1. MI 2007./2008.

① SIGNAL - funkcija nezavisnih varijabli koja nosi informaciju oblika $f: D \to K$, koja je injektivna

Prema prebrojivosti D i K postoje → kontinuirani - neprebrojiva D i K
→ vremenski diskretni - prebrojiva D
- neprebrojiva K
→ kvantizirani - neprebrojiva D
- prebrojiva K
→ digitalni - prebrojiva D i K

SUSTAV = proces koji mjeri, manipulira, pohranjuje ili prenosi signal

INFORMACIJA - podatak koji za korisnika ima značenje

② $DTFT[x[n]] = \sum_{n=-\infty}^{+\infty} x[n] e^{-j\omega k} = X(\omega)$

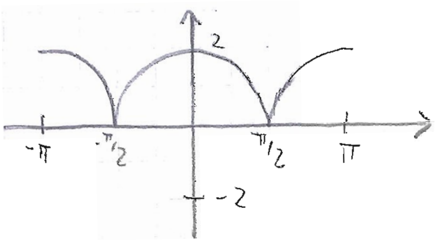
$IDTFT[X(\omega)] = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(\omega) e^{j\omega n} = x[n]$

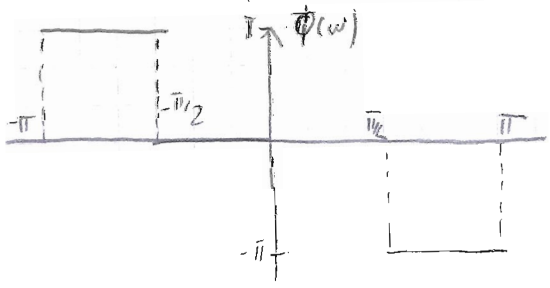
$h[n] = \delta[n-1] + \delta[n+1]$

$H(\omega) = e^{j\omega} + e^{-j\omega} = 2\cos(\omega)$

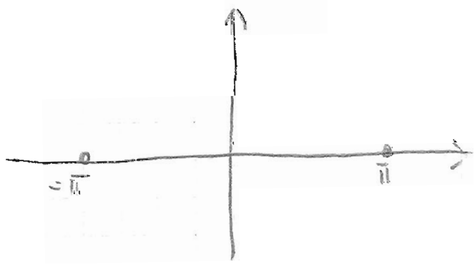
$A(\omega) = |2\cos\omega|$

$\measuredangle(\omega) = \begin{cases} 0, & -\pi/2 < \omega < \pi/2 \\ \pi, & \omega \in [-\pi, -\pi/2] \cup [\pi/2, \pi] \end{cases}$





$J(\omega) = -\frac{d}{d\omega} \measuredangle(\omega) \Rightarrow$



③ Dekonvolucija je postupak rekonstrukcije ulaznog signala $u[n]$ iz poznatog izlaznog signala $y[n]$ i impulsnog odziva $h[n]$

$$u[n] = \frac{1}{h_0}\left(y[n] - \sum_{i=1}^{n} u[n-i]\,h[i]\right)$$

$$h[n] = \{\underline{2}, 3, 2\} \qquad y[n] = \{\underline{2}, 1, 1, -1, -1, -2\}$$

$$u[0] = \frac{1}{2}\cdot 2 = 1$$

$$u[1] = \frac{1}{2}(1 - (1\cdot 3)) = \frac{1}{2}(-2) = -1$$

$$u[2] = \frac{1}{2}(1 - (-1\cdot 3 + 1\cdot 2)) = 1$$

$$u[3] = \frac{1}{2}(-1 - (1\cdot 3 + (-1)\cdot 2)) = \frac{1}{2}(-1-1) = -1$$

$$u[4] = \frac{1}{2}(-1 - (-1\cdot 3 + 1\cdot 2)) = \frac{1}{2}\cdot(0) = 0$$

$$u[5] = \frac{1}{2}(-2 - (0\cdot 3 + (-1)\cdot 2)) = \frac{1}{2}\cdot(-2+2) = 0$$

$$u[6] = \frac{1}{2}(0 - (0\cdot 3 + 0\cdot 2)) = \frac{1}{2}\cdot 0 = 0$$

$$u[n] = \{\underline{1}, -1, 1, -1\}$$

④ $H(z) = (1-2z^{-1})(3-1z^{-1}) \qquad z_1 = 2,\ z_2 = \frac{1}{3}$

$H(z)$ nema polova $\Rightarrow$ stabilan je

$z_1 > 1 \Rightarrow$ sustav nije minimalno fazni

$$H_{mf}(z) = (z^{-1}-2)(3-z^{-1}) = -6 + 5z^{-1} - z^{-2}$$

$$H_{mf}^{-1}(z) = \frac{1}{-6+5z^{-1}-z^{-2}}$$

$$H(z)\cdot H_{mf}^{-1}(z) = \frac{(1-2z^{-1})(3-z^{-1})}{(z^{-1}-2)(3-z^{-1})} = \frac{1-2z^{-1}}{z^{-1}-2} = \frac{z-2}{1-2z}$$

$$|H(e^{j\omega})H_{mf}^{-1}(e^{j\omega})| = \left|\frac{e^{j\omega}-2}{1-2e^{j\omega}}\right| = \left|\frac{\cos(\omega)-2+j\sin(\omega)}{1-2\cos(\omega)-2j\sin(\omega)}\right|$$

