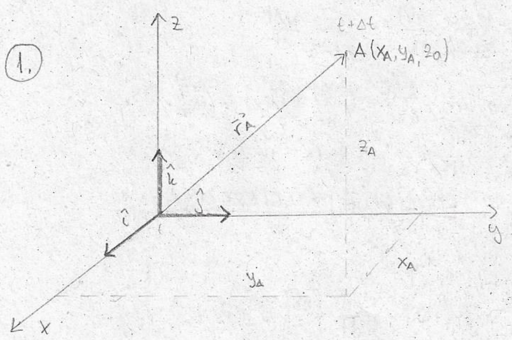
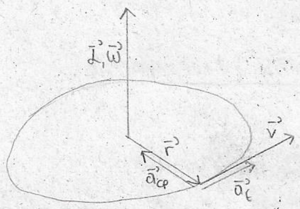
FIZIKA - TEORIJSKA PITANJA

(1.)



- pomak (Δr) - promjena vektora položaja
- brzina čestice - omjer pređenog puta i za to potrebnog vremena $\vec{v} = \frac{\vec{s}}{t}$
- akceleracija čestice - promjena brzine čestice pri nejednolikom gibanju $\vec{a} = \frac{\vec{v}}{t}$

(2.)



- Tangencijalna akceleracija ($\vec{a}_t$) je rezultat djelovanja tangencijalne sile koja mijenja brzinu po iznosu. ($\vec{a}_t = \vec{L} \times \vec{\omega}$)
- Centripetalna akceleracija ($\vec{a}_{cp}$) je akceleracija koju tijelo dobije kada sila djeluje okomito na brzinu. ($\vec{a}_{cp} = \vec{\omega} \times \vec{v}$)

(3.)

- $\vec{a}(t) = \frac{d\vec{v}(t)}{dt} \Big/ \cdot dt$

$$\vec{a}(t)\,dt = d\vec{v}(t) \Big/ \int$$

$$\int_0^t \vec{a}(t)\,dt = \int_{v_0}^{v} d\vec{v}(t)$$

$$\int_0^t \vec{a}(t)\,dt = \vec{v} - \vec{v}_0$$

$$\vec{v} = \vec{v}_0 + a\int_0^t dt = \begin{cases} t & v \\ t=0 & v_0 \end{cases}$$

- $\vec{v}(t) = \frac{d\vec{x}(t)}{dt} \Big/ \cdot dt$

$$\vec{v}(t)\,dt = d\vec{x}(t) \Big/ \int$$

$$\int_0^t \vec{v}(t)\,dt = \int_{x_0}^{x} d\vec{x}(t)$$

$$\vec{x} - \vec{x}_0 = \int_0^t \vec{v}(t)\,dt$$

$$\vec{x} = \vec{x}_0 + v\int_0^t dt = \begin{cases} t=0 & x_0 \\ t & x \end{cases}$$

- gibanje sa stalnom akceleracijom:

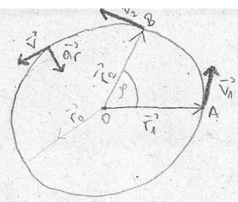
$$v = v_0 + a\int_0^t dt = v_0 + at = \frac{dx}{dt} \Big/ \cdot dt \Big/ \int$$

$$\int_{x_0}^{x} dx = \int_0^t v(t)\,dt = \int_0^t (v_0 + at)\,dt = \int_0^t v_0\,dt + \int_0^t at\,dt = v_0 t\Big|_0^t + \frac{at^2}{2}\Big|_0^t$$

$$x - x_0 = v_0 t + \frac{at^2}{2}$$

$$x = x_0 + v_0 t + \frac{at^2}{2}$$

4.



$s = r\varphi$

A:	$\vec{v}_1$	$\vec{r}_1$	t
B:	$\vec{v}_2$	$\vec{r}_2$	$t+\Delta t$

$$|\Delta\vec{v}| = v\Delta\varphi \;/ : \Delta t \cdot / \lim_{\Delta t \to 0}$$

$$\lim_{\Delta t \to 0} \frac{|\Delta\vec{v}|}{\Delta t} = \lim_{\Delta t \to 0} \frac{v\Delta\varphi}{\Delta t}$$

$$v \lim_{\Delta t \to 0} \frac{\Delta\varphi}{\Delta t} = v\frac{d\varphi}{dt} = \vec{v}\vec{\omega} = \vec{a}_r$$

$$\Rightarrow \vec{a}_r = -r\omega^2 \vec{r}_0 = -\frac{r}{v}\vec{r}_0 = \vec{\omega} \times \vec{v}$$

RADIJALNA AKCELERACIJA

$$\omega = \frac{d\varphi}{dt} = konst \;/ \cdot dt$$

$$\omega dt = d\varphi \;/ \int$$

$$\int_{\varphi_0}^{\varphi} dt = \int_0^t \omega dt = \omega \int_0^t dt$$

$$\varphi - \varphi_0 = \omega t$$

$$\varphi = \omega t + \varphi_0$$

$$\vec{a}_t = \frac{dv}{dt} = \frac{d}{dt}(r\omega) = r\frac{d\omega}{dt} \left[\frac{m}{s^2}\right] = r\alpha$$

$$\vec{a}_t = \vec{\alpha} \times \vec{r}$$

TANGENCIJALNA AKCELERACIJA

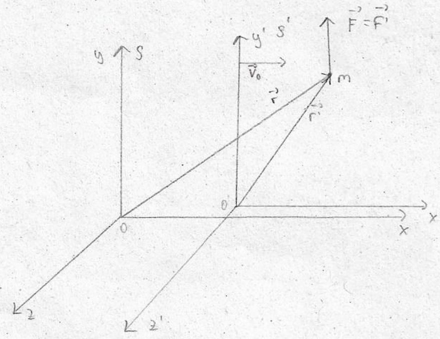
5.

- Veza između kutne i obodne brzine:

obodna: $v = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} = \lim_{\Delta t \to 0} \frac{r\Delta\varphi}{\Delta t} = r \lim_{\Delta t \to 0} \frac{\Delta\varphi}{\Delta t} = r\frac{d\varphi}{dt} = r\omega \; (\vec{v} = \vec{\omega} \times \vec{r})$

kutna: $\omega = \lim_{\Delta t \to 0} \frac{\Delta\varphi}{\Delta t} = \frac{d\varphi}{dt} \left[\frac{rad}{s}\right] \quad (\vec{\omega} = \vec{v} \times \vec{r})$

6.



$$\vec{r} = \vec{r}' + \vec{r}_0 \;/ \frac{d}{dt} \qquad \vec{v} = \vec{v}' + \vec{v}_0 \;/ \frac{d}{dt}$$

$$\frac{d\vec{r}}{dt} = \frac{d\vec{r}'}{dt} + \frac{d\vec{r}_0}{dt} \qquad \frac{d\vec{v}}{dt} = \frac{d\vec{v}'}{dt} + \frac{d\vec{v}_0}{dt}$$

$$\vec{v} = \vec{v}' + \vec{v}_0 \qquad \vec{a} = \vec{a}' + 0$$

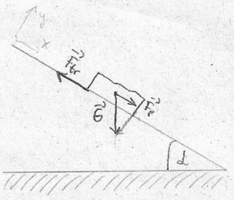
$\vec{v}$ - brzina u sustavu S

$\vec{v}'$ - brzina u sustavu S'

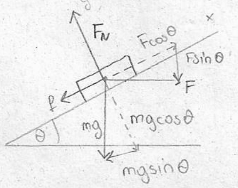
$\vec{v}_0$ - brzina sustava S' prema sustavu S

$\vec{a} = \vec{a}'$

7.



