$\boxed{3.1\text{-}3 \mid a}$

KARLO MARKOVIĆ
0036429175

1.) $y(n) - 0{,}98\, y(n-1) + 0{,}81\, y(n-2) = u(n)$

$u(n) = \delta(n)$ $\quad h(n) - 0{,}98\, h(n-1) + 0{,}81\, h(n-2) = 0$

$$z^{n-2}\left(z^2 - 0{,}98 z + 0{,}81\right) = 0$$

$$z_{1,2} = \frac{0{,}98 \pm \sqrt{0{,}98^2 - 4\cdot 0{,}81}}{2} = 0{,}49 \pm 0{,}8185 j = 0{,}954 \cdot e^{\pm j \frac{\pi}{3}}$$

$$h(n) = C\, 0{,}954^n e^{\frac{\pi}{3} j n} + C_2\, 0{,}954^n e^{-\frac{\pi}{3} j n} =$$
$$= 0{,}954^n \left(A \cos\left(\frac{\pi}{3} n\right) + B \sin\left(\frac{\pi}{3} n\right)\right)$$

$n = 0$ $\quad h(0) - 0{,}98\, h(-1) + 0{,}81\, h(-2) = \delta(0)$

$h(0) = \delta(0) = 1 = A$

$n = -1$ $\quad h(-1) = 0 = \frac{1}{0{,}954}\left(\cos\left(\frac{\pi}{3}\right) - B \sin\left(\frac{\pi}{3}\right)\right) =$

$B = 0{,}63$

$$h(n) = 0{,}954^n \left[\cos\left(\frac{\pi}{3} n\right) + 0{,}63 \sin\left(\frac{\pi}{3} n\right)\right]$$

$u(n) = \mu(n)$

$y_p(n) = K$ $\quad K - 0{,}98 K + 0{,}81 K = 1 \implies K = 1{,}075$

$$y(n) = 0{,}954^n \left(A \cos\left(\frac{\pi}{3} n\right) + B \sin\left(\frac{\pi}{3} n\right)\right) + 1{,}075$$

$y(-1) = 0{,}954^{-1}\left(A - 0{,}857 B\right) + 1{,}075 = 0$

$y(-2) = 0{,}954^{-2}\left(A + 0{,}857 B\right) + 1{,}075 = 0$

$A = -1{,}86$ $\qquad B = -0{,}9245$

$$y(n) = 0{,}954^n \left(-1{,}86 \cos\left(\frac{\pi}{3} n\right) - 0{,}9245 \left(\frac{\pi}{3} n\right)\right) \mu(n)$$

2.) $y(n) - y(n-1) - y(n-2) = u(n)$

$u(n) = \delta(n)$

$h(n) - h(n-1) + h(n-2) = 0 \qquad h(n) = z^n$

$z^{n-2}(z^2 - z + 1) = 0$

$$z_{1,2} = \frac{1 \pm \sqrt{1^2 - 4}}{2} = \frac{1}{2} \pm \frac{\sqrt{3}}{2} j = e^{\pm \frac{\pi}{3} j}$$

$$h(n) = C_1 e^{\frac{\pi}{3} j n} + C_2 e^{-\frac{\pi}{3} j n} = A \cos\left(\frac{\pi}{3} n\right) + B \sin\left(\frac{\pi}{3} n\right)$$

$n = 0$ $\qquad h(0) - h(-1) + h(-2) = \delta(0)$

$\qquad h(0) = 1 = A$

$n = -1$ $\qquad h(-1) = 0 = 0{,}5 - B \cdot 0{,}866 \rightarrow B = 0{,}57$

$$h(n) = \cos\left(\frac{\pi}{3} n\right) - 0{,}57 \sin\left(\frac{\pi}{3} n\right)$$

$u(n) = \mu(n) \qquad y_p(n) = K \qquad K - K + K = 1 \rightarrow K = 1$

$$y(n) = A \cos\left(\frac{\pi}{3} n\right) + B \sin\left(\frac{\pi}{3} n\right) + 1$$

