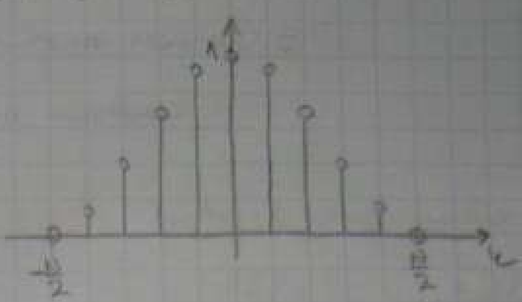
e) Blackmanov vremenski otvor

$$w[k] = 0{,}42 + 0{,}5\cdot\cos\left(\frac{2\pi k}{N+1}\right) + 0{,}08\cdot\cos\left(\frac{4\pi k}{N+1}\right), \quad -\frac{N}{2} \le k \le \frac{N}{2}$$

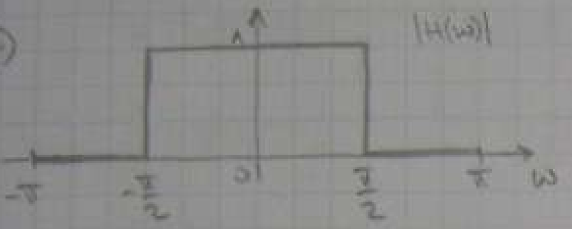
širina glavne latice: $\frac{12\pi}{N}$

gušenje prve bočne latice: 58,1

širina prijelaznog područja: $\frac{5{,}56\pi}{N/2}$

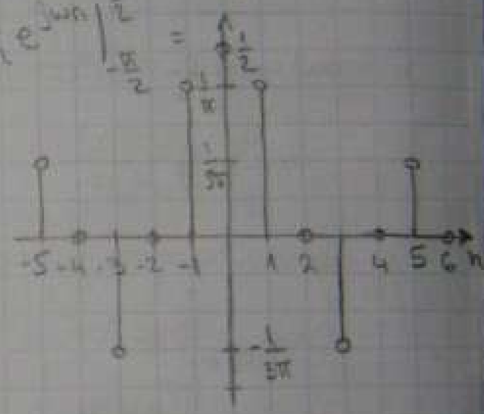


2. a)



$$\text{IDTFT} = h[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} H(\omega)e^{j\omega n}\,d\omega = \frac{1}{2\pi}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 1\cdot e^{j\omega n}\,d\omega = \frac{1}{2\pi}\cdot\frac{1}{jn}e^{j\omega n}\Big|_{-\frac{\pi}{2}}^{\frac{\pi}{2}} =$$

$$= \frac{1}{2\pi}\cdot\frac{1}{jn}\left(e^{j\frac{\pi}{2}n} - e^{-j\frac{\pi}{2}n}\right) = \frac{1}{n\pi}\sin\left(\frac{\pi}{2}n\right) \longrightarrow$$

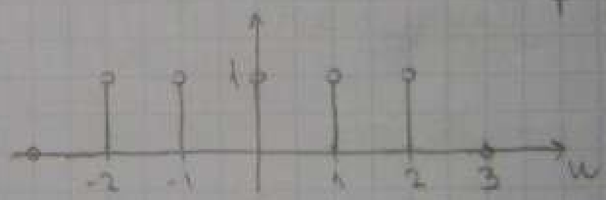


* pravokutni vremenski otvor duljine N (za neparan N):

$$w[n] = \begin{cases} 1, & -\frac{N-1}{2} \le n \le \frac{N-1}{2} \\ 0, & \text{inače} \end{cases}$$

$\boxed{N=5}$

$$w[n] = \begin{cases} 1, & -2 \le n \le 2 \\ 0, & \text{inače} \end{cases}$$



$h[-2] = h[2] = 0$

$h[-1] = h[1] = \frac{1}{\pi}$

$$h[0] = \lim_{n\to 0}\frac{\sin(\frac{\pi}{2}n)}{\pi n} \overset{L'H}{=} \lim_{n\to 0}\frac{\frac{\pi}{2}\cos(\frac{\pi}{2}n)}{\pi} = \frac{\frac{\pi}{2}\cos(0)}{\pi} = \frac{1}{2}$$

