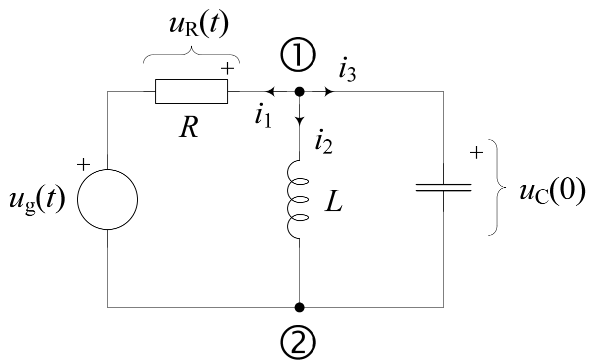
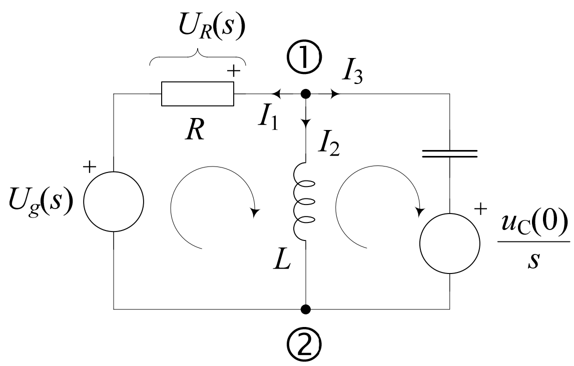
KZN-KZS

3. Za električni krug na slici izračunati napon $U_R(s)$ ako su zadane normalizirane vrijednosti elemenata R =1, L = 1,C = 1, $u_C(0)$ = 2, te $u_g(t)$ = $S(t)$. Koristiti KZS i KZN te oznake grana i čvorova prema slici.



Rješenje: Primjena $\mathcal{L}$-transformacije



$N_b = 3$ (broj grana)

$N_v = 2$ (broj čvorova)

Broj jednadžbi $\text{KZS} = N_v - 1 = 2 - 1 = 1$

Broj jednadžbi $\text{KZN} = N_b - N_v + 1 = 3 - 2 + 1 = 2$

Jednadžbe Kirchhoffovih zakona (3 jednadžbe):

1) $I_1 + I_2 + I_3 = 0$ KZS

2) $-U_1 + U_2 = 0$ KZN

3) $-U_2 + U_3 = 0$ KZN

Naponsko – strujne jednadžbe grana (3 jednadžbe):

4) $U_1 = I_1 \cdot R + U_g$

5) $U_2 = I_2 \cdot sL$

6) $U_3 = I_3 \cdot \frac{1}{sC} + \frac{u_C(0)}{s}$

Sustav ima ukupno $2N_b$=6 jednadžbi i 6 nepoznanica (sve struje i svi naponi grana)

Naponsko – strujne jednadžbe grana uvrstimo u (1) jednadžbu:

$$\left.\begin{aligned} &4)\ I_1 = \frac{U_1}{R} - \frac{U_g}{R} \\ &5)\ I_2 = \frac{1}{sL} \cdot U_2 \\ &6)\ I_3 = sC \cdot U_3 - C \cdot u_C(0) \end{aligned}\right\} \to (1)$$

$$1)'\ \frac{U_1}{R} - \frac{U_g}{R} + \frac{1}{sL} \cdot U_2 + sC \cdot U_3 - C \cdot u_C(0) = 0 \Rightarrow \frac{U_1}{R} + \frac{1}{sL} \cdot U_2 + sC \cdot U_3 = \frac{U_g}{R} + C \cdot u_C(0)$$

$$2)\ U_1 = U_2 \to (1)$$

$$3)\ U_2 = U_3 \to (1) \Rightarrow U_1\left(\frac{1}{R} + \frac{1}{sL} + sC\right) = \frac{U_g}{R} + C \cdot u_C(0)$$

$$\Rightarrow U_1 = \frac{\frac{U_g}{R} + C \cdot u_C(0)}{\left(\frac{1}{R} + \frac{1}{sL} + sC\right)}$$

$$I_1 = \frac{U_1}{R} - \frac{U_g}{R} = \frac{\frac{U_g}{R} + C \cdot u_C(0)}{R\left(\frac{1}{R} + \frac{1}{sL} + sC\right)} - \frac{U_g}{R}$$

$$= \frac{\frac{1}{s} + 2}{1 + \frac{1}{s} + s} - \frac{1}{s} = \frac{1 + 2s}{1 + s + s^2} - \frac{1}{s} = \frac{s + 2s^2 - 1 - s - s^2}{s\left(1 + s + s^2\right)} = \frac{s^2 - 1}{s\left(1 + s + s^2\right)}$$

$$U_{R1}(s) = I_1 \cdot R = \frac{s^2 - 1}{s\left(s^2 + s + 1\right)}$$

3. Za električni krug na slici izračunati napon $u_{iz}(t)$ ako su zadane normalizirane vrijednosti elemenata R =1, L = 1, C = 1, uz početne uvjete jednake nuli te $u_g(t) = S(t)$. Koristiti KZS i KZN te oznake grana i čvorova prema slici. Napisati:

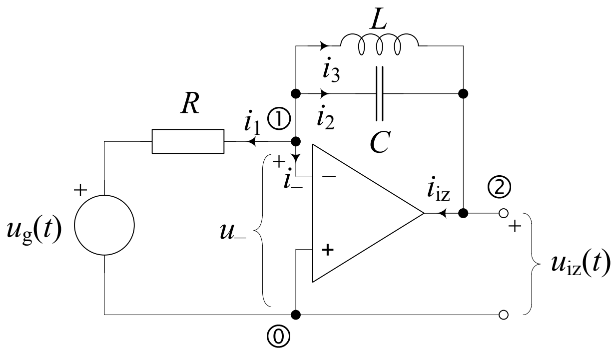
a) Broj neovisnih jednadžbi KZS i KZN (mreža ima 5 grana i 3 čvora);

b) Jednadžbe KZS;

c) Jednadžbe KZN;

d) Naponsko-strujne jednadžbe za grane;

e) Napon na izlazu $u_{iz}(t)$.



Rješenje:

N_b=5 (broj grana)

N_v=3 (broj čvorova)

Broj jednadžbi $\text{KZS} = N_v - 1 = 3 - 1 = 2$

Broj jednadžbi $\text{KZN} = N_b - N_v + 1 = 5 - 3 + 1 = 3$ (1 bod)

Jednadžbe Kirchhoffovih zakona (5 jednadžbi):

1) $I_1 + I_2 + I_3 + I_- = 0$ KZS

2) $I_{iz} - I_2 - I_3 = 0$ KZS(1 bod)

3) $U_1 - U_- = 0$ KZN

4) $-U_2 + U_3 = 0$ KZN

5) $-U_- + U_2 + U_{iz} = 0$ KZN(1 bod)

Naponsko–strujne jednadžbe grana (5 jednadžbi):

1) $U_1 = I_1 \cdot R + U_g$

2) $U_2 = I_2 \cdot \dfrac{1}{sC}$

3) $U_3 = I_3 \cdot sL$ (1 bod)

4) $U_- = 0$

5) $I_- = 0$

Sustav ima ukupno $2N_b$=10 jednadžbi i 10 nepoznanica (sve struje i svi naponi grana)

Naponsko – strujne jednadžbe grana uvrstimo u jednadžbe (1)–(5):

1) $\dfrac{U_1}{R} - \dfrac{U_g}{R} + sCU_2 + \dfrac{1}{sL}U_3 = 0$

2) $I_{iz} = sCU_2 + \dfrac{1}{sL}U_3$

3) $U_1 = U_- = 0$

4) $U_2 = U_3$

5) $U_{iz} = -U_2$

---

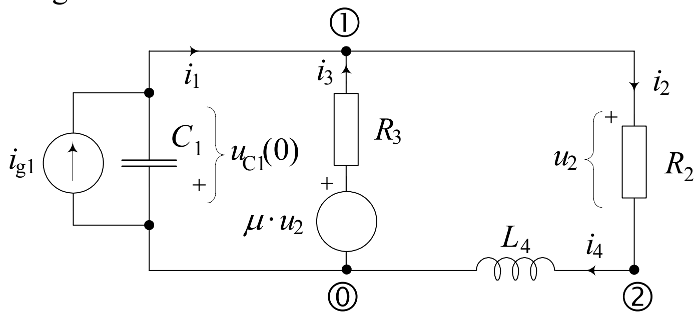
1) $\Rightarrow \left(sC + \dfrac{1}{sL}\right)U_2 = \dfrac{U_g}{R} \Rightarrow U_{iz} = -U_2 = -\dfrac{1}{R}\dfrac{U_g}{\left(sC + \dfrac{1}{sL}\right)}$

$$U_{iz}(s) = -\frac{\frac{1}{s}}{s + \frac{1}{s}} = -\frac{1}{s^2 + 1}$$

$u_{iz}(t) = -\sin(t) \cdot S(t)$ (1 bod)

MEĐUISPIT IZ ELEKTRIČNIH KRUGOVA – Rješenja – 2011

1. Za električni krug na slici i pridruženim orijentacijama grana te čvorovima zadane su normalizirane vrijednosti elemenata C_1=1, R_2=1, R_3=1, L_4=1 te μ=2, $u_{C1}(0)$=1, $i_{g1}(t)$=$S(t)$. Koristeći KZS i KZN te oznake grana i čvorova prema slici, napisati:
a) Jednadžbe KZS i KZN;
b) Naponsko-strujne jednadžbe za grane;
c) Napon na otporu R_2 $U_2(s)$;
d) Napon na otporu R_2 $u_2(t)$;
e) Da li je električni krug stabilan? Zašto?



Rješenje:

a) N_b=4 (broj grana)
N_v=3 (broj čvorova)
Broj jednadžbi $\text{KZS} = N_v - 1 = 3 - 1 = 2$
Broj jednadžbi $\text{KZN} = N_b - N_v + 1 = 4 - 3 + 1 = 2$
Slijede jednadžbe Kirchhoffovih zakona (4 jednadžbe):
1) $-I_1 + I_2 - I_3 = 0$ KZS
2) $-I_2 + I_4 = 0$ KZS
3) $U_1 - U_3 = 0$ KZN
4) $U_2 + U_3 + U_4 = 0$ KZN (1 bod)

b) Naponsko–strujne jednadžbe grana (4 jednadžbe):
1) $U_1 = \frac{1}{sC_1} \cdot I_1 - \frac{1}{sC_1} \cdot I_{g1} + \frac{u_{C1}(0)}{s}$
2) $U_2 = R_2 \cdot I_2$
3) $U_3 = R_3 \cdot I_3 - \mu U_2 = R_3 \cdot I_3 - \mu R_2 I_2$
4) $U_4 = sL_4 \cdot I_4$ (1 bod)

---

c) Sustav ima ukupno $2N_b$=8 jednadžbi i 8 nepoznanica (sve struje i svi naponi grana)
Naponsko – strujne jednadžbe grana uvrstimo u jednadžbe Kirchhoffovih zakona (1)–(4) te dobivamo:
1) $I_1 = I_2 - I_3$
2) $I_2 = I_4$
3) $\frac{1}{sC_1} \cdot I_1 - \frac{1}{sC_1} \cdot I_{g1} + \frac{u_{C1}(0)}{s} - R_3 \cdot I_3 + \mu R_2 I_2 = 0$
4) $R_2 \cdot I_2 + R_3 \cdot I_3 - \mu R_2 I_2 + sL_4 \cdot I_4 = 0$

$$1) \to 3) \Rightarrow \frac{1}{sC_1} \cdot (I_2 - I_3) - \frac{1}{sC_1} \cdot I_{g1} + \frac{u_{C1}(0)}{s} - R_3 \cdot I_3 + \mu R_2 I_2 = 0$$

$$\left(\frac{1}{sC_1} + \mu R_2\right) \cdot I_2 - \left(\frac{1}{sC_1} + R_3\right) \cdot I_3 - \frac{1}{sC_1} \cdot I_{g1} + \frac{u_{C1}(0)}{s} = 0$$

$$2) \to 4) \Rightarrow R_2 \cdot I_2 + R_3 \cdot I_3 - \mu R_2 I_2 + sL_4 \cdot I_2 = 0 \Rightarrow I_3 = -\frac{1}{R_3}\left[(1-\mu)R_2 + sL_4\right] \cdot I_2$$

$$\Rightarrow \left(\frac{1}{sC_1} + \mu R_2\right) \cdot I_2 + \left(\frac{1}{sC_1} + R_3\right) \cdot \frac{1}{R_3}\left[(1-\mu)R_2 + sL_4\right] \cdot I_2 = \frac{1}{sC_1} \cdot I_{g1} - \frac{u_{C1}(0)}{s} \Big/ \cdot sC_1$$

$$\left(1 + \mu R_2 sC_1\right) \cdot I_2 + \left(\frac{1}{R_3} + sC_1\right) \cdot \left[(1-\mu)R_2 + sL_4\right] \cdot I_2 = I_{g1} - C_1 u_{C1}(0)$$

$$I_2 = \frac{I_{g1} - C_1 u_{C1}(0)}{1 + \mu R_2 sC_1 + \left(\frac{1}{R_3} + sC_1\right) \cdot \left[(1-\mu)R_2 + sL_4\right]}; \; U_2 = R_2 I_2 = \frac{R_2 \cdot (I_{g1} - C_1 u_{C1}(0))}{1 + \mu R_2 sC_1 + \left(\frac{1}{R_3} + sC_1\right) \cdot \left[(1-\mu)R_2 + sL_4\right]}$$

$$= \frac{\frac{1}{s} - 1}{1 + 2s + (1+s) \cdot (-1+s)} = \frac{\frac{1}{s} - 1}{1 + 2s + s^2 - 1} = \frac{\frac{1}{s} - 1}{s(s+2)} = \frac{1-s}{s^2(s+2)}$$ (1 bod)

d) Odziv u vremenskoj domeni (rastav na parcijalne razlomke)

$$U_2(s) = \frac{1-s}{s^2(s+2)} = \frac{A}{s^2} + \frac{B}{s} + \frac{C}{s+2}$$

$$A(s+2) + Bs(s+2) + Cs^2 = 1 - s$$

$$(B+C)s^2 + (A+2B)s + 2A = 1 - s \Rightarrow A = \frac{1}{2}, B = -\frac{1+A}{2} = -\frac{3}{4}, C = -B = \frac{3}{4}$$

$$U_2(s) = \frac{1}{4}\left(\frac{2}{s^2} - \frac{3}{s} + \frac{3}{s+2}\right) \Rightarrow \underline{u_2(t) = \frac{1}{4}\left(2t - 3 + 3e^{-2t}\right) \cdot S(t)}$$ (1 bod)

e) Stabilnost:
NE, jer ima dvostruki pol u ishodištu
(odziv na konačnu pobudu teži u ∞ kad $t \to \infty$.) (1 bod)

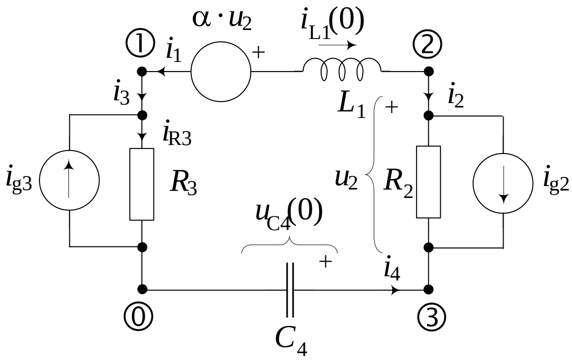
4. Za električni krug na slici i pridruženim orijentacijama grana zadane su normalizirane vrijednosti elemenata L_1=1, R_2=1, R_3=2, C_4=1/2, te α=2, $u_{C4}(0)$=2, $i_{L1}(0)$=1, $i_{g2}(t)$=$i_{g3}(t)$=$S(t)$. Koristeći KZS i KZN te oznake grana i čvorova prema slici, napisati:

a) Jednadžbe KZS i KZN;

b) Naponsko-strujne jednadžbe za grane;

c) Napon na otporu R_2: $U_2(s)$ i $u_2(t)$;

d) Struju kroz otpor R_3: $I_{R3}(s)$ i $i_{R3}(t)$;



<u>Rješenje:</u>

a) N_b=4 (broj grana)

N_v=4 (broj čvorova)

Broj jednadžbi $\text{KZS} = N_v - 1 = 4 - 1 = 3$

Broj jednadžbi $\text{KZN} = N_b - N_v + 1 = 4 - 4 + 1 = 1$

Slijede jednadžbe Kirchhoffovih zakona (4 jednadžbe):

1) $-I_1 + I_3 = 0$ KZS

2) $I_1 + I_2 = 0$ KZS

3) $-I_2 - I_4 = 0$ KZS

4) $U_1 - U_2 + U_3 + U_4 = 0$ KZN (1 bod)

b) Naponsko–strujne jednadžbe grana (4 jednadžbe):

1) $U_1 = sL_1 \cdot I_1 + L_1 i_{L1}(0) + \alpha U_2 = sL_1 \cdot I_1 + L_1 i_{L1}(0) + \alpha (I_2 - I_{g2}) R_2$

2) $U_2 = (I_2 - I_{g2}) \cdot R_2$

3) $U_3 = (I_3 + I_{g3}) \cdot R_3$

4) $U_4 = \dfrac{1}{sC_4} \cdot I_4 - \dfrac{u_{C4}(0)}{s}$ (1 bod)

---

c) Sustav ima ukupno $2N_b$=8 jednadžbi i 8 nepoznanica (sve struje i svi naponi grana)
Naponsko – strujne jednadžbe grana uvrstimo u jednadžbe Kirchhoffovih zakona (1)–(4) te dobivamo:

1) , 2), 3) $I_1 = -I_2 = I_3 = I_4$

4) $sL_1 I_1 + L_1 i_{L1}(0) + \alpha (I_2 - I_{g2}) R_2 - (I_2 - I_{g2}) R_2 + (I_3 + I_{g3}) R_3 + \dfrac{1}{sC_4} I_4 - \dfrac{u_{C4}(0)}{s} = 0$

---

1)→4) $\Rightarrow\ sL_1 I_1 + L_1 i_{L1}(0) - \alpha (I_1 + I_{g2}) R_2 + (I_1 + I_{g2}) R_2 + (I_1 + I_{g3}) R_3 + \dfrac{1}{sC_4} I_1 - \dfrac{u_{C4}(0)}{s} = 0$

$$\Rightarrow I_1 \left(sL_1 + (1-\alpha) R_2 + R_3 + \frac{1}{sC_4} \right) + L_1 i_{L1}(0) + (1-\alpha) I_{g2} R_2 + I_{g3} R_3 - \frac{u_{C4}(0)}{s} = 0$$

$$\Rightarrow \; I_1(s)=\frac{-L_1 i_{L1}(0)-(1-\alpha)I_{g2}R_2-I_{g3}R_3+\dfrac{u_{C4}(0)}{s}}{sL_1+(1-\alpha)R_2+R_3+\dfrac{1}{sC_4}}$$

$$I_1(s)=\frac{-1+\frac{1}{s}-\frac{2}{s}+\frac{2}{s}}{s-1+2+\frac{2}{s}}=\frac{-1+\frac{1}{s}}{s+1+\frac{2}{s}}=\frac{-s+1}{s^2+s+2} \quad \text{(1 bod)}$$

$$\Rightarrow \; I_2(s)=-I_1(s)=\frac{s-1}{s^2+s+2} \; \Rightarrow$$

$$U_2(s)=\left[I_2(s)-I_{g2}(s)\right]R_2=\frac{s-1}{s^2+s+2}-\frac{1}{s}=\frac{s-1}{s^2+s+\frac{1}{4}+\frac{7}{4}}-\frac{1}{s}=\frac{s+\frac{1}{2}-\frac{3}{2}}{\left(s+\frac{1}{2}\right)^2+\frac{7}{4}}-\frac{1}{s}$$

$$U_2(s)=\frac{s+\frac{1}{2}}{\left(s+\frac{1}{2}\right)^2+\left(\frac{\sqrt{7}}{2}\right)^2}-\frac{3}{\sqrt{7}}\frac{\frac{\sqrt{7}}{2}}{\left(s+\frac{1}{2}\right)^2+\left(\frac{\sqrt{7}}{2}\right)^2}-\frac{1}{s}$$

$$u_2(t)=e^{-\frac{t}{2}}\left(\cos\frac{\sqrt{7}}{2}t-\frac{3}{\sqrt{7}}\sin\frac{\sqrt{7}}{2}t\right)S(t)-S(t) \quad \text{(1 bod)}$$

---

d) Struja kroz R_3

$$\Rightarrow \; I_3(s)=I_1(s)=\frac{-s+1}{s^2+s+2} \; \Rightarrow$$

$$I_{R3}(s)=I_3(s)+I_{g3}(s)=-\frac{s-1}{s^2+s+2}+\frac{1}{s}=-\left(\frac{s-1}{s^2+s+2}-\frac{1}{s}\right)$$

$[=\text{minus izraz za } U_2(s) \text{ gore}]$

$$i_{R3}(t)=-e^{-\frac{t}{2}}\left(\cos\frac{\sqrt{7}}{2}t-\frac{3}{\sqrt{7}}\sin\frac{\sqrt{7}}{2}t\right)S(t)+S(t) \quad \text{(1 bod)}$$

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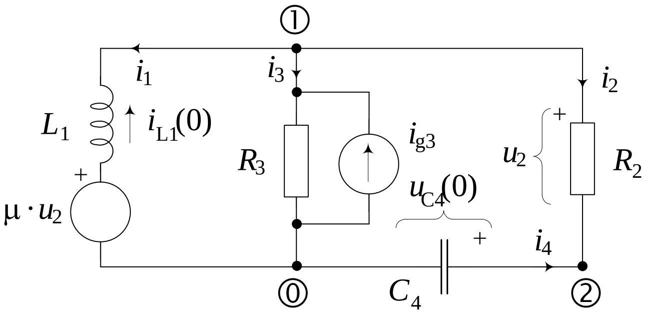
2. Za električni krug na slici i pridruženim orijentacijama grana zadane su normalizirane vrijednosti elemenata L_1=1, R_2=1, R_3=1, C_4=1, te μ=2, $u_{C4}(0)$=1, $i_{L1}(0)$=1, $i_{g3}(t)$=$S(t)$. Koristeći KZS i KZN te oznake grana i čvorova prema slici, napisati: a) Jednadžbe KZS i KZN (odabrati referentne smjerove petlji u smjeru kazaljke na satu); b) Naponsko-strujne jednadžbe za grane; c) Napon na otporu R_2 $U_2(s)$; d) Napon na otporu R_2 $u_2(t)$; e) Da li je električni krug stabilan? Zašto?

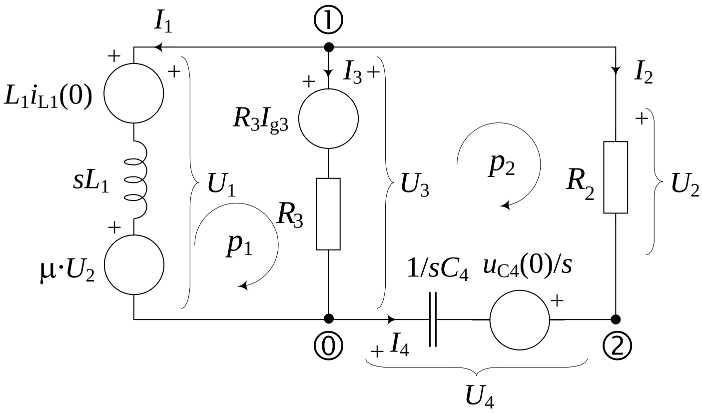


<u>Rješenje:</u> Laplaceova transformacija

a)

N_b=4 (broj grana)

N_v=3 (broj čvorova)



Broj jednadžbi

$\mathrm{KZS} = N_v - 1 = 3 - 1 = 2$

Broj jednadžbi

$\mathrm{KZN} = N_b - N_v + 1 = 4 - 3 + 1 = 2$

Slijede jednadžbe Kirchhoffovih zakona (4 jednadžbe):

1) $I_1 + I_2 + I_3 = 0$ KZS čvorište (1)

2) $-I_2 - I_4 = 0$ KZS čvorište (2)

3) $-U_1 + U_3 = 0$ KZN petlja p_1

4) $U_2 - U_3 - U_4 = 0$ KZN petlja p_2 **(1 bod)**

b) Naponsko–strujne jednadžbe grana (4 jednadžbe):

1) $U_1 = sL_1 \cdot I_1 + L_1 i_{L1}(0) + \mu U_2 = sL_1 \cdot I_1 + \mu R_2 \cdot I_2 + L_1 i_{L1}(0)$

2) $U_2 = R_2 \cdot I_2$

3) $U_3 = R_3 \cdot I_3 + I_{g3} R_3$

4) $U_4 = \dfrac{1}{sC_4} \cdot I_4 - \dfrac{u_{C4}(0)}{s}$ **(1 bod)**

c) Sustav ima ukupno $2N_b$=8 jednadžbi i 8 nepoznanica (sve struje i svi naponi grana)
Naponsko – strujne jednadžbe grana uvrstimo u jednadžbe Kirchhoffovih zakona (1)–(4) te dobivamo:

1) $I_1 = -I_2 - I_3$

2) $I_2 = -I_4$

3) $-sL_1 \cdot I_1 - \mu R_2 \cdot I_2 - L_1 i_{L1}(0) + R_3 \cdot I_3 + I_{g3} R_3 = 0$

4) $R_2 \cdot I_2 - R_3 \cdot I_3 - I_{g3} R_3 - \dfrac{1}{sC_4} \cdot I_4 + \dfrac{u_{C4}(0)}{s} = 0$

$$1) \rightarrow 3) \Rightarrow \; sL_1 \cdot (I_2 + I_3) - \mu R_2 \cdot I_2 - L_1 i_{L1}(0) + R_3 \cdot I_3 + I_{g3} R_3 = 0$$

$$2) \rightarrow 4) \Rightarrow \; R_2 \cdot I_2 - R_3 \cdot I_3 - I_{g3} R_3 + \frac{1}{sC_4} \cdot I_2 + \frac{u_{C4}(0)}{s} = 0 \; \Rightarrow$$

$$(1') \; I_2 (sL_1 - \mu R_2) + I_3 (sL_1 + R_3) = L_1 i_{L1}(0) - I_{g3} R_3$$

$$(2') \; \left(R_2 + \frac{1}{sC_4} \right) \cdot I_2 - R_3 \cdot I_3 = I_{g3} R_3 - \frac{u_{C4}(0)}{s}$$

$\Rightarrow I_2(s), \; I_3(s)$ koristimo metodu determinanti:

$$\begin{bmatrix} sL_1 - \mu R_2 & sL_1 + R_3 \\ R_2 + \dfrac{1}{sC_4} & -R_3 \end{bmatrix} \cdot \begin{bmatrix} I_2(s) \\ I_3(s) \end{bmatrix} = \begin{bmatrix} L_1 i_{L1}(0) - I_{g3} R_3 \\ I_{g3} R_3 - \dfrac{u_{C4}(0)}{s} \end{bmatrix}$$

$$\Delta = \begin{vmatrix} sL_1 - \mu R_2 & sL_1 + R_3 \\ R_2 + \dfrac{1}{sC_4} & -R_3 \end{vmatrix} = -R_3 (sL_1 - \mu R_2) - (sL_1 + R_3) \left(R_2 + \frac{1}{sC_4} \right)$$

$$\Delta = -R_3 s L_1 + \mu R_2 R_3 - sL_1 R_2 - R_3 R_2 - \frac{L_1}{C_4} - R_3 \frac{1}{sC_4} = -s + 2 - s - 1 - 1 - \frac{1}{s} = -2s - \frac{1}{s}$$

$$\Delta_2 = \begin{vmatrix} L_1 i_{L1}(0) - I_{g3} R_3 & sL_1 + R_3 \\ I_{g3} R_3 - \dfrac{u_{C4}(0)}{s} & -R_3 \end{vmatrix} = -R_3 \left(L_1 i_{L1}(0) - I_{g3} R_3 \right) - (sL_1 + R_3) \left(I_{g3} R_3 - \frac{u_{C4}(0)}{s} \right)$$

$$\Delta_2 = -R_3 L_1 i_{L1}(0) + R_3^2 I_{g3} - sL_1 I_{g3} R_3 - R_3^2 I_{g3} + sL_1 \frac{u_{C4}(0)}{s} + R_3 \frac{u_{C4}(0)}{s} =$$

$$= -R_3 L_1 i_{L1}(0) - sL_1 I_{g3} R_3 + L_1 u_{C4}(0) + R_3 \frac{u_{C4}(0)}{s} = -1 - s \cdot \frac{1}{s} + 1 + \frac{1}{s} = -1 + \frac{1}{s}$$

$$I_2(s) = \frac{\Delta_2}{\Delta} = \frac{-1 + \frac{1}{s}}{-2s - \frac{1}{s}} = \frac{1 - \frac{1}{s}}{2s + \frac{1}{s}} = \frac{s-1}{2s^2 + 1}; \quad R_2 = 1$$

$$U_2(s) = I_2(s) R_2 = \frac{s-1}{2s^2+1}$$ **(1 bod)**

d) Napon na otporu R_2 $u_2(t)$:

$$U_2(s) = \frac{1}{2} \cdot \frac{s-1}{s^2 + \frac{1}{2}} = \frac{1}{2} \cdot \frac{s}{s^2 + \frac{1}{2}} - \frac{1}{2} \cdot \frac{1}{s^2 + \frac{1}{2}} = \frac{1}{2} \cdot \frac{s}{s^2 + \left(\frac{1}{\sqrt{2}} \right)^2} - \frac{1}{\sqrt{2}} \cdot \frac{\frac{1}{\sqrt{2}}}{s^2 + \left(\frac{1}{\sqrt{2}} \right)^2}$$

$$u_2(t) = \left[\frac{1}{2} \cdot \cos\left(\frac{t}{\sqrt{2}} \right) - \frac{1}{\sqrt{2}} \cdot \sin\left(\frac{t}{\sqrt{2}} \right) \right] S(t)$$ **(1 bod)**

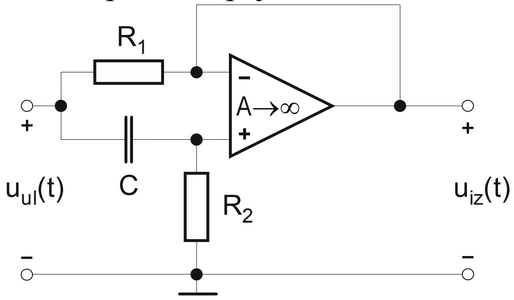
e) Stabilnost:

Električni krug je marginalno stabilan (na rubu stabilnosti).

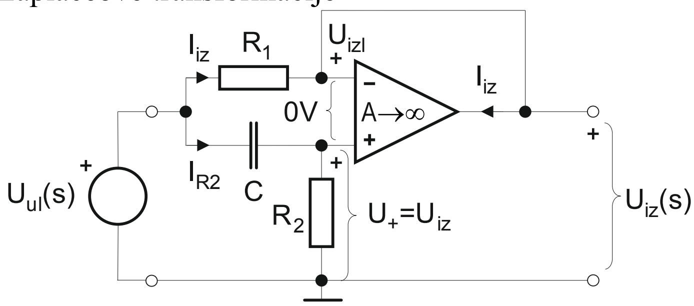
Polovi $s^2 + \frac{1}{2} = 0 \Rightarrow s_{p1,2} = \pm j \frac{\sqrt{2}}{2}$ su jednostruki i nalaze se na imaginarnoj osi.

(1 bod)

4. Odrediti odziv $u_{iz}(t)$ za mrežu prikazanu slikom ako je zadano: R_1=1kΩ, R_2=20kΩ, C=250nF te kao poticaj jedinična Step funkcija $u_{ul}(t)=S(t)$. Koji se ekvivalentni element može upotrijebiti umjesto kapaciteta C otpora R_2 i pojačala i koliko on iznosi ?



Rješenje: Primjena Laplaceove transformacije



$$U_+ \cdot \left(\frac{1}{R_2} + sC\right) - U_{ul} \cdot sC = 0 \Rightarrow U_+ = U_{ul} \cdot \frac{sC}{\frac{1}{R_2} + sC} = U_{ul} \cdot \frac{sR_2C}{1 + sR_2C} = U_{ul} \cdot \frac{R_2}{\frac{1}{sC} + R_2}$$

$$\frac{U_{iz} - U_{ul}}{R_1} = I_{iz}$$

$$(U_+ - U_{iz}) \cdot A = U_{iz} \Rightarrow U_+ = U_{iz}$$

$$U_{ul} - U_{iz} \cdot R_1 = U_{iz}$$

$$U_{ul} - I_{R2} \cdot \frac{1}{sC} = U_+$$

$$I_{R2} = \frac{U_{iz}}{\frac{1}{sC} + R_2}$$

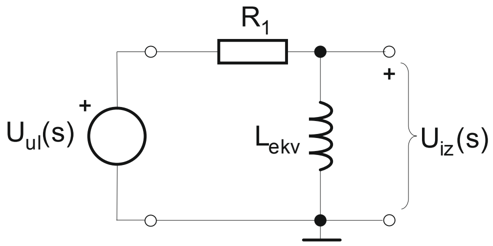
$$I_{R2} \cdot R_2 = U_+ = U_{iz}$$

$$U_{iz} = U_{ul} \cdot \frac{R_2}{\frac{1}{sC} + R_2} = U_{ul} \cdot \frac{1}{\frac{1}{sR_2C} + 1} = U_{ul} \cdot \frac{s}{s + \frac{1}{R_2C}}$$ (2 boda)

Što je isto kao i $U_{iz} = U_{ul} \cdot \frac{sL_{ekv}}{R_1 + sL_{ekv}} = U_{ul} \cdot \frac{s}{s + \frac{R_1}{L_{ekv}}}$ (vidi sliku)

Uspoređujući $\frac{R_1}{L_{ekv}} = \frac{1}{R_2C}$ slijedi $L_{ekv} = R_1R_2C = 1[\text{k}\Omega] \cdot 20[\text{k}\Omega] \cdot 250[\text{nF}] = 5[\text{H}]$

Dakle ekvivalentni element je induktivitet. (2 boda)

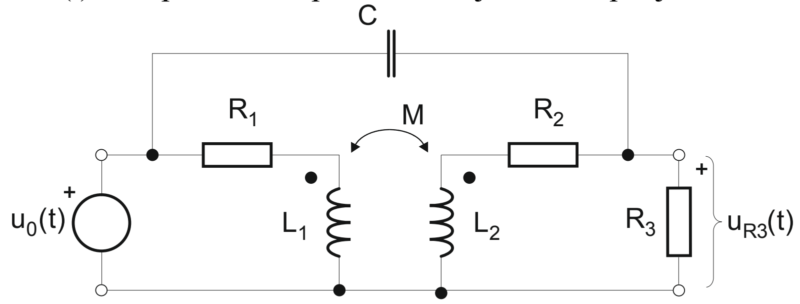


Odziv na poticaj $u_{ul}(t)=S(t)\Rightarrow U_{ul}(s)=\frac{1}{s}$ glasi: (1 bod)

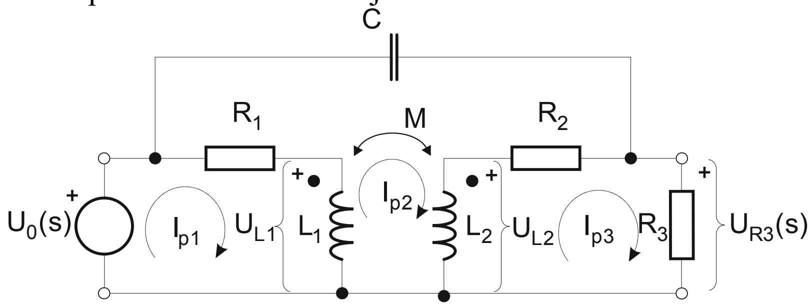
$$U_{iz}(s)=U_{ul}\cdot\frac{s}{s+\frac{R_1}{L_{ekv}}}=U_{ul}\cdot\frac{s}{s+\frac{10^3}{5}}=\frac{1}{s}\cdot\frac{s}{s+200}=\frac{1}{s+200}\Rightarrow \underline{u_{iz}(t)=e^{-200t}\cdot S(t)[\mathrm{V}]}$$

MEĐUISPIT IZ ELEKTRIČNIH KRUGOVA 2015-2016 – Rješenja

1. Za električni krug na slici zadane su normalizirane vrijednosti elemenata $R_1=R_2=R_3=1$, $C=1$, $L_1=L_2=1$, $M=1$, početne struje kroz induktivitete i početni napon na kapacitetu jednaki su nuli, te pobuda $u_0(t)=S(t)$. Primjenom Laplaceove transformacije i koristeći metodu petlji izračunati napon $u_{R3}(t)$ na otporu R_3. Napisati sustav jednadžbi petlji u matričnom obliku.



Rješenje: Primjena Laplaceove transformacije



a) Jednadžbe petlji: (1 bod)

1) $I_{p1}R_1 - I_{p2}R_1 = U_0 - U_{L1}$;

2) $-I_{p1}R_1 + I_{p2}\left(R_1 + R_2 + \frac{1}{sC}\right) - I_{p3}R_2 = U_{L1} - U_{L2}$;

3) $-I_{p2}R_2 + I_{p3}(R_2 + R_3) = U_{L2}$;

Jednadžbe vezanih induktiviteta:

4) $U_{L1} = (I_{p1} - I_{p2})sL_1 - (I_{p3} - I_{p2})sM$;

5) $U_{L2} = (I_{p1} - I_{p2})sM - (I_{p3} - I_{p2})sL_2$;

Nakon uvrštavanja 4) i 5) u 1), 2) i 3) te malo sređivanja: (1 bod)

1) $I_{p1}(R_1 + sL_1) - I_{p2}(R_1 + sL_1 - sM) - I_{p3}sM = U_0$;

2) $-I_{p1}(R_1 + sL_1 - sM) + I_{p2}\left(R_1 + R_2 + \frac{1}{sC} + sL_1 + sL_2 - 2sM\right) - I_{p3}(R_2 + sL_2 - sM) = 0$;

3) $-I_{p1}sM - I_{p2}(R_2 + sL_2 - sM) + I_{p3}(R_2 + R_3 + sL_2) = 0$;

U matričnom obliku:

$$\begin{bmatrix} R_1 + sL_1 & -(R_1 + sL_1 - sM) & -sM \\ -(R_1 + sL_1 - sM) & R_1 + R_2 + \frac{1}{sC} + sL_1 + sL_2 - 2sM & -(R_2 + sL_2 - sM) \\ -sM & -(R_2 + sL_2 - sM) & R_2 + R_3 + sL_2 \end{bmatrix} \begin{bmatrix} I_{p1} \\ I_{p2} \\ I_{p3} \end{bmatrix} = \begin{bmatrix} U_0 \\ 0 \\ 0 \end{bmatrix}$$

Na ovom mjestu se može napraviti provjera postupka ako je dobivena matrica impedancija petlji simetrična oko dijagonale. To vrijedi za recipročne mreže koje smiju sadržavati međuinduktivitete.

Uz uvrštene vrijednosti elemenata: (1 bod)

$$\begin{bmatrix} 1+s & -1 & -s \\ -1 & 2+\dfrac{1}{s} & -1 \\ -s & -1 & 2+s \end{bmatrix} \begin{bmatrix} I_{p1} \\ I_{p2} \\ I_{p3} \end{bmatrix} = \begin{bmatrix} U_0 \\ 0 \\ 0 \end{bmatrix}$$

Jadan od načina izračunavanja struje I_{p3}: $I_{p3}(s) = \dfrac{\Delta_3}{\Delta}$

Determinanta matrice npr. razvojem po prvom stupcu:

$$\Delta = \begin{vmatrix} 1+s & -1 & -s \\ -1 & 2+\dfrac{1}{s} & -1 \\ -s & -1 & 2+s \end{vmatrix} = (1+s)\cdot\begin{vmatrix} 2+\dfrac{1}{s} & -1 \\ -1 & 2+s \end{vmatrix} + 1\cdot\begin{vmatrix} -1 & -s \\ -1 & 2+s \end{vmatrix} - s\cdot\begin{vmatrix} -1 & -s \\ 2+\dfrac{1}{s} & -1 \end{vmatrix} =$$

$$= (1+s)\cdot\left[\left(2+\frac{1}{s}\right)(2+s)-1\right] + 1\cdot\left[-(2+s)-s\right] - s\cdot\left[1+s\left(2+\frac{1}{s}\right)\right] =$$

$$= (1+s)\left(2+\frac{1}{s}\right)(2+s) - (1+s) - (2+s) - s - s - s(2s+1) =$$

$$= (1+s)\left(5+2s+\frac{2}{s}\right) - 3 - 5s - 2s^2 =$$

$$= 5+2s+\frac{2}{s}+5s+2s^2+2-3-5s-2s^2 = 4+2s+\frac{2}{s} = 2\left(2+s+\frac{1}{s}\right)$$

$$\Delta_3 = \begin{vmatrix} 1+s & -1 & U_0 \\ -1 & 2+\dfrac{1}{s} & 0 \\ -s & -1 & 0 \end{vmatrix} = U_0\cdot\begin{vmatrix} -1 & -s \\ 2+\dfrac{1}{s} & -1 \end{vmatrix} = U_0\cdot\left[1+s\left(2+\frac{1}{s}\right)\right] = U_0\cdot 2(s+1)$$

uz R_3=1

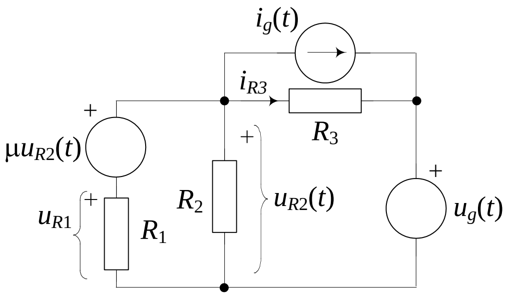
$$U_{R3}(s) = I_{p3}(s)R_3 = \frac{\Delta_3}{\Delta} = U_0(s)\cdot\frac{s+1}{2+s+\dfrac{1}{s}} = \frac{1}{s}\cdot\frac{s+1}{2+s+\dfrac{1}{s}} = \frac{s+1}{s^2+2s+1}$$ (1 bod)

Inverzna Laplaceova transformacija izlaznog napona: (1 bod)

$$U_{R3}(s) = \frac{s+1}{s^2+2s+1} = \frac{s+1}{(s+1)^2} = \frac{1}{s+1} \quad \Rightarrow \quad \underline{u_{R3}(t) = e^{-t}\cdot S(t)}$$

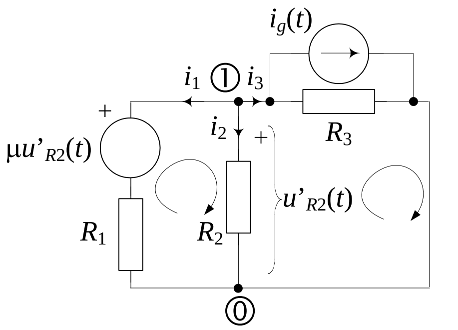
MEĐUISPIT IZ ELEKTRIČNIH KRUGOVA 2014-2015 – Rješenja

1. Za električni krug prikazan slikom primjenom metode superpozicije izračunati napon $u_{R2}(t)$. Zadane su normalizirane vrijednosti elemenata R_1=2, R_2=1, R_3=1/2 i μ=2 i pobude $u_g(t)=S(t)$ i $i_g(t)=S(t)$. Izračunati također napon $u_{R1}(t)$ na otporu R_1 i struju $i_{R3}(t)$ kroz R_3. U proračunu primijeniti Kirchhoffove zakone.



Rješenje: Primjena metode superpozicije.

a) Isključen naponski izvor u_g=0. Ovisni izvor (NONI) $\mu\, u_{R2}$ ostaje uključen.



Mreža ima N_b=3 grane i N_v=2 čvora

Jednadžbe KZN

$$\text{(p1)}\ -u_1(t)+u_2(t)=0$$

$$\text{(p2)}\ -u_2(t)+u_3(t)=0 \Rightarrow u_1(t)=u_2(t)=u_3(t),\ u'_{R2}(t)=u_2(t)$$

Naponsko-strujne relacije grana

$$\text{(g1)}\ u_1(t)=\mu u'_{R2}(t)+R_1 i_1(t) \Rightarrow i_1(t)=\frac{1}{R_1}\cdot\left[u_1(t)-\mu u_2(t)\right]$$

$$\text{(g2)}\ u'_{R2}(t)=u_2(t)=R\cdot i_2(t) \Rightarrow i_2(t)=u_2(t)\frac{1}{R_2}$$

$$\text{(g3)}\ u_3(s)=R_3\cdot\left[i_3(t)-i_g(t)\right] \Rightarrow i_3(t)=\frac{1}{R_3}u_3(t)+i_g(t)$$

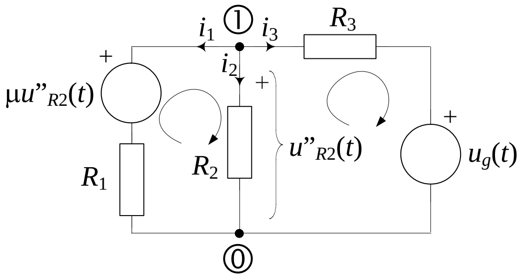
Jednadžbe KZS

$$\text{(č1)}\ i_1(t)+i_2(t)+i_3(t)=0 \Rightarrow u_1(t)\cdot\frac{1}{R_1}-\mu u_2(t)\frac{1}{R_1}+u_2(t)\frac{1}{R_2}+\frac{1}{R_3}u_3(t)+i_g(t)=0$$

$$\Rightarrow u'_{R2}(t)\cdot\left[\frac{1-\mu}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}\right]=-i_g(t)$$

$$\Rightarrow u'_{R2}(t)=\frac{-i_g(t)}{\frac{1-\mu}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}}=\frac{-S(t)}{\frac{1-2}{2}+1+2}=\frac{-S(t)}{\frac{1}{2}+\frac{4}{2}}=\frac{-2}{5}S(t)\ \text{(1 bod)}$$

b) Isključen strujni izvor $i_g(t)$=0. Ovisni izvor (NONI) μ u_{R2} ostaje uključen.



