OBRADBA INFORMACIJA

[redacted]

3. Domaća zadaća

(1)

$$X[w] = \sum_{n=-\infty}^{\infty} x[n] e^{-jwn} = DTFT[x[n]]$$

$x[n] \Rightarrow$ realan niz $\quad x(n) \in R$

Svojstva: a) $Re[X(\omega)]$ je parna f-ja od ω

$$Re[X(\omega)] = Re \sum_{n=-\infty}^{\infty} x(n) e^{-j\omega n} = \sum_{n=-\infty}^{\infty} x(n) \cdot \underbrace{Re(e^{-j\omega n})}_{\underbrace{\cos(\omega n)}_{\text{parna f-ja}}} = \sum_{n=-\infty}^{\infty} x(n) \cdot \cos(-\omega \cdot n) = \underline{X(-\omega)}$$

parna f-ja

b) $Im[X(\omega)]$ je neparna f-ja od ω

$$Im[X(\omega)] = Im \sum_{n=-\infty}^{\infty} \cdot x(n) e^{-j\omega n} = \sum_{n=-\infty}^{\infty} x(n) \cdot \underbrace{\sin(\omega n)}_{\text{neparna f-ja}} = -\sum_{n=-\infty}^{\infty} x(n) \cdot \sin(-\omega n) = -X(-\omega)$$

neparna f-ja

c) $|X(\omega)|$ je parna f-ja od ω

$$|X(\omega)| = \sqrt{Re(X(\omega))^2 + Im(X(\omega))^2} = \sqrt{(\text{parna})^2 + (\text{neparna})^2} = \sqrt{\text{parna} + \text{parna}} = \underline{\text{parna f-ja}}$$

d) $\arg(X(\omega))$ je neparna f-ja od ω

$$\arg(X(\omega)) = \text{arctg} \frac{Im(X(\omega))}{Re(X(\omega))}$$

$$\arg(-X(\omega)) = -\text{arctg} \frac{Im(X(\omega))}{Re(X(\omega))}$$

neparna f-ja.

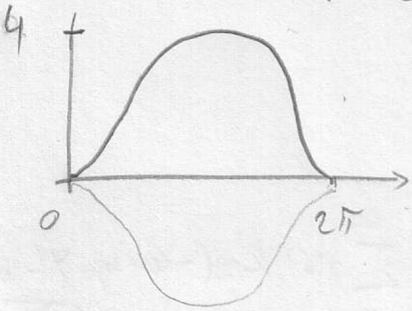
② a) DTFT i skica amp i fazne karakteristike

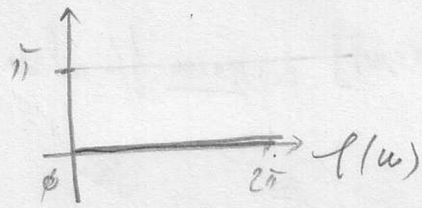
$$x[n] = \delta[n+1] - 2\delta[n] + \delta[n-1]$$

$$X[\omega] = DTFT(x[n]) = \sum_{n=-\infty}^{\infty} x(n)\cdot e^{-j\omega n} = 1\cdot e^{j\omega} - 2\cdot e^{\phi} + 1\cdot e^{j\omega}$$

$$= -2 + \cos(\omega) + \cancel{j\sin(\omega)} + \cos(\omega) - \cancel{j\sin(\omega)} = \underline{2\cos(\omega) - 2}$$

$A(\omega) = |X(\omega)| = |-2 + 2\cos(\omega)| = 2 - 2\cos(\omega)$, $\varphi(\omega) = \arctg\left(\frac{Im(X\omega)}{Re(X\omega)}\right) = \arctg(\phi) = \phi$



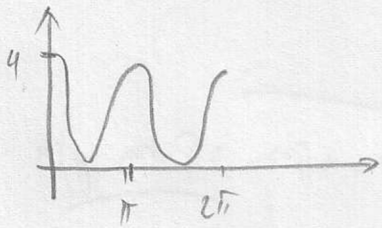


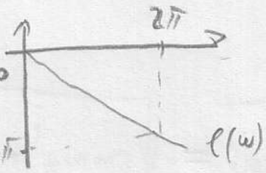
b) $x(n) = \delta(n) + 2\delta(n-2) + \delta(n-4) = \{\underline{1}, 0, 2, 0, 1\}$

$$X[\omega] = DTFT(x(n)) = 1 + \phi + 2\cdot e^{-j\omega 2} + \phi + 1\cdot e^{-j\omega 4} = 1 + 2e^{-2j\omega} + e^{-4j\omega} =$$

