Kvantna računala

① Koji od navedenih vektora nije normiran?

a) $\frac{3}{5}|0\rangle - \frac{4}{5}|1\rangle$

b) $\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$ ✓

c) $\frac{1}{2}|0\rangle - \frac{\sqrt{3}}{2}|1\rangle$

d) $|0\rangle - i|1\rangle$

e) $\frac{5}{13}|0\rangle - \frac{12}{13}|1\rangle$

$\|\Phi\| = \sqrt{\langle\Phi|\Phi\rangle}$

$\|\Phi\| = |\lambda|^2 + |\mu|^2$

↳ ako je 1 onda je normiran

② Koja 2 od navedenih stanja qubita čine ortonormiranu bazu u $\mathcal{H}^{(2)}$?

a) $\frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle$

b) $\frac{i}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle$

c) $-\frac{1}{2}|0\rangle - \frac{i}{\sqrt{2}}|1\rangle$

d) $\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$

e) $\frac{1}{\sqrt{2}}|0\rangle + \frac{i}{\sqrt{2}}|1\rangle$

$\langle x|y\rangle = 0 \qquad \langle x|x\rangle = \langle y|y\rangle = 1$

③ Koji od navedenih vektora predstavljaju isto stanje kvantnog bita?

a) $\frac{2}{\sqrt{5}}|0\rangle + \frac{1}{\sqrt{5}}|1\rangle$

b) $-\frac{2}{\sqrt{5}}|0\rangle - \frac{1}{\sqrt{5}}|1\rangle$

c) $\frac{1}{\sqrt{5}}|0\rangle + \frac{2i}{\sqrt{5}}|1\rangle$

d) $-\frac{2i}{\sqrt{5}}|0\rangle - \frac{i}{\sqrt{5}}|1\rangle$

e) $\frac{2}{\sqrt{5}}|0\rangle - \frac{i}{\sqrt{5}}|1\rangle$

→ vektori imaju isto stanje qubita ako se razlikuju samo u fazi, tj za faktor $e^{i\varphi}$

→ zadatak se rješava tako da se uzme prvi prvi vektor dok ostale množimo redom s $-1, i, -i$ i vidimo jesmo li dobili isti vektor kao prvi

④ Kvantni bit je pripremljen u stanju: $\frac{4}{5}|0\rangle + \frac{3}{5}|1\rangle$. Izračunaj vjerojatnost da taj kvantni bit bude izmjeren u stanju $\frac{4}{5}|0\rangle + \frac{3i}{5}|1\rangle$

a) $\frac{\sqrt{337}}{25}$

b) $\frac{16}{25} - \frac{9i}{25}$

c) $\frac{16}{25} + \frac{9i}{25}$

d) $\frac{337}{625}$

e) 0

$|\Phi\rangle = \frac{4}{5}|0\rangle + \frac{3}{5}|1\rangle$

$|\Psi\rangle = \frac{4}{5}|0\rangle + \frac{3i}{5}|1\rangle$

$P(\Phi\to\Psi) = ?$

$P(\Phi\to\Psi) = |\langle\Phi|\Psi\rangle|^2$

$\langle\Phi|\Psi\rangle = \Phi_1\Psi_1 + \Phi_2\Psi_2 = \frac{16}{25} + \frac{9i}{25}$

$= \left|\frac{16}{25} + \frac{9i}{25}\right|^2 = |a+bi| = \sqrt{a^2+b^2}$

$= \left(\frac{16}{25}\right)^2 + \left(\frac{9}{25}\right)^2 = \frac{337}{625}$

(5) Operator $\frac{1}{2}(|0\rangle\langle 0| - |0\rangle\langle 1| - |1\rangle\langle 0| + |1\rangle\langle 1|)$ je projektor na stanje:

a) $\frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle$

b) $\frac{1}{\sqrt{2}}|0\rangle - \frac{i}{\sqrt{2}}|1\rangle$

c) $|0\rangle$ ili $|1\rangle$

d) $\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$

e) $\frac{1}{\sqrt{2}}|0\rangle + \frac{i}{\sqrt{2}}|1\rangle$

"stanje": $|\psi| = u|0\rangle + \sigma|1\rangle = \begin{bmatrix} u \\ \sigma \end{bmatrix}$ → odredit u i σ da bi se dobilo stanje

"projektor na stanje" $P_\psi = |\psi\rangle\langle\psi| = \begin{bmatrix} u \\ \sigma \end{bmatrix}\begin{bmatrix} u^* & \sigma^* \end{bmatrix} = \begin{bmatrix} uu^* & u\sigma^* \\ \sigma u^* & \sigma\sigma^* \end{bmatrix}$

→ operator zadan u zad. prebacujemo u matrični prikaz te iz toga određujemo u i σ

$\begin{bmatrix} \frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} \end{bmatrix}$ → prebacit operator u projektor na stanje kako bismo jasno mogli iščitati u i σ

$\begin{bmatrix} \frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}} \end{bmatrix}$ $u = \frac{1}{2}$, $\sigma = -\frac{1}{2}$

