① Qubit se nalazi u stanju

$|\phi\rangle = \alpha|0\rangle + \beta|1\rangle.$

2020.

Želimo li taj qubit dovesti u stanje

$|\phi'\rangle = \beta|0\rangle + \alpha|1\rangle,$

u prikazu stanja na Blochovoj sferi moramo provesti...

a) rotaciju za π oko x-osi,
b) rotaciju za π oko y-osi,
c) rotaciju za π oko z-osi,
d) rotaciju za $\pi/2$ oko z-osi,
e) ništa od navedenog.

<u>1. način</u>

traženje operatora ("matrično")

$$\left(* \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = X \quad \pi \text{ oko x-osi} \quad \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} = Y \quad \pi \text{ oko y-osi} \quad \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} = Z \quad \pi \text{ oko z-osi} \right)$$

$|\phi'\rangle = M|\phi\rangle$

$$\begin{bmatrix} \beta \\ \alpha \end{bmatrix} = M \begin{bmatrix} \alpha \\ \beta \end{bmatrix} \rightarrow M = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \qquad \left(* \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \begin{bmatrix} 0\cdot\alpha + 1\cdot\beta \\ 1\cdot\alpha + 0\cdot\beta \end{bmatrix} = \begin{bmatrix} \beta \\ \alpha \end{bmatrix} \right)$$

↳ operator NOT(X) → odgovara rotaciji za π oko x-osi

<u>2. način</u>

Blochova sfera

$$|\phi\rangle = \cos\frac{\vartheta}{2} e^{i\frac{\varphi}{2}} |0\rangle + \sin\frac{\vartheta}{2} e^{i\frac{\varphi}{2}} |1\rangle = \alpha|0\rangle + \beta|1\rangle \rightarrow \alpha = \cos\frac{\vartheta}{2} e^{i\frac{\varphi}{2}},\ \beta = \sin\frac{\vartheta}{2} e^{i\frac{\varphi}{2}}$$
$$|\phi'\rangle = \cos\frac{\vartheta'}{2} e^{i\frac{\varphi'}{2}} |0\rangle + \sin\frac{\vartheta'}{2} e^{i\frac{\varphi'}{2}} |1\rangle = \beta|0\rangle + \alpha|1\rangle \rightarrow \alpha = \sin\frac{\vartheta'}{2} e^{i\frac{\varphi'}{2}},\ \beta = \cos\frac{\vartheta'}{2} e^{i\frac{\varphi'}{2}}$$

$\alpha \rightarrow \cos\frac{\vartheta}{2} = \sin\frac{\vartheta'}{2}$

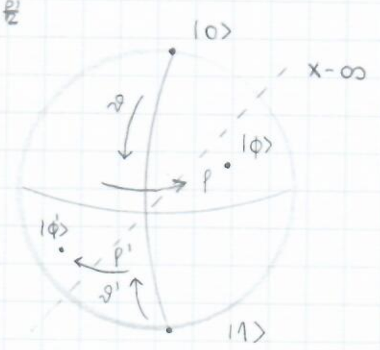
$\cos\frac{\vartheta}{2} = \cos\left(\frac{\pi}{2} - \frac{\vartheta'}{2}\right) \ /\arccos/\cdot 2$

$\vartheta = \pi - \vartheta'$

$\alpha \rightarrow e^{i\frac{\varphi}{2}} = e^{-i\frac{\varphi'}{2}}$

$\varphi = -\varphi'$

isto vrijedi za β!



② Na izlazu iz logičkog kruga

$|0\rangle$ —[H]—[Z]—[H]—

2020

stanje qubita istovjetno je stanju ...

a) $|0\rangle$,
b) $|1\rangle$,
c) $|+\rangle$,
d) $|-\rangle$,
e) ništa od navedenog.

H → Hadamardov operator
Z → Z operator

$\left(* \; H = \frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}, \; Z = \begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix}\right)$

$$H|0\rangle = \frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}\begin{bmatrix}1\\ 0\end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix}1\cdot 1 + 1\cdot 0\\ 1\cdot 1 + (-1)\cdot 0\end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix}1\\ 1\end{bmatrix} = \frac{1}{\sqrt{2}}\left(\begin{bmatrix}1\\ 0\end{bmatrix} + \begin{bmatrix}0\\ 1\end{bmatrix}\right)$$

$$= \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) = |+\rangle$$

$$ZH|0\rangle = \begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix}\cdot\frac{1}{\sqrt{2}}\left(\begin{bmatrix}1\\ 0\end{bmatrix} + \begin{bmatrix}0\\ 1\end{bmatrix}\right) = \frac{1}{\sqrt{2}}\cdot\left(\begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix}\begin{bmatrix}1\\ 0\end{bmatrix} + \begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix}\begin{bmatrix}0\\ 1\end{bmatrix}\right)$$

$$= \frac{1}{\sqrt{2}}\left(\begin{bmatrix}1\cdot 1 + 0\cdot 0\\ 0\cdot 1 + (-1)\cdot 0\end{bmatrix} + \begin{bmatrix}1\cdot 0 + 0\cdot 1\\ 0\cdot 0 + (-1)\cdot 1\end{bmatrix}\right) = \frac{1}{\sqrt{2}}\left(\begin{bmatrix}1\\ 0\end{bmatrix} + (-1)\cdot\begin{bmatrix}0\\ 1\end{bmatrix}\right)$$

$$= \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle) = |-\rangle$$

$$HZH|0\rangle = \frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}\cdot\frac{1}{\sqrt{2}}\left(\begin{bmatrix}1\\ 0\end{bmatrix} - \begin{bmatrix}0\\ 1\end{bmatrix}\right) = \frac{1}{2}\cdot\left(\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}\cdot\begin{bmatrix}1\\ 0\end{bmatrix} - \begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}\begin{bmatrix}0\\ 1\end{bmatrix}\right)$$

