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Identity is Irreducibly Relational

A Critique of Primitive Identity from ZFC to Homotopy Type Theory

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Abstract

This paper argues that identity is irreducibly relational: the statement $A = A$ presupposes that A is defined, and definition requires distinction from a background. I develop this thesis at three levels: *conceptual* (definition requires distinction), *formal* (examining how set-theoretic and type-theoretic foundations treat identity), and *historical* (engaging the literature from Leibniz through Kripke). Against the standard view that identity is primitive in first-order logic, I argue that this primitiveness reflects a genuine conceptual difficulty that modern foundations—particularly Homotopy Type Theory and Univalent Foundations—have begun to resolve by treating identity as constituted by structural equivalence. The extensionality axiom of set theory already makes identity relational for sets; the univalence axiom generalizes this insight. I conclude that the trajectory of foundational mathematics vindicates a relational conception of identity, with implications for metaphysics, philosophy of mathematics, and the "hard problems" that arise when transformation fails to preserve structure.

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Note on Prior Work

This paper is version 2.3.0. Version 2.0.0 revised the initial draft following peer review, refining the Referential Set definition to avoid circularity by invoking a primitive distinguishability relation. Version 2.1.0 addresses further referee comments by: (1) clarifying the metatheoretic status of the δ-framework (Remark 2.5); (2) acknowledging the metatheoretic character of the central theorem's proof; (3) engaging the Hilbert-Bernays definition of identity in higher-order logic; and (4) correcting the treatment of $R(\emptyset)$ to avoid improper class-theoretic claims. Version 2.2.0 adds a new objection addressing the conflation of syntactic primitiveness with ontological independence, clarifying that the paper concerns the nature of identity as a dynamic, not the vocabulary status of the symbol '='. Version 2.3.0 addresses remaining peer review critiques by: (1) adding a subsection on free logic, showing formal precedent for the existence presupposition in identity claims (Lambert, Bencivenga); (2) hedging claims about HoTT identity types to distinguish syntactic primitiveness from the model-theoretic path interpretation; (3) adding scope conditions for the univalence axiom regarding universe levels and structure encodings; and (4) recasting the "no intrinsic properties" consequence in conditional form to clarify its dependence on the relational thesis.

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All errors, omissions, and interpretive limitations remain the author's responsibility.

Methodologies: Research methodologies and reproducibility practices are documented at `farzulla.org/methodologies`.

1 Introduction: The Presupposition in $A = A$

The law of identity—$A = A$—appears to be the most primitive and unassailable claim in logic. It seems to state something that requires no justification, admits no exception, and presupposes nothing.

This paper argues that appearances deceive. The statement $A = A$ presupposes something: that A is defined. And definition is not primitive. To define A is to distinguish A from what is not A. This requires reference to something external to A.

Therefore: identity presupposes reference. The law of identity is not foundational but derivative. Something more fundamental underlies it: *relational structure*.

1.1 Distinguishing the Thesis

Before proceeding, I must distinguish my thesis from nearby positions:

- **Not Geach's relative identity.** Geach (1967) argued that there is no absolute identity—only identity relative to a sortal. I maintain that identity is *absolute* (not sortal-relative) but *relational* (constituted by referential structure). Two things that are the same F are simply the same, period—but what makes them the same is their relational constitution, not an intrinsic property of "sameness."
- **Not mere sortalism.** Wiggins (2001) argues that sortals govern identity criteria without relativizing identity itself. I go further: the need for sortals to *apply* identity claims reveals that identity is itself relational, not merely that our *access* to identity is sortal-governed.
- **Not epistemological.** The claim is not that we need reference to *know* what A is, but that A needs reference to *be defined at all*. This is an ontological thesis about what identity consists in.

1.2 The Argument in Brief

1. For $A = A$ to be meaningful, A must be defined.
2. For A to be defined, A must be distinguishable from something.
3. Distinguishability requires reference to what A is not.
4. Therefore, identity presupposes reference: $(A = A) \implies R(A) \neq \emptyset$.

1.3 Structure of the Paper

Section 2 presents the formal thesis. Section 3 engages the philosophical literature on identity, including Kit Fine's grounding framework. Section 4 examines how ZFC treats identity, arguing that extensionality already makes identity relational for sets. Section 5 develops the category-theoretic perspective, from Lawvere's structural set theory through the Yoneda lemma to higher categories. Section 6 shows how Homotopy Type Theory vindicates the relational thesis by treating identity as path-structure. Section 7 addresses the identity problem in mathematical structuralism and shows how HoTT dissolves it. Section 8 surveys contemporary analytic metaphysics, showing how multiple traditions converge on relational identity. Section 9 addresses objections. Section 10 draws out consequences. Section 11 synthesizes the convergence argument.

2 The Formal Thesis

Definition 2.1 (Referential Set)**.** Let $\delta(A, x)$ be a primitive relation of distinguishability between A and x. For any entity A, the **referential set** $R(A)$ is the set of entities from which A is distinguishable:

$$R(A) = \{x : \delta(A, x)\}$$

Axiom 2.2 (Irreflexivity of Distinguishability). No entity is distinguishable from itself:

$$\forall A\, \neg\delta(A, A)$$

Axiom 2.3 (Non-Triviality)**.** For any defined entity A, there exists some x from which A is distinguishable:

$$\mathrm{Def}(A) \implies \exists x\, \delta(A, x)$$

Remark 2.4 (Avoiding Circularity)*.* This formulation avoids presupposing identity in the definition of $R(A)$. The earlier version included "$x \neq A$" in the set-builder, which circularly invoked non-identity. Here, the condition $x \neq A$ is *derived* from Axiom 2.2: since $\neg\delta(A, A)$, $A \notin R(A)$ follows automatically. The primitive distinguishability relation δ thus does the work without presupposing the identity relation it aims to ground.

