OSNOVE ELEKTROTEHNIKE

MASOVNE

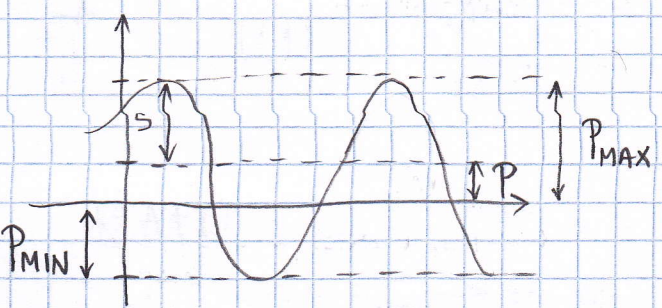
$p(t) = u(t) \cdot i(t)$

$$\dot{S} = \dot{U} \cdot \dot{I}^* = U \cdot I \cdot \angle\varphi_u - \varphi_i = U \cdot I \angle\varphi$$

$$\dot{Z} = \frac{\dot{U}}{\dot{I}} = \frac{U}{I} \angle\varphi_u - \varphi_i = \frac{U}{I} \angle\varphi$$

$$\dot{S} = P + jQ$$

→ SREDNJA SNAGA = RADNA SNAGA

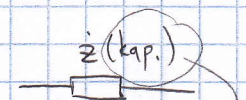


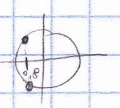
2) Na nekoj impedanciji $\dot{z}$ je $P_{MAX} = 900$ VA, $P_{MIN} = 100$ VA, a $U = 50$ V. $\dot{z} = ?$

$$P = \frac{P_{MAX} + P_{MIN}}{2} = 400 \text{ W}$$

$$S = P_{MAX} - P = 500 \text{ VA}$$

$$Q = \sqrt{S^2 - P^2} = 300 \text{ VAr}$$





$\cos\varphi = 0{,}8 \rightarrow \pm 36{,}87°$

$$\cos\varphi = \frac{P}{S} = 0{,}8 \rightarrow \varphi = 36{,}87° \text{ (NE)} \rightarrow \varphi = -36{,}87°$$

$$S = U \cdot I$$

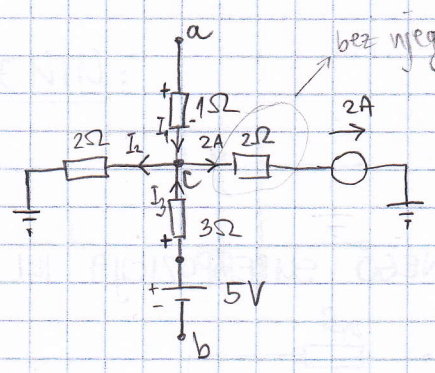
$$I = \frac{S}{U} = 10 \text{ A}$$

$$z = \frac{U}{I} = 5\,\Omega$$

$$\dot{z} = 5 \angle -36{,}87° = 4 - j3\,\Omega$$

$$P = I^2 R = I^2 Z \cdot \cos\varphi = \frac{U^2}{Z}\cos\varphi = \frac{U^2}{R}$$ → paziti kada se koja koristi!

Zad.)



$\varphi_a = 10\ V$

$\varphi_b = 12\ V$

$\varphi_c = ?$

$I_1 + I_3 = I_2 + 2$

$$\frac{\varphi_a - \varphi_c}{1} + \frac{\varphi_b + 5 - \varphi_c}{3} = \frac{\varphi_c}{2} + 2$$

: GRUPIRANJEM ↓

! POTENCIJALI ČVOROVA → SJETI SE !!

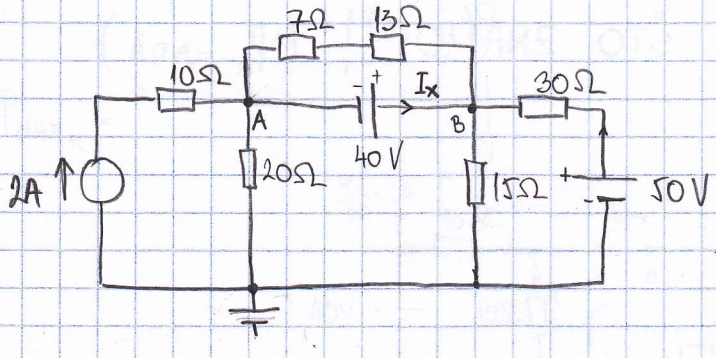
$$\varphi_c\left[\frac{1}{1} + \frac{1}{3} + \frac{1}{2}\right] - \varphi_a \cdot \frac{1}{1} - \varphi_b \cdot \frac{1}{3} = \frac{5}{3} + 2$$

(1/1 + 1/3 + 1/2 — vodljivosti u okolnoj grani, ali bez otpora koji u svojoj grani ima strujni izvor)

5/3 + 2 — odlazi iz ⓒ struje

→ želimo doći do struja (napon · otpor), a u toj grani već imamo struju!

Zad.)



