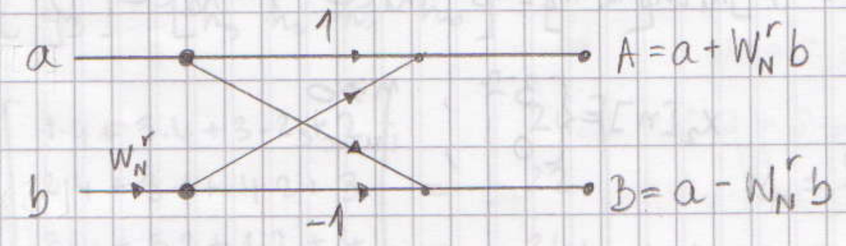
OBRADA INFORMACIJA - 2008/2009 NATALIJA NAJHART

2, 4b, 5, 6b, 7, 9 0015027891

1

DOMAĆA ZADAĆA 2.

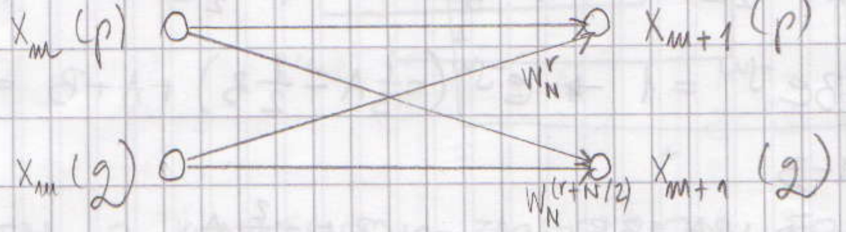
2. Nacrtajte graf toka signala za diskretnu Fourierovu transformaciju u dvije točke (DFT_2). Što je DFT-leptir?



} pojednostavljen DFT-leptir račun

DVA ULAZA I DVA IZLAZA.

Naziv ovakve strukture je LEPTIR. Tako da se graf zove DFT-leptir. To je način izračunavanja DFT-a.
DFT LEPTIR = DFT od 2 uzorka.



} DFT_2 - temeljni DFT-leptir račun

4.

LINEARNA KONVOLUCIJA: $x[n] * y[n] = \sum_{k=-\infty}^{n} x[n-k]\, y[k]$

b) $x[n] = \{\underline{1}, 2, 3, 4\}$, $y[n] = \{\underline{4}, 3, 2, 1\}$ $u[n] = x[n] * y[n]$

$n=0$ $u[0] = x[0] \cdot y[0] = 4$

$n=1$ $u[1] = x[1] \cdot y[0] + x[0] \cdot y[1] = 2 \cdot 4 + 1 \cdot 3 = 11$

$n=2$ $u[2] = x[2] \cdot y[0] + x[1] \cdot y[1] + x[0] \cdot y[2] = 3 \cdot 4 + 2 \cdot 3 + 1 \cdot 2 = 20$

$n=3$ $u[3] = x[3] \cdot y[0] + x[2] \cdot y[1] + x[1] \cdot y[2] + x[0] \cdot y[3] = 4 \cdot 4 + 3 \cdot 3 + 2 \cdot 2 + 1 \cdot 1 = 30$

$n=4$ $u[4] = \underbrace{\cancel{x[4] \cdot y[0]}}_{=0} + x[3] \cdot y[1] + x[2] \cdot y[2] + x[1] y[3] + x[0] \cdot y[4] = 12 + 6 + 2 = 20$

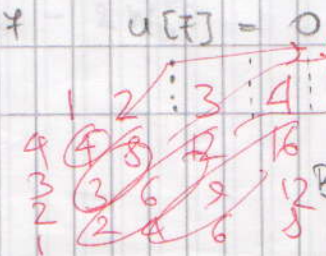
$n=5$ $u[5] = \cancel{x[5] \cdot y[0]} + \cancel{x[4] \cdot y[1]} + x[3] \cdot y[2] + x[2] \cdot y[3] + \cancel{x[1] \cdot y[4]} + \cancel{x[0] \cdot y[5]} = 11$

$n=6$ $u[6] = x[3] \cdot y[3] = 4$

$n=7$ $u[7] = 0$

⋮

LAKŠE TOČNO



$u[n] = \{\underline{4}, 11, 20, 30, 20, 11, 4\}$

BROJ UZORAKA ≠ 0 : $N = L_x + L_y - 1 = 4 + 4 - 1 = 7$

2

(5.) KONVOLUCIJA ZA VREMENSKI DISKRETNU FOURIEROVU TRANSFORMACIJU: $x_1[n] * x_2[n] \multimap X_1[Z] \cdot X_2[Z]$

