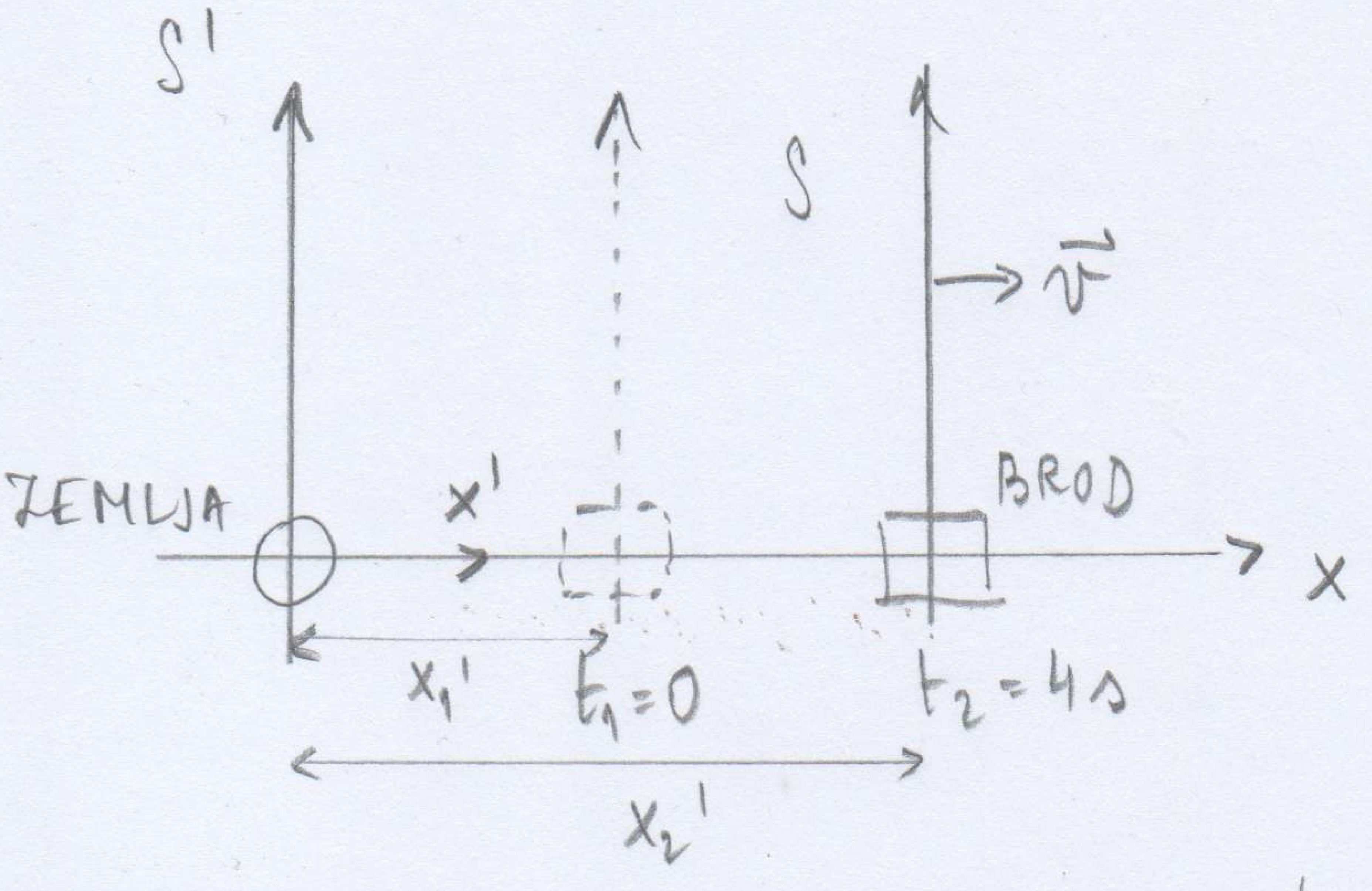
Zad. 1.