$$= \sqrt{\frac{\cos^2(\omega)-4\cos(\omega)+4-\sin^2(\omega)}{1-4\cos(\omega)+4\cos^2(\omega)+4\sin^2(\omega)}} = \sqrt{}$$

$$= \sqrt{\frac{5-4\cos\omega}{5-4\cos\omega}}$$

$$= 1$$

(5) $DFT_N[x[n]] = \sum_{n=0}^{N-1} x[n] W_N^{nk} = X[k]$

$$W_N^{nk} = e^{-2\pi j k \frac{n}{N}}$$

$$W_4 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} = \left[W_n^{(i-1)(j-1)} \right]$$

$$x[n] = \{0, \underline{1}, 0, 0\} \circ\!\!-\!\!\bullet\; X(z) = z^{-1}$$

$$DFT_4[x[n]] = W_4 \cdot x[n] = \{1, -j, -1, j\}$$

$$x[n] \to \boxed{H_0 = 1 + z + z^2 + z^3} \to Y_0[n]$$

$$x[n] \to \boxed{H_1 = 1 - jz - z^2 + jz^3} \to Y_1[n]$$

$$x[n] \to \boxed{H_2 = 1 - z + z^2 - z^3} \to Y_2[n]$$

$$x[n] \to \boxed{H_3 = 1 + jz - z^2 - jz^3} \to Y_3[n]$$

$$Y_0(z) = H_0(z) \cdot X(z) = z^{-1} + 1 + z + z^2$$

$$Y_1(z) = H_1(z) \cdot X(z) = z^{-1} - j - z + jz^2$$

$$Y_2(z) = H_2(z) \cdot X(z) = z^{-1} - 1 + z - z^2$$

$$Y_3(z) = H_3(z) \cdot X(z) = z^{-1} + j - z - jz^2$$

$$Y_k[0] = \{1, -j, -1, j\}$$

$$X_k[0] = \{1, -j, -1, j\}$$

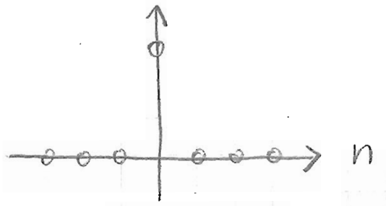
$$\Downarrow$$

$$Y_k[0] = X[k]$$

1. MI 2008./2009.

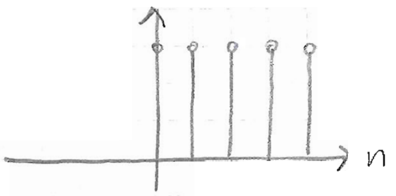
1. Diskretni signal $x: \mathbb{Z} \to \mathbb{C}$ je kauzalan ako i samo ako $x[n]=0, \forall n<0$

Kroenecker delta – kauzalna



$$\delta[n] = \begin{cases} 1, & n=0 \\ 0, & inače \end{cases}$$

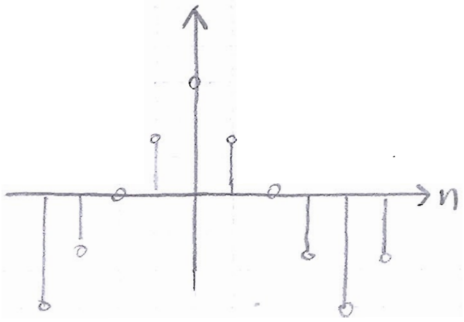
Jedinična stepenica – kauzalna



$$\mu[n] = \begin{cases} 1, & n \geq 0 \\ 0, & inače \end{cases}$$

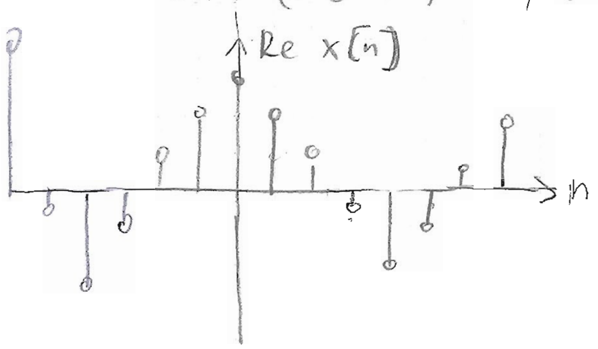
Diskretna harmonijska stepenica – nije kauzalna

$x[n] = e^{j(2\pi F n + p)}$, $F \in \mathbb{N}, p \in \mathbb{R}$



Diskretna kompleksna eksponencijala – nije kauzalna

$x[n] = (\rho e^{j\Omega})^n$, $\rho, \Omega \in \mathbb{R}$



②

$$X(e^{j\omega}) = \begin{cases} -j, & 0 < \omega < \pi \\ 0, & \omega = 0 \\ j, & -\pi < \omega < 0 \end{cases}$$

$$\text{DTFT}\left[x[n]\right] = X(\omega) = \sum_{n=-\infty}^{+\infty} x[n]e^{-j\omega n}$$

$$\text{IDTFT}\left[X(\omega)\right] = x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(\omega)\, e^{j\omega n}\, d\omega$$

