AUDITORNE VJEŽBE

KRIVO POSTAVLJEN ZAD!

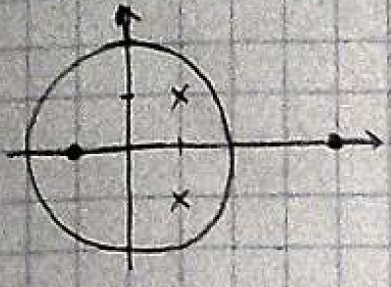
1. $H(z)=\dfrac{2z^{-2}-3z^{-1}-2}{2z^{-2}-2z^{-1}+1}=2z^{-2}+z^{-1}-4z^{-1}-2=\dfrac{z^{-1}(2z^{-1}+1)-2(2z^{-1}+1)}{2z^{-2}-2z^{-1}+1}$

$$z^{-1}_{1,2}=\frac{2\pm\sqrt{4-8}}{4}=\frac{2\pm 2j}{4} \qquad z^{-1}_1=\frac{1}{2}+\frac{1}{2}j \qquad z^{-1}_2=\frac{1}{2}-\frac{1}{2}j$$

polovi ?? z^{-1}

$$z^{-1}_{1,2}=\frac{3\pm\sqrt{9+16}}{4}=\frac{3\pm 5}{4} \qquad z^{-1}_1=-\frac{1}{2} \qquad z^{-1}_2=2$$

nule



stabilan: DA (uistinu vrijedi...)

minimalno fazni: NE

recipročna vrijednost nula

$$H_{mf}(z)=\frac{(2z^{-1}+1)(2z^{-1}-1)}{2z^{-2}-2z^{-1}+1}$$

$$H_{mf}^{-1}(z)=\frac{2z^{-2}-2z^{-1}+1}{(2z^{-1}+1)(2z^{-1}-1)}$$

$$H(z)\cdot H_{mf}^{-1}(z)=\frac{(2z^{-1}+1)(z^{-1}-2)}{(2z^{-1}+1)(2z^{-1}-1)}=\frac{z^{-1}-2}{2z^{-1}-1}$$

$z=e^{jw}$

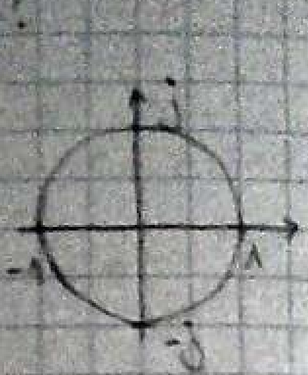
$$H(e^{jw})\cdot H_{mf}^{-1}(e^{jw})=\frac{e^{-jw}-2}{2z^{-jw}-1}=\frac{e^{-j\frac{w}{2}}\left(e^{-j\frac{w}{2}}-2e^{j\frac{w}{2}}\right)}{e^{-j\frac{w}{2}}\left(2e^{-j\frac{w}{2}}-e^{j\frac{w}{2}}\right)}=$$

$$=\frac{\cos\left(\frac{w}{2}\right)-j\sin\left(\frac{w}{2}\right)-2\cos\left(\frac{w}{2}\right)-2j\sin\left(\frac{w}{2}\right)}{2\cos\left(\frac{w}{2}\right)-2j\sin\left(\frac{w}{2}\right)-\cos\left(\frac{w}{2}\right)-j\sin\left(\frac{w}{2}\right)}=\frac{-\cos\left(\frac{w}{2}\right)-3j\sin\left(\frac{w}{2}\right)}{\cos\left(\frac{w}{2}\right)-3j\sin\left(\frac{w}{2}\right)}$$

$$\left|H(e^{jw})H_{mf}^{-1}(e^{jw})\right|=\sqrt{\frac{\cos^2\left(\frac{w}{2}\right)+9\sin^2\left(\frac{w}{2}\right)}{\cos^2\left(\frac{w}{2}\right)+9\sin^2\left(\frac{w}{2}\right)}}=1$$

