JIR 2013.

1) $x(t) = e^{4t}\left(u(t+2) - u(t-2)\right)$

a) CTFT

$$X(j\omega) = \int_{-\infty}^{+\infty} x(t)e^{-j\omega t}\,dt = \int_{-2}^{+2} e^{4t} \cdot e^{-j\omega t}\,dt =$$

$$= \int_{-2}^{+2} e^{t(4-j\omega)}\,dt = \left.\frac{e^{t(4-j\omega)}}{4-j\omega}\right|_{-2}^{+2} = \frac{1}{4-j\omega}\left(e^{2(4-j\omega)} - e^{-2(4-j\omega)}\right) =$$

$$= \frac{1}{4-j\omega}\left(e^{8-4j\omega} - e^{-8+4j\omega}\right)$$

b) ENG

$$E = \int_{-\infty}^{+\infty} |e^{4t}|^2\,dt = \int_{-2}^{2} e^{8t}\,dt = \left.\frac{e^{8t}}{8}\right|_{-2}^{2} =$$

$$= \frac{1}{8}\left(e^{16} - e^{-16}\right) = 111076311.5$$

$$P = \lim_{T \to \infty} \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} x(t) = \frac{1}{\infty} \cdot \text{broj} = 0$$

2. $x(n) = 2\cos\left(\frac{\pi}{8}n\right) + 4\sin\left(\frac{\pi}{2}n + \frac{\pi}{4}\right)$

а) ДВФ   Врста сигнала периодичан

$f: \mathbb{Z} \to \mathbb{R} \quad \forall n \in \mathbb{Z},\ N_0 \in \mathbb{Z}^+$ дефиниција

$f(n) = f(n+N)$ периодичан

б)

$$x(n+N) = 2\cos\left(\frac{\pi}{8}n + \frac{\pi}{8}N\right) + 4\sin\left(\frac{\pi}{2}n + \frac{\pi}{2}N + \frac{\pi}{4}\right) =$$

$$= \frac{2\pi}{\frac{\pi}{8}} = 16 \qquad \frac{2\pi}{\frac{\pi}{2}} = 4$$

ПАЗИ $\longrightarrow \frac{2\pi}{16} = \frac{\pi}{8}$ ПАЗИ

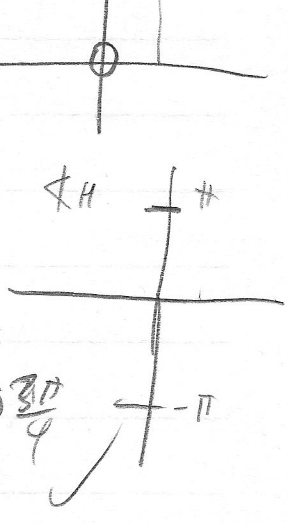
Периодичан за 16

в) DTFS

$$x(n) = 2\cdot\frac{1}{2}\left(e^{j\frac{\pi}{8}n} + e^{-j\frac{\pi}{8}n}\right) + 4\cdot\frac{1}{2j}\left(e^{j\frac{\pi}{2}n}\cdot e^{j\frac{\pi}{4}} - e^{-j\frac{\pi}{2}n}\cdot e^{-j\frac{\pi}{4}}\right)$$

$$= e^{j\frac{\pi}{8}n} + e^{-j\frac{\pi}{8}n} + 2\cdot e^{j\frac{\pi}{2}n}\cdot e^{-j\frac{\pi}{4}} - 2e^{-j\frac{\pi}{2}n}\cdot e^{-j\frac{3\pi}{4}}$$

$$= e^{j\pi n \Omega_0} + e^{-j\pi n \Omega_0} + 2e^{j\pi 4\Omega n}\cdot e^{-j\frac{\pi}{4}} - 2e^{-j\pi 4\Omega n}\cdot e^{-j\frac{3\pi}{4}}$$



$X_1 = 1$, $X_{-1} = 1$, $X_4 = 2e^{-j\frac{\pi}{4}}$, $X_{-4} = -2e^{-j\frac{3\pi}{4}}$

3. $y'(t) + 3y(t) = v'(t) + 2v(t)$ $y(0^-) = -1$ $v(t) = e^{-2t}\mu(t)$

Stabilnost sustava

a) prijenosna funkcija

$sY(s) + 3Y(s) = sV(s) + 2V(s)$

$$H(s) = \frac{Y(s)}{V(s)} = \frac{s+2}{s+3}$$

b) polovi $s=-3 \rightarrow Re\{s\}<0$, stabilan sustav

nule $s=-2$

c) $$H(j\omega) = \frac{j\omega+2}{j\omega+3} \cdot \frac{j\omega-3}{j\omega-3} = \frac{-\omega^2 - j\omega + 6}{-\omega^2 - 9} = \frac{-\omega^2-6}{-\omega^2-9} - \frac{j\omega}{-\omega^2-9}$$

$$|H(j\omega)| = \left|\frac{\omega^2+4}{\omega^2+9}\right| \qquad \measuredangle H(j\omega) = \mathrm{Arctg}\left(\frac{\omega}{2}\right) - \mathrm{Arctg}\left(\frac{\omega}{3}\right)$$

$$= \frac{\omega^2+6}{\omega^2+9} + \frac{j\omega}{\omega^2+9}$$

$$|H(j\omega)| = \left|\frac{\omega^4 + 12\omega^2 + 36}{\omega^4 + 18\omega + 81} + \frac{\omega^2}{\omega^4+18\omega^2+81}\right|$$

$$\measuredangle H(j\omega) = \mathrm{Arctg}\left(\frac{\omega}{\omega^2+6}\right) - \left(\frac{0}{\omega^2+9}\right) = \mathrm{Arctg}\left(\frac{\omega}{\omega^2+6}\right)$$