$y(-1) = A \cdot 0{,}5 - B \cdot 0{,}866 + 1 = 0$

$y(-2) = -A\, 0{,}5 - B\, 0{,}866 + 1 = 0$

$$A = 0 \qquad B = \frac{1}{0{,}866} = 1{,}154$$

$$y(n) = 1{,}154 \sin\left(\frac{\pi}{3} n\right) + 1$$

KARLO MARKOVIĆ
0036428175

3) $y(n) - 2y(n-1) + y(n-2) = u(n)$

$u(n) = \delta(n)$ $\quad h(n) - 2h(n-1) + h(n-2) = 0$

$$z^{n-2}(z^2 - 2z + 1) = 0$$

$z_1 = z_2 = 1$

$h(n) = C_1 + nC_2(1)^n$

$n=0$ $\quad h(0) = C_1 = 1$

$n=-1$ $\quad h(-1) = 0 = C_1 - C_2 \Rightarrow C_2 = 1$

$$h(n) = 1 + n$$

$y_p(n) = K$ $\quad K - 2K + K = 0 \Rightarrow K = 0$

$$y(n) = 1 + n$$

4) $y(n) - 1{,}38\, y(n-1) + 1{,}42\, y(n-2) = u(n)$

$u(n) = \delta(n)$ $\quad h(n) - 1{,}38\, h(n-1) + 1{,}42\, h(n-2) = 0$

$$z^{n-2}(z^2 - 1{,}38z + 1{,}42) = 0$$

$$z_{1,2} = \frac{1{,}38 \pm \sqrt{1{,}38^2 - 4 \cdot 1{,}42}}{2} = 0{,}69 \pm 0{,}97j = 1{,}191\, e^{\pm 54{,}62^\circ}$$

$h(n) = 1{,}191^n \cdot (A\cos(54{,}62n) + B\sin(54{,}62n))$

$$0{,}4862 + B \cdot (-0{,}875) = 0$$

$$B = 0{,}5464$$

$h(n) = 1{,}191^n \cdot (\cos(54{,}62n) + 0{,}5924\sin(54{,}62n))$

$u(n) = \mu(n)$ $\qquad y_p(n) = K \qquad K - 1{,}38K + 1{,}42K = 1$

$K = 0{,}96$

$y(n) = 1{,}191^{n} \cdot \left(A\cos(54{,}62n)\ B\sin(54{,}62n) \right) + 0{,}96$

$y(-1) = 1{,}191^{-1} \cdot 0{,}58 \cdot A - 1{,}191^{-1} \cdot 0{,}82 \cdot B + 0{,}96 = 0$

$y(-2) = -(1{,}191)^{-2} \cdot 0{,}33 \cdot A - 1{,}191^{-2} \cdot 0{,}94 \cdot B + 0{,}96 = 0$

$A = 0{,}06 \qquad B = 1{,}47$

$y(n) = 1{,}191^{n} \left[0{,}06\cos(54{,}62n) + 1{,}47\sin(54{,}62) \right] + 0{,}96$

$\boxed{3.1-4\ a}$

$u(n) = A\cos(w_0 n) \quad y_p(n) = C_1 \cdot \cos(w_0 n) + C_2 \cdot \sin(w_0 n)$