$h[n]\cdot w[n] = \left\{\frac{1}{\pi}, \underline{\frac{1}{2}}, \frac{1}{\pi}\right\}$ → nije kauzalan, treba ga pomaknuti u desno za 1

$$H(z)\cdot W(z) = \frac{1}{\pi}z + \frac{1}{2}z^0 + \frac{1}{\pi}\cdot z^{-1} \quad \Big|\cdot z^{-1}$$

$$= \frac{1}{\pi} + \frac{1}{2}z^{-1} + \frac{1}{\pi}z^{-2}$$

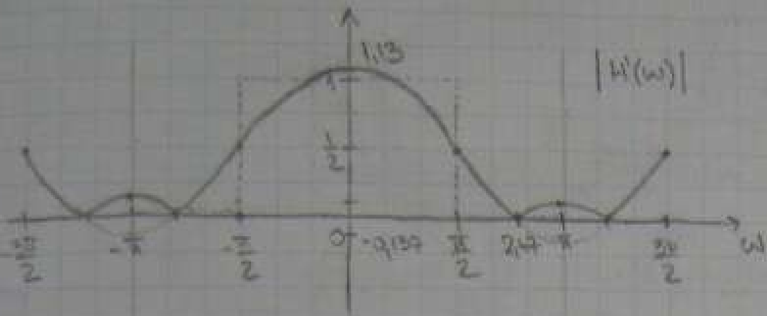
DTFT

$$H(e^{j\omega})\cdot W(e^{j\omega}) = \frac{1}{\pi} + \frac{1}{2}e^{-j\omega} + \frac{1}{\pi}e^{-2j\omega} = e^{-j\omega}\left(\frac{1}{\pi}e^{j\omega} + \frac{1}{2} + \frac{1}{\pi}e^{-j\omega}\right) = e^{-j\omega}\left(\frac{1}{2} + \frac{2\cos(\omega)}{\pi}\right)$$

$H'(\omega) = e^{-j\omega}\left(\frac{1}{2} + \frac{2\cos(\omega)}{\pi}\right)$

$|\cos(\omega)^2 + (-j\sin(\omega))^2| = 1$

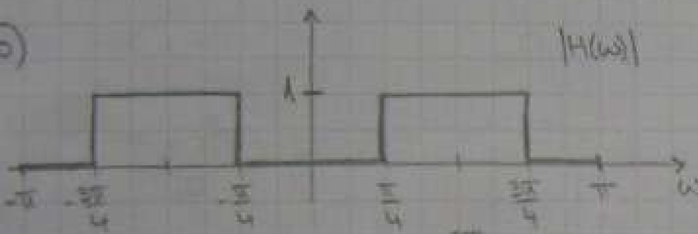
$|H'(\omega)| = |e^{-j\omega}| \cdot \left|\frac{1}{2} + \frac{2\cos(\omega)}{\pi}\right| = \left|\frac{1}{2} + \frac{2\cos(\omega)}{\pi}\right|$



$\frac{1}{2} + \frac{2}{\pi} \approx 1{,}137$

$\frac{1}{2} + \frac{2}{\pi}(-1) \approx -0{,}137$

b)



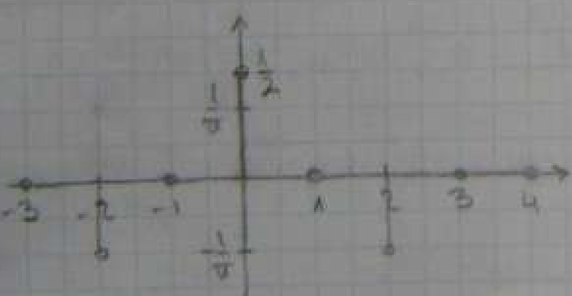
$$h[n] = \frac{1}{2\pi}\left(\int_{-\frac{3\pi}{4}}^{-\frac{\pi}{4}} 1 \cdot e^{j\omega n}\,d\omega + \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} 1 \cdot e^{j\omega n}\,d\omega\right) = \frac{1}{2\pi}\left(\frac{1}{jn}e^{j\omega n}\Big|_{-\frac{3\pi}{4}}^{-\frac{\pi}{4}} + \frac{1}{jn}e^{j\omega n}\Big|_{\frac{\pi}{4}}^{\frac{3\pi}{4}}\right) =$$

$$= \frac{1}{2j\pi n}\left(e^{-j\frac{\pi}{4}n} - e^{-j\frac{3\pi}{4}n} + e^{j\frac{3\pi}{4}n} - e^{j\frac{\pi}{4}n}\right) = \frac{1}{n\pi}\left(\frac{e^{j\frac{3\pi}{4}n} - e^{-j\frac{3\pi}{4}n}}{2j} - \frac{e^{j\frac{\pi}{4}n} - e^{-j\frac{\pi}{4}n}}{2j}\right) =$$

$$= \frac{1}{n\pi}\left(\sin\left(\frac{3\pi}{4}n\right) - \sin\left(\frac{\pi}{4}n\right)\right)$$

$n = 2 \quad \frac{1}{2\pi}\left(\sin\left(\frac{3\pi}{2}\right) - \sin\left(\frac{\pi}{2}\right)\right) = \frac{1}{2\pi}\cdot(-2) = -\frac{1}{\pi}$

