Signali i sustavi

Prvi međuispit (grupa A) – 24. ožujka 2011.

1. Totalna snaga vremenski kontinuiranog signala $x(t) = 2 + 4\sin(t)$ je:

a) 2 b) 4 c) 6 d) 12 e) 20

2. Energija vremenski diskretnog signala $x(n) = \left(\frac{1}{5}\right)^{2n}\mu(n)$ je:

a) $\frac{24}{25}$ b) $\frac{624}{625}$ c) $\frac{25}{24}$ d) $\frac{625}{624}$ e) $+\infty$

3. Totalna snaga vremenski diskretnog signala $x(n) = 4 + 2\sin(\frac{\pi}{3}n)$ je:

a) 4 b) 6 c) 16 d) 18 e) 20

4. Koji od zadanih signala NIJE periodičan?

a) $\cos(\pi t)$ b) $\cos(3\pi t) + \sin(5\pi t)$ c) $\cos(3\pi t) + \sin(3t)$ d) $\cos(3t) + \cos(5t)$ e) $\mathrm{tg}(\frac{\pi}{5}t)$

5. Samo jedna od navednih tvrdnji NE VRIJEDI za Diracovu distribuciju $\delta(t)$. Koja?

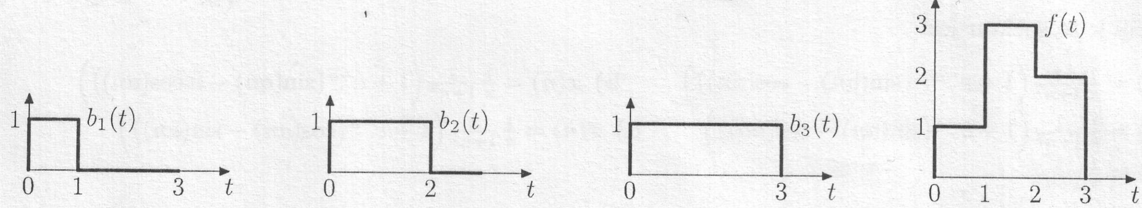
a) Za glatku $f(t) : \mathbb{R} \to \mathbb{R}$ vrijedi $\int_{-\infty}^{+\infty} f(t)\,\delta(t)\,dt = f(0)$.
b) Diracova distribucija je parna distribucija.
c) Za glatku $f(t) : \mathbb{R} \to \mathbb{R}$ vrijedi $\int_{-\infty}^{+\infty} f(t)\,\delta'(t)\,dt = -f'(0)$.
d) Generalizirana derivacija Heavisideove step funkcije je Diracova distribucija, odnosno $\mu'(t) = \delta(t)$.
e) Za glatku $f(t) : \mathbb{R} \to \mathbb{R}$ vrijedi $f(t)\,\delta(t - t_0) = f(t_0)$.

6. Generalizirana derivacija signala $f(t) = \mu(4 - t) + \mu(t) + (t - 2)^2\big(\mu(t - 2) - \mu(t - 4)\big)$ je:

a) $-\delta(t - 4) + \delta(t) + 2(t - 2)\big(\mu(t - 2) - \mu(t - 4)\big) - 4$ b) $-5\,\delta(t - 4) + \delta(t) + 2(t - 2)\big(\mu(t - 2) - \mu(t - 4)\big)$
c) $-3\,\delta(t-4) + \delta(t) + 2(t-2)\big(\mu(t-2) - \mu(t-4)\big)$ d) $2(t-2)\big(\mu(t-2) - \mu(t-4)\big)$ e) $2(t-2)\big(\mu(t-2) - \mu(t-4)\big) - 4$

7. Signal $f(t) : [0,3] \to \mathbb{R}$ prikazujemo kao linearnu kombinaciju tri osnovna signala $b_1(t) : [0,3] \to \mathbb{R}$, $b_2(t) : [0,3] \to \mathbb{R}$ i $b_3(t) : [0,3] \to \mathbb{R}$. Kako glasi linearni rastav signala $f(t)$ po osnovnim signalima?

a) $(-1, 2, 1)$ b) $(-2, 2, 1)$ c) $(1, 2, -1)$ d) $(2, -1, -2)$ e) $(-2, 1, 2)$



8. Promatramo signal $x(t) = \sin(20\pi t) + \cos(40\pi t) + \sin(60\pi t)$. Kojim periodom očitanja T_S moramo očitati taj signal da ne dođe do preklapanja spektra?