$I_{30\Omega} = ?$

Ⓐ $$\varphi_A \cdot \left[\frac{1}{20} + \frac{1}{7+13} + \frac{1}{0}\right] - \varphi_B \cdot \left(\frac{1}{0} + \frac{1}{20}\right) =$$

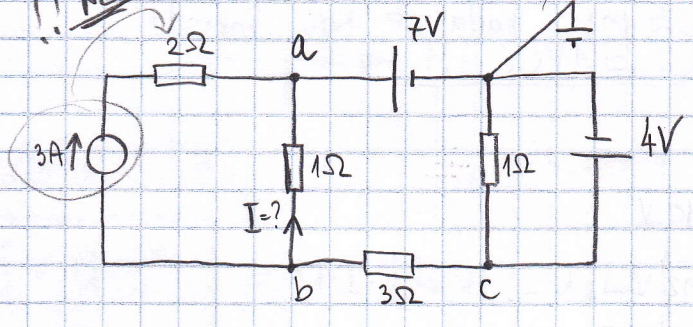
grana sa +40V

$$= 2 - \frac{40}{0} - I_x$$

Ⓑ $$\varphi_B\left[\frac{1}{15} + \frac{1}{30} + \frac{1}{\underbrace{20}_{7+13}}\right] - \varphi_A \cdot \left(\frac{1}{7+13}\right) = \frac{50}{30} + I_x$$

$$\varphi_B = \varphi_A + 40$$

Pr.) !! NE u POT. ČV. !



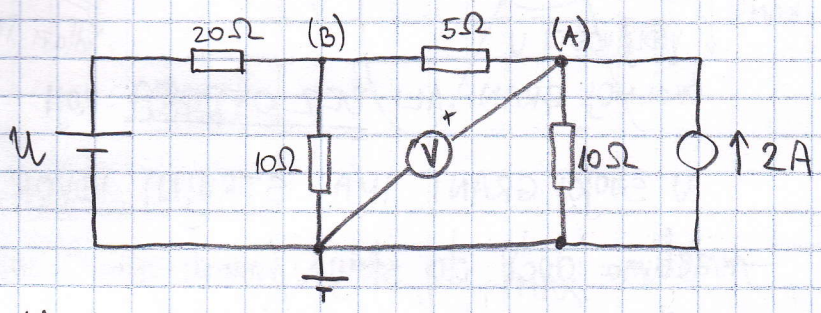
PRIKLADNO ODABRATI UZEMLJENJE!

GLEDATI POTENCIJALE ČVOROVA! BOLJE NEGO SUPERPOZICIJA ILI NEŠTO!

$\varphi_a = 7V$
$\varphi_c = 4V$
} nakon što smo uzemljili!

$$\varphi_b\left(\frac{1}{1}+\frac{1}{3}\right) - \varphi_a\frac{1}{1} - \varphi_c\cdot\frac{1}{3} = -3$$

Pr. 2)



$U_V = 10V$

$U = ?$

PONEKAD ZAPISATI JEDNADŽBU I ZA ONO ŠTO ZNAMO !! (φ_A SADA)

$\varphi_A = 10V$

$$10\cdot\left(\frac{1}{10}+\frac{1}{5}\right) - \varphi_B\cdot\frac{1}{5} = 2$$

$$\vdots$$

$\varphi_B = 5V$

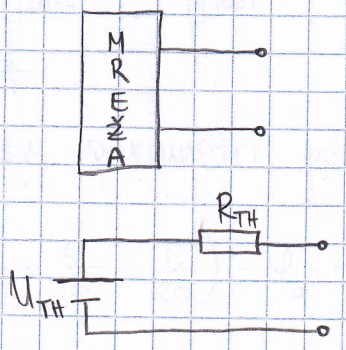
$$5\left(\frac{1}{20}+\frac{1}{10}+\frac{1}{5}\right) - 10\cdot\frac{1}{5} = \frac{U}{20}$$

$$\vdots$$

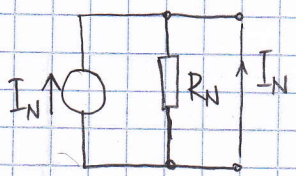
$U = 0V$

MILLMAN → ISTO KAO POTENCIJALI ČVOROVA.

THEVENIN:



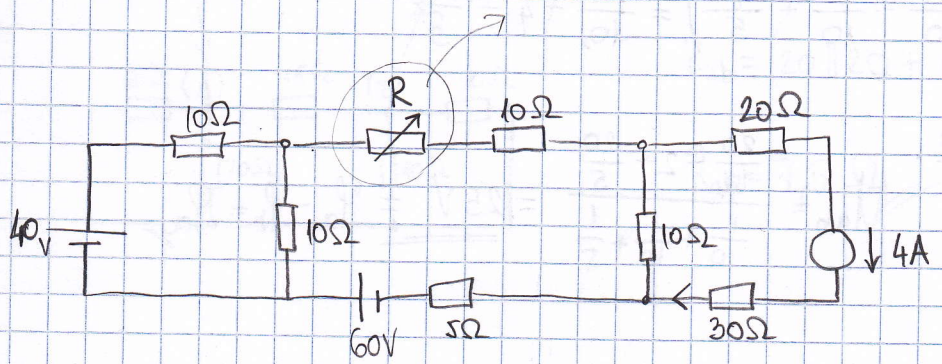
NORTON:



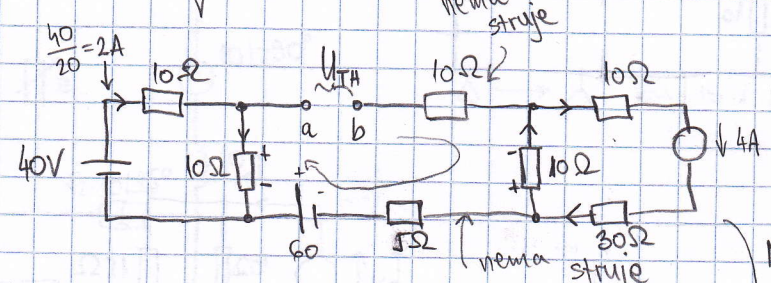
$$R_{TH} = R_N$$

$$U_{TH} = R_N \cdot I_N$$

Pr.)