$n = -2 \quad \frac{-1}{2\pi}\left(\sin\left(-\frac{3}{2}\pi\right) - \sin\left(-\frac{\pi}{2}\right)\right) = \frac{-1}{2\pi}\cdot 2 = -\frac{1}{\pi}$



$n = 0 \quad \lim_{n\to 0}\frac{\sin(\frac{3\pi}{4}n)}{n\pi} = \lim_{n\to 0}\frac{\frac{3\pi}{4}\cos(\frac{3\pi}{4}n)}{\pi} = \frac{3}{4}$

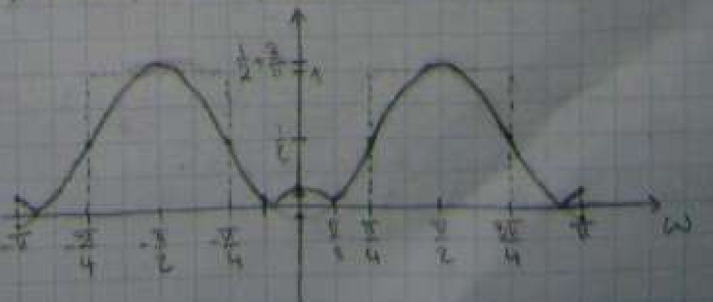
$\lim_{n\to 0}\frac{\sin(\frac{\pi}{4}n)}{n\pi} = \lim_{n\to 0}\frac{\frac{\pi}{4}\cos(\frac{\pi}{4}n)}{\pi} = \frac{1}{4}$

$h[-2] = -\frac{1}{\pi} \quad h[2] = -\frac{1}{\pi} \quad h[0] = \frac{1}{2}$

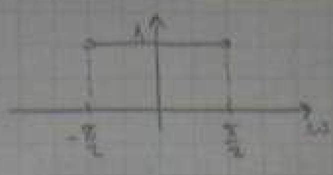
$h[n]\cdot w[n] = \left\{-\frac{1}{\pi}, \frac{1}{2}, -\frac{1}{\pi}\right\}$ → nekauzalan, treba ga pomaknuti u desno za 2

$H(e^{j\omega})\cdot W(e^{j\omega})\cdot e^{-2j\omega} = e^{-2j\omega}\left(-\frac{1}{\pi}e^{2j\omega} + \frac{1}{2} - \frac{1}{\pi}e^{-2j\omega}\right) = e^{-2j\omega}\left(\frac{1}{2} - \frac{1}{\pi}\cdot 2\cos(2\omega)\right)$

$|H(e^{j\omega})\cdot W(e^{j\omega})| = \left|\frac{1}{2} - \frac{2\cos(2\omega)}{\pi}\right|$



3.a) $A(\omega)=\begin{cases}1, & -\frac{\pi}{2}<\omega<\frac{\pi}{2}\\ 0, & \text{inače}\end{cases}$



$a[0]=\frac{1}{2\pi}\int_{-\pi}^{\pi}A(\omega)d\omega=\frac{2}{2\pi}\int_0^{\pi}A(\omega)d\omega=$

$=\frac{1}{\pi}\int_0^{\frac{\pi}{2}}1\cdot d\omega=\frac{1}{\pi}\cdot\omega\Big|_0^{\frac{\pi}{2}}=\frac{1}{\pi}\cdot\frac{\pi}{2}=\frac{1}{2}$

$a[m]=\frac{1}{\pi}\int_{-\pi}^{\pi}A(\omega)\cos(\omega m)d\omega=\frac{2}{\pi}\int_0^{\pi}A(\omega)\cos(\omega m)=\frac{2}{\pi}\int_0^{\frac{\pi}{2}}1\cdot\cos(\omega m)d\omega=$

$=\frac{2}{\pi}\cdot\frac{1}{m}\sin(\omega m)\Big|_0^{\frac{\pi}{2}}=\frac{2}{\pi}\cdot\frac{1}{m}\left(\sin\left(\frac{\pi}{2}m\right)-\sin(0)\right)=\frac{2}{m\pi}\cdot\sin\left(\frac{\pi}{2}m\right)$

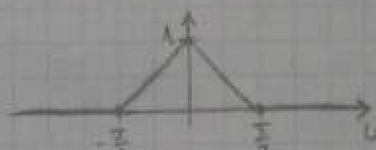
$a[1]=\frac{2}{\pi}\sin\left(\frac{\pi}{2}\right)=\frac{2}{\pi}$ $\qquad a[2]=\frac{2}{2\pi}\sin\left(\frac{\pi}{2}2\right)=0$

