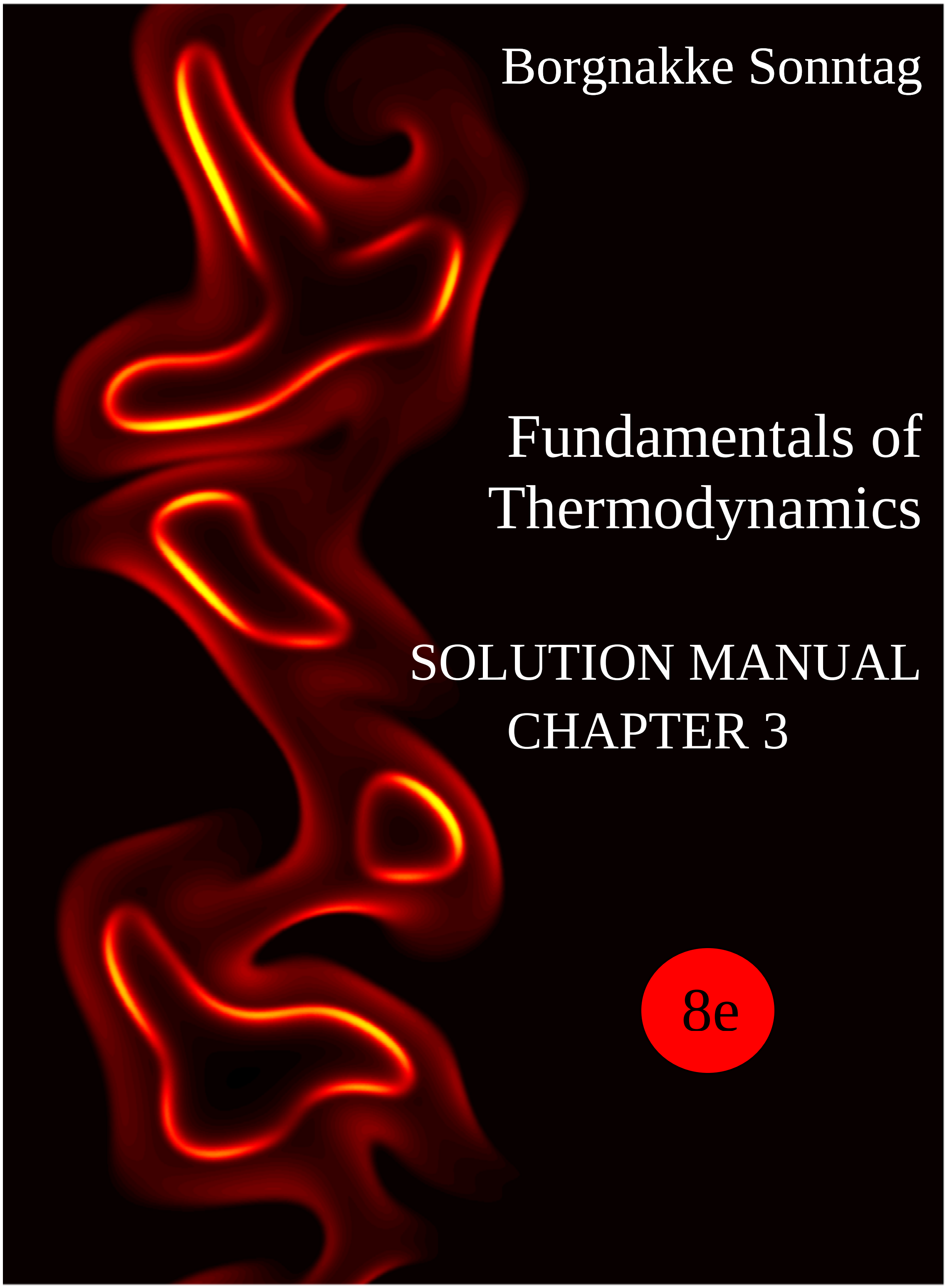


Updated June 2013

CONTENT CHAPTER 3

In-Text Concept Questions

3.a

In a complete cycle what is the net change in energy and in volume?

For a complete cycle the substance has no change in energy and therefore no storage, so the net change in energy is zero.

For a complete cycle the substance returns to its beginning state, so it has no change in specific volume and therefore no change in total volume.

3.b

Explain in words what happens with the energy terms for the stone in Example 3.3. What would happen if it were a bouncing ball falling to a hard surface?

In the beginning all the energy is potential energy associated with the gravitational force. As the stone falls the potential energy is turned into kinetic energy and in the impact the kinetic energy is turned into internal energy of the stone and the water. Finally the higher temperature of the stone and water causes a heat transfer to the ambient until ambient temperature is reached.

With a hard ball instead of the stone the impact would be close to elastic transforming the kinetic energy into potential energy (the material acts as a spring) that is then turned into kinetic energy again as the ball bounces back up. Then the ball rises up transforming the kinetic energy into potential energy (mgZ) until zero velocity is reached and it starts to fall down again. The collision with the floor is not perfectly elastic so the ball does not rise exactly up to the original height losing a little energy into internal energy (higher temperature due to internal friction) with every bounce and finally the motion will die out. All the energy eventually is lost by heat transfer to the ambient or sits in lasting deformation (internal energy) of the substance.

3.c

Make a list of at least 5 systems that store energy, explaining which form of energy.

A spring that is compressed. Potential energy $(1/2)\ kx^2$
A battery that is charged. Electrical potential energy. V Amp h
A raised mass (could be water pumped up higher) Potential energy mgH
A cylinder with compressed air. Potential (internal) energy like a spring.
A tank with hot water. Internal energy mu
A fly-wheel. Kinetic energy (rotation) $(1/2)\ I\omega^2$
A mass in motion. Kinetic energy $(1/2)\ m\mathbf{V}^2$

3.d

A constant mass goes through a process where 100 J of heat transfer comes in and 100 J of work leaves. Does the mass change state?

Yes it does.
As work leaves a control mass its volume must go up, v increases
As heat transfer comes in an amount equal to the work out means u is constant if there are no changes in kinetic or potential energy.

3.e

The electric company charges the customers per kW-hour. What is that in SI units?

Solution:

The unit kW-hour is a rate multiplied with time. For the standard SI units the rate of energy is in W and the time is in seconds. The integration in Eq.3.4 and on page 135 becomes

$$1 \text{ kW- hour} = 1000 \text{ W} \times 60 \frac{\text{min}}{\text{hour}} \text{hour} \times 60 \frac{\text{s}}{\text{min}} = 3\,600\,000 \text{ Ws}$$

$$= 3\,600\,000 \text{ J} = \mathbf{3.6\ MJ}$$

3.f

Torque and energy and work have the same units (N m). Explain the difference.

Solution:

Work = force $\times$ displacement, so units are N $\times$ m. Energy in transfer

Energy is stored, could be from work input 1 J = 1 N m

Torque = force $\times$ arm, static, no displacement needed

3.g

What is roughly the relative magnitude of the work in the process 1-2c versus the process 1-2a shown in figure 3.15?

By visual inspection the area below the curve 1-2c is roughly 50% of the rectangular area below the curve 1-2a. To see this better draw a straight line from state 1 to point f on the axis. This curve has exactly 50% of the area below it.

3.h

Helium gas expands from 125 kPa, 350 K and 0.25 m^3 to 100 kPa in a polytropic process with n = 1.667. Is the work positive, negative or zero?

The boundary work is: $W = \int P\, dV$

P drops but does V go up or down?

The process equation is: $PV^n = C$

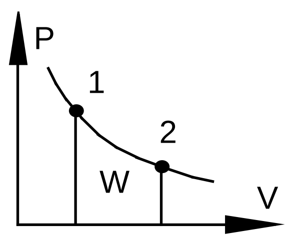
so we can solve for P to show it in a P-V diagram

$$P = CV^{-n}$$

as n = 1.667 the curve drops as V goes up we see

$V_2 > V_1$ giving $dV > 0$

and the work is then positive.

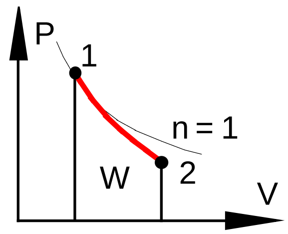


3.i

An ideal gas goes through an expansion process where the volume doubles. Which process will lead to the larger work output: an isothermal process or a polytropic process with $n = 1.25$?

The process equation is: $PV^n = C$

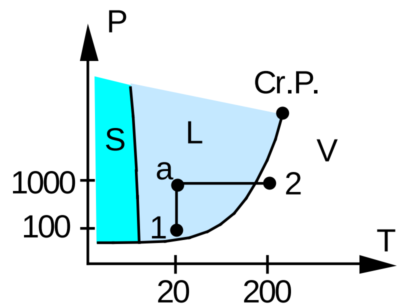
The polytropic process with n = 1.25 drops the pressure faster than the isothermal process with n = 1 and the area below the curve is then smaller.

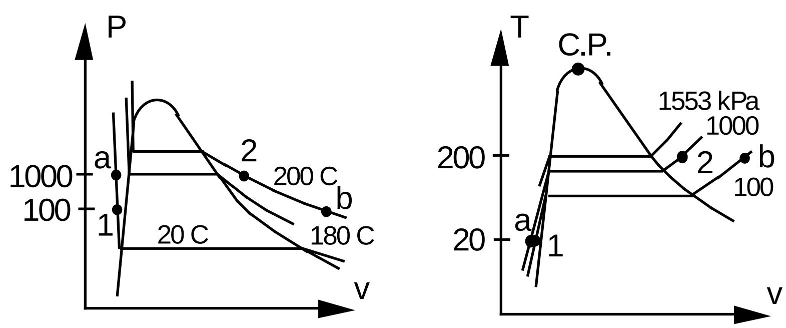


3.j

Water is heated from 100 kPa, 20^{o}C to 1000 kPa, 200^{o}C. In one case pressure is raised at T = C, then T is raised at P = C. In a second case the opposite order is done. Does that make a difference for $_1Q_2$ and $_1W_2$?

Yes it does. Both $_1Q_2$ and $_1W_2$ are process dependent. We can illustrate the work term in a P-v diagram.





In one case the process proceeds from 1 to state “a” along constant T then from “a” to state 2 along constant P.

The other case proceeds from 1 to state “b” along constant P and then from “b” to state 2 along constant T.

3.k

A rigid insulated tank A contains water at 400 kPa, 800^{o}C. A pipe and valve connect this to another rigid insulated tank B of equal volume having saturated water vapor at 100 kPa. The valve is opened and stays open while the water in the two tanks comes to a uniform final state. Which two properties determine the final state?

Continuity eq.: $m_2 - m_{1A} - m_{1B} = 0 \;\Rightarrow\; m_2 = m_{1A} + m_{1B}$

Energy eq.: $m_2u_2 - m_{1A}u_{1A} - m_{1B}u_{1B} = 0 - 0$

Process: Insulated: ${}_1Q_2 = 0$,

Rigid: $V_2 = C = V_A + V_B \;\Rightarrow\; {}_1W_2 = 0$

From continuity eq. and process: $v_2 = V_2/m_2 = \frac{m_{1A}}{m_2} v_{1A} + \frac{m_{1B}}{m_2} v_{1B}$

From energy eq.: $u_2 = \frac{m_{1A}}{m_2} u_{1A} + \frac{m_{1B}}{m_2} u_{1B}$

Final state 2: (v_2, u_2) both are the mass weighted average of the initial values.

3.l

To determine v or u for some liquid or solid, is it more important that I know P or T?

T is more important, v and u are nearly independent of P in the liquid and solid phases.

3.m

To determine v or u for an ideal gas, is it more important that I know P or T?

For v they are equally important ($v = RT/P$), but for u only T is important. For an ideal gas u is a function of T only (independent of P).

3.n

I heat 1 kg of substance at a constant pressure (200 kPa) 1 degree. How much heat is needed if the substance is water at 10^{o}C, steel at 25^{o}C, air at 325 K, or ice at –10^{o}C.

Heating at constant pressure gives (recall the analysis in Section 3.9, page 109)

$$ {}_1Q_2 = H_2 - H_1 = m(h_2 - h_1) \approx m\ C_p\ (T_2 - T_1) $$

For all cases: ${}_1Q_2 = 1\ \text{kg} \times C \times 1\ \text{K}$

Water 10^{o}C, 200 kPa (liquid)	so A.4:	C = 4.18 kJ/kg-K,	${}_1Q_2$ = 4.18 kJ
Steel 25^{o}C, 200 kPa (solid)	so A.3:	C = 0.46 kJ/kg-K	${}_1Q_2$ = 0.46 kJ
Air 325 K, 200 kPa (gas)	so A.5:	C_p = 1.004 kJ/kg-K	${}_1Q_2$ = 1.004 kJ
Ice –10^{o}C, 200 kPa (solid)	so A.3:	C = 2.04 kJ/kg-K	${}_1Q_2$ = 2.04 kJ

Comment: For liquid water we could have interpolated $h_2 - h_1$ from Table B.1.1 and for ice we could have used Table B.1.5. For air we could have used Table A.7. If the temperature is very different from those given the tables will provide a more accurate answer.

Concept Problems

3.1

What is 1 cal in SI units and what is the name given to 1 N-m?

Look in the conversion factor table A.1 under energy:

$$1 \text{ cal (Int.)} = 4.1868 \text{ J} = 4.1868 \text{ Nm} = 4.1868 \text{ kg m}^2/\text{s}^2$$

This was historically defined as the heat transfer needed to bring 1 g of liquid water from 14.5^{o}C to 15.5^{o}C, notice the value of the heat capacity of water in Table A.4

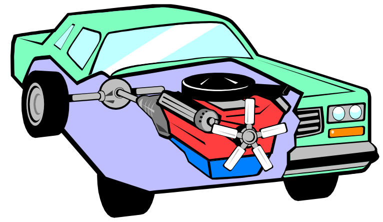
1 N-m = 1 J   or   Force times displacement = energy in Joule

3.2

A car engine is rated at 110 kW. What is the power in hp?

Solution:

The horsepower is an older unit for power usually used for car engines. The conversion to standard SI units is given in Table A.1



1 hp = 0.7355 kW = 735.5 W

1 hp = 0.7457 kW for the UK horsepower

110 kW = 110 kW / 0.7457 (kW/hp) = **147.5 hp**

3.3

Why do we write ΔE or $E_2 - E_1$ whereas we write ${}_1Q_2$ and ${}_1W_2$?

ΔE or $E_2 - E_1$ is the **change** in the stored energy from state 1 to state 2 and depends only on states 1 and 2 not upon the process between 1 and 2.

${}_1Q_2$ and ${}_1W_2$ are amounts of energy **transferred during the process** between 1 and 2 and depend on the process path. The quantities are associated with the process and they are not state properties.

3.4

If a process in a control mass increases energy $E_2 - E_1 > 0$ can you say anything about the sign for ${}_1Q_2$ and ${}_1W_2$?

No.

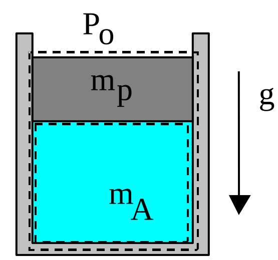
The net balance of the heat transfer and work terms from the energy equation is

$$E_2 - E_1 = {}_1Q_2 - {}_1W_2 > 0$$

but that does not separate the effect of the two terms.

3.5

In Fig. P3.5, CV A is the mass inside a piston-cylinder, CV B is that plus the piston, outside which is the standard atmosphere. Write the energy equation and work term for the two CVs assuming we have a non-zero Q between state 1 and state 2.



CV A: $E_2 - E_1 = m_A(e_2 - e_1) = m_A(u_2 - u_1) = {}_1Q_2 - {}_1WA_2$

$${}_1WA_2 = \int P\, dV = P(V_2 - V_1)$$

CV B: $E_2 - E_1 = m_A(e_2 - e_1) + m_{pist}(e_2 - e_1) = m_A(u_2 - u_1) + m_{pist}g(Z_2 - Z_1)$

$$= {}_1Q_2 - {}_1WB_2$$

$${}_1WB_2 = \int P_o\, dV = P_o(V_2 - V_1)$$

Notice how the P inside CV A is $P = P_o + m_{pist}g / A_{cyl}$ i.e. the first work term is larger than the second. The difference between the work terms is exactly equal to the potential energy of the piston sitting on the left hand side in the CV B energy Eq. The two equations are mathematically identical.

$${}_1WA_2 = P(V_2 - V_1) = [P_o + m_{pist}g / A_{cyl}]\,(V_2 - V_1)$$

$$= {}_1WB_2 + m_{pist}g(V_2 - V_1)/A_{cyl}$$

$$= {}_1WB_2 + m_{pist}g(Z_2 - Z_1)$$

3.6

A 500 W electric space heater with a small fan inside heats air by blowing it over a hot electrical wire. For each control volume: a) wire only b) all the room air and c) total room plus the heater, specify the storage, work and heat transfer terms as + 500W or -500W or 0 W, neglect any $\dot{Q}$ through the room walls or windows.

	Storage	Work	Heat transfer
Wire	0 W	-500 W	-500 W
Room air	500 W	0 W	500 W
Tot room	500 W	-500 W	0 W

3.7

Two engines provide the same amount of work to lift a hoist. One engine can provide 3 F in a cable and the other 1 F, What can you say about the motion of the point where the force F acts in the two engines?

Since the two work terms are the same we get

$$W = \int F\, dx = 3\ F\ x_1 = 1\ F\ x_2$$

$$x_2 = 3\ x_1$$

so the lower force has a larger displacement.

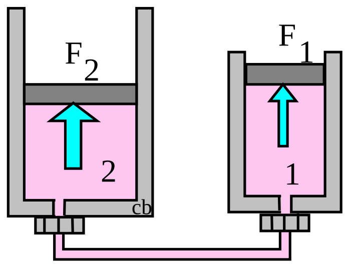
3.8

Two hydraulic piston/cylinders are connected through a hydraulic line so they have roughly the same pressure. If they have diameters of D_1 and $D_2 = 2D_1$ respectively, what can you say about the piston forces F_1 and F_2?

For each cylinder we have the total force as: $F = PA_{cyl} = P\ \pi\ D^2/4$

$$F_1 = PA_{cyl\ 1} = P\ \pi\ D_1^2/4$$

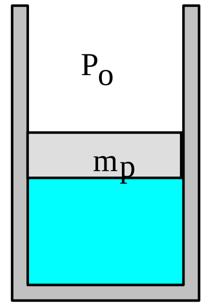
$$F_2 = PA_{cyl\ 2} = P\ \pi\ D_2^2/4 = P\ \pi\ 4\ D_1^2/4 = 4\ F_1$$



The forces are the total force acting up due to the cylinder pressure. There must be other forces on each piston to have a force balance so the pistons do not move.

3.9

Assume a physical set-up as in Fig. P3.5. We now heat the cylinder. What happens to P, T and v (up, down or constant)? What transfers do we have for Q and W (pos., neg., or zero)?



Solution:

Process: $P = P_o + m_p g/A_{cyl} = C$

Heat in so T increases, v increases and Q is positive.

As the volume increases the work is positive. ${}_1W_2 = \int P\, dV$

3.10

A drag force on an object moving through a medium (like a car through air or a submarine through water) is $F_d = 0.225\ A\ \rho\mathbf{V}^2$. Verify the unit becomes Newton.

Solution:

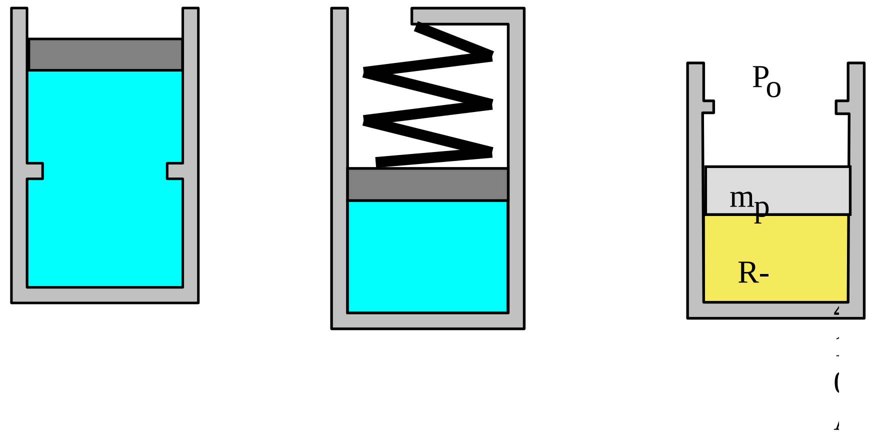
$$F_d = 0.225\ A\ \rho\mathbf{V}^2$$

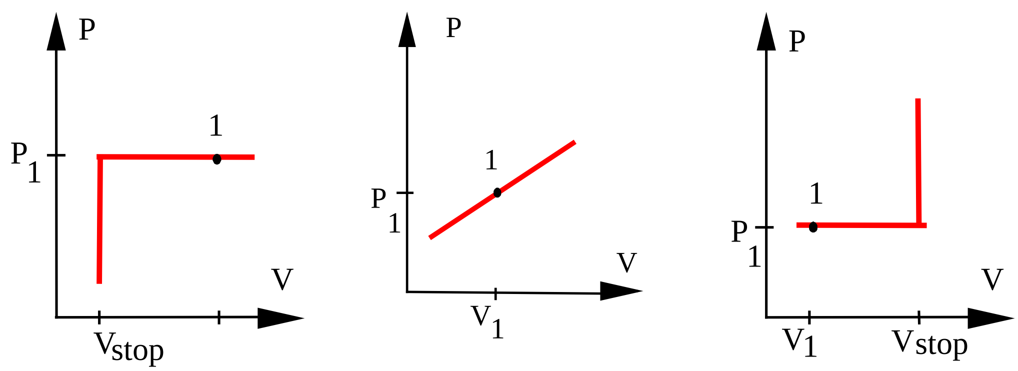
$$\text{Units} = m^2 \times (kg/m^3) \times (m^2/ s^2) = kg\ m / s^2 = N$$

3.11

The sketch shows three physical situations, show the possible process in a P-v diagram.

a) b) c)





3.12

For the indicated physical set-up in a-b and c above write a process equation and the expression for work.

a) $P = P_1$ and $V \geq V_{stop}$ or $V = V_{stop}$ and $P \leq P_1$

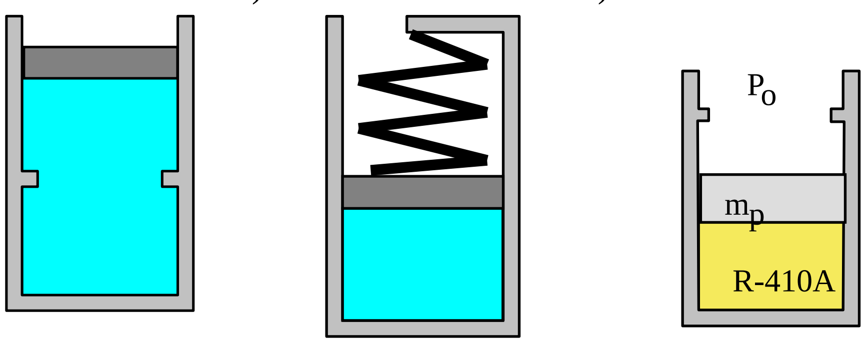
$${}_1W_2 = P_1(V_2 - V_1) \quad [\,P_1 = P_{float}\,]$$

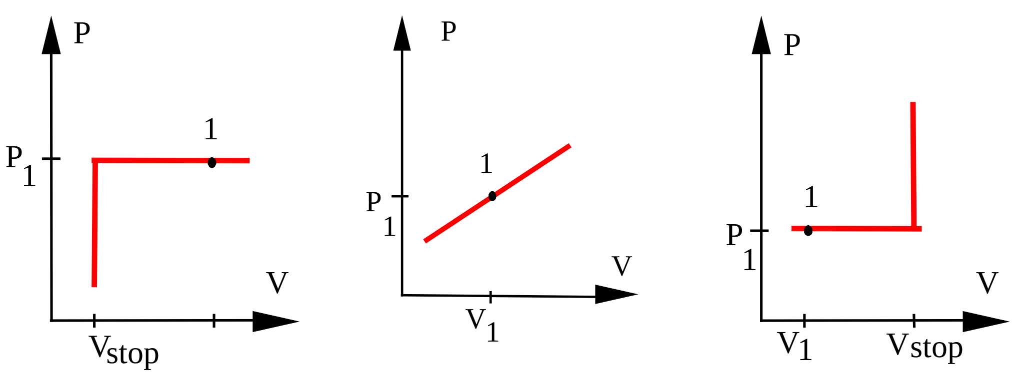
b) $P = A + BV$; ${}_1W_2 = \frac{1}{2}(P_1 + P_2)(V_2 - V_1)$

c) $P = P_1$ and $V \leq V_{stop}$ or $V = V_{stop}$ and $P \geq P_1$

$${}_1W_2 = P_1(V_2 - V_1) \quad [\,P_1 = P_{float}\,]$$

a) b) c)



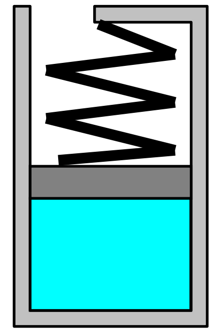


3.13

Assume the physical situation as in Fig. P3.11b; what is the work term a, b, c or d?

a: $_1w_2 = P_1(v_2 - v_1)$ b: $_1w_2 = v_1(P_2 - P_1)$

c: $_1w_2 = \frac{1}{2}(P_1 + P_2)(v_2 - v_1)$ d: $_1w_2 = \frac{1}{2}(P_1 - P_2)(v_2 + v_1)$

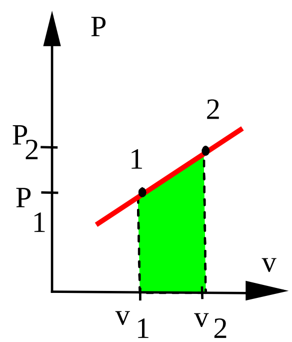


Solution:

work term is formula c, the area under the process curve in a P-v diagram.

The avg. height is $\frac{1}{2}(P_1 + P_2)$

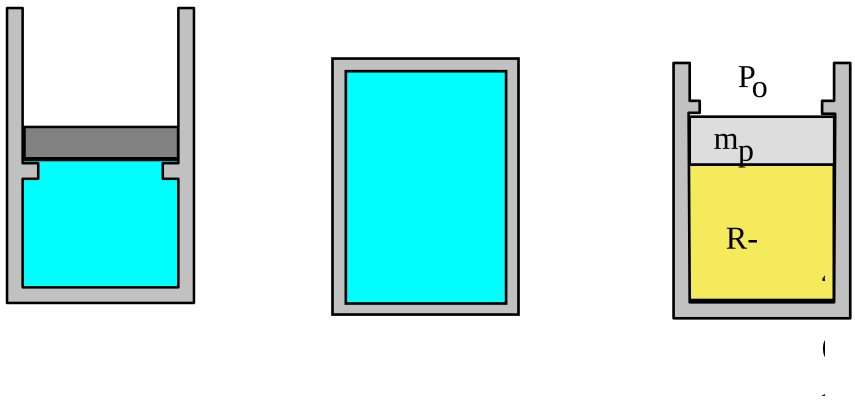
The base is $(v_2 - v_1)$



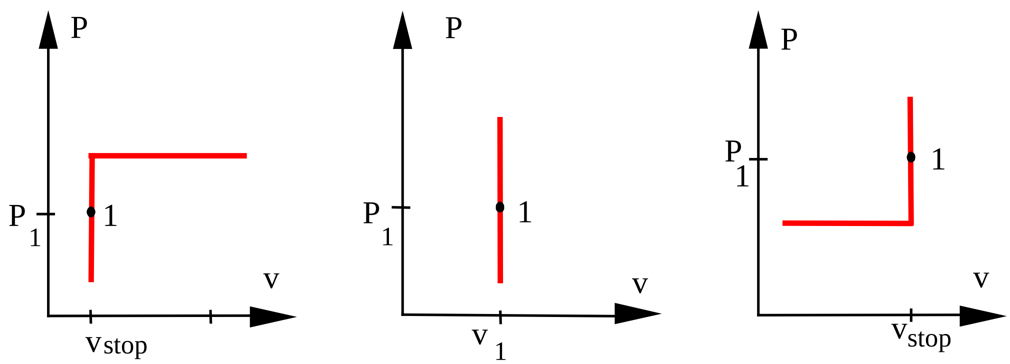
3.14

The sketch in Fig. P3.14 shows a physical situation; show the possible process in a P-v diagram.

a) b) c)



Solution:



3.15

What can you say about the beginning state of the R-410A in Fig. P3.11 versus the case in Fig. P3.14 for the same piston-cylinder?

For the case where the piston floats as in Fig. P3.11 the pressure of the R-410A must equal the equilibrium pressure that floats (balance forces on) the piston.

The situation in Fig. P3.14 is possible if the R-410A pressure equals or exceeds the float pressure.

3.16

A piece of steel has a conductivity of k = 15 W/mK and a brick has k = 1 W/mK. How thick a steel wall will provide the same insulation as a 10 cm thick brick?

The heat transfer due to conduction is from Eq. 3.23

$$\dot{Q} = - kA \frac{dT}{dx} \approx kA \frac{\Delta T}{\Delta x}$$

For the same area and temperature difference the heat transfers become the same for equal values of $(k / \Delta x)$ so

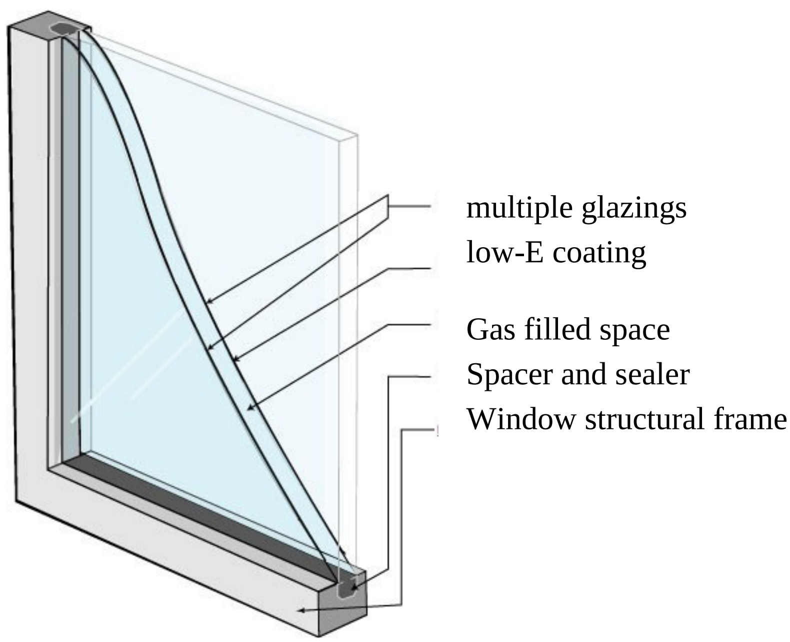
$$\left(\frac{k}{\Delta x}\right)_{brick} = \left(\frac{k}{\Delta x}\right)_{steel} \quad =>$$

$$\Delta x_{steel} = \Delta x_{brick} \frac{k_{steel}}{k_{brick}} = 0.1 \text{ m} \times \frac{15}{1} = \mathbf{1.5\ m}$$

3.17

A thermopane window, see Fig. 3.38, traps some gas between the two glass panes. Why is this beneficial?

The gas has a very low conductivity relative to a liquid or solid so the heat transfer for a given thickness becomes smaller. The gap is furthermore made so small that possible natural convection motion is reduced to a minimum. It becomes a trade off to minimize the overall heat transfer due to conduction and convection. Typically these windows can be manufactured with an E-glaze to reduce radiation loss (winter) or gain (summer).



3.18

On a chilly $10^{o}C$ fall day a house, $20^{o}C$ inside, loses 6 kW by heat transfer. What transfer happens on a $30^{o}C$ warm summer day assuming everything else is the same?

The heat transfer is $\dot{Q} = CA\ \Delta T$ where the details of the heat transfer is in the factor C. Assuming those details are the same then it is the temperature difference that changes the heat transfer so

$$\dot{Q} = CA\ \Delta T = 6\ \text{kW} = CA\ (20 - 10)\ \text{K} \;\Rightarrow\; CA = 0.6\ \frac{\text{kW}}{\text{K}}$$

Then

$$\dot{Q} = CA\ \Delta T = 0.6\ \frac{\text{kW}}{\text{K}} \times (20 - 30)\ \text{K} = -6\ \text{kW} \quad \text{(it goes in)}$$

3.19

Verify that a surface tension $\boldsymbol{S}$ with units N/m also can be called a *surface energy* with units J/m^2. The latter is useful for consideration of a liquid drop or liquid in small pores (capillary).

Units: $\quad$ N/m = Nm/m^2 = J/m^2

This is like a potential energy associated with the surface. For water in small pores it tends to keep the water in the pores rather than in a drop on the surface.

3.20

Some liquid water is heated so it becomes superheated vapor. Do I use u or h in the energy equation? Explain.

The energy equation for a control mass is: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

The storage of energy is a change in u (when we neglect kinetic and potential energy changes) and that is always so. To solve for the heat transfer we must know the work in the process and it is for a certain process (P = C) that the work term combines with the change in u to give a change in h. To avoid confusion you should always write the energy equation as shown above and substitute the appropriate expression for the work term when you know the process equation that allows you to evaluate work.

3.21

Some liquid water is heated so it becomes superheated vapor. Can I use specific heat to find the heat transfer? Explain.

NO.

The specific heat cannot give any information about the energy required to do the phase change. The specific heat is useful for single phase state changes only.

3.22

Look at the R-410A value for u_f at -50^{o}C. Can the energy really be negative? Explain.

The absolute value of *u* and *h* are arbitrary. A constant can be added to all *u* and *h* values and the table is still valid. It is customary to select the reference such that *u* for saturated liquid water at the triple point is zero. The standard for refrigerants like R-410A is that *h* is set to zero as saturated liquid at -40^{o}C, other substances as cryogenic substances like nitrogen, methane etc. may have different states at which *h* is set to zero. The ideal gas tables use a zero point for *h* as 25^{o}C or at absolute zero, 0 K.

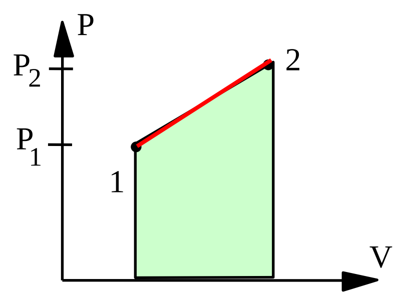
3.23

A rigid tank with pressurized air is used to a) increase the volume of a linear spring loaded piston cylinder (cylindrical geometry) arrangement and b) to blow up a spherical balloon. Assume that in both cases P = A + BV with the same A and B. What is the expression for the work term in each situation?

The expression is exactly the same; the geometry does not matter as long as we have the same relation between P and V then

$$
\begin{aligned}
{}_1W_2 &= \int P\,dV = \int (A + BV)\,dV \\
&= A(V_2 - V_1) + 0.5\,B\,(V_2^2 - V_1^2) \\
&= A(V_2 - V_1) + 0.5\,B\,(V_2 + V_1)\,(V_2 - V_1) \\
&= 0.5\,[\,A + B\,V_2 + A + B\,V_1]\,(V_2 - V_1) \\
&= 0.5\,(P_1 + P_2)\,(V_2 - V_1)
\end{aligned}
$$

Notice the last expression directly gives the area below the curve in the P-V diagram.



3.24

An ideal gas in a piston-cylinder is heated with 2 kJ during an isothermal process. How much work is involved?

Energy Eq.: $u_2 - u_1 = {}_1q_2 - {}_1w_2 = 0$ since $u_2 = u_1$ (isothermal)

Then

$${}_1W_2 = m\ {}_1w_2 = {}_1Q_2 = m\ {}_1q_2 = \mathbf{2\ kJ}$$

3.25

An ideal gas in a piston-cylinder is heated with 2 kJ during an isobaric process. Is the work pos., neg., or zero?

As the gas is heated u and T increase and since PV = mRT it follows that the volume increase and thus work goes out.

$$\mathbf{w > 0}$$

3.26

You heat a gas 10 K at P = C. Which one in Table A.5 requires most energy? Why?

A constant pressure process in a control mass gives (recall Section 3.9 and Eq.3.44)

$$ {}_1q_2 = u_2 - u_1 + {}_1w_2 = u_2 - u_1 + P_1(v_2 - v_1) $$
$$ = h_2 - h_1 \approx C_p\, \Delta T $$

The one with the highest specific heat is hydrogen, H_2. The hydrogen has the smallest mass, but the same kinetic energy per mol as other molecules and thus the most energy per unit mass is needed to increase the temperature.

3.27

You mix 20^{o}C water with 50^{o}C water in an open container. What do you need to know to determine the final temperature?

The process will take place at constant pressure (atmospheric) and you can assume there will be minimal heat transfer if the process is reasonably fast. The energy equation then becomes

$$U_2 – U_1 = 0 – {}_1W_2 = – P(V_2 – V_1)$$

Which we can write as

$$H_2 – H_1 = 0 = m_2h_2 – (m_{1\ 20C}h_{1\ 20C} + m_{1\ 50C}h_{1\ 50C})$$

You need the amount of mass at each temperature $m_{1\ 20C}$ and $m_{1\ 50C}$.

Kinetic and Potential Energy

3.28

A piston motion moves a 25 kg hammerhead vertically down 1 m from rest to a velocity of 50 m/s in a stamping machine. What is the change in total energy of the hammerhead?

Solution: C.V. Hammerhead

The hammerhead does not change internal energy (i.e. same P, T), but it does have a change in kinetic and potential energy.

$$E_2 – E_1 = m(u_2 – u_1) + m[(1/2)\mathbf{V}_2^{\,2} – 0] + mg\,(Z_2 - 0)$$

$$= 0 + 25\text{ kg} \times (1/2) \times (50\text{ m/s})^2 + 25\text{ kg} \times 9.80665\text{ m/s}^2 \times (-1)\text{ m}$$

$$= 31\,250\text{ J} – 245.17\text{ J} = 31\,005\text{ J} = \mathbf{31\ kJ}$$

3.29

A 1200 kg car is accelerated from 30 to 50 km/h in 5 s. How much work is that? If you continue from 50 to 70 km/h in 5 s; is that the same?

The work input is the increase in kinetic energy.

$$E_2 - E_1 = (1/2)m\left[\mathbf{V}_2^2 - \mathbf{V}_1^2\right] = {}_1W_2$$

$$= 0.5 \times 1200 \text{ kg } [50^2 - 30^2] \left(\frac{\text{km}}{\text{h}}\right)^2$$

$$= 600 \text{ kg} \times [\ 2500 - 900\ ] \left(\frac{1000 \text{ m}}{3600 \text{ s}}\right)^2 = 74\ 074 \text{ J} = \mathbf{74.1\ kJ}$$

The second set of conditions does not become the same

$$E_2 - E_1 = (1/2)m\left[\mathbf{V}_2^2 - \mathbf{V}_1^2\right] = 600 \text{ kg} \times [\ 70^2 - 50^2\ ] \left(\frac{1000 \text{ m}}{3600 \text{ s}}\right)^2 = \mathbf{111\ kJ}$$

3.30

The rolling resistance of a car depends on its weight as: F = 0.006 mg. How far will a car of 1200 kg roll if the gear is put in neutral when it drives at 90 km/h on a level road without air resistance?

Solution:

The car decreases its kinetic energy to zero due to the force (constant) acting over the distance.

$$m\,(1/2\mathbf{V}_2^2 - 1/2\mathbf{V}_1^2) = -{}_1W_2 = -\int F\,dx = -FL$$

$$\mathbf{V}_2 = 0, \quad \mathbf{V}_1 = 90\,\frac{\text{km}}{\text{h}} = \frac{90 \times 1000}{3600}\ \text{ms}^{-1} = 25\ \text{ms}^{-1}$$

$$-1/2\ m\mathbf{V}_1^2 = -FL = -\ 0.006\ mgL$$

$$\rightarrow \qquad L = \frac{0.5\ \mathbf{V}_1^2}{0.006\ g} = \frac{0.5 \times 25^2}{0.006 \times 9.807}\ \frac{\text{m}^2/\text{s}^2}{\text{m/s}^2} = 5311\ \text{m}$$

Remark: Over 5 km! The air resistance is much higher than the rolling resistance so this is not a realistic number by itself.

3.31

A piston of mass 2 kg is lowered 0.5 m in the standard gravitational field. Find the required force and work involved in the process.

Solution:

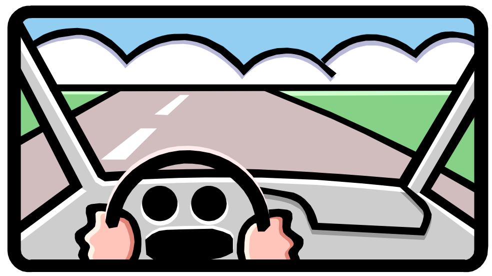
$$F = ma = 2 \text{ kg} \times 9.80665 \text{ m/s}^2 = \mathbf{19.61\ N}$$

$$W = \int F\, dx = F \int dx = F\, \Delta x = 19.61 \text{ N} \times 0.5 \text{ m} = \mathbf{9.805\ J}$$

3.32

A 1200 kg car accelerates from zero to 100 km/h over a distance of 400 m. The road at the end of the 400 m is at 10 m higher elevation. What is the total increase in the car kinetic and potential energy?

Solution:



$$\Delta KE = ½ \text{ m } (\mathbf{V}_2^2 - \mathbf{V}_1^2)$$

$$\mathbf{V}_2 = 100 \text{ km/h} = \frac{100 \times 1000}{3600} \text{ m/s}$$

$$= 27.78 \text{ m/s}$$

$$\Delta KE = ½ \times 1200 \text{ kg} \times (27.78^2 - 0^2) \text{ (m/s)}^2 = 463\ 037 \text{ J} = \mathbf{463\ kJ}$$

$$\Delta PE = mg(Z_2 - Z_1) = 1200 \text{ kg} \times 9.807 \text{ m/s}^2 \text{ (10 - 0) m} = 117684 \text{ J}$$

$$\mathbf{= 117.7\ kJ}$$

3.33

A hydraulic hoist raises a 1750 kg car 1.8 m in an auto repair shop. The hydraulic pump has a constant pressure of 800 kPa on its piston. What is the increase in potential energy of the car and how much volume should the pump displace to deliver that amount of work?

Solution: C.V. Car.

No change in kinetic or internal energy of the car, neglect hoist mass.

$$E_2 - E_1 = PE_2 - PE_1 = mg\,(Z_2 - Z_1)$$

$$= 1750 \text{ kg} \times 9.80665 \text{ m/s}^2 \times 1.8 \text{ m} = \mathbf{30\,891 \ \ J}$$

The increase in potential energy is work into car from pump at constant P.

$$W = E_2 - E_1 = \int P\,dV = P\,\Delta V \qquad \Rightarrow$$

$$\Delta V = \frac{E_2 - E_1}{P} = \frac{30891 \text{ J}}{800 \times 1000 \text{ Pa}} = \mathbf{0.0386\ m^3}$$

3.34

Airplane takeoff from an aircraft carrier is assisted by a steam driven piston/cylinder device with an average pressure of 1250 kPa. A 17500 kg airplane should be accelerated from zero to a speed of 30 m/s with 30% of the energy coming from the steam piston. Find the needed piston displacement volume.

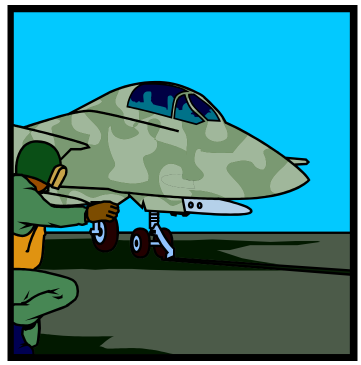
Solution: C.V. Airplane.

No change in internal or potential energy; only kinetic energy is changed.

$$E_2 - E_1 = m\ (1/2)\ (\mathbf{V}_2^2 - 0) = 17500 \text{ kg} \times (1/2) \times 30^2 \text{ (m/s)}^2$$

$$= 7\ 875\ 000 \text{ J} = 7875 \text{ kJ}$$

The work supplied by the piston is 30% of the energy increase.



$$W = \int P\ dV = P_{avg}\ \Delta V = 0.30\ (E_2 - E_1)$$

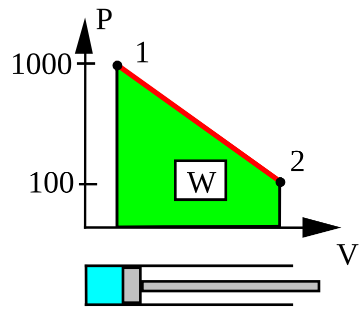
$$= 0.30 \times 7875 \text{ kJ} = 2362.5 \text{ kJ}$$

$$\Delta V = \frac{W}{P_{avg}} = \frac{2362.5}{1250}\ \frac{\text{kJ}}{\text{kPa}} = \mathbf{1.89\ m^3}$$

3.35

Solve Problem 3.34, but assume the steam pressure in the cylinder starts at 1000 kPa, dropping linearly with volume to reach 100 kPa at the end of the process.

Solution: C.V. Airplane.



$E_2 - E_1 = m\ (1/2)\ (\mathbf{V}_2^2\ - 0)$

$= 17\ 500\ \text{kg} \times (1/2) \times 30^2\ (\text{m/s})^2$

$= 7875\ 000\ \text{J} = 7875\ \text{kJ}$

$W = 0.30(E_2 - E_1) = 0.30 \times 7875 = 2362.5\ \text{kJ}$

$W = \int P\ dV = (1/2)(P_{beg} + P_{end})\ \Delta V$

$$\Delta V = \frac{W}{P_{avg}} = \frac{2362.5\ \text{kJ}}{1/2(1000 + 100)\ \text{kPa}} = \mathbf{4.29\ m^3}$$

3.36

A steel ball weighing 5 kg rolls horizontal with 10 m/s. If it rolls up an incline how high up will it be when it comes to rest assuming standard gravitation.

C.V. Steel ball.

Energy Eq.: $E_2 - E_1 = {}_1Q_2 - {}_1W_2 = 0 - 0 = 0$

$$E_1 = mu_1 + mgZ_1 + 0.5\ mV^2$$

$$E_2 = mu_2 + mgZ_2 + 0$$

We assume the steel ball does not change temperature ($u_2 = u_1$) so then the energy equation gives

$$mu_2 + mgZ_2 - mu_1 - mgZ_1 - 0.5\ mV^2 = 0$$

$$mg\ (Z_2 - Z_1) = 0.5\ mV^2$$

$$Z_2 - Z_1 = 0.5\ V^2/g = 0.5 \times 10^2\ (m^2/s^2) / (9.81\ m/s^2) = \mathbf{5.1\ m}$$

3.37

A hydraulic cylinder of area 0.01 m^2 must push a 1000 kg arm and shovel 0.5 m straight up. What pressure is needed and how much work is done?

$F = mg = 1000 \text{ kg} \times 9.81 \text{ m/s}^2$
$\quad = 9810 \text{ N} = PA$

$P = F/A = 9810 \text{ N}/ 0.01 \text{ m}^2$
$\quad = 981\,000 \text{ Pa} = \mathbf{981\ kPa}$

$W = \int F\, dx = F\, \Delta x = 9810 \text{ N} \times 0.5 \text{ m} = \mathbf{4905\ J}$

3.38

A hydraulic cylinder has a piston of cross sectional area 10 cm^2 and a fluid pressure of 2 MPa. If the piston is moved 0.25 m how much work is done?

Solution:

The work is a force with a displacement and force is constant: F = PA

$$W = \int F\,dx = \int PA\,dx = PA\,\Delta x$$

$$= 2000 \text{ kPa} \times 10 \times 10^{-4} \text{ m}^2 \times 0.25 \text{ m} = \mathbf{0.5\ kJ}$$

Units: $\text{kPa m}^2 \text{ m} = \text{kN m}^{-2} \text{ m}^2 \text{ m} = \text{kN m} = \text{kJ}$

3.39

Two hydraulic piston/cylinders are connected with a line. The master cylinder has an area of 5 cm^2 creating a pressure of 1000 kPa. The slave cylinder has an area of 3 cm^2. If 25 J is the work input to the master cylinder what is the force and displacement of each piston and the work output of the slave cylinder piston?

Solution:

$W = \int F_x\, dx = \int P\, dv = \int P\, A\, dx = P\, A\, \Delta x$

$$\Delta x_{master} = \frac{W}{PA} = \frac{25}{1000 \times 5 \times 10^{-4}}\ \frac{J}{kPa\ m^2} = 0.05\ m$$

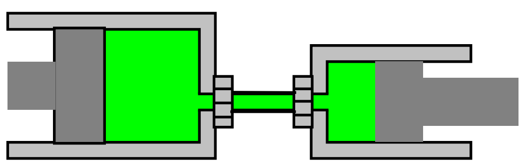
$A\Delta x = \Delta V = 5 \times 10^{-4} \times 0.05 = 2.5 \times 10^{-5}\ m^3 = \Delta V_{slave} = A\ \Delta x \rightarrow$

$\Delta x_{slave} = \Delta V/A = 2.5 \times 10^{-5}\ m^3/\ 3 \times 10^{-4}\ m^2 = 0.0083\ 33\ m$

$F_{master} = P\ A = 1000\ kPa \times 5 \times 10^{-4}\ m^2 \times 10^3\ Pa/kPa =$ **500 N**

$F_{slave} = P\ A = 1000\ kPa \times 10^3\ Pa/kPa \times 3 \times 10^{-4}\ m^2 =$ **300 N**

$W_{slave} = F\ \Delta x = 300\ N \times 0.08333\ m =$ **25 J**



Master Slave

3.40

The air drag force on a car is 0.225 A $\rho\mathbf{V}^2$. Assume air at 290 K, 100 kPa and a car frontal area of 4 m^2 driving at 90 km/h. How much energy is used to overcome the air drag driving for 30 minutes?

The formula involves density and velocity and work involves distance so:

$$\rho = \frac{1}{v} = \frac{P}{RT} = \frac{100}{0.287 \times 290} = 1.2015 \frac{kg}{m^3}$$

$$\mathbf{V} = 90 \frac{km}{h} = 90 \times \frac{1000}{3600} \frac{m}{s} = 25 \text{ m/s}$$

$$\Delta x = \mathbf{V}\, \Delta t = 25 \text{ m/s} \times 30 \text{ min} \times 60 \text{ s/min} = 45\,000 \text{ m}$$

Now

$$F = 0.225 \text{ A } \rho\, \mathbf{V}^2 = 0.225 \times 4 \text{ m}^2 \times 1.2015 \frac{kg}{m^3} \times 25^2 \frac{m^2}{s^2}$$

$$= 675.8 \text{ m}^2 \frac{kg}{m^3} \times \frac{m^2}{s^2} = \mathbf{676\ N}$$

$$W = F\, \Delta x = 676 \text{ N} \times 45\,000 \text{ m} = 30\,420\,000 \text{ J} = \mathbf{30.42\ MJ}$$

3.41

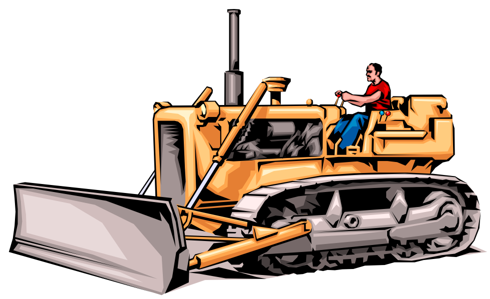
A bulldozer pushes 800 kg of dirt 100 m with a force of 1500 N. It then lifts the dirt 3 m up to put it in a dump truck. How much work did it do in each situation?

Solution:

$W = \int F\, dx = F\, \Delta x$

$= 1500 \text{ N} \times 100 \text{ m}$

$= 150\,000 \text{ J} = \mathbf{150\ kJ}$



$W = \int F\, dz = \int mg\, dz = mg\, \Delta Z$

$= 800 \text{ kg} \times 9.807 \text{ m/s}^2 \times 3 \text{ m}$

$= 23\,537 \text{ J} = \mathbf{23.5\ kJ}$

3.42

Two hydraulic cylinders maintain a pressure of 1200 kPa. One has a cross sectional area of 0.01 m^2 the other 0.03 m^2. To deliver a work of 1 kJ to the piston how large a displacement (V) and piston motion H is needed for each cylinder? Neglect P_{atm}.

Solution:

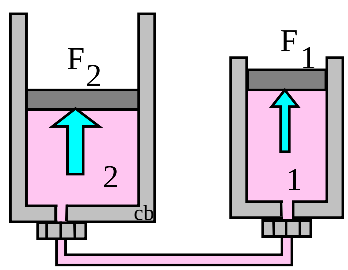
$$W = \int F\,dx = \int P\,dV = \int PA\,dx = PA^* H = P\Delta V$$

$$\Delta V = \frac{W}{P} = \frac{1 \text{ kJ}}{1200 \text{ kPa}} = \mathbf{0.000\ 833\ m^3}$$

Both cases the height is $H = \Delta V/A$

$$H_1 = \frac{0.000833}{0.01} \text{ m} = \mathbf{0.0833\ m}$$

$$H_2 = \frac{0.000833}{0.03} \text{ m} = \mathbf{0.0278\ m}$$



3.43

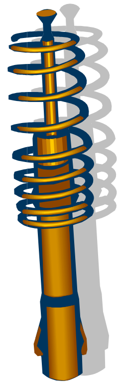
A linear spring, $F = k_s(x - x_0)$, with spring constant $k_s = 500$ N/m, is stretched until it is 100 mm longer. Find the required force and work input.

Solution:

$$F = k_s(x - x_0) = 500 \times 0.1 = \mathbf{50\ N}$$

$$W = \int F\, dx = \int k_s(x - x_0)d(x - x_0) = k_s(x - x_0)^2/2$$

$$= 500 \frac{\text{N}}{\text{m}} \times (0.1^2/2)\ \text{m}^2 = \mathbf{2.5\ J}$$



3.44

A piston of 2 kg is accelerated to 20 m/s from rest. What constant gas pressure is required if the area is 10 cm^2, the travel 10 cm and the outside pressure is 100 kPa?

C.V. Piston

$$(E_2 – E_1)_{PIST.} = m(u_2 – u_1) + m[(1/2)V_2^2 – 0] + mg (0 – 0)$$

$$= (1/2)\ m\ V_2^2 = 0.5 \times 2\ kg \times 20^2\ (m/s)^2 = 400\ J$$

Energy equation for the piston is:

$$(E_2 – E_1)_{PIST.} = W_{gas} - W_{atm} = P_{avg}\ \Delta V_{gas} – P_o\ \Delta V_{gas}$$

$$\Delta V_{gas} = A\ L = 10\ cm^2 \times 10\ cm = 0.0001\ m^3$$

$$P_{avg}\ \Delta V_{gas} = (E_2 – E_1)_{PIST.} + P_o\ \Delta V_{gas}$$

$$P_{avg} = (E_2 – E_1)_{PIST.} / \Delta V_{gas} + P_o$$

$$= 400\ J / 0.0001\ m^3 + 100\ kPa$$

$$= 4000\ kPa + 100\ kPa = \mathbf{4100\ kPa}$$

Boundary work

3.45

A 25 kg piston is above a gas in a long vertical cylinder. Now the piston is released from rest and accelerates up in the cylinder reaching the end 5 m higher at a velocity of 25 m/s. The gas pressure drops during the process so the average is 600 kPa with an outside atmosphere at 100 kPa. Neglect the change in gas kinetic and potential energy, and find the needed change in the gas volume.

Solution:

C.V. Piston

$$(E_2 - E_1)_{PIST.} = m(u_2 - u_1) + m[(1/2)V_2^2 - 0] + mg\,(H_2 - 0)$$

$$= 0 + 25\text{ kg} \times (1/2) \times 25^2 (\text{m/s})^2 + 25\text{ kg} \times 9.80665\text{ m/s}^2 \times 5\text{ m}$$

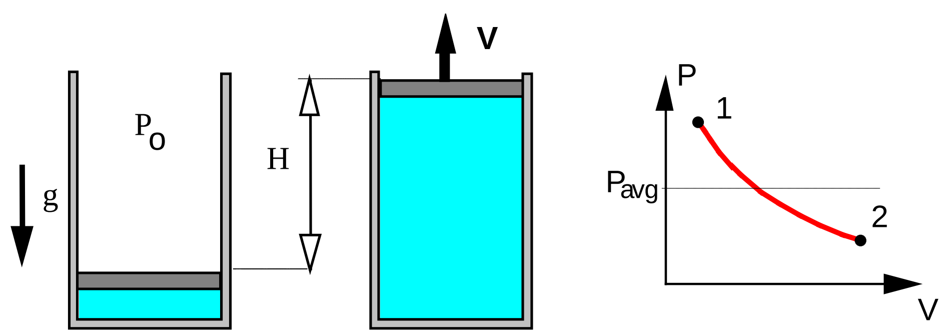
$$= 7812.5\text{ J} + 1225.8\text{ J} = 9038.3\text{ J} = 9.038\text{ kJ}$$

Energy equation for the piston is:

$$E_2 - E_1 = W_{gas} - W_{atm} = P_{avg}\ \Delta V_{gas} - P_o\ \Delta V_{gas}$$

(remark $\Delta V_{atm} = -\Delta V_{gas}$ so the two work terms are of opposite sign)

$$\Delta V_{gas} = \frac{9.038}{600 - 100}\ \frac{\text{kJ}}{\text{kPa}} = \mathbf{0.018\ m^3}$$

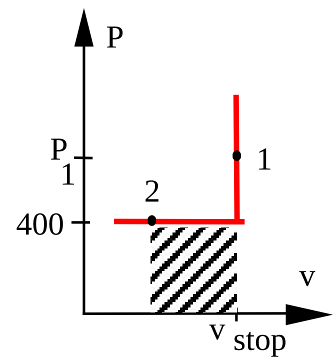


3.46

The R-410A in Problem 3.14 c is at 1000 kPa, $50^{o}C$ with mass 0.1 kg. It is cooled so the volume is reduced to half the initial volume . The piston mass and gravitation is such that a pressure of 400 kPa will float the piston. Find the work in the process.

If the volume is reduced the piston must drop and thus float with P = 400 kPa. The process therefore follows a process curve shown in the P-V diagram.

Table B.4.2: $v_1 = 0.03320\ m^3/kg$



$$_1W_2 = \int PdV = \text{area}$$
$$= P_{float}\,(V_2 - V_1) = -P_{float}\,V_1/2$$
$$= -400\ kPa \times 0.1\ kg \times 0.0332\ m^3/kg\ /2$$
$$= \mathbf{-\ 0.664\ kJ}$$

3.47

A 400-L tank A, see figure P3.47, contains argon gas at 250 kPa, 30^{o}C. Cylinder B, having a frictionless piston of such mass that a pressure of 150 kPa will float it, is initially empty. The valve is opened and argon flows into B and eventually reaches a uniform state of 150 kPa, 30^{o}C throughout. What is the work done by the argon?

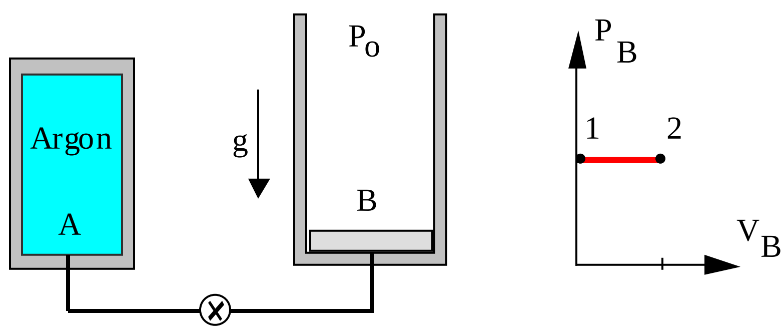
Solution:

Take C.V. as all the argon in both A and B. Boundary movement work done in cylinder B against constant external pressure of 150 kPa. Argon is an ideal gas, so write out that the mass and temperature at state 1 and 2 are the same

$$P_{A1}V_A = m_A RT_{A1} = m_A RT_2 = P_2(V_A + V_{B2})$$

$$\Rightarrow \quad V_{B2} = (P_{A1}/ P_2) V_A - V_A = \frac{250 \times 0.4}{150} - 0.4 = 0.2667 \text{ m}^3$$

$${}_1W_2 = \int_1^2 P_{ext} dV = P_{ext}(V_{B2} - V_{B1}) = 150 \text{ kPa} \times (0.2667 - 0) \text{ m}^3 = \mathbf{40\ kJ}$$



Notice there is a pressure loss in the valve so the pressure in B is always 150 kPa while the piston floats.

3.48

A piston cylinder contains 2 kg of liquid water at 20$^{\text{o}}$C and 300 kPa, as shown in Fig. P3.48. There is a linear spring mounted on the piston such that when the water is heated the pressure reaches 3 MPa with a volume of 0.1 m^3.

a) Find the final temperature

b) Plot the process in a P-v diagram.

c) Find the work in the process.

Solution:

Take CV as the water. This is a constant mass:

$$m_2 = m_1 = m \; ;$$

State 1: Compressed liquid, take saturated liquid at same temperature.

B.1.1: $v_1 = v_f(20) = 0.001002 \text{ m}^3\text{/kg}$,

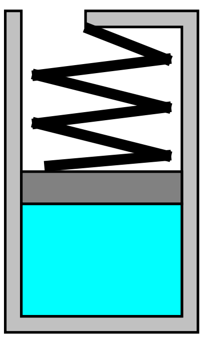
State 2: $v_2 = V_2/m = 0.1/2 = 0.05 \text{ m}^3\text{/kg}$ and $P = 3000$ kPa from B.1.2

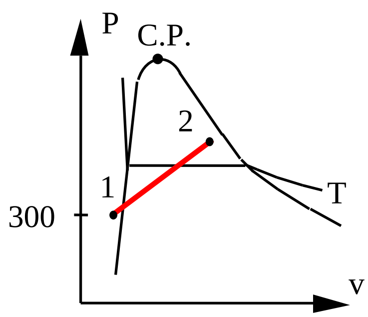
=> Two-phase: $T_2 =$ **233.9$^{\text{o}}$C**

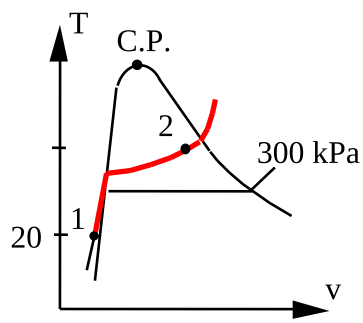
Work is done while piston moves at linearly varying pressure, so we get:

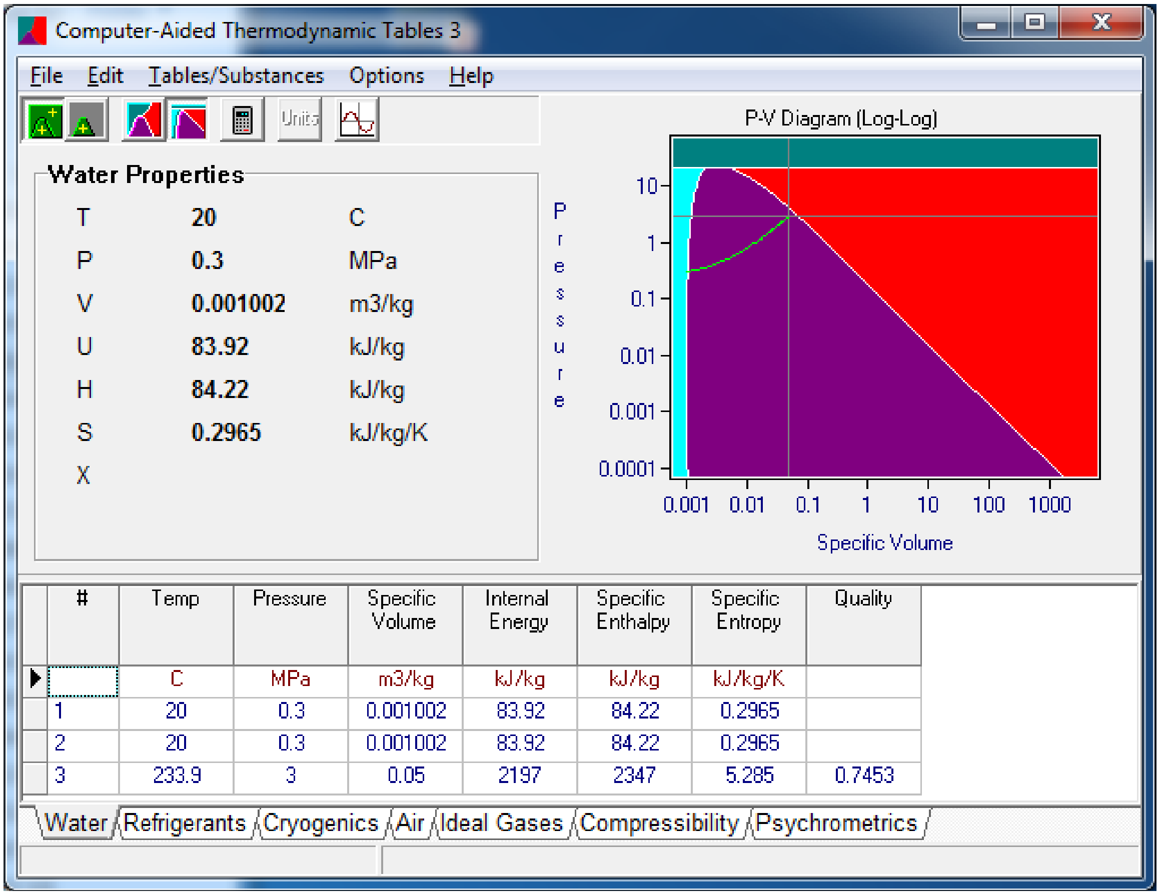
$$_1W_2 = \int P\, dV = \text{area} = P_{avg}\,(V_2 - V_1) = \frac{1}{2}(P_1 + P_2)(V_2 - V_1)$$

$$= 0.5\,(300 + 3000)\text{ kPa} \times (0.1 - 0.002)\text{ m}^3 = \mathbf{161.7\ kJ}$$









The process shown by CATT3 in a log-log diagram.

