STALNA SILA

$F = konst.$

početni uvjeti: t=0;x=x_0;$v_x = v_0$

$$m\frac{dv_x}{dt} = F \,/ : m$$

$$\frac{dv_x}{dt} = \frac{F}{m} = a$$

$$dv_x = \frac{dv_x}{dt} * dt = a * dt \,/ \int$$

$$\int dv_x = \int a * dt$$

$$v_x = at + c$$

$$v_0 = c$$

$$\boxed{v_x = at + v_0}$$

$$dx = \frac{dx}{dt} * dt = v_x * dt = (at + v_0) * dt \,/ \int$$

$$\int dx = \int (at + v_0) * dt$$

$$x = a\frac{t^2}{2} + v_0 t + c$$

$$x_0 = c$$

$$\boxed{x = a\frac{t^2}{2} + v_0 t + x_0}$$

KOSI HITAC

početni uvjeti: t=0;x=0;y=0;$v_0 = |\vec{v}_0|$

$$v_x = v_0 \cos\alpha; v_y = v_0 \sin\alpha$$

$$\vec{F}_G = -mg\vec{j}$$

$$\left.\begin{cases} \text{x-os } m\dfrac{dv_x}{dt} = 0 \\ \text{y-os } m\dfrac{dv_y}{dt} = -mg \end{cases}\right\} \textit{jednadžbe} \text{ gibanja}$$

$$\text{x-os} \Rightarrow \boxed{x = v_0 \cos\alpha * t; v_x = v_0 \cos\alpha}$$

$$\text{y-os} \Rightarrow m\frac{dv_y}{dt} = -mg \,/ : m \Rightarrow \boxed{a = -g} \Rightarrow$$

$$\Rightarrow \boxed{y = -\frac{g}{2}t^2 + v_0 \sin\alpha * t; v_y = -gt + v_0 \sin\alpha}$$

Vrijeme uspinjanja:

$$v_y = -gt + v_0 \sin\alpha$$

$$0 = -gt_H + v_0 \sin\alpha$$

$$gt_H = v_0 \sin\alpha$$

$$\boxed{t_H = \frac{v_0 \sin\alpha}{g}}$$

Ukupno trajanje hitca: $t_U = 2t_H$

Horizontalni domet:

$$y = -\frac{g}{2v_0^2 \cos^2\alpha}x^2 + xtg\alpha$$

$$0 = -\frac{g}{2v_0^2 \cos^2\alpha}D^2 + Dtg\alpha$$

$$\frac{g}{2v_0^2 \cos^2\alpha}D^{\cancel{2}} = \cancel{D}tg\alpha$$

$$\frac{g}{2v_0^2 \cos^2\alpha}D = tg\alpha$$

$$D = \frac{tg\alpha * 2v_0^2 \cos^2\alpha}{g} = \frac{\dfrac{1-\cos 2\alpha}{\sin 2\alpha}\cancel{2}v_0^2\dfrac{1+\cos 2\alpha}{\cancel{2}}}{g} =$$

$$= \frac{\dfrac{1-\cos^2 2\alpha}{\sin 2\alpha}v_0^2}{g} = \frac{\dfrac{\sin^{\cancel{2}} 2\alpha}{\cancel{\sin 2\alpha}}v_0^2}{g} = \frac{v_0^2 \sin 2\alpha}{g}$$

$$\boxed{D = \frac{v_0^2 \sin 2\alpha}{g}}$$

Vertikalni domet (maksimalna visina):

$y = H; v_y = 0; t = t_H$

$y = -\frac{g}{2}t^2 + v_0 \sin\alpha * t$

$H = -\frac{g}{2}t_H{}^2 + v_0 \sin\alpha * t_H =$

$= -\frac{\cancel{g}}{2}\frac{v_0{}^2 \sin^2\alpha}{g^{\cancel{2}}} + v_0 \sin\alpha * \frac{v_0 \sin\alpha}{g} =$

$= -\frac{v_0{}^2 \sin^2\alpha}{2g} + \frac{v_0{}^2 \sin^2\alpha}{g} = \frac{-v_0{}^2 \sin^2\alpha + 2v_0{}^2 \sin^2\alpha}{2g} =$

$= \frac{v_0{}^2 \sin^2\alpha}{2g}$

$\boxed{H = \frac{v_0{}^2 \sin^2\alpha}{2g}}$

Kut za max domet:

$\frac{dD}{d\alpha} = 0 \qquad \dot{D} = \frac{2v_0}{g}\cos 2\alpha$

$\frac{2v_0}{g}\cos 2\alpha = 0$

$\cos 2\alpha = 0$ - domet je isti za kutove α i $(90^\circ - \alpha)$

$2\alpha = 90^\circ$

$\boxed{\alpha = 45^\circ}$

HORIZONTALNI HITAC

počtni uvjeti: $t = 0; x = 0; y = h; \alpha = 0$

$v_x = v_0; v_y = 0$

$\vec{F} = -m\vec{g}\vec{j}$

$m\frac{dv_x}{dt} = 0 \Rightarrow x = v_0 t; v_x = v_0$

$\cancel{m}\frac{dv_y}{dt} = -\cancel{m}g \Rightarrow y = -g\frac{t^2}{2} + h; v_y = -gt$