Remark 2.5 (Status of the Framework)*.* The framework presented here is not itself a formal theory within a fixed logical system but a *schema* that can be instantiated within various foundational settings. In ZFC or NBG, δ might be interpreted as a definable relation on the domain of discourse; in HoTT, as a proposition-valued relation on types; in a topos, via the internal logic using a subobject classifier. The schema captures a structural insight—that definition requires distinction—while remaining neutral on which foundation implements it. The "theorems" that follow are therefore schematic: they hold in any instantiation where the axioms are satisfied. This meta-level framing is deliberate. The claim that identity presupposes definition cannot be stated *within* a system that takes identity as primitive—it is a claim *about* such systems, visible only from a metatheoretic vantage point.

Axiom 2.6 (Definition Requires Distinction)**.** An entity A is defined if and only if $R(A) \neq \emptyset$:

$$\mathrm{Def}(A) \iff R(A) \neq \emptyset$$

Together with Axiom 2.3, this entails that defined entities are distinguishable from at least one thing.

This axiom captures a fundamental insight: to define something is to say what it is by saying what it is not. A thing that cannot be contrasted with anything else is not defined—it is indistinguishable from the background, from everything, from nothing.

Theorem 2.7 (The Farzulla Identity Thesis)**.** *The identity claim $A = A$ presupposes that A is defined. Therefore:*

$$(A = A) \implies R(A) \neq \emptyset$$

Identity is not primitive. Identity is derivative of referential structure.

Proof. This is a metatheoretic argument, not a derivation within a fixed object-level formal system. The premise "$A = A$ is meaningful" is semantic: it concerns the conditions under which the formula has interpretive content rather than being an uninterpreted string.

Suppose $A = A$ is a meaningful claim. For this claim to have content—for it to say something about A—the symbol A must refer to something in the domain of discourse. That is, A must be defined. By Axiom 2.6, $\mathrm{Def}(A) \iff R(A) \neq \emptyset$. Since A is defined (otherwise $A = A$ would be a meaningless string of symbols, not a proposition), we have $R(A) \neq \emptyset$. Therefore $(A = A) \implies R(A) \neq \emptyset$.

The metatheoretic character of this argument is a feature, not a defect. The thesis concerns what is *presupposed* when identity claims are meaningful—a question that necessarily stands outside any object-level system in which identity is axiomatized. □

Corollary 2.8 (No Self-Sufficient Identity). *No entity can be self-identical in a meaningful sense without reference to something external to itself. "Pure self-identity"—identity without relation—is not false but contentless.*

3 Identity in the Philosophical Literature

3.1 The Standard View: Identity as Primitive

In first-order logic with equality, identity is typically taken as primitive. The axioms are:

$$\forall x(x = x) \qquad \text{(Reflexivity)} \tag{1}$$

$$\forall x \forall y(x = y \rightarrow (\phi(x) \rightarrow \phi(y))) \qquad \text{(Substitutivity)} \tag{2}$$

From these, symmetry and transitivity are derivable. But identity itself is "often considered a primitive notion, meaning it is not formally defined, but rather informally said to be 'a relation each thing bears to itself and nothing else' " (Gallois, 2016).

The characterization is **notably circular**: "nothing else" already invokes non-identity. This circularity is typically dismissed as unavoidable for primitive notions. I argue it is not unavoidable but *diagnostic*—it reveals that identity is not genuinely primitive but presupposes a background of distinction.

3.2 Leibniz's Principles

Leibniz proposed two principles:

- **Indiscernibility of Identicals (LL1):** If $x = y$, then for any property F, $Fx \iff Fy$.
- **Identity of Indiscernibles (LL2):** If for any property F, $Fx \iff Fy$, then $x = y$.

LL1 is uncontroversial. LL2 is disputed: Black (1952) offers the thought experiment of two qualitatively indiscernible iron spheres in an otherwise empty universe. If sound, this shows qualitative indiscernibility does not entail numerical identity.

For my purposes, the Leibnizian framework is significant because it *attempts to define identity in terms of property-sharing*—a relational characterization. The debate over LL2 concerns whether this characterization succeeds, not whether identity should be analyzed relationally at all.

3.3 The Hilbert-Bernays Definition

In higher-order logic, identity can be *defined* rather than taken as primitive (Hilbert and Bernays, 1934; Quine, 1986). The Hilbert-Bernays definition (anticipated by Leibniz) states:

$$x = y \equiv_{\text{def}} \forall P(Px \leftrightarrow Py)$$

Two objects are identical if and only if they share all properties. This eliminates identity as a primitive in favor of second-order quantification over properties.

One might think this definition already provides a relational account of identity, rendering the δ-framework superfluous. But there is a crucial difference in explanatory level. The Hilbert-Bernays definition tells us *when* two objects are identical—namely, when they are property-indiscernible. It presupposes that x and y are already well-defined terms over which properties can be predicated.

The δ-framework operates at a prior level: it concerns the conditions under which terms *enter the domain of discourse* at all. Before we can ask whether x and y share all properties, x and y must be defined—distinguishable from the background. The question "what grounds the availability of x as a subject of predication?" is not answered by the Hilbert-Bernays definition; it is presupposed.

Thus the two approaches are complementary. Hilbert-Bernays provides a relational criterion for identity among admitted objects. The δ-framework explicates what is required for objects to be admitted in the first place. The relational character of identity operates at both levels.

3.4 Free Logic and Existence Presuppositions

The δ-framework's central insight—that identity claims presuppose that their terms denote—has formal precedent in *free logic*, developed by Lambert and others (Lambert, 1991; Bencivenga, 2002). Free logics modify classical first-order logic to handle non-denoting terms systematically.

In classical logic, the schema $t = t$ (self-identity) is valid for any term t. But this validity presupposes that t denotes—that there exists something to which t refers. Free logics make this presupposition explicit. In *negative free logic*, $t = t$ fails when t is an empty term: "Pegasus = Pegasus" is not true but false or truth-valueless, because there is no Pegasus to be self-identical with.