(6) Izračunaj očekivanu vrijednost hermitskog operatora $|0\rangle\langle 0| - |1\rangle\langle 1|$ u sustavu koji se nalazi u stanju $\frac{3}{\sqrt{10}}|0\rangle + \frac{1}{\sqrt{10}}|1\rangle$

a) 0

b) $-\frac{7}{25}$

c) $-\frac{4}{5}$

d) $\frac{7}{25}$

e) $\frac{4}{5}$

$U = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ $U\phi = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \cdot \begin{bmatrix} \frac{3}{\sqrt{10}} \\ \frac{1}{\sqrt{10}} \end{bmatrix} = \begin{bmatrix} \frac{3}{\sqrt{10}} \\ -\frac{1}{\sqrt{10}} \end{bmatrix}$

$U\langle\phi\rangle = \langle\phi|U|\phi\rangle = \begin{bmatrix} \frac{3}{\sqrt{10}} & \frac{1}{\sqrt{10}} \end{bmatrix} \begin{bmatrix} \frac{3}{\sqrt{10}} \\ -\frac{1}{\sqrt{10}} \end{bmatrix} = \frac{9}{10} - \frac{1}{10} = \frac{8}{10} = \frac{4}{5}$

(7) Matrični prikaz $\begin{pmatrix} \frac{1}{2} & \frac{i}{2} \\ -\frac{i}{2} & \frac{1}{2} \end{pmatrix}$ odgovara operatoru:

a) $\frac{1}{2}(|0\rangle\langle 0| - i|0\rangle\langle 1| + i|1\rangle\langle 0| + |1\rangle\langle 1|)$

b) $\frac{1}{2}(|0\rangle\langle 0| - |0\rangle\langle 1| - |1\rangle\langle 0| + |1\rangle\langle 1|)$

c) $\frac{1}{2}(|0\rangle\langle 0| + i|0\rangle\langle 1| + i|1\rangle\langle 0| + |1\rangle\langle 1|)$

d) $\frac{1}{2}(|0\rangle\langle 0| + i|0\rangle\langle 1| - i|1\rangle\langle 0| + |1\rangle\langle 1|)$ → to

e) $\frac{1}{2}(|0\rangle\langle 0| + i|0\rangle\langle 1| + i|1\rangle\langle 0| + |1\rangle\langle 1|)$

(8) Ako je energija kvantnog bita opisana hamiltonijanom $H = \hbar w|0\rangle\langle 0|$, $w > 0$ i ako je poč. stanje kvantnog bita $\frac{1}{\sqrt{2}}|0\rangle + \frac{i}{\sqrt{2}}|1\rangle$ nakon $\Delta t = \frac{\pi}{2w}$ sustav će biti u stanju:

a) $|1\rangle$

b) $\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$

c) $\frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle$

d) $|0\rangle$

e) $\frac{1}{\sqrt{2}}|0\rangle - \frac{i}{\sqrt{2}}|1\rangle$

f) $\frac{1}{\sqrt{2}}|0\rangle + \frac{i}{\sqrt{2}}|1\rangle$

prez (2)

$w_1 = 0$

$\mu_0 = \frac{1}{\sqrt{2}}$ $\lambda_0 = \frac{i}{\sqrt{2}}$

$\frac{d\lambda}{dt} = 0$ → $\lambda(t) = \text{konst}$

$\mu(t) = \mu_0 \cdot e^{-iw_0 t} \overset{t=\frac{\pi}{2w}}{=} \mu_0 e^{-\frac{\pi}{2}} = \mu_0 \cdot (-i)$

$|\Phi(t)\rangle = \frac{-i}{\sqrt{2}}|0\rangle + \frac{i}{\sqrt{2}}|1\rangle$ $|\cdot i$ → jer ovaj nije ponuđen

(9) Alice i Bob uspostavljaju tajni enkripcijski ključ korištenjem protokola BB84. Alice odabire bazu $\oplus$ i odašilje foton u stanju 0. Kolika je vjerojatnost da Bob izmjeri vrijednost 0 ako on odabere bazu $\otimes$, a Eve prisluškuje komunikaciju?

a) $\frac{1}{4}$

b) $\frac{3}{8}$

c) $\frac{1}{2}$

d) $\frac{5}{8}$

e) $\frac{3}{4}$

$0 \quad 0 \rightarrow \frac{5}{8}$

$\otimes 1 \quad \oplus 0 \rightarrow \frac{1}{2}$

$\otimes 1 \quad \otimes 0 \rightarrow \frac{1}{4}$

$\oplus 0 \quad \oplus 0 \rightarrow \frac{3}{4}$

(za 0, 0 bez baza)

→ da nema prisluškivanja: vjerojatnost da Bob pogodi bazu 50%, da fula isto 50% (ako je fulao 50% da pogodi bit)

$P = \frac{1}{2}(\text{pog. bazu}) \cdot 1(\text{pog. bit}) + \frac{1}{2}(\text{fulao bazu}) \cdot \frac{1}{2}(\text{pog. bit}) = \frac{3}{4}$

→ prisluškivanje utječe samo na slučaj kada pogodi bazu (25%)

$P = \frac{1}{2}(\text{pog. bazu}) \cdot \frac{3}{4}(\text{Eve}) + \frac{1}{2} \cdot \frac{1}{2} = \frac{5}{8}$

(10) U kojima od navedenih stanja sustava dvaju kvantnih bitova su stanja samih bitova spregnuta?

a) $\frac{1}{2}\left(|00\rangle + |01\rangle - i|10\rangle + i|11\rangle\right)$ ✓

b) $\frac{1}{2}\left(|00\rangle - i|01\rangle + |10\rangle - i|11\rangle\right)$

c) $\frac{1}{\sqrt{2}}\left(|00\rangle - i|10\rangle\right)$

d) $\frac{1}{\sqrt{3}}\left(|00\rangle + i|01\rangle + i|10\rangle\right)$ ✓

e) $\frac{1}{2}\left(|00\rangle + |01\rangle + |10\rangle - i|11\rangle\right)$ ✓