$$= \frac{1}{2}\left(\begin{bmatrix}1\cdot 1 + 1\cdot 0\\ 1\cdot 1 + (-1)\cdot 0\end{bmatrix} - \begin{bmatrix}1\cdot 0 + 1\cdot 1\\ 1\cdot 0 + (-1)\cdot 0\end{bmatrix}\right) = \frac{1}{2}\left(\begin{bmatrix}1\\ 1\end{bmatrix} - \begin{bmatrix}1\\ -1\end{bmatrix}\right)$$

$$= \frac{1}{2}\left(\begin{bmatrix}1\\ 0\end{bmatrix} + \begin{bmatrix}0\\ 1\end{bmatrix} - \left(\begin{bmatrix}1\\ 0\end{bmatrix} - \begin{bmatrix}0\\ 1\end{bmatrix}\right)\right) = \frac{1}{2}(|0\rangle + |1\rangle - (|0\rangle - |1\rangle))$$

$$= \frac{1}{2}\cdot 2\cdot|1\rangle = |1\rangle$$

$\left(* \; |0\rangle = \begin{bmatrix}1\\ 0\end{bmatrix}, \; |1\rangle = \begin{bmatrix}0\\ 1\end{bmatrix}, \; |+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle), \; |-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)\right)$

$(* \; H|0\rangle = |+\rangle, \; H|1\rangle = |-\rangle, \; H|+\rangle = |0\rangle, \; H|-\rangle = |1\rangle, \; Z|0\rangle = |0\rangle, \; Z|1\rangle = -|1\rangle)$

② Razmatramo kvantni logički krug

$|0\rangle$ —[H]—[R(ϕ)]—[H]—[⁄]

Kolika je vjerojatnost da u mjerenju dobijemo vrijednost $|1\rangle$?

a) 0
b) $\frac{1}{2}(1+\cos\phi)$
c) $\frac{1}{2}(1-\cos\phi)$
d) $\cos\phi$,
e) $\cos\phi^2$.

2019.

$$H|0\rangle = \frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}\cdot\begin{bmatrix}1\\ 0\end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix}1\cdot 1+1\cdot 0\\ 1\cdot 1+(-1)\cdot 0\end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix}1\\ 1\end{bmatrix} = \frac{1}{\sqrt{2}}\left(\begin{bmatrix}1\\ 0\end{bmatrix}+\begin{bmatrix}0\\ 1\end{bmatrix}\right)$$

$$= \frac{1}{\sqrt{2}}(|0\rangle+|1\rangle) = |+\rangle$$

$$R(\phi)H|0\rangle = \begin{bmatrix}1 & 0\\ 0 & e^{i\phi}\end{bmatrix}\cdot\frac{1}{\sqrt{2}}\left(\begin{bmatrix}1\\ 0\end{bmatrix}+\begin{bmatrix}0\\ 1\end{bmatrix}\right) = \frac{1}{\sqrt{2}}\left(\begin{bmatrix}1 & 0\\ 0 & e^{i\phi}\end{bmatrix}\begin{bmatrix}1\\ 0\end{bmatrix}+\begin{bmatrix}1 & 0\\ 0 & e^{i\phi}\end{bmatrix}\begin{bmatrix}0\\ 1\end{bmatrix}\right)$$

$$= \frac{1}{\sqrt{2}}\left(\begin{bmatrix}1\cdot 1+0\cdot 0\\ 0\cdot 1+e^{i\phi}\cdot 0\end{bmatrix}+\begin{bmatrix}1\cdot 0+0\cdot 0\\ 0\cdot 0+e^{i\phi}\cdot 1\end{bmatrix}\right) = \frac{1}{\sqrt{2}}\left(\begin{bmatrix}1\\ 0\end{bmatrix}+e^{i\phi}\begin{bmatrix}0\\ 1\end{bmatrix}\right)$$

$$= \frac{1}{\sqrt{2}}(|0\rangle+e^{i\phi}|1\rangle)$$

$$HR(\phi)H|0\rangle = \frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}\cdot\frac{1}{\sqrt{2}}\left(\begin{bmatrix}1\\ 0\end{bmatrix}+\begin{bmatrix}0\\ e^{i\phi}\end{bmatrix}\right) = \frac{1}{2}\left(\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}\begin{bmatrix}1\\ 0\end{bmatrix}+\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}\begin{bmatrix}0\\ e^{i\phi}\end{bmatrix}\right)$$

$$= \frac{1}{2}\left(\begin{bmatrix}1\cdot 1+1\cdot 0\\ 1\cdot 1+(-1)\cdot 0\end{bmatrix}+\begin{bmatrix}1\cdot 0+1\cdot e^{i\phi}\\ 1\cdot 0+(-1)\cdot e^{i\phi}\end{bmatrix}\right) = \frac{1}{2}\left(\begin{bmatrix}1\\ 1\end{bmatrix}+e^{i\phi}\begin{bmatrix}1\\ -1\end{bmatrix}\right)$$

$|\psi\rangle$

$$= \frac{1}{2}\left(\left(\begin{bmatrix}1\\ 0\end{bmatrix}+\begin{bmatrix}0\\ 1\end{bmatrix}\right)+e^{i\phi}\left(\begin{bmatrix}1\\ 0\end{bmatrix}-\begin{bmatrix}0\\ 1\end{bmatrix}\right)\right) = \frac{1}{2}\left((|0\rangle+|1\rangle)+e^{i\phi}(|0\rangle-|1\rangle)\right)$$

$$= \frac{1}{2}\left((1+e^{i\phi})|0\rangle+(1-e^{i\phi})|1\rangle\right) = \frac{1}{2}\left((1+\cos\phi+i\sin\phi)|0\rangle+(1-\cos\phi-i\sin\phi)|1\rangle\right)$$

$$P_{\psi\to 1} = |\langle\psi|1\rangle|^2$$

$\langle\psi|$ = konj. kompleksni $\psi = \frac{1}{2}\left((1+\cos\phi-i\sin\phi)|0\rangle+(1-\cos\phi+i\sin\phi)|1\rangle\right)$