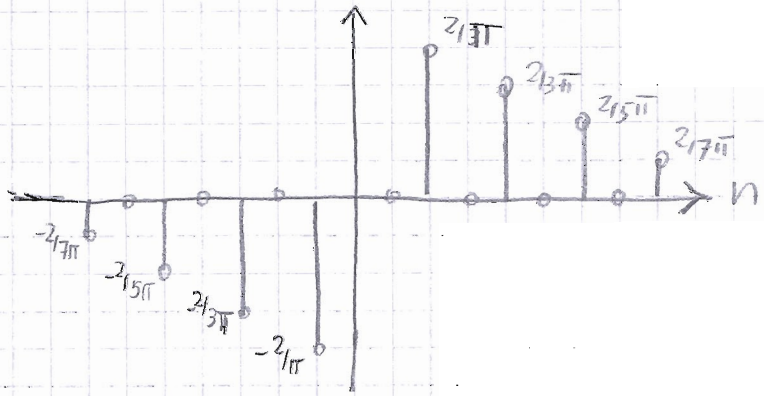
$$x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(\omega)e^{j\omega n}\,d\omega = \frac{1}{2\pi}\int_{-\pi}^{0} j e^{j\omega n}\,d\omega - \frac{1}{2\pi}\int_{0}^{\pi} j e^{j\omega n}\,d\omega$$

$$= \frac{1}{2\pi}\cdot\frac{\cancel{j}}{\cancel{j}n}\, e^{j\omega n}\Big|_{-\pi}^{0} - \frac{1}{2\pi}\,\frac{\cancel{j}}{\cancel{j}n}\, e^{j\omega n}\Big|_{0}^{\pi} = \frac{1}{2\pi n}\left(1 - e^{-j\pi n} - e^{j\pi n} + 1\right)$$

$$= \frac{1}{\pi n}\left[1 - \cos(n\pi)\right], \quad \forall n \neq 0$$

$$x[0] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(\omega)\,d\omega = \frac{1}{2\pi}\int_{-\pi}^{0} j\,d\omega - \frac{1}{2\pi}\int_{0}^{\pi} j\,d\omega = \frac{1}{2\pi}\left(j\pi - j\pi\right) = 0$$

$$x[n] = \begin{cases} \frac{1}{\pi n}\left(1 - \cos n\pi\right), & n \neq 0 \\ 0, & n = 0 \end{cases}$$



③ Dekonvolucija je postupak rekonstrukcije ulaznog signala $u[n]$ iz poznatog izlaznog signala $y[n]$ i impulsnog odziva $h[n]$

$$u[n] = \frac{1}{h[0]} \left(y[n] - \sum_{i=1}^{m} u[n-i]\, h[n] \right)$$

$$h[n] = \{ \underline{1}, -2, 1 \}$$

$$y[n] = \{ \underline{1}, -1, 1, -1, -1, 1, -1, 1 \}$$

$$u[0] = 1 \quad (1) = 1$$

$$u[1] = 1 \; (-1 - (1 \cdot (-2)) = 1(-1+2) = 1$$

$$u[2] = 1 \; (1 - (1 \cdot (-2) + 1 \cdot 1)) = 1(1-(-1)) = 2$$

$$u[3] = 1 \; (-1 - (2 \cdot (-2) + 1 \cdot 1)) = 1(-1+3) = 2$$

$$u[4] = 1 \; (-1 - (2 \cdot (-2) + 2 \cdot 1)) = 1 \cdot 1 = 1$$

$$u[5] = 1 \; (1 - (1 \cdot (-2) + 2 \cdot 1)) = 1 \cdot 1 = 1$$

$$u[6] = 1 \; (-1 - (1 \cdot (-2) + 1 \cdot 1)) = 1 \cdot (-1+1) = 0$$

$$u[7] = 1 \; (1 - (0 \cdot (-2) + 1 \cdot 1)) = 1 \cdot (1-1) = 0$$

$$u[8] = 1 \; (1 - (0 \cdot (-2) + 0 \cdot 1)) = 1$$

$h[n]$ ima samo tri elementa → ostali uzorci ulaza su 0

$$u[n] = \{ \underline{1}, 1, 2, 2, 1, 1, 0, 0 \}$$

④ $H(z) = (3-z^{-1})(1+4z^{-1}+8z^{-2})$

$z_1 = \frac{1}{3}$ $\quad z_{2,3} = \frac{-4 \pm \sqrt{16-4 \cdot 8}}{2} = -2 \pm \sqrt{-4} = -2 \pm 2j$ $\Rightarrow$ nema polova (uvijek je stabilan)