2j

d) Laplace

e) $sY(s) - Y(0^-) + 3Y(s) = sU(s) - U(0^-) + 2U(s)$

$e^{-2t}\mu_t$

$sY(s) + 1 + 3Y(s) = sU(s) + 2U(s)$

$$Y(s)(s+3) = U(s)(s+2) - 1$$

$$\frac{1}{}$$

$$Y(s) = \frac{s+2}{s+3}U(s) - \frac{1}{s+3}$$

$$Y(s) = \cancel{\frac{s+2}{s+3}} \cdot \frac{1}{\cancel{s+2}} - \cancel{\frac{1}{s+3}}$$

$$Y(s) = 0$$

$$Y(t) = 0$$

4. $y(n) + \frac{1}{4}y(n-1) - \frac{1}{4}y(n-2) = u(n)$

a) polovi nule

$$\frac{Y(z)}{z^0} + \frac{1}{4}\frac{Y(z)}{z^1} - \frac{1}{4}\frac{Y(z)}{z^2} = \frac{U(z)}{z^0}$$

$+\frac{1}{4}y(-1)$

$$Y(z)\left(1 + \frac{1}{4}z^{-1} - \frac{1}{4}z^{-2}\right) = U(z)$$

$$H(z) = \frac{Y(z)}{U(z)} = \left(\frac{1}{1+\frac{1}{4}z^{-1}-\frac{1}{4}z^{-2}}\right) = \frac{z^2}{z^2+\frac{1}{4}z-\frac{1}{4}}$$

$Z = 0, \infty$ nule

polovi $z_1 = |0.39039|$ $z_2 = |-0.64039|$

unutar 1, stabilan

Asimptotski stabilan ako je $|z|<1$, granično ako je $|z| \leq 1$

nestabilan $|z|>1$

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b) $$H(e^{j\Omega}) = \frac{e^{2j\Omega}}{e^{2j\Omega}+\frac{1}{4}e^{j\Omega}-\frac{1}{4}} = \frac{\cancel{\cos(2\Omega)+j\sin(2\Omega)}}{\cancel{\cos(2\Omega)+j\sin(2\Omega)}+\frac{1}{4}\cos(j\Omega)+\cancel{\frac{1}{4}j\sin(j\Omega)}-\frac{1}{4}}$$ 1 1 1 ne

$$|H(e^{j\Omega})| = \frac{\sqrt{\cos^2(2\Omega)+\sin^2(2\Omega)}}{\sqrt{\left(\cos(2\Omega)+\frac{1}{4}\cos(\Omega)-\frac{1}{4}\right)^2+\left(\sin(2\Omega)+\frac{1}{4}\sin(\Omega)\right)^2}}$$

$$\angle H(e^{j\Omega}) = \frac{\pi}{2} - \arctan\left(e^{2j\Omega}+\right.$$

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$$\arctan\left(\tan(2\Omega) - \arctan\left(\frac{\sin 2\omega + \frac{1}{4}\sin(2\omega)}{\cos 2\Omega + \frac{1}{4}\cos(\omega) - \frac{1}{4}}\right)\right.$$

$$u[n] = 5\sin\left(\frac{\pi}{3}n + \frac{\pi}{4}\right)$$

$$\omega = \frac{\pi}{3}$$

$$|X(e^{j\Omega})| = X\left(e^{\frac{\pi}{3}}\right) = \frac{1}{} = \frac{5}{4}$$

$$\measuredangle H(e^{j\omega}) = \measuredangle H(e^{\frac{\pi}{3}}) = \frac{2\pi}{3} + \frac{1}{3}\pi = \pi$$

$$-\frac{1\pi}{3} + \frac{1\pi}{3} = 0 \checkmark$$

$$y_p(t) = 5 \cdot \frac{5}{4} \cdot \sin\left(\frac{\pi}{3}n + \frac{\pi}{4} + \pi\right) = \frac{25}{4}\sin\left(\frac{\pi}{3} + \frac{5\pi}{4}\right)$$

$$\frac{4}{\cancel{5}}\ 5 \qquad 4\sin\left(\frac{\pi}{3} +\right.$$

5. – Nbrzowana $y_{A_p}(t) = 4 + \cos(4t)$

a) Wsp oprindu Jnoz nam noy

$A = (0^-)$

$a_0 y''(t) + a_1 y'(t) + a_2 y(t) =$

$4 + \frac{e^{j4t}}{2} + \frac{e^{-j4t}}{2}$

$\frac{4}{s} + \frac{s}{s^2+16} = \frac{4}{s} + \frac{s}{(s-4i)(s+4i)} =$

$= \frac{4}{s} + \frac{1}{2}\frac{1}{s-4i} + \frac{1}{2}\frac{1}{s+4i}$ $\frac{4i}{s-4i}$ $\frac{4i}{8i}$

$= \frac{\overset{c_1}{(4)}}{s} + \overset{c_2}{\left(\frac{1}{2}\right)}\frac{1}{s-4i} + \overset{c_3}{\left(\frac{+1}{2}\right)}\frac{1}{s+4i}$

$s=0$ $s=4i$ $s=-4i$

$s(s^2+16)$

(s^3+16s)

$c_1 + c_2 e^{-4it} + c_3 e^{4it} = u(t)$

$y'''(t) + 16y'(t) = u(t)$ $h(0) = c_1 + c_2 + c_3 = 4 + \frac{1}{2} + \frac{1}{2} = 5$

$h(0) = 5$

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