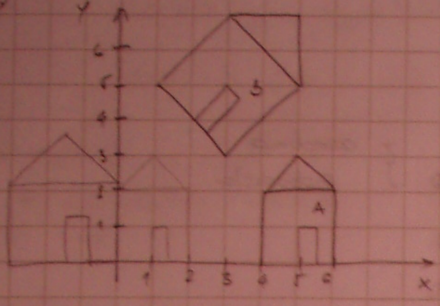
1.



1. $T_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 4 & 0 & 1 \end{bmatrix}$ translacija

2. $S = \begin{bmatrix} \sqrt{2} & 0 & 0 \\ 0 & \sqrt{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$ skaliranje & zrcaljenje po x

3. $R(45^\circ) = \begin{bmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$ rotacija

4. $T_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 3 & 3 & 1 \end{bmatrix}$

$U = T_1 \cdot S \cdot R \cdot T_2 = \begin{bmatrix} -1 & 1 & 0 \\ 1 & 1 & 0 \\ 7 & -1 & 1 \end{bmatrix}$

2. način

Poredak mora biti u 2

1 SKAL (a, b)

2 ROT (φ)

3. TRANS (x_0, y_0)

$\rightarrow a = \sqrt{2}, \; b = -\sqrt{2}$

$\varphi = \frac{3\pi}{4}$

$x_0 = 7, \; y_0 = -1$

2. LINEARNA INTERPOLACIJA

$x(t)$

$t \in [-1, 1]$

$x_0(-1) = 10$

$x_1(1) = 30$

$V = [t \;\; 1] L$

$= [t \;\; 1] \begin{bmatrix} a \\ x_0 \end{bmatrix}$

$\left.\begin{aligned} x_0 &= [-1 \;\; 1] L \\ x_1 &= [1 \;\; 1] L \end{aligned}\right\}$ $\begin{bmatrix} x_0 \\ x_1 \end{bmatrix} = \begin{bmatrix} -1 & 1 \\ 1 & 1 \end{bmatrix} L \longrightarrow L = \begin{bmatrix} -1 & 1 \\ 1 & 1 \end{bmatrix}^{-1} \begin{bmatrix} x_0 \\ x_1 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} -1 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 10 \\ 20 \end{bmatrix}$

$2, 2 \times 2, 1$

$x = [t \;\; 1] L = [t \;\; 1] \begin{bmatrix} 10 \\ 20 \end{bmatrix}$

3. A se giba po p_1

$x_{p_1} = [t \;\; 1] \begin{bmatrix} 2 & 2 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}$

B po p_2

$x_{p_2} = [t \;\; 1] \begin{bmatrix} 1 & 2 & 2 & 0 \\ 2 & 1 & -3 & 1 \end{bmatrix}$

t ... vrijeme $t = 0$

u kojem trenutku su objekti najbliže?

$d^2 = (x_{p_1} - x_{p_2})^2 + (y_{p_1} - y_{p_2})^2 + (z_{p_1} - z_{p_2})^2$

$d^2 = (2t+1 - t-2)^2 + (2t+1 - 2t-1)^2 + (t+1 - 2t+3)^2$

u ovom pr. su isti t jer je vrijeme

$d^2 = (t-1)^2 + 0^2 + (-t+4)^2$

$\frac{\partial d^2}{\partial t} = 2(t-1) - 2(-t+4) = 0$

$t = 1.5$

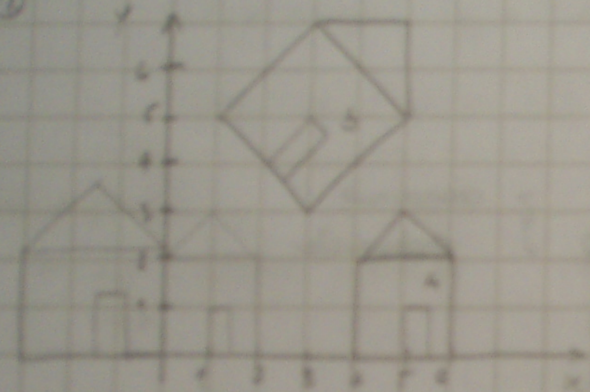
$37 \cdot 8$

636

$796 : 3 = 95$

13

1)



1.
$$T_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -4 & 0 & 1 \end{bmatrix}$$ translacija

2.
$$S = \begin{bmatrix} -\sqrt{2} & 0 & 0 \\ 0 & \sqrt{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$ skaliranje & zrcaljenje po x

3.
$$R(-\tfrac{\pi}{4}) = \begin{bmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$ rotacija

2. način:
Pomak naše ... u 2
1. SCAL(a, b)
2. ROT(φ)
3. TRANS(x_0, y_0)