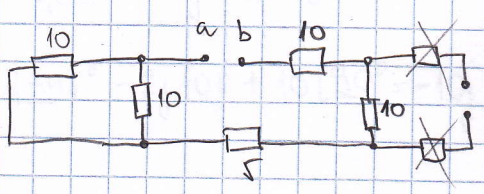
$P_{MAX_R} = ?$



$U_{TH} = U_{ab} = \varphi_a - \varphi_b = 4 \cdot 10 + 60 + 2 \cdot 10 = 120\ V$

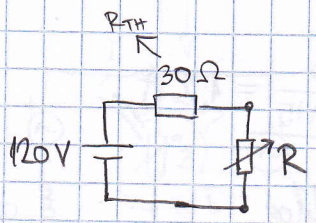
NEMOJ PRENOSITI INFORMACIJE!!

PASIVNA MREŽA (za R_{TH}):



$R_{ab} = R_T = 10 \| 10 + 5 + 10 + 10 =$

$R_{TH} = 30\ \Omega$

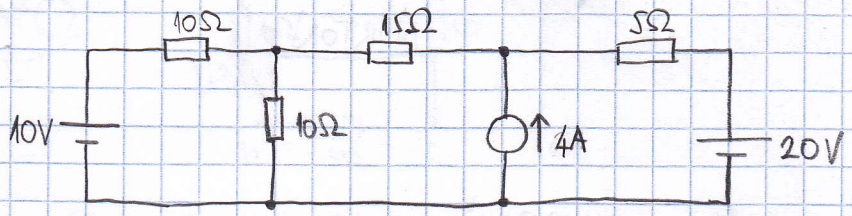


P_{MAX} → kada R bude jednak unutarnjem R_i

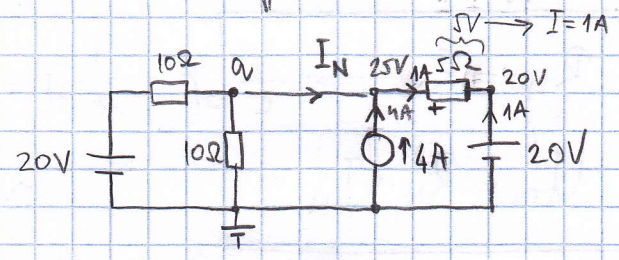
$R_i = R_{TH}$ → $R = R_{TH} = 30\ \Omega \rightarrow P_{MAX}$

$I = 2A \rightarrow P_{MAX} = I^2 \cdot R = 120\,W$

Pr. 2.)



$I_{15\Omega} = ?$



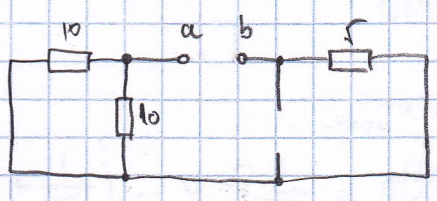
5V → I = 1A

1.kz. → $I_N = 1A - 4A = -3A$

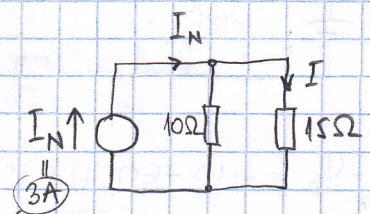
$$\varphi_a\left(\frac{1}{10}+\frac{1}{10}+\frac{1}{5}\right) = \frac{20}{10}+4+\frac{20}{5}$$

MILLMAN:

$$U_{ab} = \frac{\frac{20}{10}+2+\frac{20}{5}}{\frac{1}{10}+\frac{1}{10}+\frac{1}{5}} = 25V = \varphi_a - \varphi_b = \varphi_a$$



$R_N = 10 \parallel 10 + 5 = 10\Omega$

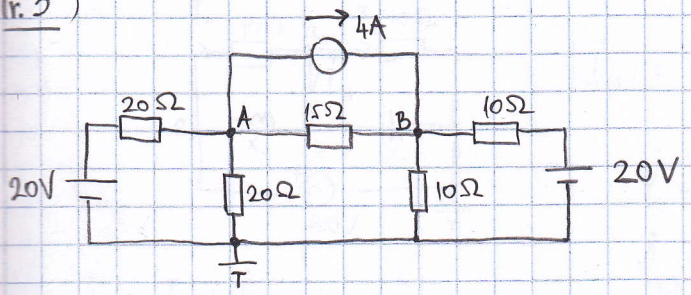


SAMO IZNOS BITAN!

$$I = I_N \cdot \frac{10}{10+15} = 1{,}2\,A$$

strujno dijelilo

Pr. 3)

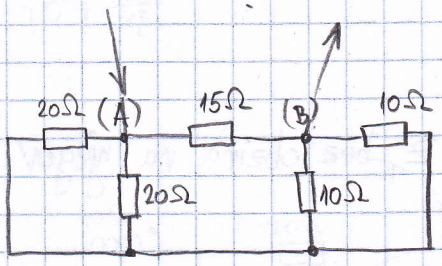


→ nadomjesni parametri po Theveninu, U_{ab}, R_{ab} = ?

$$\varphi_A\left(\frac{1}{20}+\frac{1}{15}+\frac{1}{20}\right)-\varphi_B\cdot\frac{1}{15}=\frac{20}{20}-4$$

$$\varphi_B\left(\frac{1}{10}+\frac{1}{10}+\frac{1}{15}\right)-\varphi_A\cdot\frac{1}{15}=\frac{20}{10}+4$$

