(2.) MASOVNE INSTRUKCIJE

SiS

UVOD U SUSTAVE

1) KONTINUIRANI

2) DISKRETNI

1) LINEARNI → homogenost: $f(\alpha x) = \alpha f(x)$, $\forall \alpha$, aditivnost: $f(x_1 + x_2) = f(x_1) + f(x_2)$

2) NELINEARNI

superpozicija: $f(\alpha x_1 + \beta x_2) = \alpha f(x_1) + \beta(x_2)$, $\forall \alpha, \beta$

VREMENSKA STALNOST → $S(u(t)) = y(t)$

$S(u(t-T)) = y(t-T)$ → vremenski nepromjenjiv

NEKAUZALNI → sustavi koji gledaju "u budućnost"

KAUZALNI → sustavi koji gledaju u sadašnjost i/ili prošlost

MEMORIJSKI → ako ovisi o prošloj ili budućoj vrijednosti

(6) 2010

$y(n) = \sum_{k=0}^{n} u(k)$ − memorijski (mora pamtiti sve prošle)
− nekauzalan (za negativne gleda u budućnost)

− linearnost: $u(k) = \alpha u_1(k) + \beta u_2(k)$

$y_1(n) = \sum_{k=0}^{n} u_1(k)$, $y_2(n) = \sum_{k=0}^{n} u_2(k)$

pa je $y(n) = \sum_{k=0}^{n} (\alpha u_1(k) + \beta u_2(k)) = \alpha \sum_{k=0}^{n} u_1(k) + \beta \sum_{k=0}^{n} u_2(k)$

$= \alpha y_1(n) + \beta y_2(n)$

− sustav je linearan

− vremenska stalnost: $y(n-N) = \sum_{k=0}^{n-N} u(k)$

$$y_1(n) = \sum_{k=0}^{n} u_1(k-N) = \left| \begin{matrix} k-N = a \\ k \to 0,\ a = -N \\ k \to n,\ a = n-N \end{matrix} \right|$$

$$= \sum_{a=-N}^{n-N} u(a) = \left| a = k \right| = \sum_{k=-N}^{n-N} u(k) \neq \sum_{k=0}^{n-N} u(k)$$

− vremenski je promjenjiv

7 [2010.]

$$y(n)=\sum_{k=-\infty}^{n}(n-k)u(k)$$

- memorijski (uvijek ako je Σ ili $\int$ ...)
- kauzalan (uvijek gleda u prošlost)

- linearnost: isti postupak
- sustav je linearan

- vremenska stalnost: $y(n-N)=\sum_{k=-\infty}^{n-N}(n-N-k)u(k)$

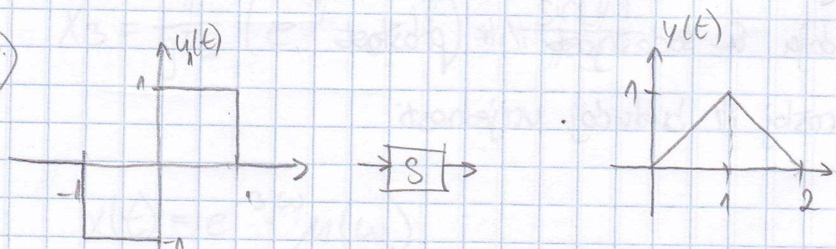
$$y_1(n)=\sum_{k=-\infty}^{n}(n-k)u(k-N)=\left|\begin{array}{l}a=k-N\\ k\to-\infty,\ a\to-\infty\\ k\to n,\ a\to n-N\end{array}\right|$$