As Nolt (2020) explains, free logics distinguish between the *outer domain* (all terms the language can form) and the *inner domain* (existing objects). Identity claims are true only when their terms denote members of the inner domain. The classical inference from $t = t$ to $\exists x(x = t)$—which would absurdly prove that anything we can name exists—is blocked.

This is precisely the formal apparatus the δ-framework invokes informally. When I argue that "$A = A$ presupposes that A is defined," I am making a claim that free logic has already formalized: the reflexivity schema requires an existence presupposition. The δ-framework adds the further claim that *being defined* (entering the inner domain) itself requires distinguishability from something—a relational condition.

The connection to free logic strengthens the thesis in two ways. First, it shows that the existence presupposition in identity claims is not a philosophical novelty but a recognized formal phenomenon (Lehmann, 2002). Second, it clarifies the paper's contribution: free logic formalizes *that* identity presupposes existence, while the δ-framework analyzes *what* existence (qua definedness) consists in—namely, relational distinguishability. The frameworks are complementary layers of the same insight.

3.5 Quine: No Entity Without Identity

Quine (1981) famously held that ontological admissibility requires clear individuation criteria: "No entity without identity." But Quine treats identity itself as primitive within first-order logic. His slogan demands criteria for *when* identity holds without analyzing *what* identity is.

This leaves a gap. Quine asks: under what conditions is x identical to y? But this presupposes we already understand what identity means. The question "when are x and y identical?" presupposes an answer to "what would it be for x and y to be identical?"

3.6 Kripke: Necessary Identity

Kripke (1980) argues that if 'a' and 'b' are rigid designators, then "$a = b$," if true, is necessarily true. This yields the *necessary a posteriori*: "Hesperus = Phosphorus" is necessary (there is no world where Hesperus is not Phosphorus) but knowable only empirically.

For my thesis, Kripke's examples are revealing. Every case of informative identity (Hesperus/Phosphorus, water/H_2O) involves discovering that two *modes of presentation* converge on the same referent. We never grasp identity directly; we discover it through the convergence of descriptions. This suggests that identity claims are always *mediated* by representational frameworks—consistent with a relational conception.

3.7 Fine: Essence, Ground, and Identity Criteria

Fine (1994) argues that essence cannot be analyzed modally. Socrates is necessarily distinct from the Eiffel Tower, but this distinctness is not *essential* to Socrates—"nothing in Socrates' nature connects him in any special way to the Eiffel Tower." What something *is*—its essence—is prior to what is necessarily true of it.

More directly relevant to our thesis, Fine (2016) argues that identity criteria are *statements of ground*. To give identity conditions for Fs is not merely to state when two Fs are the same, but to say what *grounds* their identity—what makes it the case that they are identical. This framework has profound implications: if identity requires grounding, then identity is not primitive but *derivative*. The identity of Fs is grounded in something more fundamental than identity itself.

Fine's grounding framework supports the relational thesis directly. Consider: what could ground identity if not relational/structural features? The candidates are limited. Either identity is grounded in intrinsic properties (which, as I argue, are themselves relational), or in structural position, or in equivalence relations. All three options make identity derivative of something relational. The alternative—that identity is primitive and ungrounded—conflicts with Fine's insight that identity criteria are genuinely explanatory, not merely extensionally correct.

This connection between essence, ground, and identity has been developed further by Koslicki (2020) and Correia and Skiles (2019), who explore how identity, essence, and grounding interrelate. The emerging consensus in analytic metaphysics treats identity as explicable rather than primitive—precisely the view defended here.

3.8 Wiggins: Sortals and Absolute Identity

Wiggins (2001) maintains that identity is absolute (contra Geach) but sortal-governed. When we ask whether $a = b$, we need a sortal F to ground the question: is a the same F as b?

Wiggins treats sortals as providing *criteria* for identity without making identity sortal-relative. But this raises a question: if identity *requires* sortals to be applied, and sortals relate particulars to kinds, is identity not already implicitly relational? I argue that Wiggins' framework, taken seriously, supports the relational thesis: what he treats as application-conditions are better understood as constitutive.

4 Identity in Set-Theoretic Foundations

4.1 Extensionality Already Makes Identity Relational

In ZFC, sets are identical if and only if they have the same members:

$$\forall x \forall y (x = y \iff \forall z (z \in x \iff z \in y))$$

This is the **Axiom of Extensionality**. Notice what it says: extensionality provides a *relational criterion* for identity. Two sets are the same set precisely when they stand in the same membership relations. The axiom does not "define" identity in a reductive sense—first-order logic still treats identity as primitive—but it provides the criterion by which identity is determined for sets.

This is already a relational account of identity. For any sets A and B:

$$A = B \iff \forall z (z \in A \iff z \in B)$$

Identity is not an intrinsic property that sets possess independently of their relations. Identity is *constituted* by membership relations.

One might object: but the *elements* still have primitive identity. This is true for urelements (if admitted) but not for the pure sets of ZFC, where every set is built from the empty set. And the empty set itself illustrates the relational character of identity.

4.2 The Empty Set is Relationally Defined

The Axiom of Empty Set asserts:

$$\exists x \forall y (y \notin x)$$

The empty set is defined *negatively*: it is the set with no members. But this negative definition presupposes the concept of membership. The empty set is distinguished from non-empty sets precisely by lacking what they have.

Proposition 4.1 (Empty Set is Relationally Defined)**.** *The empty set $\emptyset$ satisfies $R(\emptyset) \neq \emptyset$. That is, there exists at least one entity from which $\emptyset$ is distinguishable.*

Proof. The definition of $\emptyset$ is: $\emptyset$ is the unique set S such that $\forall x(x \notin S)$. Consider any non-empty set—for instance, $\{\emptyset\}$, whose existence is guaranteed by the Pairing axiom. The empty set is distinguishable from $\{\emptyset\}$ by the property that $\emptyset$ has no members while $\{\emptyset\}$ has one. Thus $\delta(\emptyset, \{\emptyset\})$ holds, establishing that $R(\emptyset)$ is non-empty.

