ARISTOTEL

BACON – NOVUM ORGANUM

DESCARTES

GALILEI

PODJELA FIZIKE

A)

1.) MEHANIKA

2.) TOPLINA
- MAKROSKOPSKI NIVO
- MIKROSKOPSKI NIVO

3.) OPTIKA

4.) ELEKTROMAGNETIZAM

5.) ATOMSKA FIZIKA

6.) NUKLEARNA FIZIKA

7.) FIZIKA ELEMENTARNIH ČESTICA

B)

1.) KLASIČNA

2.) MODERNA
- NERELATIVISTIČKA KVANTNA MEHANIKA
- SPECIJALNA TEORIJA RELATIVNOSTI
- RELATIV. TEORIJA KVANTNIH POLJA (KVANTNA TEORIJA POLJA)
- OPĆA TEORIJA RELATIVNOSTI

?

FIZIKALNE VELIČINE

FIZIK. VELIČINA - mjerljivo svojstvo fizikalnog stanja: procesa ili tijela

= (MJERNI BROJ) (FIZIKALNA JEDINICA

FIZIKALNE JEDINICE

SI - OSNOVNE - DULJINA - m
- MASA - kg
- VRIJEME - s
- MNOŽINA - mol
- TEMPERATURA - K
- STRUJA - A
- INT. SVJETLA - Cd
+ RADIJAN - Rd -> kut
+ STERRADIJAN - SR -> PROST. KUT

METAR - DULJINA PUTA KOJU SVJETLOST PRIJEĐE U VAKUUMU ZA VRIJEME 299 792 458-TOG DJELA SEKUNDE

KILOGRAM - MASA MEĐUNARODNE PRAMJERE (ETALONA) KILOGRAMA

SEKUNDA - TRAJANJE 9 192 631 770 PERIODA ZRAČENJA KOJE ODGOVARA PRIJELAZU IZMEĐU DVIJU HIPERFINIH RAZINA OSNOVNIH STANJA ATOMA CEZIJA 133.

KLASIČNA MEHANIKA

MEHANIKA
- KINEMATIKA
- DINAMIKA
  - STATIKA
  - KINETIKA

KINEMATIKA - NIJE BITAN UZROK GIBANJA

$\vec{r}$ - RADIJVEKTOR POLOŽAJA

$\vec{v}$

$\vec{a}$

DINAMIKA - BITAN UZROK GIBANJA

$\vec{F}$

m

- S OBZIROM NA SUSTAV TIJELA MEHANIKA SE DIJELI NA
  - MEHANIKA MATERIJALNE TOČKE
  - MEHANIKA SUSTAVA MATERIJALNIH TOČAKA
  - MEHANIKA KRUTOG TIJELA, MEHANIKA FLUIDA
  - MEHANIKA TITRANJA I VALOVA

- PROSTOR - 3D I EUKLIDSKI

KLAS. FIZ. 3D PROSTOR 1D VRIJEME

REL. FIZ. SPACETIME - 4D PROSTOR

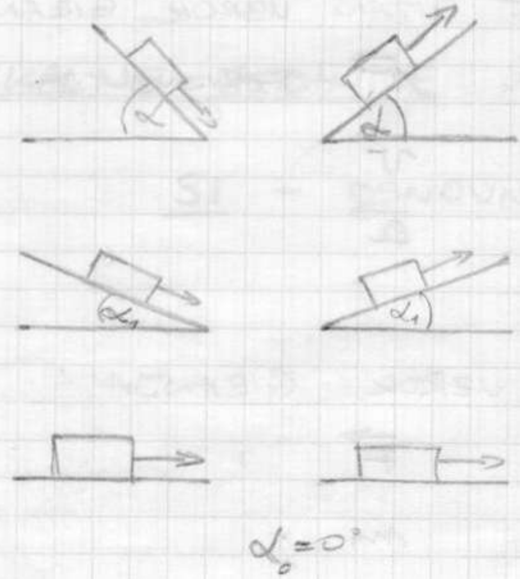
- VRIJEME - APSOLUTNO, SVUDA I ZA SVE JEDNAKO BEZ OBZIRA NA STANJE GIBANJA SUSTAVA U KOJEM SE PROMATRAČ NALAZI

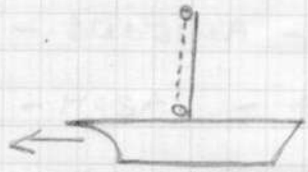
NEWTONOVI ZAKONI GIBANJA

1. NEWTONOV ZAKON

ARISTOTEL

GALILEI





TROMOST - SVOJSTVO MASE DA ŽELI OSTATI U TRENUTNOM STANJU GIBANJA

DESCARTES - PRAVOCRTNO GIBANJE

I. N.Z.

- SVAKO ĆE TIJELO OSTATI U STANJU MIROVANJA ILI JEDNOLIKOG GIBANJA PO PRAVCU SVE DOK SE POD DJELOVANJEM REZULTANTNE VANJSKE SILE RAZLIČITE OD 0 TO STANJE NE PROMIJENI

SILA - KONTAKTNE (DODIRNE)
- POLJE

MODERNA FIZIKA → POLJE JE FUNDAMENTALNO, SILA JE POSLJEDICA POLJA

→ 4 POLJA
- EM
- GRAVITACIJSKO
- JAKO - 3 KVARKA
- SLABO - $n^- \to p^+, e^-, \nu_e$

UČINAK SILE - DEFORMACIJA TIJELA
- MIJENJANJE STANJA GIBANJA TIJELA

SILA $\rightarrow \vec{F}$, N
↳ VEKTOR - SUPERPOZICIJA

- MATERIJALNA TOČKA (ČESTICA, SITNO TIJELO)
- DIMENZIJE NISU BITNE

MASA - KVANTITATIVNA MJERA TROMOSTI TIJELA

REFERENTNI SUSTAV - FIZIKALNI SUSTAVI UNUTAR KOJIH SE PROMATRA GIBANJE ČESTICE.

- INERCIJALNI REF. SUSTAVI - VRIJEDE ZAKONI KLAS. FIZ. (TJ. NEWTONOVI ZAKONI)

- NEINERCIJALNI SUSTAVI - INERC. SILE - PREFORMULIRATI NEWT. ZAKONE

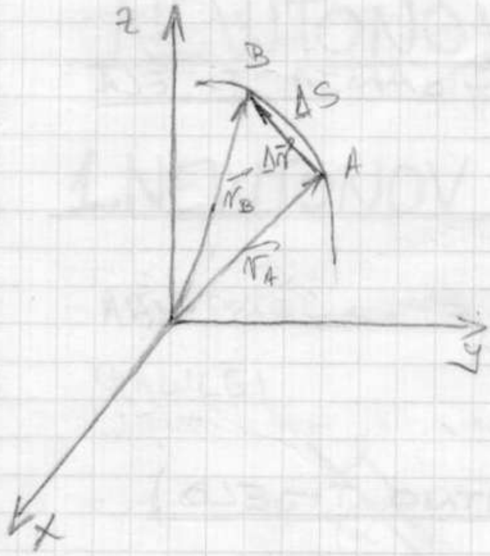
INERC. SUSTAV - 3 ČESTICE, POZADINSKO ZRAČENJE

CENTRIFUGALNO UBRZANJE NA EKVATORU $a_{CE} = 0{,}034\ m/s^2$

UBRZANJE ZEMLJE PREMA SUNCU $a_{ZS} = 0{,}006\ m/s^2$

HELIOCENTRIČNI SUSTAV $a = 10^{-10}\ m/s^2$ - SUNCE OKO SRED. GALAKSIJE - GOTOVO IDEALAN INERC. SUSTAV

6.



$\begin{cases} \text{POMAK – VEKTOR} \\ \text{PUT – SKALAR} \end{cases}$

→ JEDNAK IZNOS SAMO PRI JEDN. PRAV. GIB.

<u>PUTANJA</u> – SKUP SVIH TOČAKA KROZ KOJE PROLAZI MATERIJALNA TOČKA KOJA SE GIBA

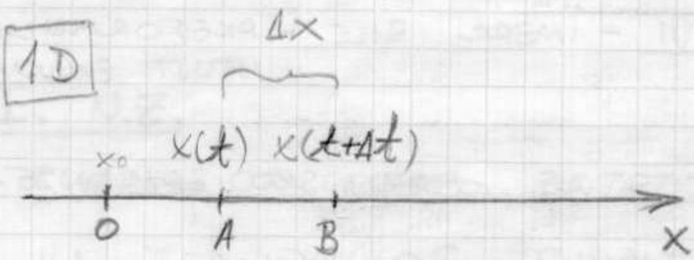
<u>PUT</u> – DIO PUTANJE KOJU TOČKA PRIJEĐE U ODREĐENOM VREMENU

<u>VEKTOR POMAKA</u> – PROMJENA VEKTORA POLOŽAJA $\vec{r}$

$$\Delta\vec{r} = \vec{r}_B - \vec{r}_A$$

I. GIBANJE PO PRAVCU

$\boxed{1D}$



$$x_A = x(t)$$
$$x_B = x(t+\Delta t)$$
$$\vec{r}_A = x(t)\,\vec{i}$$
$$\vec{r}_B = x(t+\Delta t)\,\vec{i}$$

$$\Delta\vec{r} = \vec{r}_B - \vec{r}_A = \left[\underbrace{x(t+\Delta t) - x(t)}_{\Delta x}\right]\vec{i} = \Delta x\,\vec{i}$$

– <u>SREDNJA BRZINA</u> – OMJER VEKTORA POMAKA $(\Delta\vec{r})$ I ZA TO POTREBNOG VREMENA

$$\bar{\vec{v}} = \frac{\Delta\vec{r}}{\Delta t} = \underbrace{\frac{\Delta x}{\Delta t}}_{=\bar{v}_x}\vec{i} = \bar{v}_x\,\vec{i} \Rightarrow$$ – SREDNJA BRZINA KAO VEKTOR

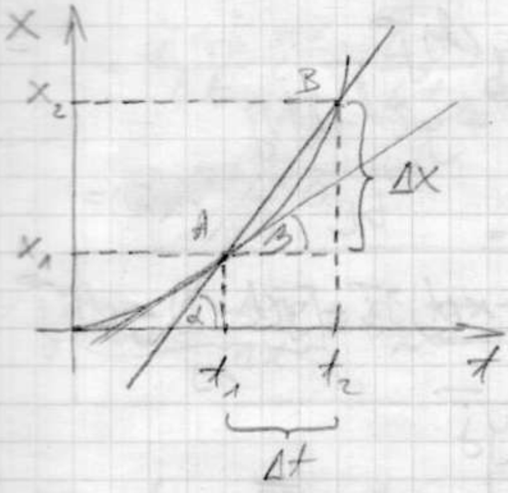
$$\bar{v} = \frac{\Delta s}{\Delta t} \rightarrow \text{PUT}$$

↳ SREDNJA BRZINA (SKALARNO)

$\vec{v}$ - VELOCITY

$|\vec{v}|$ - SPEED

$$\boxed{\bar{\vec{v}} = \frac{\Delta \vec{r}}{\Delta t} = \bar{v}_x \vec{i}}$$



→ PROSJEČNA BRZINA - SEKANTA (SREDNJA) α

→ TRENUTNA BRZINA - TANGENTA β

$$\vec{v} = \lim_{\Delta t \to 0} \bar{\vec{v}} = \lim_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta t} =$$

$$= \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} \vec{i} = \underbrace{\frac{dx}{dt}}_{= v_x} \vec{i} = v_x \vec{i} = \frac{d\vec{r}}{dt}$$

$$\boxed{\vec{v} = \frac{d\vec{r}}{dt} = \underset{= v_x \vec{i}}{\dot{\vec{r}}}}$$

TRENUTNA BRZINA - PRVA VREMENSKA DERIVACIJA VEKTORA POLOŽAJA

SREDNJA AKCELERACIJA

$$\bar{\vec{a}} = \frac{\Delta \vec{v}}{\Delta t} = \frac{\Delta v_x}{\Delta t} \vec{i} \qquad \boxed{\bar{\vec{a}} = \frac{\Delta \vec{v}}{\Delta t}}$$

$$\vec{v}_A = v(t)\vec{i}$$

$$\vec{v}_B = v(t + \Delta t)\vec{i}$$

$$\Delta \vec{v} = \vec{v}_B - \vec{v}_A =$$

$$= [\underbrace{v(t+\Delta t) - v(t)}_{\Delta v_x}]\vec{i} =$$

$$= \Delta v_x \vec{i} \qquad \Delta \vec{v} = \Delta v_x \vec{i}$$

TRENUTNA AKCELERACIJA

$$\vec{a} = \lim_{\Delta t \to 0} \bar{\vec{a}} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t} = \underbrace{\lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t}}_{= \frac{dv_x}{dt}} \vec{i} =$$

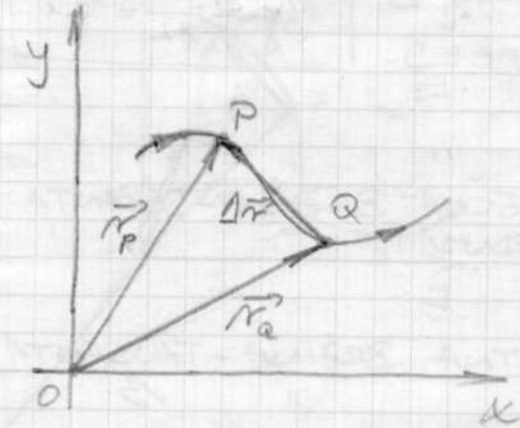
$$= \frac{dv_x}{dt} \vec{i} = \frac{d\vec{v}}{dt}$$

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{dv_x}{dt} \vec{i} = \frac{d^2 x}{dt^2} \vec{i} = \frac{d^2 \vec{r}}{dt^2}$$

$$\boxed{\vec{a} = \frac{d\vec{v}}{dt} = \frac{d^2 \vec{r}}{dt^2}}$$

8.

$\boxed{2D}$



$$P \xrightarrow{\Delta t} Q$$
$$t \qquad\qquad t+\Delta t$$

$$\vec{r}_P = x(t)\vec{i} + y(t)\vec{j}$$
$$\vec{r}_Q = x(t+\Delta t)\vec{i} + y(t+\Delta t)\vec{j}$$

$$\Delta\vec{r} = \vec{r}_Q - \vec{r}_P =$$
$$= [\underbrace{x(t+\Delta t) - x(t)}_{\Delta x}]\vec{i} + [\underbrace{y(t+\Delta t) - y(t)}_{\Delta y}]\vec{j}$$
$$\Delta\vec{r} = \Delta x\vec{i} + \Delta y\vec{j}$$

SREDNJA BRZINA

$$\bar{\vec{v}} = \frac{\Delta\vec{r}}{\Delta t} = \frac{\Delta x\vec{i} + \Delta y\vec{j}}{\Delta t} = \underbrace{\frac{\Delta x}{\Delta t}}_{=\bar{v}_x}\vec{i} + \underbrace{\frac{\Delta y}{\Delta t}}_{=\bar{v}_y}\vec{j} = \bar{v}_x\vec{i} + \bar{v}_y\vec{j}$$

$$\boxed{\bar{\vec{v}} = \frac{\Delta\vec{r}}{\Delta t} = \bar{v}_x\vec{i} + \bar{v}_y\vec{j}}$$

TRENUTNA BRZINA

$$\vec{v} = \lim_{\Delta t \to 0} \bar{\vec{v}} = \lim_{\Delta t \to 0} \frac{\Delta x\vec{i} + \Delta y\vec{j}}{\Delta t} = \lim_{\Delta t \to 0}\left(\frac{\Delta x}{\Delta t}\vec{i} + \frac{\Delta y}{\Delta t}\vec{j}\right) =$$

$$= \underbrace{\lim_{\Delta t \to 0}\frac{\Delta x}{\Delta t}}_{=\frac{dx}{dt}}\vec{i} + \underbrace{\lim_{\Delta t \to 0}\frac{\Delta y}{\Delta t}}_{=\frac{dy}{dt}}\vec{j} = \underbrace{\frac{dx}{dt}}_{=v_x}\vec{i} + \underbrace{\frac{dy}{dt}}_{=v_y}\vec{j} = v_x\vec{i} + v_y\vec{j}$$

$$\boxed{\vec{v} = \frac{d\vec{r}}{dt} = v_x\vec{i} + v_y\vec{j}}$$

SREDNJA AKCELERACIJA

$$\bar{\vec a} = \frac{\Delta \vec v}{\Delta t} = \frac{\Delta v_x \vec i + \Delta v_y \vec j}{\Delta t} =$$

$$= \underbrace{\frac{\Delta v_x}{\Delta t}}_{=\bar a_x} \vec i + \underbrace{\frac{\Delta v_y}{\Delta t}}_{=\bar a_y} \vec j =$$

$$= \bar a_x \vec i + \bar a_y \vec j$$

$$\boxed{\bar{\vec a} = \frac{\Delta \vec v}{\Delta t} = \bar a_x \vec i + \bar a_y \vec j}$$

$$\vec v_P = v_x(t)\vec i + v_y(t)\vec j$$

$$\vec v_Q = v_x(t+\Delta t)\vec i + v_y(t+\Delta t)\vec j$$

$$\Delta \vec v = \vec v_Q - \vec v_P =$$

$$= [\underbrace{v_x(t+\Delta t) - v_x(t)}_{\Delta v_x}]\vec i +$$

$$+ [\underbrace{v_y(t+\Delta t) - v_y(t)}_{\Delta v_y}]\vec j$$

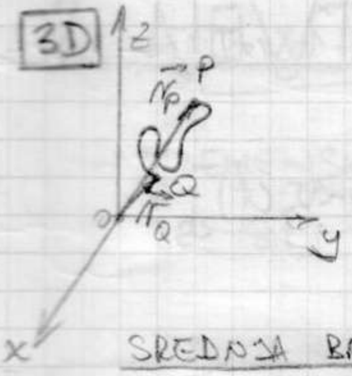
$$\Delta \vec v = \Delta v_x \vec i + \Delta v_y \vec j$$

$$\vec a = \lim_{\Delta t \to 0} \bar{\vec a} = \lim_{\Delta t \to 0}\left[\frac{\Delta v_x}{\Delta t}\vec i + \frac{\Delta v_y}{\Delta t}\vec j\right] = \underbrace{\lim_{\Delta t \to 0}\frac{\Delta v_x}{\Delta t}}_{=\frac{dv_x}{dt}}\vec i + \underbrace{\lim_{\Delta t \to 0}\frac{\Delta v_y}{\Delta t}}_{=\frac{dv_y}{dt}}\vec j =$$

$$= \underbrace{\frac{dv_x}{dt}}_{=a_x}\vec i + \underbrace{\frac{dv_y}{dt}}_{=a_y}\vec j = a_x \vec i + a_y \vec j$$

$$\boxed{\vec a = a_x \vec i + a_y \vec j}$$

3D



$$P \xrightarrow{\Delta t} Q$$

$$t \qquad t+\Delta t$$

$$\vec r_P = x(t)\vec i + y(t)\vec j$$

$$\vec r_Q = x(t+\Delta t)\vec i + y(t+\Delta t)\vec j$$

$$\Delta \vec r = \vec r_Q - \vec r_P = [\underbrace{x(t+\Delta t) - x(t)}_{\Delta x}]\vec i + [\underbrace{y(t+\Delta t) - y(t)}_{\Delta y}]\vec j$$

$$+ [\underbrace{z(t+\Delta t) - z(t)}_{\Delta z}]\vec k$$

$$\Delta \vec r = \Delta x \vec i + \Delta y \vec j + \Delta z \vec k$$

SREDNJA BRZINA

$$\bar{\vec v} = \frac{\Delta \vec r}{\Delta t} = \frac{\Delta x \vec i + \Delta y \vec j + \Delta z \vec k}{\Delta t} =$$

$$= \underbrace{\frac{\Delta x}{\Delta t}}_{=\bar v_x}\vec i + \underbrace{\frac{\Delta y}{\Delta t}}_{=\bar v_y}\vec j + \underbrace{\frac{\Delta z}{\Delta t}}_{=\bar v_z}\vec k =$$

$$= \bar v_x \vec i + \bar v_y \vec j + \bar v_z \vec k$$

$$\boxed{\bar{\vec v} = \frac{\Delta \vec r}{\Delta t} = \bar v_x \vec i + \bar v_y \vec j + \bar v_z \vec k}$$

10. TRENUTNA BRZINA

$$\vec{v} = \lim_{\Delta t \to 0} \bar{\vec{v}} = \lim_{\Delta t \to 0} \left[\frac{\Delta x \vec{i} + \Delta y \vec{j} + \Delta z \vec{k}}{\Delta t} \right] =$$

$$= \lim_{\Delta t \to 0} \left[\frac{\Delta x}{\Delta t} \vec{i} + \frac{\Delta y}{\Delta t} \vec{j} + \frac{\Delta z}{\Delta t} \vec{k} \right] =$$

$$= \underbrace{\lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}}_{= \frac{dx}{dt}} \vec{i} + \underbrace{\lim_{\Delta t \to 0} \frac{\Delta y}{\Delta t}}_{= \frac{dy}{dt}} \vec{j} + \underbrace{\lim_{\Delta t \to 0} \frac{\Delta z}{\Delta t}}_{= \frac{dz}{dt}} \vec{k} =$$

$$= \underbrace{\frac{dx}{dt}}_{= v_x} \vec{i} + \underbrace{\frac{dy}{dt}}_{= v_y} \vec{j} + \underbrace{\frac{dz}{dt}}_{= v_z} \vec{k} = v_x \vec{i} + v_y \vec{j} + v_z \vec{k} =$$

$$= \vec{v}_x + \vec{v}_y + \vec{v}_z \,//$$

$$\boxed{\vec{v} = \frac{d\vec{r}}{dt} = v_x \vec{i} + v_y \vec{j} + v_z \vec{k}}$$

TRENUTNA AKCELERACIJA

$$\vec{v}_P = v_x(t) \vec{i} + v_y(t) \vec{j} + v_z(t) \vec{k}$$

$$\vec{v}_Q = v_x(t+\Delta t) \vec{i} + v_y(t+\Delta t) \vec{j} + v_z(t+\Delta t) \vec{k}$$

$$\Delta \vec{v} = \Delta v_x \vec{i} + \Delta v_y \vec{j} + \Delta v_z \vec{k} \,//$$

$$\Delta \vec{v} = \vec{v}_Q - \vec{v}_P = \underbrace{[v_x(t+\Delta t) - v_x(t)]}_{\Delta v_x} \vec{i} + \underbrace{[v_y(t+\Delta t) - v_y(t)]}_{\Delta v_y} \vec{j} + \underbrace{[v_z(t+\Delta t) - v_z(t)]}_{\Delta v_z} \vec{k}$$

$$\bar{\vec{a}} = \frac{\Delta \vec{v}}{\Delta t} = \frac{\Delta v_x \vec{i} + \Delta v_y \vec{j} + \Delta v_z \vec{k}}{\Delta t} =$$

$$= \underbrace{\frac{\Delta v_x}{\Delta t}}_{= \bar{a}_x} \vec{i} + \underbrace{\frac{\Delta v_y}{\Delta t}}_{= \bar{a}_y} \vec{j} + \underbrace{\frac{\Delta v_z}{\Delta t}}_{= \bar{a}_z} \vec{k} =$$

$$= \bar{a}_x \vec{i} + \bar{a}_y \vec{j} + \bar{a}_z \vec{k} \,//$$

$$\boxed{\bar{\vec{a}} = \frac{\Delta \vec{v}}{\Delta t} = \bar{a}_x \vec{i} + \bar{a}_y \vec{j} + \bar{a}_z \vec{k}} \qquad \boxed{\vec{a} = \frac{d\vec{v}}{dt} = \frac{dv_x}{dt} \vec{i} + \frac{dv_y}{dt} \vec{j} + \frac{dv_z}{dt} \vec{k}}$$

$$\vec{a} = \lim_{\Delta t \to 0} \bar{\vec{a}} = \lim_{\Delta t \to 0} \left[\frac{\Delta v_x}{\Delta t} \vec{i} + \frac{\Delta v_y}{\Delta t} \vec{j} + \frac{\Delta v_z}{\Delta t} \vec{k} \right] =$$

$$= \underbrace{\lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t}}_{= \frac{dv_x}{dt}} \vec{i} + \underbrace{\lim_{\Delta t \to 0} \frac{\Delta v_y}{\Delta t}}_{= \frac{dv_y}{dt}} \vec{j} + \underbrace{\lim_{\Delta t \to 0} \frac{\Delta v_z}{\Delta t}}_{= \frac{dv_z}{dt}} \vec{k} =$$

$$= \frac{dv_x}{dt} \vec{i} + \frac{dv_y}{dt} \vec{j} + \frac{dv_z}{dt} \vec{k} = \dot{v}_x \vec{i} + \dot{v}_y \vec{j} + \dot{v}_z \vec{k}$$

- KADA SE TIJELO GIBA PO ZATVORENOJ PUTANJI NJEGOVA $\vec{a}$ UVIJEK IMA KOMPONENTU PREMA KONKAVNOJ STRANI KRIVULJE.
- AKO SE $\vec{v}$ TIJELA POVEĆAVA, NJEGOVA $\vec{a}$ UVIJEK IMA KOMPONENTU U SMJERU $\vec{v}$ A AKO SE $\vec{v}$ TIJELA SMANJUJE NJEGOVA $\vec{a}$ UVIJEK IMA KOMPONENTU U SMJERU SUPROTNOM $\vec{v}$

KOLIČINA GIBANJA

$$\boxed{\vec{p} = m \cdot \vec{v}}$$ KLASIČNA FIZIKA

$$\boxed{\vec{p} = \frac{m \cdot \vec{v}}{\sqrt{1 - \frac{v^2}{c^2}}}}$$ RELATIVISTIČKA FIZIKA

$$\boxed{c = 3 \cdot 10^8 \, m/s}$$

2. NEWTONOV ZAKON

- VREMENSKA PROMJENA KOLIČINE GIBANJA PROPORCIONALNA JE SILI I ZBIVA SE U SMJERU DJELOVANJA SILE

$$\boxed{\vec{F} = \frac{d\vec{p}}{dt}} \qquad \boxed{\vec{F} = \sum_{i=1}^{n} \vec{F}_i}$$

$$\vec{F} = \frac{d}{dt}(m\vec{v}) = \frac{dm}{dt} \cdot \vec{v} + m \cdot \frac{d\vec{v}}{dt}$$

$$\boxed{\vec{F} = \frac{dm}{dt}\vec{v} + m\frac{d\vec{v}}{dt}}$$

AKO JE $m = konst.$

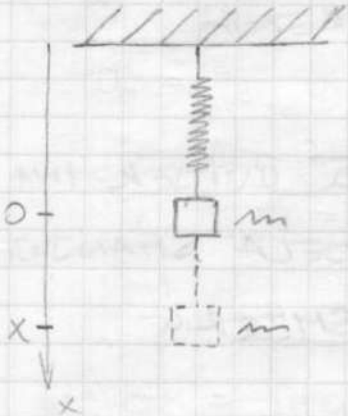
$$\boxed{\vec{F} = m\frac{d\vec{v}}{dt}}$$

- JEDNADŽBA GIBANJA

$$\boxed{\vec{F} = m\frac{d^2\vec{r}}{dt^2} = m \cdot \vec{a}}$$

12.

PRIMJER S HARMONIČKIM OSCILATOROM



$$m\frac{dv_x}{dt} = -kx \Rightarrow \text{JEDNADŽBA GIBANJA}$$

$$\frac{dv_x}{dt} = -\frac{k}{m}x$$

$$\frac{k}{m} = 1$$

$$\frac{dv_x}{dt} = -x$$

t	$x(t)$	$v_x(t)$
$t+\varepsilon$	$x(t+\varepsilon)=?$	$v_x(t+\varepsilon)$

$$x(t+\varepsilon) = x(t) + \varepsilon\, dx/dt = x(t) + \varepsilon\, v_x(t)$$

$$v_x(t+\varepsilon) = v_x(t) + \underbrace{\varepsilon\, dv_x/dt}_{\text{DINAMIKA}} = v_x(t) + \varepsilon\, a_x(t)$$

STALNA SILA

$$\vec{F} = F\vec{i}$$

x

$$F = \text{konst.} \Rightarrow a = \frac{F}{m} = \text{konst.}$$

$$a = \text{konst.}$$

$$a = \frac{F}{m}$$

$$v(t) = ?$$

$$\left.\begin{array}{l} t=0 \\ x=x_0 \\ v_x=v_0 \end{array}\right\} \text{P.U.} \qquad x(t)=? \quad v_x(t)=?$$

$$m\frac{dv_x}{dt} = F \quad \text{JEDN. GIBANJA}$$

$$\frac{dv_x}{dt} = \frac{F}{m}$$

$$\frac{dv_x}{dt} = a$$

$$dv_x = a \cdot dt \ /\int$$

$$\int dv_x = \int a\, dt$$

$$v_x = a\int dt$$

$$v_x = at + C$$

$$C = v_0$$

$$v_x = at + v_0$$

$$\boxed{v_x = at + v_0}$$

BRZINA TIJELA NA KOJEG DJELUJE STALNA SILA TJ. IMA KONSTANTNU AKCELERACIJU

$x(t) = ?$

$$dx = \frac{dx}{dt} \cdot dt \qquad \frac{dx}{dt} = v \qquad v = at + v_0$$

$$dx = (at + v_0) \cdot dt \quad / \int$$

$$x = a\frac{t^2}{2} + v_0 t + x_0$$

$$\int dx = \int (at + v_0) \cdot dt$$

$$x = \int at\,dt + v_0\,dt$$

$$x = \int at\,dt + \int v_0\,dt$$

$$x = a\int t\,dt + v_0 \int dt$$

$$x = a \cdot t^2/2 + v_0 t + c$$

$$c = x_0$$

$$\boxed{x = a\frac{t^2}{2} + v_0 t + x_0}$$

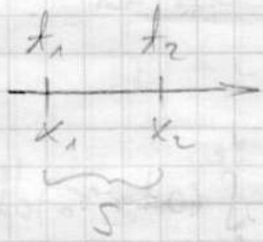
$$\boxed{\begin{aligned} x &= a\frac{t^2}{2} + v_0 t + x_0 \\ v_x &= at + v_0 \end{aligned}}$$

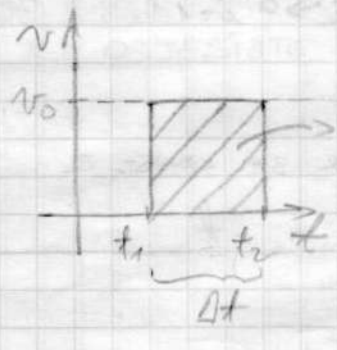
$$\boxed{\begin{aligned} &\text{Ako } F = 0 \Rightarrow a = 0 \\ &x = v_0 t + x_0 \\ &v_x = v_0 \end{aligned}}$$

PUT

I.) $a = 0$

$$\left.\begin{aligned} t &= 0 \\ x &= x_0 \\ v &= v_0 \end{aligned}\right\} \text{P.U.}$$





$\rightarrow S = v_0 \cdot \Delta t$

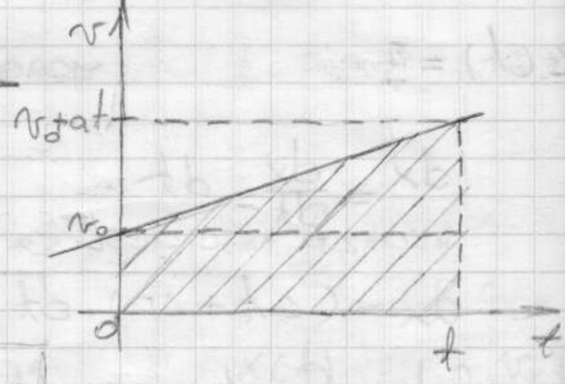
$$S = x_2 - x_1 = v_0 \underbrace{(t_2 - t_1)}_{\Delta t} = v_0 \Delta t$$

$a = 0 \quad \boxed{S = v_0 \Delta t}$

14.

II.) $a = konst.$

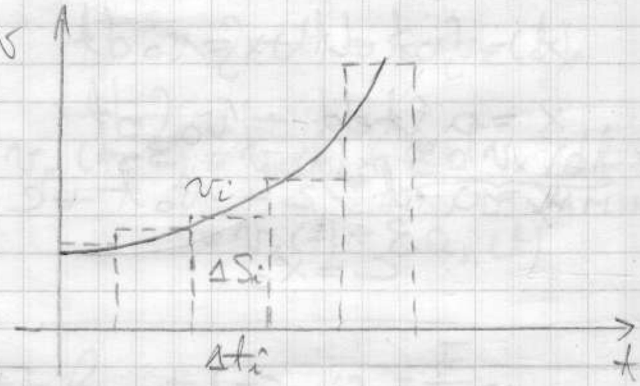
$$\left.\begin{array}{l} t = 0 \\ x = 0 \\ v = v_0 \end{array}\right\} P.U.$$



$$\boxed{S = x = a\frac{t^2}{2} + v_0 t}$$

III.) $a = f(t)$

$t_1 \longrightarrow t_2 \quad v(t)$



$n \quad \Delta t_i$

$$\Delta S_i = v_i \cdot \Delta t_i$$

$$S = \sum_{i=1}^{n} \Delta S_i \approx \sum_{i=1}^{n} v_i \Delta t_i$$

$$S = \lim_{\substack{n \to \infty \\ \Delta t_i \to 0}} \sum_{i=1}^{n} v_i \Delta t_i = \int_{t_1}^{t_2} v(t)\,dt$$

$$\boxed{S = \int_{t_1}^{t_2} v(t)\,dt}$$

– PRIMJER ZA KONST. a

$t_1 = 0, \; t_2 = t$

$$S = \int_{t_1}^{t_2} (at + v_0)\,dt = \int_{t_1}^{t_2} at\,dt + v_0\,dt =$$

$$= \int_{t_1}^{t_2} at\,dt + \int_{t_1}^{t_2} v_0\,dt = a\int_{t_1}^{t_2} t\,dt + v_0\int_{t_1}^{t_2} dt =$$

$$= a\frac{t^2}{2}\Big|_{t_1}^{t_2} + v_0 t\Big|_{t_1}^{t_2} = a\left(\frac{t^2}{2} - 0\right) + v_0 t =$$

$$= a\frac{t^2}{2} + v_0 t$$

– PRIJEĐENI PUT JEDNAK JE VREMENSKOM INTEGRALU IZNOSA VEKTORA BRZINE

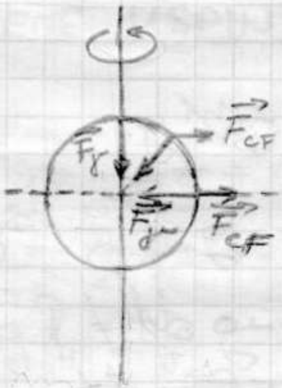
SILA TEŽA I TEŽINA

$\vec{F}_G$ ↓ y

$$\boxed{\vec{F}_G = mg\vec{j} = m\vec{g}}$$

SILA TEŽA ← $\boxed{\vec{F}_G = \vec{F}_\gamma + \vec{F}_{CF}}$ → CENTRIFUGALNA SILA

$\vec{F}_\gamma$ ↓ GRAVITACIJSKA SILA

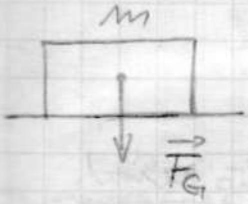


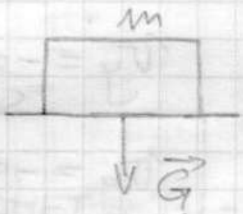
$\vec{g}$ – MIJENJA SE SA GEOG. ŠIRINOM I VISINOM

S VISINOM ↗
S ŠIRINOM ↘
(EKV – 0°)

ZG ⇒ $g = 9{,}80665\ m/s^2$

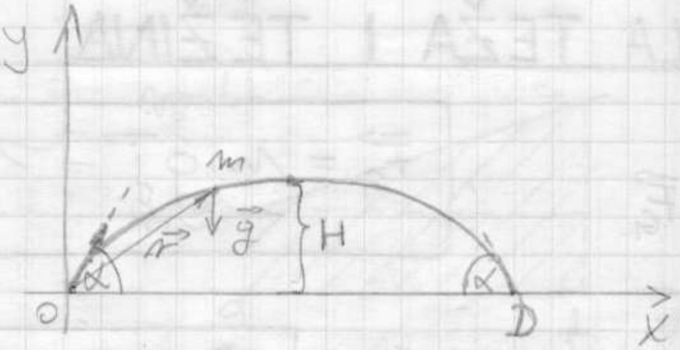
$$\boxed{g = 9{,}81\ m/s^2}$$





- **SILA TEŽA**
  - DJELUJE NA TIJELO
- **TEŽINA**
  - SILA KOJOM TIJELO DJELUJE NA PODLOGU ILI OBJESIŠTE

KOSI HITAC



$t=0$ $\quad v_x = v\cos\alpha$

$x=0$ $\quad v_y = v\sin\alpha$

$y=0$

$v_0 = |\vec{v}|$

$$\boxed{\vec{F}_G = -mg\vec{j}}$$

x - os

$$m\frac{dv_x}{dt} = 0$$

$$\frac{dv_x}{dt} = 0 \quad a = 0$$

$$v_x = konst.$$

$a = konst.$

$x = a\frac{t^2}{2} + v_0 t + x_0$ $\quad t=0$

$x = x_0$

$v_x = at + v_0$ $\quad v_x = v_0$

$a = 0$

$x = v_0 t + x_0$

$v_x = v_0$

y - os

$$m\frac{dv_y}{dt} = -mg$$

$$\frac{dv_y}{dt} = -g$$

$$dv_y = -g\,dt \;/\int$$

$$\int dv_y = \int -g\,dt$$

$$v_y = -g\int dt$$

$$\boxed{v_y = -gt + \underbrace{v_0\sin\alpha}_{c}}$$

KOSI HITAC - X KOMPONENTA

$$\boxed{\begin{aligned} x &= v_0\cos\alpha\, t \\ v_x &= v_0\cos\alpha \end{aligned}}$$