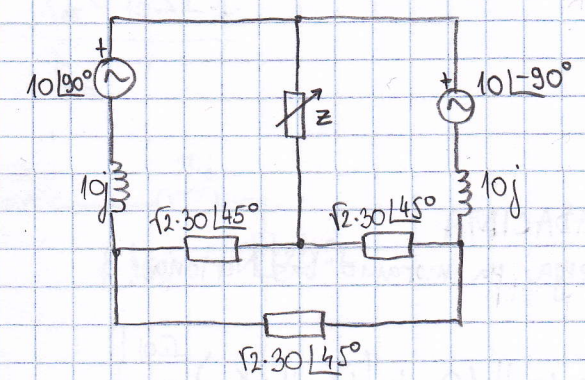
$$U_{ab}=\varphi_A-\varphi_B$$



$$R_{ab}=(20\|20+10\|10)\|15$$

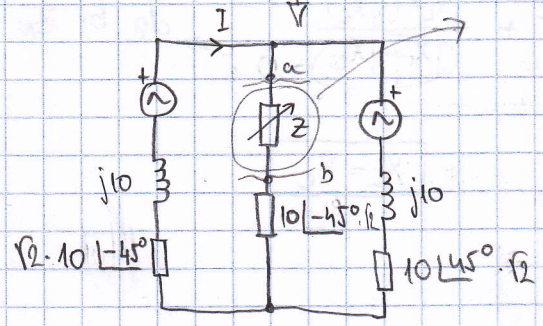
$$R_{ab}=7{,}5\ \Omega$$

Pr. 4.)



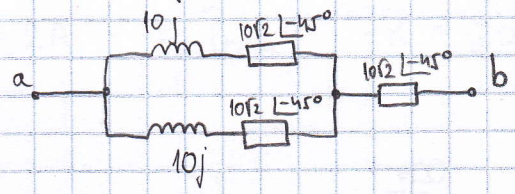
P_{MAX} na Z = ?

Δ → Y za iste 3× MANJI OTPORI

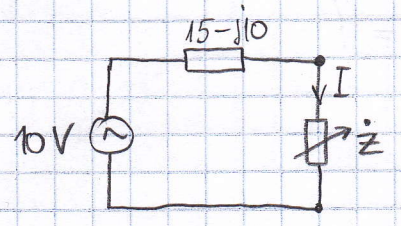


$$I=\frac{10\angle 90^\circ-10\angle -90^\circ}{10j+\underbrace{10\angle -45^\circ}_{\cdot\sqrt{2}}+\underbrace{10\angle -45^\circ}_{\cdot\sqrt{2}}+10j}=2j\ A$$

$$U_{ab}=-2j\,10\sqrt{2}\angle -45^\circ-2j\,10j+10\angle 90^\circ=-10j\ V$$



$\dot{Z}_{ab} = 15 - j10$



P_{MAX}:

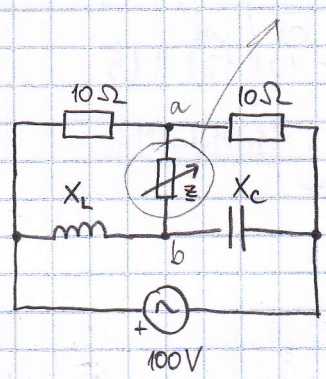
$$R = |\dot{Z}_i|$$

$$\dot{Z} = \dot{Z}_i^* = 15 + j10$$

$$Z_{UK} = 30\Omega \rightarrow I = \frac{1}{3}\,A$$

$$P_{MAX} = I^2 \cdot R = \frac{1}{9} \cdot 15 = 5/3\ W$$

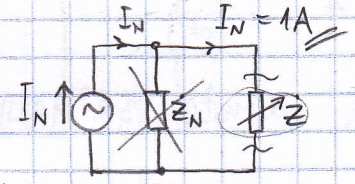
Zad.)



$I = 1A$ kroz $\dot{Z}$, bez obzira na njegov iznos

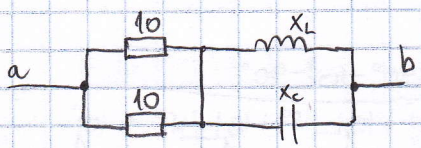
$X_L, X_C = ?$

NORTON:



!! $\dot{Z}_N \rightarrow \infty$ UVIJEK U TAKVIM ZADACIMA!

↳ po Theveninu nema rješenja, pa moramo baš Nortona!

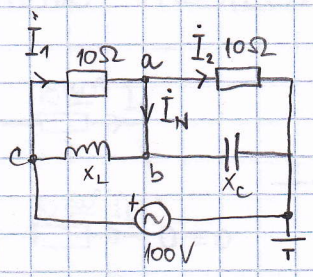


$$\dot{Z}_N = 10 \,||\, 10 + (jX_L \,||\, -jX_C) =$$

$$= 5 + \frac{jX_L \cdot (-jX_C)}{jX_L - jX_C} = 0 \rightarrow \text{da bi } \dot{Z}_N \rightarrow \infty$$

$$\Downarrow$$

$$X_L = X_C$$



$$|\dot{I}_N| = 1\ A$$

$$\dot{I}_1 = \dot{I}_N + \dot{I}_2$$

$$\dot{I}_N = \dot{I}_1 - \dot{I}_2$$

$$\dot{I}_1 = \frac{\varphi_c - \varphi_a}{10} = 5 - \frac{50}{jX_L}$$

$$\dot{I}_2 = \frac{\varphi_a}{10} = 5 + \frac{50}{jX_L}$$

$$\dot{I}_N = -\frac{100}{jX_L}$$

$$1 = \sqrt{0^2 + \frac{100^2}{X_L^2}}$$

$$\vdots$$

nema izvora između

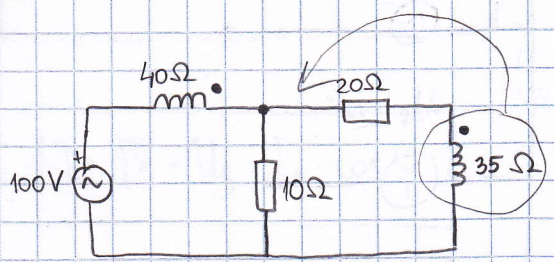
$$\dot{\varphi}_c = 100\ V$$

$$\dot{\varphi}_{a,b}\left(\frac{1}{10} + \frac{1}{jX_L} + \frac{1}{-jX_C} + \frac{1}{10}\right) - \varphi_c\left(\frac{1}{10} + \frac{1}{jX_L}\right) = 0$$