The linear relation P = A + Bv,   has B = 55.1 MPa/(m^3/kg)

3.49

Air in a spring loaded piston/cylinder has a pressure that is linear with volume, $P = A + BV$. With an initial state of P = 150 kPa, V = 1 L and a final state of 800 kPa and volume 1.5 L it is similar to the setup in Problem 3.48. Find the work done by the air.

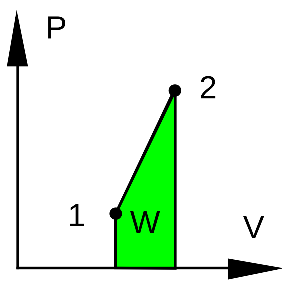
Solution:

Knowing the process equation: P = A + BV giving a linear variation of pressure versus volume the straight line in the P-V diagram is fixed by the two points as state 1 and state 2. The work as the integral of PdV equals the area under the process curve in the P-V diagram.

State 1: $P_1 = 150$ kPa $V_1 = 1 \text{ L} = 0.001 \text{ m}^3$

State 2: $P_2 = 800$ kPa $V_2 = 1.5 \text{ L} = 0.0015 \text{ m}^3$

Process: P = A + BV linear in V



$$\Rightarrow \quad {}_1W_2 = \int_1^2 PdV = \left(\frac{P_1 + P_2}{2}\right)(V_2 - V_1)$$

$$= \frac{1}{2}(150 + 800) \text{ kPa} \times (1.5 - 1) \text{ L} \times 0.001 \text{ m}^3/\text{L}$$

= 0.2375 kJ

3.50

Heat transfer to a block of 1.5 kg ice at -10^{o}C melts it to liquid at 10^{o}C in a kitchen. How much work does the water gives out?

Work is done against the atmosphere due to volume change in the process. The pressure is 101 kPa so we approximate the states as saturated

State 1: Compressed solid, B.1.5, v_1 = 0.0010891 m^3/kg

State 2: Compressed liquid B.1.1 v_2 = 0.001000 m^3/kg

$$_1W_2 = \int PdV = P_o\,(V_2 - V_1) = P_o\,m\,(v_2 - v_1)$$

$$= 101.325 \text{ kPa} \times 1.5 \text{ kg} \times (0.001 - 0.0010891) \text{ m}^3\text{/kg}$$

$$= \mathbf{-\ 0.0135\ kJ}$$

Notice the work is negative, the volume is reduced!

3.51

A cylinder fitted with a frictionless piston contains 5 kg of superheated refrigerant R-134a vapor at 1000 kPa, 140°C. The setup is cooled at constant pressure until the R-134a reaches a quality of 25%. Calculate the work done in the process.

Solution:

Constant pressure process boundary work. State properties from Table B.5.2

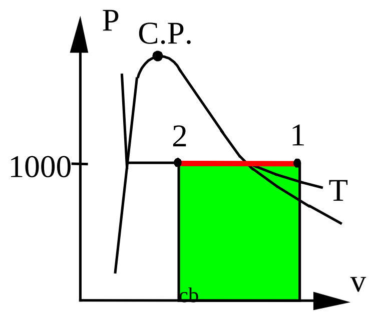
State 1: $v = 0.03150 \text{ m}^3/\text{kg}$,

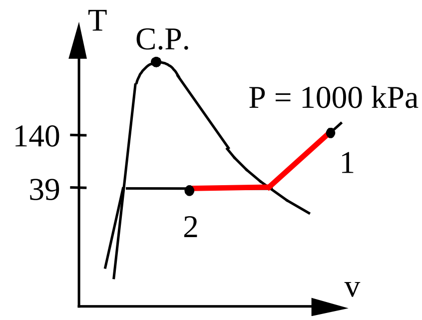
State 2: $v = 0.000871 + 0.25 \times 0.01956 = 0.00576 \text{ m}^3/\text{kg}$

Interpolated to be at 1000 kPa, numbers at 1017 kPa could have been used in which case: $v = 0.00566 \text{ m}^3/\text{kg}$

$${}_1W_2 = \int P\, dV = P\,(V_2 - V_1) = mP\,(v_2 - v_1)$$

$$= 5 \text{ kg} \times 1000 \text{ kPa} \times (0.00576 - 0.03150) \text{ m}^3/\text{kg} = \mathbf{-128.7\ kJ}$$





3.52

A piston/cylinder contains 2 kg water at 20^{o}C with volume 0.1 m^3. By mistake someone locks the piston preventing it from moving while we heat the water to saturated vapor. Find the final temperature, volume and the process work.

Solution

1: $v_1 = V/m = 0.1 \text{ m}^3/2 \text{ kg} = 0.05 \text{ m}^3/\text{kg}$ (two-phase state)

2: Constant volume: $v_2 = v_g = v_1$

$$V_2 = V_1 = \mathbf{0.1\ m^3}$$

$$_1W_2 = \int P\, dV = \mathbf{0}$$

State 2: $(v_2, x_2 = 1)$

$$T_2 = T_{sat} = 250 + 5 \frac{0.05 - 0.05013}{0.04598 - 0.05013} = \mathbf{250.2°C}$$

Polytropic process

3.53

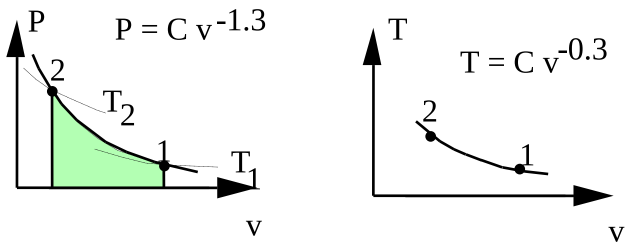
A nitrogen gas goes through a polytropic process with n = 1.3 in a piston/cylinder. It starts out at 600 K, 600 kPa and ends at 800 K. Is the work positive, negative or zero?

The work is a boundary work so it is

$$W = \int P dV = \int Pm\, dv = AREA$$

so the sign depends on the sign for dV (or dv). The process looks like the following

The actual process is on a steeper curve than n = 1.



As the temperature increases we notice the volume decreases so

$$dv < 0 \quad \Rightarrow \quad W < 0$$

Work is **negative** and goes into the nitrogen gas.

3.54

Helium gas expands from 125 kPa, 350 K and 0.25 m^3 to 100 kPa in a polytropic process with n = 1.667. How much work does it give out?

Solution:

Process equation: $PV^n = \text{constant} = P_1V_1^n = P_2V_2^n$

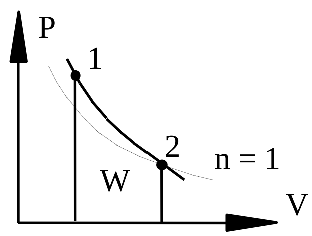
Solve for the volume at state 2

$$V_2 = V_1 (P_1/P_2)^{1/n} = 0.25 \times \left(\frac{125}{100}\right)^{0.6} = 0.2852 \text{ m}^3$$

Work from Eq.3.21

$$_1W_2 = \frac{P_2V_2 - P_1 V_1}{1-n} = \frac{100\times 0.2852 - 125\times 0.25}{1 - 1.667} \text{ kPa m}^3 = \mathbf{4.09\ kJ}$$

The actual process is on a steeper curve than n = 1.



3.55

Air goes through a polytropic process from 125 kPa, 325 K to 300 kPa and 500 K. Find the polytropic exponent n and the specific work in the process.

Solution:

Process: $Pv^n = \text{Const} = P_1 v_1^n = P_2\, v_2^n$

Ideal gas $Pv = RT$ so

$$v_1 = \frac{RT}{P} = \frac{0.287 \times 325}{125} = 0.7462 \text{ m}^3\text{/kg}$$

$$v_2 = \frac{RT}{P} = \frac{0.287 \times 500}{300} = 0.47833 \text{ m}^3\text{/kg}$$

From the process equation

$$(P_2/ P_1) = (v_1/ v_2)^n \quad => \quad \ln(P_2/ P_1) = n \ln(v_1/ v_2)$$

$$n = \ln(P_2/ P_1) \,/\, \ln(v_1/ v_2) = \frac{\ln 2.4}{\ln 1.56} = \mathbf{1.969}$$

The work is now from Eq.3.21 per unit mass and ideal gas law

$$_1w_2 = \frac{P_2v_2 - P_1v_1}{1\text{-n}} = \frac{R(T_2 - T_1)}{1\text{-n}} = \frac{0.287(500 - 325)}{1 - 1.969} \text{ (kJ/kg-K) K}$$

$$= \mathbf{-51.8 \text{ kJ/kg}}$$

3.56

A balloon behaves so the pressure is $P = C_2 V^{1/3}$, $C_2 = 100$ kPa/m. The balloon is blown up with air from a starting volume of 1 m^3 to a volume of 4 m^3. Find the final mass of air assuming it is at 25^{o}C and the work done by the air.

Solution:

The process is polytropic with exponent $n = -1/3$.

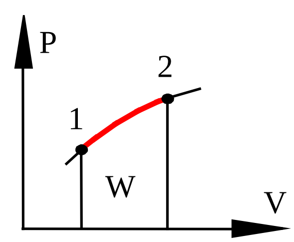
$$P_1 = C_2 V^{1/3} = 100 \times 1^{1/3} = 100 \text{ kPa}$$

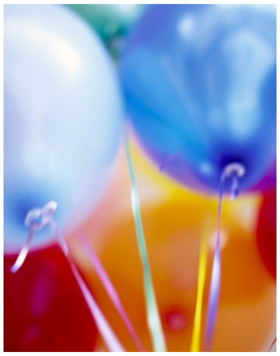
$$P_2 = C_2 V^{1/3} = 100 \times 4^{1/3} = 158.74 \text{ kPa}$$

$${}_1W_2 = \int P\, dV = \frac{P_2V_2 - P_1V_1}{1 - n} \qquad \text{(Equation 3.18 and 3.21)}$$

$$= \frac{158.74 \times 4 - 100 \times 1}{1 - (-1/3)} \text{ kPa-m}^3 = \mathbf{401.2\ kJ}$$

$$m_2 = \frac{P_2V_2}{RT_2} = \frac{158.74 \times 4}{0.287 \times 298} \frac{\text{kPa-m}^3}{\text{kJ/kg}} = \mathbf{7.424\ kg}$$





3.57

Consider a piston cylinder with 0.5 kg of R-134a as saturated vapor at -10°C. It is now compressed to a pressure of 500 kPa in a polytropic process with n = 1.5. Find the final volume and temperature, and determine the work done during the process.

Solution:

Take CV as the R-134a which is a control mass. $m_2 = m_1 = m$

Process: $Pv^{1.5}$ = constant until P = 500 kPa

1: (T, x) $v_1 = 0.09921\ m^3/kg$, $P = P_{sat} = 201.7$ kPa from Table B.5.1

2: (P, process) $v_2 = v_1 (P_1/P_2)^{(1/1.5)}$

$$= 0.09921 \times (201.7/500)^{2/3} = \mathbf{0.05416}\ m^3/kg$$

Given (P, v) at state 2 from B.5.2 it is superheated vapor at $\mathbf{T_2 = 79°C}$

Process gives $P = C\ v^{-1.5}$, which is integrated for the work term, Eq.(3.21)

$$_1W_2 = \int P\ dV = \frac{m}{1 - 1.5} (P_2v_2 - P_1v_1)$$

$$= \frac{2}{-0.5} \text{kg} \times (500 \times 0.05416 - 201.7 \times 0.09921)\ \text{kPa-m}^3/\text{kg}$$

$$= \mathbf{-7.07\ kJ}$$

Heat Transfer rates

3.58

The brake shoe and steel drum on a car continuously absorbs 75 W as the car slows down. Assume a total outside surface area of 0.1 m^2 with a convective heat transfer coefficient of 10 W/m^2 K to the air at 20°C. How hot does the outside brake and drum surface become when steady conditions are reached?

Solution :

Convection heat transfer, Eq.3.24

$$\dot{Q} = hA\Delta T \quad \Rightarrow \quad \Delta T = \frac{\dot{Q}}{hA}$$

$$\Delta T = (T_{BRAKE} - 20) = \frac{75 \text{ W}}{10 \text{ Wm}^{-2}\text{K}^{-1} \times 0.1 \text{ m}^2} = \mathbf{75\ °C}$$

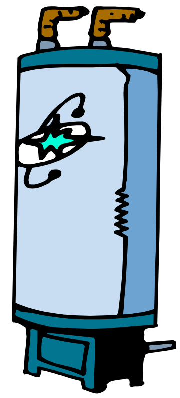
$$T_{BRAKE} = 20 + 75 = \mathbf{95°C}$$

3.59

A water-heater is covered up with insulation boards over a total surface area of 3 m^2. The inside board surface is at 75°C and the outside surface is at 18°C and the board material has a conductivity of 0.08 W/m K. How thick a board should it be to limit the heat transfer loss to 200 W ?

Solution :

Steady state conduction through a single layer board, Eq.3.23.



$$\dot{Q}_{cond} = k \;\; A \; \frac{\Delta T}{\Delta x} \qquad \Rightarrow \qquad \Delta x = k \; A \; \Delta T/\dot{Q}$$

$$\Delta x = 0.08 \, \frac{W}{m\,K} \times 3 \; m^2 \times \; \frac{75 - 18}{200} \, \frac{K}{W}$$

$$= \mathbf{0.068 \; m}$$

3.60

Find the rate of conduction heat transfer through a 1.5 cm thick hardwood board, k = 0.16 W/m K, with a temperature difference between the two sides of 20^{o}C.

One dimensional heat transfer by conduction, we do not know the area so we can find the flux (heat transfer per unit area W/m^2).

$$\dot{q} = \dot{Q}/A = k\,\frac{\Delta T}{\Delta x} = 0.16\,\frac{W}{m\,K} \times \frac{20}{0.015}\,\frac{K}{m} = \mathbf{213\ W/m^2}$$

3.61

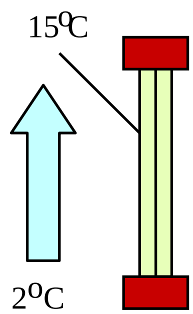
A 2 m^2 window has a surface temperature of 15^{o}C and the outside wind is blowing air at 2^{o}C across it with a convection heat transfer coefficient of h = 125 W/m^{2}K. What is the total heat transfer loss?

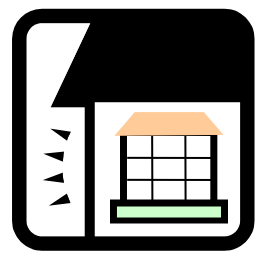
Solution:

Convection heat transfer, Eq.3.24

$$\dot{Q} = h\ A\ \Delta T = 125\ \text{W/m}^2\text{K} \times 2\ \text{m}^2 \times (15 - 2)\ \text{K} = \mathbf{3250\ W}$$

as a rate of heat transfer out.





3.62

Due to a faulty door contact the small light bulb (25 W) inside a refrigerator is kept on and limited insulation lets 50 W of energy from the outside seep into the refrigerated space. How much of a temperature difference to the ambient at 20°C must the refrigerator have in its heat exchanger with an area of 1 m^2 and an average heat transfer coefficient of 15 W/m^2 K to reject the leaks of energy.

Solution :

$$\dot{Q}_{tot} = 25 + 50 = 75 \text{ W to go out}$$

Convection heat transfer, Eq.3.24

$$\dot{Q} = hA\,\Delta T = 15 \times 1 \times \Delta T = 75 \text{ W}$$

$$\Delta T = \frac{\dot{Q}}{hA} = \frac{75 \text{ W}}{15 \text{ W/(m}^2\text{K)} \times 1 \text{ m}^2} = \mathbf{5\ °C}$$

so T must be at least **25 °C**

3.63

A large condenser (heat exchanger) in a power plant must transfer a total of 100 MW from steam running in a pipe to sea water being pumped through the heat exchanger. Assume the wall separating the steam and seawater is 4 mm of steel, conductivity 15 W/m K and that a maximum of 5°C difference between the two fluids is allowed in the design. Find the required minimum area for the heat transfer neglecting any convective heat transfer in the flows.

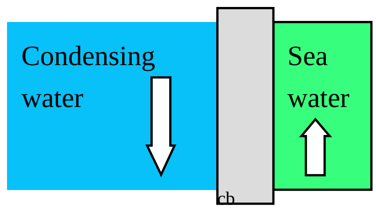
Solution :

Steady conduction through the 4 mm steel wall, Eq.3.23.

$$\dot{Q} = k\ A\ \frac{\Delta T}{\Delta x} \quad \Rightarrow \quad A = \dot{Q}\ \Delta x / k\Delta T$$

$$A = 100 \times 10^6\ \text{W} \times 0.004\ \text{m} / (15\ \text{W/mK} \times 5\ \text{K})$$

$$= \mathbf{480\ m^2}$$



3.64

The black grille on the back of a refrigerator has a surface temperature of 35°C with a total surface area of 1 m^2. Heat transfer to the room air at 20°C takes place with an average convective heat transfer coefficient of 15 W/m^2 K. How much energy can be removed during 15 minutes of operation?

Solution :

Convection heat transfer, Eq.3.24

$$\dot{Q} = hA\,\Delta T; \quad Q = \dot{Q}\,\Delta t = hA\,\Delta T\,\Delta t$$

$$Q = 15\ W/m^2\ K \times 1\ m^2 \times (35\text{-}20)\ K \times 15\ min \times 60\ s/min$$
$$= 202\ 500\ J = \mathbf{202.5\ kJ}$$

3.65

A pot of steel, conductivity 50 W/m K, with a 5 mm thick bottom is filled with 15°C liquid water. The pot has a diameter of 20 cm and is now placed on an electric stove that delivers 500 W as heat transfer. Find the temperature on the outer pot bottom surface assuming the inner surface is at 15°C.

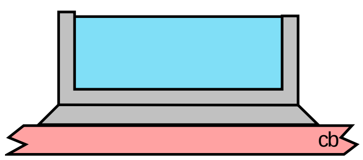
Solution :

Steady conduction, Eq.3.23, through the bottom of the steel pot. Assume the inside surface is at the liquid water temperature.

$$\dot{Q} = k\ A\ \frac{\Delta T}{\Delta x} \quad \Rightarrow \quad \Delta T = \dot{Q}\ \Delta x / kA$$

$$\Delta T = 500\ \ W \times 0.005\ m /(50\ W/m\text{-}K \times \frac{\pi}{4} \times 0.2^2\ m^2) = 1.59\ K$$

$$T = 15 + 1.59 \cong \mathbf{16.6°C}$$



3.66

A log of burning wood in the fireplace has a surface temperature of 450°C. Assume the emissivity is 1 (perfect black body) and find the radiant emission of energy per unit surface area.

Solution :

Radiation heat transfer, Eq.3.25

$$\dot{Q}/A = 1 \times \sigma \; T^4$$
$$= 5.67 \times 10^{-8}\ \text{W/m}^2\text{K}^4 \times (\,273.15 + 450)^4\ \text{K}^4$$
$$= 15\ 505\ \text{W/m}^2$$
$$= \mathbf{15.5\ kW/m^2}$$

3.67

A wall surface on a house is at 30°C with an emissivity of $\varepsilon = 0.7$. The surrounding ambient to the house is at 15°C, average emissivity of 0.9. Find the rate of radiation energy from each of those surfaces per unit area.

Solution :

Radiation heat transfer, Eq.3.25

$$\dot{Q}/A = \varepsilon\sigma A T^4, \quad \sigma = 5.67 \times 10^{-8} \text{ W/m}^2\text{K}^4$$

a) $\dot{Q}/A = 0.7 \times 5.67 \times 10^{-8} \text{ W/m}^2\text{K}^4 \times (273.15 + 30)^4 \text{ K}^4 = \mathbf{335\ W/m^2}$

b) $\dot{Q}/A = 0.9 \times 5.67 \times 10^{-8} \text{ W/m}^2\text{K}^4 \times 288.15^4 \text{ K}^4 = \mathbf{352\ W/m^2}$

3.68

A radiant heat lamp is a rod, 0.5 m long and 0.5 cm in diameter, through which 400 W of electric energy is deposited. Assume the surface has an emissivity of 0.9 and neglect incoming radiation. What will the rod surface temperature be ?

Solution :

For constant surface temperature outgoing power equals electric power.

Radiation heat transfer, Eq.3.25

$$\dot{Q}_{rad} = \varepsilon\sigma A T^4 = \dot{Q}_{el} \Rightarrow$$

$$T^4 = \dot{Q}_{el} / \varepsilon\sigma A$$

$$= 400 \text{ W} / (0.9 \times 5.67 \times 10^{-8} \text{ W/m}^2\text{K}^4 \times 0.5 \times \pi \times 0.005 \text{ m}^2)$$

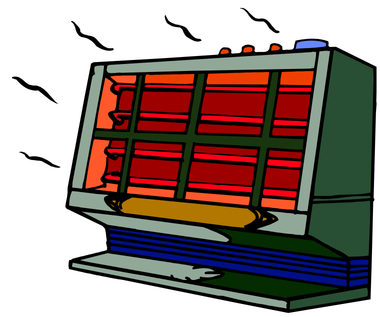
$$= 9.9803 \times 10^{11} \text{ K}^4$$

$\Rightarrow$ $T \cong$ **1000 K OR 725 °C**

3.69

A radiant heating lamp has a surface temperature of 1000 K with $\varepsilon = 0.8$. How large a surface area is needed to provide 250 W of radiation heat transfer?

Radiation heat transfer, Eq.3.25. We do not know the ambient so let us find the area for an emitted radiation of 250 W from the surface



$$\dot{Q} = \varepsilon \sigma A T^4$$

$$A = \frac{\dot{Q}}{\varepsilon \sigma T^4} = \frac{250}{0.8 \times 5.67 \times 10^{-8} \times 1000^4} \frac{\text{W}}{\text{W/m}^2}$$

$$= \mathbf{0.0055\ \ m^2}$$

Properties (u, h) from General Tables

3.70

Determine the phase of the following substances and find the values of the unknown quantities.

a. Nitrogen, P = 2000 kPa, 120 K, v = ?, Z = ?
b. Nitrogen 120 K, v = 0.0050 m^3/kg, Z = ?
c. Air, T = 100 C, v = 0.500 m^3/kg, P = ?
d. R-410A, T = 25°C, v = 0.01 m^3/kg, P = ?, h = ?

Solution:

a. B.6.2 at 2000 kPa, 120 K: v = 0.0126 m^3/kg; A.5: R = 0.2968 kJ/kgK

$$Z = \frac{Pv}{RT} = \frac{2000 \times 0.0126}{0.2968 \times 120} \frac{\text{kPa m}^3\text{/kg}}{\text{(kJ/kgK) K}} = \mathbf{0.7075}$$

b. Table B.6.1: $v_f < v < v_g = 0.00799$ m^3/kg so two-phase L + V

$$P = P_{sat} = \mathbf{2513\ kPa}$$

$$x = (v - v_f)/v_{fg} = \frac{0.005 - 0.001915}{0.00608} = 0.5074$$

$$Z = \frac{Pv}{RT} = \frac{2513 \times 0.005}{0.2968 \times 120} \frac{\text{kPa m}^3\text{/kg}}{\text{(kJ/kgK) K}} = \mathbf{0.353}$$

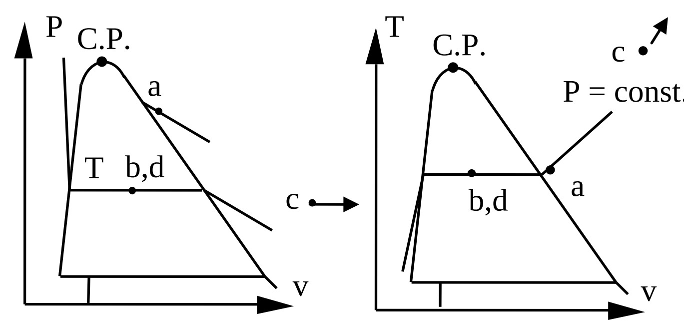
c. Ideal gas, P = RT/v = 0.287 kJ/kgK × 373.15 K / 0.5 m^3/kg = **214 kPa**

d. B.4.1 at 25°C, $v_f = 0.000944$, $v_g = 0.01514$, $v_f < v < v_g$: saturated.

$$P = \mathbf{1653.6\ kPa}, \qquad x = \frac{v - v_f}{v_{fg}} = \frac{0.01 - 0.000944}{0.0142} = 0.63775,$$

$$h = h_f + x\, h_{fg} = 97.59 + 0.63775 \times 186.43 = \mathbf{216.486\ kJ/kg}$$

States shown are placed relative to the two-phase region, not to each other.



3.71

Find the phase and the missing properties of T, P, v, u and x for water at:

a. 500 kPa, $100^{o}C$ b. 5000 kPa, u = 800 kJ/kg

c. 5000 kPa, v = 0.06 m^3/kg d. $-6^{o}C$, v = 1 m^3/kg

Solution:

a) Look in Table B.1.2 at 500 kPa

$T < T_{sat} = 151^{o}C$ => compressed liquid

Table B.1.4: v = 0.001043 m^3/kg, u = 418.8 kJ/kg

b) Look in Table B.1.2 at 5000 kPa

$u < u_f = 1147.78$ kJ/kg => compressed liquid

Table B.1.4: between $180^{o}C$ and $200^{o}C$

$$T = 180 + (200 - 180)\,\frac{800 - 759.62}{848.08 - 759.62} = 180 + 20\times0.4567 = 189.1^{o}C$$

$$v = 0.001124 + 0.4567\ (0.001153 - 0.001124) = 0.001137\ m^3/kg$$

c) Look in Table B.1.2 at 5000 kPa

$v > v_g = 0.03944\ m^3/kg$ => superheated vapor

Table B.1.3: between $400^{o}C$ and $450^{o}C$.

$$T = 400 + 50\times\frac{0.06 - 0.05781}{0.0633 - 0.05781} = 400 + 50\times0.3989 = 419.95^{o}C$$

$$u = 2906.58 + 0.3989 \times (2999.64 - 2906.58) = 2943.7\ kJ/kg$$

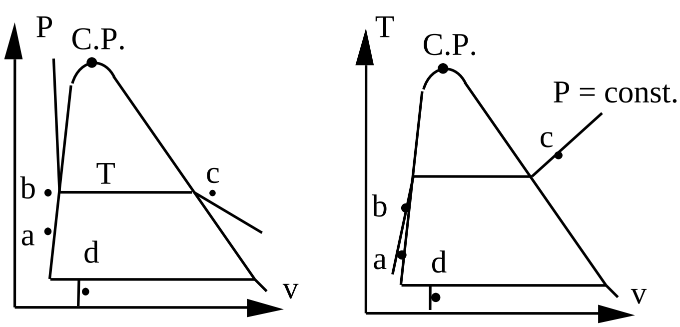
d) B.1.5: $v_i < v < v_g = 334.14\ m^3/kg$ => 2-phase, $P = P_{sat} = 887.6$ kPa,

$$x = (v - v_i) / v_{fg} = (1 - 0.0010898)/334.138 = 0.0029895$$

$$u = u_i + x\, u_{fg} = -345.91 + 0.0029895\times2712.9 = -337.8\ kJ/kg$$

3.72

States shown are placed relative to the two-phase region, not to each other.



3.73

Find the missing properties and give the phase of the ammonia, NH_3.

a. $T = 65^oC$, $P = 600$ kPa u = ? v = ?

b. $T = 20^oC$, $P = 100$ kPa u = ? v = ? x = ?

c. $T = 50^oC$, $v = 0.1185\ m^3/kg$ u = ? P = ? x = ?

Solution:

a) Table B.2.1 P < Psat => superheated vapor Table B.2.2:

$v = 0.5 \times 0.25981 + 0.5 \times 0.26888 = \mathbf{0.2645\ m^3/kg}$

$u = 0.5 \times 1425.7 + 0.5 \times 1444.3 = \mathbf{1435\ kJ/kg}$

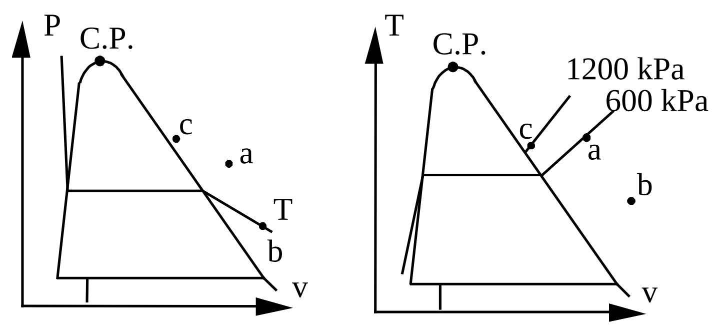
b) Table B.2.1: P < Psat => x = **undefined**, **superheated vapor**, from B.2.2:

$v = \mathbf{1.4153\ m^3/kg}$; $u = \mathbf{1374.5\ kJ/kg}$

c) Sup. vap. ($v > v_g$) Table B.2.2. P = **1200 kPa**, x = **undefined**

u = **1383 kJ/kg**

States shown are placed relative to the two-phase region, not to each other.



3.74

Find the missing properties of (P, T, v, u, h and x) and indicate the states in a P-v and T-v diagram for

a. Water at 5000 kPa, u = 1000 kJ/kg (Table B.1 reference)

b. R-134a at 20^oC, u = 300 kJ/kg

c. Nitrogen at 250 K, 200 kPa

Solution:

a) Compressed liquid: B.1.4 interpolate between 220^oC and 240^oC.

T = **233.3°C**, v = **0.001213 m³/kg**, x = **undefined**

b) Table B.5.1: $u < u_g$ => two-phase liquid and vapor

$x = (u - u_f)/u_{fg} = (300 - 227.03)/162.16 = 0.449988 = \mathbf{0.45}$

$v = 0.000817 + 0.45 \times 0.03524 = \mathbf{0.01667\ m^3/kg}$

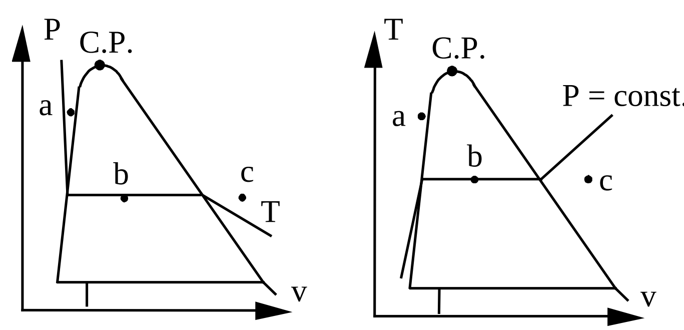
c) Table B.6.1: $T > T_{sat}$ (200 kPa) so superheated vapor in Table B.6.2

x = **undefined**

$v = 0.5(0.35546 + 0.38535) = \mathbf{0.3704\ m^3/kg}$,

$u = 0.5(177.23 + 192.14) = \mathbf{184.7\ kJ/kg}$

States shown are placed relative to the two-phase region, not to each other.



3.75

Determine the phase and the missing properties

a. H_2O 20°C, v = 0.001000 m^3/kg P = ? u = ?
b. R-410A 400 kPa, v = 0.075 m^3/kg T = ?, u = ?
c. NH_3 10°C, v = 0.1 m^3/kg P = ? u = ?
d. N_2 101.3 kPa, h = 60 kJ/kg T = ? v = ?

a) Enter Table B.1.1 with T and we see at 20°C, $v < v_f$ so compressed liquid and Table B.1.4: **P = 5000 kPa and u = 83.64 kJ/kg**.

b) Table B.4.2: P = 400 kPa $v > v_g$ so superheated, interpolate

$$T = 0 + 20 \frac{0.075 - 0.07227}{0.07916 - 0.7227} = 20 \times 0.3962 = \mathbf{7.9°C}$$

$$u = 261.51 + (276.44 - 261.51) \times 0.3962 = \mathbf{267.43\ kJ/kg},$$

c) Table B.2.1 at 10°C, v_f = 0.0016 m^3/kg, v_g = 0.20541 m^3/kg
so two-phase $P = P_{sat}$ = **615.2 kPa**

$$x = \frac{v - v_f}{v_{fg}} = \frac{0.1 - 0.0016}{0.20381} = 0.4828$$

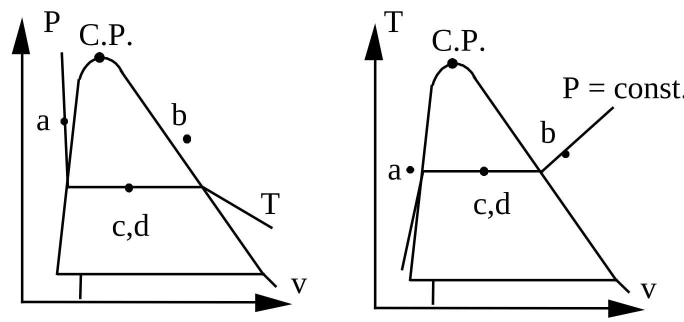
$$u = u_f + x\ u_{fg} = 225.99 + x \times 1099.7 = \mathbf{756.93\ kJ/kg}$$

d) Table B.6.1 shows that at 101.3 kPa, $h_f < h < h_g$ = 76.69 kJ/kg,
so saturated two-phase **T = 77.3 K**

$$x = (h - h_f) / h_{fg} = (60 + 122.15)/198.84 = 0.916$$

$$v = v_f + x\ v_{fg} = 0.001240 + 0.916 \times 0.21515 = \mathbf{0.1983\ m^3/kg}$$

States shown are placed relative to the two-phase region, not to each other.



3.76

Find the missing properties of (u, h, and x)

a. H_2O $T = 120°C$, $v = 0.5\ m^3/kg$

b. H_2O $T = 100°C$, $P = 10$ MPa

c. N_2 $T = 100$ K, $x = 0.75$

d. N_2 $T = 200$ K, $P = 200$ kPa

e. NH_3 $T = 100°C$, $v = 0.1\ m^3/kg$

Solution:

a) Table B.1.1: $v_f < v < v_g$ => L+V mixture, P = **198.5 kPa**,

x = (0.5 - 0.00106)/0.8908 = **0.56**,

u = 503.48 + 0.56 × 2025.76 = **1637.9 kJ/kg**

b) Table B.1.4: compressed liquid, v = 0**.001039 m^3/kg**, u = **416.1 kJ/kg**

c) Table B.6.1: 100 K, x = 0.75

v = 0.001452 + 0.75 × 0.02975 = **0.023765 m^3/kg**

u = -74.33 + 0.75 ×137.5 = **28.8 kJ/kg**

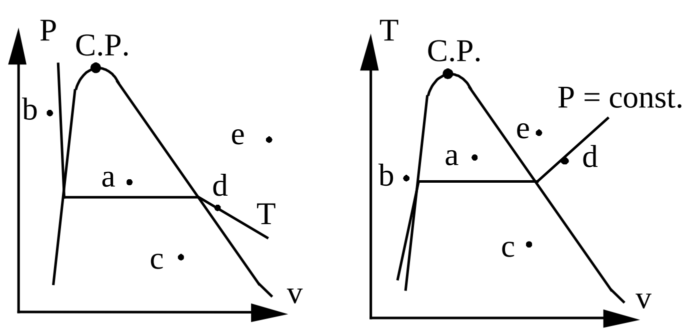
d) Table B.6.2: 200 K, 200 kPa

v = **0.29551 m^3/kg** ; u = **147.37 kJ/kg**

e) Table B.2.1: $v > v_g$ => superheated vapor, x = **undefined**

$$\text{B.2.2:}\quad P = 1600 + 400 \times \frac{0.1 - 0.10539}{0.08248 - 0.10539} = \mathbf{1694\ kPa}$$

States shown are placed relative to the two-phase region, not to each other.



3.77

Determine the phase of the following substances and find the values of the unknown quantities.

a. R-410A, T = –20°C, u = 220 kJ/kg, P = ?, x = ?
b. Ammonia, T = –20°C, v = 0.35 m^3/kg, P = ?, u = ?
c. Water, P = 400 kPa, h = 2800 kJ/kg, T = ?, v = ?

a. At –20°C in B.4.1 $u < u_g = 245.99$ kJ/kg so,

$x = (u - u_f)/u_{fg} = (220 - 27.92)/218.07 = \mathbf{0.8808}$

$P = P_{sat} = \mathbf{399.6\ kPa}$

b. At –20°C, $v_f = 0.001504$, $v_g = 0.62334$ m^3/kg, $v_f < v < v_g$: saturated.

$P = \mathbf{190.2\ kPa}$,

$$x = \frac{v - v_f}{v_{fg}} = \frac{0.35 - 0.001504}{0.62184} = 0.56043,$$

$u = u_f + x\, u_{fg} = 88.76 + 0.56043 \times 1210.7 = 767.27$ kJ/kg

c. B.1.2 at 400 kPa: $h > h_g = 2738.5$ kJ/kg so superheated vapor.

we locate it between 150 and 200°C and interpolate

$y = (2800 - 2752.82)/(2860.51 - 2752.82) = 0.43811$

$T = 150 + y\,(200 - 150) = \mathbf{171.9°C}$

$v = 0.47084 + y\,(0.53422 - 0.47084) = \mathbf{0.4986\ m^3/kg}$

3.78

Find the missing properties for CO_2 at:

a) 20^oC, 2 MPa   v = ? and h = ?

b) 10^oC, x = 0.5   P = ?, u = ?

c) 1 MPa, v = 0.05 m^3/kg, T = ?, h = ?

Solution:

a) Table B.3.1 $P < P_{sat} = 5729$ kPa so superheated vapor.

Table B.3.2: v = 0.0245 m^3/kg, h = 368.42 kJ/kg

b) Table B.3.1   since x given it is two-phase

$P = P_{sat} = 4502$ kPa

$u = u_f + x\, u_{fg} = 107.6 + 0.5 \times 169.07 = 192.14$ kJ/kg

c) Table B.3.1 $v > v_g \approx 0.0383\ m^3/kg$ so superheated vapor

Table B.3.2: Between 0 and 20^oC so interpolate.

$$T = 0 + 20 \times \frac{0.05 - 0.048}{0.0524 - 0.048} = 20 \times 0.4545 = 9.09^oC$$

$$h = 361.14 + (379.63 - 361.14) \times 0.4545 = 369.54 \text{ kJ/kg}$$

3.79

Find the missing properties among (T, P, v, u, h and x if applicable) and indicate the states in a P-v and a T-v diagram for

a. R-410A P = 500 kPa, h = 300 kJ/kg

b. R-410A T = 10^{o}C, u = 200 kJ/kg

c. R-134a T = 40^{o}C, h = 400 kJ/kg

Solution:

a) Table B.4.1: $h > h_g$ = > **superheated vapor**, look in section 500 kPa and interpolate

$$T = 0 + 20 \times \frac{300 - 287.84}{306.18 - 287.84} = 20 \times 0.66303 = \mathbf{13.26°C},$$

$v = 0.05651 + 0.66303 \times (0.06231\text{-}0.05651) = \mathbf{0.06036\ m^3/kg}$,

$u = 259.59 + 0.66303 \times (275.02 - 259.59) = \mathbf{269.82\ kJ/kg}$

b) Table B.4.1: $u < u_g$ = 255.9 kJ/kg => L+V mixture, P = **1085.7 kPa**

$$x = \frac{u - u_f}{u_{fg}} = \frac{200 - 72.24}{183.66} = \mathbf{0.6956},$$

$v = 0.000886 + 0.6956 \times 0.02295 = \mathbf{0.01685\ m^3/kg}$,

$h = 73.21 + 0.6956 \times 208.57 = \mathbf{218.3\ kJ/kg}$

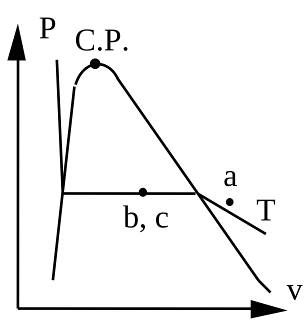
c) Table B.5.1: $h < h_g$ => **two-phase L + V,** look in B.5.1 at 40°C:

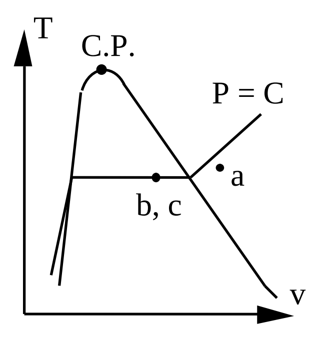
$$x = \frac{h - h_f}{h_{fg}} = \frac{400 - 256.5}{163.3} = 0.87875, \qquad P = P_{sat} = \mathbf{1017\ kPa},$$

$v = 0.000\,873 + 0.87875 \times 0.01915 = \mathbf{0.0177\ m^3/kg}$

$u = 255.7 + 0.87875 \times 143.8 = \mathbf{382.1\ kJ/kg}$

States shown are placed relative to the two-phase region, not to each other.





3.80

Saturated liquid water at 20^{o}C is compressed to a higher pressure with constant temperature. Find the changes in u and h from the initial state when the final pressure is a) 500 kPa, b) 2000 kPa

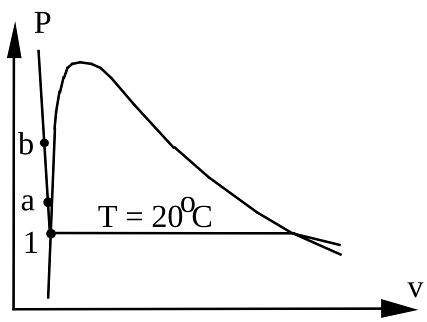
Solution:

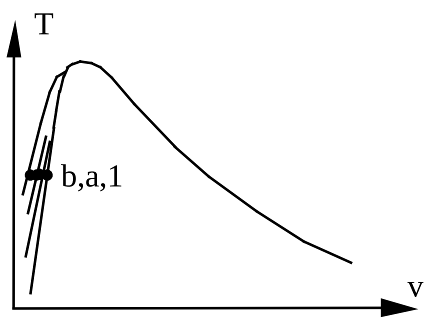
State 1 is located in Table B.1.1 and the states a-c are from Table B.1.4

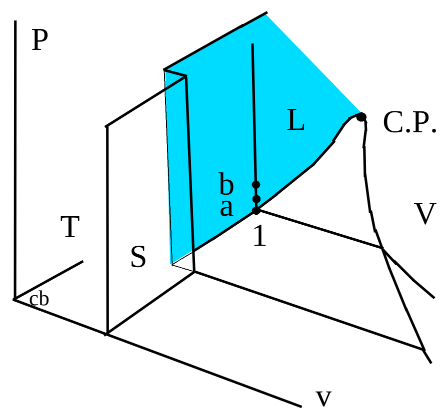
State	u [kJ/kg]	h [kJ/kg]	$\Delta u = u - u_1$	$\Delta h = h - h_1$	$\Delta(Pv)$
1	83.94	83.94			
a	83.91	84.41	-0.03	0.47	0.5
b	83.82	85.82	-0.12	1.88	2

For these states u stays nearly constant, dropping slightly as P goes up.

h varies with Pv changes.







3.81

Determine the phase of the following substances and find the values of the unknown quantities.

a. Water, P = 500 kPa, u = 2850 kJ/kg, T = ?, v = ?

b. R-134a, T = –10°C, v = 0.08 m^3/kg, P = ?, u = ?

c. Ammonia, T = –20°C, u = 1000 kJ/kg, P = ?, x = ?

a. B.1.2 at 500 kPa: $u > u_g = 2561$ kJ/kg so superheated vapor.
we locate it between 300 and 350°C and interpolate

$x = (2850 – 2802.9)/(2882.6 – 2802.9) = 0.590966$

$T = 300 + x (350 – 300) = 329.55°C$

$v = 0.52256 + x (0.57012 – 0.52256) = 0.5507\ m^3/kg$

b. B.5.1 at –10°C, $v_f = 0.000755\ m^3/kg$, $v_g = 0.09921\ m^3/kg$, $v_f < v < v_g$:

saturated, P = 201.7 kPa,

$x = (v – v_f)/v_{fg} = (0.08\text{-}0.000755)/0.09845 = 0.805$,

$u = u_f + x\ u_{fg} = 186.57 + 0.805 \times 185.7 = 336$ kJ/kg

c. B.2.1 at –20°C: $u < u_g = 1299.5$ kJ/kg so, $P = P_{sat} = 190.2$ kPa

$x = (u – u_f)/u_{fg} = (1000 – 88.76)/1210.7 = 0.7526$

Problem Analysis

3.82

Consider Problem 3.101. Take the whole room as a C.V. and write both conservation of mass and energy equations. Write some equations for the process (two are needed) and use those in the conservation equations. Now specify the four properties that determines initial (2) and final state (2), do you have them all? Count unknowns and match with equations to determine those.

C.V.: Containment room and reactor.

Mass: $m_2 - m_1 = 0$; $m_2 = m_1 = V_{reactor}/v_1$

Energy: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Process: Room volume constant $V = C$ ⇨ ${}_1W_2 = 0$

Room insulated ⇨ ${}_1Q_2 = 0$

Using these in the equation for mass and energy gives:

$m_2 = V_2/v_2 = m_1$; $m(u_2 – u_1) = 0 – 0 = 0$

State 1: P_1, T_1 so Table B.1.4 gives v_1, u_1 ⇨ m_1

State 2: P_2, ?

We do not know **one** state 2 property and the total room volume

Energy equation then gives $u_2 = u_1$ (a state 2 property)

State 2: P_2, u_2 ⇨ v_2

Now we have the room volume as

Continuity Eq.: $m_2 = V/v_2 = m_1$ so $V = m_1 v_2$

3.83

Consider a steel bottle as a CV. It contains carbon dioxide at $-20\ ^oC$, quality 20%. It has a safety valve that opens at 6 MPa. The bottle is now accidentially heated until the safety valve opens. Write the process equation that is valid until the valve opens and plot the P-v diagram for the process.

Solution:

C.V. carbon dioxide, which is a control mass of constant volume.

Energy Eq.3.5 (3.26): $u_2 - u_1 = {}_1q_2 - {}_1w_2$

Process: V = constant and m = constant

=> **v = constant** ; ${}_1w_2 = 0$

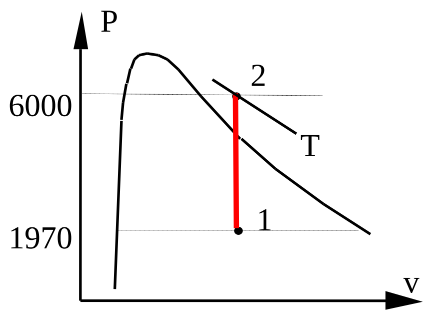
State 1: Table B.3.1, $P_1 = P_{sat} = 1970$ kPa

$v_1 = v_f + x\ v_{fg} = \ldots\ldots.$

State 2: 6000 kPa, $v_1 = v_2 = \ldots..$ => Table B.3.1 or B.3.2

$T_2 = \ldots\ ,\ u_2 = \ldots\ldots$

${}_1q_2 = (u_2 - u_1) = \ldots\ldots$



State 2 could also be in the two-phase region if

$v_1 = v_2 < v_g$ at 6 MPa

3.84

A piston/cylinder contains water with quality 75% at 200 kPa. Slow expansion is performed while there is heat transfer and the water is at constant pressure. The process stops when the volume has doubled. How do you determine the final state and the heat transfer?

CV. Water, this is a control mass, we do not know size so do all per unit mass.

Energy Eq.3.5: $u_2 - u_1 = {}_1q_2 - {}_1w_2$

Process: $P = C$; ${}_1w_2 = \int P dv = P(v_2 - v_1)$

State 1: x_1, P_1 Table B.1.2 gives T_1, u_1

State 2: $v_2 = 2v_1 = \dots$ & $P_2 = P_1$

Compare v_2 to v_g to determine if sup. vapor or not.

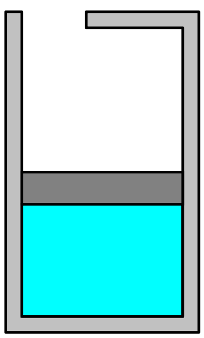
Either find x_2 or interpolate to get T_2 and u_2 in B.1.3

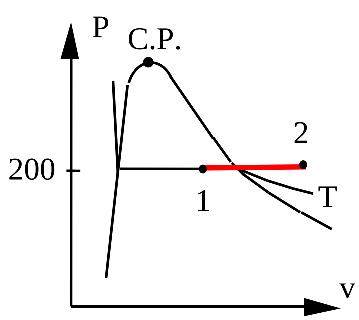
Process: $P = C$ so the work term integral is

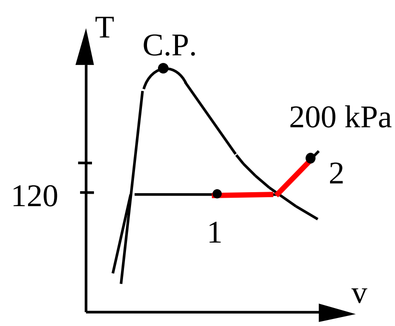
$${}_1w_2 = \int P dv = P(v_2 - v_1) = \dots\dots\dots$$

From the energy equation

$${}_1q_2 = (u_2 - u_1) + {}_1w_2 = \dots\dots..$$







3.85

Consider Problem 3.173 with the final state given but that you were not told the piston hits the stops and only told $V_{stop} = 2\ V_1$. Sketch the possible P-v diagram for the process and determine which number(s) you need to uniquely place state 2 in the diagram. There is a kink in the process curve what are the coordinates for that state? Write an expression for the work term.

C.V. R-410A. Control mass goes through process: 1 -> 2 -> 3

As piston floats pressure is constant (1 -> 2) and the volume is constant for the second part (2 -> 3). So we have: $v_3 = v_2 = 2 \times v_1$

State 2: $V_2 = V_{stop}$ $\Rightarrow$ $v_2 = 2 \times v_1 = v_3$ and $P_2 = P_1$ $\Rightarrow$ $T_2 = \ldots$

State 3: Table B.4.2 (P,T) $v_3 = 0.02249\ \mathrm{m^3/kg}$, $u_3 = 287.91$ kJ/kg

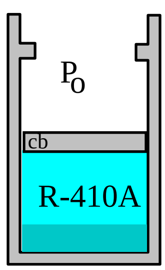
Now we can find state 1 (T_1, $v_1 = v_3/2 = 0.011245\ \mathrm{m^3/kg}$) two-phase

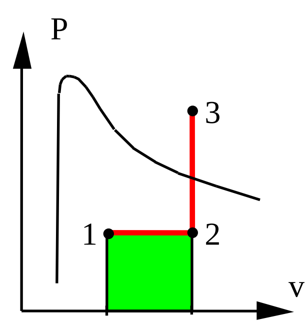
Then state 2 (the kink)

$$v_2 = v_3 = 0.02249\ \mathrm{m^3/kg}$$

$$P_2 = P_1 = 1085.7\ \mathrm{kPa} = P_{sat}$$

$$W = \int P\, dV = P(V_2 - V_1) = Pm\,(v_2 - v_1)$$

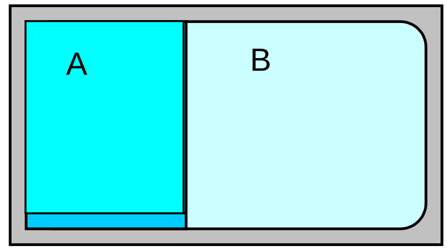




3.86

Take problem 3.210 and write the left hand side (storage change) of the conservation equations for mass and energy. How do you write m_1 and Eq. 3.5?

C.V.: Both rooms A and B in tank.



Continuity Eq.: $m_2 - m_{A1} - m_{B1} = 0$;

Energy Eq.: $m_2u_2 - m_{A1}u_{A1} - m_{B1}u_{B1} = {}_1Q_2 - {}_1W_2$

Notice how the state 1 term split into two terms

$$m_1 = \int \rho \, dV = \int (1/v) \, dV = V_A/v_{A1} + V_B/v_{B1} = m_{A1} + m_{B1}$$

and for energy as

$$m_1u_1 = \int \rho u \, dV = \int (u/v) \, dV = (u_{A1}/v_{A1})V_A + (u_{B1}/v_{B1})V_B$$
$$= m_{A1}u_{A1} + m_{B1}u_{B1}$$

Formulation continues as:

Process constant total volume: $V_{tot} = V_A + V_B$ and ${}_1W_2 = \emptyset$

$$m_2 = m_{A1} + m_{B1} \;\Rightarrow\; v_2 = V_{tot}/m_2$$

etc.

3.87

Two rigid insulated tanks are connected with a pipe and valve. One tank has 0.5 kg air at 200 kPa, 300 K and the other has 0.75 kg air at 100 kPa, 400 K. The valve is opened and the air comes to a single uniform state without any heat transfer. How do you determine the final temperature and pressure?

Solution:

C.V. Total tank. Control mass of constant volume.

Mass and volume: $m_2 = m_A + m_B$; $V = V_A + V_B$

Energy Eq.: $U_2 - U_1 = m_2 u_2 - m_A u_{A1} - m_B u_{B1} = {}_1Q_2 - {}_1W_2 = 0$

Process Eq.: $V = \text{constant} \Rightarrow {}_1W_2 = 0$; Insulated $\Rightarrow {}_1Q_2 = 0$

Ideal gas at A1: $V_A = m_A R T_{A1}/P_{A1} = \ldots..$

$u_{A1} = \ldots\ldots$ from Table A.7

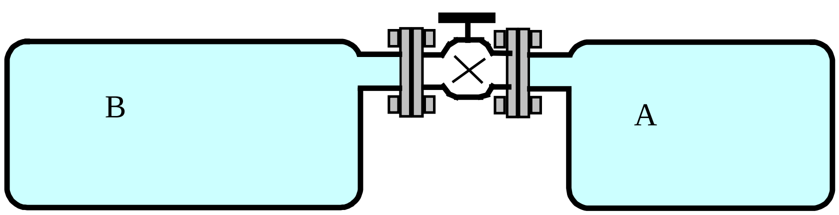
Ideal gas at B1: $V_B = m_B R T_{B1}/ P_{B1} = \ldots\ldots.$

$u_{B1} = \ldots\ldots.$ from Table A.7

State 2: $m_2 = m_A + m_B$; => $v_2 = V/ m_2 = \ldots..$

Energy Eq.: $u_2 = \dfrac{m_A u_{A1} + m_B u_{B1}}{m_2} = \ldots..$ => Table A.7.1: $\mathbf{T_2}$

$P_2 = m_2 R T_2 /V = \ldots\ldots..$



3.88

Look at problem 3.183 and plot the P-v diagram for the process. Only T_2 is given, how do you determine the 2nd property of the final state? What do you need to check and does it have an influence on the work term?

Process: $P = \text{constant} = F/A = P_1$ if $V > V\text{min}$

$V = \text{constant} = V_{1a} = V_{min}$ if $P < P_1$

State 1: (P, T) $V_1 = mRT_1/P_1 = 0.5 \times 0.287 \times 1000/2000 = 0.07175 \text{ m}^3$

The only possible P-V combinations for this system are shown in the diagram so both state 1 and 2 must be on the two lines. For state 2 we need to know if it is on the horizontal P line segment or the vertical V segment. Let us check state 1a:

State 1a: $P_{1a} = P_1$, $V_{1a} = V_{min}$, Ideal gas so $T_{1a} = T_1 \dfrac{V_{1a}}{V_1}$

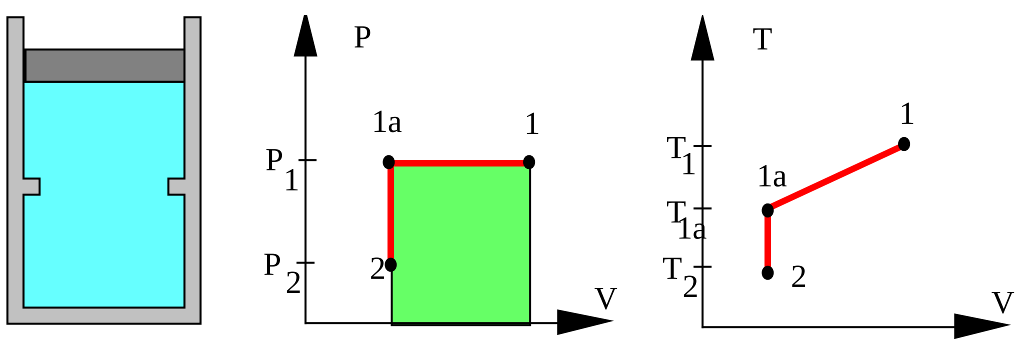
We see if $T_2 < T_{1a}$ then state 2 must have $V_2 = V_{1a} = V_{min} = 0.03 \text{ m}^3$. So state 2 is known by (T_2, v_2) and $P_2 = P_1 \times \dfrac{T_2}{T_1} \times \dfrac{V_1}{V_2}$

If it was that $T_2 > T_{1a}$ then we know state 2 as: T_2, $P_2 = P_1$ and we then have

$$V_2 = V_1 \times \frac{T_2}{T_1}$$

The work is the area under the process curve in the P-V diagram and so it does make a difference where state 2 is relative to state 1a. For the part of the process that proceeds along the constant volume V_{min} the work is zero, there is only work when the volume changes.

$${}_1W_2 = \int_1^2 P\, dV = P_1 (V_{1a} - V_1)$$



Simple processes

3.89

A 100-L rigid tank contains nitrogen (N_2) at 900 K, 3 MPa. The tank is now cooled to 100 K. What are the work and heat transfer for this process?

Solution:

C.V.: Nitrogen in tank. $m_2 = m_1$;

Energy Eq.3.5: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Process: V = constant, $v_2 = v_1 = V/m$ => $\mathbf{{}_1W_2 = 0}$

Table B.6.2: State 1: $v_1 = 0.0900\ m^3/kg$ => $m = V/v_1 = 1.111$ kg

$u_1 = 691.7$ kJ/kg

State 2: 100 K, $v_2 = v_1 = V/m$, look in Table B.6.2 at 100 K

200 kPa: $v = 0.1425\ m^3/kg$; $u = 71.7$ kJ/kg

400 kPa: $v = 0.0681\ m^3/kg$; $u = 69.3$ kJ/kg

so a linear interpolation gives:

$$P_2 = 200 + 200\ (0.09 - 0.1425)/(0.0681 - 0.1425) = 341 \text{ kPa}$$

$$u_2 = 71.7 + (69.3 - 71.7)\ \frac{0.09 - 0.1425}{0.0681 - 0.1425} = 70.0 \text{ kJ/kg},$$

$${}_1Q_2 = m(u_2 - u_1) = 1.111 \text{ kg } (70.0 - 691.7) \text{ kJ/kg} = \mathbf{-690.7\ kJ}$$

3.90

A constant pressure piston/cylinder aasembly contains 0.2 kg water as saturated vapor at 400 kPa. It is now cooled so the water occupies half the original volume. Find the work and the heat transfer in the process.

Solution:

Energy Eq.3.5: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Process: $P = C$; ${}_1W_2 = \int PdV = P(V_2 – V_1)$

Table B.1.2

State 1: $v_1 = 0.46246\ m^3/kg$; $u_1 = 2553.55\ kJ/kg$

State 2: $v_2 = v_1/ 2 = 0.23123\ m^3/kg = v_f + x\ v_{fg}$;

$$x_2 = \frac{v_2 - v_{f2}}{v_{fg2}} = \frac{0.23123 - 0.001084}{0.46138} = 0.4988$$

$$u_2 = u_{f2} + x_2\ u_{fg2} = 604.29 + x_2 \times 1949.26 = 1576.58\ kJ/kg$$

Process: $P = C$ so the work term integral is

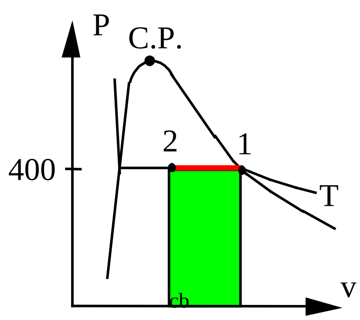
$${}_1W_2 = P(V_2\text{-}V_1) = 400\ kPa \times 0.2 \times (0.23123 – 0.46246)\ m^3$$

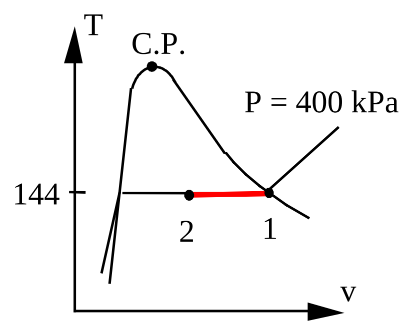
$$= \mathbf{–18.5\ kJ}$$

From the energy equation

$${}_1Q_2 = m(u_2 - u_1) + {}_1W_2 = 0.2\ (1576.58 – 2553.55) – 18.5$$

$$= \mathbf{–213.9\ kJ}$$





3.91

Saturated vapor R-410A at 0^{o}C in a rigid tank is cooled to -20^{o}C. Find the specific heat transfer.

Solution:

C.V.: R-410A in tank. $m_2 = m_1$;

Energy Eq.3.5: $(u_2 - u_1) = {}_1q_2 - {}_1w_2$

Process: V = constant, $v_2 = v_1 = V/m$ => $\mathbf{{}_1w_2 = 0}$

Table B.4.1: State 1: $u_1 = 253.0$ kJ/kg

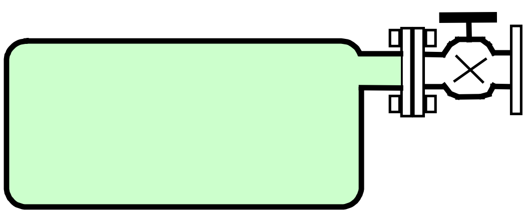
State 2: -20^{o}C, $v_2 = v_1 = V/m$, look in Table B.4.1 at -20^{o}C

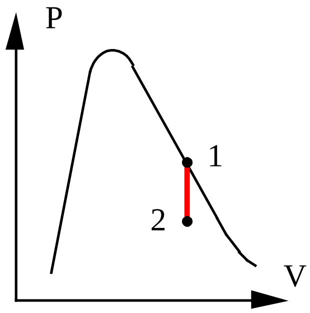
$$x_2 = \frac{v_2 - v_{f2}}{v_{fg2}} = \frac{0.03267 - 0.000803}{0.06400} = 0.4979$$

$$u_2 = u_{f2} + x_2\, u_{fg2} = 27.92 + x_2 \times 218.07 = 136.5 \text{ kJ/kg}$$

From the energy equation

$${}_1q_2 = (u_2 - u_1) = (136.5 - 253.0) = \mathbf{-116.5\ kJ/kg}$$



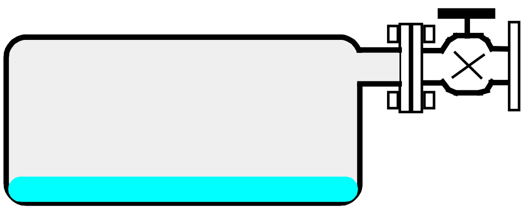


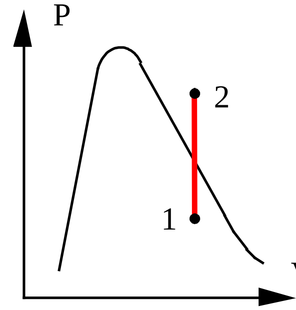
3.92

Ammonia at 0°C, quality 60% is contained in a rigid 200-L tank. The tank and ammonia is now heated to a final pressure of 1 MPa. Determine the heat transfer for the process.

Solution:

C.V.: NH_3





Continuity Eq.: $m_2 = m_1 = m$;

Energy Eq.3.5: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Process: Constant volume $\Rightarrow$ $v_2 = v_1$ & ${}_1W_2 = 0$

State 1: Table B.2.1 two-phase state.

