6:00 p.m. 16 April 20 11 FINAL EXAMINATION

DEPARTMENT & COURSE NO.: ENG 1460 TIME: 3 HOURS

EXAMINATION: Introduction to Thermal Sciences

EXAMINER(S): Dr. D.C.S. Kuhn (A01), Dr. B.-C. Wang (A02), Dr. S.J. Ormiston (A03)

Values

Instructions:

1. You are permitted to use the textbook for the course and a calculator.
2. Clear, systematic solutions are required. **Show your work.** Marks will not be assigned for problems that require unreasonable (in the opinion of the instructor(s)) effort to decipher.
3. Ask for clarification if any problem statement is unclear to you.
4. Use linear interpolation between table entries as necessary. Use constant specific heats.
5. Retain all the significant figures and units of property values from tables. Keep 4 or 5 significant figures in your intermediate results. For temperature, keep two decimal places in your final answer. Final answers must have 3 to 5 significant figures and units.
6. There are **five** questions on this exam. The weight of each problem is indicated. The exam will be marked out of 100.

---

[5] **1**. An inventor claims to have developed a heat engine that receives energy from a source at 280°C, rejects heat to a sink at 30°C, and produces 3.6 kW of power while rejecting heat at a rate of 12.6 kW. Could his claim be valid? Support your answer with the appropriate calculations.

[15] **2**. Figure 1 schematically shows a house that is heated by a geothermal heat pump (HP) during the winter season in the Winnipeg area. The HP operates between two heat sources, whose temperatures can be treated as constants over the entire winter season: the underground soil temperature is $T_L = 12°\text{C}$ and the indoor environment temperature is $T_H = 22°\text{C}$. The HP receives 12 kW heat from the underground soil (i.e., $\dot{Q}_L = 12$ kW) and its nominal coefficient of performance (COP) is $\beta' = 5$. The HP operates continuously during the entire winter season, and Manitoba Hydro charges electric energy consumption at 7 cents per kWh (i.e., \$0.07/kWh).

2 (a) Determine the rate of heat discharged by the HP to the indoor environment, $\dot{Q}_H$, in kW.

3 (b) What is the expected monthly charge (call this C for charge) shown on the Hydro bill for the 31 days in January, in dollars?

3 (c) What is the maximum possible COP (COP_{max} or β'_{max}) value for a HP under the operating conditions described above?

7 (d) One day, the owner of the house received a letter from a company advising him that he could reduce his January Hydro bill to \$23 (for operating the HP) in order to maintain the required indoor environment temperature, if he replaces his existing HP with a new-generation high-technology HP designed by the company. What is your advice to the owner of the house? Rigorously justify your answer with calculations.

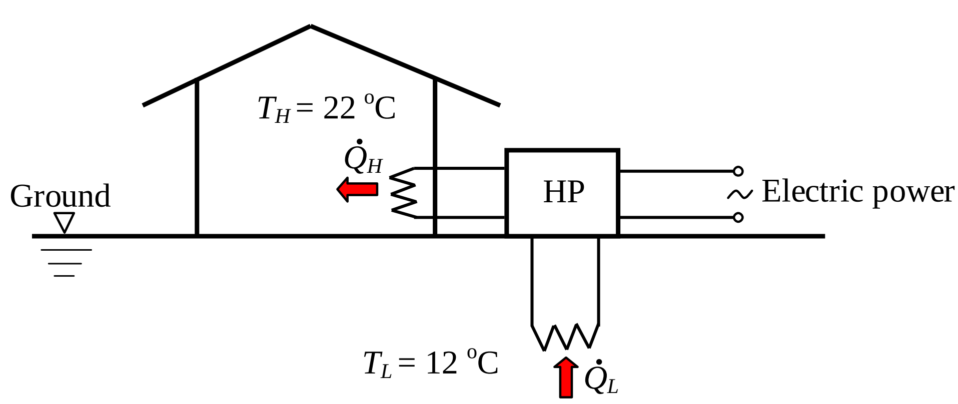


Figure 1: Figure for problem **2**

Values

3. A frictionless piston-cylinder and linear-spring assembly containing nitrogen gas is shown in Figure 2. The mass of the nitrogen gas is $m_N = 0.12$ kg. A copper plate uniformly covers the bottom of the cylinder, and this copper plate is always in **thermal equilibrium** with the nitrogen gas. The distance from the top surface of the copper plate to the bottom surface of the piston is denoted as y. At the initial state (State 1), the piston rests on the stops, with $y_1 = 0.5$ m and the pressure of the nitrogen gas being $P_1 = 110$ kPa. Then heat is transferred to the nitrogen gas and copper from the outside. At the exact moment when the piston just begins to leave the stops, the state of the nitrogen gas is referred to as State 2. At the exact moment when the piston just begins to touch the spring, the state of the nitrogen gas is referred to as State 3. At State 3, $y_3 = 1$ m. The nitrogen gas and copper continue to receive heat from the outside until the final state (State 4) is reached, when the pressure of the nitrogen gas becomes $P_4 = 220$ kPa. From the initial to the final states, the total heat received by the nitrogen gas plus copper from the surroundings is ${}_1Q_4 = 320$ kJ. The inner cross-sectional area of the cylinder is $A = 0.2$ m^2. The piston mass is $m_P = 1200$ kg. The local gravitational acceleration is $g = 9.81$ m/s^2 and the local atmospheric pressure is $P_0 = 100$ kPa. The spring-constant of the linear-spring is $k_S = 40$ kN/m. The nitrogen gas can be treated as an **ideal gas**. The volume of the stops can be ignored. $\boxed{23}$

2 (a) Determine the temperature of the nitrogen gas at State 1, T_1, in °C.

2 (b) Determine the pressure of the nitrogen gas at State 2, P_2, in kPa.

2 (c) Determine the temperature of the nitrogen gas at State 3, T_3, in °C.

3 (d) Determine the final temperature of the nitrogen gas, T_4, in °C.

5 (e) Determine the total work done by the nitrogen gas, ${}_1W_4$, in kJ.

4 (f) Determine the mass of the copper plate, m_C, in kg.

5 (g) Show all state points and all processes on a P-V (pressure-volume) diagram, and indicate the area that represents the total work ${}_1W_4$. Label the pressure and volume values for all states. Showing lines of constant temperature is not necessary (*i.e.*, it is optional).

