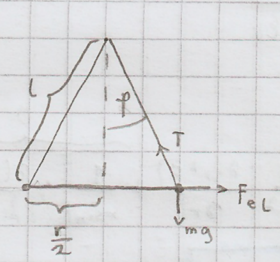
– 1. auditorna vježba –

Z1) $m_1 = m_2 = 30\ g = 0.03\ kg$

$l = 0.15\ m$

$\varphi = 5^\circ$

$Q_1 = Q_2 = ?$ u ravnoteži



$$\boxed{F_{el} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q_1 \cdot Q_2}{r^2}}$$

$$F_{el} = \frac{Q^2}{4\pi\varepsilon_0 \cdot r^2}$$

$$F_{el} = T \sin\varphi$$

$$mg = T \cos\varphi$$

$$F_{el} = mg \cdot tg\,\varphi$$

$$r = 2l \cdot \sin\varphi$$

$$F_{el} = \frac{Q^2}{4\pi\varepsilon_0 \cdot 4 \cdot l^2 \cdot \sin^2\varphi} = mg \cdot tg\,\varphi$$

$$Q_1 = Q_2 = Q = 4 l \sin\varphi \cdot \sqrt{mg \cdot \pi\varepsilon_0 \cdot tg\,\varphi}$$

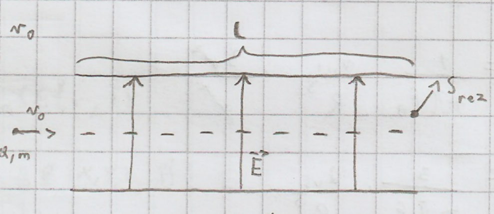
$$= 4 \cdot 0.15 \cdot \sin 5^\circ \cdot \sqrt{0.03 \cdot 9.81 \cdot \pi \cdot 8.854 \cdot 10^{-12} \cdot tg\,5^\circ}$$

$$= 4.42 \cdot 10^{-8}\ C$$

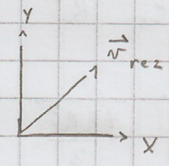
Z2) čestica m, Q, v_0

E

$\varphi = ?$



$x(0) = 0$, $y(0) = 0$



$v_y = 0$

$$\boxed{m\vec{a} = Q\vec{E}}$$

$$m(a_x\hat{x} + a_y\hat{y}) = Q \cdot E\hat{y}$$

$$x(t) = v_0 \cdot t$$

$$a_y(t) = \frac{QE}{m}$$

$$v_y(t) = \int a_y\,dt$$

$$= \frac{QE}{m} \cdot t$$

$$y(t) = \int v_y\,dt$$

$$= \frac{QE}{m} \cdot \frac{t^2}{2}$$

$$x(\tau) = l = v_0 \cdot \tau \rightarrow \tau = \frac{l}{v_0}$$

$$v_x(\tau) = v_0$$

$$v_y(\tau) = \frac{Q \cdot E}{m} \cdot \tau$$

$$= \frac{Q \cdot E}{m} \cdot \frac{l}{v_0}$$

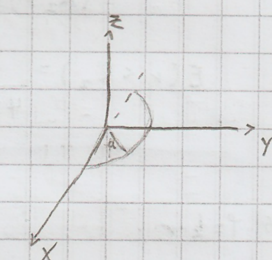
$$tg\,\varphi = \frac{v_y(\tau)}{v_x(\tau)}$$

$$= \frac{\frac{QE}{m} \cdot \frac{l}{v_0}}{v_0}$$

$$= \frac{QE \cdot l}{m \cdot v_0^2}$$

$$\varphi = arctg\left(\frac{QEl}{m \cdot v_0^2}\right)$$

Z3) $r = a$ poluprsten



Linearna gustoća naboja λ

$E_x, E_y = ?$

$$\vec{E} = \frac{1}{4\pi\varepsilon_0} \cdot \int \frac{\lambda \cdot dl}{r^2}\,\hat{r}$$

$$\hat{r} = \cos\varphi\,\hat{x} + \sin\varphi\,\hat{y}$$

$$dl = r\,d\varphi$$

$$\vec{E} = \frac{1}{4\pi\varepsilon_0} \cdot \int \frac{\lambda \cdot \cancel{r}\,d\varphi}{r^{\cancel{2}}} (\underbrace{\cos\varphi\,\hat{x}}_{0} + \sin\varphi\,\hat{y})$$