KOSI HITAC - Y KOMPONENTA

$$\boxed{\begin{aligned} y &= -g\frac{t^2}{2} + v_0\sin\alpha\, t \\ v_y &= -gt + v_0\sin\alpha \end{aligned}}$$

$$\boxed{\begin{aligned} \vec{v} &= \frac{d\vec{r}}{dt} \\ \vec{a} &= \vec{g} = \frac{d^2\vec{r}}{dt^2} \end{aligned}}$$

$$x = v_0\cos\alpha\, t \Rightarrow t = \frac{x}{v_0\cos\alpha}$$

$$y = -g\frac{t^2}{2} + v_0\sin\alpha\, t$$

$$y = -\frac{g}{2}\left(\frac{x}{v_0\cos\alpha}\right)^2 + v_0\sin\alpha\cdot\frac{x}{v_0\cos\alpha}$$

$$y = -\frac{g}{2}\frac{x^2}{v_0^2\cos^2\alpha} + x\,tg\alpha$$

$$\boxed{y = -\frac{g}{2v_0^2\cos^2\alpha}x^2 + x\,tg\alpha}$$

→ OVISNOST y O x
JEDNADŽBA PARABOLE

- **VRIJEME USPINJANJA** – t_H – VRIJEME POTREBNO DA TIJELO DOSEGNE MAX. VISINU (VERTIKALNA BRZINA – $v_y = 0$)

$$v_y = -gt + v_0\sin\alpha$$

$$0 = -gt_H + v_0\sin\alpha$$

$$gt_H = v_0\sin\alpha$$

$$t_H = \frac{v_0\sin\alpha}{g}$$

$$\boxed{t_H = \frac{v_0\sin\alpha}{g}}$$

$$\boxed{t_u = 2t_H = \frac{2v_0\sin\alpha}{g}}$$

→ UKUPNO TRAJANJE HITCA

- **VERTIKALNI DOMET** – H – NAJVEĆA VISINA KOJU TIJELO DOSEGNE (TJEME PUTANJE – y KOORDINATA)

$$y = -gt^2/2 + v_0\sin\alpha\, t \qquad t = t_H$$

$$H = -g\frac{t_H^2}{2} + v_0\sin\alpha\, t_H = -\frac{g}{2}\frac{v_0^2\sin^2\alpha}{g^2} + v_0\sin\alpha\cdot\frac{v_0\sin\alpha}{g}$$

$$= -\frac{v_0^2\sin^2\alpha}{2g} + \frac{v_0^2\sin^2\alpha}{g} = \frac{-v_0^2\sin^2\alpha + 2v_0^2\sin^2\alpha}{2g} =$$

$$= \frac{v_0^2\sin^2\alpha}{2g}$$

$$\boxed{H = \frac{v_0^2\sin^2\alpha}{2g}}$$

HORIZONTALNI DOMET – D – HORIZONTALNA UDALJENOST IZMEĐU POLAZNE I UDARNE TOČKE.

$$y = -\frac{g}{2v_0^2\cos^2\alpha}x^2 + x\,tg\alpha \qquad y(D) = 0$$

$$0 = -\frac{g}{2v_0^2\cos^2\alpha}D^2 + D\,tg\alpha$$

$$\frac{gD^{\cancel{2}}}{2v_0^2\cos^2\alpha} = \cancel{D}\,tg\alpha \quad /:D$$

$$\frac{gD}{2v_0^2\cos^2\alpha} = tg\alpha$$

$$D = \frac{tg\alpha \cdot 2v_0^2\cos^2\alpha}{g}$$

$$D = \frac{\frac{1-\cos 2\alpha}{\sin 2\alpha}\cdot \cancel{2}v_0^2\,\frac{1+\cos 2\alpha}{\cancel{2}}}{g}$$

$$D = \frac{\frac{1-\cos^2 2\alpha}{\sin 2\alpha}\cdot v_0^2}{g} \qquad 1-\cos^2 2\alpha = \sin^2 2\alpha$$

$$D = \frac{\frac{\sin^{\cancel{2}} 2\alpha}{\cancel{\sin 2\alpha}}\cdot v_0^2}{g} = \frac{v_0^2\cdot\sin 2\alpha}{g}$$

$$tg\alpha = \frac{1-\cos 2\alpha}{\sin 2\alpha}$$

$$\cos^2\alpha = \frac{1+\cos 2\alpha}{2}$$

$$\sin^2\alpha + \cos^2\alpha = 1$$

$$2\sin\alpha\cos\alpha = \sin 2\alpha$$

$$\boxed{D = \frac{v_0^2\sin 2\alpha}{g}}$$

$$x = v_0 t\cos\alpha$$

$$t_u = 2t_H = 2\frac{v_0\sin\alpha}{g}$$

$$D = v_0\,\frac{2v_0\sin\alpha}{g}\cdot\cos\alpha$$

KUT ZA MAX. DOMET

$$\frac{dD}{d\alpha} = 0 \qquad D = \frac{v_0^2\sin 2\alpha}{g} \qquad D' = \dot{D} = \frac{v_0^2}{g}\cos 2\alpha\cdot 2 = \frac{2v_0^2}{g}\cos 2\alpha$$

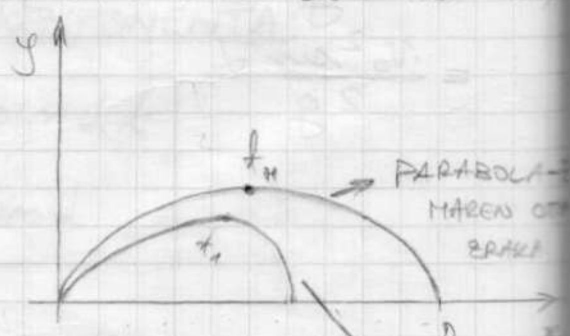
$$\frac{2v_0^2}{g}\cos 2\alpha = 0$$

$$\cos 2\alpha = 0$$

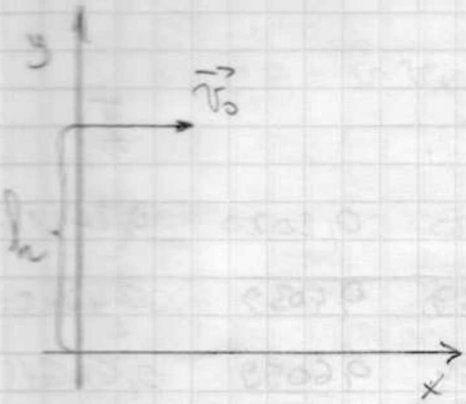
– DOMET JE ISTI ZA KUTOVE α i $(90^\circ - \alpha)$

$$2\alpha = 90^\circ$$

$$\alpha = 45^\circ$$



HORIZONTALNI HITAC



$t=0$

$x=0 \quad v_x=v_0$

$y=h \quad v_y=0$

$\alpha=0°$

$\vec{F}=-m\vec{g}$

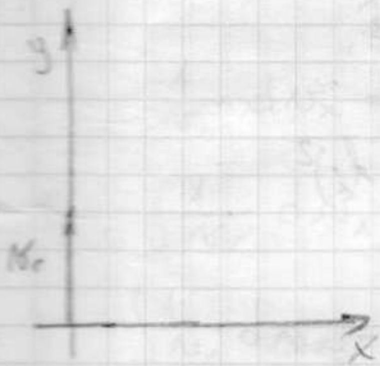
$m\frac{dv_x}{dt}=0 \qquad m\frac{dv_y}{dt}=-mg$

$x=v_0\cdot t \qquad y=-g\frac{t^2}{2}+h$

$v_x=v_0 \qquad v_y=-gt$

$$\boxed{\begin{array}{ll} x=v_0\cdot t & y=-g\frac{t^2}{2}+h \\ v_x=v_0 & v_y=-gt \end{array}}$$

VERTIKALNI HITAC



$t=0$

$x=0 \quad v_x=0$

$y=0 \quad v_y=v_0$

$\alpha=\pi/2$

$$\boxed{\begin{array}{l} y=-g\frac{t^2}{2}+v_0 t \\ v_y=-gt+v_0 \end{array}}$$

10. POKUS : PADOSTROJ

- mjerenje g u B1

	S / m		t / s			$\bar{t}$
1 : 1	0,2000	1 : 1	0,2022	0,2020	0,2020	0,2
1 : 4	0,8000	1 : 2	0,4041	0,4039	0,4039	0,4
1 : 9	1,8000	1 : 3	0,6061	0,6064	0,6059	0,6

tj. TREBALO BI BITI

$S(t) = a_2 t^2$ ili $S(t) =$

$$\Delta S_i = S_i - a_2 t_i^2$$

$$\sum_{i=1}^{n} (\Delta S_i)^2 = \min \Rightarrow \frac{\partial}{\partial a_2} \sum_{i=1}^{n} (\Delta S_i)^2$$

$$\frac{\partial}{\partial a_2} \sum_{i=1}^{n} (S_i - a_2 t_i^2)^2 = 0$$

$$2 \cdot \sum_{i=1}^{n} (S_i - a_2 t_i^2) \cdot (-t_i)^2$$

$$a_2 = \frac{\sum_{i=1}^{n} S_i t_i^2}{\sum_{i=1}^{n} t_i^4}$$

SILA LINEARNO OVISNA O BRZINI

$$\vec{F} = -b v_x \vec{i}$$

$\vec{v} = v_x \vec{i}$

$b > 0$

STOKESOV ZAKON

$$\vec{F} = -6\pi \eta R v_x \vec{i}$$

η — VISKOZNOST

ZA KUGLU

$t=0$

0

$t = 0$

$x = 0$

$v_x = v_0$

$$m \frac{dv_x}{dt} = -b v_x \quad /:m$$

$$\frac{dv_x}{dt} = -\frac{b}{m} v_x \qquad \frac{m}{b} = \tau \qquad [\tau] = s$$

$$\frac{dv_x}{dt} = -\frac{1}{\tau} v_x$$

$$dv_x = -\frac{1}{\tau} v_x \, dt$$

$$\frac{1}{v_x} dv_x = -\frac{1}{\tau} dt \quad / \int$$

$$\int \frac{1}{v_x} dv_x = \int -\frac{1}{\tau} dt$$

$$\int \frac{1}{v_x} dv_x = -\frac{1}{\tau} \int dt$$

$$\ln v_x = -\frac{t}{\tau} + c$$

$$\ln v_0 = c$$

$$\ln v_x = -\frac{t}{\tau} + \ln v_0$$

$$\ln v_x - \ln v_0 = -\frac{t}{\tau}$$

$$\ln \frac{v_x}{v_0} = -\frac{t}{\tau}$$

$$v_x = v_0 e^{-\frac{t}{\tau}}$$

$$\frac{dx}{dt} = v_0 e^{-\frac{t}{\tau}}$$

$$dx = v_0 e^{-\frac{t}{\tau}} \cdot dt \quad / \int$$

$$\int dx = \int v_0 e^{-\frac{t}{\tau}} dt$$

$$x = -v_0 \tau e^{-\frac{t}{\tau}} + c$$

$$c = v_0 \tau$$

$$x = v_0 \tau (1 - e^{-\frac{t}{\tau}})$$

$$\boxed{v_x = v_0 e^{-\frac{t}{\tau}}}$$

$$\boxed{x = v_0 \tau (1 - e^{-\frac{t}{\tau}})}$$

I.) $\boxed{t \to \infty} \Rightarrow v_x = 0$ tj: $v_x \to 0$

$\Rightarrow x = v_0 \tau$ $\qquad x = v_0 \tau \left(1 - e^{-\frac{t}{\tau}}\right) \quad v = v_0 e^{-\frac{t}{\tau}}$

tj: $x \to v_0 \tau$

II.) $\boxed{\frac{t}{\tau} \ll 1} \Rightarrow$ 1.) KRATKO VRIJEME NAKON POČETKA GIBANJA
$t \ll 1$

2.) $\tau \gg 1 \qquad \tau = \frac{m}{b} \Rightarrow b \ll 1$

tj: MALENA SILA OTPORA

$$e^{-\frac{t}{\tau}} = 1 - \frac{t}{\tau} + \frac{1}{2}\left(\frac{t}{\tau}\right)^2 + \dots$$

$$x = v_0 \tau \left[1 - 1 + \frac{t}{\tau} - \frac{1}{2}\left(\frac{t}{\tau}\right)^2\right] = v_0 t - \frac{1}{2}\underbrace{\frac{v_0}{\tau}}_{} t^2$$

$$a = \frac{v_0}{\tau}$$

$$v_x = v_0 \left(1 - \frac{t}{\tau}\right) = v_0 - \underbrace{\frac{v_0}{\tau}}_{} \cdot t$$

$$a = \frac{v_0}{\tau}$$

- TIJELO SE GIBA JEDNOLIKO USPORENO

KOSI HITAC – PRIMJER

$\alpha = 60°$

$v_0 = 44,7 \text{ m/s}$

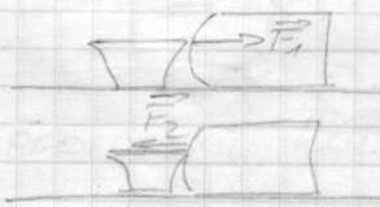
REALNO	BEZ OTPORA ZRAKA
$D = 98,5 \text{ m/s}$	$D = 177 \text{ m}$
$H = 53 \text{ m}$	$H = 76,8 \text{ m}$
$T = 6,6 \text{ s}$	$T = 7,9 \text{ s}$

	- v NA KOJOJ SE IZJEDNAČE SILE	- S ZA 95% [illegible] v NA KOJOJ SU SILE JEDNAKE
ČOVJEK	60 m/s	430 m
KIŠNA KAP	7 m/s	6 m
PING PONG	9 m/s	10 m
PADOBRANAC	5 m/s	3 m
KOŠ. LOPTA	20 m/s	47 m

3. NEWTONOV ZAKON

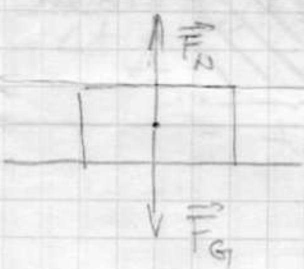
SILE AKCIJE I REAKCIJE

$$\vec{F}_{AB} = -\vec{F}_{BA}$$



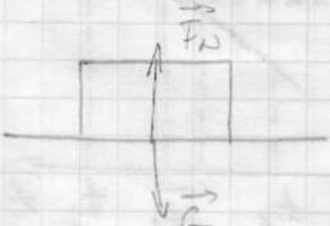
- SVAKOM DJELOVANJU (AKCIJI) UVIJEK JE SUPROTNO I JEDNAKO (PO IZNOSU) PROTUDJELOVANJE (REAKCIJA). DJELOVANJA DVA TIJELA JEDNO NA DRUGO UVIJEK SU JEDNAKA I PROTIVNOG SMJERA

-TIJELA NE MORAJU BITI U KONTAKTU = GRAVITACIJA



$$\vec{F}_N + \vec{F}_G = 0$$
$$\vec{F}_G = -\vec{F}_N$$

1. N. Z

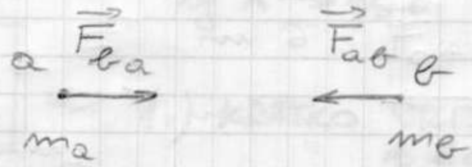


$$\vec{F}_N = -\vec{G}$$

3. N. Z.

$$-\vec{F}_G = -\vec{G} \qquad \vec{F}_G = \vec{G}$$

3. NEWTONOV ZAKON I OPERATIVNA DEFINICIJA MASE (MACH)



$$\vec{F}_{ba} = -\vec{F}_{ab}$$

$$m_a \cdot \vec{a}_a = \vec{F}_{ba}$$

$$m_b \cdot \vec{a}_b = \vec{F}_{ab}$$

$$m_a \cdot \vec{a}_a = \vec{F}_{ba} = -\vec{F}_{ab} = -m_b \cdot \vec{a}_b$$

$$m_a \cdot \vec{a}_a = -m_b \cdot \vec{a}_b \qquad |\vec{a}_a| = a \quad |\vec{a}_b| = a$$

$$m_a a_a = m_b a_b$$

$$m_a = m_b \frac{a_b}{a_a} \Big\} \text{ ODREDIMO EK}$$

$$m_b = 1\,kg$$

KOLIČINA GIBANJA I IMPULS SILE

I.) $\vec{F}$ – STALNA SILA Δt

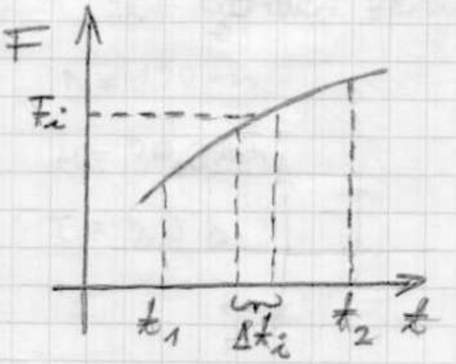
$$\boxed{\vec{I} = \vec{F} \cdot \Delta t}$$

↓ IMPULS SILE ZA STALNU SILU

II.) $\vec{F} = f(t)$ – PROMJENJIVA SILA

$$\vec{I}_i = \vec{F}_i \cdot \Delta t_i$$

$$\vec{I} \approx \sum_{i=1}^{n} \vec{F}_i \cdot \Delta t_i$$



$$\vec{I} = \lim_{\substack{\Delta t_i \to 0 \\ n \to \infty}} \sum_{i=1}^{n} \vec{F_i}\,\Delta t_i = \int_{t_1}^{t_2} \vec{F}(t)\,dt \qquad \boxed{\vec{I} = \int_{t_1}^{t_2} \vec{F}\,dt}$$

- IMPULS SILE JEDNAK JE INTEGRALU SILE PO VREMENU U KOJEMU DJELUJE TA SILA

$$\vec{F} = \frac{d\vec{p}}{dt} \qquad \frac{d\vec{p}}{dt} = \frac{d\vec{p}}{dt} \qquad \vec{I} = \int_{t_1}^{t_2} \vec{F}\,dt = \int_{p_1}^{p_2} d\vec{p} = \vec{p_2} - \vec{p_1} = \Delta\vec{p}$$

$$d\vec{p} = \frac{d\vec{p}}{dt} \cdot dt$$

$$d\vec{p} = \vec{F} \cdot dt$$

$$\boxed{\vec{I} = \int_{p_1}^{p_2} d\vec{p} = \Delta\vec{p}}$$

- IMPULS SILE JEDNAK JE PROMJENI KOLIČINE GIBANJA TIJELA NA KOJE DJELUJE TA SILA

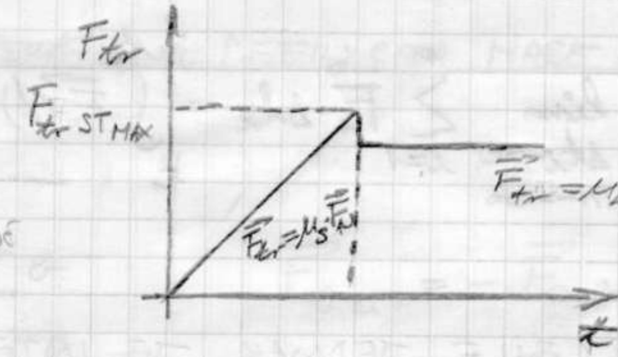
IMPULSNA SILA ($\neq$ IMPULS SILE)

- SILA KOJA VRLO KRATKO DJELUJE NA TIJELO I VRLO SE NEPRAVILNO MIJENJA $\to$ IZRAŽAVANJE PROSJEČNIM IZNOSOM
- VRLO VELIK IZNOS $\to$ DOMINANTNE (NPR. SUDAR AUTOMOBILA)

$$\boxed{\overline{\vec{F}} = \frac{1}{\Delta t}\int_{t_1}^{t_2} \vec{F}\,dt = \frac{\Delta\vec{p}}{\Delta t}}$$

$$\Delta t = t_2 - t_1$$

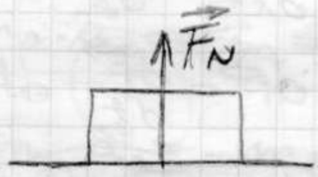
SILA TRENJA



TRENJE – VANJSKO
– UNUTRAŠNJE

VANJSKO – KLIZANJA – STATIČKO $\vec{F}_{ST}$
– KOTRLJANJA – DINAMIČKO $\vec{F}_D$, $\vec{F}_k$
(KINETIČKO)

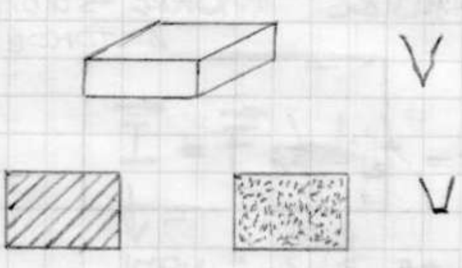
$$\boxed{\begin{gathered}\vec{F}_{ST_{MAX}} = \mu_S \cdot \vec{F}_N \\ \vec{F}_D = \mu_D \cdot \vec{F}_N\end{gathered}}$$



$$\underbrace{\mu_D < \mu_S}$$

– OVISE O MNOGIM FAKTORIMA, ALI GE-
RALNO NE OVISE O POVRŠINI (ZA
MATERIJALE OVISE – NPR. GUMA)

– DODIRNA POVRŠINA – MALA EFEKTIVNA POVRŠINA
(„ŠILJCI")



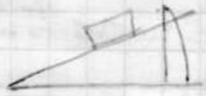
– ŠTO JE VEĆA DODIRNA POVRŠINA
MANJI JE TLAK PA SE „ŠILJCI"
DEFORMIRAJU

– ŠTO JE MANJA DODIRNA POVRŠINA
JE VEĆI, PA SE „ŠILJCI" VIŠE
DEFORMIRAJU

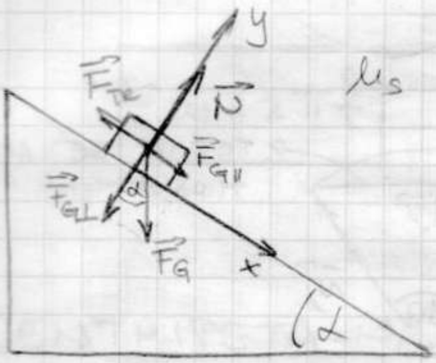
– POSLJEDICA JE TO DA SU EFEK-
TIVNE POVRŠINE ISTE U OBA SLUČA-
JA – ZATO μ NE OVISI O POVRŠ

POKUS: ODREĐIVANJE KOEFICIJENTA TRENJA (KLIZANJA)

$$\mathrm{tg}\,\alpha = \mu_S$$



- POMIČE DASKU DOK TIJELO (DRVENI KVADAR) NE POČNE KLIZATI



μ_S

$$\vec{F}_{TR} + \vec{N} + \vec{F}_G = 0$$

$\underline{x}$ $\quad mg\sin\alpha - \mu_S N = 0$

$\underline{y}$ $\quad N - mg\cos\alpha = 0$

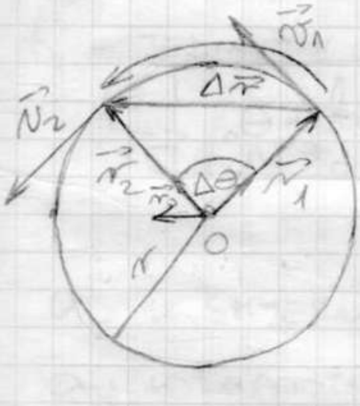
$$N = mg\cos\alpha$$

$$\mu_S = \frac{mg\sin\alpha}{N} = \frac{\cancel{mg}\sin\alpha}{\cancel{mg}\cos\alpha} = \mathrm{tg}\,\alpha$$

$$\boxed{\mu_S = \mathrm{tg}\,\alpha}$$

KRUŽNO GIBANJE

I.) JEDNOLIKO KRUŽNO GIBANJE



Δt

$S = r \cdot \theta$ → U RADIJANIMA

$\vec{r}_0$ - JEDINIČNI VEKTOR KOJI GLEDA IZ SREDIŠTA KRUŽNICE PREMA VAN

$$\vec{r}_1 = r\,\vec{r}_0$$

$$\vec{r}_2 = r\,\vec{r}_0$$

$$\Delta\vec{r} = \vec{r}_2 - \vec{r}_1$$

$$|\vec{r}_1| = |\vec{r}_2| = r \qquad |\Delta\vec{r}| \approx r\,\Delta\theta$$

$$v = \lim_{\Delta t \to 0} \frac{|\Delta\vec{r}|}{\Delta t} = \lim_{\Delta t \to 0} \frac{r\,\Delta\theta}{\Delta t} = r \underbrace{\lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t}}_{\omega} = r\omega$$

$$\boxed{v = r\omega}$$

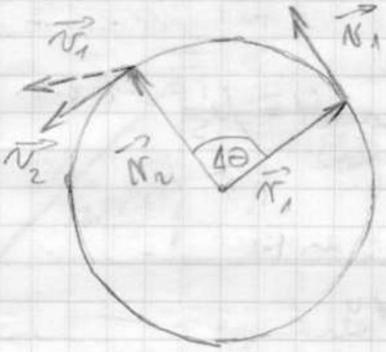
ω → KUTNA BRZINA

$\omega = 5\ s^{-1}$

v → OBODNA BRZINA

28.

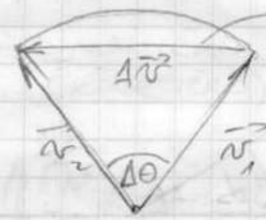
$$\boxed{\vec{v} = \vec{\omega} \times \vec{r}}$$



$$a_r = \lim_{\Delta t \to 0} \frac{|\Delta \vec{v}|}{\Delta t} =$$

$$= \lim_{\Delta t \to 0} \frac{v \Delta\theta}{\Delta t} =$$

$$= v \underbrace{\lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t}}_{\omega} = v\omega$$



$|\Delta\vec{v}| \approx v\Delta\theta$

$v = |\vec{v}_1| = |\vec{v}_2|$

$$\boxed{a_r = v\omega} \qquad \boxed{\vec{a}_r = \vec{\omega} \times \vec{v}}$$

$\theta(t) = ?$

$\omega =$ konst.

$$\boxed{\omega = \frac{d\theta}{dt}}$$

$t = 0$

$\theta = \theta_0$

$$\omega = \frac{d\theta}{dt}$$

$$\boxed{\theta = \omega t + \theta_0}$$

$$d\theta = \omega\, dt \quad / \int$$

$$\int d\theta = \int \omega\, dt$$

$$\theta = \omega \int dt$$

$$\theta = \omega t + c$$

$$c = \theta_0$$

$$\theta = \omega t + \theta_0$$

- OMJER BROJA OKRETA I VREMENA (BROJ OKRETA U SEKUNDI) JE FREKVENCIJA f, DOK JE OPHODNO VRIJEME T ONO VRIJEME KOJE JE POTREBNO DA MATERIJALNA TOČKA JEDANPUT OBIĐE KRUŽNICU

$$\boxed{\omega = 2\pi f} \qquad \boxed{T = \frac{1}{f} = \frac{2\pi}{\omega}}$$

CENTRIPETALNA SILA

$$\boxed{\vec{F}_{cp} = m \cdot \vec{a}_r = -m\frac{v^2}{r}\cdot\vec{r}_0 = -m\omega^2 r\vec{r}_0}$$

II.) NEJEDNOLIKO KRUŽNO GIBANJE

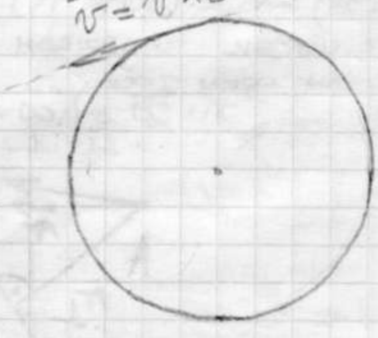
- IZNOS OBODNE BRZINE SE MIJENJA

$$a_T = \frac{dv}{dt} = \frac{d}{dt}(r\omega) = r\underbrace{\frac{d\omega}{dt}}_{\alpha} = r\alpha$$

α → UVIJEK IMA SMJER TANGENTE NA KRUŽNICU

$\vec{t}_0$ - JEDINIČNI VEKTOR TANGENTE

$$\vec{v} = v\vec{t}_0$$



a_T U SMJERU $\vec{t}_0$ - UBRZAVANJE

a_T U SUPROTNOM SMJERU OD $\vec{t}_0$ - USPORAVANJE

$\vec{\alpha}$ U SMJERU $\vec{\omega}$ - UBRZAVANJE

$\vec{\alpha}$ U SUPROTNOM SMJERU OD $\vec{\omega}$ - USPORAVANJE

$$\boxed{a_T = r\alpha} \qquad \boxed{\vec{a}_T = \vec{\alpha}\times\vec{r}} \qquad \boxed{\vec{a} = \vec{a}_r + \vec{a}_T}$$

$$\boxed{a=\sqrt{a_r^2+a_\tau^2}=\sqrt{\frac{v^4}{r^2}+r^2\alpha^2}}$$

$\alpha = \text{konst.}$

$t=0$ $\quad \omega(t)=?$

$\theta=\theta_0$ $\quad \theta(t)=?$

$\omega=\omega_0$

$$\alpha=\frac{d\omega}{dt}$$

$$d\omega=\alpha dt \,/\int$$

$$\int d\omega=\int\alpha dt$$

$$\omega=\alpha\int dt$$

$$\omega=\alpha t+c$$

$$c=\omega_0$$

$$\omega=\alpha t+\omega_0$$

$$\frac{d\theta}{dt}=\omega=\alpha t+\omega_0$$

$$\frac{d\theta}{dt}=\frac{d\theta}{dt}$$

$$d\theta=\frac{d\theta}{dt}\cdot dt$$

$$d\theta=(\alpha t+\omega_0)dt$$

$$d\theta=\alpha t\,dt+\omega_0 dt \,/\int$$

$$\int d\theta=\int\alpha t\,dt+\int\omega_0 dt$$

$$\theta=\alpha\int t\,dt+\omega_0\int dt$$

$$\theta=\alpha t^2/2+\omega_0 t+c$$

$$c=\theta_0$$

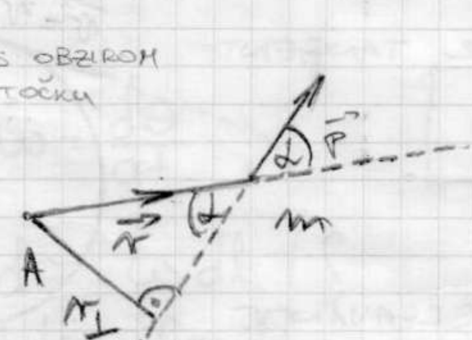
$$\theta=\alpha\frac{t^2}{2}+\omega_0 t+\theta_0$$

$$\boxed{\omega=\alpha t+\omega_0}$$

$$\boxed{\theta=\alpha\frac{t^2}{2}+\omega_0 t+\theta_0}$$

KUTNA KOLIČINA GIBANJA MATERIJALNE TOČKE

$\vec{L}_A$ → UVIJEK S OBZIROM NA NEKU TOČKU



$$\vec{p}=m\vec{v}$$

$$|\vec{p}|=p$$

$$|\vec{r}|=r$$

$$r_\perp=r\sin\alpha$$

$$\boxed{L_A=r_\perp p=r p\sin\alpha}$$

$$\boxed{\vec{L}=\vec{r}\times\vec{p}}$$ ⟹ OD $\vec{r}$ DO $\vec{p}$ KRAĆIM PUTEM

$$\frac{d\vec{L}}{dt} = \frac{d}{dt}(\vec{r}\times\vec{p}) = \underbrace{\frac{d\vec{r}}{dt}}_{=\vec{v}}\times\vec{p} + \vec{r}\times\underbrace{\frac{d\vec{p}}{dt}}_{\vec{F}=} = \vec{v}\times\vec{p}+\vec{r}\times\vec{F} = \vec{r}\times\vec{F}$$

$$\vec{v}\times\vec{p} = \vec{v}\times(m\cdot\vec{v}) = m(\underbrace{\vec{v}\times\vec{v}}_{0}) = 0$$

$$\frac{d\vec{L}}{dt} = \underbrace{\vec{r}\times\vec{F}}_{\vec{M}} \qquad \boxed{\vec{M} = \vec{r}\times\vec{F}}$$

$$\boxed{\frac{d\vec{L}}{dt} = \vec{M}}$$

$\omega = \text{konst.}$

$\vec{r}\times\vec{F}_{cp} = 0$

$\vec{M} = 0$

$\frac{d\vec{L}}{dt} = 0$

$\vec{r}, \vec{F}$ – CENTRALNE SILE

$$\vec{A}\times\vec{B}\times\vec{C} = \vec{B}\cdot(\vec{A}\cdot\vec{C}) - \vec{C}(\vec{A}\cdot\vec{B})$$

$$\vec{L} = \vec{r}\times\vec{p} = \vec{r}\times(m\cdot\vec{v}) = m(\vec{r}\times\vec{v}) = m(\vec{r}\times\vec{\omega}\times\vec{r})$$

$$\vec{r}\times\vec{\omega}\times\vec{r} = \vec{\omega}\cdot(\underbrace{\vec{r}\cdot\vec{r}}_{r^2}) - \vec{r}(\underbrace{\vec{r}\cdot\vec{\omega}}_{0}) = \vec{\omega}\cdot r^2$$

$\vec{L} = m\cdot\vec{\omega}\cdot r^2$

$\vec{L} = I\cdot\vec{\omega}$

$$\boxed{I = m\cdot r^2}$$

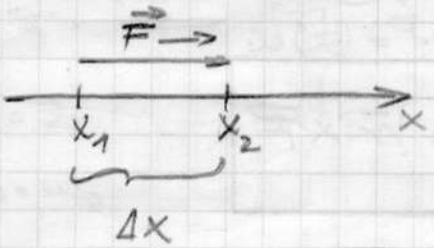
→ MOMENT TROMOSTI

$$\boxed{\vec{L} = I\cdot\vec{\omega}}$$

RAD I ENERGIJA

RAD

I.) STALNA SILA



$\vec{F} = F_x \vec{i}$ $F_x = konst$

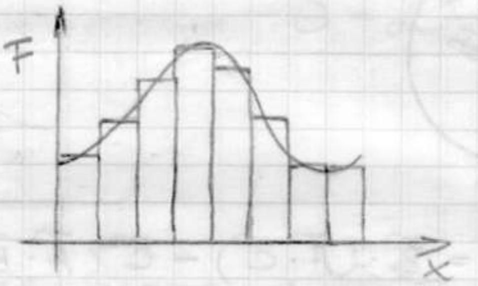
$F_x > 0$

$\Delta \vec{r} = \Delta x \vec{i}$

$$\boxed{W = F_x \Delta x}$$

- RAD JE JEDNAK UMNOŠKU SILE I POMAKU DUŽ KOJEG JE TA SILA DJELOVALA

II.) PROMJENJIVA SILA



$\vec{F} = F_x(x)\vec{i}$

$N,\ \Delta x_i,\ F_i$

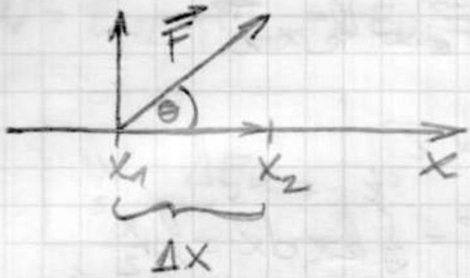
$\Delta W = F_i \cdot \Delta x_i$

$$\boxed{W = \int_{x_1}^{x_2} F_x(x)\,dx}$$

$$W \approx \sum_{i=1}^{N} F_i \cdot \Delta x_i$$

$$W = \lim_{\substack{N \to \infty \\ \Delta x_i \to 0}} \sum_{i=1}^{N} F_i \cdot \Delta x_i \doteq \int_{x_1}^{x_2} F_x(x)\,dx$$

III.) SILA DJELUJE POD KUTOM



$$\boxed{W = \vec{F} \cdot \Delta \vec{r}}$$

$$F = |\vec{F}|$$

$$\Delta \vec{r} = \Delta x \, \vec{i}$$

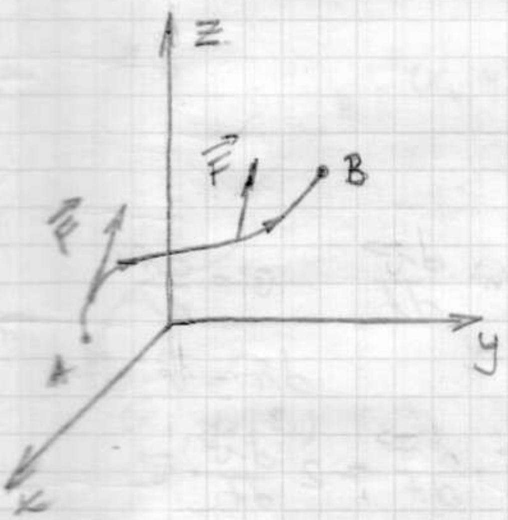
$$W = F \cdot \cos\theta \cdot \Delta x = \vec{F} \cdot \Delta \vec{r}$$

$$\theta < 90^\circ \Rightarrow W > 0$$

$$\theta = 90^\circ \Rightarrow W = 0$$

$$\theta > 90^\circ \Rightarrow W < 0$$

IV. RAD U NAJOPĆENITIJEM SLUČAJU



$$N, \; \Delta \vec{r}_i, \; \vec{F}_i$$

$$\boxed{W = \int_{\widehat{AB}} \vec{F} d\vec{r}}$$

$$W_i = \vec{F}_i \cdot \Delta \vec{r}_i$$

$$W \approx \sum_{i=1}^{N} \vec{F}_i \Delta \vec{r}_i$$

$$W = \lim_{\substack{\Delta r_i \to 0 \\ N \to \infty}} \sum_{i=1}^{N} \vec{F}_i \cdot \Delta \vec{r}_i = \int_{\widehat{AB}} \vec{F} \cdot d\vec{r}$$