2. $x[n] = \{1, 0, -1, 0\}$ $DFT_4 = ?$

a) $X[k] = \sum_{n=0}^{3} x[n] e^{-j\frac{2\pi}{N}kn}$



$X[0] = 1 + 0 - 1 + 0 = 0$

$X[1] = 1 - 0 \cdot j - 1 \cdot (-1) + 0 \cdot j = 2$

$X[2] = 1 \cdot 1 + 0 \cdot 1 - 1 \cdot 1 - 1 \cdot 0 = 0$

$X[3] = 1 \cdot 1 + 0 \cdot j - 1(-1) - 0 \cdot j = 2$

$$W_N = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix}$$

$DFT_4[x[n]] = \{0, 2, 0, 2\}$

b) $x[n] * x[n] = ?$

$= \sum_{i=-\infty}^{\infty} x[i]\,x[n-i]$

$$\begin{array}{cccccccc} & & & \underline{1} & 0 & -1 & 0 & \\ 0 & -1 & 0 & \underline{1} & & & & \longrightarrow \end{array}$$

1	0	-1	0	
1	0	-1	0	1
0	0	0	0	0
-1	0	1	0	-1
0	0	0	0	0

$\Rightarrow \{1, 0, -2, 0, 1, 0, 0\}$

$$\begin{array}{cccc} 1 & 0 & 0 & \longleftarrow + \\ \hline 2 & 0 & -2 & 0 \end{array}$$

c) $x[n] \circledast_4 x[n] = ?$

$= \sum x[i]\,x[((n-i))_4]$

$$\begin{array}{cccccccccc} & & & \underline{1} & 0 & -1 & 0 & 1 & 0 & -1 \quad 0 \\ 0 & -1 & 0 & \underline{1} & & & & & \longrightarrow & \end{array}$$

			1	0	-1	0	
			1	0	-1	0	1
		0	0	0	0	0	0
	1	0	-1	0	1	0	-1
0	0	0	0	0	0	0	0
2	0	-2	0				

$\Rightarrow \{2, 0, -2, 0\}$

$$\begin{array}{rcccc} & 1 & 0 & -2 & 0 \\ + & 1 & 0 & 0 & \\ \hline & 2 & 0 & -2 & 0 \end{array}$$

d) jesu li b) i c) jednaki?

NE

e) kada su b) i c) jednaki?

x moramo proširiti s nulama

$4+4-1=7$

$7-4=3 \rightarrow$ sa 3 nule $(N-1)$

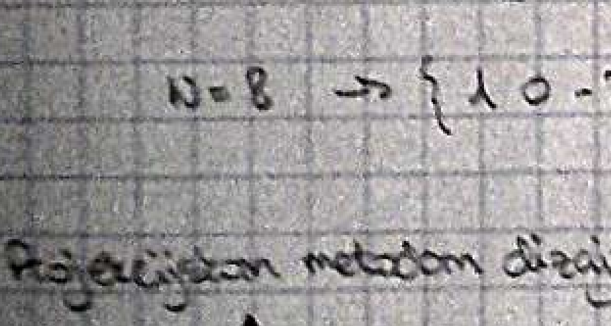
f) $IDFT_N(X \cdot X) = ?$ cirkularna ili obična konv?

$N=7 \rightarrow x * x$

$N=4 \rightarrow x \; ④ \; x$

$N=8 \rightarrow \{1 \; 0 \; -2 \; 0 \; 1 \; 0 \; 0 \; \underline{0}\}$

3. Projekcijskom metodom dizajnirati filtar zadan slikom



$$a[n] = \frac{2}{\pi} \int_0^{\pi} A_d(e^{j\omega}) \cos(\omega n) \, d\omega$$

$$a[0] = \frac{1}{\pi} \int_0^{\pi} A_d(e^{j\omega}) \, d\omega$$

A	B	C
$A_d = 1$	$A_d = \dfrac{\frac{\pi}{2} - \omega}{\frac{\pi}{2}}$	$A_d = \dfrac{\frac{3\pi}{4} - \omega}{\frac{\pi}{4}}$
0 to $\frac{\pi}{4}$	$\frac{\pi}{4}$ to $\frac{\pi}{2}$	$\frac{\pi}{2}$ to $\frac{3\pi}{4}$