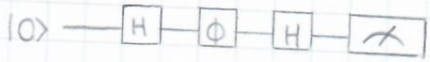
$$P_{\psi\to 1} = \left|\left(\frac{1}{2}\left((1+\cos\phi-i\sin\phi)\langle 0|+(1-\cos\phi+i\sin\phi)\langle 1|\right)\right)\cdot(0\cdot|0\rangle+1\cdot|1\rangle)\right|^2$$

= ostaje umnožak vrijednosti uz $\langle 0|\,|0\rangle$ te uz $\langle 1|\,|1\rangle$

$$= \left|\frac{1}{2}(1-\cos\phi+i\sin\phi)\right|^2 = \frac{1}{4}(1-2\cos\phi+\cos^2\phi+\sin^2\phi) = \frac{1}{4}(2-2\cos\phi)$$

$$= \frac{1}{2}(1-\cos\phi)$$

(2) Razmatramo kvantni logički krug



gdje je operator Φ definiran s $|0\rangle \to |0\rangle$ i $|1\rangle \to e^{i\phi}|1\rangle$ pri čemu je faza ϕ realan broj. Kolika je vjerojatnost da u mjerenju dobijemo vrijednost 0, tj. da je qubit u stanju $|0\rangle$?

a) 0
b) $\frac{1}{2}(1-\cos\phi)$
c) $\frac{1}{2}(1+\cos\phi)$
d) $\cos\phi$
e) $\cos^2\phi$

2018

$$H|0\rangle = |+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \qquad (*H|0\rangle = |+\rangle,\ H|1\rangle = |-\rangle,\ H|+\rangle = |0\rangle,\ H|-\rangle = |1\rangle)$$

$$\Phi H|0\rangle = \Phi\left(\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)\right) = \frac{1}{\sqrt{2}}(\Phi|0\rangle + \Phi|1\rangle) = \frac{1}{\sqrt{2}}(|0\rangle + e^{i\phi}|1\rangle)$$

$$H\Phi H|0\rangle = H\left(\frac{1}{\sqrt{2}}(|0\rangle + e^{i\phi}|1\rangle)\right) = \frac{1}{\sqrt{2}}(H|0\rangle + e^{i\phi}\cdot H|1\rangle) = \frac{1}{\sqrt{2}}(|+\rangle + e^{i\phi}|-\rangle)$$

$$= \frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) + e^{i\phi}\cdot\frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)\right) = \frac{1}{2}\left((1+e^{i\phi})|0\rangle + (1-e^{i\phi})|1\rangle\right)$$

$$= \frac{1}{2}\left((1+\cos\phi + i\sin\phi)|0\rangle + (1-\cos\phi - i\sin\phi)|1\rangle\right)$$

$$= |\psi\rangle \quad \to \quad \langle\psi| = \frac{1}{2}\left((1+\cos\phi - i\sin\phi)\langle 0| + (1-\cos\phi + i\sin\phi)\langle 1|\right)$$

$$P_{\psi\to 0} = |\langle\psi|0\rangle|^2 = \left|\frac{1}{2}\left((1+\cos\phi - i\sin\phi)\langle 0| + (1-\cos\phi + i\sin\phi)\langle 1|\right)\cdot(|0\rangle)\right|^2$$

$$= \left|\frac{1}{2}(1+\cos\phi - i\sin\phi)\right|^2 = \frac{1}{4}(1 + 2\cos\phi + \cos^2\phi + \sin^2\phi) = \frac{1}{4}(2 + 2\cos\phi)$$

$$= \frac{1}{2}(1+\cos\phi)$$

③ Ako na izlazu kvantnog logičkog kruga

$|0\rangle$ —$\boxed{H}$—$\boxed{?}$—$\boxed{Z}$—

2019.

dobivamo stanje

$\frac{1}{\sqrt{2}}(|0\rangle - i|1\rangle)$,

operator označen upitnikom je?

a) X
b) Y
c) Z
d) S
e) T

$\left(* \; X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, Y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}, Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, S = \begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}, T = \begin{bmatrix} 1 & 0 \\ 0 & e^{i\frac{\pi}{4}} \end{bmatrix} \right)$

Označimo s $|\psi\rangle$ stanje prije ulaska u nepoznati operator. Stanje $|\psi\rangle$ dobivamo djelovanjem operatora H na početno stanje $|0\rangle$.

$$|\psi\rangle = H|0\rangle \qquad (* \; H|0\rangle = |+\rangle)$$
$$= |+\rangle$$
$$= \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$

Označimo s $|\psi'\rangle$ stanje prije ulaska u operator Z. Stanje nakon djelovanja operatora Z jednako je rezultantnom stanju.

$$Z|\psi'\rangle = \frac{1}{\sqrt{2}}(|0\rangle - i|1\rangle)$$
$$Z(\alpha_{\psi'}|0\rangle + \beta_{\psi'}|1\rangle) = \frac{1}{\sqrt{2}}(|0\rangle - i|1\rangle)$$

Za operator Z vrijedi $Z|0\rangle = |0\rangle$ i $Z|1\rangle = -|1\rangle$ pomoću čega dobivamo...

$$\left.\begin{aligned} \alpha_{\psi'} &= \tfrac{1}{\sqrt{2}} \\ \beta_{\psi'} &= i\tfrac{1}{\sqrt{2}} \end{aligned}\right\} \rightarrow |\psi\rangle = \frac{1}{\sqrt{2}}(|0\rangle + i|1\rangle)$$

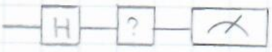
Nepoznati operator možemo dobiti pomoću stanja $|\psi\rangle$ i $|\psi'\rangle$.

$$?|\psi\rangle = |\psi'\rangle$$
$$?\left(\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)\right) = \frac{1}{\sqrt{2}}(|0\rangle + i|1\rangle)$$
$$\frac{1}{\sqrt{2}}(?|0\rangle + ?|1\rangle) = \frac{1}{\sqrt{2}}(|0\rangle + i|1\rangle)$$

Nepoznati operator trebao bi stanje $|0\rangle$ ostaviti nepromijenjeno, a stanje $|1\rangle$ pretvoriti u $i|1\rangle$. To radi operator S.