$v = 0{,}865\,c$

$t_1 = 0$

$t_2 = 4\,s$

$\Delta t' = ?$



$\left.\begin{array}{l} \text{U } t = 0 \\ \quad t' = 0 \end{array}\right\}$ Sustavi S i S' se poklapaju, gibanje broda je u smjeru +x-osi

Brod je u ishodištu sustava S pa stalno vrijedi $x = 0$.

Lorentzove transformacije: $x' = \dfrac{x + vt}{\sqrt{1-\beta^2}}$, $t' = \dfrac{t + \frac{vx}{c^2}}{\sqrt{1-\beta^2}}$, $\beta = \dfrac{v}{c}$

U trenutku odašiljanja 1. signala $t_1 = 0 \Rightarrow$ iz Lorentz. transf.

$x_1' = 0$, $t_1' = 0$

U trenutku odašiljanja 2. signala $t_2 = 4\,s \Rightarrow$ iz Lorentz. transf.

$x_2' = \dfrac{v t_2}{\sqrt{1-\beta^2}}$, $t_2' = \dfrac{t_2}{\sqrt{1-\beta^2}}$

Vremenski interval: $\Delta t' = \underbrace{t_2' - t_1'}_{\text{vremenski interval između trenutaka odašiljanja u sustavu Zemlje}} + \underbrace{\dfrac{x_2' - x_1'}{c}}_{\text{vrijeme putovanja signala zbog pomaka broda}}$

$$\Delta t' = \frac{t_2}{\sqrt{1-\beta^2}} + \frac{v t_2}{c\sqrt{1-\beta^2}} = \frac{1}{\sqrt{1-\beta^2}}\, t_2\left(1 + \frac{v}{c}\right)$$

$$= \frac{1}{\sqrt{1 - (0{,}865c/c)^2}} \cdot 4\left(1 + \frac{0{,}865c}{c}\right) = \underline{\underline{14{,}87\,s}}$$

8p

Zad. 2.

$V = 0{,}9c$ relativna brzina

u smjeru $+x$ - osi

u $t=0$ poklapanje sustava S i S'

$v_x' = 0{,}8c$

$v_y' = 0{,}5c$

$v_z' = 0$

a) iznos brzine u sustavu S'

$$v' = \sqrt{v_x'^2 + v_y'^2 + v_z'^2} = \sqrt{(0{,}8c)^2 + (0{,}5c)^2 + 0^2} = 0{,}94c$$

b) komponente brzine i iznos u sustavu S

Lorentzove transformacije:

$$v_x = \frac{v_x' + V}{1 + \frac{v_x' V}{c^2}} = \frac{0{,}8c + 0{,}9c}{1 + \frac{0{,}8c \cdot 0{,}9c}{c^2}} = 0{,}988c$$

$$v_y = \frac{v_y'}{1 + \frac{v_x' V}{c^2}} \sqrt{1 - \frac{V^2}{c^2}} = \frac{0{,}5c}{1 + \frac{0{,}8c \cdot 0{,}9c}{c^2}} \sqrt{1 - \left(\frac{0{,}9c}{c}\right)^2} = 0{,}127c$$

$v_z = 0$

$$v^2 = v_x^2 + v_y^2 + v_z^2 \Rightarrow v = \sqrt{v_x^2 + v_y^2 + v_z^2} = \sqrt{(0{,}988c)^2 + (0{,}127c)^2}$$

$$= 0{,}996c$$

c) $\operatorname{tg} \Theta' = \frac{v_y'}{v_x'} = 0{,}625 \Rightarrow \Theta' = 32{,}01°$

$\operatorname{tg} \Theta = \frac{v_y}{v_x} = 0{,}128 \Rightarrow \Theta = 7{,}31°$

SP

Zad. 3.