→ spregnuta stanja nije moguće prikazati produktom $\otimes$

$$|\phi\rangle = |\phi_A \otimes \phi_B\rangle = \alpha|00\rangle + \beta|01\rangle + \gamma|10\rangle + \delta|11\rangle$$

– mora vrijediti $\alpha\delta \neq \beta\gamma$ da bi stanja bila spregnuta

+ Gabin drugi

a) $-\frac{2i}{\sqrt{5}}|0\rangle - \frac{i}{\sqrt{5}}|1\rangle$

b) $-\frac{2}{\sqrt{5}}|0\rangle - \frac{1}{\sqrt{5}}|1\rangle$

c) $\frac{2}{\sqrt{5}}|0\rangle - \frac{i}{\sqrt{5}}|1\rangle$

d) $\frac{2}{\sqrt{5}}|0\rangle + \frac{1}{\sqrt{5}}|1\rangle$

e) $\frac{1}{\sqrt{5}}|0\rangle + \frac{2i}{\sqrt{5}}|1\rangle$

$(a|b)\ \frac{-2i}{\sqrt{5}} \cdot \frac{-2}{\sqrt{5}} + \frac{-i}{\sqrt{5}} \cdot \frac{-1}{\sqrt{5}} = \frac{4i}{5} + \frac{i}{5} = i$ ✗

$(a|c) = \frac{-2i}{\sqrt{5}} \cdot \frac{2}{\sqrt{5}} - \frac{i}{\sqrt{5}} \cdot \left(\frac{-i}{\sqrt{5}}\right)^* = \frac{-4i}{5} + \frac{1}{5}$ ✗

$(a|d) = \frac{-2i}{\sqrt{5}} \cdot \frac{2}{\sqrt{5}} - \frac{i}{\sqrt{5}} \cdot \frac{1}{\sqrt{5}} = \frac{-4i}{\sqrt{5}} - \frac{i}{\sqrt{5}} = -i$ ✗

$(a|e) = \frac{-2i}{\sqrt{5}} \cdot \frac{1}{\sqrt{5}} - \frac{i}{\sqrt{5}} \cdot \left(\frac{2i}{\sqrt{5}}\right)^* = -\frac{2i}{5} + \frac{2}{5}$ ✗

$(b|c) = \frac{-2}{\sqrt{5}} \cdot \frac{2}{\sqrt{5}} - \frac{1}{\sqrt{5}} \cdot \left(\frac{-i}{\sqrt{5}}\right)^*$ ✗

$(b|d) = -\frac{2}{\sqrt{5}} \cdot \frac{2}{\sqrt{5}} - \frac{1}{\sqrt{5}} \cdot \frac{1}{\sqrt{5}} = \frac{-4}{5} - \frac{1}{5} = -1$

$(c|e) \Rightarrow \frac{1}{\sqrt{5}} \cdot \frac{2}{\sqrt{5}} + \frac{2i}{\sqrt{5}} \cdot \left(\frac{-i}{\sqrt{5}}\right)^* = \frac{2}{5} - \frac{2}{5} = \emptyset$

MI 2018.

1. Koji vekt. nijel su norm.?

a) $\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$ → $\frac{1}{2}+\frac{1}{2}=1$ ✓

b) $\frac{1}{\sqrt{5}}|0\rangle + i\frac{2}{\sqrt{5}}|1\rangle$ $\frac{1}{5}+\left|-\frac{4}{5}\right|=1$ ✓

→ c) $\frac{3}{\sqrt{5}}|0\rangle - i\frac{2}{\sqrt{5}}|1\rangle$ $\frac{9}{5}+\left|\frac{4}{5}\right|=\frac{13}{5}$

d) $\frac{1}{\sqrt{10}}|0\rangle + \frac{3}{\sqrt{10}}|1\rangle$ $\frac{1}{10}+\frac{9}{10}=1$ ✓

e) $\frac{5}{\sqrt{29}}|0\rangle - \frac{2}{\sqrt{29}}|1\rangle$ $\frac{25}{29}+\frac{4}{29}=1$ ✓

2. Koji čine ortonor. bazu u H^2

a) $\frac{1}{\sqrt{13}}(2|0\rangle + 3|1\rangle)$ $\frac{4}{13}+\frac{9}{13}=1$ ✓

b) $\frac{1}{\sqrt{13}}(3|0\rangle + 2|1\rangle)$

→ c) $\frac{1}{\sqrt{13}}(2|0\rangle + 3i|1\rangle)$

d) $\frac{1}{\sqrt{13}}(3|0\rangle + 2i|1\rangle)$

e) $\frac{1}{\sqrt{13}}(3|0\rangle - 2i|1\rangle)$

$(a|b) = 6+6 \neq 0$ ×
$(a|d) = 6+6i \neq 0$ ×
$(a|e) = 6+6i \neq 0$ ×

$(b|c) = 6+6i \neq 0$ ×
$(c|d) = 6+6 \neq 0$ ×
$(c|e) = 6-6 = 0$ ✓

3. Koji vekt. predstavljaju isto stanje kv. bita?

→ a) $\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$

b) $\frac{1}{\sqrt{2}}(-|0\rangle - |1\rangle)$

c) $\frac{1}{\sqrt{2}}(|0\rangle - i|1\rangle)$

d) $\frac{1}{\sqrt{2}}(|0\rangle + i|1\rangle)$

→ e) $\frac{1}{\sqrt{2}}(i|0\rangle + i|1\rangle)$

4. Stanje kv. bita je: $\cos\frac{\vartheta}{2}|0\rangle + e^{i\varphi}\sin\frac{\vartheta}{2}|1\rangle$. Vjeroj. da bude u $|1\rangle$?

