ISPITI

LJIR 2014

1. $x(t)=e^{2t}\mu(-t)+e^{-3t}\mu(t)$

a) CTFT?

$$X(j\omega)=\int_{-\infty}^{0}e^{2t}e^{-j\omega t}dt+\int_{0}^{\infty}e^{-3t}e^{-j\omega t}dt=\int_{-\infty}^{0}e^{t(2-j\omega)}dt+\int_{0}^{\infty}e^{-t(3+j\omega)}dt=$$

$$=\frac{1}{2-j\omega}e^{t(2-j\omega)}\Big|_{-\infty}^{0}+\frac{1}{3+j\omega}e^{-t(3+j\omega)}\Big|_{0}^{\infty}=\frac{1}{2-j\omega}+\frac{1}{3+j\omega}=\frac{5}{(2-j\omega)(3+j\omega)}$$

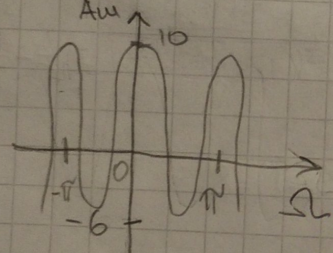
b) $E=\int_{-\infty}^{0}e^{4t}dt+\int_{0}^{\infty}e^{-6t}dt=\frac{1}{4}-\frac{1}{6}(0-1)=\frac{5}{12}$

2. $x(n)=\{4,0,\underline{2},0,4\}$

a) DTFT?

$$X(e^{j\Omega})=\sum_{n=-2}^{2}x(n)e^{-j\Omega n}=4e^{2\Omega j}+2+4e^{-2j\Omega}=2+8\cos(2\Omega)$$

b)



$\measuredangle=0$

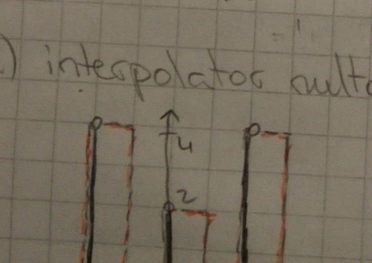
jel ovo dobro

?

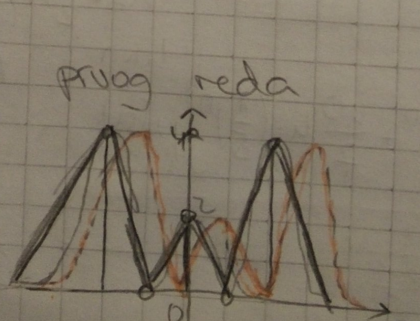
c) $x(n)=\{\underline{4},0,2,0,4\}$

$$X(e^{j\Omega})=\sum_{n=0}x(n)e^{-j\Omega n}=4+2e^{-2j\Omega}+4e^{-4j\Omega}=$$

d) interpolator nultog reda



prvog reda



3. LTI

$u \rightarrow \boxed{S_1} \xrightarrow{w} \boxed{S_2} \rightarrow y$ kaskada: $S_2 \circ S_1$

$S_1: w(n) + \frac{1}{2}w(n-1) = 2u(n)$

$S_2: y(n) - \frac{1}{5}y(n-1) = 4w(n)$

a) $h_{o1}(n) = ?$ u vrem! $\underline{w(n-1)=0}$!

$q = -\frac{1}{2}$

$h_{o1}(n) = C_1\left(-\frac{1}{2}\right)^n$

$C_1 = 2$

$h_{o1}(n) = 2\left(-\frac{1}{2}\right)^n \mu(n) \quad \bullet\!\!-\!\!\circ \quad 2 \cdot \frac{z}{z+\frac{1}{2}}$

??

b) $h_{o2}(n) = ?$ Z-dom!

$W(z) = 1$

$H(z) = \frac{4z}{z-\frac{1}{5}} \cdot 1 = 4 \cdot \frac{z}{z-\frac{1}{5}}$

$z_{1,2} = \frac{-3 \pm \sqrt{9+40}}{20}$

$Y(z) = H(z)\overset{=1}{U(z)} = 4 \cdot \frac{z}{z-\frac{1}{5}} \quad \circ\!\!-\!\!\bullet \quad 4 \cdot \left(\frac{1}{5}\right)^n \mu(n)$

c) ukupan impulsni?