$$x_1[n] * x_2[n] = X_1(e^{j\omega}) \cdot X_2(e^{j\omega})$$

$$x_1[n] = \begin{cases} 2^{-n}, & n \geq 0 \\ 0, & \text{inače} \end{cases} \qquad x_2[n] = \begin{cases} 3^{-n}, & n \geq 0 \\ 0, & \text{inače} \end{cases}$$

$$a^n \mu[n] \multimap \frac{1}{1-ae^{-j\omega}}, \quad |a|<1$$

$$X_1(e^{j\omega}) = \frac{1}{1-\frac{1}{2}e^{-j\omega}}, \quad X_2(e^{j\omega}) = \frac{1}{1-\frac{1}{3}e^{-j\omega}}$$

$$X_1(e^{j\omega}) \cdot X_2(e^{j\omega}) = \frac{1}{1-\frac{1}{2}e^{-j\omega}} \cdot \frac{1}{1-\frac{1}{3}e^{-j\omega}} = \frac{A}{1-\frac{1}{2}e^{-j\omega}} + \frac{B}{1-\frac{1}{3}e^{-j\omega}}$$

$$A - \frac{1}{3}Ae^{-j\omega} + B - \frac{1}{2}Be^{-j\omega} = 1 \rightarrow e^{-j\omega}\left(-\frac{1}{3}A - \frac{1}{2}B\right) + A + B = 1$$

$A+B=1 \qquad A = 1-B$

$\frac{1}{3}A + \frac{1}{2}B = 0 \,/\cdot 6 \rightarrow 2A + 3B = 0 \rightarrow B = -\frac{2}{3}A$

$$B = -\frac{2}{3}(1-B) = -\frac{2}{3} + \frac{2}{3}B$$

$$\frac{1}{3}B = -\frac{2}{3} \rightarrow B = -2$$

$$A = 3$$

$$X(e^{j\omega}) = \frac{3}{1-\frac{1}{2}e^{-j\omega}} + \frac{2}{1-\frac{1}{3}e^{-j\omega}}$$

$$x[n] = (3 \cdot 2^{-n} + 2 \cdot 3^{-n})\,\mu[n]$$

(6.) b) $x[n] = \{\underline{1}, 2, 3, 4\}$, $y[n] = \{\underline{4}, 3, 2, 1\}$

Cirkularnom ili kružnom konvolucijom rativamo 1 periodu periodične konvolucije:

$$y[n] = \sum_{j=0}^{N-1} u(j) \cdot h(\mathrm{mod}[n-j])$$

$$y_n = \sum_{k=0}^{N-1} h_{\mathrm{mod}(n-k)} \cdot u_k$$

$$\begin{array}{cccc} 0 & 1 & 2 & 3 \\ \{\underline{1}, & 2, & 3, & 4\} \end{array}$$

$$Y_N = \begin{bmatrix} y_0 \\ y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} h_0 & h_3 & h_2 & h_1 \\ h_1 & h_0 & h_3 & h_2 \\ h_2 & h_1 & h_0 & h_3 \\ h_3 & h_2 & h_1 & h_0 \end{bmatrix} \cdot \begin{bmatrix} u_0 \\ u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} 1 & 4 & 3 & 2 \\ 2 & 1 & 4 & 3 \\ 3 & 2 & 1 & 4 \\ 4 & 3 & 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} 4 \\ 3 \\ 2 \\ 1 \end{bmatrix} =$$

$$= \begin{bmatrix} 1\cdot 4 + 3\cdot 4 + 3\cdot 2 + 2 \\ 2\cdot 4 + 3\cdot 1 + 4\cdot 2 + 3 \\ 3\cdot 4 + 3\cdot 2 + 1\cdot 2 + 4 \\ 4\cdot 4 + 3\cdot 3 + 2\cdot 2 + 1 \end{bmatrix} = \begin{bmatrix} 24 \\ 22 \\ 24 \\ 30 \end{bmatrix} \qquad Y_m = \{\underline{24}, 22, 24, 30\}$$

USPOREDBA SA ZADATKOM 4. : BROJ UMNOŽAKA JE MANJI ZA 3.

$$Y_n = \{\underline{24}, 22, 24, 30\}$$

$$u[n] = \{\underline{4}, 11, 20, 30, 20, 11, 4\} \rightarrow \{24, 22, 24, 30\}$$

