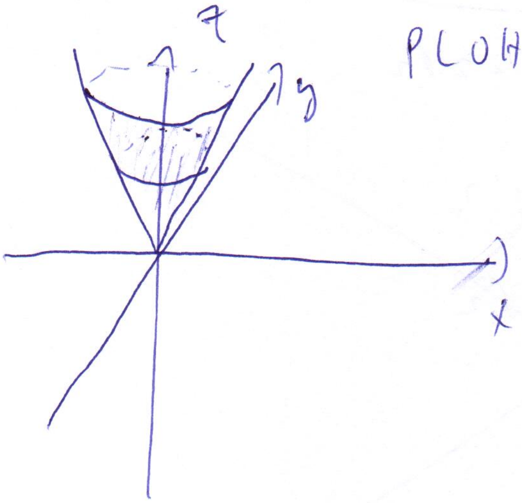
1.

a) 

PLOHA JE STOŽAC

b)

$$C \ldots \begin{cases} z = \sqrt{x^2+y^2} \\ x + z = 5 \end{cases} \qquad \boxed{z = 5 - x}$$

$$5 - x = \sqrt{x^2+y^2}$$

$$25 - 10x + x^2 = x^2 + y^2$$

$$x = \frac{25 - y^2}{10}$$

$$z = 5 - \frac{25-y^2}{10} = \frac{25+y^2}{10}$$

$y = t$

$$\vec{r}(t) = \left(\frac{25-t^2}{10}, t, \frac{25+t^2}{10} \right)$$

1.c) $\vec{r}\,'(t) = \left(-\frac{t}{5}, 1, \frac{t}{5}\right)$

$T(x_T, y_T, z_T) \ldots \quad x_T = \frac{25 - t_0^2}{10}$

$y_T = t_0$

$z_T = \frac{25 + t_0^2}{10}$

TANGENTA U TOČKI T...

$$\frac{x - \frac{25 - t_0^2}{10}}{-\frac{t_0}{5}} = \frac{y - t_0}{1} = \frac{z - \frac{25 + t_0^2}{10}}{\frac{t_0}{5}}$$

SIJEČE x-OS ⇒ $y = 0$ i $z = 0$

$$\frac{x - \frac{25 - t_0^2}{10}}{-\frac{t_0}{5}} = \boxed{-t_0 = \frac{-\frac{25 + t_0^2}{10}}{\frac{t_0}{5}}}$$

$$-\frac{t_0^2}{5} = -\frac{25 + t_0^2}{10}$$

$$2t_0^2 = 25 + t_0^2$$

$$t_0 = \pm 5$$

$$\vec{r}(5) = (0, 5, 5)$$

$$\vec{r}(-5) = (0, -5, 5)$$

2. T_1: TOČNO

$$\frac{\partial f}{\partial \vec{i}}(P) = \nabla f(P) \cdot \vec{i} = \left(\frac{\partial f}{\partial x}(P)\vec{i} + \frac{\partial f}{\partial y}(P)\vec{j}\right)\cdot \vec{i}$$

$$= \frac{\partial f}{\partial x}(P)$$

T_2: TOČNO

$\nabla f(P)$ JE OKOMIT NA NIVO KRIVULJE OD f KROZ TOČKU P

$\Rightarrow$ OKOMIT JE NA TANGENTU NA TE KRIVULJE U TOČKI P

$$\Rightarrow 0 = \nabla f(P) \cdot \vec{v} = \frac{\partial f}{\partial \vec{v}}(P)$$

T_3: NETOČNO

NA PRIMJER $f: \mathbb{R}^2 \to \mathbb{R}$, $f(x,y) = x$

$P = (0,0)$ $\quad \vec{h}_1 = \vec{i}$, $\vec{h}_2 = -\vec{i}$

$$\frac{\partial f}{\partial \vec{h}_1}(0,0) = \nabla f(0,0) \cdot \vec{i} = \vec{i} \cdot \vec{i} = 1$$

$$\frac{\partial f}{\partial \vec{h}_2}(0,0) = \nabla f(0,0) \cdot (-\vec{i}) = \vec{i} \cdot (-\vec{i}) = -1$$