4.
$$T_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 3 & 3 & 1 \end{bmatrix}$$

→ $a = \sqrt{2}$, $b = -\sqrt{2}$
$\varphi = \frac{3\pi}{2}$
$x_0 = 7$, $y_0 = -1$

$$D = T_1 \cdot S \cdot R \cdot T_2 = \begin{bmatrix} -1 & 1 & 0 \\ 1 & 1 & 0 \\ 7 & -1 & 1 \end{bmatrix}$$

2) LINEARNA INTERPOLACIJA

$x(t)$
$t \in [-1, 1]$
$x_0(-1) = 10$
$x_1(1) = 30$

$v = [t \;\; 1]\, L$
$[t \;\; 1] \begin{bmatrix} a \\ b \end{bmatrix}$

$x_0 = [-1 \;\; 1]\, L$ ... $\begin{bmatrix} x_0 \\ \end{bmatrix} = \begin{bmatrix} -1 & 1 \\ \end{bmatrix}$ → $\begin{bmatrix} -1 & 1 \\ \end{bmatrix}^{-1} \begin{bmatrix} x_0 \\ \end{bmatrix} = \frac{1}{\;} \begin{bmatrix} -1 & 1 \\ \end{bmatrix} \begin{bmatrix} x_1 \\ \end{bmatrix} = \begin{bmatrix} 10 \\ \end{bmatrix}$

2.) LINEARNA INTERPOLACIJA

$x(t)$

$t \in [-1, 1]$

$x_0(-1) = 10$

$x_1(1) = 30$

$v = [t\ 1]\,L$

$[t\ 1]\begin{bmatrix} \\ x_0 \end{bmatrix}$

$x_0 = [-1\ \ 1]\,L$

$x_1 = [1\ \ 1]\,L$

$$\begin{bmatrix} x_0 \\ x_1 \end{bmatrix} = \begin{bmatrix} -1 & 1 \\ 1 & 1 \end{bmatrix} L \longrightarrow L = \begin{bmatrix} -1 & 1 \\ 1 & 1 \end{bmatrix}^{-1} \begin{bmatrix} x_0 \\ x_1 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} -1 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 10 \\ 20 \end{bmatrix}$$

$2, 2 \times 2\ 1$

$$v = [t\ \ 1]\,L = [t\ \ 1]\begin{bmatrix} 10 \\ 20 \end{bmatrix}$$

3.) A ide po P_1

$$x_{P_1} = [t\ \ 1]\begin{bmatrix} 2 & 2 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

B po P_2

$$x_{P_2} = [t\ \ 1]\begin{bmatrix} 1 & 2 & 2 & 0 \\ 2 & 1 & -3 & 1 \end{bmatrix}$$

t ... infame t=0

u kojem trenutku su najbliže?

$$d^2 = (x_{P_1} - x_{P_2})^2 + (y_{P_1} - y_{P_2})^2 + (z_{P_1} - z_{P_2})^2$$

$$d^2 = (2t+1 - t-2)^2 + (2t+1 - 2t-1)^2 + (t+1 - 2t+3)^2$$

u ovom po tu isk – jer je infame

$$d^2 = (t-1)^2 + 0^2 + (-t+4)^2$$

$$\frac{\partial d^2}{\partial t} = 2(t-1) - 2(-t+4) = 0$$

$$t = 1.5$$

$37 \cdot 8$

$\overline{236}$

$256 : 3 = 85$

16

(3) RAVNINA $R = [2\ 5\ 3\ 1]^T$

$V = [1\ 1\ 1]$ u ravnini prostoru

odredi ortogonalnu proj. na ravninu

$p = [t\ \ 1] \begin{bmatrix} 2 & 5 & 3 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix} \longrightarrow$ pravac $\perp$ na r } odredimo

2 točke koje su ... ⊙ } probodište

$\left.\begin{aligned} x_p &= 2t+1 \\ y_p &= 5t+1 \\ z_p &= 3t+1 \end{aligned}\right\}$ točka na pravcu – uvrstimo probodište u R je 0

$$[2t+1 \quad 5t+1 \quad 3t+1] \begin{bmatrix} 2 \\ 5 \\ 3 \\ 1 \end{bmatrix} = 0$$

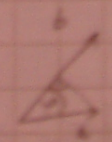
$$t = -\frac{11}{38} \longrightarrow v' = \left[\frac{16}{38} \quad -\frac{17}{38} \quad \frac{5}{38}\right]$$

(5) $\vec{a} = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} \quad \vec{b} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$