$E_x = 0$ $\qquad \mu_0 = 1{,}257 \cdot 10^{-6}\ H/m^2$

$E_y = 0$

$E_z = 0{,}3\ \frac{V}{m} \sin\left(2\pi \cdot 10^{14} s^{-1}\left(t - \frac{x}{3 \cdot 10^8 m/s}\right)\right) \Rightarrow \vec{v} = c\,\vec{i}$

$c = 3 \cdot 10^8 m/s$

$\vec{S} = ?$ $\qquad \bar{S} = ?$

$\vec{S} = \frac{1}{\mu_0}(\vec{E} \times \vec{B})$ $\qquad \vec{E} = \vec{B} \times \vec{v} \Rightarrow \vec{B} = \frac{1}{v^2}(\vec{v} \times \vec{E})$

$$\vec{B} = \frac{1}{c^2}\begin{vmatrix} \vec{i}^{\,+} & \vec{j}^{\,-} & \vec{k}^{\,+} \\ c & 0 & 0 \\ 0 & 0 & E_z \end{vmatrix} = \frac{1}{c^2}(-\vec{j})\, c \cdot E_z = -\frac{E_z}{c}\vec{j}$$

$B_x = 0, \quad B_y = -\frac{E_z}{c}, \quad B_z = 0$

$$\vec{S} = \frac{1}{\mu_0}(\vec{E} \times \vec{B}) = \frac{1}{\mu_0}\begin{vmatrix} \vec{i}^{\,+} & \vec{j}^{\,-} & \vec{k}^{\,+} \\ 0 & 0 & E_z \\ 0 & -\frac{E_z}{c} & 0 \end{vmatrix} = \frac{1}{\mu_0}\vec{i}\,\frac{E_z^2}{c}$$

$$\vec{S} = \frac{0{,}3^2}{1{,}257 \cdot 10^{-6} \cdot 3 \cdot 10^8}\ \frac{W}{m^2} \sin^2\left(2\pi \cdot 10^{14} s^{-1}\left(t - \frac{x}{3 \cdot 10^8 m/s}\right)\right)\vec{i}$$

$$\vec{S} = 2{,}39 \cdot 10^{-4}\ \frac{W}{m^2} \sin^2\left(2\pi \cdot 10^{14} s^{-1}\left(t - \frac{x}{3 \cdot 10^8 m/s}\right)\right)\vec{i}$$

$$\bar{S} = \frac{1}{2} \cdot 2{,}39 \cdot 10^{-4}\ \frac{W}{m^2} = 1{,}19 \cdot 10^{-4}\ \frac{W}{m^2}$$

SP

Zad. 4.

$f = 1{,}5 \cdot 10^{14}\ Hz$

$\vec{E} = E_0 \vec{i} \sin(\omega t - kz)$ $\quad \vec{v} = c\vec{k}$

jedi. vektor u smjeru z-osi

$E_0 = 60\ V/m$

$\mu_0 = 1{,}257 \cdot 10^{-6}\ H/m$

$c = 3 \cdot 10^8\ m/s$

$\vec{B} = ? \qquad \bar{S} = ?$

$$\vec{B} = \frac{1}{v^2}(\vec{v} \times \vec{E}) = \frac{1}{c^2}(\vec{v} \times \vec{E}) = \frac{1}{c^2}\begin{vmatrix} \overset{+}{\vec{i}} & \overset{-}{\vec{j}} & \overset{+}{\vec{k}} \\ 0 & 0 & c \\ E_0 & 0 & 0 \end{vmatrix} = \frac{1}{c^2} c E_0 = \frac{E_0}{c}\vec{j}$$

$$\vec{B} = \frac{E_0}{c}\vec{j}\sin(\omega t - kz) = \frac{60}{3 \cdot 10^8}\vec{j}\sin(\omega t - kz)$$

$$= 2 \cdot 10^{-7}\vec{j}\sin(\omega t - kz)$$

$$\vec{B} = 2 \cdot 10^{-7}\ T\ \vec{j}\sin\left(2\pi \cdot 1{,}5 \cdot 10^{14}\ s^{-1}\left(t - \frac{z}{3 \cdot 10^8\ m/s}\right)\right)$$

$$\bar{S} = \frac{E_0 B_0}{2\mu_0} = \frac{60 \cdot 2 \cdot 10^{-7}}{2 \cdot 1{,}257 \cdot 10^{-6}} = 4{,}77\ W/m^2$$