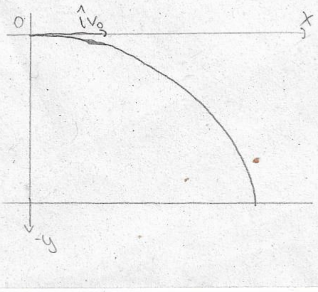
niz kosinu

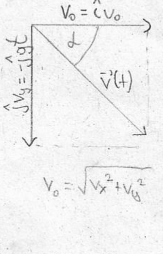


uz kosinu

$$\boxed{\begin{aligned} m\vec{a} &= \vec{N} + m\vec{g} + \vec{F}_{tr} \\ ma_x\vec{i} &= N\vec{j} + mg(\sin\alpha\,\vec{i} - \cos\alpha\,\vec{j}) \mp \mu_d N\vec{i} \quad (v_x \gtrless 0) \end{aligned}}$$

8.





$v_0 = \sqrt{v_x^2 + v_y^2}$

$$\vec{v}_0 = \hat{i}v_x + \hat{j}v_y$$

$$\vec{v} = \hat{i}v_x + \hat{j}v_y'$$

poč. uvjeti: $x(0) = 0 \quad y(0) = 0$

$\dot{x}(0) = \hat{i}v_0 \quad \dot{y}(0) = 0$

$$v_x(t) = C_1 \Rightarrow v_x(0) = v_0 = C_1$$

$$v_x(t) = \frac{dx}{dt} = v_0 \;/ \cdot dt / \int$$

$$v_0 \int dt = \int dx$$

$$v_0 t = x(t) \quad uz\; (x(0) = 0)$$

$$\Rightarrow \boxed{x(t) = v_0 t}$$

$\int dv_y = -g\int dt \Rightarrow v_y(t) = -gt + D_1$

$v_y(0) = -gt + 0$

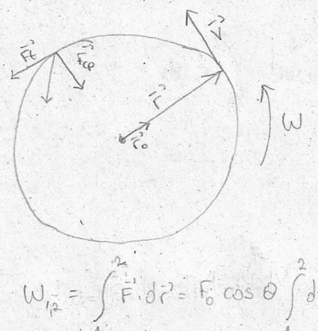
$v_y(t) = \frac{dy}{dt} = -2gt \; / \cdot dt / \int$

$-g\int t\,dt = \int dy$

$\boxed{y(t) = -g\frac{t^2}{2}}$

?

(9.)



$F_{cp} = -m\frac{v^2}{r}\cdot\vec{r_0}$

$= -mr\omega^2\vec{r_0}$

(10.) $W_{1,2} = \int_1^2 \vec{F}\cdot d\vec{r} = F_0\cos\Theta\int_1^2 dx = F_0\cos\Theta(x_2 - x_1) = F_0\cos\Theta\cdot\Delta x$

$\left(\vec{F} = \hat{i}F_0\cos\Theta + \hat{j}F_0\sin\Theta \; / \cdot dx = \hat{i}dx\right)$

$\vec{F}dx = F_0\cos\Theta\,dx$

TEOREM O RADU I KINETIČKOJ ENERGIJI:

Neka na tijelo mase m djeluje sila F duž osi x i daje tijelu akceleraciju a:

$$F = \frac{d}{dt}p = m\frac{dv}{dt} = m\frac{dv}{dt}\frac{dx}{dx} = m\frac{dv}{dx}\frac{dx}{dt} = m\frac{dv}{dx}\cdot t$$

Rad koji izvrši sila je:

$$W_{1,2} = \int_{x_1}^{x_2} F dx = \int_{x_1}^{x_2} mv\frac{dv}{dx}dx = \int_{v_1}^{v_2} mv\,dv = m\frac{v^2}{2}\Big|_{v_1}^{v_2}$$

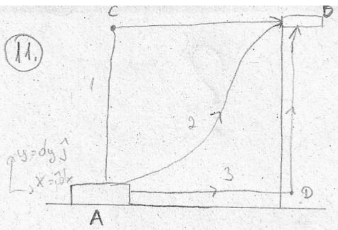
$$W_{1,2} = \underbrace{\frac{mv_2^2}{2} - \frac{mv_1^2}{2}}_{\text{razlika kinetičkih energija}} \Rightarrow \boxed{\frac{mv_2^2}{2} - \frac{mv_1^2}{2} = \int_{x_1}^{x_2} F(x)dx} \; *$$

$\Rightarrow E_k = \frac{mv^2}{2}$

(*) Ukupan rad koji izvrše sile na neku česticu jednak je promjeni kinetičke energije čestice:

$$W_{uk} = \Delta E_k = E_{k\,kon.} - E_{k\,poč.} = \frac{1}{2}mv_{kon.}^2 - \frac{1}{2}mv_{poč.}^2$$

11.



- Rad izvršen protiv sile teže ne ovisi o putu, već samo o početnoj i konačnoj točki => takvu silu nazivamo **konzervativnom silom**.

* Potencijalnu energiju mogu imati samo konzervativne sile, a njezinu promjenu definiramo kao negativan rad koji te sile izvrše.

$$\Delta U = -W_k \quad || \quad \Delta E_p = -W_k \quad \Rightarrow \quad W_k = -\Delta U = U_p - U_k = \int_{x_p}^{x_k} F_x \, dx$$

$$U_k(x) = -\int_{x_p}^{x_k} F_x \cdot dx + U_p$$

a) $E_p = \frac{kx^2}{2}$

$$\Rightarrow \vec{F} = -\nabla E_p = -\hat{i}\frac{\partial}{\partial x}\left(\frac{1}{2}kx^2\right) = -\hat{i}kx$$

b) $E_{kp} = 0$, $U_p = 0$, $E_{kk} = \frac{mv_k^2}{2}$, $U_k = -mgy_h$

$$E_{kp} + U_p = E_{kk} + U_k \qquad h = y_h - y$$

$$0 = \frac{mv_k^2}{2} + mgy_h \qquad y_h = h + y$$

$$v_k^2 = -2gy_h \qquad v_k^2 = -2g(h+y)$$

12. Mehanička energija E je zbroj kinetičke energije Ek i potencijalne energije $U \equiv E_p$
$E = E_k + U$. Ona je očuvana u situacijama u kojima kada se kinetička energija sustava poveća za neki iznos, za isti iznos se tada smanji potencijalna energija (vrijedi i obratno).

13. $\boxed{F(\vec{r}) = -\frac{dU(\vec{r})}{d\vec{r}}}$

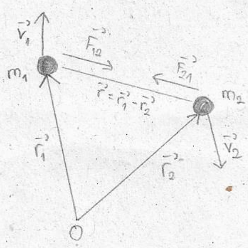
$$U(r) = -\int_{r_0} F(r')dr' = \int_{r_0}^{r_0} - \int_{r_0} F(r')dr'$$

$$= konst + U(r)$$

- ako je $U(\vec{r})$ minimum u ishodištu, tada čestica oscilira oko ravnotežnog položaja - stabilna je
- ako je $U(\vec{r})$ maksimum, tada se pomak iz ravnoteže stalno povećava - čestica je nestabilna

14. Količinu gibanja definiramo kao umnožak brzine tijela i njegove mase: $\vec{p} = m\vec{v}$.
Zakon očuvanja količine gibanja kaže da je količina gibanja sustava konstantna veličina, ako je zbroj svih vanjskih sila na sustav jednak nuli:

$$\vec{p} = konst. \iff \sum \vec{F}_{vanjske} = 0$$



$$\vec{p}_1 + \vec{p}_2 = konst. = m_1\vec{v}_1 + m_2\vec{v}_2$$

15.

