

Borgnakke Sonntag

Fundamentals of Thermodynamics

SOLUTION MANUAL CHAPTER 6



8e

Updated June 2013

CONTENT CHAPTER 6

SUBSECTION	PROB NO.
In-Text concept questions	a-j
Concept-Study Guide problems	1-16
Inequality of Clausius	17-23
Entropy of a pure substance	24-34
Reversible processes	35-59
Entropy of a liquid or solid	60-76
Entropy of ideal gases	77-99
Polytropic processes	100-114
Entropy generation	115-161
Rates or fluxes of entropy	162-172
Review	173-189
Problem solution repeated, but using the Pr and vr functions in Table A.7.2: 92, 105, an additional air problem	

The clip-art included in the solution manual is from **Microsoft**.

In-Text Concept Questions

Excerpts from this work may be reproduced by instructors for distribution on a not-for-profit basis for testing or instructional purposes only to students enrolled in courses for which this textbook has been adopted. *Any other reproduction or translation of this work beyond that permitted by Sections 107 or 108 of the 1976 United States Copyright Act without the permission of the copyright owner is unlawful.*

6.a

Does Clausius say anything about the sign for $\oint dQ/T$?

No.

The total (net) heat transfer can be coming in like in a heat engine ($W_{out} = Q_H - Q_L$) in which case it is positive. It can also be net going out like in a refrigerator or heat pump ($W_{in} = Q_H - Q_L$) in which case the sign is negative. Finally if you look at a transmission gearbox there could be no heat transfer (first approximation) in which case the integral is zero.

6.b

Does the statement of Clausius require a constant T for the heat transfer as in a Carnot cycle?

No.

The statement for a cycle involves an integral of dQ/T so T can vary, which it does during most processes in actual devices. This just means that you cannot that easily get a closed expression for the integral.

6.c

How can you change s of a substance going through a reversible process?

From the definition of entropy

$$ds = \frac{dq}{T}$$

for a reversible process. Thus only heat transfer gives a change in s, expansion/compression involving work does not give such a contribution.

6.d

A reversible process adds heat to a substance. If T is varying does that influence the change in s?

Yes.

$$\text{Reversible: } ds = \frac{dq}{T}$$

So if T goes up it means that s changes less per unit of dq, and the opposite if T decreases then s changes more per unit of dq.

6.e

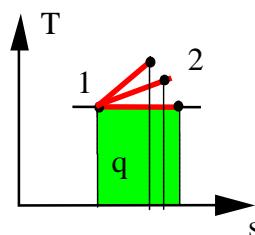
Water at 100 kPa, 150°C receives 75 kJ/kg in a reversible process by heat transfer. Which process changes s the most: constant T, constant v or constant P?

$$ds = \frac{dq}{T}$$

Look at the constant property lines in a T-s diagram, Fig. 6.5. The constant v line has a higher slope than the constant P line also at positive slope. Thus both the constant P and v processes have an increase in T. As T goes up the change in s is smaller for the same area (heat transfer) under the process curve in the T-s diagram as compared with the constant T process.

The constant T (isothermal) process therefore changes s the most.

In a reversible process the area below the process curve in the T-s diagram is the heat transfer.



6.f

A liquid is compressed in a reversible adiabatic process. What is the change in T?

If the process is reversible then s is constant, $ds = \frac{dq}{T} = 0$

Change in s for a liquid (an incompressible substance) is

$$\text{Eq. 6.10: } ds = \frac{C}{T} dT$$

From this it follows that if $ds = 0$ then **T is constant**.

6.g

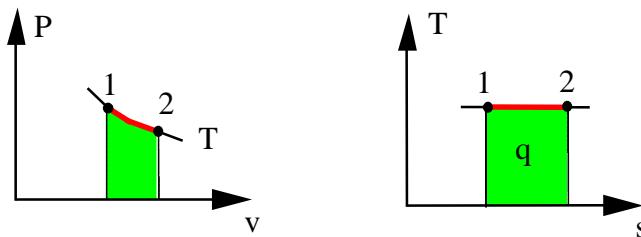
An ideal gas goes through a constant T reversible heat addition process. How do the properties (v , u , h , s , P) change (up, down or constant)?

Ideal gas: $u(T)$, $h(T)$ so they are both constant

Eq. 6.2 gives: $ds = dq/T > 0$ so s goes up by q/T

Eq. 6.12 gives: $ds = (R/v) dv$ so v increases

Eq. 6.14 gives: $ds = -(R/P) dP$ so P decreases



6.h

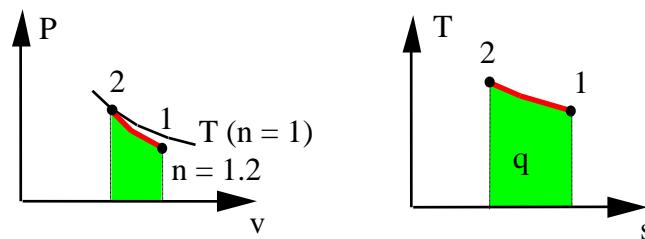
Carbon dioxide is compressed to a smaller volume in a polytropic process with $n = 1.2$. How do the properties (u , h , s , P , T) change (up, down or constant)?

For carbon dioxide Table A.5 $k = 1.289$ so we have $n < k$ and the process curve can be recognized in Figure 8.13. From this we see a smaller volume means moving to the left in the P-v diagram and thus also up.

From P-v diagram: P up, T up

From T-s diagram Since T is up then s down.

As T is up so is h and u .



6.i

A substance has heat transfer out. Can you say anything about changes in s if the process is reversible? If it is irreversible?

$$\text{Reversible: } ds = \frac{dq}{T} < 0 \quad \text{since} \quad dq < 0$$

$$\text{Irreversible: } ds = \frac{dq}{T} + ds_{\text{gen}} = ? \quad dq < 0 \quad \text{but} \quad ds_{\text{gen}} > 0$$

You cannot say, ds depends on the magnitude of dq/T versus ds_{gen}

6.j

A substance is compressed adiabatically so P and T go up. Does that change s?

$$\text{If the process is reversible then } s \text{ is constant, } ds = \frac{dq}{T} = 0$$

$$\text{If the process is irreversible then } s \text{ goes up, } ds = \frac{dq}{T} + ds_{\text{gen}} = ds_{\text{gen}} > 0$$

Borgnakke and Sonntag

Concept Problems

Excerpts from this work may be reproduced by instructors for distribution on a not-for-profit basis for testing or instructional purposes only to students enrolled in courses for which this textbook has been adopted. *Any other reproduction or translation of this work beyond that permitted by Sections 107 or 108 of the 1976 United States Copyright Act without the permission of the copyright owner is unlawful.*

6.1

When a substance has completed a cycle, v, u, h, and s are unchanged. Did anything happen? Explain.

Yes.

During various parts of the cycle work and heat transfer may be transferred. That happens at different P and T. The net work out equals the net heat transfer in (energy conservation) so dependent upon the sign it is a heat engine or a heat pump (refrigerator). The net effect is thus a conversion of energy from one storage location to another and it may also change nature, some Q was changed to W or the opposite.

6.2

Assume a heat engine with a given Q_H . Can you say anything about Q_L if the engine is reversible? If it is irreversible?

For a reversible heat engine it must be that:

$$\oint \frac{dQ}{T} = 0 = \frac{Q_H}{T_H} - \frac{Q_L}{T_L} \quad \text{or integrals if } T \text{ not constant}$$

So as T_L is lower than T_H then Q_L must be correspondingly lower than Q_H to obtain the net zero integral.

For an irreversible heat engine we have

$$\oint \frac{dQ}{T} = \frac{Q_H}{T_H} - \frac{Q_L}{T_L} < 0$$

This means that Q_L is larger than before (given Q_H and the T 's). The irreversible heat engine rejects more energy and thus gives less out as work.

6.3

CV A is the mass inside a piston/cylinder, CV B is that plus part of the wall out to a source of ${}_1Q_2$ at T_s . Write the entropy equation for the two control volumes assuming no change of state of the piston mass or walls.

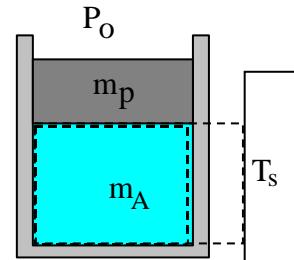


Fig. P6.3

The general entropy equation for a control mass is Eq.6.37

$$S_2 - S_1 = \int_1^2 \frac{dQ}{T} + {}_1S_{2\text{ gen}}$$

The left hand side is storage so that depends of what is inside the C.V. and the integral is summing the dQ/T that crosses the control volume surface while the process proceeds from 1 to 2.

$$\text{C.V. A: } m_A (s_2 - s_1) = \int_1^2 \frac{dQ}{T_A} + {}_1S_{2\text{ gen CV A}}$$

$$\text{C.V. B: } m_A (s_2 - s_1) = \int_1^2 \frac{dQ}{T_s} + {}_1S_{2\text{ gen CV B}}$$

In the first equation the temperature is that of mass m_A which possibly changes from 1 to 2 whereas in the second equation it is the reservoir temperature T_s . The two entropy generation terms are also different the second one includes the first one plus any s generated in the walls that separate the mass m_A from the reservoir and there is a Q over a finite temperature difference. When the storage effect in the walls are neglected the left hand sides of the two equations are equal.

6.4

Consider the previous set-up with the mass m_A and the piston cylinder of mass m_p starting out at two different temperatures. After a while the temperature becomes uniform without any external heat transfer. Write the entropy equation storage term ($S_2 - S_1$) for the total mass.

In this case the storage effect must be summed up over all the mass inside the control volume.

$$\begin{aligned} S_2 - S_1 &= m_A (s_2 - s_1)_A + m_p (s_2 - s_1)_P \\ &\approx m_A (s_2 - s_1)_A + m_p C_p \ln \left(\frac{T_2}{T_{1P}} \right) \end{aligned}$$

The last equation assumed a constant specific heat for the solid material of the piston, a common assumption. There is only a single temperature T_2 , but there are two different temperatures for state 1: T_{1A} and T_{1P} . The temperature T_2 would be found from the energy equation.

6.5

Water at 100°C , quality 50% in a rigid box is heated to 110°C . How do the properties (P , v , x , u and s) change? (increase, stay about the same, or decrease)

A fixed mass in a rigid box give a constant v process. So

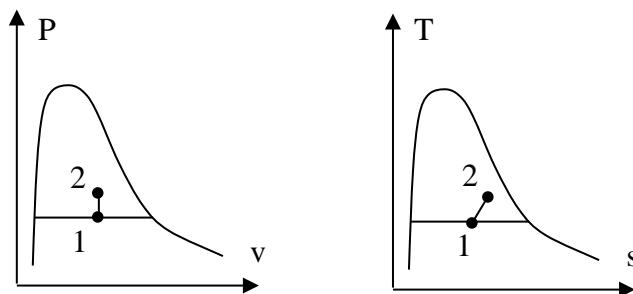
P goes up (in the two-phase region $P = P_{\text{sat}}$ at given T)

v stays constant.

x goes up (we get closer to the saturated vapor state see P - v diagram)

u goes up (Q in and no work)

s goes up (Q in)



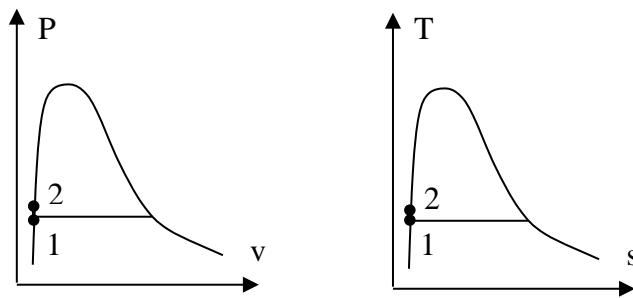
6.6

Liquid water at 20°C , 100 kPa is compressed in a piston/cylinder without any heat transfer to a pressure of 200 kPa. How do the properties (T , v , u , and s) change?
(increase, stay about the same, or decrease)

$$\begin{array}{ll} \text{Adiabatic } dq = 0: & dq = T ds = 0; \\ \text{Incompressible } dv = 0: & dw = P dv = 0 \end{array}$$

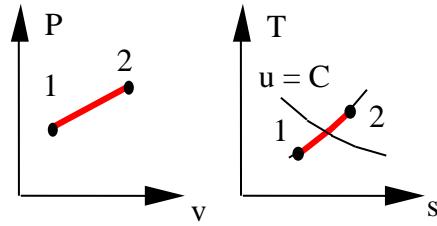
(T , v , u , and s) they are all constant.
Only the pressure and enthalpy goes up.

In the T - s diagram the two states are in the same location as T does not go up for v constant.



6.7

A reversible process in a piston/cylinder is shown in Fig. P6.7. Indicate the storage change $u_2 - u_1$ and transfers $_{1w_2}$ and $_{1q_2}$ as positive, zero, or negative

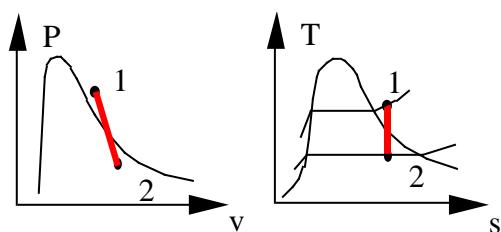


$$_{1w_2} = \int P dv > 0 ; \quad _{1q_2} = \int T ds > 0$$

$u_2 - u_1 > 0$ from general shape of the constant u curves. Further out in the ideal gas region the constant u curve become horizontal ($u = fct(T)$ only).

6.8

A reversible process in a piston/cylinder is shown in Fig. P6.8. Indicate the storage change $u_2 - u_1$ and transfers w_2 and q_2 as positive, zero, or negative



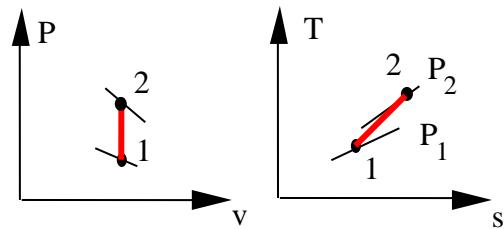
$$w_2 = \int P \, dv > 0 ; \quad q_2 = \int T \, ds = 0$$

$$u_2 - u_1 = q_2 - w_2 < 0$$

6.9

Air at 290 K, 100 kPa in a rigid box is heated to 325 K. How do the properties (P , v , u and s) change? (increase, stay about the same, or decrease)

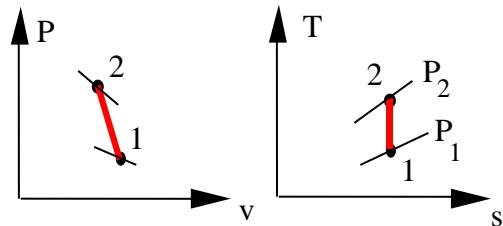
Rigid box: $v = \text{constant}$, $(P, u, \text{ and } s)$ all increases.



6.10

Air at 20°C , 100 kPa is compressed in a piston/cylinder without any heat transfer to a pressure of 200 kPa. How do the properties (T , v , u and s) change? (increase, about the same or decrease)

T goes up,
 v goes down
 u goes up (work in, $q = 0$)
 s = constant



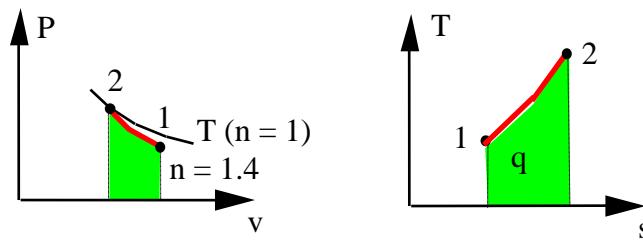
6.11

Carbon dioxide is compressed to a smaller volume in a polytropic process with $n = 1.4$. How do the properties (u , h , s , P , T) change (up, down or constant)?

For carbon dioxide Table A.5 $k = 1.289$ so we have $n > k$ and the process curve can be recognized in Figure 6.13. From this we see a smaller volume means moving to the left in the P-v diagram and thus also up.

P up, T up. As T is up so is h and u .

From the T-s diagram as $n > k$ then we move to larger T means s is up.



6.12

Process A: Air at 300 K, 100 kPa is heated to 310 K at constant pressure.
 Process B: Heat air at 1300 K to 1310 K at constant 100 kPa.
 Use the table below to compare the property changes.

	Property	$\Delta_A > \Delta_B$	$\Delta_A \approx \Delta_B$	$\Delta_A < \Delta_B$
a	$\Delta = v_2 - v_1$		✓	
b	$\Delta = h_2 - h_1$			✓
c	$\Delta = s_2 - s_1$	✓		

- a. Ideal gas $Pv = RT$ so v goes with absolute T
 $\Delta v = (R/P) \Delta T$ thus the same
- b. Since $dh = C_P dT$ and C_P increases with T
- c. At constant P: $ds = (C_P/T) dT$
 C_P is only 15% higher at 1300 K compared to 300 K (see Fig. 3.11)

6.13

Why do we write ΔS or $S_2 - S_1$ whereas we write $\int dQ/T$ and ${}_1S_{2\text{ gen}}$?

This is similar to the terms in the continuity equation $m_2 - m_1$ versus m_i, m_e and the energy equation $E_2 - E_1$ versus ${}_1Q_2, {}_1W_2$. The first part is the **change in the storage** $S_2 - S_1$ of entropy in the mass and the second part is the transfer or generation during the process from 1 to 2. The storage terms correspond to the left hand side of the balance equation for entropy. The integral $\int dQ/T$ represents a **transfer** of entropy **across the control volume surface** during the process from 1 to 2 and the ${}_1S_{2\text{ gen}}$ expresses the total amount of entropy being generated **inside** the control volume and both are on the right hand side of the balance equation.

6.14

A reversible heat pump has a flux of s entering as \dot{Q}_L/T_L . What can you say about the exit flux of s at T_H ?

For the entropy equation 6.3 and 6.42, the rate of storage is zero and the generation term is zero. Thus the entropy equation becomes

$$0 = \frac{\dot{Q}_L}{T_L} - \frac{\dot{Q}_H}{T_H} + 0$$

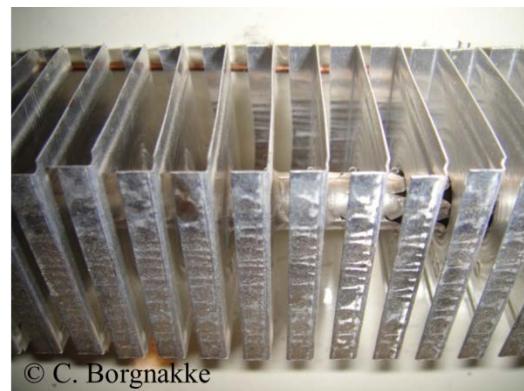
$$\text{So: } \frac{\dot{Q}_L}{T_L} = \frac{\dot{Q}_H}{T_H} = \text{flux of } s$$

We have the same flux of s in as out matching the result in chapter 5.

6.15

An electric baseboard heater receives 1500 W of electrical power that heats the room air which loses the same amount out through the walls and windows. Specify exactly where entropy is generated in that process.

- a Electrical heating wire (electrical work turned into internal energy,
leaving as heat transfer).
- b Heat transfer from hot wire to cooler room air, i.e. in the wire coverings
- c Room air to walls
- d Inside walls and windows, heat transfer over a finite ΔT
- e from outside wall surface to ambient T



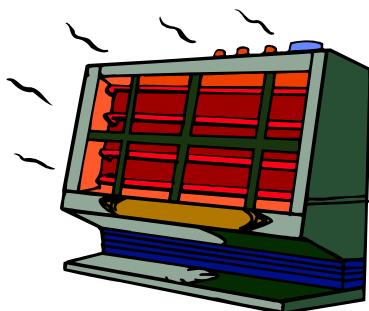
The electric wire is inside the pipe and surrounded by sand to electrically isolate it. The pipe has fins mounted to increase the surface area.

6.16

A 500 W electric space heater with a small fan inside heats air by blowing it over a hot electrical wire. For each control volume: a) wire at T_{wire} only b) all the room air at T_{room} and c) total room plus the heater, specify the storage, entropy transfer terms and entropy generation as rates (neglect any \dot{Q} through the room walls or windows).

	Storage	\dot{Q} / T	\dot{S}_{gen}
C.V. Wire only	0	$\frac{-500 \text{ W}}{T_{\text{wire}}}$	$\frac{500 \text{ W}}{T_{\text{wire}}}$
C.V. Room air	$\frac{500 \text{ W}}{T_{\text{room}}}$	$\frac{500 \text{ W}}{T_{\text{wire}}}$	$500 \text{ W} \left(\frac{1}{T_{\text{room}}} - \frac{1}{T_{\text{wire}}} \right)$
C.V. Total room	$\frac{500 \text{ W}}{T_{\text{room}}}$	0	$\frac{500 \text{ W}}{T_{\text{room}}}$

Remark: Room only receives the electrical power input of 500 W.



Some of the heaters can be radiant heaters in which case the fan is not needed.

Inequality of Clausius

Excerpts from this work may be reproduced by instructors for distribution on a not-for-profit basis for testing or instructional purposes only to students enrolled in courses for which this textbook has been adopted. *Any other reproduction or translation of this work beyond that permitted by Sections 107 or 108 of the 1976 United States Copyright Act without the permission of the copyright owner is unlawful.*

6.17

Consider the steam power plant in Example 4.7 and assume an average T in the line between 1 and 2. Show that this cycle satisfies the inequality of Clausius.

Solution:

$$\text{Show Clausius: } \int \frac{dQ}{T} \leq 0$$

For this problem we have three heat transfer terms:

$$q_b = 2831 \text{ kJ/kg}, \quad q_{\text{loss}} = 21 \text{ kJ/kg}, \quad q_c = 2173.3 \text{ kJ/kg}$$

$$\begin{aligned} \int \frac{dq}{T} &= \frac{q_b}{T_b} - \frac{q_{\text{loss}}}{T_{\text{avg } 1-2}} - \frac{q_c}{T_c} \\ &= \frac{2831}{573} - \frac{21}{568} - \frac{2173.3}{318} \\ &= -1.93 \text{ kJ/kg K} < 0 \quad \mathbf{OK} \end{aligned}$$

6.18

A heat engine receives 6 kW from a 250°C source and rejects heat at 30°C. Examine each of three cases with respect to the inequality of Clausius.

- a. $\dot{W} = 6 \text{ kW}$ b. $\dot{W} = 0 \text{ kW}$ c. Carnot cycle

Solution:

$$T_H = 250 + 273 = 523 \text{ K}; \quad T_L = 30 + 273 = 303 \text{ K}$$

Case a)
$$\int \frac{d\dot{Q}}{T} = \frac{6000}{523} - \frac{0}{303} = 11.47 \text{ kW/K} > 0 \quad \textbf{Impossible}$$

b)
$$\int \frac{d\dot{Q}}{T} = \frac{6000}{523} - \frac{6000}{303} = -8.33 \text{ kW/K} < 0 \quad \textbf{OK}$$

c)
$$\int \frac{d\dot{Q}}{T} = 0 = \frac{6000}{523} - \frac{\dot{Q}_L}{303} \quad \Rightarrow$$

$$\dot{Q}_L = \frac{303}{523} \times 6 \text{ kW} = 3.476 \text{ kW}$$

$$\dot{W} = \dot{Q}_H - \dot{Q}_L = 2.529 \text{ kW}$$

6.19

Use the inequality of Clausius to show that heat transfer from a warm space towards a colder space without work is a possible process i.e. a heat engine with no work output.

$$\text{Clausius: } \int \frac{dQ}{T} \leq 0 \quad \text{or} \quad \int \frac{d\dot{Q}}{T} \leq 0$$

Take C.V. as the space separating the warm and cold space. It is the same Q that crosses each surface (from energy equation) so

$$\int \frac{dQ}{T} = \frac{Q}{T_{\text{warm}}} - \frac{Q}{T_{\text{cold}}} = Q \left(\frac{1}{T_{\text{warm}}} - \frac{1}{T_{\text{cold}}} \right) < 0 \quad \text{OK}$$

6.20

Use the inequality of Clausius to show that heat transfer from a cold space towards a warmer space without work is an impossible process i.e. a heat pump with no work input.

$$\text{Clausius: } \int \frac{dQ}{T} \leq 0 \quad \text{or} \quad \int \frac{d\dot{Q}}{T} \leq 0$$

Take C.V. as the space separating the warm and cold space. It is the same Q that crosses each surface (from energy equation) so

$$\int \frac{dQ}{T} = \frac{Q}{T_{\text{cold}}} - \frac{Q}{T_{\text{warm}}} = Q \left(\frac{1}{T_{\text{cold}}} - \frac{1}{T_{\text{warm}}} \right) > 0 \quad \text{Impossible!}$$

6.21

Assume the heat engine in Problem 5.34 has a high temperature of 1000 K and a low temperature of 400 K. What does the inequality of Clausius say about each of the four cases?

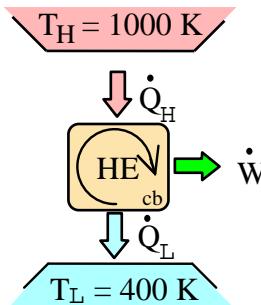
Solution:

$$\text{Cases a)} \int \frac{d\dot{Q}}{T} = \frac{6}{1000} - \frac{4}{400} = -0.004 \text{ kW/K} < 0 \quad \text{OK}$$

$$\text{b)} \int \frac{d\dot{Q}}{T} = \frac{6}{1000} - \frac{0}{400} = 0.006 \text{ kW/K} > 0 \quad \text{Impossible}$$

$$\text{c)} \int \frac{d\dot{Q}}{T} = \frac{6}{1000} - \frac{2}{400} = 0.001 \text{ kW/K} > 0 \quad \text{Impossible}$$

$$\text{d)} \int \frac{d\dot{Q}}{T} = \frac{6}{1000} - \frac{6}{400} = -0.009 \text{ kW/K} < 0 \quad \text{OK}$$



6.22

Let the steam power plant in Problem 5.30 have 700°C in the boiler and 40°C during the heat rejection in the condenser. Does that satisfy the inequality of Clausius? Repeat the question for the cycle operated in reverse as a refrigerator.

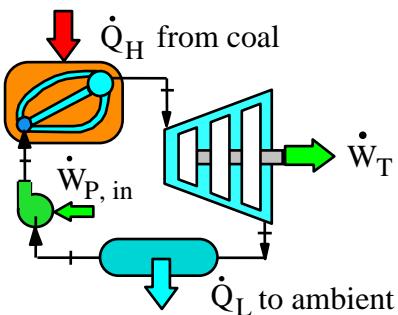
Solution:

$$\dot{Q}_H = 1 \text{ MW} \quad \dot{Q}_L = 0.58 \text{ MW}$$

$$\int \frac{d\dot{Q}}{T} = \frac{1000}{973} - \frac{580}{313} = -0.82 \text{ kW/K} < 0 \quad \text{OK}$$

Refrigerator

$$\int \frac{d\dot{Q}}{T} = \frac{580}{313} - \frac{1000}{973} = 0.82 > 0 \quad \text{Cannot be possible}$$



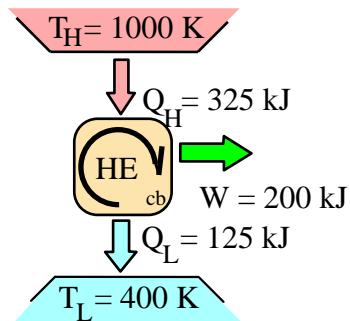
6.23

Examine the heat engine given in Problem 5.74 to see if it satisfies the inequality of Clausius.

Solution:

$$\begin{aligned} Q_H &= 325 \text{ kJ} \quad \text{at} \quad T_H = 1000 \text{ K} \\ Q_L &= 125 \text{ kJ} \quad \text{at} \quad T_L = 400 \text{ K} \end{aligned}$$

$$\int \frac{dQ}{T} = \frac{325}{1000} - \frac{125}{400} = 0.0125 \text{ kJ/K} > 0 \quad \text{Impossible}$$



Entropy of a pure substance

Excerpts from this work may be reproduced by instructors for distribution on a not-for-profit basis for testing or instructional purposes only to students enrolled in courses for which this textbook has been adopted. *Any other reproduction or translation of this work beyond that permitted by Sections 107 or 108 of the 1976 United States Copyright Act without the permission of the copyright owner is unlawful.*

6.24

Determine the entropy for these states:

- Nitrogen, $P = 2000 \text{ kPa}$, 120 K
- Nitrogen, 120 K , $v = 0.005 \text{ m}^3/\text{kg}$
- R-410A, $T = 25^\circ\text{C}$, $v = 0.01 \text{ m}^3/\text{kg}$

Solution:

- a) Table B.6.1: $P < P_{\text{sat}} = 2513 \text{ kPa}$ so superheated vapor.

$$\text{B.6.2 at } 2000 \text{ kPa: } s = \mathbf{4.8116 \text{ kJ/kg-K}}$$

- b) Table B.6.1: $v_f < v < v_g = 0.00799 \text{ m}^3/\text{kg}$ so two-phase L + V

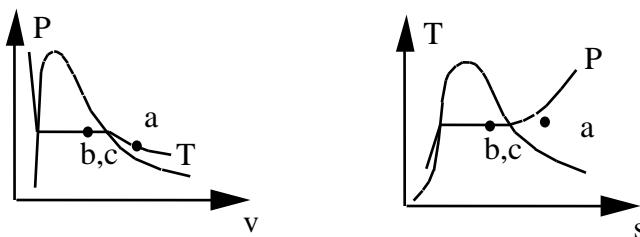
$$x = (v - v_f)/v_{fg} = \frac{0.005 - 0.001915}{0.00608} = 0.5074$$

$$s = s_f + x s_{fg} = 3.8536 + x 0.7659 = \mathbf{4.2422 \text{ kJ/kg K}}$$

- c) Table B.4.1 $v_f < v < v_g = 0.01514 \text{ m}^3/\text{kg}$ so two-phase L + V

$$x = (v - v_f)/v_{fg} = \frac{0.01 - 0.000944}{0.01420} = 0.63775$$

$$s = s_f + x s_{fg} = 0.3631 + x 0.6253 = \mathbf{0.7619 \text{ kJ/kg K}}$$



6.25

Determine the missing property among P, T, s, and x for R-410A at:

- a. $T = -20^\circ\text{C}$, $v = 0.1377 \text{ m}^3/\text{kg}$
- b. $T = 20^\circ\text{C}$, $v = 0.01377 \text{ m}^3/\text{kg}$
- c. $P = 200 \text{ kPa}$, $s = 1.409 \text{ kJ/kgK}$

a) B.4.1: $v > v_g = 0.0648 \text{ m}^3/\text{kg} \Rightarrow$

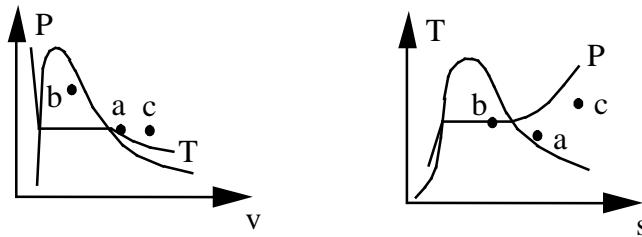
B.4.2 superheated vapor so x is undefined
very close to 200 kPa, $s = 1.1783 \text{ kJ/kg-K}$

b) B.4.1: $0.000923 = v_f < v < v_g = 0.01758 \text{ m}^3/\text{kg} \Rightarrow$ Two-phase
 $P = P_{\text{sat}} = 1444.2 \text{ kPa}$

$$x = (v - v_f)/v_{fg} = \frac{0.01377 - 0.000923}{0.01666} = 0.77113$$

$$s = s_f + x s_{fg} = 0.3357 + 0.77113 \times 0.6627 = 0.8467 \text{ kJ/kg-K}$$

c) Table B.4.2 at 200 kPa, $s > s_g$ so superheated vapor,
x is undefined, and we find the state at $T = 60^\circ\text{C}$.



6.26

Find the missing properties of P , v , s and x for ammonia (NH_3) at

- a. $T = 65^\circ\text{C}$, $P = 600 \text{ kPa}$
- b. $T = 20^\circ\text{C}$, $u = 800 \text{ kJ/kg}$
- c. $T = 50^\circ\text{C}$, $v = 0.1185 \text{ m}^3/\text{kg}$

- a) B.2.2 average between 60°C and 70°C

$$v = (0.25981 + 0.26999)/2 = 0.26435 \text{ m}^3/\text{kg}$$

$$s = (5.6383 + 5.7094)/2 = 5.6739 \text{ kJ/kgK}$$

- b) B.2.1: $u < u_g = 1332.2 \text{ kJ/kg} \Rightarrow P = P_{\text{sat}} = 857.5 \text{ kPa}$

$$x = (u - u_f)/u_{fg} = \frac{800 - 272.89}{1059.3} = 0.49666$$

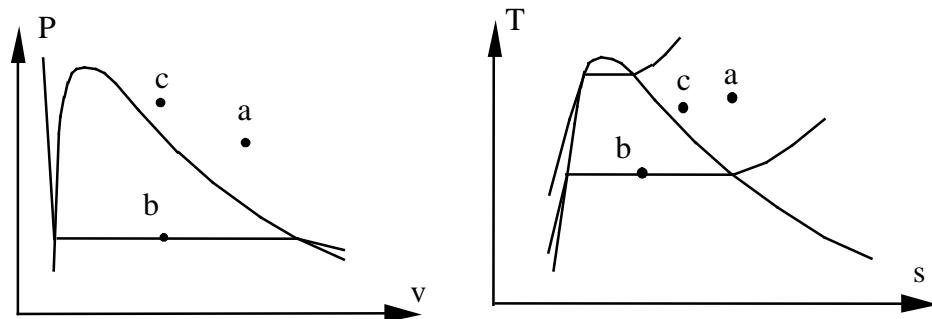
$$v = 0.001638 + x \times 0.14758 = 0.07494 \text{ m}^3/\text{kg},$$

$$s = 1.0408 + x \times 4.0452 = 3.04989 \text{ kJ/kg-K}$$

- c) B.2.1: $v > v_g = 0.06337 \text{ m}^3/\text{kg} \Rightarrow$

B.2.2 superheated vapor so x is undefined

very close to 1200 kPa, $s = 5.1497 \text{ kJ/kgK}$



6.27

Find the entropy for the following water states and indicate each state on a T-s diagram relative to the two-phase region.

a. 250°C , $v = 0.02 \text{ m}^3/\text{kg}$

b. 250°C , 2000 kPa

c. -2°C , 100 kPa

Solution:

a) Table B.1.1: $0.001251 = v_f < v < v_g = 0.05013 \text{ m}^3/\text{kg} \Rightarrow$ Two-phase

$$x = \frac{0.02 - 0.001251}{0.04887} = 0.38365$$

$$s = s_f + x s_{fg} = 2.7927 + 0.38365 \times 3.2802 = \mathbf{4.05 \text{ kJ/kg K}}$$

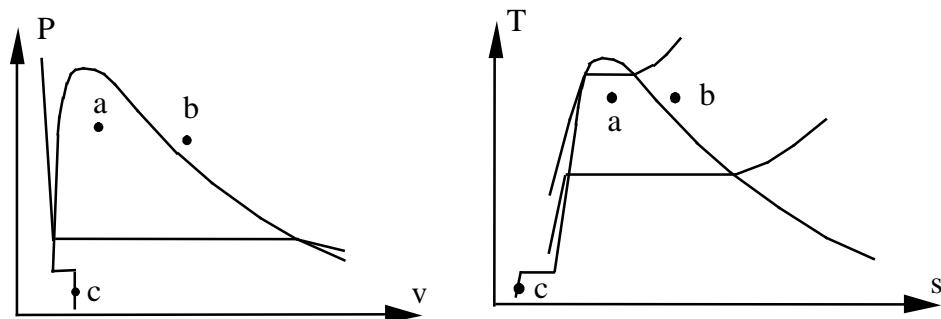
b) Table B.1.1: $P < P_{\text{sat}} = 3973 \text{ kPa} \Rightarrow$ superheated vapor B.1.3

$$s = \mathbf{6.5452 \text{ kJ/kg K}}$$

c) Table B.1.1 $T < T_{\text{triple}} = 0.01^{\circ}\text{C}$ so goto B.1.5

Table B.1.5: $P > P_{\text{sat}} = 0.5177 \text{ kPa}$ so compressed solid

$$s = \mathbf{-1.2369 \text{ kJ/kg K}}$$



6.28

Determine the missing property among P, v, s, and x for CO₂ and indicate each state on a T-s diagram relative to the two-phase region.

- a. $T = -20^\circ\text{C}$, $P = 2000 \text{ kPa}$
- b. $T = 20^\circ\text{C}$, $s = 1.49 \text{ kJ/kgK}$
- c. $T = -10^\circ\text{C}$, $s = 1 \text{ kJ/kgK}$

a) Table B.3.1 at -20°C : $P > P_{\text{sat}} = 1969.6 \text{ kPa} \Rightarrow$ Compressed liquid

$$v = v_f = 0.000969 \text{ m}^3/\text{kg}$$

$$s = s_f = 0.1672 \text{ kJ/kg-K}, \quad x \text{ is undefined}$$

b) Table B.3.1: $s > s_g = 1.0406 \text{ kJ/kg-K} \Rightarrow$ superheated vapor

Table B.3.2 located between 1400 kPa and 2000 kPa

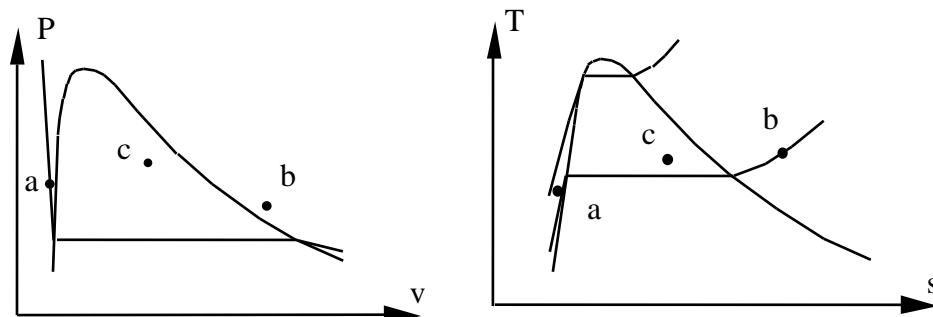
$$P = 1400 + 600 \frac{1.49 - 1.5283}{1.4438 - 1.5283} = 1400 + 600 \times 0.4532 = 1672 \text{ kPa}$$

$$v = 0.03648 + (0.02453 - 0.03648) 0.4532 = 0.03106 \text{ m}^3/\text{kg}$$

c) Table B.3.1: $0.2501 = s_f < s < s_g = 1.2328 \text{ kJ/kgK} \Rightarrow$ Two-phase

$$x = \frac{s - s_f}{s_{fg}} = \frac{1 - 0.2501}{0.9828} = 0.763; \quad P = P_{\text{sat}} = 2648.7 \text{ kPa}$$

$$v = v_f + x v_{fg} = 0.001017 + 0.763 \times 0.01303 = 0.01096 \text{ m}^3/\text{kg}$$



6.29

Determine the missing property among (T, P, v, s)

- a. H₂O 20°C, v = 0.01000 m³/kg
- b. R-410A 400 kPa, s = 1.17 kJ/kgK
- c. NH₃ 10°C, v = 0.1 m³/kg
- d. N₂ 101.3 kPa, s = 3.5 kJ/kg-K

a) Table B.1.1 at 20°C: v < v_f = 0.001002 m³/kg => Compr. Liquid

B.1.4: at about **5000 kPa**, s = **0.2955 kJ/kg-K**

b) Table B.4.1: s > s_g = 1.0779 kJ/kg-K => Sup. vapor

$$\text{Table B.4.2: } T = 0 + 20 \times \frac{1.17 - 1.1483}{1.2108 - 1.1483} = 20 \times 0.3472 = \mathbf{6.94^\circ C}$$

$$v = 0.07227 + (0.07916 - 0.07227) 0.3472 = \mathbf{0.07466 \text{ m}^3/\text{kg}}$$

c) Table B.2.1: v_f < v < v_g = 0.20541 so two-phase L + V, P = **615.2 kPa**

$$x = \frac{v - v_f}{v_{fg}} = \frac{0.1 - 0.0016}{0.20381} = 0.4828$$

$$s = s_f + x s_{fg} = 0.8778 + x \times 4.3266 = \mathbf{2.9667 \text{ kJ/kg-K}}$$

d) Table B.6.1: s_f < s < s_g = 5.4033 kJ/kgK so two-phase, T = **77.3 K**

$$x = \frac{s - s_f}{s_{fg}} = \frac{3.5 - 2.8326}{2.5707} = 0.25962$$

$$v = v_f + x v_{fg} = 0.00124 + x \times 0.21515 = 0.0571 \text{ m}^3/\text{kg}$$

6.30

Two kg water at 120°C with a quality of 25% has its temperature raised 20°C in a constant volume process. What are the new quality and specific entropy?

Solution:

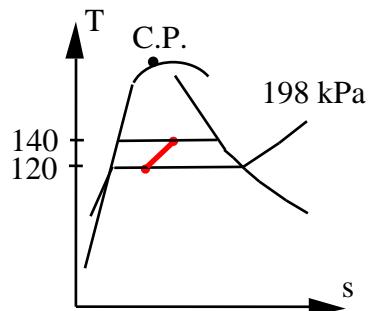
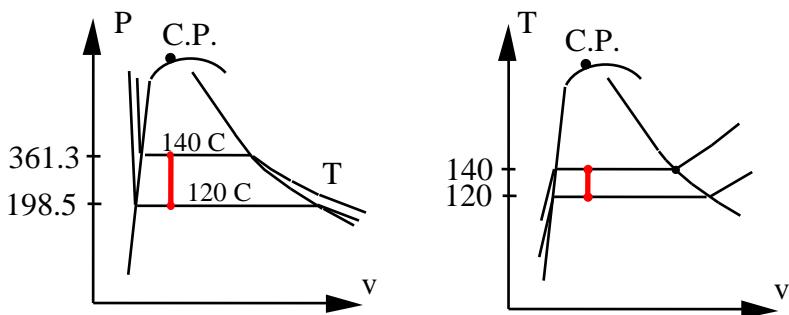
State 1 from Table B.1.1 at 120°C

$$v = v_f + x v_{fg} = 0.001060 + 0.25 \times 0.8908 = 0.22376 \text{ m}^3/\text{kg}$$

State 2 has same v at 140°C also from Table B.1.1

$$x = \frac{v - v_f}{v_{fg}} = \frac{0.22376 - 0.00108}{0.50777} = \mathbf{0.4385}$$

$$s = s_f + x s_{fg} = 1.739 + 0.4385 \times 5.1908 = \mathbf{4.015 \text{ kJ/kg K}}$$



6.31

Two kg water at 400 kPa with a quality of 25% has its temperature raised 20°C in a constant pressure process. What is the change in entropy?

Solution:

State 1 from Table B.1.2 at 400 kPa

$$s = s_f + x s_{fg} = 1.7766 + 0.25 \times 5.1193 = 3.0564 \text{ kJ/kg}$$

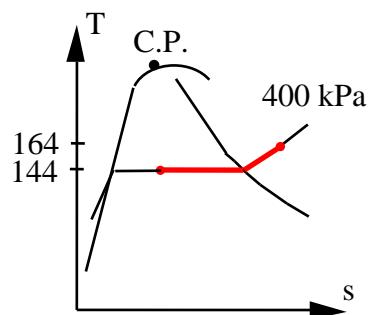
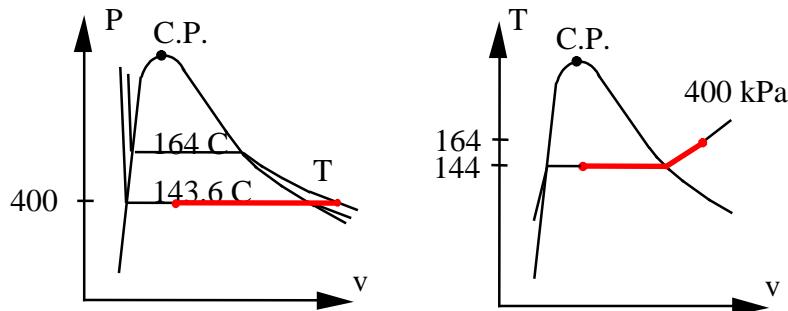
State 2 has same P from Table B.1.2 at 400 kPa

$$T_2 = T_{\text{sat}} + 20 = 143.63 + 20 = 163.63^\circ\text{C}$$

so state is superheated vapor look in B.1.3 and interpolate between 150 and 200 C in the 400 kPa superheated vapor table.

$$s_2 = 6.9299 + (7.1706 - 6.9299) \frac{163.63 - 150}{200 - 150} = 6.9955 \text{ kJ/kgK}$$

$$s_2 - s_1 = 6.9955 - 3.0564 = \mathbf{3.9391 \text{ kJ/kgK}}$$



6.32

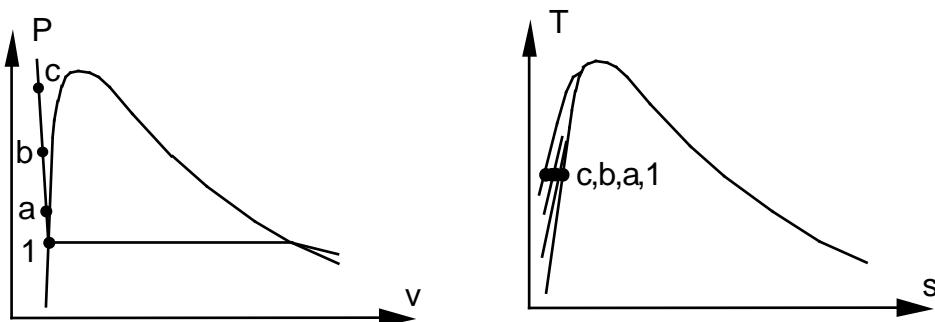
Saturated liquid water at 20°C is compressed to a higher pressure with constant temperature. Find the changes in u and s when the final pressure is

- a. 500 kPa b. 2000 kPa c. 20 000 kPa

Solution:

	kJ/kg	kJ/kg K		
B.1.1:	$u_1 = 83.94$	$s_1 = 0.2966$		
B.1.4:	$u_a = 83.91$	$s_a = 0.2965$	$\Delta u = -0.03$	$\Delta s = -0.0001$
B.1.4:	$u_b = 83.82$	$s_b = 0.2962$	$\Delta u = -0.12$	$\Delta s = -0.0004$
B.1.4:	$u_c = 82.75$	$s_c = 0.2922$	$\Delta u = -1.19$	$\Delta s = -0.0044$

Nearly constant u and s , incompressible media



6.33

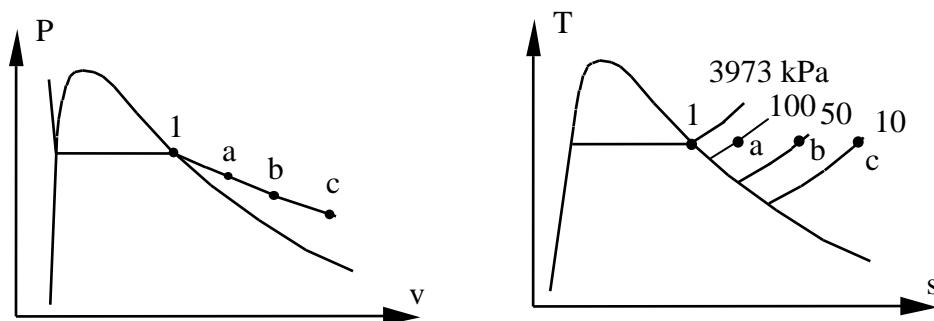
Saturated vapor water at 250°C is expanded to a lower pressure with constant temperature. Find the changes in u and s when the final pressure is

- a. 100 kPa b. 50 kPa c. 10 kPa

Solution:

Table B.1.1 for the first state then B.1.3 for the a, b and c states.

kJ/kg	kJ/kg K	kJ/kg	kJ/kg K
$u_1 = 2602.37$	$s_1 = 6.0729$		
$u_a = 2733.73$	$s_a = 8.0332$	$\Delta u = 131.36$	$\Delta s = 1.9603$
$u_b = 2734.97$	$s_b = 8.3555$	$\Delta u = 132.6$	$\Delta s = 2.2826$
$u_c = 2735.95$	$s_c = 9.1002$	$\Delta u = 133.58$	$\Delta s = 3.0273$



Remark: You approach ideal gas as P drops so u is $u(T)$ but s is still $s(T,P)$.

6.34

Determine the missing property among P , T , s , x for the following states:

- Ammonia 25°C , $v = 0.10 \text{ m}^3/\text{kg}$
- Ammonia 1000 kPa , $s = 5.2 \text{ kJ/kg K}$
- R-410A 500 kPa , $s = 1.4 \text{ kJ/kg K}$
- R-410A 50°C , $s = 0.8 \text{ kJ/kg K}$

Solution:

	Table	$P \text{ kPa}$	$T ^{\circ}\text{C}$	$s \text{ kJ/kg K}$	x
a)	B2.1	1003	25	4.1601	0.7776
b)	B2.2	1000	42.53	5.2	----
c)	B4.2	500	100	1.4	----
d)	B4.1	3065	50	0.8	0.6973

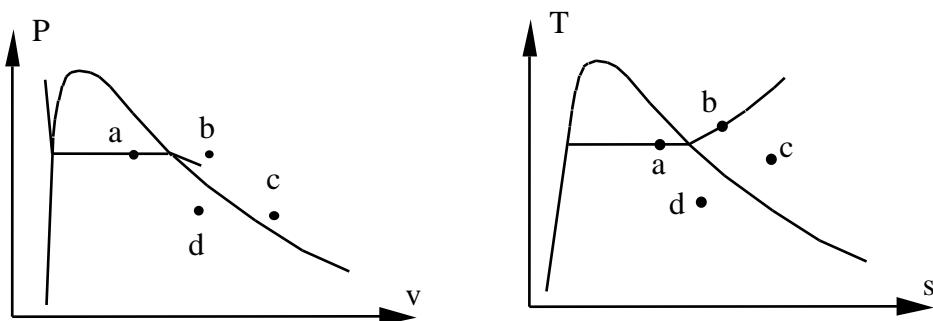
a) $x = (0.1 - 0.001658)/0.12647 = \mathbf{0.7776}$

$$s = s_f + x s_{fg} = 1.121 + x \times 3.9083 = \mathbf{4.1601 \text{ kJ/kg K}}$$

b) $T = 40 + 10 \times (5.2 - 5.1778)/(5.2654 - 5.1778) = \mathbf{42.53^{\circ}\text{C}}$
superheated vapor so x is undefined

c) $s > s_g = 1.0647$ so superheated vapor found close to 100°C

d) $s_f < s < s_g$ so two-phase $P = P_{\text{sat}} = 3065.2 \text{ kPa}$
 $x = (0.8 - 0.5067)/0.4206 = 0.69734$



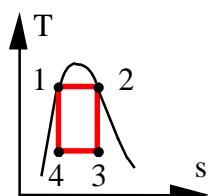
Reversible processes

Excerpts from this work may be reproduced by instructors for distribution on a not-for-profit basis for testing or instructional purposes only to students enrolled in courses for which this textbook has been adopted. *Any other reproduction or translation of this work beyond that permitted by Sections 107 or 108 of the 1976 United States Copyright Act without the permission of the copyright owner is unlawful.*

6.35

In a Carnot engine with ammonia as the working fluid, the high temperature is 60°C and as Q_H is received, the ammonia changes from saturated liquid to saturated vapor. The ammonia pressure at the low temperature is 190 kPa. Find T_L , the cycle thermal efficiency, the heat added per kilogram, and the entropy, s , at the beginning of the heat rejection process.

Solution:



Constant $T \Rightarrow$ constant P from 1 to 2, Table B.2.1

$$\begin{aligned} q_H &= \int T ds = T(s_2 - s_1) = T s_{fg} \\ &= h_2 - h_1 = h_{fg} = \mathbf{997.0 \text{ kJ/kg}} \end{aligned}$$

States 3 & 4 are two-phase, Table B.2.1

$$\Rightarrow T_L = T_3 = T_4 = T_{\text{sat}}(P) = \mathbf{-20^\circ C}$$

$$\eta_{\text{cycle}} = 1 - \frac{T_L}{T_H} = 1 - \frac{253.2}{333.2} = \mathbf{0.24}$$

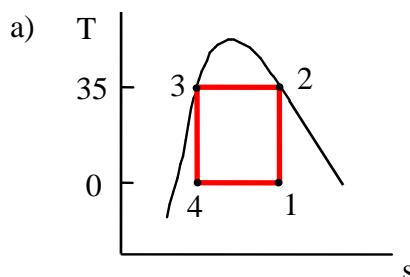
Table B.2.1: $s_3 = s_2 = s_g(60^\circ C) = \mathbf{4.6577 \text{ kJ/kg K}}$

6.36

Consider a Carnot-cycle heat pump with R-410A as the working fluid. Heat is rejected from the R-410A at 35°C, during which process the R-410A changes from saturated vapor to saturated liquid. The heat is transferred to the R-410A at 0°C.

- Show the cycle on a $T-s$ diagram.
- Find the quality of the R-410A at the beginning and end of the isothermal heat addition process at 0°C.
- Determine the COP for the cycle.

Solution:



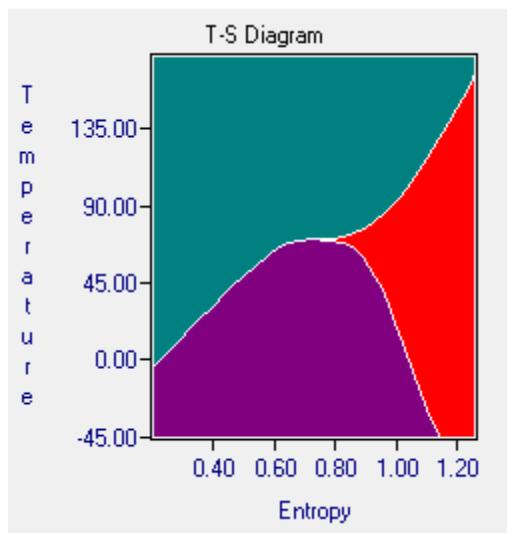
b) From Table B.4.1, state 3 is saturated liquid

$$\begin{aligned}s_4 &= s_3 = 0.4189 \text{ kJ/kg K} \\ &= 0.2264 + x_4(0.8104) \\ \Rightarrow x_4 &= \mathbf{0.2375}\end{aligned}$$

State 2 is saturated vapor so from Table B.4.1

$$\begin{aligned}s_1 &= s_2 = 0.9671 \text{ kJ/kg K} = 0.2264 + x_1(0.8104) \\ \Rightarrow x_1 &= \mathbf{0.914}\end{aligned}$$

c) $\beta' = \frac{q_H}{w_{IN}} = \frac{T_H}{T_H - T_L} = \frac{308.15}{35} = \mathbf{8.8}$



T-s diagram
from CATT3
for R-410A

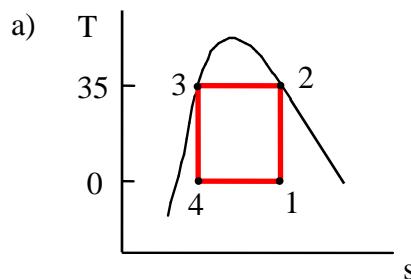
6.37

Do Problem 6.36 using refrigerant R-134a instead of R-410A.

Consider a Carnot-cycle heat pump with R-410A as the working fluid. Heat is rejected from the R-410A at 35°C, during which process the R-410A changes from saturated vapor to saturated liquid. The heat is transferred to the R-410A at 0°C.

- Show the cycle on a $T-s$ diagram.
- Find the quality of the R-410A at the beginning and end of the isothermal heat addition process at 0°C.
- Determine the coefficient of performance for the cycle.

Solution:



b) From Table B.5.1, state 3 is
saturated liquid

$$s_4 = s_3 = 1.1673 \text{ kJ/kg K} \\ = 1.0 + x_4(0.7262)$$

$$\Rightarrow x_4 = \mathbf{0.2303}$$

State 2 is saturated vapor so from Table B.5.1

$$s_1 = s_2 = 1.7139 \text{ kJ/kg K} = 1.0 + x_1(0.7262)$$

$$\Rightarrow x_1 = \mathbf{0.983}$$

c)

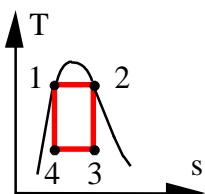
$$\beta' = \frac{q_H}{w_{IN}} = \frac{T_H}{T_H - T_L} = \frac{308.15}{35} = \mathbf{8.8}$$

6.38

Water is used as the working fluid in a Carnot cycle heat engine, where it changes from saturated liquid to saturated vapor at 200°C as heat is added. Heat is rejected in a constant pressure process (also constant T) at 20 kPa. The heat engine powers a Carnot cycle refrigerator that operates between -15°C and +20°C. Find the heat added to the water per kg water. How much heat should be added to the water in the heat engine so the refrigerator can remove 1 kJ from the cold space?

Solution:

Carnot cycle heat engine:



Constant T \Rightarrow constant P from 1 to 2, Table B.2.1

$$\begin{aligned} q_H &= \int T ds = T(s_2 - s_1) = T s_{fg} = h_{fg} \\ &= 473.15 (4.1014) = \mathbf{1940 \text{ kJ/kg}} \end{aligned}$$

States 3 & 4 are two-phase, Table B.2.1

$$\Rightarrow T_L = T_3 = T_4 = T_{\text{sat}}(P) = 60.06^\circ\text{C}$$

Carnot cycle refrigerator (T_L and T_H are different from above):

$$\beta_{\text{ref}} = \frac{Q_L}{W} = \frac{T_L}{T_H - T_L} = \frac{273 - 15}{20 - (-15)} = \frac{258}{35} = 7.37$$

$$W = \frac{Q_L}{\beta} = \frac{1}{7.37} = 0.136 \text{ kJ}$$

The needed work comes from the heat engine

$$W = \eta_{\text{HE}} Q_{H \text{ H}_2\text{O}} ; \quad \eta_{\text{HE}} = 1 - \frac{T_L}{T_H} = 1 - \frac{333}{473} = 0.296$$

$$Q_{H \text{ H}_2\text{O}} = \frac{W}{\eta_{\text{HE}}} = \frac{0.136}{0.296} = \mathbf{0.46 \text{ kJ}}$$

6.39

Water at 1 MPa, 250°C is expanded in a piston/cylinder to 200 kPa, $x = 1.0$ in a reversible process. Find the sign for the work and the sign for the heat transfer.

Solution:

The process is not specified, but the beginning and end states are and we assume a unidirectional process so v keeps decreasing.

State 1: Table B.1.3:

$$v_1 = 0.23268 \text{ m}^3/\text{kg}; \quad u_1 = 2709.9 \text{ kJ/kg}; \quad s_1 = 6.9246 \text{ kJ/kg K}$$

State 2: Table B.1.1:

$$v_2 = 0.8857 \text{ m}^3/\text{kg}; \quad u_2 = 2529.5 \text{ kJ/kg}; \quad s_2 = 7.1271 \text{ kJ/kg K}$$

Reversible process: $dw = P dv$

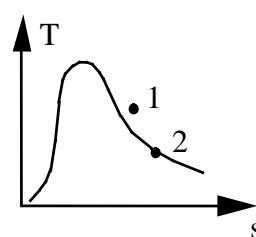
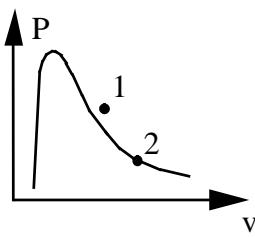
$$v_2 > v_1 \Rightarrow$$

$$s_2 > s_1 \Rightarrow$$

$$dq = T ds$$

$$_1 w_2 = \int P dv > 0$$

$$_1 q_2 = \int T ds > 0$$



6.40

R-410A at 1 MPa and 60°C is expanded in a piston cylinder to 500 kPa, 40°C in a reversible process. Find the sign for both the work and the heat transfer for this process.

Solution:

The process is not specified, but the beginning and end states are and we assume a unidirectional process so v keeps increasing.

$$_1 w_2 = \int P \, dv \quad \text{so sign } dv$$

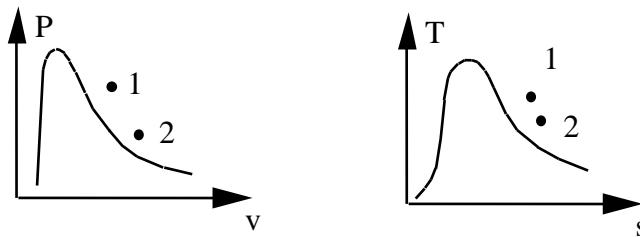
$$_1 q_2 = \int T \, ds \quad \text{so sign } ds$$

State 1: B.4.2 $v_1 = 0.03470 \text{ m}^3/\text{kg}$ $s_1 = 1.2019 \text{ kJ/kg K}$

State 2: B.4.2 $v_2 = 0.06775 \text{ m}^3/\text{kg}$ $s_2 = 1.2398 \text{ kJ/kg K}$

$$dv > 0 \quad \Rightarrow \quad \mathbf{w \text{ is positive}}$$

$$ds > 0 \quad \Rightarrow \quad \mathbf{q \text{ is positive}}$$



6.41

A piston/cylinder compressor takes R-410A as saturated vapor 500 kPa and compresses it in a reversible adiabatic process to 3000 kPa. Find the final temperature and the specific compression work.

CV R-410A, this is a control mass

$$\text{Energy Eq.3.5: } u_2 - u_1 = 1q_2 - 1w_2$$

$$\text{Entropy Eq.6.3: } s_2 - s_1 = \int dq/T = 1q_2 /T$$

Process: Adiabatic and reversible $\Rightarrow 1q_2 = 0$ so then $s_2 = s_1$

State 1: $P_1, x = 1, u_1 = 248.29 \text{ kJ/kg}, s_1 = 1.0647 \text{ kJ/kgK}$

State 2: $P_2, s_2 = s_1$

$$T_2 = 60 + 20 \times \frac{1.0647 - 0.9933}{1.0762 - 0.9933} = 60 + 20 \times 0.8613 = 77.2^\circ\text{C}$$

$$u_2 = 274.96 + (298.38 - 274.96) \times 0.8613 = 295.13 \text{ kJ/kg}$$

Now the work becomes

$$1w_2 = u_1 - u_2 = 248.29 - 295.13 = -46.84 \text{ kJ/kg}$$

6.42

A piston/cylinder receives R-410A at 500 kPa and compresses it in a reversible adiabatic process to 1800 kPa, 60°C. Find the initial temperature.

CV R-410A, this is a control mass

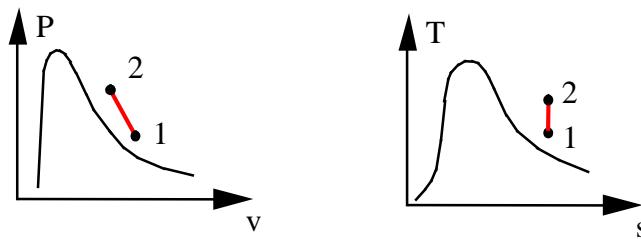
$$\text{Energy Eq.3.5: } u_2 - u_1 = 1q_2 - 1w_2$$

$$\text{Entropy Eq.6.3: } s_2 - s_1 = \int dq/T = 1q_2 / T$$

Process: Adiabatic and reversible $\Rightarrow 1q_2 = 0$ so then $s_2 = s_1$

State 1: $P_1, s_1 = s_2 = 1.1076 \text{ kJ/kgK} \Rightarrow$

$$T_1 = -13.89 + 13.89 \times \frac{1.1076 - 1.0647}{1.1155 - 1.0647} = -2.16^\circ\text{C}$$



6.43

Compression and heat transfer brings carbon dioxide in a piston/cylinder from 1400 kPa, 20°C to saturated vapor in an isothermal process. Find the specific heat transfer and the specific work.

Solution:

$$m = \text{constant}$$

$$\text{Energy Eq.3.5: } u_2 - u_1 = q_1 - w_1$$

$$\text{Entropy Eq.6.3: } s_2 - s_1 = \int dq/T = q_1/T$$

$$\text{Process: } T = C \text{ and assume reversible} \Rightarrow q_1 = T(s_2 - s_1)$$

State 1: Table B.4.2:

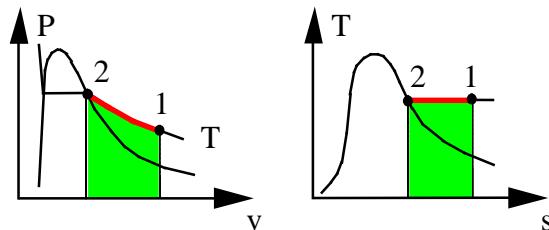
$$u_1 = 259.18 \text{ kJ/kg},$$

$$s_1 = 1.0057 \text{ kJ/kg K}$$

State 2: Table B.4.1

$$u_2 = 258.16 \text{ kJ/kg},$$

$$s_2 = 0.9984 \text{ kJ/kg K}$$



$$q_1 = (273 + 20) \times (0.9984 - 1.0057) = -2.14 \text{ kJ/kg}$$

$$w_1 = q_1 + u_1 - u_2 = -2.14 + 259.18 - 258.16$$

$$= -1.12 \text{ kJ/kg}$$

6.44

A piston cylinder maintaining constant pressure contains 0.1 kg saturated liquid water at 100°C. It is now boiled to become saturated vapor in a reversible process. Find the work term and then the heat transfer from the energy equation. Find the heat transfer from the entropy equation, is it the same?

$$\text{Energy Eq.: } m(u_2 - u_1) = _1Q_2 - _1W_2$$

$$\text{Entropy Eq.: } m(s_2 - s_1) = \int \frac{dq}{T} + 0 = \frac{1}{T} Q_2$$

$$\text{Process: } P = C \Rightarrow _1W_2 = m P(v_2 - v_1) = m P v_{fg}$$

$$= 0.1 \text{ kg} \times 101.3 \text{ kPa} \times 1.67185 \text{ m}^3/\text{kg} = \mathbf{16.936 \text{ kJ}}$$

From the energy equation we get

$$_1Q_2 = m(u_2 - u_1) + _1W_2 = m u_{fg} + _1W_2$$

$$= 0.1 \times 2087.58 + 16.936 = 225.7 \text{ kJ}$$

$$\text{or } = m(h_2 - h_1) = m h_{fg} = 0.1 \times 2257.03 = \mathbf{225.7 \text{ kJ}}$$

From the entropy equation we can get

$$_1Q_2 = mT(s_2 - s_1) = m T s_{fg} = 0.1 \times 373.15 \times 6.048 = \mathbf{225.68 \text{ kJ}}$$

So they are equal to within round off errors.

