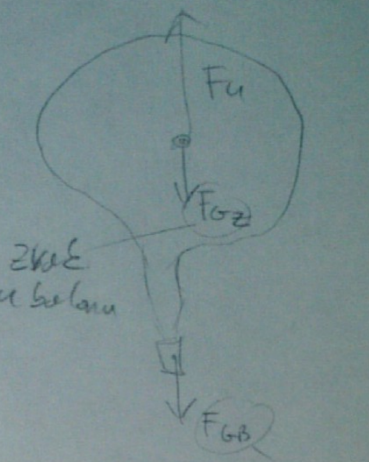
1

$m_b = 600\ kg$

$V_b = 2800\ m^3$

$T_0 = 20°C$

$\rho_0 = 1{,}2\ \frac{kg}{m^3}$



$F_U = F_{GZ} + F_{GB}$

$F_U = \rho_0 \cdot V_B \cdot g$

$\rho = \frac{m}{V} = \frac{n \cdot M}{V} = \frac{pM}{RT}$

$pV = nRT$

$\rho \cdot T = konst.$

$\rho_0 \cdot T_0 = \rho_B \cdot T_B$

$\rho_0 = \frac{T_B}{T_0} \rho_B$

$m_z \cdot g + m_B \cdot g = \rho_0 \cdot V_B \cdot g$

m_z - masa zraka u balonu

$m_z + m_B = \rho_0 \cdot V_B$

$m_z = \rho_B \cdot V_B$

$= \frac{\rho_0 T_0}{T_B} \cdot V_B$

$\frac{\rho_0 \cdot T_0}{T_B} \cdot V_B + m_B = \rho_0 \cdot V_B \quad / \cdot T_B$

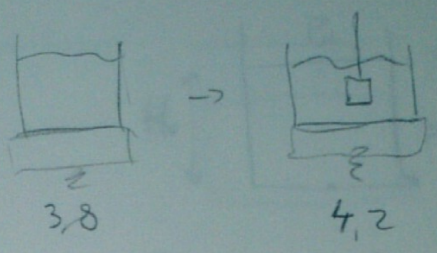
$T_B(\rho_0 \cdot V_B - m_B) = \rho_0 \cdot T_0 \cdot V_B$

$T_B = \frac{\rho_0 T_0 V_B}{\rho_0 V_B - m_B} \quad / : \rho_0 V_B \quad / : \rho_0 V_B = \frac{T_0}{1 - \frac{m_B}{\rho_0 V_B}}$

2) $m_1 = 3{,}8\ kg$

$m_2 = 4{,}2\ kg$

$m_t = 1{,}1\ kg$



$m_2 \cdot g = m_1 \cdot g + \cancel{m_t \cdot g} + F_u$

↑ tijelo ne dira dno

$V_t \cdot \rho_v \cdot g = m_2 g - m_1 g$

$\frac{m_t}{\rho_t} \cdot \rho_v = m_2 - m_1$

$\rho_t = \frac{\rho_v \cdot m_t}{m_2 - m_1}$

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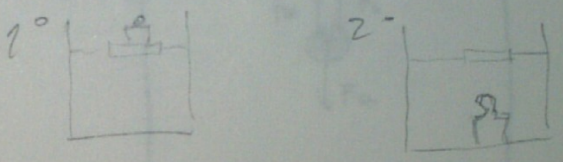
3) $m_{Al} = 0{,}2\ kg$

$S = 100 : 100^2 = 0{,}01\ m^2$

$\Delta h = ?$



Volumen istisnute vode za 1° slučaj:

$V_1 = \frac{m_1}{\rho_v} = \frac{m_{Al} - m_{st}}{\rho_v}$

2° slučaj:

$V_2 = V_{Al} + \frac{m_{st}}{\rho_v} = \frac{m_{Al}}{\rho_{Al}} + \frac{m_{st}}{\rho_v}$

$\Delta V = V_2 - V_1 = \frac{m_{Al}}{\rho_{Al}} + \frac{m_{st}}{\rho_v} - \frac{m_{Al}}{\rho_v} - \frac{m_{st}}{\rho_v} = m_{Al}\left(\frac{1}{\rho_{Al}} - \frac{1}{\rho_v}\right)$

$\Delta V = m_{Al} \cdot \left(\frac{1}{\rho_{Al}} - \frac{1}{\rho_v}\right)$

$\Delta h = \frac{\Delta V}{S} = \frac{m_{Al}\left(\frac{1}{\rho_{Al}} - \frac{1}{\rho_v}\right)}{S}$