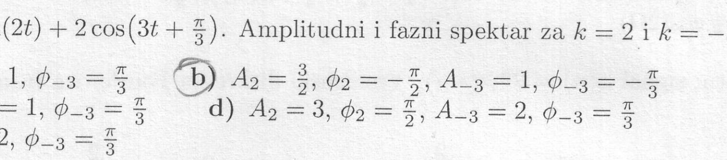
a) $T_S > 20$ b) $T_S > 60$ c) $T_S < 1/20$ d) $T_S < 1/60$ e) Ne postoji takav period T_S!

9. Zadan je signal $x(t) = 3\sin(2t) + 2\cos(3t + \frac{\pi}{3})$. Amplitudni i fazni spektar za $k = 2$ i $k = -3$ su:

a) $A_2 = \frac{3}{2}$, $\phi_2 = \frac{\pi}{2}$, $A_{-3} = 1$, $\phi_{-3} = \frac{\pi}{3}$ b) $A_2 = \frac{3}{2}$, $\phi_2 = -\frac{\pi}{2}$, $A_{-3} = 1$, $\phi_{-3} = -\frac{\pi}{3}$
c) $A_2 = \frac{3}{2}$, $\phi_2 = -\frac{\pi}{2}$, $A_{-3} = 1$, $\phi_{-3} = \frac{\pi}{3}$ d) $A_2 = 3$, $\phi_2 = \frac{\pi}{2}$, $A_{-3} = 2$, $\phi_{-3} = \frac{\pi}{3}$
e) $A_2 = 3$, $\phi_2 = 0$, $A_{-3} = 2$, $\phi_{-3} = \frac{\pi}{3}$

10. Za vremenski kontinuirani i periodčan signal $x(t)$ perioda 4 zadan slikom izračunaj NULTI i DRUGI član rastava u Fourierov red.

a) $(X_0, X_2) = (1, -\frac{2j}{\pi^2})$ b) $(X_0, X_2) = (0, -\frac{1}{2\pi})$ c) $(X_0, X_2) = (0, -\frac{j}{2\pi})$ d) $(X_0, X_2) = (0, -\frac{j}{\pi^2})$
e) $(X_0, X_2) = (1, \frac{2}{\pi^2})$



11. Snaga signala iz prethodnog zadatka je:

a) 0 b) $\frac{1}{9}$ c) $\frac{1}{6}$ d) $\frac{2}{3}$ e) 1

12. Izračunaj vremenski kontinuiranu Fourierovu transformaciju (CTFT) signala $f(t) = e^{-2t}\,\mu(t) + e^{3t}\,\mu(-t)$.

a) $F(j\omega) = \dfrac{-5}{6+\omega^2+j\omega}$ b) $F(j\omega) = \dfrac{5}{6+\omega^2-j\omega}$ c) $F(j\omega) = \dfrac{5}{6+\omega^2+j\omega}$ d) $F(j\omega) = \dfrac{5}{\sqrt{(6+\omega)^2+\omega^2}}$
e) $F(j\omega) = \dfrac{1}{2+j\omega}$

13. Zadan je spektar $X(j\omega) = 4\big(\mu(\omega+2\pi) - \mu(\omega-2\pi)\big)$. Signal čiji je to spektar je:

a) $x(t) = \frac{8}{t}\sin(2\pi t)$ b) $x(t) = -\frac{4}{\pi t}\sin(2\pi t)$ c) $x(t) = \frac{4}{\pi t}\sin(2\pi t)$ d) $x(t) = \frac{4}{\pi t}\cos(2\pi t)$
e) $x(t) = 4\,\delta(t) - \frac{4}{j\pi t}\cos(2\pi t)$

14. Energija signala iz prethodnog zadatka je:

a) 8 b) 32 c) 16π d) 64π e) $+\infty$

15. Zadan je vremenski diskretan periodičan signal $x(n) = \sin(\frac{\pi}{55}n)$. Temeljni period signala N i temeljni period spektra K su:

a) $(N,K) = (55,55)$ b) $(N,K) = (55,110)$ c) $(N,K) = (110,110)$ d) $(N,K) = (220,110)$
e) $(N,K) = (110,220)$