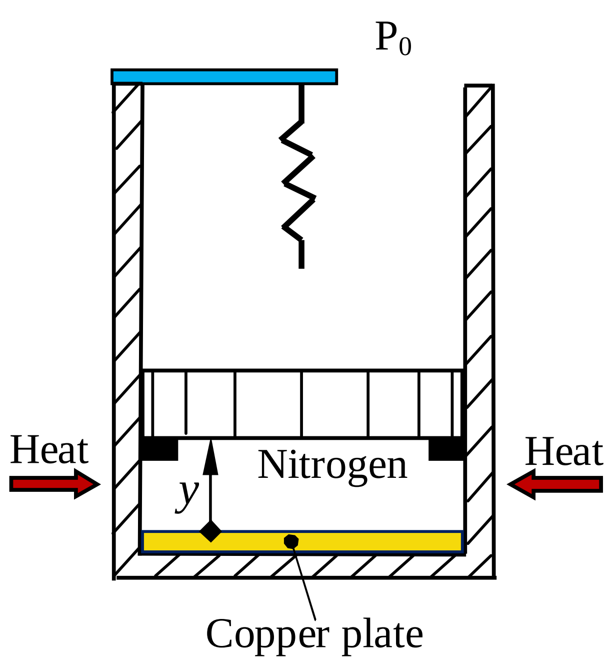


Figure 2: Piston-cylinder arrangement for problem **3**

Values

4. A dual loop heat-powered refrigeration cycle that uses **R-134a** (Refrigerant 134a) as the working fluid is shown in Figure 3. Saturated vapour at 3.0 MPa leaves the boiler and expands in the adiabatic turbine. The flow leaves the turbine at the condenser pressure (the condenser pressure is P_1). Saturated vapour at $-20°$C leaves the evaporator, enters the adiabatic compressor, and is compressed to the condenser pressure and a temperature of 50°C. The ratio of mass flows through the two loops is such that the turbine produces just enough power to drive the compressor. The two streams exiting from the turbine and the compressor are mixed together in an insulated mixing chamber; the stream leaving the mixing chamber is at a temperature of 40°C. The rate of heat transferred from the condenser is 122 kW. The stream leaving the condenser is saturated liquid at 1.0 MPa; this stream is then separated into two streams in the
29 necessary proportions. Neglect changes in potential and kinetic energies.

Note: The following values have been calculated by interpolation for you to use in the solution:
h_f(at 1 MPa) = 255.60 kJ/kg
v_f(at 39.37°C) = 0.000871 m^3/kg

4 (a) Calculate the mass flow rate through the condenser, $\dot{m}_1$, in kg/s.

8 (b) Calculate the mass flow rate through the power loop, $\dot{m}_P$, in kg/s. (**Hint:** Apply the first law to the turbine, the compressor, and the mixing chamber).

3 (c) Calculate the power **input** to the pump, $\dot{W}_P$, in kW.

6 (d) Calculate the ratio $\dot{Q}_L/\dot{Q}_H$.

8 (e) On a T-v (temperature–specific volume) diagram, draw a process representation for the working fluid in both loops (*i.e.*, for all states). On the diagram, clearly indicate and number the state points and show the process paths (use a dashed line if the path is unknown). Also show the constant pressure lines that pass through the state points and their corresponding saturation temperature values. Indicate state T values and the values of v for States 3, 4, 7, and 8. Do any additional work necessary to label the diagram.

