asimptotski stabilan $|z|<1$
granično stabilan $|z|=1$
nestabilan $|z|>1$

Obrada informacije - Prva domaća zadaća

(10) Sustav je relacija (funkcija) koja ulaznom signalu (funkciji) pridružuje izlazni signal (funkciju).

Sustav f je linearan ako vrijedi:

$$f(a \cdot u) = a \cdot f(u)$$

$$f(u+v) = f(u) + f(v)$$

$$f(u) = a f_1(u) + b f_2(u)$$

a) $y(t) = 5u(t) \Rightarrow y(t) = a y_1(t) + b y_2(t)$ linearan sustav!

$y(t) = a \cdot 5u_1(t) + b \cdot 5u_2(t) = 5[a u_1(t) + b u_2(t)] = 5u(t)$ ✓ sustav je linearan

b) $y(t) = 5u(t) + 2$

$y(t) = a[5u_1(t) + 2] + b[5u_2(t) + 2] = 5a u_1(t) + 2a + 5b u_2(t) + 2b = 5[a u_1(t) + b u_2(t)] + 2(a+b)$

$= 5u(t) + 2(a+b) \neq 5u(t) + 2$ sustav nije linearan

c) $y(t) = 5u(t)^2$

$y(t) = a[5u_1(t)^2] + b[5u_2(t)^2] = 5a u_1(t)^2 + 5b u_2(t)^2 = 5[a u_1(t)^2 + b u_2(t)^2] \neq 5u(t)^2$

jer je $u(t)^2 = [a u_1(t) + b u_2(t)]^2 = a^2 u_1(t)^2 + 2ab u_1(t) u_2(t) + b^2 u_2(t)^2 \neq a u_1(t)^2 + b u_2(t)^2$

sustav nije linearan

d) $y(t) = 5t u(t) + 10t^2 u(t-2)$

$y(t) = a[5t u_1(t) + 10t^2 u_1(t-2)] + b[5t u_2(t) + 10t^2 u_2(t-2)]$

$= 5at u_1(t) + 10at^2 u_1(t-2) + 5bt u_2(t) + 10bt^2 u_2(t-2) = 5t[a u_1(t) + b u_2(t)] + 10t^2[a u_1(t-2) + b u_2(t-2)]$

$= 5t u(t) + 10t^2 u(t-2)$ ✓ sustav je linearan

(13) $X(\omega) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n} = DTFT[x[n]]$

vremenski diskretna Fourierova transformacija realnog niza $x[n]$

c) zadovoljava svojstvo $|X(\omega)|$ je parna funkcija od ω

$X(-\omega) = \sum_{n=-\infty}^{\infty} x[n] e^{j\omega n}$

$x^*(-\omega) = \sum_{n=-\infty}^{\infty} x^*[n] e^{-j\omega n}$

$e^{j\omega n} = \cos(\omega n) + j\sin(\omega n)$

$(e^{j\omega n})^* = \cos(\omega n) - j\sin(\omega n) = e^{-j\omega n}$

$e^{-j\omega n} = \cos(-\omega n) + j\sin(-\omega n) = \cos(\omega n) - j\sin(\omega n) = (e^{j\omega n})^*$ ✓

za realni niz $x[n]$ vrijedi $x^*[n] = x[n]$ pa slijedi:

$X^*(-\omega) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n} = X(\omega)$

$X(\omega) = Re\{X(\omega)\} + Im\{X(\omega)\} = |X(\omega)| e^{j\angle\{X(\omega)\}}$

$X^*(-\omega) = Re\{X(-\omega)\} - Im\{X(-\omega)\} = |X(-\omega)| e^{-j\{X(\omega)\}}$

amplitudni spektar $= \sqrt{Re^2 + Im^2}$

$|X(\omega)| = \sqrt{Re\{X(\omega)\}^2 + Im\{X(\omega)\}^2}$

$|X(-\omega)| = \sqrt{Re\{X(-\omega)\}^2 + Im\{X(-\omega)\}^2}$

fazni spektar $: tg\,\varphi = \frac{Im}{Re}$

$tg\,\varphi_1 = \frac{Im\{X(\omega)\}}{Re\{X(\omega)\}} = \angle\{X(\omega)\}$

$tg\,\varphi_2 = \frac{-Im\{X(-\omega)\}}{Re\{X(-\omega)\}} = -\angle\{X(-\omega)\}$