$v_1 = 0.001566 + x_1 \times 0.28763 = 0.17414 \text{ m}^3\text{/kg}$

$u_1 = 179.69 + 0.6 \times 1138.3 = 862.67 \text{ kJ/kg}$

$m = V/v_1 = 0.2 \text{ m}^3/0.17414 \text{ m}^3\text{/kg} = 1.148 \text{ kg}$

State 2: P_2 , $v_2 = v_1$ superheated vapor Table B.2.2

$\Rightarrow T_2 \cong 100°C$, $u_2 \cong 1490.5$ kJ/kg

So solve for heat transfer in the energy equation

${}_1Q_2 = m(u_2 - u_1) = 1.148 \text{ kg } (1490.5 - 862.67) \text{ kJ/kg} = \mathbf{720.75 \text{ kJ}}$

3.93

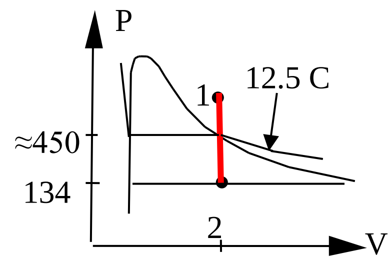
A rigid tank contains 1.5 kg of R-134a at 40°C, 500 kPa. The tank is placed in a refrigerator that brings it to -20°C. Find the process heat transfer and show the process in a *P-v* diagram.

CV the R-134a.

Energy Eq.: $U_2 - U_1 = {}_1Q_2 - {}_1W_2 = {}_1Q_2$,
Process: Rigid tank V = C => v = constant & ${}_1W_2 = \int_1^2 PdV = 0$

State 1: $v_1 = 0.04656\ m^3/kg$,
$u_1 = 407.44$ kJ/kg

State 2: **T, v** => two-phase (straight down in P-v diagram from state 1)



$$x_2 = (v - v_f)/v_{fg} = (0.04656 - 0.000738)/0.14576 = 0.31437$$
$$u_2 = u_f + x_2\ u_{fg} = 173.65 + 0.31437 \times 192.85 = 234.28\ kJ/kg$$

From the energy equation

$${}_1Q_2 = m(u_2 - u_1) = 1.5\ kg\ (234.28 - 407.44)\ kJ/kg = \mathbf{-259.7\ kJ}$$

3.94

A piston cylinder contains air at 600 kPa, 290 K and a volume of 0.01 m^3. A constant pressure process gives 54 kJ of work out. Find the final volume, the temperature of the air and the heat transfer.

C.V AIR control mass

Continuity Eq.: $m_2 – m_1 = 0$

Energy Eq.: $m (u_2 – u_1) = {}_1Q_2 - {}_1W_2$

Process: $P = C$ so ${}_1W_2 = \int P\, dV = P(V_2 – V_1)$

State 1 : P_1 , T_1,V_1 State 2 : $P_1 = P_2$, ?

$$m_1 = P_1V_1/RT_1 = \frac{600 \text{ kPa} \times 0.01 \text{ m}^3}{0.287 \text{ kJ/kg-K} \times 290 \text{ K}} = 0.0721 \text{ kg}$$

$${}_1W_2 = P(V_2 – V_1) = 54 \text{ kJ} \rightarrow$$

$$V_2 – V_1 = {}_1W_2 / P = 54 \text{ kJ} / 600 \text{ kPa} = 0.09 \text{ m}^3$$

$$V_2 = V_1 + {}_1W_2 / P = 0.01 + 0.09 = 0.1 \text{ m}^3$$

Ideal gas law : $P_2V_2 = mRT_2$

$$T_2 = P_2V_2 / mR = \frac{P_2V_2}{P_1V_1} T_1 = \frac{0.10}{0.01} \times 290 \text{ K} = \mathbf{2900 \text{ K}}$$

Energy equation with u's from table A.7.1

$${}_1Q_2 = m (u_2 – u_1) + {}_1W_2$$

$$= 0.0721 \text{ kg} \times (2563.8 – 207.19) \text{ kJ/kg} + 54 \text{ kJ}$$

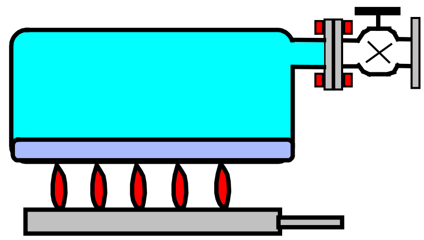
$$= \mathbf{223.9 \text{ kJ}}$$

3.95

Two kg water at 120°C with a quality of 25% has its temperature raised 20°C in a constant volume process as in Fig. P3.95. What are the heat transfer and work in the process?

Solution:

C.V. Water. This is a control mass



Energy Eq.: $m (u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Process : $V = \text{constant}$

$\rightarrow \quad {}_1W_2 = \int P\, dV = \mathbf{0}$

State 1: T, x_1 from Table B.1.1

$v_1 = v_f + x_1 v_{fg} = 0.00106 + 0.25 \times 0.8908 = 0.22376 \text{ m}^3/\text{kg}$

$u_1 = u_f + x_1 u_{fg} = 503.48 + 0.25 \times 2025.76 = 1009.92 \text{ kJ/kg}$

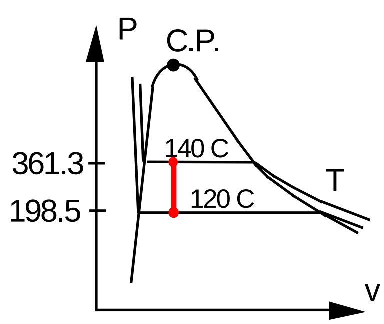
State 2: $T_2,\ v_2 = v_1 < v_{g2} = 0.50885 \text{ m}^3/\text{kg}$ so two-phase

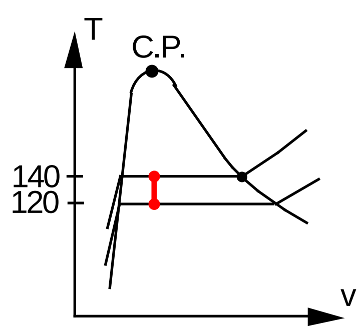
$$x_2 = \frac{v_2 - v_{f2}}{v_{fg2}} = \frac{0.22376 - 0.00108}{0.50777} = 0.43855$$

$u_2 = u_{f2} + x_2 u_{fg2} = 588.72 + x_2 \times 1961.3 = 1448.84 \text{ kJ/kg}$

From the energy equation

${}_1Q_2 = m(u_2 - u_1) = 2 \text{ kg} \times (1448.84 - 1009.92) \text{ kJ/kg} = \mathbf{877.8\ kJ}$





3.96

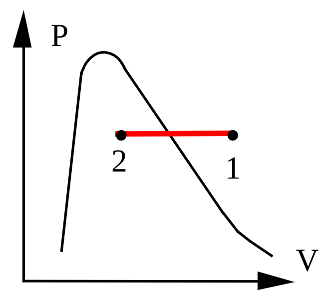
A cylinder fitted with a frictionless piston contains 2 kg of superheated refrigerant R-134a vapor at 350 kPa, 100^oC. The cylinder is now cooled so the R-134a remains at constant pressure until it reaches a quality of 75%. Calculate the heat transfer in the process.

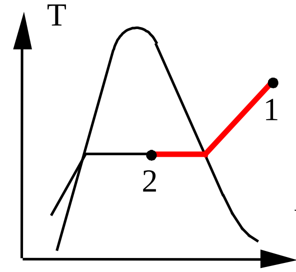
Solution:

C.V.: R-134a $\quad m_2 = m_1 = m;$

Energy Eq.3.5 $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Process: P = const. $\Rightarrow {}_1W_2 = \int PdV = P\Delta V = P(V_2 - V_1) = Pm(v_2 - v_1)$





State 1: Table B.5.2 $\quad h_1 = (490.48 + 489.52)/2 = 490$ kJ/kg

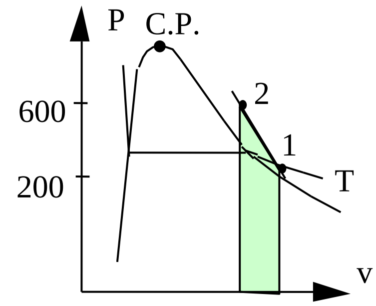
State 2: Table B.5.1 $\quad h_2 = 206.75 + 0.75 \times 194.57 = 352.7$ kJ/kg (350.9 kPa)

$${}_1Q_2 = m(u_2 - u_1) + {}_1W_2 = m(u_2 - u_1) + Pm(v_2 - v_1) = m(h_2 - h_1)$$

$${}_1Q_2 = 2 \text{ kg} \times (352.7 - 490) \text{ kJ/kg} = \mathbf{-274.6 \ kJ}$$

3.97

A piston cylinder contains 1.5 kg water at 200 kPa, 150°C. It is now heated in a process where pressure is linearly related to volume to a state of 600 kPa, 350°C. Find the final volume, the work and the heat transfer in the process.



Take as CV the 1.5 kg of water.

$m_2 = m_1 = m$;

Process Eq.: $P = A + BV$ (linearly in V)

State 1: (P, T) => $v_1 = 0.95964$ m^3/kg,

$u_1 = 2576.87$ kJ/kg

State 2: (P, T) => $v_2 = 0.47424$ m^3/kg, $u_2 = 2881.12$ kJ/kg

$V_2 = mv_2 =$ **0.7114 m³**

From process eq.:

$$_1W_2 = \int P\,dV = \text{area} = \frac{m}{2}(P_1 + P_2)(v_2 - v_1)$$

$$= \frac{1.5}{2}\text{ kg }(200 + 600)\text{ kPa }(0.47424 - 0.95964)\text{ m}^3\text{/kg} = \mathbf{-291.24\ kJ}$$

Notice volume is reduced so work is negative.

Energy Eq.: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

$${}_1Q_2 = m(u_2 - u_1) + {}_1W_2$$

$$= 1.5\text{ kg }(2881.12 - 2576.87)\text{ kJ/kg} - 291.24\text{ kJ}$$

$$= \mathbf{165.1\ kJ}$$

3.98

A piston/cylinder contains 50 kg of water at 200 kPa with a volume of 0.1 m^3. Stops in the cylinder are placed to restrict the enclosed volume to a maximum of 0.5 m^3. The water is now heated until the piston reaches the stops. Find the necessary heat transfer.

Solution:

C.V. H_2O m = constant

Energy Eq.3.5: $m(e_2 - e_1) = m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Process : P = constant (forces on piston constant)

$$\Rightarrow {}_1W_2 = \int P\, dV = P_1 (V_2 - V_1)$$

Properties from Table B.1.1

State 1: $v_1 = 0.1/50 = 0.002\ m^3/kg$ => 2-phase as $v_1 < v_g$

$$x_1 = \frac{v_1 - v_f}{v_{fg}} = \frac{0.002 - 0.001061}{0.88467} = 0.001061$$

$$h_1 = 504.68 + 0.001061 \times 2201.96 = 507.02\ kJ/kg$$

State 2: $v_2 = 0.5/50 = 0.01\ m^3/kg$ also 2-phase same P

$$x_2 = \frac{v_2 - v_f}{v_{fg}} = \frac{0.01 - 0.001061}{0.88467} = 0.01010$$

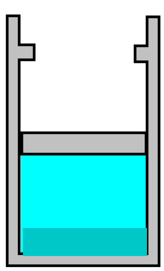
$$h_2 = 504.68 + 0.01010 \times 2201.96 = 526.92\ kJ/kg$$

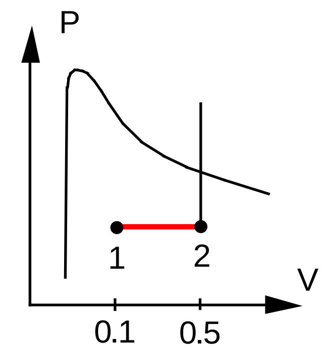
Find the heat transfer from the energy equation as

$${}_1Q_2 = m(u_2 - u_1) + {}_1W_2 = m(h_2 - h_1)$$

$${}_1Q_2 = 50\ kg \times (526.92 - 507.02)\ kJ/kg = \mathbf{995\ kJ}$$

[Notice that $_1W_2 = P_1 (V_2 - V_1) = 200\ kPa \times (0.5 - 0.1)\ m^3 = 80\ kJ$]





3.99

Ammonia (0.5 kg) is in a piston cylinder at 200 kPa, -10^{o}C is heated in a process where the pressure varies linear with the volume to a state of 120^{o}C, 300 kPa. Find the work and the heat transfer for the ammonia in the process.

Solution:

Take CV as the Ammonia, constant mass.

Continuity Eq.: $m_2 = m_1 = m$;

Process: $P = A + BV$ (linear in V)

State 1: Superheated vapor $v_1 = 0.6193$ m^3/kg, $u_1 = 1316.7$ kJ/kg

State 2: Superheated vapor $v_2 = 0.63276$ m^3/kg, $u_2 = 1542.0$ kJ/kg

Work is done while piston moves at increasing pressure, so we get

$$ {}_1W_2 = \int P\,dV = \text{area} = P_{avg}(V_2 - V_1) = \frac{1}{2}(P_1 + P_2)m(v_2 - v_1) $$

$$ = ½(200 + 300)\text{ kPa} \times 0.5\text{ kg }(0.63276 - 0.6193)\text{ m}^3\text{/kg} $$

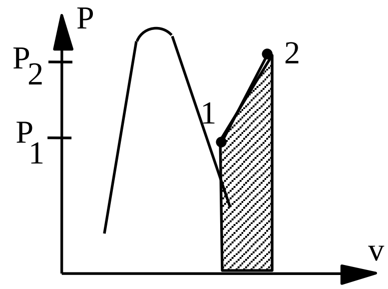
$$ = \mathbf{1.683\ kJ} $$

Energy Eq.: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

$$ {}_1Q_2 = m(u_2 - u_1) + {}_1W_2 $$

$$ = 0.5\text{ kg }(1542.0 - 1316.7)\text{ kJ/kg} + 1.683\text{ kJ} $$

$$ = \mathbf{114.3\ kJ} $$



3.100

A piston/cylinder contains 1 kg water at 20^{o}C with volume 0.1 m^3. By mistake someone locks the piston preventing it from moving while we heat the water to saturated vapor. Find the final temperature and the amount of heat transfer in the process.

Solution:

C.V. Water. This is a control mass

Energy Eq.: $m\ (u_2 - u_1\ ) = {}_1Q_2 - {}_1W_2$

Process : V = constant $\rightarrow$ ${}_1W_2 = 0$

State 1: T, $v_1 = V_1/m = 0.1\ m^3/kg > v_f$ so two-phase

$$x_1 = \frac{v_1 - v_f}{v_{fg}} = \frac{0.1\text{-}0.001002}{57.7887} = 0.0017131$$

$$u_1 = u_f + x_1\ u_{fg} = 83.94 + x_1 \times 2318.98 = 87.913\ kJ/kg$$

State 2: $v_2 = v_1 = 0.1$ & $x_2 = 1$

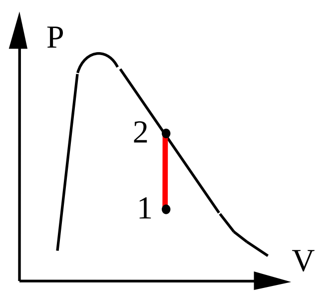
$\rightarrow$ found in Table B.1.1 between 210°C and 215° C

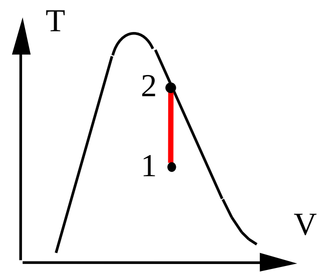
$$T_2 = 210 + 5 \times \frac{0.1\text{-}0.10441}{0.09479\text{-}0.10441} = 210 + 5 \times 0.4584 = 212.3°C$$

$$u_2 = 2599.44 + 0.4584\ (2601.06 - 2599.44) = 2600.2\ kJ/kg$$

From the energy equation

$${}_1Q_2 = m(u_2 - u_1) = 1\ kg\ (\ 2600.2 - 87.913)\ kJ/kg = \mathbf{2512.3\ kJ}$$





3.101

A water-filled reactor with volume of 1 m^3 is at 20 MPa, 360°C and placed inside a containment room as shown in Fig. P3.101. The room is well insulated and initially evacuated. Due to a failure, the reactor ruptures and the water fills the containment room. Find the minimum room volume so the final pressure does not exceed 200 kPa.

Solution:

C.V.: Containment room and reactor.

Mass: $m_2 = m_1 = V_{reactor}/v_1 = 1/0.001823 = 548.5$ kg

Energy: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2 = 0 - 0 = 0$

State 1: Table B.1.4 $v_1 = 0.001823$ m^3/kg, $u_1 = 1702.8$ kJ/kg

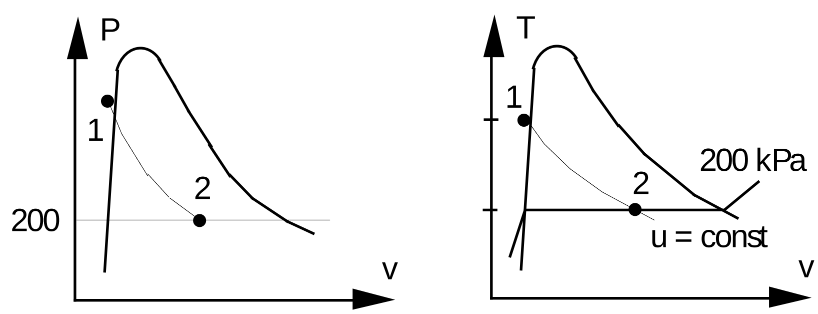
Energy equation then gives $u_2 = u_1 = 1702.8$ kJ/kg

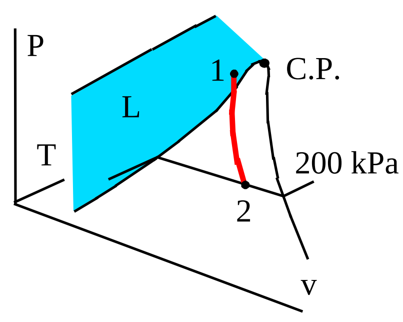
State 2: $P_2 = 200$ kPa, $u_2 < u_g$ => Two-phase Table B.1.2

$x_2 = (u_2 - u_f)/ u_{fg} = (1702.8 – 504.47)/2025.02 = 0.59176$

$v_2 = 0.001061 + 0.59176 \times 0.88467 = 0.52457$ m^3/kg

$V_2 = m_2 v_2 = 548.5$ kg $\times$ 0.52457 m^3/kg = $\mathbf{287.7\ m^3}$





3.102

A rigid tank holds 0.75 kg ammonia at 70°C as saturated vapor. The tank is now cooled to 20°C by heat transfer to the ambient. Which two properties determine the final state. Determine the amount of work and heat transfer during the process.

C.V. The ammonia, this is a control mass.

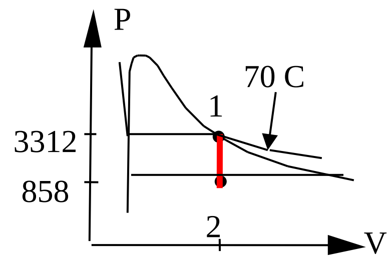
Process: Rigid tank V = C => v = constant & $_1W_2 = \int_1^2 PdV = \mathbf{0}$

Energy Eq.: $U_2 - U_1 = {}_1Q_2 - {}_1W_2 = {}_1Q_2$,

State 1: $v_1 = 0.03787\ m^3/kg$,

$u_1 = 1338.9$ kJ/kg

State 2: **T, v** => two-phase (straight down in P-v diagram from state 1)



$x_2 = (v - v_f)/v_{fg} = (0.03787 - 0.001638)/0.14758 = 0.2455$

$u_2 = u_f + x_2\ u_{fg} = 272.89 + 0.2455 \times 1059.3 = 532.95$ kJ/kg

$_1Q_2 = m(u_2 - u_1) = 0.75$ kg $(532.95 - 1338.9)$ kJ/kg $=$ **-604.5 kJ**

3.103

Water in a 150-L closed, rigid tank is at 100°C, 90% quality. The tank is then cooled to –10°C. Calculate the heat transfer during the process.

Solution:

C.V.: Water in tank. $m_2 = m_1$;

Energy Eq.3.5: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Process: V = constant, $v_2 = v_1$, ${}_1W_2 = 0$

State 1: Two-phase L + V look in Table B.1.1

$v_1 = 0.001044 + 0.9 \times 1.6719 = 1.5057 \text{ m}^3\text{/kg}$

$u_1 = 418.94 + 0.9 \times 2087.6 = 2297.8 \text{ kJ/kg}$

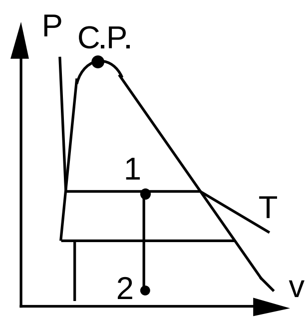
State 2: T_2, $v_2 = v_1$ $\Rightarrow$ mix of saturated solid + vapor Table B.1.5

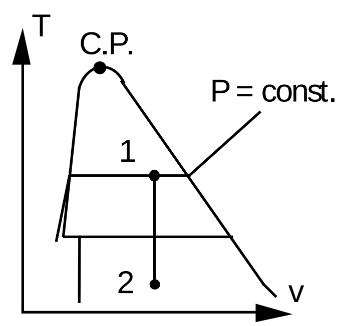
$v_2 = 1.5057 = 0.0010891 + x_2 \times 466.7$ => $x_2 = 0.003224$

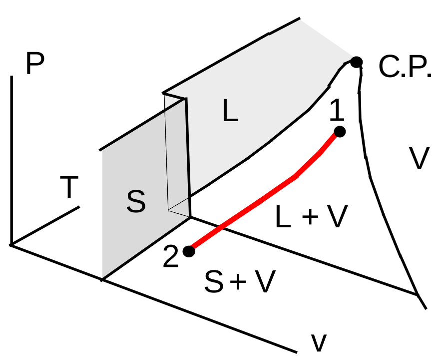
$u_2 = -354.09 + 0.003224 \times 2715.5 = -345.34 \text{ kJ/kg}$

$m = V/v_1 = 0.15/1.5057 = 0.09962 \text{ kg}$

${}_1Q_2 = m(u_2 - u_1) = 0.09962 \text{ kg } (-345.34 - 2297.8) \text{ kJ/kg} = \mathbf{-263.3 \text{ kJ}}$







3.104

A 25 kg mass moves with 25 m/s. Now a brake system brings the mass to a complete stop with a constant deceleration over a period of 5 seconds. The brake energy is absorbed by 0.5 kg water initially at 20^{o}C, 100 kPa. Assume the mass is at constant P and T. Find the energy the brake removes from the mass and the temperature increase of the water, assuming P = C.

Solution:

C.V. The mass in motion.

$$E_2 - E_1 = \Delta E = 0.5\ m\mathbf{V}^2 = 0.5 \times 25 \times 25^2/1000 = \mathbf{7.8125\ kJ}$$

C.V. The mass of water.

$$m(u_2 - u_1)_{H_2O} = \Delta E = 7.8125\ \text{kJ}$$

$$\Rightarrow \quad u_2 - u_1 = 7.8125\ \text{kJ}/\ 0.5\ \text{kg} = 15.63\ \text{kJ/kg}$$

$$u_2 = u_1 + 15.63 = 83.94 + 15.63 = 99.565\ \text{kJ/kg}$$

Assume $u_2 = u_f$ then from Table B.1.1: $T_2 \cong 23.7^{o}$C, $\Delta T = \mathbf{3.7^{o}C}$

We could have used $u_2 - u_1 = C\Delta T$ with C from Table A.4: C = 4.18 kJ/kg K giving $\Delta T = 15.63/4.18 =$ **3.7^{o}C**.

3.105

A cylinder having a piston restrained by a linear spring (of spring constant 15 kN/m) contains 0.5 kg of saturated vapor water at 120°C, as shown in Fig. P3.105. Heat is transferred to the water, causing the piston to rise. If the piston cross-sectional area is 0.05 m^2, and the pressure varies linearly with volume until a final pressure of 500 kPa is reached. Find the final temperature in the cylinder and the heat transfer for the process.

Solution:

C.V. Water in cylinder.

Continuity: $m_2 = m_1 = m$;

Energy Eq.3.5: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

State 1: (T, x) Table B.1.1 => $v_1 = 0.89186\ m^3/kg$, $u_1 = 2529.2$ kJ/kg

Process: $P_2 = P_1 + \frac{k_s m}{A_p^2}(v_2 - v_1) = 198.5 + \frac{15 \times 0.5}{(0.05)^2}(v_2 - 0.89186)$

State 2: $P_2 = 500$ kPa and on the process curve (see above equation).

=> $v_2 = 0.89186 + (500 - 198.5) \times (0.05^2/7.5) = 0.9924\ m^3/kg$

(P, v) Table B.1.3 => $T_2 = \mathbf{803°C}$; $u_2 = 3668$ kJ/kg

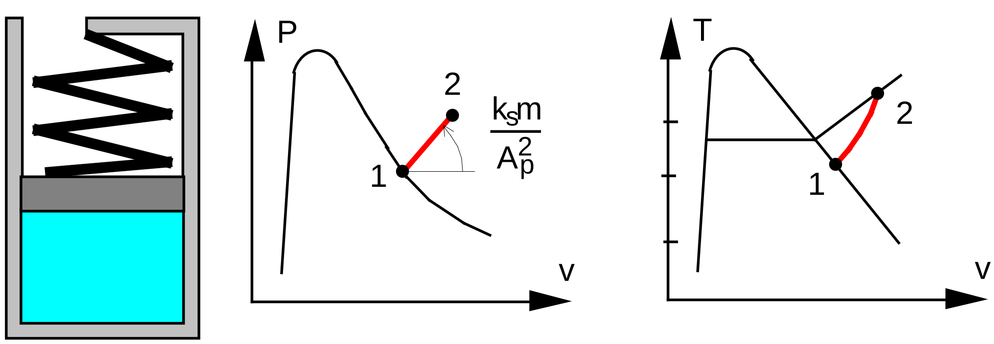
The process equation allows us to evaluate the work

$${}_1W_2 = \int PdV = \left(\frac{P_1 + P_2}{2}\right) m(v_2 - v_1)$$

$$= \left(\frac{198.5 + 500}{2}\right) \text{kPa} \times 0.5 \text{ kg} \times (0.9924 - 0.89186)\ m^3/kg = 17.56 \text{ kJ}$$

Substitute the work into the energy equation and solve for the heat transfer

$${}_1Q_2 = m(u_2 - u_1) + {}_1W_2 = 0.5 \text{ kg} \times (3668 - 2529.2) \text{ kJ/kg} + 17.56 \text{ kJ} = \mathbf{587\ kJ}$$



3.106

A piston cylinder arrangement with a linear spring similar to Fig. P3.105 contains R-134a at 15^{o}C, x = 0.4 and a volume of 0.02 m^3. It is heated to 60^{o}C at which point the specific volume is 0.03002 m^3/kg. Find the final pressure, the work and the heat transfer in the process.

Take CV as the R-134a.

$$m_2 = m_1 = m \quad ; \qquad m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$$

State 1: T_1, x_1 => Two phase so Table B.5.1: $P_1 = P_{sat} = 489.5$ kPa

$$v_1 = v_f + x_1 v_{fg} = 0.000805 + 0.4 \times 0.04133 = 0.01734 \text{ m}^3\text{/kg}$$

$$u_1 = u_f + x_1 u_{fg} = 220.1 + 0.4 \times 166.35 = 286.64 \text{ kJ/kg}$$

$$m = V_1/v_1 = 0.02 \text{ m}^3 / 0.01734 \text{ m}^3\text{/kg} = 1.1534 \text{ kg}$$

State 2: (T, v) Superheated vapor, Table B.5.2.

$$P_2 = 800 \text{ kPa}, \; v_2 = 0.03002 \text{ m}^3\text{/kg}, \; u_2 = 421.2 \text{ kJ/kg}$$

$$V_2 = m\, v_2 = 1.1534 \text{ kg} \times 0.03002 \text{ m}^3\text{/kg} = 0.03463 \text{ m}^3$$

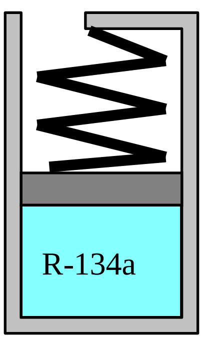
Work is done while piston moves at linearly varying pressure, so we get

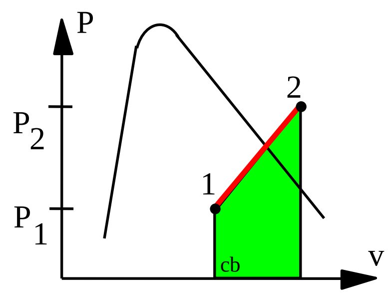
$${}_1W_2 = \int P\, dV = \text{area} = P_{avg} (V_2 - V_1) = 0.5(P_2 + P_1)(V_2 - V_1)$$

$$= 0.5 \times (489.5 + 800) \text{ kPa } (0.03463 - 0.02) \text{ m}^3 = \mathbf{9.433\ kJ}$$

Heat transfer is found from the energy equation

$${}_1Q_2 = m(u_2 - u_1) + {}_1W_2 = 1.1534 \times (421.2 - 286.64) + 9.43 = \mathbf{164.6\ kJ}$$





3.107

A 10-m high open cylinder, A_{cyl} = 0.1 m^2, contains 20°C water above and 2 kg of 20°C water below a 198.5-kg thin insulated floating piston, shown in Fig. P3.107. Assume standard *g*, *Po*. Now heat is added to the water below the piston so that it expands, pushing the piston up, causing the water on top to spill over the edge. This process continues until the piston reaches the top of the cylinder. Find the final state of the water below the piston (*T*, *P*, *v*) and the heat added during the process.

Solution:

C.V. Water below the piston.

Piston force balance at initial state: $F\uparrow = F\downarrow = P_A A = m_p g + m_B g + P_0 A$

State $1_{A,B}$: Comp. Liq. $\Rightarrow$ $v \cong v_f = 0.001002$ m^3/kg; $u_{1A} = 83.95$ kJ/kg

$V_{A1} = m_A v_{A1} = 0.002$ m^3; $m_{tot} = V_{tot}/v = 1/0.001002 = 998$ kg

mass above the piston $m_{B1} = m_{tot} - m_A = \mathbf{996\ kg}$

$$P_{A1} = P_0 + (m_p + m_B)g/A = 101.325 + \frac{(198.5 + 996) \times 9.807}{0.1 \times 1000} = \mathbf{218.5\ kPa}$$

State 2_A: $P_{A2} = P_0 + \dfrac{m_p g}{A} = \mathbf{120.8\ kPa}$; $v_{A2} = V_{tot}/\, m_A = \mathbf{0.5\ m^3/kg}$

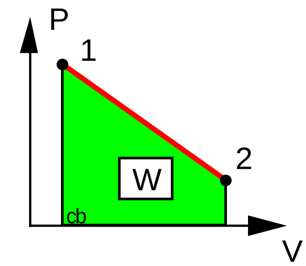
$x_{A2} = (0.5 - 0.001047)/1.4183 = 0.352$; $T_2 = \mathbf{105°C}$

$u_{A2} = 440.0 + 0.352 \times 2072.34 = 1169.5$ kJ/kg

Continuity eq. in A: $m_{A2} = m_{A1}$

Energy: $m_A(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Process: P linear in V as m_B is linear with V



$${}_1W_2 = \int PdV = \frac{1}{2}(218.5 + 120.82) \text{ kPa } (1 - 0.002) \text{ m}^3$$

$= \mathbf{169.32\ kJ}$

${}_1Q_2 = m_A(u_2 - u_1) + {}_1W_2 = 2170.1 + 169.3 = \mathbf{2340.4\ kJ}$

3.108

Assume the same setup as in Problem 3.101, but the room has a volume of 100 m^3. Show that the final state is two-phase and find the final pressure by trial and error.

C.V.: Containment room and reactor.

Mass: $m_2 = m_1 = V_{reactor}/v_1 = 1/0.001823 = 548.5$ kg

Energy: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2 = 0 - 0 = 0 \Rightarrow\ u_2 = u_1 = 1702.8$ kJ/kg

Total volume and mass => $v_2 = V_{room}/m_2 = 0.1823\ m^3/kg$

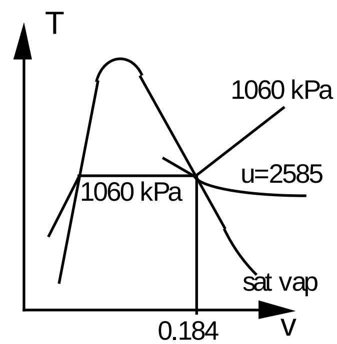
State 2: u_2 , v_2 Table B.1.1 see Figure.

Note that in the vicinity of $v = 0.1823\ m^3/kg$ crossing the saturated vapor line the internal energy is about 2585 kJ/kg. However, at the actual state 2, u = 1702.8 kJ/kg. Therefore state 2 must be in the two-phase region.

Trial & error $v = v_f + xv_{fg}$; $u = u_f + xu_{fg}$

$$\Rightarrow u_2 = 1702.8 = u_f + \frac{v_2 - v_f}{v_{fg}} u_{fg}$$

Compute RHS for a guessed pressure P_2:



$P_2 = 600$ kPa: RHS $= 669.88 + \dfrac{0.1823-0.001101}{0.31457} \times 1897.52 = 1762.9$ too large

$P_2 = 550$ kPa: RHS $= 655.30 + \dfrac{0.1823-0.001097}{0.34159} \times 1909.17 = 1668.1$ too small

Linear interpolation to match u = 1702.8 gives $P_2 \cong$ **568.5 kPa**

3.109

A piston cylinder contains carbon dioxide at -20^{o}C and quality 75%. It is compressed in a process where pressure is linear in volume to a state of 3 MPa and 20^{o}C. Find the specific heat transfer.

CV Carbon dioxide out to the source, both $_1Q_2$ and $_1W_2$

Energy Eq.3.5: $m(u_2 - u_1) = {_1Q_2} - {_1W_2}$

Process: $P = A + BV$ => $_1W_2 = \int P\, dV = ½\, m(P_1 + P_2)\,(v_2 - v_1)$

State 1: Table B.3.1 P = 1969.6 kPa

$v_1 = 0.000969 + 0.75 \times 0.01837 = 0.01475$ m^3/kg,

$u_1 = 39.64 + 0.75 \times 246.25 = 224.33$ kJ/kg,

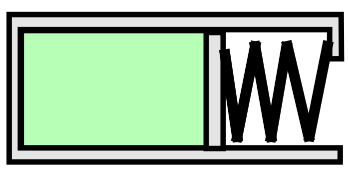
State 2: Table B.3 $v_2 = 0.01512$ m^3/kg, $u_2 = 310.21$ kJ/kg,

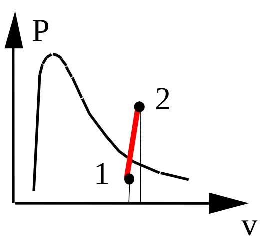
$_1w_2 = ½\,(P_1 + P_2)(v_2 - v_1)$

$= ½ \times (1969.6 + 3000)$ kPa $\times$ (0.01512 – 0.01475) m^3/kg

= **0.92 kJ/kg**

$_1q_2 = u_2 - u_1 + {_1w_2} = 310.21 - 224.33 + 0.92$ **= 86.8 kJ/kg**





3.110

A rigid steel tank of mass 2.5 kg contains 0.5 kg R-410A at 0°C with specific volume 0.01m/kg. The whole system is now heated to to a room temperature of 25°C.

a. Find the volume of the tank.
b. Find the final P.
c. Find the process heat transfer.

C.V. R-410A and steel tank. Control mass goes through process: 1 -> 2

Continuity Eq.: $m_2 - m_{R410a} - m_{st} = 0$

Energy Eq.: $m_{R410a}(u_2 - u_1) + m_{st}(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Process: $V = C$ so ${}_1W_2 = 0$

State 1: $T_1 = 0°C$, $v_1 = 0.01\ m^3/kg$,

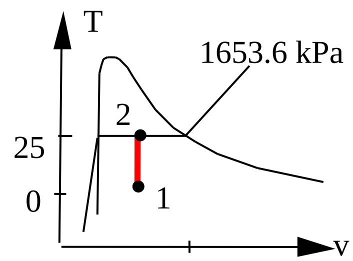
$V = mv_1 = 0.005\ m^3$

$x_1 = (v - v_f)/v_{fg} = (0.01 - 0.000855)/0.03182$

$= 0.28758$

$u_1 = u_f + x_1\ u_{fg} = 57.07 + x_1\ 195.95$

$= 113.42\ kJ/kg$



State 2: **(T, v)** => sup-vapor (straight up in T-v diagram from state 1)

B.4.1 at 25°C, $v_f = 0.000944\ m^3/kg$, $v_g = 0.01514\ m^3/kg$, $v_f < v < v_g$: saturated.

$P = 1653.6$ kPa, $x = \frac{v - v_f}{v_{fg}} = \frac{0.01 - 0.000944}{0.01420} = 0.63775$,

$u_2 = u_f + x_2\ u_{fg} = 96.03 + x_2\ 162.95 = 199.95\ kJ/kg$

From the energy Eq.:

${}_1Q_2 = m_{R410a}(u_2 - u_1) + m_{st}\ C_{st}(T_2 - T_1)$

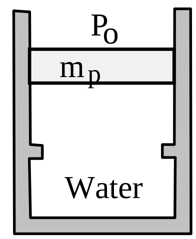
$= 0.5\ kg \times (199.95 - 113.42)\ kJ/kg + 2.5\ kg \times 0.46\ kJ/kgK \times (25–0)\ K$

$=$ **72.0 kJ**

3.111

The piston/cylinder in Fig.P3.111 contains 0.1 kg water at 500°C, 1000 kPa. The piston has a stop at half the original volume. The water now cools to room temperature 25°C

(a) Sketch the possible water states in a P-v diagram.
(b) Find the final pressure and volume
(c) Find the heat transfer and work in the process



Energy Eq.: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

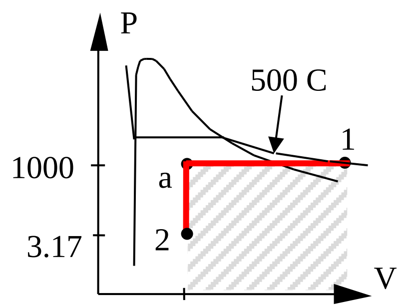
Process Eq: $P = C$ if $v > v_{stop}$; $V = C$ if $P < P_{float}$

State 1: $v_1 = 0.35411\ m^3/kg$, $u_1 = 3124.34$ kJ/kg

State a: $v_a = v_1/2 = 0.177055\ m^3/kg < v_{g\ 1000\ kPa}$ so $T_a = T_{sat\ 1000\ kPa} = 179.9°C$

The possible state 2 (P,V) combinations are shown. State "a" is (1000 kPa, v_a) so it is

two-phase with $T_a = 180°C > T_2$



$P_2 = P_{sat\ 25\ C} =$ **3.169 kPa** and $v_2 = v_a$

$x_2 = (v_2 - v_f)/v_{fg} = (0.177 - 0.001003)/43.358$
$= 0.0040604$

$u_2 = u_f + x_2\ u_{fg} = 104.86 + x_2\ 2304.9 = 114.219$ kJ/kg

$V_2 = mv_2 = 0.1\ kg \times 0.177055\ m^3/kg = \mathbf{0.0177\ m^3}$

${}_1W_2 = m \int P\,dv = m P_1 (v_2 - v_1)$ [see area below process curve in figure]
$= 0.1\ kg \times 1000\ kPa \times (0.177055 - 0.35411)\ m^3/kg =$ **– 17.706 kJ**

${}_1Q_2 = m(u_2 - u_1) + {}_1W_2 = 0.1$ kg $(114.219 - 3124.34)$ kJ/kg $- 17.706$ kJ
$=$ **–318.72 kJ**

3.112

A spring loaded piston/cylinder assembly contains 1 kg water at 500°C, 3 MPa. The setup is such that the pressure is proportional to volume, P = CV. It is now cooled until the water becomes saturated vapor. Sketch the P-v diagram and find the final state, the work and heat transfer in the process.

Solution :

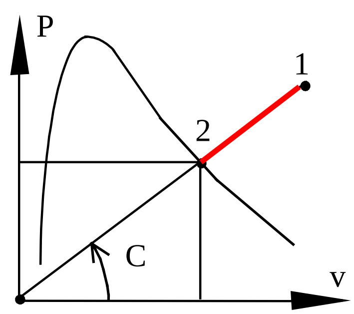
State 1: Table B.1.3: $v_1 = 0.11619\ m^3/kg$, $u_1 = 3107.92\ kJ/kg$

Process: m is constant and $P = C_0V = C_0m\ v = C\ v$

polytropic process with n = –1

$P = Cv \quad \Rightarrow \quad C = P_1/v_1 = 3000/0.11619 = 25\ 820\ kPa\ kg/m^3$

State 2: $x_2 = 1$ & $P_2 = Cv_2$ (on process line)



Trial & error on T_{2sat} or P_{2sat}:

Here from B.1.2:

at 2 MPa $v_g = 0.09963 \Rightarrow C = P/v_g = 20074$ (low)

2.5 MPa $v_g = 0.07998 \Rightarrow C = P/v_g = 31258$ (high)

2.25 MPa $v_g = 0.08875 \Rightarrow C = P/v_g = 25352$ (low)

Now interpolate to match the right slope C:

$$P_2 = 2250 + 250\ \frac{25\ 820 - 25\ 352}{31\ 258 - 25\ 352} = 2270\ kPa,$$

$v_2 = P_2/C = 2270/25820 = 0.0879\ m^3/kg$, $u_2 = 2602.07\ kJ/kg$

P is linear in V so the work becomes (area in P-v diagram)

$$_1W_2 = \int P\ dv = m\ \frac{1}{2}(P_1 + P_2)(v_2 - v_1)$$

$$= 1\ kg \times \frac{1}{2}\ (3000 + 2270)\ kPa\ (0.0879 - 0.11619)\ m^3 = \mathbf{-\ 74.5\ kJ}$$

From the energy Eq.:

$$_1Q_2 = m(u_2 - u_1) + {_1W_2} = 1\ (2602.07 - 3107.92) - 74.5$$

$$= \mathbf{-1250.85\ kJ}$$

3.113

A piston cylinder contains 1.5 kg water at 600 kPa, 350^{o}C. It is now cooled in a process where pressure is linearly related to volume to a state of 200 kPa, 150^{o}C. Plot the P-v diagram for the process and find both the work and the heat transfer in the process.

Take as CV the 1.5 kg of water.

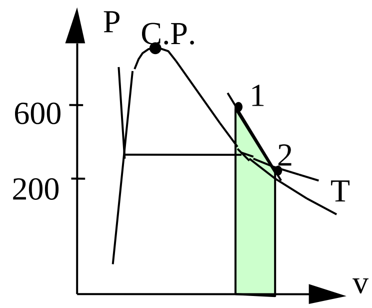
$$m_2 = m_1 = m \;;$$

Energy Eq.3.5: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Process Eq.: $P = A + BV$ (linearly in V)

State 1: (P, T) => $v_1 = 0.47424 \ m^3/kg$,

$u_1 = 2881.12$ kJ/kg



State 2: (P, T) => $v_2 = 0.95964 \ m^3/kg$, $u_2 = 2576.87$ kJ/kg

From process eq.: ${}_1W_2 = \int P\,dV = \text{area} = \frac{m}{2}(P_1 + P_2)(v_2 - v_1)$

$$= \frac{1.5}{2} \text{ kg } (200 + 600) \text{ kPa } (0.95964 - 0.47424) \text{ m}^3\text{/kg}$$

$= \mathbf{291.24\ kJ}$

From energy eq.: ${}_1Q_2 = m(u_2 - u_1) + {}_1W_2$

$= 1.5$ kg $(2576.87 - 2881.12)$ kJ/kg $+ 291.24$ kJ

$= \mathbf{-165.14\ kJ}$

3.114

Superheated refrigerant R-134a at 20°C, 0.5 MPa is cooled in a piston/cylinder arrangement at constant temperature to a final two-phase state with quality of 50%. The refrigerant mass is 5 kg, and during this process 500 kJ of heat is removed. Find the initial and final volumes and the necessary work.

Solution:

C.V. R-134a, this is a control mass.

Continuity: $m_2 = m_1 = m$;

Energy Eq.3.5: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2 = -500 \text{ kJ} - {}_1W_2$

State 1: T_1 , P_1 Table B.5.2, $v_1 = 0.04226 \text{ m}^3\text{/kg}$; $u_1 = 390.52 \text{ kJ/kg}$

=> $V_1 = mv_1 = \mathbf{0.211 \ m^3}$

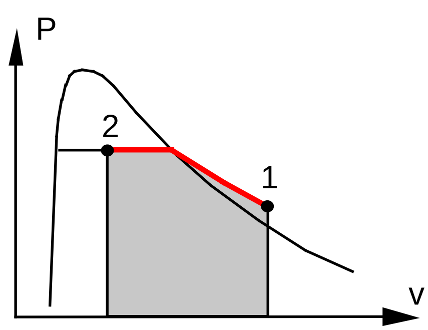
State 2: T_2 , x_2 $\Rightarrow$ Table B.5.1

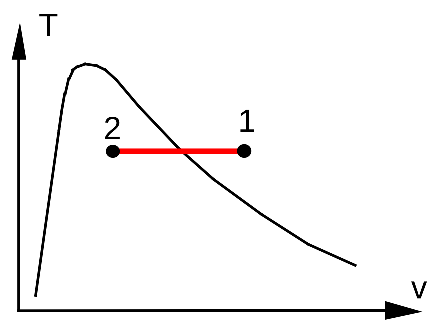
$u_2 = 227.03 + 0.5 \times 162.16 = 308.11 \text{ kJ/kg}$,

$v_2 = 0.000817 + 0.5 \times 0.03524 = 0.018437 \text{ m}^3\text{/kg}$ => $V_2 = mv_2 = \mathbf{0.0922 \ m^3}$

${}_1W_2 = -500 \text{ kJ} - m(u_2 - u_1) = -500 \text{ kJ} – 5 \text{ kg} \times (308.11 - 390.52) \text{ kJ/kg}$

$= \mathbf{-87.9 \ kJ}$





3.115

Two kilograms of nitrogen at 100 K, *x* = 0.5 is heated in a constant pressure process to 300 K in a piston/cylinder arrangement. Find the initial and final volumes and the total heat transfer required.

Solution:

Take CV as the nitrogen.

Continuity Eq.: $m_2 = m_1 = m$;

Energy Eq.3.5: $m(u_2 – u_1) = {}_1Q_2 - {}_1W_2$

Process: P = constant $\Rightarrow$ ${}_1W_2 = \int PdV = Pm(v_2 - v_1)$

State 1: Table B.6.1

$v_1 = 0.001452 + 0.5 \times 0.02975 = 0.01633\ m^3/kg$, $V_1 = \mathbf{0.0327\ m^3}$

$h_1 = -73.20 + 0.5 \times 160.68 = 7.14$ kJ/kg

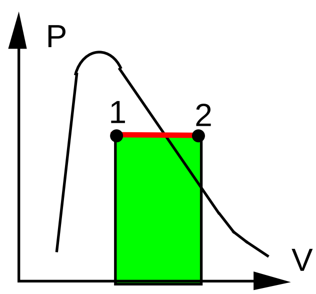
State 2: (P = 779.2 kPa , 300 K) => sup. vapor interpolate in Table B.6.2

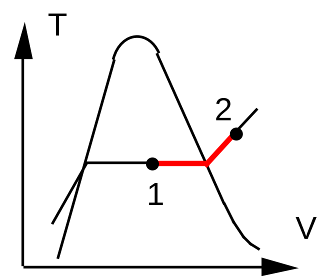
$v_2 = 0.14824 + (0.11115-0.14824)\times 179.2/200 = 0.115\ m^3/kg$, $V_2 = \mathbf{0.23\ m^3}$

$h_2 = 310.06 + (309.62-310.06) \times 179.2/200 = 309.66$ kJ/kg

Now solve for the heat transfer from the energy equation

${}_1Q_2 = m(u_2 - u_1) + {}_1W_2 = m(h_2 - h_1) = 2\ kg \times (309.66 - 7.14)\ kJ/kg = \mathbf{605\ kJ}$





Energy Equation: Solids and Liquids

3.116

In a sink 5 liters of water at 70^{o}C is combined with 1 kg aluminum pots, 1 kg of flatware (steel) and 1 kg of glass all put in at 20^{o}C. What is the final uniform temperature neglecting any heat loss and work?

Energy Eq.: $U_2 - U_1 = \sum m_i(u_2 - u_1)_i = {}_1Q_2 - {}_1W_2 = 0$

For the water: $v_f = 0.001023$ m^3/kg, V = 5 L = 0.005 m^3; m = V/v = 4.8876 kg

For the liquid and the metal masses we will use the specific heats (Tbl A.3, A.4) so

$$\sum m_i(u_2 - u_1)_i = \sum m_iC_{v\,i}\,(T_2 - T_1)_i = T_2\sum m_iC_{v\,i} - \sum m_iC_{v\,i}T_{1\,i}$$

noticing that all masses have the same T_2 but not same initial T.

$$\sum m_iC_{v\,i} = 4.8876 \times 4.18 + 1 \times 0.9 + 1 \times 0.46 + 1 \times 0.8 = 22.59 \text{ kJ/K}$$

Energy Eq.: $22.59\ T_2 = 4.8876 \times 4.18 \times 70 + (1 \times 0.9 + 1 \times 0.46 + 1 \times 0.8) \times 20$

$= 1430.11 + 43.2 = 1473.3$ kJ

$T_2 =$ **65.2^{o}C**

3.117

A computer CPU chip consists of 50 g silicon, 20 g copper, 50 g polyvinyl chloride (plastic). It heats from 15^oC to 70^oC as the computer is turned on. How much energy does the heating require?

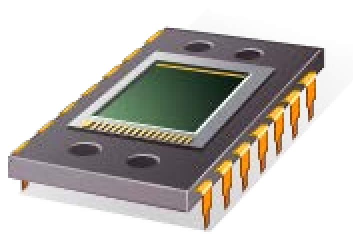
Energy Eq.: $U_2 - U_1 = \sum m_i (u_2 - u_1)_i = {}_1Q_2 - {}_1W_2$

For the solid masses we will use the specific heats, Table A.3, and they all have the same temperature so

$$\sum m_i(u_2 - u_1)_i = \sum m_i C_{v\,i} (T_2 - T_1)_i = (T_2 - T_1)\sum m_i C_{v\,i}$$

$$\sum m_i C_{v\,i} = 0.05 \times 0.7 + 0.02 \times 0.42 + 0.05 \times 0.96 = 0.0914 \text{ kJ/K}$$

$$U_2 - U_1 = 0.0914 \text{ kJ/K} \times (70 - 15) \text{ K} = \mathbf{5.03\ kJ}$$



3.118

A copper block of volume 1 L is heat treated at 500°C and now cooled in a 200-L oil bath initially at 20°C, shown in Fig. P3.118. Assuming no heat transfer with the surroundings, what is the final temperature?

Solution:

C.V. Copper block and the oil bath.

Also assume no change in volume so the work will be zero.

Energy Eq.: $U_2 - U_1 = m_{met}(u_2 - u_1)_{met} + m_{oil}(u_2 - u_1)_{oil} = {}_1Q_2 - {}_1W_2 = 0$

Properties from Table A.3 and A.4

$$m_{met} = V\rho = 0.001 \text{ m}^3 \times 8300 \text{ kg/m}^3 = 8.3 \text{ kg},$$

$$m_{oil} = V\rho = 0.2 \text{ m}^3 \times 910 \text{ kg/m}^3 = 182 \text{ kg}$$

Solid and liquid Eq.3.32: $\Delta u \cong C_V \,\Delta T$,

Table A.3 and A.4: $C_{V\ met} = 0.42 \dfrac{\text{kJ}}{\text{kg K}}$, $C_{V\ oil} = 1.8 \dfrac{\text{kJ}}{\text{kg K}}$

The energy equation for the C.V. becomes

$$m_{met}C_{V\ met}(T_2 - T_{1,met}) + m_{oil}C_{V\ oil}(T_2 - T_{1,oil}) = 0$$

$$8.3 \text{ kg} \times 0.42 \frac{\text{kJ}}{\text{kg K}} (T_2 - 500 \text{ C}) + 182 \text{ kg} \times 1.8 \frac{\text{kJ}}{\text{kg K}} (T_2 - 20 \text{ C}) = 0$$

$$331.09\ T_2 - 1743 - 6552 = 0$$

$$\Rightarrow\ T_2 = \mathbf{25\ °C}$$

3.119

A 1 kg steel pot contains 1 kg liquid water both at 15^{o}C. It is now put on the stove where it is heated to the boiling point of the water. Neglect any air being heated and find the total amount of energy needed.

Solution:

Energy Eq.: $U_2 - U_1 = {}_1Q_2 - {}_1W_2$

The steel does not change volume and the change for the liquid is minimal, so ${}_1W_2 \cong 0$.

State 2: $T_2 = T_{sat}$ (1atm) = 100^{o}C

Tbl B.1.1 : u_1 = 62.98 kJ/kg, u_2 = 418.91 kJ/kg

Tbl A.3 : C_{st} = 0.46 kJ/kg K

Solve for the heat transfer from the energy equation

$$ {}_1Q_2 = U_2 - U_1 = m_{st} (u_2 - u_1)_{st} + m_{H2O} (u_2 - u_1)_{H2O} $$
$$ = m_{st}C_{st} (T_2 - T_1) + m_{H2O} (u_2 - u_1)_{H2O} $$

$$ {}_1Q_2 = 1 \text{ kg} \times 0.46 \frac{\text{kJ}}{\text{kg K}} \times (100 - 15) \text{ K} + 1 \text{ kg} \times (418.91 - 62.98) \text{ kJ/kg} $$
$$ = 39.1 + 355.93 = \mathbf{395\ kJ} $$

3.120

I have 2 kg of liquid water at 20^{o}C, 100 kPa. I now add 20 kJ of energy at a constant pressure. How hot does it get if it is heated? How fast does it move if it is pushed by a constant horizontal force? How high does it go if it is raised straight up?

a) Heat at 100 kPa.

Energy equation:

$$E_2 - E_1 = {}_1Q_2 - {}_1W_2 = {}_1Q_2 - P(V_2 - V_1) = H_2 - H_1 = m(h_2 - h_1)$$

$$h_2 = h_1 + {}_1Q_2/m = 83.94 + 20/2 = 94.04 \text{ kJ/kg}$$

Back interpolate in Table B.1.1: $T_2 = \mathbf{22.5^oC}$

[We could also have used $\Delta T = {}_1Q_2/mC = 20 / (2*4.18) = 2.4^oC$]

b) Push at constant P. It gains kinetic energy.

$$0.5 \text{ m } \mathbf{V}_2^2 = {}_1W_2$$

$$\mathbf{V}_2 = \sqrt{2\ {}_1W_2/m} = \sqrt{2 \times 20 \times 1000 \text{ J/2 kg}} = \mathbf{141.4 \text{ m/s}}$$

c) Raised in gravitational field

$$m\ g\ Z_2 = {}_1W_2$$

$$Z_2 = {}_1W_2/m\ g = \frac{20\ 000 \text{ J}}{2 \text{ kg} \times 9.807 \text{ m/s}^2} = \mathbf{1019 \text{ m}}$$

Comment: Notice how fast (500 km/h) and how high it should be to have the same energy as raising the temperature just 2 degrees. I.e. in most applications we can disregard the kinetic and potential energies unless we have very high **V** or Z.

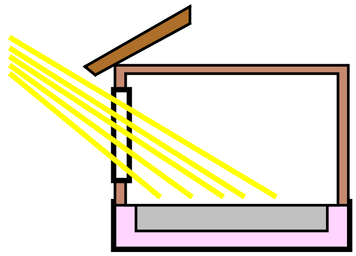
3.121

A house is being designed to use a thick concrete floor mass as thermal storage material for solar energy heating. The concrete is 30 cm thick and the area exposed to the sun during the daytime is 4 m × 6 m. It is expected that this mass will undergo an average temperature rise of about 3°C during the day. How much energy will be available for heating during the nighttime hours?

Solution:

C.V.: Control mass concrete.

$$V = 4 \text{ m} \times 6 \text{ m} \times 0.3 \text{ m} = 7.2 \text{ m}^3$$



Concrete is a solid with some properties listed in Table A.3

$$m = \rho V = 2200 \text{ kg/m}^3 \times 7.2 \text{ m}^3 = 15\,840 \text{ kg}$$

Energy Eq.: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2 = {}_1Q_2$

The available heat transfer is the change in U. From Eq.3.33 and C from table A.3

$$\Delta U = m\, C\, \Delta T = 15\,840 \text{ kg} \times 0.88 \frac{\text{kJ}}{\text{kg K}} \times 3 \text{ K} = 41\,818 \text{ kJ} = \mathbf{41.82 \text{ MJ}}$$

3.122

Because a hot water supply must also heat some pipe mass as it is turned on so it does not come out hot right away. Assume 80^{o}C liquid water at 100 kPa is cooled to 45^{o}C as it heats 15 kg of copper pipe from 20 to 45^{o}C. How much mass (kg) of water is needed?

Solution:

C.V. Water and copper pipe. No external heat transfer, no work.

Energy Eq.3.5: $U_2 - U_1 = \Delta U_{cu} + \Delta U_{H_2O} = 0 - 0$

From Eq.3.33 and Table A.3:

$$\Delta U_{cu} = mC\ \Delta T = 15\text{ kg} \times 0.42\ \frac{\text{kJ}}{\text{kg K}} \times (45 - 20)\text{ K} = 157.5\text{ kJ}$$

From the energy equation

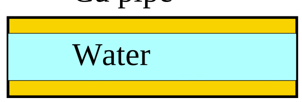
$$m_{H_2O} = -\ \Delta U_{cu}\ /\ \Delta u_{H_2O}$$

$$m_{H_2O} = \ \Delta U_{cu}\ /\ C_{H_2O}(-\ \Delta T_{H_2O}) = \frac{157.5}{4.18 \times 35}\ = \mathbf{1.076\ kg}$$

or using Table B.1.1 for water

$$m_{H_2O} = \ \Delta U_{cu}\ /\ (\ u_1 - u_2) = \frac{157.5}{334.84 - 188.41}\ \frac{\text{kJ}}{\text{kJ/kg}} = \mathbf{1.076\ kg}$$

Cu pipe



The real problem involves a flow and is not analyzed by this simple process.

3.123

A car with mass 1275 kg drives at 60 km/h when the brakes are applied quickly to decrease its speed to 20 km/h. Assume the brake pads are 0.5 kg mass with heat capacity of 1.1 kJ/kg K and the brake discs/drums are 4.0 kg steel. Further assume both masses are heated uniformly. Find the temperature increase in the brake assembly.

Solution:

C.V. Car. Car loses kinetic energy and brake system gains internal u.

No heat transfer (short time) and no work term.

m = constant;

Energy Eq.3.5: $E_2 - E_1 = 0 - 0 = m_{car} \frac{1}{2}(V_2^2 - V_1^2) + m_{brake}(u_2 - u_1)$

The brake system mass is two different kinds so split it, also use C_v from Table A.3 since we do not have a u table for steel or brake pad material.

$$m_{steel}\, C_v\, \Delta T \;+ m_{pad}\, C_v\, \Delta T \;=\; m_{car}\, 0.5\, (60^2 - 20^2) \left(\frac{1000}{3600}\right)^2 \text{ m}^2/\text{s}^2$$

$$(4 \times 0.46 + 0.5 \times 1.1)\, \frac{\text{kJ}}{\text{K}}\, \Delta T \;= 1275 \text{ kg} \times 0.5 \times (3200 \times 0.077\,16) \text{ m}^2/\text{s}^2$$

$$= 157\,406 \text{ J} = 157.4 \text{ kJ}$$

$$\Rightarrow \Delta T \;= \mathbf{65.9\ °C}$$

3.124

A piston cylinder (0.5 kg steel altogether) maintaining a constant pressure has 0.2 kg R-134a as saturated vapor at 150 kPa. It is heated to 40$^{\mathrm{o}}$C and the steel is at the same temperature as the R-134a at any time. Find the work and heat transfer for the process.

C.V. The R-134a plus the steel. Constant total mass

$m_2 = m_1 = m$;

$U_2 - U_1 = m_{R134a}(u_2 - u_1)_{R134a} + m_{steel}(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

State 1: B.5.2 sat. vapor $v_1 = 0.13139\ m^3/kg$, $u_1 = 368.06\ kJ/kg$

State 2: B.5.2 sup. vapor $v_2 = 0.16592\ m^3/kg$, $u_2 = 411.59\ kJ/kg$

$V_1 = mv_1 = 0.2 \times 0.13139 = 0.02628\ m^3$

$V_2 = mv_2 = 0.2 \times 0.16592 = 0.03318\ m^3$

Steel: A.3, $C_{steel} = 0.46$ kJ/kg-K

Process: P = C for the R134a and constant volume for the steel =>

$${}_1W_2 = \int P\,dV = P_1(V_2 - V_1) = 150\ kPa \times (0.03318 - 0.02628)\ m^3$$
$$= \mathbf{1.035\ kJ}$$

$${}_1Q_2 = m_{R134a}\,(u_2 - u_1) + m_{steel}\,(u_2 - u_1) + {}_1W_2$$
$$= m_{R134a}\,(u_2 - u_1) + m_{steel}C_{steel}\,(T_2 - T_1) + {}_1W_2$$
$$= 0.2 \times (411.59 - 368.06) + 0.5 \times 0.46 \times [40 - (-17.29)] + 1.035$$
$$= 8.706 + 13.177 + 1.035 = \mathbf{22.92\ kJ}$$

3.125

A 25 kg steel tank initially at -10^{o}C is filled up with 100 kg of milk (assume properties as water) at 30^{o}C. The milk and the steel come to a uniform temperature of +5 ^{o}C in a storage room. How much heat transfer is needed for this process?

Solution:

C.V. Steel + Milk. This is a control mass.

Energy Eq.3.5: $U_2 - U_1 = {}_1Q_2 - {}_1W_2 = {}_1Q_2$

Process: V = constant, so there is no work

$${}_1W_2 = 0.$$

Use Eq.3.33 and values from A.3 and A.4 to evaluate changes in u

$${}_1Q_2 = m_{steel}\,(u_2 - u_1)_{steel} + m_{milk}(u_2 - u_1)_{milk}$$

$$= 25 \text{ kg} \times 0.46 \frac{\text{kJ}}{\text{kg K}} \times [5 - (-10)] \text{ K} + 100 \text{ kg} \times 4.18 \frac{\text{kJ}}{\text{kg K}} \times (5 - 30) \text{ K}$$

$$= 172.5 - 10450 = \mathbf{-10277 \text{ kJ}}$$

3.126

An engine consists of a 100 kg cast iron block with a 20 kg aluminum head, 20 kg steel parts, 5 kg engine oil and 6 kg glycerine (antifreeze). Everything begins at 5^o C and as the engine starts we want to know how hot it becomes if it absorbs a net of 7000 kJ before it reaches a steady uniform temperature.

Energy Eq.: $U_2 - U_1 = {}_1Q_2 - {}_1W_2$

Process: The steel does not change volume and the change for the liquid is minimal, so ${}_1W_2 \cong 0$.