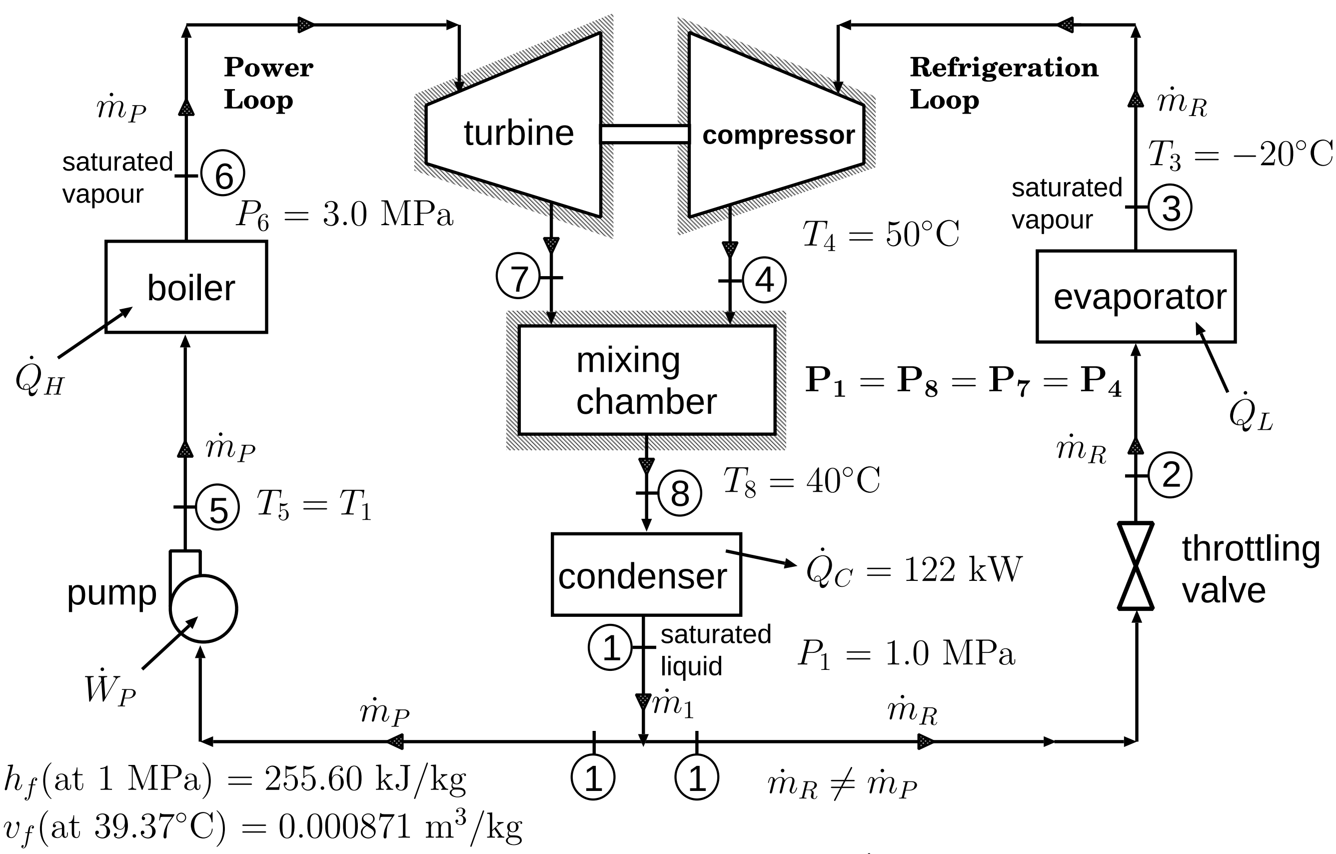


Figure 3: Figure for problem **4**

Values

5. A closed rigid cylindrical chamber of total volume 4.0 m^3 is divided by a piston that separates air (A) and water (W) as shown in Figure 4. In the water there is a solid aluminum (S) brick of mass 5.0 kg. The cylindrical side wall of the chamber is perfectly insulated. The top of the chamber is maintained at constant temperature of 30°C and is always in **thermal equilibrium** with the air inside the chamber. Heat may be transferred to or from the top of the chamber to maintain the temperature. The piston is perfectly insulated, free to move without friction and can be assumed to have negligible mass and volume. Initially (State 1) the air has pressure $P_{A,1} = 200$ kPa and has mass $m_A = 7$ kg while the water is a saturated mixture with $x_1 = 0.5$. Heat is transferred to the water plus aluminum side such that at the final state (State 2) the air pressure is $P_{A,2} = 400$ kPa. Assume the air is an **ideal gas**.

28

3 (a) Determine the volume, in m^3, of the aluminum, V_S , and the initial volume of the air, $V_{A,1}$, and the water, $V_{W,1}$.

6 (b) For the air side, determine the final state volume, $V_{A,2}$, in m^3, and the work and heat transfer during the process from the initial to final state, $({}_1W_2)_A$ and $({}_1Q_2)_A$, respectively, in kJ.

15 (c) For the water plus aluminum side, determine the final state volume of the water, $V_{W,2}$, in m^3, and the work and heat transfer during the process from the initial to final state, $({}_1W_2)_{W+S}$ and $({}_1Q_2)_{W+S}$, respectively, in kJ.

4 (d) On two separate figures, draw pressure versus volume diagrams for the air and for the water. On each diagram, clearly indicate the state points and the process paths (use a dashed line if the path is unknown). Label all T, P and V values as appropriate. Also indicate the appropriate lines of constant temperature on both diagrams.

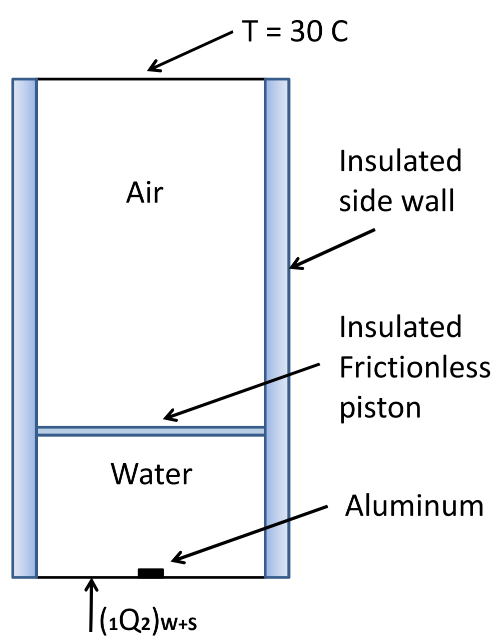
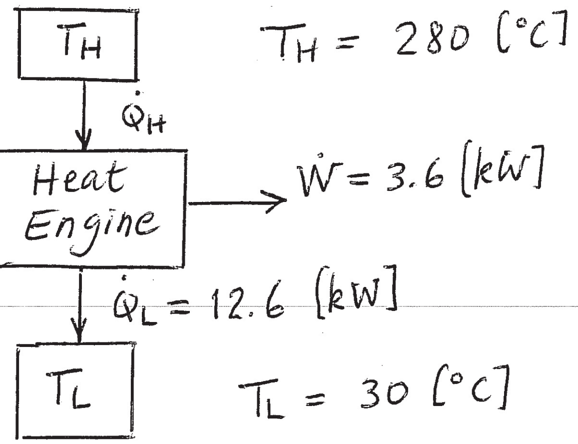


Figure 4: Figure for problem **5**

ENG 1460 (W11) Final Exam Solution

1.



The maximum possible efficiency is $\eta_{th,rev}$:

$$\eta_{th,rev} = 1 - \frac{T_L}{T_H} = 1 - \frac{(30 + 273.15)}{(280 + 273.15)} = 1 - \frac{303.15}{553.15} = 0.4520$$

The actual efficiency is $\eta_{th,act}$:

$$\eta_{th,act} = \frac{\dot{W}}{\dot{Q}_H} = \frac{\dot{W}}{\dot{Q}_L + \dot{W}} = \frac{3.6}{12.6 + 3.6} = \frac{3.6}{16.2} = 0.2222$$

$\eta_{th,act} < \eta_{th,rev}$ $\quad\therefore$ The claim could be <u>valid</u>.

Problem 2

(a) $\beta' = \dfrac{\dot{Q}_H}{\dot{W}_C} = \dfrac{\dot{Q}_H}{\dot{Q}_H - \dot{Q}_L}$

$5 = \dfrac{\dot{Q}_H}{\dot{Q}_H - 12}$

$\therefore \dot{Q}_H = 15$ (kW)

(b) $\dot{W}_C = \dot{Q}_H - \dot{Q}_L = 15 - 12 = 3$ kW

January Energy: $E = \dot{W}_C \times 24 \times 31 = 3 \times 24 \times 31 = 2232$ (kWh)

January Bill: $B = E \cdot 0.07 = 2232 \times 0.07 = 156.24$ (\$)

(c) The Carnot HP has the highest COP

$\beta'_{max} = \beta'_{Carnot} = \dfrac{T_H}{T_H - T_L} = \dfrac{273.15+22}{22-12} = 29.515$

(d) In the best scenario, the new HP is a Carnot machine,

$\dot{W}_C = \dot{Q}_H / \beta'_{max} = 15/29.515 = 0.5082$ (kW)

January Energy: $E_{Best} = \dot{W}_C \times 24 \times 31 = 0.5082 \times 24 \times 31 = 378.113$ (kWh)

January Bill: $B_{Best} = E_{Best} \times 0.07 = 378.113 \times 0.07 = 26.47$ (\$)

Claimed amount $= \$23 < B_{Best}$

$\therefore$ Impossible. Do not buy.