$\vec{F}_{12} = -\vec{F}_{21} \Rightarrow m_1 \frac{d^2}{dt^2}\vec{r}_1 = \vec{F}_{V1} + \vec{F}_{12}$

$m_2 \frac{d^2}{dt^2}\vec{r}_2 = \vec{F}_{V2} + \vec{F}_{21}$

$$\Rightarrow m_1 \frac{d^2}{dt^2}\vec{r}_1 + m_2 \frac{d^2}{dt^2}\vec{r}_2 = \vec{F}_{V1} + \vec{F}_{V2}$$

$$m_1 \frac{d}{dt}\vec{v}_1 + m_2 \frac{d}{dt}\vec{v}_2 = \frac{d}{dt}(m_1\vec{v}_1 + m_2\vec{v}_2) = (m_1+m_2)\frac{d}{dt}\frac{m_1\vec{v}_1 + m_2\vec{v}_2}{m_1+m_2}$$

$$= M\frac{d}{dt}\frac{m_1\vec{v}_1 + m_2\vec{v}_2}{M} = M\frac{d}{dt}\left[\frac{d}{dt}\underbrace{\left(\frac{m_1\vec{r}_1 + m_2\vec{r}_2}{m_1+m_2}\right)}_{\vec{r}_{CM}}\right]$$

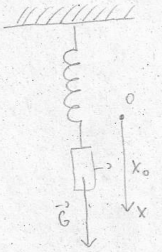
$$= M\frac{d^2}{dt^2}\vec{r}_{CM} = M\frac{d}{dt}\vec{v}_{CM} = M\vec{a}_{CM}$$

$$\Rightarrow \boxed{\vec{r}_{CM} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2}{m_1+m_2}} \Rightarrow \boxed{\vec{v}_{CM} = \frac{m_1\vec{v}_1 + m_2\vec{v}_2}{m_1+m_2} = \text{konst.}}$$

$$\boxed{F_{vanjske} = \frac{d}{dt}\vec{p} = m\frac{dv_{cm}}{dt} = m a_{cm}}$$

$$\vec{p}_1 + \vec{p}_2 = \text{konst.} = m_1\vec{v}_1 + m_2\vec{v}_2$$

16.



$F_0 = -kx$

$m\frac{d^2\vec{r}}{dt^2} = 0 = mg\hat{i} - kx_0\hat{i}$

$$\frac{d^2}{dt^2}\vec{r}_{CM} = \frac{1}{M}\vec{F}_V = \vec{a}_{CM}$$

$m\ddot{x} = -kx$

$(\omega_0 = \sqrt{k/m})$

$\ddot{x} + \omega_0^2 x = 0$

početni uvjeti: $x(0) = A$, $\dot{x}(0) = 0$

$x(t) = X_0 e^{\lambda t}$

$m X_0 \lambda^2 e^{\lambda t} + k X_0 e^{\lambda t} = 0 \quad /: m X_0 e^{\lambda t}$

$\lambda^2 = -\frac{k}{m} \Rightarrow \lambda = \pm i\frac{k}{m} \equiv \pm i\omega_0^2$

$x(t) = X_0 e^{\pm i\omega_0 t}$

općenitije: $x(t) = X_1 e^{-i\omega_0 t} + X_2 e^{i\omega_0 t}$

$x(0) = A = X_1 + X_2$

$\dot{x}(0) = 0 = -i\omega_0 X_1 + i\omega_0 X_2 \Rightarrow X_1 = X_2 = \frac{A}{2}$

$x(t) = \frac{A}{2}(e^{-i\omega_0 t} + e^{i\omega_0 t})$

$$\boxed{x(t) = A\cos\omega_0 t \quad , \quad v(t) = -A\omega_0 \sin\omega_0 t}$$

17.

$m\ddot{x} = F_x = -kx - \beta\dot{x} \quad /:m$

$(\omega_0 = \sqrt{k/m})$

$\left(2\delta = \frac{\beta}{m}\right)$

$\ddot{x} + 2\delta\dot{x} + \omega_0^2 x = 0$

- $\delta < \omega_0$: $x(t) = Ae^{-\delta t}\cos[\omega t + \phi]$, $\omega^2 = \omega_0^2 - \delta^2 > 0$ — podkritično
- $\delta > \omega_0$: $x(t) = x(0)e^{-\delta t}\left(\operatorname{ch} gt + \frac{\delta}{g}\operatorname{sh} gt\right) + \frac{1}{g}\dot{x}[0]e^{-\delta t}\operatorname{sh} gt$, $g^2 = \delta^2 - \omega_0^2 > 0$ — nadkritično
- $\delta = \omega_0$: $x(t) = x[0]e^{-\omega_0 t}(1 + \omega_0 t) + \dot{x}(0)e^{-\omega_0 t}$, $\delta = \omega_0$ — kritično

(18.) $E_k = \frac{1}{2} m\dot{x}^2$ - u trenucima $E_k = 0$, $E = U > 0 \Rightarrow x(t) \simeq Ae^{-\delta t}$

$U = \frac{kx^2}{2}$

$$E = U \simeq \frac{1}{2} kA^2 e^{-2\delta t}$$

?

$E = E_k + U$

$$\left.\begin{aligned} E_k &= \frac{mv^2}{2} = \frac{m}{2} A^2 \omega^2 \cos^2(\omega t + \varphi_0) \\ \omega &= \sqrt{k/m} \end{aligned}\right\} E_k = \frac{k}{2} A^2 \cos^2(\omega t + \varphi_0) = \frac{k}{2} A^2 (1 - \sin^2(\omega t + \varphi_0)) = \frac{k}{2} (A^2 - s^2)$$

$$U = -W = -\int_0^s (-ks)\, ds = \frac{ks^2}{2} = \frac{k}{2} A^2 \sin^2(\omega t + \varphi_0)$$

$$E = E_k + U$$

$$E = \underbrace{\frac{k}{2} A^2 \cos^2(\omega t + \varphi_0)}_{v^2} + \frac{k}{2} A^2 \sin^2(\omega t + \varphi_0) = \frac{kA^2}{2}$$

$\downarrow$ $\downarrow$

(19.) - omjer dvaju susjednih amplituda: $\frac{a_1}{a_2} = \frac{a(t)}{a(t+T)} = \frac{Ae^{-\delta t}}{Ae^{-\delta t} e^{-\delta T}} = e^{\delta T}$

- logaritamski dekrement prigušenja λ je: $\lambda = \ln \frac{a(t)}{a(t+T)} = \ln(e^{\delta T}) = \delta T$
- konstanta trenja $b = 2m\delta$

$\boxed{\text{Q faktor:}}$ $Q = \frac{\pi}{\lambda} = \frac{\pi}{\delta T_0} = \frac{\omega_0}{2\delta}$

$$Q = 2\pi \frac{\bar{E}}{\Delta E} \quad ; \quad \bar{E} = \frac{1}{4} k(a_1^2 + a_2^2) \quad , \quad \Delta E = \frac{1}{2} k (a_1^2 - a_2^2)$$

$$Q = 2\pi \frac{\frac{1}{4} k(A^2 e^{-2\delta t} + A^2 e^{-2\delta(t+T)})}{\frac{1}{2} k(A^2 e^{-2\delta t} - A^2 e^{-2\delta(t+T)})} = \pi \frac{(e^{-2\delta t} + e^{-2\delta t} e^{-2\delta T})}{(e^{-2\delta t} - e^{2\delta t} e^{-2\delta T})}$$

$$= \pi \frac{(1 + e^{-2\delta T})}{1 - e^{-2\delta T}} = \frac{\pi(1 + e^{-\delta T} e^{-\delta T})}{1 - e^{-\delta T} e^{-\delta T}} = \frac{\pi e^{-\delta T}}{e^{-\delta T}} \frac{(e^{\delta T} + e^{-\delta T})}{e^{\delta T} - e^{-\delta T}}$$

$$Q = \pi \frac{e^{\lambda} + e^{-\lambda}}{e^{\lambda} - e^{-\lambda}} \quad \text{uz} \quad \text{tgh}\,\lambda = \frac{e^{\lambda} + e^{-\lambda}}{e^{\lambda} - e^{-\lambda}}$$

$$Q = \frac{\pi}{\text{tgh}\,\lambda} \quad ; \text{ za mali } \text{tgh}\,\lambda \approx \lambda \Rightarrow Q = \frac{\pi}{\lambda}$$