$|z_{2,3}| > 1 \Rightarrow$ sustav nije minimalno fazni

$H_{mF}(z) = (3-z^{-1})(z^{-2}+4z^{-1}+8)$

$H_{mF}^{-1}(z) = \frac{1}{3-z^{-1}} \cdot \frac{1}{z^{-2}+4z^{-1}+8}$

$|H_{mF}^{-1}(e^{j\omega}) H(e^{j\omega})| = 1$

$H_{mF}^{-1}(z) \cdot H(z) = \frac{3-z^{-1}}{3-z^{-1}} \cdot \frac{1+4z^{-1}+8z^{-2}}{z^{-2}+4z^{-1}+8} = \frac{1+4z^{-1}+8z^{-2}}{z^{-2}+4z^{-1}+8}$

$$|H_{mF}^{-1}(e^{j\omega}) H(e^{j\omega})| = \left| \frac{1+4e^{-j\omega}+8e^{-2j\omega}}{e^{-2j\omega}+4e^{-j\omega}+8} \right| = \left| \frac{1}{e^{-2j\omega}} \cdot \frac{1+4e^{-j\omega}+8e^{-2j\omega}}{1+4e^{j\omega}+8e^{2j\omega}} \right|$$

$$= \left| \frac{1+4\cos(\omega) - j4\sin(\omega) + 8\sin(2\omega) - j8\sin(2\omega)}{1+4\cos(\omega)+j4\sin(\omega)+8\cos(2\omega)+j8\sin(2\omega)} \right|$$

$$= \sqrt{\frac{(1+4\cos(\omega)+8\cos(2\omega))^2 + (4\sin(\omega)+8\sin(2\omega))^2}{(1+4\cos(\omega)+8\cos(2\omega))^2 + (4\sin(\omega)+8\sin(2\omega))^2}} = 1$$

5

$x[n] = \{\underline{0}, 1, 0, 0\} \circ\!\!-\!\!\bullet X(z) = 1z^{-1}$

$Y_0(z) = X(z) \cdot H_0(z) = z^{-1} + z^{-2} + z^{-3} + z^{-4} \bullet\!\!-\!\!\circ\, y_0[n] = \{\underline{0}, 1, 1, 1, 1\}$

$Y_1(z) = X(z) \cdot H_1(z) = z^{-1} - jz^{-2} + z^{-3} + jz^{-4} \bullet\!\!-\!\!\circ\, y_1[n] = \{\underline{0}, 1, -j, 1, j\}$

$Y_2(z) = X(z) \cdot H_2(z) = z^{-1} - z^{-2} + z^{-3} - z^{-4} \bullet\!\!-\!\!\circ\, y_2[n] = \{0, 1, -1, 1, -1\}$

$Y_3(z) = X(z) \cdot H_3(z) = z^{-1} + jz^{-2} - z^{-3} - jz^{-4} \bullet\!\!-\!\!\circ\, y_3[n] = \{0, 1, j, -1, -j\}$

$$DFT_4[x[n]] = W_4 \cdot \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ -j \\ -1 \\ j \end{bmatrix}$$

$y_k[3] = \{1, 1, 1, -1\}$

$X[k] = \{1, -j, -1, j\}$

$\Rightarrow y_k[3] \neq X[k]$

Pobudom s $x[3-n] = \{0, 0, 1, 0\} \circ\!\!-\!\!\bullet X(z) = z^{-2}$

dobivamo odzive

$y_0[n] = \{\underline{0}, 0, 1, 1, 1, 1\}$

$y_1[n] = \{\underline{0}, 0, 1, -j, 1, j\}$

$y_2[n] = \{\underline{0}, 0, 1, -1, 1, -1\}$

$y_3[n] = \{\underline{0}, 0, 1, j, -1, -j\}$

$y_k[3] = \{1, -j, -1, j\} = X[k] = \{1, -j, -1, j\}$

1. MI 2009./2010.

① $DTFT\{x[n]\} = X(\omega) = \sum_{n=-\infty}^{+\infty} x[n] e^{-j\omega k}$

$IDTFT(X(\omega)) = x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(\omega) e^{j\omega n}$

$h[n] = -\frac{1}{\pi}\delta[n-3] - \frac{2}{\pi}\delta[n-1] + \frac{2}{\pi}\delta[n+1] + \frac{1}{\pi}\delta[n+3]$

$= \{\frac{1}{\pi}, 0, \frac{2}{\pi}, 0, -\frac{2}{\pi}, 0, -\frac{1}{\pi}\}$

$H(\omega) = \frac{1}{\pi} e^{3j\omega} + \frac{2}{\pi} e^{j\omega} - \frac{2}{\pi} e^{-j\omega} - \frac{1}{\pi} e^{-3j\omega}$

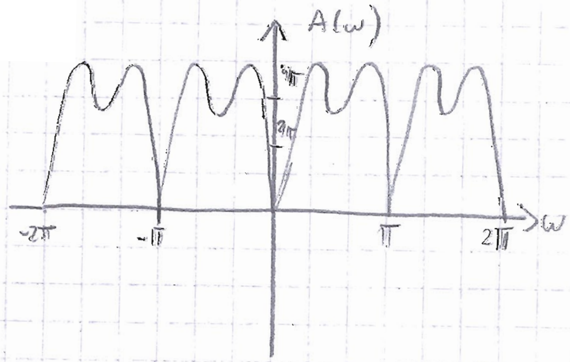
$= \frac{2j}{\pi} \sin(3\omega) + \frac{4j}{\pi} \sin(\omega)$

