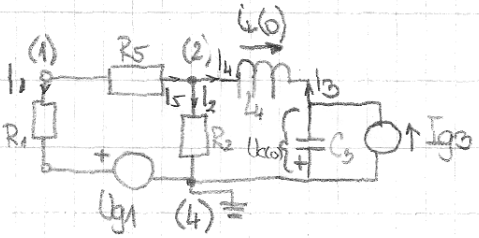
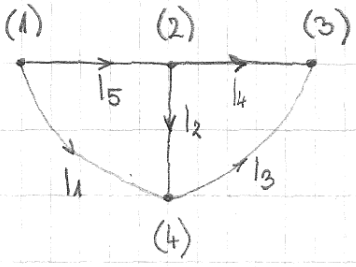
Matrice

- izgled kruga: A, S, Q → reza
  - A: incid. term.
  - S: petlji



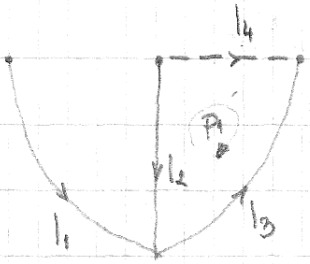


$(\check{C}-1)\times G$

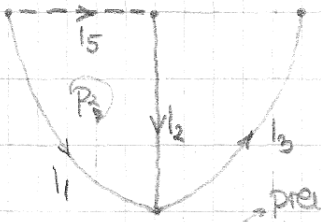
$$A = \begin{array}{c} \\ \check{C}_1 \\ \check{C}_2 \\ \check{C}_3 \end{array} \begin{bmatrix} G_1 & G_2 & G_3 & G_4 & G_5 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & -1 \\ 0 & 0 & -1 & -1 & 0 \end{bmatrix}$$

struja ↗ u čvor
- ako ulazi onda je minus

stablo:



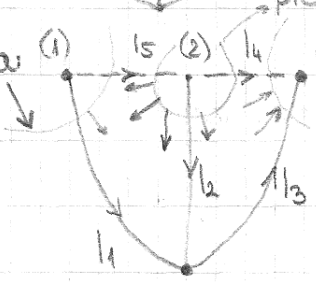
- smjer petlje i smjer grane
- broj petlji jednak broju spona



$$S = \begin{array}{c} \\ P_1 \\ P_2 \end{array} \left[\begin{array}{ccc:cc} G_1 & G_2 & G_3 & G_4 & G_5 \\ 0 & -1 & -1 & 1 & 0 \\ -1 & 1 & 0 & 0 & 1 \end{array}\right]$$

jed. matrica

Matrica reza:



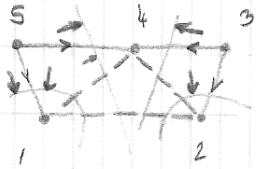
prerezavanje

- rez: ima smjer stabla (stablene grane)
- rez tako da presijecemo 1 granu

$$Q = \begin{array}{c} \\ \Gamma_1 \\ \Gamma_2 \\ \Gamma_3 \end{array} \left[\begin{array}{ccc:cc} G_1 & G_2 & G_3 & G_4 & G_5 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & -1 \\ 0 & 0 & 1 & 1 & 0 \end{array}\right]$$

broj rezova = broj stablenih grana

pr.



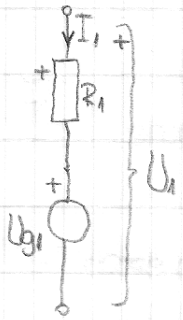
smjer reza

$$\left.\begin{aligned} U_b &= Z_b \cdot I_b + U_{ob} \\ I_b &= Y_b \cdot U_b + I_{ob} \end{aligned}\right\} \quad U\text{-}I \text{ jednadžbe}$$