16. Jedan period periodičnog signala perioda $N = 6$ je $x(n) = \begin{cases} 2\sqrt{3}n, & n \in \{-2,-1,0,1,2\} \\ 6, & n = 3 \end{cases}$. Prva dva člana spektra su:

a) $X_0 = 0,\ X_1 = 6$ b) $X_0 = 1,\ X_1 = -1-3j$ c) $X_0 = 1,\ X_1 = -1+3j$ d) $X_0 = 1,\ X_1 = -1+j$
e) $X_0 = 1,\ X_1 = -1-j$

17. Zadan je vremenski diskretan periodički signal $x(n) = \cos(\frac{\pi}{12}n) - \sin(\frac{3\pi}{4}n)$. Petnaesti član spektra je:

a) $X_{15} = \frac{1}{2}e^{-j\pi/2}$ b) $X_{15} = \frac{1}{2}e^{j\pi/2}$ c) $X_{15} = 0$ d) $X_{15} = e^{j\pi/2}$ e) $X_{15} = e^{-j\pi/2}$

18. Jedan period spektra vremenski diskretne Fourierove transformacije (DTFT) je $X(e^{j\Omega}) = \begin{cases} e^{-|\Omega|}, & \Omega \in [-a,a] \\ 0, & \Omega \in \langle -\pi,-a\rangle \cup \langle a,\pi] \end{cases}$.

Signal čiji je to spektar jest:

a) $x(n) = \frac{1}{\pi}\frac{1}{1+n^2}\Big(1 + e^{-a}\big(n\sin(an) - \cos(an)\big)\Big)$ b) $x(n) = \frac{1}{\pi}\frac{1}{1+n^2}\Big(1 + e^{-a}\big(\sin(an) - n\cos(an)\big)\Big)$
c) $x(n) = \frac{1}{\pi}\frac{1}{1+n^2}\Big(1 + e^{-a}\big(\sin(an) - \cos(an)\big)\Big)$ d) $x(n) = \frac{1}{\pi}\frac{1}{1+n^2}\Big(1 + e^{-a}\big(\cos(an) - \sin(an)\big)\Big)$
e) $x(n) = \frac{1}{\pi}\frac{1}{1+n^2}$

19. Promatramo vremenski diskretan signal čiji jedini uzorci različiti od nule su $\{1,3,\underline{4},3,1\}$ (podcrtani član je uzorak za korak $n = 0$). Vremenski diskretna Fourierova transformacija zadanog signala je:

a) $X(e^{j\Omega}) = \frac{2}{\pi} + \frac{3}{\pi}\cos(\Omega) + \frac{1}{\pi}\cos(2\Omega)$ b) $X(e^{j\Omega}) = 4 + 6\cos(\Omega) + 2\cos(2\Omega)$
c) $X(e^{j\Omega}) = 4 + 6j\sin(\Omega) + 2j\sin(2\Omega)$ d) $X(e^{j\Omega}) = \frac{2}{\pi} + \frac{3j}{\pi}\cos(\Omega) + \frac{j}{\pi}\cos(2\Omega)$
e) $X(e^{j\Omega}) = 4 + 3\cos(\Omega) + \cos(2\Omega)$

20. Zadan je vremenski diskretni signal $x(n) = 2^n\,\mu(-n)$. Vremenski diskretna Fourierova transformacija (DTFT) zadanog signala je:

a) $X(e^{j\Omega}) = \dfrac{1}{1+2e^{-j\Omega}}$ b) $X(e^{j\Omega}) = \dfrac{2}{2+e^{-j\Omega}}$ c) $X(e^{j\Omega}) = \dfrac{2}{1-e^{-j\Omega}}$ d) $X(e^{j\Omega}) = \dfrac{1}{1-2e^{j\Omega}}$
e) $X(e^{j\Omega}) = \dfrac{2}{2-e^{j\Omega}}$

PRVI MEĐUISPIT – 2011.

