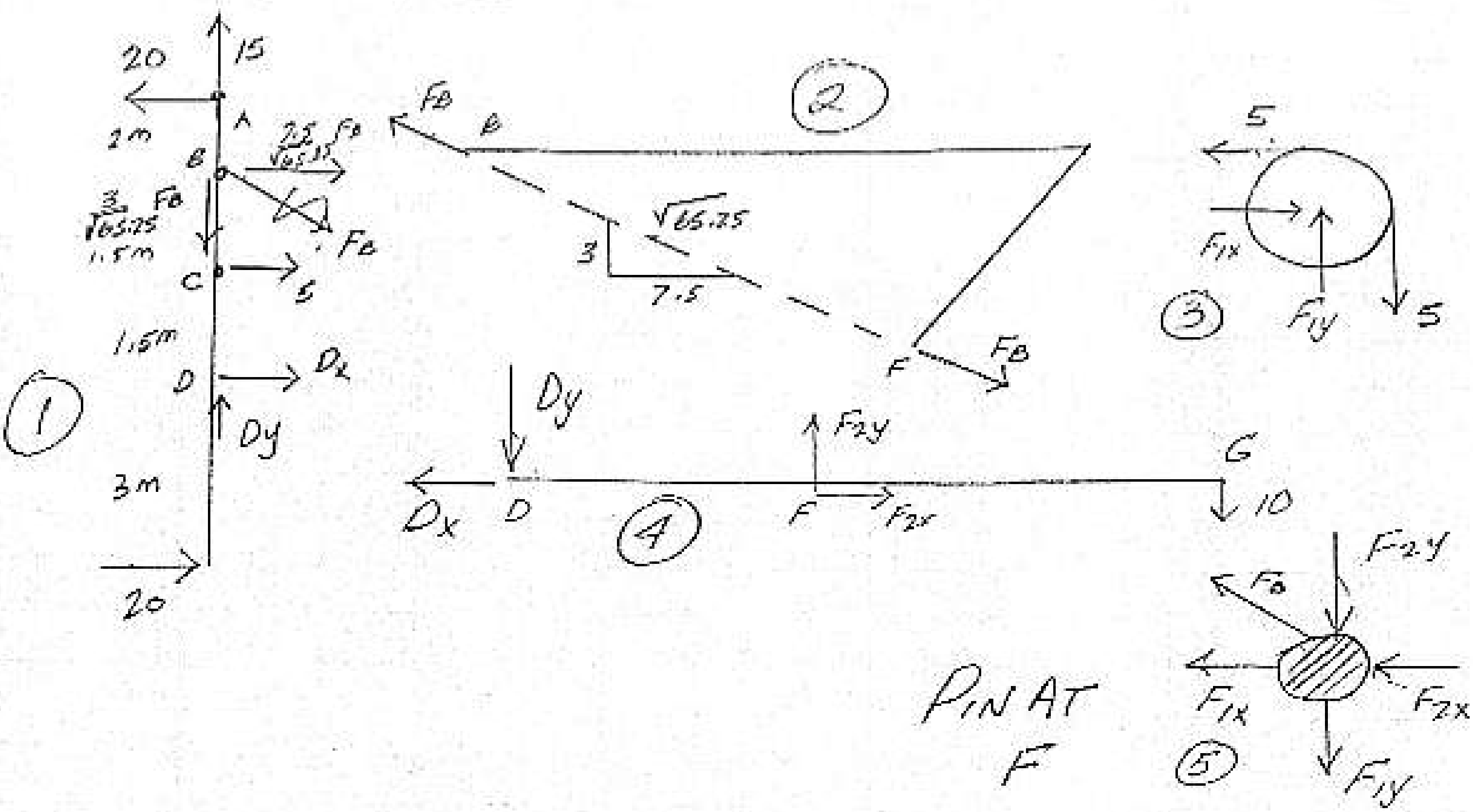
$$\Sigma M_A = 0$$

$$E_x(8) - 5(9) - 10(11.5) = 0 \qquad E_x = +20\text{kN} \;\therefore\; \vec{E}_x = 20\text{kN} \rightarrow$$

$$\Sigma F_x = 0 \qquad A_x + 20 = 0 \qquad A_x = -20\text{kN} \;\therefore\; \vec{A}_x = 20\text{kN} \leftarrow$$

$$\Sigma F_y = 0 \qquad A_y - 5 - 10 = 0 \qquad A_y = +15\text{kN} \;\therefore\; \vec{A}_y = 15\text{kN} \uparrow$$

Substructure:



52 422 2/2

From (3) $\Sigma F_x = 0 \quad F_{1x} - 5 = 0$

$F_{1x} = +5\text{ kN} \therefore \vec{F}_{1x} = 5\text{ kN} \rightarrow$ on the pulley

$\Sigma F_y = 0 \quad F_{1y} - 5 = 0 \quad F_{1y} = +5\text{ kN}$

$\therefore \vec{F}_{1y} = 5\text{ kN} \uparrow$ on the pulley

From (1) $\Sigma M_D = 0$

$$20(5) - \frac{7.5}{\sqrt{65.25}} F_B(3) - 5(1.5) + 20(3) = 0$$

$F_B = +54.75\text{ kN} \therefore \vec{F}_B = 54.75\text{ kN}$ (slope 3 : 7.5, ↘) on ABCDE

$\Sigma F_x = 0$

$$-20 + \frac{7.5}{\sqrt{65.25}}(54.75) + 5 + D_x + 20 = 0$$

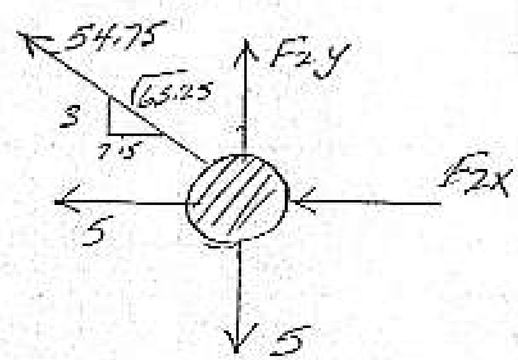
$D_x = -55.83\text{ kN} \therefore \vec{D}_x = 55.83\text{ kN}$ on ABCDE

$\Sigma F_y = 0$

$$15 - \frac{3}{\sqrt{65.25}}(54.75) + D_y = 0$$

$D_y = +5.33\text{ kN} \therefore \vec{D}_y = 5.33\text{ kN} \uparrow$ on ABCDE

From (5) We redraw pin



$\Sigma F_x = 0 \quad -5 - \frac{7.5}{\sqrt{65.25}}(54.75) - F_{2x} = 0$

$F_{2x} = -55.83\text{ kN}$

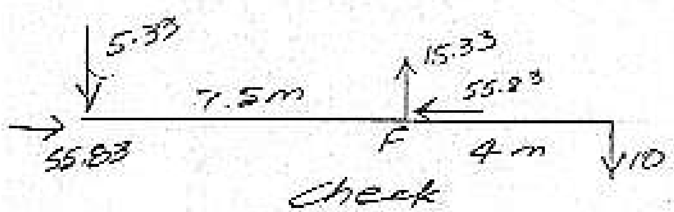
$\vec{F}_{2x} = 55.83\text{ kN} \rightarrow$ on the pin at F

$\Sigma F_y = 0$

$$\frac{3}{\sqrt{65.25}}(54.75) - 5 + F_{2y} = 0$$

$F_{2y} = -15.33\text{ kN}$

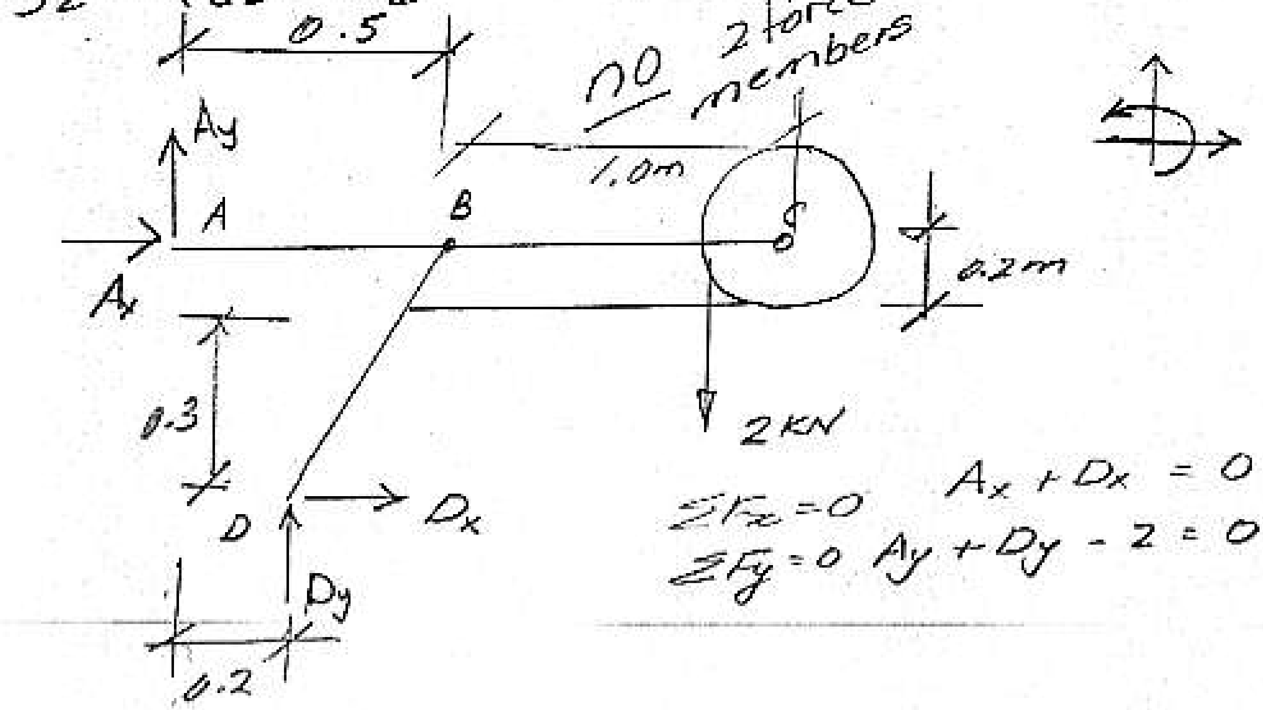
$\therefore \vec{F}_{2y} = 15.33\text{ kN} \downarrow$ on the pin at F



$\Sigma M_D = 0 \quad 15.33(7.5) - 10(11.5) = 0$

$\approx 0.03 = 0 \quad$ ✓ OK

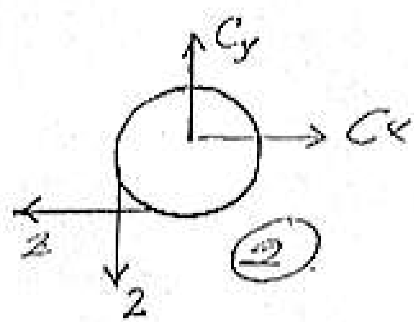
SZ-423 1/2

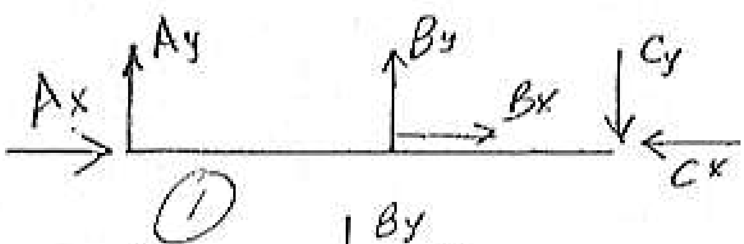


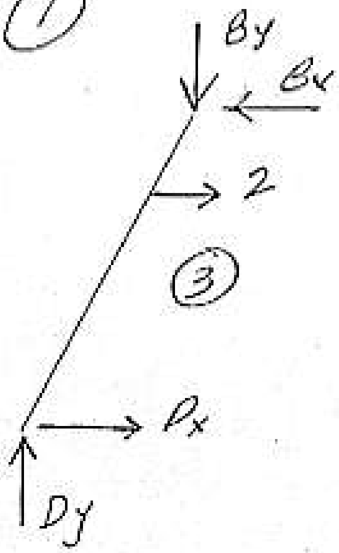
$\Sigma F_x = 0 \quad A_x + D_x = 0$ (1)

$\Sigma F_y = 0 \quad A_y + D_y - 2 = 0$ (2)

Substructure:







From (2):

$\Sigma F_x = 0 \quad -2 + C_x = 0$

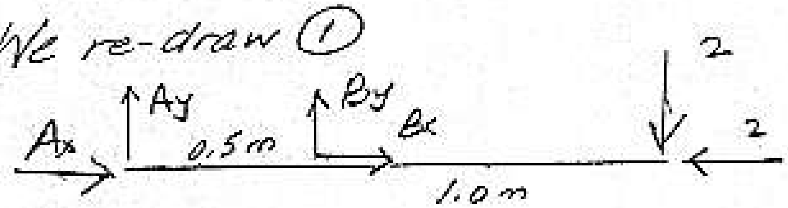
$C_x = +2\,kN \therefore \vec{C_x} = 2\,kN \rightarrow$ on the pulley

$\Sigma F_y = 0$

$C_y - 2 = 0$

$C_y = +2\,kN \therefore \vec{C_y} = 2\,kN \uparrow$ on the pulley

We re-draw (1)



S2-423 2/2

$\Sigma M_A = 0$ ↺

$B_y(0.5) - 2(1.5) = 0$

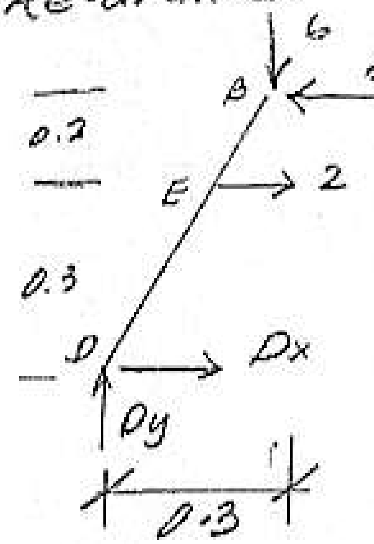
$B_y = \frac{3}{0.5} = +6\text{ kN}$ $\therefore$ $\vec{B_y} = 6\text{ kN}$ ↑ on ABC

$\Sigma F_y = 0$ $\quad A_y + B_y - 2 = 0$ $\quad A_y = -4\text{ kN}$

$A_y + 6 - 2 = 0$

$\therefore A_y = 4\text{ kN}$ ↓

Re-draw ②



$\Sigma F_y = 0$

$-6 + D_y = 0$ $\quad D_y = +6\text{ kN}$

$\therefore \vec{D_y} = 6\text{ kN}$ ↑

$\Sigma M_D = 0$ $\quad -6(0.3) + B_x(0.5) - 2(0.3) = 0$

$B_x = +4.8\text{ kN}$

$\therefore \vec{B_x} = 4.8\text{ kN} \leftarrow$ on DEB

$\Sigma F_x = 0$

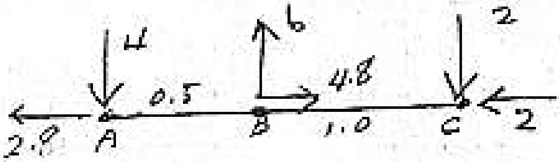
$D_x + 2 - 4.8 = 0$

$D_x = +2.8$ $\quad \vec{D_x} = 2.8\text{ kN} \rightarrow$

From ① $\quad A_x + D_x = 0$ $\quad A_x = -2.8\text{ kN}$ $\therefore \vec{A_x} = 2.8\text{ kN} \leftarrow$

$A_x + 2.8 = 0$

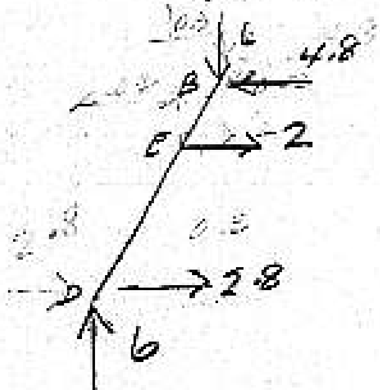
check.



$\Sigma M_B = 0$

$4(0.5) - 2(1) = 0$

$0 = 0$ ✓

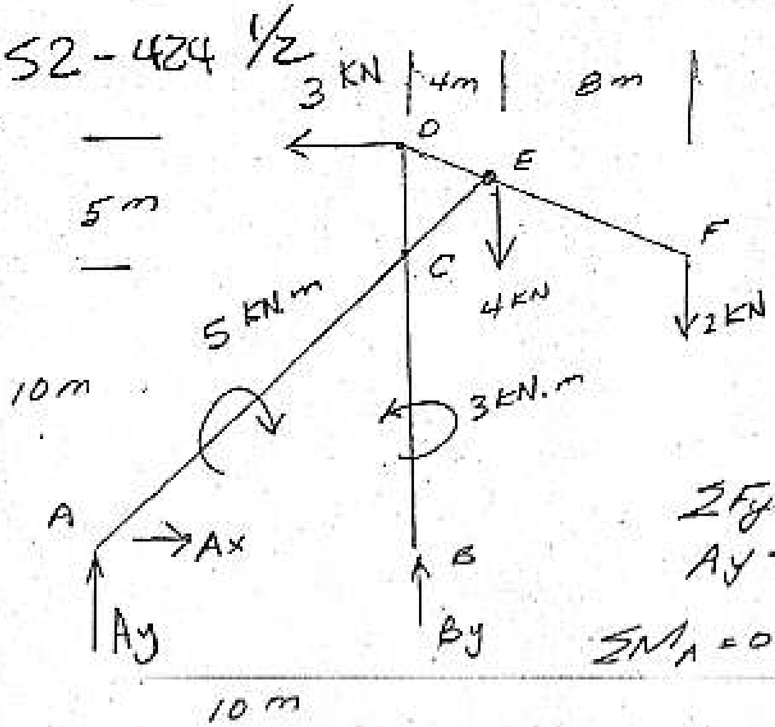


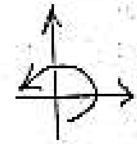
$\Sigma M_B = 0$

$-2(0.2) + 2.8(0.5) - 6(0.3) = 0$

$0 = 0$ ✓

52-424 1/2





$\Sigma F_x = 0$ (1)

$A_x - 3 = 0$

$A_x = +3\,kN \therefore \vec{A}_x = 3\,kN \rightarrow$

$\Sigma F_y = 0$

$A_y - 4 - 2 + B_y = 0$ (2)

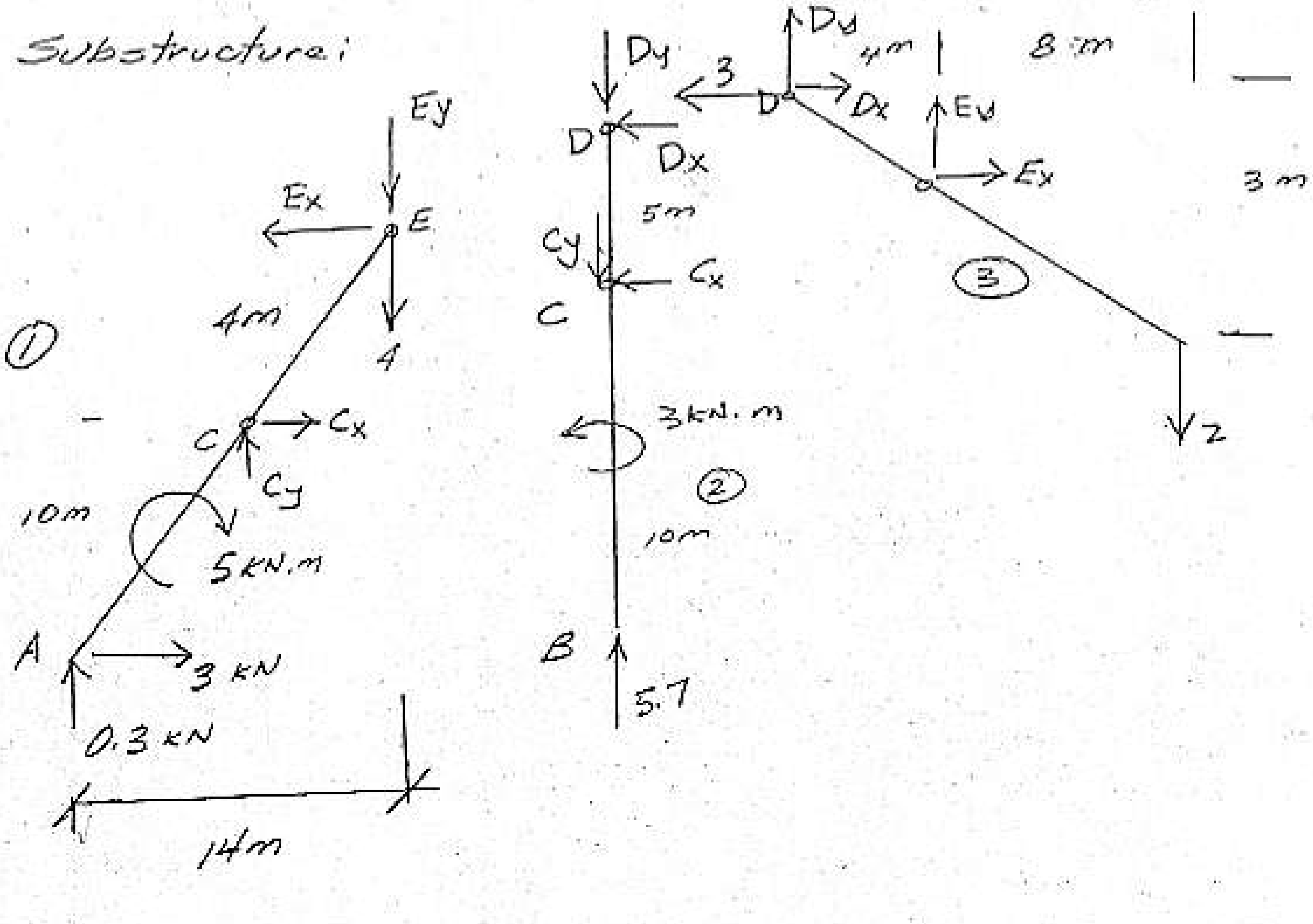
$\Sigma M_A = 0 \quad -5 + 3 + 3(15) - 4(14) - 2(22) + B_y(10) = 0$ (3)

$B_y = +5.7\,kN \therefore \vec{B}_y = 5.7\,kN \uparrow$

From (2) $\quad A_y - 4 - 2 + 5.7 = 0 \rightarrow$

$A_y = +0.3\,kN \quad \vec{A}_y = 0.3\,kN \uparrow$

Substructure:



52-424 2/2

From ② $\Sigma M_D = 0 \quad +3 - C_x(5) = 0$

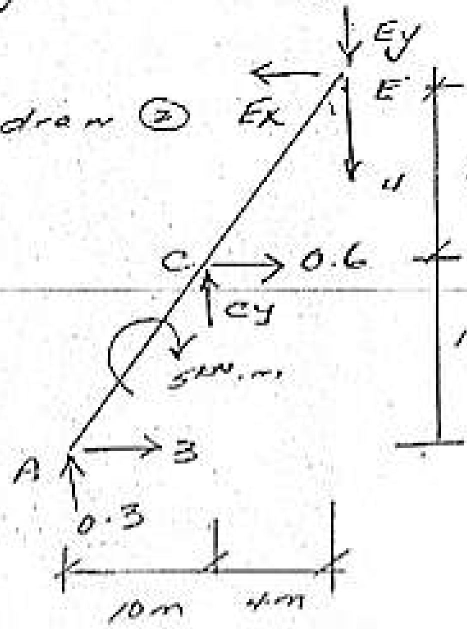
$C_x = +0.6\text{ kN} \quad \vec{C}_x = 0.6\text{ kN} \leftarrow$ on BCD