$+ \quad + \quad +$

7. TEOREM O KONVOLUCIJI ZA DISKRETNU FOURIEROVU TRANSFORM U N TOČAKA.

$$X[k] = \sum x(n) e^{2j\pi kn\frac{1}{N}} \qquad Y[k] = \sum y(n) \cdot e^{2j\pi kn\frac{1}{N}} \qquad X(k)\cdot Y(k) = U(k)$$

$$x[n] = \{\underline{2}, 1, 0, 1\} \qquad y[n] = \{\underline{2}, -1, 0, -1\}$$

$$X[k] = 2e^{2j\pi\frac{0\cdot k}{4}} + 1\cdot e^{2j\pi\frac{1\cdot k}{4}} + 0\cdot e^{2j\pi\frac{2\cdot k}{4}} + 1\, e^{2j\pi\frac{3\cdot k}{4}}$$

$$= 2\, W_4^{0k} + 1\cdot W_4^{1k} + 0\cdot W_4^{2k} + 1\cdot W_4^{3k}$$

$$Y[k] = 2\, W_4^{0k} - 1\, W_4^{1k} + 0\cdot W_4^{2k} - 1\, W_4^{3k}$$

$$X[k]\cdot Y[k] = \left(2 + W_4^{1k} + W_4^{3k}\right)\left(2 - W_4^{1k} - W_4^{3k}\right) =$$

$$= 4 + \cancel{2W_4^{1k}} + \cancel{2W_4^{3k}} - \cancel{2W_4^{1k}} - W_4^{1k}\cdot W_4^{1k} - W_4^{3k} W_4^{1k} - \cancel{2W_4^{3k}} - W_4^{1k} W_4^{3k} - W_4^{3k} W_4^{3k}$$

$$= 4 - W_4^{1k}\cdot W_4^{1k} - 2\, W_4^{3k} W_4^{1k} - W_4^{3k} W_4^{3k} = 4 - W_4^{2k} - 2\, W_4^{4k} - W_4^{6k}$$

$$= 2 - 2\, W_4^{2k}$$

$$u(n) = \{\overset{0}{2}, \overset{1}{0}, \overset{2}{-2}, \overset{3}{0}\}$$

9. $x_1[n] = \{\overset{0}{\underline{5}}, \overset{1}{4}, \overset{2}{3}, \overset{3}{2}, \overset{4}{1}\}$, $x_2[n] = \{\overset{0}{\underline{1}}, \overset{1}{2}, \overset{2}{3}, \overset{3}{4}, \overset{4}{5}\}$

$x_1[n] * x_2[n] \Rightarrow$ KONVOLUCIJA:

$n=0, \quad x[0] = 5 \cdot 1 = 5$

$n=1, \quad x[1] = 4 \cdot 1 + 5 \cdot 2 = 14$

$n=2, \quad x[2] = 3 \cdot 1 + 4 \cdot 2 + 5 \cdot 3 = 26$

$n=3, \quad x[3] = 2 \cdot 1 + 3 \cdot 2 + 4 \cdot 3 + 5 \cdot 4 = 40$

$n=4, \quad x[4] = 1 \cdot 1 + 2 \cdot 2 + 3 \cdot 3 + 4 \cdot 4 + 5 \cdot 5 = 55$

$n=5, \quad x[5] = 1 \cdot 2 + 2 \cdot 3 + 3 \cdot 4 + 4 \cdot 5 = 40$

$n=6, \quad x[6] = 1 \cdot 3 + 8 + 15 = 26$

$n=7, \quad x[7] = 1 \cdot 4 + 2 \cdot 5 = 14$

$n=8, \quad x[8] = 1 \cdot 5 = 5$

$$\begin{bmatrix} y_0 \\ y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} = \begin{bmatrix} 5 & 1 & 2 & 3 & 4 \\ 4 & 5 & 1 & 2 & 3 \\ 3 & 4 & 5 & 1 & 2 \\ 2 & 3 & 4 & 5 & 1 \\ 1 & 2 & 3 & 4 & 5 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{bmatrix} = \begin{bmatrix} 5+2+6+12+20 \\ 4+10+3+8+15 \\ 3+8+15+4+10 \\ 2+6+12+20+5 \\ 1+4+9+16+25 \end{bmatrix} = \begin{bmatrix} 45 \\ 40 \\ 40 \\ 45 \\ 55 \end{bmatrix}$$

Linearnom konvolucijom dobiti samo 9 članova. Ako i cirkularnom konvolucijom želimo dobiti 9 članova onda oba signala moramo proširiti. Bitno je da prvom dodamo N−1 nula i onda još nadopunimo nulama ako je potrebno do N=9.

$x_1[n] = \{\underline{5}, 4, 3, 2, 1, 0, 0, 0, 0\}$ $x_2[n] = \{\underline{1}, 2, 3, 4, 5, 0, 0, 0, 0\}$

