Zad. 1.

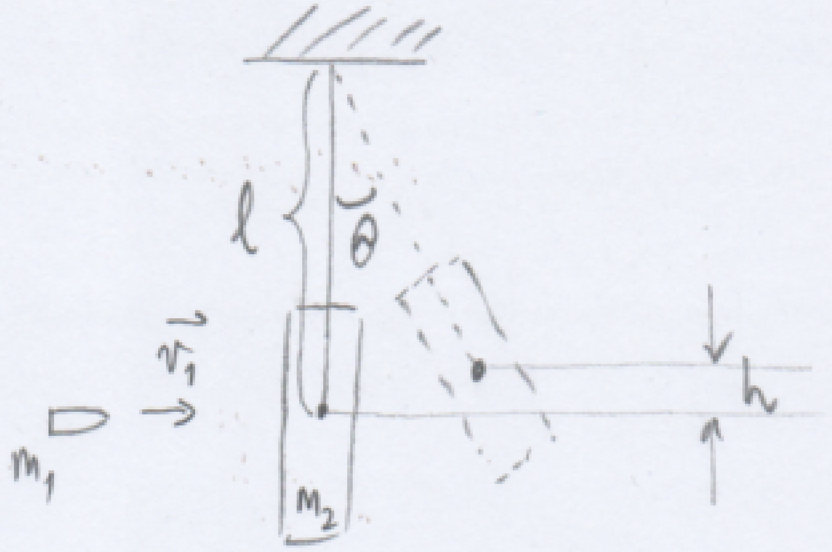
$m_1 = 50\,g = 0{,}05\,kg$

$m_2 = 100\,kg$

$l = 5\,m$

$\theta = 1{,}25°$

$v_1 = ?$

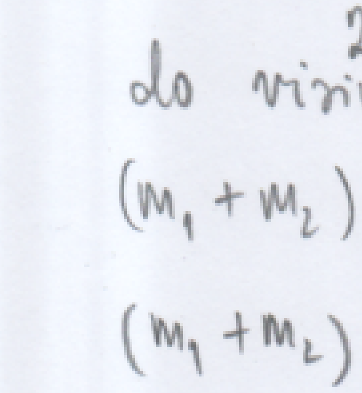


Neelastičan sudar - zakon očuvanja količine gibanja

$m_1 v_1 = (m_1 + m_2) v \quad \Rightarrow v = \dfrac{m_1 v_1}{m_1 + m_2}$

Kad se metak zaustavi u vreći, vreća se počne ponašati kao njihalo $\Rightarrow$ vreća dobije kinetičku energiju $(m_1+m_2)\dfrac{v^2}{2}$, koja kad se vreća zanjiše za kut θ, odn. do visine h, se pretvori u potencijalnu energiju $(m_1+m_2)gh$:

$$(m_1+m_2)\frac{v^2}{2} = (m_1+m_2)gh \Rightarrow v^2 = 2gh = \frac{m_1^2 v_1^2}{(m_1+m_2)^2}$$



$\cos\theta = \dfrac{l-h}{l}$

$l\cos\theta = l - h$

$h = l(1-\cos\theta)$

$$v_1^2 = \frac{2(m_1+m_2)^2 gh}{m_1^2}$$

$$v_1^2 = \frac{2(m_1+m_2)^2 g l (1-\cos\theta)}{m_1^2}$$

$$v_1 = \frac{m_1+m_2}{m_1}\sqrt{2gl(1-\cos\theta)} = \frac{0{,}05+100}{0{,}05}\sqrt{2\cdot 9{,}81\cdot 5\,(1-\cos 1{,}25°)}$$

$$= 306\,m/s$$

sp

Zad. 2. $m_1 = 30\,g = 0{,}03\,kg$

$v_1 = 8\,m/s$

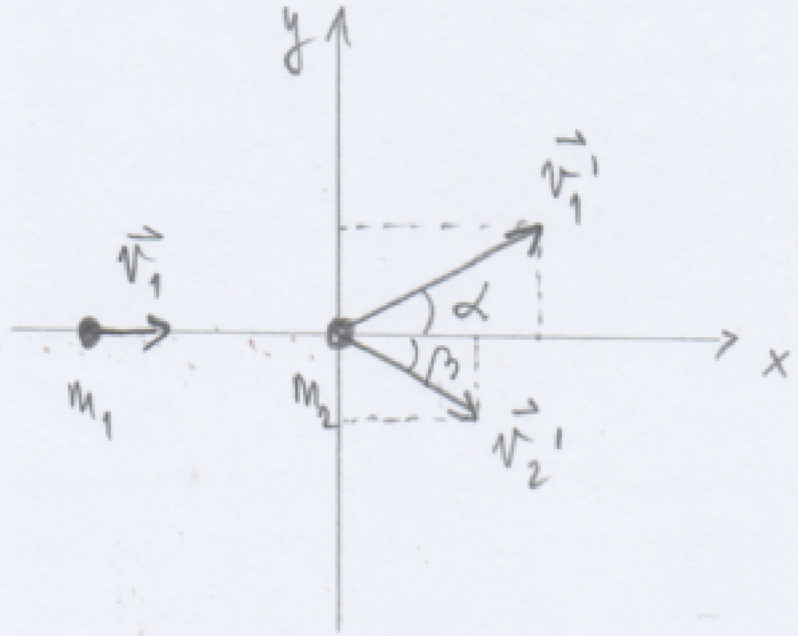
$m_2 = 200\,g = 0{,}2\,kg$

$\alpha = 30^\circ$

β

$v_1' = 3\,m/s$

$Q = ?$



Zakon očuvanja količine gibanja:

$$\left.\begin{aligned} m_1 v_1 &= m_1 v_1' \cos\alpha + m_2 v_2' \cos\beta \\ 0 &= m_1 v_1' \sin\alpha - m_2 v_2' \sin\beta \end{aligned}\right\} \quad \left.\begin{aligned} m_2 v_2' \sin\beta &= m_1 v_1' \sin\alpha \\ m_2 v_2' \cos\beta &= m_1 v_1 - m_1 v_1' \cos\alpha \end{aligned}\right\} :$$

$$\mathrm{tg}\,\beta = \frac{\cancel{m_1} v_1' \sin\alpha}{\cancel{m_1} v_1 - \cancel{m_1} v_1' \cos\alpha} \Rightarrow \mathrm{tg}\,\beta = \frac{3 \cdot \sin 30^\circ}{8 - 3 \cdot \cos 30^\circ} = 0{,}278$$

$$\beta = 15{,}52^\circ$$

$$v_2' = \frac{m_1 v_1' \sin\alpha}{m_2 \sin\beta} = \frac{0{,}03 \cdot 3 \cdot \sin 30^\circ}{0{,}2 \cdot \sin 15{,}52^\circ} = 0{,}84\,\frac{m}{s}$$

$$\frac{m_1 v_1^2}{2} = \frac{m_1 v_1'^2}{2} + \frac{m_2 v_2'^2}{2} + Q$$

$$Q = \frac{m_1 v_1^2}{2} - \frac{m_1 v_1'^2}{2} - \frac{m_2 v_2'^2}{2}$$

$$Q = \frac{0{,}03 \cdot 8^2}{2} - \frac{0{,}03 \cdot 3^2}{2} - \frac{0{,}2 \cdot 0{,}84^2}{2} = 0{,}754\,J$$

8P

Zad. 3.