① $\int k\cos(wm)\,dw = k\int \cos(wm)\,dw = \frac{k}{m}\sin(wm)$

② $\int \underbrace{w}_{u}\underbrace{\cos(wm)\,dw}_{v'} = \frac{w}{m}\sin(wm) + \frac{1}{m}\int \sin(wm)\,dw = \quad uv - \int v\,du$

$$= \frac{w}{m}\sin(wm) + \frac{1}{m^2}\cos(wm)$$

$$a[m] = \frac{2}{\pi}\left[\underbrace{\int_0^{\frac{\pi}{4}} \cos(wm)\,dw}_{A} + \underbrace{\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \left(\frac{1}{2} + \frac{2w}{\pi}\right)\cos(wm)\,dw}_{B} + \underbrace{\int_{\frac{\pi}{2}}^{\frac{3\pi}{4}} \left(2 - \frac{2w}{\pi}\right)\cos(wm)\,dw}_{C}\right.$$

$$A \to \frac{1}{m}\sin(wm)\Big|_0^{\frac{\pi}{4}} = \frac{1}{\pi}\sin\frac{m\pi}{4}$$

$$B \to \frac{1}{2m}\sin(wm)\Big|_{\frac{\pi}{4}}^{\frac{\pi}{2}} + \frac{2}{\pi}\left(\frac{w}{m}\sin(wm) + \frac{1}{m^2}\cos(wm)\right)\Big|_{\frac{\pi}{4}}^{\frac{\pi}{2}} =$$

$$= \frac{1}{2m}\left(\sin\frac{m\pi}{2} - \sin\frac{m\pi}{4}\right) + \frac{2}{\pi m}\left(\frac{\pi}{2}\sin\frac{m\pi}{2} - \frac{\pi}{4}\sin\frac{m\pi}{4}\right) + \frac{2}{\pi m^2}\left(\cos\frac{m\pi}{2} - \cos\frac{m\pi}{4}\right.$$

$$C \to \frac{2}{m}\sin(wm)\Big|_{\frac{\pi}{2}}^{\frac{3\pi}{4}} - \frac{2}{\pi}\left(\frac{w}{m}\sin(wm) + \frac{1}{m}\cos(wm)\right)\Big|_{\frac{\pi}{2}}^{\frac{3\pi}{4}} =$$

$$= \frac{2}{m}\left(\sin\frac{3\pi m}{4} - \sin\frac{\pi}{2}m\right) - \frac{2}{\pi m}\left(\frac{3\pi}{4}m\sin\frac{3\pi m}{4} - \frac{\pi}{2}m\sin\frac{\pi}{2}m\right) -$$

$$- \frac{2}{\pi m^2}\left(\cos\frac{3\pi}{4}m - \cos\frac{\pi}{2}m\right)$$

$$a[m] = [A + B + C]\cdot\frac{2}{\pi}$$

$$a[n] = \ldots\ldots \frac{2}{\pi}(2 - 3\sqrt{2})\left(\frac{1}{\pi} - 1\right) + \frac{2\sqrt{2}}{\pi}$$

(DCT - II)

4. Napisati matricu DCT transformacije. Naći inverznu matricu. Skicirati analizirajući i razlagajući slog. Ispisati sve impulsne odzive.

$$X[k] = \sum_{n=0}^{N-1} x(n) W_k \cos\left[\frac{\pi k}{N}\left(n+\frac{1}{2}\right)\right]$$

$$x[n] = \sum_{k=0}^{N-1} X(k) W_k \cos\left[\frac{\pi k}{N}\left(n+\frac{1}{2}\right)\right]$$

$$W_k = \begin{cases} \sqrt{\frac{1}{N}} & k=0 \\ \sqrt{\frac{2}{N}} & k\neq 0 \end{cases}$$