$$\begin{bmatrix} y_0 \\ y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \\ y_6 \\ y_7 \\ y_8 \\ y_9 \end{bmatrix} = \begin{bmatrix} 5 & 0 & 0 & 0 & 0 & 1 & 2 & 3 & 4 \\ 4 & 5 & 0 & 0 & 0 & 0 & 1 & 2 & 3 \\ 3 & 4 & 5 & 0 & 0 & 0 & 0 & 1 & 2 \\ 2 & 3 & 4 & 5 & 0 & 0 & 0 & 0 & 1 \\ 1 & 2 & 3 & 4 & 5 & 0 & 0 & 0 & 0 \\ 0 & 1 & 2 & 3 & 4 & 5 & 0 & 0 & 0 \\ 0 & 0 & 1 & 2 & 3 & 4 & 5 & 0 & 0 \\ 0 & 0 & 0 & 1 & 2 & 3 & 4 & 5 & 0 \\ 0 & 0 & 0 & 0 & 1 & 2 & 3 & 4 & 5 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 5+0 \\ 4+10 \\ 3+12+15 \\ 2+6+12+20 \\ 1+4+9+16+25 \\ 2+6+12+20 \\ 3+8+15 \\ 4+10 \\ 5 \end{bmatrix} = \begin{bmatrix} 5 \\ 14 \\ 30 \\ 40 \\ 55 \\ 40 \\ 26 \\ 14 \\ 5 \end{bmatrix}$$

10c), 11c), 12, 13b), 14g, 15b, 15f

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10. c) HANNOV VREMENSKI OTVOR

$$w[k] = 0.5 + 0.5 \cdot \cos\left(\frac{2\pi \cdot k}{N+1}\right) \qquad -\frac{N}{2} \le k \le \frac{N}{2}$$

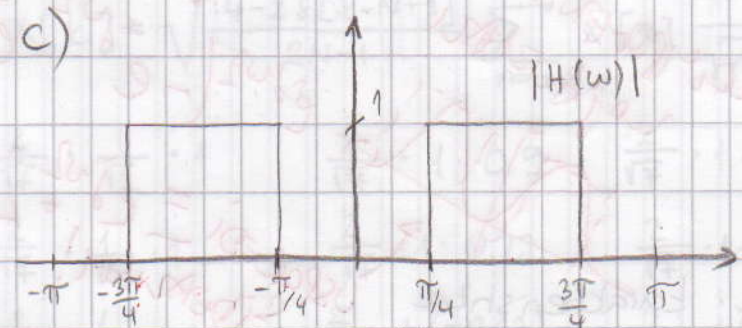
ŠIRINA GLAVNE LATICE : $8\pi / N$

GUŠENJE PRVE BOČNE LATICE : 31.5

ŠIRINA PRIJELAZNOG PODRUČJA : $\frac{3{,}11\pi}{N/2}$

Što je broj uzoraka otvora veći to je širina glavne latice manja.

11. c)



$$w[n] = \begin{cases} 1, & -\frac{N-1}{2} \le n \le \frac{N-1}{2} \\ 0, & \text{inače} \end{cases}$$

$$x[k] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{jw}) e^{jw}\, dw \leftarrow \text{IDFT}$$

$$|H(w)| = \begin{cases} 1, & \left(-\frac{3\pi}{4}, -\frac{\pi}{4}\right) \cup \left(\frac{\pi}{4}, \frac{3\pi}{4}\right) \\ 0, & \text{inače} \end{cases}$$

$$X[k] = \frac{1}{2\pi}\int_{-\frac{3\pi}{4}}^{-\frac{\pi}{4}} 1 \cdot e^{jwk}\, dw + \frac{1}{\frac{\pi}{2}}\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} 1 \cdot e^{jwk}\, dw = \frac{1}{2\pi}\left[\frac{e^{jwk}}{jk}\Big|_{-\frac{3\pi}{4}}^{-\frac{\pi}{4}} + \frac{e^{jwn}}{jk}\Big|_{\frac{\pi}{4}}^{\frac{3\pi}{4}}\right]$$

$$= \frac{1}{2\pi jk}\left(e^{-\frac{\pi}{4}jk} - e^{-\frac{3\pi}{4}jk} + e^{\frac{3\pi}{4}jk} - e^{\frac{\pi}{4}jk}\right) = \frac{1}{k}\left(\sin\frac{3\pi}{4}k - \sin\frac{\pi}{4}k\right)$$

$$\lim_{k \to \cancel{\infty}} \frac{\sin k}{k} = 1$$

PAZI NA NULU!!

$X[0] = \cancel{0}$ S.V. ili D.C.

$X[1] = 0$

$X[2] = -1$

$X[3] = 0$

$X[4] = 0$

$\frac{1}{2\pi} \cdot \left(1 \cdot \frac{\pi}{2} + 1 \cdot \frac{\pi}{2}\right)$

$X[-1] = 0$

$X[-2] = -1$

$X[-3] = 0$

$N = 5$

$$w[n] = \begin{cases} 1, & -2 \le n \le 2 \\ 0, & \text{inače} \end{cases}$$

$y[-2] = -1,\ y[-1] = 0,\ \cancel{y[0] = 0},\ y[1] = 0,\ y[2] = -1$ ne!