Jednadžbe KZN (iste kao i u slučaju a)

(p1) $-u_1(t)+u_2(t)=0$

(p2) $-u_2(t)+u_3(t)=0 \Rightarrow u_1(t)=u_2(t)=u_3(t)\,,\ u''_{R2}(t)=u_2(t)$

Naponsko-strujne relacije grana (g1 i g2 iste kao i u slučaju a)

(g1) $u_1(t)=\mu u'_{R2}(t)+R_1 i_1(t) \Rightarrow i_1(t)=\frac{1}{R_1}\cdot[u_1(t)-\mu u_2(t)]$

(g2) $u''_{R2}(t)=u_2(t)=R\cdot i_2(t) \Rightarrow i_2(t)=u_2(t)\frac{1}{R_2}$

(g3) $u_3(s)=R_3\cdot i_3(t)+u_g(t) \Rightarrow i_3(t)=\frac{1}{R_3}[u_3(t)-u_g(t)]$

Jednadžbe KZS

(č1) $i_1(t)+i_2(t)+i_3(t)=0 \Rightarrow u_1(t)\cdot\frac{1}{R_1}-\mu u_2(t)\frac{1}{R_1}+u_2(t)\frac{1}{R_2}+\frac{1}{R_3}[u_3(t)-u_g(t)]=0$

$$\Rightarrow u''_{R2}(t)\cdot\left[\frac{1-\mu}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}\right]=\frac{1}{R_3}u_g(t)$$

$$\Rightarrow u''_{R2}(t)=\frac{\frac{1}{R_3}u_g(t)}{\frac{1-\mu}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}}=\frac{2S(t)}{\frac{1-2}{2}+1+2}=\frac{2S(t)}{\frac{1}{2}+\frac{4}{2}}=\frac{4}{5}S(t)$$ (1 bod)

c) Superpozicija:

$$u_{R2}(t)=u'_{R2}(t)+u''_{R2}(t)=\frac{-i_g(t)+\frac{1}{R_3}u_g(t)}{\frac{1-\mu}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}}=\frac{-S(t)+2S(t)}{\frac{1-2}{2}+1+2}=\frac{S(t)}{\frac{1}{2}+\frac{4}{2}}=\frac{2}{5}S(t)$$ (1 bod)

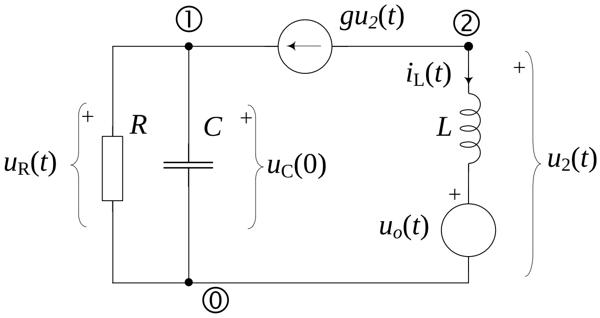
d) Napon u_{R1}

$$u_1=u_{R2}-\mu\cdot u_{R2}=(1-\mu)u_{R2}=-u_{R2}=-\frac{2}{5}S(t)$$ (1 bod)

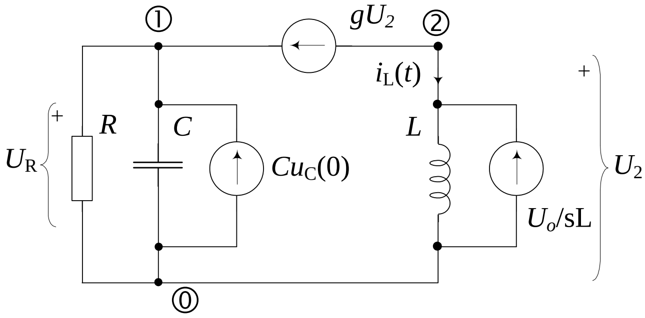
e) Struja i_{R3}

$$i_{R3}=\frac{u_{R2}-u_g}{R_3}=2\left(\frac{2}{5}-1\right)S(t)=-\frac{6}{5}S(t)$$ (1 bod)

3. Za električni krug na slici zadane su normalizirane vrijednosti elemenata R=1, C=1, L=2, g=2, početni napon na kapacitetu $u_C(0)$=1, te pobuda $u_o(t)=S(t)$. Primjenom Laplaceove transformacije i koristeći metodu čvorišta izračunati napon $u_R(t)$ na otporu R i struju $i_L(t)$.



Rješenje: Primjena Laplaceove transformacije



(1 bod)

$$U_1(s)\left(\frac{1}{R}+sC\right)=Cu_C(0)+g\cdot U_2(s)\;\Rightarrow\quad U_1\left(\frac{1}{R}+sC\right)-g\cdot U_2=Cu_C(0)$$

$$U_2(s)\frac{1}{sL}=-g\cdot U_2(s)+\frac{1}{sL}\cdot U_o(s)\quad\Rightarrow\quad U_2\left(\frac{1}{sL}+g\right)=\frac{U_o}{sL}\Rightarrow\; U_2=\frac{U_o}{1+sgL}\;\text{(1 bod)}$$

$$U_1\frac{1}{R}(1+sCR)=\frac{gU_o}{1+sgL}+Cu_C(0)\quad\Rightarrow\quad U_1=\frac{R(gU_o+Cu_C(0)(1+sgL))}{(1+sCR)(1+sgL)}\;\text{(1 bod)}$$

$$U_1=\frac{\frac{2}{s}+1+4s}{(1+s)(1+4s)}=\frac{s^2+\frac{s}{4}+\frac{1}{2}}{s(s+1)\left(s+\frac{1}{4}\right)}=\frac{A}{s}+\frac{B}{s+1}+\frac{C}{s+\frac{1}{4}}$$

$$s^2+\frac{s}{4}+\frac{1}{2}=A(s+1)\left(s+\frac{1}{4}\right)+Bs\left(s+\frac{1}{4}\right)+Cs(s+1)$$

$$A+B+C=1$$

$$\frac{5A}{4}+\frac{B}{4}+C=\frac{1}{4}$$

$$\frac{A}{4}=\frac{1}{2}\Rightarrow\qquad A=2\;\Rightarrow\qquad B+C=1-A=-1$$

$$\frac{5A}{4}+\frac{B}{4}+C=\frac{1}{4}\quad\Rightarrow\quad\frac{3}{4}C=\frac{1}{4}-\frac{5A}{4}-\frac{B+C}{4}=\frac{1}{4}-\frac{10}{4}-\frac{(-1)}{4}\quad\Rightarrow\quad C=-\frac{8}{3}$$

$$B = -1 - C = \frac{5}{3}$$

$$U_1 = \frac{2}{s} + \frac{5}{3} \cdot \frac{1}{s+1} - \frac{8}{3} \cdot \frac{1}{s + \frac{1}{4}} \qquad \Rightarrow \qquad u_R = u_1 = \left(2 + \frac{5}{3} \cdot e^{-t} - \frac{8}{3} \cdot e^{-t/4} \right) S(t)$$ (1 bod)

$$I_L = \frac{U_2 - U_o}{sL} = \frac{U_o}{sL}\left(\frac{1}{1+sgL} - 1 \right) = \frac{-gU_o}{1+sgL} = \frac{-\frac{2}{s}}{1+4s} = -\frac{1}{2} \cdot \frac{1}{s(s+1/4)}$$

$$I_L = -\frac{1}{2} \cdot \frac{1}{s(s+1/4)} = \frac{A}{s} + \frac{B}{s+1/4}$$

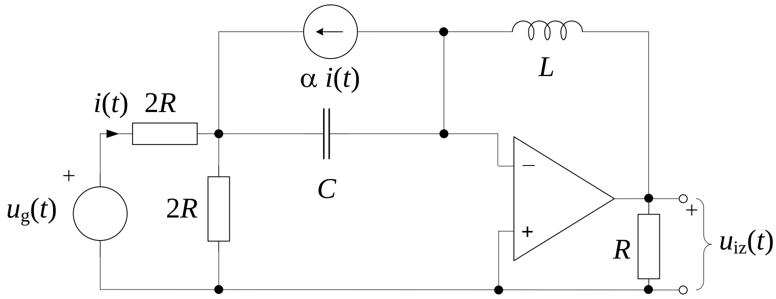
$$-\frac{1}{2} = A(s+1/4) + Bs$$

$$A + B = 0$$

$$\frac{A}{4} = -\frac{1}{2} \qquad \Rightarrow \qquad A = -2 \qquad \Rightarrow \qquad B = 2$$

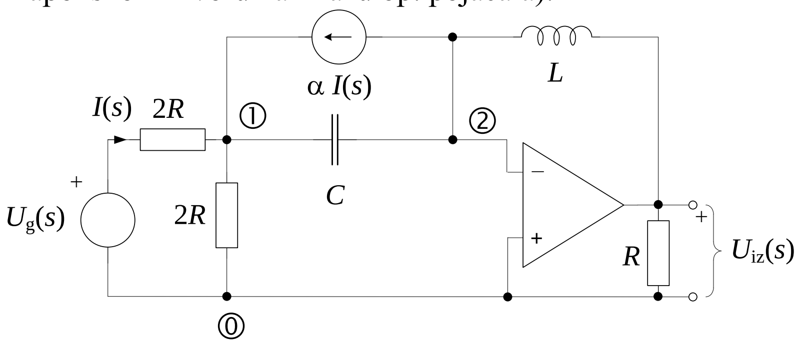
$$I_L = -\frac{2}{s} + \frac{2}{s+1/4} \qquad \Rightarrow \qquad i_L = -2\left(1 - e^{-t/4}\right) S(t)$$ (1 bod)

4. Za električni krug prikazan slikom izračunati odziv $u_{iz}(t)$ na pobudu $u_g(t)=S(t)$. Zadane su normalizirane vrijednosti elemenata R=1, C=1 i L=2; te konstanta ovisnog izvora α=1. Operacijsko pojačalo je idealno. Početni uvjeti su jednaki nuli. Traženi odziv treba odrediti primjenom jednadžbi čvorišta.



Rješenje:

a) Primjenom Laplaceove transformacije dobivamo slijedeći električni krug u *frekvencijskoj* domeni. Postavimo jednadžbe čvorišta (otpor R na izlazu op. pojačala se zanemaruje, jer je paralelno spojen naponskom izvoru na izlazu op. pojačala):



1) $U_1\left(\dfrac{1}{2R}+\dfrac{1}{2R}+sC\right)-U_2sC=\alpha I(s)+\dfrac{U_g(s)}{2R}$; $\Rightarrow$ $I(s)=\dfrac{U_g(s)-U_1(s)}{2R}$;

2) $-U_1sC+U_2\left(sC+\dfrac{1}{sL}\right)=U_{iz}(s)\dfrac{1}{sL}-\alpha I(s)$;

Virtualni kratki spoj $\Rightarrow$ $U_2=0$ $\Rightarrow$

1) $U_1\left(\dfrac{2}{2R}+sC\right)=\alpha\dfrac{U_g(s)-U_1(s)}{2R}+\dfrac{U_g(s)}{2R}$;

2) $-U_1sC=U_{iz}(s)\dfrac{1}{sL}-\alpha\dfrac{U_g(s)-U_1(s)}{2R}$;

Nakon malo sređivanja:

1) $U_1\left(\dfrac{2+\alpha}{2R}+sC\right)=\dfrac{U_g(s)}{2R}(1+\alpha)$

$$\Rightarrow U_1(s)=\frac{\frac{1}{2R}(1+\alpha)}{\frac{2+\alpha}{2R}+sC}U_g(s)=\frac{1+\alpha}{2+\alpha+s(2RC)}U_g(s);$$

2) $U_1\left(\dfrac{\alpha}{2R}+sC\right)=-U_{iz}(s)\dfrac{1}{sL}+U_g(s)\dfrac{\alpha}{2R}$;

$$1),\ 2)\ \Rightarrow\ \frac{1+\alpha}{2+\alpha+s(2RC)}\left(\frac{\alpha}{2R}+sC\right)U_g(s)=-U_{iz}(s)\frac{1}{sL}+U_g(s)\frac{\alpha}{2R};$$

$$U_{iz}(s)\frac{1}{sL}=-\frac{1+\alpha}{2+\alpha+s(2RC)}\left(\frac{\alpha}{2R}+sC\right)U_g(s)+\frac{\alpha}{2R}U_g(s)$$

$$U_{iz}(s)=-sL\left[\frac{1+\alpha}{2+\alpha+s(2RC)}\left(\frac{\alpha}{2R}+sC\right)-\frac{\alpha}{2R}\right]U_g(s)$$ (3 boda)

b) Uz uvrštene vrijednosti elemenata: R=1, C=1, L=2; $U_g(s)$=1/s i α=1

$$U_{iz}(s)=-2s\left[\frac{1+1}{2+1+2s}\left(\frac{1}{2}+s\right)-\frac{1}{2}\right]\cdot\frac{1}{s}=-2\left[\frac{2}{3+2s}\left(\frac{1+2s}{2}\right)-\frac{1}{2}\right]$$

$$U_{iz}(s)=-2\left[\frac{1+2s}{3+2s}-\frac{1}{2}\right]$$

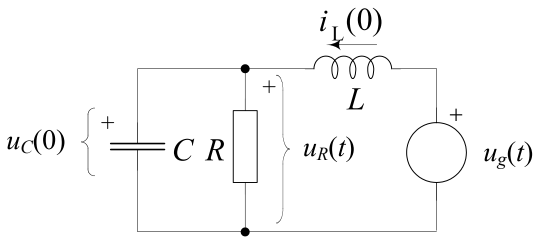
$$U_{iz}(s)=-2\left[\frac{3+2s-2}{3+2s}-\frac{1}{2}\right]=-2\left[1-\frac{2}{3+2s}-\frac{1}{2}\right]=-2\left[\frac{1}{2}-\frac{1}{s+\frac{3}{2}}\right]=-1+\frac{2}{s+\frac{3}{2}}$$ (1 bod)

c) Inverzna Laplaceova transformacija izlaznog napona:

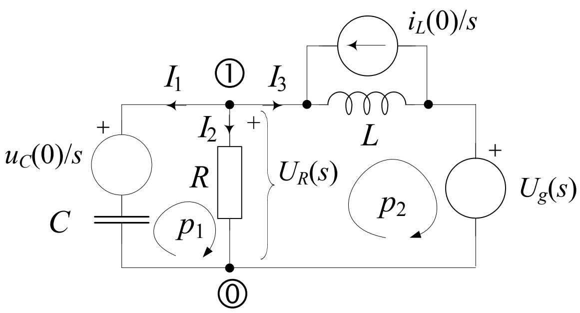
$$\underline{u_{iz}(t)=L^{-1}[U_{iz}(s)]=-\delta(t)+2e^{-3/2t}\cdot S(t)}$$ (1 bod)

MEĐUISPIT IZ ELEKTRIČNIH KRUGOVA 2013-2014 – Rješenja

1. Za električni krug prikazan slikom primjenom Kirchhoffovih zakona izračunati valni oblik napona $u_R(t)$ kao odziv, ako je zadana pobuda $u_g(t)=\delta(t)$. Zadane su normalizirane vrijednosti elemenata C=1, R=1, L=1/2, te početni uvjeti $u_C(0)$=8 i $i_L(0)$=2.



Rješenje: Primjena Laplaceove transformacije



(1 bod)

Mreža ima N_b=3 grane i N_v=2 čvora

Jednadžbe KZN

$$\text{(p1)}\ -U_1(s)+U_2(s)=0$$

$$\text{(p2)}\ -U_2(s)+U_3(s)=0$$

Jednadžbe KZS

$$\text{(č1)}\ I_1(s)+I_2(s)+I_3(s)=0 \quad \text{(1 bod)}$$

Naponsko-strujne relacije grana

$$\text{(g1)}\ U_1(s)=\frac{1}{sC}I_1(s)+\frac{u_C(0)}{s} \Big/ \cdot sC$$

$$\text{(g2)}\ U_2(s)=R\cdot I_2(s)$$

$$\text{(g3)}\ U_3(s)=sL\cdot\left[I_3(s)+\frac{i_L(0)}{s}\right]+U_g(s) \quad \text{(1 bod)}$$

$$\text{(g1)}\Rightarrow\ I_1(s)=sC\cdot U_1(s)-Cu_C(0)$$

$$\text{(g2)}\Rightarrow\ I_2(s)=U_2(s)\frac{1}{R}$$

$$\text{(g3)}\Rightarrow\ U_3(s)=sL\cdot I_3(s)+Li_L(0)+U_g(s)\ \Rightarrow\ I_3(s)=\frac{1}{sL}U_3(s)-\frac{i_L(0)}{s}-\frac{1}{sL}U_g(s)$$

$$\text{(č1)}\Rightarrow\ sC\cdot U_1(s)-Cu_C(0)+U_2(s)\frac{1}{R}+\frac{1}{sL}U_3(s)-\frac{i_L(0)}{s}-\frac{1}{sL}U_g(s)=0$$

$$\text{(p1), (p2)}\Rightarrow\ U_1(s)=U_2(s)=U_3(s),\ U_R(s)=U_2(s)$$

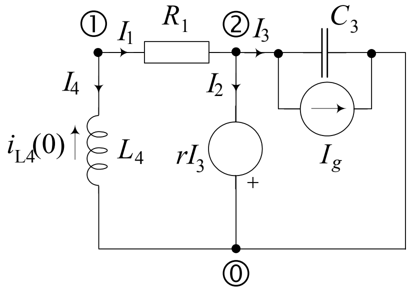
$$\Rightarrow \left(sC+\frac{1}{R}+\frac{1}{sL}\right)\cdot U_R(s)=Cu_C(0)+\frac{i_L(0)}{s}+\frac{1}{sL}U_g(s)$$

$$\Rightarrow U_R(s)=\frac{Cu_C(0)+\frac{i_L(0)}{s}+\frac{1}{sL}U_g(s)}{sC+\frac{1}{R}+\frac{1}{sL}}=\frac{8+\frac{2}{s}+\frac{2}{s}}{s+1+\frac{2}{s}}=\frac{8+\frac{4}{s}}{s+1+\frac{2}{s}}=\frac{8s+4}{s^2+s+2}$$ (1 bod)

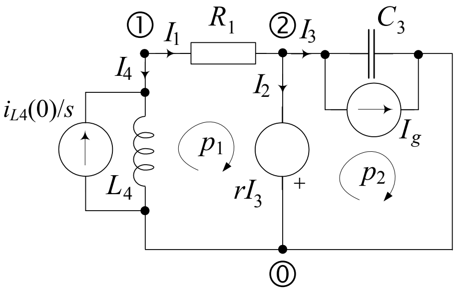
$$U_R(s)=8\cdot\frac{s+\frac{1}{2}}{\left(s+\frac{1}{2}\right)^2+\frac{7}{4}}=8\cdot\frac{s+\frac{1}{2}}{\left(s+\frac{1}{2}\right)^2+\left(\frac{\sqrt{7}}{2}\right)^2}$$

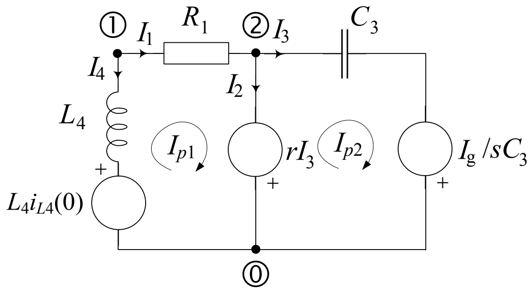
$$\Rightarrow \underline{u_R(t)=8\cdot e^{-\frac{t}{2}}\cos\left(\frac{\sqrt{7}}{2}t\right)\cdot S(t)}$$ (1 bod)

2. Za električni krug na slici zadane su normalizirane vrijednosti elemenata R_1=1, C_3=1/2, L_4=1, r=2, $u_{C3}(0)$=0, $i_{L4}(0)$=1, te pobuda $i_g(t)$=$S(t)$. Koristeći metodu petlji te oznake grana i čvorova prema slici kao odziv izračunati napon grane 1 $u_1(t)$.



Rješenje: Primjena Laplaceove transformacije





(1 bod)

Vidljivo je:

$$I_1(s)=I_{p1}(s); \quad I_2(s)=I_{p1}(s)-I_{p2}(s); \quad I_3(s)=I_{p2}(s)$$

Jednadžbe petlji

$$1)\ I_{p1}(s)(R_1+sL_4)=L_4 i_{L4}(0)+r\cdot I_{p2}(s)$$

$$2)\ I_{p2}(s)\frac{1}{sC_3}=-r\cdot I_{p2}(s)+\frac{1}{sC_3}\cdot I_g(s)$$ (1 bod)

$$2)\Rightarrow\ I_{p2}(s)\left(\frac{1}{sC_3}+r\right)=\frac{1}{sC_3}\cdot I_g(s)\ \Rightarrow\ I_{p2}(s)=\frac{\frac{1}{sC_3}}{\frac{1}{sC_3}+r}\cdot I_g(s)\Rightarrow\ I_{p2}(s)=\frac{1}{1+rsC_3}\cdot I_g(s)$$

$$1)\Rightarrow\ I_1(s)=I_{p1}(s)=\frac{L_4 i_{L4}(0)+r\cdot I_{p2}(s)}{R_1+sL_4}=\frac{L_4 i_{L4}(0)+r\cdot\frac{1}{1+rsC_3}\cdot I_g(s)}{R_1+sL_4}$$

Uz uvrštene vrijednosti elemenata:

$$I_1(s)=\frac{1+2\cdot\frac{1}{1+s}\cdot\frac{1}{s}}{1+s}=\frac{1}{1+s}+\frac{2}{s}\cdot\frac{1}{(1+s)^2};$$

$$\Rightarrow U_1(s)=R_1\cdot I_1(s)=\frac{1}{1+s}+\frac{2}{s}\cdot\frac{1}{(1+s)^2}$$ (1 bod)

Rastav na parcijalne razlomke:

$$\frac{2}{s}\cdot\frac{1}{(s+1)^2}=\frac{A}{s}+\frac{B}{s+1}+\frac{C}{(s+1)^2}$$

$$2=A(s+1)^2+Bs(s+1)+Cs$$

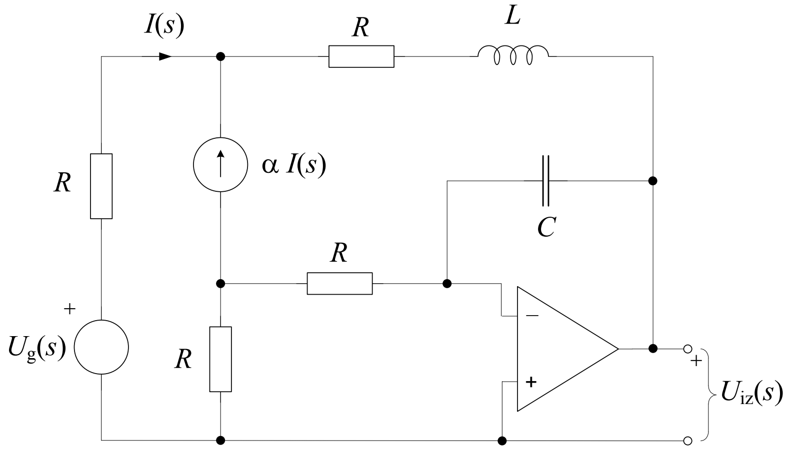
$$2 = As^2 + 2As + A + Bs^2 + Bs + Cs$$
$$2 = (A+B)s^2 + (2A+B+C)s + A$$

$$A + B = 0 \Rightarrow B = -A = -2$$
$$2A + B + C = 0 \Rightarrow C = -2A - B = -4 + 2 = -2$$
$$\underline{A = 2}$$

$$U_1(s) = \frac{1}{1+s} + \frac{2}{s} \cdot \frac{1}{(1+s)^2} = \frac{1}{1+s} + \frac{A}{s} + \frac{B}{s+1} + \frac{C}{(s+1)^2}$$
$$U_1(s) = \frac{1}{1+s} + \frac{2}{s} \cdot \frac{1}{(1+s)^2} = \frac{1}{1+s} + \frac{2}{s} - \frac{2}{s+1} - \frac{2}{(s+1)^2}$$
$$U_1(s) = \frac{2}{s} - \frac{1}{s+1} - \frac{2}{(s+1)^2}$$ (1 bod)

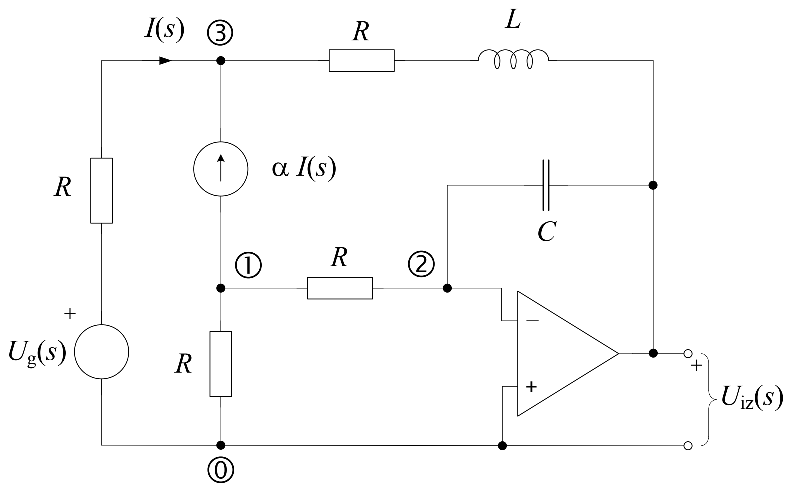
$\Rightarrow \underline{u_1(t) = \left(2 - e^{-t} - 2 \cdot t \cdot e^{-t}\right) \cdot S(t)}$ (1 bod)

4. Za električni krug prikazan slikom izračunati odziv u frekvencijskoj domeni $U_{iz}(s)$ na pobudu $U_g(s)=E/s$. Zadane su normalizirane vrijednosti elemenata R=4, C=0.1 i L=1.25; te konstante E=6 i α=3. Operacijsko pojačalo je idealno. Početni uvjeti: $u_C(0)=0$, $i_L(0)=0$. Traženi odziv treba odrediti primjenom jednadžbi čvorišta.



Rješenje:

Primjenom Laplaceove transformacije dobivamo slijedeći električni krug u *frekvencijskoj* domeni. Postavimo jednadžbe čvorišta:



$$1)\ U_1\left(\frac{1}{R}+\frac{1}{R}\right)-U_2\frac{1}{R}=-\alpha I(s);\ \Rightarrow\ I(s)=\frac{U_g(s)-U_3(s)}{R};$$

$$2)\ -U_1\frac{1}{R}+U_2\left(sC+\frac{1}{R}\right)=U_{iz}(s)sC;$$

$$3)\ U_3\left(\frac{1}{R+sL}\right)=U_{iz}\frac{1}{R+sL}+\alpha\cdot I(s)+I(s);$$ (1 bod)

$$U_2=0\ \Rightarrow$$

$$1)\ U_1\frac{2}{R}=-\alpha\frac{U_g(s)-U_3(s)}{R};$$

$$2)\ -U_1\frac{1}{R}=U_{iz}(s)sC;$$

$$3)\ U_3(s)\frac{1}{R+sL}=(1+\alpha)\cdot\frac{U_g(s)-U_3(s)}{R}+U_{iz}(s)\frac{1}{R+sL};$$ (1 bod)

Nakon malo sređivanja:

$$1)\ 2U_1 - \alpha U_3 = -\alpha U_g(s) \Rightarrow U_1 = \frac{\alpha}{2}U_3(s) - \frac{\alpha}{2}U_g(s);$$

$$2)\ U_{iz}(s) = -\frac{1}{sRC}U_1;$$

$$3)\ U_3(s)\left[\frac{1}{R+sL} + (1+\alpha)\cdot\frac{1}{R}\right] = U_g(s)\cdot\frac{1+\alpha}{R} + U_{iz}(s)\frac{1}{R+sL}$$

$$3) \Rightarrow U_3(s)\left[R + (1+\alpha)(R+sL)\right] = U_g(s)\cdot(1+\alpha)(R+sL) + U_{iz}(s)R;$$

$$\Rightarrow U_3(s) = U_g(s)\cdot\frac{(1+\alpha)(R+sL)}{R+(1+\alpha)(R+sL)} + U_{iz}(s)\frac{R}{R+(1+\alpha)(R+sL)};$$

$$2) \Rightarrow U_{iz}(s) = -\frac{1}{sRC}U_1 = -\frac{1}{sRC}\frac{\alpha}{2}\left[U_3(s) - U_g(s)\right]$$

$$U_{iz}(s) = -\frac{1}{sRC}\frac{\alpha}{2}\left[\frac{(1+\alpha)(R+sL)}{R+(1+\alpha)(R+sL)}\cdot U_g(s) - U_g(s) + \frac{R}{R+(1+\alpha)(R+sL)}\cdot U_{iz}(s)\right]$$

$$-U_{iz}(s)\frac{2sRC}{\alpha} = \frac{-R}{R+(1+\alpha)(R+sL)}\cdot U_g(s) + \frac{R}{R+(1+\alpha)(R+sL)}\cdot U_{iz}(s)$$

$$U_{iz}(s)\left[\frac{2sRC}{\alpha} + \frac{R}{R+(1+\alpha)(R+sL)}\right] = \frac{R}{R+(1+\alpha)(R+sL)}\cdot U_g(s)$$

$$U_{iz}(s) = \frac{\dfrac{R}{R+(1+\alpha)(R+sL)}}{\dfrac{2sRC}{\alpha} + \dfrac{R}{R+(1+\alpha)(R+sL)}}\cdot U_g(s) = \frac{R\alpha}{2sRC\left[R+(1+\alpha)(R+sL)\right] + R\alpha}\cdot U_g(s)$$

(2 boda)

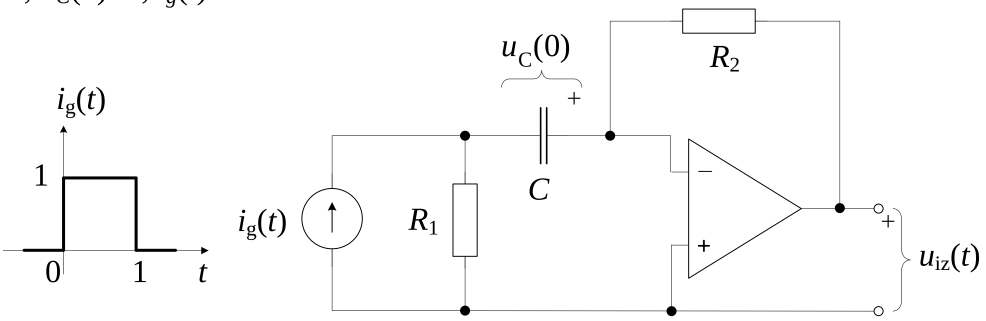
Uz uvrštene vrijednosti elemenata:

$$U_{iz}(s) = \frac{12}{2s0.4\left[4+(1+3)(4+s1.25)\right]+3\cdot 4}\cdot\frac{6}{s} = \frac{12}{0.8s\left[5s+20\right]+12}\cdot\frac{6}{s} = \frac{12}{4s^2+16s+12}\cdot\frac{6}{s}$$

$$\underline{U_{iz}(s) = \frac{3}{s^2+4s+3}\cdot\frac{6}{s} = \frac{18}{s\left(s^2+4s+3\right)}}$$ (1 bod)

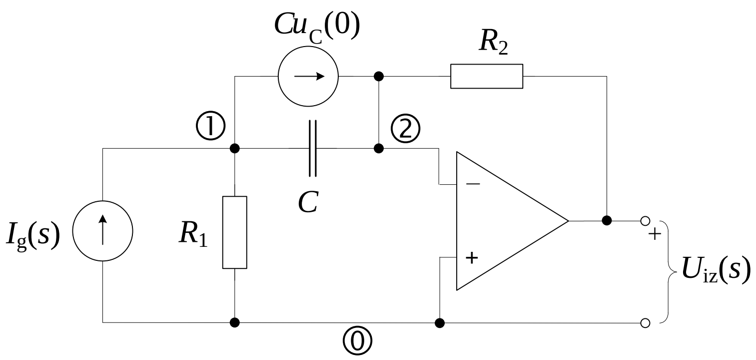
MEĐUISPIT IZ ELEKTRIČNIH KRUGOVA 2012-2013 – Rješenja

1. Za električni krug prikazan slikom izračunati: a) napon na izlazu operacijskog pojačala $U_{iz}(s)$; b) valni oblik napona $u_{iz}(t)$. Zadane su normalizirane vrijednosti elemenata: $R_1=1$, $R_2=1/2$, $C=2$, $u_C(0)=1$, $i_g(t)$ zadan slikom.



Rješenje:

a) Jednadžbe čvorišta:



$$1)\ U_1\left(sC+\frac{1}{R_1}\right)-U_2sC=I_g(s)-Cu_C(0)$$

$$2)\ -U_1sC+U_2\left(sC+\frac{1}{R_2}\right)=\frac{U_{iz}(s)}{R_2}+Cu_C(0)$$

$$U_2=0$$

$$1)\ U_1\left(sC+\frac{1}{R_1}\right)=I_g(s)-Cu_C(0)$$

$$2)\ -U_1sC=\frac{U_{iz}(s)}{R_2}+Cu_C(0)\Rightarrow U_1=-\frac{U_{iz}(s)}{sCR_2}-\frac{u_C(0)}{s}$$ (1 bod)

$$\Rightarrow\ -\left(\frac{U_{iz}(s)}{sCR_2}+\frac{u_C(0)}{s}\right)\left(sC+\frac{1}{R_1}\right)=I_g(s)-Cu_C(0)$$

$$\Rightarrow\ U_{iz}(s)=-\frac{I_g(s)-Cu_C(0)}{sC+\frac{1}{R_1}}sCR_2-Cu_C(0)R_2$$ (1 bod)

$$i_g(t)=S(t)-S(t-1)\Rightarrow I_g(s)=\frac{1}{s}-\frac{1}{s}e^{-s}=\frac{1}{s}(1-e^{-s})$$ (1 bod)

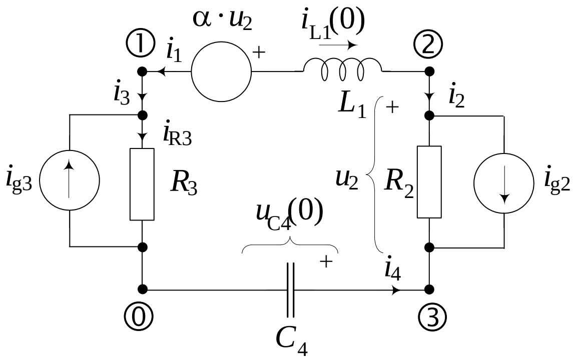
$$U_{iz}(s) = -\frac{\frac{1}{s}(1-e^{-s})-2}{2s+1}s - 1 = \frac{2s-1}{2s+1} + \frac{e^{-s}}{2s+1} - 1 =$$

$$= \frac{2s+1-2}{2s+1} + \frac{e^{-s}}{2s+1} - 1 = 1 + \frac{-2}{2s+1} + \frac{e^{-s}}{2s+1} - 1 = \frac{-2}{2s+1} + \frac{e^{-s}}{2s+1} = -\frac{1}{s+\frac{1}{2}} + \frac{\frac{1}{2}}{s+\frac{1}{2}} \cdot e^{-s}$$

(1 bod)

$\Rightarrow \; u_{iz}(t) = -e^{-t/2}S(t) + \frac{1}{2}e^{-(t-1)/2}S(t-1)$ (1 bod)

4. Za električni krug na slici i pridruženim orijentacijama grana zadane su normalizirane vrijednosti elemenata L_1=1, R_2=1, R_3=2, C_4=1/2, te α=2, $u_{C4}(0)$=2, $i_{L1}(0)$=1, $i_{g2}(t)$=$i_{g3}(t)$=$S(t)$. Koristeći KZS i KZN te oznake grana i čvorova prema slici, napisati:
a) Jednadžbe KZS i KZN;
b) Naponsko-strujne jednadžbe za grane;
c) Napon na otporu R_2: $U_2(s)$ i $u_2(t)$;
d) Struju kroz otpor R_3: $I_{R3}(s)$ i $i_{R3}(t)$;



<u>Rješenje:</u>

a) N_b=4 (broj grana)
N_v=4 (broj čvorova)
Broj jednadžbi KZS $= N_v - 1 = 4 - 1 = 3$
Broj jednadžbi KZN $= N_b - N_v + 1 = 4 - 4 + 1 = 1$
Slijede jednadžbe Kirchhoffovih zakona (4 jednadžbe):
1) $-I_1 + I_3 = 0$ KZS
2) $I_1 + I_2 = 0$ KZS
3) $-I_2 - I_4 = 0$ KZS
4) $U_1 - U_2 + U_3 + U_4 = 0$ KZN (1 bod)

b) Naponsko–strujne jednadžbe grana (4 jednadžbe):
1) $U_1 = sL_1 \cdot I_1 + L_1 i_{L1}(0) + \alpha U_2 = sL_1 \cdot I_1 + L_1 i_{L1}(0) + \alpha (I_2 - I_{g2}) R_2$
2) $U_2 = (I_2 - I_{g2}) \cdot R_2$
3) $U_3 = (I_3 + I_{g3}) \cdot R_3$
4) $U_4 = \frac{1}{sC_4} \cdot I_4 - \frac{u_{C4}(0)}{s}$ (1 bod)

---

c) Sustav ima ukupno $2N_b$=8 jednadžbi i 8 nepoznanica (sve struje i svi naponi grana)
Naponsko – strujne jednadžbe grana uvrstimo u jednadžbe Kirchhoffovih zakona (1)–(4) te dobivamo:

1) , 2), 3) $I_1 = -I_2 = I_3 = I_4$

4) $sL_1 I_1 + L_1 i_{L1}(0) + \alpha (I_2 - I_{g2}) R_2 - (I_2 - I_{g2}) R_2 + (I_3 + I_{g3}) R_3 + \frac{1}{sC_4} I_4 - \frac{u_{C4}(0)}{s} = 0$

$$1)\rightarrow 4) \Rightarrow sL_1 I_1 + L_1 i_{L1}(0) - \alpha (I_1 + I_{g2}) R_2 + (I_1 + I_{g2}) R_2 + (I_1 + I_{g3}) R_3 + \frac{1}{sC_4} I_1 - \frac{u_{C4}(0)}{s} = 0$$

$$\Rightarrow I_1 \left(sL_1 + (1-\alpha) R_2 + R_3 + \frac{1}{sC_4} \right) + L_1 i_{L1}(0) + (1-\alpha) I_{g2} R_2 + I_{g3} R_3 - \frac{u_{C4}(0)}{s} = 0$$

$$\Rightarrow\ I_1(s)=\frac{-L_1 i_{L1}(0)-(1-\alpha)I_{g2}R_2-I_{g3}R_3+\dfrac{u_{C4}(0)}{s}}{sL_1+(1-\alpha)R_2+R_3+\dfrac{1}{sC_4}}$$

$$I_1(s)=\frac{-1+\dfrac{1}{s}-\dfrac{2}{s}+\dfrac{2}{s}}{s-1+2+\dfrac{2}{s}}=\frac{-1+\dfrac{1}{s}}{s+1+\dfrac{2}{s}}=\frac{-s+1}{s^2+s+2}$$ (1 bod)

$$\Rightarrow\ I_2(s)=-I_1(s)=\frac{s-1}{s^2+s+2}\ \Rightarrow$$

$$U_2(s)=\left[I_2(s)-I_{g2}(s)\right]R_2=\frac{s-1}{s^2+s+2}-\frac{1}{s}=\frac{s-1}{s^2+s+\frac{1}{4}+\frac{7}{4}}-\frac{1}{s}=\frac{s+\frac{1}{2}-\frac{3}{2}}{\left(s+\frac{1}{2}\right)^2+\frac{7}{4}}-\frac{1}{s}$$

$$U_2(s)=\frac{s+\frac{1}{2}}{\left(s+\frac{1}{2}\right)^2+\left(\frac{\sqrt{7}}{2}\right)^2}-\frac{3}{\sqrt{7}}\frac{\frac{\sqrt{7}}{2}}{\left(s+\frac{1}{2}\right)^2+\left(\frac{\sqrt{7}}{2}\right)^2}-\frac{1}{s}$$

$$u_2(t)=e^{-\frac{t}{2}}\left(\cos\frac{\sqrt{7}}{2}t-\frac{3}{\sqrt{7}}\sin\frac{\sqrt{7}}{2}t\right)S(t)-S(t)$$ (1 bod)

---

d) Struja kroz R_3

$$\Rightarrow\ I_3(s)=I_1(s)=\frac{-s+1}{s^2+s+2}\ \Rightarrow$$

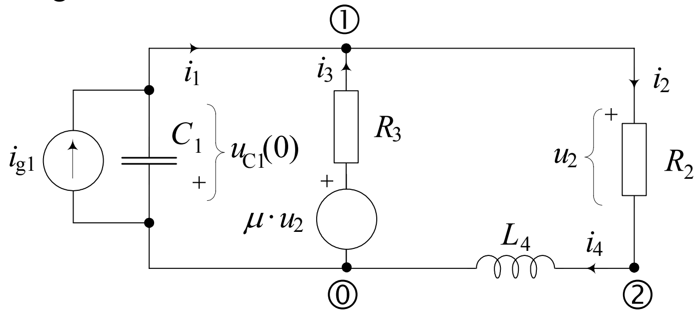
$$I_{R3}(s)=I_3(s)+I_{g3}(s)=-\frac{s-1}{s^2+s+2}+\frac{1}{s}=-\left(\frac{s-1}{s^2+s+2}-\frac{1}{s}\right)$$

[= minus izraz za $U_2(s)$ gore]

$$i_{R3}(t)=-e^{-\frac{t}{2}}\left(\cos\frac{\sqrt{7}}{2}t-\frac{3}{\sqrt{7}}\sin\frac{\sqrt{7}}{2}t\right)S(t)+S(t)$$ (1 bod)

MEĐUISPIT IZ ELEKTRIČNIH KRUGOVA – Rješenja – 2011

1. Za električni krug na slici i pridruženim orijentacijama grana te čvorovima zadane su normalizirane vrijednosti elemenata C_1=1, R_2=1, R_3=1, L_4=1 te μ=2, $u_{C1}(0)$=1, $i_{g1}(t)$=$S(t)$. Koristeći KZS i KZN te oznake grana i čvorova prema slici, napisati:
a) Jednadžbe KZS i KZN;
b) Naponsko-strujne jednadžbe za grane;
c) Napon na otporu R_2 $U_2(s)$;
d) Napon na otporu R_2 $u_2(t)$;
e) Da li je električni krug stabilan? Zašto?



Rješenje:

a) N_b=4 (broj grana)
N_v=3 (broj čvorova)
Broj jednadžbi $\text{KZS} = N_v - 1 = 3 - 1 = 2$
Broj jednadžbi $\text{KZN} = N_b - N_v + 1 = 4 - 3 + 1 = 2$
Slijede jednadžbe Kirchhoffovih zakona (4 jednadžbe):
1) $-I_1 + I_2 - I_3 = 0$ KZS
2) $-I_2 + I_4 = 0$ KZS
3) $U_1 - U_3 = 0$ KZN
4) $U_2 + U_3 + U_4 = 0$ KZN (1 bod)

b) Naponsko–strujne jednadžbe grana (4 jednadžbe):
1) $U_1 = \frac{1}{sC_1} \cdot I_1 - \frac{1}{sC_1} \cdot I_{g1} + \frac{u_{C1}(0)}{s}$
2) $U_2 = R_2 \cdot I_2$
3) $U_3 = R_3 \cdot I_3 - \mu U_2 = R_3 \cdot I_3 - \mu R_2 I_2$
4) $U_4 = sL_4 \cdot I_4$ (1 bod)

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c) Sustav ima ukupno $2N_b$=8 jednadžbi i 8 nepoznanica (sve struje i svi naponi grana)
Naponsko – strujne jednadžbe grana uvrstimo u jednadžbe Kirchhoffovih zakona (1)–(4) te dobivamo:
1) $I_1 = I_2 - I_3$
2) $I_2 = I_4$
3) $\frac{1}{sC_1} \cdot I_1 - \frac{1}{sC_1} \cdot I_{g1} + \frac{u_{C1}(0)}{s} - R_3 \cdot I_3 + \mu R_2 I_2 = 0$
4) $R_2 \cdot I_2 + R_3 \cdot I_3 - \mu R_2 I_2 + sL_4 \cdot I_4 = 0$

$$1)\to 3)\Rightarrow \frac{1}{sC_1}\cdot(I_2-I_3)-\frac{1}{sC_1}\cdot I_{g1}+\frac{u_{C1}(0)}{s}-R_3\cdot I_3+\mu R_2 I_2=0$$

$$\left(\frac{1}{sC_1}+\mu R_2\right)\cdot I_2-\left(\frac{1}{sC_1}+R_3\right)\cdot I_3-\frac{1}{sC_1}\cdot I_{g1}+\frac{u_{C1}(0)}{s}=0$$

$$2)\to 4)\Rightarrow R_2\cdot I_2+R_3\cdot I_3-\mu R_2 I_2+sL_4\cdot I_2=0\ \Rightarrow\ I_3=-\frac{1}{R_3}\left[(1-\mu)R_2+sL_4\right]\cdot I_2$$

$$\Rightarrow\left(\frac{1}{sC_1}+\mu R_2\right)\cdot I_2+\left(\frac{1}{sC_1}+R_3\right)\cdot\frac{1}{R_3}\left[(1-\mu)R_2+sL_4\right]\cdot I_2=\frac{1}{sC_1}\cdot I_{g1}-\frac{u_{C1}(0)}{s}\ \Big/\cdot sC_1$$

$$\left(1+\mu R_2 sC_1\right)\cdot I_2+\left(\frac{1}{R_3}+sC_1\right)\cdot\left[(1-\mu)R_2+sL_4\right]\cdot I_2=I_{g1}-C_1u_{C1}(0)$$

$$I_2=\frac{I_{g1}-C_1u_{C1}(0)}{1+\mu R_2sC_1+\left(\frac{1}{R_3}+sC_1\right)\cdot\left[(1-\mu)R_2+sL_4\right]};\ U_2=R_2I_2=\frac{R_2\cdot(I_{g1}-C_1u_{C1}(0))}{1+\mu R_2sC_1+\left(\frac{1}{R_3}+sC_1\right)\cdot\left[(1-\mu)R_2+sL_4\right]}$$

$$=\frac{\frac{1}{s}-1}{1+2s+(1+s)\cdot(-1+s)}=\frac{\frac{1}{s}-1}{1+2s+s^2-1}=\frac{\frac{1}{s}-1}{s(s+2)}=\frac{1-s}{s^2(s+2)}\quad \text{(1 bod)}$$

d) Odziv u vremenskoj domeni (rastav na parcijalne razlomke)

$$U_2(s)=\frac{1-s}{s^2(s+2)}=\frac{A}{s^2}+\frac{B}{s}+\frac{C}{s+2}$$

$$A(s+2)+Bs(s+2)+Cs^2=1-s$$

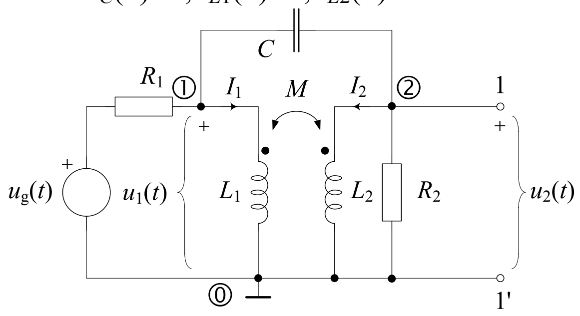
$$(B+C)s^2+(A+2B)s+2A=1-s\Rightarrow A=\frac{1}{2},B=-\frac{1+A}{2}=-\frac{3}{4},C=-B=\frac{3}{4}$$

$$U_2(s)=\frac{1}{4}\left(\frac{2}{s^2}-\frac{3}{s}+\frac{3}{s+2}\right)\Rightarrow \underline{u_2(t)=\frac{1}{4}\left(2t-3+3e^{-2t}\right)\cdot S(t)}\quad \text{(1 bod)}$$

e) Stabilnost:
NE, jer ima dvostruki pol u ishodištu
(odziv na konačnu pobudu teži u ∞ kad $t\to\infty$.) (1 bod)

ZAVRŠNI ISPIT IZ ELEKTRIČNIH KRUGOVA – Rješenja – 2011

1. Za električni krug na slici izračunati odziv $u_2(t)$ na prilazu 1–1', ako je zadan poticaj $u_g(t)=e^{-t}\cdot S(t)$. Zadane su normalizirane vrijednosti elemenata: $R_1=R_2=1$, $L_1=L_2=1$, $M=1$, $C=1$. Početni uvjeti su jednaki nula: $u_C(0)=0$, $i_{L1}(0)=0$, $i_{L2}(0)=0$.