$\Sigma F_x = 0 \quad -0.6 - D_x = 0$

$D_x = -0.6\text{ kN} \therefore \vec{D}_x = 0.6\text{ kN} \rightarrow$ on BCD

$\Sigma F_y = 0 \quad 5.7 - C_y - D_y = 0$

We redraw ②



$\Sigma M_E = 0$

$-0.3(14) + 3(14) - 5$

$-C_y(4) + 0.6(4) = 0$

$C_y = +8.8\text{ kN}$

$\therefore \vec{C}_y = 8.8\text{ kN} \uparrow$ on ACE

$\Sigma F_x = 0$

$3 + 0.6 - E_x = 0$

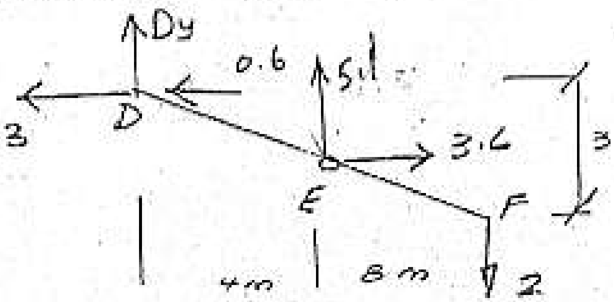
$E_x = +3.6\text{ kN}$

$\therefore \vec{E}_x = 3.6\text{ kN} \leftarrow$ on ACE

$\Sigma F_y = 0 \quad 0.3 + 8.8 - 4 - E_y = 0 \quad E_y = +4.5\text{ kN}$

$\vec{E}_y = 5.1\text{ kN} \downarrow$ on ACE

We redraw ③

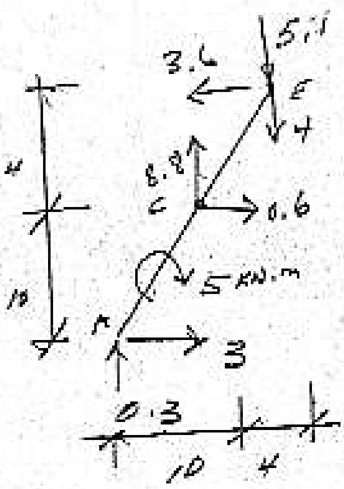


$\Sigma F_y = 0$

$D_y + 5.1 - 2 = 0$

$D_y = +3.1\text{ kN}$

$\therefore \vec{D}_y = 3.1\text{ kN} \downarrow$ on DEF

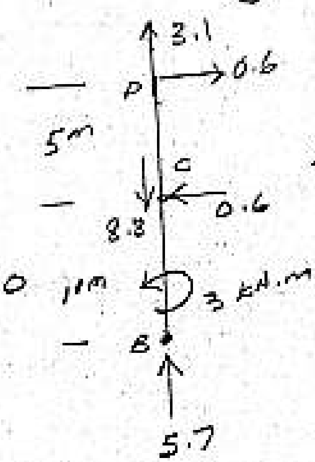


$\Sigma M_C = 0$

$-0.3(10) + 3(10) - 5$

$+3.6(4) - 9.1(4) = 0$

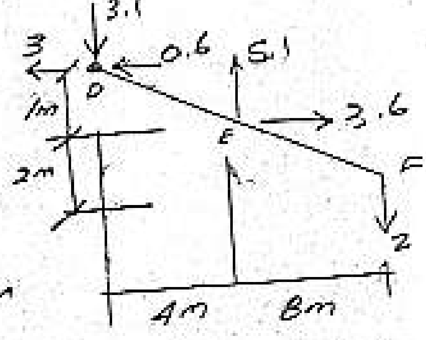
$0 = 0$



$\Sigma M_C = 0$

$-0.6(5) + 3 = 0$

$0 = 0$



$\frac{y}{4} = \frac{3}{12} \quad y = 1\text{ m}$

$\Sigma M_E = 0$

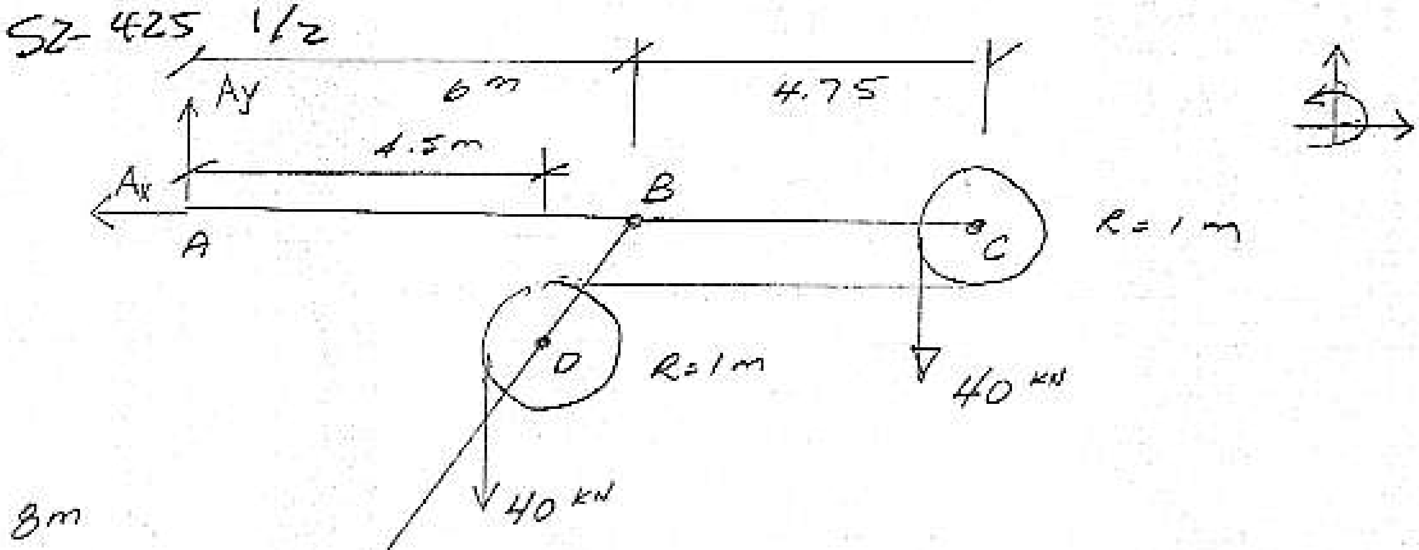
$3(1) + 3.1(4) + 0.6(1)$

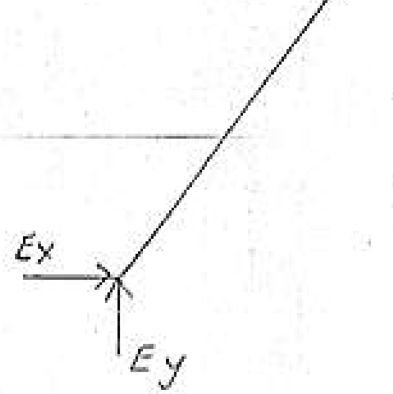
$-2(8) = 0$

$16 - 16 = 0$

$0 = 0$ ✓

SZ-425 1/2





$$\Sigma M_A = 0$$

$$E_x(8) - 40(3.5) - 40(9.75) = 0$$

$$E_x = +66.25 \text{ kN}$$

$$\therefore \vec{E}_x = 66.25 \text{ kN} \rightarrow$$

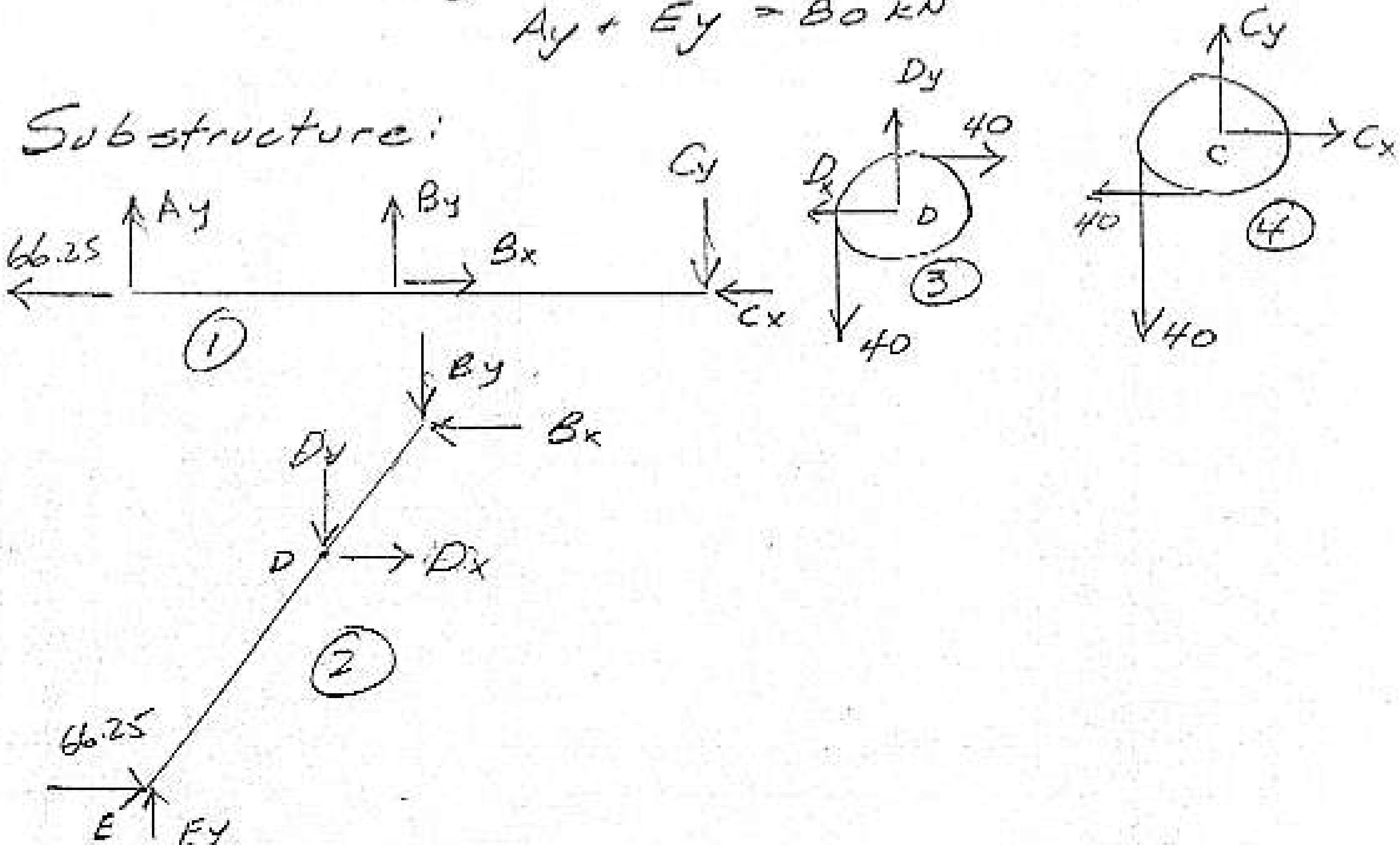
$$\Sigma F_x = 0 \qquad -A_x + 66.25 \text{ kN} = 0$$

$$A_x = +66.25 \text{ kN} \quad \therefore \vec{A}_x = 66.25 \text{ kN} \leftarrow$$

$$\Sigma F_y = 0 \qquad A_y + E_y - 40 - 40 = 0$$

$$A_y + E_y = 80 \text{ kN}$$

Substructure:



S2-425 2/2

FROM ③ $\Sigma F_x = 0 \quad 40 - D_x = 0$

$D_x = +40\,kN \quad \therefore \vec{D_x} = 40\,kN \leftarrow$ on the pulley

$\Sigma F_y = 0 \quad D_y - 40 = 0$

$D_y = +40\,kN \quad \therefore \vec{D_y} = 40\,kN \uparrow$ on the pulley

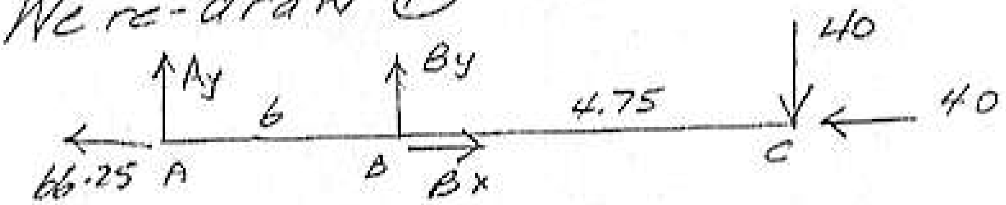
From ④ $\Sigma F_x = 0$

$C_x - 40 = 0 \quad C_x = +40\,kN \quad \therefore \vec{C_x} = 40\,kN \rightarrow$ on the pulley

$\Sigma F_y = 0 \quad C_y - 40 = 0$

$C_y = +40\,kN \quad \therefore \vec{C_y} = 40\,kN \uparrow$ on the pulley

We re-draw ①



$\Sigma F_x = 0 \quad -66.25 + B_x - 40 = 0$

$B_x = +106.25\,kN$

$\vec{B_x} = 106.25\,kN \rightarrow$ on ABC

$\Sigma M_B = 0 \quad -A_y(6) - 40(4.75) = 0$

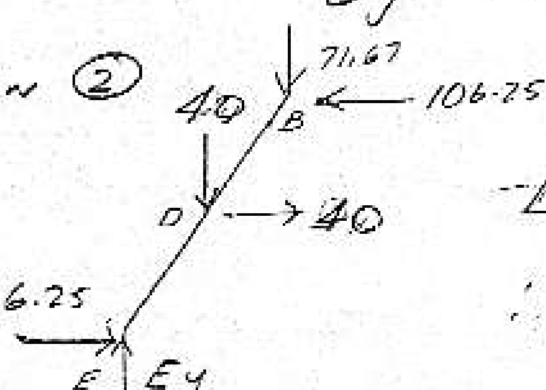
$A_y = -31.67\,kN$

$\therefore \vec{A_y} = 31.67\,kN \downarrow$ on ABC

$\Sigma F_y = 0 \quad -31.67 + B_y - 40 = 0$

$B_y = +71.67\,kN \quad \vec{B_y} = 71.67\,kN \uparrow$ on ABC

We redraw ②



$\Sigma F_y = 0$

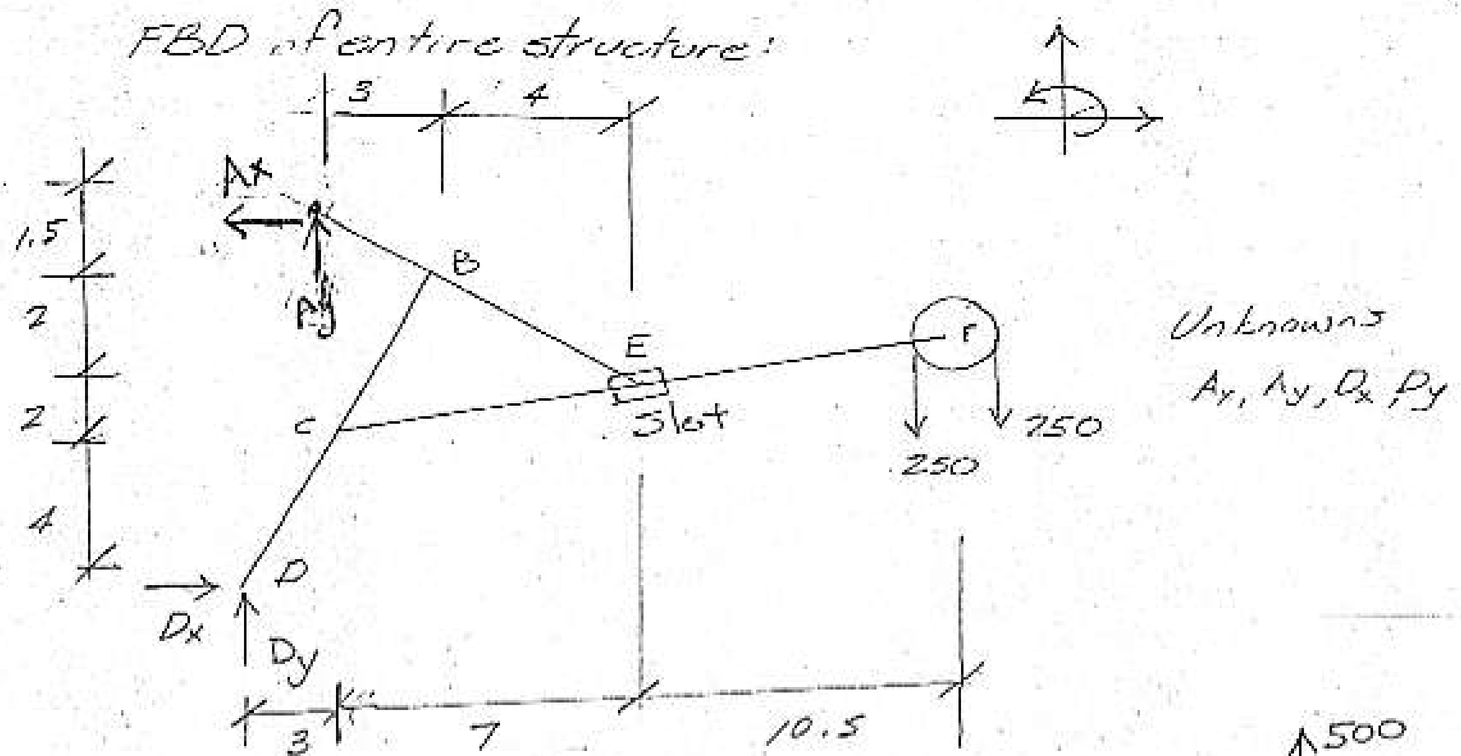
$-E_y - 40 - 71.67 = 0$

$E_y = +111.67\,kN$

$\therefore \vec{E_y} = 111.67\,kN \uparrow$ on BDE

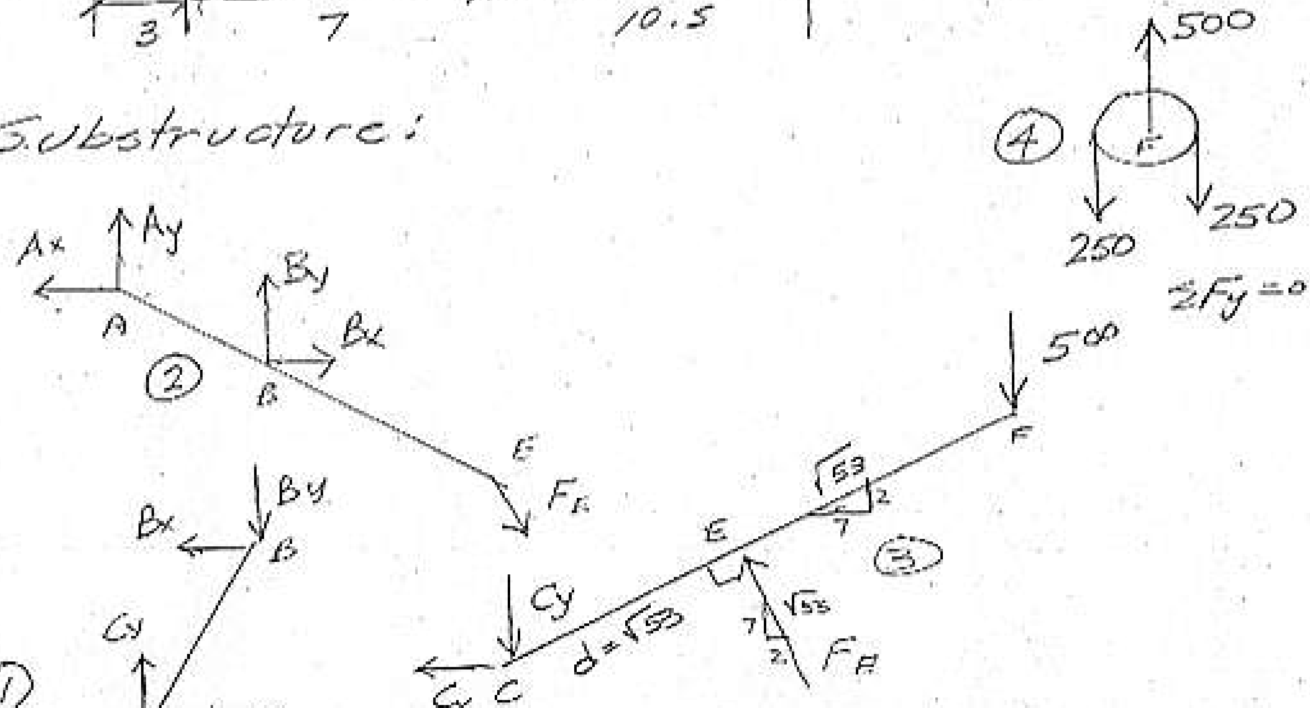
52 - 426 1/4

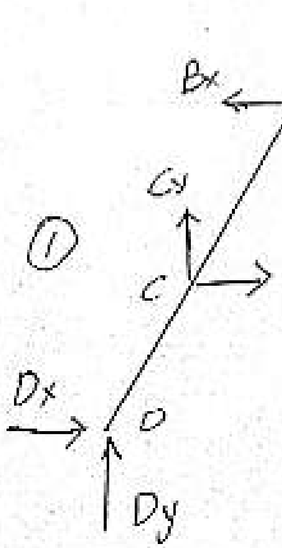
FBD of entire structure:



Unknowns
A_x, A_y, D_x, D_y

Substructure:





From 3

$\Sigma M_C = 0 \qquad F_E(\sqrt{53}) - 500(17.5) = 0$

$F_E = +1201.9\ kN$

$F_E = 1201.9\ kN$ on CEF

$\Sigma F_x = 0 \qquad -C_x - \frac{2}{\sqrt{53}}(1201.9) = 0$

$C_x = -330.19\ kN \quad \therefore \vec{C_x} = 330.19\ kN \rightarrow$ on CEF

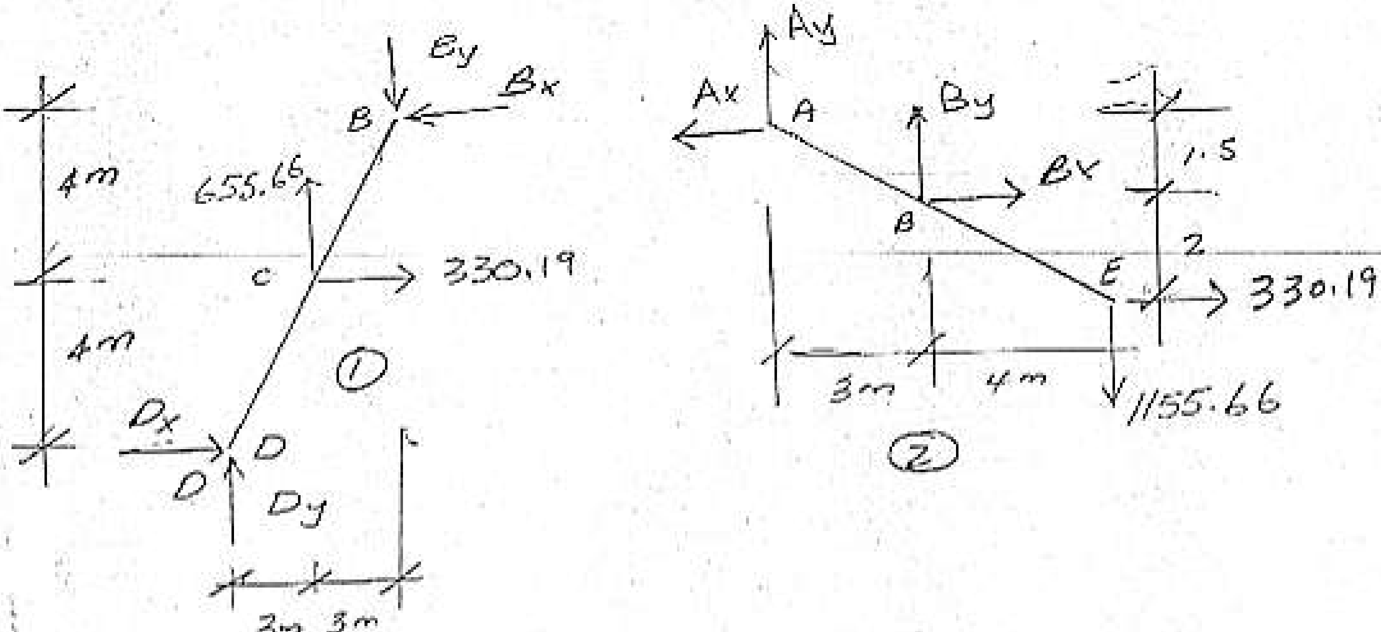
52-426 2/4

$$\Sigma F_y = -C_y + \frac{7}{\sqrt{53}}(1201.9) - 500 = 0$$

$$C_y = +655.66\,kN$$

$$\vec{C_y} = 655.66\,kN \downarrow \text{ on CEF}$$

We re-draw ① + ②



From① $\Sigma M_D = 0$ $B_x(8) - B_y(6) + 330.19(4) + 655.66(3) = 0$

$$8B_x - 6B_y = -3287.74$$

From② $\Sigma M_A = 0$ $B_x(1.5) + B_y(3) - 1155.66(7) + 330.19(3.5) = 0$

$$1.5B_x + 3B_y = 6933.96$$

$$8B_x - 6B_y = -3287.74$$
$$3B_x + 6B_y = 13867.91$$
$$11B_x = 10580.17 \qquad B_x = +961.83\,kN$$

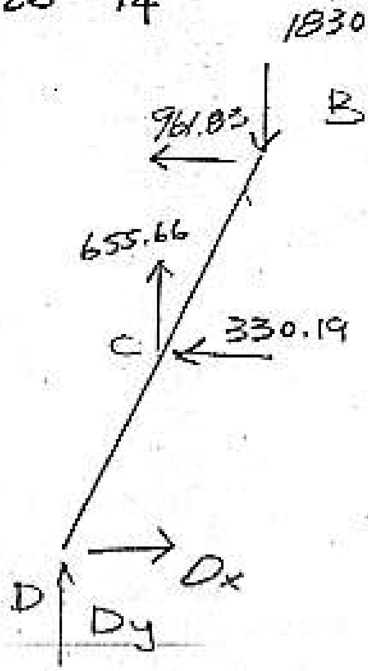
$$\vec{B_x} = 961.83 \leftarrow \text{ on DCB}$$
$$\rightarrow \text{ on ABE}$$

$$1.5(961.83) + 3B_y = 6933.96$$

$$B_y = +1830.41$$

$$B_y = 1830.41 \downarrow \text{ on DCB}$$
$$\uparrow \text{ on ABE}$$

52-426 3/4



$\sum F_x = 0$

$D_x - 961.83 - 330.19$

$D_x = +1292.02 \text{ kN}$

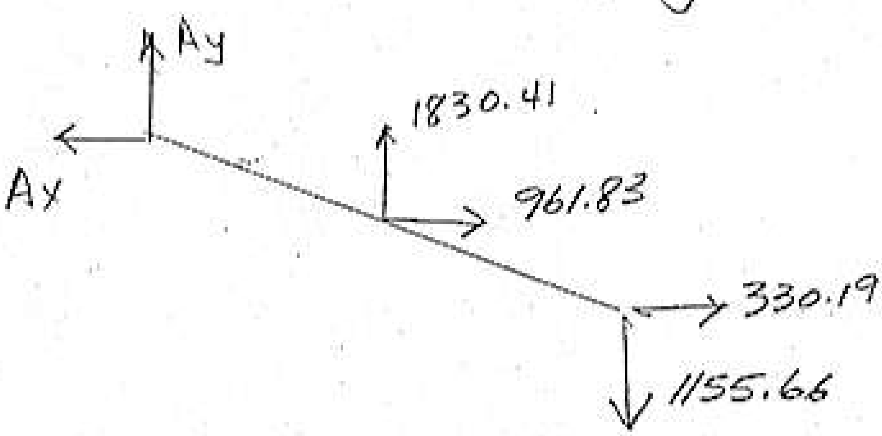
$\vec{D}_x = 1292.02 \text{ kN} \rightarrow$

$\sum F_y = 0$

$D_y + 655.66 - 1830.41 = 0$

$D_y = +1174.75 \text{ kN}$

$\vec{D}_y = 1174.75 \text{ kN} \uparrow$



$\sum F_x = 0 \quad -A_x + 961.83 + 330.19 = 0$

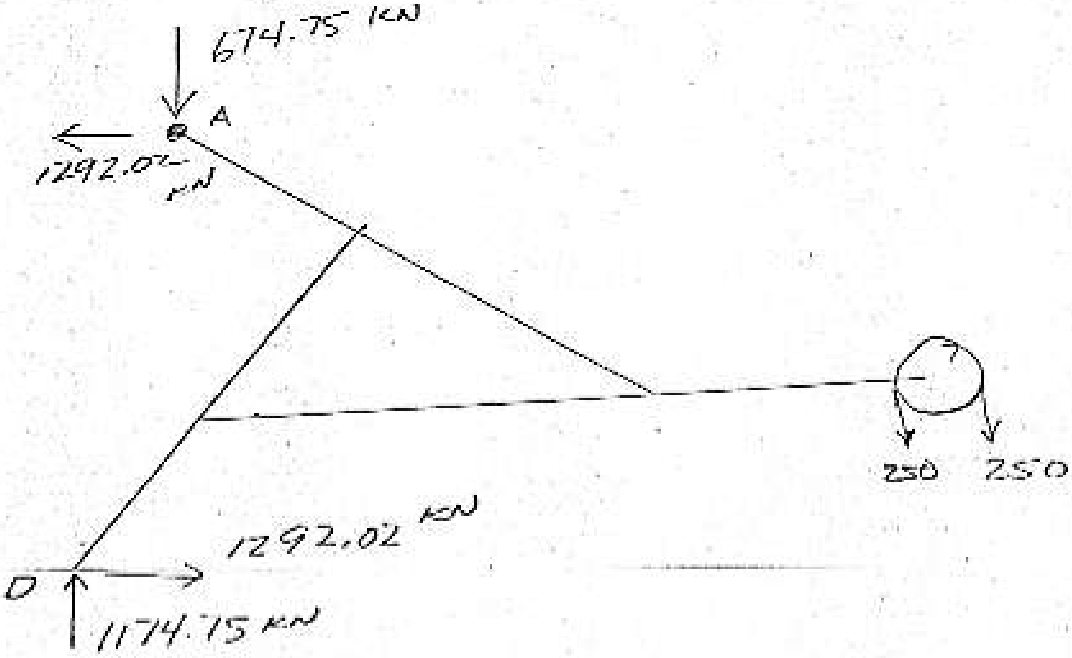
$A_x = +1292.02$

$\vec{A}_x = 1292.02 \text{ kN} \leftarrow$

$\sum F_y = 0 \quad A_y + 1830.41 - 1155.66 = 0$

$A_y = -674.75 \text{ kN}$

$\vec{A}_y = 674.75 \text{ kN} \downarrow$



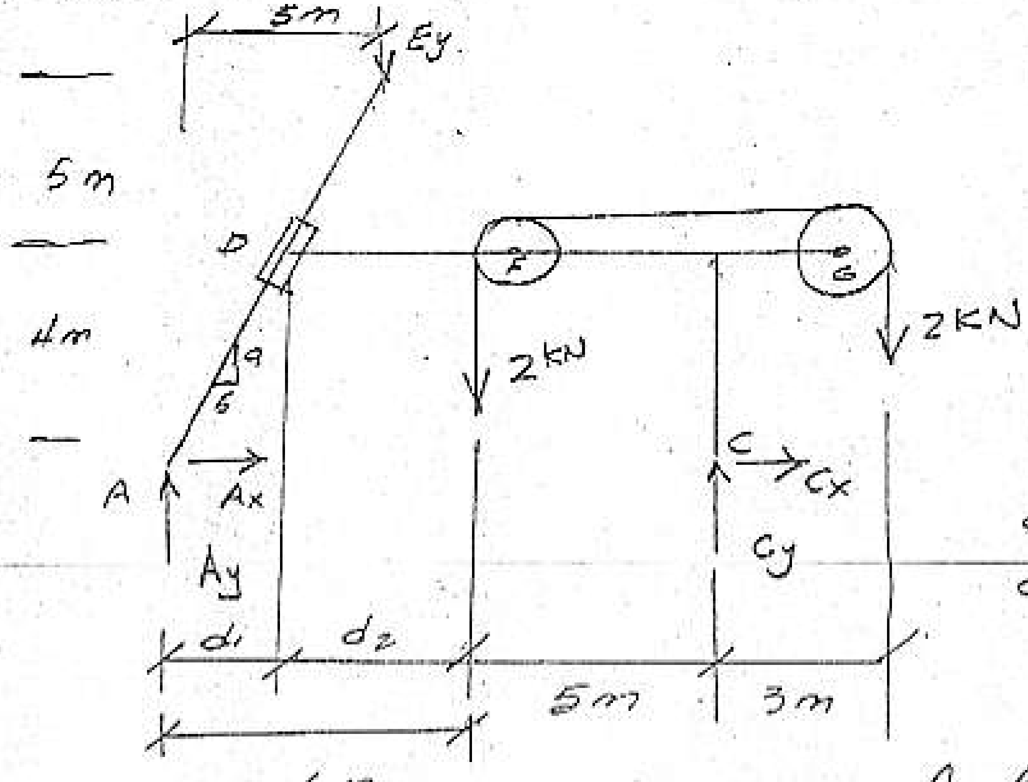
$\Sigma M_A = 0$

$-1174.75(3) + 1292.02(9.5) - 500(17.5) = 0$

$-0.06 = 0 \quad$ OK.

