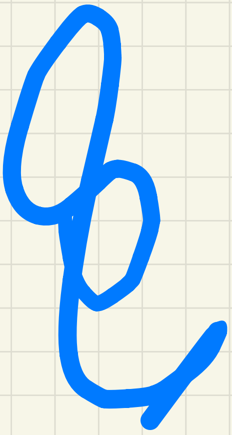
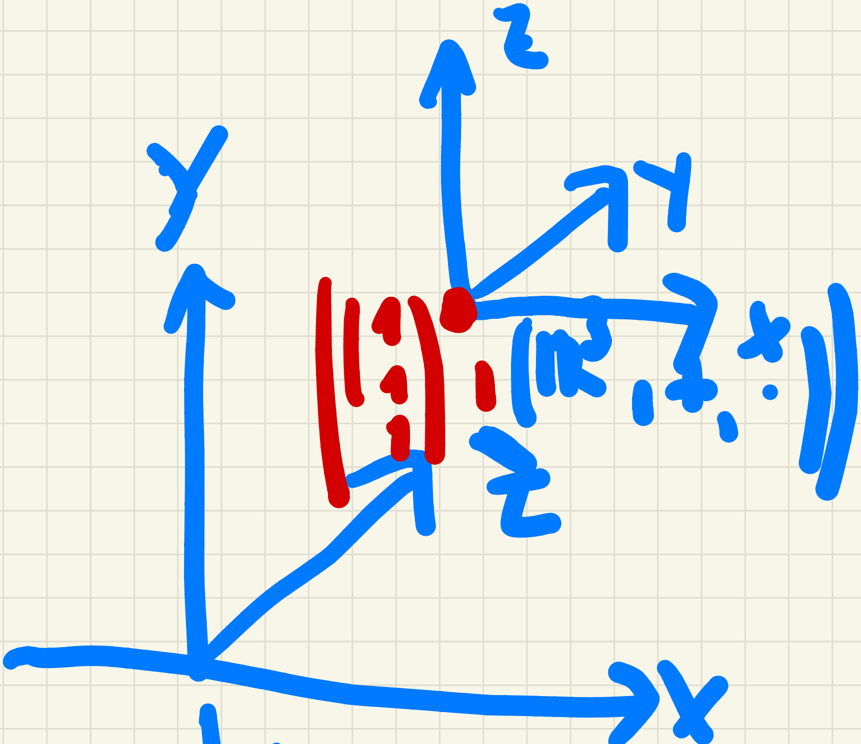
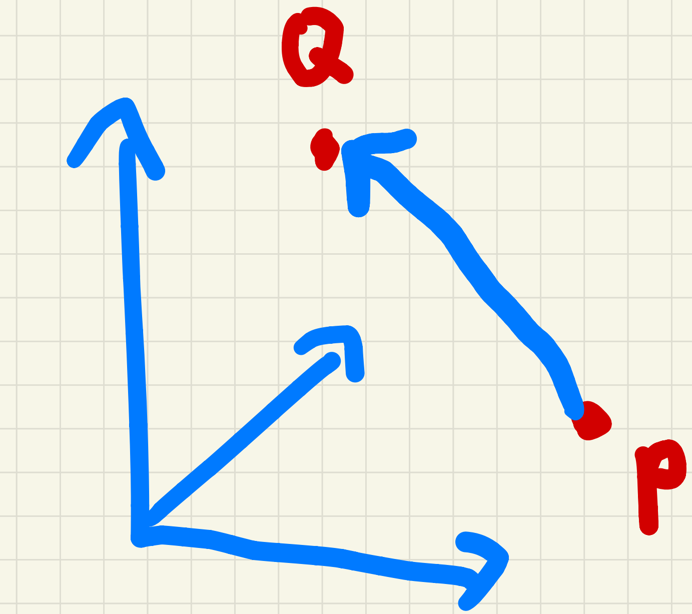




$\left| \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \right. = 0$



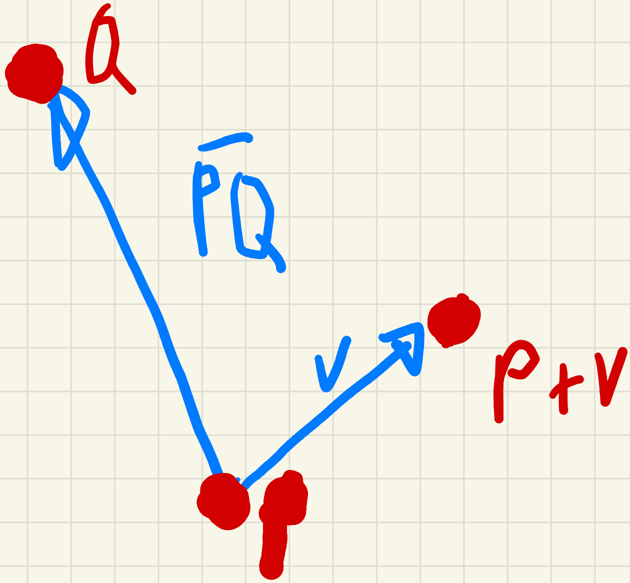
$$\overline{PQ} := Q - P$$

$\overline{}$: Punkte $\rightarrow$

$$\begin{pmatrix} q_1 \\ q_2 \\ q_3 \end{pmatrix} - \begin{pmatrix} p_1 \\ p_2 \\ p_3 \end{pmatrix}$$

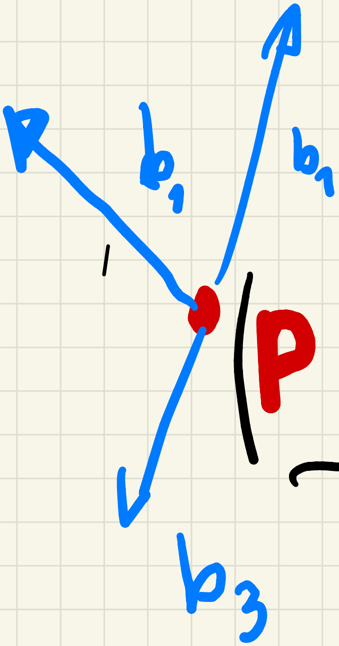
Vektor

$$= \begin{pmatrix} q_1 - p_1 \\ q_2 - p_2 \end{pmatrix}$$



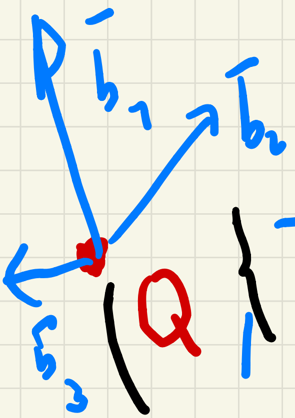
$$\boxed{P + \overline{PQ} = P + (Q - P) = Q}$$

(P, B)

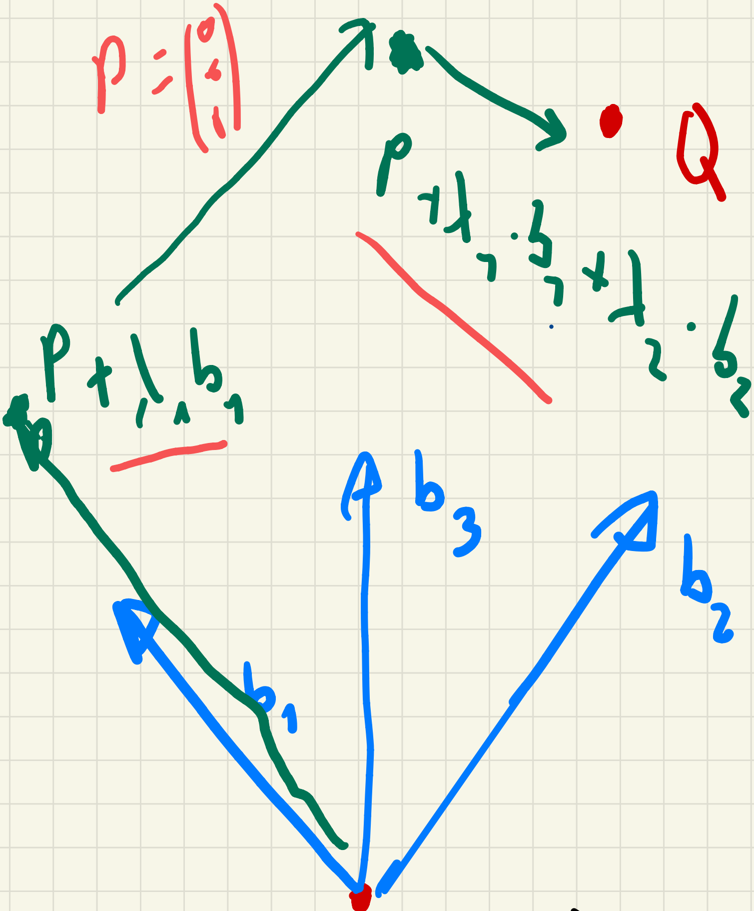


$$\left(P, \overbrace{\{b_1, b_2, b_3\}}^{B}\right)$$

Affine Basis



$$\underbrace{\left(Q, \overbrace{\{\bar{b}_1, \bar{b}_2, \bar{b}_3\}}^{\bar{B}}\right)}_{(Q, \bar{B})}$$



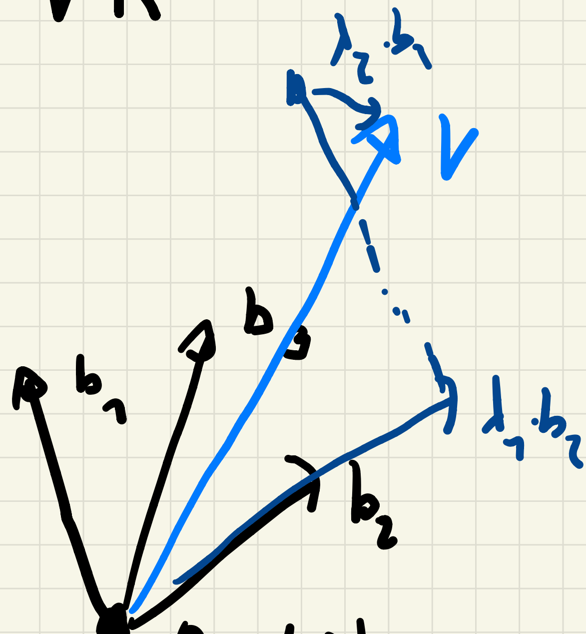
$Q = P + \lambda_1 b_1 + \lambda_2 b_2 + \lambda_3 \cdot b_3$

(P, B)

$\bullet = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

$\Theta_{(P,B)}(Q) = \begin{pmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \end{pmatrix}$

VR



$$0 = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\Theta_B(v) = \begin{pmatrix} b_1 & b_2 & b_3 \end{pmatrix}^{-1} \cdot v = \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \end{bmatrix}$$

$$\Rightarrow \quad v = \lambda_1 \cdot b_1 + \lambda_2 \cdot b_2 + \lambda_3$$

$$Q = 1$$

$$Q = P + \lambda_1 \cdot b_1 + \lambda_2 b_2 + \lambda_3 b_3 \iff$$

$$Q - P = \lambda_1 \cdot b_1 + \lambda_2 \cdot b_2 + \lambda_3 \cdot b_3$$

Basis

$$Q - P = \begin{pmatrix} b_1 & b_2 & b_3 \end{pmatrix} \cdot \begin{pmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \end{pmatrix}$$

$$\iff \quad \begin{pmatrix} b_1 & b_2 & b_3 \end{pmatrix}^{-2} \cdot \begin{pmatrix} Q - P \end{pmatrix} = \begin{pmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \end{pmatrix}$$

$$\Theta_{(P,B)}(Q) =$$

$$\lambda = \begin{pmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \end{pmatrix}$$

$$Q - p = \begin{pmatrix} b_1 & b_2 & b_3 \end{pmatrix} \lambda$$

$$\begin{pmatrix} b_1 & b_2 & b_3 \end{pmatrix}^{-1} (Q - p) = \underbrace{\begin{pmatrix} b_1 & b_2 & b_3 \end{pmatrix}^{-1} \begin{pmatrix} b_1 & b_2 & b_3 \end{pmatrix}}_{\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}} \cdot \lambda$$

gesucht Φ mit

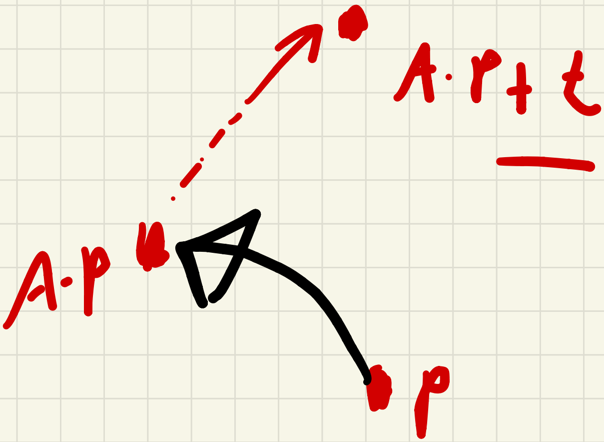
$$\Phi\big(\underbrace{A \cdot p + t}_{\text{Überlegung}}\big) = p$$

$$A^{-1}(A \cdot p + t) = A^{-1} \cdot A \cdot p + A^{-1} \cdot t$$

$$= p + A^{-1} \cdot t$$

$$- A^{-1} \cdot t$$

$$A^{-1} \cdot p - A^{-1} \cdot t = A^{-1}(p - t)$$



$\cdot 0$

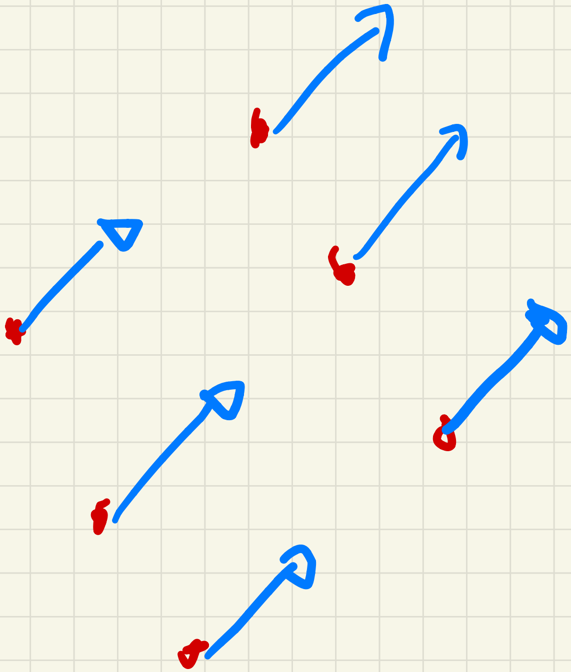
$(u+v) \mapsto A(u+v)+t$

$=$

$(0+$

$\underline{A\cdot u + A\cdot v} + t$

$u+v \mapsto \underline{A\cdot u+t} + \underline{A\cdot v+t}$

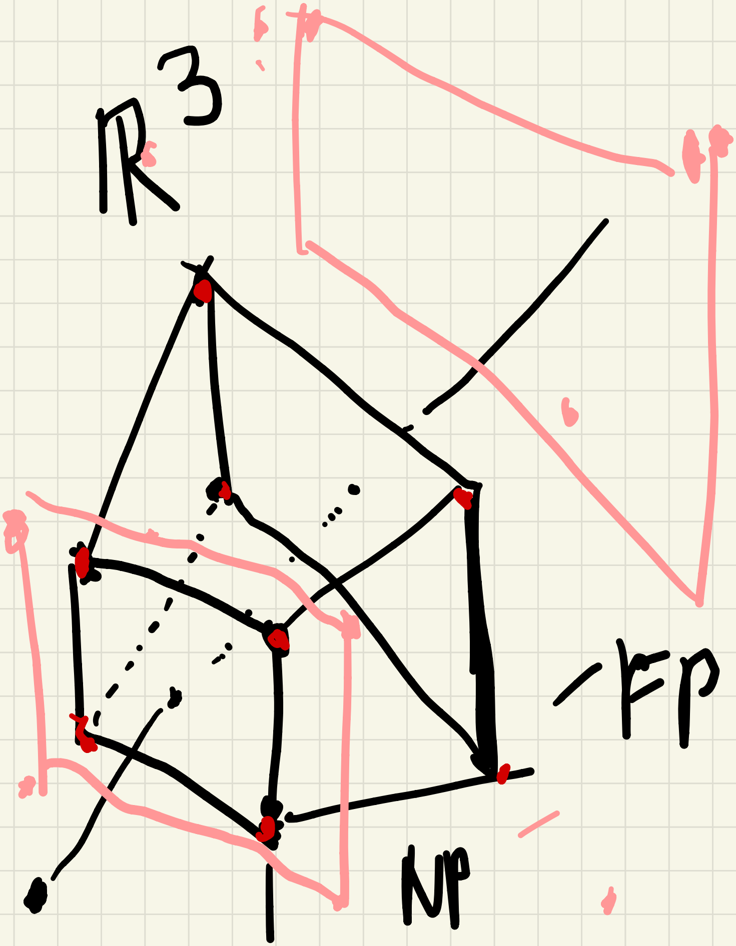


$$\begin{tikzcd}
\mathbb{R}^3 & & \mathbb{R}^3 \\
v \arrow[rr, mapsto] \arrow[d] & & A \cdot v + t \\
\mathbb{P}^3 \quad \begin{pmatrix} v \\ 1 \end{pmatrix} \arrow[rr, mapsto] & & \overset{?}{M} \cdot \begin{pmatrix} v \\ 1 \end{pmatrix} \quad \mathbb{P}^3 \arrow[u, "/w"']
\end{tikzcd}$$