6.45

A piston cylinder contains 0.25 kg of R-134a at 100 kPa. It will be compressed in an adiabatic reversible process to 400 kPa and should be 70°C. What should the initial temperature be?

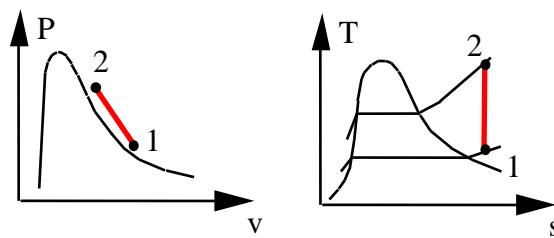
C.V. R-134a which is a control mass.

$$\text{Entropy Eq.6.3: } m(s_2 - s_1) = \int dQ/T = 0$$

$$\text{State 2: } s_2 = s_1 = 1.9051 \text{ kJ/kgK}$$

Work backwards from state 2 to state 1

$$\text{State 1: 100 kPa \& } s_1 \Rightarrow T_1 = 26.4^\circ\text{C}$$



6.46

A piston/cylinder contains 0.5 kg of water at 200 kPa, 300°C, and it now cools to 150°C in an isobaric process. The heat goes into a heat engine that rejects heat to the ambient at 25°C (shown in Fig. P.6.46), and the whole process is assumed to be reversible. Find the heat transfer out of the water and the work given out by the heat engine.

C.V. H₂O

$$\text{Energy Eq.3.5: } m(u_2 - u_1) = _1Q_2 - _1W_2$$

$$\text{Entropy Eq.6.3: } m(s_2 - s_1) = \int dQ/T + 0$$

$$\text{Process: } P = C \Rightarrow W = \int P dV = P(V_2 - V_1) = m P (v_2 - v_1)$$

$$\text{State 1: B.1.3 } s_1 = 7.8926 \text{ kJ/kg-K}, h_1 = 3071.79 \text{ kJ/kg}$$

$$\text{State 2: B.1.3 } s_2 = 7.2795 \text{ kJ/kg K}, h_2 = 2768.8 \text{ kJ/kg}$$

From the process equation and the energy equation

$$\begin{aligned} _1Q_2 &= m(u_2 - u_1) + _1W_2 = m(h_2 - h_1) = 0.5(2768.8 - 3071.79) \\ &= \mathbf{-151.495 \text{ kJ}} \end{aligned}$$

CV Total

$$\text{Energy Eq.3.5: } m(u_2 - u_1) = -Q_L - _1W_2 - W_{HE}$$

$$\text{Entropy Eq.6.3: } m(s_2 - s_1) = -Q_L/T_{amb} + 0$$

$$\begin{aligned} Q_L &= mT_{amb}(s_1 - s_2) = 0.5 \text{ kg } 298.15 \text{ K } (7.8926 - 7.2795) \text{ kJ/kgK} \\ &= 91.398 \text{ kJ} \end{aligned}$$

Now the energy equation for the heat engine gives

$$W_{HE} = -_1Q_2 - Q_L = 151.495 - 91.398 = \mathbf{60.1 \text{ kJ}}$$

6.47

A cylinder fitted with a piston contains ammonia at 50°C, 20% quality with a volume of 1 L. The ammonia expands slowly, and during this process heat is transferred to maintain a constant temperature. The process continues until all the liquid is gone. Determine the work and heat transfer for this process.

Solution:

C.V. Ammonia in the cylinder.

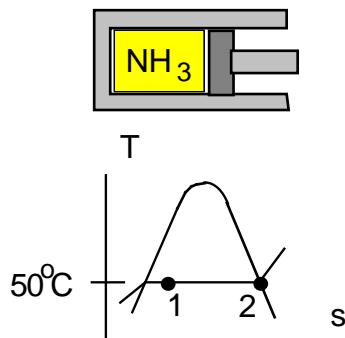
$$\text{Energy Eq.3.5: } m(u_2 - u_1) = _1Q_2 - _1W_2$$

$$\text{Entropy Eq.6.3: } m(s_2 - s_1) = \int dQ/T + 0$$

$$\text{Process: } T = \text{constant to } x_2 = 1.0, \quad P = \text{constant} = 2.033 \text{ MPa}$$

$$\Rightarrow W = \int P dV = P(V_2 - V_1) = m P (v_2 - v_1)$$

$$\Rightarrow \int dQ/T = _1Q_2 / T$$



$$\text{Table B.2.1: } T_1 = 50^\circ\text{C}, \quad x_1 = 0.20, \quad V_1 = 1 \text{ L}$$

$$v_1 = 0.001777 + 0.2 \times 0.06159 = 0.014095 \text{ m}^3/\text{kg}$$

$$s_1 = 1.5121 + 0.2 \times 3.2493 = 2.1620 \text{ kJ/kg K}$$

$$m = V_1/v_1 = 0.001/0.014095 = 0.071 \text{ kg}$$

$$v_2 = v_g = 0.06336 \text{ m}^3/\text{kg},$$

$$s_2 = s_g = 4.7613 \text{ kJ/kg K}$$

$$_1W_2 = Pm(v_2 - v_1) = 2033 \times 0.071 \times (0.06336 - 0.014095) = \mathbf{7.11 \text{ kJ}}$$

From the entropy equation

$$\begin{aligned} _1Q_2 &= Tm(s_2 - s_1) = 323.2 \text{ K} \times 0.071 \text{ kg} (4.7613 - 2.1620) \text{ kJ/kg-K} \\ &= \mathbf{59.65 \text{ kJ}} \end{aligned}$$

$$\text{or } _1Q_2 = m(u_2 - u_1) + _1W_2 = m(h_2 - h_1)$$

$$h_1 = 421.48 + 0.2 \times 1050.01 = 631.48 \text{ kJ/kg}, \quad h_2 = 1471.49 \text{ kJ/kg}$$

$$_1Q_2 = 0.071 \text{ kg} (1471.49 - 631.48) \text{ kJ/kg} = \mathbf{59.65 \text{ kJ}}$$

6.48

Water in a piston/cylinder at 400°C , 2000 kPa is expanded in a reversible adiabatic process. The specific work is measured to be 415.72 kJ/kg out. Find the final P and T and show the P-v and the T-s diagram for the process.

Solution:

C.V. Water, which is a control mass. Adiabatic so: $q_2 = 0$

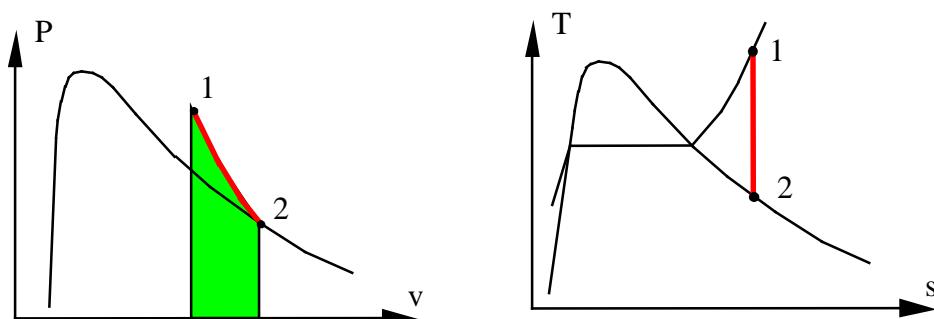
Energy Eq.3.5: $u_2 - u_1 = q_2 - w_2 = -w_2$

Entropy Eq.6.3: $s_2 - s_1 = \int dq/T = 0$ (= since reversible)

State 1: Table B.1.3 $u_1 = 2945.21 \text{ kJ/kg}$; $s_1 = 7.127 \text{ kJ/kg K}$

State 2: (s, u) : $u_2 = u_1 - w_2 = 2945.21 - 415.72 = 2529.49 \text{ kJ/kg}$

$$\Rightarrow \text{sat. vapor } 200 \text{ kPa}, \quad T = 120.23^{\circ}\text{C}$$



6.49

A piston/cylinder has 2 kg water at 1000 kPa, 200°C which is now cooled with a constant loading on the piston. This isobaric process ends when the water has reached a state of saturated liquid. Find the work and heat transfer and sketch the process in both a P-v and a T-s diagram.

Solution:

C.V. H₂O

$$\text{Energy Eq.3.5: } m(u_2 - u_1) = _1Q_2 - _1W_2$$

$$\text{Entropy Eq.6.3: } m(s_2 - s_1) = \int dQ/T$$

$$\text{Process: } P = C \Rightarrow W = \int P dV = P(V_2 - V_1)$$

$$\text{State 1: B.1.3 } v_1 = 0.20596 \text{ m}^3/\text{kg}, s_1 = 6.6939 \text{ kJ/kg-K}, u_1 = 2621.90 \text{ kJ/kg}$$

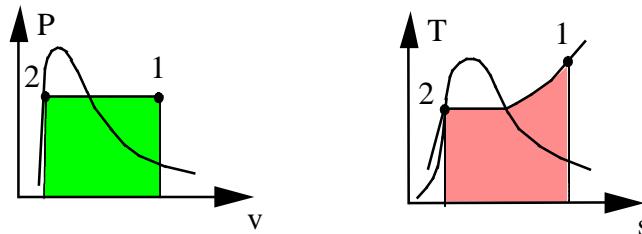
$$\text{State 2: B.1.2 } v_2 = 0.001127 \text{ m}^3/\text{kg}, s_2 = 2.1386 \text{ kJ/kg K}, u_2 = 761.67 \text{ kJ/kg}$$

From the process equation

$$_1W_2 = m P (v_2 - v_1) = 2 \times 1000 (0.001127 - 0.20596) = \mathbf{-409.7 \text{ kJ}}$$

From the energy equation we get

$$_1Q_2 = m(u_2 - u_1) + _1W_2 = 2 (761.67 - 2621.90) - 409.7 = \mathbf{-4130.2 \text{ kJ}}$$



6.50

One kilogram of water at 300°C expands against a piston in a cylinder until it reaches ambient pressure, 100 kPa, at which point the water has a quality of 90.2%. It may be assumed that the expansion is reversible and adiabatic. What was the initial pressure in the cylinder and how much work is done by the water?

Solution:

C.V. Water. Process: Rev., Q = 0

$$\text{Energy Eq.3.5: } m(u_2 - u_1) = _1Q_2 - _1W_2 = - _1W_2$$

$$\text{Entropy Eq.6.3: } m(s_2 - s_1) = \int dQ/T$$

Process: Adiabatic Q = 0 and reversible => s₂ = s₁

State 2: P₂ = 100 kPa, x₂ = 0.902 from Table B.1.2

$$s_2 = 1.3026 + 0.902 \times 6.0568 = 6.7658 \text{ kJ/kg K}$$

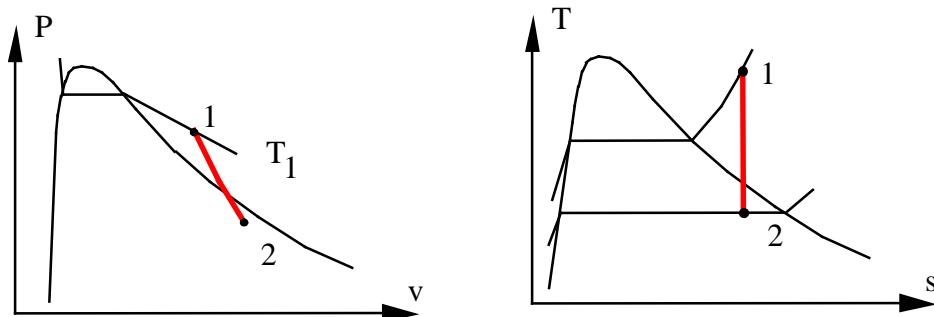
$$u_2 = 417.36 + 0.902 \times 2088.7 = 2301.4 \text{ kJ/kg}$$

State 1 At T₁ = 300°C, s₁ = 6.7658 Find it in Table B.1.3

$$\Rightarrow P_1 = 2000 \text{ kPa}, u_1 = 2772.6 \text{ kJ/kg}$$

From the energy equation

$$_1W_2 = m(u_1 - u_2) = 1(2772.6 - 2301.4) = 471.2 \text{ kJ}$$



6.51

Water at 1000 kPa, 250°C is brought to saturated vapor in a rigid container, shown in Fig. P8.54. Find the final T and the specific heat transfer in this isometric process.

Solution:

$$\text{Energy Eq.3.5: } u_2 - u_1 = \int q_2 - \int w_2$$

$$\text{Entropy Eq.6.3: } s_2 - s_1 = \int dq/T$$

$$\text{Process: } v = \text{constant} \Rightarrow \int w_2 = 0$$

$$\text{State 1: (T, P) Table B.1.3 } u_1 = 2709.91 \text{ kJ/kg}, \quad v_1 = 0.23268 \text{ m}^3/\text{kg}$$

$$\text{State 2: } x = 1 \text{ and } v_2 = v_1 \quad \text{so from Table B.1.1 we see } P_2 \approx 800 \text{ kPa}$$

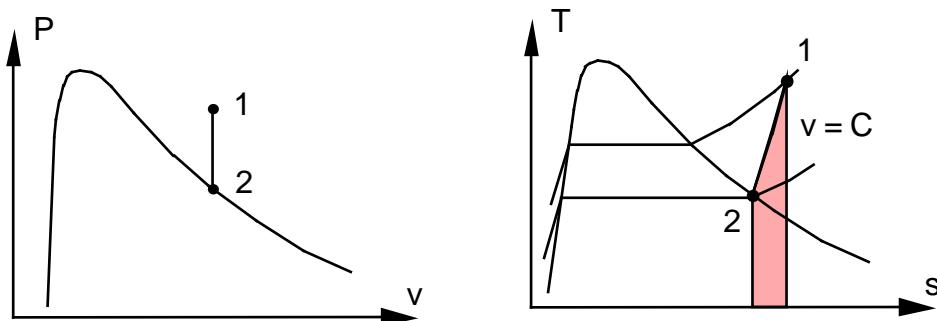
$$T_2 = 170 + 5 \times (0.23268 - 0.24283)/(0.2168 - 0.24283)$$

$$= 170 + 5 \times 0.38993 = \mathbf{171.95^\circ C}$$

$$u_2 = 2576.46 + 0.38993 \times (2580.19 - 2576.46) = 2577.9 \text{ kJ/kg}$$

From the energy equation

$$\int q_2 = u_2 - u_1 = 2577.9 - 2709.91 = \mathbf{-132 \text{ kJ/kg}}$$



Notice to get $\int q_2 = \int T ds$ we must know the function $T(s)$ which we do not readily have for this process.

6.52

Estimate the specific heat transfer from the area in the T-s diagram and compare it to the correct value for the states and process in Problem 6.51.

Solution:

$$\text{Energy Eq.3.5: } u_2 - u_1 = q_1 - w_1$$

$$\text{Entropy Eq.6.3: } s_2 - s_1 = \int dq/T$$

$$\text{Process: } v = \text{constant} \Rightarrow w_1 = 0$$

$$\text{State 1: (T, P) Table B.1.3 } u_1 = 2709.91 \text{ kJ/kg, } v_1 = 0.23268 \text{ m}^3/\text{kg},$$

$$s_1 = 6.9246 \text{ kJ/kg K}$$

$$\text{State 2: } x = 1 \text{ and } v_2 = v_1 \text{ so from Table B.1.1 we see } P_2 \approx 800 \text{ kPa}$$

$$T_2 = 170 + 5 \times (0.23268 - 0.24283)/(0.2168 - 0.24283)$$

$$= 170 + 5 \times 0.38993 = 171.95^\circ\text{C}$$

$$u_2 = 2576.46 + 0.38993 \times (2580.19 - 2576.46) = 2577.9 \text{ kJ/kg}$$

$$s_2 = 6.6663 + 0.38993 (6.6256 - 6.6663) = 6.6504 \text{ kJ/kg K}$$

From the energy equation

$$q_1 = u_2 - u_1 = 2577.9 - 2709.91 = -132 \text{ kJ/kg}$$

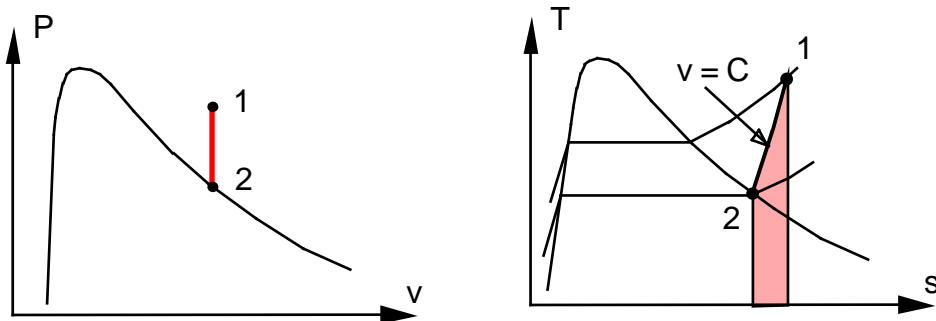
Assume a linear variation of T versus s.

$$q_1 = \int T ds = \text{area} \approx \frac{1}{2}(T_1 + T_2)(s_2 - s_1)$$

$$= \frac{1}{2}(171.95 + (2 \times 273.15) + 250) \text{ K} (6.6504 - 6.9246) \text{ kJ/kg-K}$$

$$= -132.74 \text{ kJ/kg}$$

very close i.e. the $v = C$ curve is close to a straight line in the T-s diagram. Look at the constant v curves in Fig. E.1. In the two-phase region they curve slightly and more so in the region above the critical point.



6.53

A closed tank, $V = 10 \text{ L}$, containing 5 kg of water initially at 25°C , is heated to 150°C by a heat pump that is receiving heat from the surroundings at 25°C . Assume that this process is reversible. Find the heat transfer to the water and the change in entropy.

Solution:

C.V.: Water from state 1 to state 2.

$$\text{Energy Eq.3.5: } m(u_2 - u_1) = _1Q_2 - _1W_2$$

$$\text{Entropy Eq.6.3: } m(s_2 - s_1) = \int dQ/T$$

Process: constant volume (reversible isometric) so $_1W_2 = 0$

State 1: $v_1 = V/m = 0.002$ from Table B.1.1

$$x_1 = (0.002 - 0.001003)/43.358 = 0.000023$$

$$u_1 = 104.86 + 0.000023 \times 2304.9 = 104.93 \text{ kJ/kg}$$

$$s_1 = 0.3673 + 0.000023 \times 8.1905 = 0.36759 \text{ kJ/kg K}$$

Continuity eq. (same mass) and $V = C$ fixes v_2

State 2: $T_2, v_2 = v_1$ so from Table B.1.1

$$x_2 = (0.002 - 0.001090)/0.39169 = 0.0023233$$

$$u_2 = 631.66 + 0.0023233 \times 1927.87 = 636.14 \text{ kJ/kg}$$

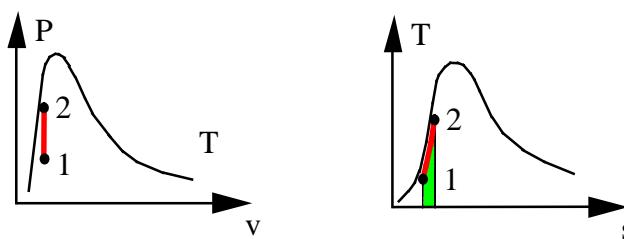
$$s_2 = 1.8417 + 0.0023233 \times 4.9960 = 1.8533 \text{ kJ/kg K}$$

Energy eq. has $W = 0$, thus provides heat transfer as

$$_1Q_2 = m(u_2 - u_1) = \mathbf{2656.05 \text{ kJ}}$$

The entropy change becomes

$$m(s_2 - s_1) = 5(1.8533 - 0.36759) = \mathbf{7.4286 \text{ kJ/K}}$$



Notice we do not perform the integration $\int dQ/T$ to find change in s as the equation for the dQ as a function of T is not known.

6.54

A piston/cylinder has 2 kg of R-410A at 60°C, 100 kPa which is compressed to 1000 kPa. The process happens so slowly that the temperature is constant. Find the heat transfer and work for the process assuming it to be reversible.

Solution:

CV : R-410A Control Mass

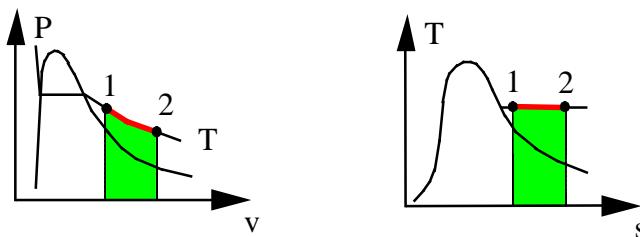
$$\text{Energy Eq.3.5: } m(u_2 - u_1) = \dot{Q}_2 - \dot{W}_2 ;$$

$$\text{Entropy Eq.6.3: } m(s_2 - s_1) = \int dQ/T$$

Process: T = constant and assume reversible process

$$\begin{aligned} 1: (T, P), \text{ Table B.4.2: } v_1 &= 0.37833 \text{ m}^3/\text{kg}, \quad u_1 = 309.4 \text{ kJ/kg,} \\ s_1 &= 1.4910 \text{ kJ/kg K} \end{aligned}$$

$$\begin{aligned} 2: (T, P), \text{ Table B.4.2: } v_2 &= 0.03470 \text{ m}^3/\text{kg}, \quad u_2 = 301.04 \text{ kJ/kg,} \\ s_2 &= 1.2019 \text{ kJ/kg K} \end{aligned}$$



From the entropy equation (2nd law)

$$\begin{aligned} \dot{Q}_2 &= mT(s_2 - s_1) = 2 \text{ kg} \times 333.15 \text{ K} \times (1.2019 - 1.4910) \text{ kJ/kg-K} \\ &= \mathbf{-192.63 \text{ kJ}} \end{aligned}$$

From the energy equation

$$\begin{aligned} \dot{W}_2 &= \dot{Q}_2 - m(u_2 - u_1) = -192.63 \text{ kJ} - 2 \text{ kg} \times (301.04 - 309.4) \text{ kJ/kg} \\ &= \mathbf{-175.9 \text{ kJ}} \end{aligned}$$

6.55

A heavily insulated cylinder/piston contains ammonia at 1200 kPa, 60°C. The piston is moved, expanding the ammonia in a reversible process until the temperature is -20°C. During the process 200 kJ of work is given out by the ammonia. What was the initial volume of the cylinder?

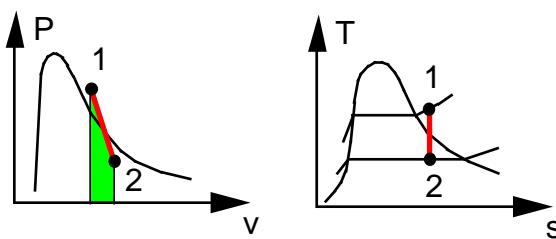
C.V. ammonia. Control mass with no heat transfer.

State 1: Table B.2.2 $v_1 = 0.1238 \text{ m}^3/\text{kg}$, $s_1 = 5.2357 \text{ kJ/kg K}$

$$u_1 = 1404.8 \text{ kJ/kg}$$

$$\text{Entropy Eq.: } m(s_2 - s_1) = \int dQ/T + _1S_{2 \text{ gen}}$$

$$\text{Process: reversible } (_1S_{2 \text{ gen}} = 0) \text{ and adiabatic } (dQ = 0) \Rightarrow s_2 = s_1$$



$$\text{State 2: } T_2, s_2 \Rightarrow x_2 = (5.2357 - 0.3657)/5.2498 = 0.928$$

$$u_2 = 88.76 + 0.928 \times 1210.7 = 1211.95 \text{ kJ/kg}$$

$$_1Q_2 = 0 = m(u_2 - u_1) + _1W_2 = m(1211.95 - 1404.8) + 200$$

$$\Rightarrow m = 1.037 \text{ kg}$$

$$V_1 = mv_1 = 1.037 \times 0.1238 = \mathbf{0.1284 \text{ m}^3}$$

6.56

Water at 1000 kPa, 250°C is brought to saturated vapor in a piston/cylinder with an isothermal process. Find the specific work and heat transfer. Estimate the specific work from the area in the P-v diagram and compare it to the correct value.

Solution:

Continuity Eq.: $m_2 = m_1 = m$;

Energy Eq.3.5 $m(u_2 - u_1) = _1Q_2 - _1W_2$

Entropy Eq.6.3: $m(s_2 - s_1) = \int dQ/T$

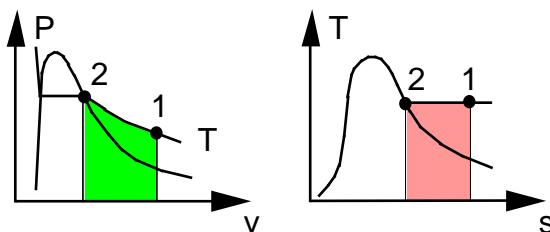
Process: $T = \text{constant}$, reversible

State 1: Table B.1.3:

$$v_1 = 0.23268 \text{ m}^3/\text{kg}; \quad u_1 = 2709.91 \text{ kJ/kg}; \quad s_1 = 6.9246 \text{ kJ/kg K}$$

State 2: (T, x) Table B.1.1 $P_2 = 3973 \text{ kPa}$

$$v_2 = 0.05013 \text{ m}^3/\text{kg}, \quad u_2 = 2602.37 \text{ kJ/kg}, \quad s_2 = 6.0729 \text{ kJ/kg K}$$



From the entropy equation

$$_1q_2 = \int T ds = T(s_2 - s_1) = (250 + 273)(6.0729 - 6.9246) = -445.6 \text{ kJ/kg}$$

From the energy equation

$$_1w_2 = _1q_2 + u_1 - u_2 = -445.6 + 2709.91 - 2602.37 = -338 \text{ kJ/kg}$$

Estimation of the work term from the area in the P-v diagram

$$\begin{aligned} _1w_{\text{area}} &\approx \frac{1}{2}(P_1 + P_2)(v_2 - v_1) = \frac{1}{2}(1000 + 3973)(0.05013 - 0.23268) \\ &= -454 \text{ kJ/kg} \end{aligned}$$

Not extremely accurate estimate; P-v curve not linear more like $Pv = \text{constant}$ as curve has positive curvature the linear variation over-estimates area.

6.57

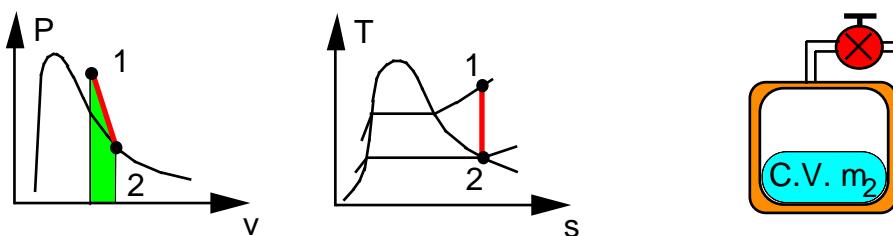
A rigid, insulated vessel contains superheated vapor steam at 3 MPa, 400°C. A valve on the vessel is opened, allowing steam to escape. The overall process is irreversible, but the steam remaining inside the vessel goes through a reversible adiabatic expansion. Determine the fraction of steam that has escaped, when the final state inside is saturated vapor.

C.V.: steam remaining inside tank. Rev. & Adiabatic (inside only)

Continuity Eq.: $m_2 = m_1 = m$;

Energy Eq.: $m(u_2 - u_1) = \dot{Q}_2 - \dot{W}_2$

Entropy Eq.: $m(s_2 - s_1) = \int dQ/T + \dot{S}_{gen}$



Rev ($\dot{S}_{gen} = 0$) Adiabatic ($Q = 0$) $\Rightarrow s_2 = s_1 = 6.9212 = s_G$ at T_2

$$\Rightarrow T_2 = 141^\circ\text{C}, v_2 = v_g \text{ at } T_2 = 0.4972 \text{ m}^3/\text{kg}$$

$$\frac{m_e}{m_1} = \frac{m_1 - m_2}{m_1} = 1 - \frac{m_2}{m_1} = 1 - \frac{v_1}{v_2} = 1 - \frac{0.09936}{0.4972} = \mathbf{0.80}$$

6.58

Water at 100 kPa, 25°C is brought to the boiling point in a piston/cylinder with an isobaric process. The heat is supplied by a heat pump with the cold side at the ambient temperature of 25°C. Assume that the whole process is reversible and find the work input to the heat pump per kg of water.

C.V. H₂O

$$\text{Energy Eq.3.5: } u_2 - u_1 = q_2 - w_2$$

$$\text{Entropy Eq.6.3: } s_2 - s_1 = \int dq/T + 0$$

$$\text{Process: } P = C \Rightarrow w = \int P dv = P(v_2 - v_1)$$

$$\text{State 1: B.1.1 } s_1 = 0.3673 \text{ kJ/kg-K}, h_1 = 104.87 \text{ kJ/kg}$$

$$\text{State 2: B.1.2 } s_2 = 1.3025 \text{ kJ/kg K}, h_2 = 417.44 \text{ kJ/kg}$$

From the process equation and the energy equation

$$q_2 = u_2 - u_1 + w_2 = h_2 - h_1 = 417.44 - 104.87 = 312.57 \text{ kJ/kg}$$

CV Total

$$\text{Energy Eq.3.5: } u_2 - u_1 = q_L - q_{HP} + w_{HP}$$

$$\text{Entropy Eq.6.3: } s_2 - s_1 = q_L/T_{amb} + 0$$

$$q_L = T_{amb}(s_2 - s_1) = 298.15 \text{ K} (1.3025 - 0.3673) \text{ kJ/kgK}$$

$$= 278.83 \text{ kJ/kg}$$

Now the energy equation for the heat engine gives

$$w_{HP} = q_2 - q_L = 312.57 - 278.83 = \mathbf{33.74 \text{ kJ/kg}}$$

6.59

Water at 1000 kPa, 200°C is brought to saturated vapor in a piston/cylinder with an isobaric process. Find the specific work and heat transfer. Estimate the specific heat transfer from the area in the T-s diagram and compare it to the correct value.

Solution:

C.V. H₂O

$$\text{Energy Eq.3.5: } u_2 - u_1 = q_1 - w_1$$

$$\text{Entropy Eq.6.3: } s_2 - s_1 = \int dq/T$$

$$\text{Process: } P = C \Rightarrow w = \int P dv = P(v_2 - v_1)$$

$$\text{State 1: B.1.3 } v_1 = 0.20596 \text{ m}^3/\text{kg}, s_1 = 6.6939 \text{ kJ/kg-K}, u_1 = 2621.90 \text{ kJ/kg}$$

$$\text{State 2: B1.3 } v_2 = 0.19444 \text{ m}^3/\text{kg}, s_2 = 6.5864 \text{ kJ/kg K}, u_2 = 2583.64 \text{ kJ/kg}$$

$$T_2 = 179.91^\circ\text{C}$$

From the process equation

$$w_1 = P(v_2 - v_1) = 1000 \text{ kPa} (0.19444 - 0.20596) \text{ m}^3/\text{kg} = -11.52 \text{ kJ/kg}$$

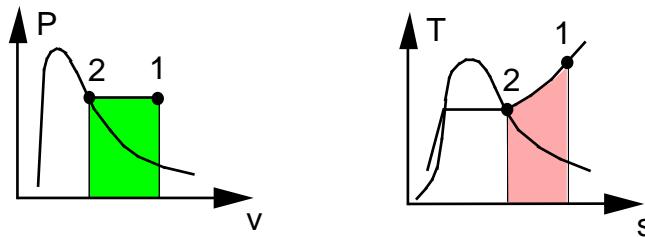
From the energy equation

$$q_1 = u_2 - u_1 + w_1 = 2583.64 - 2621.90 - 11.52 = -49.78 \text{ kJ/kg}$$

Now estimate the heat transfer from the T-s diagram.

$$\begin{aligned} q_1 &= \int T ds = \text{AREA} \approx \frac{1}{2} (T_1 + T_2)(s_2 - s_1) \\ &= \frac{1}{2} (200 + 179.91 + 2 \times 273.15) \text{ K} \times (6.5864 - 6.6939) \text{ kJ/kg-K} \\ &= 463.105 \times (-0.1075) = -49.78 \text{ kJ/kg} \end{aligned}$$

very close approximation. The $P = C$ curve in the T-s diagram is nearly a straight line. Look at the constant P curves on Fig.E.1. Up over the critical point they curve significantly.



Entropy of a liquid or a solid

Excerpts from this work may be reproduced by instructors for distribution on a not-for-profit basis for testing or instructional purposes only to students enrolled in courses for which this textbook has been adopted. Any other reproduction or translation of this work beyond that permitted by Sections 107 or 108 of the 1976 United States Copyright Act without the permission of the copyright owner is unlawful.

6.60

Two 5 kg blocks of steel, one at 250°C the other at 25°C, come in thermal contact. Find the final temperature and the change in entropy of the steel?

C.V. Both blocks, no external heat transfer, C from Table A.3.

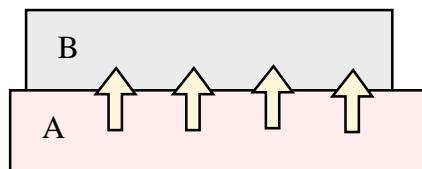
$$\begin{aligned}\text{Energy Eq.: } U_2 - U_1 &= m_A(u_2 - u_1)_A + m_B(u_2 - u_1)_B = 0 - 0 \\ &= m_A C(T_2 - T_{A1}) + m_B C(T_2 - T_{B1})\end{aligned}$$

$$T_2 = \frac{m_A T_{A1} + m_B T_{B1}}{m_A + m_B} = \frac{1}{2} T_{A1} + \frac{1}{2} T_{B1} = 137.5^{\circ}\text{C}$$

$$\text{Entropy Eq.6.37: } S_2 - S_1 = m_A(s_2 - s_1)_A + m_B(s_2 - s_1)_B = _1S_2 \text{ gen}$$

Entropy changes from Eq.6.11

$$\begin{aligned}S_2 - S_1 &= m_A C \ln \frac{T_2}{T_{A1}} + m_B C \ln \frac{T_2}{T_{B1}} \\ &= 5 \times 0.46 \ln \frac{137.5 + 273.15}{250 + 273.15} + 5 \times 0.46 \ln \frac{137.5 + 273.15}{298.15} \\ &= -0.5569 + 0.7363 = \mathbf{0.1794 \text{ kJ/K}}$$



Heat transfer over a finite temperature difference is an irreversible process

6.61

A rigid tank of 1.2 kg steel contains 1.5 kg of R-134a at 40°C, 500 kPa. The tank is placed in a refrigerator that brings it to -20°C. Find the process heat transfer and the combined steel and R-134a change in entropy.

C.V. The steel tank and the R-134a.

The energy equation, Eq. 3.5 now becomes (summing over the mass)

$$m_{st} (u_2 - u_1)_{st} + m_{R134a} (u_2 - u_1)_{R134a} = \dot{Q}_2 - 0$$

Process: No change in volume so no work as used above.

Use specific heat from Table A.3 for steel and Table B.5 for R-134a

$$\text{R-134a: } v_1 = 0.04656 \text{ m}^3/\text{kg}, \quad u_1 = 407.44 \text{ kJ/kg}, \quad s_1 = 1.7971 \text{ kJ/kgK}$$

$$\text{State 2: } v_2 = v_1 < v_g \Rightarrow x_2 = (v_2 - v_f)/v_{fg} = \frac{0.04656 - 0.000738}{0.14576} = 0.314366$$

$$u_2 = u_f + x_2 u_{fg} = 173.65 + x_2 \times 192.85 = 234.275 \text{ kJ/kg}$$

$$s_2 = s_f + x_2 s_{fg} = 0.9007 + x_2 \times 0.8388 = 1.16439 \text{ kJ/kg-K}$$

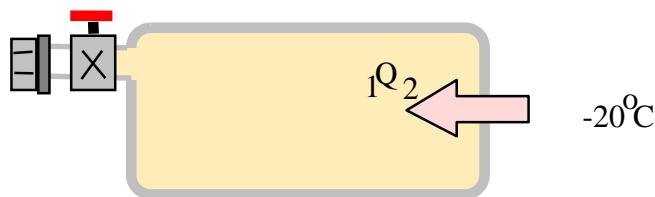
Now the heat transfer from the energy equation

$$\begin{aligned} \dot{Q}_2 &= m_{R134a}(u_2 - u_1)_{R134a} + m_{st} C_{st} (T_2 - T_1) \\ &= 1.5 \times (234.275 - 407.44) + 1.2 \times 0.46 (-20 - 40) = \mathbf{-292.87 \text{ kJ}} \end{aligned}$$

$$\text{Steel: } m_{st}(s_2 - s_1)_{st} = m_{st} C_{st} \ln(T_2/T_1) = 1.2 \times 0.46 \ln \frac{253.15}{313.15} = -0.11741 \text{ kJ/K}$$

Entropy change for the total control volume steel and R-134a

$$\begin{aligned} S_2 - S_1 &= m_{st} (s_2 - s_1)_{st} + m_{R134a} (s_2 - s_1)_{R134a} \\ &= -0.11741 + 1.5(1.16439 - 1.7971) = \mathbf{-1.066 \text{ kJ/K}} \end{aligned}$$



6.62

A large slab of concrete, $5 \times 8 \times 0.3$ m, is used as a thermal storage mass in a solar-heated house. If the slab cools overnight from 23°C to 18°C in an 18°C house, what is the net entropy change associated with this process?

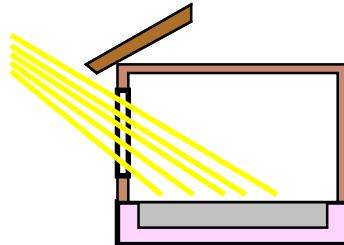
Solution:

C.V.: Control mass concrete.

$$V = 5 \times 8 \times 0.3 = 12 \text{ m}^3$$

$$m = \rho V = 2200 \times 12 = 26400 \text{ kg}$$

$$\text{Energy Eq.: } m(u_2 - u_1) = _1Q_2 - _1W_2$$



$$\text{Entropy Eq.: } m(s_2 - s_1) = \frac{1}{T_0} Q_2 + S_{2 \text{ gen}}$$

$$\text{Process: } V = \text{constant} \text{ so } _1W_2 = 0$$

Use heat capacity (Table A.3) for change in u of the slab

$$_1Q_2 = mC\Delta T = 26400 \text{ kg} \times 0.88 \text{ kJ/kg-K} \times (-5) \text{ K} = -116160 \text{ kJ}$$

We add all the storage changes as in Eq.6.39:

$$\begin{aligned} \Delta S_{\text{slab}} &= m(s_2 - s_1) = m C \ln \frac{T_2}{T_1} \\ &= 26400 \text{ kg} \times 0.88 \text{ kJ/kg-K} \times \ln \frac{291.2}{296.2} = -395.5 \text{ kJ/K} \end{aligned}$$

6.63

A foundry form box with 25 kg of 200°C hot sand is dumped into a bucket with 50 L water at 15°C. Assuming no heat transfer with the surroundings and no boiling away of liquid water, calculate the net entropy change for the mass.

Solution:

C.V. Sand and water, constant pressure process

$$\begin{aligned} m_{\text{sand}}(u_2 - u_1)_{\text{sand}} + m_{\text{H}_2\text{O}}(u_2 - u_1)_{\text{H}_2\text{O}} &= -P(V_2 - V_1) \\ \Rightarrow m_{\text{sand}}\Delta h_{\text{sand}} + m_{\text{H}_2\text{O}}\Delta h_{\text{H}_2\text{O}} &= 0 \end{aligned}$$

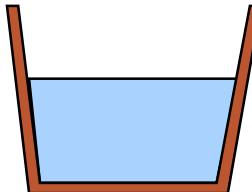
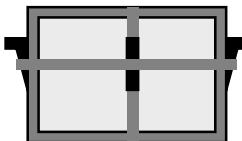
For this problem we could also have said that the work is nearly zero as the solid sand and the liquid water will not change volume to any measurable extent. Now we get changes in u 's instead of h 's. For these phases $C_V = C_P = C$ which is a consequence of the incompressibility. Now the energy equation becomes

$$\begin{aligned} m_{\text{sand}}C_{\text{sand}}\Delta T_{\text{sand}} + m_{\text{H}_2\text{O}}C_{\text{H}_2\text{O}}\Delta T_{\text{H}_2\text{O}} &= 0 \\ 25 \text{ kg} \times 0.8 \text{ kJ/kg-K} \times (T_2 - 200) \text{ K} \\ + (50 \times 10^{-3} \text{ m}^3 / 0.001001 \text{ m}^3/\text{kg}) \times 4.184 \text{ kJ/kg-K} \times (T_2 - 15) \text{ K} &= 0 \end{aligned}$$

$$T_2 = 31.2^\circ\text{C}$$

$$\begin{aligned} S_2 - S_1 &= m_{\text{sand}}(s_2 - s_1) + m_{\text{H}_2\text{O}}(s_2 - s_1) \\ &= m_{\text{sand}}C_{\text{sand}}\ln(T_2/T_1) + m_{\text{H}_2\text{O}}C_{\text{H}_2\text{O}}\ln(T_2/T_1) \\ &= 25 \times 0.8 \ln\left(\frac{304.3}{473.15}\right) + 49.95 \times 4.184 \ln\left(\frac{304.3}{288.15}\right) = \mathbf{2.57 \text{ kJ/K}} \end{aligned}$$

Box holds the sand for
form of the cast part



6.64

Heat transfer to a block of 1.5 kg ice at -10°C melts it to liquid at 10°C in a kitchen. Find the entropy change of the water.

Water changes state from nearly saturated solid to nearly saturated liquid. The pressure is 101 kPa but we approximate the state properties with saturated state at the same temperature.

State 1: Compressed (saturated) solid, B.1.5, $s_1 = -1.2995 \text{ kJ/kg-K}$

State 2: Compressed (saturated) liquid B.1.1 $s_2 = 0.1510 \text{ kJ/kg-K}$

The entropy change is

$$\Delta s = s_2 - s_1 = 0.151 - (-1.2995) = \mathbf{1.4505 \text{ kJ/kg-K}}$$

6.65

In a sink 5 liters of water at 70°C is combined with 1 kg aluminum pots, 1 kg of flatware (steel) and 1 kg of glass all put in at 20°C. What is the final uniform temperature and change in stored entropy neglecting any heat loss and work?

$$\text{Energy Eq.: } U_2 - U_1 = \sum m_i(u_2 - u_1)_i = 1Q_2 - 1W_2 = 0$$

$$\text{Entropy Eq.: } S_2 - S_1 = \int dQ/T + 1S_{2\text{ gen}}$$

For the water: $v_f = 0.001023 \text{ m}^3/\text{kg}$, $V = 5 \text{ L} = 0.005 \text{ m}^3$; $m = V/v = 4.8876 \text{ kg}$

For the liquid and the metal masses we will use the specific heats (Tbl A.3, A.4) so

$$\sum m_i(u_2 - u_1)_i = \sum m_i C_v i (T_2 - T_1)_i = T_2 \sum m_i C_v i - \sum m_i C_v i T_1 i$$

noticing that all masses have the same T_2 but not same initial T .

$$\sum m_i C_v i = 4.8876 \times 4.18 + 1 \times 0.9 + 1 \times 0.46 + 1 \times 0.8 = 22.59 \text{ kJ/K}$$

$$\begin{aligned} \text{Energy Eq.: } 22.59 T_2 &= 4.8876 \times 4.18 \times 70 + (1 \times 0.9 + 1 \times 0.46 + 1 \times 0.8) \times 20 \\ &= 1430.11 + 43.2 \end{aligned}$$

$$T_2 = \mathbf{65.2^\circ C}$$

$$\begin{aligned} S_2 - S_1 &= \sum m_i(s_2 - s_1)_i = \sum m_i C_i \ln \frac{T_2}{T_{i1}} \\ &= 4.8876 \times 4.18 \times \ln \frac{65.22 + 273.15}{70 + 273.15} \\ &\quad + 1 \times (0.9 + 0.46 + 0.8) \ln \frac{65.22 + 273.15}{20 + 273.15} \\ &= -0.28659 + 0.30986 = \mathbf{0.02327 \text{ kJ/K}} \end{aligned}$$

6.66

A piston cylinder has constant pressure of 2000 kPa with water at 20°C. It is now heated up to 100°C. Find the heat transfer and the entropy change using the steam tables. Repeat the calculation using constant heat capacity and incompressibility.

Solution:

C.V. Water. Constant pressure heating.

$$\text{Energy Eq.3.5: } u_2 - u_1 = q_1 - w_1$$

$$\text{Entropy Eq.6.37: } s_2 - s_1 = q_1 / T_{\text{SOURCE}} + s_2 \text{ gen}$$

$$\text{Process: } P = P_1 \Rightarrow w_1 = P(v_2 - v_1)$$

The energy equation then gives the heat transfer as

$$q_1 = u_2 - u_1 + w_1 = h_2 - h_1$$

$$\text{Steam Tables B.1.4: } h_1 = 85.82 \text{ kJ/kg; } s_1 = 0.2962 \text{ kJ/kg K}$$

$$h_2 = 420.45 \text{ kJ/kg; } s_2 = 1.3053 \text{ kJ/kg K}$$

$$q_1 = h_2 - h_1 = -85.82 + 420.45 = \mathbf{334.63 \text{ kJ/kg}}$$

$$s_2 - s_1 = 1.3053 - 0.2962 = \mathbf{1.0091 \text{ kJ/kg K}}$$

Now using values from Table A.4: Liquid water $C_p = 4.18 \text{ kJ/kg K}$

$$h_2 - h_1 \approx C_p(T_2 - T_1) = 4.18 \times 80 = \mathbf{334.4 \text{ kJ/kg}}$$

$$s_2 - s_1 \approx C_p \ln(T_2/T_1) = 4.18 \ln \frac{373.15}{293.15} = \mathbf{1.0086 \text{ kJ/kg K}}$$

Approximations are very good

6.67

A 4 L jug of milk at 25°C is placed in your refrigerator where it is cooled down to the refrigerator's inside constant temperature of 5°C. Assume the milk has the property of liquid water and find the entropy change of the milk.

Solution:

C.V. Jug of milk. Control mass at constant pressure.

Continuity Eq.: $m_2 = m_1 = m$;

Energy Eq.3.5: $m(u_2 - u_1) = \int Q_2 - \int W_2$

Entropy Eq.6.37: $m(s_2 - s_1) = \int dQ/T + \int S_{gen}$

State 1: Table B.1.1: $v_1 \approx v_f = 0.001003 \text{ m}^3/\text{kg}$, $s_f = 0.3673 \text{ kJ/kg K}$

$$m = V/v = 0.004 \text{ m}^3 / 0.001003 \text{ (m}^3/\text{kg)} = 3.988 \text{ kg}$$

State 2: Table B.1.1: $s = s_f = 0.0761 \text{ kJ/kg K}$

The change of entropy becomes

$$\begin{aligned} S_2 - S_1 &= m(s_2 - s_1) = 3.988 \text{ kg} (0.0761 - 0.3673) \text{ kJ/kg-K} \\ &= \mathbf{-1.1613 \text{ kJ/K}} \end{aligned}$$

6.68

A constant pressure container of 1.2 kg steel contains 1.5 kg of R-134a at 40°C, 500 kPa. The container is placed in a refrigerator that brings it to -20°C. Find the process heat transfer and the combined steel and R-134a change in entropy.

C.V. The steel container and the R-134a.

The energy equation, Eq. 3.5 now becomes (summing over the mass)

$$m_{st}(u_2 - u_1)_{st} + m_{R134a}(u_2 - u_1)_{R134a} = Q_2 - W_2$$

Process: $P = C$ so $W_2 = P(V_2 - V_1) = P m_{R134a}(v_2 - v_1)_{R134a}$

Substitute the work into the energy equation and we combine the R-134a terms

$$Q_2 = m_{R134a}(h_2 - h_1)_{R134a} + m_{st} C_{st} (T_2 - T_1)$$

Use specific heat from Table A.3 for steel and Table B.5 for R-134a

R-134a: $v_1 = 0.04656 \text{ m}^3/\text{kg}$, $h_1 = 430.72 \text{ kJ/kg}$, $s_1 = 1.7971 \text{ kJ/kgK}$

State 2: 500 kPa, -20°C compressed liquid.

$$v_2 = 0.000738, s_2 = s_f = 0.9007 \text{ kJ/kg-K}$$

$$h_2 = h_f + \Delta Pv = 173.74 + (500-133.7) \times 0.000738 = 174.0 \text{ kJ/kg}$$

notice how the correction for P higher than Psat is small

Now the heat transfer from the energy equation

$$\begin{aligned} Q_2 &= m_{R134a}(h_2 - h_1)_{R134a} + m_{st} C_{st} (T_2 - T_1) \\ &= 1.5 \times (174.0 - 430.72) + 1.2 \times 0.46 (-20 - 40) = -418.2 \text{ kJ} \end{aligned}$$

$$\text{Steel: } m_{st}(s_2 - s_1)_{st} = m_{st}C_{st} \ln(T_2/T_1) = 1.2 \times 0.46 \ln \frac{253.15}{313.15} = -0.11741 \text{ kJ/kgK}$$

Entropy change for the total control volume steel and R-134a

$$\begin{aligned} S_2 - S_1 &= m_{st}(s_2 - s_1)_{st} + m_{R134a}(s_2 - s_1)_{R134a} \\ &= -0.11741 + 1.5(0.9007 - 1.7971) = -1.462 \text{ kJ/K} \end{aligned}$$

6.69

A 10-kg steel container is cured at 500°C. An amount of liquid water at 15°C, 100 kPa is added to the container so a final uniform temperature of the steel and the water becomes 50°C. Neglect any water that might evaporate during the process and any air in the container. How much water should be added and how much was the entropy changed?

CV. The steel and the water no external heat transfer nor any work.

$$\text{Energy Eq.: } m_{\text{H}_2\text{O}}(u_2 - u_1)_{\text{H}_2\text{O}} + m_{\text{st}}(u_2 - u_1) = 0$$

$$m_{\text{H}_2\text{O}}(209.3 - 62.98) + m_{\text{st}}C(T_2 - T_1) = 0$$

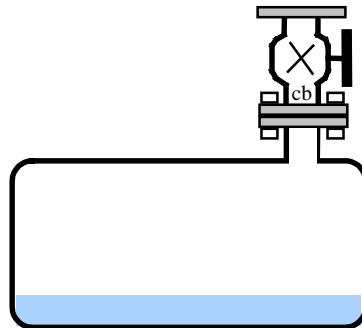
$$m_{\text{H}_2\text{O}} \times 146.32 \text{ kJ/kg} + 10 \times 0.46 \times (50 - 500) \text{ kJ} = 0$$

$$m_{\text{H}_2\text{O}} = 2070/146.32 = \mathbf{14.147 \text{ kg}}$$

$$\text{Entropy Eq. 6.37: } m_{\text{H}_2\text{O}}(s_2 - s_1) + m_{\text{st}}(s_2 - s_1) = \emptyset + S_{2 \text{ gen}}$$

$$S_2 - S_1 = 14.147 \text{ kg} (0.7037 - 0.2245) \text{ kJ/kg-K} + 10 \text{ kg} \times 0.46 \text{ kJ/kg-K} \times \ln \frac{50 + 273}{773}$$

$$= 6.7792 - 4.0141 = \mathbf{2.7651 \text{ kJ/K}}$$



6.70

A pan in an autoshop contains 5 L of engine oil at 20°C, 100 kPa. Now 3 L of hot 100°C oil is mixed into the pan. Neglect any work term and find the final temperature and the entropy change.

Solution:

Since we have no information about the oil density, we assume the same for both from Table A.4: $\rho = 885 \text{ kg/m}^3$

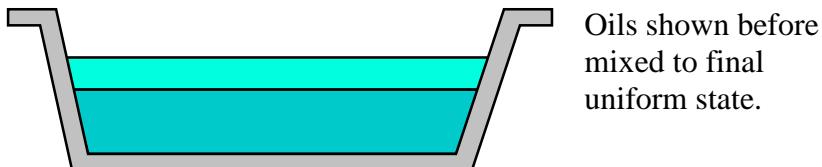
$$\text{Energy Eq.: } m_2 u_2 - m_A u_A - m_B u_B \approx 0 - 0$$

$$\Delta u \approx C_v \Delta T \text{ so same } C_v = 1.9 \text{ kJ/kg K for all oil states.}$$

$$T_2 = \frac{m_A}{m_2} T_A + \frac{m_B}{m_2} T_B = \frac{5}{8} \times 20 + \frac{3}{8} \times 100 = 50.0^\circ\text{C} = \mathbf{323.15 \text{ K}}$$

$$\begin{aligned} S_2 - S_1 &= m_2 s_2 - m_A s_A - m_B s_B = m_A(s_2 - s_A) + m_B(s_2 - s_B) \\ &= 0.005 \times 885 \times 1.9 \ln \frac{323.15}{293.15} + 0.003 \times 885 \times 1.9 \ln \frac{323.15}{373.15} \\ &= 0.8192 - 0.7257 = + \mathbf{0.0935 \text{ kJ/K}} \end{aligned}$$

Entropy generation is the total change in S, recall Eq.6.39, no external Q



6.71

A computer CPU chip consists of 50 g silicon, 20 g copper, 50 g polyvinyl chloride (plastic). It heats from 15°C to 75°C as the computer is turned on. How much did the entropy increase?

C.V. CPU chip. The process has electrical work input and no heat transfer.

$$\text{Entropy Eq.: } S_2 - S_1 = \sum m_i(s_2 - s_1)_i = \int dQ/T + _1S_2 \text{ gen} = _1S_2 \text{ gen}$$

For the solid masses we will use the specific heats, Table A.3, and they all have the same temperature so

$$\begin{aligned}\sum m_i(s_2 - s_1)_i &= \sum m_i C_i \ln(T_2 / T_1)_i = \ln(T_2 / T_1) \sum m_i C_i \\ \sum m_i C_i &= 0.05 \times 0.7 + 0.02 \times 0.42 + 0.05 \times 0.96 = 0.0914 \text{ kJ/K}\end{aligned}$$

$$S_2 - S_1 = 0.0914 \text{ kJ/K} \times \ln(348.15 / 288.15) = \mathbf{0.0173 \text{ kJ/K}}$$

6.72

A 5-kg aluminum radiator holds 2 kg of liquid R-134a at -10°C . The setup is brought indoors and heated with 220 kJ. Find the final temperature and the change in entropy of all the mass.

Solution:

C.V. The aluminum radiator and the R-134a.

$$\text{Energy Eq.3.5: } m_2 u_2 - m_1 u_1 = \dot{Q}_2 - 0$$

Process: No change in volume so no work as used above.

The energy equation now becomes (summing over the mass)

$$m_{\text{al}}(u_2 - u_1)_{\text{al}} + m_{\text{R134a}}(u_2 - u_1)_{\text{R134a}} = \dot{Q}_2$$

Use specific heat from Table A.3 and A.4

$$m_{\text{al}}C_{\text{al}}(T_2 - T_1) + m_{\text{R134a}}C_{\text{R134a}} \ln(T_2/T_1) = \dot{Q}_2$$

$$T_2 - T_1 = \dot{Q}_2 / [m_{\text{al}}C_{\text{al}} + m_{\text{R134a}}C_{\text{R134a}}]$$

$$= 220 \text{ kJ} / [5 \times 0.9 + 2 \times 1.43] \text{ kJ/K} = 29.89^{\circ}\text{C}$$

$$T_2 = -10 + 29.89 = 19.89^{\circ}\text{C}$$

Entropy change for solid (A.3) and liquid (A.4) from Eq.6.11

$$\begin{aligned} S_2 - S_1 &= m_{\text{al}}(s_2 - s_1)_{\text{al}} + m_{\text{R134a}}(s_2 - s_1)_{\text{R134a}} \\ &= m_{\text{al}}C_{\text{al}} \ln(T_2/T_1) + m_{\text{R134a}}C_{\text{R134a}} \ln(T_2/T_1) \\ &= (5 \times 0.9 + 2 \times 1.43) \text{ kJ/K} \ln \frac{(19.89 + 273.15)}{-10 + 273.15} \\ &= \mathbf{0.792 \text{ kJ/K}} \end{aligned}$$



6.73

A 12 kg steel container has 0.2 kg superheated water vapor at 1000 kPa, both at 200°C. The total mass is now cooled to ambient temperature 30°C. How much heat transfer was taken out and what is the steel-water entropy change?

Solution:

C.V.: Steel and the water, control mass of constant volume.

$$\text{Energy Eq.3.5: } U_2 - U_1 = \dot{Q}_2 - \dot{W}_2$$

$$\text{Process: } V = \text{constant} \Rightarrow \dot{W}_2 = 0$$

$$\text{State 1: H}_2\text{O Table B.1.3: } u_1 = 2621.9 \text{ kJ/kg, } v_1 = 0.20596 \text{ m}^3/\text{kg, } s_1 = 6.6939 \text{ kJ/kg K}$$

$$\text{State 2: H}_2\text{O: } T_2, v_2 = v_1 \Rightarrow \text{from Table B.1.1}$$

$$x_2 = \frac{v - v_f}{v_{fg}} = \frac{0.20596 - 0.001004}{32.8922} = 0.006231$$

$$u_2 = 125.77 + x_2 \times 2290.81 = 140.04 \text{ kJ/kg}$$

$$s_2 = 0.4369 + x_2 \times 8.0164 = 0.48685 \text{ kJ/kg K}$$

$$\begin{aligned} \dot{Q}_2 &= m(u_2 - u_1) = m_{\text{steel}} C_{\text{steel}} (T_2 - T_1) + m_{\text{H}_2\text{O}} (u_2 - u_1)_{\text{H}_2\text{O}} \\ &= 12 \text{ kg} \times 0.46 \text{ kJ/kg-K} (30 - 200) \text{ K} + 0.2 \text{ kg} (140.04 - 2621.9) \text{ kJ/kg} \\ &= \mathbf{-1434.8 \text{ kJ}} \end{aligned}$$

Entropy changes from Eq.6.11 and the water tables

$$\begin{aligned} S_2 - S_1 &= m_2 s_2 - m_1 s_1 = m_{\text{steel}} C_{\text{steel}} \ln \left(\frac{T_2}{T_1} \right) + m_{\text{H}_2\text{O}} (s_2 - s_1)_{\text{H}_2\text{O}} \\ &= 12 \times 0.46 \text{ kJ/K} \times \ln \left(\frac{303.15}{473.15} \right) + 0.2 \text{ kg} (0.48685 - 6.6939) \text{ kJ/kg-K} \\ &= -2.4574 - 1.2414 \\ &= \mathbf{-3.699 \text{ kJ/K}} \end{aligned}$$

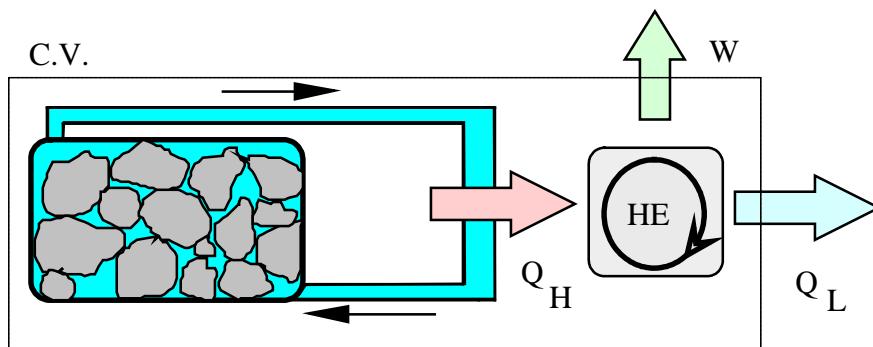
6.74

Find the total work the heat engine can give out as it receives energy from the rock bed as described in Problem 5.60 (see Fig.P 6.74). Hint: write the entropy balance equation for the control volume that is the combination of the rock bed and the heat engine.

Solution:

To get the work we must integrate over the process or do the 2nd law for a control volume around the whole setup out to T_0

C.V. Heat engine plus rock bed out to T_0 . W and Q_L goes out.



$$\text{Energy Eq.3.5: } (U_2 - U_1)_{\text{rock}} = -Q_L - W$$

$$\text{Entropy Eq.6.3, 11: } (S_2 - S_1)_{\text{rock}} = -\frac{Q_L}{T_0} = mC \ln \left(\frac{T_2}{T_1} \right)$$

$$= 5500 \times 0.89 \ln \frac{290}{400} = -1574.15 \text{ kJ/K}$$

$$Q_L = -T_0 (S_2 - S_1)_{\text{rock}} = -290 (-1574.15) = 456\,504 \text{ kJ}$$

The energy drop of the rock $-(U_2 - U_1)_{\text{rock}}$ equals Q_H into heat engine

$$(U_2 - U_1)_{\text{rock}} = mC (T_2 - T_1) = 5500 \times 0.89 (290 - 400) = -538\,450 \text{ kJ}$$

$$W = -(U_2 - U_1)_{\text{rock}} - Q_L = 538\,450 - 456\,504 = \mathbf{81\,946 \text{ kJ}}$$

6.75

Consider problem 6.60 if the two blocks of steel exchange energy through a heat engine similar to the setup in Problem 6.74. Find the work output of the heat engine.

C.V. Both blocks, no external heat transfer, C from Table A.3.

$$\begin{aligned}\text{Energy Eq.: } U_2 - U_1 &= m_A(u_2 - u_1)_A + m_B(u_2 - u_1)_B = 0 - W \\ &= m_A C(T_2 - T_{A1}) + m_B C(T_2 - T_{B1})\end{aligned}$$

$$\text{Entropy Eq.6.37: } S_2 - S_1 = m_A(s_2 - s_1)_A + m_B(s_2 - s_1)_B = \dot{S}_{2 \text{ gen}}$$

Process: Assume reversible then $\dot{S}_{2 \text{ gen}} = 0$

Entropy changes from Eq.6.11

$$S_2 - S_1 = m_A C \ln \frac{T_2}{T_{A1}} + m_B C \ln \frac{T_2}{T_{B1}} = 0$$

Now solve for T_2 by combining the “ln” terms as

$$0 = \ln \frac{T_2}{T_{A1}} + \ln \left(\frac{T_2}{T_{B1}} \right)^{m_B/m_A} = \ln \left[\frac{T_2}{T_{A1}} \left(\frac{T_2}{T_{B1}} \right)^{m_B/m_A} \right]$$

So the factors inside the ln function equal to one. The rewrite as

$$\frac{T_2}{T_{A1}} \left(\frac{T_2}{T_{B1}} \right)^{m_B/m_A} = 1 \quad \Rightarrow \quad T_2 = T_{A1}^{x_A} T_{B1}^{x_B}$$

$$x_A = m_A / (m_A + m_B); \quad x_B = m_B / (m_A + m_B)$$

In the actual case the two masses are the same so $x_A = x_B = 1/2$ and the result is

$$T_2 = \sqrt{T_{A1} T_{B1}} = \sqrt{523.15 \times 298.15} = 394.94 \text{ K} = 121.8^\circ\text{C}$$

The work output comes from the energy equation

$$\begin{aligned}W &= m_A C(T_{A1} - T_2) + m_B C(T_{B1} - T_2) \\ &= 5 \text{ kg} \times 0.46 \text{ kJ/kg-K} \times [(250 - 121.8) + (25 - 121.8)] \text{ K} \\ &= \mathbf{72.2 \text{ kJ}}\end{aligned}$$

6.76

Two kg of liquid lead initially at 400°C is poured into a form. It then cools at constant pressure down to room temperature of 20°C as heat is transferred to the room. The melting point of lead is 327°C and the enthalpy change between the phases, h_{if} , is 24.6 kJ/kg. The specific heats are in Tables A.3 and A.4. Calculate the net entropy change for the mass.

Solution:

C.V. Lead, constant pressure process

$$m_{Pb}(u_2 - u_1)_{Pb} = Q_2 - P(V_2 - V_1)$$

We need to find changes in enthalpy ($u + Pv$) for each phase separately and then add the enthalpy change for the phase change.

Consider the process in several steps:

Cooling liquid to the melting temperature

Solidification of the liquid to solid, recall $s_{if} = h_{if}/T$, see page 268

Cooling of the solid to the final temperature

$$\begin{aligned} Q_2 &= m_{Pb}(h_2 - h_1) = m_{Pb}(h_2 - h_{327,sol} - h_{if} + h_{327,f} - h_{400}) \\ &= 2 \text{ kg} \times [0.138 \times (20 - 327) - 24.6 + 0.155 \times (327 - 400)] \text{ kJ/kg} \\ &= -84.732 - 49.2 - 22.63 = -156.56 \text{ kJ} \end{aligned}$$

$$\begin{aligned} S_2 - S_1 &= m_{Pb}[C_p \ln(T_2/600) - (h_{if}/600) + C_p \ln(600/T_1)] \\ &= 2 \times \left[0.138 \ln \frac{293.15}{600} - \frac{24.6}{600} + 0.155 \ln \frac{600}{673.15} \right] = -0.315 \text{ kJ/K} \end{aligned}$$



Entropy of ideal gases

Excerpts from this work may be reproduced by instructors for distribution on a not-for-profit basis for testing or instructional purposes only to students enrolled in courses for which this textbook has been adopted. Any other reproduction or translation of this work beyond that permitted by Sections 107 or 108 of the 1976 United States Copyright Act without the permission of the copyright owner is unlawful.

6.77

Air inside a rigid tank is heated from 300 to 350 K. Find the entropy increase $s_2 - s_1$? What if it is from 1300 to 1350 K?

Process: $V = C \rightarrow _1 W_2 = \emptyset$

Entropy change from Eq.6.17:

$$a) \quad s_2 - s_1 = C_{vo} \ln \left(\frac{T_2}{T_1} \right) = 0.717 \ln \left(\frac{350}{300} \right) = \mathbf{0.1105 \text{ kJ/kgK}}$$

$$b) \quad s_2 - s_1 = C_{vo} \ln \left(\frac{T_2}{T_1} \right) = 0.717 \ln \left(\frac{1350}{1300} \right) = \mathbf{0.02706 \text{ kJ/kgK}}$$

From A.7:

case a) $C_v \approx \Delta u / \Delta T = 36/50 = 0.72 \text{ kJ/kg K}$, see A.5

case b) $C_v \approx \Delta u / \Delta T = 45.2/50 = 0.904 \text{ kJ/kg K}$ (25 % higher)

so result should have been 0.0341 kJ/kgK

6.78

A rigid tank contains 1 kg methane at 500 K, 1500 kPa. It is now cooled down to 300 K. Find the heat transfer and the change in entropy using ideal gas.