$k = 7\ N/m$

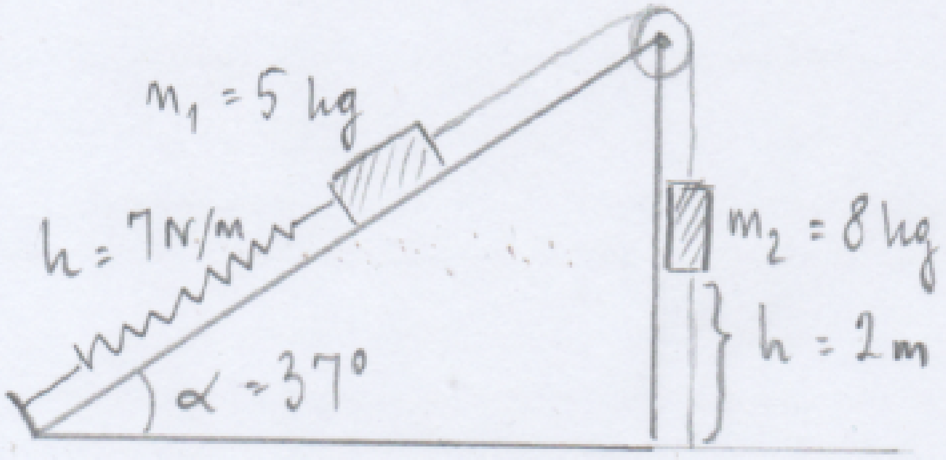
$m_1 = 5\ kg$

$m_2 = 8\ kg$

$h = 2\ m$

$\alpha = 37^\circ$

$v = ?$



Zakon o očuvanju energije:

$$m_2 g h = \frac{m_2 v^2}{2} + m_1 g h \sin\alpha + \frac{k h^2}{2} + \frac{m_1 v^2}{2}$$

- $m_2 g h$ → potencijalna energija od m_2
- $\frac{m_2 v^2}{2}$ → kinetička energija od m_2
- $m_1 g h \sin\alpha$ → promjena potencijalne energije od m_1
- $\frac{k h^2}{2}$ → potencijalna energija opruge
- $\frac{m_1 v^2}{2}$ → kinetička energija od m_1

$$\frac{m_1 + m_2}{2} v^2 = m_2 g h - m_1 g h \sin\alpha - \frac{k h^2}{2}$$

$$v^2 = \frac{2}{m_1 + m_2}\left[g h (m_2 - m_1 \sin\alpha) - \frac{k h^2}{2}\right]$$

$$v = \sqrt{\frac{2}{m_1 + m_2}\left[g h (m_2 - m_1 \sin\alpha) - \frac{k h^2}{2}\right]}$$

$$v = \sqrt{\frac{2}{5+8}\left[9{,}81 \cdot 2\,(8 - 5 \sin 37^\circ) - \frac{7 \cdot 2^2}{2}\right]}$$

$$\underline{\underline{v = 3{,}6\ m/s}}$$

Zad. 4.

$m = 5\,kg$

$v_0 = 4\,m/s$

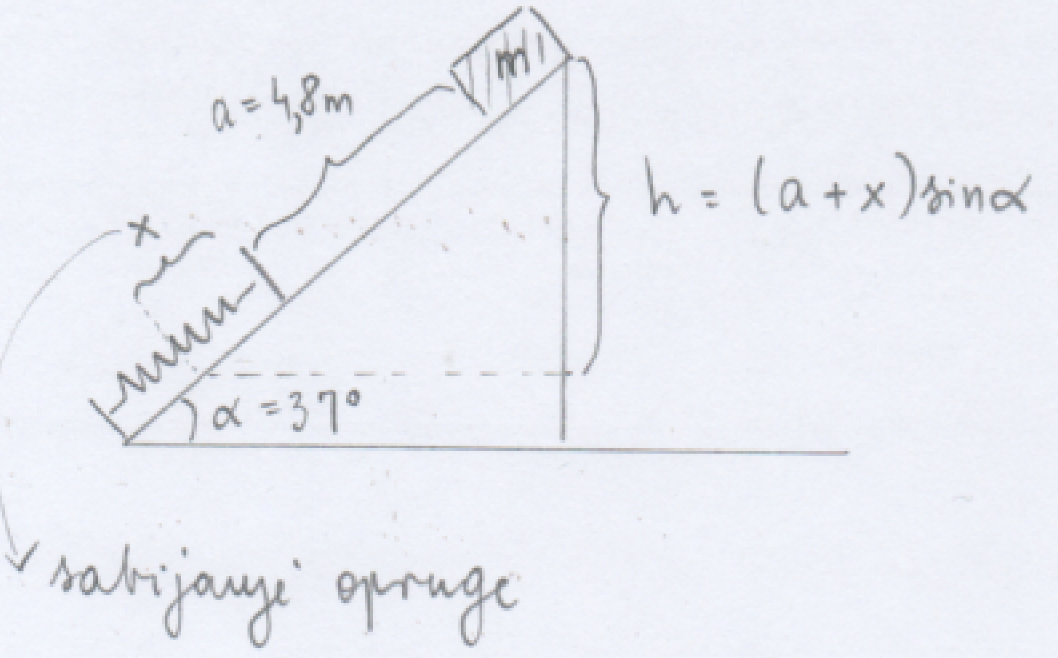
$a = 4{,}8\,m$

$\alpha = 37°$

$F_{tr} = 8\,N$

$x = 20\,cm = 0{,}2\,m$

$k = ?$



Kinetička i potencijalna energija tijela mase m troše se na savladavanje sile trenja na putu do opruge (a) i dok se opruga sabija (x) i na rad pri sabijanju opruge:

$$\frac{m v_0^2}{2} + mgh = F_{tr}(a+x) + \frac{k x^2}{2}$$

$$k = \frac{2}{x^2}\left[\frac{m v_0^2}{2} + mg(a+x)\sin\alpha - F_{tr}(a+x)\right]$$

$$k = \frac{2}{0{,}2^2}\left[\frac{5\cdot 4^2}{2} + 5\cdot 9{,}81\,(4{,}8+0{,}2)\sin 37° - 8\cdot(4{,}8+0{,}2)\right]$$

$$k = 7380\ N/m$$

8P

Zad. 5.

M, v_0 raketa $m_0 = M$

m, u plin $v_p = u = konst.$

$v_n = ?$

Neka je brzina rakete nakon k-te sekunde $= v_k$

Zakon očuvanja količine gibanja

$$\underbrace{(M-km)}_{\text{masa rakete}} \underbrace{v_k}_{\text{brzina rakete}} = [M-(k+1)m]v_{k+1} + m(\underbrace{-u}_{\text{u suprotnom smjeru}} + v_k)$$

$$(M-km)v_k = (M-km)v_{k+1} - mv_{k+1} - mu + mv_k$$

$$(M-km)(v_{k+1} - v_k) - m(v_{k+1} - v_k) = mu$$

$$(v_{k+1} - v_k)(M - km - m) = mu$$

$$\underbrace{v_{k+1} - v_k}_{\text{promjena brzine rakete u 1 sekundi}} = \frac{mu}{M-km-m} = \frac{mu}{M-(k+1)m}$$

Brzina na kraju n-te sekunde:

$$v_n = v_0 + u\left(\frac{m}{M-m} + \frac{m}{M-2m} + \dots + \frac{m}{M-nm}\right)$$