- RAD JE KRIVULJNI INTEGRAL SILE UZDUŽ PUTANJE TIJELA OD POČETNE DO KRAJNJE TOČKE

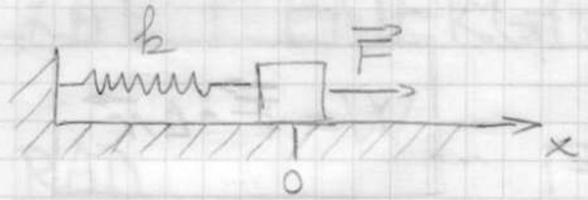
$$\vec{F}_G = -mg\vec{j}$$

$$\vec{F}_D = -\vec{F}_G = mg\vec{j}$$

$$d\vec{r} = dy\,\vec{j}$$

$$W = \int_0^h mg\vec{j}\, dy\vec{j} = mg\int_0^h dy = mgh$$

$$W_G = -mgh$$



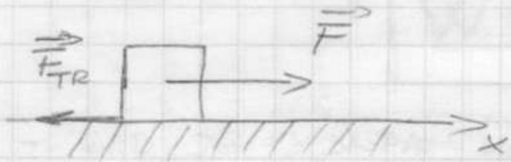
$$\vec{F}_E = -kx\vec{i}$$

$$\vec{F} = -\vec{F}_E = kx\vec{i}$$

$$d\vec{r} = dx\vec{i}$$

$$W = \int_0^x \vec{F}\cdot d\vec{r} = \int_0^x kx\vec{i}\cdot dx\vec{i} = \int_0^x kx\,dx = k\frac{x^2}{2}$$

$$W_E = -k\frac{x^2}{2}$$

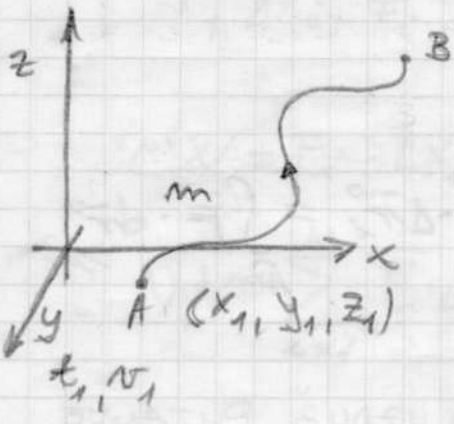


$$\vec{F} = -\vec{F}_{TR}$$

$$\vec{F} = -\vec{F}_{TR} = \mu\cdot\vec{F}_N = \mu\cdot m\vec{g}$$

$$W = \mu m g \Delta x$$

KINETIČKA ENERGIJA



$$\vec{F} = m\frac{d\vec{v}}{dt} \qquad \vec{v} = \frac{d\vec{r}}{dt}$$

$$d\vec{r} = \vec{v}\cdot dt$$

$$\frac{d}{dt}(\vec{v}\cdot\vec{v}) = \frac{d\vec{v}}{dt}\cdot\vec{v} + \vec{v}\cdot\frac{d\vec{v}}{dt} = 2\frac{d\vec{v}}{dt}\cdot\vec{v}$$

$$\frac{d}{dt}\cdot(\vec{v}^2) = 2\frac{d\vec{v}}{dt}\cdot\vec{v} \Rightarrow \frac{d\vec{v}}{dt}\cdot\vec{v} = \frac{1}{2}\frac{d}{dt}(\vec{v}^2)$$

$$W = \int_{\widehat{AB}}\vec{F}\cdot d\vec{r} = \int_{\widehat{AB}} m\cdot\frac{d\vec{v}}{dt}\cdot\vec{v}\,dt = \int_{\widehat{AB}} m\frac{1}{2}\frac{d(\vec{v}^2)}{\cancel{dt}}\cdot\cancel{dt} = m\frac{1}{2}\int_{\widehat{AB}} d(\vec{v}^2)$$

$$= m\frac{1}{2}\Delta v^2 = m\frac{1}{2}(v_2^2 - v_1^2) = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$$

$$\boxed{W = \frac{1}{2}m\Delta v^2 = \frac{1}{2}m(v_2^2 - v_1^2) = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2}$$

$$W = \frac{1}{2} m v_2^2 - \frac{1}{2} m v_1^2$$

$$\boxed{E_k = \frac{1}{2} m v^2}$$

$$W = E_{k2} - E_{k1} = \Delta E_k$$

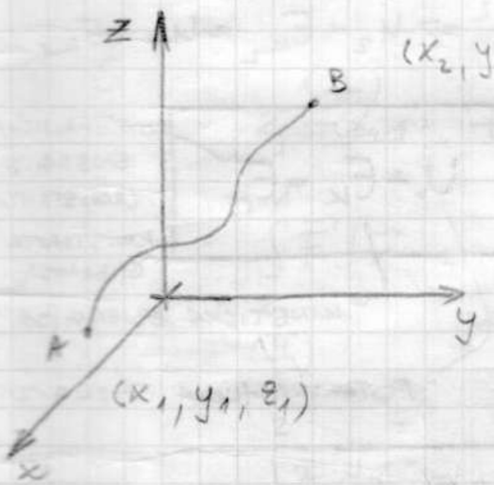
$$E_{k2} = \frac{1}{2} m v_2^2 \quad E_{k1} = \frac{1}{2} m v_1^2$$

$$\boxed{W = \Delta E_k}$$

ZAKON O RADU I KINETIČKOJ ENERGIJI - PROMJENA KINETIČKE ENERGIJE JEDNAKA JE RADU ŠTO JE NA ČESTICI IZVRŠI REZULTANTNA SILA

POTENCIJALNA ENERGIJA

GRAVITACIJSKA POTENCIJALNA ENERGIJA



$$\vec{F}_G = -mg\vec{k}$$

$$d\vec{r} = dx\vec{i} + dy\vec{j} + dz\vec{k}$$

$$W = \int_{\widehat{AB}} \vec{F}_G d\vec{r} = \int_{\widehat{AB}} -mg\vec{k} \cdot (dx\vec{i} + dy\vec{j} + dz\vec{k}) =$$

$$= \int_{\widehat{AB}} -\underbrace{mg\vec{k}dx\vec{i}}_{0} - \underbrace{mg\vec{k}dy\vec{j}}_{0} - mg\vec{k}dz\vec{k} =$$

$$= \int_{\widehat{AB}} -mg\,dz = -mg \int_{\widehat{AB}} dz = -mg\Delta z =$$

$$= -mg(z_2 - z_1) = -mgz_2 + mgz_1$$

$$U = mgz$$

$$U_1 = mgz_1$$

$$U_2 = mgz_2$$

$$W = -mgz_2 + mgz_1 = -U_2 + U_1 = -(U_2 - U_1) = -\Delta U$$

$$\boxed{W = -\Delta U}$$

ZAKON O RADU I POTENCIJALNOJ ENERGIJI - PROMJENA GRAV. POTENC. ENER. IZMEĐU POČETNE I KONAČNE TOČKE JEDNAK JE NEGATIVNOM RADU SILE TEŽE NA ČESTICU KOJA SE KREĆE IZMEĐU TE 2 TOČKE

$U = mgz + c \qquad z = \infty, \; U = 0 \; ?$

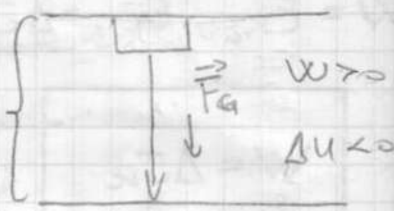
$z = 0, \; U = 0, \; c = 0$

$\vec{F}_G = -mg\vec{j} \qquad d\vec{r} = dh\vec{j}$

$W = \int_h^0 \vec{F}_G \cdot d\vec{r} = \int_h^0 -mg\vec{j} \cdot dh\vec{j}$

$= \int_h^0 -mg\,dh = -mg\int_h^0 dh = -mgh\Big|_h^0$

$= -(-mgh) = mgh \qquad W = mgh \qquad W = -\Delta U$



1.) $(z_1, v_1) \qquad W = -\Delta U \qquad U = mgh$

2.) $(z_2, v_2) \qquad W = \Delta E_k \qquad E_k = \frac{1}{2}mv^2$

$\Delta U = mgh_2 - mgh_1 = -W = -\Delta E_k = -(\frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2)$

$mgh_2 - mgh_1 = -\frac{1}{2}mv_2^2 + \frac{1}{2}mv_1^2$

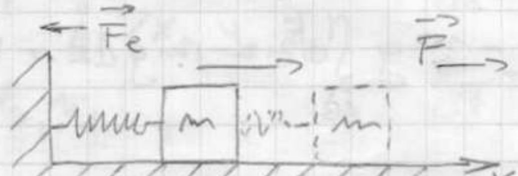
$mgh_2 + \frac{1}{2}mv_2^2 = mgh_1 + \frac{1}{2}mv_1^2 \Rightarrow U_2 + E_{k_2} = U_1 + E_{k_1}$

$\boxed{U_2 + E_{k_2} = U_1 + E_{k_1}}$ $\Rightarrow$ ZAKON OČUVANJA ENERGIJE ZA GRAVITACIJSKU SILU

$\boxed{U + E_k = E_M}$ — MEHANIČKA ENERGIJA (KONSTANTNA) — KONSTANTA GIBANJA

E_k → KINETIČKA ENERGIJA

U → POTENCIJALNA ENERGIJA



$W_e = -\int_{x_1}^{x_2} kx\vec{i}\,dx\vec{i} = -k\int_{x_1}^{x_2} x\,dx =$

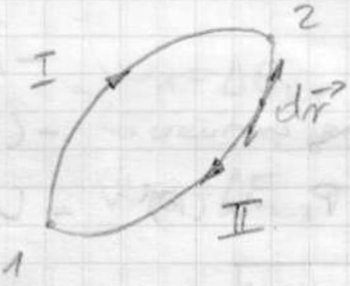
$= -k\frac{x^2}{2}\Big|_{x_1}^{x_2} = -k\frac{x_2^2}{2} + k\frac{x_1^2}{2} =$

$= -\frac{k}{2}\left(\frac{x_2^2}{2} - \frac{x_1^2}{2}\right)$

$W = -\Delta U$

KONZERVATIVE SILE

- SILE ČIJI RAD OVISI SAMO O POLOŽAJU POČETNE I KONAČNE TOČKE, A NE O OBLIKU PUTANJE IZMEĐU NJIH



- GRAVITACIJA
- ELASTIČNA SILA

- SILE ČIJI JE RAD PO ZATVORENOJ KRIVULJI JEDNAK NULI.

$$\mathrm{I}\quad \underset{(\mathrm{I})}{\int_1^2} \vec{F}\,d\vec{r} = -\underset{(\mathrm{I})}{\int_2^1} \vec{F}\,d\vec{r} \qquad \mathrm{II}\quad \underset{(\mathrm{II})}{\int_1^2} \vec{F}\,d\vec{r} = -\underset{(\mathrm{II})}{\int_2^1} \vec{F}\,d\vec{r}$$

$$\overset{(\mathrm{II})}{2 \to 1} + \overset{(\mathrm{I})}{1 \to 2} = 0$$

$$\underset{(\mathrm{II})}{\int_2^1} \vec{F}\,d\vec{r} + \underset{(\mathrm{I})}{\int_1^2} \vec{F}\,d\vec{r} = 0$$

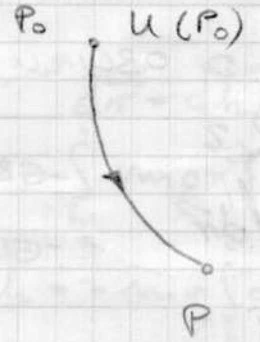
$$\underset{(\mathrm{I})}{\int_1^2} \vec{F}\,d\vec{r} = -\underset{(\mathrm{II})}{\int_2^1} \vec{F}\,d\vec{r}$$

$$\underset{(\mathrm{I})}{\int_1^2} \vec{F}\,d\vec{r} = \underset{(\mathrm{II})}{\int_1^2} \vec{F}\,d\vec{r}$$

- NA MIKROSKOPSKOM NIVOU SVE SU SILE KONZERVATIVNE

POTENCIJALNE ENERGIJE ZA KONZERVATIVNE SILE

$\vec{F}$



$$W = -\Delta U \qquad W = \int_{AB} \vec{F}\,d\vec{r}$$

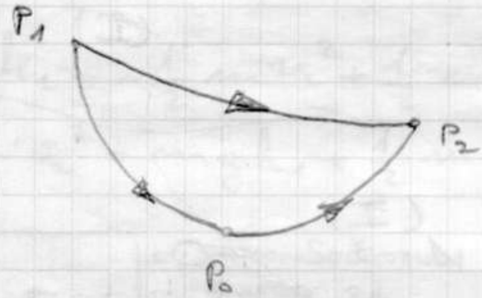
$$-\Delta U = W$$

$$-(U(P) - U(P_0)) = W$$

$$-U(P) + U(P_0) = \int_{P_0}^{P} \vec{F}\,d\vec{r}$$

$$U(P) = -\int_{P_0}^{P} \vec{F}\,d\vec{r} + U(P_0)$$

$$\boxed{U(P_x) = -\int_{P_0}^{P_x} \vec{F}\,d\vec{r} + U(P_0)}$$



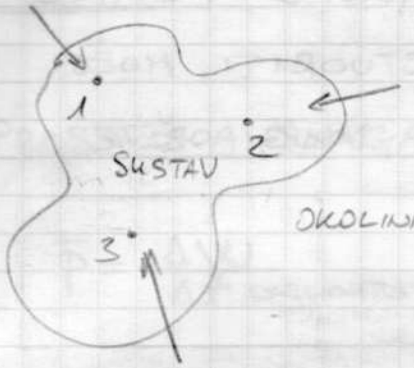
$$\underbrace{U(P_2) - U(P_1)}_{\Delta U} = -\int_{P_0}^{P_2} \vec{F}\,d\vec{r} + U(P_0) - \left(-\int_{P_0}^{P_1} \vec{F}\,d\vec{r} + U(P_0)\right) =$$

$$= -\int_{P_0}^{P_2} \vec{F}\,d\vec{r} + \cancel{U(P_0)} + \int_{P_0}^{P_1} \vec{F}\,d\vec{r} - \cancel{U(P_0)} =$$

$$= -\int_{P_0}^{P_2} \vec{F}\,d\vec{r} - \int_{P_1}^{P_0} \vec{F}\,d\vec{r} = -\left(\int_{P_1}^{P_0} \vec{F}\,d\vec{r} + \int_{P_0}^{P_2} \vec{F}\,d\vec{r}\right) =$$

$$= -\int_{P_1}^{P_2} \vec{F}\,d\vec{r} = -W \qquad \Delta U = -W$$

ZAKON OČUVANJA ENERGIJE



$$W = W_{KS} + W_{VS}$$

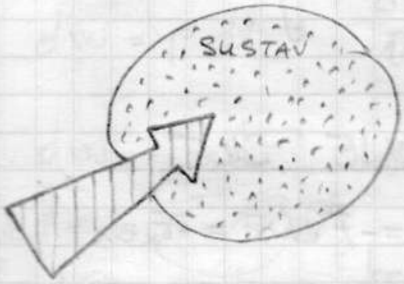
$$W = W_1 + W_2 + W_3$$

$$W_1 = \Delta E_{K1} = W_{1KS} + W_{1VS} = -\Delta U_1 + W_{1VS}$$

↳ UKUPAN RAD NA 1. ČESTICU

$$W_2 = \Delta E_{K2} = \ldots = -\Delta U_2 + W_{2VS}$$

$$W_3 = \Delta E_{K3} = \ldots = -\Delta U_3 + W_{3VS}$$



W_{KS} - RAD KONZERVATIVNIH SILA UNUTAR SUSTAVA

W - UKUPAN RAD

W_{VS} - RAD VANJSKIH SILA

$$\Delta E_{K1} + \Delta E_{K2} + \Delta E_{K3} = -\Delta U_1 + W_{1VS} - \Delta U_2 + W_{2VS} - \Delta U_3 + W_{3VS}$$

$$\Delta E_{K1} + \Delta U_1 + \Delta E_{K2} + \Delta U_2 + \Delta E_{K3} + \Delta U_3 = W_{1VS} + W_{2VS} + W_{3VS}$$

$$\Delta(\underbrace{E_{K1} + U_1}_{E_{M1}}) + \Delta(\underbrace{E_{K2} + U_2}_{E_{M2}}) + \Delta(\underbrace{E_{K3} + U_3}_{E_{M3}}) = W_{VS}$$

$$\Delta E_{M1} + \Delta E_{M2} + \Delta E_{M3} = W_{VS}$$

$$\boxed{\Delta E_{MH} = W_{VS}}$$

$$W_{VS} > 0 \iff \Delta E_{MH} < 0$$

$$W_{VS} < 0 \iff \Delta E_{MH} > 0$$

- MEHANIČKA ENERGIJA ZATVORENOG SUSTAVA ČESTICA U KOJEM DJELUJU SAMO KONZERVATIVNE SILE JE STALNA

$$\boxed{\Delta E = 0}$$

40.

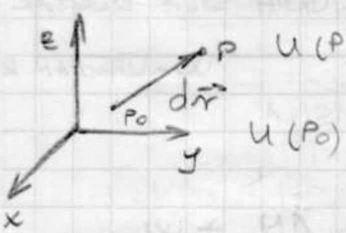
ZAKON OČUVANJA ENERGIJE - UKUPNA ENERGIJA NE MOŽE SE UNIŠTITI, NITI IZ ČEGA STVORITI. MOŽE SE SAMO PRETVARATI IZ JEDNOG U DRUGI OBLIK

BETA RASPAD

$n \rightarrow p + e^- + \nu_e$ → NEUTRINO ELEKTRONSKI

$$[W] = [E] = J \qquad 1\,eV = 1{,}6 \cdot 10^{-19}\,J$$

PRORAČUN ENERGIJE IZ KONZERVATIVNE SILE



$$U(P) - U(P_0) = dU = -dW = -\vec{F}\,d\vec{r} = -F_x dx - F_y d\ldots$$

$$-\vec{F} = -F_x\vec{i} - F_y\vec{j} - F_z\vec{k}$$

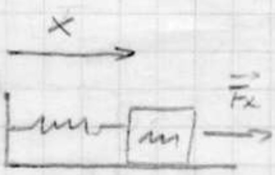
$$d\vec{r} = dx\,\vec{i} + dy\,\vec{j} + dz\,\vec{k}$$

- KAD SE POMNOŽ... PREŽIVI

$-F_x dx - F_y dy - \ldots$

$$-\vec{F}\,d\vec{r} = -F_x dx - F_y dy - F_z dz$$

$$F_x = -\frac{dU}{dx}\Big|_{y,z = konst.}$$



$$dU = -F_x dx$$

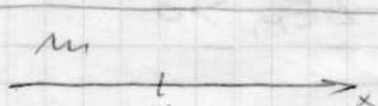
$$dU = \frac{\partial U}{\partial x}\,dx$$

$$= -\frac{\partial U}{\partial x} \qquad F_x = -\frac{\partial U}{\partial x}$$

$$F_y = -\frac{\partial U}{\partial y}$$

$$F_z = -\frac{\partial U}{\partial z}$$

$$F_x = -\frac{\partial}{\partial x}\left(\frac{1}{2}kx^2\right) = -kx$$



$U(x)$

$$E = \frac{1}{2}mv^2 + U(x)$$

$$E - U(x) = \frac{1}{2}mv^2 \Rightarrow v^2 = \frac{2}{m}(E - U(x))$$

$$\boxed{v^2 = \frac{2}{m}(E - U(x))}$$

$$\boxed{v = \sqrt{\frac{2}{m}(E - U(x))}}$$

$$v^2 \geq 0$$

$$E - U(x) \geq 0$$

$$\left.\begin{aligned} v &= 0 \\ E - U(x) &= 0 \end{aligned}\right\}\ \text{TOČKE OKRETA}$$

SNAGA - OMJER RADA I VREMENA

PROSJEČNA SNAGA

$$\boxed{\overline{P} = \frac{\Delta W}{\Delta t}}$$

TRENUTNA SNAGA

$$P = \lim_{\Delta t \to 0} \frac{\Delta W}{\Delta t} = \frac{dW}{dt}$$

$$\boxed{P = \frac{dW}{dt}} \quad [P] = W$$

$$dW = \frac{dW}{dt} \cdot dt = P\,dt$$

$$dW = P\,dt \qquad \vec{v} = \frac{d\vec{r}}{dt}$$

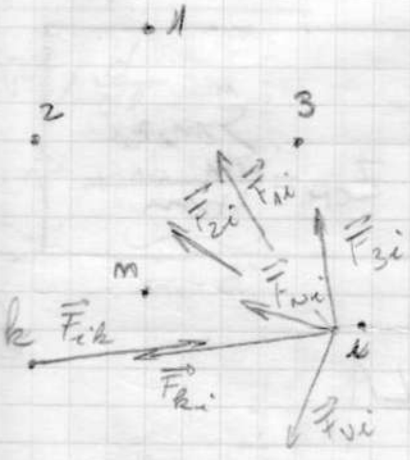
$$dW = \vec{F} \cdot d\vec{r} \qquad d\vec{r} = \vec{v}\,dt$$

$$dW = \vec{F} \cdot \vec{v}\,dt$$

$$\frac{dW}{dt} = \vec{F} \cdot \vec{v}$$

$$P = \vec{F} \cdot \vec{v} \qquad \boxed{P = \vec{F} \cdot \vec{v}}$$

SUSTAV MATERIJALNIH TOČAKA



$$\vec{F}_{ki} = -\vec{F}_{ik}$$

$$\frac{d\vec{p}_1}{dt} = \vec{F}_{v_1} + \vec{F}_{21} + \vec{F}_{31} + \dots + \vec{F}_{n1}$$

$$\frac{d\vec{p}_2}{dt} = \vec{F}_{v_2} + \vec{F}_{12} + \vec{F}_{32} + \dots + \vec{F}_{n2}$$

$$\vdots$$

$$\sum_{i=1}^{n} \frac{d\vec{p}_i}{dt} = \sum_{i=1}^{n} \vec{F}_{vi} + \underbrace{\sum_{\substack{i,j=1 \\ i \neq j}}^{n} \vec{F}_{ij}}_{\vec{F}_{ij} + \vec{F}_{ji} = 0}$$

$$\boxed{\sum_{\substack{i,j=1 \\ i \neq j}}^{n} \vec{F}_{ij} = 0}$$

$$\boxed{\sum_{i=1}^{n} \frac{d\vec{P}_i}{dt} = \vec{F}_v} \qquad \boxed{\vec{F}_v = \sum_{i=1}^{n} \vec{F}_{v_i}}$$

CENTAR MASE - SREDNJI POLOŽAJ MASE SUSTAVA

n

1 $\quad m_1 \; (x_1, y_1, z_1)$

2 $\quad m_2 \; (x_2, y_2, z_2)$

$\vdots$

$n \quad m_n \; (x_n, y_n, z_n)$

$$x_{CM} = \frac{m_1 x_1 + m_2 x_2 + \ldots + m_n x_n}{\underbrace{m_1 + m_2 + \ldots + m_n}_{m}} =$$

$$y_{CM} = \frac{\sum_{i=1}^{n} m_i y_i}{m} \qquad z_{CM} = \frac{\sum_{i=1}^{n} m_i}{m}$$

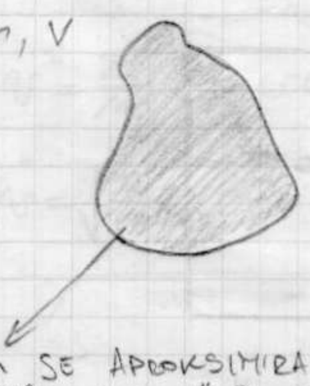
$$\boxed{\vec{r}_{CM} = \frac{\sum_{i=1}^{n} m_i \vec{r}_i}{m}}$$

$$U_i = m_i \cdot g z_i \qquad U = \sum_{i=1}^{n} m_i \cdot g z_i = g \sum_{i=1}^{n} m_i z_i = m g z_{CM}$$

$$\boxed{x_{CM} = \frac{\sum_{i=1}^{n} m_i x_i}{m}} \qquad \boxed{y_{CM} = \frac{\sum_{i=1}^{n} m_i y_i}{m}} \qquad \boxed{z_{CM} = \frac{\sum_{i=1}^{n} m_i z_i}{m}}$$

- CENTAR MASE ČVRSTOG TIJELA

m, V



$\Delta V, \; \Delta m$

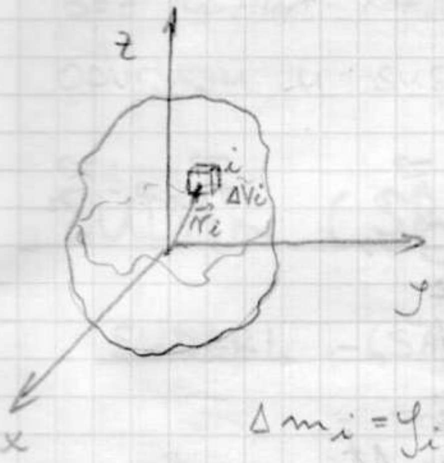
$$\boxed{\bar{\rho} = \frac{\Delta m}{\Delta V}} \qquad \rho = \lim_{\Delta V \to 0} \frac{\Delta m}{\Delta V} = \frac{dm}{dV}$$

$$dm = \rho \, dV \qquad \boxed{m = \int_V \rho \, dV}$$

$$m = \int_V \rho \, dV$$

m SE APROKSIMIRA KAO DA JE „RAZMAZANA" PO TIJELU, A NE DISKRETNO RASPODIJELJENA

V, m, ρ $\quad N, \Delta V_i, \Delta m_i, \rho_i$ $\quad \Delta m = \rho \cdot \Delta V$



$$\vec{r}_{CM} \approx \frac{\sum_{i=1}^{n} \Delta m_i \cdot \vec{r}_i}{m} \approx \frac{\sum_{i=1}^{n} \rho_i \vec{r}_i \cdot \Delta V_i}{m}$$

$$\vec{r}_{CM} = \lim_{\Delta V_i \to 0} \frac{\sum_{i=1}^{n} \rho_i \vec{r}_i \cdot \Delta V_i}{m} = \frac{\int_V \rho \vec{r} \, dV}{m}$$

$\Delta m_i = \rho_i \Delta V_i$

$$\boxed{\vec{r}_{CM} = \frac{\int_V \rho \vec{r} \, dV}{m}}$$

$m_i = \text{konst.}$

$$\sum_{i=1}^{N} m_i \vec{a}_i = \vec{F}_v$$

$$\Updownarrow$$

$m \cdot \vec{r}_i \,/\, \frac{d}{dt}$

$\underbrace{\frac{dm}{dt} \cdot \vec{r}}_{0} + \underbrace{m \cdot \frac{d\vec{r}}{dt}}_{m \cdot \vec{v}}$

$m \cdot \vec{v}$

$$m \vec{r}_{CM} = \sum_{i=1}^{n} m_i \vec{r}_i \,/\, \frac{d}{dt}$$

$$m \cdot \vec{v}_{CM} = \sum_{i=1}^{n} m_i \cdot \vec{v}_i \,/\, \frac{d}{dt}$$

$$m \cdot \vec{a}_{CM} = \sum_{i=1}^{n} m_i \cdot \vec{a}_i = \vec{F}_v$$

$m \vec{v} \,/\, \frac{d}{dt}$

$\underbrace{\frac{dm}{dt} \cdot \vec{v}}_{0} + \underbrace{m \cdot \frac{d\vec{v}}{dt}}_{m \cdot \vec{a}}$

$m \cdot \vec{a}$

$$\boxed{m \cdot \vec{a}_{CM} = \vec{F}_v}$$

→ JEDNADŽBA GIBANJA ZA CENTAR MASE SUSTAVA OD N ČESTICA

$\vec{F}_v = 0 \qquad m \cdot \vec{a}_{CM} = 0$

$\vec{v}_0 = 0$

KRETANJE C.M.

$$\boxed{\begin{aligned} \vec{v}_{CM} &= \vec{v}_0 \\ \vec{r}_{CM} &= \vec{v}_0 t + \vec{r}_0 \end{aligned}}$$

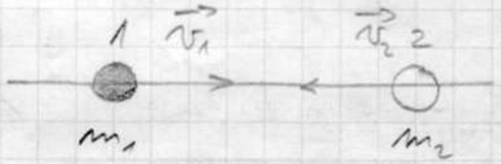
$\Downarrow$

CENTAR MASE NE MIJENJA PUTANJU (NPR. PRI EKSPLOZIJI)

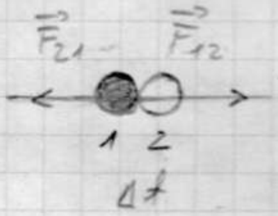
ZAKON OČUVANJA KOLIČINE GIBANJA

ZATVORENI, CENTRALNI SRAZ

1.)



2.)



$\vec{F}_{21} = -\vec{F}_{12}$

3.)

$\vec{I}_1 = \vec{F}_{21} \cdot \Delta t$

$\vec{I}_2 = \vec{F}_{12} \cdot \Delta t$

$\vec{I}_1 = -\vec{I}_2$

$\vec{I} = \Delta \vec{p} = (\vec{p}_2 - \vec{p}_1) = m\vec{v}' - m\vec{v}$

$\vec{I}_1 = m_1 \vec{v}_1' - m_1 \vec{v}_1$

$\vec{I}_2 = m_2 \vec{v}_2' - m_2 \vec{v}_2$

$$m_1 \vec{v}_1' - m_1 \vec{v}_1 = -(m_2 \vec{v}_2' - m_2 \vec{v}_2)$$

$$m_1 \vec{v}_1' - m_1 \vec{v}_1 = -m_2 \vec{v}_2' + m_2 \vec{v}_2$$

$$m_1 \vec{v}_1' + m_2 \vec{v}_2' = m_2 \vec{v}_2 + m_2 \vec{v}_1$$

$$\boxed{m_1 \vec{v}_1 + m_2 \vec{v}_2 = m_1 \vec{v}_1' + m_2 \vec{v}_2'}$$

n, ZATVOREN

$\vec{F}_v = 0$

$\sum_{i=1}^{n} \frac{d\vec{p}_i}{dt} = 0$

$\frac{d}{dt} \underbrace{\sum_{i=1}^{n} \vec{p}_i}_{\vec{p}} = 0$

$\frac{d}{dt} \vec{p} = 0$

$\frac{d\vec{p}}{dt} = 0$

$$\boxed{\frac{d\vec{p}}{dt} = 0}$$

- AKO JE SUSTAV ZATVOREN UKUPNA KOLIČINA GIBANJA JE KONSTANTNA

- UKUPNA KOLIČINA GIBANJA SUSTAVA KONSTANTNA JE BEZ OBZIRA NA TO KAKVI SE PROCESI MEĐUDJELOVANJA ODVIJAJU U SUSTAVU

SUDAR (SRAZ)

SUDARI - (SAVRŠENO) ELASTIČNI
- TOTALNO NEELASTIČNI
- DJELOMIČNO ELASTIČNI

} ZAKON OČUVANJA p

→ NIJE NUŽNO DA DOĐE DO FIZIČKOG DODIRA

I.) (SAVRŠENO) ELASTIČNI SUDAR

- VRIJEDE ZAKONI - OČUVANJA $\vec{p}$ A)
- OČUVANJA E_k B)

- CENTRALNI SUDAR (POJEDNOSTAVLJENJE) - SVI VEKTORI POLOŽAJA SU NA ISTOM PRAVCU PRIJE I POSLIJE SUDARA)

A.) $m_1\vec{v}_1 + m_2\vec{v}_2 = m_1\vec{v}_1{}' + m_2\vec{v}_2{}'$

$$m_1\vec{v}_1 - m_1\vec{v}_1{}' = m_2\vec{v}_2{}' - m_2\vec{v}_2$$

$$m_1(\vec{v}_1 - \vec{v}_1{}') = m_2(\vec{v}_2{}' - \vec{v}_2)$$

$$m_1(\vec{v}_1 - \vec{v}_1{}') = -m_2(\vec{v}_2 - \vec{v}_2{}')$$

B.) $m_1\frac{v_1^2}{2} + m_2\frac{v_2^2}{2} = m_1\frac{v_1'^2}{2} + m_2\frac{v_2'^2}{2} \quad /\cdot 2$

$$m_1 v_1^2 + m_2 v_2^2 = m_1 v_1'^2 + m_2 v_2'^2$$

$$m_1 v_1^2 - m_1 v_1'^2 = m_2 v_2'^2 - m_2 v_2^2$$

$$m_1(v_1^2 - v_1'^2) = m_2(v_2'^2 - v_2^2)$$

$$m_1(v_1^2 - v_1'^2) = -m_2(v_2^2 - v_2'^2) \qquad a^2 - b^2 = (a-b)(a+b)$$

$$m_1(\vec{v}_1 - \vec{v}_1{}')(\vec{v}_1 + \vec{v}_1{}') = -m_2(\vec{v}_2 - \vec{v}_2{}')(\vec{v}_2 + \vec{v}_2{}')$$

$$-m_2(\vec{v}_2 - \vec{v}_2{}') = m_1(\vec{v}_1 - \vec{v}_1{}')$$

$$\cancel{m_1}(\vec{v}_1 - \vec{v}_1{}')(\vec{v}_1 + \vec{v}_1{}') = \cancel{m_1}(\vec{v}_1 - \vec{v}_1{}')(\vec{v}_2 + \vec{v}_2{}')$$

$$(\vec{v}_1+\vec{v}_1')\cdot(\vec{v}_1-\vec{v}_1') = (\vec{v}_2+\vec{v}_2')\cdot(\vec{v}_1-\vec{v}_1')$$

$$(\vec{v}_1+\vec{v}_1')\cdot(\vec{v}_1-\vec{v}_1') - (\vec{v}_2+\vec{v}_2')(\vec{v}_1-\vec{v}_1') = 0 \Rightarrow (\vec{v}_1-\vec{v}_1')(\vec{v}_1+\vec{v}_1'-\vec{v}_2-\vec{v}_2') = 0$$

$$\boxed{(\vec{v}_1-\vec{v}_1')(\vec{v}_1+\vec{v}_1'-\vec{v}_2-\vec{v}_2') = 0}$$

A) $\vec{v}_1-\vec{v}_1' = 0$

$\vec{v}_1 = \vec{v}_1' \Rightarrow$ SUDAR SE NIJE NITI DOGODIO

B) $\vec{v}_1+\vec{v}_1'-\vec{v}_2-\vec{v}_2' = 0$

$\vec{v}_1-\vec{v}_2 = \vec{v}_2'-\vec{v}_1'$

$\vec{v}_1-\vec{v}_2 = -(\vec{v}_1'-\vec{v}_2')$

$$\boxed{\vec{v}_1-\vec{v}_2 = -(\vec{v}_1'-\vec{v}_2')}$$

$\vec{v} = \vec{v}_2$

$S \xrightarrow{\vec{v}} S'$ REFERENTNO TIJELO (SUSTAV)

$\vec{v}_1 \qquad \vec{v}_1' = \vec{v}_1-\vec{v} = \vec{v}_1-\vec{v}_2$

$\vec{v}_2 \qquad \vec{v}_2' = \vec{v}_2-\vec{v}$

- RELATIVNA BRZINA PRIMICANJA KUGLICE PRIJE SUDARA JEDNAKA JE PO IZNOSU I SUPROTNA PO SMJERU RELATIVNOJ BRZINI ODMICANJA KUGLICE POSLIJE SUDARA
- RELATIVNE BRZINE PROMIJENILE SU SMJER.

$$m_1\vec{v}_1 + m_2\vec{v}_2 = m_1\vec{v}_1' + m_2\vec{v}_2'$$

$$\vec{v}_1-\vec{v}_2 = -(\vec{v}_1'-\vec{v}_2') \Rightarrow \vec{v}_1-\vec{v}_2 = -\vec{v}_1'+\vec{v}_2'$$

$$\vec{v}_1' = \vec{v}_2'+\vec{v}_2-\vec{v}_1$$

$$m_2\vec{v}_2' = m_1\vec{v}_1+m_2\vec{v}_2-m_1\vec{v}_1' \quad /:m_2$$

$$\vec{v}_2' = \frac{m_1\vec{v}_1+m_2\vec{v}_2-m_1\vec{v}_1'}{m_2}$$

$$\vec{v}_1' = \frac{m_1\vec{v}_1+m_2\vec{v}_2-m_1\vec{v}_1'}{m_2} + \vec{v}_2 - \vec{v}_1$$

$$= \frac{m_1\vec{v}_1+m_2\vec{v}_2-m_1\vec{v}_1'+m_2\vec{v}_2-m_2\vec{v}_1}{m_2}$$

$$= \frac{(m_1-m_2)\vec{v}_1+2m_2\vec{v}_2}{m_2} - \frac{m_1}{m_2}\vec{v}_1'$$

$$\vec{v}_1' + \frac{m_1}{m_2}\vec{v}_1' = \frac{(m_1 - m_2)\vec{v}_1 + 2m_2\vec{v}_2}{m_2}$$

$$\left(1+\frac{m_1}{m_2}\right)\vec{v}_1' = \frac{(m_1 - m_2)\vec{v}_1 + 2m_2\vec{v}_2}{m_2} \qquad 1+\frac{m_1}{m_2} = \frac{m_2+m_1}{m_2}$$

$$\vec{v}_1'\frac{m_2+m_1}{\cancel{m_2}} = \frac{(m_1 - m_2)\vec{v}_1 + 2m_2\vec{v}_2}{\cancel{m_2}}$$

$$\vec{v}_1' = \frac{(m_1 - m_2)\vec{v}_1 + 2m_2\vec{v}_2}{m_2+m_1} \quad //$$

$$\vec{v}_2' = \vec{v}_1' - \vec{v}_2 + \vec{v}_1$$

$$\vec{v}_2' = \frac{(m_1 - m_2)\vec{v}_1 + 2m_2\vec{v}_2}{m_2+m_1} - \vec{v}_2 + \vec{v}_1$$

$$= \frac{(m_1 - m_2)\vec{v}_1 + 2m_2\vec{v}_2 - (m_1+m_2)\vec{v}_2 + (m_1+m_2)\vec{v}_1}{m_1+m_2}$$

$$= \frac{(m_1 - \cancel{m_2} + m_1 + \cancel{m_2})\vec{v}_1 + (2m_2 - m_1 - \cancel{m_2})\vec{v}_2}{m_1+m_2}$$

$$= \frac{2m_1\vec{v}_1 + (m_2-m_1)\vec{v}_2}{m_1+m_2}$$

$$\boxed{\begin{aligned} \vec{v}_1' &= \frac{(m_1 - m_2)\vec{v}_1 + 2m_2\vec{v}_2}{m_1+m_2} \\ \vec{v}_2' &= \frac{(m_2 - m_1)\vec{v}_2 + 2m_1\vec{v}_1}{m_1+m_2} \end{aligned}}$$

POSEBNI SLUČAJEVI

1.) $m_1 = m_2 = m$

$$\vec{v}_1' = \frac{2\cancel{m}\vec{v}_2}{2\cancel{m}} = \vec{v}_2 \qquad \vec{v}_2' = \frac{2\cancel{m}\vec{v}_1}{2\cancel{m}} = \vec{v}_1$$

$$v_2 = 0 \Rightarrow \vec{v}_1' = 0 \qquad \vec{v}_2' = \vec{v}_1$$

2.) $m_1 \ll m_2 \qquad \frac{m_1}{m_2} \ll 1 \qquad \vec{v}_1' = \frac{\left(\frac{m_1}{m_2}-1\right)\vec{v}_1 + 2\vec{v}_2}{\frac{m_1}{m_2}+1} \approx -\vec{v}_1 + 2\vec{v}_2$

$$v_2 = 0 \Rightarrow \vec{v}_1' \approx -\vec{v}_1$$

$$\Rightarrow \vec{v}_2' \approx 0 \qquad \vec{v}_2' = \frac{\left(1-\frac{m_1}{m_2}\right)\vec{v}_2 + 2\frac{m_1}{m_2}\vec{v}_1}{\frac{m_1}{m_2}+1} \approx \vec{v}_2$$

3.) $m_1 >> m_2$

$\frac{m_2}{m_1} << 1$

$$\vec{v}_1' = \frac{\left(1-\frac{m_2}{m_1}\right)\vec{v}_1 + 2\frac{m_2}{m_1}\vec{v}_2}{1+\frac{m_2}{m_1}} \approx \vec{v}_1$$

$\vec{v}_2 = 0 \Rightarrow \vec{v}_1' \approx \vec{v}_1$

$\vec{v}_2' \approx 2\vec{v}_1$

$$\vec{v}_2' = \frac{\left(\frac{m_2}{m_1}-1\right)\cdot\vec{v}_2 + 2\vec{v}_1}{1+\frac{m_2}{m_1}} \approx -\vec{v}_2 + 2\vec{v}_1$$

II.) TOTALNO (SAVRŠENO) NEELASTIČNI SUDAR

- VRIJEDI ZAKON OČUVANJA $\vec{p}$

$\left.\begin{array}{l} m_1, v_1 \\ m_2, v_2 \end{array}\right\}$ PRIJE SUDARA

$(m_1+m_2)v'$ - POSLIJE SUDARA

$$m_1\vec{v}_1 + m_2\vec{v}_2 = (m_1+m_2)\vec{v}'$$

$$\vec{v}' = \frac{m_1\vec{v}_1 + m_2\vec{v}_2}{m_1+m_2}$$

$$\boxed{\vec{v}' = \frac{m_1\vec{v}_1 + m_2\vec{v}_2}{m_1+m_2}}$$

$$\boxed{g = E_k' - E_k}$$

$$g = \frac{1}{2}(m_1+m_2)v'^2 - \left(\frac{1}{2}m_1\cdot v_1^2 + \frac{1}{2}m_2 v_2^2\right) =$$

$$= \frac{1}{2}(m_1+m_2)\left(\frac{m_1\vec{v}_1+m_2\vec{v}_2}{m_1+m_2}\right)^2 - \frac{1}{2}m_1\vec{v}_1^2 - \frac{1}{2}m_2\vec{v}_2^2 =$$

$$= \frac{1}{2}\cancel{(m_1+m_2)}\frac{(m_1\vec{v}_1+m_2\vec{v}_2)^2}{(m_1+m_2)^{\cancel{2}}} - \frac{1}{2}(m_1\vec{v}_1^2 + m_2\vec{v}_2^2) =$$

$$= -\frac{1}{2}\left[-\frac{m_1^2\vec{v}_1^2 + 2m_1m_2\vec{v}_1^2\vec{v}_2^2 + m_2^2\vec{v}_2^2}{m_1+m_2} + m_1\vec{v}_1^2 + m_2\vec{v}_2^2\right] =$$

$$= -\frac{1}{2}\left[\frac{-m_1^2\vec{v}_1^2 - 2m_1m_2\vec{v}_1^2\vec{v}_2^2 - m_2^2\vec{v}_2^2 + (m_1+m_2)m_1\vec{v}_1^2 + (m_1+m_2)m_2\vec{v}_2^2}{m_1+m_2}\right]$$

$$= -\frac{1}{2}\left[\frac{-\cancel{m_1^2\vec{v}_1^2} - 2m_1m_2\vec{v}_1^2\vec{v}_2^2 - \cancel{m_2^2\vec{v}_2^2} + \cancel{m_1^2\vec{v}_1^2} + m_1m_2\vec{v}_1^2 + m_1m_2\vec{v}_2^2 + \cancel{m_2^2\vec{v}_2^2}}{m_1+m_2}\right]$$

$$= -\frac{1}{2}\left[\frac{-2m_1m_2\vec{v}_1^2\vec{v}_2^2 + m_1m_2\vec{v}_1^2 + m_1m_2\vec{v}_2^2}{m_1+m_2}\right] =$$

$$= -\frac{1}{2}\left[\frac{m_1m_2(-2\vec{v}_1^2\vec{v}_2^2 + \vec{v}_1^2 + \vec{v}_2^2)}{m_1+m_2}\right] = -\frac{1}{2}\frac{m_1m_2}{m_1+m_2}\left(\vec{v}_1^2 - 2\vec{v}_1^2\vec{v}_2^2 + \vec{v}_2^2\right)$$

$$= -\frac{m_1 m_2}{2(m_1+m_2)}(\vec{v}_1 - \vec{v}_2)^2$$

$$\boxed{g = -\frac{m_1 m_2}{2(m_1+m_2)}(\vec{v}_1 - \vec{v}_2)^2}$$

POSEBNI SLUČAJEVI

1.) $m_1 = m_2 = m$ $\qquad \vec{v}' = \frac{\cancel{m}(\vec{v}_1 + \vec{v}_2)}{2\cancel{m}} = \frac{\vec{v}_1 + \vec{v}_2}{2}$

$\vec{v}_2 = 0 \Rightarrow \vec{v}' = \frac{1}{2}\vec{v}_1$

$\vec{v}_1 = -\vec{v}_2 \Rightarrow \vec{v}' = 0$

2.) $m_1 << m_2$ $\qquad \frac{m_1}{m_2} << 1$ $\qquad \vec{v}' = \frac{\frac{m_1}{m_2}\vec{v}_1 + \vec{v}_2}{\frac{m_1}{m_2} + 1} \approx \vec{v}_2$

$\vec{v}_2 = 0 \Rightarrow \vec{v}' \approx 0$

3.) $m_1 >> m_2$ $\qquad \frac{m_2}{m_1} << 1$ $\qquad \vec{v}' = \frac{\vec{v}_1 + \frac{m_2}{m_1}\vec{v}_2}{1 + \frac{m_2}{m_1}} \approx \vec{v}_1$

POKUS: SAVRŠENO ELASTIČNI SUDAR

- 5 KUGLICA (ISTE MASE)

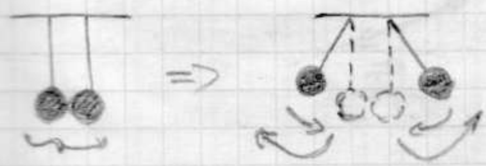
- ZAKON OČUVANJA KOLIČINE GIBANJA

SPECIJALNI SLUČAJ KAD SU MASE ISTE

$m_1 = m_2 = m$

$\vec{v}_1, \vec{v}_2 = 0 \Rightarrow \vec{v}_2' = \vec{v}_1, \vec{v}_1' = 0$

- OTKLONIMO JEDNU KUGLICU S KRAJA NIZA, PUSTIMO DA UDARI OSTALE, 1 KUGLICA NA SUPROTNOM KRAJU NIZA ODSKOČI
- AKO OTKLONIMO 2 KUGLICE I PUSTIMO IH, KAD UDARE U OSTALE KUGLICE NA SUPROTNOM KRAJU ODSKOČE TAKOĐE 2 KUGLICE.