(1.) $x(t) = 2 + 4\sin(t)$, $\omega_0 = 1$

$$X_0 = 2 \quad X_1 = \frac{2}{j} \quad X_{-1} = -\frac{2}{j}$$

$$P = \sum_{k=-\infty}^{\infty} |X_k|^2 = 2^2 + 2^2 + 2^2 = 12 \quad \text{(PARSEVALOVA RELACIJA)}$$

(2.) $x(n) = \left(\frac{1}{5}\right)^{2n} \mu(n)$

$$E = \lim_{N\to\infty} \sum_{n=-N}^{N} |x(n)|^2 = \lim_{N\to\infty} \sum_{n=0}^{N} \left(\left(\frac{1}{25}\right)^n\right)^2 = \sum_{n=0}^{\infty} \left(\frac{1}{25^2}\right)^n = \dots \text{ GEOMETRIJSKI RED} =$$

$$= \frac{1}{1-\frac{1}{25^2}} = \frac{25^2}{25^2-1} = \frac{625}{624}$$

(3.) $x(n) = 4 + 2\sin\left(\frac{\pi}{3}n\right)$, $\omega_0 = \frac{\pi}{3} \to N_0 = 6$

$$x(n) = 4 + \frac{1}{j}e^{j1\omega_0 n} - \frac{1}{j}e^{j(-1)\omega_0 n} =$$

$$= 4 + e^{j\frac{3\pi}{2}}e^{j1\omega_0 n} + e^{j\frac{\pi}{2}}e^{j5\omega_0 n}$$

$$X_0 = 4 \quad X_1 = e^{j\frac{3\pi}{2}} \quad X_5 = e^{j\frac{\pi}{2}} \to P = 4^2 + 1^2 + 1^2 = 18$$

(4.) (a) $\cos(\pi t)$ $T = 2$

(b) $\cos(3\pi t) + \sin(5\pi t)$ $T = 2$

(c) $\cos(3\pi t) + \sin(3t)$ – APERIODIČAN

(d) $\cos(3t) + \cos(5t)$ $T = 2\pi$

(e) $tg\left(\frac{\pi}{5}t\right)$ $T = 5$

(5.) (e) ZA GLATKU $f(t): \mathbb{R} \to \mathbb{R}$ VRIJEDI $f(t)\delta(t-t_0) = f(t_0)$

(6.) $f(t) = \mu(4-t) + \mu(t) + (t-2)^2(\mu(t-2) - \mu(t-4))$

$$\frac{d}{dt}f(t) = -\delta(t-4) + \delta(t) + 2(t-2)(\mu(t-2) - \mu(t-4)) + (t-2)^2(\delta(t-2) - \delta(t-4))$$

$$= -\delta(t-4) + \delta(t) - 4\delta(t-4) + 2(t-2)(\mu(t-2) - \mu(t-4))$$

$$= -5\delta(t-4) + \delta(t) + 2(t-2)(\mu(t-2) - \mu(t-4))$$

7. OČIGLEDNO VRIJEDI:

$$f(t) = -2b_1(t) + b_2(t) + 2b_3(t) \rightarrow (-2,1,2)$$

8. $x(t) = \sin(20\pi t) + \cos(40\pi t) + \sin(60\pi t)$

$$\Omega_{max} = 60\pi \rightarrow \Omega_s > 2\Omega_{max} \rightarrow \Omega_s > 120\pi$$

$$\frac{2\pi}{\Omega_1} = T_s \rightarrow \frac{2\pi}{T_s} > 120\pi \rightarrow T_s < \frac{1}{60}$$

9. $x(t) = 3\sin(2t) + 2\cos(3t + \frac{\pi}{3}) \qquad \omega_0 = 1$

$$= \frac{3}{2j} e^{j2\omega_0 t} - \frac{3}{2j} e^{j(-2)\omega_0 t} + e^{j\frac{\pi}{3}} e^{j3\omega_0 t} + e^{-j\frac{\pi}{3}} e^{j(-3)\omega_0 t}$$

$$X_2 = \frac{3}{2j} = (-j)\frac{3}{2} = \frac{3}{2} e^{j\frac{3\pi}{2}} \rightarrow |X_2| = \frac{3}{2}, \quad \measuredangle X_2 = \frac{3\pi}{2} = -\frac{\pi}{2}$$

$$X_3 = e^{-j\frac{\pi}{3}} \rightarrow |X_3| = 1, \quad \measuredangle X_3 = -\frac{\pi}{3}$$

10. $$\tilde{x}(t) = \begin{cases} t, & -1 \le t \le 1 \\ 0, & -2 \le t \le -1 \,,\; 1 \le t \le 2 \end{cases} \qquad T_0 = 4$$

