11. auditorna vježba

- Faradejev zakon indukcije

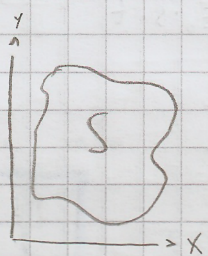
$$\boxed{\vec{\nabla} \times \vec{E} = -\frac{\partial B}{\partial t}} \quad \Big/ \int d\vec{S}$$

$$\int \vec{\nabla} \times \vec{E}\, d\vec{S} = -\frac{\partial}{\partial t} \cdot \int \vec{B}\, d\vec{S}$$

$$\boxed{\int \vec{B}\, d\vec{S} = \Phi} \quad \boxed{\int \vec{\nabla} \times \vec{E}\, d\vec{S} = \int E\, dl}$$

$$\boxed{\varepsilon = \int E\, dl = -(N) \cdot \frac{d\Phi}{dt}}$$

z1) $S = 0.65\ m^2$



$z = 0$

$\varepsilon_{max} = ?$

$\vec{B} = \frac{B_0 \cdot \cos(wt)(\hat{y} + \hat{z})}{\sqrt{2}}$

$B_0 = 0.05\ T$

$w = 1000\ rad/s$

$\varepsilon = -\frac{d\Phi}{dt}$

$\Phi = \int \vec{B}\, d\vec{S}$

$S \in \{x, y, z\}$ ili;	$S \in \{r, \varphi\}$
$B \in \{t\}$, tj.	$B \notin \{x, y, z, r, \varphi\}$ → ≈konst.

$\Phi = \vec{B} \cdot \int d\vec{S}$

$= \vec{B} \cdot \vec{S}$

$d\vec{S} = \hat{n}\, dS \quad / \int$

$\boxed{\vec{S} = \hat{n} \cdot S}$

$\vec{S} = \hat{z} \cdot S$

$\Phi = \frac{B_0 \cdot \cos(wt)(\hat{y} + \hat{z})}{\sqrt{2}} \cdot S\, \hat{z}$

$|\hat{y} \cdot \hat{z} = 0 \quad \hat{z} \cdot \hat{z} = 1|$

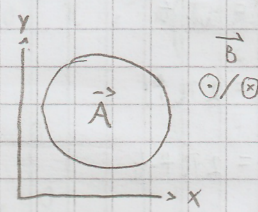
$\Phi = \frac{B_0 \cdot \cos(wt) \cdot S}{\sqrt{2}}$

$\varepsilon = -\frac{B_0 \cdot S \cdot [-\sin(wt)] \cdot w}{\sqrt{2}}$

$= \frac{B_0 \cdot S \cdot w}{\sqrt{2}} \cdot \sin(wt)$

$\varepsilon_{max} = \frac{B_0 \cdot S \cdot w}{\sqrt{2}} \cdot 1$

z2) $S = A$



$B(t) = B_0 \cdot e^{-at}$

$a \Rightarrow$ konst

$\varepsilon(t) = ?$

$\varepsilon = -\frac{\partial \Phi}{\partial t}$

$\Phi = \int \vec{B}\, d\vec{S}$

$$\boxed{\Phi = \int B \cdot dS \cdot \cos(\measuredangle B, S)}$$

$\measuredangle B, S = 0 \rightarrow \cos(\measuredangle B, S) = 1$

B homogeno $\longrightarrow \Phi = B \cdot \int dS$

$\Phi = B \cdot S$

$= |S = A|$

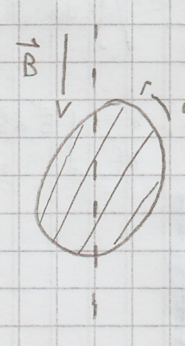
$= B \cdot A$

$= B_0 \cdot e^{-at} \cdot A$

$\varepsilon = -B_0 \cdot A \cdot e^{-at} \cdot (-a)$

$\varepsilon(t) = A \cdot a \cdot B_0 \cdot e^{-at}$

z3) $N = 8$



$S = 0.09\ m^2$

$R_{uk} = 12\ \Omega$

$B = 0.5\ T$

$f = 60\ Hz$

a) $\varepsilon_{max} = ?$

$\varepsilon = -N \cdot \frac{d\Phi}{dt}$

$E, B = ?$

$E_x(z,t) = E_0 \cdot \sin(wt - kz)$

$\vec{S} = \frac{\vec{E} \times \vec{B}}{\mu_0}$

$S = \frac{EB}{\mu_0}$

$\left| \vec{B} = \hat{k} \cdot \frac{\vec{E}}{c} \right|$

$S = \frac{E^2}{\mu_0 \cdot c}$

$\left| c = \frac{1}{\sqrt{\mu_0 \cdot \varepsilon_0}} \right|$

$S = E^2 \cdot \sqrt{\frac{\varepsilon_0}{\mu_0}}$

$\bar{S} = \sqrt{\frac{\varepsilon_0}{\mu_0}} \cdot \overline{E^2} = I$

$= \sqrt{\frac{\varepsilon_0}{\mu_0}} \cdot \overline{E_0^2} \cdot \overbrace{\sin^2(wt - kz)}^{\frac{1}{2}}$