$$\dot{\varphi}_{a,b}\cdot\frac{1}{5} = 10 + \frac{100}{jX_L}$$

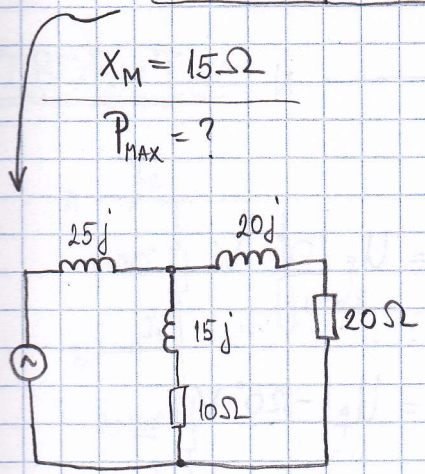
$$\dot{\varphi}_{ab} = 50 + \frac{500}{jX_L}$$

Zad2.)

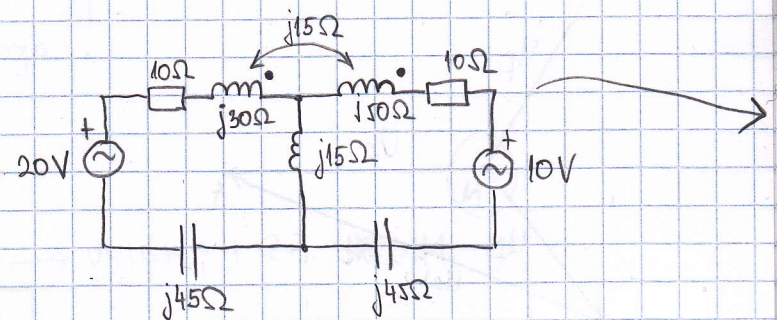


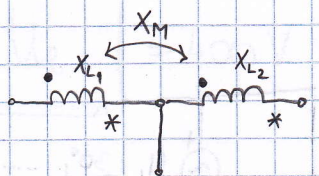
$X_M = 15\Omega$

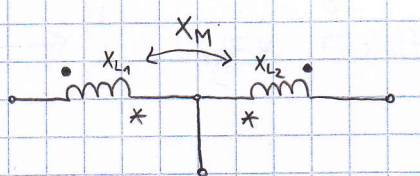
$P_{MAX} = ?$

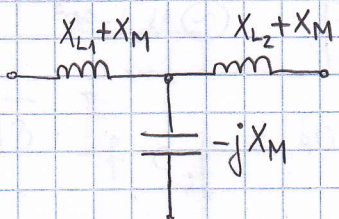


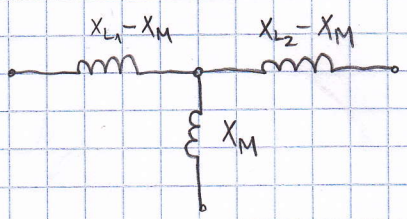
$$\vdots$$

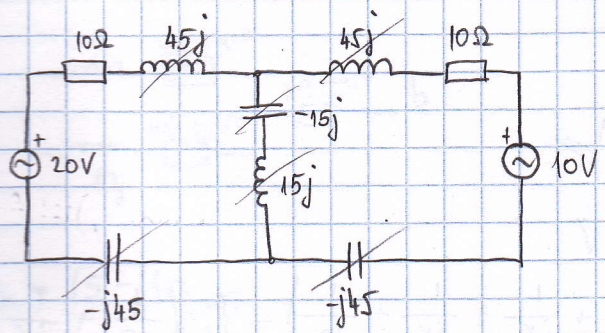


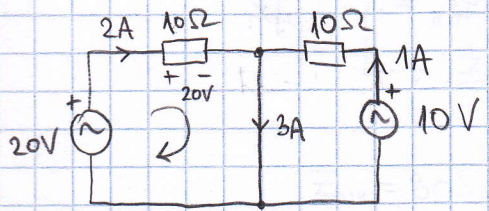






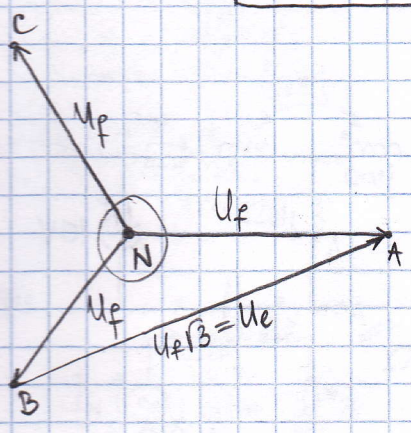


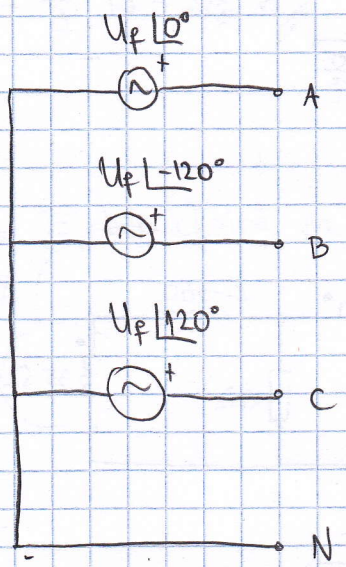


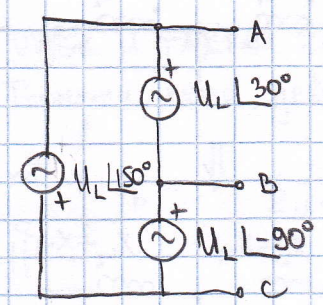


TROFAZNI:

$$\frac{U_f}{U_e} = \frac{230}{400} \rightarrow \boxed{U_f = \frac{U_e}{\sqrt{3}}}$$