$X_0 = 0$ – SREDNJA VRIJEDNOST SIGNALA

$$X_2 = \frac{1}{4}\int_{-1}^{1} t e^{-j\pi t} dt = \left| \begin{matrix} u = t & du = dt \\ v = \frac{(-1)}{j\pi} e^{-j\pi t} & \end{matrix} \right| = u \cdot v - \int v\, du =$$

$$= \frac{1}{4}\left\{ \frac{(-1)}{j\pi} t e^{-j\pi t}\Big|_{-1}^{1} - \frac{(-1)}{j\pi}\int_{-1}^{1} e^{-j\pi t} dt \right\} = \frac{1}{4}\frac{(-1)}{j\pi}\left\{ e^{-j\pi} + e^{j\pi} - \frac{(-1)}{j\pi}\underbrace{\left(e^{-j\pi} - e^{j\pi}\right)}_{0} \right\} =$$

$$= \ldots = \frac{1}{4}\frac{(-1)}{j\pi}(-2) = \frac{-j}{2\pi}$$

11. P = ENERGIJA U JEDNOM PERIODU : DULJINA TRAJANJA PERIODA

$$P = \frac{1}{4}\int_{-1}^{1} t^2 dt = \frac{1}{4}\frac{1}{3} t^3\Big|_{-1}^{1} = \frac{1}{6}$$

(12) $f(t) = e^{-2t}\mu(t) + e^{3t}\mu(-t)$

$x_1(t) = e^{-2t}\mu(t) \circ\!\!-\!\!\bullet\ X_1(j\omega) = \frac{1}{2+j\omega}$

$x_2(t) = e^{3t}\mu(-t)$

$y_2(t) = e^{-3t}\mu(t) \circ\!\!-\!\!\bullet\ Y_2(j\omega) = \frac{1}{3+j\omega}$

$x_2(t) = y_2(-t) \circ\!\!-\!\!\bullet\ \frac{1}{|-1|} Y_2(j\frac{\omega}{-1}) = Y_2(-j\omega) = \frac{1}{3-j\omega} = X_2(j\omega)$

$F(j\omega) = X_1(j\omega) + X_2(j\omega) = \frac{1}{2+j\omega} + \frac{1}{3-j\omega} = \frac{5}{6+\omega^2+j\omega}$

(13) $X(j\omega) = 4\left(\mu(\omega+2\pi) - \mu(\omega-2\pi)\right)$

$x(t) = \frac{1}{2\pi}\int_{-2\pi}^{2\pi} 4e^{j\omega t}\,d\omega = \frac{2}{\pi}\frac{1}{jt}e^{j\omega t}\Big|_{-2\pi}^{2\pi} = \frac{2}{\pi t}\frac{2}{2j}\left(e^{j2\pi t} - e^{-j2\pi t}\right) =$

$= \frac{4}{\pi t}\sin(2\pi t)$

(14) $E = \frac{1}{2\pi}\int_{-2\pi}^{2\pi} 16\,d\omega = \frac{8}{\pi}\cdot 4\pi = 32$

(15) $x(n) = \sin(\frac{\pi}{55}n) \rightarrow \omega_0 = \frac{\pi}{55} \rightarrow N_0 = 110 \rightarrow K = N_0 = 110$

(16)

$\tilde{x}(n) = \begin{cases} 2\sqrt{3}n, & n \in \{-2,-1,0,1,2\} \\ 6, & n = 3 \end{cases}$

$x(n) = \{\overset{-2}{-4\sqrt{3}}, \overset{-1}{-2\sqrt{3}}, \underset{\uparrow}{\overset{0}{0}}, \overset{1}{2\sqrt{3}}, \overset{2}{4\sqrt{3}}, \overset{3}{6}, \overset{4}{-4\sqrt{3}}, \overset{5}{-2\sqrt{3}}, \overset{6}{0}, \dots\}$

$X_0 = \frac{1}{6}\sum_{n=0}^{5} x(n)e^{0} = \frac{1}{6}\sum_{n=0}^{5} x(n) = 1$

$X_1 = \frac{1}{6}\sum_{n=0}^{5} x(n)e^{-j\overset{=1}{k}n\frac{\pi}{3}} = \frac{1}{6}\left\{0 + 2\sqrt{3}e^{-j\frac{\pi}{3}} + 4\sqrt{3}e^{-j\frac{2\pi}{3}} + 6e^{-j\pi} - 4\sqrt{3}e^{-j\frac{4\pi}{3}} -\right.$

$\left. - 2\sqrt{3}e^{-j\frac{5\pi}{3}}\right\} = \frac{1}{6}\left\{0 + (\sqrt{3}-3j) + (-2\sqrt{3}-6j) + (-6) +\right.$