$X(\omega) = X^*(-\omega) \Rightarrow \sqrt{Re\{X(\omega)\}^2 + Im\{X(\omega)\}^2} = \sqrt{Re\{X(-\omega)\}^2 + Im\{X(-\omega)\}^2}$

parna funkcija ✓

(14) c) $x[n] = \delta[n+1] - 2\delta[n] + \delta[n-1]$

odredi DTFT

skiciraj pripadnu amplitudnu i faznu karakteristiku

$$X(\omega) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n} = \sum_{n=-\infty}^{\infty} (\delta[n+1] - 2\delta[n] + \delta[n-1]) e^{-j\omega n}$$

$$= \sum_{n=-\infty}^{\infty} \delta[n+1] e^{-j\omega n} - 2\sum_{n=-\infty}^{\infty} \delta[n] e^{-j\omega n} + \sum_{n=-\infty}^{\infty} \delta[n-1] e^{-j\omega n} = e^{-j\omega(-1)} - 2e^{-j\omega \cdot 0} + e^{-j\omega \cdot 1}$$

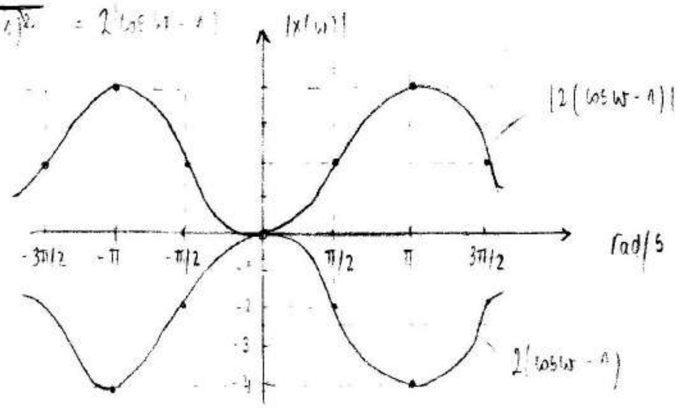
$$= e^{j\omega} - 2 \cdot e^{0} + e^{-j\omega} = e^{j\omega} + e^{-j\omega} - 2 \cdot 1 = \cos\omega + j\sin\omega + \cos(-\omega) + j\sin(-\omega) - 2 =$$

$$= \cos\omega + \cancel{j\sin\omega} + \cos\omega - \cancel{j\sin\omega} - 2 = 2\cos\omega - 2 = 2(\cos\omega - 1)$$

amplitudna karakteristika:

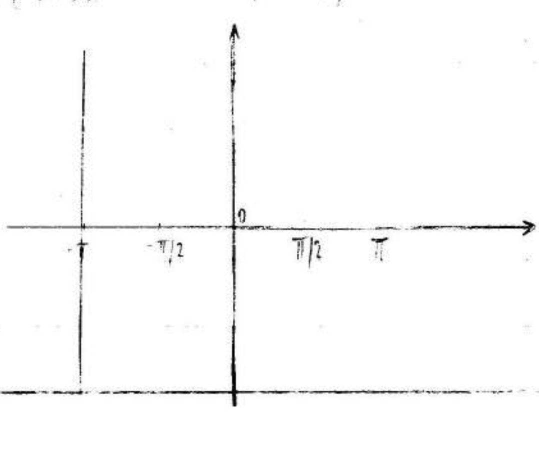
$$|X(\omega)| = \sqrt{Re\{X(\omega)\}^2 + Im\{X(\omega)\}^2} = \sqrt{(2\cos\omega - 2)^2 + 0^2} = \sqrt{4\cos^2\omega - 8\cos\omega + 4} = \sqrt{4(\cos^2\omega - 2\cos\omega + 1)}$$

$$= 2\sqrt{(\cos\omega - 1)^2} = 2|\cos\omega - 1|$$



fazna karakteristika:

$$\angle\{X(\omega)\} = \arctan\frac{Im\{X(\omega)\}}{Re\{X(\omega)\}} = \arctan\frac{0}{2(\cos\omega - 1)} = \arctan 0 = -\pi$$



(11) a) $H(z) = \frac{1-r}{1+r} \cdot \frac{1-rz^{-1}}{1+rz^{-1}}$ [illegible]

odredi položaj polova i nula u z ravnini

ispitaj stabilnost sustava

ispitaj postoje li vrijednosti parametra r koje zadani sustav čine minimalno-faznim

mule: $1 - rz^{-1} = 0$

$1 - \frac{r}{z} = 0 \;/ \cdot z$

$z - r = 0$

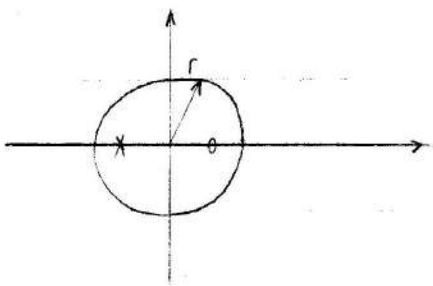
$z = r$

poloni: $1 + rz^{-1} = 0$

$1 + \frac{r}{z} = 0 \;/ z$

$z + r = 0$

$z = -r$



sustav je stabilan jer su poloni i mule unutar jedinične kružnice !

