Kvantna računala, završni ispit, 28. siječnja 2022.

Ime, prezime i JMBAG:

Uputa:

- Ispit se sastoji od 10 zadataka u obliku pitanja s ponuđenim odgovorima.
- Odgovore koje smatrate točnima označite (zacrnite) na posebnom obrascu. Mogu se pojaviti zadaci u kojima je potrebno označiti više od jednog ponuđenog odgovora.
- U praznom prostoru pored zadatka ili na dodatnim papirima napišite obrazloženje ili računski postupak koji vas je doveo do rješenja koje smatrate točnim.
- Točno riješeni zadatak donosi 4 boda. Kazneni (negativni) bodovi se ne obračunavaju.

Notacija i terminologija:

- Vektori $|0\rangle = \left(\begin{smallmatrix}1\\0\end{smallmatrix}\right)$ i $|1\rangle = \left(\begin{smallmatrix}0\\1\end{smallmatrix}\right)$ čine ortonormiranu bazu u $\mathcal{H}^{(2)}$.
- Pri realizaciji kvantnog bita projekcijom spina čestice spinskog kvantnog broja $s = 1/2$ na os z uzimamo da $|0\rangle$ i $|1\rangle$ odgovarju projekcijama $\hbar/2$ i $-\hbar/2$.
- Računalnu bazu u prostoru stanja dvaju qubitova obilježavamo s $\{|ij\rangle = |i\rangle \otimes |j\rangle \,; i, j = 0, 1\}$.
- Gornji (najviši) qubit na grafičkom prikazu logičkog kruga jest lijevi (najlijeviji) qubit u prikazu stanja u računalnoj bazi.

1 Koji od navedenih operatora je Hadamardov operator H? // *Which of the following operators is the Hadamard operator H?*

(a) $\frac{1}{\sqrt{2}}\big(\,|0\rangle\langle 0| - |1\rangle\langle 1|\,\big)$

(b) $\frac{1}{\sqrt{2}}\big(\,|0\rangle\langle 1| + |1\rangle\langle 0|\,\big)$

(c) $\frac{1}{\sqrt{2}}\big(\,|0\rangle\langle 1| - |1\rangle\langle 0|\,\big)$

(d) $\frac{1}{\sqrt{2}}\big((|0\rangle + |1\rangle)\langle 0| - (|0\rangle + |1\rangle)\langle 1|\,\big)$

(e) — Nijedan od navedenih // *None of the above* **točno**

2 Koji od navedenih vektora *nije* svojstveni vektor operatora ctrl-NOT? // *Which of the following vectors is* not *an eigenvector of the ctrl-NOT operator?*

(a) $|00\rangle + |01\rangle$

(b) $|00\rangle - |01\rangle$

(c) $|10\rangle + |11\rangle$

(d) $|10\rangle - |11\rangle$

(e) $|01\rangle + |10\rangle$ **točno**

3 Operator $S = R[\pi/2]$ u računalnoj bazi ima matrični prikaz: // *In the computational basis, the matrix representation of the operator $S = R[\pi/2]$ is:*

$$\begin{pmatrix} 1 & 0 \\ 0 & \mathrm{i} \end{pmatrix}$$

Koje od navedenih tvrdnji su istinite? // *Which of the following statements are true?*

(a) S je unitaran operator. // *S is a unitary operator.* **točno**

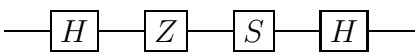
(b) S je hermitski operator. // *S is a Hermitean operator.*

(c) Na Blochovoj sferi, S rotira stanje kvantnog bita za π oko x-osi. // *On the Bloch sphere, S rotates the state of a qubit by π about the x-axis*

(d) S rotira stanje za $\pi/2$ oko z-osi. // *S rotates the state by $\pi/2$ about z-axis.* **točno**

(e) S rotira stanje za $\pi/2$ oko x-osi. // *S rotates the state by $\pi/2$ about x-axis.*

4 Na izlazu iz logičkog kruga // *At the output of the logical circuit*



stanje qubita istovjetno je stanju // *the state of the qubit is equivalent to the state*

(a) $|0\rangle$

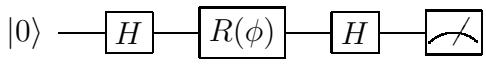
(b) $|1\rangle$

(c) $|+\rangle$

(d) $|-\rangle$

(e) nijednom od navedenih **točno**

5 Razmatramo kvantni logički krug // *Consider the following quantum logical circuit*



Kolika je vjerojatnost da u mjerenju dobijemo vrijednost 1 (tj. da qubit bude izmjeren u stanju $|1\rangle$)? // *What is the probability that in the measurement we get the value* 1 *(ie. that the qubit is in the state* $|1\rangle$*)?*

