① Koji od vektora su normirani?

HI 2020

ⓐ $\frac{\sqrt{2}}{\sqrt{3}}|0\rangle + \frac{i}{\sqrt{3}}|1\rangle$ ✓ $\|\vec{a}\| = 1$

b) $\frac{1}{\sqrt{3}}|0\rangle + \frac{2}{\sqrt{3}}|1\rangle$ ✗ $\|\vec{a}\| \neq 1$

ⓒ $\frac{2}{\sqrt{5}}|0\rangle + \frac{i}{\sqrt{5}}|1\rangle$ ✓ $\|\vec{a}\| = 1$

d) $\frac{3}{5}|0\rangle - \frac{2i}{5}|1\rangle$ ✗

ⓔ $\frac{3}{\sqrt{25}}|0\rangle + \frac{4}{\sqrt{25}}|1\rangle$ ✓

Vektor je NORMIRAN ako mu je modul $\|\vec{a}\| == 1$

npr. $\|\vec{a}\| = \sqrt{\left(\sqrt{\frac{2}{3}}\right)^2 + \left(\frac{i}{\sqrt{3}}\right)^2} = 1$

② Koja 2 od navedenih vektora čine ortonomiranu bazu u $\mathcal{H}^{(2)}$?

a) $\frac{4}{5}|0\rangle - \frac{3}{5}|1\rangle$

b) $\frac{3}{5}|0\rangle - \frac{4}{5}|1\rangle$

c) $\frac{4}{5}|0\rangle + \frac{3}{5}i|1\rangle$

ⓓ $\frac{\sqrt{3}}{2}|0\rangle - \frac{1}{2}|1\rangle$ ✓

ⓔ $\frac{1}{2}|0\rangle + \frac{\sqrt{3}}{2}|1\rangle$

Da bi vektori činili ortonomiranu bazu u $\mathcal{H}^{(2)}$, mora vrijediti:

$\vec{a} \cdot \vec{b} = a_x b_x + a_y b_y = 0$

(jer $\cos 90° = 0$)

$\vec{d} \cdot \vec{e} = \frac{\sqrt{3}}{2} \cdot \frac{1}{2} + \left(-\frac{1}{2}\right) \cdot \frac{\sqrt{3}}{2} = 0$

③ Koji od navedenih predstavljaju isto stanje kvantnog bita?

ⓐ $\frac{1}{\sqrt{2}}(+|0\rangle + |1\rangle)$

ⓑ $\frac{1}{\sqrt{2}}(+i|0\rangle + i|1\rangle)$ ✓

ⓒ $\frac{1}{\sqrt{2}}(-|0\rangle - |1\rangle)$

d) $\frac{1}{\sqrt{2}}(|0\rangle - i|1\rangle)$

e) $\frac{1}{\sqrt{2}}(|0\rangle + i|1\rangle)$

Da bi bili isto stanje trebaju...

=> isti broj ispred zagrade

=> isti "ukupni" predznak $+ \cdot + = +$ $- \cdot + = -$ $- \cdot - = +$

=> isti "raspored" imaginarnih / realnih brojeva

↓ u ovom slučaju ili oba člana imaju i ili niti jedan član nema i

④ Kvantni bit nalazi se u stanju $\frac{4}{\sqrt{25}}|0\rangle + \frac{3}{\sqrt{25}}|1\rangle$

U kojem od ponuđenih stanja je vjerojatnost nalaženja tog qubita MAX?

a) $|0\rangle$

$p=0.98$ b) $\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$ ✓

c) $\frac{1}{\sqrt{2}}(|0\rangle + i|1\rangle)$

d) $\frac{1}{\sqrt{2}}(|0\rangle - i|1\rangle)$

e) $|1\rangle$

Samo pomnožimo zadani vektor skalarno s mogućim rješenjima (amplituda) pa zatim kvadriramo rješenje (vjerojatnost).

MAX – najveće koje smo dobili

MIN – najmanje koje smo dobili

$$\left(\frac{4}{5}|0\rangle + \frac{3}{5}|1\rangle\right) \cdot |0\rangle = \frac{4}{5}\cdot 1 + \cancel{\frac{3}{5}\cdot 0} = \frac{4}{5} \xrightarrow{\ |^2\ } \frac{16}{25} = 0.64$$

$$\left(\frac{4}{5}|0\rangle + \frac{3}{5}|1\rangle\right) \cdot \left(\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle\right) = \frac{4}{5\sqrt{2}} + \frac{3}{5\sqrt{2}} = \frac{7}{5\sqrt{2}} \xrightarrow{\ |^2\ } \frac{49}{50} = 0.98$$