$\boxed{\begin{array}{l} x = v_0 t; v_x = v_0 \\ y = -g\frac{t^2}{2} + h; v_y = -gt \end{array}}$

VERTIKALNI HITAC

početni uvjeti: t=0; x=0; y=0; $\mathrm{v_x} = 0$; $\mathrm{v_y} = v_0$

$\vec{F} = -mg\vec{j} \qquad \alpha = \frac{\pi}{2}$

$\boxed{\begin{array}{l} x = 0; \ \mathrm{v_x} = 0 \\ y = -\frac{g^2}{2}t + v_0 t; \ v_y = -gt + v_0 \end{array}}$

SILA OTPORA

početni uvjeti: t=0; x=0; $v_x = v_0$

$\vec{F} = -bv_x\vec{i}$

$m\frac{dv_x}{dt} = -bv_x / : m$

$\frac{dv_x}{dt} = -\frac{b}{m}v_x \quad (\tau = \frac{\mathrm{m}}{\mathrm{b}} \quad \tau = s)$

$\frac{dv_x}{dt} = -\frac{1}{\tau}v_x /* dt$

$dv_x = -\frac{1}{\tau}v_x dt / : v_x$

$\frac{1}{v_x}dv_x = -\frac{1}{\tau}dt / \int$

$\int\frac{1}{v_x}dv_x = -\frac{1}{\tau}\int dt$

$\ln v_x = -\frac{t}{\tau} + c$

$\ln v_0 = c$

$\ln v_x - \ln v_0 = -\frac{t}{\tau}$

$\ln\frac{v_x}{v_0} = -\frac{t}{\tau}$

$\boxed{v_x = v_0 e^{-\frac{t}{\tau}}}$

$\frac{dx}{dt} = v_0 e^{-\frac{t}{\tau}} /* dt / \int$

$\int dx = \int v_0 e^{-\frac{t}{\tau}} dt$

$x = -v_0 \tau e^{-\frac{t}{\tau}} + c$

$0 = -v_0\tau + c \Rightarrow v_0\tau = c$

$\boxed{x = v_0\tau(-e^{-\frac{t}{\tau}} + 1)}$

KOLIČINA GIBANJA I IMPULS SILE

$\vec{F} = konst$

$$\boxed{\vec{I} = \vec{F}\Delta t}$$

$$\vec{I} \approx \sum_{i=1}^{N} \vec{F}_i \Delta t_i$$

$$\vec{I} = \lim_{\Delta t \to o} \sum_{i=1}^{N} \vec{F}_i \Delta t = \int_{t_1}^{t_2} \vec{F} dt$$

$$\boxed{\vec{I} = \int_{t_1}^{t_2} \vec{F} dt}$$

$$d\vec{p} = \frac{d\vec{p}}{dt} dt = \vec{F} dt$$

$$\vec{I} = \int_{t_1}^{t_2} \vec{F} dt = \int_{p_1}^{p_2} d\vec{p} = \vec{p}_2 - \vec{p}_1 = \Delta\vec{p}$$

$$\boxed{\vec{I} = \Delta\vec{p}}$$

SILA TRENJA

$F_{ST} = \mu_S F_N \qquad \mu_D < \mu_S$

$F_D = \mu_D F_N$

$\vec{F}_{tr} + \vec{N} + \vec{F}_G = 0$

$$x - os \quad mg\sin\alpha - \mu_s N = 0 \Rightarrow \mu_s = \frac{mg\sin\alpha}{N} = \frac{\cancel{mg}\sin\alpha}{\cancel{mg}\cos\alpha} = tg\alpha$$

$$\boxed{\mu_s = tg\alpha}$$

$y - os \quad$ N-mgcos$\alpha = 0$

$$\boxed{N = mg\cos\alpha}$$

JEDNOLIKO KRUŽNO GIBANJE

$\vec{r}_1 = r\vec{r}_0;\ \vec{r}_2 = r\vec{r}_0;\ |\vec{r}_1| = |\vec{r}_2| = r$

$\Delta\vec{r} = \vec{r}_2 - \vec{r}_1;\ |\Delta\vec{r}| \approx r\Delta\theta$

$$v = \lim_{\Delta t \to 0} \frac{|\Delta\vec{r}|}{\Delta t} = \lim_{\Delta t \to 0} \frac{r\Delta\theta}{\Delta t} = r \underbrace{\lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t}}_{\omega} = r\omega$$

$$\boxed{v = r\omega}$$

$\vec{v} = \vec{\omega} \times \vec{r}$

$$a_r = \lim_{\Delta t \to 0} \frac{|\Delta\vec{v}|}{\Delta t} = \lim_{\Delta t \to 0} \frac{v\Delta\theta}{\Delta t} = v \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = v\omega$$

$$\boxed{a_r = v\omega}$$

$$\vec{a}_r = -v\omega\vec{r}_0 = -\frac{v^2}{r}\vec{r}_0 = -r\omega^2\vec{r}_0$$

$$\boxed{\vec{a}_r = \vec{\omega} \times \vec{v}}$$

$\theta(t) = ? \quad \omega$=konst.