So sum over the various parts of the left hand side in the energy equation

$$m_{Fe}\,(u_2 - u_1) + m_{Al}\,(u_2 - u_1)_{Al} + m_{st}\,(u - u_1)_{st}$$
$$+ m_{oil}\,(u_2 - u_1)_{oil} + m_{gly}\,(u_2 - u_1)_{gly} = {}_1Q_2$$

Table A.3 : $C_{Fe} = 0.42$, $C_{Al} = 0.9$, $C_{st} = 0.46$ all units of kJ/kg K

Table A.4 : $C_{oil} = 1.9$, $C_{gly} = 2.42$ all units of kJ/kg K

So now we factor out $T_2 - T_1$ as $u_2 - u_1 = C(T_2 - T_1)$ for each term

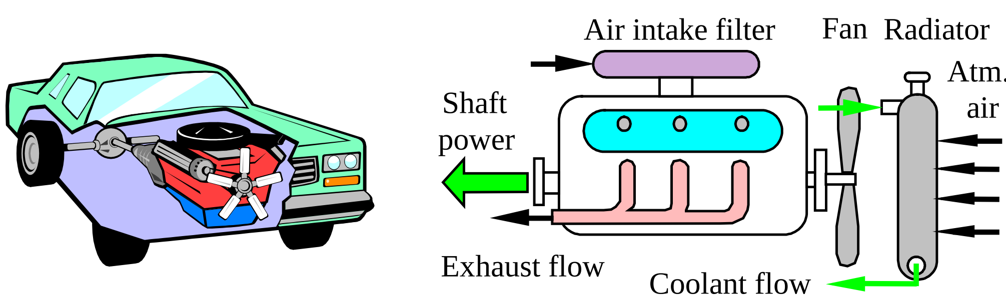
$$[\, m_{Fe}C_{Fe} + m_{Al}C_{Al} + m_{st}C_{st} + m_{oil}C_{oil} + m_{gly}C_{gly} \,]\,(T_2 - T_1) = {}_1Q_2$$

$$T_2 - T_1 = {}_1Q_2 \,/\, \Sigma m_i\, C_i$$

$$= \frac{7000}{100\times 0.42 + 20\times 0.9 + 20\times 0.46 + 5\times 1.9 + 6\times 2.42}\,\frac{\text{kJ}}{\text{kJ/K}}$$

$$= \frac{7000}{93.22}\text{ K} = 75\text{ K}$$

$$T_2 = T_1 + 75^oC = 5 + 75 = \mathbf{80^oC}$$



Properties (u, h, C_v and C_p), Ideal Gas

3.127

An ideal gas is heated from 500 to 1500 K. Find the change in enthalpy using constant specific heat from Table A.5 (room temperature value) and discuss the accuracy of the result if the gas is

a. Argon b. Oxygen c. Carbon dioxide

Solution:

$$T_1 = 500 \text{ K}, T_2 = 1500 \text{ K}, \quad \Delta h = C_{P0}(T_2\text{-}T_1)$$

a) Ar : $\Delta h = 0.520$ kJ/kg-K $\times$ (1500-500) K = 520 kJ/kg

Monatomic inert gas very good approximation.

b) O_2 : $\Delta h = 0.922$ kJ/kg-K $\times$ (1500-500) K = 922 kJ/kg

Diatomic gas approximation is OK with some error.

c) CO_2: $\Delta h = 0.842$ kJ/kg-K $\times$ (1500-500) K = 842 kJ/kg

Polyatomic gas heat capacity changes, see figure 3.26

See also appendix C for more explanation.

3.128

Use the ideal gas air table A.7 to evaluate the heat capacity C_p at 300 K as a slope of the curve h(T) by Δh/ΔT. How much larger is it at 1000 K and 1500 K.

Solution :

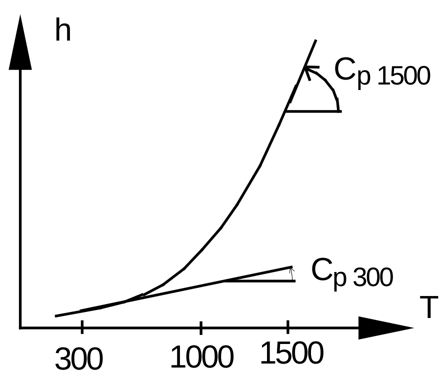
From Eq.3.39:

$$C_p = \frac{dh}{dT} = \frac{\Delta h}{\Delta T} = \frac{h_{320} - h_{290}}{320 - 290} = \mathbf{1.005\ kJ/kg\ K}$$

$$1000\text{ K} \quad C_p = \frac{\Delta h}{\Delta T} = \frac{h_{1050} - h_{950}}{1050 - 950} = \frac{1103.48 - 989.44}{100} = \mathbf{1.140\ kJ/kg\ K}$$

$$1500\text{ K} \quad C_p = \frac{\Delta h}{\Delta T} = \frac{h_{1550} - h_{1450}}{1550 - 1450} = \frac{1696.45 - 1575.4}{100} = \mathbf{1.21\ kJ/kg\ K}$$

Notice an increase of 14%, 21% respectively.



3.129

Estimate the constant specific heats for R-134a from Table B.5.2 at 100 kPa and 125^{o}C. Compare this to table A.5 and explain the difference.

Solution:

Using values at 100 kPa for h and u at 120^{o}C and 130^{o}C from Table B5.2, the approximate specific heats at 125^{o}C are

$$C_p \approx \frac{\Delta h}{\Delta T} = \frac{521.98 - 511.95}{130 - 120} = 1.003 \text{ kJ/kg K}$$

compared with 0.852 kJ/kg K for the ideal-gas value at 25^{o}C from Table A.5.

$$C_v \approx \frac{\Delta u}{\Delta T} = \frac{489.36 - 480.16}{130 - 120} = 0.920 \text{ kJ/kg K}$$

compared with 0.771 kJ/kg K for the ideal-gas value at 25^{o}C from Table A.5.

There are two reasons for the differences. First, R-134a is not exactly an ideal gas at the given state, 125^{o}C and 100 kPa. Second and by far the biggest reason for the differences is that R-134a, chemically CF_3CH_2 , is a polyatomic molecule with multiple vibrational mode contributions to the specific heats (see Appendix C), such that they are strongly dependent on temperature. Note that if we repeat the above approximation for C_p in Table B.5.2 at 25^{o}C, the resulting value is 0.851 kJ/kg K.

3.130

We want to find the change in u for carbon dioxide between 600 K and 1200 K.

a) Find it from a constant C_{vo} from table A.5

b) Find it from a C_{vo} evaluated from equation in A.6 at the average T.

c) Find it from the values of u listed in table A.8

Solution :

a) $\Delta u \cong C_{vo}\, \Delta T = 0.653$ kJ/kg-K × (1200 – 600) K = **391.8 kJ/kg**

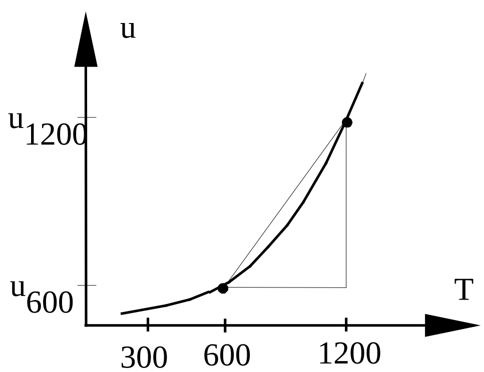
b) $T_{avg} = \frac{1}{2}(1200 + 600) = 900, \quad \theta = \frac{T}{1000} = \frac{900}{1000} = 0.9$

$C_{po} = 0.45 + 1.67 \times 0.9 - 1.27 \times 0.9^2 + 0.39 \times 0.9^3 = 1.2086$ kJ/kg K

$C_{vo} = C_{po} – R = 1.2086 – 0.1889 = 1.0197$ kJ/kg K

$\Delta u = 1.0197 \times (1200 – 600) =$ **611.8 kJ/kg**

c) $\Delta u = 996.64 – 392.72 =$ **603.92 kJ/kg**



3.131

Nitrogen at 300 K, 3 MPa is heated to 500 K. Find the change in enthalpy using a) Table B.6, b) Table A.8, and c) Table A.5.

B.6.2 $h_2 - h_1 = 519.29 - 304.94 = 214.35$ kJ/kg

A.8 $h_2 - h_1 = 520.75 - 311.67 = 209.08$ kJ/kg

A.5 $h_2 - h_1 = C_{po} (T_2 - T_1) = 1.042$ kJ/kg-K × (500 – 300) K

$= 208.4$ kJ/kg

Comment: The results are listed in order of accuracy (B.6.2 best).

3.132

We want to find the change in u for carbon dioxide between 50^{o}C and 200^{o}C at a pressure of 10 MPa. Find it using ideal gas and Table A.5 and repeat using the B section table.

Solution:

Using the value of C_{vo} for CO_2 from Table A.5,

$\Delta u = C_{vo} \Delta T = 0.653$ kJ/kg-K × (200 – 50) K = **97.95 kJ/kg**

Using values of u from Table B3.2 at 10 000 kPa, with linear interpolation between 40^{o}C and 60^{o}C for the 50^{o}C value,

$\Delta u = u_{200} - u_{50} = 437.6 - 230.9 =$ **206.7 kJ/kg**

Note: Since the state 50^{o}C, 10 000 kPa is in the dense-fluid supercritical region, a linear interpolation is quite inaccurate. The proper value for u at this state is found from the CATT software to be 245.1 instead of 230.9. This results is

$\Delta u = u_{200} - u_{50} = 437.6 - 245.1 =$ **192.5 kJ/kg**

3.133

We want to find the change in u for oxygen gas between 600 K and 1200 K.

a) Find it from a constant C_{vo} from table A.5

b) Find it from a C_{vo} evaluated from equation in A.6 at the average T.

c) Find it from the values of u listed in table A.8

Solution:

a) $\Delta u \cong C_{vo}\,\Delta T = 0.662$ kJ/kg-K × (1200 – 600) K = **397.2 kJ/kg**

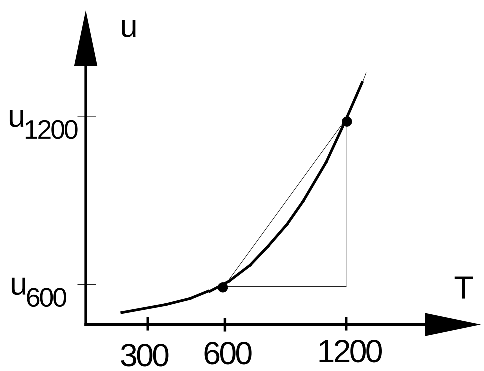
b) $T_{avg} = \frac{1}{2}(1200 + 600) = 900$ K, $\theta = \frac{T}{1000} = \frac{900}{1000} = 0.9$

$C_{po} = 0.88 - 0.0001 \times 0.9 + 0.54 \times 0.9^2 - 0.33 \times 0.9^3 = 1.0767$ kJ/kgK

$C_{vo} = C_{po} - R = 1.0767 - 0.2598 = 0.8169$ kJ/kg-K

$\Delta u = 0.8169 \times (1200 - 600) =$ **490.1 kJ/kg**

c) $\Delta u = 889.72 - 404.46 =$ **485.3 kJ/kg**



3.134

For a special application we need to evaluate the change in enthalpy for carbon dioxide from 30°C to 1500°C at 100 kPa. Do this using constant specific heat from Table A.5 and repeat using Table A.8. Which is the more accurate one?

Solution:

Using constant specific heat:

$$\Delta h = C_{po}\Delta T = 0.842\ (1500 - 30) = \mathbf{1237.7\ kJ/kg}$$

Using Table A.8:

$$30°\text{C} = 303.15\text{ K} \;\; => \;\; h = 214.38 + \frac{3.15}{50}\,(257.9 - 214.38) = 217.12\text{ kJ/kg}$$

$$1500°\text{C} = 1773.15\text{ K} \;\; =>$$

$$h = 1882.43 + \frac{73.15}{100}\,(2017.67 - 1882.43) = 1981.36\text{ kJ/kg}$$

$$\Delta h = 1981.36 - 217.12 = \mathbf{1764.2\ kJ/kg}$$

The result from A.8 is best. For large ΔT or small ΔT at high T_{avg}, constant specific heat is poor approximation or it must be evaluated at a higher T (A.5 is at 25°C).

3.135

Water at 150°C, 400 kPa, is brought to 1200°C in a constant pressure process. Find the change in the specific internal energy, using a) the steam tables, b) the ideal gas water table A.8, and c) the specific heat from A.5.

Solution:

a)

State 1: Table B.1.3 Superheated vapor $u_1 = 2564.48$ kJ/kg

State 2: Table B.1.3 $u_2 = 4467.23$ kJ/kg

$$u_2 - u_1 = 4467.23 - 2564.48 = \mathbf{1902.75\ kJ/kg}$$

b)

Table A.8 at 423.15 K: $u_1 = 591.41$ kJ/kg

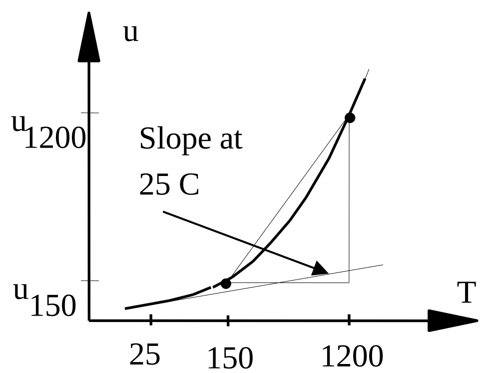
Table A.8 at 1473.15 K: $u_2 = 2474.25$ kJ/kg

$$u_2 - u_1 = 2474.25 - 591.41 = \mathbf{1882.8\ kJ/kg}$$

c) Table A.5 : $C_{vo} = 1.41$ kJ/kgK

$$u_2 - u_1 = 1.41 \text{ kJ/kgK} \times (1200 - 150) \text{ K} = \mathbf{1480.5\ kJ/kg}$$

Notice how the average slope from 150°C to 1200 °C is higher than the one at 25°C (= C_{vo})



3.136

Repeat the previous problem but use a constant specific heat at the average temperature from equation in Table A.6 and also integrate the equation in Table A.6 to get the change in enthalpy.

$T_{ave} = \frac{1}{2}(30 + 1500) + 273.15 = 1038.15$ K; $\theta = T/1000 = 1.0382$

Table A.6 $\Rightarrow$ $C_{po} = 1.2513$ kJ/kg-K

$\Delta h = C_{po,ave}\, \Delta T = 1.2513$ kJ/kg-K $\times$ 1470 K = **1839 kJ/kg**

For the entry to Table A.6:

$30°C = 303.15\text{ K} \;\Rightarrow\; \theta_1 = 0.30315$

$1500°C = 1773.15\text{ K} \;\Rightarrow\; \theta_2 = 1.77315$

$$\Delta h = h_2 - h_1 = \int C_{po}\, dT$$

$$= [0.45\,(\theta_2 - \theta_1) + 1.67 \times \frac{1}{2}(\theta_2^2 - \theta_1^2)$$

$$-1.27 \times \frac{1}{3}(\theta_2^3 - \theta_1^3) + 0.39 \times \frac{1}{4}(\theta_2^4 - \theta_1^4)]$$

= **1762.8 kJ/kg**

3.137

Water at 20°C, 100 kPa, is brought to 200 kPa, 1500°C. Find the change in the specific internal energy, using the water table and the ideal gas water table in combination.

Solution:

State 1: Table B.1.1 $u_1 \cong u_f = 83.95$ kJ/kg

State 2: Highest T in Table B.1.3 is 1300°C

Using a Δu from the ideal gas tables, A.8, we get

$u_{1500} = 3139$ kJ/kg $\qquad u_{1300} = 2690.72$ kJ/kg

$u_{1500} - u_{1300} = 448.26$ kJ/kg

We now add the ideal gas change at low P to the steam tables, B.1.3, u_X = 4683.23 kJ/kg as the reference.

$$u_2 - u_1 = (u_2 - u_X)_{ID.G.} + (u_X - u_1)$$

$$= 448.28 + 4683.23 - 83.95 = \mathbf{5048\ kJ/kg}$$

3.138

Reconsider Problem 3.134 and examine if also using Table B.3 would be more accurate and explain.

Table B.3 does include non-ideal gas effects, however at 100 kPa these effects are extremely small so the answer from Table A.8 is accurate.

Table B.3. does not cover the 100 kPa superheated vapor states as the saturation pressure is below the triple point pressure. Secondly Table B.3 does not go up to the high temperatures covered by Table A.8 and A.9 at which states you do have ideal gas behavior. Table B.3 covers the region of states where the carbon dioxide is close to the two-phase region and above the critical point (dense fluid) which are all states where you cannot assume ideal gas.

Specific Heats Ideal Gas

3.139

Air is heated from 300 to 350 K at V = C. Find ${}_1q_2$. What if from 1300 to 1350 K?

Process: V = C $\rightarrow$ ${}_1W_2 = \emptyset$

Energy Eq.: $u_2 - u_1 = {}_1q_2 - 0 \rightarrow {}_1q_2 = u_2 - u_1$

Read the u-values from Table A.7.1

a) ${}_1q_2 = u_2 - u_1 = 250.32 - 214.36 =$ **36.0 kJ/kg**

b) ${}_1q_2 = u_2 - u_1 = 1067.94 - 1022.75 =$ **45.2 kJ/kg**

case a) $C_v \approx 36/50 = 0.72$ kJ/kg K , see A.5

case b) $C_v \approx 45.2/50 = 0.904$ kJ/kg K (25 % higher)

3.140

A rigid container has 2 kg of carbon dioxide gas at 100 kPa, 1200 K that is heated to 1400 K. Solve for the heat transfer using a. the heat capacity from Table A.5 and b. properties from Table A.8

Solution:

C.V. Carbon dioxide, which is a control mass.

Energy Eq.3.5: $U_2 - U_1 = m (u_2 - u_1) = {}_1Q_2 - {}_1W_2$

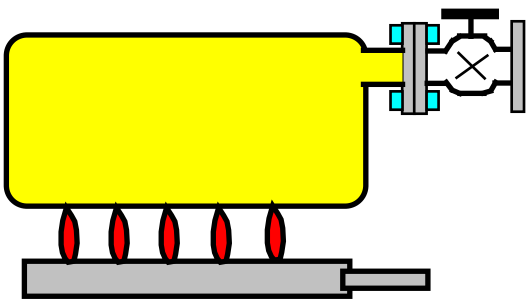
Process: $\Delta V = 0 \;\; \Rightarrow {}_1W_2 = 0$

a) For constant heat capacity we have: $u_2 - u_1 = C_{vo} (T_2 - T_1)$ so

$${}_1Q_2 \cong mC_{vo} (T_2 - T_1) = 2 \text{ kg} \times 0.653 \text{ kJ/kg-K} \times (1400 - 1200) \text{ K} = \mathbf{261.2 \text{ kJ}}$$

b) Taking the u values from Table A.8 we get

$${}_1Q_2 = m (u_2 - u_1) = 2 \text{ kg} \times (1218.38 - 996.64) \text{ kJ/kg} = \mathbf{443.5 \text{ kJ}}$$



3.141

Do the previous problem for nitrogen, N_2, gas.

A rigid container has 2 kg of carbon dioxide gas at 100 kPa, 1200 K that is heated to 1400 K. Solve for the heat transfer using a. the heat capacity from Table A.5 and b. properties from Table A.8

Solution:

C.V. Nitrogen gas, which is a control mass.

Energy Eq.3.5: $U_2 – U_1 = m (u_2 - u_1) = {}_1Q_2 – {}_1W_2$

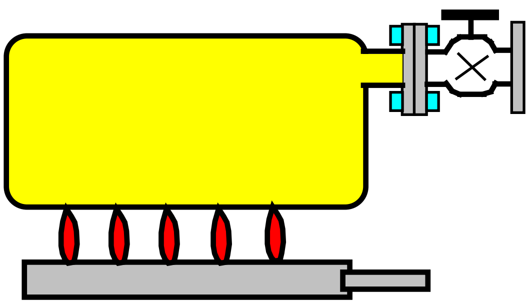
Process: $\Delta V = 0 \Rightarrow {}_1W_2 = 0$

a) For constant heat capacity we have: $u_2 - u_1 = C_{vo} (T_2 - T_1)$ so

$${}_1Q_2 \cong mC_{vo} (T_2 - T_1) = 2 \text{ kg} \times 0.745 \text{ kJ/kg-K} \times (1400 – 1200) \text{ K} = \mathbf{298 \text{ kJ}}$$

b) Taking the u values from Table A.8, we get

$${}_1Q_2 = m (u_2 - u_1) = 2 \text{ kg} \times (1141.35 – 957) \text{ kJ/kg} = \mathbf{368.7 \text{ kJ}}$$



3.142

Air, 3 kg, is in a piston/cylinder similar to Fig. P.3.5 at 27^{o}C, 300 kPa. It is now heated to 500 K. Plot the process path in a P-v diagram, and find the work and heat transfer in the process.

Solution:

CV Air, so this is a control mass.

Energy Eq.3.5: $U_2 - U_1 = m (u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Process: $P = C$ so ${}_1W_2 = \int PdV = P_1(V_2 - V_1)$

State 1: T_1, P_1 ideal gas so $P_1V_1 = mRT_1$

$V_1 = mR\,T_1 / P_1 = 3 \text{ kg} \times 0.287 \text{ kJ/kg-K} \times 300.15 \text{ K} / 300 \text{ kPa}$

$= 0.86143 \text{ m}^3$

State 2: $T_2, P_2 = P_1$ and ideal gas so $P_2V_2 = mRT_2$

$V_2 = mR\,T_2 / P_2 = 3 \times 0.287 \times 500/300 = 1.435 \text{ m}^3$

From the process

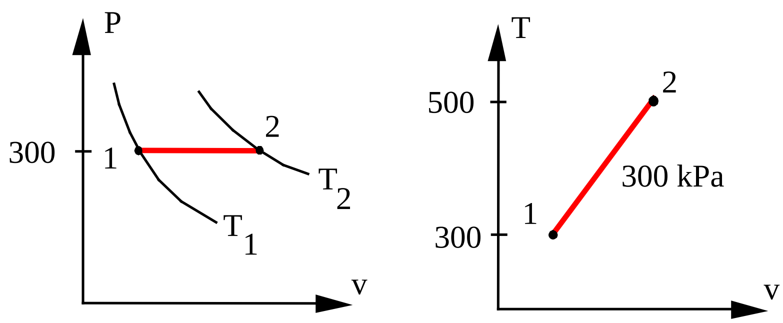
$${}_1W_2 = \int PdV = P (V_2 - V_1) = 300 \text{ kPa}(1.435 - 0.86143) \text{ m}^3 = \mathbf{172.1\ kJ}$$

From the energy equation

$${}_1Q_2 = m (u_2 - u_1) + {}_1W_2 = mC_{vo} (T_2 - T_1) + {}_1W_2$$

$= 3 \text{ kg} \times 0.717 \text{ kJ/kg-K} \times (500 - 300) \text{ K} + 172.1 \text{ kJ}$

$= \mathbf{602.3\ kJ}$



3.143

A closed rigid container is filled with 1.5 kg water at 100 kPa, 55^oC, 1 kg of stainless steel and 0.5 kg of PVC (polyvinyl chloride) both at 20^oC and 0.1 kg of air at 400 K, 100 kPa. It is now left alone with no external heat transfer and no water vaporizes. Find the final temperature and air pressure.

CV. Container.

Process: V = constant => ${}_1W_2 = 0$ and also given ${}_1Q_2 = 0$

Energy Eq.: $U_2 - U_1 = \sum m_i(u_2 - u_1)_i = {}_1Q_2 - {}_1W_2 = 0$

For the liquid and the metal masses we will use the specific heats (Tbl A.3, A.4) so

$$\sum m_i(u_2 - u_1)_i = \sum m_iC_{v\,i}\,(T_2 - T_1)_i = T_2 \sum m_iC_{v\,i} - \sum m_iC_{v\,i}T_{1\,i}$$

noticing that all masses have the same T_2 but not same initial T.

$$\sum m_iC_{v\,i} = 1.5 \times 4.18 + 1 \times 0.46 + 0.5 \times 0.96 + 0.1 \times 0.717 = 7.282 \text{ kJ/K}$$

The T for air must be converted to ^oC like the others.

Energy Eq.: $T_2 \sum m_iC_{v\,i} = \sum m_iC_{v\,i}T_{1\,i}$

$$7.282\ T_2 = 1.5 \times 4.18 \times 55 + (1 \times 0.46 + 0.5 \times 0.96) \times 20$$

$$+ 0.1 \times 0.717 \times (400\text{-}273.15) = 372.745 \text{ kJ}$$

$$T_2 = \mathbf{51.2^oC}$$

The volume of the air is constant so from PV = mRT it follows that P varies with T

$$P_2 = P_1\ T_2/T_{1\,air} = 100 \text{ kPa} \times 324.34 \text{ K}/\ 400 \text{ K} = \mathbf{81\ kPa}$$

3.144

A 250 L rigid tank contains methane at 500 K, 1500 kPa. It is now cooled down to 300 K. Find the mass of methane and the heat transfer using ideal gas.

Solution:

C.V. Methane gas, which is a control mass.

Energy Eq.3.5: $U_2 – U_1 = m (u_2 - u_1) = {}_1Q_2 – {}_1W_2$

Process: $\Delta V = 0 \Rightarrow {}_1W_2 = 0$

State 1: Ideal gas so

$$m = P_1V/RT_1 = \frac{1500 \times 0.25}{0.5183 \times 500} \frac{\text{kPa m}^3}{\text{(kJ/kg-K)} \times \text{K}} = \mathbf{1.447\ kg}$$

For constant heat capacity A.5, we have: $u_2 - u_1 = C_{vo} (T_2 - T_1)$ so

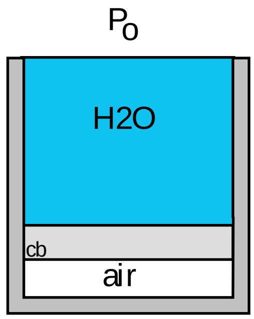
$$u_2 - u_1 = C_v (T_2 – T_1) = 1.736 \text{ kJ/kg-K} \times (300 – 500) \text{ K} = –347.2 \text{ kJ/kg}$$

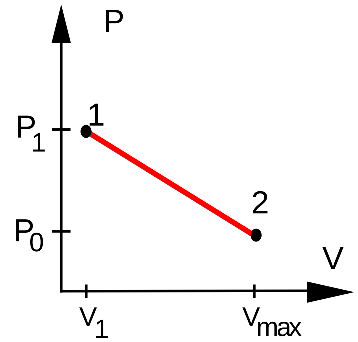
$${}_1Q_2 = m(u_2 - u_1) = 1.447 \text{ kg } (-347.2) \text{ kJ/kg} = \mathbf{–502.4\ kJ}$$

3.145

A 10-m high cylinder, cross-sectional area 0.1 m^2, has a massless piston at the bottom with water at 20°C on top of it, shown in Fig. P3.145. Air at 300 K, volume 0.3 m^3, under the piston is heated so that the piston moves up, spilling the water out over the side. Find the total heat transfer to the air when all the water has been pushed out.

Solution:





The water on top is compressed liquid and has volume and mass

$$V_{H_2O} = V_{tot} - V_{air} = 10 \times 0.1 - 0.3 = 0.7 \text{ m}^3$$

$$m_{H_2O} = V_{H_2O}/v_f = 0.7 \text{ m}^3/\ 0.001002 \text{ m}^3\text{/kg} = 698.6 \text{ kg}$$

The initial air pressure is then

$$P_1 = P_0 + m_{H_2O}g/A = 101.325 + \frac{698.6 \times 9.807}{0.1 \times 1000} = \mathbf{169.84\ kPa}$$

and then $\quad m_{air} = PV/RT = \dfrac{169.84 \times 0.3}{0.287 \times 300}\ \dfrac{\text{kPa m}^3}{\text{(kJ/kg-K)} \times \text{K}} = 0.592 \text{ kg}$

State 2: No liquid over piston: $\quad P_2 = P_0 = 101.325$ kPa, $\quad V_2 = 10{\times}0.1 = 1 \text{ m}^3$

State 2: $P_2, V_2 \quad \Rightarrow \quad T_2 = \dfrac{T_1P_2V_2}{P_1V_1} = \dfrac{300{\times}101.325{\times}1}{169.84{\times}0.3} = 596.59$ K

The process line shows the work as an area

$$_1W_2 = \int PdV = \frac{1}{2}(P_1 + P_2)(V_2 - V_1) = \frac{1}{2}(169.84 + 101.325)(1 - 0.3) = 94.91 \text{ kJ}$$

The energy equation solved for the heat transfer becomes

$$_1Q_2 = m(u_2 - u_1) + {_1W_2} \cong mC_v(T_2 - T_1) + {_1W_2}$$

$$= 0.592 \text{ kg} \times 0.717 \text{ kJ/kg-K} \times (596.59 - 300) \text{ K} + 94.91 \text{ kJ} = \mathbf{220.7\ kJ}$$

Remark: we could have used u values from Table A.7:

$u_2 - u_1 = 432.5 - 214.36 = 218.14$ kJ/kg $\quad$ versus 212.5 kJ/kg with C_v.

3.146

A cylinder with a piston restrained by a linear spring contains 2 kg of carbon dioxide at 500 kPa, 400°C. It is cooled to 40°C, at which point the pressure is 300 kPa. Calculate the heat transfer for the process.

Solution:

C.V. The carbon dioxide, which is a control mass.

Continuity Eq.: $m_2 – m_1 = 0$

Energy Eq.: $m (u_2 – u_1) = {}_1Q_2 - {}_1W_2$

Process Eq.: $P = A + BV$ (linear spring)

$${}_1W_2 = \int PdV = \frac{1}{2}(P_1 + P_2)(V_2 - V_1)$$

Equation of state: $PV = mRT$ (ideal gas)

State 1: $V_1 = mRT_1/P_1 = 2 \times 0.18892 \times 673.15 /500 = 0.5087 \text{ m}^3$

State 2: $V_2 = mRT_2/P_2 = 2 \times 0.18892 \times 313.15 /300 = 0.3944 \text{ m}^3$

$${}_1W_2 = \frac{1}{2}(500 + 300)(0.3944 - 0.5087) = -45.72 \text{ kJ}$$

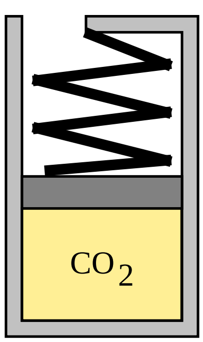
To evaluate $u_2 - u_1$ we will use the specific heat at the average temperature.

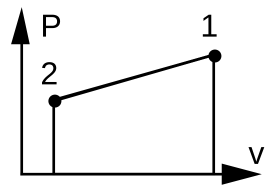
From Figure 3.26: $C_{po}(T_{avg}) \approx 5.4 \text{ R} = 1.02 \Rightarrow C_{vo} = 0.83 = C_{po} - R$

For comparison the value from Table A.5 at 300 K is $C_{vo} = 0.653$ kJ/kg K

$${}_1Q_2 = m(u_2 - u_1) + {}_1W_2 = mC_{vo}(T_2 - T_1) + {}_1W_2$$

$$= 2 \times 0.83(40 - 400) - 45.72 = \mathbf{-643.3 \text{ kJ}}$$





Remark: We could also have used the ideal gas table in A.8 to get $u_2 - u_1$.

3.147

Water at 100 kPa, 400 K is heated electrically adding 700 kJ/kg in a constant pressure process. Find the final temperature using

a) The water tables B.1 b) The ideal gas tables A.8

c) Constant specific heat from A.5

Solution :

Energy Eq.3.5: $u_2 - u_1 = {}_1q_2 - {}_1w_2$

Process: $P = \text{constant} \Rightarrow {}_1w_2 = P (v_2 - v_1)$

Substitute this into the energy equation to get

$${}_1q_2 = h_2 - h_1$$

Table B.1:

$$h_1 \cong 2675.46 + \frac{126.85 - 99.62}{150 - 99.62} \times (2776.38 – 2675.46) = 2730.0 \text{ kJ/kg}$$

$$h_2 = h_1 + {}_1q_2 = 2730 + 700 = 3430 \text{ kJ/kg}$$

$$T_2 = 400 + (500 – 400) \times \frac{3430 - 3278.11}{3488.09 - 3278.11} = \mathbf{472.3°C}$$

Table A.8:

$$h_2 = h_1 + {}_1q_2 = 742.4 + 700 = 1442.4 \text{ kJ/kg}$$

$$T_2 = 700 + (750 – 700) \times \frac{1442.4 - 1338.56}{1443.43 - 1338.56} = 749.5 \text{ K} = \mathbf{476.3°C}$$

Table A.5

$$h_2 - h_1 \cong C_{po} (T_2 - T_1)$$

$$T_2 = T_1 + {}_1q_2 / C_{po} = 400 + 700 / 1.872 = 773.9\text{K} = \mathbf{500.8°C}$$

3.148

A constant pressure container is filled with 1.5 kg water at 100 kPa, 55^{o}C, 1 kg of stainless steel and 0.5 kg of PVC (polyvinyl chloride) both at 20^{o}C and 0.1 kg of air at 400 K, 100 kPa. It is now left alone with no external heat transfer and no water vaporizes. Find the final temperature and process work.

Energy Eq.: $U_2 - U_1 = \sum m_i(u_2 - u_1)_i = {}_1Q_2 - {}_1W_2 = 0$

Process: $P = \text{constant} \Rightarrow {}_1W_2 = P(V_2 - V_1)$ and given ${}_1Q_2 = 0$

For the liquid and the metal masses we will use the specific heats (Tbl A.3, A.4) so

$$\sum m_i(u_2 - u_1)_i = \sum m_iC_{v\,i}\,(T_2 - T_1)_i = T_2\sum m_iC_{v\,i} - \sum m_iC_{v\,i}T_{1\,i}$$

noticing that all masses have the same T_2 but not same initial T. Since it is only the air that changes volume then the work term and internal energy change for air combines to give changes in enthalpy as

$$m_{air}\,(u_2 - u_1) + {}_1W_2 = m_{air}\,(h_2 - h_1) = m_{air}C_p\,(T_2 - T_1)$$

$$\sum m_iC_{v\,i} = 1.5 \times 4.18 + 1 \times 0.46 + 0.5 \times 0.96 + 0.1 \times 1.004 = 7.3104 \text{ kJ/K}$$

Energy Eq.: $7.3104\ T_2 = 1.5 \times 4.18 \times 55 + (1 \times 0.46 + 0.5 \times 0.96) \times 20$

$$+ 0.1 \times 1.004 \times (400 - 273.15) = 376.386 \text{ kJ}$$

$T_2 = \mathbf{51.48^oC}$

$$V_1 = mRT_1/P_1 = 0.1 \text{ kg} \times 0.287 \text{ kJ/kgK} \times 400 \text{ K} /100 \text{ kPa} = 0.1148 \text{ m}^3$$

$$V_2 = V_1 \times T_2/T_{1air} = 0.1148\ \ 324.63 / 400 = 0.09317 \text{ m}^3$$

$${}_1W_2 = P(V_2 - V_1) = 100 \text{ kPa } (0.09317 - 0.1148) \text{ m}^3 = \mathbf{-2.16\ kJ}$$

3.149

A spring loaded piston/cylinder contains 1.5 kg of air at 27C and 160 kPa. It is now heated to 900 K in a process where the pressure is linear in volume to a final volume of twice the initial volume. Plot the process in a P-v diagram and find the work and heat transfer.

Take CV as the air.

$m_2 = m_1 = m$; $\quad m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Process: $P = A + BV \Rightarrow {}_1W_2 = \int P\,dV = \text{area} = 0.5(P_1 + P_2)(V_2 - V_1)$

State 1: Ideal gas. $\quad V_1 = mRT_1/P_1 = 1.5\times 0.287 \times 300/160 = 0.8072\ m^3$

Table A.7 $\quad u_1 = u(300) = 214.36$ kJ/kg

State 2: $P_2V_2 = mRT_2$ $\quad$ so ratio it to the initial state properties

$$P_2V_2 / P_1V_1 = P_2 2 / P_1 = mRT_2 / mRT_1 = T_2 / T_1 \Rightarrow$$

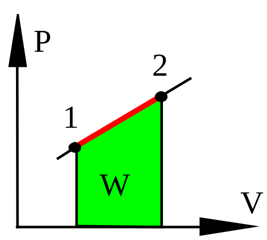
$$P_2 = P_1 (T_2 / T_1)(1/2) = 160 \times (900/300) \times (1/2) = 240 \text{ kPa}$$

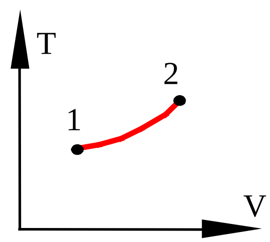
Work is done while piston moves at linearly varying pressure, so we get

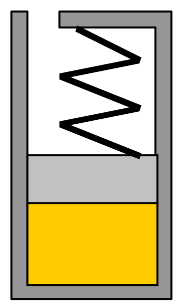
$${}_1W_2 = 0.5(P_1 + P_2)(V_2 - V_1) = 0.5\times(160 + 240) \text{ kPa} \times 0.8072 \text{ m}^3 = \mathbf{161.4\ kJ}$$

Heat transfer is found from energy equation

$${}_1Q_2 = m(u_2 - u_1) + {}_1W_2 = 1.5\times(674.824 - 214.36) + 161.4 = \mathbf{852.1\ kJ}$$







3.150

A constant pressure piston cylinder contains 0.5 kg air at 300 K, 400 kPa. Assume the piston cylinder has a total mass of 1 kg steel and is at the same temperature as the air at any time. The system is now heated to 1600 K by heat transfer.

(a) Find the heat transfer using constant specific heats for air.
(b) Find the heat transfer **NOT** using constant specific heats for air.

C.V. Air and Steel.

Energy Eq.: $U_2 - U_1 = m_{air}(u_2 - u_1) + m_{st}(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Process: P = C => ${}_1W_2 = \int_1^2 PdV = P\,(V_2 - V_1) = P\,m_{air}\,(v_2 - v_1)$

$${}_1Q_2 = m_{air}(u_2 - u_1)_{air} + m_{st}(u_2 - u_1)_{st} + {}_1W_2 = m_{air}(h_2 - h_1)_{air} + m_{st}(u_2 - u_1)_{st}$$

Use A.3: $(u_2 - u_1)_{st} = C\,(T_2 - T_1) = 0.46\text{ kJ/kgK} \times (1600 - 300)\text{ K} = 598\text{ kJ/kg}$

Use A.5: $(h_2 - h_1)_{air} = C_p(T_2 - T_1) = 1.004\text{ kJ/kgK} \times (1600 - 300)\text{ K} = 1305.2\text{ kJ/kg}$

$${}_1Q_2 = m_{air}(h_2 - h_1)_{air} + m_{st}(u_2 - u_1)_{st}$$
$$= 0.5\text{ kg} \times 1305\text{ kJ/kg} + 1\text{ kg} \times 598\text{ kJ/kg} = \mathbf{1250.6\ kJ}$$

Use air tables A.7: $(h_2 - h_1)_{air} = 1757.33 - 300.47 = 1456.86\text{ kJ/kg}$

$${}_1Q_2 = m_{air}(h_2 - h_1)_{air} + m_{st}(u_2 - u_1)_{st}$$
$$= 0.5\text{ kg} \times 1456.86\text{ kJ/kg} + 1\text{ kg} \times 598\text{ kJ/kg} = \mathbf{1326.43\ kJ}$$

Comment: we could also have computed the work explicitly

$${}_1W_2 = P\,m_{air}\,(v_2 - v_1) = m_{air}\,R(T_2 - T_1)$$
$$= 0.5\text{ kg} \times 0.287\text{ kJ/kgK} \times (1600 - 300)\text{ K} = 186.55\text{ kJ}$$
$$(u_2 - u_1)_{air} = 1298.08 - 214.36 = 1083.72\text{ kJ/kg}$$
$${}_1Q_2 = m_{air}(u_2 - u_1)_{air} + m_{st}(u_2 - u_1)_{st} + {}_1W_2$$
$$= 0.5 \times 1083.72 + 598 + 186.55 = 1139.86 + 186.55 = 1326.4\text{ kJ}$$

3.151

An insulated cylinder is divided into two parts of 1 m^3 each by an initially locked piston, as shown in Fig. P3.151. Side A has air at 200 kPa, 300 K, and side B has air at 1.0 MPa, 1000 K. The piston is now unlocked so it is free to move, and it conducts heat so the air comes to a uniform temperature $T_A = T_B$. Find the mass in both A and B, and the final *T* and *P*.

C.V. A + B Force balance on piston: $P_A A = P_B A$

So the final state in A and B is the same.

State 1A: Table A.7 u_{A1} = 214.364 kJ/kg,

$m_A = P_{A1}V_{A1}/RT_{A1}$ = 200 kPa × 1 m^3/(0.287 kJ/kg-K × 300 K) = **2.323 kg**

State 1B: Table A.7 u_{B1} = 759.189 kJ/kg,

$m_B = P_{B1}V_{B1}/RT_{B1}$ = 1000 kPa × 1 m^3/(0.287 kJ/kg-K × 1000 K) = **3.484 kg**

For chosen C.V. ${}_1Q_2 = 0$, ${}_1W_2 = 0$ so the energy equation becomes

$$m_A(u_2 - u_1)_A + m_B(u_2 - u_1)_B = 0$$

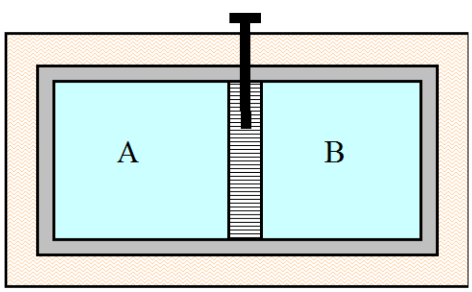
$$(m_A + m_B)u_2 = m_A u_{A1} + m_B u_{B1}$$

$$= 2.323 \times 214.364 + 3.484 \times 759.189 = 3143 \text{ kJ}$$

$$u_2 = 3143 \text{ kJ} / (3.484 + 2.323) \text{ kg} = 541.24 \text{ kJ/kg}$$

From interpolation in Table A.7: $\Rightarrow$ $\mathbf{T_2 = 736\ K}$

$$P = (m_A + m_B)RT_2/V_{tot} = 5.807 \text{ kg} \times 0.287 \frac{\text{kJ}}{\text{kg K}} \times 736 \text{ K}/ 2 \text{ m}^3 = \mathbf{613\ kPa}$$



3.152

Air in a piston cylinder is at 1800 K, 7 MPa and expands in a polytropic process with n = 1.5 to a volume 8 times larger. Find the specific work and the specific heat transfer in the process and draw the P-v diagram. Use constant specific heat to solve the problem.

C.V. Air of constant mass $m_2 = m_1 = m$.

Energy Eq.3.5: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Process: $Pv^{1.50}$ = constant, $v_2/v_1 = 8$

State 1: $P_1 = 7$ MPa, $T_1 = 1800$ K

State 2: $(v_2 = 8v_1, ?)$ Must be on process curve so

$$P_2 = P_1 (v_1/v_2)^n = 7000\ (1/8)^{1.50} = 309.36 \text{ kPa}$$

$$T_2 = T_1 \frac{P_2 v_2}{P_1 v_1} = T_1 (v_1/v_2)^{n-1} = 1800\ (1/8)^{0.5} = 636.4 \text{ K}$$

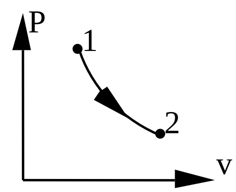
Work from the process expressed in Eq.3.21

$$_1w_2 = \int P dv = \frac{P_2 v_2 - P_1 v_1}{1 - n} = \frac{R}{1 - n}(T_2 - T_1) = \frac{0.287\ (636.4 - 1800)}{1 - 1.5}$$

$$= \mathbf{667.9\ kJ/kg}$$

Heat transfer from the energy equation

$$_1q_2 = (u_2 - u_1) + {}_1w_2 = 0.717\ (636.4 - 1800) + 667.9 = \mathbf{-166.4\ kJ/kg}$$



Notice:
n = 1.5, k = 1.4

n > k

3.153

Do the previous problem but do **not** use constant specific heats.
Air in a piston cylinder is at 1800 K, 7 MPa and expands in a polytropic process with n = 1.5 to a volume 8 times larger. Find the specific work and the specific heat transfer in the process and draw the P-v diagram.

C.V. Air of constant mass $m_2 = m_1 = m$.

Energy Eq.3.5: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Process: $Pv^{1.50}$ = constant, $v_2/v_1 = 8$

State 1: P_1 = 7 MPa, T_1 = 1800 K

State 2: $(v_2 = 8v_1, ?)$ Must be on process curve so

$$P_2 = P_1 (v_1/v_2)^n = 7000\ (1/8)^{1.50} = 309.36 \text{ kPa}$$

$$T_2 = T_1 \frac{P_2v_2}{P_1v_1} = T_1(v_1/v_2)^{n-1} = 1800\ (1/8)^{0.5} = 636.4 \text{ K}$$

Table A.7: u_1 = 1486.33 kJ/kg and interpolate u_2 = 463.06 kJ/kg

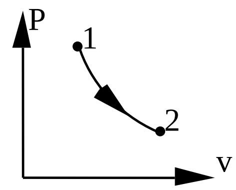
Work from the process expressed in Eq.3.21

$${}_1w_2 = \int P dv = \frac{P_2v_2 - P_1v_1}{1 - n} = \frac{R}{1 - n}(T_2 - T_1) = \frac{0.287\ (636.4 - 1800)}{1 - 1.5}$$

$$= \mathbf{667.9\ kJ/kg}$$

Heat transfer from the energy equation

$${}_1q_2 = (u_2 - u_1) + {}_1w_2 = (463.06 - 1486.33) + 667.9 = \mathbf{-355.4\ kJ/kg}$$



Notice:
n = 1.5, k = 1.4

n > k

3.154

Helium gas expands from 125 kPa, 350 K and 0.25 m^3 to 100 kPa in a polytropic process with n = 1.667. How much heat transfer is involved?

Solution:

C.V. Helium gas, this is a control mass.

Energy equation: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Process equation: $PV^n = \text{constant} = P_1V_1^n = P_2V_2^n$

Ideal gas (A.5): $m = PV/RT = \dfrac{125 \times 0.25}{2.0771 \times 350} = 0.043 \text{ kg}$

Solve for the volume at state 2

$$V_2 = V_1 (P_1/P_2)^{1/n} = 0.25 \times \left(\frac{125}{100}\right)^{0.6} = 0.2852 \text{ m}^3$$

$$T_2 = T_1 P_2V_2/(P_1V_1) = 350 \text{ K} \frac{100 \times 0.2852}{125 \times 0.25} = 319.4 \text{ K}$$

Work from Eq.3.21

$${}_1W_2 = \frac{P_2V_2 - P_1 V_1}{1-n} = \frac{100\times 0.2852 - 125\times 0.25}{1 - 1.667} \text{ kPa m}^3 = 4.09 \text{ kJ}$$

Use specific heat from Table A.5 to evaluate $u_2 - u_1$**,** $C_v = 3.116$ kJ/kg K

$${}_1Q_2 = m(u_2 - u_1) + {}_1W_2 = m\, C_v (T_2 - T_1) + {}_1W_2$$

$$= 0.043 \text{ kg} \times 3.116 \text{ kJ/kg-K} \times (319.4 - 350) \text{ K} + 4.09 \text{ kJ} = \mathbf{-0.01\ kJ}$$

3.155

A piston cylinder contains 0.1 kg air at 300 K and 100 kPa. The air is now slowly compressed in an isothermal (T = C) process to a final pressure of 250 kPa. Show the process in a P-V diagram and find both the work and heat transfer in the process.

Solution :

Process: T = C & ideal gas $\Rightarrow$ PV = mRT = constant

The work was found as in Eq.3.22

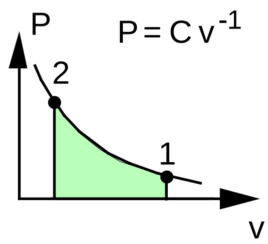
$$_1W_2 = \int PdV = \int \frac{mRT}{V} dV = mRT \ln \frac{V_2}{V_1} = mRT \ln \frac{P_1}{P_2}$$

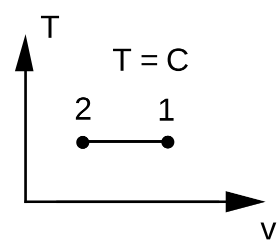
$$= 0.1 \text{ kg} \times 0.287 \text{ kJ/kg-K} \times 300 \text{ K} \times \ln (100 / 250) = \mathbf{-7.89 \text{ kJ}}$$

since $T_1 = T_2 \Rightarrow u_2 = u_1$

The energy equation thus becomes

$$_1Q_2 = m \times (u_2 - u_1) + {_1W_2} = {_1W_2} = \mathbf{-7.89 \text{ kJ}}$$





3.156

A gasoline engine has a piston/cylinder with 0.1 kg air at 4 MPa, 1527^{o}C after combustion and this is expanded in a polytropic process with n = 1.5 to a volume 10 times larger. Find the expansion work and heat transfer using Table A.5 heat capacity.

Take CV as the air. $m_2 = m_1 = m$;

Energy Eq.3.5 $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Process Eq.: $Pv^n = \text{Constant}$ (polytropic)

From the ideal gas law and the process equation we can get:

State 2: $P_2 = P_1 \left(v_2 / v_1\right)^{-n} = 4000 \times 10^{-1.5} = 126.5 \text{ kPa}$

$$T_2 = T_1 \; (P_2v_2 / P_1v_1) = (1527 + 273) \frac{126.5 \times 10}{4000} = 569.3 \text{ K}$$

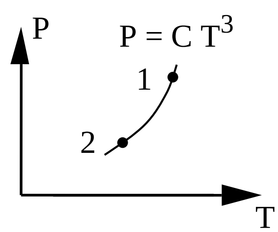
From process eq.: $${}_1W_2 = \int P\, dV = \frac{m}{1-n} (P_2v_2 - P_1v_1) = \frac{mR}{1-n} (T_2 - T_1)$$

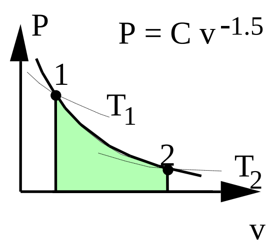
$$= \frac{0.1 \times 0.287}{1 - 1.5} (569.3 - 1800) = \mathbf{70.64 \text{ kJ}}$$

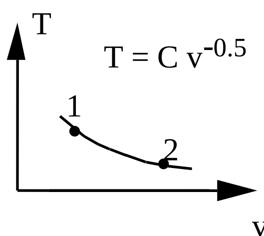
From energy eq.: ${}_1Q_2 = m(u_2 - u_1) + {}_1W_2 = mC_v(T_2 - T_1) + {}_1W_2$

$= 0.1 \text{ kg} \times 0.717 \text{ kJ/kg-K} \times (569.3 - 1800) \text{ K} + 70.64 \text{ kJ}$

$= \mathbf{-17.6 \text{ kJ}}$







3.157

Solve the previous problem using Table A.7

Take CV as the air. $m_2 = m_1 = m$;

Energy Eq.3.5 $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Process Eq.: $Pv^n = \text{Constant}$ (polytropic)

From the ideal gas law and the process equation we can get:

State 2: $P_2 = P_1 \left(v_2 / v_1\right)^{-n} = 4000 \times 10^{-1.5} = 126.5 \text{ kPa}$

$$T_2 = T_1 \; (P_2v_2 / P_1v_1) = (1527 + 273) \frac{126.5 \times 10}{4000} = 569.3 \text{ K}$$

From process eq.: $${}_1W_2 = \int P\, dV = \frac{m}{1-n} (P_2v_2 - P_1v_1) = \frac{mR}{1-n} (T_2 - T_1)$$

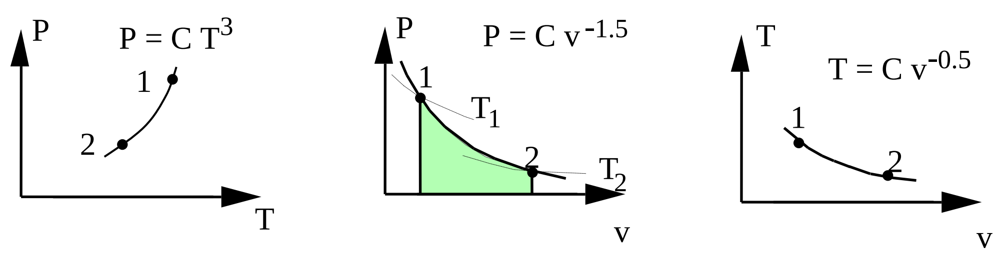
$$= \frac{0.1 \times 0.287}{1 - 1.5} (569.3 - 1800) = \mathbf{70.64\ kJ}$$

From energy eq.: ${}_1Q_2 = m(u_2 - u_1) + {}_1W_2$

$= 0.1 \text{ kg } (411.78 - 1486.33) \text{ kJ/kg} + 70.64 \text{ kJ}$

$= \mathbf{-36.8\ kJ}$

The only place where Table A.7 comes in is for values of u_1 and u_2



3.158

Find the specific heat transfer in Problem 3.55.

Air goes through a polytropic process from 125 kPa, 325 K to 300 kPa and 500 K. Find the polytropic exponent n and the specific work in the process.

Solution:

Energy Eq.: $u_2 - u_1 = {}_1q_2 - {}_1w_2$

Process: $Pv^n = \text{Const} = P_1 v_1^n = P_2\, v_2^n$

Ideal gas $Pv = RT$ so

$$v_1 = \frac{RT}{P} = \frac{0.287 \times 325}{125} = 0.7462 \text{ m}^3\text{/kg}$$

$$v_2 = \frac{RT}{P} = \frac{0.287 \times 500}{300} = 0.47833 \text{ m}^3\text{/kg}$$

From the process equation

$$(P_2/\,P_1) = (v_1/\,v_2)^n \quad => \quad \ln(P_2/\,P_1) = n \ln(v_1/\,v_2)$$

$$n = \ln(P_2/\,P_1) \,/\, \ln(v_1/\,v_2) = \frac{\ln 2.4}{\ln 1.56} = \mathbf{1.969}$$

The work is now from Eq.3.21 per unit mass and ideal gas law

$${}_1w_2 = \frac{P_2v_2\text{-}P_1v_1}{1\text{-}n} = \frac{R(T_2 - T_1)}{1\text{-}n} = \frac{0.287(500 - 325)}{1\text{-}1.969} \text{ (kJ/kg-K) K}$$

$$= \mathbf{-51.8 \text{ kJ/kg}}$$

From the energy equation

$${}_1q_2 = u_2 - u_1 + {}_1w_2 = C_v\,(T_2 - T_1) + {}_1w_2$$

$$= 0.717 \text{ kJ/kg-K } (500 - 325) \text{ K} - 51.5 \text{ kJ/kg}$$

$$= \mathbf{73.98 \text{ kJ/kg}}$$

3.159

A piston/cylinder has nitrogen gas at 750 K and 1500 kPa. Now it is expanded in a polytropic process with n = 1.2 to P = 750 kPa. Find the final temperature, the specific work and specific heat transfer in the process.

C.V. Nitrogen. This is a control mass going through a polytropic process.

Continuty: $m_2 = m_1$

Energy Eq.3.5: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Process: $Pv^n = \text{constant}$

Substance ideal gas: $Pv = RT$

$$T_2 = T_1 \, (P_2/P_1)^{\frac{n-1}{n}} = 750 \left(\frac{750}{1500}\right)^{\frac{0.2}{1.2}} = 750 \times 0.8909 = \mathbf{668\ K}$$

The work is integrated as in Eq.3.21

$${}_1w_2 = \int P dv = \frac{1}{1 - n} (P_2 v_2 - P_1 v_1) = \frac{R}{1 - n} (T_2 - T_1)$$

$$= \frac{0.2968}{1 - 1.2} \text{ kJ/kg-K } (668 - 750) \text{ K} = \mathbf{121.7\ kJ/kg}$$

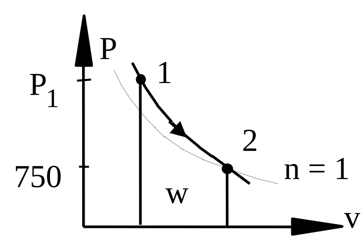
The energy equation with values of u from Table A.8 is

$${}_1q_2 = u_2 - u_1 + {}_1w_2 = 502.8 - 568.45 + 121.7 = \mathbf{56.0\ kJ/kg}$$

If constant specific heat is used from Table A.5

$${}_1q_2 = C_V(T_2 - T_1) + {}_1w_2 = 0.745(668 - 750) + 121.7 = \mathbf{60.6\ kJ/kg}$$

The actual process is on a steeper curve than n = 1.



3.160

A piston/cylinder has 1 kg propane gas at 700 kPa, 40°C. The piston cross-sectional area is 0.5 m^2, and the total external force restraining the piston is directly proportional to the cylinder volume squared. Heat is transferred to the propane until its temperature reaches 700°C. Determine the final pressure inside the cylinder, the work done by the propane, and the heat transfer during the process.

Solution:

C.V. The 1 kg of propane.

Energy Eq.3.5: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Process: $P = P_{ext} = CV^2 \Rightarrow PV^{-2} = \text{constant}$, polytropic $n = -2$

Ideal gas: $PV = mRT$, and process yields

$$P_2 = P_1(T_2/T_1)^{\frac{n}{n-1}} = 700 \left(\frac{700+273.15}{40+273.15}\right)^{2/3} = \mathbf{1490.7\ kPa}$$

The work is integrated as Eq.3.21

$${}_1W_2 = \int_1^2 PdV = \frac{P_2V_2 - P_1V_1}{1 - n} = \frac{mR(T_2 - T_1)}{1 - n}$$

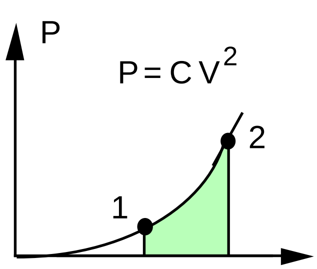
$$= \frac{1 \times 0.18855 \times (700 - 40)}{1 - (-2)} = \mathbf{41.48\ kJ}$$

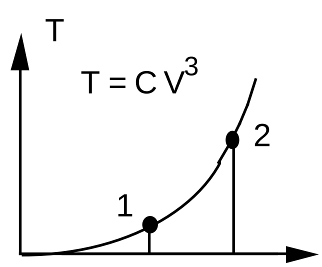
The energy equation with specific heat from Table A.5 becomes

$${}_1Q_2 = m(u_2 - u_1) + {}_1W_2 = mC_v(T_2 - T_1) + {}_1W_2$$

$$= 1 \text{ kg} \times 1.490 \text{ kJ/kg-K} \times (700 - 40) \text{ K} + 41.48 \text{ kJ}$$

$$= \mathbf{1024.9\ kJ}$$





3.161

A piston/cylinder arrangement of initial volume 0.025 m^3 contains saturated water vapor at 180°C. The steam now expands in a polytropic process with exponent *n* = 1 to a final pressure of 200 kPa, while it does work against the piston. Determine the heat transfer in this process.

Solution:

C.V. Water. This is a control mass.

State 1: Table B.1.1 P = 1002.2 kPa, $v_1 = 0.19405\ m^3/kg$, $u_1 = 2583.7$ kJ/kg ,

$$m = V/v_1 = 0.025/0.19405 = 0.129 \text{ kg}$$

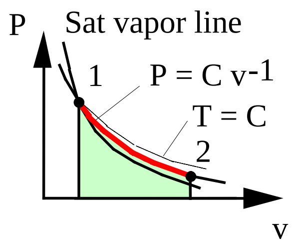
Process: $Pv = \text{const.} = P_1v_1 = P_2v_2$; polytropic process n = 1.

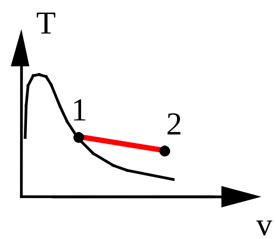
$$\Rightarrow v_2 = v_1P_1/P_2 = 0.19405 \times 1002.1/200 = 0.9723\ m^3/kg$$

State 2: P_2, v_2 $\Rightarrow$ Table B.1.3 $T_2 \cong 155°C$, $u_2 = 2585$ kJ/kg

$$_1W_2 = \int PdV = P_1V_1 \ln\frac{v_2}{v_1} = 1002.2 \text{ kPa} \times 0.025\ m^3 \ln\frac{0.9723}{0.19405} = 40.37 \text{ kJ}$$

$$_1Q_2 = m(u_2 - u_1) + {}_1W_2 = 0.129(2585 - 2583.7) + 40.37 = \mathbf{40.54\ kJ}$$





Notice T drops, it is not an ideal gas.

3.162

A piston cylinder contains pure oxygen at ambient conditions 20°C, 100 kPa. The piston is moved to a volume that is 7 times smaller than the initial volume in a polytropic process with exponent n = 1.25. Use constant heat capacity to find the final pressure and temperature, the specific work and the specific heat transfer.

Energy Eq.: $u_2 - u_1 = {}_1q_2 - {}_1w_2$

Process Eq: $Pv^n = C$; $P_2 = P_1 (v_1/v_2)^n = 100\,(7)^{1.25} = 1138.6$ kPa

From the ideal gas law and state 2 (P, v) we get

$$T_2 = T_1(P_2/P_1)(v_1/v_2) = 293 \times \frac{1138.6}{100} \times (1/7) = 476.8\text{ K}$$

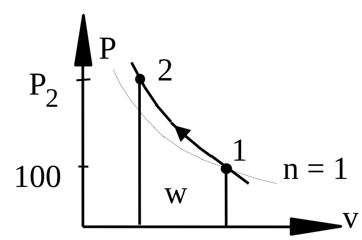
We could also combine process eq. and gas law to give: $T_2 = T_1 (v_1/v_2)^{n-1}$

Polytropic work Eq. 3.21: ${}_1w_2 = \frac{1}{1-n}(P_2v_2 - P_1v_1) = \frac{R}{1-n}(T_2 - T_1)$

$${}_1w_2 = \frac{0.2598}{1 - 1.25}\ \frac{\text{kJ}}{\text{kg K}} \times (476.8 - 293.2)\text{ K} = \mathbf{-190.88\ kJ/kg}$$

$${}_1q_2 = u_2 - u_1 + {}_1w_2 = C_v (T_2 - T_1) + {}_1w_2$$
$$= 0.662\,(476.8 - 293.2) - 190.88 = \mathbf{-69.3\ kJ/kg}$$

The actual process is on a steeper curve than n = 1.



3.163

A piston/cylinder in a car contains 0.2 L of air at 90 kPa, 20°C, shown in Fig. P3.163. The air is compressed in a quasi-equilibrium polytropic process with polytropic exponent n = 1.25 to a final volume six times smaller. Determine the final pressure, temperature, and the heat transfer for the process.

Solution:

C.V. Air. This is a control mass going through a polytropic process.

Continuty: $m_2 = m_1$

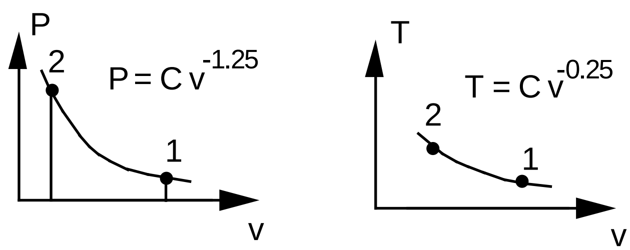
Energy Eq.3.5: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Process: $Pv^n = \text{const.}$

$P_1v_1^n = P_2v_2^n \Rightarrow P_2 = P_1(v_1/v_2)^n = 90 \text{ kPa} \times 6^{1.25} = \mathbf{845.15\ kPa}$

Substance ideal gas: $Pv = RT$

$T_2 = T_1(P_2v_2/P_1v_1) = 293.15 \text{ K } (845.15/90 \times 6) = \mathbf{458.8\ K}$



$$m = \frac{PV}{RT} = \frac{90 \times 0.2{\times}10^{-3}}{0.287 \times 293.15} = 2.14{\times}10^{-4} \text{ kg}$$

The work is integrated as in Eq.3.21

$${}_1w_2 = \int P dv = \frac{1}{1 - n}(P_2v_2 - P_1v_1) = \frac{R}{1 - n}(T_2 - T_1)$$

$$= \frac{0.287}{1 - 1.25} \text{ kJ/kg-K } (458.8 - 293.15) \text{ K} = -190.17 \text{ kJ/kg}$$

The energy equation with values of u from Table A.7 is

$${}_1q_2 = u_2 - u_1 + {}_1w_2 = 329.4 - 208.03 - 190.17 = -68.8 \text{ kJ/kg}$$

${}_1Q_2 = m\ {}_1q_2 = \mathbf{-0.0147\ kJ}$ (i.e a heat loss)

3.164

An air pistol contains compressed air in a small cylinder, shown in Fig. P3.164. Assume that the volume is 1 cm^3, pressure is 1 MPa, and the temperature is 27°C when armed. A bullet, *m* = 15 g, acts as a piston initially held by a pin (trigger); when released, the air expands in an isothermal process (*T* = constant). If the air pressure is 0.1 MPa in the cylinder as the bullet leaves the gun, find

a. The final volume and the mass of air.

b. The work done by the air and work done on the atmosphere.

c. The work to the bullet and the bullet exit velocity.

Solution:

C.V. Air.

Air ideal gas: $m_{air} = P_1V_1/RT_1 = 1000 \times 10^{-6}/(0.287 \times 300) = \mathbf{1.17{\times}10^{-5}\ kg}$

Process: $PV = \text{const} = P_1V_1 = P_2V_2 \;\Rightarrow\; V_2 = V_1P_1/P_2 = \mathbf{10\ cm^3}$

$${}_1W_2 = \int PdV = \int \frac{P_1V_1}{V}\, dV = P_1V_1 \ln (V_2/V_1) = \mathbf{2.303\ J}$$

$${}_1W_{2,ATM} = P_0(V_2 - V_1) = 101 \times (10 - 1) \times 10^{-6}\ \text{kJ} = \mathbf{0.909\ J}$$

$$W_{bullet} = {}_1W_2 - {}_1W_{2,ATM} = 1.394\ \text{J} = \frac{1}{2}\, m_{bullet}(V_{exit})^2$$

$$V_{exit} = (2W_{bullet}/m_B)^{1/2} = (2 \times 1.394/0.015)^{1/2} = \mathbf{13.63\ m/s}$$

3.165

Air goes through a polytropic process with n = 1.3 in a piston/cylinder setup. It starts at 200 kPa, 300 K and ends with a pressure of 2200 kPa. Find the expansion ratio (v_2/v_1), the specific work, and the specific heat transfer.