$$y[n] = -\delta[n-2] - \delta[n+2]$$

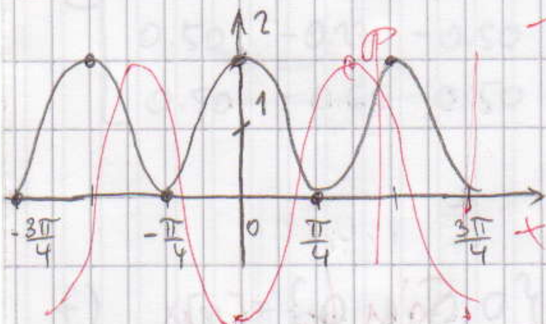
6

$y[0]=-1,\ y[1]=0,\ y[2]=0,\ y[3]=0,\ y[4]=-1$

$$Y(e^{j\omega})=\sum_{k=-\infty}^{\infty}x[n]e^{-j\omega k}=-1e^{-j\omega 0}-1\cdot e^{-j\omega 4}=-1-e^{-j\omega 4}$$

$$=-1-[\cos(4\omega)-j\sin(4\omega)]=-1-\cos(4\omega)+j\sin(4\omega)$$

$$|Y(e^{j\omega})|=\sqrt{(-1-\cos(4\omega))^2+\sin^2(4\omega)}=\sqrt{1+2\cos 4\omega+1}=\sqrt{2+2\cos 4\omega}$$



+ de komponenta koja fali

$e^{j\omega 2}(-e^{+j\omega 2}-e^{-j\omega 2})$

usporedi s izlazom!

$= e^{j\omega 2}\cdot 2\cos$

(12.) $A_s(\omega)$ - idealna ampl. karakteristika

$A(\omega)$ - stvarna ampl. karakteristika

p-udaljenost $\sqrt[p]{\int (A(\omega)-A_s(\omega))^p d\omega}$ → ČEBIŠEVA UDALJENOST

$p=1$ $\int (A(\omega)-A_s(\omega))d\omega$ → MANHATAN UDALJENOST
- apsolutna pogreška

$p=2$ $\sqrt{\int (A(\omega)-A_s(\omega))^2 d\omega}$ → EUKLIDSKA UDALJENOST
- kvadratna pogreška

$W(e^{j\omega})$ - težinska funkcija

(13.) $A(\omega)=\begin{cases}1, & -\frac{\pi}{4}<\omega<\frac{\pi}{4}\\ 0, & \text{inače}\end{cases}$

$$a[m]=a_m=\frac{1}{\pi}\int_{-\pi}^{\pi}A(\omega)\cos\omega n\,d\omega$$

$$a[0]=\frac{a_0}{2}=\frac{1}{2\pi}\int_{-\pi}^{\pi}A(\omega)\,d\omega$$

$$a_m=\frac{1}{\pi}\int_{-\frac{\pi}{4}}^{\pi/4}\cos\omega n\,d\omega=\frac{1}{\pi}\left.\frac{\sin\omega n}{n}\right|_{-\frac{\pi}{4}}^{\frac{\pi}{4}}=\frac{1}{\pi n}\left(\sin\frac{\pi}{4}n-\sin\left(-\frac{\pi}{4}n\right)\right)$$

$$=\frac{2}{\pi n}\sin\frac{\pi}{4}n$$

$$a[0] = \frac{a_0}{2} = \frac{1}{2\pi} \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} 1\, d\omega = \frac{1}{2\pi}\, \omega \Big|_{-\frac{\pi}{4}}^{\frac{\pi}{4}} = \frac{1}{2\pi}\left(\frac{\pi}{4} + \frac{\pi}{4}\right) = \frac{1}{4}$$

$$a[1] = \frac{\sqrt{2}}{\pi} \qquad a[2] = \frac{1}{\pi}$$

$$H(\omega) = e^{-\frac{j\omega 5}{2}}\left(\frac{1}{2} + \frac{\sqrt{2}}{\pi}\cos\omega + \frac{1}{\pi}\cos(2\omega)\right)$$

FALI IMPULSNI ODZIV!!!

$$h[n] = \left\{ \frac{1}{2\pi}, \frac{\sqrt{2}}{2\pi}, \frac{1}{4}, \frac{\sqrt{2}}{2\pi}, \frac{1}{2\pi} \right\}$$

14. g) DCT-VII $N=4$, $x[n] = \{1, 0, 0, 0\}$

$$X[k] = \sqrt{\frac{4-2\delta[k-N+1]}{2N-1}} \sum_{n=0}^{N-1} x[n] \frac{1}{\sqrt{1+\delta[n]}} \cos\frac{n(2k+1)\pi}{2N-1}$$

