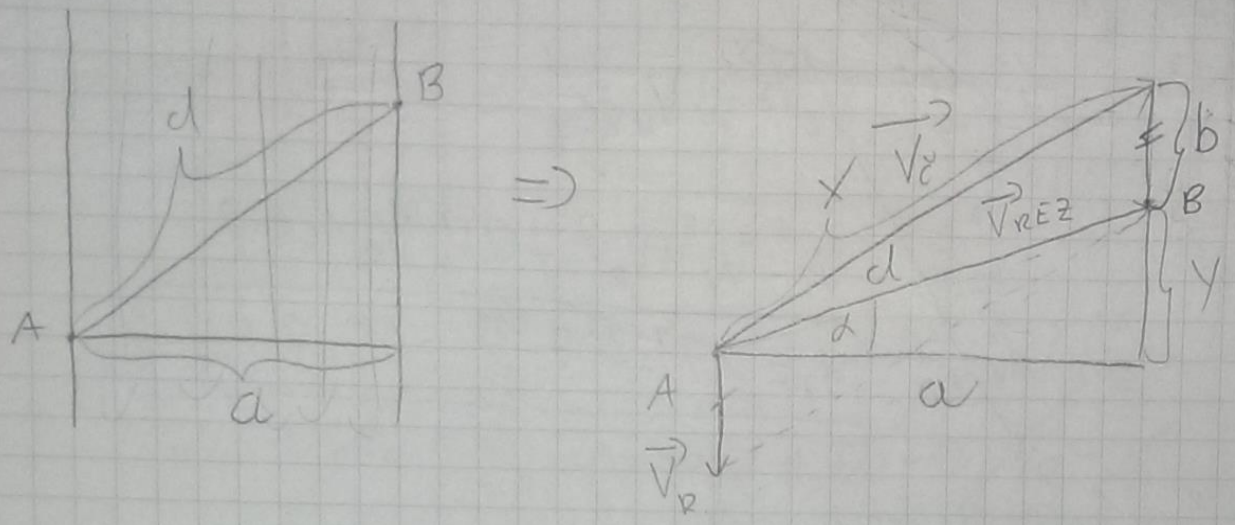
Kulišić - zadaci:

(2.1.) $d=\sqrt{5}\,km$, $a=1\,km$, $V_č=5\frac{km}{h}$, $V_R=2\frac{km}{h}\Rightarrow t=?$



$V_R \cdot t = b$
$V_č \cdot t = x$
$V_{REZ} \cdot t = \sqrt{5}$

$V_{REZ} \cdot t = \sqrt{5}$
$5 \cdot t = x$
$2 \cdot t = b$

$t = \frac{x}{5}$
$t = \frac{b}{2}$

$\frac{x}{5} = \frac{b}{2}$, $b = \frac{2}{5}x$, $x = \frac{5}{2}b$

$y = \sqrt{d^2 - a^2} = \sqrt{5-1} = 2\,km$

$(y+b)^2 + a^2 = x^2$
$(2+b)^2 = x^2 - 1 \;/\sqrt{\ }$
$2+b = \sqrt{x^2-1}$
$\frac{2}{5}x + 2 = \sqrt{x^2-1}\;/^2$
$\frac{4}{25}x^2 + \frac{8}{5}x + 4 - x^2 + 1 = 0 \;/\cdot 25$
$4x^2 + 40x + 100 - 25x^2 + 25 = 0$
$-21x^2 + 40x + 125 = 0$
$21x^2 - 40x - 125 = 0$

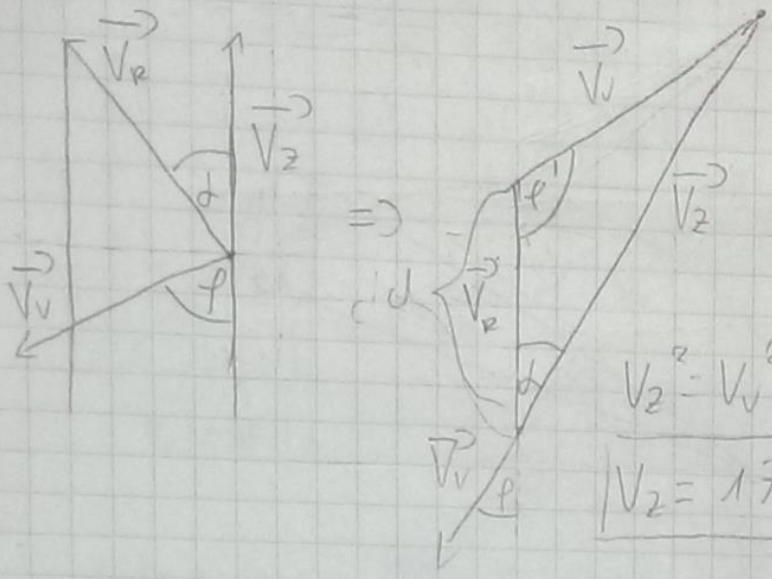
$x_{1,2} = \begin{cases} \boxed{x_1 = \frac{25}{7}} \Rightarrow x = \frac{25}{7}\,km = \\ x_2 = \cancel{-\frac{5}{3}} \end{cases} = 3{,}57\,km$

$V_č = \frac{x}{t}$

$t = \frac{x}{V_č} = \frac{3.57}{5} = 0.714\,h \Rightarrow$

$\Rightarrow \boxed{t = 43\,min}$

2.2 $\varphi = 30°$, $t = 2h$, $d = 300km$, $V_v = 27 \frac{km}{h} \Rightarrow V_z, \alpha = ?$



$\varphi' = 180° - 30° = 150°$

$V_R = \frac{d}{t} = 150 \frac{km}{h}$

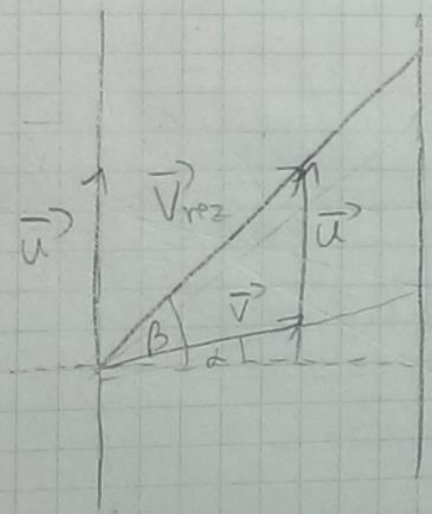
$V_z^2 = V_v^2 + V_R^2 - 2 V_v V_R \cos\varphi'$

$\boxed{V_z = 173.907 \frac{km}{h} = 48.31 \frac{m}{s}}$

$V_v^2 = V_R^2 + V_z^2 - 2 V_R V_z \cos\alpha$

$\cos\alpha = \frac{V_R^2 + V_z^2 - V_v^2}{2 V_R V_z} \Rightarrow \boxed{\alpha = \arccos \frac{V_R^2 + V_z^2 - V_v^2}{2 V_R V_z}} = 4.453°$

2.3 $V\ 1 \frac{m}{s}$, $u = 1.5 \frac{m}{s}$, $\alpha = 15° \Rightarrow \beta = ?$

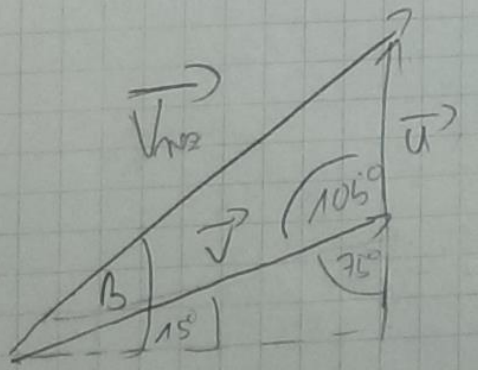


$u^2 = v^2 + V_{rez}^2 - 2 V V_{rez} \cos\varphi$

$V_{rez}^2 = v^2 + u^2 - 2 \cdot v \cdot u \cdot \cos 105°$

$V_{rez} = 2 \frac{m}{s}$

$\varphi = \arccos \frac{v^2 + V_{rez}^2 - u^2}{2 V V_{rez}}$

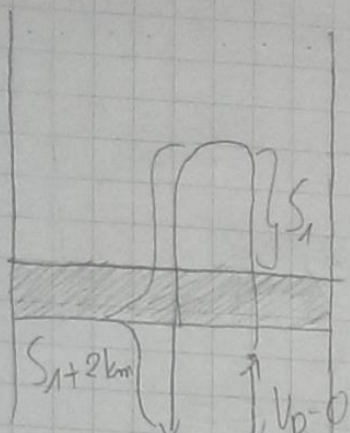


$\varphi = 65.47°$

$\beta = \varphi + \alpha = 61.57°$

$\varphi = \alpha + \beta$

2.4



$t = 0.5h$, $S = 2000m = 2km \Rightarrow V_R = ?$

$V_p = \frac{S}{t} = 4 \frac{km}{h}$, $t_p = \frac{S}{V_R} = \frac{2}{V_R}$

$S_1 = (V_c - V_R) \cdot t = (V_c - V_R) \cdot 0.5$

$S_2 = S_1 + 2 \Rightarrow S_1 + 2 = (V_c + V_R)\left(\frac{2}{V_R} - 0.5\right)$