Ideal gas, constant volume so there is no work.

$$\text{Energy Eq. 3.5: } m(u_2 - u_1) = {}_1Q_2 - {}_1W_2 = {}_1Q_2$$

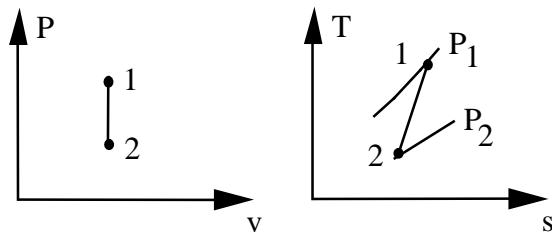
Use specific heat from Table A.5

$$u_2 - u_1 = C_v (T_2 - T_1) = 1.736 (300 - 500) = -347.2 \text{ kJ/kg}$$

$${}_1Q_2 = m(u_2 - u_1) = 1 (-347.2) = \mathbf{-347.2 \text{ kJ}}$$

The change in s for an ideal gas, Eqs.6.16-17 and $v_2 = v_1$ gives

$$\begin{aligned} m(s_2 - s_1) &= m [C_{vo} \ln \frac{T_2}{T_1} + R \ln \frac{v_2}{v_1}] = m C_{vo} \ln \frac{T_2}{T_1} \\ &= 1 \text{ kg} \times 1.736 \text{ kJ/kg-K} \ln \frac{300}{500} = \mathbf{-0.8868 \text{ kJ/K}} \end{aligned}$$



6.79

Three kg of air is in a piston/cylinder keeping constant pressure at 27°C, 300 kPa. It is now heated to 500 K. Plot the process path in a T-s diagram and find the heat transfer in the process.

Solution:

CV Air, so this is a control mass.

$$\text{Energy Eq.3.5: } U_2 - U_1 = m(u_2 - u_1) = _1Q_2 - _1W_2$$

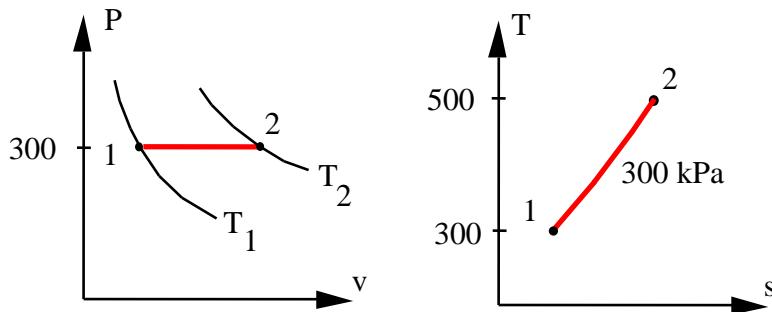
$$\text{Process: } P = C \text{ so } _1W_2 = \int P dV = P_1(V_2 - V_1) = m(Pv_2 - Pv_1)$$

$$\text{State 1: } T_1, P_1 \quad \text{ideal gas so } P_1V_1 = mRT_1$$

$$\text{State 2: } T_2, P_2 = P_1 \quad \text{and ideal gas so } P_2V_2 = mRT_2$$

From the energy equation

$$\begin{aligned} _1Q_2 &= m(u_2 - u_1) + m(Pv_2 - Pv_1) = m(h_2 - h_1) = mC_{P_0}(T_2 - T_1) \\ &= 3 \text{ kg} \times 1.004 \text{ kJ/kg-K} \times (500 - 300) \text{ K} = \mathbf{602.4 \text{ kJ}} \end{aligned}$$



If we were to find the change in entropy we get

$$s_2 - s_1 = C_{P_0} \ln(T_2 / T_1) = 1.004 \ln(500/300) = 0.5129 \text{ kJ/kgK}$$

The process curve in the T-s diagram is: $T = T_1 \exp[(s - s_1)/C_{P_0}]$

6.80

A piston/cylinder setup contains air at 100 kPa, 400 K which is compressed to a final pressure of 1000 kPa. Consider two different processes (i) a reversible adiabatic process and (ii) a reversible isothermal process. Show both processes in P-v and a T-s diagram. Find the final temperature and the specific work for both processes.

Solution:

C.V. Air, control mass of unknown size and mass.

$$\text{Energy Eq.3.5: } u_2 - u_1 = q_1 - w_1$$

$$\text{Entropy Eq.6.3: } s_2 - s_1 = \int dq/T$$

$$\text{i) } dq = 0 \text{ so } q_1 = 0$$

$$\text{ii) } T = C \text{ so } \int dq/T = q_1/T$$

i) For this process the entropy equation reduces to:

$$s_2 - s_1 = 0 \text{ so we have constant } s, \text{ an isentropic process.}$$

The relation for an ideal gas, constant s and k becomes Eq.6.23

$$T_2 = T_1 \left(\frac{P_2}{P_1} \right)^{\frac{k-1}{k}} = 400 \left(\frac{1000}{100} \right)^{\frac{0.4}{1.4}} = 400 \times 10^{0.28575} = 772 \text{ K}$$

From the energy equation we get the work term

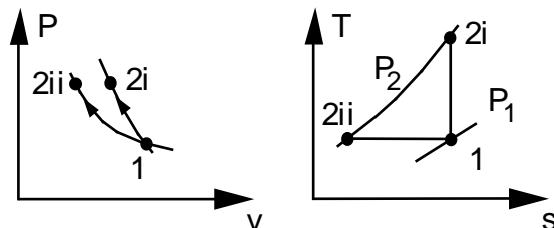
$$w_1 = u_1 - u_2 = C_v(T_1 - T_2) = 0.717(400 - 772) = -266.7 \text{ kJ/kg}$$

ii) For this process $T_2 = T_1$ so since ideal gas we get

$$u_2 = u_1 \text{ also } s_{T2}^o = s_{T1}^o \Rightarrow \text{Energy Eq.: } w_1 = q_1$$

Now from the entropy equation we solve for q_1

$$\begin{aligned} w_1 &= q_1 = T(s_2 - s_1) = T[s_{T2}^o - s_{T1}^o - R \ln \frac{P_2}{P_1}] = -RT \ln \frac{P_2}{P_1} \\ &= -0.287 \times 400 \ln 10 = -264 \text{ kJ/kg} \end{aligned}$$



6.81

Prove that the two relations for changes in s , Eqs.6.16 and 6.17 are equivalent once we assume constant specific heat. Hint: recall the relation for specific heat in Eq. 3.42.

From Eq. 3.42:

$$C_{po} = C_{vo} + R$$

$$\text{Start with Eq.6.16: } s_2 - s_1 = C_{po} \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1}$$

Now substitute Eq.3.42 to get:

$$\begin{aligned} s_2 - s_1 &= (C_{vo} + R) \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1} \\ &= C_{vo} \ln \frac{T_2}{T_1} - R \left[\ln \frac{P_2}{P_1} - \ln \frac{T_2}{T_1} \right] \\ &= C_{vo} \ln \frac{T_2}{T_1} - R \ln \left(\frac{P_2 T_1}{P_1 T_2} \right) \end{aligned}$$

Use the ideal gas law $Pv = RT$ for both states to get the ratio

$$\frac{P_2 v_2}{R T_2} = \frac{P_1 v_1}{R T_1} \Rightarrow \frac{P_2}{P_1} \frac{T_1}{T_2} = \frac{v_1}{v_2}$$

so then we get to Eq. 6.17 as

$$\begin{aligned} s_2 - s_1 &= C_{vo} \ln \frac{T_2}{T_1} - R \ln \frac{v_1}{v_2} \\ &= C_{vo} \ln \frac{T_2}{T_1} + R \ln \frac{v_2}{v_1} \end{aligned}$$

6.82

A closed rigid container is filled with 1.5 kg water at 100 kPa, 55°C, 1 kg of stainless steel and 0.5 kg of PVC (polyvinyl chloride) both at 20°C and 0.1 kg of hot air at 400 K, 100 kPa. It is now left alone, with no external heat transfer and no water vaporizes. Find the final temperature and the change in entropy of the masses.

$$\text{Energy Eq.: } U_2 - U_1 = \sum m_i(u_2 - u_1)_i = Q_2 - W_2 = 0$$

$$\text{Process: } Q_2 = 0; \quad W_2 = 0$$

For the liquid and the metal masses we will use the specific heats (Tbl A.3, A.4) so

$$\sum m_i(u_2 - u_1)_i = \sum m_i C_{v,i} (T_2 - T_1)_i = T_2 \sum m_i C_{v,i} - \sum m_i C_{v,i} T_1$$

noticing that all masses have the same T_2 but not same initial T .

$$\sum m_i C_{v,i} = 1.5 \times 4.18 + 1 \times 0.46 + 0.5 \times 0.96 + 0.1 \times 0.717 = 7.282 \text{ kJ/K}$$

$$\begin{aligned} \text{Energy Eq.: } 7.282 T_2 &= 1.5 \times 4.18 \times 55 + (1 \times 0.46 + 0.5 \times 0.96) \times 20 \\ &\quad + 0.1 \times 0.717 \times (400 - 273.15) = 372.745 \text{ kJ} \end{aligned}$$

$$T_2 = 51.2^\circ\text{C} = 324.3 \text{ K}$$

The change in entropy for the solids and liquid follow Eq.6.11 and that for the ideal gas is from Eq.6.16 or 6.17 or 6.19, we use here 6.17 since $v_2 = v_1$

$$\begin{aligned} S_2 - S_1 &= \sum m_i (s_2 - s_1)_i = \sum C_{v,i} m_i [\ln(T_2 / T_1)_i] \\ &= 1.5 \times 4.18 \ln \frac{324.3}{328.15} + 1 \times 0.46 \ln \frac{324.3}{293.15} \\ &\quad + 0.5 \times 0.96 \ln \frac{324.3}{293.15} + 0.1 \times 0.717 \ln \frac{324.3}{400} \\ &= 0.00588 \text{ kJ/K} \end{aligned}$$

6.83

Water at 150°C, 400 kPa, is brought to 1200°C in a constant pressure process. Find the change in the specific entropy, using a) the steam tables, b) the ideal gas water Table A.8, and c) the specific heat from A.5.

Solution:

a)

State 1: Table B.1.3 Superheated vapor $s_1 = 6.9299 \text{ kJ/kgK}$

State 2: Table B.1.3 $s_2 = 9.7059 \text{ kJ/kgK}$

$$s_2 - s_1 = 9.7059 - 6.9299 = \mathbf{2.776 \text{ kJ/kgK}}$$

b)

Table A.8 at 423.15 K: $s_{T1}^o = 11.13891 \text{ kJ/kgK}$

Table A.8 at 1473.15 K: $s_{T2}^o = 13.86383 \text{ kJ/kgK}$

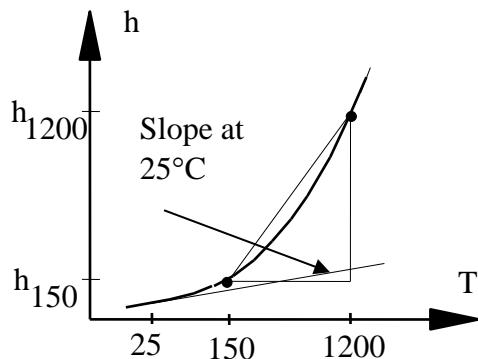
$$s_2 - s_1 = s_{T2}^o - s_{T1}^o - R \ln \frac{P_2}{P_1} = s_{T2}^o - s_{T1}^o$$

$$s_{T2}^o - s_{T1}^o = 13.86383 - 11.13891 = \mathbf{2.72492 \text{ kJ/kgK}}$$

c) Table A.5: $C_{po} = 1.872 \text{ kJ/kgK}$

$$s_2 - s_1 \approx C_{po} \ln \left(\frac{T_2}{T_1} \right) = 1.872 \ln \left(\frac{1473.15}{423.15} \right) = \mathbf{2.3352 \text{ kJ/kgK}}$$

Notice how the average slope from 150°C to 1200°C is higher than the one at 25°C ($= C_{po}$)



6.84

R-410A at 400 kPa is brought from 20°C to 120°C in a constant pressure process. Evaluate the change in specific entropy using Table B.4 and using ideal gas with $C_p = 0.81 \text{ kJ/kgK}$.

$$\begin{aligned} \text{Table B.4.2} \quad s_1 &= 1.2108 \text{ kJ/kg-K}, \quad s_2 = 1.4788 \text{ kJ/kg-K} \\ s_2 - s_1 &= 1.4788 - 1.2108 = \mathbf{0.268 \text{ kJ/kg-K}} \end{aligned}$$

$$\text{Eq. 6.16: } s_2 - s_1 \approx C_{p0} \ln \frac{T_2}{T_1} = 0.81 \ln \frac{393.15}{293.15} = \mathbf{0.238 \text{ kJ/kg-K}}$$

Two explanations for the difference are as the average temperature is higher than 25°C we could expect a higher value of the specific heat and secondly it is not an ideal gas (if you calculate $Z = Pv/RT = 0.94$).

6.85

R-410A at 300 kPa, 20°C is brought to 200°C in a constant volume process. Evaluate the change in specific entropy using Table B.4 and using ideal gas with $C_v = 0.695 \text{ kJ/kgK}$.

Table B.4.2 $s_1 = 1.2485 \text{ kJ/kg-K}$, $v_1 = 0.10720 \text{ m}^3/\text{kg}$
 $s_2 = 1.6413 \text{ kJ/kg-K}$, $v_2 = 0.10714 \text{ m}^3/\text{kg}$ at 500 kPa
(very close, we could have interpolated between 400 and 500 kPa)
 $s_2 - s_1 = 1.6413 - 1.2485 = \mathbf{0.3928 \text{ kJ/kg-K}}$

$$\text{Eq. 6.17: } s_2 - s_1 \approx C_{vo} \ln \frac{T_2}{T_1} = 0.695 \ln \frac{473.15}{293.15} = \mathbf{0.333 \text{ kJ/kg-K}}$$

Two explanations for the difference are as the average temperature is higher than 25°C we could expect a higher value of the specific heat and secondly it is not an ideal gas (if you calculate $Z = P_v/RT = 0.96$).

6.86

Consider a small air pistol with a cylinder volume of 1 cm³ at 250 kPa, 27°C. The bullet acts as a piston initially held by a trigger. The bullet is released so the air expands in an adiabatic process. If the pressure should be 120 kPa as the bullet leaves the cylinder find the final volume and the work done by the air.

Solution:

C.V. Air. Assume a reversible, adiabatic process.

$$\text{Energy Eq.3.5: } u_2 - u_1 = 0 - \dot{W}_2 ;$$

$$\text{Entropy Eq.6.37: } s_2 - s_1 = \int dq/T + \dot{s}_{2 \text{ gen}} = 0$$

State 1: (T₁, P₁) State 2: (P₂, ?)

So we realize that one piece of information is needed to get state 2.

Process: Adiabatic $\dot{q}_2 = 0$ Reversible $\dot{s}_{2 \text{ gen}} = 0$

With these two terms zero we have a zero for the entropy change. So this is a constant s (isentropic) expansion process giving s₂ = s₁. From Eq.6.23

$$T_2 = T_1 \left(\frac{P_2}{P_1} \right)^{\frac{k-1}{k}} = 300 \left(\frac{120}{250} \right)^{\frac{0.4}{1.4}} = 300 \times 0.48^{0.28575} = 243.24 \text{ K}$$

The ideal gas law PV = mRT at both states leads to

$$V_2 = V_1 P_1 T_2 / P_2 T_1 = 1 \times 250 \times 243.24 / 120 \times 300 = \mathbf{1.689 \text{ cm}^3}$$

The work term is from Eq.6.29 or Eq.4.4 with polytropic exponent n = k

$$\begin{aligned} \dot{W}_2 &= \frac{1}{1-k} (P_2 V_2 - P_1 V_1) = \frac{1}{1-1.4} (120 \times 1.689 - 250 \times 1) \times 10^{-6} \text{ kPa m}^3 \\ &= \mathbf{0.118 \text{ J}} \end{aligned}$$

6.87

Oxygen gas in a piston cylinder at 300 K, 100 kPa with volume 0.1 m³ is compressed in a reversible adiabatic process to a final temperature of 700 K. Find the final pressure and volume using Table A.5 and repeat the process with Table A.8.

Solution:

C.V. Air. Assume a reversible, adiabatic process.

$$\text{Energy Eq.3.5: } u_2 - u_1 = 0 - w_1 ;$$

$$\text{Entropy Eq.6.37: } s_2 - s_1 = \int dq/T + s_{\text{gen}} = 0$$

$$\text{Process: Adiabatic } q_1 = 0 \quad \text{Reversible } s_{\text{gen}} = 0$$

$$\text{Properties: Table A.5: } k = 1.393$$

With these two terms zero we have a zero for the entropy change. So this is a constant s (isentropic) expansion process. From Eq.6.23

$$P_2 = P_1 \left(\frac{T_2}{T_1} \right)^{\frac{k}{k-1}} = 100 \left(\frac{700}{300} \right)^{\frac{1.393}{1-1.393}} = 2015 \text{ kPa}$$

Using the ideal gas law to eliminate P from this equation leads to Eq.6.24

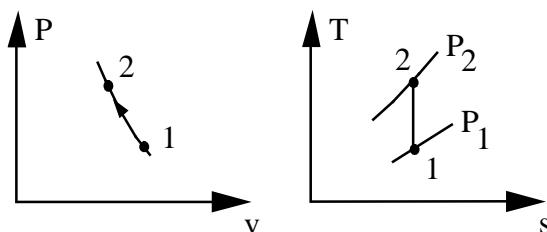
$$V_2 = V_1 \left(\frac{T_2}{T_1} \right)^{\frac{1}{k-1}} = 0.1 \times \left(\frac{700}{300} \right)^{\frac{1}{1-1.393}} = 0.0116 \text{ m}^3$$

Using the ideal gas tables A.8 we get

$$s_2 - s_1 = s_{T2}^o - s_{T1}^o - R \ln(P_2/P_1) = 0 \quad \text{or} \quad P_2 = P_1 \exp[(s_{T2}^o - s_{T1}^o)/R]$$

$$P_2 = 100 \exp[(7.2336 - 6.4168)/0.2598] = 2319.5 \text{ kPa}$$

$$V_2 = V_1 (T_2 / T_1)(P_1 / P_2) = 0.1 \text{ m}^3 \times \frac{700}{300} \times \frac{100}{2319.5} = 0.010 \text{ m}^3$$



6.88

Air in a rigid tank is at 100 kPa, 300 K with a volume of 0.75 m³. The tank is heated to 400 K, state 2. Now one side of the tank acts as a piston, letting the air expand slowly at constant temperature to state 3 with a volume of 1.5 m³. Find the entropy at states 1, 2 and 3.

Solution:

$$\text{State 1: } (P_1, T_1) \quad m = P_1 V_1 / RT_1 = \frac{100 \times 0.75}{0.287 \times 300} \frac{\text{kPa m}^3}{\text{kJ/kg}} = 0.871 \text{ kg}$$

Process 1 to 2: Constant volume heating, $dV = 0 \Rightarrow \dot{W}_2 = 0$

$$\text{State 2: } T_2, P_2 = P_1 T_2 / T_1 = 100 \times 400 / 300 = 133.3 \text{ kPa}$$

Process 2 to 3: Isothermal expansion, $dT = 0 \Rightarrow u_3 = u_2$ and

$$\text{State 3: } T_3, P_3 = P_2 V_2 / V_3 = 133.3 \times 0.75 / 1.5 = 66.67 \text{ kPa}$$

The entropy is generally used as a relative value with different reference states in different tables. We can use here the reference as in Table A.7 so

$$s = s_T^\circ - R \ln(P/P_0)$$

where s_T° is absolute entropy from A.7 at T and $P_0 = 100$ kPa

$$s_1 = 6.86926 \text{ kJ/kgK} \quad \text{which is at 300 K, 100 kPa}$$

$$s_2 = 7.15926 - 0.287 \ln(133.3/100) = 7.07677 \text{ kJ/kgK}$$

$$s_3 = 7.15926 - 0.287 \ln(66.67/100) = 7.27561 \text{ kJ/kgK}$$

To get total entropy for the given mass multiply with $m = 0.871$ kg, which will give **(5.9831, 6.1639, 6.3371)** all in kJ/K.

6.89

An insulated piston/cylinder setup contains carbon dioxide gas at 800 kPa, 300 K which is then compressed to 6 MPa in a reversible adiabatic process. Calculate the final temperature and the specific work using a) ideal gas tables A.8 and b) using constant specific heats A.5.

Solution:

C.V. CO₂, a control mass undergoing a reversible, adiabatic process.

$$\text{Energy Eq.3.5: } u_2 - u_1 = 0 - \dot{w}_2 ;$$

$$\text{Entropy Eq.6.37: } s_2 - s_1 = \int dq/T + \dot{s}_{2\text{ gen}} = 0$$

$$\text{Process: Adiabatic } \dot{q}_2 = 0 \quad \text{Reversible } \dot{s}_{2\text{ gen}} = 0$$

State 1: (300 K, 800 kPa) State 2: (6000 kPa, ?)

With two terms zero in the entropy equation we have a zero for the entropy change. So this is a constant s (isentropic) expansion process, $s_2 = s_1$.

a) Table A.8 for CO₂ and Eq.6.19

$$s_2 - s_1 = 0 = s_{T2}^o - s_{T1}^o - R \ln(P_2/P_1)$$

$$s_{T2}^o = s_{T1}^o + R \ln \frac{P_2}{P_1} = 4.8631 + 0.1889 \ln \frac{6000}{800} = 5.2437 \text{ kJ/kg-K}$$

Now interpolate in A.8 to find T₂

$$T_2 = 450 + 50 (5.2437 - 5.2325)/(5.3375 - 5.2325) = \mathbf{455.3 \text{ K}}$$

$$\dot{w}_2 = -(u_2 - u_1) = -(271 - 157.7) = \mathbf{-113.3 \text{ kJ/kg}}$$

b) Table A.5: $k = 1.289$, $C_{V0} = 0.653 \text{ kJ/kg K}$ and now Eq.6.23

$$T_2 = T_1 \left(\frac{P_2}{P_1} \right)^{\frac{k-1}{k}} = 300 \left(\frac{6000}{800} \right)^{0.224} = \mathbf{471.1 \text{ K}}$$

$$\dot{w}_2 = -C_{V0}(T_2 - T_1) = -0.653 \text{ kJ/kg-K} (471.1 - 300) \text{ K} = \mathbf{-111.7 \text{ kJ/kg}}$$

6.90

Extend the previous problem to solve using Table B.3.

C.V. CO₂, a control mass undergoing a reversible, adiabatic process.

$$\text{Energy Eq.3.5: } u_2 - u_1 = 0 - \dot{w}_2 ;$$

$$\text{Entropy Eq.6.37: } s_2 - s_1 = \int dq/T + \dot{s}_{2\text{ gen}} = 0$$

$$\text{Process: Adiabatic } \dot{q}_2 = 0 \quad \text{Reversible } \dot{s}_{2\text{ gen}} = 0$$

State 1: (300 K, 800 kPa) State 2: (6000 kPa, ?)

With two terms zero in the entropy equation we have a zero for the entropy change. So this is a constant s (isentropic) expansion process, $s_2 = s_1$.

From Table B.3.2: $s_1 = 1.8240 \text{ kJ/kg-K}$, $u_1 = 333.5 \text{ kJ/kg}$

$$T_2 = 260.7^\circ\text{C} = \mathbf{533.9 \text{ K}}, \quad u_2 = 502.9 \text{ kJ/kg}$$

$$\dot{w}_2 = -(u_2 - u_1) = -(502.9 - 333.5) = \mathbf{-169.4 \text{ kJ/kg}}$$

6.91

A handheld pump for a bicycle has a volume of 25 cm^3 when fully extended. You now press the plunger (piston) in while holding your thumb over the exit hole so that an air pressure of 300 kPa is obtained. The outside atmosphere is at P_0, T_0 . Consider two cases: (1) it is done quickly ($\sim 1 \text{ s}$), and (2) it is done very slowly ($\sim 1 \text{ h}$).

- State assumptions about the process for each case.
- Find the final volume and temperature for both cases.

Solution:

C.V. Air in pump. Assume that both cases result in a reversible process.

$$\text{State 1: } P_0, T_0 \quad \text{State 2: } 300 \text{ kPa, ?}$$

One piece of information must resolve the ? for a state 2 property.

Case I) Quickly means no time for heat transfer

$Q = 0$, so a reversible adiabatic compression.

$$u_2 - u_1 = -w_2 ; \quad s_2 - s_1 = \int dq/T + s_{\text{gen}} = 0$$

With constant s and constant heat capacity we use Eq.6.23

$$T_2 = T_1 \left(\frac{P_2}{P_1} \right)^{\frac{k-1}{k}} = 298 \left(\frac{300}{101.325} \right)^{\frac{0.4}{1.4}} = 405.3 \text{ K}$$

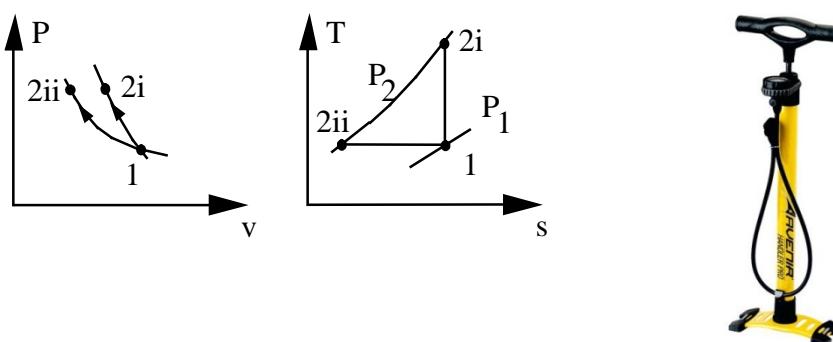
Use ideal gas law $PV = mRT$ at both states so ratio gives

$$\Rightarrow V_2 = P_1 V_1 T_2 / T_1 P_2 = 11.48 \text{ cm}^3$$

Case II) Slowly, time for heat transfer so $T = \text{constant} = T_0$.

The process is then a reversible isothermal compression.

$$T_2 = T_0 = 298 \text{ K} \quad \Rightarrow \quad V_2 = V_1 P_1 / P_2 = 8.44 \text{ cm}^3$$



6.92

A piston/cylinder, shown in Fig. P6.92, contains air at 1380 K, 15 MPa, with $V_1 = 10 \text{ cm}^3$, $A_{\text{cyl}} = 5 \text{ cm}^2$. The piston is released, and just before the piston exits the end of the cylinder the pressure inside is 200 kPa. If the cylinder is insulated, what is its length? How much work is done by the air inside?

Solution:

C.V. Air, Cylinder is insulated so adiabatic, $Q = 0$.

Continuity Eq.: $m_2 = m_1 = m$,

Energy Eq.3.5: $m(u_2 - u_1) = _1Q_2 - _1W_2 = - _1W_2$

Entropy Eq.6.37: $m(s_2 - s_1) = \int dQ/T + _1S_{2 \text{ gen}} = 0 + _1S_{2 \text{ gen}}$

State 1: (T_1, P_1) State 2: $(P_2, ?)$

So one piece of information is needed for the ?, assume reversible process.

$$_1S_{2 \text{ gen}} = 0 \Rightarrow s_2 - s_1 = 0 \text{ (which is also Eq.6.3)}$$

State 1: Table A.7: $u_1 = 1095.2 \text{ kJ/kg}$, $s_{T1}^\circ = 8.5115 \text{ kJ/kg K}$

$$m = P_1 V_1 / RT_1 = \frac{15000 \times 10 \times 10^{-6}}{0.287 \times 1380} = 0.000379 \text{ kg}$$

State 2: P_2 and from Entropy eq.: $s_2 = s_1$ so from Eq.6.19

$$s_{T2}^\circ = s_{T1}^\circ + R \ln \frac{P_2}{P_1} = 8.5115 + 0.287 \ln \left(\frac{200}{15000} \right) = 7.2724 \text{ kJ/kg K}$$

Now interpolate in Table A.7 to get T_2

$$T_2 = 440 + 20 (7.2724 - 7.25607) / (7.30142 - 7.25607) = 447.2 \text{ K}$$

$$u_2 = 315.64 + (330.31 - 315.64) 0.36 = 320.92 \text{ kJ/kg}$$

$$V_2 = V_1 \frac{T_2 P_1}{T_1 P_2} = \frac{10 \times 447.2 \times 15000}{1380 \times 200} = 243 \text{ cm}^3$$

$$\Rightarrow L_2 = V_2 / A_{\text{cyl}} = 243/5 = \mathbf{48.6 \text{ cm}}$$

$$_1W_2 = u_1 - u_2 = 774.3 \text{ kJ/kg}, \quad _1W_2 = m_1 w_2 = \mathbf{0.2935 \text{ kJ}}$$

6.93

Argon in a light bulb is at 90 kPa and 20°C when it is turned on and electric input now heats it to 60°C. Find the entropy increase of the argon gas.

Solution:

C.V. Argon gas. Neglect any heat transfer.

Energy Eq.3.5: $m(u_2 - u_1) = \dot{W}_2$ electrical in

Entropy Eq.6.37: $s_2 - s_1 = \int dq/T + s_{2\text{ gen}} = s_{2\text{ gen}}$

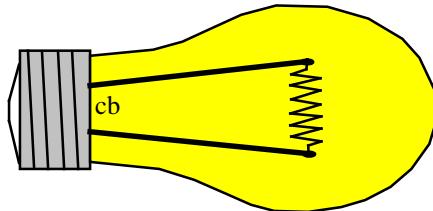
Process: $v = \text{constant}$ and ideal gas $\Rightarrow P_2/P_1 = T_2/T_1$

Evaluate changes in s from Eq.6.16 or 8.17

$$s_2 - s_1 = C_p \ln(T_2/T_1) - R \ln(P_2/P_1) \quad \text{Eq.6.16}$$

$$= C_p \ln(T_2/T_1) - R \ln(T_2/T_1) = C_v \ln(T_2/T_1) \quad \text{Eq.6.17}$$

$$= 0.312 \text{ kJ/kg-K} \times \ln \left[\frac{60 + 273}{20 + 273} \right] = \mathbf{0.04 \text{ kJ/kg K}}$$



Since there was no heat transfer but work input all the change in s is generated by the process (irreversible conversion of W to internal energy)

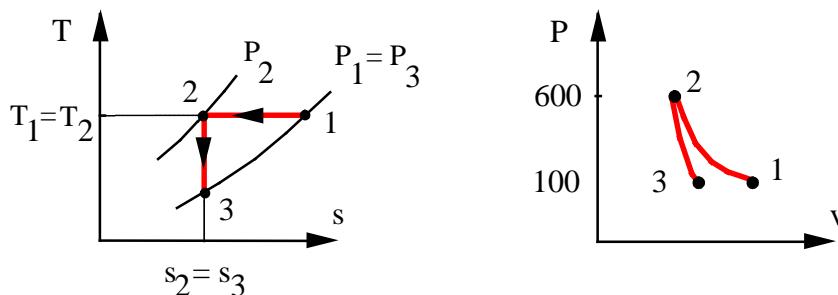
6.94

We wish to obtain a supply of cold helium gas by applying the following technique. Helium contained in a cylinder at ambient conditions, 100 kPa, 20°C, is compressed in a reversible isothermal process to 600 kPa, after which the gas is expanded back to 100 kPa in a reversible adiabatic process.

- Show the process on a $T-s$ diagram.
- Calculate the final temperature and the net work per kilogram of helium.

Solution:

a)



- The adiabatic reversible expansion gives constant s from the entropy equation Eq.6.37. With ideal gas and constant specific heat this gives relation in Eq.6.23

$$T_3 = T_2 (P_3/P_2)^{\frac{k-1}{k}} = 293.15 \times (100/600)^{0.4} = \mathbf{143.15 \text{ K}}$$

The net work is summed up over the two processes. The isothermal process has work as Eq.6.31

$$\begin{aligned} {}_1w_2 &= -RT_1 \ln(P_2/P_1) = -2.0771 \text{ kJ/kg-K} \times 293.15 \text{ K} \times \ln(600/100) \\ &= -1091.0 \text{ kJ/kg} \end{aligned}$$

The adiabatic process has a work term from energy equation with no q

$${}_2w_3 = C_{V0}(T_2 - T_3) = 3.116 \text{ kJ/kg-K} (293.15 - 143.15) \text{ K} = +467.4 \text{ kJ/kg}$$

The net work is the sum

$$w_{\text{NET}} = -1091.0 + 467.4 = \mathbf{-623.6 \text{ kJ/kg}}$$

6.95

A 1-m³ insulated, rigid tank contains air at 800 kPa, 25°C. A valve on the tank is opened, and the pressure inside quickly drops to 150 kPa, at which point the valve is closed. Assuming that the air remaining inside has undergone a reversible adiabatic expansion, calculate the mass withdrawn during the process.

Solution:

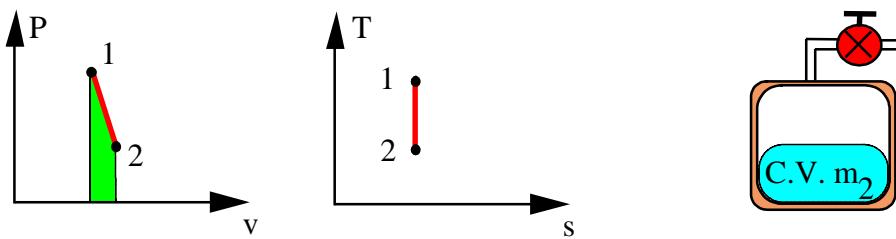
C.V.: Air remaining inside tank, m_2 .

Cont.Eq.: $m_2 = m$;

Energy Eq.3.5: $m(u_2 - u_1) = _1Q_2 - _1W_2$

Entropy Eq.6.37: $m(s_2 - s_1) = \int dQ/T + _1S_{2\text{ gen}}$

Process: adiabatic $_1Q_2 = 0$ and reversible $_1S_{2\text{ gen}} = 0$



Entropy eq. then gives $s_2 = s_1$ and ideal gas gives the relation in Eq.6.23

$$T_2 = T_1(P_2/P_1)^{\frac{k-1}{k}} = 298.2 \text{ K} (150/800)^{0.286} = \mathbf{184.8 \text{ K}}$$

$$m_1 = P_1 V / RT_1 = (800 \text{ kPa} \times 1 \text{ m}^3) / (0.287 \text{ kJ/kgK} \times 298.2 \text{ K}) = 9.35 \text{ kg}$$

$$m_2 = P_2 V / RT_2 = (150 \text{ kPa} \times 1 \text{ m}^3) / (0.287 \text{ kJ/kgK} \times 184.8 \text{ K}) = 2.83 \text{ kg}$$

$$m_e = m_1 - m_2 = \mathbf{6.52 \text{ kg}}$$

6.96

Two rigid, insulated tanks are connected with a pipe and valve. One tank has 0.5 kg air at 200 kPa, 300 K and the other has 0.75 kg air at 100 kPa, 400 K. The valve is opened and the air comes to a single uniform state without any heat transfer. Find the final temperature and the change in entropy of the air.

Solution:

C.V. Total tank. Control mass of constant volume.

$$\text{Mass and volume: } m_2 = m_A + m_B; \quad V = V_A + V_B$$

$$\text{Energy Eq.: } U_2 - U_1 = m_2 u_2 - m_A u_{A1} - m_B u_{B1} = 1Q_2 - 1W_2 = 0$$

$$\text{Process Eq.: } V = \text{constant} \Rightarrow 1W_2 = 0; \quad \text{Insulated} \Rightarrow 1Q_2 = 0$$

$$\text{Ideal gas at A1: } V_A = m_A R T_{A1} / P_{A1} = 0.5 \times 0.287 \times 300 / 200 = 0.2153 \text{ m}^3$$

$$\text{Ideal gas at B1: } V_B = m_B R T_{B1} / P_{B1} = 0.75 \times 0.287 \times 400 / 100 = 0.861 \text{ m}^3$$

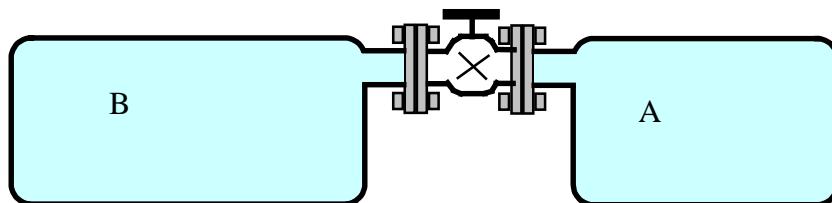
$$\text{State 2: } m_2 = m_A + m_B = 1.25 \text{ kg}; \quad V_2 = V_A + V_B = 1.0763 \text{ m}^3$$

$$\text{Energy Eq.: } u_2 = \frac{m_A u_{A1} + m_B u_{B1}}{m_2} \quad \text{and use constant specific heat}$$

$$T_2 = \frac{m_A}{m_2} T_{A1} + \frac{m_B}{m_2} T_{B1} = \frac{0.5}{1.25} 300 + \frac{0.75}{1.25} 400 = 360 \text{ K}$$

$$P_2 = m_2 R T_2 / V = 1.25 \text{ kg} \times 0.287 \text{ kJ/kgK} \times 360 \text{ K} / 1.0763 \text{ m}^3 = 120 \text{ kPa}$$

$$\begin{aligned} S_2 - S_1 &= m_A [C_P \ln T_2 / T_{A1} - R \ln(P_2 / P_{A1})] + m_B [C_P \ln T_2 / T_{B1} - R \ln(P_2 / P_{B1})] \\ &= 0.5 \left[1.004 \ln \frac{360}{300} - 0.287 \ln \frac{120}{200} \right] + 0.75 \left[1.004 \ln \frac{360}{400} - 0.287 \ln \frac{120}{100} \right] \\ &= 0.5 \times 1.3514 + 0.75 \times (-0.1581) = 0.5571 \text{ kJ/K} \end{aligned}$$



6.97

Two rigid tanks, shown in Fig. P6.97, each contain 10 kg N₂ gas at 1000 K, 500 kPa. They are now thermally connected to a reversible heat pump, which heats one and cools the other with no heat transfer to the surroundings. When one tank is heated to 1500 K the process stops. Find the final (P, T) in both tanks and the work input to the heat pump, assuming constant heat capacities.

Solution:

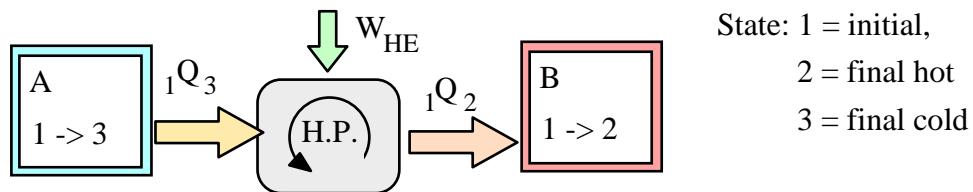
Control volume of hot tank B,

Process = constant volume & mass so no work

Energy equation Eq.3.5 and specific heat in Eq.5.20 gives

$$U_2 - U_1 \cong mC_v(T_2 - T_1) = 1Q_2 = 10 \text{ kg} \times 0.7448 \text{ kJ/kg-K} \times 500 \text{ K} = 3724 \text{ kJ}$$

$$P_2 = P_1 T_2 / T_1 = 1.5(P_1) = \mathbf{750 \text{ kPa}}$$



To fix temperature in cold tank, C.V.: total

For this CV only W_{HP} cross the control surface no heat transfer. The entropy equation Eq.6.37 for a reversible process becomes

$$(S_2 - S_1)_{\text{tot}} = 0 = m_{\text{hot}}(s_2 - s_1) + m_{\text{cold}}(s_3 - s_1)$$

Use specific heats to evaluate the changes in s from Eq.6.16 and division by m

$$C_{p,\text{hot}} \ln(T_2 / T_1) - R \ln(P_2 / P_1) + C_{p,\text{cold}} \ln(T_3 / T_1) - R \ln(P_3 / P_1) = 0$$

$$P_3 = P_1 T_3 / T_1 \quad \text{and} \quad P_2 = P_1 T_2 / T_1$$

Now everything is in terms of T and $C_p = C_v + R$, so

$$C_{v,\text{hot}} \ln(T_2 / T_1) + C_{v,\text{cold}} \ln(T_3 / T_1) = 0$$

$$\text{same } C_v: \quad T_3 = T_1(T_1/T_2) = \mathbf{667 \text{ K}}, \quad P_3 = \mathbf{333 \text{ kPa}}$$

$$Q_{\text{cold}} = -1Q_3 = m C_v (T_3 - T_1) = -2480 \text{ kJ},$$

$$W_{HP} = 1Q_2 + Q_{\text{cold}} = 1Q_2 - 1Q_3 = \mathbf{1244 \text{ kJ}}$$

6.98

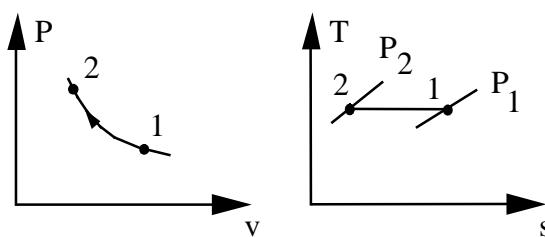
A hydrogen gas in a piston/cylinder assembly at 300 K, 100 kPa with a volume of 0.1 m³ is now slowly compressed to a volume of 0.01 m³ while cooling it in a reversible isothermal process. What is the final pressure, the heat transfer and the change in entropy?

Solution:

C.V. Hydrogen, control mass.

$$\text{Energy Eq. 3.5: } m(u_2 - u_1) = \int Q_2 - \int W_2$$

$$\text{Process: } T = \text{constant} \text{ so with ideal gas} \Rightarrow u_2 = u_1$$



From the process equation ($T_2 = T_1$) and ideal gas law we get

$$P_2 = P_1 (V_1/V_2) = 10 P_1 = \mathbf{1000 \text{ kPa}}$$

we can calculate the work term as in Eq.3.22

$$\begin{aligned} \int Q_2 - \int W_2 &= \int P dV = P_1 V_1 \ln(V_2/V_1) \\ &= 100 \text{ kPa} \times 0.1 \text{ m}^3 \times \ln(1/10) = \mathbf{-23.0 \text{ kJ}} \end{aligned}$$

The change of entropy from the entropy equation Eq.6.3 is

$$m(s_2 - s_1) = \int Q_2/T_1 = \frac{-23 \text{ kJ}}{300 \text{ K}} = \mathbf{-0.07667 \text{ kJ/K}}$$

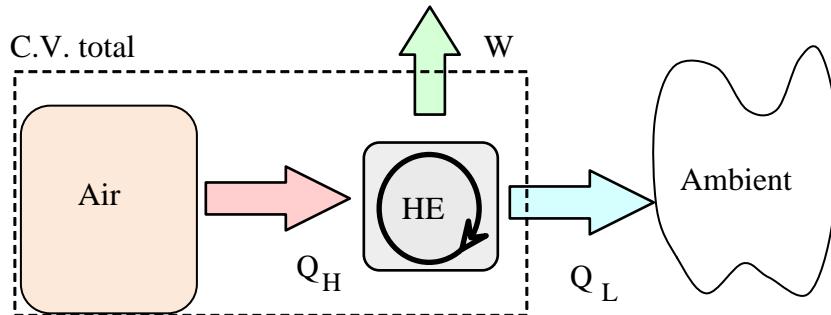
If instead we use Eq.6.17 we would get

$$\begin{aligned} \Delta S &= m(s_2 - s_1) = m(C_{vo} \ln \frac{T_2}{T_1} + R \ln \frac{V_2}{V_1}) = m R \ln \frac{V_2}{V_1} \\ &= (P_1 V_1 / T_1) \ln \left(\frac{V_2}{V_1} \right) = \int Q_2/T_1 \end{aligned}$$

consistent with the above result.

6.99

A rigid tank contains 4 kg air at 300°C, 4 MPa which acts as the hot energy reservoir for a heat engine with its cold side at 20°C shown in Fig. P.6.99. Heat transfer to the heat engine cools the air down in a reversible process to a final 20°C and then stops. Find the final air pressure and the work output of the heat engine.



$$\text{Energy Eq.3.5: } m(u_2 - u_1) = \int Q_2 - \int W_2$$

$$\text{Entropy Eq.6.3: } m(s_2 - s_1) = \int dQ/T \quad (\text{T not constant})$$

$$\text{Process: } v = \text{constant} \Rightarrow \int W_2 = 0$$

$$\text{State 2: } T_2 \text{ and } v_2 = v_1,$$

$$P_2 = P_1 T_2 / T_1 = 4000 \times 293.15 / 573.15 = \mathbf{2045.9 \text{ kPa}}$$

From the energy equation

$$\begin{aligned} Q_H &= -\int Q_2 = -m(u_2 - u_1) = -m C_{vo} (T_2 - T_1) \\ &= -4 \text{ kg} \times 0.717 \text{ kJ/kg-K} (293.15 - 573.15) \text{ K} = \mathbf{803.0 \text{ kJ}} \end{aligned}$$

Take now CV total as the air plus heat engine out to ambient

$$\text{Entropy Eq.6.3: } m(s_2 - s_1) = -\frac{\int Q_L}{T_{amb}} \Rightarrow$$

$$Q_L = -m T_{amb} (s_2 - s_1) = -m T_{amb} C_{vo} \ln \frac{T_2}{T_1}$$

$$= -4 \text{ kg} \times 293.15 \text{ K} \times 0.717 \text{ kJ/kg-K} \ln(293.15 / 573.15) = 563.7 \text{ kJ}$$

Now the CV heat engine can give the engine work from the energy equation

$$\text{Energy H.E.: } W_{HE} = Q_H - Q_L = 803.0 - 563.7 = \mathbf{239.3 \text{ kJ}}$$

Notice to get $\int Q_2 = \int T ds$ we must know the function $T(s)$ which we do not readily have for this process.

Borgnakke and Sonntag

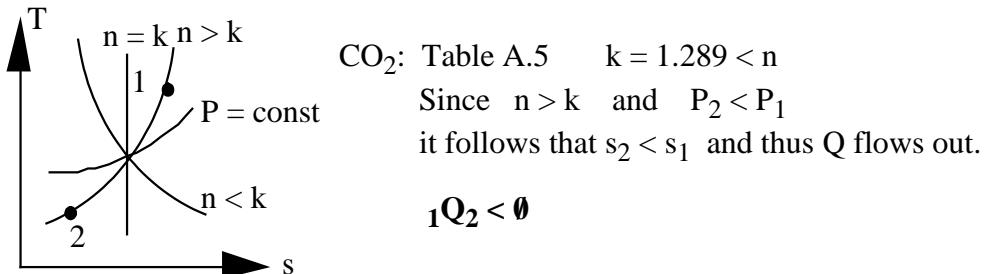
Polytropic processes

Excerpts from this work may be reproduced by instructors for distribution on a not-for-profit basis for testing or instructional purposes only to students enrolled in courses for which this textbook has been adopted. Any other reproduction or translation of this work beyond that permitted by Sections 107 or 108 of the 1976 United States Copyright Act without the permission of the copyright owner is unlawful.

6.100

An ideal gas having a constant specific heat undergoes a reversible polytropic expansion with exponent, $n = 1.4$. If the gas is carbon dioxide will the heat transfer for this process be positive, negative, or zero?

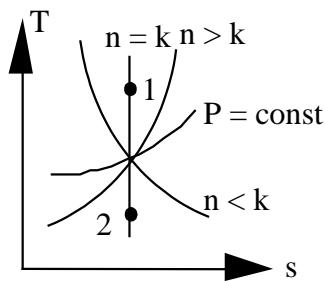
Solution:



6.101

Repeat the previous problem for the gas carbon monoxide, CO.

Solution:

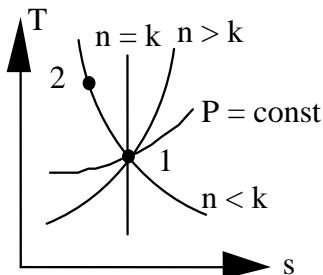


CO: Table A.5 $k = 1.399 = n$
 Since $n \approx k$ and $P_2 < P_1$
 it follows that $s_2 \approx s_1$ and thus adiabatic.

$$1Q_2 \approx 0$$

6.102

A nitrogen gas goes through a polytropic process with $n = 1.3$ in a piston/cylinder arrangement. It starts out at 600 K, 600 kPa and ends at 800 K. Is the heat transfer positive, negative or zero?



N_2 : Table A.5 $k = 1.40$ and $n = 1.3 < k$

Since $n < k$ and $T_2 > T_1$

process goes up on the $n < k$ curve

$s_2 < s_1$ and thus q must go out.

q is negative

6.103

A cylinder/piston contains 1 kg methane gas at 100 kPa, 300 K. The gas is compressed reversibly to a pressure of 800 kPa. Calculate the work required if the process is adiabatic.

Solution:

C.V. Methane gas of constant mass $m_2 = m_1 = m$ and reversible process.

$$\text{Energy Eq.3.5: } m(u_2 - u_1) = _1Q_2 - _1W_2$$

$$\text{Entropy Eq.6.3 (37): } m(s_2 - s_1) = \int dQ/T + _1S_2 \text{ gen} = \int dQ/T$$

$$\text{Process: } _1Q_2 = 0 \Rightarrow s_2 = s_1$$

thus isentropic process $s = \text{constant}$ and ideal gas gives the relation in Eq.6.23 with $k = 1.299$ from Table A.5

$$T_2 = T_1 (P_2/P_1)^{\frac{k-1}{k}} = 300 \text{ K} \left(\frac{800}{100}\right)^{0.230} = 483.99 \text{ K}$$

$$\begin{aligned} _1W_2 &= -mC_{V0}(T_2 - T_1) = -1 \text{ kg} \times 1.736 \text{ kJ/kg-K} (483.99 - 293.15) \text{ K} \\ &= \mathbf{-331.3 \text{ kJ}} \end{aligned}$$

6.104

Do the previous problem but assume the process is isothermal.

Solution:

C.V. Methane gas of constant mass $m_2 = m_1 = m$ and reversible process.

$$\text{Energy Eq.3.5: } m(u_2 - u_1) = _1Q_2 - _1W_2$$

$$\text{Entropy Eq.6.37: } m(s_2 - s_1) = \int dQ/T + _1S_2 \text{ gen} = \int dQ/T$$

Process: $T = \text{constant}$. For ideal gas then $u_2 = u_1$ and $s_{T2}^\circ = s_{T1}^\circ$

$$\text{Energy eq. gives } _1W_2 = _1Q_2 \text{ and } \int dQ/T = _1Q_2/T$$

with the entropy change found from Eq.6.16

$$\begin{aligned} \Rightarrow _1W_2 &= _1Q_2 = mT(s_2 - s_1) = -mRT \ln(P_2/P_1) \\ &= -0.5183 \text{ kJ/K} \times 300 \text{ K} \ln\left(\frac{800}{100}\right) = \mathbf{-323.3 \text{ kJ}} \end{aligned}$$

6.105

A piston/cylinder contains air at 300 K, 100 kPa. It is now compressed in a reversible adiabatic process to a volume 7 times as small. Use constant heat capacity and find the final pressure and temperature, the specific work and specific heat transfer for the process.

Solution:

$$\text{Expansion ratio: } v_2/v_1 = 1/7$$

Process eq.: Rev. adiabatic and ideal gas gives $Pv^n = C$, with $n = k$

$$P_2/P_1 = (v_2/v_1)^{-k} = 7^{1.4} = 15.245$$

$$P_2 = P_1 (7^{1.4}) = 100 \times 15.245 = \mathbf{1524.5 \text{ kPa}}$$

$$T_2 = T_1 (v_1/v_2)^{k-1} = 300 \times 7^{0.4} = \mathbf{653.4 \text{ K}}$$

$$q_2 = \mathbf{0 \text{ kJ/kg}}$$

Polytropic process work term from Eq.6.29

$$w_2 = \frac{R}{1-k} (T_2 - T_1) = \frac{0.287}{-0.4} \text{ kJ/kg-K} (653.4 - 300) \text{ K} = \mathbf{-253.6 \text{ kJ/kg}}$$

Notice: $C_v = R/(k-1)$ so the work term is also the change in u consistent with the energy equation.

6.106

A piston/cylinder contains pure oxygen at 500 K, 600 kPa. The piston is moved to a volume such that the final temperature is 700 K in a polytropic process with exponent $n = 1.25$. Use ideal gas approximation and constant heat capacity to find the final pressure, the specific work, and the heat transfer.

Take CV as the oxygen. $m_2 = m_1 = m$;

$$\text{Energy Eq.3.5} \quad m(u_2 - u_1) = _1Q_2 - _1W_2$$

$$\text{Process Eq.:} \quad Pv^n = \text{Constant} \quad (\text{polytropic})$$

From the ideal gas law and the process equation we can get:

$$\text{State 2: } P_2 = P_1 (v_2 / v_1)^{-n} \quad \text{and} \quad Pv = RT \Rightarrow T_2 / T_1 = (v_2 / v_1)^{1-n}$$

$$P_2 = P_1 \left(\frac{T_2}{T_1} \right)^{n/(n-1)} = 600 \left(\frac{700}{500} \right)^{1.25/0.25} = 3226.9 \text{ kPa}$$

From process eq.:

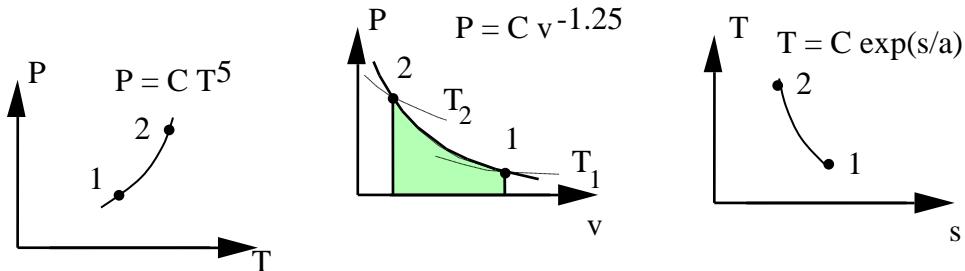
$$\begin{aligned} _1W_2 &= \int P \, dv = \text{area} = \frac{1}{1-n} (P_2 v_2 - P_1 v_1) = \frac{R}{1-n} (T_2 - T_1) \\ &= \frac{0.2598}{1-1.25} (700 - 500) = -207.8 \text{ kJ/kg} \end{aligned}$$

From the energy equation and Table A.8

$$_1q_2 = u_2 - u_1 + _1W_2 = (480.18 - 331.72 - 207.8) \text{ kJ/kg} = -59.34 \text{ kJ/kg}$$

From the energy equation and constant specific heat from Table A.5

$$\begin{aligned} _1q_2 &= u_2 - u_1 + _1W_2 = C_v (T_2 - T_1) + _1W_2 \\ &= 0.662 \text{ kJ/kg-K} (700 - 500) \text{ K} - 207.8 \text{ kJ/kg} = -75.4 \text{ kJ/kg} \end{aligned}$$



The function $T(s)$ can be done with constant C_P : $a = C_P - nR/(n-1) = -0.377$

6.107

Do Problem 6.103 and assume the process is polytropic with $n = 1.15$

Process: $Pv^n = \text{constant}$ with $n = 1.15$;

The T-P relation is given in Eq.6.28

$$T_2 = T_1 (P_2/P_1)^{\frac{n-1}{n}} = 300 \left(\frac{800}{100}\right)^{0.130} = 393.1 \text{ K}$$

and the work term is given by Eq.6.29

$$\begin{aligned} _1W_2 &= \int mP dv = m(P_2v_2 - P_1v_1)/(1 - n) = mR (T_2 - T_1)/(1 - n) \\ &= 1 \text{ kg} \times 0.5183 \frac{\text{kJ}}{\text{kg-K}} \times \frac{393.1 - 293.15}{1 - 1.15} \text{ K} = \mathbf{-345.4 \text{ kJ}} \end{aligned}$$

6.108

Hot combustion air at 2000 K expands in a polytropic process to a volume 6 times as large with $n = 1.3$. Find the specific boundary work and the specific heat transfer.

$$\text{Energy Eq.: } u_2 - u_1 = q_1 - w_1$$

$$\text{Reversible work Eq. 6.29: } w_1 = \frac{1}{1-n} (P_2 v_2 - P_1 v_1) = \frac{R}{1-n} (T_2 - T_1)$$

$$\text{Process Eq: } Pv^n = C; \quad T_2 = T_1 (v_1/v_2)^{n-1} = 2000 \left(\frac{1}{6}\right)^{0.3} = 1168.4 \text{ K}$$

Properties from Table A.7.1: $u_1 = 1677.52 \text{ kJ/kg}$, $u_2 = 905.46 \text{ kJ/kg}$

$$w_1 = \frac{0.287}{1 - 1.3} \text{ kJ/kg-K} (1168.4 - 2000) \text{ K} = \mathbf{795.6 \text{ kJ/kg}}$$

$$q_1 = u_2 - u_1 + w_1 = 905.46 - 1677.52 + 795.6 = \mathbf{23.5 \text{ kJ/kg}}$$

6.109

Air in a piston/cylinder is at 1800 K, 7 MPa and expands in a polytropic process with $n = 1.5$ to a volume eight times larger. Find the specific work and specific heat transfer in the process using Table A.7 and draw the T-s diagram.

C.V. Air of constant mass $m_2 = m_1 = m$.

$$\text{Energy Eq.3.5: } m(u_2 - u_1) = _1Q_2 - _1W_2$$

$$\text{Process: } Pv^{1.50} = \text{constant}, \quad v_2/v_1 = 8$$

$$\text{State 1: } P_1 = 7 \text{ MPa, } T_1 = 1800 \text{ K}$$

$$\text{State 2: } (v_2 = 8v_1, ?) \quad \text{Must be on process curve so}$$

$$P_2 = P_1 (v_1/v_2)^n = 7000 (1/8)^{1.50} = 309.36 \text{ kPa}$$

$$T_2 = T_1 \frac{P_2 v_2}{P_1 v_1} = T_1 (v_1/v_2)^{n-1} = 1800 (1/8)^{0.5} = 636.4 \text{ K}$$

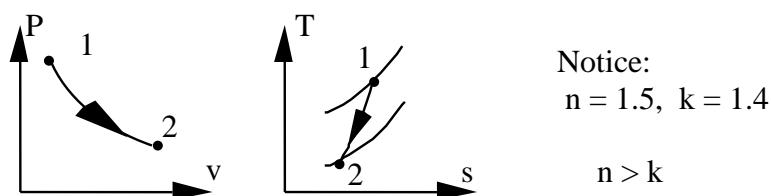
$$\text{Table A.7: } u_1 = 1486.33 \text{ kJ/kg and interpolate } u_2 = 463.06 \text{ kJ/kg}$$

Work from the process expressed in Eq.3.21, 6.29

$$\begin{aligned} _1w_2 &= \int P dv = \frac{P_2 v_2 - P_1 v_1}{1-n} = \frac{R}{1-n} (T_2 - T_1) = \frac{0.287 (636.4 - 1800)}{1 - 1.5} \\ &= \mathbf{667.9 \text{ kJ/kg}} \end{aligned}$$

Heat transfer from the energy equation

$$_1q_2 = (u_2 - u_1) + _1w_2 = (463.06 - 1486.33) + 667.9 = \mathbf{-355.4 \text{ kJ/kg}}$$



6.110

Helium in a piston/cylinder at 20°C, 100 kPa is brought to 400 K in a reversible polytropic process with exponent $n = 1.25$. You may assume helium is an ideal gas with constant specific heat. Find the final pressure and both the specific heat transfer and specific work.

Solution:

C.V. Helium

Continuity Eq.: $m_2 = m_1 = m$;

Energy Eq.3.5: $m(u_2 - u_1) = _1Q_2 - _1W_2$

Process: $Pv^n = C$ & $Pv = RT \Rightarrow Tv^{n-1} = C$

Table A.5: $C_v = 3.116 \text{ kJ/kg K}$, $R = 2.0771 \text{ kJ/kg K}$

From the process equation and $T_1 = 293.15$, $T_2 = 400 \text{ K}$

$$T_1 v^{n-1} = T_2 v^{n-1} \Rightarrow v_2 / v_1 = (T_1 / T_2)^{1/n-1} = 0.2885$$

$$P_2 / P_1 = (v_1 / v_2)^n = 4.73 \Rightarrow P_2 = \mathbf{473 \text{ kPa}}$$

The work is from Eq.6.29 per unit mass

$$\begin{aligned} _1w_2 &= \int P \, dv = \int C v^{-n} \, dv = [C / (1-n)] \times (v_2^{1-n} - v_1^{1-n}) \\ &= \frac{1}{1-n} (P_2 v_2 - P_1 v_1) = \frac{R}{1-n} (T_2 - T_1) \\ &= \frac{2.0771}{1 - 1.25} \text{ kJ/kg-K} \times (400 - 293.15) \text{ K} = \mathbf{-887.7 \text{ kJ/kg}} \end{aligned}$$

The heat transfer follows from the energy equation

$$_1q_2 = u_2 - u_1 + _1w_2 = C_v (T_2 - T_1) + (-887.7) = \mathbf{-554.8 \text{ kJ/kg}}$$

6.111

The power stroke in an internal combustion engine can be approximated with a polytropic expansion. Consider air in a cylinder volume of 0.2 L at 7 MPa, 1800 K, shown in Fig. P6.111. It now expands in a reversible polytropic process with exponent, $n = 1.5$, through a volume ratio of 10:1. Show this process on $P-v$ and $T-s$ diagrams, and calculate the work and heat transfer for the process.

Solution:

C.V. Air of constant mass $m_2 = m_1 = m$.

$$\text{Energy Eq.3.5: } m(u_2 - u_1) = _1Q_2 - _1W_2$$

$$\text{Entropy Eq.6.3 (37): } m(s_2 - s_1) = \int dQ/T + _1S_{2\text{ gen}} = \int dQ/T$$

$$\text{Process: } PV^{1.50} = \text{constant}, \quad V_2/V_1 = 10$$

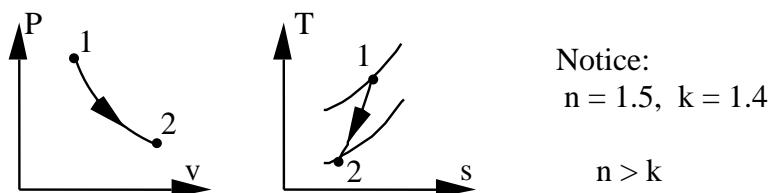
$$\text{State 1: } P_1 = 7 \text{ MPa}, \quad T_1 = 1800 \text{ K}, \quad V_1 = 0.2 \text{ L}$$

$$m_1 = \frac{P_1 V_1}{RT_1} = \frac{7000 \times 0.2 \times 10^{-3}}{0.287 \times 1800} = 2.71 \times 10^{-3} \text{ kg}$$

State 2: $(v = V_2/m, ?)$ Must be on process curve so Eq.6.24 gives

$$T_2 = T_1 (V_1/V_2)^{n-1} = 1800 (1/10)^{0.5} = 569.2 \text{ K}$$

Table A.7: $u_1 = 1486.33 \text{ kJ/kg}$ and interpolate $u_2 = 411.707 \text{ kJ/kg}$



Work from the process expressed in Eq.6.29

$$\begin{aligned} _1W_2 &= \int PdV = mR(T_2 - T_1)/(1 - n) \\ &= \frac{2.71 \times 10^{-3} \times 0.287(569.2 - 1800)}{1 - 1.5} = \mathbf{1.91 \text{ kJ}} \end{aligned}$$

Heat transfer from the energy equation

$$\begin{aligned} _1Q_2 &= m(u_2 - u_1) + _1W_2 \\ &= 2.71 \times 10^{-3} \text{ kg} \times (411.707 - 1486.33) \text{ kJ/kg} + 1.91 \text{ kJ} = \mathbf{-1.00 \text{ kJ}} \end{aligned}$$

6.112

A cylinder/piston contains saturated vapor R-410A at 10°C; the volume is 10 L. The R-410A is compressed to 2 MPa, 60°C in a reversible (internally) polytropic process. Find the polytropic exponent n and calculate the work and heat transfer.

