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Quantum computing

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The main features of quantum computing are described in the framework of spin resonance methods. Stress is put on the fact that quantum computing is itself nothing but a reinterpretation (fruitful indeed) of well-known concepts. The role of the two basic operations, one-spin rotation and controlled-NOT gates, is analyzed, and some exercises are proposed. © 1998 American Association of Physics Teachers.

I. INTRODUCTION

Quantum computing (QC) is one of the latest booms in science. The first detailed paper on QC was published by Deutsch in 1985,¹ but it is only in 1994 that Shor showed that “it should work.”² Since that date, scientific reviews have been filled (and continue to be) with articles related to this topic; and an almost entirely new area of theoretical physics has been born: the theory of “quantum error correcting codes” (for a simple protocol, see Ref. 3). Presently we still don’t know if a quantum computer can be built; but, whatever the end of the story may be, I believe that it is worthwhile working out a simple model of quantum computation, and letting undergraduate students put it to work on their paper.

A. QC: a new reading of an old book

One of the striking features of the idea of the quantum computer is the fact that it contains nothing really new: it is “nothing but” a reinterpretation of very well-known mathematical objects, mainly the theory of quantum two-level systems. In what follows, we shall focus on spins $\frac{1}{2}$, although all that follows could be carried out for any two-level system. Other examples of two-level systems which may be of interest can be found in Refs. 4 and 5. Here you have the translational recipe:

- (1) First of all, rename the eigenstates of your two-level system as “0” and “1” (instead of, for instance, “spin up” and “spin down”): your “two-level system” has become a “qubit” (the standard shortcut for “quantum binary digit”).
- (2) Of course, you must act on your system; don’t call it a “perturbation,” but a “logic gate.”

If you have done this, your spin device has been transformed in a true one-qubit quantum computer! Now, a one-bit computer is nothing exciting. To obtain an N -qubit computer, you have to take N spins, and to be able to address them in a selective way (i.e., you must be able to turn spin number 5, then to couple spins number 3, 5, and 9, and so on). Nowadays physicists are able to address single quantum states and to work on them, but on systems whose *decoherence times* are very short. Decoherence is a mechanism we are beginning to understand, since some experimental results have been obtained;⁶ in a very first approach, decoherence is the modification of the quantum state of the system due to interaction with an environment. In other words, an irreversible loss of information takes place because the system is not perfectly isolated.⁷ In the case of a QC, such a mechanism

can cause the value of a qubit, or the correlation between qubits, to change during the calculation, in a way we cannot control. Decoherence is the most fundamental obstacle to date preventing us from building a QC.

Experimental realizations being forbidden up to now, what about calculations? It would seem that calculating a quantum computer is a task for a computer (a classical one), since one should understand, in principle, how an N -qubit logic gate works. However, Barenco *et al.*⁸ have shown that any possible N -qubit quantum computer operations can be described in terms of two basic operations: the rotation of *one* spin, and an operation involving *two* spins, called controlled-NOT (CNOT) or exclusive-OR (XOR) gate. This means that from the theorist’s viewpoint, once you have understood these two simple operations, you know everything about how a quantum computer works. And, of course, that is exactly what we are going to do in the following section.

B. How to rotate one spin

In this section, we give some basic elements of spin rotation, inspired by pulsed nuclear magnetic resonance (NMR) techniques.^{9–11}

First of all, we define the matrix representation we are going to work with, by defining the Pauli matrices as follows:

$$\sigma_x \doteq \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_y \doteq \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_z \doteq \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}; \quad (1)$$

the eigenvectors of σ_z are written as $|+\rangle_z \overset{\text{QC}}{=} |0\rangle \doteq \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $|-\rangle_z \overset{\text{QC}}{=} |1\rangle \doteq \begin{pmatrix} 0 \\ 1 \end{pmatrix}$. I find it better to keep the NMR notation $|+\rangle_z$ until the end of Sec. II, to avoid introducing curious terms like “rotating a qubit around an axis.” The reader will be invited to translate these notations into QC notations at the beginning of Sec. III.