$\omega = \dfrac{d\theta}{dt} \qquad$ t=0; $\theta = \theta_0$

$d\theta = \omega dt / \int$

$\theta = \omega t + c$

$c = \theta_0$

$\boxed{\theta = \omega t + \theta_0} \rightarrow$ promjena kuta u vremenu pri jednolikom gibanju po kružnici

$$\boxed{\omega = 2\pi f} \quad \boxed{T = \frac{1}{f}} \quad \boxed{T = \frac{2\pi}{\omega}}$$

CENTRIPETALNA SILA

$$\vec{F}_{CP} = m\vec{a}_r = -m\frac{v^2}{r}\vec{r}_0 = -m\omega^2 r\vec{r}_0$$

$$a_T = \frac{dv}{dt} = \frac{d}{dt}(r\omega) = r\underbrace{\frac{d\omega}{dt}}_{\alpha} \quad (\alpha \rightarrow \text{kutna akceleracija})$$

$$\boxed{a_T = r\alpha} \quad \boxed{\vec{a}_T = \vec{\alpha}\times\vec{r}}$$

$$\vec{a} = \vec{a}_r + \vec{a}_T$$

$$a = \sqrt{a^2{}_r + a^2{}_T} = \sqrt{\frac{v^4}{r^2} + r^2\alpha^2}$$

$\alpha = konst.$ t=0; $\theta = \theta_0$; $\omega = \omega_0$

$$\alpha = \frac{d\omega}{dt}$$

$$d\omega = \alpha dt \,/\int$$

$$\omega = \alpha t + c$$

$$\omega_0 = c$$

$$\boxed{\omega = \alpha t + \omega_0}$$

$$\omega = \frac{d\theta}{dt}$$

$$d\theta = \omega dt \,/\int$$

$$\int d\theta = \int(\alpha t + \omega_0)dt$$

$$\theta = \alpha\frac{t^2}{2} + \omega_0 t + c$$

$$\theta_0 = c$$

$$\boxed{\theta = \alpha\frac{t^2}{2} + \omega_0 t + \theta_0}$$

KUTNA KOLIČINA GIBANJA ČESTICE

$$\vec{p} = \omega\vec{v}$$

$$r_\perp = r\sin\alpha$$

$$L_A = r_\perp p = rp\sin\alpha$$

$$\boxed{\vec{L}_A = \vec{r}\times\vec{p}}$$

$$\frac{d\vec{L}}{dt} = \frac{d}{dt}(\vec{r}\times\vec{p}) = \underbrace{\frac{d\vec{r}}{dt}}_{v}\times\vec{p} + \vec{r}\times\frac{d\vec{p}}{dt} = \vec{r}\times\vec{F}$$

$$(\vec{v}\times(m\vec{v}) = m\underbrace{(\vec{v}\times\vec{v})}_{0} = 0)$$

$$\vec{r}\times\vec{F}_{CP} = 0$$

$$\vec{r} \parallel \vec{F}_{CP}$$

$$\frac{d\vec{L}}{dt} = 0$$

$$\boxed{\vec{M} = \vec{r}\times\vec{F}} \quad \boxed{\frac{d\vec{L}}{dt} = \vec{M}}$$

$$\vec{L} = \vec{r}\times\vec{p} = m(\vec{r}\times\vec{v}) = m(\vec{r}\times\vec{\omega}\times\vec{r})$$

$$\vec{L} = m(\vec{r}\times\vec{\omega}\times\vec{r}) = m(\vec{\omega}\underbrace{(\vec{r}\cdot\vec{r})}_{r^2} - \vec{r}\underbrace{(\vec{r}\cdot\vec{\omega})}_{0}) = \underbrace{mr^2}_{I}\omega = I\vec{\omega}$$

$$\boxed{\vec{L} = I\vec{\omega}}$$

RAD I ENERGIJA

1. slučaj (stalna sila)

$$\vec{F} = F_x \vec{i}$$

$$\Delta\vec{r} = \Delta x\vec{i} \qquad \boxed{W = F_x \Delta x}$$

$$\underbrace{F_x}_{konst.} > 0$$

2. slučaj (promjenjiva sila)

$$\vec{F} = F_x(x)\vec{i}$$

$$W_i = F_i \Delta x_i$$

$$W \approx \sum_{i=1}^{N} F_i \Delta x_i$$

$$W = \lim_{\Delta x_i \to 0} \sum_{i=1}^{N} F_i \Delta x_i \Rightarrow \boxed{W = \int_{x_1}^{x_2} F_x(x)dx}$$