Solution:

C.V. R-410A of constant mass $m_2 = m_1 = m$ out to ambient.

$$\text{Energy Eq.3.5: } m(u_2 - u_1) = _1Q_2 - _1W_2$$

$$\text{Entropy Eq.6.37: } m(s_2 - s_1) = \int dQ/T + _1S_2 \text{ gen} = _1Q_2/T_{\text{amb}} + _1S_2 \text{ gen}$$

$$\text{Process: } P_1 v_1^n = P_2 v_2^n \quad \text{Eq.6.36}$$

$$\text{State 1: } (T_1, x_1) \quad \text{Table B.4.1} \quad P_1 = 1085.7 \text{ kPa}, v_1 = 0.02383 \text{ m}^3/\text{kg}$$

$$m = V_1/v_1 = 0.01/0.02383 = 0.4196 \text{ kg}$$

$$\text{State 2: } (T_2, P_2) \quad \text{Table B.4.2} \quad v_2 = 0.01536 \text{ m}^3/\text{kg}$$

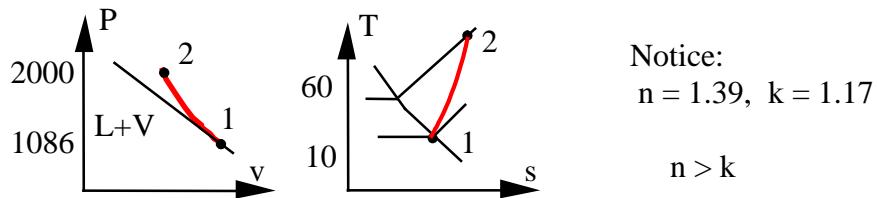
$$\text{From process eq. } P_2/P_1 = \frac{2000}{1085.7} = \left(\frac{0.02383}{0.01536}\right)^n \Rightarrow n = 1.39106$$

The work is from Eq.6.29

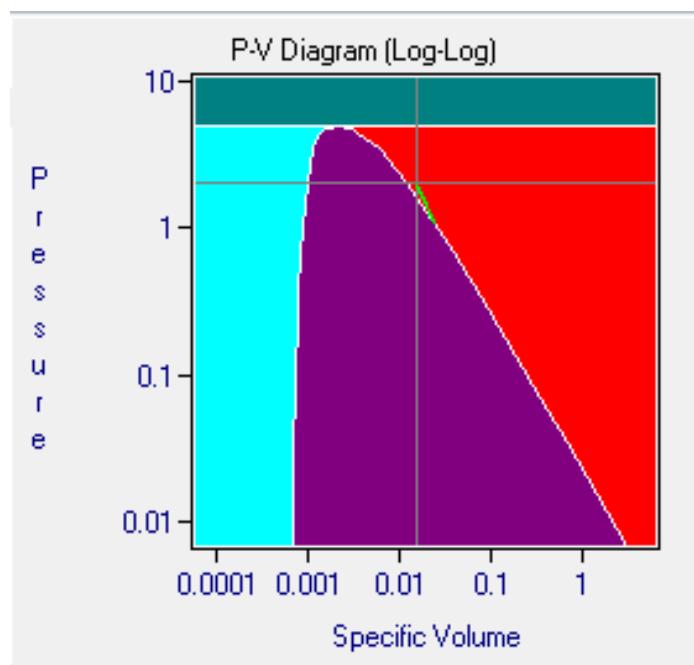
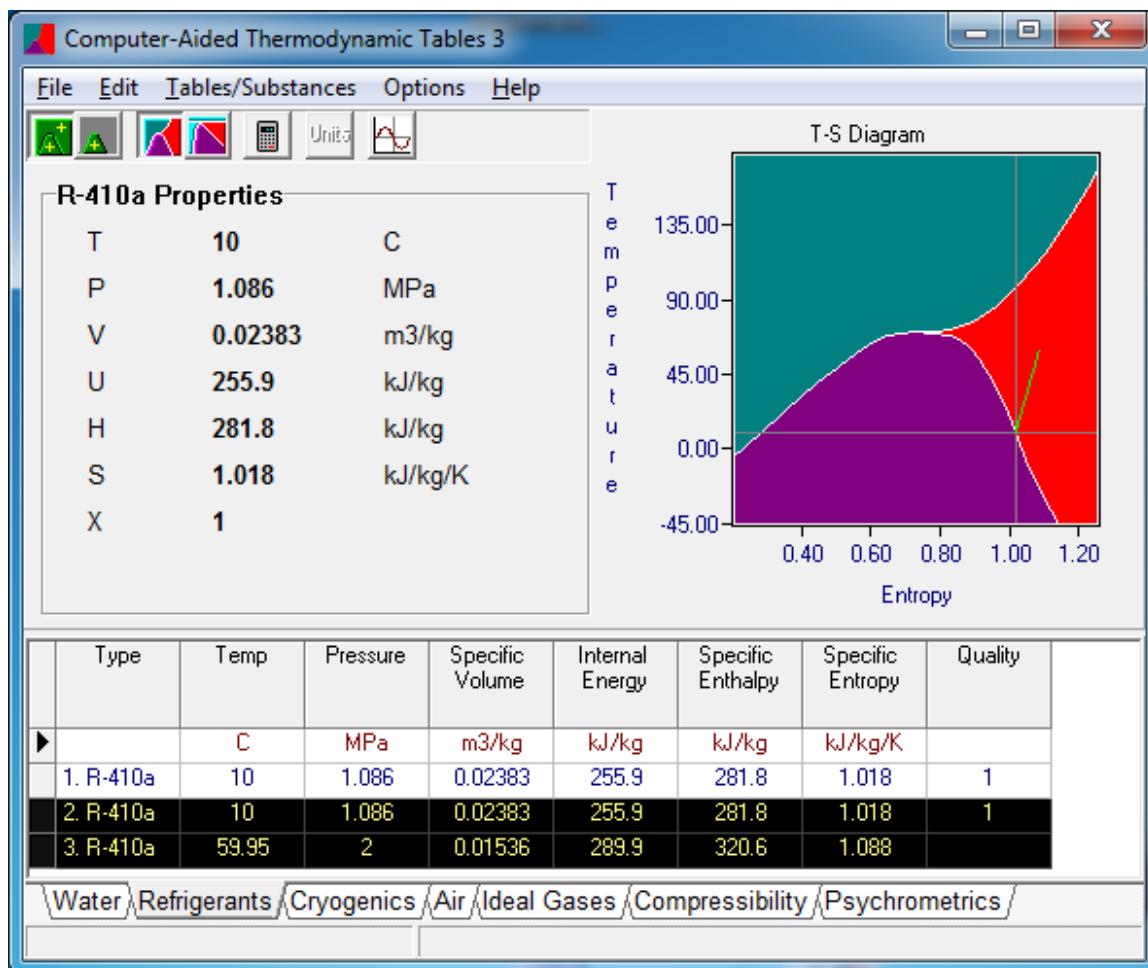
$$_1W_2 = \int P dV = m \frac{P_2 v_2 - P_1 v_1}{1-n} = 0.4196 \frac{2000 \times 0.01536 - 1085.7 \times 0.02383}{1 - 1.39106} \\ = -5.20 \text{ kJ}$$

Heat transfer from energy equation

$$_1Q_2 = m(u_2 - u_1) + _1W_2 = 0.4196 (289.9 - 255.9) - 5.20 = 9.07 \text{ kJ}$$



The two curves are shown below from the CATT3 process plot function.



Excerpts from this work may be reproduced by instructors for distribution on a not-for-profit basis for testing or instructional purposes only to students enrolled in courses for which this textbook has been adopted. Any other reproduction or translation of this work beyond that permitted by Sections 107 or 108 of the 1976 United States Copyright Act without the permission of the copyright owner is unlawful.

6.113

Air goes through a polytropic process with $n = 1.3$ in a piston/cylinder setup. It starts at 200 kPa, 300 K and ends with a pressure of 2200 kPa. Find the expansion ratio v_2/v_1 , the specific work, and the specific heat transfer.

Take CV as the air. $m_2 = m_1 = m$;

$$\text{Energy Eq.3.5} \quad m(u_2 - u_1) = 1Q_2 - 1W_2$$

$$\text{Process Eq.:} \quad Pv^n = \text{Constant} \quad (\text{polytropic})$$

From the ideal gas law and the process equation we can get:

$$\text{State 2: } (v_2/v_1) = (P_2/P_1)^{-1/n} \quad \text{and} \quad Pv = RT \Rightarrow T_2/T_1 = (v_2/v_1)^{1-n}$$

$$(v_2/v_1) = (P_2/P_1)^{-1/n} = (2200 / 200)^{-1/1.3} = \mathbf{0.1581}$$

$$T_2 = T_1 (P_2/P_1)^{(n-1)/n} = 300 \left(\frac{2200}{200}\right)^{0.3/1.3} = \mathbf{521.7 \text{ K}}$$

From process eq.:

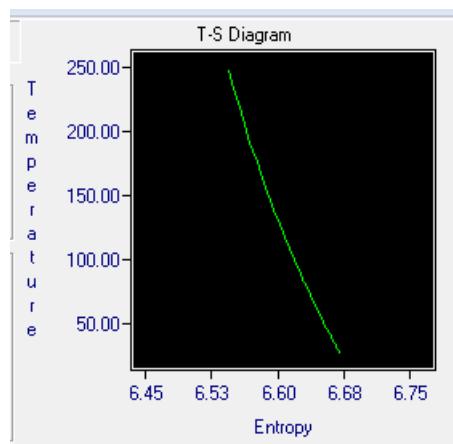
$$\begin{aligned} 1W_2 &= \int P \, dv = \text{area} = \frac{1}{1-n} (P_2 v_2 - P_1 v_1) = \frac{R}{1-n} (T_2 - T_1) \\ &= \frac{0.287}{1-1.3} (521.7 - 300) = \mathbf{-212.09 \text{ kJ/kg}} \end{aligned}$$

From the energy equation and constant specific heat from Table A.5

$$\begin{aligned} 1q_2 &= u_2 - u_1 + 1w_2 = C_v (T_2 - T_1) + 1w_2 \\ &= 0.717 \text{ kJ/kg-K} (521.7 - 300) \text{ K} - 212.09 \text{ kJ/kg} = \mathbf{-53.13 \text{ kJ/kg}} \end{aligned}$$

From the energy equation and Table A.7

$$\begin{aligned} 1q_2 &= u_2 - u_1 + 1w_2 \\ &= 376.0 - 214.36 - 212.09 = \mathbf{-50.45 \text{ kJ/kg}} \end{aligned}$$



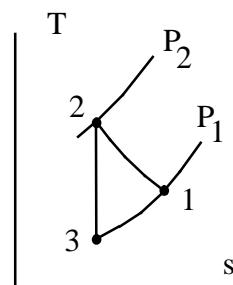
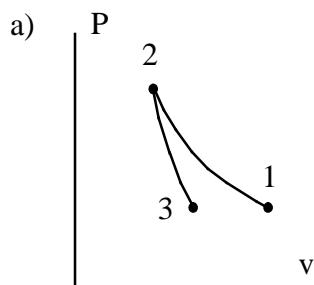
process plotted from CATT3

6.114

A cylinder/piston contains air at ambient conditions, 100 kPa and 20°C with a volume of 0.3 m³. The air is compressed to 800 kPa in a reversible polytropic process with exponent, $n = 1.2$, after which it is expanded back to 100 kPa in a reversible adiabatic process.

- a. Show the two processes in $P-v$ and $T-s$ diagrams.
- b. Determine the final temperature and the net work.

Solution:



$$\begin{aligned} m &= P_1 V_1 / RT_1 \\ &= \frac{100 \times 0.3}{0.287 \times 293.2} \\ &= 0.3565 \text{ kg} \end{aligned}$$

- b) The process equation is expressed in Eq.6.28

$$T_2 = T_1 (P_2/P_1)^{\frac{n-1}{n}} = 293.2 \left(\frac{800}{100}\right)^{0.167} = 414.9 \text{ K}$$

The work is from Eq.6.29

$$w_2 = \int_1^2 P dv = \frac{P_2 v_2 - P_1 v_1}{1-n} = \frac{R(T_2 - T_1)}{1-n} = \frac{0.287(414.9 - 293.2)}{1-1.20} = -174.6 \text{ kJ/kg}$$

Isentropic relation is from Eq.6.23

$$T_3 = T_2 (P_3/P_2)^{\frac{k-1}{k}} = 414.9 \left(\frac{100}{800}\right)^{0.286} = 228.9 \text{ K}$$

With zero heat transfer the energy equation gives the work

$$w_3 = C_v (T_2 - T_3) = 0.717(414.9 - 228.9) = +133.3 \text{ kJ/kg}$$

$$w_{NET} = 0.3565(-174.6 + 133.3) = -14.7 \text{ kJ}$$

Entropy generation

Excerpts from this work may be reproduced by instructors for distribution on a not-for-profit basis for testing or instructional purposes only to students enrolled in courses for which this textbook has been adopted. *Any other reproduction or translation of this work beyond that permitted by Sections 107 or 108 of the 1976 United States Copyright Act without the permission of the copyright owner is unlawful.*

6.115

Consider a heat transfer of 100 kJ from 1500 K hot gases to a steel container at 750 K that has a heat transfer of the 100 kJ out to some air at 375 K. Determine the entropy generation in each of the control volumes indicated in Fig. P6.115.

There is no change in energy or entropy in the indicated control volumes, so no storage effect. There is a transfer of energy in and out of each CV and an associated transfer of entropy.

Take CV1

Energy Eq.:

$$0 = Q - Q$$

Entropy Eq.:

$$0 = \frac{Q}{T_H} - \frac{Q}{T_M} + S_{\text{gen}} \text{ CV1}$$

$$\begin{aligned} S_{\text{gen}} \text{ CV1} &= \frac{Q}{T_M} - \frac{Q}{T_H} \\ &= \frac{100}{750} - \frac{100}{1500} = \mathbf{0.0667 \text{ kJ/K}} \end{aligned}$$

Take CV2

Energy Eq.:

$$0 = Q - Q$$

Entropy Eq.:

$$0 = \frac{Q}{T_M} - \frac{Q}{T_L} + S_{\text{gen}} \text{ CV2}$$

$$\begin{aligned} S_{\text{gen}} \text{ CV2} &= \frac{Q}{T_L} - \frac{Q}{T_M} \\ &= \frac{100}{375} - \frac{100}{750} = \mathbf{0.133 \text{ kJ/K}} \end{aligned}$$

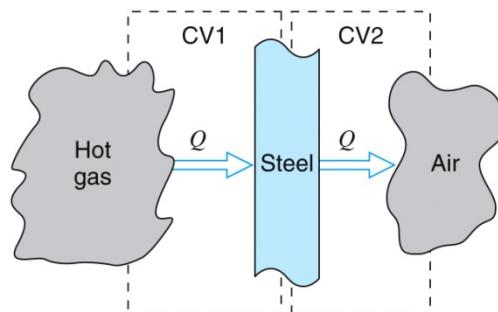


Figure P6.115
© John Wiley & Sons, Inc. All rights reserved.

6.116

A rigid tank has 0.1 kg saturated vapor R-410A at 0°C that is cooled to -20°C by a -20°C heat sink. Show the process in a T-s diagram; find the change in entropy of the R-410A, the heat sink, and the total entropy generation.

Solution:

C.V.: R-410A in tank + out to the sink. $m_2 = m_1$;

$$\text{Energy Eq.3.5: } m(u_2 - u_1) = _1Q_2 - _1W_2$$

$$\text{Entropy Eq. 6.37: } m(s_2 - s_1) = _1Q_2 / T_{\text{sink}} + _1S_{2 \text{ gen}}$$

$$\text{Process: } V = \text{constant}, v_2 = v_1 = V/m \Rightarrow _1W_2 = 0$$

Table B.4.1: State 1: $u_1 = 253.02 \text{ kJ/kg}$, $s_1 = 1.0368 \text{ kJ/kgK}$

State 2: -20°C, $v_2 = v_1 = V/m$, look in Table B.4.1 at -20°C

$$x_2 = \frac{v_2 - v_{f2}}{v_{fg2}} = \frac{0.03267 - 0.000803}{0.06400} = 0.4979$$

$$u_2 = u_{f2} + x_2 u_{fg2} = 27.92 + x_2 \times 218.07 = 136.5 \text{ kJ/kg}$$

$$s_2 = s_{f2} + x_2 s_{fg2} = 0.1154 + x_2 \times 0.9625 = 0.5946 \text{ kJ/kgK}$$

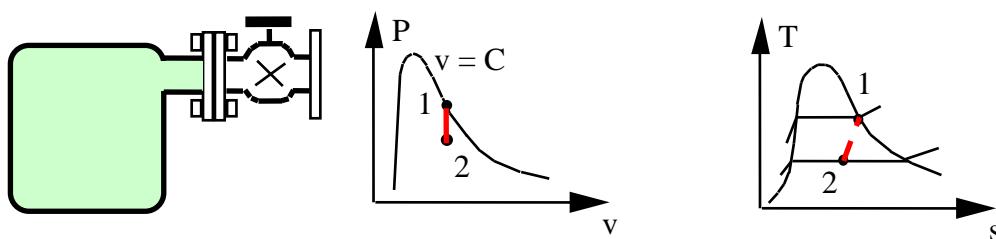
From the energy equation

$$_1Q_2 = m(u_2 - u_1) = 0.1 \text{ kg} (136.5 - 253.0) \text{ kJ/kg} = -11.65 \text{ kJ}$$

$$(S_2 - S_1)_{\text{R-410a}} = m(s_2 - s_1) = 0.1 (0.5946 - 1.0368) = -0.0442 \text{ kJ/K}$$

$$(S_2 - S_1)_{\text{sink}} = -_1Q_2 / T_{\text{sink}} = 11.65 \text{ kJ}/253.15 \text{ K} = 0.046 \text{ kJ/K}$$

$$_1S_{2 \text{ gen}} = m(s_2 - s_1) - _1Q_2 / T_{\text{sink}} = -0.0442 + 0.046 = 0.0018 \text{ kJ/K}$$



6.117

One kg water at 500°C and 1 kg saturated water vapor both at 200 kPa are mixed in a constant pressure and adiabatic process. Find the final temperature and the entropy generation for the process.

Solution:

$$\text{Continuity Eq.: } m_2 - m_A - m_B = 0$$

$$\text{Energy Eq.3.5: } m_2 u_2 - m_A u_A - m_B u_B = -W_2$$

$$\text{Entropy Eq.6.37: } m_2 s_2 - m_A s_A - m_B s_B = \int dQ/T + S_{2 \text{ gen}}$$

$$\begin{aligned} \text{Process: } P &= \text{Constant} \Rightarrow W_2 = \int P dV = P(V_2 - V_1) \\ Q &= 0 \end{aligned}$$

Substitute the work term into the energy equation and rearrange to get

$$m_2 u_2 + P_2 V_2 = m_2 h_2 = m_A u_A + m_B u_B + PV_1 = m_A h_A + m_B h_B$$

where the last rewrite used $PV_1 = PV_A + PV_B$.

State A1: Table B.1.3 $h_A = 3487.03 \text{ kJ/kg}$, $s_A = 8.5132 \text{ kJ/kg K}$

State B1: Table B.1.2 $h_B = 2706.63 \text{ kJ/kg}$, $s_B = 7.1271 \text{ kJ/kg K}$

Energy equation gives:

$$h_2 = \frac{m_A}{m_2} h_A + \frac{m_B}{m_2} h_B = \frac{1}{2} 3487.03 + \frac{1}{2} 2706.63 = 3096.83 \text{ kJ/kg}$$

State 2: $P_2, h_2 = 3096.83 \text{ kJ/kg} \Rightarrow s_2 = 7.9328 \text{ kJ/kg K}; T_2 = 312.2^\circ\text{C}$

With the zero heat transfer we have

$$\begin{aligned} S_{2 \text{ gen}} &= m_2 s_2 - m_A s_A - m_B s_B \\ &= 2 \times 7.9328 - 1 \times 8.5132 - 1 \times 7.1271 = 0.225 \text{ kJ/K} \end{aligned}$$

6.118

A car uses an average power of 25 hp for a one hour round trip. With a thermal efficiency of 35% how much fuel energy was used? What happened to all the energy? What change in entropy took place if we assume ambient at 20°C?

Since it is a round trip, there are no changes in storage of energy for the car after it has cooled down again. All the energy is given out to the ambient in the form of exhaust flow (hot air) and heat transfer from the radiator and underhood air flow.

$$W = \int \dot{W} dt = 25 \text{ hp} \times 0.7457 \text{ (kW/hp)} \times 3600 \text{ s} = 67\ 113 \text{ kJ} = \eta Q$$

Fuel energy used to deliver the W

$$Q = E / \eta = 67\ 113 \text{ kJ} / 0.35 = \mathbf{191\ 751 \text{ kJ}}$$

$$\Delta S = Q / T = 191\ 751 \text{ kJ} / 293.15 \text{ K} = \mathbf{654.1 \text{ kJ/K}}$$

All the energy (Q) ends up in the ambient at the ambient temperature.

6.119

A computer chip dissipates 2 kJ of electric work over time and rejects that as heat transfer from its 50°C surface to 25°C air. How much entropy is generated in the chip? How much if any is generated outside the chip?

C.V.1 Chip with surface at 50°C, we assume chip state is constant.

$$\text{Energy: } U_2 - U_1 = 0 = {}_1Q_2 - {}_1W_2 = W_{\text{electrical in}} - Q_{\text{out 1}}$$

$$\text{Entropy: } S_2 - S_1 = 0 = - \frac{Q_{\text{out 1}}}{T_{\text{surf}}} + {}_1S_{2 \text{ gen1}}$$

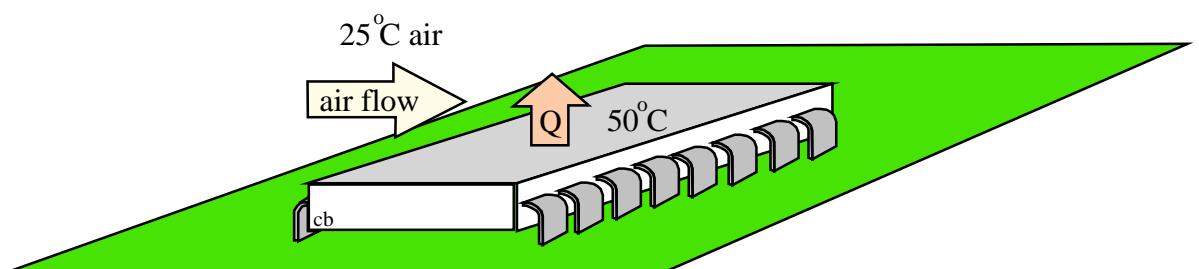
$${}_1S_{2 \text{ gen1}} = \frac{Q_{\text{out 1}}}{T_{\text{surf}}} = \frac{W_{\text{electrical in}}}{T_{\text{surf}}} = \frac{2 \text{ kJ}}{323.15 \text{ K}} = \mathbf{6.19 \text{ J/K}}$$

C.V.2 From chip surface at 50°C to air at 25°C, assume constant state.

$$\text{Energy: } U_2 - U_1 = 0 = {}_1Q_2 - {}_1W_2 = Q_{\text{out 1}} - Q_{\text{out 2}}$$

$$\text{Entropy: } S_2 - S_1 = 0 = \frac{Q_{\text{out 1}}}{T_{\text{surf}}} - \frac{Q_{\text{out 2}}}{T_{\text{air}}} + {}_1S_{2 \text{ gen2}}$$

$${}_1S_{2 \text{ gen2}} = \frac{Q_{\text{out 2}}}{T_{\text{air}}} - \frac{Q_{\text{out 1}}}{T_{\text{surf}}} = \frac{2 \text{ kJ}}{298.15 \text{ K}} - \frac{2 \text{ kJ}}{323.15 \text{ K}} = \mathbf{0.519 \text{ J/K}}$$



6.120

An insulated cylinder/piston contains R-134a at 1 MPa, 50°C, with a volume of 100 L. The R-134a expands, moving the piston until the pressure in the cylinder has dropped to 100 kPa. It is claimed that the R-134a does 190 kJ of work against the piston during the process. Is that possible?

Solution:

C.V. R-134a in cylinder. Insulated so assume $Q = 0$.

State 1: Table B.5.2, $v_1 = 0.02185 \text{ m}^3/\text{kg}$, $u_1 = 409.39 \text{ kJ/kg}$,

$$s_1 = 1.7494 \text{ kJ/kg K}, \quad m = V_1/v_1 = 0.1/0.02185 = 4.577 \text{ kg}$$

Energy Eq.3.5: $m(u_2 - u_1) = _1Q_2 - _1W_2 = 0 - 190 \Rightarrow$

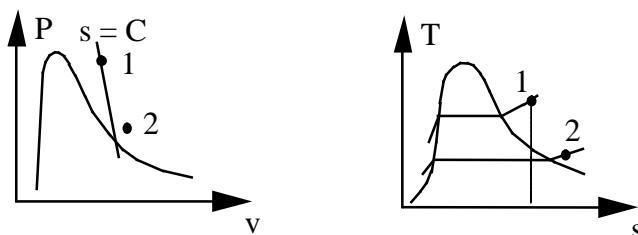
$$u_2 = u_1 - _1W_2/m = 367.89 \text{ kJ/kg}$$

State 2: $P_2, u_2 \Rightarrow$ Table B.5.2: $T_2 = -19.25^\circ\text{C}$; $s_2 = 1.7689 \text{ kJ/kg K}$

Entropy Eq.6.37: $m(s_2 - s_1) = \int dQ/T + _1S_{2,\text{gen}} = _1S_{2,\text{gen}}$

$$_1S_{2,\text{gen}} = m(s_2 - s_1) = 0.0893 \text{ kJ/K}$$

This is possible since $_1S_{2,\text{gen}} > 0$



6.121

A rigid tank holds 0.75 kg ammonia at 70°C as saturated vapor. The tank is now cooled to 20°C by heat transfer to the ambient at 20°C. Determine the amount of entropy generation during the process.

C.V. The ammonia out to 20°C, this is a control mass.

$$\text{Process: Rigid tank } V = C \Rightarrow v = \text{constant} \quad \& \quad {}_1W_2 = \int_1^2 PdV = 0$$

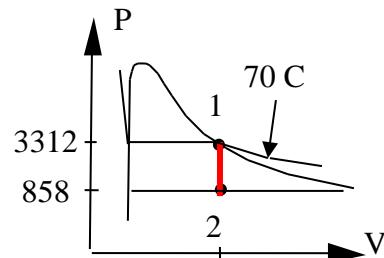
$$\text{Energy Eq.: } m(u_2 - u_1) = {}_1Q_2 - {}_1W_2 = {}_1Q_2,$$

$$\text{Entropy Eq.6.37: } m(s_2 - s_1) = \int dQ/T + {}_1S_{2,\text{gen}} = {}_1Q_2/T_0 + {}_1S_{2,\text{gen}}$$

$$\text{State 1: } v_1 = 0.03787 \text{ m}^3/\text{kg},$$

$$u_1 = 1338.9 \text{ kJ/kg, } s_1 = 4.3533 \text{ kJ/kgK}$$

State 2: $T, v \Rightarrow$ two-phase (straight down in P-v diagram from state 1)



$$x_2 = (v - v_f)/v_{fg} = (0.03787 - 0.001638)/0.14758 = 0.2455$$

$$u_2 = u_f + x_2 u_{fg} = 272.89 + 0.2455 \times 1059.3 = 532.95 \text{ kJ/kg}$$

$$s_2 = s_f + x_2 s_{fg} = 1.0408 + 0.2455 \times 4.0452 = 2.0339 \text{ kJ/kgK}$$

$${}_1Q_2 = m(u_2 - u_1) = 0.75 \text{ kg} (532.95 - 1338.9) \text{ kJ/kg} = -604.5 \text{ kJ}$$

$$\begin{aligned} {}_1S_{2,\text{gen}} &= m(s_2 - s_1) - {}_1Q_2/T_0 = 0.75(2.0339 - 4.3533) + 604.5/293.15 \\ &= \mathbf{0.322 \text{ kJ/K}} \end{aligned}$$

6.122

The unrestrained expansion of the reactor water in Problem 3.101 has a final state in the two-phase region. Find the entropy generated in the process.

A water-filled reactor with volume of 1 m^3 is at 20 MPa , 360°C and placed inside a containment room as shown in Fig. P3.101. The room is well insulated and initially evacuated. Due to a failure, the reactor ruptures and the water fills the containment room. Find the minimum room volume so the final pressure does not exceed 200 kPa .

Solution:

C.V.: Containment room and reactor.

$$\text{Mass: } m_2 = m_1 = V_{\text{reactor}}/v_1 = 1/0.001823 = 548.5 \text{ kg}$$

$$\text{Energy Eq.3.5: } m(u_2 - u_1) = {}_1Q_2 - {}_1W_2 = 0 - 0 = 0$$

$$\text{Entropy Eq.6.37: } m(s_2 - s_1) = \int dQ/T + {}_1S_2 \text{ gen}$$

$$\text{State 1: (T, P) Table B.1.4 } u_1 = 1702.8 \text{ kJ/kg, } s_1 = 3.877$$

$$\text{Energy equation implies } u_2 = u_1 = 1702.8 \text{ kJ/kg}$$

$$\text{State 2: } P_2 = 200 \text{ kPa, } u_2 < u_g \Rightarrow \text{Two-phase Table B.1.2}$$

$$x_2 = (u_2 - u_f)/u_{fg} = (1702.8 - 504.47)/2025.02 = 0.59176$$

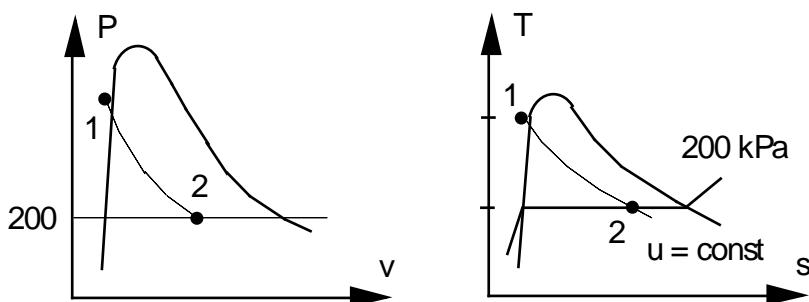
$$v_2 = 0.001061 + 0.59176 \times 0.88467 = 0.52457 \text{ m}^3/\text{kg}$$

$$s_2 = s_f + x_2 s_{fg} = 1.53 + 0.59176 \times 5.597 = 4.8421 \text{ kJ/kg K}$$

$$V_2 = m_2 v_2 = 548.5 \times 0.52457 = 287.7 \text{ m}^3$$

From the entropy equation the generation is

$${}_1S_2 \text{ gen} = m(s_2 - s_1) = 548.5 (4.8421 - 3.877) = 529.4 \text{ kJ/K}$$



Entropy is generated due to the unrestrained expansion. No work was taken out as the volume goes up.

6.123

Heat transfer from a 20°C kitchen to a block of 1.5 kg ice at -10°C melts it to liquid at 10°C. How much entropy is generated?

Water changes state from nearly saturated solid to nearly saturated liquid. The pressure is 101 kPa but we approximate the state properties with saturated state at the same temperature.

CV Ice out to the 20°C kitchen air

$$\text{Energy Eq.3.5: } m(u_2 - u_1) = {}_1Q_2 - {}_1W_2 = {}_1Q_2$$

$$\text{Entropy Eq.6.37: } m(s_2 - s_1) = \int dQ/T + {}_1S_2 \text{gen} = {}_1Q_2/T_0 + {}_1S_2 \text{gen}$$

State 1: Compressed (saturated) solid, B.1.5,

$$u_1 = -354.09 \text{ kJ/kg}, \quad s_1 = -1.2995 \text{ kJ/kg-K}$$

State 2: Compressed (saturated) liquid B.1.1

$$u_2 = 41.99 \text{ kJ/kg}, \quad s_2 = 0.1510 \text{ kJ/kg-K}$$

Heat transfer from the energy Eq.

$${}_1Q_2 = m(u_2 - u_1) = 1.5 [41.99 - (-354.09)] = 594.12 \text{ kJ}$$

From the entropy Eq.

$$\begin{aligned} {}_1S_2 \text{gen} &= m(s_2 - s_1) - {}_1Q_2/T_0 \\ &= 1.5 [0.151 - (-1.2995)] - 594.12/293.15 = \mathbf{0.149 \text{ kJ/K}} \end{aligned}$$

6.124

Ammonia is contained in a rigid sealed tank unknown quality at 0°C. When heated in boiling water to 100°C its pressure reaches 1200 kPa. Find the initial quality, the heat transfer to the ammonia and the total entropy generation.

Solution:

C.V. Ammonia, which is a control mass of constant volume.

$$\text{Energy Eq.3.5: } u_2 - u_1 = q_2 - w_2$$

$$\text{Entropy Eq.6.37: } s_2 - s_1 = \int dq/T + s_{\text{gen}}$$

State 2: 1200 kPa, 100°C => Table B.2.2

$$s_2 = 5.5325 \text{ kJ/kg K}, \quad v_2 = 0.14347 \text{ m}^3/\text{kg}, \quad u_2 = 1485.8 \text{ kJ/kg}$$

State 1: $v_1 = v_2 \Rightarrow$ Table B.2.1

$$x_1 = (0.14347 - 0.001566)/0.28763 = 0.49336$$

$$u_1 = 741.28 \text{ kJ/kg}, \quad s_1 = 0.7114 + x_1 \times 4.6195 = 2.9905 \text{ kJ/kg K}$$

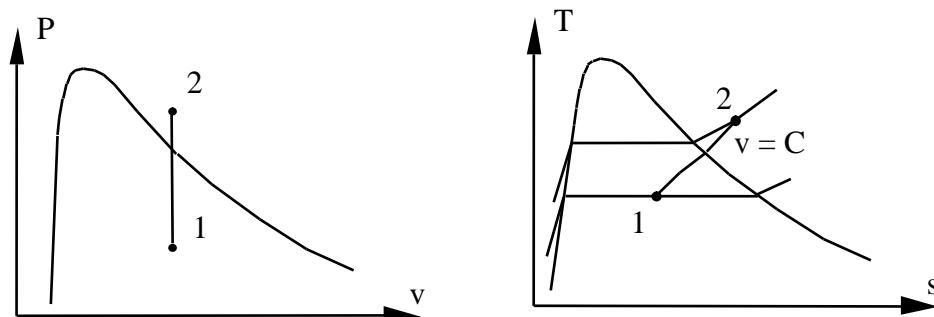
Process: $V = \text{constant} \Rightarrow w_2 = 0$

$$q_2 = (u_2 - u_1) = 1485.8 - 741.28 = 744.52 \text{ kJ/kg}$$

To get the total entropy generation take the C.V. out to the water at 100°C.

$$s_{\text{gen}} = s_2 - s_1 - q_2/T = 5.5325 - 2.9905 - 744.52/373.15$$

$$= 0.547 \text{ kJ/kg K}$$



6.125

Water in a piston/cylinder is at 101 kPa, 25°C, and mass 0.5 kg. The piston rests on some stops, and the pressure should be 1000 kPa to float the piston. We now heat the water from a 200°C reservoir, so the volume becomes 5 times the initial volume. Find the total heat transfer and the entropy generation.

Solution:

Take CV as the water out to the reservoir.

Continuity Eq.: $m_2 = m_1 = m$;

Energy Eq.: $m(u_2 - u_1) = \dot{Q}_2 - \dot{W}_2$

Entropy Eq.6.37: $m(s_2 - s_1) = \int dQ/T + \dot{S}_{2\text{ gen}} = \dot{Q}_2/T_{\text{res}} + \dot{S}_{2\text{ gen}}$

Process: $v = \text{constant}$, then $P = C = P_{\text{float}}$.

Volume does go up so we get $v_2 = 5 v_1$

State 1: $v_1 = 0.001003 \text{ m}^3/\text{kg}$, $u_1 = 104.86 \text{ kJ/kg}$, $s_1 = 0.3673 \text{ kJ/kgK}$

State 2: $P_2 = P_{\text{float}}$, $v_2 = 5 \times 0.001003 = 0.005015 \text{ m}^3/\text{kg}$; $T_2 = 179.91^\circ\text{C}$

$$x_2 = (v_2 - v_f)/v_{fg} = (0.005015 - 0.001127)/0.19332 = 0.02011,$$

$$u_2 = 761.67 + x_2 \times 1821.97 = 798.31 \text{ kJ/kg}$$

$$s_2 = 2.1386 + x_2 \times 4.4478 = 2.2280 \text{ kJ/kgK}$$

From the process equation (see P-V diagram) we get the work as

$$\dot{W}_2 = P_{\text{float}}(v_2 - v_1) = 1000 \text{ kPa} (0.005015 - 0.001003) \text{ m}^3/\text{kg} = 4.012 \text{ kJ/kg}$$

From the energy equation we solve for the heat transfer

$$\dot{Q}_2 = m[u_2 - u_1 + \dot{W}_2] = 0.5 \times [798.31 - 104.86 + 4.012] = \mathbf{348.7 \text{ kJ}}$$

$$\dot{S}_{2\text{ gen}} = m(s_2 - s_1) - \dot{Q}_2/T_{\text{res}} = 0.5(2.2280 - 0.3673) - 348.7/473.15$$

$$= \mathbf{0.1934 \text{ kJ/K}}$$

6.126

Do Problem 6.125 assuming the piston/cylinder is 1.5 kg of steel and has the same temperature as the water at any time.

Solution:

Take CV as the water and steel out to the reservoir.

$$\text{Continuity Eq.: } m_2 = m_1 = m_{\text{H}_2\text{O}} + m_{\text{steel}}$$

$$\text{Energy Eq.: } m_{\text{H}_2\text{O}}(u_2 - u_1)_{\text{H}_2\text{O}} + m_{\text{steel}}(u_2 - u_1)_{\text{steel}} = 1Q_2 - 1W_2$$

$$\text{Entropy Eq.6.37: } m_{\text{H}_2\text{O}}(s_2 - s_1)_{\text{H}_2\text{O}} + m_{\text{steel}}(s_2 - s_1)_{\text{steel}} = 1Q_2/T_{\text{res}} + 1S_2 \text{ gen}$$

$$\text{Process: } v = \text{constant, then } P = C = P_{\text{float}}.$$

$$\text{Volume does go up so we get } v_2 = 5 v_1$$

$$\text{State 1: } v_1 = 0.001003 \text{ m}^3/\text{kg}, u_1 = 104.86 \text{ kJ/kg}, s_1 = 0.3673 \text{ kJ/kgK}$$

$$\text{State 2: } P_2 = P_{\text{float}}, v_2 = 5 \times 0.001003 = 0.005015 \text{ m}^3/\text{kg}; T_2 = 179.91^\circ\text{C}$$

$$x_2 = (v_2 - v_f)/v_{fg} = (0.005015 - 0.001127)/0.19332 = 0.02011,$$

$$u_2 = 761.67 + x_2 \times 1821.97 = 798.31 \text{ kJ/kg}$$

$$s_2 = 2.1386 + x_2 \times 4.4478 = 2.2280 \text{ kJ/kgK}$$

There is only work when piston moves and then $P = P_{\text{float}}$ so the work is

$$1W_2 = P_{\text{float}}(V_2 - V_1) = 1000 \text{ kPa} \times 0.5 (0.005015 - 0.001003) \text{ m}^3 = 2.006 \text{ kJ}$$

From the energy equation we solve for the heat transfer

$$\begin{aligned} 1Q_2 &= U_2 - U_1 + 1W_2 \\ &= 0.5 \times [798.31 - 104.86] + 1.5 \times 0.46 \times (179.91 - 25) + 2.006 \\ &= \mathbf{455.6 \text{ kJ}} \end{aligned}$$

For the entropy change we use B.1.1 for water and A.5 and Eq.6.11 for steel

$$\begin{aligned} 1S_2 \text{ gen} &= S_2 - S_1 - 1Q_2/T_{\text{res}} \\ &= 0.5(2.2280 - 0.3673) + 1.5 \times 0.46 \times \ln \frac{453.06}{298.15} - \frac{455.6}{473.15} \\ &= \mathbf{0.256 \text{ kJ/K}} \end{aligned}$$

6.127

A cylinder fitted with a movable piston contains water at 3 MPa, 50% quality, at which point the volume is 20 L. The water now expands to 1.2 MPa as a result of receiving 600 kJ of heat from a large source at 300°C. It is claimed that the water does 124 kJ of work during this process. Is this possible?

Solution:

C.V.: H₂O in Cylinder

$$\text{Energy Eq.: } m(u_2 - u_1) = _1Q_2 - _1W_2;$$

$$\text{Entropy Eq.6.37: } m(s_2 - s_1) = \frac{1}{T_{\text{source}}} Q_2 + _1S_{2,\text{gen}}; \quad T_{\text{source}} = 300^\circ\text{C},$$

$$\text{Process: } _1Q_2 = 600 \text{ kJ, } _1W_2 = 124 \text{ kJ ?}$$

$$\text{State 1: } 3 \text{ MPa, } x_1 = 0.5, \quad \text{Table B.1.2: } T_1 = 233.9^\circ\text{C}$$

$$v_1 = v_f + x_1 v_{fg} = 0.001216 + 0.5 \times 0.06546 = 0.033948 \text{ m}^3/\text{kg}$$

$$u_1 = u_f + x_1 u_{fg} = 1804.5 \text{ kJ/kg, } \quad s_1 = s_f + x_1 s_{fg} = 4.4162 \text{ kJ/kg-K}$$

$$m_1 = V_1/v_1 = 0.02 \text{ m}^3 / [0.033948 \text{ m}^3/\text{kg}] = 0.589 \text{ kg}$$

Now solve for u_2 from the energy equation

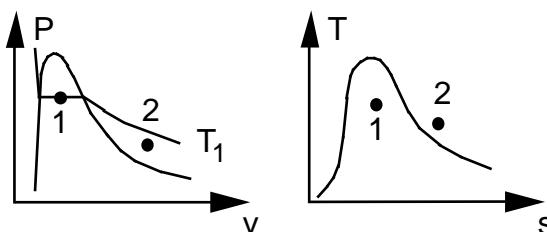
$$\begin{aligned} u_2 &= u_1 + (_1Q_2 - _1W_2)/m_1 \\ &= 1804.5 + (600 - 124)/0.589 = 2612.6 \text{ kJ/kg} \end{aligned}$$

$$\text{State 2: } P_2 = 1.2 \text{ MPa : } u_2 = 2612.6 \text{ kJ/kg} \quad \text{Table B.1.3}$$

$$T_2 \approx 200^\circ\text{C, } s_2 = 6.5898 \text{ kJ/kgK}$$

From the entropy equation

$$\begin{aligned} _1S_{2,\text{gen}} &= m(s_2 - s_1) - \frac{1}{T_{\text{source}}} Q_2 \\ &= 0.589 \text{ kg} (6.5898 - 4.4162) \text{ kJ/kg-K} - \frac{600}{300 + 273} \text{ kJ/K} \\ &= 0.2335 \text{ kJ/K} \geq 0; \quad \text{Process is possible} \end{aligned}$$



6.128

A piston/cylinder device keeping a constant pressure has 1 kg water at 20°C and 1 kg of water at 100°C both at 500 kPa separated by a thin membrane. The membrane is broken and the water comes to a uniform state with no external heat transfer. Find the final temperature and the entropy generation for the process.

Solution:

$$\text{Continuity Eq.: } m_2 - m_A - m_B = 0$$

$$\text{Energy Eq.3.5: } m_2 u_2 - m_A u_A - m_B u_B = -W_2$$

$$\text{Entropy Eq.6.37: } m_2 s_2 - m_A s_A - m_B s_B = \int dQ/T + S_{2\text{ gen}}$$

$$\begin{aligned} \text{Process: } P &= \text{Constant} \Rightarrow W_2 = \int P dV = P(V_2 - V_1) \\ Q &= 0 \end{aligned}$$

Substitute the work term into the energy equation and rearrange to get

$$m_2 u_2 + P_2 V_2 = m_2 h_2 = m_A u_A + m_B u_B + PV_1 = m_A h_A + m_B h_B$$

where the last rewrite used $PV_1 = PV_A + PV_B$.

$$\text{State A1: Table B.1.4 } h_A = 84.41 \text{ kJ/kg} \quad s_A = 0.2965 \text{ kJ/kg K}$$

$$\text{State B1: Table B.1.4 } h_B = 419.32 \text{ kJ/kg} \quad s_B = 1.3065 \text{ kJ/kg K}$$

Energy equation gives:

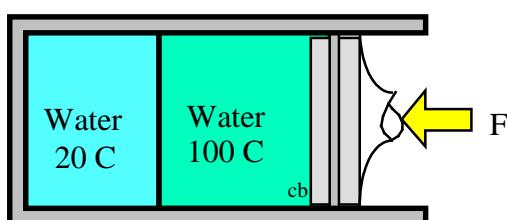
$$h_2 = \frac{m_A}{m_2} h_A + \frac{m_B}{m_2} h_B = \frac{1}{2} 84.41 + \frac{1}{2} 419.32 = 251.865 \text{ kJ/kg}$$

$$\text{State 2: } h_2 = 251.865 \text{ kJ/kg} \text{ & } P_2 = 500 \text{ kPa from Table B.1.4}$$

$$T_2 = 60.085^\circ\text{C}, \quad s_2 = 0.83184 \text{ kJ/kg K}$$

With the zero heat transfer we have

$$\begin{aligned} S_{2\text{ gen}} &= m_2 s_2 - m_A s_A - m_B s_B \\ &= 2 \times 0.83184 - 1 \times 0.2965 - 1 \times 1.3065 = \mathbf{0.0607 \text{ kJ/K}} \end{aligned}$$



6.129

Reconsider Problem 3.109 where CO₂ is compressed from -20°C, x = 0.75 to a state of 3 MPa, 20°C in a piston/cylinder where pressure is linear in volume. Assume heat transfer is from a reservoir at 100°C and find the specific entropy generation in the process (external to the CO₂).

CV Carbon dioxide out to the source, both ${}_1Q_2$ and ${}_1W_2$

$$\text{Energy Eq.3.5: } m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$$

$$\text{Entropy Eq.6.37: } S_2 - S_1 = \int dQ/T + {}_1S_{2,\text{gen}} = {}_1Q_2/T_{\text{res}} + {}_1S_{2,\text{gen}}$$

$$\text{Process: } P = A + BV \Rightarrow {}_1W_2 = \int P dV = \frac{1}{2} m(P_1 + P_2)(v_2 - v_1)$$

State 1: Table B.3.1 P = 1969.6 kPa

$$v_1 = 0.000969 + 0.75 \times 0.01837 = 0.01475 \text{ m}^3/\text{kg},$$

$$u_1 = 39.64 + 0.75 \times 246.25 = 224.33 \text{ kJ/kg},$$

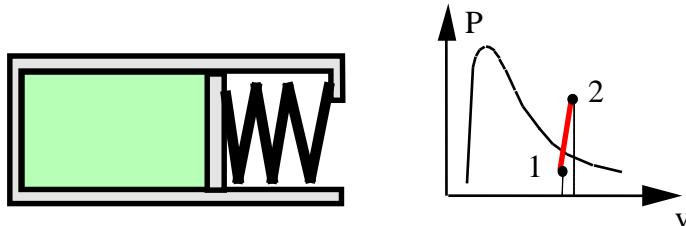
$$s_1 = 0.1672 + 0.75 \times 1.1157 = 1.004 \text{ kJ/kg-K}$$

State 2: Table B.3 $v_2 = 0.01512 \text{ m}^3/\text{kg}$, $u_2 = 310.21 \text{ kJ/kg}$, $s_2 = 1.3344 \text{ kJ/kg-K}$

$$\begin{aligned} {}_1W_2 &= \frac{1}{2}(P_1 + P_2)(v_2 - v_1) = \frac{1}{2} \times (1969.6 + 3000)(0.01512 - 0.01475) \\ &= 0.92 \text{ kJ/kg} \end{aligned}$$

$${}_1q_2 = u_2 - u_1 + {}_1W_2 = 310.21 - 224.33 + 0.92 = 86.8 \text{ kJ/kg}$$

$${}_1S_{2,\text{gen}} = s_2 - s_1 - {}_1q_2/T_{\text{res}} = 1.3344 - 1.004 - \frac{86.8}{373.15} = \mathbf{0.098 \text{ kJ/kg-K}}$$



6.130

A piston/cylinder contains 1 kg water at 150 kPa, 20°C. The piston is loaded so pressure is linear in volume. Heat is added from a 600°C source until the water is at 1 MPa, 500°C. Find the heat transfer and the total change in entropy.

Solution:

CV H₂O out to the source, both ${}_1Q_2$ and ${}_1W_2$

Energy Eq.3.5: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Entropy Eq.6.37: $m(s_2 - s_1) = {}_1Q_2 / T_{\text{SOURCE}} + {}_1S_{2 \text{ gen}}$

Process: $P = A + BV \Rightarrow {}_1W_2 = \int P dV = \frac{1}{2} (P_1 + P_2) (V_2 - V_1)$

State 1: B.1.1 Compressed liquid use saturated liquid at same T:

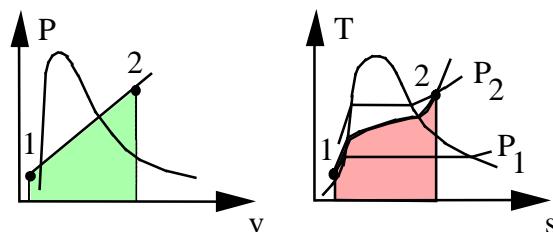
$$v_1 = 0.001002 \text{ m}^3/\text{kg}; \quad u_1 = 83.94 \text{ kJ/kg}; \quad s_1 = 0.2966 \text{ kJ/kg K}$$

State 2: Table B.1.3 sup. vap.

$$v_2 = 0.35411 \text{ m}^3/\text{kg}$$

$$u_2 = 3124.3 \text{ kJ/kg};$$

$$s_2 = 7.7621 \text{ kJ/kg K}$$



$${}_1W_2 = \frac{1}{2} (1000 + 150) \text{ kPa} \times 1 \text{ kg} (0.35411 - 0.001002) \text{ m}^3/\text{kg} = 203 \text{ kJ}$$

$${}_1Q_2 = 1(3124.3 - 83.94) + 203 = \mathbf{3243.4 \text{ kJ}}$$

$$m(s_2 - s_1) = 1 \text{ kg} (7.7621 - 0.2968) \text{ kJ/kg-K} = 7.4655 \text{ kJ/K}$$

$${}_1Q_2 / T_{\text{source}} = 3.7146 \text{ kJ/K} \quad (\text{for source } Q = -{}_1Q_2)$$

$${}_1S_{2 \text{ gen}} = m(s_2 - s_1) - {}_1Q_2 / T_{\text{SOURCE}}$$

$$= \Delta S_{\text{H}_2\text{O}} + \Delta S_{\text{source}} = 7.4655 - 3.7146 = \mathbf{3.751 \text{ kJ/K}}$$

Remark: This is an external irreversible process (delta T to the source)

6.131

A closed rigid container is filled with 1.5 kg water at 100 kPa, 55°C, 1 kg of stainless steel and 0.5 kg of PVC (polyvinyl chloride) both at 20°C and 0.1 kg of air at 400 K, 100 kPa. It is now left alone with no external heat transfer and no water vaporizes. Find the final temperature and the entropy generation for the process.

CV. Container.

Process: $V = \text{constant} \Rightarrow {}_1W_2 = 0$ and also given ${}_1Q_2 = 0$

$$\text{Energy Eq.: } U_2 - U_1 = \sum m_i(u_2 - u_1)_i = {}_1Q_2 - {}_1W_2 = 0$$

$$\text{Entropy Eq.6.37: } S_2 - S_1 = \sum m_i(s_2 - s_1)_i = 0 + {}_1S_{2,\text{gen}}$$

For the liquid and the metal mass we will use the specific heat (Tbl A.3, A.4) so

$$\sum m_i(u_2 - u_1)_i = \sum m_i C_v i (T_2 - T_1)_i = T_2 \sum m_i C_v i - \sum m_i C_v i T_1 i$$

$$\sum m_i(s_2 - s_1)_i = \sum m_i C_v i \ln(T_2/T_1)_i$$

noticing that all masses have the same T_2 but not same initial T.

$$\sum m_i C_v i = 1.5 \times 4.18 + 1 \times 0.46 + 0.5 \times 0.96 + 0.1 \times 0.717 = 7.282 \text{ kJ/K}$$

$$\text{Energy Eq.: } 7.282 T_2 = 1.5 \times 4.18 \times 55 + (1 \times 0.46 + 0.5 \times 0.96) \times 20$$

$$+ 0.1 \times 0.717 \times (400 - 273.15) = 372.745 \text{ kJ}$$

$$T_2 = \mathbf{51.2^\circ C}$$

The volume of the air is constant so entropy change from Eq.6.17 is the same expression as for the solids and liquids given above.

$$\begin{aligned} {}_1S_{2,\text{gen}} &= S_2 - S_1 = \sum m_i(s_2 - s_1)_i = 1.5 \times 4.18 \times \ln(324.35/328.15) \\ &\quad + (1 \times 0.46 + 0.5 \times 0.96) \times \ln\left(\frac{324.35}{293.15}\right) + 0.1 \times 0.717 \times \ln\left(\frac{324.35}{400}\right) \\ &= -0.07304 \text{ kJ/K} + 0.09507 \text{ kJ/K} - 0.01503 \text{ kJ/K} = \mathbf{0.007 \text{ kJ/K}} \end{aligned}$$

6.132

A cylinder/piston contains water at 200 kPa, 200°C with a volume of 20 L. The piston is moved slowly, compressing the water to a pressure of 800 kPa. The loading on the piston is such that the product PV is a constant. Assuming that the room temperature is 20°C, show that this process does not violate the second law.

Solution:

C.V.: Water + cylinder out to room at 20°C

$$\text{Energy Eq.3.5: } m(u_2 - u_1) = _1Q_2 - _1W_2$$

$$\text{Entropy Eq.6.37: } m(s_2 - s_1) = _1Q_2 / T_{\text{room}} + _1S_2 \text{ gen}$$

$$\text{Process: } PV = \text{constant} = Pmv \Rightarrow v_2 = P_1v_1/P_2$$

$$_1W_2 = \int Pdv = P_1v_1 \ln(v_2/v_1)$$

$$\text{State 1: Table B.1.3, } v_1 = 1.0803 \text{ m}^3/\text{kg}, \quad u_1 = 2654.4 \text{ kJ/kg,}$$

$$s_1 = 7.5066 \text{ kJ/kg K}$$

$$\text{State 2: } P_2, v_2 = P_1v_1/P_2 = 200 \times 1.0803/800 = 0.2701 \text{ m}^3/\text{kg}$$

$$\text{Table B.1.3: } u_2 = 2655.0 \text{ kJ/kg, } s_2 = 6.8822 \text{ kJ/kg K}$$

$$_1W_2 = 200 \times 1.0803 \ln\left(\frac{0.2701}{1.0803}\right) = -299.5 \text{ kJ/kg}$$

$$_1q_2 = u_2 - u_1 + _1W_2 = 2655.0 - 2654.4 - 299.5 = -298.9 \text{ kJ/kg}$$

$$_1S_{2,\text{gen}} = s_2 - s_1 - \frac{_1q_2}{T_{\text{room}}} = 6.8822 - 7.5066 + \frac{298.9}{293.15}$$

$$= 0.395 \text{ kJ/kg K} > 0 \quad \text{satisfies entropy eq.}$$

6.133

A rigid steel tank of mass 2.5 kg contains 0.5 kg R-410A at 0°C with specific volume 0.01m/kg. The whole system heats up to the room temperature, 25°C. Find the process heat transfer and the entropy generation.

C.V. R-410A and steel tank out to room T. Control mass.

$$\text{Continuity Eq.: } m_2 - m_{R410a} - m_{st} = 0$$

$$\text{Energy Eq.: } m_{R410a}(u_2 - u_1) + m_{st}(u_2 - u_1) = Q_2 - W_2$$

$$\text{Entropy Eq.: } m_{R410a}(s_2 - s_1) + m_{st}(s_2 - s_1) = Q_2/T_{\text{room}} + S_{2,\text{gen}}$$

$$\text{Process: } V = C \quad \text{so} \quad W_2 = 0$$

$$\text{State 1: } T_1 = 0^\circ\text{C}, v_1 = 0.01 \text{ m}^3/\text{kg},$$

$$V = mv_1 = 0.005 \text{ m}^3$$

$$x_1 = (v - v_f)/v_{fg} = (0.01 - 0.000855)/0.03182$$

$$= 0.28758$$

$$u_1 = u_f + x_1 u_{fg} = 57.07 + x_1 195.95$$

$$= 113.42 \text{ kJ/kg}$$

$$s_1 = 0.2264 + x_1 0.8104 = 0.45945 \text{ kJ/kgK}$$

State 2: (T, v) \Rightarrow sup-vapor (straight up in T-v diagram from state 1)

B.4.1 at 25°C, $v_f = 0.000944 \text{ m}^3/\text{kg}$, $v_g = 0.01514 \text{ m}^3/\text{kg}$, $v_f < v < v_g$: saturated.

$$P = 1653.6 \text{ kPa}, \quad x = \frac{v - v_f}{v_{fg}} = \frac{0.01 - 0.000944}{0.01420} = 0.63775,$$

$$u_2 = u_f + x_2 u_{fg} = 96.03 + x_2 162.95 = 199.95 \text{ kJ/kg}$$

$$s_2 = s_f + x_2 s_{fg} = 0.3631 + x_2 0.6253 = 0.7619 \text{ kJ/kgK}$$

From the energy Eq.:

$$Q_2 = m_{R410a}(u_2 - u_1) + m_{st} C_{st}(T_2 - T_1)$$

$$= 0.5 \text{ kg} \times (199.95 - 113.42) \text{ kJ/kg} + 2.5 \text{ kg} \times 0.46 \text{ kJ/kgK} \times (25 - 0) \text{ K}$$

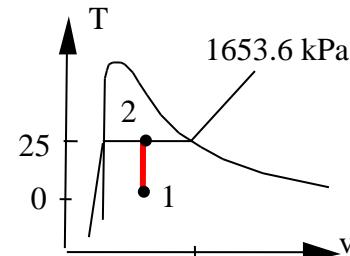
$$= \mathbf{72.0 \text{ kJ}}$$

$$S_{2,\text{gen}} = m_{R410a}(s_2 - s_1) + m_{st} C_{st} \ln(T_2/T_1) - Q_2/T_{\text{room}}$$

$$= 0.5 \text{ kg} \times (0.7619 - 0.45945) \frac{\text{kJ}}{\text{kgK}} + 2.5 \text{ kg} \times 0.46 \frac{\text{kJ}}{\text{kgK}} \times \ln \frac{298.15}{273.15}$$

$$- (72/298.15) \text{ kJ/K}$$

$$= 0.1512 \text{ kJ/K} + 0.1007 \text{ kJ/K} - 0.2415 \text{ kJ/K} = \mathbf{0.0104 \text{ kJ/K}}$$



6.134

A piston/cylinder has ammonia at 2000 kPa, 80°C with a volume of 0.1 m³. The piston is loaded with a linear spring and outside ambient is at 20°C, shown in Fig. P6.134. The ammonia now cools down to 20°C at which point it has a quality of 15%. Find the work, the heat transfer and the total entropy generation in the process.

CV Ammonia out to the ambient, both ${}_1Q_2$ and ${}_1W_2$

$$\text{Energy Eq.3.5: } m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$$

$$\text{Entropy Eq.6.37: } m(s_2 - s_1) = {}_1Q_2 / T_{\text{ambient}} + {}_1S_{2 \text{ gen}}$$

$$\text{Process: } P = A + BV \Rightarrow {}_1W_2 = \int P dV = \frac{1}{2} m(P_1 + P_2)(v_2 - v_1)$$

State 1: Table B.2.2

$$v_1 = 0.07595 \text{ m}^3/\text{kg}, \quad u_1 = 1421.6 \text{ kJ/kg}, \quad s_1 = 5.0707 \text{ kJ/kg K}$$

$$m = V_1/v_1 = 0.1/0.07595 = 1.31665 \text{ kg}$$

State 2: Table B.2.1

$$v_2 = 0.001638 + 0.15 \times 0.14758 = 0.023775 \text{ m}^3/\text{kg}$$

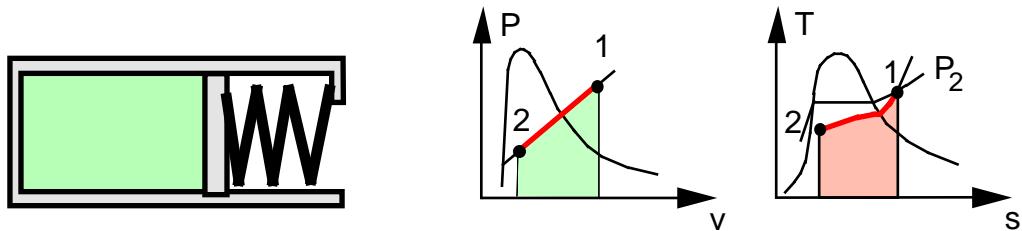
$$u_2 = 272.89 + 0.15 \times 1059.3 = 431.785 \text{ kJ/kg}$$

$$s_2 = 1.0408 + 0.15 \times 4.0452 = 1.64758 \text{ kJ/kg K}$$

$$\begin{aligned} {}_1W_2 &= \frac{1}{2} m(P_1 + P_2)(v_2 - v_1) \\ &= \frac{1}{2} \times 1.31665 \text{ kg} (2000 + 857.5) \text{ kPa} (0.023775 - 0.07595) \text{ m}^3/\text{kg} \\ &= - \mathbf{98.15 \text{ kJ}} \end{aligned}$$

$$\begin{aligned} {}_1Q_2 &= m(u_2 - u_1) + {}_1W_2 = 1.31665 (431.785 - 1421.6) - 98.15 \\ &= - \mathbf{1401.39 \text{ kJ}} \end{aligned}$$

$$\begin{aligned} {}_1S_{2 \text{ gen}} &= m(s_2 - s_1) - ({}_1Q_2 / T_{\text{amb}}) \\ &= 1.31665 (1.64758 - 5.0707) - \frac{-1401.39}{293.15} \\ &= -4.507051 + 5.78045 = \mathbf{0.273 \text{ kJ/k}} \end{aligned}$$



6.135

One kilogram of ammonia (NH_3) is contained in a spring-loaded piston/cylinder, Fig. P6.135, as saturated liquid at -20°C . Heat is added from a reservoir at 100°C until a final condition of $800 \text{ kPa}, 70^\circ\text{C}$ is reached. Find the work, heat transfer, and entropy generation, assuming the process is internally reversible.

Solution:

C.V. = NH_3 out to the reservoir.

$$\text{Continuity Eq.:} \quad m_2 = m_1 = m$$

$$\text{Energy Eq.3.5:} \quad E_2 - E_1 = m(u_2 - u_1) = _1Q_2 - _1W_2$$

$$\text{Entropy Eq.6.37:} \quad S_2 - S_1 = \int dQ/T + _1S_{2,\text{gen}} = _1Q_2/T_{\text{res}} + _1S_{2,\text{gen}}$$

Process: $P = A + BV$ linear in $V \Rightarrow$

$$_1W_2 = \int PdV = \frac{1}{2}(P_1 + P_2)(V_2 - V_1) = \frac{1}{2}(P_1 + P_2)m(v_2 - v_1)$$

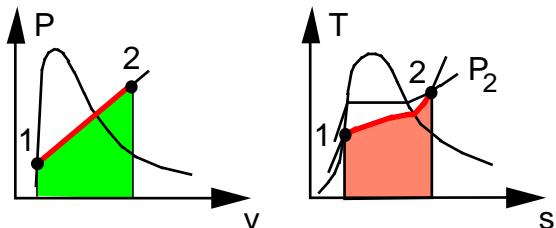
State 1: Table B.2.1

$$P_1 = 190.08 \text{ kPa},$$

$$v_1 = 0.001504 \text{ m}^3/\text{kg}$$

$$u_1 = 88.76 \text{ kJ/kg},$$

$$s_1 = 0.3657 \text{ kJ/kg K}$$



State 2: Table B.2.2 sup. vapor

$$v_2 = 0.199 \text{ m}^3/\text{kg}, \quad u_2 = 1438.3 \text{ kJ/kg}, \quad s_2 = 5.5513 \text{ kJ/kg K}$$

$$_1W_2 = \frac{1}{2}(190.08 + 800) \text{ kPa} \times 1 \text{ kg} (0.1990 - 0.001504) \text{ m}^3/\text{kg} = \mathbf{97.768 \text{ kJ}}$$

$$_1Q_2 = m(u_2 - u_1) + _1W_2 = 1(1438.3 - 88.76) + 97.768 = \mathbf{1447.3 \text{ kJ}}$$

$$_1S_{2,\text{gen}} = m(s_2 - s_1) - _1Q_2/T_{\text{res}} = 1(5.5513 - 0.3657) - \frac{1447.3}{373.15} = \mathbf{1.307 \text{ kJ/K}}$$

6.136

A 5 kg aluminum radiator holds 2 kg of liquid R-134a both at -10°C . The setup is brought indoors and heated with 220 kJ from a heat source at 100°C . Find the total entropy generation for the process assuming the R-134a remains a liquid.

Solution:

C.V. The aluminum radiator and the R-134a.

$$\text{Energy Eq.3.5: } m_2u_2 - m_1u_1 = _1Q_2 - 0$$

Process: No change in volume so no work as used above.

The energy equation now becomes (summing over the mass)

$$m_{\text{al}}(u_2 - u_1)_{\text{al}} + m_{\text{R134a}}(u_2 - u_1)_{\text{R134a}} = _1Q_2$$

Use specific heat from Table A.3 and A.4

$$m_{\text{al}}C_{\text{al}}(T_2 - T_1) + m_{\text{R134a}}C_{\text{R134a}} \ln(T_2/T_1) = _1Q_2$$

$$T_2 - T_1 = _1Q_2 / [m_{\text{al}}C_{\text{al}} + m_{\text{R134a}}C_{\text{R134a}}]$$

$$= 220 / [5 \times 0.9 + 2 \times 1.43] = 29.89^{\circ}\text{C}$$

$$T_2 = -10 + 29.89 = 19.89^{\circ}\text{C}$$

Entropy generation from Eq.6.37

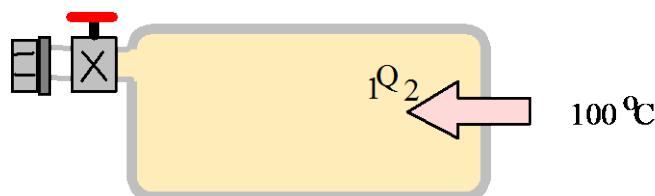
$$_1S_{2\text{ gen}} = m(s_2 - s_1) - _1Q_2/T$$

$$= m_{\text{al}}C_{\text{al}} \ln(T_2/T_1) + m_{\text{R134a}}C_{\text{R134a}} \ln(T_2/T_1) - \frac{_1Q_2}{T_{\text{amb}}}$$

$$= (5 \times 0.9 + 2 \times 1.43) \ln \frac{(19.89 + 273.15)}{-10 + 273.15} - \frac{220}{373.15}$$

$$= 0.7918 - 0.5896$$

$$= \mathbf{0.202 \text{ kJ/K}}$$



6.137

A piston/cylinder of total 1 kg steel contains 0.5 kg ammonia at 1600 kPa both masses at 120°C. Some stops are placed so a minimum volume is 0.02 m³, shown in Fig. P6.137. Now the whole system is cooled down to 30°C by heat transfer to the ambient at 20°C, and during the process the steel keeps same temperature as the ammonia. Find the work, the heat transfer and the total entropy generation in the process.

$$\text{Energy Eq.3.5: } m(u_2 - u_1) + m_{st}(u_2 - u_1) = 1Q_2 - 1W_2$$

$$\text{Entropy Eq.6.37: } m(s_2 - s_1) + m_{st}(s_2 - s_1) = 1Q_2/T_{amb} + 1S_{gen}$$

$$\text{State 1: } v_1 = 0.11265 \text{ m}^3/\text{kg}, \quad u_1 = 1516.6 \text{ kJ/kg}, \quad s_1 = 5.5018 \text{ kJ/kg K}$$

$$V_1 = mv_1 = 0.05634 \text{ m}^3$$

$$\text{Stop 1a: } v_{stop} = V/m = 0.02/0.5 = 0.04 \text{ m}^3/\text{kg}, \quad P_{stop} = P_1 \Rightarrow T \sim 42^\circ\text{C}$$

$$\text{State 2: } 30^\circ\text{C} < T_{stop} \text{ so } v_2 = v_{stop} = 0.04 \text{ m}^3/\text{kg}$$

$$x_2 = \frac{(v_2 - v_f)}{v_{fg}} = \frac{0.04 - 0.00168}{0.10881} = 0.35217$$

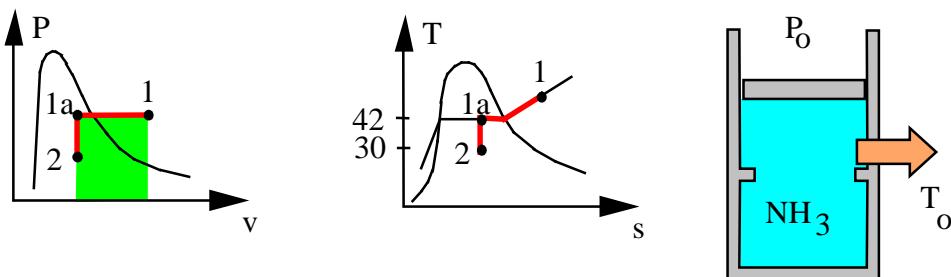
$$u_2 = 320.46 + x_2 \times 1016.9 = 678.58 \text{ kJ/kg}$$

$$s_2 = 1.2005 + x_2 \times 3.7734 = 2.5294 \text{ kJ/kg K}$$

$$1W_2 = \int P dV = P_1 m (v_2 - v_1) = 1600 \times 0.5 (0.004 - 0.11268) = -58.14 \text{ kJ}$$

$$\begin{aligned} 1Q_2 &= m(u_2 - u_1) + m_{st}(u_2 - u_1) + 1W_2 \\ &= 0.5(678.58 - 1516.6) + 1 \times 0.46(30 - 120) - 58.14 \\ &= -419.01 - 41.4 - 58.14 = \mathbf{-518.55 \text{ kJ}} \end{aligned}$$

$$\begin{aligned} 1S_{gen} &= m(s_2 - s_1) + m_{st}(s_2 - s_1) - 1Q_2/T_{amb} \\ &= 0.5(2.5294 - 5.5018) + 1 \times 0.46 \ln \frac{273+30}{273+120} - \frac{-518.5}{293.15} \\ &= -1.4862 - 0.1196 + 1.6277 = \mathbf{0.02186 \text{ kJ/K}} \end{aligned}$$



6.138

A piston/cylinder contains 0.1 kg water at 500°C, 1000 kPa. The piston has a stop at half the original volume, similar to Fig. 6.137. The water now cools to room temperature 25°C. Find the heat transfer and the entropy generation.