It is a matter of evidence that any rotation can be decomposed using only rotations around \hat{e}_z and (say) \hat{e}_x (a more precise statement is given in Ref. 8, Lemma 4.1). So we will simply give one-spin Hamiltonians allowing performance of these two rotations. Here we remind the general form of rotation matrices in our representation, with the convention that clockwise rotation is positive [we give also $R_y(\theta)$, which may be useful in practice]:

$$R_x(\theta) \doteq \begin{pmatrix} \cos \theta & i \sin \theta \\ i \sin \theta & \cos \theta \end{pmatrix}, \quad R_y(\theta) \doteq \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix},$$

$$R_z(\theta) \doteq \begin{pmatrix} e^{i\theta} & 0 \\ 0 & e^{-i\theta} \end{pmatrix}. \quad (2)$$

We consider now a spin $\frac{1}{2}$ in a static uniform magnetic field $\mathbf{B}_0 = B_0 \hat{e}_z$; we suppose that this spin feels another (much weaker) interaction, such that the total Hamiltonian at equilibrium is

$$H_0 = -\frac{\hbar}{2} (\omega_0 + \omega_1) \sigma_z \quad (3)$$

with $\omega_0 = \gamma B_0$ the Larmor frequency associated with the external field. The external field plays here a most trivial role: basically, we need it to lift by a sufficient amount the degeneracy of spin levels, so it is customary to work in a rotating frame in which the contribution of \mathbf{B}_0 cancels out. This frame is defined by

$$\hat{e}_{x'} = \cos \omega_0 t \hat{e}_x + \sin \omega_0 t \hat{e}_y, \quad (4)$$

$$\hat{e}_{y'} = -\sin \omega_0 t \hat{e}_x + \cos \omega_0 t \hat{e}_y, \quad (5)$$

$$\hat{e}_{z'} = \hat{e}_z, \quad (6)$$

and the static Hamiltonian becomes simply

$$H'_0 = -\frac{\hbar}{2} \omega_1 \sigma_z. \quad (7)$$

Under H'_0 , the most general spin state $\alpha|+\rangle_z + \beta|-\rangle_z$ evolves according to

$$|\psi\rangle(t) \doteq \begin{pmatrix} \alpha(t) \\ \beta(t) \end{pmatrix} = \begin{pmatrix} e^{i\omega_1 t/2} & 0 \\ 0 & e^{-i\omega_1 t/2} \end{pmatrix} \begin{pmatrix} \alpha(0) \\ \beta(0) \end{pmatrix} \doteq R_z\left(\frac{\omega_1 t}{2}\right) |\psi\rangle(0), \quad (8)$$

so the “free” evolution gives us the possibility of performing rotations around \hat{e}_z . Rotations around $\hat{e}_{x'}$ can be obtained by applying the time-dependent Hamiltonian

$$H_{\text{pert}} = -\frac{\hbar}{2} \omega_p [\cos(\omega_0 + \omega_1)t \sigma_x + \sin(\omega_0 + \omega_1)t \sigma_y], \quad (9)$$

which yields for any practical purpose¹²

$$|\psi\rangle(t) \doteq \begin{pmatrix} \alpha(t) \\ \beta(t) \end{pmatrix} = \begin{pmatrix} \cos \omega_p t/2 & i \sin \omega_p t/2 \\ i \sin \omega_p t/2 & \cos \omega_p t/2 \end{pmatrix} \begin{pmatrix} \alpha(0) \\ \beta(0) \end{pmatrix} \doteq R_{x'}\left(\frac{\omega_p t}{2}\right) |\psi\rangle(0). \quad (10)$$

Before turning to the model for QC, it is important to notice the following.

- (1) The energy levels separation at equilibrium plays an important role; by adjusting the *frequency* of the perturbation (above, $\omega_0 + \omega_1$), one can select one transition in a multilevel system.
- (2) A rotation of the state¹³ by an angle θ around $\hat{e}_{x'}$ is obtained by applying the perturbation during a time $\tau_\theta = 2\theta/\omega_p$, depending on the *intensity* of the perturbation.

We shall make extensive use of the first remark in what follows. Here we must tell something more on the second remark. We have said that spin rotation is obtained by applying during a well-defined time τ a pulse having a well-defined frequency ω_r . Such a pulse does not excite only resonance at ω_r ; it excites a frequency band $\omega_r \pm \Delta\omega$, where basically $\Delta\omega \sim \tau^{-1}$.¹⁴ This means that by shortening the pulse (i.e., by increasing the intensity of the perturbation), one can be more selective, and vice versa.