Take CV as the air. $m_2 = m_1 = m$;

Energy Eq.3.5 $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Process Eq.: $Pv^n = \text{Constant}$ (polytropic)

From the ideal gas law and the process equation we can get:

State 2: $(v_2/v_1) = (P_2/P_1)^{-1/n}$ and $Pv = RT$ => $T_2/T_1 = (v_2 / v_1)^{1 - n}$

$$(v_2/v_1) = (P_2/P_1)^{-1/n} = (2200 / 200)^{-1/1.3} = \mathbf{0.1581}$$

$$T_2 = T_1 (P_2/P_1)^{(n-1)/n} = 300 \left(\frac{2200}{200}\right)^{0.3/1.3} = \mathbf{521.7\ K}$$

From process eq.:

$${}_1w_2 = \int P\, dv = \text{area} = \frac{1}{1-n} (P_2v_2 - P_1v_1) = \frac{R}{1-n} (T_2 - T_1)$$

$$= \frac{0.287}{1-1.3} \text{ kJ/kg-K} \times (521.7 - 300) \text{ K} = \mathbf{-212.09\ kJ/kg}$$

From the energy equation and constant specific heat from Table A.5

$${}_1q_2 = u_2 - u_1 + {}_1w_2 = C_v (T_2 - T_1) + {}_1w_2$$

$$= 0.717 \text{ kJ/kg-K} \times (521.7 - 300) \text{ K} - 212.09 \text{ kJ/kg} = \mathbf{-53.13\ kJ/kg}$$

3.166

Nitrogen gas goes through a polytropic process with n = 1.3 in a piston/cylinder arrangement. It starts out at 600 K and 600 kPa and ends with 800 K. Find the final pressure, the process specific work and heat transfer.

Take CV as the nitrogen. $m_2 = m_1 = m$;

Energy Eq.3.5 $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Process Eq.: $Pv^n = \text{Constant}$ (polytropic)

From the ideal gas law and the process equation we can get:

State 2: $P_2 = P_1 \left(v_2 / v_1\right)^{-n}$ and $Pv = RT$ => $T_2/T_1 = (v_2 / v_1)^{1 - n}$

$$P_2 = P_1 \left(T_2 / T_1\right)^{n/(n-1)} = 600 \left(\frac{800}{600}\right)^{1.3/0.3} = \mathbf{2087\ kPa}$$

From process eq.:

$$_1w_2 = \int P\, dv = \text{area} = \frac{1}{1-n}(P_2v_2 - P_1v_1) = \frac{R}{1-n}(T_2 - T_1)$$

$$= \frac{0.2968}{1-1.3}(800 - 600) = \mathbf{-197.9\ kJ/kg}$$

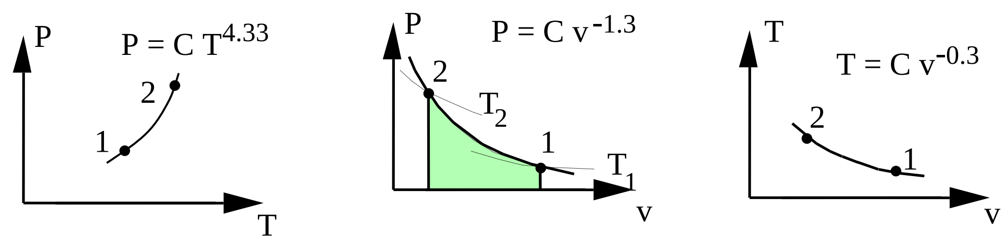
From the energy equation and Table A.8

$$_1q_2 = u_2 - u_1 + {}_1w_2 = (609.41 - 449.16 - 197.9)\text{ kJ/kg} = \mathbf{-37.65\ kJ/kg}$$

From the energy equation and constant specific heat from Table A.5

$$_1q_2 = u_2 - u_1 + {}_1w_2 = C_v (T_2 - T_1) + {}_1w_2$$

$$= 0.745\text{ kJ/kg-K} \times (800 - 600)\text{ K} - 197.9\text{ kJ/kg} = -48.9\text{ kJ/kg}$$



3.167

A piston/cylinder contains pure oxygen at 500 K, 600 kPa. The piston is moved to a volume such that the final temperature is 700 K in a polytropic process with exponent n = 1.25. Use ideal gas approximation and constant heat capacity to find the final pressure, the specific work and heat transfer.

Energy Eq.: $u_2 - u_1 = {}_1q_2 - {}_1w_2$

Process Eq: $Pv^n = C$ and ideal gas $Pv = RT$ gives

$$P_2 = P_1(v_1/v_2)^n = P_1(T_2/T_1)^{n/(n-1)} = 600(7/5)^5 = \mathbf{3227\ kPa}$$

Reversible work Eq. 3.21: ${}_1w_2 = \frac{1}{1-n}(P_2v_2 - P_1v_1) = \frac{R}{1-n}(T_2 - T_1)$

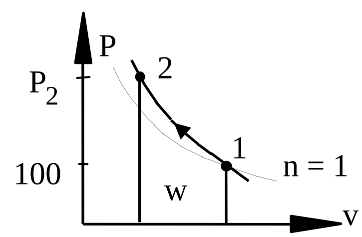
$${}_1w_2 = \frac{0.2598}{1 - 1.25}\ \frac{kJ}{kg\ K} \times (700 - 500)\ K = \mathbf{-207.84\ kJ/kg}$$

$${}_1q_2 = u_2 - u_1 + {}_1w_2 = C_v\,(T_2 - T_1) + {}_1w_2$$

$$= 0.662\ kJ/kg\text{-}K \times (700 - 500)\ K - 207.84\ kJ/kg$$

$$= \mathbf{-\ 69.3\ kJ/kg}$$

The actual process is on a steeper curve than n = 1.



3.168

Calculate the heat transfer for the process described in Problem 3.57.

Consider a piston cylinder with 0.5 kg of R-134a as saturated vapor at -10°C. It is now compressed to a pressure of 500 kPa in a polytropic process with n = 1.5. Find the final volume and temperature, and determine the work done during the process.

Solution:

Take CV as the R-134a which is a control mass

Continuity: $m_2 = m_1 = m$; Energy: $m(u_2 - u_1) = {_1Q_2} - {_1W_2}$

Process: $Pv^{1.5}$ = constant. Polytropic process with n = 1.5

1: (T, x) P = Psat = 201.7 kPa from Table B.5.1

$v_1 = 0.09921\ m^3/kg$, $u_1 = 372.27$ kJ/kg

2: (P, process)

$$v_2 = v_1 (P_1/P_2)^{(1/1.5)} = 0.09921 \times (201.7/500)^{0.667} = 0.05416\ m^3/kg$$

=> Table B.5.2 superheated vapor, $T_2 = 79°C$, $V_2 = mv_2 = 0.027\ m^3$

$u_2 = 440.9$ kJ/kg

Process gives $P = C\ v^{(-1.5)}$, which is integrated for the work term, Eq.3.21

$${_1W_2} = \int P\ dV = m(P_2v_2 - P_1v_1)/(1-1.5)$$

$$= -2 \times 0.5\ kg \times (500 \times 0.05416 - 201.7 \times 0.09921)\ kJ/kg = -7.07\ kJ$$

$${_1Q_2} = m(u_2 - u_1) + {_1W_2} = 0.5(440.9 - 372.27) + (-7.07) = \mathbf{27.25\ kJ}$$

3.169

A piston cylinder shown in Fig. P3.169 contains 0.5 m^3 of R-410A at 2 MPa, 150^{o}C. The piston mass and atmosphere gives a pressure of 450 kPa that will float the piston. The whole setup cools in a freezer maintained at -20^{o}C. Find the heat transfer and show the P-v diagram for the process when T_2 = -20^{o}C.

C.V.: R-410A. Control mass.

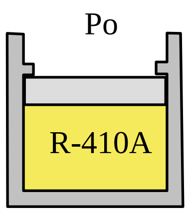
Continuity: m = constant,

Energy Eq.3.5: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Process: $F\downarrow = F\uparrow = P\,A = P_{air}A + F_{stop}$

if $V < V_{stop} \Rightarrow F_{stop} = \emptyset$

This is illustrated in the P-v diagram shown below.



State 1: $v_1 = 0.02247\ m^3/kg$, $u_1 = 373.49$ kJ/kg

$\Rightarrow$ m = V/v = 22.252 kg

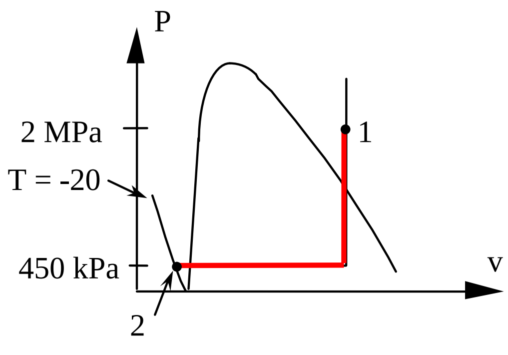
State 2: T_2 and on line $\Rightarrow$ compressed liquid, see figure below.

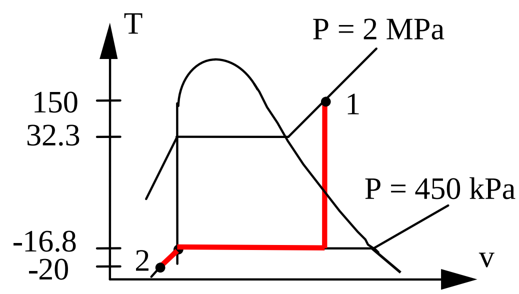
$v_2 \cong v_f = 0.000\ 803\ m^3/kg \Rightarrow V_2 = 0.01787\ m^3$; $u_2 = u_f = 27.92$ kJ/kg

${}_1W_2 = \int PdV = P_{lift}(V_2 - V_1) = 450\ kPa\ (0.01787 - 0.5)\ m^3 = -217.0$ kJ ;

Energy eq. $\Rightarrow$

${}_1Q_2 = 22.252\ kg \times (27.92 - 373.49)\ kJ/kg - 217.9\ kJ =$ **-7906.6 kJ**

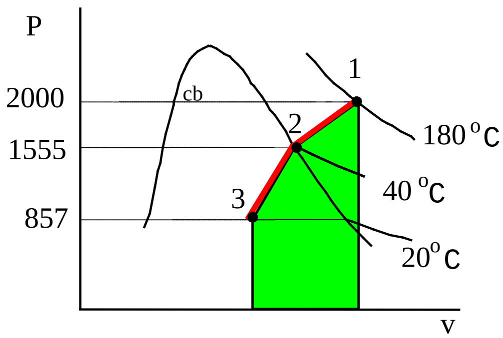




3.170

A cylinder containing 1 kg of ammonia has an externally loaded piston. Initially the ammonia is at 2 MPa, 180°C and is now cooled to saturated vapor at 40°C, and then further cooled to 20°C, at which point the quality is 50%. Find the total work and the heat transfer for the process, assuming a piecewise linear variation of *P* versus *V*.

Solution:



State 1: (T, P) Table B.2.2

$v_1 = 0.10571 \ m^3/kg$

$u_1 = 1630.6$ kJ/kg

State 2: (T, x) Table B.2.1 sat. vap.

$P_2 = 1555$ kPa,

$v_2 = 0.08313 \ m^3/kg$

$u_2 = 1341.0$ kJ/kg

State 3: (T, x) $P_3 = 857$ kPa, $v_3 = (0.001638 + 0.14922)/2 = 0.07543 \ m^3/kg$

$u_3 = (272.89 + 1332.2)/2 = 802.55$ kJ/kg

Sum the work as two integrals each evaluated by the area in the P-v diagram.

$${}_1W_3 = \int_1^3 PdV \approx \left(\frac{P_1 + P_2}{2}\right) m(v_2 - v_1) + \left(\frac{P_2 + P_3}{2}\right) m(v_3 - v_2)$$

$$= \frac{2000 + 1555}{2} \text{ kPa} \times 1 \text{ kg} \times (0.08313 - 0.10571) \text{ m}^3/\text{kg}$$

$$+ \frac{1555 + 857}{2} \text{ kPa} \times 1 \text{ kg} \times (0.07543 - 0.08313) \text{ m}^3/\text{kg}$$

$= \mathbf{-49.4\ kJ}$

From the energy equation

$${}_1Q_3 = m(u_3 - u_1) + {}_1W_3 = 1 \text{ kg} \times (802.55 - 1630.6) \text{ kJ/kg} - 49.4 \text{ kJ}$$

$= \mathbf{-877.5\ kJ}$

3.171

10 kg of water in a piston cylinder arrangement exists as saturated liquid/vapor at 100 kPa, with a quality of 50%. It is now heated so the volume triples. The mass of the piston is such that a cylinder pressure of 200 kPa will float it, as in Fig. 3.171. Find the final temperature and the heat transfer in the process.

Solution:

Take CV as the water.

Continuity Eq.: $m_2 = m_1 = m$;

Energy Eq.: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Process: v = constant until $P = P_{lift}$, then P is constant.

State 1: Two-phase so look in Table B.1.2 at 100 kPa

$u_1 = 417.33 + 0.5 \times 2088.72 = 1461.7$ kJ/kg,

$v_1 = 0.001043 + 0.5 \times 1.69296 = 0.8475$ m^3/kg

State 2: v_2, $\mathbf{P_2 \le P_{lift}}$ => $v_2 = 3 \times 0.8475 = 2.5425$ m^3/kg ;

Interpolate: $\mathbf{T_2 = 829°C}$, $u_2 = 3718.76$ kJ/kg

=> $V_2 = mv_2 = 25.425$ m^3

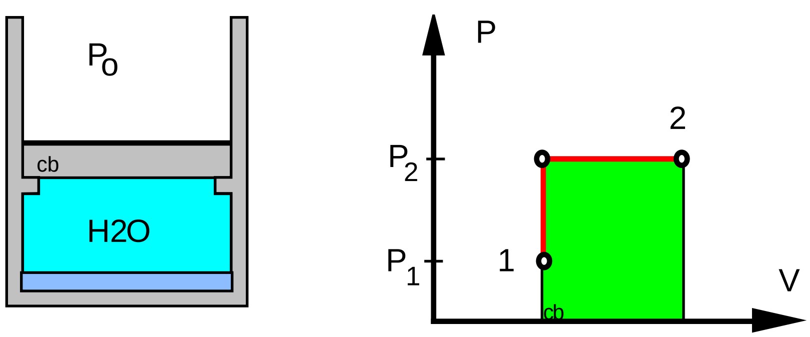
From the process equation (see P-V diagram) we get the work as

$${}_1W_2 = P_{lift}(V_2 - V_1) = 200 \text{ kPa} \times 10 \text{ kg } (2.5425 - 0.8475) \text{ m}^3\text{/kg} = 3390 \text{ kJ}$$

From the energy equation we solve for the heat transfer

$${}_1Q_2 = m(u_2 - u_1) + {}_1W_2 = 10 \text{ kg} \times (3718.76 - 1461.7) \text{ kJ/kg} + 3390 \text{ kJ}$$

$$= \mathbf{25\ 961\ kJ}$$



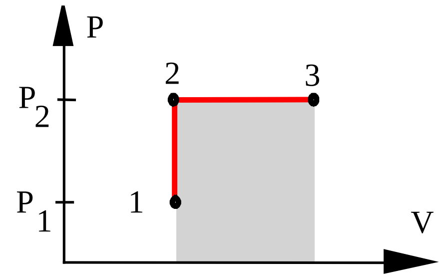
3.172

A helium gas is heated at constant volume from a state of 100 kPa, 300 K to 500 K. A following process expands the gas at constant pressure to three times the initial volume. What are the specific work and the specific heat transfer in the combined process?

The two processes are:

1 -> 2: Constant volume $V_2 = V_1$

2 -> 3: Constant pressure $P_3 = P_2$



Use ideal gas approximation for helium.

State 1: T, P => $v_1 = RT_1/P_1$

State 2: $V_2 = V_1$ => $P_2 = P_1 (T_2/T_1)$

State 3: $P_3 = P_2$ => $V_3 = 3V_2$; $T_3 = T_2\ v_3/v_2 = 500 \times 3 = 1500$ K

We find the work by summing along the process path.

$${}_1w_3 = {}_1w_2 + {}_2w_3 = {}_2w_3 = P_3(v_3 - v_2) = R(T_3 - T_2)$$

$$= 2.0771 \text{ kJ/kg-K} \times (1500 - 500) \text{ K} = \mathbf{2077\ kJ/kg}$$

$${}_1q_3 = u_3 – u_1 + {}_1w_3 = C_v (T_3 – T_1) + {}_1w_3$$

$$= 3.116 \text{ kJ/kg-K} \times (1500 – 300) \text{ K} + 2077 \text{ kJ/kg} = \mathbf{5816\ kJ/kg}$$

3.173

A vertical cylinder fitted with a piston contains 5 kg of R-410A at 10°C, shown in Fig. P3.173. Heat is transferred to the system, causing the piston to rise until it reaches a set of stops at which point the volume has doubled. Additional heat is transferred until the temperature inside reaches 50°C, at which point the pressure inside the cylinder is 1.4 MPa.

a. What is the quality at the initial state?

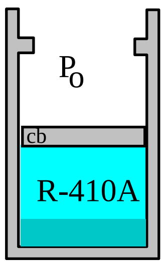
b. Calculate the heat transfer for the overall process.

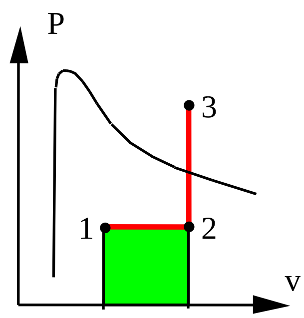
Solution:

C.V. R-410A. Control mass goes through process: 1 -> 2 -> 3

As piston floats pressure is constant (1 -> 2) and the volume is constant for the second part (2 -> 3). So we have: $v_3 = v_2 = 2 \times v_1$

State 3: Table B.4.2 (P,T) $v_3 = 0.02249\ \text{m}^3/\text{kg}$, $u_3 = 287.91$ kJ/kg





So we can then determine state 1 and 2 Table B.4.1:

$v_1 = 0.011245 = 0.000886 + x_1 \times 0.02295$ => $x_1 = \mathbf{0.4514}$

b) $u_1 = 72.24 + 0.4514 \times 183.66 = 155.14$ kJ/kg

State 2: $v_2 = 0.02249\ \text{m}^3/\text{kg}$, $P_2 = P_1 = 1086$ kPa this is still 2-phase.

We get the work from the process equation (see P-V diagram)

$$_1W_3 = {}_1W_2 = \int_1^2 P dV = P_1(V_2 - V_1) = 1086 \text{ kPa} \times 5 \text{ kg} \times 0.011245 \text{ m}^3/\text{kg}$$

$$= 61.1 \text{ kJ}$$

The heat transfer from the energy equation becomes

$$_1Q_3 = m(u_3 - u_1) + {}_1W_3 = 5 \text{ kg} \times (287.91 - 155.14) \text{ kJ/kg} + 61.1 \text{ kJ}$$

$$= \mathbf{725.0\ kJ}$$

3.174

Water in a piston/cylinder (Fig. P3.174) is at 101 kPa, 25°C, and mass 0.5 kg. The piston rests on some stops, and the pressure should be 1000 kPa to float the piston. We now heat the water, so the piston just reaches the end of the cylinder. Find the total heat transfer.

Solution:

Take CV as the water.

Continuity Eq.: $m_2 = m_1 = m$;

Energy Eq.: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Process: v = constant until $P = P_{float}$, then P is constant.

Volume does go up so we get $v_2 = 5\ v_1$

State 1: $v_1 = 0.001003\ m^3/kg$, $u_1 = 104.86\ kJ/kg$

State 2: v_2, $P_2 = P_{float}$ so $v_2 = 5 \times 0.001003 = 0.005015\ m^3/kg$;

$x_2 = (v_2 - v_f)/v_{fg} = (0.005015 - 0.001127)/0.19332 = 0.02011$,

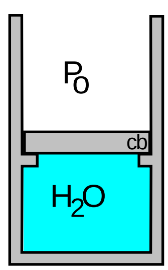
$u_2 = 761.67 + x_2 \times 1821.97 = 798.31\ kJ/kg$

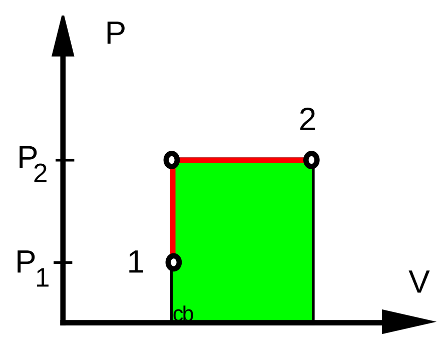
From the process equation (see P-V diagram) we get the work as

$${}_1w_2 = P_{float}(v_2 - v_1) = 1000\ kPa\ (0.005015 - 0.001003)\ m^3/kg = 4.012\ kJ/kg$$

From the energy equation we solve for the heat transfer

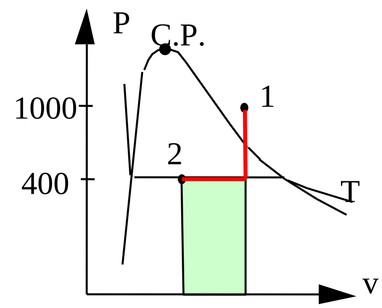
$${}_1Q_2 = m[u_2 - u_1 + {}_1w_2] = 0.5\times[798.31 - 104.86 + 4.012] = \mathbf{348.7\ kJ}$$





3.175

A setup as in Fig. P3.169 has the R-410A initially at 1000 kPa, 50^{o}C of mass 0.1 kg. The balancing equilibrium pressure is 400 kPa and it is now cooled so the volume is reduced to half the starting volume. Find the work and heat transfer for the process.



Take as CV the 0.1 kg of R-410A.

Continuity Eq.: $m_2 = m_1 = m$;

Energy Eq.3.5 $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Process Eq.: $P = P_{float}$ or $v = C = v_1$,

State 1: (P, T) => $v_1 = 0.0332$ m^3/kg,

$u_1 = 292.695$ kJ/kg

State 2: (P, v) => $v_2 = v_1/2 = 0.0166$ m^3/kg < v_g, so it is two-phase.

$x_2 = (v_2 - v_f)/v_{fg} = (0.0166 - 0.000803)/0.064 = 0.2468$

$u_2 = u_f + x_2\, u_{fg} = 27.92 + x_2\, 218.07 = 81.746$ kJ/kg

From process eq.: ${}_1W_2 = \int P\, dV = \text{area} = mP_2\,(v_2 - v_1)$

$= 0.1 \text{ kg} \times 400 \text{ kPa}\ (0.0166 - 0.0332) \text{ m}^3\text{/kg} =$ -0.**664 kJ**

From energy eq.: ${}_1Q_2 = m(u_2 - u_1) + {}_1W_2 = 0.1\times (81.746 - 292.695) - 0.664$

= **-21.8 kJ**

3.176

A piston/cylinder contains 1 kg of liquid water at 20°C and 300 kPa. Initially the piston floats, similar to the setup in Problem 3.173, with a maximum enclosed volume of 0.002 m^3 if the piston touches the stops. Now heat is added so a final pressure of 600 kPa is reached. Find the final volume and the heat transfer in the process.

Solution:

Take CV as the water which is a control mass: $m_2 = m_1 = m$;

Energy Eq.: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Table B.1.1: 20°C => $P_{sat} = 2.34$ kPa

State 1: Compressed liquid $v = v_f(20) = 0.001002\ m^3/kg$, $u = 83.94$ kJ/kg

State 1a: $v_{stop} = 0.002\ m^3/kg$, 300 kPa

State 2: Since $P_2 = 600$ kPa $> P_{lift}$ then piston is pressed against the stops

$$v_2 = v_{stop} = 0.002\ m^3/kg \text{ and } V = \mathbf{0.002\ m^3}$$

For the given P : $v_f < v < v_g$ so 2-phase $T = T_{sat} = 158.85$ °C

$$x = (v – v)/v = (0.002 – 0.001101)/0.31457 = 0.002858$$

$$u = 669.88 + x\ 1897.52 = 675.3 \text{ kJ/kg}$$

Work is done while piston moves at P_{lift} = constant = 300 kPa so we get

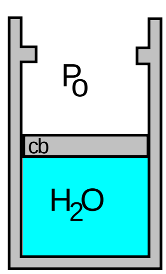
$${}_1W_2 = \int P\,dV = m\,P_{lift}(v_2 - v_1) = 1 \text{ kg} \times 300 \text{ kPa} \times (0.002 – 0.001002)\ m^3$$

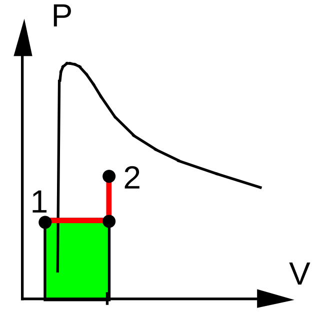
$$= 0.30 \text{ kJ}$$

The heat transfer is from the energy equation

$${}_1Q_2 = m(u_2 - u_1) + {}_1W_2 = 1 \text{ kg} \times (675.3 – 83.94) \text{ kJ/kg} + 0.30 \text{ kJ}$$

$$= \mathbf{591.7\ kJ}$$





3.177

A cylinder/piston arrangement contains 5 kg of water at 100°C with x = 20% and the piston, m_P = 75 kg, resting on some stops, similar to Fig. P3.171. The outside pressure is 100 kPa, and the cylinder area is A = 24.5 cm^2. Heat is now added until the water reaches a saturated vapor state. Find the initial volume, final pressure, work, and heat transfer terms and show the *P–v* diagram.

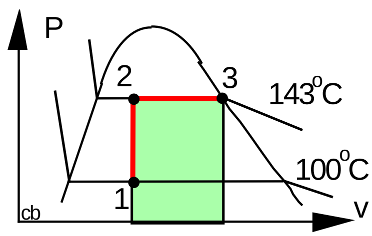
Solution:

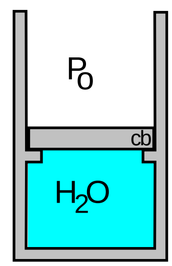
C.V. The 5 kg water.

Continuty: $m_2 = m_1 = m$; Energy: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Process: V = constant if $P < P_{lift}$ otherwise $P = P_{lift}$ see P-v diagram.

$$P_3 = P_2 = P_{lift} = P_0 + m_p\, g / A_p = 100 + \frac{75 \times 9.807}{0.00245 \times 1000} = \mathbf{400\ kPa}$$





State 1: (T,x) Table B.1.1

$v_1 = 0.001044 + 0.2 \times 1.6719$ m^3/kg, $V_1 = mv_1 = 5 \times 0.3354 = \mathbf{1.677\ m^3}$

$u_1 = 418.91 + 0.2 \times 2087.58 = 836.4$ kJ/kg

State 3: (P, x = 1) Table B.1.2 => $v_3 = 0.4625 > v_1$, $u_3 = 2553.6$ kJ/kg

Work is seen in the P-V diagram (if volume changes then $P = P_{lift}$)

${}_1W_3 = {}_2W_3 = P_{ext}m(v_3 - v_2) = 400$ kPa × 5 kg (0.46246 - 0.3354) m^3/kg

= **254.1 kJ**

Heat transfer is from the energy equation

${}_1Q_3 = 5$ kg × (2553.6 - 836.4) kJ/kg + 254.1 kJ = **8840 kJ**

3.178

A piston cylinder setup similar to Problem 3.171 contains 0.1 kg saturated liquid and vapor water at 100 kPa with quality 25%. The mass of the piston is such that a pressure of 500 kPa will float it. The water is heated to 300°C. Find the final pressure, volume, work, ${}_1W_2$ and ${}_1Q_2$.

Solution:

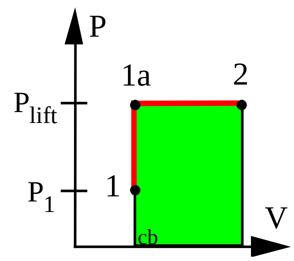
Take CV as the water: $m_2 = m_1 = m$

Process: v = constant until $P = P_{lift}$

To locate state 1: Table B.1.2

$v_1 = 0.001043 + 0.25\times1.69296 = 0.42428\ m^3/kg$

$u_1 = 417.33 + 0.25\times2088.72 = 939.51\ kJ/kg$



1a: $v_{1a} = v_1 = 0.42428\ m^3/kg > v_g$ at 500 kPa,

state 1a is superheated vapor $T_{1a} = 200°C$

State 2 is 300°C so heating continues after state 1a to 2 at constant P =>

2: T_2, $P_2 = P_{lift}$ = **500 kPa** =>

Table B.1.3 $v_2 = 0.52256\ m^3/kg$; $u_2 = 2802.91\ kJ/kg$

$V_2 = mv_2 = \mathbf{0.05226\ m^3}$

${}_1W_2 = P_{lift}(V_2 - V_1) = 500\ kPa \times (0.05226 - 0.04243)\ m^3 = \mathbf{4.91\ kJ}$

The heat transfer is from the energy equation

${}_1Q_2 = m(u_2 - u_1) + {}_1W_2 = 0.1\ kg \times (2802.91 – 939.51)\ kJ/kg + 4.91\ kJ$

$= \mathbf{191.25\ kJ}$

3.179

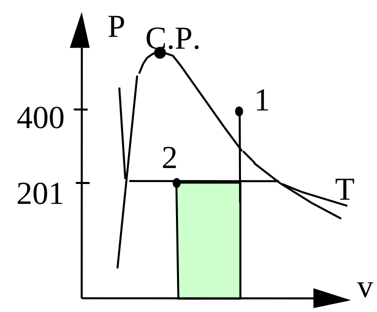
A piston/cylinder contains 0.1 kg R-410A at 600 kPa, 60^{o}C. It is now cooled, so the volume is reduced to half the initial volume. The piston has upper stops mounted and the piston mass and gravitation is such that a floating pressure is 400 kPa.

a) Find the final temperature

b) How much work is involved.

c) What is the heat transfer in the process.

d) Show the process path in a P-v diagram.



Take as CV the 0.1 kg of R-410A. $m_2 = m_1 = m$;

Energy Eq.3.5 $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Process Eq.: $P = P_{float}$ or $v = C = v_1$,

State 1: (P, T) => $v_1 = 0.06023$ m^3/kg,

$u_1 = 304.91$ kJ/kg

State 2: (P, v) => $v_2 = v_1/2 = 0.030115$ m^3/kg $< v_g$, so it is two-phase.

$x_2 = (v_2 - v_f) / v_{fg} = (0.030115 - 0.000803)/0.064 = 0.458$

$u_2 = u_f + x_2 u_{fg} = 27.92 + x_2\, 218.07 = 127.8$ kJ/kg

From process eq.: ${}_1W_2 = \int P\, dV = \text{area} = mP_2 (v_2 - v_1)$

$= 0.1$ kg × 400 kPa (0.030115 – 0.06023) m^3/kg = **- 1.2046 kJ**

From energy eq.: ${}_1Q_2 = m(u_2 - u_1) + {}_1W_2$

= 0.1 kg × (127.8 – 304.91) kJ/kg - 1.2046 kJ

= **-18.9 kJ**

3.180

A piston cylinder contains air at 1000 kPa, 800 K with a volume of 0.05 m^3. The piston is pressed against the upper stops, see Fig. P3.14c and it will float at a pressure of 750 kPa. Now the air is cooled to 400 K. What is the process work and heat transfer?

CV. Air, this is a control mass

Energy Eq.3.5 $\quad m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Process Eq.: $\quad P = P_{float} \quad$ or $\quad v = C = v_1,$

State 1: $\;$ u = 592.58 kJ/kg, $\;$ m = PV/RT = 1000 × 0.05 /(0.287 × 800) = 0.2178 kg

We need to find state 2. Let us see if we proceed past state 1a during the cooling.

$$T_{1a} = T_1\, P_{float} / P_1 = 800 \times 750 / 100 = 600 \text{ K}$$

so we do cool below T_{1a}. That means the piston is floating. Write the ideal gas law for state 1 and 2 to get

$$V_2 = \frac{mRT_2}{P_2} = \frac{P_1V_1T_2}{P_2T_1} = \frac{1000 \times 0.05 \times 400}{750 \times\ 800} \text{ m}^3 = 0.0333 \text{ m}^3$$

$${}_1W_2 = {}_{1a}W_2 = \int P\, dV = P_2\, (V_2 - V_1)$$

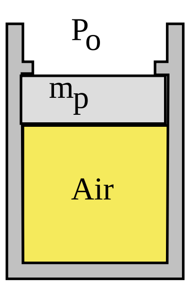
$$= 750 \text{ kPa} \times (0.0333 - 0.05) \text{ m}^3 = \mathbf{-12.5\ kJ}$$

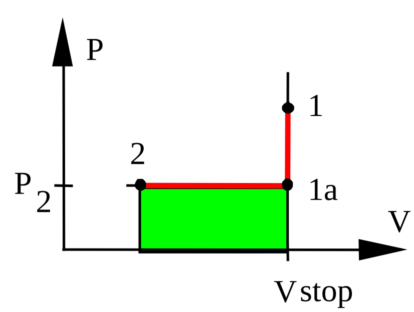
From energy eq.:

$${}_1Q_2 = m(u_2 - u_1) + {}_1W_2$$

$$= 0.2178 \text{ kg} \times (286.49 - 592.58) \text{ kJ/kg} - 12.5 \text{ kJ}$$

$$= \mathbf{-79.2\ kJ}$$





3.181

The piston/cylinder arrangement in Fig. P3.181 contains 10 g ammonia at 20°C with a volume of 1 L. There are some stops so if the piston is at the stops the volume is 1.4 L. The ammonia is now heated to 200°C. The piston and cylinder is made of 0.5 kg aluminum. Assume that the mass has the same temperature as the ammonia at any time. Find the final volume and the total heat transfer and plot the P-V diagram for the process.

C.V. NH3. Control mass goes through process: 1 -> 2 -> 3

Energy Eq.: $U_3 - U_1 = m_{NH3} (u_3 - u_1) + m_{Alu} (u_3 - u_1) = {}_1Q_3 - {}_1W_3$

As piston floats pressure is constant (1 -> 2) and the volume is constant for the second part (2 -> 3) if we go this far. So we have at stop: $v_3 = v_2 = 1.4 \times v_1$

State 1: B.2.1 $v_1 = V/m = 0.001 / 0.01 = 0.1 \text{ m}^3/\text{kg} < v$ so 2-phase P = 857.5 kPa

$$x_1 = (v - v_f)/ v_{fg} = (0.1 - 0.001638)/ 0.14758 = 0.6665$$

$$u_1 = u_f + x_1 u_{fg} = 272.89 + x_1 \times 1059.3 = 978.91 \text{ kJ/kg}$$

State 2: $v_2 = 1.4 \times v_1 = 0.14 \text{ m}^3/\text{kg}$ & P = 857.5 kPa still 2-phase so $T_2 = 20^o\text{C}$

State 3: 200^oC & $v_3 = v_2 = 0.14 \text{ m}^3/\text{kg}$, => P = 1600 kPa, V = **1.4 L**, $u_3 = 1676.5$ kJ/kg

We get the work from the process equation (see P-V diagram)

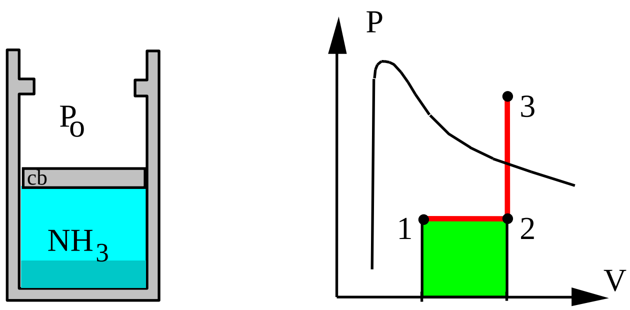
$${}_1W_3 = {}_1W_2 = \int_1^2 PdV = P_1(V_2 - V_1) = 857.5 \text{ kPa} \times (0.0014 - 0.001) \text{ m}^3 = 0.343 \text{ kJ}$$

The heat transfer from the energy equation becomes

$${}_1Q_3 = m_{NH3} (u_3 - u_1) + m_{Alu}C_{Alu} (T_3 - T_1) + {}_1W_3$$

$$= 0.01 \text{ kg } (1676.5 - 978.91) \text{ kJ/kg} + 0.5 \text{ kg} \times 0.9 \text{ kJ/kg-K } (200 - 20) \text{ K} + 0.343 \text{ kJ}$$

$$= \mathbf{88.32 \text{ kJ}}$$



3.182

Air in a piston/cylinder at 200 kPa, 600 K, is expanded in a constant-pressure process to twice the initial volume (state 2), shown in Fig. P3.182. The piston is then locked with a pin and heat is transferred to a final temperature of 600 K. Find *P*, *T*, and *h* for states 2 and 3, and find the work and heat transfer in both processes.

Solution:

C.V. Air. Control mass $m_2 = m_3 = m_1$

Energy Eq.3.5: $u_2 - u_1 = {}_1q_2 - {}_1w_2$;

Process 1 to 2: P = constant => ${}_1w_2 = \int P\,dv = P_1(v_2 - v_1) = R(T_2 - T_1)$

Ideal gas Pv = RT $\Rightarrow$ $T_2 = T_1v_2/v_1 = 2T_1 = \mathbf{1200\ K}$

$P_2 = P_1 = 200$ kPa, ${}_1w_2 = RT_1 = \mathbf{172.2\ kJ/kg}$

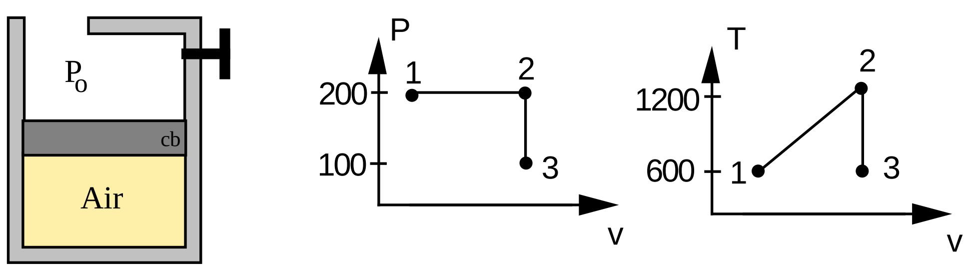
Table A.7 $\mathbf{h_2 = 1277.8\ kJ/kg,}$ $\mathbf{h_3 = h_1 = 607.3\ kJ/kg}$

${}_1q_2 = u_2 - u_1 + {}_1w_2 = h_2 - h_1 = 1277.8 - 607.3 = \mathbf{670.5\ kJ/kg}$

Process 2→3: $v_3 = v_2 = 2v_1$ $\Rightarrow$ $\mathbf{{}_2w_3 = 0,}$

$P_3 = P_2T_3/T_2 = P_1T_1/2T_1 = P_1/2 = \mathbf{100\ kPa}$

${}_2q_3 = u_3 - u_2 = 435.1 - 933.4 = \mathbf{-498.3\ kJ/kg}$



3.183

A piston/cylinder has 0.5 kg air at 2000 kPa, 1000 K as shown in Fig P3.183. The cylinder has stops so $V_{min} = 0.03\ m^3$. The air now cools to 400 K by heat transfer to the ambient. Find the final volume and pressure of the air (does it hit the stops?) and the work and heat transfer in the process.

We recognize this is a possible two-step process, one of constant P and one of constant V. This behavior is dictated by the construction of the device.

Continuity Eq.: $m_2 - m_1 = 0$

Energy Eq.3.5: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Process: $P = \text{constant} = F/A = P_1 \quad \text{if} \quad V > V\text{min}$

$V = \text{constant} = V_{1a} = V_{min} \quad \text{if} \quad P < P_1$

State 1: (P, T) $V_1 = mRT_1/P_1 = 0.5 \times 0.287 \times 1000/2000 = 0.07175\ m^3$

The only possible P-V combinations for this system is shown in the diagram so both state 1 and 2 must be on the two lines. For state 2 we need to know if it is on the horizontal P line segment or the vertical V segment. Let us check state 1a:

State 1a: $P_{1a} = P_1,\ V_{1a} = V_{min}$

$$\text{Ideal gas so } T_{1a} = T_1 \frac{V_{1a}}{V_1} = 1000 \times \frac{0.03}{0.07175} = 418\ K$$

We see that $T_2 < T_{1a}$ and state 2 must have $V_2 = V_{1a} = V_{min} = 0.03\ m^3$.

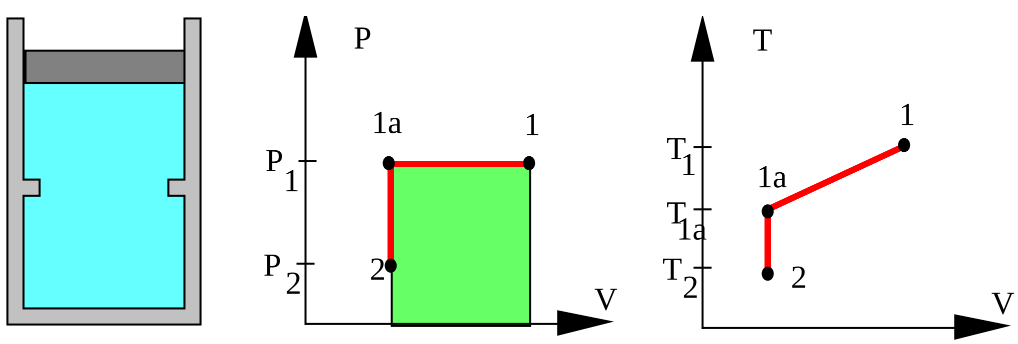
$$P_2 = P_1 \times \frac{T_2}{T_1} \times \frac{V_1}{V_2} = 2000 \times \frac{400}{1000} \times \frac{0.07175}{0.03} = 1913.3\ kPa$$

The work is the area under the process curve in the P-V diagram

$${}_1W_2 = \int_1^2 P\,dV = P_1 (V_{1a} - V_1) = 2000\ kPa\ (0.03 - 0.07175)\ m^3 = \mathbf{-\ 83.5\ kJ}$$

Now the heat transfer is found from the energy equation, u's from Table A.7.1,

$${}_1Q_2 = m(u_2 - u_1) + {}_1W_2 = 0.5\ (286.49 - 759.19) - 83.5 = \mathbf{-319.85\ kJ}$$



3.184

Air in a rigid tank is at 100 kPa, 300 K with a volume of 0.75 m^3. The tank is heated to 400 K, state 2. Now one side of the tank acts as a piston letting the air expand slowly at constant temperature to state 3 with a volume of 1.5 m^3. Find the pressures at states 2 and 3, Find the total work and total heat transfer.

State 1: $m = P_1V_1 / RT_1 = \frac{100 \times 0.75}{0.287 \times 300} \frac{\text{kPa m}^3}{\text{kJ/kg}} = 0.871 \text{ kg}$

Process 1 to 2: Constant volume heating, $dV = 0 \;\Rightarrow\; {}_1W_2 = 0$

$P_2 = P_1 T_2 / T_1 = 100 \times 400 / 300 = \mathbf{133.3 \text{ kPa}}$

Process 2 to 3: Isothermal expansion, $dT = 0 \;\Rightarrow\; u_3 = u_2$ and

$P_3 = P_2 V_2 / V_3 = 133.3 \times 0.75 / 1.5 = \mathbf{66.67 \text{ kPa}}$

$${}_2W_3 = \int_2^3 P\, dV = P_2V_2 \ln\left(\frac{V_3}{V_2}\right) = 133.3 \times 0.75 \ln(2) = 69.3 \text{ kJ}$$

The overall process:

${}_1W_3 = {}_1W_2 + {}_2W_3 = {}_2W_3 = \mathbf{69.3 \text{ kJ}}$

From the energy equation

$${}_1Q_3 = m(u_3 - u_1) + {}_1W_3 = m\, C_v\, (T_3 - T_1) + {}_1W_3$$
$$= 0.871 \text{ kg} \times 0.717 \text{ kJ/kg-K} \times (400 - 300) \text{ K} + 69.3 \text{ kJ}$$
$$= \mathbf{131.8 \text{ kJ}}$$

3.185

A 100 hp car engine has a drive shaft rotating at 2000 RPM. How much torque is on the shaft for 25% of full power?

Solution:

Power = 0.25 × 100 hp = 0.25 × 73.5 kW (if SI hp) = 18.375 kW = $T\omega$

ω = angular velocity (rad/s) = RPM 2 π /60

$$T = \text{Power}/\omega = \frac{\text{power} \times 60}{\text{RPM} \times 2\pi} = \frac{18.375 \text{ kW} \times 60 \text{ s/min}}{2000 \times 2\pi \text{ rad/min}} = \mathbf{87.73\ Nm}$$

We could also have used UK hp to get 0.25 × 74.6 kW. then T = 89 Nm.

3.186

A crane use 2 kW to raise a 100 kg box 20 m. How much time does it take?

$$\text{Power} = \dot{W} = F\mathbf{V} = mg\mathbf{V} = mg\frac{L}{t}$$

$$t = \frac{mgL}{\dot{W}} = \frac{100 \text{ kg } 9.807 \text{ m/s}^2 \text{ } 20 \text{ m}}{2000 \text{ W}} = \mathbf{9.81 \text{ s}}$$

3.187

An escalator raises a 100 kg bucket of sand 10 m in 1 minute. Determine the rate of work done during the process.

Solution:

The work is a force with a displacement and force is constant: $F = mg$

$$W = \int F\,dx = F \int dx = F\,\Delta x = 100\text{ kg} \times 9.80665\text{ m/s}^2 \times 10\text{ m} = 9807\text{ J}$$

The rate of work is work per unit time

$$\dot{W} = \frac{W}{\Delta t} = \frac{9807\text{ J}}{60\text{ s}} = \mathbf{163\ W}$$

3.188

A piston/cylinder of cross sectional area 0.01 m^2 maintains constant pressure. It contains 1 kg water with a quality of 5% at 150^{o}C. If we heat so 1 g/s liquid turns into vapor what is the rate of work out?

$V_{vapor} = m_{vapor}\, v_g\,, \quad V_{liq} = m_{liq}\, v_f$

m_{tot} = constant = $m_{vapor}\, m_{liq}$

$V_{tot} = V_{vapor} + V_{liq}$

$\dot{m}_{tot} = 0 = \dot{m}_{vapor} + \dot{m}_{liq} \quad \Rightarrow \quad \dot{m}_{liq} = -\dot{m}_{vapor}$

$\dot{V}_{tot} = \dot{V}_{vapor} + \dot{V}_{liq} = \dot{m}_{vapor} v_g + \dot{m}_{liq} v_f$

$= \dot{m}_{vapor}\, (v_g - v_f\,) = \dot{m}_{vapor}\, v_{fg}$

$\dot{W} = P\dot{V} = P\, \dot{m}_{vapor}\, v_{fg}$

$= 475.9$ kPa $\times$ 0.001 kg/s $\times$ 0.39169 m^3/kg **= 0.1864 kW**
= 186 W

3.189

A pot of water is boiling on a stove supplying 325 W to the water. What is the rate of mass (kg/s) vaporizing assuming a constant pressure process?

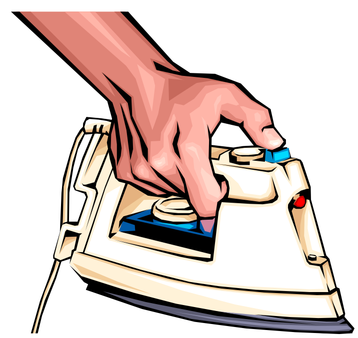
To answer this we must assume all the power goes into the water and that the process takes place at atmospheric pressure 101 kPa, so T = 100^{o}C.

Energy equation

$$dQ = dE + dW = dU + PdV = dH = h_{fg}\, dm$$

$$\frac{dQ}{dt} = h_{fg} \frac{dm}{dt}$$

$$\frac{dm}{dt} = \frac{\dot{Q}}{h_{fg}} = \frac{325 \text{ W}}{2257 \text{ kJ/kg}} = \mathbf{0.144 \ g/s}$$



The volume rate of increase is

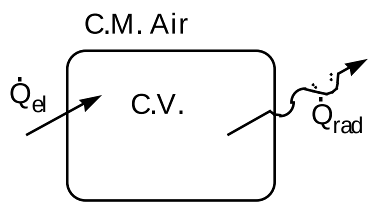
$$\frac{dV}{dt} = \frac{dm}{dt} v_{fg} = 0.144 \text{ g/s} \times 1.67185 \text{ m}^3\text{/kg}$$

$$= 0.24 \times 10^{-3} \text{ m}^3\text{/s} = 0.24 \text{ L/s}$$

3.190

The heaters in a spacecraft suddenly fail. Heat is lost by radiation at the rate of 100 kJ/h, and the electric instruments generate 75 kJ/h. Initially, the air is at 100 kPa, 25°C with a volume of 10 m^3. How long will it take to reach an air temperature of –20°C?

Solution:



Continuity Eq: $\dfrac{dM}{dt} = 0$ $\qquad \dot{W} = 0$

Energy Eq: $\dfrac{dE}{dt} = \dot{Q}_{el} - \dot{Q}_{rad}$ $\qquad \dot{KE} = 0 \qquad \dot{PE} = 0$

$$\dot{E} = \dot{U} = \dot{Q}_{el} - \dot{Q}_{rad} = \dot{Q}_{net} \;\Rightarrow\; U_2 - U_1 = m(u_2 - u_1) = \dot{Q}_{net}(t_2 - t_1)$$

$$\text{Ideal gas: } m = \frac{P_1V_1}{RT_1} = \frac{100 \times 10}{0.287 \times 298.15} = 11.688 \text{ kg}$$

$$m(u_2 - u_1) = m\, C_{v0}(T_2 - T_1) = 11.688 \times 0.717\,(-20 - 25) = -377.1 \text{ kJ}$$

$$t_2 - t_1 = m\,(u_2 - u_1)/\dot{Q}_{net} = (-377.1 \text{ kJ}) /(-25 \text{ kJ/h}) = \mathbf{15.08\ h}$$

3.191

As fresh poured concrete hardens, the chemical transformation releases energy at a rate of 2 W/kg. Assume the center of a poured layer does not have any heat loss and that it has an average specific heat of 0.9 kJ/kg-K. Find the temperature rise during 1 hour of the hardening (curing) process.

Solution:

$$\dot{U} = (m\dot{u}) = mC_V\dot{T} = \dot{Q} = m\dot{q}$$

$$\dot{T} = \dot{q}/C_V = 2\times10^{-3}\ \text{kW}/\ 0.9\ \text{kJ/kg-K}$$

$$= 2.222 \times 10^{-3}\ \ °\text{C/sec}$$

$$\Delta T = \dot{T}\Delta t = 2.222 \times 10^{-3} \times 3600 = \mathbf{8\ °C}$$

3.192

A pot of 1.2 kg water at 20^{o}C is put on a stove supplying 1250 W to the water. After how long time can I expect it to come to a boil (100^{o}C)?

Energy Equation on a rate form: $\frac{dE_{water}}{dt} = \frac{dU_{water}}{dt} = \dot{Q} - \dot{W} = \dot{Q} - P\dot{V}$

$$\dot{Q} = \frac{dU_{water}}{dt} + P\dot{V} = \frac{dH_{water}}{dt} = m_{water}C_p \frac{dT_{water}}{dt}$$

Integrate over time

$$Q = \dot{Q}\ \Delta t = \Delta H = m_{water}\ (h_2 - h_1) \approx m_{water}C_p\ (T_2 - T_1)$$

$$\Delta t = m_{water}\ (h_2 - h_1)/\dot{Q} \approx m_{water}C_p\ (T_2 - T_1)/\dot{Q}$$
$$= 1.2\ (419.02 - 83.94)/1.25 \ \approx\ 1.2 \times 4.18\ (100 - 20)/1.25$$
$$= \mathbf{321.7\ s} \ \approx\ 5.5 \text{ min}$$

Comment: Notice how close the two results are, i.e. use of constant C_p is OK.

3.193

A computer in a closed room of volume 200 m^3 dissipates energy at a rate of 10 kW. The room has 50 kg wood, 25 kg steel and air, with all material at 300 K, 100 kPa. Assuming all the mass heats up uniformly, how long will it take to increase the temperature 10°C?

Solution:

C.V. Air, wood and steel. $m_2 = m_1$; no work

Energy Eq.3.5: $U_2 - U_1 = {}_1Q_2 = \dot{Q}\Delta t$

The total volume is nearly all air, but we can find volume of the solids.

$V_{wood} = m/\rho = 50/510 = 0.098\ m^3$; $V_{steel} = 25/7820 = 0.003\ m^3$

$V_{air} = 200 - 0.098 - 0.003 = 199.899\ m^3$

$$m_{air} = PV/RT = 101.325 \times 199.899/(0.287 \times 300) = 235.25\ kg$$

We do not have a u table for steel or wood so use heat capacity from A.3.

$$\Delta U = [m_{air}\, C_v + m_{wood}\, C_v + m_{steel}\, C_v\,]\Delta T$$

$$= (235.25 \times 0.717 + 50 \times 1.38 + 25 \times 0.46)\ kJ/K \times 10\ K$$

$$= 1686.7 + 690 + 115 = 2492\ kJ = \dot{Q} \times \Delta t = 10\ kW \times \Delta t$$

=> $\Delta t = 2492\ kJ/10\ kW =$ **249.2 sec = 4.2 minutes**

3.194

The rate of heat transfer to the surroundings from a person at rest is about 400 kJ/h. Suppose that the ventilation system fails in an auditorium containing 100 people. Assume the energy goes into the air of volume 1500 m^3 initially at 300 K and 101 kPa. Find the rate (degrees per minute) of the air temperature change.

Solution:

$$\dot{Q} = n\,\dot{q} = 100 \times 400 = 40\,000 \text{ kJ/h} = 666.7 \text{ kJ/min}$$

$$\frac{dE_{air}}{dt} = \dot{Q} = m_{air} C_v \frac{dT_{air}}{dt}$$

$$m_{air} = PV/RT = 101 \text{ kPa} \times 1500 \text{ m}^3 / (0.287 \text{ kJ/kg-K} \times 300 \text{ K}) = 1759.6 \text{ kg}$$

$$\frac{dT_{air}}{dt} = \dot{Q}\,/mC_v = 666.7 \text{ kJ/min} / (1759.6 \text{ kg} \times 0.717 \text{ kJ/kg-K})$$

$$= \mathbf{0.53°C/min}$$

3.195

A steam generating unit heats saturated liquid water at constant pressure of 800 kPa in a piston cylinder. If 1.5 kW of power is added by heat transfer find the rate (kg/s) of saturated vapor that is made.

Solution:

Energy equation on a rate form making saturated vapor from saturated liquid

$$\dot{U} = (\dot{m}u) = \dot{m}\Delta u = \dot{Q} - \dot{W} = \dot{Q} - P\dot{V} = \dot{Q} - P\,\dot{m}\Delta v$$

Rearrange to solve for heat transfer rate

$$\dot{Q} = \dot{m}(\Delta u + \Delta vP) = \dot{m}\,\Delta h = \dot{m}\,h_{fg}$$

So now

$$\dot{m} = \dot{Q}/\,h_{fg} = 1.5\text{ kW}/\ 2048.04\text{ kJ/kg} = \mathbf{0.732\ g/s}$$

3.196

A 500 Watt heater is used to melt 2 kg of solid ice at –10^{o}C to liquid at +5^{o}C at a constant pressure of 150 kPa.

a) Find the change in the total volume of the water.

b) Find the energy the heater must provide to the water.

c) Find the time the process will take assuming uniform T in the water.

Solution:

Take CV as the 2 kg of water. $m_2 = m_1 = m$;

Energy Eq.3.5 $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

State 1: Compressed solid, take saturated solid at same temperature.

$v = v_i(-10) = 0.0010891 \text{ m}^3/\text{kg}$, $h = h_i = -354.09 \text{ kJ/kg}$

State 2: Compressed liquid, take saturated liquid at same temperature

$v = v_f = 0.001$, $h = h_f = 20.98 \text{ kJ/kg}$

Change in volume:

$$V_2 - V_1 = m(v_2 - v_1) = 2(0.001 - 0.0010891) = \mathbf{-0.000178\ m^3}$$

Work is done while piston moves at constant pressure, so we get

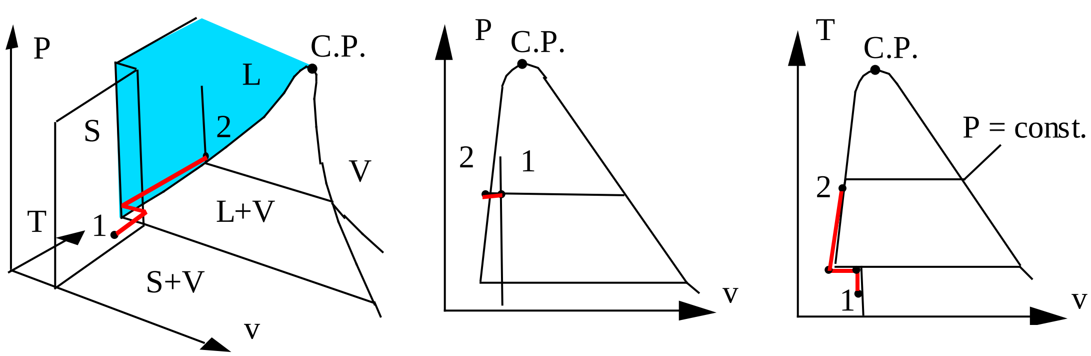
$${}_1W_2 = \int P\, dV = \text{area} = P(V_2 - V_1) = -150 \times 0.000178 = -0.027 \text{ kJ} = -27 \text{ J}$$

Heat transfer is found from energy equation

$${}_1Q_2 = m(u_2 - u_1) + {}_1W_2 = m(h_2 - h_1) = 2 \times [20.98 - (-354.09)] = \mathbf{750\ kJ}$$

The elapsed time is found from the heat transfer and the rate of heat transfer

$$t = {}_1Q_2/\dot{Q} = (750 \text{ kJ} /500 \text{ W}) \times 1000 \text{ J/kJ} = 1500 \text{ s} = \mathbf{25\ min}$$



3.197

A drag force on a car, with frontal area A = 2 m^2, driving at 80 km/h in air at 20^o C is $F_d = 0.225\ A\ \rho_{air}\mathbf{V}^2$. How much power is needed and what is the traction force?

$$\dot{W} = F\mathbf{V}$$

$$\mathbf{V} = 80\ \frac{km}{h} = 80 \times \frac{1000}{3600}\ ms^{-1} = 22.22\ ms^{-1}$$

$$\rho_{AIR} = \frac{P}{RT} = \frac{101}{0.287 \times 293} = 1.20\ kg/m^3$$

$$F_d = 0.225\ A\rho\mathbf{V}^2 = 0.225 \times 2 \times 1.2 \times 22.22^2 = \mathbf{266.61\ N}$$

$$\dot{W} = F\mathbf{V} = 266.61\ N \times 22.22\ m/s = 5924\ W = \mathbf{5.92\ kW}$$

3.198

A mass of 3 kg nitrogen gas at 2000 K, V = C, cools with 500 W. What is dT/dt?

Process: V = C → ${}_1W_2 = 0$

$$\frac{dE}{dt} = \frac{dU}{dt} = m\frac{dU}{dt} = mC_v\frac{dT}{dt} = \dot{Q} - W = \dot{Q} = -500 \text{ W}$$

$$C_{v\ 2000} = \frac{du}{dT} = \frac{\Delta u}{\Delta T} = \frac{u_{2100} - u_{1900}}{2100\text{-}1900} = \frac{1819.08 - 1621.66}{200} = 0.987 \text{ kJ/kg K}$$

$$\frac{dT}{dt} = \frac{\dot{Q}}{mC_v} = \frac{-500 \text{ W}}{3 \times 0.987 \text{ kJ/K}} = \mathbf{-0.17\ \frac{K}{s}}$$

Remark: Specific heat from Table A.5 has $C_{v\ 300}$ = 0.745 kJ/kg K which is nearly 25% lower and thus would over-estimate the rate with 25%.

3.199

Consider the pot in Problem 3.119. Assume the stove supplies 1 kW of heat. How much time does the process take?

A 1 kg steel pot contains 1 kg liquid water both at 15^{o}C. It is now put on the stove where it is heated to the boiling point of the water. Neglect any air being heated and find the total amount of energy needed.

Solution:

Energy Eq.: $U_2 - U_1 = {}_1Q_2 - {}_1W_2$

The steel does not change volume and the change for the liquid is minimal, so ${}_1W_2 \cong 0$.

State 2: $T_2 = T_{sat}$ (1atm) = 100^{o}C

Tbl B.1.1 : u_1 = 62.98 kJ/kg, u_2 = 418.91 kJ/kg

Tbl A.3 : C_{st} = 0.46 kJ/kg K

Solve for the heat transfer from the energy equation

$$ {}_1Q_2 = U_2 - U_1 = m_{st}\,(u_2 - u_1)_{st} + m_{H2O}\,(u_2 - u_1)_{H2O} $$
$$ = m_{st}C_{st}\;(T_2 - T_1) + m_{H2O}\,(u_2 - u_1)_{H2O} $$

$$ {}_1Q_2 = 1\text{ kg} \times 0.46\,\frac{\text{kJ}}{\text{kg K}} \times (100 - 15)\text{ K} + 1\text{ kg} \times (418.91 - 62.98)\text{ kJ/kg} $$
$$ = 39.1 + 355.93 = 395\text{ kJ} $$

To transfer that amount of heat with a rate of 1 kW we get the relation

$$ {}_1Q_2 = \int \dot{Q}\,dt = \dot{Q}\,\Delta t \quad => \quad \Delta t = {}_1Q_2 / \dot{Q} = 395\text{ kJ} /1\text{ (kJ/s)} = \mathbf{395\ s} $$

General work

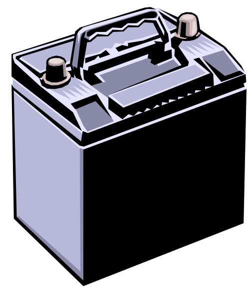
3.200

Electric power is volts times ampere (P = V i). When a car battery at 12 V is charged with 6 amp for 3 hours how much energy is delivered?

Solution:

Work term integrated as

$$W = \int \dot{W}\, dt = \dot{W}\, \Delta t = V\, i\, \Delta t$$
$$= 12\ \text{V} \times 6\ \text{Amp} \times 3 \times 3600\ \text{s}$$
$$= 777\ 600\ \text{J} = \mathbf{777.6\ kJ}$$



Remark: Volt times ampere is also watts, $1\ \text{W} = 1\ \text{V} \times 1\ \text{Amp.} = 1\ \text{J/s}$

3.201

A copper wire of diameter 2 mm is 10 m long and stretched out between two posts. The normal stress (pressure) $\sigma = E(L - L_o)/L_o$, depends on the length L versus the un-stretched length L_o and Young's modulus $E = 1.1 \times 10^6$ kPa. The force is $F = A\sigma$ and measured to be 110 N. How much longer is the wire and how much work was put in?

Solution:

$F = As = AE\ \Delta L/\ L_o$ and $\Delta L = FL_o\ /AE$

$$A = \frac{\pi}{4}D^2 = \frac{\pi}{4} \times 0.002^2 = 3.142 \times 10^{-6}\ m^2$$

$$\Delta L = \frac{110\ N \times 10\ m}{3.142 \times 10^{-6}\ m^2 \times 1.1 \times 10^6 \times 10^3\ Pa} = \mathbf{0.318\ m}$$

$${}_1W_2 = \int F\ dx = \int A\ s\ dx = \int AE \frac{x}{L_o}\ dx$$

$$= \frac{AE}{L_o}\ ½\ x^2 \qquad \text{where } x = L - L_o$$

$$= \frac{3.142 \times 10^{-6} \times 1.1 \times 10^6 \times 10^3}{10} \times ½ \times 0.318^2 = \mathbf{17.47\ J}$$

3.202

A film of ethanol at 20°C has a surface tension of 22.3 mN/m and is maintained on a wire frame as shown in Fig. P3.202. Consider the film with two surfaces as a control mass and find the work done when the wire is moved 10 mm to make the film 20 × 40 mm.

Solution :

Assume a free surface on both sides of the frame, i.e., there are two surfaces 20 × 30 mm

$$W = -\int S\, dA = -22.3\times 10^{-3}\ \text{N/m} \times 2\ (800 - 600) \times 10^{-6}\ \text{m}^2$$

$$= -8.92\times 10^{-6}\ \text{J} = \mathbf{-8.92\ \mu J}$$

3.203

A 10-L rigid tank contains R-410A at –10°C, 80% quality. A 10-A electric current (from a 6-V battery) is passed through a resistor inside the tank for 10 min, after which the R-410A temperature is 40°C. What was the heat transfer to or from the tank during this process?