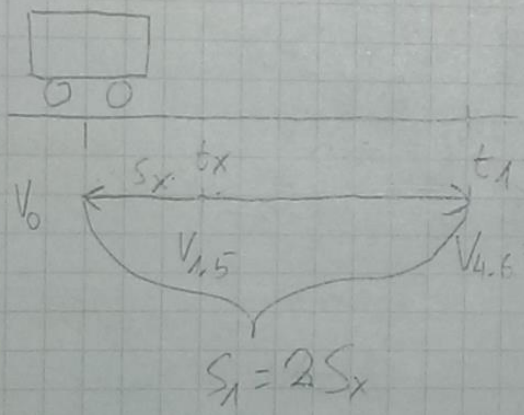
$S_1 = (V_c + V_R)\left(\frac{2}{V_R} - 0.5\right) - 2$

$0.5V_c - 0.5V_R = \frac{2}{V_R} V_c - 0.5V_c + 2 - 0.5V_R - 2$

$V_c = \frac{2V_R}{V_R} \Rightarrow \boxed{V_R = 2 \frac{km}{h} = 0.56 \frac{m}{s}}$

2.5 $V_0 = 40 \frac{km}{h} = 11.11 \frac{m}{s}$, $t_1 = 4.6s$, $S_1 = 2S_x \Rightarrow a = ?$, $t_x = 1.5$

↳ kočí – jednoliko usporeno gibanje, $a < 0$



$S_x = V_0 t_x - \frac{a}{2} t_x^2 / \cdot 2 \Rightarrow 2S_x = 2V_0 t_x - a t_x^2$

$S_1 = V_0 t_1 - \frac{a}{2} t_1^2$

$2S_x = 11.11 \cdot 3 - a \cdot 1.5^2 = 33.33 - 2.25a$

$S_1 = 11.11 \cdot 4.6 - \frac{a}{2} \cdot 4.6^2 = 51.106 - 10.58a$

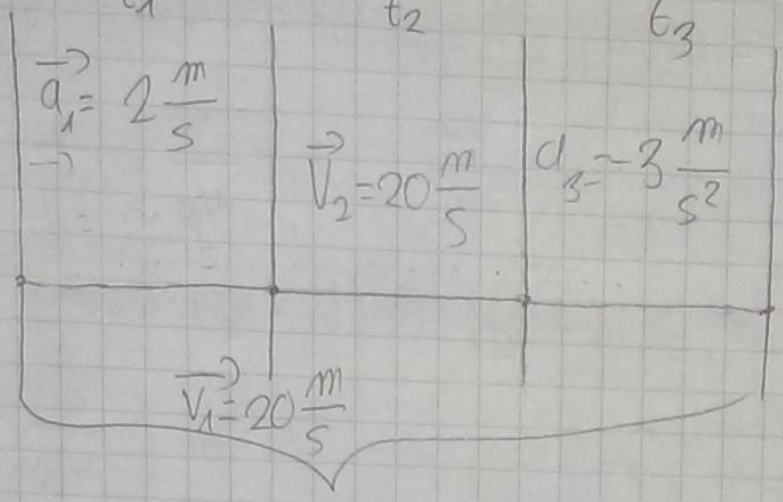
$33.33 - 2.25a = 51.106 - 10.58a$

$8.33a = 17.776$

$\boxed{a = -2.134 \frac{m}{s^2}}$

↳ zbog kočenja

2.6



$\Rightarrow s = ?$

$t_{uk.} = t_1 + t_2 + t_3 = 20\,s$

t_1: $V_1 = V_0 + a t_1 = a t_1$ ($V_0 = 0$)

$t_1 = \frac{V_1}{a} = 10\,s$

t_3: $V_3 = V_0 + a t_3$ ($V_0 = V_2$)

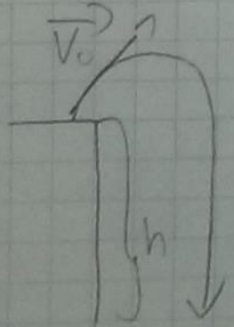
$t_3 = \frac{V_3 - V_2}{a} = \frac{-10}{-3} = 3.33\,s$

$t_2 = t_{uk} - t_1 - t_3 = 6.67\,s$

$s_1 = V_0 t_1 + \frac{a_1 t_1^2}{2} = \frac{a_1 t_1^2}{2} = 100\,m$ ($V_0 = 0$), $s_2 = V_2 t_2 = 133.4\,m$

$s_3 = V_0 t_3 + \frac{a_3 t_3^2}{2} = 16.67\,m$ ($V_0 = V_2$), $\boxed{s = s_1 + s_2 + s_3 = 250.07\,m}$

2.9 $h = 15\,m$, $V_0 = 10\frac{m}{s} \Rightarrow t, v, t' = ?$, $V_k = 0$



$V_k = V_0 - gt$ ($V_k = 0$) → u trenutku baranja

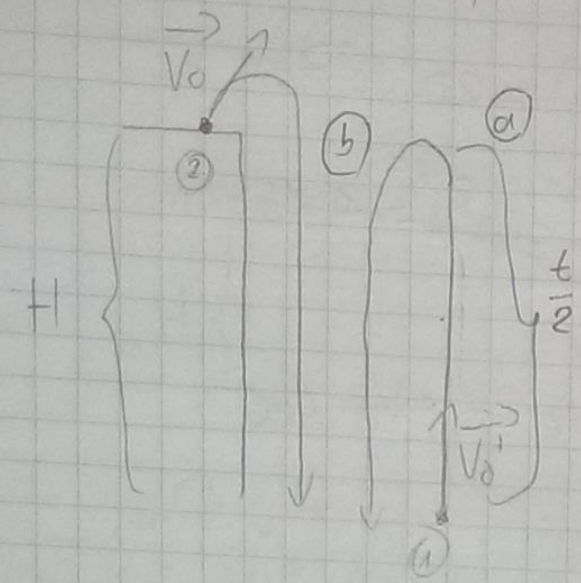
$V_0 = gt \Rightarrow t = \frac{V_0}{g} = \frac{10}{9.81} = 1.02\,s$

$V = V_0 + gt = 20\frac{m}{s}$ → u trenutku udara

slob. pad.: $t' \Rightarrow V_0 = 0$

$h = V_0 t + \frac{g}{2} t'^2 \Rightarrow t' = \sqrt{\frac{2h}{g}} = 1.75\,s$ ($V_0 = 0$)

2.10. $V_0 = 10\frac{m}{s}$, $H = 21m$, $V_0' = 5\frac{m}{s} \Rightarrow h_1, V_{0,min.} = ?$



gore:

$$h_1 = V_0 t - \frac{g}{2}t^2$$

dolje

$$h_2 = h_0 - V_0' t - \frac{g}{2}t^2 = H - \frac{V_0}{2}t - \frac{g}{2}t^2$$

$$h_1 = h_2$$

$$V_0 t - \frac{g}{2}t^2 = H - \frac{V_0}{2}t - \frac{g}{2}t^2$$

$$H = \frac{3V_0 t}{2} \Rightarrow t = \frac{2}{3}\frac{H}{V_0} = 1.4s$$

$$h_1 = V_0 t - \frac{g}{2}t^2 = 10 \cdot 1.4 - \frac{9.81}{2} \cdot 1.4^2 = 4.39m$$