$u(n) = A\cos(w_0 n)$

- KADA SE FREKVENCIJA POBUDE POKLOPI S VLASTITOM FREKVENCIJOM SUSTAVA, SUSTAV POSTAJE NESTABILAN

- TA SE POJAVA ZOVE REZONANCIJA

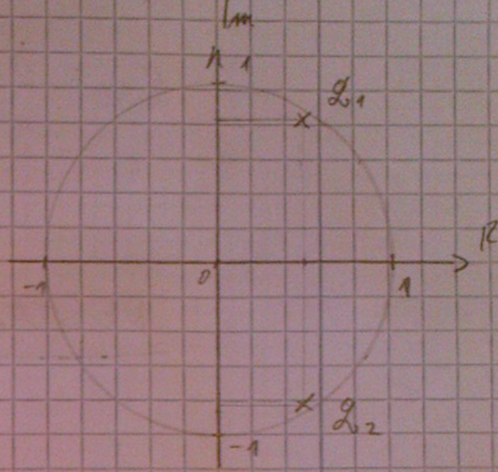
$\boxed{3.1\text{-}5\,a}$

KARLO MARKOVIĆ
0036423176

1.) $y(n) - 0{,}98\, y(n-1) + 0{,}91\, y(n-2) = u(n)$

$$z^2 - 0{,}98\, z + 0{,}91 = 0$$

$$z_{1,2} = 0{,}49 \pm 0{,}8185\, j$$

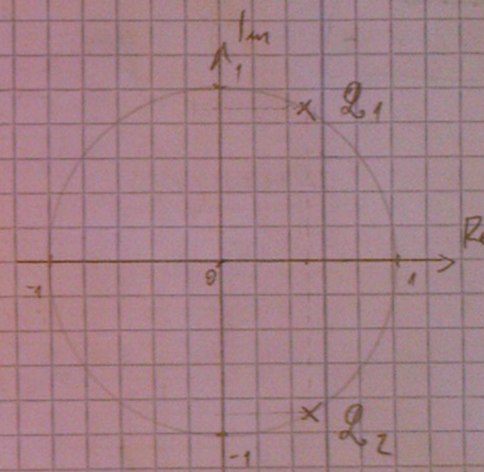


STABILAN

2.) $y(n) - y(n-1) - y(n-2) = u_n$

$$z^2 - z - 1 = 0$$

$$z_{1,2} = \frac{1}{2} \pm \frac{\sqrt{3}}{2}\, j$$

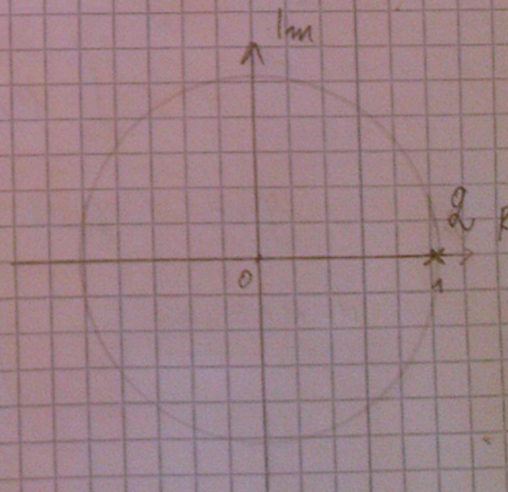


MARGINALNO STABILAN

3.) $y(n) - 2y(n-1) + y(n-2) = u(n)$

$$z^2 - 2z + 1 = 0$$

$$z = 1$$

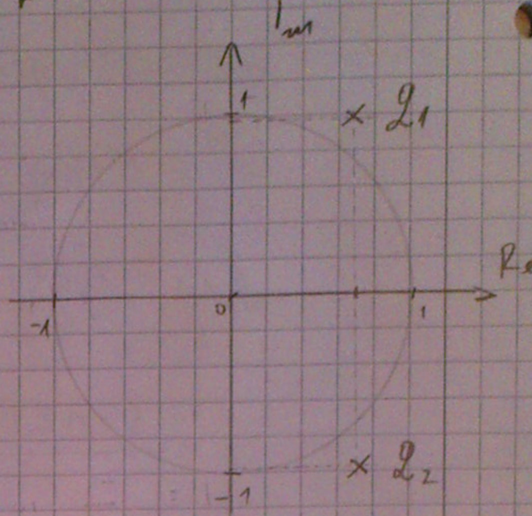


NESTABILAN

4.) $y(n) - 1{,}38\, y(n-1) + 1{,}42\, y(n-2) = u_n$

$$z^2 - 1{,}38\, z + 1{,}42 = 0$$

$$z_{1,2} = 0{,}69 \pm 0{,}97\, j$$



NESTABILAN

$\boxed{32\text{-}1\ a}$ 1.) $y(t) = \int_{-\infty}^{t} u(\tau)\, d\tau$

LINEARNOST:

$$u(t) = \alpha u_1(t) + \beta u_2(t)$$

$$y_1(t) = \int_{-\infty}^{t} \alpha\, u_1(\tau)\, d\tau$$

$$y_2(t) = \int_{-\infty}^{t} \beta\, u_2(\tau)\, d\tau$$

$$y(t) = \int_{-\infty}^{t} u(\tau)\, d\tau = \int_{-\infty}^{t} \left[\alpha u_1(\tau) + \beta u_2(\tau)\right] d\tau =$$

$$= \alpha \int_{-\infty}^{t} u_1(\tau)\, d\tau + \beta \int_{-\infty}^{t} u_2(\tau)\, d\tau$$

$y(t) = \alpha y_1(t) + \beta y_2(t)$ $\longrightarrow$ LINEARAN

MEMORIJA:

$$y(t) = \int_{-\infty}^{t} u(\tau)\, d\tau = \int_{-\infty}^{t_0} u(\tau)\, d\tau + \int_{t_0}^{t} u(\tau)\, d\tau$$

$$y(t) = y(t_0) + \int_{t_0}^{t} u(\tau)\, d\tau$$

MEMORIJSKI

KARLO MARKOVIĆ
0036429175

KAUZALNOST:

$$\forall t \in R \qquad y(t) = F(u_{(-\infty, t)})\, t$$

, a to je upravo INTEGRATOR $\quad y(t) = \int_{-\infty}^{t} u(\tau)\, d\tau$

KAUZALAN

2.) $\quad y(t) = \frac{d}{dt} u(t)$

LINEARNOST: $\quad u(t) = \alpha u_1(t) + \beta u_2(t)$

$$y(t) = \frac{d}{dt}\left[\alpha u_1(t) + \beta u_2(t)\right]$$

$$y(t) = \alpha \frac{d}{dt} u_1(t) + \beta \frac{d}{dt} u_2(t)$$

$$y(t) = \alpha y_1(t) + \beta y_2(t) \longrightarrow \text{LINEARAN}$$

$$y(t) = \frac{d}{dt} u(t) = \lim_{\Delta h \to 0} \frac{u(t + \Delta h) - u(t)}{\Delta h}$$

MEMORIJSKI , NEKAUZALAN

- ZATO JER ZA TRENUTNU VRIJEDNOST
MORAMO ZNATI NJEGOVU BUDUĆU VRIJEDNOST
$(t + \Delta h)$

$\boxed{3.2-2\ a}$

1.) $y''(t) + 2y'(t) + 20y(t) = u(t)$ $\qquad u(t) = \delta(t)$

$s^2 + 2s + 20 = 0$ $\qquad y(0^+) = 1$

$\qquad y(0^-) = 0$

$$s_{1,2} = \frac{-2 \pm \sqrt{4 - 4 \cdot 20}}{2}$$

$$= \frac{-2 \pm 2\sqrt{19}\,j}{2} = -1 \pm \sqrt{19}\,j$$

$$h_A(t) = C_1 e^{(-1 \pm \sqrt{19}j)t} + C_2 e^{(-1 \pm \sqrt{19}j)t} =$$

$$= e^{-t}\left[A\cos(\sqrt{19}\,t) + B\sin(\sqrt{19}\,t)\right]$$

$$h_A(0^+) = A = 0$$

$$h'_A(t) = -e^{-t}\left[A\cos(\sqrt{19}\,t) + B\sin(\sqrt{19}\,t)\right] + e^{-t}\left[-\sqrt{19}\cdot A\sin(\sqrt{19}t) + \sqrt{19}\cdot B\cos(\sqrt{19}t)\right]$$