S2-427 1/2

FBD entire system



By similar triangles

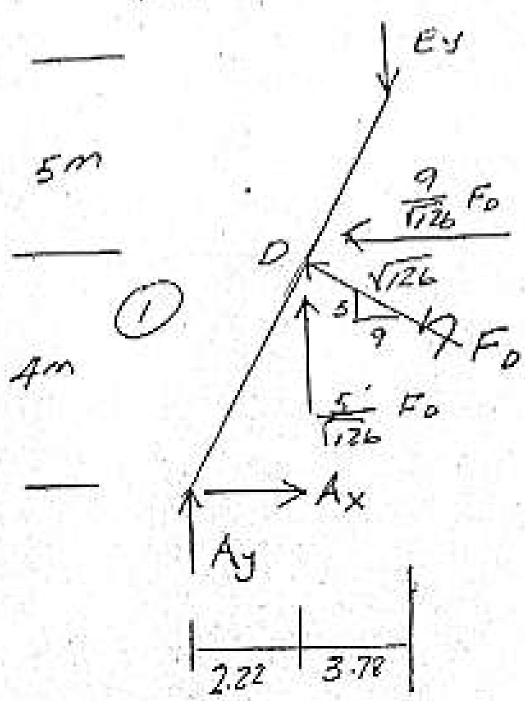
$$\frac{d_1}{4} = \frac{5}{9}$$

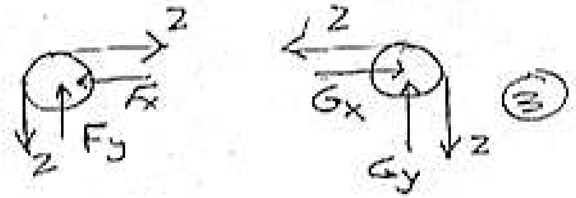
$d_1 = 2.22\,m$

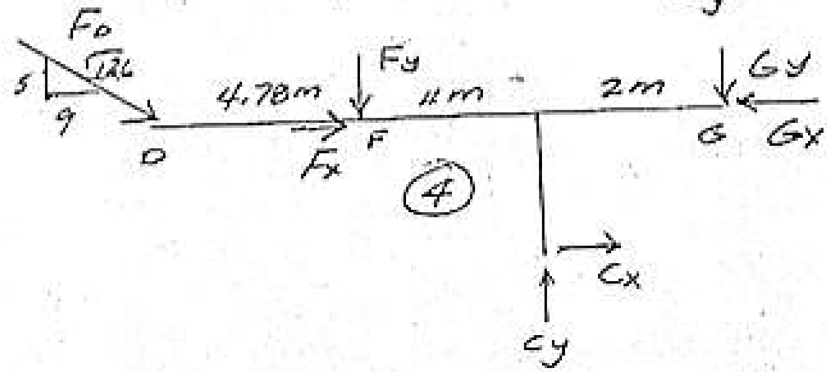
$d_2 = 6 - 2.22 = 3.78\,m$

Unknowns: A_x, A_y, E_y, C_x, C_y

We substructure:







From ② $\Sigma F_x = 0 \quad 2 - F_x = 0$

$F_x = +2\,kN \quad \vec{F_x} = 2\,kN \leftarrow$ on the pulley

$\Sigma F_y = 0 \quad F_y - 2 = 0 \quad F_y = +2\,kN$

$\therefore \vec{F_y} = 2\,kN \uparrow$ on the pulley

From ③

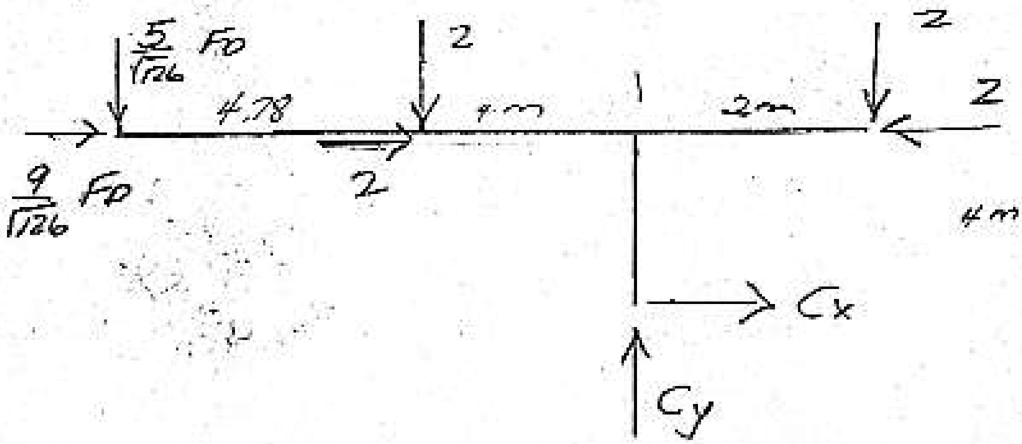
$\Sigma F_x = 0 \quad G_x - 2 = 0$

$G_x = +2\,kN \quad \therefore \vec{G_x} = 2\,kN \rightarrow$ on the pulley

$\Sigma F_y = 0 \quad G_y - 2 = 0$

$G_y = +2\,kN \quad \vec{G_y} = 2\,kN \uparrow$ on the pulley

We redraw ④ 52-427 2/2



$\Sigma M_C = 0$

$$\frac{5}{\sqrt{126}} F_D (8.78) - \frac{9}{\sqrt{126}} F_D (4) + 2(4) - 2(4) - 2(2) + 2(4) = 0$$

$$0.704 F_D + 4 = 0$$

$$\therefore F_D = -5.68 \text{ kN} \quad \vec{F_D} = 5.68 \text{ kN}$$ on DFG

$\Sigma F_x = 0 \quad \frac{9}{\sqrt{126}} F_D + 2 + C_x - 2 = 0$

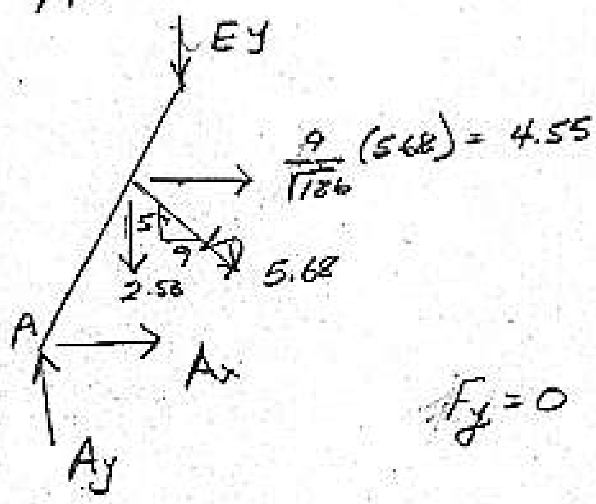
$$\frac{9}{\sqrt{126}}(-5.68) + 2 + C_x - 2 = 0$$

$$C_x = +4.56 \text{ kN} \quad \therefore \vec{C_x} = 4.56 \text{ kN} \rightarrow$$

$\Sigma F_y = 0 \quad \frac{-5}{\sqrt{126}}(-5.68) - 2 + C_y - 2 = 0$

$$C_y = +1.47 \text{ kN} \quad \vec{C_y} = 1.47 \text{ kN} \uparrow$$

We Redraw ①



$\Sigma F_x = 0 \quad 4.55 + A_x = 0$

$$A_x = -4.55 \text{ kN}$$

$$\therefore \vec{A_x} = 4.55 \text{ kN} \leftarrow$$

$\Sigma M_A = 0 \quad -E_y(5) - 2.53(2.22) - 4.55(4) = 0$

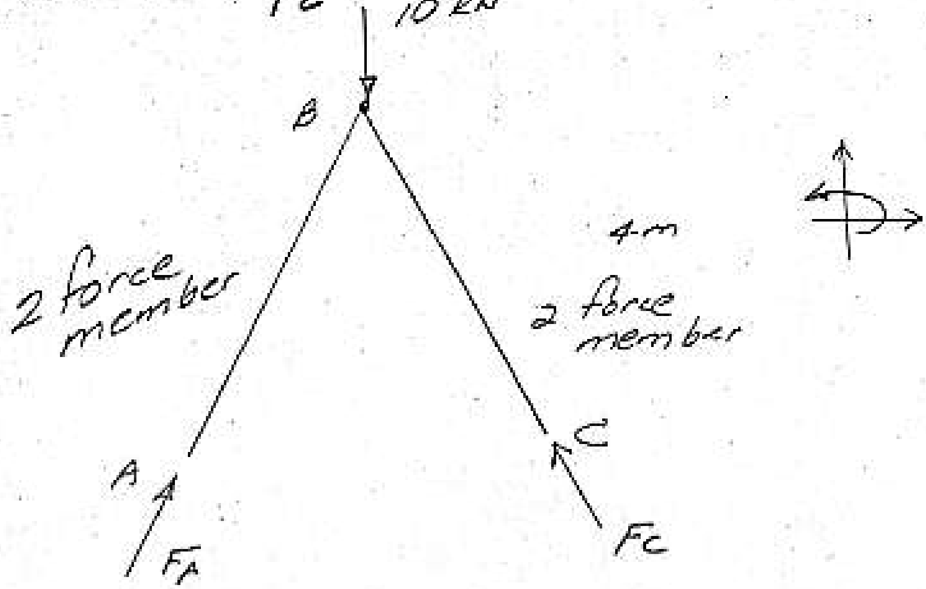
$$E_y = -4.76 \text{ kN} \quad \therefore \vec{E_y} = 4.76 \text{ kN} \uparrow$$

$\Sigma F_y = 0 \quad -E_y - 2.53 + A_y = 0$

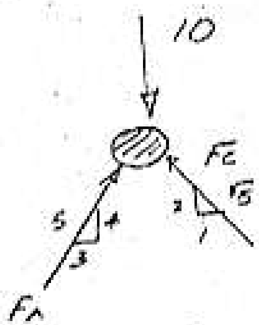
$$-(-4.76) - 2.53 + A_y = 0$$

$$A_y = -2.23 \text{ kN} \quad \vec{A_y} = 2.23 \text{ kN} \downarrow$$

52-428 1/2



a)



$$\Sigma F_x = 0$$

$$\frac{3}{5} F_A - \frac{1}{\sqrt{5}} F_C = 0 \qquad (1)$$

$$\therefore F_A = \frac{5}{3}\left(\frac{1}{\sqrt{5}}\right) F_C = 0.745 F_C$$

$$\Sigma F_y = 0 \qquad \frac{4}{5} F_C + \frac{2}{\sqrt{5}} F_C - 10 = 0 \qquad (2)$$

$$\frac{4}{5} F_C + \frac{2}{\sqrt{5}}(0.745 F_C) = 10$$

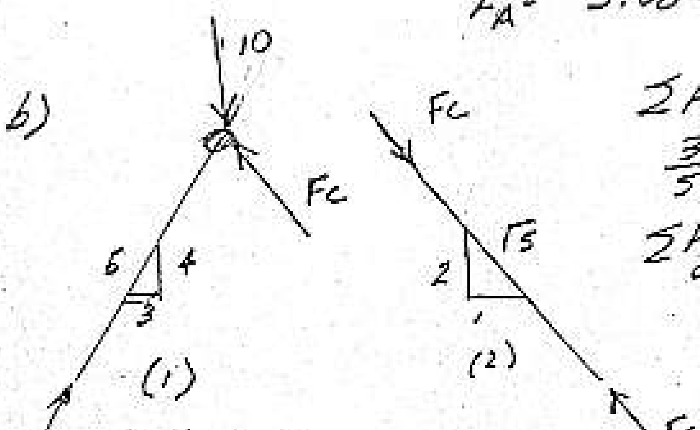
$$F_C = +6.82 \text{ kN}$$

$\vec{F_C}$: 6.82 kN on the pin

$$\frac{3}{5} F_A = \frac{1}{\sqrt{5}}(6.82) \qquad F_A = +5.08 \text{ kN}$$

$\vec{F_A}$ = 5.08 kN on the pin

b)



Pin left attached to member AB

$$\Sigma F_x = 0$$

$$\frac{3}{5} F_A - \frac{1}{\sqrt{5}} F_C = 0 \qquad (1)$$

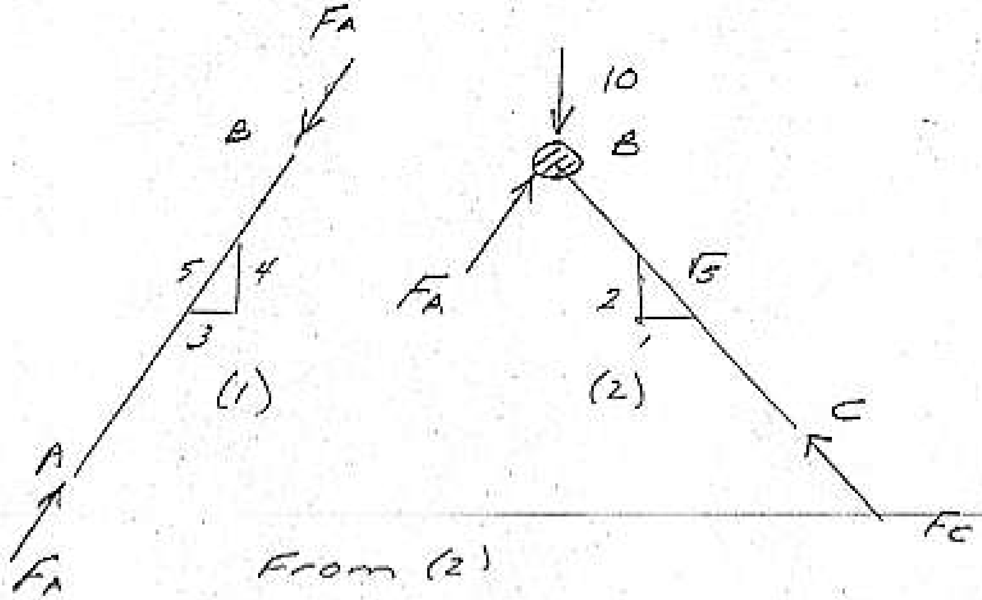
$$\Sigma F_y = 0$$

$$\frac{4}{5} F_A + \frac{2}{\sqrt{5}} F_C - 10 = 0 \qquad (2)$$

same equations as in (a)

SZ-428 2/2

c) Pin left attached to BC



From (2)

$$\Sigma F_x = 0 \qquad \frac{3}{5} F_A - \frac{1}{\sqrt{5}} F_C = 0 \qquad (1)$$

$$\Sigma F_y = 0 \qquad \frac{4}{5} F_A + \frac{2}{\sqrt{5}} F_C - 10 = 0 \qquad (2)$$

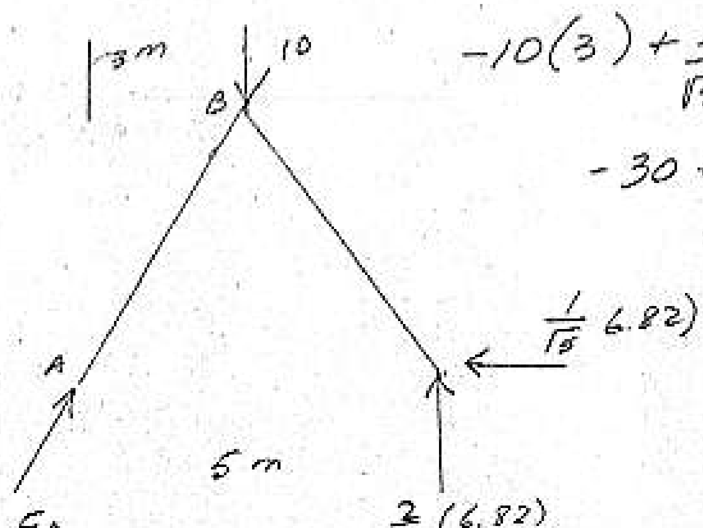
same equations as in (a)

check $\Sigma M_A = 0$

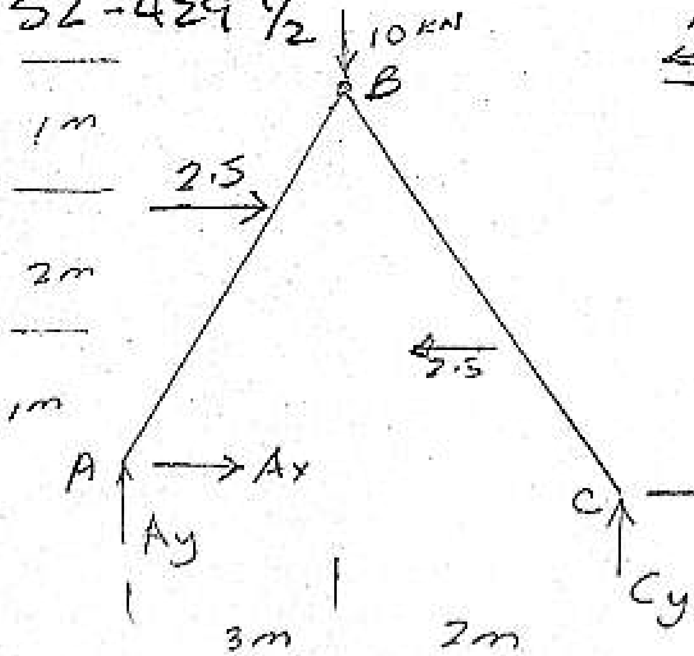
$$-10(3) + \frac{2}{\sqrt{5}}(6.82)(5) = 0$$

$$-30 + 30.5 = 0$$

$$0.5 \approx 0$$



52-429 1/2



$\Sigma F_x = 0$

$A_x + 2.5 - 2.5 + C_x = 0 \quad (1)$

$\Sigma F_y = 0$

$A_y - 10 + C_y = 0 \quad (2)$

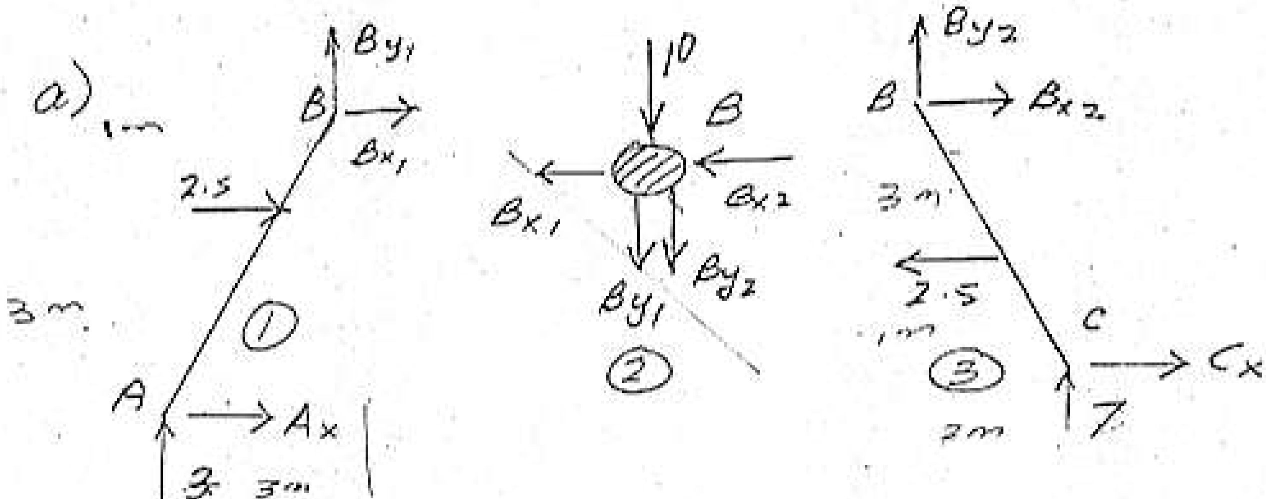
$\Sigma M_A = 0$

$-10(3) - 2.5(3) + 2.5(1) + C_y(5) = 0 \quad (3)$

$C_y = +7 \therefore \vec{C_y} = 7\text{kN} \uparrow$

From (2) $A_y - 10 + 7 = 0 \quad A_y = +3\text{kN} \therefore \vec{A_y} = 3\text{kN} \uparrow$

a)



From (1) $\Sigma F_x = 0 \quad A_x + 2.5 + B_{x_1} = 0 \quad (1)$

$\Sigma F_y = 0 \quad 3 + B_{y_1} = 0 \quad B_{y_1} = -3\text{kN} \quad (2)$

$\therefore \vec{B}_{y_1} = 3\text{kN} \downarrow$ on AB

$\Sigma M_B = 0 \quad -3(3) + A_x(4) + 2.5(1) = 0 \quad (3)$

$A_x = +1.625\text{kN} \therefore \vec{A}_x = 1.625\text{kN} \rightarrow$

subst in (1)

$1.625 + 2.5 + B_{x_1} = 0 \quad B_{x_1} = -4.125\text{kN}$

$\vec{B}_{x_1} = 4.125\text{kN} \leftarrow$ on AB

From (3) $\Sigma F_x = 0 \quad B_{x_2} - 2.5 + C_x = 0$

$\Sigma F_y = 0 \quad B_{y_2} + 7 = 0 \quad B_{y_2} = -7\text{kN}$

$\therefore \vec{B}_{y_2} = 7\text{kN} \downarrow$ on BC

SZ-429 2/2

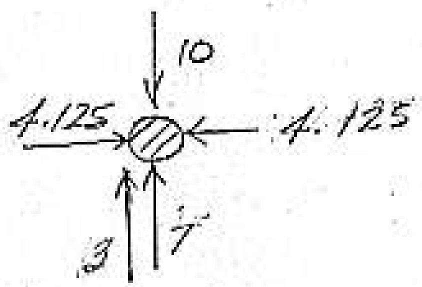
$\sum M_B = 0 \quad -2.5(3) + 7(2) + C_x(4) = 0$

$C_x = -1.625\text{ kN} \quad \vec{C}_x = 1.625\text{ kN} \leftarrow$

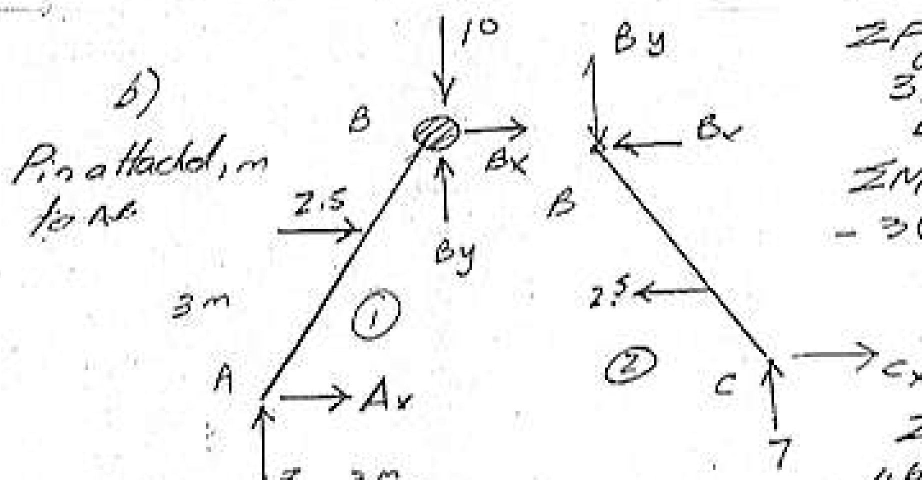
From eqn (1)

$B_{x2} - 2.5 + (-1.625) = 0$

$B_{x2} = +4.125\text{ kN} \quad \vec{B}_{x2} = 4.625\text{ kN} \rightarrow$ on BC



b) Pin attached to AB



From (1)

$\sum F_y = 0$

$3 - 10 + B_y = 0$

$B_y = +7\text{ kN} \quad B_y = 7\text{ kN} \uparrow$ on AB

$\sum M_B = 0$

$-3(3) + A_x(4) + 2.5(1) = 0$

$A_x = +1.625\text{ kN}$

$\therefore \vec{A}_x = 1.625\text{ kN} \rightarrow$

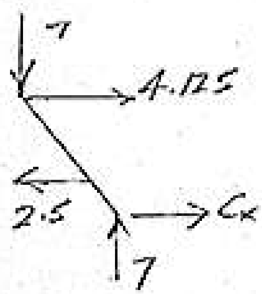
$\sum F_x = 0$

$1.625 + 2.5 + B_x = 0$

$B_x = -4.125\text{ kN}$

$\therefore \vec{B}_x = 4.125\text{ kN} \leftarrow$ on AB

From (2)

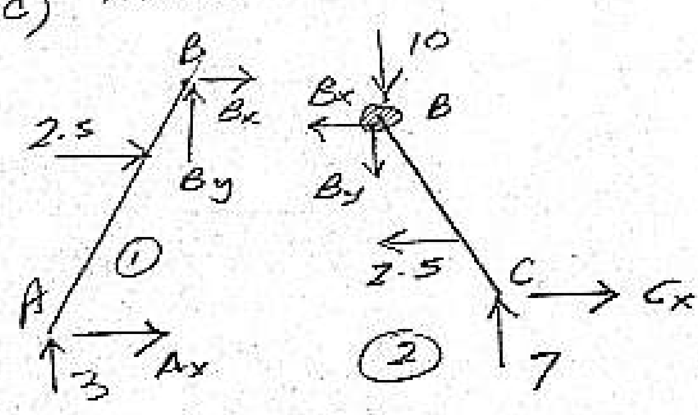


$\sum F_x = 0 \quad 4.125 + C_x = 2.5$

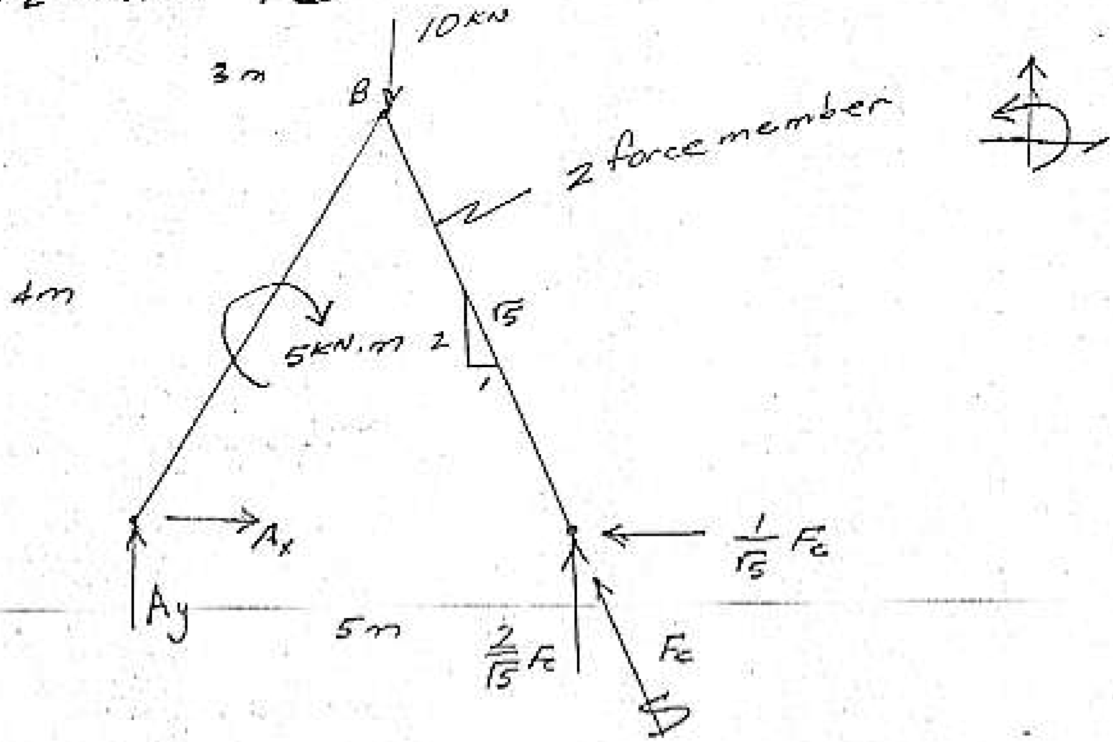
$C_x = -1.625\text{ N}$

$\therefore \vec{C}_x = 1.625\text{ kN} \leftarrow$

c) Pin attached to BC



S2-430 1/2



$\Sigma F_x = 0$

$$A_x - \frac{1}{\sqrt{5}} F_C = 0 \qquad (1)$$

$\Sigma F_y = 0 \quad A_y - 10 + \frac{2}{\sqrt{5}} F_C = 0 \qquad (2)$

$\Sigma M_A = 0$

$$-5 - 10(3) + \frac{2}{\sqrt{5}} F_C (5) = 0 \quad (3) \qquad F_C = +7.83 \text{ kN}$$

$$\therefore \vec{F_C} = 7.83 \text{ kN}$$

From (1)

$$A_x - \frac{1}{\sqrt{5}}(7.83) = 0 \qquad A_x = +3.5 \text{ kN}$$

$$\therefore \vec{A_x} = 3.5 \text{ kN} \rightarrow$$

From (2)

$$A_y - 10 + \frac{2}{\sqrt{5}}(7.83) = 0 \qquad A_y = +3 \text{ kN}$$

$$\therefore \vec{A_y} = 3 \text{ kN} \uparrow$$

52-430 2/2

From ① $\Sigma F_y = 0 \quad 3 + B_y = 0$

$B_y = -3^{kN} \quad \therefore \vec{B}_y = 3^{kN} \downarrow$ on AB

$\Sigma M_O = 0 \quad -3(3) + 2.5(1) + A_x(4) = 0$

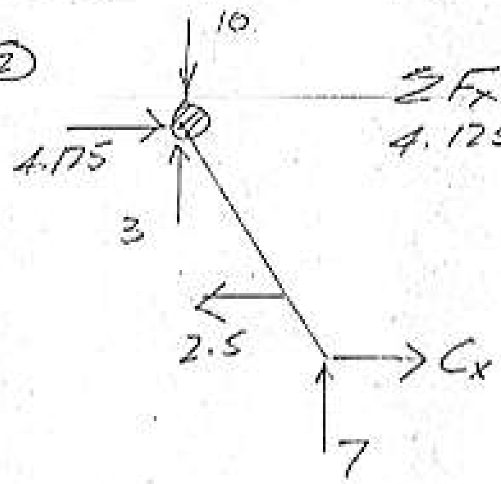
$A_x = +1.625^{kN} \therefore \vec{A}_x = 1.625^{kN} \rightarrow$

$\Sigma F_x = 0 \quad 1.625 + 2.5 + B_x = 0$

$B_x = -4.125^{kN}$

$\vec{B}_x = 4.125^{kN} \leftarrow$ on AB

From ②



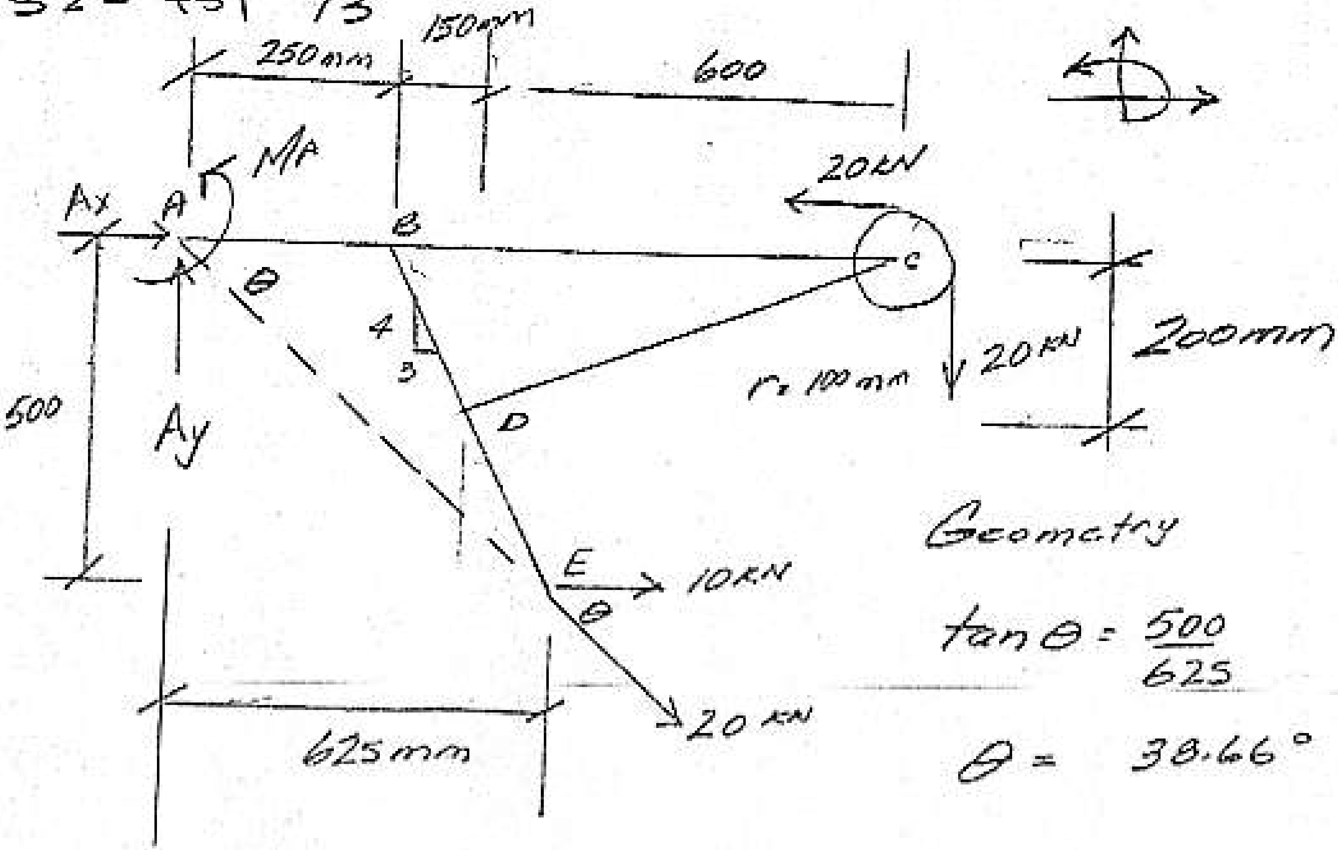
$\Sigma F_x = 0$

$4.125 - 2.5 + C_x = 0$

$C_x = -1.625^{kN}$

$\therefore \vec{C}_x = 1.625^{kN} \rightarrow$

52 - 431 1/3



Geometry

$\tan\theta = \frac{500}{625}$

$\theta = 38.66°$

$\Sigma M_A = 0$

$M_A + 20(0.1) - 20(1.1) + 10(0.5)$
$+ 20\cos 38.66°(0.5) - 20\sin 38.66°(0.625) = 0$