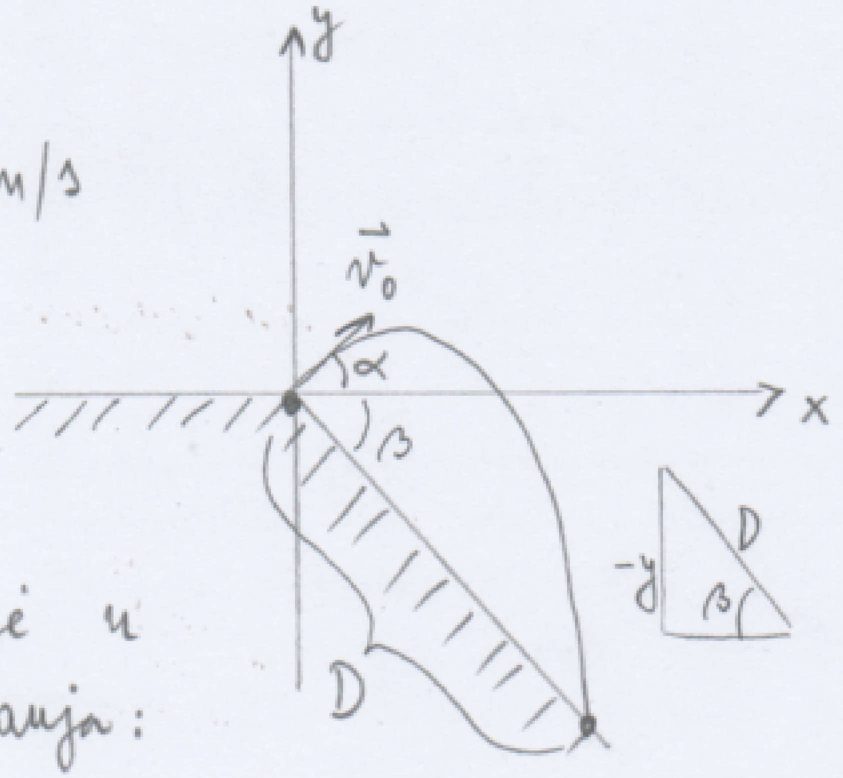
Zad. 6.

$v_0 = 58\ km/h = 58 \cdot \frac{1000}{3600} = 16{,}11\ m/s$

$\alpha = 20^\circ$

$\beta = 50^\circ$

$D = ?$



Ishodište koordinatnog sustava je u točki polijetanja → jednadžbe gibanja:

$$\left.\begin{aligned} x &= v_0 t \cdot \cos\alpha \\ y &= v_0 t \sin\alpha - \frac{g}{2} t^2 \end{aligned}\right\} y = x\,tg\,\alpha - \frac{g x^2}{2 v_0^2 \cos^2\alpha}$$

Ravnina (padina) se nalazi u 4. kvadrantu koordinatnog sustava pa su koordinate doskočišta $(x, -y)$, a jednadžba ravnine na koju doskače skijaš je: $-\frac{y}{x} = tg\beta \Rightarrow y = -x\,tg\beta$

Iz slike se vidi da je $D = \frac{x}{\cos\beta}$

$$-x\,tg\beta = x\,tg\alpha - \frac{g x^2}{2 v_0^2 \cos^2\alpha} \Rightarrow \cancel{x}(tg\alpha + tg\beta) = \frac{g x^{\cancel{2}}}{2 v_0^2 \cos^2\alpha}$$

$$x = \frac{2 v_0^2 \cos^2\alpha}{g}(tg\alpha + tg\beta)$$

$$D = \frac{x}{\cos\beta} = \frac{2 v_0^2 \cos^2\alpha}{g \cos\beta}(tg\alpha + tg\beta)$$

$$D = \frac{2 \cdot 16{,}11^2 \cdot \cos^2 20^\circ}{9{,}81 \cdot \cos 50^\circ}(tg\,20^\circ + tg\,50^\circ)$$

$$D = 113{,}08\ m$$

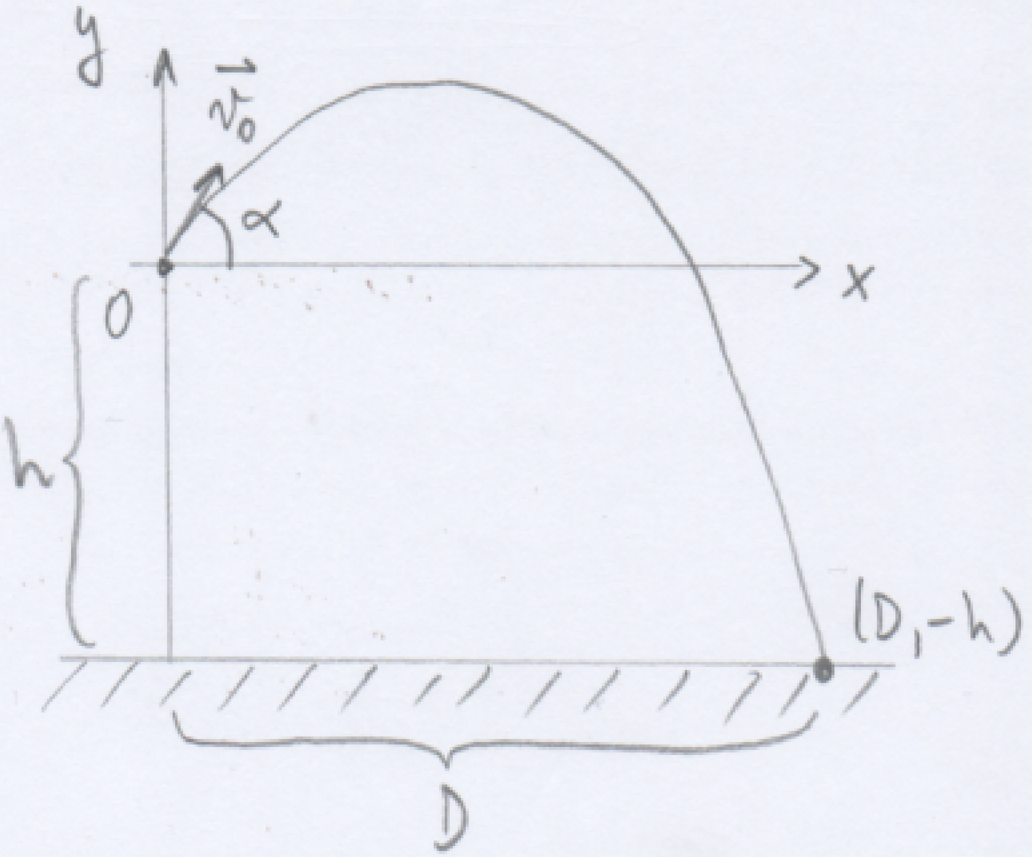
Zad. 7.

$\alpha = 60^\circ$

$h = 11\,m$

$v_0 = 10\,m/s$

$D = ?$



Jednadžba putanje:

$$y = x\,tg\alpha - \frac{g x^2}{2 v_0^2 \cos^2\alpha}$$

$y = -h$, $x = D = ?$

$$\frac{g}{2 v_0^2 \cos^2\alpha} x^2 - tg\alpha\, x - h = 0$$

$$\frac{9{,}81}{2 \cdot 10^2 \cos^2 60^\circ} x^2 - tg\,60^\circ\, x - 11 = 0$$

$$0{,}1962\, x^2 - 1{,}7321 x - 11 = 0$$

$$x_{1,2} = \frac{1{,}7321 \pm \sqrt{1{,}7321^2 + 4 \cdot 10 \cdot 0{,}1962}}{2 \cdot 0{,}1962} = \frac{1{,}7321 \pm 3{,}4107}{0{,}3924}$$

$x_1 = 13{,}11\,m$ $\qquad$ $\cancel{x_2 = -4{,}28\,m}$

$D = 13{,}11\,m$

8P

Zad. 8.