$$C_{N=4} = \frac{1}{\sqrt{N}} \begin{array}{cc} \begin{array}{cccc} n=0 & n=1 & n=2 & n=3 \end{array} & \\ \begin{bmatrix} \cos 0 & \cos 0 & \cos 0 & \cos 0 \\ \sqrt{2}\cos\frac{\pi}{8} & \sqrt{2}\cos\left(\frac{\pi}{4}\cdot\frac{3}{2}\right) & \sqrt{2}\cos\left(\frac{\pi}{4}\cdot\frac{5}{2}\right) & \sqrt{2}\cos\left(\frac{\pi}{4}\cdot\frac{7}{2}\right) \\ \sqrt{2}\cos\frac{\pi}{4} & \sqrt{2}\cos\left(\frac{\pi}{2}\cdot\frac{3}{2}\right) & \sqrt{2}\cos\left(\frac{\pi}{2}\cdot\frac{5}{2}\right) & \sqrt{2}\cos\left(\frac{\pi}{2}\cdot\frac{7}{2}\right) \\ \sqrt{2}\cos\frac{3\pi}{8} & \sqrt{2}\cos\left(\frac{3\pi}{4}\cdot\frac{3}{2}\right) & \sqrt{2}\cos\left(\frac{3\pi}{4}\cdot\frac{5}{2}\right) & \sqrt{2}\cos\left(\frac{3\pi}{4}\cdot\frac{7}{2}\right) \end{bmatrix} & \begin{array}{c} k=0 \\ k=1 \\ k=2 \\ k=3 \end{array} \end{array}$$

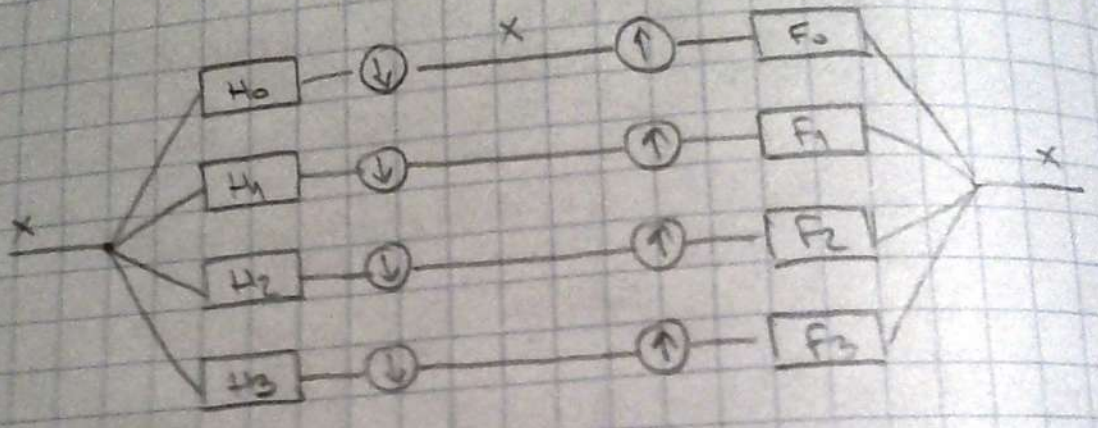
$$C_4^{-1} = C_4^T \qquad \underset{C_N}{C} \cdot \underset{C_N^T}{C^{-1}} = I$$

$$X = C_4 \cdot x$$

$$\begin{bmatrix} X[0] \\ X[1] \\ X[2] \\ X[3] \end{bmatrix} = \begin{bmatrix} & C_4 & \end{bmatrix} \cdot \begin{bmatrix} x[0] = x[3]\cdot z^{-3} \\ x[1] = x[3]\cdot z^{-2} \\ x[2] = x[3]\cdot z^{-1} \\ x[3] = x[3]\cdot z^{0} \end{bmatrix}$$

$$X[0] = \frac{1}{2}\big(\underbrace{z^{-3}+z^{-2}+z^{-1}+1}_{H_0}\big)\, x[0]$$

$$X[1] = \frac{1}{2}\sqrt{2}\,\frac{\sqrt{2}}{2}\big(\underbrace{\sqrt{2}z^{-3}-z^{-2}-z^{-1}+ \text{[illegible]}}_{H_1}\big)\, x[1]$$



x
H0
H1
H2
H3
x
F0
F1
F2
F3
x