$M_A - 15 = 0 \therefore \vec{M_A} = 15\text{ kN.m}$

$\Sigma F_x = 0 \quad A_x - 20 + 10 + 20\cos 38.66° = 0$

$A_x = -5.62\text{ kN}$

$\therefore \vec{A_x} = 5.62\text{ kN} \leftarrow$

$\Sigma F_y = 0 \quad A_y - 20 - 20\sin 38.66° = 0$

$A_y = 32.49\text{ kN} \therefore \vec{A_y} = 32.49\text{ kN} \uparrow$

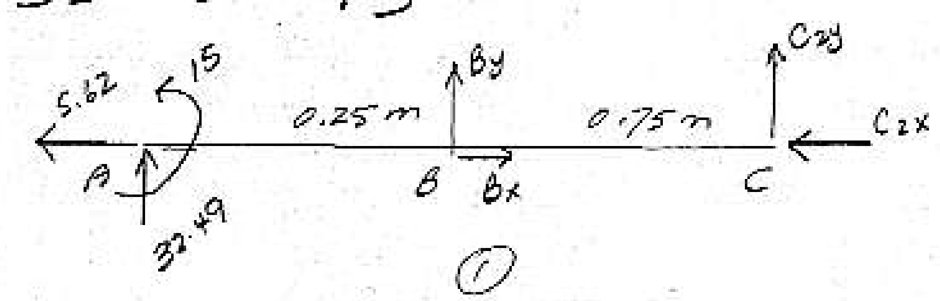
Check

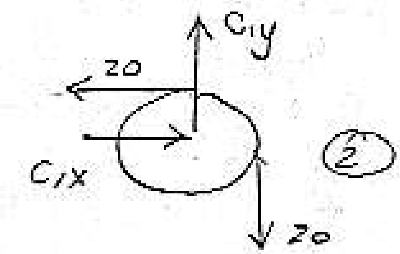
$\Sigma M_E = 0$

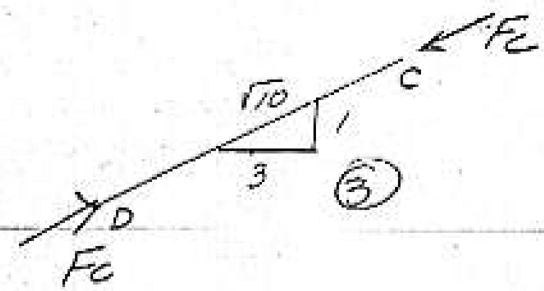
$5.62(0.5) - 32.49(0.625) + 15 + 20(0.6) - 20(0.475) = 0$

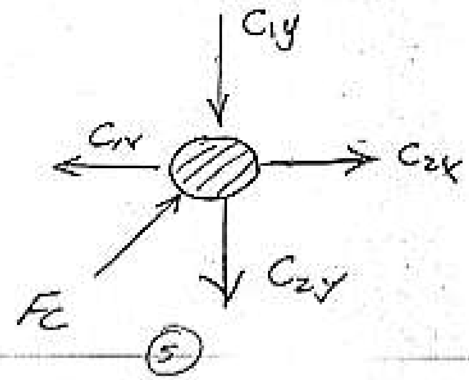
$0 = 0$ ✓

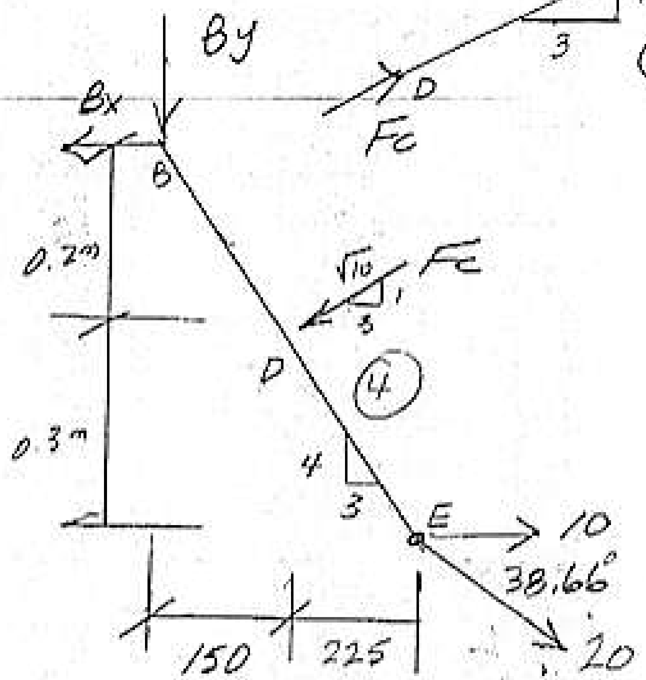
52-431 2/3











From ②

$\Sigma F_x = 0 \quad C_{1x} - 20 = 0$

$C_{1x} = +20 \text{ kN}$

$\therefore \vec{C}_{1x} = 20 \text{ kN} \rightarrow$ on the pulley

$\Sigma F_y = 0 \quad C_{1y} - 20 = 0$

$C_{1y} = +20 \text{ kN}$

$\vec{C}_{1y} = 20 \text{ kN} \uparrow$ on the pulley

From ④ $\Sigma M_B = 0$

$$-\frac{3}{\sqrt{10}} F_C (0.2) - \frac{1}{\sqrt{10}} F_C (0.15) + 10(0.5)$$

$$+20\cos 38.66°(0.5) - 20\sin 38.66°(0.375) = 0$$

$-0.2372 F_C + 8.1234 = 0 \quad \therefore F_C = 34.25 \text{ kN}$ on BDE

$$\Sigma F_x = 0 \quad -B_x - \frac{3}{\sqrt{10}}(34.25) + 10 + 20\cos 38.66 = 0$$

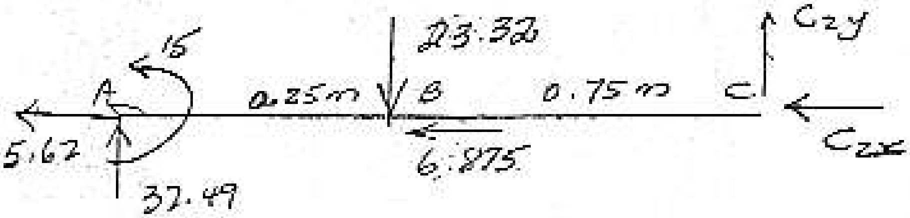
$B_x = +6.875 \text{ kN} \quad \vec{B}_x = 6.875 \text{ kN} \rightarrow$ on BDE

$$\Sigma F_y = 0 \quad -B_y - \frac{1}{\sqrt{10}}(34.25) - 20\sin 38.66° = 0$$

$B_y = -23.32 \text{ kN}$

$\therefore \vec{B}_y = 23.32 \text{ kN} \uparrow$ on BDE

$$\Sigma M_E = 0 \quad -23.32(0.375) + 6.875(0.5) + \frac{3}{\sqrt{10}}(34.25)(0.3) + \frac{1}{\sqrt{10}}(34.25)(0.225) = 0$$



$\sum F_x = 0 \quad -5.62 - 6.875 + C_{2x} = 0 \quad C_{2x} = -12.5$

$\therefore \vec{C}_{2x} = 12.5 \text{ kN} \rightarrow$ on ABC

$\sum F_y = 0 \quad 32.49 - 23.32 + C_{2y} = 0$

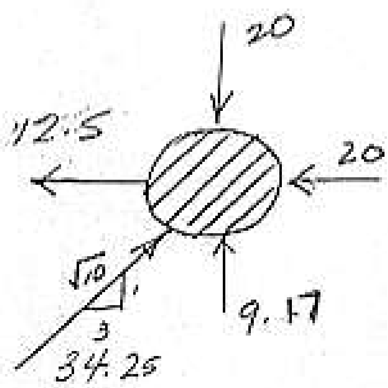
$C_{2y} = -9.17 \text{ kN}$

$\therefore \vec{C}_{2y} = 9.17 \text{ kN} \downarrow$ on ABC

$\sum M_A = 0$

$15 - 23.22(0.25) - 9.17(1.0) = 0$

$-0.025 = 0$ ✓ ok.



$\sum F_x = 0$

$34.25\left(\frac{3}{\sqrt{10}}\right) - 12.55 - 20 = 0$

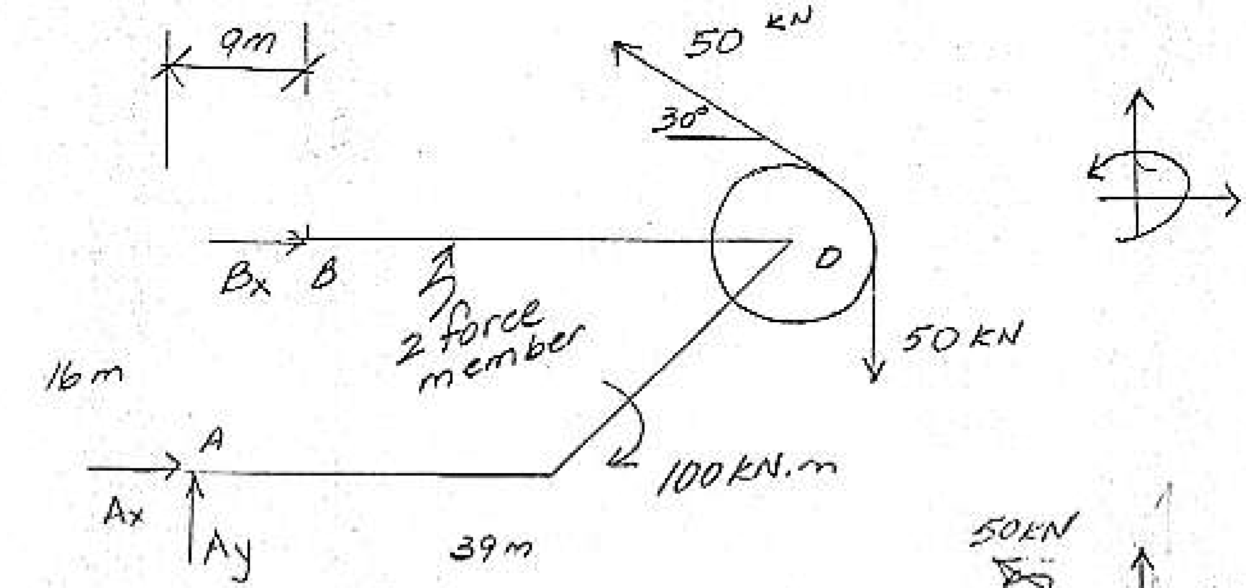
$-0.05 = 0$ ok

$\sum F_y = 0$

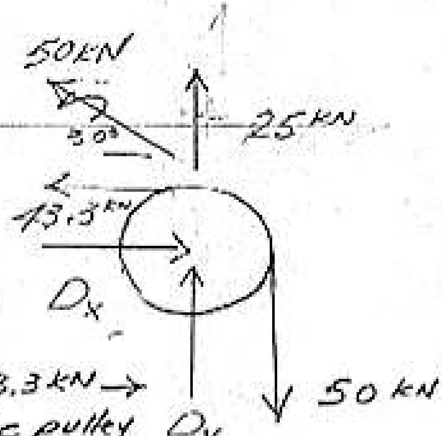
$-20 + \frac{1}{\sqrt{10}}(34.25) + 9.17 = 0$

$0 = 0$ ✓

S2-432 1/2



Draw FBD of Pulley



For pulley:

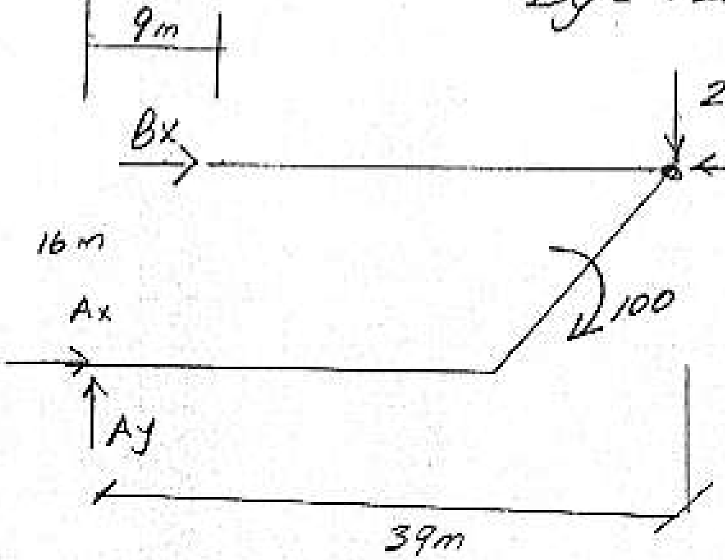
$\Sigma F_x = 0 \quad D_x - 43.3 = 0$

$D_x = +43.3\text{ kN} \therefore \vec{D_x} = 43.3\text{ kN} \rightarrow$ on the pulley

$\Sigma F_y = 0$

$25 + D_y - 50 = 0$

$D_y = +25\text{ kN} \quad \therefore \vec{D_y} = 25\text{ kN} \uparrow$ on the pulley



$\Sigma M_A = 0$

$-100 - B_x(16) - 25(39) + 43.3(16) = 0$

$-382.2 - B_x(16) = 0$

$B_x = -23.89\text{ kN}$

$\therefore \vec{B_x} = 23.89\text{ kN} \leftarrow$

$\Sigma F_x = 0$

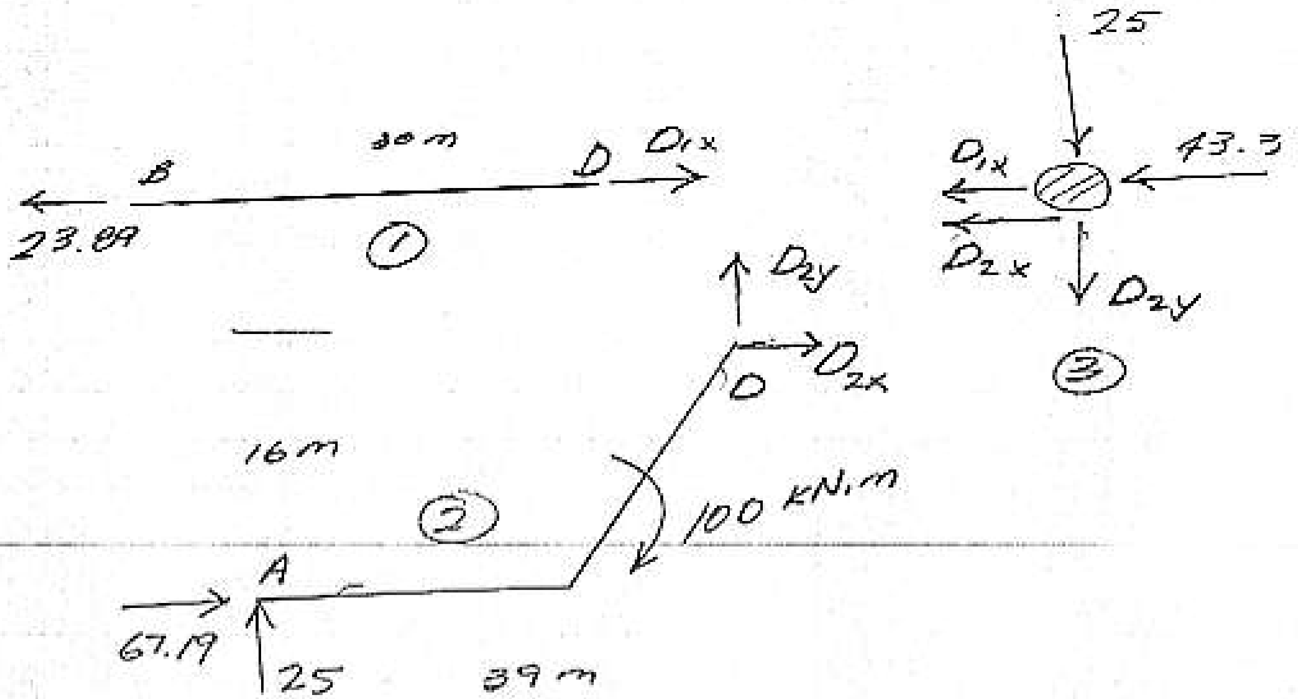
$-23.89 - 43.3 + A_x = 0$

$A_x = +67.19\text{ kN} \quad \therefore \vec{A_x} = 67.19\text{ kN} \rightarrow$

$\Sigma F_y = 0 \quad A_y - 25 = 0 \quad A_y = +25\text{ kN} \quad \vec{A_y} = 25\text{ kN} \uparrow$

52-432 2/2

Substructure:



FROM ① $\Sigma F_x = 0 \quad -23.89 + D_{1x} = 0$

$\vec{D}_{1x} = 23.89\text{ kN} \longrightarrow$ on BD

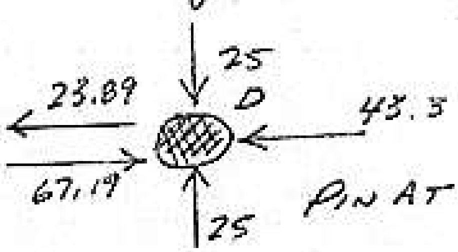
FROM ② $\Sigma F_x = 0 \quad 67.19 + D_{2x} = 0$

$D_{2x} = -67.19$

$\therefore \vec{D}_{2x} = 67.19 \longleftarrow$ on AD

$\Sigma F_y = 25 + D_{2y}$

$D_{2y} = -25\text{ kN} \quad \therefore \vec{D}_{2y} = 25\text{ kN} \downarrow$ on AD

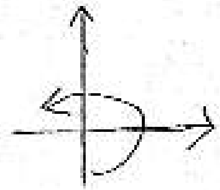


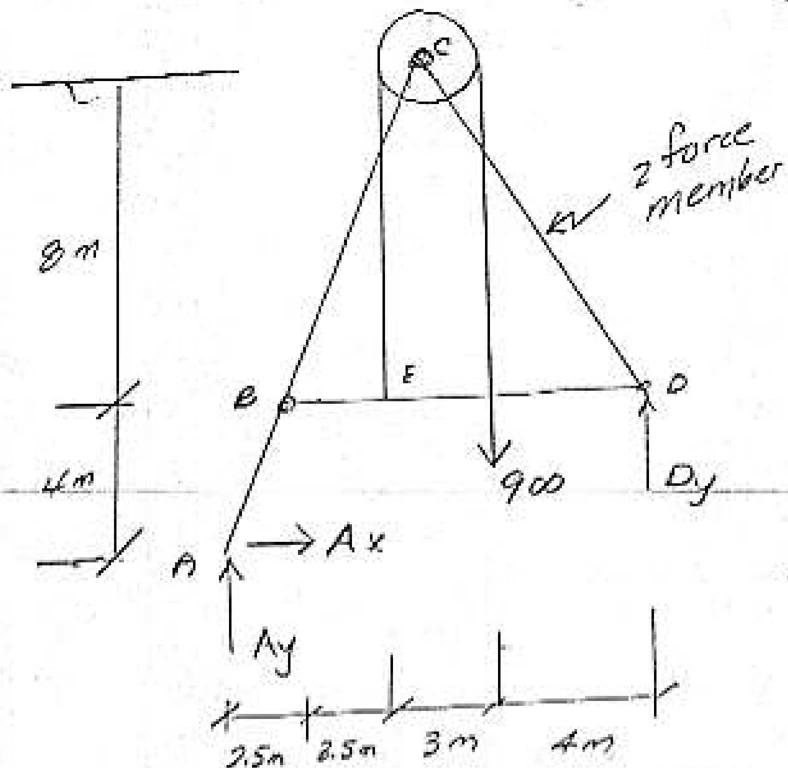
PIN AT D

$\Sigma M_D = 0 \quad 67.19(16) - 25(39) - 100 = 0$

$0.04 = 0$ OK

52-433 1/3





$\Sigma M_A = 0$

$-900(9) + D_y(13) = 0$

$D_y = +623.08\,N$

$\therefore \vec{D_y} = 623.08\,N \uparrow$

$\Sigma F_x = 0 \quad \vec{A_x} = 0$

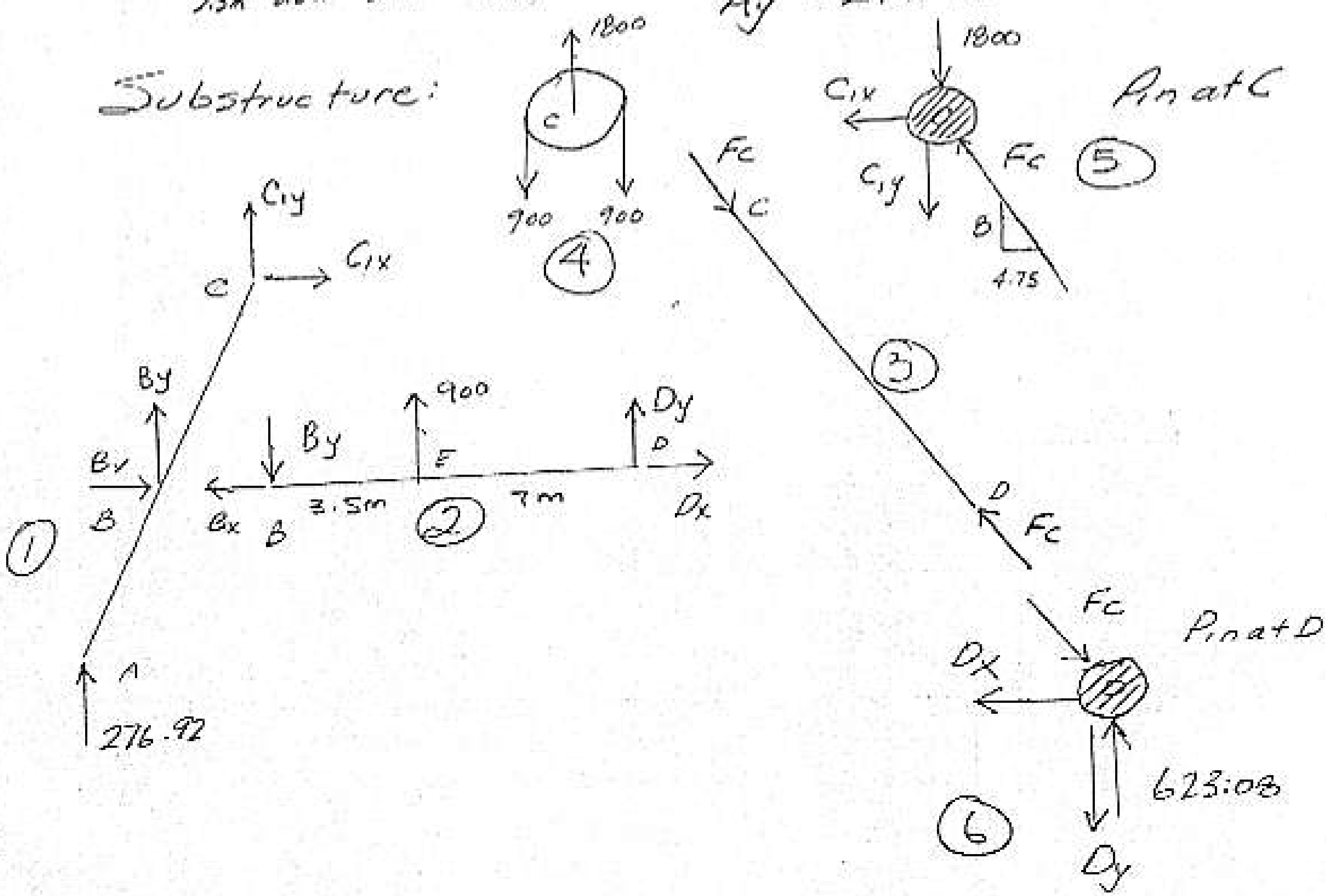
$\Sigma F_y = 0$

$A_y - 900 + 623.08 = 0$

$A_y = +276.92\,N$

$\vec{A_y} = 276.92\,N \uparrow$

Substructure:



S2-433 2/3

FROM ② $\Sigma M_C = 0$ $\quad 900(3.5) + D_y(10.5) = 0$

$D_y = -300\ N$

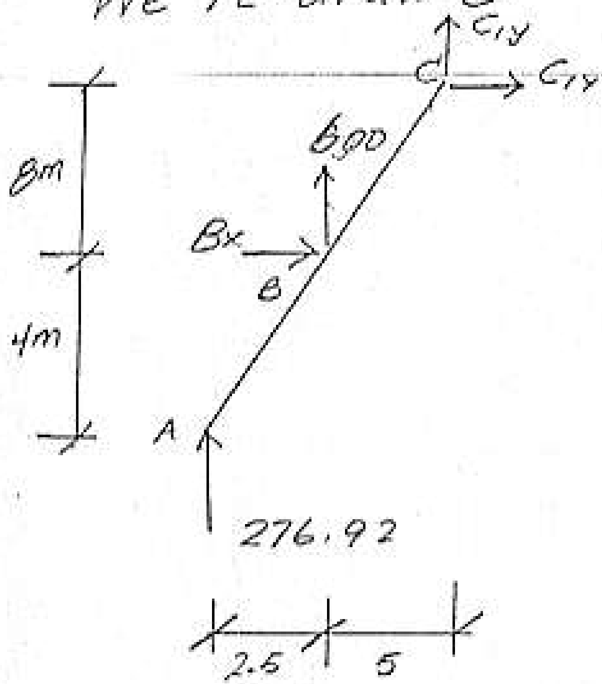
$\therefore \vec{D_y} = 300N \downarrow$ on BED

$\Sigma F_y = 0$ $\quad -B_y + 900 + (-300) = 0$

$B_y = +600\ N$

$\therefore \vec{B_y} = 600N \downarrow$ on BED

We re-draw ①



$\Sigma M_C = 0$

$-276.92(7.5) + B_x(8)$

$-600(5) = 0$

$B_x = +634.61$

$\therefore \vec{B_x} = 634.61^N \rightarrow$ on ABC

$\Sigma F_x = 0$

$634.61 + C_{1x} = 0$

$C_{1x} = -634.61^N$

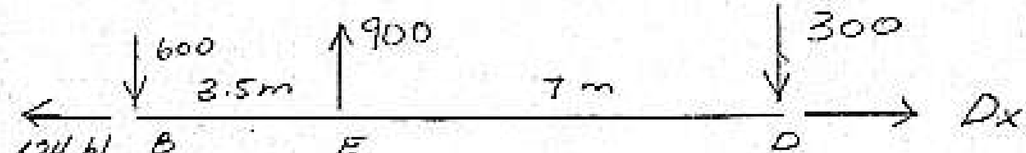
$\therefore \vec{C_{1x}} = 634.61\ N \leftarrow$ on ABC

$\Sigma F_y = 0$ $\quad 276.92 + 600 + C_{1y}$

$C_{1y} = -876.92^N$

$\vec{C_{1y}} = 876.92^N \downarrow$ on ABC

We re-draw ②

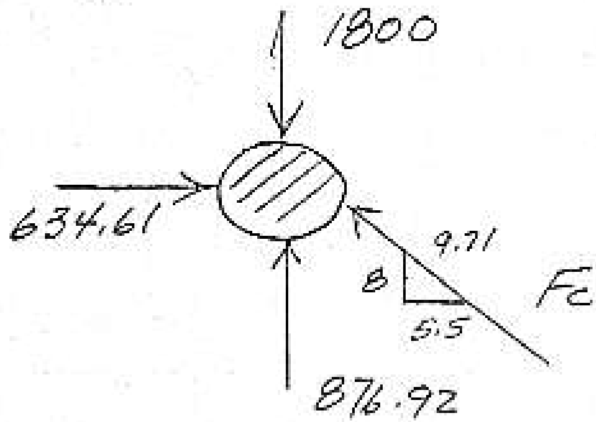


$\Sigma F_x = 0$ $\quad -634.61 + D_x = 0$

$D_x = +634.61^N$ $\quad \therefore \vec{D_x} = 634.61N \rightarrow$ on BED

52-433 3/3

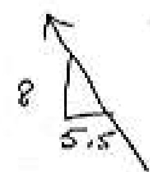
Pin at C



$$\Sigma F_x = 0 \quad 634.61 - \frac{5.5}{9.71} F_C = 0$$

$$F_C = +1120.17^N$$

$$\vec{F_C} = 1120.17^N$$

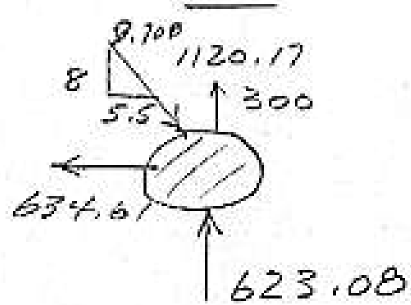


$$\Sigma F_y = 0$$

$$-1800 + 876.92 + \frac{8}{9.71}(1120.17) = 0$$

$$-0.18 = 0 \quad \underline{\text{OK}}$$

Pin AT D:

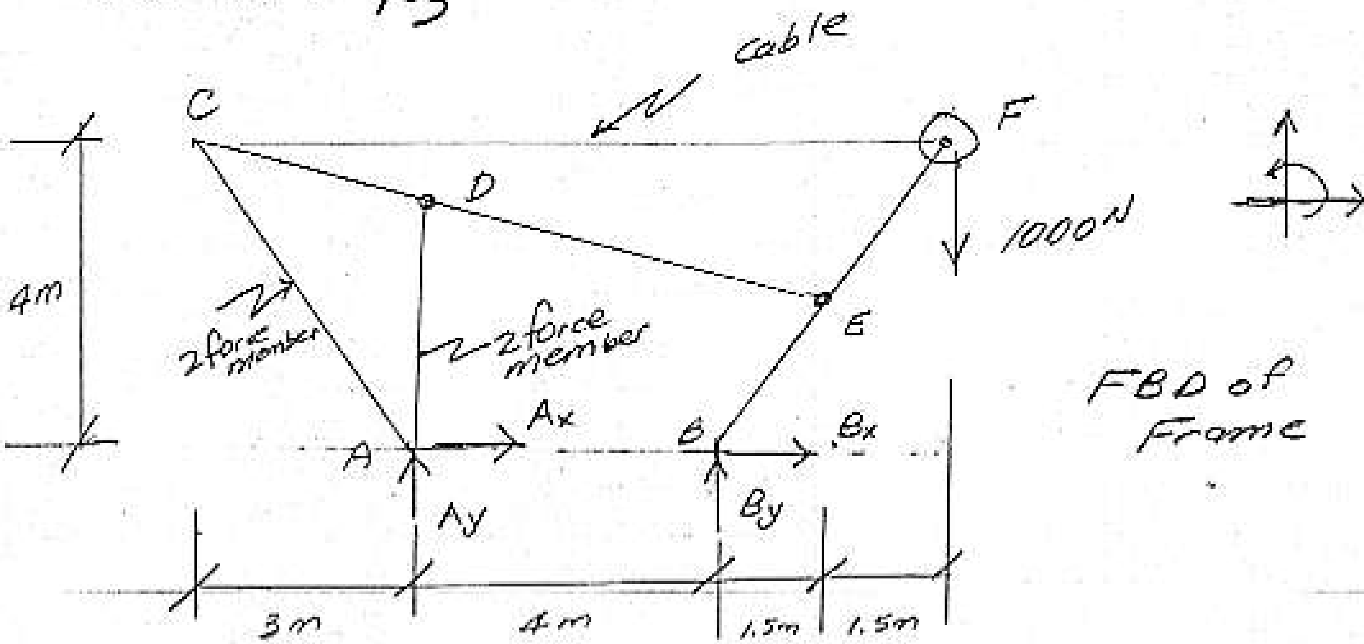


$$\Sigma F_x \quad -634.61 + \frac{5.5}{9.708}(1120.17) = 0.01 \text{ OK}$$

$$\Sigma F_y = \quad 623.08 - \frac{8}{9.708}(1120.17) + 300 = 0$$

$$= -0.01 \quad \text{OK}$$

52-434 1/3



$\sum M_A = 0 \qquad B_y(4) - 1000(7) = 0 \qquad (1)$

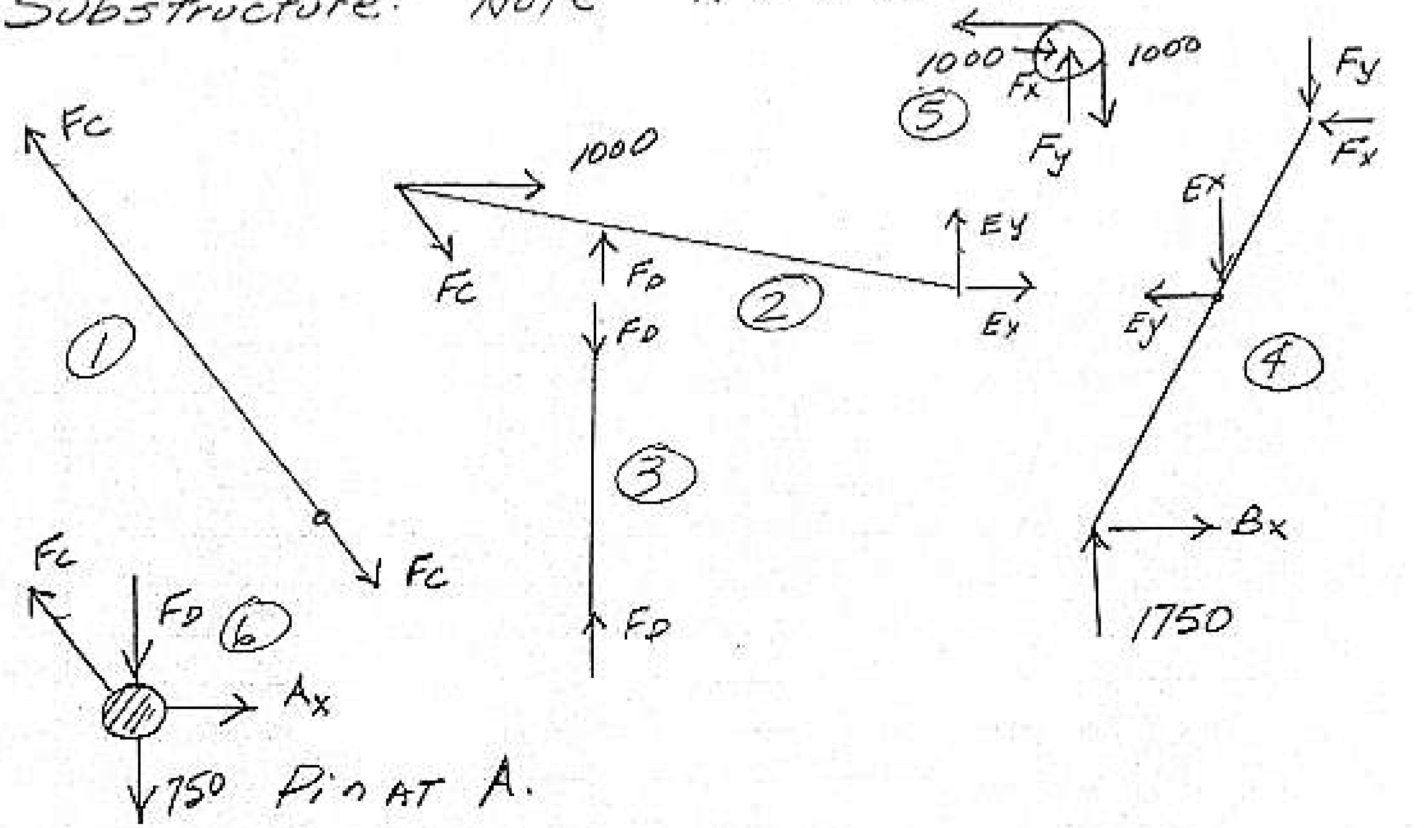
$\vec{B_y} = 1750\,N \uparrow$

$\sum F_x = 0 \qquad A_x + B_x = 0 \qquad (2) \qquad A_y = -750\,N$

$\sum F_y = 0 \qquad A_y - 1000 + 1750 = 0 \qquad (3)$

$\therefore \vec{A_y} = 750\,N \downarrow$

Substructure: Note AC & AB are 2-force members



52-434 2/3

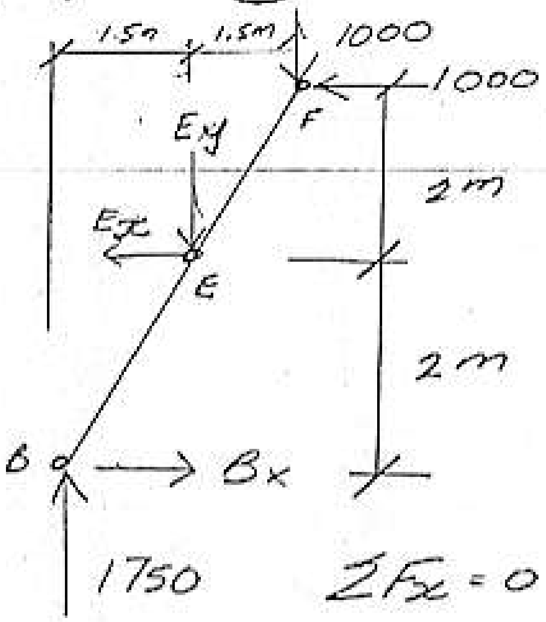
FROM ③ $\sum F_x = 0 \quad -1000 + F_x = 0$

$\vec{F_x} = 1000\,N \rightarrow$ on the pulley

$\sum F_y = 0 \quad F_y - 1000 = 0$

$\vec{F_y} = 1000\,N \uparrow$ on the pulley

We re-draw ④



$\sum M_E = 0$

$B_x(2) - 1750(1.5)$
$-1000(1.5) + 1000(2) = 0$

$2B_x - 2125 = 0$

$B_x = +1062.5\,N$

$\therefore \vec{B_x} = 1062.5\,N \rightarrow$

$\sum F_x = 0 \quad 1062.5 - E_x - 1000 = 0$

$E_x = +62.5\,N$

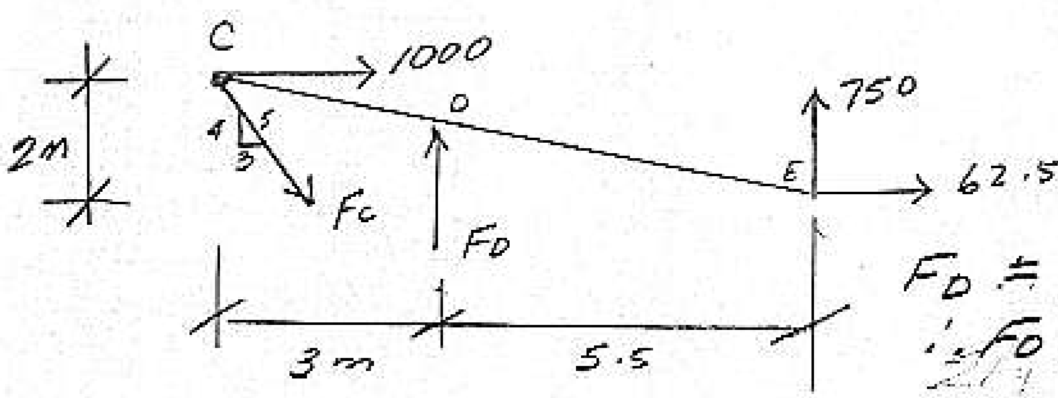
$\therefore \vec{E_x} = 62.5\,N \leftarrow$ on BEF

$\sum F_y = 0 \quad 1750 - E_y - 1000 = 0$

$E_y = +750\,N$

$\therefore \vec{E_y} = 750\,N \downarrow$ on BEF

We redraw ②



$\sum M_C = 0$

$F_D(3) + 750(8.5)$
$+62.5(2) = 0$

$F_D = -2166.67\,N$

$\therefore F_D = 2166.67\,N \downarrow$ on CDE

$\sum F_y = 0 \quad -\frac{4}{5}F_C + (-2166.67) + 750 = 0 \quad F_C = -1770.83\,N$

$\therefore \vec{F_C} = 1770.83\,N$ (4/3 slope) on CDE

S2-434 3/3

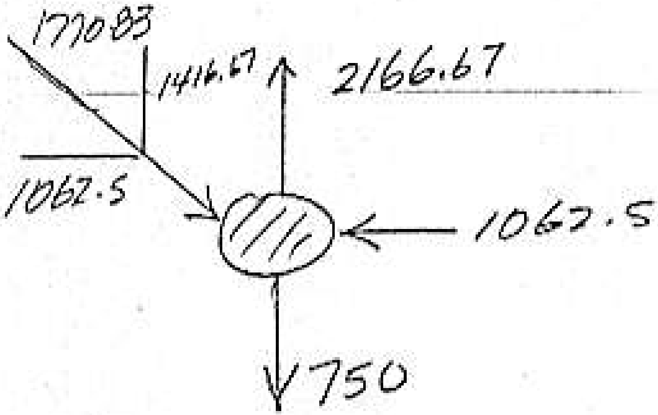
From Equation 1

$A_x + B_x = 0$

$A_x + 1062.5 = 0$

$A_x = -1062.5N \quad \therefore \; \vec{A}_x = 1062.5^N \leftarrow$

We re-draw Pin at A



$\Sigma F_x = 0$

$1062.5 - 1062.5 = 0$

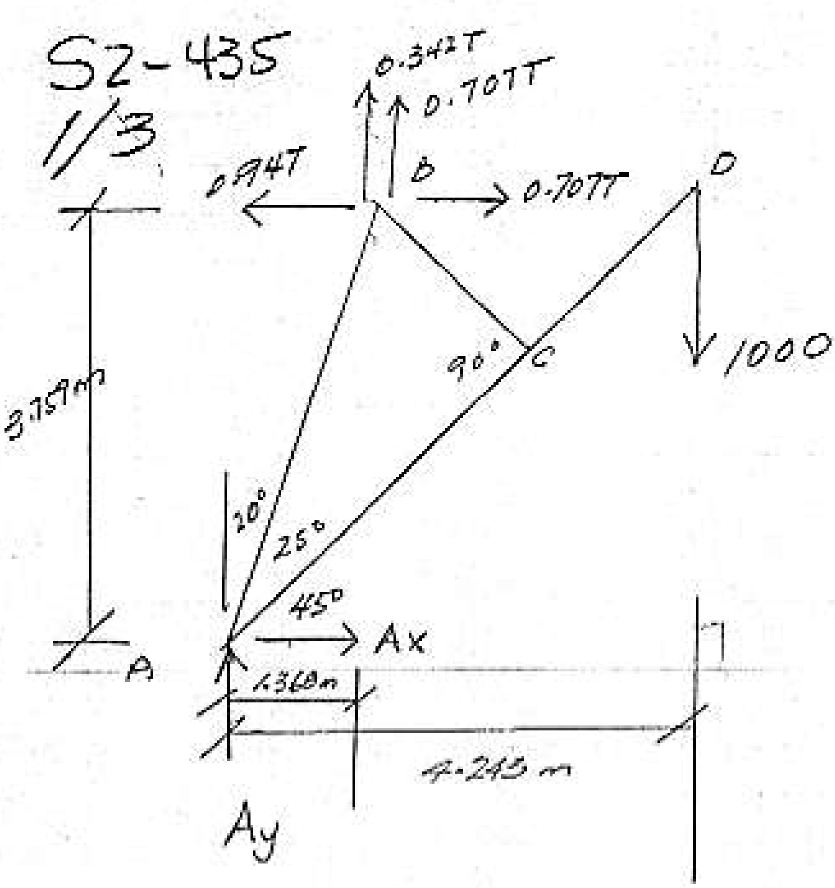
$0 = 0$ ✓

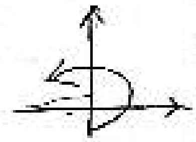
$\Sigma F_y = 0$

$1416.67 + 750 - 2166.67$

$0 = 0$ ✓

S2-435
1/3





AB = 4m
AD = 6m

AC = 4cos25°
= 3.625m

AB & BC
2-force members

$\Sigma M_A = 0 \quad (1.049T)(1.368) + 0.94T(3.759) - (0.707T)(3.759) - 1000(4.243) = 0$

$2.311T = 4243 \quad T = 1836.1\ N$

$\Sigma F_x = 0 \quad A_x - 0.94(1836.1) + 0.707(1836.1) = 0$

$A_x = +427.81N$

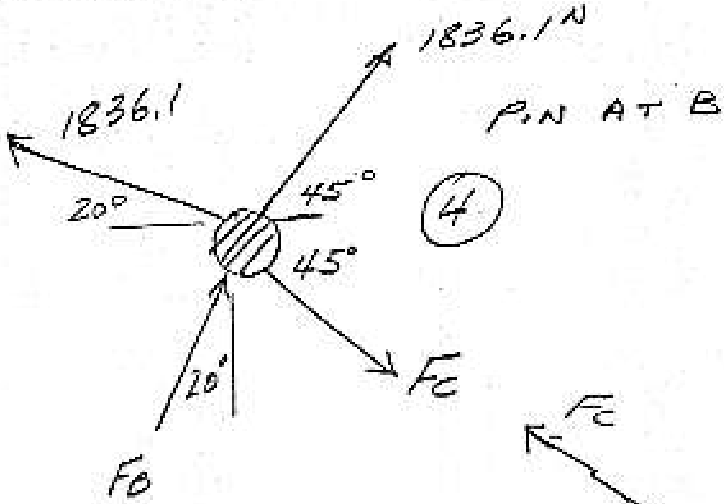
$\therefore \vec{A}_x = 427.81^N \rightarrow$

$\Sigma F_y = 0 \quad A_y + 0.342(1836.1) + 0.707(1836.1) - 1000 = 0$

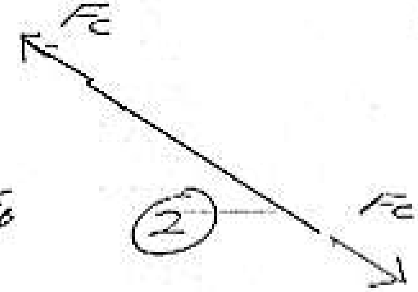
$A_y = -926.1N$

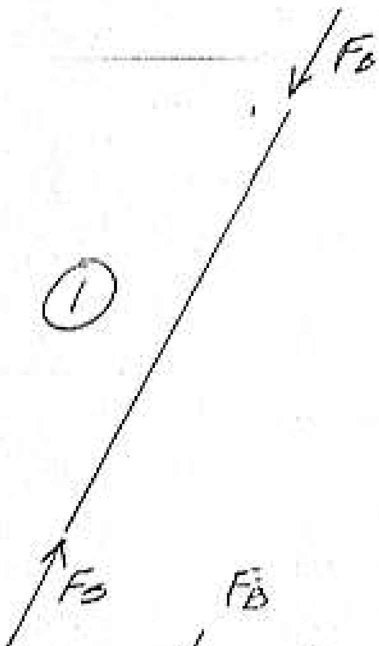
$\therefore \vec{A}_y = 926.1^N \downarrow$

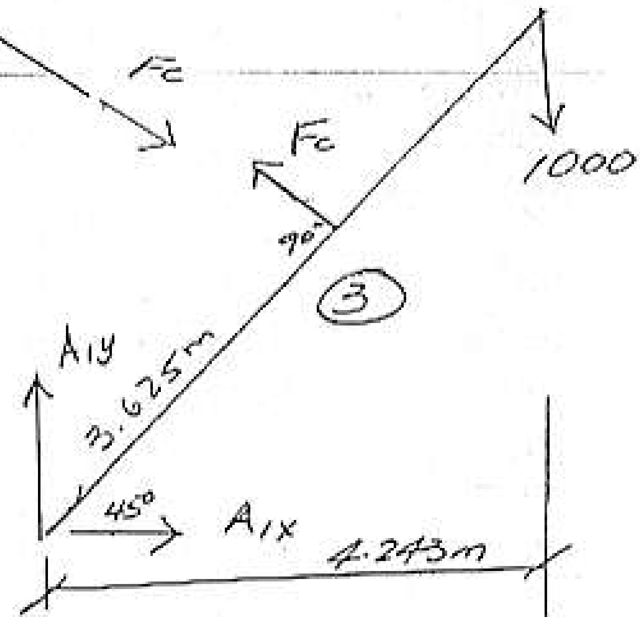
substructure

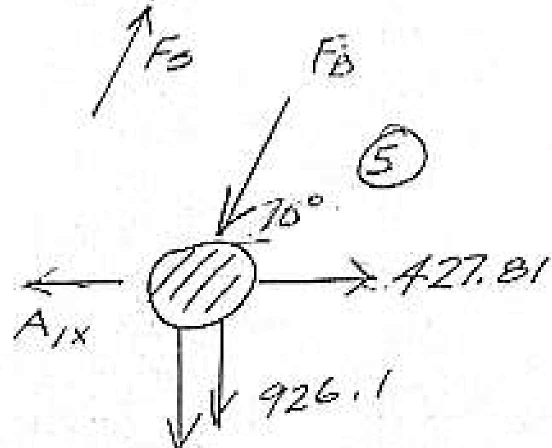


$AC = 4\cos 25°$
$= 3.625^m$









Pin AT A

From (3)

$\Sigma M_A = 0$

$F_C(3.625) - 1000(4.243) = 0$

$F_C = +1170.483^N$

$\therefore \vec{F_C} = 1170.483^N \; 45°$ on ACD

$\Sigma F_x = 0 \quad A_{1x} - 1170.483\cos 45° = 0$

$A_{1x} = +827.66^N$

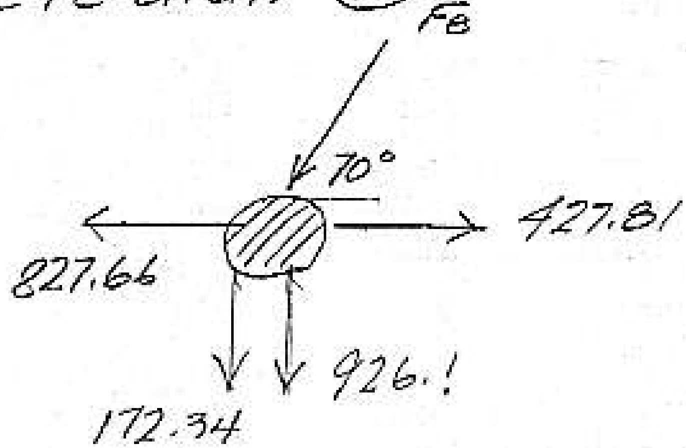
$\therefore \vec{A_{1x}} = 827.66^N \rightarrow$ on ACD

$\Sigma F_y = 0 \quad A_{1y} + 1170.483\sin 45° - 1000 = 0$

$A_{1y} = +172.34^N \; \therefore \; \vec{A_{1y}} = 172.34\,N \uparrow$ on ACD

52-435 3/3

We re-draw (5)



$\Sigma F_y = 0 \quad -F_B \sin 70° - 172.34 - 926.1 = 0$

$F_B = -1168.94$

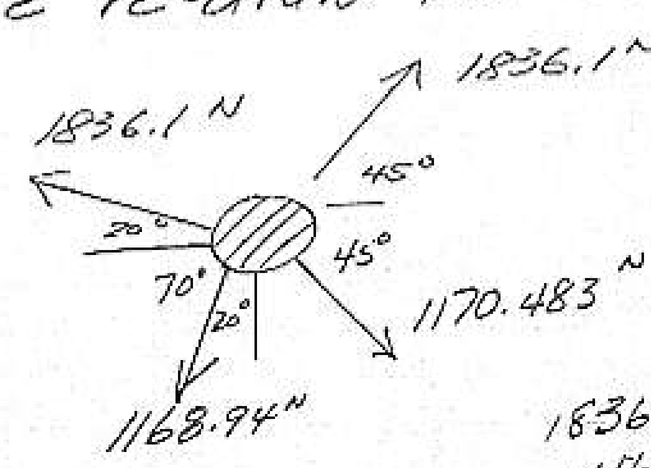
$\therefore \quad \vec{F_B} = 1168.94 \; \underline{\nearrow 70°}$ on pin AT A

check $\Sigma F_x = 0$

$-827.66 + 1168.94 \cos 70° + 427.81 = 0$

$-0.05 = 0$ OK

We re-draw pin AT B



$\Sigma F_x = 0$

$-1836.1 \cos 20° - 1168.94 \sin 20°$
$+1836.1 \cos 45° + 1170.483 \cos 45° = 0$

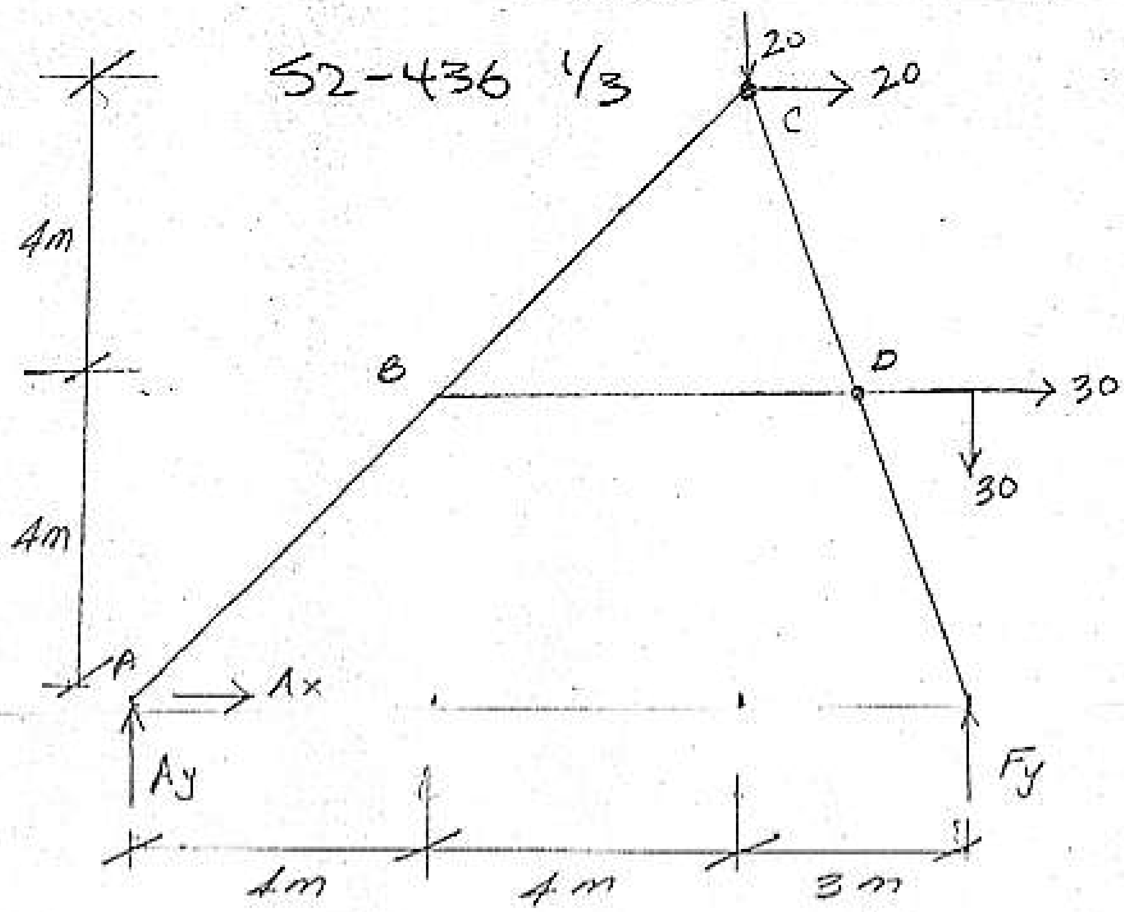
$0.8 = 0$ OK.