II. PUTTING A QUANTUM COMPUTER TO WORK!

A. The model

The system we work with are two spins $\frac{1}{2}$ (two qubits); the Hilbert space describing such a system is $\mathcal{H} = \mathbb{C}^2 \otimes \mathbb{C}^2$, the tensor product of two copies of \mathbb{C}^2 , each describing one spin.¹⁵ The static Hamiltonian will be taken as

$$H_0 = -\frac{\hbar}{2} [\Omega_1(\sigma_z^1 \otimes \mathbb{1}) + \Omega_2(\mathbb{1} \otimes \sigma_z^2) + \omega_c(\sigma_z^1 \otimes \sigma_z^2)] \quad (11)$$

whose eigenstates are the four products of two Pauli matrices eigenstates. In all that follows we shall use notations like $|++\rangle$ as shortcuts for $|+\rangle_z \otimes |+\rangle_z$. We use the notation $\Omega_i = \omega_0 + \omega_i$. We choose the following representation: $|++\rangle \doteq e_1$, $|+-\rangle \doteq e_2$, $|+--\rangle \doteq e_3$, $|---\rangle \doteq e_4$, where of course e_i is the column four-tuple whose elements are 1 at the i th place and 0 at the others. We have then

$$H_0 \doteq -\frac{\hbar}{2} \begin{pmatrix} \Omega_1 + \Omega_2 + \omega_c & & & \\ & -\omega_1 + \omega_2 - \omega_c & & \\ & & \omega_1 - \omega_2 - \omega_c & \\ & & & -\Omega_1 - \Omega_2 + \omega_c \end{pmatrix}. \quad (12)$$

We choose $\omega_1 > \omega_2$, and we assume that $\omega_c \ll \omega_0$ (weak coupling) and that $\omega_1 - \omega_2 \geq 4\omega_c$ (the reason for this is given in a subsequent discussion). The energy level diagram is immediately drawn, whence we can easily derive the transition

frequency spectrum, drawn in Fig. 1 (the low-frequency transition $|+-\rangle \leftrightarrow |-\rangle$ and the high-frequency transition $|++\rangle \leftrightarrow |--\rangle$ are omitted, since we are not interested in transitions involving both spins).

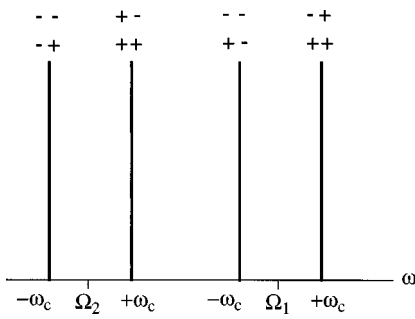


Fig. 1. Transition frequency spectrum for Hamiltonian (11). At the top of each line the corresponding transition is indicated.

Let's discuss some conditions:

- (1) To perform *one-spin rotation* around an axis lying in the (\hat{e}_x, \hat{e}_y) plane (remember that rotations around \hat{e}_z are obtained by letting the system evolve under the static Hamiltonian, and do not involve any resonance technique) on spin 1, we must be able to address *both* $|++\rangle \leftrightarrow |+-\rangle$ and $|+-\rangle \leftrightarrow |--\rangle$ without exciting any other transition; an analogous requirement must be satisfied for one-spin rotation on spin 2. This yields a condition on the physical parameters, namely $\omega_1 - \omega_2 > 2\omega_c$ (to work more comfortably, when we anticipated this condition we took $4\omega_c$ as upper bound) and an upper limit for τ_θ at fixed θ (for this operation, the pulse must not be too selective).
- (2) We shall see that to perform all the possible *CNOT operations* means the possibility of addressing each transition separately; this yields a lower bound for τ_θ at fixed θ (for this operation, we need selective pulses).

In all that follows, we suppose that we are able to control the frequency and intensity of each pulse, in order to address the chosen transition with the desired selectivity.