Rješenje:

Napomena: ako se odmah uvrste numeričke vrijednosti (što se jednako priznaje za točno rješenje) tada je postupak znatno jednostavniji i kraći.

Postavimo jednadžbe čvorova:

$$(1)\ U_1\cdot\left(\frac{1}{R_1}+sC\right)-U_2\cdot sC=\frac{U_g}{R_1}-I_1$$

$$(2)\ -U_1\cdot sC+U_2\cdot\left(\frac{1}{R_2}+sC\right)=-I_2$$

$$\begin{matrix} U_1=sL_1\cdot I_1+sM\cdot I_2 \\ U_2=sM\cdot I_1+sL_2\cdot I_2 \end{matrix}\quad \text{(jednadžbe vezanih induktiviteta)}$$

(2 boda)

Uvrstimo vrijednosti elemenata:

$$(1)\ U_1\cdot(1+s)-U_2\cdot s=\frac{1}{s+1}-I_1$$

$$(2)\ -U_1\cdot s+U_2\cdot(1+s)=-I_2$$

$$(3)\ U_1=s\cdot I_1+s\cdot I_2$$

$$(4)\ U_2=s\cdot I_1+s\cdot I_2$$

(1 bod)

$$(3),\ (4)\Rightarrow U_1=U_2$$

$$(1)\ I_1=\frac{1}{s+1}-U_1\cdot(1+s)+U_1\cdot s=\frac{1}{s+1}-U_1$$

$$(2)\ I_2=-U_2$$

$$(4)\ U_2=s\cdot\overbrace{\left(\frac{1}{s+1}-U_1\right)}^{I_1}+s\cdot I_2=s\cdot\left(\frac{1}{s+1}-U_2\right)-s\cdot U_2=\frac{s}{s+1}-2s\cdot U_2$$

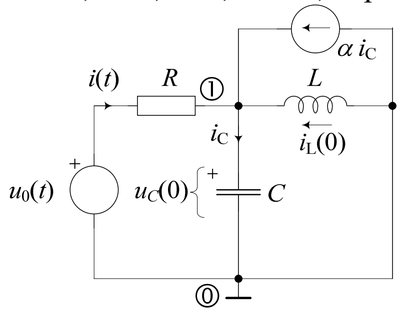
$$U_2(1+2s)=\frac{s}{s+1}$$

$$U_2(s)=\frac{s}{(s+1)(2s+1)}=\frac{s}{2(s+1)(s+1/2)}=\frac{A}{s+1}+\frac{B}{s+1/2}\Rightarrow A=1,\quad B=-\frac{1}{2}$$ (1 bod)

$$\Rightarrow u_2(t)=\left(e^{-t}-\frac{1}{2}e^{-\frac{t}{2}}\right)\cdot S(t)$$ (1 bod)

PISMENI ISPIT IZ ELEKTRIČNIH KRUGOVA – Rješenja – 2011

1. Za električni krug prikazan slikom odrediti odziv $i(t)$ ako je zadan poticaj $u_0(t)=\cos(t)S(t)$. Zadani su normalizirani elementi $R=1$, $C=1$, $L=2$, $\alpha=1/2$, te početni uvjeti $u_C(0)=1$, $i_L(0)=1/2$.



Rješenje:

Metoda napona čvorova:

$$(1)\ U_1 \cdot \left(\frac{1}{R} + sC + \frac{1}{sL}\right) = \frac{U_0}{R} + \frac{i_L(0)}{s} + \alpha I_C + Cu_C(0) \quad \text{(1 bod)}$$

$$U_1 = I_C \frac{1}{sC} + \frac{u_C(0)}{s} \Rightarrow I_C = sC\left(U_1 - \frac{u_C(0)}{s}\right) \quad \text{(1 bod)}$$

Uz uvrštene vrijednosti elemenata: $I_C = s\left(U_1 - \frac{1}{s}\right) = sU_1 - 1$

$$U_1 \cdot \left(1 + s + \frac{1}{2s}\right) = \frac{s}{s^2+1} + \frac{1}{2s} + \frac{s}{2}U_1 - \frac{1}{2} + 1$$

$$U_1 \cdot \left(1 + s + \frac{1}{2s} - \frac{s}{2}\right) = \frac{s}{s^2+1} + \frac{1}{2s} + \frac{1}{2}$$

$$U_1 \cdot \left(1 + \frac{s}{2} + \frac{1}{2s}\right) = \frac{2s^2 + s^2 + 1 + s^3 + s}{2s(s^2+1)}$$

$$U_1 \frac{1 + 2s + s^2}{2s} = \frac{s^3 + 3s^2 + s + 1}{2s(s^2+1)} \quad \Rightarrow \quad U_1 = \frac{s^3 + 3s^2 + s + 1}{(s+1)^2(s^2+1)} \quad \text{(1 bod)}$$

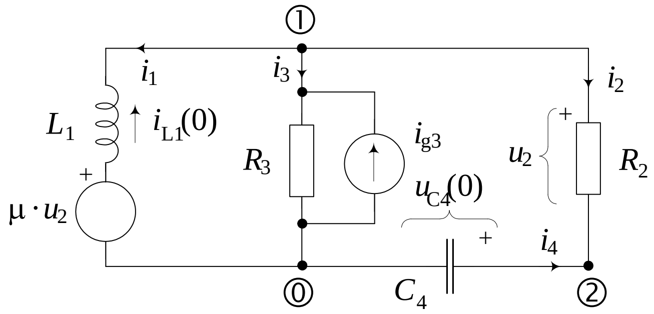
$$U_1 = U_0 - IR \Rightarrow I = U_0 - U_1$$

$$I = \frac{s}{s^2+1} - \frac{s^3 + 3s^2 + s + 1}{(s+1)^2(s^2+1)} = \frac{s(s^2 + 2s + 1) - (s^3 + 3s^2 + s + 1)}{(s+1)^2(s^2+1)}$$

$$= \frac{-(s^2+1)}{(s+1)^2(s^2+1)} = \frac{-1}{(s+1)^2} \quad \text{(1 bod)}$$

$$I(s) = \frac{-1}{(s+1)^2} \Rightarrow i(t) = -te^{-t}S(t) \quad \text{(1 bod)}$$

2. Za električni krug na slici i pridruženim orijentacijama grana zadane su normalizirane vrijednosti elemenata L_1=1, R_2=1, R_3=1, C_4=1, te μ=2, $u_{C4}(0)$=1, $i_{L1}(0)$=1, $i_{g3}(t)=S(t)$. Koristeći KZS i KZN te oznake grana i čvorova prema slici, napisati: a) Jednadžbe KZS i KZN (odabrati referentne smjerove petlji u smjeru kazaljke na satu); b) Naponsko-strujne jednadžbe za grane; c) Napon na otporu R_2 $U_2(s)$; d) Napon na otporu R_2 $u_2(t)$; e) Da li je električni krug stabilan? Zašto?

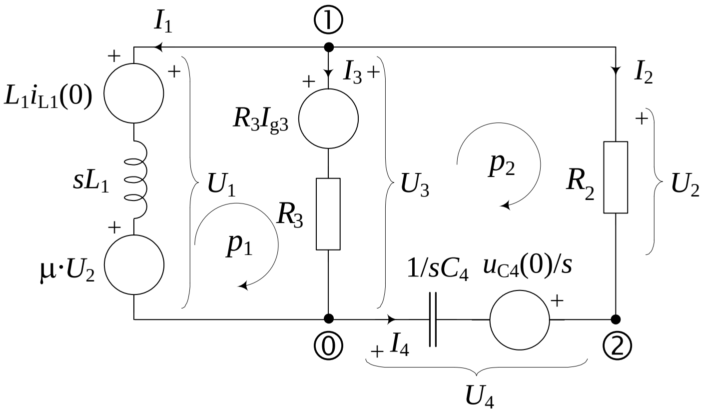


<u>Rješenje:</u> Laplaceova transformacija

a)

N_b=4 (broj grana)

N_v=3 (broj čvorova)



Broj jednadžbi

$\text{KZS} = N_v - 1 = 3 - 1 = 2$

Broj jednadžbi

$\text{KZN} = N_b - N_v + 1 = 4 - 3 + 1 = 2$

Slijede jednadžbe Kirchhoffovih zakona (4 jednadžbe):

1) $I_1 + I_2 + I_3 = 0$ KZS čvorište (1)

2) $-I_2 - I_4 = 0$ KZS čvorište (2)

3) $-U_1 + U_3 = 0$ KZN petlja p_1

4) $U_2 - U_3 - U_4 = 0$ KZN petlja p_2 **(1 bod)**

b) Naponsko–strujne jednadžbe grana (4 jednadžbe):

1) $U_1 = sL_1 \cdot I_1 + L_1 i_{L1}(0) + \mu U_2 = sL_1 \cdot I_1 + \mu R_2 \cdot I_2 + L_1 i_{L1}(0)$

2) $U_2 = R_2 \cdot I_2$

3) $U_3 = R_3 \cdot I_3 + I_{g3} R_3$

4) $U_4 = \dfrac{1}{sC_4} \cdot I_4 - \dfrac{u_{C4}(0)}{s}$ **(1 bod)**

c) Sustav ima ukupno $2N_b$=8 jednadžbi i 8 nepoznanica (sve struje i svi naponi grana)
Naponsko – strujne jednadžbe grana uvrstimo u jednadžbe Kirchhoffovih zakona (1)–(4) te dobivamo:

1) $I_1 = -I_2 - I_3$

2) $I_2 = -I_4$

3) $-sL_1 \cdot I_1 - \mu R_2 \cdot I_2 - L_1 i_{L1}(0) + R_3 \cdot I_3 + I_{g3} R_3 = 0$

4) $R_2 \cdot I_2 - R_3 \cdot I_3 - I_{g3} R_3 - \dfrac{1}{sC_4} \cdot I_4 + \dfrac{u_{C4}(0)}{s} = 0$

$$1) \to 3) \Rightarrow \; sL_1 \cdot (I_2 + I_3) - \mu R_2 \cdot I_2 - L_1 i_{L1}(0) + R_3 \cdot I_3 + I_{g3} R_3 = 0$$

$$2) \to 4) \Rightarrow \; R_2 \cdot I_2 - R_3 \cdot I_3 - I_{g3} R_3 + \frac{1}{sC_4} \cdot I_2 + \frac{u_{C4}(0)}{s} = 0 \; \Rightarrow$$

$$(1') \; I_2 (sL_1 - \mu R_2) + I_3 (sL_1 + R_3) = L_1 i_{L1}(0) - I_{g3} R_3$$

$$(2') \; \left(R_2 + \frac{1}{sC_4} \right) \cdot I_2 - R_3 \cdot I_3 = I_{g3} R_3 - \frac{u_{C4}(0)}{s}$$

$\Rightarrow I_2(s), \; I_3(s)$ koristimo metodu determinanti:

$$\begin{bmatrix} sL_1 - \mu R_2 & sL_1 + R_3 \\ R_2 + \frac{1}{sC_4} & -R_3 \end{bmatrix} \cdot \begin{bmatrix} I_2(s) \\ I_3(s) \end{bmatrix} = \begin{bmatrix} L_1 i_{L1}(0) - I_{g3} R_3 \\ I_{g3} R_3 - \frac{u_{C4}(0)}{s} \end{bmatrix}$$

$$\Delta = \begin{vmatrix} sL_1 - \mu R_2 & sL_1 + R_3 \\ R_2 + \frac{1}{sC_4} & -R_3 \end{vmatrix} = -R_3 (sL_1 - \mu R_2) - (sL_1 + R_3) \left(R_2 + \frac{1}{sC_4} \right)$$

$$\Delta = -R_3 s L_1 + \mu R_2 R_3 - sL_1 R_2 - R_3 R_2 - \frac{L_1}{C_4} - R_3 \frac{1}{sC_4} = -s + 2 - s - 1 - 1 - \frac{1}{s} = -2s - \frac{1}{s}$$

$$\Delta_2 = \begin{vmatrix} L_1 i_{L1}(0) - I_{g3} R_3 & sL_1 + R_3 \\ I_{g3} R_3 - \frac{u_{C4}(0)}{s} & -R_3 \end{vmatrix} = -R_3 \left(L_1 i_{L1}(0) - I_{g3} R_3 \right) - (sL_1 + R_3) \left(I_{g3} R_3 - \frac{u_{C4}(0)}{s} \right)$$

$$\Delta_2 = -R_3 L_1 i_{L1}(0) + R_3^2 I_{g3} - sL_1 I_{g3} R_3 - R_3^2 I_{g3} + sL_1 \frac{u_{C4}(0)}{s} + R_3 \frac{u_{C4}(0)}{s} =$$

$$= -R_3 L_1 i_{L1}(0) - sL_1 I_{g3} R_3 + L_1 u_{C4}(0) + R_3 \frac{u_{C4}(0)}{s} = -1 - s \cdot \frac{1}{s} + 1 + \frac{1}{s} = -1 + \frac{1}{s}$$

$$I_2(s) = \frac{\Delta_2}{\Delta} = \frac{-1 + \frac{1}{s}}{-2s - \frac{1}{s}} = \frac{1 - \frac{1}{s}}{2s + \frac{1}{s}} = \frac{s - 1}{2s^2 + 1}; \quad R_2 = 1$$

$$U_2(s) = I_2(s) R_2 = \frac{s-1}{2s^2+1}$$ **(1 bod)**

d) Napon na otporu R_2 $u_2(t)$:

$$U_2(s) = \frac{1}{2} \cdot \frac{s-1}{s^2 + \frac{1}{2}} = \frac{1}{2} \cdot \frac{s}{s^2 + \frac{1}{2}} - \frac{1}{2} \cdot \frac{1}{s^2 + \frac{1}{2}} = \frac{1}{2} \cdot \frac{s}{s^2 + \left(\frac{1}{\sqrt{2}} \right)^2} - \frac{1}{\sqrt{2}} \cdot \frac{\frac{1}{\sqrt{2}}}{s^2 + \left(\frac{1}{\sqrt{2}} \right)^2}$$

$$u_2(t) = \left[\frac{1}{2} \cdot \cos\left(\frac{t}{\sqrt{2}} \right) - \frac{1}{\sqrt{2}} \cdot \sin\left(\frac{t}{\sqrt{2}} \right) \right] S(t)$$ **(1 bod)**

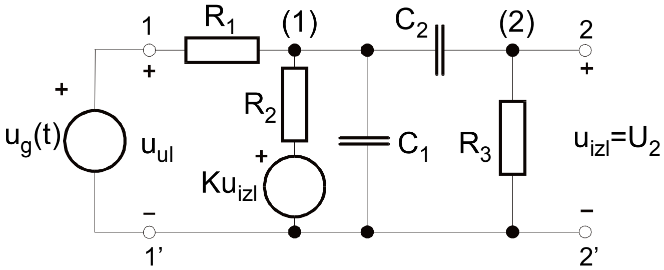
e) Stabilnost:

Električni krug je marginalno stabilan (na rubu stabilnosti).

Polovi $s^2 + \frac{1}{2} = 0 \Rightarrow s_{p1,2} = \pm j \frac{\sqrt{2}}{2}$ su jednostruki i nalaze se na imaginarnoj osi.

(1 bod)

4. Odrediti prijenosnu funkciju $T(s)=U_{iz}(s)/U_{ul}(s)$ za mrežu prikazanu slikom (koristiti metodu čvorova ili metodu petlji). Izračunati omjer amplituda te razliku u fazi napona na ulazu i izlazu mreže ako je zadano: napon generatora na ulazu $u_g(t)$= 10 sin t, i normirane vrijednosti elemenata R_1=R_2=2, R_3=1, C_1=C_2=1 i K=2.



Rješenje: Metoda čvorišta:

$$(1)\qquad U_1\left(\frac{1}{R_1}+\frac{1}{R_2}+sC_1+sC_2\right)-U_2sC_2=\frac{U_g}{R_1}+\frac{KU_{iz}}{R_2}$$

$$(2)\qquad -U_1sC_2+U_2\left(sC_2+\frac{1}{R_3}\right)=0$$

$$(2)\Rightarrow U_1=U_2\left(1+\frac{1}{sR_3C_2}\right)\rightarrow(1)$$

$$(1)\qquad U_2\left[\left(1+\frac{1}{sR_3C_2}\right)\left(\frac{1}{R_1}+\frac{1}{R_2}+sC_1+sC_2\right)-sC_2-\frac{K}{R_2}\right]=\frac{U_g}{R_1}$$

$$U_2\left[\left(\frac{1}{R_1}+\frac{1}{R_2}+sC_1+sC_2\right)+\frac{1}{sR_1R_3C_2}+\frac{1}{sR_2R_3C_2}+\frac{C_1}{R_3C_2}+\frac{1}{R_3}-sC_2-K\frac{1}{R_2}\right]=\frac{U_g}{R_1}\Big/R_1R_2R_3C_2s$$

$$U_2\left[sR_2R_3C_2+sR_1R_3C_2+s^2C_1C_2R_1R_2R_3+R_2+R_1+sR_1R_2C_1+sR_1R_2C_2-sKR_1R_3C_2\right]=U_gR_2R_3C_2s$$

Prijenosna funkcija napona:

$$T(s)=\frac{U_2}{U_g}=\frac{sR_2R_3C_2}{s^2C_1C_2R_1R_2R_3+s\left[R_2R_3C_2+R_1R_2(C_1+C_2)+R_1R_3C_2(1-K)\right]+R_1+R_2}$$

Uz uvrštene vrijednosti:

$$T(s)=\frac{s\cdot 2}{s^2 4+s\left[2+8-2\right]+4}=\frac{1}{4}\cdot\frac{2\cdot s}{s^2+2s+1}$$

Signal: $\omega_g=1\quad U_g=10\angle 0^o$

$$\left|T(j\omega)\right|_{\omega=1}=\frac{1}{2}\frac{|j\omega|}{\left|-\omega^2+2j\omega+1\right|}=\frac{1}{2}\frac{\omega}{\sqrt{(1-\omega^2)^2+(2\omega)^2}}$$

$$\left|T(j\omega)\right|_{\omega=1}=\frac{1}{2}\frac{1}{\sqrt{0+2^2}}=\frac{1}{2}\cdot\frac{1}{2}=\frac{1}{4}$$

$$T(j\omega)=\frac{1}{2}\cdot\frac{\left[(1-\omega^2)-j(2\omega)\right]j\omega}{(1-\omega^2)^2+(2\omega)^2}=\frac{1}{2}\cdot\frac{2\omega^3+j\omega(1-\omega^2)}{(1-\omega^2)^2+4\omega^2}$$

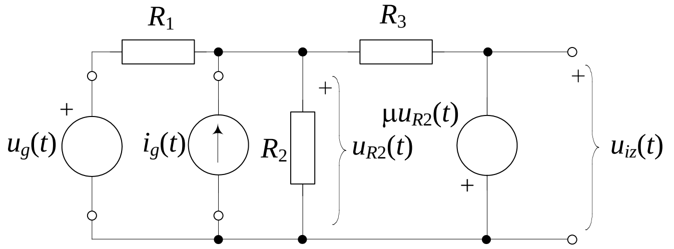
$$\varphi(\omega)=\arctan\left[\frac{\operatorname{Im}\{T(j\omega)\}}{\operatorname{Re}\{T(j\omega)\}}\right]=\arctan\left[\frac{\omega(1-\omega^2)}{2\omega^3}\right]=\arctan\left[\frac{1-\omega^2}{2\omega^2}\right]$$

$$\varphi(1)=\arctan\left(\frac{1-1}{2\cdot 1}\right)=\arctan(0)=0^o$$

Odgovor: Omjer amplituda iz-ul signala je: 1:4. Razlika u fazi iz-ul signala je: 0

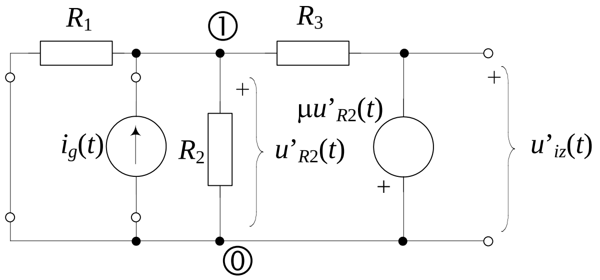
ZAVRŠNI ISPIT IZ ELEKTRIČNIH KRUGOVA 2014-2015 – Rješenja

1. Za električni krug prikazan slikom primjenom metode superpozicije izračunati valni oblik napona $u_{iz}(t)$ kao odziv, ako je zadana pobuda $u_g(t)=6S(t)$ i $i_g(t)=3\delta(t)$. Zadane su normalizirane vrijednosti elemenata $R_1=1$, $R_2=1$, $R_3=2$ i a) $\mu=1$; b) $\mu=\infty$.



<u>Rješenje:</u> Primjena metode superpozicije.

a) Isključen je naponski izvor: $u_g(t)=0$ (umjesto isključenog naponskog izvora je kratki spoj). Ovisni izvor (NONI) s parametrom μ (naponsko pojačanje) ostaje uključen.



Jednadžbe čvorišta (samo jedan čvor)

$$(1)\ u'_1(t)\left(\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}\right)=i_g(t)+\frac{u'_{iz}(t)}{R_3}$$

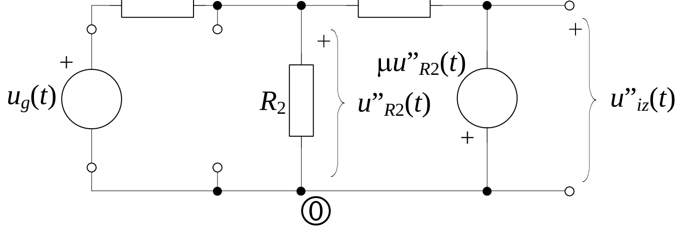
$$(2)\ u'_{iz}(t)=-\mu u'_{R2}(t)=-\mu u'_1(t)\ \Rightarrow\ u'_1(t)=-\frac{u'_{iz}(t)}{\mu}$$

$$\Rightarrow\qquad -\frac{u'_{iz}(t)}{\mu}\left(\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}\right)=i_g(t)+\frac{u'_{iz}(t)}{R_3}$$

$$\frac{u'_{iz}(t)}{\mu}\frac{1}{R_1}+\frac{u'_{iz}(t)}{\mu}\frac{1}{R_2}+\frac{u'_{iz}(t)}{R_3}\left(1+\frac{1}{\mu}\right)=-i_g(t)$$

$$u'_{iz}(t)=\frac{-i_g(t)}{\dfrac{1}{\mu R_1}+\dfrac{1}{\mu R_2}+\dfrac{1}{R_3}\left(1+\dfrac{1}{\mu}\right)}$$

b) Isključen je strujni izvor: $i_g(t)=0$ (umjesto isključenog strujnog izvora je prazni hod). Ovisni izvor (NONI) s parametrom μ (naponsko pojačanje) ostaje uključen.



Jednadžbe čvorišta (samo jedan čvor)

$$(1)\;\; u''_1(t)\left(\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}\right)=\frac{u_g(t)}{R_1}+\frac{u''_{iz}(t)}{R_3}$$

$$(2)\;\; u''_{iz}(t)=-\mu u''_{R2}(t)=-\mu u''_1(t)\;\Rightarrow\; u''_1(t)=-\frac{u''_{iz}(t)}{\mu}$$

$$\Rightarrow \qquad -\frac{u''_{iz}(t)}{\mu}\left(\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}\right)=\frac{u_g(t)}{R_1}+\frac{u''_{iz}(t)}{R_3}$$

$$\frac{u''_{iz}(t)}{\mu}\frac{1}{R_1}+\frac{u''_{iz}(t)}{\mu}\frac{1}{R_2}+\frac{u''_{iz}(t)}{R_3}\left(1+\frac{1}{\mu}\right)=-\frac{u_g(t)}{R_1}$$

$$u''_{iz}(t)=\frac{-\frac{u_g(t)}{R_1}}{\frac{1}{\mu R_1}+\frac{1}{\mu R_2}+\frac{1}{R_3}\left(1+\frac{1}{\mu}\right)}$$

(3 boda)

c) Superpozicija:

$$u_{iz}(t)=u'_{iz}(t)+u''_{iz}(t)=\frac{-i_g(t)-\frac{u_g(t)}{R_1}}{\frac{1}{\mu R_1}+\frac{1}{\mu R_2}+\frac{1}{R_3}\left(1+\frac{1}{\mu}\right)}$$

(1 bod)

Uz uvrštene vrijednosti elemenata R_1=1, R_2=1, R_3=2 i μ=1 slijedi:

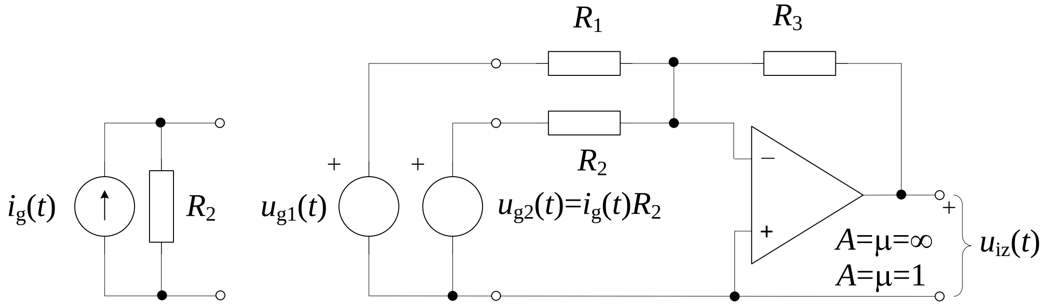
$$u_{iz}(t)=u'_{iz}(t)+u''_{iz}(t)=\frac{-i_g(t)-\frac{u_g(t)}{R_1}}{\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}\left(1+\frac{1}{1}\right)}=\frac{-i_g(t)-u_g(t)}{3}=-\delta(t)-2S(t)$$

Uz uvrštene vrijednosti elemenata R_1=1, R_2=1, R_3=2 i μ=∞ slijedi:

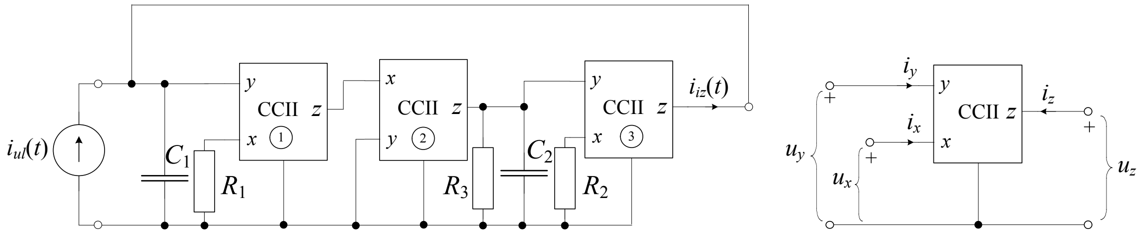
$$u_{iz}(t)=u'_{iz}(t)+u''_{iz}(t)=\frac{-i_g(t)-\frac{u_g(t)}{R_1}}{\frac{1}{R_3}}=-R_3 i_g(t)-\frac{R_3}{R_1}u_g(t)=-6\delta(t)-12S(t)$$

(1 bod)

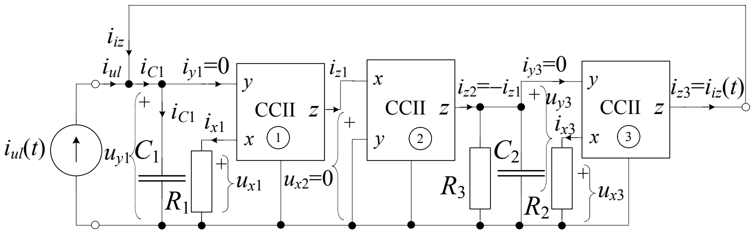
Električni krug u ovom zadatku također možemo prikazati pomoću operacijskog pojačala:



5. Za električni krug prikazan slikom izračunati valni oblik struje $i_{iz}(t)$ za $t>0$ kao odziv, ako je zadana pobuda $i_{ul}(t)=\delta(t)$[A]. Zadane su normalizirane vrijednosti elemenata $R_1=1$, $R_2=1$, $R_3=4$, $C_1=1$, $C_2=1/16$. Za strujni prijenosnik druge generacije (CCII) vrijede slijedeće definicijske jednadžbe: $u_x=u_y$, $i_y=0$, $i_z=i_x$ uz referentna usmjerenja struja i napona prilaza prikazana na slici.



Rješenje:



a) Za prvi CCII vrijedi: (1 bod)

$$U_{x1}=U_{y1}=U_{C1}=I_{C1}\cdot\frac{1}{sC_1},\; I_{y1}=0$$

$$I_{z1}=I_{x1}=\frac{U_{x1}}{R_1}=I_{C1}\frac{1}{sR_1C_1}\Rightarrow\boxed{I_{z1}=I_{C1}\frac{1}{sR_1C_1}}$$

b) Za drugi CCII vrijedi: (1 bod)

$$I_{z2}=I_{x2}=-I_{z1}\Rightarrow\boxed{I_{z2}=-I_{z1}=-I_{C1}\frac{1}{sR_1C_1}}$$

$$U_{x2}=0,\; U_{y2}=0$$

c) Za treći CCII vrijedi: (1 bod)

$$U_{y3}=I_{z2}\cdot\frac{R_3\cdot 1/(sC_2)}{R_3+1/(sC_2)}=I_{z2}\cdot\frac{R_3}{sR_3C_2+1},$$

$$I_{iz}=I_{z3}=I_{x3}=\frac{U_{x3}}{R_2}=\frac{U_{y3}}{R_2}=I_{z2}\cdot\frac{R_3}{R_2}\cdot\frac{1}{sR_3C_2+1}\Rightarrow\boxed{I_{z3}=I_{z2}\cdot\frac{R_3}{R_2}\cdot\frac{1}{sR_3C_2+1}}$$

d) Nakon sređivanja do sada napisanih izraza: (1 bod)

$$I_{C1}=I_{ul}+I_{iz};\quad I_{iz}=I_{z3}=-(I_{ul}+I_{iz})\cdot\frac{1}{sR_1C_1}\cdot\frac{R_3}{R_2}\cdot\frac{1}{sR_3C_2+1}$$

$$I_{iz}\cdot\left[1+\frac{R_3}{sR_1C_1\cdot R_2\cdot(sR_3C_2+1)}\right]=-I_{ul}\cdot\frac{R_3}{sR_1C_1\cdot R_2\cdot(sR_3C_2+1)}$$

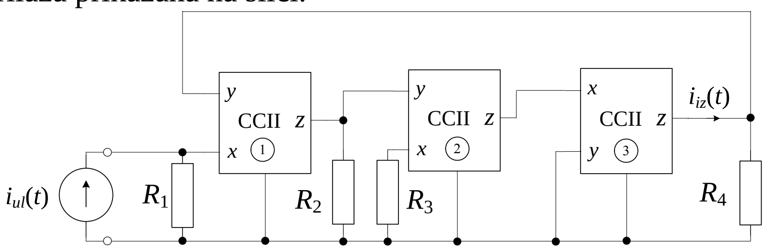
$$I_{iz}=-I_{ul}\cdot\frac{R_3}{s^2R_1R_2R_3C_1C_2+sR_1R_2C_1+R_3}=-I_{ul}\cdot\frac{1}{s^2R_1R_2C_1C_2+sR_1R_2C_1/R_3+1}$$

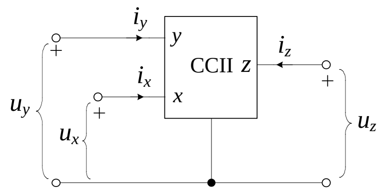
e) Uz uvrštene vrijednosti elemenata: (1 bod)

$$I_{iz}(s)=-I_{ul}\cdot\frac{16}{s^2+4s+16}=-1\cdot\frac{16}{(s+2)^2+12}=-\frac{16}{\sqrt{12}}\cdot\frac{\sqrt{12}}{(s+2)^2+(\sqrt{12})^2}$$

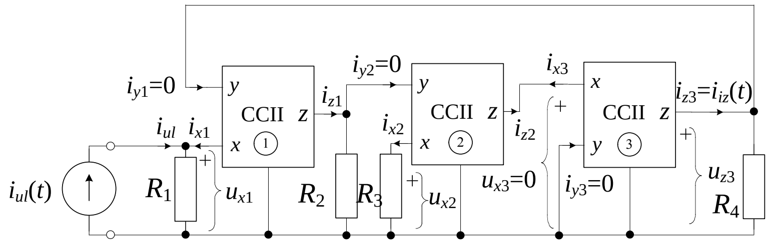
$$\Rightarrow i_{iz}(t)=-\frac{8\sqrt{3}}{3}\cdot e^{-2t}\cdot\sin(2\sqrt{3})\cdot S(t)[A]$$

5. Za električni krug prikazan slikom izračunati valni oblik struje $i_{iz}(t)$ za $t>0$ kao odziv, ako je zadana pobuda $i_{ul}(t)=E\cdot S(t)$[A]. Zadane su normalizirane vrijednosti elemenata $R_1=1$, $R_2=2$, $R_3=3$, $R_4=4$, te konstanta $E=5{,}5$. Za strujni prijenosnik druge generacije (CCII) vrijede slijedeće definicijske jednadžbe: $u_x=u_y$, $i_y=0$, $i_z=i_x$ uz referentna usmjerenja struja i napona prilaza prikazana na slici.





Rješenje:



a) Za prvi CCII vrijedi: (1 bod)

$$u_{x1}=u_{y1}=u_{z3},\ i_{y1}=0,\ i_{x1}+i_{ul}=\frac{u_{x1}}{R_1},\ i_{z1}=i_{x1}$$

b) Za drugi CCII vrijedi: (1 bod)

$$u_{x2}=u_{y2}=i_{z1}R_2,\ i_{x2}=\frac{u_{x2}}{R_3},\ i_{z2}=i_{x2}$$

$$i_{z2}=i_{x2}=\frac{u_{x2}}{R_3}=\frac{u_{y2}}{R_3}=i_{z1}\frac{R_2}{R_3}\Rightarrow\boxed{i_{z2}=i_{z1}\frac{R_2}{R_3}}$$

c) Za treći CCII vrijedi: (1 bod)

$$u_{x3}=u_{y3}=0,\ i_{z3}=i_{x3}=-i_{z2}\Rightarrow\boxed{i_{z3}=-i_{z2}}$$

$$u_{z3}=i_{z3}R_4$$

d) Nakon sređivanja do sada napisanih izraza: (1 bod)

$$\boxed{i_{iz}=i_{z3}},\ \boxed{i_{z3}=-i_{z1}\frac{R_2}{R_3}},\ i_{z1}=i_{x1}=\frac{u_{x1}}{R_1}-i_{ul}=\frac{u_{z3}}{R_1}-i_{ul}=i_{z3}\frac{R_4}{R_1}-i_{ul}$$

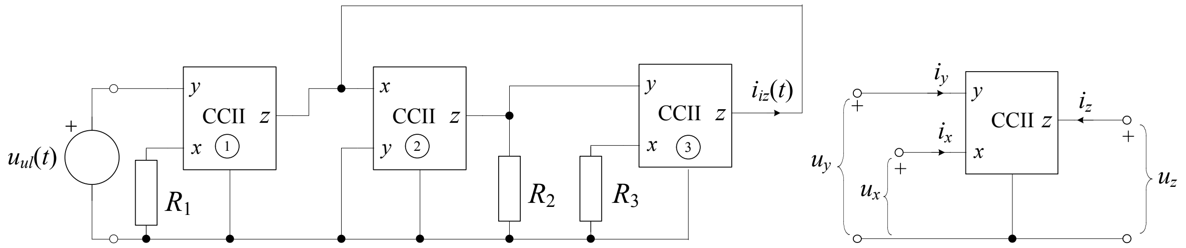
$$\Rightarrow i_{iz}=-\left(i_{iz}\frac{R_4}{R_1}-i_{ul}\right)\frac{R_2}{R_3}\Rightarrow i_{iz}\left(1+\frac{R_2R_4}{R_1R_3}\right)=i_{ul}\frac{R_2}{R_3}\Rightarrow\boxed{i_{iz}=\frac{\frac{R_2}{R_3}}{1+\frac{R_2R_4}{R_1R_3}}i_{ul}}$$

e) Uz uvrštene vrijednosti elemenata: (1 bod)

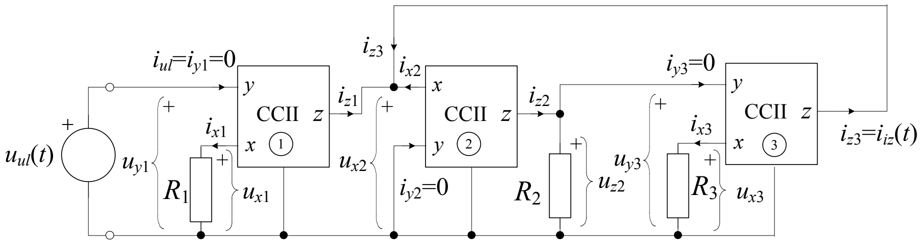
$$i_{iz}(t)=\frac{\frac{2}{3}}{1+\frac{2\cdot4}{1\cdot3}}i_{ul}(t)=\frac{2}{3+8}i_{ul}(t)=\frac{2}{11}i_{ul}(t)=\frac{2\cdot5{,}5}{11}S(t)=1\cdot S(t)[A]$$

$$\Rightarrow \underline{i_{iz}(t)=1\cdot S(t)[A]}$$

5. Za električni krug prikazan slikom izračunati valni oblik struje $i_{iz}(t)$ za t>0 kao odziv, ako je zadana pobuda $u_{ul}(t)=E\cdot S(t)$. Zadane su normalizirane vrijednosti elemenata $R_1=2$, $R_2=4$, $R_3=1$, te konstanta $E=10$. Za strujni prijenosnik druge generacije (CCII) vrijede slijedeće definicijske jednadžbe: $u_x=u_y$, $i_y=0$, $i_z=i_x$ uz referentna usmjerenja struja i napona prilaza prikazana na slici.



Rješenje:



Za prvi CCII vrijedi:

$$u_{x1}=u_{y1}=u_{ul}\,,\; i_{y1}=0\,,\; i_{x1}=\frac{u_{x1}}{R_1}=\frac{u_{ul}}{R_1}\,,\; i_{z1}=i_{x1}=\frac{u_{ul}}{R_1}\text{ (1 bod)}$$

Za drugi CCII vrijedi:

$$u_{x2}=u_{y2}=0\,,\; i_{x2}=-i_{z1}-i_{z3}\,,\; i_{z2}=i_{x2}=-(i_{z1}+i_{z3})\text{ (1 bod)}$$

$$u_{z2}=R_2 i_{z2}=-R_2(i_{z1}+i_{z3})$$

Za treći CCII vrijedi:

$$u_{y3}=u_{z2}\,,\; i_{y3}=0\,,\; u_{x3}=u_{y3}=R_3 i_{x3}\text{ (1 bod)}$$

$$i_{z3}=i_{x3}=\frac{u_{y3}}{R_3}=\frac{-(i_{z1}+i_{z3})R_2}{R_3}$$

$$i_{z3}=-i_{z1}\frac{R_2}{R_3}-i_{z3}\frac{R_2}{R_3}\Rightarrow i_{z3}+i_{z3}\frac{R_2}{R_3}=-i_{z1}\frac{R_2}{R_3}\Rightarrow i_{z3}\left(1+\frac{R_2}{R_3}\right)=-i_{z1}\frac{R_2}{R_3}$$

$$\Rightarrow i_{iz}=i_{z3}=-i_{z1}\cdot\frac{\frac{R_2}{R_3}}{1+\frac{R_2}{R_3}}=-\frac{u_{ul}}{R_1}\cdot\frac{\frac{R_2}{R_3}}{1+\frac{R_2}{R_3}}$$

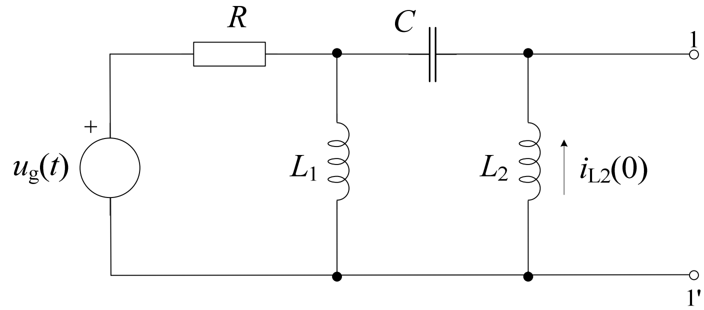
Uz uvrštene vrijednosti elemenata:

$$i_{iz}=-\frac{10}{2}\cdot\frac{\frac{4}{1}}{1+\frac{4}{1}}=-\frac{10}{2}\cdot\frac{4}{5}=-4\text{ (1 bod)}$$

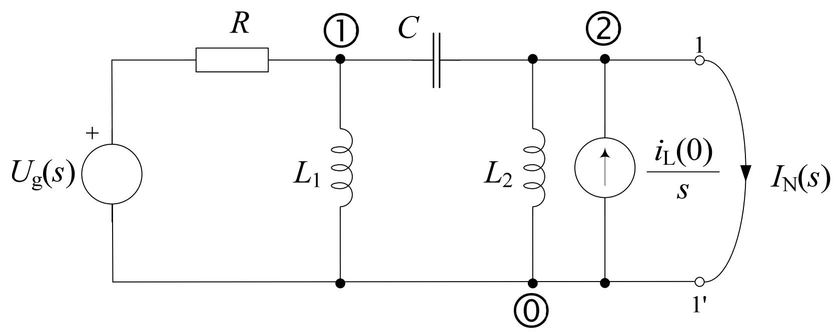
$$\Rightarrow \underline{i_{iz}(t)=-4\cdot S(t)[A]}\text{ (1 bod)}$$

NORTON

2. Za električni krug na slici pomoću teorema superpozicije odrediti parametre nadomjesnog kruga po Nortonu s obzirom na priključnice 1-1': a) I_N(s); b) $Y_N(s)$. Zadane su normalizirane vrijednosti elemenata: $R = 1$, $L_1 = L_2 = 1$, $C = 2$ te $i_{L2}(0) = 1$, $u_g(t) = S(t)$. Koristiti jednadžbe čvorova.

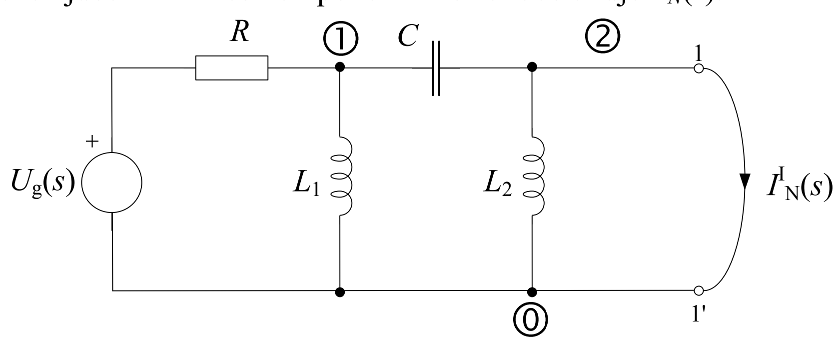


Rješenje: Primjena $\mathscr{L}$-transformacije na električni krug:



a) Nortonova struja $I_N(s)$:

a.1) početni uvjet je isključen i traži se komponenta Nortonove struje $I^I_N(s)$.