$\Sigma F_y = 0$

$1836.1 \sin 20° + 1836.1 \sin 45°$
$-1168.94 \cos 20° - 1170.483 \sin 45° = 0$

$0.2 = 0$ OK

52-436 1/3



$\Sigma F_x = 0 \quad A_x + 20 + 30 = 0$

$A_x = -50kN \quad \therefore \vec{A_x} = 50kN \leftarrow$

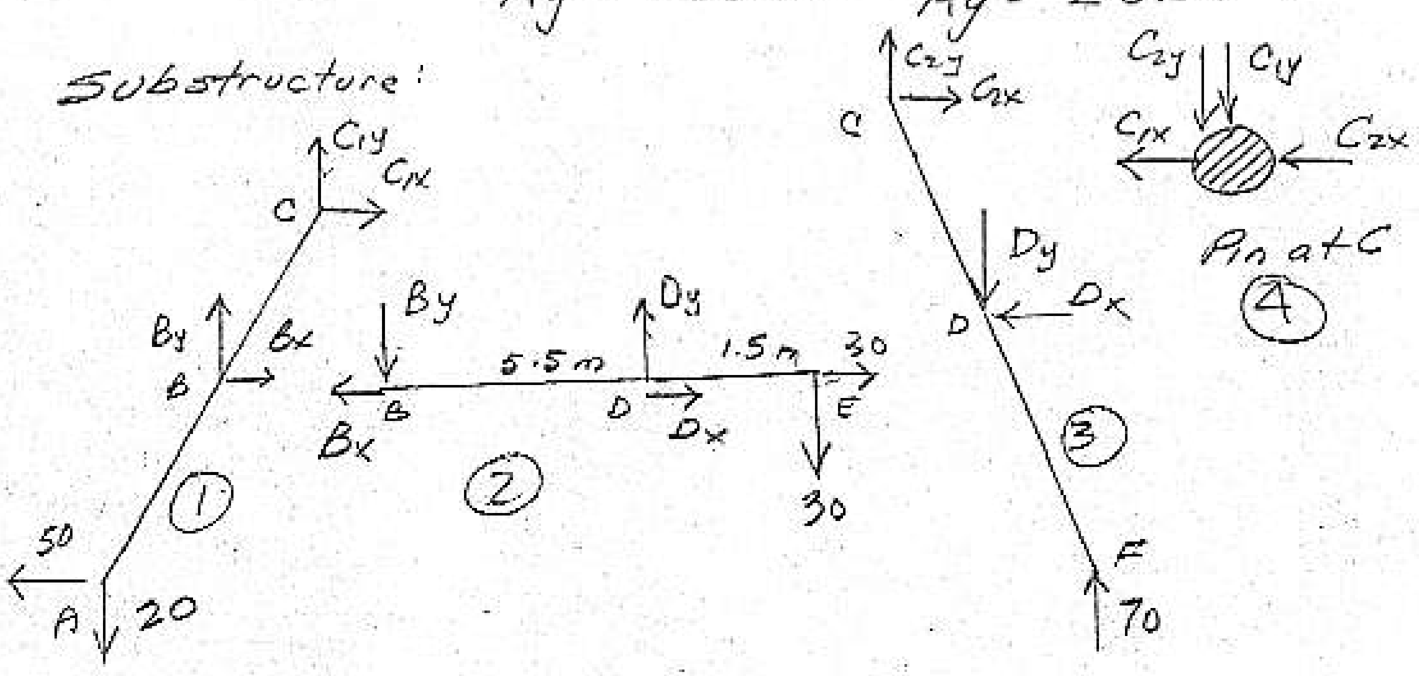
$\Sigma M_A = 0 \quad -20(8) - 20(8) - 30(11) - 30(4) + F_y(11) = 0$

$F_y = +70kN \quad \vec{F_y} = 70kN \uparrow$

$\Sigma F_y = 0 \quad A_y - 20 - 30 + 70 = 0$

$A_y = -20kN \quad \vec{A_y} = 20kN \downarrow$

Substructure:



FROM ② 52-436 2/3

$\Sigma M_B = 0 \quad D_y(5.5) - 30(7) = 0$

$D_y = +38.18^{kN}$

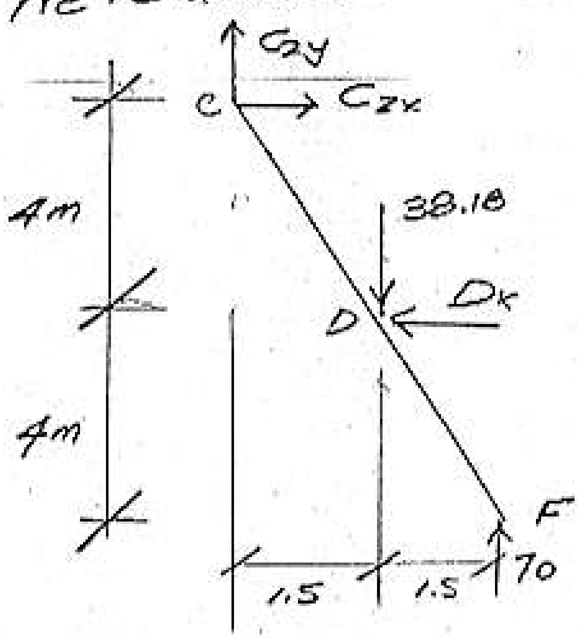
$\vec{D}_y = 38.18^{kN} \uparrow$ on BDE

$\Sigma F_y = 0$

$-B_y + 38.18 - 30 = 0$

$B_y = +8.18\,kN \quad \therefore \vec{B}_y = 8.18^{kN} \downarrow$ on BDE

We re-draw ③



$\Sigma M_C = 0$

$-38.18(1.5) - D_x(4) + 70(3) = 0$

$D_x = +38.18$ kN

$\vec{D}_x = 38.18^{kN} \leftarrow$ on CDF

$\Sigma F_x = 0$

$C_{2x} - 38.18 = 0$

$\vec{C}_{2x} = 38.18\,kN \rightarrow$ on CDF

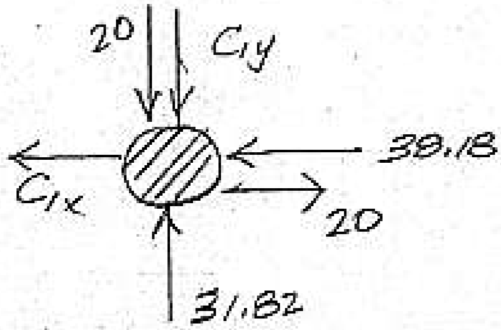
$\Sigma F_y = 0$

$C_{2y} - 38.18 + 70 = 0$

$C_{2y} = -31.82^{kN}$

$\therefore \vec{C}_{2y} = 31.82^{kN} \downarrow$ on CDF

We redraw ④



$\Sigma F_x = 0$

$-C_{1x} - 38.18 + 20 = 0$

$C_{1x} = -8.18^{kN}$

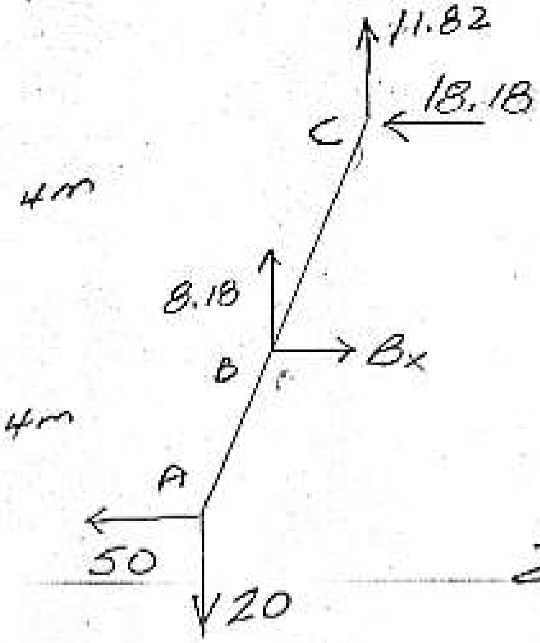
$\therefore \vec{C}_{1x} = 18.18^{kN} \rightarrow$ on PIN

$\Sigma F_y = 0$

$-C_{1y} + 31.82 - 20 = 0 \quad C_{1y} = 11.82^{kN}$

$\therefore \vec{C}_{1y} = 11.82^{kN} \downarrow$ on PIN

We re-draw ①   52-436 3/3



$\Sigma F_x = 0$

$-18.18 + B_x - 50 = 0$

$B_x = 68.18$

$B_x = 68.18 \text{ kN} \rightarrow$
on ABC

$\Sigma F_y = 0$

$11.82 + 8.18 - 20 = 0$

$0 = 0$ ✓

check $\Sigma M_B = 0$

$-50(4) + 20(4) + 18.18(4) + 11.82(4) = 0$

$0 = 0$ ✓

52-501

$$\vec{F}_{OA} = 1000\hat{j}\ N$$

$$\vec{F}_{OB} = 700\,\vec{\lambda}_{OB}$$

$$\vec{\lambda}_{OB} = \frac{\vec{OB}}{OB} \qquad \vec{OB} = 7\hat{i} - 2\hat{j} - 2\hat{k}$$

$$OB = \sqrt{(7)^2 + (-2)^2 + (-2)^2} = \sqrt{57}$$

$$\vec{F}_{OB} = 700\left(\frac{7\hat{i} - 2\hat{j} - 2\hat{k}}{\sqrt{57}}\right)$$

$$\vec{F}_{OB} = (649.02\hat{i} - 185.43\hat{j} - 185.43\hat{k})\ N$$

$$\vec{F}_{OC} = 500\,\vec{\lambda}_{OC} \qquad \vec{\lambda}_{OC} = \frac{\vec{OC}}{OC}$$

$$\vec{OC} = 3\hat{i} - 4\hat{j} + 6\hat{k}$$

$$OC = \sqrt{(3)^2 + (-4)^2 + (6)^2} = \sqrt{61}$$

$$\therefore\ \vec{F}_{OC} = 500\left(\frac{3\hat{i} - 4\hat{j} + 6\hat{k}}{\sqrt{61}}\right)$$

$$\vec{F}_{OC} = (192.06\hat{i} - 257.07\hat{j} + 384.11\hat{k})\ N$$

$$\vec{R} = (649.02 + 192.06)\hat{i} + (1000 - 185.43 - 257.07)\hat{j} + (-185.43 + 384.11)\hat{k}$$

$$\vec{R} = (841.08\hat{i} + 557.5\hat{j} + 198.68\hat{k})\ N$$

$$R = \sqrt{(841.08)^2 + (557.5)^2 + (198.68)^2} = 1028.44\ N$$

$$\cos\theta_x = \frac{841.08}{1028.44} = 0.8178 \Rightarrow \theta_x = 35.13^\circ$$

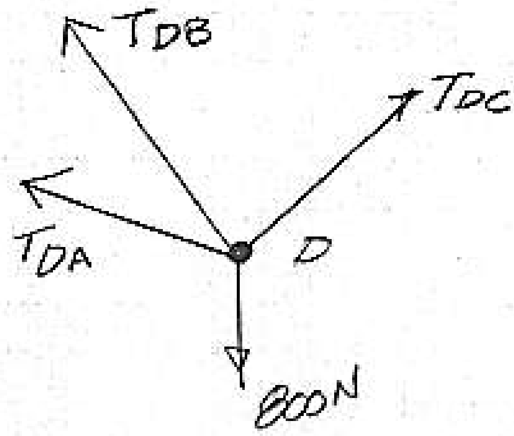
$$\cos\theta_y = \frac{557.5}{1028.44} = 0.542 \Rightarrow \theta_y = 57.17^\circ$$

$$\cos\theta_z = \frac{198.68}{1028.44} = 0.1932 \Rightarrow \theta_z = 78.86^\circ$$

S2-502 1/2

Coordinates:

A : (-8, 5, 4)
B : (-8, 5, -4)
C : (6, 5, 0)
D : (0, 0, 0)



FBD

$$\vec{T}_{DA} = T_{DA}\vec{\lambda}_{DA} \qquad \lambda_{DA} = \frac{\vec{DA}}{DA} \qquad \vec{DA} = -8\hat{i} + 5\hat{j} + 4\hat{k}$$

$$DA = \sqrt{(-8)^2 + (5)^2 + (4)^2} = \sqrt{105}$$

$$\vec{T}_{DA} = T_{DA}\left(\frac{-8\hat{i} + 5\hat{j} + 4\hat{k}}{\sqrt{105}}\right)$$

$$\vec{T}_{DB} = T_{DB}\vec{\lambda}_{DB} \qquad \vec{\lambda}_{DB} = \frac{\vec{DB}}{DB} \qquad \vec{DB} = -8\hat{i} + 5\hat{j} - 4\hat{k}$$

$$DB = \sqrt{(-8)^2 + (5)^2 + (-4)^2} = \sqrt{105}$$

$$\vec{T}_{DB} = T_{DB}\left(\frac{-8\hat{i} + 5\hat{j} - 4\hat{k}}{\sqrt{105}}\right)$$

$$\vec{T}_{DC} = T_{DC}\vec{\lambda}_{DC} \qquad \vec{\lambda}_{DC} = \frac{\vec{DC}}{DC} \qquad \vec{DC} = 6\hat{i} + 5\hat{j}$$

$$DC = \sqrt{(6)^2 + (5)^2} = \sqrt{61}$$

$$\vec{T}_{DC} = \frac{T_{DC}(6\hat{i} + 5\hat{j})}{\sqrt{61}}$$

$$-800\hat{j}$$

52-502 2/2

Equilibrium at D:

$$\Sigma F_x = \frac{-8}{\sqrt{105}} T_{DA} - \frac{8}{\sqrt{105}} T_{DB} + \frac{6}{\sqrt{61}} T_{DC} = 0 \quad (1)$$

$$\Sigma F_y = 0 \quad \frac{5}{\sqrt{105}} T_{DA} + \frac{5}{\sqrt{105}} T_{DB} + \frac{5}{\sqrt{61}} T_{DC} - 800 = 0 \quad (2)$$

$$\Sigma F_z = 0 \quad \frac{4}{\sqrt{105}} T_{DA} - \frac{4}{\sqrt{105}} T_{DB} = 0 \quad (3)$$

From (3) $T_{DA} = T_{DB}$

Subst in (1) & (2)

$$5* \quad \frac{-16}{\sqrt{105}} T_{DB} + \frac{6}{\sqrt{61}} T_{DC} = 0$$

$$6* \quad \frac{10}{\sqrt{105}} T_{DB} + \frac{5}{\sqrt{61}} T_{DC} = 800$$

$$\frac{-80}{\sqrt{105}} T_{DB} + \frac{30}{\sqrt{61}} T_{DC} = 0$$

$$+\frac{60}{\sqrt{105}} T_{DB} + \frac{30}{\sqrt{61}} T_{DC} = 4800$$

$$-\frac{140}{\sqrt{105}} T_{DB} = -4800$$

$$T_{DB} = 351.32\text{ N}$$

$$T_{DA} = 351.32\text{ N}$$

$$\frac{-8}{\sqrt{105}}(351.32) - \frac{8}{\sqrt{105}}(351.32) + \frac{6}{\sqrt{61}} T_{DC} = 0$$

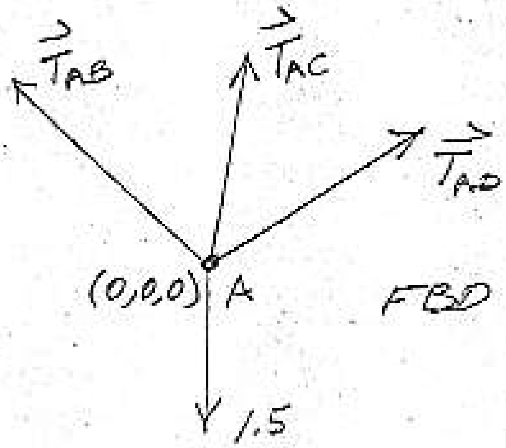
$$T_{DC} = 714.08\text{ N}$$

$$\frac{5}{\sqrt{105}}(351.324) + \frac{5}{\sqrt{105}}(351.32) + \frac{5}{\sqrt{61}}(714.08) = 800$$

$$800 = 800 \checkmark$$

S2 - 503 1/2

A(0,0,0)
B(-3,5,4)
C(0,5,-3)
D(4,3,0)



$-1.5\hat{j}$

$\vec{T}_{AD} = T_{AD}\vec{\lambda}_{AD}$ $\quad \vec{\lambda}_{AD} = \dfrac{\vec{AD}}{AD}$ $\quad \vec{AD} = 4\hat{i} + 3\hat{j} + 0\hat{k}$

$AD = \sqrt{4^2 + 3^2} = 5$

$$\vec{T}_{AD} = T_{AD}\left(\frac{4\hat{i} + 3\hat{j}}{5}\right) = 0.8T_{AD}\hat{i} + 0.6T_{AD}\hat{j}$$

$\vec{T}_{AC} = T_{AC}\vec{\lambda}_{AC}$ $\quad \vec{\lambda}_{AC} = \dfrac{\vec{AC}}{AC}$ $\quad \vec{AC} = 0\hat{i} + 5\hat{j} - 3\hat{k}$

$AC = \sqrt{(5)^2 + (-3)^2} = \sqrt{34}$

$$\vec{T}_{AC} = T_{AC}\left(\frac{0\hat{i} + 5\hat{j} - 3\hat{k}}{\sqrt{34}}\right) = 0.857T_{AC}\hat{j} - 0.514T_{AC}\hat{k}$$

$\vec{T}_{AB} = T_{AB}\vec{\lambda}_{AB}$ $\quad \vec{\lambda}_{AB} = \dfrac{\vec{AB}}{AB}$ $\quad \vec{AB} = -3\hat{i} + 5\hat{j} + 4\hat{k}$

$AB = \sqrt{(-3)^2 + (5)^2 + (4)^2} = \sqrt{50}$

$$\therefore \quad \vec{T}_{AB} = T_{AB}\left(\frac{-3\hat{i} + 5\hat{j} + 4\hat{k}}{\sqrt{50}}\right)$$

$$\vec{T}_{AB} = -0.424T_{AB}\hat{i} + 0.707T_{AB}\hat{j} + 0.566T_{AB}\hat{k}$$

$\Sigma F_x = 0 \qquad 0.8T_{AD} - 0.424T_{AB} = 0 \qquad (1)$

$\Sigma F_y = 0 \qquad -1.5 + 0.6T_{AD} + 0.857T_{AC} + 0.707T_{AB} = 0 \qquad (2)$

$\Sigma F_z = 0 \qquad -0.514T_{AC} + 0.566T_{AB} = 0 \qquad (3)$

S2-503 2/2

From (1) $T_{AD} = \dfrac{0.424\,T_{AB}}{0.8} = 0.53\,T_{AB}$

From (3) $T_{AC} = \dfrac{0.566\,T_{AB}}{0.514} = 1.101\,T_{AB}$

Subst in (2)

$$0.6(0.53\,T_{AB}) + 0.857(1.101\,T_{AB}) + 0.707\,T_{AB} = 1.5$$

$$1.969\,T_{AB} = 1.5\ \text{kN}$$

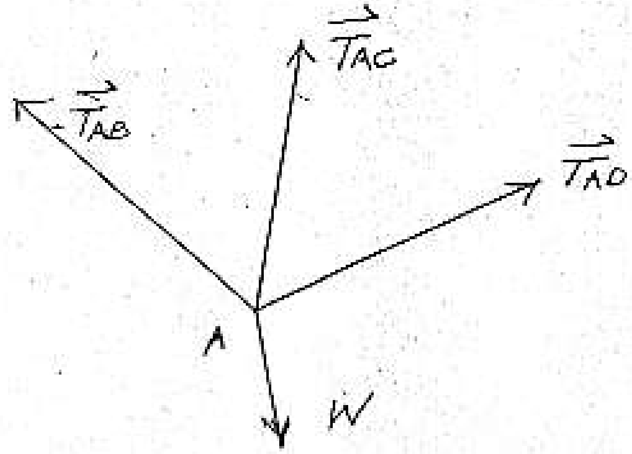
$$T_{AB} = 0.762\ \text{kN}$$

$$T_{AD} = 0.53(0.762) = 0.404\ \text{kN}$$

$$T_{AC} = 1.101(0.762) = 0.839\ \text{kN}$$

S2-504 1/2

A: (0, 0, 0)
B: (-3, 5, 4)
C: (0, 5, -3)
D: (4, 3, 0)



$-W\hat{j}$

$\vec{T}_{AD} = T_{AD}\vec{\lambda}_{AD}$

$\vec{\lambda}_{AD} = \dfrac{\vec{AD}}{AD}$ $\quad \vec{AD} = 4\hat{i} + 3\hat{j}$

$AD = \sqrt{4^2 + 3^2} = 5$

$\vec{T}_{AD} = T_{AD}\left(\dfrac{4\hat{i}+3\hat{j}}{5}\right) = 0.8\,T_{AD}\,\hat{i} + 0.6\,T_{AD}\,\hat{j}$

$\vec{\lambda}_{AC} = \dfrac{\vec{AC}}{AC}$ $\quad \vec{AC} = 5\hat{j} - 3\hat{k}$

$\vec{T}_{AC} = T_{AC}\vec{\lambda}_{AC}$ $\quad AC = \sqrt{5^2 + (-3)^2}$

$\vec{T}_{AC} = T_{AC}\left(\dfrac{5\hat{j} - 3\hat{k}}{\sqrt{34}}\right)$

$\vec{T}_{AC} = 0.857\,T_{AC}\,\hat{j} - 0.514\,T_{AC}\,\hat{k}$

$\vec{\lambda}_{AB} = \dfrac{\vec{AB}}{AB}$ $\quad \vec{AB} = -3\hat{i} + 5\hat{j} + 4\hat{k}$

$\vec{T}_{AB} = T_{AB}\vec{\lambda}_{AB}$

$AB = \sqrt{(-3)^2 + 5^2 + 4^2} = \sqrt{50}$

$\vec{T}_{AB} = T_{AB}\left(\dfrac{-3\hat{i} + 5\hat{j} + 4\hat{k}}{\sqrt{50}}\right)$

$\vec{T}_{AB} = 0.424\,T_{AB}\,\hat{i} + 0.707\,T_{AB}\,\hat{j} + 0.566\,T_{AB}\,\hat{k}$

$\Sigma F_x = 0 \quad 0.8\,T_{AD} + 0.424\,T_{AB} = 0$

$T_{AD} = \dfrac{0.424}{0.8}\,T_{AB} = 0.53\,T_{AB}$

$\Sigma F_y = 0 \quad 0.6\,T_{AD} + 0.857\,T_{AC} + 0.707\,T_{AB} - W = 0$

$\Sigma F_z = 0 \quad -0.514\,T_{AC} + 0.566\,T_{AB} = 0$

$T_{AC} = \dfrac{0.566}{0.514}\,T_{AB} = 1.101\,T_{AB}$

52 504 2/2

Let $T_{AC} = 1.2\ kN$ Governs

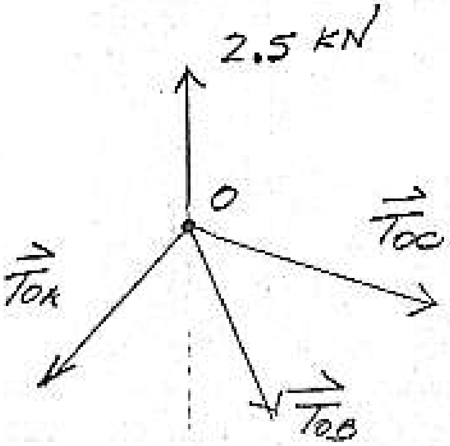
$\therefore T_{AB} = \frac{1.2}{1.101} = 1.09\ kN < 1.2$

$T_{AD} = 0.53\ T_{AB} = 0.53(1.09) = 0.578\ kN$
< 1.2

From 2

$0.6(0.578) + 0.857(1.2) + 0.707(1.09) = W$

$W = 2.15\ kN$



$\vec{T}_{OA} = T_{OA}\vec{\lambda}_{OA}$ $\quad \vec{\lambda}_{OA} = \dfrac{\vec{OA}}{OA}$

$\vec{OA} = -1.5\hat{i} - 1.0\hat{j} + 0.75\hat{k}$

$OA = \sqrt{(-1.5)^2 + (-1.0)^2 + (0.75)^2} = 1.953$

$\vec{T}_{OA} = T_{OA}\left(\dfrac{-1.5\hat{i} - 1.0\hat{j} + 0.75\hat{k}}{1.953}\right) =$

$\vec{T}_{OA} = -0.768\,T_{OA}\hat{i} - 0.512\,T_{OA}\hat{j} + 0.384\,T_{OA}\hat{k}$

$\vec{T}_{OB} = T_{OB}\vec{\lambda}_{OB}$ $\quad \vec{\lambda}_{OB} = \dfrac{\vec{OB}}{OB}$

$\vec{OB} = 2\hat{i} - 1.5\hat{j} + \hat{k}$

$OB = \sqrt{(2)^2 + (-1.5)^2 + (1)^2} = \sqrt{7.25} = 2.693$

$\vec{T}_{OB} = T_{OB}\left(\dfrac{2\hat{i} - 1.5\hat{j} + \hat{k}}{2.693}\right) =$

$= 0.743\,T_{OB}\hat{i} - 0.557\,T_{OB}\hat{j} + 0.371\,T_{OB}\hat{k}$

$\vec{T}_{OC} = T_{OC}\vec{\lambda}_{OC}$ $\quad \vec{\lambda}_{OC} = \dfrac{\vec{OC}}{OC}$

$\vec{OC} = 0\hat{i} - 0.5\hat{j} - 2\hat{k}$ $\quad OC = \sqrt{(-0.5)^2 + (-2)^2} = 2.062$

$\vec{T}_{OC} = T_{OC}\left(\dfrac{-0.5\hat{j} - 2\hat{k}}{2.062}\right) = -0.242\,T_{OC}\hat{j} - 0.97\,T_{OC}\hat{k}$

Weight:
$+2.5\hat{j}$

52-505

For equilibrium

$\Sigma F_x = 0$

$$-0.768\,T_{OA} + 0.743\,T_{OB} = 0 \quad (1)$$

$\Sigma F_y = 0$

$$-0.512\,T_{OA} - 0.557\,T_{OB} - 0.242\,T_{OC} + 2.5 = 0 \quad (2)$$

$\Sigma F_z = 0$

$$0.384\,T_{OA} + 0.371\,T_{OB} - 0.97\,T_{OC} = 0 \quad (3)$$

From (1) $\quad T_{OB} = \dfrac{0.768\,T_{OA}}{0.743} = 1.034\,T_{OA}$

From (2) & (3)

$$-0.512\,T_{OA} - 0.557(1.034\,T_{OA}) - 0.242\,T_{OC} = -2.5$$
$$0.384\,T_{OA} + 0.371(1.034\,T_{OA}) - 0.97\,T_{OC} = 0$$

$$-1.088\,T_{OA} - 0.242\,T_{OC} = -2.5$$
$$0.768\,T_{OA} - 0.97\,T_{OC} = 0$$

$$-0.836\,T_{OA} - 0.186\,T_{OC} = -1.92$$
$$0.836\,T_{OA} - 1.055\,T_{OC} = 0$$

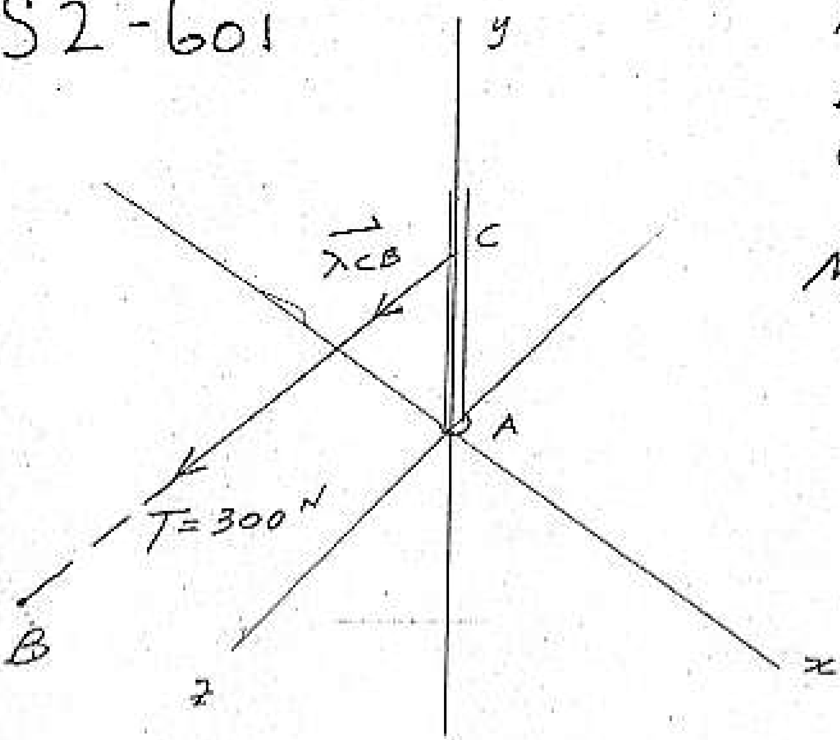
$$-1.241\,T_{OC} = -1.92$$

$$\therefore T_{OC} = 1.547 \text{ kN}$$

$$T_{OA} = \frac{0.97(1.547)}{0.768} = 1.954 \text{ kN}$$

$$T_{OB} = 1.034(1.954) = 2.021 \text{ kN}$$

S2-601



A:(0,0,0)
B:(-5,0,6)
C:(0,6,0)

$$\vec{M}_A = \vec{r}_{AC} \times \vec{T}$$
$$= \vec{r}_{AB} \times \vec{T}$$

$$\vec{T} = T\vec{\lambda}_{CB} = 300\,\lambda_{CB}$$

$$\vec{\lambda}_{CB} = \frac{\vec{CB}}{CB} \qquad \vec{CB} = -5\hat{i} - 6\hat{j} + 6\hat{k}$$

$$CB = \sqrt{(-5)^2+(-6)^2+6^2} = \sqrt{97}$$

$$\vec{T} = 300\left(\frac{-5\hat{i} - 6\hat{j} + 6\hat{k}}{\sqrt{97}}\right) = (-152.3\hat{i} - 182.76\hat{j} + 182.76\hat{k})\,N$$

$$\vec{r}_{AC} = 0\hat{i} + 6\hat{j} + 0\hat{k} \qquad \vec{r}_{AB} = -5\hat{i} + 0\hat{j} + 6\hat{k}$$

$$\vec{M}_A = \begin{vmatrix} i & j & k \\ 0 & 6 & 0 \\ -152.3 & -182.76 & +182.76 \end{vmatrix} \begin{matrix} i & j \\ 0 & 6 \\ -152.3 & -182.76 \end{matrix}$$

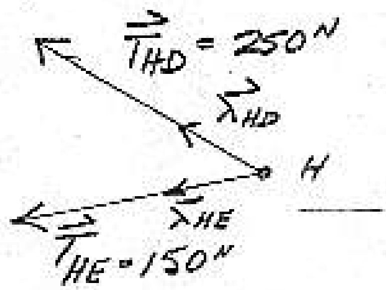
$$= [1096.56\hat{i}] - [-913.8\hat{k}] = (1096.56\hat{i} + 913.8\hat{k})\,N.m$$

$$\vec{M}_A = \begin{vmatrix} i & j & k \\ -5 & 0 & 6 \\ -152.3 & -182.76 & 182.76 \end{vmatrix} \begin{matrix} i & j \\ -5 & 0 \\ -152.3 & -182.76 \end{matrix}$$

$$= [-913.8\hat{j} + 913.8\hat{k}] - [-1096.56\hat{i} - 913.8\hat{j}]$$

$$= (1096.56\hat{i} + 913.8\hat{k})\,N.m$$

S2-602 1/2



H(70, 40, −20)
D(30, 115, 0)
E(30, 100, 50)
B(30, 0, 50)
M(95, 0, −20)

a) $\vec{T}_{HD} = T_{HD}\vec{\lambda}_{HD} = 250\,\lambda_{HD}$

$\vec{\lambda}_{HD} = \dfrac{\vec{HD}}{HD}$ $\quad HD = -40\hat{i} + 75\hat{j} + 20\hat{k}$