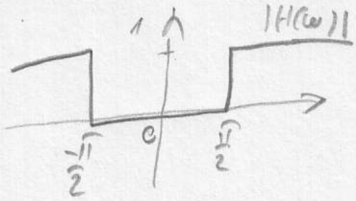
$$= 1 + 2\cdot e^{-2j\omega} + e^{-2j\omega}\cdot e^{-2j\omega} = e^{-2j\omega}\cdot\left(e^{2j\omega} + 2\cdot e^{-2j\omega}\right) = e^{-2j\omega}\cdot\left(2 + 2\cos(2\omega)\right)$$

$A(\omega) = \sqrt{Re^2 + Im^2} = 2 + 2\cos(2\omega)$, $\varphi(\omega) = -2\omega$





3) a)



$h(n) = ?$ ako pretpostavka nulte faze i da je faza oblika $e^{-2j\omega}$ i $e^{2j\omega}$

$$|H(\omega)| = \begin{cases} 1, & -\infty < \omega < -\frac{\pi}{2}, \ \frac{\pi}{2} < \omega < \infty \\ 0, & -\frac{\pi}{2} < \omega < \frac{\pi}{2} \end{cases}$$

Inverzna DTFT

$$x(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(\omega)\cdot e^{j\omega n}\, d\omega = \frac{1}{2\pi}\cdot\left(\int_{-\pi}^{-\pi/2} \underbrace{X(\omega)}_{1} e^{j\omega n}\, d\omega + \int_{\pi/2}^{\pi} \underbrace{X(\omega)}_{1}\cdot e^{j\omega n}\, d\omega\right) =$$

$$= \frac{1}{2\pi}\cdot\left(\frac{1}{jn}\cdot e^{j\omega n}\Big|_{-\pi}^{-\pi/2} + \frac{1}{jn}\cdot e^{j\omega n}\Big|_{\pi/2}^{\pi}\right) = \frac{1}{2\pi j n}\cdot\left(e^{-\frac{jn\pi}{2}} - e^{-jn\pi} + e^{j\pi n} - e^{+\frac{jn\pi}{2}}\right) =$$

$$= \frac{1}{\pi n}\cdot\left(-\sin\left(\frac{\pi}{2}n\right) + \sin(\pi n)\right)$$ za nultu fazu

$$H(\omega)\cdot e^{\pm 2j\omega} \bullet\!\!-\!\!\circ \frac{1}{(n\pm 2)\pi}\cdot\left(\sin(\pi\cdot(n\pm 2)) - \sin\left(\frac{\pi}{2}\cdot(n\pm 2)\right)\right)$$

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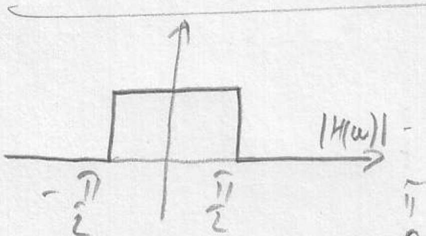
b)

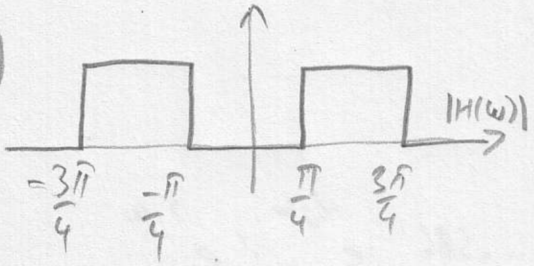


$$h(n) = DTFT^{-1}(H(\omega)) = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(\omega) e^{j\omega n}\, d\omega = \frac{1}{2\pi}\cdot\int_{-\frac{\pi}{2}}^{\pi/2} 1\cdot e^{j\omega n}\, d\omega = \frac{1}{2\pi}\cdot\frac{1}{jn}\cdot e^{j\omega n}\Big|_{-\frac{\pi}{2}}^{\pi/2} =$$

$$= \frac{1}{\pi n}\cdot\frac{1}{2j}\cdot\left(e^{\frac{\pi}{2}jn} - e^{-\frac{\pi}{2}jn}\right) = \frac{1}{\pi n}\cdot\sin\left(\frac{\pi}{2}n\right)$$

