Kvantna računala, završni ispit, 1. veljače 2021.

Ime, prezime i JMBAG:

Uputa:

- Ispit se sastoji od 10 zadataka u obliku pitanja s ponuđenim odgovorima.
- Odgovore koje smatrate točnima označite (zacrnite) na posebnom obrascu. Mogu se pojaviti zadaci u kojima je potrebno označiti više od jednog ponuđenog odgovora.
- U praznom prostoru pored zadatka ili na dodatnim papirima napišite obrazloženje ili računski postupak koji vas je doveo do rješenja koje smatrate točnim.
- Točno riješeni zadatak donosi 4 boda. Kazneni (negativni) bodovi se ne obračunavaju.

Notacija i terminologija:

- Vektori $|0\rangle = \left(\begin{smallmatrix}1\\0\end{smallmatrix}\right)$ i $|1\rangle = \left(\begin{smallmatrix}0\\1\end{smallmatrix}\right)$ čine ortonormiranu bazu u $\mathcal{H}^{(2)}$.
- Pri realizaciji kvantnog bita projekcijom spina čestice spinskog kvantnog broja $s = 1/2$ na os z uzimamo da $|0\rangle$ i $|1\rangle$ odgovarju projekcijama $\hbar/2$ i $-\hbar/2$.
- Računalnu bazu u prostoru stanja dvaju qubitova obilježavamo s $\{|ij\rangle = |i\rangle \otimes |j\rangle\,; i,j = 0,1\}$.

1 Koji od navedenih operatora je operator T (prema definiciji $T = R(\pi/4)$)? // *Which of the following operators is the T operator (defined as $T = R(\pi/4)$)?*

(a) $|0\rangle\langle 0| + \frac{1+\mathrm{i}}{\sqrt{2}}|1\rangle\langle 1|$ **točno**

(b) $|0\rangle\langle 0| + \frac{1-\mathrm{i}}{\sqrt{2}}|1\rangle\langle 1|$

(c) $|0\rangle\langle 0| - \frac{1+\mathrm{i}}{\sqrt{2}}|1\rangle\langle 1|$

(d) $|0\rangle\langle 0| - \frac{1-\mathrm{i}}{\sqrt{2}}|1\rangle\langle 1|$

(e) Ništa od navedenog // *None of the above*

2 Koji od navedenih vektora *nije* svojstveni vektor operatora ctrl-NOT? // *Which of the following vectors is* not *an eigenvector of the ctrl-NOT operator?*

(a) $|00\rangle + |01\rangle$

(b) $|00\rangle - |01\rangle$

(c) $|10\rangle + |11\rangle$

(d) $|10\rangle - |11\rangle$

(e) $|01\rangle + |10\rangle$ **točno**

3 Početno stanje kvantnog bita je // *The initial state of a qubit is*

$$|\phi\rangle = \alpha|0\rangle + \beta|1\rangle .$$

Želimo li taj kvantni bit dovesti u stanje // *If we are to bring this qubit into the state*

$$|\phi'\rangle = \beta|0\rangle - \alpha|1\rangle ,$$

u prikazu stanja na Blochovoj sferi moramo provesti rotaciju // *on the Bloch sphere we must apply a rotation*

(a) za π oko osi x // *of π about x-axis.*

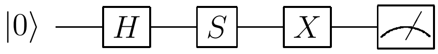
(b) za π oko osi y // *of π about y-axis.* **točno**

(c) za π oko osi z // *of π about z-axis.*

(d) za $\pi/2$ oko osi z // *of $\pi/2$ about z-axis.*

(e) — ništa od gore navedenog // *none of the above.*

4 Razmatramo kvantni logički krug: // *Consider the following quantum logical circuit:*



Kolika je vjerojatnost da u mjerenju dobijemo vrijednost 0 (tj. da qubit bude izmjeren u stanju $|0\rangle$)? // *What is the probability that in the measurement we get the value* 0 *(ie. that the qubit is measured in the state* $|0\rangle$*)?*