$a_t = 0{,}7\ m/s^2$

$d = 2r = 84\ m \Rightarrow r = 42\ m$

$\mu = 0{,}25$

$t = 0,\ v = 0$

$s = ?$

$F_{tr} = \mu m g$, $F = ma$ $\Big\}$ $ma = \mu m g$, $a = \mu g$

$a = \sqrt{a_r^2 + a_t^2} = \sqrt{\frac{v^4}{r^2} + a_t^2} = \mu g \Big/^2$

$\frac{v^4}{r^2} + a_t^2 = \mu^2 g^2 \Rightarrow$

$v^2 = r\sqrt{\mu^2 g^2 - a_t^2}$

$t = 0,\ v = 0 \Rightarrow a_t = \frac{\Delta v}{\Delta t} = \frac{v}{t} \Rightarrow t = \frac{v}{a_t}$

$s = r\theta = r\frac{\alpha}{2}t^2 = r\frac{a_t}{r}\frac{t^2}{2} = \frac{a_t}{2}t^2 = \frac{a_t}{2}\frac{v^2}{a_t^2} = \frac{v^2}{2a_t}$

$s = \frac{r}{2a_t}\sqrt{\mu^2 g^2 - a_t^2}$

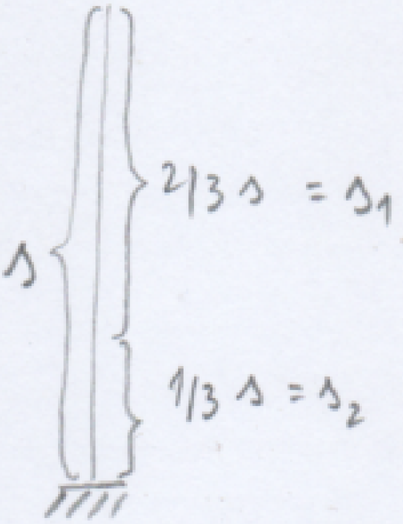
$s = \frac{42}{2 \cdot 0{,}7}\sqrt{0{,}25^2 \cdot 9{,}81^2 - 0{,}7^2} = 70{,}51\ m$

Zad. 9. $v_0 = 4{,}5\ m/s$

$t_p = 2{,}5\ s$

$s_2 = 1/3\ s$

$t = ?$



t = ukupno trajanje leta

t_1 = trajanje leta do posljednje 2,5 s

$t = t_1 + t_p$

Put prevaljen u vremenu t_1:

$$s(t_1) = v_0 t_1 + \frac{g}{2} t_1^2 = s_1 = \frac{2}{3} s$$

Ukupni prevaljeni put u vremenu t:

$$s(t) = v_0 t + \frac{g}{2} t^2 = s$$

$$\frac{2}{3} v_0 (t_1 + t_p) + \frac{\cancel{2}}{3} \frac{g}{\cancel{2}} (t_1 + t_p)^2 = v_0 t_1 + \frac{g}{2} t_1^2$$

$$\frac{2}{3} v_0 t_1 + \frac{2}{3} v_0 t_p + \frac{g}{3} t_1^2 + 2 \frac{g}{3} t_1 t_p + \frac{g}{3} t_p^2 = v_0 t_1 + \frac{g}{2} t_1^2$$

$$t_1^2 \left(\frac{g}{3} - \frac{g}{2}\right) + t_1 \left(\frac{2}{3} v_0 + \frac{2g}{3} t_p - v_0\right) + \frac{2}{3} v_0 t_p + \frac{g}{3} t_p^2 = 0$$

$$t_1^2 \left(-\frac{9{,}81}{6}\right) + t_1 \left(\frac{2}{3} \cdot 4{,}5 + 2 \cdot \frac{9{,}81}{3} \cdot 2{,}5 - 4{,}5\right) + \frac{2}{3} \cdot 4{,}5 \cdot 2{,}5 + \frac{9{,}81}{3} \cdot 2{,}5^2 = 0$$

$$1{,}635\, t_1^2 - 14{,}85\, t_1 - 27{,}94 = 0$$

$$t_1^2 - 9{,}08\, t_1 - 17{,}09 = 0$$

$$t_1 = \frac{9{,}08 \pm \sqrt{9{,}08 + 4 \cdot 17{,}09}}{2} = \frac{9{,}08 \pm 12{,}28}{2}$$

$t_1 = 10{,}68\ s$

~~$t_1 = -1{,}6\ s$~~

$$t = t_1 + t_p = 10{,}68 + 2{,}5 = 13{,}18\ s$$

Zad. 10.

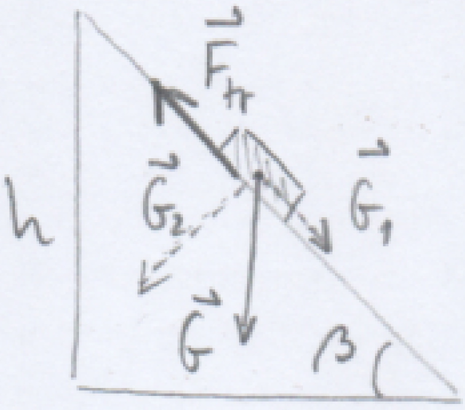
$h = 2{,}5\ \mathrm{s}$

$\alpha = 50^\circ$

$\beta = 45^\circ$

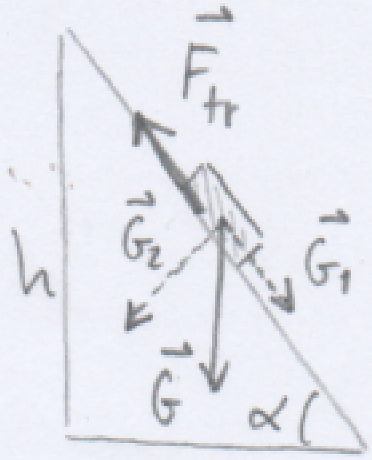
$\mu_k = 0{,}85\,\mu_s$

$t = ?$



$G_2 = G\cos\beta$

$F_{tr} = \mu_s G_2 = \mu_s G\cos\beta = G\sin\beta$



$G_1 - F_{tr} = ma$

$G_1 = G\sin\alpha = mg\sin\alpha$

$F_{tr} = \mu_k G_2 = \mu_k G\cos\alpha = \mu_k mg\cos\alpha$

$mg\sin\alpha - \mu_k mg\cos\alpha = ma$

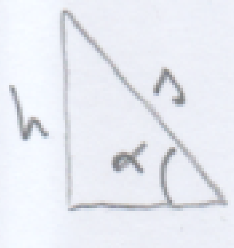
$a = g\sin\alpha - \mu_k g\cos\alpha$

$a = g\sin\alpha - 0{,}85\,\mu_s g\cos\alpha$

$a = g(\sin\alpha - 0{,}85\,\mathrm{tg}\,\beta\cos\alpha)$

Sila trenja je izjednačena s komponentom težine jer tijelo miruje na kosini

$$\mu_s = \frac{G\sin\beta}{G\cos\beta} = \mathrm{tg}\,\beta$$



$$s = \frac{h}{\sin\alpha} = \frac{a}{2}t^2$$

$$t^2 = \frac{2s}{a} = \frac{2h}{a\sin\alpha}$$

$$t = \sqrt{\frac{2h}{g\sin\alpha(\sin\alpha - 0{,}85\,\mathrm{tg}\,\beta\cos\alpha)}} = \sqrt{\frac{2\cdot 2{,}5}{9{,}81\cdot\sin 50^\circ(\sin 50^\circ - 0{,}85\,\mathrm{tg}\,45^\circ\cos 50^\circ)}}$$

$t = 1{,}74\ \mathrm{s}$

sp

Zad. 11.