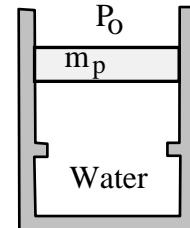
$$\text{Energy Eq.: } m(u_2 - u_1) = _1Q_2 - _1W_2$$

$$\text{Entropy Eq.: } m(s_2 - s_1) = _1Q_2/T_{\text{room}} + _1S_{2,\text{gen}}$$

$$\text{Process Eq: } P = C \text{ if } v > v_{\text{stop}}; \quad V = C \text{ if } P < P_{\text{float}}$$

$$\text{State 1: } v_1 = 0.35411 \text{ m}^3/\text{kg}, \quad u_1 = 3124.34 \text{ kJ/kg}$$

$$s_1 = 7.7621 \text{ kJ/kgK}$$



$$\text{State a: } v_a = v_1/2 = 0.177055 \text{ m}^3/\text{kg} < v_g \text{ at } 1000 \text{ kPa} \text{ so } T_a = T_{\text{sat}} \text{ at } 1000 \text{ kPa} = 179.9^\circ\text{C}$$

The possible state 2 (P,V) combinations are shown. State "a" is (1000 kPa, v_a) so it is

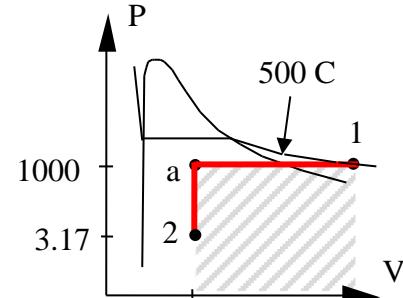
two-phase with $T_a = 180^\circ\text{C} > T_2$

$$P_2 = P_{\text{sat}} \text{ at } 25^\circ\text{C} = 3.169 \text{ kPa} \quad \text{and} \quad v_2 = v_a$$

$$x_2 = (v_2 - v_f)/v_{fg} = (0.177 - 0.001003)/43.358 \\ = 0.0040604$$

$$u_2 = u_f + x_2 u_{fg} = 104.86 + x_2 2304.9 = 114.219 \text{ kJ/kg}$$

$$s_2 = s_f + x_2 s_{fg} = 0.3673 + x_2 8.1905 = 0.40056 \text{ kJ/kgK}$$



$$_1W_2 = m \int P \, dv = m P_1 (v_2 - v_1) \quad [\text{see area below process curve in figure}]$$

$$= 0.1 \text{ kg} \times 1000 \text{ kPa} \times (0.177055 - 0.35411) \text{ m}^3/\text{kg} = -17.706 \text{ kJ}$$

$$_1Q_2 = m(u_2 - u_1) + _1W_2 = 0.1 \text{ kg} (114.219 - 3124.34) \text{ kJ/kg} - 17.706 \text{ kJ}$$

$$= -318.72 \text{ kJ}$$

$$_1S_{2,\text{gen}} = m(s_2 - s_1) - _1Q_2/T_{\text{room}}$$

$$= 0.1 \text{ kg}(0.40056 - 7.7621) \text{ kJ/kgK} + 318.72 \text{ kJ}/298.15 \text{ K}$$

$$= -0.73615 \text{ kJ/K} + 1.06899 \text{ kJ/K} = 0.333 \text{ kJ/K}$$

6.139

A hollow steel sphere with a 0.5-m inside diameter and a 2-mm thick wall contains water at 2 MPa, 250°C. The system (steel plus water) cools to the ambient temperature, 30°C. Calculate the net entropy change of the system and surroundings for this process.

C.V.: Steel + water out to ambient T_0 . This is a control mass.

$$\text{Energy Eq.: } U_2 - U_1 = m_{H_2O}(u_2 - u_1) + m_{\text{steel}}(u_2 - u_1) = Q_2 - W_2$$

$$\text{Entropy Eq.: } S_2 - S_1 = \int dQ/T + S_{\text{gen}} = Q_2/T_0 + S_{\text{gen}}$$

$$\text{Process: } V = \text{constant} \Rightarrow W_2 = 0$$

$$m_{\text{steel}} = (\rho V)_{\text{steel}} = 8050 \times (\pi/6)[(0.504)^3 - (0.5)^3] = 12.746 \text{ kg}$$

$$V_{H_2O} = (\pi/6)(0.5)^3, \quad m_{H_2O} = V/v = 6.545 \times 10^{-2} / 0.11144 = 0.587 \text{ kg}$$

$$v_2 = v_1 = 0.11144 = 0.001004 + x_2 \times 32.889 \Rightarrow x_2 = 3.358 \times 10^{-3}$$

$$u_2 = 125.78 + 3.358 \times 10^{-3} \times 2290.8 = 133.5 \text{ kJ/kg}$$

$$s_2 = 0.4639 + 3.358 \times 10^{-3} \times 8.0164 = 0.4638 \text{ kJ/kg K}$$

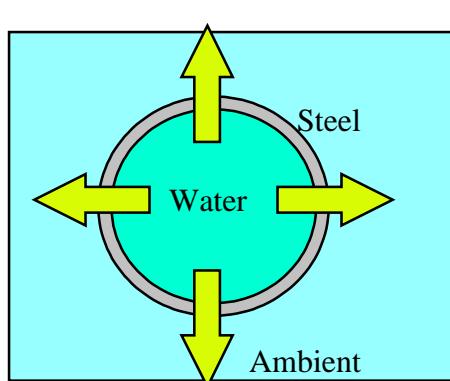
$$\begin{aligned} Q_2 &= m_{H_2O}(u_2 - u_1) + m_{\text{steel}}(u_2 - u_1) \\ &= 0.587(133.5 - 2679.6) + 12.746 \times 0.48(30 - 250) \\ &= -1494.6 + (-1346) = -2840.6 \text{ kJ} \end{aligned}$$

$$S_2 - S_1 = m_{H_2O}(s_2 - s_1) + m_{\text{steel}}(s_2 - s_1) = 0.587(0.4638 - 6.545)$$

$$+ 12.746 \times 0.48 \ln (303.15 / 523.15) = -6.908 \text{ kJ/K}$$

$$\Delta S_{\text{SURR}} = -Q_2/T_0 = +2840.6 / 303.2 = +9.370 \text{ kJ/K}$$

$$S_{\text{gen}} = S_2 - S_1 - Q_2/T_0 = -6.908 + 9.370 = +2.462 \text{ kJ/K}$$



6.140

A cylinder/piston arrangement contains 10 g ammonia at 20°C with a volume of 1 L. There are some stops so if the piston is at the stops the volume is 1.4 L. The ammonia is now heated to 200°C by a 240°C source. The piston and cylinder is made of 0.5 kg aluminum, and assume that the mass has the same temperature as the ammonia at any time. Find the total heat transfer and the total entropy generation

C.V. NH₃. Control mass, goes out to source

$$\text{Energy Eq.: } U_3 - U_1 = m_{\text{NH}_3} (u_3 - u_1) + m_{\text{Alu}} (u_3 - u_1) = _1Q_3 - _1W_3$$

$$\text{Entropy Eq.: } m_{\text{NH}_3} (s_3 - s_1) + m_{\text{Alu}} (s_3 - s_1) = _1Q_3/T_{\text{source}} + _1S_{3,\text{gen}}$$

$$\text{State 1: B.2.1 } v_1 = V/m = 0.001 / 0.01 = 0.1 \text{ m}^3/\text{kg} < v \text{ so 2-phase } P = 857.5 \text{ kPa}$$

$$x_1 = (v - v_f) / v_{fg} = (0.1 - 0.001638) / 0.14758 = 0.6665$$

$$u_1 = u_f + x_1 u_{fg} = 272.89 + x_1 \times 1059.3 = 978.91 \text{ kJ/kg}$$

$$s_1 = s_f + x_1 s_{fg} = 1.0408 + x_1 \times 4.0452 = 3.73693 \text{ kJ/kgK}$$

$$\text{State 2: } v_2 = 1.4 \times v_1 = 0.14 \text{ m}^3/\text{kg} \text{ & } P = 857.5 \text{ kPa} \text{ still 2-phase so } T_2 = 20^\circ\text{C}$$

$$\text{State 3: } 200^\circ\text{C} \text{ & } v_3 = v_2, \Rightarrow P = 1600 \text{ kPa}, u_3 = 1676.5 \text{ kJ/kg}, s_3 = 5.9734 \text{ kJ/kgK}$$

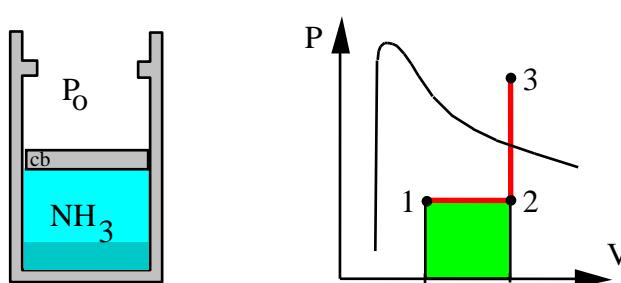
We get the work from the process equation (see P-V diagram)

$$_1W_3 = _1W_2 = P_1 m(v_2 - v_1) = 857.5 \text{ kPa} \times 0.01 (0.14 - 0.1) \text{ m}^3 = 0.343 \text{ kJ}$$

The energy equation and the entropy equation give heat transfer and entropy generation

$$\begin{aligned} _1Q_3 &= m_{\text{NH}_3} (u_3 - u_1) + m_{\text{Alu}} (u_3 - u_1) + _1W_3 \\ &= 0.01 (1676.5 - 978.91) + 0.5 \times 0.9 (200 - 20) + 0.343 = \mathbf{88.32 \text{ kJ}} \end{aligned}$$

$$\begin{aligned} _1S_{3,\text{gen}} &= m_{\text{NH}_3} (s_3 - s_1) + m_{\text{Alu}} (s_3 - s_1) - _1Q_3/T_{\text{source}} \\ &= 0.01 (5.9734 - 3.73693) + 0.5 \times 0.9 \ln\left(\frac{473.15}{293.15}\right) - \frac{88.32}{513.15} \\ &= 0.02236 + 0.21543 - 0.1721 = \mathbf{0.0657 \text{ kJ/K}} \end{aligned}$$



6.141

A cylinder/piston arrangement contains 0.1 kg R-410A of quality $x = 0.2534$ and at -20°C . Stops are mounted so $V_{\text{stop}} = 3V_1$, similar to Fig. P6.140. The system is now heated to the final temperature of 20°C by a 50°C source. Find the total entropy generation.

C.V. The R-410A mass out to source.

$$\text{Energy Eq.3.5: } m(u_2 - u_1) = _1Q_2 - _1W_2$$

$$\text{Entropy Eq.: } S_2 - S_1 = \int dQ/T + _1S_{2 \text{ gen}} = _1Q_2/T_{\text{source}} + _1S_{2 \text{ gen}}$$

$$\text{Process: } P = \text{Constant if } V < V_{\text{stop}}; \quad V = V_{\text{stop}} \text{ if } P > P_1$$

$$\text{State 1: } u_1 = 27.92 + x_1 218.07 = 83.18 \text{ kJ/kg}, \quad P_1 = P_{\text{sat}} = 399.6 \text{ kPa}$$

$$v_1 = 0.000803 + x_1 0.064 = 0.01702 \text{ m}^3/\text{kg}$$

$$s_1 = 0.1154 + x_1 0.9625 = 0.2439 \text{ kJ/kgK}$$

$$\text{State 1a: } v_{\text{stop}} = 3 v_1 = 0.05106 \text{ m}^3/\text{kg} < v_g \text{ at } P_1$$

$$\text{State 2: } \text{at } 20^\circ\text{C} > T_1 : \quad v_{\text{stop}} > v_g = 0.01758 \text{ m}^3/\text{kg} \text{ so superheated vapor.}$$

$$\text{Table B.4.2: } P_2 = 600 \text{ kPa}, u_2 = 273.56 \text{ kJ/kg}, s_2 = 1.1543 \text{ kJ/kgK}$$

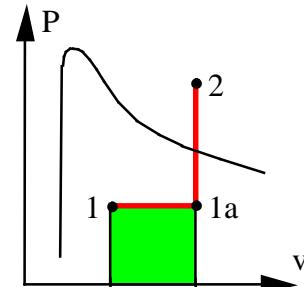
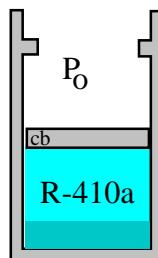
$$_1W_2 = \int PdV = P_1 m(v_2 - v_1) = 399.6 \times 0.1 (0.051 - 0.017) = 1.36 \text{ kJ}$$

$$_1Q_2 = m(u_2 - u_1) + _1W_2 = 0.1(273.56 - 83.18) + 1.36 = 20.398 \text{ kJ}$$

$$_1S_{2,\text{gen}} = m(s_2 - s_1) - _1Q_2/T_{\text{source}}$$

$$= 0.1 (1.1543 - 0.2439) - \frac{20.398}{323.15} = \mathbf{0.0279 \text{ kJ/K}}$$

See the work term from the process in the P-v diagram



6.142

One kg of air at 300 K is mixed with 2 kg air at 400 K in a process at a constant 100 kPa and $Q = 0$. Find the final T and the entropy generation in the process.

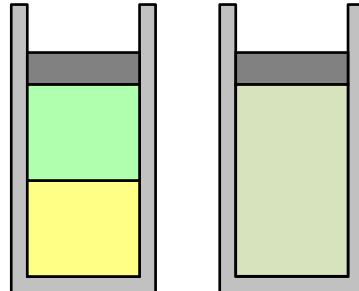
C.V. All the air.

$$\text{Energy Eq.: } U_2 - U_1 = 0 - W$$

$$\text{Entropy Eq.: } S_2 - S_1 = 0 + \dot{S}_{\text{gen}}$$

$$\text{Process Eq.: } P = C; \quad W = P(V_2 - V_1)$$

Substitute W into energy Eq.



$$U_2 - U_1 + W = U_2 - U_1 + P(V_2 - V_1) = H_2 - H_1 = 0$$

Due to the low T let us use constant specific heat

$$\begin{aligned} H_2 - H_1 &= m_A(h_2 - h_1)_A + m_B(h_2 - h_1)_B \\ &= m_A C_p(T_2 - T_{A1}) + m_B C_p(T_2 - T_{B1}) = 0 \end{aligned}$$

$$T_2 = \frac{m_A T_{A1} + m_B T_{B1}}{m_A + m_B} = \frac{1}{3} T_{A1} + \frac{2}{3} T_{B1} = \mathbf{366.67 \text{ K}}$$

Entropy change is from Eq. 6.16 with no change in P

$$\begin{aligned} \dot{S}_{\text{gen}} &= S_2 - S_1 = m_A C_p \ln \frac{T_2}{T_{A1}} + m_B C_p \ln \frac{T_2}{T_{B1}} \\ &= 1 \times 1.004 \ln \frac{366.67}{300} + 2 \times 1.004 \ln \frac{366.67}{400} \\ &= 0.20148 - 0.17470 = \mathbf{0.0268 \text{ kJ/K}} \end{aligned}$$

Remark: If you check, the volume does not change and there is no work.

6.143

Air in a rigid tank is at 900 K, 500 kPa and it now cools to the ambient temperature of 300 K by heat loss to the ambient. Find the entropy generation.

C.V. Air out to ambient. No size given so do it per unit mass.

$$\text{Energy Eq.3.5: } u_2 - u_1 = C_v(T_2 - T_1) = \dot{q}_2 - \dot{w}_2$$

$$\text{Entropy Eq.6.37: } s_2 - s_1 = \dot{q}_2/T_{\text{amb}} + \dot{s}_{\text{gen tot}}$$

$$\text{Process: } V = \text{constant} \Rightarrow v_2 = v_1 \text{ also } \dot{W}_2 = 0$$

$$\text{Ideal gas: } P_2 = P_1 T_2 / T_1 = 500 \times 300/900 = 166.67 \text{ kPa}$$

$$\text{From Table A.7: } u_1 = 674.82 \text{ kJ/kg; } s_{T1} = 8.01581 \text{ kJ/kg-K;}$$

$$u_2 = 214.36 \text{ kJ/kg; } s_{T2} = 6.86926 \text{ kJ/kg-K}$$

$$\dot{q}_2 = u_2 - u_1 = 214.36 - 674.82 = -460.46 \text{ kJ/kg}$$

$$\begin{aligned} \dot{s}_{\text{gen tot}} &= s_2 - s_1 - \frac{\dot{q}_2}{T_{\text{amb}}} = s_{T2} - s_{T1} - R \ln\left(\frac{P_2}{P_1}\right) - \frac{\dot{q}_2}{T_{\text{amb}}} \\ &= 6.86926 - 8.01581 - 0.287 \ln\left(\frac{166.67}{500}\right) - \frac{-460.46}{300} \\ &= \mathbf{0.661 \text{ kJ/kg-K}} \end{aligned}$$

We could also have used constant specific heat being slightly less accurate.

6.144

Two rigid insulated tanks are connected with a pipe and valve. One tank has 0.5 kg air at 200 kPa, 300 K and the other has 0.75 kg air at 100 kPa, 400 K. The valve is opened and the air comes to a single uniform state without external heat transfer. Find the final T and P and the entropy generation.

Solution:

C.V. Total tank. Control mass of constant volume.

$$\text{Mass and volume: } m_2 = m_A + m_B; \quad V = V_A + V_B$$

$$\text{Energy Eq.: } U_2 - U_1 = m_2 u_2 - m_A u_{A1} - m_B u_{B1} = 1Q_2 - 1W_2 = 0$$

$$\text{Entropy Eq.: } S_2 - S_1 = m_2 s_2 - m_A s_{A1} - m_B s_{B1} = 1Q_2/T + 1S_{\text{gen}}$$

$$\text{Process Eq.: } V = \text{constant} \Rightarrow 1W_2 = 0; \quad \text{Insulated} \Rightarrow 1Q_2 = 0$$

$$\text{Ideal gas at A1: } V_A = m_A R T_{A1} / P_{A1} = 0.5 \times 0.287 \times 300 / 200 = 0.2153 \text{ m}^3$$

$$\text{Ideal gas at B1: } V_B = m_B R T_{B1} / P_{B1} = 0.75 \times 0.287 \times 400 / 100 = 0.861 \text{ m}^3$$

$$\text{State 2: } m_2 = m_A + m_B = 1.25 \text{ kg}; \quad V_2 = V_A + V_B = 1.0763 \text{ m}^3$$

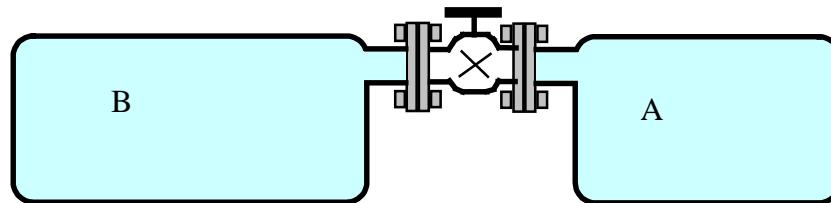
$$\text{Energy Eq.: } u_2 = \frac{m_A u_{A1} + m_B u_{B1}}{m_2} \quad \text{and use constant specific heat}$$

$$T_2 = \frac{m_A}{m_2} T_{A1} + \frac{m_B}{m_2} T_{B1} = \frac{0.5}{1.25} 300 + \frac{0.75}{1.25} 400 = 360 \text{ K}$$

$$P_2 = m_2 R T_2 / V = 1.25 \text{ kg} \times 0.287 \text{ kJ/kgK} \times 360 \text{ K} / 1.0763 \text{ m}^3 = 120 \text{ kPa}$$

$$\begin{aligned} S_2 - S_1 &= m_A [C_P \ln T_2 / T_{A1} - R \ln(P_2 / P_{A1})] + m_B [C_P \ln T_2 / T_{B1} - R \ln(P_2 / P_{B1})] \\ &= 0.5 \left[1.004 \ln \frac{360}{300} - 0.287 \ln \frac{120}{200} \right] + 0.75 \left[1.004 \ln \frac{360}{400} - 0.287 \ln \frac{120}{100} \right] \\ &= 0.5 \times 1.3514 + 0.75 \times (-0.1581) = 0.5571 \text{ kJ/K} \end{aligned}$$

$$1S_{\text{gen}} = S_2 - S_1 = 0.5571 \text{ kJ/K}$$



6.145

One kg of air at 100 kPa is mixed with 2 kg air at 200 kPa, both at 300 K, in a rigid insulated tank. Find the final state (P, T) and the entropy generation in the process.

C.V. All the air.

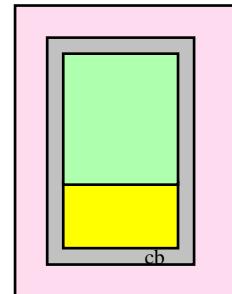
$$\text{Energy Eq.: } U_2 - U_1 = Q - W$$

$$\text{Entropy Eq.: } S_2 - S_1 = Q/T + \Delta S_{\text{gen}}$$

$$\text{Process Eqs.: } V = C; \quad W = 0, \quad Q = 0$$

$$\text{States A1, B1: } u_{A1} = u_{B1}$$

$$V_A = m_A R T_1 / P_{A1}; \quad V_B = m_B R T_1 / P_{B1}$$



$$U_2 - U_1 = m_2 u_2 - m_A u_{A1} - m_B u_{B1} = 0 \Rightarrow u_2 = (u_{A1} + 2u_{B1})/3 = u_{A1}$$

$$\text{State 2: } T_2 = T_1 = 300 \text{ K} \text{ (from } u_2\text{); } \quad m_2 = m_A + m_B = 3 \text{ kg;}$$

$$V_2 = m_2 R T_1 / P_2 = V_A + V_B = m_A R T_1 / P_{A1} + m_B R T_1 / P_{B1}$$

Divide with $m_A R T_1$ and get

$$3/P_2 = 1/P_{A1} + 2/P_{B1} = \frac{1}{100} + \frac{2}{200} = 0.02 \text{ kPa}^{-1} \Rightarrow P_2 = 150 \text{ kPa}$$

Entropy change from Eq. 6.16 with the same T, so only P changes

$$\begin{aligned} \Delta S_{\text{gen}} &= S_2 - S_1 = -m_A R \ln \frac{P_2}{P_{A1}} - m_B R \ln \frac{P_2}{P_{B1}} \\ &= -0.287 [1 \times \ln \frac{150}{100} + 2 \times \ln \frac{150}{200}] \\ &= -0.287 (0.4055 - 0.5754) = 0.049 \text{ kJ/K} \end{aligned}$$

6.146

A rigid storage tank of 1.5 m^3 contains 1 kg argon at 30°C . Heat is then transferred to the argon from a furnace operating at 1300°C until the specific entropy of the argon has increased by 0.343 kJ/kg K . Find the total heat transfer and the entropy generated in the process.

Solution:

C.V. Argon out to 1300°C . Control mass. , $m = 1 \text{ kg}$

Argon is an ideal gas with constant heat capacity.

$$\text{Energy Eq.3.5: } m(u_2 - u_1) = m C_v(T_2 - T_1) = _1Q_2 - _1W_2$$

$$\text{Entropy Eq.6.37: } m(s_2 - s_1) = _1Q_2/T_{\text{res}} + _1S_2 \text{ gen tot}$$

$$\text{Process: } V = \text{constant} \Rightarrow v_2 = v_1 \text{ also } _1W_2 = 0$$

$$\text{Properties: Table A.5 } R = 0.20813, C_v = 0.312 \text{ kJ/kg K}$$

$$\text{State 1: } (T_1, v_1 = V/m) \quad P_1 = mRT_1/V = 42.063 \text{ kPa}$$

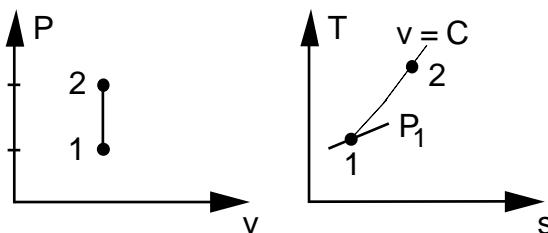
$$\text{State 2: } s_2 = s_1 + 0.343, \text{ and change in } s \text{ from Eq.6.16 or Eq.6.17}$$

$$s_2 - s_1 = C_p \ln(T_2 / T_1) - R \ln(v_2 / v_1) = C_v \ln(T_2 / T_1)$$

$$T_2 / T_1 = \exp\left[\frac{s_2 - s_1}{C_v}\right] = \exp\left[\frac{0.343}{0.312}\right] = \exp(1.09936) = 3.0$$

$$Pv = RT \Rightarrow (P_2 / P_1)(v_2 / v_1) = T_2 / T_1 = P_2 / P_1$$

$$T_2 = 3.0 \times T_1 = 909.45 \text{ K}, \quad P_2 = 3.0 \times P_1 = 126.189 \text{ kPa}$$



Heat transfer from energy equation

$$_1Q_2 = 1 \times 0.312 (909.45 - 303.15) = \mathbf{189.2 \text{ kJ}}$$

Entropy generation from entropy equation

$$\begin{aligned} _1S_2 \text{ gen tot} &= m(s_2 - s_1) - _1Q_2/T_{\text{res}} \\ &= 1 \times 0.343 - 189.2 / (1300 + 273) = \mathbf{0.223 \text{ kJ/K}} \end{aligned}$$

6.147

Argon in a light bulb is at 110 kPa, 90°C. The light is turned off so the argon cools to the ambient 20°C. Disregard the glass and any other mass and find the specific entropy generation.

Solution:

C.V. Argon gas. Neglect any heat transfer.

$$\text{Energy Eq.3.5: } m(u_2 - u_1) = \dot{Q}_2$$

$$\text{Entropy Eq.6.37: } s_2 - s_1 = \int dq/T + \dot{s}_{2 \text{ gen}} = \dot{q}_2/T_{\text{room}} + \dot{s}_{2 \text{ gen}}$$

$$\text{Process: } v = \text{constant and ideal gas} \Rightarrow P_2/P_1 = T_2/T_1$$

$$\dot{q}_2 = u_2 - u_1 = C_v(T_2 - T_1) = 0.312(20 - 90) = -21.84 \text{ kJ/kg}$$

Evaluate changes in s from Eq.6.16 or 8.17

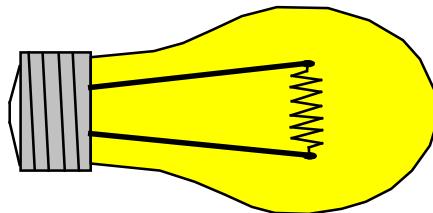
$$s_2 - s_1 = C_p \ln(T_2/T_1) - R \ln(P_2/P_1) \quad \text{Eq.6.16}$$

$$= C_p \ln(T_2/T_1) - R \ln(T_2/T_1) = C_v \ln(T_2/T_1) \quad \text{Eq.6.17}$$

$$= 0.312 \ln[(20 + 273)/(90 + 273)] = -0.06684 \text{ kJ/kg K}$$

$$\dot{s}_{2 \text{ gen}} = s_2 - s_1 - \dot{q}_2/T_{\text{room}} = -0.06684 + 21.84 / 293.15$$

$$= \mathbf{0.00766 \text{ kJ/kgK}}$$



6.148

A rigid tank contains 2 kg of air at 200 kPa and ambient temperature, 20°C. An electric current now passes through a resistor inside the tank. After a total of 100 kJ of electrical work has crossed the boundary, the air temperature inside is 80°C. Is this possible?

Solution:

C.V.: Air in tank out to ambient;

$$\text{Energy Eq.3.5: } m(u_2 - u_1) = \dot{Q}_2 - \dot{W}_2, \quad \dot{W}_2 = -100 \text{ kJ}$$

$$\text{Entropy Eq.6.37: } m(s_2 - s_1) = \int dQ/T + \dot{S}_{\text{gen}} = \dot{Q}_2/T_{\text{amb}} + \dot{S}_{\text{gen}}$$

Process: Constant volume and mass so $v_2 = v_1$

State 1: $T_1 = 20^\circ\text{C}$, $P_1 = 200 \text{ kPa}$, $m_1 = 2 \text{ kg}$

State 2: $T_2 = 80^\circ\text{C}$, $v_2 = v_1$

Ideal gas, Table A.5: $R = 0.287 \text{ kJ/kg-K}$, $C_V = 0.717 \text{ kJ/kg-K}$

Assume constant specific heat then energy equation gives

$$\dot{Q}_2 = mC_V(T_2 - T_1) + \dot{W}_2 = 2 \times 0.717(80 - 20) - 100 = -14.0 \text{ kJ}$$

Change in s from Eq.6.17 (since second term drops out)

$$s_2 - s_1 = C_V \ln(T_2/T_1) + R \ln \frac{v_2}{v_1}; \quad v_2 = v_1, \quad \ln \frac{v_2}{v_1} = 0$$

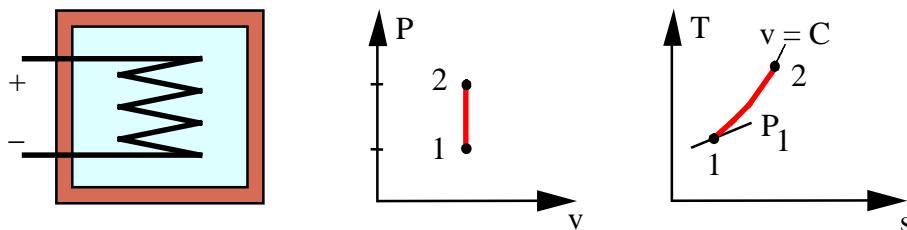
$$s_2 - s_1 = C_V \ln(T_2/T_1) = 0.1336 \text{ kJ/kg-K}$$

Now Eq.6.37

$$\dot{S}_{\text{gen}} = m(s_2 - s_1) - \dot{Q}_2/T_{\text{amb}} = 2 \times 0.1336 + \frac{14}{293} = 0.315 \text{ kJ/K} \geq 0,$$

Process is Possible

Note: $P_2 = P_1 \frac{T_2}{T_1}$ in Eq.6.16 $s_2 - s_1 = C_p \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1}$, results in the same answer as Eq.6.17.



6.149

A piston/cylinder system contains 50 L of air at 300°C, 100 kPa, with the piston initially on a set of stops. A total external constant force acts on the piston so a balancing pressure inside should be 200 kPa. The cylinder is made of 2 kg of steel initially at 1300°C. The system is insulated so that heat transfer occurs only between the steel cylinder and the air. The system comes to equilibrium. Find the final temperature, and the entropy generation.

C.V.: Steel + water out to ambient T_0 . This is a control mass.

$$\text{Energy Eq.: } U_2 - U_1 = m_{\text{air}}(u_2 - u_1) + m_{\text{st}}(u_2 - u_1) = 1Q_2 - 1W_2$$

$$\text{Entropy Eq.: } S_2 - S_1 = m_{\text{air}}(s_2 - s_1) + m_{\text{st}}(s_2 - s_1) = \int dQ/T + 1S_2 \text{ gen} = 1S_2 \text{ gen}$$

Process: $1Q_2 = 0$ and must be on P-V diagram shown

$$m_{\text{air}} = P_1 V_1 / RT_1 = 100 \times 0.05 / (0.287 \times 573.15) = 0.0304 \text{ kg}$$

$$\text{Since } V_{1a} = V_1 \text{ then } T_{1a} = T_1(P_{\text{float}}/P_1) = 573.15 \times 200/100 = 1146.3 \text{ K}$$

$$\text{Use constant } C_v \text{ for air at 900 K: } C_v = \Delta u / \Delta T = 0.833 \text{ kJ/kgK} \text{ (from A.7)}$$

$$\text{To reach state 1a: } \Delta U_{\text{air}} = mC_v \Delta T = 0.0304 \times 0.833 \times (1146 - 573) = 14.5 \text{ kJ}$$

$$\Delta U_{\text{st}} = mC_v \Delta T = 2 \times 0.46 \times (1146 - 1573) = -392.8 \text{ kJ}$$

Conclusion from this is: T_2 is higher than $T_{1a} = 1146 \text{ K}$, piston lifts, $P_2 = P_{\text{float}}$

Write the work as $1W_2 = P_2(V_2 - V_1)$ and use constant C_v in the energy Eq. as

$$m_{\text{air}} C_v (T_2 - T_1) + m_{\text{st}} C_{\text{st}} (T_2 - T_1) = -P_2 m_{\text{air}} v_2 + P_2 V_1$$

now $P_2 v_2 = RT_2$ for the air, so isolate T_2 terms as

$$[m_{\text{air}} (C_v + R) + m_{\text{st}} C_{\text{st}}] T_2 = m_{\text{air}} C_v T_{1 \text{ air}} + m_{\text{st}} C_{\text{st}} T_{1 \text{ st}} + P_2 V_1$$

$$[0.0304 \times 1.12 + 2 \times 0.46] T_2 = 0.0304 \times 0.833 \times 573.15$$

$$+ 2 \times 0.46 \times 1573.15 + 200 \times 0.05$$

Solution gives: $T_2 = 1542.7 \text{ K}$

$$\begin{aligned} 1S_2 \text{ gen} &= S_2 - S_1 = m_{\text{air}}(s_2 - s_1) + m_{\text{st}}(s_2 - s_1) \\ &= m_{\text{air}} [C_P \ln(T_2/T_{1 \text{ air}}) - R \ln(P_2/P_1)] + m_{\text{st}} C_{\text{st}} \ln(T_2/T_{1 \text{ st}}) \\ &= 0.0304 [1.12 \ln \frac{1542.7}{573.15} - 0.287 \ln \frac{200}{100}] + 2 \times 0.46 \ln \left(\frac{1542.7}{1573.15} \right) \\ &= 0.027665 - 0.017982 = \mathbf{0.0097 \text{ kJ/K}} \end{aligned}$$

6.150

A spring loaded piston cylinder contains 1.5 kg air at 27°C and 160 kPa. It is now heated in a process where pressure is linear in volume, $P = A + BV$, to twice the initial volume where it reaches 900 K. Find the work, the heat transfer and the total entropy generation assuming a source at 900 K.

Solution:

C.V. Air out to the 900 K source. Since air T is lower than the source temperature we know that this is an irreversible process.

Continuity Eq.: $m_2 = m_1 = m$,

Energy Eq.3.5: $m(u_2 - u_1) = _1Q_2 - _1W_2$

Entropy Eq.6.37: $m(s_2 - s_1) = \int dQ/T + _1S_{2\text{ gen}} = _1Q_2/T_{\text{source}} + _1S_{2\text{ gen}}$

Process: $P = A + BV$

State 1: (T_1, P_1) Table A.7 $u_1 = 214.36 \text{ kJ/kg}$

$$V_1 = mRT_1/P_1 = (1.5 \times 0.287 \times 300) \text{ kJ} / 160 \text{ kPa} = 0.8072 \text{ m}^3$$

State 2: $(T_2, v_2 = 2v_1)$ Table A.7 $u_2 = 674.824 \text{ kJ/kg}$

$$\begin{aligned} P_2 &= RT_2/v_2 = RT_2/2v_1 = T_2 P_1 / 2T_1 = P_1 T_2 / 2 T_1 \\ &= 160 \text{ kPa} \times 900 \text{ K} / (2 \times 300 \text{ K}) = 240 \text{ kPa} \end{aligned}$$

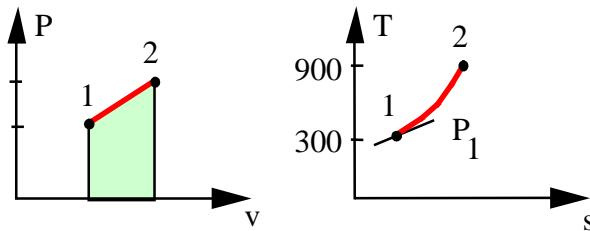
From the process equation we can express the work as

$$\begin{aligned} _1W_2 &= \int P dV = 0.5 \times (P_1 + P_2)(V_2 - V_1) = 0.5 \times (P_1 + P_2)V_1 \\ &= 0.5 \times (160 + 240) \text{ kPa} \times 0.8072 \text{ m}^3 = \mathbf{161.4 \text{ kJ}} \end{aligned}$$

$$_1Q_2 = 1.5 \times (674.824 - 214.36) + 161.4 = \mathbf{852.1 \text{ kJ}}$$

Change in s from Eq.6.19 and Table A.7 values

$$\begin{aligned} _1S_{2\text{ gen}} &= m(s_{T2}^o - s_{T1}^o - R \ln \frac{P_2}{P_1}) - _1Q_2/T_{\text{SOURCE}} \\ &= 1.5 \times [8.0158 - 6.8693 - 0.287 \ln (\frac{240}{160})] - (\frac{852.1}{900}) \\ &= 1.545 - 0.947 = \mathbf{0.598 \text{ kJ/K}} \end{aligned}$$



6.151

A rigid container with volume 200 L is divided into two equal volumes by a partition, shown in Fig. P6.151. Both sides contain nitrogen, one side is at 2 MPa, 200°C, and the other at 200 kPa, 100°C. The partition ruptures, and the nitrogen comes to a uniform state at 70°C. Assume the temperature of the surroundings is 20°C, determine the work done and the net entropy change for the process.

Solution:

$$\text{C.V. : A + B no change in volume. } \mathbf{W}_2 = \mathbf{0}$$

$$m_{A1} = P_{A1}V_{A1}/RT_{A1} = (2000 \times 0.1)/(0.2968 \times 473.2) = 1.424 \text{ kg}$$

$$m_{B1} = P_{B1}V_{B1}/RT_{B1} = (200 \times 0.1)/(0.2968 \times 373.2) = 0.1806 \text{ kg}$$

$$P_2 = m_{\text{TOT}}RT_2/V_{\text{TOT}} = (1.6046 \times 0.2968 \times 343.2)/0.2 = 817 \text{ kPa}$$

From Eq.6.16

$$\begin{aligned} S_2 - S_1 &= m_{A1}(s_2 - s_1)_{A1} + m_{B1}(s_2 - s_1)_{B1} \\ &= 1.424 \text{ kg} \left[1.042 \ln \frac{343.2}{473.2} - 0.2968 \ln \frac{817}{2000} \right] \text{ kJ/kgK} \\ &\quad + 0.1806 \text{ kg} \left[1.042 \ln \frac{343.2}{373.2} - 0.2968 \ln \frac{817}{200} \right] \text{ kJ/kgK} = -0.1894 \text{ kJ/K} \end{aligned}$$

$$\begin{aligned} {}_1Q_2 &= U_2 - U_1 = 1.424 \times 0.745(70 - 200) + 0.1806 \times 0.745(70 - 100) \\ &= -141.95 \text{ kJ} \end{aligned}$$

From Eq.6.37

$$\begin{aligned} {}_1S_{2 \text{ gen}} &= S_2 - S_1 - {}_1Q_2/T_0 = -0.1894 \text{ kJ/K} + 141.95 \text{ kJ /293.2 K} \\ &= -0.1894 + 0.4841 = \mathbf{+0.2947 \text{ kJ/K}} \end{aligned}$$

6.152

A constant pressure piston/cylinder contains 0.5 kg air at 300 K, 400 kPa. Assume the piston/cylinder has a total mass of 1 kg steel and is at the same temperature as the air at any time. The system is now heated to 1600 K by heat transfer from a 1700 K source. Find the entropy generation using constant specific heat for air.

C.V. Air and Steel.

$$\text{Energy Eq.: } U_2 - U_1 = m_{\text{air}}(u_2 - u_1) + m_{\text{st}}(u_2 - u_1) = {}_1Q_2 - {}_1W_2$$

$$\text{Entropy Eq.6.37: } m_{\text{air}}(s_2 - s_1) + m_{\text{st}}(s_2 - s_1) = {}_1Q_2/T_{\text{source}} + {}_1S_{2 \text{ gen}}$$

$$\text{Process: } P = C \Rightarrow {}_1W_2 = \int_1^2 P dV = P(V_2 - V_1) = P m_{\text{air}}(v_2 - v_1)$$

$${}_1Q_2 = m_{\text{air}}(u_2 - u_1)_{\text{air}} + m_{\text{st}}(u_2 - u_1)_{\text{st}} + {}_1W_2 = m_{\text{air}}(h_2 - h_1)_{\text{air}} + m_{\text{st}}(u_2 - u_1)_{\text{st}}$$

$$\text{Use A.3: } (u_2 - u_1)_{\text{st}} = C(T_2 - T) = 0.46 \text{ kJ/kgK} \times (1600 - 300) \text{ K} = 598 \text{ kJ/kg}$$

$$\text{Use A.5: } (h_2 - h_1)_{\text{air}} = C_p(T_2 - T_1) = 1.004 \text{ kJ/kgK} \times (1600 - 300) \text{ K} = 1305.2 \text{ kJ/kg}$$

$$\begin{aligned} {}_1Q_2 &= m_{\text{air}}(h_2 - h_1)_{\text{air}} + m_{\text{st}}(u_2 - u_1)_{\text{st}} \\ &= 0.5 \text{ kg} \times 1305 \text{ kJ/kg} + 1 \text{ kg} \times 598 \text{ kJ/kg} = 1250.6 \text{ kJ} \end{aligned}$$

$$\begin{aligned} S_2 - S_1 &= m_{\text{air}}(s_2 - s_1) + m_{\text{st}}(s_2 - s_1) \\ &= 0.5 \text{ kg} \times 1.004 \text{ kJ/kgK} \times \ln \frac{1600}{300} + 1 \text{ kg} \times 0.46 \text{ kJ/kgK} \times \ln \frac{1600}{300} \\ &= 1.6104 \text{ kJ/K} \\ {}_1S_{2 \text{ gen}} &= S_2 - S_1 - {}_1Q_2/T_{\text{source}} = 1.6104 - 1250.6/1700 = \mathbf{0.875 \text{ kJ/K}} \end{aligned}$$

6.153

Do Problem 6.152 using Table A.7.

C.V. Air and Steel.

$$\text{Energy Eq.: } U_2 - U_1 = m_{\text{air}}(u_2 - u_1) + m_{\text{st}}(u_2 - u_1) = 1Q_2 - 1W_2$$

$$\text{Entropy Eq.6.37: } m_{\text{air}}(s_2 - s_1) + m_{\text{st}}(s_2 - s_1) = 1Q_2/T_{\text{source}} + 1S_2 \text{ gen}$$

$$\text{Process: } P = C \Rightarrow 1W_2 = \int_1^2 P dV = P(V_2 - V_1) = P m_{\text{air}}(v_2 - v_1)$$

$$1Q_2 = m_{\text{air}}(u_2 - u_1)_{\text{air}} + m_{\text{st}}(u_2 - u_1)_{\text{st}} + 1W_2 = m_{\text{air}}(h_2 - h_1)_{\text{air}} + m_{\text{st}}(u_2 - u_1)_{\text{st}}$$

$$\text{Use air tables A.7: } (h_2 - h_1)_{\text{air}} = 1757.33 - 300.47 = 1456.86 \text{ kJ/kg}$$

$$(s_2 - s_1)_{\text{air}} = 8.69051 - 6.86926 + 0 = 1.82125 \text{ kJ/kgK}$$

No pressure correction as $P_2 = P_1$

$$\text{Use A.3: } (u_2 - u_1)_{\text{st}} = C(T_2 - T_1) = 0.46 \text{ kJ/kgK} \times (1600 - 300) \text{ K} = 598 \text{ kJ/kg}$$

$$\begin{aligned} 1Q_2 &= m_{\text{air}}(h_2 - h_1)_{\text{air}} + m_{\text{st}}(u_2 - u_1)_{\text{st}} \\ &= 0.5 \text{ kg} \times 1456.86 \text{ kJ/kg} + 1 \text{ kg} \times 598 \text{ kJ/kg} = \mathbf{1326.43 \text{ kJ}} \end{aligned}$$

$$S_2 - S_1 = m_{\text{air}}(s_2 - s_1) + m_{\text{st}}(s_2 - s_1)$$

$$= 1.82125 + 1 \text{ kg} \times 0.46 \text{ kJ/kgK} \times \ln \frac{1600}{300} = 2.59128 \text{ kJ/K}$$

$$1S_2 \text{ gen} = S_2 - S_1 - 1Q_2/T_{\text{source}} = 2.59128 - 1326.43/1700 = \mathbf{1.811 \text{ kJ/K}}$$

6.154

Nitrogen at 200°C, 300 kPa is in a piston cylinder, volume 5 L, with the piston locked with a pin. The forces on the piston require a pressure inside of 200 kPa to balance it without the pin. The pin is removed and the piston quickly comes to its equilibrium position without any heat transfer. Find the final P, T and V and the entropy generation due to this partly unrestrained expansion.

Solution:

C.V. Nitrogen gas.

$$\text{Energy Eq.3.5: } m(u_2 - u_1) = _1Q_2 - _1W_2 = - \int P_{\text{eq}} dV = -P_2(V_2 - V_1)$$

$$\text{Entropy Eq.6.37: } m(s_2 - s_1) = 0 + _1S_{2\text{ gen}}$$

Process: $_1Q_2 = 0$ (already used), $P = P_{\text{eq}}$ after pin is out.

State 1: 200 °C, 300 kPa State 2: $P_2 = P_{\text{eq}} = \mathbf{200 \text{ kPa}}$

$$m = P_1 V_1 / RT_1 = 300 \times 0.005 / 0.2968 \times 473.15 = 0.01068 \text{ kg}$$

The energy equation becomes

$$mu_2 + P_2 V_2 = mu_1 + P_2 V_1 = mh_2 \Rightarrow$$

$$h_2 = u_1 + P_2 V_1 / m = u_1 + P_2 V_1 RT_1 / P_1 V_1 = u_1 + (P_2 / P_1) RT_1$$

Solve using constant C_p , C_v

$$C_p T_2 = C_v T_1 + (P_2 / P_1) RT_1$$

$$T_2 = T_1 [C_v + (P_2 / P_1) R] / C_p$$

$$= 473.15 [0.745 + (200 / 300) \times 0.2368] / 1.042$$

$$= \mathbf{428.13 \text{ K}}$$

$$V_2 = V_1 (T_2 / T_1) \times (P_1 / P_2) = 0.005 \times \frac{428.13}{473.15} \times \frac{300}{200}$$

$$= 0.00679 \text{ m}^3$$

$$_1S_{2\text{ gen}} = m(s_2 - s_1) \cong m[C_p \ln(T_2/T_1) - R \ln(P_2/P_1)]$$

$$= P_1 V_1 / RT_1 [C_p \ln(T_2/T_1) - R \ln(P_2/P_1)]$$

$$= 0.01068 [1.042 \times \ln(428.13/473.15) - 0.2968 \times \ln(200 / 300)]$$

$$= \mathbf{0.000173 \text{ kJ/K}}$$

6.155

The air in the tank of Problem 6.88 receives the heat transfer from a reservoir at 450 K. Find the entropy generation due to the process from 1 to 3.

C.V. Air out to reservoir.

$$\text{Energy eq.: } m(u_3 - u_1) = _1Q_3 - _1W_3$$

$$\text{Entropy eq.: } m(s_3 - s_1) = _1Q_3 / T_{\text{res}} + _1S_{3 \text{ gen}}$$

$$\text{State 1: } m = P_1 V_1 / RT_1 = \frac{100 \times 0.75}{0.287 \times 300} \frac{\text{kPa m}^3}{\text{kJ/kg}} = 0.871 \text{ kg}$$

Process 1 to 2: Constant volume heating, $dV = 0 \Rightarrow _1W_2 = 0$

$$P_2 = P_1 T_2 / T_1 = 100 \times 400 / 300 = 133.3 \text{ kPa}$$

Process 2 to 3: Isothermal expansion, $dT = 0 \Rightarrow u_3 = u_2$ and

$$P_3 = P_2 V_2 / V_3 = 133.3 \times 0.75 / 1.5 = 66.67 \text{ kPa}$$

$$_2W_3 = \int_2^3 P dV = P_2 V_2 \ln \left(\frac{V_3}{V_2} \right) = 133.3 \times 0.75 \ln(2) = 69.3 \text{ kJ}$$

The overall process:

$$_1W_3 = _1W_2 + _2W_3 = _2W_3 = 69.3 \text{ kJ}$$

From the energy equation

$$\begin{aligned} _1Q_3 &= m(u_3 - u_1) + _1W_3 = m C_v (T_3 - T_1) + _1W_3 \\ &= 0.871 \times 0.717 (400 - 300) + 69.3 = 131.8 \text{ kJ} \end{aligned}$$

$$\begin{aligned} _1S_{3 \text{ gen}} &= m(s_3 - s_1) - _1Q_3 / T_{\text{res}} \\ &= m \left(C_p \ln \frac{T_3}{T_1} - R \ln \frac{P_3}{P_1} \right) - _1Q_3 / T_{\text{res}} \\ &= 0.871 \left[1.004 \ln \frac{400}{300} - 0.287 \ln \frac{66.67}{100} \right] - \frac{131.8}{450} \\ &= \mathbf{0.060 \text{ kJ/K}} \end{aligned}$$

6.156

One kg of carbon dioxide at 100 kPa, 400 K is mixed with two kg carbon dioxide at 200 kPa, 2000 K, in a rigid insulated tank. Find the final state (P, T) and the entropy generation in the process using constant heat capacity from Table A.5.

C.V. All the carbon dioxide.

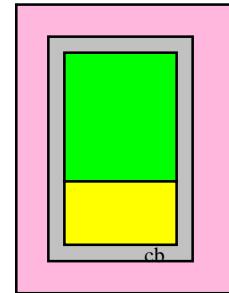
$$\text{Continuity: } m_2 = m_A + m_B = 3 \text{ kg}$$

$$\text{Energy Eq.: } U_2 - U_1 = 0 - 0$$

$$\text{Entropy Eq.: } S_2 - S_1 = 0 + S_{\text{gen}}$$

$$\text{Process Eqs.: } V = C; \quad W = 0, \quad Q = 0$$

$$V_A = m_A R T_1 / P_{A1}; \quad V_B = m_B R T_1 / P_{B1}$$



$$\begin{aligned} U_2 - U_1 &= m_2 u_2 - m_A u_{A1} - m_B u_{B1} = 0 \\ &= m_2 C_v T_2 - m_A C_v T_{A1} - m_B C_v T_{B1} \\ \Rightarrow T_2 &= (m_A T_{A1} + m_B T_{B1}) / m_2 = \frac{1}{3} \times 400 + \frac{2}{3} \times 2000 = \mathbf{1466.7 \text{ K}} \end{aligned}$$

$$\begin{aligned} \text{State 2: } V_2 &= m_2 R T_2 / P_2 = V_A + V_B = m_A R T_{A1} / P_{A1} + m_B R T_{B1} / P_{B1} \\ &= 1 \times R \times 400 / 100 + 2 \times R \times 2000 / 200 = 24 \text{ R} \end{aligned}$$

Substitute m_2 , T_2 and solve for P_2

$$P_2 = 3 \text{ R } T_2 / 24 \text{ R} = 3 \times 1466.7 / 24 = \mathbf{183.3 \text{ kPa}}$$

Entropy change from Eq. 6.16

$$\begin{aligned} (s_2 - s_1)_A &= C_p \ln \frac{T_2}{T_{A1}} - R \ln \frac{P_2}{P_{A1}} = 0.842 \ln \frac{1466.7}{400} - 0.1889 \ln \frac{183.3}{100} \\ &= 0.97955 \text{ kJ/kgK} \end{aligned}$$

$$\begin{aligned} (s_2 - s_1)_B &= C_p \ln \frac{T_2}{T_{B1}} - R \ln \frac{P_2}{P_{B1}} = 0.842 \ln \frac{1466.7}{2000} - 0.1889 \ln \frac{183.3}{200} \\ &= -0.24466 \text{ kJ/kgK} \end{aligned}$$

$$\begin{aligned} S_{\text{gen}} &= S_2 - S_1 = m_A (s_2 - s_1)_A + m_B (s_2 - s_1)_B \\ &= 1 \times 0.97955 + 2 (-0.24466) = \mathbf{0.49 \text{ kJ/K}} \end{aligned}$$

6.157

One kg of carbon dioxide at 100 kPa, 400 K is mixed with two kg carbon dioxide at 200 kPa, 2000 K, in a rigid insulated tank. Find the final state (P, T) and the entropy generation in the process using table A.8.

C.V. All the carbon dioxide.

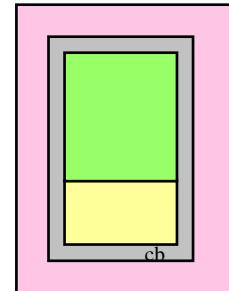
$$\text{Continuity: } m_2 = m_A + m_B = 3 \text{ kg}$$

$$\text{Energy Eq.: } U_2 - U_1 = 0 - 0$$

$$\text{Entropy Eq.: } S_2 - S_1 = 0 + S_{\text{gen}}$$

$$\text{Process Eqs.: } V = C; \quad W = 0, \quad Q = 0$$

$$V_A = m_A R T_{A1} / P_{A1}; \quad V_B = m_B R T_{B1} / P_{B1}$$



$$U_2 - U_1 = m_2 u_2 - m_A u_{A1} - m_B u_{B1} = 0$$

$$u_2 = (m_A u_{A1} + m_B u_{B1}) / m_2 = \frac{1}{3} \times 228.19 + \frac{2}{3} \times 1912.67 = 1351.18 \text{ kJ/kg}$$

$$\text{Interpolate in Table A.8: } T_2 = 1517.2 \text{ K}, \quad s_{T2}^{\circ} = 6.6542 \text{ kJ/kgK}$$

$$\begin{aligned} \text{State 2: } V_2 &= m_2 R T_2 / P_2 = V_A + V_B = m_A R T_{A1} / P_{A1} + m_B R T_{B1} / P_{B1} \\ &= 1 \times R \times 400 / 100 + 2 \times R \times 2000 / 200 = 24 \text{ R} \end{aligned}$$

Substitute m_2 , T_2 and solve for P_2

$$P_2 = 3 \text{ R } T_2 / 24 \text{ R} = 3 \times 1517.2 / 24 = \mathbf{189.65 \text{ kPa}}$$

Entropy change from Eq. 6.19

$$\begin{aligned} (s_2 - s_1)_A &= s_{T2}^{\circ} - s_{T1}^{\circ} - R \ln \frac{P_2}{P_{A1}} = 6.6542 - 5.1196 - 0.1889 \ln \frac{189.65}{100} \\ &= 1.4137 \text{ kJ/kgK} \end{aligned}$$

$$\begin{aligned} (s_2 - s_1)_B &= s_{T2}^{\circ} - s_{T1}^{\circ} - R \ln \frac{P_2}{P_{B1}} = 6.6542 - 7.0278 - 0.1889 \ln \frac{189.65}{200} \\ &= -0.36356 \text{ kJ/kgK} \end{aligned}$$

$$\begin{aligned} S_{\text{gen}} &= S_2 - S_1 = m_A (s_2 - s_1)_A + m_B (s_2 - s_1)_B \\ &= 1 \times 1.4137 + 2 (-0.36356) = \mathbf{0.687 \text{ kJ/K}} \end{aligned}$$

6.158

Nitrogen at 600 kPa, 127°C is in a 0.5 m³ insulated tank connected to a pipe with a valve to a second insulated initially empty tank of volume 0.25 m³, shown in Fig. P6.158. The valve is opened and the nitrogen fills both tanks at a uniform state. Find the final pressure and temperature and the entropy generation this process causes. Why is the process irreversible?

Solution:

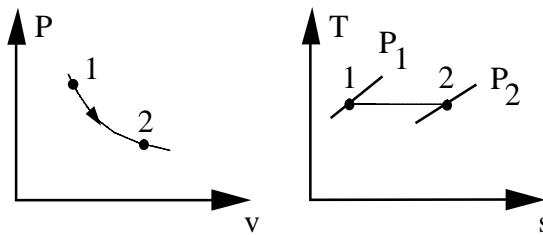
$$\text{CV Both tanks + pipe + valve Insulated : } Q = 0 \quad \text{Rigid: } W = 0$$

$$\text{Energy Eq.3.5: } m(u_2 - u_1) = 0 - 0 \Rightarrow u_2 = u_1 = u_{a1}$$

$$\text{Entropy Eq.6.37: } m(s_2 - s_1) = \int dQ/T + _1S_2 \text{ gen} = _1S_2 \text{ gen} \quad (dQ = 0)$$

$$\begin{aligned} 1: P_1, T_1, V_a &\Rightarrow m = PV/RT = (600 \times 0.25) / (0.2968 \times 400) = 1.2635 \\ &\text{kg} \end{aligned}$$

$$2: V_2 = V_a + V_b ; \text{ uniform state} \quad v_2 = V_2 / m ; \quad u_2 = u_{a1}$$



$$\text{Ideal gas } u(T) \Rightarrow u_2 = u_{a1} \Rightarrow T_2 = T_{a1} = 400 \text{ K}$$

$$P_2 = mR T_2 / V_2 = (V_1 / V_2) P_1 = \frac{1}{2} \times 600 = 300 \text{ kPa}$$

From entropy equation and Eq.6.19 for entropy change

$$\begin{aligned} S_{\text{gen}} &= m(s_2 - s_1) = m[s_{T2} - s_{T1} - R \ln(P_2 / P_1)] \\ &= m[0 - R \ln(P_2 / P_1)] = -1.2635 \times 0.2968 \ln \frac{1}{2} = 0.26 \text{ kJ/K} \end{aligned}$$

Irreversible due to unrestrained expansion in valve $P \downarrow$ but no work out.

6.159

A cylinder/piston contains carbon dioxide at 1 MPa, 300°C with a volume of 200 L. The total external force acting on the piston is proportional to V^3 . This system is allowed to cool to room temperature, 20°C. What is the total entropy generation for the process?

Solution:

C.V. Carbon dioxide gas of constant mass $m_2 = m_1 = m$ out to ambient.

$$\text{Energy Eq.3.5: } m(u_2 - u_1) = _1Q_2 - _1W_2$$

$$\text{Entropy Eq.6.37: } m(s_2 - s_1) = \int dQ/T + _1S_2 \text{ gen} = _1Q_2/T_{\text{amb}} + _1S_2 \text{ gen}$$

Process: $P = CV^3$ or $PV^{-3} = \text{constant}$, which is polytropic with $n = -3$

$$\text{State 1: } (T, P) \Rightarrow m = P_1 V_1 / RT_1 = \frac{1000 \times 0.2}{0.18892 \times 573.2} = 1.847 \text{ kg}$$

State 2: $(T, ?)$ state must be on process curve and ideal gas leads to Eq.6.28

$$\Rightarrow P_2 = P_1 (T_2/T_1)^{\frac{n}{n-1}} = 1000 (293.2/573.2)^{3/4} = 604.8 \text{ kPa}$$

$$V_2 = V_1 (T_1/T_2)^{\frac{1}{n-1}} = 0.16914 \text{ m}^3$$

$$\begin{aligned} _1W_2 &= \int P dV = (P_2 V_2 - P_1 V_1)/(1-n) \\ &= [604.8 \times 0.16914 - 1000 \times 0.2] / [1 - (-3)] = -24.4 \text{ kJ} \end{aligned}$$

$$\begin{aligned} _1Q_2 &= m(u_2 - u_1) + _1W_2 \\ &= 1.847 \times 0.653 (20 - 300) - 24.4 = -362.1 \text{ kJ} \end{aligned}$$

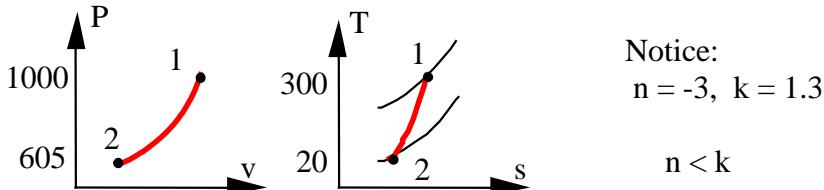
From Eq.6.16

$$\begin{aligned} m(s_2 - s_1) &= 1.847 \left[0.842 \ln \frac{293.2}{573.2} - 0.18892 \ln \frac{604.8}{1000} \right] \\ &= 1.847[-0.4694] = -0.87 \text{ kJ/K} \end{aligned}$$

$$\Delta S_{\text{SURR}} = -_1Q_2/T_{\text{amb}} = +362.1 / 293.2 = +1.235 \text{ kJ/K}$$

From Eq.6.37 or 8.39

$$\begin{aligned} _1S_2 \text{ gen} &= m(s_2 - s_1) - _1Q_2/T_{\text{amb}} = \Delta S_{\text{NET}} = \Delta S_{\text{CO}_2} + \Delta S_{\text{SURR}} \\ &= -0.87 + 1.235 = +0.365 \text{ kJ/K} \end{aligned}$$



6.160

The air in the engine cylinder of Problem 3.156 loses the heat to the engine coolant at 100°C. Find the entropy generation (external to the air) using constant specific heat.

Take CV as the air. $m_2 = m_1 = m$;

$$\text{Energy Eq.3.5 } m(u_2 - u_1) = _1Q_2 - _1W_2$$

$$\text{Entropy Eq.6.37: } m(s_2 - s_1) = \int dQ/T + _1S_2 \text{ gen} = _1Q_2/T_{\text{amb}} + _1S_2 \text{ gen}$$

$$\text{Process Eq.: } Pv^n = \text{Constant} \quad (\text{polytropic})$$

From the ideal gas law and the process equation we can get:

$$\text{State 2: } P_2 = P_1 (v_2 / v_1)^{-n} = 4000 \times 10^{-1.5} = 126.5 \text{ kPa}$$

$$T_2 = T_1 (P_2 v_2 / P_1 v_1) = (1527 + 273) \frac{126.5 \times 10}{4000} = 569.3 \text{ K}$$

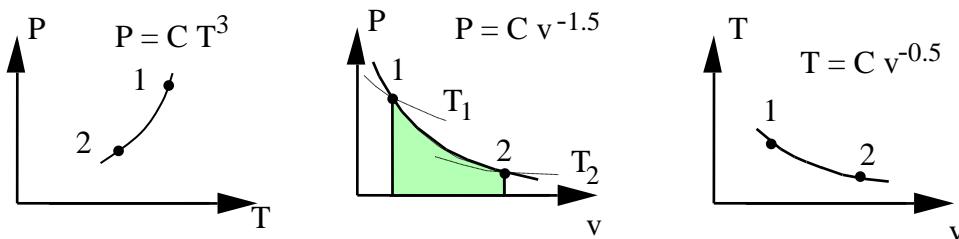
$$\text{From process eq.: } _1W_2 = \int P dV = \frac{m}{1-n} (P_2 v_2 - P_1 v_1) = \frac{mR}{1-n} (T_2 - T_1)$$

$$= \frac{0.1 \times 0.287}{1 - 1.5} (569.3 - 1800) = 70.64 \text{ kJ}$$

$$\begin{aligned} \text{From energy eq.: } _1Q_2 &= m(u_2 - u_1) + _1W_2 = mC_v(T_2 - T_1) + _1W_2 \\ &= 0.1 \times 0.717(569.3 - 1800) + 70.64 = -17.6 \text{ kJ} \end{aligned}$$

$$\begin{aligned} m(s_2 - s_1) &= m [C_p \ln(T_2/T_1) - R \ln(P_2/P_1)] \\ &= 0.1 [1.004 \ln \frac{569.3}{1800} - 0.287 \ln \frac{126.5}{4000}] = -0.01645 \text{ kJ/K} \end{aligned}$$

$$\begin{aligned} _1S_2 \text{ gen} &= m(s_2 - s_1) - _1Q_2/T_{\text{amb}} \\ &= -0.01645 + 17.6/373.15 = \mathbf{0.0307 \text{ kJ/K}} \end{aligned}$$



6.161

A cylinder/piston contains 100 L of air at 110 kPa, 25°C. The air is compressed in a reversible polytropic process to a final state of 800 kPa, 500 K. Assume the heat transfer is with the ambient at 25°C and determine the polytropic exponent n and the final volume of the air. Find the work done by the air, the heat transfer and the total entropy generation for the process.