$$h'_A(0^+) = -A + \sqrt{19}\,B = 1 \implies B = \frac{1}{\sqrt{19}}$$

$$\boxed{h(t) = \frac{e^{-t}}{\sqrt{19}}\sin(\sqrt{19}\cdot t)\cdot \mu(t)}$$

2.) $y''(t) + 26y(t) = u(t)$ $\qquad u(t) = \delta(t)$

$s^2 + 26 = 0$ $\qquad y(0^+) = 1$

$\qquad y(0^-) = 0$

$s_{1,2} = \pm\sqrt{26}\,j$

$$h_A(t) = A\cos(\sqrt{26}\,t) + B\sin(\sqrt{26}\,t)$$

$$h_A(0^+) = A = 0$$

$$h'_A(t) = -\sqrt{26}\cdot A\cdot\sin(\sqrt{26}\,t) + \sqrt{26}\,B\cdot\cos(\sqrt{26}\cdot t)$$

$$h'_A(0^+) = \sqrt{26}\,B = 1 \longrightarrow B = \frac{1}{\sqrt{26}}$$

$$\boxed{h(t) = \frac{1}{\sqrt{26}}\cdot\sin(\sqrt{26}\,t)}$$

KARLO MARKOVIĆ
0036423175

3.) $y''(t) = \mu(t)$

$s^2 = 0$

$s_{1,2} = 0 \qquad h(t) = C_1 e^{0 \cdot t} + t C_2 e^{0 \cdot t}$

$h_1(t) = C_1 + C_2 t \qquad h_1(t) = t$

$h_1(0^+) = C_1 = 0$

$h'_1 = C_2 = 1$

$$\boxed{h(t) = t\,\mu(t)}$$

4.) $y''(t) - 2y'(t) + 26y(t) = \mu(t)$

$s^2 - 2s + 26 = 0$

$$s_{1,2} = \frac{2 \pm \sqrt{4 - 4 \cdot 26}}{2} = 1 \pm 5j$$

$$h_1(t) = e^t \left[A\cos(5t) + B\sin(5t) \right]$$

$h_1(0^+) = A = 0$

$h'_1(t) = e^t \left[A\cos(5t) + B\sin(5t) \right] + e^t \left[-5A\sin(5t) + 5B\cos(5t) \right]$

$h'_1(0^+) = A + 5B = 1 \longrightarrow B = \frac{1}{5}$

$$h_1(t) = e^t \frac{1}{5} \sin(5t)$$

$$\boxed{h(t) = \frac{e^t}{5} \sin(5t) \cdot \mu(t)}$$

$\boxed{3.2-2\ h}$

1.) $s^2 + 2s + 20 = 0$

2.) $s^2 + 26 = 0$

3.) $s^2 = 0$

4.) $s^2 - 2s + 26 = 0$

$\boxed{3.2-5\ a}$

$$\frac{d}{dt} u_{ix}(t) + \frac{1}{RC} u_{ix}(t) = \frac{d}{dt} u_{ul}(t)$$

$$y'(t) + \frac{1}{RC} y(t) = u'(t)$$

$$u(t) = \begin{cases} \sin(t) & , \quad -\infty < t < 0^- \\ 1 & , \quad 0^+ < t < +\infty \end{cases}$$

$$u(0^-) = \sin(0) = 0 \qquad\qquad u'(t) = \cos(t)$$

$$u(0^+) = 1$$

$$y_p = K_1 \cos(t) + K_2 \sin(t)$$

$$y'_p = -K_1 \sin(t) + K_2 \cos(t)$$

$$-K_1 \sin(t) + K_2 \cos(t) + \frac{1}{RC}\left(K_1 \cos(t) + K_2 \sin(t)\right) = \cos(t)$$

$$\frac{1}{RC} = \frac{1}{10^3 \cdot 10 \cdot 10^{-6}} = 100$$

$$\cos(t)\left[K_2 + 100 K_1\right] + \sin t \left[K_2 - 100 K_1\right] = \cos(t)$$