(a) 0

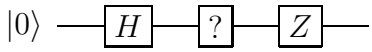
(b) $\frac{1}{2}(1+\cos\phi)$

(c) $\frac{1}{2}(1-\cos\phi)$ **točno**

(d) $\cos\phi$

(e) $\cos^2\phi$

6 Ako na izlazu iz kvantnog logičkog kruga // *If at the output of the quantum logical circuit*



dobivamo stanje // *the state is*

$$\frac{1}{\sqrt{2}}\big(\,|0\rangle - \mathrm{i}\,|1\rangle\,\big),$$

operator označen upitnikom je // *the operator indicated with the question mark is*

(a) X

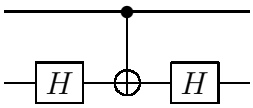
(b) Y

(c) Z

(d) S **točno**

(e) T

7 Kvantni logički krug prikazan slikom // *Quantum logical circuit shown below*



jest implementacija operatora // *implements the operator*

$$U_f \left|x\right\rangle = \mathrm{e}^{\mathrm{i}\phi}(-1)^{f[x]} \left|x\right\rangle, \qquad \phi \in \mathbb{R}, \qquad x = 00, 01, 10, 11,$$

gdje je $f[x] = 0$ za svaki x osim za $x = w$, za koji vrijedi $f[w] = 1$. Odredi w. // *where* $f[x] = 0$ *for each* x *except for* $x = w$, *for which* $f[w] = 1$. *Find* w.

(a) $w = 00$

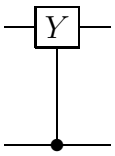
(b) $w = 01$

(c) $w = 10$

(d) $w = 11$ **točno**

(e) Ništa od navedenog (nema rješenja). // *None of the above (no solution).*

8 Matrični prikaz operatora // *Matrix representation of the operator*



je: // *is*:

(a) $\begin{pmatrix} 0 & -\mathrm{i} & 0 & 0 \\ \mathrm{i} & 0 & 0 & 0 \\ 0 & 0 & 0 & -\mathrm{i} \\ 0 & 0 & \mathrm{i} & 0 \end{pmatrix}$

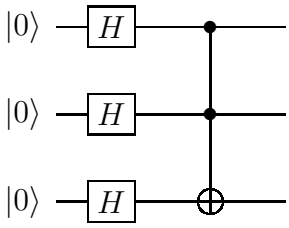
(b) $\begin{pmatrix} 0 & -\mathrm{i} & 0 & 0 \\ \mathrm{i} & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

(c) $\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -\mathrm{i} \\ 0 & 0 & \mathrm{i} & 0 \end{pmatrix}$

(d) $\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\mathrm{i} \\ 0 & 0 & 1 & 0 \\ 0 & \mathrm{i} & 0 & 0 \end{pmatrix}$ **točno**

(e) $\begin{pmatrix} 0 & 0 & 0 & -\mathrm{i} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \mathrm{i} & 0 & 0 & 0 \end{pmatrix}$

9 Na izlazu iz logičkog kruga // *At the output of the quantum logical circuit*



stanje ciljnog bita je // *the state of the target bit is:*

(a) $\frac{1}{\sqrt{2}}\big(\,|0\rangle + |1\rangle\,\big)$ **točno**

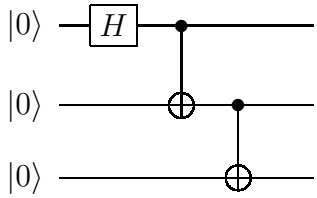
(b) $\frac{1}{\sqrt{2}}\big(\,|0\rangle - |1\rangle\,\big)$

(c) $\frac{1}{\sqrt{2}}\big(\,|0\rangle + \mathrm{i}\,|1\rangle\,\big)$

(d) $\frac{1}{\sqrt{2}}\big(\,|0\rangle - \mathrm{i}\,|1\rangle\,\big)$

(e) ništa od navedenog (stanje nije moguće prikazati vektorom stanja). // none of the above *(state is not representable by a state vector).*

10 Na izlazu iz kvantnog logičkog kruga // *at the output of the quantum logical circuit*



stanje sustava je: // *the state of the system is:*

(a) $\frac{1}{\sqrt{2}}\big(\,|000\rangle + |100\rangle\,\big)$

(b) $\frac{1}{\sqrt{2}}\big(\,|000\rangle + |110\rangle\,\big)$

(c) $\frac{1}{\sqrt{2}}\big(\,|000\rangle + |111\rangle\,\big)$ **točno**

(d) $|000\rangle$

(e) $|111\rangle$