8. auditorna vježba

z1) $v = 0.6\,c$

$t_1 = 1250\ s$

$t' = ?$

$s = c \cdot t_1$

$= 3 \cdot 10^8 \cdot 1250 = 3.75 \cdot 10^{11}\ m$

$t = \frac{s}{v} = \frac{s}{0.6c}$

$= \frac{3.75 \cdot 10^{11}}{0.6 \cdot 3 \cdot 10^8} = 2083.3\dot{3}\ s$

$$t' = t \cdot \sqrt{1 - \frac{v^2}{c^2}}$$

$= 2083.3\dot{3} \cdot \sqrt{1 - \frac{0.6^2 c^2}{c^2}} = 1666.6\dot{6}\ s$

z2) $p_e = ?$

$$E_{uk} = E_{kin} + E_0$$

$E_{kin} = E_0$

$$E_0 = mc^2$$

$E_{uk} = 2 \cdot E_0$

$$E_{uk}^2 = m^2c^4 + p^2c^2$$

$(2mc^2)^2 = m^2c^4 + p^2c^2$

$3m^2c^4 = p^2c^2$

$p^2 = 3m^2c^2$

$p = \sqrt{3} \cdot m \cdot c$

$= \sqrt{3} \cdot 9.10938 \cdot 10^{-31} \cdot 3 \cdot 10^8 = 4.73 \cdot 10^{-22}\ kg \cdot m/s$

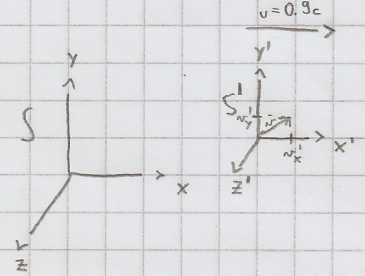
z3) $v = 0.9\,c$

$v_x' = 0.8\,c$

$v_y' = 0.5\,c$

$v_z' = 0$

$v_x, v_y, v_z, |v| = ?$



$$|v| = \sqrt{v_x^2 + v_y^2 + v_z^2}$$

$v_z = 0\ m/s$

$$v_x = \frac{v_x' + u'}{1 + \frac{v_x' \cdot u'}{c^2}}$$

$$v_x = \frac{0.8c - 0.9c}{1 - \frac{0.8c \cdot 0.9c}{c^2}} = 0.9884\,c$$

$$v_{y,z} = \frac{v_{y,z}'}{\gamma \cdot \left(1 + \frac{v_x' \cdot u}{c^2}\right)} = \frac{v_{y,z}' \cdot \sqrt{1 - \frac{u^2}{c^2}}}{1 + \frac{v_x' \cdot u}{c^2}}$$

$$\gamma = \frac{1}{\sqrt{1 - \beta^2}} = \frac{1}{\sqrt{1 - \frac{u^2}{c^2}}}$$

$$\beta = \frac{u}{c}$$

$$v_y = \frac{0.5c \cdot \sqrt{1 - \frac{0.9^2 c^2}{c^2}}}{1 + \frac{0.8c \cdot 0.9c}{c^2}} = 0.1267\,c$$

$|v| = \sqrt{0.9884^2 c^2 + 0.1267^2 c^2 + 0}$

$= \sqrt{c^2} \cdot \sqrt{0.9884^2 + 0.1267^2}$

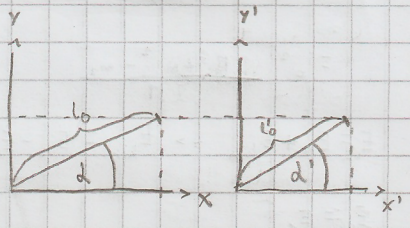
$= 0.9965\,c$

z4) $l_0 = 5\ m$

$\alpha = 30°$

$v_x' = 0.5\,c$

$l_0', \alpha' = ?$



$$l' = l \cdot \sqrt{1 - \frac{v^2}{c^2}}$$

$\sin\alpha = \frac{l_y}{l_0} \rightarrow l_y = l_0 \cdot \sin\alpha = 2.5\ m$

$\cos\alpha = \frac{l_x}{l_0} \rightarrow l_x = l_0 \cdot \cos\alpha = 4.33\ m$

$l_y' = l_y = 2.5\ m$

$l_x' = 5 \cdot \sqrt{1 - 0.5^2} = 3.75\ m$

$$(L_0')^2 = (L_x')^2 + (L_y')^2$$

$$L_0' = 4.51 \text{ m}$$

$$\operatorname{tg} \alpha' = \frac{L_y'}{L_x'}$$

$$\alpha' = 33^\circ 41' 24.24''$$

z5) $E^2 = M^2c^4 + p^2c^2$

$E = Mc^2$

$E_1^2 = m_1^2c^4 + p_1^2c^2$

$E_2^2 = m_2^2c^4 + p_2^2c^2$

$E_{kin1}, E_{kin2} = ?$

$p_1 + p_2 = 0 \rightarrow p_1 = -p_2$

$E = E_1 + E_2 \rightarrow E_2 = E - E_1$

$(E - E_1)^2 = m_2^2c^4 + p_2^2c^2$

$E^2 - 2 \cdot E \cdot E_1 + E_1^2 = m_2^2c^4 + p_1^2c^2$

$E^2 - 2EE_1 + E_1^2 = m_2^2c^4 + E_1^2 - m_1^2c^4$

$$E_1 = \frac{E^2 + m_1^2c^4 - m_2^2c^4}{2E} \quad |E = Mc^2|$$

$$= \frac{M^2c^4 + m_1^2c^4 - m_2^2c^4}{2Mc^2}$$

$$= c^2 \frac{M^2 + m_1^2 - m_2^2}{2M}$$

$E_{uk} = E_0 + E_{kin} \rightarrow E_{kin} = E_{uk} - E_0$

$E_{uk} = E_1$

$E_{kin} = E_1 - m_1c^2$

$$= c^2 \cdot \frac{M^2 + m_1^2 - m_2^2}{2M} - m_1c^2$$

$$= c^2 \cdot \frac{M^2 - 2M \cdot m_1 + m_1^2 - m_2^2}{2M}$$

$$= c^2 \cdot \frac{(M - m_1)^2 - m_2^2}{2M}$$

$$E_{kin2} = c^2 \frac{(M - m_2)^2 - m_1^2}{2M}$$