$E_x = 0$

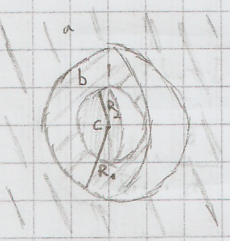
$$E_y = \frac{1}{4\pi\varepsilon_0} \cdot \int \frac{\lambda \cdot \sin\varphi\,d\varphi}{r}$$

$$= \frac{1}{4\pi\varepsilon_0} \cdot \frac{\lambda}{a} \cdot \int_0^\pi \sin\varphi\,d\varphi$$

$$= \frac{\lambda}{4\pi\varepsilon_0 \cdot a} \cdot -\cos\varphi\Big|_0^\pi$$

$$= \frac{\lambda}{\cancel{4}\pi\varepsilon_0 \cdot a} \cdot 2 = \frac{\lambda}{2\pi\varepsilon_0 \cdot a}$$

Z4) $E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{r^2}$



a) $r > R_1$

$Q_{obuhvaćeno} = Q_1 + Q_2$

$E(r > R_1) = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q_1 + Q_2}{r^2}$

b) $R_1 > r > R_2$

$Q_{ob} = Q_2$

$E(R_1 > r > R_2) = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q_2}{r^2}$

c) $r < R_2$

$\rho = \frac{Q_2}{V_2} = \frac{Q_2}{\frac{4\pi \cdot R_2^3}{3}}$

$= \frac{Q_{ob}}{\frac{4\pi r^3}{3}}$

$Q_{ob} = Q_2 \cdot \left(\frac{r}{R_2}\right)^3$

$E(r < R_2) = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q_2 \cdot r}{R_2^3}$

Z5) $\rho = k \cdot r$

$k \Rightarrow$ konst.

$E = ?$

$Q_{ob} = 2\pi L \cdot k \cdot \frac{r^3}{3}$

$\oint \vec{E} \cdot d\vec{A} = \oint E\, dA$

$= E \cdot 2\pi r \cdot L$

$\frac{Q_{ob}}{\varepsilon_0} = E \cdot 2\pi r \cdot L$

$E(r) = \frac{k r^2}{3\varepsilon_0}$

Z6) $\mathrm{div}\, \vec{E} = ?$

kugla R naboja Q

$\mathrm{div}\, \vec{E} = \frac{\rho}{\varepsilon_0}$

$r < R$

$\vec{\nabla} \cdot \vec{E} = \frac{1}{r^2} \frac{\partial (r^2 E)}{\partial r} + \overbrace{\frac{1}{r \sin\varphi} \cdot \frac{\partial (E_\varphi \sin\varphi)}{\partial \varphi}}^{0} + \overbrace{\frac{1}{r \sin\theta} \cdot \frac{\partial E_\theta}{\partial \theta}}^{0}$

$\boxed{\oint \vec{E} \cdot d\vec{A} = \frac{Q_{ob}}{\varepsilon_0}}$

$Q_{ob} = \int \rho\, dV$

$= \rho \cdot \frac{4\pi r^3}{3}$

$= \frac{Q_{uk}}{\frac{4\pi R^3}{\cancel{3}}} \cdot \frac{4\pi r^3}{\cancel{3}}$

$= Q_{uk} \cdot \left(\frac{r}{R}\right)^3$

$\oint \vec{E} \cdot d\vec{A} = E \cdot 4\pi r^2$

$E \cdot 4\pi r^2 = \frac{Q_{uk} \cdot \left(\frac{r}{R}\right)^3}{\varepsilon_0}$

$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q_{uk} \cdot r}{R_3}$

$\vec{\nabla} \cdot \vec{E} = \frac{1}{r^2} \cdot \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q_{uk}}{R^3} \cdot \frac{dr^3}{dr}$

$= \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q_{uk}}{R^3} \cdot \frac{3\cancel{r^2}}{\cancel{r^2}}$