20. $m\frac{d^2s}{dt^2} = -ks - b\frac{ds}{dt} + F_0\sin\omega t$ ($F = -ks$ harmonička, $F_{tr} = -bv = -b\frac{ds}{dt}$ trenja, $F_v = F_0\sin\omega t$ vanjska)

uz $\frac{b}{m} = 2\delta$, $\omega_0^2 = \frac{k}{m}$, $A_0 = \frac{F_0}{m}$ slijedi:

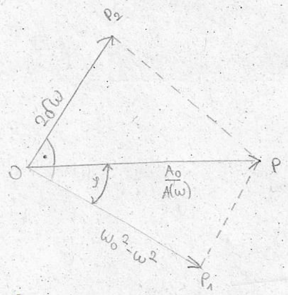
$\frac{d^2s}{dt^2} + 2\delta\frac{ds}{dt} + \omega_0^2 s = A_0\sin\omega t$ (rješenje oblika $s(t) = A(\omega)\sin(\omega t - \varphi)$)

$$s(t) = A(\omega)\sin(\omega t - \varphi)$$
$$\frac{ds}{dt} = A(\omega)\omega\cos(\omega t - \varphi)$$
$$\frac{d^2s}{dt^2} = -A(\omega)\omega^2\sin(\omega t - \varphi)$$

$$-A(\omega)\omega^2\sin(\omega t - \varphi) + 2\delta A(\omega)\omega\cos(\omega t - \varphi) - \omega_0^2 A(\omega)\sin(\omega t - \varphi) = A_0\sin\omega t \; / : A(\omega)$$

$$(\omega_0^2 - \omega^2)\sin(\omega t - \varphi) + 2\delta\omega\cos(\omega t - \varphi) = \frac{A_0}{A(\omega)}\sin\omega t$$

$$\underbrace{(\omega_0^2 - \omega^2)\sin(\omega t - \varphi) + 2\delta\omega\sin\left(\omega t - \varphi + \frac{\pi}{2}\right)}_{\text{međusobno okomita titranja}} = \frac{A_0}{A(\omega)}\sin(\omega t)$$



$$\operatorname{tg}\varphi = \frac{2\delta\omega}{\omega_0^2 - \omega^2}$$

$$\left(\frac{A_0}{A(\omega)}\right)^2 = (\omega_0^2 - \omega^2)^2 + (2\delta\omega)^2 \; / \sqrt{}$$

$$\frac{A_0}{A(\omega)} = \sqrt{(\omega_0^2 - \omega^2)^2 + (2\delta\omega)^2} \Rightarrow A(\omega) = \frac{A_0}{\sqrt{(\omega_0^2 - \omega^2)^2 + (2\delta\omega)^2}}$$

$$\boxed{A(\omega) = \frac{A_0}{\omega_0^2\sqrt{\left(1 - \frac{\omega^2}{\omega_0^2}\right)^2 + \left(\frac{2\delta\omega}{\omega_0^2}\right)^2}}}$$

- REZONANCIJA

$$\frac{d}{d\omega}\left[(\omega_0^2 - \omega^2)^2 + 4\delta^2\omega^2\right]^{1/2} = \frac{1}{2}\left[(\omega_0^2 - \omega^2)^2 + (2\delta\omega)^2\right]^{-1/2} \cdot \left(2(\omega_0^2 - \omega^2)\cdot(-2\omega) + 4\delta^2\cdot 2\omega\right) = 0$$

$$\left(2(\omega_0^2 - \omega^2)\cdot(-2\omega) + 4\delta^2\cdot 2\omega\right) = 0$$

$$\left(-(\omega_0^2 - \omega^2) + 2\delta^2\right) = 0$$

$$\omega^2 = \omega_0^2 - 2\delta^2$$

$$\boxed{\omega_r = \sqrt{\omega_0^2 - 2\delta^2}}$$

$s_1(t) = Ae^{-\delta t}\sin(\omega_p t + \varphi_0)$ $\omega_p = \sqrt{\omega_0^2 - \delta^2}$

$s_2(t) = A(\omega)\sin(\omega t - \varphi)$

$$\left.\right\} s(t) = s_1(t) + s_2(t) = \underbrace{A_1 e^{-\delta t}\sin(\omega_p t + \varphi_0)}_{\text{iščezava zbog } e^{-\delta t}} + A(\omega)\sin(\omega t - \varphi)$$

$$\boxed{s(t) = s_2(t) = A(\omega)\sin(\omega t - \varphi)}$$

(21.)

$$\left.\begin{aligned} m_1 \frac{d^2 s_1}{dt^2} &= -k_1 s_1 + k(s_2 - s_1) \\ m_2 \frac{d^2 s_2}{dt^2} &= -k_2 s_2 - k(s_2 - s_1) \end{aligned}\right\} \text{ deriviramo}$$

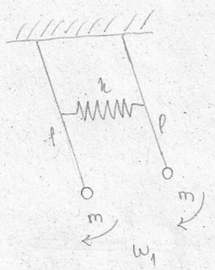
P: $m_1 = m_2$, $k_1 = k_2$, $s_1(t) = A\sin(\omega_1 t + \varphi_{01})$, $s_2(t) = B\sin(\omega_2 t + \varphi_{02})$

$$\frac{ds_1}{dt} = A\omega_1 \cos(\omega_1 t + \varphi_{01}) \Rightarrow \frac{d^2 s_1}{dt^2} = -A\omega_1^2 \sin(\omega_1 t + \varphi_{01})$$

$$\frac{ds_2}{dt} = B\omega_2 \cos(\omega_2 t + \varphi_{02}) \Rightarrow \frac{d^2 s_2}{dt^2} = -B\omega_2^2 \sin(\omega_2 t + \varphi_{02})$$

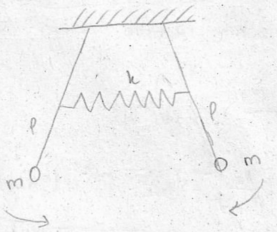
$$-m_1 A\omega_1^2 \sin(\omega_1 t + \varphi_{01}) = -k_1 A\sin(\omega_1 t + \varphi_{01}) + k\left[B\sin(\omega_2 t + \varphi_{02}) - A\sin(\omega_1 t + \varphi_{01})\right]$$

$$-m_2 B\omega_2^2 \sin(\omega_2 t + \varphi_{02}) = -k_2 B\sin(\omega_2 t + \varphi_{02}) - k\left[B\sin(\omega_2 t + \varphi_{02}) - A\sin(\omega_1 t + \varphi_{01})\right]$$



ω_1

u fazi



ω_2

u protufazi

$A = B = A_1$, $\omega_1 = \omega_0 = \sqrt{\frac{k}{m}}$

$s_1(t) = A_1 \sin(\omega_1 t + \varphi_{01})$

$s_2(t) = A_1 \sin(\omega_1 t + \varphi_{01})$

$A = -B = A_2$, $\omega_2 = \sqrt{\frac{k_1}{m_1} + \frac{2k}{m_1}} = \sqrt{\omega_0^2 + \frac{2k}{m_1}}$

$s_1(t) = A_2 \sin(\omega_2 t + \varphi_{02})$

$s_2(t) = -A_2 \sin(\omega_2 t + \varphi_{02})$

- općenito rješenje je zbroj ovih dvaju osnovnih načina titranja

- ako su amplitude jednake, što pretpostavljamo:

$$s_1 = 2A\cos\left(\frac{\omega_1 - \omega_2}{2} t + \frac{\varphi_{01} - \varphi_{02}}{2}\right) \sin\left(\frac{\omega_1 + \omega_2}{2} t + \frac{\varphi_{01} + \varphi_{02}}{2}\right)$$

$$s_2 = 2A\sin\left(\frac{\omega_1 - \omega_2}{2} t + \frac{\varphi_{01} - \varphi_{02}}{2}\right) \cos\left(\frac{\omega_1 + \omega_2}{2} t + \frac{\varphi_{01} + \varphi_{02}}{2}\right)$$

- amplituda se mijenja od max. 2A do 0 i varira u vremenu frekvencijom $f_a = (f_1 - f_2)/2$ $\Rightarrow$ amplituda je MODULIRANA

22. $x_1(t) = A\cos(\omega_A t + \phi_A) - B\cos(\omega_B t + \phi_B)$

$x_2(t) = A\cos(\omega_A t + \phi_A) + B\cos(\omega_B t + \phi_B)$

POČ. UVJETI:

$x_1(0) \neq 0$ $\quad x_2(0) = 0$

$v_1(0) = 0$ $\quad v_2(0) = 0$

?