$HD = \sqrt{(-40)^2 + (75)^2 + (20)^2} = 87.32$

$\vec{\lambda}_{HD} = \dfrac{-40\hat{i} + 75\hat{j} + 20\hat{k}}{87.32}$

$\vec{T}_{HD} = 250\left(\dfrac{-40\hat{i} + 75\hat{j} + 20\hat{k}}{87.32}\right) = (-114.52\hat{i} + 214.73\hat{j} + 57.26\hat{k})\,N$

$\vec{T}_{HE} = T_{HE}\vec{\lambda}_{HE} = 150\,\vec{\lambda}_{HE}$ $\quad \lambda_{HE} = \dfrac{\vec{HE}}{HE}$

$\vec{HE} = -40\hat{i} + 60\hat{j} + 70\hat{k}$

$HE = \sqrt{(-40)^2 + (60)^2 + (70)^2} = 100.5$

$\vec{\lambda}_{HE} = \dfrac{-40\hat{i} + 60\hat{j} + 70\hat{k}}{100.5}$

$\vec{T}_{HE} = 150\left(\dfrac{-40\hat{i} + 60\hat{j} + 70\hat{k}}{100.5}\right) = (-59.7\hat{i} + 89.55\hat{j} + 104.48\hat{k})\,N$

$\vec{R} = \vec{T}_{HD} + \vec{T}_{HE}$

$\vec{R} = (-174.22\hat{i} + 304.28\hat{j} + 161.74\hat{k})$

b) $\cos\theta = \vec{\lambda}_{HD} \cdot \vec{\lambda}_{HE}$

$\cos\theta = \dfrac{1}{(87.32)(100.5)}\left[(-40)(-40) + (75)(60) + (20)(70)\right]$

$\cos\theta = 0.85464 \Longrightarrow \theta = 31.28°$

c) $\vec{M}_B = \vec{r}_{BH} \times \vec{R}$ $\quad \vec{r}_{BH} = 40\hat{i} + 40\hat{j} - 70\hat{k}$

$\vec{M}_B = \begin{vmatrix} i & j & k \\ 40 & 40 & -70 \\ -174.22 & 304.28 & 161.74 \end{vmatrix}\begin{matrix} i & j \\ 40 & 40 \\ -174.22 & 304.28 \end{matrix} = [6469.6\hat{i} + 12195.4\hat{j} + 12171.2\hat{k}] - [-6968.8\hat{k} - 21299.6\hat{i} + 6469.6\hat{j}]$

$\vec{M}_B = (27769.2\hat{i} + 5701.6\hat{j} + 19140\hat{k})\ N.m$

d) $M_{BM} = \vec{\lambda}_{BM} \cdot \vec{M}_B$ 52-602 2/2

$\vec{\lambda}_{BM} = \dfrac{\vec{BM}}{BM}$ $BM = 65\hat{i} + 0\hat{j} - 70\hat{k}$

$BM = \sqrt{(65)^2 + (-70)^2} = \sqrt{9125}$

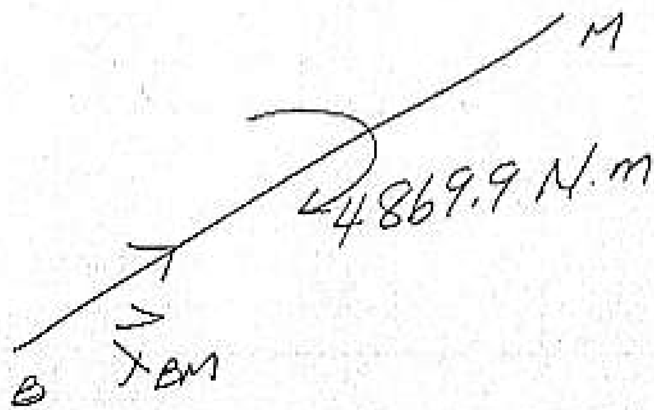
$= 95.52$

$$\vec{\lambda}_{BM} = \frac{65\hat{i} + 0\hat{j} - 70\hat{k}}{\sqrt{9125}}$$

$$\vec{\lambda}_{BM} \cdot \vec{M}_B = \left(\frac{65\hat{i} + 0\hat{j} - 70\hat{k}}{\sqrt{9125}}\right) \cdot \left(27769.2\hat{i} + 5701.6\hat{j} + 19140\hat{k}\right)$$

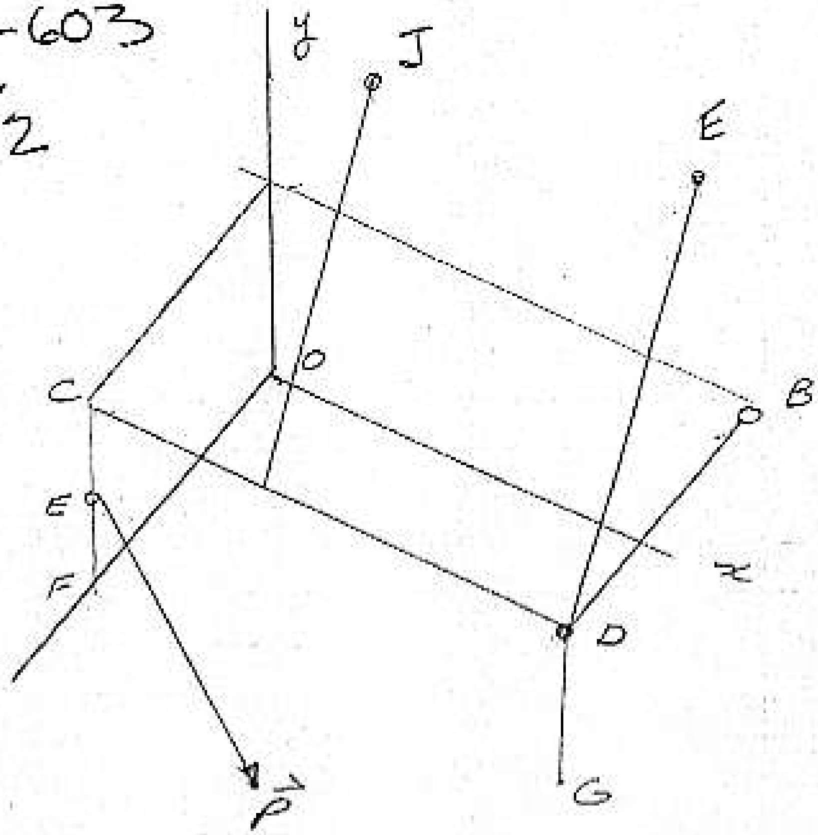
$$= \frac{1}{\sqrt{9125}}\left[(65)(27769.2) + (-70)(19140)\right]$$

$$= +4869.9 \text{ N.m}$$



S2-603

1/2



A: (0, 3.6, 0)
B: (5, 3.6, 0)
C: (0, 3.6, 1.5)
D: (5, 3.6, 1.5)
E: (0, 1.8, 1.5)
F: (0, 0, 1.5)
G: (5, 0, 1.5)
H: (5, 0, 4)
J: (1.5, 4.1, 0)
K: (5, 4.1, 0)

a) $\vec{M}_D = \vec{r}_{DE} \times \vec{P} = \vec{r}_{DH} \times \vec{P}$

$$\vec{r}_{DE} = -5\hat{i} - 1.8\hat{j} + 0\hat{k}$$

$$\vec{P} = \vec{\lambda}_{EH} \times \vec{P} \qquad \vec{\lambda}_{EH} = \frac{\vec{EH}}{EH}$$

$$\vec{EH} = 5\hat{i} - 1.8\hat{j} + 2.5\hat{k}$$

$$EH = \sqrt{(5)^2 + (-1.8)^2 + (2.5)^2} = \sqrt{34.49}$$

$$\vec{\lambda}_{EH} = \frac{5\hat{i} - 18\hat{j} + 2.5\hat{k}}{\sqrt{34.49}}$$

$$\vec{P} = 2600\left(\frac{5\hat{i} - 18\hat{j} + 2.5\hat{k}}{\sqrt{34.49}}\right) = 2213.59\hat{i} - 7968.92\hat{j} + 1106.79\hat{k}$$

$$\vec{M}_D = \left|\begin{array}{ccc|cc} i & j & k & i & j \\ -5 & -1.8 & 0 & -5 & -1.8 \\ 2213.59 & -7968.92 & +1106.79 & 2213.59 & -7968.92 \end{array}\right|$$

$= [-1992.22\hat{i} + 39844.6\hat{k}] - [17708.72\hat{k} - 5533.95\hat{j}]$

$= (-1992.22\hat{i} + 5533.95\hat{j} + 22135.88\hat{k})$ N.m

S2-603 2/2

b) $M_{OD} = \vec{\lambda}_{OD} \cdot (\vec{M}_O)$

$$\vec{\lambda}_{OD} = \frac{\vec{OD}}{OD}$$

$$\vec{OD} = 5\hat{i} + 3.6\hat{j} + 1.5\hat{k}$$

$$OD = \sqrt{(5)^2 + (3.6)^2 + (1.5)^2} = \sqrt{40.21}$$

$$\vec{\lambda}_{OD} = \frac{5\hat{i} + 3.6\hat{j} + 1.5\hat{k}}{\sqrt{40.21}}$$

$$M_{OD} = \left(\frac{5}{\sqrt{40.21}}\right)(-1992.22) + \left(\frac{3.6}{\sqrt{40.21}}\right)(5533.95) + \left(\frac{1.5}{\sqrt{40.21}}\right)(22135.88) = 6807.13\,N.m$$

c) $M = Fd$

$$M_O = \sqrt{(-1992.22)^2 + (5533.95)^2 + (22135.88)^2}$$

$$22903.95\ N.m$$

$$22903.95 = 2600\,d$$

$$d = 8.81\,m$$

e) $M_{EK} = 0$ Line of action P passes through line EK

S2-604 1/2

$\vec{T}_{AB}$ & $\vec{T}_{CB}$ have lines of action that pass through the Line AB

$\therefore$ the moment of $\vec{T}_{AB}$ & $\vec{T}_{CB}$ about Line AB = 0

$$\therefore M_{AB} = \vec{\lambda}_{AB} \cdot (\vec{r}_{AC} \times \vec{T}_{CE}) + \vec{\lambda}_{AB} \cdot (\vec{r}_{AD} \times \vec{T}_{DF})$$

A: (0, 10, 10)
B: (10, 0, 40)
C: (10, 10, 10)
D: (10, 10, 0)
E: (20, 0, 10)
F: (10, 0, -30)

$$\vec{\lambda}_{AB} = \frac{\vec{AB}}{AB}$$

$$\vec{AB} = 10\hat{i} - 10\hat{j} + 30\hat{k}$$

$$AB = \sqrt{(10)^2 + (-10)^2 + 30^2} = \sqrt{1100}$$

$$\vec{\lambda}_{AB} = \frac{10\hat{i} - 10\hat{j} + 30\hat{k}}{\sqrt{1100}}$$

$$\vec{r}_{AC} = 10\hat{i} + 0\hat{j} + 0\hat{k}$$

$$\vec{T}_{CE} = T_{CE}\vec{\lambda}_{CE} = 30\vec{\lambda}_{CE}$$

$$\vec{\lambda}_{CE} = \frac{\vec{CE}}{CE} \qquad \vec{CE} = 10\hat{i} - 10\hat{j} + 0\hat{k}$$

$$CE = \sqrt{10^2 + (-10)^2} = \sqrt{200}$$

$$\vec{\lambda}_{CE} = \frac{10\hat{i} - 10\hat{j}}{\sqrt{200}}$$

$$\vec{T}_{CE} = 30\left(\frac{10\hat{i} - 10\hat{j}}{\sqrt{200}}\right) = 21.21\hat{i} - 21.21\hat{j}$$

$$\vec{r}_{AD} = 10\hat{i} + 0\hat{j} - 10\hat{k}$$

$$\vec{T}_{DF} = T_{DF}\vec{\lambda}_{DF} = 50\vec{\lambda}_{DF}$$

$$\vec{\lambda}_{DF} = \frac{\vec{DF}}{DF} \qquad \vec{DF} = 0\hat{i} - 10\hat{j} - 30\hat{k}$$

$$DF = \sqrt{(-10)^2 + (-30)^2} = \sqrt{1000}$$

$$\vec{T}_{DF} = 50\left(\frac{-10\hat{j} - 30\hat{k}}{\sqrt{1000}}\right) = -15.81\hat{j} - 47.43\hat{k}$$

S2-604 2/2

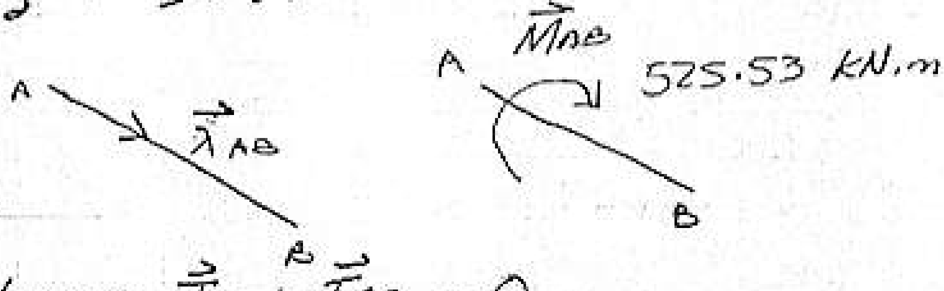
$$M_{AB} = \frac{1}{\sqrt{1100}} \left| \begin{array}{ccc|cc} 10 & -10 & 30 & 10 & -10 \\ 10 & 0 & 0 & 10 & 0 \\ 21.21 & -21.21 & 0 & 21.21 & -21.21 \end{array} \right|$$

$$+ \frac{1}{\sqrt{1100}} \left| \begin{array}{ccc|cc} 10 & -10 & 30 & 10 & -10 \\ 10 & 0 & -10 & 10 & 0 \\ 0 & -15.81 & -47.43 & 0 & -15.81 \end{array} \right|$$

$$= \frac{1}{\sqrt{1100}} \left\{ [-6363] - [0] \right\}$$

$$+ \frac{1}{\sqrt{1100}} \left\{ [-4743] - [1581 + 4743] \right\}$$

$$= -191.85 - 333.68 = -525.53 \text{ kN.m}$$



b) Angle between $\vec{T}_{CB}$ & $\vec{T}_{CE} = \theta$

$\cos\theta = \vec{\lambda}_{CB} \cdot \vec{\lambda}_{CE}$ $\quad \vec{\lambda}_{CB} = \frac{\vec{CB}}{CB}$

$\vec{CB} = 0\hat{i} - 10\hat{j} + 30k$ $\quad CB = \sqrt{(-10)^2 + (30)^2} = \sqrt{1000}$

$\vec{\lambda}_{CB} = \frac{-10\hat{j} + 30\hat{k}}{\sqrt{1000}}$

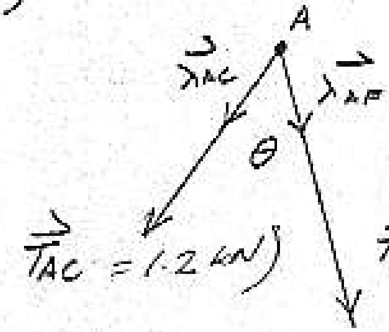
$\vec{\lambda}_{CE} = \frac{10\hat{i} - 10\hat{j}}{\sqrt{200}}$ $\quad \theta = \left(\frac{-10\hat{j} + 30\hat{k}}{\sqrt{1000}}\right) \cdot \left(\frac{10\hat{i} - 10\hat{j}}{\sqrt{200}}\right)$

$\cos\theta = \left(\frac{-10}{\sqrt{1000}}\right)\left(\frac{-10}{\sqrt{200}}\right) = 0.223606$

$\Rightarrow \theta = 77.08°$

S2-605

a)



$$\cos\theta = \vec{\lambda}_{AC} \cdot \vec{\lambda}_{AF}$$

$$\vec{\lambda}_{AC} = \frac{\vec{AC}}{AC}$$

$$\vec{AC} = -5\hat{i} - 4\hat{j} + \hat{k}$$

$$AC = \sqrt{(-5)^2 + (-4)^2 + (1)^2} = \sqrt{42}$$

$$\vec{\lambda}_{AC} = \frac{-5\hat{i} - 4\hat{j} + \hat{k}}{\sqrt{42}}$$

$$\vec{\lambda}_{AF} = \frac{\vec{AF}}{AF} \qquad \vec{AF} = 3\hat{i} - 6\hat{j} + 6\hat{k}$$

$$AF = \sqrt{(3)^2 + (-6)^2 + (6)^2} = 9$$

$$\vec{\lambda}_{AF} = \frac{3\hat{i} - 6\hat{j} + 6\hat{k}}{9}$$

$$\cos\theta = \vec{\lambda}_{AC} \cdot \vec{\lambda}_{AF} = \left(\frac{-5}{\sqrt{42}}\right)\left(\frac{3}{9}\right) + \left(\frac{-4}{\sqrt{42}}\right)\left(\frac{-6}{9}\right) + \left(\frac{1}{\sqrt{42}}\right)\left(\frac{6}{9}\right)$$

$$\cos\theta = 0.25717 \Rightarrow \theta = 75.1^\circ$$

b) $$M_{OD} = \vec{\lambda}_{OD} \cdot (\vec{r}_{OA} \times \vec{T}_{AC})$$

$$\vec{\lambda}_{OD} = \frac{\vec{OD}}{OD} \qquad \vec{OD} = 5\hat{i} - 3\hat{j} + 7\hat{k}$$

$$OD = \sqrt{(5)^2 + (-3)^2 + (7)^2} = \sqrt{83}$$

$$\vec{\lambda}_{OD} = \frac{5\hat{i} - 3\hat{j} + 7\hat{k}}{\sqrt{83}} = 0.549\hat{i} - 0.329\hat{j} + 0.768\hat{k}$$

$$\vec{r}_{OA} = 0\hat{i} + 3\hat{j} + 4\hat{k}$$

$$\vec{T}_{AC} = T_{AC}\vec{\lambda}_{AC} = 1.2\left(\frac{-5\hat{i} - 4\hat{j} + \hat{k}}{\sqrt{42}}\right) = -0.926\hat{i} - 0.741\hat{j} + 0.185\hat{k}$$

$$M_{OD} = \left|\begin{array}{ccc|cc} 0.549 & -0.329 & 0.768 & 0.549 & -0.329 \\ 0 & 3 & 4 & 0 & 3 \\ -0.926 & -0.741 & 0.185 & -0.926 & -0.741 \end{array}\right| = 5.32\ kN.m$$

c) $M_{CD} = 0$ Line of Action of T_{AC} passes thru CD.

52-606 1/2

a) Equivalent force couple at O
(We use right hand rule)

$$\vec{R}_O = \vec{F}_1 + \vec{F}_2$$

O: (0, 0, 0)
A: (0, 1, 0.5)
B: (0.5, 1, 0.5)
C: (0.5, 1, -0.5)
D: (1, 4, -4.5)
E: (3.5, 0, 4.5)

$$\vec{F}_1 = -5\hat{i}$$

$$\vec{F}_2 = F_2 \vec{\lambda}_{BE} = 8\vec{\lambda}_{BE}$$

$$\vec{\lambda}_{BE} = \frac{\vec{BE}}{BE} \quad \vec{BE} = 3\hat{i} - 1\hat{j} + 4\hat{k}$$

$$BE = \sqrt{(3)^2 + (-1)^2 + 4^2} = \sqrt{26}$$

$$\vec{F}_2 = 8\left(\frac{3\hat{i} - 1\hat{j} + 4\hat{k}}{\sqrt{26}}\right) = 4.71\hat{i} - 1.57\hat{j} + 6.28\hat{k}$$

$$\vec{R}_O = -5\hat{i} + (4.71\hat{i} - 1.57\hat{j} + 6.28\hat{k}) = (-0.29\hat{i} - 1.57\hat{j} + 6.28\hat{k})\ kN$$

$$\vec{M}_{R_O} = \vec{M}_1 + \vec{M}_2 + \vec{M}_3 + \vec{M}_4 + \vec{r}_{OA} \times \vec{F}_1 + \vec{r}_{OB} \times \vec{F}_2$$

$$\vec{M}_1 = +4k\ kN.m \qquad \vec{M}_2 = -3\hat{j}\ kN.m$$

$$\vec{M}_3 = +5\hat{j}\ kN.m \quad \vec{M}_4 = M_4 \vec{\lambda}_{CD} = 6\vec{\lambda}_{CD}$$

$$\vec{\lambda}_{CD} = \frac{\vec{CD}}{CD} \qquad \vec{CD} = 0.5\hat{i} + 3\hat{j} - 4\hat{k}$$

$$CD = \sqrt{(0.5)^2 + (3)^2 + (-4)^2} = \sqrt{25.25}$$

$$\vec{M}_4 = 6\left(\frac{0.5\hat{i} + 3\hat{j} - 4\hat{k}}{\sqrt{25.25}}\right) = (0.6\hat{i} + 3.58\hat{j} - 4.78\hat{k})\ kN.m$$

$$\vec{r}_{OA} = 0\hat{i} + 1\hat{j} + 0.5\hat{k} \qquad \vec{r}_{OB} = 0.5\hat{i} + 1\hat{j} + 0.5\hat{k}$$

$$\vec{r}_{OA} \times \vec{F}_1 = \begin{vmatrix} i & j & k \\ 0 & 1 & 0.5 \\ -5 & 0 & 0 \end{vmatrix} \begin{matrix} i & j \\ 0 & 1 \\ -5 & 0 \end{matrix} = [-2.5\hat{j}] - [-5\hat{k}]$$

$$= (-2.5\hat{j} + 5\hat{k})\ kN.m$$

52-606 2/2

$$\vec{r}_{OB} \times \vec{F}_2 = \left| \begin{array}{ccc|cc} \hat{i} & \hat{j} & \hat{k} & \hat{i} & \hat{j} \\ 0.5 & 1 & 0.5 & 0.5 & 1 \\ 4.71 & -1.57 & 6.28 & 4.71 & -1.57 \end{array} \right|$$

$$= \left[6.28\hat{i} + 2.36\hat{j} - 0.79\hat{k}\right] - \left[4.71\hat{k} - 0.79\hat{i} + 3.14\hat{j}\right]$$

$$\left(7.07\hat{i} - 0.76\hat{j} - 5.5\hat{k}\right) \text{ kN.m}$$

$$\therefore \vec{M}_{R_O} = 4\hat{k} - 3\hat{j} + 5\hat{j} + \left(10.6\hat{i} + 3.58\hat{j} - 4.78\hat{k}\right) + \left(-2.5\hat{j} + 5\hat{k}\right) + \left(7.07\hat{i} - 0.76\hat{j} - 5.5\hat{k}\right)$$

$$\vec{M}_{R_O} = \left(7.67\hat{i} + 2.32\hat{j} - 1.28\hat{k}\right) \text{ kN.m}$$

$$M_{R_O} = \sqrt{(7.67)^2 + (2.32)^2 + (-1.28)^2} = \sqrt{65.85} = 8.11 \text{ kN.m}$$

b)

$$\cos\theta_x = \frac{7.67}{\sqrt{65.85}} \Rightarrow \theta_x = 19.06^\circ$$

$$\cos\theta_y = \frac{2.32}{\sqrt{65.85}} \Rightarrow \theta_y = 73.39^\circ$$

$$\cos\theta_z = -\frac{1.28}{\sqrt{65.85}} \Rightarrow \theta_z = 99.08^\circ$$

c) $M = Fd$

$$M_{F_2} = \sqrt{(7.07)^2 + (-0.76)^2 + (-5.5)^2}$$

$$M_{F_2} = 8.99 \text{ kN.m}$$

$$F_2 = 8 \text{ kN} \quad \therefore d = \frac{8.99}{8} = 1.12 \text{ m}$$

S2-607 1/3

Using the "Right Hand Rule" and $\vec{M} = \vec{r} \times \vec{F}$

a) Equivalent Force-Couple at D:

$$\vec{M}_D = +80\hat{i} - 145\hat{j} - 110\hat{k} - 110\hat{k} + \vec{r}_{DB} \times \vec{F}_1 + \vec{r}_{DO} \times \vec{F}_2 + \vec{M}_1$$

$$\vec{M}_D = 80\hat{i} - 145\hat{j} - 220\hat{k} + \vec{r}_{DB} \times \vec{F}_1 + \vec{r}_{DO} \times \vec{F}_2 + \vec{M}_1$$

$\vec{r}_{DB} = 10\hat{i} + 3\hat{j} + 0\hat{k}$ $\quad \vec{F}_1 = 4\hat{i} - 10\hat{j} + 3\hat{k}$

$\vec{r}_{DO} = 0\hat{i} + 3\hat{j} + 0\hat{k}$ $\quad \vec{F}_2 = -5\hat{i}$

$$\vec{r}_{DB} \times \vec{F}_1 = \left|\begin{array}{ccc|cc} i & j & k & i & j \\ 10 & 3 & 0 & 10 & 3 \\ 4 & -10 & 3 & 4 & -10 \end{array}\right|$$

$$= [9\hat{i} - 100\hat{k}] - [12\hat{k} + 30\hat{j}]$$

$$= (9\hat{i} - 30\hat{j} - 112\hat{k})\ kN.m$$

$$\vec{r}_{DO} \times \vec{F}_2 = \left|\begin{array}{ccc|cc} i & j & k & i & j \\ 0 & 3 & 0 & 0 & 3 \\ -5 & 0 & 0 & -5 & 0 \end{array}\right|$$

$$= [\ 0\ ] - [-15\hat{k}] = (15k)\ kN.m$$

$\vec{M}_1 = M_1 \vec{\lambda}_{DE} = 210\,\lambda_{DE}$ $\quad \vec{\lambda}_{DE} = \dfrac{\vec{OE}}{OE}$

$\vec{OE} = 4\hat{i} + 6\hat{j} - 4\hat{k}$ $\quad OE = \sqrt{(4)^2 + (6)^2 + (-4)^2}$

$= \sqrt{68}$

$$\vec{M}_1 = 210\left(\frac{4\hat{i} + 6\hat{j} - 4\hat{k}}{\sqrt{68}}\right) = (101.86\hat{i} + 152.8\hat{j} + 101.86\hat{k})\ kN.m$$

$$\therefore \vec{M}_D = (80\hat{i} - 145\hat{j} - 220\hat{k}) + (9\hat{i} - 30\hat{j} - 112\hat{k}) + (15\hat{k})$$
$$+ (101.86\hat{i} + 152.8\hat{j} + 101.86\hat{k})$$

$$\vec{M}_D = \left(190.86\hat{i} - 22.2\hat{j} - 215.14\hat{k}\right) kN.m$$

$$\vec{R}_D = \vec{F_1} + \vec{F_2}$$

$$\vec{R}_D = \left(4\hat{i} - 10\hat{j} + 3\hat{k}\right) - 5\hat{i}$$

$$\vec{R}_D = \left(-\hat{i} - 10\hat{j} + 3\hat{k}\right) kN$$

$\therefore$ Equivalent Force-Couple at D:

$$\vec{R}_o = \left(-\hat{i} - 10\hat{j} + 3\hat{k}\right) kN$$

$$\vec{M}_o = \left(190.86\hat{i} - 22.2\hat{j} - 215.14\hat{k}\right) kN.m$$

b) Direction of the resultant moment at D:

$$M_D = \sqrt{(190.86)^2 + (-22.2)^2 + (-215.14)^2} = 288.454 \text{ kN.m}$$

$$\cos\theta_x = \frac{190.86}{288.454} \Rightarrow \theta_x = 48.57^\circ$$

$$\cos\theta_y = \frac{-22.2}{288.454} \Rightarrow \theta_y = 94.41^\circ$$

$$\cos\theta_z = \frac{-215.14}{288.454} \Rightarrow \theta_z = 138.23^\circ$$

c) Direction of Resultant force at D:

$$R_D = \sqrt{(-1)^2 + (-10)^2 + (3)^2} = \sqrt{110}$$

$$\cos\theta_x = \frac{-1}{\sqrt{110}} \Rightarrow \theta_x = 95.47^\circ$$

$$\cos\theta_y = \frac{-10}{\sqrt{110}} \Rightarrow \theta_y = 162.45^\circ$$

$$\cos\theta_z = \frac{3}{\sqrt{110}} \Rightarrow \theta_z = 73.38^\circ$$

d) $M = F_1 d$ 52-607 3/3

$\vec{M}_{F_1} = (9\hat{i} - 30\hat{j} - 112\hat{k})$ kN.m

$M_{F_1} = \sqrt{(9)^2 + (-30)^2 + (-112)^2} = 116.3$ kN.m

$\vec{F}_1 = (4\hat{i} - 10\hat{j} + 3\hat{k})$ kN

$F_1 = \sqrt{(4)^2 + (-10)^2 + (3)^2} = \sqrt{125} = 11.18$ kN

$116.3 = 11.18\, d$ $\qquad d = 10.4$ m

e) Moment of $\vec{F}_1$ about Line AC

$M_{AC} = \vec{\lambda}_{AC} \cdot (\vec{r}_{AB} \times \vec{F}_1)$

$\vec{\lambda}_{AC} = \frac{\vec{AC}}{AC} \qquad \vec{AC} = 7\hat{i} - 1.5\hat{j} + 0\hat{k}$

$AC = \sqrt{(7)^2 + (-1.5)^2} = \sqrt{51.25}$

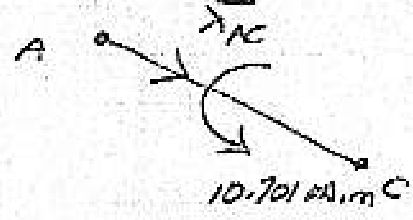
$\vec{\lambda}_{AC} = \frac{7\hat{i} - 1.5\hat{j} + 0\hat{k}}{\sqrt{51.25}} = 0.978\hat{i} - 0.21\hat{j}$