Solution:

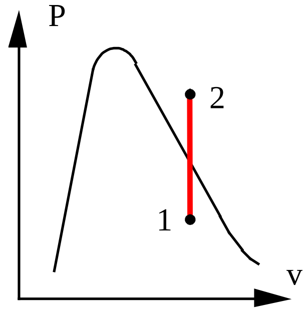
C.V. R-410A in tank. Control mass at constant V.

Continuity Eq.: $m_2 = m_1 = m$;

Energy Eq.: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Process: Constant V $\Rightarrow$ $v_2 = v_1$

=> no boundary work, but electrical work



State 1 from table B.4.1

$v_1 = 0.000827 + 0.8 \times 0.04470 = 0.03659 \text{ m}^3\text{/kg}$

$u_1 = 42.32 + 0.8 \times 207.36 = 208.21 \text{ kJ/kg}$

$m = V/v = 0.010 \text{ m}^3 /0.03659 \text{ m}^3\text{/kg} = 0.2733 \text{ kg}$

State 2: Table B.4.2 at 40°C and $v_2 = v_1 = 0.03659 \text{ m}^3\text{/kg}$

=> superheated vapor, so use linear interpolation to get

$P_2 = 800 + 200 \times (0.03659 – 0.04074)/(0.03170 – 0.04074)$

$= 800 + 200 \times 0.45907 = 892 \text{ kPa}$

$u_2 = 286.83 + 0.45907\times (284.35 – 286.83) = 285.69 \text{ kJ/kg}$

$${}_1W_{2\ \text{elec}} = –\text{power} \times \Delta t = –\text{Amp} \times \text{volts} \times \Delta t = –\frac{10 \times 6 \times 10 \times 60}{1000} = –36 \text{ kJ}$$

${}_1Q_2 = m(u_2 – u_1) + {}_1W_2 = 0.2733 \text{ kg} (285.69 – 208.21) \text{ kJ/kg} – 36 \text{ kJ}$

$= \mathbf{–14.8\ kJ}$

3.204

A battery is well insulated while being charged by 12.3 V at a current of 6 A. Take the battery as a control mass and find the instantaneous rate of work and the total work done over 4 hours.

Solution :

Battery thermally insulated $\Rightarrow$ Q = 0

For constant voltage *E* and current i,

Power = *E* i = 12.3 × 6 = **73.8 W** [Units V × A = W]

$$W = \int \text{power } dt = \text{power } \Delta t$$

$$= 73.8 \text{ W} \times 4 \text{ h} \times 3600 \text{ (s/h)} = 1\,062\,720 \text{ J} = \mathbf{1062.7 \text{ kJ}}$$

3.205

A sheet of rubber is stretched out over a ring of radius 0.25 m. I pour liquid water at 20^{o}C on it, as in Fig. P3.205, so the rubber forms a half sphere (cup). Neglect the rubber mass and find the surface tension near the ring?

Solution:

$F\uparrow = F\downarrow$; $F\uparrow =$ SL

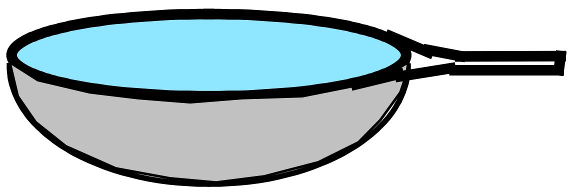
The length is the perimeter, 2πr, and there is two surfaces

$$S \times 2 \times 2\pi r = m_{H2o}\, g = \rho_{H2o}\, Vg = \rho_{H2o} \times \frac{1}{12}\,\pi\,(2r)^{3} g = \rho_{H2o} \times \pi\,\frac{2}{3}\,r^{3}$$

$$S = \rho_{H2o}\,\frac{1}{6}\,r^{2}\,g$$

$$= 997\ kg/m^{3} \times \frac{1}{6} \times 0.25^{2}\ m^{2} \times 9.81\ m/s^{2}$$

= **101.9 N/m**



3.206

Assume we fill a spherical balloon from a bottle of helium gas. The helium gas provides work $\int PdV$ that stretches the balloon material $\int S\, dA$ and pushes back the atmosphere $\int P_o\, dV$. Write the incremental balance for $dW_{helium} = dW_{stretch} + dW_{atm}$ to establish the connection between the helium pressure, the surface tension S and P_o as a function of radius.

$$W_{He} = \int P\, dV = \int S\, dA + \int P_o\, dV$$

$$dW_{He} = P\, dV = S\, dA + P_o\, dV$$

$$dV = d\left(\frac{\pi}{6} D^3 \right) = \frac{\pi}{6} \times 3D^2\, dD$$

$$dA = d\left(2 \times \pi \times D^2 \right) = 2\pi\, (2D)\, dD$$

$$P \frac{\pi}{2} D^2\, dD = S\, (4\pi) D\, dD + P_o \frac{\pi}{2} D^2\, dD$$

Divide by $\frac{\pi}{2} D^2$ to recognize

$$\mathbf{P_{He} = P_o + 8 \frac{S}{D} = P_o + 4 \frac{S}{r}}$$

3.207

Assume a balloon material with a constant surface tension of S = 2 N/m. What is the work required to stretch a spherical balloon up to a radius of r = 0.5 m? Neglect any effect from atmospheric pressure.

Assume the initial area is small, and that we have 2 surfaces inside and out

$$W = -\int S\, dA = -S\,(A_2 - A_1)$$

$$= -\,S(A_2) = -S(\,2\times \pi\, \mathbf{D}_2^2\,)$$

$$= -2\text{ N/m} \times 2 \times \pi \times 1\text{ m}^2 = -12.57\text{ J}$$

$W_{in} = -W =$ **12.57 J**

3.208

A soap bubble has a surface tension of S = 3 × 10^{-4} N/cm as it sits flat on a rigid ring of diameter 5 cm. You now blow on the film to create a half sphere surface of diameter 5 cm. How much work was done?

$$_1W_2 = \int F\ dx = \int S\ dA = S\ \Delta A$$

$$= 2 \times S \times (\frac{\pi}{2} D^2 - \frac{\pi}{4} D^2)$$

$$= 2 \times 3 \times 10^{-4} \text{ N/cm} \times 100 \text{ cm/m} \times \frac{\pi}{2} 0.05^2 \text{ m}^2 (1- 0.5)$$

$$= \mathbf{1.18 \times 10^{-4}\ J}$$

Notice the bubble has 2 surfaces.

$$A_1 = \frac{\pi}{4} D^2 ,$$

$$A_2 = ½ \pi D^2$$

3.209

A 0.5-m-long steel rod with a 1-cm diameter is stretched in a tensile test. What is the required work to obtain a relative strain of 0.1%? The modulus of elasticity of steel is 2×10^8 kPa.

Solution :

$$-{}_1W_2 = \frac{AEL_0}{2}(e)^2, \quad A = \frac{\pi}{4}(0.01)^2 = 78.54 \times 10^{-6}\ m^2$$

$$-{}_1W_2 = \frac{1}{2} \times 78.54 \times 10^{-6}\ m^2 \times 2{\times}10^8\ kPa \times 0.5\ m\ (10^{-3})^2$$

$$= \mathbf{3.93\ J}$$

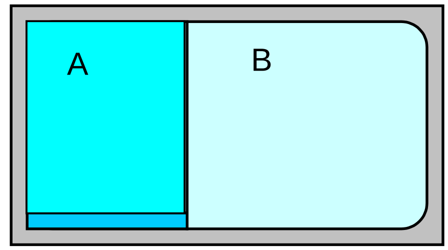
More Complex Devices

3.210

A rigid tank is divided into two rooms by a membrane, both containing water, shown in Fig. P3.210. Room A is at 200 kPa, $v = 0.5\ m^3/kg$, $V_A = 1\ m^3$, and room B contains 3.5 kg at 0.5 MPa, 400°C. The membrane now ruptures and heat transfer takes place so the water comes to a uniform state at 100°C. Find the heat transfer during the process.

Solution:

C.V.: Both rooms A and B in tank.



Continuity Eq.: $m_2 = m_{A1} + m_{B1}$;

Energy Eq.: $m_2u_2 - m_{A1}u_{A1} - m_{B1}u_{B1} = {}_1Q_2 - {}_1W_2$

State 1A: (P, v) Table B.1.2, $m_{A1} = V_A/v_{A1} = 1/0.5 = 2\ kg$

$$x_{A1} = \frac{v - v_f}{v_{fg}} = \frac{0.5 - 0.001061}{0.88467} = 0.564$$

$$u_{A1} = u_f + x\ u_{fg} = 504.47 + 0.564 \times 2025.02 = 1646.6\ kJ/kg$$

State 1B: Table B.1.3, $v_{B1} = 0.6173$, $u_{B1} = 2963.2$, $V_B = m_{B1}v_{B1} = 2.16\ m^3$

Process constant total volume: $V_{tot} = V_A + V_B = 3.16\ m^3$ and ${}_1W_2 = \emptyset$

$$m_2 = m_{A1} + m_{B1} = 5.5\ kg \quad => \quad v_2 = V_{tot}/m_2 = 0.5746\ m^3/kg$$

State 2: T_2 , v_2 $\Rightarrow$ Table B.1.1 two-phase as $v_2 < v_g$

$$x_2 = \frac{v_2 - v_f}{v_{fg}} = \frac{0.5746 - 0.001044}{1.67185} = 0.343\ ,$$

$$u_2 = u_f + x\ u_{fg} = 418.91 + 0.343 \times 2087.58 = 1134.95\ kJ/kg$$

Heat transfer is from the energy equation

$${}_1Q_2 = m_2u_2 - m_{A1}u_{A1} - m_{B1}u_{B1}$$
$$= (5.5 \times 1134.95 - 2 \times 1646.6 - 3.5 \times 2963.2)\ kg \times kJ/kg$$
$$= \mathbf{-7421\ kJ}$$

3.211

A piston cylinder has a water volume separated in $V_A = 0.2\ m^3$ and $V_B = 0.3\ m^3$ by a stiff membrane. The initial state in A is 1000 kPa, x = 0.75 and in B it is 1600 kPa and 250°C. Now the membrane ruptures and the water comes to a uniform state at 200°C. What is the final pressure? Find the work and the heat transfer in the process.

Take the water in A and B as CV.

Continuity: $m_2 - m_{1A} - m_{1B} = 0$

Energy: $m_2u_2 - m_{1A}u_{1A} - m_{1B}u_{1B} = {}_1Q_2 - {}_1W_2$

Process: $P_2 = P_{eq} = \text{constant} = P_{1A}$ as piston floats and m_p, P_o do not change

State 1A: Two phase. Table B.1.2

$v_{1A} = 0.001127 + 0.75 \times 0.19332 = 0.146117\ m^3/kg$,

$u_{1A} = 761.67 + 0.75 \times 1821.97 = 2128.15\ kJ/kg$

State 1B: Table B.1.3 $v_{1B} = 0.14184\ m^3/kg$, $u_{1B} = 2692.26\ kJ/kg$

=> $m_{1A} = V_{1A}/v_{1A} = 1.3688\ kg$, $m_{1B} = V_{1B}/v_{1B} = 2.115\ kg$

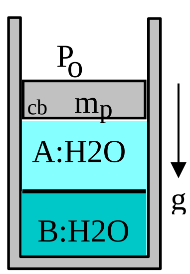
State 2: 1000 kPa, 200°C sup. vapor => $v_2 = 0.20596\ m^3/kg$, $u_2 = 2621.9\ kJ/kg$

$m_2 = m_{1A} + m_{1B} = 3.4838\ kg$ => $V_2 = m_2v_2 = 3.4838 \times 0.20596 = 0.7175\ m^3$

Piston moves at constant pressure

$${}_1W_2 = \int P\, dV = P_{eq}(V_2 - V_1) = 1000\ kPa \times (0.7175 - 0.5)\ m^3 = \mathbf{217.5\ kJ}$$

$${}_1Q_2 = m_2u_2 - m_{1A}u_{1A} - m_{1B}u_{1B} + {}_1W_2$$
$$= 3.4838 \times 2621.9 - 1.3688 \times 2128.15 - 2.115 \times 2692.26 + 217.5 = \mathbf{744\ kJ}$$



3.212

The cylinder volume below the constant loaded piston has two compartments A and B filled with water. A has 0.5 kg at 200 kPa, 150^{o}C and B has 400 kPa with a quality of 50% and a volume of 0.1 m^3. The valve is opened and heat is transferred so the water comes to a uniform state with a total volume of 1.006 m^3.

a) Find the total mass of water and the total initial volume.

b) Find the work in the process

c) Find the process heat transfer.

Solution:

Take the water in A and B as CV.

Continuity: $m_2 - m_{1A} - m_{1B} = 0$

Energy: $m_2u_2 - m_{1A}u_{1A} - m_{1B}u_{1B} = {}_1Q_2 - {}_1W_2$

Process: $P = \text{constant} = P_{1A}$ if piston floats

$(V_A$ positive) i.e. if $V_2 > V_B = 0.1$ m^3

State A1: Sup. vap. Table B.1.3 $v = 0.95964$ m^3/kg, $u = 2576.9$ kJ/kg

=> $V = mv = 0.5 \text{ kg} \times 0.95964 \text{ m}^3\text{/kg} = 0.47982$

State B1: Table B.1.2 $v = (1-x) \times 0.001084 + x \times 0.4625 = 0.2318$ m^3/kg

=> $m = V/v = 0.4314$ kg

$u = 604.29 + 0.5 \times 1949.3 = 1578.9$ kJ/kg

State 2: 200 kPa, $v_2 = V_2/m = 1.006/0.9314 = 1.0801$ m^3/kg

Table B.1.3 => close to $T_2 = 200^o$C and $u_2 = 2654.4$ kJ/kg

So now

$V_1 = 0.47982 + 0.1 =$ **0.5798 m^3**, $m_1 = 0.5 + 0.4314 =$ **0.9314 kg**

Since volume at state 2 is larger than initial volume piston goes up and the pressure then is constant (200 kPa which floats piston).

$${}_1W_2 = \int P\, dV = P_{lift}\,(V_2 - V_1) = 200 \text{ kPa } (1.006 - 0.57982) \text{ m}^3 = \mathbf{85.24\ kJ}$$

$${}_1Q_2 = m_2u_2 - m_{1A}u_{1A} - m_{1B}u_{1B} + {}_1W_2$$
$$= 0.9314 \times 2654.4 - 0.5 \times 2576.9 - 0.4314 \times 1578.9 + 85.24 = \mathbf{588\ kJ}$$

3.213

Water in a tank A is at 250 kPa with a quality of 10% and mass 0.5 kg. It is connected to a piston cylinder holding constant pressure of 200 kPa initially with 0.5 kg water at 400°C. The valve is opened and enough heat transfer takes place to have a final uniform temperature of 150°C. Find the final P and V, the process work and the process heat transfer.

C.V. Water in A and B. Control mass goes through process: 1 -> 2

Continuity Eq.: $m_2 - m_{A1} - m_{B1} = 0$ => $m_2 = m_{A1} + m_{B1} = 0.5 + 0.5 = 1$ kg

Energy Eq.: $U_2 - U_1 = {}_1Q_2 - {}_1W_2$

State A1: $v_{A1} = 0.001067 + x_{A1} \times 0.71765 = 0.072832$; $V_{A1} = mv = 0.036416$ m^3

$u_{A1} = 535.08 + 0.1 \times 2002.14 = 735.22$ kJ/kg

State B1: $v_{B1} = 1.5493$ m^3/kg; $u_{B1} = 2966.69$ kJ/kg

=> $V_{B1} = m_{B1}v_{B1} = 0.77465$ m^3

State 2: If $V_2 > V_{A1}$ then P_2 = **200 kPa** that is the piston floats.

For (T_2, P_2) = (150°C, 200 kPa) => superheated vapor $u_2 = 2576.87$ kJ/kg

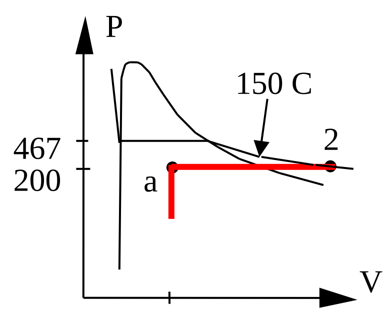
$v_2 = 0.95964$ m^3/kg $V_2 = m_2v_2$ = **0.95964 m³** $> V_{A1}$ checks OK.

The possible state 2 (P,V) combinations are shown. State "a" is at 200 kPa and

$$v_a = \frac{V_{A1}}{m_2} = 0.036\ m^3/kg$$

and thus two-phase

$T_a = 120°C < T_2$



Process: ${}_1W_2 = P_2 (V_2 - V_1) = 200\ kPa \times (0.95964 - 0.77465 - 0.036416)\ m^3$

= **29.72 kJ**

From the energy Eq.:

${}_1Q_2 = m_2u_2 - m_{A1}u_{A1} - m_{B1}u_{B1} + {}_1W_2$

$= 1 \times 2576.87 - 0.5 \times 735.222 - 0.5 \times 2966.69 + 29.72$ = **755.6 kJ**

3.214

Two rigid tanks are filled with water. Tank A is 0.2 m^3 at 100 kPa, 150^{o}C and tank B is 0.3 m^3 at saturated vapor 300 kPa. The tanks are connected by a pipe with a closed valve. We open the valve and let all the water come to a single uniform state while we transfer enough heat to have a final pressure of 300 kPa. Give the two property values that determine the final state and find the heat transfer.

State A1: u = 2582.75 kJ/kg, v = 1.93636 m^3/kg

=> m_{A1} = V/v = 0.2/1.93636 = **0.1033 kg**

State B1: u = 2543.55 kJ/kg, v = 0.60582 m^3/kg

=> m_{B1} = V/v = 0.3 / 0.60582 = **0.4952 kg**

The total volume (and mass) is the sum of volumes (mass) for tanks A and B.

$m_2 = m_{A1} + m_{B1} = 0.1033 + 0.4952 = 0.5985$ kg,

$V_2 = V_{A1} + V_{B1} = 0.2 + 0.3 = 0.5\ m^3$

=> $v_2 = V_2/m_2 = 0.5 / 0.5985 = \mathbf{0.8354\ m^3/kg}$

State 2: $[P_2, v_2]$ = [300 kPa, 0.8354 m^3/kg]

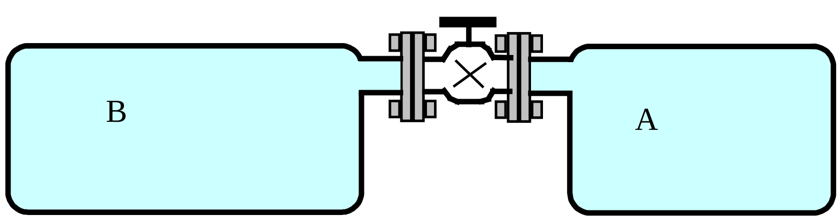
=> T_2 = 274.76°C and u_2 = 2767.32 kJ/kg

The energy equation is (neglecting kinetic and potential energy)

$m_2 u_2 - m_A u_{A1} - m_B u_{B1} = {}_1Q_2 - {}_1W_2 = {}_1Q_2$

${}_1Q_2 = (0.5985 \times 2767.32 - 0.1033 \times 2582.75 - 0.4952 \times 2543.55)$ kg ×kJ/kg

= **129.9 kJ**



3.215

A tank has a volume of 1 m^3 with oxygen at 15^{o}C, 300 kPa. Another tank contains 4 kg oxygen at 60^{o}C, 500 kPa. The two tanks are connected by a pipe and valve which is opened allowing the whole system to come to a single equilibrium state with the ambient at 20^{o}C. Find the final pressure and the heat transfer.

C.V. Both tanks of constant volume.

Continuity Eq.: $m_2 - m_{1A} - m_{1B} = 0$

Energy Eq.: $m_2u_2 - m_{1A}u_{1A} - m_{1B}u_{1B} = {}_1Q_2 - {}_1W_2$

Process Eq.: $V_2 = V_A + V_B = \text{constant}, \quad {}_1W_2 = 0$

State 1A: $m_{1A} = \dfrac{P_{1A}V_A}{RT_{1A}} = \dfrac{300 \text{ kPa} \times 1 \text{ m}^3}{0.2598 \text{ kJ/kg-K} \times 288.15 \text{ K}} = 4.007 \text{ kg}$

State 1B: $V_B = \dfrac{m_{1B}RT_{1B}}{P_{1B}} = \dfrac{4 \text{ kg} \times 0.2598 \text{ kJ/kg-K} \times 333.15 \text{ K}}{500 \text{ kPa}} = 0.6924 \text{ m}^3$

State 2: $(T_2, v_2 = V_2/m_2) \quad V_2 = V_A + V_B = 1 + 0.6924 = 1.6924 \text{ m}^3$

$m_2 = m_{1A} + m_{1B} = 4.007 + 4 = 8.007 \text{ kg}$

$P_2 = \dfrac{m_2RT_2}{V_2} = \dfrac{8.007 \text{ kg} \times 0.2598 \text{ kJ/kg-K} \times 293.15 \text{ K}}{1.6924 \text{ m}^3} = \mathbf{360.3 \text{ kPa}}$

Heat transfer from energy equation

$${}_1Q_2 = m_2u_2 - m_{1A}u_{1A} - m_{1B}u_{1B} = m_{1A}(u_2 - u_{1A}) + m_{1B}(u_2 - u_{1B})$$

$$= m_{1A}C_v (T_2 - T_{1A}) + m_{1B} C_v (T_2 - T_{1B})$$

$$= 4.007 \text{ kg}\times 0.662 \text{ kJ/kg-K}\times(20 - 15) \text{ K} + 4 \text{ kg}\times 0.662 \text{ kJ/kg-K} \times(20 - 60) \text{ K}$$

$$= \mathbf{-92.65 \text{ kJ}}$$

3.216

A rigid insulated tank is separated into two rooms by a stiff plate. Room A of 0.5 m^3 contains air at 250 kPa, 300 K and room B of 1 m^3 has air at 500 kPa, 1000 K. The plate is removed and the air comes to a uniform state without any heat transfer. Find the final pressure and temperature.

Solution:

C.V. Total tank. Control mass of constant volume.

Mass and volume: $m_2 = m_A + m_B$; $V = V_A + V_B = 1.5\ m^3$

Energy Eq.: $U_2 - U_1 = m_2 u_2 - m_A u_{A1} - m_B u_{B1} = Q - W = 0$

Process Eq.: $V = \text{constant} \Rightarrow W = 0$; Insulated $\Rightarrow Q = 0$

Ideal gas at 1: $m_A = P_{A1}V_A/RT_{A1} = 250 \times 0.5/(0.287 \times 300) = 1.452$ kg

$u_{A1} = 214.364$ kJ/kg from Table A.7

Ideal gas at 2:

$m_B = P_{B1}V_B/RT_{B1} = 500\ \text{kPa} \times 1\ m^3/(0.287\ \text{kJ/kg-K} \times 1000\ \text{K}) = 1.742$ kg

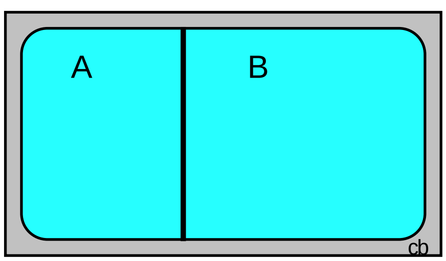
$u_{B1} = 759.189$ kJ/kg from Table A.7

$m_2 = m_A + m_B = 3.194$ kg

$$u_2 = \frac{m_A u_{A1} + m_B u_{B1}}{m_2} = \frac{1.452 \times 214.364 + 1.742 \times 759.189}{3.194} = 511.51\ \text{kJ/kg}$$

=> Table A.7.1: $\mathbf{T_2 = 698.6\ K}$

$P_2 = m_2 RT_2 /V = 3.194\ \text{kg} \times 0.287\ \text{kJ/kg-K} \times 698.6\ \text{K} /1.5\ m^3 = \mathbf{426.9\ kPa}$



3.217

A rigid tank A of volume 0.6 m^3 contains 3 kg water at 120^oC and the rigid tank B is 0.4 m^3 with water at 600 kPa, 200^oC. They are connected to a piston cylinder initially empty with closed valves. The pressure in the cylinder should be 800 kPa to float the piston. Now the valves are slowly opened and heat is transferred so the water reaches a uniform state at 250^oC with the valves open. Find the final volume and pressure and the work and heat transfer in the process.

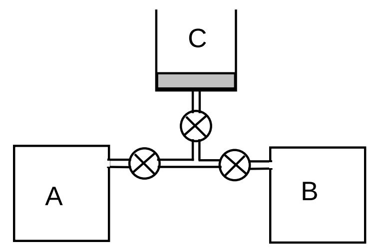
C.V.: A + B + C.

Only work in C, total mass constant.

$m_2 - m_1 = 0 \quad => \quad m_2 = m_{A1} + m_{B1}$

$U_2 - U_1 = {}_1Q_2 - {}_1W_2$;

${}_1W_2 = \int PdV = P_{lift} (V_2 - V_1)$



1A: $v = 0.6/3 = 0.2\ m^3/kg \quad => \quad x_{A1} = (0.2 - 0.00106)/0.8908 = 0.223327$

$u = 503.48 + 0.223327 \times 2025.76 = 955.89$ kJ/kg

1B: $v = 0.35202\ m^3/kg \quad => \quad m_{B1} = 0.4/0.35202 = 1.1363$ kg ; $u = 2638.91$ kJ/kg

$m_2 = 3 + 1.1363 = 4.1363$ kg and $V_1 = V_A + V_B = 1\ m^3$

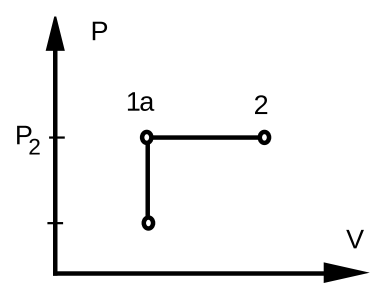
$V_2 = V_A + V_B + V_C = 1\ m^3 + V_C$

Locate state 2: Must be on P-V lines shown

State 1a: 800 kPa,

$$v_{1a} = \frac{V_A+V_B}{m} = 0.24176\ m^3/kg$$

800 kPa, v_{1a} => T = 173°C too low.



Assume 800 kPa: 250°C => $v = 0.29314\ m^3/kg > v_{1a}$ OK

$V_2 = m_2 v_2 = 4.1363$ kg $\times$ 0.29314 m^3/kg = **1.2125 m^3**

Final state is : **800 kPa**; 250°C => $u_2 = 2715.46$ kJ/kg

${}_1W_2 = P_{lift} (V_2 - V_1) = 800\ kPa \times (1.2125 - 1)\ m^3 =$ **170 kJ**

${}_1Q_2 = m_2u_2 - m_1u_1 + {}_1W_2 = m_2u_2 - m_{A1}u_{A1} - m_{B1}u_{B1} + {}_1W_2$

$= 4.1363 \times 2715.46 - 3 \times 955.89 - 1.1363 \times 2638.91 + 170$

$= 11\ 232 - 2867.7 - 2998.6 + 170 =$ **5536 kJ**

Review Problems

3.218

Ten kilograms of water in a piston/cylinder setup with constant pressure is at 450°C and a volume of 0.633 m^3. It is now cooled to 20°C. Show the *P–v* diagram and find the work and heat transfer for the process.

Solution:

C.V. The 10 kg water.

Energy Eq.3.5: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

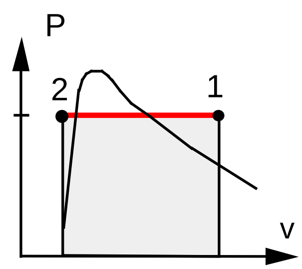
Process: $P = C \quad \Rightarrow \quad {}_1W_2 = mP(v_2 - v_1)$

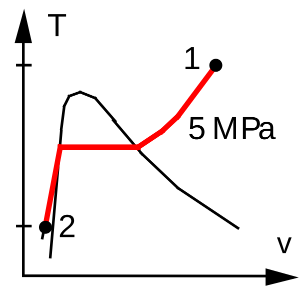
State 1: (T, $v_1 = 0.633/10 = 0.0633\ m^3/kg$) Table B.1.3

$P_1 = 5$ MPa, $h_1 = 3316.2$ kJ/kg

State 2: (P = P = 5 MPa, 20°C) $\Rightarrow$ Table B.1.4

$v_2 = 0.000\ 999\ 5\ m^3/kg$; $h_2 = 88.65$ kJ/kg





The work from the process equation is found as

$${}_1W_2 = 10\ kg \times 5000\ kPa \times (0.0009995 - 0.0633)\ m^3/kg = \mathbf{-3115\ kJ}$$

The heat transfer from the energy equation is

$${}_1Q_2 = m(u_2 - u_1) + {}_1W_2 = m(h_2 - h_1)$$

$${}_1Q_2 = 10\ kg \times (88.65 - 3316.2)\ kJ/kg = \mathbf{-32\ 276\ kJ}$$

3.219

A piston/cylinder (Fig. P3.171) contains 1 kg of water at 20°C with a volume of 0.1 m^3. Initially the piston rests on some stops with the top surface open to the atmosphere, P_o and a mass so a water pressure of 400 kPa will lift it. To what temperature should the water be heated to lift the piston? If it is heated to saturated vapor find the final temperature, volume and the work, $_1W_2$.

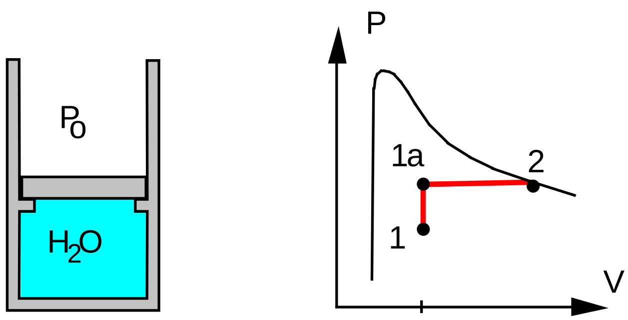
Solution:

(a) State to reach lift pressure of P = 400 kPa, v = V/m = 0.1 m^3/kg

Table B.1.2: $v_f < v < v_g = 0.4625$ m^3/kg

=> $T = T_{sat}$ = **143.63°C**

(b) State 2 is saturated vapor at 400 kPa since state 1a is two-phase.



$v_2 = v_g = 0.4625$ m^3/kg , $V_2 = m\, v_2 = 0.4625$ m^3,

Pressure is constant as volume increase beyond initial volume.

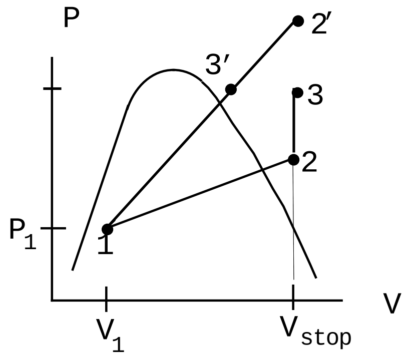
$_1W_2 = \int P\, dV = P\,(V_2 - V_1) = P_{lift}\,(V_2 - V_1)$

= 400 kPa × (0.4625 – 0.1) m^3 = **145 kJ**

3.220

Two kilograms of water is contained in a piston/cylinder (Fig. P3.220) with a massless piston loaded with a linear spring and the outside atmosphere. Initially the spring force is zero and $P_1 = P_o$ = 100 kPa with a volume of 0.2 m^3. If the piston just hits the upper stops the volume is 0.8 m^3 and T = 600°C. Heat is now added until the pressure reaches 1.2 MPa. Find the final temperature, show the *P–V* diagram and find the work done during the process.

Solution:



State 1: $v_1 = V/m = 0.2 / 2 = 0.1\ m^3/kg$

Process: $1 \rightarrow 2 \rightarrow 3$ or $1 \rightarrow 3'$

State at stops: 2 or 2'

$v_2 = V_{stop}/m = 0.4\ m^3/kg$ & $T_2 = 600°C$

Table B.1.3 $\Rightarrow$ $P_{stop} = 1\ MPa < P_3$

since $P_{stop} < P_3$ the process is as $1 \rightarrow 2 \rightarrow 3$

State 3: $P_3 = 1.2\ MPa$, $v_3 = v_2 = 0.4\ m^3/kg$ $\Rightarrow$ $T_3 \cong$ **770°C**

$$W_{13} = W_{12} + W_{23} = \frac{1}{2}(P_1 + P_2)(V_2 - V_1) + 0$$

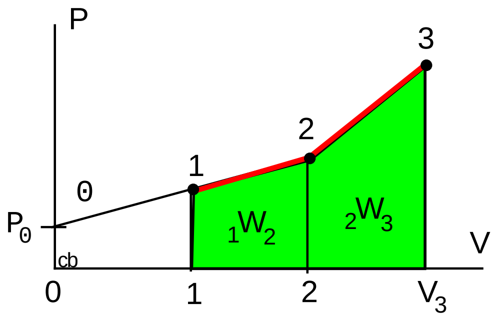
$$= \frac{1}{2}(100 + 1000)\ kPa \times (0.8 - 0.2)\ m^3$$

= **330 kJ**

3.221

Two springs with same spring constant are installed in a massless piston/cylinder with the outside air at 100 kPa. If the piston is at the bottom, both springs are relaxed and the second spring comes in contact with the piston at $V = 2\ \text{m}^3$. The cylinder (Fig. P3.221) contains ammonia initially at −2°C, $x = 0.13$, $V = 1\ \text{m}^3$, which is then heated until the pressure finally reaches 1200 kPa. At what pressure will the piston touch the second spring? Find the final temperature, the total work done by the ammonia and the heat transfer.

Solution :



State 1: P = 399.7 kPa Table B.2.1

$v = 0.00156 + 0.13 \times 0.3106 = 0.0419\ \text{m}^3/\text{kg}$

$u = 170.52 + 0.13 \times 1145.78 = 319.47\ \text{kJ/kg}$

$m = V/v = 1/0.0419 = 23.866\ \text{kg}$

At bottom state 0: $0\ \text{m}^3$, 100 kPa

State 2: $V = 2\ \text{m}^3$ and on line 0-1-2

Final state 3: 1200 kPa, on line segment 2.

Slope of line 0-1-2: $\Delta P/\Delta V = (P_1 - P_0)/\Delta V = (399.7-100)/1 = 299.7\ \text{kPa/m}^3$

$$P_2 = P_1 + (V_2 - V_1)\Delta P/\Delta V = 399.7 + (2-1)\times 299.7 = \mathbf{699.4\ kPa}$$

State 3: Last line segment has twice the slope.

$$P_3 = P_2 + (V_3 - V_2)2\Delta P/\Delta V \quad \Rightarrow \quad V_3 = V_2 + (P_3 - P_2)/(2\Delta P/\Delta V)$$

$$V_3 = 2 + (1200-699.4)/599.4 = 2.835\ \text{m}^3$$

$$v_3 = v_1 V_3/V_1 = 0.0419 \times 2.835/1 = 0.1188 \quad \Rightarrow \quad T_3 = \mathbf{51°C}$$

$$u_3 = 1384.39\ \text{kJ/kg} = 1383 + (1404.8-1383)\frac{0.1188 - 0.11846}{0.12378 - 0.11846} \quad \text{from B.2.2}$$

$$_1W_3 = {}_1W_2 + {}_2W_3 = \frac{1}{2}(P_1 + P_2)(V_2 - V_1) + \frac{1}{2}(P_3 + P_2)(V_3 - V_2)$$

$$= 549.6 + 793.0 = \mathbf{1342.6\ kJ}$$

The energy equation gives the heat transfer as

$$_1Q_3 = m(u_3 - u_1) + {}_1W_3 = 23.866\ \text{kg}\ (1384.39 - 319.47)\ \text{kJ/kg} + 1342.6\ \text{kJ}$$

$$= \mathbf{26\ 758\ kJ}$$

3.222

Ammonia, NH_3, is contained in a sealed rigid tank at 0°C, *x* = 50% and is then heated to 100°C. Find the final state P_2, u_2 and the specific work and heat transfer.

Solution:

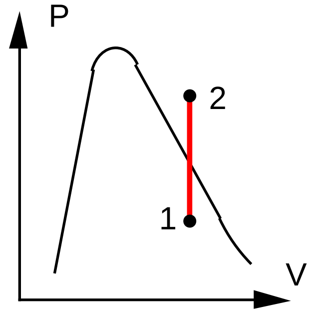
Continuity Eq.: $m_2 = m_1$;

Energy Eq.3.5: $E_2 - E_1 = {}_1Q_2$; $\left({}_1W_2 = \emptyset\right)$

Process: $V_2 = V_1 \Rightarrow v_2 = v_1 = 0.001566 + 0.5 \times 0.28783 = 0.14538 \text{ m}^3\text{/kg}$

Table B.2.2: v_2 & $T_2 \Rightarrow$ between 1000 kPa and 1200 kPa

$$P_2 = 1000 + 200 \frac{0.14538 - 0.17389}{0.14347 - 0.17389} = \mathbf{1187\ kPa}$$



$u_2 = 1490.5 + (1485.8 - 1490.5) \times 0.935$

$= 1485.83$ kJ/kg

$u_1 = 179.69 + 0.5 \times 1138.3 = 748.84$ kJ/kg

Process equation gives no displacement: ${}_1w_2 = \mathbf{0}$;

The energy equation then gives the heat transfer as

$${}_1q_2 = u_2 - u_1 = 1485.83 - 748.84 = \mathbf{737\ kJ/kg}$$

3.223

A piston/cylinder system contains 50 L of air at 300°C, 100 kPa, with the piston initially on a set of stops as shown. A total external constant force acts on the piston so a balancing pressure inside should be 200 kPa. The cylinder is made of 2 kg of steel initially at 1300°C. The system is insulated so that heat transfer only occurs between the steel cylinder and the air. The system comes to equilibrium. Find the final temperature and the work done by the air in the process and plot the process P-V diagram.

C.V.: Steel + water out to ambient T_0. This is a control mass.

Energy Eq.: $U_2 - U_1 = m_{air}(u_2 - u_1) + m_{st}(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Entropy Eq.: $S_2 - S_1 = m_{air}(s_2 - s_1) + m_{st}(s_2 - s_1) = \int dQ/T + {}_1S_{2\ gen} = {}_1S_{2\ gen}$

Process: ${}_1Q_2 = 0$ and must be on P-V diagram shown

$$m_{air} = P_1V_1 / RT_1 = 100 \times 0.05 /(0.287 \times 573.15) = 0.0304 \text{ kg}$$

Since $V_{1a} = V_1$ then $T_{1a} = T_1(P_{float}/P_1) = 573.15 \times 200/100 = 1146.3$ K

Use constant C_v for air at 900 K: $C_v = \Delta u/\Delta T = 0.833$ kJ/kgK (from A.7)

To reach state 1a: $\Delta U_{air} = mC_v\Delta T = 0.0304 \times 0.833 \times (1146 - 573) = 14.5$ kJ

$$\Delta U_{st} = mC_v\Delta T = 2 \times 0.46 \times (1146 - 1573) = -392.8 \text{ kJ}$$

Conclusion from this is: T_2 is higher than T_{1a} **=** 1146 K, piston lifts, $P_2 = P_{float}$

Write the work as ${}_1W_2 = P_2 (V_2 - V_1)$ and use constant C_v in the energy Eq. as

$$m_{air}\, C_v (T_2 - T_1) + m_{st}\, C_{st} (T_2 - T_1) = - P_2 m_{air} v_2 + P_2 V_1$$

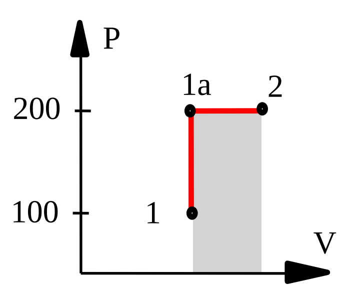
now $P_2 v_2 = RT_2$ for the air, so isolate T_2 terms as

$$[m_{air} (C_v + R) + mC_{st}]\, T_2 = m_{air}\, C_v T_{1\ air} + mC_{st}\, T_{1\ st} + P_2V_1$$

$$[0.0304 \times 1.12 + 2 \times 0.46]\, T_2 = 0.0304 \times 0.833 \times 573.15$$
$$+ 2 \times 0.46 \times 1573.15 + 200 \times 0.05$$

Solution gives: $T_2 =$ **1542.7 K**, $V_2 = 0.0304 \times 0.287 \times 1542.7/200 = 0.0673$ m^3

$${}_1W_2 = P_2 (V_2 - V_1) = 200 (0.0673 - 0.05) = \mathbf{3.46\ kJ}$$



3.224

A piston/cylinder arrangement has the piston loaded with outside atmospheric pressure and the piston mass to a pressure of 150 kPa. It contains water at −2°C, which is then heated until the water becomes saturated vapor. Find the final temperature and specific work and heat transfer for the process.

Solution:

C.V. Water in the piston cylinder.

Continuity: $m_2 = m_1$,

Energy Eq.3.5 per unit mass: $u_2 - u_1 = {}_1q_2 - {}_1w_2$

Process: P = constant = P_1, => ${}_1w_2 = \int_1^2 P\,dv = P_1(v_2 - v_1)$

State 1: T_1 , P_1 => Table B.1.5 compressed solid, take as saturated solid.

$v_1 = 1.09\times10^{-3}$ m^3/kg, u_1 = -337.62 kJ/kg

State 2: x = 1, $P_2 = P_1$ = 150 kPa due to process => Table B.1.2

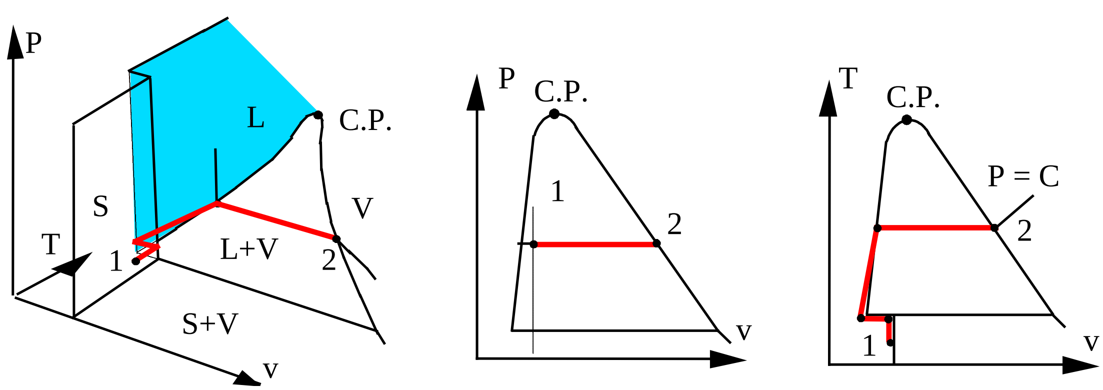
$v_2 = v_g(P_2) = 1.1593$ m^3/kg, T_2 = **111.4°C** ; u_2 = 2519.7 kJ/kg

From the process equation

${}_1w_2 = P_1(v_2 - v_1)$ = 150 kPa (1.1593 -1.09×10^{-3}) m^3/kg = **173.7 kJ/kg**

From the energy equation

${}_1q_2 = u_2 - u_1 + {}_1w_2$ = 2519.7 - (-337.62) + 173.7 = **3031 kJ/kg**



3.225

A piston/cylinder contains 1 kg of ammonia at 20°C with a volume of 0.1 m^3, shown in Fig. P3.225. Initially the piston rests on some stops with the top surface open to the atmosphere, *P*o, so a pressure of 1400 kPa is required to lift it. To what temperature should the ammonia be heated to lift the piston? If it is heated to saturated vapor find the final temperature, volume, and the heat transfer.

Solution:

C.V. Ammonia which is a control mass.

$$m_2 = m_1 = m \; ; \qquad m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$$

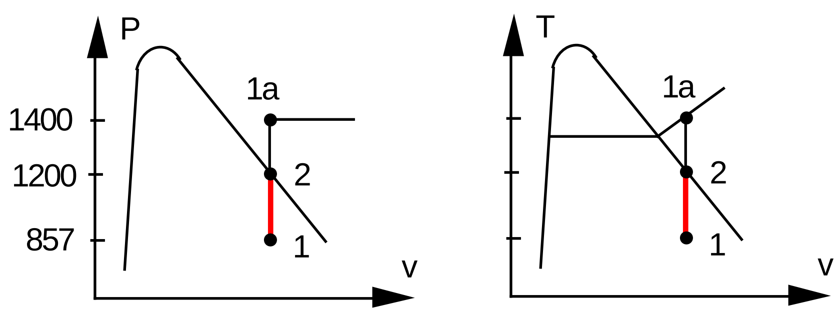
State 1: 20°C; $v_1 = 0.10 < v_g \;\Rightarrow\; x_1 = (0.1 - 0.001638)/0.14758 = 0.6665$

$$u_1 = u_f + x_1\, u_{fg} = 272.89 + 0.6665 \times 1059.3 = 978.9 \text{ kJ/kg}$$

Process: Piston starts to lift at state 1a (P_{lift}, v_1)

State 1a: 1400 kPa, v_1 Table B.2.2 (superheated vapor)

$$T_a = 50 + (60 - 50)\,\frac{0.1 - 0.09942}{0.10423 - 0.09942} = 51.2 \text{ °C}$$



State 2: x = 1.0, $v_2 = v_1$ => $V_2 = mv_2 = \mathbf{0.1\ m^3}$

$T_2 = 30 + (0.1 - 0.11049) \times 5/(0.09397 - 0.11049) = \mathbf{33.2\ °C}$

$u_2 = 1338.7$ kJ/kg; ${}_1W_2 = 0$;

${}_1Q_2 = m_1q_2 = m(u_2 - u_1) = 1 \text{ kg} \times (1338.7 - 978.9) \text{ kJ/kg} = \mathbf{359.8\ kJ}$

3.226

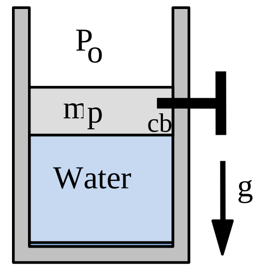
A piston held by a pin in an insulated cylinder, shown in Fig. P3.226, contains 2 kg water at 100°C, quality 98%. The piston has a mass of 102 kg, with cross-sectional area of 100 cm^2, and the ambient pressure is 100 kPa. The pin is released, which allows the piston to move. Determine the final state of the water, assuming the process to be adiabatic.

Solution:

C.V. The water. This is a control mass.

Continuity Eq.: $m_2 = m_1 = m$;

Energy Eq.3.5: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$



Process in cylinder: $P = P_{float}$ (if piston not supported by pin)

$$P_2 = P_{float} = P_0 + m_p g/A = 100 + \frac{102 \times 9.807}{100 \times 10^{-4} \times 10^3} = 200 \text{ kPa}$$

We thus need one more property for state 2 and we have one equation namely the energy equation. From the equilibrium pressure the work becomes

$$_1W_2 = \int P_{float}\, dV = P_2\, m(v_2 - v_1)$$

With this work the energy equation gives per unit mass

$$u_2 - u_1 = {}_1q_2 - {}_1w_2 = 0 - P_2(v_2 - v_1)$$

or with rearrangement to have the unknowns on the left hand side

$$u_2 + P_2 v_2 = h_2 = u_1 + P_2 v_1$$

$$h_2 = u_1 + P_2 v_1 = 2464.8 \text{ kJ/kg} + 200 \text{ kPa} \times 1.6395 \text{ m}^3\text{/kg} = 2792.7 \text{ kJ/kg}$$

State 2: (P_2 , h_2) Table B.1.3 => $T_2 \cong$ **161.75°C**

3.227

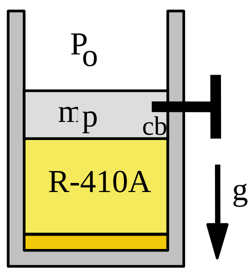
A vertical cylinder (Fig. P3.227) has a 61.18-kg piston locked with a pin trapping 10 L of R-410A at 10°C, 90% quality inside. Atmospheric pressure is 100 kPa, and the cylinder cross-sectional area is 0.006 m^2. The pin is removed, allowing the piston to move and come to rest with a final temperature of 10°C for the R-410A. Find the final pressure, final volume and the work done and the heat transfer for the R-410A.

Solution:

CV R-410A, this is a control mass.

Energy Eq.: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Process in cylinder: $P = P_{float}$ (if piston not supported by pin)



State 1: (T, x) from table B.4.1

$v_1 = 0.000886 + 0.9 \times 0.02295 = 0.021541\ m^3/kg$

$u_1 = 72.24 + 0.9 \times 183.66 = 131.64$ kJ/kg

$m = V_1/v_1 = 0.010\ m^3/0.021541\ m^3/kg = 0.464$ kg

Force balance on piston gives the equilibrium pressure

$$P_2 = P_0 + m_P g/ A_P = 100 + \frac{61.18 \times 9.807}{0.006 \times 1000} = \mathbf{200\ kPa}$$

State 2: (T,P) in Table B.4.2 $v_2 = (0.1507 + 0.16322)/2 = 0.15696\ m^3/kg$

$u_2 = (265.06 + 279.13)/2 = 272.095$ kJ/kg

$V_2 = mv_2 = 0.464\ kg \times 0.15696\ m^3/kg = 0.0728\ m^3 = \mathbf{72.8\ L}$

${}_1W_2 = \int P_{equil}\, dV = P_2(V_2 – V_1) = 200\ kPa\ (0.0728 - 0.010)\ m^3 = \mathbf{12.56\ kJ}$

${}_1Q_2 = m(u_2 - u_1) + {}_1W_2$

$= 0.464\ kg\ (272.095 – 131.64)\ kJ/kg + 12.56\ kJ = \mathbf{77.73\ kJ}$

3.228

A cylinder having an initial volume of 3 m^3 contains 0.1 kg of water at 40°C. The water is then compressed in an isothermal quasi-equilibrium process until it has a quality of 50%. Calculate the work done in the process splitting it into two steps. Assume the water vapor is an ideal gas during the first step of the process.

Solution: C.V. Water

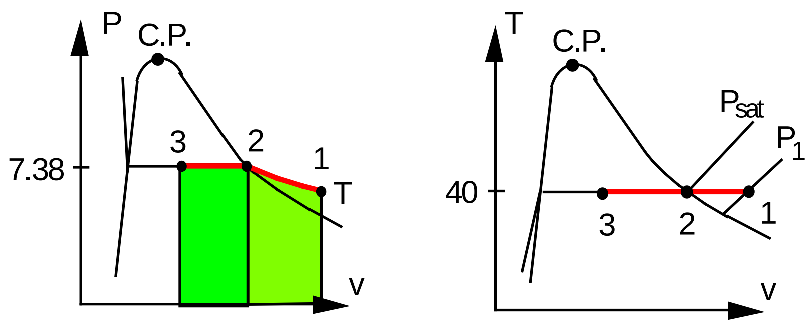
State 2: (40°C, x = 1) Tbl B.1.1 => P_G = 7.384 kPa, v_G = 19.52 m^3/kg

State 1: $v_1 = V_1/m = 3 / 0.1 = 30\ m^3/kg \quad (> v_G)$

so H_2O ~ ideal gas from 1-2 so since constant T

$$P_1 = P_G \frac{v_G}{v_1} = 7.384 \times \frac{19.52}{30} = 4.8 \text{ kPa}$$

$$V_2 = mv_2 = 0.1 \times 19.52 = 1.952 \text{ m}^3$$



Process T = C: and ideal gas gives work from Eq.3.21

$${}_1W_2 = \int_1^2 PdV = P_1V_1 \ln\frac{V_2}{V_1} = 4.8 \times 3.0 \times \ln\frac{1.952}{3} = -6.19 \text{ kJ}$$

$v_3 = 0.001008 + 0.5 \times 19.519 = 9.7605 \quad => \quad V_3 = mv_3 = 0.976\ m^3$

$P = C = P_g$: This gives a work term as

$${}_2W_3 = \int_2^3 PdV = P_g\,(V_3 - V_2) = 7.384 \text{ kPa } (0.976 - 1.952) \text{ m}^3 = -7.21 \text{ kJ}$$

Total work:

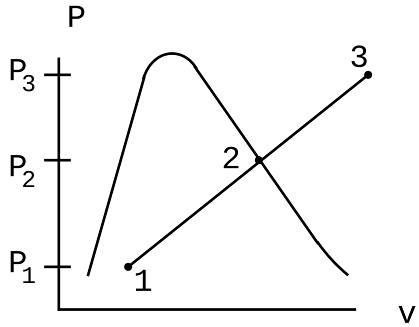
$${}_1W_3 = {}_1W_2 + {}_2W_3 = -6.19 - 7.21 = \mathbf{-13.4\ kJ}$$

3.229

A spring-loaded piston/cylinder arrangement contains R-134a at 20°C, 24% quality with a volume 50 L. The setup is heated and thus expands, moving the piston. It is noted that when the last drop of liquid disappears the temperature is 40°C. The heating is stopped when $T = 130°C$. Verify the final pressure is about 1200 kPa by iteration and find the work done in the process.

Solution:

C.V. R-134a. This is a control mass.



State 1: Table B.5.1 =>

$v_1 = 0.000817 + 0.24\times 0.03524 = 0.009274$

$P_1 = 572.8$ kPa,

$m = V/ v_1 = 0.050 / 0.009274 = 5.391$ kg

Process: Linear Spring

$P = A + Bv$

State 2: $x_2 = 1$, T_2 $\Rightarrow$ $P_2 = 1.017$ MPa, $v_2 = 0.02002$ m^3/kg

Now we have fixed two points on the process line so for final state 3:

$$P_3 = P_1 + \frac{P_2 - P_1}{v_2 - v_1}(v_3 - v_1) = \text{RHS}$$ Relation between P_3 and v_3

State 3: T_3 and on process line $\Rightarrow$ iterate on P_3 given T_3

at $P_3 = 1.2$ MPa => $v_3 = 0.02504$ => P_3 - RHS = -0.0247

at $P_3 = 1.4$ MPa => $v_3 = 0.02112$ => P_3 - RHS = 0.3376

Linear interpolation gives :

$$P_3 \cong 1200 + \frac{0.0247}{0.3376 + 0.0247}(1400 - 1200) = 1214 \text{ kPa}$$

$$v_3 = 0.02504 + \frac{0.0247}{0.3376 + 0.0247}(0.02112 - 0.02504) = 0.02478 \text{ m}^3\text{/kg}$$

$$W_{13} = \int P\, dV = \frac{1}{2}(P_1 + P_3)(V_3 - V_1) = \frac{1}{2}(P_1 + P_3)\, m\, (v_3 - v_1)$$

$$= \frac{1}{2} \times 5.391 \text{ kg } (572.8 + 1214) \text{ kPa} \times (0.02478 - 0.009274) \text{ m}^3\text{/kg}$$

$= \mathbf{74.7\ kJ}$

3.230

Water in a piston/cylinder, similar to Fig. P3.225, is at 100^{o}C, x = 0.5 with mass 1 kg and the piston rests on the stops. The equilibrium pressure that will float the piston is 300 kPa. The water is heated to 300^{o}C by an electrical heater. At what temperature would all the liquid be gone? Find the final (P,v), the work and heat transfer in the process.

C.V. The 1 kg water.

Continuity: $m_2 = m_1 = m$; Energy: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Process: V = constant if $P < P_{lift}$ otherwise $P = P_{lift}$ see P-v diagram.

State 1: (T,x) Table B.1.1

$$v_1 = 0.001044 + 0.5 \times 1.6719 = 0.83697 \text{ m}^3/\text{kg}$$

$$u_1 = 418.91 + 0.5 \times 2087.58 = 1462.7 \text{ kJ/kg}$$

State 1a: (300 kPa, $v = v_1 > v_{g\ 300\ kPa} = 0.6058 \text{ m}^3/\text{kg}$) so superheated vapor

Piston starts to move at state 1a, ${}_1W_{1a} = 0$, $u_{1a} = 2768.82$ kJ/kg

$${}_1Q_{1a} = m(u - u) = 1 \text{ kg } (2768.82 - 1462.7) \text{ kJ/kg} = 1306.12 \text{ kJ}$$

State 1b: reached before state 1a so $v = v_1 = v_g$ see this in B.1.1

$$T_{1b} = 120 + 5\ (0.83697 - 0.8908)/(0.76953 - 0.8908) = \mathbf{122.2^{o}C}$$

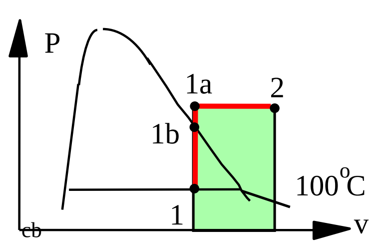
State 2: $(T_2 > T_{1a})$ Table B.1.3 => $v_2 = 0.87529$, $u_2 = 2806.69$ kJ/kg

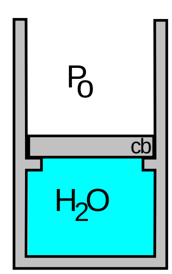
Work is seen in the P-V diagram (when volume changes $P = P_{lift}$)

$${}_1W_2 = {}_{1a}W_2 = P_2\, m(v_2 - v_1) = 300 \times 1(0.87529 - 0.83697) = \mathbf{11.5\ kJ}$$

Heat transfer is from the energy equation

$${}_1Q_2 = 1 \text{ kg } (2806.69 - 1462.7) \text{ kJ/kg} + 11.5 \text{ kJ} = \mathbf{1355.5\ kJ}$$





3.231

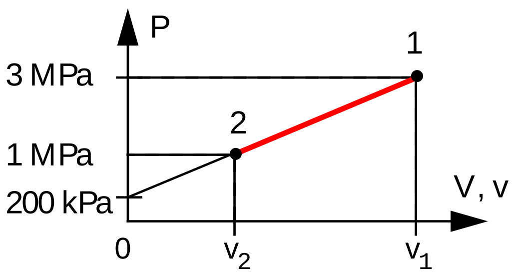
A piston/cylinder arrangement has a linear spring and the outside atmosphere acting on the piston, shown in Fig. P3.231. It contains water at 3 MPa, 400°C with the volume being 0.1 m^3. If the piston is at the bottom, the spring exerts a force such that a pressure of 200 kPa inside is required to balance the forces. The system now cools until the pressure reaches 1 MPa. Find the heat transfer for the process.

Solution:

C.V. Water.

Continuity Eq.: $m_2 = m_1 = m$;

Energy Eq.3.5: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$



State 1: Table B.1.3

$v_1 = 0.09936$ m^3/kg, $u_1 = 2932.8$ kJ/kg

$m = V/v_1 = 0.1/0.09936 = 1.006$ kg

Process: Linear spring so P linear in v.

$$P = P_0 + (P_1 - P_0)v/v_1$$

$$v_2 = \frac{(P_2 - P_0)v_1}{P_1 - P_0} = \frac{(1000 - 200)0.09936}{3000 - 200} = 0.02839 \text{ m}^3\text{/kg}$$

State 2: P_2 , v_2 $\Rightarrow$ $x_2 = (v_2 - 0.001127)/0.19332 = 0.141$, $T_2 = 179.91$°C,

$u_2 = 761.62 + x_2 \times 1821.97 = 1018.58$ kJ/kg

Process => $${}_1W_2 = \int PdV = \frac{1}{2}m(P_1 + P_2)(v_2 - v_1)$$

$$= \frac{1}{2}1.006 \text{ kg } (3000 + 1000) \text{ kPa } (0.02839 - 0.09936) \text{ m}^3\text{/kg}$$

$$= -142.79 \text{ kJ}$$

Heat transfer from the energy equation

${}_1Q_2 = m(u_2 - u_1) + {}_1W_2 = 1.006$ kg (1018.58 - 2932.8) kJ/kg - 142.79 kJ

= **-2068.5 kJ**

3.232

A 1 m^3 tank containing air at 25^{o}C and 500 kPa is connected through a valve to another tank containing 4 kg of air at 60^{o}C and 200 kPa. Now the valve is opened and the entire system reaches thermal equilibrium with the surroundings at 20^{o}C. Assume constant specific heat at 25^{o}C and determine the final pressure and the heat transfer.

Control volume all the air. Assume air is an ideal gas.

Continuity Eq.: $m_2 - m_{A1} - m_{B1} = 0$

Energy Eq.: $U_2 - U_1 = m_2u_2 - m_{A1}u_{A1} - m_{B1}u_{B1} = {}_1Q_2 - {}_1W_2$

Process Eq.: $V = \text{constant} \quad \Rightarrow \quad {}_1W_2 = 0$

State 1:

$$m_{A1} = \frac{P_{A1}V_{A1}}{RT_{A1}} = \frac{(500\text{ kPa})(1\text{m}^3)}{(0.287\text{ kJ/kgK})(298.2\text{ K})} = 5.84\text{ kg}$$

$$V_{B1} = \frac{m_{B1}RT_{B1}}{P_{B1}} = \frac{(4\text{ kg})(0.287\text{ kJ/kgK})(333.2\text{ K})}{(200\text{ kN/m}^2)} = 1.91\text{ m}^3$$

State 2: $T_2 = 20°\text{C}$, $v_2 = V_2/m_2$

$$m_2 = m_{A1} + m_{B1} = 4 + 5.84 = 9.84\text{ kg}$$

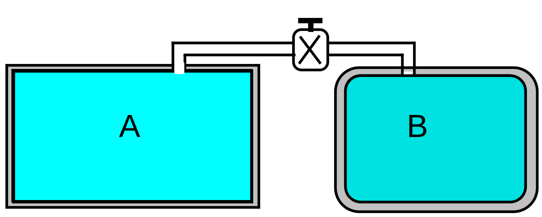
$$V_2 = V_{A1} + V_{B1} = 1 + 1.91 = 2.91\text{ m}^3$$

$$P_2 = \frac{m_2RT_2}{V_2} = \frac{(9.84\text{ kg})(0.287\text{ kJ/kgK})(293.2\text{ K})}{2.91\text{ m}^3} = \mathbf{284.5\text{ kPa}}$$

Energy Eq.5.5 or 5.11:

$${}_1Q_2 = U_2 - U_1 = m_2u_2 - m_{A1}u_{A1} - m_{B1}u_{B1}$$
$$= m_{A1}(u_2 - u_{A1}) + m_{B1}(u_2 - u_{B1})$$
$$= m_{A1}C_{v0}(T_2 - T_{A1}) + m_{B1}C_{v0}(T_2 - T_{B1})$$
$$= 5.84 \times 0.717\ (20 - 25) + 4 \times 0.717\ (20 - 60) = \mathbf{-135.6\text{ kJ}}$$

The air gave energy out.



3.233

A rigid container has two rooms filled with water, each 1 m^3 separated by a wall (see Fig. P3.210). Room A has P = 200 kPa with a quality x = 0.80. Room B has P = 2 MPa and T = 400°C. The partition wall is removed and the water comes to a uniform state, which after a while due to heat transfer has a temperature of 200°C. Find the final pressure and the heat transfer in the process.

Solution:

C.V. A + B. Constant total mass and constant total volume.

Continuity: $m_2 - m_{A1} - m_{B1} = 0$; $V_2 = V_A + V_B = 2\ m^3$

Energy Eq.3.5: $U_2 - U_1 = m_2u_2 - m_{A1}u_{A1} - m_{A1}u_{A1} = {}_1Q_2 - {}_1W_2 = {}_1Q_2$

Process: $V = V_A + V_B$ = constant => ${}_1W_2 = 0$

State 1A: Table B.1.2 $u_{A1} = 504.47 + 0.8 \times 2025.02 = 2124.47$ kJ/kg,

$v_{A1} = 0.001061 + 0.8 \times 0.88467 = 0.70877\ m^3/kg$

State 1B: Table B.1.3 $u_{B1} = 2945.2$, $v_{B1} = 0.1512$

$m_{A1} = 1/v_{A1} = 1.411$ kg $m_{B1} = 1/v_{B1} = 6.614$ kg

State 2: T_2, $v_2 = V_2/m_2 = 2/(1.411 + 6.614) = 0.24924\ m^3/kg$

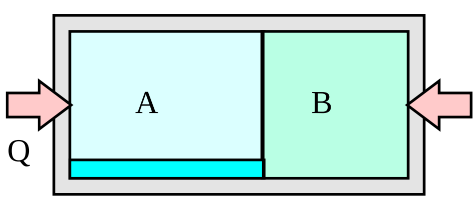
Table B.1.3 superheated vapor. 800 kPa < P_2 < 1 MPa

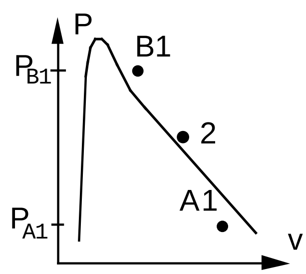
Interpolate to get the proper v_2

$$P_2 \cong 800 + \frac{0.24924-0.2608}{0.20596-0.2608} \times 200 = \mathbf{842\ kPa} \qquad u_2 \cong 2628.8\ \text{kJ/kg}$$

From the energy equation

$${}_1Q_2 = 8.025 \times 2628.8 - 1.411 \times 2124.47 - 6.614 \times 2945.2 = \mathbf{-\ 1381\ kJ}$$





3.234

Consider the piston/cylinder arrangement shown in Fig. P3.234. A frictionless piston is free to move between two sets of stops. When the piston rests on the lower stops, the enclosed volume is 400 L. When the piston reaches the upper stops, the volume is 600 L. The cylinder initially contains water at 100 kPa, 20% quality. It is heated until the water eventually exists as saturated vapor. The mass of the piston requires 300 kPa pressure to move it against the outside ambient pressure. Determine the final pressure in the cylinder, the heat transfer and the work for the overall process.

