JIR 2014

1. $x(t) = \sin\left(\frac{2\pi}{7}t\right)$ $\qquad T_0 = \frac{2\pi}{\frac{2\pi}{7}} = 7$

a) CTFS?

$$x(t) = \frac{1}{2j}\left(e^{j\frac{2\pi}{7}t} - e^{-j\frac{2\pi}{7}t}\right) = \frac{1}{2}e^{-j\frac{\pi}{2}} \cdot e^{j\frac{2\pi}{7}t} + \frac{1}{2}e^{j\frac{\pi}{2}}e^{-j\frac{2\pi}{7}t}$$

jel ok

$$X_1 = \frac{1}{2}e^{-j\frac{\pi}{2}} \qquad X_{-1} = \frac{1}{2}e^{j\frac{\pi}{2}}$$

b) CTFT

$$X(j\omega) = \frac{1}{2}\int_0^7 \left(e^{-j\frac{\pi}{2}} \cdot e^{j\frac{2\pi}{7}t} e^{-j\omega t} + e^{j\frac{\pi}{2}} e^{-j\frac{2\pi}{7}t} e^{-j\omega t}\right) dt =$$

$$= \frac{1}{2}e^{-j\frac{\pi}{2}}\int_0^7 e^{t\left(j\frac{2\pi}{7}-\omega\right)} dt + \frac{1}{2}e^{j\frac{\pi}{2}}\int_0^7 e^{+t\left(j\frac{2\pi}{7}+\omega\right)} dt = \frac{e^{-j\frac{\pi}{2}}}{2} \cdot \frac{1}{j\frac{2\pi}{7}\omega}\left(e^{7\left(j\frac{2\pi}{7}-\omega\right)} - 1\right) +$$

$$+ \frac{e^{j\frac{\pi}{2}}}{2} \cdot \frac{1}{-j\frac{2\pi}{7}-\omega}\left(e^{7\left(-j\frac{2\pi}{7}-\omega\right)} - 1\right) = \frac{e^{-j\frac{\pi}{2}} \cdot e^{7j\frac{2\pi}{7}} \cdot e^{-7\omega} - e^{-j\frac{\pi}{2}}}{j\frac{4\pi}{7} - 2\omega} + \frac{e^{j\frac{\pi}{2}} \cdot e^{-7j\frac{2\pi}{7}} e^{-7\omega} - e^{j\frac{\pi}{2}}}{-j\frac{4\pi}{7} - 2\omega} =$$

iz šalabahtera!

2. $x(t) = t\,[\mu(t+3.5) - \mu(t-3.5)]$

a) $E = ?$

$$E = \int_{-3.5}^{3.5} t^2\,dt = \frac{t^3}{3}\Big|_{-3.5}^{3.5} = 28{,}58$$

b) $T_s = 1$

$$x(nT_s) = n\,[\mu(n+3) - \mu(n-3)]$$

c) $E = \sum_{n=-3}^{3} n^2 = 28$

d) DTFT = ? $X(e^{j\Omega}) = \sum_{n=-3}^{3} n e^{-j\Omega n} = -3e^{j\Omega 3} - 2e^{j\Omega 2} - e^{j\Omega} + e^{-j\Omega} + 2e^{-j\Omega 2} + 3e^{-j\Omega 3} = \ldots$

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3. $y(n) + \frac{1}{16}\,y(n-2) = u(n) - \frac{1}{4}\,u(n-1)$

a) imp. u vrem.? $q_1 = \left|\begin{matrix} r = \sqrt{a^2+b^2} \\ \varphi = \operatorname{arctg}\frac{b}{a} \end{matrix}\right| = \frac{1}{4} e^{j\varphi}$

b) imp. u

c) odziv na ulaz (?) , $u(n) = \left\{ \underline{1}, 0, \frac{1}{16} \right\}$

$y(n) + \frac{1}{16} y(n-2) = u(n) - \frac{1}{4} u(n-1)$

$y(n-2) = 0$

$y(n) = u(n) - \frac{1}{4} u(n-1) - \frac{1}{16} y(n-2)$

$\underline{n=0} \Rightarrow y(0) = 1$

$n=1 \Rightarrow y(1) = 0 - \frac{1}{4} - 0 = -\frac{1}{4}$

$n=2 \Rightarrow y(2) = \frac{1}{16} - 0 - \frac{1}{16} = 0$

$\underline{n=3} \quad y(3) = 0 - \frac{1}{4} \cdot \frac{1}{16} + \frac{1}{16} \cdot \frac{1}{4} = 0$

$n=4 \quad y(4) = 0 \ \ldots$

$\underline{y(n) = \left\{ \underline{1}, -\frac{1}{4}, 0, \ldots \right\}}$

$16z^4 - \frac{1}{4}z^3 + z^2 - \frac{1}{4}z : 16z^4 + z^2 = 1$

$-16z^4 \quad - z^2$

$-\frac{1}{4}z^3 - \frac{1}{4}z$

d) u Z-domeni $\quad U(z) = 1 + \frac{1}{16} z^{-2}$

$Y_u = H(z) \cdot U(z) = \frac{z^2 - \frac{1}{4}z}{z^2 + \frac{1}{16}} \cdot \frac{16z^2 + 1}{16z^2} = \frac{16z^4 + z^2 - \frac{1}{4}z^3 - \frac{1}{4}z}{16z^4 + z^2} = 1 - \frac{\frac{1}{4}z^3 - \frac{1}{4}z}{16z^4 + z^2} = 1 - \frac{\frac{1}{4}z^2 + \frac{1}{4}}{16z^3 + z}$