23. $$\frac{\partial^2}{\partial t^2} y[x,t] - \frac{T}{\mu}\frac{\partial^2}{\partial x^2} y[x,t] = 0$$

$$\ddot{y} - v^2 y'' = 0, \quad v^2 = \frac{T}{\mu}, \quad \mu = \frac{\Delta m}{\Delta x}$$

$$v = \sqrt{T/\mu}$$

$$\Delta m \ddot{y}_i = T\frac{y_{i+1} - y_i}{\Delta x} + T\frac{y_{i-1} - y_i}{\Delta x}$$

$(\Delta m = \mu \Delta x)$

$$\ddot{y}_i = \frac{T}{\mu}\frac{y_{i+1} - 2y_i + y_{i-1}}{(\Delta x)^2}$$

$$\frac{\partial^2}{\partial t^2} y[x_i, t] = \frac{T}{\mu}\frac{y[x_i + \Delta x, t] - 2y[x_i, t] + y[x_i - \Delta x, t]}{(\Delta x)^2}$$

24. $\Delta m = \rho S \Delta x$

$$\Delta m \ddot{\xi}_i = -k(\xi_i - \xi_{i+1}) - k(\xi_i - \xi_{i-1})$$

$$\ddot{\xi}_i = \frac{E}{\rho}\frac{\xi_{i+1} - 2\xi_i + \xi_{i-1}}{(\Delta x)^2}$$

$$\frac{\partial^2}{\partial t^2}\xi[x,t] = \frac{E}{\rho}\frac{\xi[x_i + \Delta x, t] - 2\xi[x_i, t] + \xi[x_i - \Delta x, t]}{(\Delta x)^2}$$

$$\ddot{\xi} - v^2 \xi'' = 0, \quad v^2 = \frac{E}{\rho}$$

25. $s = s(x,y,z,t)$ općenito, $s = s(x,t)$ 1D

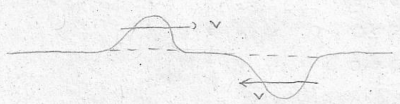
- izvor vala u ishodištu: $s(x=0,t) = A\sin\omega t = A\sin\frac{2\pi}{T}t$
  - širi se do točke P do trenutka $t' = \frac{x}{v}$
  - val dođe do čestice x: $s(x,t) = A\sin\omega(t - t') = A\sin\omega(t - \frac{x}{v})$
- $\omega(t - \frac{x}{v}) = \text{konst.}$, $t - \frac{x}{v} = \text{konst.} \Rightarrow x = vt + \text{konst.}$
- $\omega = 2\pi f = \frac{2\pi}{T}$ i $v = \lambda f \Rightarrow s(x,t) = A\sin\omega(t - \frac{x}{v}) = A\sin 2\pi(\frac{t}{T} - \frac{x}{\lambda})$
- $k = \frac{2\pi}{\lambda} = \frac{2\pi}{vT} = \frac{\omega}{v} \Rightarrow s(x,t) = A\sin(\omega t - kx)$ → val se širi u pozitivnom djelu x osi
- ako val ide u negativnom smjeru: $x = -vt + \text{konst.} \Rightarrow s(x,t) = A\sin\omega(t + \frac{x}{v}) = A\sin 2\pi(\frac{t}{T} + \frac{x}{\lambda})$

  $s(x,t) = A\sin(\omega t + kx)$
- val je oblika: $s(x,t) = A\sin(\omega t - kx + \varphi_0)$
- ako se val giba po nekom drugom pravcu smjera $\vec{u}_0$:

  $s(\vec{r},t) = A\sin(k\vec{u}_0 \cdot \vec{r} - \omega t + \varphi_0) = A\sin(\vec{k}\cdot\vec{r} - \omega t + \varphi_0)$

  $\vec{k} = k\vec{u}_0$ (valni vektor), $\vec{k} = \frac{2\pi}{\lambda}\vec{u}_0$

(26.)



$$y[x,t] = f[x-vt] - f[x+vt]$$

$$\text{za } t=0 \Rightarrow y[x,t]=0$$

$$2E = \int \frac{1}{2} (\dot{y}[x,0])^2\, dm$$

$$\dot{y}[x,t] = \frac{d}{dt} y[x,t] = -vf'[x-vt] - vf'[x+vt]$$

$$\text{za } t=0 \Rightarrow \dot{y}[x,t] = -2vf'[x]$$

$$dm = \mu dx$$

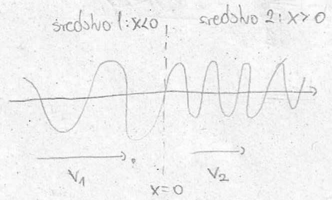
$$E = \mu v^2 \int (f'[x])^2 dx \quad \Rightarrow \text{ općenito}$$

- prosječna:

$$\left.\begin{aligned} f[x] &= A\cos kx \\ f'[x] &= -kA\sin kx \end{aligned}\right\} E = \mu v^2 \int (-kA\sin kx)^2 dx = \mu v^2 k^2 A^2 \int \sin^2 kx\, dx = \mu \omega^2 A^2 \int \sin^2 kx\, dx$$

$$\left\langle \frac{dE}{dx} \right\rangle = \frac{1}{2}\mu\omega^2 A^2$$

(27.)



$$k_{1,2} = \frac{\omega}{v_{1,2}} = \omega\sqrt{\frac{\mu_{1,2}}{T}} = \frac{2\pi}{\lambda_{1,2}}$$

$$A_u\cos[k_1x - \omega t] \longrightarrow$$

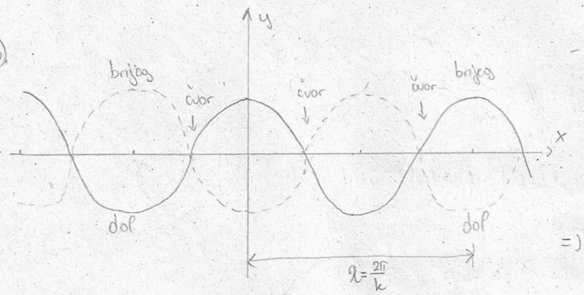
$$\longleftarrow A_r\cos[-(k_1x + \omega t + \phi_r)]$$

$$A_t\cos[k_2x - \omega t + \phi_t] \longrightarrow$$

$x=0$

$$\left.\begin{aligned} &\text{1. spojni uvjet: } A_u + A_r e^{-i\phi_r} = A_t e^{i\phi_t} \\ &\text{2. spojni uvjet: } k_1A_u - k_1A_r e^{-i\phi_r} = k_2 A_t e^{i\phi_t} \end{aligned}\right\} A_t e^{i\phi_t} = \frac{2k_1}{k_1+k_2}A_u,\ A_r e^{i\phi_r} = \frac{k_1-k_2}{k_1+k_2}A_u$$

(28.)