B. Rotations and CNOT (XOR) gates

The discussion of *one-spin rotations* is merely a matter of rewriting, since we know everything thereabout. Writing

$$R_u(\theta) \doteq \begin{pmatrix} r_{11} & r_{12} \\ r_{21} & r_{22} \end{pmatrix} \quad (13)$$

one obtains immediately for our representation on $\mathbb{C}^2 \otimes \mathbb{C}^2$:

$$R_u(\theta) \otimes \mathbb{1} \doteq \begin{pmatrix} R_u(\theta) & 0 \\ 0 & R_u(\theta) \end{pmatrix}; \quad (14)$$

$$\mathbb{1} \otimes R_u(\theta) \doteq \begin{pmatrix} r_{11}\mathbb{1} & r_{12}\mathbb{1} \\ r_{21}\mathbb{1} & r_{22}\mathbb{1} \end{pmatrix}. \quad (15)$$

Thus we must now focus our attention on *CNOT gates*. We recall that this stands for *controlled NOT*, and means that we flip one spin according to the state of the other. The following matrix is a CNOT, in which if spin 2 is in the state $|-\rangle$, then the state of spin 1 is flipped:

$$C_{2-}^1 \doteq \begin{pmatrix} 1 & 0 & & \\ 0 & 1 & & \\ & & 0 & 1 \\ & & 1 & 0 \end{pmatrix}. \quad (16)$$

It is not hard to describe this operation using our tools: All we need to do, is to address uniquely the $|+-\rangle \leftrightarrow |--\rangle$ transition, with a pulse whose length τ is defined by [see (10)] $\omega_p \tau/2 = \pi/2$. The reader is invited to write down the other three possible CNOT gates.

Note that one-spin rotations are intrinsically nonclassical, since (in general) they generate superposition states. On the contrary, the CNOT operation is in itself classical; however, in a QC we want to perform such an operation on arbitrary states, and this assumes highly nonclassical features. So, for instance,

$$\begin{aligned} C_{2-}^1 \left(\frac{1}{\sqrt{2}} |++\rangle + \frac{1}{\sqrt{2}} |--\rangle \right) &= \frac{1}{\sqrt{2}} |++\rangle + \frac{1}{\sqrt{2}} |+-\rangle \\ &= |+\rangle \otimes \left(\frac{1}{\sqrt{2}} |+\rangle + \frac{1}{\sqrt{2}} |-\rangle \right), \end{aligned} \quad (17)$$

which means *disentanglement*. Some short exercises follow, whose purpose is twofold: understanding the different role of one-spin rotations and CNOT gates in a quantum computation and “feeling” (a general proof is not our purpose here) that with these two operations one can simulate any quantum calculation.

III. EXERCISES

Even though this is totally trivial, the reader is invited to “translate” spin states into “qubits” using the standard rules of binary calculations; thus, for instance,

$$\begin{aligned} |++\rangle &= |00\rangle = |0\rangle, & |+-\rangle &= |10\rangle = |1\rangle, \\ |+-\rangle &= |01\rangle = |2\rangle, & |--\rangle &= |11\rangle = |3\rangle. \end{aligned}$$

This translation is used in the exercises in Secs. III C and III D.

A. Three-spins maximally entangled state (GHZ)

Give an algorithm using only one-spin rotations and CNOT gates to transform the fundamental three-spins state $|+++\rangle$ into the maximally entangled GHZ state $1/\sqrt{2}(|+++\rangle + |--\rangle)$.

Imagine now you *don't know* the input state: do you have any hope of building a “universal GHZ preparator,” i.e., an algorithm that transforms *any input state whatsoever* into the GHZ state?

Solution: Here is a possible sequence starting from $|+++\rangle$:

$$\left[\mathbb{1} \otimes \mathbb{1} \otimes R_y \left(\frac{\pi}{4} \right) \right] |+++\rangle = \frac{1}{\sqrt{2}} (|+++\rangle + |++-\rangle),$$

$$[\mathbb{1} \otimes C_{3-}^2] \frac{1}{\sqrt{2}} (|+++\rangle + |++-\rangle)$$

$$= \frac{1}{\sqrt{2}} (|+++\rangle + |--\rangle),$$

$$[C_{2-}^1 \otimes 1] \frac{1}{\sqrt{2}} (|+++ \rangle + |+- - \rangle)$$

$$= \frac{1}{\sqrt{2}} (|+++ \rangle + |-- - \rangle).$$

Of course, it is *not* possible to find an algorithm that gives the same output state (GHZ, or whatever else) for any input state: QC is concerned with unitary evolution, thus in particular orthogonal input states must give orthogonal output states.