$$\boxed{h = h_1 = 4.39m}$$

$V_{0,min.} = ?$ $\Rightarrow$ 1. dio gibanja ⓐ

$V_k = 0$, $t_1 = \frac{t}{2}$

$V_k = V_0 - g\frac{t}{2}$

$V_0 = g\frac{t}{2} \Rightarrow t = \frac{2V_0}{g}$

2. dio gibanja $\Rightarrow h_2 = 0$

$$H = \frac{V_0}{2}t + \frac{g}{2}t^2$$

$$H = \frac{V_0}{2}\cdot\frac{2V_0}{g} + \frac{g}{2}\cdot\frac{4V_0^2}{g^2}$$

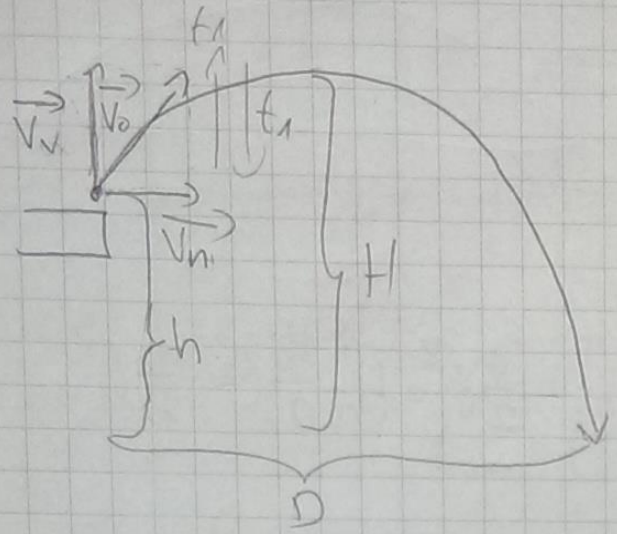
$$H = \frac{V_0^2}{g} + \frac{2V_0^2}{g} \quad / \cdot g$$

$$3V_0^2 = Hg$$

$$V_0 = \sqrt{\frac{Hg}{3}} = 8.29\frac{m}{s}$$

$$V_0 > 8.29\frac{m}{s}$$

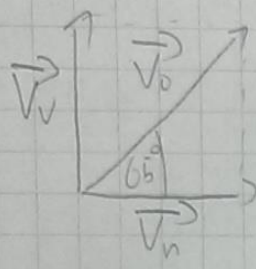
2.13 $V_0 = 10\frac{m}{s}$, $\alpha = 65°$, $h = 16m \Rightarrow D, H = ?$



$H = h + h' = 20.187m$

$h' = \frac{V_0^2}{2g}\sin^2\alpha = 4.187m$

$D = \frac{V_0^2}{g}\sin 2\alpha = V_h \cdot t_{uk}$



$\Rightarrow V_h = V_0\cos 65° = 4.23\frac{m}{s}$

$V_v = V_0\sin 65° = 9.06\frac{m}{s}$

vertik. hitac:

$t_1 \Rightarrow V_k = 0,\ V_p = 9.06\frac{m}{s}$; $V_k = V_p - gt \Rightarrow V_p = gt_1,\ t_1 = \frac{V_p}{g} = 0.924s$

$V_k^2 = V_p^2 - 2gs \Rightarrow (s = h_x) \Rightarrow h_x = \frac{V_p^2}{2g} = 4.184m \Rightarrow H = h + h_x =$

$= 20.187m$

$s = V_0 t + \frac{g}{2}t^2 \,/\cdot 2$; $V_0 = V_p$

$D = V_h \cdot t_{uk} = 12.49m$

$2V_0 t + gt^2 - 2h = 0$

$gt^2 + 2V_0 t - 2h = 0$

$9.81t^2 + 18.12t - 32 = 0$

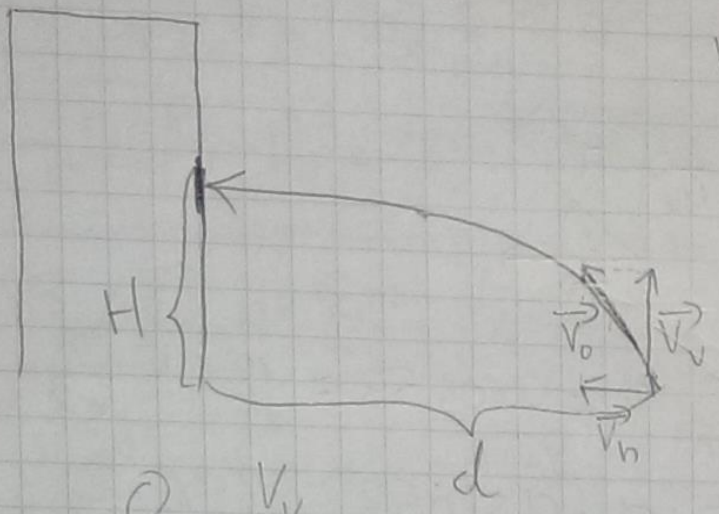
$t_2 = 1.105s$

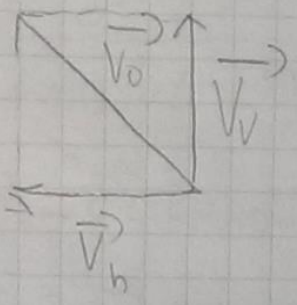
$t_{uk} = 2t_1 + t_2 = 2.953s$

2.14 $d = 100\,m \Rightarrow D = 200\,m$, $H = 50\,m \Rightarrow V_0, \alpha = ?$

$\downarrow$

$V_v = ?$,





$V_0 = 0$, $V_p = V_v$

v.h.: $V_k^2 = V_p^2 - 2gH \Rightarrow V_v = \sqrt{2gH} = 31.32\,\frac{m}{s}$

$V_k = 0$

$V_k = V_p - gt \Rightarrow t = \frac{V_v}{g} = 3.19\,s \Rightarrow t_{uk.} = 2t = 6.38\,s$

↳ vrnjemo kad će postići maksimum

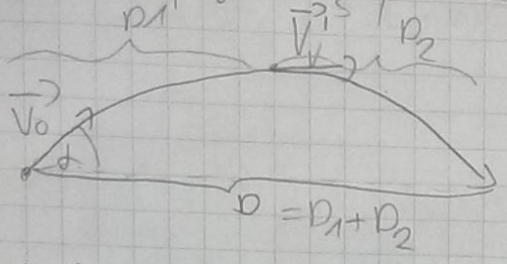
$D = V_h \cdot t_{uk.} \Rightarrow V_h = \frac{D}{t_{uk}} = 31.35\,\frac{m}{s}$

$$\boxed{V_0 = \sqrt{V_h^2 + V_v^2} = 44.31\,\frac{m}{s}}$$

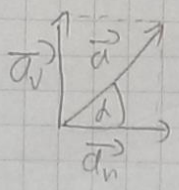
$$D = \frac{V_0^2}{g}\sin 2\alpha \Rightarrow 2\alpha = \arcsin\frac{Dg}{V_0^2} \Rightarrow \alpha = \frac{\arcsin\frac{Dg}{V_0^2}}{2}$$

$$\boxed{\alpha = 43.93^\circ \approx 44^\circ}$$

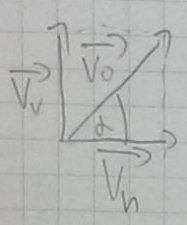
2.15. $\alpha = 53°$, $V_0 = 100\frac{m}{s}$, $a = 30\frac{m}{s^2}$, $t = 3s \Rightarrow D = ?$



dijeljenje akceleracije:



dijeljenje brzine:



$V_h = V_0 \cos 53° = 60.18\frac{m}{s}$ $\quad a_h = a\cos 53° = 18.05\frac{m}{s^2}$

$V_v = V_0 \sin 53° = 79.86\frac{m}{s}$ $\quad a_v = a\sin 53° = 23.96\frac{m}{s^2}$

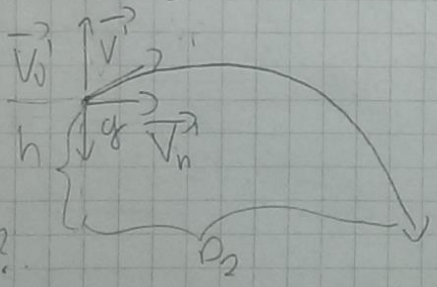
1) $t_1 = t = 3s \Big\uparrow \begin{matrix}\vec{a_v}\\ \vec{g}\end{matrix} \Rightarrow a_{uk} = 23.96\frac{m}{s}$, $V_0 = V_v = 79.86\frac{m}{s}$, $V_k = V_0'$, $s = D_1$

kvantikalni hitac!