- 2 KUGLICE (ISTE MASE)

- OTKLONIMO OBJE, PUSTIMO, ODBIJU SE JEDNA O DRUGU (BRZINE, TJ. KOLIČINE GIBANJA PROMIJENE SMJER)

$\vec{v}_1 = -\vec{v}_2$ $\qquad \vec{v}_1' = \vec{v}_2$

$\vec{v}_1' = -\vec{v}_2'$ $\qquad \vec{v}_2' = \vec{v}_1$

- GUMENA LOPTICA BAČENA NA ZID - SPECIJALNI SLUČAJ ($m_2 >> m_1$) - LOPTICA IMA BRZINU KOJA JE PO IZNOSU ISTA BRZINI KOJOM JE BAČENA NA ZID (ILI BAREM OTPRILIKE), A SUPROTNA PO SMJERU.

POKUS: TOTALNO NEELASTIČNI SUDAR

- 2 KUGLE OD ILOVAČE - 1 OTKLONIMO PA PUSTIMO KUGLE SE PRILJEPE I TITRAJU KAO JEDNO TIJELO

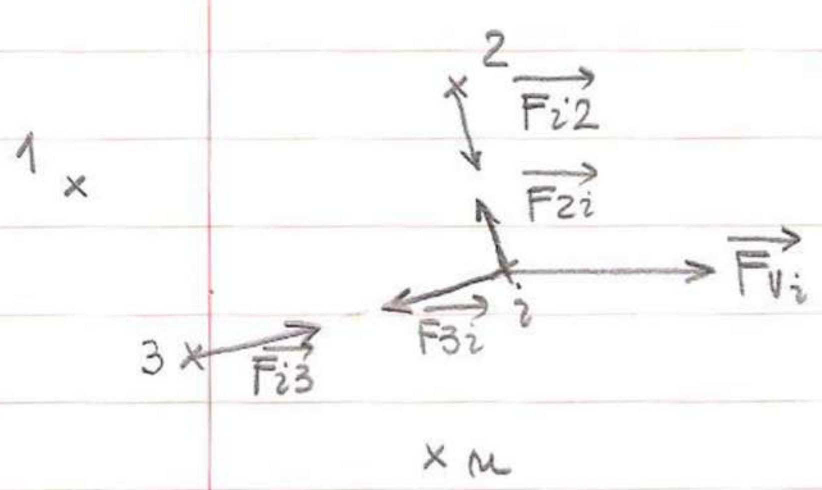
2. CIKLUS

P

31.3.2008.

SUSTAV MATERIJALNIH TOČAKA

- n čestica u prostoru
- $m_1, m_2, m_3, \dots, m_n$



$$\vec{F}_{i1} = -\vec{F}_{1i}$$
$$\vec{F}_{i2} = -\vec{F}_{2i}$$

$$\vec{F}_{ij} = -\vec{F}_{ji}$$

$$\frac{d\vec{P}_1}{dt} = \vec{F}_{V1} + \vec{F}_{21} + \vec{F}_{31} + \dots + \vec{F}_{i1} + \dots + \vec{F}_{n1}$$

$$\frac{d\vec{P}_2}{dt} = \vec{F}_{V2} + \vec{F}_{12} + \vec{F}_{32} + \dots + \vec{F}_{i2} + \dots + \vec{F}_{n2}$$

$$\frac{d\vec{P}_3}{dt} = \vec{F}_{V3} + \vec{F}_{13} + \vec{F}_{23} + \dots + \vec{F}_{i3} + \dots + \vec{F}_{n3}$$

$$\vdots$$

$$\sum_{i=1}^{n} \frac{d\vec{P}_i}{dt} = \sum_{i=1}^{n} \vec{F}_{Vi} + (\vec{F}_{12} + \vec{F}_{21}) + (\vec{F}_{13} + \vec{F}_{31}) + (\vec{F}_{32} + \vec{F}_{23}) + \dots + (\vec{F}_{ij} + \vec{F}_{ji}) + \dots$$

$$\boxed{\sum_{i=1}^{n} \frac{d\vec{P}_i}{dt} = \sum_{i=1}^{n} \vec{F}_{Vi}}$$

- zbroj unutrašnjih sila je 0 $\boxed{\sum_{\substack{j,i=1 \\ i \neq j}}^{n} \vec{F}_{ij} = 0}$

CENTAR MASE

1. (x_1, y_1, z_1)
2. (x_2, y_2, z_2)

$\vdots$

$i \quad (x_i, y_i, z_i)$

$\vdots$

$n \quad (x_n, y_n, z_n)$

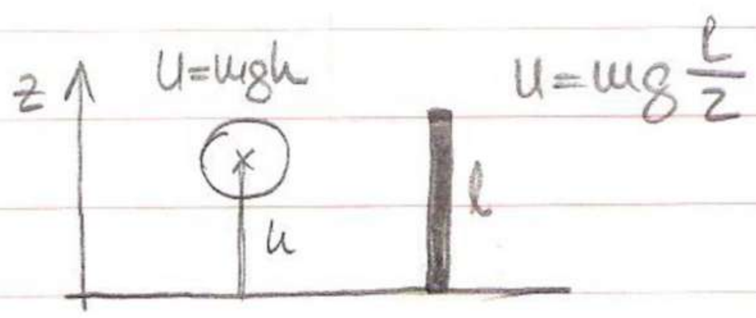
$$m = \sum_{i=1}^{n} m_i$$

$$x_{CM} = \frac{m_1 x_1 + m_2 x_2 + \ldots + m_i x_i + \ldots + m_n x_n}{m} = \frac{\sum_{i=1}^{n} m_i x_i}{m}$$

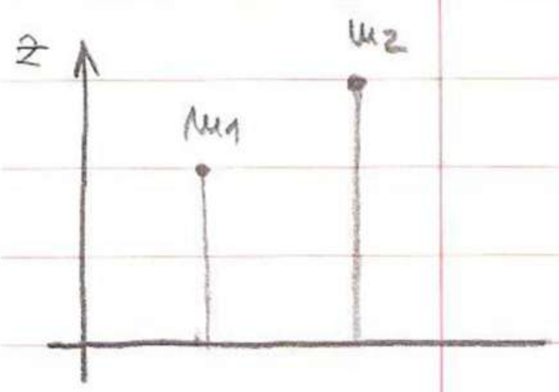
$$y_{CM} = \frac{\sum_{i=1}^{n} m_i y_i}{m}$$

$$z_{CM} = \frac{\sum_{i=1}^{n} m_i z_i}{m}$$

$$\boxed{\vec{r}_{CM} = \frac{\sum_{i=1}^{n} m_i \vec{r}_i}{m}}$$



* PRIMJENA

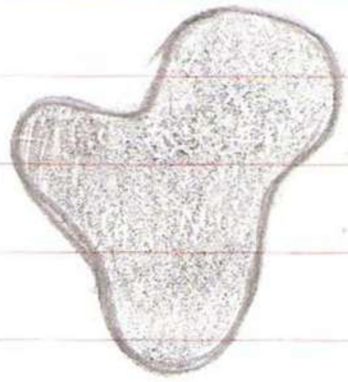


$$U = m_1 z_1 g + m_2 z_2 g + \ldots$$

$$U = \sum_{i=1}^{n} m_i z_i g = g \sum_{i=1}^{n} m_i z_i = g m z_{CM}$$

$$U = g m z_{CM}$$

CENTAR MASE ČVRSTOG TIJELA

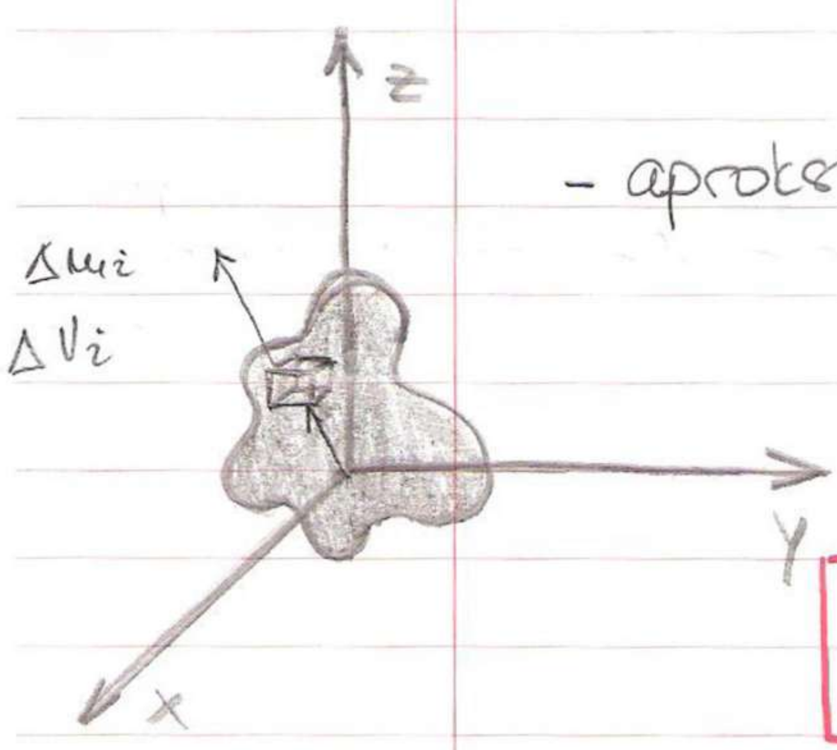


- PRETPOSTAVKA: masa je kontrolirano razmazana po površini

$$\overline{\rho} = \frac{\Delta m}{\Delta V} \qquad \boxed{\overline{\rho} = \frac{\Delta m}{\Delta V}}$$

$$\rho = \lim_{\Delta V \to 0} \frac{\Delta m}{\Delta V} = \frac{dm}{dV} \qquad \boxed{\rho = \frac{dm}{dV}}$$

$$dm = \frac{dm}{dV} \cdot dV \qquad \boxed{m = \int_V \rho \, dV}$$



- aproksimacija volumena na n dijelova

$$x_{CM} = \frac{\sum_{i=1}^{n} x_i \Delta m_i}{m} = \lim_{\substack{\Delta m_i \to 0 \\ n \to \infty}} \frac{\sum_{i=1}^{n} x_i \Delta m_i}{m}$$

$$\boxed{x_{CM} = \frac{1}{m} \int x \rho \, dV}$$

y_{CM}, z_{CM} ... jednako

- ρ = konst. $\quad x_{CM} = \frac{1}{\rho V} \rho \int x \, dV = \frac{1}{V} \int x \, dV$

$$\boxed{x_{CM} = \frac{1}{V} \int x \, dV}$$

GIBANJE CENTRA MASE

$$\sum_{i=1}^{n} \frac{dp_i}{dt} = \sum_{i=1}^{n} m_i \frac{dv_i}{dt} = \sum_{i=1}^{n} m_i \frac{d^2\vec{r_i}}{dt^2} = \sum_{i=1}^{n} m_i \vec{a_i} = \sum_{i=1}^{n} \vec{F_{Vi}}$$

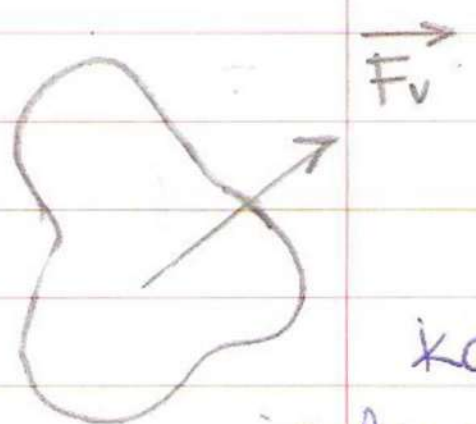
$$\vec{v_{CM}} = \frac{d\vec{r_{CM}}}{dt} = \frac{\sum\limits_{i=1}^{n} m_i \frac{d\vec{r_i}}{dt}}{m}$$

$$\vec{a_{CM}} = \frac{d\vec{v_{CM}}}{dt} = \frac{\sum\limits_{i=1}^{n} \frac{d^2\vec{r_{CM}}}{dt^2}}{m}$$

$$\boxed{m\vec{a_{CM}} = \sum_{i=1}^{n} \vec{F_{Vi}}}$$

II. Newtonov aksiom za gibanje centra mase (m = konst.)

Centar mase sustava giba se kao da je u njemu potencirana ukupna masa sustava i kao da sve vanjske sile djeluju u toj točki.

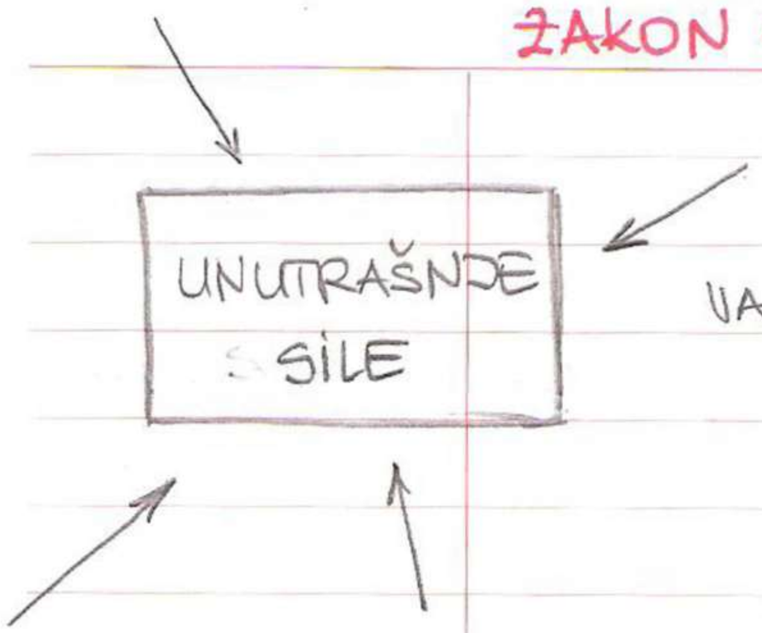


$$\vec{F_V} = 0$$

$$m\vec{a_{CM}} = 0 \longrightarrow \vec{a_{CM}} = 0$$

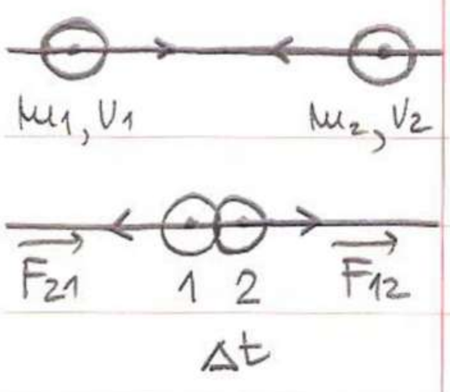
Kad je rezultanta svih vanjskih sila jednaka nuli, centar mase ili miruje ili se giba stalnom brzinom.

ZAKON OČUVANJA KOLIČINE GIBANJA



VANJSKE SILE

$$\vec{F_V} = 0$$



CENTRALNI, ELASTIČAN SRAZ (SUDAR)

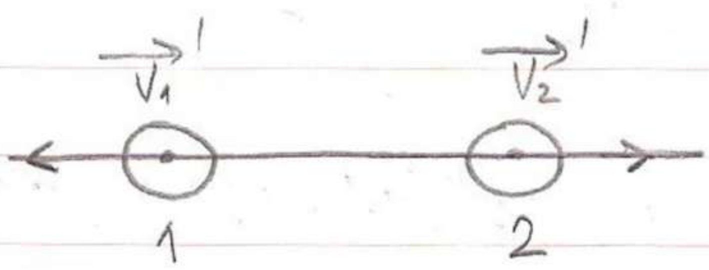
$\vec{I_1} = \vec{F_{21}}\Delta t$

$\vec{I_2} = \vec{F_{21}}\Delta t$

$\Delta\vec{P_1} = \vec{I_1}$

$\Delta\vec{P_2} = \vec{I_2}$

$\vec{I_1} = -\vec{I_2}$



$\Delta\vec{P_1} = m_1\vec{v_1}' - m_1\vec{v_1}$

$\Delta\vec{P_2} = m_2\vec{v_2}' - m_2\vec{v_2}$

$$m_1\vec{v_1}' - m_1\vec{v_1} = \vec{I_1} = -\vec{I_2} = -\Delta\vec{P_2} = -(m_2\vec{v_2}' - m_2\vec{v_2})$$

$$\boxed{m_1\vec{v_1} + m_2\vec{v_2} = m_1\vec{v_1}' + m_2\vec{v_2}'}$$

$$\sum_{i=1}^{n}\frac{d\vec{P_i}}{dt} = \sum_{i=1}^{n}\vec{F_{V_i}} = \vec{F_V}$$

$$\frac{d}{dt}\sum_{i=1}^{n} m_i\vec{v_i} = \vec{F_V} = 0$$ IZOLIRANI SUSTAV

$$\boxed{\frac{d}{dt}\sum_{i=1}^{n} m_i v_i = 0}$$ zakon očuvanja količine gibanja za zatvoreni sustav

Ukupna količina gibanja zatvorenog sustava konstantna je bez obzira na to kakvi se procesi i međudjelovanja događaju unutar sustava.

SUDARI i SRAZOVI

$$\vec{v_1}, \vec{v_2} \longrightarrow \vec{v_1}', \vec{v_2}'$$

$$m_1\vec{v_1} + m_2\vec{v_2} = m_1\vec{v_1}' + m_2\vec{v_2}'$$

$$\frac{m_1\vec{v_1}^2}{2} + \frac{m_2\vec{v_2}^2}{2} = \frac{m_1\vec{v_1}'^2}{2} + \frac{m_2\vec{v_2}'^2}{2}$$

$$m_1(v_1^2 - v_1'^2) = m_2(v_2'^2 - v_2^2)$$

$$m_1(\vec{v_1} - \vec{v_1}')(\vec{v_1} + \vec{v_1}') = m_2(\vec{v_2}' - \vec{v_2})(\vec{v_2}' + \vec{v_2})$$

$$m_1(\vec{v_1} - \vec{v_1}') = m_2(\vec{v_2}' - \vec{v_2})$$

$$\cancel{m_1}(\vec{v_1} - \vec{v_1}')(\vec{v_1} + \vec{v_1}') = \cancel{m_1}(\vec{v_1} - \vec{v_1}')(\vec{v_2} + \vec{v_2}')$$

$$(\vec{v_1} - \vec{v_1}')\left[(\vec{v_1} + \vec{v_1}') - (\vec{v_2} + \vec{v_2}')\right] = 0$$

$$(\vec{v_1} - \vec{v_1}')\underbrace{(\vec{v_1} + \vec{v_1}' - \vec{v_2} - \vec{v_2}')}_{0} = 0$$

... jedini slučaj koji nas zanima

- svi su na istom pravcu mogućnosti zato što imamo centralni sraz

$$\boxed{\vec{v_1} - \vec{v_2} = -(\vec{v_1}' - \vec{v_2}')}$$

Relativna brzina primicanja kuglica prije sudara jednaka je po iznosu i suprotna po smjeru relativnoj brzini odmicanja kuglica poslije sudara. Relativne brzine promijenile su samo smjer, a ne iznos.

$$\vec{V_1}' = \vec{V_2}' \implies \vec{V_1} = \vec{V_2}$$
$$\vec{V_2}' = \vec{V_1} - \vec{V_2} + \vec{V_1}'$$

$$m_1\vec{V_1} + m_2\vec{V_2} = m_1\vec{V_1}' + m_2\left(\vec{V_1} - \vec{V_2} + \vec{V_1}'\right)$$
$$m_1\vec{V_1} + m_2\vec{V_2} = m_1\vec{V_1}' + m_2\vec{V_1} - m_2\vec{V_2} + m_2\vec{V_1}'$$

$$(m_1 + m_2)\vec{V_1}' = (m_1 - m_2)\vec{V_1} + 2m_2\vec{V_2}$$

$$\boxed{\vec{V_1}' = \frac{(m_1 - m_2)\vec{V_1} + 2m_2\vec{V_2}}{m_1 + m_2}}$$

$$\vec{V_1}' = \vec{V_2}' - \vec{V_1} + \vec{V_2}$$

$$m_1\vec{V_1} + m_2\vec{V_2} = m_1\left(\vec{V_2}' - \vec{V_1} + \vec{V_2}\right) + m_2\vec{V_2}'$$
$$m_1\vec{V_1} + m_2\vec{V_2} = m_1\vec{V_2}' - m_1\vec{V_1} + m_1\vec{V_2} + m_2\vec{V_2}'$$

$$(m_1 + m_2)\vec{V_2}' = (m_2 - m_1)\vec{V_2} + 2m_1\vec{V_1}$$

$$\boxed{\vec{V_2}' = \frac{(m_2 - m_1)\vec{V_2} + 2m_1\vec{V_1}}{(m_1 + m_2)}}$$

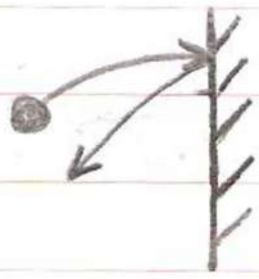
2.4.2008.

* SAVRŠENO ELASTIČAN SUDAR *

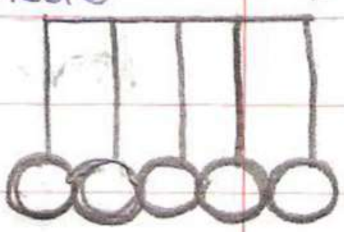
1. POKUS
   - u 1D
   - $m_1 = m_2$

   nakon sudara $|\vec{v}_1'| = |\vec{v}_2'|$

2. POKUS
   - $m_1 << m_2$
   - koliki $\vec{I}$ dobije zid ?



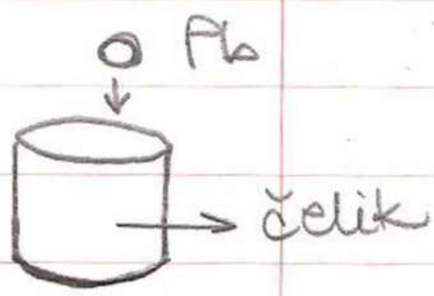
3. POKUS → NEWTONOVO NJIHALO



- izbaci se E_k zadnjoj kuglici

* SAVRŠENO NEELASTIČAN SUDAR *

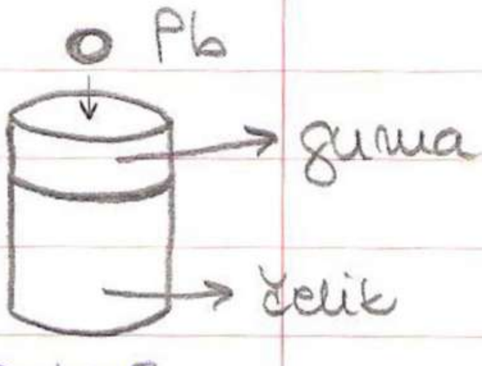
1. Pokus



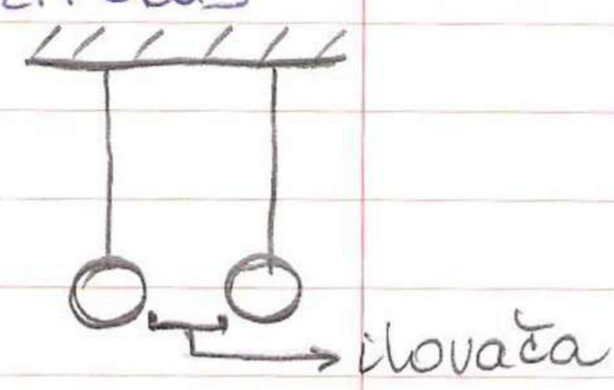
- ne vrijedi zakon očuvanja E_k
- dio E se pretvori u toplinu

* DJELOMIČNO ELASTIČAN SUDAR *

1. POKUS

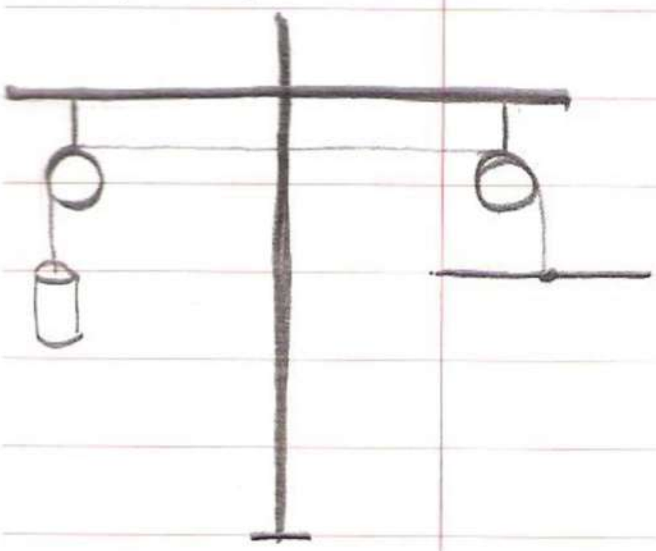


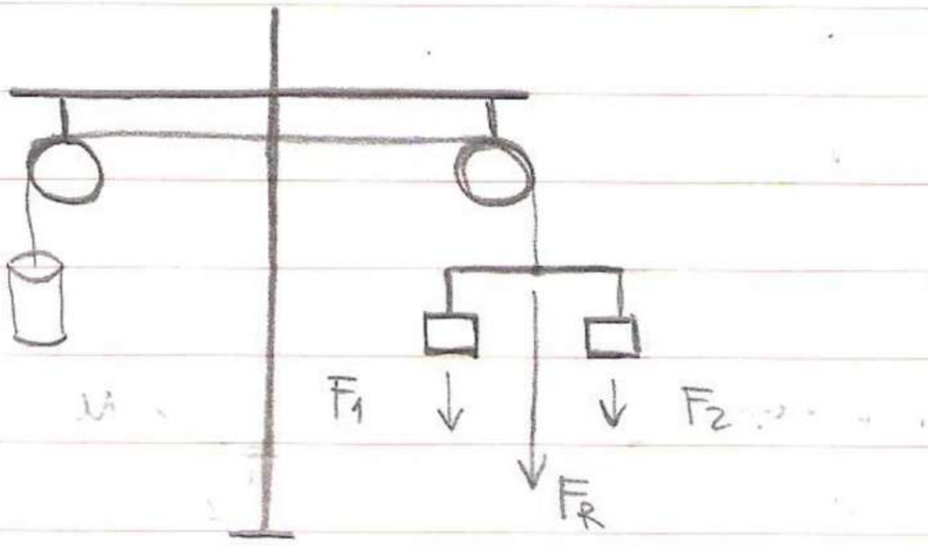
2. POKUS



- $m_1 = m_2$
- $\vec{v}_1' = \frac{1}{2}\vec{v}_1 = \vec{v}_2'$

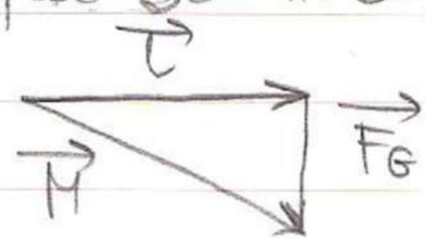
* RAVNOTEŽA *





- $F_1 + F_2 + \dots + F_n = 0$
- $M_1 + M_2 + \dots + M_n = 0$

najkraći put od 1. do 2. vektora

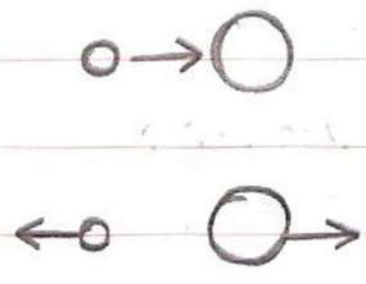


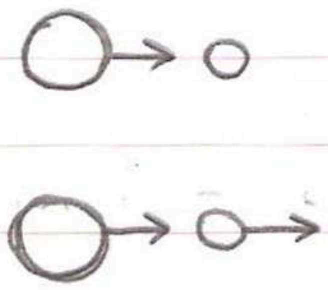
Težište se nalazi na vertikali kroz objesište

(*) ZADATAK: ljestve prislonjene uza zid

- $m_1 = m_2 = 0$
  $\vec{V_1},\ \vec{V_2} = 0$
  $\vec{V_1}' = \vec{V_2}$
  $\vec{V_2}' = \vec{V_1}$

- $m_1 < m_2$ | $m_1 > m_2$
  $\vec{V_2} = 0$ | $\vec{V_2} = 0$





- $m_1 \gg m_2$
  $\vec{V_2} = 0$

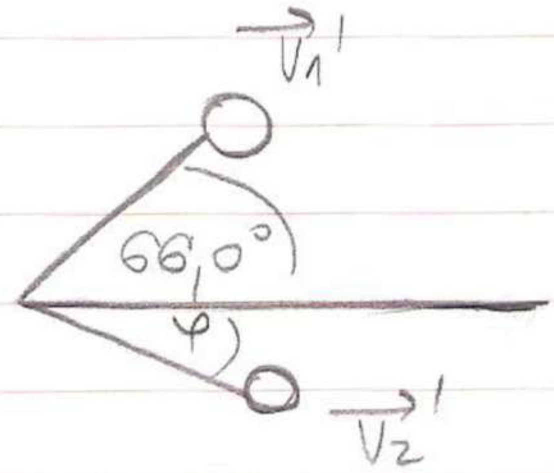
$$\vec{V_1}' = \frac{\left(1 - \frac{m_2}{m_1}\right)\vec{V_1}}{(1 + m_2/m_1)} = \vec{V_1}$$

$$\vec{V_2}' = \frac{2\,\frac{m_1}{m_1}\,V_1}{(1 + m_2/m_1)} = 2\vec{V_1}$$

⊛ ZADATAK:

$m_1 = m_2$
$\vec{V_1} = 1{,}5\ m/s$
$\vec{V_2} = 0$
$\vec{V_1}' = 0{,}610\ m/s$

$V_2 = ?$



SAVRŠENO NEELASTIČAN SRAZ

- E_k se nije očuvala $\longrightarrow$ dio se pretvori u unutrašnju E

- $m_1, \vec{v_1}$
- $m_2, \vec{v_2}$

$$m_1\vec{v_1} + m_2\vec{v_2} = (m_1+m_2)\vec{v}\,'$$

$$\boxed{\vec{v}\,' = \frac{m_1\vec{v_1} + m_2\vec{v_2}}{m_1+m_2}}$$

brzina CM prije i poslije sudara nije se mijenjala

$$\boxed{E_k' - E_k = Q}$$

$$\frac{(m_1+m_2)(\vec{v}\,')^2}{2} - \frac{m_1\vec{v_1}^2}{2} - \frac{m_2\vec{v_2}^2}{2} = \frac{(m_1\vec{v_1}+m_2\vec{v_2})^2}{2(m_1+m_2)} - \frac{m_1\vec{v_1}^2}{2} - \frac{m_2\vec{v_2}^2}{2} =$$

$\longrightarrow E_{k_{CM}} = E_k'$

ni kinetička E se ne promijeni nakon sudara

$$= \frac{-m_1 m_2}{2(m_1+m_2)}(\vec{v_1}-\vec{v_2})^2$$

$$E_k = E_{kCM} - Q = E_{kCM} + \frac{m_1 m_2}{2(m_1+m_2)}(\vec{v_1}-\vec{v_2})^2$$

$$\boxed{Q = -\frac{m_1 m_2}{2(m_1+m_2)}(\vec{v_1}-\vec{v_2})^2}$$

UNUTRAŠNJA E koju čestica ima bez obzira na CM

- ako je sraz elastičan:

$$= E_{kCM} + \frac{m_1 m_2}{2(m_1+m_2)}(\vec{v_1}\,'-\vec{v_2}\,')^2$$

$$\boxed{|\vec{v_1}-\vec{v_2}\,'| = |\vec{v_1}\,'-\vec{v_2}|}$$

relativna brzina čestica prije i poslije sudara ostaje sačuvana

- $m_1 = m_2 = m$

$$\vec{v}\,' = \frac{\vec{v_1} + \vec{v_2}}{2}$$

$$\vec{v_2} = 0 \longrightarrow \vec{v}\,' = \frac{\vec{v_1}}{2}$$

$$\vec{v_1} = -\vec{v_2} \longrightarrow \vec{v}\,' = 0$$

- $m_1 \ll m_2$

$\vec{v_2} = 0$

$$\vec{v}\,' = \frac{\frac{m_1}{m_2}\,\vec{v_1}}{\frac{m_1}{m_2} + 1} = 0$$

STATIKA

Statika je dio mehanike koji proučava zakone slaganja sila koje djeluju na ravnotežu tijela.