$|\theta\rangle = \cos\frac{\vartheta}{2}|0\rangle + e^{i\varphi}\sin\frac{\vartheta}{2}|1\rangle$

$P(\theta\to 1) = |\langle 1|\theta\rangle|^2 = |\langle 1|\theta\rangle|^2 = \left|e^{i\varphi}\sin\frac{\vartheta}{2}\right|^2$

→ vjerojatnost ovisi samo o ϑ jer je $|e^{\varphi i}| = \sqrt{\cos^2\varphi + \sin^2\varphi} = 1$

5. Koje stanje ima najveću vjerojatnost nalaženja u $|0\rangle$?

a) $\frac{1}{13}(12i|0\rangle + 5|1\rangle)$

b) $\frac{1}{\sqrt{5}}(2|0\rangle - i|1\rangle)$

c) $\frac{1}{\sqrt{3}}(|0\rangle - \sqrt{2}|1\rangle)$

d) $\frac{1}{\sqrt{7}}(\sqrt{3}|0\rangle + 2i|1\rangle)$

e) $\frac{1}{3}(\sqrt{5}|0\rangle - 2i|1\rangle)$

$P(\theta\to 0) = |\langle 0|\theta\rangle|^2 =$ a) $= \frac{144}{169}$ ✓

$b = \frac{4}{5}$ $d = \frac{3}{7}$

$c = \frac{1}{3}$ $e = \frac{5}{9}$

6. Stanje kvantnog bita je $|\psi\rangle = e^{-i\varphi/2}\cos\frac{\vartheta}{2}|0\rangle + e^{i\varphi/2}\sin\frac{\vartheta}{2}|1\rangle$
Djelovanje operatora Pauli-Z na to stanje istovjetno je zamjeni:
σ_z ili σ_3

a) $\varphi \to \varphi \pm 2\pi$

→ b) $\varphi \to \varphi \pm \pi$

c) $\vartheta \to \vartheta \pm 2\pi$

d) $\vartheta \to \vartheta \pm \pi$

e) $\vartheta \to \pi - \vartheta$

$\sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

Pauli-Z
→ uzrokuje rotaciju oko z-osi gledajući Blochovu sferu to je istovjetno zamjeni

$|0\rangle$ isto
$|1\rangle$ u $-|1\rangle$

2 prez

$e^{ix} = \cos(x) + \sin(x)$
$e^{-ix} = \cos(x) - \sin(x)$

$\sin(\theta+\frac{\pi}{2}) = \cos\theta$ $\sin(\theta+\pi) = -\sin\theta$
$\cos(\theta+\frac{\pi}{2}) = -\sin\theta$ $\cos(\theta+\pi) = -\cos\theta$

⑦ Operator projekcije na stanje $\frac{1}{\sqrt{2}}(|0\rangle - i|1\rangle)$ odgovara matrica
② prez

$\frac{1}{\sqrt{2}}(|0\rangle - i|1\rangle)$

stanje $\Longrightarrow$ $|\psi| = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{-i}{\sqrt{2}} \end{bmatrix}$

projektor na stanje $\Longrightarrow$ $P_\psi = |\psi\rangle\langle\psi| = \begin{bmatrix} \frac{1}{2} & \frac{i}{2} \\ -\frac{i}{2} & \frac{1}{2} \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 1 & i \\ -i & 1 \end{bmatrix}$ ✓

⑧ Ako je hamiltonijan kvantnog bita $H = \frac{\hbar w_0}{2}|0\rangle\langle 0| - \frac{\hbar w_1}{2}|1\rangle\langle 1|$ te ako je poč. stanje kv. bita $\frac{1}{\sqrt{5}}(2|0\rangle + |1\rangle)$ taj će se kv. naći u stanju $\frac{1}{\sqrt{5}}(|0\rangle + 2|1\rangle)$ nakon vremena:

$w_0 > w_1$

$|\Phi[t]\rangle = \mu[t]|0\rangle + \lambda[t]|1\rangle \qquad \mu[0] = \mu_0 \xrightarrow{} \frac{2}{\sqrt{5}} \qquad \lambda[0] = \lambda_0 \xrightarrow{} \frac{1}{\sqrt{5}}$

$$i\hbar\left(\frac{d}{dt}\mu[t]|0\rangle + \frac{d}{dt}\lambda[t]|1\rangle\right) = \hbar\frac{w_0}{2}\mu[t]|0\rangle + \hbar\frac{w_1}{2}\lambda[t]|1\rangle$$

$$\frac{d\mu}{\mu} = -i\frac{w_0}{2}dt \qquad \frac{d\lambda}{\lambda} = -i\frac{w_1}{2}dt$$

$$\mu[t] = \mu_0 e^{-i\frac{w_0 t}{2}} \qquad \lambda[t] = \lambda_0 e^{-i\frac{w_1 t}{2}}$$

$$|\Phi[t]\rangle = \mu_0 e^{-i\frac{w_0 t}{2}}|0\rangle + \lambda_0 e^{-i\frac{w_1 t}{2}}|1\rangle$$

$$\frac{1}{\sqrt{5}}|0\rangle + \frac{2}{\sqrt{5}}|1\rangle = \frac{2}{\sqrt{5}}e^{-\frac{w_0}{2}t}|0\rangle + \lambda_0 e^{-i\frac{w_1}{2}t}|1\rangle$$