$= \frac{1}{2} \cdot \sqrt{\frac{\varepsilon_0}{\mu_0}} \cdot E_0^2$

$E_0 = \sqrt{2\bar{S}\sqrt{\frac{\mu_0}{\varepsilon_0}}}$

$= \sqrt{2 \cdot 1 \cdot \sqrt{\frac{4\pi \cdot 10^{-7}}{8.854 \cdot 10^{-12}}}} = 27.45 \text{ V/m}$

$w = 2\pi f$

$= 2\pi \cdot 5 \cdot 10^{14} = \pi \cdot 10^{15} \text{ s}^{-1}$

$k = \frac{w}{c}$

$= \frac{\pi \cdot 10^{15}}{3 \cdot 10^8} = 1.047 \cdot 10^7 \text{ m}^{-1}$

$E_x = 27.45 \text{ V/m} \cdot \sin(\pi \cdot 10^{15} \text{ s}^{-1} \cdot t - 1.047 \cdot 10^7 \text{ m}^{-1} \cdot z)$

$B_y = B_0 \cdot \sin(wt - kz)$

$\boxed{B_0 = \frac{E_0}{c}}$

$= \frac{27.45 \text{ V/m}}{3 \cdot 10^8 \text{ m/s}} = 9.15 \cdot 10^{-8} \text{ T}$

$B_y = 9.15 \cdot 10^{-8} \text{ T} \cdot \sin(\pi \cdot 10^{15} \text{ s}^{-1} \cdot t - 1.047 \cdot 10^7 \text{ m}^{-1} \cdot z)$

4) $\lambda, \hat{k} = \hat{x}, \vec{E}_0, \hat{y} + \hat{z}$

$E_x = 0, \ E_y, E_z \neq 0$

$\vec{B}, \vec{S} = ?$

$\vec{S} = \frac{\vec{E} \times \vec{B}}{\mu_0}$

$E_y = E_{0y} \cdot \sin(wt - kx)$

$E_z = E_{0z} \cdot \sin(wt - kx)$

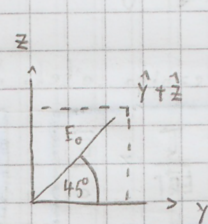
$\lambda = \frac{2\pi}{k} \to k = \frac{2\pi}{\lambda}$

$k = \frac{w}{c} \to w = \frac{2\pi c}{\lambda}$

$E_0^2 = \cancel{E_{0x}}^{\to 0}{}^2 + E_{0y}^2 + E_{0z}^2$

$= E_{0y}^2 + E_{0z}^2$

$E_{0y} = E_{0z} = \frac{E_0}{\sqrt{2}}$



$E(x,t) = \frac{E_0}{\sqrt{2}} (\hat{y} + \hat{z}) \cdot \sin\left[\frac{2\pi c}{\lambda}\left(t - \frac{x}{c}\right)\right]$

$\vec{B} = \hat{k} \times \frac{\vec{E}}{c}$

$= \hat{x} \times (\hat{y} + \hat{z})$

$\left.\begin{array}{l} \hat{x} \times \hat{y} = \hat{z} \\ \hat{x} \times \hat{z} = -\hat{y} \end{array}\right\} \vec{B} \parallel (\hat{z} - \hat{y})$

$B(x,t) = \frac{E_0}{\sqrt{2} \cdot c} (\hat{z} - \hat{y}) \cdot \sin\left[\frac{2\pi c}{\lambda}\left(t - \frac{x}{c}\right)\right]$

$\vec{S} = \frac{\vec{E} \times \vec{B}}{\mu_0}$

$= (\hat{y} + \hat{z}) \times (\hat{z} - \hat{y})$

$= \hat{y} \times \hat{z} - 0 + 0 + \hat{z} \times (-\hat{y})$

$= \hat{x} + \hat{x}$

$= 2\hat{x}$

$\vec{S} = \frac{E_0^2}{\mu_0 \cdot 2c} \cdot 2\hat{x} \sin^2\left[\frac{2\pi c}{\lambda}\left(t - \frac{x}{c}\right)\right]$

$\vec{S} = \frac{E_0^2}{\mu_0 \cdot c} \hat{x} \cdot \sin^2\left[\frac{2\pi c}{\lambda}\left(t - \frac{x}{c}\right)\right]$