$(* \; |L\rangle = \frac{1}{\sqrt{2}}(|0\rangle - i|1\rangle), \; |R\rangle = \frac{1}{\sqrt{2}}(|0\rangle + i|1\rangle))$

③ Mjerenjem stanja qubita na izlazu iz logičkog kruga

$|0\rangle$ 

vrijednost 0 dobivamo u 50% slučajeva. Iz toga možemo zaključiti da operator označen upitnikom zasigurno nije operator...

a) X,
b) Y,
c) Z,
d) H,
e) S.

2018.

Operator H za ulazno stanje $|0\rangle$ vraća izlazno stanje $|+\rangle$, što je ujedno i ulazno stanje nepoznatog operatora. Za operator H također vrijedi $H|+\rangle = |0\rangle$ što znači da bi mjerenjem stanja qubita uvijek dobili vrijednost 0. Zbog toga, operator H sigurno nije nepoznati operator.

(4) Na izlazu iz logičkog kruga

$|0\rangle$ —[H]—[Z]—[S]—[H]—

stanje qubita istovjetno je stanju ...

2018.

a) $|0\rangle$,
b) $|1\rangle$,
c) $|+\rangle$,
d) $|-\rangle$,
e) ništa od navedenog.

$H|0\rangle = |+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$

$ZH|0\rangle = Z(\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)) = \frac{1}{\sqrt{2}}(Z|0\rangle + Z|1\rangle) = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle) = |-\rangle$

$SZH|0\rangle = S(\frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)) = \frac{1}{\sqrt{2}}(S|0\rangle - S|1\rangle) = \frac{1}{\sqrt{2}}(|0\rangle - i|1\rangle)$

$HSZH|0\rangle = H(\frac{1}{\sqrt{2}}(|0\rangle - i|1\rangle)) = \frac{1}{\sqrt{2}}(H|0\rangle - iH|1\rangle)$

$= \frac{1}{\sqrt{2}}(\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) - i\frac{1}{\sqrt{2}}(|0\rangle - |1\rangle))$

$= \frac{1}{2}((1-i)|0\rangle + (1+i)|1\rangle)$

(* $H|0\rangle = |+\rangle$, $H|1\rangle = |-\rangle$, $Z|0\rangle = |0\rangle$, $Z|1\rangle = -|1\rangle$, $S|0\rangle = |0\rangle$, $S|1\rangle = i|1\rangle$)

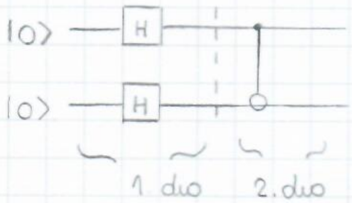
(* $|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$, $|-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$)

③ Na desnoj (izlaznoj) strani kvantnog logičkog kruga

2020.

dobivamo stanje...

a) $\frac{1}{2}(|00\rangle + |01\rangle + |10\rangle + |11\rangle)$,
b) $\frac{1}{2}(|00\rangle - |01\rangle + |10\rangle - |11\rangle)$,
c) $\frac{1}{2}(|00\rangle - |01\rangle - |10\rangle + |11\rangle)$,
d) $\frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)$,
e) $\frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$.



1. dio $\rightarrow H|0\rangle \otimes H|0\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \otimes \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$
$= \frac{1}{2}(|00\rangle + |01\rangle + |10\rangle + |11\rangle)$
$= |\psi_1\rangle$

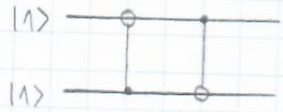
2. dio $\rightarrow CNOT|\psi_1\rangle = CNOT\left(\frac{1}{2}(|00\rangle + |01\rangle + |10\rangle + |11\rangle)\right)$
$= \frac{1}{2}(CNOT|00\rangle + CNOT|01\rangle + CNOT|10\rangle + CNOT|11\rangle)$
$= \frac{1}{2}(|00\rangle + |01\rangle + |11\rangle + |10\rangle)$
$= \frac{1}{2}(|00\rangle + |01\rangle + |10\rangle + |11\rangle)$

$(* (\lambda_1|0\rangle + \mu_1|1\rangle) \otimes (\lambda_2|0\rangle + \mu_2|1\rangle) = \lambda_1\lambda_2|00\rangle + \lambda_1\mu_2|01\rangle + \mu_1\lambda_2|10\rangle + \mu_1\mu_2|11\rangle)$

$$\left(* \ |00\rangle = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, |01\rangle = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, |10\rangle = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}, |11\rangle = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \ CNOT = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \right)$$

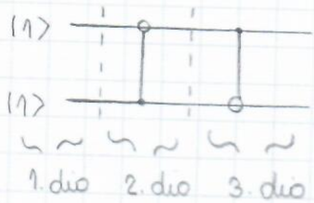
$(* \ CNOT|00\rangle = |00\rangle, CNOT|01\rangle = |01\rangle, CNOT|10\rangle = |11\rangle, CNOT|11\rangle = |10\rangle)$

4) Stanje sustava na izlaznoj strani kvantnog logičkog kruga



je...

a) $|01\rangle$,
b) $|10\rangle$,
c) $\frac{1}{\sqrt{2}}(|01\rangle+|10\rangle)$,
d) $\frac{1}{\sqrt{2}}(|01\rangle-|10\rangle)$,
e) $|11\rangle$.