$$=\sum_{a=-\infty}^{n-N}(n-(a+N))u(a)=|a=k|$$

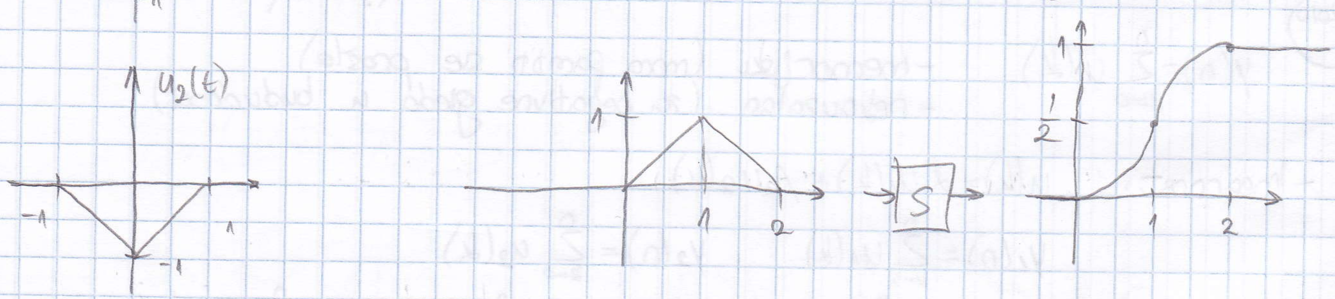
$$=\sum_{k=-\infty}^{n-N}(n-k-N)u(k)=\sum_{k=-\infty}^{n-N}(n-N-k)u(k)$$

- sustav je vremenski nepromjenjiv

8 [2010.]



$$S(u(t))=\int_{-\infty}^{t}-u(\tau-1)\,d\tau$$



koef. smjera: + ∪

− ∩

KONVOLUCIJA

$$y(t) = u(t) * h(t) = \int_{-\infty}^{\infty} u(\tau) h(t-\tau) d\tau$$

$$y(n) = u(n) * h(n) = \sum_{m=-\infty}^{\infty} u(m) h(n-m)$$

1) KOMUTATIVNOST $\quad x * y = y * x$

2) DISTRIBUTIVNOST $\quad (x_1 * (x_2 + x_3)) = (x_1 * x_2 + x_1 * x_3)$

3) ASOCIJATIVNOST $\quad (x_1 * (x_2 * x_3)) = ((x_1 * x_2) * x_3)$

4) POMAK $\quad x(t) * y(t) = z(t)$

$$x(t-T_1) * y(t-T_2) = z(t-T_1-T_2)$$

5) KONVOLUCIJA S IMPULSOM $\quad x * \delta = x$

12 2010.

$$(\sin(n) * \delta(n+1)) \cdot \delta(n-2) = \sin(n+1) \cdot \delta(n-2) = \sin(2+1) \cdot \delta(n-2)$$
$$= \sin(3) \delta(n-2)$$

9 2010.

$x(t) = e^{-3t} \mu(t)$

$y(t) = e^{-2t} \mu(t)$

$$x(t) * y(t) = \int_{-\infty}^{\infty} x(\tau) y(t-\tau) d\tau$$
$$= \int_{-\infty}^{\infty} e^{-3\tau} \mu(\tau) \cdot e^{-2t+2\tau} \mu(t-\tau) d\tau$$
$$= \left| \mu(\tau) = \begin{cases} 1, & \tau \geq 0 \\ 0, & \tau < 0 \end{cases}, \quad \mu(t-\tau) = \begin{cases} 1, & \tau \leq t \\ 0, & \tau > t \end{cases} \right|$$
$$= \int_0^t e^{-3\tau} \cdot e^{-2t} \cdot e^{2\tau} d\tau = e^{-2t} \int_0^t e^{-\tau} d\tau$$
$$= e^{-2t} (-1) e^{-\tau} \Big|_0^t = e^{-2t} (1 - e^{-t}) = (e^{-2t} - e^{-3t}) \mu(t)$$

11 2010.

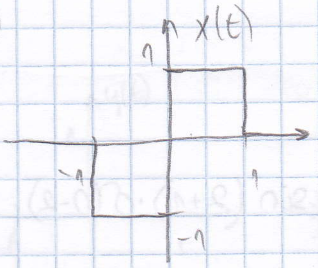
$$h(n) = n\mu(n)$$
$$u(n) = \mu(n)$$

$$y(n) = h(n) * u(n) = \sum_{m=-\infty}^{\infty} h(m)u(n-m) = \sum_{m=\infty}^{\infty} m\mu(m)\cdot\mu(n-m)$$

$$= \sum_{m=0}^{n} m = \frac{n(n+1)}{2} = y(n)$$

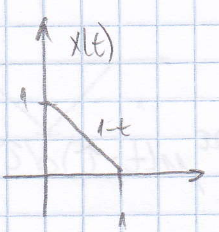
$$\frac{n(n+1)}{2} = 2019045 \quad , \; n_1 = 2009 \checkmark \quad (n \geq 0)$$
$$n_2 = -2010 \; \times$$