$w_0 = w_1 = w$ $\quad t = ?$

$\rightarrow$ ao se nikad neće dogodit jer

$$\frac{2}{\sqrt{5}}e^{-\frac{w}{2}t} \neq \frac{1}{\sqrt{5}} \quad \text{za bilo koji } t$$

9. Sustav dvaju kv. bitova ostvaren je projekcijama spinova dviju čestica spinskog kvantnog broja $s=\frac{1}{2}$ na z-os. Matrični prikaz hermitskog operatora koji opisuje zbroj projekcija na z-os je

$$\hbar\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}.$$ Radi se o operatoru?

4 prezentacija

a) $\frac{\hbar}{2}\sigma_z \otimes 1 = \frac{1}{2}\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$ ×

b) $1 \otimes \frac{\hbar}{2}\sigma_z = \frac{1}{2}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$ ×

c) a)+b) $= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$ ✓

10 Op. (gustoće) stanja kvantnog bita je $\rho = \frac{1}{3}|0\rangle\langle 0| + \frac{2}{3}|1\rangle\langle 1|$
očekivana vrijednost operatora σ_z za taj kv. bit je

$$\begin{bmatrix} \frac{1}{3} & 0 \\ 0 & \frac{2}{3} \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} = \frac{1}{3} + 0 + 0 - \frac{2}{3} = -\frac{1}{3}$$

MI 2017

1 Koji od navedenih vektora su jedinični vektori? → vektori čiji je modul 1

a) $\frac{1}{\sqrt{2}}(|0\rangle + i|1\rangle)$

b) $\frac{\sqrt{3}}{2}(|0\rangle + \frac{1}{2}i|1\rangle)$

c) $\frac{1}{3}|0\rangle - \frac{2\sqrt{2}}{3}|1\rangle$

d) $\frac{1}{\sqrt{5}}(2|0\rangle + |1\rangle)$

e) $\frac{\sqrt{3}}{2}(|0\rangle - \frac{1}{2}i|1\rangle)$

a) $\left|\frac{1}{\sqrt{2}} + i\right| = \sqrt{\frac{1}{2} + 1} = \sqrt{\frac{3}{2}}$

b) $\sqrt{\frac{3}{4} + \frac{1}{4}} = 1$

c) $\sqrt{\frac{1}{9} + \frac{8}{9}} = 1$

d) $\sqrt{\frac{4}{5} + \frac{1}{5}} = 1$

e) $\sqrt{\frac{3}{4} + \frac{3}{16}} = \sqrt{\frac{15}{16}}$

$$|a + bi| = \sqrt{a^2 + b^2}$$

2) Koja 2 od 5 vektora čine ortonormiranu bazu u $\mathcal{H}^2$?

	$\cdot(-1)$	$\cdot i$	$\cdot(-i)$
X a)	$-\frac{1}{\sqrt2}\|0\rangle - \frac{1}{\sqrt2}\|1\rangle$	$\frac{1}{\sqrt2}i\|0\rangle + \frac{1}{\sqrt2}i\|1\rangle$	$-\frac{1}{\sqrt2}i\|0\rangle - \frac{i}{\sqrt2}\|1\rangle$
X b)	$-\frac{1}{\sqrt2}\|0\rangle + i\|1\rangle$	$-\frac{i}{\sqrt2}\|0\rangle - \|1\rangle$	$-\frac{i}{\sqrt2}\|0\rangle + \|1\rangle$
c)	$-\frac{1}{\sqrt2}\|0\rangle - \frac12(1+i)\|1\rangle$	$-\frac{i}{\sqrt2}\|0\rangle + \frac12(1-i)\|1\rangle$	$\frac{i}{\sqrt2}\|0\rangle - \frac12(1-i)\|1\rangle$
d)	$-\frac{1}{\sqrt2}\|0\rangle - \frac12(1-i)\|1\rangle$	$-\frac{i}{\sqrt2}\|0\rangle + \frac12(1+i)\|1\rangle$	$\frac{i}{\sqrt2}\|0\rangle + \frac12(1+i)\|1\rangle$
e)	$-\frac{1}{\sqrt2}\|0\rangle + \frac12(1+i)\|1\rangle$	$-\frac{i}{\sqrt2}\|0\rangle - \frac12(1-i)\|1\rangle$	$\frac{i}{\sqrt2}\|0\rangle + \frac12(1-i)\|1\rangle$

smotana sam

a) $\frac{1}{\sqrt2}(|0\rangle + |1\rangle)$ $\qquad \frac12 + \frac12 = 1$ ✓

b) $\frac{1}{\sqrt2}(|0\rangle - i|1\rangle)$ $\qquad \frac12 + \frac12 = 1$ ✓

c) $\frac{1}{\sqrt2}|0\rangle + \frac12(1+i)|1\rangle$ $\qquad \frac12 + \frac24 = 1$ ✓

d) $\frac{1}{\sqrt2}|0\rangle + \frac12(1-i)|1\rangle$ $\qquad \frac12 + \frac24 = 1$ ✓

e) $\frac{1}{\sqrt2}|0\rangle - \frac12(1+i)|1\rangle$ $\qquad \frac12 + \frac24 = 1$ ✓

a-b) $\frac12 + \frac12 i \neq 0$

c-d) $\frac12 + \frac14(1-i)(1-i) = \frac12 + \frac14(1-i-i-1) \neq 0$

c-e) $\frac12 - \frac14(1-i)(1+i) = \frac12 - \frac14(1+i-i+1) = \frac12 - \frac24 = \emptyset$ ✓