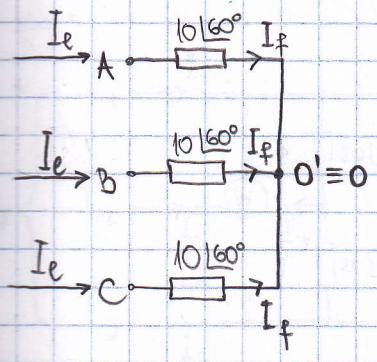
$$\dot{\varphi}_A = U_f \angle 0° \text{ V}$$

$$\dot{\varphi}_B = U_f \angle -120° \text{ V}$$

$$\dot{\varphi}_C = U_f \angle 120° \text{ V}$$

$$\dot{\varphi}_N = 0 \text{ V}$$

SIMETRIČNO TROŠILO: ZVIJEZDA



→ ako postoji NULVOD njegova je struja NULA !

$$I_0 = 0\ A$$

$$U_f = \frac{U_l}{\sqrt{3}} \rightarrow \text{ZVIJEZDA}$$

$$I_f = I_l \rightarrow \text{I ZA NESIM}$$

$U_f = 230\ V$

bez kuteva, svi su pomaknuti...

$$I_f = \frac{230}{10} = 23\ A$$

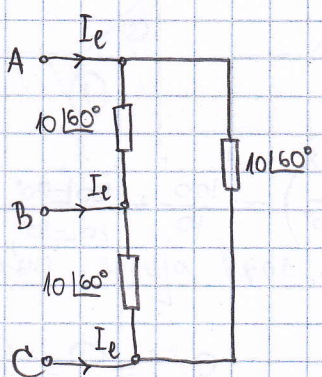
$$P_{\lambda\Delta} = 3 \cdot U_f \cdot I_f \cdot \cos\varphi$$

$$P_{\lambda\Delta} = \sqrt{3} \cdot U_l \cdot I_l \cdot \cos\varphi$$

} ZA OBA SPOJA !

$P = \sqrt{3} \cdot 400 \cdot 23 \cos 60°$ (naš primjer)

TROKUT



→ UVIJEK, I ZA NESIM

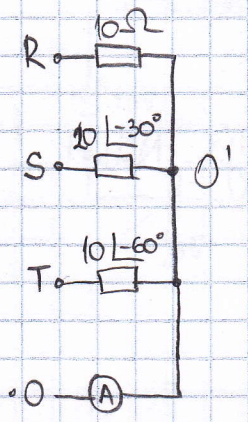
$$U_f = U_l = 400\ V \rightarrow U_l \text{ UVIJEK JEDNAK!}$$

$$I_f = \frac{U_l}{Z} = \frac{400}{10} = 40\ A$$

$$I_l = \sqrt{3} \cdot I_f = 40\sqrt{3}\ A$$

$$P_\Delta = 3P_\lambda$$

Zad.)



spojeno na $U_l = 73\,V$

$\dot{\varphi}_{O'} = \dot{\varphi}_O = 0\,V$ → zbog nulvoda

$\dot{\varphi}_R = 100 \angle 0°$

$\dot{\varphi}_S = 100 \angle -120°$

$\dot{\varphi}_T = 100 \angle 120°$

$$\dot{I}_R = \frac{\dot{\varphi}_O - \dot{\varphi}_{O'}}{10} = 10\,A$$

$$\dot{I}_S = \frac{\dot{\varphi}_S - \dot{\varphi}_{O'}}{20 \angle -30°} = 5 \angle -90°\,A$$

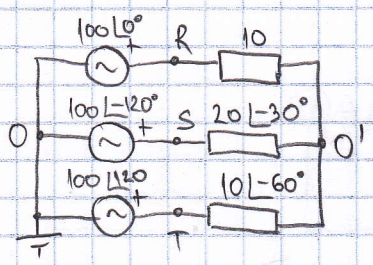
$$\dot{I}_T = \frac{\dot{\varphi}_T - \dot{\varphi}_{O'}}{10 \angle -60°} = 10 \angle 180° = -10\,A$$

$$\dot{I}_O = \dot{I}_R + \dot{I}_S + \dot{I}_T = 5j$$

$$I_A = 5\,A$$ //

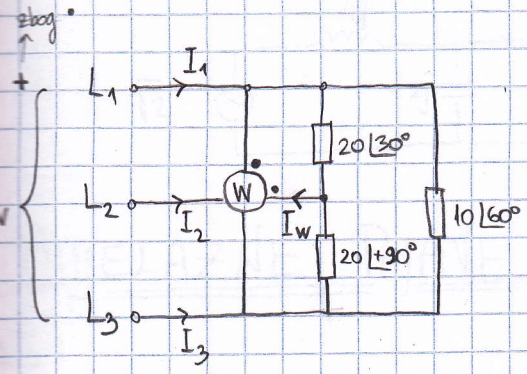
→ umjesto (A) došao (V)

→ moramo naći O'



$$\dot{\varphi}_O\left(\frac{1}{10} + \frac{1}{20 \angle -30°} + \frac{1}{10 \angle -60°}\right) = \frac{100}{10} + \frac{100 \angle -120°}{20 \angle -30°} + \frac{100 \angle 120°}{10 \angle -60°}$$