- Superpozicija dvaju harmonijskih valova valne duljine $\lambda = \frac{2\pi}{k}$, f ω i ampl. $\frac{A}{2}$ koji se šire u suprotnim smjerovima, pišemo kao:

$$y[x,t] = \frac{A}{2}\cos[kx-\omega t] + \frac{A}{2}\cos[kx+\omega t] = \ldots = A\cos kx \cos\omega t$$

$\Rightarrow$ valna duljina $\lambda = \frac{2\pi}{k}$

(29.)

$$L = n\frac{\lambda}{2},\ n=1,2,\ldots,\quad \lambda\omega = 2\pi v$$

$$\lambda_n = \frac{2L}{n},\ \omega_n = \frac{2\pi v}{\lambda_n} = n\frac{\pi v}{L},\ n=1,2,\ldots$$

$$2\pi f = \omega_1 = \frac{\pi v}{L} = \frac{\pi}{L}\sqrt{\frac{T}{\mu}} = \frac{\pi}{L}\sqrt{\frac{T}{\rho S}} = \frac{\pi}{L}\sqrt{\frac{T}{\rho r^2 \pi}}$$

$$f = \frac{1}{2L}\sqrt{\frac{T}{\rho r^2 \pi}}$$

30. a) $\lambda' = \lambda - Tv_i = \frac{v}{f_i} - \frac{v_i}{f_i}$, $f_p = \frac{v}{\lambda'} = \frac{f_i v}{v - v_i}$, $f_p > f_i$ $\Rightarrow$ $f_p = \frac{v}{v-v_i} f_i$

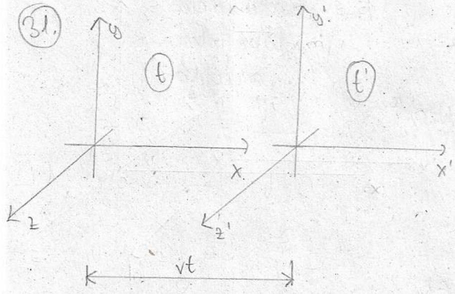
b) $v' = v + v_p$

$v' = \frac{\lambda}{T'} = \lambda f_p$, $f_p = \frac{v'}{\lambda} = \frac{v+v_p}{\lambda} \cdot \frac{v}{v} = \frac{v+v_p}{v} f_i$, $f_p > f_i$ $\Rightarrow$ $f_p = \frac{v \pm v_p}{v} f_i$

c) $f_p = \frac{v+v_p}{v-v_i} f_i$

(izvodi komplicirani $\Rightarrow$ dodatak G (Ilijić)

31.



· Lorentzove transformacije koordinata:

$$x' = \gamma(x - vt), \quad y' = y, \quad z' = z, \quad t' = \gamma\left(t - \frac{vx}{c^2}\right)$$

$$\gamma = \frac{1}{\sqrt{1-\beta^2}}, \quad \beta = \frac{v}{c} < 1$$

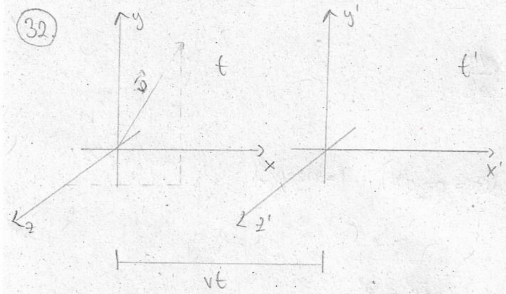
zakon širenja svjetlosti: $r^2 = x^2 + y^2 + z^2 = (ct)^2$

$$\gamma^2(x' + vt')^2 + y'^2 + z'^2 = c^2\gamma^2(t' + vx'/c^2)^2$$

$$x'^2 + y'^2 + z'^2 = (ct')^2$$

- Kad je brzina gibanja S' u odnosu na S znatno manja od brzine svjetlosti tj. kad $v \to 0 \Rightarrow \beta \to ?$ i $\gamma \to 0$, a Lorentzove transformacije se time svode na Galilejeve.

32.



t'

$u: u_x = \frac{dx}{dt}, \quad u_y = \frac{dy}{dt}, \quad u_z = \frac{dz}{dt}$

$u': u'_x = \frac{dx'}{dt'}, \quad u'_y = \frac{dy'}{dt'}, \quad u'_z = \frac{dz'}{dt'}$

$$u'_x = \frac{u_x - v}{1 - u_x v/c^2}, \quad u'_y = \frac{u_y}{\gamma(1 - u_x v/c^2)}, \quad u'_z = \frac{u_z}{\gamma(1 - u_x v/c^2)}$$

$$u_x = \frac{u'_x + v}{1 + u'_x v/c^2}, \quad u_y = \frac{u'_y}{\gamma(1 + u'_x v/c^2)}, \quad u_z = \frac{u'_z}{\gamma(1 + u'_x v/c^2)}$$

33. Vrijeme koje protječe za opažača koji miruje u nekom inercijskom referentnom okviru zovemo njegovim vlastitim vremenom. Ako mirni opažač u referentnom okviru S' u nekom trenutku pokrene i ubrzo nakon toga zaustavi zaporni sat, vremenski razmak $\Delta t'$ među tim događajima koje on izmjeri odgovara njegovom vlastitom vremenu. Pomoću Lorentzovih transformacija možemo izračunati vremenski razmak među istim događajima u referentnom okviru S.

$$\Delta t = \gamma \Delta t' = \frac{\Delta t'}{\sqrt{1 - (v/c)^2}}$$ (dilatacija vremena)

To nam pokazuje da je vremenski razmak u referentnom okviru S dulji od vremenskog razmaka u S'.

34. $\vec{p} = m\vec{v}$, $E_k = \frac{1}{2} m v^2$ } $\vec{p} = \gamma m \vec{v}$, $\gamma = \frac{1}{\sqrt{1-(v/c)^2}}$ relativistička količina gibanja

$$E = \gamma m c^2 = mc^2 + E_k$$ relativistička energija

$$dE_k = dW = \vec{F} \cdot d\vec{r} = \frac{d\vec{p}}{dt} \cdot d\vec{r} = \frac{d\vec{r}}{dt} \cdot d\vec{p} = \vec{v} \cdot d\vec{p}$$

$$dE_k = v\,dp = v\,d\left(\frac{mv}{\sqrt{1-(v/c)^2}}\right) = \dots = \frac{mv\,dv}{(1-(v/c)^2)^{3/2}}$$

$$E_k = \int dE_k = \int_0^v \frac{mv'\,dv'}{(1-(v'/c)^2)^{3/2}} = \dots = (\gamma - 1)mc^2$$

relativistička kinetička energija

35. $\vec{F}_m = Q\vec{v} \times \vec{B} \rightarrow \vec{F}_{L/m}$

$F_m = QvB\sin\theta$

$$\vec{F}_L = \vec{F}_{L/e} + \vec{F}_{L/m} = Q\vec{E} + Q\vec{v} \times \vec{B}$$ (LORENTZOVA SILA)

$$\vec{F}_{L/m} \cdot d\vec{l} = \vec{F}_{L/m} \cdot \vec{v}\,dt = Q(\vec{v} \times \vec{B}) \cdot \vec{v}\,dt = 0$$

$\vec{F}_m$ - centripetalna sila (okomita na brzinu) $\Rightarrow a_c = \frac{F_m}{m} = \frac{qvB}{m}$

$$a_c = \frac{v^2}{R} \Rightarrow \frac{v^2}{R} = \frac{qvB}{m}$$

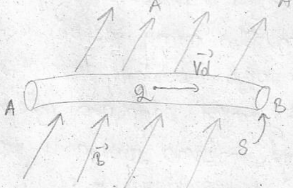
$$R = \frac{v^2 m}{qvB} = \frac{vm}{qB}$$

