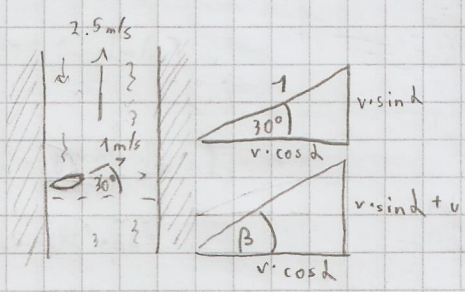
1. auditorna vježba

z1) $v = 1\ m/s$

$v = 2.5\ m/s$

$\alpha = 30°$

$\beta = ?$



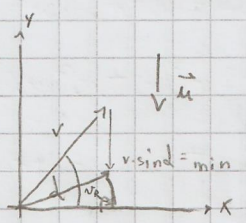
$$\beta = \operatorname{arctg}\left(\frac{v\cdot\sin\alpha + v}{v\cdot\cos\alpha}\right)$$

$$= \operatorname{arctg}\left(\frac{1\cdot\sin 30° + 2.5}{1\cdot\cos 30°}\right) = 73°53'52.39''$$

z2) $v = 2v \quad [m/s]$

$\alpha = ?$

$\frac{d\beta}{d\alpha} = 0 \quad \alpha_{min}$



$$\vec{v} = v\cdot\cos\alpha\,\vec{i} + v\sin\alpha\,\vec{j}$$

$$\vec{v} = -v\vec{j} = -2v\,\vec{j}$$

$$v_R = \vec{v} + \vec{v}$$

$$= v\cdot\cos\alpha\,\vec{i} + (v\cdot\sin\alpha - 2v)\,\vec{j}$$

$$\vec{v_R} = v_R\cdot\cos\beta\,\vec{i} + v_R\cdot\sin\beta\,\vec{j}$$

$$\operatorname{tg}\beta = \frac{v_{Ry}}{v_{Rx}}$$

$$= \frac{v\sin\alpha - 2v}{v\cdot\cos\alpha}$$

$$\beta = \operatorname{arctg}\left(\frac{\sin\alpha - 2}{\cos\alpha}\right) \quad /'$$

$$\beta'(\alpha) = \frac{1}{1 + \left(\frac{\sin\alpha - 2}{\cos\alpha}\right)^2} \cdot \frac{\cos\alpha\cdot\cos\alpha + (\sin\alpha - 2)(\sin\alpha)}{\cos^2\alpha} = 0$$

$$= (\ >0\ ) \cdot \frac{1 - 2\sin\alpha}{\cos^2\alpha} = 0$$

$$1 - 2\sin\alpha = 0$$

$$\sin\alpha = \frac{1}{2}$$

$$\alpha = 30°$$

z3) $r(t) = 2t\,\vec{i} + \sin\left(\frac{4\pi t}{5}\right)\vec{j} \qquad v_{max},\ a_{max} = ?$

$$v(t) = r'(t)$$

$$= 2\,\vec{i} + \cos\left(\frac{4\pi t}{5}\right)\cdot\frac{4\pi}{5}\,\vec{j}$$

$$v_{max} = |\cos 0| = |t = 0|$$

$$= 2\,\vec{i} + \frac{4\pi}{5}\,\vec{j}$$

$$\|v_{max}\| = \sqrt{2^2 + \left(\frac{4\pi}{5}\right)^2} = 3.212\ m/s$$

$$a(t) = v'(t) = r''(t)$$

$$= -\sin\left(\frac{4\pi t}{5}\right)\cdot\frac{4\pi}{5}\cdot\frac{4\pi}{5}\,\vec{j}$$

$$a_{max} = \left|\sin\frac{3\pi}{2}\right| = \left|\frac{4\pi}{5}t = \frac{3\pi}{2}\right| = |t = 15|$$