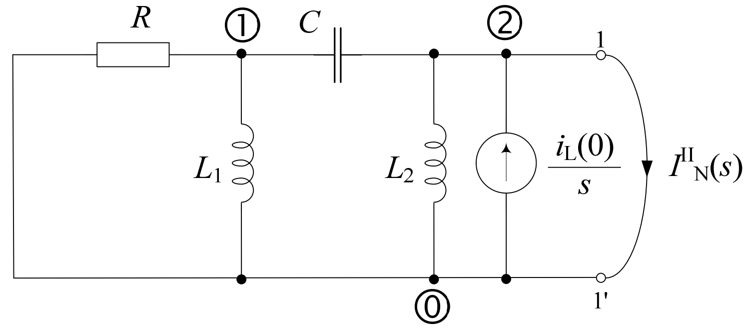
$$(1)\quad \frac{U_g(s)}{R} = U_1\left(\frac{1}{R} + \frac{1}{sL_1} + sC\right) - U_2 sC$$

$$(2)\quad -I_N^I(s) = -U_1 sC + U_2\left(\frac{1}{sL_2} + sC\right)$$

$$U_2 = 0$$

$$(1) \Rightarrow U_1 = \frac{U_g(s)}{R\left(\frac{1}{R} + \frac{1}{sL_1} + sC\right)}\ ;\ (2) \Rightarrow I_N^I(s) = \frac{U_g(s)}{R\left(\frac{1}{R} + \frac{1}{sL_1} + sC\right)} sC$$

a.2) naponski izvor je isključen i traži se komponenta Nortonove struje $I^{II}_{N}(s)$.



(1) $0 = U_1\left(\frac{1}{R} + \frac{1}{sL_1} + sC\right) - U_2 sC$

(2) $\frac{i_{L2}(0)}{s} - I_N^{II}(s) = -U_1 sC + U_2\left(\frac{1}{sL_2} + sC\right)$

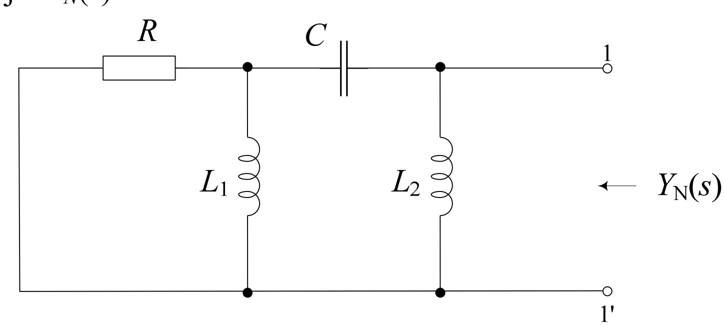
$U_2 = 0 \Rightarrow U_1 = 0$

(2) $\Rightarrow I_N^{II}(s) = \frac{i_{L2}(0)}{s}$

Ukupna Nortonova struja je:

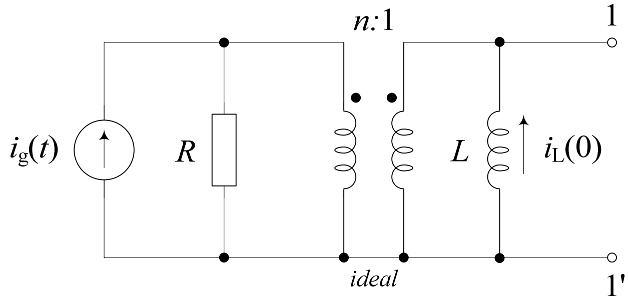
$$I_N(s) = I_N^{I}(s) + I_N^{II}(s) = \frac{U_g(s)}{R\left(\frac{1}{R} + \frac{1}{sL_1} + sC\right)} sC + \frac{i_{L2}(0)}{s} = \frac{\frac{1}{s}2s}{1\left(1 + \frac{1}{s} + 2s\right)} + \frac{1}{s} = \frac{4s^2 + s + 1}{s\left(2s^2 + s + 1\right)}$$

b) Nortonova admitancija $Y_N(s)$:

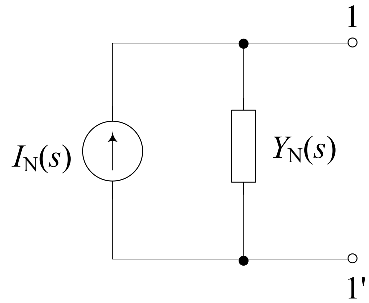


$$Y_N(s) = \frac{1}{sL_2} + \frac{1}{\frac{1}{sC} + \frac{RsL_1}{R + sL_1}} = \frac{2s^3 + 4s^2 + s + 1}{s(2s^2 + s + 1)}$$

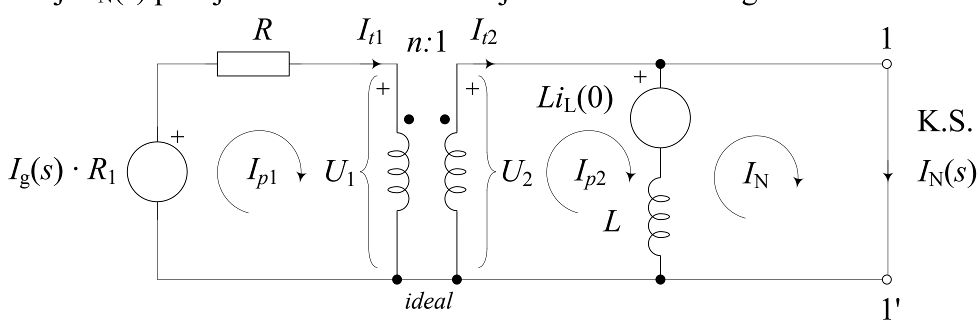
4. Za električni krug na slici izračunati parametre nadomjesnog kruga po Nortonu s obzirom na polove 1–1': $I_N(s)$ i $Y_N(s)$. Zadane su normalizirane vrijednosti elemenata: R=1, L=1, $i_L(0)$=1, n=2, $i_g(t)$=$S(t)$. (Koristiti bilo koju metodu u izračunu; preporučuje se metoda petlji.) Napisati:
a) Nortonovu struju $I_N(s)$ uz uvrštene vrijednosti elemenata;
b) Nortonovu admitanciju $Y_N(s)$ uz uvrštene vrijednosti elemenata;
c) Struju kroz otpor R.



Rješenje:



a) Nortonova struja $I_N(s)$ primjenom $\mathcal{L}$-transformacije na električni krug:



Jednadžbe idealnog transformatora:

$$U_1 = n \cdot U_2 \Rightarrow U_2 = \frac{U_1}{n}$$

$$I_{t1} = \frac{1}{n} \cdot I_{t2} \Rightarrow I_{t2} = n \cdot I_{t1}$$

1) $I_{p1}R = -U_1 + I_g R$ $\qquad I_{p1} = I_{t1}$

2) $(I_{p2} - I_N)sL + Li_L(0) = U_2$ $\qquad I_{p2} = I_{t2}$

Nakon sređivanja jednadžbe glase:

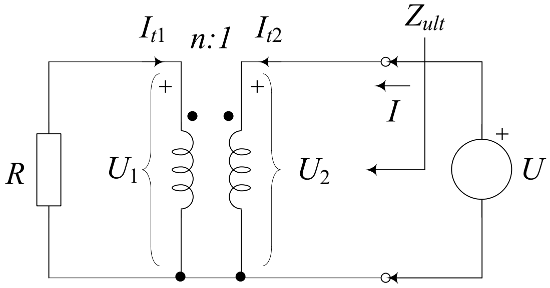
$$\left.\begin{array}{l} 1)\ I_{p1}R + nU_2 = I_g R \\ 2)\ I_N = \dfrac{1}{sL}(I_{p2}sL + Li_L(0) - U_2) \end{array}\right\} \quad \left.\begin{array}{l} 1)\ I_{p1}R + nU_2 = I_g R \\ 2)\ I_N = \dfrac{1}{sL}\left[nI_{p1}sL + Li_L(0) - U_2\right] \end{array}\right\}$$

U_2=0

$$\left.\begin{array}{l} 1)\ I_{p1}R = I_g R \\ \underline{2)\ I_N = \frac{1}{sL}\left[nI_{p1}sL + Li_L(0)\right]} \end{array}\right\} \quad I_N = \frac{1}{sL}\left[nI_g sL + Li_L(0)\right]$$

$$I_N = \frac{1}{s}\left[2\frac{1}{s}s + 1\right] = \frac{3}{s}$$ (1 bod)

b) Nortonova admitancija $Y_N(s)$:
Najjednostavnije je izračunati ulaznu impedanciju u transformator zaključen s R. Označimo je s Z_{ult}.



$$Z_{ult} = \frac{1}{n^2}R$$

$$I_{t1} = -\frac{U_1}{R_1} \Rightarrow \frac{U_1}{I_{t1}} = -R$$

$$U = U_2$$

$$I = I_{t2}$$

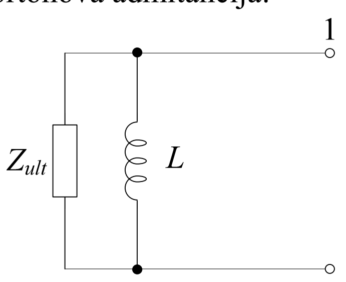
Jednadžbe transformatora su:

$$U_1 = n \cdot U_2 \Rightarrow U_2 = \frac{U_1}{n}$$

$$\underline{I_{t1} = -\frac{1}{n} \cdot I_{t2} \Rightarrow I_{t2} = -n \cdot I_{t1}}$$

$$Z_{ult} = \frac{U}{I} = \frac{U_2}{I_{t2}} = \frac{\frac{U_1}{n}}{-n \cdot I_{t1}} = -\frac{\frac{U_1}{I_{t1}}}{n^2} = -\frac{-R}{n^2} = \frac{R}{n^2}$$

Tada je Nortonova admitancija:

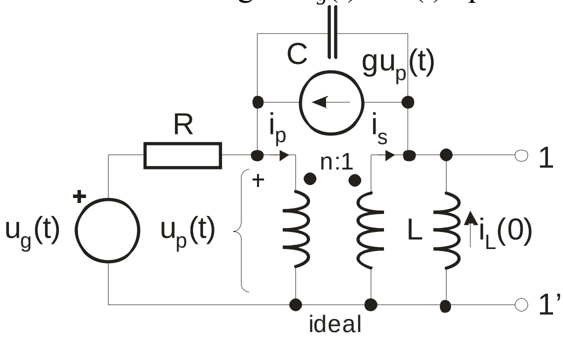


$$\leftarrow Y_N(s) = \frac{1}{Z_{ult}} + \frac{1}{sL}$$

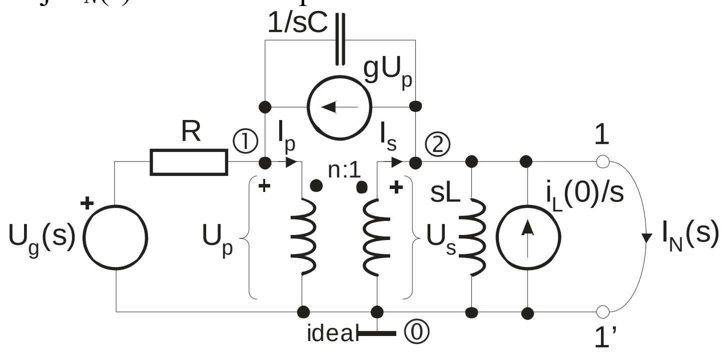
$$Y_N(s) = \frac{n^2}{R} + \frac{1}{sL} = 4 + \frac{1}{s}$$ (1 bod)

c) Za izračun Nortonove struje $I_N(s)$, polovi dvopola 1–1' trebaju biti kratko spojeni.
Struja kroz R je nula jer sva struja iz izvora I_g teče kroz primar idealnog transformatora, odn. I_g=I_{t1}. To je zato jer kratki spoj na sekundaru transformatora uzrokuje kratki spoj na primaru transformatora, a cjelokupna struja uvijek teče kroz kratki spoj, a ništa ne teče kroz R. (1 bod)

3. Za mrežu prikazanu slikom odrediti nadomjesne parametre mreže po Nortonu $I_N(s)$ i $Y_N(s)$ s obzirom na polove 1–1'. Koristiti metodu napona čvorova u proračunu. Zadane su normalizirane vrijednosti elemenata: L=1, C=1, R=1, g=2, n=2, $i_L(0)$=1, $u_C(0)$=0 te izvor $u_g(t)$=$S(t)$. Napisati: a) Jednadžbe čvorova za izračun struje $I_N(s)$; b) Jednadžbe čvorova za izračun $Y_N(s)$. Uz uvrštene vrijednosti elemenata: c) Nortonovu struju $I_N(s)$; d) Nortonovu admitanciju $Y_N(s)$; e) Nortonovu struju $i_N(t)$ ako je pobuda stacionarni sinusni signal $u_g(t)$=sin(t) i početni uvjeti jednaki nula.



Rješenje: Nortonova struja $I_N(s)$ metodom napona čvorova:



U_1=U_p, U_2=U_s

(1) $$U_1\left(\frac{1}{R}+sC\right)-U_2sC=gU_1+\frac{U_g}{R}-I_p \qquad U_s=(1/n)\cdot U_p$$

(2) $$-U_1sC+U_2\left(sC+\frac{1}{sL}\right)=-gU_1+I_s+\frac{i_L(0)}{s}-I_N \qquad I_s=n\cdot I_p$$

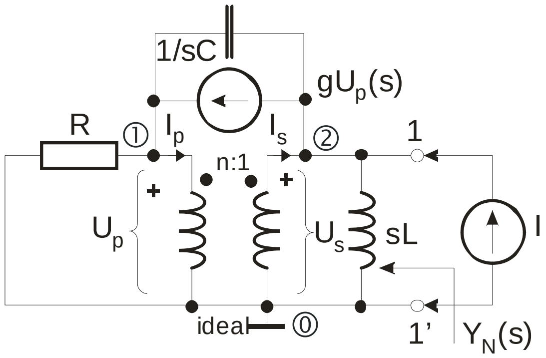
U_2=U_s=0 $\Rightarrow$ U_1=U_p=0

(1) $$I_p=\frac{U_g}{R}\ \Rightarrow\ I_s=nI_p=n\frac{U_g}{R}$$

(2) $$I_N=I_s+\frac{i_L(0)}{s}=n\frac{U_g}{R}+\frac{i_L(0)}{s}$$ **(1 bod)**

$$\Rightarrow\ I_N(s)=n\frac{U_g}{R}+\frac{i_L(0)}{s}=\frac{2}{s}+\frac{1}{s}=\frac{3}{s}$$ **(1 bod)**

Nortonova admitancija $Y_N(s)$ (isključeni su početni uvjeti i neovisni izvori):



$Y_N(s) = \frac{I}{U_2}$, $U_1 = U_p$, $U_2 = U_s$

$$(1) \qquad U_1\left(\frac{1}{R} + sC\right) - U_2 sC = gU_1 - I_p \qquad\qquad U_s = (1/n)\cdot U_p$$

$$(2) \qquad -U_1 sC + U_2\left(sC + \frac{1}{sL}\right) = -gU_1 + I_s + I \qquad\qquad I_s = n\cdot I_p \qquad \textbf{(1 bod)}$$

$$(1) \qquad U_1\left(\frac{1}{R} + sC - \frac{1}{n}sC - g\right) = -I_p$$

$$(2) \qquad U_1\left[-sC + g + \frac{1}{n}\left(sC + \frac{1}{sL}\right)\right] = nI_p + I$$

$$(1) \qquad \Rightarrow nU_2\left(-\frac{1}{R} + \frac{1}{n}sC - sC + g\right) = I_p$$

$$(2) \qquad \Rightarrow nU_2\left(-sC + g + \frac{1}{n}sC + \frac{1}{nsL}\right) - nI_p = I$$

$(1) \rightarrow (2) \Rightarrow$

$$I = nU_2\left(-sC + g + \frac{1}{n}sC + \frac{1}{nsL}\right) - n^2U_2\left(-\frac{1}{R} + \frac{1}{n}sC - sC + g\right)$$

$$Y_N(s) = \frac{I}{U_2} = -nsC + ng + sC + \frac{1}{sL} + n^2\frac{1}{R} - nsC + n^2sC - n^2g =$$

$$= -2s + 4 + s + \frac{1}{s} + 4 - 2s + 4s - 8 = s + \frac{1}{s} \qquad \textbf{(1 bod)}$$

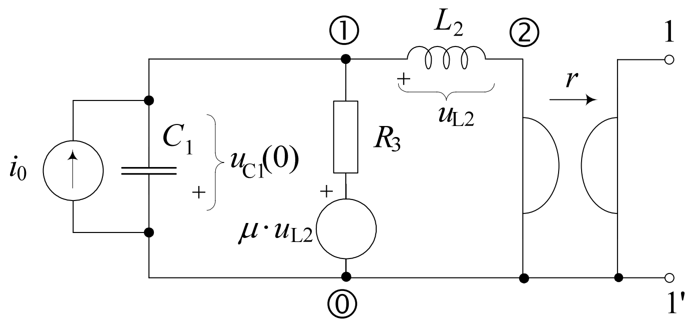
e) Nortonova struja $i_N(t)$ ako je pobuda stacionarni sinusni signal $u_g(t)=\sin(t)$ i početni uvjeti jednaki nula.

$$I_N(j\omega) = \frac{n}{R}U_g(j\omega) = 2\cdot 1\angle 0° = 2\angle 0°$$

$$\Rightarrow i_N(t) = 2\sin(t) \qquad \textbf{(1 bod)}$$

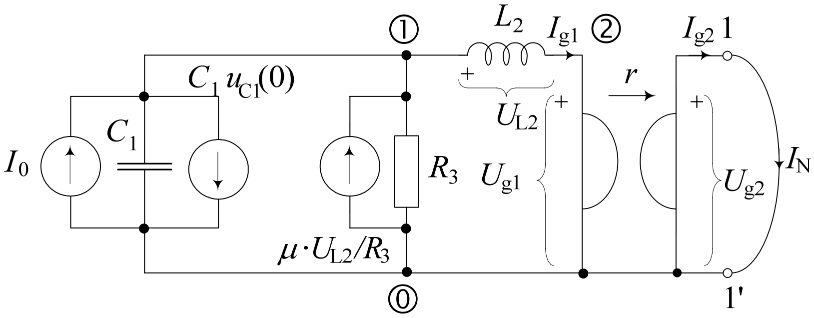
2. Za električni krug na slici zadane su normalizirane vrijednosti elemenata C_1=1, L_2=1, R_3=1 te μ=2, r=1, $u_{C1}(0)$=1, $i_0(t)=S(t)$. Odrediti nadomjesne parametre mreže po Nortonu s obzirom na polove 1–1'. Koristiti metodu napona čvorišta (čvorište ⓪ je referentno). U zadatku je potrebno:

a) Nacrtati sklop za izračunavanje Nortonove struje, postaviti jednadžbe napona za čvorišta ① i ②;
b) Odrediti Nortonovu struju $I_N(s)$;
c) Nacrtati sklop za izračunavanje Nortonove admitancije, postaviti jednadžbe napona za čvorišta ① i ②;
d) Odrediti Nortonovu admitanciju $Y_N(s)$.
e) Da li je električni krug recipročan? Zašto?



Rješenje:

a) Jednadžbe napona za čvorišta ① i ②:



$$1)\ U_1\left(sC_1+\frac{1}{sL_2}+\frac{1}{R_3}\right)-U_2\frac{1}{sL_2}=I_0(s)-C_1u_{C1}(0)+\frac{\mu U_{L2}}{R_3}$$

$$2)\ U_{L2}\frac{1}{sL_2}=I_{g1}(s)\,,\ U_{L2}(s)=U_1(s)-U_2(s)$$

$$3)\ I_{g2}=-\frac{1}{r}U_{g1}\,,\ U_{g1}=U_2\,,\ I_{g2}=I_N\ \Rightarrow\ I_N=-\frac{1}{r}U_2$$

$$4)\ I_{g1}=-\frac{1}{r}U_{g2}\,,\ U_{g2}=0\ \Rightarrow\ I_{g1}=0\ \Rightarrow\ U_{L2}=0$$

$$1)\ U_1\left(sC_1+\frac{1}{sL_2}+\frac{1}{R_3}\right)-U_2\frac{1}{sL_2}=I_0(s)-C_1u_{C1}(0)$$

$$2)\ -U_1\frac{1}{sL_2}+U_2\frac{1}{sL_2}=0\ \Rightarrow\ U_1=U_2\ \Rightarrow\ I_N=-\frac{1}{r}U_1$$ (1 bod)

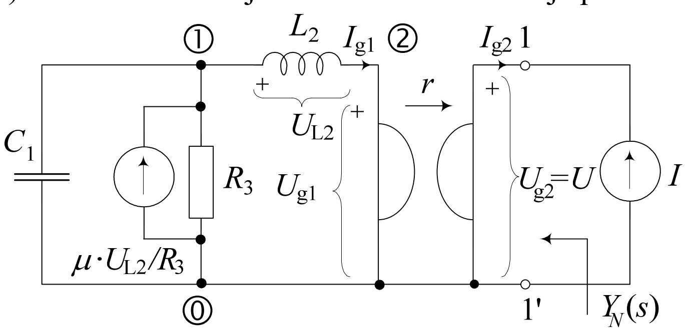
---

b) Nortonova struja $I_N(s)$:

$$U_1\left(sC_1+\frac{1}{R_3}\right)=I_0(s)-C_1u_{C1}(0)\ \Rightarrow U_1=\frac{I_0(s)-C_1u_{C1}(0)}{sC_1+1/R_3}$$

$$I_N(s)=-\frac{1}{r}U_1(s)=-\frac{1}{r}\frac{I_0(s)-C_1u_{C1}(0)}{sC_1+1/R_3}\ \Rightarrow I_N(s)=-\frac{1/s-1}{s+1}=\frac{1-1/s}{s+1}=\frac{s-1}{s(s+1)}$$ (1 bod)

c) Izračunavanje Nortonove admitancije pomoću jednadžbi napona čvorišta ① i ②



$$\Rightarrow Y_N(s)=\frac{I(s)}{U(s)}$$

1) $U_1\left(sC_1+\frac{1}{sL_2}+\frac{1}{R_3}\right)-U_2\frac{1}{sL_2}=\frac{\mu U_{L2}}{R_3}$

2) $U_{L2}\frac{1}{sL_2}=I_{g1}(s)$, $U_{L2}(s)=U_1(s)-U_2(s)$

3) $I_{g2}=-\frac{1}{r}U_{g1}$, $U_{g1}=U_2$, $I_{g2}=-I$

4) $I_{g1}=-\frac{1}{r}U_{g2}$, $U_{g2}=U$

---

1) $U_1\left(sC_1+\frac{1}{sL_2}+\frac{1-\mu}{R_3}\right)-U_2\left(\frac{1}{sL_2}-\frac{\mu}{R_3}\right)=0$

2) $-U_1\frac{1}{sL_2}+U_2\frac{1}{sL_2}=-I_{g1}=\frac{1}{r}U$

3) $I=\frac{1}{r}U_2\ \Rightarrow U_2=r\cdot I$ (1 bod)

---

d) Nortonova admitancija $Y_N(s)$:

$$2)\Rightarrow\ -U_1+U_2=\frac{sL_2}{r}U\ \rightarrow 1)\ \left(U_2-\frac{sL_2}{r}U\right)\left(sC_1+\frac{1}{sL_2}+\frac{1-\mu}{R_3}\right)-U_2\left(\frac{1}{sL_2}-\frac{\mu}{R_3}\right)=0$$

$$U_2\left(sC_1+\frac{1}{sL_2}+\frac{1-\mu}{R_3}-\frac{1}{sL_2}+\frac{\mu}{R_3}\right)-\frac{sL_2}{r}U\left(sC_1+\frac{1}{sL_2}+\frac{1-\mu}{R_3}\right)=0$$

$$I\cdot r\cdot\left(sC_1+\frac{1}{R_3}\right)=\frac{sL_2}{r}U\left(sC_1+\frac{1}{sL_2}+\frac{1-\mu}{R_3}\right)\Rightarrow$$

$$Y_N(s)=\frac{I(s)}{U(s)}=\frac{\frac{sL_2}{r}\left(sC_1+\frac{1}{sL_2}+\frac{1-\mu}{R_3}\right)}{r\left(sC_1+1/R_3\right)}$$

$$Y_N(s)=\frac{I(s)}{U(s)}=\frac{s\left(s+1/s-1\right)}{s+1}=\frac{s^2-s+1}{s+1}=\frac{(s+1)^2-3s}{s+1}=s+1-\frac{3s}{s+1}$$ (1 bod)

e) Da li je električni krug recipročan? Zašto?
NE, električni krug nije recipročan jer sadrži ovisni izvor i girator. (1 bod)

ZAVRŠNI ISPIT IZ ELEKTRIČNIH KRUGOVA – Rješenja – 2012-2013

1. Za električni krug na slici zadane su normalizirane vrijednosti elemenata C=1, L=1, R=1 te μ=2, r=1, $i_g(t)=S(t)$. Odrediti nadomjesne parametre mreže po Northonu s obzirom na polove 1–1'. Koristiti metodu napona čvorišta. U zadatku je potrebno:

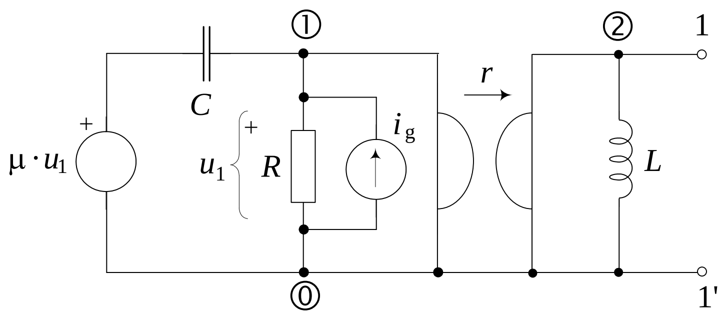
   a) Nacrtati sklop za izračunavanje Nortonove struje,
      postaviti jednadžbe napona za čvorišta ① i ②;

   b) Odrediti Nortonovu struju $I_N(s)$;

   c) Nacrtati sklop za izračunavanje Nortonove admitancije,
      postaviti jednadžbe napona za čvorišta ① i ②;

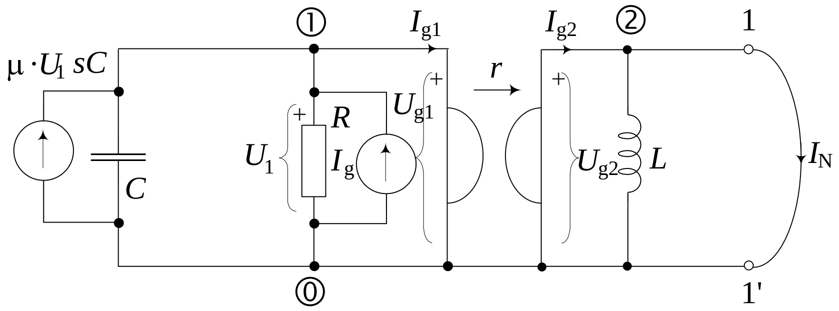
   d) Odrediti Nortonovu admitanciju $Y_N(s)$.

   e) Da li je električni krug recipročan? Zašto?



Rješenje:

a) Jednadžbe napona za čvorišta ① i ②:



1) $U_1\left(sC+\dfrac{1}{R}\right)=I_g(s)+\mu U_1 sC-I_{g1}$

2) $U_2\dfrac{1}{sL}=I_{g2}(s)-I_N(s)$

3) $I_{g2}=-\dfrac{1}{r}U_1$

4) $I_{g1}=-\dfrac{1}{r}U_2$

$U_2=0,\ I_{g1}=0$

1) $U_1\left(sC-\mu sC+\dfrac{1}{R}\right)=I_g(s)\ \Rightarrow\ U_1=\dfrac{I_g(s)}{sC(1-\mu)+1/R}$

2) $I_N(s)=I_{g2}(s)=-\dfrac{1}{r}U_1(s)$ (1 bod)

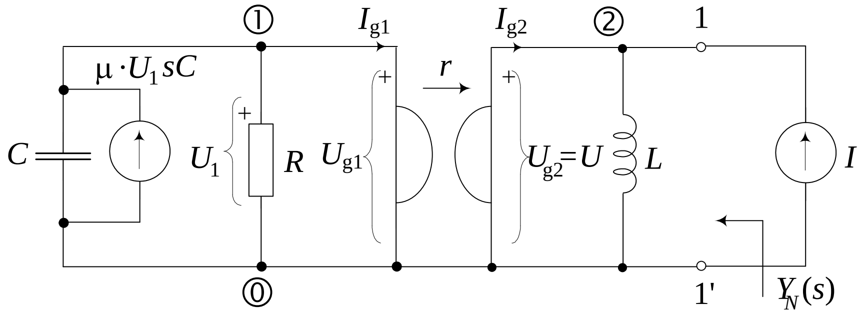
---

b) Nortonova struja $I_N(s)$:

$$1) \rightarrow 2) \Rightarrow I_N(s) = -\frac{1}{r}\cdot\frac{I_g(s)}{sC(1-\mu)+1/R}$$

$$I_N(s) = -\frac{1}{1}\cdot\frac{\frac{1}{s}}{-s+1} = -\frac{1}{s(1-s)} = \frac{1}{s(s-1)} \Rightarrow I_N(s) = \frac{1}{s(s-1)}$$ (1 bod)

c) Izračunavanje Nortonove admitancije pomoću jednadžbi napona čvorišta ① i ②



$$\Rightarrow Y_N(s) = \frac{I(s)}{U(s)}$$

1) $U_1\left(sC+\frac{1}{R}\right) = \mu U_1 sC - I_{g1}$ 3) $I_{g2} = -\frac{1}{r}U_1$

2) $U_2\frac{1}{sL} = I_{g2}(s) + I(s)$ 4) $I_{g1} = -\frac{1}{r}U_2,\ U_2 = U$

$$1)\ U_1\left[sC(1-\mu)+\frac{1}{R}\right] = \frac{1}{r}U_2 \Rightarrow U_1 = \frac{1}{r}\cdot\frac{U_2}{sC(1-\mu)+1/R}$$

$$2)\ U_2\frac{1}{sL} = -\frac{1}{r}U_1 + I(s) \Rightarrow U_2\frac{1}{sL} = -\frac{1}{r^2}\cdot\frac{U_2}{sC(1-\mu)+1/R} + I(s)$$ (1 bod)

d) Nortonova admitancija $Y_N(s)$:

$$I(s) = U(s)\left[\frac{1}{sL}+\frac{1}{r^2}\cdot\frac{1}{sC(1-\mu)+1/R}\right] \Rightarrow Y_N(s) = \frac{I(s)}{U(s)} = \frac{1}{sL}+\frac{1}{r^2}\cdot\frac{1}{sC(1-\mu)+1/R}$$

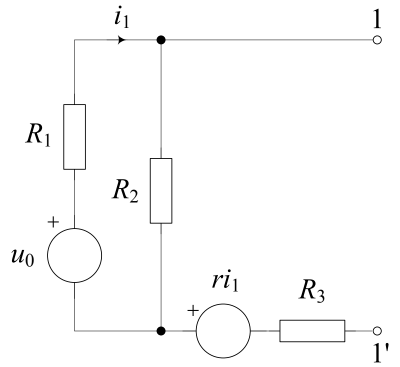
$$Y_N(s) = \frac{I(s)}{U(s)} = \frac{1}{s}+\frac{1}{-s+1} = \frac{-s+1+s}{s(-s+1)} = \frac{1}{s(1-s)}$$ (1 bod)

e) Da li je električni krug recipročan? Zašto?
NE, električni krug nije recipročan jer sadrži ovisni izvor i girator. (1 bod)

3. Za električni krug na slici obzirom na priključnice 1–1' odrediti:

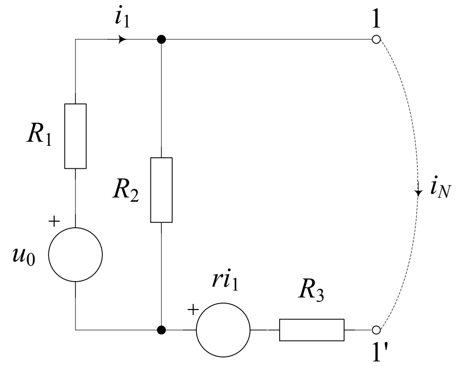
a) izraz za Nortonovu struju i_N;
b) izraz za Nortonovu admitanciju G_N;
c) iznos konstante r ako je G_N=1S;
d) struju i_1 kad se na priključnice 1–1' spoji otpor R=2Ω;
e) iznos konstante r za koji je G_N=1/2.

Zadano je: u_0=2V, R_1=3Ω, R_2=2Ω, R_3=1Ω.



<u>Rješenje:</u>

a) Nortonova struja i_N:



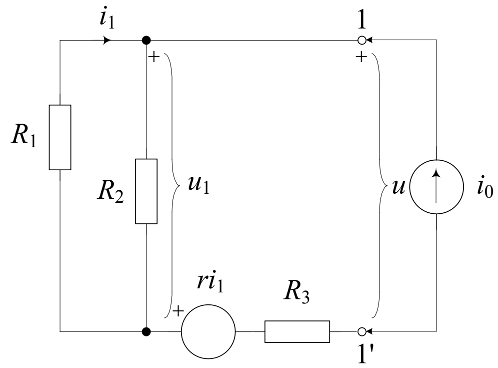
$$u_0 = i_1 \cdot (R_1 + R_2) - i_N \cdot R_2$$

$$0 = -i_1 \cdot R_2 - r \cdot i_1 + i_N \cdot (R_2 + R_3) \;\Rightarrow\; i_1 = i_N \cdot \frac{R_2 + R_3}{r + R_2}$$

$$u_0 = i_N \cdot \frac{(R_1 + R_2)(R_2 + R_3)}{r + R_2} - i_N \cdot R_2$$

$$\boxed{i_N = u_0 \cdot \frac{r + R_2}{R_1 R_2 + (R_1 + R_2) R_3 - r R_2}} \text{ (1 bod)}$$

b) Nortonova admitancija G_N:



$$i_1 = -\frac{u_1}{R_1} = -i_0 \frac{R_1 R_2}{R_1 + R_2} \cdot \frac{1}{R_1} = -i_0 \frac{R_2}{R_1 + R_2}$$

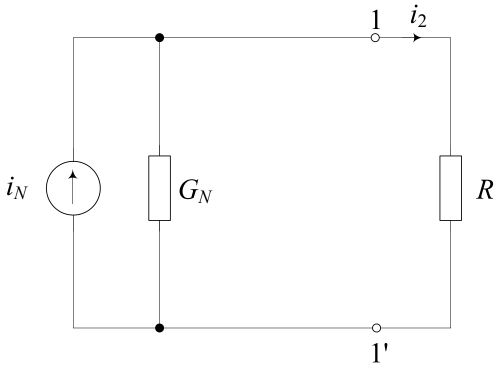
$$u = u_1 + ri_1 + i_0 R_3 = i_0 \frac{R_1 R_2}{R_1 + R_2} - ri_0 \frac{R_2}{R_1 + R_2} + i_0 R_3$$

$$\frac{u}{i_0} = \frac{R_1 R_2 + R_1 R_3 + R_2 R_3 - rR_2}{R_1 + R_2} \quad \Rightarrow \boxed{G_N = \frac{R_1 + R_2}{R_1 R_2 + (R_1 + R_2) R_3 - rR_2}} \text{ (1 bod)}$$

c) iznos konstante r ako je $G_N = 1S$.

$$G_N = \frac{5}{11 - 2r} = 1 \quad \Rightarrow \boxed{r = 3} \text{ (1 bod)}$$

d) struju i_1 kad se na priključnice 1–1' spoji otpor $R = 2\Omega$



$$i_N = u_0 \frac{r + R_2}{R_1 R_2 + (R_1 + R_2) R_3 - rR_2} \quad \Rightarrow \quad i_N = 2 \cdot \frac{3 + 2}{3 \cdot 2 + (3 + 2) \cdot 1 - 3 \cdot 2} = 2 \cdot \frac{5}{5} = 2$$

$$i_2 \cdot R = \frac{i_N - i_2}{G_N} \Rightarrow i_2 = i_N \frac{1}{R \cdot G_N + 1} = \frac{i_N}{3} \quad \Rightarrow \quad i_2 = \frac{2}{3}$$

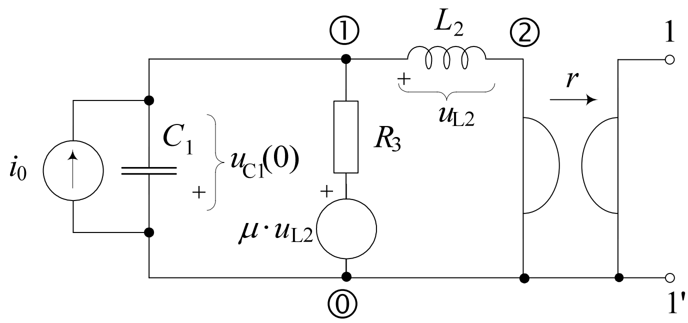
$$i_1 = i_2 \cdot \frac{R_2 + R_3 + R}{r + R_2} = \frac{2}{3} \cdot \frac{5}{5} = \frac{2}{3} \text{ (1 bod)}$$

e) iznos konstante r za koji je $G_N = 1/2$.

$$G_N = \frac{5}{11 - 2r} = \frac{1}{2} \Rightarrow r = \frac{1}{2} \text{ (1 bod)}$$

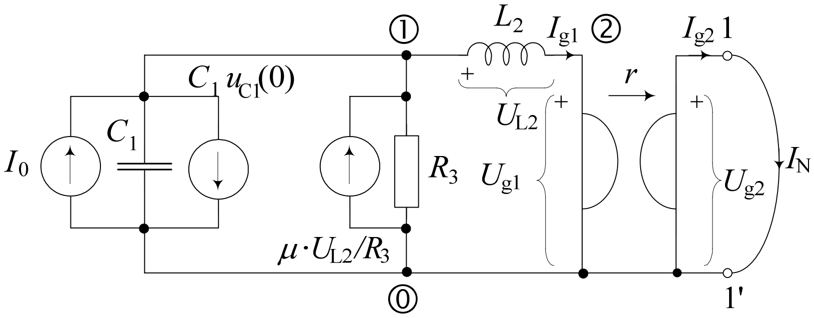
2. Za električni krug na slici zadane su normalizirane vrijednosti elemenata C_1=1, L_2=1, R_3=1 te μ=2, r=1, $u_{C1}(0)$=1, $i_0(t)$=$S(t)$. Odrediti nadomjesne parametre mreže po Nortonu s obzirom na polove 1–1'. Koristiti metodu napona čvorišta (čvorište ⓪ je referentno). U zadatku je potrebno:

a) Nacrtati sklop za izračunavanje Nortonove struje, postaviti jednadžbe napona za čvorišta ① i ②;
b) Odrediti Nortonovu struju $I_N(s)$;
c) Nacrtati sklop za izračunavanje Nortonove admitancije, postaviti jednadžbe napona za čvorišta ① i ②;
d) Odrediti Nortonovu admitanciju $Y_N(s)$.
e) Da li je električni krug recipročan? Zašto?



Rješenje:

a) Jednadžbe napona za čvorišta ① i ②:



$$1)\ U_1\left(sC_1+\frac{1}{sL_2}+\frac{1}{R_3}\right)-U_2\frac{1}{sL_2}=I_0(s)-C_1u_{C1}(0)+\frac{\mu U_{L2}}{R_3}$$

$$2)\ U_{L2}\frac{1}{sL_2}=I_{g1}(s)\,,\ U_{L2}(s)=U_1(s)-U_2(s)$$

$$3)\ I_{g2}=-\frac{1}{r}U_{g1}\,,\ U_{g1}=U_2\,,\ I_{g2}=I_N\ \Rightarrow\ I_N=-\frac{1}{r}U_2$$

$$4)\ I_{g1}=-\frac{1}{r}U_{g2}\,,\ U_{g2}=0\ \Rightarrow\ I_{g1}=0\ \Rightarrow\ U_{L2}=0$$

$$1)\ U_1\left(sC_1+\frac{1}{sL_2}+\frac{1}{R_3}\right)-U_2\frac{1}{sL_2}=I_0(s)-C_1u_{C1}(0)$$

$$2)\ -U_1\frac{1}{sL_2}+U_2\frac{1}{sL_2}=0\ \Rightarrow\ U_1=U_2\ \Rightarrow\ I_N=-\frac{1}{r}U_1$$ (1 bod)

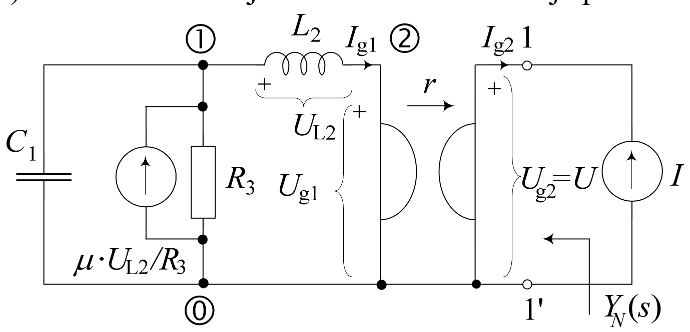
---

b) Nortonova struja $I_N(s)$:

$$U_1\left(sC_1+\frac{1}{R_3}\right)=I_0(s)-C_1u_{C1}(0) \;\Rightarrow\; U_1=\frac{I_0(s)-C_1u_{C1}(0)}{sC_1+1/R_3}$$

$$I_N(s)=-\frac{1}{r}U_1(s)=-\frac{1}{r}\frac{I_0(s)-C_1u_{C1}(0)}{sC_1+1/R_3} \;\Rightarrow\; I_N(s)=-\frac{1/s-1}{s+1}=\frac{1-1/s}{s+1}=\frac{s-1}{s(s+1)}$$ (1 bod)

c) Izračunavanje Nortonove admitancije pomoću jednadžbi napona čvorišta ① i ②



$$\Rightarrow Y_N(s)=\frac{I(s)}{U(s)}$$

1) $U_1\left(sC_1+\frac{1}{sL_2}+\frac{1}{R_3}\right)-U_2\frac{1}{sL_2}=\frac{\mu U_{L2}}{R_3}$

2) $U_{L2}\frac{1}{sL_2}=I_{g1}(s)\,,\; U_{L2}(s)=U_1(s)-U_2(s)$

3) $I_{g2}=-\frac{1}{r}U_{g1}\,,\; U_{g1}=U_2\,,\; I_{g2}=-I$

4) $I_{g1}=-\frac{1}{r}U_{g2}\,,\; U_{g2}=U$

---

1) $U_1\left(sC_1+\frac{1}{sL_2}+\frac{1-\mu}{R_3}\right)-U_2\left(\frac{1}{sL_2}-\frac{\mu}{R_3}\right)=0$

2) $-U_1\frac{1}{sL_2}+U_2\frac{1}{sL_2}=-I_{g1}=\frac{1}{r}U$

3) $I=\frac{1}{r}U_2 \;\Rightarrow\; U_2=r\cdot I$ (1 bod)

---

d) Nortonova admitancija $Y_N(s)$:

$$2)\Rightarrow\; -U_1+U_2=\frac{sL_2}{r}U \;\rightarrow 1)\; \left(U_2-\frac{sL_2}{r}U\right)\left(sC_1+\frac{1}{sL_2}+\frac{1-\mu}{R_3}\right)-U_2\left(\frac{1}{sL_2}-\frac{\mu}{R_3}\right)=0$$

$$U_2\left(sC_1+\frac{1}{sL_2}+\frac{1-\mu}{R_3}-\frac{1}{sL_2}+\frac{\mu}{R_3}\right)-\frac{sL_2}{r}U\left(sC_1+\frac{1}{sL_2}+\frac{1-\mu}{R_3}\right)=0$$

$$I\cdot r\cdot\left(sC_1+\frac{1}{R_3}\right)=\frac{sL_2}{r}U\left(sC_1+\frac{1}{sL_2}+\frac{1-\mu}{R_3}\right) \Rightarrow$$

$$Y_N(s)=\frac{I(s)}{U(s)}=\frac{\frac{sL_2}{r}\left(sC_1+\frac{1}{sL_2}+\frac{1-\mu}{R_3}\right)}{r(sC_1+1/R_3)}$$

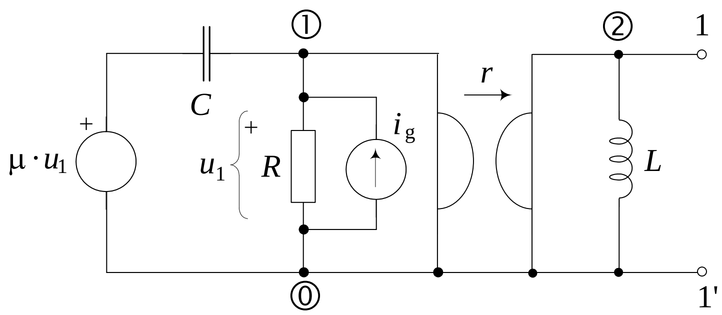
$$Y_N(s)=\frac{I(s)}{U(s)}=\frac{s(s+1/s-1)}{s+1}=\frac{s^2-s+1}{s+1}=\frac{(s+1)^2-3s}{s+1}=s+1-\frac{3s}{s+1}$$ (1 bod)

e) Da li je električni krug recipročan? Zašto?
NE, električni krug nije recipročan jer sadrži ovisni izvor i girator. (1 bod)

ZAVRŠNI ISPIT IZ ELEKTRIČNIH KRUGOVA – Rješenja – 2012-2013

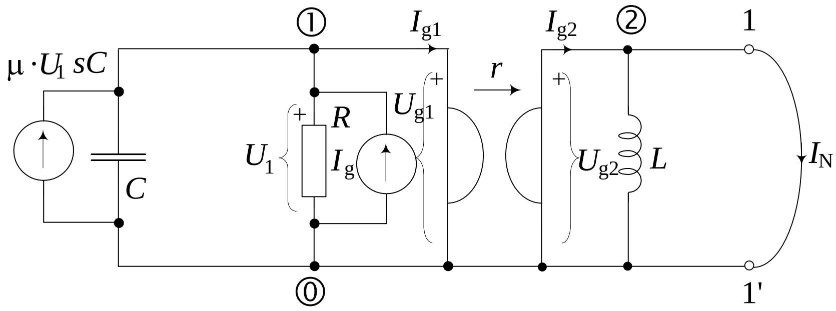
1. Za električni krug na slici zadane su normalizirane vrijednosti elemenata C=1, L=1, R=1 te μ=2, r=1, $i_g(t)=S(t)$. Odrediti nadomjesne parametre mreže po Northonu s obzirom na polove 1–1'. Koristiti metodu napona čvorišta. U zadatku je potrebno:

   a) Nacrtati sklop za izračunavanje Nortonove struje, postaviti jednadžbe napona za čvorišta ① i ②;
   b) Odrediti Nortonovu struju $I_N(s)$;
   c) Nacrtati sklop za izračunavanje Nortonove admitancije, postaviti jednadžbe napona za čvorišta ① i ②;
   d) Odrediti Nortonovu admitanciju $Y_N(s)$.
   e) Da li je električni krug recipročan? Zašto?