4) $D = 0{,}4\ m$

$d = 0{,}01\ m$

$H_0 = 0{,}2\ m$

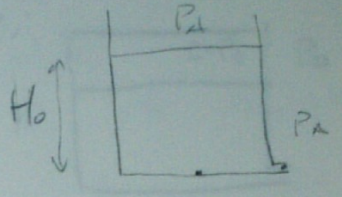
$t = ?$

Bernoullijeva:

$$p_A + \rho_v \cdot g \cdot H_0 + \frac{\rho_v \cdot v_1^2}{2} = p_A + \frac{\rho_v \cdot v_2^2}{2}$$

$$p_A + \rho_v g H_0 = p_A + \frac{\rho_v \cdot v_2^2}{2}$$

$$v_2 = \sqrt{2 g H_0}$$



Pošto je $d \ll D$ po jedn. kontin. $\Rightarrow v_1 \ll v_2$ pa možemo zanemariti v_1

Jer voda „pada"

$$v_1 = -\frac{dh}{dt} = \left(\frac{d}{D}\right)^2 \sqrt{2 g H_0}$$

$$v_1 \cdot S_1 = v_2 \cdot S_2$$

$$v_1 = \frac{S_2}{S_1} \cdot v_2 = \frac{d^2}{D^2} v_2$$

← Krate se $\frac{\pi}{2}$

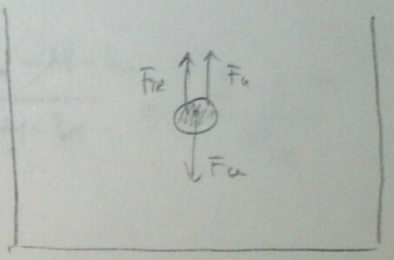
Separacija varijabli

$$-\frac{dh}{\sqrt{h}} = \frac{d^2}{D^2}\sqrt{2g} \cdot dt \quad / \int$$

$$-\int_{H_0}^{0} \frac{dh}{\sqrt{h}} = \frac{d^2}{D^2}\sqrt{2g} \cdot \int_0^T dt$$

$$-2\sqrt{h}\Big|_{H_0}^{0} = \frac{d^2}{D^2}\sqrt{2g} \cdot t\Big|_0^T$$

$$T = \frac{D^2}{d^2}\sqrt{\frac{2H_0}{g}}$$



$$F = F_u - F_{IR} - F_u$$

$v = \text{const}$ ⇓ $F = 0$

$$F_u = F_{IR} + F_u$$

5) $m = 20\ g = 0{,}02\ kg$

$2r = 2\ cm$

$r = 0{,}01\ cm$

$v = 2\ \frac{cm}{s} = 0{,}02\ \frac{m}{s}$

$\rho_m = 1{,}36\ \frac{g}{cm^3} = 1360\ \frac{kg}{m^3}$

$\eta = ?$

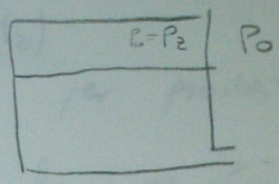
$$m \cdot g = 6\pi\eta \cdot r \cdot v + \rho_m \cdot V \cdot g$$

$$\eta = \frac{m \cdot g - \rho_m \cdot \frac{4}{3} r^3 \pi \cdot g}{6\pi \cdot r \cdot v}$$

6 – NIJE RJEŠAVAN

7

Voda se isticati dok je unutarnji tlak veći od vanjskog:



$$p_z + \rho_v \cdot g \cdot h > p_0$$

$H = 5\,m$

$h_0 = 3\,m$

$\rho_v = .$

$p_0 = p_{atm} = .$

$pV = nRT = konst$

$p_z \cdot V_z = p_0 \cdot V_0$

početno stanje u cilindru

$$p_0 = p_z + \rho_v g \cdot h$$

$$p_0 = p_0 \cdot \frac{H-h_0}{H-h} + \rho \cdot g \cdot h$$

$$\frac{p_z}{p_0} = \frac{V_0}{V_z} = \frac{(H-h_0)\cdot S}{(H-h)\cdot S}$$

$$p_0\left(1 - \frac{H-h_0}{H-h}\right) = \rho \cdot g \cdot h$$

$$1 - \frac{H-h_0}{H-h} = \frac{\rho \cdot g \cdot h}{p_0}$$

$$h_0 - h = \frac{\rho g h H}{p_0} - \frac{\rho g h^2}{p_0}$$

$$\frac{H-h-H+h_0}{H-h} = \frac{\rho g h}{p_0}$$

$$\frac{\rho g}{p_0} \cdot h^2 - \left(\frac{\rho g H}{p_0} + 1\right) \cdot h + h_0 = 0$$

$h_1 \neq 12{,}93$ h_1 otpada jer je veće od h_0

$h_2 = 2{,}396\,m$

8| $t_1 = -10\,^\circ C$

$t_2 = 20\,^\circ C$

$m_1 = ?\quad m_2 = ?$

$t_k = 5\,^\circ C$

$m_{uk} = 10\,kg$

nema gubitka energije

$\Delta Q_v = m_v \cdot c_v \cdot (t_k - t_2)$

mora biti < 0 jer predaje toplinu

$\Delta Q_L = m_L \cdot c_L \cdot (t_0 - t_1) + m_L \cdot l_L + m_L \cdot c_v \cdot (t_k - t_1)$

$\Delta Q_L > 0$

zagrijavanje do $0\,^\circ C$ | prelazak u tek. | zagrijavanje otopljenog leda do kon. temp.

