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1. $y''(t) + 5y'(t) + 6y(t) = u(t)$

$u(t) = 5\cos(t)$

$y(0) = 0, \quad y'(0) = 1$

- OPĆA HOMOGENA JEDNADŽBA

$$y_h(t) = Ce^{st}$$

$$Ce^{st}(s^2 + 5s + 6) = 0, \quad Ce^{st} \neq 0$$

$$s^2 + 5s + 6 = 0 \rightarrow s_1 = -3, \; s_2 = -2$$

$$y_h(t) = C_1 e^{-3t} + C_2 e^{-2t}$$

SUSTAV JE STABILAN.

- PRIJENOSNA FUNKCIJA

$$H(s) = \frac{1}{s^2 + 5s + 6}$$

- FREKVENCIJSKA KARAKTERISTIKA

$s = jw$

$$H(jw) = \frac{1}{(jw)^2 + 5jw + 6} = \frac{1}{6 - w^2 + j5w}$$

$$|H(jw)| = \frac{1}{\sqrt{(6-w^2)^2 + (5w)^2}} = \frac{1}{\sqrt{w^4 + 13w^2 + 36}}$$

$$\angle H(jw) = -\operatorname{arctg}\frac{5w}{6-w^2}$$

$$\angle H(jw) = \operatorname{arctg}\frac{5w}{w^2-6}$$

- PARTIKULARNA JEDNADŽBA

$$u(t) = 5\cos(t) \rightarrow y_p(t) = K\cos(t+\Theta)$$

$w = 1$

$$K = 5 \cdot |H(jw)|\Big|_{w=1} \qquad \Theta = \Theta_0 + \angle H(jw)\Big|_{w=1}$$

$$K = 5 \cdot \frac{1}{\sqrt{50}} \qquad \Theta = 0^\circ + (-45^\circ)$$

$$K = \frac{\sqrt{2}}{2} \qquad \Theta = -45^\circ$$

$$y_p(t) = \frac{\sqrt{2}}{2}\cos(t-45^\circ), \quad t \geq 0$$

- TOTALNI ODZIV

$$y(t) = y_h(t) + y_p(t)$$

$$y(0) = 0, \quad y'(0) = 1$$

$$y(t) = C_1 e^{-3t} + C_2 e^{-2t} + \frac{\sqrt{2}}{2}\cos(t-45^\circ)$$

$$y'(t) = -3C_1 e^{-3t} - 2C_2 e^{-2t} - \frac{\sqrt{2}}{2}\sin(t-45^\circ)$$

$$\left.\begin{aligned} y(0) &= C_1 + C_2 + \frac{1}{2} = 0 \\ y'(0) &= -3C_1 - 2C_2 + \frac{1}{2} = 1 \end{aligned}\right\} \quad C_1 = \frac{1}{2}, \; C_2 = -1$$

$$y(t) = \frac{1}{2}e^{-3t} - e^{-2t} + \frac{\sqrt{2}}{2}\cos(t-45^\circ), \quad t \geq 0$$

- STABILNOST SUSTAVA

IZ VLASTITIH FREKVENCIJA SUSTAVA MOŽEMO ZAKLJUČITI DA JE SUSTAV STABILAN.
NA TEMELJU TOGA, DOLAZIMO DO ZAKLJUČKA DA ĆE U $t \gg 0$, TOTALNI ODZIV BITI JEDNAK SAMO PARTIKULARNOM RJEŠENJE, JER ĆE HOMOGENA JEDNADŽBA BITI JEDNAKA NULI

$$\lim_{t\to\infty} Ce^{st} = 0, \quad \forall s < 0$$

2. $y(n) - 2y(n-1) + y(n-2) = u(n)$

$u(n) = 5$

$y(-2) = 0, \; y(-1) = 1$

- OPĆA HOMOGENA JEDNADŽBA

$y_h(n) = Cz^n$

$Cz^n - 2Cz^{n-1} + Cz^{n-2} = 0$

$Cz^{n-2}(z^2 - 2z + 1) = 0, \quad Cz^{n-2} \neq 0$

$z^2 - 2z + 1 = 0 \rightarrow z_{1,2} = 1$

IZ VLASTITE FREKVENCIJE SUSTAVA ZAKLJUČUJEMO DA JE SUSTAV NESTABILAN. S OBZIROM NA TO, ZAKLJUČUJEMO KAKO SUSTAV <u>NEMA</u> FREKVENCIJSKU KARAKTERISTIKU. ZBOG TOGA, PARTIKULARNO RJEŠENJE TRAŽIT ĆEMO NA KLASIČAN NAČIN.