Alternative Method:

For the claimed January Hydro Bill $B = \$23$,

the corresponding energy is: $E = B/0.07 = 23/0.07 = 328.57$ (kWh)

and the corresponding compressor power is:

$\dot{W}_C = E/(24 \times 31) = 328.57/(24 \times 31) = 0.4416$ (kW)

the claimed COP is: $\beta'_{claimed} = \dot{Q}_H / \dot{W}_C = 15/0.4416 = 33.965$

$\beta'_{claimed} > \beta'_{max} = 29.515$

$\therefore$ Impossible. Do not buy.

Problem 3

For the nitrogen gas, from Table A.5:

$R = 0.2968 \ \mathrm{kJ/kg \cdot K}$, $C_{v0} = 0.745 \ \mathrm{kJ/kg \cdot K}$

For the Copper plate, from Tabe A.3:

$C_{cu} = 0.42 \ \mathrm{kJ/kg \cdot K}$

(a) $V_1 = A \cdot y_1 = 0.2 \times 0.5 = 0.1 \ (m^3)$

$$T_1 = \frac{P_1 V_1}{mR} = \frac{110 \times 0.1}{0.12 \times 0.2968} = 308.85 \ (K)$$

or, $35.70 \ ^\circ C$

(b) $P_2 = P_0 + \frac{m_p g}{A} = 100 + \frac{1200 \times 9.81}{0.2} \times \frac{1}{1000} = 158.86 \ (kPa)$

$(V_2 = V_1 = 0.1 \ m^3)$

(c) Constant pressure process between states 2 & 3

$\therefore P_3 = P_2 = 158.86 \ kPa$

$V_3 = A y_3 = 0.2 \times 1 = 0.2 \ (m^3)$

$$T_3 = \frac{P_3 V_3}{mR} = \frac{158.86 \times 0.2}{0.12 \times 0.2968} = 892.07 \ (K)$$

or, $618.92 \ (^\circ C)$

(d) $P_4 = P_3 + \frac{k_s}{A^2}(V_4 - V_3)$

$220 = 158.86 + \frac{40}{0.2^2}(V_4 - 0.2)$

$\therefore V_4 = 0.26114 \ (m^3)$

(Correspondingly, $y_4 = V_4 / A = 0.26114 / 0.2 = 1.3057 \ m$)

$$T_4 = \frac{P_4 V_4}{mR} = \frac{220 \times 0.26114}{0.12 \times 0.2968} = 1613.06 \ (K)$$

or, $1339.91 \ ^\circ C$

(e) ${}_1W_4 = \cancel{{}_1W_2} + {}_2W_3 + {}_3W_4$

${}_2W_3 = P_2 (V_3 - V_2) = 158.86 \times (0.2 - 0.1) = 15.886 \ (kJ)$

${}_3W_4 = \frac{1}{2}(P_3 + P_4)(V_4 - V_3) = \frac{1}{2}(158.86 + 220) \times (0.26114 - 0.2) = 11.582 \ (kJ)$

$\therefore {}_1W_4 = 15.886 + 11.582 = 27.468\ (kJ)$

(f) ${}_1Q_4 = {}_1W_4 + \Delta U$

$$= {}_1W_4 + (\Delta U_{N_2} + \Delta U_{Cu})$$

$$= {}_1W_4 + [m_{N_2} \cdot C_{v0}(T_4 - T_1) + m_{Cu} C_{Cu}(T_4 - T_1)]$$

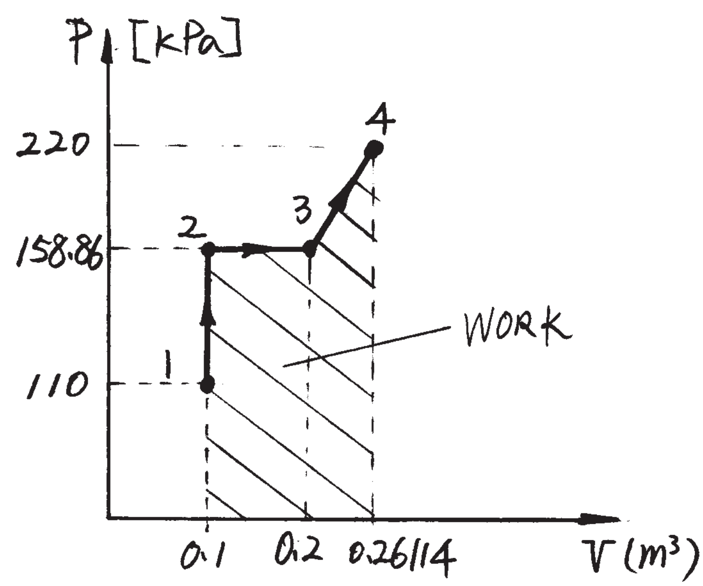
$$= {}_1W_4 + (m_{N_2} C_{v0} + m_{Cu} C_{Cu})(T_4 - T_1)$$

substitute the numbers into this equation,

$$320 = 27.468 + (0.12 \times 0.745 + m_{Cu} \times 0.42) \times (1339.91 - 35.70)$$

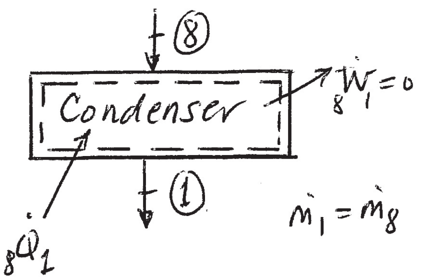
$\therefore m_{Cu} = 0.3212\ (kg)$

(g)