$\frac{dE}{dt} = 0 \Rightarrow \Delta Q_v + \Delta Q_L = 0$

$m_{uk} = m_v + m_L$

$m_v = m_{uk} - m_L$

$(m_{uk} - m_L) \cdot c_v \cdot (t_k - t_2) + m_L \cdot c_L \cdot (t_0 - t_1) + m_L \cdot l_L + m_L \cdot c_v \cdot (t_k - t_1) = 0$

$m_L \cdot c_v \cdot (t_k - t_2) - m_L \cdot c_L (t_0 - t_1) - m_L \cdot l_L - m_L \cdot c_v (t_k - t_1) = m_{uk} \cdot c_v (t_k - t_2)$

$m_L \left(c_v (t_k - t_2) - c_L (t_0 - t_1) - l_L - c_v (t_k - t_1) \right) = m_{uk} \cdot c_v \cdot (t_k - t_2)$

$m_L = \frac{m_{uk} \cdot c_v (t_k - t_2)}{c_v (t_k - t_2) - c_L (t_0 - t_1) - l_L - c_v (t_k - t_1)}$ $\quad c_v (t_k - t_2 - t_k + t_1)$

$m_L = \frac{m_{uk} \cdot c_v \cdot (t_k - t_2)}{c_v (t_1 - t_2) - c_L (t_0 - t_1) - l_L} = \frac{m_{uk} \cdot c_v \cdot (t_2 - t_k)}{c_L (t_0 - t_1) + l_L + c_v (t_2 - t_1)}$

POŠTO JE RIJEČ O PROMJENI TEMP., MOŽE SE UVRŠTAVATI U °C

9) $T = \text{konst.}$

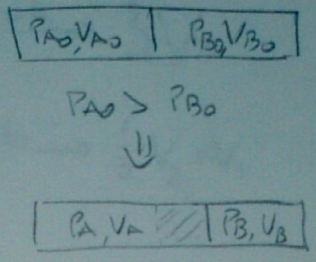
$\varkappa = \frac{7}{5}$

$p_{A0} = 200 \cdot 10^3\ Pa$

$p_{B0} = 100 \cdot 10^3\ Pa$

$V_{A0} = 1 \cdot dm^3 = 1 \cdot 10^{-3}\ m^3$

$V_{B0} = V_{A0} = \frac{1}{1000}\ m^3$



Proces je adiabatski $\Rightarrow p \cdot V^{\varkappa} = \text{konst.}$

A: $p_{A0} \cdot V_{A0}^{\varkappa} = p_A \cdot V_A^{\varkappa} = p_A (V_{A0} + \Delta V)^{\varkappa}$

B: $p_{B0} \cdot V_{B0}^{\varkappa} = p_B \cdot V_B^{\varkappa} = p_B (V_{B0} - \Delta V)^{\varkappa}$

$p_A = p_B$

$V_{A0} = V_{B0} = V_0$

$$\frac{p_{A0}}{p_{B0}} = \frac{p_A (V_{A0} + \Delta V)^{\varkappa}}{p_B (V_{B0} - \Delta V)^{\varkappa}} = \left(\frac{1 + \frac{\Delta V}{V_0}}{1 - \frac{\Delta V}{V_0}} \right)^{\varkappa} \dots \frac{\Delta V}{V_0} = \frac{\left(\frac{p_{A0}}{p_{B0}}\right)^{\frac{1}{\varkappa}} - 1}{\left(\frac{p_{A0}}{p_{B0}}\right)^{\frac{1}{\varkappa}} + 1}$$

$$\Delta V = \frac{\left(\frac{p_{A0}}{p_{B0}}\right)^{\frac{1}{\varkappa}} - 1}{\left(\frac{p_{A0}}{p_{B0}}\right)^{\frac{1}{\varkappa}} + 1} \cdot V_0 \qquad V_{A/B} = V_0 \pm \Delta V = V_0 \left(1 \pm \frac{\left(\frac{p_{A0}}{p_{B0}}\right)^{\frac{1}{\varkappa}} - 1}{\left(\frac{p_{A0}}{p_{B0}}\right)^{\frac{1}{\varkappa}} + 1} \right)$$

$$p_{A0} \cdot V_{A0}^{\varkappa} = p_A \cdot V_A^{\varkappa}$$

$$p_A = p_{A0} \left(\frac{V_{A0}}{V_A} \right)^{\varkappa} = p_{A0} \left(\frac{V_A - \Delta V}{V_A} \right)^{\varkappa} = p_{A0} \left(1 - \frac{\Delta V}{V_A} \right) \approx p_{A0} \left(1 - \frac{\Delta V}{V_0} \right)$$