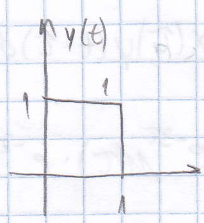
10 2010.



$$x(t) * \mu(t) = \int_{-\infty}^{\infty} x(\tau)\mu(t-\tau)\,d\tau$$
$$= \int_{-\infty}^{t} x(\tau)\,d\tau$$

14 2009.





$$x(t) = (1-t)\big(\mu(t) - \mu(t-1)\big)$$

$$y(t) = 1\cdot\big(\mu(t) - \mu(t-1)\big)$$

$$x(t) * y(t) = \int_{-\infty}^{\infty} x(\tau)y(t-\tau)\,d\tau = \int_{-\infty}^{\infty}(1-\tau)\big(\mu(\tau)-\mu(\tau-1)\big)\big(\mu(t-\tau)-\mu(t-\tau-1)\big)d\tau$$

$$= \int_{-\infty}^{\infty}(1-\tau)\mu(\tau)\mu(t-\tau)\,d\tau - \int_{-\infty}^{\infty}(1-\tau)\mu(\tau)\mu(t-\tau-1)\,d\tau$$

$$- \int_{-\infty}^{\infty}(1-\tau)\mu(\tau-1)\mu(t-\tau)\,d\tau + \int_{-\infty}^{\infty}(1-\tau)\mu(\tau-1)\mu(t-\tau-1)\,d\tau$$

$$= \int_{0}^{t}(1-\tau)\,d\tau - \int_{0}^{t-1}(1-\tau)\,d\tau - \int_{1}^{t}(1-\tau)\,d\tau + \int_{1}^{t-1}(1-\tau)\,d\tau$$

$$= \left(t-\tfrac{1}{2}t^2\right)\mu(t) - \left(t-1-\tfrac{1}{2}t^2+t-\tfrac{1}{2}\right)\mu(t-1) - \left(t-\tfrac{1}{2}t^2-1+\tfrac{1}{2}\right)\mu(t-1) + \left(t-1-\tfrac{1}{2}t^2 + \ldots\right.$$

JEDNADŽBE DIFERENCIJA

- odzivi:
  - mirni
  - nepobuđeni
  - totalni
  - prirodni
  - prisilni
  - impulsni

17,18,19,20 2008

$$y(n) - 6y(n-1) + 8y(n-2) = 4u(n)$$

$$u(n) = (1-3n)\mu(n)$$

- poč. uvjeti $y(-1)=2$, $y(-2)=1$

- opća homogena jednadžba:

$$y(n) - 6y(n-1) + 8y(n-2) = 0$$

$$y_h(n) - 6y_h(n-1) + 8y_h(n-2) = 0$$

$$\boxed{y_h(n) = Cg^n} \rightarrow Cg^n - 6Cg^{n-1} + 8Cg^{n-2} = 0$$

pretpostavljeni homogeni oblik

$$Cg^{n-2}(g^2 - 6g + 8) = 0$$

$g^2 - 6g + 8 = 0$ → karakteristična jednadžba sustava

$g_1 = 4$ $g_2 = 2$ → karakteristične frekvencije sustava

$$\boxed{y_h(n) = C_1 4^n + C_2 2^n}$$

(ako je npr. $g_1 = 2$, $g_2 = g_3 = g_4 = 5$, $g_5 = 6$
onda je $y_h(n) = C_1 2^n + (C_2 + C_3 n + C_4 n^2) 5^n + C_5 6^n$)

