PRIPREMA-2.LAB-OBRINF

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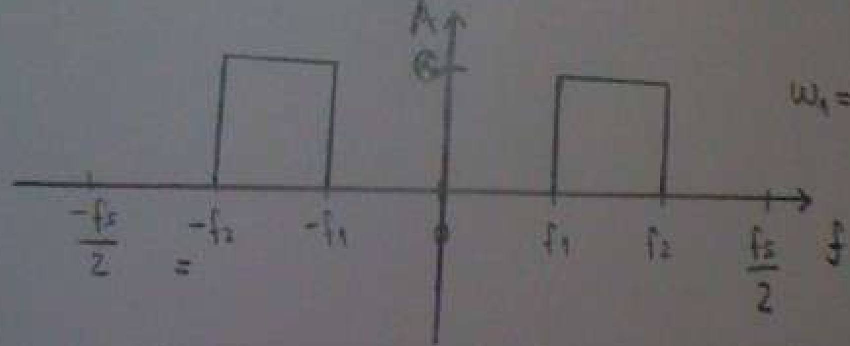
3.1-2 a)

N=red filtra	širina gl. latice	gušenje 1. bočne latice	širina prijelaznog područja
PRAVOKUTNI	$\frac{4\pi}{N+1}$	13,3	$0{,}92\pi / \frac{N}{2}$
BARLETTOV	$\frac{8\pi}{N}$	25	
HANNOV	$\frac{8\pi}{N}$	31,5	$3{,}11\pi / \frac{N}{2}$
HAMMINGOV	$\frac{8\pi}{N}$	42,7	$3{,}32\pi / \frac{N}{2}$
BLACKMANOV	$\frac{12\pi}{N}$	58,1	$5{,}36\pi / \frac{N}{2}$

3.2-1 a) $a(m) = ?$ $h(n) = ?$



$$\omega = \frac{2\pi f}{f_s}$$

$$\omega_1 = \frac{2\pi f_1}{f_s} \qquad \omega_2 = \frac{2\pi f_2}{f_s}$$

pojasno propusni filtar koji propušta frekvencije od f_1 do f_2

$$a_m = \frac{1}{\pi} \left\{ \int_{-\omega_2}^{-\omega_1} \underbrace{A_d(\omega)}_{\text{konstantna } \bar{z}i = C} \cos(\omega m)\, d\omega + \int_{\omega_1}^{\omega_2} \underbrace{A_d(\omega)}_{\text{konstantna } \bar{z}i = C} \cos(\omega m)\, d\omega \right\}$$

$$= \frac{1}{\pi} \cdot C \left\{ \left. \frac{\sin(\omega m)}{m} \right|_{-\omega_2}^{-\omega_1} + \left. \frac{\sin(\omega m)}{m} \right|_{\omega_1}^{\omega_2} \right\}$$

$$= \frac{C}{m\pi} \left\{ \sin(-\omega_1 m) - \sin(-\omega_2 m) + \sin(\omega_2 m) - \sin(\omega_1 m) \right\}$$

$$= \frac{C}{m\pi} \left\{ -\sin(\omega_1 m) + \sin(\omega_2 m) + \sin(\omega_2 m) - \sin(\omega_1 m) \right\}$$

$$= \frac{2C}{m\pi} \left\{ \sin(\omega_2 m) - \sin(\omega_1 m) \right\}$$

$$h(n) = \begin{cases} \frac{1}{\pi} \cdot \frac{C}{\frac{N}{2} - n} \cdot \left(\sin\left(w_2 \cdot \left(\frac{N}{2} - n\right)\right) - \sin\left(w_1 \cdot \left(\frac{N}{2} - n\right)\right)\right), & 0 \leq n < \frac{N}{2} \\ \frac{C}{\pi} (w_2 - w_1), & n = \frac{N}{2} \\ \frac{1}{\pi} \cdot \frac{C}{n - \frac{N}{2}} \cdot \left(\sin\left(w_2 \cdot \left(n - \frac{N}{2}\right)\right) - \sin\left(w_1 \left(n - \frac{N}{2}\right)\right)\right), & \frac{N}{2} < n \leq N \end{cases}$$

$$a[0] = \frac{1}{2\pi} \left(\int_{-w_2}^{-w_1} C \, dw + \int_{w_1}^{w_2} C \, dw \right)$$

$$= \frac{1}{2\pi} \cdot C \left[\underbrace{(-w_1 + w_2) + (w_2 - w_1)}_{-2w_1 + 2w_2} \right] = \frac{C}{\pi} \cdot (w_2 - w_1)$$

3.2-2 a)

$$\omega_D = \frac{2\pi f_D}{f_s} = \frac{2\pi \cdot 22050}{44100} = \pi$$

$$\left| H_1(e^{j\omega}) + H_2(e^{j\omega}) + H_3(e^{j\omega}) + H_4(e^{j\omega}) \right| = 1$$

$H_1(e^{j\omega})$ $= a_0 + a_1 e^{-j\omega} + a_2 e^{-2j\omega} + \dots a_N e^{-Nj\omega}$

$$a_0 = \frac{1}{2\pi} \int_{-\omega_A}^{\omega_A} d\omega = \frac{1}{2\pi} \left\{ \omega_A + \omega_A \right\} = \frac{\omega_A}{\pi}$$

$$a_1 = \frac{1}{\pi} \int_{-\omega_A}^{\omega_A} \cos \omega \, d\omega$$

$$a_2 = \frac{1}{\pi} \int_{-\omega_A}^{\omega_A} \cos 2\omega \, d\omega$$

...

$$a_{N/2} = \frac{1}{\pi} \int_{-\omega_A}^{\omega_A} \cos \frac{N}{2}\omega \, d\omega$$