4-1/4

4. (a)



$\dot{m}_1 = \dot{m}_8$

State 8

$P_8 = P_1 = 1.0\,[MPa]$

$T_8 = 40\,[°C]$

$T_{sat}(1.0\,[MPa]) = 39.37\,[°C]$ (Table B.5.2)

$T_8 > T_{sat}(P_8)$

$\Rightarrow$ super heated vapour

$h_8 = 420.25\,\left[\frac{kJ}{kg}\right]$

(Table B.5.2)

State 1

$P_1 = 1.0\,(MPa)$

Saturated liquid

$h_1 = h_f(1.0\,(MPa))$

$h_1 = 255.60\,\left[\frac{kJ}{kg}\right]$

(given)

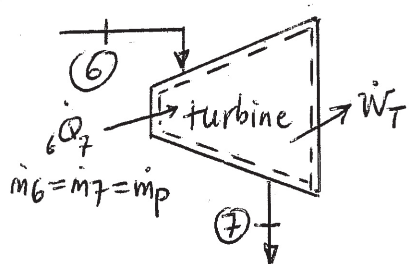
$$_8\dot{Q}_1 = \dot{m}_1(h_1 - h_8)$$

$$_8\dot{Q}_1 = -\dot{Q}_c$$ as per the diagram on the test paper

$$-\dot{Q}_c = \dot{m}_1(h_1 - h_8)$$

$$\dot{m}_1 = \frac{-\dot{Q}_c}{(h_1 - h_8)} = \frac{-122}{(255.60 - 420.25)} = 0.7410\,\left[\frac{kg}{s}\right] \leftarrow$$

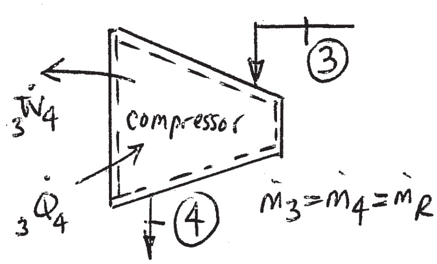
(b)



$\dot{m}_6 = \dot{m}_7 = \dot{m}_p$

$$\cancel{_6\dot{Q}_7}^{\,0} - \dot{W}_T = \dot{m}_p(h_7 - h_6)$$

$$\boxed{-\dot{W}_T = \dot{m}_p(h_7 - h_6)} \quad (1)$$



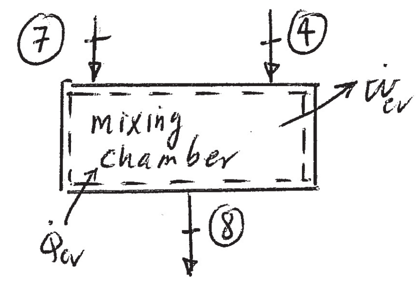
$\dot{m}_3 = \dot{m}_4 = \dot{m}_R$

$$\cancel{_3\dot{Q}_4}^{\,0} - {_3\dot{W}_4} = \dot{m}_R(h_4 - h_3)$$

$$\boxed{-{_3\dot{W}_4} = \dot{m}_R(h_4 - h_3)} \quad (2)$$

Compress input is $-{_3\dot{W}_4}$, which is equal to turbine output, so

$$\boxed{-{_3\dot{W}_4} = \dot{W}_T} \quad (3)$$



$$\cancel{\dot{Q}_{cv}}^{\,0} - \cancel{\dot{W}_{cv}}^{\,0} = \dot{m}_8 h_8 - (\dot{m}_7 h_7 + \dot{m}_4 h_4)$$

$\dot{m}_7 = \dot{m}_p$, $\dot{m}_4 = \dot{m}_R$

$$\boxed{\dot{m}_8 h_8 = \dot{m}_p h_7 + \dot{m}_R h_4} \quad (4)$$

mass conservation

$$\dot{m}_p + \dot{m}_R = \dot{m}_8$$

or $$\boxed{\dot{m}_R = \dot{m}_8 - \dot{m}_p} \quad (5)$$

Combine Equations (1) and (2) using (3):

$$-\dot{m}_p(h_7 - h_6) = \dot{m}_R(h_4 - h_3) \quad (6)$$

$$\Rightarrow \quad -\dot{m}_p h_7 = -\dot{m}_p h_6 + \dot{m}_R(h_4 - h_3) \quad (7)$$

Equation (4) gives

$$\dot{m}_p h_7 = \dot{m}_8 h_8 - \dot{m}_R h_4 \quad (8)$$

Alternative: (all 3 devices at once plus mass conservation)

$$\dot{m}_p h_6 + \dot{m}_R h_3 = \dot{m}_8 h_8$$

$$\dot{m}_p h_6 + (\dot{m}_8 - \dot{m}_p) h_3 = \dot{m}_8 h_8$$

$$\dot{m}_p(h_6 - h_3) = \dot{m}_8(h_8 - h_3)$$

$$\dot{m}_p = \dot{m}_8 \frac{(h_8 - h_3)}{(h_6 - h_3)}$$

4-2/4

Add Equations (7) & (8)

$$0 = \dot{m}_8 h_8 - \dot{m}_P h_6 - \dot{m}_R h_3 \quad (9)$$

Substitute Equation (5) into Equation (9)

$$0 = \dot{m}_8 h_8 - \dot{m}_P h_6 - (\dot{m}_8 - \dot{m}_P) h_3$$

Collect terms and solve for $\dot{m}_P$:

$$\boxed{\dot{m}_P = \dot{m}_8 \frac{(h_8 - h_3)}{(h_6 - h_3)} \quad (10)}$$

State 3

Saturated vapour

$T_3 = -20\ [°C]$

$h_3 = 386.08 \left(\frac{kJ}{kg}\right)$

(Table B.5.1)

State 6

Saturated vapour

$P_6 = 3.0\ [MPa]$

$h_6 = 427.67 \left(\frac{kJ}{kg}\right)$

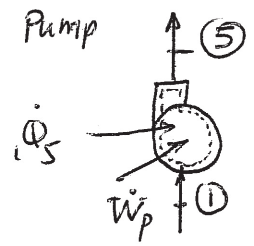
(Table B.5.2)