$\vec{r}_{AB} = 10\hat{i} + 1.5\hat{j} + 0\hat{k}$

$\vec{F}_1 = 4\hat{i} - 10\hat{j} + 3\hat{k}$

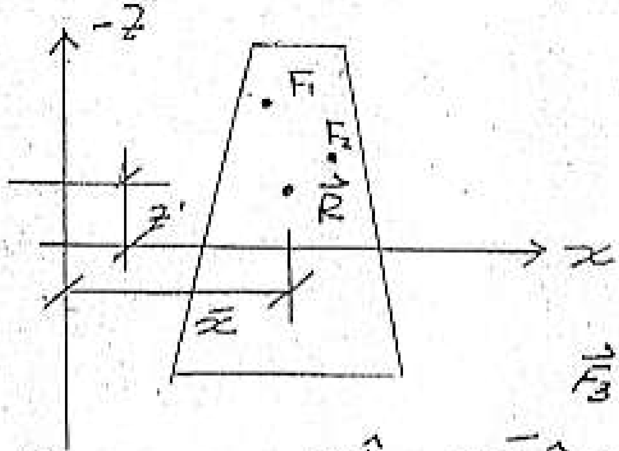
$$M_{AC} = \left| \begin{array}{ccc|cc} 0.978 & -0.21 & 0 & 0.978 & -0.21 \\ 10 & 1.5 & 0 & 10 & 1.5 \\ 4 & -10 & 3 & 4 & -10 \end{array} \right|$$

$= [4.401] - [-6.3] = 10.701$ kN.m



52-608 $\overset{k}{\underset{i\ \ j}{\circlearrowleft}}$ $-\hat{i}$

Force	$\vec{F}_j$	$\vec{r}$	$\vec{M} = \vec{r} \times \vec{F}$
F1	$-10j$	$9\hat{i} - 6k$	$-90\hat{k} - 60\hat{i}$
F2	$-5j$	$12\hat{i} - 3k$	$-60\hat{k} - 15\hat{i}$
R	$-15j$	$\bar{x}\hat{i} + \bar{z}k$	$-15\bar{x}\hat{k} + 15\bar{z}\hat{i}$



$\vec{F}_3 = 7\,kN$

$\vec{F}_3 = F_3 \vec{\lambda}_{QP}$

$\lambda_{QP} = \dfrac{\vec{QP}}{QP}$ $\vec{QP} = 2\hat{i} - 4\hat{j} + 4\hat{k}$

$QP = \sqrt{(2)^2 + (-4)^2 + (-4)^2} = 6$

$\vec{F}_3 = 7\left(\dfrac{2\hat{i} - 4\hat{j} + 4\hat{k}}{6}\right) = \dfrac{14\hat{i} - 28\hat{j} + 28\hat{k}}{6}$

a) $-15\bar{x}\hat{k} + 15\bar{z}\hat{i} = -150\hat{k} - 75\hat{i}$

$-15\bar{x} = -150$ $\bar{x} = 10$

$15\bar{z} = -75$ $\bar{z} = -5$

b) Force-Couple at O:

$\vec{R} = -15j$ $\vec{r}_{OP} = -10\hat{i} + 0\hat{j} + 6k$

$\vec{M}_{R_O} = \vec{r}_{OP} \times \vec{F}_3 + (-90\hat{k} - 60\hat{i}) + (-60\hat{k} - 15\hat{i})$

$$\begin{vmatrix} i & j & k \\ -10 & 0 & 6 \\ 2.33 & -4.67 & 4.67 \end{vmatrix} \begin{matrix} i & j \\ -10 & 0 \\ 2.33 & -4.67 \end{matrix} = [14\hat{j} + 46.7\hat{k}] - [-28\hat{i} - 46.7\hat{j}]$$

$= 28\hat{i} + 60.7\hat{j} + 46.7k$

$\vec{M}_{R_O} = (28\hat{i} + 60.7\hat{j} + 46.7\hat{k}) + (-90\hat{k} - 60\hat{i}) + (-60\hat{k} - 15\hat{i})$

$\vec{M}_{R_O} = -47\hat{i} + 60.7\hat{j} - 103.3\hat{k}$

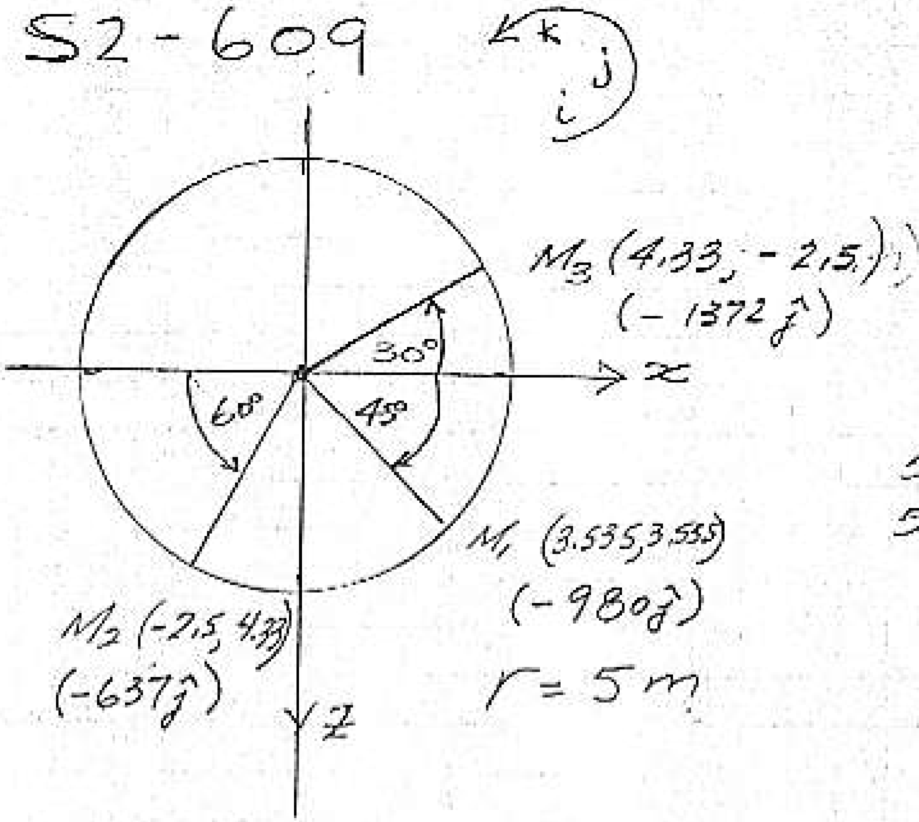
c) $\vec{\lambda}_{PS} = \dfrac{\vec{PS}}{PS}$ $\vec{PS} = \hat{i} + 5\hat{j} - 8\hat{k}$ $PS = \sqrt{(1)^2 + (5)^2 + (-8)^2}$
$= \sqrt{90}$

$\vec{\lambda}_{PS} = \dfrac{\hat{i} + 5\hat{j} - 8\hat{k}}{\sqrt{90}}$

Projection $= \vec{F}_3 \cdot \vec{\lambda}_{PS} = \left(\dfrac{14}{6}\right)\left(\dfrac{1}{\sqrt{90}}\right) + \left(\dfrac{-28}{6}\right)\left(\dfrac{5}{\sqrt{90}}\right) + \left(\dfrac{+28}{6}\right)\left(\dfrac{-8}{\sqrt{90}}\right)$

$4.469\ kN$

S2-609



$5\cos 30° = 4.33\,m$
$5\sin 30° = 2.5\,m$

$M_1 = 100\,kg = 100(9.8) = 980\,N \quad \vec{F}_1 = -980\hat{j}\,N$

$M_2 = 65\,kg = 65(9.8) = 637\,N \quad \vec{F}_2 = -637\hat{j}\,N$

$M_3 = 140\,kg = 140(9.8) = 1372\,N \quad \vec{F}_3 = -1372\hat{j}\,N$

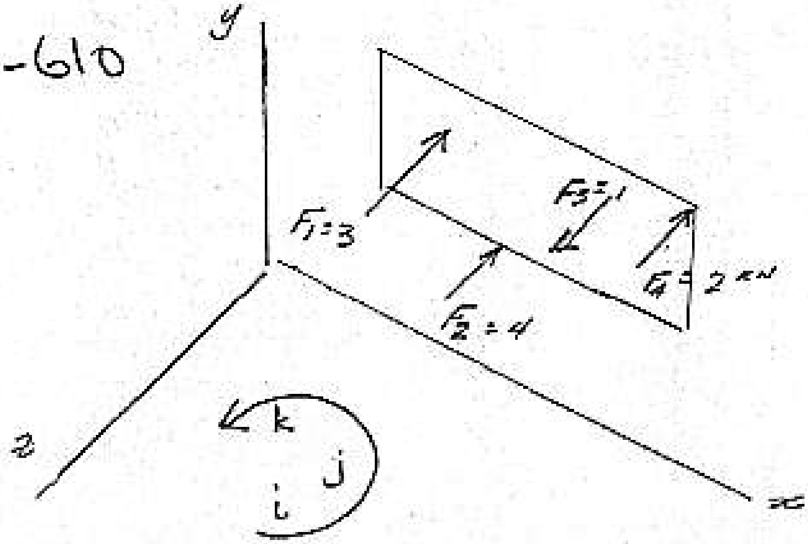
Force	$\vec{r}$ (meters)	$\vec{F}$, (N)	$\vec{M}_o = \vec{r}\times\vec{F}$
F_1	$3.535\hat{i} + 3.535\hat{k}$	$-980\hat{j}$	$-3464.3\hat{k} + 3464.3\hat{i}$
F_2	$-2.5\hat{i} + 4.33\hat{k}$	$-637\hat{j}$	$+1592.5\hat{k} + 2758.21\hat{i}$
F_3	$4.33\hat{i} - 2.5\hat{k}$	$-1372\hat{j}$	$-5940.76\hat{k} - 3430\hat{i}$
R	$x\hat{i} + z\hat{k}$	$-2989\hat{j}$	$-7821.56\hat{k} + 2792.51\hat{i}$

$$(x\hat{i} + z\hat{k}) \times (-2989\,\hat{j}) = -7821.56\hat{k} + 2792.51\hat{i}$$

$$-2989x\,\hat{k} = -7821.56\hat{k} \qquad x = 2.62\,m$$

$$2989z\,\hat{i} = 2792.51\hat{i} \qquad z = 0.934\,m$$

52-610



A: (3, 12, 0)
B: (6, 10, 0)
C: (8, 13, 0)
D: (12, 14, 0)

Force	$\vec{r}$	$\vec{F}$	$\vec{M}_o = \vec{r} \times \vec{F}$
$\vec{F}_1$	$3\hat{i} + 12\hat{j}$	$-3\hat{k}$	$9\hat{j} - 36\hat{i}$
$\vec{F}_2$	$6\hat{i} + 10\hat{j}$	$-4\hat{k}$	$24\hat{j} - 40\hat{i}$
$\vec{F}_3$	$8\hat{i} + 13\hat{j}$	$+1\hat{k}$	$-8\hat{j} + 13\hat{i}$
$\vec{F}_4$	$12\hat{i} + 14\hat{j}$	$-2\hat{k}$	$24\hat{j} - 28\hat{i}$
$\vec{R}$	$x\hat{i} + y\hat{j}$	$-8\hat{k}$	$\Sigma\ 49\hat{j} - 91\hat{i}$

$$(x\hat{i} + y\hat{j}) \times (-8\hat{k}) = 49\hat{j} - 91\hat{i}$$

$$8x\hat{j} - 8y\hat{i} = 49\hat{j} - 91\hat{i}$$

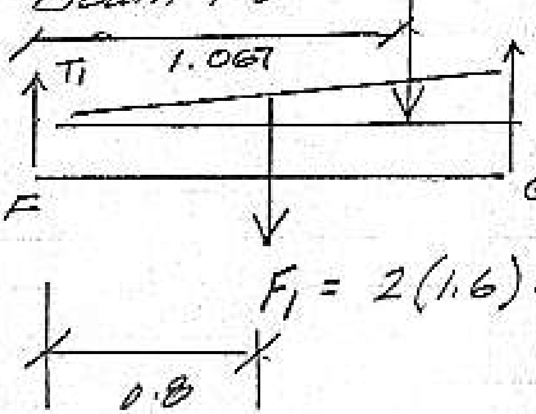
$$\therefore\ 8x = 49 \qquad x = 6.125\,m$$

$$-8y = -91 \qquad y = 11.375\,m$$

SZ-611 1/2

Beam FG

$$F_2 = \frac{(4-2)(1.6)}{2} = 1.6\ \text{kN}$$



$$F_1 = 2(1.6) = 3.2\ \text{kN}$$

a)

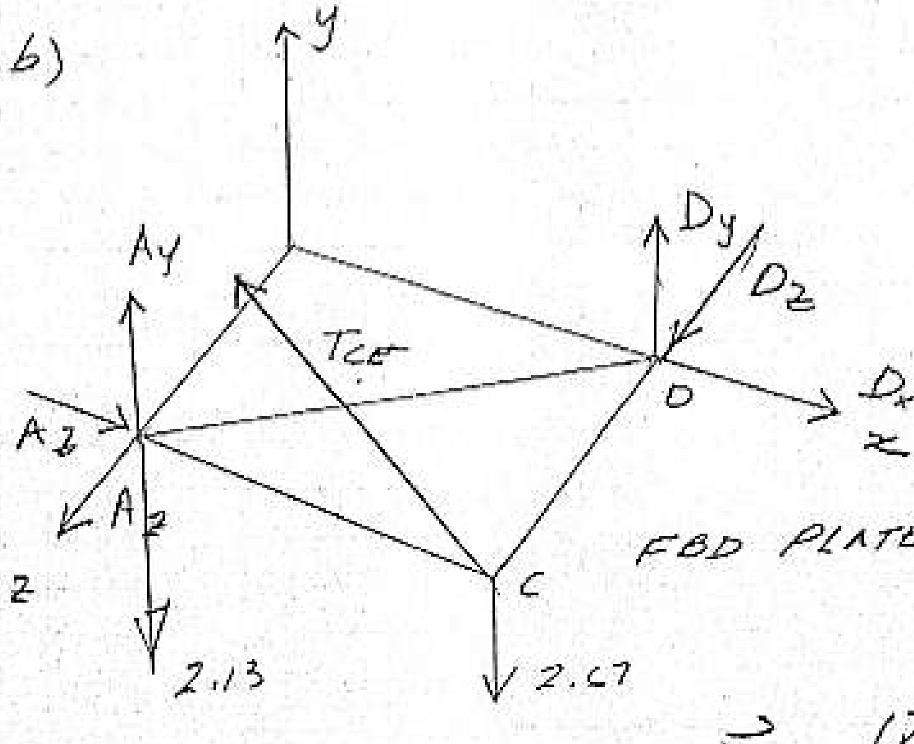
$$\Sigma M_F = 0 \quad -3.2(0.8) - 1.6(1.067) + T_2(1.6) = 0$$

$$T_2 = 2.67$$

$$\Sigma F_y = 0 \quad T_1 - 3.2 - 1.6 + 2.67 = 0$$

$$T_1 = 2.13$$

b)



A: (0, 0, 1.2)
C: (1.6, 0, 1.2)
D: (1.6, 0, 0)
~~E: (0, 1.498, 0.56)~~
E: (0, 1.498, 0.56)

FBD PLATE ADC

$$\Sigma M_{AD} = 0 = \vec{\lambda}_{AD} \cdot (\vec{r}_{AC} \times \vec{T}_{CE}) + \vec{\lambda}_{AD} \cdot (r_{AC} \times -2.67\hat{j})$$

S2-611 2/2

$$\vec{\lambda}_{AD} = \frac{\vec{AD}}{AD} \qquad \vec{AD} = 1.6\hat{i} + 0\hat{j} - 1.2\hat{k}$$

$$AD = \sqrt{(1.6)^2 + (-1.2)^2} = 2$$

$$\vec{\lambda}_{AD} = \frac{1.6\hat{i} + 0\hat{j} - 1.2\hat{k}}{2} = 0.8\hat{i} + 0\hat{j} - 0.6\hat{k}$$

$$\vec{r}_{AC} = 1.6\hat{i} + 0\hat{j} + 0\hat{k}$$

$$\vec{T}_{CE} = T_{CE}\vec{\lambda}_{CE} \qquad \vec{\lambda}_{CE} = \frac{\vec{CE}}{CE}$$

$$\vec{CE} = -1.6\hat{i} + 1.48\hat{j} - 0.64\hat{k}$$

$$CE = \sqrt{(-1.6)^2 + (1.48)^2 + (-0.64)^2} = \sqrt{5.16}$$

$$\vec{T}_{CE} = T_{CE}\left(\frac{-1.6\hat{i} + 1.48\hat{j} - 0.64\hat{k}}{\sqrt{5.16}}\right.$$

$$M_{AD} = 0$$

$$\frac{T_{CE}}{\sqrt{5.16}}\begin{vmatrix} 0.8 & 0 & -0.6 \\ 1.6 & 0 & 0 \\ -1.6 & 1.48 & -0.64 \end{vmatrix} + \begin{vmatrix} 0.8 & 0 & -0.6 \\ 1.6 & 0 & 0 \\ 0 & -2.67 & 0 \end{vmatrix}$$

$$\frac{T_{CE}}{\sqrt{5.16}}\left|\begin{array}{ccc|cc} 0.8 & 0 & -0.6 & 0.8 & 0 \\ 1.6 & 0 & 0 & 1.6 & 0 \\ -1.6 & 1.48 & -0.64 & -1.6 & 1.48 \end{array}\right|$$

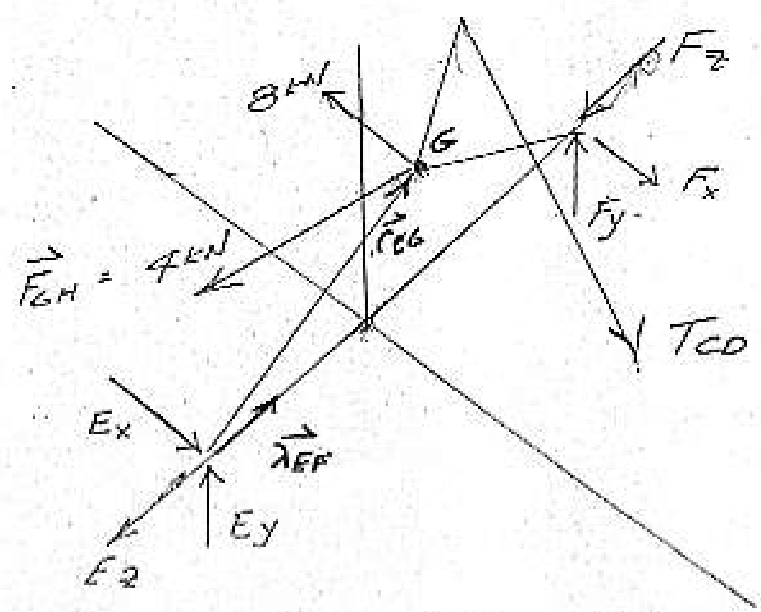
$$+ \left|\begin{array}{ccc|cc} 0.8 & 0 & -0.6 & 0.8 & 0 \\ 1.6 & 0 & 0 & 1.6 & 0 \\ 0 & -2.67 & 0 & 0 & -2.67 \end{array}\right]$$

$$\frac{T_{CE}}{\sqrt{5.16}}\left\{[-1.4208] - [0]\right\} + \left\{[2.5632] - [0]\right\}$$

$$T_{CE} = \frac{2.5632\sqrt{5.16}}{1.4208} = 4.09\text{ kN}$$

5Z-612 1/2

Draw the FBD of the frame:



Unknowns

$E_x, E_y, E_z, F_x, F_y, F_z, T_{CD}$

7 unknowns

6 equilibrium equations

If we take moments about the line EF only unknown is T_{CD}

$M_{EF} = 0$ for equilibrium

$$M_{EF} = \vec{\lambda}_{EF} \cdot (\vec{r}_{EG} \times \vec{F}_{GH}) + \vec{\lambda}_{EF} \cdot (\vec{r}_{OB} \times -8\hat{i}) + \vec{\lambda}_{EF} \cdot (\vec{r}_{ED} \times \vec{T}_{CD})$$

$\vec{\lambda}_{EF} = -\hat{k}$ $\quad \vec{r}_{EG} = \hat{i} + 2\hat{j} - 2\hat{k}$

$\vec{F}_{GH} = 4\vec{\lambda}_{GH}$ $\quad \vec{\lambda}_{GH} = \dfrac{\vec{GH}}{GH}$ $\quad \vec{GH} = -1.75\hat{i} - 2\hat{j} + 3.5\hat{k}$

$GH = \sqrt{(-1.75)^2 + (-2)^2 + (3.5)^2} = 4.395$

$F_{GH} = 4\left(\dfrac{-1.75\hat{i} - 2\hat{j} + 3.5\hat{k}}{4.395}\right) = -1.593\hat{i} - 1.82\hat{j} + 3.185\hat{k}$

$\vec{r}_{OB} = 2\hat{i} + 4\hat{j}$ $\quad \vec{CD} = 3\hat{i} - 6\hat{j} - 3\hat{k}$

$\vec{T}_{CD} = T_{CD}\vec{\lambda}_{CD}$ $\quad \vec{\lambda}_{CD} = \dfrac{\vec{CD}}{CD}$, $\quad CD = \sqrt{(3)^2 + (-6)^2 + (-3)^2} = \sqrt{54}$

$\vec{T}_{CD} = T_{CD}\left(\dfrac{3\hat{i} - 6\hat{j} - 3\hat{k}}{\sqrt{54}}\right) = 0.408T_{CD}\hat{i} - 0.816T_{CD}\hat{j} - 0.408T_{CD}\hat{k}$

$\vec{r}_{ED} = 6\hat{i} + 0\hat{j} - 5\hat{k})$

$$M_{EF} = \begin{vmatrix} 0 & 0 & -1 \\ 1 & 2 & -2 \\ -1.593 & -1.82 & 3.185 \end{vmatrix} + \begin{vmatrix} 0 & 0 & -1 \\ 2 & 4 & 0 \\ -8 & 0 & 0 \end{vmatrix} + \begin{vmatrix} 0 & 0 & -1 \\ 6 & 0 & -5 \\ T_{CD}\,0.408 & -0.816 & -0.408 \end{vmatrix}$$

$= 0$

52-612 2/2

$$\begin{vmatrix} 0 & 0 & -1 \\ 1 & 2 & -2 \\ -1.593 & -1.82 & 3.185 \end{vmatrix} \begin{matrix} 0 & 0 \\ 1 & 2 \\ -1.593 & -1.82 \end{matrix} = [1.82] - [3.186]$$

$$= -1.366$$

$$\begin{vmatrix} 0 & 0 & -1 \\ 2 & 4 & 0 \\ -8 & 0 & 0 \end{vmatrix} \begin{matrix} 0 & 0 \\ 2 & 4 \\ -8 & 0 \end{matrix} = [0] - [32]$$

$$= -32$$

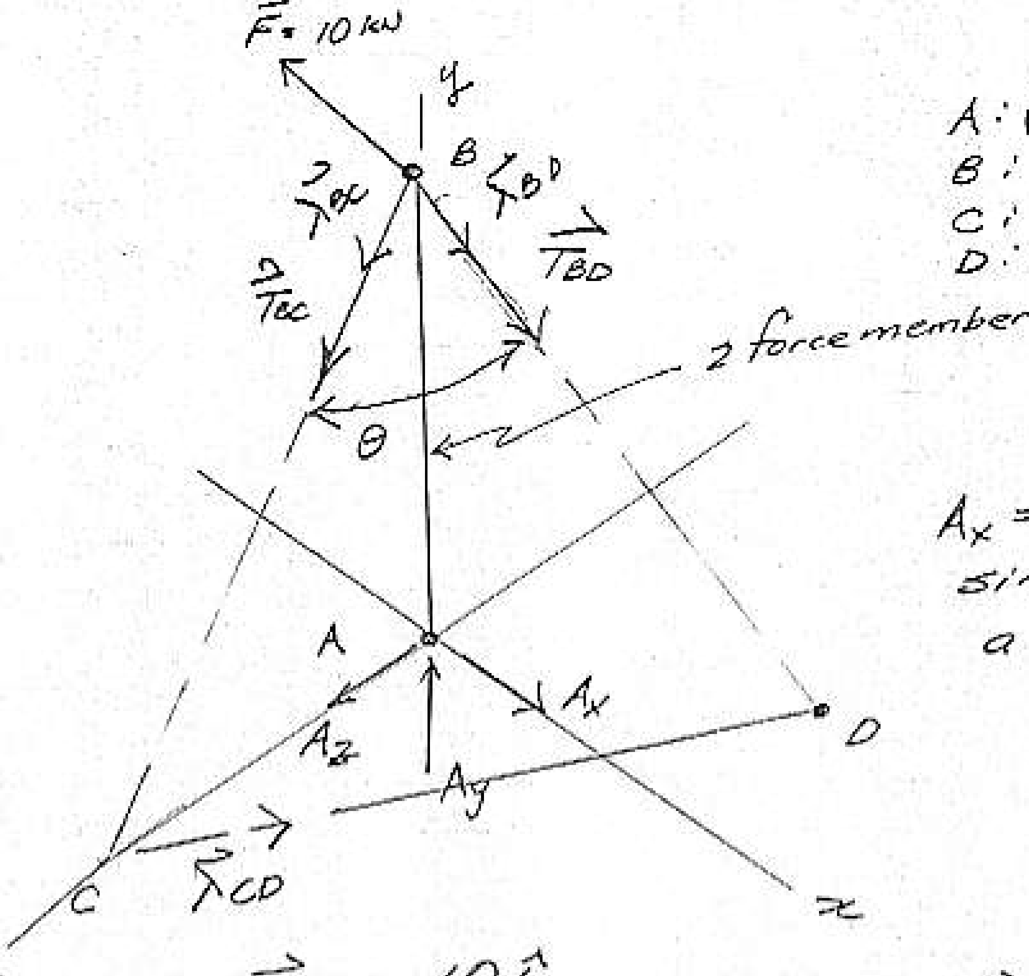
$$T_{CD} \begin{vmatrix} 0 & 0 & -1 \\ 6 & 0 & -5 \\ 0.408 & -0.816 & -0.408 \end{vmatrix} \begin{matrix} 0 & 0 \\ 6 & 0 \\ 0.408 & -0.816 \end{matrix}$$

$$= T_{CD}\left\{[4.896] - [0]\right\} = 4.896\, T_{CD}$$

$$4.896\, T_{CD} = 32 + 1.366$$

$$T_{CD} = \frac{33.366}{4.896} = 6.815\ kN$$

S2-613 1/2



A : (0, 0, 0)
B : (0, 6, 0)
C : (0, 0, 6)
D : (6, 0, −3)

$A_x = A_y = 0$
since AB is
a 2-force
member

$\vec{F} = -10\hat{i}$

a) $\vec{T}_{BD} = T_{BD}\,\lambda_{BD}$ $\quad \vec{\lambda}_{BD} = \dfrac{\vec{BD}}{BD}$ $\quad \vec{BD} = 6\hat{i} - 6\hat{j} - 3\hat{k}$

$BD = \sqrt{(6)^2 + (-6)^2 + (-3)^2} = 9$

$\vec{T}_{BD} = T_{BD}\left(\dfrac{6\hat{i} - 6\hat{j} - 3\hat{k}}{9}\right)$

$\vec{T}_{BD} = 0.67T_{BD}\hat{i} - 0.67\,T_{BD}\hat{j} - 0.33T_{BD}\hat{k}$

$\vec{T}_{BC} = T_{BC}\vec{\lambda}_{BC}$ $\quad \vec{\lambda}_{BC} = \dfrac{\vec{BC}}{BC}$ $\quad \vec{BC} = 0\hat{i} - 6\hat{j} + 6\hat{k}$

$BC = \sqrt{(-6)^2 + (6)^2} = \sqrt{72}$

$\vec{T}_{BC} = T_{BC}\left(\dfrac{-6\hat{j} + 6\hat{k}}{\sqrt{72}}\right) = -0.707\,T_{BC}\hat{j} + 0.707T_{BC}\hat{k}$

$\Sigma F_x = 0 \quad -10 + 0.67\,T_{BD} = 0 \quad T_{BD} = 14.93\text{ kN}$

$\Sigma F_y = 0 \quad A_y - 0.67(14.93) - 0.707T_{BC} = 0$

$\Sigma F_z = 0 \quad -0.33(14.93) + 0.707\,T_{BC} = 0$

$T_{BC} = 6.97\text{ kN}$

b) 52-613 2/2

$$A_y - 0.67(14.93) - 0.707(6.97) = 0$$

$$A_y = +14.93$$

$$\vec{A_y} = 14.93\ \text{kN} \uparrow$$

$$\vec{A_x} = 0$$

$$\vec{A_y} = 0$$

c) $$\vec{\lambda}_{BC} \cdot \vec{\lambda}_{BD} = \cos\theta$$

$$\cos\theta = \left(\frac{6}{9}\right)(0) + \left(-\frac{6}{9}\right)\left(-\frac{6}{\sqrt{72}}\right) + \left(-\frac{3}{9}\right)\left(\frac{6}{\sqrt{72}}\right)$$

$$\cos\theta = 0.23570$$

$$\theta = 76.37^\circ$$

d) $$M_{CD} = \vec{\lambda}_{CD} \cdot (\vec{r}_{CB} \times -10\hat{i})$$

$$\vec{\lambda}_{CD} = \frac{\vec{CD}}{CD} \qquad \vec{CD} = 6\hat{i} + 0\hat{j} - 9\hat{k}$$

$$CD = \sqrt{(6)^2 + (-9)^2} = \sqrt{127}$$

$$\vec{\lambda}_{CD} = \frac{6\hat{i} + 0\hat{j} - 9\hat{k}}{\sqrt{127}} \qquad \vec{r}_{CB} = 0\hat{i} + 6\hat{j} + 6\hat{k}$$

$$M_{CD} = \frac{1}{\sqrt{127}} \left| \begin{array}{ccc|cc} 6 & 0 & -9 & 6 & 0 \\ 0 & 6 & 6 & 0 & 6 \\ -10 & 0 & 0 & -10 & 0 \end{array} \right|$$

$$= \frac{1}{\sqrt{127}} \left\{ [0] - [540] \right\} = -47.92\ \text{kN.m}$$

M_{CD}