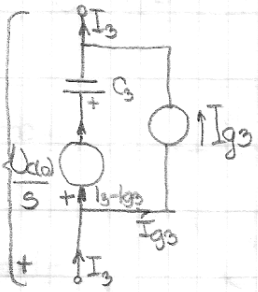
$$Z_p \cdot I_p = U_{op}$$
$$Y_v \cdot U_v = I_{ov}$$
$$Y_r \cdot U_r = I_{or}$$

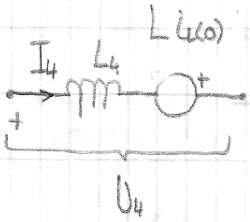
$$Z_p = S \cdot Z_b \cdot S^T$$

grana 1:



$$U_1 = I_1 R_1 + U_{g1}$$
$$U_2 = I_2 R_2$$
$$U_3 = \frac{U_{c(0)}}{s} + \frac{1}{sC_3} \cdot (I_3 - I_{g3})$$
$$U_4 = I_4 s L_4 - L i_{(0)}$$
$$U_5 = I_5 R_5$$

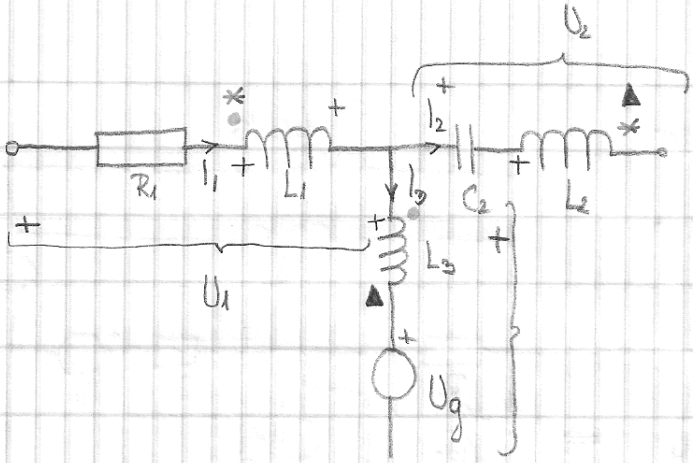




napon i g. otpornika strujni g.

$U_b = Z_b \cdot I_b + U_{ob}$ → naponi g. u kojima su sadržani početni uvjeti i izvori

$$\begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \\ U_5 \end{bmatrix} = \begin{bmatrix} R_1 & 0 & 0 & 0 & 0 \\ 0 & R_2 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{sC_3} & 0 & 0 \\ 0 & 0 & 0 & sL_4 & 0 \\ 0 & 0 & 0 & 0 & R_5 \end{bmatrix} \cdot \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \end{bmatrix} + \begin{bmatrix} U_{g1} \\ 0 \\ -\frac{I_{g3}}{sC_3} + \frac{U_{c(0)}}{s} \\ -L i_{(0)} \\ 0 \end{bmatrix}$$



- kada I_3 prolazi kroz L_3 stvara napon i na L_1

(+ na mjestu točke pa +)

● M_{13}
✱ M_{12}
▲ M_{23}

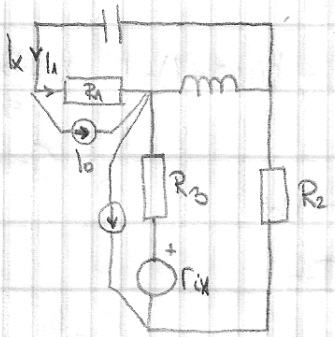
$$U_1 = I_1 R_1 + I_1 s L_1 + I_3 s M_{13} - I_2 M_{12}$$