=> sustav s nulama unutar jedinične kružnice naziva se sustav s minimalnom fazom

to $0 < r < 1$ je sustav minimalno fazni

(13) c) $u[n] = \frac{1}{h[0]}\left\{ y[n] - \sum_{i=1}^{n} u[n-k]h[i] \right\}$ $\qquad H(z) = \frac{Y(z)}{U(z)} \Rightarrow U(z) = \frac{Y(z)}{H(z)}$

$y[n] = \{1, 1, 2, 1, 2, 1, 1\}$, $h[n] = \{1, -1, 1\}$

odredi $u[n]$ prema [illegible] i dijeljenjem [illegible] u z domeni

$u[0] = 1 - \{u[0-1]h[1]\} = 1$

$u[1] = 1 - \{u[1-1]h[1]\} = 1 - u[0]h[1] = 1 - 1\cdot(-1) = 1+1 = 2$

$u[2] = 2 - \{u[2-1]h[1] + u[2-2]h[2]\} = 2 - u[1]h[1] - u[0]h[2] = 2 - 2\cdot(-1) - 1\cdot 1 = 2+2-1 = 3$

$u[3] = 1 - \{u[2]h[1] + u[1]h[2] + u[0]h[3]\} = 1 - 3\cdot(-1) - 2\cdot 1 - 1\cdot 0 = 1+3-2 = 2$

$u[4] = 2 - \{u[3]h[1] + u[2]h[2] + u[1]h[3] + u[0]h[4]\} = 2 - 2\cdot(-1) - 3\cdot 1 - 2\cdot 0 - 1\cdot 0 = 2+2-3 = 1$

$u[5] = 1 - \{u[4]h[1] + u[3]h[2] + u[2]h[3] + u[1]h[4] + u[0]h[5]\} = 1 - 1\cdot(-1) - 2\cdot 1 - 3\cdot 0 - 2\cdot 0 - 1\cdot 0 = 1+1-2 = 0$

$u[6] = 1 - \{u[5]h[1] + u[4]h[2] + u[3]h[3] + u[2]h[4] + u[1]h[5] + u[0]h[6]\} = 1 - 0 - 1 - 0 - 0 - 0 - 0 = 0$

$u[7] = 0 - \{u[6]h[1] + u[5]h[2] + u[4]h[3] + u[3]h[4] + u[2]h[5] + u[1]h[6] + u[0]h[7]\} = 0$

$\vdots$

sve dalje su nule $\Rightarrow u[n] = \{1, 2, 3, 2, 1\}$

dijeljenje polinoma u z domeni

$U(z) = \frac{Y(z)}{H(z)} = \frac{1\cdot z^{0} + 1\cdot z^{-1} + 2\cdot z^{-2} + 1\cdot z^{-3} + 2\cdot z^{-4} + 1\cdot z^{-5} + 1\cdot z^{-6}}{1\cdot z^{0} + (-1)\cdot z^{-1} + 1\cdot z^{-2}} = \frac{1 + z^{-1} + 2z^{-2} + z^{-3} + 2z^{-4} + z^{-5} + z^{-6}}{1 - z^{-1} + z^{-2}}$

$(1 + z^{-1} + 2z^{-2} + z^{-3} + 2z^{-4} + z^{-5} + z^{-6}) : (1 - z^{-1} + z^{-2}) = 1 + 2z^{-1} + 3z^{-2} + 2z^{-3} + z^{-4}$