$\vec{a} \cdot \vec{b} = a_1 b_1 + a_2 b_2 + a_3 b_3$

$\|\vec{a}\| =$

$\cos\varphi = \dfrac{\vec{a} \cdot \vec{b}}{\|\vec{a}\| \|\vec{b}\|}$



<u>projekcija</u> $\vec{a}$ na $\vec{b}$

$$\cos\varphi \|\vec{a}\| = \frac{\vec{a} \cdot \vec{b}}{\|\vec{b}\|}$$

<u>projekcija</u> $\vec{b}$ na $\vec{a}$

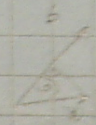
$t = -\frac{11}{38} \longrightarrow v' = \left[\frac{13}{19} \quad -\frac{15}{19} \quad \frac{5}{19}\right]$

④ $\vec{a} = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} \quad \vec{b} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$

$\vec{a} \cdot \vec{b} = a_1 b_1 + a_2 b_2 + a_3 b_3$

$\|\vec{a}\| =$

$\cos\varphi = \frac{\vec{a} \cdot \vec{b}}{\|\vec{a}\|\|\vec{b}\|}$



Projekcija $\vec{a}$ na $\vec{b}$

$\cos\varphi \|\vec{a}\| = \frac{\vec{a} \cdot \vec{b}}{\|\vec{b}\|}$

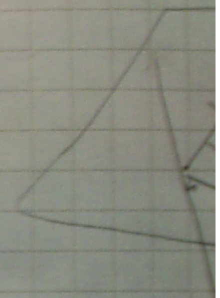
Projekcija $\vec{b}$ na $\vec{a}$

$\cos\varphi \|\vec{b}\| = \frac{\vec{b} \vec{a}}{\|\vec{a}\|}$

⑦ $v_0 = [1$

$v_1 = [0$

$v_2 = [$



⑤ $R = [2 \quad 5 \quad 3 \quad 1]^T$

$v = [1 \quad 1 \quad 1 \quad 1]$

odredi udaljenost ①

$R = [a \quad b \quad c \quad d]$

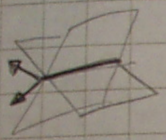
$f = \sqrt{a^2+b^2+c^2}$

$\frac{a}{f}x + \frac{b}{f}y + \frac{c}{f}z + \frac{d}{f} = 0$

$R = \left[\frac{2}{\sqrt{38}} \quad \frac{5}{\sqrt{38}} \quad \frac{3}{\sqrt{38}} \quad \frac{1}{\sqrt{38}}\right]$

$d = 1.78$

⑥ $R_1 = [-3 \quad 2 \quad 1 \quad 10]^T$
$R_2 = [3 \quad 3 \quad 5 \quad 10]$



$n_1 = [-3 \quad 2 \quad 1]$
$n_2 = [3 \quad 3 \quad 5]$

$n_1 \times n_2 = \begin{bmatrix} 7 \\ 18 \\ -15 \end{bmatrix}$

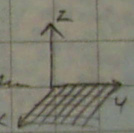
$x = [t \quad 1] \begin{bmatrix} 7 & 18 & -15 & 0 \\ \frac{2}{3} & -4 & 0 & 1 \end{bmatrix}$

↳ bilo koja može biti

kalkulator s pridruživanjem za inverz

$-3x_1 + 2x_2 + x_3 + 10x_4 = 0$
$3x_1 + 3x_2 + 5x_3 + 10x_4 = 0$

odaberemo $x_4 = 1$ ~ homogeno komp.
uzmemo $x_3 = 0$ ~ probodište pravca kroz xy ravninu
(ako ne postoji odaberemo neku drugu kao 0)

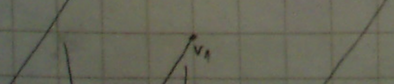


$\left.\begin{matrix} x_3 = 0 \\ x_4 = 1 \end{matrix}\right\} \quad \begin{matrix} x_1 = \frac{2}{3} \\ x_2 = -4 \end{matrix}$

② 2 točke ~ pravac
③ jednu proglasimo parametrom ...

⑦ $\left.\begin{matrix} v_0 = [10 \quad 20 \quad 10] \\ v_1 = [0 \quad 20 \quad 20] \end{matrix}\right\}$ dužina
$v_2 = [20 \quad 20 \quad 20]$

rotirati tako da sve 3 leže na pravcu!!



rotiramo oko v_0 - treba odrediti p kroz v_0 $\perp$ na R

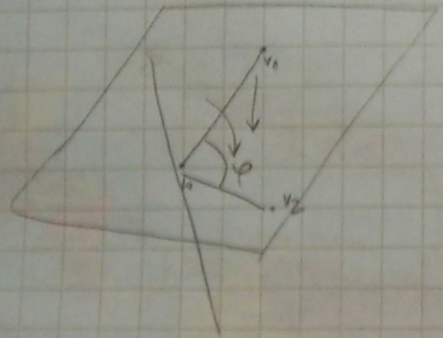
drugu kao o)

$y_2 = 0 \quad x_1 = \frac{?}{8}$
$y_1 = 1 \quad x_2 = -4$

1) 2 točke + pravac
2) jednu proglasimo parametrom ...