$$U_2 = I_2 \frac{1}{sC_2} + I_2 s L_2 - I_1 s M_{12} + I_2 s M_{23}$$

$$U_3 = I_3 s L_3 + U_g + I_1 s M_{13} + I_2 s M_{23}$$

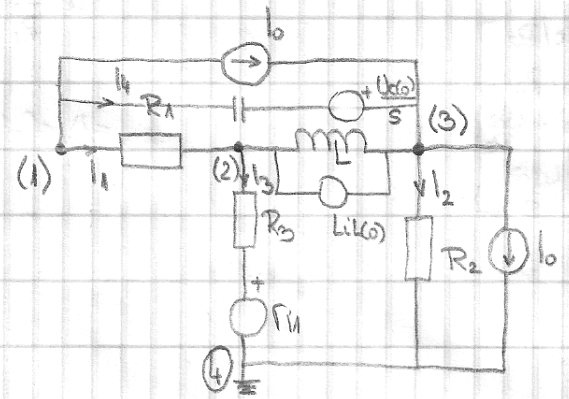
10.) iz zad. za vježbu (grafovi)

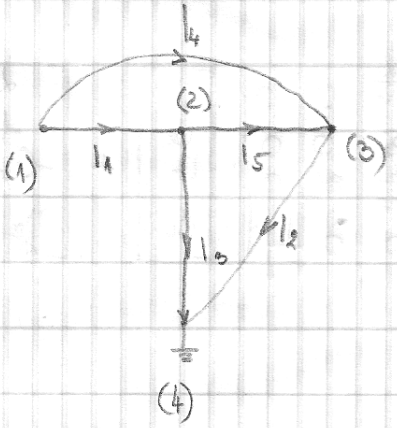
1. način



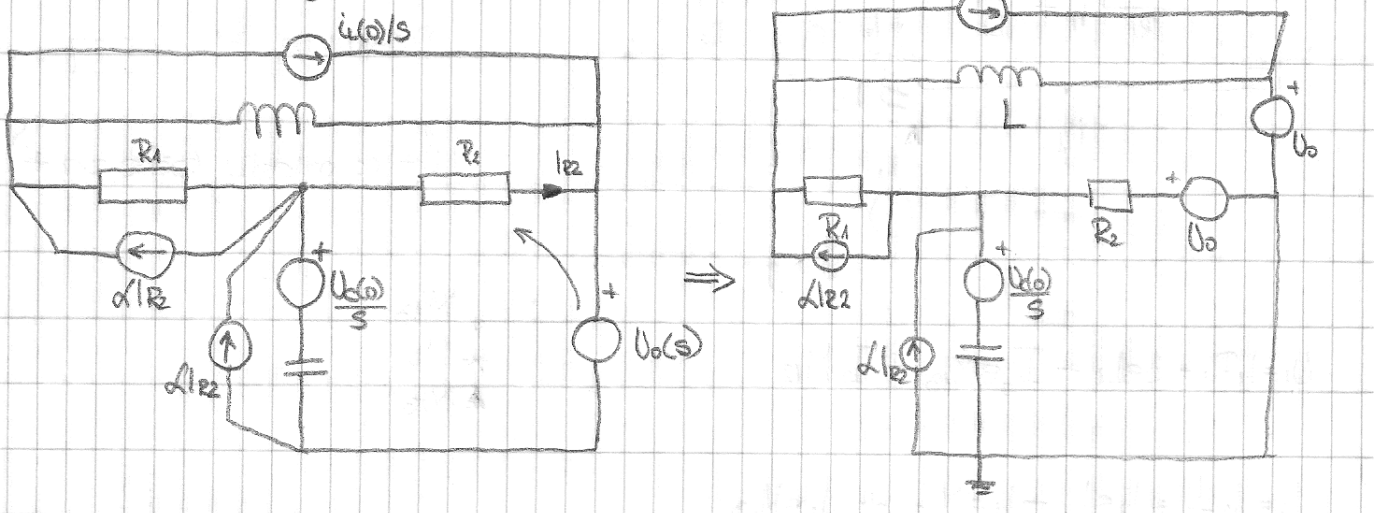
$I_1 = I_k - I_o$

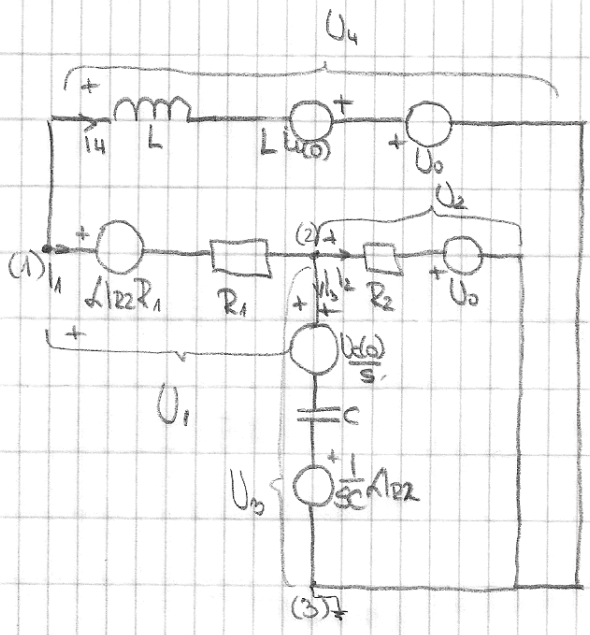
2. način





9. -pomičemo i strujni i naponski izvor





$U_1 = \alpha I_{R2} R_1 + I_1 R_1$

$U_2 = I_2 R_2 + U_0$

$U_3 = \frac{U_c(0)}{s} + \frac{I_3}{sC} + \frac{1}{sC}\alpha I_{R2}$

$U_4 = I_4 sL - Li_L(0) + U_0$

$U_0 = Z_0 I_0 + U_{0b}$

$I_0 = Y_0 U_0 + I_{0b}$

$I_1 = \frac{U_1}{R_1} - \alpha I_{R2}$

$I_2 = \frac{U_2}{R_2} - \frac{U_0}{R_2}$

$I_3 = U_3 sC - (U_c(0) - \alpha I_{R2})$

$I_4 = \frac{U_4}{sL} - \frac{U_0}{sL} + \frac{i_L(0)}{s}$