$80z^2 : (10z^2 + 3z - 1) = 8$

$-80 - 24z + 8$

$24z + 8$

$H_1(z) \cdot H_2(z) = H(z)$

$$\frac{2z}{\frac{2z+1}{2}} \cdot \frac{4z}{\frac{5z-1}{5}} = \frac{4z \cdot 20z}{(2z+1)(5z-1)} = \frac{80z^2}{10z^2+3z-1} = 8 + \frac{24z+8}{10z^2+3z-1} = \frac{A}{z-\frac{1}{5}} + \frac{B}{z+\frac{1}{2}}$$

$Az + \frac{1}{2}A + Bz - \frac{1}{5}B = 24z - 8$

$A = \frac{128}{7} \quad B = \frac{40}{7}$

$$H(z) = \left[8 + \frac{128}{7} \cdot 2\left(-\frac{1}{2}\right)^n + \frac{40}{7} 4 \cdot \left(\frac{1}{5}\right)^n\right]\mu(n) \quad /:10$$

$$H(z) = \left[\frac{4}{5} + \frac{64}{35} \cdot 2\left(-\frac{1}{2}\right)^n + \frac{4}{7} \cdot 4\left(\frac{1}{5}\right)^n\right]\mu(n)$$

4. $y''(t) + 11y'(t) + 30y(t) = 2u'(t) + 6u(t)$

$u(t) = 3e^{-5t}\mu(t)$ $\qquad$ $u'(t) = -15e^{-5t}$

$\left.\begin{matrix} y(0^-) = 3 \\ y'(0^-) = -40 \end{matrix}\right| = y(0^+)$ $\qquad$ $y'(0^+) + 40 = 2u(0^+)$

$\underline{y'(0^+) = -34}$

$q^2 + 11q + 30 = 0$

$y_{tot}(t) = C_1 + C_2 = 3$

$-5C_1 - 6C_2 - 12 = -34$ $\Big\}$ $C_1 = -4$, $C_2 = 7$

$q_1 = \underline{-5}$

$q_2 = -6$

$\underline{y_h(t) = C_1 e^{-5t} + Ce^{-6t}}$

$y_p(t) = K_0 e^{-5t} + K_1 te^{-5t}$

$y_p'(t) = -5K_0 e^{-5t} + K_1 e^{-5t} - 5K_1 te^{-5t}$

$y''(t) = 25K_0 e^{-5t} - 5K_1 e^{-5t} + 25te^{-5t} - 5K_1 e^{-5t}$

$25K_0 - 5K_1 - 55K_0 + 11K_1 + 30K_0 - 5K_1 = -12$

$K_1 = -12$

$y_p(t) = -12te^{-5t}\mu(t)$ $\qquad$ $y_p'(t) = -12e^{-5t} + 60te^{-5t}$

$y_{tot}(t) = (\underbrace{-4e^{-5t} + 7e^{-5t}}_{\text{pri.}} - \underbrace{12te^{-5t}}_{\text{pris.}})\mu(t)$

y_{mirni} : $pu = 0 \Rightarrow y(0^+) = 0$

$y'(0^+) = 6$

$y_{mir}(0) = C_1 + C_2 = 0$

$-5C_1 - 6C_2 - 12 = 6$ $\Big\}$ $C_1 = 18$, $C_2 = -18$

$y_m(t) = (18e^{-5} - 18e^{-6t} - 12te^{-5t})\mu(t)$

y_{nep} : y_h uz pu bez u: $\quad C_1 + C_2 = 3$

$-5C_1 - 6C_2 = -40$ $\Big\}$ $C_1 = -22$, $C_2 = 25$

$y_{nep}(t) = (-22e^{-5} + 25e^{-6})\mu(t)$

4. $y''(t)+11y'(t)+30y(t)=2u'(t)+6u(t)$ $\qquad s^2+11s+30 \Rightarrow \begin{matrix} s_1=-5 \\ s_2=-6 \end{matrix}$

$u(t)=3e^{-5t}\mu(t) \quad \circ\!\!-\!\!\bullet \quad 3\frac{1}{s+5}$

$y(0^-)=3$

$y'(0^-)=-40$

$$s^2Y(s)-sy(0^-)-y'(0^-)+11sY(s)-11y(0^-)+30Y(s)=2sU(s)-2u(0^-)+6U(s)$$

$$Y(s)(s^2+11s+30)=3s-7+U(s)(2s+6)$$

$$Y(s)=\underbrace{\frac{3s-7}{s^2+11s+30}}_{Y_{nep}}+\underbrace{\frac{2s+6}{s^2+11s+30}U(s)}_{Y_{mirni}}$$

$$\frac{3s-7}{(s+5)(s+6)}=\frac{A}{s+5}+\frac{B}{s+6}=-\frac{22}{s+5}+\frac{25}{s+6} \quad \bullet\!\!-\!\!\circ \quad y_{nep}=(-22e^{-5t}+25e^{-6t})\mu(t)$$