(a) 0

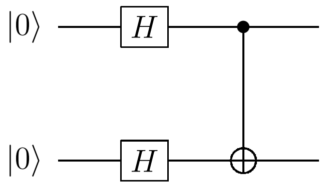
(b) $1/4$

(c) $1/2$ **točno**

(d) $1/\sqrt{2}$

(e) 1

5 Stanje sustava na izlaznoj strani kvantnog logičkog kruga // *The state of the system at the output of the quantum logical circuit*



je // *is*

(a) $|00\rangle$

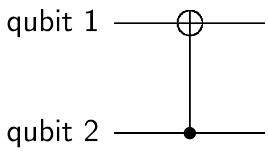
(b) $|11\rangle$

(c) $\frac{1}{\sqrt{2}}\big(\,|01\rangle + |10\rangle\,\big)$

(d) $\frac{1}{\sqrt{2}}\big(\,|01\rangle - |10\rangle\,\big)$

(e) $\frac{1}{2}\big(\,|00\rangle + |01\rangle + |10\rangle + |11\rangle\,\big)$ **točno**

6 Matrični prikaz "preokrenutog" ctrl-NOT operatora // *Matrix representing the "upside–down" ctrl-NOT operator*



je: // *is:*

(a) $\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$

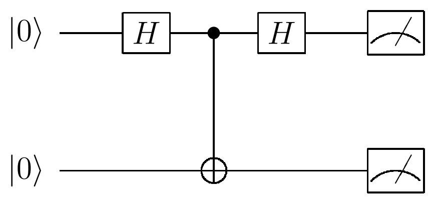
(b) $\begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

(c) $\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

(d) $\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$ **točno**

(e) $\begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}$

7 Kolika je vjerojatnost da na izlazu iz kvantnog logičkog kruga // *What is the probability that at the output of the quantum logical circuit*



sustav izmjerimo u stanju $|10\rangle$? // *the system is measured in the state* $|10\rangle$*?*

(a) 0

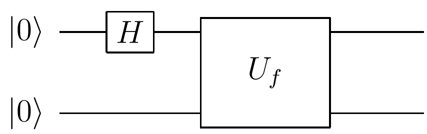
(b) $\frac{1}{4}$ **točno**

(c) $\frac{1}{2}$

(d) $\frac{1}{\sqrt{2}}$

(e) 1

8 U kvantnom logičkom krugu // *In the quantum logical circuit*



vrata U_f predstavljaju implementaciju uravnotežene funkcije $f(0) = 0,\ f(1) = 1.$ Stanje prvog (gornjeg) kvantnog bita na izlaznoj (desnoj) strani je // *the gate* U_f *implements the balanced function* $f(0) = 0,\ f(1) = 1.$ *The state of the first (upper) qubit at the output (right) side is*

(a) $\frac{1}{\sqrt{2}}\big(\,|0\rangle + |1\rangle\,\big)$

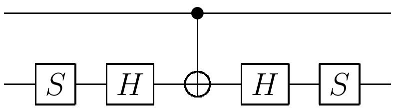
(b) $\frac{1}{\sqrt{2}}\big(\,|0\rangle - |1\rangle\,\big)$

(c) $\frac{1}{\sqrt{2}}\big(\,|0\rangle + \mathrm{i}\,|1\rangle\,\big)$

(d) $\frac{1}{\sqrt{2}}\big(\,|0\rangle - \mathrm{i}\,|1\rangle\,\big)$

(e) — nije moguće prikazati vektorom stanja // *can not be given by a state vector* **točno**

9 Kvantni logički krug prikazan slikom // *The quantum logical circuit shown below*



jest implementacija operatora // *implements the operator*

$$U_f\,|x\rangle = \mathrm{e}^{\mathrm{i}\phi}(-1)^{f[x]}\,|x\rangle\,, \qquad \phi \in \mathbb{R}, \qquad x = 00, 01, 10, 11,$$

gdje je $f[x] = 0$ za svaki x osim za $x = w$, za koji vrijedi $f[w] = 1$. Odredi w. // *where $f[x] = 0$ for all x except for $x = w$ for which $f[w] = 1$. Find w.*

(a) $w = 00$

(b) $w = 01$ **točno**

(c) $w = 10$

(d) $w = 11$

(e) Ništa od navedenog (nema rješenja). // None of the above (no solution).

10 Koliko će se puta (približno) produljiti vrijeme potrebno za pretragu nestrukturirane baze podataka ako se veličina baze poveća 2^{16} puta, a pretražujemo ju Groverovim algoritmom? // *How many times (approximately) will the time required to search an unstructured data base become longer if the size of the data base increases by factor 2^{16} and Grover's algorithm is being used?*

(a) $2\times$

(b) $8\times$

(c) $16\times$

(d) $256\times$ **točno**

(e) $2^{15}\times$