$$M = \underbrace{\begin{array}{cc} 3\times 3 & 3\times 1 \\ \left(\begin{array}{cc} \boxed{A} & t \\ 0 & 1 \end{array}\right) \end{array}}_{4\times 4} \; 3\times 4$$

$$\begin{pmatrix} A \cdot v + t \\ 1 \end{pmatrix}$$

$\mathbb{R}^3$



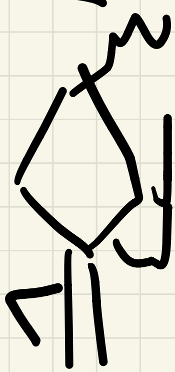
FP

NP

A

Bildebene

Clipping

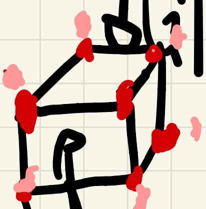


A'

$\mathbb{P}^3$

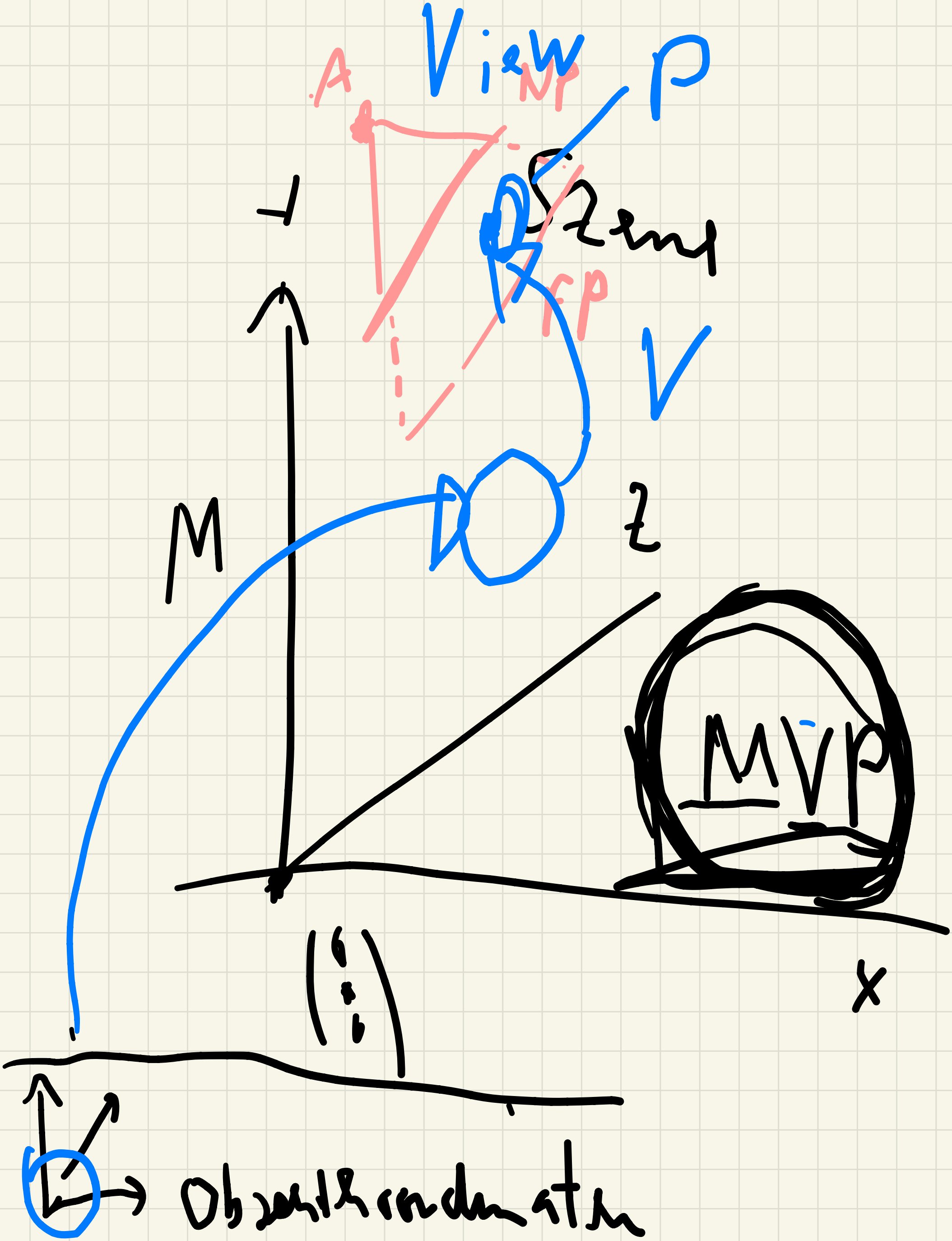
Zentralprojektion

A : Zentralproj. $\longmapsto \mathbb{R}^3$



$\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$

$\begin{pmatrix} -1 \\ -1 \\ -1 \end{pmatrix}$



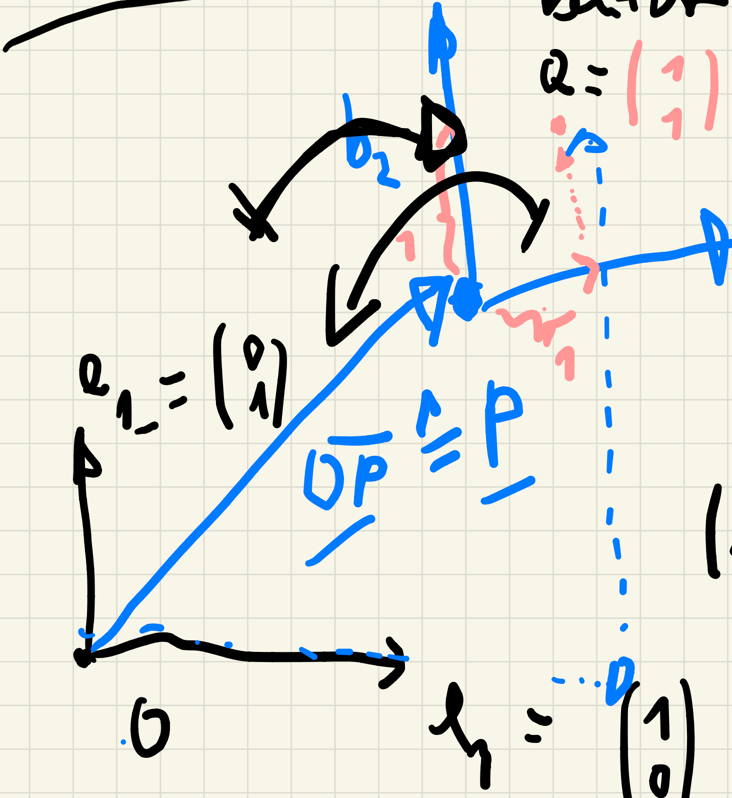
View
A
NP
P
Szene
FP
V
y
M
z
MVP
x
Objektkoordinaten

Matrix: Produkt + Vektor

$$(b_1, b_2)\cdot Q + \overline{OP}$$

Aff.

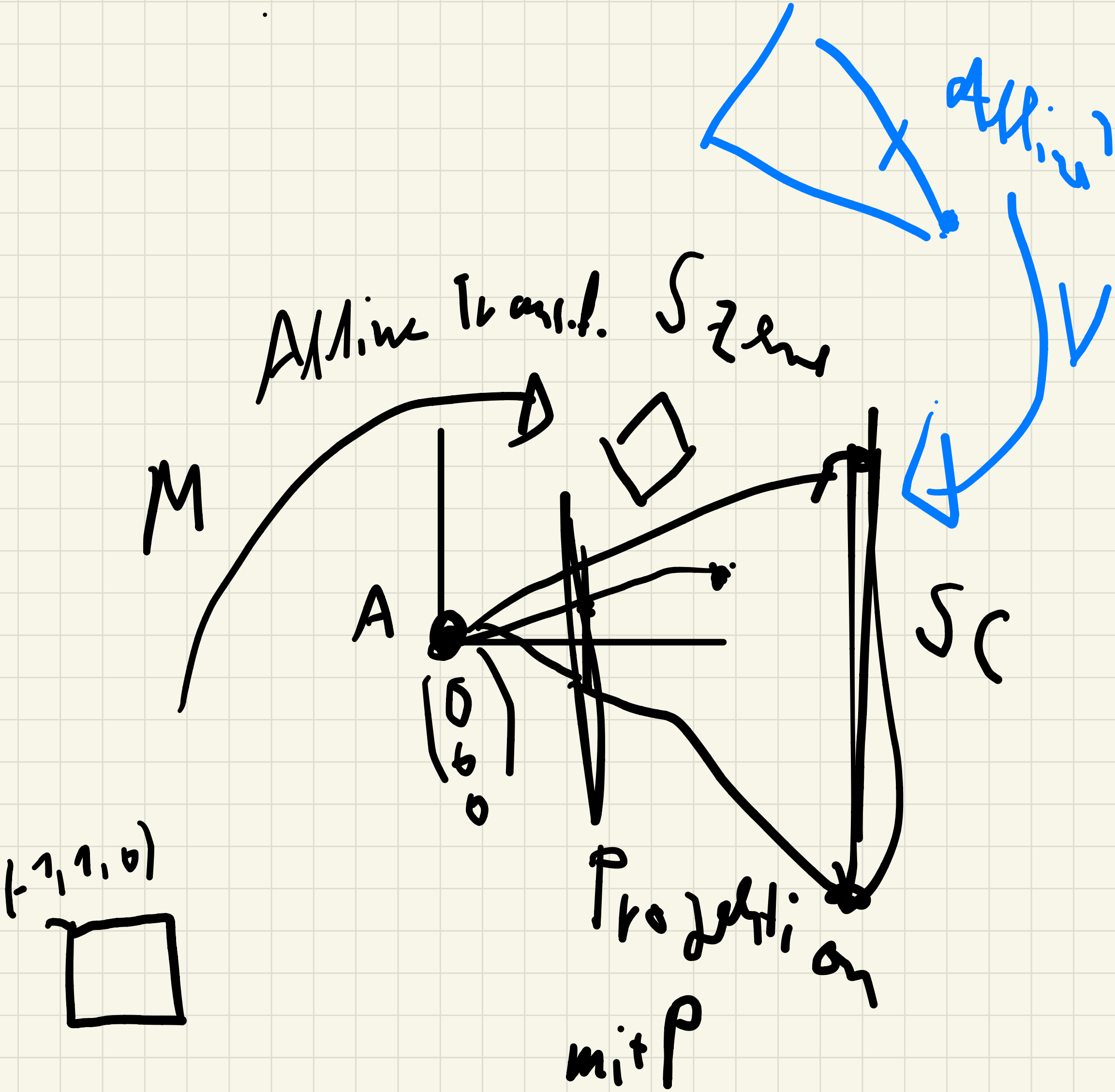
Vektor − Trans.



$(b_1, b_2)\cdot Q + 1$

Affine

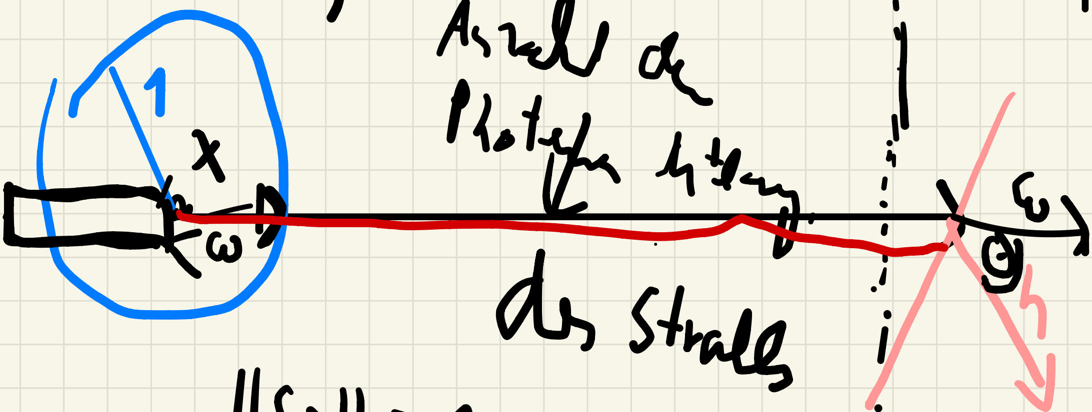
Transf.