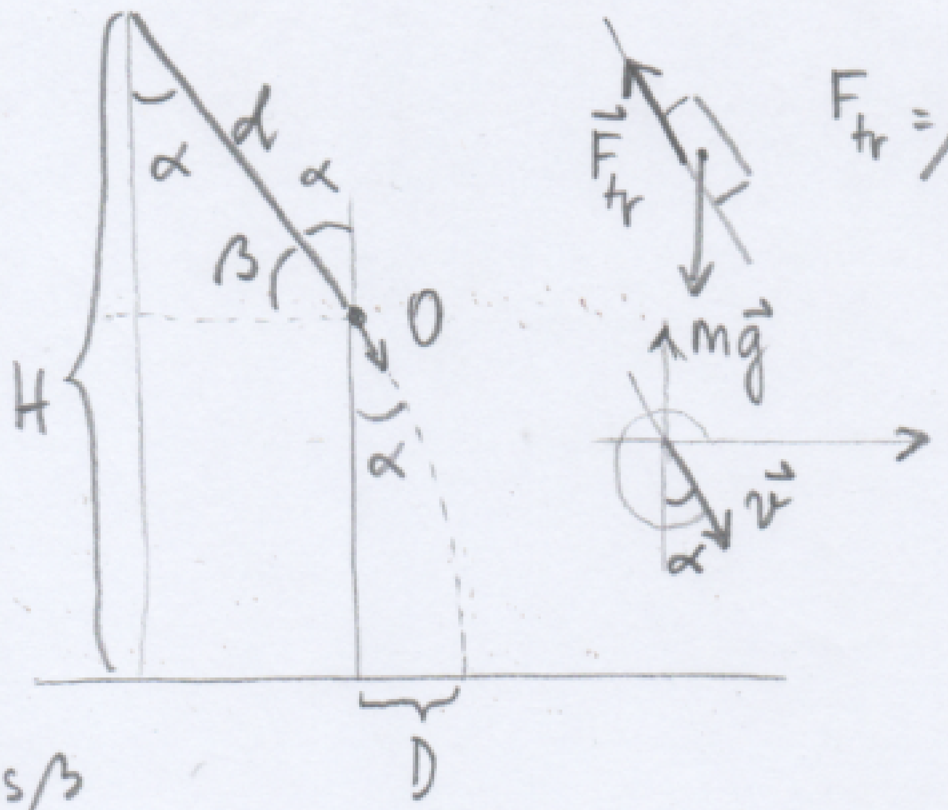
$H = 20\ m$

$d = 8\ m$

$\alpha = 35°$, $\beta = 55°$

$\mu = 0{,}45$

$t, D = ?$



$F_{tr} = \mu mg \cos\beta$

$ma = mg\sin\beta - \mu mg\cos\beta$

$a = g(\sin\beta - \mu\cos\beta) = 9{,}81\,(\sin 55° - 0{,}45\cos 55°) = 5{,}5\ \frac{m}{s^2}$

$d = \frac{a}{2}t^2 \Rightarrow t = \sqrt{\frac{2d}{a}} = \sqrt{\frac{2\cdot 8}{5{,}5}} = 1{,}71\ s$ → vrijeme klizanja niz krov

Kosi hitac: $v_0 = at = 5{,}5 \cdot 1{,}71 = 9{,}405\ m/s$

kut elevacije: $\theta = 305°$

Parametarske jednadžbe: $x = v_0 \cos\alpha\ t$

$y = -\frac{g}{2}t^2 + v_0 \sin\alpha\ t$

$x = D$

$y = -(H - d\sin\beta) = -(H - d\cos\alpha) = -(20 - 8\cdot\cos 35°) = -13{,}45\ m$

↓ jer smo ishodište stavili na rub krova

$-13{,}45 = -\frac{9{,}81}{2}t^2 + 9{,}405\sin 305°\ t \Rightarrow 4{,}905t^2 + 7{,}7t - 13{,}45 = 0$

$t_{1,2} = \frac{-7{,}7 \pm \sqrt{7{,}7^2 + 4\cdot 4{,}905\cdot 13{,}45}}{2\cdot 4{,}905} = \frac{-7{,}7 \pm 18}{9{,}81}$ $\quad t_1 = 1{,}05\ s$ $\quad \cancel{t_2 = -2{,}62\ s}$

Ukupno vrijeme $= t + t_1 = 1{,}71 + 1{,}05 = \underline{\underline{2{,}76\ s}}$

$D = v_0\cos\alpha\ t = 9{,}405\cdot\cos 305°\cdot 1{,}05 = \underline{\underline{5{,}6\ m}}$

SP

Zad. 12.

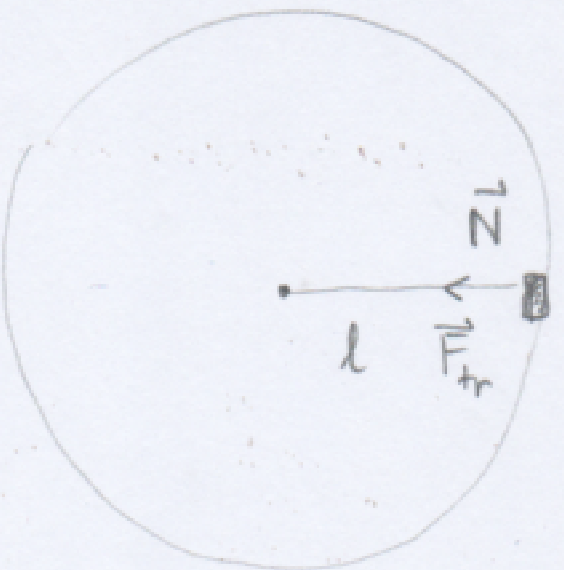
$m_1 = 4\ kg$

$l = 0{,}3\ m$

$m_2 = 10\ kg$

$\mu = 0{,}6$

$\omega = ?$



$\vec{N}$ napetost

$\vec{F}_{tr}$ sila trenja

$$N + F_{tr} = \frac{m_1 v^2}{l} = m_1 \omega^2 l \quad \Rightarrow \quad \omega^2 = \frac{N + F_{tr}}{m_1 l}$$

$$N_{max} = m_2 g$$

$$F_{tr} = \mu m_1 g$$

$$\omega = \sqrt{\frac{N + F_{tr}}{m_1 l}} = \sqrt{\frac{m_2 g + \mu m_1 g}{m_1 l}}$$

$$\omega = \sqrt{\frac{10 \cdot 9{,}81 + 0{,}6 \cdot 4 \cdot 9{,}81}{4 \cdot 0{,}3}} = 10{,}1\ s^{-1}$$

8p

Zad. 13. $a = 12\ m/s^2$

$v_z = 340\ m/s$

$t = 9\ s$

$f_0 = 1600\ Hz$

$f = ?$

z - zvuk , i - izvor

$$\left.\begin{array}{l} s_z = v_z t_z \\ s_i = \frac{a}{2} t_i^2 \end{array}\right\} \Rightarrow \quad s_z = s_i$$

$$v_z t_z = \frac{a}{2} t_i^2$$

$$t = t_i + t_z$$

$$t_z = \frac{a}{2v_z} t_i^2$$

$$t - t_i = \frac{a}{2v_z} t_i^2$$

$$\frac{a}{2v_z} t_i^2 + t_i - t = 0 \quad / \cdot \frac{2v_z}{a}$$

$$t_i^2 + \frac{2v_z}{a} t_i - \frac{2v_z}{a} t = 0$$

$$t_i^2 + \frac{2 \cdot 340}{12} t_i - \frac{2 \cdot 340}{12} \cdot 9 = 0$$

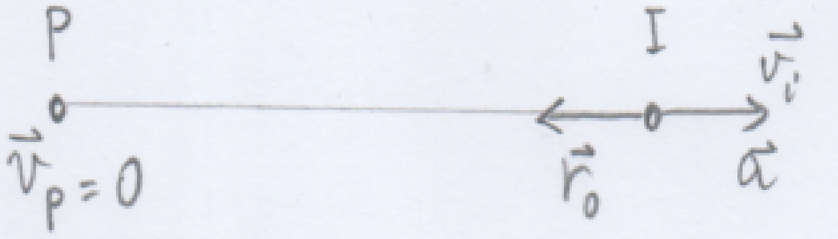
$$t_i^2 + 56{,}67\, t_i - 510 = 0$$

$$t_{i_{1,2}} = \frac{-56{,}67 \pm \sqrt{56{,}67^2 + 4 \cdot 510}}{2} = \frac{-56{,}67 \pm 72{,}47}{2}$$

$t_{i_1} = 7{,}9\ s$

~~$t_{i_2} = -64{,}57\ s$~~

Brzina izvora: $v_i = a t_i = 12 \cdot 7{,}9 = 94{,}8\ m/s$



Doppler:

$$f = f_0 \frac{v_z - \vec{r}_0 \cdot \vec{v}_p}{v_z - \vec{r}_0 \cdot \vec{v}_i}$$

$$f = f_0 \frac{v_z}{v_z + v_i} = 1600 \cdot \frac{340}{340 + 94{,}8} = 1251\ Hz$$

SP

Zad. 14.