36. sila na naboj $\rightarrow q\vec{v}_d \times \vec{B}$

sila na vodič $\Rightarrow$ $d\vec{F} = (q\vec{v}_d \times \vec{B})\,nS\,dl$; $(dQ = nq\,dV)$, $I = nqv_dS$

$$d\vec{F} = I\,d\vec{l} \times \vec{B}$$

$$\vec{F} = I\int_A^B d\vec{l} \times \vec{B} = \int_A^B (\vec{I} \times \vec{B})\,dl = I\,\vec{AB} \times \vec{B}$$



(37.) $\vec{F} = k\dfrac{Q_1 Q_2}{r^2}\hat{r} = Q_2 \vec{E_1}$

$\vec{E_1} = \dfrac{1}{4\pi\varepsilon_0}\dfrac{Q_1}{r^2}\hat{r}$

Iz relacije $\vec{E} = \dfrac{1}{4\pi\varepsilon_0}\sum_{i=1}^{N}\dfrac{Q_i}{r_i^2}\hat{r_i}$ ili općenitije $\vec{E} = \dfrac{1}{4\pi\varepsilon_0}\sum_i \dfrac{Q_i(\vec{r}-\vec{r_i})}{|\vec{r}-\vec{r_i}|^3}$

- za linearni raspored naboja linearne gustoće $\lambda = dQ/dl$ imamo:

$$\vec{E_l} = \frac{1}{4\pi\varepsilon_0}\int \frac{\lambda dl\,\hat{r}}{r^2}$$

- za površinski raspored naboja površinske gustoće $\sigma = dQ/dS$ imamo:

$$\vec{E_\sigma} = \frac{1}{4\pi\varepsilon_0}\int \frac{\sigma dS\,\hat{r}}{r^2}$$

- za prostorni raspored naboja volumne gustoće $\rho = dQ/dV$ imamo:

$$\vec{E_V} = \frac{1}{4\pi\varepsilon_0}\int \frac{\rho dV\,\hat{r}}{r^2} \quad \text{ili} \quad \vec{E_V}(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\int_V \rho(\vec{r}')\frac{\vec{r}-\vec{r}'}{|\vec{r}-\vec{r}'|^3}dV'$$

- $\phi_E = \dfrac{Q_{unutrašnje}}{\varepsilon_0} \rightarrow \oint_S \vec{E}\cdot d\vec{S} = \dfrac{Q_{unutrašnje}}{\varepsilon_0}$

$$\oint_S \vec{E}\cdot d\vec{S} = \int_V \nabla\vec{E}\,dV = \int_V \frac{\rho dV}{\varepsilon_0}$$

$$\nabla\vec{E} = \frac{\rho}{\varepsilon_0}$$

Gaussov zakon za el. polje (prva Maxwellova jednadžba)

- polje unutar kugle: $\oint_{S<} \vec{E_<}\cdot d\vec{S} = \dfrac{Q_<}{\varepsilon_0}$

$$\rho = \frac{Q_{ukupno}}{V_{ukupno}} = \frac{Q}{4R^3\frac{\pi}{3}} = \frac{Q_<}{V_<} = \frac{Q_<}{4r_<^3\frac{\pi}{3}}$$

$$Q_< = Q\cdot\frac{r_<^3}{R^3}$$

$$E_<\oint_{S<} dS = E_<\cdot 4r_<^2\pi = \frac{Q_<}{\varepsilon_0} = \frac{1}{\varepsilon_0}Q\cdot\frac{r_<^3}{R^3}$$

$$E_<(r) = k\cdot\frac{Q}{R^3}\cdot r \quad (r<R)$$

izvan kugle: $E_>(r) = k\cdot\dfrac{Q}{r_>^3}\cdot r \quad (r>R)$

- linijska žica: $\oint \vec{E}\cdot d\vec{S} = \dfrac{Q_{un.}}{\varepsilon_0} = \int_{baza}\vec{E}\cdot d\vec{S} + \int_{plašt}\vec{E}\cdot d\vec{S}$

$$\int_{plašt}\vec{E}\cdot d\vec{S} = E\cdot\int_{plašt} dS = E\cdot 2r\pi h$$

$$Q_{unutr} = Q_u = \lambda\cdot h = \frac{Qh}{t}$$

$$\oint\vec{E}\cdot d\vec{S} = E\cdot\int_{plašt} dS = E\cdot 2r\pi h = \frac{Q_u}{\varepsilon_0} = \frac{\lambda\cdot h}{\varepsilon_0}$$

$$E = \frac{\lambda}{2r\pi\varepsilon_0} = k\cdot\frac{2\lambda}{r}, \quad k = \frac{1}{4\pi\varepsilon_0}$$

- jednoliko nabijena ploha:

$$\oint\vec{E}\cdot d\vec{S} = \frac{Q_{unutr.s}}{\varepsilon_0} = \int_{2baze}\vec{E}\cdot d\vec{S} + \int_{plašt}\vec{E}\cdot d\vec{S} = E\cdot 2S = \frac{Q_u}{\varepsilon_0} = \frac{\sigma\cdot S}{\varepsilon_0}$$

$$E = \frac{\sigma}{2\varepsilon_0}$$

(38) $F_e = qE = F_m = qvB$ ili $E = vB$

$$U = El = Blv$$

$$W = Fl = qvBl \Rightarrow \mathcal{E}_i = \frac{W}{q} \text{ ili } \boxed{\mathcal{E}_i = Blv}$$

(39) $\oint \vec{B} \cdot dl \to 0 \Rightarrow \oint \vec{J} \cdot d\vec{S} = 0$

$$I = -\frac{dq}{dt} \Rightarrow \oint_S \vec{J} \cdot d\vec{S} = -\frac{dq}{dt}$$

$$\nabla \vec{J} = -\frac{\partial}{\partial t} \rho_1$$

$$\nabla \vec{J} = -\frac{\partial}{\partial t}(\varepsilon_0 \nabla \vec{E}) = -\nabla\left(\varepsilon_0 \frac{\partial \vec{E}}{\partial t}\right)$$

$$\oint \vec{J}_p \cdot d\vec{S} = -\frac{\partial}{\partial t} Q_{unutras}$$

$$-\frac{\partial}{\partial t} Q_{unutras} = -\varepsilon_0 \frac{\partial}{\partial t} \oint \vec{E} \cdot d\vec{S}$$

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 (I - I_p) = \mu_0 \int (\vec{J} - \vec{J}_p) d\vec{S}$$

$$= \mu_0 \int \vec{J} \cdot d\vec{S} + \mu_0 \varepsilon_0 \frac{\partial}{\partial t} \int \vec{E} \cdot d\vec{S}$$

$$\boxed{\oint \vec{B} \cdot d\vec{l} = \mu_0 \int \vec{J} \cdot d\vec{S} + \mu_0 \varepsilon_0 \frac{\partial}{\partial t} \int \vec{E} \cdot d\vec{S}}$$

$$\boxed{\nabla \times \vec{B} = \mu_0 \vec{J} + \mu_0 \varepsilon_0 \frac{\partial}{\partial t} \vec{E}}$$

AMPERE-MAXWELLOV ZAKON
(3. MAXWELLOVA JEDNADŽBA)