3. a) $\exists \lambda \in \mathbb{R}$ T.D

$$\nabla(f(T) + \lambda \varphi(T)) = \vec{0}$$

b) LAGRANGOVA FUNKCIJA

$$L(x, y, \lambda) = x^2 + xy + y^2 + \lambda(4x^2 + 4xy + y^2 - 1)$$

$L'_x = 0 \qquad 2x + y + 8\lambda x + 4\lambda y = 0$

$L'_y = 0 \qquad x + 2y + 4\lambda x + 2\lambda y = 0 \quad \Big\} (\cdot -2) +$

$\varphi = 0 \qquad 4x^2 + 4xy + y^2 = 1$

$$2x + y + 8\lambda x + 4\lambda y - 2(x + 2y + 4x\lambda + 2\lambda y) = 0$$

$$-3y = 0 \Rightarrow y = 0$$

$4x^2 = 1 \qquad 2x + 8\lambda x = 0 \qquad$ uz $x \neq 0$

$x = \pm \frac{1}{2} \qquad \lambda = -\frac{1}{4}$

STACIONARNE TOČKE

$T_1(\frac{1}{2}, 0, -\frac{1}{4})$

$T_2(-\frac{1}{2}, 0, -\frac{1}{4})$

$L''_{xx} = 2 + 8\lambda$

$L''_{xy} = 1 + 4\lambda$

$L''_{yy} = 2 + 2\lambda$

DIFERENCIJAL UVJETA

$(8x + 4y)\,dx + (4x + 2y)\,dy = 0$

$$dy = -\frac{8x + 4y}{4x + 2y}\,dx$$

$$dy = -2\,dx$$

3. b) $d^2L = (2+8\lambda)(dx)^2 + 2(1+4\lambda)\,dx\,dy + (2+2\lambda)(dy)^2$

$d^2L = (2+8\lambda)(dx)^2 - 4(1+4\lambda)(dx)^2 + 4(2+2\lambda)(dx)^2$

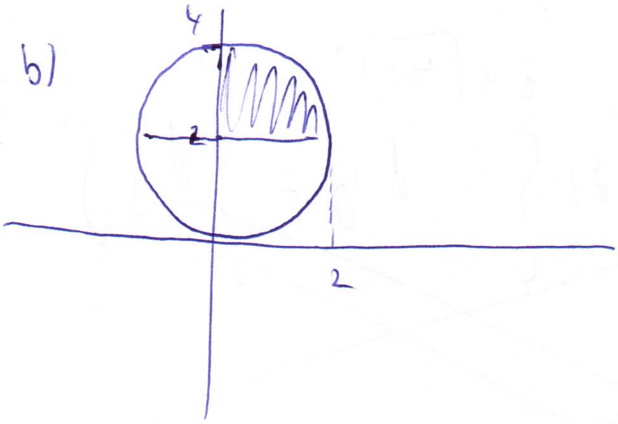
$d^2L = [2+8\lambda-4-16\lambda+8+8\lambda](dx)^2$

$d^2L = 6(dx)^2 > 0$ ZA $(dx, dy) \neq (0, 0)$

T_1 I T_2 SU LOKALNI UVJETNI MINIMUMI

4. a) KNJIGA

b)



$$\int_0^2 x\,dx \int_2^{\sqrt{4-x^2}} dy = \int_2^4 dy \int_0^{\sqrt{4y-y^2}} x\,dx$$

c)
$$\int_2^4 dy \int_0^{\sqrt{4y-y^2}} x\,dx = \int_2^4 dy \; \frac{x^2}{2}\Big|_0^{\sqrt{4y-y^2}} = \int_2^4 2y - \frac{y^2}{2}\,dy$$

$$= y^2 - \frac{y^3}{6}\Big|_2^4 = 16 - \frac{64}{6} - \left(4 - \frac{8}{6}\right)$$

$$= \frac{8}{3}$$

5.