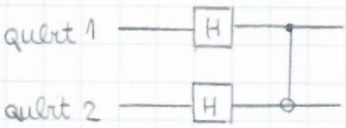


1. dio $\rightarrow |1\rangle \otimes |1\rangle = |11\rangle = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$

2. dio $\rightarrow$ prekrenuti CNOT$|11\rangle = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} = |01\rangle$

3. dio $\rightarrow$ CNOT$|01\rangle = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} = |01\rangle$

5) Shvatimo li logički krug



2019.

Kao jedan operator, njegov matrični prikaz je...

a) $\frac{1}{\sqrt{2}}\begin{bmatrix}1&0&1&0\\0&1&0&1\\0&1&0&-1\\1&0&-1&0\end{bmatrix}$,

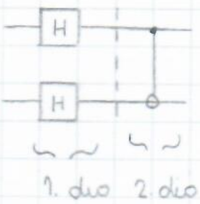
b) $\frac{1}{\sqrt{2}}\begin{bmatrix}1&0&0&1\\0&1&1&0\\1&0&0&-1\\0&1&-1&0\end{bmatrix}$,

c) $\frac{1}{\sqrt{2}}\begin{bmatrix}1&1&0&0\\1&-1&0&0\\0&0&1&-1\\0&0&1&1\end{bmatrix}$,

d) $\frac{1}{\sqrt{2}}\begin{bmatrix}1&1&0&0\\1&-1&0&0\\0&0&1&1\\0&0&-1&1\end{bmatrix}$,

e) $\frac{1}{2}\begin{bmatrix}1&1&1&1\\1&-1&1&-1\\1&-1&-1&1\\1&1&-1&-1\end{bmatrix}$.

Promatramo sljedeći dio kruga kao jedan operator...

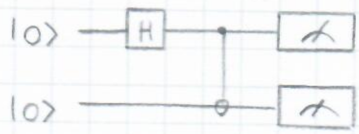


1. dio $\rightarrow$ $H \otimes H = \frac{1}{\sqrt{2}}\begin{bmatrix}1&1\\1&-1\end{bmatrix} \otimes \frac{1}{\sqrt{2}}\begin{bmatrix}1&1\\1&-1\end{bmatrix} = \frac{1}{2}\begin{bmatrix}1&1&1&1\\1&-1&1&-1\\1&1&-1&-1\\1&-1&-1&1\end{bmatrix}$

2. dio $\rightarrow$ $CNOT(H \otimes H) = \begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&0&1\\0&0&1&0\end{bmatrix}\begin{bmatrix}1&1&1&1\\1&-1&1&-1\\1&1&-1&-1\\1&-1&-1&1\end{bmatrix} = \begin{bmatrix}1&1&1&1\\1&-1&1&-1\\1&-1&-1&1\\1&1&-1&-1\end{bmatrix}$

$\longrightarrow$ $\downarrow$

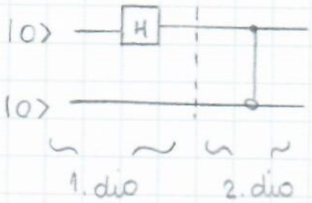
(6) Kolika je vjerojatnost da na izlazu iz kvantnog logičkog kruga



sustav izmjerimo u stanju $|01\rangle$?

2019.

a) 0
b) $\frac{1}{4}$
c) $\frac{1}{2}$
d) $\frac{1}{\sqrt{2}}$
e) 1



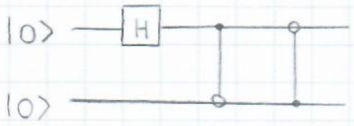
1. dio → $H|0\rangle \otimes |0\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \otimes |0\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |10\rangle)$

2. dio → $CNOT(H|0\rangle \otimes |0\rangle) = CNOT\left(\frac{1}{\sqrt{2}}(|00\rangle + |10\rangle)\right)$
$= \frac{1}{\sqrt{2}}(CNOT|00\rangle + CNOT|10\rangle)$
$= \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$

↳ vjerojatnost za $|01\rangle$ je 0!

(* $CNOT|00\rangle = |00\rangle$, $CNOT|01\rangle = |01\rangle$, $CNOT|10\rangle = |11\rangle$, $CNOT|11\rangle = |10\rangle$)

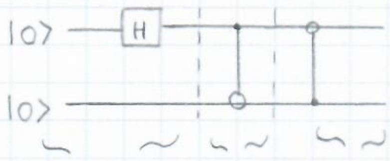
⑤ Stanje sustava na izlaznoj (desnoj) strani kvantnog logičkog kruga



2018.

je...

a) $\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$,
b) $\frac{1}{\sqrt{2}}(|00\rangle - |11\rangle)$,
c) $\frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)$,
d) $\frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$,
e) $\frac{1}{\sqrt{2}}(|00\rangle + |01\rangle)$.



1. dio 2. dio 3. dio

1. dio $\rightarrow$ $H|0\rangle \otimes |0\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \otimes |0\rangle$
$= \frac{1}{\sqrt{2}}(|00\rangle + |10\rangle)$

2. dio $\rightarrow$ $CNOT(H|0\rangle \otimes |0\rangle) = CNOT(\frac{1}{\sqrt{2}}(|00\rangle + |10\rangle))$
$= \frac{1}{\sqrt{2}}(CNOT|00\rangle + CNOT|10\rangle)$
$= \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$

3. dio $\rightarrow$ preokrenuti $CNOT(CNOT(H|0\rangle \otimes |0\rangle)) = p.CNOT(\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$
$= \frac{1}{\sqrt{2}}(p.CNOT|00\rangle + p.CNOT|11\rangle)$
$= \frac{1}{\sqrt{2}}(|00\rangle + |01\rangle)$

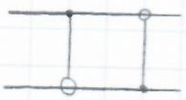
(* $CNOT|00\rangle = |00\rangle$, $CNOT|01\rangle = |01\rangle$, $CNOT|10\rangle = |11\rangle$, $CNOT|11\rangle = |10\rangle$)

