(1) $T = \begin{bmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{bmatrix} \quad X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \quad H = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$

$$[T, X] = TX - XT = \begin{bmatrix} 0 & 1 \\ e^{i\pi/4} & 0 \end{bmatrix} - \begin{bmatrix} 0 & e^{i\pi/4} \\ 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 - e^{i\pi/4} \\ e^{i\pi/4} - 1 & 0 \end{bmatrix}$$

$$H|0\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$H|1\rangle = \frac{|0\rangle - |1\rangle}{\sqrt{2}} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$[T, X]\, H|0\rangle = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 - e^{i\pi/4} \\ e^{i\pi/4} - 1 \end{bmatrix}$$

$$[T, X]\, H|1\rangle = \frac{1}{\sqrt{2}} \begin{bmatrix} e^{i\pi/4} - 1 \\ e^{i\pi/4} - 1 \end{bmatrix}$$

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$$|\psi\rangle = \frac{|00\rangle - |11\rangle}{\sqrt{2}} \qquad |\phi\rangle = \frac{|00\rangle + |11\rangle}{\sqrt{2}}$$

$$X \otimes I\, |\psi\rangle = \frac{1}{\sqrt{2}} \left(X|0\rangle \otimes I|0\rangle - X|1\rangle \otimes I|1\rangle \right) =$$

$$= \frac{|10\rangle - |01\rangle}{\sqrt{2}}$$

$$X \otimes I\, |\phi\rangle = \frac{1}{\sqrt{2}} \left(X|0\rangle \otimes I|0\rangle + X|1\rangle \otimes I|1\rangle \right) =$$

$$= \frac{|10\rangle + |01\rangle}{\sqrt{2}}$$

$$Z \otimes I\, |\psi\rangle = \frac{1}{\sqrt{2}} \left(Z|0\rangle \otimes I|0\rangle - Z|1\rangle \otimes I|1\rangle \right) =$$

$$= \frac{|0\rangle + |1\rangle}{\sqrt{2}}$$

$$Z \otimes I\, |\phi\rangle = \frac{1}{\sqrt{2}} \left(Z|0\rangle \otimes I|0\rangle + Z|1\rangle \otimes I|1\rangle \right) =$$

$$= \frac{|0\rangle - |1\rangle}{\sqrt{2}}$$

(3) a) $H H = I \Rightarrow |a\rangle \rightarrow |a\rangle$

b) $|c\rangle = (H \otimes H)(|ab\rangle) = H|a\rangle \otimes H|b\rangle =$

$$= \frac{(-1)^a |a\rangle + |\bar{a}\rangle}{\sqrt{2}} \otimes \frac{(-1)^b |b\rangle + |\bar{b}\rangle}{\sqrt{2}} =$$

$$= \frac{1}{2}\left((-1)^{a+b} |ab\rangle + (-1)^a |a\bar{b}\rangle + (-1)^b |\bar{a}b\rangle + |\bar{a}\bar{b}\rangle \right)$$

(4) a) $|\varphi_1\rangle = (H \otimes I)(|ab\rangle) =$

$$= \frac{(-1)^a |a\rangle + |\bar{a}\rangle}{\sqrt{2}} \otimes |b\rangle =$$

$$= \frac{1}{\sqrt{2}}\left((-1)^a |ab\rangle + |\bar{a}b\rangle \right)$$

$$|\varphi_2\rangle = CNOT|\varphi_1\rangle = \frac{1}{\sqrt{2}}\left((-1)^a |a, a \oplus b\rangle + |\bar{a}, \bar{a} \oplus b\rangle \right)$$

b) $|\varphi_1\rangle = (H \otimes I)(|ab\rangle) = \frac{(-1)^a |a\rangle + |\bar{a}\rangle}{\sqrt{2}} \otimes |b\rangle$

$$\frac{(-1)^a |a\rangle}{\sqrt{2}} \otimes \left(\left(\frac{(-1)^b |b\rangle + |\bar{b}\rangle}{\sqrt{2}} \right) \cdot a + (|b\rangle) \cdot \bar{a} \right) +$$

$$+ \frac{|\bar{a}\rangle}{\sqrt{2}} \left(\left(\frac{(-1)^b |b\rangle + |\bar{b}\rangle}{\sqrt{2}} \right) \cdot \bar{a} + (|b\rangle) \cdot a \right)$$