SP

Zad. 5.

$$\vec{B} = 3 \cdot 10^{-9}\,\mathrm{T}\,\vec{k}\,\sin\left(\pi \cdot 10^{15}\,\mathrm{s}^{-1}\left(t - \frac{x}{c}\right)\right)$$

$\mu_0 = 1{,}257 \cdot 10^{-6}\ \mathrm{H/m}$

$c = 3 \cdot 10^{8}\ \mathrm{m/s}$

$\vec{v} = c\,\vec{i}$

jed. vektor u smjeru x-osi

$\vec{S} = ?$

$$\vec{S} = \frac{1}{\mu_0}\left(\vec{E} \times \vec{B}\right) \qquad \vec{E} = \vec{B} \times \vec{v}$$

$$\vec{E} = \vec{B} \times \vec{v} = \begin{vmatrix} \vec{i}^{\,+} & \vec{j}^{\,-} & \vec{k}^{\,+} \\ 0 & 0 & B_z \\ c & 0 & 0 \end{vmatrix} = B_z c\,\vec{j}$$

$$\vec{S} = \frac{1}{\mu_0}\left(\vec{E} \times \vec{B}\right) = \frac{1}{\mu_0}\begin{vmatrix} \vec{i}^{\,+} & \vec{j}^{\,-} & \vec{k}^{\,+} \\ 0 & B_z c & 0 \\ 0 & 0 & B_z \end{vmatrix} = \frac{1}{\mu_0} B_z^2 c\,\vec{i}$$

$$\vec{S} = \frac{1}{1{,}257 \cdot 10^{-6}}\left(3 \cdot 10^{-9}\right)^2 \cdot 3 \cdot 10^{8}\,\vec{i}\,\sin^2\left(\pi \cdot 10^{15}\,\mathrm{s}^{-1}\left(t - \frac{x}{3 \cdot 10^{8}}\right)\right)\vec{i}$$

$$\vec{S} = 2{,}15 \cdot 10^{-3}\,\frac{\mathrm{W}}{\mathrm{m}^2}\sin^2\left(\pi \cdot 10^{15}\,\mathrm{s}^{-1}\left(t - \frac{x}{3 \cdot 10^{8}}\right)\right)\vec{i}$$

JP

Zad. 6.