$h = V_v t + \frac{a_{uk}}{2}t^2 = 347.4\,m$; $V_v' = V_v + a_{uk}\cdot t = 151.74\frac{m}{s}$

$D_1 = V_0 t + \frac{a}{2}t^2 = V_h t + \frac{a_h}{2}t^2 = 261.77\,m$ ($V_h = V_0$, $a = a_h$)

$V_h' = V_h + a_h t = 114.33\frac{m}{s}$



vertik. hitac:

$V_k^2 = V_p^2 - 2gh \Rightarrow V_v'^2 = 2gh$

$H = h = \frac{V_v'^2}{2g} = 1173.55\,m$

$t_{uk} = ?$

$V_k = 0 = V_v' - gt_2 \Rightarrow V_v' = gt_2$

$V_{1k} = 0$ $\quad t_{uk} = 2t_2 + t_3$ $\quad t_2 = \frac{V_v'}{g} = 15.47\,s$

$h = V_0 t + \frac{g}{2}t^2 = V_v' t_3 + \frac{g}{2}t_3^2$

$347.4 = 151.74\,t_3 + 4.905\,t_3^2$

$4.905\,t_3^2 - 151.74\,t_3 - 347.4 = 0$

$t_3 = 2.14\,s$

$t_{uk} = 2t_2 + t_3 = 33.08\,s$

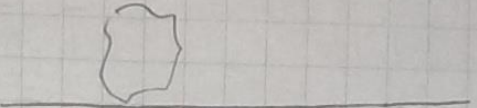
$D_2 = V_h' \cdot t_{uk} = 3782.04\,m$

$D = D_1 + D_2 = 261.77 + 3782.04 = 4043.81\,m \approx 4044\,m$

KULIŠIĆ ZBIRKA:

- (3.1.) $V_E = 300 \frac{kg}{s}$, $V_t = 2 \frac{m}{s}$

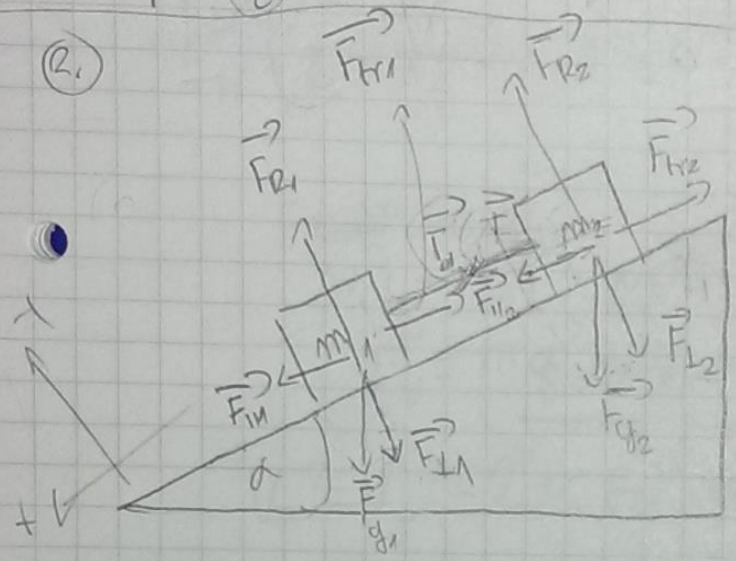
$F = ?$



$\dot{m} = 300\ kg$ $\quad F = m \cdot a = m \cdot \frac{v}{t}$

$F = \left(\frac{m}{t}\right)^{\dot{m}} \cdot V_t = \dot{m} \cdot V_E = 600\ N$

(2.)



$m_1 = 4\ kg$, $m_2 = 8\ kg$, $\alpha = 30°$, $\mu_1 = 0.1$, $\mu_2 = 0.2 \Rightarrow T = ?$

$\vec{F}_{g_1} = m_1 g = 39.24\ N$

$\vec{F}_{g_2} = m_2 g = 78.48\ N$

$\vec{F}_{t1} = m_1 g \cos\alpha = 33.98\ N$, $\vec{F}_{t2} = m_2 g \cos\alpha = 67.966\ N$

$\vec{F}_{11} = m_1 g \sin\alpha = 19.62\ N$, $\vec{F}_{12} = m_2 g \sin\alpha = 39.24\ N$

- $\vec{F}_{tr1} = \mu_1 \vec{N}_1 = \mu_1 \vec{F}_{t1} = 3.398\ N$, $\vec{F}_{tr2} = \mu \vec{F}_{t2} = 13.5932\ N$

$F = (m_1 + m_2) a = \vec{F}_{11} + \vec{F}_{12} - \vec{F}_{tr1} - \vec{F}_{tr2} + \cancel{\vec{T}} - \cancel{\vec{T}}$

$a = \frac{\vec{F}_{11} + \vec{F}_{12} - \vec{F}_{tr1} - \vec{F}_{tr2}}{m_1 + m_2} = 3.489 \frac{m}{s^2}$

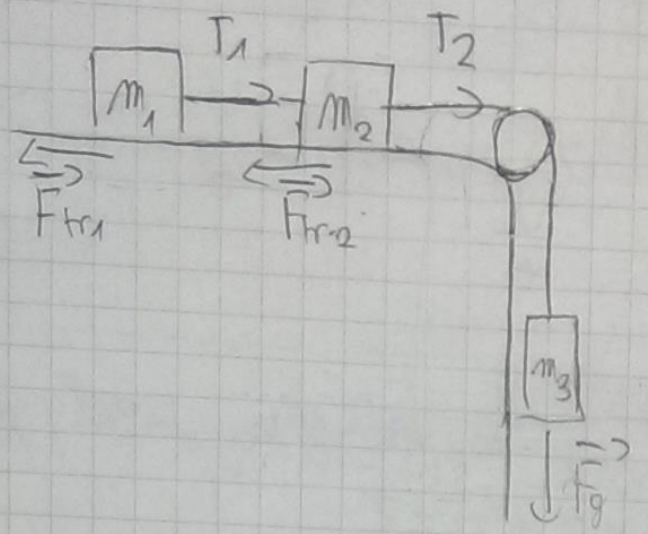
$\vec{F}_{tr1} \parallel \vec{T}$

m_1: $m_1 a = \vec{F}_{11} - \vec{T}_{tr} - \vec{T}$

$\vec{T} = \vec{F}_{11} - \vec{F}_{tr1} - m_1 a = 2.266\ N \approx 2.3\ N$

3.4. $m_1 = 0.2\ kg$, $m_2 = 0.3\ kg$, $m_3 = 0.6\ kg \Rightarrow a = ?$

a) $\mu = 0$, b) $\mu = 0.4$, c) $T_1, T_2 = ?$ ako je $\mu = 0$



a) $m_3 g = (m_1 + m_2 + m_3)a$

$$a = \frac{m_3}{m_1 + m_2 + m_3} g$$

$$a = 5.35 \frac{m}{s^2}$$

b) $\mu = 0.4$ $\quad (m_1 + m_2 + m_3)a = m_3 g - \vec{F}_{tr_1} - \vec{F}_{tr_2}$

$$a = \frac{m_3 g - \mu m_1 g - \mu m_2 g}{m_1 + m_2 + m_3}$$

$$a = 3.57\ m s^{-1}$$

c) $T_1, T_2 = ?$ ako je $\mu = 0$

$m_1 \vec{a} = \vec{T}_1 - \vec{F}_{tr_1}$

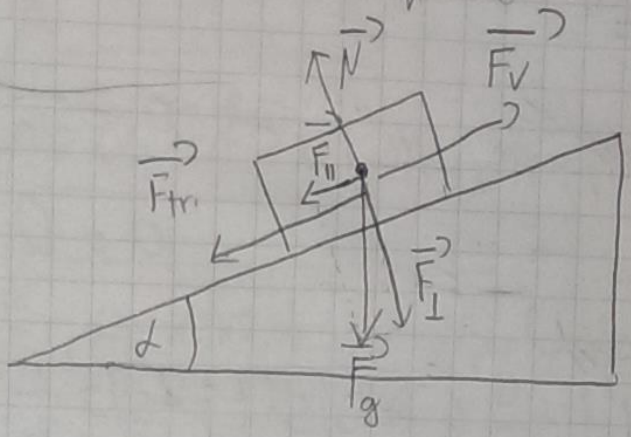
$\vec{T}_1 = m_1 \vec{a} = 1.07\ N$

$(m_1 + m_2)a = \vec{T}_2 - \vec{F}_{tr_2}$

$\vec{T}_2 = (m_1 + m_2)\vec{a} = 2.675\ N$

(3.5.) $G = 8000\,N$, $\alpha = 35°$, $s = 40\,m \Rightarrow$ od $V_0 = 20\,\frac{km}{h}$ do $V = 40\,\frac{km}{h}$

$\mu = 0.15 \rightarrow F = ?$



$\vec{F_g} = 8000\,N$

$\vec{F_\perp} = \vec{F_g}\cos\alpha = 6553.22\,N$

$\vec{F_{||}} = \vec{F_g}\sin\alpha = 4588.61\,N$

$\vec{F_{tr.}} = \mu\vec{N} = \mu\vec{F_\perp} = 982.983\,N$

$V_0 = 5.56\,\frac{m}{s}$

$V = 11.11\,\frac{m}{s}$

$\vec{F} = m\vec{a} = \vec{F_V} - \vec{F_{||}} - \vec{F_{tr.}}$

$\vec{F_V} = m\vec{a} + \vec{F_{||}} + \vec{F_{tr.}}$

$m = \frac{G}{g} = 815.5\,kg$

$V_k^2 = V_p^2 + 2as \Rightarrow a = \frac{V^2 - V_0^2}{2s} = 1.156\,\frac{m}{s^2}$

$\boxed{\vec{F_V} = m\vec{a} + \vec{F_{||}} + \vec{F_{tr.}}} = 6513.1\,N$

(3.12.) $m_1 = 0.05\,kg$, $m_2 = 0.01\,kg$, $m_3 = 0.015\,kg$, $t = 0$: $A_1(3,4,5)$,

$A_2(-2,4,-6)$, $A_3(0,0,0) \Rightarrow F = 0.05\,N$, položaj?