$\dot{m}_8 = \dot{m}_1$

$$\Rightarrow \dot{m}_P = 0.7410 \frac{(420.25 - 386.08)}{(427.67 - 386.08)} = 0.7410(0.8216)$$

$$\dot{m}_P = 0.6088 \left[\frac{kg}{s}\right] \leftarrow$$

(c) Pump



$\dot{m}_1 = \dot{m}_5 = \dot{m}_P$

$\boxed{P_5 = P_6}$

Assume the pump is adiabatic

$$\cancel{\dot{Q}_5}^{\,0} + \dot{W}_P = \dot{m}_P (h_5 - h_1)$$

$$\dot{W}_P = \dot{m}_P (h_5 - h_1)$$

Alternative treatment of work: Choose $\dot{W}_{cv}$ in and then use $\dot{W}_P = -\dot{W}_{cv}$

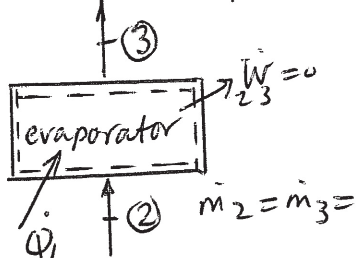
For $T_1 = T_5$ $(h_5 - h_1) = v_f|_{T_1} (P_5 - P_1)$

$$\dot{W}_P = \dot{m}_P\, v_f|_{T_1} (P_5 - P_1)$$

$$v_f|_{T_1} = 0.000871 \left(\frac{m^3}{kg}\right) \text{ (given)}$$

$$\dot{W}_P = 0.6088(0.000871)(3000 - 1000) = 1.061 [kW] \leftarrow$$

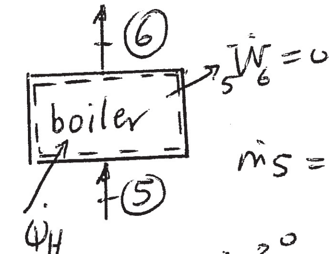
(d)



$\dot{m}_2 = \dot{m}_3 = \dot{m}_R$

$$\dot{Q}_L - \cancel{{}_2\dot{W}_3}^{\,0} = \dot{m}_R (h_3 - h_2)$$

$$\dot{Q}_L = \dot{m}_R (h_3 - h_2)$$



$\dot{m}_5 = \dot{m}_6 = \dot{m}_P$

$$\dot{Q}_H - \cancel{{}_5\dot{W}_6}^{\,0} = \dot{m}_P (h_6 - h_5)$$

$$\dot{Q}_H = \dot{m}_P (h_6 - h_5)$$

4-3/4

$$\frac{\dot{Q}_L}{\dot{Q}_H} = \frac{\dot{m}_R (h_3 - h_2)}{\dot{m}_P (h_6 - h_5)}$$

$\dot{m}_R = \dot{m}_8 - \dot{m}_P$

$\dot{m}_R = 0.7410 - 0.6088$

$\dot{m}_R = 0.1322 \left(\frac{kg}{s}\right)$

State 2

$h_2 = h_1$

$h_2 = 255.60 \left[\frac{kJ}{kg}\right]$

$$\frac{\dot{Q}_L}{\dot{Q}_H} = \frac{0.1322(386.08 - 255.60)}{0.6088(427.67 - 257.35)}$$

State 5

$\dot{W}_P = \dot{m}_P (h_5 - h_1) \Rightarrow h_5 = h_1 + \frac{\dot{W}_P}{\dot{m}_P}$

$h_5 = 255.60 + \frac{1.063}{0.6088} = 257.35 \left[\frac{kJ}{kg}\right]$

$$\frac{\dot{Q}_L}{\dot{Q}_H} = 0.1664 \leftarrow$$

(e) $v_3 = v_g(-20[°C]) = 0.14649 \ [m^3/kg]$ (Table B.5.1)

$v_4 = v(1.0 [MPa], 50[°C]) = 0.02185 \ [m^3/kg]$ (Table B.5.2)

Must define State 7 to get v_7 $\quad P_7 = 1.0 [MPa]$; need h_7

h_7 can be calculated from Equation (8) (or Equation (4))

$$h_7 = \frac{\dot{m}_8}{\dot{m}_P} h_8 - \frac{\dot{m}_R}{\dot{m}_P} h_4$$

State 4

$P_4 = 1.0 \ (MPa)$

$T_4 = 50(°C)$

$h_4 = 431.24 \left(\frac{kJ}{kg}\right)$ (Table B.5.2)

$$h_7 = \left(\frac{0.7410}{0.6088}\right) 420.25 - \left(\frac{0.1322}{0.6088}\right) 431.24$$

$$h_7 = 417.86 \left[\frac{kJ}{kg}\right]$$

At $1.0 [MPa]$ $\quad h_f = 255.60 \left(\frac{kJ}{kg}\right)$ (given)

$h_g = 419.54 \left[\frac{kJ}{kg}\right]$ (Table B.5.2)

$h_f|_{P_7} < h_7 < h_g|_{P_7} \Rightarrow$ saturated mixture

$$x_7 = \frac{(h_7 - h_f)}{(h_g - h_f)} = \frac{(417.86 - 255.60)}{(419.54 - 255.60)} = 0.9898$$