ravnina $y = 0 \Rightarrow xz$ ravnina

$S = 0{,}5\ m^2$ površina

$\vec{S} = S\vec{j}$ - vektor u smjeru normale na xz ravninu

$\vec{j}$ → jed. vektor u smjeru y-osi

$B_x = B_y = 0{,}06\,T \cos(10^3 s^{-1} t)$

$B_z = 0$

$U_{ind} = ?$

$U_{ind} = \dfrac{d\phi}{dt}$ $\qquad \phi = \vec{B} \cdot \vec{S} = B_y S$

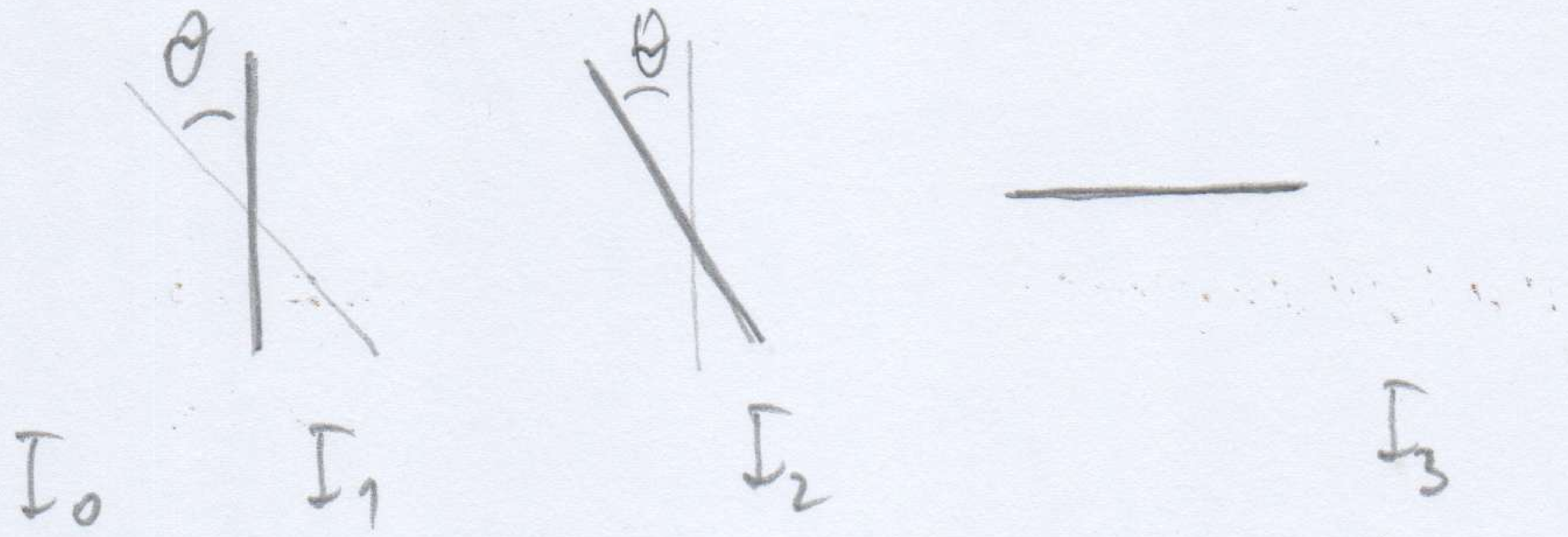
$U_{ind} = \dfrac{d}{dt}\left(0{,}06 \cdot 0{,}5 \cos(10^3 t)\right)$

$= 0{,}06 \cdot 0{,}5 \cdot 10^3 \sin(10^3 t)$

$U_{ind} = 30\,V \sin(10^3 s^{-1} t)$

SP

Zad. 7.



I_0

$I_1 = I_0 \cos^2\theta$

$I_2 = I_1 \cos^2\theta$

$$I_3 = I_2 \cos^2(90^\circ - \theta) = I_2 \sin^2\theta = I_1 \cos^2\theta \sin^2\theta =$$

$$= I_0 \cos^2\theta \cos^2\theta \sin^2\theta = I_0 \cos^4\theta (1 - \cos^2\theta)$$

$$= I_0 (\cos^4\theta - \cos^6\theta)$$

Maksimum nalazimo iz derivacije $\frac{dI_3}{d\theta} = 0$

$$4\cos^3\theta \cdot \cancel{(-\sin\theta)} - 6\cos^5\theta \cdot \cancel{(-\sin\theta)} = 0$$