$H(z) = \frac{1 - \frac{1}{4}z^{-1}}{1 + \frac{1}{16}z^{-2}}$

4. $y''(t) + 7y'(t) + 10y(t) = u'(t)$

$u(t) = 10\cos(2t)\,\mu(t)$

$u'(t) = -20\sin(2t)\mu(t) + 10\cos(2t)\delta(t)$
$= -20\sin(2t)\mu(t) + 10\delta(t)$

$y(0^-) = 1$
$y'(0^-) = 1$

a) $H(s) = \dfrac{s}{s^2+7s+10}$ $\quad s_{1,2} = \dfrac{-7 \pm \sqrt{49-40}}{2}$ $\quad s_1 = -2$, $s_2 = -5$ $\quad s_i < 0$ stabilan

b) $H(j\omega) = \dfrac{j\omega}{-\omega^2+7j\omega+10} \cdot \dfrac{10-\omega^2-7j\omega}{(10-\omega^2)-7j\omega} = \dfrac{10j\omega - j\omega^3 + 7\omega^2}{100+\omega^4+49\omega^2-20\omega^2}$

$|H(j\omega)| = \sqrt{\dfrac{49\omega^4+(10\omega-\omega^3)^2}{(100+\omega^4+49\omega^2)^2}} = \dfrac{\omega}{\sqrt{\underbrace{(10-\omega^2)^2}_{-20\omega^2} + \underbrace{(7\omega)^2}_{49\omega^2}}} = \dfrac{\omega}{\sqrt{100+29\omega^2+\omega^4}}$

$\dfrac{2}{\sqrt{100+29\cdot 4+16}}$

$\measuredangle H(j\omega) = \operatorname{arctg} \dfrac{-\omega^3+10\omega}{7\omega^2}$

c) $y_{pris} = ?$

$\omega = 2$

$|H(2j)| = \dfrac{\sqrt{58}}{58}$

$\measuredangle H(2j) = 0.405$

$y_{pris} = \dfrac{-20\sqrt{58}}{58}\sin(2t+0.405)\mu(t)$

rj. s formula: $\boxed{\dfrac{\sqrt{58}}{58}}$

d) $y_{tot} = ?$

$y_h(t) = C_1e^{-2t} + C_2e^{-5t}$

$y_{tot}(t) = C_1e^{-2t} + C_2e^{-5t} - \dfrac{10\sqrt{58}}{39}\sin(2t+0.405)\mu(t)$

$y'(0^+) - y'(0^-) = u(0^+) - 0$

$y'(0^+) = 10 + 1 = 11$

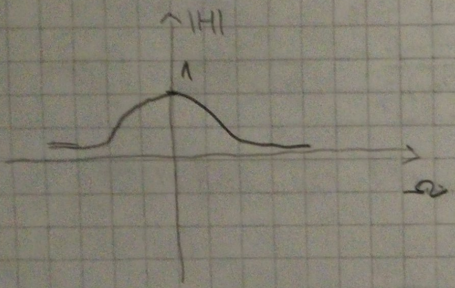
$y'_{tot}(t) = -2C_1e^{-2t} - 5C_2e^{-5t} - \dfrac{20\sqrt{58}}{39}\cos(2t+0.405)\mu(t) - \dfrac{10\sqrt{58}}{39}\sin(2t+0.405)\overset{=0}{\delta(t)}$

5. $y(n)+\frac{1}{2}y(n-1)=u(n)-\frac{1}{2}u(n-1)$

a) $H(z)=\frac{z-\frac{1}{2}}{z+\frac{1}{2}}$

b) $H(e^{j\Omega})=\frac{1-\frac{1}{2}e^{-j\Omega}}{1+\frac{1}{2}e^{-j\Omega}}=\frac{1-\frac{1}{2}(\cos\Omega-j\sin\Omega)}{1+\frac{1}{2}(\cos\Omega-j\sin\Omega)}=\frac{1-\frac{1}{2}\cos\Omega+\frac{1}{2}j\sin\Omega}{1+\frac{1}{2}\cos\Omega-\frac{1}{2}j\sin\Omega}$

$$|H(e^{j\Omega})|=\sqrt{\frac{(1-\frac{1}{2}\cos\Omega)^2+\frac{1}{4}\sin^2\Omega}{(1+\frac{1}{2}\cos\Omega)^2+\frac{1}{4}\sin^2\Omega}}=\sqrt{\frac{1-\cos\Omega+\frac{1}{4}\cos^2\Omega+\frac{1}{4}\sin^2\Omega}{1+\cos\Omega+\frac{1}{4}\cos^2\Omega+\frac{1}{4}\sin^2\Omega}}=\sqrt{\frac{\frac{5}{4}-\cos\Omega}{\frac{5}{4}+\cos\Omega}}$$



c) $u(n)=2\cos(\frac{\pi}{2}n)\mu(n)$

$|H(e^{j\frac{\pi}{2}})|=1 \qquad y_{pris}=2\cos(\frac{\pi}{2}n)\mu(n)$