We need not (and should not) attempt to *collect* all entities from which $\emptyset$ is distinguishable into a set. In ZFC, the "collection" of all non-empty sets would be a proper class, not a set. But the δ-framework does not require $R(A)$ to be a set in the ZFC sense—only that $R(A) \neq \emptyset$, which requires merely exhibiting a witness. In foundational settings where proper classes are first-class citizens (NBG, MK) or where size issues do not arise (HoTT with universe polymorphism), $R(A)$ can be given a more robust interpretation. The key claim—that $\emptyset$ is defined by its distinguishability from something—holds regardless. □

4.3 ZFC's Design Choices

A referee has noted that ZFC avoids the universal set and that complements are always relative to a given set. This is correct. But these are *design choices*—motivated by the need to avoid Russell's paradox—not forced by logic itself.

The point is not that ZFC is "circular" (a charge I retract from earlier formulations) but that ZFC's design choices *obscure* the relational character of identity while not eliminating it. Consider:

1. **No universal set**: ZFC avoids U to prevent $\{x : x \notin x\}$ from being well-formed. This is a restriction on comprehension, not a vindication of primitive identity.
2. **Relative complements**: The complement of A is always relative to some $B \supseteq A$. This means "what A is not" is always contextual—*reinforcing* rather than undermining the relational thesis.
3. **Extensionality as criterion**: Set identity is *defined* via membership. The primitiveness of identity in ZFC applies only to urelements (if admitted) and to the membership relation itself.

The upshot: ZFC does not refute the relational thesis. If anything, extensionality *instantiates* it for the domain of sets. The question is whether this relational character generalizes.

5 Category Theory and Structural Identity

Category theory provides a powerful framework for understanding identity relationally. Where set theory asks "what are the members?", category theory asks "how does this object relate to others?" This shift in perspective reveals identity as fundamentally structural.

5.1 Lawvere's Structural Set Theory

In 1964, F. William Lawvere proposed the Elementary Theory of the Category of Sets (ETCS), which axiomatizes set theory categorically rather than membership-theoretically (Lawvere, 1964). In ETCS, sets are characterized not by their elements but by their *universal properties*—how they relate to other sets via morphisms.

As Lawvere later reflected: "What my program discarded was instead the idea of elementhood as a primitive." This is a direct instantiation of the relational thesis: rather than defining sets by intrinsic membership facts, ETCS defines them by their categorical role. A set is what it does, not what it contains.

The empty set in ETCS is the *initial object*—the unique (up to unique isomorphism) object with exactly one morphism to every other object. The singleton is the *terminal object*—the unique object with exactly one morphism from every other object. These characterizations are purely relational; they say nothing about "internal structure" and everything about external relations.

5.2 The Yoneda Lemma

The Yoneda lemma formalizes a profound insight: an object in a category is *completely determined, up to isomorphism,* by its morphisms to and from other objects. More precisely, the functor $\mathrm{Hom}(A, -)$ that sends each object B to the set of morphisms from A to B fully determines A up to unique isomorphism.

This is relational identity in mathematical form. There is no "intrinsic nature" of A beyond its relational structure. What A *is*, is exhausted by how A relates to everything else. The Yoneda lemma proves that this relational characterization is complete: any two objects with the same relational profile are isomorphic.

Marquis (2013) develops the philosophical implications: category theory offers "foundations without foundationalism." Rather than building mathematics on primitive elements with primitive identity, categorical foundations build on morphisms (relations) as primary. Objects are, in a precise sense, secondary—they are nodes in a web of relations.

5.3 The Principle of Equivalence

In categorical practice, properties and constructions should be *isomorphism-invariant*: if $A \cong B$, then any categorically meaningful property of A holds of B and vice versa. This principle is so fundamental that violations are informally termed "evil" in the category theory community.

The principle of equivalence embodies a methodological commitment to relational identity. To ask whether two isomorphic objects are "really the same" is, categorically speaking, a malformed question. They are equivalent, and equivalence is all the sameness that matters. The intuition that there might be some "residual difference" between isomorphic objects reflects set-theoretic habits, not categorical reality.

This connects to Awodey (1996)'s argument that category theory instantiates mathematical structuralism: mathematical objects simply *are* positions in structures, characterized by their structural relations. The principle of equivalence is structuralism operationalized.

5.4 Higher Categories and the Richness of Identity

The development of higher category theory, particularly ∞-categories, reveals that identity has *rich internal structure* (Lurie, 2009). In an ∞-category:

- Objects are related by morphisms (1-cells).
- Morphisms are related by 2-morphisms (transformations between morphisms).

- 2-morphisms are related by 3-morphisms, and so on.
- This hierarchy continues to infinity.

At each level, "sameness" becomes equivalence at that level: isomorphism of objects, natural isomorphism of functors, equivalence of 2-morphisms, and so forth. The result is that identity is not a simple binary relation but a structure with infinitely many layers.

This undermines any conception of identity as primitive. Primitive identity would be structureless—a bare fact of sameness. But categorical identity has internal structure all the way down (or up). The identity relation between two objects in an ∞-category is itself an ∞-groupoid, which can be arbitrarily complex.

5.5 Categorical Identity Supports the Thesis

Category theory supports the relational thesis at multiple levels:

1. **ETCS**: Sets can be characterized without primitive membership, via universal properties.
2. **Yoneda**: Objects are completely determined by their morphisms—relational data exhausts identity.
3. **Equivalence**: Isomorphism-invariance treats equivalence as the natural notion of sameness.
4. **Higher categories**: Identity has rich internal structure, incompatible with primitiveness.