Solution:

C.V. Air of constant mass $m_2 = m_1 = m$ out to ambient.

$$\text{Energy Eq.3.5: } m(u_2 - u_1) = _1Q_2 - _1W_2$$

$$\text{Entropy Eq.6.37: } m(s_2 - s_1) = \int dQ/T + _1S_{2\text{ gen}} = _1Q_2/T_0 + _1S_{2\text{ gen}}$$

$$\text{Process: } P_1 v_1^n = P_2 v_2^n \quad \text{Eq.6.27}$$

$$\text{State 1: } (T_1, P_1) \quad \text{State 2: } (T_2, P_2)$$

Thus the unknown is the exponent n .

$$m = P_1 V_1 / (RT_1) = 110 \times 0.1 / (0.287 \times 298.15) = 0.1286 \text{ kg}$$

The relation from the process and ideal gas is in Eq.6.28

$$T_2/T_1 = (P_2/P_1)^{\frac{n-1}{n}} \Rightarrow \frac{500}{298.15} = \left(\frac{800}{110}\right)^{\frac{n-1}{n}} \Rightarrow \frac{n-1}{n} = 0.260573$$

$$n = 1.3524, \quad V_2 = V_1 (P_1/P_2)^{\frac{1}{n}} = 0.1 \left(\frac{110}{800}\right)^{0.73943} = 0.02306 \text{ m}^3$$

The work is from Eq.6.29

$$_1W_2 = \int P dV = \frac{P_2 V_2 - P_1 V_1}{1-n} = \frac{800 \times 0.02306 - 110 \times 0.1}{1 - 1.3524} = -21.135 \text{ kJ}$$

Heat transfer from the energy equation

$$\begin{aligned} _1Q_2 &= mC_v(T_2 - T_1) + _1W_2 \\ &= 0.1286 \times 0.717 \times (500 - 298.15) - 21.135 = -2.523 \text{ kJ} \end{aligned}$$

Entropy change from Eq.6.16

$$\begin{aligned} s_2 - s_1 &= C_{P0} \ln(T_2/T_1) - R \ln(P_2/P_1) \\ &= 1.004 \ln\left(\frac{500}{298.15}\right) - 0.287 \ln\left(\frac{800}{110}\right) = -0.0504 \frac{\text{kJ}}{\text{kg K}} \end{aligned}$$

From the entropy equation (also Eq.6.37)

$$\begin{aligned} _1S_{2\text{ gen}} &= m(s_2 - s_1) - _1Q_2/T_0 \\ &= 0.1286 \times (-0.0504) + (2.523/298.15) = 0.00198 \text{ kJ/K} \end{aligned}$$

Rates or fluxes of entropy

Excerpts from this work may be reproduced by instructors for distribution on a not-for-profit basis for testing or instructional purposes only to students enrolled in courses for which this textbook has been adopted. Any other reproduction or translation of this work beyond that permitted by Sections 107 or 108 of the 1976 United States Copyright Act without the permission of the copyright owner is unlawful.

6.162

A room at 22°C is heated electrically with 1500 W to keep steady temperature. The outside ambient is at 5°C . Find the flux of S ($= \dot{Q}/T$) into the room air, into the ambient and the rate of entropy generation.

CV. The room and walls out to the ambient T, we assume steady state

$$\text{Energy Eq.: } 0 = \dot{W}_{\text{el in}} - \dot{Q}_{\text{out}} \Rightarrow \dot{Q}_{\text{out}} = \dot{W}_{\text{el in}} = 1500 \text{ W}$$

$$\text{Entropy Eq.: } 0 = -\dot{Q}_{\text{out}}/T + \dot{S}_{\text{gen tot}}$$

$$\text{Flux of S into room air at } 22^{\circ}\text{C: } \dot{Q}/T = 1500 / 295.15 = 5.08 \text{ W/K}$$

$$\text{Flux of S into ambient air at } 5^{\circ}\text{C: } \dot{Q}/T = 1500 / 278.15 = 5.393 \text{ W/K}$$

$$\text{Entropy generation: } \dot{S}_{\text{gen tot}} = \dot{Q}_{\text{out}}/T = 1500 / 278.15 = \mathbf{5.393 \text{ W/K}}$$

Comment: The flux of S into the outside air is what leaves the control volume and since the control volume did not receive any S it was all made in the process. Notice most of the generation is done in the heater, the room heat loss process generates very little S ($5.393 - 5.08 = 0.313 \text{ W/K}$)

6.163

A mass of 3 kg nitrogen gas at 2000 K, $V = C$, cools with 500 W. What is dS/dt ?

Assume that we do not generate any s in the nitrogen then

$$\text{Entropy Eq. 6.42: } \dot{S}_{cv} = \frac{\dot{Q}}{T} = -\frac{500 \text{ W}}{2000 \text{ K}} = -0.25 \text{ W/K}$$

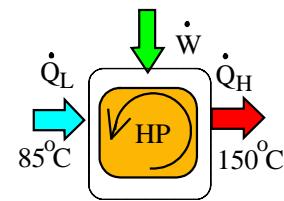
6.164

A heat pump, see problem 5.49, should upgrade 5 MW of heat at 85°C to heat delivered at 150°C. For a reversible heat pump what are the fluxes of entropy in and out of the heat pump?

C.V.TOT. Assume reversible Carnot cycle.

$$\text{Energy Eq.: } \dot{Q}_L + \dot{W} = \dot{Q}_H$$

$$\text{Entropy Eq.: } 0 = \frac{\dot{Q}_L}{T_L} - \frac{\dot{Q}_H}{T_H} \Rightarrow \frac{\dot{Q}_L}{T_L} = \frac{\dot{Q}_H}{T_H}$$



The fluxes of entropy become the same as

$$\frac{\dot{Q}_H}{T_H} = \frac{\dot{Q}_L}{T_L} = \frac{5}{273.15 + 85} \frac{\text{MW}}{\text{K}} = \mathbf{0.01396 \text{ MW/K}}$$

This is what constitutes a reversible process (flux of S in = flux out, no generation).

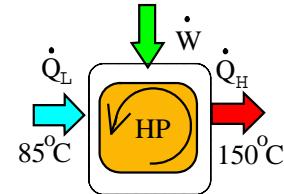
6.165

Reconsider the heat pump in the previous problem and assume it has a COP of 2.5. What are the fluxes of entropy in and out of the heat pump and the rate of entropy generation inside it?

C.V. TOT.

$$\text{Energy Eq.: } \dot{Q}_L + \dot{W} = \dot{Q}_H$$

$$\text{Entropy Eq.: } 0 = \frac{\dot{Q}_L}{T_L} - \frac{\dot{Q}_H}{T_H} + \dot{S}_{\text{gen tot}}$$



$$\text{Definition of COP: } \beta_{\text{HP}} = \frac{\dot{Q}_H}{\dot{W}_{\text{in}}} = 2.5; \quad \beta_{\text{REF}} = \beta_{\text{HP}} - 1 = \frac{\dot{Q}_L}{\dot{W}_{\text{in}}} = 1.50$$

$$\dot{W}_{\text{in}} = \dot{Q}_L / \beta_{\text{REF}} = 5 / 1.50 = 3.333 \text{ MW}$$

$$\dot{Q}_H = \dot{Q}_L + \dot{W} = 5 \text{ MW} + 3.333 \text{ MW} = 8.333 \text{ MW}$$

$$\frac{\dot{Q}_L}{T_L} = \frac{5}{273.15 + 85} \frac{\text{MW}}{\text{K}} = \mathbf{0.01396 \text{ MW/K}}$$

$$\frac{\dot{Q}_H}{T_H} = \frac{8.333}{273.15 + 85} \frac{\text{MW}}{\text{K}} = \mathbf{0.01969 \text{ MW/K}}$$

From the entropy equation

$$\dot{S}_{\text{gen tot}} = \frac{\dot{Q}_H}{T_H} - \frac{\dot{Q}_L}{T_L} = (0.01969 - 0.01396) \frac{\text{MW}}{\text{K}} = \mathbf{5.73 \text{ kW/K}}$$

6.166

A radiant heating lamp powered by electricity has a surface temperature of 1000 K emitting 500 W. The radiation is absorbed by surfaces at the ambient 18°C. Find the total entropy generation and specify where it is made, including how much.

CV1 Radiant element.

$$\text{Energy Eq.: } 0 = \dot{W}_{\text{el in}} - \dot{Q}_{\text{out}} \quad \text{Entropy Eq.: } 0 = -\dot{Q}_{\text{out}}/T_{\text{surf}} + \dot{S}_{\text{gen CV1}}$$

$$\dot{Q}_{\text{out}} = \dot{W}_{\text{el in}} = 500 \text{ W}; \quad \dot{S}_{\text{gen CV1}} = \dot{Q}_{\text{out}}/T_{\text{surf}} = \frac{500}{1000} = \mathbf{0.5 \text{ W/K}}$$

CV2 Space between radiant element and 18°C surfaces (this is the room air).

$$\text{Energy Eq.: } 0 = \dot{Q}_{\text{in}} - \dot{Q}_{\text{out}} \quad \text{Entropy Eq.: } 0 = \frac{\dot{Q}_{\text{in}}}{T_{\text{in}}} - \frac{\dot{Q}_{\text{out}}}{T_{\text{amb}}} + \dot{S}_{\text{gen CV2}}$$

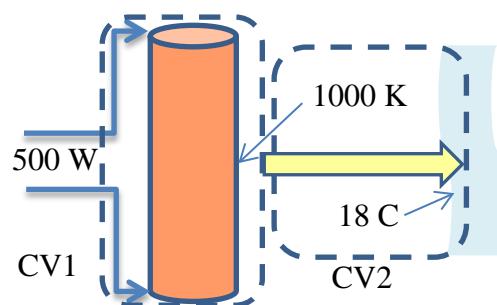
$$\begin{aligned} \dot{Q}_{\text{in}} &= \dot{Q}_{\text{out}} = 500 \text{ W}; & \dot{S}_{\text{gen CV2}} &= \frac{\dot{Q}_{\text{out}}}{T_{\text{amb}}} - \frac{\dot{Q}_{\text{in}}}{T_{\text{in}}} = \frac{500}{291.15} - \frac{500}{1000} \\ &&&= \mathbf{1.217 \text{ W/K}} \end{aligned}$$

The total entropy generation is the sum of the two contributions above, which also matches with a total control volume that is the element and the room air.

CV Radiant element and space out to ambient 18°C.

$$\text{Energy Eq.: } 0 = \dot{W}_{\text{el in}} - \dot{Q}_{\text{out}} \quad \text{Entropy Eq.: } 0 = -\dot{Q}_{\text{out}}/T_{\text{amb}} + \dot{S}_{\text{gen CV tot}}$$

$$\dot{Q}_{\text{out}} = \dot{W}_{\text{el in}} = 500 \text{ W}; \quad \dot{S}_{\text{gen CV tot}} = \frac{\dot{Q}_{\text{out}}}{T_{\text{amb}}} = \frac{500}{291.15} = \mathbf{1.717 \text{ W/K}}$$



6.167

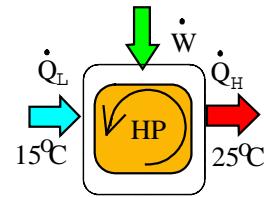
A heat pump with COP = 4 uses 1 kW of power input to heat a 25°C room, drawing energy from the outside at 15°C. Assume the high/low temperatures in the heat pump are 45°C /0°C. Find the total rates of entropy into and out of the heat pump, the rate from the outside at 15°C, and the rate to the room at 25°C?

Solution:

C.V.TOT.

$$\text{Energy Eq.: } \dot{Q}_L + \dot{W} = \dot{Q}_H$$

$$\text{Entropy Eq.: } 0 = \frac{\dot{Q}_L}{T_L} - \frac{\dot{Q}_H}{T_H} + \dot{S}_{\text{gen CV tot}}$$



$$\text{From definition of COP: } \dot{Q}_H = \text{COP } \dot{W} = 4 \times 1 \text{ kW} = 4 \text{ kW}$$

$$\text{From energy equation: } \dot{Q}_L = \dot{Q}_H - \dot{W} = (4 - 1) \text{ kW} = 3 \text{ kW}$$

$$\text{Flux into heat pump at } 0^\circ\text{C: } \frac{\dot{Q}_L}{T_{L-HP}} = \frac{3}{273.15} \frac{\text{kW}}{\text{K}} = \mathbf{0.0110 \text{ kW/K}}$$

$$\text{Flux out of heat pump at } 45^\circ\text{C: } \frac{\dot{Q}_H}{T_{H-HP}} = \frac{4}{318.15} \frac{\text{kW}}{\text{K}} = \mathbf{0.0126 \text{ kW/K}}$$

$$\text{Flux out into room at } T_H = 25^\circ\text{C: } \frac{\dot{Q}_H}{T_H} = \frac{4}{298.15} \frac{\text{kW}}{\text{K}} = \mathbf{0.0134 \text{ kW/K}}$$

$$\text{Flux from outside at } 15^\circ\text{C: } \frac{\dot{Q}_L}{T_L} = \frac{3}{288.15} \frac{\text{kW}}{\text{K}} = \mathbf{0.0104 \text{ kW/K}}$$

Comment: Following the flow of energy notice how the flux from the outside at 15°C grows a little when it arrives at 0°C this is due to entropy generation in the low T heat exchanger. The flux out of the heat pump at 45°C is larger than the flux in which is due to entropy generation in the heat pump cycle (COP is smaller than Carnot COP) and finally this flux increases due to entropy generated in the high T heat exchanger as the energy arrives at room T.

6.168

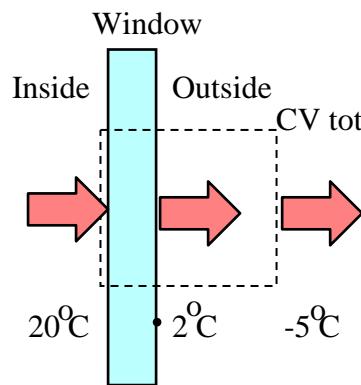
A window receives 500 W of heat transfer at the inside surface of 20°C and transmits the 500 W from its outside surface at 2°C continuing to ambient air at -5°C. Find the flux of entropy at all three surfaces and the window's rate of entropy generation.

$$\text{Flux of entropy: } \dot{S} = \frac{\dot{Q}}{T}$$

$$\dot{S}_{\text{inside}} = \frac{500}{293.15} \frac{\text{W}}{\text{K}} = 1.7056 \text{ W/K}$$

$$\dot{S}_{\text{win}} = \frac{500}{275.15} \frac{\text{W}}{\text{K}} = 1.8172 \text{ W/K}$$

$$\dot{S}_{\text{amb}} = \frac{500}{268.15} \frac{\text{W}}{\text{K}} = 1.8646 \text{ W/K}$$



$$\text{Window only: } \dot{S}_{\text{gen win}} = \dot{S}_{\text{win}} - \dot{S}_{\text{inside}} = 1.8172 - 1.7056 = \mathbf{0.112 \text{ W/K}}$$

If you want to include the generation in the outside air boundary layer where T changes from 2°C to the ambient -5°C then chose the control volume as CV tot and it becomes

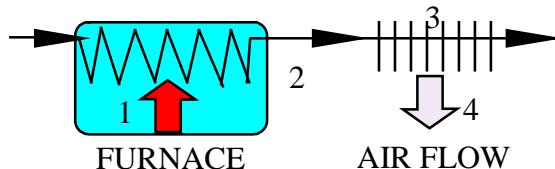
$$\dot{S}_{\text{gen tot}} = \dot{S}_{\text{amb}} - \dot{S}_{\text{inside}} = 1.8646 - 1.7056 = 0.159 \text{ W/K}$$

6.169

An amount of power, say 1000 kW, comes from a furnace at 800°C going into water vapor at 400°C. From the water the power goes to a solid metal at 200°C and then into some air at 70°C. For each location calculate the flux of s through a surface as (\dot{Q}/T) . What makes the flux larger and larger?

Solution:

$$\begin{array}{cccc} T_1 & \Rightarrow & T_2 & \Rightarrow \\ \text{furnace} & & \text{vapor} & \end{array} \quad \begin{array}{c} T_3 \Rightarrow T_4 \\ \text{metal} \quad \text{air} \end{array}$$



$$\text{Flux of } s: \quad F_s = \dot{Q}/T \quad \text{with } T \text{ as absolute temperature.}$$

$$\begin{aligned} F_{s1} &= 1000/1073.15 = 0.932 \text{ kW/K}, & F_{s2} &= 1000/673.15 = 1.486 \text{ kW/K} \\ F_{s3} &= 1000/473.15 = 2.11 \text{ kW/K}, & F_{s4} &= 1000/343.15 = 2.91 \text{ kW/K} \end{aligned}$$

T	800	400	200	70	(°C)
T _{amb}	1073	673	476	646	K
Q/T	0.932	1.486	2.114	2.915	kW/K

$1S_2$ gen for every change in T

Q over ΔT is an irreversible process

6.170

Room air at 23°C is heated by a 2000 W space heater with a surface filament temperature of 700 K, shown in Fig. P6.170. The room at steady state loses the power to the outside which is at 7°C. Find the rate(s) of entropy generation and specify where it is made.

Solution:

For any C.V at steady state the entropy equation as a rate form is Eq.6.43

$$\frac{dS_{c.v.}}{dt} = 0 = \int d\dot{Q}/T + \dot{S}_{gen}$$

C.V. Heater Element

$$\dot{S}_{gen} = -\int d\dot{Q}/T = -(-2000/700) = \mathbf{2.857 \text{ W/K}}$$

C.V. Space between heater 700 K and room 23°C

$$\dot{S}_{gen} = -\int d\dot{Q}/T = (-2000 / 700) - [-2000 / (23+273)] = \mathbf{3.9 \text{ W/K}}$$

C.V. Wall between 23°C inside and 7°C outside

$$\dot{S}_{gen} = -\int d\dot{Q}/T = [-2000 / (23+273)] - [2000 / (7 + 273)] = \mathbf{0.389 \text{ W/K}}$$

Notice biggest \dot{S}_{gen} is for the largest change in 1/T.



6.171

A car engine block receives 2 kW at its surface of 450 K from hot combustion gases at 1500 K. Near the cooling channel the engine block transmits 2 kW out at its 400 K surface to the coolant flowing at 370 K. Finally, in the radiator the coolant at 350 K delivers the 2 kW to air which is at 25 C. Find the rate of entropy generation inside the engine block, inside the coolant and in the radiator/air combination.

For a C.V at steady state we have the entropy equation as a rate form as Eq.6.43

$$\frac{dS_{c.v.}}{dt} = 0 = \int d\dot{Q}/T + \dot{S}_{gen}$$

CV1. Engine block receives 2 kW at 450 K and it leaves at 400 K

$$\dot{S}_{gen1} = -\int d\dot{Q}/T = \frac{-2000}{450} - \left[\frac{-2000}{400} \right] = \mathbf{0.555 \text{ W/K}}$$

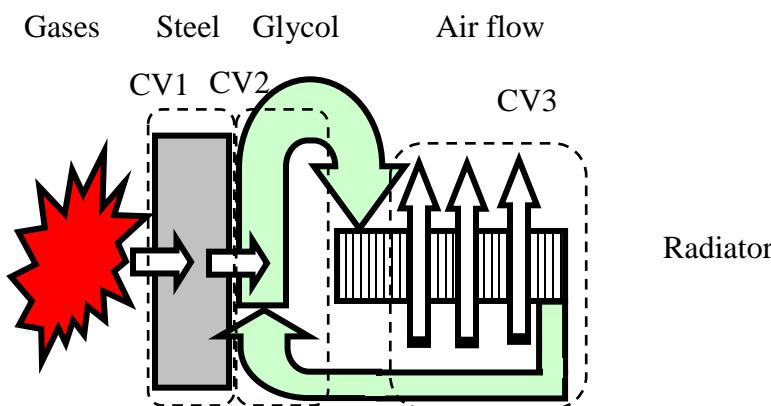
CV2. The coolant receives 2 kW at 370 K and gives it out at 350 K

$$\dot{S}_{gen2} = -\int d\dot{Q}/T = \left[\frac{-2000}{370} \right] - \left[\frac{-2000}{350} \right] = \mathbf{0.309 \text{ W/K}}$$

CV3 Radiator to air heat transfer.

$$\dot{S}_{gen3} = -\int d\dot{Q}/T = \left[\frac{-2000}{350} \right] - \left[\frac{-2000}{298.15} \right] = \mathbf{0.994 \text{ W/K}}$$

Notice the biggest \dot{S}_{gen} is for the largest change $\Delta[1/T]$



Remark: The flux of S is \dot{Q}/T flowing across a surface. Notice how this flux increases as the heat transfer flows towards lower and lower T.

T [K]	1500	450	370	298.15
\dot{Q}/T [W/K]	1.33	4.44	5.40	6.71

6.172

A farmer runs a heat pump using 2 kW of power input. It keeps a chicken hatchery at a constant 30°C while the room loses 10 kW to the colder outside ambient at 10°C. Find the COP of the heat pump, the rate of entropy generated in the heat pump and its heat exchangers, and the rate of entropy generated in the heat loss process?

Solution:

C.V. Hatchery, steady state.

To have steady state at 30°C for the hatchery

$$\text{Energy Eq.: } 0 = \dot{Q}_H - \dot{Q}_{\text{Loss}} \quad \Rightarrow \quad \dot{Q}_H = \dot{Q}_{\text{Loss}} = 10 \text{ kW}$$

$$\text{COP: } \text{COP} = \dot{Q}_H / \dot{W} = 10 / 2 = 5$$

C.V. Heat pump, steady state

$$\text{Energy eq.: } 0 = \dot{Q}_L + \dot{W} - \dot{Q}_H \quad \Rightarrow \quad \dot{Q}_L = \dot{Q}_H - \dot{W} = 8 \text{ kW}$$

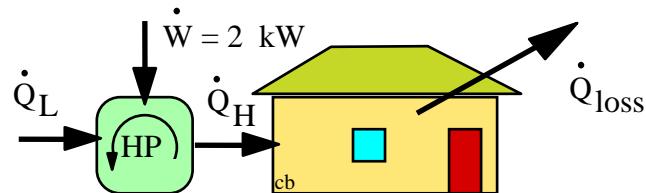
$$\text{Entropy Eq.: } 0 = \frac{\dot{Q}_L}{T_L} - \frac{\dot{Q}_H}{T_H} + \dot{S}_{\text{gen HP}}$$

$$\dot{S}_{\text{gen HP}} = \frac{\dot{Q}_H}{T_H} - \frac{\dot{Q}_L}{T_L} = \frac{10}{273 + 30} - \frac{8}{273 + 10} = 0.00473 \text{ kW/K}$$

C.V. From hatchery at 30°C to the ambient 10°C. This is typically the walls and the outer thin boundary layer of air. Through this goes \dot{Q}_{Loss} .

$$\text{Entropy Eq.: } 0 = \frac{\dot{Q}_{\text{Loss}}}{T_H} - \frac{\dot{Q}_{\text{Loss}}}{T_{\text{amb}}} + \dot{S}_{\text{gen walls}}$$

$$\dot{S}_{\text{gen walls}} = \frac{\dot{Q}_{\text{Loss}}}{T_{\text{amb}}} - \frac{\dot{Q}_{\text{Loss}}}{T_H} = \frac{10}{283} - \frac{10}{303} = 0.00233 \text{ kW/K}$$



Review problems

Excerpts from this work may be reproduced by instructors for distribution on a not-for-profit basis for testing or instructional purposes only to students enrolled in courses for which this textbook has been adopted. *Any other reproduction or translation of this work beyond that permitted by Sections 107 or 108 of the 1976 United States Copyright Act without the permission of the copyright owner is unlawful.*

6.173

An insulated cylinder/piston has an initial volume of 0.15 m^3 and contains steam at 400 kPa , 200°C . The steam is expanded adiabatically, and the work output is measured very carefully to be 30 kJ . It is claimed that the final state of the water is in the two-phase (liquid and vapor) region. What is your evaluation of the claim?

Solution:

C.V. Water.

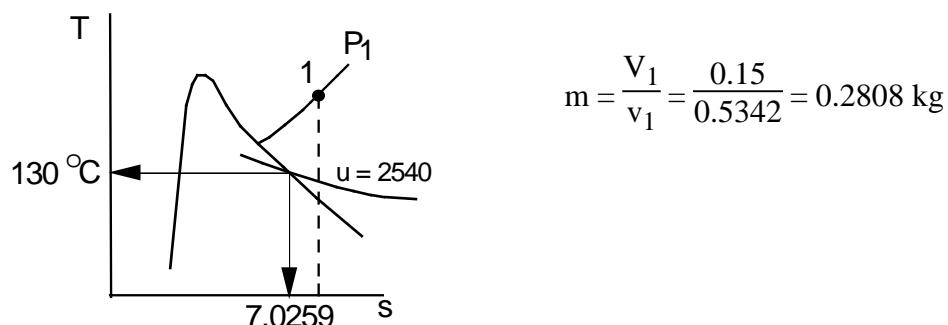
$$\text{Energy Eq.3.5: } m(u_2 - u_1) = _1Q_2 - _1W_2$$

$$\text{Entropy Eq.6.3: } m(s_2 - s_1) = \int dQ/T$$

Process: $_1Q_2 = 0$ and reversible

State 1: (T, P) Table B.1.3

$$v_1 = 0.5342, u_1 = 2646.8, s_1 = 7.1706 \text{ kJ/kg K}$$



$$m = \frac{V_1}{v_1} = \frac{0.15}{0.5342} = 0.2808 \text{ kg}$$

With the assumed reversible process we have from entropy equation

$$s_2 = s_1 = 7.1706 \text{ kJ/kg K}$$

and from the energy equation

$$u_2 = u_1 - _1W_2/m = 2646.8 - \frac{30}{0.2808} = 2540.0 \text{ kJ/kg}$$

State 2 given by (u, s) check Table B.1.1: s_G (at $u_G = 2540$) = $7.0259 < s_1$

\Rightarrow **State 2 must be in superheated vapor region.**

6.174

A piston cylinder has a water volume separated in $V_A = 0.2 \text{ m}^3$ and $V_B = 0.3 \text{ m}^3$ by a stiff membrane. The initial state in A is 1000 kPa, $x = 0.75$ and in B it is 1600 kPa and 250°C . Now the membrane ruptures and the water comes to a uniform state at 200°C with heat transfer from a 250°C source. Find the work, the heat transfer, and the total entropy generation in the process.

Take the water in A and B as CV.

$$\text{Continuity: } m_2 - m_{1A} - m_{1B} = 0$$

$$\text{Energy: } m_2 u_2 - m_{1A} u_{1A} - m_{1B} u_{1B} = 1 Q_2 - 1 W_2$$

$$\text{Entropy Eq.: } m_2 s_2 - m_A s_{A1} - m_B s_{B1} = 1 Q_2 / T_{\text{res}} + 1 S_{2 \text{ gen}}$$

$$\text{Process: } P_2 = P_{\text{eq}} = \text{constant} = P_{1A} \text{ as piston floats and } m_p, P_o \text{ are constant}$$

State 1A: Two phase. Table B.1.2

$$v_{1A} = 0.001127 + 0.75 \times 0.19332 = 0.146117 \text{ m}^3/\text{kg},$$

$$u_{1A} = 761.67 + 0.75 \times 1821.97 = 2128.15 \text{ kJ/kg}$$

$$s_{1A} = 2.1386 + 0.75 \times 4.4478 = 5.47445 \text{ kJ/kg-K}$$

State 1B: $v_{1B} = 0.14184 \text{ m}^3/\text{kg}$, $u_{1B} = 2692.26 \text{ kJ/kg}$, $s_{1B} = 6.6732 \text{ kJ/kg-K}$

$$\Rightarrow m_{1A} = V_{1A}/v_{1A} = 1.3688 \text{ kg}, \quad m_{1B} = V_{1B}/v_{1B} = 2.115 \text{ kg}$$

State 2: 1000 kPa, 200°C sup. vapor \Rightarrow

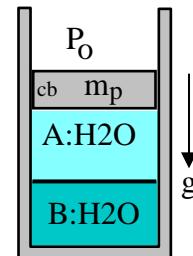
$$v_2 = 0.20596 \text{ m}^3/\text{kg}, \quad u_2 = 2621.9 \text{ kJ/kg}, \quad s_2 = 6.6939 \text{ kJ/kgK}$$

$$m_2 = m_{1A} + m_{1B} = 3.4838 \text{ kg} \Rightarrow V_2 = m_2 v_2 = 3.4838 \times 0.20596 = 0.7175 \text{ m}^3$$

So now

$$\begin{aligned} 1 W_2 &= \int P dV = P_{\text{eq}} (V_2 - V_1) = 1000 (0.7175 - 0.5) \\ &= \mathbf{217.5 \text{ kJ}} \end{aligned}$$

$$\begin{aligned} 1 Q_2 &= m_2 u_2 - m_{1A} u_{1A} - m_{1B} u_{1B} + 1 W_2 \\ &= 3.4838 \times 2621.9 - 1.3688 \times 2128.15 \\ &\quad - 2.115 \times 2692.26 + 217.5 = \mathbf{744 \text{ kJ}} \end{aligned}$$



$$\begin{aligned} 1 S_{2 \text{ gen}} &= m_2 s_2 - m_A s_{A1} - m_B s_{B1} - 1 Q_2 / T_{\text{res}} \\ &= 3.4838 \times 6.6939 - 1.3688 \times 5.47445 - 2.115 \times 6.6732 - \frac{744}{523.15} \\ &= \mathbf{0.2908 \text{ kJ/K}} \end{aligned}$$

6.175

The water in the two tanks of Problem 3.214 receives the heat transfer from a reservoir at 300°C. Find the total entropy generation due to this process.

Two rigid tanks are filled with water. Tank A is 0.2 m³ at 100 kPa, 150°C and tank B is 0.3 m³ at saturated vapor 300 kPa. The tanks are connected by a pipe with a closed valve. We open the valve and let all the water come to a single uniform state while we transfer enough heat to have a final pressure of 300 kPa. Give the two property values that determine the final state and heat transfer.

Take CV total A + B out to reservoir (neglect kinetic and potential energy)

$$\text{Energy Eq.: } m_2 u_2 - m_A u_{A1} - m_B u_{B1} = 1Q_2 - 1W_2 = 1Q_2$$

$$\text{Entropy Eq.: } m_2 s_2 - m_A s_{A1} - m_B s_{B1} = 1Q_2/T_{\text{res}} + 1S_2 \text{ gen}$$

$$\text{State A1: } u = 2582.75 \text{ kJ/kg}, v = 1.93636 \text{ m}^3/\text{kg}, s = 7.6133 \text{ kJ/kg-K}$$

$$\Rightarrow m_{A1} = V/v = 0.2/1.93636 = 0.1033 \text{ kg}$$

$$\text{State B1: } u = 2543.55 \text{ kJ/kg}, v = 0.60582 \text{ m}^3/\text{kg}, s = 6.9918 \text{ kJ/kg-K}$$

$$\Rightarrow m_{B1} = V/v = 0.3 / 0.60582 = 0.4952 \text{ kg}$$

The total volume (and mass) is the sum of volumes (mass) for tanks A and B.

$$m_2 = m_{A1} + m_{B1} = 0.1033 + 0.4952 = 0.5985 \text{ kg},$$

$$V_2 = V_{A1} + V_{B1} = 0.2 + 0.3 = 0.5 \text{ m}^3$$

$$\Rightarrow v_2 = V_2/m_2 = 0.5 / 0.5985 = 0.8354 \text{ m}^3/\text{kg}$$

$$\text{State 2: } [P_2, v_2] = [300 \text{ kPa}, 0.8354 \text{ m}^3/\text{kg}]$$

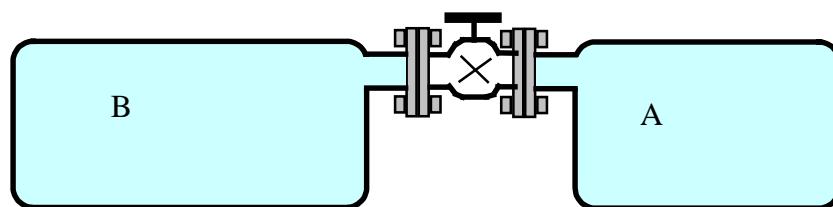
$$\Rightarrow T_2 = 274.76^\circ\text{C} \text{ and } u_2 = 2767.32 \text{ kJ/kg}, s = 7.60835 \text{ kJ/kgK}$$

From energy eq.

$$1Q_2 = 0.5985 \times 2767.32 - 0.1033 \times 2582.75 - 0.4952 \times 2543.55 = 129.9 \text{ kJ}$$

From entropy equation

$$\begin{aligned} 1S_2 \text{ gen} &= m_2 s_2 - m_A s_{A1} - m_B s_{B1} - 1Q_2/T_{\text{res}} \\ &= 0.5985 \times 7.60835 - 0.1033 \times 7.6133 - 0.4952 \times 6.9918 \\ &\quad - 129.9 / (273.15 + 300) = \mathbf{0.0782 \text{ kJ/K}} \end{aligned}$$



6.176

A steel piston/cylinder of 1 kg contains 2.5 kg ammonia at 50 kPa, -20°C. Now it is heated to 50°C at constant pressure through the bottom of the cylinder from external hot gas at 200°C, and we assume the steel has the same temperature as the ammonia. Find the heat transfer from the hot gas and the total entropy generation.

Solution:

C.V. Ammonia plus space out to the hot gas.

$$\text{Energy Eq.3.5: } m_{\text{NH}_3}(u_2 - u_1) + m_{\text{steel}}(u_2 - u_1) = 1Q_2 - 1W_2$$

$$\text{Entropy Eq.6.37: } S_2 - S_1 = \int dQ/T + 1S_{2,\text{gen}} = 1Q_2/T_{\text{gas}} + 1S_{2,\text{gen}}$$

$$S_2 - S_1 = m_{\text{NH}_3}(s_2 - s_1) + m_{\text{steel}}(s_2 - s_1)$$

$$\text{Process: } P = C \Rightarrow 1W_2 = Pm_{\text{NH}_3}(v_2 - v_1)_{\text{NH}_3}$$

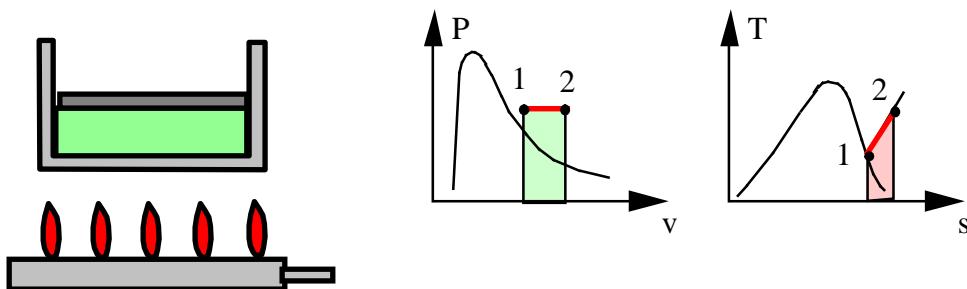
$$\text{State 1: (B.2.2)} \quad v_1 = 2.4463 \text{ m}^3/\text{kg}, \quad h_1 = 1434.6 \text{ kJ/kg}, \quad s_1 = 6.3187 \text{ kJ/kg K}$$

$$\text{State 2: (B.2.2)} \quad v_2 = 3.1435 \text{ m}^3/\text{kg}, \quad h_2 = 1583.5 \text{ kJ/kg}, \quad s_2 = 6.8379 \text{ kJ/kg K}$$

Substitute the work into the energy equation and solve for the heat transfer

$$\begin{aligned} 1Q_2 &= m_{\text{NH}_3}(h_2 - h_1) + m_{\text{steel}}(u_2 - u_1) \\ &= 2.5 (1583.5 - 1434.6) + 1 \times 0.46 [50 - (-20)] = \mathbf{404.45 \text{ kJ}} \end{aligned}$$

$$\begin{aligned} 1S_{2,\text{gen}} &= m_{\text{NH}_3}(s_2 - s_1) + m_{\text{steel}}(s_2 - s_1) - 1Q_2/T_{\text{gas}} \\ &= 2.5 (6.8379 - 6.3187) + 1 \times 0.46 \ln\left(\frac{323.15}{253.15}\right) - \frac{404.45}{473.15} \\ &= \mathbf{0.555 \text{ kJ/K}} \end{aligned}$$

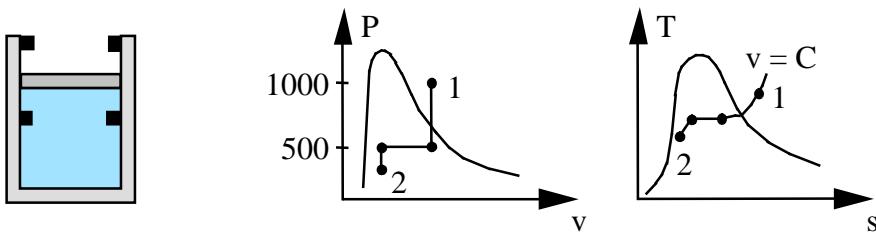


6.177

Water in a piston/cylinder is at 1 MPa, 500°C. There are two stops, a lower one at which $V_{\min} = 1 \text{ m}^3$ and an upper one at $V_{\max} = 3 \text{ m}^3$. The piston is loaded with a mass and outside atmosphere such that it floats when the pressure is 500 kPa. This setup is now cooled to 100°C by rejecting heat to the surroundings at 20°C. Find the total entropy generated in the process.

C.V. Water.

Initial state: Table B.1.3: $v_1 = 0.35411 \text{ m}^3/\text{kg}$, $u_1 = 3124.3$, $s_1 = 7.7621$
 $m = V/v_1 = 3/0.35411 = 8.472 \text{ kg}$



Final state: 100°C and on line in P-V diagram.

Notice the following: $v_g(500 \text{ kPa}) = 0.3749 > v_1$, $v_1 = v_g(154^\circ\text{C})$

$T_{\text{sat}}(500 \text{ kPa}) = 152^\circ\text{C} > T_2$, so now piston hits bottom stops.

State 2: $v_2 = v_{\text{bot}} = V_{\text{bot}}/m = 0.118 \text{ m}^3/\text{kg}$,

$$x_2 = (0.118 - 0.001044)/1.67185 = 0.0699,$$

$$u_2 = 418.91 + 0.0699 \times 2087.58 = 564.98 \text{ kJ/kg},$$

$$s_2 = 1.3068 + 0.0699 \times 6.048 = 1.73 \text{ kJ/kg K}$$

Now we can do the work and then the heat transfer from the energy equation

$$_1W_2 = \int P dV = 500(V_2 - V_1) = -1000 \text{ kJ} \quad (_1w_2 = -118 \text{ kJ/kg})$$

$$_1Q_2 = m(u_2 - u_1) + _1W_2 = -22683.4 \text{ kJ} \quad (_1q_2 = -2677.5 \text{ kJ/kg})$$

Take C.V. total out to where we have 20°C:

$$m(s_2 - s_1) = _1Q_2/T_0 + S_{\text{gen}} \Rightarrow$$

$$\begin{aligned} S_{\text{gen}} &= m(s_2 - s_1) - _1Q_2/T_0 = 8.472 (1.73 - 7.7621) + 22683 / 293.15 \\ &= \mathbf{26.27 \text{ kJ/K}} \quad (= \Delta S_{\text{water}} + \Delta S_{\text{sur}}) \end{aligned}$$

6.178

A piston/cylinder contains air at 300 K, 100 kPa. A reversible polytropic process with $n = 1.3$ brings the air to 500 K. Any heat transfer if it comes in is from a 325°C reservoir and if it goes out it is to the ambient at 300 K. Sketch the process in a P-v and a T-s diagram. Find the specific work and specific heat transfer in the process. Find the specific entropy generation (external to the air) in the process.

Solution:

Process : $Pv^n = C$

$$P_2/P_1 = (T_2/T_1)^{n/(n-1)} = (500/300)^{1.3/0.3} = 9.148$$

$$\begin{aligned} {}_1w_2 &= \int P \, dv = \left(\frac{P_2 v_2 - P_1 v_1}{1-n} \right) = \frac{R}{1-n} (T_2 - T_1) \\ &= \frac{0.287}{1-1.3} (500 - 300) = \mathbf{-191.3 \text{ kJ/kg}} \end{aligned}$$

Energy equation

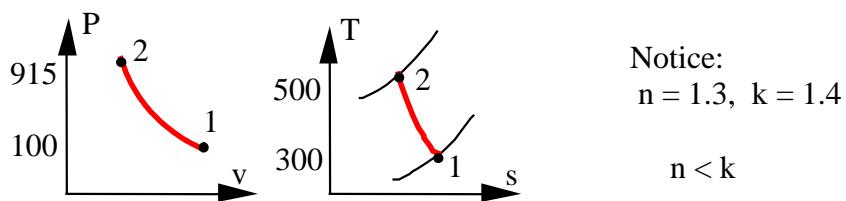
$$\begin{aligned} {}_1q_2 &= u_2 - u_1 + {}_1w_2 = C_v (T_2 - T_1) + {}_1w_2 \\ &= 0.717 (500 - 300) - 191.3 = \mathbf{-47.93 \text{ kJ/kg}} \end{aligned}$$

The ${}_1q_2$ is negative and thus goes out. Entropy is generated between the air and ambient.

$$s_2 - s_1 = {}_1q_2/T_{\text{amb}} + {}_1s_2 \text{ gen}$$

$${}_1s_2 \text{ gen} = s_2 - s_1 - {}_1q_2/T_{\text{amb}} = C_p \ln(T_2/T_1) - R \ln(P_2/P_1) - {}_1q_2/T_{\text{amb}}$$

$$\begin{aligned} {}_1s_2 \text{ gen} &= 1.004 \ln\left(\frac{500}{300}\right) - 0.287 \ln 9.148 - \left(\frac{-47.93}{300}\right) \\ &= 0.51287 - 0.635285 + 0.15977 \\ &= \mathbf{0.03736 \text{ kJ/kg K}} \end{aligned}$$



6.179

Assume the heat transfer in problem 3.213 came from a 200°C reservoir. What is the total entropy generation in the process?

C.V. Water in A and B. Control mass goes through process: 1 → 2

$$\text{Continuity Eq.: } m_2 - m_{A1} - m_{B1} = 0 \Rightarrow m_2 = m_{A1} + m_{B1} = 0.5 + 0.5 = 1 \text{ kg}$$

$$\text{Energy Eq.: } U_2 - U_1 = Q_2 - W_2$$

$$\text{Entropy Eq.: } m_2 s_2 - m_A s_{A1} - m_B s_{B1} = Q_2/T_{\text{res}} + S_{\text{gen}}$$

$$\text{State A1: } v_{A1} = 0.001067 + x_{A1} \times 0.71765 = 0.072832; \quad V_{A1} = mv = 0.036416 \text{ m}^3$$

$$u_{A1} = 535.08 + 0.1 \times 2002.14 = 735.22 \text{ kJ/kg};$$

$$s_{A1} = 1.6072 + 0.1 \times 5.4455 = 2.15175 \text{ kJ/kgK}$$

$$\text{State B1: } v_{B1} = 1.5493 \text{ m}^3/\text{kg}; \quad u_{B1} = 2966.69 \text{ kJ/kg}; \quad s_{B1} = 8.2217 \text{ kJ/kg-K}$$

$$V_{B1} = (mv)_{B1} = 0.77465 \text{ m}^3$$

State 2: If $V_2 > V_{A1}$ then $P_2 = 200 \text{ kPa}$ that is the piston floats.

For $(T_2, P_2) = (150^\circ\text{C}, 200 \text{ kPa}) \Rightarrow$ superheated vapor

$$u_2 = 2576.87 \text{ kJ/kg}; \quad v_2 = 0.95964 \text{ m}^3/\text{kg}, \quad s_2 = 7.2795 \text{ kJ/kgK}$$

$$V_2 = m_2 v_2 = 0.95964 \text{ m}^3 > V_{A1} \text{ checks OK.}$$

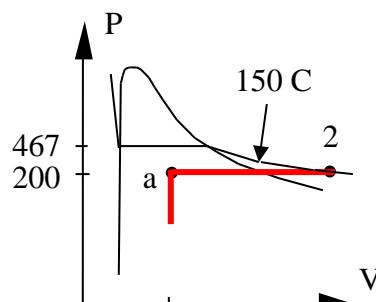
$$\text{Process: } W_2 = P_2 (V_2 - V_1) = 200 (0.95964 - 0.77465 - 0.036416) = 29.715 \text{ kJ}$$

From the energy and entropy equations:

$$\begin{aligned} Q_2 &= m_2 u_2 - m_{A1} u_{A1} - m_{B1} u_{B1} + W_2 \\ &= 1 \times 2576.87 - 0.5 \times 735.222 - 0.5 \times 2966.69 + 29.715 = 755.63 \text{ kJ} \end{aligned}$$

$$\begin{aligned} S_{\text{gen}} &= m_2 s_2 - m_A s_{A1} - m_B s_{B1} - Q_2/T_{\text{res}} \\ &= 1 \times 7.2795 - 0.5 \times 2.15175 - 0.5 \times 8.2217 - 755.63/473.15 \\ &= \mathbf{0.496 \text{ kJ/K}} \end{aligned}$$

The possible state 2 (P,V) combinations are shown. State a is 200 kPa, $v_a = V_{A1}/m_2 = 0.036$ and thus two-phase $T_a = 120^\circ\text{C}$ less than T_2



6.180

A closed tank, $V = 10 \text{ L}$, containing 5 kg of water initially at 25°C , is heated to 150°C by a heat pump that is receiving heat from the surroundings at 25°C . Assume that this process is reversible. Find the heat transfer to the water and the work input to the heat pump.

C.V.: Water from state 1 to state 2.

Process: constant volume (reversible isometric)

$$\text{State 1: } v_1 = V/m = 0.002 \Rightarrow x_1 = (0.002 - 0.001003)/43.358 = 0.000023$$

$$u_1 = 104.86 + 0.000023 \times 2304.9 = 104.93 \text{ kJ/kg}$$

$$s_1 = 0.3673 + 0.000023 \times 8.1905 = 0.36759 \text{ kJ/kg K}$$

Continuity eq. (same mass) and $V = C$ fixes v_2

$$2: T_2, v_2 = v_1 \Rightarrow$$

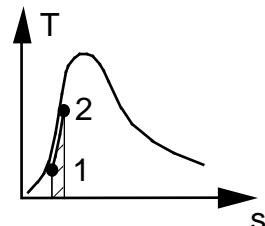
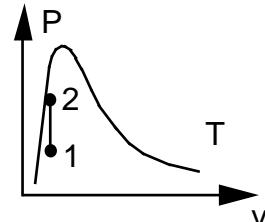
$$x_2 = (0.002 - 0.001090)/0.39169 = 0.0023233$$

$$u_2 = 631.66 + 0.0023233 \times 1927.87 = 636.14 \text{ kJ/kg}$$

$$s_2 = 1.8417 + 0.0023233 \times 4.9960 = 1.8533 \text{ kJ/kg K}$$

Energy eq. has $W = 0$, thus provides heat transfer as

$$1Q_2 = m(u_2 - u_1) = \mathbf{2656 \text{ kJ}}$$



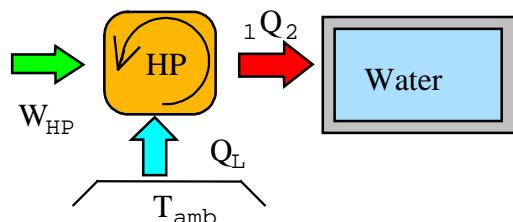
Entropy equation for the total (tank plus heat pump) control volume gives for a reversible process:

$$m(s_2 - s_1) = Q_L/T_0 \Rightarrow$$

$$Q_L = mT_0(s_2 - s_1) = 5 \times 298.15 \times (1.8533 - 0.36759) = 2214.8 \text{ kJ}$$

and then the energy equation for the heat pump gives

$$W_{HP} = 1Q_2 - Q_L = 2656 - 2214.8 = \mathbf{441.2 \text{ kJ}}$$



6.181

A resistor in a heating element is a total of 0.5 kg with specific heat of 0.8 kJ/kgK. It is now receiving 500 W of electric power so it heats from 20°C to 180°C. Neglect external heat loss and find the time the process took and the entropy generation.

C.V. Heating element.

$$\text{Energy Eq.: } m(u_2 - u_1) = \dot{W}_{\text{electrical in}} = \dot{W}_{\text{electrical in}} / \Delta t$$

$$\text{Entropy Eq.: } m(s_2 - s_1) = 0 + \dot{S}_{\text{gen}} \quad (\text{no heat transfer})$$

$$\begin{aligned} \Delta t &= m(u_2 - u_1) / \dot{W}_{\text{electrical in}} = m C (T_2 - T_1) / \dot{W}_{\text{electrical in}} \\ &= 0.5 \text{ kg} \times 800 \text{ J/kg-K} \times (180 - 20) \text{ K} / 500 \text{ (J/s)} = \mathbf{128 \text{ s}} \end{aligned}$$

$$\begin{aligned} \dot{S}_{\text{gen}} &= m(s_2 - s_1) = m C \ln \frac{T_2}{T_1} = 0.5 \text{ kg} \times 0.8 \text{ kJ/kg-K} \ln \left(\frac{180 + 273}{20 + 273} \right) \\ &= \mathbf{0.174 \text{ kJ/K}} \end{aligned}$$

6.182

Two tanks contain steam, and they are both connected to a piston/cylinder as shown in Fig. P6.182. Initially the piston is at the bottom and the mass of the piston is such that a pressure of 1.4 MPa below it will be able to lift it. Steam in A is 4 kg at 7 MPa, 700°C and B has 2 kg at 3 MPa, 350°C. The two valves are opened, and the water comes to a uniform state. Find the final temperature and the total entropy generation, assuming no heat transfer.

Solution:

Control mass: All water $m_A + m_B$.

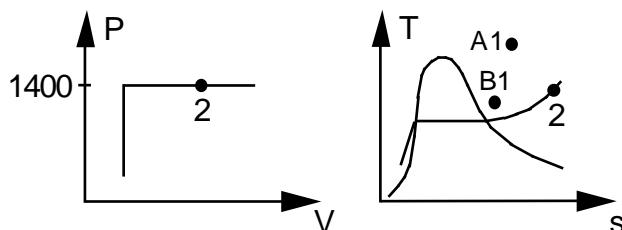
Continuity Eq.: $m_2 = m_A + m_B = 6 \text{ kg}$

Energy Eq.3.5: $m_2 u_2 - m_A u_{A1} - m_B u_{B1} = 1 Q_2 - 1 W_2 = - 1 W_2$

Entropy Eq.6.37: $m_2 s_2 - m_A s_{A1} - m_B s_{B1} = 1 S_{2 \text{ gen}}$

B.1.3: $v_{A1} = 0.06283, u_{A1} = 3448.5, s_{A1} = 7.3476, V_A = 0.2513 \text{ m}^3$

B.1.3: $v_{B1} = 0.09053, u_{B1} = 2843.7, s_{B1} = 6.7428, V_B = 0.1811 \text{ m}^3$



The only possible P, V combinations for state 2 are on the two lines.

Assume $V_2 > V_A + V_B \Rightarrow P_2 = P_{\text{lift}}, 1 W_2 = P_2(V_2 - V_A - V_B)$

Substitute into energy equation:

$$\begin{aligned} m_2 h_2 &= m_A u_{A1} + m_B u_{B1} + P_2(V_A + V_B) \\ &= 4 \times 3448.5 + 2 \times 2843.7 + 1400 \times 0.4324 = 20\,086.8 \text{ kJ} \end{aligned}$$

State 2: $h_2 = m_2 h_2 / m_2 = 20\,086.8 \text{ kJ} / 6 \text{ kg} = 3347.8 \text{ kJ/kg}, P_2 = 1400 \text{ kPa}, v_2 = 0.2323 \text{ m}^3/\text{kg}, s_2 = 7.433 \text{ kJ/kg-K}, T_2 = 441.9^\circ\text{C},$

Check assumption: $V_2 = m_2 v_2 = 1.394 \text{ m}^3 > V_A + V_B \quad \text{OK.}$

$$1 S_{2 \text{ gen}} = 6 \times 7.433 - 4 \times 7.3476 - 2 \times 6.7428 = 1.722 \text{ kJ/K}$$

6.183

Assume the heat source in Problem 3.217 is at 300°C in a setup similar to Fig. P6.182. Find the heat transfer and the entropy generation.

C.V.: A + B + C.

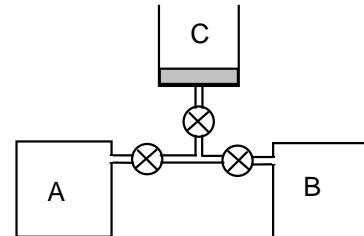
Only work in C, total mass constant.

$$m_2 - m_1 = 0 \Rightarrow m_2 = m_{A1} + m_{B1}$$

$$U_2 - U_1 = Q_2 - W_2;$$

$$S_2 - S_1 = Q_2/T_{\text{res}} + S_{\text{gen}}$$

$$W_2 = \int P dV = P_{\text{lift}} (V_2 - V_1)$$



$$1A: v = 0.6/3 = 0.2 \text{ m}^3/\text{kg} \Rightarrow x_{A1} = (0.2 - 0.00106)/0.8908 = 0.223327$$

$$u = 503.48 + x \times 2025.76 = 955.89 \text{ kJ/kg}, s = 1.5275 + x \times 5.602 = 2.7786 \text{ kJ/kgK}$$

$$1B: v = 0.35202 \text{ m}^3/\text{kg} \Rightarrow ; u = 2638.91 \text{ kJ/kg}, s = 6.9665 \text{ kJ/kgK}$$

$$m_{B1} = 0.4/0.35202 = 1.1363 \text{ kg} \text{ and } m_2 = 3 + 1.1363 = 4.1363 \text{ kg}$$

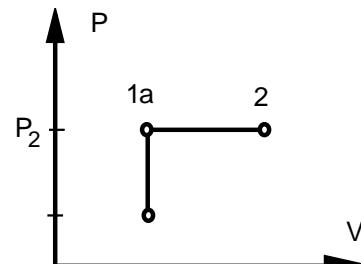
$$V_2 = V_A + V_B + V_C = 1 \text{ m}^3 + V_C$$

Locate state 2: Must be on P-V lines shown

State 1a: 800 kPa,

$$v_{1a} = \frac{V_A + V_B}{m} = 0.24176 \text{ m}^3/\text{kg}$$

$$800 \text{ kPa}, v_{1a} \Rightarrow T = 173^\circ\text{C} \text{ too low.}$$



$$\text{Assume 800 kPa: } 250^\circ\text{C} \Rightarrow v = 0.29314 \text{ m}^3/\text{kg} > v_{1a} \text{ OK}$$

$$V_2 = m_2 v_2 = 4.1363 \text{ kg} \times 0.29314 \text{ m}^3/\text{kg} = 1.2125 \text{ m}^3$$

Final state is : **800 kPa**; **250°C** $\Rightarrow u_2 = 2715.46 \text{ kJ/kg}, s_2 = 7.0384 \text{ kJ/kgK}$

$$W_2 = P_{\text{lift}} (V_2 - V_1) = 800 \text{ kPa} \times (1.2125 - 1) \text{ m}^3 = 170 \text{ kJ}$$

$$\begin{aligned} Q_2 &= m_2 u_2 - m_1 u_1 + W_2 = m_2 u_2 - m_{A1} u_{A1} - m_{B1} u_{B1} + W_2 \\ &= 4.1363 \times 2715.46 - 3 \times 955.89 - 1.1363 \times 2638.91 + 170 \\ &= 11232 - 2867.7 - 2998.6 + 170 = 5536 \text{ kJ} \end{aligned}$$

$$\begin{aligned} S_{\text{gen}} &= S_2 - S_1 - Q_2/T_{\text{res}} = m_2 s_2 - m_{A1} s_{A1} - m_{B1} s_{B1} - Q_2/T_{\text{res}} \\ &= 4.1363 \times 7.0384 - 3 \times 2.7786 - 1.1363 \times 6.9665 - 5536/573.15 \\ &= 3.202 \text{ kJ/K} \end{aligned}$$

6.184

A cylinder fitted with a piston contains 0.5 kg of R-134a at 60°C, with a quality of 50 percent. The R-134a now expands in an internally reversible polytropic process to ambient temperature, 20°C at which point the quality is 100 percent. Any heat transfer is with a constant-temperature source, which is at 60°C. Find the polytropic exponent n and show that this process satisfies the second law of thermodynamics.

Solution:

C.V.: R-134a, Internally Reversible, Polytropic Expansion: $PV^n = \text{Const.}$

Cont.Eq.: $m_2 = m_1 = m$; Energy Eq.: $m(u_2 - u_1) = _1Q_2 - _1W_2$

Entropy Eq.: $m(s_2 - s_1) = \int dQ/T + _1S_{\text{gen}}$

State 1: $T_1 = 60^\circ\text{C}$, $x_1 = 0.5$, Table B.5.1: $P_1 = P_g = 1681.8 \text{ kPa}$,

$$v_1 = v_f + x_1 v_{fg} = 0.000951 + 0.5 \times 0.010511 = 0.006207 \text{ m}^3/\text{kg}$$

$$s_1 = s_f + x_1 s_{fg} = 1.2857 + 0.5 \times 0.4182 = 1.4948 \text{ kJ/kg K},$$

$$u_1 = u_f + x_1 u_{fg} = 286.19 + 0.5 \times 121.66 = 347.1 \text{ kJ/kg}$$

State 2: $T_2 = 20^\circ\text{C}$, $x_2 = 1.0$, $P_2 = P_g = 572.8 \text{ kPa}$, Table B.5.1

$$v_2 = v_g = 0.03606 \text{ m}^3/\text{kg}, s_2 = s_g = 1.7183 \text{ kJ/kg K}$$

$$u_2 = u_g = 389.19 \text{ kJ/kg}$$

$$\text{Process: } PV^n = \text{Const.} \Rightarrow \frac{P_1}{P_2} = \left(\frac{v_2}{v_1}\right)^n \Rightarrow n = \ln \frac{P_1}{P_2} / \ln \frac{v_2}{v_1} = \mathbf{0.6122}$$

$$\begin{aligned} _1W_2 &= \int PdV = \frac{P_2 V_2 - P_1 V_1}{1-n} \\ &= 0.5(572.8 \times 0.03606 - 1681.8 \times 0.006207)/(1 - 0.6122) = 13.2 \text{ kJ} \end{aligned}$$

2nd Law for C.V.: R-134a plus wall out to source:

$$_1S_{\text{gen}} = m(s_2 - s_1) - \frac{Q_H}{T_H}; \quad \text{Check } \Delta S_{\text{net}} \geq 0$$

$$Q_H = _1Q_2 = m(u_2 - u_1) + _1W_2 = 34.2 \text{ kJ}$$

$$_1S_{\text{gen}} = 0.5(1.7183 - 1.4948) - 34.2/333.15 = 0.0092 \text{ kJ/K},$$

$_1S_{\text{gen}} > 0$ Process Satisfies 2nd Law

6.185

A device brings 2 kg of ammonia from 150 kPa, -20°C to 400 kPa, 80°C in a polytropic process. Find the polytropic exponent, n, the work and the heat transfer. Find the total entropy generated assuming a source at 100°C.

Solution:

C.V. Ammonia of constant mass $m_2 = m_1 = m$ out to source.

$$\text{Energy Eq.3.5: } m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$$

$$\text{Entropy Eq.6.37: } m(s_2 - s_1) = \int dQ/T + {}_1S_{2,\text{gen}} = {}_1Q_2/T + {}_1S_{2,\text{gen}}$$

$$\text{Process: } P_1v_1^n = P_2v_2^n \quad \text{Eq. (8.27)}$$

State 1: Table B.2.2

$$v_1 = 0.79774 \text{ m}^3/\text{kg}, \quad s_1 = 5.7465 \text{ kJ/kg K}, \quad u_1 = 1303.3 \text{ kJ/kg}$$

State 2: Table B.2.2

$$v_2 = 0.4216 \text{ m}^3/\text{kg}, \quad s_2 = 5.9907 \text{ kJ/kg K}, \quad u_2 = 1468.0 \text{ kJ/kg}$$

$$\ln(P_2/P_1) = \ln(v_1/v_2)^n = n \times \ln(v_1/v_2)$$

$$\ln\left(\frac{400}{150}\right) = n \times \ln\left(\frac{0.79774}{0.4216}\right) = 0.98083 = n \times 0.63773$$

$$\Rightarrow n = 1.538$$

The work term is integration of PdV as done in text leading to Eq.6.29

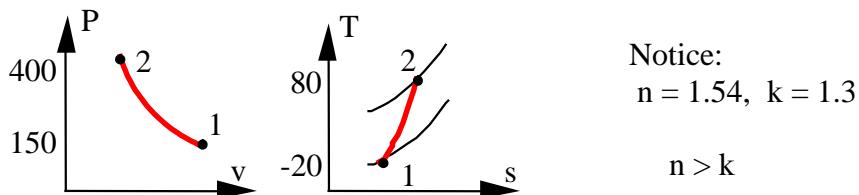
$$\begin{aligned} {}_1W_2 &= \frac{m}{1-n} (P_2v_2 - P_1v_1) \\ &= \frac{2}{1-1.538} \times (400 \times 0.4216 - 150 \times 0.79774) = -182.08 \text{ kJ} \end{aligned}$$

Notice we did not use $Pv = RT$ as we used the ammonia tables.

$${}_1Q_2 = m(u_2 - u_1) + {}_1W_2 = 2(1468 - 1303.3) - 182.08 = 147.3 \text{ kJ}$$

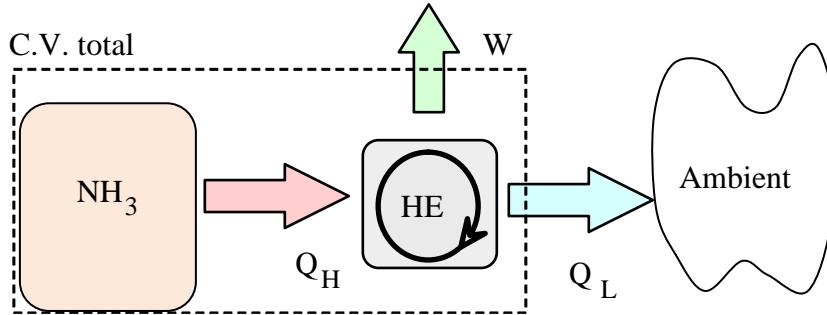
From Eq.6.37

$$\begin{aligned} {}_1S_{2,\text{gen}} &= m(s_2 - s_1) - {}_1Q_2/T = 2(5.9907 - 5.7465) - \frac{147.3}{373.15} \\ &= 0.0936 \text{ kJ/K} \end{aligned}$$



6.186

A rigid tank with 0.5 kg ammonia at 1600 kPa, 160°C is cooled in a reversible process by giving heat to a reversible heat engine that has its cold side at ambient 20°C, shown in Fig. P6.186. The ammonia eventually reaches 20°C and the process stops. Find the heat transfer from the ammonia to the heat engine and the work output of the heat engine.



C.V. Ammonia

$$\text{Energy Eq.3.5: } m(u_2 - u_1) = \int Q_2 - \int W_2$$

$$\text{Entropy Eq.6.3: } m(s_2 - s_1) = \int dQ/T \quad (\text{T not constant})$$

$$\text{Process: } v = \text{constant} \Rightarrow \int W_2 = 0$$

$$\text{State 1: (T, P) Table B.2.2: } u_1 = 1596.1 \text{ kJ/kg, } v_1 = 0.12662 \text{ m}^3/\text{kg},$$

$$s_1 = 5.7485 \text{ kJ/kgK}$$

$$\text{State 2: } T_2 \text{ and } v_2 = v_1 \text{ Table B.2.1 as } v_2 < v_g$$

$$\Rightarrow \text{2-phase, } P_2 = P_{\text{sat}} = 857.5 \text{ kPa}$$

$$x_2 = (v_2 - v_f)/v_{fg} = (0.12662 - 0.001638)/0.14758 = 0.846876$$

$$u_2 = u_f + x_2 u_{fg} = 272.89 + 0.846876 \times 1059.3 = 1170 \text{ kJ/kg}$$

$$s_2 = s_f + x_2 s_{fg} = 1.0408 + 0.846876 \times 4.0452 = 4.4666 \text{ kJ/kgK}$$

From the energy equation

$$Q_H = -\int Q_2 = -m(u_2 - u_1) = -0.5(1170 - 1596.1) = \mathbf{213.05 \text{ kJ}}$$

Take now CV total ammonia plus heat engine out to ambient

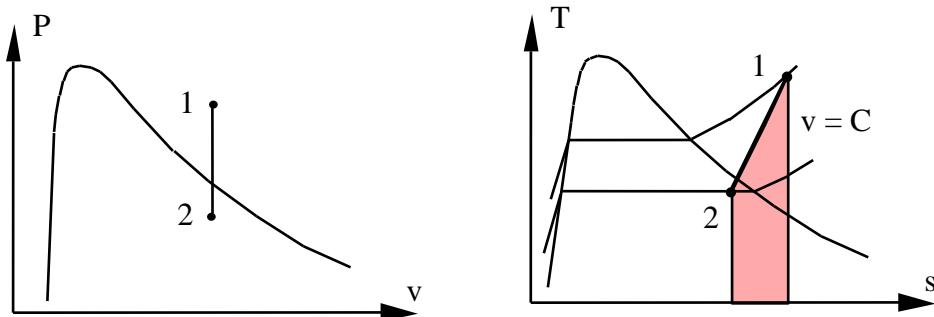
$$\text{Entropy Eq.6.3: } m(s_2 - s_1) = -\frac{Q_L}{T_{\text{amb}}} \Rightarrow$$

$$Q_L = -mT_{\text{amb}}(s_2 - s_1) = -0.5 \times 293.15 (4.4666 - 5.7485)$$

$$= 187.89 \text{ kJ}$$

Now the CV heat engine can give the engine work from the energy equation

$$\text{Energy H.E.: } W_{\text{HE}} = Q_H - Q_L = 213.05 - 187.89 = \mathbf{25.2 \text{ kJ}}$$



Notice to get $q_2 = \int T ds$ we must know the function $T(s)$ which we do not readily have for this process.

6.187

A piston/cylinder with constant loading of piston contains 1 L water at 400 kPa, quality 15%. It has some stops mounted so the maximum possible volume is 11 L. A reversible heat pump extracting heat from the ambient at 300 K, 100 kPa heats the water to 300°C. Find the total work and heat transfer for the water and the work input to the heat pump.