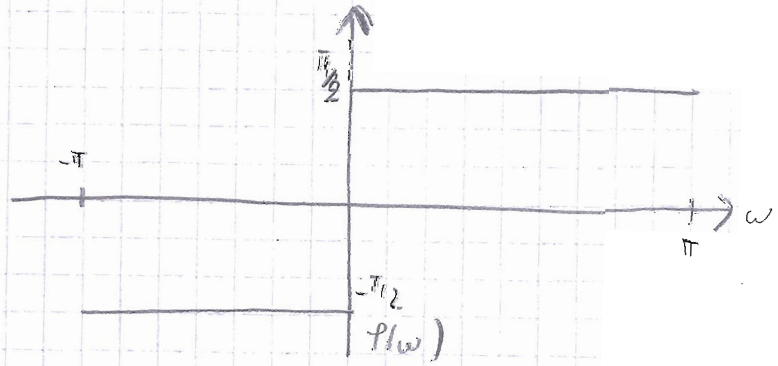
$= e^{j\frac{\pi}{2}} \left(\frac{2}{\pi} \sin 3\omega + \frac{4}{\pi} \sin \omega\right)$

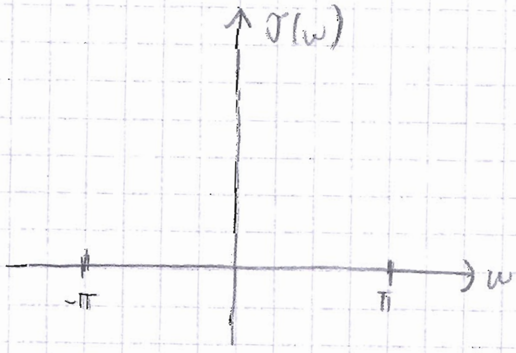
$A(\omega) = \left|\frac{2}{\pi} \sin 3\omega + \frac{4}{\pi} \sin \omega\right|$

$\phi(\omega) = \frac{\pi}{2}$, $\varphi(\omega) = \begin{cases} \pi/2 & , 0 < \omega < \pi \\ -\pi/2 & , -\pi < \omega < 0 \end{cases}$

$\tau(\omega) = -\frac{d}{d\omega}\phi(\omega) = 0$







2. $h[n] = \{-2, 0, 2\}$

$y[n] = \{-2, -8, -12, 4, 14, 8, 10, -16, -28, 12, 18\}$

Dekonvolucija je postupak rekonstrukcije ulaznog signala $u[n]$ iz poznatog izlaznog signala $y[n]$ i impulsnog odziva $h[n]$

$$u[n] = \frac{1}{h[0]}\left(y[n] - \sum_{i=1}^{n} u[n-i]\,h[i]\right)$$

$\{-2, -8, -12, 4, 14, 8, 10, -16, -28, 12, 18\} : -2\ 0\ 2 = 1, 4, 7, 2, 0, -2, -5, 6, 9$

```
-2  0 -2
 0 -8 -14  4
    8   0 -8
    0 -14 -4 14
      14   0 -14
       0  -4  0  8
           4  0 -4
           0  0  4 10
             -0  0  0
              0  4 10 -16
                -4  0   4
                 0 10 -12 -28
                  -10   0 +10
                      -12 -18  12
                       12   0 -12
                          -18   0  18
```

(Dekonvolucija dijeljenjem)

③ $H(z) = (2-z^{-1})(1+7z^{-1}+10z^{-2})$

$z_1 = \frac{1}{2}$ $\quad z_{2,3} = \frac{-7 \pm \sqrt{49-40}}{2} = \frac{-7 \pm 3}{2} \Rightarrow z_2 = -2, \ z_3 = -5$

$H(z)$ nema polove (stabilan je)

$|z_2|, |z_3| > 1 \rightarrow$ sustav nije minimalno fazni

$H(z)^{-1} = \frac{1}{(2-z^{-1})(1+7z^{-1}+10z^{-2})}$

$H_{mF}^{-1}(z) = \frac{1}{(2-z^{-1})(z^{-2}+7z^{-1}+10z)}$

Ako područje konvergencije obuhvaća $z=\infty$,

$H^{-1}(z)$ je nestabilan kauzalan sustav

a $H_{mF}^{-1}(z)$ je stabilan kauzalan sustav

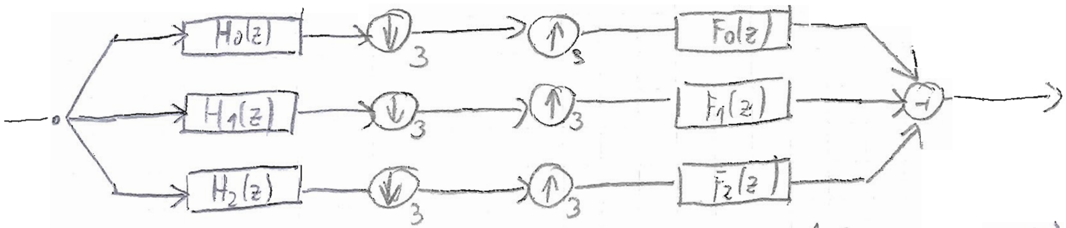
Mijenjanjem područja konvergencije tako da obuhvaća $z=e^{j\omega}$,

$H^{-1}(z)$ postaje stabilan nekauzalan sustav

④ W_N matrica je kvadratna matrica dimenzija $N \times N$ koja u i-tom retku i n-tom stupcu ima element $e^{2\pi j \frac{(i-1)(j-1)}{N}}$

$$W_3 = \begin{bmatrix} W_3^{00} & W_3^{01} & W_3^{02} \\ W_3^{10} & W_3^{11} & W_3^{12} \\ W_3^{20} & W_3^{21} & W_3^{22} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & -\frac{1}{2}-\frac{\sqrt{3}}{2}j & -\frac{1}{2}+\frac{\sqrt{3}}{2}j \\ 1 & -\frac{1}{2}+\frac{\sqrt{3}}{2}j & -\frac{1}{2}-\frac{\sqrt{3}}{2}j \end{bmatrix}$$