5) $|a\rangle = |1\rangle \qquad |b\rangle = |1\rangle$

LIJEVI: $CX|11\rangle = |10\rangle$

DESNI: $|\varphi_1\rangle = I \otimes H|11\rangle = |1\rangle \otimes \frac{|0\rangle - |1\rangle}{\sqrt{2}}$

$|\varphi_2\rangle = CZ|\varphi_1\rangle = |1\rangle \otimes \frac{|0\rangle + |1\rangle}{\sqrt{2}}$

$|\varphi_3\rangle = I \otimes H|\varphi_2\rangle = |1\rangle \otimes |0\rangle = |10\rangle$

VIDIMO DA SE DOBIJE ISTO, U ZADATKU PIŠE DA SAMI BIRAMO ULAZ, MOGLI SMO UZET I DRUGU KOMBINACIJU $|a\rangle$ I $|b\rangle$

6) $|\phi_1\rangle = (H \otimes I)|00\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}} \otimes |0\rangle = \frac{|00\rangle + |10\rangle}{\sqrt{2}}$

$U_f|x, y\rangle = |x, y \oplus f(x)\rangle \qquad |\phi_2\rangle = U_f|\phi_1\rangle$

AKO JE $f(x)$ URAVNOTEŽENA a) $f(0) = 0 \quad f(1) = 1$

b) $f(0) = 1 \quad f(1) = 0$

$|\phi_2\rangle_a = U_f|\phi_1\rangle = \frac{|00\rangle + |11\rangle}{\sqrt{2}} \qquad |\phi_2\rangle_b = U_f|\phi_1\rangle = \frac{|01\rangle + |10\rangle}{\sqrt{2}}$

AKO JE $f(x)$ KONSTANTNA c) $f(0)=0$ $f(1)=0$

d) $f(0)=1$ $f(1)=1$

$$|\phi_2\rangle_c = U_f|\phi_1\rangle = \frac{|00\rangle + |10\rangle}{\sqrt{2}} \qquad |\phi_2\rangle_d = U_f|\phi_1\rangle = \frac{|01\rangle + |11\rangle}{\sqrt{2}}$$

VIDIMO DA JE MOGUĆE GENERIRATI BELLOVA STANJA ZA URAVNOTEŽENU FUNKCIJU

DODATNO (AKO SE STIGNE)

$$|\phi_3\rangle_a = H\otimes I\,|\phi_2\rangle_a = \frac{|0\rangle+|1\rangle}{\sqrt{2}}\otimes\frac{|0\rangle}{\sqrt{2}} + \frac{|0\rangle-|1\rangle}{\sqrt{2}}\otimes\frac{|1\rangle}{\sqrt{2}} =$$

$$= \frac{1}{2}\left(|00\rangle + |10\rangle + |01\rangle - |11\rangle\right)$$

$$|\phi_3\rangle_b = H\otimes I\,|\phi_2\rangle_b = \frac{|0\rangle+|1\rangle}{\sqrt{2}}\otimes\frac{|1\rangle}{\sqrt{2}} + \frac{|0\rangle-|1\rangle}{\sqrt{2}}\otimes\frac{|0\rangle}{\sqrt{2}} =$$

$$= \frac{1}{2}\left(|01\rangle + |11\rangle + |00\rangle - |10\rangle\right)$$

$$|\phi_3\rangle_c = H\otimes I\,|\phi_2\rangle_c = |00\rangle \quad \text{JER} \quad |\phi_2\rangle_c = \frac{|0\rangle+|1\rangle}{\sqrt{2}}\otimes|0\rangle$$

$$|\phi_3\rangle_d = H\otimes I\,|\phi_2\rangle_d = |01\rangle \quad \text{JER} \quad |\phi_2\rangle_d = \frac{|0\rangle+|1\rangle}{\sqrt{2}}\otimes|1\rangle$$