$p_A = p_B$

10] $p_1 = 100\ kPa$
$V_1 = 10\ dm^3$
$\varkappa = \frac{7}{5}$

1° $V_1 \xrightarrow{adiab.} 3V_1$

2° $V_2 \xrightarrow{izob.} \frac{1}{3}V_2 = V_1$

$$1°\ W_{12} = \frac{nR}{\varkappa - 1}(T_1 - T_2) = \frac{nRT_1}{\varkappa - 1}\left(1 - \frac{T_2}{T_1}\right) = \frac{p_1 V_1}{\varkappa - 1}\left(1 - \left(\frac{V_1}{V_2}\right)^{\varkappa - 1}\right) = 889{,}014\ J$$

$$nRT_1 = p_1 V_1 \qquad \frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\varkappa - 1}$$

$$2°\ W_{23} = p_2 \Delta V = p_2 (V_2 - V_1)$$

$$= p_1 \left(\frac{V_1}{V_2}\right)^{\varkappa} (V_2 - V_1) = p_1 V_1 \left(\frac{V_1}{V_2}\right)^{\varkappa} \left(\frac{V_2}{V_1} - 1\right) = 429{,}596\ J$$

POŠTO SE PLIN TLAČI, NJEGOV RAD JE NEGATIVAN!

$$\Rightarrow W_{23} = p_1 V_1 \left(\frac{V_1}{V_2}\right)^{\varkappa} \left(1 - \frac{V_2}{V_1}\right) = -429{,}596\ J$$

$$W = W_{12} + W_{23} = 889{,}014 - 429{,}596 = 459{,}42\ J$$

11] Plin u cilindru s pomičnim klipom:

(0) κ, V_0, T_0, p_0 —odvrab→ (1) $\kappa = \kappa$ $V_1 = \frac{V_0}{2}$, T_1, p_1

$$W_{01} = \frac{nR}{\kappa-1}(T_0 - T_1) = \frac{nR}{\kappa-1} T_0 \left(1 - \frac{T_1}{T_0}\right) = \frac{nRT_0}{\kappa-1}\left(1 - \left(\frac{V_0}{V_1}\right)^{\kappa-1}\right) = \frac{p_0 V_0}{\kappa-1}\left(1 - 2^{\kappa-1}\right)$$

$$W_{12} = [V = \text{konst}] = 0$$

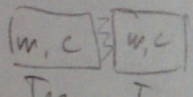
$$W_{20} = nRT_0 \ln \frac{V_0}{V_2} = p_0 V_0 \ln 2$$

W_{01} i W_{20} su radovi koje obavlja plin, što znači da klip obavi $-W_{01}$ i $-W_{20}$ radove

$$W_{uk} = -W_{01} - W_{20} = p_0 V_0 \left(\frac{2^{\kappa-1} - 1}{\kappa - 1} - \ln 2\right)$$

12, 13 → web

14



$$dQ = 0 \Rightarrow dQ_1 + dQ_2 = 0$$

$$dQ = mc\,dT_1 + mc\,dT_2 \;/\; S$$

$$0 = \int_{T_{10}}^{T_k} mc \cdot dT_1 + \int_{T_{20}}^{T_k} m \cdot c \cdot dT_2$$

$$0 = mc(T_k - T_{10}) + mc(T_k - T_{20})$$

$$T_k = \frac{T_{10} + T_{20}}{2}$$

u ovisnosti o vremenu

$$dS = dS_1 + dS_2 = \frac{dQ_1}{T_1\,[+]} + \frac{dQ_2}{T_2\,[+]}$$

$$= \frac{m \cdot c \cdot dT_1}{T_1} + \frac{m \cdot c \cdot dT_2}{T_2} \Big/ \int$$

$$\Delta S = mc \int_{T_{10}}^{T_k} \frac{dT_1}{T_1} + mc \int_{T_{20}}^{T_k} \frac{dT_2}{T_2}$$

$$\Delta S = m \cdot c \cdot \left[\ln \frac{T_k}{T_{10}} + \ln \frac{T_k}{T_{20}}\right]$$

$$\Delta S = m \cdot c \ln \frac{\left(1 + \frac{T_{20}}{T_{10}}\right)\left(1 + \frac{T_{20}}{T_{10}}\right)}{4}$$