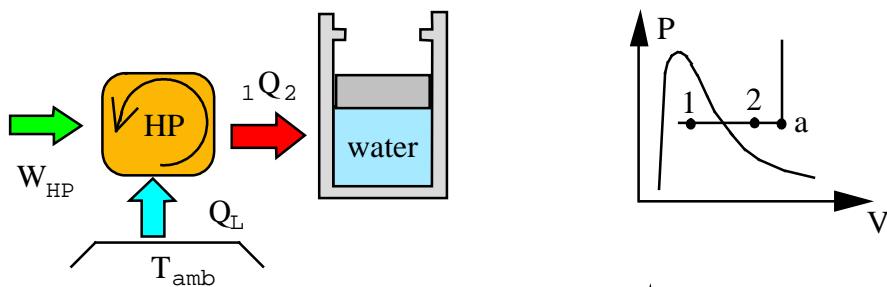
Solution: Take CV around the water and check possible P-V combinations.

$$\text{State 1: } v_1 = 0.001084 + 0.15 \times 0.46138 = 0.07029 \text{ m}^3/\text{kg}$$

$$u_1 = 604.29 + 0.15 \times 1949.26 = 896.68 \text{ kJ/kg}$$

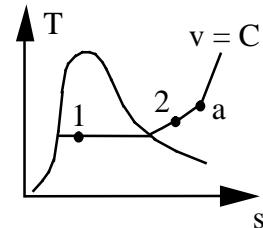
$$s_1 = 1.7766 + 0.15 \times 5.1193 = 2.5445 \text{ kJ/kg K}$$

$$m_1 = V_1/v_1 = 0.001/0.07029 = 0.0142 \text{ kg}$$



$$\text{State a: } v = 11 \text{ m}^3/\text{kg}, \\ 400 \text{ kPa}$$

$$\Rightarrow \text{Sup. vapor} \quad T_a = 400^\circ\text{C} > T_2$$



State 2: Since $T_2 < T_a$ then piston is not at stops but floating so $P_2 = 400 \text{ kPa}$.

$$(T, P) \Rightarrow v_2 = 0.65484 \text{ m}^3/\text{kg} \Rightarrow V_2 = (v_2/v_1) \times V_1 = 9.316 \text{ L}$$

$$W_2 = \int P dV = P(V_2 - V_1) = 400 \text{ kPa} (9.316 - 1) \times 0.001 \text{ m}^3 = 3.33 \text{ kJ}$$

$$Q_2 = m(u_2 - u_1) + W_2 = 0.0142 (2804.8 - 896.68) + 3.33 = 30.43 \text{ kJ}$$

Take CV as water plus the heat pump out to the ambient.

$$m(s_2 - s_1) = Q_L/T_o \Rightarrow$$

$$Q_L = mT_o(s_2 - s_1) = 0.0142 \text{ kg} \times 300 \text{ K} \times (7.5661 - 2.5445) \text{ kJ/kg-K} \\ = 21.39 \text{ kJ}$$

$$W_{HP} = Q_2 - Q_L = 30.43 - 21.39 = 9.04 \text{ kJ}$$

6.188

An un-insulated cylinder fitted with a piston contains air at 500 kPa, 200°C, at which point the volume is 10 L. The external force on the piston is now varied in such a manner that the air expands to 150 kPa, 25 L volume. It is claimed that in this process the air produces 70% of the work that would have resulted from a reversible, adiabatic expansion from the same initial pressure and temperature to the same final pressure.

Room temperature is 20°C.

- What is the amount of work claimed?
- Is this claim possible?

Solution:

$$\text{C.V.: Air; } R = 0.287 \text{ kJ/kg-K}, C_p = 1.004 \text{ kJ/kg K}, C_v = 0.717 \text{ kJ/kg K}$$

$$\text{State 1: } T_1 = 200^\circ\text{C}, P_1 = 500 \text{ kPa}, V_1 = 10 \text{ L} = 0.01 \text{ m}^3;$$

$$m_1 = V_1/v_1 = P_1 V_1 / RT_1 = 0.0368 \text{ kg}$$

$$\text{State 2: } P_2 = 150 \text{ kPa}, V_2 = 25 \text{ L} = 0.025 \text{ m}^3$$

$\eta_s = 70\%$; Actual Work is 70% of Isentropic Work

- Assume Reversible and Adiabatic Process; $s_1 = s_{2s}$

$$T_{2s} = T_1 \left(\frac{P_2}{P_1} \right)^{\frac{k-1}{k}} = 473.15 \left(\frac{150}{500} \right) = 335.4 \text{ K}$$

$$\text{Energy Eq.: } Q_{2s} = m(u_{2s} - u_1) + W_{2s}; \quad Q_{2s} = 0$$

Assume constant specific heat

$$W_{2s} = mC_v(T_1 - T_{2s}) = 3.63 \text{ kJ}$$

$$W_{2ac} = 0.7 \times W_{2s} = 2.54 \text{ kJ}$$

- Use Ideal Gas Law; $T_{2ac} = T_1 P_2 V_2 / P_1 V_1 = 354.9 \text{ K}$

$$\text{Energy Eq.: } Q_{2ac} = mC_v(T_{2ac} - T_1) + W_{2ac} = -0.58 \text{ kJ}$$

$$\text{2nd Law: } S_{2gen} = m(s_2 - s_1) - \frac{Q_{cv}}{T_0}; \quad Q_{cv} = Q_{2ac}, \quad T_0 = 20^\circ\text{C}$$

$$s_2 - s_1 = C_p \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1} = 0.0569 \text{ kJ/kg-K}$$

$$S_{2gen} = 0.00406 \text{ kJ/K} \geq 0; \quad \text{Process is Possible}$$

6.189

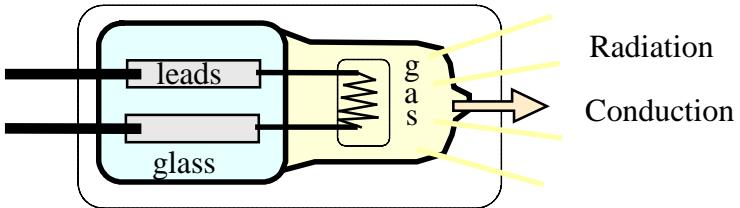
A small halogen light bulb receives an electrical power of 50 W. The small filament is at 1000 K and gives out 20% of the power as light and the rest as heat transfer to the gas, which is at 500 K; the glass is at 400 K. All the power is absorbed by the room walls at 25°C. Find the rate of generation of entropy in the filament, in the total bulb including glass and the total room including bulb.

Solution:

$$\dot{W}_{el} = 50 \text{ W}$$

$$\dot{Q}_{RAD} = 10 \text{ W}$$

$$\dot{Q}_{COND} = 40 \text{ W}$$



We will assume steady state and no storage in the bulb, air or room walls.

C.V. Filament steady-state

$$\text{Energy Eq.3.31: } dE_{c.v.}/dt = 0 = \dot{W}_{el} - \dot{Q}_{RAD} - \dot{Q}_{COND}$$

$$\text{Entropy Eq.6.42: } dS_{c.v.}/dt = 0 = -\frac{\dot{Q}_{RAD}}{T_{FILA}} - \frac{\dot{Q}_{COND}}{T_{FILA}} + \dot{S}_{gen}$$

$$\dot{S}_{gen} = (\dot{Q}_{RAD} + \dot{Q}_{COND})/T_{FILA} = \dot{W}_{el}/T_{FILA} = \frac{50}{1000} = \mathbf{0.05 \text{ W/K}}$$

C.V. Bulb including glass

$$\dot{Q}_{RAD} \text{ leaves at } 1000 \text{ K}$$

$$\dot{Q}_{COND} \text{ leaves at } 400 \text{ K}$$

$$\dot{S}_{gen} = \int d\dot{Q}/T = -(-10/1000) - (-40/400) = \mathbf{0.11 \text{ W/K}}$$

C.V. Total room. All energy leaves at 25°C

$$\text{Eq.5.31: } dE_{c.v.}/dt = 0 = \dot{W}_{el} - \dot{Q}_{RAD} - \dot{Q}_{COND}$$

$$\text{Eq.6.42: } dS_{c.v.}/dt = 0 = -\frac{\dot{Q}_{TOT}}{T_{WALL}} + \dot{S}_{gen}$$

$$\dot{S}_{gen} = \frac{\dot{Q}_{TOT}}{T_{WALL}} = 50/(25+273) = \mathbf{0.168 \text{ W/K}}$$

Solutions using the Pr and vr functions in Table A.7.2

If you would like to see more of these please let me know (claus@umich.edu) and I can prepare more of the problem solutions using these functions.

6.92 uses P_r function

A piston/cylinder, shown in Fig. P6.92, contains air at 1380 K, 15 MPa, with $V_1 = 10 \text{ cm}^3$, $A_{\text{cyl}} = 5 \text{ cm}^2$. The piston is released, and just before the piston exits the end of the cylinder the pressure inside is 200 kPa. If the cylinder is insulated, what is its length? How much work is done by the air inside?

Solution:

C.V. Air, Cylinder is insulated so adiabatic, $Q = 0$.

Continuity Eq.: $m_2 = m_1 = m$,

Energy Eq.3.5: $m(u_2 - u_1) = _1Q_2 - _1W_2 = - _1W_2$

Entropy Eq.6.37: $m(s_2 - s_1) = \int dQ/T + _1S_2 \text{gen} = 0 + _1S_2 \text{gen}$

State 1: (T_1, P_1) State 2: $(P_2, ?)$

So one piece of information is needed for the ?, assume reversible process.

$$_1S_2 \text{gen} = 0 \Rightarrow s_2 - s_1 = 0$$

State 1: Table A.7.1: $u_1 = 1095.2 \text{ kJ/kg}$,

Table A.7.2: $P_{r1} = 340.53$, $v_{r1} = 2.7024$

$$m = P_1 V_1 / RT_1 = \frac{15000 \times 10 \times 10^{-6}}{0.287 \times 1380} = 0.000379 \text{ kg}$$

State 2: P_2 and from Entropy eq.: $s_2 = s_1$

$$\Rightarrow P_{r2} = P_{r1} P_2 / P_1 = 340.53 \times 200 / 15000 = 4.5404$$

Interpolate in A.7.2 to match the P_{r2} value

$$T_2 = 447 \text{ K}, \quad u_2 = 320.85 \text{ kJ/kg}, \quad v_{r2} = 65.67$$

$$\Rightarrow V_2 = V_1 v_{r2} / v_{r1} = 10 \times 65.67 / 2.7024 = 243 \text{ cm}^3$$

$$\Rightarrow L_2 = V_2 / A_{\text{cyl}} = 243 / 5 = 48.6 \text{ cm}$$

$$\Rightarrow _1W_2 = u_1 - u_2 = 774.4 \text{ kJ/kg}, \quad _1W_2 = m_1 w_2 = 0.2935 \text{ kJ}$$

We could also have done $V_2 = V_1 (T_2 P_1 / T_1 P_2)$ from ideal gas law and thus did not need the v_r function for this problem

6.105 uses v_r function

A piston/cylinder contains air at 300 K, 100 kPa. It is now compressed in a reversible adiabatic process to a volume 7 times as small. Use constant heat capacity and find the final pressure and temperature, the specific work and specific heat transfer for the process.

Solution: Here we use the v_r function from Table A.7.2

$$\text{Expansion ratio: } v_2/v_1 = 1/7$$

Process eq.: Rev. adiabatic and ideal gas gives $Pv^n = C$, with $n = k$

Since we know the v ratio and s is constant we use the v_r function

$$v_{r1} = 179.49 \Rightarrow v_{r2} = v_{r1} v_2/v_1 = 179.49/7 = 25.641$$

Table A.7.2: Interpolate $T_2 = \mathbf{640.7 \text{ K}}$

$$P_2 = P_1 \times (T_2 / T_1) \times (v_1/v_2) = 100 \times (640.7/300) \times 7 = \mathbf{1495 \text{ kPa}}$$

Adiabatic: $q_2 = \mathbf{0 \text{ kJ/kg}}$

Polytropic process work term from the energy equation

$$w_2 = -(u_2 - u_1) = -(466.37 - 214.36) = \mathbf{-252.0 \text{ kJ/kg}}$$

6. additional problem uses P_r function

A mass of 1 kg of air contained in a cylinder at 1.5 MPa, 1000 K, expands in a reversible adiabatic process to 100 kPa. Calculate the final temperature and the work done during the process, using

- a. Constant specific heat, value from Table A.5
- b. The ideal gas tables, Table A.7

Solution:

C.V. Air.

Continuity Eq.: $m_2 = m_1 = m$;

Energy Eq.3.5: $m(u_2 - u_1) = _1Q_2 - _1W_2$

Entropy Eq.6.37: $m(s_2 - s_1) = \int dQ/T + _1S_2 \text{ gen}$

Process: $_1Q_2 = 0$, $_1S_2 \text{ gen} = 0 \Rightarrow s_2 = s_1$

a) Using constant C_p from Table A.5 gives the power relation Eq.6.23.

$$T_2 = T_1(P_2/P_1)^{\frac{k-1}{k}} = 1000 \left(\frac{0.1}{1.5}\right)^{0.286} = 460.9 \text{ K}$$

$$\begin{aligned} _1W_2 &= -(U_2 - U_1) = mC_{V0}(T_1 - T_2) \\ &= 1 \text{ kg} \times 0.717 \text{ kJ/kg-K} (1000 - 460.9) \text{ K} = 386.5 \text{ kJ} \end{aligned}$$

b) Use the tabulated **reduced pressure function** that

includes variable heat capacity from A.7.2 so since $s_2 = s_1$ we have

$$P_{r2} = P_{r1} \times P_2/P_1 = 91.65 \times \frac{0.1}{1.5} = 6.11$$

Interpolation gives $T_2 = 486 \text{ K}$ and $u_2 = 349.4 \text{ kJ/kg}$

$$_1W_2 = m(u_1 - u_2) = 1 \text{ kg} (759.2 - 349.4) \text{ kJ/kg} = 409.8 \text{ kJ}$$

ENGLISH UNIT PROBLEMS

Borgnakke Sonntag

Fundamentals of
Thermodynamics

SOLUTION MANUAL
CHAPTER 6
English units

8e

UPDATED JULY 2013

CHAPTER 6

SUBSECTION	PROB NO.
Concept-Study Guide Problems	190-191
Entropy, Clausius	192-195
Reversible Processes	196-206
Entropy of a Liquid or Solid	207-212
Entropy of Ideal Gases	213-219
Polytropic Processes	220-221
Entropy Generation	222-238
Rates or Fluxes of Entropy	239-242
Review problems	243-245

Excerpts from this work may be reproduced by instructors for distribution on a not-for-profit basis for testing or instructional purposes only to students enrolled in courses for which this textbook has been adopted. Any other reproduction or translation of this work beyond that permitted by Sections 107 or 108 of the 1976 United States Copyright Act without the permission of the copyright owner is unlawful.

Concept Problems

Excerpts from this work may be reproduced by instructors for distribution on a not-for-profit basis for testing or instructional purposes only to students enrolled in courses for which this textbook has been adopted. Any other reproduction or translation of this work beyond that permitted by Sections 107 or 108 of the 1976 United States Copyright Act without the permission of the copyright owner is unlawful.

6.190E

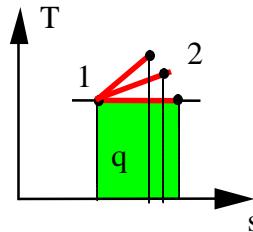
Water at 20 psia, 240 F receives 40 Btu/lbm in a reversible process by heat transfer. Which process changes s the most: constant T, constant v or constant P?

$$ds = \frac{dq}{T}$$

Look at the constant property lines in a T-s diagram, Fig. 6.5. The constant v line has a higher slope than the constant P line also at positive slope. Thus both the constant P and v processes have an increase in T. As T goes up the change in s is smaller for the same area (heat transfer) under the process curve in the T-s diagram as compared with the constant T process.

The constant T (isothermal) process therefore changes s the most.

In a reversible process the area below the process curve in the T-s diagram is the heat transfer.



6.191E

Saturated water vapor at 20 psia is compressed to 60 psia in a reversible adiabatic process. Find the change in v and T.

Process adiabatic: $dq = 0$

Process reversible: $ds_{gen} = 0$

Change in s: $ds = dq/T + ds_{gen} = 0 + 0 = 0$ thus s is constant

Table F.7.2: $T_1 = 227.96 \text{ F}$, $v_1 = 20.091 \text{ ft}^3/\text{lbm}$, $s_1 = 1.732 \text{ Btu/lbm R}$

Table F.7.2 at 60 psia and $s = s_1 = 1.732 \text{ Btu/lbm R}$

$$T = 400 + 40 \frac{1.732 - 1.7134}{1.736 - 1.7134} = 400 + 40 \times 0.823 = \mathbf{432.9 \text{ F}}$$

$$v = 8.353 + (8.775 - 8.353) \times 0.823 = \mathbf{8.700 \text{ ft}^3/\text{lbm}}$$

Entropy, Clausius

Excerpts from this work may be reproduced by instructors for distribution on a not-for-profit basis for testing or instructional purposes only to students enrolled in courses for which this textbook has been adopted. Any other reproduction or translation of this work beyond that permitted by Sections 107 or 108 of the 1976 United States Copyright Act without the permission of the copyright owner is unlawful.

6.192E

Consider the steam power plant in Problem 5.133E and show that this cycle satisfies the inequality of Clausius.

Solution:

$$\text{Show Clausius: } \int \frac{dQ}{T} \leq 0$$

For this problem we have two heat transfer terms:

Boiler: 1000 Btu/s at 1200 F = 1660 R

Condenser: 580 Btu/s at 100 F = 560 R

$$\int \frac{dQ}{T} = \frac{Q_H}{T_H} - \frac{Q_L}{T_L} = \frac{1000}{1660} - \frac{580}{560}$$

$$= 0.6024 - 1.0357 = -0.433 \text{ Btu/s R} < 0 \quad \text{OK}$$

6.193E

Find the missing properties of P, v, s and x for ammonia, NH₃.

- a. T = 190 F, P = 100 psia
- b. T = 80 F, h = 650 Btu/lbm
- c. T = 120 F, v = 1.6117 ft³/lbm

a) Table F.8.1, P < P_g = 708 psia, superheated vapor so x = undefined,

$$v = (3.8804 + 4.0188)/2 = 3.9496 \text{ ft}^3/\text{lbm}$$

$$s = (1.3658 + 1.3834)/2 = 1.3746 \text{ Btu/lbm R}$$

b) Table F.8.2 h > h_g = 629.62 Btu/lbm so superheated vapor

x = undefined, found between 50 and 60 psia

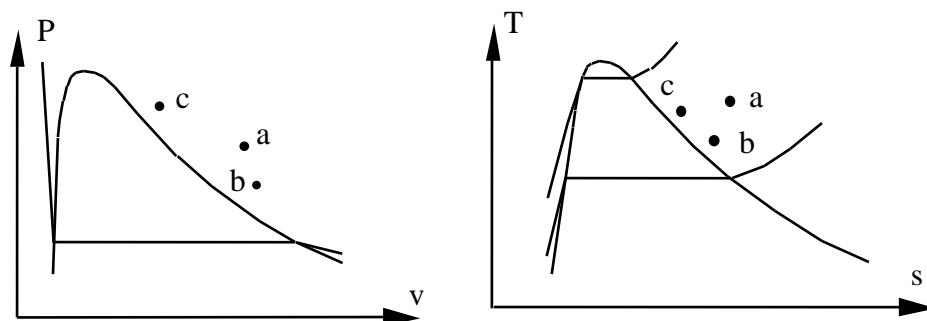
$$P = 50 + 10 (650 - 651.49)/(649.57 - 651.49) = 57.76 \text{ psia}$$

$$v = 6.5573 + 0.77604 (5.4217 - 6.5573) = 5.676 \text{ ft}^3/\text{lbm}$$

$$s = 1.3588 + 0.77604 (1.3348 - 1.3588) = 1.3402 \text{ Btu/lbm R}$$

c) Table F.8.1: v > v_g = 1.0456 ft³/lbm so superheated vapor

Table F.8.2: x = undefined, P = 200 psia, s = 1.2052 Btu/lbm R



6.194E

Find the missing properties and give the phase of the substance

- a. H_2O $s = 1.75 \text{ Btu/lbm R}$, $T = 150 \text{ F}$ $h = ? P = ? x = ?$
 b. H_2O $u = 1350 \text{ Btu/lbm}$, $P = 1500 \text{ lbf/in.}^2$ $T = ? x = ? s = ?$

a) Table F.7.1: $s < s_g = 1.8683 \text{ Btu/lbm-R}$, so 2 phase

$$P = P_{\text{sat}}(T) = 3.722 \text{ lbf/in.}^2$$

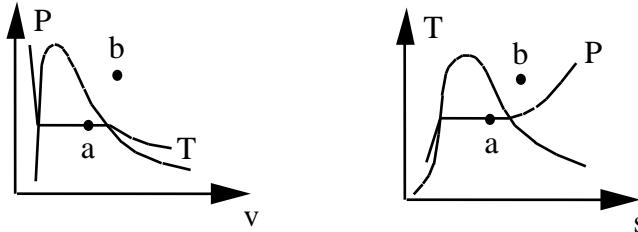
$$x = (s - s_f)/s_{fg} = (1.75 - 0.215)/1.6533 = 0.92845$$

$$h = 117.95 + 0.92845 \times 1008.1 = 1053.9 \text{ Btu/lbm}$$

b) Table F.7.2, found between 1000 F and 1100 F, $x = \text{undefined}$,

$$T = 1000 + 100 (1350 - 1340.43)/(1387.16 - 1340.43) = 1020.5 \text{ F},$$

$$s = 1.6001 + 0.2048 (1.6398 - 1.6001) = 1.6083 \text{ Btu/lbm R}$$



6.195E

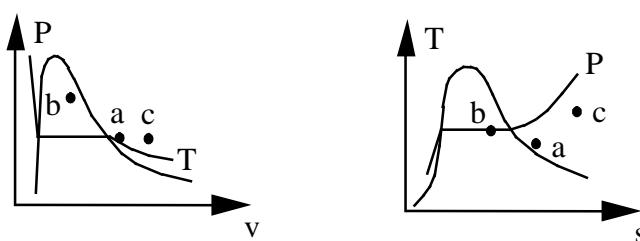
Determine the missing property among P, T, s, and x for R-410A at:

- a. $T = -20 \text{ F}$, $v = 3.1214 \text{ ft}^3/\text{lbm}$
- b. $T = 60 \text{ F}$, $v = 0.3121 \text{ ft}^3/\text{lbm}$
- c. $P = 30 \text{ psia}$, $s = 0.3425 \text{ Btu/lbm-R}$

a) F.9.1: $v > v_g = 1.4522 \text{ ft}^3/\text{lbm} \Rightarrow$
 F.9.2 superheated vapor so x is undefined
 very close to 20 psia, $s = 0.2862 \text{ Btu/lbm-R}$

b) F.9.1: $0.01451 = v_f < v < v_g = 0.3221 \text{ ft}^3/\text{lbm} \Rightarrow$ Two-phase
 $P = P_{\text{sat}} = 184.98 \text{ psia}$
 $x = (v - v_f)/v_{fg} = \frac{0.3121 - 0.01451}{0.3076} = 0.96746$
 $s = s_f + x s_{fg} = 0.0744 + 0.96746 \times 0.1662 = 0.2352 \text{ Btu/lbm-R}$

c) Table F.9.2 at 30 psia, $s > s_g$ so superheated vapor,
 x is undefined, and we find the state at $T = 160 \text{ F}$.

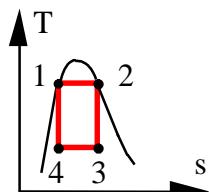


Reversible Processes

Excerpts from this work may be reproduced by instructors for distribution on a not-for-profit basis for testing or instructional purposes only to students enrolled in courses for which this textbook has been adopted. Any other reproduction or translation of this work beyond that permitted by Sections 107 or 108 of the 1976 United States Copyright Act without the permission of the copyright owner is unlawful.

6.196E

In a Carnot engine with water as the working fluid, the high temperature is 500 F and as Q_L is received, the water changes from saturated liquid to saturated vapor. The water pressure at the low temperature is 14.7 lbf/in.². Find T_L , cycle thermal efficiency, heat added per pound-mass, and entropy, s , at the beginning of the heat rejection process.



Constant T \Rightarrow constant P from 1 to 2 Table F.7.1

$$q_H = \int T ds = T(s_2 - s_1) = T s_{fg}$$

$$= h_2 - h_1 = h_{fg} = \mathbf{714.76 \text{ Btu/lbm}}$$

States 3 & 4 are two-phase Table F.7.1

$$\Rightarrow T_L = T_3 = T_4 = \mathbf{212 \text{ F}}$$

$$\eta_{cycle} = 1 - T_L/T_H = 1 - \frac{212 + 459.67}{500 + 459.67} = \mathbf{0.300}$$

$$\text{Table F.8.1: } s_3 = s_2 = s_g(T_H) = \mathbf{1.4335 \text{ Btu/lbm R}}$$

6.197E

Consider a Carnot-cycle heat pump with R-410A as the working fluid. Heat is rejected from the R-410A at 110 F, during which process the R-410A changes from saturated vapor to saturated liquid. The heat is transferred to the R-410A at 30 F.

- Show the cycle on a $T-s$ diagram.
- Find the quality of the R-410A at the beginning and end of the isothermal heat addition process at 30 F.
- Determine the coefficient of performance for the cycle.

a)

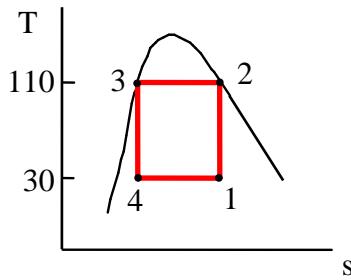


Table F.9.1

b) State 3 is saturated liquid

$$s_4 = s_3 = 0.1115 \text{ Btu/lbm R} \\ = 0.0526 + x_4(0.1955)$$

$$x_4 = \mathbf{0.30128}$$

State 2 is saturated vapor

$$s_1 = s_2 = 0.2261 \text{ Btu/lbm R} \\ = 0.0526 + x_1(0.1955)$$

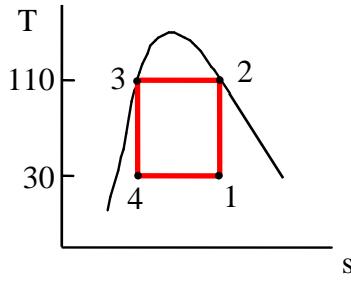
$$x_1 = \mathbf{0.8875}$$

c) $\beta' = \frac{q_H}{w_{IN}} = \frac{T_H}{T_H - T_L} = \frac{569.67}{110 - 30} = \mathbf{7.121}$ (depends only on the temperatures)

6.198E

Do Problem 6.197E using refrigerant R-134a instead of R-410A.

a)



b) Table F.10.1

State 3 is saturated liquid

$$s_4 = s_3 = 0.2882 \text{ Btu/lbm R}$$

$$= 0.2375 + x_4(0.1749)$$

$$x_4 = \mathbf{0.2899}$$

State 2 is saturated vapor

$$s_1 = s_2 = 0.4087 \text{ Btu/lbm R}$$

$$= 0.2375 + x_1(0.1749)$$

$$x_1 = \mathbf{0.9788}$$

c) $\beta' = \frac{q_H}{w_{IN}} = \frac{T_H}{T_H - T_L} = \frac{569.67}{110 - 30} = \mathbf{7.121}$

6.199E

R-410A at 150 psia and 140 F is expanded in a piston cylinder to 75 psia, 80 F in a reversible process. Find the sign for both the work and the heat transfer for this process.

Solution:

$$_1 w_2 = \int P \, dv \quad \text{so sign } dv$$

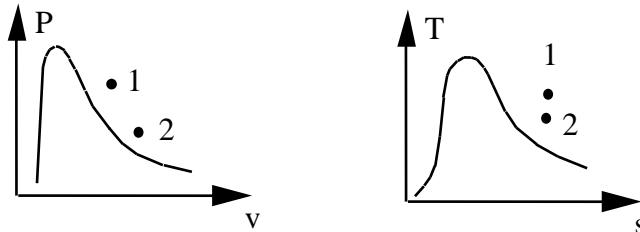
$$_1 q_2 = \int T \, ds \quad \text{so sign } ds$$

$$\text{F.9.2} \quad v_1 = 0.5356 \text{ ft}^3/\text{lbm} \quad s_1 = 0.2859 \text{ Btu/lbm-R}$$

$$\text{F.9.2} \quad v_2 = 0.9911 \text{ ft}^3/\text{lbm} \quad s_2 = 0.2857 \text{ Btu/lbm-R}$$

$$dv > 0 \quad \Rightarrow \quad \mathbf{w \text{ is positive}}$$

$$ds < 0 \quad (\approx 0) \quad \Rightarrow \quad \mathbf{q \text{ is negative (nearly zero)}}$$



6.200E

A piston/cylinder receives R-410A at 75 psia and compresses it in a reversible adiabatic process to 300 psia, 160 F. Find the initial temperature.

CV R-410A, this is a control mass

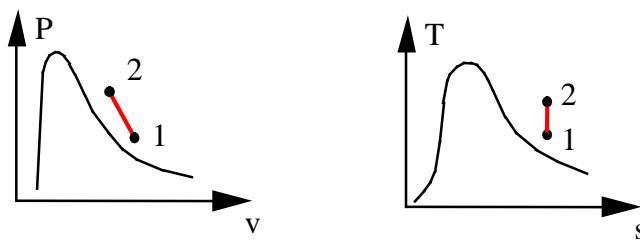
$$\text{Energy Eq.3.5: } u_2 - u_1 = q_1 - w_1$$

$$\text{Entropy Eq.6.3: } s_2 - s_1 = \int dq/T = q_1 / T$$

Process: Adiabatic and reversible $\Rightarrow q_1 = 0$ so then $s_2 = s_1$

State 1: $P_1, s_1 = s_2 = 0.2673 \text{ Btu/lbm-R}$ \Rightarrow

$$T_1 = 20 + (40 - 20) \times \frac{0.2673 - 0.2595}{0.2688 - 0.2595} = 36.77 \text{ F}$$



6.201E

A piston/cylinder contains 1 lbm of water at 40 psia, 600 F and it now cools to 280 F in an isobaric process. The heat goes into a heat engine which rejects heat to the ambient at 77 F shown in Fig. P6.46 and the whole process is assumed reversible. Find the heat transfer out of the water and the work given out by the heat engine.

C.V. H₂O

$$\text{Energy Eq.3.5: } m(u_2 - u_1) = _1Q_2 - _1W_2$$

$$\text{Entropy Eq.6.3: } m(s_2 - s_1) = \int dQ/T + 0$$

$$\text{Process: } P = C \Rightarrow W = \int P dV = P(V_2 - V_1) = m P (v_2 - v_1)$$

$$\text{State 1: F.7.2 } s_1 = 1.8621 \text{ Btu/lbm-R}, h_1 = 1333.43 \text{ Btu/lbm}$$

$$\text{State 2: F.7.3 } s_2 = 1.6857 \text{ Btu/lbm-R}, h_2 = 1176.59 \text{ Btu/lbm}$$

From the process equation and the energy equation

$$\begin{aligned} _1Q_2 &= m(u_2 - u_1) + _1W_2 = m(h_2 - h_1) = 1 \text{ lbm} (1176.59 - 1333.43) \text{ Btu/lbm} \\ &= \mathbf{-156.84 \text{ Btu}} \end{aligned}$$

CV Total

$$\text{Energy Eq.3.5: } m(u_2 - u_1) = -Q_L - _1W_2 - W_{HE}$$

$$\text{Entropy Eq.6.3: } m(s_2 - s_1) = -Q_L/T_{amb} + 0$$

$$\begin{aligned} Q_L &= mT_{amb}(s_1 - s_2) = 1 \text{ lbm} \times 536.7 \text{ R} \times (1.8621 - 1.6857) \text{ Btu/lbmR} \\ &= 94.67 \text{ Btu} \end{aligned}$$

Now the energy equation for the heat engine gives

$$W_{HE} = -_1Q_2 - Q_L = 156.84 - 94.67 = \mathbf{62.17 \text{ Btu}}$$

6.202E

A cylinder fitted with a piston contains ammonia at 120 F, 20% quality with a volume of 60 in.³. The ammonia expands slowly, and during this process heat is transferred to maintain a constant temperature. The process continues until all the liquid is gone. Determine the work and heat transfer for this process.

C.V. Ammonia in the cylinder.

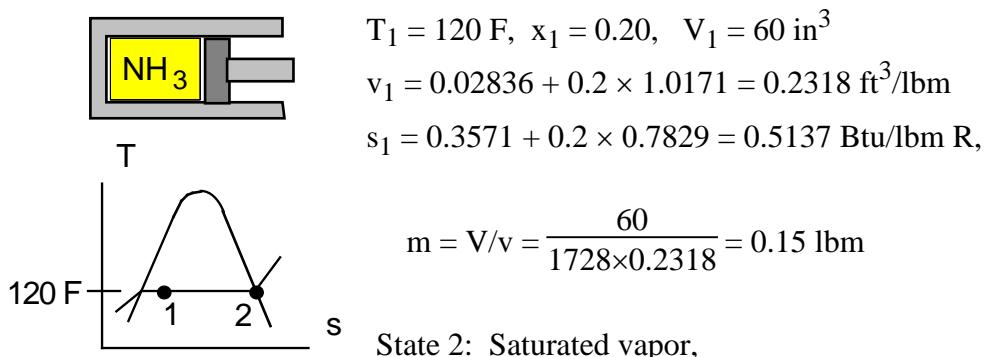
$$\text{Energy Eq.3.5: } m(u_2 - u_1) = _1Q_2 - _1W_2$$

$$\text{Entropy Eq.6.3: } m(s_2 - s_1) = \int dQ/T + 0$$

$$\text{Process: } T = \text{constant to } x_2 = 1.0, \quad \text{Table F.8.1: } P = 286.5 \text{ lbf/in}^2$$

$$\Rightarrow W = \int P dV = P(V_2 - V_1) = m P (v_2 - v_1)$$

$$\Rightarrow \int dQ/T = _1Q_2 / T$$



$$_1W_2 = \frac{286.5 \times 144}{778} \times 0.15 \times (1.045 - 0.2318) = \mathbf{6.47 \text{ Btu}}$$

From the entropy equation

$$\begin{aligned} _1Q_2 &= T m (s_2 - s_1) \\ &= 579.7 \text{ R} \times 0.15 \text{ lbm} \times (1.1400 - 0.5137) \text{ Btu/lbm-R} \\ &= \mathbf{54.46 \text{ Btu}} \end{aligned}$$

$$\text{- or - } h_1 = 178.79 + 0.2 \times 453.84 = 269.56 \text{ Btu/lbm}; \quad h_2 = 632.63 \text{ Btu/lbm}$$

$$_1Q_2 = m(h_2 - h_1) = 0.15(632.63 - 269.56) = \mathbf{54.46 \text{ Btu}}$$

6.203E

One pound-mass of water at 600 F expands against a piston in a cylinder until it reaches ambient pressure, 14.7 lbf/in.², at which point the water has a quality of 90%. It may be assumed that the expansion is reversible and adiabatic.

- What was the initial pressure in the cylinder?
- How much work is done by the water?

Solution:

C.V. Water. Process: Rev., $Q = 0$

$$\text{Energy Eq.3.5: } m(u_2 - u_1) = _1Q_2 - _1W_2 = - _1W_2$$

$$\text{Entropy Eq.6.3: } m(s_2 - s_1) = \int dQ/T$$

Process: Adiabatic $Q = 0$ and reversible $\Rightarrow s_2 = s_1$

State 2: $P_2 = 14.7 \text{ lbf/in}^2$, $x_2 = 0.90$ from Table F.7.1

$$s_2 = 0.3121 + 0.9 \times 1.4446 = 1.6123 \text{ Btu/lbm R}$$

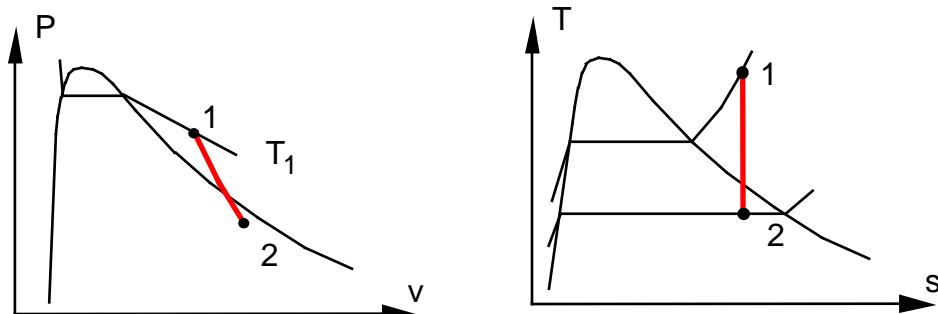
$$u_2 = 180.1 + 0.9 \times 897.5 = 987.9 \text{ Btu/lbm}$$

State 1 Table F.7.2: at $T_1 = 600 \text{ F}$, $s_1 = s_2$

$$\Rightarrow P_1 = 335 \text{ lbf/in}^2 \quad u_1 = 1201.2 \text{ Btu/lbm}$$

From the energy equation

$$_1W_2 = m(u_1 - u_2) = 1 \text{ lbm}(1201.2 - 987.9) \text{ Btu/lbm} = 213.3 \text{ Btu}$$



6.204E

A closed tank, $V = 0.35 \text{ ft}^3$, containing 10 lbm of water initially at 77 F is heated to 350 F by a heat pump that is receiving heat from the surroundings at 77 F.

Assume that this process is reversible. Find the heat transfer to the water and the work input to the heat pump.

C.V.: Water from state 1 to state 2.

Process: constant volume (reversible isometric)

$$1: v_1 = V/m = 0.35/10 = 0.035 \text{ ft}^3/\text{lbf} \Rightarrow x_1 = 2.692 \times 10^{-5}$$

$$u_1 = 45.11 \text{ Btu/lbm}, \quad s_1 = 0.08779 \text{ Btu/lbm R}$$

Continuity eq. (same mass) and constant volume fixes v_2

$$\text{State 2: } T_2, v_2 = v_1 \Rightarrow x_2 = (0.035 - 0.01799) / 3.3279 = 0.00511$$

$$u_2 = 321.35 + 0.00511 \times 788.45 = 325.38 \text{ Btu/lbm}$$

$$s_2 = 0.5033 + 0.00511 \times 1.076 = 0.5088 \text{ Btu/lbm R}$$

Energy eq. has zero work, thus provides heat transfer as

$$_1Q_2 = m(u_2 - u_1) = 10(325.38 - 45.11) = \mathbf{2802.7 \text{ Btu}}$$

Entropy equation for the total control volume gives
for a reversible process:

$$m(s_2 - s_1) = Q_L/T_0$$

\Rightarrow

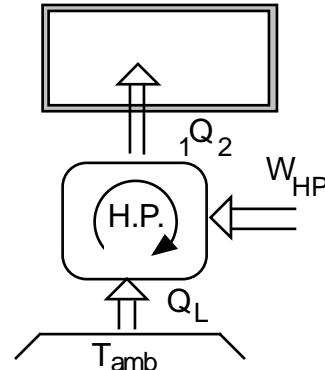
$$Q_L = mT_0(s_2 - s_1)$$

$$= 10 \text{ lbm} (77 + 459.67)R (0.5088 - 0.08779) \text{ Btu/lbm-R}$$

$$= 2259.4 \text{ Btu}$$

and the energy equation for the heat pump gives

$$W_{HP} = _1Q_2 - Q_L = 2802.7 - 2259.4 = \mathbf{543.3 \text{ Btu}}$$



6.205E

A cylinder containing R-134a at 60 F, 30 lbf/in.², has an initial volume of 1 ft³. A piston compresses the R-134a in a reversible, isothermal process until it reaches the saturated vapor state. Calculate the required work and heat transfer to accomplish this process.

Solution:

C.V. R-134a.

Continuity Eq.: $m_2 = m_1 = m$;

Energy Eq.:3.5 $m(u_2 - u_1) = _1Q_2 - _1W_2$

Entropy Eq.6.3: $m(s_2 - s_1) = \int dQ/T + _1S_{2\text{ gen}}$

Process: $T = \text{constant}$, reversible so $_1S_{2\text{ gen}} = 0$

State 1: (T, P) Table F.10.2 $u_1 = 168.41 \text{ Btu/lbm}$, $s_1 = 0.4321 \text{ Btu/lbm R}$

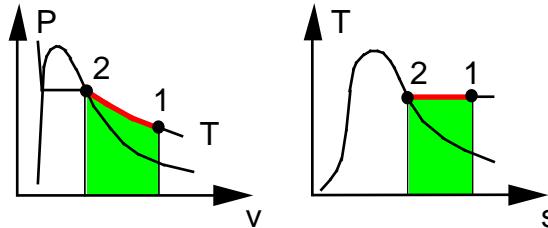
$$m = V/v_1 = 1/1.7367 = 0.5758 \text{ lbm}$$

State 2: (60 F sat. vapor)

Table F.10.1

$$u_2 = 166.28 \text{ Btu/lbm},$$

$$s_2 = 0.4108 \text{ Btu/lbm R}$$



As T is constant we can find Q by integration as

$$\begin{aligned} _1Q_2 &= \int T ds = mT(s_2 - s_1) = 0.5758 \text{ lbm} \times 519.7 \text{ R} \times (0.4108 - 0.4321) \text{ Btu/lbm-R} \\ &= \mathbf{-6.374 \text{ Btu}} \end{aligned}$$

The work is then from the energy equation

$$\begin{aligned} _1W_2 &= m(u_1 - u_2) + _1Q_2 = 0.5758 \text{ lbm} \times (168.41 - 166.28) \text{ Btu/lbm} - 6.374 \text{ Btu} \\ &= \mathbf{-5.15 \text{ Btu}} \end{aligned}$$

6.206E

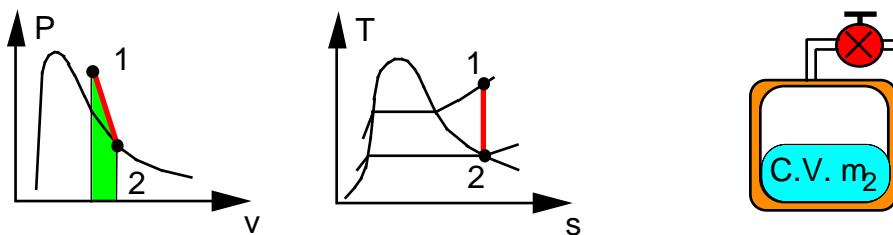
A rigid, insulated vessel contains superheated vapor steam at 400 lbf/in.², 700 F.

A valve on the vessel is opened, allowing steam to escape. It may be assumed that the steam remaining inside the vessel goes through a reversible adiabatic expansion. Determine the fraction of steam that has escaped, when the final state inside is saturated vapor.

C.V.: steam remaining inside tank. Rev. & Adiabatic (inside only)

Cont.Eq.: $m_2 = m_1 = m$; Energy Eq.: $m(u_2 - u_1) = _1Q_2 - _1W_2$

Entropy Eq.: $m(s_2 - s_1) = \int dQ/T + _1S_{2\text{ gen}} = 0 + 0$



State 1: Table F.7.2 $v_1 = 1.6503 \text{ ft}^3/\text{lbfm}$, $s_1 = 1.6396 \text{ Btu/lbm R}$

State 2: Table F.7.1 $s_2 = s_1 = 1.6396 \text{ Btu/lbm R} = s_g$ at P_2

\Rightarrow Interpolate in F.7.1 $P_2 = 63.861 \text{ lbf/in}^2$, $v_2 = v_g = 6.800 \text{ ft}^3/\text{lbfm}$

$$\frac{m_e}{m_1} = \frac{m_1 - m_2}{m_1} = 1 - \frac{m_2}{m_1} = 1 - \frac{v_1}{v_2} = 1 - \frac{1.6503}{6.800} = \mathbf{0.757}$$

Entropy of a Liquid or Solid

Excerpts from this work may be reproduced by instructors for distribution on a not-for-profit basis for testing or instructional purposes only to students enrolled in courses for which this textbook has been adopted. Any other reproduction or translation of this work beyond that permitted by Sections 107 or 108 of the 1976 United States Copyright Act without the permission of the copyright owner is unlawful.

6.207E

Two 5 lbm blocks of steel, one at 500 F, the other at 80 F, come in thermal contact. Find the final temperature and the change in the entropy of the steel.

C.V. Both blocks, no external heat transfer, $C = 0.11 \text{ Btu/lbm-R}$ from Table F.2.

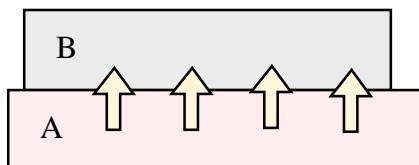
$$\begin{aligned}\text{Energy Eq.: } U_2 - U_1 &= m_A(u_2 - u_1)_A + m_B(u_2 - u_1)_B = 0 - 0 \\ &= m_A C(T_2 - T_{A1}) + m_B C(T_2 - T_{B1})\end{aligned}$$

$$T_2 = \frac{m_A T_{A1} + m_B T_{B1}}{m_A + m_B} = \frac{1}{2} T_{A1} + \frac{1}{2} T_{B1} = 290 \text{ F}$$

$$\text{Entropy Eq.6.37: } S_2 - S_1 = m_A(s_2 - s_1)_A + m_B(s_2 - s_1)_B = 1S_2 \text{ gen}$$

Entropy changes from Eq.6.11

$$\begin{aligned}S_2 - S_1 &= m_A C \ln \frac{T_2}{T_{A1}} + m_B C \ln \frac{T_2}{T_{B1}} \\ &= 5 \times 0.11 \ln \frac{290 + 459.67}{500 + 459.67} + 5 \times 0.11 \ln \frac{290 + 459.67}{80 + 459.67} \\ &= -0.13583 + 0.18077 = \mathbf{0.0449 \text{ Btu/R}}$$



Heat transfer over a finite temperature difference is an irreversible process

6.208E

A foundry form box with 50 lbm of 400 F hot sand is dumped into a bucket with 2 ft³ water at 60 F. Assuming no heat transfer with the surroundings and no boiling away of liquid water, calculate the net entropy change for the masses.

C.V. Sand and water, P = const.

$$\text{Energy Eq.: } m_{\text{sand}}(u_2 - u_1)_{\text{sand}} + m_{\text{H}_2\text{O}}(u_2 - u_1)_{\text{H}_2\text{O}} = -P(V_2 - V_1)$$

$$\Rightarrow m_{\text{sand}}\Delta h_{\text{sand}} + m_{\text{H}_2\text{O}}\Delta h_{\text{H}_2\text{O}} = 0, \quad m_{\text{H}_2\text{O}} = \frac{2}{0.016035} = 124.73 \text{ lbm}$$

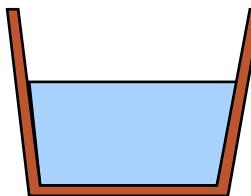
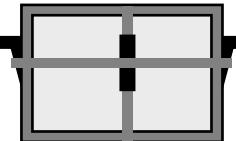
$$50 \text{ lbm} \times 0.19 \text{ Btu/lbm-R} \times (T_2 - 400 \text{ F})$$

$$+ 124.73 \text{ lbm} \times 1.0 \text{ Btu/lbm-R} \times (T_2 - 60 \text{ F}) = 0,$$

$$T_2 = 84 \text{ F}$$

$$\begin{aligned} S_2 - S_1 &= m_{\text{sand}}(s_2 - s_1) + m_{\text{H}_2\text{O}}(s_2 - s_1) \\ &= m_{\text{sand}} C_{\text{sand}} \ln(T_2/T_1) + m_{\text{H}_2\text{O}} C_{\text{H}_2\text{O}} \ln(T_2/T_1) \\ &= 50 \times 0.19 \times \ln\left(\frac{544}{860}\right) + 124.73 \times 1.0 \times \ln\left(\frac{544}{520}\right) \\ &= \mathbf{1.293 \text{ Btu/R}} \end{aligned}$$

Box holds the sand for
form of the cast part



6.209E

Heat transfer to a block of 3 lbm ice at 15 F melts it to liquid at 50 F in a kitchen.
Find the entropy change of the water.

Water changes state from nearly saturated solid to nearly saturated liquid. The pressure is 1 atm, but we approximate the state properties with saturated state at the same temperature.

State 1: Compressed (saturated) solid, F.7.4, $s_1 = -0.3093 \text{ Btu/lbm-R}$

State 2: Compressed (saturated) liquid F.7.1 $s_2 = 0.0361 \text{ Btu/lbm-R}$

The entropy change is

$$\begin{aligned}\Delta S &= m(s_2 - s_1) = 3 \text{ lbm} [0.0361 - (-0.3093)] \text{ Btu/lbm-R} \\ &= \mathbf{1.0362 \text{ Btu/R}}\end{aligned}$$

6.210E

A rigid tank of 1.2 lbm steel contains 1.5 lbm of R-134a at 100 F, 80 psia. The tank is placed in a refrigerator that brings it to 0 F. Find the process heat transfer and the combined steel and R-134a change in entropy.

C.V. The steel tank and the R-134a

The energy equation, Eq. 3.5 now becomes (summing over the mass)

$$m_{st} (u_2 - u_1)_{st} + m_{R134a} (u_2 - u_1)_{R134a} = 1Q_2 - 0$$

Process: No change in volume so no work as used above.

Use specific heat from Table F.2 for steel and Table F.10 for R-134a

$$\text{R-134a: } v_1 = 0.6617 \text{ ft}^3/\text{lbm}, \quad u_1 = 174.06 \text{ Btu/lbm}, \quad s_1 = 0.4252 \text{ Btu/lbmR}$$

$$\text{State 2: } v_2 = v_1 < v_g \Rightarrow x_2 = (v_2 - v_f)/v_{fg} = \frac{0.6617 - 0.01187}{2.1340} = 0.304513$$

$$u_2 = u_f + x_2 u_{fg} = 75.88 + x_2 \times 82.24 = 100.923 \text{ Btu/lbm}$$

$$s_2 = s_f + x_2 s_{fg} = 0.2178 + x_2 \times 0.1972 = 0.27785 \text{ Btu/lbmR}$$

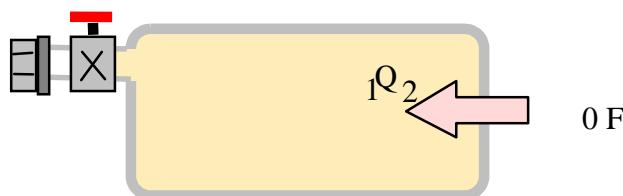
Now the heat transfer from the energy equation

$$\begin{aligned} 1Q_2 &= m_{R134a}(u_2 - u_1)_{R134a} + m_{st} C_{st} (T_2 - T_1) \\ &= 1.5 \times (100.923 - 174.06) + 1.2 \times 0.11 (0 - 100) = -122.91 \text{ Btu} \end{aligned}$$

$$\text{Steel: } m_{st}(s_2 - s_1)_{st} = m_{st} C_{st} \ln(T_2/T_1) = 1.2 \times 0.11 \ln \frac{459.7}{559.7} = -0.02598 \text{ Btu/lbmR}$$

Entropy change for the total control volume steel and R-134a

$$\begin{aligned} S_2 - S_1 &= m_{st} (s_2 - s_1)_{st} + m_{R134a} (s_2 - s_1)_{R134a} \\ &= -0.02598 + 1.5(0.27785 - 0.4252) = -0.247 \text{ Btu/R} \end{aligned}$$



6.211E

A 5-lbm aluminum radiator holds 2 lbm of liquid R-134a at 10 F. The setup is brought indoors and heated with 220 Btu. Find the final temperature and the change in entropy of all the mass.

Solution:

C.V. The aluminum radiator and the R-134a.

$$\text{Energy Eq.3.5: } m_2 u_2 - m_1 u_1 = \dot{Q}_2 - 0$$

Process: No change in volume so no work as used above.

The energy equation now becomes (summing over the mass)

$$m_{\text{al}}(u_2 - u_1)_{\text{al}} + m_{\text{R134a}}(u_2 - u_1)_{\text{R134a}} = \dot{Q}_2$$

Use specific heat from Table F.2 and F.3

$$m_{\text{al}}C_{\text{al}}(T_2 - T_1) + m_{\text{R134a}}C_{\text{R134a}} \ln(T_2 - T_1) = \dot{Q}_2$$

$$T_2 - T_1 = \dot{Q}_2 / [m_{\text{al}}C_{\text{al}} + m_{\text{R134a}}C_{\text{R134a}}]$$

$$= 220 / [5 \times 0.215 + 2 \times 0.34] = 125 \text{ F}$$

$$T_2 = 10 + 125 = 135 \text{ F}$$

Entropy change for solid (F.2) and liquid (F.3) from Eq.6.11

$$\begin{aligned} S_2 - S_1 &= m_{\text{al}}(s_2 - s_1)_{\text{al}} + m_{\text{R134a}}(s_2 - s_1)_{\text{R134a}} \\ &= m_{\text{al}}C_{\text{al}} \ln(T_2/T_1) + m_{\text{R134a}}C_{\text{R134a}} \ln(T_2/T_1) \\ &= (5 \times 0.215 + 2 \times 0.34) \ln \frac{(135 + 459.67)}{10 + 459.67} \\ &= \mathbf{0.414 \text{ Btu/R}} \end{aligned}$$



6.212E

Four pounds of liquid lead at 750 F are poured into a form. It then cools at constant pressure down to room temperature at 68 F as heat is transferred to the room. The melting point of lead is 620 F and the enthalpy change between the phases h_{if} is 10.6 Btu/lbm. The specific heats are in Table F.2 and F.3. Calculate the net entropy change for the lead.

Solution:

C.V. Lead, constant pressure process

$$m_{Pb}(u_2 - u_1)_{Pb} = 1Q_2 - P(V_2 - V_1)$$

We need to find changes in enthalpy ($u + Pv$) for each phase separately and then add the enthalpy change for the phase change.

$$C_{liq} = 0.038 \text{ Btu/lbm R}, \quad C_{sol} = 0.031 \text{ Btu/lbm R}$$

Consider the process in several steps:

Cooling liquid to the melting temperature

Solidification of the liquid to solid

Cooling of the solid to the final temperature

$$\begin{aligned} 1Q_2 &= m_{Pb}(h_2 - h_1) = m_{Pb}(h_2 - h_{620,sol} - h_{if} + h_{620,f} - h_{750}) \\ &= 4 \times [0.031 \times (68 - 620) - 10.6 + 0.038 \times (620 - 750)] \\ &= -68.45 - 42.4 - 19.76 = -130.61 \text{ Btu} \end{aligned}$$

$$\begin{aligned} S_2 - S_1 &= m_{Pb}[C_{p\ sol}\ln(T_2/1079.7) - (h_{if}/1079.7) + C_{p\ liq}\ln(1079.7/T_1)] \\ &= 4 \times [0.031 \ln \frac{527.7}{1079.7} - \frac{10.6}{1079.7} + 0.038 \ln \frac{1079.6}{1209.7}] = \mathbf{-0.145 \text{ Btu/R}} \end{aligned}$$



Entropy of Ideal Gases

Excerpts from this work may be reproduced by instructors for distribution on a not-for-profit basis for testing or instructional purposes only to students enrolled in courses for which this textbook has been adopted. Any other reproduction or translation of this work beyond that permitted by Sections 107 or 108 of the 1976 United States Copyright Act without the permission of the copyright owner is unlawful.

6.213E

Air inside a rigid tank is heated from 550 to 600 R. Find the entropy increase $s_2 - s_1$? What if it is from 2300 to 2350 R?

Process: $V = C \rightarrow W_2 = \emptyset$

Entropy change from Eq.6.17:

$$a) \quad s_2 - s_1 = C_{vo} \ln \left(\frac{T_2}{T_1} \right) = 0.171 \ln \left(\frac{600}{550} \right) = \mathbf{0.015 \text{ Btu/lbm-R}}$$

$$b) \quad s_2 - s_1 = C_{vo} \ln \left(\frac{T_2}{T_1} \right) = 0.171 \ln \left(\frac{2350}{2300} \right) = \mathbf{0.00368 \text{ Btu/lbm-R}}$$

From F.5:

case a) $C_v \approx \Delta u / \Delta T = 6.868/40 = 0.1717 \text{ Btu/lbm-R}$, see F.4 (0.171)

case b) $C_v \approx \Delta u / \Delta T = 21.53/100 = 0.215 \text{ Btu/lbm-R}$ (25 % higher)
so result should have been 0.00463 Btu/lbm-R

6.214E

R-410A at 60 psia is brought from 60 F to 240 F in a constant pressure process. Evaluate the change in specific entropy using Table F.9 and using ideal gas with $C_p = 0.1935 \text{ Btu/lbmR}$.

$$\begin{aligned} \text{Table F.9.2} \quad s_1 &= 0.2849 \text{ Btu/lbm-R}, \quad s_2 = 0.3496 \text{ Btu/lbm-R} \\ s_2 - s_1 &= 0.3496 - 0.2849 = \mathbf{0.0647 \text{ Btu/lbm-R}} \end{aligned}$$

$$\text{Eq. 6.16: } s_2 - s_1 \approx C_{po} \ln \frac{T_2}{T_1} = 0.1935 \ln \frac{240 + 460}{60 + 460} = \mathbf{0.0575 \text{ Btu/lbm-R}}$$

Two explanations for the difference are as the average temperature is higher than 77 F we could expect a higher value of the specific heat and secondly it is not an ideal gas (if you calculate $Z = Pv/RT = 0.94$).

6.215E

Oxygen gas in a piston/cylinder at 500 R and 1 atm with a volume of 1 ft³ is compressed in a reversible adiabatic process to a final temperature of 1000 R. Find the final pressure and volume using constant heat capacity from Table F.4 and repeat with Table F.6.

Solution:

C.V. Air. Assume a reversible, adiabatic process.

$$\text{Energy Eq.3.5: } u_2 - u_1 = 0 - \dot{w}_2 ;$$

$$\text{Entropy Eq.6.37: } s_2 - s_1 = \int dq/T + \dot{s}_{\text{gen}} = 0$$

$$\text{Process: Adiabatic } \dot{q}_2 = 0 \quad \text{Reversible } \dot{s}_{\text{gen}} = 0$$

$$\text{Properties: Table F.4: } k = 1.393$$

With these two terms zero we have a zero for the entropy change. So this is a constant s (isentropic) expansion process. From Eq.6.23

$$P_2 = P_1 (T_2 / T_1)^{\frac{k}{k-1}} = 14.7 (1000/500)^{3.5445} = \mathbf{171.5 \text{ psia}}$$

Using the ideal gas law to eliminate P from this equation leads to Eq.6.24

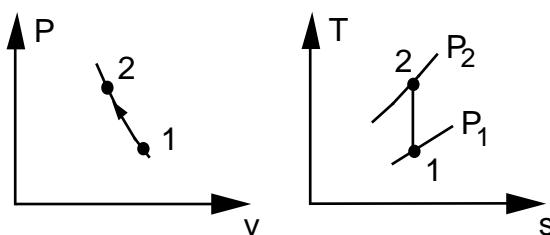
$$V_2 = V_1 (T_2 / T_1)^{\frac{1}{1-k}} = 1 \times \left(\frac{1000}{500}\right)^{\frac{1}{1-1.393}} = \mathbf{0.171 \text{ ft}^3}$$

Using the ideal gas tables F.6 we get

$$s_2 - s_1 = s_{T2}^o - s_{T1}^o - R \ln(P_2/P_1) \quad \text{or} \quad P_2 = P_1 \exp[(s_{T2}^o - s_{T1}^o)/R]$$

$$P_2 = 14.7 \text{ psia} \times \exp[(53.475 - 48.4185)/1.98589] = \mathbf{187.55 \text{ psia}}$$

$$V_2 = V_1 (T_2 / T_1)(P_1/P_2) = 1 \text{ ft}^3 \times \frac{1000}{500} \times \frac{14.7}{187.55} = \mathbf{0.157 \text{ ft}^3}$$



6.216E

A handheld pump for a bicycle has a volume of 2 in.³ when fully extended. You now press the plunger (piston) in while holding your thumb over the exit hole so an air pressure of 45 lbf/in.² is obtained. The outside atmosphere is at P_o , T_o . Consider two cases: (1) it is done quickly (~1 s), and (2) it is done slowly (~1 h).

- State assumptions about the process for each case.
- Find the final volume and temperature for both cases.

Solution:

C.V. Air in pump. Assume that both cases result in a reversible process.

$$\text{State 1: } P_0, T_0 \quad \text{State 2: } 45 \text{ lbf/in.}^2, ?$$

One piece of information must resolve the ? for a state 2 property.

Case I) Quickly means no time for heat transfer

$Q = 0$, so a reversible adiabatic compression.

$$u_2 - u_1 = -1w_2 ; \quad s_2 - s_1 = \int dq/T + 1s_2 \text{ gen} = 0$$

With constant s and constant heat capacity we use Eq.6.23

$$T_2 = T_1 \left(\frac{P_2}{P_1} \right)^{\frac{k-1}{k}} = 536.7 \left(\frac{45}{14.696} \right)^{\frac{0.4}{1.4}} = \mathbf{738.9 \text{ R}}$$

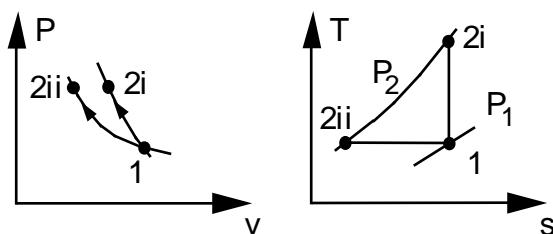
Use ideal gas law $PV = mRT$ at both states so ratio gives

$$\Rightarrow V_2 = P_1 V_1 T_2 / T_1 P_2 = \mathbf{0.899 \text{ in}^3}$$

Case II) Slowly, time for heat transfer so $T = \text{constant} = T_0$.

The process is then a reversible isothermal compression.

$$T_2 = T_0 = \mathbf{536.7 \text{ R}} \quad \Rightarrow \quad V_2 = V_1 P_1 / P_2 = \mathbf{0.653 \text{ in}^3}$$



6.217E

A piston/cylinder contains air at 2500 R, 2200 lbf/in.², with $V_1 = 1 \text{ in.}^3$, $A_{\text{cyl}} = 1 \text{ in.}^2$ as shown in Fig. P6.92. The piston is released and just before the piston exits the end of the cylinder the pressure inside is 30 lbf/in.². If the cylinder is insulated, what is its length? How much work is done by the air inside?

Solution:

C.V. Air, Cylinder is insulated so adiabatic, $Q = 0$.

Continuity Eq.: $m_2 = m_1 = m$,

Energy Eq.3.5: $m(u_2 - u_1) = _1Q_2 - _1W_2 = - _1W_2$

Entropy Eq.6.14: $m(s_2 - s_1) = \int dQ/T + _1S_2 \text{ gen} = 0 + _1S_2 \text{ gen}$

State 1: (T_1, P_1) State 2: $(P_2, ?)$

So one piece of information is needed for the ?, assume reversible process.

$$_1S_2 \text{ gen} = 0 \Rightarrow s_2 - s_1 = 0$$

State 1: Table F.5: $u_1 = 474.33 \text{ Btu/lbm}$, $s_{T_1}^\circ = 2.03391 \text{ Btu/lbm R}$

$$m = P_1 V_1 / RT_1 = \frac{2200 \times 1.0}{53.34 \times 2500 \times 12} = 1.375 \times 10^{-3} \text{ lbm}$$

State 2: P_2 and from Entropy eq.: $s_2 = s_1$ so from Eq.6.19

$$s_{T_2}^\circ = s_{T_1}^\circ + R \ln \frac{P_2}{P_1} = 2.03391 + \frac{53.34}{778} \ln \left(\frac{30}{2200} \right) = 1.73944 \text{ Btu/lbm R}$$

Now interpolate in Table F.5 to get T_2

$$T_2 = 840 + 40 (1.73944 - 1.73463) / (1.74653 - 1.73463) = 816.2 \text{ R}$$

$$u_2 = 137.099 + (144.114 - 137.099) 0.404 = 139.93 \text{ Btu/lbm}$$

$$V_2 = V_1 \frac{T_2 P_1}{T_1 P_2} = \frac{1 \times 816.2 \times 2200}{2500 \times 30} = 23.94 \text{ in}^3$$

$$\Rightarrow L_2 = V_2 / A_{\text{cyl}} = 23.94 / 1 = 23.94 \text{ in}$$

$$_1W_2 = m(u_1 - u_2) = 1.375 \times 10^{-3} (474.33 - 139.93) = 0.46 \text{ Btu}$$

6.218E

A 25-ft³ insulated, rigid tank contains air at 110 lbf/in.², 75 F. A valve on the tank is opened, and the pressure inside quickly drops to 15 lbf/in.², at which point the valve is closed. Assuming that the air remaining inside has undergone a reversible adiabatic expansion, calculate the mass withdrawn during the process.

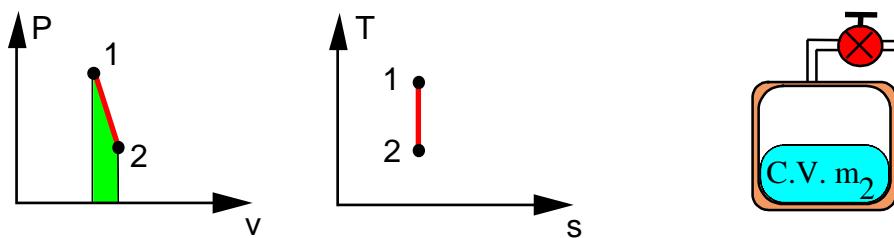
C.V.: Air remaining inside tank, m_2 .

Cont.Eq.: $m_2 = m$;

Energy Eq.3.5: $m(u_2 - u_1) = _1Q_2 - _1W_2$

Entropy Eq.6.37: $m(s_2 - s_1) = \int dQ/T + _1S_{2\text{ gen}}$

Process: adiabatic $_1Q_2 = 0$ and reversible $_1S_{2\text{ gen}} = 0$



Entropy eq. then gives $s_2 = s_1$ and ideal gas gives the relation in Eq.6.23

$$T_2 = T_1(P_2/P_1)^{\frac{k-1}{k}} = 535 \text{ R} (15/110)^{0.286} = 302.6 \text{ R}$$

$$\begin{aligned} m_1 &= P_1 V / RT_1 = 110 \text{ psia} \times 144 \frac{\text{in}^2}{\text{ft}^2} \times 25 \text{ ft}^3 / (53.34 \frac{\text{ft-lbf}}{\text{lbm-R}} \times 535 \text{ R}) \\ &= 13.88 \text{ lbm} \end{aligned}$$

$$\begin{aligned} m_2 &= P_2 V / RT_2 = 15 \text{ psia} \times 144 \frac{\text{in}^2}{\text{ft}^2} \times 25 \text{ ft}^3 / (53.34 \frac{\text{ft-lbf}}{\text{lbm-R}} \times 302.6 \text{ R}) \\ &= 3.35 \text{ lbm} \end{aligned}$$

$$m_e = m_1 - m_2 = \mathbf{10.53 \text{ lbm}}$$

Polytropic Processes

Excerpts from this work may be reproduced by instructors for distribution on a not-for-profit basis for testing or instructional purposes only to students enrolled in courses for which this textbook has been adopted. Any other reproduction or translation of this work beyond that permitted by Sections 107 or 108 of the 1976 United States Copyright Act without the permission of the copyright owner is unlawful.