(* $p.CNOT|00\rangle = |00\rangle$, $p.CNOT|01\rangle = |11\rangle$, $p.CNOT|10\rangle = |10\rangle$, $p.CNOT|11\rangle = |01\rangle$)

⑥ Shvatimo li kvantni logički krug

qubit 1

qubit 2



2018.

kao jedan operator, njegov matrični prikaz je...

a) $\begin{bmatrix} 1&0&0&0\\ 0&0&1&0\\ 0&1&0&0\\ 0&0&0&1 \end{bmatrix}$,

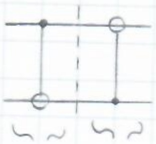
b) $\begin{bmatrix} 1&0&0&0\\ 0&1&0&0\\ 0&0&0&1\\ 0&0&1&0 \end{bmatrix}$,

ⓒ $\begin{bmatrix} 1&0&0&0\\ 0&0&1&0\\ 0&0&0&1\\ 0&1&0&0 \end{bmatrix}$,

d) $\begin{bmatrix} 0&1&0&0\\ 1&0&0&0\\ 0&0&0&1\\ 0&0&1&0 \end{bmatrix}$,

e) $\begin{bmatrix} 0&0&1&0\\ 0&0&0&1\\ 1&0&0&0\\ 0&1&0&0 \end{bmatrix}$

Promatramo sljedeći dio kruga kao jedan operator...



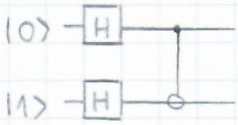
1. dio 2. dio

1. dio → $\text{CNOT} = \begin{bmatrix} 1&0&0&0\\ 0&1&0&0\\ 0&0&0&1\\ 0&0&1&0 \end{bmatrix}$

2. dio → prekrenuti $\text{CNOT}(\text{CNOT}) = \begin{bmatrix} 1&0&0&0\\ 0&0&0&1\\ 0&0&1&0\\ 0&1&0&0 \end{bmatrix}\begin{bmatrix} 1&0&0&0\\ 0&1&0&0\\ 0&0&0&1\\ 0&0&1&0 \end{bmatrix} = \begin{bmatrix} 1&0&0&0\\ 0&0&1&0\\ 0&0&0&1\\ 0&1&0&0 \end{bmatrix}$

→ ↓

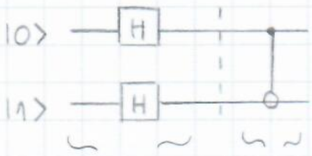
7. Na desnoj (izlaznoj) strani kvantnog logičkog kruga



2018

dobivamo stanje...

a) $|01\rangle$,
b) $\frac{1}{2}(|00\rangle - |01\rangle + |10\rangle - |11\rangle)$,
c) $\frac{1}{2}(|00\rangle - |01\rangle - |10\rangle + |11\rangle)$,
d) $\frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)$,
e) $\frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$.



1. dio 2. dio

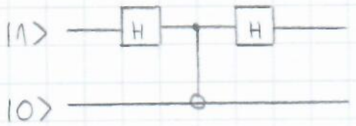
1. dio $\rightarrow H|0\rangle \otimes H|1\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \otimes \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$
$= \frac{1}{2}(|00\rangle - |01\rangle + |10\rangle - |11\rangle)$

2. dio $\rightarrow CNOT(H|0\rangle \otimes H|1\rangle) = CNOT(\frac{1}{2}(|00\rangle - |01\rangle + |10\rangle - |11\rangle))$
$= \frac{1}{2}(CNOT|00\rangle - CNOT|01\rangle + CNOT|10\rangle - CNOT|11\rangle)$
$= \frac{1}{2}(|00\rangle - |01\rangle + |11\rangle - |10\rangle)$
$= \frac{1}{2}(|00\rangle - |01\rangle - |10\rangle + |11\rangle)$

(* $H|0\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$, $H|1\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$)

(* $CNOT|00\rangle = |00\rangle$, $CNOT|01\rangle = |01\rangle$, $CNOT|10\rangle = |11\rangle$, $CNOT|11\rangle = |10\rangle$)

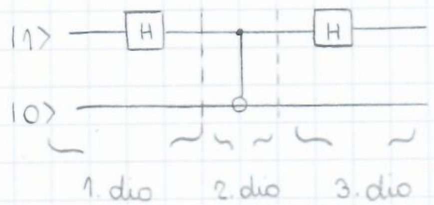
(11) Na desnoj (izlaznoj) strani kvantnog logičkog kruga



2017.

dobivamo stanje...

a) $\frac{1}{2}(|00\rangle - |01\rangle + |10\rangle - |11\rangle)$,
b) $\frac{1}{2}(|00\rangle + |01\rangle + |10\rangle - |11\rangle)$,
c) $\frac{1}{2}(|00\rangle + |01\rangle - |10\rangle + |11\rangle)$,
d) $\frac{1}{2}(|00\rangle - |01\rangle + |10\rangle + |11\rangle)$,
e) $\frac{1}{2}(|00\rangle - |01\rangle - |10\rangle - |11\rangle)$.



1. dio $\rightarrow H|1\rangle \otimes |0\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle) \otimes |0\rangle = \frac{1}{\sqrt{2}}(|00\rangle - |10\rangle)$

2. dio $\rightarrow CNOT(H|1\rangle \otimes |0\rangle) = CNOT\left(\frac{1}{\sqrt{2}}(|00\rangle - |10\rangle)\right)$
$= \frac{1}{\sqrt{2}}(CNOT|00\rangle - CNOT|10\rangle)$
$= \frac{1}{\sqrt{2}}(|00\rangle - |11\rangle)$