a) KNJIGA

b) AKO JE $f(x) = \sum_{n=0}^{\infty} c_n (x-a)^n$ GDJE RED KONVERGIRA NA $\langle a-R, a+R \rangle$, TADA VRIJEDI $f'(x) = \sum_{n=0}^{\infty} c_n n (x-a)^{n-1}$ S ISTIM POLUMJEROM R

c) $$\sum_{n=1}^{\infty} \frac{3n}{2^{3n}} = \sum_{n=1}^{\infty} 3n \left(\frac{1}{8}\right)^n$$

$$\sum_{n=0}^{\infty} x^n = \frac{1}{1-x} \Big| \frac{d}{dx} \quad |x|<1$$

$$\sum_{n=1}^{\infty} n x^{n-1} = -\frac{1}{(1-x)^2} \cdot (-1) \Big/ \cdot x$$

$$\sum_{n=1}^{\infty} n x^n = \frac{x}{(1-x)^2}$$

$$x = \frac{1}{8}$$

$$\sum_{n=1}^{\infty} n \left(\frac{1}{8}\right)^n = \frac{\frac{1}{8}}{\left(1-\frac{1}{8}\right)^2} = \frac{\frac{1}{8}}{\frac{49}{64}} = \frac{8}{49}$$

PA JE

$$\sum_{n=1}^{\infty} 3n \left(\frac{1}{8}\right)^n = 3 \cdot \frac{8}{49} = \frac{24}{49}$$

6. a) $y' + f(x)y = g(x)$

KNJIGA 6.2.2

b) $y' + 2xy = e^{-x^2}$

$f(x) = 2x$ $\int f(x)dx = \int 2x\,dx = x^2$

$g(x) = e^{-x^2}$

$y(x) = e^{-\int f(x)dx}\left[C + \int g(x)e^{\int f(x)dx}dx\right]$ $C \in \mathbb{R}$

$y(x) = e^{-x^2}\left[C + \int e^{-x^2}e^{x^2}dx\right]$

$y(x) = e^{-x^2}\left[C + \int dx\right]$

$y(x) = e^{-x^2}\left[C + x\right]$

7.

$$\frac{d\mu}{dx} + \frac{1}{Q}\left[\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right]\mu = 0$$

$$Q = y$$

$$P = x^2 + y^2 + x$$

$$\frac{1}{Q}\left[\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right] = \frac{1}{y}\left[0 - 2y\right] = -2$$

$$\frac{d\mu}{dx} - 2\mu = 0$$

$$\frac{d\mu}{dx} = 2\mu$$

$$\frac{d\mu}{\mu} = 2\,dx$$

$$\ln|\mu| = 2x$$

$$\mu = e^{2x}$$

MNOŽENJEM S MULTIPLIKATOROM JEDNADŽBA POSTAJE

$$e^{2x}(x^2+y^2+x)\,dx + e^{2x}y\,dy = 0$$

$$\frac{\partial P}{\partial y} = 2ye^{2x} = \frac{\partial Q}{\partial x}$$ ✓ egzaktna

$$U(x,y)=\int_{x_0}^{x} P(x,y_0)\,dx+\int_{y_0}^{y} Q(x,y)\,dy + C$$

$(x_0,y_0)=(0,0)$

$$=\int_0^x e^{2x}(x^2+x)\,dx+\int_0^y e^{2x}y\,dy + C$$

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$$\left|\begin{array}{ll} u=x^2+x & dv=e^{2x}dx \\ du=(2x+1)dx & v=\frac{1}{2}e^{2x}\end{array}\right| = \frac{1}{2}e^{2x}(x^2+x)-\int \underbrace{\left(x+\tfrac{1}{2}\right)}_{u}\underbrace{e^{2x}dx}_{dv}$$

$$=\frac{1}{2}e^{2x}(x^2+\cancel{x})-\cancel{\left(x+\tfrac{1}{2}\right)\tfrac{1}{2}e^{2x}}+\cancel{\tfrac{1}{4}e^{2x}}=\frac{1}{2}e^{2x}\cdot x^2$$