$$\underline{PVM} \cdot v$$

Strahldichte

$L(x, \omega)$

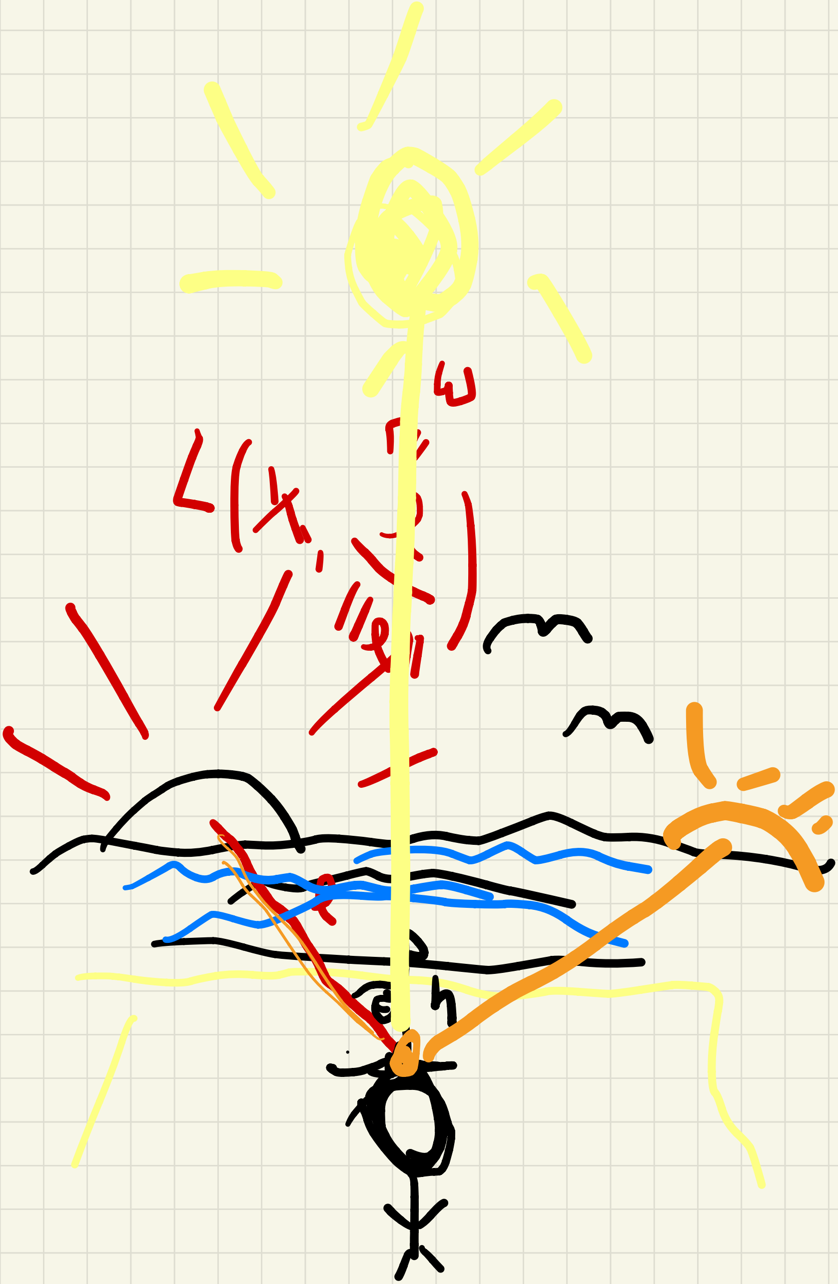


$\|\omega\| = 1$

pro Zeit

$L(x, \omega) \cdot \cos(\Theta)$

Anzahl der Photonen auf Oberfläche / pro Zeit



?

Punkt Mittel

Ebene

?

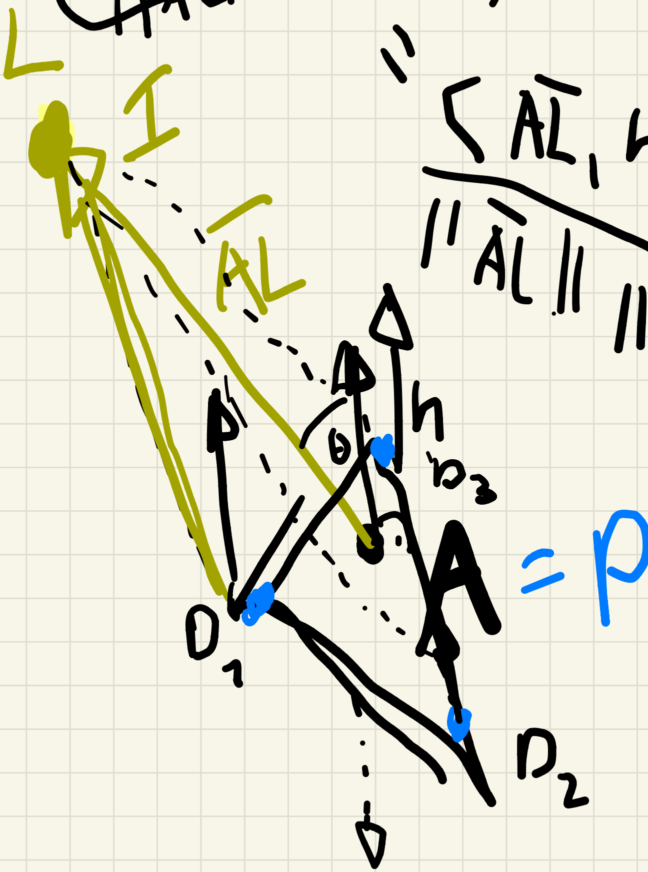
1530

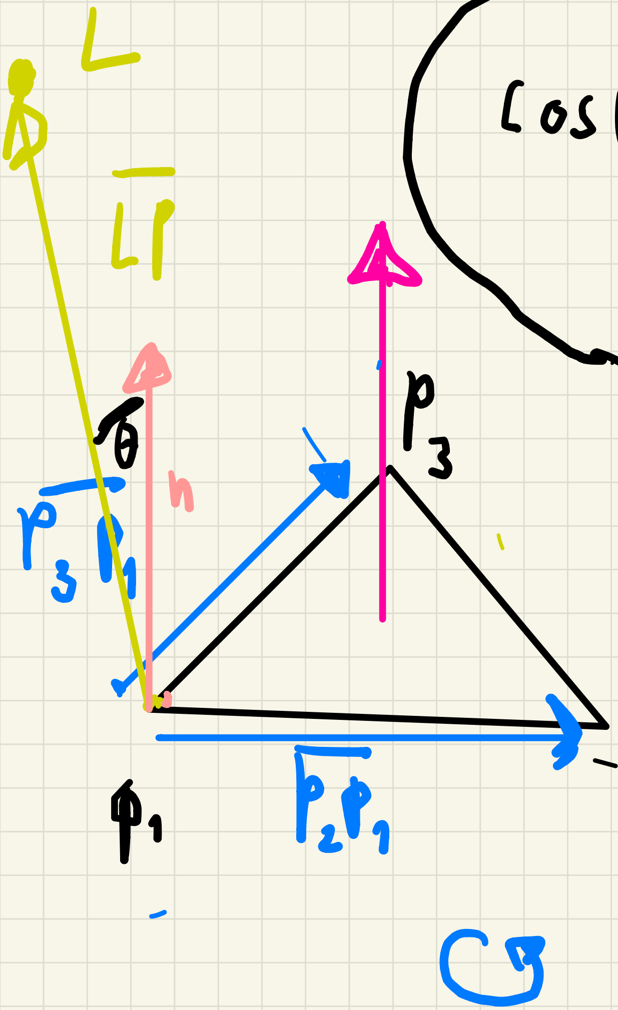
Mathematik

Photomythische Grundsatz

$$I = \frac{(L(x, \vec{AL}))}{\cancel{\|AL\|}} \cos(\theta)$$

$$\cos(\theta) = \frac{\langle \vec{AL}, n \rangle}{\|\vec{AL}\| \, \|n\|}$$





$$\boxed{\cos(\theta) = \frac{\langle \overline{LP}, n \rangle}{\|\overline{LP}\| \cdot \|n\|}}$$

d.h.: $\|n\| = 1$

$$\overline{p_2p_1} \times \overline{p_3p_1} = n$$

$$I(p) = \underline{\cos(\theta)} \cdot I_L$$

$$\text{normalize}(v) := \frac{v}{\|v\|}$$

$$\left\| \frac{v}{\|v\|} \right\| = \Big|\underbrace{\frac{1}{\|v\|}}_{>0}\Big| \cdot \|v\|$$

$$= \frac{1}{\cancel{\|v\|}} \cdot \cancel{\|v\|} = 1$$

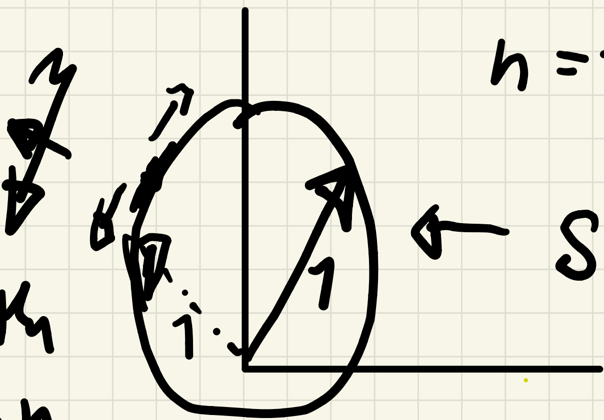
$n = 2, 3$

$$S^{n-1} = \{ v \in \mathbb{R}^n \mid \|v\| = 1 \}$$

Richtungen

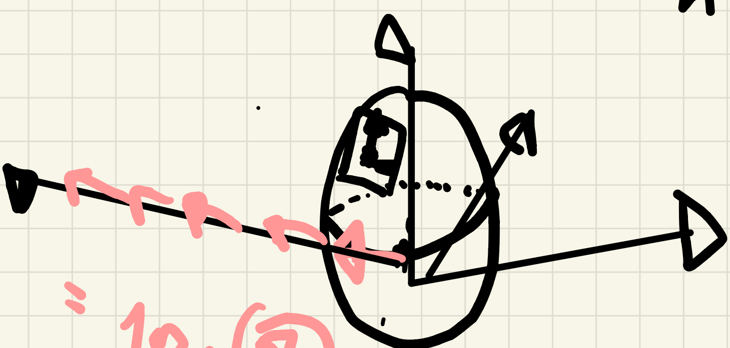
im $\mathbb{R}^n$

$n = 2$



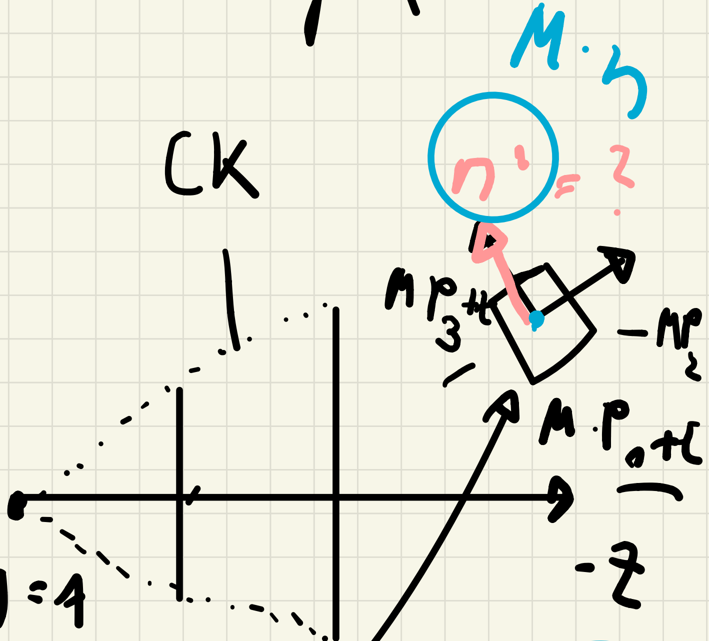
$S^1 \subseteq \mathbb{R}^2$

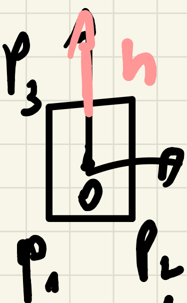
$n = 3$



$= 10 \cdot \mathbb{R}$

World Space





Model space

nP

FP

$\Phi(P) = MP + t$

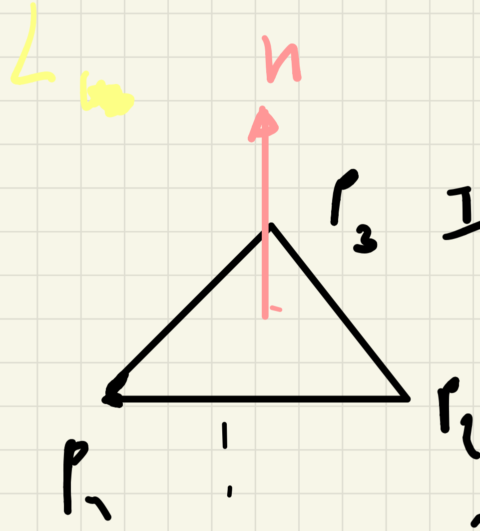
$M(\overline{PQ}) = MP - MQ = MP + t - MQ - t$

$n = \overline{P_1P_2} \times \overline{P_1P_3}$

$\phi(\overline{PQ}) = \phi(P - Q) = \phi(P) - \phi(Q)$

1)

VS :



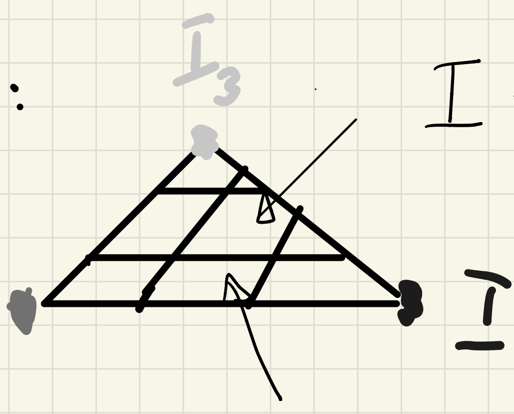
$I(p_3) = I_3$

$I(p_2) = I_2$

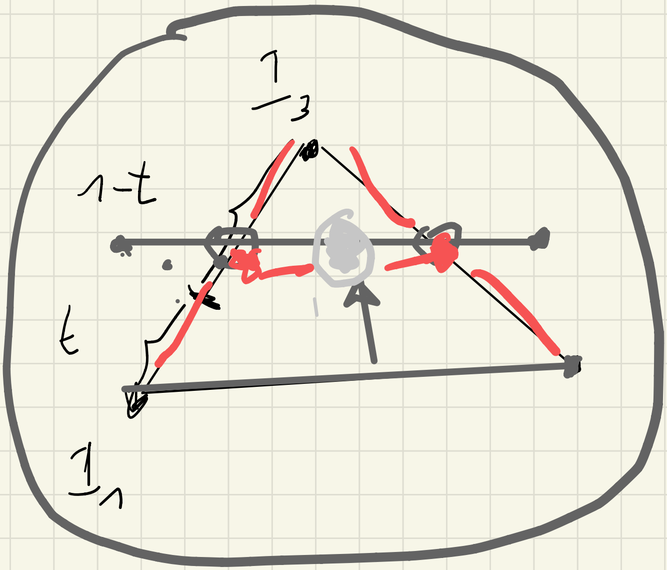
$I_1 = I(p_1) =$

$$I(p) = \cos\left(\frac{\langle p, n \rangle}{\|p\| \, \|n\|}\right)$$

FS :



$I = \mathrm{Int}(I_1, I_2, \ldots)$

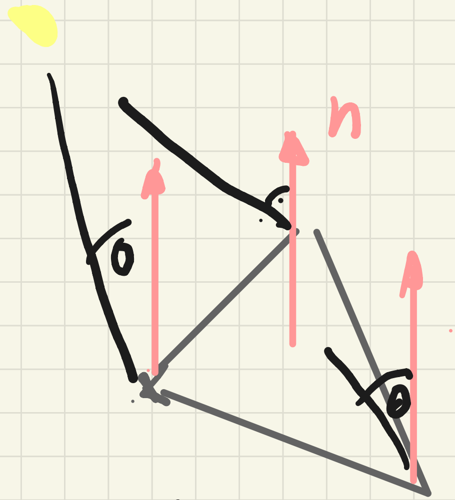


$$t \cdot I_1 + (1-t)\, I_3$$

Lin. Int. entl. Strecke

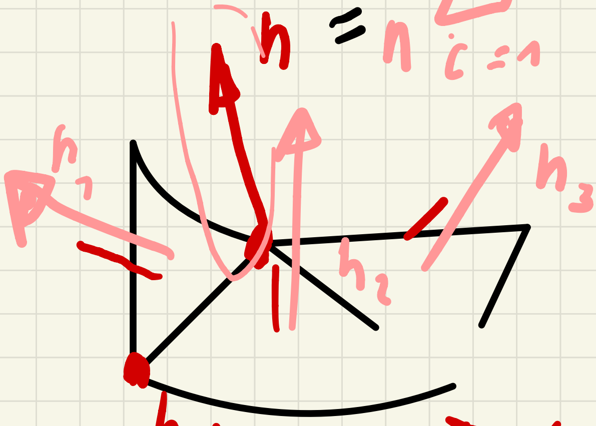
Lin. Int

Int ≙ 3 · mal Lin. Int.