$= \frac{3}{4\pi\varepsilon_0} \cdot \frac{Q_{uk}}{R^3}$

$= \frac{Q_{uk}}{\varepsilon_0} \cdot \frac{1}{\frac{4\pi R^3}{3}}$

$$\Phi = \int \vec{B} \cdot d\vec{S}$$

$$= \int B \, dS \cdot \cos(\measuredangle B, S)$$

$$|\measuredangle B, S = \varphi|$$

$$\boxed{w = \frac{d\varphi}{dt}}$$

$$d\varphi = w \, dt \quad / \int$$

$$\varphi = w \cdot t$$

$$|w = 2\pi f|$$

$$\varphi = 2\pi f \cdot t$$

|B homogeno|

$$\Phi = B \cdot \cos(2\pi f \cdot t) \cdot \int dS$$

$$= B \cdot \cos(2\pi f \cdot t) \cdot S$$

$$\varepsilon = -N \cdot BS \left[-\sin(2\pi f \cdot t)\right] \cdot 2\pi f$$

$$= NBS \cdot 2\pi f \cdot \sin(2\pi f \cdot t)$$

$$\varepsilon_{max} = NBS \cdot 2\pi f$$

$$= 8 \cdot 0.5 \cdot 0.09 \cdot 2 \cdot \pi \cdot 60 = 135.72 \text{ V}$$

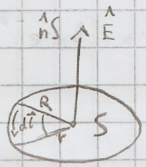
b) $I_{ind\,max} = ?$

$$\boxed{I_{ind\,max} = \frac{\varepsilon_{max}}{R_{uk}}}$$

$$= \frac{135.72}{12} = 11.31 \text{ A}$$

z4) $E(t) = E_0 \cdot t^2$

$$B(r < R) = ?$$



$$\vec{\nabla} \cdot \vec{B} = \frac{1}{c^2} \frac{\partial \vec{E}}{\partial t} \quad / \int d\vec{S}$$

$$\int \vec{\nabla} \cdot \vec{B} \, d\vec{S} = \frac{1}{c^2} \cdot \frac{\partial}{\partial t} \cdot \int \vec{E} \, d\vec{S}$$

$$\boxed{\int \vec{E} \, d\vec{S} = \Phi}$$

$$\boxed{\int d\vec{l} = \int r \, d\varphi}$$

$$\int \vec{B} \, d\vec{l} = \frac{1}{c^2} \cdot \frac{\partial \Phi}{\partial t}$$

$$\Phi = \int E \, dS \cdot \cos(0^\circ)$$

$$= E \cdot S$$

$$\frac{1}{c^2} \cdot \frac{\partial \Phi}{\partial t} = \frac{1}{c^2} \cdot \frac{\partial (E_0 \cdot t^2 \cdot S)}{\partial t}$$

$$= \frac{1}{c^2} \, 2 E_0 \cdot S \cdot t$$

$$\int \vec{B} \, d\vec{l} = \int B \, dl$$

$$= \int_0^{2\pi} B r \, d\varphi$$

$$= B r \cdot \varphi \Big|_0^{2\pi}$$

$$\cancel{2}\pi r B = \frac{\cancel{2}}{c^2} E_0 \cdot S \cdot t$$

$$|S = r^2 \cdot \pi|$$

$$B = \frac{E_0 \cdot r \cdot t}{c^2}$$

$$\boxed{c = \frac{1}{\sqrt{\mu_0 \cdot \varepsilon_0}}}$$

$$B = \mu_0 \cdot \varepsilon_0 \cdot E_0 \cdot r \cdot t$$

$$\vec{B} = \mu_0 \cdot \varepsilon_0 \cdot E_0 \cdot r \cdot t \; \hat{l}$$

z5) l, w, B

$$\varepsilon = ?$$

$$\varepsilon = -\frac{\partial \Phi}{\partial t}$$

$$\Phi = \int \vec{B} \, d\vec{S}$$

$$= B \int\int l \, dl \, d\varphi(l)$$

$$w = \frac{d\varphi}{dt} \rightarrow d\varphi = w \, dt$$

$$\Phi = B \cdot \frac{l^2}{2} \cdot w t$$

$$\varepsilon = \frac{B \cdot l^2 \cdot w}{2}$$