(1) $|0\rangle \Rightarrow 60\% \Rightarrow \frac{3}{5} = \alpha^2 \quad \alpha = \sqrt{\frac{3}{5}}$

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

$$|\alpha|^2 + |\beta|^2 = 1 \Rightarrow \beta^2 = \frac{2}{5} \Rightarrow \beta = \sqrt{\frac{2}{5}}$$

$$|\psi\rangle = \sqrt{\frac{3}{5}}\,|0\rangle + \sqrt{\frac{2}{5}}\,|1\rangle$$

(2) $|a\rangle = 2i\,|e_1\rangle - |e_2\rangle$

$|b\rangle = i\,|e_1\rangle + 2\,|e_2\rangle$

$$\langle a|a\rangle = 5 \qquad \langle b|b\rangle = 5$$

$$|\bar{a}\rangle = \frac{|a\rangle}{\sqrt{\langle a|a\rangle}} \qquad |\bar{b}\rangle = \frac{|b\rangle}{\sqrt{\langle b|b\rangle}}$$

$$|\bar{a}\rangle = \frac{2i}{\sqrt{5}}\,|e_1\rangle - \frac{1}{\sqrt{5}}\,|e_2\rangle$$

$$|\bar{b}\rangle = \frac{i}{\sqrt{5}}\,|e_1\rangle + \frac{2}{\sqrt{5}}\,|e_2\rangle$$

ŽELIMO $\langle \bar{a}|\bar{b}\rangle = 0$ VRIJEDI $\langle e_i|e_j\rangle = \delta_{ij}$

$\langle \bar{a}|\bar{b}\rangle = \frac{2}{\sqrt{5}} - \frac{2}{\sqrt{5}} = 0$ ✓ $\mathcal{B}\{\bar{a}, \bar{b}\}$

3

$$|x\rangle = \begin{bmatrix} 1-i \\ i \end{bmatrix} = 1-i \begin{bmatrix} 1 \\ 0 \end{bmatrix} + i \begin{bmatrix} 0 \\ 1 \end{bmatrix} =$$

$$= 1-i\,|\uparrow\rangle + i\,|\downarrow\rangle$$

$$|\bar{x}\rangle = \frac{|x\rangle}{\sqrt{\langle x|x\rangle}} = \frac{1-i}{\sqrt{3}}|\uparrow\rangle + \frac{i}{\sqrt{3}}|\downarrow\rangle$$

$$\left|\frac{1-i}{\sqrt{3}}\right|^2 = \frac{2}{3} \Rightarrow 66{,}6\,\%$$

4

$$\sigma_z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \qquad \sigma_x = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$\hat{A} = \frac{1}{2}(\hat{1} + \sigma_z) + \sigma_x$$

$$A = \frac{1}{2}\left(\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\right) + \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$\hat{A} = \frac{1}{2}\begin{bmatrix} 2 & 1 \\ 1 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & \frac{3}{2} \\ \frac{3}{2} & 0 \end{bmatrix}$$

a) $\hat{A}^+ = \begin{bmatrix} 1 & \frac{3}{2} \\ \frac{3}{2} & 0 \end{bmatrix}$ $\qquad A = A^+$ ✓

b) $\det(A - \lambda I) = 0$

$$\begin{vmatrix} 1-\lambda & \frac{3}{2} \\ \frac{3}{2} & -\lambda \end{vmatrix} = 0 \qquad \lambda^2 - \lambda - \frac{9}{4} = 0$$

$$\lambda_{12} = \frac{1 \pm \sqrt{10}}{2}$$

$$\lambda_1 = \frac{1+\sqrt{10}}{2}$$

$$\lambda_2 = \frac{1-\sqrt{10}}{2}$$

$$\hat{A} |v_1\rangle = \lambda_1 |v_1\rangle$$

$$\begin{bmatrix} 1 & \frac{3}{2} \\ \frac{3}{2} & 0 \end{bmatrix} \begin{bmatrix} a_1 \\ b_1 \end{bmatrix} = \frac{1+\sqrt{10}}{2} \begin{bmatrix} a_1 \\ b_1 \end{bmatrix}$$

$$a_1 + \frac{3}{2} b_1 = \frac{1+\sqrt{10}}{2} a_1$$

$$\frac{3}{2} a_1 = \frac{1+\sqrt{10}}{2} b_1 \Rightarrow a_1 = \frac{1+\sqrt{10}}{3} b_1$$