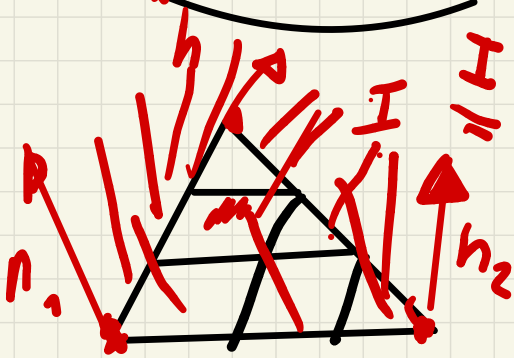
$I(p, n)$

$$n = \frac{1}{n}\sum_{i=1}^{n} n_i$$



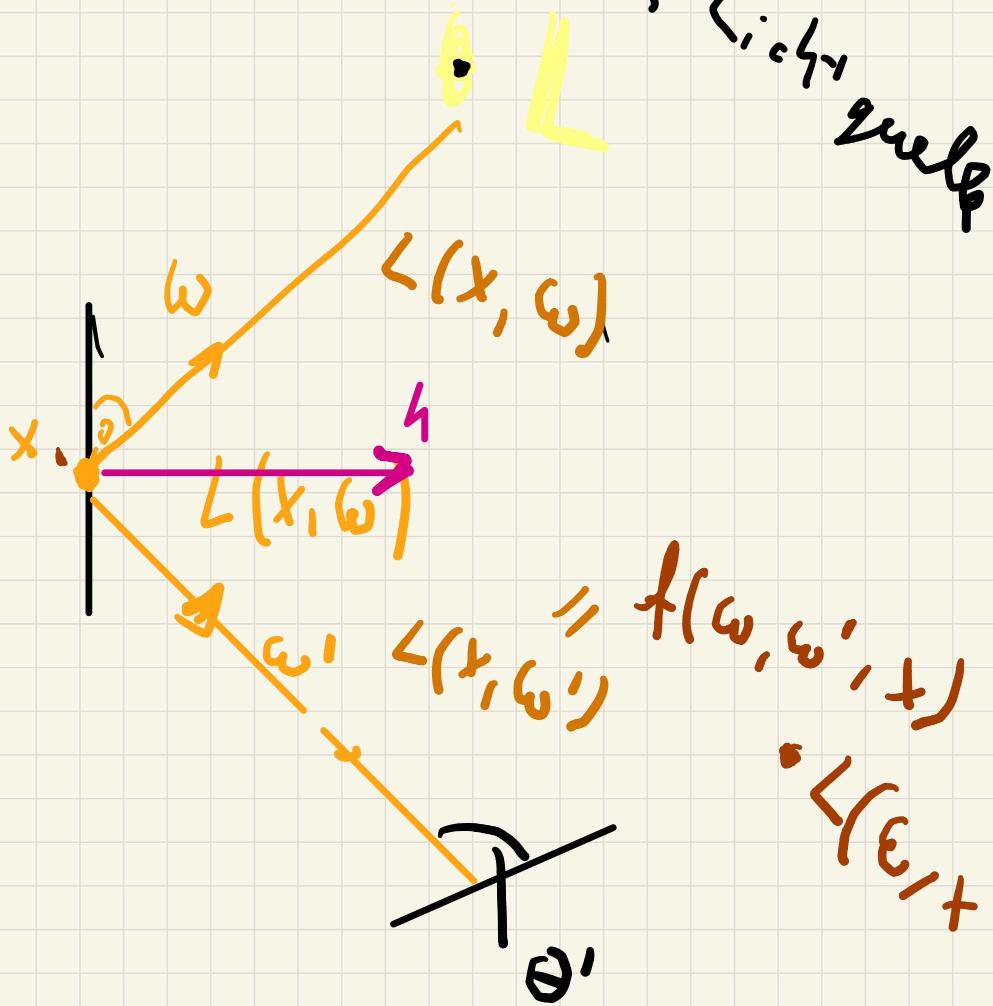
$\mathrm{Int}(p_1, p_2, p_3)$

FS



$I = (p, \mathrm{Int}(n_1, n_2, n_3))$

1 Punktförmige Lichtquelle

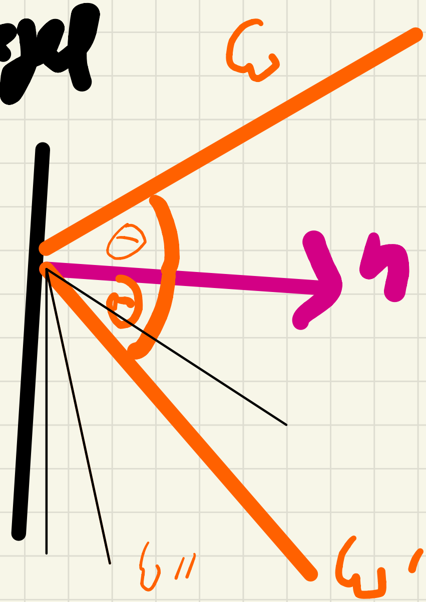


Schwarzes Loch

$f(\omega, \omega', x) = 0$

Ideal

Spiegel



$\langle \omega, n \rangle = \langle \omega', n \rangle$

und $\omega \neq \omega'$

$$f(x, \omega, \omega') = \begin{cases} \text{[illegible]} \\ 0 \text{ sonst} \end{cases}$$

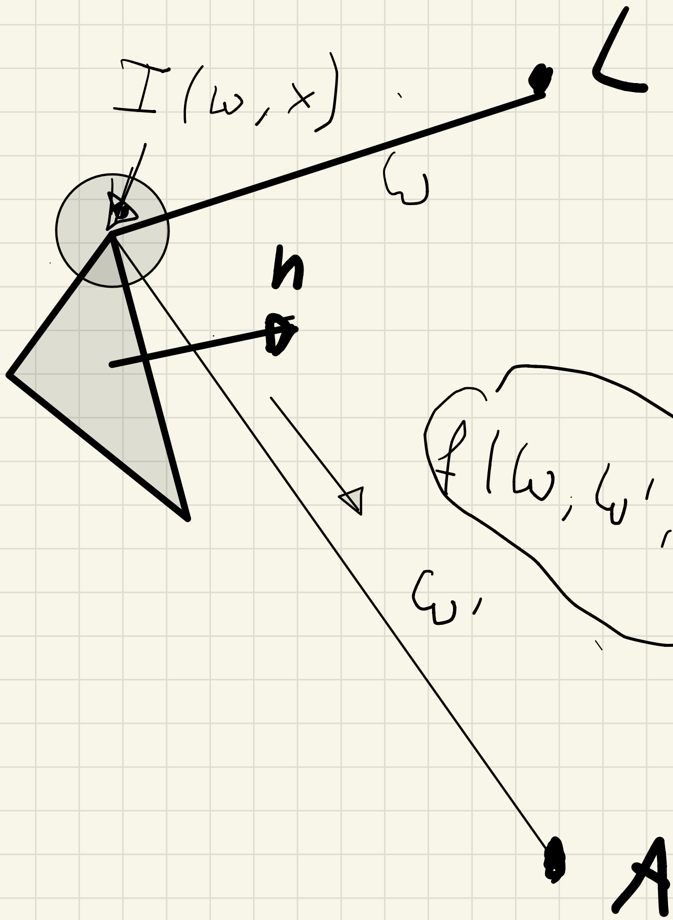
Brotpapier

$\hat{=}$ Difuse

$$f(x, \omega, \omega') = \text{const}$$

$< \rho$

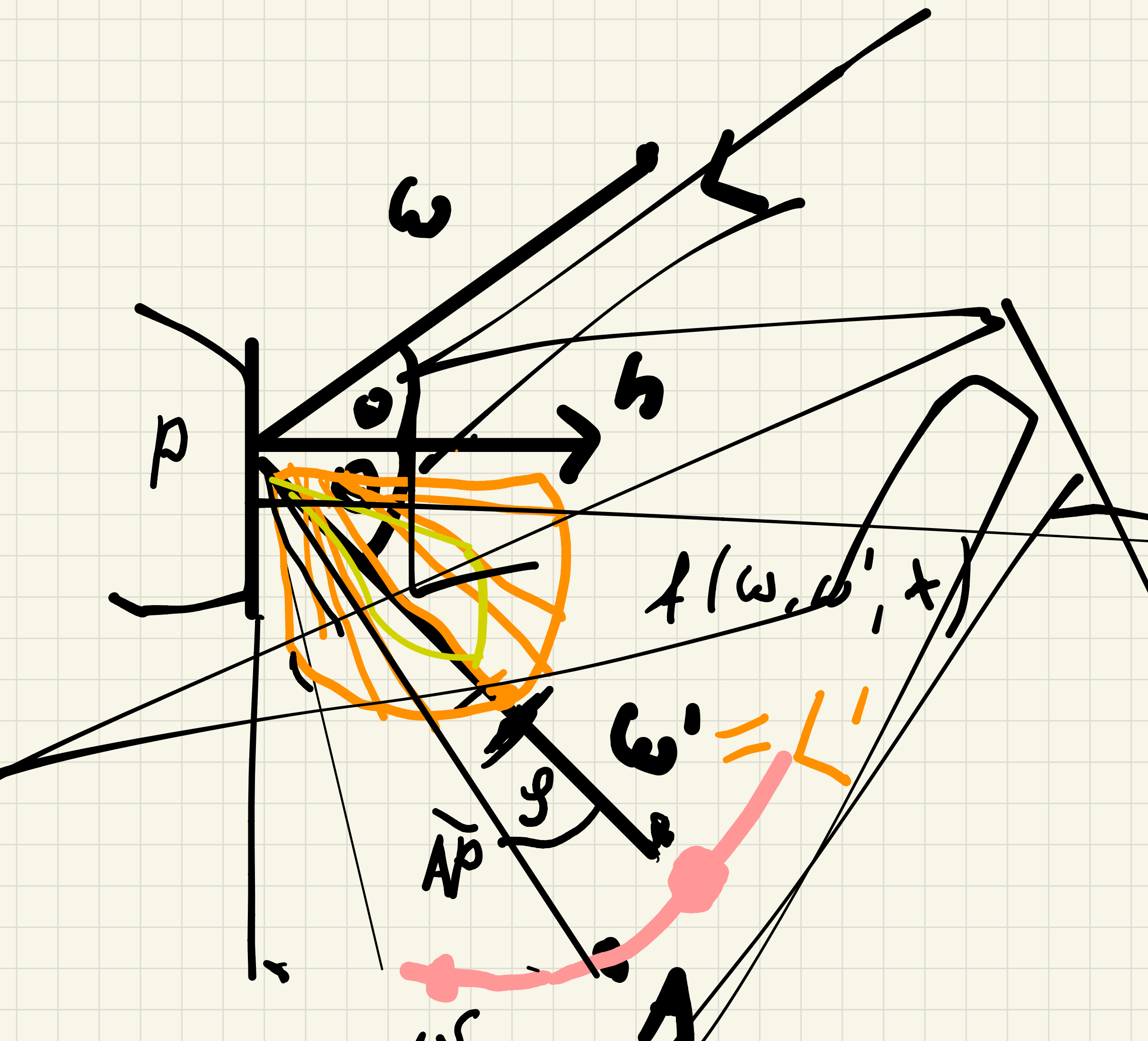
Lambert ≙ BRDF konstant



BR

$f(\omega, \omega', x)$ = const = BRDF

≙ Difusen Oberfl.



$$I_D \times s \cdot I_S$$

$$\cos(\varphi)$$

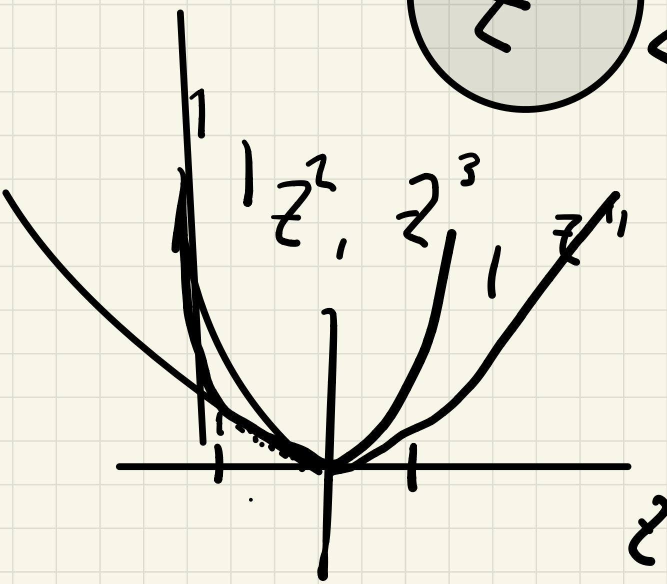
$$= s \cdot \cos(\angle \vec{Ap}, L') \cdot I_L$$

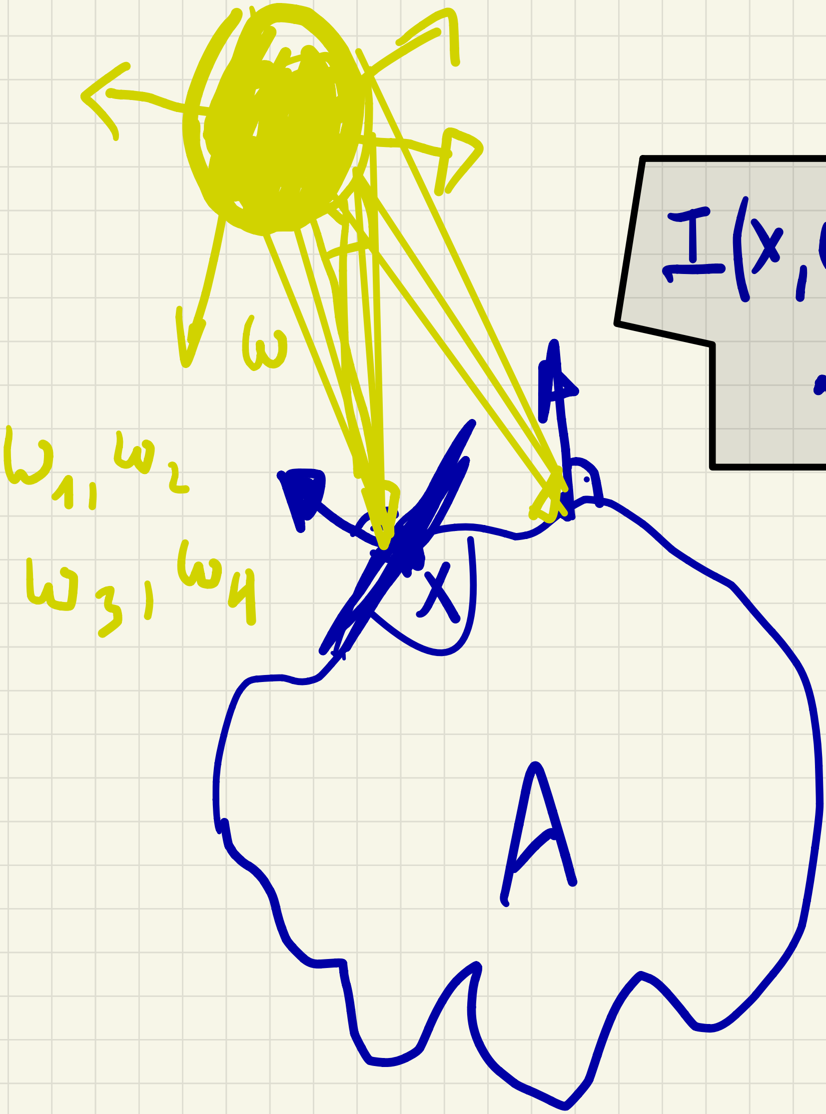
$$\|Ap\| \, \|L'\|$$

$0 \le \varphi \le 90^\circ$

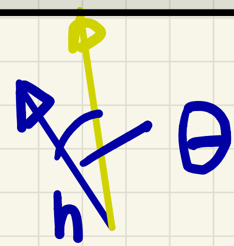
$0 \le \cos(\varphi)^n \le 1 \qquad n \in [0, 1, 2, 3, 4, \dots$