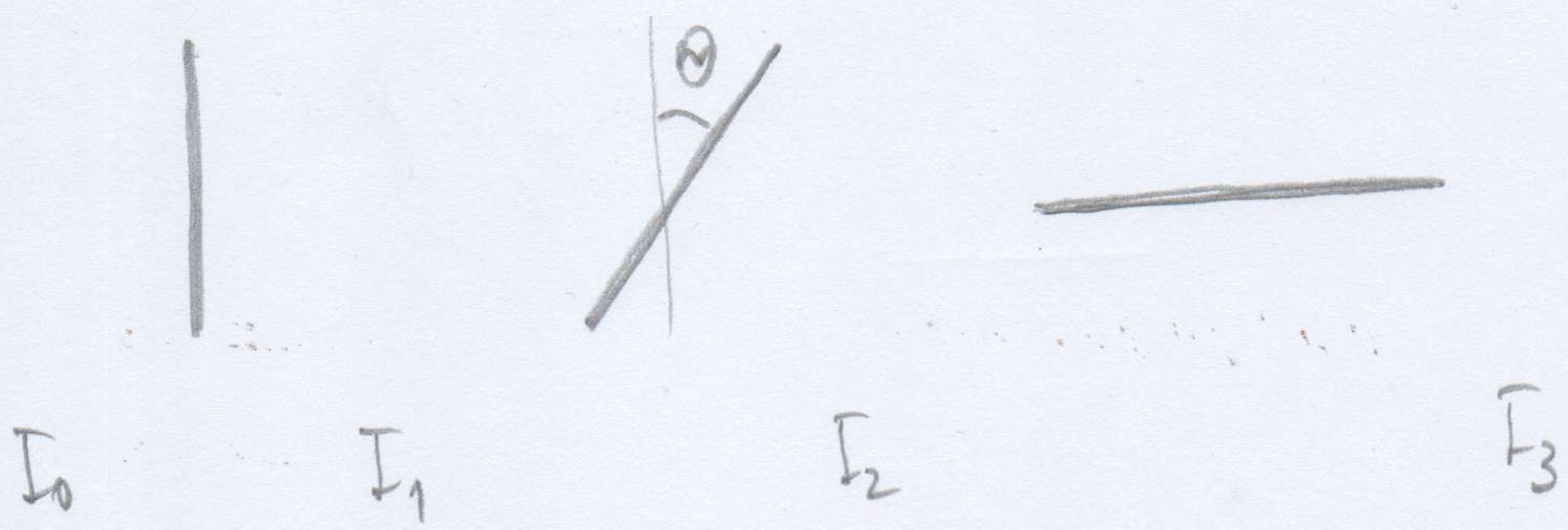
$$4\cos^3\theta = 6\cos^5\theta \quad / : 2\cos^3\theta$$

$$2 = 3\cos^2\theta$$

$$\cos^2\theta = \frac{2}{3} \Rightarrow \cos\theta = \sqrt{\frac{2}{3}} \Rightarrow \theta = 35{,}26^\circ$$

SP

Zad. 8. $\theta = 15^\circ$



nepolarizirano $\left(\frac{1}{2}\right)$

$$I_1 = \frac{I_0}{2}$$

$$I_2 = I_1 \cos^2\theta = \frac{I_0}{2}\cos^2\theta$$

$$I_3 = I_2 \cos^2(90^\circ - \theta) = I_2 \sin^2\theta = I_1 \cos^2\theta \sin^2\theta$$

$$= \frac{1}{2} I_0 (\cos\theta \sin\theta)^2 = \frac{1}{2} I_0 \left(\frac{\sin 2\theta}{2}\right)^2 = \frac{1}{2} I_0 \frac{\sin^2 2\theta}{4}$$

$$= \frac{1}{8} I_0 \sin^2 2\theta$$

$$I_3 = \frac{1}{8} I_0 \sin^2 2\cdot 15^\circ = 0{,}03125\, I_0$$

fp

Zad. 9.