$$\begin{bmatrix} \frac{2}{\sqrt{7}}\cdot\frac{1}{\sqrt{2}}\cdot 1 & \frac{2}{\sqrt{7}}\cdot 1\cdot 0.9 & \frac{2}{\sqrt{7}}\cdot 1\cdot 0.62 & \frac{2}{\sqrt{7}}\cdot 1\cdot 0,22 \\ \frac{2}{\sqrt{7}}\cdot\frac{1}{\sqrt{2}}\cdot 1 & \frac{2}{\sqrt{7}}\cdot 1\cdot 0,22 & \frac{2}{\sqrt{7}}\cdot 1\cdot(-0.9) & \frac{2}{\sqrt{7}}\cdot 1\cdot(-0.62) \\ \frac{2}{\sqrt{7}}\cdot\frac{1}{\sqrt{2}}\cdot 1 & \frac{2}{\sqrt{7}}\cdot 1(-0.62) & \frac{2}{\sqrt{7}}\cdot 1(-0.22) & \frac{2}{\sqrt{7}}\cdot 1\cdot(0.9) \\ \frac{\sqrt{2}}{\sqrt{7}}\cdot\frac{1}{\sqrt{2}}\cdot 1 & \frac{\sqrt{2}}{\sqrt{7}}\cdot 1\cdot(-1) & \frac{\sqrt{2}}{\sqrt{7}}\cdot 1\cdot 1 & \frac{\sqrt{2}}{\sqrt{7}}\cdot 1\cdot(-1) \end{bmatrix}$$

$$\begin{bmatrix} 0.53 & 0.68 & 0.47 & 0.17 \\ 0.53 & 0.17 & -0.68 & -0.47 \\ 0.53 & -0.47 & -0.17 & 0.68 \\ 0.38 & -0.53 & 0.53 & -0.53 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0.53 \\ 0.53 \\ 0.53 \\ 0.38 \end{bmatrix}$$

15. b) $x[n] = \left\{ \frac{1}{2}\sqrt{1+\sqrt{2}/2}, \frac{1}{2}\sqrt{1-\sqrt{2}/2}, -\frac{1}{2}\sqrt{1-\sqrt{2}/2}, -\frac{1}{2}\sqrt{1+\sqrt{2}/2} \right\}$

DCT: $X(k) = w_k \cdot \sum_{n=0}^{N-1} x(n)\cdot\cos\frac{\pi}{N}\left(n+\frac{1}{2}\right)k \qquad w_k = \begin{cases} \sqrt{1/N} & k=0 \\ \sqrt{2/N} & k\neq 0 \end{cases}$ između 2 uzorka

Za $N=4$ je matrica transformacije dimenzija 4×4, koja u n-tom stupcu i k-tom retku ima element:

$$\begin{bmatrix} \sqrt{1/N}\ x(n) & \sqrt{1/N}\ x(n) & \sqrt{1/N}\ x(n) & \sqrt{1/N}\ x(n) \\ \sqrt{2/N}\ x(n)\cos\frac{\pi}{2N} & \sqrt{2/N}\ x(n)\frac{\pi}{N}\frac{3}{2} & \sqrt{2/N}\ x(n)\frac{\pi}{N}\frac{5}{2} & \sqrt{2/N}\ x(n)\frac{\pi}{N}\cdot\frac{7}{2} \\ \sqrt{2/N}\ x(n)\cos\frac{\pi}{N}2 & \sqrt{2/N}\ x(n)\frac{\pi}{N}3 & \sqrt{2/N}\ x(n)\frac{\pi}{N}5 & \sqrt{2/N}\ x(n)\frac{\pi}{N}7 \\ \sqrt{2/N}\ x(n)\cos\frac{\pi}{N}3 & \sqrt{2/N}\ x(n)\frac{\pi}{N}\frac{9}{2} & \sqrt{2/N}\ x(n)\frac{\pi}{N}\frac{15}{2} & \sqrt{2/N}\ x(n)\frac{\pi}{N}\frac{21}{2} \end{bmatrix}$$

$DCT_4[x[n]]$

$$\begin{bmatrix} X[0] \\ X[1] \\ X[2] \\ X[3] \end{bmatrix} = \begin{bmatrix} 0.5 & 0.5 & 0.5 & 0.5 \\ 0.65 & 0.27 & -0.27 & -0.65 \\ 0.50 & -0.50 & -0.50 & 0.50 \\ 0.27 & -0.65 & 0.65 & -0.27 \end{bmatrix} \begin{bmatrix} 0.65 \\ 0.27 \\ -0.27 \\ -0.65 \end{bmatrix} = \begin{bmatrix} 0 \\ 0.9908 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

$IDCT\,[X[k]]$

$$\begin{bmatrix} 0.50 & 0.65 & 0.50 & 0.27 \\ 0.50 & 0.27 & -0.50 & -0.65 \\ 0.50 & -0.27 & -0.50 & 0.65 \\ 0.50 & -0.65 & 0.50 & -0.27 \end{bmatrix} \begin{bmatrix} 0 \\ 0.9908 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0.644 \\ 0.268 \\ -0.268 \\ -0.644 \end{bmatrix}$$

f) $x[n] = \{\underline{0}, 1, 0, 0\}$

$DCT_4[x[n]]$

$$\begin{bmatrix} 0.5 & 0.5 & 0.5 & 0.5 \\ 0.65 & 0.27 & -0.27 & -0.65 \\ 0.50 & -0.50 & -0.50 & 0.50 \\ 0.27 & -0.65 & 0.65 & -0.27 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0.5 \\ 0.27 \\ -0.5 \\ -0.65 \end{bmatrix}$$