$$K_2 + 100 K_1 = 1 \qquad K_2 + 10\,000 K_2 = 1 \implies K_2 = -0{,}0001$$

$$100 K_2 - K_1 = 0 \qquad K_1 = -0{,}01$$

$$y_p(t) = -0{,}01 \cos(t) - 0{,}0001 \sin(t)$$

$$\boxed{y(0^-) = -0{,}01}$$

$$y(0^+) - y(0^-) = h_0 \left(u(0^+)\right)$$

$$y(0^+) = 1 - 0{,}01$$

$$\boxed{y(0^+) = 0{,}99}$$

KARLO MARKOVIĆ
0036423175

$\boxed{3.2-7\ a}$

$\zeta = -0{,}125$
$\zeta = 0{,}25$
$\zeta = 1$

$$y''(t) + 2\zeta\Omega_n y'(t) + \Omega_n^2 y(t) = A\Omega_n^2 \mu(t)$$

$\Omega_n = 0{,}4$

$A = \Omega_n^{-2} = 6{,}25$

$$H(s) = \frac{A\Omega_n^2}{s^2 + 2\zeta\Omega_n s + \Omega_n^2}$$

1.) $\zeta = -0{,}125$

$$H(s) = \frac{6{,}25 \cdot 0{,}4^2}{s^2 + 2(-0{,}125)\cdot 0{,}4 s + 0{,}4^2} = \frac{1}{s^2 - 0{,}1 s + 0{,}16}$$

$s = j\omega$

$$H(j\omega) = \frac{1}{-\omega^2 - 0{,}1 j\omega + 0{,}16} = \frac{1}{0{,}16 - \omega^2 - 0{,}1 j\omega}$$

$$\boxed{|H(j\omega)| = \frac{1}{\sqrt{(0{,}16-\omega^2)^2 + (0{,}1\omega)^2}}}$$

2.) $\zeta = 0{,}25$

$$H(s) = \frac{1}{s^2 + 0{,}2 s + 0{,}16} \qquad s = j\omega$$

$$H(j\omega) = \frac{1}{0{,}16 - \omega^2 + 0{,}2 j\omega}$$

$$\boxed{|H(j\omega)| = \frac{1}{\sqrt{(0{,}16-\omega^2)^2 + (0{,}2\omega)^2}}}$$

3.) $\zeta = 1$

$$H(s) = \frac{1}{s^2 + 0{,}8s + 0{,}16} \qquad s = j\omega$$

$$H(j\omega) = \frac{1}{-\omega^2 + 0{,}8j\omega + 0{,}16}$$

$$\boxed{|H(j\omega)| = \frac{1}{\sqrt{(0{,}16-\omega^2)^2 + (0{,}8\omega)^2}}}$$

$\boxed{3.2\text{-}7\ c}$

$A \sin 3t$

a) $A = |H(j\omega)|_{\omega=3} \cdot 3 = 3 \cdot \dfrac{1}{\sqrt{(0{,}16-9)^2 + (0{,}1\cdot 3)^2}} = 0{,}339$

$$\varphi = \operatorname{arctg} \frac{0{,}1\cdot 3}{0{,}16-9} = \operatorname{arctg} \frac{0{,}3}{-8{,}84} = -1{,}94^\circ + 180^\circ = 178{,}06^\circ$$

$$\boxed{0{,}339 \sin(3t + 178{,}06^\circ)}$$

b) $A = 3 \cdot \dfrac{1}{\sqrt{(0{,}16-9)^2 + (0{,}2\cdot 3)^2}} = 0{,}339$

$$\varphi = \operatorname{arctg} \frac{0{,}2\cdot 3}{0{,}16-9} = \operatorname{arctg} \frac{0{,}6}{-8{,}84} = -3{,}88 + 180^\circ = 176{,}12^\circ$$

$$\boxed{0{,}339 \sin(3t + 176{,}12^\circ)}$$

c) $A = 3 \cdot \dfrac{1}{\sqrt{(0{,}16-9)^2 + (0{,}8\cdot 3)^2}} = 0{,}328$

$$\varphi = \operatorname{arctg} \frac{0{,}8\cdot 3}{0{,}16-9} = \operatorname{arctg} \frac{2{,}4}{-8{,}84} = 164{,}81^\circ$$

$$\boxed{0{,}328 \sin(3t + 164{,}81^\circ)}$$