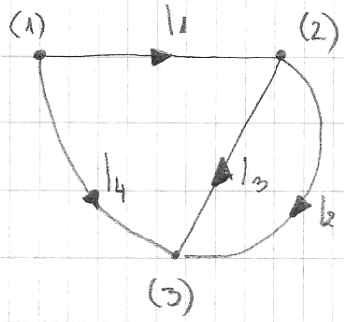
$(I_{R2} = I_2)$

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix} = \begin{bmatrix} \frac{1}{R_1} & -\frac{\alpha}{R_2} & 0 & 0 \\ 0 & \frac{1}{R_2} & 0 & 0 \\ 0 & -\frac{\alpha}{R_2} & sC & 0 \\ 0 & 0 & 0 & \frac{1}{sL} \end{bmatrix} \cdot \begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \end{bmatrix} + \begin{bmatrix} \alpha U_0 / R_2 \\ -U_0 / R_2 \\ -(U_c(0) + \alpha \frac{U_0}{R_2}) \\ -\frac{U_0}{sL} + \frac{i_L(0)}{s} \end{bmatrix}$$

$$I_1 = \frac{U_1}{R_1} - \alpha\left(\frac{U_2}{R_2} - \frac{U_0}{R_2}\right)$$

$$I_3 = U_3 sC - (U_c(0) - \alpha\left(\frac{U_2}{R_2} - \frac{U_0}{R_2}\right)$$

$$Y_b = A \cdot Y_v \cdot A^T$$



$$A = \begin{bmatrix} 1 & 0 & 0 & 1 \\ -1 & 1 & 1 & 0 \\ 0 & -1 & -1 & -1 \end{bmatrix}$$