Rješenje:

a) Jednadžbe napona za čvorišta ① i ②:



1) $U_1\left(sC+\frac{1}{R}\right)=I_g(s)+\mu U_1 sC-I_{g1}$

2) $U_2\frac{1}{sL}=I_{g2}(s)-I_N(s)$

3) $I_{g2}=-\frac{1}{r}U_1$

4) $I_{g1}=-\frac{1}{r}U_2$

$U_2=0,\ I_{g1}=0$

1) $U_1\left(sC-\mu sC+\frac{1}{R}\right)=I_g(s)\ \Rightarrow\ U_1=\frac{I_g(s)}{sC(1-\mu)+1/R}$

2) $I_N(s)=I_{g2}(s)=-\frac{1}{r}U_1(s)$ (1 bod)

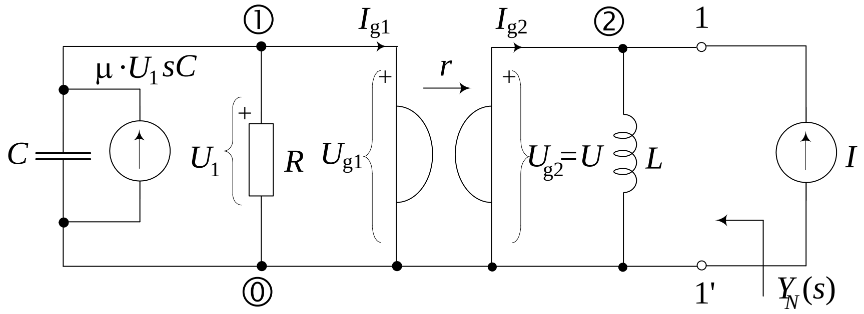
---

b) Nortonova struja $I_N(s)$:

$$1)\to 2) \Rightarrow I_N(s) = -\frac{1}{r}\cdot\frac{I_g(s)}{sC(1-\mu)+1/R}$$

$$I_N(s) = -\frac{1}{1}\cdot\frac{\frac{1}{s}}{-s+1} = -\frac{1}{s(1-s)} = \frac{1}{s(s-1)} \Rightarrow I_N(s) = \frac{1}{s(s-1)}$$ (1 bod)

c) Izračunavanje Nortonove admitancije pomoću jednadžbi napona čvorišta ① i ②



$$\Rightarrow Y_N(s) = \frac{I(s)}{U(s)}$$

1) $U_1\left(sC+\frac{1}{R}\right) = \mu U_1 sC - I_{g1}$

2) $U_2\frac{1}{sL} = I_{g2}(s) + I(s)$

3) $I_{g2} = -\frac{1}{r}U_1$

4) $I_{g1} = -\frac{1}{r}U_2$, $U_2 = U$

---

1) $U_1\left[sC(1-\mu)+\frac{1}{R}\right] = \frac{1}{r}U_2 \Rightarrow U_1 = \frac{1}{r}\cdot\frac{U_2}{sC(1-\mu)+1/R}$

2) $U_2\frac{1}{sL} = -\frac{1}{r}U_1 + I(s) \Rightarrow U_2\frac{1}{sL} = -\frac{1}{r^2}\cdot\frac{U_2}{sC(1-\mu)+1/R} + I(s)$ (1 bod)

d) Nortonova admitancija $Y_N(s)$:

$$I(s) = U(s)\left[\frac{1}{sL}+\frac{1}{r^2}\cdot\frac{1}{sC(1-\mu)+1/R}\right] \Rightarrow Y_N(s) = \frac{I(s)}{U(s)} = \frac{1}{sL}+\frac{1}{r^2}\cdot\frac{1}{sC(1-\mu)+1/R}$$

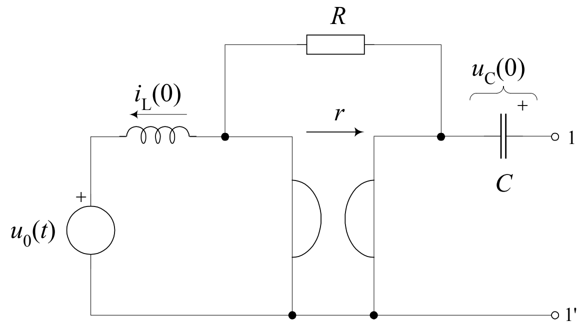
$$Y_N(s) = \frac{I(s)}{U(s)} = \frac{1}{s}+\frac{1}{-s+1} = \frac{-s+1+s}{s(-s+1)} = \frac{1}{s(1-s)}$$ (1 bod)

---

e) Da li je električni krug recipročan? Zašto?
NE, električni krug nije recipročan jer sadrži ovisni izvor i girator. (1 bod)

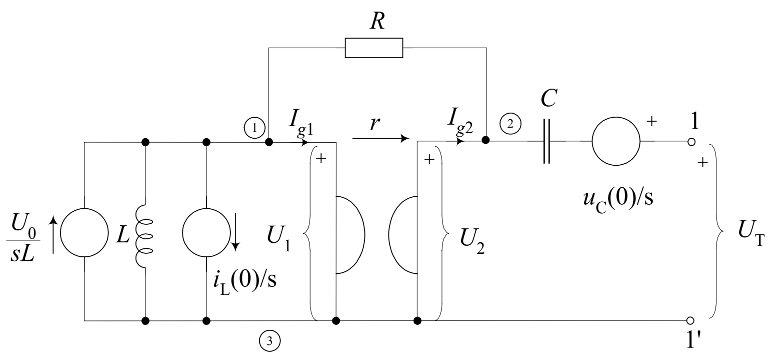
THEVENIN

5. Za mrežu prikazanu slikom odrediti nadomjesnu shemu po Theveninu obzirom na priključnice 1-1', koristeći postupak jednadžbi čvorišta, ako je pobuda $u_0(t)=\delta(t)$. Zadane su normirane vrijednosti elemenata: R=0.5, L=1, C=1, r=1 i početni uvjeti $u_C(0)$=1, $i_L(0)$=0.5.



Rješenje: Primjena Laplaceove transformacije

a) Teveninov napon $U_T(s)$:



(1) $$\frac{U_0}{sL}-\frac{i_L(0)}{s}-I_{g1}=U_1\left(\frac{1}{sL}+\frac{1}{R}\right)-U_2\frac{1}{R} \qquad I_{g1}=-U_2\frac{1}{r}$$

(2) $$I_{g2}=-U_1\frac{1}{R}+U_2\frac{1}{R} \qquad I_{g2}=-U_1\frac{1}{r}$$

---

(1) $$\frac{U_0}{sL}-\frac{i_L(0)}{s}=U_1\left(\frac{1}{sL}+\frac{1}{R}\right)-U_2\left(\frac{1}{R}+\frac{1}{r}\right)$$

(2) $$0=-U_1\left(\frac{1}{R}-\frac{1}{r}\right)+U_2\frac{1}{R}$$

---

(2) $$\Rightarrow \qquad U_1=\frac{U_2\frac{1}{R}}{\frac{1}{R}-\frac{1}{r}}$$

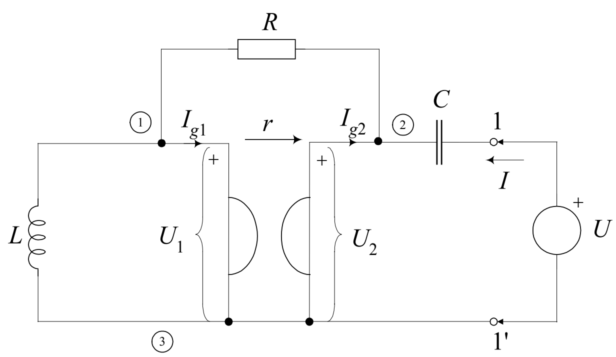
(1) $$\frac{U_0}{sL}-\frac{i_L(0)}{s}=U_2\frac{\frac{1}{R}}{\frac{1}{R}-\frac{1}{r}}\left(\frac{1}{sL}+\frac{1}{R}\right)-U_2\left(\frac{1}{R}+\frac{1}{r}\right) \qquad \Big/ \cdot\left(\frac{1}{R}-\frac{1}{r}\right)$$

$$U_2=\frac{\left(\frac{1}{R}-\frac{1}{r}\right)\left(\frac{U_0}{sL}-\frac{i_L(0)}{s}\right)}{\frac{1}{sRL}+\frac{1}{r^2}}=\frac{\left(r^2-Rr\right)\left(U_0-Li_L(0)\right)}{r^2+sRL}$$

$$U_T(s)=U_2(s)+\frac{u_C(0)}{s}=\frac{\left(r^2-Rr\right)\left(U_0-Li_L(0)\right)}{r^2+sRL}+\frac{u_C(0)}{s}=\frac{(1-0.5)(1-0.5)}{1+0.5s}+\frac{1}{s}$$

$$U_T(s)=\frac{0.5}{s+2}+\frac{1}{s}=\frac{1.5s+2}{s(s+2)}$$

a) Teveninova impedancija $Z_T(s)$:



$$Z_T(s)=\frac{U}{I},\qquad I=(U-U_2)sC\qquad\Rightarrow\qquad Z_T(s)=\frac{U_2}{I}+\frac{1}{sC}$$

(1) $\quad -I_{g1}=U_1\left(\frac{1}{sL}+\frac{1}{R}\right)-U_2\frac{1}{R}\qquad\qquad I_{g1}=-U_2\frac{1}{r}$

(2) $\quad I_{g2}+I=-U_1\frac{1}{R}+U_2\frac{1}{R}\qquad\qquad I_{g2}=-U_1\frac{1}{r}$

---

(1) $\quad 0=U_1\left(\frac{1}{sL}+\frac{1}{R}\right)-U_2\left(\frac{1}{R}+\frac{1}{r}\right)\qquad\Rightarrow\qquad U_1=\frac{\frac{1}{R}+\frac{1}{r}}{\frac{1}{sL}+\frac{1}{R}}U_2$

(2) $\quad I=-U_1\left(\frac{1}{R}-\frac{1}{r}\right)+U_2\frac{1}{R}$

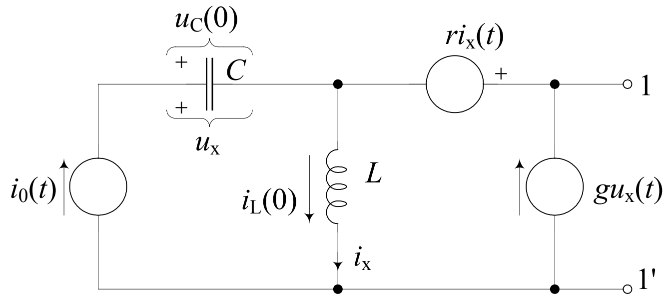
---

(2) $\quad I=-\frac{\frac{1}{R}+\frac{1}{r}}{\frac{1}{sL}+\frac{1}{R}}\left(\frac{1}{R}-\frac{1}{r}\right)U_2+U_2\frac{1}{R}=\frac{\frac{1}{sLR}+\frac{1}{r^2}}{\frac{1}{sL}+\frac{1}{R}}U_2=\frac{r^2+sLR}{r^2(sL+R)}U_2$

$$Z_T(s)=\frac{U_2}{I}+\frac{1}{sC}=\frac{r^2(sL+R)}{r^2+sLR}+\frac{1}{sC}=\frac{1(s+0.5)}{1+0.5s}+\frac{1}{s}=\frac{2s+1}{s+2}+\frac{1}{s}$$

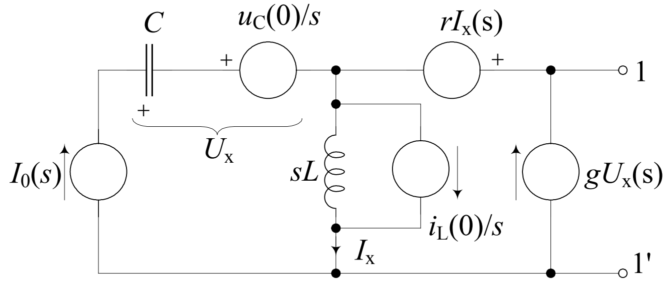
$$Z_T(s)=\frac{2(s^2+s+1)}{s(s+2)}$$

3. Za krug prikazan slikom odrediti nadomjesnu shemu po Theveninu obzirom na priključnice 1-1', ako je pobuda $i_0(t)=S(t)$. Zadane su normirane vrijednosti elemenata: R=1, L=1, C=1, r=0.5, g=0.5, a početni uvjeti su $u_C(0)=1$ i $i_L(0)=1$.



Rješenje: Primjena Laplaceove transformacije

a) Theveninov napon $U_T(s)$



$$U_x(s) = I_0\frac{1}{sC} + \frac{u_C(0)}{s}$$

$$I_x(s) = I_0(s) + gU_x(s) = I_0(s) + g\left(I_0\frac{1}{sC} + \frac{u_C(0)}{s}\right)$$

$$U_T(s) = U_L(s) + rI_x(s) = sLI_x(s) - Li_L(0) + rI_x(s) = (sL+r)I_x(s) - Li_L(0)$$

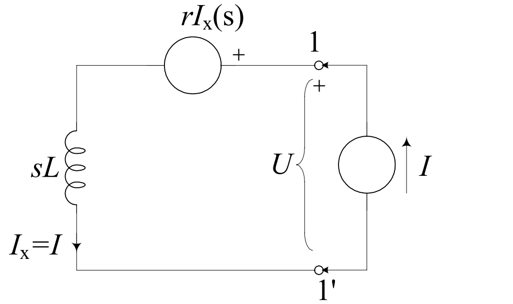
$$U_T(s) = (sL+r)\left[I_0(s) + g\left(I_0\frac{1}{sC} + \frac{u_C(0)}{s}\right)\right] - Li_L(0)$$

$$U_T(s) = (sL+r)\left(1+\frac{g}{sC}\right)I_0(s) + (sL+r)g\frac{u_C(0)}{s} - Li_L(0)$$

$$U_T(s) = \left(s+\frac{1}{2}\right)\left(1+\frac{1}{2s}\right)\frac{1}{s} + \left(s+\frac{1}{2}\right)\frac{1}{2s} - 1 = \frac{2s^2+5s+1}{4s^2} = \frac{1}{2} + \frac{5}{4s} + \frac{1}{4s^2}$$

$$u_T(t) = \frac{1}{2}\delta(t) + \frac{5}{4}S(t) + \frac{1}{4}tS(t)$$

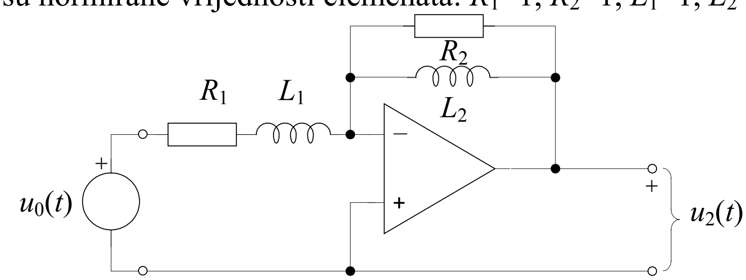
b) Theveninova impedancija $Z_T(s)$



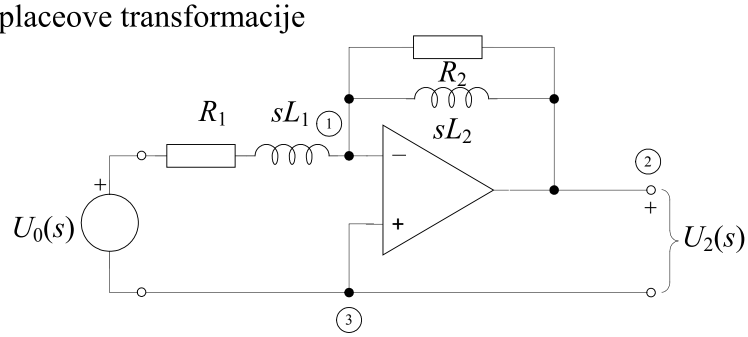
$$U = I_x sL + r \cdot I_x, I_x = I \Rightarrow U = I(sL + r)$$

$$Z_T(s) = \frac{U}{I} = sL + r = s + 1/2$$

4. Za krug prikazan slikom odrediti napon na izlazu operacijskog pojačala $u_2(t)$, ako je pobuda $u_0(t)=S(t)$. Zadane su normirane vrijednosti elemenata: $R_1=1$, $R_2=1$, $L_1=1$, $L_2=1/2$, $A\to\infty$.



Rješenje: Primjena Laplaceove transformacije



(1) $$\frac{U_0}{sL_1 + R_1} = U_1\left(\frac{1}{sL_1 + R_1} + \frac{1}{sL_2} + \frac{1}{R_2}\right) - U_2\left(\frac{1}{sL_2} + \frac{1}{R_2}\right)$$

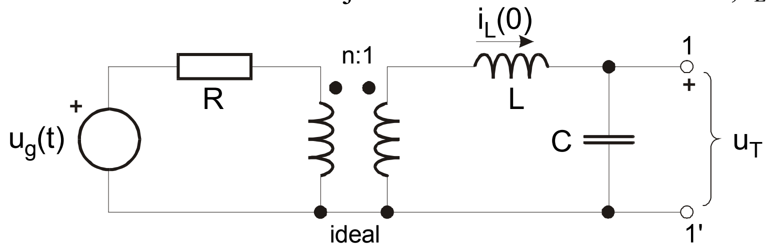
$U_1 = 0$, jer $A \to \infty$

(1) $\Rightarrow$ $$U_2 = -\frac{U_0}{(sL_1 + R_1)\left(\frac{1}{sL_2} + \frac{1}{R_2}\right)} = -\frac{U_0}{\frac{L_1}{sR_2}\left(s + \frac{R_1}{L_1}\right)\left(s + \frac{R_2}{L_2}\right)}$$

$$U_2 = -\frac{\frac{R_2}{L_1} s}{\left(s + \frac{R_1}{L_1}\right)\left(s + \frac{R_2}{L_2}\right)} U_0 = -\frac{1 \cdot s \cdot \frac{1}{s}}{(s+1)(s+2)} = \frac{-1}{(s+1)(s+2)}$$

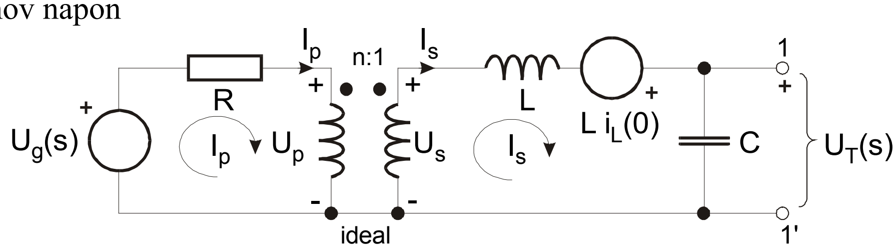
PONOVLJENI ZAVRŠNI ISPIT IZ ELEKTRIČNIH KRUGOVA

1. Za električni krug prikazan slikom odrediti $U_T(s)$ i $Z_T(s)$ nadomjesne sheme po Teveninu s obzirom na stezaljke 1-1'. Zadane su normalizirane vrijednosti elemenata: L=C=R=1, $i_L(0)$=1, n=2, $u_g(t)$=$S(t)$.



Rješenje:

a) Theveninov napon



Jednadžbe transformatora:

$$U_p = n \cdot U_s \quad \Rightarrow \quad U_s = \frac{U_p}{n}$$

$$I_p = \frac{1}{n} \cdot I_s \quad \Rightarrow \quad I_s = nI_p$$

Jednadžbe petlji :

$$(1) \qquad U_g(s) = I_p \cdot R + U_p(s)$$

$$(2) \qquad U_s = I_s sL + I_s \frac{1}{sC} - Li_L(0)$$

$$(3) \qquad U_T(s) = I_s \cdot \frac{1}{sC}$$

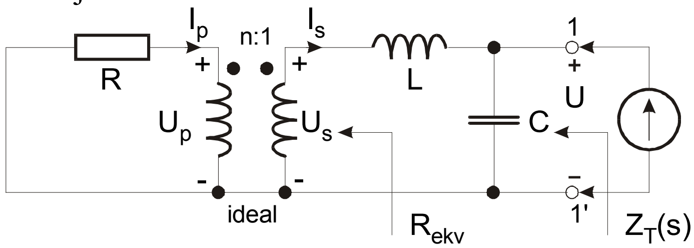
$$(1) \Rightarrow \quad U_g(s) = \frac{1}{n} I_s R + nU_s; \quad (2) \to (1) \Rightarrow \quad U_g(s) = \frac{1}{n} I_s R + n\left(I_s sL + I_s \frac{1}{sC} - Li_L(0) \right) \Big/ : n$$

$$\Rightarrow \quad \frac{U_g(s)}{n} + Li_L(0) = I_s \left(\frac{R}{n^2} + sL + \frac{1}{sC} \right); \quad \Rightarrow \quad I_s = \frac{\dfrac{U_g(s)}{n} + Li_L(0)}{\dfrac{R}{n^2} + sL + \dfrac{1}{sC}}$$

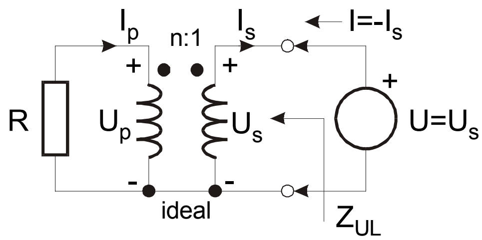
$$\Rightarrow \quad U_T(s) = I_s \cdot \frac{1}{sC} = \frac{\dfrac{U_g(s)}{n} + Li_L(0)}{\dfrac{R}{n^2} + sL + \dfrac{1}{sC}} \cdot \frac{1}{sC} = \frac{\dfrac{U_g(s)}{n} + Li_L(0)}{s^2 LC + sC \dfrac{R}{n^2} + 1}$$

$$U_T(s) = \frac{\dfrac{1}{2s} + 1}{s^2 + \dfrac{1}{4} s + 1} = \frac{\dfrac{2}{s} + 4}{4s^2 + s + 4} = \frac{2(2s+1)}{s(4s^2 + s + 4)}$$

b) Teveninova impedancija

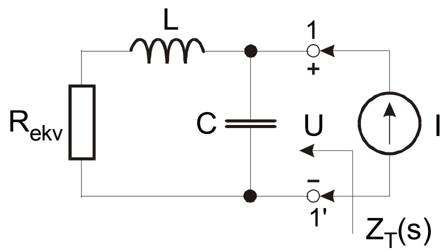


Izračunajmo najprije R_{ekv} :



$$Z_{ul} = \frac{U}{I} = \frac{U_s}{-I_s} = -\frac{\frac{U_p}{n}}{nI_p} = \frac{1}{n^2}\cdot\left(-\frac{U_p}{I_p}\right); \qquad -\frac{U_p}{I_p} = R; \quad \Rightarrow \quad Z_{ul} = \frac{R}{n^2} = R_{ekv}; \quad R_{ekv} = \frac{R}{n^2} = \frac{1}{4}$$

Teveninova impedancija :

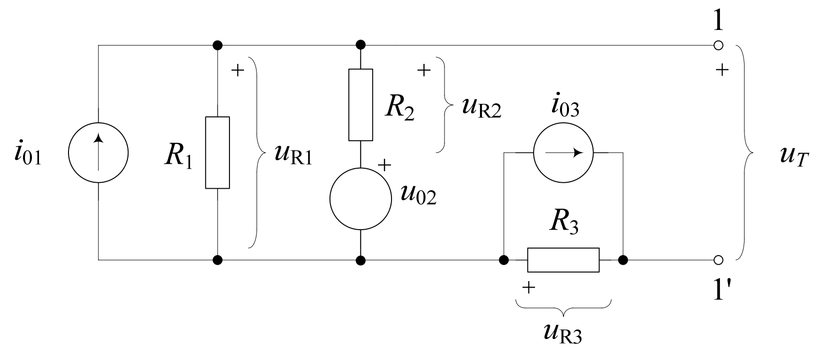


$$Z_T = \frac{U}{I} = \frac{\frac{1}{sC}\left(R_{ekv} + sL\right)}{\frac{1}{sC} + R_{ekv} + sL} = \frac{R_{ekv} + sL}{1 + sCR_{ekv} + s^2CL} = \frac{s + \frac{1}{4}}{s^2 + \frac{1}{4}s + 1} = \frac{4s+1}{4s^2 + s + 4}$$

$$Z_T(s) = \frac{4s+1}{4s^2 + s + 4}$$

PRVI MEĐUISPIT IZ ELEKTRIČNIH KRUGOVA – Rješenja – 2009

1. Za električni krug na slici odrediti: a) Theveninov napon $u_T(t)$; b) Theveninov otpor R_T; c) napon na otporu R_3; d) napon na otporu R_2 i e) napon na otporu R_1. Zadano je: R_1 = 5kΩ, R_2 = 10kΩ, R_3 = 10kΩ, i_{01} = 3mA, u_{02} = 3V, i_{03} = 1mA.



Rješenje:

$$u_T = \frac{i_{01}R_2 + u_{02}}{R_1 + R_2} R_1 - i_{03}R_3 = i_{01} \cdot 3{,}333 \cdot 10^{-3} + u_{02} \cdot 3{,}333 - i_{03} \cdot 10^4$$

$$u_T = 10\text{V} + 1\text{V} - 10\text{V} = 1\text{V}$$

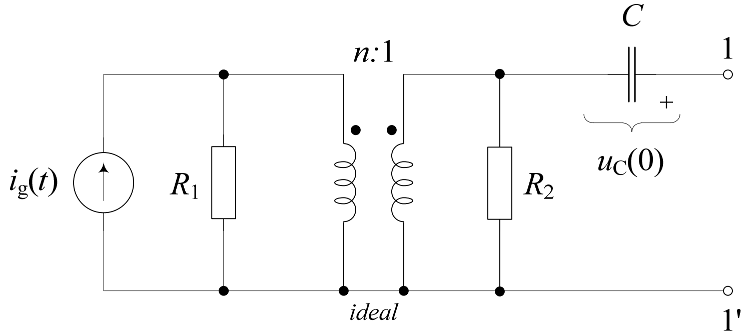
$$R_T = \frac{R_1 R_2}{R_1 + R_2} + R_3 = \frac{5 \cdot 10}{5 + 10} + 10 = 13{,}333\text{k}\Omega$$

$$u_{R3} = -i_{03} \cdot R_3 = -10^{-3} \cdot 10 \cdot 10^3 = -10\text{V}$$

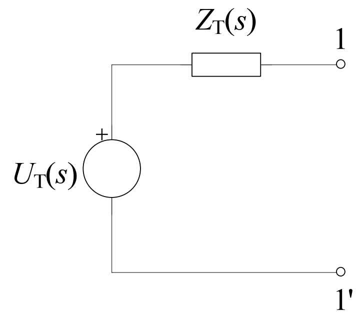
$$u_{R1} = u_T - u_{R3} = 1 - (-10) = 11\text{V}$$

$$u_{R2} = u_T - u_{R3} - u_{02} = 11 - 3 = 8\text{V}$$

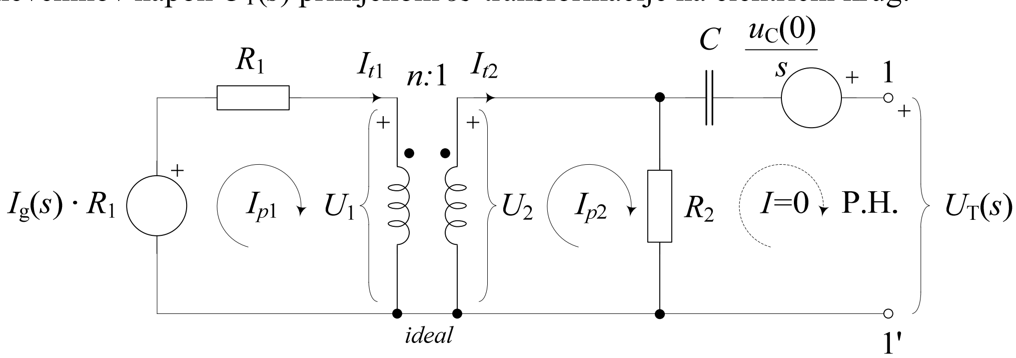
4. Za električni krug na slici izračunati nadomjesne parametre nadomjesnog kruga po Theveninu s obzirom na stezaljke 1 – 1': a) $U_T(s)$ i b) $Z_T(s)$. Zadane su normalizirane vrijednosti elemenata: $R_1 = R_2 = 1$, $C = 1$, $u_C(0) = 1$, $n = 2$, $i_g(t) = S(t)$. Koristiti metodu petlji.



Rješenje:



a) Theveninov napon $U_T(s)$ primjenom $\mathcal{L}$-transformacije na električni krug:



Jednadžbe idealnog transformatora:

$$U_1 = n \cdot U_2 \Rightarrow U_2 = \frac{U_1}{n}$$

$$I_{t1} = \frac{1}{n} \cdot I_{t2} \Rightarrow I_{t2} = n \cdot I_{t1}$$

1) $I_{p1}R_1 = -U_1 + I_g R_1$ $\qquad I_{p1} = I_{t1}$

2) $I_{p2}R_2 = U_2$ $\qquad I_{p2} = I_{t2}$

Nakon sređivanja jednadžbe glase:

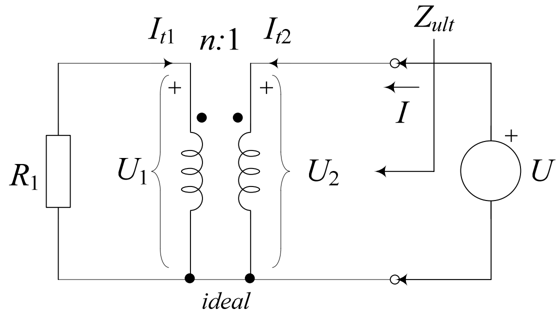
$$\left.\begin{array}{l} 1)\ I_{p1}R_1 + nU_2 = I_g R_1 \\ 2)\ -nI_{p1}R_2 + U_2 = 0 \end{array}\right\} \quad \left.\begin{array}{l} I_{p1}R_1 + nU_2 = I_g R_1 \\ I_{p1} = \dfrac{U_2}{nR_2} \end{array}\right\} \quad \left.\dfrac{U_2}{nR_2}R_1 + nU_2 = I_g R_1 \right\}$$

$$U_2\left(\frac{1}{nR_2}R_1+n\right)=I_gR_1 \quad\Bigg\} \qquad U_2=\frac{I_gR_1}{\dfrac{n^2R_2+R_1}{nR_2}}=I_g\cdot\frac{nR_1R_2}{n^2R_2+R_1}$$

$$U_T(s)=U_2(s)+\frac{u_C(0)}{s}=I_g\cdot\frac{nR_1R_2}{n^2R_2+R_1}+\frac{u_C(0)}{s}=\frac{1}{s}\cdot\frac{2}{5}+\frac{1}{s}=\frac{1}{s}\cdot\frac{7}{5}$$

b) Theveninova impedancija $Z_T(s)$:

1. način (pojednostavljen): Najprije izračunati ulaznu impedanciju u transformator zaključen s R_1. Označimo je s Z_{ult}.



$$Z_{ult}=\frac{1}{n^2}R_1$$

$$I_{t1}=-\frac{U_1}{R_1}\Rightarrow\frac{U_1}{I_{t1}}=-R_1$$

$$U=U_2$$

$$I=I_{t2}$$

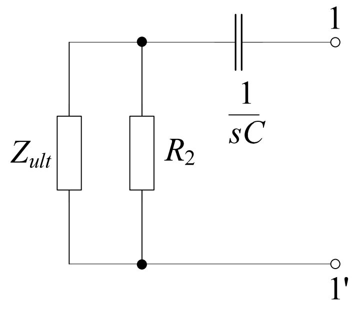
Jednadžbe od transformatora su (obratiti pažnju na referentno usmjerenje I_{t2}):

$$U_1=n\cdot U_2\Rightarrow U_2=\frac{U_1}{n}$$

$$I_{t1}=-\frac{1}{n}\cdot I_{t2}\Rightarrow I_{t2}=-n\cdot I_{t1}$$

$$Z_{ult}=\frac{U}{I}=\frac{U_2}{I_{t2}}=\frac{\dfrac{U_1}{n}}{-n\cdot I_{t1}}=-\frac{\dfrac{U_1}{I_{t1}}}{n^2}=-\frac{-R_1}{n^2}=\frac{R_1}{n^2}$$

Tada je Teveninova impedancija:



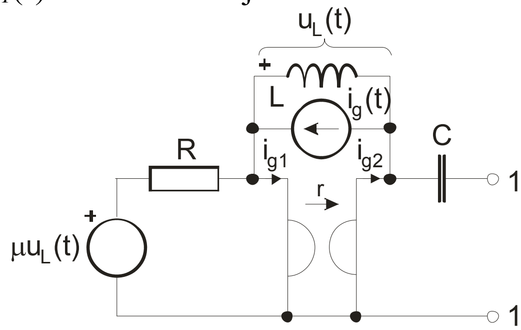
$$\leftarrow Z_T(s)=\frac{1}{sC}+R_2\parallel Z_{ult}$$

$$Z_T(s)=\frac{1}{sC}+\frac{\dfrac{R_1}{n^2}\cdot R_2}{\dfrac{R_1}{n^2}+R_2}=\frac{1}{s}+\frac{\dfrac{1}{4}}{\dfrac{1}{4}+1}=\frac{1}{s}+\frac{\dfrac{1}{4}}{\dfrac{5}{4}}=\frac{1}{s}+\frac{1}{5}$$

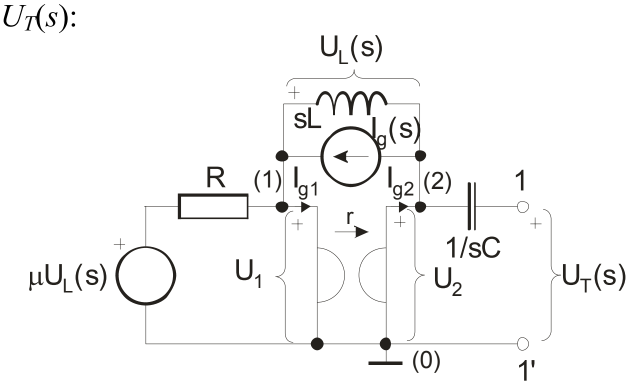
2. način: metodom petlji (nije ovdje prikazan, ali se priznaje u potpunosti).

2. Za mrežu prikazanu slikom odrediti nadomjesne parametre mreže po Theveninu $U_T(s)$ i $Z_T(s)$ s obzirom na polove 1–1'. Koristiti metodu napona čvorova u proračunu. Zadane su normalizirane vrijednosti elemenata: L=1, C=1, R=1, μ=2, r=2 te izvor $i_g(t)$=$S(t)$. Napisati:

a) Jednadžbu za čvor (1) za izračun $U_T(s)$;
b) Jednadžbu za čvor (2) za izračun $U_T(s)$;
c) Theveninov napon $U_T(s)$ uz uvrštene vrijednosti elemenata;
d) Theveninovu impedanciju $Z_T(s)$ uz uvrštene vrijednosti elemenata.



Rješenje: Metodom napona čvorova:
Theveninov napon $U_T(s)$:



$$U_T(s) = U_2(s)$$

(1) $$U_1\left(\frac{1}{R}+\frac{1}{sL}\right)-U_2\frac{1}{sL}=\mu\frac{U_L(s)}{R}+I_g(s)-I_{g1}(s) \text{ (1 bod)}$$

(2) $$-U_1\frac{1}{sL}+U_2\frac{1}{sL}=-I_g(s)+I_{g2}(s) \text{ (1 bod)}$$

$$U_2=-r\cdot I_{g1}$$
$$U_1=-r\cdot I_{g2} \qquad U_L(s)=U_1(s)-U_2(s)$$

---

(1) $$U_1\left(\frac{1}{R}+\frac{1}{sL}\right)-U_2\frac{1}{sL}=\mu\frac{U_1(s)-U_2(s)}{R}+I_g(s)+\frac{U_2}{r}$$

(2) $$-U_1\frac{1}{sL}+U_2\frac{1}{sL}=-I_g(s)-\frac{U_1}{r}$$

$$(2)\Rightarrow U_2\frac{1}{sL}+I_g(s)=U_1\left(\frac{1}{sL}-\frac{1}{r}\right) \Rightarrow U_1=\frac{U_2\frac{1}{sL}+I_g(s)}{\frac{1}{sL}-\frac{1}{r}} \rightarrow (1)$$

$$(1) \Rightarrow U_1\left(\frac{1-\mu}{R}+\frac{1}{sL}\right)=U_2\left(\frac{1}{sL}-\frac{\mu}{R}+\frac{1}{r}\right)+I_g$$

$$(1),\,(2) \Rightarrow \frac{U_2\frac{1}{sL}+I_g(s)}{\frac{1}{sL}-\frac{1}{r}}\left(\frac{1-\mu}{R}+\frac{1}{sL}\right)=U_2\left(\frac{1}{sL}-\frac{\mu}{R}+\frac{1}{r}\right)+I_g$$

$$\left[U_2\frac{1}{sL}+I_g(s)\right]\left(\frac{1-\mu}{R}+\frac{1}{sL}\right)=U_2\left(\frac{1}{sL}-\frac{\mu}{R}+\frac{1}{r}\right)\left(\frac{1}{sL}-\frac{1}{r}\right)+I_g\left(\frac{1}{sL}-\frac{1}{r}\right)$$

$$U_2\frac{1}{sL}\left(\frac{1}{R}-\frac{\mu}{R}+\frac{1}{sL}\right)-U_2\frac{1}{sL}\left(\frac{1}{sL}-\frac{\mu}{R}+\frac{1}{r}\right)+U_2\frac{1}{r}\left(\frac{1}{sL}-\frac{\mu}{R}+\frac{1}{r}\right)=$$

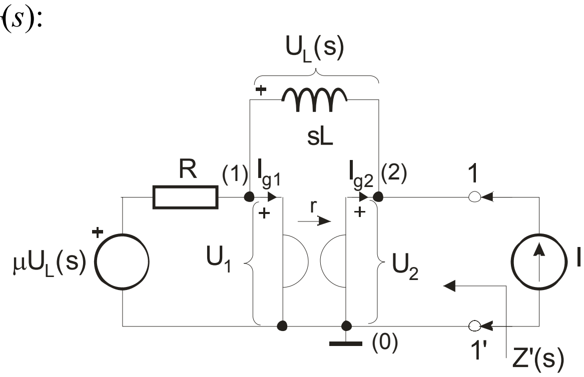
$$=I_g\left(\frac{1}{sL}-\frac{1}{r}\right)-I_g(s)\left(\frac{1}{R}-\frac{\mu}{R}+\frac{1}{sL}\right)$$

$$U_2\left(\frac{1}{sL}\frac{1}{R}+\frac{1}{r^2}-\frac{\mu}{rR}\right)=-I_g(s)\left(\frac{1}{r}+\frac{1-\mu}{R}\right)$$

$$U_2\left(\frac{1}{s}+\frac{1}{4}-1\right)=-I_g(s)\left(\frac{1}{2}-1\right)$$

$$U_2\left(\frac{1}{s}-\frac{3}{4}\right)=\frac{1}{2}I_g(s) \;\Rightarrow\; U_T(s)=U_2=\frac{\frac{1}{2}}{\frac{1}{s}-\frac{3}{4}}I_g(s)=\frac{\frac{1}{2}}{\frac{1}{s}-\frac{3}{4}}\cdot\frac{1}{s}=\frac{2}{4-3s}$$ (1 bod)

Theveninova impedancija $Z_T(s)$:



$$(1) \qquad U_1\left(\frac{1}{R}+\frac{1}{sL}\right)-U_2\frac{1}{sL}=\mu\frac{U_L(s)}{R}-I_{g1}(s)$$

$$(2) \qquad -U_1\frac{1}{sL}+U_2\frac{1}{sL}=I(s)+I_{g2}(s)$$

$$U_2=-r\cdot I_{g1}$$

$$U_1=-r\cdot I_{g2} \qquad U_L(s)=U_1(s)-U_2(s)$$

$$(1) \qquad U_1\left(\frac{1}{R}+\frac{1}{sL}\right)-U_2\frac{1}{sL}=\mu\frac{U_1(s)-U_2(s)}{R}+\frac{U_2(s)}{r}$$

$$(2) \qquad -U_1\frac{1}{sL}+U_2\frac{1}{sL}=I(s)-\frac{U_1(s)}{r}$$

$$(1)\qquad U_1\left(\frac{1-\mu}{R}+\frac{1}{sL}\right)-U_2\left(\frac{1}{sL}-\frac{\mu}{R}+\frac{1}{r}\right)=0$$

$$(2)\qquad -U_1\left(\frac{1}{sL}-\frac{1}{r}\right)+U_2\,\frac{1}{sL}=I(s)$$

$$(1)\Rightarrow U_1(s)=\frac{\frac{1}{sL}-\frac{\mu}{R}+\frac{1}{r}}{\frac{1-\mu}{R}+\frac{1}{sL}}U_2(s)$$

$$(1),(2)\Rightarrow -\frac{\frac{1}{sL}-\frac{\mu}{R}+\frac{1}{r}}{\frac{1-\mu}{R}+\frac{1}{sL}}U_2\left(\frac{1}{sL}-\frac{1}{r}\right)+U_2\,\frac{1}{sL}=I(s)$$

$$Z'(s)=\frac{U_2(s)}{I(s)}=\frac{\frac{1-\mu}{R}+\frac{1}{sL}}{\left(\frac{1}{sL}-\frac{\mu}{R}+\frac{1}{r}\right)\left(\frac{1}{r}-\frac{1}{sL}\right)+\frac{1}{sL}\left(\frac{1-\mu}{R}+\frac{1}{sL}\right)}=\frac{-1+\frac{1}{s}}{\left(\frac{1}{s}-2+\frac{1}{2}\right)\left(\frac{1}{2}-\frac{1}{s}\right)+\frac{1}{s}\left(-1+\frac{1}{s}\right)}=$$

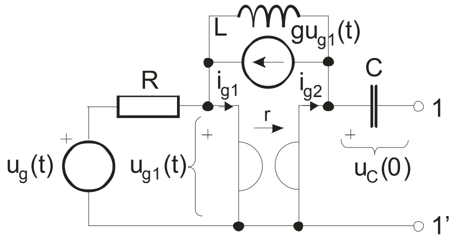
$$=\frac{-1+\frac{1}{s}}{\frac{1}{2s}-\frac{1}{s^2}-1+\frac{2}{s}+\frac{1}{4}-\frac{1}{2s}-\frac{1}{s}+\frac{1}{s^2}}=\frac{-1+\frac{1}{s}}{-\frac{3}{4}+\frac{1}{s}}=\frac{4s-4}{3s-4}\qquad\Rightarrow$$

$$Z_T(s)=Z'(s)+\frac{1}{sC}=\frac{4s-4}{3s-4}+\frac{1}{s}=\frac{4s^2-s-4}{s(3s-4)}$$ (1 bod)

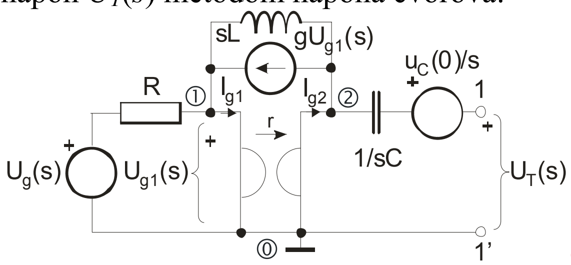
PONOVLJENI ZAVRŠNI ISPIT IZ ELEKTRIČNIH KRUGOVA 2010/11

Rješenja i bodovi (svaki zadatak je bodovan od 0 do 5 bodova):

1. Za mrežu prikazanu slikom odrediti nadomjesne parametre mreže po Theveninu $U_T(s)$ i $Z_T(s)$ s obzirom na polove 1–1'. Koristiti metodu napona čvorova u proračunu. Zadane su normalizirane vrijednosti elemenata: L=1, C=1, R=1, g=2, r=2, $u_C(0)$=1 te izvor $u_g(t)$=$S(t)$. Nacrtati i napisati: a) Slike za izračun $U_T(s)$ i $Z_T(s)$; b) Jednadžbe čvorova za izračun $U_T(s)$; c) Jednadžbe čvorova za izračun $Z_T(s)$. Uz uvrštene vrijednosti elemenata: d) Theveninov napon $U_T(s)$; e) Theveninovu impedanciju $Z_T(s)$.