$$H(\omega)\cdot e^{\pm 2j\omega} \bullet\!\!-\!\!\circ \frac{1}{\pi\cdot(n\pm 2)}\cdot\sin\left(\frac{\pi}{2}\cdot(n\pm 2)\right)$$ za fazu $\pm 2\omega$

c)



$$h(n) = ? = IDTFT(H(\omega)) = \frac{1}{2\pi}\cdot\int_{-\pi}^{\pi} X(\omega)\cdot e^{j\omega n}\,d\omega =$$

$$= \frac{1}{2\pi}\left(\int_{-\frac{3\pi}{4}}^{-\pi/4} e^{j\omega n}\,d\omega + \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} e^{j\omega n}\,d\omega\right) = \frac{1}{2\pi}\cdot\left(\frac{1}{jn}\cdot e^{j\omega n}\Big|_{-\frac{3\pi}{4}}^{-\pi/4} + \frac{1}{jn}\cdot e^{j\omega n}\Big|_{\frac{\pi}{4}}^{\frac{3\pi}{4}}\right) =$$

$$= \frac{1}{\pi n}\cdot\frac{1}{2j}\cdot\left(e^{+\frac{3\pi}{4}jn} - e^{-\frac{3\pi}{4}jn}\right) + \frac{1}{\pi n}\cdot\frac{1}{2j}\cdot\left(e^{-\frac{\pi}{4}jn} - e^{\frac{\pi}{4}jn}\right) =$$

$$= \frac{1}{\pi n}\cdot\left((1)\cdot\sin\left(\frac{3}{4}\pi n\right) - \sin\left(\frac{\pi}{4}n\right)\right)$$ za nultu fazu

za faze $\pm 2\omega$

$$h(n\pm2) = \frac{1}{\pi(n\pm2)}\cdot\left(\sin\left(\frac{3}{4}\pi\cdot(n\pm2)\right) - \sin\left(\frac{\pi}{4}\cdot(n\pm2)\right)\right)$$

(4) a) DFT_N transformacija:

definicija

$$X[k] = \sum_{n=0}^{N-1} x(n)\, e^{-j\frac{2\pi}{N}nk} = \sum_{n=0}^{N-1} x(n)\cdot W_N^{+nk}, \quad W_N^{nk} = e^{-2\pi j\frac{nk}{N}}$$

W_N matrica: $\left[W_N^{kn}\right]$, $W_N = e^{-j\frac{2\pi}{N}}$

$$W_N \begin{array}{c|l} & n\rightarrow \\ \hline k\downarrow & \begin{bmatrix} 1 & 1 & 1 & 1 & \dots \\ 1 & \ddots & & & \\ 1 & & W_N^{kn} & & \\ \vdots & & & & \end{bmatrix} \end{array}$$

(4) b) Izračun spektra 4 signala koji odgovaraju vektorima DFT_4 matrice transformacije.

$$X[k] = DFT_4(x(n)) = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} x(0) \\ x(1) \\ x(2) \\ x(3) \end{bmatrix}$$

$x_1 = \{1, 1, 1, 1\}$

$$X_1(k) = DFT_4(x(n)) = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$x_2 = \{1, -j, -1, j\}$

$$X_2(k) = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 1 \\ -j \\ -1 \\ j \end{bmatrix} = \begin{bmatrix} \phi \\ \phi \\ \phi \\ 4 \end{bmatrix}$$

$$X_3(k) = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \end{bmatrix} = \begin{bmatrix} \phi \\ \phi \\ 4 \\ \phi \end{bmatrix}$$

$$X_4(k) \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 1 \\ j \\ -1 \\ -j \end{bmatrix} = \begin{bmatrix} \phi \\ 4 \\ \phi \\ \phi \end{bmatrix}$$

c) Za koji indeks k je spektar različit od ϕ

Za $X_1(k)$ je za $k = 1$ različit spektar od ϕ odnosno za $X_i(k)$ je za $k = i$ spektar različit od ϕ jer je to zapravo apsolutna vrijednost ili modul tog broja. To je zbog međusobne ortogonalnosti redaka matrice.

