Prva domaća zadaća iz kvantnih računala (11. studenog 2016.)

Ime i prezime: [redacted]

Rok za predaju zadaće: na predavanju 18. studenog.

Uputa: Ako gledate u elektronički dokument, isprintajte ga. Odgovore je potrebno *označiti (zaokružiti) na ovom papiru*. Osim toga, u praznom prostoru pored ponuđenih odgovora ili na dodatnim praznim papirima, potrebno je napisati *kratko obrazloženje ili račun* za svaki zadatak. Točno riješeni zadaci donose po jedan bod (nema "negativnih bodova").

Notacija: Uzimamo da vektori $|0\rangle = \binom{1}{0}$ i $|1\rangle = \binom{0}{1}$ čine ortonormiranu bazu u $\mathcal{H}^{(2)}$. Kad se radi o stanjima polarizacije fotona, koristimo $|0\rangle \to |x\rangle$, $|0\rangle \to |y\rangle$, bazu $\{|x\rangle, |y\rangle\}$ obilježavamo simbolom $\oplus$, a bazu $\{\frac{1}{\sqrt{2}}(|x\rangle \pm |y\rangle)\}$ obilježavamo simbolom $\otimes$.

Zadaci:

1 Koji od navedenih vektora *nije* "normiran na jedinicu"?

(a) $|0\rangle - \mathrm{i}\,|1\rangle$

(b) $\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$

$|\langle a|a\rangle| = \left|(1 \;\; i)\binom{1}{-i}\right| = |1+1| = 2 \neq 1$

(c) $\frac{1}{2}|0\rangle - \frac{\sqrt{3}}{2}\mathrm{i}\,|1\rangle$

(d) $\frac{3}{5}|0\rangle + \frac{4}{5}|1\rangle$

(e) $\frac{5}{13}|0\rangle - \frac{12}{13}\mathrm{i}\,|1\rangle$

2 Koja dva od pet navedenih vektora čine ortonormiranu bazu u $\mathcal{H}^{(2)}$?

(a) $\frac{1}{\sqrt{2}}(|0\rangle - \mathrm{i}\,|1\rangle)$

$|\langle e|e\rangle| = \left|\left(\frac{\sqrt{3}}{2} \;\; \frac{-1}{2}\right)\begin{pmatrix}\frac{\sqrt{3}}{2}\\ \frac{-1}{2}\end{pmatrix}\right| = \left|\frac{3}{4} + \frac{1}{4}\right| = 1$

(b) $\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$

$|\langle c|c\rangle| = \left|\left(\frac{1}{2} \;\; \frac{\sqrt{3}}{2}\right)\begin{pmatrix}\frac{1}{2}\\ \frac{\sqrt{3}}{2}\end{pmatrix}\right| = \left|\frac{1}{4} + \frac{3}{4}\right| = 1$ normirani!

(c) $\frac{1}{2}|0\rangle + \frac{\sqrt{3}}{2}|1\rangle$

(d) $\frac{1}{2}|0\rangle - \frac{\sqrt{3}}{2}|1\rangle$

$\langle c|e\rangle = \left(\frac{1}{2} \;\; \frac{\sqrt{3}}{2}\right)\begin{pmatrix}\frac{\sqrt{3}}{2}\\ -\frac{1}{2}\end{pmatrix} = \frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{4} = 0$

(e) $\frac{\sqrt{3}}{2}|0\rangle - \frac{1}{2}|1\rangle$

ortogonalni!

3 Qubit se nalazi u stanju $\phi\ \frac{1}{\sqrt{2}}(|0\rangle - \mathrm{i}\,|1\rangle)$. Izračunajte amplitudu vjerojatnosti nalaženja tog qubita u stanju $\frac{3}{5}\mathrm{i}\,|0\rangle + \frac{4}{5}|1\rangle$ ψ.

(a) $\frac{1}{\sqrt{2}}\left(\frac{3}{5} + \frac{4}{5}\mathrm{i}\right)$

(b) $\frac{1}{\sqrt{2}}\left(\frac{3}{5}\mathrm{i} + \frac{4}{5}\right)$

$a(\phi \to \psi) = \langle \psi|\phi\rangle = \left(\frac{-3}{5}i \;\; \frac{4}{5}\right)\begin{pmatrix}\frac{1}{\sqrt{2}}\\ \frac{-i}{\sqrt{2}}\end{pmatrix} = \frac{-3i}{5\sqrt{2}} - \frac{4}{5\sqrt{2}}i$

(c) $\frac{1}{\sqrt{2}}\left(\frac{3}{5} - \frac{4}{5}\mathrm{i}\right)$

(d) $\frac{1}{\sqrt{2}}\left(\frac{3}{5}\mathrm{i} - \frac{4}{5}\right)$

$= \frac{-7}{5\sqrt{2}}i$

(e) $\frac{1}{5\sqrt{2}}\mathrm{i}$

NIŠTA OD PONUĐENOG

4 Qubit se nalazi u stanju $\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$. Kolika je vjerojatnost da taj qubit bude izmjeren u stanju $\frac{1}{\sqrt{2}}(|0\rangle + \mathrm{i}\,|1\rangle)$?