$\underline{-1 + z^{-1} - z^{-2}}$

$2z^{-1} + z^{-2} + z^{-3}$

$\underline{-2z^{-1} + 2z^{-2} - 2z^{-3}}$

$3z^{-2} - z^{-3} + 2z^{-4}$

$\underline{-3z^{-2} + 3z^{-3} - 3z^{-4}}$

$2z^{-3} - z^{-4} + z^{-5}$

$\underline{-2z^{-3} + 2z^{-4} - 2z^{-5}}$

$z^{-4} - z^{-5} + z^{-6}$

$\underline{-z^{-4} + z^{-5} - z^{-6}}$

0 ✓

$U(z) = 1 + 2z^{-1} + 3z^{-2} + 2z^{-3} + z^{-4}$

$\Rightarrow u[n] = \{1, 2, 3, 2, 1\}$ ✓

(24.) a) definirajte IDFT$_N$ transformaciju

odredite i skicirajte IDFT$_N$ spektra

$X[k] = \{2, 1, 0, 1\} \quad (N = 4)$

Definicija inverzne diskretne Fourierove transformacije

$$x[m] = \frac{1}{N} \sum_{k=0}^{N-1} X[k]\, e^{j \frac{2\pi}{N} mk}$$

$$x[m] = \frac{1}{4}\left[2e^{j \frac{\cancel{2}\pi}{\cancel{4}_2} m\cdot 0} + 1e^{j \frac{\cancel{2}\pi}{\cancel{4}_2} m\cdot 1} + \underbrace{0e^{j \frac{2\pi}{4} m\cdot 2}}_{0} + 1e^{j \frac{\cancel{2}\pi}{\cancel{4}_2} m\cdot 3}\right] = \frac{1}{4}\left[2\cdot 1 + e^{j\frac{m\pi}{2}} + e^{j\frac{3m\pi}{2}}\right]$$

$$= \frac{1}{2} + \frac{1}{4} e^{j\frac{m\pi}{2}} + \frac{1}{4} e^{j\frac{3m\pi}{2}} = \frac{1}{2} + \frac{1}{4}\cos\left(\frac{m\pi}{2}\right) + \frac{1}{4} j \sin\left(\frac{m\pi}{2}\right) + \frac{1}{4}\cos\left(\frac{3m\pi}{2}\right) + \frac{1}{4} j \sin\left(\frac{3m\pi}{2}\right)$$

$$= \frac{1}{2} + \frac{1}{4}\cos\left(\frac{m\pi}{2}\right) + \frac{1}{4}\cos\left(\frac{3m\pi}{2}\right) + j\left[\frac{1}{4}\sin\left(\frac{m\pi}{2}\right) + \frac{1}{4}\sin\left(\frac{3m\pi}{2}\right)\right]$$

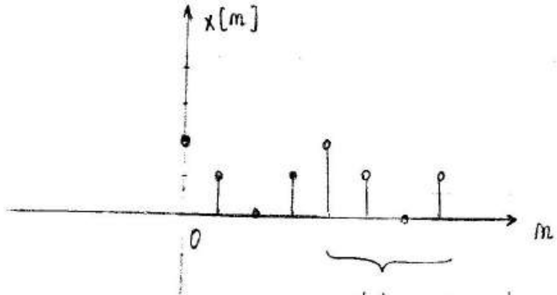
$$x[0] = \frac{1}{2} + \frac{1}{4}\cdot 1 + \frac{1}{4}\cdot 1 + j\left[\frac{1}{4}\cdot 0 + \frac{1}{4}\cdot 0\right] = \frac{1}{2} + \frac{2}{4} = 1$$

$$x[1] = \frac{1}{2} + \frac{1}{4}\cdot 0 + \frac{1}{4}\cdot 0 + j\left[\frac{1}{4}\cdot 1 + \frac{1}{4}\cdot(-1)\right] = \frac{1}{2} + j\left[\cancel{\frac{1}{4}} - \cancel{\frac{1}{4}}\right] = \frac{1}{2}$$

$$x[2] = \frac{1}{2} + \frac{1}{4}\cdot(-1) + \frac{1}{4}\cdot(-1) + j\left[\frac{1}{4}\cdot 0 + \frac{1}{4}\cdot 0\right] = \frac{1}{2} - \frac{1}{4} - \frac{1}{4} + 0 = \frac{1}{2} - \frac{2}{4} = 0$$

$$x[3] = \frac{1}{2} + \frac{1}{4}\cdot 0 + \frac{1}{4}\cdot 0 + j\left[\frac{1}{4}\cdot(-1) + \frac{1}{4}\cdot 1\right] = \frac{1}{2} + j\left[\cancel{-\frac{1}{4}} + \cancel{\frac{1}{4}}\right] = \frac{1}{2}$$

$$\Rightarrow x[m] = \left\{\underline{1}, \frac{1}{2}, 0, \frac{1}{2}\right\}$$



ostalo nisu nule nego se ponavlja

odnosno $x[4], x[5], x[6], \ldots$ nije sve 0, već redom opet $1, \frac{1}{2}, 0, \ldots$