3. slučaj (sila djeluje pod kutem)

$$\mathrm{F} = |\vec{\mathrm{F}}|$$

$$W = F\cos\theta \Delta x = \vec{F}\cdot\Delta\vec{r} \Rightarrow \boxed{W = \vec{F}\cdot\Delta\vec{r}}$$

4.slučaj (rad u najopćenitijem slučaju)

$$W_i = \vec{F}_i \cdot \Delta\vec{r}_i$$

$$W \approx \sum_{i=1}^{N} \vec{F}_i \cdot \Delta\vec{r}_i$$

$$W = \lim_{\Delta x_i \to 0} \sum_{i=1}^{N} \vec{F}_i \cdot \Delta\vec{r}_i = \int_{\overset{\frown}{AB}} \vec{F}d\vec{r} \Rightarrow \boxed{W = \int_{\overset{\frown}{AB}} \vec{F}d\vec{r}}$$

KINETIČKA ENERGIJA

$$W = \int_{\overset{\frown}{AB}} \vec{F}d\vec{r} = \int_{\overset{\frown}{AB}} m\frac{d\vec{v}}{dt}d\vec{r} = \quad d\vec{r} = \vec{v}dt \quad =$$

$$= \int_{\overset{\frown}{AB}} m\frac{d\vec{v}}{dt}\vec{v}dt = \int_{\overset{\frown}{AB}} m\frac{1}{2}\frac{d}{dt}(\vec{v}^2)dt = \frac{1}{2}m(\vec{v}_2^2 - \vec{v}_1^2)$$

$$\boxed{W = \frac{1}{2}m\vec{v}_2^2 - \frac{1}{2}m\vec{v}_1^2} \Rightarrow \boxed{W = E_{K2} - E_{K1}} \Rightarrow$$

$$\Rightarrow \underbrace{\boxed{W = \Delta E_K}}_{\text{Teorem o radu i kinetičkoj energiji}}$$

$$\boxed{E_K = \frac{1}{2}m\vec{v}^2} \rightarrow \text{Kinetička energija}$$

GRAVITACIJSKA POTENCIJALNA ENERGIJA

$$\vec{F} = -mg\vec{k}$$

$$W = \int_{\overset{\frown}{AB}} \vec{F}d\vec{r} = -mg\int_{\overset{\frown}{AB}} dz = -mg(z_2 - z_1) =$$

$$= -mgz_2 + mgz_1 = -(U_2 - U_1) = -\Delta U$$

$$\underbrace{\boxed{U = mgz}}_{\text{Gravitacijska pot-encijalna energija}} \qquad \underbrace{\boxed{W = -\Delta U}}_{\text{Teorem o radu i grav. pot. energiji}}$$

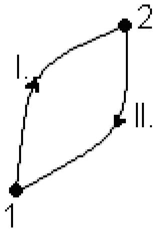
$$\Delta U = mgz_2 - mgz_1 = -W = -\Delta E_K = (m\frac{v_2^2}{2} - m\frac{v_1^2}{2})$$

$$mgz_2 + m\frac{v_2^2}{2} = mgz_1 + m\frac{v_1^2}{2}$$

$\boxed{U_2 + E_{K2} = U_1 + E_{K1}}$ -Zakon očuvanja energije za gravitacijsku silu

$\boxed{U + E_K = E_M}$ -Mehanička energija čestica

KONZERVATIVNE SILE (DISIPATIVNE SILE)



$$\text{I.} \int_{1\,(I.)}^{2} \vec{F}d\vec{r} = -\int_{2\,(I.)}^{1} \vec{F}d\vec{r} \qquad \text{II.} \int_{1\,(II.)}^{2} \vec{F}d\vec{r} = -\int_{2\,(II.)}^{1} \vec{F}d\vec{r}$$

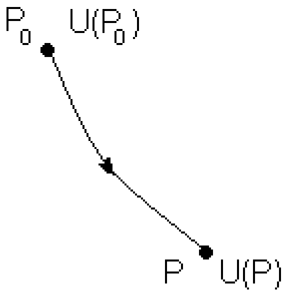
$$2 \underset{(II.)}{\rightarrow} 1 + 1 \underset{(I.)}{\rightarrow} 2 = 0$$