(gibose u smjeru x-osi)

1) $X_{CM} = \frac{X_1 + X_2 + X_3}{3} = \frac{1}{3}$, $Y_{CM} = \frac{Y_1 + Y_2 + Y_3}{3} = \frac{8}{3}$, $Z_{CM} = \frac{Z_1 + Z_2 + Z_3}{3} = -\frac{1}{3}$

$T\left(\frac{1}{3}, \frac{8}{3}, -\frac{1}{3}\right)$ $\qquad Y_{cm} = 3.2\,cm$

2) $X_{cm} = \frac{X_1 m_1 + X_2 m_2 + X_3 m_3}{m_1 + m_2 + m_3} = 1.73\,cm$, $Z_{cm} = 2.53\,cm$

↳ gibanje u smjeru x-osi ⇒ samo se X_{cm} mijenja!

3) $X'_{cm} = ?$

$F = (m_1 + m_2 + m_3)\, a \rightarrow a = \dfrac{F}{m_1 + m_2 + m_3}$

$a = 0{,}67 \dfrac{m}{s^2}$

$S = V_0 t + \dfrac{a}{2} t^2 \Rightarrow s(t=2s) = \dfrac{a}{2} t^2 = 1{,}34\ cm$ ($V_0 = 0$)

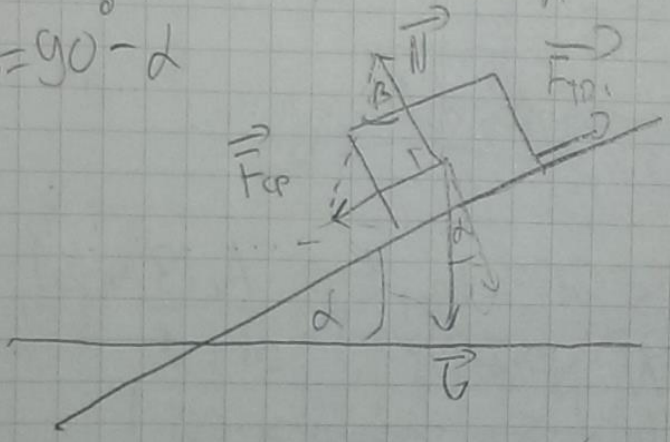
$\boxed{X'_{cm} = x_{cm} + S = 1{,}73 + 1{,}34 = 3{,}07\ cm}$

$\boxed{Y'_{cm} = Y_{cm} = 3{,}2\ cm}$

$\boxed{Z'_{cm} = Z_{cm} = 2{,}53\ cm}$

3.16. a) $R = 100\ m$, $v = 60 \dfrac{km}{h} = 16{,}67 \dfrac{m}{s}$, u zanem. $\Rightarrow \alpha = ?$

$\beta = 90^\circ - \alpha$



$\vec{F}_{cp} = \dfrac{mv^2}{R} = N \sin\alpha$

$\vec{G} = N \cos\alpha$

$\vec{F}_{tr.} = \vec{F}_{cp} \Rightarrow mg\cos\alpha = \dfrac{mv^2}{R} \sin\alpha$

$tg\beta = \dfrac{Rg}{v^2}$

$\beta = \operatorname{arctg} \dfrac{Rg}{v^2} = 74{,}18^\circ$

$\boxed{\alpha = 90^\circ - \beta = 15{,}82^\circ}$

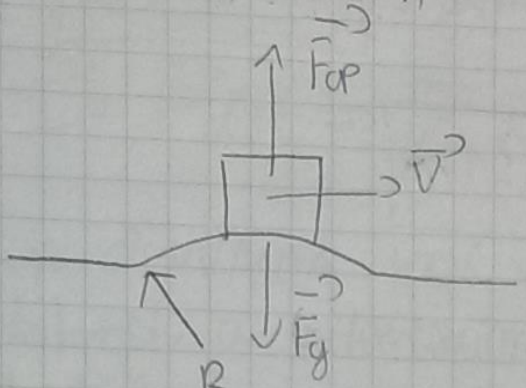
b) $\alpha = 0^\circ \Rightarrow \mu mg = \frac{mv^2}{R}$

$\mu = ?$

$$\boxed{\mu = \frac{v^2}{gR} = 0{,}283}$$

(3.17.) $m = 2000\,kg$, $V = 36\,\frac{km}{h} = 10\,\frac{m}{s}$, $R = 100\,m \Rightarrow$ a) $\vec{F_p} = ?$, b) $\vec{F_p}, \vec{v} = ?$

a)



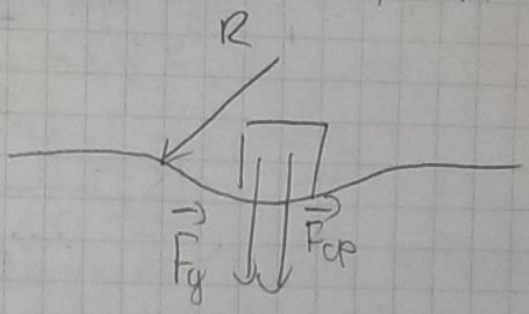
$$\vec{F_p} = \vec{F_g} - \vec{F_{cp}}$$

$$\vec{F_g} = mg = 1962\,N$$

$$\vec{F_{cp}} = \frac{mv^2}{R} = 2000\,N$$

$$\boxed{\vec{F_p} = |1962 - 2000| = 1762\,N}$$

b)



$$\boxed{\vec{F_p} = \vec{F_g} + \vec{F_{cp}} = 21620\,N}$$

$\vec{v}$

$\rightarrow$ da sila iščezne: $F_p = 0$

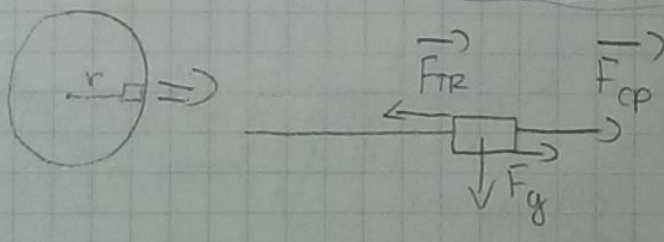
$$\vec{F_g} - \vec{F_{cp}} = 0$$

$$\vec{F_g} = \vec{F_{cp}} \Rightarrow mg = \frac{mv^2}{R}$$

$$\boxed{v = \sqrt{gR} = 31{,}32\,\frac{m}{s} = 112{,}76\,\frac{km}{h}}$$

(3.18.) $m = 4\,kg$, $r = 0{,}8\,m$, $F_N = 10\,kg = 98{,}1\,N$

$\mu = 0{,}6 \Rightarrow \omega = ?$



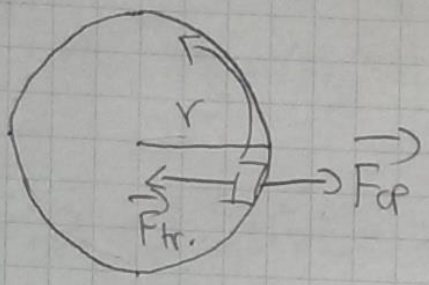
$$\vec{F_N} = \vec{F_{cp}} - \vec{F_{TR}} = \mu \vec{F_g}$$

$$\vec{F_{cp}} = \vec{F_N} + \mu mg = 121{,}644\,N$$

$$v = \omega \cdot r \Rightarrow F_{cp} = \frac{mv^2}{r} = \frac{m\omega^2 r^2}{r}$$

$$\omega^2 = \frac{F_{cp}}{mr} \Rightarrow \boxed{\omega = \sqrt{\frac{F_{cp}}{mr}} = 10{,}07\,s^{-1}}$$