$$v_7 = (1 - x_7) v_f + x_7 v_g$$

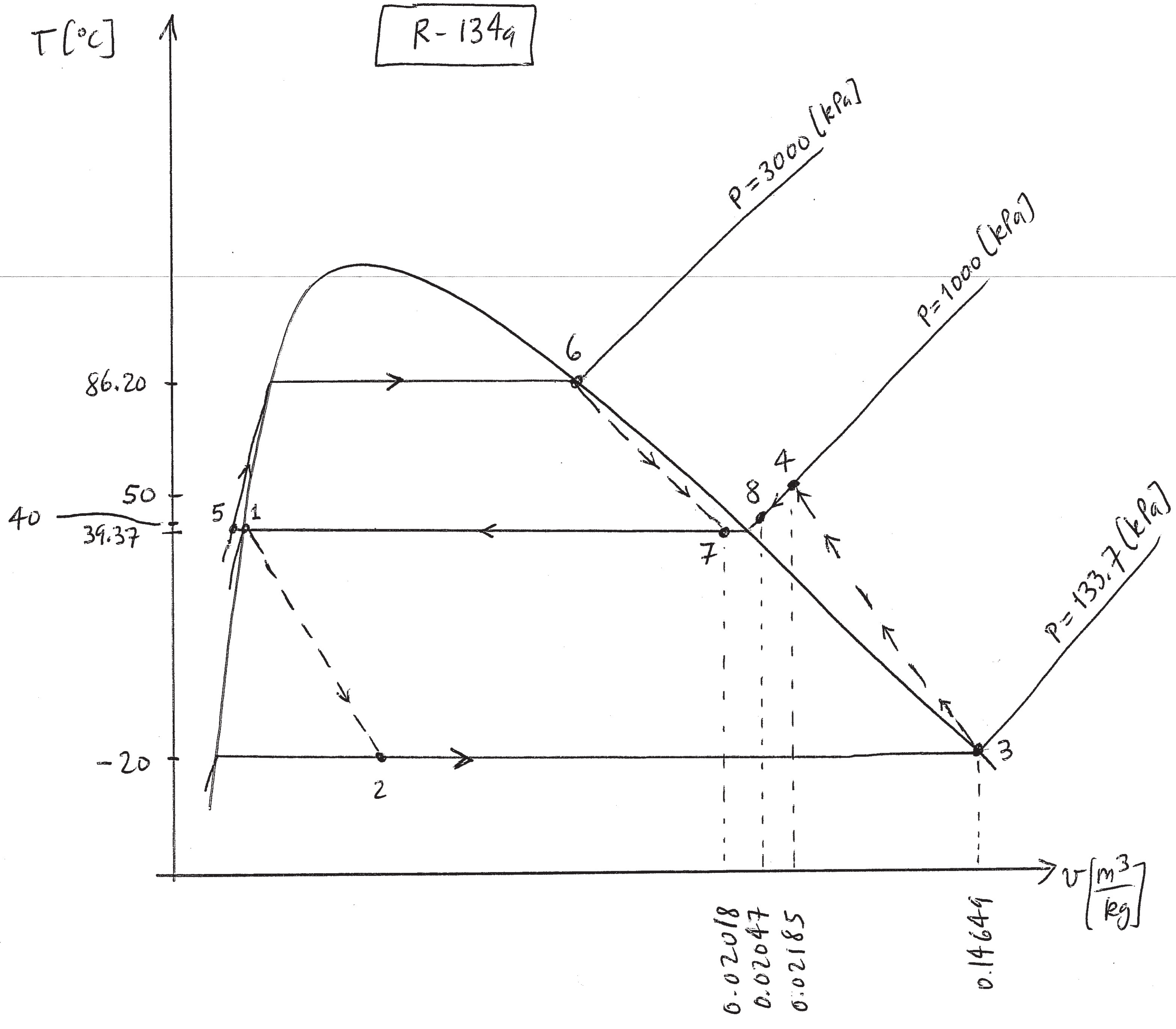
$v_f|_{1 MPa} = v_f|_{39.37°C}$ (given)

$v_7 = (1 - 0.9898)\, 0.000871 + (0.9898)\, 0.02038$ $\quad v_g|_{1 MPa} = 0.02038 \left(\frac{m^3}{kg}\right)$ (Table B.5.2)

$v_7 = 0.02018 \ [m^3/kg]$

$v_8 = v(1.0 [MPa], 40[°C]) = 0.02047 \ (m^3/kg)$ (Table B.5.2)

4-4/4


R-134a
T [°C]
v [m3/kg]
P=3000 [kPa]
P=1000 [kPa]
P=133.7 [kPa]
86.20
50
40
39.37
-20
1
2
3
4
5
6
7
8
0.02018
0.02047
0.02185
0.14649

1/1

PROBLEM: 5

a) $V_{Al\,s} = m_{Al\,s} / \rho_{Al}$ Aluminum

$= 5\ [kg] \times 2700\ [kg/m^3]$

$= 0.001852\ m^3$ ρ TABLE A.3

$$V_{A,1} = \frac{m_A R T_{A,1}}{P_{A,1}}$$ TABLE A.5

$$= \frac{7 \times 0.287 \times (30 + 273.15)}{200}$$

$= 3.04514\ m^3$

$$V_{W,1} = V_T - V_{A,1} - V_s$$

$= 4 - 3.04514 - 0.001852$

$= 0.95301\ m^3$

b) $V_{A,2}$, ${}_1W_{A,2}$ ${}_1Q_{A,2}$

$$V_{A,2} = \frac{P_{A,1} V_{A,1}}{P_{A,2}}$$ Ideal Gas Const. Temp.

$= 1.52257\ m^3$

$${}_1W_{A,2} = P_{A,1} V_{A,1} \ln \frac{V_{A,2}}{V_{A,1}}$$

$$= (200 \times 3.04514) \times \ln \frac{1.52257}{3.04514}$$

$= -422.1460$ kJ

~~${}_1Q_{A,2} =$~~ :

$${}_1Q_{A,2} - {}_1W_{A,2} = m_A C_v (\cancel{T_2 - T_1})^{0,\ T_2 = T_1}$$

$${}_1Q_{A,2} = {}_1W_{A,2} = -422.1460\ kJ$$

2/4

c) $V_{w,2}$, ${}_1W_{w,2}$, ${}_1Q_{w,2}$

$$V_{w,2} = V_T - V_{A,2} - V_s$$
$$= 4 - 1.52257 - 0.001852$$
$$= 2.47558 \ m^3$$

$${}_1W_{w,2} = - {}_1W_{A,2} = 422.1460 \ kJ$$

$C_v = C_p$

$${}_1Q_{w,2} - {}_1W_{w,2} = m_w (u_{w,2} - u_{w,1}) + m_s C_v (T_2 - T_1)$$

STATE 1: @ $P = 200$ kPa , $x = 0.5$, $T_{SAT} = T_{w,1} = 120.23$ °C
$u_f = 504.47$, $u_{fg} = 2025.02$ kJ/kg.

$$u_{w,1} = u_f + x\, u_{fg}$$
$$= 504.47 + 0.5\,(2025.02)$$
$$= 1516.98 \ kJ/kg$$