$$Y_v = A Y_b A^T$$

$$= \underbrace{\begin{bmatrix} 1 & 0 & 0 & 1 \\ -1 & 1 & 1 & 0 \end{bmatrix}}_{2\times 4} \cdot \underbrace{\begin{bmatrix} \frac{1}{R_1} & -\frac{\alpha}{R_2} & 0 & 0 \\ 0 & \frac{1}{R_2} & 0 & 0 \\ 0 & -\frac{\alpha}{R_2} & sC & 0 \\ 0 & 0 & 0 & \frac{1}{sL} \end{bmatrix}}_{4\times 4} = \underbrace{\begin{bmatrix} \frac{1}{R_1} & -\frac{\alpha}{R_2} & 0 & \frac{1}{sL} \\ -\frac{1}{R_1} & \frac{1}{R_2} & sC & 0 \end{bmatrix}}_{2\times 4} \begin{bmatrix} 1 & -1 \\ 0 & 1 \\ 0 & 1 \\ 1 & 0 \end{bmatrix} = \underbrace{\begin{bmatrix} \frac{1}{R_1}+\frac{1}{sL} & \frac{1}{R_1}-\frac{\alpha}{R_2} \\ -\frac{1}{R_1} & \frac{1}{R_2}+sC \end{bmatrix}}_{Y_v}$$

$I_{ov} = A \cdot Y_b \, U_{ob}$ b-grane, črtkani-v, o-poč.vozlišči, p-petlje, r-rezovi

$I_{ov} = -A \, I_{ob}$

$Y_v \, U_v = I_{ov}$

$U_{op} = -S U_{ob}$

$Z_p I_p = U_{op}$	$Y_v U_v = I_{ov}$	$Y_r \cdot U_r = I_{or}$	$U_b = Z_b I_b + U_{ob}$
$Z_p = S Z_b S^T$	$Y_v = A Y_b A^T$	$Y_r = Q Y_b Q^T$	$I_b = Y_b \cdot U_b + I_{ob}$
$U_{op} = -S U_{ob}$	$I_{ov} = -A I_{ob}$	$I_{or} = Q I_{ob}$	
$I_p = \begin{bmatrix} I_{p1} \\ I_{p2} \\ \vdots \end{bmatrix}$	$U_v = \begin{bmatrix} U_{v1} \\ U_{v2} \\ \vdots \end{bmatrix}$	$U_r = \begin{bmatrix} U_{r1} \\ U_{r2} \\ \vdots \end{bmatrix}$	

Pr. Yv preko Zb

$Y_b = Z_b^{-1}$

$$\left[\begin{array}{ccccc|ccccc} R_1 & 0 & \mu & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & sL & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & R_3 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{sC_1} & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{sC_2} & 0 & 0 & 0 & 0 & 1 \end{array}\right]$$

Z_b

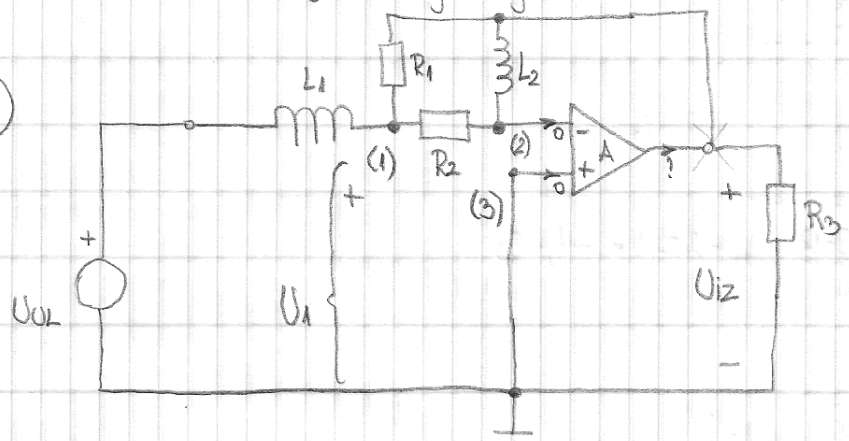
$$\left[\begin{array}{ccc|ccc} R_1 & 0 & \mu & 1 & 0 & 0 \\ 0 & sL & 0 & 0 & 1 & 0 \\ 0 & 0 & R_3 & 0 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & \frac{\mu}{R_1} & \frac{1}{R_1} & 0 & 0 \\ 0 & 1 & 0 & 0 & \frac{1}{sL} & 0 \\ 0 & 0 & 1 & 0 & 0 & \frac{1}{R_3} \end{array}\right] \begin{array}{l} \\ \\ -\frac{\mu}{R_1} \end{array} \quad +$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{1}{R_1} & 0 & -\frac{\mu}{R_1 R_3} \\ 0 & 1 & 0 & 0 & \frac{1}{sL} & 0 \\ 0 & 0 & 1 & 0 & 0 & \frac{1}{R_3} \end{array}\right]$$