$A(\omega)=\frac{1}{2}+\frac{2}{\pi}\cos(\omega)+0\cdot\cos(2\omega)$ $\qquad \frac{2}{\pi}\cdot\frac{e^{j\omega}+e^{-j\omega}}{2}$

$H(\omega)=e^{-2j\omega}\left(\frac{1}{2}+\frac{2}{\pi}\cos(\omega)\right)=\frac{1}{2}e^{-2j\omega}+\frac{1}{\pi}e^{-j\omega}+\frac{1}{\pi}e^{-3j\omega}$

$h[n]=\left\{\underline{0},\frac{1}{\pi},\frac{1}{2},\frac{1}{\pi},0\right\}$

b) $A(\omega)=\begin{cases}1-\frac{2}{\pi}|\omega|, & -\frac{\pi}{2}<\omega<\frac{\pi}{2}\\ 0, & \text{inače}\end{cases}$



$a[0]=\frac{1}{\pi}\int_0^{\pi}A(\omega)d\omega=\frac{1}{\pi}\int_0^{\frac{\pi}{2}}\left(1-\frac{2}{\pi}\omega\right)d\omega=$ $\qquad \int u\,dv=uv-\int v\,du$

$=\frac{1}{\pi}\left(\int_0^{\frac{\pi}{2}}d\omega-\int_0^{\frac{\pi}{2}}\frac{2}{\pi}\omega d\omega\right)=\frac{1}{\pi}\cdot\omega\Big|_0^{\frac{\pi}{2}}-\frac{1}{\pi}\cdot\frac{2}{\pi}\cdot\frac{1}{2}\omega^2\Big|_0^{\frac{\pi}{2}}=\frac{1}{\pi}\left(\frac{\pi}{2}-0\right)-\frac{1}{\pi^2}\left(\frac{\pi^2}{4}-0\right)=\frac{1}{2}-\frac{1}{4}=\frac{1}{4}$

$a[m]=\frac{2}{\pi}\int_0^{\pi}A(\omega)\cos(m\omega)d\omega=\frac{2}{\pi}\int_0^{\frac{\pi}{2}}\left(1-\frac{2}{\pi}|\omega|\right)\cos(m\omega)d\omega=\frac{2}{\pi}\left(\int_0^{\frac{\pi}{2}}\cos(m\omega)d\omega-\frac{2}{\pi}\int_0^{\frac{\pi}{2}}\omega\cos(m\omega)d\omega\right)=$

$=\frac{2}{\pi}\left(\frac{1}{m}\sin(m\omega)\Big|_0^{\frac{\pi}{2}}-\frac{2}{\pi}\left(\frac{\omega}{m}\sin(m\omega)\Big|_0^{\frac{\pi}{2}}-\int_0^{\frac{\pi}{2}}\frac{1}{m}\sin(m\omega)d\omega\right)\right)=$

$=\frac{2}{\pi}\left(\frac{1}{m}\sin\left(\frac{\pi}{2}m\right)-\frac{2}{\pi}\left(\frac{1}{m}\frac{\pi}{2}\sin\left(\frac{\pi}{2}m\right)+\frac{1}{m^2}\cos(m\omega)\Big|_0^{\frac{\pi}{2}}\right)\right)=\frac{2}{\pi}\left(\frac{1}{m}\sin\left(\frac{\pi}{2}m\right)-\frac{1}{m}\sin\left(\frac{\pi}{2}m\right)-\frac{2}{\pi m^2}\left(\cos\left(\frac{\pi}{2}m-1\right)\right)\right)=$

$=\frac{4}{\pi^2m^2}\left(1-\cos\left(\frac{\pi}{2}m\right)\right)\Rightarrow a[1]=\frac{4}{\pi^2}$ $\qquad a[2]=\frac{4}{4\pi^2}\cdot(1-(-1))=\frac{2}{\pi^2}$

$H(\omega)=e^{-2j\omega}\left(\frac{1}{4}+\frac{4}{\pi^2}\cos(\omega)+\frac{2}{\pi^2}\cos(2\omega)\right)=e^{-2j\omega}\left(\frac{1}{4}+\frac{2}{\pi^2}e^{j\omega}+\frac{2}{\pi^2}e^{-j\omega}+\frac{1}{\pi^2}e^{2j\omega}+\frac{1}{\pi^2}e^{-2j\omega}\right)$

$=\frac{1}{4}e^{-2j\omega}+\frac{2}{\pi^2}e^{-j\omega}+\frac{2}{\pi^2}e^{-3j\omega}+\frac{1}{\pi^2}+\frac{1}{\pi^2}e^{-4j\omega}$

$h[n]=\left\{\frac{1}{\pi^2},\frac{2}{\pi^2},\frac{1}{4},\frac{2}{\pi^2},\frac{1}{\pi^2}\right\}$