(7.) $v_0 = [10 \;\; 20 \;\; 10]$
$v_1 = [0 \;\; 20 \;\; 20]$ } dužina
$v_2 = [30 \;\; 20 \;\; 30]$

rotirati tako da sve 3 leže na pravcu!!



rotiramo oko v_0 – treba odrediti p tako $v_0 \perp$ na R

$v_1 - v_0 = [-10 \;\; 0 \;\; 10]$
$v_2 - v_0 = [10 \;\; 0 \;\; 10]$

$[v_1 - v_0][v_2 - v_0] = \begin{bmatrix} 0 \\ 200 \\ 0 \end{bmatrix}$ ~ vektorski produkt

$x = [t \;\; 1] \begin{bmatrix} 0 & 200 & 0 & 0 \\ 10 & 20 & 10 & 1 \end{bmatrix}$ ~ os rotacije

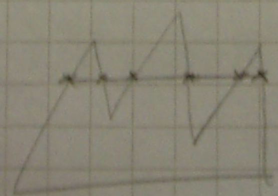
translacija nije u kr. trenutku → iste točke izražavamo u razl. koor. sustavima

$\cos\varphi = \dfrac{\overrightarrow{v_1 - v_0} \; \overrightarrow{v_2 - v_0}}{\|v_1 - v_0\| \, \|v_2 - v_0\|} = \dfrac{0}{\sqrt{200}\sqrt{200}} = 0 \rightarrow \varphi = 90^\circ$ ~ kut rotacije

??

$\sqrt{10^2 + 10^2}$

treba paziti & na smjer
imamo 2 rješenja!!



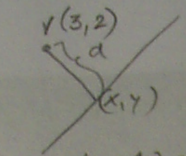
①

$$G_1 = \begin{bmatrix} -1 \\ 4 \\ -10 \end{bmatrix}$$

$$x_1 = [3, 2, 1]$$

odredi udaljenost točke do pravca!

$\boxed{\text{1. način}}$



$$-x + 4y - 10 = 0$$
$$x = 4y - 10$$

$$d^2 = (x-3)^2 + (y-2)^2$$

MIN d!!

$$d^2 = (4y - 13)^2 + (y-2)^2$$

$$\frac{\partial d}{\partial y} = 2(4y - 13) \cdot 4 + 2(y-2) = 0$$

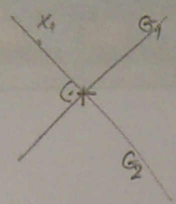
$$17y = 54$$

$$y = \frac{54}{17}$$

$$x = 4y - 10 = \frac{46}{17}$$

$$d = \sqrt{(x-3)^2 + (y-2)^2} = \frac{5\sqrt{17}}{17} = 1.213$$

$\boxed{\text{2. način}}$



$$G_1 = \begin{bmatrix} -1 \\ 4 \\ -10 \end{bmatrix}$$

$$G_2 = \begin{bmatrix} 4 \\ 1 \\ c \end{bmatrix} !$$

$$4x + y + c = 0$$
$$c = -14$$

$$G_2 = \begin{bmatrix} 4 \\ 1 \\ -14 \end{bmatrix}$$

$$x_1 = [3 \;\; 2 \;\; 1]$$

$$x = G_1 \times G_2 = \begin{bmatrix} \vec{i} & \vec{j} & \vec{k} \\ -1 & 4 & -10 \\ 4 & 1 & -14 \end{bmatrix} = \begin{bmatrix} -46 \\ -54 \\ -17 \end{bmatrix}$$

$$x = -\frac{46}{-17}$$

$$y = \frac{-54}{-17}$$

$\boxed{\text{3. način}}$ ~ implicitni oblik

$$ax + by + c = 0 \, / \sqrt{a^2 + b^2} \rightarrow \text{normirani oblik}$$

$$G_1 = \begin{bmatrix} -1 \\ 4 \\ -10 \end{bmatrix} = \begin{bmatrix} -\frac{1}{\sqrt{17}} \\ \frac{4}{\sqrt{17}} \\ -\frac{10}{\sqrt{17}} \end{bmatrix}$$

$$x = [3, 2, 1]$$

$$d = -\frac{x}{\sqrt{17}} + \frac{4y}{\sqrt{17}} - \frac{10}{\sqrt{17}} = \ominus \frac{5}{\sqrt{17}}$$

↳ ispod pravca

→ isto vrijedi i za ravninu !!!