Solution:

C.V. Water. Check to see if piston reaches upper stops.

Energy Eq.3.5: $m(u_4 - u_1) = {}_1Q_4 - {}_1W_4$

Process: If $P < 300$ kPa then $V = 400$ L, line 2-1 and below

If $P > 300$ kPa then $V = 600$ L, line 3-4 and above

If $P = 300$ kPa then $400\text{ L} < V < 600\text{ L}$ line 2-3

State 1: $v_1 = 0.001043 + 0.2 \times 1.693 = 0.33964;\quad m = \frac{V_1}{v_1} = \frac{0.4}{0.33964} = 1.178$ kg

$u_1 = 417.36 + 0.2 \times 2088.7 = 835.1$ kJ/kg

State 3: $v_3 = \frac{0.6}{1.178} = 0.5095 < v_G = 0.6058$ at $P_3 = 300$ kPa

$\Rightarrow$ Piston does reach upper stops to reach sat. vapor.

State 4: $v_4 = v_3 = 0.5095\ m^3/kg = v_G$ at P_4 From Table B.1.2

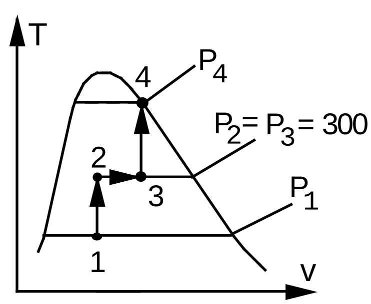
=> $P_4 =$ **361 kPa,** $u_4 = 2550.0$ kJ/kg

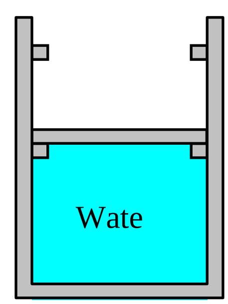
${}_1W_4 = {}_1W_2 + {}_2W_3 + {}_3W_4 = 0 + {}_2W_3 + 0$

${}_1W_4 = P_2(V_3 - V_2) = 300 \times (0.6 - 0.4) =$ **60 kJ**

${}_1Q_4 = m(u_4 - u_1) + {}_1W_4 = 1.178(2550.0 - 835.1) + 60 =$ **2080 kJ**

The three lines for process parts are shown in the P-V diagram, and is dictated by the motion of the piston (force balance).





3.235

Ammonia (2 kg) in a piston/cylinder is at 100 kPa, -20^{o}C and is now heated in a polytropic process with n = 1.3 to a pressure of 200 kPa. Do not use ideal gas approximation and find T_2, the work and heat transfer in the process.

Take CV as the Ammonia, constant mass.

Energy Eq.3.5: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Process: Pv^n = constant (n = 1.3)

State 1: Superheated vapor table B.2.2.

$$v_1 = 1.2101 \text{ m}^3\text{/kg}, \quad u_1 = 1307.8 \text{ kJ/kg}$$

Process gives: $v_2 = v_1 (P_1/P_2)^{1/n} = 1.2101 (100/200)^{1/1.3} = 0.710 \text{ m}^3\text{/kg}$

State 2: Table B.2.2 at 200 kPa interpolate: $u_2 = 1376.49$ kJ/kg, $T_2 = 24^{o}$C

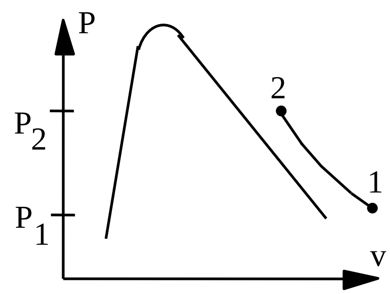
Work is done while piston moves at increasing pressure, so we get

$$_1W_2 = \frac{m}{1 - n}(P_2v_2 - P_1v_1) = \frac{2}{1 - 1.3} \text{ kg } (200\times0.71 - 100\times1.2101) \text{ kJ/kg}$$

$$= \mathbf{-139.9 \text{ kJ}}$$

Heat transfer is found from the energy equation

$$_1Q_2 = m(u_2 - u_1) + {}_1W_2$$
$$= 2\ (1376.49 - 1307.8) - 139.9$$
$$= \mathbf{-2.52 \text{ kJ}}$$



3.236

A small flexible bag contains 0.1 kg ammonia at –10^{o}C and 300 kPa. The bag material is such that the pressure inside varies linear with volume. The bag is left in the sun with with an incident radiation of 75 W, loosing energy with an average 25 W to the ambient ground and air. After a while the bag is heated to 30^{o}C at which time the pressure is 1000 kPa. Find the work and heat transfer in the process and the elapsed time.

Take CV as the Ammonia, constant mass.

Continuity Eq.: $m_2 = m_1 = m$;

Energy Eq.3.5: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Process: $P = A + BV$ (linear in V)

State 1: Compressed liquid $P > P_{sat}$, take saturated liquid at same temperature.

$$v_1 = v_{f\,-10} = 0.001534 \text{ m}^3\text{/kg}, \qquad u_1 = u_f = 133.96 \text{ kJ/kg}$$

State 2: Table B.2.1 at 30^{o}C : $P < P_{sat}$ so superheated vapor

$$v_2 = 0.13206 \text{ m}^3\text{/kg}, \;\; u_2 = 1347.1 \text{ kJ/kg}, \qquad V_2 = mv_2 = \mathbf{0.0132\ m^3}$$

Work is done while piston moves at increacing pressure, so we get

$${}_1W_2 = ½(300 + 1000) \text{ kPa}*0.1 \text{ kg}(0.13206 - 0.001534) \text{ m}^3\text{/kg} = \mathbf{8.484\ kJ}$$

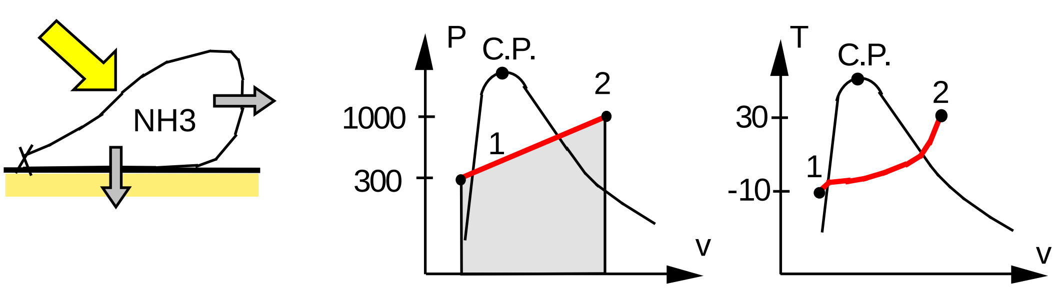
Heat transfer is found from the energy equation

$${}_1Q_2 = m(u_2 - u_1) + {}_1W_2 = 0.1 \text{ kg } (1347.1 - 133.96) \text{ kJ/kg} + 8.484 \text{ kJ}$$
$$= 121.314 + 8.484 = \mathbf{129.8\ kJ}$$

$$\dot{Q}_{net} = 75 - 25 = 50 \text{ Watts}$$

Assume the constant rate $\dot{Q}_{net} = dQ/dt = {}_1Q_2 / t$, so the time becomes

$$t = {}_1Q_2 / \dot{Q}_{net} = \frac{129800}{50} \text{ (J/W)} = \mathbf{2596\ s = 43.3\ min}$$



3.237

A cylinder/piston arrangement contains 0.1 kg R-410A of quality x = 0.2534 and at −20°C. Stops are mounted so $V_{stop} = 3V_1$, see Fig. P3.237. The system is now heated to the final temperature of 20°C. Find the work and heat transfer in the process and draw the P-v diagram.

C.V. The R-410A mass.

Energy Eq.5.11: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Process: $P = \text{Constant if } V < V_{stop}$; $V = V_{stop}$ if $P > P_1$

State 1: $u_1 = 27.92 + x_1\ 218.07 = 83.18$ kJ/kg, $P_1 = P_{sat} = 399.6$ kPa

$v_1 = 0.000803 + x_1\ 0.064 = 0.01702\ \text{m}^3\text{/kg}$

State 1a: $v_{stop} = 3\ v_1 = 0.05106\ \text{m}^3\text{/kg} < v_g$ at P_1

State 2: at 20°C > T_1 **:** $v_{stop} > v_g = 0.01758\ \text{m}^3\text{/kg}$ so superheated vapor.

Table B.4.2: Find it at $P_2 =$ **600 kPa**, $u_2 = 273.56$ kJ/kg

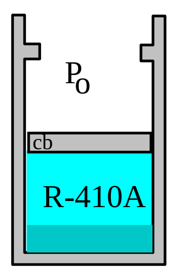
$V_1 = mv_1 = 0.0017\ \text{m}^3$, $V_2 = mv_2 = 0.0051\ \text{m}^3$

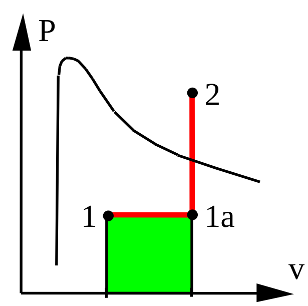
$${}_1W_2 = \int PdV = P_1 (V_2 - V_1) = 399.6 \text{ kPa } (0.0051 - 0.0017) \text{ m}^3 = \mathbf{1.36\ kJ}$$

$${}_1Q_2 = m(u_2 - u_1) + {}_1W_2 = 0.1 \text{ kg } (273.56 - 83.18) \text{ kJ/kg} + 1.36 \text{ kJ}$$

$$= \mathbf{20.398\ kJ}$$

See the work term from the process in the P-v diagram





3.238

A piston/cylinder, shown in Fig. P3.238, contains R-410A at – 20°C, *x* = 20%. The volume is 0.2 m^3. It is known that V_{stop} = 0.4 m^3, and if the piston sits at the bottom, the spring force balances the other loads on the piston. It is now heated up to 20°C. Find the mass of the fluid and show the *P–v* diagram. Find the work and heat transfer.

Solution:

C.V. R-410A, this is a control mass. Properties in Table B.4.

Continuity Eq.: $m_2 = m_1$

Energy Eq.3.5: $E_2 - E_1 = m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Process: $P = A + BV$, $V < 0.4\ m^3$, $A = 0$ (at $V = 0$, $P = 0$)

State 1: $v_1 = 0.000803 + 0.2 \times 0.0640 = 0.0136\ m^3/kg$

$u_1 = 27.92 + 0.2 \times 218.07 = 71.5\ kJ/kg$

$m = m_1 = V_1/v_1 = 14.706\ kg$

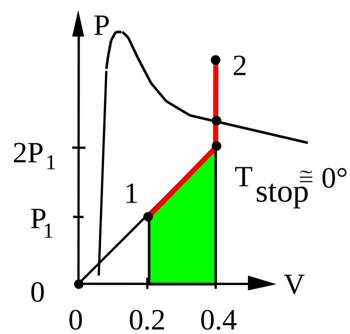
System: on line

$V \le V_{stop}$;

P_1 = 399.6 kPa

$P_{stop} = 2P_1$ = 799.2 kPa

$v_{stop} = 2v_1 = 0.0272\ m^3/kg$



State stop: (P,v) $\Rightarrow T_{stop} \cong 0°C$ TWO-PHASE STATE

Since $T_2 > T_{stop}$ $\Rightarrow$ $v_2 = v_{stop} = 0.0272\ m^3/kg$

State 2: (T_2 , v_2) Table B.4.2: Interpolate between 1000 and 1200 kPa

P_2 = 1035 kPa ; u_2 = 366.5 kJ/kg

From the process curve, see also area in P-V diagram, the work is

$${}_1W_2 = \int PdV = \frac{1}{2}(P_1 + P_{stop})(V_{stop} - V_1) = \frac{1}{2}(399.6 + 799.2)0.2 = \mathbf{119.8\ kJ}$$

From the energy equation

$${}_1Q_2 = m(u_2 - u_1) + {}_1W_2 = 14.706(266.5 - 71.5) + 119.8 = \mathbf{2987.5\ kJ}$$

3.239

A spherical balloon contains 2 kg of R-410A at 0°C, 30% quality. This system is heated until the pressure in the balloon reaches 1 MPa. For this process, it can be assumed that the pressure in the balloon is directly proportional to the balloon diameter. How does pressure vary with volume and what is the heat transfer for the process?

Solution:

C.V. R-410A which is a control mass.

$$m_2 = m_1 = m \;;$$

Energy Eq.3.5: $\quad m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

State 1: 0°C, x = 0.3. Table B.4.1 gives $P_1 = 798.7$ kPa

$$v_1 = 0.000855 + 0.3 \times 0.03182 = 0.01040 \text{ m}^3\text{/kg}$$

$$u_1 = 57.07 + 0.3 \times 195.95 = 115.86 \text{ kJ/kg}$$

Process: $P \propto D,\quad V \propto D^3 \quad => \quad PV^{-1/3} = \text{constant, polytropic} \quad n = -1/3.$

$$=> \quad V_2 = mv_2 = V_1\,(P_2 /P_1)^3 = mv_1\,(P_2 /P_1)^3$$

$$v_2 = v_1\,(P_2 /P_1)^3 = 0.01040 \times (1000 / 798.7)^3 = 0.02041 \text{ m}^3\text{/kg}$$

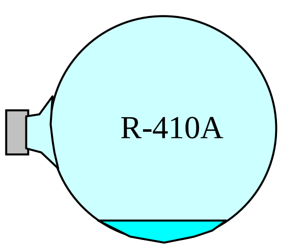
State 2: $P_2 = 1$ MPa, process : $v_2 = 0.02041 \;\rightarrow$ Table B.4.2, $T_2 = 7.25$°C (sat)

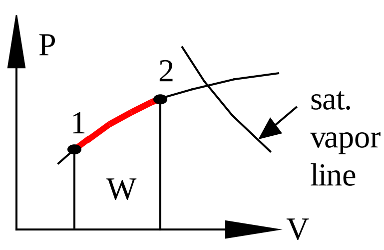
$$v_f = 0.000877,\ v_{fg} = 0.02508 \text{ m}^3\text{/kg},\ u_f = 68.02,\ u_{fg} = 187.18 \text{ kJ/kg}$$

$$x_2 = 0.7787,\quad u_2 = 213.7 \text{ kJ/kg},$$

$$_1W_2 = \int P\,dV = m\,\frac{P_2v_2 - P_1v_1}{1 - n} = 2\,\frac{1000 \times 0.02041 - 798.7 \times 0.0104}{1 - (-1/3)} = 18.16 \text{ kJ}$$

$$_1Q_2 = m(u_2 - u_1) + {}_1W_2 = 2\,(213.7 - 115.86) + 18.16 = \mathbf{213.8\ kJ}$$





Notice: The R-410A is not an ideal gas at any state in this problem.

3.240

A piston/cylinder arrangement B is connected to a 1-m^3 tank A by a line and valve, shown in Fig. P3.240. Initially both contain water, with A at 100 kPa, saturated vapor and B at 400°C, 300 kPa, 1 m^3. The valve is now opened and, the water in both A and B comes to a uniform state.

a. Find the initial mass in A and B.

b. If the process results in T_2 = 200°C, find the heat transfer and work.

Solution:

C.V.: A + B. This is a control mass.

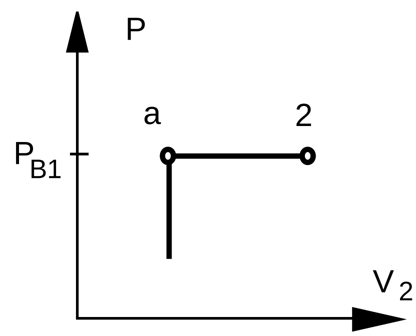
Continuity equation: $m_2 - (m_{A1} + m_{B1}) = 0$;

Energy: $m_2u_2 - m_{A1}u_{A1} - m_{B1}u_{B1} = {}_1Q_2 - {}_1W_2$

System: if $V_B \geq 0$ piston floats $\Rightarrow$ $P_B = P_{B1}$ = const.

if $V_B = 0$ then $P_2 < P_{B1}$ and $v = V_A/m_{tot}$ see P-V diagram

$${}_1W_2 = \int P_B dV_B = P_{B1}(V_2 - V_1)_B = P_{B1}(V_2 - V_1)_{tot}$$



State A1: Table B.1.1, x = 1

v_{A1} = 1.694 m^3/kg, u_{A1} = 2506.1 kJ/kg

$m_{A1} = V_A/v_{A1}$ = **0.5903 kg**

State B1: Table B.1.2 sup. vapor

v_{B1} = 1.0315 m^3/kg, u_{B1} = 2965.5 kJ/kg

$m_{B1} = V_{B1}/v_{B1}$ = **0.9695 kg**

$m_2 = m_{TOT}$ = 1.56 kg

* At (T_2 , P_{B1}) $v_2 = 0.7163 > v_a = V_A/m_{tot} = 0.641$ so $V_{B2} > 0$

so now state 2: $P_2 = P_{B1}$ = 300 kPa, T_2 = 200 °C

=> u_2 = 2650.7 kJ/kg and $V_2 = m_2 v_2 = 1.56 \times 0.7163 = 1.117$ m^3

(we could also have checked T_a at: 300 kPa, 0.641 m^3/kg => T = 155 °C)

${}_1W_2 = P_{B1}(V_2 - V_1)$ = **-264.82 kJ**

${}_1Q_2 = m_2u_2 - m_{A1}u_{A1} - m_{B1}u_{B1} + {}_1W_2$ = **-484.7 kJ**

3.241

Consider the system shown in Fig. P3.241. Tank A has a volume of 100 L and contains saturated vapor R-134a at 30°C. When the valve is cracked open, R-134a flows slowly into cylinder B. The piston mass requires a pressure of 200 kPa in cylinder B to raise the piston. The process ends when the pressure in tank A has fallen to 200 kPa. During this process heat is exchanged with the surroundings such that the R-134a always remains at 30°C. Calculate the heat transfer for the process.

Solution:

C.V. The R-134a. This is a control mass.

Continuity Eq.: $m_2 = m_1 = m$;

Energy Eq.3.5: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Process in B: If $V_B > 0$ then $P = P_{float}$ (piston must move)

$$\Rightarrow \quad {}_1W_2 = \int P_{float}\, dV = P_{float} m(v_2 - v_1)$$

Work done in B against constant external force (equilibrium P in cyl. B)

State 1: 30°C, x = 1. Table B.5.1: $v_1 = 0.02671\ m^3/kg$, $u_1 = 394.48$ kJ/kg

$$m = V/v_1 = 0.1 / 0.02671 = 3.744 \text{ kg}$$

State 2: 30°C, 200 kPa superheated vapor Table B.5.2

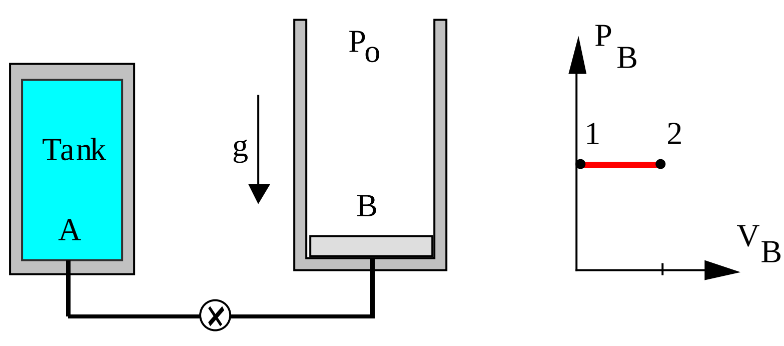
$$v_2 = 0.11889\ m^3/kg, \quad u_2 = 403.1 \text{ kJ/kg}$$

From the process equation

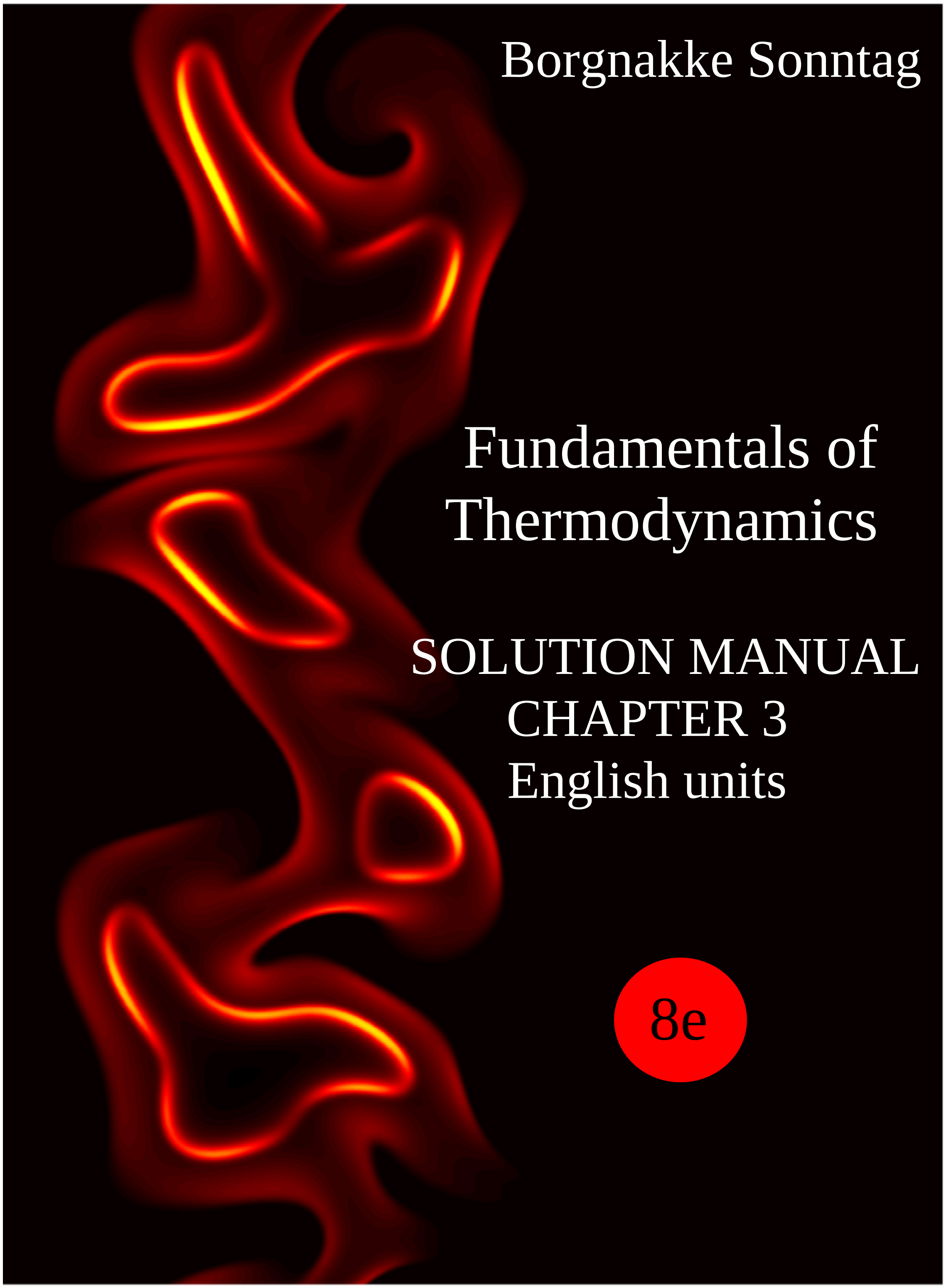
${}_1W_2 = P_{float} m(v_2 - v_1) = 200 \times 3.744 \times (0.11889 - 0.02671) = 69.02$ kJ

From the energy equation

${}_1Q_2 = m(u_2 - u_1) + {}_1W_2 = 3.744 \times (403.1 - 394.48) + 69.02 =$ **101.3 kJ**



ENGLISH UNIT PROBLEMS



CHAPTER 3

Concept Problems

3.242E

What is 1 cal in english units, what is 1 Btu in ft-lbf?

Look in Table A.1 for the conversion factors under energy

1 Btu = **778.1693 lbf-ft**

$$1 \text{ cal} = 4.1868 \text{ J} = \frac{4.1868}{1055} \text{ Btu} = \mathbf{0.00397\ Btu}$$

$$= 0.00397 \times 778.1693 \text{ lbf-ft} = \mathbf{3.088\ lbf\text{-}ft}$$

3.243E

Work as F Δx has units of lbf-ft, what is that in Btu?

Look in Table A.1 for the conversion factors under energy

1 lbf-ft = 1 lbf-ft / (778.1693 ft-lbf/Btu) = 1.28507×10^{-3} Btu

3.244E

Work in the expression Eq. 3.18 or Eq. 3.22 involves PV. For P in psia and V in ft^3, how does that become Btu?

Units: psia × ft^3 = (lbf/in.^{2}) × ft^3 = lbf-ft × (ft/in.)^2 = 144 lbf-ft

= (144/778.1693) Btu

= 144 × 1.28507 × 10^{-3} Btu

= 0.18509 Btu

3.245E

Look at the R-410A value for u_f at -60 F. Can the energy really be negative? Explain.

The absolute value of *u* and *h* are arbitrary. A constant can be added to all *u* and *h* values and the table is still valid. It is customary to select the reference such that *u* for saturated liquid water at the triple point is zero. The standard for refrigerants like R-410A is that *h* is set to zero as saturated liquid at -40 F, other substances as cryogenic substances like nitrogen, methane etc. may have different states at which *h* is set to zero. The ideal gas tables use a zero point for *h* as 77 F or at absolute zero, 0 R.

3.246E

An ideal gas in a piston-cylinder is heated with 2 Btu during an isothermal process. How much work is involved?

Energy Eq.: $u_2 - u_1 = {}_1q_2 - {}_1w_2 = 0$ since $u_2 = u_1$ (isothermal)

Then

$${}_1W_2 = {}_1Q_2 = m\ {}_1w_2 = \mathbf{2\ Btu}$$

3.247E

You heat a gas 20 R at P = C. Which one in table F.4 requires most energy? Why?

A constant pressure process in a control mass gives (recall p. 109 and Eq. 3.39)

$$_1q_2 = u_2 - u_1 + {}_1w_2 = h_2 - h_1 \approx C_p \, \Delta T$$

The one with the highest specific heat is hydrogen, H_2. The hydrogen has the smallest mass, but the same kinetic energy per mol as other molecules and thus the most energy per unit mass is needed to increase the temperature.

3.248E

The air drag force on a car is 0.225 A $\rho\mathbf{V}^2$. Verify that the unit becomes lbf.

$$F_d = 0.225\ A\ \rho\mathbf{V}^2$$

Units: $ft^2 \times (lbm/ft^3) \times (ft/s)^2 = lbm\text{-}ft / s^2 = (1/32.174)\ lbf$

Recall the result from Newtons law p. 9.

Kinetic and Potential Energy

3.249E

An escalator raises a 200 lbm bucket of sand 30 ft in 1 minute. Determine the amount of work done during the process.

Solution:

$$W = \int F dx = F \int dx = F\ \Delta x = mgH$$

$$= 200 \text{ lbm} \times 32.174 \text{ ft/s}^2 \times 30 \text{ ft}$$

$$= 200 \text{ lbf} \times 30 \text{ ft} = 6000 \text{ ft lbf}$$

$$= (6000/778) \text{ Btu}$$

$$= \mathbf{7.71\ Btu}$$

3.250E

A hydraulic hoist raises a 3650 lbm car 6 ft in an auto repair shop. The hydraulic pump has a constant pressure of 100 lbf/in.^{2} on its piston. What is the increase in potential energy of the car and how much volume should the pump displace to deliver that amount of work?

Solution: C.V. Car.

No change in kinetic or internal energy of the car, neglect hoist mass.

$$E_2 - E_1 = PE_2 - PE_1 = mg\,(Z_2 - Z_1)$$

$$= \frac{3650 \times 32.174 \times 6}{32.174} = \mathbf{21\ 900\ lbf\text{-}ft}$$

The increase in potential energy is work into car from pump at constant P.

$$W = E_2 - E_1 = \int P\,dV = P\,\Delta V \qquad \Rightarrow$$

$$\Delta V = \frac{E_2 - E_1}{P} = \frac{21\ 900}{100 \times 144} = \mathbf{1.52\ ft^3}$$

3.251E

A piston motion moves a 50 lbm hammerhead vertically down 3 ft from rest to a velocity of 150 ft/s in a stamping machine. What is the change in total energy of the hammerhead?

Solution: C.V. Hammerhead

The hammerhead does not change internal energy i.e. same P,T

$$E_2 - E_1 = m(u_2 - u_1) + m(\frac{1}{2}V_2^2 - 0) + mg\,(h_2 - 0)$$

$$= 0 + [\ 50 \times (1/2) \times 150^2 + 50 \times 32.174 \times (-3)]\ \text{lbm-(ft/s)}^2$$

$$= [562\ 500 - 4826]/32.174 = 17\ 333\ \text{lbf-ft}$$

$$= (\frac{17\ 333}{778})\ \text{Btu} = \mathbf{22.28\ Btu}$$

Recall that 1 lbf-ft = 32.174 lbm-$(\text{ft/s})^2$

3.252E

Airplane takeoff from an aircraft carrier is assisted by a steam driven piston/cylinder with an average pressure of 200 psia. A 38 500 lbm airplane should be accelerated from zero to a speed of 100 ft/s with 30% of the energy coming from the steam piston. Find the needed piston displacement volume.

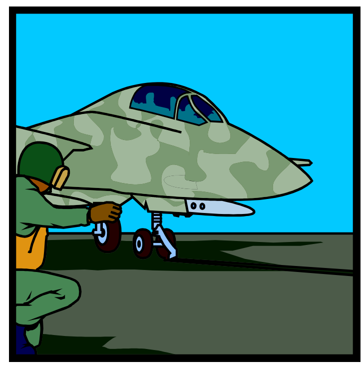
Solution: C.V. Airplane.

No change in internal or potential energy; only kinetic energy is changed.

$$E_2 - E_1 = m\ (1/2)\ (\mathbf{V}_2^2 - 0) = 38\ 500\ \text{lbm} \times (1/2) \times 100^2\ (\text{ft/s})^2$$

$$= 192\ 500\ 000\ \text{lbm-(ft/s)}^2 = 5\ 983\ 092\ \text{lbf-ft}$$

The work supplied by the piston is 30% of the energy increase.



$$W = \int P\ dV = P_{avg}\ \Delta V = 0.30\ (E_2 - E_1)$$
$$= 0.30 \times 5\ 983\ 092\ \text{lbf-ft} = 1\ 794\ 928\ \text{lbf-ft}$$

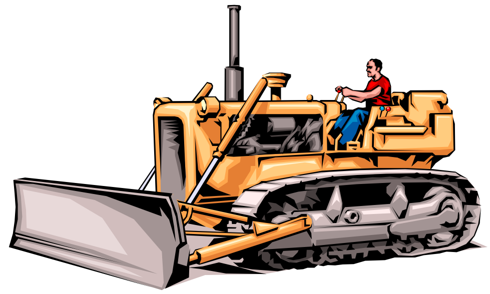
$$\Delta V = \frac{W}{P_{avg}} = \frac{1\ 794\ 928}{200}\ \frac{\text{lbf-ft}}{144\ \text{lbf/ft}^2} = \mathbf{62.3\ ft^3}$$

3.253E

A bulldozer pushes 1000 lbm of dirt 300 ft with a force of 400 lbf. It then lifts the dirt 10 ft up to put it in a dump truck. How much work did it do in each situation?

Solution:

$W = \int F\,dx = F\,\Delta x$
$= 400\text{ lbf} \times 300\text{ ft}$
$= 120\,000\text{ lbf-ft} =$ **154 Btu**



$W = \int F\,dz = \int mg\,dz = mg\,\Delta Z$
$= 1000\text{ lbm} \times 32.174\text{ ft/s}^2 \times 10\text{ ft} / (32.174\text{ lbm-ft / s}^2\text{-lbf})$
$= 10\,000\text{ lbf-ft} =$ **12.85 Btu**

3.254E

Two hydraulic cylinders maintain a pressure of 175 psia. One has a cross sectional area of 0.1 ft^2 the other 0.3 ft^2. To deliver a work of 1 Btu to the piston how large a displacement (V) and piston motion H is needed for each cylinder? Neglect P_{atm}

Solution:

$$W = \int F\, dx = \int P\, dV = \int PA\, dx = PA\times H = P\, \Delta V$$

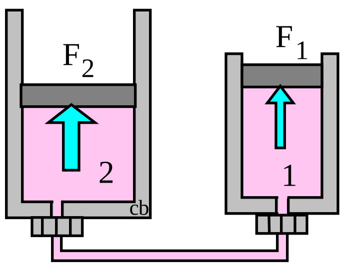
$$W = 1 \text{ Btu} = 778.17 \text{ lbf-ft}$$

$$\Delta V = \frac{W}{P} = \frac{778.17 \text{ lbf-ft}}{175 \times 144 \text{ lbf/ft}^2} = \mathbf{0.030\ 873\ ft^3}$$

Both cases the height is $H = \Delta V/A$

$$H_1 = \frac{0.030873}{0.1} = \mathbf{0.3087\ ft}$$

$$H_2 = \frac{0.030873}{0.3} = \mathbf{0.1029\ ft}$$



3.255E

A linear spring, $F = k_s(x - x_0)$, with spring constant k_s = 35 lbf/ft, is stretched until it is 2.5 in. longer. Find the required force and work input.

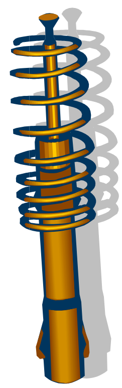
Solution:

$$F = k_s(x - x_0) = 35 \text{ lbf/ft} \times (2.5/12) \text{ ft} = 7.292 \text{ lbf}$$

$$W = \int F dx = \int k_s(x - x_0) d(x - x_0) = \frac{1}{2} k_s(x - x_0)^2$$

$$= \frac{1}{2} \times 35 \text{ lbf/ft} \times (2.5/12)^2 \text{ ft}^2 = 0.76 \text{ lbf-ft}$$

$$= \mathbf{9.76 \times 10^{-4} \ Btu}$$



3.256E

A cylinder fitted with a frictionless piston contains 10 lbm of superheated refrigerant R-134a vapor at 100 lbf/in.^{2}, 300 F. The setup is cooled at constant pressure until the R-134a reaches a quality of 25%. Calculate the work done in the process.

Solution:

Constant pressure process boundary work. State properties from Table F.10

State 1: Table F.10.2 $\quad v_1 = 0.76629\ \text{ft}^3/\text{lbm}$;

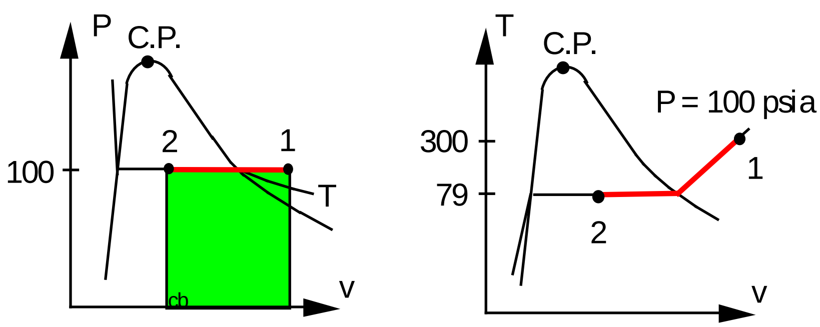
State 2: Table F.10.1 $\quad v_2 = 0.013331 + 0.25 \times 0.46652 = 0.12996\ \ \text{ft}^3/\text{lbm}$

Interpolated to be at 100 psia, numbers at 101.5 psia could have been used.

$$ {}_1W_2 = \int P\, dV = P\,(V_2 - V_1) = mP\,(v_2 - v_1) $$

$$ = 10\ \text{lbm} \times 100\ \text{lbf/in.}^2 \times \frac{144}{778}\ \frac{(\text{in/ft})^2}{\text{lbf-ft/Btu}} \times (0.12996 - 0.76629)\ \text{ft}^3/\text{lbm} $$

$= \mathbf{-117.78\ Btu}$



3.257E

A piston of 4 lbm is accelerated to 60 ft/s from rest. What constant gas pressure is required if the area is 4 in^2, the travel 4 in. and the outside pressure is 15 psia?

C.V. Piston

$$(E_2 – E_1)_{PIST.} = m(u_2 – u_1) + m[(1/2)V_2^2 – 0] + mg\,(0 – 0)$$

$$= (1/2)\, m\, V_2^2 = 0.5 \times 4 \text{ lbm} \times 60^2 \text{ (ft/s)}^2$$

$$= 7200 \text{ lbm-ft}^2/\text{s}^2 = \frac{7200}{32.174} \text{ ft-lbf}$$

Energy equation for the piston is:

$$(E_2 – E_1)_{PIST.} = W_{gas} - W_{atm} = P_{avg}\ \Delta V_{gas} – P_o\ \Delta V_{gas}$$

$$\Delta V_{gas} = A\,L = 4 \text{ in}^2 \times 4 \text{ in} = 16 \text{ in}^3$$

$$P_{avg}\ \Delta V_{gas} = (E_2 – E_1)_{PIST.} + P_o\ \Delta V_{gas}$$

$$P_{avg} = (E_2 – E_1)_{PIST.} / \Delta V_{gas} + P_o$$

$$= \frac{7200 \text{ lbf-ft}}{32.174 \times 16 \text{ in}^3} + 15 \text{ lbf/in}^2$$

$$= 167.84 \text{ psia} + 15 \text{ psia} = \mathbf{182.8\ psia}$$

Properties General Tables

3.258E

Find the missing properties and give the phase of the substance.

a. H_2O u = 1000 Btu/lbm, T = 270 F h = ? v = ? x = ?

b. H_2O u = 450 Btu/lbm, P = 1500 lbf/in.^{2} T = ? x = ? v = ?

c. R-410A T = 30 F, P = 120 lbf/in.^{2} h = ? x = ?

Solution:

a) Table F.7.1: $u_f < u < u_g$ => 2-phase mixture of liquid and vapor

$x = (u – u_f)/ u_{fg}$ = (1000 – 238.81)/854.14 = **0.8912**

$v = v_f + x\, v_{fg}$ = 0.01717 + 0.8912 × 10.0483 = **8.972 ft^3/lbm**

$h = h_f + x\, h_{fg}$ = 238.95 + 0.8912 × 931.95 = **1069.5 Btu/lbm**

(h = u + Pv = 1000 + 41.848 × 8.972 × 144/778)

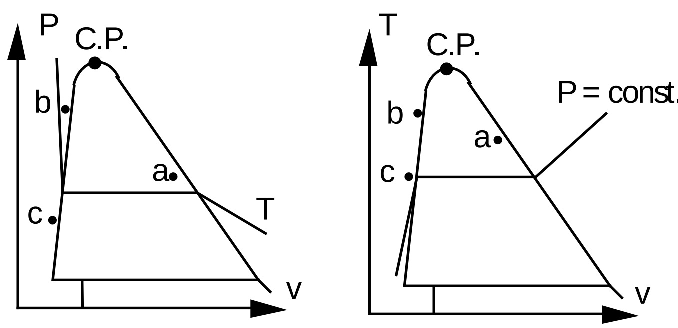
b) Table F.7.1: $u < u_f$ so compressed liquid B.1.3, x = **undefined**

T = **471.8 F**, v = **0.019689 ft^3/lbm**

c) Table F.9.1: P > Psat => x = **undef**, **compr. liquid**

Approximate as saturated liquid at same T, $h \cong h_f$ = **24.11 Btu/lbm**

States shown are placed relative to the two-phase region, not to each other.



3.259E

Find the missing properties among (*P*, *T*, *v*, *u*, *h*) together with *x*, if applicable, and give the phase of the substance.

a. R-410A $T = 50$ F, $u = 85$ Btu/lbm

b. H_2O $T = 600$ F, $h = 1322$ Btu/lbm

c. R-410A $P = 150$ lbf/in.^{2}, $h = 135$ Btu/lbm

Solution:

a) Table F.9.1: $u < u_g$ => L+V mixture, P = **157.473 lbf/in^2**

$x = (85 - 31.06)/ 78.96 = \mathbf{0.6831}$

$v = 0.014 + 0.6831 \times 0.3676 = \mathbf{0.2653\ ft^3/lbm}$

$h = 31.47 + 0.6831 \times 89.67 = \mathbf{92.72\ Btu/lbm}$

b) Table F.7.1: $h > h_g$ => superheated vapor follow 600 F in F.7.2

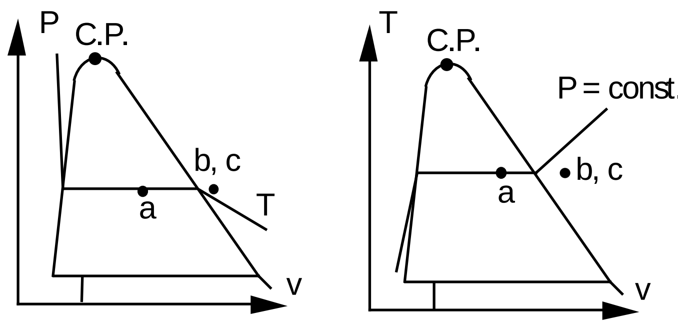
$P \cong$ **200 lbf/in^2** ; v = **3.058 ft^3/lbm ;** u = **1208.9 Btu/lbm**

c) Table F.9.1: $h > h_g$ => superheated vapor so in F.9.2

$T \cong$ **100 F** ; v **= 0.483 ft^3/lbm**

u = **121.25 Btu/lbm**

States shown are placed relative to the two-phase region, not to each other.



3.260E

Find the missing properties among (P, T, v, u, h) together qwith x if applicable and give the phase of the substance.

a.	R-134a	$T = 140$ F, $h = 185$ Btu/lbm
b.	NH_3	$T = 170$ F, $P = 60$ lbf/in.^{2}
c.	R-134a	$T = 100$ F, $u = 175$ Btu/lbm

Solution:

a) Table F.10.1: $h > h_g$ => x = **undef**, **superheated vapor** F.10.2,

find it at given T between saturated 243.9 psi and 200 psi to match h:

$$v \cong 0.1836 + (0.2459 - 0.1836)\times \frac{185 - 183.63}{186.82 - 183.63} = \mathbf{0.2104\ ft^3/lbm}$$

$$P \cong 243.93 + (200 - 243.93)\times \frac{185 - 183.63}{186.82 - 183.63} = \mathbf{225\ lbf/in^2}$$

b) Table F.8.1: $P < P_{sat}$ $\Rightarrow$ x = **undef.** **superheated vapor** F.8.2,

$v = (6.3456 + 6.5694)/ 2 = \mathbf{6.457\ ft^3/lbm}$

$u = h - Pv = (1/2)(694.59 + 705.64) – 60 \times 6.4575 \times (144/778)$

$= 700.115 – 71.71 = \mathbf{628.405\ Btu/lbm}$

c) Table F.10.1: $u > u_g$ => **sup. vapor,**

Interpolate between 40 and 60 psia tables in F.10.2

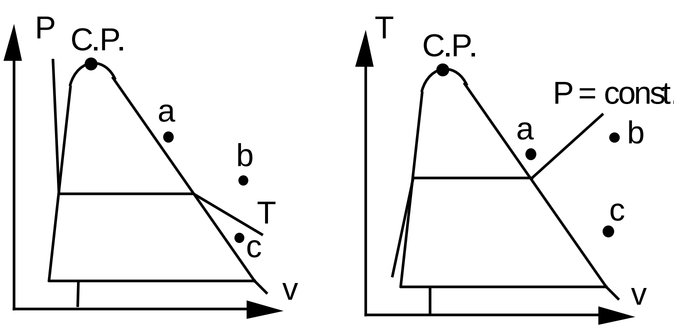
$P = 40 + (40 – 20)(175 – 175.57)/(174.85 – 175.57)$

$= 40 + 20 \times 0.791667 = \mathbf{55.8\ lbf/in^2}$;

$v = 1.4015 + (0.9091 – 1.4015) \times 0.791667 = \mathbf{1.0117\ ft^3/lbm;}$

$h = 185.95 + (184.94 – 185.95) \times 0.791667 = \mathbf{185.15\ Btu/lbm}$

States shown are placed relative to the two-phase region, not to each other.



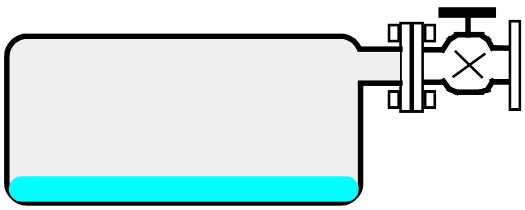
Simple Processes

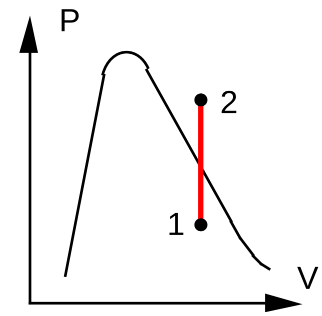
3.261E

Ammonia at 30 F, quality 60% is contained in a rigid 8-ft^3 tank. The tank and ammonia are now heated to a final pressure of 150 lbf/in.^{2}. Determine the heat transfer for the process.

Solution:

C.V.: NH_3





Continuity Eq.: $m_2 = m_1 = m$;

Energy Eq.3.5: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Process: Constant volume $\Rightarrow$ $v_2 = v_1$ & ${}_1W_2 = 0$

State 1: Table F.8.1 two-phase state.

$v_1 = 0.02502 + 0.6 \times 4.7978 = 2.904 \text{ ft}^3/\text{lbm}$

$u_1 = 75.06 + 0.6 \times 491.17 = 369.75 \text{ Btu/lbm}$

$m = V/v_1 = 8/2.904 = 2.755 \text{ lbm}$

State 2: P_2, $v_2 = v_1$ superheated vapor Table F.8.2

$\Rightarrow$ $T_2 \cong 258$ F, $u_2 \cong 661.42$ Btu/lbm

So solve for heat transfer in the energy equation

${}_1Q_2 = 2.755 \text{ lbm} \times (661.42 - 369.75) \text{ Btu/lbm} =$ **803.6 Btu**

3.262E

Saturated vapor R-410A at 30 F in a rigid tank is cooled to 0 F. Find the specific heat transfer.

Solution:

C.V.: R-410A in tank. $m_2 = m_1$;

Energy Eq.3.5: $(u_2 - u_1) = {}_1q_2 - {}_1w_2$

Process: V = constant, $v_2 = v_1 = V/m$ => ${}_1\mathbf{w}_2 = \mathbf{0}$

Table F.9.1: State 1: $v_1 = 0.5426\ \text{ft}^3/\text{lbm}$, $u_1 = 108.63$ Btu/lbm

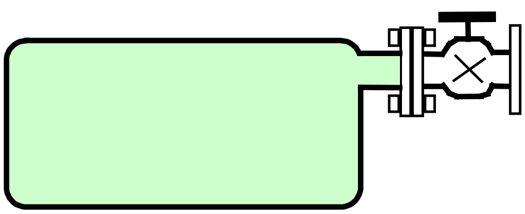
State 2: 0 F, $v_2 = v_1 = V/m$, look in Table F.9.1 at 0 F

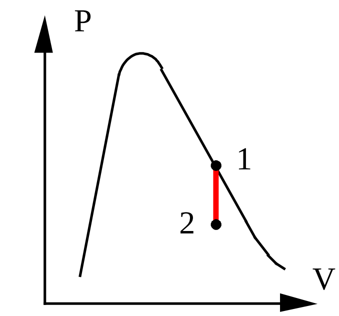
$$x_2 = \frac{v_2 - v_{f2}}{v_{fg2}} = \frac{0.5426 - 0.01295}{0.9448} = 0.560595$$

$$u_2 = u_{f2} + x_2\, u_{fg2} = 13.37 + x_2 \times 92.75 = 65.365 \text{ Btu/lbm}$$

From the energy equation

$${}_1q_2 = (u_2 - u_1) = (65.365 - 108.63) = \mathbf{-43.26\ Btu/lbm}$$





3.263E

Saturated vapor R-410A at 100 psia in a constant pressure piston cylinder is heated to 70 F. Find the specific heat transfer.

Solution:

C.V. R-410A: $m_2 = m_1 = m;$

Energy Eq.3.5 $(u_2 - u_1) = {}_1q_2 - {}_1w_2$

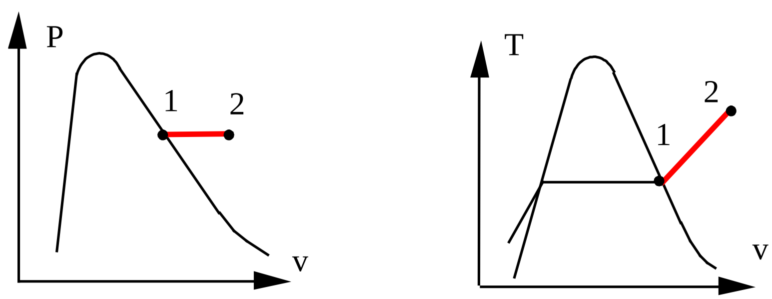
Process: P = const. $\Rightarrow {}_1w_2 = \int P dv = P\Delta v = P(v_2 - v_1)$

State 1: Table F.9.2 (or F.9.1) $h_1 = 119.38$ Btu/lbm

State 2: Table F.9.2 $h_2 = 130.44$ Btu/lbm

$${}_1q_2 = (u_2 - u_1) + {}_1w_2 = (u_2 - u_1) + P(v_2 - v_1) = (h_2 - h_1)$$

$${}_1q_2 = 130.44 - 119.38 = \mathbf{11.06\ Btu/lbm}$$



3.264E

A rigid tank holds 1.5 lbm R-410A at 100 F as saturated vapor. The tank is now cooled to 60 F by heat transfer to the ambient. Which two properties determine the final state. Determine the amount of work and heat transfer during the process.

C.V. The R-410A, this is a control mass.

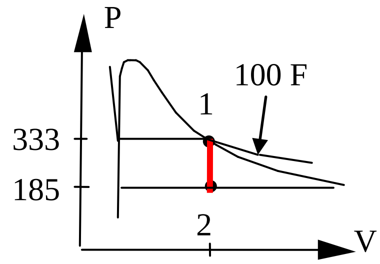
Process: Rigid tank V = C => v = constant & $_1W_2 = \int_1^2 PdV = \mathbf{0}$

Energy Eq.: $U_2 - U_1 = {_1Q_2} - {_1W_2} = {_1Q_2}$,

State 1: $v_1 = 0.1657$ ft^3/lbm,

$u_1 = 111.7$ Btu/lbm

State 2: **T, v** => two-phase (straight down in P-v diagram from state 1)



$x_2 = (v - v_f)/v_{fg} = (0.1657 - 0.01451)/0.3076 = 0.49151$

$u_2 = u_f + x_2\, u_{fg} = 34.78 + 0.49151 \times 75.82 = 72.046$ Btu/lbm

$_1Q_2 = m(u_2 - u_1) = 1.5$ lbm $(72.046 - 111.7)$ Btu/lbm = **-59.5 Btu**

3.265E

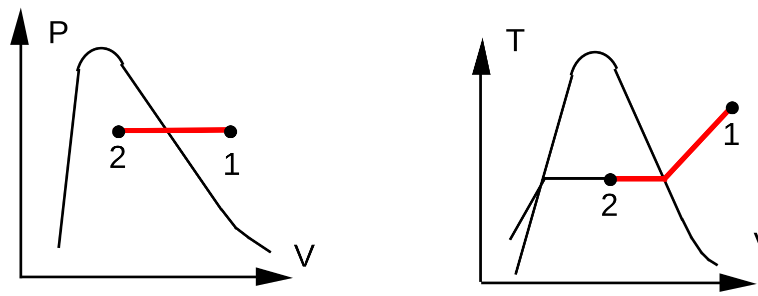
A cylinder fitted with a frictionless piston contains 4 lbm of superheated refrigerant R-134a vapor at 400 lbf/in.^{2}, 200 F. The cylinder is now cooled so the R-134a remains at constant pressure until it reaches a quality of 75%. Calculate the heat transfer in the process.

Solution:

C.V.: R-134a $\quad m_2 = m_1 = m;$

Energy Eq.3.5 $\quad m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Process: P = const. $\Rightarrow {}_1W_2 = \int PdV = P\Delta V = P(V_2 - V_1) = Pm(v_2 - v_1)$



State 1: Table F.10.2 $\quad h_1 = 192.92$ Btu/lbm

State 2: Table F.10.1 $\quad h_2 = 140.62 + 0.75 \times 43.74 = 173.425$ Btu/lbm

$${}_1Q_2 = m(u_2 - u_1) + {}_1W_2 = m(u_2 - u_1) + Pm(v_2 - v_1) = m(h_2 - h_1)$$

$$= 4 \text{ lbm} \times (173.425 - 192.92) \text{ Btu/lbm} = \mathbf{-77.98\ Btu}$$

3.266E

A water-filled reactor with volume of 50 ft^3 is at 2000 lbf/in.^{2}, 560 F and placed inside a containment room, as shown in Fig. P3.101. The room is well insulated and initially evacuated. Due to a failure, the reactor ruptures and the water fills the containment room. Find the minimum room volume so the final pressure does not exceed 30 lbf/in.^{2}.

C.V.: Containment room and reactor.

Mass: $m_2 = m_1 = V_{reactor}/v_1 = 50/0.02172 = 2295.7$ lbm

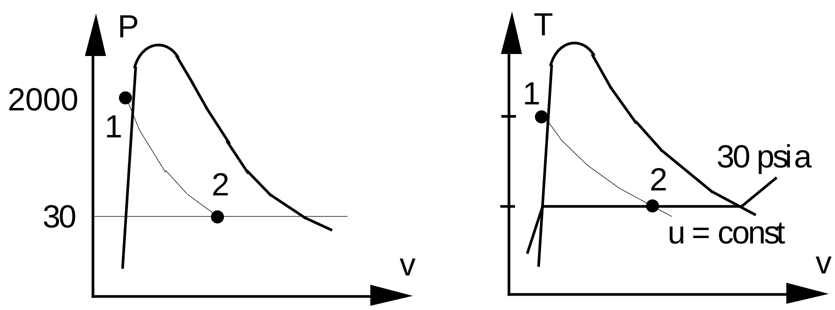
Energy Eq.: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2 = \emptyset \;\Rightarrow\; u_2 = u_1 = 552.5$ Btu/lbm

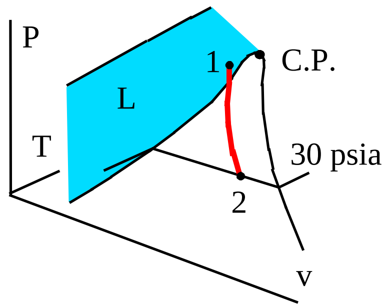
State 2: 30 lbf/in.^{2}, $u_2 < ug \;\Rightarrow\;$ 2 phase Table F.7.1

$$u = 552.5 = 218.48 + x_2\, 869.41 \quad\Rightarrow\quad x_2 = 0.3842$$

$$v_2 = 0.017 + 0.3842 \times 13.808 = 5.322 \text{ ft}^3\text{/lbm}$$

$$V_2 = mv_2 = 2295.7 \text{ lbm} \times 5.322 \text{ ft}^3\text{/lbm} = \mathbf{12\ 218\ ft^3}$$





3.267E

Ammonia (1 lbm) is in a piston cylinder at 30 psia, 20 F is heated in a process where the pressure varies linear with volume to a state of 240 F, 40 psia. Find the work and the heat transfer in the process.

Solution:

Take CV as the Ammonia, constant mass.

Continuity Eq.: $m_2 = m_1 = m$;

Process: $P = A + BV$ (linear in V)

State 1: Superheated vapor $v_1 = 9.7206\ \text{ft}^3/\text{lbm}$,

$$u_1 = 622.39 - 30 \times 9.7206 \times \frac{144}{778} = 568.4\ \text{Btu/lbm}$$

State 2: Superheated vapor $v_2 = 10.9061\ \text{ft}^3/\text{lbm}$,

$$u_2 = 741.4 - 40 \times 10.9061 \times 144/778 = 660.65\ \text{Btu/lbm}$$

Work is done while piston moves at increasing pressure, so we get

$$_1W_2 = \int P\, dV = \text{area} = P_{avg}\,(V_2 - V_1) = \frac{1}{2}(P_1 + P_2)m(v_2 - v_1)$$

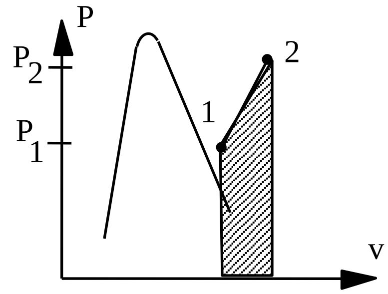
$$= ½(30 + 40)\ \text{psi} \times 1\ \text{lbm}\ (10.9061 - 9.7206)\ \text{ft}^3/\text{lbm}$$

$$= 41.493\ \text{psi-ft}^3 = 41.493 \times 144\ \text{lbf-ft} = 5974.92\ \text{lbf-ft} = \mathbf{7.68\ Btu}$$

(see conversions in A.1, p. 755)

Heat transfer from the energy equation

$$_1Q_2 = m(u_2 - u_1) + {_1W_2} = 1\ (660.65 - 568.4) + 7.68 = \mathbf{99.93\ Btu}$$



3.268E

A piston cylinder arrangement with a linear spring similar to Fig. P3.105 contains R-134a at 60 F, x = 0.6 and a volume of 0.7 ft^3. It is heated to 140 F at which point the specific volume is 0.4413 ft^3/lbm. Find the final pressure, the work and the heat transfer in the process.

Take CV as the R-134a.

$$m_2 = m_1 = m \quad ; \qquad m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$$

State 1: T_1, x_1 => Two phase so Table F.10.1: $P_1 = P_{sat} = 72.271$ psia

$$v_1 = v_f + x_1 v_{fg} = 0.01291 + 0.6 \times 0.6503 = 0.40309 \ ft^3/lbm$$

$$u_1 = u_f + x_1 u_{fg} = 98.27 + 0.6 \times 69.31 = 139.856 \text{ Btu/lbm}$$

$$m = V_1/v_1 = 0.7 \ ft^3/ \ 0.40309 \ ft^3/lbm = 1.7366 \text{ lbm}$$

State 2: (T, v) Superheated vapor, Table F.10.2.

$$P_2 = 125 \text{ psia}, \quad v_2 = 0.4413 \ ft^3/lbm, \qquad u_2 = 180.77 \text{ Btu/lbm}$$

$$V_2 = m\, v_2 = 1.7366 \times 0.4413 = 0.76636 \ ft^3$$

Work is done while piston moves at linearly varying pressure, so we get

$${}_1W_2 = \int P\, dV = \text{area} = P_{avg} (V_2 - V_1) = 0.5(P_2 + P_1)(V_2 - V_1)$$

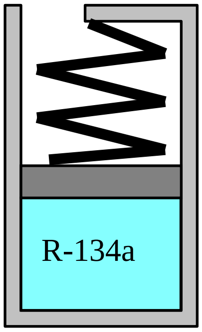
$$= 0.5 \times (72.271 + 125) \text{ psia } (0.76636 - 0.7) \ ft^3$$

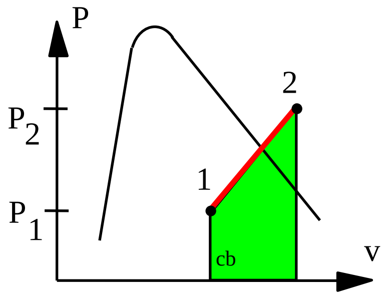
$$= 6.54545 \times (144/778) \text{ Btu} = \mathbf{1.212 \ Btu}$$

Heat transfer is found from the energy equation

$${}_1Q_2 = m(u_2 - u_1) + {}_1W_2 = 1.7366 \times (180.77 - 139.856) + 1.212$$

$$= \mathbf{72.26 \ Btu}$$





3.269E

Water in a 6-ft^3 closed, rigid tank is at 200 F, 90% quality. The tank is then cooled to 20 F. Calculate the heat transfer during the process.

Solution:

C.V.: Water in tank. $m_2 = m_1$; $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Process: V = constant, $v_2 = v_1$, ${}_1W_2 = 0$

State 1: Two-phase L + V look in Table F.7.1

$v_1 = 0.01663 + 0.9 \times 33.6146 = 30.27 \text{ ft}^3/\text{lbm}$

$u_1 = 168.03 + 0.9 \times 906.15 = 983.6 \text{ Btu/lbm}$

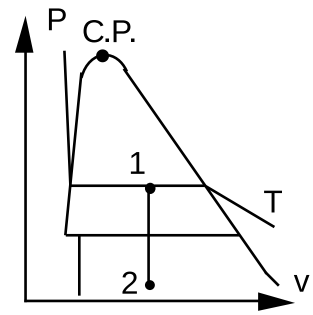
State 2: T_2, $v_2 = v_1$ $\Rightarrow$ mix of sat. solid + vap. Table F.7.4

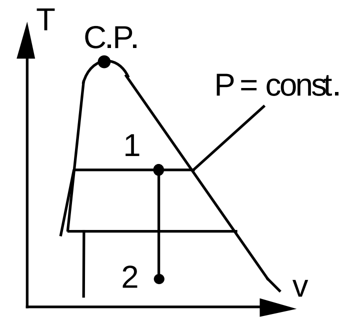
$v_2 = 30.27 = 0.01744 + x_2 \times 5655$ => $x_2 = 0.00535$

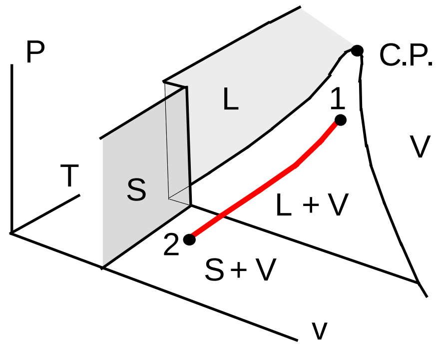
$u_2 = -149.31 + 0.00535 \times 1166.5 = -143.07 \text{ Btu/lbm}$

$m = V/v_1 = 6 \text{ ft}^3 / 30.27 \text{ ft}^3/\text{lbm} = 0.198 \text{ lbm}$

${}_1Q_2 = m(u_2 - u_1) = 0.198 \text{ lbm } (-143.07 - 983.6) \text{ Btu/lbm} = \mathbf{-223 \text{ Btu}}$







3.270E

A constant pressure piston/cylinder has 2 lbm water at 1100 F and 2.26 ft^3. It is now cooled to occupy 1/10 of the original volume. Find the heat transfer in the process.