- 3 uvjeta ravnoteže tijela:
  1. $\vec{F_R} = 0$
  2. $\vec{M_R} = 0$
  3. tijelo rotira konst. brzinom

- gibanje krutog tijela se svodi na translaciju i rotaciju oko CM

SUSTAV SILA - skup svih sila koje djeluju na česticu

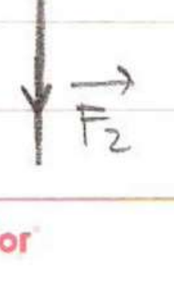
HVATIŠTE SILE - točka u kojoj sile djeluju

KONKURENTNE SILE - sile čiji se pravci djelovanja sijeku u istoj točki

RAVNOTEŽA MATERIJALNE TOČKE

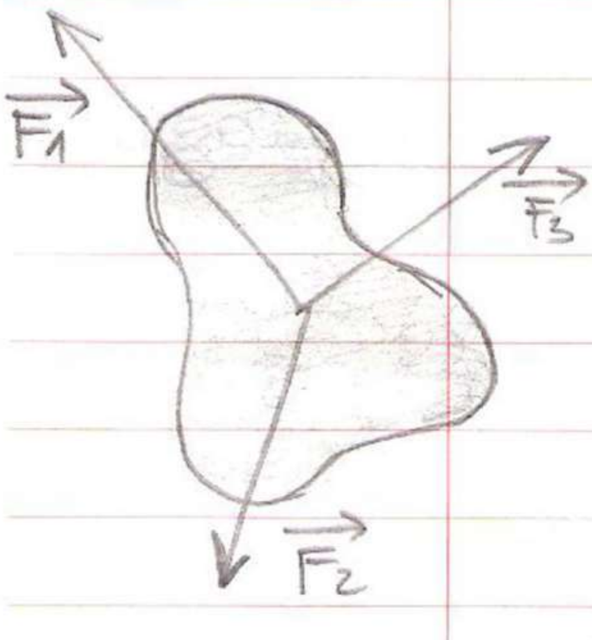
$\rightarrow \vec{F_R} = 0$

$\boxed{\vec{R} = \sum_{i=1}^{n} \vec{F_i} = 0}$ čestica (materijalna točka) je u ravnoteži kad je zatvoren vektorski poligon sila koje djeluju na nju.



$\vec{F_1} = -\vec{F_2}$

DJELOVANJE KONKURENTNIH SILA NA KRUTO TIJELO



$$\vec{R} = \vec{F_1} + \vec{F_2} + \vec{F_3} = 0$$

Nužan i dovoljan uvjet da bi kruto tijelo u tom slučaju bilo u ravnoteži je da bude zatvoren vektorski poligon tih sila.

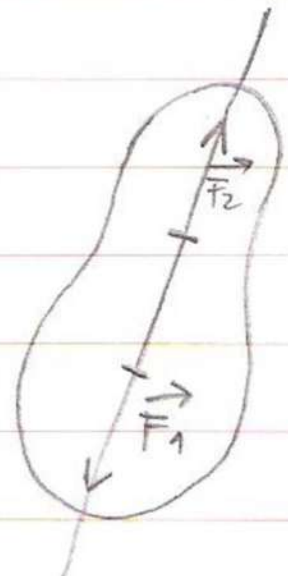
* OSNOVNI AKSIOM STATIKE *

1. Ako postoje 2 sile koje djeluju na neko tijelo, nalaze se na istom pravcu, jednakog iznosa i suprotnog predznaka ($\vec{F_1} = -\vec{F_2}$), kruto tijelo će tada biti u ravnoteži.

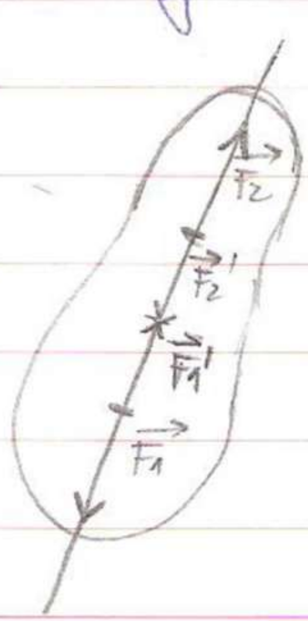
2. Djelovanje sustava na kruto tijelo se neće promijeniti ako tom sustavu dodamo dvije sile koje djeluju na istom pravcu, tako da vrijedi $\vec{F_1} = -\vec{F_2}$.

⟶ TEOREM (KOROLAR):

Ako sila djeluje na kruto tijelo, onda njeno hvatište možemo premještati duž pravca na kojem djeluje ta sila, bez da se djelovanje te sile na kruto tijelo promijeni.
Tada kažemo da je sila KLIZNI VEKTOR.

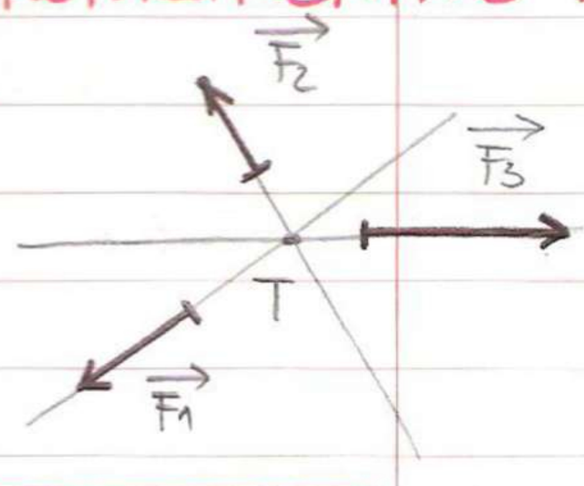


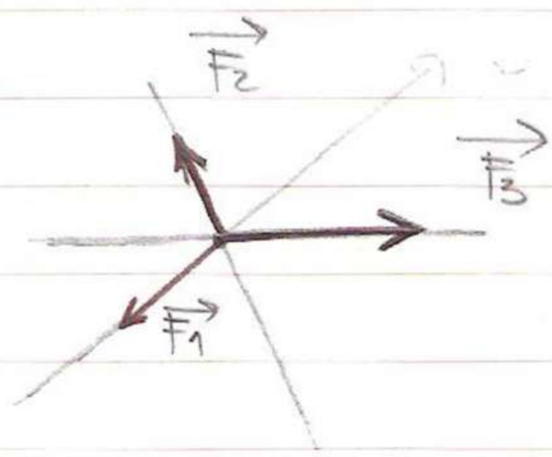
$\vec{F_1} = -\vec{F_2}$



$\vec{F_1}' = -\vec{F_2}'$

KONKURENTNE SILE





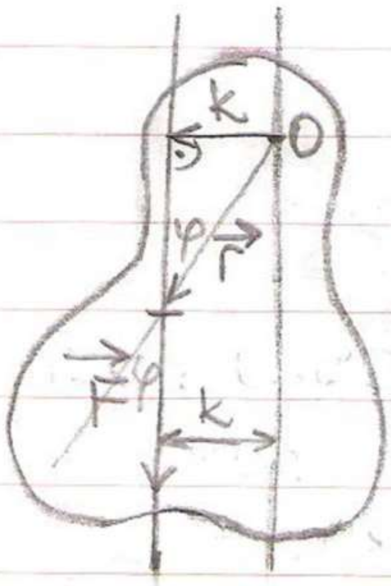
$$\boxed{\vec{R} = \sum_{i=1}^{n} \vec{F_i} = 0}$$

nužan, ali ne i dovoljan uvjet za kruto tijelo

7.4.2008.

P

MOMENT SILE



k... krak sile

→ pravci se ne smiju podudarati – tada nema rotacije tijela oko točke O

$\vec{M}$, $|\vec{M}| = kF$

$|\vec{F}| = F$

$k = r\sin\varphi$
$M = rF\sin\varphi$

$$\boxed{M = kF = rF\sin\varphi}$$

$\vec{M} = \vec{r_c} \times \vec{R}$ $\qquad\qquad$ $\vec{R} = R\vec{\mu}$

↳ radij-vektor položaja hvatišta rezultantne sile

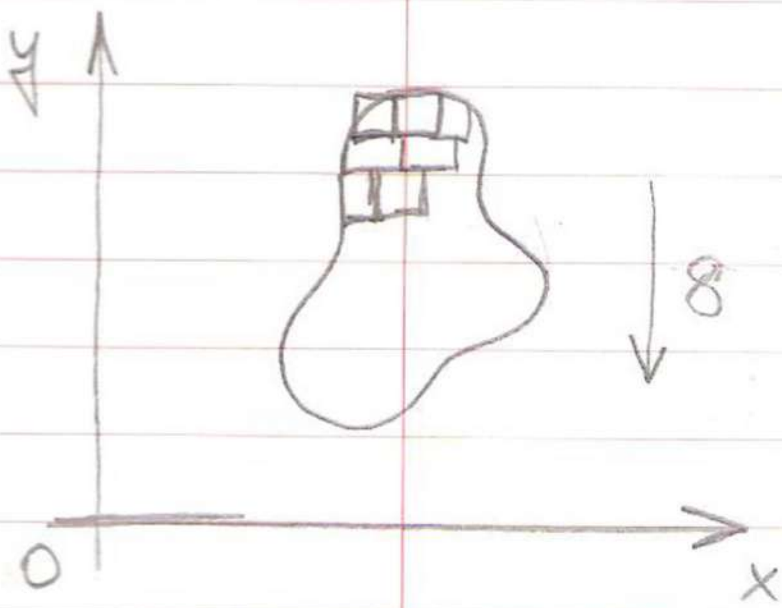
$= \vec{r_c} \times \vec{R} \cdot \vec{\mu} = R\vec{r_c} \times \vec{\mu}$

$(F_1\vec{r_1} + F_2\vec{r_2} + \dots + F_n\vec{r_n}) \times \vec{\mu} = R\vec{r_c} \times \vec{\mu}$

$F_1\vec{r_1} + F_2\vec{r_2} + \dots + F_n\vec{r_n} = R\vec{r_c}$

$$\boxed{\vec{r_c} = \frac{F_1\vec{r_1} + F_2\vec{r_2} + \dots + F_n\vec{r_n}}{R}}$$

TEŽIŠTE TIJELA



- podijeliti tijelo na n dijelova

→ HOMOGENO GRAVITACIJSKO POLJE

→ u svim točkama je jednak smjer i iznos djelovanja sile gravitacije

Δm_i, r_i ⟶ položaj i-te mase

ΔF_i ⟶ iznos sile koja djeluje na masu Δm_i = $\underline{\underline{\Delta m_i g}}$

$$\vec{r}_T = \frac{\sum_{i=1}^{n} g \Delta m_i \vec{r}_i}{Mg}$$

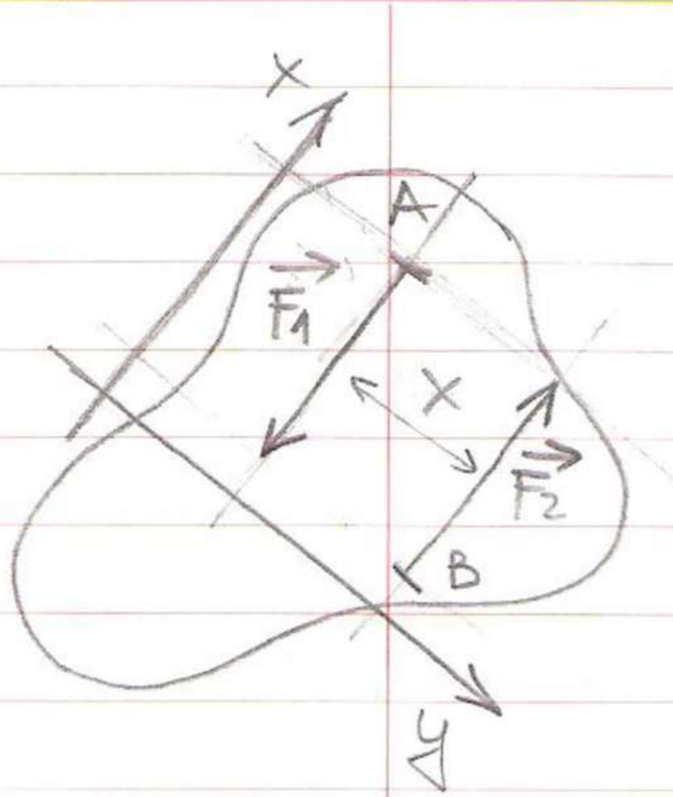
$$\vec{r}_T = \lim_{\substack{\Delta m_i \to 0 \\ n \to \infty}} \frac{\sum_{i=1}^{n} g \Delta m_i \vec{r}_i}{Mg} = \frac{\int g \vec{r}\, dm}{Mg} = \frac{\cancel{g} \int \vec{r}\, dm}{M\cancel{g}}$$

$$\boxed{\vec{r}_T = \frac{\int \vec{r}\, dm}{M}}$$

T = CM

$$x_T = \frac{\int x\, dm}{M}$$

PAR SILA



$$\vec{F_1} = -\vec{F_2}$$
$$\vec{M} = \vec{M_1} + \vec{M_2}$$

$$\vec{M_1} = x_1 F_1 \; (-\vec{k})$$
$$\vec{M_2} = x_2 F_2 \; (\vec{k})$$

$$\vec{M} = \vec{M_1} + \vec{M_2}$$
$$= (x_2 F_2 - x_1 F_1)\,\vec{k} \qquad |\vec{F_1}| = |\vec{F_2}| = F$$
$$= (x_2 - x_1)\, F \vec{k}$$

$$\boxed{\vec{M} = Fx\vec{k}}$$

Moment para sila okomit je na ravninu u kojoj leže sile, a po iznosu je jednak umnošku jedne od sila i udaljenosti pravaca njihovih djelovanja.

→ uravnotežiti tijelo ⟶ poništiti djelovanje para sila

RAVNOTEŽA KRUTOG TIJELA

1. tijelo miruje
2. $v = konst.$
3. rotira $\omega = konst.$

*** 1. UVJET RAVNOTEŽE ***

Rezultanta sila na kruto tijelo u ravnoteži jednak je nuli.

$$M\overrightarrow{a_{CM}} = \overrightarrow{F_V} \qquad \vec{R} = 0$$

$$\overrightarrow{RF_V}$$

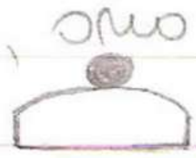
*** 2. UVJET RAVNOTEŽE ***

Rezultat svih vanjskih momenata (s obzirom na bilo koju točku) što djeluje na neuravnoteženo kruto tijelo mora biti 0.

$$\alpha \approx M$$

$$\alpha = 0 \Rightarrow \overrightarrow{M_V} = 0$$

VRSTE RAVNOTEŽA

1. STABILNA 
   - kada tijelo malo pomaknemo iz položaja ravnoteže, ono se samo vrati u taj položaj
2. LABILNA 
   - kada tijelo malo pomaknemo iz položaja ravnoteže, ono se ne vraća natrag
3. INDIFERENTNA 
   - kada tijelo malo pomaknemo iz položaja ravnoteže, tijelo je i u novom položaju u ravnoteži

ROTACIJA KRUTOG TIJELA

- djelovanje sila ⟶ DEFORMACIJA
  ⟶ GIBANJE

TRANSLACIJA

Tijelo se giba translatorno ako linija koja povezuje bilo koje 2 njegove čestice zadržava svoj smjer u prostoru.

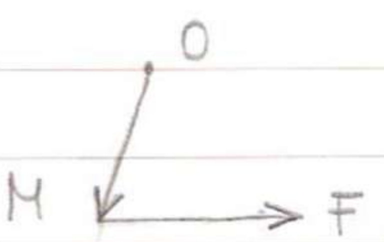
ROTACIJA

Kruto tijelo rotira kada se sve njegove čestice gibaju istom kutnom brzinom po kružnicama čija središta leže na pravcu koji se zove OS ROTACIJE.

ROTACIJA KRUTOG TIJELA OKO NEPOMIČNE OSI

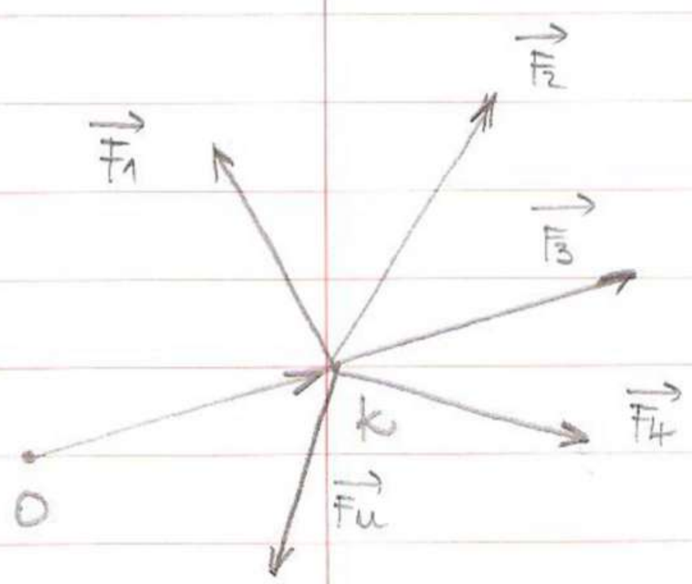
Ako zbog djelovanja sile dolazi do rotacije u smjeru suprotnom od smjera kazaljke na satu, $\vec{M} > 0$; i obrnuto.

Moment sile s obzirom na os rotacije jednak je umnošku iznosa komponente sile koja leži u ravnini okomitoj na os rotacije i okomito udaljenosti od osi do pravca djelovanja sile.



AKSIJALNI VEKTOR

$$\boxed{\vec{M} = \vec{r} \times \vec{F}}$$

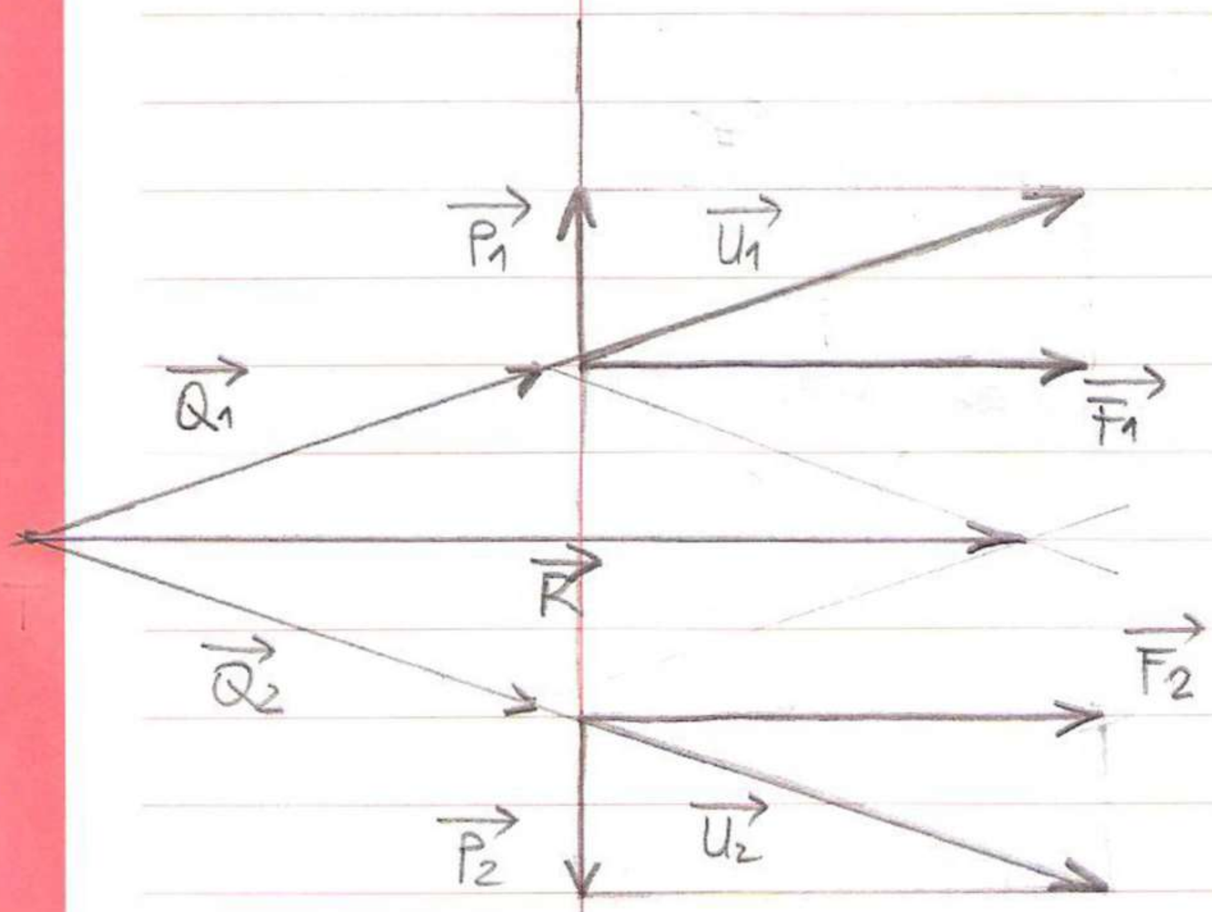


$n \ldots \vec{F_1}, \vec{F_2}, \vec{F_3}, \ldots, \vec{F_n}$

$$\vec{M} = \vec{r} \times \vec{F_1} + \vec{r} \times \vec{F_2} + \vec{r} \times \vec{F_3} + \ldots + \vec{r} \times \vec{F_n}$$
$$= \vec{r} \times \underbrace{(\vec{F_1} + \vec{F_2} + \vec{F_3} + \ldots + \vec{F_n})}_{\vec{R}}$$

$$\boxed{\vec{M} = \vec{r} \times \vec{R}}$$

DJELOVANJE NEKONKURENTNIH SILA NA KRUTO TIJELO



$\vec{F_1} \parallel \vec{F_2}$

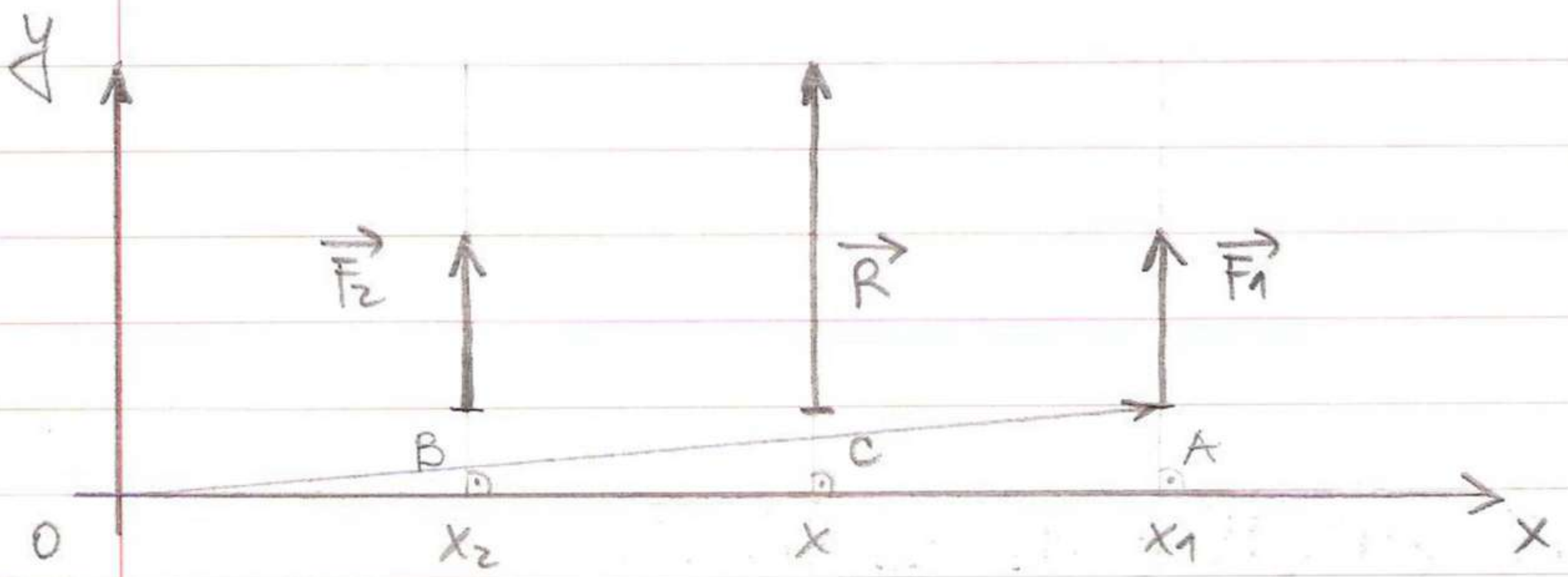
$\vec{P_1} = -\vec{P_2}$

$\vec{U_1} = \vec{F_1} + \vec{P_1} = \vec{Q_1}$

$\vec{U_2} = \vec{F_2} + \vec{P_2} = \vec{Q_2}$

$$\vec{R} = \vec{Q_1} + \vec{Q_2} = \vec{F_1} + \cancel{\vec{P_1}} + \vec{F_2} + \cancel{\vec{P_2}} = \vec{F_1} + \vec{F_2}$$

$$\boxed{\vec{R} = \vec{F_1} + \vec{F_2}}$$



- os z ide iz papira prema van (nama)

$M_1 = x_1 F_1 \qquad \vec{M_1} = x_1 F_1 \vec{k}$

$M_2 = x_2 F_2 \qquad \vec{M_2} = x_2 F_2 \vec{k}$

$$\vec{M} = \vec{M_1} + \vec{M_2} = (x_1 F_1 + x_2 F_2)\vec{k}$$
$$= (\vec{F_1} + \vec{F_2}) x \vec{k}$$

$$(x_1 F_1 + x_2 F_2)\cancel{\vec{k}} = (\vec{F_1} + \vec{F_2})\cancel{\vec{k}}\, x$$
$$x_1 F_1 + x_2 F_2 = (F_1 + F_2)\, x$$

$$\boxed{x = \frac{x_1 F_1 + x_2 F_2}{F_1 + F_2}}$$

* n paralelnih sila

$R = F_1 + F_2 + F_3 + \ldots + F_n$

$\vec{R} = \vec{F_1} + \vec{F_2} + \vec{F_3} + \ldots + \vec{F_n}$

$\vec{r_1}, \vec{r_2}, \vec{r_3}, \ldots, \vec{r_n}$

$\vec{M} = \vec{r_1} \times \vec{F_1} + \vec{r_2} \times \vec{F_2} + \vec{r_3} \times \vec{F_3} + \ldots + \vec{r_n} \times \vec{F_n}$

$\vec{F_1} = F_1 \vec{n}$

$\vec{F_2} = F_2 \vec{n}$

$\vec{F_3} = F_3 \vec{n}$ $\qquad$ $\vec{n}$ ... jedinični vektor koji gleda u smjeru F

$\vdots$

$\vec{F_n} = F_n \vec{n}$

$$\begin{aligned} \vec{M} &= \vec{r_1} \times (F_1 \vec{n}) + \vec{r_2} \times (F_2 \vec{n}) + \vec{r_3} \times (F_3 \vec{n}) + \ldots + \vec{r_n} \times (F_n \vec{n}) \\ &= \vec{r_1} \times F_1 \vec{n} + \vec{r_2} \times F_2 \vec{n} + \vec{r_3} \times F_3 \vec{n} + \ldots + \vec{F_n} \times F_n \vec{n} \\ &= (F_1 \vec{r_1} + F_2 \vec{r_2} + F_3 \vec{r_3} + \ldots + F_n \vec{r_n}) \times \vec{n} \end{aligned}$$

$$\boxed{\vec{M} = (F_1 \vec{r_1} + \ldots + F_n \vec{r_n}) \times \vec{n}}$$

$M = F r_\perp = F r \sin\varphi$ $\qquad$ $r_\perp = r \sin\varphi$

$F \sin\varphi = F_t$

9.4. 2008.

Pokusi

$\varphi / 2\pi$	t/s	$\alpha / rad\,s^{-2}$
1	8,11	0,19
	16,00	0,2

$f = \frac{\alpha}{2} t^2 \quad \Rightarrow \quad \alpha = \frac{2f}{t^2}$

$r = 0{,}01\ m$
$R = 0{,}125\ m$
$m = 0{,}6\ kg$
$m_1 = 0{,}02\ kg$

stalni moment, stalna akceleracija

TRANSLACIJA

$\varphi / 2\pi$	t/s	$\alpha / rad\,s^{-2}$
1	5,42	0,42
2		

$m_1 = 2 \times 0{,}02\ kg$

$\alpha \sim M$ ... 2x veći moment, 2x veći α

$\alpha \sim \frac{1}{R^2} \qquad \alpha \sim \frac{M}{rR^2}$

$\alpha = \frac{M}{I}$ → moment tromosti

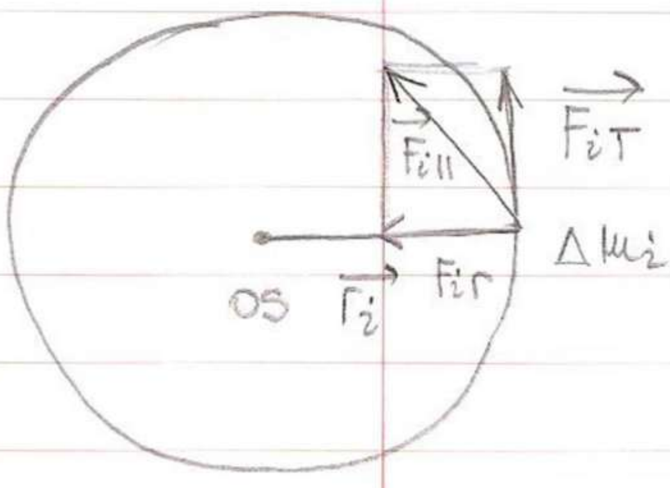
$\vec{L} = \vec{r} \times \vec{p}$

$\frac{d\vec{L}}{dt} = \frac{d\vec{r}}{dt} \times \vec{p} + \vec{r}\, \frac{d\vec{p}}{dt} = \cancel{\vec{v} \times \vec{p}} + \vec{r} \times \vec{F}$

$\boxed{\vec{M} = \frac{d\vec{L}}{dt}}$

P

$$\boxed{M_z = I_z \alpha}$$



$$\vec{F_i} = \vec{F_{i\perp}} + \vec{F_{i\parallel}}$$

$$M_{iz} = F_{ir}\, r_i$$

$$F_{iT} = a_{iT} \cdot \Delta m_i$$

$$a_{iT} = r_i \alpha$$

$$M_{iz} = a_{iT}\, \Delta m_i\, r_i = r_i^2 \Delta m_i \alpha$$

$$M_z = \sum_{i=1}^{n} r_i^2 \Delta m_i \alpha$$

$$= \lim_{\substack{n\to\infty \\ \Delta m_i \to 0}} \alpha \sum_{i=1}^{n} r_i^2 \Delta m_i = \alpha \lim_{\substack{n\to\infty \\ \Delta m_i \to 0}} \sum_{i=1}^{n} r_i^2 \Delta m_i$$

$$\boxed{M_z = \alpha \int r^2\, dm = \alpha I_z}$$

I_z ... moment tromosti

$\boxed{M_z = \alpha I_z}$ jednadžba gibanja; 1. NA za kruto tijelo

Može se dokazati da je moment unutrašnjih sila za kruto tijelo jednak 0.

Moment svih vanjskih sila s obzirom na os rotacije jednak je umnošku momenata tromosti tijela s obzirom na tu istu os i kutne akceleracije tijela.

KARAKTERISTIČNA SITUACIJA

$M_z = 0 \rightarrow \alpha = 0$

$M_z = konst. \Rightarrow \alpha = konst.$

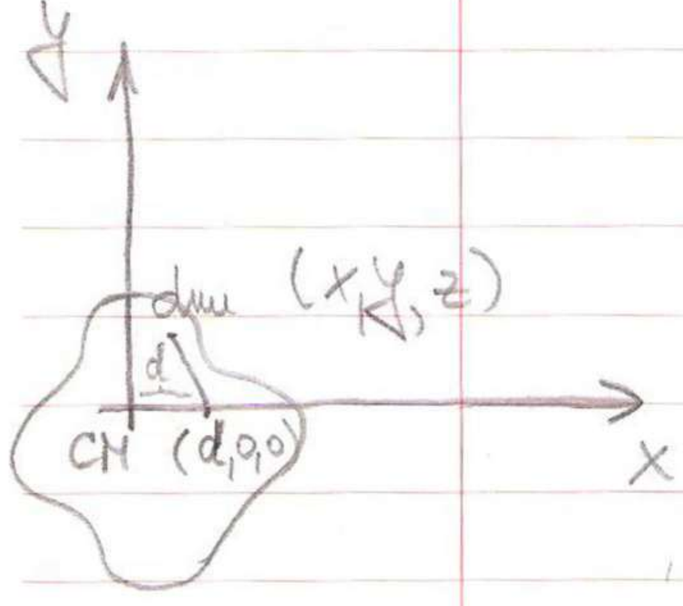
MOMENT TROMOSTI

$$\boxed{I_z = \int r^2 dm}$$

$$dm = \rho dV$$

$$\boxed{I = \int r^2 \rho dV}$$

STEINEROV STAVAK



$$I_{CM} = \int (x^2+y^2)\, dm$$

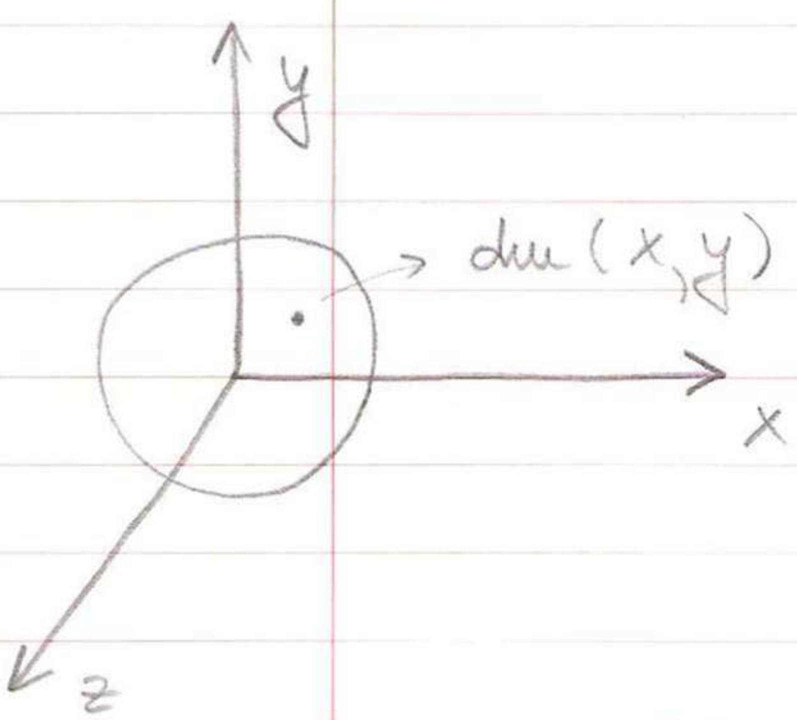
$$I = \int \left[(x-d)^2 + y^2\right] dm$$

$$= \underbrace{\int (x^2+y^2)\, dm}_{I_{CM}} - \cancel{2d\int x\, dm} + d^2\int dm$$

$$\boxed{I = I_{CM} + md^2}$$

Moment tromosti s obzirom na neku os jednak je momentu tromosti s obzirom na paralelnu os kroz CM uvećan za umnožak mase tijela i kvadrata udaljenosti tih dviju osi.

POUČAK O MEĐUSOBNO OKOMITIM OSIMA



$$I_x = \int y^2 dm$$

$$I_y = \int x^2 dm$$

$$I_z = \int x^2 dm + \int y^2 dm$$

$$\boxed{I_z = I_y + I_x}$$

Moment tromosti ravne krute ploče s obzirom na os okomitu na njezinu ravninu jednak je zbroju momenata tromosti oko bilo koje dvije međusobno okomite osi koje leže u ravnini ploče i presijecaju se okomito na ravninu.

14.4.2008.

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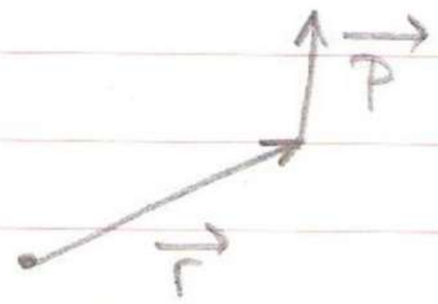
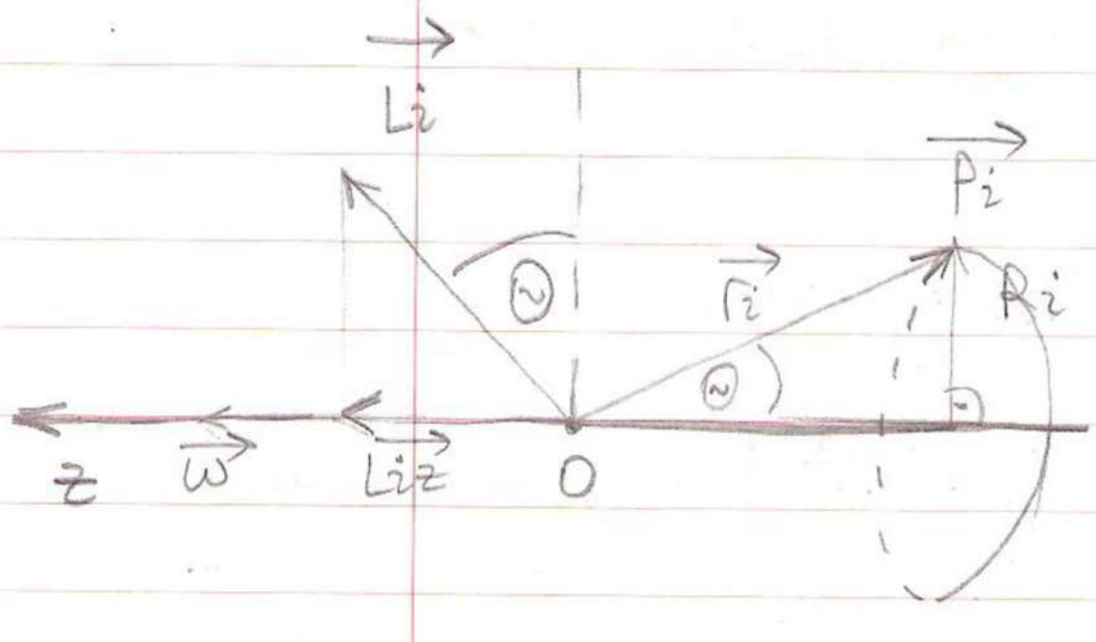
KUTNA KOLIČINA GIBANJA

$\vec{p}, \vec{r}$

$$\boxed{\vec{L} = \vec{r} \times \vec{p}}$$



$m, \Delta m_i$



$$\vec{L_i} = \vec{r_i} \times \vec{p_i}$$

$$\vec{p_i} = \Delta m_i \cdot \vec{v_i}$$

$v_i = R_i \omega$ → ne treba ω_i jer po pretpostavci svaki dio ima jednaku brzinu ω

$$L_i = r_i p_i = r_i \Delta m_i v_i = r_i \Delta m_i R_i \omega$$

$$L_{iz} = L_i \cos\left(\frac{\pi}{2} - \theta\right) = L_i \sin\theta$$

$$L_{iz} = \omega \Delta m_i R_i r_i \sin\theta = \omega \Delta m_i R_i^2$$

$$\frac{R_i}{r_i} = \sin\theta \qquad R_i = r_i \sin\theta$$

$$L_z = \lim_{\substack{n \to \infty \\ \Delta m_i \to 0}} \left(\sum_{i=1}^{n} R_i^2 \Delta m_i \right) \omega = I_z \omega$$

↳ moment tromosti s obzirom na os z

$$\boxed{L_z = I_z \omega}$$

Za tijela koja rotiraju oko svoje osi simetrije možemo zapisati u vektorskom obliku:

$$\boxed{\vec{L} = I\vec{\omega}}$$

GLAVNE OSI – kada tijelo rotira oko njih i vrijedi gornja relacija

$$\frac{d\vec{L_i}}{dt} = \vec{r_i} \times \vec{F_i} = \vec{M_i}$$

$$\boxed{\frac{d\vec{L}}{dt} = \vec{M}}$$

2. NA za rotaciju krutog tijela oko nepomične ~~točke~~ linije ili učvršćene u 1 točki

$M_z = I_z \alpha$

$L_z = I_z \omega$

$$\frac{d\vec{L}}{dt} = \vec{M}$$

$$\frac{dL_z}{dt} = M_z$$

$$\frac{d(I_z\omega)}{dt} = I_z\alpha = M_z$$

ZAKON OČUVANJA KUTNE KOLIČINE GIBANJA

$\vec{M} = 0 \longrightarrow \frac{d\vec{L}}{dt} = 0 \longrightarrow L = konst.$

$I_z \omega = konst.$

$I_1 \omega_1 = I_2 \omega_2 \qquad I_1 > I_2 \Rightarrow \omega_1 < \omega_2$

⟶ objašnjenje pokusa vrtnje na stolcu

GIBANJE ZVRKA

Zvrk je rotaciono simetrično tijelo koje se vrlo brzo vrti oko svoje osi simetrije, pri čemu je stalno učvršćeno u 1 točki koja leži na toj osi.

