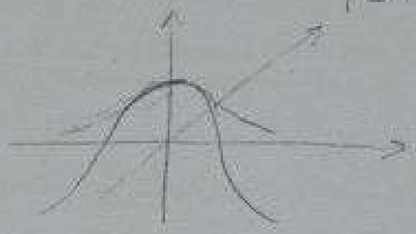
$$F(\omega_1) = \frac{1 + j\omega_1 + 1 - j\omega_1}{1+\omega_1^2} = \frac{2}{1+\omega_1^2}$$

$$F(\omega_2) = \frac{2}{1+\omega_2^2}$$

$$F(\omega_1)\cdot F(\omega_2) = \frac{2}{1+\omega_1^2}\cdot\frac{2}{1+\omega_2^2} = \frac{4}{(1+\omega_1)^2(1+\omega_2^2)} = F(\omega_1,\omega_2)$$

$$A(\omega_1,\omega_2) = |F(\omega_1,\omega_2)| = \left|\frac{4}{(1+\omega_1)^2(1+\omega_2)^2}\right| = \frac{4}{|(1+\omega_1)^2(1+\omega_2)^2|}$$



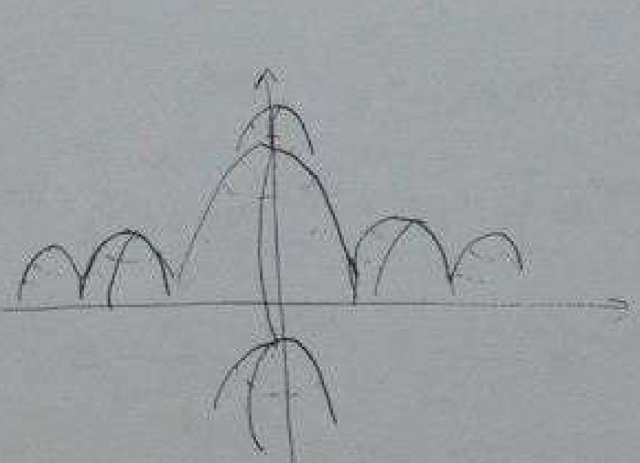
2.) $h(x,y) = \mathrm{rect}(x,y) = \mathrm{rect}(x)\,\mathrm{rect}(y) = \begin{cases} 1 & |x| \le \frac{1}{2} \\ 0 & |x| > \frac{1}{2} \end{cases}$

$$H(\omega_1,\omega_2) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(x,y)\, e^{-j(x\omega_1 + y\omega_2)}\,dx\,dy = H_1(\omega_1)\,H_2(\omega_2)$$

$$F(\omega) = \int_{-\infty}^{\infty} f(x)\, e^{-j\omega x}\,dx = \int_{-\frac{1}{2}}^{\frac{1}{2}} e^{-j\omega x}\,dx = \frac{e^{-j\omega x}}{j\omega}\Bigg|_{-\frac{1}{2}}^{\frac{1}{2}} = \frac{-e^{-\frac{1}{2}j\omega} + e^{\frac{1}{2}j\omega}}{j\omega} = \frac{\sin\left(\frac{\omega}{2}\right)}{\omega}$$

$$H(\omega_1,\omega_2) = \frac{\sin\left(\frac{\omega_1}{2}\right)\sin\left(\frac{\omega_2}{2}\right)}{\omega_1\,\omega_2} \qquad \lim_{x\to 0}\frac{\sin x}{x} = 1$$

$$\mathrm{OTF} = \frac{H(\omega_1,\omega_2)}{H(0,0)} = \frac{\sin\left(\frac{\omega_1}{2}\right)\sin\left(\frac{\omega_2}{2}\right)}{\omega_1\,\omega_2} \qquad \mathrm{MTF} = |\mathrm{OTF}| = \left|\frac{\sin\left(\frac{\omega_1}{2}\right)\sin\left(\frac{\omega_2}{2}\right)}{\omega_1\,\omega_2}\right|$$



Domaća zadaća iz
obradbe informacija

Irena [illegible]
0036452137

1.) a) $f(x,y) = \mathrm{sinc}(x)\,\mathrm{sinc}(y)$

dualnost

$x(t) \circ\!\!-\!\!\bullet\ 2\pi\, x(-\omega)$

$x(t) \circ\!\!-\!\!\bullet\ X(\omega)$

separabilnost: $f_1(x) f_2(x) \circ\!\!-\!\!\bullet\ F_1(\omega) F_2(\omega)$

$$F(\omega) = \int_{-\infty}^{\infty} f(t)\, e^{-j\omega t}\, dt$$

$$f(x) = \mathrm{sinc}(x) = \frac{\sin \pi x}{\pi x}$$

$$g(x) = \begin{cases} 1 & |x| \le \frac{\tau}{2} \\ 0 & \text{inače} \end{cases}$$

$$G(\omega) = \int_{-\tau/2}^{\tau/2} e^{-j\omega t}\, dt = \frac{e^{-j\omega t}}{-j\omega}\Bigg|_{-\frac{\tau}{2}}^{\frac{\tau}{2}} = \frac{-e^{-j\omega\frac{\tau}{2}} + e^{j\omega\frac{\tau}{2}}}{j\omega} = \tau\, \frac{\sin\left(\omega\frac{\tau}{2}\right)}{\omega\frac{\tau}{2}}$$