5) Definicija $IDFT_N$:

a)
$$x(n) = \frac{1}{N}\sum_{k=0}^{N-1} X(k)\cdot e^{j\frac{2\pi}{N}\cdot nk}$$

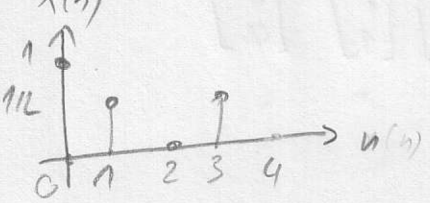
$$W_N^{-1} = \frac{1}{N}\cdot\left[e^{-j\frac{2\pi}{N}\cdot kn}\right]$$ da je $N\times N$ matrica koja u i-tom stupcu i j-tom retku

ima element $\frac{1}{N}W_N^{-(i-1)(j-1)}$

b) $IDFT_4$ za $X(k) = \{\underline{2}, 1, 0, 1\}$

$$x(n) = \frac{1}{N}W_N^{-1}\cdot X[k] = \frac{1}{4}\cdot\begin{bmatrix}1 & 1 & 1 & 1\\ 1 & j & -1 & -j\\ 1 & -1 & 1 & -1\\ 1 & -j & -1 & j\end{bmatrix}\begin{bmatrix}2\\1\\0\\1\end{bmatrix} = \frac{1}{4}\cdot\begin{bmatrix}4\\2\\0\\2\end{bmatrix} = \begin{bmatrix}1\\1/2\\0\\1/2\end{bmatrix}$$

$$x(n) = \left\{\underline{1}, \frac{1}{2}, 0, \frac{1}{2}\right\}$$



c) Pokaži da za $N=4$ $\quad W_N\cdot W_N^H = N\cdot I$

$$W_4 = \begin{bmatrix}1 & 1 & 1 & 1\\ 1 & -j & -1 & j\\ 1 & -1 & 1 & -1\\ 1 & j & -1 & -j\end{bmatrix} \qquad W_4^H = \begin{bmatrix}1 & 1 & 1 & 1\\ 1 & j & -1 & -j\\ 1 & -1 & 1 & -1\\ 1 & -j & -1 & j\end{bmatrix}$$

$$W_4\cdot W_4^H = \begin{bmatrix}1 & 1 & 1 & 1\\ 1 & -j & -1 & j\\ 1 & -1 & +1 & -1\\ 1 & j & -1 & -j\end{bmatrix}\begin{bmatrix}1 & 1 & 1 & 1\\ 1 & j & -1 & -j\\ 1 & -1 & 1 & -1\\ 1 & -j & -1 & j\end{bmatrix} = \begin{bmatrix}4 & 0 & 0 & 0\\ 0 & 4 & 0 & 0\\ 0 & 0 & 4 & 0\\ 0 & 0 & 0 & 4\end{bmatrix} = \frac{1}{4}\cdot I$$

(6)

$$X[k] = \sum_{n=0}^{N-1} x(n) \cdot W_N^{nk} = DFT_N[x(n)] \quad , \quad x(n) \in R$$

Zadovoljava relaciju $X(k) = X^*[N-k]$

$$X[k] = \sum_{n=0}^{N-1} x(n) \cdot W_N^{nk} \quad , \quad X^*[N-k] = \sum_{n=0}^{N-1} x(n) \cdot \left(W_N^{n \cdot (N-k)}\right)^*$$

$$\rightarrow \sum_{n=0}^{N-1} x(n) \cdot \left(W_N^{n \cdot (-k) + nN}\right)^* = \qquad \boxed{W_N^{nk} = e^{-\frac{2\pi}{N} n \cdot k \cdot j}}$$

$$\rightarrow \sum_{n=0}^{N-1} x(n) \cdot \left(e^{+\frac{2\pi}{N} \cdot nk \cdot j - \frac{2\pi}{N} \cdot n \cdot N \cdot j}\right)^* = \sum_{n=0}^{N-1} x(n) \cdot e^{-\frac{2\pi}{N} \cdot nk \cdot j} \cdot e^{2\pi n \cdot j} = \sum_{n=0}^{N-1} x(n) \cdot e^{-2\pi j \cdot n\left(k - \frac{1}{N}\right)} =$$

$$= \sum_{n=0}^{N-1} x(n) \cdot e^{-\frac{2\pi}{N} j n (k - N)} = \sum_{n=0}^{N-1} x(n) \cdot W_N^{n(k-N)}$$

S obzirom da je $X[k]^* = \sum_{n=0}^{N-1} x(n) \cdot W_N^{-nk}$ i W_N periodička s periodom N

vrijedi $W_N^{-nk} = W^{-nk + n \cdot N} = W^{n(N-k)}$

i stoga je $X(k) = x^*(N-k)$