The next section shows how Homotopy Type Theory synthesizes these categorical insights into a foundational framework where, for types in a universe, identity *just is* equivalence.

6 Homotopy Type Theory: Identity as Path-Structure

Homotopy Type Theory (HoTT) and Univalent Foundations provide a rigorous framework in which identity is *explicitly* relational. This section argues that HoTT vindicates the Identity Thesis.

6.1 Two Levels of Identity

HoTT distinguishes two related but distinct notions of identity:

- **Identity of elements**: For terms $a, b : A$, the identity type $\mathrm{Id}_A(a, b)$ captures when a and b are identical as elements of A.
- **Identity of types**: For types $A, B : \mathcal{U}$ in a universe $\mathcal{U}$, the identity type $\mathrm{Id}_{\mathcal{U}}(A, B)$ captures when A and B are identical as types.

This distinction matters because the univalence axiom (discussed below) specifically concerns identity of types within a universe, not identity of elements within a type. The two notions interact but should not be conflated.

6.2 Identity Types as Paths

In HoTT, identity between terms a and b of type A is captured by the **identity type** $\mathrm{Id}_A(a, b)$. Syntactically, identity types are primitive: Martin-Löf type theory introduces them via formation, introduction, and elimination rules without appeal to paths. However, the homotopy-theoretic *interpretation* views identity types as spaces of paths from a to b in the space A.

- Types are *interpreted* as spaces (homotopy types).
- Terms are *interpreted* as points in those spaces.
- Identity proofs are *interpreted* as paths between points.
- Higher identity proofs are homotopies—paths between paths.

This path interpretation is model-theoretic: it tells us what identity types *denote* in the simplicial set or ∞-groupoid models, not what they *are* syntactically. Nevertheless, the interpretation is not arbitrary—it is *sound*, meaning anything provable in the syntax holds in the models. The philosophical significance lies in what this soundness reveals: a formal system with syntactically primitive identity types admits models where identity has rich relational structure.

An identity proof is not a binary truth-value but a *witness*—a constructive term showing *how* a and b are identified. On the path interpretation, this witness specifies a relational route from a to b, not an intrinsic property of "sameness."

6.3 The Philosophy of Identity Types

Ladyman and Presnell (2015) and Ladyman and Presnell (2016) provide a comprehensive philosophical analysis of identity in HoTT. They argue that the identity type's distinctive features—particularly path induction—can be justified on pre-mathematical philosophical grounds, not merely as formal conveniences.

Their central insight is that, under the homotopy interpretation, identity in HoTT is *constituted* by paths, not merely represented by them. When we prove $a = b$ by exhibiting a term of type $\mathrm{Id}_A(a, b)$, we are not discovering a pre-existing identity fact; we are *constructing* the identity through the witness itself. Multiple distinct terms of the identity type (when they exist) represent genuinely different ways of identifying a and b—the identity type has internal structure.

This supports the relational thesis directly, at least within the homotopy-theoretic interpretation. If identity were primitive in the sense of structureless, there would be exactly one "sameness fact" between identical objects. But in HoTT, the identity type $\mathrm{Id}_A(a, b)$ can be inhabited by multiple non-equal terms (in higher types). Identity is not a binary yes/no question but a *type* of identifications, each representing—on the path interpretation—a different relational route. This rich structure is incompatible with primitive, structureless identity.

Chen (2024) connects these insights to ontic structural realism and philosophy of physics, arguing that univalent foundations provide the mathematical framework for a thoroughgoing structuralism about mathematical and physical objects. The HoTT treatment of identity is not merely a technical convenience but a philosophical stance: identity is constituted by structure.

6.4 The Univalence Axiom

The univalence axiom, due to Vladimir Voevodsky, concerns types within a fixed universe $\mathcal{U}$. For types $A, B : \mathcal{U}$, it states:

$$(A =_{\mathcal{U}} B) \simeq (A \simeq B)$$

That is: for types in a universe, the type of identities between A and B is equivalent to the type of equivalences between them. Two types (in a universe) are identical precisely when they are *structurally equivalent*.

The qualification "in a universe" matters: univalence governs identity of types *qua* elements of a universe type. This is not a limitation but a precise specification of scope. For practical purposes in HoTT, types always live in some universe, so univalence applies broadly.

A technical note on scope: structure identity principles like univalence require certain hypotheses to hold. Specifically, the univalence axiom applies to types within a fixed universe $\mathcal{U}$; extending it across universe levels requires universe polymorphism or explicit transport. Furthermore, deriving that specific mathematical structures (groups, rings, categories) satisfy the univalence principle for their domains requires encoding those structures via appropriate record types and verifying that the induced notion of equivalence matches the mathematical one. These conditions are typically satisfied in practice but should not be elided in foundational discussions.

As Awodey (2014) puts it: "Univalence embodies mathematical structuralism." The axiom does not merely say equivalent things *should be treated as* identical; it says they *are* identical, in the foundational sense.

Ahrens et al. (2021) generalize this insight into *The Univalence Principle*, proving that in any univalent foundation, equivalent mathematical structures are *indistinguishable*—there is no property that can distinguish them. This is a theorem, not a philosophical interpretation: structural equivalence exhausts identity. The Univalence Principle demonstrates that the relational treatment of identity is not merely consistent but *forced* by the univalent framework.

This is the strongest possible formulation of relational identity. Identity just *is* structural equivalence. There is no residual "primitive identity" over and above the relational structure. Tsementzis (2017) argues that univalent foundations thereby constitute the first mathematically rigorous implementation of philosophical structuralism.

6.5 HoTT Vindicates the Identity Thesis

The development of HoTT shows that:

1. **Identity can be formalized relationally.** The identity type $\mathrm{Id}_A(a, b)$ treats identity as constituted by paths—relational structure—rather than an intrinsic property.
2. **Univalence makes structuralism foundational.** The univalence axiom identifies equivalence with equality, embedding a relational conception of identity into the foundations.
3. **Higher identity structure emerges.** Paths can have paths between them (homotopies), revealing that identity has internal relational structure invisible on the classical view.
4. **This is not a philosophical preference but a mathematical discovery.** HoTT is consistent and has been verified in proof assistants. The relational treatment of identity *works*.

