Kvantna računala, međuispit, 25. studenog 2020.

Ime, prezime i JMBAG:

Uputa:

- Ispit se sastoji od 10 zadataka u obliku pitanja s ponuđenim odgovorima.
- Odgovore koje smatrate točnima označite (zacrnite) na posebnom obrascu. Mogu se pojaviti zadaci u kojima je potrebno označiti više od jednog ponuđenog odgovora.
- U praznom prostoru pored zadatka ili na dodatnim papirima napišite obrazloženje ili računski postupak koji vas je doveo do rješenja koje smatrate točnim.
- Točno riješeni zadatak donosi 4 boda. Kazneni (negativni) bodovi se ne obračunavaju.

Notacija i terminologija:

- Vektori $|0\rangle = \left(\begin{smallmatrix}1\\0\end{smallmatrix}\right)$ i $|1\rangle = \left(\begin{smallmatrix}0\\1\end{smallmatrix}\right)$ čine ortonormiranu bazu u $\mathcal{H}^{(2)}$.
- Pri realizaciji qubita projekcijom spina čestice spinskog kvantnog broja $s = {}^1\!/_2$ na z-os uzimamo da $|0\rangle$ i $|1\rangle$ odgovarju projekcijama $\hbar/2$ i $-\hbar/2$.
- Računalnu bazu u prostoru stanja dvaju qubitova obilježavamo s $\{|ij\rangle = |i\rangle \otimes |j\rangle \,; i, j = 0, 1\}$.

1 Koji od navedenih vektora su normirani? // *Which of the following vectors are normalized?*

(a) $\sqrt{\frac{2}{3}}\,|0\rangle + \frac{\mathrm{i}}{\sqrt{3}}\,|1\rangle$ **točno**

(b) $\frac{1}{\sqrt{3}}\,|0\rangle + \frac{2}{\sqrt{3}}\,|1\rangle$

(c) $\frac{2}{\sqrt{5}}\,|0\rangle + \frac{\mathrm{i}}{\sqrt{5}}\,|1\rangle$ **točno**

(d) $\frac{3}{5}\,|0\rangle - \mathrm{i}\frac{2}{5}\,|1\rangle$

(e) $\frac{3}{\sqrt{25}}\,|0\rangle + \frac{4}{\sqrt{25}}\,|1\rangle$ **točno**

2 Koja dva od navedenih vektora čine ortonormiranu bazu u $\mathcal{H}^{(2)}$? // *Which two of the following vectors comprise an orthonormal basis in* $\mathcal{H}^{(2)}$*?*

(a) $\frac{4}{5}\,|0\rangle - \frac{3}{5}\,|1\rangle$

(b) $\frac{3}{5}\,|0\rangle - \frac{4}{5}\,|1\rangle$

(c) $\frac{4}{5}\,|0\rangle + \frac{3}{5}\mathrm{i}\,|1\rangle$

(d) $\frac{\sqrt{3}}{2}\,|0\rangle - \frac{1}{2}\,|1\rangle$ **točno**

(e) $\frac{1}{2}\,|0\rangle + \frac{\sqrt{3}}{2}\,|1\rangle$ **točno**

3 Koji (dva ili više) od navedenih vektora predstavljaju isto stanje kvantnog bita? // *Which (two or more) of the following kets represent the same qubit state?*

(a) $\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$ **točno**

(b) $\frac{1}{\sqrt{2}}(\mathrm{i}\,|0\rangle + \mathrm{i}\,|1\rangle)$ **točno**

(c) $\frac{1}{\sqrt{2}}(-\,|0\rangle - |1\rangle)$ **točno**

(d) $\frac{1}{\sqrt{2}}(|0\rangle - \mathrm{i}\,|1\rangle)$

(e) $\frac{1}{\sqrt{2}}(|0\rangle + \mathrm{i}\,|1\rangle)$

4 Kvantni bit se nalazi u stanju // *Qubit is in the state*

$$\frac{4}{\sqrt{25}}\,|0\rangle + \frac{3}{\sqrt{25}}\,|1\rangle\,.$$

U kojem od ponuđenih stanja je vjerojatnost nalaženja tog kvantnog bita najveća? // *In which of the following states are we most likely to find this qubit?*