3.19. $a_t = 0.62 \frac{m}{s^2}$, $r = 40m$, $\mu = 0.2 \Rightarrow s = ?$, $V_0 = 0$



$$\vec{F_{tr.}} = \vec{F_{cp}}$$

$$\mu mg = \frac{mv^2}{r} \Rightarrow v = \sqrt{\mu r g}$$

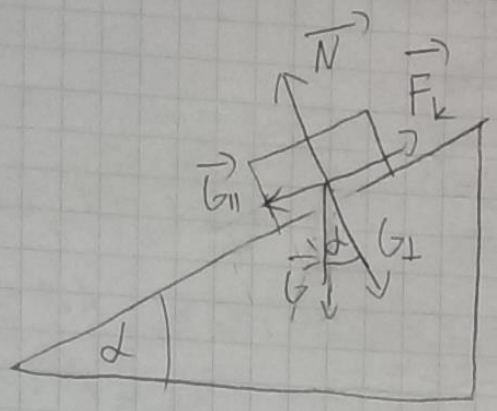
$$\boxed{v = 8.86 \frac{m}{s}}$$

$$V_k^2 = V_p^2 + 2as \quad (V_k = 0)$$

$$\boxed{s = \frac{V^2}{2a_t} = 63.31\,m}$$

4. RAD I ENERGIJA. SUDARI

4.1.



$\alpha = 30°$
$m = 2 \cdot 10^3\ kg$
$v_p = 20\ \frac{m}{s}$, $v_k = 0$ (kada se zaustavi)
$s = 100\ m$
$\vec{F}_k = ?$

$\vec{G}_\perp = \vec{G}\cos\alpha$
$\vec{G}_{11} = \vec{G}\sin\alpha$

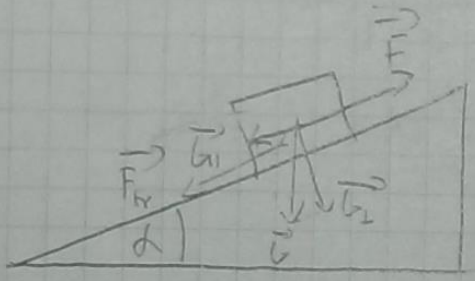
$\vec{F} = \vec{G}_{11} - \vec{F}_k$
$ma = mg\sin\alpha - F_k$
$a = 0$

$v_k^2 = v_p^2 + 2as \Rightarrow v_p^2 = -2as$

$a = \frac{-v_p^2}{2s} = -2\ \frac{m}{s^2}$

$F_k = m(g\sin\alpha - a) = 13.81\ kN$

4.3.



$m = 1000\ kg$, $v = 72\ \frac{km}{h} = 20\ \frac{m}{s}$,
$p = 10\% = 0.1$, $\mu = 0.05$, $P = ?$

$P = \frac{W}{t} = F \cdot v = 1469.05 \cdot 20 =$
$= 29381\ W = 29.4\ kW$

$\sin\alpha = 0.1 \Rightarrow \cos\alpha = \sqrt{1 - \sin^2\alpha} = 0.995$

$\vec{F} = \vec{G}_{11} + \vec{F}_{tr.} = mg\sin\alpha + \mu mg\cos\alpha =$
$= mg(\sin\alpha + \mu\cos\alpha) = 1469.05\ N$

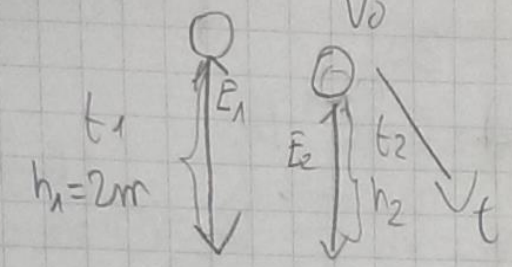
$P = F \cdot \bar{v} =$
$= 61726.10\ W =$
$= 61.73\ kW$

$P' = ?$, $a = 1\ \frac{m}{s^2}$
$v = 108\ \frac{km}{h} = 30\ \frac{m}{s}$
$V_0 = 20\ \frac{m}{s}$
$\bar{v} = \frac{v + v_0}{2} = 25\ \frac{m}{s}$

$\vec{F} = ma = \vec{F}_v - \vec{F}_{tr} - \vec{G}_{11}$

$\vec{F}_v = ma + \vec{F}_{tr} + \vec{G}_{11} =$
$= ma + \mu mg\cos\alpha + mg\sin\alpha =$
$= m(a + g(\mu\cos\alpha + \sin\alpha)) =$
$= 2469.05\ N$

4.4.



$h_1 = 2m$

$t = 1.8s$

$k = ?$

$t_1 + t_2 + t_2 = 1.8s$

$h_1 = V_0 t_1 + \frac{g}{2} t_1^2 \overset{V_0=0}{\Rightarrow} t_1 = \sqrt{\frac{2h_1}{g}} = 0.6386s$

$2t_2 = 1.8 - 0.6386$

$t_2 = 0.5807s$

$h_2 = \frac{g t_2^2}{2} = 1.65m$

$V_0' = g t_2 = 5.7 \frac{m}{s}$

$V_{1k} = V_0 + g t_1$ ($V_0 = 0$)

$V_{1k} = g t_1 = 6.265 \frac{m}{s}$

$k = \frac{V_0'}{V_{1k}} = \frac{5.7}{6.265} = 0.91$

4.6 $m_1 = m$

$m_2 = \frac{m}{2}$

$V_1, V_2 = 0$

$m_1 V_1 + m_2 V_2 \overset{0}{=} m_1 V_1' + m_2 V_2'$

$\cancel{m} V_1 = \cancel{m} V_1' + \frac{m}{2} V_2'$

$V_1 = V_1' + \frac{1}{2} V_2'$

a) $k = 1$

$V_1' = \frac{(m - \frac{m}{2}) V_1}{m + \frac{m}{2}} = \frac{\frac{m}{2} V_1}{\frac{3m}{2}} = \frac{1}{3} V_1$

$V_2' = \frac{2mV_1}{\frac{3m}{2}} = \frac{4}{3} V_1$

$Q = 0$

b) $k=0.6$

$$V_1' = \frac{mV_1}{\frac{3}{2}m} - 0.6 \cdot \left[\frac{mV_1}{\frac{3}{2}m}\right] = \frac{2}{3}V_1 - \frac{1}{5}V_1 = \frac{7}{15}V_1$$

$$V_2' = \frac{2}{3}V_1 + 0.6\,\frac{mV_1}{\frac{3}{2}m} = \frac{2}{3}V_1 + \frac{2}{5}V_1 = \frac{16}{15}V_1$$

$$Q = -0.64\,\frac{\frac{m^2}{2}V_1^2}{2\cdot\frac{3}{2}m} = mV_1^2\left(\frac{-0.64\cdot\frac{1}{2}}{3}\right) =$$

$$= -0.1067\,mV_1^2 \quad \Delta Q = 0.11\,mV^2$$

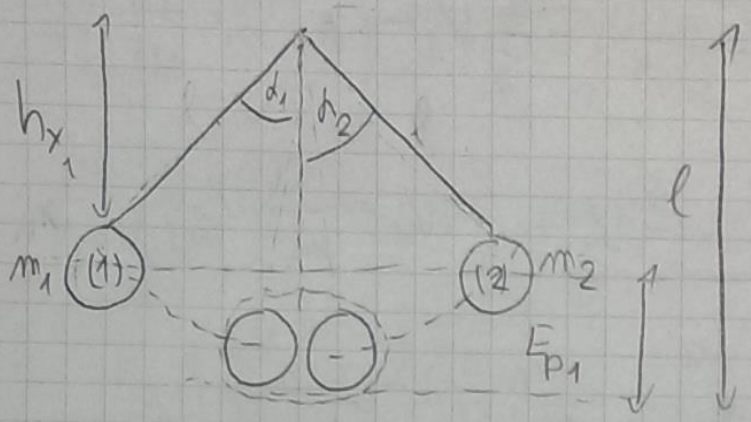
c) $k=0$

$$V_1' = V_2' = \frac{mV_1}{\frac{3}{2}m} = \frac{2}{3}V_1$$

$$Q = -\frac{\frac{m^2}{2}V_1^2}{2\cdot\frac{3}{2}m} = mV_1^2\left(\frac{-\frac{1}{2}}{3}\right) = -\frac{1}{6}mV^2$$

$$\Delta Q = \frac{1}{6}mV^2$$

(4.10.) $m_1 = 0.3\ kg$, $m_2 = 0.2\ kg$, $\alpha_1 = 40°$, $\alpha_2 = 20° \Rightarrow \alpha = ?$



drugi smjer

$$m_1 V_1 - m_2 V_2 = (m_1 + m_2) V$$

(1)

$E_{pot_1} = m_1 g h_1$

$E_{kin_1} = \frac{m_1 V_1^2}{2}$

Z.O.E.