Rješenje: Theveninov napon $U_T(s)$ metodom napona čvorova:



$$U_T(s) = U_2(s) - \frac{u_C(0)}{s}$$

(0,5 boda)

(1) $$U_1\left(\frac{1}{R}+\frac{1}{sL}\right) - U_2\frac{1}{sL} = gU_1 + \frac{U_g(s)}{R} - I_{g1}(s)$$ $$U_2 = -r\cdot I_{g1} \Rightarrow I_{g1} = -\frac{U_2}{r}$$

(2) $$-U_1\frac{1}{sL} + U_2\frac{1}{sL} = -gU_1(s) + I_{g2}(s) \quad \textbf{(1 bod)}$$ $$U_1 = -r\cdot I_{g2} \Rightarrow I_{g2} = -\frac{U_1}{r}$$

---

(1) $$U_1\left(\frac{1}{R}+\frac{1}{sL}\right) - U_2\frac{1}{sL} = gU_1 + \frac{U_g(s)}{R} + \frac{U_2}{r} \Rightarrow U_1\left(\frac{1}{R}+\frac{1}{sL}-g\right) - U_2\left(\frac{1}{sL}+\frac{1}{r}\right) = \frac{U_g(s)}{R}$$

(2) $$-U_1\frac{1}{sL} + U_2\frac{1}{sL} = -gU_1(s) - \frac{U_1(s)}{r}$$

$$(2) \Rightarrow -U_1\left(\frac{1}{sL} - g - \frac{1}{r}\right) + U_2\frac{1}{sL} = 0 \Rightarrow U_1 = \frac{1/sL}{1/sL - g - 1/r}\cdot U_2 \rightarrow (1)$$

$$(1),(2) \Rightarrow U_2\frac{\frac{1}{sL}}{\frac{1}{sL} - g - \frac{1}{r}}\left(\frac{1}{R}+\frac{1}{sL}-g\right) - U_2\left(\frac{1}{sL}+\frac{1}{r}\right) = \frac{U_g}{R}$$

$$U_2\frac{1}{sL}\left(\frac{1}{R}+\frac{1}{sL}-g\right) - U_2\left(\frac{1}{sL}+\frac{1}{r}\right)\left(\frac{1}{sL}-g-\frac{1}{r}\right) = \frac{U_g}{R}\left(\frac{1}{sL}-g-\frac{1}{r}\right)$$

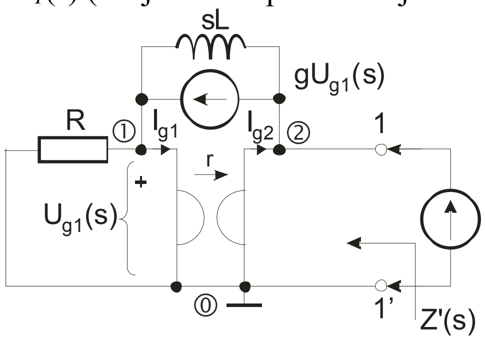
$$U_2\left(\frac{1}{sLR}+\frac{1}{(sL)^2}-\frac{g}{sL}-\frac{1}{(sL)^2}+\frac{g}{sL}+\frac{1}{rsL}-\frac{1}{rsL}+\frac{g}{r}+\frac{1}{r^2}\right) = \frac{U_g}{R}\left(\frac{1}{sL}-g-\frac{1}{r}\right)$$

$$U_2\left(\frac{1}{sLR}+\frac{g}{r}+\frac{1}{r^2}\right) = \frac{U_g}{R}\left(\frac{1}{sL}-g-\frac{1}{r}\right)$$

$$U_2(s)=\frac{\frac{1}{R}\left(\frac{1}{sL}-g-\frac{1}{r}\right)}{\frac{1}{sLR}+\frac{g}{r}+\frac{1}{r^2}}U_g(s)==\frac{\frac{1}{s}-2-\frac{1}{2}}{\frac{1}{s}+1+\frac{1}{4}}\cdot\frac{1}{s}=\frac{\frac{1}{s}-\frac{5}{2}}{\frac{1}{s}+\frac{5}{4}}\cdot\frac{1}{s}=\frac{\frac{1}{s}-\frac{5}{2}}{1+\frac{5}{4}s}=\frac{4\frac{1}{s}-10}{4+5s}=\frac{-10s+4}{s(5s+4)}$$

$$\Rightarrow\ U_T(s)=U_2(s)-\frac{u_C(0)}{s}=\frac{-10s+4}{s(5s+4)}-\frac{1}{s}=\frac{-10s+4-(5s+4)}{s(5s+4)}=\frac{-15}{5s+4}$$ **(1 bod)**

Theveninova impedancija $Z_T(s)$ (isključeni su početni uvjeti i neovisni izvori):



(0,5 boda)

$$Z_T(s)=Z'(s)+\frac{1}{sC}$$

(1) $$U_1\left(\frac{1}{R}+\frac{1}{sL}\right)-U_2\frac{1}{sL}=gU_1-I_{g1}(s)$$

(2) $$-U_1\frac{1}{sL}+U_2\frac{1}{sL}=-gU_1(s)+I_{g2}(s)+I(s)$$ **(1 bod)**

---

(1) $$U_1\left(\frac{1}{R}+\frac{1}{sL}\right)-U_2\frac{1}{sL}=gU_1+\frac{U_2}{r}\ \Rightarrow\ U_1\left(\frac{1}{R}+\frac{1}{sL}-g\right)-U_2\left(\frac{1}{sL}+\frac{1}{r}\right)=0$$

(2) $$-U_1\frac{1}{sL}+U_2\frac{1}{sL}=-gU_1(s)-\frac{U_1(s)}{r}+I(s)$$

$$(2)\Rightarrow\ -U_1\left(\frac{1}{sL}-g-\frac{1}{r}\right)+U_2\frac{1}{sL}=I(s)\ \ \Rightarrow\ \ U_1=\frac{\frac{1}{sL}}{\frac{1}{sL}-g-\frac{1}{r}}\cdot U_2-\frac{I(s)}{\frac{1}{sL}-g-\frac{1}{r}}\ \rightarrow(1)$$

$$(1),(2)\Rightarrow\left[\frac{\frac{1}{sL}}{\frac{1}{sL}-g-\frac{1}{r}}\cdot U_2-\frac{I(s)}{\frac{1}{sL}-g-\frac{1}{r}}\right]\left(\frac{1}{R}+\frac{1}{sL}-g\right)-U_2\left(\frac{1}{sL}+\frac{1}{r}\right)=0$$

$$\left[\frac{1}{sL}\cdot U_2-I(s)\right]\left(\frac{1}{R}+\frac{1}{sL}-g\right)=U_2\left(\frac{1}{sL}+\frac{1}{r}\right)\left(\frac{1}{sL}-g-\frac{1}{r}\right)$$

$$U_2\frac{1}{sL}\cdot\left(\frac{1}{R}+\frac{1}{sL}-g\right)-I(s)\left(\frac{1}{R}+\frac{1}{sL}-g\right)=U_2\left(\frac{1}{sL}+\frac{1}{r}\right)\left(\frac{1}{sL}-g-\frac{1}{r}\right)$$

$$U_2\frac{1}{sL}\left(\frac{1}{R}+\frac{1}{sL}-g\right)-U_2\left(\frac{1}{sL}+\frac{1}{r}\right)\left(\frac{1}{sL}-g-\frac{1}{r}\right)=I(s)\left(\frac{1}{R}+\frac{1}{sL}-g\right)$$

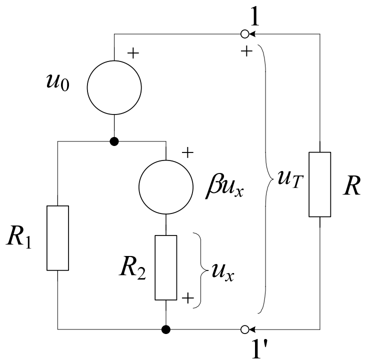
$$U_2\left(\frac{1}{sLR}+\frac{g}{r}+\frac{1}{r^2}\right)=I(s)\left(\frac{1}{R}+\frac{1}{sL}-g\right)\Rightarrow Z'(s)=\frac{U_2(s)}{I(s)}=\frac{\frac{1}{R}+\frac{1}{sL}-g}{\frac{1}{sLR}+\frac{g}{r}+\frac{1}{r^2}}=\frac{1+\frac{1}{s}-2}{\frac{1}{s}+1+\frac{1}{4}}=\frac{4-4s}{5s+4}$$

$$Z_T(s)=Z'(s)+\frac{1}{sC}=\frac{4-4s}{5s+4}+\frac{1}{s}=\frac{-4s^2+9s+4}{s(5s+4)}$$ **(1 bod)**

4. Za krug prikazan slikom isključiti otpor R i obzirom na priključnice 1–1' odrediti:

a) Theveninov napon u_T;
b) Theveninov otpor R_T;
c) iznos konstante β za koji je R_T=R;
d) napon u_x uz uključen otpor R [β iz zadatka c)].
e) Za koji iznos konstante β je R_T=∞?

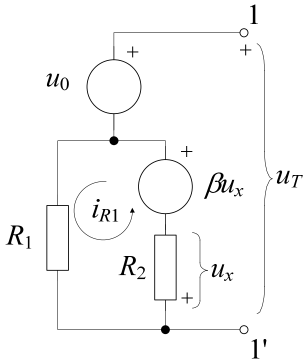
Zadano je: pobuda u_0=2 V i vrijednosti elemenata R_1=R_2=2 Ω, R=4 Ω.



Rješenje:

Isključiti otpor R i odrediti nadomjesni spoj po Theveninu obzirom na priključnice 1–1';

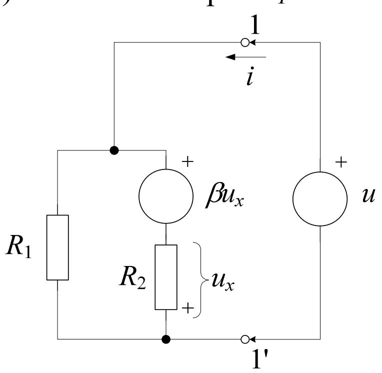
a) Theveninov napon u_T:

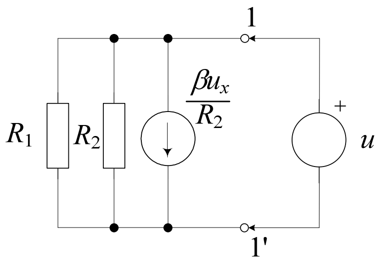


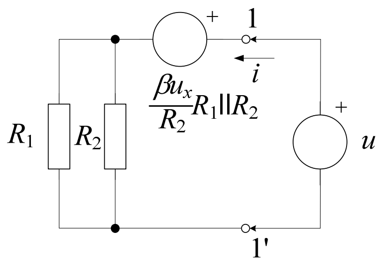
$$u_T = u_0 + i_{R1}\cdot R_1;\qquad i_{R1} = \frac{\beta u_x}{R_1+R_2};\qquad u_x = i_{R1}\cdot R_2$$

$$i_{R1} = \frac{\beta\cdot i_{R1}R_2}{R_1+R_2} \Rightarrow i_{R1}\left(1-\frac{\beta\cdot R_2}{R_1+R_2}\right) = 0 \Rightarrow i_{R1} = 0 \Rightarrow u_T = u_0 = 2\text{V}$$ (1 bod)

b) Theveninov otpor R_T:







$$-u_x = u - \beta\cdot u_x \Rightarrow u_x(1-\beta) = -u \Rightarrow u_x = \frac{u}{\beta-1}$$

$$i=\frac{u}{R_1}+\frac{u-\beta\cdot u_x}{R_2}=\frac{u}{R_1}+\frac{u-\dfrac{\beta}{\beta-1}u}{R_2} \quad\Rightarrow\quad R_T=\frac{u}{i}=\frac{1}{\dfrac{1}{R_1}+\dfrac{\beta-1-\beta}{R_2(\beta-1)}}=\frac{R_1R_2}{R_2-R_1\dfrac{1}{\beta-1}}$$

$$R_T=\frac{R_1R_2(\beta-1)}{R_2(\beta-1)-R_1}=\frac{R_1R_2(1-\beta)}{R_2(1-\beta)+R_1} \quad \text{(1 bod)}$$

c) odrediti iznos konstante β za koji je $R_T=R$.

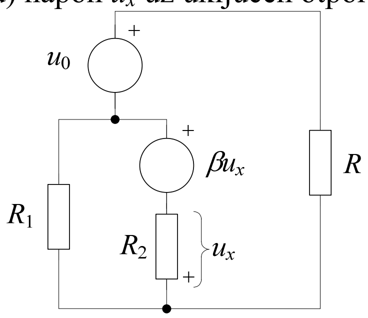
$$\left[R_2(1-\beta)+R_1\right]R_T=R_1R_2(1-\beta) \quad\Rightarrow\quad (1-\beta)(R_T-R_1)R_2=-R_1R_T$$

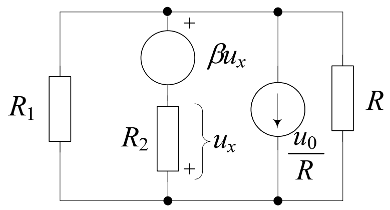
$$1-\beta=\frac{R_1R_T}{(R_1-R_T)R_2} \quad\Rightarrow\quad \beta=1-\frac{R_1R_T}{(R_1-R_T)R_2}$$

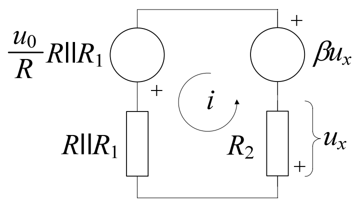
$$\beta=1-\frac{2\cdot 4}{(2-4)\cdot 2}=1+\frac{4}{2}=3 \quad \text{(1 bod)}$$

Provjera: $R_T=\dfrac{R_1R_2(1-\beta)}{R_2(1-\beta)+R_1}=\dfrac{2\cdot 2\cdot(1-3)}{2\cdot(1-3)+2}=\dfrac{2\cdot(-2)}{-2+1}=4\,\Omega$

d) napon u_x uz uključen otpor R







$$\frac{u_0}{R}=\frac{2}{4}=\frac{1}{2}\text{A}$$

$$\frac{u_0}{R}\cdot\frac{RR_1}{R+R_1}+\beta\cdot u_x=i\cdot\left(R_2+\frac{RR_1}{R+R_1}\right)$$

$$u_0\cdot R_1+\beta\cdot u_x(R+R_1)=i\cdot\left[R_2(R+R_1)+RR_1\right]$$

$$u_x=i\cdot R_2$$

$$u_0\cdot R_1+\beta\cdot u_x(R+R_1)=\frac{u_x}{R_2}\cdot\left[R_2(R+R_1)+RR_1\right]$$

$$u_0\cdot R_1=(1-\beta)\cdot u_x(R+R_1)+u_x\frac{RR_1}{R_2}$$

$$u_x=\frac{u_0\cdot R_1}{(1-\beta)(R+R_1)+\dfrac{RR_1}{R_2}}=\frac{2\cdot 2}{(1-3)\cdot(4+2)+\dfrac{4\cdot 2}{2}}=-\frac{1}{2}V \quad \text{(1 bod)}$$

e) Za koji iznos konstante β je $R_T=\infty$? Za $\beta=2$. (1 bod)

$$R_T=\frac{R_1R_2(1-\beta)}{R_2(1-\beta)+R_1}=\frac{2\cdot 2\cdot(1-2)}{2\cdot(1-2)+2}=\frac{-4}{-2+2}=\frac{-4}{0}=\infty$$

$$1)\ U_1\left(sC+\frac{1}{R_1}+\frac{1}{sL}\right)-U_2\frac{1}{sL}=-\frac{rI_s(s)}{sL}-I_p(s)$$

$$2)\ -U_1\frac{1}{sL}+U_2\left(\frac{1}{R_2}+\frac{1}{sL}\right)=\frac{rI_s(s)}{sL}+I_s(s)+I(s)$$

$$3)\ U_2=\frac{1}{n}U_1$$

$$4)\ I_s=nI_p$$

$$1)\ nU_2\left(sC+\frac{1}{R_1}+\frac{1}{sL}\right)-U_2\frac{1}{sL}=-\frac{rnI_p(s)}{sL}-I_p(s)$$

$$2)\ -nU_2\frac{1}{sL}+U_2\left(\frac{1}{R_2}+\frac{1}{sL}\right)=\frac{rnI_p(s)}{sL}+nI_p(s)+I(s)$$ (1 bod)

---

d) Theveninova impedancija $Z_T(s)=U_2(s)/I(s)$:
Uvrstimo vrijednosti: $C=1/2$, $L=2$, $R_1=R_2=1$ te $n=2$, $r=4$, $u_C(0)=1$, $i_L(0)=1$, $u_g(t)=S(t)$.

$$1)\ 2U_2\left(\frac{s}{2}+1+\frac{1}{2s}\right)-U_2\frac{1}{2s}=-\frac{4I_p(s)}{s}-I_p(s)\ \Rightarrow$$

$$U_2\left(s+2+\frac{1}{2s}\right)=-I_p(s)\left(\frac{4}{s}+1\right)\ \Rightarrow\ I_p(s)=\frac{-U_2\left(s+2+\frac{1}{2s}\right)}{\left(\frac{4}{s}+1\right)}$$

$$2)\ -U_2\frac{1}{s}+U_2\left(1+\frac{1}{2s}\right)=\frac{4I_p(s)}{s}+2I_p(s)+I(s)$$

$$\underline{U_2\left(1-\frac{1}{2s}\right)=2I_p(s)\left(1+\frac{2}{s}\right)+I(s)}$$

$$1),2)\Rightarrow U_2\left(1-\frac{1}{2s}\right)=2\frac{-U_2\left(s+2+\frac{1}{2s}\right)}{\left(\frac{4}{s}+1\right)}\left(1+\frac{2}{s}\right)+I(s)=\frac{-U_2 2[s+2+1/(2s)]}{(s+4)}(s+2)+I(s)$$

$$\Rightarrow U_2(2s-1)=2s\frac{-U_2 2(s+2+1/(2s))}{(s+4)}(s+2)+2sI(s)$$

$$\Rightarrow U_2(2s-1)(s+4)=-U_2 4s\left[(s+2)^2+(s+2)/(2s)\right]+2s(s+4)I(s)$$

$$\Rightarrow U_2\left[(2s-1)(s+4)+4s(s+2)^2+4s(1/2+1/s)\right]=2s(s+4)I(s)$$

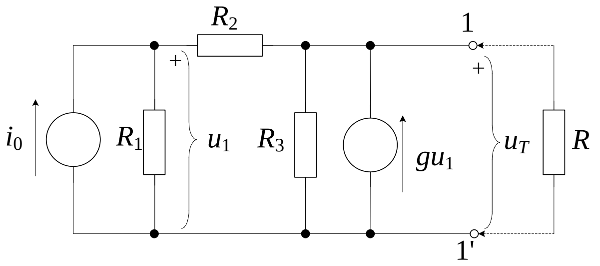
$$\Rightarrow U_2\left[2s^2-s+8s-4+4s^3+16s^2+16s+2s+4\right]=\left[2s^2+8s\right]I(s)$$

$$Z_T(s)=\frac{U_2(s)}{I(s)}=\frac{2s^2+8s}{4s^3+18s^2+25s}=\frac{2(s+4)}{4s^2+18s+25}$$ (1 bod)

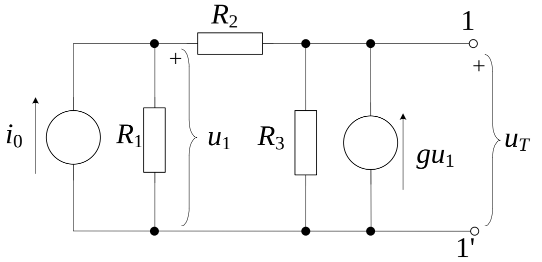
e) Da li je električni krug recipročan? Zašto?
NE jer ima strujno ovisni naponski izvor. (1 bod)

2. Za krug na slici obzirom na priključnice 1–1' i isključen otpor R odrediti:
a) Theveninov napon u_T; b) Theveninov otpor R_T; c) iznos konstante g za koji je $R_T=R$;
d) napon u_1 uz uključen otpor R [g iz zadatka c)]; e) iznos konstante g za koji je $R_T=\infty$.

Zadano je: $i_0=2$ A i $R_1=1\Omega$, $R_1=\frac{1}{2}\Omega$ i $R_3=R=\frac{1}{3}\Omega$.



Rješenje: a) Theveninov napon u_T:

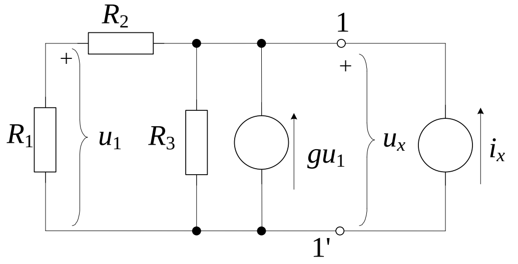


$$u_1(G_1+G_2)-u_2G_2=i_0$$

$$-u_1G_2+u_2(G_2+G_3)=gu_1 \quad \Rightarrow \quad u_1=u_2\frac{G_2+G_3}{g+G_2}$$

$$u_T=u_2=\frac{g+G_2}{G_3(G_1+G_2)+G_1G_2-gG_2}i_0=\frac{2g+4}{11-2g}$$ (1 bod)

b) Theveninov otpor R_T:



$$u_1=u_x\frac{G_2}{G_1+G_2}$$

$$i_x+gu_1=u_x\left(G_3+\frac{G_1G_2}{G_1+G_2}\right) \quad \Rightarrow \quad i_x=u_x\left(-g\frac{G_2}{G_1+G_2}+G_3+\frac{G_1G_2}{G_1+G_2}\right)$$

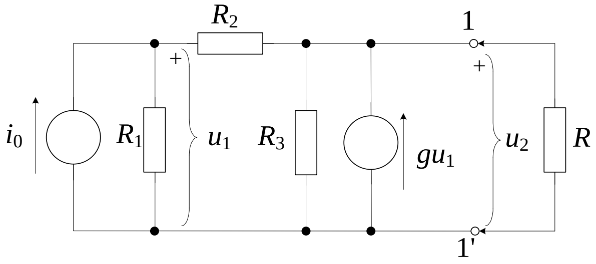
$$R_T=\frac{u_x}{i_x}=\frac{G_1+G_2}{G_3(G_1+G_2)+G_1G_2-gG_2}=\frac{3}{11-2g}$$ (1 bod)

c) odrediti iznos konstante g za koji je $R_T=R$.

$$R_T=R=\frac{1}{3}=\frac{G_1+G_2}{G_3(G_1+G_2)+G_1G_2-gG_2} \quad \Rightarrow \quad \frac{1}{3}=\frac{1+2}{3(1+2)+2-2g} \quad \Rightarrow \quad g=1\ \Omega^{-1}$$

(1 bod)

d) napon u_1 uz uključen otpor R



$$-u_1G_2 + u_2(G_2 + G_3 + G) = gu_1$$

$$u_1 = u_2\frac{G_2 + G_3 + G}{g + G_2} = \frac{u_T}{2}\cdot\frac{G_2 + G_3 + G}{g + G_2} = \frac{1}{2}\cdot\frac{G_2 + G_3 + G}{g + G_2}\cdot\frac{g + G_2}{G_3(G_1 + G_2) + G_1G_2 - gG_2}i_0 = \frac{8}{9}\,\mathrm{V}$$

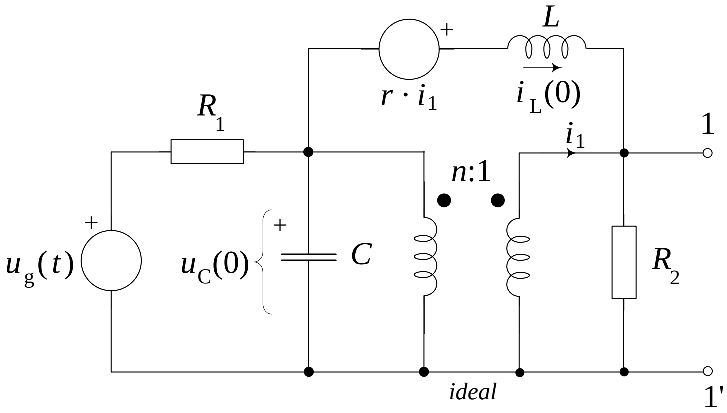
(1 bod)

e) iznos konstante g za koji je $R_T=\infty$?

$$R_T = \frac{u_x}{i_x} = \frac{G_1 + G_2}{G_3(G_1 + G_2) + G_1G_2 - gG_2}\,\Omega$$

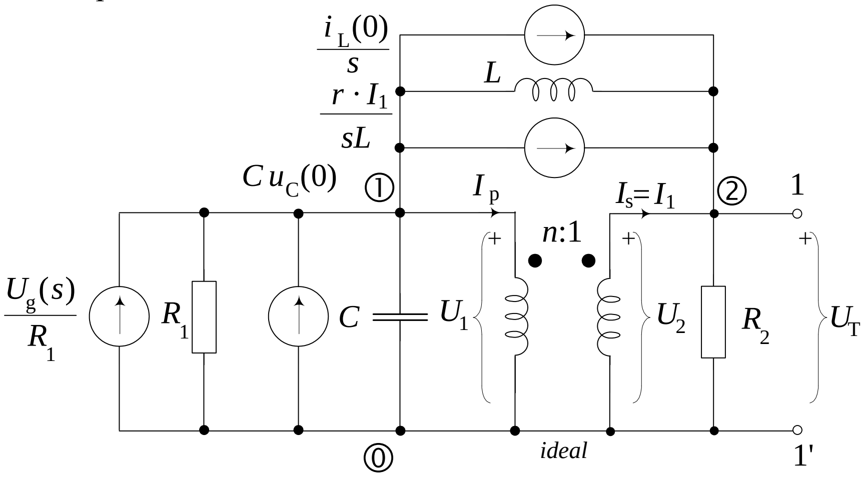
$$G_3(G_1 + G_2) + G_1G_2 - gG_2 = 0 \Rightarrow \qquad g = \frac{G_3(G_1 + G_2)}{G_2} + G_1 = \frac{11}{2}\ \Omega^{-1}$$ (1 bod)

5. Za električni krug na slici zadane su normalizirane vrijednosti elemenata C=1/2, L=2, R_1=R_2=1 te n=2, r=4, $u_C(0)$=1, $i_L(0)$=1, $u_g(t)$=$S(t)$. Odrediti nadomjesne parametre mreže po Theveninu s obzirom na polove 1–1'. Koristiti metodu napona čvorišta. U zadatku je potrebno: a) Nacrtati sklop za izračunavanje Theveninovog napona i postaviti jednadžbe napona za čvorišta ① i ②; b) Odrediti Theveninov napon $U_T(s)$; c) Nacrtati sklop za izračunavanje Theveninove impedancije i postaviti jednadžbe napona za čvorišta ① i ②; d) Odrediti Theveninovu impedanciju $Z_T(s)$. e) Da li je električni krug recipročan? Zašto?



Rješenje:

a) Jednadžbe napona za čvorišta ① i ②:



$$1)\ U_1\left(sC+\frac{1}{R_1}+\frac{1}{sL}\right)-U_2\frac{1}{sL}=\frac{U_g(s)}{R_1}+Cu_C(0)-\frac{rI_s(s)}{sL}-\frac{i_L(0)}{s}-I_p(s)$$

$$2)\ -U_1\frac{1}{sL}+U_2\left(\frac{1}{R_2}+\frac{1}{sL}\right)=\frac{rI_s(s)}{sL}+\frac{i_L(0)}{s}+I_s(s)$$

$$3)\ U_2=\frac{1}{n}U_1$$

$$4)\ I_s=nI_p$$

$$1)\ nU_2\left(sC+\frac{1}{R_1}+\frac{1}{sL}\right)-U_2\frac{1}{sL}=\frac{U_g(s)}{R_1}+Cu_C(0)-\frac{rnI_p(s)}{sL}-\frac{i_L(0)}{s}-I_p(s)$$

$$2)\ -nU_2\frac{1}{sL}+U_2\left(\frac{1}{R_2}+\frac{1}{sL}\right)=\frac{rnI_p(s)}{sL}+\frac{i_L(0)}{s}+nI_p(s)$$ (1 bod)

---

b) Theveninov napon $U_T(s)=U_2(s)$:

Uvrstimo vrijednosti: $C=1/2$, $L=2$, $R_1=R_2=1$ te $n=2$, $r=4$, $u_C(0)=1$, $i_L(0)=1$, $u_g(t)=S(t)$.

$$1)\;\; 2U_2\left(\frac{s}{2}+1+\frac{1}{2s}\right)-U_2\frac{1}{2s}=\frac{1}{s}+\frac{1}{2}-\frac{4I_p(s)}{s}-\frac{1}{s}-I_p(s)\;\Rightarrow$$

$$U_2\left(s+2+\frac{1}{2s}\right)=\frac{1}{2}-I_p(s)\left(\frac{4}{s}+1\right)\;\Rightarrow\; I_p(s)=\frac{\frac{1}{2}-U_2\left(s+2+\frac{1}{2s}\right)}{\left(\frac{4}{s}+1\right)}$$

$$2)\;\; -U_2\frac{1}{s}+U_2\left(1+\frac{1}{2s}\right)=\frac{4I_p(s)}{s}+\frac{1}{s}+2I_p(s)$$

$$U_2\left(1-\frac{1}{2s}\right)=\frac{1}{s}+2I_p(s)\left(1+\frac{2}{s}\right)$$

1), 2) $\Rightarrow$

$$U_2\left(1-\frac{1}{2s}\right)=\frac{1}{s}+2\frac{\frac{1}{2}-U_2\left(s+2+\frac{1}{2s}\right)}{\left(\frac{4}{s}+1\right)}\left(1+\frac{2}{s}\right)=\frac{1}{s}+\frac{1-U_2 2\left(s+2+\frac{1}{2s}\right)}{(s+4)}(s+2)$$

$$\Rightarrow U_2(2s-1)=2+2s\frac{1-U_2 2(s+2+1/(2s))}{(s+4)}(s+2)$$

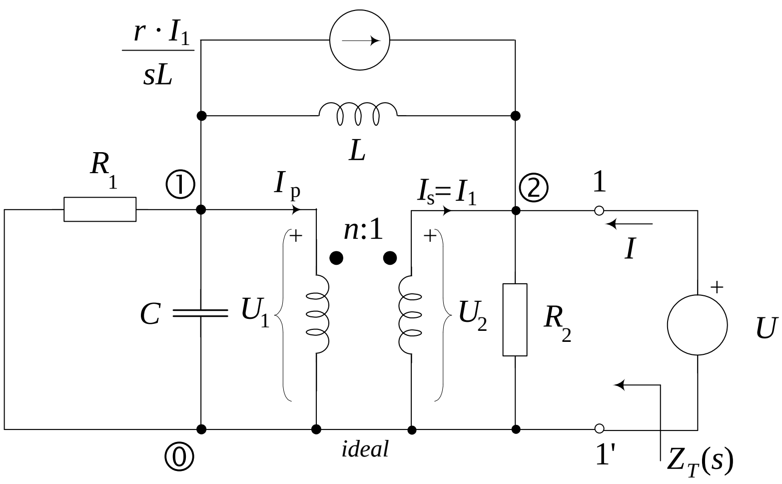
$$\Rightarrow U_2(2s-1)(s+4)=2(s+4)+2s(s+2)-U_2 4s\left[(s+2)^2+\frac{1}{2s}(s+2)\right]$$

$$\Rightarrow U_2\left[(2s-1)(s+4)+4s\left(s^2+4s+4+\frac{1}{2}+\frac{1}{s}\right)\right]=2(s+4)+2s(s+2)$$

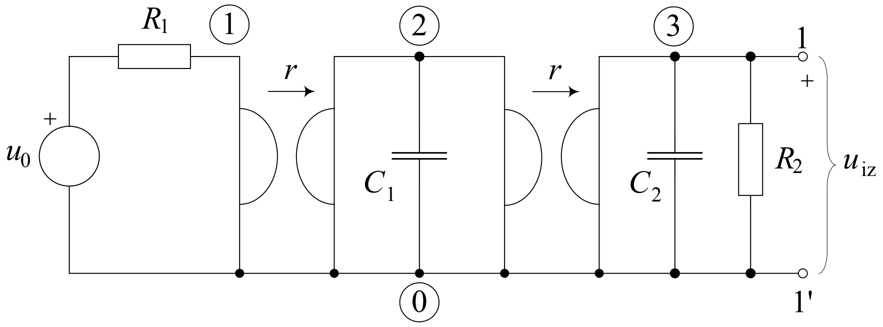
$$\Rightarrow U_2\left[2s^2-s+8s-4+4s^3+16s^2+16s+2s+4\right]=2s+8+2s^2+4s$$

$$U_T(s)=U_2(s)=\frac{2s^2+6s+8}{4s^3+18s^2+25s}=\frac{2(s^2+3s+4)}{s(4s^2+18s+25)}$$ (1 bod)

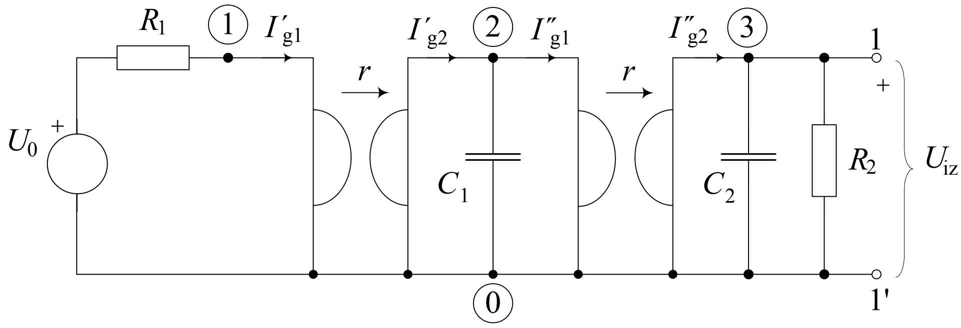
c) Izračunavanje Theveninove impedancije pomoću jednadžbi napona čvorišta ① i ②



3. Za električni krug na slici zadane su normalizirane vrijednosti elemenata $C_1 = C_2 = \sqrt{2}$, $R_1 = R_2 = 1$, te $r = 1$. Odrediti: nadomjesne parametre mreže po Theveninu obzirom na polove 1–1': a) Theveninov napon U_T i b) Theveninovu impedanciju Z_T. Također izračunati: c) napon u_1; d) napon u_2. Koristiti metodu napona čvorišta (čvorište 0 je referentno).



Rješenje: a) Jednadžbe napona za čvorišta 1, 2 i 3 (1 bod)



a) Theveninov napon

$$\frac{U_0}{R_1} - I'_{g1} = \frac{U_1}{R_1} \qquad\qquad I'_{g1} = -\frac{U_2}{r}, \quad I'_{g2} = -\frac{U_1}{r}$$

$$I'_{g2} - I''_{g1} = U_2 s C_1 \qquad\qquad I''_{g1} = -\frac{U_3}{r}, \quad I''_{g2} = -\frac{U_2}{r}$$

$$I''_{g2} = U_3\left(sC_2 + \frac{1}{R_2}\right)$$

$$\frac{U_0}{R_1} = \frac{U_1}{R_1} - \frac{U_2}{r}$$

$$0 = \frac{U_1}{r} + U_2 s C_1 - \frac{U_3}{r} \quad \Rightarrow \quad U_1 = -rU_2 sC_1 + U_3$$

$$0 = \frac{U_2}{r} + U_3\left(sC_2 + \frac{1}{R_2}\right) \quad \Rightarrow \quad U_2 = -rU_3\left(sC_2 + \frac{1}{R_2}\right)$$

$$U_1 = -r\left(-rU_3\left(sC_2 + \frac{1}{R_2}\right)\right)sC_1 + U_3 = \left(r^2\left(sC_2 + \frac{1}{R_2}\right)sC_1 + 1\right)U_3$$

$$\frac{U_0}{R_1}=\frac{U_1}{R_1}-\frac{U_2}{r}=\frac{U_3}{R_1}\cdot\left(r^2\left(sC_2+\frac{1}{R_2}\right)sC_1+1\right)+U_3\left(sC_2+\frac{1}{R_2}\right)$$

$$U_3=\frac{U_0}{\left(r^2\left(sC_2+\frac{1}{R_2}\right)sC_1+1\right)+R_1\left(sC_2+\frac{1}{R_2}\right)}$$

$$U_T=U_3=\frac{U_0R_2}{s^2r^2R_2C_1C_2+s\left(r^2C_1+R_1R_2C_2\right)+R_1+R_2}$$

$$U_T=\frac{1}{2}\cdot\frac{U_0}{s^2+\sqrt{2}s+1}\ \text{(1 bod)}$$

b) Theveninova impedancija

$$Z_{iz1}=\frac{r^2}{R_1}$$

$$Z_{iz2}=\frac{r^2\left(Z_{iz1}+\frac{1}{sC_2}\right)}{Z_{iz1}\frac{1}{sC_2}}=\frac{r^2\left(Z_{iz1}sC_2+1\right)}{Z_{iz1}}=r^2sC_2+R_1$$

$$Z_T=\frac{Z_{iz2}R_2}{Z_{iz2}\left(sC_3R_2+1\right)+R_2}=\frac{\left(r^2sC_1+R_1\right)R_2}{s^2r^2C_1C_2R_2+s\left(R_1R_2C_2+r^2C_1\right)+R_1+R_2}=\frac{\sqrt{2}s+1}{2\left(s^2+\sqrt{2}s+2\right)}\ \text{(1 bod)}$$

c) Napon U_1

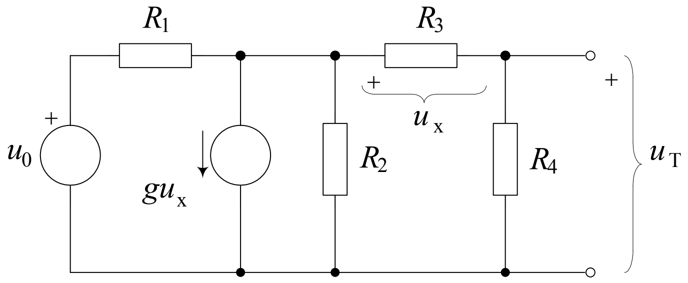
$$U_1=\left(r^2\left(sC_2+\frac{1}{R_2}\right)sC_1+1\right)U_3=\frac{1}{R_2}\left(r^2\left(sR_2C_2+1\right)sC_1+R_2\right)U_3$$

$$U_1=\frac{U_0\left(r^2\left(sC_2R_2+1\right)sC_1+R_2\right)}{s^2r^2R_2C_1C_2+s\left(r^2C_1+R_1R_2C_2\right)+R_1+R_2}=\frac{U_0\left(2s^2+\sqrt{2}s+1\right)}{2s^2+2\sqrt{2}s+2}\ \text{(1 bod)}$$

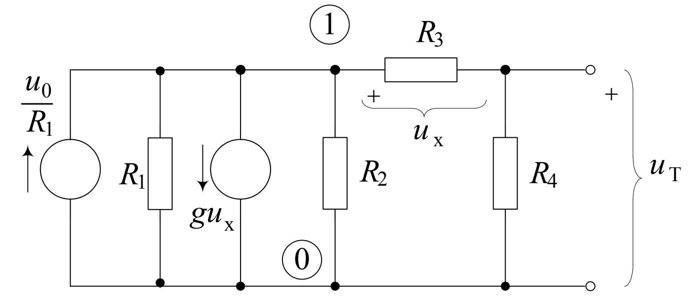
d) Napon U_2

$$U_2=\frac{-r\left(sC_2R_2+1\right)U_0}{s^2r^2C_1C_2R_1+s\left(r^2C_1+C_2R_1R_2\right)+R_1+R_2}=\frac{-\left(s\sqrt{2}+1\right)U_0}{2s^2+2\sqrt{2}s+2}\ \text{(1 bod)}$$

2. Za električni krug na slici odrediti ekvivalentni dvopol po Theveninu. Zadane su vrijednosti elemenata: $R_1 = 1/6\ \text{k}\Omega$, $R_2 = 1/4\ \text{k}\Omega$, $R_3 = 1/3\ \text{k}\Omega$, $R_4 = 1\ \text{k}\Omega$, $g = 2\ \text{mS}$, te $u_0 = 2{,}5\ V$. Odrediti: a) Theveninov napon u_T i b) Theveninov otpor R_T, c) napon u_x, d) napon na otporu R_2 i e) struju kroz R_2.



Rješenje: a) Theveninov napon



$$\frac{u_0}{R_1} - g u_x = u_1\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right) - u_T\frac{1}{R_3}$$

$$\frac{u_T}{R_4} = \frac{u_x}{R_3} \Rightarrow u_x = \frac{u_T R_3}{R_4}$$

$$u_1 = u_x + u_T = u_T\left(1 + \frac{R_3}{R_4}\right)$$

$$\frac{u_0}{R_1} - g\frac{u_T R_3}{R_4} = u_T\left(1 + \frac{R_3}{R_4}\right)\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right) - u_T\frac{1}{R_3}$$

$$\frac{u_0}{R_1} = u_T\left(g\frac{R_3}{R_4} + \left(1 + \frac{R_3}{R_4}\right)\left(\frac{1}{R_1} + \frac{1}{R_2}\right) + \frac{1}{R_4}\right)$$

$$u_T = \frac{\frac{1}{R_1}u_0}{g\frac{R_3}{R_4} + \left(1 + \frac{R_3}{R_4}\right)\left(\frac{1}{R_1} + \frac{1}{R_2}\right) + \frac{1}{R_4}} = \frac{6 \cdot 2{,}5}{\frac{2}{3} + \left(1 + \frac{1}{3}\right)(6+4) + 1} = \frac{15}{\frac{2}{3} + \frac{4}{3}10 + 1} = 1\ \text{V}$$

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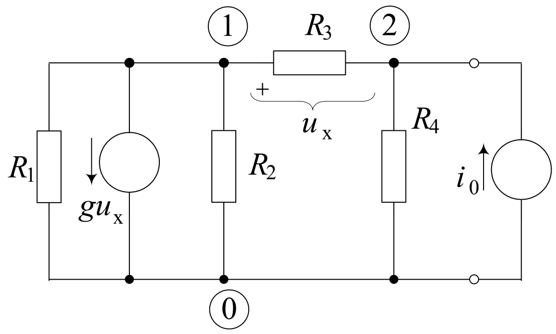
b) Theveninov otpor



$$-gu_x = u_1\left(\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}\right)-u_2\frac{1}{R_3} \qquad u_x = u_1 - u_2$$

$$i_0 = -u_1\frac{1}{R_3}+u_2\left(\frac{1}{R_3}+\frac{1}{R_4}\right)$$

$$-g(u_1-u_2)=u_1\left(\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}\right)-u_2\frac{1}{R_3} \quad\Rightarrow\quad u_1 = u_2\frac{g+\frac{1}{R_3}}{g+\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}}$$

$$i_0 = -u_2\frac{g+\frac{1}{R_3}}{g+\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}}\frac{1}{R_3}+u_2\left(\frac{1}{R_3}+\frac{1}{R_4}\right)=u_2\frac{\frac{1}{R_3}\left(\frac{1}{R_1}+\frac{1}{R_2}\right)+\frac{1}{R_4}\left(g+\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}\right)}{g+\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}}$$

$$R_T = \frac{u_2}{i_0} = \frac{g+\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}}{\frac{1}{R_3}\left(\frac{1}{R_1}+\frac{1}{R_2}\right)+\frac{1}{R_4}\left(g+\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}\right)} = \frac{2+6+4+3}{3(6+4)+1(2+6+4+3)} = \frac{1}{3}\ \mathrm{k\Omega}$$

Bodovi: a) + b) = (3 boda)

c) napon u_x

$$u_x = \frac{u_T R_3}{R_4} = \frac{1}{3}\ V$$

d) napon na R_2

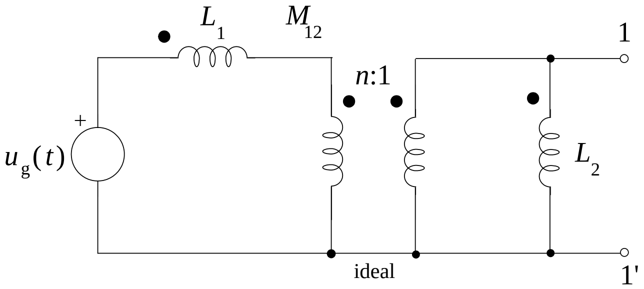
$$u_{R2} = u_1 = u_x + u_T = \frac{4}{3}\ V$$

e) struja kroz R_2

$$i_{R2} = \frac{u_{R2}}{R_2} = \frac{4}{3}\cdot 4 = \frac{16}{3}\ mA$$

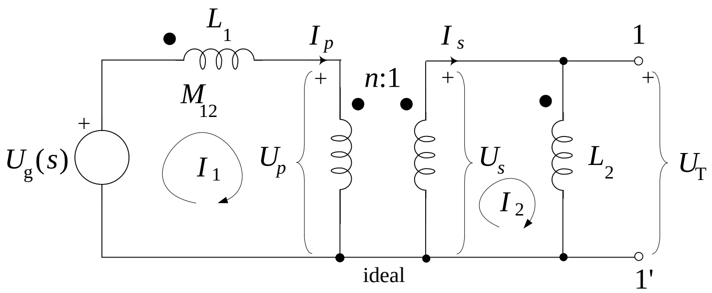
Bodovi: c) + d) + e) = (2 boda)

2. Za električni krug na slici zadane su normalizirane vrijednosti elemenata L_1=2, L_2=1, M_{12}=1 te n=2, $u_g(t)$=$S(t)$. Odrediti nadomjesne parametre mreže po Theveninu s obzirom na polove 1–1'. Koristiti metodu struja petlji. U zadatku je potrebno: a) Nacrtati sklop za izračunavanje Theveninovog napona i postaviti jednadžbe petlji; b) Odrediti Theveninov napon $U_T(s)$; c) Nacrtati sklop za izračunavanje Theveninove impedancije i postaviti jednadžbe petlji; d) Odrediti Theveninovu impedanciju $Z_T(s)$. e) Da li je električni krug recipročan? Zašto?