$z^4 < \cancel{z}$





$$I(x,\omega) = L(x,\omega) \cdot \cos(\Theta)$$

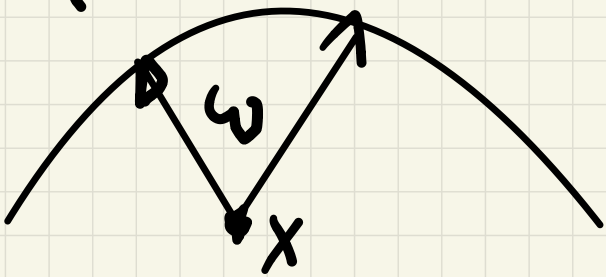


$$L \to A\,?$$

$$\approx \sum_{i=1}^{N} I(x, \omega_i)$$

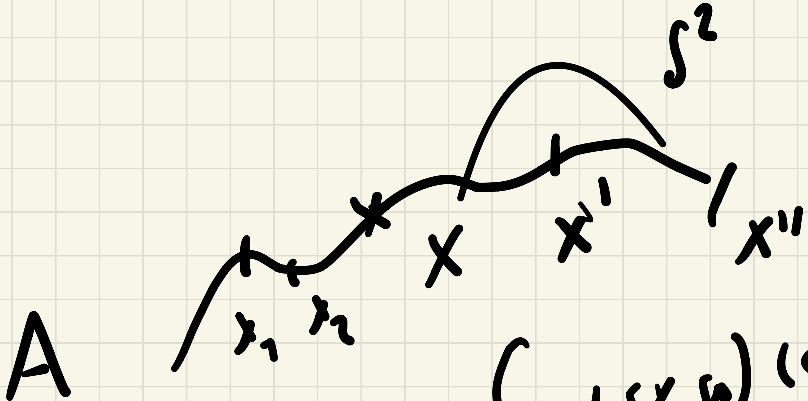
$$\|\omega\| = 1$$

$$S^2 := \{ \omega \in \mathbb{R}^3 \mid \|\omega\| = \}$$



$$\sum_{i=1}^{N} I(x, \omega_i) \overset{N \to \infty}{\triangleq} \int_{S^2} I(x, \omega)\, d\omega$$

$$= \int_{S^2} L(x, \omega) \cdot \cos(\Theta_\omega) \cdot d\omega$$



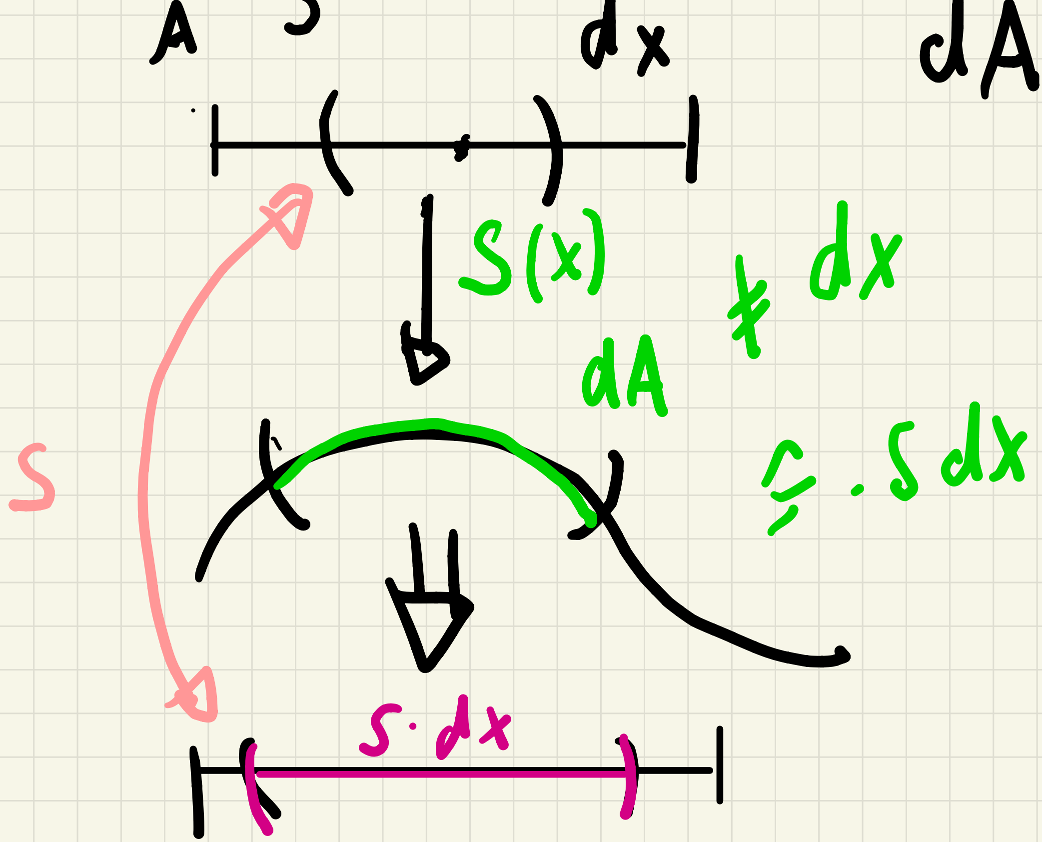
$$I(x) = \int_{S^2} L(x,\omega)\cos(\theta)\, d\omega$$

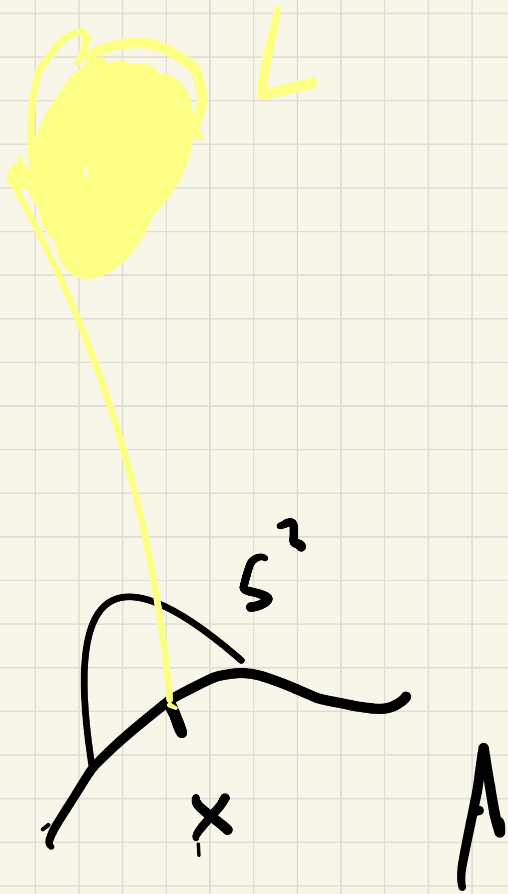
$$I(x') = \int_{S^2} L(x',\omega)\cdot\cos(\theta)\, d\omega$$

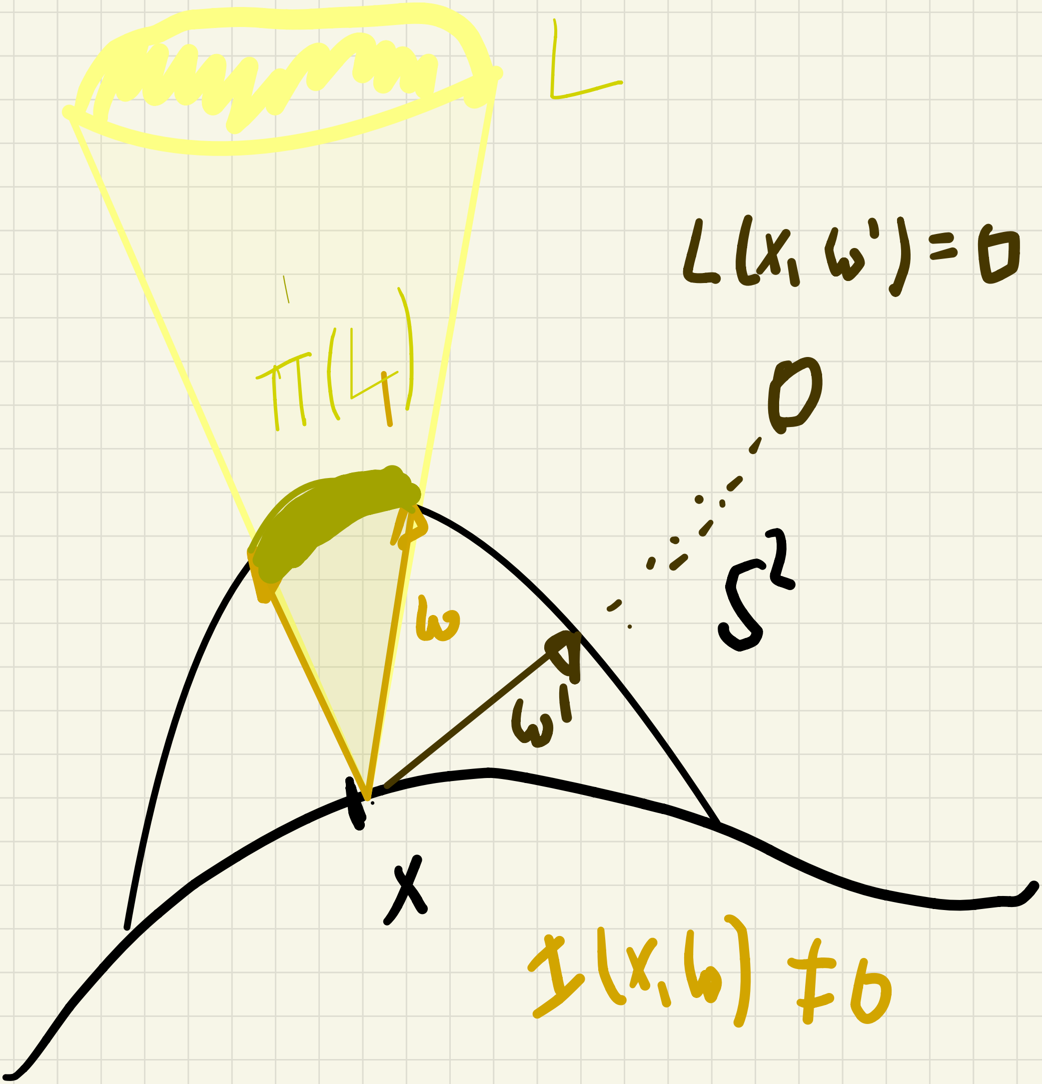
$$I(A) \approx \sum_{i=1}^{M} I(x_i) = \sum_i \int_{S^2} \ldots$$

$$\sum_{i=1}^{M} I(x_i) \overset{M\to\infty}{=} \int_A I(x)\, dA$$

$$I(A) = \int_A \int_{S^2} L(x,\omega)\cdot\cos(\theta(x,\omega))\, d\omega\, dA$$