$E_{pot_1} = E_{kin_1}$

$g h_1 = \frac{1}{2} V_1^2$

$V_1 = \sqrt{2 g h_1}$

$h_1 = \ell - h_{x_1}$

$h_{x_1} = \ell \cos 40° \Rightarrow h_1 = \ell - \ell \cos 40°$

$h_1 = \ell (1 - \cos 40°) \Rightarrow$

h_2 analogno

$h_2 = \ell (1 - \cos 20°) \Rightarrow$

$\Rightarrow V_1 = \sqrt{2 g \ell (1 - \cos 40°)}$

$V_2 = \sqrt{2 g \ell (1 - \cos 20°)}$

$V = \sqrt{2 g \ell (1 - \cos \alpha)}$

$$m_1 \sqrt{2g\ell (1 - \cos 40°)} - m_2 \sqrt{2g\ell (1 - \cos 20°)} = (m_1 + m_2) \sqrt{2g\ell (1 - \cos \alpha)}$$

0.23396 $\qquad$ 0.06031

$0.1451 - 0.04912 = 0.09598 = (m_1 + m_2) \sqrt{1 - \cos \alpha}$

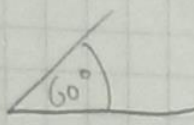
$$\sqrt{1 - \cos \alpha} = \frac{0.09598}{\underbrace{m_1 + m_2}_{0.5}} = 0.019196 \Big/^2$$

$1 - \cos \alpha = 0.03685 \Rightarrow \cos \alpha = 0.9632$

$\alpha = 15.6°$

(4.11.) $m_1 = 0.4\ kg$, $m_2 = 0.2\ kg$, $V_1 = 10\ \frac{m}{s}$, $V_2 = 8\ \frac{m}{s}$, $\alpha = 60° \Rightarrow Q =$

$k = 0$



$$\Rightarrow Q = -(1-k^2)\frac{m_1 m_2 |\vec{V_1} - \vec{V_2}|^2}{2(m_1+m_2)}$$

$$\vec{V_1} = 10\vec{i},\ \vec{V_2} = 8\cos 60°\vec{i} + 8\sin 60°\vec{j}$$

$$\vec{V_2} = 4\vec{i} + 4\sqrt{3}\vec{j}$$

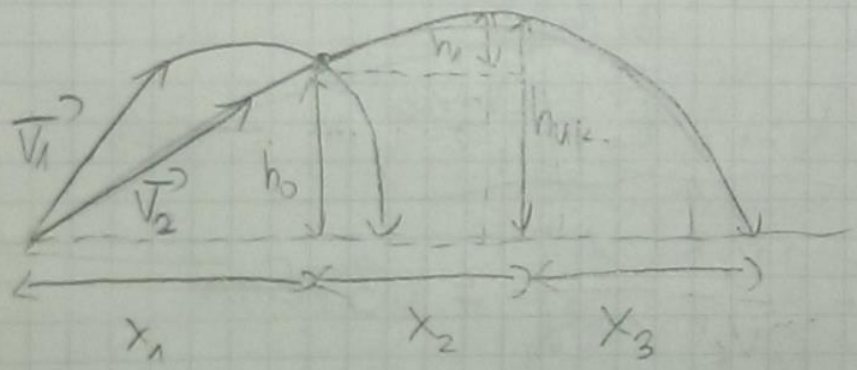
$$|\vec{V_1} - \vec{V_2}| = |6\vec{i} - 4\sqrt{3}\vec{j}| = 9.165\ \frac{m}{s}$$

$$Q = -\frac{m_1 m_2 \cdot 9.165^2}{2(m_1+m_2)} = \ominus 5.6\ J$$

↳ oslobodila

⊕ dala

(4.12.) $\alpha_1 = 60°$, $V_1 = 6\ \frac{m}{s}$, $\alpha_2 = 45°$, $V_2 = 10\ \frac{m}{s}$, $m_1 = m_2 = m$, $X = ?$



$$y_1(x) = x\,tg\alpha_1 - \frac{gx^2}{2V_1^2}\frac{1}{\cos^2\alpha_1},\quad y_2(x) = x\,tg\alpha_2 - \frac{gx^2}{2V_2^2}\frac{1}{\cos^2\alpha_2}$$

$$y_1(x) = y_2(x)$$

$$x\,tg\alpha_1 - \frac{gx^2}{2V_1^2}\frac{1}{\cos^2\alpha_1} = x\,tg\alpha_2 - \frac{gx^2}{2V_2^2}\frac{1}{\cos^2\alpha_2}$$

$$\sqrt{3} - 0.545x = 1 - 0.0981x$$

$0.4469x = \sqrt{3} - 1$

$\underline{x_1 = 1.638\ m}$

$x_1(t) = V_1 t_1 \cos\alpha_1 \Rightarrow t_1 = 0.546\ s$

$6t_1 \cos 60° = 1.638$

$\vec{V_1}(t) = 6\cos 60°\,\vec{i} + (6\sin 60° - 9.81 \cdot 0.546)\vec{j}$

$\vec{V_1}(t) = 3\vec{i} - 0.16\vec{j}$

$\vec{V_2}(t) = 10\cos 45°\,\vec{i} + (10\sin 45° - 9.81 \cdot 0.546)\vec{j}$

$\vec{V_2}(t) = 7.07\vec{i} + 1.715\vec{j}$

$\vec{V_p}(t) = \frac{\vec{V_1} + \vec{V_2}}{2} = 5.035\vec{i} + 0.7775\vec{j}$

$V_k = V_p - gt_2 \Rightarrow t_2 = \frac{V_y}{g} = 0.0793\ s$

($V_k = 0$, $V_y = V_y$)

najveću visinu tražimo

$\underline{x_2 = V_n \cdot t_2 = 0.399\ m}$

$h_0 = y(x_1) = x_1 \operatorname{tg}\alpha_1 - \frac{g x_1^2}{2V_1^2} \frac{1}{\cos^2\alpha_1} = 1.375\ m$

$V_k^2 = V_p^2 - 2gh_1 \Rightarrow h_1 = \frac{V_y^2}{2g} = \frac{0.7775^2}{2 \cdot 9.81} = 0.031\ m$

($V_k = 0$, $V_y = V_y$)

$h_{uk} = h_0 + h_1 = 1.406\ m$, $h_{uk} = \frac{gt^2}{2} \Rightarrow t_3 = \sqrt{\frac{2h_{uk}}{g}}$

$t_3 = 0.535\ s$

$t_{uk} = 1.1603\ s$

$\underline{x = V_n \cdot t_{uk} = 5.84\ m}$

(4.13.) $m_2 = 4.5\,kg$, $k = 500\,\frac{N}{m}$, $m_1 = 0.01\,kg$, $v_1 = 600\,\frac{m}{s}$, $x = ?$.

$$m_1 v_1 = (m_1 + m_2) v_2'$$

$$v_2' = \frac{m_1 v_1}{m_1 + m_2} = 1.33\,\frac{m}{s}$$

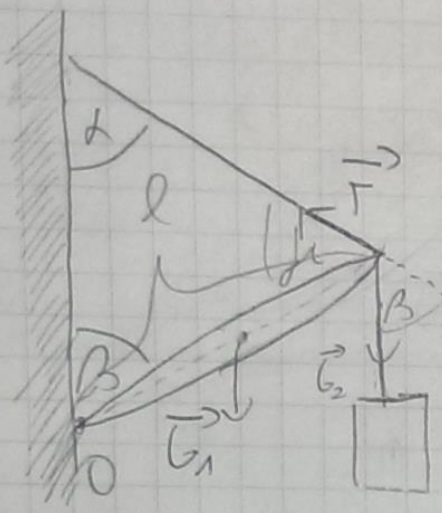
$E_{kin.} = E_{pot.} \Rightarrow \frac{mv^2}{2} = \frac{kx^2}{2}$

$$x = \sqrt{\frac{mv^2}{k}} = \sqrt{\frac{m_1 + m_2}{k}}\, v_2' = 0.126\,m$$

ZADACI STATIKA - KULIŠIĆ:

- (5.2.) $m_s = 10\,kg$, $m_u = 20\,kg$, $\alpha = 60°$, $\beta = 53°$; $T = ?$

$\gamma = 67°$ $\quad 90 - \beta = 37$



$\vec{T} = T \sin\gamma = T \sin 67°$

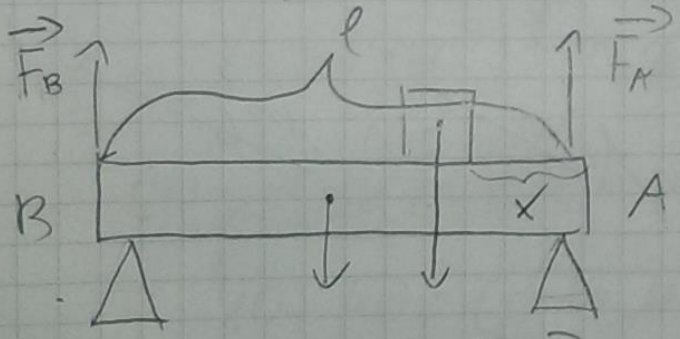
$\vec{T} \cdot l = \vec{G_1} \cdot \frac{l}{2} + \vec{G_2} \cdot l$

$\vec{T} = \vec{G_2} + \frac{1}{2}\vec{G_1}$

- $T \sin 67° = m_u g \cos(90-\beta) + m_s g \cos(90-\beta) \cdot \frac{1}{2}$

$$T = \frac{20g\cos 37° + 5g\cos 37°}{\sin 67°} = 204.21\,N$$

(5.3.) $l = 12\,m$, $m_g = 100\,kg$, $m_v = 200\,kg$, $x = 3\,m$, $\vec{F_A}, \vec{F_B} = ?$



1) $\sum_i \vec{F_i} = 0$

$\vec{F_A} + \vec{F_B} = m_g \cdot g + m_v g =$

$= 2943\,N$

2) $\sum_i M_i = 0 \quad \vec{M_A} = \vec{M_B}$

$\vec{F_B} \cdot l - F_G \frac{l}{2} - F_V \cdot \left(\frac{l}{2} - 3\right) = \vec{F_A} \cdot l - F_G \frac{l}{2} - F_V \left(\frac{l}{2} + 3\right)$

$12\vec{F_B} - 3\vec{F_V} = 12\vec{F_A} - 3\vec{F_V}$

$\vec{F_A} - \vec{F_B} = \frac{F_V}{2} = \frac{m_v \cdot g}{2} = 981\,N$

$\vec{F_A} = \vec{F_B} + 981$

$\vec{F_A} + \vec{F_B} = 2943$

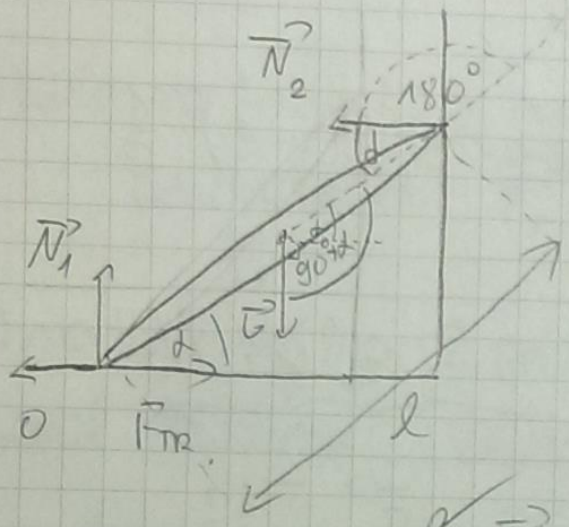
$2\vec{F_B} + 981 = 2943$

$\vec{F_B} = \frac{1962}{2} = 981\ N$

$\vec{F_A} = 981 + 981$

$\vec{F_A} = 1962\ N$

(5.4.) $\alpha = 45^\circ$, $m = 10\,kg$, $F_{tr.} = ?$



$\sum_i \vec{F_i} = 0$

$x: \vec{F_{tr.}} = \vec{N_2}$

$\mu \vec{N_1} = \vec{N_2}$

$y: \vec{G} = \vec{N_1}$

$\Rightarrow \vec{N_2} = \mu mg$

$mg = \vec{N_1}$

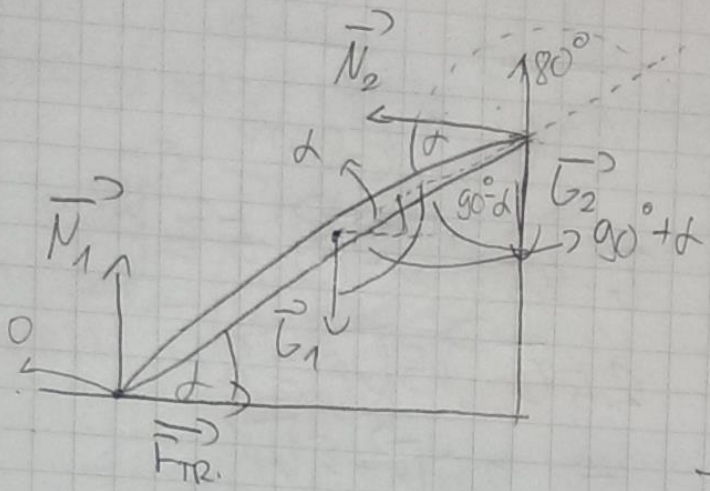
$\sum_i M_i = 0 \Rightarrow \frac{l}{2} \vec{G} \sin(90^\circ + \alpha) = l \cdot \vec{N_2} \cdot \sin(180^\circ - \alpha)$

$\frac{1}{2} mg \sin 135^\circ = \mu mg \sin 135^\circ$

$\mu = \frac{1}{2}$

$F_{tr.} = \mu mg = 49.05\ N$

5.5. $m_1 = 40\,kg$, $\alpha = 60°$, $m_2 = 80\,kg \Rightarrow \mu = ?$



$\sum_i \vec{F_i} = 0$

x: $\vec{N_2} = \vec{F_{TR}}$

$\vec{N_2} = \mu \vec{N_1}$

y: $\vec{N_1} = \vec{G_1} + \vec{G_2} = m_1 g + m_2 g$

$\vec{N_1} = g(m_1 + m_2)$

$\Rightarrow \vec{N_2} = \mu g(m_1 + m_2)$

$\sum_i \vec{M_i} = 0$

$\frac{\ell}{2} \cdot \vec{G_1} \cdot \sin(90° + \alpha) + \ell \cdot \vec{G_2} \cdot \sin(90° - \alpha) = \ell \cdot \vec{N_2} \cdot \sin(180° - \alpha)$

$\frac{1}{2} m_1 g \sin(150°) + m_2 g \sin(30°) = \mu g (m_1 + m_2) \sin(120°)$

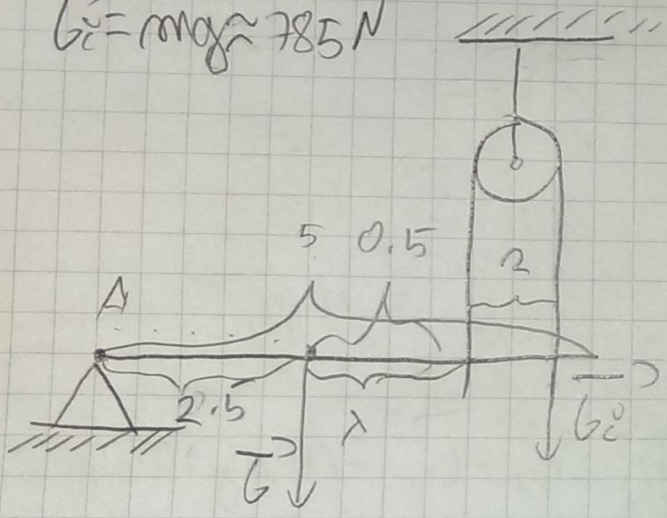
$20 \sin(150°) + 80 \sin(30°) = 120 \mu \sin(120°)$

$10 + 40 = 60\sqrt{3}\,\mu$

$\mu = \frac{50}{60\sqrt{3}} = 0.4811$

(5.7.) $m = 80\ kg$, $l = 5\ m$, $G = 400\ N$, $r = 1\ m$, $F = ?$

$\vec{G_c} = mg \approx 785\ N$



$\vec{M_{dulje}} = \vec{M_{gore}}$

$\vec{G} \cdot 2{,}5 + (\vec{G_c} - \vec{F}) \cdot 5 = \vec{F} \cdot 3$

$2{,}5\vec{G} + 5\vec{G_c} = 8\vec{F}$

$\vec{F} = \dfrac{2{,}5\vec{G} + 5\vec{G_c}}{8}$

$\vec{F} = 615{,}625\ N$