DFT_3 slog s maksimalnom decimacijom se sastoji od sva tri filtra u analizirajućem dijelu sloga i sva tri filtra u rekonstrukcijskom dijelu sloga.



$$H_0(z) = 1 + z + z^2$$
$$H_1(z) = 1 + \left(-\frac{1}{2}-\frac{\sqrt{3}}{2}j\right)z + \left(-\frac{1}{2}+\frac{\sqrt{3}}{2}j\right)z^2$$
$$H_2(z) = 1 + \left(-\frac{1}{2}+\frac{\sqrt{3}}{2}j\right)z + \left(-\frac{1}{2}-\frac{\sqrt{3}}{2}j\right)z^2$$

$\Downarrow$

elementi odgovaraju retcima matrice W_3

$$F_0(z) = \frac{1}{3}\left(1 + z^{-1} + z^{-2}\right)$$
$$F_1(z) = \frac{1}{3}\left(1 + \left(-\frac{1}{2}+\frac{\sqrt{3}}{2}j\right)z^{-1} + \left(-\frac{1}{2}+\frac{\sqrt{3}}{2}j\right)z^{-2}\right)$$
$$F_2(z) = \frac{1}{3}\left(1 + \left(-\frac{1}{2}-\frac{\sqrt{3}}{2}j\right)z^{-1} + \left(-\frac{1}{2}-\frac{\sqrt{3}}{2}j\right)z^{-2}\right)$$

$\Downarrow$

elementi odgovaraju stupcima matrice W_3

(5)

$$A(\omega)=\begin{cases} \omega/\pi, & 0<\omega<\pi \\ -\omega/\pi, & -\pi<\omega<0 \end{cases}$$

$$a[0]=\frac{1}{2\pi}\int_{-\pi}^{\pi} A(\omega)\,d\omega = \frac{1}{2\pi}\left(\int_{-\pi}^{0} -\frac{\omega}{\pi}\,d\omega + \int_{0}^{\pi}\frac{\omega}{\pi}\,d\omega\right)$$

$$=\frac{1}{2\pi}\left(\frac{\pi}{2}+\frac{\pi}{2}\right)=\frac{1}{2}$$

$$a[m]=\frac{1}{\pi}\int_{-\pi}^{\pi} A(\omega)\,d\omega$$

$$=\frac{2}{\pi}\int_{0}^{\pi} A(\omega)\,d\omega$$

$$=\frac{2}{\pi^2}\left(\omega\cdot m\cdot \sin(\omega m)\Big|_0^{\pi} - \int_0^{\pi} m\sin(\omega m)\,d\omega\right.$$

$$=\frac{2}{\pi^2}\left(m\pi\sin(\pi m)-0+m^2\cos(\omega m)\Big|_0^{\pi}\right)$$

$$=\frac{2m^2}{\pi^2}\left(\cos(m\pi)-1\right)$$

Za filter s 3 uzorka trebamo $a[0]$ i $a[1]$

$$a[0]=\frac{1}{2}$$

$$a[1]=-\frac{4}{\pi^2}$$

$$H(\omega)=\frac{1}{2}-\frac{4}{\pi^2}\cos(\omega)$$

$$h[n]=\left\{-\frac{2}{\pi^2},\ \underline{\frac{1}{2}},\ -\frac{2}{\pi^2}\right\}$$

1. PMI 2009./2010.

1) SIGNAL je funkcija nezavisnih varijabli koja nosi informaciju, oblika $f: D \to K$, koja je injektivna

Obzirom na prebrojivost D i K razlikujemo:

1) KONTINUIRANI SIGNAL - neprebrojiva D i K
2) VREMENSKI DISKRETNI SIGNAL - prebrojiva D, neprebrojiva K
3) KVANTIZIRANI SIGNAL - neprebrojiva D, prebrojiva K
4) DIGITALNI SIGNAL - prebrojiva D i K

a) Kroeneckerov delta-impuls

$f[n] = \delta[n]$ -

$f: \mathbb{Z} \to \mathbb{R}$ je vremenski diskretni signal

b) $x(t) = \begin{cases} 0, & t < 0 \\ 1, & t \geq 0 \end{cases}$

$f: \mathbb{R} \to \mathbb{R}$ je kontinuirani signal

$f: \mathbb{R} \to \{0, 1\}$ je kvantizirani signal

c) $x(t) = \sin(t)$

$f: \mathbb{R} \to \mathbb{R}$ je kontinuirani signal

d) $x[n] = e^{j\pi \frac{n}{3}}$

$f: \mathbb{Z} \to \mathbb{R}$ je vremenski diskretni signal

$f: \mathbb{Z} \to \{\pm 1, \frac{1}{2} \pm \frac{\sqrt{3}}{2}j, -\frac{1}{2} \pm \frac{\sqrt{3}}{2}j\}$ je digitalni signal

(2) $\text{DTFT}\,[x[n]] = X(\omega) = \sum_{n=-\infty}^{+\infty} x[n]\, e^{-j\omega n}$