NESIMETRIČNO, TROKUT:



izvor $U_f = 400\ V$

$\dot{U}_f$

$\dot{\varphi}_1 = 230\angle 0°$

$\dot{\varphi}_2 = 230\angle -120°$

$\dot{\varphi}_3 = 230\angle 120°$

$\dot{I}_w = -\dot{I}_2$

$\dot{U}_w = \dot{U}_{13} = \dot{\varphi}_1 - \dot{\varphi}_3 = 400\angle -30°$

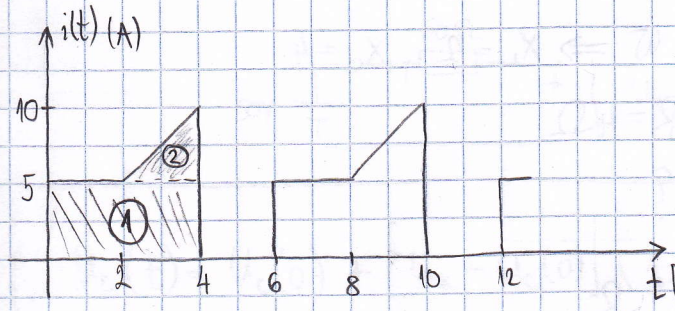
$$P_w = Re\{\dot{U}_w \cdot \dot{I}_w^*\} = U \cdot I \cdot \cos(\alpha_u - \alpha_i)$$

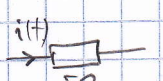
$$\dot{I}_w = \dot{I}_{12} + \dot{I}_{32} = \frac{\dot{\varphi}_1 - \dot{\varphi}_2}{20\angle 30°} + \frac{\dot{\varphi}_3 - \dot{\varphi}_2}{20\angle +90°} = 20 + 20 = 40\ A$$

$P_w = 400 \cdot 40 \cos(-30° - 0°)$

NESINUSNI VALNI OBLICI:

2.)

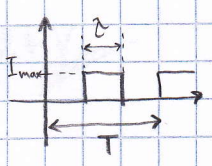




→ takva struja kroz otpor $R = 5\ \Omega$, $P_{SR} = ?$

$P_{SR} = P = I^2 \cdot R$

$T = 6\ ms$

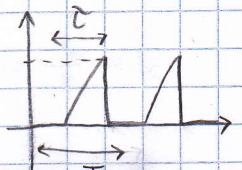


$I_{SR} = I_m \cdot \frac{\tau}{T}$

$I_{ef} = I_m \cdot \sqrt{\frac{\tau}{T}}$

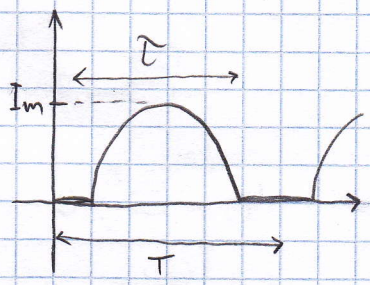
$I_{SR} = \frac{1}{T} \int_T i(t)\,dt$

$I_{EF} = \sqrt{\frac{1}{T} \int_T i^2(t)\,dt}$



$I_{SR} = \frac{I_m}{2} \cdot \frac{\tau}{T}$

$I_{ef} = \frac{I_m}{\sqrt{3}} \cdot \sqrt{\frac{\tau}{T}}$



$$I_{SR} = \frac{I_m}{\pi} \cdot \frac{\tau}{T}$$

$$I_{ef} = \frac{I_m}{\sqrt{2}} \cdot \sqrt{\frac{\tau}{T}}$$

$$I_{EF} = \sqrt{I_{EF_1}^2 + I_{EF_2}^2 + \dots}$$

$$I_{SR} = I_{1SR} + I_{2SR}$$

① $I_{1m} = 5$, $\tau_1 = 4\,ms$ $\Rightarrow$ $I_{ef_1} = 5 \cdot \sqrt{\frac{4}{6}}$

② $I_{2m} = 5$, $\tau_2 = 2\,ms$ $\Rightarrow$ $I_{ef_2} = \frac{5}{\sqrt{3}} \cdot \sqrt{\frac{2}{6}}$

preko integrala:

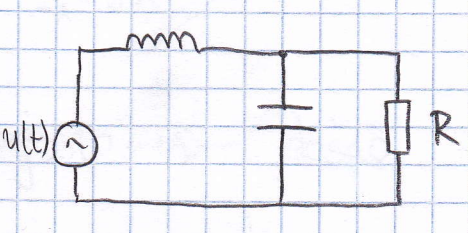
$$I_{ef} = \sqrt{\frac{1}{6\,ms}\left[\int_{0ms}^{2ms} 5^2\,dt + \int_{2ms}^{4ms} \left(\frac{5}{2}t\right)^2 dt + \int_{4ms}^{6ms} 0^2\,dt\right]}$$

$$y - 5 = \frac{5}{2}(t-2)$$

$$y = \frac{5}{2}t$$

Pr.2)