(a) $|0\rangle$

(b) $\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$ **točno**

(c) $\frac{1}{\sqrt{2}}(|0\rangle + \mathrm{i}\,|1\rangle)$

(d) $\frac{1}{\sqrt{2}}(|0\rangle - \mathrm{i}\,|1\rangle)$

(e) $|1\rangle$

5 Operatoru projekcije na stanje // *Projector onto the state*

$$\frac{1}{\sqrt{2}}\big(\,|0\rangle + \mathrm{i}\,|1\rangle\,\big)$$

odgovara matrica // has the matrix representation

(a) $\frac{1}{2}\begin{pmatrix} 1 & -\mathrm{i} \\ \mathrm{i} & 1 \end{pmatrix}$. **točno**

(b) $\frac{1}{2}\begin{pmatrix} 1 & \mathrm{i} \\ -\mathrm{i} & 1 \end{pmatrix}$.

(c) $\begin{pmatrix} 1 & 0 \\ 0 & -\mathrm{i} \end{pmatrix}$.

(d) $\frac{1}{2}\begin{pmatrix} 0 & -\mathrm{i} \\ \mathrm{i} & 0 \end{pmatrix}$.

(e) $\frac{1}{2}\begin{pmatrix} 0 & \mathrm{i} \\ -\mathrm{i} & 0 \end{pmatrix}$.

6 Očekivana vrijednost operatora // *The expectation value of the operator*

$$|0\rangle\langle 0| - |1\rangle\langle 1|$$

za kvantni bit u stanju // *for the qubit in the state*

$$\frac{1}{\sqrt{2}}\left(|0\rangle + \mathrm{i}\,|1\rangle\right)$$

je: // *is:*

(a) -1

(b) $-1/2$

(c) 0 **točno**

(d) $1/2$

(e) 1

7 Ako je hamiltonijan kvantnog bita // *If the Hamiltonian of a qubit is*

$$H = \frac{\hbar\omega}{2}|0\rangle\langle 0| - \frac{\hbar\omega}{2}|1\rangle\langle 1|,$$

te ako je početno stanje kvantnog bita // *and if the initial state of the qubit is*

$$\frac{1}{\sqrt{5}}(2|0\rangle + |1\rangle),$$

taj će se kvantni naći u stanju // *that qubit will find itself in the state*

$$\frac{1}{\sqrt{5}}(|0\rangle + 2|1\rangle)$$

nakon vremena // *after the time*

(a) $\frac{\pi}{2\omega}$

(b) $\frac{\pi}{\omega}$

(c) $\frac{3\pi}{2\omega}$

(d) $\frac{2\pi}{\omega}$

(e) — to se neće dogotiti. // — *that will not happen.* **točno**

8 U kojima od pet navedenih stanja je sustav dvaju kvantnih bitova u spregnutom stanju? // *In which of the following states is the system of two quantum bits in the entangled state?*

(a) $\frac{1}{2}(|00\rangle + |01\rangle + |10\rangle + |11\rangle)$

(b) $\frac{1}{2}(|00\rangle - |01\rangle + |10\rangle - |11\rangle)$

(c) $\frac{1}{\sqrt{2}}(|00\rangle + \mathrm{i}\,|11\rangle)$ **točno**

(d) $\frac{1}{\sqrt{2}}(|01\rangle + \mathrm{i}\,|10\rangle)$ **točno**

(e) $\frac{1}{\sqrt{2}}(|00\rangle + |01\rangle)$

9 Matrica // *The matrix*

$$\frac{1}{2}\begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 \end{pmatrix}$$

prikazuje operator projekcije na stanje // *represents the projection operator onto the state*

(a) $\frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)$

(b) $\frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$

(c) $\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$ **točno**

(d) $\frac{1}{\sqrt{2}}(|00\rangle - |11\rangle)$

(e) — nijedno od navedenih.

10 Sustav dvaju kvantnih bitova realiziran je projekcijama spinova dviju čestica ($s = 1/2$) os z. Sustav se nalazi u stanju // *The system of two quantum bits is realized by projections of spins ($s = 1/2$) onto the z-axis. The state of the system is*

$$\frac{1}{\sqrt{2}}\big(\,|00\rangle + |01\rangle\,\big).$$

Očekivana vrijednost projekcije spina prve čestice na z-os iznosi // *The expectation value of the projection of the spin of the first particle onto the z-axis is*

(a) $-\hbar$

(b) $-\hbar/2$

(c) 0

(d) $+\hbar/2$ **točno**

(e) $+\hbar$