⑤ Operator projekcije na stanje $\frac{1}{\sqrt{2}}(|0\rangle + i|1\rangle)$ odgovara matrici...?

a) $\frac{1}{2}\begin{bmatrix} 1 & -i \\ i & 1 \end{bmatrix}$ ✓

b) $\frac{1}{2}\begin{bmatrix} 1 & i \\ -i & 1 \end{bmatrix}$

c) $\begin{bmatrix} 1 & 0 \\ 0 & -i \end{bmatrix}$

d) $\frac{1}{2}\begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$

e) $\frac{1}{2}\begin{bmatrix} 0 & i \\ -i & 0 \end{bmatrix}$

Prvo vektor zadanog stanja pomnožimo s njegovim konjugirano-kompleksnim vektorom:

$(a|0\rangle + b|1\rangle)\cdot(a^*|0\rangle + b^*|1\rangle)$

I dobijemo vektor oblika

$\alpha|00\rangle + \beta|01\rangle + \gamma|10\rangle + \delta|11\rangle$

Što je matrično $\begin{bmatrix} \alpha & \beta \\ \gamma & \delta \end{bmatrix}$

$$\frac{1}{\sqrt{2}}(|0\rangle + i|1\rangle) \longrightarrow \left(\frac{1}{\sqrt{2}}|0\rangle + \frac{i}{\sqrt{2}}|1\rangle\right)$$

Conj-cmplx: $\left(\frac{1}{\sqrt{2}}|0\rangle - \frac{i}{\sqrt{2}}|1\rangle\right)$

$$\left(\frac{1}{\sqrt{2}}|0\rangle + \frac{i}{\sqrt{2}}\right)\left(\frac{1}{\sqrt{2}}|0\rangle - \frac{i}{\sqrt{2}}|1\rangle\right) = \frac{1}{2}|00\rangle - \frac{i}{2}|01\rangle + \frac{i}{2}|10\rangle + \frac{1}{2}|11\rangle$$

$$\Rightarrow \frac{1}{2}\cdot\begin{bmatrix} 1 & -i \\ i & 1 \end{bmatrix}$$

⑥ Očekivana vrijednost operatora $|0\rangle\langle 0| - |1\rangle\langle 1|$ za qubit stanja $\frac{1}{\sqrt{2}}(|0\rangle + i|1\rangle)$ je:

a) -1

b) $-1/2$

c) 0 ✓

d) $1/2$

e) 1

$$u = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \qquad \phi = \begin{bmatrix} 1/\sqrt{2} \\ i/\sqrt{2} \end{bmatrix}$$

$$u\phi = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \cdot \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{i}{\sqrt{2}} \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{-i}{\sqrt{2}} \end{bmatrix}$$

$$u\langle\phi\rangle = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{i}{\sqrt{2}} \end{bmatrix} \cdot \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{-i}{\sqrt{2}} \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & \frac{-i}{2} \\ \frac{i}{2} & \frac{1}{2} \end{bmatrix}$$

$$\det(u\langle\phi\rangle) = \frac{1}{2}\cdot\frac{1}{2} - \frac{i}{2}\cdot\frac{-i}{2} = \frac{1}{4} - \frac{1}{4} = 0$$

⑦ Ako je hamiltonijan qubita $H = \frac{\hbar\omega}{2}|0\rangle\langle 0| - \frac{\hbar\omega}{2}|1\rangle\langle 1|$

te ako je početno stanje qubita $\frac{1}{\sqrt{5}}(2|0\rangle + |1\rangle)$

taj će se qubit naći u stanju $\frac{1}{\sqrt{5}}(|0\rangle + 2|1\rangle)$ nakon vremena ...?

a) $\frac{\pi}{2\omega}$

b) $\frac{\pi}{\omega}$

c) $\frac{3\pi}{2\omega}$

d) $\frac{2\pi}{\omega}$

e) To se neće dogoditi ✓

$\omega_0 > \omega_1 \qquad \mu_0 = \frac{2}{\sqrt{5}} \qquad \eta_0 = \frac{1}{\sqrt{5}}$

$$|\phi[t]\rangle = \mu[t]|0\rangle + \eta[t]|1\rangle$$

$$\frac{d\mu}{\mu} = \frac{-i\omega_0 dt}{2} \qquad \frac{d\eta}{\eta} = \frac{-i\omega_1 dt}{2}$$

$$\mu[t] = \mu_0 e^{\frac{-i\omega_0 t}{2}} \qquad \eta[t] = \eta_0 e^{-\frac{i\omega_1 t}{2}}$$

$$|\phi[t]\rangle = \mu_0 e^{\frac{-i\omega_0 t}{2}}|0\rangle + \eta_0 e^{-\frac{i\omega_1 t}{2}}|1\rangle$$

formula nakon izvoda

$$i\hbar\left(\frac{d}{dt}\mu[t]|0\rangle + \frac{d}{dt}\eta[t]|1\rangle\right) = \frac{\hbar\omega_0}{2}\mu[t] + \frac{\hbar\omega_1}{2}\eta[t]|1\rangle$$