$$\int_{2\,(II.)}^{1} \vec{F}d\vec{r} + \int_{1\,(I.)}^{2} \vec{F}d\vec{r} = 0$$

$$\int_{1\,(I.)}^{2} \vec{F}d\vec{r} = -\int_{2\,(II.)}^{1} \vec{F}d\vec{r}$$

$$\int_{1\,(I.)}^{2} \vec{F}d\vec{r} = \int_{1\,(II.)}^{2} \vec{F}d\vec{r}$$

POTENCIJALNE ENERGIJE KONZERVATIVNE SILE



$$W = -\Delta U \qquad W = \int_{AB} \vec{F} d\vec{r}$$

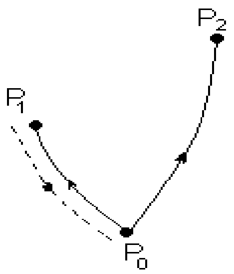
$$-\Delta U = W$$

$$-(U(P) - U(P_0)) = W$$

$$-U(P) + U(P_0) = \int_{P_0}^{P} \vec{F} d\vec{r}$$

$$U(P) = -\int_{P_0}^{P} \vec{F} d\vec{r} + U(P_0)$$

$$\boxed{U(P_x) = -\int_{P_0}^{P_x} \vec{F} d\vec{r} + U(P_0)}$$



$$U(P_2) - U(P_1) =$$

$$= -\int_{P_0}^{P_2} \vec{F} d\vec{r} + U(P_0) - \left(-\int_{P_0}^{P_1} \vec{F} d\vec{r} + U(P_0)\right) =$$

$$= -\int_{P_0}^{P_2} \vec{F} d\vec{r} + \cancel{U(P_0)} + \int_{P_0}^{P_1} \vec{F} d\vec{r} - \cancel{U(P_0)} =$$

$$= -\int_{P_0}^{P_2} \vec{F} d\vec{r} - \int_{P_1}^{P_0} \vec{F} d\vec{r} = -\left(\int_{P_1}^{P_0} \vec{F} d\vec{r} + \int_{P_0}^{P_2} \vec{F} d\vec{r}\right) =$$

$$= -\int_{P_1}^{P_2} \vec{F} d\vec{r} = -W$$

$$\boxed{\Delta U = -W}$$

ZAKON OČUVANJA ENERGIJE

$$W = W_{KS} + W_{VS}$$

$$W = W_1 + W_2 + W_3$$

$$W_1 = \Delta E_{K1} = W_{1KS} + W_{1VS} = -\Delta U_1 + W_{1VS}$$

$$W_2 = \Delta E_{K2} = W_{2KS} + W_{2VS} = -\Delta U_2 + W_{2VS}$$

$$W_3 = \Delta E_{K3} = W_{3KS} + W_{3VS} = -\Delta U_3 + W_{3VS}$$

$$\Delta E_{K1} + \Delta E_{K2} + \Delta E_{K3} = -\Delta U_1 + W_{1VS} - \Delta U_2 + W_{2VS} -$$
$$-\Delta U_3 + W_{3VS}$$

$$\Delta E_{K1} + \Delta U_1 + \Delta E_{K2} + \Delta U_2 + \Delta E_{K3} + \Delta U_3 =$$
$$= W_{1VS} + W_{2VS} + W_{3VS}$$

$$\Delta\underbrace{(E_{K1} + U_1)}_{E_{M1}} + \Delta\underbrace{(E_{K2} + U_2)}_{E_{M2}} + \Delta\underbrace{(E_{K3} + U_3)}_{E_{M3}} = W_{VS}$$

$$\Delta E_{M1} + \Delta E_{M2} + \Delta E_{M3} = W_{VS}$$

$$\boxed{\Delta E_{MH} = W_{VS}} \Rightarrow W_{VS} > 0 \Leftrightarrow \Delta E_{MH} > 0$$
$$W_{VS} < 0 \Leftrightarrow \Delta E_{MH} < 0$$
$$W_{VS} = 0 \Leftrightarrow \Delta E_{MH} = 0$$

$$\boxed{\Delta E = 0}$$

PRORAČUN SILE IZ POTENCIJALNE ENERGIJE

$$U(P)-U(P_0)=dU=-dW=-\vec{F}d\vec{r}=$$
$$=-F_x dx-F_y dy-F_z dz$$
$$dU=-F_x dx$$

x-os

$$dU=\text{-}F_x dx$$
$$dU=\frac{dU}{dx}\Big|dx(y,z\ \text{kons.})$$
$$\mathrm{F_x}=\frac{\partial U}{\partial x}$$
$$\mathrm{F_y}=\frac{\partial U}{\partial y}$$
$$\mathrm{F_z}=\frac{\partial U}{\partial z}$$
$$U(x)$$
$$E=\frac{1}{2}mv^2+U(x)$$
$$\frac{1}{2}mv^2=E-U(x)$$
$$\boxed{\begin{array}{l} v^2=\frac{2}{m}(E-U(x)) \\ v=\sqrt{\frac{2}{m}(E-U(x))}\end{array}}$$
$$\left.\begin{cases}\mathrm{v=0} \\ E-U(x)=0\end{cases}\right\}\ \text{točke okreta}$$

SNAGA

prosječna sanga:

$$\boxed{\overline{P}=\frac{\Delta W}{\Delta t}}$$

Trenutna snaga:

$$P=\lim_{\Delta t\to 0}\frac{\Delta W}{\Delta t}=\frac{dW}{dt}$$
$$\boxed{P=\frac{dW}{dt}}$$
$$dW=\frac{dW}{dt}dt=pdt$$
$$dW=\vec{F}d\vec{r}=\vec{F}\vec{v}dt$$
$$\frac{dW}{dt}=\vec{F}\vec{v}$$
$$\boxed{P=\vec{F}\vec{v}}$$