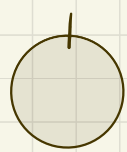


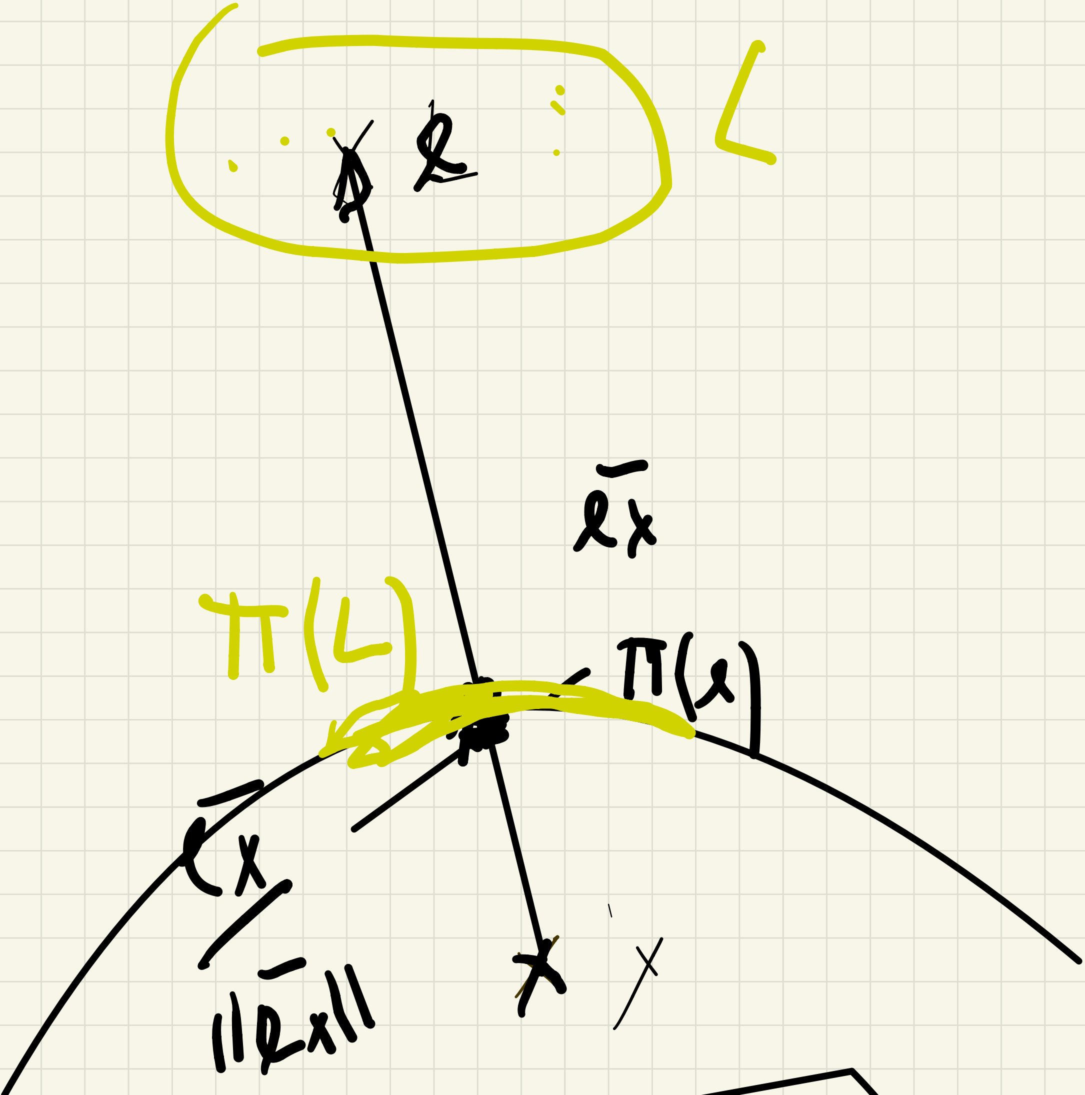




$I(x, \omega') = 0$

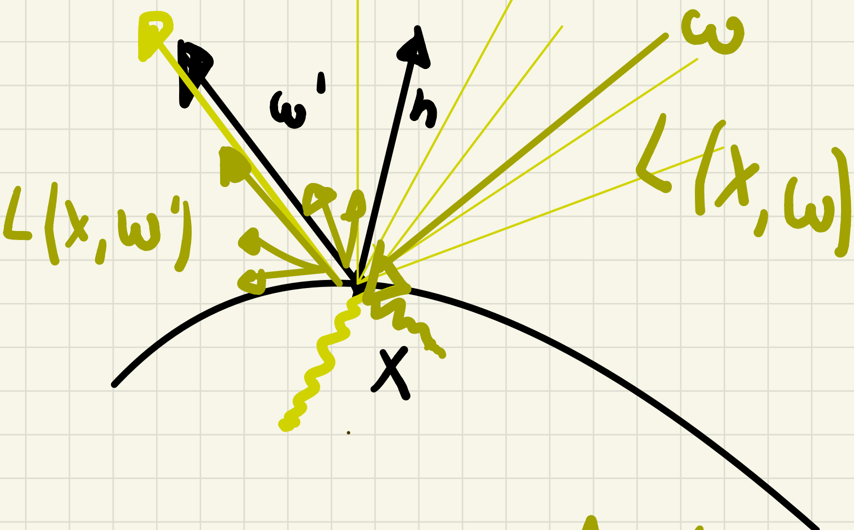
Raumwinkel





$$\boxed{\pi(\ell) = x + \frac{\bar{\ell}_x}{\|\bar{\ell}_x\|}}$$

$L(x,\omega') \overset{?}{=} L_e(x,\omega') +$



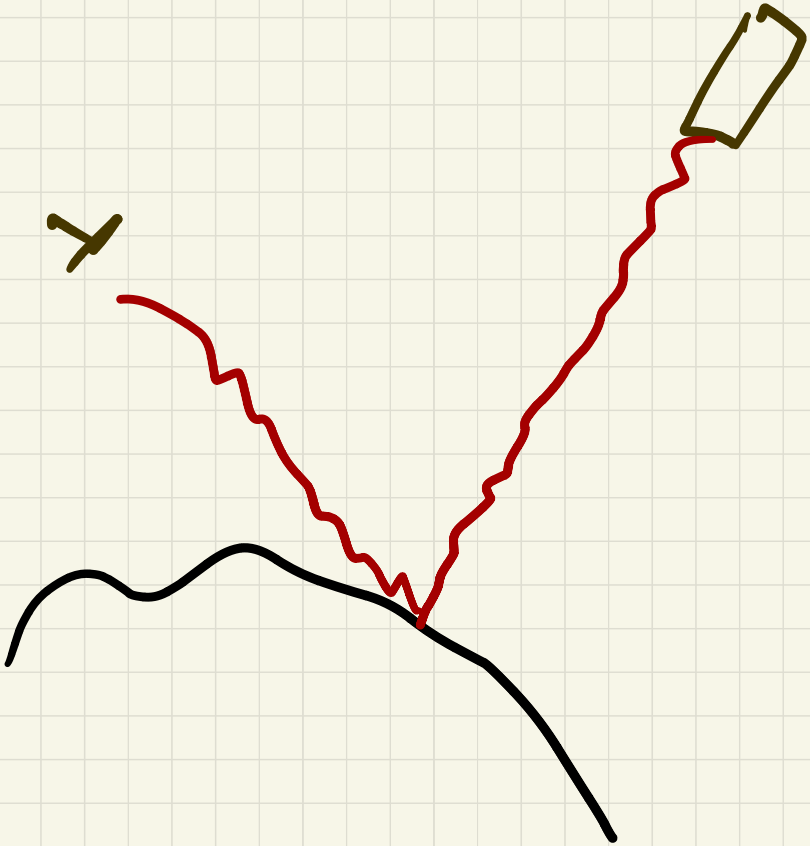
$$f_r(x,\omega,\omega') \overset{!}{=} \frac{L_o(x,\omega)}{L_I(x,\omega')}$$

$$f_r(x,\omega,\omega') \cdot L_I = L_o$$

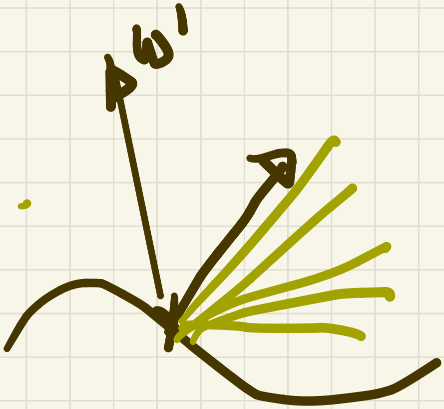
Phong

$$f_r(x,\omega,\omega') = \begin{cases} \overset{\text{Diffuse}}{\overset{\shortparallel}{\text{const}}} + \\ f_{spec}(x,\omega,\omega') \end{cases}$$

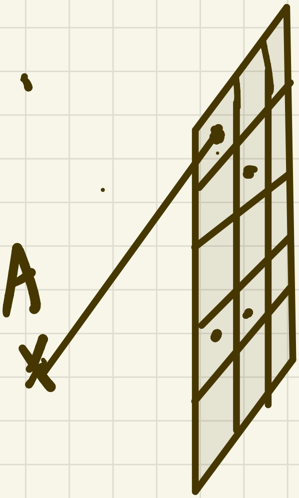
$$f_{spec} = \max(0, \cos(\theta(\omega,\omega'))^n)$$



$$L_r(x, \omega') = \int_{S^2} f_r(x, \omega, \omega') \cdot L_I(x, \omega) \cdot \cos(\theta(\omega, n)) \, d\omega$$



$$L\left(p_i, \frac{\overrightarrow{p_i A}}{\|\overrightarrow{p_i A}\|}\right)$$

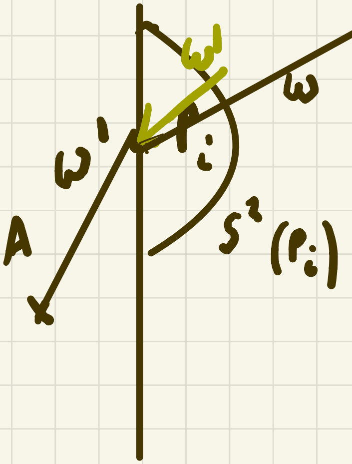


$$\|(\underline{R}, \underline{G}, \underline{B})$$

Alle Formeln

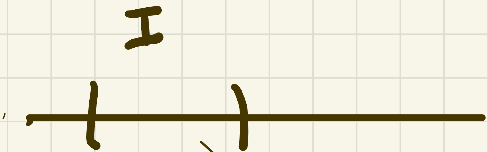
für Licht einer Wellenlänge

$$L(P_i, \omega') = \int_{S^2} L(x, \omega) \cdot \cos(\Theta(\omega, \omega'))\, d\omega$$

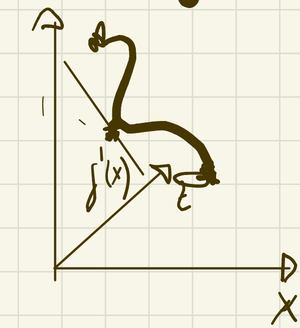


$$\bar{L}(x, \omega) = L_0(x', \omega) = L_0(x', \omega) = L_e(x', \omega) + L_R(x', \omega)$$

$\widehat{=}$ (exp Laufzeit)



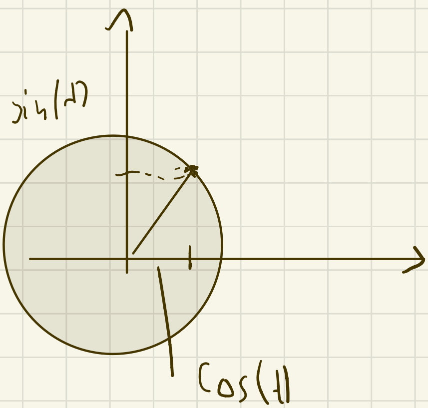
$$\gamma(x) := \begin{pmatrix} f(x) \\ g(x) \\ h(x) \end{pmatrix}$$

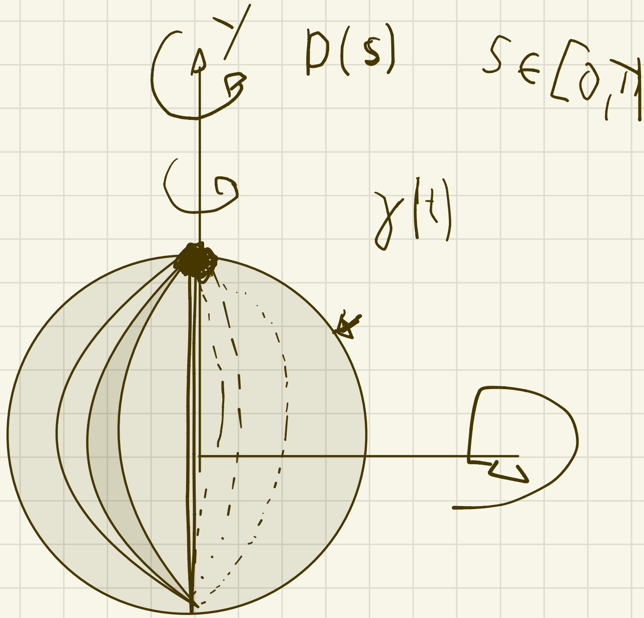


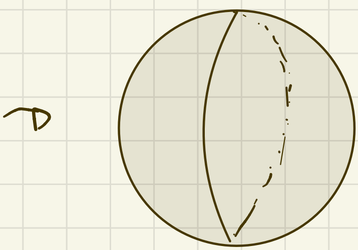
f, g, h stetig
diff'bar

$$\gamma'(x) := \begin{pmatrix} \frac{\partial}{\partial x} f(x) \\ \frac{\partial}{\partial x} g(x) \\ \vdots \end{pmatrix}$$

$$\gamma(t) = \begin{pmatrix} \cos(t) \\ \sin(t) \\ 0 \end{pmatrix} \qquad t \in [0, 2\pi]$$



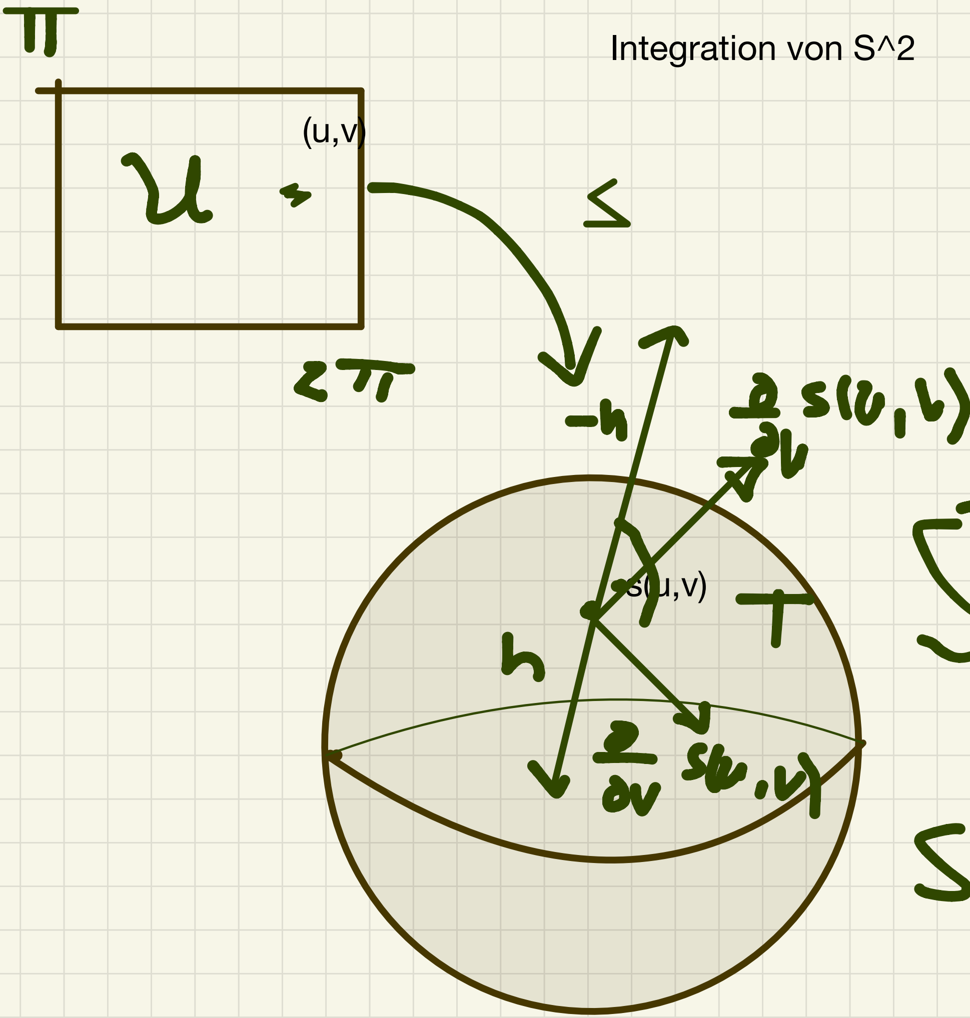




$$D(s) = \begin{pmatrix} \cos(s) & 0 & \sin(s) \\ 0 & 1 & 0 \\ -\sin(s) & 0 & \cos(s) \end{pmatrix}$$

$$S^2 = D(s) \cdot \gamma(t)$$

Integration von S^2



$$S^2 = S(\mathcal{U})$$

$$\int_{S^2} f \, dA$$

f

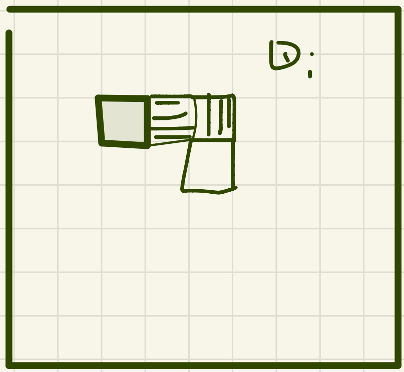
$\mathbb{R}$

z.B. 1

$x^2 + y^2 + z^2$

Integration R^2

$\mathbb{R}^2$

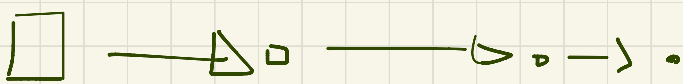


$d(u,v)$

$\xrightarrow{f} \mathbb{R}$

U

Q_i



N

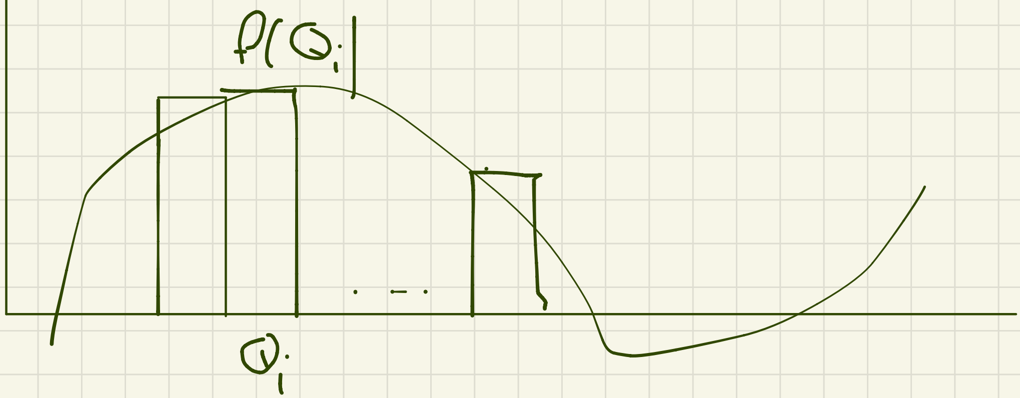
$$\sum_{i=1}^{N} Q_i \longrightarrow \int_U 1 \, d(u,v)$$