STATE 2: @ $P = 400$ kPa

$$v_{w,2} = \frac{V_{w,2}}{m_w} = \cancel{\frac{2.4755577}{2.149335}}$$

$$m_w = \frac{V_{w,1}}{v_{w,1}}$$

$v_{w,1}$: $P = 200$ kPa $x = 0.5$
$v_f = 0.001061 \ m^3/kg$
$v_{fg} = 0.88467 \ m^3/kg.$

$$v_{w,1} = v_f + x\, v_{fg}$$
$$= 0.001061 + 0.5\,(0.88467)$$
$$= 0.443396 \ m^3/kg$$

$$m_w = \frac{V_{w,1}}{v_{w,1}}$$
$$= \frac{0.95301 \ m^3}{0.443396 \ m^3/kg}$$
$$= 2.149335 \ kg.$$

3/4

$$v_{w,2} = \frac{V_{w,2}}{m_w}$$

$$= \frac{2.475577}{2.149335}$$

$$= 1.151788 \quad m^3/kg$$

Now: $P_{w,2} = 400$ kPa & $v_{w,2} = 1.151788$ m³/kg

@ $P_{w,2} = 400$ kPa $\quad v_{g\,SAT} \big|_{P=400}$ ~~VAPOR~~ $= 0.46246$ m³/kg TABLE B1.2

$\therefore\ v_g \big|_{P=400} < v_{w,2}\ \therefore$ superheated vapour.

TABLE B.1.3. @ 400 kPa.

v [m³/kg]	T [°C]	u [kJ/kg]
1.12147	700	3477.95
1.151788	~~726.1924~~ $T_{w,2}$	$u_{w,2}$
1.23722	800	3662.51

By Interpolation $u_{w,2} = 3526.291$ kJ/kg. ✓

By INTERPOLATION $T_{w,2} = 726.1924$ °C

$${}_1Q_{w,2} = {}_1W_{w,2} + m_w(u_{w,2} - u_{w,1}) + m_s C (T_2 - T_1)$$

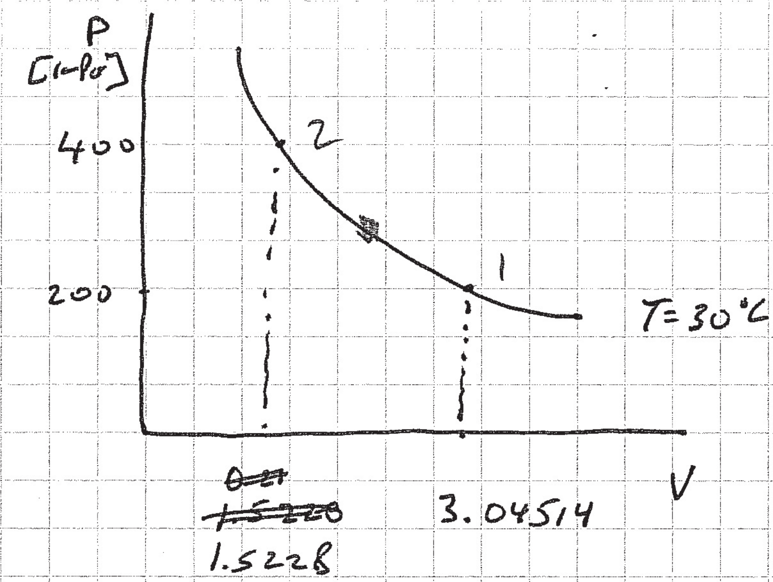
$$= 422.1460 + 2.149335(3526.291 - 1516.98) + 5 \times 0.9 \times (726.1924 - 120.23)$$

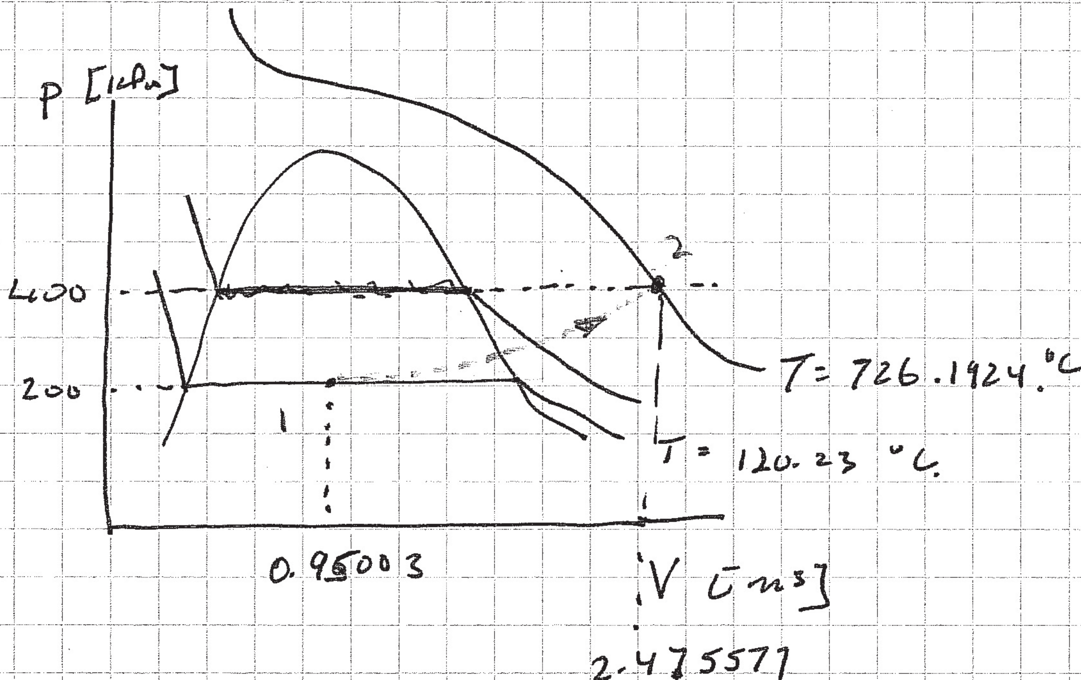
$$= 7467.658 \text{ kJ}$$

4/4.

AIR.

d)


P [kPa]
400
200
2
1
T=30°C
V
~~0.~~
~~1.5220~~
3.04514
1.5228


P [kPa]
400
200
2
1
T= 726.1924 °C
T = 120.23 °C
0.95003
V [m³]
2.475577