LJIR 2013

(1) $x(t) = 8\cos\left(6\pi t + \frac{3\pi}{4}\right) + 6\sin\left(8\pi t + \frac{\pi}{6}\right)$

a) $\left.\begin{array}{l}\omega_1 = 6\pi \\ \omega_2 = 8\pi\end{array}\right\} \omega_0 = 2\pi$

$$x(t) = \cancel{8}^{4}\frac{e^{j6\pi t}\cdot e^{j\frac{3\pi}{4}} + e^{-j6\pi t}e^{j\frac{3\pi}{4}}}{\cancel{2}} + \cancel{6}^{3}\frac{e^{j8\pi t}e^{j\frac{\pi}{6}} - e^{-j8\pi t}e^{j\frac{\pi}{6}}}{\cancel{2}j}$$

$$= 4e^{j3\omega_0 t}e^{j\frac{3\pi}{4}} + 4e^{-j3\omega_0 t}e^{j\frac{3\pi}{4}} + 3e^{j4\omega_0 t}e^{j\left(\frac{\pi}{6}-\frac{\pi}{2}\right)} - 3e^{-j4\omega_0 t}e^{j\left(\frac{\pi}{6}-\frac{\pi}{2}\right)}$$

$X_3 = 4e^{j\frac{3\pi}{4}}$

$X_{-3} = 4e^{j\frac{3\pi}{4}}$

$X_4 = 3e^{j\left(\frac{\pi}{6}-\frac{\pi}{2}\right)}$ $\quad -\frac{\pi}{3}$

$X_{-4} = -3e^{j\left(\frac{\pi}{6}-\frac{\pi}{2}\right)}$

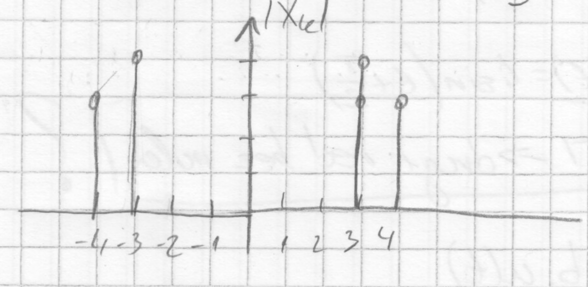
b) $|X_3| = 4 \qquad \measuredangle X_3 = \frac{3\pi}{4}$

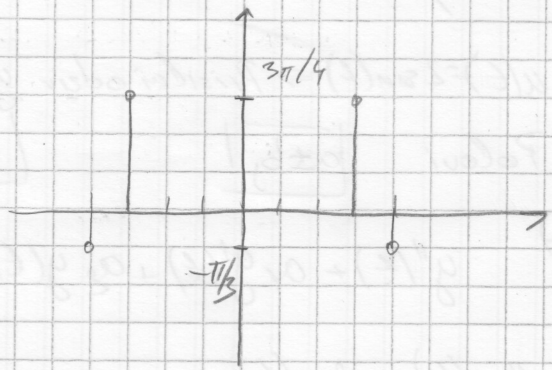
$|X_{-3}| = 4 \qquad \measuredangle X_3 = \frac{3\pi}{4}$

$|X_4| = 3 \qquad \measuredangle X_4 = -\frac{\pi}{3}$

$|X_4| = -3 \qquad \measuredangle X_{-4} = -\frac{\pi}{3}$

c)





d) $P = 16 + 16 + 9 + 9 = 50$

(2.) $x(n) = 6^{n}\mu(-n) + 6^{-n}\mu(n)$

a) $$X(e^{j\Omega}) = \sum_{n=-\infty}^{+\infty} x(n)e^{-j\Omega n} = \sum_{n=-\infty}^{0} 6^{n} e^{-j\Omega n} + \sum_{n=0}^{\infty} 6^{-n} e^{-j\Omega n} =$$

$$= 2\sum_{n=0}^{\infty}\left(6^{-1}e^{-j\Omega}\right)^{n} = 2 \cdot \frac{1}{1-\frac{1}{6e^{j\Omega}}} = \frac{12e^{j\Omega}}{6e^{j\Omega}-1} =$$

$$= \frac{12(\cos(\Omega)+j\sin(\Omega))}{6(\cos(\Omega)+j\sin(\Omega))-1}$$

b) $$E = \sum_{n=-\infty}^{\infty}\left|6^{n}\mu(-n)+6^{-n}\mu(n)\right|^{2} = \sum_{n=-\infty}^{\infty}\left(6^{2n}\mu(-n)+2\,6^{n-n}\mu(-n)\mu(n)+6^{-2n}\mu(n)\right)$$

$$= 2\sum_{n=0}^{\infty} 36^{-n} + 2\sum_{n=0}^{0} 1 = 2 \cdot \frac{1}{1-\frac{1}{36}} + 2 = \frac{72}{35} + 2 = \frac{142}{35} = 4{,}06$$