B. The NOT logic gate

Write down the matrix representation of the NOT logic gate (inversion of all spins) for a two-spins system. What are its eigenstates? Can such a gate modify entanglements?

Solution: The NOT logic gate is

$$N \doteq \begin{pmatrix} & 0 & 1 \\ & 1 & 0 \\ 0 & 1 & \\ 1 & 0 & \end{pmatrix}. \quad (18)$$

One can write it down either by direct reasoning on the four basis states, or by calculating the product of two one-spin NOTs. Since $N = -(R_x(\pi/2) \otimes 1)(1 \otimes R_x(\pi/2))$ is (up to an overall phase factor) the product of two one-spin rotations, such a gate cannot modify entanglements. Its eigenstates form the so-called Bell basis:

$$|\Phi^\pm\rangle = \frac{1}{\sqrt{2}} (|++\rangle \pm |--\rangle),$$

$$|\Psi^\pm\rangle = \frac{1}{\sqrt{2}} (|+-\rangle \pm |-+\rangle). \quad (19)$$

C. Readout of Bell states

This exercise is inspired by Ref. 16: had Zeilinger's group had a suitable logic gate for their polarized photons, the readout would have been by far easier! Imagine thus you have done an experiment whose result is one of the four Bell states (19). However, your detectors' eigenstates are *not* Bell states, but the standard basis states $|++\rangle$, etc. Write down a logic gate which would permit you to make the translation, in matrix representation and as a product of basic operations (suggestion: first destroy entanglement, then superpositions).

Solution: We choose the following translation:

$$|\Phi^+\rangle \Rightarrow |0\rangle, \quad |\Psi^+\rangle \Rightarrow |1\rangle,$$

$$-|\Phi^-\rangle \Rightarrow |2\rangle, \quad -|\Psi^-\rangle \Rightarrow |3\rangle.$$

Thus the logic gate will be

$$T \doteq \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ -1 & 0 & 0 & 1 \\ 0 & -1 & 1 & 0 \end{pmatrix}. \quad (20)$$

It is not difficult to decompose this gate into a product of the basic operations. One possible solution is $T = (1 \otimes R_y(\pi/4))C_{2-}^1$.

D. Quantum Fourier transform

The operator known as *quantum Fourier transform* plays an important role in Shor's algorithm.^{2,4} For a system of n spins...sorry, n qubits, it is defined as (we write $Q=2^n$)

$$F = \frac{1}{\sqrt{Q}} \sum_{x,k=0}^{Q-1} |k\rangle e^{2\pi i k x / Q} \langle x|. \quad (21)$$

Verify that F is unitary. Write down the matrix representation of F for $n=2$.

Solution: One has

$$F^\dagger F = \frac{1}{Q} \sum_{x,k,x',k'} |x'\rangle e^{-2\pi i k' x' / Q} \underbrace{\langle k' | k \rangle}_{\delta_{k,k'}} e^{2\pi i k x / Q} \langle x|$$

$$= \sum_{x,x'} |x'\rangle \underbrace{\left(\frac{1}{Q} \sum_{k=0}^{Q-1} \exp(2\pi i k (x-x')/Q) \right)}_{\langle x' | F^\dagger F | x \rangle} \langle x|.$$

If $x=x'$, then $\langle x' | F^\dagger F | x \rangle = 1$. Otherwise,

$$\langle x' | F^\dagger F | x \rangle = \frac{1}{Q} \frac{1 - e^{2\pi i (x-x')}}{1 - e^{2\pi i (x-x')/Q}} = 0$$

since $N \ni |x-x'| < Q$. Thus $F^\dagger F = 1$.

For $n=2$ one has

$$F \doteq \frac{1}{2} \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & i & -1 & -i \\ 1 & -1 & 1 & -1 \\ 1 & -i & -1 & i \end{pmatrix}. \quad (22)$$

IV. CONCLUSION

In this paper, the reader has found a self-contained description of a quantum computer based on well-known elements of undergraduate quantum mechanics. I wanted to stress that, while the connected field of quantum error correcting codes is something for specialists (and that's why it was totally neglected here), the basic idea of quantum computation is something very simple; so simple that most readers have probably already made many "quantum calculations" without calling them by this name!