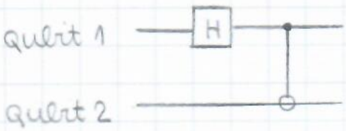
3. dio $\rightarrow (H \otimes 1)(CNOT(H|1\rangle \otimes |0\rangle)) = \left(\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \otimes \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right) \cdot \left(\frac{1}{\sqrt{2}}(|00\rangle - |11\rangle)\right)$

$$= \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \end{bmatrix} \cdot \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 0 \\ 0 \\ -1 \end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} 1 \\ -1 \\ 1 \\ 1 \end{bmatrix}$$

$$= \frac{1}{2}(|00\rangle - |01\rangle + |10\rangle + |11\rangle)$$

(10) Shvatimo li kvantni logički krug



2017.

kao jedan operator, njegov matrični prikaz je...

a) $\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & -1 \\ 1 & 0 & -1 & 0 \end{bmatrix}$,

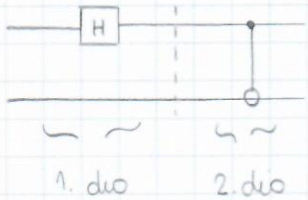
b) $\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & -1 \\ 0 & 1 & -1 & 0 \end{bmatrix}$,

c) $\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$,

d) $\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & -1 & 1 \end{bmatrix}$,

e) $\frac{1}{2}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & -1 & -1 & 1 \\ 1 & 1 & -1 & -1 \end{bmatrix}$.

Promatramo sljedeći dio kruga kao jedan operator...



1. dio → $H \otimes I = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \otimes \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \end{bmatrix}$

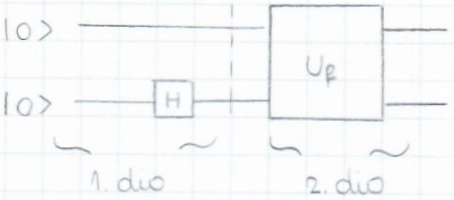
2. dio → $CNOT(H \otimes I) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & -1 \\ 1 & 0 & -1 & 0 \end{bmatrix}$

(4) U kvantnom logičkom krugu na dva vrata U_f predstavlja implementaciju uravnotežene funkcije $f:\{0,1\} \rightarrow \{0,1\}$.

Stanje drugog (donjeg) bita na izlaznoj (desnoj) strani je...

a) $|0\rangle$,
b) $|1\rangle$,
c) $\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$,
d) $\frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$,
e) nije moguće prikazati vektorom stanja.

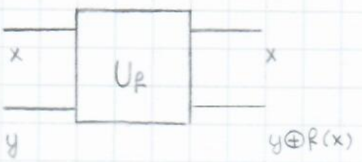
2020



1. dio $\rightarrow$ $|0\rangle \otimes H|0\rangle = |0\rangle \otimes \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$
$= \frac{1}{\sqrt{2}}(|00\rangle + |01\rangle)$

2. dio $\rightarrow$ $U_f(|0\rangle \otimes H|0\rangle) = U_f(\frac{1}{\sqrt{2}}(|00\rangle + |01\rangle))$
$= \frac{1}{\sqrt{2}}(U_f|00\rangle + U_f|01\rangle)$

Za U_f vrijedi...



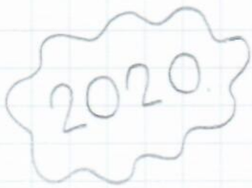
→ prvi element se ne mijenja.
→ drugi element se zbraja s $f(x)$ po modulu 2

$$
\begin{aligned}
U_f(|0\rangle \otimes H|0\rangle) &= \frac{1}{\sqrt{2}}(U_f|00\rangle + U_f|01\rangle) \\
&= \frac{1}{\sqrt{2}}(|0\rangle \otimes |0 \oplus f(0)\rangle + |0\rangle \otimes |1 \oplus f(0)\rangle) \\
&= |0\rangle \otimes \frac{1}{\sqrt{2}}(|0 \oplus f(0)\rangle + |1 \oplus f(0)\rangle) \\
&= |0\rangle \otimes \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)
\end{aligned}
$$

suprotne vrijednosti, jedna je sigurno $|0\rangle$, a druga $|1\rangle$

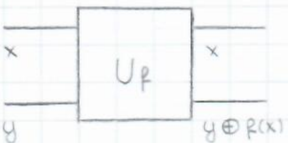
⑤ Ako vrata U_f predstavljaju implementaciju funkcije f sa svojstvima $f(0)=1$ i $f(1)=1$ te ako na izlaznoj (desnoj) strani kvantnog logičkog kruga

imamo stanje $|00\rangle$, možemo zaključiti da na ulazu u krug imamo stanje ...



a) $|00\rangle$,
b) $|01\rangle$,
c) $|10\rangle$,
d) $|11\rangle$,
e) takva situacija nije moguća.

Za implementaciju funkcije f vrijedi...



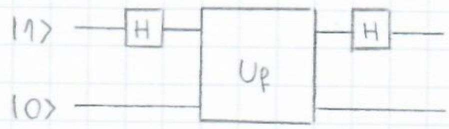
Na temelju zadanog kruga zaključujemo...

↳ $|x\rangle = |0\rangle$
↳ $|y \oplus f(x)\rangle = |0\rangle$
$|y \oplus 1\rangle = |0\rangle \rightarrow |y\rangle = |1\rangle$

ulazno stanje $\rightarrow |x\rangle \otimes |y\rangle = |0\rangle \otimes |1\rangle = |01\rangle$

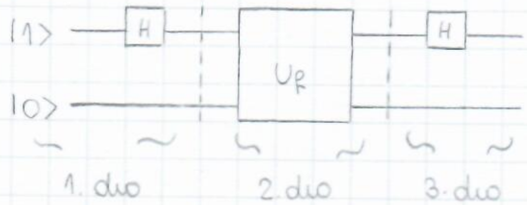
(6) U kvantnom logičkom krugu na slici vrata U_f su implementacija funkcije f za koju vrijedi $f(0) = f(1) = 1$.

2020



Stanje sustava na izlaznoj (desnoj) strani kruga je...

a) $|00\rangle$,
b) $|01\rangle$,
c) $|10\rangle$,
d) $|11\rangle$,
e) $\frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)$.