$\text{IDTFT}\,[X(\omega)] = x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(\omega)\, e^{j\omega n}\, d\omega$

$h[n] = \{\underline{1}, 2, 0, -2, -1\}$

$$H(\omega) = \sum_{n=-\infty}^{+\infty} h[n] e^{-j\omega n} = 1 + 2e^{-j\omega} - 2e^{-3j\omega} - 1e^{-4j\omega}$$

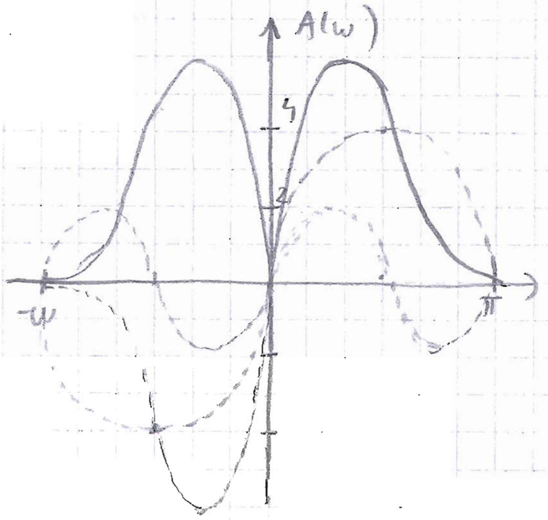
$$= e^{-2j\omega}\left(e^{2j\omega} + 2e^{j\omega} - 2e^{-j\omega} - e^{-2j\omega}\right)$$

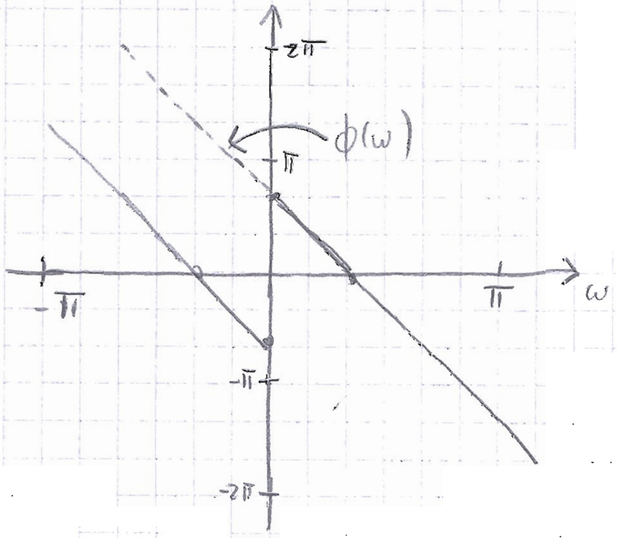
$$= e^{-2j\omega} \cdot j\left[2\sin(2\omega) + 4\sin(\omega)\right]$$

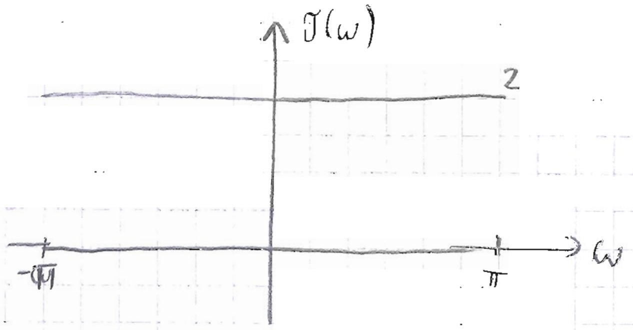
$A(\omega) = |H(\omega)| = |2\sin(2\omega) + 4\sin(\omega)|$

$\phi(\omega) = \measuredangle H(\omega) = -2\omega + \frac{\pi}{2}$

$\tau(\omega) = -\frac{d}{d\omega}\phi(\omega) = 2$







③ $h = \{1, -2, 1\}$, $y[n] = \{2, 6, -8, -5, 8, -10, 10, 2, -6, -7, 8\}$

Dekonvolucija je postupak rekonstrukcije ulaznog signala $v[n]$ iz poznatog izlaznog signala $y[n]$ i impulsnog odziva $h[n]$.

$$v[n] = \frac{1}{h[0]} \left(y[n] - \sum_{i=1}^{n} v[n-i]\, h[i] \right)$$

2 6 -8 -5 8 -10 10 2 -6 -7 8 : 1, -2, 1 = {2, 10, 10, 5, 8, 1, 4, 9, 8}
-2 4 -2
10 -10 -5
-10 20 -10
10 -15 8
-10 20 -10
5 -2 -10
-5 +10 -5
8 -15 10
-8 16 -8
1 2 2
-1 2 -1
4 1 -6
-4 8 -4
9 -10 -7
-9 18 -9
8 -16 8
-8 16 -8
0 //