* SLOBODNI ZVRK *

A... točka u kojoj je zvrk poduprt (A ≡ T) težište

$\vec{F_G}$, $\vec{M_G} = 0$ $\vec{L} = konst.$

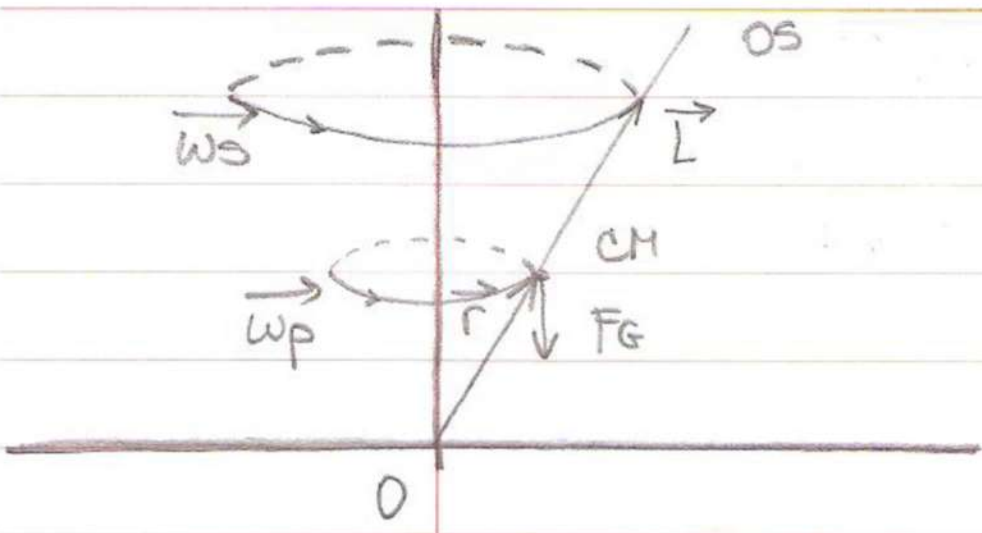
$\vec{L} = I\vec{\omega}$ $\qquad \vec{\omega}$ leži na osi rotacije

$\vec{L} \parallel \vec{\omega}$

Os rotacije za slobodan zvrk ne mijenja se u prostoru.

* PRECESIJA ZVRKA *

$A \neq T$



$\omega_s >> \omega_p$ → kutna brzina precesije

$\vec{L} = \vec{L_{oCM}} + \vec{L_{CM}}$

↳ kutna količina gibanja rotacije oko CM
↳ puno više doprinosi u ukupnoj sumi

$|\vec{L_{oCM}}| \approx \omega_s$

$\vec{L_{CM}} = \vec{r} \times \vec{P_{CM}}$ $\quad |\vec{P_{CM}}| \approx \vec{V_{CM}} \approx \omega_p$

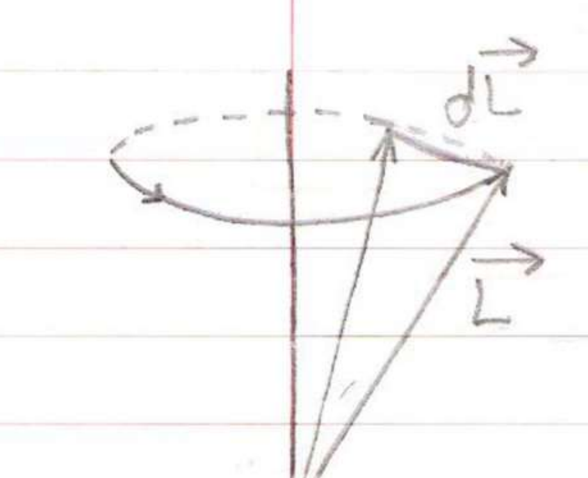
$|\vec{L_{CM}}| \approx \omega_p$

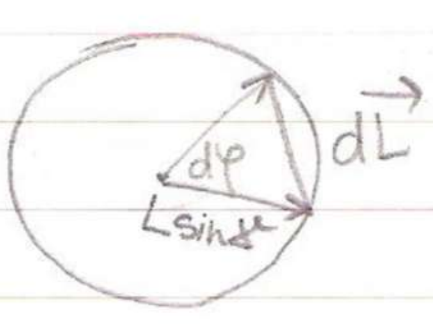
$|\vec{L_{oCM}}| >> |\vec{L_{CM}}|$

$\boxed{\vec{L} = \vec{L_{oCM}}}$

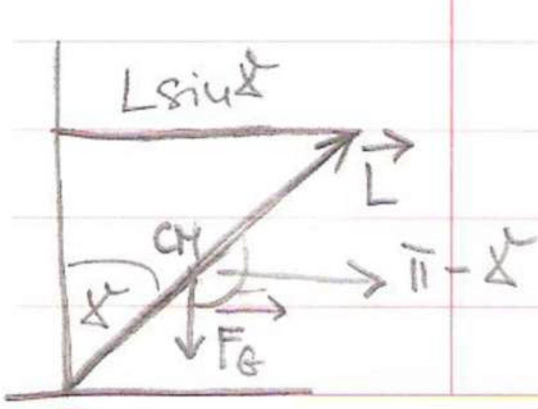
$\vec{M} = \vec{r} \times \vec{F_G}$

$\frac{d\vec{L}}{dt} = \vec{M}$ $\quad \vec{M} \perp \vec{L}$





* primjena II. NA *



$d\vec{L} = \frac{d\vec{L}}{dt} dt = \vec{M} dt$

*geometrijski *

$d\vec{L} = L\sin\alpha \, d\varphi = |\vec{M}| \, dt$

$$|\vec{M}| = rmg\sin(\pi - \vartheta) = rmg\sin\vartheta$$

$$L\cancel{\sin\vartheta}\, d\varphi = rmg\cancel{\sin\vartheta}\, dt \quad (/ : dt)$$

$$L\omega = rmg$$

$$L = \frac{rmg}{\omega_p} \qquad \boxed{L = \frac{rmg}{\omega_p}}$$

$$L\omega dt = rmg\, dt$$

$$\omega_p = \frac{rmg}{L} \qquad \boxed{\omega_p = \frac{rmg}{L}}$$

$$L = L_{OCM} \qquad L_{OCM} = I_s \cdot \omega_s$$

$$\omega_p = \frac{rmg}{I_s \omega_s} \qquad \boxed{\omega_p = \frac{rmg}{I_s \omega_s}}$$

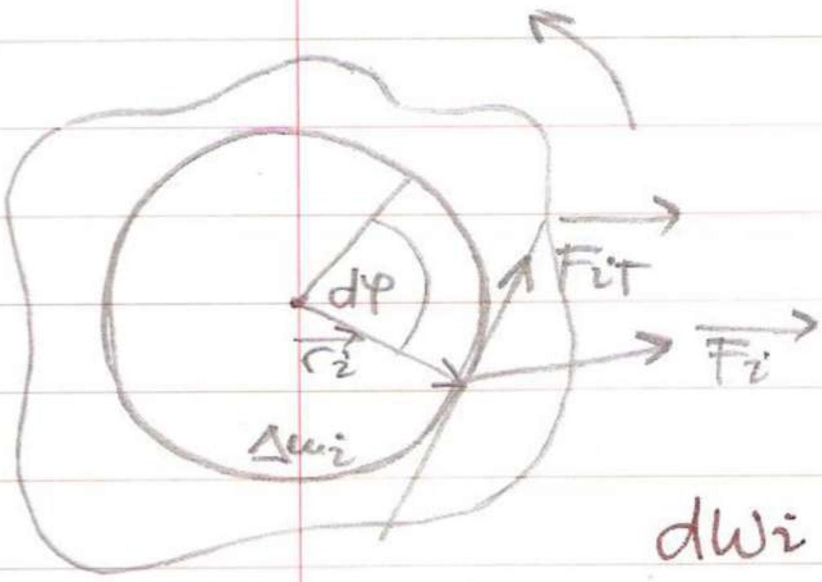
→ odgovara početnom uvjetu $\omega_s >> \omega_p$

$$L\sin\vartheta\, \omega_p\, dt = M dt \qquad \boxed{M = \omega_p L \sin\vartheta}$$

$$\cancel{M = \omega_p L \sin\vartheta}$$

$$\boxed{\vec{M} = \vec{\omega_p} \times \vec{L}}$$

RAD I KINETIČKA ENERGIJA PRI ROTACIJI OKO NEPOMIČNE OSI



$dW, d\varphi$

$\mu, \Delta m_i$

$$dW_i = \vec{F_i} \cdot d\vec{r_i} = F_{iT} \cdot dr_i =$$
$$= F_{iT}\, r_i\, d\varphi = M_{iz}\, d\varphi$$

$$dW = M_z d\varphi$$

$$\boxed{W = \int_0^{\varphi} M_z\, d\varphi}$$

16.4.2008.

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RAD i SNAGA

$dW = M_z\, d\varphi$

$M_z = \dfrac{dW}{d\varphi}$ $\qquad\qquad W = \int_0^{\varphi} M_z\, d\varphi$

$dE_k = dW = M_z \underbrace{d\varphi}_{\omega dt} = I_z \underbrace{\alpha\, \omega\, dt}_{d\omega} = I_z \omega\, d\omega$

$$\boxed{dE_k = I_z \omega\, d\omega}$$

$E_k = \int_0^{\omega} I_z \omega\, d\omega = I_z \dfrac{\omega^2}{2}$ $\qquad\qquad \boxed{E_k = I_z \dfrac{\omega^2}{2}}$

SNAGA PRI ROTACIJI KRUTOG TIJELA OKO FIKSNE OSI

$P = \dfrac{dW}{dt} = \underbrace{\dfrac{dW}{d\varphi}}_{M_z} \cdot \underbrace{\dfrac{d\varphi}{dt}}_{\omega} = M_z \omega$

$$\boxed{P = M_z \omega}$$ $\qquad \cancel{\vec{P} =}$

$$\boxed{dW = \vec{M}\, d\vec{\varphi}}$$

$$\boxed{d\vec{\varphi} = d\varphi \dfrac{\vec{\omega}}{\omega}}$$

PRINCIP VIRTUALNOG RADA

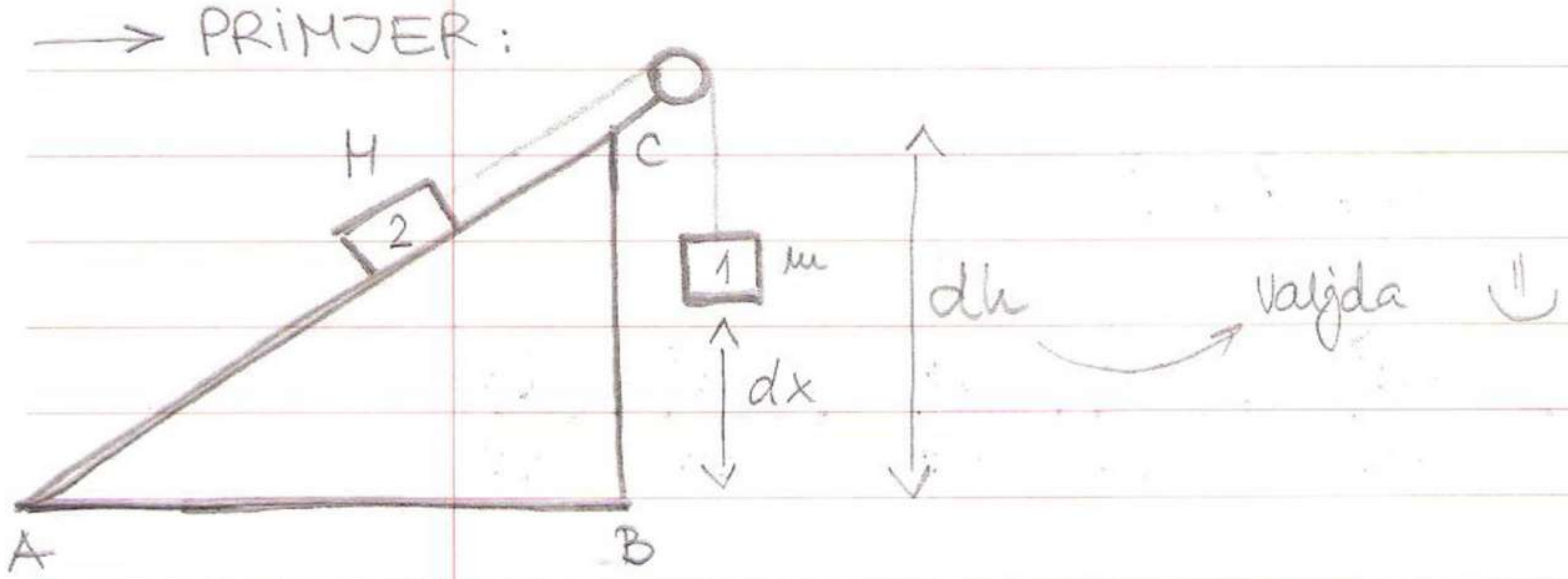
$$dW = \sum_{i=1}^{n} \vec{F_i}\, d\vec{r} + \vec{M}\, d\vec{\varphi}$$

Tijelo je u ravnoteži ako je ukupni rad pri bilo kakvoj zamišljenoj (virtualnoj) promjeni položaja jednak nuli.

$$dW = 0 = \sum_{i=1}^{n} \vec{F_i}\, \underbrace{\delta\vec{r}}_{\text{VIRTUALNI POMAK}} + \vec{M}\,\delta\vec{\varphi} \longrightarrow \text{VIRTUALNI ZAKRET}$$

$$\sum_{i=1}^{n} \vec{F_i} = 0 \qquad \vec{M} = 0 \qquad dW = 0$$

→ PRIMJER:



$$\frac{\overline{BC}}{\overline{AC}} = \frac{1}{2}$$

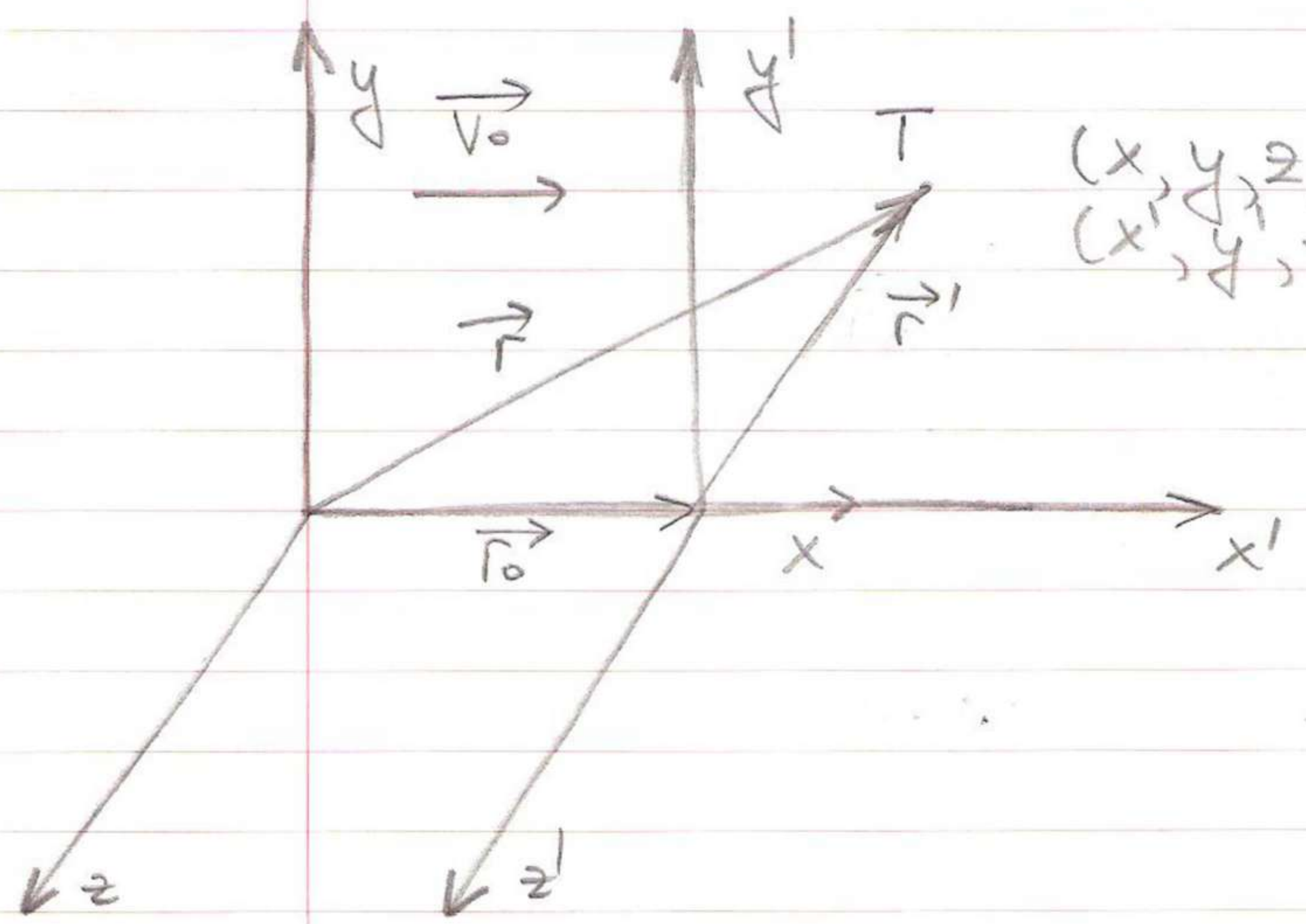
$$\frac{m}{M} = \frac{\delta h}{\delta x} = \text{sličnost trokuta} = \frac{\overline{BC}}{\overline{AC}} = \frac{1}{2}$$

$$dW = 0 \qquad dW = mg\,\delta x - Mg\,\delta h = 0$$

$$\delta W_2 = \delta U_2 = -Mg\,\delta h$$

INERCIJALNI i NEINERCIJALNI SUSTAVI

INERCIJALNI SUSTAVI $\longrightarrow$ Galilejeve transformacije



(x, y, z, t)
(x', y', z', t')

$t = t'$
$v \ll c$

$\vec{r_0} + \vec{r}' = \vec{r}$ $\qquad$ $\vec{r}' = \vec{r} - \vec{r_0}$

$$\vec{v}' = \frac{d\vec{r}'}{dt} = \frac{d\vec{r}}{dt} - \frac{d\vec{r_0}}{dt} = \frac{d\vec{r}}{dt} \cdot \underbrace{\frac{dt}{dt'}}_{=1} - \frac{d\vec{r_0}}{dt} \cdot \underbrace{\frac{dt}{dt'}}_{=1}$$

$\vec{v}' = \vec{v} - \vec{v_0}$ $\qquad$ $\vec{v} = \vec{v}' + \vec{v_0}$

$$\vec{a}' = \frac{d\vec{v}'}{dt} = \underbrace{\frac{d\vec{v}}{dt}}_{=\vec{a}} - \underbrace{\frac{d\vec{v_0}}{dt}}_{=0} = \vec{a}$$

$\vec{a}' = \vec{a}$

$\vec{r}' = \vec{r} - \vec{r_0}$

$$\boxed{\vec{r}\,' = \vec{r} - \vec{r}_0}$$

$$\boxed{\begin{aligned} \vec{v}\,' &= \vec{v} - \vec{v}_0 \\ \vec{a}\,' &= \vec{a} \end{aligned}}$$

$$\boxed{\begin{aligned} x' &= x - v_0 t \\ y' &= y \\ z' &= z \end{aligned}}$$

$$\boxed{\begin{aligned} v_x' &= v_x - v_0 \\ v_y' &= v_y \\ v_z' &= v_z \end{aligned}}$$

GALILEJEVE TRANSFORMACIJE

$$\vec{F} = \vec{F}\,'$$

$$\vec{F} = m\vec{a} = m\vec{a}\,' = \vec{F}\,'$$

Svi zakoni klasične mehanike imaju isti oblik u svim inercijalnim sustavima.
Dakle, nema apsolutno mirnog sustava.

NEWTONOVA (OPĆA) TEORIJA GRAVITACIJE

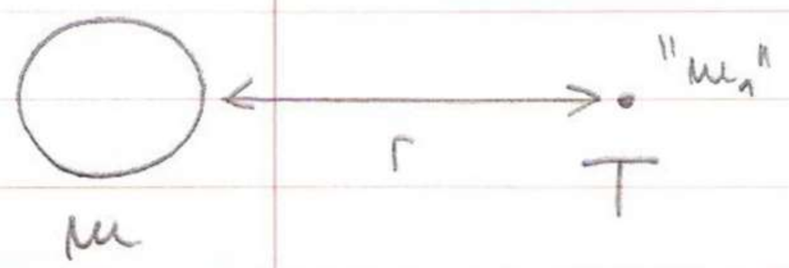
1. SILA

$$\boxed{\vec{F} = -G\frac{m_1 m_2}{r_{12}}\hat{r}_{12}}$$

$\hat{r}_{12}$ → jedinični vektor

$G = 6{,}67 \cdot 10^{-11}\ Nm^2/kg^2$

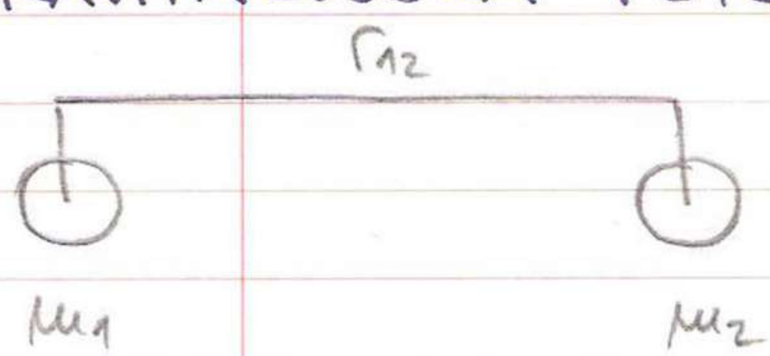
2. GRAVITACIJSKO POLJE



$\vec{\gamma} = \frac{\vec{F}}{m_1}$

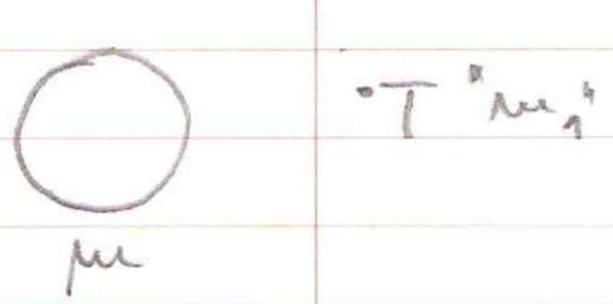
$$\boxed{\vec{\gamma} = -G\frac{m}{r^2}\vec{r}_0}$$

3. GRAVITACIJSKA POTENCIJALNA E



$$\boxed{U = -G\frac{m_1 m_2}{r_{12}}}$$

4. POTENCIJAL



$\phi = \frac{U}{m_1}$ $\quad E = -\frac{d\phi}{dr}$ $\quad F = -\frac{dU}{dr}$

$$\boxed{\phi = -G\frac{m}{r}}$$

21.4.2008.

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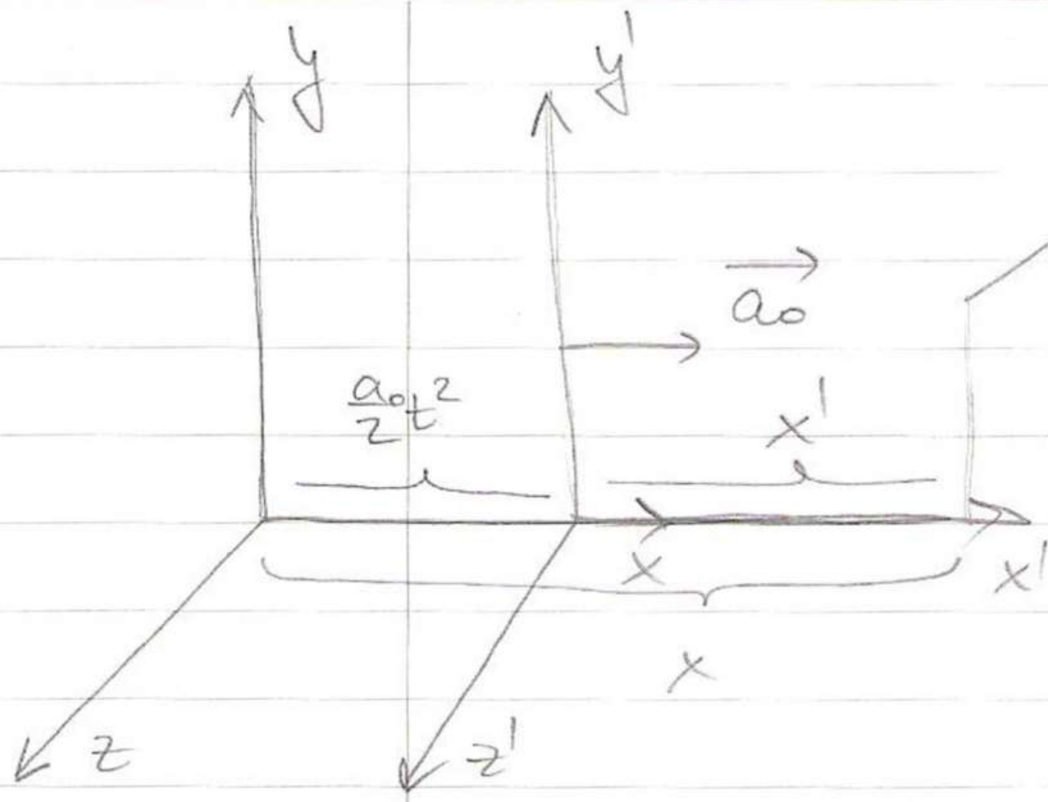
JEDNOLIKO UBRZANI SUSTAVI; INERCIJALNE SILE



(x, y, z)

(x', y', z')

$\vec{F} = \vec{F}'$

$$x' = x - \frac{a_0}{2}t^2$$

$y' = y$

$z' = z$

$v'_x = v_x - a_0 t$

$v'_y = v_y$

$v'_z = v_z$

$a'_x = a_x - a_0$

$a'_y = a_y$

$a'_z = a_z$

$$\vec{a}' = \vec{a} - \vec{a_0}$$

SUSTAV S

$$\boxed{\vec{F} = m\vec{a}}$$

SUSTAV S'

$$\boxed{\begin{gathered} m\vec{a}' = m\vec{a} - m\vec{a_0} = \vec{F}' - m\vec{a_0} \\ \vec{F} = \vec{F}' = 0 \end{gathered}}$$

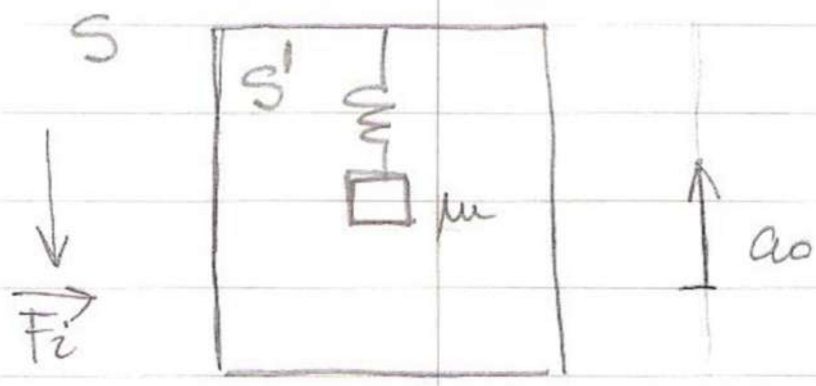
$$m\vec{a}' = -m\vec{a_0}$$

$$\boxed{\vec{F_i} = -m\vec{a_0}}$$

Inercijalne sile javljaju se samo u neinercijalnim sustavima.

$$\boxed{m\vec{a}' = \vec{F}' + \vec{F_i}}$$

U sustavu koji se giba ubrzano (jednoliko) uvesti inercijalne sile



PERSPEKTIVA IZ SUSTAVA S' (neinercijalni)

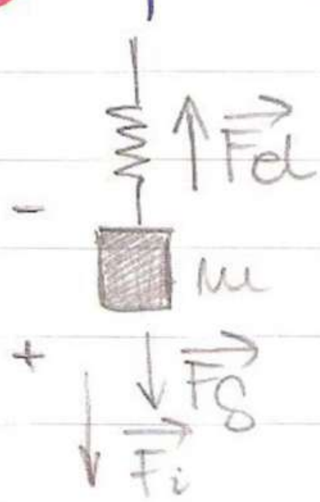
1. Lift se giba prema gore $v_0 = konst.$

$$\vec{Fel} + \vec{F_g} = 0$$

$$\vec{Fel} = \vec{Fg} = m\vec{g}$$

$$\vec{G} = -\vec{Fel} = \vec{F_g} = mg$$

2. Lift se giba prema gore $a_0 = konst.$



$$\vec{G} = -\vec{Fel}$$

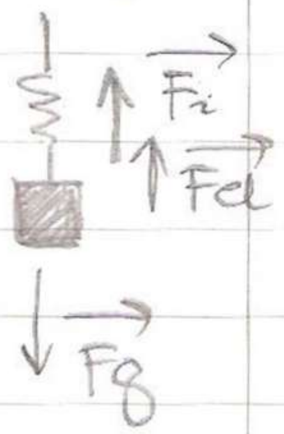
$$\vec{Fel} + \vec{Fg} + \vec{Fi} = 0$$

$$mg + ma_0 - Fel = 0$$

$$Fel = mg + ma_0$$

$$G = -Fel \Rightarrow G + Fel = mg + ma_0$$

(3.) lift se giba prema dolje $a_0 = konst.$



$$\vec{G} = \vec{Fel} + \vec{Fi}$$

$$\vec{Fel} + \vec{Fi} + \vec{F_G} = 0$$

$$-Fel - ma_0 + mg = 0$$

$$Fel = mg - ma_0$$

$$G + Fel = mg - ma_0$$

* kada je $a_0 = g \Rightarrow G = 0 \Rightarrow$ bestežinsko stanje

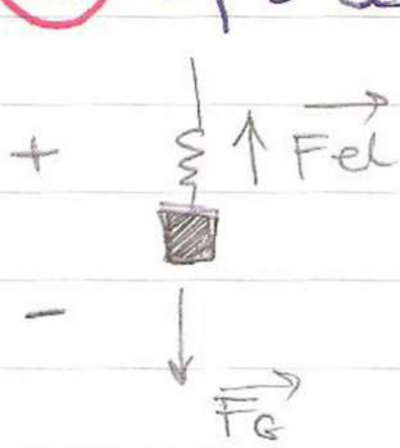
PERSPEKTIVA IZ SUSTAVA S (inercijalni)

(1.) lift se giba prema gore $V_0 = konst.$

$$\vec{Fel} + \vec{Fg} = 0$$

$$G = mg$$

(2.) lift se giba prema gore $a_0 = konst.$



$$ma_0 = \vec{F_G} + \vec{Fel}$$

$$ma_0 = -mg + Fel$$

$$Fel = ma_0 + mg$$

$$G + Fel = ma_0 + mg$$

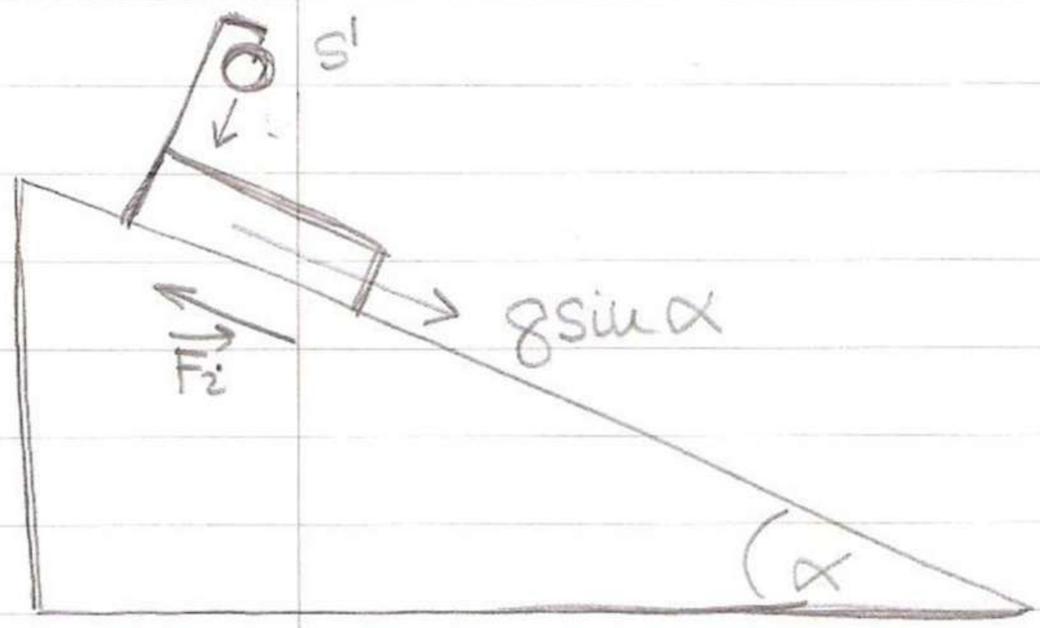
3. Lift se giba prema dole $a_0 = \text{konst.}$

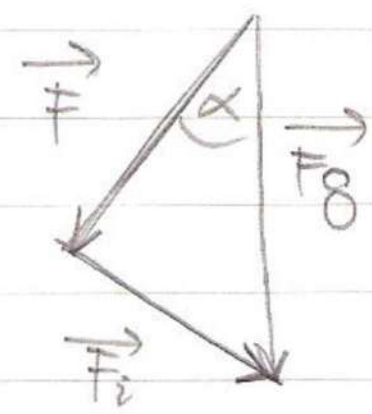
$$m\vec{a_0} = \vec{F_{el}} + \vec{F_g}$$

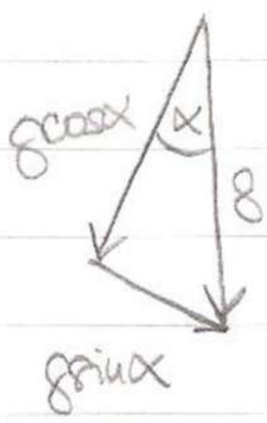
$$ma_0 = -F_{el} + mg$$

$$F_{el} = mg - ma_0$$

$$G + F_{el} = mg - ma_0$$



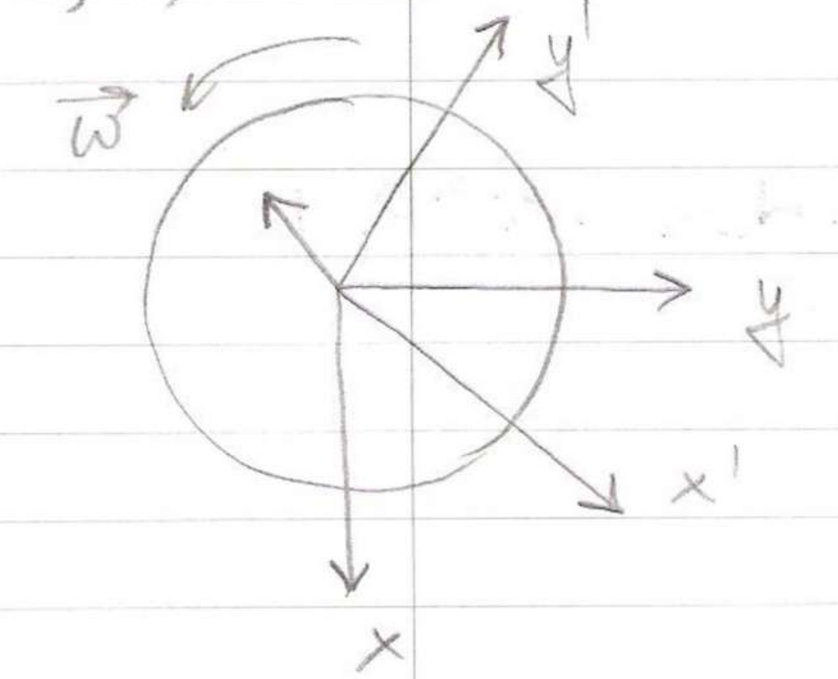




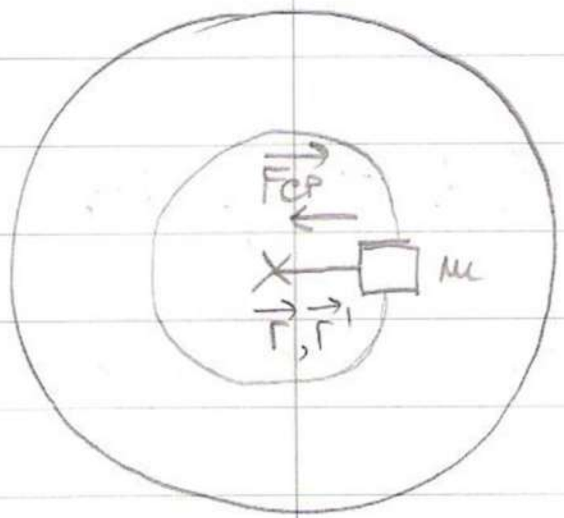
$$|\vec{F_i}| = mg\sin\alpha$$

ROTIRAJUĆI SUSTAV, CENTRIFUGALNA SILA i CORIOLISOVA SILA

$S, S', \vec{\omega} = konst.$



$\vec{r} = \vec{r}'$



$S: \vec{F_{cp}} = -m\omega^2\vec{r}$

$\vec{F_{cp}} = \vec{F_{el}}$

$S': \vec{F_{el}} + \vec{F_i} = 0$

$\vec{F_i} = -\vec{F_{el}} = m\omega^2\vec{r}$ CF sila (centrifugalna)

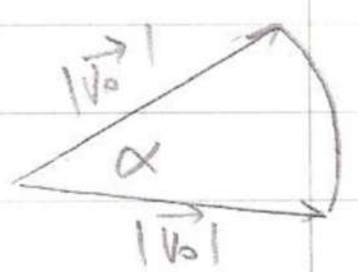
CORIOLIS – SUSTAV S

$\vec{v} = \vec{v_0} + \vec{\omega} \times \vec{R}$

$t \Rightarrow \quad \vec{R}$

$t + \Delta t \Rightarrow \quad \vec{R} + d\vec{R}$

1. T $\quad (d\vec{v})_1 \qquad d\alpha = \omega dt$



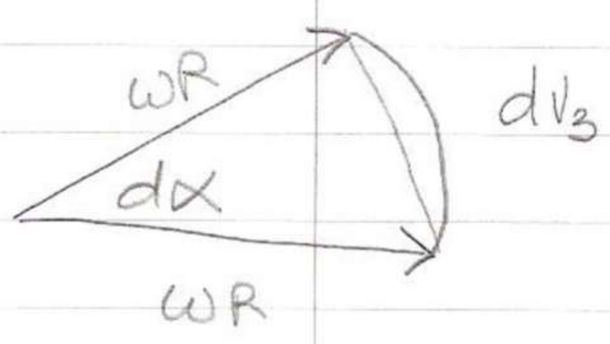
$|d\vec{v}|_1 = dr_1$

$dv_1 = v_0 d\alpha = v_0 \omega dt$ promjena smjera vektora zbog rotacije

2. T $\quad (d\vec{v_2})_2 \qquad dv_2 = (\omega(R + dR)) - \omega R = \omega dR = \omega v_0 dt$

→ promjena zbog toga što se čestica giba od ishodišta prema nama $v_0 = konst.$

3. R $(d\vec{v})_3$



$$dv_3 = \omega R \cdot d\alpha = \omega^2 R\, dt$$

$$d\vec{v} = (d\vec{v})_1 + (d\vec{v})_2 + (d\vec{v})_3$$

T... $(d\vec{v})_T = (d\vec{v})_1 + (d\vec{v})_2 = (d\vec{\omega})_T\, dt$

promjena u tangencijalnom smjeru uzrokovana a_T

$$(da)_T = 2 v_0 \omega$$

$$\boxed{\vec{a}_T = 2(\vec{\omega} \times \vec{v}_0) = -2(\vec{v}_0 \times \vec{\omega})}$$

R... $(d\vec{v})_R = (d\vec{a})_R\, dt$

$$(da)_R = \omega^2 R$$

$$\boxed{\vec{a}_R = -\omega^2 \vec{R}}$$ centripetalna akceleracija

$$\boxed{\begin{aligned} \vec{F}_T &= m\vec{a}_T = -2m(\vec{v}_0 \times \vec{\omega}) \\ \vec{F}_R &= m\vec{a}_R = -m\omega^2 \vec{R} \end{aligned}}$$

SUSTAV S'

- na disku

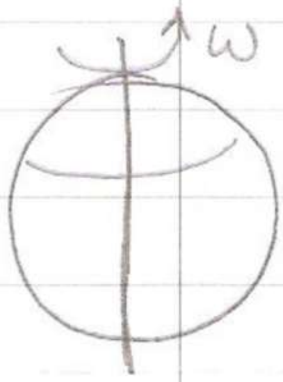
$$\vec{F_T} + \vec{F_R} + \vec{F_{cf}} + \vec{F_c} = 0$$

⟶ coriolisova sila

$$-2m(\vec{v_0} \times \vec{\omega}) - m\omega^2\vec{R} + m\omega^2\vec{R} + \vec{F_c} = 0$$

$$\boxed{\vec{F_c} = 2m(\vec{v_0} \times \vec{\omega})}$$

Javlja se u rotacijskom sustavu ako se neko tijelo giba u tom rotacijskom sustavu.



u našoj polukugli skreće desno

23.4.2008.