$\omega = 25 s^{-1}$

$t = 0$, $v = 0$

$s(t=0) = \dfrac{A}{1{,}02}$

$\delta = ?$

$s(t) = A e^{-\delta t} \sin(\omega t + \varphi) \Big/ \dfrac{d}{dt}$

$v(t) = \dfrac{ds}{dt} = A(-\delta) e^{-\delta t} \sin(\omega t + \varphi) + A e^{-\delta t} \omega \cos(\omega t + \varphi)$

$s(0) = A e^{-\delta \cdot 0} \sin(\omega \cdot 0 + \varphi) = A \cdot 1 \sin\varphi = A \sin\varphi = \dfrac{A}{1{,}02}$

$\sin\varphi = \dfrac{1}{1{,}02} = 0{,}98 \Rightarrow \varphi = 78{,}6^\circ$

$v(0) = -A\delta e^{-\delta \cdot 0} \sin(\omega \cdot 0 + \varphi) + A e^{-\delta \cdot 0} \omega \cos(\omega \cdot 0 + \varphi)$

$= -A\delta \cdot 1 \cdot \sin\varphi + A \cdot 1 \cdot \omega \cos\varphi = 0$

$\cancel{A}\delta \sin\varphi = \cancel{A}\omega \cos\varphi$

$\delta = \omega \dfrac{\cos\varphi}{\sin\varphi} = \omega \operatorname{ctg}\varphi = 25 \cdot \operatorname{ctg} 78{,}6^\circ$

$\delta = 5{,}04 s^{-1}$

SP

Zad. 15.

$t = 9\,s$

E_0

$E_t = 0{,}4 E_0 \quad (40\%)$

$T = 1{,}5\,s$

$\lambda = \delta T = ?$

$$x(t) = A e^{-\delta t} \sin(\omega t + \varphi)$$

$$\left.\begin{aligned} E_0 &= \frac{k x_0^2}{2} \\ E_t &= \frac{k x_t^2}{2} \end{aligned}\right\} \quad \frac{E_t}{E_0} = \frac{\cancel{k} x_t^2 / \cancel{2}}{\cancel{k} x_0^2 / \cancel{2}} = \left(\frac{x_t}{x_0}\right)^2 = 0{,}4 \Rightarrow x_t = \sqrt{0{,}4}\, x_0$$

$$\frac{x_t}{x_0} = e^{-\delta t} = \sqrt{0{,}4} \Rightarrow e^{-9\delta} = \sqrt{0{,}4} \quad / \ln$$

$$-9\delta = \ln \sqrt{0{,}4}$$

$$\delta = \frac{\ln \sqrt{0{,}4}}{-9} = 0{,}051\,s^{-1}$$

$$\lambda = \delta T = 0{,}051 \cdot 1{,}5 = 0{,}076$$

JP

Zad. 16.

$s(t) = 10 e^{-t} \sin(4t + \varphi)$ [cm]

$t = 0,\ v = 0$

$t = 2s,\ v = ?$

$A = 10 cm$

$\delta = 1 s^{-1}$

$\omega = 4 \frac{rad}{s} = 229{,}2 \frac{°}{s}$

$1 rad : x = \pi\ rad : 180°$

$x = 57{,}3°$

$s(t) = A e^{-\delta t} \sin(\omega t + \varphi)$

$v(t) = \frac{ds}{dt} = A \cdot (-\delta) e^{-\delta t} \sin(\omega t + \varphi) + A e^{-\delta t} \omega \cos(\omega t + \varphi)$

$= A e^{-\delta t} (-\delta \sin(\omega t + \varphi) + \omega \cos(\omega t + \varphi))$

$v(0) = 10 \cdot e^{-\delta \cdot 0} (-\delta \sin(\omega \cdot 0 + \varphi) + \omega \cos(\omega \cdot 0 + \varphi))$

$= 10 \cdot 1 (-1 \cdot \sin\varphi + 4 \cos\varphi) = 0 \Rightarrow \sin\varphi = 4 \cos\varphi$

$tg\,\varphi = \frac{\sin\varphi}{\cos\varphi} = 4$

$\varphi = 75{,}96°$

$\varphi = 1{,}33\ rad$

$v(t = 2s) = 10 \cdot e^{-1 \cdot 2} (-1 \sin(229{,}2 \cdot 2 + 75{,}96) + 4 \cos(229{,}2 \cdot 2 + 75{,}96))$

$= 10 \cdot 0{,}135 (-\sin(534{,}36°) + 4 \cos(534{,}36°))$

$= 1{,}35 (-0{,}098 + 4 \cdot (-0{,}995)) = -5{,}5 cm/s$

SP

Zad. 17.

$s_1 = 4\,\text{cm} \sin\left(3s^{-1}t - \frac{x}{7\,\text{cm}}\right)$

čvor $x = 5\,\text{cm}$

$s_1 + s_2 = s$ stojni val

$s_2 = ?$

$$\left.\begin{aligned} s_1 &= 4 \sin\left(3t - \frac{x}{7}\right) \\ s_2 &= 4 \sin\left(3t + \frac{x}{7} + \varphi\right) \end{aligned}\right\} \quad s = \underset{\rightarrow}{s_1} + \underset{\leftarrow}{s_2}$$

$\sin\alpha + \sin\beta = 2 \sin\frac{\alpha+\beta}{2} \cos\frac{\alpha-\beta}{2}$

$$s = 2 \cdot 4 \sin\left(\frac{3t - \cancel{\frac{x}{7}} + 3t + \cancel{\frac{x}{7}} + \varphi}{2}\right) \cos\left(\frac{\cancel{3t} - \frac{x}{7} - \cancel{3t} - \frac{x}{7} - \varphi}{2}\right)$$

$$s = 8 \sin\left(\frac{6t + \varphi}{2}\right) \cos\left(\frac{-\frac{2x}{7} - \varphi}{2}\right) = 8 \sin\left(3t + \frac{\varphi}{2}\right) \cos\left(-\frac{x}{7} - \frac{\varphi}{2}\right)$$

Uvjet za čvor stojnog vala:

$$\cos\left(-\frac{x}{7} - \frac{\varphi}{2}\right) = 0 \qquad \cos(-\alpha) = \cos\alpha$$

$$\cos\left(\frac{x}{7} + \frac{\varphi}{2}\right) = 0 \qquad \cos\left[(2n+1)\frac{\pi}{2}\right] = 0, \quad n = 0, 1, 2 \ldots$$

$$\cos\left(\frac{x}{7} + \frac{\varphi}{2}\right) = \cos\left((2n+1)\frac{\pi}{2}\right) \Rightarrow \frac{x}{7} + \frac{\varphi}{2} = \frac{\pi}{2}$$

$$\frac{\varphi}{2} = \frac{\pi}{2} - \frac{x}{7} \Rightarrow \varphi = \pi - \frac{2x}{7} = \pi - \frac{2 \cdot 5}{7} = \pi - \frac{10}{7} = 1{,}71\ \text{rad}$$

$$s_2 = 4\,\text{cm} \sin\left(3s^{-1}t + \frac{x}{7\,\text{cm}} + 1{,}71\right)$$

SP

Zad. 18.