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$$\Rightarrow\ =\frac{1}{2}e^{2x}\cdot x^2\Big|_0^x+e^{2x}\frac{y^2}{2}\Big|_0^y+C$$

$$=\frac{1}{2}e^{2x}(x^2+y^2)+C$$

Rješenje je dano u implicitnom obliku $U(x,y)=C$

tj. $e^{2x}(x^2+y^2)=C$

Poč. uvjet $y(0)=1 \Rightarrow 1(1+0)=C$

$C=1$

Konačno rješenje: $e^{2x}(x^2+y^2)=1$

8. a) UVRŠTAVANJEM $y_1(x) = e^{3x}$

$$27y_1 + 9a y_1 + 3y_1 + 3b y_1 = 0$$

$$10 + 3a + b = 0$$

UVRŠTAVANJEM $y_2 = \cos x$

$y_2''' = -y_2'$

$y_2'' = -y_2$

$$-y_2' - a y_2 + y_2' + 3y_2 b = 0$$

$$-a + 3b = 0$$

SUSTAV

$$3a + b = -10$$
$$-a + 3b = 0$$

$a = -3$

$b = -1$

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2. način: nultočke karakt. polinoma $r_1 = 3$, $r_{2,3} = \pm i$

$$\Rightarrow \quad (r-3)(r^2+1) = 0$$

$$r^3 - 3r^2 + r - 3 = 0$$

$$a = -3, \; b = -1$$

$$\Rightarrow \quad y''' - 3y'' + y' - 3y = 0$$

b) Rješenje homogene jedn. (iz a) dijela):

$$y_h = C_1 e^{3x} + C_2 \cos x + C_3 \sin x$$

Partikularna rješenja:

1) $f_1(x) = 3x^2 \Rightarrow y_{p1} = Ax^2 + Bx + C$

$y_{p1}' = 2Ax + B, \quad y_{p1}'' = 2A, \quad y_{p1}''' = 0$

$\Rightarrow -3 \cdot 2A + 2Ax + B - 3(Ax^2 + Bx + C) = 3x^2$

$$\left.\begin{array}{l} -3A = 3 \rightarrow A = -1 \\ 2A - 3B = 0 \rightarrow B = -\frac{2}{3} \\ -6A + B - 3C = 0 \rightarrow C = \frac{16}{9} \end{array}\right\} y_{p1} = -x^2 - \frac{2}{3}x + \frac{16}{9}$$

2) $f_2(x) = 2e^{3x} \rightarrow y_{p2} = Ae^{3x} \cdot x$

$y_{p2}' = 3Axe^{3x} + Ae^{3x}, \quad y_{p2}'' = 3Ae^{3x} + 9Axe^{3x} + 3Ae^{3x}$

$= e^{3x}(6A + 9Ax)$

$y_{p2}''' = 3e^{3x}(6A + 9Ax) + e^{3x}(0 + 9A) = e^{3x}(27A + 27Ax)$

$\Rightarrow e^{3x}(27A + \cancel{27Ax}) - 3e^{3x}(6A + \cancel{9Ax}) + e^{3x}(\cancel{3Ax} + A)$

$- 3\cancel{Ax}e^{3x} = 2e^{3x}$

$\Rightarrow 10A = 2 \rightarrow A = \frac{1}{5} \quad y_{p2} = \frac{1}{5}xe^{3x}$

Rj: $y = y_h + y_{p1} + y_{p2} = C_1 e^{3x} + C_2 \cos x + C_3 \sin x$

$- x^2 - \frac{2}{3}x + \frac{16}{9} + \frac{1}{5}xe^{3x}$