- pretpostavljeni partikularni oblik ovisi o korijenima i pobudi sustava

	POBUDA	PRETPOSTAVLJENI OBLIK
1)	A	K
2)	$A\lambda^n$	$K\lambda^n n^m$
3)	n^M	$K_0 + K_1 n + \ldots + K_M n^M$
4)	$n^M \lambda^n$	$(K_0 + K_1 n + \ldots + K_M n^M)\lambda^n n^m$
5)	$A\sin(\omega_0 n + \psi)$	$K\sin(\omega_0 n + \psi)$

m - koliko se puta λ pojavljuje kao karakteristična frekv. sustava

1) $u(n) = 4\mu(n)$

$g_1 = 0{,}1 \;,\; g_2 = 0{,}2$

$y_p(n) = K \;,\; n \geq 0$

- part. rj. živi samo na intervalu na kojem živi pobuda koja ga je uzrokovala

2) $u(n) = 4 \cdot (0{,}3)^n \mu(n)$

$g_1 = -0{,}3 \quad g_2 = 0{,}1$

$y_p(n) = K \cdot (0{,}3)^n \cdot n^0 \;,\; n \geq 0$

3) $u(n) = 4 \cdot (0{,}3)^n \mu(n)$

$g_1 = -0{,}3 \;,\; g_2 = 0{,}3$

$y_p(n) = K \cdot (0{,}3)^n \cdot n^1 \;,\; n \geq 0$

$\boxed{\text{PRIMJERI}}$

4) $u(n) = (n^2 + n)\mu(n)$

$g_1 = -0{,}3 \;,\; g_2 = 0{,}2$

$y_p(n) = K_0 + K_1 n + K_2 n^2 \;,\; n \geq 0$

- partikularno rješenje:

$$u(n) = (1-3n)\mu(n)$$

$$q_1 = 4 \quad , \quad q_2 = 2$$

$$y_p = (k_0 + k_1 n) \quad , \quad n \geq 0$$

$$y_p(n) - 6y_p(n-1) + 8y_p(n-2) = 4u(n)$$

$$(k_0 + k_1 n) - 6(k_0 + k_1(n-1)) + 8(k_0 + k_1(n-2)) = 4(1-3n)$$

$$(3k_0 - 10k_1) + (3k_1)n = 4 - 12n$$

$$\left.\begin{aligned} 3k_0 - 10k_1 &= 4 \\ 3k_1 &= -12 \end{aligned}\right\} \begin{aligned} k_0 &= -12 \\ k_1 &= -4 \end{aligned}$$

$$\boxed{y_p(n) = (-12 - 4n) \quad , n \geq 0}$$

- mirni odziv: $y_m(n) = y_h(n) + y_p(n) = C_1 \cdot 4^n + C_2 2^n + (-12-4n) \; , \; n \geq 0$

$$\cancel{y(-1) = 2} \rightarrow y(-1) = 0$$

$$\cancel{y(-2) = 1} \rightarrow y(-2) = 0$$

$$y(n) = 4u(n) + 6y(n-1) - 8y(n-2)$$

$$y(0) = 4u(0) + 6y(-1) - 8y(-2) = 4$$

$$y(1) = 4u(1) + 6y(0) - 8y(-1) = 16$$

$$\left.\begin{aligned} y_m(0) &= C_1 + C_2 - 12 = 4 \\ y_m(1) &= 4C_1 + 2C_2 - 16 = 16 \end{aligned}\right\} \begin{aligned} C_1 &= 0 \\ C_2 &= 16 \end{aligned}$$

$$\boxed{y_m(n) = 16 \cdot 2^n + (-12 - 4n) \, , \, n \geq 0}$$

- nepobuđeni odziv: $y_n(n) = y_h(n) = C_1 \cdot 4^n + C_2 2^n$

$y(-1) = 2 \quad , \; y(-2) = 1$

$y_n(-1) = C_1 4^{-1} + C_2 2^{-1} = 2 \rightarrow C_1 + 2C_2 = 8$
$y_n(-2) = C_2 4^{-2} + C_2 2^{-2} = 1 \rightarrow C_1 + 4C_2 = 16$
$C_1 = 0$, $C_2 = 4$

$$\boxed{y_n(n) = 4 \cdot 2^n}$$

- totalni odziv: 1) $y_t(n) = y_m(n) + y_n(n) = \underbrace{20 \cdot 2^n}_{\text{prirodni}} + \underbrace{(-12 - 4n)}_{\text{prisilni}}, \; n \geq 0$

2 načina 2) isto ko i kod mirnog samo sa zadanim poč. uvjetima

- prisilni odziv: jednak partikularnom rješenju
- prirodni odziv: totalni bez prisilnog