$r = 4\,cm = 0{,}04\,m$
$m = 1\,kg$
$\eta = 5\,Ns/m^2$
$k = 25\,N/m$
$F(t) = F_0 \cos\omega t$
$F_0 = 1\,N$

$\omega,\ x_{0max} = ?$

Na kuglu djeluju:
- elastična sila $F_e = -kx$
- sila trenja: $F_{tr} = -6\pi\eta r v$
- vanjska sila: $F(t) = F_0 \cos\omega t$

Jednadžba gibanja je:

$$m\frac{d^2x}{dt^2} + 6\pi\eta r\frac{dx}{dt} + kx = F_0\cos\omega t$$

$$\frac{d^2x}{dt^2} + \frac{6\pi\eta r}{m}\frac{dx}{dt} + \frac{k}{m}x = \frac{F_0}{m}\cos\omega t$$

Uz $a = \frac{6\pi\eta r}{m}$, $b = \frac{k}{m} = \omega_0^2$, $c = -\frac{F_0}{m}$ slijedi:

$$\frac{d^2x}{dt^2} + a\frac{dx}{dt} + bx + c\cos\omega t = 0 \Rightarrow \text{rješenje oblika: } x = x_0\cos(\omega t + \varphi)$$

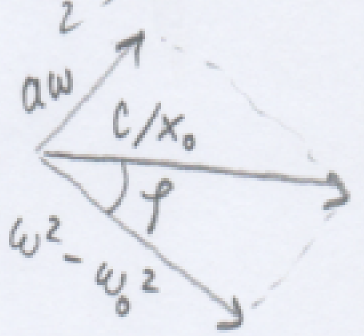
Konstante x_0 i φ možemo izračunati uvrštavanjem tog rješenja u diferencijalnu jednadžbu:

$$\frac{dx}{dt} = -x_0\omega\sin(\omega t + \varphi) \Rightarrow \frac{d^2x}{dt^2} = -x_0\omega^2\cos(\omega t + \varphi)$$

$$-x_0\omega^2\cos(\omega t + \varphi) - a x_0\omega\sin(\omega t + \varphi) + b x_0\cos(\omega t + \varphi) + c\cos\omega t = 0$$

$$-x_0\omega^2\cos(\omega t + \varphi) + x_0\omega_0^2\cos(\omega t + \varphi) - x_0 a\omega\cos(\omega t + \varphi + \tfrac{\pi}{2}) = -c\cos\omega t$$

$$(\omega^2 - \omega_0^2)\cos(\omega t + \varphi) + a\omega\cos(\omega t + \varphi + \tfrac{\pi}{2}) = \frac{c}{x_0}\cos\omega t$$



$$x_0 = \frac{c}{\sqrt{(\omega^2 - \omega_0^2)^2 + a^2\omega^2}} = -\frac{F_0}{m}\left((\omega^2 - \omega_0^2)^2 + a^2\omega^2\right)^{-1/2}$$

Položaj maksimuma: $\frac{dx_0}{d\omega} = 0 \Rightarrow -\frac{1}{2}(\quad)^{-3/2}\left[2(\omega^2 - \omega_0^2)\cdot 2\omega + a^2\cdot 2\omega\right] = 0$

$$4\omega(\omega^2 - \omega_0^2) + 2\omega a^2 = 0 \Rightarrow 2(\omega^2 - \omega_0^2) + a^2 = 0 \Rightarrow \omega^2 = \omega_0^2 - \frac{a^2}{2}$$

$$\omega = \sqrt{\omega_0^2 - \frac{a^2}{2}} = \sqrt{\left(\frac{k}{m}\right) - \left(\frac{6\pi\eta r}{m}\right)^2\cdot\frac{1}{2}} = \sqrt{\left(\frac{25}{1}\right) - \frac{1}{2}\left(\frac{6\pi\cdot 5\cdot 0{,}04}{1}\right)^2} = \underline{\underline{4{,}2\ rad/s}}$$

$$x_{0max} = \frac{F_0}{m}\left[(\omega^2 - \omega_0^2)^2 + \left(\frac{6\pi\eta r}{m}\right)^2\omega^2\right]^{-1/2} = \frac{1}{1}\left[\left(4{,}2^2 - \frac{25}{1}\right)^2 + \left(\frac{6\pi\cdot 5\cdot 0{,}04}{1}\right)^2\cdot 4{,}2^2\right]^{-1/2} = \underline{\underline{0{,}057\,m}}$$

Zad. 19.

$A = 0{,}05\ m$

$T = 2\ s$

$\lambda = 5\ m$

$\varphi = \frac{\pi}{10}$

$\pm$ x-os smjer širenja

$v_{max}, a_{max} = ?$

$s(t) = A \sin(\omega t - kx + \varphi)$ +x-os

$s(t) = A \sin(\omega t + kx + \varphi)$ −x-os

$\omega = \frac{2\pi}{T}$ $k = \frac{2\pi}{\lambda}$

$\omega = \frac{2\pi}{2} = \pi s^{-1}$, $k = \frac{2\pi}{5} = 0{,}4\pi\ m^{-1}$

+x-os: $s(t) = 0{,}05 \sin\left(\pi t - 0{,}4\pi x + \frac{\pi}{10}\right)$

−x-os: $s(t) = 0{,}05 \sin\left(\pi t + 0{,}4\pi x + \frac{\pi}{10}\right)$

$v(t) = \frac{ds}{dt} = A\omega \cos(\omega t - kx + \varphi)$

$a(t) = \frac{dv}{dt} = \frac{d^2 s}{dt^2} = -A\omega^2 \sin(\omega t - kx + \varphi)$

$v = v_{max}$ kad je $\cos(\omega t - kx + \varphi) = max = 1$

$v_{max} = A\omega = A\frac{2\pi}{T} = 0{,}05 \cdot \frac{2\pi}{2} = 0{,}157\ m/s$

$a = a_{max}$ kad je $\sin(\omega t - kx + \varphi) = max = 1$

$a_{max} = A\omega^2 = A \cdot \frac{4\pi^2}{T^2} = 0{,}05 \cdot \frac{4 \cdot \pi^2}{4} = 0{,}493\ m/s^2$

SP

Zad. 20.

$f = 50\,Hz$

$v = 300\,m/s$

$x_1 = 2m$

$x_2 = 8m$

$\Delta\varphi = ?$

$v = \lambda f$

$\lambda = \frac{v}{f} = \frac{300}{50} = 6m$

Razlika u fazi titranja čestice udaljene od izvora vala za x i titranja čestice u izvoru vala je

$\Delta\varphi = \omega\Delta t = kx = \frac{2\pi}{\lambda}x$

Ako je izvor u ishodištu, $x_0 = 0$

$\varphi_1 = \frac{2\pi}{\lambda}x_1 = \frac{2\pi}{6}\cdot 2 = \frac{4\pi}{6}$

$\varphi_2 = \frac{2\pi}{\lambda}x_2 = \frac{2\pi}{6}\cdot 8 = \frac{16\pi}{6}$

$\Delta\varphi = \varphi_2 - \varphi_1 = \frac{16\pi}{6} - \frac{4\pi}{6} = \frac{12\pi}{6} = 2\pi$

Zad. 21. F

$4\mu_1 = \mu_2$

$s_u = A_u \sin(\omega t - k_1 x)$ upadni val

v_1

refleksija i transmisija

rezultanta = ?