One might object that HoTT is just one foundational system among many. True, but HoTT resolves long-standing problems (like Benacerraf's problem about the nature of mathematical objects) that plagued set-theoretic foundations. It represents the cutting edge of foundational mathematics, and its treatment of identity as relational suggests this is the direction of travel.

7 The Identity Problem in Mathematical Structuralism

The debate over identity in mathematical structuralism provides a natural test case for the relational thesis. Structuralism holds that mathematical objects are positions in structures, not independently existing entities. But this raises a sharp question: how can structuralism account for the identity and distinctness of mathematical objects?

7.1 Benacerraf's Problem

Benacerraf (1965) posed a foundational puzzle: natural numbers can be identified with multiple, mutually incompatible set-theoretic reductions. In von Neumann's construction, $2 = \{\emptyset, \{\emptyset\}\}$; in Zermelo's, $2 = \{\{\emptyset\}\}$. Both work equally well for arithmetic, yet they are different sets.

Benacerraf's conclusion: numbers are not "really" any particular sets. They are, at best, *positions in the natural number structure*. What matters is that something plays the "2-role"—succeeds 1, precedes 3, is even—not what intrinsic properties it has.

This is already a relational conception of identity. The number 2 is not defined by what it *is* but by where it *sits* in the structure. Identity is positional, not intrinsic.

7.2 The Identity Problem

Keranen (2001) pressed a sharp objection to structuralism. If mathematical objects are mere positions in structures, how can structuralism distinguish structurally indiscernible objects?

Consider the complex numbers i and $-i$. They are *structurally indiscernible*: any structural property true of one is true of the other. They are both square roots of -1, related symmetrically to all other complex numbers. If identity is purely structural, what distinguishes them?

This is the *identity problem for realist structuralism*. The structuralist seems forced to either:

1. Deny that $i \neq -i$ (absurd—they are demonstrably distinct);
2. Admit non-structural facts about mathematical objects (abandoning structuralism);
3. Find some structural ground for their distinctness.

7.3 Proposed Responses

Several responses have been offered:

Weak Discernibility (Ladyman, 2005): Objects can be discerned through *irreflexive relations*. If there is a relation R such that $R(i, -i)$ but not $R(i, i)$, then i and $-i$ are weakly discernible. For complex numbers, consider: $i + (-i) = 0$ while $i + i \neq 0$. This provides a structural, if indirect, ground for distinctness.

Primitive Identity (Shapiro, 2008): Shapiro (1997) initially held that structuralism can invoke primitive identity for positions. Objects are identical or distinct as a brute fact, irreducible to structural properties. This preserves structuralism about the *nature* of mathematical objects while conceding that *identity* is not reducible.

HoTT Resolution (Awodey, 2014): In univalent foundations, the identity problem does not arise in its classical form. Identity is structural equivalence, full stop. The question "are i and $-i$ the same?" is answered by asking whether there is a path between them in the relevant type. There is not: complex conjugation swaps them, but this is an automorphism, not an identity path.

7.4 HoTT Dissolves the Problem

The HoTT resolution is not merely another response but a *dissolution* of the problem. The identity problem arises from a gap between "structural sameness" and "numerical identity." HoTT closes this gap by definition: identity *just is* structural equivalence. There is no "residual" identity question once equivalence is settled.

This completes an arc in the philosophy of mathematics:

1. **Early structuralism**: Objects are positions, but identity remains primitive.
2. **Identity problem**: Exposes tension in combining structural objects with primitive identity.
3. **HoTT resolution**: Identity = equivalence; no gap remains for the problem to exploit.

The relational thesis predicts this trajectory. If identity is constituted by relational structure, then any framework that treats identity as primitive while making objects structural will face tensions. HoTT resolves these tensions by making identity relational at the foundational level.

8 Contemporary Metaphysics of Identity

The relational thesis finds support not only in mathematical foundations but in contemporary analytic metaphysics. Multiple research programs converge on treating identity as grounded, contextual, or structural rather than primitive.

8.1 Identity Criteria as Ground

Fine (2016) argues that identity criteria are *statements of ground*—they say what makes it the case that two things are identical. To give identity conditions for Fs is not merely to describe when Fs are the same, but to explain *why* they are the same.

This has a crucial implication: if identity has grounds, identity is not primitive. The grounding framework in contemporary metaphysics (Correia and Skiles, 2019) treats explanatory relations as fundamental. If identity is grounded in other facts (about essence, structure, or relations), then identity is derivative.

What grounds identity? The candidates Fine considers—qualitative properties, individual essences, structural roles—are all relational in the relevant sense. Identity is grounded in how things relate to other things (including properties, kinds, and structures), not in a primitive "sameness" fact.

8.2 Against Primitive Haecceities

Koslicki (2020) systematically evaluates proposals for cross-world identity criteria. She considers haecceities (primitive "thisnesses"), individual essences, qualitative properties, and structural/formal features.

Koslicki argues against haecceities on multiple grounds: they are explanatorily vacuous, multiply ontological commitments unnecessarily, and conflict with our best scientific understanding. Instead, she defends *individual forms*—structural/formal features that ground identity without invoking primitive thisness.

This is a vote for relational identity from within mainstream analytic metaphysics. If individual forms—essentially structural features—ground identity, then identity is constituted relationally.

8.3 Ontic Structural Realism

French and Ladyman (2003) develop *ontic structural realism* (OSR) as a response to challenges from physics, particularly quantum mechanics. OSR holds that structure is ontologically fundamental; objects (if they exist at all) have only "contextual identity" emerging from their structural positions.