$As+6A+Bs+5B=3s-7$

$A=-22$

$B=25$

$$\frac{(2s+6)\cdot 3}{(s+5)^2(s+6)}=\frac{A}{(s+5)^2}+\frac{B}{s+5}+\frac{C}{s+6} \quad /\cdot(s+5)^2(s+6)$$

$$As+6A+Bs^2+11Bs+30B+Cs^2+10Cs+25C=6s+18$$

$$\left.\begin{matrix} B+C=0 \\ A+11B+10C=6 \\ 6A+30B+25C=18 \end{matrix}\right\} \begin{matrix} A=-12 \\ B=18 \\ C=-18 \end{matrix}$$

$$(-12te^{-5t}+18e^{-5t}-18e^{-6t})\mu(t)=y_{mirni}(t)$$

5. LTI $\quad y(n)+a_1y(n-1)+a_2y(n-2)=b_0u(n)$

$u_1(n)=1^n \Rightarrow y_1(n)=1^n$ $\qquad u(n)=Uz^n \Rightarrow y(n)=H(z)Uz^n$

$u_2(n)=\left(\frac{1}{2}\right)^n \Rightarrow y_2(n)=2\left(\frac{1}{2}\right)^n$ $\qquad H(z)=z\ ?$

$u_3(n)=\left(\frac{1}{4}\right)^n \Rightarrow y_3(n)=4\left(\frac{1}{4}\right)^n$

$$H(z)=\frac{b_0z^2}{z^2+a_1z+a_2}=b_0+\frac{b_0}{a_1}z+\frac{b_0}{a_2}z^2=z^2\left(b_0z^{-2}+\frac{b_0}{a_1}z^{-1}+\frac{b_0}{a_2}z^0\right)$$

$$Y(z)+a_1z^{-1}Y(z)+a_1y(-1)+a_2z^{-2}Y(z)=b_0U(z)$$

$$\sum_{n=-2}^{0}h(n)z^{-n}$$

?

$Y(z)=H(z)U(z)$

$h(0)=b_0$

$h(1)=\frac{b_0}{a_1}$

$h(2)=\frac{b_0}{a_2}$

$\frac{z}{z-1}=b_0\frac{z}{z-1} \Rightarrow b_0=1$ $\qquad a_2=\frac{1}{4}$

$\frac{2z}{z-\frac{1}{2}}=\frac{1}{a_1}\frac{z}{z-\frac{1}{2}} \Rightarrow a_1=\frac{1}{2}$

5.

$$H(z) = \frac{b_0}{1 + a_1 z^{-1} + a_2 z^{-2}} = \frac{z^2 b_0}{z^2 + a_1 z + a_2}$$

$$H(z) = \frac{Y(z)}{U(z)}\Big| = 1 = \frac{b_0}{1 + a_1 + a_2} \Rightarrow b_0 = 1 + a_1 + a_2$$

$$H\left(\frac{1}{2}\right) = 2 = \frac{b_0}{1 + 2a_1 + 4a_2} \Rightarrow 2 + 4a_1 + 8a_2 = 1 + a_1 + a_2 \Rightarrow 3a_1 + 7a_2 = -1 \qquad a_1 = -\frac{1}{2}$$

$$\Rightarrow 15a_1 + 63a_2 = -3 \qquad a_2 = \frac{1}{14}$$

$$H\left(\frac{1}{4}\right) = 4 = \frac{b_0}{1 + 4a_1 + 16a_2} \Rightarrow 4 + 16a_1 + 64a_2 = 1 + a_1 + a_2$$

$$b_0 = \frac{4}{7}$$

b) $u(n) = \left(\frac{1}{8}\right)^n \circ\!\!-\!\!\bullet \frac{z}{z - \frac{1}{8}}$

$$y(n) = H\left(\frac{1}{8}\right) \cdot \left(\frac{1}{8}\right)^n$$

$$H\left(\frac{1}{8}\right) = \frac{\frac{4}{7}}{1 + 8 \cdot \frac{-1}{2} + \frac{1}{14} 64} = \frac{4}{11} \Rightarrow y(n) = \frac{4}{11}\left(\frac{1}{8}\right)^n$$

c) y_{pris}? $u(n) = \left(\frac{1}{8}\right)^n \mu(n)$

?

$$y_p(n) = K \cdot \left(\frac{1}{8}\right)^n$$

$$y_p(n-1) = K \cdot \left(\frac{1}{8}\right)^n \cdot 8$$

$$y_p(n-2) = 64K\left(\frac{1}{8}\right)^n$$

$$K\left(\frac{1}{8}\right)^n - 4K\left(\frac{1}{8}\right)^n + \frac{32}{7}K\left(\frac{1}{8}\right)^n = \frac{4}{7}\left(\frac{1}{8}\right)^n$$

$$\frac{11}{7}K = \frac{4}{7} \Rightarrow K = \frac{4}{11}$$

$$y_{pris}(n) = \frac{4}{11}\left(\frac{1}{8}\right)^n$$