$$= \frac{16\pi^2}{25}\,\vec{j}$$

$$\|a_{max}\| = \sqrt{\left(\frac{16\pi^2}{25}\right)^2} = 6.317\ m/s^2$$

z4) $s = 360\ m$

$t = 14\ s$

$a \sim t$

$v(14) = ?$

$a(t) = k\cdot t$

$a = \frac{d^2x}{dt^2}$

$v(0) = v_0 = 0$

$x(0) = x_0 = 0$

$$a = \frac{dv}{dt} = k\cdot t \quad /\int$$

$$\int_{v_0}^{v(t)} dv = \int_0^t k\,t\cdot dt$$

$$v(t) = \frac{k t^2}{2} - 0 + v_0$$

$$v(t) = \frac{k t^2}{2}$$

$$\frac{dx}{dt} = \frac{k t^2}{2} \quad / dt$$

$$\int_{x_0}^{x(t)} dx = \int_0^t \frac{k t^2}{2}\, dt$$

$$x(t) - x_0 = \frac{k \cdot t^3}{6}$$

$$x(t) = \frac{k \cdot t^3}{6}$$

$$360 = \frac{k \cdot 14^3}{6}$$

$$k = 0.787172 \ m/s^3$$

$$v(14) = \frac{0.787 \cdot 14^2}{2} = 77.1428 \ m/s$$

25) $v(x) = b\sqrt{x}$

$$b = 2 \ m^{1/2} \cdot s^{-1}$$

$$x(t=0) = 0$$

$$v(t=2) = ?$$

$$v(x) = 2\sqrt{x}$$

$$v(0) = 0$$

$$\frac{dx}{dt} = b\sqrt{x} \quad / \cdot \frac{dt}{\sqrt{x}}$$

$$\frac{dx}{\sqrt{x}} = b\, dt \quad / \int$$

$$\int_0^{x(t)} x^{-\frac{1}{2}} dx = b \int_0^t dt$$

$$2\sqrt{x(t)} = b \cdot t \quad / ^2$$

$$x(t) = \frac{b^2 \cdot t^2}{4} = \frac{2^2 \cdot 2^2}{4} = 4 \ m$$

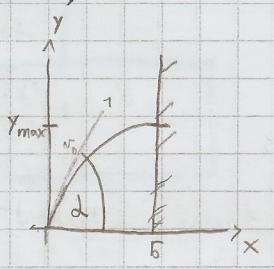
$$v(t) = x'(t) = \frac{b^2 \cdot 2t}{4} = \frac{b^2 \cdot t}{2} = \frac{2^2 \cdot 2}{2} = 4 \ m/s$$

$$a(t) = v'(t) = \frac{b^2}{2} = \frac{2^2}{2} = 2 \ m/s^2$$

26) $v_0 = 10 \ m/s$

$d = 5 \ m = x$

$\alpha(Y_{max}) = ?$



$$0 = v_0 - \frac{g \cdot t^2}{2}$$

$$\frac{dY}{d\alpha} = 0$$

$$Y(t) = Y_0 + v_0 \cdot \sin\alpha \cdot t - \frac{g \cdot t^2}{2}$$

$$x(t) = v_0 \cdot \cos\alpha \cdot t \ \rightarrow \ t = \frac{x}{v_0 \cdot \cos\alpha}$$

$$Y(x) = Y_0 + \frac{v_0 \cdot \sin\alpha \cdot x}{v_0 \cdot \cos\alpha} - \frac{g \cdot x^2}{2 \cdot v_0^2 \cdot \cos^2\alpha}$$

$$= 5\,tg\alpha - 1.22625 \cdot \cos^{-2}\alpha$$

$$\frac{dY}{d\alpha} = \frac{5}{\cos^2\alpha} + 2.4525 \cdot \cos^{-3}\alpha \cdot (-\sin\alpha) = 0 \quad / \cdot \cos^2\alpha$$

$$0 = 5 - 2.4525\,tg\alpha$$

$$tg\alpha = \frac{5}{2.4525}$$

$$\alpha = 53°52'13.38''$$