3) Neka se qubit nalazi u stanju $|\phi\rangle = \frac15(3|0\rangle + 4i|1\rangle)$. U kojem od 5 navedenih stanja je vjerojatnost nalaženja tog qubita min?

a) $\frac{1}{13}(12i|0\rangle + 5|1\rangle)$

b) $\frac{1}{\sqrt5}(2|0\rangle - i|1\rangle)$

c) $\frac{1}{\sqrt3}(|0\rangle - \sqrt2|1\rangle)$

d) $\frac{1}{\sqrt7}(\sqrt3|0\rangle + 2i|1\rangle)$

e) $\frac13(\sqrt5|0\rangle - 2i|1\rangle)$

$|\phi\rangle = \frac15(3|0\rangle + 4i|1\rangle)$

$|\psi\rangle = a, b, c, d, e$

$P(\phi \to \psi) = |\langle\phi|\psi\rangle|^2$

a) $= \left|\frac35 \cdot \frac{12i}{13} + \frac{4i}{5}\cdot\frac{5}{13}\right|^2 = \left|\frac{56}{65}\right|^2 = 0{,}74$

b) $= \left|\frac35 \cdot \frac{2}{\sqrt5} + \frac45 \cdot \frac{1}{\sqrt5}\right|^2 = \left|\frac{10}{5\sqrt5}\right|^2 = 0.8$

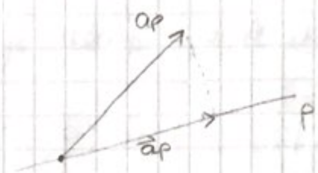
c) $= \left|\frac35 \frac{1}{\sqrt3} + \frac{4i}{5}\cdot\frac{\sqrt2}{\sqrt3}\right|^2 = \left|\frac{3}{5\sqrt3} + \frac{4\sqrt2}{5\sqrt3} i\right|^2 = 0.546$

d) $= \left|\frac35 \cdot \frac{\sqrt3}{\sqrt7} + \frac{4i}{5}\cdot\frac{2i}{\sqrt7}\right|^2 = \left|\frac{3\sqrt3}{5\sqrt7} - \frac{8}{5\sqrt7}\right|^2 = 0.045$

e) $= \left|\frac35 \cdot \frac{\sqrt5}{3} + \frac{4i}{5}\cdot\frac{2i}{3}\right|^2 = \left|\frac{3\sqrt5 - 8}{15}\right|^2 = 0.0074$

④ Vektor $\cos\frac{\vartheta}{2}|0\rangle + e^{i\varphi}\sin\frac{\vartheta}{2}|1\rangle$ opisuje stanje qubita. Stanje koje je ortogonalno tom dobijemo zamjenom

a) $\varphi \to \varphi + 2\pi$, $\vartheta \to 2\pi - \vartheta$
b) $\varphi \to \varphi + \pi$, $\vartheta \to \pi - \vartheta$ $\longrightarrow$
c) $\varphi \to \varphi + \pi$, $\vartheta \to \vartheta + \pi$
d) $\varphi \to \varphi + 2\pi$, $\vartheta \to \vartheta + \pi$
e) $\varphi \to \varphi + 2\pi$, $\vartheta \to \vartheta + 2\pi$



⑤ Koja 2 od navedenih operatora su Hermitski operatori?

a) $i|1\rangle\langle 1|$
b) $|0\rangle\langle 1|$
c) $|1\rangle\langle 1|$
d) $i(|0\rangle\langle 0| + |1\rangle\langle 1|)$
e) $|1\rangle\langle 1| - |0\rangle\langle 0|$

② prez

$$M = \sum_{n=1}^{N} a_n P_n = \sum_{n=1}^{N} a_n |n\rangle\langle n|$$

(ako su im svojstvene vrijednosti realne

⑥ Koja od 5 jednakosti ne vrijedi?

a) $[\sigma_1, \sigma_2] = 2i\sigma_3$ ✓
b) $[\sigma_2, \sigma_1] = -2i\sigma_3$ ✓
c) $[\sigma_2, \sigma_3] = 2i\sigma_1$ ✓
d) $[\sigma_3, \sigma_2] = -2i\sigma_1$ ✓
e) $[\sigma_3, \sigma_1] = -2i\sigma_2$ ✗

$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ $\sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$ $\sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

$\sigma_1\sigma_2 = i\sigma_3$ $\sigma_3\sigma_1 = i\sigma_2$ $\sigma_2\sigma_3 = i\sigma_1$

⑦ Koji od navedenih vektora su svojstveni vektori operatora prikazanog Paulijevom mat σ_2?

a) $|0\rangle + |1\rangle$
b) $|0\rangle - |1\rangle$
c) $|0\rangle + i|1\rangle$
d) $i|0\rangle + |1\rangle$
e) $|0\rangle$