$H_2(e^{j\omega})$ $= a_0 + a_1 e^{-j\omega} + a_2 e^{-2j\omega} + \dots + a_N e^{-Nj\omega}$

$$a_0 = \frac{1}{2\pi} \left\{ \int_{-\omega_B}^{-\omega_A} d\omega + \int_{\omega_A}^{\omega_B} d\omega \right\} = \frac{1}{2\pi} \left\{ -\omega_A + \omega_B + \omega_B - \omega_A \right\} = \frac{\omega_B - \omega_A}{\pi}$$

$$a_1 = \frac{1}{\pi} \left\{ \int_{-\omega_B}^{-\omega_A} \cos \omega \, d\omega + \int \cos \omega \, d\omega \right\}$$

$$a_2 = \frac{1}{\pi} \left\{ \int_{-\omega_B}^{-\omega_A} \cos 2\omega \, d\omega + \int_{\omega_A}^{\omega_B} \cos 2\omega \, d\omega \right\}$$

...

$$a_{N/2} = \frac{1}{\pi} \left\{ \int_{-\omega_B}^{-\omega_A} \cos \frac{N}{2}\omega \, d\omega + \int_{\omega_A}^{\omega_B} \cos \frac{N}{2}\omega \, d\omega \right\}$$

$H_3(e^{j\omega})$ $= a_0 + a_1 e^{-j\omega} + a_2 e^{-2j\omega} + \dots + a_N e^{-Nj\omega}$

$$a_0 = \frac{\omega_C - \omega_B}{\pi}$$

$$a_1 = \frac{1}{\pi} \left\{ \int_{-\omega_C}^{-\omega_B} \cos \omega \, d\omega + \int_{\omega_B}^{\omega_C} \cos \omega \, d\omega \right\}$$

$$a_2 = \frac{1}{\pi} \left\{ \int_{-\omega_C}^{-\omega_B} \cos 2\omega \, d\omega + \int_{\omega_B}^{\omega_C} \cos \omega \, d\omega \right\}$$

...

$$a_{\frac{N}{2}} = \frac{1}{\pi}\left\{ \int_{-\omega_C}^{-\omega_B} \cos \tfrac{N}{2}\omega \, d\omega + \int_{\omega_B}^{\omega_C} \cos \tfrac{N}{2}\omega \, d\omega \right\}$$

$$H_4(e^{j\omega}) = a_0 + a_1 e^{-j\omega} + a_2 e^{-2j\omega} + \dots + a_N e^{-Nj\omega}$$

$$a_0 = \frac{\omega_D - \omega_C}{\pi}$$

$$a_1 = \frac{1}{\pi}\left\{ \int_{-\omega_D}^{-\omega_C} \cos \omega \, d\omega + \int_{\omega_C}^{\omega_D} \cos \omega \, d\omega \right\}$$

$$a_2 = \frac{1}{\pi}\left\{ \int_{-\omega_D}^{-\omega_C} \cos 2\omega \, d\omega + \int_{\omega_C}^{\omega_D} \cos \omega \, d\omega \right\}$$

...

$$a_{\frac{N}{2}} = \frac{1}{\pi}\left\{ \int_{-\omega_D}^{-\omega_C} \cos \tfrac{N}{2}\omega \, d\omega + \int_{\omega_C}^{\omega_D} \cos \tfrac{N}{2}\omega \, d\omega \right\}$$

$$H_1(e^{j\omega}) + H_2(e^{j\omega}) + H_3(e^{j\omega}) + H_4(e^{j\omega}) =$$

a_0 članovi: $\frac{\omega_A}{\pi} + \frac{\omega_B - \omega_A}{\pi} + \frac{\omega_C - \omega_B}{\pi} + \frac{\omega_D - \omega_C}{\pi} = \frac{\cancel{\omega_A} + \cancel{\omega_B} - \cancel{\omega_A} + \cancel{\omega_C} - \cancel{\omega_B} + \omega_D - \cancel{\omega_C}}{\pi} = \frac{\omega_D}{\pi}$

a_1 članovi: $\frac{1}{\pi}\left\{ \int_{-\omega_A}^{\omega_A} \cos \omega \, d\omega + \int_{-\omega_B}^{-\omega_A} \cos \omega \, d\omega + \int_{\omega_A}^{\omega_B} \cos \omega \, d\omega + \int_{-\omega_C}^{-\omega_B} \cos \omega \, d\omega + \right.$

$\left. \int_{\omega_B}^{\omega_C} \cos \omega \, d\omega + \int_{-\omega_D}^{-\omega_C} \cos \omega \, d\omega + \int_{\omega_C}^{\omega_D} \cos \omega \, d\omega \right\} = \frac{1}{\pi} \int_{-\omega_D = -\pi}^{\omega_D = \pi} \cos \omega \, d\omega = 0$

zbroj a_2 članova je također 0 zbog $\int_{-\pi}^{\pi} \cos 2\omega \, d\omega = 0$

zbroj $a_{\frac{N}{2}}$ članova je također 0 zbog $\int_{-\pi}^{\pi} \cos \tfrac{N}{2}\omega \, d\omega = 0$

ostaje samo $\frac{\omega_D}{\pi} = \frac{\pi}{\pi} = 1$ //

($\omega_D = \pi$ → piše na poč. zadatka)

Dodatno dodamo kašnjenje $e^{-\frac{N}{2}j\omega}$ jer radimo s kauzalnim filtrima.

$$\boxed{H_1(e^{j\omega}) + H_2(e^{j\omega}) + H_3(e^{j\omega}) + H_4(e^{j\omega}) = e^{\frac{N}{2}j\omega}}$$