$$u(t) = 4 + 10\sin(\omega t) + 2\sin(2\omega t) = u_1 + u_2 + u_3$$



na $\omega \Rightarrow X_L = 2$, $X_C = 4$

$R = 4\Omega$

$P = ?$

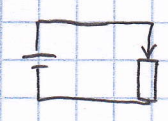
$P = I^2 \cdot R$

$U_1 = 4$

$\omega = 0$

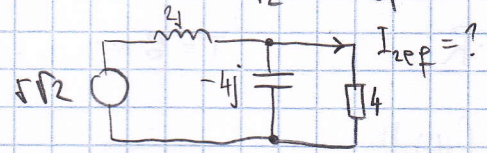
$X_L = 0$

$X_C = \infty$

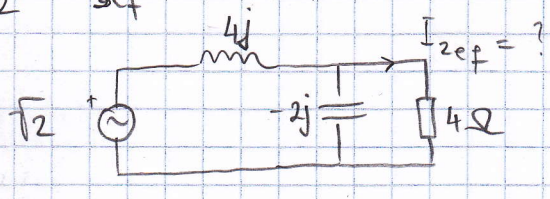


$I_1 = I_{SR} = I_{1ef} = 1A$

$u_2 = 10\sin(\omega t) \rightarrow U_2 = \frac{10}{\sqrt{2}} = U_{2ef}$

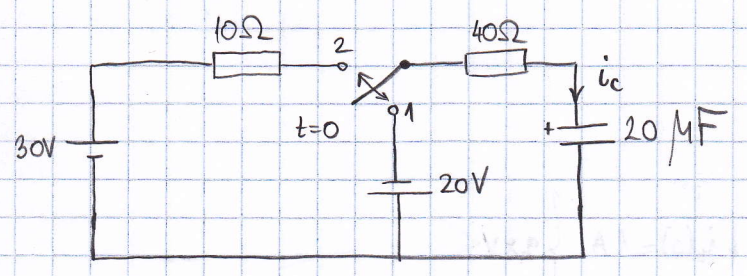


$u_3 = \frac{2}{\sqrt{2}} = u_{3ef}$



PRIJELAZNE POJAVE:

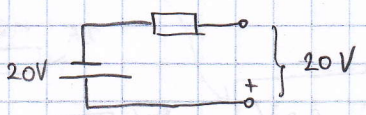
Zad.)



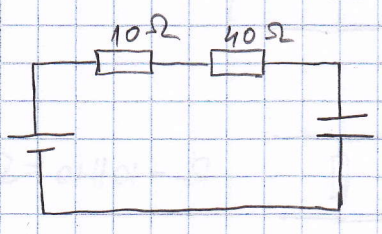
$(t = 0{,}01\ ms)$

$u_c(0) = -20\,V$

$t < 0 \rightarrow \omega = 0 \rightarrow X_c = \infty$

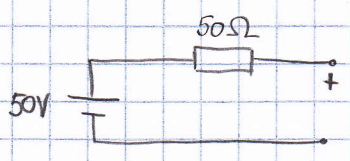


$t \geq 0$



$R = 50\,\Omega$
$C = 20\,\mu F$
$\Big\} \tau = RC = 100\,\mu s = 0{,}1\ ms$

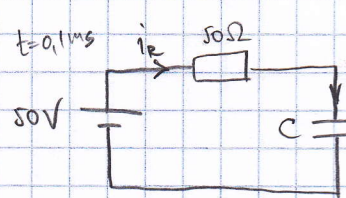
$\Downarrow \sim 5\tau$



$$u_c(t) = u_c(0) + \left[u_k - u_c(0)\right]\left(1 - e^{-\frac{t}{\tau}}\right)$$

$u_c(t) = -20 + 70\left(1 - e^{-1}\right)$

$u_c(t = 0{,}1\ ms) = -20 + 70\left(1 - e^{-1}\right) = 24\,V$



$u_R = 26\,V$

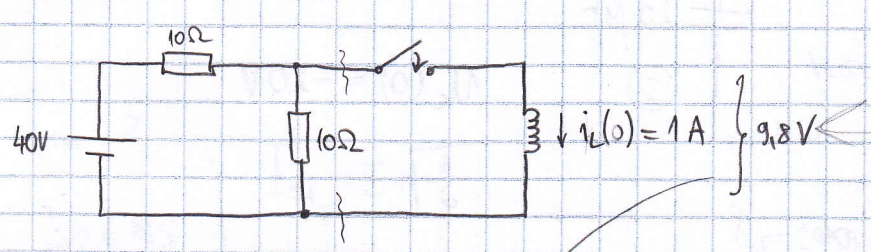
$i(t) = \frac{u_R}{R} = i_c$

RL:

$$\tau = \frac{L}{R}$$

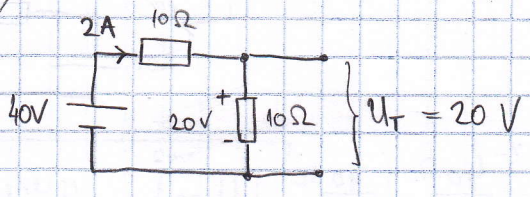
$$u(t) = u(0) + \left[I_K - i_k(0)\right]\left(e^{-\frac{t}{\tau}}\right)$$

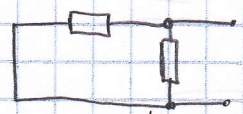
Zad.)



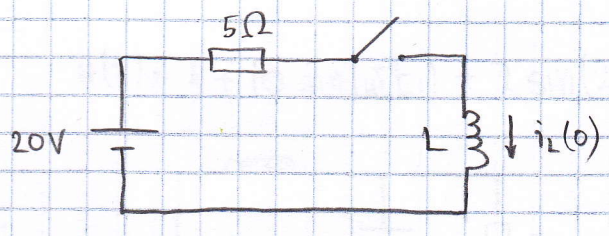
$L = 50\,mH$

$u_{R2}(t = 0{,}01\,s) = 9{,}8\,V$



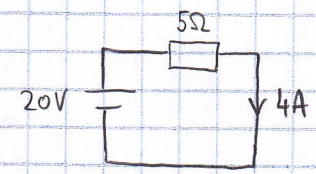


$R_T = 10 \| 10 = 5\Omega$



$$\tau = \frac{L}{R}$$

nakon $> 5\tau$



$$u(t) = 1 + 3e^{-\frac{t}{\tau}}$$

$$u(0{,}01\,ms) = 1 + 3e^{-1} = 2{,}1\,A$$