$\sigma_2 = \underset{2\times 2}{\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}}$ $A\vec{x} = \lambda\vec{x}$ ($\lambda \to$ realni broj)

$|0\rangle = \underset{2\times 1}{\begin{pmatrix} 1 \\ 0 \end{pmatrix}}$ $|1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

a) $\to \begin{pmatrix} -i \\ i \end{pmatrix}$

b $\to \begin{pmatrix} i \\ -i \end{pmatrix}$

c $\to \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}\begin{pmatrix} 1 \\ i \end{pmatrix} = \begin{pmatrix} 1 \\ i \end{pmatrix}$ W

d $\to \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}\begin{pmatrix} i \\ 1 \end{pmatrix} = \begin{pmatrix} -i \\ -1 \end{pmatrix}$ W

⑧ Alice i Bob + Eve

$\left.\begin{array}{l} A \to \oplus\ 0 \\ B \to \oplus\ 1 \end{array}\right\} \frac{1}{4}$

~~⑨ Hamiltonijan nekog~~ $\to$

(9) Hamiltonijan nekog qubita dan je s $H=\hbar\omega|0\rangle\langle 0|$ gdje je $\omega>0$ konstanta. Ako je poč. $\frac{1}{\sqrt{2}}(|0\rangle+|1\rangle)$ on će se nakon $\frac{\pi}{\omega}$ naći u

$w_1=0$

$$\mu[0]=\frac{1}{\sqrt{2}}$$

$\lambda=\text{konst}$

$\hbar w \mu[t]|0\rangle$

$$\frac{d\mu}{\mu}=-i\omega dt$$

$$\mu[t]=\frac{1}{\sqrt{2}}e^{-i\omega t}$$

$$\hookrightarrow \frac{1}{\sqrt{2}}\cdot e^{-i\pi}=-\frac{1}{\sqrt{2}}$$

$$|\phi[t]\rangle=-\frac{1}{\sqrt{2}}|0\rangle+\frac{1}{\sqrt{2}}|1\rangle \;/\cdot i^2$$

$$=\frac{1}{\sqrt{2}}|0\rangle-\frac{i}{\sqrt{2}}|1\rangle$$

(10) Sustav dvaju qubitova je realiziran projekcijama spinova dviju čestica $(s=1/2)$ na z-os, a nalazi se u stanju $\frac{1}{\sqrt{2}}(|00\rangle+|01\rangle)$. Očekivana vrijednost projekcije spina prve čestice na z-osi iznosi

$\rightarrow$ Kad se radi o čestici spinskog kvantnog broja $s=1/2$ moguće su samo 2 projekcije spina na z-os

$$S_z=\pm\frac{\hbar}{2}$$

$$\begin{bmatrix}\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}\\ 0 & 0\end{bmatrix}$$

prve čestice $=\frac{\hbar}{2}$

(11) Operator kojim u sustavu dvaju qubitova realiziranih orijentacije spinova čestica $(s=1/2)$ na z-os opisujemo zbroj projekcija spinova na z-os je $(\sigma_0=1)$

(4) riješen na prezi

$$S_z=\frac{\hbar}{2}(\sigma_3\otimes\sigma_0+\sigma_0\otimes\sigma_3)$$

(14) Matrični prikaz operatora M je $\begin{pmatrix}1 & 0\\ 0 & e^{i\phi}\end{pmatrix}$ gdje je ϕ realan broj. Šta je točno

a) M je hermitski operator

b) M je unitaran operator $\rightarrow$

c) M je mogući operator stanja qubita koji opisuje čisto stanje

d) M je mogući op stanja qubita koji opisuje miješano stanje

e) Ništa

MI 2016

(1.) Neka vektori $|\Phi\rangle$ i $|\Psi\rangle$ prikazuju stanja nekog kvantnog sustava. Koja od navedenih tvrdnji je istinita

a) Veličina $\langle\Psi|\Phi\rangle$ je uvijek realan broj u int $[0,1]$
b) $\langle\Psi|\Phi\rangle$ je općenito kompl. broj čiji modul može bit proizvoljno velik
c) $\langle\Psi|\Phi\rangle$ je općenito kompleksan broj čiji je modul u int $[0,1]$
d) Ako $\langle\Psi|\Phi\rangle = 0$ onda $\langle\Phi|\Psi\rangle = 1$
e) $\langle\Psi|\Phi\rangle = i\langle\Psi|\Phi\rangle^*$

(2) Kvantni sustav može iz stanja opisanog vekt. $|\alpha\rangle$ stići u $|\gamma\rangle$ jedino ako pritom prođe kroz $|\beta\rangle$. Ako je sustav poč u $|\alpha\rangle$ y. da bude izmjeren u $|\gamma\rangle$ je?

a) $|\langle\gamma|\beta\rangle|^2$
b) $|\langle\gamma|\alpha\rangle|^2$
c) $|\langle\gamma|\beta\rangle|^2 + |\langle\beta|\alpha\rangle|^2$
d) $|\langle\gamma|\beta\rangle + \langle\beta|\alpha\rangle|^2$
$\rightarrow$ e) $|\langle\gamma|\beta\rangle\langle\beta|\alpha\rangle|^2$

(3) Koji od navedenih vekt. nije „normiran na jedinicu"?