6.219E

A cylinder/piston contains 0.1 lbm methane gas at 15 psia, 70 F. The gas is compressed reversibly to a pressure of 120 psia. Calculate the work required if the process is adiabatic.

Solution:

C.V. Methane gas of constant mass $m_2 = m_1 = m$ and reversible process.

$$\text{Energy Eq.3.5: } m(u_2 - u_1) = _1Q_2 - _1W_2$$

$$\text{Entropy Eq.6.3 (37): } m(s_2 - s_1) = \int dQ/T + _1S_2 \text{ gen} = \int dQ/T$$

$$\text{Process: } _1Q_2 = 0 \Rightarrow s_2 = s_1$$

thus isentropic process $s = \text{const}$ and ideal gas gives relation in Eq.6.23

$$T_2 = T_1 (P_2/P_1)^{\frac{k-1}{k}} = 529.7 \text{ R} \left(\frac{120}{15}\right)^{0.230} = 854.6 \text{ R}$$

$$\begin{aligned} _1W_2 &= -mC_{V0}(T_2 - T_1) = -0.1 \text{ lbm} \times 0.415 \text{ Btu/lbm-R} (854.6 - 529.7) \text{ R} \\ &= \mathbf{-13.48 \text{ Btu}} \end{aligned}$$

6.220E

Helium in a piston/cylinder at 70 F, 20 psia is brought to 720 R in a reversible polytropic process with exponent $n = 1.25$. You may assume helium is an ideal gas with constant specific heat. Find the final pressure and both the specific heat transfer and specific work.

Solution:

C.V. Helium, control mass. $C_v = 0.744 \text{ Btu/lbm R}$, $R = 386 \text{ ft lbf/lbm R}$

$$\text{Process} \quad Pv^n = C \quad \& \quad Pv = RT \quad \Rightarrow T v^{n-1} = C$$

$$\text{From the process equation and } T_1 = 70 + 460 = 530 \text{ R}, \quad T_2 = 720 \text{ R}$$

$$T_1 v^{n-1} = T_2 v^{n-1} \quad \Rightarrow \quad v_2 / v_1 = (T_1 / T_2)^{1/n-1} = 0.2936$$

$$P_2 / P_1 = (v_1 / v_2)^n = 4.63 \quad \Rightarrow P_2 = \mathbf{69.4 \text{ lbf/in.}^2}$$

The work is from Eq.6.29 per unit mass

$$\begin{aligned} {}_1w_2 &= \int P \, dv = \int C v^{-n} \, dv = \frac{1}{1-n} (P_2 v_2 - P_1 v_1) = \frac{R}{1-n} (T_2 - T_1) \\ &= \frac{386}{778 \times (-0.25)} \text{ Btu/lbm-R} (720 - 530) \text{ R} = \mathbf{-377 \text{ Btu/lbm}} \end{aligned}$$

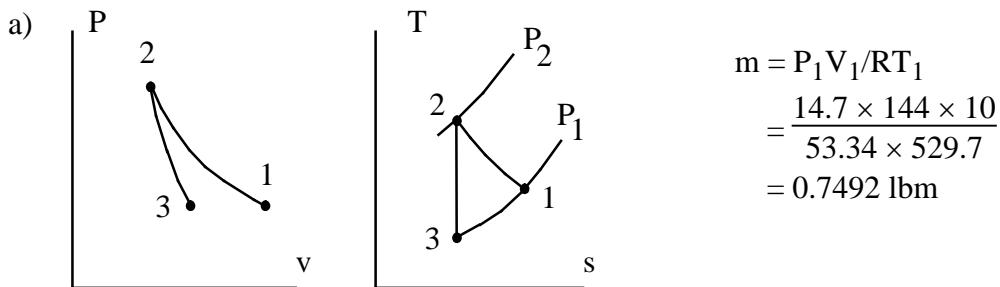
The heat transfer follows from the energy equation

$$\begin{aligned} {}_1q_2 &= u_2 - u_1 + {}_1w_2 = C_v (T_2 - T_1) + {}_1w_2 \\ &= 0.744(720 - 530) + (-377) = \mathbf{-235.6 \text{ Btu/lbm}} \end{aligned}$$

6.221E

A cylinder/piston contains air at ambient conditions, 14.7 lbf/in.² and 70 F with a volume of 10 ft³. The air is compressed to 100 lbf/in.² in a reversible polytropic process with exponent, $n = 1.2$, after which it is expanded back to 14.7 lbf/in.² in a reversible adiabatic process.

- Show the two processes in $P-v$ and $T-s$ diagrams.
- Determine the final temperature and the net work.



- b) The process equation is expressed in Eq.6.28

$$T_2 = T_1 (P_2/P_1)^{\frac{n-1}{n}} = 529.7 \left(\frac{100}{14.7} \right)^{0.167} = 729.6 \text{ R}$$

The work is from Eq.6.29

$$\begin{aligned} w_1 &= \int_1^2 P dv = \frac{P_2 v_2 - P_1 v_1}{1-n} = \frac{R(T_2 - T_1)}{1-n} = \frac{53.34(729.6 - 529.7)}{778(1 - 1.20)} \\ &= -68.5 \text{ Btu/lbm} \end{aligned}$$

Isentropic relation is from Eq.6.23

$$T_3 = T_2 (P_3/P_2)^{\frac{k-1}{k}} = 729.7 \text{ R} \left(\frac{14.7}{100} \right)^{0.286} = 421.6 \text{ R}$$

With zero heat transfer the energy equation gives the work

$$w_2 = C_V \theta (T_2 - T_3) = 0.171(729.6 - 421.6) = +52.7 \text{ Btu/lbm}$$

$$w_{NET} = 0.7492(-68.5 + 52.7) = -11.8 \text{ Btu}$$

Entropy Generation

Excerpts from this work may be reproduced by instructors for distribution on a not-for-profit basis for testing or instructional purposes only to students enrolled in courses for which this textbook has been adopted. Any other reproduction or translation of this work beyond that permitted by Sections 107 or 108 of the 1976 United States Copyright Act without the permission of the copyright owner is unlawful.

6.222E

Consider a heat transfer of 100 Btu from 2400 R hot gases to a steel container at 1200 R which has a heat transfer of the 100 Btu out to some air at 600 R. Determine the entropy generation in each of the control volumes indicated in Fig. P6.115.

There is no change in energy or entropy in the indicated control volumes, so no storage effect. There is a transfer of energy in and out of each CV and an associated transfer of entropy.

Take CV1

Energy Eq.:

$$0 = Q - Q$$

Entropy Eq.:

$$0 = \frac{Q}{T_H} - \frac{Q}{T_M} + S_{gen\ CV1}$$

$$S_{gen\ CV1} = \frac{Q}{T_M} - \frac{Q}{T_H}$$

$$= \frac{100}{1200} - \frac{100}{2400} = \mathbf{0.0417\ Btu/R}$$

Take CV2

Energy Eq.:

$$0 = Q - Q$$

Entropy Eq.:

$$0 = \frac{Q}{T_M} - \frac{Q}{T_L} + S_{gen\ CV2}$$

$$S_{gen\ CV2} = \frac{Q}{T_L} - \frac{Q}{T_M}$$

$$= \frac{100}{600} - \frac{100}{1200} = \mathbf{0.083\ Btu/R}$$

6.223E

A computer chip dissipates 2 Btu of electric work over time and rejects that as heat transfer from its 125 F surface to 70 F air. How much entropy is generated in the chip? How much if any is generated outside the chip?

C.V.1 Chip with surface at 125 F, we assume chip state is constant.

$$\text{Energy: } U_2 - U_1 = 0 = Q_2 - W_2 = W_{\text{electrical in}} - Q_{\text{out 1}}$$

$$\text{Entropy: } S_2 - S_1 = 0 = -\frac{Q_{\text{out 1}}}{T_{\text{surf}}} + S_{2 \text{ gen1}}$$

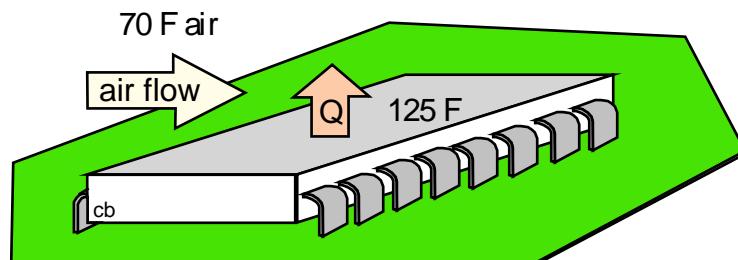
$$S_{2 \text{ gen1}} = \frac{Q_{\text{out 1}}}{T_{\text{surf}}} = \frac{W_{\text{electrical in}}}{T_{\text{surf}}} = \frac{2 \text{ Btu}}{(125 + 459.7) \text{ R}} = 0.0034 \text{ Btu/R}$$

C.V.2 From chip surface at 125 F to air at 70 F, assume constant state.

$$\text{Energy: } U_2 - U_1 = 0 = Q_2 - W_2 = Q_{\text{out 1}} - Q_{\text{out 2}}$$

$$\text{Entropy: } S_2 - S_1 = 0 = \frac{Q_{\text{out 1}}}{T_{\text{surf}}} - \frac{Q_{\text{out 2}}}{T_{\text{air}}} + S_{2 \text{ gen2}}$$

$$S_{2 \text{ gen2}} = \frac{Q_{\text{out 2}}}{T_{\text{air}}} - \frac{Q_{\text{out 1}}}{T_{\text{surf}}} = \frac{2 \text{ Btu}}{529.7 \text{ R}} - \frac{2 \text{ Btu}}{584.7 \text{ R}} = 0.00036 \text{ Btu/R}$$



6.224E

An insulated cylinder/piston contains R-134a at 150 lbf/in.², 120 F, with a volume of 3.5 ft³. The R-134a expands, moving the piston until the pressure in the cylinder has dropped to 15 lbf/in.². It is claimed that the R-134a does 180 Btu of work against the piston during the process. Is that possible?

Solution:

C.V. R-134a in cylinder. Insulated so assume $Q = 0$.

State 1: Table F.10.2, $v_1 = 0.3332 \text{ ft}^3/\text{lbm}$, $u_1 = 175.33 \text{ Btu/lbm}$,

$$s_1 = 0.41586 \text{ Btu/lbm R}, \quad m = V_1/v_1 = 3.5/0.3332 = 10.504 \text{ lbm}$$

Energy Eq.3.5: $m(u_2 - u_1) = _1Q_2 - _1W_2 = 0 - 180 \Rightarrow$

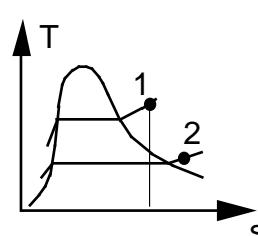
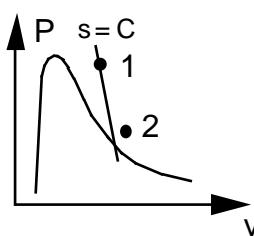
$$u_2 = u_1 - _1W_2/m = 158.194 \text{ Btu/lbm}$$

State 2: $P_2, u_2 \Rightarrow$ Table F.10.2: $T_2 = -2 \text{ F}$; $s_2 = 0.422 \text{ Btu/lbm R}$

Entropy Eq.6.37: $m(s_2 - s_1) = \int dQ/T + _1S_{2,\text{gen}} = _1S_{2,\text{gen}}$

$$_1S_{2,\text{gen}} = m(s_2 - s_1) = 10.504 (0.422 - 0.41586) = \mathbf{0.0645 \text{ Btu/R}}$$

This is **possible since $_1S_{2,\text{gen}} > 0$**



6.225E

Heat transfer from a 70 F kitchen to a block of 3 lbm ice at 15 F melts it to liquid at 50 F. Find the entropy generation.

Water changes state from nearly saturated solid to nearly saturated liquid. The pressure is 1 atm, but we approximate the state properties with saturated state at the same temperature.

CV Ice out to the 20°C kitchen air

$$\text{Energy Eq.3.5: } m(u_2 - u_1) = {}_1Q_2 - {}_1W_2 = {}_1Q_2$$

$$\text{Entropy Eq.6.37: } m(s_2 - s_1) = \int dQ/T + {}_1S_2 \text{gen} = {}_1Q_2/T_0 + {}_1S_2 \text{gen}$$

State 1: Compressed (saturated) solid, F.7.4,

$$u_1 = -151.75 \text{ Btu/lbm}, \quad s_1 = -0.3093 \text{ Btu/lbm-R}$$

State 2: Compressed (saturated) liquid F.7.1

$$u_2 = 38.09 \text{ Btu/lbm}, \quad s_2 = 0.0746 \text{ Btu/lbm-R}$$

Heat transfer from the energy Eq.

$${}_1Q_2 = m(u_2 - u_1) = 3 [38.09 - (-151.75)] = 569.52 \text{ Btu}$$

From the entropy Eq.

$$\begin{aligned} {}_1S_2 \text{gen} &= m(s_2 - s_1) - {}_1Q_2/T_0 \\ &= 3 [0.0746 - (-0.3093)] - 569.52/529.7 = \mathbf{0.0765 \text{ Btu/R}} \end{aligned}$$

6.226E

A mass and atmosphere loaded piston/cylinder contains 4 lbm of water at 500 lbf/in.², 200 F. Heat is added from a reservoir at 1200 F to the water until it reaches 1200 F. Find the work, heat transfer, and total entropy production for the system and surroundings.

Solution:

C.V. Water out to surroundings at 1200 F. This is a control mass.

$$\text{Energy Eq.3.5: } U_2 - U_1 = \dot{Q}_2 - \dot{W}_2$$

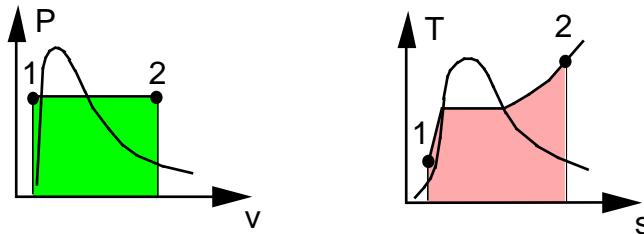
$$\text{Entropy Eq.6.37: } m(s_2 - s_1) = \int dQ/T + \dot{S}_{\text{gen}} = \dot{Q}_2/T_{\text{res}} + \dot{S}_{\text{gen}}$$

$$\text{Process: } P = \text{constant} \quad \text{so} \quad \dot{W}_2 = P(V_2 - V_1) = mP(v_2 - v_1)$$

$$\text{State 1: Table F.7.3, } v_1 = 0.01661 \text{ ft}^3/\text{lbm}$$

$$h_1 = 169.18 \text{ Btu/lbm, } s_1 = 0.2934 \text{ Btu/lbm R}$$

$$\begin{aligned} \text{State 2: Table F.7.2, } v_2 &= 1.9518 \text{ ft}^3/\text{lbm, } h_2 = 1629.8 \text{ Btu/lbm,} \\ s_2 &= 1.8071 \text{ Btu/lbm R} \end{aligned}$$



Work is found from the process (area in P-V diagram)

$$\dot{W}_2 = mP(v_2 - v_1) = 4 \times 500(1.9518 - 0.01661) \frac{144}{778} = \mathbf{716.37 \text{ Btu}}$$

The heat transfer from the energy equation is

$$\dot{Q}_2 = U_2 - U_1 + \dot{W}_2 = m(u_2 - u_1) + mP(v_2 - v_1) = m(h_2 - h_1)$$

$$\dot{Q}_2 = 4(1629.8 - 169.18) = \mathbf{5842.48 \text{ Btu}}$$

Entropy generation from entropy equation (Eq.6.37)

$$\dot{S}_{\text{gen}} = m(s_2 - s_1) - \frac{\dot{Q}_2}{T_{\text{res}}} = 4(1.8071 - 0.2934) - \frac{5842.48}{1659.67} = \mathbf{2.535 \text{ Btu/R}}$$

6.227E

A 1 gallon jug of milk at 75 F is placed in your refrigerator where it is cooled down to the refrigerators inside temperature of 40 F. Assume the milk has the properties of liquid water and find the entropy generated in the cooling process.

Solution:

C.V. Jug of milk. Control mass at constant pressure.

Continuity Eq.: $m_2 = m_1 = m$;

Energy Eq.3.5: $m(u_2 - u_1) = _1Q_2 - _1W_2$

Entropy Eq.6.37: $m(s_2 - s_1) = \int dQ/T + _1S_{2\text{ gen}}$

State 1: Table F.7.1: $v_1 \approx v_f = 0.01606 \text{ ft}^3/\text{lbfm}$, $h_1 = h_f = 43.085 \text{ Btu/lbm}$;
 $s_f = 0.08395 \text{ Btu/lbm R}$

State 2: Table F.7.1: $h_2 = h_f = 8.01 \text{ Btu/lbm}$, $s_2 = s_f = 0.0162 \text{ Btu/lbm R}$

Process: $P = \text{constant} = 14.7 \text{ psia} \Rightarrow _1W_2 = mP(v_2 - v_1)$

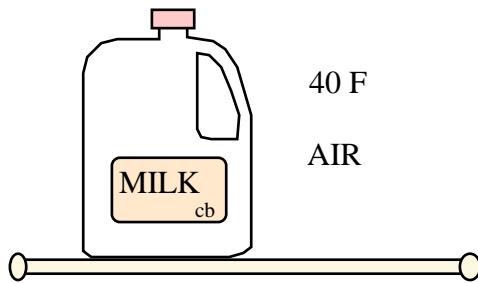
$$V_1 = 1 \text{ Gal} = 231 \text{ in}^3 \Rightarrow m = 231 / 0.01606 \times 12^3 = 8.324 \text{ lbm}$$

Substitute the work into the energy equation and solve for the heat transfer

$$_1Q_2 = m(h_2 - h_1) = 8.324 \text{ lbm} (8.01 - 43.085) \text{ Btu/lbm} = -292 \text{ Btu}$$

The entropy equation gives the generation as

$$\begin{aligned} _1S_{2\text{ gen}} &= m(s_2 - s_1) - _1Q_2/T_{\text{refrig}} \\ &= 8.324 (0.0162 - 0.08395) - (-292 / 500) \\ &= -0.564 + 0.584 = \mathbf{0.02 \text{ Btu/R}} \end{aligned}$$



6.228E

A cylinder/piston contains water at 30 lbf/in.², 400 F with a volume of 1 ft³. The piston is moved slowly, compressing the water to a pressure of 120 lbf/in.². The loading on the piston is such that the product PV is a constant. Assuming that the room temperature is 70 F, show that this process does not violate the second law.

Solution:

C.V.: Water + cylinder out to room at 70 F

$$\text{Energy Eq.3.5: } m(u_2 - u_1) = \dot{Q}_2 - \dot{W}_2$$

$$\text{Entropy Eq.6.37: } m(s_2 - s_1) = \dot{Q}_2 / T_{\text{room}} + \dot{S}_{2 \text{ gen}}$$

$$\text{Process: } PV = \text{constant} = Pmv \Rightarrow v_2 = P_1 v_1 / P_2$$

$$\dot{W}_2 = \int P dv = P_1 v_1 \ln(v_2/v_1)$$

$$\text{State 1: Table B.1.3, } v_1 = 16.891 \text{ ft}^3/\text{lbm}, \quad u_1 = 1144 \text{ Btu/lbm},$$

$$s_1 = 1.7936 \text{ Btu/lbm R}$$

$$\text{State 2: } P_2, v_2 = P_1 v_1 / P_2 = 30 \times 16.891 / 120 = 4.223 \text{ ft}^3/\text{lbm}$$

$$\text{Table F.7.3: } T_2 = 425.4 \text{ F}, \quad u_2 = 1144.4 \text{ Btu/lbm}, \quad s_2 = 1.6445 \text{ Btu/lbmR}$$

$$\dot{W}_2 = 30 \times 16.891 \times \frac{144}{778} \ln \left(\frac{4.223}{16.891} \right) = -130.0 \text{ Btu}$$

$$\dot{Q}_2 = u_2 - u_1 + \dot{W}_2 = 1144.4 - 1144 - 130 = -129.6 \text{ Btu/lbm}$$

$$\dot{S}_{2,\text{gen}} = s_2 - s_1 - \frac{\dot{Q}_2}{T_{\text{room}}} = 1.6445 - 1.7936 + \frac{129.6}{529.67}$$

$$= 0.0956 \text{ Btu/lbm R} > 0 \quad \text{satisfies entropy equation.}$$

6.229E

One pound mass of ammonia (NH_3) is contained in a linear spring-loaded piston/cylinder as saturated liquid at 0 F. Heat is added from a reservoir at 225 F until a final condition of 200 lbf/in.², 160 F is reached. Find the work, heat transfer, and entropy generation, assuming the process is internally reversible.

Solution:

C.V. = NH_3 out to the reservoir.

$$\text{Continuity Eq.: } m_2 = m_1 = m$$

$$\text{Energy Eq.3.5: } E_2 - E_1 = m(u_2 - u_1) = _1Q_2 - _1W_2$$

$$\text{Entropy Eq.6.37: } S_2 - S_1 = \int dQ/T + _1S_{2,\text{gen}} = _1Q_2/T_{\text{res}} + _1S_{2,\text{gen}}$$

Process: $P = A + BV$ linear in V \Rightarrow

$$_1W_2 = \int P dV = \frac{1}{2}(P_1 + P_2)(V_2 - V_1) = \frac{1}{2}(P_1 + P_2)m(v_2 - v_1)$$

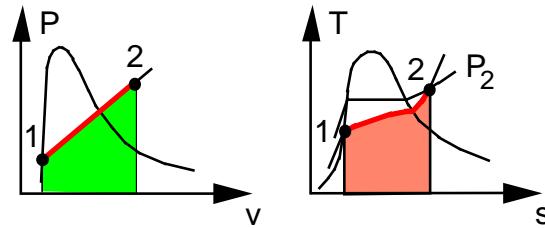
State 1: Table F.8.1

$$P_1 = 30.4 \text{ psia},$$

$$v_1 = 0.0242 \text{ ft}^3/\text{lbfm}$$

$$u_1 = 42.46 \text{ Btu/lbm},$$

$$s_1 = 0.0967 \text{ Btu/lbm R}$$



State 2: Table F.8.2 sup. vap.

$$v_2 = 1.7807 \text{ ft}^3/\text{lbfm}, \quad u_2 = 677.36 \text{ Btu/lbm},$$

$$s_2 = 1.2514 \text{ Btu/lbm R}$$

$$_1W_2 = \frac{1}{2}(30.4 + 200)1(1.7807 - 0.0242) \times 144/778 = \mathbf{37.45 \text{ Btu}}$$

$$_1Q_2 = m(u_2 - u_1) + _1W_2 = 1(677.36 - 42.46) + 37.45 = \mathbf{672.35 \text{ Btu}}$$

$$S_{\text{gen}} = m(s_2 - s_1) - _1Q_2/T_{\text{res}} = 1(1.2514 - 0.0967) - \frac{672.35}{684.7} = \mathbf{0.173 \text{ Btu/R}}$$

6.230E

A hollow steel sphere with a 2-ft inside diameter and a 0.1-in. thick wall contains water at 300 lbf/in.², 500 F. The system (steel plus water) cools to the ambient temperature, 90 F. Calculate the net entropy change of the system and surroundings for this process.

C.V.: Steel + water out to ambient T_0 . This is a control mass.

$$\text{Energy Eq.: } U_2 - U_1 = \dot{Q}_2 - \dot{W}_2 = m_{H_2O}(u_2 - u_1) + m_{\text{steel}}(u_2 - u_1)$$

$$\text{Entropy Eq.: } S_2 - S_1 = \int dQ/T + \dot{S}_{\text{gen}} = \dot{Q}_2/T_0 + \dot{S}_{\text{gen}}$$

$$\text{Process: } V = \text{constant} \Rightarrow \dot{W}_2 = 0$$

$$V_{\text{steel}} = \frac{\pi}{6} [2.0083^3 - 2^3] = 0.0526 \text{ ft}^3$$

$$m_{\text{steel}} = (\rho V)_{\text{steel}} = 490 \times 0.0526 = 25.763 \text{ lbm}$$

$$V_{H_2O} = \pi/6 \times 2^3 = 4.189 \text{ ft}^3, \quad m_{H_2O} = V/v = 2.372 \text{ lbm}$$

$$v_2 = v_1 = 1.7662 = 0.016099 + x_2 \times 467.7 \Rightarrow x_2 = 3.74 \times 10^{-3}$$

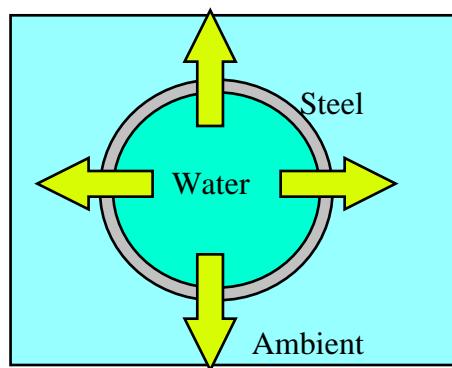
$$u_2 = 61.745 \text{ Btu/lbm}, \quad s_2 = 0.1187 \text{ Btu/lbm R}$$

$$\begin{aligned} \dot{Q}_2 &= \Delta U_{\text{steel}} + \Delta U_{H_2O} = (mC)_{\text{steel}}(T_2 - T_1) + m_{H_2O}(u_2 - u_1) \\ &= 25.763 \times 0.107(90 - 500) + 2.372(61.74 - 1159.5) \\ &= -1130 - 2603.9 = -3734 \text{ Btu} \end{aligned}$$

$$\begin{aligned} S_2 - S_1 &= m_{\text{steel}}(s_2 - s_1) + m_{H_2O}(s_2 - s_1) = 25.763 \times 0.107 \times \ln(550/960) \\ &\quad + 2.372(0.1187 - 1.5701) = -4.979 \text{ Btu/R} \end{aligned}$$

$$\Delta S_{\text{SUR}} = -\dot{Q}_{12}/T_0 = 3734/549.67 = 6.793 \text{ Btu/R}$$

$$\begin{aligned} \dot{S}_{\text{gen}} &= S_2 - S_1 - \dot{Q}_2/T_0 = \Delta S_{\text{SYS}} + \Delta S_{\text{SUR}} \\ &= -4.979 + 6.793 = \mathbf{1.814 \text{ Btu/R}} \end{aligned}$$



6.231E

One lbm of air at 540 R is mixed with two lbm air at 720 R in a process at a constant 15 psia and $Q = 0$. Find the final T and the entropy generation in the process.

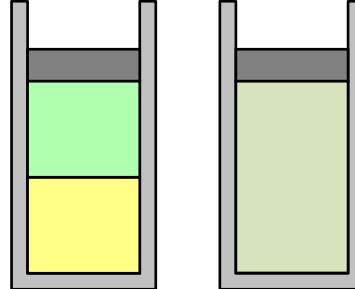
C.V. All the air.

$$\text{Energy Eq.: } U_2 - U_1 = 0 - W$$

$$\text{Entropy Eq.: } S_2 - S_1 = 0 + \Delta S_{\text{gen}}$$

$$\text{Process Eq.: } P = C; \quad W = P(V_2 - V_1)$$

Substitute W into energy Eq.



$$U_2 - U_1 + W = U_2 - U_1 + P(V_2 - V_1) = H_2 - H_1 = 0$$

Due to the low T let us use constant specific heat

$$\begin{aligned} H_2 - H_1 &= m_A(h_2 - h_1)_A + m_B(h_2 - h_1)_B \\ &= m_A C_p(T_2 - T_{A1}) + m_B C_p(T_2 - T_{B1}) = 0 \end{aligned}$$

$$T_2 = \frac{m_A T_{A1} + m_B T_{B1}}{m_A + m_B} = \frac{1}{3} T_{A1} + \frac{2}{3} T_{B1} = 660 \text{ R}$$

Entropy change is from Eq. 6.16 with no change in P and Table F.4 for C_p

$$\begin{aligned} \Delta S_{\text{gen}} &= S_2 - S_1 = m_A C_p \ln \frac{T_2}{T_{A1}} + m_B C_p \ln \frac{T_2}{T_{B1}} \\ &= 1 \times 0.24 \ln \frac{660}{540} + 2 \times 0.24 \ln \frac{660}{720} \\ &= 0.0482 - 0.0418 = 0.0064 \text{ Btu/R} \end{aligned}$$

Remark: If you check the volume does not change and there is no work.

6.232E

One lbm of air at 15 psia is mixed with two lbm air at 30 psia, both at 540 R, in a rigid insulated tank. Find the final state (P, T) and the entropy generation in the process.

C.V. All the air.

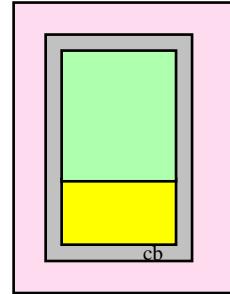
$$\text{Energy Eq.: } U_2 - U_1 = 0 - 0$$

$$\text{Entropy Eq.: } S_2 - S_1 = 0 + \dot{S}_{\text{gen}}$$

$$\text{Process Eqs.: } V = C; \quad W = 0, \quad Q = 0$$

$$\text{States A1, B1: } u_{A1} = u_{B1}$$

$$V_A = m_A RT_1 / P_{A1}; \quad V_B = m_B RT_1 / P_{B1}$$



$$U_2 - U_1 = m_2 u_2 - m_A u_{A1} - m_B u_{B1} = 0 \Rightarrow u_2 = (u_{A1} + 2u_{B1})/3 = u_{A1}$$

$$\text{State 2: } T_2 = T_1 = 540 \text{ R} \text{ (from } u_2); \quad m_2 = m_A + m_B = 3 \text{ kg;}$$

$$V_2 = m_2 RT_1 / P_2 = V_A + V_B = m_A RT_1 / P_{A1} + m_B RT_1 / P_{B1}$$

Divide with $m_A RT_1$ and get

$$3/P_2 = 1/P_{A1} + 2/P_{B1} = \frac{1}{15} + \frac{2}{30} = 0.133 \text{ psia}^{-1} \Rightarrow P_2 = 22.5 \text{ psia}$$

Entropy change from Eq. 6.16 with the same T, so only P changes

$$\begin{aligned} \dot{S}_{\text{gen}} &= S_2 - S_1 = -m_A R \ln \frac{P_2}{P_{A1}} - m_B R \ln \frac{P_2}{P_{B1}} \\ &= -1 \times 53.34 \left[\ln \frac{22.5}{15} + 2 \ln \frac{22.5}{30} \right] \\ &= -53.34 (0.4055 - 0.5754) = 9.06 \text{ lbf-ft/R} = 0.0116 \text{ Btu/R} \end{aligned}$$

6.233E

A rigid container with volume 7 ft³ is divided into two equal volumes by a partition. Both sides contain nitrogen, one side is at 300 lbf/in.², 400 F, and the other at 30 lbf/in.², 200 F. The partition ruptures, and the nitrogen comes to a uniform state at 160 F. Assume the temperature of the surroundings is 68 F, determine the work done and the net entropy change for the process.

Solution:

$$\text{C.V.: A + B Control mass no change in volume } \Rightarrow \mathbf{1W_2 = 0}$$

$$m_{A1} = P_{A1}V_{A1}/RT_{A1} = 300 \times 144 \times 3.5 / (55.15 \times 859.7) = 3.189 \text{ lbm}$$

$$m_{B1} = P_{B1}V_{B1}/RT_{B1} = 30 \times 144 \times 3.5 / (55.15 \times 659.7) = 0.416 \text{ lbm}$$

$$P_2 = m_{\text{TOT}}RT_2/V_{\text{TOT}} = 3.605 \times 55.15 \times 619.7 / (144 \times 7) = 122.2 \text{ lbf/in}^2$$

From Eq.6.16

$$\begin{aligned} S_2 - S_1 &= m_{A1}(s_2 - s_1)_{A1} + m_{B1}(s_2 - s_1)_{B1} \\ &= 3.189 [0.249 \ln \frac{619.7}{859.7} - \frac{55.15}{778} \ln \frac{122.2}{300}] \\ &\quad + 0.416 [0.249 \ln \frac{619.7}{659.7} - \frac{55.15}{778} \ln \frac{122.2}{30}] \\ &= -0.0569 - 0.0479 = -0.1048 \text{ Btu/R} \end{aligned}$$

$$\begin{aligned} {}_1Q_2 &= m_{A1}(u_2 - u_1) + m_{B1}(u_2 - u_1) \\ &= 3.189 \times 0.178(160 - 400) + 0.416 \times 0.178(160 - 200) = -139.2 \text{ Btu} \end{aligned}$$

From Eq.6.37

$$\begin{aligned} {}_1S_{2 \text{ gen}} &= S_2 - S_1 - {}_1Q_2/T_0 = -0.1048 \text{ Btu/R} + 139.2 \text{ Btu} / 527.7 \text{ R} \\ &= \mathbf{+0.159 \text{ Btu/R}} \end{aligned}$$

6.234E

A constant pressure piston/cyl. is 2 lbm of steel and it contains 1 lbm of air 540 R, 60 psia. The system is now heated to 2600 R by a 2800 R source and the steel has the same temperature as the air. Find the entropy generation using constant specific heats.

C.V. Air and Steel.

$$\text{Energy Eq.: } U_2 - U_1 = m_{\text{air}}(u_2 - u_1) + m_{\text{st}}(u_2 - u_1) = _1Q_2 - _1W_2$$

$$\text{Entropy Eq.6.37: } m_{\text{air}}(s_2 - s_1) + m_{\text{st}}(s_2 - s_1) = _1Q_2/T_{\text{source}} + _1S_2 \text{ gen}$$

$$\text{Process: } P = C \Rightarrow _1W_2 = \int_1^2 P dV = P(V_2 - V_1) = P m_{\text{air}}(v_2 - v_1)$$

$$_1Q_2 = m_{\text{air}}(u_2 - u_1)_{\text{air}} + m_{\text{st}}(u_2 - u_1)_{\text{st}} + _1W_2 = m_{\text{air}}(h_2 - h_1)_{\text{air}} + m_{\text{st}}(u_2 - u_1)_{\text{st}}$$

$$\begin{aligned} \text{Use F.2: } (u_2 - u_1)_{\text{st}} &= C(T_2 - T_1) = 0.11 \text{ Btu/lbm-R} \times (2600 - 540) \text{ R} \\ &= 226.6 \text{ Btu/lbm} \end{aligned}$$

$$\begin{aligned} \text{Use F.4: } (h_2 - h_1)_{\text{air}} &= C_p(T_2 - T_1) = 0.24 \text{ Btu/lbm-R} \times (2600 - 540) \text{ R} \\ &= 494.4 \text{ Btu/lbm} \end{aligned}$$

$$\begin{aligned} _1Q_2 &= m_{\text{air}}(h_2 - h_1)_{\text{air}} + m_{\text{st}}(u_2 - u_1)_{\text{st}} \\ &= 1 \text{ lbm} \times 494.4 \text{ Btu/lbm} + 2 \text{ lbm} \times 226.6 \text{ Btu/lbm} = 947.6 \text{ Btu} \end{aligned}$$

$$\begin{aligned} S_2 - S_1 &= m_{\text{air}}(s_2 - s_1) + m_{\text{st}}(s_2 - s_1) \\ &= 1 \text{ lbm} \times 0.24 \text{ Btu/lbm-R} \times \ln \frac{2600}{540} \\ &\quad + 2 \text{ lbm} \times 0.11 \text{ Btu/lbm-R} \times \ln \frac{2600}{540} = 0.723 \text{ Btu/R} \end{aligned}$$

$$_1S_2 \text{ gen} = S_2 - S_1 - _1Q_2/T_{\text{source}} = 0.723 - 947.6/2800 = \mathbf{0.385 \text{ Btu/R}}$$

6.235E

Do Problem 6.234E using Table F.5.

C.V. Air and Steel.

$$\text{Energy Eq.: } U_2 - U_1 = m_{\text{air}}(u_2 - u_1) + m_{\text{st}}(u_2 - u_1) = _1Q_2 - _1W_2$$

$$\text{Entropy Eq.6.37: } m_{\text{air}}(s_2 - s_1) + m_{\text{st}}(s_2 - s_1) = _1Q_2/T_{\text{source}} + _1S_2 \text{ gen}$$

$$\text{Process: } P = C \Rightarrow _1W_2 = \int_1^2 P dV = P(V_2 - V_1) = P m_{\text{air}}(v_2 - v_1)$$

$$_1Q_2 = m_{\text{air}}(u_2 - u_1)_{\text{air}} + m_{\text{st}}(u_2 - u_1)_{\text{st}} + _1W_2 = m_{\text{air}}(h_2 - h_1)_{\text{air}} + m_{\text{st}}(u_2 - u_1)_{\text{st}}$$

$$\text{Use air tables F.5: } (h_2 - h_1)_{\text{air}} = 674.421 - 129.18 = 545.24 \text{ Btu/lbm}$$

$$(s_2 - s_1)_{\text{air}} = 2.04517 - 1.63979 + 0 = 0.40538 \text{ Btu/lbmR}$$

No pressure correction as $P_2 = P_1$

$$\begin{aligned} \text{Use F.2: } (u_2 - u_1)_{\text{st}} &= C(T_2 - T_1) = 0.11 \text{ Btu/lbm-R} \times (2600 - 540) \text{ R} \\ &= 226.6 \text{ Btu/lbm} \end{aligned}$$

$$\begin{aligned} _1Q_2 &= m_{\text{air}}(h_2 - h_1)_{\text{air}} + m_{\text{st}}(u_2 - u_1)_{\text{st}} \\ &= 1 \text{ lbm} \times 545.24 \text{ Btu/lbm} + 2 \text{ lbm} \times 226.6 \text{ Btu/lbm} = \mathbf{998.44 \text{ Btu}} \end{aligned}$$

$$\begin{aligned} S_2 - S_1 &= m_{\text{air}}(s_2 - s_1) + m_{\text{st}}(s_2 - s_1) \\ &= 0.40538 + 2 \text{ lbm} \times 0.11 \text{ kJ/kgK} \times \ln \frac{2600}{540} = 0.75115 \text{ Btu/lbm} \end{aligned}$$

$$_1S_2 \text{ gen} = S_2 - S_1 - _1Q_2/T_{\text{source}} = 0.75115 - 998.44/2800 = \mathbf{0.395 \text{ Btu/R}}$$

6.236E

Nitrogen at 90 lbf/in.², 260 F is in a 20 ft³ insulated tank connected to a pipe with a valve to a second insulated initially empty tank of volume 10 ft³. The valve is opened and the nitrogen fills both tanks. Find the final pressure and temperature and the entropy generation this process causes. Why is the process irreversible?

CV Both tanks + pipe + valve Insulated : $Q = 0$ Rigid: $W = 0$

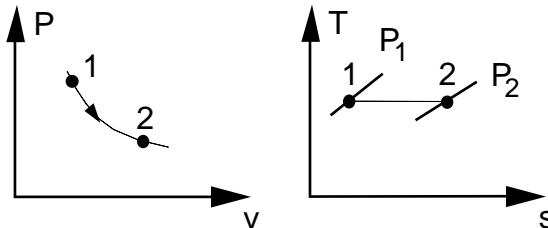
$$\text{Energy Eq.3.5: } m(u_2 - u_1) = 0 - 0 \Rightarrow u_2 = u_1 = u_{a1}$$

$$\text{Entropy Eq.6.37: } m(s_2 - s_1) = \int dQ/T + _1S_2 \text{ gen} = _1S_2 \text{ gen} \quad (dQ = 0)$$

State 1: $P_1, T_1, V_a \Rightarrow$ Ideal gas

$$m = PV/RT = (90 \times 20 \times 144) / (55.15 \times 720) = 6.528 \text{ lbm}$$

$$\text{State 2: } V_2 = V_a + V_b; \text{ uniform final state } v_2 = V_2 / m; \quad u_2 = u_{a1}$$



$$\text{Ideal gas } u(T) \Rightarrow u_2 = u_{a1} \Rightarrow T_2 = T_{a1} = 720 \text{ R}$$

$$P_2 = mR T_2 / V_2 = (V_1 / V_2) P_1 = (2/3) \times 90 = 60 \text{ lbf/in.}^2$$

From entropy equation and Eq.6.19 for entropy change

$$\begin{aligned} S_{\text{gen}} &= m(s_2 - s_1) = m(s_{T_2} - s_{T_1} - R \ln(P_2 / P_1)) \\ &= m(0 - R \ln(P_2 / P_1)) = -6.528 \times 55.15 \times (1/778) \ln 2/3 \\ &= 0.187 \text{ Btu/R} \end{aligned}$$

Irreversible due to unrestrained expansion in valve $P \downarrow$ but no work out.

If not a uniform final state then flow until $P_{2b} = P_{2a}$ and valve is closed.

Assume no Q between A and B

$$m_{a2} + m_{b2} = m_{a1} ; \quad m_{a2} v_{a2} + m_{b2} v_{b2} = m_{a1} v_{a1}$$

$$m_{a2} s_{a2} + m_{b2} s_{b2} - m_{a1} s_{a1} = 0 + 1S_2 \text{gen}$$

Now we must assume m_{a2} went through rev adiabatic expansion

$$1) V_2 = m_{a2} v_{a2} + m_{b2} v_{b2} ; \quad 2) P_{b2} = P_{a2} ;$$

$$3) s_{a2} = s_{a1} ; \quad 4) \text{Energy eqs.}$$

$$4 \text{ Eqs} \quad 4 \text{ unknowns : } P_2, T_{a2}, T_{b2}, x = m_{a2} / m_{a1}$$

$$V_2 / m_{a1} = x v_{a2} + (1 - x) v_{b2} = x \times (R T_{a2} / P_2) + (1 - x) (R T_{b2} / P_2)$$

$$\text{Energy Eq.: } m_{a2} (u_{a2} - u_{a1}) + m_{b2} (u_{b2} - u_{a1}) = 0$$

$$x C_V (T_{a2} - T_{a1}) + (1 - x) C_V (T_{b2} - T_{a1}) = 0$$

$$x T_{a2} + (1 - x) T_{b2} = T_{a1}$$

$$\text{Vol constraint: } P_2 V_2 / m_{a1} R = x T_{a2} + (1 - x) T_{b2} = T_{a1}$$

$$P_2 = m_{a1} R T_{a1} / V_2 = m_{a1} R T_{a1} / (1.5 V_{a1}) = 2/3 P_{a1} = 60 \text{ lbf/in.}^2$$

$$s_{a2} = s_{a1} \Rightarrow T_{a2} = T_{a1} (P_2 / P_{a1})^{k-1/k} = 720 \times (2/3)^{0.2857} = 641.24 \text{ R}$$

Now we have final state in A

$$v_{a2} = R T_{a2} / P_2 = 4.0931 \text{ ft/lbm; } m_{a2} = V_a / v_{a2} = 4.886 \text{ lbm}$$

$$x = m_{a2} / m_{a1} = 0.7485 \quad m_{b2} = m_{a1} - m_{a2} = 1.642 \text{ lbm}$$

Substitute into energy equation

$$T_{b2} = (T_{a1} - x T_{a2}) / (1 - x) = 954.4 \text{ R}$$

$$1S_2 \text{gen} = m_{b2} (s_{b2} - s_{a1}) = m_{b2} [C_p \ln(T_{b2} / T_{a1}) - R \ln(P_2 / P_{a1})]$$

$$= 1.642 [0.249 \ln(954.4/720) - (55.15/778) \ln(2/3)]$$

$$= 0.1624 \text{ Btu/R}$$

Comment: This shows less entropy generation as the mixing part is out compared with the previous solution.

6.237E

A cylinder/piston contains carbon dioxide at 150 lbf/in.², 600 F with a volume of 7 ft³. The total external force acting on the piston is proportional to V³. This system is allowed to cool to room temperature, 70 F. What is the total entropy generation for the process?

C.V. Carbon dioxide gas of constant mass $m_2 = m_1 = m$ out to ambient.

$$\text{Energy Eq.3.5: } m(u_2 - u_1) = _1Q_2 - _1W_2$$

$$\text{Entropy Eq.6.37: } m(s_2 - s_1) = \int dQ/T + _1S_{\text{gen}} = _1Q_2/T_{\text{amb}} + _1S_{\text{gen}}$$

Process: $P = CV^3$ or $PV^{-3} = \text{constant}$, which is polytropic with $n = -3$

State 1: $P_1 = 150 \text{ lbf/in}^2$, $T_1 = 600 \text{ F} = 1060 \text{ R}$, $V_1 = 7 \text{ ft}^3$ Ideal gas

$$m = \frac{P_1 V_1}{RT_1} = \frac{150 \times 144 \times 7}{35.10 \times 1060} = 4.064 \text{ lbm}$$

Process: $P = CV^3$ or $PV^{-3} = \text{const.}$ polytropic with $n = -3$.

$$P_2 = P_1(T_2/T_1)^{\frac{n}{n-1}} = 150 \left(\frac{530}{1060} \right)^{0.75} = 89.2 \text{ lbf/in}^2$$

$$\& V_2 = V_1(T_1/T_2)^{\frac{1}{n-1}} = V_1 \times \frac{P_1}{P_2} \times \frac{T_2}{T_1} = 7 \times \frac{150}{89.2} \times \frac{530}{1060} = 5.886$$

$$_1W_2 = \int PdV = \frac{P_2 V_2 - P_1 V_1}{1-n} = \frac{(89.2 \times 5.886 - 150 \times 7)}{1+3} \times \frac{144}{778} = -24.3 \text{ Btu}$$

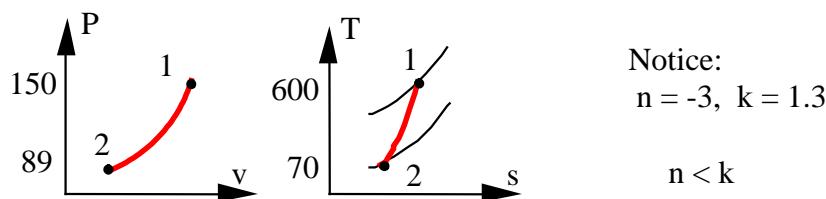
$$_1Q_2 = 4.064 \times 0.158 \times (530 - 1060) - 24.3 = -346.6 \text{ Btu}$$

$$m(s_2 - s_1) = 4.064 \times \left[0.203 \times \ln\left(\frac{530}{1060}\right) - \frac{35.10}{778} \ln\left(\frac{89.2}{150}\right) \right] = -0.4765 \text{ Btu/R}$$

$$\Delta S_{\text{SURR}} = -_1Q_2/T_{\text{amb}} = 364.6 / 530 = +0.6879 \text{ Btu/R}$$

From Eq.6.37 or 6.39

$$\begin{aligned} _1S_{\text{gen}} &= m(s_2 - s_1) - _1Q_2/T_{\text{amb}} = \Delta S_{\text{NET}} = \Delta S_{\text{CO}_2} + \Delta S_{\text{SURR}} \\ &= -0.4765 \text{ Btu/R} + 0.6879 \text{ Btu/R} = +0.2114 \text{ Btu/R} \end{aligned}$$



6.238E

A cylinder/piston contains 4 ft³ of air at 16 lbf/in.², 77 F. The air is compressed in a reversible polytropic process to a final state of 120 lbf/in.², 400 F. Assume the heat transfer is with the ambient at 77 F and determine the polytropic exponent n and the final volume of the air. Find the work done by the air, the heat transfer and the total entropy generation for the process.

Solution:

C.V. Air of constant mass $m_2 = m_1 = m$ out to ambient.

$$\text{Energy Eq.3.5: } m(u_2 - u_1) = _1Q_2 - _1W_2$$

$$\text{Entropy Eq.6.37: } m(s_2 - s_1) = \int dQ/T + _1S_{2\text{ gen}} = _1Q_2/T_0 + _1S_{2\text{ gen}}$$

$$\text{Process: } P_1 v_1^n = P_2 v_2^n \quad \text{Eq.6.27}$$

$$\text{State 1: } (T_1, P_1) \quad \text{State 2: } (T_2, P_2)$$

Thus the unknown is the exponent n .

$$m = (P_1 V_1) / (RT_1) = (16 \times 4 \times 144) / (53.34 \times 537) = 0.322 \text{ lbm}$$

The relation from the process and ideal gas is in Eq.6.28

$$T_2/T_1 = (P_2/P_1)^{\frac{n-1}{n}} \Rightarrow \frac{n-1}{n} = \ln(T_2 / T_1) / \ln(P_2 / P_1) = 0.2337$$

$$n = 1.305, \quad V_2 = V_1 (P_1/P_2)^{1/n} = 4 \times (16/20)^{1/1.305} = 0.854 \text{ ft}^3$$

$$_1W_2 = \int P dV = \frac{P_2 V_2 - P_1 V_1}{1 - n} \\ = [(120 \times 0.854 - 16 \times 4) (144 / 778)] / (1 - 1.305) = -23.35 \text{ Btu / lbm}$$

$$_1Q_2 = m(u_2 - u_1) + _1W_2 = mC_V(T_2 - T_1) + _1W_2 \\ = 0.322 \times 0.171 \times (400 - 77) - 23.35 = -5.56 \text{ Btu / lbm}$$

$$s_2 - s_1 = C_p \ln(T_2 / T_1) - R \ln(P_2 / P_1) \\ = 0.24 \ln(860/537) - (53.34/778) \ln(120/16) = -0.0251 \text{ Btu/lbm R}$$

$$_1S_{2\text{ gen}} = m(s_2 - s_1) - _1Q_2/T_0 \\ = 0.322 \times (-0.0251) + (5.56/537) = 0.00226 \text{ Btu/R}$$

Rates or Fluxes of Entropy

Excerpts from this work may be reproduced by instructors for distribution on a not-for-profit basis for testing or instructional purposes only to students enrolled in courses for which this textbook has been adopted. Any other reproduction or translation of this work beyond that permitted by Sections 107 or 108 of the 1976 United States Copyright Act without the permission of the copyright owner is unlawful.

6.239E

A room at 72 F is heated electrically with 1500 W to keep steady temperature. The outside ambient is at 40 F. Find the flux of S ($= \dot{Q}/T$) into the room air, into the ambient and the rate of entropy generation.

CV. The room and walls out to the ambient T, we assume steady state

$$\text{Energy Eq.: } 0 = \dot{W}_{\text{el in}} - \dot{Q}_{\text{out}} \Rightarrow \dot{Q}_{\text{out}} = \dot{W}_{\text{el in}} = 1500 \text{ W}$$

$$\text{Entropy Eq.: } 0 = -\dot{Q}_{\text{out}}/T + \dot{S}_{\text{gen tot}}$$

$$\text{Flux of S into room air at } 22^{\circ}\text{C: } \dot{Q}/T = 1500 / 531.7 = 2.82 \text{ W/R}$$

$$\text{Flux of S into ambient air at } 5^{\circ}\text{C: } \dot{Q}/T = 1500 / 499.7 = 3.0 \text{ W/R}$$

$$\text{Entropy generation: } \dot{S}_{\text{gen tot}} = \dot{Q}_{\text{out}}/T = 1500 / 499.7 = \mathbf{3.0 \text{ W/R}}$$

Comment: The flux of S into the outside air is what leaves the control volume and since the control volume did not receive any S it was all made in the process. Notice most of the generation is done in the heater, the room heat loss process generates very little S ($3.0 - 2.82 = 0.18 \text{ W/R}$)

6.240E

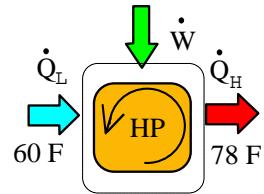
A heat pump with COP = 4 uses 1 kW of power input to heat a 78 F room, drawing energy from the outside at 60 F. Assume the high/low temperatures in the HP are 120 F/32 F. Find the total rates of entropy in and out of the heat pump, the rate from the outside at 60 F and to the room at 78 F.

Solution:

C.V.TOT.

$$\text{Energy Eq.: } \dot{Q}_L + \dot{W} = \dot{Q}_H$$

$$\text{Entropy Eq.: } 0 = \frac{\dot{Q}_L}{T_L} - \frac{\dot{Q}_H}{T_H} + \dot{S}_{\text{gen CV tot}}$$



$$\text{From definition of COP: } \dot{Q}_H = \text{COP } \dot{W} = 4 \times 1 \text{ kW} = 4 \text{ kW}$$

$$\text{From energy equation: } \dot{Q}_L = \dot{Q}_H - \dot{W} = (4 - 1) \text{ kW} = 3 \text{ kW}$$

$$\text{Flux into heat pump at } 32 \text{ F: } \frac{\dot{Q}_L}{T_{L-HP}} = \frac{3}{491.7} \frac{\text{kW}}{\text{R}} = \mathbf{0.0061 \text{ kW/R}}$$

$$\text{Flux out of heat pump at } 120 \text{ F: } \frac{\dot{Q}_H}{T_{H-HP}} = \frac{4}{579.7} \frac{\text{kW}}{\text{R}} = \mathbf{0.0069 \text{ kW/R}}$$

$$\text{Flux out into room at } T_H = 78 \text{ F: } \frac{\dot{Q}_H}{T_H} = \frac{4}{537.7} \frac{\text{kW}}{\text{R}} = \mathbf{0.00744 \text{ kW/R}}$$

$$\text{Flux from outside at } 60 \text{ F: } \frac{\dot{Q}_L}{T_L} = \frac{3}{519.7} \frac{\text{kW}}{\text{R}} = \mathbf{0.00577 \text{ kW/R}}$$

Comment: Following the flow of energy notice how the flux from the outside at 60 F grows a little when it arrives at 32 F this is due to entropy generation in the low T heat exchanger. The flux out of the heat pump at 120 F is larger than the flux in which is due to entropy generation in the heat pump cycle (COP is smaller than Carnot COP) and finally this flux increases due to entropy generated in the high T heat exchanger as the energy arrives at room T.

6.241E

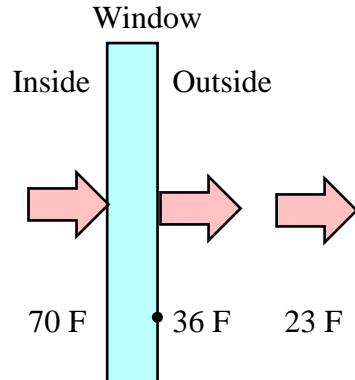
A window receives 800 Btu/h of heat transfer at the inside surface of 70 F and transmits the 800 Btu/h from its outside surface at 36 F continuing to ambient air at 23 F. Find the flux of entropy at all three surfaces and the window's rate of entropy generation.

$$\text{Flux of entropy: } \dot{S} = \frac{\dot{Q}}{T}$$

$$\dot{S}_{\text{inside}} = \frac{800}{529.7} \frac{\text{Btu}}{\text{h-R}} = 1.51 \text{ Btu/h-R}$$

$$\dot{S}_{\text{win}} = \frac{800}{495.7} \frac{\text{Btu}}{\text{h-R}} = 1.614 \text{ Btu/h-R}$$

$$\dot{S}_{\text{amb}} = \frac{800}{482.7} \frac{\text{Btu}}{\text{h-R}} = 1.657 \text{ Btu/h-R}$$



$$\text{Window only: } \dot{S}_{\text{gen win}} = \dot{S}_{\text{win}} - \dot{S}_{\text{inside}} = 1.614 - 1.51 = \mathbf{0.104 \text{ Btu/h-R}}$$

If you want to include the generation in the outside air boundary layer where T changes from 36 F to the ambient 23 F then it is

$$\dot{S}_{\text{gen tot}} = \dot{S}_{\text{amb}} - \dot{S}_{\text{inside}} = 1.657 - 1.51 = 0.147 \text{ Btu/h-R}$$

6.242E

A farmer runs a heat pump using 2.5 hp of power input. It keeps a chicken hatchery at a constant 86 F while the room loses 20 Btu/s to the colder outside ambient at 50 F. What is the rate of entropy generated in the heat pump? What is the rate of entropy generated in the heat loss process?

Solution:

C.V. Hatchery, steady state.

$$\text{Power: } \dot{W} = 2.5 \text{ hp} = 2.5 \frac{2544.4}{3600} = 1.767 \text{ Btu/s}$$

To have steady state at 86 F for the hatchery

$$\text{Energy Eq.: } 0 = \dot{Q}_H - \dot{Q}_{\text{Loss}} \Rightarrow \dot{Q}_H = \dot{Q}_{\text{Loss}} = 20 \text{ Btu/s}$$

C.V. Heat pump, steady state

$$\text{Energy eq.: } 0 = \dot{Q}_L + \dot{W} - \dot{Q}_H \Rightarrow \dot{Q}_L = \dot{Q}_H - \dot{W} = 18.233 \text{ Btu/s}$$

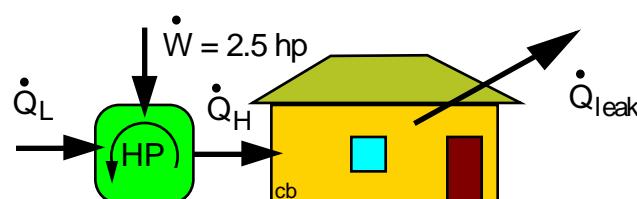
$$\text{Entropy Eq.: } 0 = \frac{\dot{Q}_L}{T_L} - \frac{\dot{Q}_H}{T_H} + \dot{S}_{\text{gen HP}}$$

$$\dot{S}_{\text{gen HP}} = \frac{\dot{Q}_H}{T_H} - \frac{\dot{Q}_L}{T_L} = \frac{20}{545.7} - \frac{18.233}{509.7} = \mathbf{0.000\ 878 \frac{Btu}{s\ R}}$$

C.V. From hatchery at 86 F to the ambient 50 F. This is typically the walls and the outer thin boundary layer of air. Through this goes \dot{Q}_{Loss} .

$$\text{Entropy Eq.: } 0 = \frac{\dot{Q}_{\text{Loss}}}{T_H} - \frac{\dot{Q}_{\text{Loss}}}{T_{\text{amb}}} + \dot{S}_{\text{gen walls}}$$

$$\dot{S}_{\text{gen walls}} = \frac{\dot{Q}_{\text{Loss}}}{T_{\text{amb}}} - \frac{\dot{Q}_{\text{Loss}}}{T_H} = \frac{20}{509.7} - \frac{20}{545.7} = \mathbf{0.00259 \frac{Btu}{s\ R}}$$



Review Problems

Excerpts from this work may be reproduced by instructors for distribution on a not-for-profit basis for testing or instructional purposes only to students enrolled in courses for which this textbook has been adopted. Any other reproduction or translation of this work beyond that permitted by Sections 107 or 108 of the 1976 United States Copyright Act without the permission of the copyright owner is unlawful.

6.243E

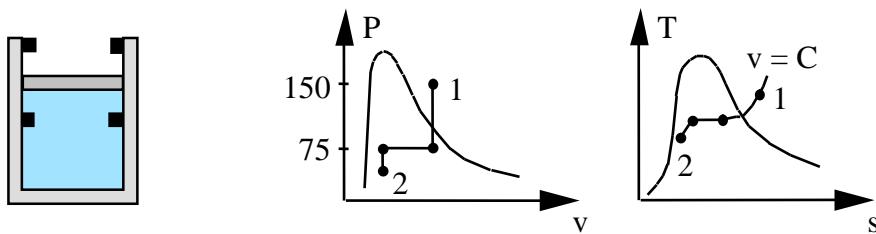
Water in a piston/cylinder is at 150 lbf/in.², 900 F, as shown in Fig. P6.177. There are two stops, a lower one at which $V_{\min} = 35 \text{ ft}^3$ and an upper one at $V_{\max} = 105 \text{ ft}^3$. The piston is loaded with a mass and outside atmosphere such that it floats when the pressure is 75 lbf/in.². This setup is now cooled to 210 F by rejecting heat to the surroundings at 70 F. Find the total entropy generated in the process.

C.V. Water.

$$\text{State 1: Table F.7.2 } v_1 = 5.3529 \text{ ft}^3/\text{lbm}, \quad u_1 = 1330.2 \text{ btu/lbm},$$

$$s_1 = 1.8381 \text{ Btu/lbm}$$

$$m = V/v_1 = 105/5.353 = 19.615 \text{ lbm}$$



State 2: 210 F and on line in P-v diagram.

$$\text{Notice the following: } v_g(P_{\text{float}}) = 5.818 \text{ ft}^3/\text{lbm}, \quad v_{\text{bot}} = V_{\min}/m = 1.7843$$

$$T_{\text{sat}}(P_{\text{float}}) = 307.6 \text{ F}, \quad T_2 < T_{\text{sat}}(P_{\text{float}}) \Rightarrow V_2 = V_{\min}$$

$$\text{State 2: } 210 \text{ F, } v_2 = v_{\text{bot}} \Rightarrow x_2 = (1.7843 - 0.0167)/27.796 = 0.06359$$

$$u_2 = 178.1 + 0.06359 \times 898.9 = 235.26 \text{ btu/lbm},$$

$$s_2 = 0.3091 + 0.06359 \times 1.4507 = 0.4014 \text{ Btu/lbm R}$$

Now we can do the work and then the heat transfer from the energy equation

$$W_2 = \int P dV = P_{\text{float}}(V_2 - V_1) = 75(35 - 105) \frac{144}{778} = -971.72 \text{ Btu}$$

$$Q_2 = m(u_2 - u_1) + W_2 = 19.615(235.26 - 1330.2) - 971.72 = -22449 \text{ Btu}$$

Take C.V. total out to where we have 70 F:

$$m(s_2 - s_1) = Q_2/T_0 + S_{\text{gen}} \Rightarrow$$

$$S_{\text{gen}} = m(s_2 - s_1) - Q_2/T_0 = 19.615(0.4014 - 1.8381) + \frac{22449}{529.67}$$

$$= 14.20 \text{ Btu/R} \quad (= \Delta S_{\text{water}} + \Delta S_{\text{sur}})$$

6.244E

A cylinder/piston contains 5 lbm of water at 80 lbf/in.², 1000 F. The piston has cross-sectional area of 1 ft² and is restrained by a linear spring with spring constant 60 lbf/in. The setup is allowed to cool down to room temperature due to heat transfer to the room at 70 F. Calculate the total (water and surroundings) change in entropy for the process.

State 1: Table F.7.2 $v_1 = 10.831 \text{ ft}^3/\text{lbm}$, $u_1 = 1372.3 \text{ btu/lbm}$,

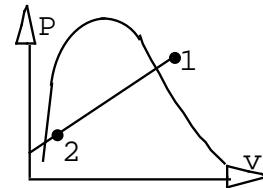
$$s_1 = 1.9453 \text{ Btu/lbm R}$$

State 2: T_2 & on line in P-v diagram.

$$P = P_1 + (k_s/A_{\text{cyl}}^2)(V - V_1)$$

Assume state 2 is two-phase,

$$\Rightarrow P_2 = P_{\text{sat}}(T_2) = 0.3632 \text{ lbf/in}^2$$



$$v_2 = v_1 + (P_2 - P_1)A_{\text{cyl}}^2/mk_s = 10.831 + (0.3632 - 80)1 \times 12/5 \times 60$$

$$= 7.6455 \text{ ft}^3/\text{lbm} = v_f + x_2 v_{fg} = 0.01605 + x_2 867.579$$

$$x_2 = 0.008793, u_2 = 38.1 + 0.008793 \times 995.64 = 46.85 \text{ btu/lbm},$$

$$s_2 = 0.0746 + 0.008793 \times 1.9896 = 0.0921 \text{ Btu/lbm R}$$

$${}_1W_2 = \frac{1}{2}(P_1 + P_2)m(v_2 - v_1)$$

$$= \frac{5}{2}(80 + 0.3632)(7.6455 - 10.831)\frac{144}{778} = -118.46 \text{ Btu}$$

$${}_1Q_2 = m(u_2 - u_1) + {}_1W_2 = 5(46.85 - 1372.3) - 118.46 = -6746 \text{ Btu}$$

$$\Delta S_{\text{tot}} = S_{\text{gen tot}} = m(s_2 - s_1) - {}_1Q_2/T_{\text{room}}$$

$$= 5(0.0921 - 1.9453) + 6746/529.67 = \mathbf{3.47 \text{ Btu/R}}$$

6.245E

A cylinder with a linear spring-loaded piston contains carbon dioxide gas at 300 lbf/in.² with a volume of 2 ft³. The device is of aluminum and has a mass of 8 lbm. Everything (Al and gas) is initially at 400 F. By heat transfer the whole system cools to the ambient temperature of 77 F, at which point the gas pressure is 220 lbf/in.². Find the total entropy generation for the process.

Solution:

$$\text{CO}_2: \quad m = P_1 V_1 / RT_1 = 300 \times 2 \times 144 / (35.10 \times 860) = 2.862 \text{ lbm}$$

$$V_2 = V_1 (P_1 / P_2) (T_2 / T_1) = 2(300/220)(537/860) = 1.703 \text{ ft}^3$$

$$W_{\text{CO}_2} = \int P dV = 0.5(P_1 + P_2)(V_2 - V_1)$$

$$= [(300 + 220)/2] (1.703 - 2) \frac{144}{778} = -14.29 \text{ Btu}$$

$$Q_{\text{CO}_2} = m C_V (T_2 - T_1) + W_{\text{CO}_2} = 0.156 \times 2.862 (77 - 400) - 14.29 = -158.5 \text{ Btu}$$

$$Q_{\text{Al}} = m C (T_2 - T_1) = 8 \times 0.21 (77 - 400) = -542.6 \text{ Btu}$$

System: CO₂ + Al

$$\text{Entropy Eq.: } m_{\text{CO}_2}(s_2 - s_1)_{\text{CO}_2} + m_{\text{AL}}(s_2 - s_1)_{\text{AL}} = Q_2/T_0 + S_{\text{gen tot}}$$

$$Q_2 = -542.6 - 158.5 = -701.14 \text{ Btu}$$

$$m_{\text{CO}_2}(s_2 - s_1)_{\text{CO}_2} + m_{\text{AL}}(s_2 - s_1)_{\text{AL}} =$$

$$2.862[0.201 \ln(537/860) - (35.10/778) \ln(220/300)]$$

$$+ 8 \times 0.21 \ln(537/860) = -0.23086 - 0.79117 = -1.022 \text{ Btu/R}$$

$$S_{\text{gen tot}} = m_{\text{CO}_2}(s_2 - s_1)_{\text{CO}_2} + m_{\text{AL}}(s_2 - s_1)_{\text{AL}} - Q_2/T_0$$

$$= -1.022 + \frac{701.14}{537} = +0.2837 \text{ Btu/R}$$