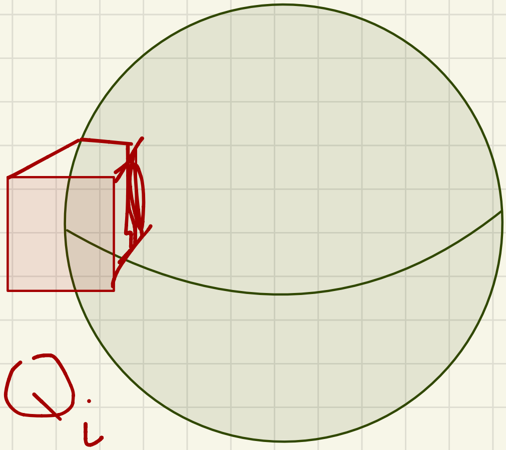
$$\sum f(Q_i) \cdot Q_i \longrightarrow \int_U f \, d(u,v)$$

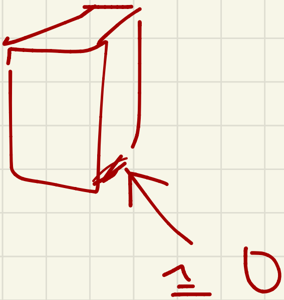
$f(Q_i) = \max f$ auf Q_i

Integration R

Riemannintegral

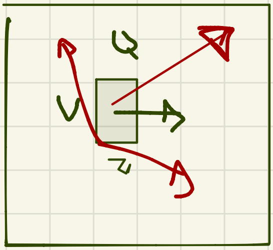



Q_i

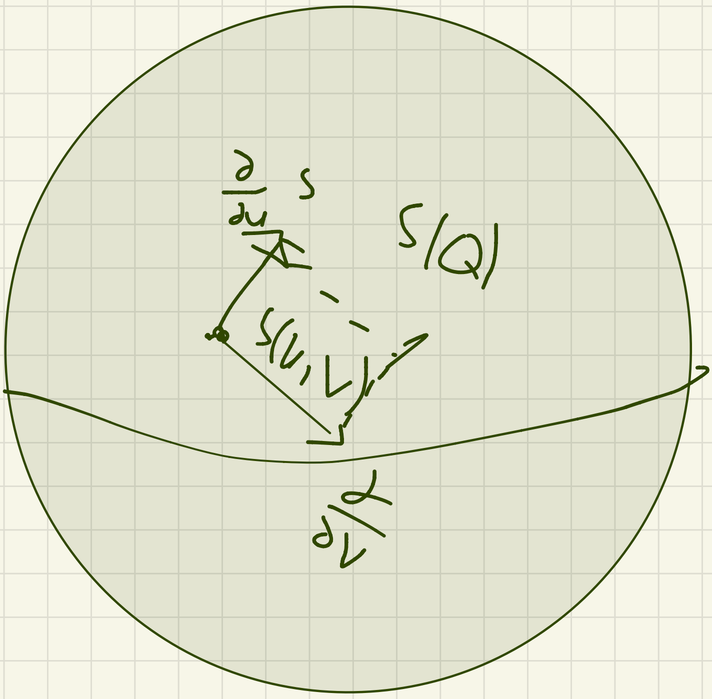


Lokal Approx.

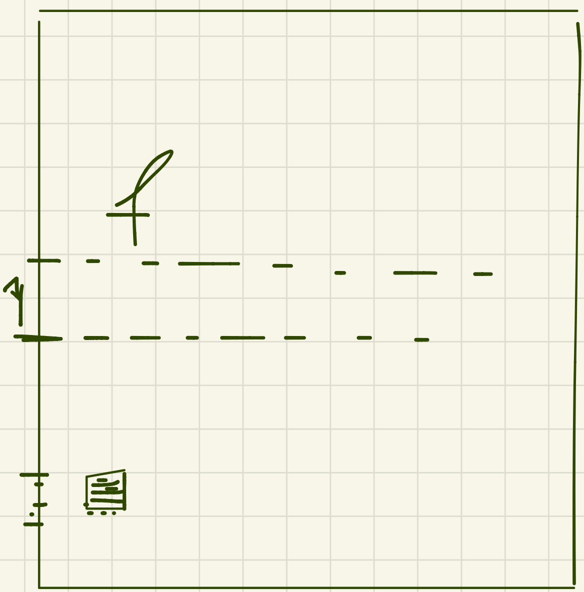
Taylor



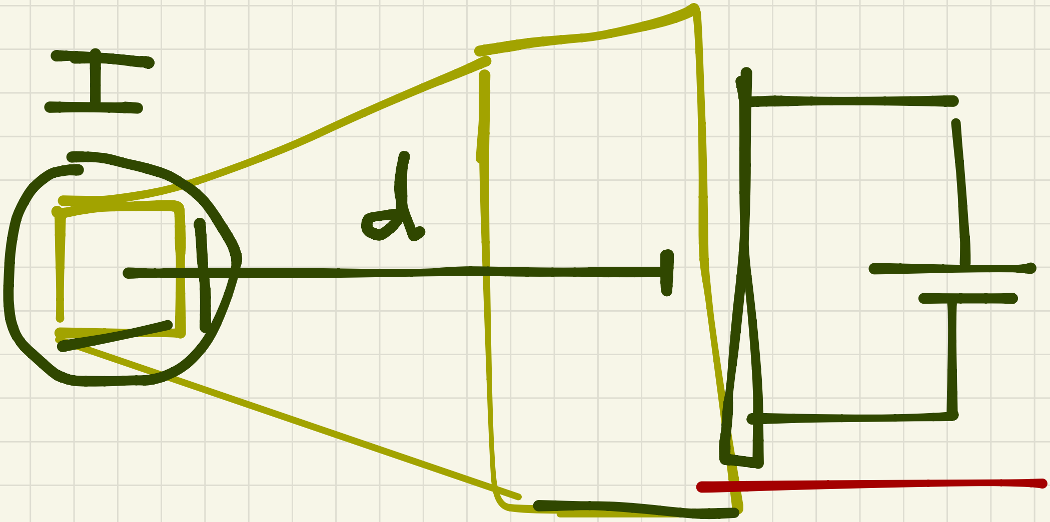
$$S \approx dS$$

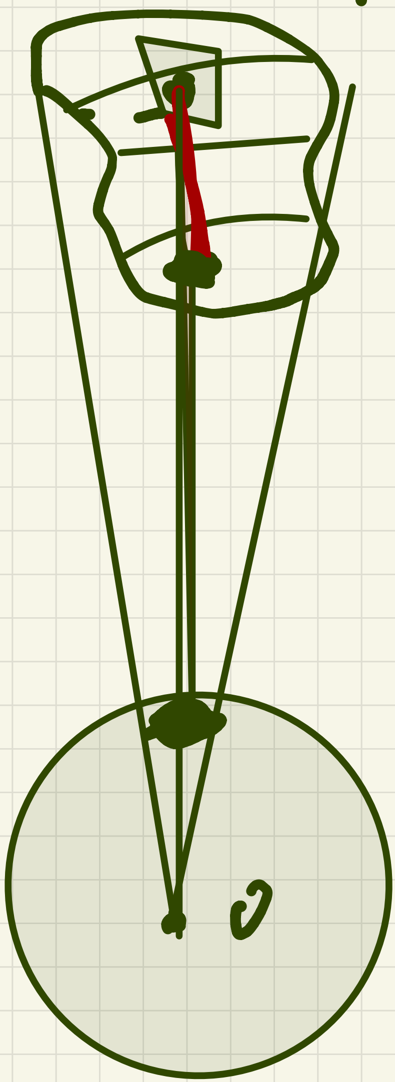


$$\left\| \frac{\partial}{\partial u} s(u,v) \times \frac{\partial}{\partial v} s(u,v) \right\|$$