SUSTAV MATERIJALNIH TOČAKA

$$\vec{F}_{ki}=-\vec{F}_{ik}$$
$$\frac{d\vec{p}_1}{dt}=\vec{F}_{v1}+\vec{F}_{21}+\vec{F}_{31}+\ldots+\vec{F}_{n1}$$
$$\frac{d\vec{p}_2}{dt}=\vec{F}_{v2}+\vec{F}_{12}+\vec{F}_{32}+\ldots+\vec{F}_{n2}$$
$$\sum_{i=1}^{n}\frac{d\vec{p}_i}{dt}=\sum_{i=1}^{n}\vec{F}_{vi}+\underbrace{\sum_{\substack{i,j=1\\ i\neq j}}^{n}\vec{F}_{ij}}_{\vec{F}_{ij}+\vec{F}_{ji}=0}$$
$$\boxed{\sum_{\substack{i,j=1\\ i\neq j}}^{n}\vec{F}_{ij}=0}\quad\boxed{\sum_{i=1}^{n}\frac{d\vec{p}_i}{dt}=\vec{F}_v}\quad\boxed{\vec{F}_v=\sum_{i=1}^{n}\vec{F}_{vi}}$$

CENTAR MASE

1 $m_1(x_1, y_1, z_1)$

2 $m_2(x_2, y_2, z_2)$

$\vdots$

n $m_n(x_n, y_n, z_n)$

$$x_{CM} = \frac{m_1x_1 + m_2x_2 + \ldots + m_nx_n}{m} = \frac{\sum_{i=1}^{n} m_ix_i}{m}$$

$$\boxed{x_{CM} = \frac{\sum_{i=1}^{n} m_ix_i}{m}} \quad \boxed{y_{CM} = \frac{\sum_{i=1}^{n} m_iy_i}{m}} \quad \boxed{z_{CM} = \frac{\sum_{i=1}^{n} m_iz_i}{m}}$$

$$\boxed{\vec{r}_{CM} = \frac{\sum_{i=1}^{n} m_i\vec{r}_i}{m}}$$

CENTAR MASE ČVRSTOG TIJELA

$$\bar{\varphi} = \frac{\Delta m}{\Delta V}$$

$$\varphi = \lim_{\Delta V \to 0} \frac{\Delta m}{\Delta V} = \frac{dm}{dV}$$

$$dm = \varphi dV$$

$$\boxed{m = \int_V \varphi dV}$$

$$\vec{r}_{CM} = r_{CM} \approx \frac{\sum_{i=1}^{n} \Delta m_i r_i}{m} = \frac{\sum_{i=1}^{n} \varphi r_i \Delta V_i}{m}$$

$$\vec{r}_{CM} = \lim_{\Delta V_i \to 0} \frac{\sum_{i=1}^{n} \varphi_i \vec{r}_i \Delta V_i}{m} = \frac{\int_V \varphi \vec{r} dV}{m}$$

$$\boxed{\vec{r}_{CM} = \frac{1}{m} \int_V \varphi \vec{r} dV}$$ -Centar mase

$$m_i = konst.$$

$$\sum_{i=1}^{n} m_i \vec{a}_i = \vec{F}_v$$

$$m\vec{r}_{CM} = \sum_{i=1}^{n} m_i \vec{r}_i \; / \frac{d}{dt}$$

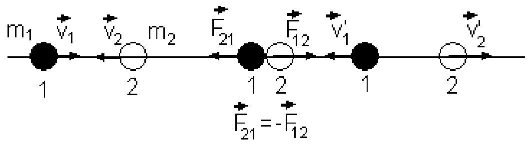
$$m\vec{v}_{CM} = \sum_{i=1}^{n} m_i \vec{v}_i \; / \frac{d}{dt}$$

$$m\vec{a}_{CM} = \sum_{i=1}^{n} m_i \vec{a}_i = \vec{F}_v$$

$\boxed{\vec{F}_v = m\vec{a}_{CM}}$ -jedn. gibanja centra mase od N čestica

Ako je $\vec{F}_v = 0$ tijelo se giba jednoliko po pravcu

ZATVORENI CENTRALNI SRAZ



$$\vec{I}_1 = \vec{F}_{21}\Delta t$$
$$\vec{I}_2 = \vec{F}_{12}\Delta t$$
$$\vec{I}_1 = -\vec{I}_2$$
$$\vec{I} = \Delta\vec{p} = (\vec{p}_2 - \vec{p}_1) = m\vec{v}' - m\vec{v}$$
$$\vec{I}_1 = m_1\vec{v}_1' - m_1\vec{v}_1$$
$$\vec{I}_2 = m_2\vec{v}_2' - m_2\vec{v}_2$$
$$m_1\vec{v}_1' - m_1\vec{v}_1 = -(m_2\vec{v}_2' - m_2\vec{v}_2)$$
$$m_1\vec{v}_1' - m_1\vec{v}_1 = -m_2\vec{v}_2' + m_2\vec{v}_2$$
$$\boxed{m_1\vec{v}_1' + m_2\vec{v}_2' = m_1\vec{v}_1 + m_2\vec{v}_2}$$