$IDCT_4[X[k]]$

$$\begin{bmatrix} 0.5 & 0.65 & 0.5 & 0.27 \\ 0.5 & 0.27 & -0.5 & -0.65 \\ 0.5 & -0.27 & -0.5 & 0.65 \\ 0.5 & -0.65 & 0.5 & -0.27 \end{bmatrix} \begin{bmatrix} 0.5 \\ 0.27 \\ -0.5 \\ -0.65 \end{bmatrix} = \begin{bmatrix} 0 \\ 0.9954 \\ 0.00046 \\ 0 \end{bmatrix}$$

17. DCT-II ; $X[k] = \sqrt{\frac{2-\delta[k]}{N}} \sum_{n=0}^{N-1} x[n] \cos \frac{(2n+1)k\pi}{2N}$

$$X_{DFT_{4N}}(k) = \sum_{n=0}^{N-1} x[n]\, W_{4N}^{nk}$$

$$\downarrow$$

$$\cos\left(2\pi \frac{nk}{4N}\right) + j \sin\left(2\pi \frac{nk}{4N}\right)$$

NATALIJA NAJHART
0015027891

$W_{4N}^{(4N-n)k}$

$x[n] = \{0, 1\} \rightarrow 4N = 8$

$2 \cdot 0 + 1 = 1$
$2 \cdot 1 + 1 = 3$
$2 \cdot 2 + 1 = 5$
$2 \cdot 3 + 1 = 7$

UZORCI IDU NA OVA MJESTA

$DFT_{4N} = \{0, 0, 0, 3, 0, 3, 0, 0\}$

$x_{4N}[n] = \{0, 0, 0, 1, 0, 1, \emptyset, \emptyset\}$
0 2 4 6 8

OSTALO SU NULE!

ZRCALNO (PARNO

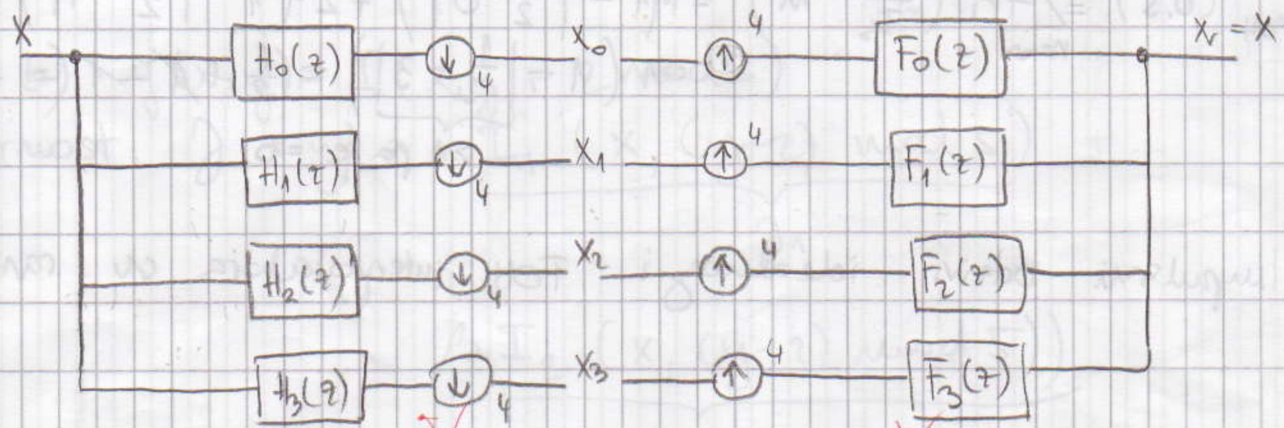
$DFT_{4N}[x_{4N}] = \{2, -\sqrt{2}, 0, \sqrt{2}, -2, \sqrt{2}, 0, -\sqrt{2}\}$

$$\sqrt{\frac{2-\delta[k]}{2}} \cdot \frac{1}{2} DFT_{4N}[x_{4N}] = DCT_{II}[x_n]_{2e}$$