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{I \, d\vec{S} \times \vec{r}}{r^2}$$

$$\vec{B} = \frac{\mu_0 I}{4\pi} \int \frac{d\vec{s} \times \hat{r}}{r^2}$$

$$\nabla \times \vec{B} = \nabla \times (\nabla \times \vec{A}) = \nabla(\nabla \vec{A}) - \Delta \vec{A} = -\Delta \vec{A}$$

$$\Delta A_i = -\mu_0 \hat{j} \quad (i = x, y, z)$$

$$V(\vec{r}) = \frac{1}{4\pi\varepsilon_0} \int_V \frac{\rho(\vec{r}')dV'}{|\vec{r} - \vec{r}'|}$$

$$A_i(\vec{r}) = \frac{\mu_0}{4\pi} \int_V \frac{j_i(\vec{r}')dV'}{|\vec{r} - \vec{r}'|} \quad (i = x, y, z)$$

$$E(\vec{r}) = \frac{1}{4\pi\varepsilon_0} \int \rho(\vec{r}') \frac{\vec{r} - \vec{r}'}{|\vec{r} - \vec{r}'|^3} dV'$$

$$\boxed{B(\vec{r}) = \frac{\mu_0}{4\pi} \int \frac{\vec{j}(\vec{r}') \times (\vec{r} - \vec{r}')}{|\vec{r} - \vec{r}'|^3} dV'}$$

$$dV' = dl$$

BIOT-SAVARTOV ZAKON

(40) MAXWELLOVE JEDNADŽBE U VAKUUMU:

I $\nabla \vec{E} = \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z} = 0$ tj. $\frac{\partial E_x}{\partial x} = 0$

II $\nabla \vec{B} = 0$

III $\nabla \times \vec{E} = -\frac{d\vec{B}}{dt} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & E_y & E_z \end{vmatrix} = -\frac{d}{dt}(B_x \hat{i} + B_y \hat{j} + B_z \hat{k}) \Rightarrow$

$$0 = -\frac{\partial B_x}{\partial t}$$

$$-\frac{\partial E_z}{\partial x} = -\frac{\partial B_y}{\partial t}$$

$$\frac{\partial E_y}{\partial x} = -\frac{\partial B_z}{\partial t}$$

IV $c^2 \nabla \times \vec{B} = \frac{d\vec{E}}{dt}$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ B_x & B_y & B_z \end{vmatrix} = \frac{1}{c^2} \frac{d}{dt}(E_x \hat{i} + E_y \hat{j} + E_z \hat{k}) \Rightarrow$$

$$0 = \frac{1}{c^2} \frac{\partial E_x}{\partial t}$$

$$-\frac{\partial B_z}{\partial x} = \frac{1}{c^2} \frac{\partial E_y}{\partial t}$$

$$\frac{\partial B_y}{\partial x} = \frac{1}{c^2} \frac{\partial E_z}{\partial t}$$

$$\frac{\partial^2 f}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 f}{\partial t^2}$$

c

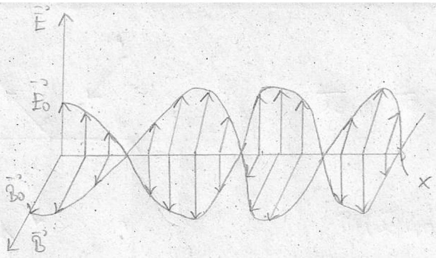
$$E_y = E_0 \sin \omega\left(t - \frac{x}{v}\right)$$

$$B_z = B_0 \sin \omega\left(t - \frac{x}{v}\right)$$

(41.)

$E_y = E_0 \sin\omega\left(t - \frac{x}{v}\right)$

$B_z = B_0 \sin\omega\left(t - \frac{x}{v}\right)$



(42.)

$B_0 = \frac{1}{c} E_0$

$\vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B} \Rightarrow S = \frac{1}{\mu_0} E_0 B_0$

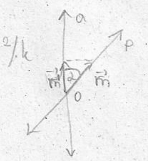
lijevo:

desno:

$E_y = a_1 \cos(2\pi f t + \delta_1) \Rightarrow$ $E_y = a\cos 2\pi f t$ $\quad E_y = a \sin 2\pi f t$

$E_z = a_2 \cos(2\pi f + \delta_2) \Rightarrow$ $E_z = a \sin 2\pi f t$ $\quad E_z = a\cos 2\pi f t$

$m' = m\cos\alpha \;/^2 / k$



$I = k m'^2$

$k m'^2 = k m^2 \cos^2\alpha$

$\boxed{I = I_0 \cos\alpha}$ Malusov zakon

(43) $\vec{E}(x,t) = E_0 \vec{j} \cdot \cos(\omega t - kx)$

?

$\boxed{\vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B}}$

Poyntingov vektor

(44.) $E_p = \left\{ 2E_0 \cos\left[\frac{\omega}{2c}(n_1 x_1 - n_2 x_2)\right]\right\} \cdot \cos\left[\omega t_0 - \frac{\omega}{2c}(n_1 x_1 + n_2 x_2)\right]$

$n_1 = n_2 \simeq 1$

$\delta = n \cdot |x_1 - x_2| = \Delta$

$\Delta = d \sin\theta$

$d \sin\theta_m = m \cdot \lambda \;(m = 0, \pm 1, \pm 2, \dots) \Rightarrow$ maks. u P

$\mathrm{tg}\,\theta = \frac{y}{D}$

$\mathrm{tg}\,\theta \simeq \sin\theta \simeq \theta$ tj. $\frac{\Delta}{d} \simeq \frac{y}{D}$

$y_{maks} = 0, \pm 1 \cdot \frac{\lambda D}{d}, \pm 2 \cdot \frac{\lambda D}{d}, \dots$

$= m \cdot \frac{\lambda D}{d}, \;(m = 0, \pm 1, \pm 2, \dots)$

(45.)

$d \sin\theta_m = (2m+1) \cdot \frac{\lambda}{2} \;(m = 0, \pm 1, \pm 2, \dots) \Rightarrow$ min. u P.

$y_{min} = 0, \pm 1 \frac{\lambda D}{d}, \pm 2 \frac{\lambda D}{d}, \pm 3 \frac{\lambda D}{d} \dots$

$= m \cdot \frac{\lambda D}{d} \;(m = 0, \pm 1, \pm 2, \dots)$

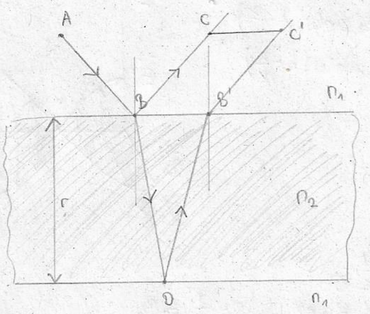
(46.)

$$L_1 = n_1 \cdot \overline{AB} + n_1 \cdot \overline{BC}$$

$$L_2 = n_1 \cdot \overline{AB} + n_2 \cdot \overline{BD} + n_2 \cdot \overline{DB'} + n_1 \cdot \overline{B'C'}$$

$$\overline{BD} = \overline{DB'} = t$$

$$\overline{BC} \simeq \overline{B'C'}$$



$$\phi_1 = \omega t - \frac{2\pi}{\lambda}\left(n_1 \cdot \overline{AB} + n_1 \cdot \overline{BC}\right) + \pi$$

$$\phi_2 = \omega t - \frac{2\pi}{\lambda}\left(n_1 \cdot \overline{AB} + n_2 \cdot 2t + n_1 \overline{B'C'}\right)$$

$$\Delta\phi = \phi_2 - \phi_1 = -\left(\frac{2\pi}{\lambda} n_2 \cdot 2t + \pi\right)$$

$$E_{0\,lrez} = \left\{2E_0 \cos\left[\frac{\omega}{2c}\left(n_1 x_1 - n_2 x_2\right)\right]\right\} = 2E_0 \cos\left(\frac{\Delta\phi}{2}\right)$$

$$\frac{1}{2}\left(\frac{2\pi}{\lambda} \cdot n_2 \cdot 2t + \pi\right) = m\pi \quad (m = 1, 2, 3 \ldots)$$

$$\text{b)}\; \boxed{t_{maks.} = \frac{2m-1}{2n_2} \cdot \frac{\lambda}{2}}$$

(47)

$$\boxed{t_{min} = \frac{m}{2n_2} \cdot \lambda}$$