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NEWTONOV ZAKON GRAVITACIJE

KEPLEROVI ZAKONI

1. Planeti se gibaju u elipsama u čijem se jednom žarištu nalazi Sunce.

2. Planeti se gibaju tako da pravac koji spaja položaj planeta sa Suncem opisuje u jednakim vremenima jednake površine.

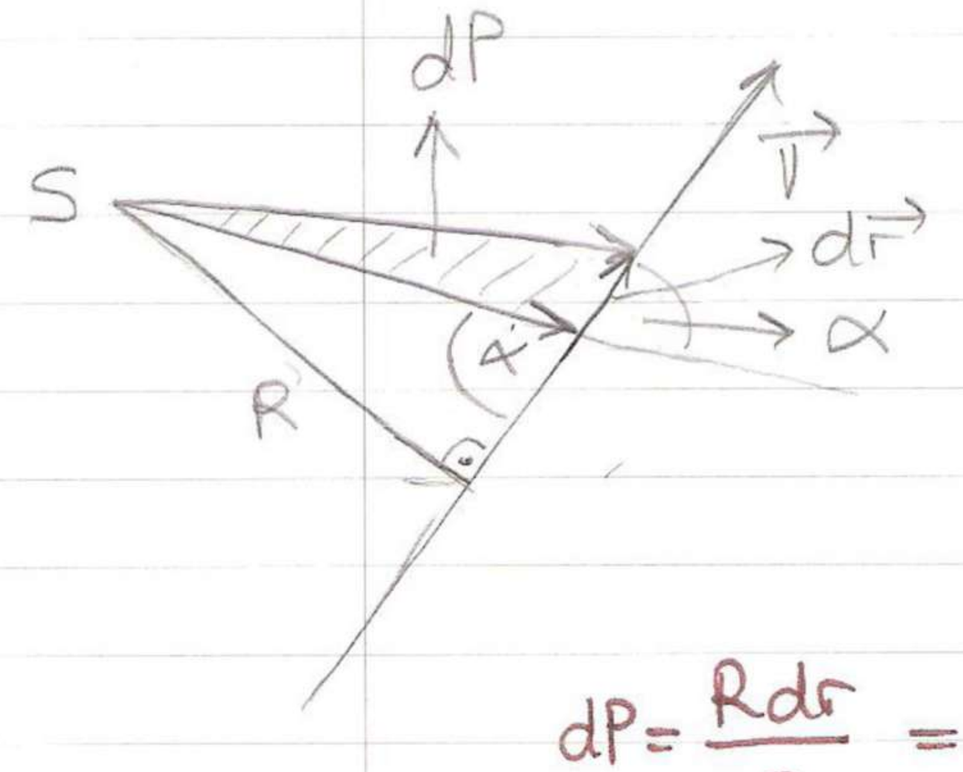
3. Kvadrati vremena ophodnje planeta oko Sunca odnose se kao kubovi nj. srednjih udaljenosti od Sunca.

$$\boxed{\frac{T^2}{R^3} = konst}$$

$$R = \frac{r_1 + r_2}{2} = a$$

r_1 ... min. udaljenost
r_2 ... max. udaljenost

→ velika poluos elipse



$$d\vec{r} = \vec{v}\,dt$$

$$L = rmv\sin\alpha = mRv$$

$$R = r\sin\alpha$$

$$dP = \frac{R\,dr}{2} = \frac{R}{2}v\,dt$$

PLOŠNA BRZINA jednaka je površini koju radijus-vektor (radij-vektor) prekrije u jedinici vremena

$$\frac{dP}{dt}$$

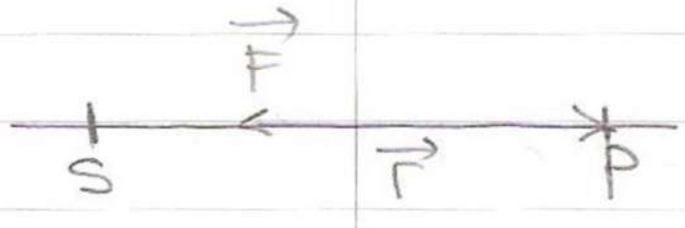
iz $L = m(RV)$

$$dP = \frac{dP}{dt}\,dt = \frac{VR}{2}\,dt = \text{konst.}$$

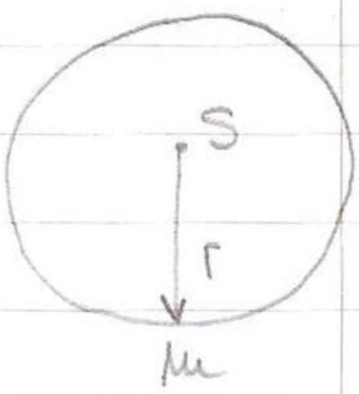
2. keplerov z.

$L = \text{konst.}$

$$\frac{d\vec{L}}{dt} = \vec{M} = \underbrace{\vec{r} \times \vec{F}}_{\text{kolinearni}} = 0$$



PRETPOSTAVKA radi jednostavnosti: planeti se gibaju po kružnici polumjera r, planeti su mase m i stalna brzina v



$$F_{cp} = \frac{mv^2}{r} \qquad v = \frac{2r\pi}{T}$$

$$\frac{T^2}{r^3} = k$$

$$F_{cp} = \frac{m}{r}\left(\frac{2r\pi}{T}\right)^2 = \frac{4mr^2\pi^2}{T^2 r} \cdot \frac{r}{r} = \frac{4mr^3\pi^2}{T^2 r^2} = \frac{4\pi^2 m}{k} \cdot \frac{1}{r^2}$$

$\frac{1}{k}$

NEWTONOV ZAKON UNIVERZALNE GRAVITACIJE

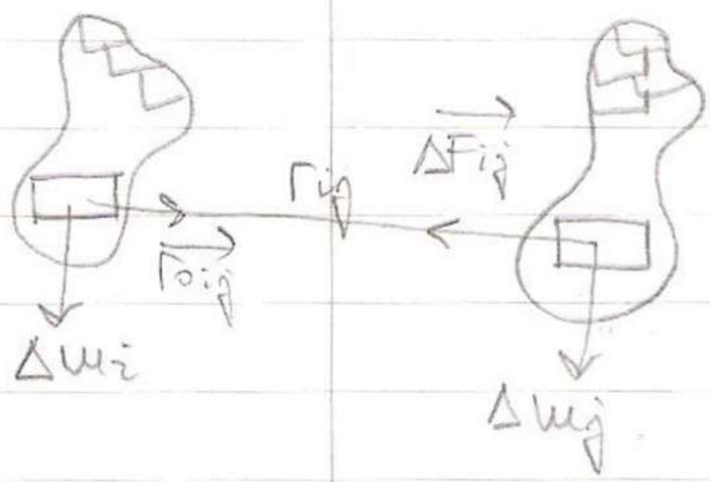
Svaka materijalna čestica privlači svaku drugu česticu silom koja je proporcionalna produktu masa tijela, a obrnuto proporcionalna kvadratu udaljenosti među njima

Privlačne sile djeluju u smjeru spojnica čestica.

$\vec{F_{12}} = -\vec{F_{21}}$ 3. NA

$$\vec{F_{12}} = -G\,\frac{m_1 m_2}{r^2}\,\vec{r_0}$$

$G = 6{,}67 \cdot 10^{-11}\, m^2N/kg^2$



n dijelova

$$\vec{\Delta F_{ij}} = -G\,\frac{\Delta m_i\,\Delta m_j}{r_{ij}^2}\,\vec{r_{0ij}}$$

$$\vec{F} = -\sum_{i,j=1}^{n} G\,\frac{\Delta m_i\,\Delta m_j}{r_{ij}^2}\,\vec{r_{0ij}}$$

HOMOGENA KUGLA

1. $\rho = konst.$ ⇒ masa je koncentrirana u središtu

2. gustoća se mijenja radijalno od središta ⇒ masa je koncentrirana u središtu

GRAVITACIJSKO POLJE

Masa u prostoru stvara svoje gravitacijsko polje

→ INTENZITET POLJA

$$\vec{\gamma} = \frac{\vec{F}}{m_2}$$

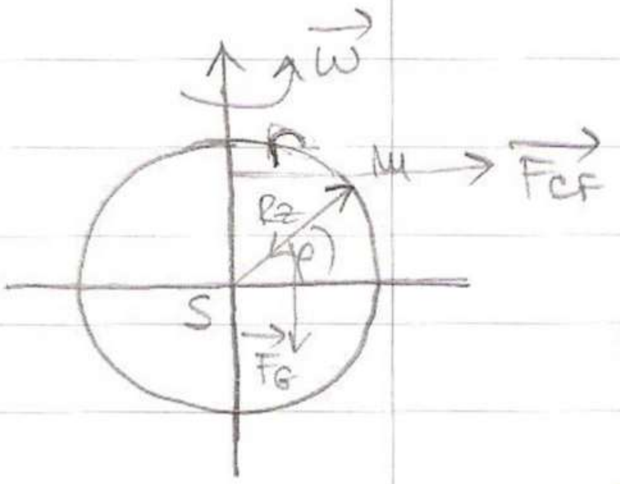
točkasta masa

$$\vec{\gamma} = -G\frac{m}{r^2}\vec{r_0}$$

- n masa u prostoru:

$$\vec{F} = M\gamma_1 + M\gamma_2 + \ldots + M\gamma_n = M(\gamma_1 + \gamma_2 + \ldots + \gamma_n)$$

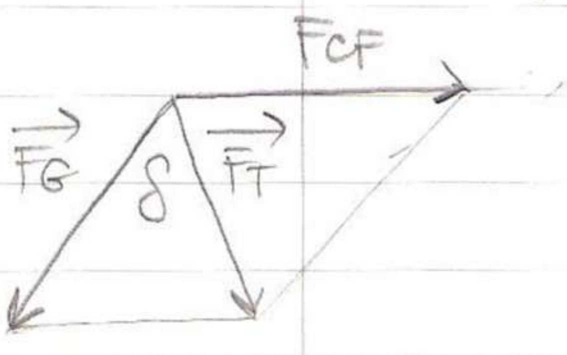
GRAVITACIJSKO POLJE ZEMLJE



$$[\gamma] = m/s^2$$

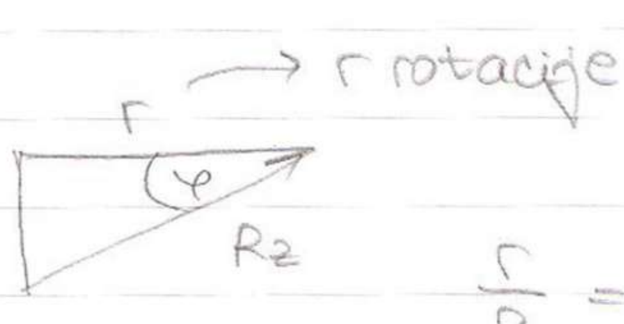
F_g ... grav. sila na površini zemlje

$$\vec{F_g} + \vec{F_{CF}} = \vec{F_T}$$



$\delta = 5{,}9°$ (izračunati!)

$$\vec{F_{CF}} = m\omega^2\vec{r} = m\omega^2 R_Z \cos\varphi$$



$$\frac{r}{R_Z} = \cos\varphi$$

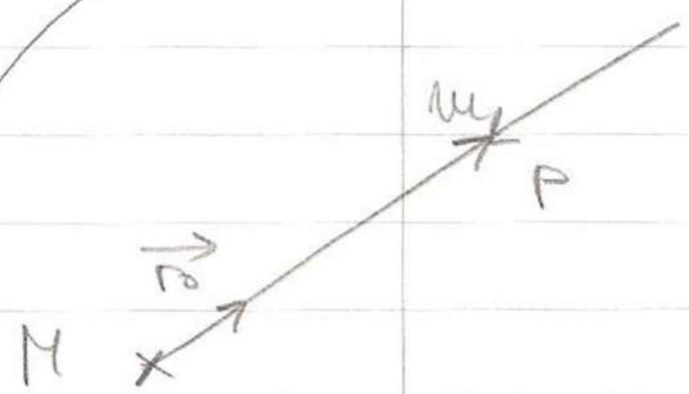
$$\vec{F_G} = -G\frac{mM_Z}{R_Z^2}\vec{r_0}$$

GRAVITACIJSKA POTENCIJALNA E

P_0 $\qquad$ $P_0 = \infty$

$U(P_0)$ $\qquad$ $U(P_0) = 0$

$$U(r) = U(P_0) - \int_{P_0}^{P} \vec{F} \, d\vec{r}$$



dolazimo radijalno iz ∞ u P

$d\vec{r} = dr \cdot \vec{r_0}$

↳ u ishodištu

$$= -\int \left(-G \frac{Mm}{r^2} \vec{r_0}\right) dr \cdot \vec{r_0} = \dots \text{ IZVESTI !}$$

$$= \frac{-GmM}{r^2}$$

. CIKLUS

P

12.5.2008.

RELATIVISTIČKA MEHANIKA

- brzine puno veće od brzine svjetlosti

- MICHELSON, MORLEY $\Rightarrow$ eter je apsolutno mirni sustav, u odnosu na njega $c = 3 \cdot 10^8$ m/s ; $V_{ZEMLJE} = 3 \cdot 10^4$ m/s $= \frac{c}{10^4}$ m/s

- Maxwellove jednadžbe se ne poklapaju s Galilejevim transformacijama (mogu se poklapati jedino ako se odustane od Newtonove teze da je vrijeme apsolutno)

EINSTEINOVI POSTULATI:

1. PRINCIP KONSTANTE BRZINE SVJETLOSTI
   Brzina svjetlosti u vakuumu jednaka je u svim inercijalnim referentnim sustavima i ne ovisi o gibanju detektora ili izvora svjetlosti ($c = 2{,}998 \cdot 10^8$ m/s)
2. Svi prirodni postulati imaju isti oblik u svim inercijalnim sustavima.

LORENTZOVE TRANSFORMACIJE

$S \xrightarrow{v} S'$

$S \quad O, x, y, z$
$S' \quad O, x', y', z'$

$S \quad (x, y, z, t)$
$S' \quad (x', y', z', t')$

$S \quad c^2t^2 = x^2 + y^2 + z^2$
$S' \quad c^2t'^2 = x'^2 + y'^2 + z'^2$

$$x^2 + y^2 + z^2 - c^2t^2 = x'^2 + y'^2 + z'^2 - c^2t'^2$$

- iz homogenosti i izotropnosti prostora $\Rightarrow$ ove transformacije se mogu međusobno povezati najopćenitijom linearnom transformacijom

$S, S' \quad y = y' \; ; \; z = z'$

$$x' = Ax + Bt + E$$
$$t' = Cx + Dt + F$$

$x = 0, \; t = 0 \quad \Rightarrow E = 0$
$x' = 0, \; t' = 0 \quad \Rightarrow F = 0$

$x' = Ax + Bt$
$t' = Cx + Dt$

$S' : x' = 0$
$S : t, x$
$x = vt$

$$0 = Ax + B\frac{x}{v} = x\left(A + \frac{B}{v}\right) \quad \Rightarrow A = -\frac{B}{v} \quad B = -Av$$

$$x' = A(x - vt)$$
$$t' = D\left(t + \frac{C}{D}x\right) \qquad a = -\frac{C}{D}$$
$$t' = D(t - ax)$$

$S: \; x = 0$

$S': \; x', t'$

$$\left.\begin{array}{l} x' = -vt' \\ x' = -Avt \end{array}\right\} \cancel{v}t' = A\cancel{v}t$$

$$t' = Dt \longrightarrow At = Dt \Rightarrow A = D$$

$$x' = A(x - vt)$$
$$t' = A(t - ax)$$

$$x^2 + \cancel{y^2} + \cancel{z^2} - c^2t^2 = A^2(x - vt)^2 + \cancel{y'^2} + \cancel{z'^2} - c^2A^2(t - ax)^2$$
$$= A^2\left[x^2 - 2xvt + v^2t^2 - c^2t^2 + 2c^2axt - c^2a^2x^2\right]$$
$$= A^2\left[2xt\left(-v + c^2a\right) + x^2\left(1 - c^2a^2\right) + v^2t^2 - c^2t^2\right]$$

$$2xt\left(-v + c^2a\right) = 0 \Rightarrow a = \frac{v}{c^2}$$

$$x^2 = A^2x^2\left(1 - a^2c^2\right)$$
$$1 = A^2\left(1 - a^2c^2\right)$$
$$A^2 = \frac{1}{1 - a^2c^2} = \frac{1}{1 - \frac{v^2}{c^2}}$$

$$\boxed{x' = \frac{x - vt}{\sqrt{1 - \frac{v^2}{c^2}}}} \; ; \quad \boxed{t' = \frac{t - \frac{v}{c^2}x}{\sqrt{1 - \frac{v^2}{c^2}}}} \qquad y' = y \; ; \; z' = z$$

iz $S \longrightarrow S'$

ako uvrstimo $v \ll c$ dobijemo GALILEJEVE TRANSFORMACIJE inverzne transformacije   iz $S' \rightarrow S$

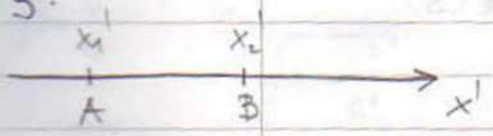
$$\boxed{x = \frac{x' + vt'}{\sqrt{1 - \frac{v^2}{c^2}}}} \quad ; \quad \boxed{t = \frac{t' + \frac{v}{c^2} x^2}{\sqrt{1 - \frac{v^2}{c^2}}}}$$

$y = y'$; $z = z'$

IZVESTI ! P

KONTRAKCIJA DUŽINE

S':



$d_0 = x_2' - x_1'$

S: $A(t, x_1)$

$B(t, x_2)$

$d = x_2 - x_1$

$$d_0 = x_2' - x_1' = \frac{x_2 - vt}{\sqrt{1-\frac{v^2}{c^2}}} - \frac{x_1 - vt}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{x_2 - x_1}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$\boxed{d = d_0\sqrt{1-\frac{v^2}{c^2}}}$$

d_0 ... vlastita duljina

$d \leq d_0$

S': t_1' A

t_2' B

$$t_2' - t_1' = \frac{t - \frac{v}{c^2}x_2}{\sqrt{1-\frac{v^2}{c^2}}} - \frac{t - \frac{v}{c^2}x_1}{\sqrt{1-\frac{v^2}{c^2}}} = -\frac{\frac{v}{c^2}(x_2 - x_1)}{\sqrt{1-\frac{v^2}{c^2}}} < 0$$

DILATACIJA VREMENA

- pr. svemirski brod (S'), svemirska postaja (S)

S': $A(t_1', x') \longrightarrow B(t_2', x')$

$\Delta t' = t_2' - t_1'$

S: $t_1 \longrightarrow t_2$

$\Delta t = t_2 - t_1$

⊛ iz $S' \longrightarrow S$ => INVERZNE LORENTZOVE TRANSFORMACIJE

$$\Delta t = t_2 - t_1 = \frac{t_2' + \frac{v}{c^2}x' - t_1' - \frac{v}{c^2}x'}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{t_2' - t_1'}{\sqrt{1-\frac{v^2}{c^2}}}$$

$\Delta t > \Delta t'$

$\Delta t' = \Delta \tau$

$$\boxed{\Delta t = \frac{\Delta \tau}{\sqrt{1-\frac{v^2}{c^2}}}}$$

- svi događaji traju dulje u odnosu na sustav koji miruje

RELATIVISTIČKO SLAGANJE BRZINA

$\beta = \frac{v}{c}$ $\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$

$S \longrightarrow S'$

$S: A(t_1, x_1, y_1, z_1) \longrightarrow B(t_2, x_2, y_2, z_2)$ $\vec{v}(v_x, v_y, v_z)$

$S': A(t_1', x_1', y_1', z_1') \longrightarrow B(t_2', x_2', y_2', z_2')$ $\vec{v}'(v_x', v_y', v_z')$

$$v_x = \frac{dx}{dt} = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}$$

$$v_x' = \frac{dx'}{dt'} = \lim_{\Delta t' \to 0} \frac{\Delta x'}{\Delta t'}$$

$$\Delta x' = x_2' - x_1' = \gamma(x_2 - vt_2) - \gamma(x_1 - vt_1) = \gamma\Big[\underbrace{x_2 - x_1}_{\Delta x} - v\underbrace{(t_2 - t_1)}_{\Delta t}\Big] =$$

$$= \gamma\big[\Delta x - v\Delta t\big]$$

$$\Delta t' = t_2' - t_1' = \gamma\left(t_2 - \frac{v}{c^2}x_2\right) - \gamma\left(t_1 - \frac{v}{c^2}x_1\right) = \gamma\left[\Delta t - \frac{v}{c^2}\Delta x\right]$$

$$\frac{\Delta x'}{\Delta t'} = \frac{\Delta x - v\Delta t}{\Delta t - \frac{v}{c^2}\Delta x} : \frac{\Delta t}{\Delta t} = \frac{\frac{\Delta x}{\Delta t} - v}{1 - \frac{v}{c^2}\frac{\Delta x}{\Delta t}}$$

$$\Rightarrow v_x' = \lim_{\Delta t' \to 0} \frac{\frac{\Delta x}{\Delta t} - v}{1 - \frac{v}{c^2}\frac{\Delta x}{\Delta t}} = \frac{v_x - v}{1 - \frac{v}{c^2}v_x}$$

$$\boxed{v_x' = \frac{v_x - v}{1 - \frac{v}{c^2}v_x}}$$

$$v_y = \lim_{\Delta t \to 0} \frac{\Delta y}{\Delta t}$$

$$v_y' = \lim_{\Delta t' \to 0} \frac{\Delta y'}{\Delta t'} = \lim_{\Delta t' \to 0} \frac{\Delta y}{\Delta t'}$$ $(\Delta y' = \Delta y)$

$$v_y' = \lim_{\Delta t \to 0} \frac{\Delta y}{\gamma\left[\Delta t - \frac{v}{c^2}\Delta x\right]} : \frac{\Delta t}{\Delta t} = \lim_{\Delta t \to 0} \frac{\frac{\Delta y}{\Delta t}}{\gamma\left[1 - \frac{v}{c^2}\frac{\Delta x}{\Delta t}\right]} =$$

$$= \frac{v_y'}{\gamma\left[1 - \frac{v}{c^2} v_x\right]}$$

$$\boxed{v_y' = \frac{v_y\sqrt{1 - \frac{v^2}{c^2} v_x}}{1 - \frac{v}{c^2} v_x}}$$

$$\boxed{v_z' = \frac{v_z\sqrt{1 - \frac{v^2}{c^2} v_x}}{1 - \frac{v}{c^2} v_x}}$$

- PRIMJER: Zraka svjetlosti koja se kreće

$v_x = c$

$\longrightarrow x$

$$v_x' = \frac{c - v}{1 - \frac{v}{c^2} c} = \frac{c - v}{\frac{c - v}{c}} = \underline{\underline{c}}$$

(*) Brzina svjetlosti ista je za sve inercijalne sustave.

* za male brzine dobijemo Galilejeve transformacije

RELATIVISTIČKA DINAMIKA

$\vec{p} = m\vec{v}$ ... klasična fizika

$$\boxed{\vec{p} = \frac{m\vec{v}}{\sqrt{1-\frac{v^2}{c^2}}}}$$

... u relativističkoj fizici ne vrijedi isti izraz kao u klasičnoj, jer dobijemo da ne vrijedi zakon očuvanja p

$$\boxed{\vec{F} = \frac{d\vec{p}}{dt}\left(\frac{m\vec{v}}{\sqrt{1-\frac{v^2}{c^2}}}\right)}$$

$\Rightarrow$ AKCELERACIJA i SILA NEMAJU ISTI SMJER

KINETIČKA ENERGIJA

A $t=0,\ x=0,\ v=0$

$\downarrow$

B $t,\ x,\ v$

$$W = \int_0^x F\,dx = \int_0^x \frac{d}{dt}\left(\frac{mv}{\sqrt{1-\frac{v^2}{c^2}}}\right)\underbrace{dx}_{dx = v\,dt} = \int_0^t \underbrace{\frac{d}{dt}\left(\frac{mv}{\sqrt{1-\frac{v^2}{c^2}}}\right)}_{z} v\,dt = (*)$$

$$z = \left[\frac{m}{\sqrt{1-\frac{v^2}{c^2}}} + \frac{m\frac{v^2}{c^2}}{\left(1-\frac{v^2}{c^2}\right)^{3/2}}\right]\frac{dv}{dt} = \left[\frac{m}{\left(1-\frac{v^2}{c^2}\right)^{3/2}}\left(1-\frac{v^2}{c^2}+\frac{v^2}{c^2}\right)\right]\frac{dv}{dt}$$

$$= \frac{m}{\left(1-\frac{v^2}{c^2}\right)^{3/2}}\frac{dv}{dt}$$

$$(*) = \int_0^t \frac{mv}{\left(1-\frac{v^2}{c^2}\right)^{3/2}}\frac{dv}{dt}\cdot dt = \int_0^t \frac{d}{dt}\left(\frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}}\right)dt = \left.\frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}}\right|_0^t =$$

LeColor

$$= \frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}} - mc^2$$

$$\boxed{E_k = \frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}} - mc^2}$$

$$\boxed{E_u = \frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}}}$$ ... E_u čestice koja se giba brzinom v

$\boxed{E_c = mc^2}$ ... $v=0$; vlastita E čestice (mirovanja)

* pretpostavka : $v \ll c$

$$E_k = mc^2\left[\cancel{1} + \frac{v^2}{2c^2} + \ldots - \cancel{1}\right] = \frac{mv^2}{2}$$ KLASIČNA FIZIKA

$$E = \frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}} \qquad p = \frac{mv^2}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$E^2 - c^2p^2 = \frac{m^2c^4}{1-\frac{v^2}{c^2}} - \frac{m^2v^4c^2}{1-\frac{v^2}{c^2}} = \frac{m^2c^4}{\cancel{1-\frac{v^2}{c^2}}}\cancel{\left(1-\frac{v^2}{c^2}\right)} = m^2c^4$$

$$E^2 - c^2p^2 = m^2c^4$$
$$E^2 = m^2c^4 + c^2p^2$$

- ako je $m=0 \Rightarrow E^2 = c^2p^2 \Rightarrow p = \frac{E}{c}$

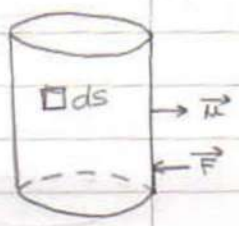
(WTF ?!? Oo)

STATIKA FLUIDA

Zbog sudara čestica sa stijenkama posude nastaje sila koju zovemo PRITISAK (TLAK).

$$\boxed{\overline{p} = \frac{\Delta F}{\Delta S}}$$

$$\boxed{p = \lim_{\Delta S \to 0} \frac{\Delta F}{\Delta S} = \frac{dF}{dS}}$$



$d\vec{S} = dS\,\vec{n}$

$d\vec{F} = -p\,d\vec{S}$

$[p] = N/m^2 = Pa$ $\quad$ $bar = 10^5\ Pa$

$$\boxed{\kappa = -\frac{1}{V}\left(\frac{dV}{dp}\right)_T}$$ ... KOEFICIJENT KOMPRESIBILNOSTI

IDEALNI PLIN

$V = \frac{c}{p}$

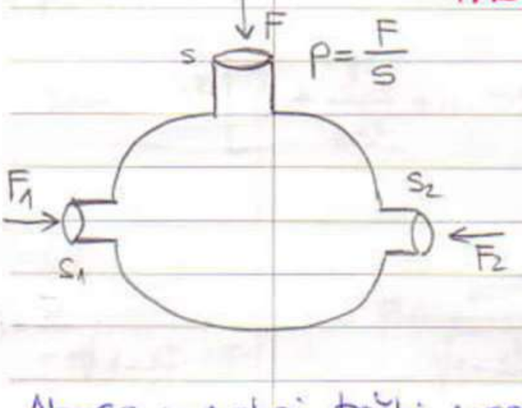
$T = \text{konst.}$

$pV = \text{konst.} = c$

$$\left(\frac{dV}{dp}\right)_T = \frac{d}{dp}\left(\frac{c}{p}\right) = -\frac{c}{p^2} = \frac{-pV}{p^2} = -\frac{V}{p}$$

$$K = -\frac{1}{V}\left(-\frac{V}{p}\right) = \frac{1}{p}$$ ... za idealni plin

PASCALOV ZAKON



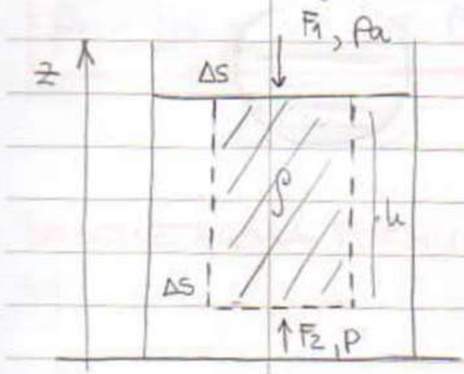
$$p = \frac{F}{S} = \frac{F_1}{S_1} = \frac{F_2}{S_2}$$

... tlak se prenosi

Ako se u nekoj točki nestlačivog mirnog fluida tlak promijeni za neki iznos, onda se tlak promijeni za taj isti iznos svuda u fluidu.

HIDROSTATSKI TLAK

→ uzrokovan težinom samog fluida
→ nastaje u gravitacijskom polju kad djeluje sila teža na sve čestice tijela (VOLUMNA SILA)



$$F_1 = p_a \Delta S$$
$$F_2 = p \Delta S$$

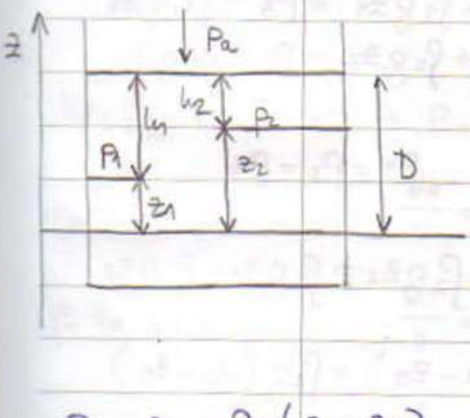
$$F_2 - F_1 = \rho g \Delta S h$$
$$p\cancel{\Delta S} - p_a\cancel{\Delta S} = \rho g \cancel{\Delta S} h$$

$$p - p_a = \rho g h$$

$$\boxed{p = p_a + \rho g h}$$

$\boxed{p_h = \rho g h}$ ... HIDROSTATSKI TLAK

NESTLAČIVI HOMOGENI FLUID



$$p_1 = p_a + \rho g h_1$$
$$p_2 = p_a + \rho g h_2$$

$$p_1 - p_2 = \rho g h_1 - \rho g h_2 = \rho g (h_1 - h_2)$$

$$h_1 - h_2 = D - z_1 - D + z_2 = z_2 - z_1$$

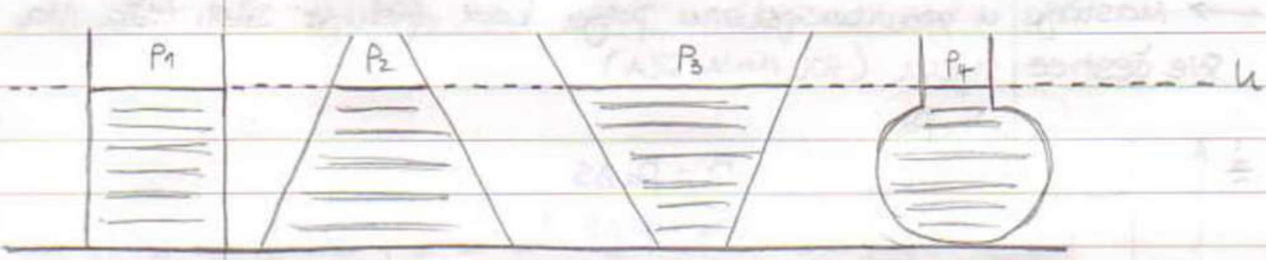
$$p_1 - p_2 = \rho g (z_2 - z_1)$$

$$\boxed{\begin{gathered} p_1 + \rho g z_1 = p_2 + \rho g z_2 \\ p + \rho g z = \text{KONST.} \end{gathered}}$$

U mirnom nestlačivom fluidu zbroj tlaka i umnoška gustoće s visinom od čvrste horizontalne ravnine je konst.