Prijenosne funkcije

10.



$U_2 = U_3 = 0$

$U_{iz} = U_{ul} \cdot -$

(1) $U_1\left[\frac{1}{sL_1} + \frac{1}{R_2} + \frac{1}{R_1}\right] - U_{ul}\cdot\frac{1}{sL} - U_2\frac{1}{R_2} - U_{iz}\frac{1}{R_1} = 0$ ($U_2 \to 0$)

(2) $U_2\left(\frac{1}{R_2} + \frac{1}{sL_2}\right) - U_1\frac{1}{R_2} - U_{iz}\frac{1}{sL_2} = 0$ ($\frac{1}{R_2} \to 0$)

$U_1 = -U_{iz}\frac{R_2}{sL_2}$

(1) $U_{iz} = \frac{s\sqrt{2}}{s^2+\sqrt{2}s+1}\cdot U_{ul}$

$T(s) = \frac{U_{iz}}{U_{ul}} = \frac{\sqrt{2}s}{s^2+\sqrt{2}s+1}$

$T(s) = K\frac{(s-s_{01})(s-s_{02})\cdots}{(s-s_{p1})(s-s_{p2})\cdots}$

nula: brojnik=0

polovi: nazivnik=0

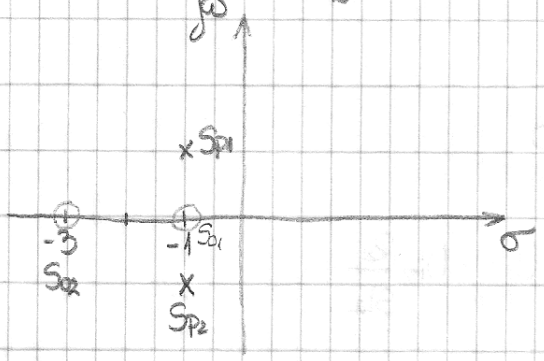
primjer: $T(s) = 4\,\frac{s^2+4s+3}{s^2+2s+2}$

s_0 ... (nule) → $s^2+4s+3=0$

$s_{01,2} = \frac{-4 \pm \sqrt{16-12}}{2} = -2 \pm 1 = -1, -3$

s_p ... (polovi) → $s^2+2s+2=0$

$s_{p1,2} = \frac{-2 \pm \sqrt{4-8}}{2} = -1 \pm j$



- ako jedan pol u nuli (granično stabilan)
- desno (nestabilan)

↑ stabilan sustav

$s = j\omega$

$T(j\omega) = 4\,\frac{-\omega^2+4j\omega+3}{-\omega^2+2j\omega+2}$

$A(\omega) = |T(j\omega)| = 4 \cdot \frac{\sqrt{(3-\omega^2)^2+(4\omega)^2}}{\sqrt{(2-\omega^2)^2+(2\omega)^2}}$

↓ amplituda

$\varphi(\omega) = \sphericalangle T(j\omega) = 0 + tg^{-1}\frac{4\omega}{3-\omega^2} - tg^{-1}\frac{2\omega}{2-\omega^2}$

$U_{ul} = 3\cos(4t+20°)$, $\omega = 4$

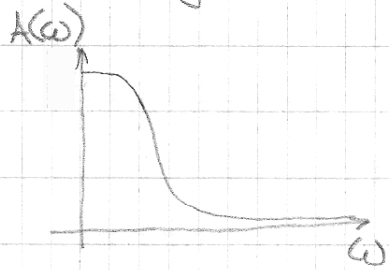
$U_{iz} = 3 \cdot A(4) \cdot \cos(4t + (20° + \varphi(4))$