(3.) $y'(t) + 3y(t) = 2u'(t) \qquad y(0^-) = 2 = y(0^+)$

a) $u(t) = \alpha u_1(t) + \beta u_2(t)$

$u'(t) = \alpha u_1'(t) + \beta u_2'(t)$

$$\left.\begin{array}{l} y'(t) = 2u'(t) - 3y(t) = 2\alpha u_1'(t) + 2\beta u_2'(t) - 3y(t) \\ \left.\begin{array}{l} y_1'(t) = 2\alpha u_1'(t) - 3y(t) \\ y_2'(t) = 2\beta u_1'(t) - 3y(t) \end{array}\right\} + = 2\alpha u_1'(t) + 2\beta u_2'(t) - 6y(t) \end{array}\right\} \neq$$

b) $y'(t) = 2u'(t) - 3y(t)$

$u_1'(t) = u(t-M)$

$$\left.\begin{array}{l} y_1'(t) = 2u'(t-M) - 3y(t) \\ y'(t-M) = 2u'(t-M) - 3y(t-M) \end{array}\right\} \neq \text{ nije vremenski stalan}$$

c)/d) $sY(s) + 3Y(s) = 2sU(s)$

?

$H(s) = \frac{2s}{s+3} \qquad s = -3 < 0 \Rightarrow$ sustav je stabilan

e) $u(t) = \mu(t) \Rightarrow u'(t) = \delta(t)$

$y'(t) + 3y(t) = 2u'(t)$ $\quad y(0^-) = 2$

$b_0 = 2$

$y(0^+) - y(0^-) = 2\mu(0) \Rightarrow \boxed{y(0^+) = 4}$

$y_h(t) = C_1 e^{-3t}$

$y_h(0^+) = C_1 = 4$

$\Rightarrow \boxed{y_{TOT}(t) = 4e^{-3t}\mu(t)}$

4. $6y(n) - y(n-1) - y(n-2) = 7u(n) - u(n-1)$

$u(n) = 2\cos\left(\frac{\pi}{2}n - \frac{\pi}{3}\right)\mu(n)$ $\quad y(-1) = -\sqrt{3}$

$y(-2) = \sqrt{3} - 1$

a) $6Y(z) - z^{-1}Y(z) - z^{-2}Y(z) = 7U(z) - z^{-1}U(z)$

$6z^2Y(z) - zY(z) - Y(z) = 7z^2U(z) - zU(z)$

$H(z) = \dfrac{7z^2 - z}{6z^2 - z - 1}$ $\quad z_1 = \frac{1}{2} \quad z_2 = -\frac{1}{3} \rightarrow$ polovi

$\boxed{z_1 = 0 \quad z_2 = \frac{1}{7}} \rightarrow$ nule

b) $H(z) = \dfrac{7z^2 - z}{6z^2 - z - 1}$

c) $$H(e^{j\omega}) = \frac{7e^{2j\omega} - e^{j\omega}}{6e^{2j\omega} - e^{j\omega} - 1}$$

d) $$H(e^{j\omega}) = \frac{7(\cos(2\omega) + j\sin(2\omega)) - (\cos(\omega) + j\sin(\omega))}{6(\cos(2\omega) + j\sin(2\omega)) - (\cos(\omega) + j\sin(\omega)) - 1}$$

$$= \frac{7\cos(2\omega) - \cos(\omega) + j(\sin(2\omega) - \sin(\omega))}{6\cos(2\omega) - \cos(\omega) - 1 + j(\sin(2\omega) - \sin(\omega))}$$

$$|H(e^{j\omega})| = \sqrt{(7\cos(2\omega) - \cos(\omega))^2 + (\sin(2\omega) - \sin(\omega))^2}$$