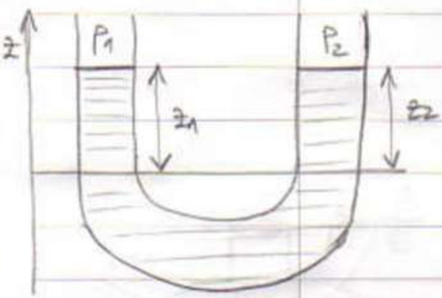
HIDROSTATSKI PARADOKS

HORVAT – Fizika 1; slika 2.9 (str. 8-12)

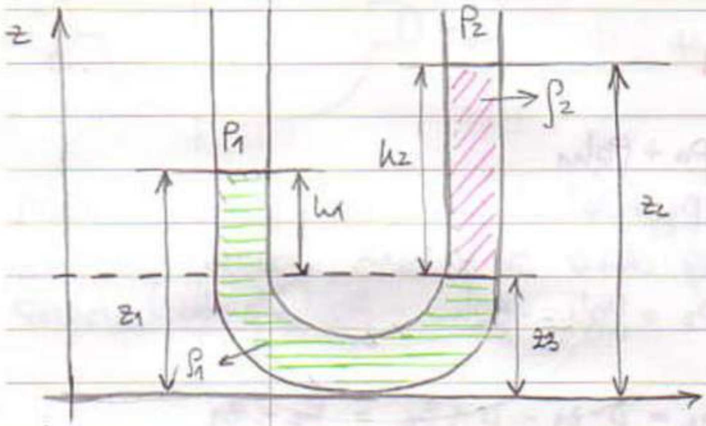


$p_1 = p_2 = p_3 = p_4$

SPOJENE POSUDE



$$\cancel{p_1} + \rho g z_1 = \cancel{p_2} + \rho g z_2$$
$$\rho g z_1 = \rho g z_2$$
$$z_1 = z_2$$



① $p_1 + \rho_1 g z_1 = p_3 + \rho_1 g z_3$

② $p_2 + \rho_2 g z_2 = p_3 + \rho_2 g z_3$

$$p_1 = p_2 = p_a$$

$$\rho_1 g z_1 - \rho_2 g z_2 = \rho_1 g z_3 - \rho_2 g z_3$$

$$\rho_1 g (z_1 - z_3) = \rho_2 g (z_2 - z_3)$$

$$\cancel{g} \rho_1 h_1 = \cancel{g} \rho_2 h_2$$

$$\boxed{\rho_2 = \frac{\rho_1 h_1}{h_2}}$$

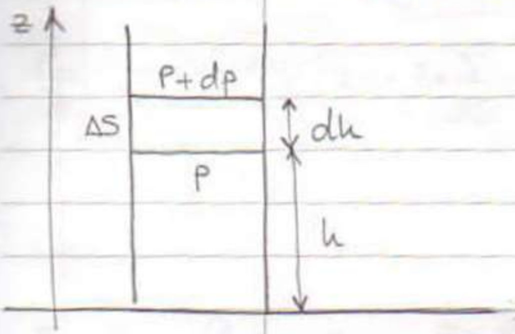
ATMOSFERSKI TLAK

TORRICHELLIJEV POKUS

- na površini mora, $h = 0{,}76\,m \longrightarrow \rho_z = 13{,}595 \cdot 10^3\ kg/m^3$

$$\boxed{p_a = \rho g h = 101\ 325\ Pa}$$

BAROMETARSKA FORMULA



$T = konst$

11 km → 20 km

$$p + \rho g h = p + \rho g (h + dh) + dp$$

$$p + \rho g h = p + dp + \rho g h + \rho g dh$$

$$dp = \rho g dh$$

$$h_0, p_0, \rho_0 \longrightarrow h, p, \rho$$

$$p_0 V_0 = p V \qquad \frac{V_0}{V} = \frac{p}{p_0}$$

$$\rho_0 V_0 = \rho V \qquad \frac{V_0}{V} = \frac{\rho}{\rho_0}$$

$$\boxed{\frac{p}{p_0} = \frac{\rho}{\rho_0}}$$

$$\rho = \frac{p}{p_0} \rho_0$$

$$dp = -\frac{p}{p_0} g\, dh\, \rho_0$$

Le Color

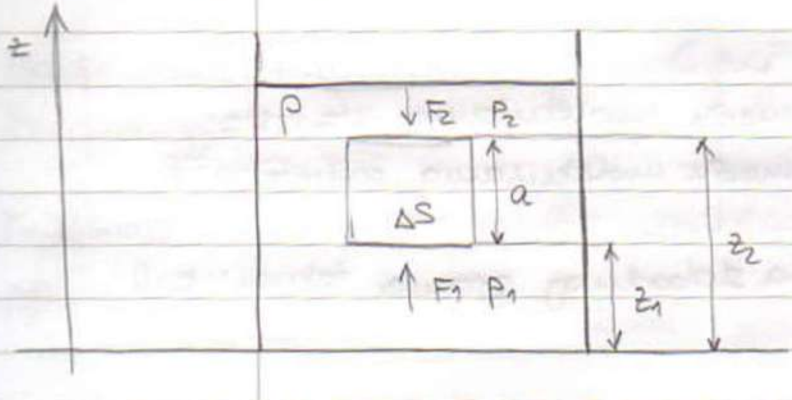
$$\int_{p_0}^{p} \frac{1}{p}\,dp = \int_{h_0}^{h} -\frac{\rho_0 g}{p_0}\,dh$$

$$\ln p \Big|_{p_0}^{p} = -\frac{\rho_0 g}{p_0}\, h \Big|_{h_0}^{h}$$

$$\ln \frac{p}{p_0} = -\frac{\rho_0 g}{p_0}(h - h_0)$$

$$\boxed{p = p_0\, e^{-\frac{\rho_0 g}{p_0}(h-h_0)}}$$

UZGON



$$p_1 + \rho g z_1 = p_2 + \rho g z_2$$

$$(p_1 - p_2) = \rho g (z_2 - z_1)$$

$$= \rho g a$$

$$F_1 - F_2 = \rho g a \, \Delta S = \rho g V \qquad \boxed{U = \rho g V}$$

Uzgon je sila koja djeluje prema gore i po iznosu je jednak težini istisnutog fluida na mjestu uronjenog tijela => ARHIMEDOV ZAKON

Težina tijela uronjenog u fluid smanjuje se za iznos težine istisnutog fluida.

FANGEOVA VAGA

→ sapun smanjuje površinsku napetost
→ sile površinske napetosti nastoje smanjiti površinu =>
- KAPILARNA ELEVACIJA
- KAPILARNA DEPRESIJA

NAPETOST POVRŠINE

- molekularne sile
  - * KOHEZIJSKE = među molekulama iste vrste
  - * ADHEZIJSKE = među molekulama različite vrste

- površinska napetost na slobodnoj granici tekućine

$F, \Delta x$

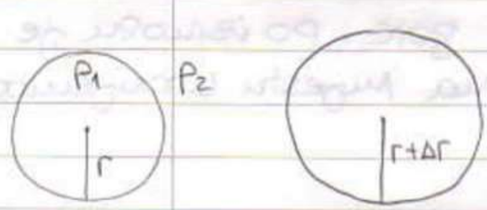
$\Delta W = F\Delta x$

$\Delta S = L\Delta x \, 2$

$\sigma = \frac{F\Delta x}{2L\Delta x} = \frac{F}{2L}$

$$\boxed{\sigma = \frac{\Delta W}{\Delta S} = \frac{F}{2L}} \qquad [\sigma] = \frac{J}{m^2} = \frac{N}{m}$$

TLAK ISPOD ZAKRIVLJENE POVRŠINE TEKUĆINE



$\Delta p = p_1 - p_2$

$dW = \sigma dS$ $\qquad S = 4r^2\pi \qquad dS = 8r\pi dr$

$dW = 2\sigma \cdot 8r\pi dr$

$dW = \Delta p \, S \, dr = \Delta p \, 4r^2\pi \, dr$

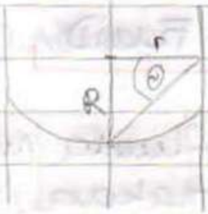
$2\sigma \cdot 8r\pi dr = \Delta p \, 4r^2\pi dr$

$$\boxed{\Delta p = \frac{4\sigma}{r}}$$

* poseban slučaj => mjehurić u vodi => $\Delta p = \frac{2\sigma}{r}$

KAPILARNA ELEVACIJA

r... polumjer kapilare
R... polumjer



$r = R\cos\Theta$

$$\Delta p = \frac{2\sigma}{R}$$

$p_v = p_u + \rho g h$
$\Delta p = p_v + p_u = \rho g h$

$$\frac{2\sigma}{R} = \rho g h$$

$$\boxed{h = \frac{2\sigma\cos\Theta}{\rho g h}}$$

HIDRODINAMIKA - DINAMIKA FLUIDA

IDEALNI NESTLAČIVI FLUID

- gibanje tekućine ili fluida nazivamo STRUJANJE
- nastaje zbog razlike tlakova, težine fluida

$\vec{v}(\vec{r},t)$ ... vektorsko polje brzine

- STRUJNICE - zamišljene linije u fluidu čija tangenta u svakoj točki pokazuje smjer brzine

- dio fluida omeđenog strujnicama nazivamo STRUJNOM CIJEVI

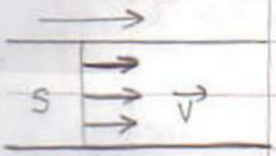
→ STRUJANJE:

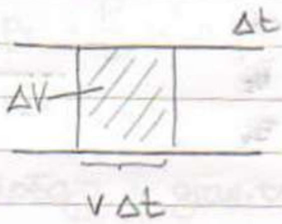
a) stacionarno
- brzina i tlak su FUNKCIJE POLOŽAJA (ne mijenjaju se u vremenu)
- $\vec{v}(\vec{r})$, $\vec{p}(\vec{r})$

b) nestacionarno
- $\vec{v}(\vec{r},t)$, $\vec{p}(\vec{r},t)$
- $v < 100$ m/s ⇒ možemo reći da se plinovi ponašaju nestlačivo

JEDNADŽBA KONTINUITETA

STRUJANJE IDEALNOG NESTLAČIVOG FLUIDA



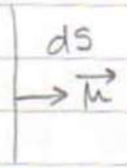


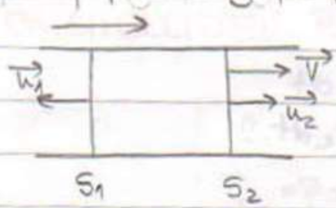
$\Delta V = v S \Delta t$

$q_v = \frac{\Delta V}{\Delta t} = vS$ $\qquad \boxed{q_v = vS}$ VOLUMNI PROTOK

$S_1 v_1 = S_2 v_2$ $\qquad \boxed{S v = konst.}$

→ ako se brzina duž poprečnog presjeka mijenja:





$\boxed{q_v = \int_S \vec{v}\, d\vec{S}}$

$d\vec{S} = dS \cdot \vec{n}$ ... gleda iz volumena prema van

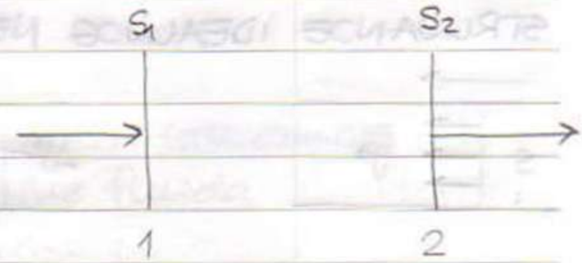
$\int_{S_1} \vec{v}\, d\vec{S} + \int_{S_2} \vec{v}\, d\vec{S} = 0$ ... ako gledamo rubne plohe

$S = S_1 \cup S_2$ ... + bočne plohe

$$\boxed{\oint_S \vec{v}\, d\vec{S} = 0}$$

BERNOULLIJEVA JEDNADŽBA

1 – ULAZ	2 – IZLAZ
p_1	p_2
v_1	v_2
S_1	S_2
	dt



$F_1 = p_1 S_1$
$F_2 = p_2 S_2$

$v_1 S_1 = v_2 S_2$

$dr_1 = v_1\, dt$
$dr_2 = v_2\, dt$

$dW_1 = F_1\, dr_1 = F_1 v_1\, dt = p_1 S_1 v_1\, dt$
$dW_2 = -F_2\, dr_2 = -F_2 v_2\, dt = -p_2 S_2 v_2\, dt$

$dW = dW_1 + dW_2 = p_1 S_1 v_1\, dt - p_2 S_2 v_2\, dt$

$dV_1 = S_1\, dr_1 = dV_2 = S_2\, dr_2 = dV$

$S_1 v_1\, dt = S_2 v_2\, dt$
$dV_1 = dV_2$

$dW = (p_1 - p_2)\, dV$
$dm = \rho\, dV$

$dE = \frac{1}{2} v_2^2\, dm + g y_2\, dm - \frac{1}{2} v_1^2\, dm - g y_1\, dr$

$dW = dE$
$(p_1 - p_2)\, dV = \frac{1}{2}(v_2^2 - v_1^2)\, dm + g(y_2 - y_1)\, dm \quad \Big| \rho$

$\rho(p_1 - p_2)\, dV = \frac{\rho}{2}(v_2^2 - v_1^2)\, dm + \rho g\, (y_2 - y_1)\, dm$

$$(p_1 - p_2) = \frac{\rho}{2}(v_2^2 - v_1^2) + \rho g (y_2 - y_1)$$

$$\boxed{p_1 + \frac{\rho}{2} v_1^2 + \rho g y_1 = p_2 + \frac{\rho}{2} v_2^2 + \rho g y_2}$$

dinamički

statički

zbog položaja u gravitacijskom polju

$$\boxed{p + \frac{\rho}{2} v^2 + \rho g y = \text{konst.}}$$

$y_1 - y_2$ $\qquad p_1 - p_2 = \frac{\rho}{2}(v_2^2 - v_1^2)$

TORICHELLIJEV POKUS

1.

$p_1 = p_{at}$

$v_1 = 0$

$S_1 >> S_2$

$y_1 = h$

2.

$p_2 = p_{at}$

v_2

$y_2 = 0$

$$p_{at} + \rho g h = p_{at} + \frac{\rho}{2} v_2^2$$

$$v_2 = v = \sqrt{2gh}$$

$$Q_V = kS\sqrt{2gh}$$

SILA VISKOZNOG TRENJA

$$\boxed{F_{TD} = \mu S \frac{dv}{dz}}$$

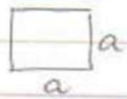
REYNOLDS $\qquad v = v_K$

$\vec{v}(\vec{r})$

$\vec{v}(\vec{r}, t)$

$$R_e = \frac{\rho v l}{\mu} \qquad \gamma = \frac{\mu}{\rho}$$

$l = 2r$



$l = a$

$R_e \longrightarrow R_{ek}$

$L \longrightarrow T$

$\rho_1, \mu_1, l_1, v_1 \qquad \rho_2, \mu_2, l_2, v_2$

$R_e \qquad R_e$

DINAMIČKA SLIČNOST

$l_1 v_1 = l_2 v_2 \qquad \frac{l_1}{l_2} = 25$

$\frac{v_2}{v_1} = 25$

PROTJECANJE REALNOG FLUIDA KROZ CIJEV

$\vec{F}$ ... sila zbog razlike tlakova
$\vec{F}_{TR}$ ... viskozna sila

$$\vec{F} + \vec{F}_{TR} = 0 \qquad \vec{F} = \vec{F}_{TR}$$

$$\vec{F} = (p_1 - p_2) r^2 \pi$$
$$\vec{F}_{TR} = -\mu S \frac{dv}{dr} \qquad S = 2r\pi L$$

... uvjeti $r = R$, $v = 0$

$$(p_1 - p_2) r^2 \pi = -\mu 2 r \pi l \frac{dv}{dr}$$ integriramo

$$\frac{r^2}{2}(p_1 - p_2) = -2\mu l v + c$$

iz uvjeta $C = \frac{R^2}{2}(p_1 - p_2)$

$$v = \frac{(p_1 - p_2)}{4\mu l}(R^2 - r^2)$$

$$q_v = \int v \, dS = \int_0^R \frac{p_1 - p_2}{4\mu l}(R^2 - r^2) 2r\pi dr = \frac{\pi}{8\mu} \frac{p_1 - p_2}{l} R^4$$

↳ VOLUMNI PROTOK

$S = r^2 \pi$
$dS = 2r\pi dr$

$$v = v_0 \left(1 - \frac{r^2}{R^2}\right)$$

$$q_v = \frac{\pi}{8\mu} \frac{p_1 - p_2}{l} R^4$$

$$r = 0 \rightarrow v_0 = \frac{p_1 - p_2}{4\mu L} R^2 \qquad v = \frac{p_1 - p_2}{4\mu l}(R^2 - r^2)$$

$$F_{OT} = (p_1 - p_2) R^2 \pi = \frac{q_v - 8\mu L}{R^2} = \frac{8\mu L}{R^2} \cdot q_v$$

$$\overline{v} = \frac{q_v}{R^2 \pi}$$

$$q_v = \overline{v} R^2 \pi$$

$$F_{OT} = \frac{8\mu l}{R^2} \overline{v} R^2 \pi = 8\pi \mu L \overline{v}$$

MAGNUSOV EFEKT

$$F_{OT} = 6\pi \mu r v$$

$$F_{OT} = \frac{1}{2} C_{OT} \rho S v^2$$

$\hookrightarrow$ otporni broj

... anomalija vode

$$\gamma = \frac{1}{V} \lim \frac{\Delta V}{\Delta t} = \frac{1}{V} \frac{dV}{dt}$$

TOPLINA i TEMPERATURA

Toplinski sustav je konačni dio ~~[illegible]~~ prostora koji sadrži fizikalne objekte koji se istražuju i razmatraju sa stanovišta njihovih toplinskih svojstava.

 gibanje klipa bez trenja

Kažemo da su dva tijela (sustava) u toplinskom dodiru ako mogu izmjenjivati E bez da jedan vrši rad nad onim drugim.

Dva su tijela (sustava) u toplinskoj ravnoteži ako postoji toplinski dodir, ali nema izmjene E.

Temperatura je fizikalna veličina koja karakterizira stupanj zagrijanosti nekog tijela.

0. ZAKON TERMODINAMIKE
Ako su 2 tijela (sustava) odvojena u toplinskoj ravnoteži s trećim sustavom, onda su i oni međusobno u toplinskoj ravnoteži.

TERMOMETRIJA

- termometrijska svojstva
- temperaturna skala
- jedinice temperature

- Celzijusova skala
- termodinamička ili apsolutna temperaturna skala

PLINSKI ZAKONI

- zanemaruje se međudjelovanje između čestica plina, volumen čestica u odnosu na volumen posude plina

ZAKONI

1. BOYLE - MARIOTTEOV ZAKON
   - $T = konst.$ IZOTERMNI
   - $\boxed{p_1 V_1 = p_2 V_2}$
   - $\boxed{pV = konst.}$

2. GAY - LUSSAC
   - $p = konst.$ IZOBARAN
   - $V = V_0 (1 + \alpha t)$ $\qquad \alpha = \frac{1}{273{,}15}\ K^{-1}$
   - $V = V_0 \left(\frac{273{,}15 + t}{273{,}15} \right) = V_0 \frac{T}{T_0}$
   - $\boxed{\frac{V}{T} = \frac{V_0}{T_0}}$

3. CHARLESOV ZAKON
   - $V = konst.$ IZOHORAN
   - $p = p_0 (1 + \beta \Delta t)$ $\qquad \beta = \alpha$
   - $\boxed{\frac{p}{T} = \frac{p_0}{T_0}}$

JEDNADŽBA STANJA IDEALNOG PLINA

A — B

V_0 — V

$T_0 = 273{,}15$ K — T

$p_0 = 101\,325$ Pa — p

$$A \xrightarrow{p_0 \text{ konst.}} p_0, T, V' \xrightarrow{T} B$$

$$\frac{V_0}{T_0} = \frac{V'}{T}$$

$$p_0 V' = pV$$

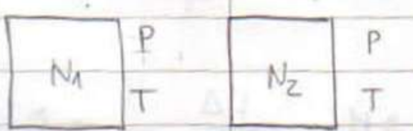
$$V' = T\frac{V_0}{T_0} = V\frac{p}{p_0}$$

$$\gamma = \frac{V_0}{T_0} = V\frac{p}{p_0}$$

$$\boxed{\frac{p_0 V_0}{T_0} = p\frac{V}{T}}$$

AVOGADRO

Jednaki volumeni svih plinova pri istoj T i p imaju jednak broj čestica.



množina, količina tvari

$_{12}C$ $\quad U = 1{,}661 \cdot 10^{-27}$ kg

$$N_0 = \frac{M_1}{m_1} = \frac{M}{m_u} \cdot \frac{1}{1{,}661 \cdot 10^{-27}}$$

$$N_0 = 6{,}022 \cdot 10^{23}$$

$V_0 = 22{,}4 \cdot 10^{-3}\ m^3/mol$ molarni volumen plina pri standardnim uvjetima

$$V_0 = n V_m$$

$$p \frac{V}{T} = p_0 \frac{V_m}{T_0} n$$ plinska konstanta

$$R = 8{,}314\ J/K\,mol$$

$\boxed{pV = nRT}$ CLAPEYRONOVA J. STANJA IDEALNOG PLINA

$$m = nM$$

$$pV = \frac{m}{M} RT$$

$$pV = \frac{N}{N_0} RT$$

$$K = \frac{R}{N_0} = 1{,}38 \cdot 10^{-32}\ Jk$$

$$pV = KNT$$

TOPLINSKI KAPACITETI

$c = \frac{\Delta Q}{\Delta T}$ ... specifični toplinski kapacitet

→ Toplina potrebna da se tijelu temp. povisi za 1 stupanj

$c = \frac{1}{m} \frac{\Delta Q}{\Delta T}$ $[c] = J/kgK$

$c = \frac{1}{m} \lim_{\Delta T \to 0} \frac{\Delta Q}{\Delta T} = \frac{1}{m} \frac{dQ}{dT}$

$\Delta Q = cm\Delta T = \int_{T_1}^{T_2} cm\,dT$

molarni toplinski kapacitet

$\boxed{C = \frac{1}{\mu} \frac{\Delta Q}{\Delta T}}$ ... molarni toplinski kapacitet

... veza između c i C

$m = \mu M$

$\frac{1}{\mu} = \frac{M}{m}$

$C = \frac{M}{m} \frac{\Delta Q}{\Delta T} = Mc$

$C = Mc$

$$c_p = \frac{1}{m}\left(\frac{dq}{dT}\right)$$ P... konst

$$c_v = \frac{1}{m}\left(\frac{dq}{dT}\right)$$ V... konst.

NERNST - LICHLEMANNOVA RELACIJA

$$C_v = C_p\left[1 - a\,\frac{M c_p T}{T_m}\right]$$ $a = 0{,}0051$ Mol K / J

$T_m \rightarrow$ T taljenja za a

PROMJENA AGREGATNIH STANJA

FAZA je homogeni dio nekog sustava koji u svim svojim dijelovima ima ista svojstva i koji je određen granicom odvojen od ostalih dijelova sustava.

$q = mL$ ... LATENTNA TOPLINA

FAZNI DIJAGRAMI

$$v = \frac{V}{m}$$

$$V_m = \frac{V}{n}$$

PRIJENOS TOPLINE

1. VOĐENJE
   - kruta tijela
2. STRUJANJE
   - tekućine
3. ZRAČENJE
   - Sunce

FOURIEROV ZAKON VOĐENJA

$$Q = -\lambda \frac{\Delta T}{\Delta x} S t$$

T_1, T_2 $\Delta T = T_2 - T_1$

λ ... koeficijent toplinske vodljivosti materijala

$\frac{\Delta T}{\Delta x}$ ... gradijent temperature

TOPLINSKI TOK

$$\phi = \frac{Q}{t} = -\lambda \frac{\Delta T}{\Delta x} S$$

$$q = -\frac{\phi}{S} = -\lambda \frac{\Delta T}{\Delta x}$$

↳ gustoća toplinskog toka

$$\phi = -\frac{\Delta T}{\frac{\Delta x}{\lambda S}} = -\frac{\Delta T}{R} = \frac{|\Delta T|}{R}$$

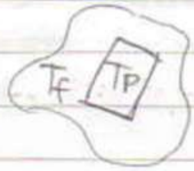
$$R = \frac{\Delta x}{\lambda S} \qquad [R] = K/W$$

↳ toplinski otpor

PRIJENOS TOPLINE STRUJANJEM

$$q = h_c (T_P - T_F)$$

↳ koeficijent konvekcije



RADIJACIJA

$$I = \sigma T^4 \qquad \sigma = 5,67 \cdot 10^{-8}\ W s^{-2} K^{-4}$$

Crno tijelo emitira 4x veću toplinu; najbolji apsorber i emiter topline

ε... koef. emisivnosti ... $\langle 0,1 \rangle$

$$I = \varepsilon \sigma T^4$$

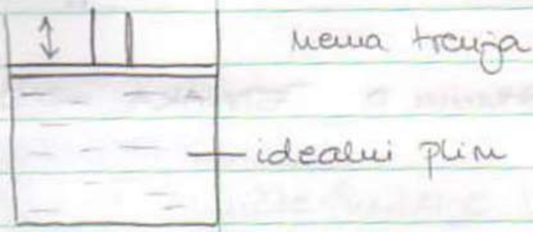
TERMODINAMIKA

MAKROSKOPSKI SUSTAV

Objekt koji je velik u usporedbi sa dimenzijama atoma i molekula. On sam sastoji se od velikog broja molekula i atoma.

MAKROSKOPSKA VELIČINA

Ona fizikalna veličina koja opisuje neposredno mjerljivo svojstvo makroskopskog sustava. (p, V, T)



Stanje sustava za čiji opis se rabe fizikalne veličine određene makroskopskim mjerenjima bez uzimanja u obzir mikroskopskih svojstava sustava ⟶ MAKROSKOPSKO STANJE

Stanje sustava koje je određeno potpunim opisom prema zakonima mehanike svih čestica sustava. ⟶ MIKROSKOPSKO STANJE

TERMODINAMIČKI PROCESI

1. REVERZIBILNI
2. IREVERZIBILNI

Termodinamički sustav je u ravnoteži ako:

- je postignuta meh. ravnoteža unutar sustava i prema okolini
- temperatura svih dijelova sustava je jednaka. Ako

TOPLINSKI TOK

$$\phi = \frac{Q}{t} = -\lambda \frac{\Delta T}{\Delta x} S$$

$$q = -\frac{\phi}{S} = -\lambda \frac{\Delta T}{\Delta x}$$

↳ gustoća toplinskog toka

$$\phi = -\frac{\Delta T}{\frac{\Delta x}{\lambda S}} = -\frac{\Delta T}{R} = \frac{|\Delta T|}{R}$$

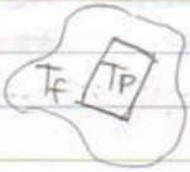
$$R = \frac{\Delta x}{\lambda S} \qquad [R] = K/W$$

↳ toplinski otpor

PRIJENOS TOPLINE STRUJANJEM

$$q = h_c (T_P - T_F)$$

↳ koeficijent konvekcije



RADIJACIJA

$$I = \sigma T^4 \qquad \sigma = 5{,}67 \cdot 10^{-8}\ W s^{-2} k^{-4}$$

Crno tijelo emitira 4x veću toplinu ; Najbolji apsorber i emiter topline

ε... koef. emisivnosti $\langle 0,1 \rangle$

$$I = \varepsilon \sigma T^4$$

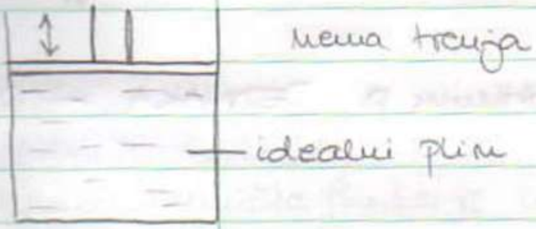
TERMODINAMIKA

MAKROSKOPSKI SUSTAV

Objekt koji je velik u usporedbi sa dimenzijama atoma i molekula. On sam sastoji se od velikog broja molekula i atoma.

MAKROSKOPSKA VELIČINA

Ona fizikalna veličina koja opisuje neposredno mjerljivo svojstvo makroskopskog sustava. (p, V, T)



Stanje sustava za čiji opis se rabe fizikalne veličine određene makroskopskim mjerilima bez uzimanja u obzir mikroskopskih svojstava sustava → MAKROSKOPSKO STANJE

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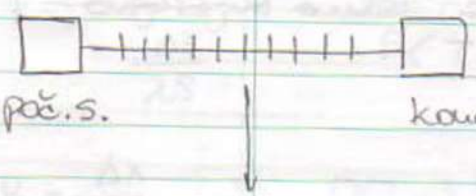
sustav može razmjenjivati toplinu s okolinom
da bi bio u ravnoteži mora mu temperatura
biti beskonačno malo različita od temperature
okoline.

- Sustav je u kem. ravnoteži, što znači da mu se kem. sastav ne mijenja s vremenom.



poč.s. konačno.s.

međustanja ⇒ u reverzibilnim p. JEDNAKA → preko istih

PRIJENOS ENERGIJE

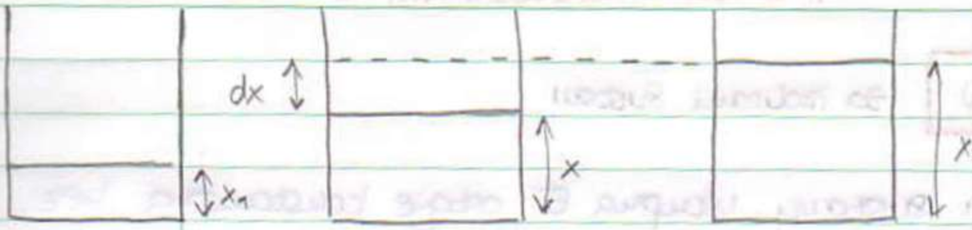
UNUTRAŠNJA E – kad sustav međudjeluje s okolinom, on tada povećava|smanjuje svoju unutrašnju E

$\boxed{U = E_k + E_p}$

IDEALNI PLIN $U = E_k$; zanemarimo sile između čestica $E_p = 0$

IREVERZIBILNI PROCESI

- bez trenja
- zagrijavamo klip, te nakon nekog vremena otpustimo
- niz nejednakih međustanja, ne možemo istim putem iz konačnog doći u početno bez vanjske E



P.S. M.S. K.S.

$$dW = F dx = pS dx = p dV$$

$$W = \int_{V_1}^{V_2} p\, dV$$ mehanički rad termodinamičkih procesa

FUNKCIJE PROCESA
- termodinamičke funkcije kojima dolazimo iz početnog u konačno stanje

FUNKCIJA STANJA
- ovisi samo o početnom i konačnom stanju, ne i o načinu na koji smo došli do njega
- unutrašnja E

DIGRESIJA:
- da bi se f-ja $P(x,y)\,dx + Q(x,y)\,dy = 0$ mogla prikazati kao totalni diferencijal, nužan i dovoljan uvjet:

$$\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}$$

- ako vrijedi df $\qquad \oint df = 0$
- ako ne vrijedi $đf$ $\qquad \oint đf \neq 0$

1. ZAKON TERMODINAMIKE

$\boxed{\Delta Q = \Delta U + W}$ za izolirani sustav

U izoliranom sustavu ukupna E ostaje konstantna bez obzira na događaje u sustavu.

- ako sustavu dovodimo E $\Delta Q > 0$
- ako sustavu odvodimo E $\Delta Q < 0$

- ako sustav obavlja rad na okolinu $W > 0$
- ako okolina obavlja rad na sustav $W < 0$

$\boxed{đQ = dU + đW}$ uvijek vrijedi

V=konst. $đW = 0$

$dU = đQ = \mu C_v dT$

$\boxed{đQ = \mu C_v dT + đW}$ vrijedi za idealne plinove

$\boxed{\begin{matrix} \Delta U = 0 \\ W = \Delta Q \end{matrix}}$ kružni proces

→ nije moguć perpetuum mobile 1. vrste ⇒ ne možemo dobiti više nego što smo uložili

HVALA!

→ Malome od skenera ⇒ ljubman

→ Sanči na skriptama

(fali srijeda, 11.6. → Poisson, Carnotov proces, kin. teorija plinova)

11.6.2008.

$\Delta Q = U + \Delta W$

$dQ = \mu C_v \, dT + p dV$

<u>V = konst.</u>

$W = \int_p^k p dV = 0 \Rightarrow \boxed{\Delta Q = U}$

<u>p = konst.</u>

$W = \int_{V_1}^{V_2} p dV \boxed{= p(V_2 - V_1)}$

<u>T = konst.</u>

$W = \int_{V_1}^{V_2} p dV = \int_{V_1}^{V_2} \frac{\mu R T}{V} dV = \mu R T \int_{V_1}^{V_2} \frac{dV}{V} = \mu R T \ln \frac{V_2}{V_1} = \mu R T \ln \frac{p_1}{p_2}$

$\Delta U = 0 \Rightarrow \boxed{\Delta Q = \Delta W}$

POISSONOVE JEDNADŽBE

MAYER

K ... adijabatski kapacitet [kapa]

$$\boxed{K = \frac{C_p}{C_v}}$$

$p, V_1, T_1 \longrightarrow p, V_2, T_2$

$dQ = \mu C_v dT + p dV$

$pV = \mu RT$

$dQ = \mu C_p dT$

$p dV = \mu R dT$

$\mu C_p dT = \mu C_v dT + \mu R dT$

$\mu [C_p - C_v] dT = \mu R dT$

$$\boxed{C_p - C_v = R}$$

$$c_p - c_v = \frac{R}{M}$$

$C_p = M c_p$

$$\boxed{C_p = \frac{KR}{K-1}}$$

$$\boxed{C_v = \frac{R}{K-1}}$$

POISSON

- povezuje parove termodinamičkih varijabli T-V, T-p, p-V u adijabatskim procesima promjene stanja

$\Delta Q = 0$ ... nema izmjene topline s okolinom

T-V

$nC_v dT = -p dV$

$nC_v dT = -nRT \frac{dV}{V}$ $\qquad \frac{C_v}{R} = konst.$

$\frac{C_v}{R}\int_{T_1}^{T_2} \frac{dT}{T} = -\int_{V_1}^{V_2} \frac{dV}{V}$

$\frac{1}{\kappa-1} \ln \frac{T_2}{T_1} = -\ln \frac{V_2}{V_1} = \ln \frac{V_1}{V_2}$

$$\boxed{\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\kappa-1}} \qquad \boxed{TV^{\kappa-1} = konst.}$$

$\Rightarrow \frac{T_2}{T_1} = \frac{p_2 V_2}{p_1 V_1}$

p-V

$$\boxed{\frac{p_2}{p_1} = \left(\frac{V_1}{V_2}\right)^{\kappa}} \qquad \boxed{pV^{\kappa} = konst.}$$

T - p

I. $\frac{V_1}{V_2} =$

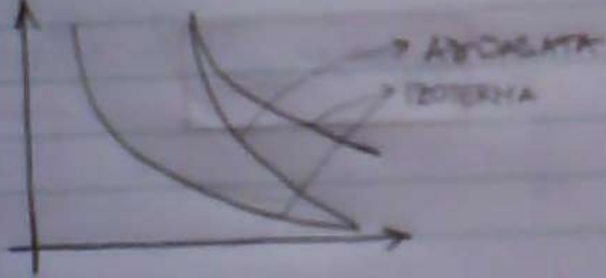
II. $\frac{V_1}{V_2} =$

$$\boxed{\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{k-1}{k}}}$$

$$\boxed{T^k p^{1-k} = konst.}$$

$$\boxed{W_a = \mu \frac{R}{k-1}(T_p - T_k)}$$

graf adijabate je strmiji od izoterme

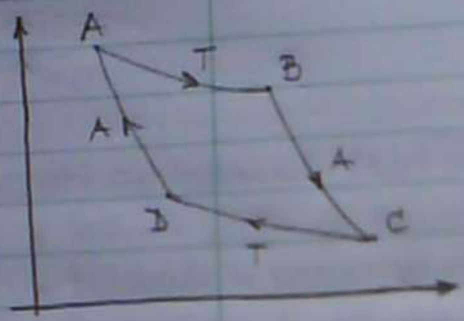


Nemoguć je proces u kojem bi toplina spontano prelazila iz spremnika niže u spremnik više temperature.

KELVIN - PLANCKOVA FORMULACIJA

Nemoguće je napraviti toplinski stroj koji bi, ponavljajući kružni proces, svu toplinu uzetu iz jednog spremnika pretvorio u rad. Ako se iz topline želi dobiti rad, uvijek dio te Q mora prijeći u okolinu (perpetuum mobile 2. vrste)

CARNOTOV PROCES



$$A \xrightarrow{\text{IZOTERMA}} B \xrightarrow{\text{ADIJABATA}} C \xrightarrow{\text{IZOTERMA}} D \xrightarrow{\text{ADIJABATA}} A$$

p_1, V_1, T_1 (1) $\quad p_2, V_2, T_1$ (2) $\quad p_3, V_3, T_2$ (3) $\quad p_4, V_4, T_2$ (4) $\quad p_1, V_1, T_1$ (1)

$$W_{12} = \mu R T_1 \ln \frac{V_2}{V_1}$$

$$Q_1 = W_{12}$$

$$W_{23} = \mu \frac{R}{K-1} (T_1 - T_2)$$

$$W_{34} = \mu R T_2 \ln \frac{V_4}{V_3}$$

$$Q_2 = W_{34}$$

$$W_{41} = \mu \frac{R}{K-1} (T_2 - T_1)$$

$$W_{23} = -W_{41}$$

$$W = W_{12} + W_{23} + W_{34} + W_{41} = W_{12} + W_{34} = Q_1 + Q_2 = |Q_1| - |Q_2|$$

$$\boxed{W = |Q_1| - |Q_2|}$$

$$\mu = \frac{|Q_1| - |Q_2|}{|Q_1|} = 1 - \frac{|Q_2|}{|Q_1|} = 1 - \frac{T_2}{T_1}$$

$$\boxed{\mu = 1 - \frac{|Q_2|}{|Q_1|} = 1 - \frac{T_2}{T_1}}$$

$T = konst.$

$$\frac{|Q_2|}{|Q_1|} = \frac{-\mu R T_2 \ln \frac{V_4}{V_3}}{\mu R T_1 \ln \frac{V_2}{V_1}} = \frac{\mu R T_2 \ln \frac{V_3}{V_4}}{\mu R T_1 \ln \frac{V_2}{V_1}}$$

$\Delta Q = 0$ ... adijabatski proces

$$T_1 V_2^{k-1} = T_2 V_3^{k-1}$$
$$T_1 V_1^{k-1} = T_2 V_4^{k-1}$$

$$\boxed{\frac{V_2}{V_1} = \frac{V_3}{V_4}}$$

ENTROPIJA

$$\frac{Q_1}{T_1} + \frac{Q_2}{T_2} = 0$$

reducirana toplina $\frac{Q}{T}$

$$\frac{dQ}{T} = dS$$

$$S_K - S_P = \int_P^K \frac{dQ}{T}$$

$$\boxed{dS = 0}$$

KINETIČKA TEORIJA PLINOVA

$\overline{\vec{v}} = 0$

UVJETI ZA IDEALNI PLIN:

- Volumen molekula je zanemariv u odnosu na volumen posude plina
- Molekule u savršeno el. sudarima izmjenjuju E, vrijede z. očuvanja
- Raspodjela molekula po brzinama je jednaka
- Međudjelovanja molekula su zanemariva ($U = E_k$, $E_P = 0$)
- Srednja brzina pojedine molekule je 0

N čestica

$\longrightarrow m_0, v_x, a$ (put)

$\Delta P_{ix} = 2 m_0 v_{ix}$

$\frac{2a}{v_{ix}} \longrightarrow \frac{v_{ix}}{2a}$

$F_i \Delta t = \frac{v_{ix}}{2a} \cdot 2 m_0 v_{ix} \Delta t$

$F_i = \frac{m_0}{a} v_{ix}^2$

$$\boxed{F = \sum_{i=1}^{n} \frac{m_0}{a} v_{ix}^2}$$

$$p = \frac{F}{a^2} = \frac{m}{a^3} \sum_{i=1}^{n} v_{ix}^2 = \boxed{\frac{m_0}{V} \sum_{i=1}^{n} v_{ix}^2}$$

$\overline{v_x^2}$ ... srednja kvadratična brzina

$$\overline{v_x^2} = \frac{\sum_{i=1}^{n} v_{ix}^2}{N} \Rightarrow \boxed{\sum_{i=1}^{n} v_{ix}^2 = N \overline{v_x^2}}$$

$$\boxed{p = \frac{N}{V} m_0 \overline{v_x^2}}$$

$$\overline{v_x^2} = \overline{v_y^2} = \overline{v_z^2}$$

$$\overline{v^2} = \overline{v_x^2} + \overline{v_y^2} + \overline{v_z^2} \Rightarrow \boxed{\overline{v_x^2} = \frac{1}{3} \overline{v^2}}$$

$$\boxed{p = \frac{N}{V} m_0 \frac{1}{3} \overline{v^2}} \qquad \boxed{p = \frac{2}{3} \frac{N}{V} \overline{E_k}}$$

$$\boxed{E_{k_i} = \frac{1}{2} m_0 \overline{v_i^2}}$$

$$pV = NkT = \frac{2}{3} N \overline{E_k}$$

$$\boxed{\overline{E_k} = \frac{3}{2} kT}$$