- primjer, nacrtati $A(\omega)$

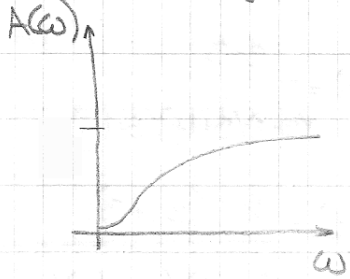
$$T(s)=\frac{a}{s^2+bs+c}$$

$$T(s)=\frac{as^2}{s^2+bs+c}$$

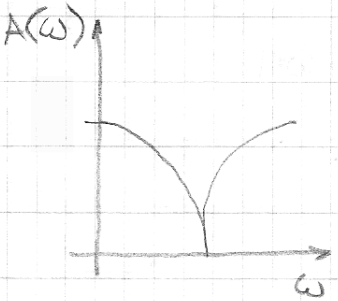
- niskopropusni filter



- visokopropusni filter

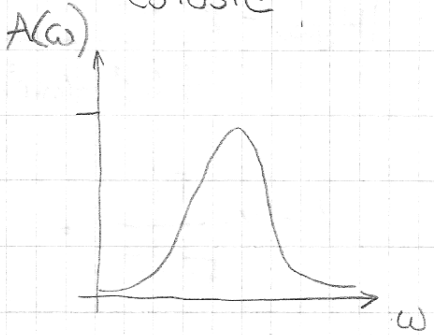


$$T(s)=\frac{as^2+b}{cs^2+ds+e}$$



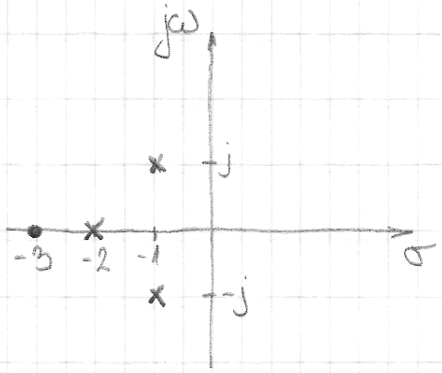
- pojasna brana (filter)

$$T(s)=\frac{s}{cs^2+ds+e}$$



- pojasno propusni filter

4. zad. 2MI (2006.)



$$H(s) = \frac{K\,(s+3)}{(s+2)(s-(-1-j))(s-(-1+j))}$$

$$H(s) = \frac{s+3}{(s+2)(s+1+j)(s+1-j)}$$

$|H(j\omega)| = 1, \quad \omega = 1$

$$H(j\omega) = \frac{K\,(j\omega+3)}{(j\omega+2)(j\omega+1+j)(j\omega+1-j)}$$

$$|H(j\omega)| = K \cdot \frac{\sqrt{3^2+\omega^2}}{\sqrt{\omega^2+2^2}\,\sqrt{(\omega+1)^2+1}\,\sqrt{(\omega-1)^2+1}}\Bigg|_{\omega=1}$$

$$1 = K \cdot \frac{\sqrt{10}}{\sqrt{5}\sqrt{5}\cdot 1} =$$

$$K = \frac{\sqrt{5}\cdot\sqrt{5}}{\sqrt{10}} = \frac{5}{\sqrt{10}} \cdot \frac{\sqrt{10}}{\sqrt{10}} = \frac{5\sqrt{10}}{10} = \frac{\sqrt{10}}{2}$$