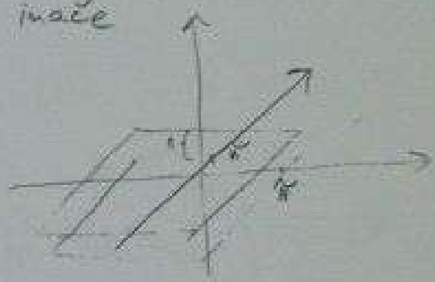
svojstvo dualnosti

$$f(x) = \tau\, \frac{\sin\left(\frac{\tau}{2} x\right)}{x \frac{\tau}{2}} \circ\!\!-\!\!\bullet\ F(\omega) = \begin{cases} 4\pi^{2} & |\omega| \le \frac{\tau}{2} \\ 0 & \text{inače} \end{cases}$$

za $\tau = 2\pi$

$$F_1(\omega_1) F_2(\omega_2) = \begin{cases} 2\pi & |\omega_1| \le \pi,\ |\omega_2| \le \pi \\ 0 & \text{inače} \end{cases}$$

$$F_1(\omega_1) F_2(\omega_2) = 4\pi^{2} \text{ za } |\omega_1| \le \pi,\ |\omega_2| \le \pi$$



b) $f(x,y) = e^{-|x|-|y|}$

$$F(\omega_1,\omega_2) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\, e^{-j\omega_1 x}\, e^{-j\omega_2 y}\, dx\, dy$$

$$f(x,y) = f(x) f(y)$$

$$F(\omega_1,\omega_2) = \int_{-\infty}^{\infty} f_x(x)\, e^{-j\omega_1 x}\, dx \int_{-\infty}^{\infty} f_y(y)\, e^{-j\omega_2 y}\, dy = F(\omega_1) F(\omega_2)$$

$$f(x,y) = e^{-|x|} e^{-|y|} = f(x) f(y) \Rightarrow F(\omega_1,\omega_2) = \int_{-\infty}^{\infty} e^{-|x|} e^{-j\omega_1 x}\, dx \cdot \int_{-\infty}^{\infty} e^{-|y|} e^{-j\omega_2 y}\, dy$$

$$= \left[\int_{-\infty}^{0} e^{x} e^{-j\omega_1 x}\, dx + \int_{0}^{\infty} e^{-x} e^{-j\omega_1 x}\, dx\right]\left[\int_{-\infty}^{0} e^{y} e^{-j\omega_2 y}\, dy + \int_{0}^{\infty} e^{-y} e^{-j\omega_2 y}\, dy\right]$$

$$F(\omega_1) = \int_{-\infty}^{0} e^{(1-j\omega_1)x}\, dx + \int_{0}^{\infty} e^{-(1+j\omega_1)x}\, dx = \frac{1}{1-j\omega_1} e^{(1-j\omega_1)x}\Bigg|_{-\infty}^{0} - \frac{1}{1+j\omega_1} e^{-(1+j\omega_1)x}\Bigg|_{0}^{\infty}$$

$$= \frac{1}{1-j\omega_1} - \frac{1}{1-j\omega_1} \lim e^{(1-j\omega_1)a} - \frac{1}{1+j\omega_1} \lim e^{(1+j\omega_1)(-b)} + \frac{1}{1+j\omega_1} = \frac{1}{1-j\omega_1} + \frac{1}{1+j\omega_1}$$

4/ a) $W_N F W_M^T$

$$F = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$W_N^{nk} = e^{-j\frac{2\pi}{N}nk}$$

$$W_3 = \begin{bmatrix} 1 & 1 & 1 \\ 1 & e^{-j\frac{2\pi}{3}} & e^{-j\frac{4\pi}{3}} \\ 1 & e^{-j\frac{4\pi}{3}} & e^{-j\frac{8\pi}{3}} \end{bmatrix}$$

$$W_3 F = \begin{bmatrix} 1 & 1 & 1 \\ 1 & e^{-j\frac{2\pi}{3}} & e^{-j\frac{4\pi}{3}} \\ 1 & e^{-j\frac{4\pi}{3}} & e^{-j\frac{8\pi}{3}} \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & e^{-j\frac{2\pi}{3}} & e^{-j\frac{4\pi}{3}} \\ 1 & e^{-j\frac{4\pi}{3}} & e^{-j\frac{8\pi}{3}} \end{bmatrix}$$