The moderate version maintains objects as metaphysically "thin"—real but not fundamental. The radical version eliminates objects entirely in favor of pure structural relations. Both versions treat identity as derivative: objects are individuated by their structural role, not by primitive identity.

French and Krause (2006) extend this analysis to quantum particles, arguing that the identity and individuality of particles cannot be understood on classical models. Particles may lack primitive identity altogether, or their identity may be "transcendentally constituted" by structural position.

8.4 Identity as Invariance

Simons (2000) offers a distinctive account: continuants (persisting objects) are "invariants among occurrents under equivalence relations." What persists through time is what remains *invariant* under transformations.

This is explicitly a relational/structural account. Identity through time is not a primitive sameness but a structural fact: the object at t_1 and the object at t_2 are identical because they are related by an appropriate equivalencepreserving transformation. Identity is invariance under equivalence.

Sider (2020) explores "structuralism about individuals"—the view that individuals are secondary, patterns of relations primary. On this view, the fundamental level contains only relational structure; objects are derived from structural nodes. Sider considers this a live metaphysical option, noting that much of our best physics points in a structural direction.

8.5 Convergence in Metaphysics

The convergence is striking. From different starting points and methodological traditions, contemporary metaphysics arrives at similar conclusions:

- **Grounding**: Identity criteria are grounding claims; identity is derivative (Fine).
- **Anti-haecceitism**: Primitive thisness is rejected; structural forms individuate (Koslicki).
- **OSR**: Structure is fundamental; objects have contextual identity (French, Ladyman).
- **Invariance**: Identity is invariance under transformation (Simons).
- **Structuralism**: Individuals are secondary to relational structure (Sider).

This convergence provides abductive support for the relational thesis. When multiple independent research programs reach similar conclusions, the most plausible explanation is that they have tracked something real. The relational character of identity is not a philosophical prejudice but a discovery.

9 Objections and Replies

9.1 Objection: Identity Remains Primitive as a Logical Symbol

"You've shown semantic preconditions for meaningful identity claims. But this doesn't make the identity predicate itself non-primitive—'=' remains in the primitive vocabulary of first-order logic."

Reply: This objection conflates syntactic primitiveness with ontological independence. The symbol '=' appears in the primitive vocabulary—granted. But what does this symbol *denote*? Not a thing, but a *dynamic*: a relation between the terms flanking it.

Consider an analogy: '+' appears primitively in arithmetic. Does addition therefore exist as an ontological entity, independent of quantities being added? Obviously not. '+' represents the dynamic of combining; it has no content apart from the operands it relates. The same holds for '='. The symbol represents the dynamic of identity *between* things. Without relata, there is nothing for '=' to represent.

The objection treats "primitive in the formal vocabulary" as if it entailed "ontologically independent of what it relates." But these are entirely different claims. Syntactic primitiveness means only that the symbol is not *defined via other symbols* in the language. It says nothing about whether the *concept* the symbol expresses is independent or relational. The identity symbol is syntactically primitive; the identity *dynamic* it represents is constitutively relational. These claims are perfectly compatible.

To insist that identity is "really" primitive because '=' is listed in the vocabulary is to mistake the menu for the meal. The paper is not about symbols; it is about what identity *is*—and what it is, is a dynamic between things, not a thing itself.

9.2 Objection: This Conflates Semantic and Syntactic Levels

"You move between claims about what terms mean (semantics) and claims about formal axioms (syntax) without being clear about which level you're operating at."

Reply: The thesis operates primarily at the semantic level: it concerns what is required for identity claims to be meaningful, not just formally derivable. But the semantic claim has syntactic consequences: if identity requires reference, then any formal system that treats identity as primitive is implicitly relying on a background semantic interpretation that supplies the referential structure.

The point is that the standard characterization of identity ("a relation each thing bears to itself and nothing else") is *circular at the semantic level.* This circularity is not a syntactic defect but a diagnostic of the fact that identity is not genuinely primitive.

9.3 Objection: Extensionality Solves the Problem

"In set theory, extensionality defines identity for sets. So identity is already relational. What more do you want?"

Reply: This objection *supports* the thesis rather than refuting it. Extensionality shows that for sets, identity is indeed relational. The question is whether this generalizes beyond sets.

For pure mathematics, HoTT shows it does: univalence generalizes extensionality to all types. For metaphysics, the question is whether the relational character of mathematical identity reflects something about identity as such. I argue it does: the success of relational treatments in foundations suggests that relationality is not a peculiarity of sets but a feature of identity itself.

9.4 Objection: Identity in First-Order Logic is Primitive by Design

"First-order logic treats identity as primitive for good reason: it enables a simple, well-understood framework. Your philosophical complaints don't undermine this."

Reply: The question is not whether treating identity as primitive is *useful* but whether it is *correct*—whether it captures what identity actually is. I grant that primitive identity is a useful idealization for many purposes. But the emergence of HoTT, where identity is explicitly relational and the framework is still well-understood and usable, shows that the primitive treatment is not forced on us.

When a useful idealization conflicts with a more accurate picture, we should note the idealization while preferring the accurate picture for foundational purposes.

9.5 Objection: What About Necessary Beings?

"Couldn't there be a necessary being whose identity is primitive—a being that just IS, without needing definition through contrast?"

Reply: Necessity of existence does not entail primitiveness of identity. Even a necessary being must be distinguished from contingent beings to be identified *as* necessary. The concept of "necessary existence" is defined against "contingent existence." The necessary being, if it exists, is defined by what it is not: not contingent, not dependent, not mutable. These negations constitute its identity.

9.6 Objection: This is Just Epistemology

"You've shown that we need reference to know what A is. But that's about knowledge, not being."

Reply: The thesis is ontological. Consider: what would it mean for A to be self-identical if A is not defined? Self-identity of an undefined entity is not a deep metaphysical truth—it is a meaningless string of symbols.