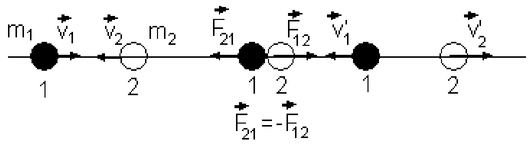
N zatvoreni (zatvoreni sustav s N čestica)

$$\vec{F}_v = 0$$
$$\sum_{i=1}^{N}\frac{d\vec{p}_i}{dt} = 0$$
$$\frac{d}{dt}\sum_{i=1}^{N}\vec{p}_i = 0$$

$\boxed{\frac{d\vec{p}_i}{dt} = 0}$ - ako je sustav zatvoren ukupna

količina gibanja je konstantna

(SAVRŠENO) ELASTIČNI SUDAR



očuvanje $\vec{p}$

$$m_1\vec{v}_1 + m_2\vec{v}_2 = m_1\vec{v}_1' + m_2\vec{v}_2'$$
$$m_1\vec{v}_1 - m_1\vec{v}_1' = m_2\vec{v}_2' - m_2\vec{v}_2$$
$$m_1(\vec{v}_1 - \vec{v}_1') = m_2(\vec{v}_2' - \vec{v}_2)$$
$$\boxed{m_1(\vec{v}_1 - \vec{v}_1') = -m_2(\vec{v}_2 - \vec{v}_2')}$$

očuvanje $\vec{E}_K$

$$m_1\frac{v_1^2}{2} + m_2\frac{v_2^2}{2} = m_1\frac{v_1'^2}{2} + m_2\frac{v_2'^2}{2} \;/:2$$
$$m_1v_1^2 + m_2v_2^2 = m_1v_1'^2 + m_2v_2'^2$$
$$m_1v_1^2 - m_1v_1'^2 = m_2v_2'^2 - m_2v_2^2$$
$$m_1(v_1^2 - v_1'^2) = m_2(v_2'^2 - v_2^2)$$
$$m_1(v_1^2 - v_1'^2) = -m_2(v_2^2 - v_2'^2)$$
$$m_1(v_1 - v_1')(v_1 + v_1') = -m_2(v_2 - v_2')(v_2 + v_2')$$
$$-m_2(\vec{v}_2 - \vec{v}_2') = m_1(\vec{v}_1 - \vec{v}_1')$$
$$\cancel{m_1}(\vec{v}_1 - \vec{v}_1')(\vec{v}_1 + \vec{v}_1') = \cancel{m_1}(\vec{v}_2 + \vec{v}_2')(\vec{v}_1 - \vec{v}_1')$$
$$\underbrace{(\vec{v}_1 - \vec{v}_1')}_{\substack{0 \\ \vec{v}_1 = \vec{v}_1'}}\underbrace{(\vec{v}_1 + \vec{v}_1' - \vec{v}_2 - \vec{v}_2')}_{\substack{0 \\ \vec{v}_1 + \vec{v}_1' - \vec{v}_2 - \vec{v}_2' = 0 \\ \vec{v}_1 - \vec{v}_2 = -(\vec{v}_1' - \vec{v}_2')}} = 0$$

$$m_2\vec{v}_2' = m_1\vec{v}_1 + m_2\vec{v}_2 - m_1\vec{v}_1' \;/:m_2$$
$$\vec{v}_1' = \vec{v}_2' + \vec{v}_2 - \vec{v}_1$$
$$\vec{v}_2' = \frac{m_1\vec{v}_1 + m_2\vec{v}_2 - m_1\vec{v}_1'}{m_2}$$
$$\vec{v}_1' = \frac{m_1\vec{v}_1 + m_2\vec{v}_2 - m_1\vec{v}_1'}{m_2} + \vec{v}_2 - \vec{v}_1 =$$
$$= \frac{m_1\vec{v}_1 + m_2\vec{v}_2 - m_1\vec{v}_1' + m_2\vec{v}_2 - m_2\vec{v}_1}{m_2} =$$
$$= \frac{(m_1 - m_2)\vec{v}_1 + 2m_2\vec{v}_2}{m_2} - \frac{m_1}{m_2}\vec{v}_1'$$
$$\vec{v}_1' + \frac{m_1}{m_2}\vec{v}_1' = \frac{(m_1 - m_2)\vec{v}_1 + 2m_2\vec{v}_2}{m_2}$$
$$(1 + \frac{m_1}{m_2})\vec{v}_1' = \frac{(m_1 - m_2)\vec{v}_1 + 2m_2\vec{v}_2}{m_2}$$

$$\vec v_1' \frac{m_2+m_1}{\cancel{m_2}} = \frac{(m_1-m_2)\vec v_1 + 2m_2\vec v_2}{\cancel{m_2}}$$