$$|v_1\rangle = b_1 \begin{bmatrix} \frac{1+\sqrt{10}}{3} \\ 1 \end{bmatrix}$$ ODABEREMO $b_1 = 1$

$$|v_1\rangle = \begin{bmatrix} \frac{1+\sqrt{10}}{3} \\ 1 \end{bmatrix}$$

$$\hat{A}\,|v_2\rangle = \lambda_2\,|v_2\rangle$$

$$\begin{bmatrix} 1 & \frac{3}{2} \\ \frac{3}{2} & 0 \end{bmatrix}\begin{bmatrix} a_2 \\ b_2 \end{bmatrix} = \frac{1-\sqrt{10}}{2}\begin{bmatrix} a_2 \\ b_2 \end{bmatrix}$$

$$a_2 + \frac{3}{2} b_2 = \frac{1-\sqrt{10}}{2} a_2$$

$$\frac{3}{2} a_2 = \frac{1-\sqrt{10}}{2} b_2 \Rightarrow a_2 = \frac{1-\sqrt{10}}{3} b_2$$

$$|v_2\rangle = b_2 \begin{bmatrix} \frac{1-\sqrt{10}}{3} \\ 1 \end{bmatrix}$$ ODABERIMO $b_2 = 1$

$$|v_2\rangle = \begin{bmatrix} \frac{1-\sqrt{10}}{3} \\ 1 \end{bmatrix}$$

c) $Tr(\hat{A}) = 1 + 0 = 1$

d) $$P = \begin{bmatrix} \frac{1+\sqrt{10}}{3} & \frac{1-\sqrt{10}}{3} \\ 1 & 1 \end{bmatrix}$$

$$P^{+} A P = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} = \begin{bmatrix} \frac{1+\sqrt{10}}{2} & 0 \\ 0 & \frac{1-\sqrt{10}}{2} \end{bmatrix}$$

2) $[\hat{A}, \sigma_y] = \hat{A}\sigma_y - \sigma_y \hat{A} =$

$$= \begin{bmatrix} 1 & \frac{3}{2} \\ \frac{3}{2} & 0 \end{bmatrix} \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} - \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \begin{bmatrix} 1 & \frac{3}{2} \\ \frac{3}{2} & 0 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{3i}{2} & -i \\ 0 & -\frac{3i}{2} \end{bmatrix} + \begin{bmatrix} -\frac{3i}{2} & 0 \\ i & \frac{3i}{2} \end{bmatrix} = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} = \sigma_y$$

(1) $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle \qquad \alpha, \beta \in \mathbb{C}$

$$|\alpha|^2 + |\beta|^2 = 1$$

$|\alpha|^2$ - vjerojatnost da je nakon mjerenja ustanovljeno da se nalazi u stanju $|0\rangle$

$|\beta|^2$ - vjerojatnost da je nakon mjerenja ustanovljeno da se nalazi u stanju $|1\rangle$

q-bit => osnovni element preko kojega prenosimo informacije u kvantnom računalu

(2) $E^2 = (mc^2)^2 + (pc)^2 \quad / \sqrt{\ }$

$$E = mc^2\sqrt{1 + \frac{p^2}{m^2c^2}} \approx mc^2\left(1 + \frac{p^2}{2m^2c^2}\right)$$

$$E = mc^2 + \frac{p^2}{2m} \qquad E = \hbar\omega \qquad p = \frac{h}{\lambda} = \hbar k$$

$$\hbar\omega = mc^2 + \frac{(\hbar k)^2}{2m} \quad / : \hbar$$

$$\omega = \frac{mc^2}{\hbar} + \frac{\hbar k^2}{2m}$$

(4) $$-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2}+\underbrace{E_p(x)}_{=\,0}\psi(x)=E\psi(x)$$

ЗА СЛОБОДНУ
ЧЕСТИЦУ

$$-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2}=E\psi(x)$$

$$\psi(x)=Ae^{ax}$$

$$-\frac{\hbar^2}{2m}a^2\psi(x)=E\psi(x)$$

$$a^2=-\frac{2mE}{\hbar^2}$$

$$E=\frac{p^2}{2m}=p^2=2mE$$

$$a=i\sqrt{\frac{2mE}{\hbar^2}}$$

$$p=\frac{h}{\lambda}=\hbar k$$

$$a=i\frac{p}{\hbar}$$

$$a=i\frac{h}{\hbar\lambda}=i\frac{2\pi}{\lambda}$$

$$\psi(x)=Ae^{i\frac{2\pi}{\lambda}}$$

(13) непрекинута, диференцијабилна, нормализирана