$\left. + (2\sqrt{3}-6j) + (-\sqrt{3}-3j)\right\} = -1-3j$

(17.) $x(n) = \cos\left(\frac{\pi}{12}n\right) - \sin\left(\frac{3\pi}{4}n\right)$ $\qquad \omega_0 = \frac{\pi}{12} \rightarrow N_0 = 24$

$\omega_1 = \frac{\pi}{12} \qquad \omega_2 = \frac{9\pi}{12}$

$$x(n) = \frac{1}{2}e^{j1\omega_0 n} + \frac{1}{2}e^{j(-1)\omega_0 n} - \frac{1}{2j}e^{j9\omega_0 n} + \frac{1}{2j}e^{j(-9)\omega_0 n} =$$

$$= \frac{1}{2}e^{j1\omega_0 n} + \frac{1}{2}e^{j23\omega_0 n} - \frac{1}{2j}e^{j9\omega_0 n} + \frac{1}{2j}e^{j15\omega_0 n}$$

$$X_{15} = \frac{1}{2j} = -\frac{j}{2} = \frac{1}{2}e^{j\frac{3\pi}{2}}$$

(18.)

$$X(e^{j\omega}) = \begin{cases} e^{-|\Omega|}, & \Omega \in (-a, a) \\ 0, & \Omega \in \langle -\pi, a \rangle \cup \langle a, \pi) \end{cases}$$

$$x(n) = \frac{1}{2\pi}\int_{-a}^{a} X(e^{j\omega})e^{j\omega n}d\omega = \frac{1}{2\pi}\left\{\int_{-a}^{0} e^{\omega}e^{j\omega n}d\omega + \int_{0}^{a} e^{-\omega}e^{j\omega n}d\omega\right\} =$$

$$= \frac{1}{2\pi}\left\{\int_{-a}^{0} e^{\omega(1+jn)}d\omega + \int_{0}^{a} e^{\omega(-1+jn)}d\omega\right\} =$$

$$= \frac{1}{2\pi}\left\{\frac{1}{1+jn}e^{\omega(1+jn)}\Big|_{-a}^{0} + \frac{(-1)}{1-jn}e^{-\omega(1-jn)}\Big|_{0}^{a}\right\} =$$

$$= \frac{1}{2\pi}\frac{1}{1+n^2}\left\{(1-jn)(1-e^{-a(1+jn)}) - (1+jn)(e^{-a(1-jn)} - 1)\right\} =$$

$$= \frac{1}{2\pi}\frac{1}{1+n^2}\left\{(1\cancel{-jn}+1\cancel{+jn}) + jne^{-a}e^{-jan} - e^{-a}e^{-jan} - e^{-a}e^{jan} - jne^{-a}e^{jan}\right\} =$$

$$= \frac{1}{\cancel{2}\pi}\frac{1}{1+n^2}\left\{\cancel{2} - jne^{-a}\frac{\cancel{2}j}{2j}(e^{jan} - e^{-jan}) - e^{-a}\frac{\cancel{2}}{2}(e^{jan} - e^{-jan})\right\} =$$

$$= \frac{1}{\pi}\frac{1}{1+n^2}\left\{1 + e^{-a}(n\sin(an) - \cos(an))\right\}$$

(19.) $x(n) = \{\overset{-2}{1}, \overset{-1}{3}, \overset{0}{\underline{4}}, \overset{1}{3}, \overset{2}{1}\}$

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x(n)e^{-j\omega n} = 1e^{2j\omega} + 3e^{j\omega} + 4 + 3e^{-j\omega} + 1e^{-2j\omega} =$$

$$= 4 + 6\cos(\omega) + 2\cos(2\omega)$$

(20.) $x(n) = 2^n u(-n)$

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x(n)e^{-j\omega n} = \sum_{n=-\infty}^{0} 2^n e^{-j\omega n} = \left| \begin{array}{ll} n=-k & \\ n\to-\infty & k\to\infty \\ n\to 0 & k\to 0 \end{array} \right| = \sum_{k=\infty}^{0} 2^{-k}e^{j\omega k} =$$

$$= \sum_{k=0}^{\infty}(2^{-1}e^{j\omega})^k = \frac{1}{1-\frac{1}{2}e^{j\omega}} = \frac{2}{2-e^{j\omega}}$$