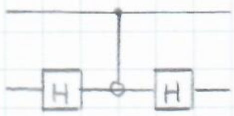
1. dio $\rightarrow$ $H|1\rangle \otimes |0\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle) \otimes |0\rangle = \frac{1}{\sqrt{2}}(|00\rangle - |10\rangle)$

2. dio $\rightarrow$ $U_f(H|1\rangle \otimes |0\rangle) = U_f(\frac{1}{\sqrt{2}}(|00\rangle - |10\rangle))$
$= \frac{1}{\sqrt{2}}(U_f|00\rangle - U_f|10\rangle)$
$= \frac{1}{\sqrt{2}}(|01\rangle - |11\rangle)$

$U_f|00\rangle = |0\rangle \otimes |0 + f(0)\rangle$
$= |0\rangle \otimes |1\rangle$
$= |01\rangle$

3. dio $\rightarrow$ $(H \otimes I)(U_f(H|1\rangle \otimes |0\rangle)) = (H \otimes I) \cdot \frac{1}{\sqrt{2}}(|01\rangle - |11\rangle)$
$= (H \otimes I) \cdot \frac{1}{\sqrt{2}}(|0\rangle \otimes |1\rangle - |1\rangle \otimes |1\rangle)$
$= \frac{1}{\sqrt{2}}(H|0\rangle \otimes |1\rangle - H|1\rangle \otimes |1\rangle)$
$= \frac{1}{\sqrt{2}}(\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \otimes |1\rangle - \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle) \otimes |1\rangle)$
$= \frac{1}{2}(|01\rangle + |11\rangle - |01\rangle - (-|11\rangle))$
$= |11\rangle$

7. Kvantni logički krug prikazan slikom



2020

jest implementacija operatora

$U_f|x\rangle = e^{i\phi}(-1)^{f(x)}|x\rangle$, $\phi \in \mathbb{R}$, $x = 00, 01, 10, 11$

gdje je $f(x) = 0$ za svaki x osim za $x = w$ za koji vrijedi $f(w) = 1$. Odredi w.

a) $w = 00$
b) $w = 01$
c) $w = 10$
d) $w = 11$
e) ništa od navedenog

$$U_f|x\rangle = \underbrace{e^{i\phi}}_{\text{nebitno (fazni faktor)}} \underbrace{(-1)^{f(x)}|x\rangle}_{\text{Groverov algoritam, tražimo winner-a}}$$

Izračun matričnog prikaza...

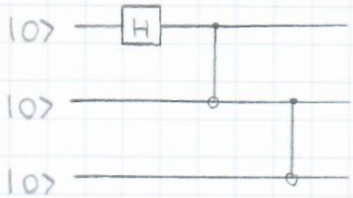
$$(I \otimes H)\,\mathrm{CNOT}\,(I \otimes H) = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & -1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & -1 \end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & -1 \end{bmatrix}\begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & -2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \rightarrow w = 11$$

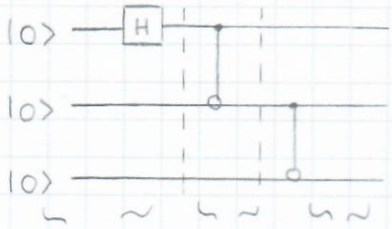
Ako rezultantna matrica ima samo jedinice u dijagonali, winner se čitava pomoću retka u kojem je (jedina) negativna vrijednost.

(8.) Na ulazu u kvantnog logičkog kruga



stanje sustava je...

a) $|000\rangle$,
b) $|111\rangle$,
c) $\frac{1}{\sqrt{2}}(|000\rangle+|111\rangle)$,
d) $\frac{1}{\sqrt{2}}(|000\rangle+|100\rangle)$,
e) $\frac{1}{\sqrt{2}}(|000\rangle+|110\rangle)$.



1. dio 2. dio 3. dio

1. dio $\rightarrow$ $H|0\rangle \otimes |0\rangle \otimes |0\rangle = \frac{1}{\sqrt{2}}(|0\rangle+|1\rangle) \otimes |0\rangle \otimes |0\rangle$
$= \frac{1}{\sqrt{2}}(|000\rangle+|100\rangle)$

2. dio $\rightarrow$ CNOT (1. za 2. bit) $(H|0\rangle \otimes |0\rangle \otimes |0\rangle) = \frac{1}{\sqrt{2}}(|000\rangle+|110\rangle)$

3. dio $\rightarrow$ CNOT (2. za 3 bit) $(\text{CNOT}(H|0\rangle \otimes |0\rangle \otimes |0\rangle)) = \frac{1}{\sqrt{2}}(|000\rangle+|111\rangle)$

(10) Za pohranu matričnog prikaza općenitog operatora 16-qubitnog kvantnog računala u memoriji (npr. pri simulaciji izvođenja kvantnog algoritma) potrebno je približno (uzmite da je svaki skalar matrice kompleksan broj te da za pohranu jednog realnog broja koristimo 8 bajtova)...

a) 0,5 MB,
b) 1 MB,
c) 35 GB,
d) 70 GB,
e) još (znatno) više.

2020

$m = 16$
$N = 2^m = 2^{16}$
matrični prikaz operatora → matrica $N \times N$ → mat. $2^{16} \times 2^{16}$

↳ uzimajući u obzir da je za pohranu jednog kompleksnog broja potrebno 16 bajtova, onda je ukupno potrebno...

$2^{16} \cdot 2^{16} \cdot 16 = 68{,}72 \cdot 10^9$ B ≈ 70 GB

(?) Pretražujemo li bazu veličine 10^{10} Groverom, potrebno je računalo s približno koliko qubita?

$\log_2(10^{10}) = 33{,}22$ → uzima se 33 ili 34

(?) Pretražujemo li bazu veličine 10^6, Groverov operator mora djelovati koliko puta?

$m = \log_2(10^6) = 19{,}93 \approx 20$
$k = \sqrt{2^m} = 1024 \approx 1000$