Ulazni signal $v[n] = \{2, 10, 10, 5, 8, 1, 4, 9, 8\}$

4 $H(z) = 2 + 25z^{-1} + 52z^{-2} + 20z^{-3}$

$n_1 = -\frac{1}{2}$ $\quad 1 = -\frac{1}{2z}$ $\quad -2 = z^{-1} \Rightarrow -2 - z^{-1} = 0 \Rightarrow (2 + z^{-1}) = 0$

$$(2 + 25z^{-1} + 52z^{-2} + 20z^{-3}) : (2 + z^{-1}) = 1 + 12z^{-1} + 20z^{-2}$$

$$\begin{array}{lll} +24z^{-1} & +52z^{-2} & \\ -24z^{-1} & -12z^{-2} & \\ \hline 0 & +40z^{-2} & +20z^{-3} \end{array}$$

$$H(z) = (2 + z^{-1})(1 + 12z^{-1} + 20z^{-2})$$

$$n_{2,3} = \frac{-12 \pm \sqrt{144-80}}{2} = -6 \pm \frac{8}{2} = -6 \pm 4 < \begin{matrix} n_2 = -2 \\ n_3 = -10 \end{matrix}$$

$|n_1| < 1,\ |n_2| > 1,\ |n_3| > 1 \Rightarrow$ Sustav nije minimalno fazni

Sustav nema polova $\Rightarrow$ sustav je stabilan

Da dobijemo minimalno fazni sustav, nule koje nisu unutar jedinične kružnice zamijene svojim recipročnim vrijednostima.

$$H_{mf}(z) = (2 + z^{-1})(z^{-2} + 12z^{-1} + 20)$$

$$H_{mf}^{-1} = \frac{1}{(2+z^{-1})(z^{-2} + 12z^{-1} + 20)} = \frac{1}{(2+z^{-1})(z^{-1}+10)(z^{-1}+2)}$$

$$|H(e^{j\omega}) H_{mf}^{-1}(e^{j\omega})| = 1 \ ?$$

$$= \left| \frac{(2+e^{-j\omega})(1 + 12e^{-j\omega} + 20e^{-2j\omega})}{(2+e^{-j\omega})(e^{-2j\omega} + 12e^{-j\omega} + 20)} \right| = \left| \frac{1 + 10e^{-j\omega}}{e^{-j\omega} + 10} \right| \cdot \left| \frac{1 + 2e^{-j\omega}}{e^{-j\omega} + 2} \right|$$

$$= \left| \frac{1 + 10\cos(\omega) - 10j\sin(\omega)}{\cos(\omega) - j\sin(\omega) + 10} \right| \cdot \left| \frac{1 + 2\cos(\omega) - 2j\sin(\omega)}{\cos(\omega) - j\sin(\omega) + 2} \right|$$

$$= \sqrt{\frac{(1+10\cos(\omega))^2 + 10^2\sin^2(\omega)}{(\cos(\omega)+10)^2 + \sin^2\omega}} \cdot \sqrt{\frac{(1+2\cos(\omega))^2 + 4\sin^2(\omega)}{(2+\cos(\omega))^2 + \sin^2(\omega)}}$$

$$= \sqrt{\frac{101 + 20\cos(\omega)}{101 + 20\cos(\omega)}} \cdot \sqrt{\frac{5 + 4\cos(\omega)}{5 + 4\cos(\omega)}}$$

$$= 1 \ //$$

(5) $A(\omega)=\begin{cases} 1, & -\frac{3\pi}{4} < \omega < -\frac{\pi}{4} \\ 1, & \frac{\pi}{4} < \omega < \frac{3\pi}{4} \\ 0, & \text{inače} \end{cases}$

$$a[0] = \frac{1}{2\pi}\int_{-\pi}^{\pi} A(\omega)\,d\omega = \frac{1}{2\pi}\int_{-\frac{3\pi}{4}}^{\frac{\pi}{4}} 1\,d\omega + \int_{+\frac{\pi}{4}}^{\frac{3\pi}{4}} 1\,d\omega$$

$$= \frac{1}{2\pi}\left(\frac{\pi}{2}+\frac{\pi}{2}\right) = \frac{1}{2}$$

$$a[m] = \frac{1}{\pi}\int_{-\pi}^{\pi} A(\omega)\cos(m\omega)\,d\omega = \frac{1}{\pi}\left(\int_{-\frac{3\pi}{4}}^{-\frac{\pi}{4}} \cos(m\omega)\,d\omega + \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \cos(m\omega)\,d\omega\right)$$

$$= \frac{1}{m\pi}\left(\sin\left(\frac{-m\pi}{4}\right) - \sin\left(-\frac{3m\pi}{4}\right) + \sin\left(\frac{3\pi m}{4}\right) - \sin\left(\frac{\pi}{4}m\right)\right)$$

$$= \frac{1}{m\pi}\left(2\sin\left(\frac{3\pi}{4}m\right) - 2\sin\left(\frac{\pi}{4}m\right)\right)$$

Za 5 uzoraka trebamo $a[1]$ i $a[2]$

$$a[1] = \frac{1}{\pi}\left[2\sin\left(\frac{3\pi}{4}\right) - 2\sin\left(\frac{\pi}{4}\right)\right] = 0$$

$$a[2] = \frac{1}{2\pi}\left[2\sin\left(\frac{6\pi}{4}\right) - 2\sin\left(\frac{2\pi}{4}\right)\right] = -\frac{4}{2\pi} = -\frac{2}{\pi}$$

$$h[n] = \left\{-\frac{1}{\pi}, 0, \underline{\frac{1}{2}}, 0, -\frac{1}{\pi}\right\}$$

$$H(\omega) = e^{-2j\omega}\left[\frac{1}{2} - \frac{2}{\pi}\cos(2\omega)\right]$$