C.V.: Water $\quad m_2 = m_1 = m;$

Energy Eq.3.5 $\quad m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Process: P = const. $\Rightarrow {}_1W_2 = \int PdV = P\Delta V = P(V_2 - V_1) = Pm(v_2 - v_1)$

State 1: Table F.7.2 $\quad (T, v_1 = V/m = 2.26/2 = 1.13 \text{ ft}^3\text{/lbm})$

$P_1 = 800$ psia, $\quad h_1 = 1567.81$ Btu/lbm

State 2: Table F.7.2 $\quad (P, v_2 = v_1/10 = 0.113 \text{ ft}^3\text{/lbm})$ $\quad$ two-phase state

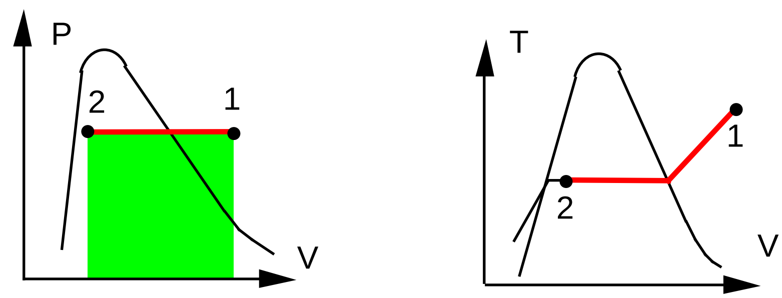
$T_2 = 510 + (520 – 510)(800 – 743.53)/(811.48 – 743.53) = 518.3$ F

$x_2 = (v_2 – v_f)/v_{fg} = (0.113 – 0.02087)/0.5488 = 0.1679$

$h_2 = h_f + x_2\, h_{fg} = 509.63 + x_2\, 689.62 = 625.42$ Btu/lbm

${}_1Q_2 = m(u_2 - u_1) + {}_1W_2 = m(h_2 - h_1)$

$= 2\ (625.42 – 1567.81) =$ **-1884.8 Btu**



Solids and Liquids

3.271E

I have 4 lbm of liquid water at 70 F, 15 psia. I now add 20 Btu of energy at a constant pressure. How hot does it get if it is heated? How fast does it move if it is pushed by a constant horizontal force? How high does it go if it is raised straight up?

a) Heat at 15 psia.
Energy equation:

$$E_2 - E_1 = {}_1Q_2 - {}_1W_2 = {}_1Q_2 - P(V_2 - V_1) = H_2 - H_1 = m(h_2 - h_1)$$

$$h_2 = h_1 + {}_1Q_2/m = 38.09 + 20/4 = 43.09 \text{ Btu/lbm}$$

Back interpolate in Table F.7.1: $T_2 = \mathbf{75\ F}$

(We could also have used $\Delta T = {}_1Q_2/mC = 20 / (4\times1.00) = 5$ F)

b) Push at constant P. It gains kinetic energy.

$$0.5 \text{ m } \mathbf{V}_2^2 = {}_1W_2$$

$$\mathbf{V}_2 = \sqrt{2\ {}_1W_2/m} = \sqrt{2 \times 20 \times 778.17 \text{ lbf-ft/4 lbm}}$$

$$= \sqrt{2 \times 20 \times 778.17 \times 32.174 \text{ lbm-(ft/s)}^2 \text{ /4 lbm}} = \mathbf{500\ ft/s}$$

c) Raised in gravitational field

$$m\ g\ Z_2 = {}_1W_2$$

$$Z_2 = {}_1W_2/m\ g = \frac{20 \times 778.17 \text{ lbf-ft}}{4 \text{ lbm} \times 32.174 \text{ ft/s}^2} \times 32.174 \frac{\text{lbm-ft/s}^2}{\text{lbf}} = \mathbf{3891\ ft}$$

Comment: Notice how fast (500 ft/s) and how high it should be to have the same energy as raising the temperature just 5 degrees. I.e. in most applications we can disregard the kinetic and potential energies unless we have very high **V** or Z.

3.272E

A computer cpu chip consists of 0.1 lbm silicon, 0.05 lbm copper, 0.1 lbm polyvinyl chloride (plastic). It now heats from 60 F to 160 F as the computer is turned on. How much energy did the heating require?

Energy Eq.: $U_2 - U_1 = \sum m_i(u_2 - u_1)_i = {}_1Q_2 - {}_1W_2$

For the solid masses we will use the specific heats, Table F.2, and they all have the same temperature so

$$\sum m_i(u_2 - u_1)_i = \sum m_i C_{v\,i} (T_2 - T_1)_i = (T_2 - T_1)\sum m_i C_{v\,i}$$

$$\sum m_i C_{v\,i} = 0.1 \times 0.167 + 0.05 \times 0.1 + 0.1 \times 0.229 = 0.0446 \text{ Btu/R}$$

$$U_2 - U_1 = 0.0446 \text{ Btu/R} \times (160 - 60) \text{ R} = \mathbf{4.46\ Btu}$$

3.273E

A copper block of volume 60 in.^{3} is heat treated at 900 F and now cooled in a 3-ft^3 oil bath initially at 70 F. Assuming no heat transfer with the surroundings, what is the final temperature?

C.V. Copper block and the oil bath.

Also assume no change in volume so the work will be zero.

Energy Eq.: $U_2 - U_1 = m_{met}(u_2 - u_1)_{met} + m_{oil}(u_2 - u_1)_{oil} = {}_1Q_2 - {}_1W_2 = 0$

Solid and liquid $\Delta u \cong C_V \Delta T$, C_V from Table F.2 and F.3

$$m_{met} = V\rho = 60 \times 12^{-3} \text{ ft}^3 \times 518 \text{ lbm/ft}^3 = 17.986 \text{ lbm}$$

$$m_{oil} = V\rho = 3.0 \text{ ft}^3 \times 57 \text{ lbm/ft}^3 = 171 \text{ lbm}$$

$$C_{V\ met} = 0.10 \text{ Btu/lbm-R}; \qquad C_{V\ oil} = 0.43 \text{ Btu/lbm-R}$$

Energy equation becomes

$$m_{met} C_{V\ met} (T_2 - T_{1,met}) + m_{oil} C_{V\ oil} (T_2 - T_{1,oil}) = 0$$

$$17.986 \text{ lbm} \times 0.10 \text{ Btu/lbm-R} (T_2 - 900 \text{ F})$$

$$+ 171 \text{ lbm} \times 0.43 \text{ Btu/lbm-R} (T_2 - 70 \text{ F}) = 0$$

$$\Rightarrow T_2 = \mathbf{89.8\ F}$$

Ideal Gas

3.274E

Estimate the constant specific heats for R-134a from Table F.10.2 at 15 psia and 150 F. Compare this to table F.4 and explain the difference.

Solution:

Using values at 15 psia for h and u at 140 F and 160 F from Table F.10.2, the approximate specific heats at 150 F are

$$C_p \approx \frac{\Delta h}{\Delta T} = \frac{199.95 - 195.59}{160 - 140} = 0.218 \text{ Btu/lbm R}$$

compared with 0.203 Btu/lbm-R for the ideal-gas value at 77 F from Table F.4.

$$C_v \approx \frac{\Delta u}{\Delta T} = \frac{188.03 - 184.08}{160 - 140} = 0.198 \text{ Btu/lbm R}$$

compared with 0.184 Btu/lbm-R for the ideal-gas value at 77 F from Table F.4.

There are two reasons for the differences. First, R-134a is not exactly an ideal gas at the given state, 150 F and 15 psia. Second and by far the biggest reason for the differences is that R-134a, chemically CF_3CH_2 , is a polyatomic molecule with multiple vibrational mode contributions to the specific heats (see Appendix C), such that they are strongly dependent on temperature. Note that if we repeat the above approximation for C_p in Table F.10.2 at 77 F, the resulting value is 0.203 Btu/lbm R

3.275E

Air is heated from 540 R to 640 R at V = C. Find ${}_1q_2$? What if from 2400 to 2500 R?

Process: V = C $\rightarrow$ ${}_1W_2 = \emptyset$

Energy Eq.: $u_2 - u_1 = {}_1q_2 - 0$ $\rightarrow$ ${}_1q_2 = u_2 - u_1$

Read the u-values from Table F.5

a) ${}_1q_2 = u_2 - u_1 = 109.34 - 92.16 =$ **17.18 Btu/lbm**

b) ${}_1q_2 = u_2 - u_1 = 474.33 - 452.64 =$ **21.7 Btu/lbm**

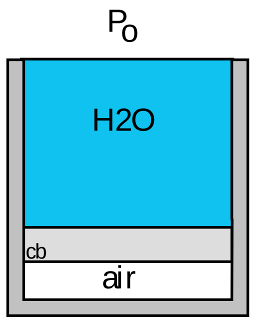
case a) $C_v \approx 17.18/100 = 0.172$ Btu/lbm-R, see F.4

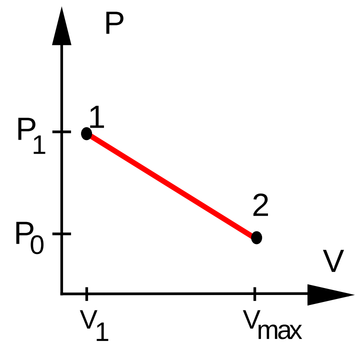
case b) $C_v \approx 21.7/100 = 0.217$ Btu/lbm-R (26 % higher)

3.276E

A 30-ft high cylinder, cross-sectional area 1 ft^2, has a massless piston at the bottom with water at 70 F on top of it, as shown in Fig. P3.107. Air at 540 R, volume 10 ft^3 under the piston is heated so that the piston moves up, spilling the water out over the side. Find the total heat transfer to the air when all the water has been pushed out.

Solution





The water on top is compressed liquid and has mass

$$V_{H2O} = V_{tot} - V_{air} = 30 \times 1 - 10 = 20 \text{ ft}^3$$

$$m_{H2O} = V_{H2O}/v_f = 20/0.016051 = 1246 \text{ lbm}$$

Initial air pressure is: $P_1 = P_0 + m_{H2O}g/A = 14.7 + \dfrac{g}{1 \times 144} = 23.353$ psia

and then $\quad m_{air} = \dfrac{PV}{RT} = \dfrac{23.353 \times 10 \times 144}{53.34 \times 540} = 1.1675$ lbm

State 2: $P_2 = P_0 = 14.7$ lbf/in^2, $\quad V_2 = 30 \times 1 = 30$ ft^3

$$_1W_2 = \int PdV = \frac{1}{2}(P_1 + P_2)(V_2 - V_1)$$

$$= \frac{1}{2}(23.353 + 14.7)(30 - 10) \times 144 = 54\,796 \text{ lbf-ft} = 70.43 \text{ Btu}$$

State 2: $P_2, V_2 \;\Rightarrow\; T_2 = \dfrac{T_1P_2V_2}{P_1V_1} = \dfrac{540 \times 14.7 \times 30}{23.353 \times 10} = 1019.7$ R

$$_1Q_2 = m(u_2 - u_1) + {_1W_2} = 1.1675 \times 0.171\,(1019.7 - 540) + 70.43$$

$$= \mathbf{166.2\ Btu}$$

3.277E

A closed rigid container is filled with 3 lbm water at 1 atm, 130 F, 2 lbm of stainless steel and 1 lbm of PVC (polyvinyl chloride) both at 70 F and 0.2 lbm of air at 700 R, 1 atm. It is now left alone with no external heat transfer and no water vaporizes. Find the final temperature and air pressure.

CV. Container.

Process: V = constant => ${}_1W_2 = 0$ and also given ${}_1Q_2 = 0$

Energy Eq.: $U_2 - U_1 = \sum m_i(u_2 - u_1)_i = {}_1Q_2 - {}_1W_2 = 0$

For the liquid and the metal masses we will use the specific heats (Tbl F.3, F.4) so

$$\sum m_i(u_2 - u_1)_i = \sum m_i C_{v\,i} (T_2 - T_1)_i = T_2 \sum m_i C_{v\,i} - \sum m_i C_{v\,i} T_{1\,i}$$

noticing that all masses have the same T_2 but not same initial T.

$$\sum m_i C_{v\,i} = 3 \times 1.0 + 2 \times 0.11 + 1 \times 0.229 + 0.2 \times 0.171 = 3.4832 \text{ Btu/R}$$

The T for air must be converted to F like the others.

Energy Eq.: $3.4832\, T_2 = 3 \times 1.0 \times 130 + (2 \times 0.11 + 1 \times 0.229) \times 70$

$$+ 0.2 \times 0.171 \times (700 - 459.67) = 429.649 \text{ Btu}$$

$$T_2 = \mathbf{123.35\ F}$$

The volume of the air is constant so from PV = mRT it follows that P varies with T

$$P_2 = P_1\, T_2/T_{1\,\text{air}} = 1 \text{ atm} \times (123.35 + 459.67) / 700 = \mathbf{0.833\ atm}$$

3.278E

An engine consists of a 200 lbm cast iron block with a 40 lbm aluminum head, 40 lbm steel parts, 10 lbm engine oil and 12 lbm glycerine (antifreeze). Everything begins at 40 F and as the engine starts, it absorbs a net of 7000 Btu before it reaches a steady uniform temperature. We want to know how hot it becomes.

Energy Eq.: $U_2 - U_1 = {}_1Q_2 - {}_1W_2$

Process: The steel does not change volume and the change for the liquid is minimal, so ${}_1W_2 \cong 0$.

So sum over the various parts of the left hand side in the energy equation

$$m_{Fe}\,(u_2 - u_1) + m_{Al}\,(u_2 - u_1)_{Al} + m_{st}\,(u - u_1)_{st}$$
$$+\, m_{oil}\,(u_2 - u_1)_{oil} + m_{gly}\,(u_2 - u_1)_{gly} = {}_1Q_2$$

Tbl F.2 : $C_{Fe} = 0.1$, $C_{Al} = 0.215$, $C_{st} = 0.11$ all units of Btu/lbm-R

Tbl F.3 : $C_{oil} = 0.46$, $C_{gly} = 0.58$ all units of Btu/lbm-R

So now we factor out $T_2 - T_1$ as $u_2 - u_1 = C(T_2 - T_1)$ for each term

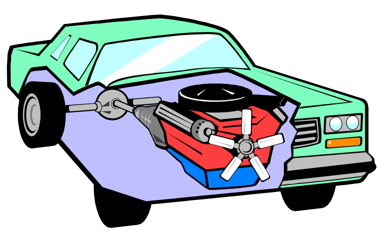
$$[\, m_{Fe}C_{Fe} + m_{Al}C_{Al} + m_{st}C_{st} + m_{oil}C_{oil} + m_{gly}C_{gly}\,]\,(T_2 - T_1) = {}_1Q_2$$

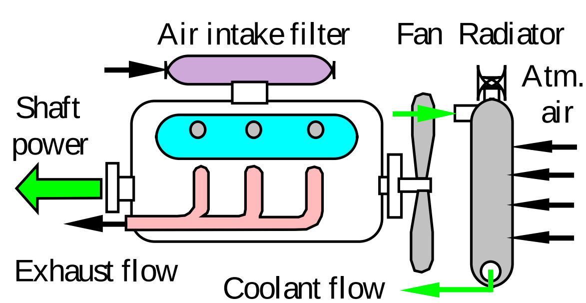
$$T_2 - T_1 = {}_1Q_2 \,/\, \Sigma m_i\, C_i$$

$$= \frac{7000}{200\times 0.1 + 40\times 0.215 + 40\times 0.11 + 10 \times 0.46 + 12 \times 0.58}$$

$$= \frac{7000}{44.56}\ \text{R} = 157\ \text{R}$$

$T_2 = T_1 + 157$ F $= 40 + 157 =$ **197 F**





3.279E

A car with mass 3250 lbm drives with 60 mi/h when the brakes are applied to quickly decrease its speed to 20 mi/h. Assume the brake pads are 1 lbm mass with heat capacity of 0.2 Btu/lbm R and the brake discs/drums are 8 lbm steel where both masses are heated uniformly. Find the temperature increase in the brake assembly.

C.V. Car. Car looses kinetic energy and brake system gains internal u.

No heat transfer (short time) and no work term.

m = constant;

Energy Eq.3.5: $E_2 - E_1 = 0 - 0 = m_{car} \frac{1}{2}(V_2^2 - V_1^2) + m_{brake}(u_2 - u_1)$

The brake system mass is two different kinds so split it, also use C_v since we do not have a u table for steel or brake pad material.

$$m_{steel}\, C_v\, \Delta T + m_{pad}\, C_v\, \Delta T = m_{car} \frac{1}{2}(V_2^2 - V_1^2)$$

$(8\times0.11 + 1\times0.2)$ Btu/R $\times \Delta T$

$= 3250 \times 0.5 \times (3600 - 400) \times 1.46667^2 / (32.174 \times 778)$ Btu

$= 446.9$ Btu

=> $\Delta T =$ **414 F**

3.280E

Water at 60 psia is brought from 320 F to 1800 F. Evaluate the change in specific internal energy using a) the steam tables, b) the ideal gas Table F.6, and the specific heat F.4

Solution:

a)

State 1: Table F.7.3 Superheated vapor $u_1 = 1109.46$ Btu/lbm

State 2: Table F.7.3 $u_2 = 1726.69$ Btu/lbm

$$u_2 - u_1 = 1726.69 - 1109.46 = \mathbf{617.23\ Btu/lbm}$$

b)

Table F.6 at 780 R: $u_1 = 1978.7 - 1.98589 \times 780 = 429.71$ Btu/lbmol

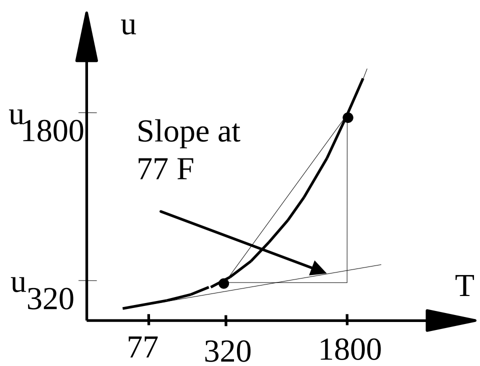
Table F.6 at 2260 R: $u_2 = 15894 - 1.98589 \times 2260 = 11\,406$ Btu/lbmol

$$u_2 - u_1 = (11\,406 - 429.71)/18.013 = \mathbf{609.35\ Btu/lbm}$$

c) Table F.4 : $C_{vo} = 0.337$ Btu/lbm-R

$$u_2 - u_1 = 0.337 \text{ Btu/lbm-R } (1800 - 320) \text{ R} = \mathbf{498.8\ Btu/lbm}$$

Notice how the average slope from 320 F to 1800 F is higher than the one at 77 F (= C_{vo})



3.281E

A 65 gallons rigid tank contains methane gas at 900 R, 200 psia. It is now cooled down to 540 R. Assume ideal gas and find the needed heat transfer.

Solution:

Ideal gas and recall from Table A.1 that 1 gal = 231 in^3,

$$m = P_1V/RT_1 = \frac{200 \text{ psi} \times 65 \text{ gal} \times 231 \text{ in}^3\text{/gal}}{96.35 \text{ lbf-ft/lbm-R} \times 900 \text{ R} \times 12 \text{ in/ft}} = 2.886 \text{ lbm}$$

Process: $V = \text{constant} = V_1 \quad => \quad {}_1W_2 = 0$

Use specific heat from Table F.4

$$u_2 - u_1 = C_v (T_2 – T_1) = 0.415 \text{ Btu/lbm-R } (900 – 540) \text{ R}$$
$$= –149.4 \text{ Btu/lbm}$$

Energy Equation

$${}_1Q_2 = m(u_2 - u_1) = 2.886 (-149.4) = \mathbf{–431.2\ Btu}$$

Polytropic Processes

3.282E

Oxygen at 50 lbf/in.^{2}, 200 F is in a piston/cylinder arrangement with a volume of 4 ft^3. It is now compressed in a polytropic process with exponent, *n* = 1.2, to a final temperature of 400 F. Calculate the heat transfer for the process.

Continuity: $m_2 = m_1$; Energy: $E_2 - E_1 = m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

State 1: T, P and ideal gas, small change in T, so use Table F.4

$$\Rightarrow \quad m = \frac{P_1V_1}{RT_1} = \frac{50 \text{ psi} \times 4 \text{ ft}^3 \times 144 \text{ in}^2/\text{ft}^2}{48.28 \text{ lbf-ft/lbm-R} \times 659.67 \text{ R}} = 0.9043 \text{ lbm}$$

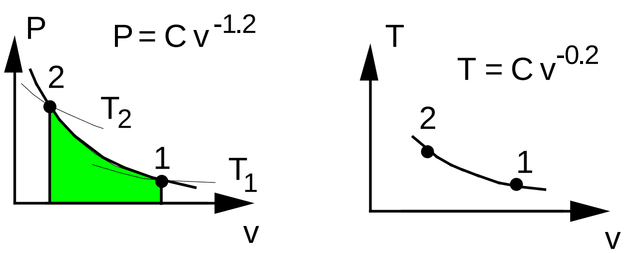
Process: PV^n = constant

$${}_1W_2 = \frac{1}{1-n}(P_2V_2 - P_1V_1) = \frac{mR}{1-n}(T_2 - T_1) = \frac{0.9043 \times 48.28}{1 - 1.2} \times \frac{400 - 200}{778}$$

$$= -\ 56.12 \text{ Btu}$$

$${}_1Q_2 = m(u_2 - u_1) + {}_1W_2 \cong mC_v(T_2 - T_1) + {}_1W_2$$

$$= 0.9043 \times 0.158\ (400 - 200) - 56.12 = \mathbf{-\ 27.54\ Btu}$$



3.283E

An air pistol contains compressed air in a small cylinder, as shown in Fig. P3.164. Assume that the volume is 1 in.^{3}, pressure is 10 atm, and the temperature is 80 F when armed. A bullet, m = 0.04 lbm, acts as a piston initially held by a pin (trigger); when released, the air expands in an isothermal process (T = constant). If the air pressure is 1 atm in the cylinder as the bullet leaves the gun, find

a. The final volume and the mass of air.

b. The work done by the air and work done on the atmosphere.

c. The work to the bullet and the bullet exit velocity.

C.V. Air. Air ideal gas:

$$m_{air} = \frac{P_1V_1}{RT_1} = \frac{10 \text{ atm} \times 14.7 \text{ psi/atm} \times 1 \text{ in}^3}{53.34 \text{ lbf-ft/lbm-R} \times 539.67 \text{ R} \times 12 \text{ in/ft}} = \mathbf{4.26{\times}10^{-5} \text{ lbm}}$$

Process: $PV = \text{const} = P_1V_1 = P_2V_2 \Rightarrow V_2 = V_1P_1/P_2 = \mathbf{10 \text{ in}^3}$

$${}_1W_2 = \int PdV = P_1V_1 \int (1/V)\, dV = P_1V_1 \ln(V_2/V_1)$$

$$= 147 \text{ psi} \times 1 \text{ in}^3 \times \ln(10) = 28.2 \text{ lbf-ft} = \mathbf{0.0362 \text{ Btu}}$$

$${}_1W_{2,ATM} = P_0(V_2 - V_1) = 14.7 \text{ psi} \times 9 \text{ in}^3 = 11.03 \text{ lbf-ft} = \mathbf{0.0142 \text{ Btu}}$$

$$W_{bullet} = {}_1W_2 - {}_1W_{2,ATM} = 0.022 \text{ Btu} = \frac{1}{2} m_{bullet}(\mathbf{V}_{ex})^2$$

$$\mathbf{V}_{ex} = (2W_{bullet}/m_B)^{1/2} = (2{\times}0.022{\times}778{\times}32.174 / 0.04)^{1/2} = \mathbf{165.9 \text{ ft/s}}$$

3.284E

Helium gas expands from 20 psia, 600 R and 9 ft^3 to 15 psia in a polytropic process with n = 1.667. How much heat transfer is involved?

Solution:

C.V. Helium gas, this is a control mass.

Energy equation: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Process equation: $PV^n = \text{constant} = P_1V_1^n = P_2V_2^n$

Ideal gas (F.4): $m = PV/RT = \dfrac{20 \text{ psi} \times 9 \text{ ft}^3 \times 144 \text{ in}^2/\text{ft}^2}{386 \text{ lbf-ft/lbm-R} \times 600 \text{ R}} = 0.112 \text{ lbm}$

Solve for the volume at state 2

$$V_2 = V_1 (P_1/P_2)^{1/n} = 9 \times \left(\frac{20}{15}\right)^{0.6} = 10.696 \text{ ft}^3$$

$$T_2 = T_1 P_2V_2/(P_1V_1) = 600 \text{ R} \frac{15 \times 10.696}{20 \times 9} = 534.8 \text{ R}$$

Work from Eq.3.21

$$_1W_2 = \frac{P_2V_2 - P_1 V_1}{1-n} = \frac{15 \times 10.696 - 20 \times 9}{1 - 1.667} \text{ psi ft}^3 = 29.33 \text{ psia ft}^3$$
$$= 4223 \text{ lbf-ft} = 5.43 \text{ Btu}$$

Use specific heat from Table F.4 to evaluate $u_2 - u_1$**,** $C_v = 0.744$ Btu/lbm R

$$_1Q_2 = m(u_2 - u_1) + {}_1W_2 = m\, C_v (T_2 - T_1) + {}_1W_2$$
$$= 0.112 \text{ lbm} \times 0.744 \text{ Btu/lbm-R} \times (534.8 - 600) \text{ R} + 5.43 \text{ Btu}$$
$$= \mathbf{-0.003\ Btu}$$

More Complex Devices

3.285E

Water in a tank A is at 270 F with a quality of 10% and mass 1 lbm. It is connected to a piston cylinder holding constant pressure of 40 psia initially with 1 lbm water at 700 F. The valve is opened and enough heat transfer takes place to have a final uniform temperature of 280 F. Find the final P and V, the process work and the process heat transfer.

Solution:

C.V. Water in A and B. Control mass goes through process: 1 -> 2

Continuity Eq.: $m_2 - m_{A1} - m_{B1} = 0 \quad \Rightarrow \quad m_2 = m_{A1} + m_{B1} = 1.0 + 1.0 = 2$ lbm

Energy Eq.: $U_2 - U_1 = {}_1Q_2 - {}_1W_2$

State A1: $v_{A1} = 0.01717 + x_{A1} \times 10.0483 = 1.022$; $V_{A1} = mv = 1.022$ ft^3

$u_{A1} = 238.81 + 0.1 \times 854.14 = 324.22$ Btu/lbm

State B1: $v_{B1} = 17.196$ ft^3/lbm; $u_{B1} = 1255.14$ Btu/lbm

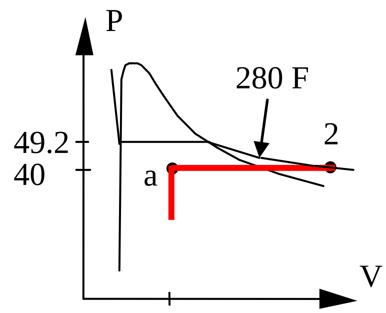
$\Rightarrow \quad V_{B1} = m_{B1}v_{B1} = 17.196$ ft^3

State 2: If $V_2 > V_{A1}$ then $P_2 =$ **40 psia** that is the piston floats.

For $(T_2, P_2) = (280$ F, 40 psia$)$ => superheated vapor $u_2 = 1097.31$ Btu/lbm

$v_2 = 10.711$ ft^3/lbm $\quad V_2 = m_2v_2 =$ **21.422 ft^3** $> V_{A1}$ checks OK.

The possible state 2 (P,V) combinations are shown. State a is 40 psia, $v_a = V_{A1}/m_2 = 0.511$ and thus two-phase $T_a = 267.3$ F less than T_2



Process: ${}_1W_2 = P_2 (V_2 - V_1) = 40 (21.422 - 1.022 - 17.196)\frac{144}{778} =$ **23.72 Btu**

From the energy Eq.:

$${}_1Q_2 = m_2u_2 - m_{A1}u_{A1} - m_{B1}u_{B1} + {}_1W_2$$

$$= 2 \times 1097.31 - 1.0 \times 324.22 - 1.0 \times 1255.14 + 23.72$$

$$= \mathbf{638.98\ Btu}$$

3.286E

Two rigid tanks are filled with water as shown in Fig. P.3.214. Tank A is 7 ft^3 at 1 atm, 280 F and tank B is 11 ft^3 at saturated vapor 40 psia. The tanks are connected by a pipe with a closed valve. We open the valve and let all the water come to a single uniform state while we transfer enough heat to have a final pressure of 40 psia. Give the two property values that determine the final state and find the heat transfer.

Solution:

State A1: $u = 1102.4$ Btu/lbm, $v = 29.687\ ft^3/lbm$

$\Rightarrow\ m_{A1} = V/v = 7.0/29.687 = 0.236$ lbm

State B1: $u = 1092.3$ Btu/lbm, $v = 19.501\ ft^3/lbm$

$\Rightarrow\ m_{B1} = V/v = 11.0 / 19.501 = 0.564$ lbm

The total volume (and mass) is the sum of volumes (mass) for tanks A and B.

$$m_2 = m_{A1} + m_{B1} = 0.236 + 0.564 = 0.800 \text{ lbm},$$

$$V_2 = V_{A1} + V_{B1} = 7.0 + 11.0 = 18.0 \text{ ft}^3$$

$$\Rightarrow v_2 = V_2/m_2 = 18.0 /0.800 = \mathbf{22.5\ ft^3/lbm}$$

State 2: $[P_2, v_2] = [40 \text{ psia}, 22.5 \text{ ft}^3/\text{lbm}]$

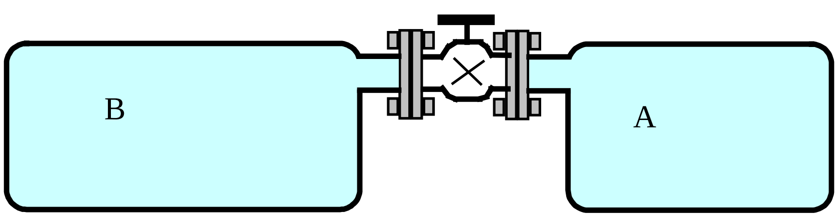
$$\Rightarrow\ T_2 = 1053.5 \text{ F} \ \text{ and } \ u_2 = 1395.1 \text{ Btu/lbm}$$

The energy equation is (neglecting kinetic and potential energy)

$$m_2 u_2 - m_A u_{A1} - m_B u_{B1} = {}_1Q_2 - {}_1W_2 = {}_1Q_2$$

$${}_1Q_2 = 0.800 \times 1395.1 - 0.236 \times 1102.4 - 0.564 \times 1092.3$$

$$= \mathbf{239.9\ Btu}$$



3.287E

A vertical cylinder fitted with a piston contains 10 lbm of R-410A at 50 F, shown in Fig. P3.173. Heat is transferred to the system causing the piston to rise until it reaches a set of stops at which point the volume has doubled. Additional heat is transferred until the temperature inside reaches 120 F, at which point the pressure inside the cylinder is 200 lbf/in.^{2}.

a. What is the quality at the initial state?

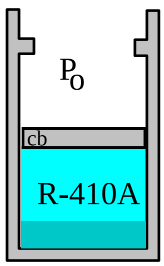
b. Calculate the heat transfer for the overall process.

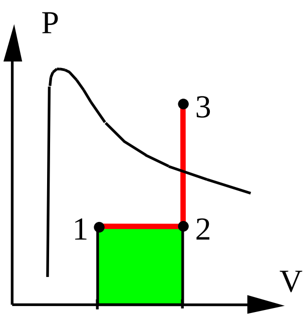
Solution:

C.V. R-410A. Control mass goes through process: 1 -> 2 -> 3

As piston floats pressure is constant (1 -> 2) and the volume is constant for the second part (2 -> 3). So we have: $v_3 = v_2 = 2 \times v_1$

State 3: Table F.9.2 (P,T) $v_3 = 0.3652$ ft^3/lbm, $u_3 = 123.5$ Btu/lbm





So we can determine state 1 and 2 Table F.9.1:

$$v_1 = 0.1826 = 0.01420 + x_1(0.3636) \quad => \quad x_1 = \mathbf{0.463}$$

$$u_1 = 31.06 + 0.463 \times 78.96 = 67.6 \text{ Btu/lbm}$$

State 2: $v_2 = 0.3652$ ft^3/lbm, $P_2 = P_1 = 157.5$ psia, this is still 2-phase.

We get the work from the process equation (see P-V diagram)

$$_1W_3 = {}_1W_2 = \int_1^2 PdV = P_1(V_2 - V_1)$$

$$= 157.5 \text{ psia} \times 10 \text{ lbm } (0.3652 - 0.1826) \text{ ft}^3/\text{lbm} \times 144 \text{ (in/ft)}^2$$

$$= 41\,413.7 \text{ lbf-ft} = 53.2 \text{ Btu}$$

The heat transfer from the energy equation becomes

$$_1Q_3 = m(u_3 - u_1) + {}_1W_3 = 10(123.5 - 67.6) + 53.2 = \mathbf{612.2\ Btu}$$

3.288E

A piston cylinder shown in Fig. P3.169 contains 18 ft^3 of R-410A at 300 psia, 300 F. The piston mass and atmosphere gives a pressure of 70 psia that will float the piston. The whole setup cools in a freezer maintained at 0 F. Find the heat transfer and show the P-v diagram for the process when T_2 = 0 F.

Solution:

C.V.: R-410A. Control mass.

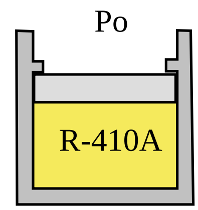
Continuity: m = constant,

Energy Eq.3.5: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Process: $F\downarrow = F\uparrow = P\,A = P_{air}A + F_{stop}$

if $V < V_{stop} \Rightarrow F_{stop} = \emptyset$

This is illustrated in the P-v diagram shown below.



State 1: $v_1 = 0.3460\ ft^3/lbm$, $u_1 = 159.95$ Btu/lbm

$\Rightarrow$ m = V/v = 52.023 lbm

State 2: T_2 and on line $\Rightarrow$ compressed liquid, see figure below.

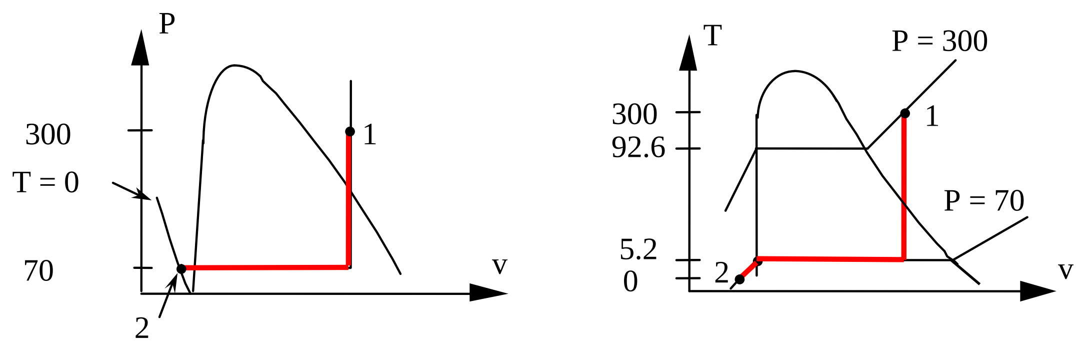
$v_2 \cong v_f = 0.01295\ ft^3/lbm \Rightarrow V_2 = 0.674\ ft^3$; $u_2 = u_f = 13.37$ Btu/lbm

$${}_1W_2 = \int PdV = P_{lift}(V_2 - V_1) = 70\ psi\ (0.674 - 18)\ ft^3 \times 144\ in^2/ft^2$$

$$= -\ 174\ 646\ lbf\text{-}ft = -224.5\ Btu$$

Energy eq. $\Rightarrow$

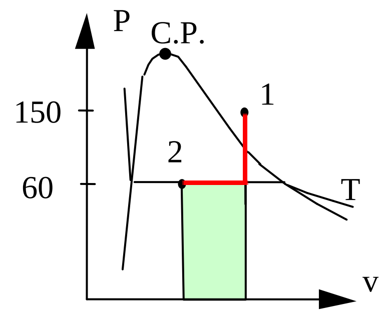
$${}_1Q_2 = 52.023\ (13.37 - 159.95) - 224.5 = \mathbf{-7850\ Btu}$$



3.289E

A setup as in Fig. P3.169 has the R-410A initially at 150 psia, 120 F of mass 0.2 lbm. The balancing equilibrium pressure is 60 psia and it is now cooled so the volume is reduced to half the starting volume. Find the heat transfer for the process.

Solution:



Take as CV the 0.2 lbm of R-410A.

Continuity Eq.: $m_2 = m_1 = m$;

Energy Eq.3.5 $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Process Eq.: $P = P_{float}$ or $v = C = v_1$,

State 1: (P, T) => $v_1 = 0.5099\ ft^3/lbm$

$u_1 = 125.28$ Btu/lbm

State 2: (P, v) => $v_2 = v_1/2 = 0.2550\ ft^3/lbm < v_g$, so it is two-phase.

$x_2 = (v_2 - v_f)/v_{fg} = (0.255 - 0.0129)/0.9911 = 0.2443$

$u_2 = u_f + x_2\, u_{fg} = 12.57 + x_2\, 93.33 = 35.37$ Btu/lbm

From process eq.: ${}_1W_2 = \int P\, dV = \text{area} = mP_2 (v_2 - v_1)$

$= 0.2\ lbm \times 60\ psi\ (0.255 - 0.5099)\ ft^3/lbm \times 144\ in^2/ft^2$

$= 440.5$ lbf-ft = - 0.57 Btu

From energy eq.: ${}_1Q_2 = m(u_2 - u_1) + {}_1W_2 = 0.2\times (35.37 - 125.28) - 0.57$

$=$ **-18.55 Btu**

3.290E

A piston cylinder contains air at 150 psia, 1400 R with a volume of 1.75 ft^3. The piston is pressed against the upper stops, see Fig. P3.14-c and it will float at a pressure of 110 psia. Now the air is cooled to 700 R. What is the process work and heat transfer?

CV. Air, this is a control mass

Energy Eq.3.5 $\quad m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Process Eq.: $\quad P = P_{float} \quad$ or $\quad v = C = v_1$,

State 1: u = 247.04 Btu/lbm,

$$m = PV/RT = 150 \text{ psi} \times 1.75 \text{ ft}^3/(53.34 \text{ lbf-ft/lbm-R} \times 1400 \text{ R}) = 0.506 \text{ lbm}$$

We need to find state 2. Let us see if we proceed past state 1a during the cooling.

$$T_{1a} = T_1 P_{float} / P_1 = 1400 \text{ R} \times 110 / 150 = 1026.67 \text{ R}$$

so we do cool below T_{1a}. That means the piston is floating. Write the ideal gas law for state 1 and 2 to get

$$V_2 = \frac{mRT_2}{P_2} = \frac{P_1V_1T_2}{P_2T_1} = \frac{150 \times 1.75 \times 700}{110 \times 1400} = 1.1932 \text{ ft}^3$$

$${}_1W_2 = {}_{1a}W_2 = \int P\, dV = P_2 (V_2 - V_1)$$

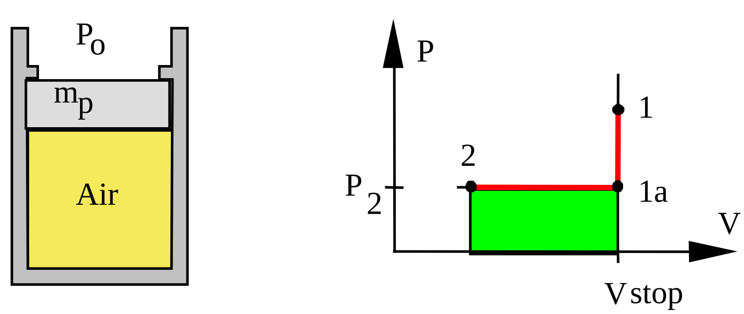
$$= 110 \text{ psia} \times (1.1932 - 1.75) \text{ ft}^3 = \mathbf{-8819.7 \text{ lbf-ft} = -11.34 \text{ Btu}}$$

From the energy equation

$${}_1Q_2 = m(u_2 - u_1) + {}_1W_2$$

$$= 0.506 \text{ lbm} \times (119.7 - 247.04) \text{ Btu/lbm} - 11.34 \text{ Btu}$$

$$= \mathbf{-75.8 \text{ kJ}}$$



3.291E

A mass of 6 lbm nitrogen gas at 3600 R, V = C, cools with 1 Btu/s. What is dT/dt?

Process: V = C $\rightarrow$ ${}_1W_2 = 0$

$$\frac{dE}{dt} = \frac{dU}{dt} = m\frac{dU}{dt} = mC_v\frac{dT}{dt} = \dot{Q} - W = \dot{Q} = -1 \text{ Btu/s}$$

$$C_{p\ 3600} = \frac{dh}{dT} = \frac{\Delta h}{\Delta T} = \frac{h_{3800} - h_{3400}}{3800-3400} = \frac{25857 - 22421}{400 \times 28.013} = 0.3066 \text{ Btu/lbm-R}$$

$$C_{v\ 3600} = C_{p\ 3600} - R = 0.3066 - 55.15 / 778 = 0.2357 \text{ Btu/lbm-R}$$

$$\frac{dT}{dt} = \frac{\dot{Q}}{mC_v} = \frac{-1 \text{ Btu/s}}{6 \times 0.2357 \text{ Btu/R}} = \mathbf{-0.71\ \frac{R}{s}}$$

Remark: Specific heat from Table F.4 has $C_{v\ 300} = 0.178$ Btu/lbm-R which is nearly 25% lower and thus would over-estimate the rate with 25%.

3.292E

A crane use 7000 Btu/h to raise a 200 lbm box 60 ft. How much time does it take?

$$\text{Power} = \dot{W} = F\mathbf{V} = mg\mathbf{V} = mg\frac{L}{t}$$

$$F = mg = 200\,\frac{32.174}{32.174}\ \text{lbf} = 200\ \text{lbf}$$

$$t = \frac{FL}{\dot{W}} = \frac{200\ \text{lbf} \times 60\ \text{ft}}{7000\ \text{Btu/h}} = \frac{200 \times 60 \times 3600}{7000 \times 778.17}\ \text{s}$$

$$= \mathbf{7.9\ s}$$

Recall Eq. on page 9: 1 lbf = 32.174 lbm ft/s^2, 1 Btu = 778.17 lbf-ft (A.1)

3.293E

A computer in a closed room of volume 5000 ft^3 dissipates energy at a rate of 10 kW. The room has 100 lbm of wood, 50 lbm of steel and air, with all material at 540 R, 1 atm. Assuming all the mass heats up uniformly how long time will it take to increase the temperature 20 F?

C.V. Air, wood and steel. $m_2 = m_1$; no work

Energy Eq.3.5: $U_2 - U_1 = {}_1Q_2 = \dot{Q}\Delta t$

The total volume is nearly all air, but we can find volume of the solids.

$$V_{wood} = m/\rho = 100/44.9 = 2.23 \text{ ft}^3 \text{ ; } V_{steel} = 50/488 = 0.102 \text{ ft}^3$$

$$V_{air} = 5000 - 2.23 - 0.102 = 4997.7 \text{ ft}^3$$

$$m_{air} = PV/RT = 14.7 \times 4997.7 \times 144/(53.34 \times 540) = 367.3 \text{ lbm}$$

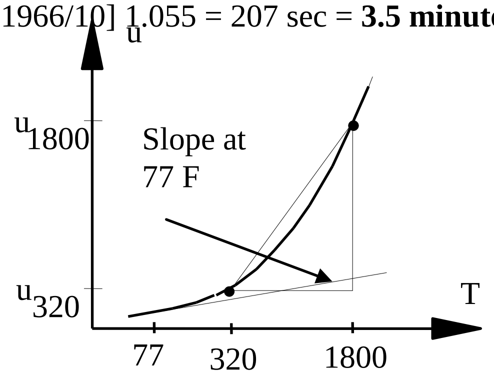
We do not have a u table for steel or wood so use specific heat.

$$\Delta U = [m_{air}\, C_v + m_{wood}\, C_v + m_{steel}\, C_v\,]\, \Delta T$$

$$= (367.3 \times 0.171 + 100 \times 0.3 + 50 \times 0.11) \text{ Btu/F} \times 20 \text{ F}$$

$$= 1256.2 + 600 + 110 = 1966 \text{ Btu} = \dot{Q} \times \Delta t = (10/1.055) \times \Delta t$$

=> $\Delta t = [1966/10]\ 1.055 = 207$ sec = **3.5 minutes**



3.294E

Water is in a piston cylinder maintaining constant P at 330 F, quality 90% with a volume of 4 ft^3. A heater is turned on heating the water with 10 000 Btu/h. What is the elapsed time to vaporize all the liquid?

Solution:

Control volume water.

Continuity Eq.: $m_{tot} = \text{constant} = m_{vapor} + m_{liq}$

on a rate form: $\dot{m}_{tot} = 0 = \dot{m}_{vapor} + \dot{m}_{liq} \quad \Rightarrow \quad \dot{m}_{liq} = -\dot{m}_{vapor}$

Energy equation: $\dot{U} = \dot{Q} - \dot{W} = \dot{m}_{vapor}\, u_{fg} = \dot{Q} - P\, \dot{m}_{vapor}\, v_{fg}$

Rearrange to solve for $\dot{m}_{vapor}$

$$\dot{m}_{vapor}\,(u_{fg} + Pv_{fg}) = \dot{m}_{vapor}\, h_{fg} = \dot{Q}$$

From table F.7.1

$h_{fg} = 887.5$ Bt/lbm, $v_1 = 0.01776 + 0.9\ 4.2938 = 3.8822$ ft^3/lbm

$m_1 = V_1/v_1 = 4/3.8822 = 1.0303$ lbm, $m_{liq} = (1-x_1)m_1 = 0.10303$ lbm

$$\dot{m}_{vapor} = \dot{Q}/h_{fg} = \frac{10\ 000}{887.5}\ \frac{\text{Btu/h}}{\text{Btu/lbm}} = 11.2676 \text{ lbm/h} = 0.00313 \text{ lbm/s}$$

$$\Delta t = m_{liq} / \dot{m}_{vapor} = 0.10303 / 0.00313 = \mathbf{32.9\ s}$$

3.295E

A piston/cylinder has 2 lbm of R-134a at state 1 with 200 F, 90 lbf/in.^{2}, and is then brought to saturated vapor, state 2, by cooling while the piston is locked with a pin. Now the piston is balanced with an additional constant force and the pin is removed. The cooling continues to a state 3 where the R-134a is saturated liquid. Show the processes in a P-V diagram and find the work and heat transfer in each of the two steps, 1 to 2 and 2 to 3.

Solution :

C.V. R-134a   This is a control mass. Properties from table F.10.1 and F.10.2

State 1: (T,P)   => $v = 0.7239$ ft^3/lbm, $u = 194.605$ Btu/lbm

State 2 given by fixed volume and $x_2 = 1.0$, $v_2 = v_1 = v_g$ => $\mathbf{{}_1W_2 = 0}$

$$T_2 = 50 + 10 \times \frac{0.7239 - 0.7921}{0.6632 - 0.7921} = 55.3 \text{ F},$$

$$P_2 = 60.311 + (72.271 - 60.311) \times 0.5291 = 66.64 \text{ psia}$$

$$u_2 = 164.95 + (166.28 - 164.95) \times 0.5291 = 165.65 \text{ Btu/lbm}$$

From the energy equation $_1Q_2 = m(u_2 - u_1) + {}_1W_2 = m(u_2 - u_1)$

$= 2\,(165.65 - 194.605) = \mathbf{-57.91\ Btu}$

State 3 reached at constant P (F = constant) state 3: $P_3 = P_2$ and

$$v_3 = v_f = 0.01271 + (0.01291 - 0.01271) \times 0.5291 = 0.01282 \text{ ft}^3\text{/lbm}$$

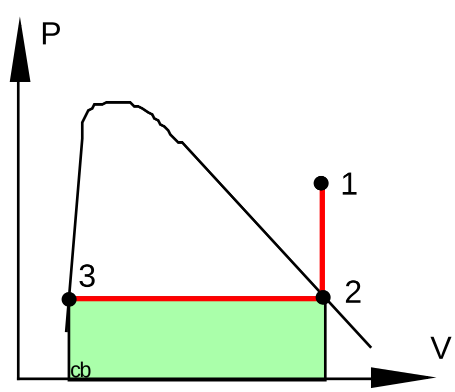
$$u_3 = u_f = 91.68 + (94.95 - 91.68) \times 0.5291 = 93.41 \text{ Btu/lbm}$$

$$_1W_3 = {}_1W_2 + {}_2W_3 = 0 + {}_2W_3 = \int P\, dV = P(V_3 - V_2) = mP(v_3 - v_2)$$

$$= 2\times 66.64\,(0.01282 - 0.7239)\,\frac{144}{778} = \mathbf{-\,17.54\ Btu}$$

From the energy equation

$$_2Q_3 = m(u_3 - u_2) + {}_2W_3 = 2(93.41 - 165.65) - 17.54 = \mathbf{-162.02\ Btu}$$



Rates of Work

3.296E

A force of 300 lbf moves a truck with 40 mi/h up a hill. What is the power?

Solution:

$$\dot{W} = F\,\mathbf{V} = 300\text{ lbf} \times 40\text{ (mi/h)}$$
$$= 12\,000 \times \frac{1609.3 \times 3.28084}{3600}\ \frac{\text{lbf-ft}}{\text{s}}$$
$$= 17\,600\ \frac{\text{lbf-ft}}{\text{s}} = \mathbf{22.62\ Btu/s}$$

Heat Transfer Rates

3.297E

Find the rate of conduction heat transfer through a 1.5 cm thick hardwood board, k = 0.09 Btu/h-ft-R, with a temperature difference between the two sides of 40 F.

One dimensional heat transfer by conduction, we do not know the area so we can find the flux (heat transfer per unit area Btu/ft^{2}h).

$$t = 1.5 \text{ cm} = 0.59 \text{ in.} = 0.0492 \text{ ft}$$

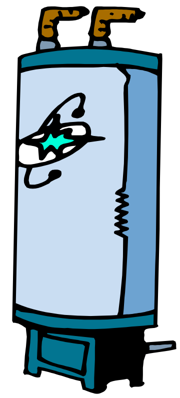
$$\dot{q} = \dot{Q}/A = k\ \frac{\Delta T}{\Delta x} = 0.09 \frac{\text{Btu}}{\text{h-ft-R}} \times \frac{40}{0.0492} \frac{\text{R}}{\text{ft}} = \mathbf{73.2\ Btu/ft^2\text{-}h}$$

3.298E

A water-heater is covered up with insulation boards over a total surface area of 30 ft^2. The inside board surface is at 175 F and the outside surface is at 70 F and the board material has a conductivity of 0.05 Btu/h ft F. How thick a board should it be to limit the heat transfer loss to 720 Btu/h ?

Solution:

Steady state conduction through a single layer board.



$$\dot{Q}_{cond} = k\ A\ \frac{\Delta T}{\Delta x} \quad \Rightarrow \quad \Delta x = k\ A\ \Delta T/\dot{Q}$$

$$\Delta x = 0.05 \frac{\text{Btu}}{\text{h-ft-R}} \times 30\ \text{ft}^2\ (175\text{-}70)\ \text{R}/\ 720\ \text{Btu/h}$$

$$= 0.219\ \text{ft} = \mathbf{2.6\ in}$$

3.299E

The sun shines on a 1500 ft^2 road surface so it is at 115 F. Below the 2 inch thick asphalt, average conductivity of 0.035 Btu/h-ft-F, is a layer of compacted rubbles at a temperature of 60 F. Find the rate of heat transfer to the rubbles.

Solution:

$$\dot{Q} = k\ A\ \frac{\Delta T}{\Delta x}$$

$$= 0.035 \frac{\text{Btu}}{\text{h-ft-R}} \times 1500\ \ \text{ft}^2 \times \frac{115 - 60}{2/12} \frac{\text{R}}{\text{ft}}$$

$$= \mathbf{17325\ Btu/h}$$

Review Problems

3.300E

A cylinder having an initial volume of 100 ft^3 contains 0.2 lbm of water at 100 F. The water is then compressed in an isothermal quasi-equilibrium process until it has a quality of 50%. Calculate the work done in the process assuming water vapor is an ideal gas.

Solution:

State 1: T_1, $v_1 = V/m = \frac{100}{0.2} = 500\ ft^3/lbm \quad (> v_g)$

since P_g = 0.95 psia, very low so water is an ideal gas from 1 to 2.

$$P_1 = P_g \times \frac{v_g}{v_1} = 0.950 \times \frac{350}{500} = 0.6652\ lbf/in^2$$

$$V_2 = mv_2 = 0.2\ lbm *350\ ft^3/lbm = 70\ ft^3$$

$$v_3 = 0.01613 + 0.5\times(350 - 0.01613) = 175.0\ ft^3/lbm$$

For ideal gas and constant T the work term follows Eq. 3.21

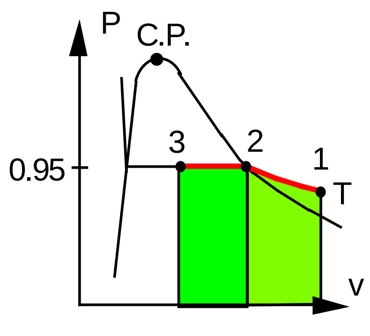
$$_1W_2 = \int PdV = P_1V_1 \ln \frac{V_2}{V_1} = 0.6652 \times \frac{144}{778} \times 100 \ln \frac{70}{100} = -4.33\ Btu$$

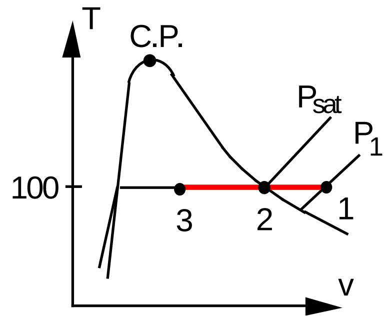
For the constant pressure part of the process the work becomes

$$_2W_3 = P_2\, m(v_3 - v_2) = 0.95\ psi \times 0.2\ lbm \times(175 - 350)\ ft^3/lbm \times 144\ in^2/ft^2$$

$$= -4788\ lbf\text{-}ft = -6.15\ Btu$$

$$_1W_3 = -\ 6.15 - 4.33 = \mathbf{-10.48\ Btu}$$





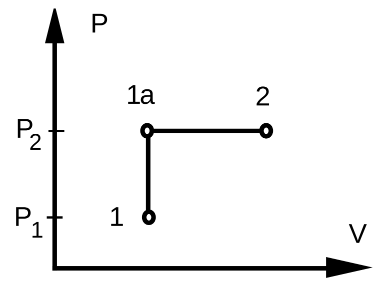
3.301E

A piston/cylinder contains 2 lbm of water at 70 F with a volume of 0.1 ft^3, shown in Fig. P3.225. Initially the piston rests on some stops with the top surface open to the atmosphere, *P*o, so a pressure of 40 lbf/in.^{2} is required to lift it. To what temperature should the water be heated to lift the piston? If it is heated to saturated vapor find the final temperature, volume, and the heat transfer.

Solution:

C.V. Water. This is a control mass.

$$m_2 = m_1 = m \; ; \quad m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$$



State 1: 20 C, $v_1 = V/m = 0.1/2 = 0.05$ ft^3/lbm

$x = (0.05 - 0.01605)/867.579 = 0.0003913$

$u_1 = 38.09 + 0.0003913 \times 995.64 = 38.13$ Btu/lbm

To find state 2 check on state 1a:

P = 40 psia, $v = v_1 = 0.05$ ft^3/lbm

Table F.7.1: $v_f < v < v_g = 10.501$, $x_{1a} > 0$

State 2 is saturated vapor at 40 psia as state 1a is two-phase. $T_2 = 267.3$ F

$v_2 = vg = 10.501$ ft^3/lbm , $V_2 = m\, v_2 = 21.0$ ft^3, $u_2 = ug = 1092.27$ Btu/lbm

Pressure is constant as volume increase beyond initial volume.

$${}_1W_2 = \int P\, dV = P_{lift}\,(V_2 - V_1) = 40\,(21.0 - 0.1) \times 144 / 778 = 154.75 \text{ Btu}$$

$${}_1Q_2 = m(u_2 - u_1) + {}_1W_2 = 2\,(1092.27 - 38.13) + 154.75 = 2263 \text{ Btu}$$

3.302E

A twenty pound-mass of water in a piston/cylinder with constant pressure is at 1100 F and a volume of 22.6 ft^3. It is now cooled to 100 F. Show the *P–v* diagram and find the work and heat transfer for the process.

Solution:

C.V. Water

Energy Eq.3.5: $\quad {}_1Q_2 = m(u_2 – u_1) + {}_1W_2 = m(h_2 - h_1)$

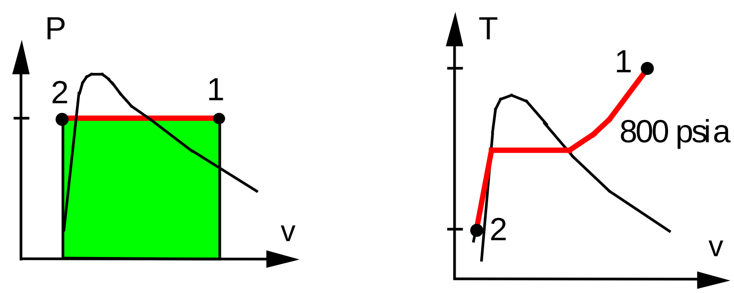
Process Eq.: Constant pressure $\Rightarrow \quad {}_1W_2 = mP(v_2 - v_1)$

Properties from Table F.7.2 and F.7.3

State 1: T_1, $v_1 = 22.6/20 = 1.13$ ft^3/lbm, $P_1 = 800$ lbf/in^2 , $h_1 = 1567.8$ Btu/lbm

State 2: 800 lbf/in^2, 100 F

$\Rightarrow \quad v_2 = 0.016092$ ft^3/lbm, $h_2 = 70.15$ Btu/lbm



The work from the process equation is found as

$${}_1W_2 = 20 \text{ lbm} \times 800 \text{ psi} \times (0.016092 - 1.13) \text{ ft}^3\text{/lbm} \times 144 \text{ in}^2\text{/ft}^2$$

$$= -2\,566\,444 \text{ lbf-ft} = \mathbf{-3299\ Btu}$$

The heat transfer from the energy equation is

$${}_1Q_2 = 20 \text{ lbm} \times (70.15 - 1567.8) \text{ Btu/lbm} = \mathbf{-29\,953\ Btu}$$

3.303E

A cylinder fitted with a frictionless piston contains R-134a at 100 F, 80% quality, at which point the volume is 3 Gal. The external force on the piston is now varied in such a manner that the R-134a slowly expands in a polytropic process to 50 lbf/in.^{2}, 80 F. Calculate the work and the heat transfer for this process.

Solution:

C.V. The mass of R-134a. Properties in Table F.10.1

$$v_1 = v_f + x_1 v_{fg} = 0.01387 + 0.8 \times 0.3278 = 0.2761 \text{ ft}^3/\text{lbm}$$

$$u_1 = 108.51 + 0.8 \times 62.77 = 158.73 \text{ Btu/lbm}; \quad P_1 = 138.926 \text{ psia}$$

$$m = V/v_1 = 3 \times 231 \times 12^{-3} / 0.2761 = 0.401/ 0.2761 = 1.4525 \text{ lbm}$$

State 2: $v_2 = 1.1035 \text{ ft}^3/\text{lbm}$ (sup.vap.); $u_2 = 171.32$ Btu/lbm

(linear interpolation is not so accurate as v is more like ~1/P)

Process: $$n = \ln \frac{P_1}{P_2} / \ln \frac{V_2}{V_1} = \ln \frac{138.926}{50} / \ln \frac{1.1035}{0.2761} = 0.7376$$

$${}_1W_2 = \int P\, dV = \frac{P_2 V_2 - P_1 V_1}{1- n}$$

$$= \frac{50 \times 1.1035 - 138.926 \times 0.2761}{1 - 0.7376} \times 1.4525 \times \frac{144}{778} = \mathbf{17.23\ Btu}$$

$${}_1Q_2 = m(u_2 - u_1) + {}_1W_2 = 1.4525\,(171.32 - 158.73) + 17.23 = \mathbf{35.5\ Btu}$$

3.304E

Ammonia, NH_3, is contained in a sealed rigid tank at 30 F, x = 50% and is then heated to 200 F. Find the final state P_2, u_2 and the specific work and heat transfer.

Solution:

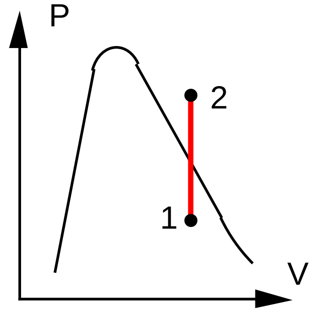
Continuity Eq.: $m_2 = m_1$;

Energy Eq.3.5: $E_2 - E_1 = {}_1Q_2$; $\left({}_1W_2 = \emptyset\right)$

Process: $V_2 = V_1 \Rightarrow v_2 = v_1 = 0.02502 + 0.5 \times 4.7945 = 2.422\ \text{ft}^3/\text{lbm}$

State 1: Table F.8.1, $u_1 = 75.06 + 0.5 \times 491.17 = 320.65$ Btu/lbm

Table F.8.2: v_2 & $T_2 \Rightarrow$ between 150 psia and 175 psia



$P_2 = 163\ \text{lbf/in}^2$, $u_2 = 633.5$ Btu/lbm

linear interpolation

Process equation gives no displacement: ${}_1w_2 = \mathbf{0}$;

The energy equation then gives the heat transfer as

$${}_1q_2 = u_2 - u_1 = 633.5 - 320.65 = \mathbf{312.85\ Btu/lbm}$$

3.305E

Water in a piston/cylinder, similar to Fig. P3.225, is at 212 F, x = 0.5 with mass 1 lbm and the piston rests on the stops. The equilibrium pressure that will float the piston is 40 psia. The water is heated to 500 F by an electrical heater. At what temperature would all the liquid be gone? Find the final (P,v), the work and heat transfer in the process.

C.V. The 1 lbm water.

Continuty: $m_2 = m_1 = m$; Energy: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Process: V = constant if $P < P_{lift}$ otherwise $P = P_{lift}$ see P-v diagram.

State 1: (T,x) Table F.7.1

$$v_1 = 0.01672 + 0.5 \times 26.7864 = 13.4099 \text{ ft}^3\text{/lbm}$$

$$u_1 = 180.09 + 0.5 \times 897.51 = 628.845 \text{ Btu/lbm}$$

State 1a: (40 psia, $v = v_1 > v_{g\ 40\ psia} = 10.501$ ft^3/lbm) so superheated vapor

Piston starts to move at state 1a, ${}_1W_{1a} = 0$,

State 1b: reached before state 1a so $v = v_1 = v_g$ see this in F.7.1

$$T_{1b} = 250 + 10\ (13.40992 – 13.8247)/(11.7674 – 13.8247) = \mathbf{252\ F}$$

State 2: $(T_2 > T_{1a})$ Table F.7.2 => $v_2 = 14.164$, $u_2 = 1180.06$ Btu/lbm

Work is seen in the P-V diagram (when volume changes $P = P_{lift}$)

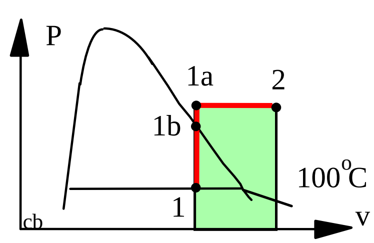
$${}_1W_2 = {}_{1a}W_2 = P_2\ m(v_2 - v_1)$$

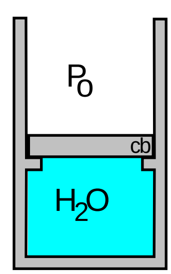
$$= 40 \text{ psi} \times 1 \text{ lbm } (14.164 - 13.4099) \text{ ft}^3\text{/lbm} \times 144 \text{ in}^2\text{/ft}^2$$

$$= 4343.5 \text{ lbf-ft} = \mathbf{5.58\ Btu}$$

Heat transfer is from the energy equation

$${}_1Q_2 = 1 \text{ lbm } (1180.06 - 628.845) \text{ Btu/lbm} + 5.58 \text{ Btu} = \mathbf{556.8\ Btu}$$





3.306E

An insulated cylinder is divided into two parts of 10 ft^3 each by an initially locked piston. Side A has air at 2 atm, 600 R and side B has air at 10 atm, 2000 R as shown in Fig. P3.151. The piston is now unlocked so it is free to move, and it conducts heat so the air comes to a uniform temperature $T_A = T_B$. Find the mass in both A and B and also the final T and P.

C.V. A + B . Then $_1Q_2 = \emptyset$, $_1W_2 = \emptyset$.

Force balance on piston: $P_A A = P_B A$, so final state in A and B is the same.

State 1A: u_{A1} = 102.457 Btu/lbm;

$$m_A = \frac{PV}{RT} = \frac{29.4 \text{ psi} \times 10 \text{ ft}^3 \times 144 \text{ in}^2/\text{ft}^2}{53.34 \text{ lbf-ft/lbm-R} \times 600 \text{ R}} = \mathbf{1.323\ lbm}$$

State 1B: u_{B1} = 367.642 Btu/lbm ;

$$m_B = \frac{PV}{RT} = \frac{147 \text{ psi} \times 10 \text{ ft}^3 \times 144 \text{ in}^2/\text{ft}^2}{53.34 \text{ lbf-ft/lbm-R} \times 2000 \text{ R}} = \mathbf{1.984\ lbm}$$

For chosen C.V. $_1Q_2 = 0$, $_1W_2 = 0$ so the energy equation becomes

$$m_A(u_2 - u_1)_A + m_B(u_2 - u_1)_B = 0$$

$$(m_A + m_B)u_2 = m_A u_{A1} + m_B u_{B1}$$

$$= 1.323 \times 102.457 + 1.984 \times 367.642 = 864.95 \text{ Btu}$$

u_2 = 864.95 Btu/3.307 lbm = 261.55 Btu/lbm $\Rightarrow$ T_2 = **1475 R**

$$P = m_{tot}RT_2/V_{tot} = \frac{3.307 \text{ lbm} \times 53.34 \text{ lbf-ft/lbm-R} \times 1475 \text{ R}}{20 \text{ ft}^3 \times 144 \text{ in}^2/\text{ft}^2}$$

$$= \mathbf{90.34\ lbf/in^2}$$