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¹D. Deutsch, "Quantum theory, the Church-Turing principle and the universal quantum computer," *Proc. R. Soc. London Ser. A* **400**, 97-117 (1985). Deutsch was impressed by "quantum parallelism": it seems as if many parallel computers were performing calculations, and that all these calculations contribute to the final result (the advised reader recognizes here the quantum computing version of the superposition principle). We read in the abstract of the paper: "The intuitive explanation of these properties places an intolerable strain on all interpretations of quantum theory other than Everett's." I feel that this was a very important topic in Deutsch's mind, possibly the main reason for him for proposing a new reading of quantum mechanics. Of course, the "intuitive explanation" proposed in the paper is not compelling at all; but we cannot avoid stressing that once again a great advance in science is due to somebody who tries to *think* physics, not only to make it.

²P. W. Shor, "Algorithms for Quantum Computation: Discrete Logarithms and Factoring," *Proceedings of the 35th Annual Symposium on the Foundations of Computer Science* (IEEE Computer Society, Los Alamitos, 1994), pp. 124-134.

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- ⁵H. Weinfurter and A. Zeilinger, "Informationsübertragung und Informationsverarbeitung in der Quanten-welt," Phys. Bl. **52**, 219–224 (1996).
- ⁶M. Brune *et al.*, "Observing the Progressive Decoherence of the "Meter" in a Quantum Measurement," Phys. Rev. Lett. **77**, 4887–4890 (1996); a good simplified article written by the very authors of the experiment: S. Haroche *et al.*, "Le chat de Schrödinger se prête à l'expérience," La Recherche **301**, 50–55 (Septembre 1997).
- ⁷In other words, the origin of decoherence lies in the possibility of obtaining information about the system by "performing a measurement" on the environment; a "measurement" being in fact any irreversible event, independent of the effective possibility for a human observer of detecting it. For a didactic approach, French-speaking readers can refer to J.-L. Basdevant; *Problèmes de Mécanique Quantique* (Ellipse, Paris, 1996), problème 5; in this model, decoherence occurs because of energy exchange, and it is shown that a single quantum of energy exchanged between the system and environment is enough to erase almost all information.
- ⁸A. Barenco *et al.*, "Elementary gates for quantum computation," Phys. Rev. A **52**, 3457–3467 (1995).
- ⁹Here you have three good reasons for this choice: (1) NMR is a standard experimental technique, contrarily, for instance, to cavity QED: many physicists and chemists are used to NMR, and those who are not can find excellent descriptions in many standard books (see, for instance, Ref. 10). (2) As discussed by Gershenfeld and Chuang (Ref. 11), NMR is up to now the only proposed method for QC involving very long decoherence times. (3) NMR is my own research field, and I am accustomed to explaining it to undergraduate students!
- ¹⁰C. Cohen-Tannoudji, B. Diu, and F. Laloë, *Quantum Mechanics* (Wiley, Paris, 1977), compl. F_{IV} .
- ¹¹N. A. Gershenfeld and I. L. Chuang, "Bulk Spin-Resonance Quantum Computation," Science **275**, 350–356 (1997).
- ¹²This is rigorously true in another frame, rotating with angular frequency $\omega_0 + \omega_1$. But we assume that $\omega_p \gg \omega_1$, i.e., the induced rotation around \hat{e}_x is much more rapid than the "free" rotation around \hat{e}_z . Since we will be interested in small angles ($\omega_p t = \pi$ at most), we can consider that no free evolution takes place while the perturbation is applied.
- ¹³Let's remind the reader as an example that the spin flip $|+\rangle_z \leftrightarrow |-\rangle_z$ is obtained by rotating the state vector around an axis perpendicular to \hat{e}_z by $\pi/2$, and not by π .
- ¹⁴The reader can check it out as a complementary exercise by calculating the Fourier transform of a rectangular pulse:
- $$p(t) = \begin{cases} \sin \omega_p t, & 0 < t < \tau, \\ 0, & \text{otherwise.} \end{cases}$$
- ¹⁵We admit that readers know the reasons why systems are described by Hilbert spaces in quantum mechanics. Tensor product of Hilbert spaces is the mathematical structure one needs to describe a system S composed by subsystems S_i , under the hypothesis that *the whole* system S admits the superposition principle.
- ¹⁶K. Mattle *et al.*, "Dense Coding in Experimental Quantum Communication," Phys. Rev. Lett. **76**, 4656–4659 (1996).