(a) 0

(b) 1/4

(c) 1/2

(d) $1/\sqrt{2}$

(e) 1

$P(\phi \to \psi) = |\alpha(\phi \to \psi)|^2 = |\langle \psi | \phi \rangle|^2 = \left| \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{-i}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix} \right|^2 = \left| \frac{1}{2} - \frac{1}{2} i \right|^2 =$

$= \sqrt{\frac{1}{4} + \frac{1}{4}}^{\,2} = \frac{1}{2}$

5 Koja dva od pet navedenih vektora predstavljaju (na Blochovoj sferi) isto stanje qubita?

(a) $\frac{1}{\sqrt{2}}(\mathrm{i}\,|0\rangle + \mathrm{i}\,|1\rangle)$

(b) $\frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$

(c) $\frac{1}{\sqrt{2}}(|0\rangle - \mathrm{i}\,|1\rangle)$

(d) $\frac{1}{\sqrt{2}}(\mathrm{i}\,|0\rangle - \mathrm{i}\,|1\rangle)$

(e) $\frac{1}{\sqrt{2}}(\mathrm{i}\,|0\rangle + |1\rangle)$

Promjena faze ne utječe na stanje

6 Koja dva od pet navedenih operatora su hermitski operatori?

(a) $|0\rangle\langle 0|$

(b) $|0\rangle\langle 1|$

(c) $\mathrm{i}\,|1\rangle\langle 1|$

(d) $|0\rangle\langle 0| - |1\rangle\langle 1|$

(e) $|0\rangle\langle 0| + \mathrm{i}\,|1\rangle\langle 1|$

a) - po definiciji $|0\rangle\langle 0| = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$

d) $\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} - \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

7 Projekcija stanja qubita $\frac{3}{5}|0\rangle + \frac{4}{5}|1\rangle$ na stanje $\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$ je:

(a) $\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$

(b) $\frac{3}{5\sqrt{2}}|0\rangle + \frac{4}{5\sqrt{2}}|0\rangle$

(c) $\frac{4}{5\sqrt{2}}|0\rangle + \frac{3}{5\sqrt{2}}|0\rangle$

(d) $\frac{4}{5}|0\rangle + \frac{3}{5}|1\rangle$

(e) $\frac{7}{10}(|0\rangle + |1\rangle)$

$|\phi\rangle = \frac{3}{5}|0\rangle + \frac{4}{5}|1\rangle$

$|\psi\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$

$|P_\psi \phi\rangle = |\psi\rangle\langle\psi|\phi\rangle = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix} \otimes \left(\begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} \frac{3}{5} \\ \frac{4}{5} \end{pmatrix} \right) =$

$= \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix} \otimes \frac{7}{5\sqrt{2}} = \begin{pmatrix} \frac{7}{10} \\ \frac{7}{10} \end{pmatrix} = \frac{7}{10}(|0\rangle + |1\rangle)$

8 Matrični prikaz

$$\begin{pmatrix} 0 & i \\ 0 & 0 \end{pmatrix}$$

odgovara operatoru:

$\begin{pmatrix} i \\ 0 \end{pmatrix} \otimes (0 \ \ 1) = \begin{pmatrix} 0 & i \\ 0 & 0 \end{pmatrix}$

(a) $|0\rangle\langle 0|$

(b) $|1\rangle\langle 0|$

(c) $\mathrm{i}\,|0\rangle\langle 1|$

(d) $|0\rangle\langle 0| - |1\rangle\langle 1|$

(e) $|0\rangle\langle 0| + \mathrm{i}\,|1\rangle\langle 1|$

9 Očekivana vrijednost operatora prikazanog Paulijevom matricom σ_3 u sustavu koji se nalazi u stanju $\frac{1}{\sqrt{2}}(|0\rangle + \mathrm{i}\,|1\rangle)$ je:

(a) 1

(b) $1/\sqrt{2}$

(c) 0

(d) $-1/\sqrt{2}$

(e) -1

$\langle\psi|M|\psi\rangle = \frac{1}{\sqrt{2}}(\langle 0| - i\langle 1|)(|0\rangle\langle 0| - |1\rangle\langle 1|)\frac{1}{\sqrt{2}}(|0\rangle + i|1\rangle) =$

$= \frac{1}{2}(\underbrace{\langle 0|0\rangle}_{1}\langle 0| - \underbrace{\langle 0|1\rangle}_{0}\langle 1| - i\underbrace{\langle 1|0\rangle}_{0}\langle 0| + i\underbrace{\langle 1|1\rangle}_{1}\langle 1|)(|0\rangle + i|1\rangle) =$

$= \frac{1}{2}(\langle 0| + i\langle 1|)(|0\rangle + i|1\rangle) = \frac{1}{2}(\langle 0|0\rangle + i\langle 0|1\rangle + i\langle 1|0\rangle - \langle 1|1\rangle)$

$= \frac{1}{2} \cdot 0 = 0$

10 Tablica prikazuje uspostavljanje tajnog ključa protokolom BB84. Označite stupac u kojem možemo uočiti da je komunikacija bila prisluškivana.

Alice:	1	1	0	1	0	1	1	0	1	...
	⊗	⊗	⊗	⊕	⊕	⊗	⊗	⊕	⊕	...
	⊘	⊘	⦸	⊖	⦶	⊘	⊘	⦶	⊖	...
Bob:	⊕	⊕	⊕	⊕	⊗	⊗	⊕	⊕	⊕	...
	1	0	1	1	1	0	0	0	1	...