a) $\frac{1}{\sqrt{2}}(|0\rangle + i|1\rangle)$
b) $\frac{4}{5}|0\rangle - \frac{3}{5}|1\rangle$
c) $\frac{1}{\sqrt{5}}(2i|0\rangle + |1\rangle)$
$\rightarrow$ d) $\frac{\sqrt{3}}{2}(|0\rangle + \frac{1}{2}|1\rangle)$
e) $\frac{1}{3}|0\rangle - \frac{2\sqrt{2}}{3}|1\rangle$

⋮

(9) Svojstveni vektori i odgovarajuće svojstvene vrijednosti operatora $|0\rangle\langle 1| + |1\rangle\langle 0|$ su (2 odg):

$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

a) vekt $|0\rangle + i|1\rangle$, vrijednost 1
b) -||- $|0\rangle - |1\rangle$, -||- -1
c) -||- $|0\rangle - |1\rangle$, -||- 1
d) -||- $|0\rangle + |1\rangle$, -||- 1
e) -||- $|0\rangle + |1\rangle$, -||- -1

a) $\rightarrow \begin{pmatrix} 1 \\ i \end{pmatrix}$ ×
b) $\rightarrow \begin{pmatrix} 1 \\ -1 \end{pmatrix} \rightarrow \begin{pmatrix} -1 \\ +1 \end{pmatrix}$ ✗ $\rightarrow -1$ ✓
c) $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$ ×
d) $\begin{pmatrix} 1 \\ 1 \end{pmatrix} \rightarrow \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ ✓ $\rightarrow 1$ ✓

(10) Operator projekcije na stanje ~~je~~ $\frac{1}{\sqrt{2}}(|0\rangle + i|1\rangle)$ je:

a) $\frac{1}{2}(|0\rangle\langle 0| + i|1\rangle\langle 0| - i|0\rangle\langle 1| + |1\rangle\langle 1|)$

stanje $\Rightarrow$ $|\Psi| = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ i\frac{1}{\sqrt{2}} \end{bmatrix}$

proj. $\rightarrow$ $P_\psi = |\psi\rangle\langle\psi| = \begin{pmatrix} \left(\frac{1}{\sqrt{2}}\right)^2 & -\frac{i}{\sqrt{2}}\cdot\frac{1}{\sqrt{2}} \\ \frac{i}{2} & \frac{1}{2} \end{pmatrix} = \begin{pmatrix} \frac{1}{2} & -\frac{i}{2} \\ \frac{i}{2} & \frac{1}{2} \end{pmatrix}$

(11) Projekcija stanja qubita $\frac{1}{\sqrt{2}}(|0\rangle+|1\rangle)$ na stanje $\frac{1}{\sqrt{2}}(|0\rangle+i|1\rangle)$ je:

e) $\frac{1}{2\sqrt{2}}\left((1-i)|0\rangle+(1+i)|1\rangle\right)$

$$|\psi\rangle\langle\psi|\phi\rangle=\begin{pmatrix}\frac{1}{\sqrt{2}}\\ \frac{i}{\sqrt{2}}\end{pmatrix}\left(\begin{pmatrix}\frac{1}{\sqrt{2}} & -\frac{i}{\sqrt{2}}\end{pmatrix}\begin{pmatrix}\frac{1}{\sqrt{2}}\\ \frac{1}{\sqrt{2}}\end{pmatrix}\right)=\begin{pmatrix}\left|\frac{1}{\sqrt{2}}\right|^2\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}\cdot-\frac{i}{\sqrt{2}}\cdot\frac{1}{\sqrt{2}}\\ \frac{i}{\sqrt{2}}\cdot\frac{1}{\sqrt{2}}\cdot\frac{1}{\sqrt{2}}+\frac{1}{2}\cdot\frac{1}{\sqrt{2}}\end{pmatrix}$$

$$=\begin{pmatrix}\frac{1}{2\sqrt{2}}-\frac{i}{2\sqrt{2}}\\ \frac{i}{2\sqrt{2}}+\frac{1}{2\sqrt{2}}\end{pmatrix}=\frac{1}{2\sqrt{2}}\left((1-i)|0\rangle+(i+1)|1\rangle\right)$$

(13) Očekivana vrijednost operatora $|0\rangle\langle 0|-|1\rangle\langle 1|$ u sustavu koji se nalazi u stanje $\cos\left[\frac{\vartheta}{2}\right]e^{-i\varphi/2}|0\rangle+\sin\left[\frac{\vartheta}{2}\right]e^{i\varphi/2}|1\rangle$ je:

a) $\cos\vartheta$

$$U=\begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix}$$

$$U\phi=\begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix}\cdot\begin{bmatrix}\cos\left[\frac{\vartheta}{2}\right]e^{-i\varphi/2}\\ \sin\left[\frac{\vartheta}{2}\right]e^{i\varphi/2}\end{bmatrix}$$

$$U\phi=\begin{bmatrix}\cos\left[\frac{\vartheta}{2}\right]e^{-i\varphi/2}\\ -\sin\left[\frac{\vartheta}{2}\right]e^{i\varphi/2}\end{bmatrix}$$

$$U\langle\phi\rangle=\langle\phi|U\phi\rangle=\cos^2\left[\frac{\vartheta}{2}\right]e^{-i\varphi/2}-\sin^2\left[\frac{\vartheta}{2}\right]e^{i\varphi/2}$$

$$=\cos\left(\cancel{2}\cdot\frac{\vartheta}{\cancel{2}}\right)$$

$$=\cos\vartheta$$