$$W_M^T = W_3^T = \begin{bmatrix} 1 & 1 & 1 \\ 1 & e^{-j\frac{2\pi}{3}} & e^{-j\frac{4\pi}{3}} \\ 1 & e^{-j\frac{4\pi}{3}} & e^{-j\frac{8\pi}{3}} \end{bmatrix}$$

$$G = W_3 F W_3^T = \begin{bmatrix} 1 & 1 & 1 \\ 1 & e^{-j\frac{2\pi}{3}} & e^{-j\frac{4\pi}{3}} \\ 1 & e^{-j\frac{4\pi}{3}} & e^{-j\frac{8\pi}{3}} \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & e^{-j\frac{2\pi}{3}} & e^{-j\frac{4\pi}{3}} \\ 1 & e^{-j\frac{4\pi}{3}} & e^{-j\frac{8\pi}{3}} \end{bmatrix}$$

$$\cos\frac{2\pi}{3} - j\sin\frac{2\pi}{3} + \cos\frac{4\pi}{3} - j\sin\frac{4\pi}{3} = -\frac{1}{2} - j\frac{\sqrt{3}}{2} - \frac{1}{2} + j\frac{\sqrt{3}}{2} = -1$$

$$= \begin{bmatrix} 3 & 1+e^{-j\frac{2\pi}{3}}+e^{-j\frac{4\pi}{3}} & 1+e^{-j\frac{4\pi}{3}}+e^{-j\frac{8\pi}{3}} \\ 1+e^{-j\frac{2\pi}{3}}+e^{-j\frac{4\pi}{3}} & 1+e^{-j\frac{4\pi}{3}}+e^{-j\frac{8\pi}{3}} & 3 \\ 1+\text{[illegible]}+e^{-j\frac{8\pi}{3}} & 3 & 1+e^{-j\frac{8\pi}{3}}+e^{-j\frac{16\pi}{3}} \end{bmatrix} = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 0 & 3 \\ 0 & 3 & 0 \end{bmatrix}$$

$$1 + \cos\frac{2\pi}{3} - j\sin\frac{2\pi}{3} + \cos\frac{4\pi}{3} - j\sin\frac{4\pi}{3} = 1 - \frac{1}{2}\cdot 2 = 0$$

$$1 + \cos\frac{4\pi}{3} - j\sin\frac{4\pi}{3} + \cos\frac{8\pi}{3} - j\sin\frac{8\pi}{3} = 0$$

$$1 + \cos\frac{8\pi}{3} - j\sin\frac{8\pi}{3} + \cos\frac{16\pi}{3} - j\sin\frac{16\pi}{3} = 0$$

$\Rightarrow)$

$$G = W_N F W_M^T \qquad F = C \cdot (W_N^H)^T G\, W_M^H$$

2DFT $\rightarrow$ Matrice W_N i W_M su simetrične $W_N^{nk} = e^{-j\frac{2\pi}{N}nk}$

$W_M^{nk} = e^{-j\frac{2\pi}{M}nk}$

$$W_N^{-1} = W_N^H = (W_N^*)^T = W_N^*$$

$$W_M^{-1} = W_M^H = (W_M^*)^T = W_M^*$$

$$G = W_N F W_M^T \;/\cdot W_N^{-1}$$

$$W_N^{-1} \cdot G = F \cdot W_M^T \;/\cdot W_M^{-1} \qquad W_M^T = W_M$$

$$W_N^{-1} G\, W_M^{-1} = F \qquad F = W_N^H \cdot G \cdot W_M^H$$

$$F = W_N^H G\, W_M^H \quad i \quad F = C(W_N^H)^T G\, W_M^H$$

$$C\,(W_N^H)^T = W_N^H$$

$$C\left((W_N^*)^T\right)^T = (W_N^*)^T$$

$$C\, W_N^* = W_N^* \quad \Rightarrow \quad C = 1 + 0j$$

$$F = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad \begin{matrix} N = 6, \\ M = 3 \end{matrix}$$

$$W_3 = \begin{bmatrix} 1 & 1 & 1 \\ 1 & e^{j\frac{2\pi}{3}} & e^{j\frac{4\pi}{3}} \\ 1 & e^{j\frac{4\pi}{3}} & e^{j\frac{8\pi}{3}} \end{bmatrix} \qquad W_6 = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & e^{j\frac{\pi}{3}} & e^{j\frac{2\pi}{3}} & -1 & e^{j\frac{4\pi}{3}} & e^{j\frac{5\pi}{3}} \\ 1 & e^{j\frac{2\pi}{3}} & e^{j\frac{4\pi}{3}} & 1 & e^{j\frac{2\pi}{3}} & e^{j\frac{4\pi}{3}} \\ 1 & -1 & 1 & -1 & 1 & -1 \\ 1 & e^{j\frac{4\pi}{3}} & e^{j\frac{2\pi}{3}} & 1 & e^{j\frac{4\pi}{3}} & e^{j\frac{2\pi}{3}} \\ 1 & e^{j\frac{5\pi}{3}} & e^{j\frac{4\pi}{3}} & -1 & e^{j\frac{2\pi}{3}} & e^{j\frac{\pi}{3}} \end{bmatrix}$$