$v = \sqrt{\frac{F}{\mu}}$ $\quad v = \frac{\omega}{k} \Rightarrow k = \frac{\omega}{v}$

$v_1 = \sqrt{\frac{F}{\mu_1}}$ $\quad k_1 = \omega\sqrt{\frac{\mu_1}{F}}$

$v_2 = \sqrt{\frac{F}{\mu_2}}$ $\quad k_2 = \omega\sqrt{\frac{\mu_2}{F}}$

Promjena gustoće u $x = 0$

Reflektirani val $s_r = A_r \sin(\omega t + k_1 x)$

Transmitirani val $s_t = A_t \sin(\omega t - k_2 x)$, brzina v_2

Rubni uvjeti u $x = 0$:

$s_u(x=0,t) + s_r(x=0,t) = s_t(x=0,t)$ $\quad A_u + A_r = A_t$

$\frac{\partial}{\partial x}(s_u + s_r) = \frac{\partial}{\partial x} s_t$ $\quad k_1(A_u - A_r) = k_2 A_t$

$A_r = \frac{v_2 - v_1}{v_1 + v_2} A_u$, $A_t = \frac{2v_2}{v_1 + v_2} A_u$ $\quad \frac{A_u}{v_1} - \frac{A_r}{v_1} = \frac{A_t}{v_2}$

$$k = \frac{\omega}{v} = \omega\sqrt{\frac{\mu}{F}} \Rightarrow \frac{k_1}{k_2} = \sqrt{\frac{\mu_1}{\mu_2}} = \sqrt{\frac{\cancel{\mu_1}}{4\cancel{\mu_1}}} = \frac{1}{2} \Rightarrow k_2 = 2k_1$$

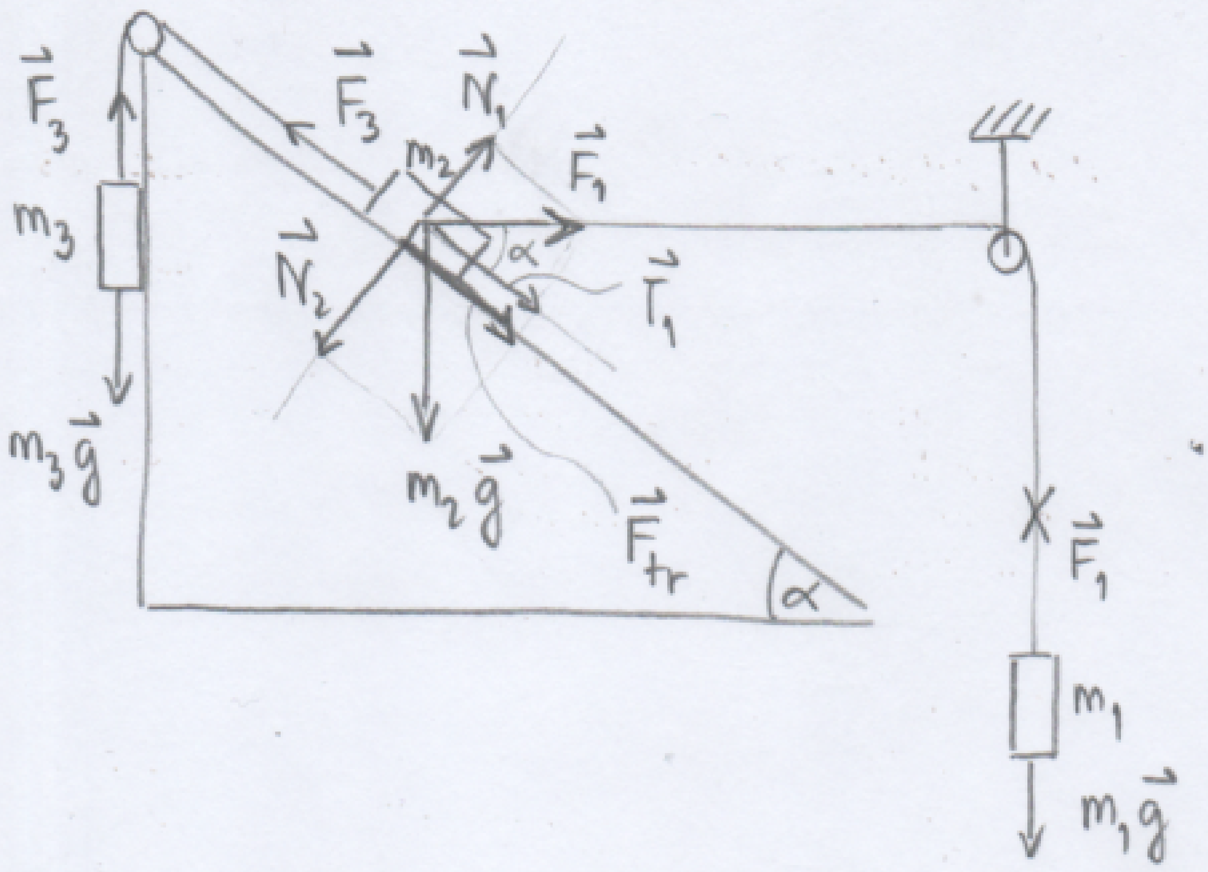
$$A_r = \frac{v_2 - v_1}{v_1 + v_2} A_u = \frac{\frac{\omega}{k_2} - \frac{\omega}{k_1}}{\frac{\omega}{k_1} + \frac{\omega}{k_2}} A_u = \frac{k_1 - k_2}{k_1 + k_2} A_u = \frac{k_1 - 2k_1}{k_1 + 2k_1} A_u = -\frac{1}{3} A_u$$

$$A_t = \frac{2v_2}{v_1 + v_2} A_u = \frac{2\frac{\omega}{k_2} A_u}{\frac{\omega}{k_1} + \frac{\omega}{k_2}} = \frac{2k_1 A_u}{k_1 + k_2} = \frac{2k_1 A_u}{k_1 + 2k_1} = \frac{2}{3} A_u$$

$s_t = \frac{2}{3} A_u \sin(\omega t - 2k_1 x)$ kroz desnu stranu

$$s_u + s_r = A_u \sin(\omega t - k_1 x) + A_r \sin(\omega t + k_1 x) = A_u \sin(\omega t - k_1 x) +$$
$$+ A_u \sin(\omega t + k_1 x) + (A_r - A_u)\sin(\omega t + k_1 x)$$
$$= 2A_u \cos k_1 x \sin \omega t + (A_r - A_u)\sin(\omega t + k_1 x)$$
$$= 2A_u \cos k_1 x \sin \omega t - \frac{4}{3} A_u \sin(\omega t + k_1 x)$$ kroz lijevu stranu

Zad. 22.



Kad bi sustav mirovao, zbroj sila bi morao biti jednak 0 :

$$F_3 - F_{tr} - F_1 \cos\alpha - m_2 g \sin\alpha = 0$$

Sila trenja je : $F_{tr} = \mu\,(N_2 - N_1) = \mu\,(m_2 g \cos\alpha - F_1 \sin\alpha)$

Kako je $F_1 = m_1 g$ i $F_3 = m_3 g$, slijedi :

$$m_3 \cancel{g} - \mu\, m_2 \cancel{g} \cos\alpha + \mu\, m_1 \cancel{g} \sin\alpha - m_1 \cancel{g} \cos\alpha - m_2 \cancel{g} \sin\alpha = 0$$

$$m_3 = m_1 (\cos\alpha - \mu \sin\alpha) + m_2 (\sin\alpha + \mu \cos\alpha)$$

sp