$$\vec v_1' = \frac{(m_1-m_2)\vec v_1+2m_2\vec v_2}{m_2+m_1}$$

$$\vec v_2' = \vec v_1' - \vec v_2 + \vec v_1$$

$$\vec v_2' = \frac{(m_1-m_2)\vec v_1+2m_2\vec v_2}{m_2+m_1} - \vec v_2+\vec v_1 =$$

$$= \frac{(m_1-m_2)\vec v_1 + 2m_2\vec v_2 - (m_2+m_1)\vec v_2 + (m_2+m_1)\vec v_1}{m_2+m_1} =$$

$$= \frac{(m_1 - \cancel{m_2} + \cancel{m_2} + m_1)\vec v_1 + (\cancel{2}m_2 - \cancel{m_2} - m_1)\vec v_2}{m_2+m_1} =$$

$$\vec v_2' = \frac{2m_1\vec v_1 + (m_2-m_1)\vec v_2}{m_2+m_1}$$

$$\boxed{\begin{aligned} \vec v_1' &= \frac{(m_1-m_2)\vec v_1+2m_2\vec v_2}{m_1+m_2} \\ \vec v_2' &= \frac{(m_2-m_1)\vec v_2+2m_1\vec v_1}{m_1+m_2} \end{aligned}}$$

Posebni slučajevi:

1.) $m_1 = m_2 = m$

$\boxed{\vec v_1' = \vec v_2} \Rightarrow \vec v_2 = 0 \Leftrightarrow \vec v_1' = 0$

$\boxed{\vec v_2' = \vec v_1} \Rightarrow \vec v_1 = 0 \Leftrightarrow \vec v_2' = 0$

2.) $m_1 \ll m_2$

$\vec v_2 = 0 \Rightarrow \boxed{\vec v_1' \approx -\vec v_1}$

$\Rightarrow \boxed{\vec v_2' \approx 0}$

3.) $m_1 \gg m_2 \quad v_2 = 0$

$$\boxed{\begin{aligned} \vec v_1' &\approx \vec v_1 \\ \vec v_2' &\approx 2\vec v_1 \end{aligned}}$$

TOTALNO (SAVRŠENO)NEELASTIČNI SUDAR

vrijedi zakon očuvanja $\vec{p}$

$\left.\begin{matrix} m_1, \vec v_1 \\ m_2, \vec v_2 \end{matrix}\right\}$ prije sudara

$(m_1+m_2)\vec v'$ -poslije sudara

$$m_1\vec v_1 + m_2\vec v_2 = (m_1+m_2)\vec v'$$

$$\boxed{\vec v' = \frac{m_1\vec v_1 + m_2\vec v_2}{m_1+m_2}} \qquad \boxed{q = E_K' - E_K}$$

$$q = \frac{1}{2}(m_1+m_2)v'^2 - \frac{1}{2}m_1v_1^2 - \frac{1}{2}m_2v_2^2 =$$

$$= \frac{1}{2}(m_1+m_2)\left(\frac{m_1v_1+m_2v_2}{m_1+m_2}\right)^2 - \frac{1}{2}m_1v_1^2 - \frac{1}{2}m_2v_2^2$$

$$= \frac{1}{2}\cancel{(m_1+m_2)}\left(\frac{m_1^2v_1^2 + 2m_1v_1m_2v_2 + m_2^2v_2^2}{(m_1+m_2)^{\cancel{2}}}\right) - \frac{1}{2}m_1v_1^2 - \frac{1}{2}m_2v_2^2 =$$

$$= \frac{\cancel{m_1^2v_1^2} + 2m_1v_1m_2v_2 + \cancel{m_2^2v_2^2} - \cancel{m_1^2v_1^2} - m_2m_1v_1^2 - \cancel{m_2^2v_2^2} - m_1m_2v_2^2}{2(m_1+m_2)} =$$

$$= \frac{2m_1v_1m_2v_2 - m_2m_1v_1^2 - m_1m_2v_2^2}{2(m_1+m_2)} =$$

$$= \frac{m_1m_2(-v_1^2 + 2v_1v_2 - v_2^2)}{2(m_1+m_2)} = -\frac{m_1m_2}{2(m_1+m_2)}(v_1-v_2)^2$$

$$\boxed{q = -\frac{m_1m_2}{2(m_1+m_2)}(v_1-v_2)^2}$$

Posebni slučaj:

1.) $m_1 = m_2 = m$

$$\vec v' = \frac{1}{2}(\vec v_1 + \vec v_2)$$

$$\vec v_2 = 0 \Rightarrow \vec v' = \frac{1}{2}\vec v_1$$

$$\vec v_1 = -\vec v_2 \Rightarrow \vec v' = 0$$

2.) $m_1 \ll m_2$

$$\vec v' = \frac{\frac{m_1}{m_2}\vec v_1}{1+\frac{m_1}{m_2}} = 0$$