<u>Rješenje:</u>

a) Jednadžbe petlji:



Iz sheme je vidljivo da vrijedi: $I_p = I_1,\ \ I_s = I_2,\ \ U_s = U_T$

1) $I_1 sL_1 + I_2 sM_{12} + U_p(s) = U_g(s)$

2) $I_1 sM_{12} + I_2 sL_2 = U_s(s)$

3) $U_s = \dfrac{1}{n} U_p \ \Rightarrow\ U_p = nU_s$

4) $I_s = nI_p \ \Rightarrow\ I_2 = nI_1$

---

1) $I_1 sL_1 + I_2 sM_{12} + nU_s(s) = U_g(s)$

2) $I_1 sM_{12} + I_2 sL_2 = U_s(s)$ (1 bod)

---

b) Theveninov napon $U_T(s)$=$U_s(s)$:

1), 2) $\Rightarrow$ $I_1 sL_1 + I_2 sM_{12} + n\left(I_1 sM_{12} + I_2 sL_2\right) = U_g(s)$

$I_1 sL_1 + nI_1 sM_{12} + n\left(I_1 sM_{12} + nI_1 sL_2\right) = U_g(s)$

$I_1\left(sL_1 + 2nsM_{12} + n^2 sL_2\right) = U_g(s)$

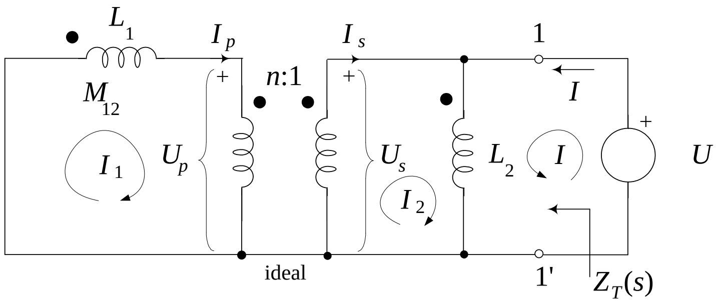
Uvrstimo vrijednosti: L_1=2, L_2=1, M_{12}=1, n=2, $u_g(t)$=$S(t)$.

$$I_1(s) = \frac{U_g(s)}{s\left(L_1 + 2nM_{12} + n^2 L_2\right)} = \frac{1}{s}\cdot\frac{1}{s(2+4+4)} = \frac{1}{s}\cdot\frac{1}{s(2+4+4)} = \frac{1}{10s^2}$$

$$I_2(s) = n\cdot I_1(s) = 2\cdot\frac{1}{10s^2} = \frac{1}{5s^2}$$

$$U_T(s) = U_s(s) = sM_{12}I_1(s) + sL_2 I_2(s) = s\frac{1}{10s^2} + s\frac{2}{10s^2} = \frac{3}{10s} \quad \text{(1 bod)}$$

c) Izračunavanje Theveninove impedancije pomoću jednadžbi petlji



Iz sheme je vidljivo da vrijedi: $I_p = I_1, \quad I_s = I_2, U(s) = U_s(s), \; Z_T(s) = \dfrac{U(s)}{I(s)}$

1) $I_1 sL_1 + (I_2 + I)sM_{12} + U_p(s) = 0$

2) $I_1 sM_{12} + (I_2 + I)sL_2 = U_s(s)$

3) $U_s(s) = U(s)$

4) $U_s = \dfrac{1}{n}U_p \;\Rightarrow\; U_p = nU_s$

5) $I_s = nI_p \;\Rightarrow\; I_2 = nI_1$

---

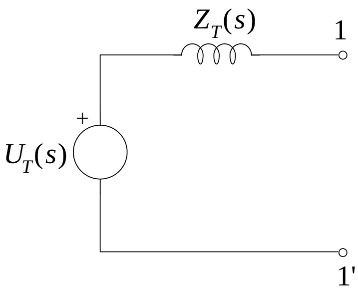
1) $I_1 sL_1 + (nI_1 + I)sM_{12} + nU_s(s) = 0$

2) $I_1 sM_{12} + (nI_1 + I)sL_2 = U_s(s)$

---

1) $I_1(sL_1 + nsM_{12}) + IsM_{12} + nU(s) = 0$

2) $I_1(sM_{12} + nsL_2) + IsL_2 = U(s)$ (1 bod)



d) Theveninova impedancija $Z_T(s)=U(s)/I(s)$:

$$2) \Rightarrow I_1 = \frac{U(s) - IsL_2}{sM_{12} + nsL_2} \quad \rightarrow 1) \; \frac{U(s) - IsL_2}{sM_{12} + nsL_2}(sL_1 + nsM_{12}) + IsM_{12} + nU(s) = 0$$

$$U(s)\frac{L_1 + nM_{12}}{M_{12} + nL_2} + nU(s) = I(s)sL_2\frac{L_1 + nM_{12}}{M_{12} + nL_2} - I(s)sM_{12}$$

$$Z_T(s) = \frac{U(s)}{I(s)} = \frac{sL_2\dfrac{L_1 + nM_{12}}{M_{12} + nL_2} - sM_{12}}{\dfrac{L_1 + nM_{12}}{M_{12} + nL_2} + n} = \frac{s\dfrac{2+2}{1+2} - s}{\dfrac{2+2}{1+2} + 2} = \frac{s\dfrac{4}{3} - s}{\dfrac{4}{3} + \dfrac{6}{3}} = \frac{4s - 3s}{10} = \frac{s}{10} = sL_T$$

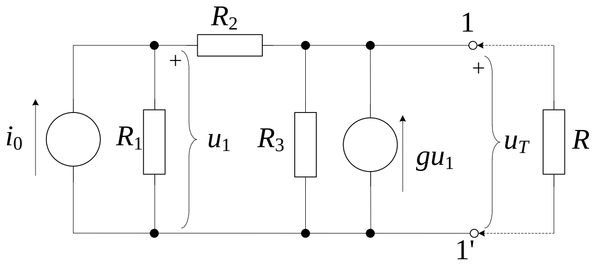
(1 bod)

e) Da li je električni krug recipročan? Zašto?

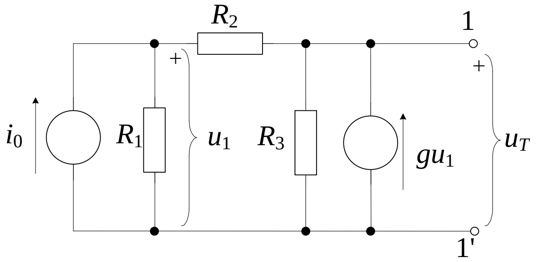
DA! Jer ima idealni transformator i vezane induktivitete (nema ovisne izvore ili girator). (1 bod)

2. Za krug na slici obzirom na priključnice 1–1' i isključen otpor R odrediti:
a) Theveninov napon u_T; b) Theveninov otpor R_T; c) iznos konstante g za koji je $R_T=R$; d) napon u_1 uz uključen otpor R [g iz zadatka c)]; e) iznos konstante g za koji je $R_T=\infty$.

Zadano je: $i_0=2$ A i $R_1=1\Omega$, $R_1=\frac{1}{2}\Omega$ i $R_3=R=\frac{1}{3}\Omega$.



Rješenje: a) Theveninov napon u_T:

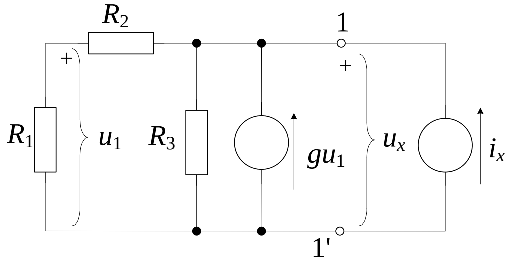


$$u_1(G_1+G_2)-u_2G_2=i_0$$

$$-u_1G_2+u_2(G_2+G_3)=gu_1 \quad \Rightarrow \quad u_1=u_2\frac{G_2+G_3}{g+G_2}$$

$$u_T=u_2=\frac{g+G_2}{G_3(G_1+G_2)+G_1G_2-gG_2}i_0=\frac{2g+4}{11-2g}$$ (1 bod)

b) Theveninov otpor R_T:



$$u_1=u_x\frac{G_2}{G_1+G_2}$$

$$i_x+gu_1=u_x\left(G_3+\frac{G_1G_2}{G_1+G_2}\right) \quad \Rightarrow \quad i_x=u_x\left(-g\frac{G_2}{G_1+G_2}+G_3+\frac{G_1G_2}{G_1+G_2}\right)$$

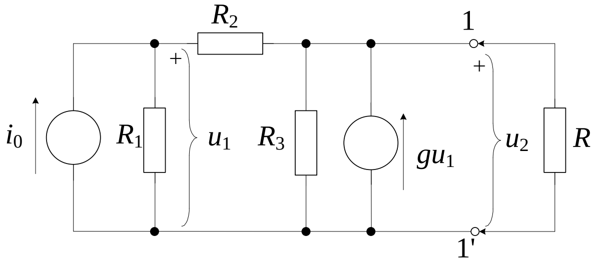
$$R_T=\frac{u_x}{i_x}=\frac{G_1+G_2}{G_3(G_1+G_2)+G_1G_2-gG_2}=\frac{3}{11-2g}$$ (1 bod)

c) odrediti iznos konstante g za koji je $R_T=R$.

$$R_T=R=\frac{1}{3}=\frac{G_1+G_2}{G_3(G_1+G_2)+G_1G_2-gG_2} \quad \Rightarrow \quad \frac{1}{3}=\frac{1+2}{3(1+2)+2-2g} \quad \Rightarrow \quad g=1\ \Omega^{-1}$$

(1 bod)

d) napon u_1 uz uključen otpor R



$$-u_1G_2+u_2(G_2+G_3+G)=gu_1$$

$$u_1=u_2\frac{G_2+G_3+G}{g+G_2}=\frac{u_T}{2}\cdot\frac{G_2+G_3+G}{g+G_2}=\frac{1}{2}\cdot\frac{G_2+G_3+G}{g+G_2}\cdot\frac{g+G_2}{G_3(G_1+G_2)+G_1G_2-gG_2}i_0=\frac{8}{9}\text{V}$$

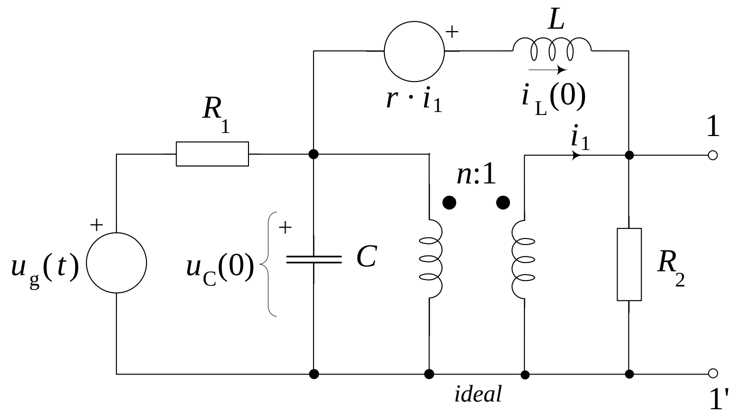
(1 bod)

e) iznos konstante g za koji je R_T=∞?

$$R_T=\frac{u_x}{i_x}=\frac{G_1+G_2}{G_3(G_1+G_2)+G_1G_2-gG_2}\Omega$$

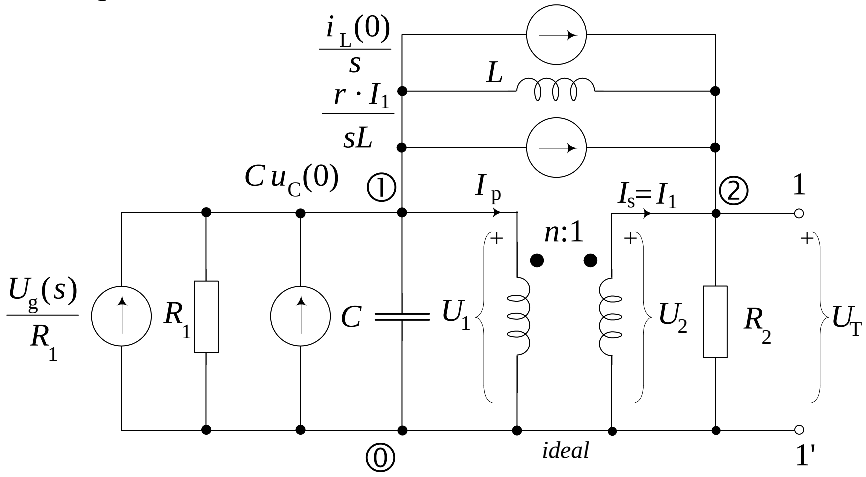
$$G_3(G_1+G_2)+G_1G_2-gG_2=0\Rightarrow \qquad g=\frac{G_3(G_1+G_2)}{G_2}+G_1=\frac{11}{2}\ \Omega^{-1}$$ (1 bod)

5. Za električni krug na slici zadane su normalizirane vrijednosti elemenata C=1/2, L=2, R_1=R_2=1 te n=2, r=4, $u_C(0)$=1, $i_L(0)$=1, $u_g(t)$=$S(t)$. Odrediti nadomjesne parametre mreže po Theveninu s obzirom na polove 1–1'. Koristiti metodu napona čvorišta. U zadatku je potrebno: a) Nacrtati sklop za izračunavanje Theveninovog napona i postaviti jednadžbe napona za čvorišta ① i ②; b) Odrediti Theveninov napon $U_T(s)$; c) Nacrtati sklop za izračunavanje Theveninove impedancije i postaviti jednadžbe napona za čvorišta ① i ②; d) Odrediti Theveninovu impedanciju $Z_T(s)$. e) Da li je električni krug recipročan? Zašto?



Rješenje:

a) Jednadžbe napona za čvorišta ① i ②:



$$1)\ U_1\left(sC+\frac{1}{R_1}+\frac{1}{sL}\right)-U_2\frac{1}{sL}=\frac{U_g(s)}{R_1}+Cu_C(0)-\frac{rI_s(s)}{sL}-\frac{i_L(0)}{s}-I_p(s)$$

$$2)\ -U_1\frac{1}{sL}+U_2\left(\frac{1}{R_2}+\frac{1}{sL}\right)=\frac{rI_s(s)}{sL}+\frac{i_L(0)}{s}+I_s(s)$$

$$3)\ U_2=\frac{1}{n}U_1$$

$$4)\ I_s=nI_p$$

$$1)\ nU_2\left(sC+\frac{1}{R_1}+\frac{1}{sL}\right)-U_2\frac{1}{sL}=\frac{U_g(s)}{R_1}+Cu_C(0)-\frac{rnI_p(s)}{sL}-\frac{i_L(0)}{s}-I_p(s)$$

$$2)\ -nU_2\frac{1}{sL}+U_2\left(\frac{1}{R_2}+\frac{1}{sL}\right)=\frac{rnI_p(s)}{sL}+\frac{i_L(0)}{s}+nI_p(s)$$ (1 bod)

---

b) Theveninov napon $U_T(s)=U_2(s)$:

Uvrstimo vrijednosti: $C=1/2$, $L=2$, $R_1=R_2=1$ te $n=2$, $r=4$, $u_C(0)=1$, $i_L(0)=1$, $u_g(t)=S(t)$.

$$1)\ 2U_2\left(\frac{s}{2}+1+\frac{1}{2s}\right)-U_2\frac{1}{2s}=\frac{1}{s}+\frac{1}{2}-\frac{4I_p(s)}{s}-\frac{1}{s}-I_p(s)\ \Rightarrow$$

$$U_2\left(s+2+\frac{1}{2s}\right)=\frac{1}{2}-I_p(s)\left(\frac{4}{s}+1\right)\ \Rightarrow\ I_p(s)=\frac{\frac{1}{2}-U_2\left(s+2+\frac{1}{2s}\right)}{\left(\frac{4}{s}+1\right)}$$

$$2)\ -U_2\frac{1}{s}+U_2\left(1+\frac{1}{2s}\right)=\frac{4I_p(s)}{s}+\frac{1}{s}+2I_p(s)$$

$$U_2\left(1-\frac{1}{2s}\right)=\frac{1}{s}+2I_p(s)\left(1+\frac{2}{s}\right)$$

1), 2) $\Rightarrow$

$$U_2\left(1-\frac{1}{2s}\right)=\frac{1}{s}+2\frac{\frac{1}{2}-U_2\left(s+2+\frac{1}{2s}\right)}{\left(\frac{4}{s}+1\right)}\left(1+\frac{2}{s}\right)=\frac{1}{s}+\frac{1-U_2 2\left(s+2+\frac{1}{2s}\right)}{(s+4)}(s+2)$$

$$\Rightarrow U_2(2s-1)=2+2s\frac{1-U_2 2(s+2+1/(2s))}{(s+4)}(s+2)$$

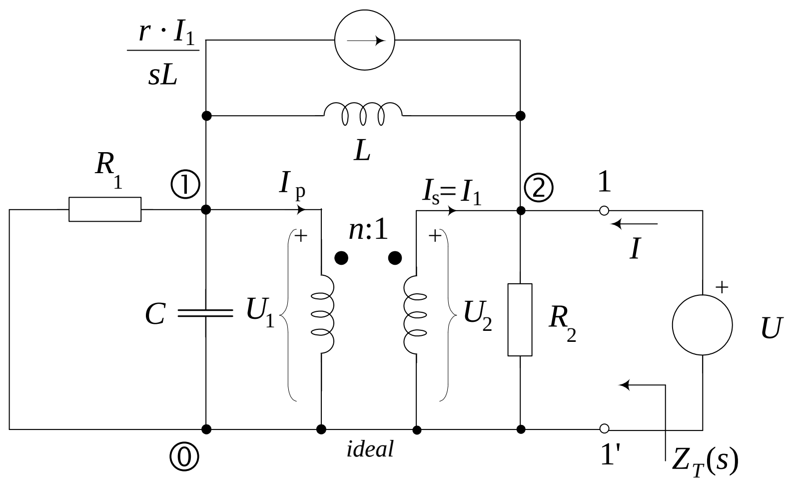
$$\Rightarrow U_2(2s-1)(s+4)=2(s+4)+2s(s+2)-U_2 4s\left[(s+2)^2+\frac{1}{2s}(s+2)\right]$$

$$\Rightarrow U_2\left[(2s-1)(s+4)+4s\left(s^2+4s+4+\frac{1}{2}+\frac{1}{s}\right)\right]=2(s+4)+2s(s+2)$$

$$\Rightarrow U_2\left[2s^2-s+8s-4+4s^3+16s^2+16s+2s+4\right]=2s+8+2s^2+4s$$

$$U_T(s)=U_2(s)=\frac{2s^2+6s+8}{4s^3+18s^2+25s}=\frac{2(s^2+3s+4)}{s(4s^2+18s+25)}$$ (1 bod)

c) Izračunavanje Theveninove impedancije pomoću jednadžbi napona čvorišta ① i ②



$$1)\ U_1\left(sC+\frac{1}{R_1}+\frac{1}{sL}\right)-U_2\frac{1}{sL}=-\frac{rI_s(s)}{sL}-I_p(s)$$

$$2)\ -U_1\frac{1}{sL}+U_2\left(\frac{1}{R_2}+\frac{1}{sL}\right)=\frac{rI_s(s)}{sL}+I_s(s)+I(s)$$

$$3)\ U_2=\frac{1}{n}U_1$$

$$4)\ I_s=nI_p$$

$$1)\ nU_2\left(sC+\frac{1}{R_1}+\frac{1}{sL}\right)-U_2\frac{1}{sL}=-\frac{rnI_p(s)}{sL}-I_p(s)$$

$$2)\ -nU_2\frac{1}{sL}+U_2\left(\frac{1}{R_2}+\frac{1}{sL}\right)=\frac{rnI_p(s)}{sL}+nI_p(s)+I(s)$$ (1 bod)

---

d) Theveninova impedancija $Z_T(s)=U_2(s)/I(s)$:
Uvrstimo vrijednosti: $C=1/2$, $L=2$, $R_1=R_2=1$ te $n=2$, $r=4$, $u_C(0)=1$, $i_L(0)=1$, $u_g(t)=S(t)$.

$$1)\ 2U_2\left(\frac{s}{2}+1+\frac{1}{2s}\right)-U_2\frac{1}{2s}=-\frac{4I_p(s)}{s}-I_p(s)\ \Rightarrow$$

$$U_2\left(s+2+\frac{1}{2s}\right)=-I_p(s)\left(\frac{4}{s}+1\right)\ \Rightarrow\ I_p(s)=\frac{-U_2\left(s+2+\frac{1}{2s}\right)}{\left(\frac{4}{s}+1\right)}$$

$$2)\ -U_2\frac{1}{s}+U_2\left(1+\frac{1}{2s}\right)=\frac{4I_p(s)}{s}+2I_p(s)+I(s)$$

$$U_2\left(1-\frac{1}{2s}\right)=2I_p(s)\left(1+\frac{2}{s}\right)+I(s)$$

$$1),\ 2)\Rightarrow U_2\left(1-\frac{1}{2s}\right)=2\frac{-U_2\left(s+2+\frac{1}{2s}\right)}{\left(\frac{4}{s}+1\right)}\left(1+\frac{2}{s}\right)+I(s)=\frac{-U_2 2[s+2+1/(2s)]}{(s+4)}(s+2)+I(s)$$

$$\Rightarrow U_2(2s-1)=2s\frac{-U_2 2(s+2+1/(2s))}{(s+4)}(s+2)+2sI(s)$$

$$\Rightarrow U_2(2s-1)(s+4)=-U_2 4s\left[(s+2)^2+(s+2)/(2s)\right]+2s(s+4)I(s)$$

$$\Rightarrow U_2\left[(2s-1)(s+4)+4s(s+2)^2+4s(1/2+1/s)\right]=2s(s+4)I(s)$$

$$\Rightarrow U_2\left[2s^2-s+8s-4+4s^3+16s^2+16s+2s+4\right]=\left[2s^2+8s\right]I(s)$$

$$Z_T(s)=\frac{U_2(s)}{I(s)}=\frac{2s^2+8s}{4s^3+18s^2+25s}=\frac{2(s+4)}{4s^2+18s+25}$$ (1 bod)

e) Da li je električni krug recipročan? Zašto?
NE jer ima strujno ovisni naponski izvor. (1 bod)

4. Za krug prikazan slikom isključiti otpor R i obzirom na priključnice 1–1' odrediti:

a) Theveninov napon u_T;

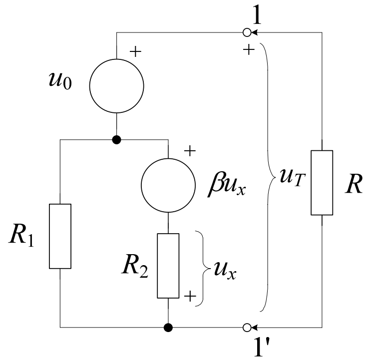
b) Theveninov otpor R_T;

c) iznos konstante β za koji je $R_T=R$;

d) napon u_x uz uključen otpor R [β iz zadatka c)].

e) Za koji iznos konstante β je $R_T=\infty$?

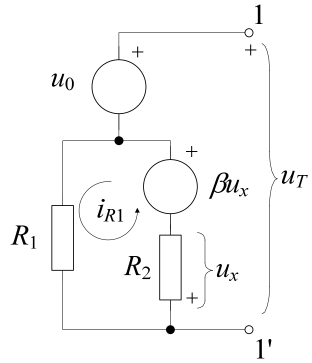
Zadano je: pobuda u_0=2 V i vrijednosti elemenata $R_1=R_2=2\ \Omega$, R=4 Ω.



Rješenje:

Isključiti otpor R i odrediti nadomjesni spoj po Theveninu obzirom na priključnice 1–1';

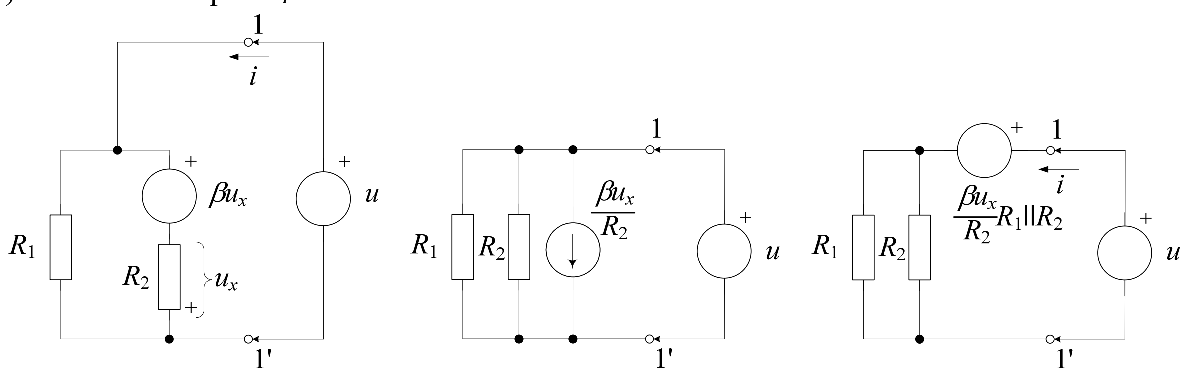
a) Theveninov napon u_T:



$$u_T = u_0 + i_{R1}\cdot R_1; \qquad i_{R1} = \frac{\beta u_x}{R_1+R_2}; \qquad u_x = i_{R1}\cdot R_2$$

$$i_{R1} = \frac{\beta\cdot i_{R1}R_2}{R_1+R_2} \Rightarrow i_{R1}\left(1-\frac{\beta\cdot R_2}{R_1+R_2}\right) = 0 \Rightarrow i_{R1} = 0 \Rightarrow u_T = u_0 = 2\text{V} \ \ (1\ \text{bod})$$

b) Theveninov otpor R_T:



$$-u_x = u - \beta\cdot u_x \Rightarrow u_x(1-\beta) = -u \Rightarrow u_x = \frac{u}{\beta - 1}$$

$$i=\frac{u}{R_1}+\frac{u-\beta\cdot u_x}{R_2}=\frac{u}{R_1}+\frac{u-\frac{\beta}{\beta-1}u}{R_2}\quad\Rightarrow\quad R_T=\frac{u}{i}=\frac{1}{\frac{1}{R_1}+\frac{\beta-1-\beta}{R_2(\beta-1)}}=\frac{R_1R_2}{R_2-R_1\frac{1}{\beta-1}}$$

$$R_T=\frac{R_1R_2(\beta-1)}{R_2(\beta-1)-R_1}=\frac{R_1R_2(1-\beta)}{R_2(1-\beta)+R_1}\ \text{(1 bod)}$$

c) odrediti iznos konstante β za koji je $R_T=R$.

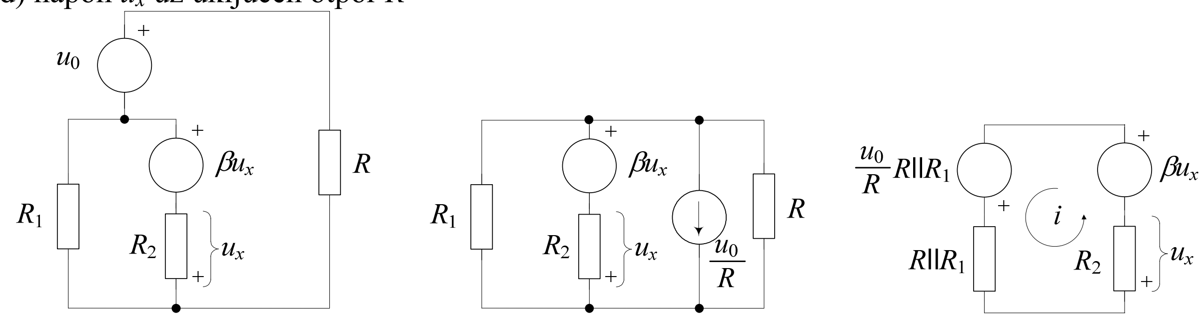
$$\left[R_2(1-\beta)+R_1\right]R_T=R_1R_2(1-\beta)\quad\Rightarrow\quad(1-\beta)(R_T-R_1)R_2=-R_1R_T$$

$$1-\beta=\frac{R_1R_T}{(R_1-R_T)R_2}\quad\Rightarrow\quad\beta=1-\frac{R_1R_T}{(R_1-R_T)R_2}$$

$$\beta=1-\frac{2\cdot4}{(2-4)\cdot2}=1+\frac{4}{2}=3\ \text{(1 bod)}$$

Provjera: $$R_T=\frac{R_1R_2(1-\beta)}{R_2(1-\beta)+R_1}=\frac{2\cdot2\cdot(1-3)}{2\cdot(1-3)+2}=\frac{2\cdot(-2)}{-2+1}=4\,\Omega$$

d) napon u_x uz uključen otpor R



$$\frac{u_0}{R}=\frac{2}{4}=\frac{1}{2}\text{A}$$

$$\frac{u_0}{R}\cdot\frac{RR_1}{R+R_1}+\beta\cdot u_x=i\cdot\left(R_2+\frac{RR_1}{R+R_1}\right)$$

$$u_0\cdot R_1+\beta\cdot u_x(R+R_1)=i\cdot\left[R_2(R+R_1)+RR_1\right]$$

$$u_x=i\cdot R_2$$

$$u_0\cdot R_1+\beta\cdot u_x(R+R_1)=\frac{u_x}{R_2}\cdot\left[R_2(R+R_1)+RR_1\right]$$

$$u_0\cdot R_1=(1-\beta)\cdot u_x(R+R_1)+u_x\frac{RR_1}{R_2}$$

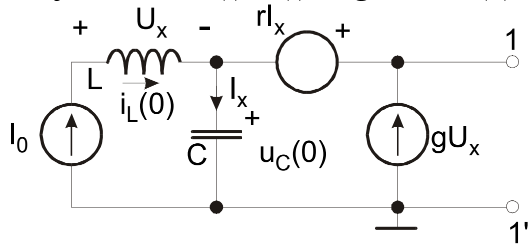
$$u_x=\frac{u_0\cdot R_1}{(1-\beta)(R+R_1)+\frac{RR_1}{R_2}}=\frac{2\cdot2}{(1-3)\cdot(4+2)+\frac{4\cdot2}{2}}=-\frac{1}{2}V\ \text{(1 bod)}$$

e) Za koji iznos konstante β je $R_T=\infty$? Za $\beta=2$. (1 bod)

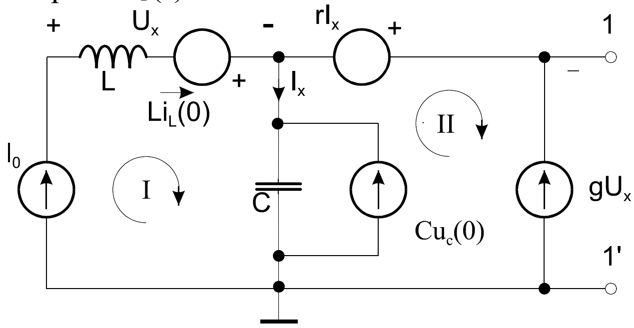
$$R_T=\frac{R_1R_2(1-\beta)}{R_2(1-\beta)+R_1}=\frac{2\cdot2\cdot(1-2)}{2\cdot(1-2)+2}=\frac{-4}{-2+2}=\frac{-4}{0}=\infty$$

DEKANSKI ISPIT IZ ELEKTRIČNIH KRUGOVA Jesen 2014 – Rješenja

1. Za mrežu prikazanu slikom odrediti $U_T(s)$ i $Z_T(s)$ nadomjesne sheme po Teveninu s obzirom na stezaljke 1–1' ako je zadano: $i_0(t)=S(t)$, $r=g=1/2$, $u_C(0)=i_L(0)=1$, $L=C=1$.

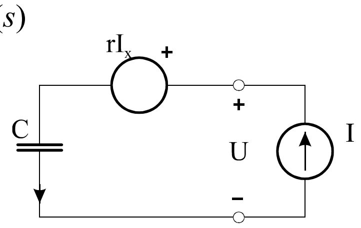


Rješenje: a) Teveninov napon: $U_T(s)$



$$I_I = I_0$$
$$I_{II} = -gU_x$$
$$U_x = I_0 sL - Li_L(0)$$
$$I_x = I_0 + gU_x$$
$$U_1 = \left[I_x + Cu_c(0)\right]\frac{1}{sC}$$
$$\underline{U_T = U_1 + rI_x}$$
$$I_x = I_0 + gI_0 sL - gLi_L(0)$$
$$I_x = I_0(1+gsL) - gLi_L(0)$$
$$U_1 = \frac{I_x}{sC} + \frac{u_C(0)}{s}$$
$$U_T = \frac{I_x}{sC} + \frac{u_C(0)}{s} + rI_x = I_x\left(r + \frac{1}{sC}\right) + \frac{u_C(0)}{s}$$
$$U_T = \left[I_0 + I_0 gsL - gLi_L(0)\right]\left(r + \frac{1}{sC}\right) + \frac{u_C(0)}{s}$$
$$U_T = I_0 r + I_0 rgsL - grLi_L(0) + \frac{I_0}{sC} + I_0 g\frac{L}{C} - gLi_L(0)\frac{1}{sC} + \frac{u_C(0)}{s}$$
$$= \frac{1}{2s} + \frac{1}{4} - \frac{1}{4} + \frac{1}{s^2} + \frac{1}{2s} - \frac{1}{2s} + \frac{1}{s} = \frac{1}{s^2} + \frac{3}{2s} = \frac{3s+2}{2s^2}$$

b) Theveninova impedancija: $Z_T(s)$



$$I = I_x$$

$$U = I\frac{1}{sC} + rI$$

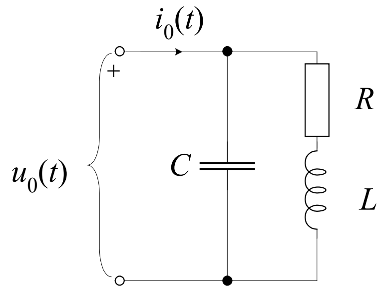
$$Z_T = \frac{U}{I} = \frac{1}{sC} + r = \frac{1}{s} + \frac{1}{2} = \frac{2+s}{2s}$$

OSTALO

PRVI MEĐUISPIT IZ ELEKTRIČNIH KRUGOVA

1. Na priključnice dvopola sastavljenog od paralelnoga spoja kapaciteta C=1 nF i serijske kombinacije otpora R=1000 Ω i induktiviteta L=1 mH, djeluje strujni izvor $i_0(t)=\delta(t)$. Normirati elemente dvopola na frekvenciju $\omega_0=10^6$ rad/s i na otpor R_0=1000 Ω. Odrediti napon $u(t)$ na priključnicama tog dvopola.

Rješenje: Normiranje elemenata zatim primjena Laplaceove transformacije



$$R_n = \frac{R}{R_0} = \frac{1000}{1000} = 1$$

$$Z_{C_n} = \frac{1}{sCR_0} = \frac{1}{\frac{s}{\omega_0}\underbrace{\omega_0 CR_0}_{C_n}} \qquad \Rightarrow \qquad C_n = \omega_0 CR_0 = 10^6 \cdot 10^{-9} \cdot 10^3 = 1$$

$$Z_{L_n} = \frac{sL}{R_0} = \frac{s}{\omega_0} \cdot \underbrace{\frac{\omega_0 L}{R_0}}_{L_n} \qquad \Rightarrow \qquad L_n = \frac{\omega_0 L}{R_0} = \frac{10^6 \cdot 10^{-3}}{10^3} = 1$$

$$Z_n(s) = \frac{\frac{1}{sC}(R+sL)}{\frac{1}{sC}+R+sL} = \frac{R+sL}{1+sCR+s^2LC}$$

Uz uvrštene normirane vrijednosti elemenata impedancija dvopola glasi:

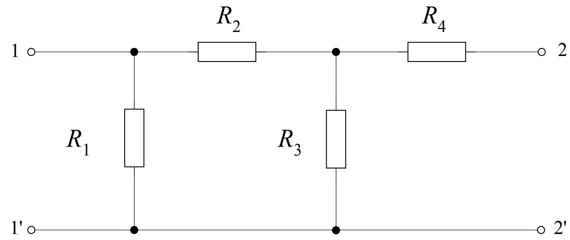
$$Z_n(s) = \frac{1+s}{s^2+s+1} = \frac{s+\frac{1}{2}+\frac{1}{2}}{\left(s+\frac{1}{2}\right)^2+\frac{3}{4}}$$

Odnosno:

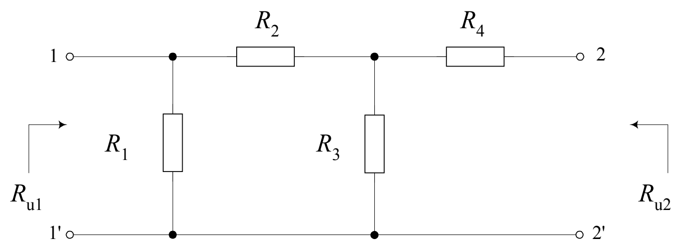
$$U_0(s) = I_0(s)\cdot Z_n(s) = \frac{s+\frac{1}{2}}{\left(s+\frac{1}{2}\right)^2+\frac{3}{4}} + \frac{\frac{1}{2}\cdot\frac{\sqrt{3}}{2}\cdot\frac{2}{\sqrt{3}}}{\left(s+\frac{1}{2}\right)^2+\frac{3}{4}} = \frac{s+\frac{1}{2}}{\left(s+\frac{1}{2}\right)^2+\frac{3}{4}} + \frac{1}{\sqrt{3}}\frac{\frac{\sqrt{3}}{2}}{\left(s+\frac{1}{2}\right)^2+\frac{3}{4}}$$

pa je $u_0(t) = \mathscr{L}^{-1}\left[U_0(s)\right] = e^{-\frac{t}{2}}\left(\cos\frac{\sqrt{3}}{2}t + \frac{1}{\sqrt{3}}\sin\frac{\sqrt{3}}{2}t\right)S(t)$

2. Za krug prikazan slikom odrediti otpor R_1 tako da ukupni otpor gledan sa priključnica 1-1' bude jednak otporu gledanome s priključnica 2-2'. Zadano je: R_2= 40 Ω, R_3=30 Ω i R_4=20 Ω.



<u>Rješenje:</u>



$$R_{u1}=\frac{1}{\dfrac{1}{R_1}+\dfrac{1}{R_2+R_3}}=\frac{R_1(R_2+R_3)}{R_1+R_2+R_3}=\frac{R_1\cdot 70}{R_1+70}$$

$$R_{u2}=R_4+\frac{1}{\dfrac{1}{R_3}+\dfrac{1}{R_1+R_2}}=R_4+\frac{R_3(R_1+R_2)}{R_1+R_2+R_3}=20+\frac{30(R_1+40)}{R_1+70}$$

$$R_{u1}=R_{u2}$$

$$\frac{R_1\cdot 70}{R_1+70}=20+\frac{30(R_1+40)}{R_1+70}$$

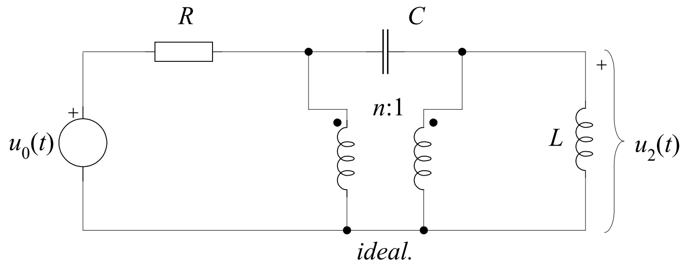
$$\frac{R_1\cdot 70}{R_1+70}=\frac{20(R_1+70)+30(R_1+40)}{R_1+70}$$

$$R_1\cdot 70-R_1\cdot 50=20\cdot 70+30\cdot 40$$

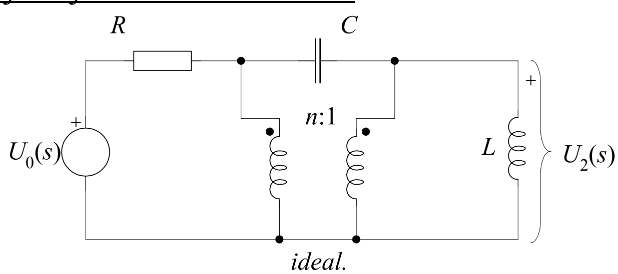
$$R_1=\frac{20\cdot 70+30\cdot 40}{20}=130\,\Omega$$

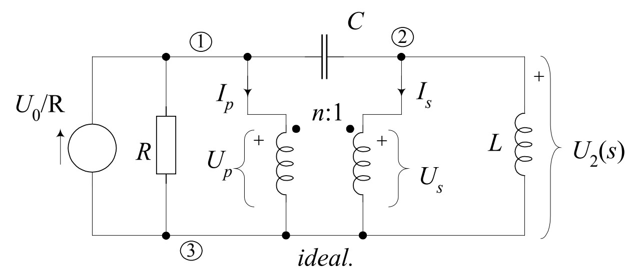
DRUGI MEĐUISPIT IZ ELEKTRIČNIH KRUGOVA

1. Za krug prikazan slikom izračunati odziv $u_2(t)$ ako je zadana pobuda $u_0(t)=S(t)$, prijenosni omjer $n=1/2$, a normirane vrijednosti elemenata su: $R=2$, $L=1$ i $C=1$. Početni uvjeti su jednaki nuli.



Rješenje: Jednadžbe čvorišta





$$(1) \qquad \frac{U_0(s)}{R} - I_p(s) = \left(\frac{1}{R} + sC\right)U_1(s) - sCU_2(s) \qquad\qquad U_p(s) = nU_s(s) \Rightarrow U_1(s) = nU_2(s)$$

$$(2) \qquad -I_s(s) = -sCU_1(s) + \left(sC + \frac{1}{sL}\right)U_2(s) \qquad\qquad I_p(s) = -\frac{1}{n}I_s(s)$$

$$(1) \qquad \frac{U_0}{R} + \frac{I_s}{n} = \left(\frac{1}{R} + sC\right)nU_2 - sCU_2$$

$$(2) \qquad -I_s(s) = -sCnU_2 + \left(sC + \frac{1}{sL}\right)U_2(s) \qquad \Rightarrow \qquad I_s(s) = \left(sCn - sC - \frac{1}{sL}\right)U_2(s)$$

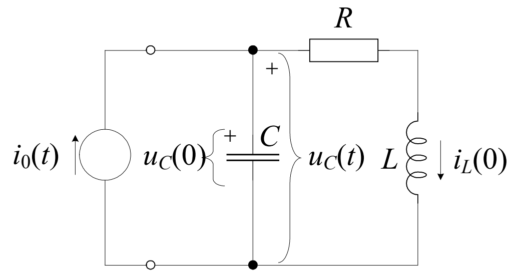
$$\frac{U_0}{R} = \left(-sC + \frac{1}{n}\left(sC + \frac{1}{sL}\right)\right)U_2 + \left(\frac{1}{R} + sC\right)nU_2 - sCU_2$$

$$\frac{U_0}{R} = \left(\frac{1}{n}\left(sC + \frac{1}{sL}\right) + \left(\frac{1}{R} + sC\right)n - 2sC\right)U_2$$

$$U_2(s) = \frac{nU_0}{R\left(\left(sC + \frac{1}{sL}\right) + \left(\frac{1}{R} + sC\right)n^2 - 2nsC\right)} = \frac{snU_0}{RC\left(s^2\left(1 + n^2 - 2n\right) + s\frac{n^2}{RC} + \frac{1}{LC}\right)} = \frac{2}{2s^2 + s + 8}$$

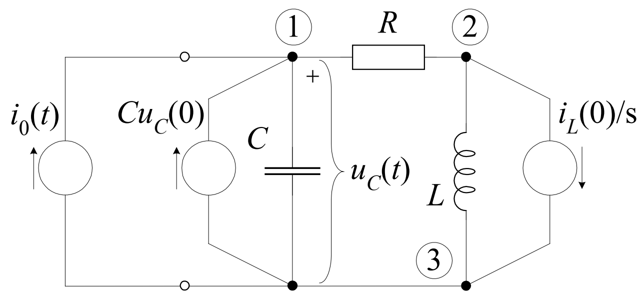
$$U_2(s) = \frac{1}{s^2 + \frac{s}{2} + 4} = \frac{1}{\left(s + \frac{1}{4}\right)^2 + \frac{63}{16}} = \frac{4}{\sqrt{63}} \cdot \frac{\frac{\sqrt{63}}{4}}{\left(s + \frac{1}{4}\right)^2 + \left(\frac{\sqrt{63}}{4}\right)^2} \qquad u_2(t) = \frac{4}{3\sqrt{7}} \cdot e^{-t/4} \sin\left(\frac{3\sqrt{7}}{4}\right)S(t)$$

3. Odrediti odziv $u_C(t)$ mreže prema slici za $t \geq 0$, ako su zadane normirane vrijednosti elemenata: $R=1$, $C=2$ i $L=1$, početni uvjeti u mreži $u_C(0)=1/5$ i $i_L(0)=-1/5$, a pobuda je $i_0(t)=S(t)$.



Rješenje:

Za $t \geq 0$: Primjena Laplaceove transformacije → Jednadžbe čvorišta



$$I_0(s)+Cu_C(0)=\left(sC+\frac{1}{R}\right)U_1(s)-\frac{1}{R}U_2(s)$$

$$\frac{-i_L(0)}{s}=-\frac{1}{R}U_1(s)+\left(\frac{1}{sL}+\frac{1}{R}\right)U_2(s) \qquad \Rightarrow \qquad U_2(s)=\frac{sL}{(R+sL)}U_1(s)-i_L(0)\frac{RL}{(R+sL)}$$

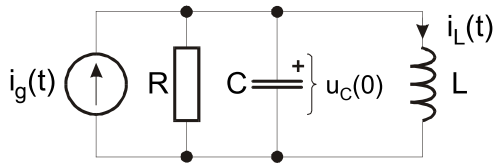
$$U_1(s)=\frac{(R+sL)I_0(s)+C(R+sL)u_C(0)-Li_L(0)}{s^2LC+sRC+1}=\frac{(1+s)+2s(1+s)u_C(0)-si_L(0)}{s(2s^2+2s+1)}$$

$$U_1(s)=\frac{1}{5s}\cdot\frac{5(1+s)+2s(1+s)+s}{2s^2+2s+1}=\frac{1}{5s}\cdot\frac{2s^2+8s+5}{2s^2+2s+1}=\frac{1}{5}\left(\frac{5}{s}-4\frac{s+0,5}{(s+0,5)^2+0,25}+2\frac{0,5}{(s+0,5)^2+0,25}\right)$$

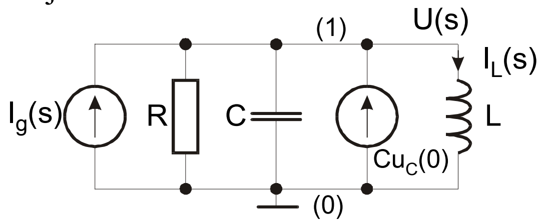
$$u_1(t)=\frac{1}{5}\left(5-4e^{-0,5t}\cos(0,5t)+2e^{-0,5t}\sin(0,5t)\right)S(t)$$

ZAVRŠNI ISPIT IZ ELEKTRIČNIH KRUGOVA

1. U električnom krugu na slici odrediti odziv struje kroz induktivitet $i_L(t)$ ako su zadane normalizirane vrijednosti elemenata: C=2, R=1/2, L=1/4, početni uvjeti $i_L(0)$=0, $u_C(0)$=1 te struja pobude $i_g(t)=e^{-2t}\cdot S(t)$.