$$G = W_6 F W_3^T$$

$$W_6 \cdot F\, W_3^T = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & e^{j\frac{\pi}{3}} & e^{j\frac{2\pi}{3}} & -1 & e^{j\frac{4\pi}{3}} & e^{j\frac{5\pi}{3}} \\ 1 & e^{j\frac{2\pi}{3}} & e^{j\frac{4\pi}{3}} & 1 & e^{j\frac{2\pi}{3}} & e^{j\frac{4\pi}{3}} \\ 1 & -1 & 1 & -1 & -1 & 1 \\ 1 & e^{j\frac{4\pi}{3}} & e^{j\frac{2\pi}{3}} & 1 & e^{j\frac{4\pi}{3}} & e^{j\frac{2\pi}{3}} \\ 1 & e^{j\frac{5\pi}{3}} & e^{j\frac{4\pi}{3}} & -1 & e^{j\frac{2\pi}{3}} & e^{j\frac{\pi}{3}} \end{bmatrix} \cdot \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \cdot W_3^T =$$

$$= \begin{bmatrix} 1 & 1 & 1 \\ e^{j\frac{4\pi}{3}} & e^{j\frac{2\pi}{3}} & 1 \\ e^{j\frac{2\pi}{3}} & e^{j\frac{4\pi}{3}} & 1 \\ 1 & 1 & 1 \\ e^{j\frac{4\pi}{3}} & e^{j\frac{2\pi}{3}} & 1 \\ e^{j\frac{2\pi}{3}} & e^{j\frac{4\pi}{3}} & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 & 1 \\ 1 & e^{j\frac{2\pi}{3}} & e^{j\frac{4\pi}{3}} \\ 1 & e^{j\frac{4\pi}{3}} & e^{j\frac{8\pi}{3}} \end{bmatrix} = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 0 & 3 \\ 0 & 3 & 0 \\ 3 & 0 & 0 \\ 0 & 0 & 3 \\ 0 & 3 & 0 \end{bmatrix}$$

a)

$$C_3 = \begin{bmatrix} \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{2}} & \frac{0}{\sqrt{2}} & \frac{-1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & \frac{-2}{\sqrt{6}} & \frac{1}{\sqrt{6}} \end{bmatrix}$$

Za unitarne matrice vrijedi: $C_3^H \cdot C_3$

$U^H \cdot U^H$

$U \cdot U^H = U U^H = I = U^H \cdot U$

$$C_3^H = (C_3^*)^T = \begin{bmatrix} \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & \frac{0}{\sqrt{2}} & \frac{-1}{\sqrt{2}} \\ \frac{1}{\sqrt{3}} & \frac{-2}{\sqrt{6}} & \frac{1}{\sqrt{6}} \end{bmatrix}$$

$$C_3 \cdot C_3^H = \begin{bmatrix} \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{2}} & \frac{0}{\sqrt{2}} & \frac{-1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & \frac{-2}{\sqrt{6}} & \frac{1}{\sqrt{6}} \end{bmatrix} \cdot \begin{bmatrix} \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & \frac{0}{\sqrt{2}} & \frac{-1}{\sqrt{2}} \\ \frac{1}{\sqrt{3}} & \frac{-2}{\sqrt{6}} & \frac{1}{\sqrt{6}} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Matrica C_3 je unitarna.

b)

$$S_3 = \begin{bmatrix} \frac{1}{\sqrt{6}} & \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{2}} & 0 & \frac{-1}{\sqrt{2}} \\ \frac{1}{\sqrt{3}} & \frac{-1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \end{bmatrix} \qquad S_3^H = (S_3^*)^T = \begin{bmatrix} \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{3}} \\ \frac{2}{\sqrt{6}} & 0 & \frac{-1}{\sqrt{3}} \\ \frac{1}{\sqrt{6}} & \frac{-1}{\sqrt{2}} & \frac{1}{\sqrt{3}} \end{bmatrix}$$

$$S_3 \cdot S_3^H = \begin{bmatrix} \frac{1}{\sqrt{6}} & \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{2}} & 0 & \frac{-1}{\sqrt{2}} \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{3}} \\ \frac{2}{\sqrt{6}} & 0 & \frac{-1}{\sqrt{3}} \\ \frac{1}{\sqrt{6}} & \frac{-1}{\sqrt{2}} & \frac{1}{\sqrt{3}} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Matrica S_3 je unitarna.