$$\int_u f \, d(u,v) = \int_u \int_V \underline{f \, dx} \; dy$$

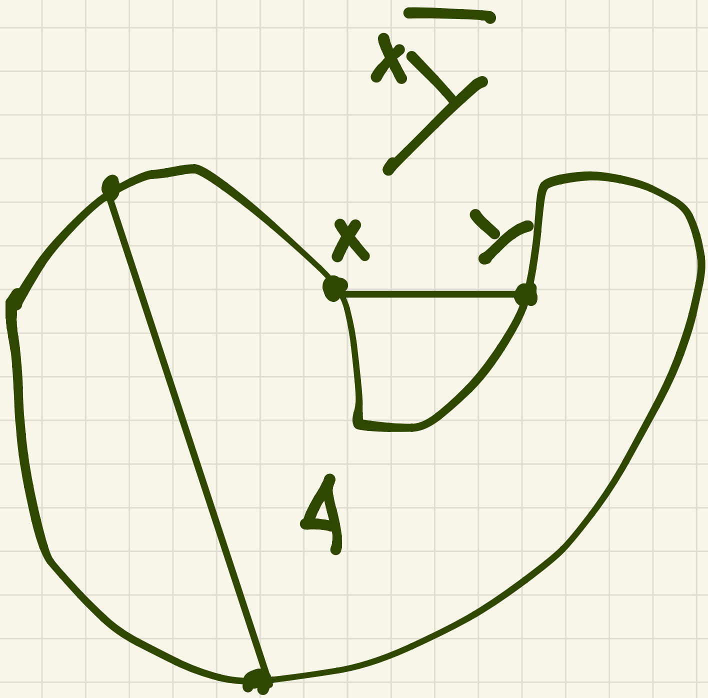

d



A

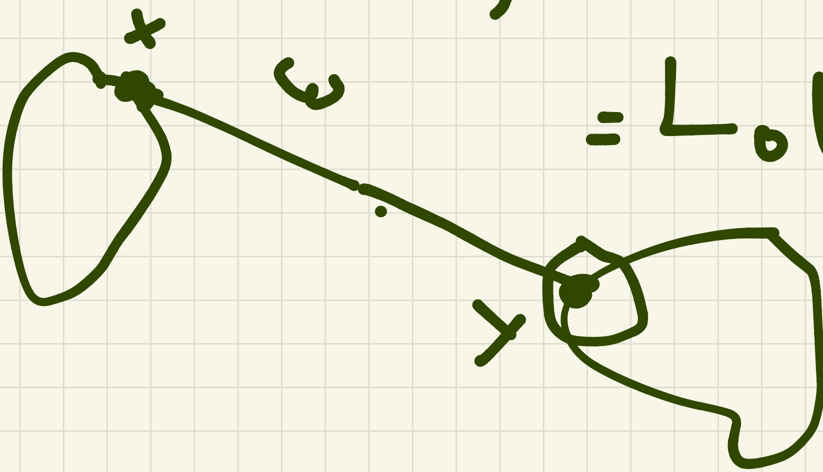
$\int_A$

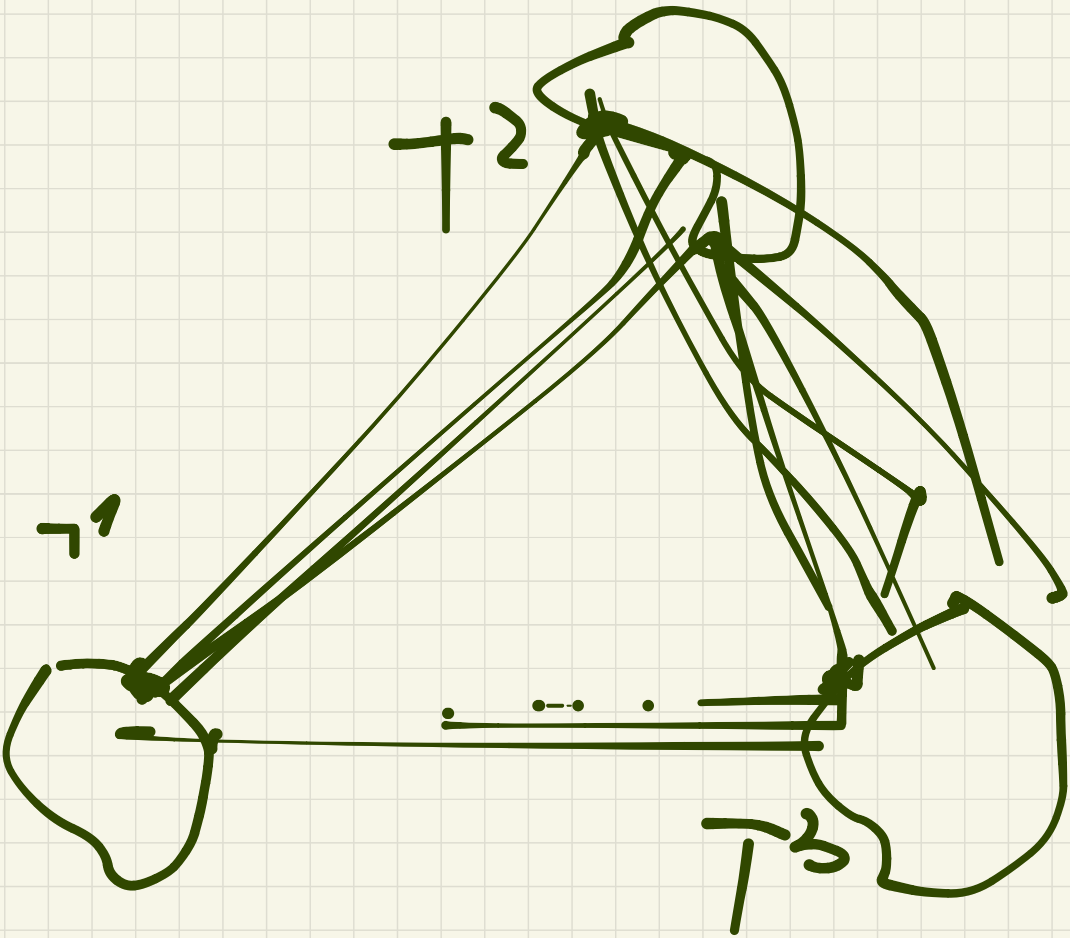
$\int_{\pi(A)}$

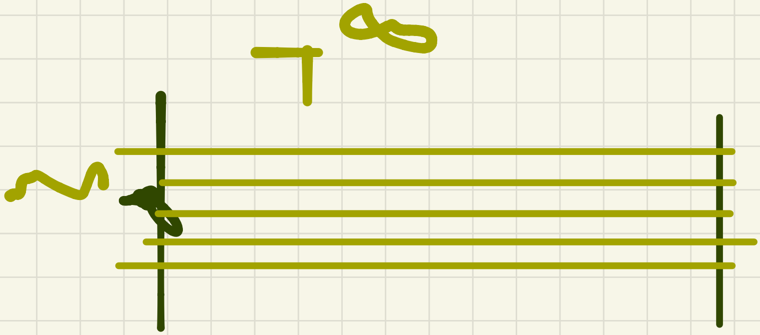


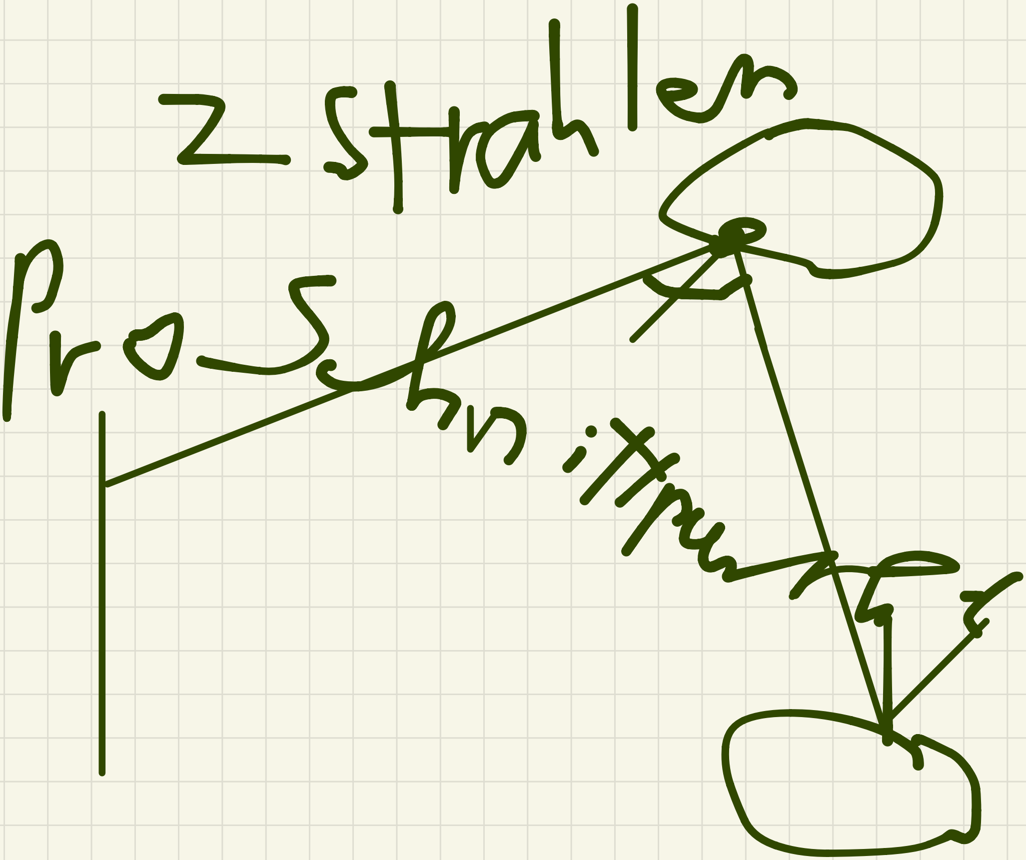
$\overline{xy}$
x
y
A

$$L_i(x, \omega) = L_o(y, -\omega)$$



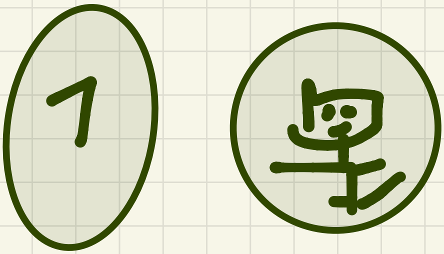

T^1
T^2
T^3


T^∞



$$\int \hat{=} \text{ 2 Strahlen}$$

1 0



N

1 1 0 0 1 0 1 1 1 0 ...

X_1 X_2 X_i

Samples

?

$$\frac{1}{N}\sum_{i=1}^{N} X_i \xrightarrow[N \to \infty]{} \frac{1}{2}$$

$$= \boxed{\sum x_i P(x_i) =} \mathbb{E}(X)$$

$$N \to \infty$$

$$\frac{1}{N} \sum_i X_i \longrightarrow \mathbb{E}(X)$$

Münzwurf = 1/2

Würfel = 3,5

$X : \{ \boxed{1},$ ☺ $\} \longrightarrow \{1, 0\}$

ZV

$$X : \mathbb{R}^n \longrightarrow \mathbb{R}$$

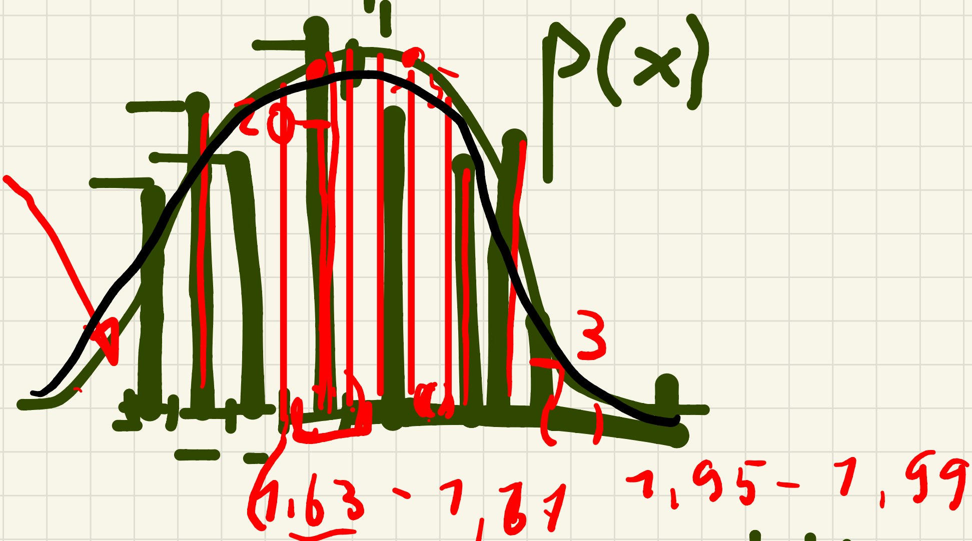
$$\mathbb{E}(X) = \int_{\mathbb{R}^n} X(x) \cdot \underline{p(x)} \, dx$$

$$\boxed{\frac{1}{N} \sum_{i=1}^{N} X_i} \;\underset{N \to \infty}{\hat{=}}\; \mathbb{E}(X)$$

<u>Samples</u>

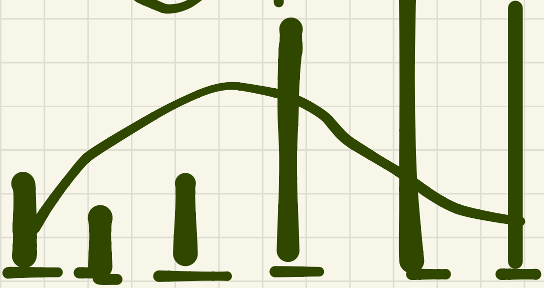
NV

<— 1,65 —>



EV

BINS



$$g(x) = \frac{f(x)}{p(x)}$$

ZE g muss durchführbar sein.

$$\boxed{\int f(x)\,dx \mathrel{\hat{=}} \frac{1}{n}\sum_{i=1}^{n} g(x_i)}$$

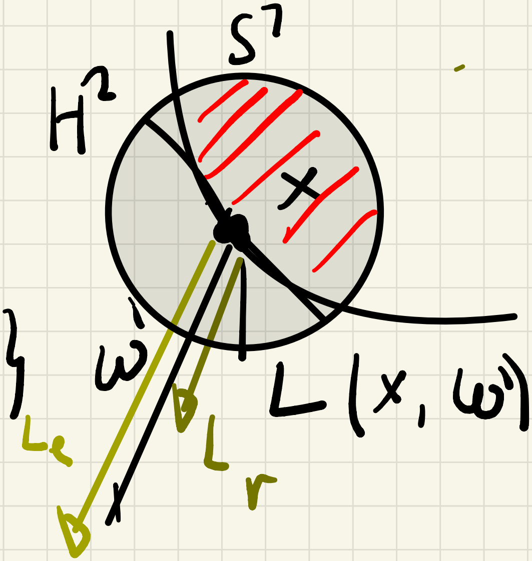
$p(x)$ Dichte

$$\int p(x)\,dx = 1$$

$$\int f(x)\cdot p(x) = \mathbb{E}(f(x))$$

$$\mathrel{\hat{=}} \frac{1}{n}\sum_{i=1}^{n} f(x_i)$$

Gleichverteilung lässt sich gut sampeln auf Computer (Signale auf Motherboard sind quasi zufällig -> Gleichverteilung)



$$\Omega = \{\Omega_1, \dots, \Omega_N\}$$

alle Oberflächen

$$L(x,\omega') = L_e(x,\omega') + L_r(x,\omega')$$

$$\| $$

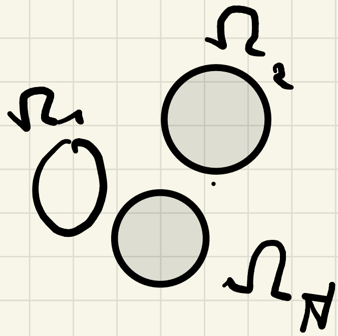
$$\int_{H^2} f_r(\omega, \omega'') \cdot \cos(\theta)\, d\omega$$

$$= \int_\Omega f_r\left(\omega, \frac{\overrightarrow{xy}}{\|xy\|}\right) \cdot g(x,y) \cdot dy$$

x fest

$$\int_{\Omega} f_r(y) \cdot g(y) \cdot dy$$

$$\Omega_i \cap \Omega_j = \emptyset$$



$=$

$$\int_{\Omega_1} f_{r_1}(y) \cdot g_1(y)\, dy + \dots \int_{\Omega_n} f_{r_n}(y)\, g\, dy$$

Fazit

müssen Lösen Oberfl. Int.

$$\int_{O} f(y) \cdot g(y)\, dy$$

$$\int_0 f(y)\cdot g(y)\,dy$$

$T:$ 

$$\boxed{= \int_a^b\int_c^d f(T(y_1,y_2))\cdot g(y_1,y_2)\cdot \underbrace{|\det T'(y)|}_{\text{inv. El.}}\,dy}$$

$$= \int_a^b\int_c^d h(y_1,y_2)\,dy_1\,dy_2$$

$$\boxed{\approx \frac{1}{N+M}\sum_{i=1}^{b}\sum_{j=1}^{m}\frac{h(y_i,y_j)}{p(y_i,y_j)}}$$

p frei wählbar

Dichte:

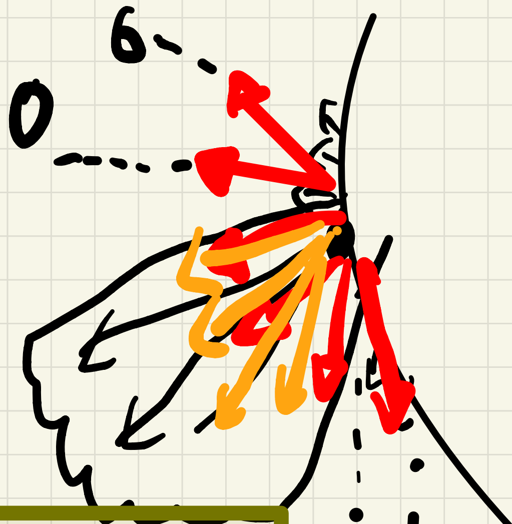
$$\int_a^b \int_c^d p(y_1, y_2)\, dy_1\, dy_2 = 1$$

p $\hat{=}$ gleichvert. auf

$(a,b) \times (c,d)$

$$\frac{1}{(b-a)\cdot(d-c)} \cdot 1_{(a,b)\times(c,d)}(x)$$

$p = $ gleichvert.



$$\boxed{p(y) \triangleq \frac{f_r(y)}{c}}$$

$c = \int f_r \ldots$

$p(y) \sim f_r(y)$

import. Sampl.

Echtzeit:

1) wenig Samples

+ OpenGL

2) wenig Samples

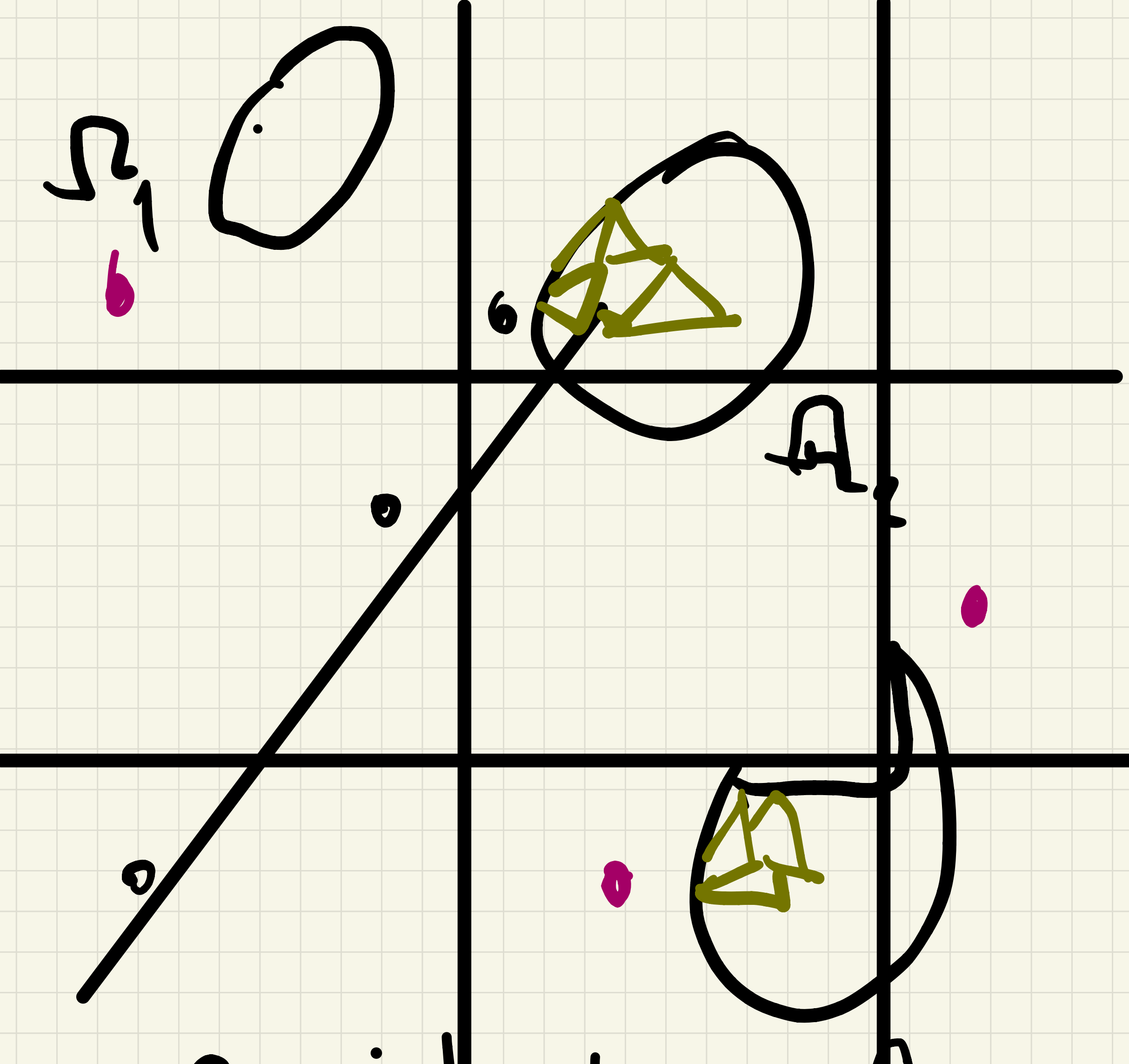
+ Rauschfilter

Bildfilter

CNN

3) Radiosity + OpenGL

4) Hardware based.



Bereichsabfrage Ω_N

→ Datenstruktur

kD-Tree