Rješenje:
Primjena Laplaceove transformacije:



$$U(s)\cdot\left(sC+\frac{1}{R}+\frac{1}{sL}\right)-C\cdot u_C(0)-I_g(s)=0$$

$$U(s)=\frac{I_g(s)+C\cdot u_C(0)}{sC+\frac{1}{R}+\frac{1}{sL}}$$

$$I_L(s)=U(s)\cdot\frac{1}{sL}=\frac{I_g(s)+C\cdot u_C(0)}{s^2LC+s\frac{L}{R}+1}$$

$$i_g(t)=e^{-2t}\cdot S(t)\ \Rightarrow\ I_g(s)=\frac{1}{s+2}$$

$$I_L(s)=\frac{\frac{1}{s+2}+2}{\frac{s^2}{2}+\frac{s}{2}+1}=\frac{2(1+2s+4)}{(s+2)(s^2+s+2)}=\frac{4s+10}{(s+2)(s^2+s+2)}$$

Slijedi rastav na parcijalne razlomke izraza $\dfrac{4s+10}{(s+2)(s^2+s+2)}=\dfrac{A}{s+2}+\dfrac{Bs+C}{s^2+s+2}$

$$As^2+As+2A+Bs^2+2Bs+Cs+2C=4s+10$$

$$(A+B)s^2+(A+2B+C)s+(2A+2C)=4s+10$$

(1) $\quad A+B=0$

(2) $\quad A+2B+C=4$

(3) $\quad 2(A+C)=10$

(1)$\Rightarrow\ B=-A$, (3)$\Rightarrow\ C=5-A$, (2) $\Rightarrow\ A+2B+5-A=4$

$$\Rightarrow \quad 2B=-1 \;\Rightarrow\quad B=-\frac{1}{2},\; A=\frac{1}{2},\; C=5-A=5-\frac{1}{2}=\frac{10}{2}-\frac{1}{2}=\frac{9}{2}$$

$$\Rightarrow I_L(s)=\frac{4s+10}{(s+2)(s^2+s+2)}=\frac{1}{2}\cdot\frac{1}{s+2}-\frac{1}{2}\cdot\frac{s}{s^2+s+2}+\frac{9}{2}\cdot\frac{1}{s^2+s+2}=$$

$$=\frac{1}{2}\cdot\frac{1}{s+2}-\frac{1}{2}\cdot\frac{s+\frac{1}{2}-\frac{1}{2}}{\left(s+\frac{1}{2}\right)^2+\left(\frac{\sqrt{7}}{2}\right)^2}+\frac{9}{2}\cdot\frac{1}{\left(s+\frac{1}{2}\right)^2+\left(\frac{\sqrt{7}}{2}\right)^2}=$$

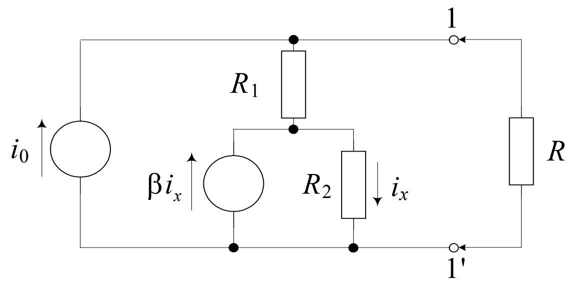
$$=\frac{1}{2}\cdot\frac{1}{s+2}-\frac{1}{2}\cdot\frac{s+\frac{1}{2}}{\left(s+\frac{1}{2}\right)^2+\left(\frac{\sqrt{7}}{2}\right)^2}+\frac{19}{4}\cdot\frac{\frac{2}{\sqrt{7}}\cdot\frac{\sqrt{7}}{2}}{\left(s+\frac{1}{2}\right)^2+\left(\frac{\sqrt{7}}{2}\right)^2}=$$

$$=\frac{1}{2}\cdot\frac{1}{s+2}-\frac{1}{2}\cdot\frac{s+\frac{1}{2}}{\left(s+\frac{1}{2}\right)^2+\left(\frac{\sqrt{7}}{2}\right)^2}+\frac{19}{2\sqrt{7}}\cdot\frac{\frac{\sqrt{7}}{2}}{\left(s+\frac{1}{2}\right)^2+\left(\frac{\sqrt{7}}{2}\right)^2}=$$

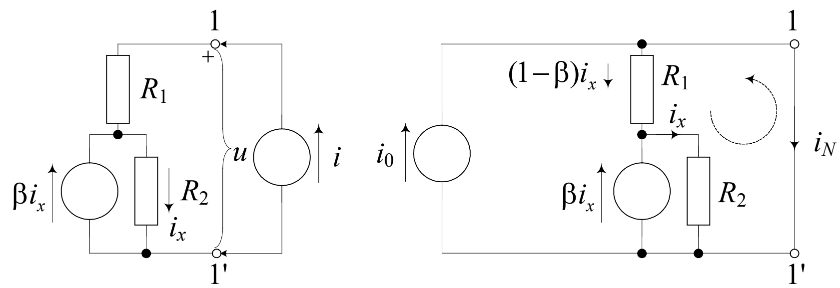
$$i_L(t)=\mathscr{L}\{I_L(s)\} \quad\Rightarrow\quad \underline{i_L(t)=\left(\frac{1}{2}e^{-2t}-\frac{1}{2}e^{-\frac{t}{2}}\cos\frac{\sqrt{7}}{2}t+\frac{19}{2\sqrt{7}}e^{-\frac{t}{2}}\sin\frac{\sqrt{7}}{2}t\right)\cdot S(t)}$$

PRVI MEĐUISPIT IZ ELEKTRIČNIH KRUGOVA

1. Za krug prikazan slikom:
   a) isključiti otpor R i odrediti nadomjesni spoj po Nortonu obzirom na priključnice 1–1';
   b) odrediti iznos konstante β za koji je $Y_N(s)=1/R$.
   c) uz uključen otpor R primjenom transformacija izvora i Kirchhoffovih zakona odrediti struju i_x;
   Zadana je pobuda i_0=2 A, i vrijednosti elemenata $R_1=R_2=4\ \Omega$, $R=16\ \Omega$.



Rješenje:
a) isključiti otpor R i odrediti nadomjesni spoj po Nortonu obzirom na priključnice 1–1';



Nortonova admitancija: $i_x=\beta i_x+i \Rightarrow i_x(1-\beta)=i \Rightarrow i_x=\dfrac{i}{1-\beta}$, $u=iR_1+\dfrac{i}{1-\beta}R_2 \Rightarrow Z_T=\dfrac{u}{i}=R_1+\dfrac{R_2}{1-\beta}$,

$$Y_N=\frac{1}{Z_T}=\frac{1}{R_1+\dfrac{R_2}{1-\beta}}=\frac{1-\beta}{R_1(1-\beta)+R_2}$$

Nortonova struja: $i_N=(\beta-1)i_x+i_0$,

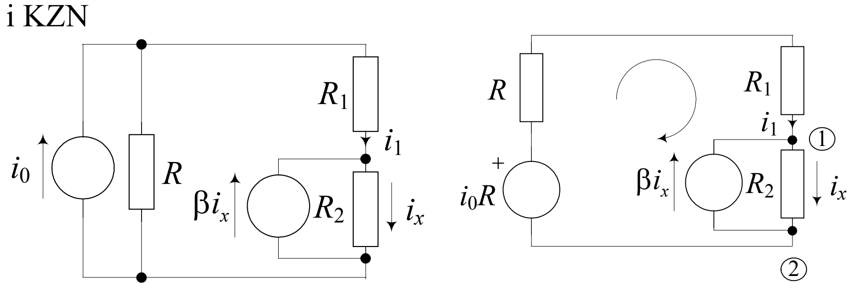
$$(1-\beta)i_xR_1+i_xR_2=0 \;\Rightarrow\; (\beta-1)i_xR_1=i_xR_2 \Rightarrow i_x=0 \Rightarrow i_N=i_0=2A$$

b) odrediti iznos konstante β za koji je $Y_N(s)=1/R$.

$$Z_T=R_1+\frac{R_2}{1-\beta}=R \Rightarrow 4+\frac{4}{1-\beta}=16 \Rightarrow \frac{4}{1-\beta}=12 \Rightarrow \frac{1}{1-\beta}=3 \Rightarrow 1-\beta=\frac{1}{3} \Rightarrow \beta=1-\frac{1}{3}=\frac{2}{3},$$

konačno je: $Y_N=1/Z_T$= 1/16.

c) Jednadžbe KZS i KZN



KZS za čvor (1): $0 = -i_1 - \beta i_x + i_x \quad \Rightarrow \quad i_1 = -(\beta - 1) i_x = (1-\beta) i_x$

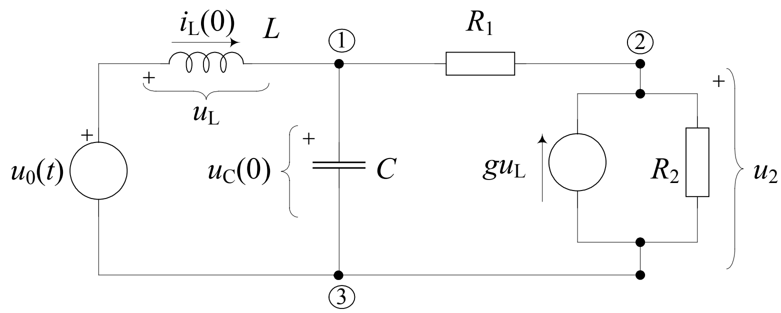
KZN za petlju: $i_1 (R + R_1) + i_x R_2 = i_0 R$

$$\Rightarrow \quad (1-\beta) i_x (R + R_1) + i_x R_2 = i_0 R \quad \Rightarrow$$

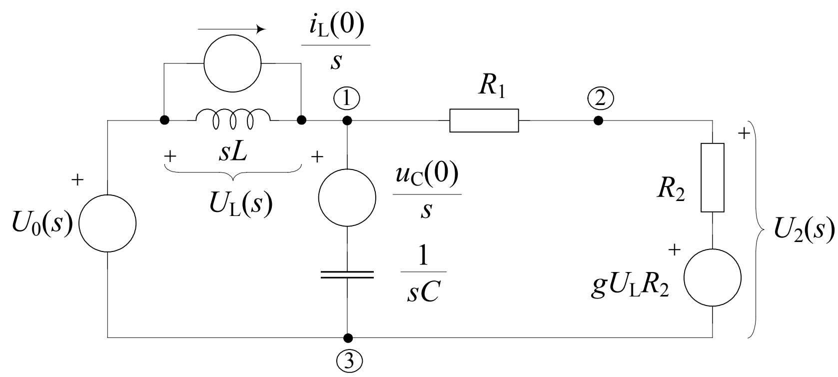
$$i_x = \frac{i_0 R}{(1-\beta)(R+R_1)+R_2}$$

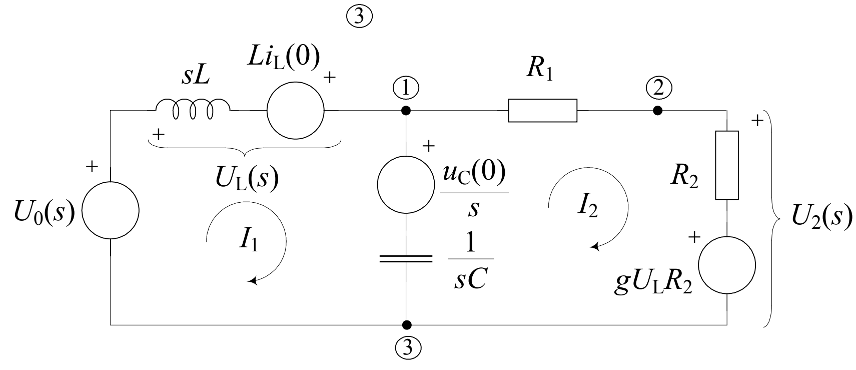
$$i_x = \frac{2 \cdot 16}{(1-2/3)(16+4)+4} = \frac{32}{(1/3)\cdot 20 + 4} = \frac{3 \cdot 32}{20+12} = \frac{96}{32} = 3A$$

2. Za krug prikazan slikom napisati jednadžbe petlji. Izračunati napon $U_2(s)$, ako je zadana pobuda $u_0(t) = 2S(t)$, konstanta $g=1$, normirane vrijednosti elemenata su: $R_1 = R_2 = 1$, $L=1$ i $C=1$, a početni uvjeti su $u_C(0)=1$, $i_L(0)=1$.



Rješenje: Primjena Laplaceove transformacije





Jednadžbe petlji:

(1) $$I_1(s) sL + [I_1(s) - I_2(s)] \frac{1}{sC} = U_0(s) + L i_L(0) - \frac{u_C(0)}{s}$$

(2) $$-[I_1(s) - I_2(s)] \frac{1}{sC} + I_2(s) R_1 + I_2(s) R_2 = -g U_L(s) R_2 + \frac{u_C(0)}{s}$$

$$U_L(s) = I_1(s)sL - Li_L(0)$$
$$U_2(s) = I_2(s)R_2 + gU_L(s)R_2 = I_2(s)R_2 + g[I_1(s)sL - Li_L(0)]R_2$$

$$(1) \qquad I_1(s)sL + [I_1(s) - I_2(s)]\frac{1}{sC} = U_0(s) + Li_L(0) - \frac{u_C(0)}{s}$$
$$(2) \qquad -[I_1(s) - I_2(s)]\frac{1}{sC} + I_2(s)R_1 + I_2(s)R_2 = -g[I_1(s)sL - Li_L(0)]R_2 + \frac{u_C(0)}{s}$$

$$(1) \qquad I_1(s)\left(sL + \frac{1}{sC}\right) - I_2(s)\frac{1}{sC} = U_0(s) + Li_L(0) - \frac{u_C(0)}{s}$$
$$(2) \qquad -I_1(s)\left(\frac{1}{sC} - gR_2sL\right) + I_2(s)\left(\frac{1}{sC} + R_1 + R_2\right) = gR_2Li_L(0) + \frac{u_C(0)}{s}$$

Uvrstimo vrijednosti elemenata:

$$(1) \qquad I_1(s)\left(s + \frac{1}{s}\right) - I_2(s)\frac{1}{s} = \frac{2}{s} + 1 - \frac{1}{s} = 1 + \frac{1}{s} \Big/ \cdot s$$
$$(2) \qquad -I_1(s)\left(\frac{1}{s} - s\right) + I_2(s)\left(\frac{1}{s} + 2\right) = 1 + \frac{1}{s} \Big/ \cdot s$$

$$(1) \qquad I_1(s)(s^2 + 1) - I_2(s) = s + 1$$
$$(2) \qquad I_1(s)(s^2 - 1) + I_2(s)(2s + 1) = s + 1$$

$$\Delta = \begin{vmatrix} s^2+1 & -1 \\ s^2-1 & 2s+1 \end{vmatrix} = (s^2+1)(2s+1) + s^2 - 1 = 2s^3 + 2s + s^2 + 1 + s^2 - 1 = 2s^3 + 2s^2 + 2s$$
$$\Delta_1 = \begin{vmatrix} s+1 & -1 \\ s+1 & 2s+1 \end{vmatrix} = (s+1)(2s+1) + s + 1 = 2s^2 + 2s + s + 1 + s + 1 = 2s^2 + 4s + 2$$
$$\Delta_2 = \begin{vmatrix} s^2+1 & s+1 \\ s^2-1 & s+1 \end{vmatrix} = (s^2+1)(s+1) - (s^2-1)(s+1) = (s+1)(s^2+1-s^2+1) = 2(s+1)$$
$$I_1 = \frac{\Delta_1}{\Delta} = \frac{2s^2+4s+2}{2s^3+2s^2+2s} = \frac{2(s^2+2s+1)}{2(s^3+s^2+s)} = \frac{s^2+2s+1}{s^3+s^2+s}$$
$$I_2 = \frac{\Delta_2}{\Delta} = \frac{2(s+1)}{2(s^3+s^2+s)} = \frac{s+1}{s^3+s^2+s}$$

$$U_2(s) = I_2(s)R_2 + g[I_1(s)sL - Li_L(0)]R_2 = \frac{s+1}{s^3+s^2+s} + \frac{s^2+2s+1}{s^3+s^2+s}s - 1$$
$$U_2(s) = \frac{s+1+s^3+2s^2+s-s^3-s^2-s}{s^3+s^2+s} = \frac{s^2+s+1}{s^3+s^2+s} = \frac{s^2+s+1}{s(s^2+s+1)} = \frac{1}{s}$$

Rastav na parcijalne razlomke:

$$U_2(s)=\frac{-1}{(s+1)(s+2)}=\frac{A}{s+1}+\frac{B}{s+2}=\frac{A(s+2)+B(s+1)}{(s+1)(s+2)}=\frac{(A+B)s+(2A+B)}{(s+1)(s+2)}$$

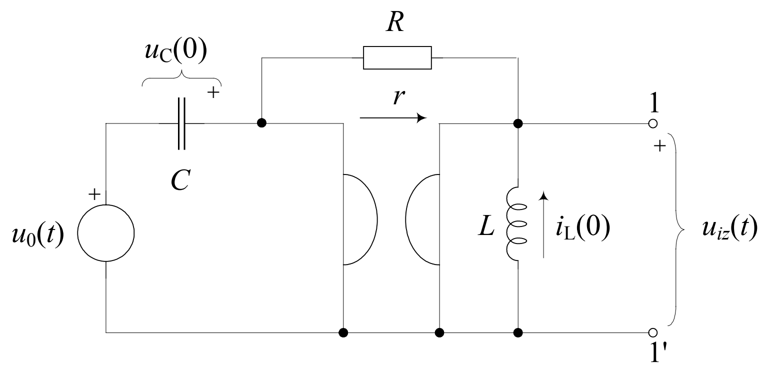
$$A+B=0 \Rightarrow B=-A$$

$$2A+B=-1 \Rightarrow 2A-A=-1 \Rightarrow A=-1 \Rightarrow B=1$$

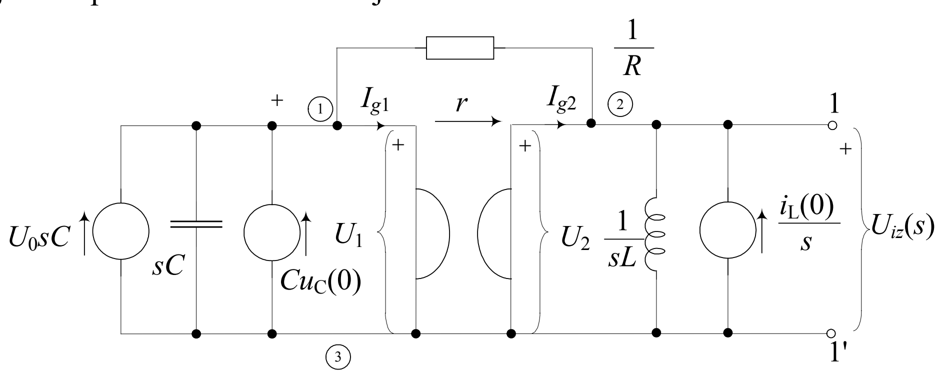
$$U_2(s)=\frac{-1}{s+1}+\frac{1}{s+2}$$

Konačno je: $u_2(t)=\left(-e^{-t}+e^{-2t}\right)\cdot S(t)$

5. Za krug prikazan slikom odrediti napon $u_{iz}(t)$ na priključnicama 1-1', koristeći postupak jednadžbi čvorišta, ako je pobuda $u_0(t)= S(t)$. Zadane su normirane vrijednosti elemenata: R=0.5, L=1, C=1, r=1 i početni uvjeti $u_C(0)$=2, $i_L(0)$=1.



Rješenje: Primjena Laplaceove transformacije



(1) $\quad U_1\left(sC+\frac{1}{R}\right)-U_2\frac{1}{R}=U_0(s)sC+Cu_C(0)-I_{g1} \qquad I_{g1}=-U_2\frac{1}{r}$

(2) $\quad -U_1\frac{1}{R}+U_2\left(\frac{1}{R}+\frac{1}{sL}\right)=\frac{i_L(0)}{s}+I_{g2} \qquad I_{g2}=-U_1\frac{1}{r}$

---

(1) $\quad U_1\left(sC+\frac{1}{R}\right)-U_2\left(\frac{1}{R}+\frac{1}{r}\right)=U_0(s)sC+Cu_C(0)$

(2) $$-U_1\left(\frac{1}{R}-\frac{1}{r}\right)+U_2\left(\frac{1}{R}+\frac{1}{sL}\right)=\frac{i_L(0)}{s}$$

---

(2) $\Rightarrow$ $$U_1=\frac{U_2\left(\frac{1}{R}+\frac{1}{sL}\right)-\frac{i_L(0)}{s}}{\frac{1}{R}-\frac{1}{r}}$$

(1) $$\frac{U_2\left(\frac{1}{R}+\frac{1}{sL}\right)-\frac{i_L(0)}{s}}{\frac{1}{R}-\frac{1}{r}}\left(sC+\frac{1}{R}\right)-U_2\left(\frac{1}{R}+\frac{1}{r}\right)=U_0sC+Cu_C(0) \qquad \Big/\cdot\left(\frac{1}{R}-\frac{1}{r}\right)$$

$$\left[U_2\left(\frac{1}{R}+\frac{1}{sL}\right)-\frac{i_L(0)}{s}\right]\left(sC+\frac{1}{R}\right)-U_2\left(\frac{1}{R}+\frac{1}{r}\right)\left(\frac{1}{R}-\frac{1}{r}\right)=\left[U_0sC+Cu_C(0)\right]\left(\frac{1}{R}-\frac{1}{r}\right)$$

$$U_2\left(\frac{1}{R}+\frac{1}{sL}\right)\left(sC+\frac{1}{R}\right)-U_2\left(\frac{1}{R^2}-\frac{1}{r^2}\right)=\left[U_0sC+Cu_C(0)\right]\left(\frac{1}{R}-\frac{1}{r}\right)+\frac{i_L(0)}{s}\left(sC+\frac{1}{R}\right)$$

$$U_2\left(\frac{sC}{R}+\frac{sC}{sL}+\frac{1}{R^2}+\frac{1}{RsL}-\frac{1}{R^2}+\frac{1}{r^2}\right)=\left[U_0sC+Cu_C(0)\right]\left(\frac{1}{R}-\frac{1}{r}\right)+\frac{i_L(0)}{s}\left(sC+\frac{1}{R}\right)$$

$$U_2(s)=\frac{\left[U_0sC+Cu_C(0)\right]\left(\frac{1}{R}-\frac{1}{r}\right)+\frac{i_L(0)}{s}\left(sC+\frac{1}{R}\right)}{\frac{sC}{R}+\frac{sC}{sL}+\frac{1}{RsL}+\frac{1}{r^2}}$$

$$U_2(s)=\frac{\left[U_0sC+Cu_C(0)\right]\left(\frac{1}{R}-\frac{1}{r}\right)+\frac{i_L(0)}{s}\left(sC+\frac{1}{R}\right)}{\frac{sC}{R}+\frac{sC}{sL}+\frac{1}{RsL}+\frac{1}{r^2}}$$

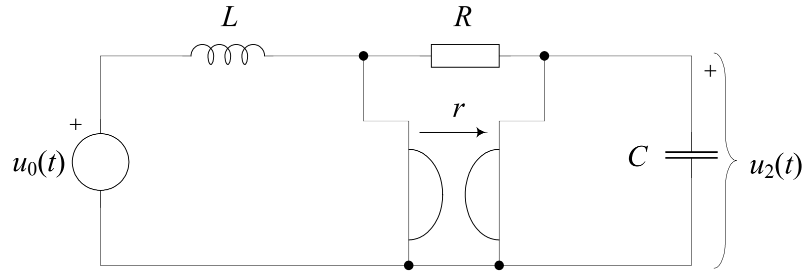
$$U_2(s)=\frac{\left[\frac{1}{s}s+1\cdot 2\right](2-1)+\frac{1}{s}(s+2)}{2s+1+\frac{2}{s}+1}=\frac{3+\frac{2}{s}+1}{2s+2+\frac{2}{s}}=\frac{4+\frac{2}{s}}{2s+2+\frac{2}{s}}=\frac{2+\frac{1}{s}}{s+1+\frac{1}{s}}=\frac{2s+1}{s^2+s+1}$$

$$U_2(s)=\frac{2s+1}{s^2+s+1}=2\frac{s+\frac{1}{2}}{\left(s+\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}$$

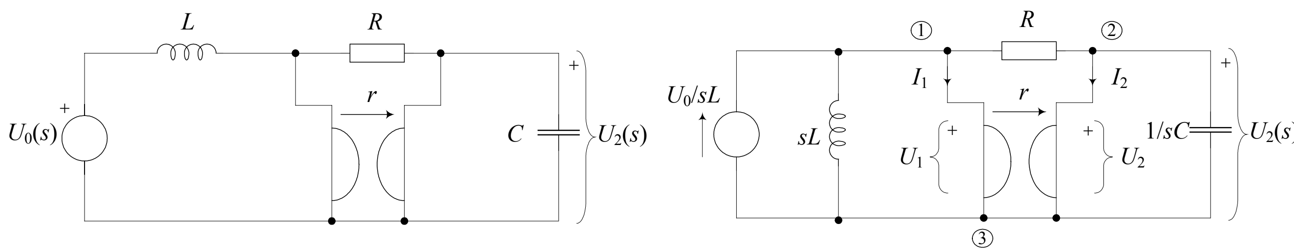
$$u_2(t)=2e^{-\frac{t}{2}}\cos\left(\frac{\sqrt{3}}{2}\right)\cdot S(t)$$

DRUGI MEĐUISPIT IZ ELEKTRIČNIH KRUGOVA

1. Za krug prikazan slikom izračunati odziv $u_2(t)$ ako je zadana pobuda $u_0(t)=2\delta(t)$, konstanta giratora r=2, a normirane vrijednosti elemenata su: R=1, L=2 i C=1/2. Početni uvjeti su jednaki nuli.



<u>Rješenje: Jednadžbe čvorišta</u>



(1) $$\frac{U_0(s)}{sL}-I_1(s)=\left(\frac{1}{sL}+\frac{1}{R}\right)U_1(s)-\frac{1}{R}U_2(s)$$ $$U_1(s)=rI_2(s)\quad\Rightarrow\quad I_2(s)=\frac{1}{r}U_1(s)$$

(2) $$-I_2(s)=-\frac{1}{R}U_1(s)+\left(sC+\frac{1}{R}\right)U_2(s)$$ $$U_2(s)=-rI_1(s)\quad\Rightarrow\quad I_1(s)=-\frac{1}{r}U_2(s)$$

---

(1) $$\frac{U_0(s)}{sL}+\frac{U_2(s)}{r}=\left(\frac{1}{sL}+\frac{1}{R}\right)U_1(s)-\frac{1}{R}U_2(s)$$

(2) $$-\frac{U_1(s)}{r}=-\frac{1}{R}U_1(s)+\left(sC+\frac{1}{R}\right)U_2(s)$$

---

(1) $$\frac{U_0(s)}{sL}=\left(\frac{1}{sL}+\frac{1}{R}\right)U_1(s)-\left(\frac{1}{R}+\frac{1}{r}\right)U_2(s)$$

(2) $$0=-\left(\frac{1}{R}-\frac{1}{r}\right)U_1(s)+\left(sC+\frac{1}{R}\right)U_2(s)$$

$$\Delta=\begin{vmatrix}\frac{1}{sL}+\frac{1}{R} & -\left(\frac{1}{R}+\frac{1}{r}\right)\\ -\left(\frac{1}{R}-\frac{1}{r}\right) & sC+\frac{1}{R}\end{vmatrix}=\left(\frac{1}{sL}+\frac{1}{R}\right)\left(sC+\frac{1}{R}\right)-\left(\frac{1}{R}+\frac{1}{r}\right)\left(\frac{1}{R}-\frac{1}{r}\right)=\frac{sC}{sL}+\frac{sC}{R}+\frac{1}{RsL}+\frac{1}{R^2}-\frac{1}{R^2}+\frac{1}{r^2}$$

$$=\frac{sC}{sL}+\frac{sC}{R}+\frac{1}{RsL}+\frac{1}{r^2}$$

$$\Delta_2 = \begin{vmatrix} \frac{1}{sL}+\frac{1}{R} & \frac{U_0}{sL} \\ -\left(\frac{1}{R}-\frac{1}{r}\right) & 0 \end{vmatrix} = \frac{U_0}{sL}\left(\frac{1}{R}-\frac{1}{r}\right)$$

Uvrstimo vrijednosti: r=2, R=1, L=2 i C=1/2, $U_0(s)$=2.

$$\Delta = \frac{1}{4}+\frac{1}{2}s+\frac{1}{2}\cdot\frac{1}{s}+\frac{1}{4} = \frac{1}{2}+\frac{1}{2}s+\frac{1}{2s}$$

$$\Delta_2 = \frac{2}{2s}\left(1-\frac{1}{2}\right) = \frac{1}{2s}$$

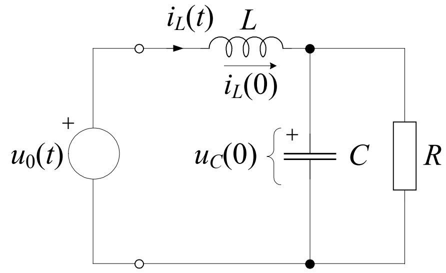
$$U_2(s) = \frac{\Delta_2}{\Delta} = \frac{\frac{1}{2s}}{\frac{1}{2}+\frac{1}{2}s+\frac{1}{2s}} = \frac{1}{s^2+s+1}$$

$$s^2+s+1=0 \Rightarrow s_{1,2} = \frac{-1\pm\sqrt{1-4}}{2} = -\frac{1}{2}\pm j\frac{\sqrt{3}}{2}$$

$$U_2(s) = \frac{1}{s^2+s+\frac{1}{4}+\frac{3}{4}} = \frac{2}{\sqrt{3}}\cdot\frac{\frac{\sqrt{3}}{2}}{\left(s+\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}$$

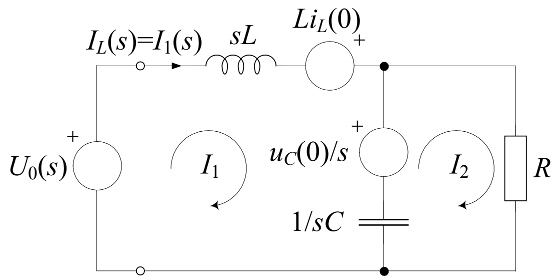
$$u_2(t) = \mathscr{L}^{-1}[U_2(s)] = \frac{2}{\sqrt{3}}\cdot e^{-\frac{1}{2}t}\cos\left(\frac{\sqrt{3}}{2}t\right)\cdot S(t)$$

3. Odrediti odziv $i_L(t)$ mreže prema slici za $t \geq 0$, ako su zadane normirane vrijednosti elemenata: $R=1$, $L=2$ i $C=1$, početni uvjeti u mreži $u_C(0)=-1/5$ i $i_L(0)=1/5$, a pobuda je $u_0(t)=S(t)$.



<u>Rješenje:</u>

Za $t \geq 0$: Primjena Laplaceove transformacije → Jednadžbe petlji



(1) $U_0(s)+Li_L(0)-\dfrac{u_C(0)}{s}=\left(sL+\dfrac{1}{sC}\right)I_1(s)-\dfrac{1}{sC}I_2(s)$

(2) $\dfrac{u_C(0)}{s}=-\dfrac{1}{sC}I_1(s)+\left(\dfrac{1}{sC}+R\right)I_2(s)$

---

(2) $\Rightarrow \quad I_2(s)\dfrac{1+sRC}{sC}=\dfrac{1}{sC}I_1(s)+\dfrac{u_C(0)}{s} \quad \Rightarrow \quad I_2(s)=\dfrac{1}{1+sRC}I_1(s)+\dfrac{Cu_C(0)}{1+sRC} \quad \rightarrow \quad (1) \quad \Rightarrow$

$$U_0(s)+Li_L(0)-\frac{u_C(0)}{s}=\frac{s^2LC+1}{sC}I_1(s)-\frac{1}{sC}\cdot\frac{I_1(s)+Cu_C(0)}{1+sRC}\Big/\cdot sC(1+sRC)$$

$$Cu_C(0)+(1+sRC)\big(sCU_0(s)+sCLi_L(0)-Cu_C(0)\big)=I_1(s)\left[(s^2LC+1)(1+sRC)-1\right]$$

$$I_1(s)=\frac{(1+sRC)\left[U_0(s)+Li_L(0)\right]-RCu_C(0)}{s^2RLC+sL+R}=\frac{U_0(s)\left(\frac{1}{R}+sC\right)+Li_L(0)\left(\frac{1}{R}+sC\right)-Cu_C(0)}{s^2LC+s\frac{L}{R}+1}$$

$$I_1(s)=\frac{\frac{1}{s}(1+s)+\frac{2}{5}(1+s)+\frac{1}{5}}{2s^2+2s+1}=\frac{1}{5s}\cdot\frac{5(1+s)+2s(1+s)+s}{2s^2+2s+1}$$

$$I_1(s)=\frac{1}{5s}\cdot\frac{2s^2+8s+5}{2s^2+2s+1}=\frac{1}{5}\left(\frac{5}{s}-4\frac{s+0{,}5}{(s+0{,}5)^2+0{,}25}+2\frac{0{,}5}{(s+0{,}5)^2+0{,}25}\right)$$

$$i_L(t)=i_1(t)=\frac{1}{5}\left(5-4e^{-0{,}5t}\cos(0{,}5t)+2e^{-0{,}5t}\sin(0{,}5t)\right)\cdot S(t)$$

4. Odziv neke mreže na pobudu $x(t)$=S(t) glasi: $y(t)$=e^{-3t}ch($2t$)·S(t). Odrediti funkciju mreže i fazor odziva na pobudu $x(t)$=2 cos($3t$+45˚).

Rješenje:

$$X(s)=\frac{1}{s}$$

$$Y(s)=\frac{s+3}{(s+3)^2-4}=\frac{s+3}{s^2+6s+5}$$

$$H(s)=\frac{s(s+3)}{(s+3)^2-4}=\frac{s(s+3)}{s^2+6s+5}$$

Fazori:

$$H(\mathrm{j}\omega)=\frac{\mathrm{j}\omega(\mathrm{j}\omega+3)}{(\mathrm{j}\omega)^2+6\mathrm{j}\omega+5}$$

$$X_1(\mathrm{j}\omega)=2e^{\mathrm{j}\pi/4}$$

$$Y_1(\mathrm{j}\omega)=H(\mathrm{j}\omega)X_1(\mathrm{j}\omega)=\frac{\mathrm{j}\omega(\mathrm{j}\omega+3)}{(\mathrm{j}\omega)^2+6\mathrm{j}\omega+5}X_1(\mathrm{j}\omega)=\frac{\mathrm{j}\omega(\mathrm{j}\omega+3)}{(\mathrm{j}\omega)^2+6\mathrm{j}\omega+5}2e^{\mathrm{j}\pi/4}$$

$$\omega=3$$

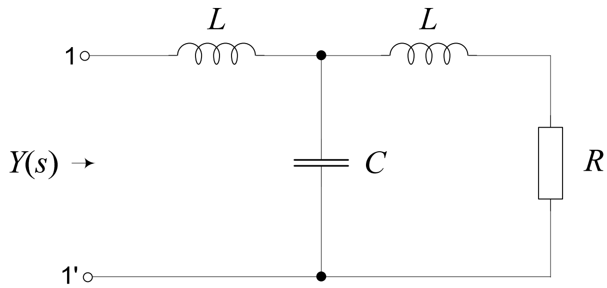
$$Y_1(\mathrm{j}3)=\frac{\mathrm{j}3\cdot(\mathrm{j}3+3)}{(\mathrm{j}3)^2+18\mathrm{j}+5}\cdot 2\cdot\left(\frac{\sqrt{2}}{2}+j\frac{\sqrt{2}}{2}\right)$$

$$Y_1(\mathrm{j}3)=\frac{9\sqrt{2}}{85}\cdot(2+j9)$$

$$y_1(t)=\frac{9\sqrt{2}}{\sqrt{85}}\cos(3t+77.47°)$$

PRVI MEĐUISPIT IZ ELEKTRIČNIH KRUGOVA – Rješenja – 2010

1. Zadan je dvopol sastavljen od normiranih elemenata R=1, C=2, L=1.
   a) Izračunati ulaznu admitanciju $Y(s)$ na priključnicama 1-1' tog dvopola;
   b) Denormirati elemente dvopola na frekvenciju ω_0=10^6 rad/s i na otpor R_0=1000 Ω;
   c) Odrediti denormiranu ulaznu admitanciju $Y(s)$;
   d) Koliki je iznos denormirane ulazne admitancije $Y(s)$ na frekvenciji nula $Y(0)$?



Rješenje:

a) Ulazna admitancija:

$$Y_n(s)=\frac{1}{sL+\dfrac{1}{sC+\dfrac{1}{sL+R}}}=\frac{1}{sL+\dfrac{sL+R}{sC(sL+R)+1}}=\frac{sC(sL+R)+1}{sL[sC(sL+R)+1]+sL+R}=$$

$$=\frac{s^2LC+sCR+1}{sL[s^2LC+sCR+1]+sL+R}=\frac{s^2LC+sCR+1}{s^3L^2C+s^2LCR+s2L+R}=\frac{2s^2+2s+1}{2s^3+2s^2+2s+1}\ \text{(1 bod)}$$

b) Denormiranje elemenata: (1 bod)

$$R=R_0\cdot R_n=1000\cdot 1=1000\Omega=1k\Omega$$

$$Z_{C_n}=\frac{1}{sCR_0}=\frac{1}{\dfrac{s}{\omega_0}\underbrace{\omega_0 CR_0}_{C_n}}\qquad\Rightarrow\qquad C=\frac{C_n}{\omega_0 R_0}=\frac{2}{10^6\cdot 10^3}=2\cdot 10^{-9}F=2nF$$

$$Z_{L_n}=\frac{sL}{R_0}=\frac{s}{\omega_0}\cdot\underbrace{\frac{\omega_0 L}{R_0}}_{L_n}\qquad\Rightarrow\qquad L=\frac{L_nR_0}{\omega_0}=\frac{1\cdot 10^3}{10^6}=1\cdot 10^{-3}H=1mH$$

c) Uz uvrštene denormirane vrijednosti elemenata admitancija dvopola glasi:

$$Y(s)=\frac{s^2LC+sCR+1}{s^3L^2C+s^2LCR+s2L+R}=\frac{s^2\cdot 10^{-3}\cdot 2\cdot 10^{-9}+s\cdot 2\cdot 10^{-9}\cdot 10^3+1}{s^3\cdot(10^{-3})^2\cdot 2\cdot 10^{-9}+s^2\cdot 10^{-3}\cdot 2\cdot 10^{-9}\cdot 10^3+s\cdot 2\cdot 10^{-3}+10^3}=$$

$$=\frac{s^2\cdot 2\cdot 10^{-12}+s\cdot 2\cdot 10^{-6}+1}{s^3\cdot 2\cdot 10^{-15}+s^2\cdot 2\cdot 10^{-9}+s\cdot 2\cdot 10^{-3}+10^3}=\frac{s^2\cdot\dfrac{2\cdot 10^{-12}}{2\cdot 10^{-15}}+s\cdot\dfrac{2\cdot 10^{-6}}{2\cdot 10^{-15}}+\dfrac{1}{2\cdot 10^{-15}}}{s^3+s^2\cdot\dfrac{2\cdot 10^{-9}}{2\cdot 10^{-15}}+s\cdot\dfrac{2\cdot 10^{-3}}{2\cdot 10^{-15}}+\dfrac{10^3}{2\cdot 10^{-15}}}=$$

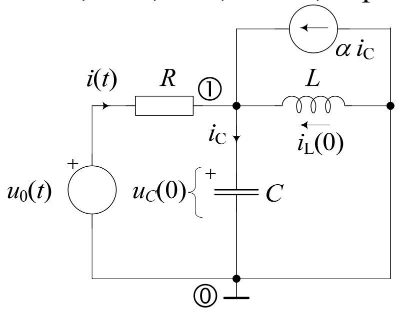
$$=\frac{s^2\cdot 10^3+s\cdot 10^9+5\cdot 10^{14}}{s^3+s^2\cdot 10^6+s\cdot 10^{12}+5\cdot 10^{17}}\ \text{(1 bod)}$$

d) Uz uvrštene denormirane vrijednosti elemenata admitancija dvopola na frekvenciji nula glasi:

$$Y(0)=\frac{5\cdot 10^{14}}{5\cdot 10^{17}}=10^{-3}\ \text{(1 bod)}$$

PISMENI ISPIT IZ ELEKTRIČNIH KRUGOVA – Rješenja – 2011

1. Za električni krug prikazan slikom odrediti odziv $i(t)$ ako je zadan poticaj $u_0(t)=\cos(t)S(t)$. Zadani su normalizirani elementi R=1, C=1, L=2, α=1/2, te početni uvjeti $u_C(0)$=1, $i_L(0)$=1/2.



Rješenje:

Metoda napona čvorova:

$$(1)\ U_1 \cdot \left(\frac{1}{R} + sC + \frac{1}{sL}\right) = \frac{U_0}{R} + \frac{i_L(0)}{s} + \alpha I_C + Cu_C(0) \quad \text{(1 bod)}$$

$$U_1 = I_C \frac{1}{sC} + \frac{u_C(0)}{s} \Rightarrow I_C = sC\left(U_1 - \frac{u_C(0)}{s}\right) \quad \text{(1 bod)}$$

Uz uvrštene vrijednosti elemenata: $I_C = s\left(U_1 - \frac{1}{s}\right) = sU_1 - 1$

$$U_1 \cdot \left(1 + s + \frac{1}{2s}\right) = \frac{s}{s^2+1} + \frac{1}{2s} + \frac{s}{2}U_1 - \frac{1}{2} + 1$$

$$U_1 \cdot \left(1 + s + \frac{1}{2s} - \frac{s}{2}\right) = \frac{s}{s^2+1} + \frac{1}{2s} + \frac{1}{2}$$

$$U_1 \cdot \left(1 + \frac{s}{2} + \frac{1}{2s}\right) = \frac{2s^2 + s^2 + 1 + s^3 + s}{2s(s^2+1)}$$

$$U_1 \frac{1+2s+s^2}{2s} = \frac{s^3+3s^2+s+1}{2s(s^2+1)} \quad \Rightarrow \quad U_1 = \frac{s^3+3s^2+s+1}{(s+1)^2(s^2+1)} \quad \text{(1 bod)}$$

$$U_1 = U_0 - IR \Rightarrow I = U_0 - U_1$$

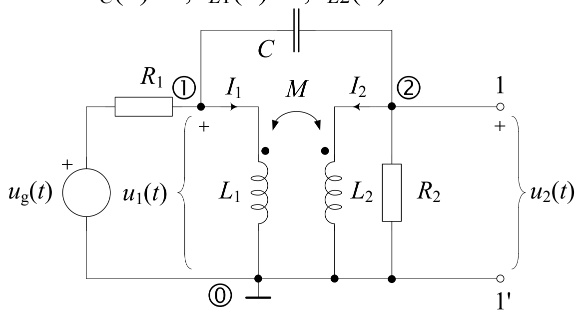
$$I = \frac{s}{s^2+1} - \frac{s^3+3s^2+s+1}{(s+1)^2(s^2+1)} = \frac{s(s^2+2s+1) - (s^3+3s^2+s+1)}{(s+1)^2(s^2+1)}$$

$$= \frac{-(s^2+1)}{(s+1)^2(s^2+1)} = \frac{-1}{(s+1)^2} \quad \text{(1 bod)}$$

$$I(s) = \frac{-1}{(s+1)^2} \Rightarrow i(t) = -te^{-t}S(t) \quad \text{(1 bod)}$$

ZAVRŠNI ISPIT IZ ELEKTRIČNIH KRUGOVA – Rješenja – 2011

1. Za električni krug na slici izračunati odziv $u_2(t)$ na prilazu 1–1', ako je zadan poticaj $u_g(t)=e^{-t}\cdot S(t)$. Zadane su normalizirane vrijednosti elemenata: $R_1=R_2=1$, $L_1=L_2=1$, $M=1$, $C=1$. Početni uvjeti su jednaki nula: $u_C(0)=0$, $i_{L1}(0)=0$, $i_{L2}(0)=0$.



Rješenje:

Napomena: ako se odmah uvrste numeričke vrijednosti (što se jednako priznaje za točno rješenje) tada je postupak znatno jednostavniji i kraći.

Postavimo jednadžbe čvorova:

$$(1)\ U_1\cdot\left(\frac{1}{R_1}+sC\right)-U_2\cdot sC=\frac{U_g}{R_1}-I_1$$

$$(2)\ -U_1\cdot sC+U_2\cdot\left(\frac{1}{R_2}+sC\right)=-I_2$$

$$\begin{aligned}U_1&=sL_1\cdot I_1+sM\cdot I_2\\U_2&=sM\cdot I_1+sL_2\cdot I_2\end{aligned}\quad\text{(jednadžbe vezanih induktiviteta)}$$

(2 boda)

Uvrstimo vrijednosti elemenata:

$$(1)\ U_1\cdot(1+s)-U_2\cdot s=\frac{1}{s+1}-I_1$$

$$(2)\ -U_1\cdot s+U_2\cdot(1+s)=-I_2$$

$$(3)\ U_1=s\cdot I_1+s\cdot I_2$$

$$(4)\ U_2=s\cdot I_1+s\cdot I_2$$

(1 bod)

$$(3),(4)\Rightarrow U_1=U_2$$

$$(1)\ I_1=\frac{1}{s+1}-U_1\cdot(1+s)+U_1\cdot s=\frac{1}{s+1}-U_1$$

$$(2)\ I_2=-U_2$$

$$(4)\ U_2=s\cdot\overbrace{\left(\frac{1}{s+1}-U_1\right)}^{I_1}+s\cdot I_2=s\cdot\left(\frac{1}{s+1}-U_2\right)-s\cdot U_2=\frac{s}{s+1}-2s\cdot U_2$$

$$U_2(1+2s)=\frac{s}{s+1}$$

$$U_2(s)=\frac{s}{(s+1)(2s+1)}=\frac{s}{2(s+1)(s+1/2)}=\frac{A}{s+1}+\frac{B}{s+1/2}\Rightarrow A=1,\quad B=-\frac{1}{2}$$ (1 bod)

$$\Rightarrow u_2(t)=\left(e^{-t}-\frac{1}{2}e^{-\frac{t}{2}}\right)\cdot S(t)$$ (1 bod)