$u_{UL}(t) = \cos(3t + 20°)$

$u_{IZ}(t) = |H(j\omega)|_{\omega_1} \cos(\omega t + 20° + \varphi)$

Naša prijen. funkc. $H(s) = \frac{\sqrt{10}}{2} \frac{s+3}{(s+2)(s^2+2s+2)}$

$$H(j\omega) = \frac{\sqrt{10}}{2} \frac{j\omega+3}{(j\omega+2)(-\omega^2+2j\omega+2)}$$

$$|H(j\omega)| = \frac{\sqrt{10}}{2} \frac{\sqrt{\omega^2+9}}{\sqrt{\omega^2+4}\sqrt{(2-\omega^2)^2+4\omega^2}} \qquad \varphi(j\omega) = \operatorname{arctg}\frac{\omega}{3} - \operatorname{arctg}\frac{\omega}{2} - \operatorname{arctg}\frac{2\omega}{2-\omega^2}$$

- zadana $\omega_1 = 3$

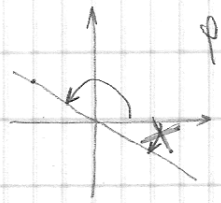
$u_{iz}(t) = |H(j\omega)|_{\omega_1} \cos(3t + 20° + \varphi)$, izračunamo uvrstivši zadani $\omega_1 = 3$

$= 0.201 \cos(3t + 20° - 150.7°)$

$= 0.201\, e^{-130.7°j}$ $\quad Ae^{j\varphi} = A\cos(\omega t + \varphi)$

$\varphi = \operatorname{tg}^{-1}\frac{2\omega}{2-\omega^2} = \operatorname{tg}^{-1}\frac{6}{-7} = -40.6°$

$z = -7 + j6$



(3. zad) 2M1, 2006.

- pojačalo

$U_1\left(sC_1 + \frac{1}{R_1} + \frac{1}{R_2}\right) - U_2\frac{1}{R_2} - U_{ul}sC_1 - U_{iz}\frac{1}{R_1} = 0$

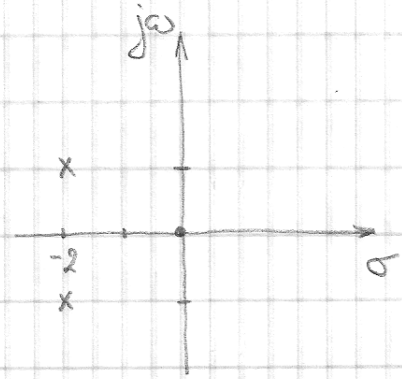
$U_2\left(\frac{1}{R_2} + \frac{1}{R_3} + sC_2\right) - U_1\frac{1}{R_2} - U_3\frac{1}{R_3} = 0$ ($U_3 = 0$)

$U_3\left(\frac{1}{R_3} + \frac{1}{R_4}\right) - U_2\frac{1}{R_3} - U_{iz}\frac{1}{R_4} = 0$

$U_3 = 0$ (iz jedn. oper. poj.)

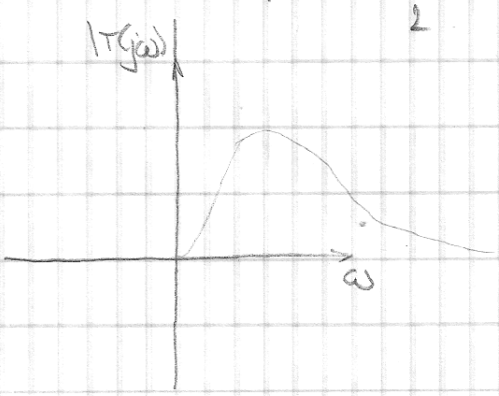
$T(s) = \frac{U_{iz}}{U_{ul}} = \frac{-2s}{s^2 + 4s + 5}$

...



NULE: $s_0 = 0$

POLOVI: $s^2 + 4s + 5 = 0$, $s_{p1,2} = \frac{-4 \pm \sqrt{16-20}}{2} = -2 \pm j$



$T(j\omega) = \frac{-2j\omega}{-\omega^2 + 4j\omega + 5}$ $\quad |T(j\omega)| = \frac{2}{\sqrt{(5-\omega^2)^2 + (4\omega)^2}}$