$\Rightarrow$ uvrstimo μ_0 i η_0 u $|\phi[t]\rangle = \ldots$

$$\frac{1}{\sqrt{5}}|0\rangle + \frac{2}{\sqrt{5}}|1\rangle = \frac{2}{\sqrt{5}}e^{-\frac{\omega_0 t}{2}}|0\rangle + \frac{1}{5}e^{-\frac{i\omega_1 t}{2}}|1\rangle$$

|u⟩

$\omega_0 = \omega_1 = \omega \rightarrow t = ?$

$$\frac{2}{\sqrt{5}}e^{-\frac{\omega t}{2}} \neq \frac{1}{\sqrt{5}} \quad \text{za bilo koji } t$$

$\Rightarrow$ nikad se neće dogoditi (N.R.)

⑧ U kojima od 5 navedenih stanja je sustav dvaju kvantnih bitova spregnut?

a) $\frac{1}{2}(|00\rangle + |01\rangle + |10\rangle + |11\rangle)$

b) $\frac{1}{2}(|00\rangle - |01\rangle + |10\rangle - |11\rangle)$

c) $\frac{1}{\sqrt{2}}(|00\rangle + i|11\rangle)$ ✓

d) $\frac{1}{\sqrt{2}}(|01\rangle + i|10\rangle)$

e) $\frac{1}{\sqrt{2}}(|00\rangle + |01\rangle)$

Vektori oblika:

$a|00\rangle + b|01\rangle + c|10\rangle + d|11\rangle$

Uvjet spregnutosti:

$ad - bc \neq 0$

npr. $\frac{1}{2}(|00\rangle + |01\rangle + |10\rangle + |11\rangle)$

$a = b = c = d = \frac{1}{2}$

$ad - bc = \frac{1}{2}\cdot\frac{1}{2} - \frac{1}{2}\cdot\frac{1}{2} = 0$ nije spregnut

npr. $\frac{1}{\sqrt{2}}(|00\rangle + i|11\rangle)$

$a = \frac{1}{\sqrt{2}}$ $d = \frac{i}{\sqrt{2}}$ $c = b = 0$

$ad - bc = \frac{1}{\sqrt{2}}\cdot\frac{i}{\sqrt{2}} - 0 = \frac{i}{2} \neq 0$ spregnut

⑨ Matrica $\frac{1}{2}\cdot\begin{bmatrix}1&0&0&1\\0&0&0&0\\0&0&0&0\\1&0&0&1\end{bmatrix}$ prikazuje operator projekcije na stanje...?

a) $\frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)$

b) $\frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$

c) $\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$ ✓

d) $\frac{1}{\sqrt{2}}(|00\rangle - |11\rangle)$

e) Ništa od navedenog

$\frac{1}{2} \xrightarrow{\sqrt{\ }} \frac{1}{\sqrt{2}}$

$$\begin{array}{c|cccc} & 00 & 01 & 10 & 11 \\ \hline 00 & 1 & 0 & 0 & 1 \\ 01 & 0 & 0 & 0 & 0 \\ 10 & 0 & 0 & 0 & 0 \\ 11 & 1 & 0 & 0 & 1 \end{array}$$

$|00\rangle = \begin{bmatrix}1\\0\\0\\1\end{bmatrix}$ + $|01\rangle = \begin{bmatrix}0\\0\\0\\0\end{bmatrix}$ → sve 0 nema ga

$|10\rangle = \begin{bmatrix}0\\0\\0\\0\end{bmatrix}$ $|11\rangle = \begin{bmatrix}1\\0\\0\\1\end{bmatrix}$ +

(10) Sustav dvaju kvantnih bitova realiziran je projekcijama spinova dviju čestica ($s = 1/2$) os z. Sustav se nalazi u stanju $\frac{1}{\sqrt{2}}(|00\rangle + |01\rangle)$

Očekivana vrijednost projekcije spina prve čestice na z osi iznosi:

a) $-\hbar$

b) $-\hbar/2$

c) 0

d) $\hbar/2$ ✓

e) $\hbar$

Separabilno stanje $|0\rangle|+\rangle \Rightarrow \frac{\hbar}{2}$