$k = 0, 1$

$$[\sqrt{2} \quad -\sqrt{2}] = [\sqrt{2} \quad -\sqrt{2}] \ !!$$

20. DFT filtarski slog s decimacijom $N=4$



$$H_0(z) = 1 + zW_4^{0\cdot 1} + z^2 W_4^{0\cdot 2} + z^3 W_4^{0\cdot 3}$$
$$H_1(z) = 1 + zW_4^{1\cdot 1} + z^2 W_4^{1\cdot 2} + z^3 W_4^{1\cdot 3}$$
$$H_2(z) = 1 + zW_4^{2\cdot 1} + z^2 W_4^{2\cdot 2} + z^3 W_4^{2\cdot 3}$$
$$H_3(z) = 1 + zW_4^{3\cdot 1} + z^2 W_4^{3\cdot 2} + z^3 W_4^{3\cdot 3}$$

$$F_k(z) = 1 + z^{-1} W_4^{-k\cdot 1} + z^{-2} W_4^{-k\cdot 2} + z^{-3} W_4^{-k\cdot 3}$$

25. $x[n] = \{\underline{1}, 2, 0, -1\}$

$t_1 = 0.5s$, $T_s = 1s$

$$h_{I}(t) = \sum_{n=-\infty}^{\infty} x[n] \frac{\sin\left(\frac{\pi}{T}(t-nT)\right)}{\frac{\pi}{T}(t+nT)} = \sum_{n=-\infty}^{\infty} x[n] \operatorname{sinc}\left(\frac{t}{T} - n\right)$$

$$h_{II}(0.5) = \operatorname{sinc}\left(\frac{1}{2}\right) + 2\operatorname{sinc}\left(\frac{1}{2}-1\right) - 1\cdot\operatorname{sinc}\left(\frac{1}{2}-3\right) =$$
$$= \frac{2}{\pi} + 2\cdot\left(-\frac{2}{\pi}\right) - \frac{2}{5\pi} = \frac{10-20-2}{5\pi} = -\frac{12}{5\pi}$$

$$\operatorname{tri}(x) = \begin{cases} 1-|x|, & |x|<1 \quad \checkmark \quad x=\frac{1}{2} \\ 0, & |x|>1 \end{cases}$$

$$h_{FOH}(0.5) = \sum_{n=-\infty}^{\infty} \operatorname{tri}\left(\frac{t}{T_s} - m\right) = 1\cdot\left(1 - \left|\frac{1}{2}-0\right|\right) + 2\cdot\left(1 - \left|\frac{1}{2}-1\right|\right)$$
$$-1\left(1 - \underbrace{\left|\frac{1}{2}-3\right|}_{>1 \text{ pa je } =0}\right) = \frac{1}{2} + \cancel{1} - \cancel{1} = \frac{1}{2}$$

Impulsni odzivi idealnog i FOH interpolatora su različiti.

26. Slika $f(x,y)$ je dvodimenzionalna funkcija intenziteta svjetla $f(x,y)$, $0 < f(x,y) < \infty$, gdje su x i y prostorne koordinate, a vrijednost funkcije predstavlja svjetlinu slike u toj točki.

U digitalnom obliku slika se oblikuje kao matrica čiji elementi predstavljaju svjetlinu slike i zovu se pikseli (od engl. pixel).

Matrica (bitmap) je dvodimenzionalna.

Svaki element prikazuje 1 točkicu na slici.

Digitalna slika je kvantizirana funkcija 2 varijable koja je generirana optičkim sredstvima, odmjerena u jednako razmaknutim točkama i kvantizirana jednakim intervalima amplituda.

DOMENA - kvantizirana cjelobrojna vrijednost svjetline

KODOMENA - pixeli (tj. pridružena vrijednost svjetline)

SIVE SLIKE - vrijednost svakog piksela određena je 1 uzorkom vrijednosti

SLIKE U BOJI - vrijednost svakog piksela je određena sa 3 uzorka vrijednosti (RGB)

27. b) $\mathcal{J}(x,y) = I(x, y + 2 \bmod 5)$

linearnost: $\mathcal{J}(x,y) = \alpha \underbrace{I_1(x, (y+2) \bmod 5)}_{\mathcal{J}_1} +$

$$\beta \underbrace{I_2(x, (y+2) \bmod 5))}_{\mathcal{J}_2}$$

$$= \alpha \mathcal{J}_1 + \beta \mathcal{J}_2$$

LINEARAN SUSTAV !

$$g(x,y) = \begin{array}{|c|c|c|c|c|} \hline 2 & 3 & 4 & 5 & 6 \\ \hline 1 & 2 & 3 & 4 & 5 \\ \hline 5 & 6 & 7 & 8 & 9 \\ \hline 4 & 5 & 6 & 7 & 8 \\ \hline 3 & 4 & 5 & 6 & 7 \\ \hline \end{array}$$

28 $g(x,y) = \frac{1}{4+4+7}\left(0.29894\, r(x,y) + 0.58704\, g(x,y) + 0.11402\, b(x,y)\right)$

0.206	0.234	0.262	0.289
0.160	0.188	0.215	0.243
0.113	0.141	0.168	0.196
0.067	0.094	0.122	0.149