If one insists that A could be self-identical while being undefined, one must explain what "A" refers to in the claim "$A = A$." If it refers to nothing, the claim says nothing. If it refers to something, that something is defined, and definition requires distinction.

10 Consequences

10.1 On Intrinsic Properties

If identity is constituted relationally as argued above, *then* a strong consequence follows for the metaphysics of properties: there can be no properties that entities have "in themselves," wholly independent of their relations. This is a conditional claim, not a freestanding theorem—it depends on accepting the relational thesis developed in the preceding sections.

The reasoning runs as follows. Suppose an entity's identity is constituted by its relational position. Then any property P that individuates or partially determines what the entity *is* must figure in that relational constitution. A property that played no role in any relation—that was completely "invisible" to the entity's environment—would be epiphenomenal with respect to identity. It could vary without affecting what the entity is. But then it would not be a property of the entity in any robust sense; it would be an idle wheel.

One might resist this consequence by distinguishing identity-constituting relations from other relations, allowing "intrinsic" properties that do not bear on identity. This is a coherent position, but it requires explaining why some properties escape the relational web while others do not. The burden of argument shifts to the defender of intrinsic properties.

This consequence might seem to conflict with physics, which speaks of "intrinsic properties" like mass and charge. But mass is defined operationally by gravitational and inertial behavior—relations to other masses and to spacetime. Charge is defined by electromagnetic interactions. What appear to be intrinsic properties are dispositional: capacities for certain relations. The physics case thus supports rather than undermines the conditional: if identity is relational, so are the properties physics attributes to entities.

10.2 Reality is Self-Referential

The Identity Thesis implies that reality is a closed system of mutual reference. There is no "outside" from which entities get their identity. Everything defines everything else in a web of relational constitution.

This is not solipsism or idealism. It is **relational realism**: relations exist, and what we call "entities" are stable patterns in the web of relations.

10.3 Foundation for Transformation Theory

When identity is relational, the central question becomes: *when does transformation preserve identity?* If A is defined by its relations, and those relations change, at what point does A cease to be A?

The answer: **identity is preserved under transformation iff the transformation respects the equivalence relations constituting that identity.** This principle—which I develop elsewhere as the *Preservation Principle*—governs:

- Coarse-graining in statistical mechanics
- Scale transition in institutional dynamics
- Temporal sampling in physics
- Nominalization in language

In each case, apparent complexity at the coarse-grained level—memory terms, aliasing, pseudo-depth—emerges as an artifact of transformation failure. The structure was there; the transformation failed to preserve it.

11 Conclusion

The law of identity is not foundational. $A = A$ presupposes that A is defined, and definition requires reference to what A is not. Identity is therefore derivative of referential structure.

11.1 The Convergence of Four Traditions

This thesis finds support from four independent research programs, each with different methodologies, starting points, and concerns:

Homotopy Type Theory. Identity types are path spaces; identity is constituted by the paths connecting points, not by an intrinsic sameness property. The univalence axiom makes structural equivalence identical to equality—literally, not merely analogically. As Ladyman and Presnell (2016) emphasize, this is not a philosophical interpretation but a mathematical fact.

Category Theory. The Yoneda lemma proves that objects are completely determined by their morphisms. ETCS (Lawvere, 1964) axiomatizes set theory without primitive membership. The principle of equivalence treats isomorphism as the natural notion of sameness. Marquis (2013) draws the conclusion: categorical foundations treat relational structure as primary, objects as secondary.

Mathematical Structuralism. The trajectory from Benacerraf's problem through Keranen's identity problem to the HoTT resolution traces a path toward relational identity. Hellman and Shapiro (2018) survey this landscape, noting that the most promising structuralist foundations are those that treat identity as constituted by structure rather than primitive.

Analytic Metaphysics. Fine's grounding framework, Koslicki's rejection of haecceities, ontic structural realism, and Simons' invariance account all converge on treating identity as derivative. As Sider (2020) notes, structuralism about individuals—the view that relational structure is primary and individuals secondary—is a live metaphysical option supported by our best physics.

This convergence is significant. These traditions developed largely independently, with different motivations:

- HoTT emerged from computer science and homotopy theory.
- Category theory from algebraic topology and universal algebra.
- Structuralism from philosophy of mathematics.
- OSR from philosophy of physics and quantum mechanics.
- Grounding metaphysics from debates about fundamentality.

Yet they reach similar conclusions about identity. When independent inquiries converge, the most plausible explanation is that they have tracked something real. The relational character of identity is not an artifact of any particular methodology but a feature of identity itself.

11.2 The Trajectory of Foundations

The historical trajectory confirms this conclusion. Foundational mathematics has moved steadily from treating identity as primitive toward treating it as constituted by structure:

- **1889**: Peano's axioms treat identity primitively.
- **1908**: Zermelo's axiomatization uses extensionality—relational identity for sets.
- **1964**: Lawvere's ETCS drops primitive membership entirely.
- **2009**: Higher category theory reveals identity has rich internal structure.
- **2013**: HoTT makes univalence foundational—identity *is* equivalence.

This is not random drift but directed progress. Each step makes identity more explicitly relational, because that is what works mathematically. The relational thesis predicted this trajectory; the trajectory confirms the thesis.

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11.3 Consequences and Future Directions

If identity is relational, then apparent complexity in transformed descriptions—memory terms in coarse-grained dynamics, aliasing in sampled signals, pseudo-depth in nominalized processes—are artifacts of transformation failure. The structure was always there. The transformation failed to preserve it.

This insight opens several research directions:

- **Transformation theory**: When does transformation preserve identity? The Preservation Principle provides a framework.
- **Hard problems**: Consciousness, personal identity, institutional persistence—these may be problems of transformation failure, not metaphysical mysteries.
- **AI and formalization**: Systems that treat identity primitively may face systematic difficulties that relational approaches avoid.

Identity is relational. Transformation that respects relation preserves identity. The rest is artifact.

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