$l = 1\,m$

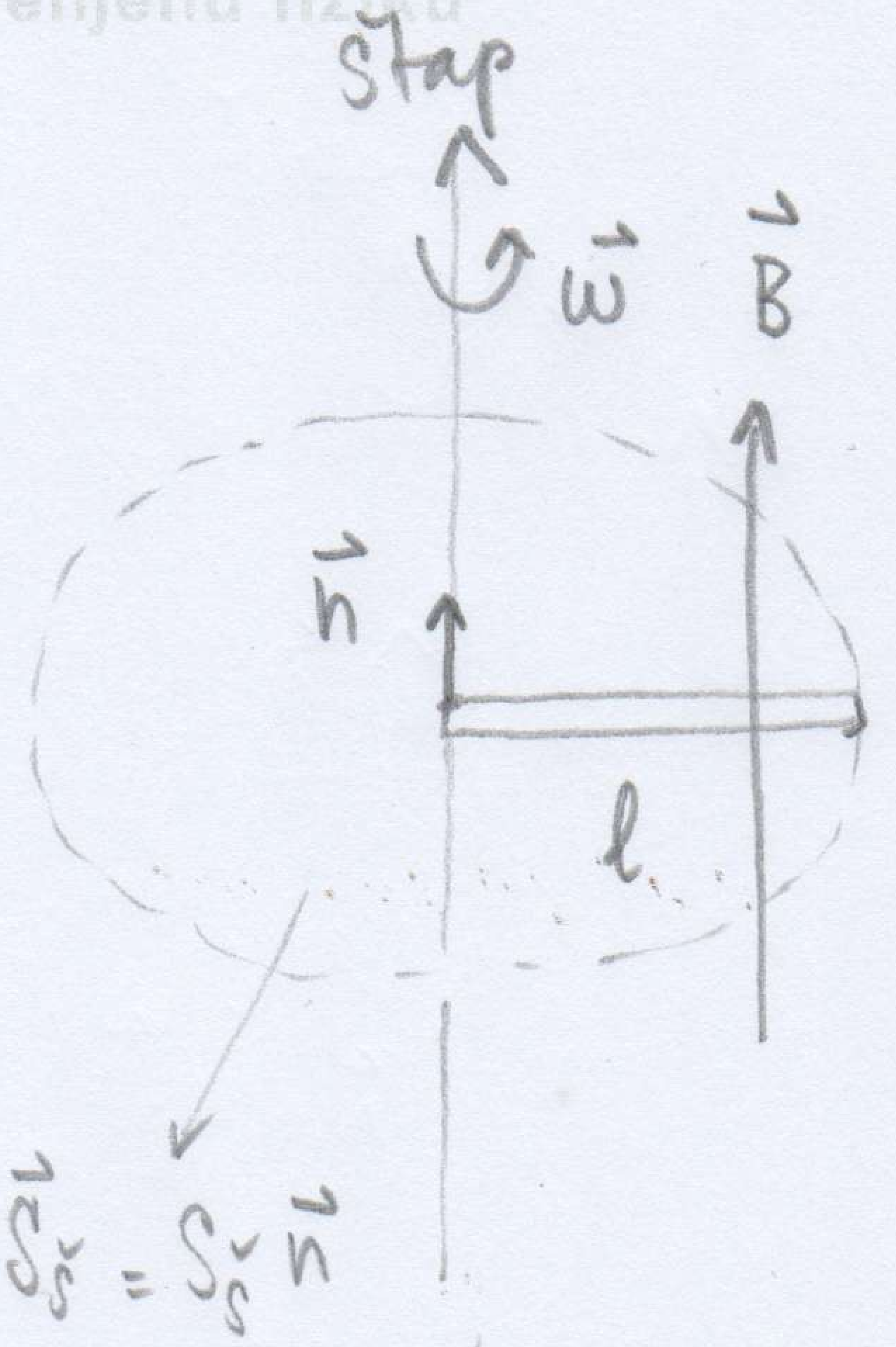
$\omega = 2\pi f$

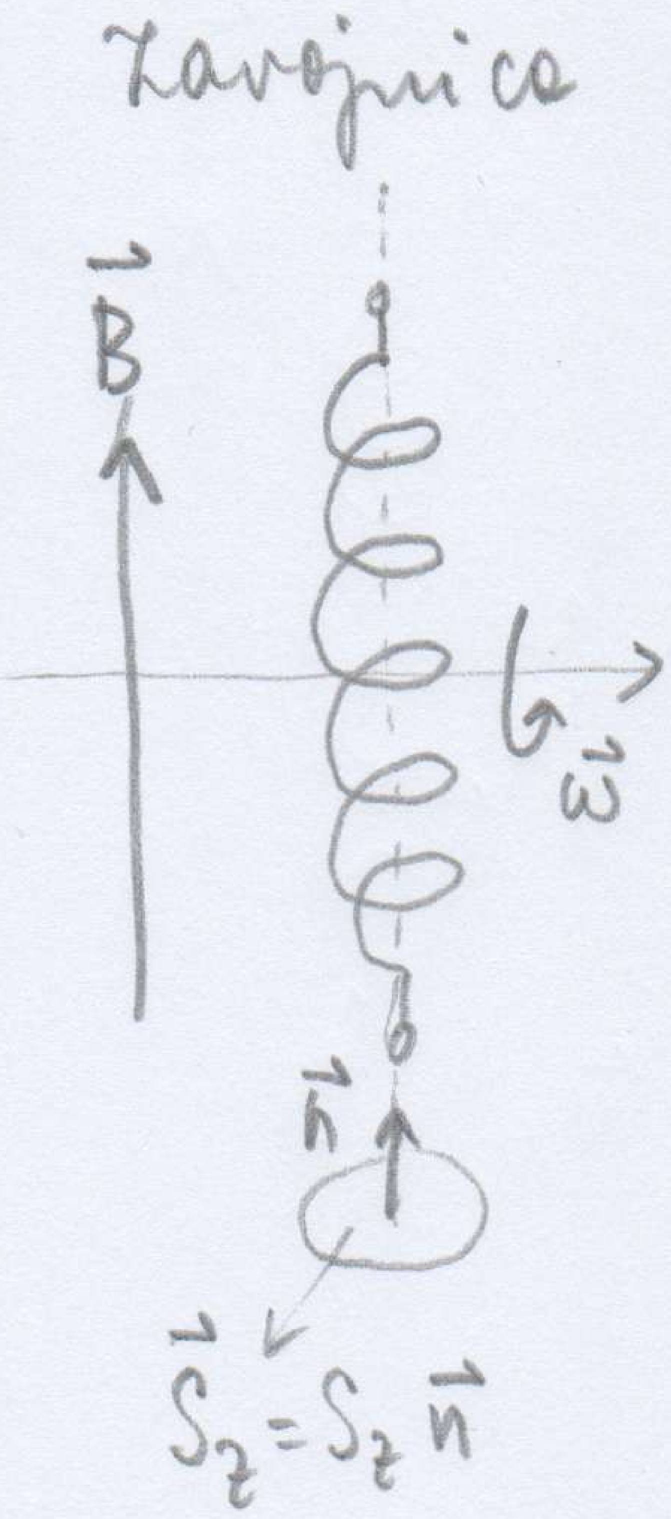
B

$N = 100$ zavoja

$S_z = 100\,cm^2 = 100 \cdot 10^{-4}\,m^2$

$U_{ind\check{s}} / U_{ind z} = ?$





Magnetski tok koji štap obuhvaća svojom vrtnjom:

$$\phi_{\check{s}} = \vec{B} \cdot \vec{S}_{\check{s}} = B \cdot l^2 \pi \cos 0^\circ = B l^2 \pi \qquad \measuredangle(\vec{B}, \vec{S}_{\check{s}}) = 0^\circ$$

Inducirani napon na krajevima vodiča:

$$U_{ind\check{s}} = \frac{d\phi_{\check{s}}}{dt} = B l^2 \pi f = B l^2 \pi \frac{\omega}{2\pi}$$

Najveći napon induciran na krajevima zavojnice:

$$U_{ind z} = \phi_z \frac{d\varphi}{dt} = \phi_z \omega = B S_z N \omega$$

Omjer induciranih napona:

$$\frac{U_{ind\check{s}}}{U_{ind z}} = \frac{\cancel{B} l^2 \cancel{\pi} \frac{\cancel{\omega}}{2\cancel{\pi}}}{\cancel{B} S_z N \cancel{\omega}} = \frac{l^2}{2 S_z N} = \frac{1^2}{2 \cdot 100 \cdot 10^{-4} \cdot 100} = \underline{\underline{\frac{1}{2}}}$$

SP