$y_h(n) = (C_1 + C_2 n)(1)^n$

$y_h(n) = C_1 + C_2 n$

- PARTIKULARNA JEDNADŽBA

$u(n) = 5 \cdot (1)^n \rightarrow y_p(n) = K \cdot (1)^n \cdot n^2$

$y_p(n) = Kn^2$

$Kn^2 - 2K(n-1)^2 + K(n-2)^2 = 5$

$K(n^2 - 2n^2 + 4n - 2 + n^2 - 4n + 4) = 5 \rightarrow K = \frac{5}{2}$

$y_p(n) = \frac{5}{2} n^2, \; n \geq 0$

- TOTALNI ODZIV

$$y(n) = y_h(n) + y_p(n)$$

$$y(-2) = 0 \,,\; y(-1) = 1 \rightarrow y(0) = 7 \,,\; y(1) = 18$$

$$y(n) = C_1 + C_2 n + \frac{5}{2} n^2$$

$$\left.\begin{array}{l} y(0) = C_1 = 7 \\ y(1) = C_1 + C_2 + \frac{5}{2} = 18 \end{array}\right\} C_1 = 7 \,,\; C_2 = \frac{17}{2}$$

$$y(n) = 7 + \frac{17}{2} n + \frac{5}{2} n^2 \,,\quad n \geq 0$$

- $n >> 0$

U TRENUTKU $n>>0$, ODZIV SUSTAVA TEŽI K BESKONAČNO (RADI NESTABILNOSTI SUSTAVA).

3. $y''(t) + 2y'(t) + 5y(t) = u(t)$

$u(t) = \sin(t) \,,\; t < 0$

$u(t) = 2\sin(2t) \,,\; t > 0$

- OPĆA HOMOGENA JEDNADŽBA

$$y_h(t) = Ce^{st}$$

$$Ce^{st}(s^2 + 2s + 5) = 0 \,,\; Ce^{st} \neq 0$$

$$s^2 + 2s + 5 = 0 \rightarrow s_1 = -1 + j2 \,,\; s_2 = -1 - j2$$

$$y_h(t) = Ae^{(-1+j2)t} + Be^{(-1-j2)t}$$

$$y_h(t) = e^{-t}(Ae^{2jt} + Be^{-2jt})$$

$$y_h(t) = e^{-t}(C_1\cos(2t) + C_2\sin(2t))$$

- PRIJENOSNA FUNKCIJA

$$H(s) = \frac{1}{s^2+2s+5}$$

- FREKVENCIJSKA KARAKTERISTIKA

$$s = jw$$

$$H(jw) = \frac{1}{(jw)^2+2(jw)+5} = \frac{1}{5-w^2+j2w}$$

$$|H(jw)| = \frac{1}{\sqrt{(5-w^2)^2+(2w)^2}} = \frac{1}{\sqrt{w^4-6w^2+25}}$$

$$\angle H(jw) = -\mathrm{arctg}\frac{2w}{5-w^2}$$

$$\angle H(jw) = \mathrm{arctg}\frac{2w}{w^2-5}$$

- PARTIKULARNA JEDNADŽBA

$$u_1(t) = \sin(t) \rightarrow y_{P_1}(t) = K\sin(t+\Theta)$$

$$w=1$$

$$K = 1\cdot |H(jw)|\Big|_{w=1} \qquad \Theta = \Theta_0 + \angle H(jw)\Big|_{w=1}$$

$$K = \frac{\sqrt{5}}{10} \qquad \Theta = -26.56^\circ$$

$$y_{P_1}(t) = \frac{\sqrt{5}}{10}\sin(t-26.56^\circ),\ t<0$$

$$u_2(t) = 2\sin(2t) \rightarrow y_{P_2}(t) = K\sin(2t+\Theta)$$

$$w=2$$

$$K = 2\cdot |H(jw)|\Big|_{w=2} \qquad \Theta = \Theta_0 + \angle H(jw)\Big|_{w=2}$$

$$K = \frac{2\sqrt{17}}{17} \qquad \Theta = -75.96^\circ$$

$$y_{P_2}(t) = \frac{2\sqrt{17}}{17}\sin(2t-76.96^\circ),\ t>0$$

- TOTALNI ODZIV

$$y(t) = \begin{cases} y_{p_1}(t) & , t<0 \\ y_h(t) + y_{p_2}(t) & , t>0 \end{cases}$$

* $t<0$

$$y(t) = \frac{\sqrt{5}}{10} \sin(t - 26.56^\circ)$$

$$y'(t) = \frac{\sqrt{5}}{10} \cos(t - 26.56^\circ)$$

$$y(0^-) = -0.1$$

$$y'(0^-) = 0.2$$

* POČETNI UVJETI U 0^+

$$b_0 = b_1 = 0$$

$$b_2 = 1$$

$$y(0^+) - y(0^-) = b_0 u(0^+)$$

$$y(0^+) = y(0^-) = -0.1$$

$$y'(0^+) - y'(0^-) + a_1(y(0^+) - y(0^-)) = b_0 u'(0^+) + b_1 u(0^+)$$

$$y'(0^+) = y'(0^-) = 0.2$$

* $t>0$

$$y(t) = e^{-t}(C_1 \cos(2t) + C_2 \sin(2t)) + \frac{2\sqrt{17}}{17} \sin(2t - 75.96^\circ)$$

$$y'(t) = e^{-t}(C_1(-\cos(2t) - 2\sin(2t)) + C_2(-\sin(2t) + 2\cos(2t))) + \frac{4\sqrt{17}}{17} \cos(2t - 75.96^\circ)$$

$$\left.\begin{aligned} y(0^+) &= C_1 - 0.47 = -0.1 \\ y'(0^+) &= -C_1 + 2C_2 + 0.235 = 0.2 \end{aligned}\right\} C_1 = 0.37,\ C_2 = 0.17$$

$$y(t) = \begin{cases} \frac{\sqrt{5}}{10} \sin(t - 26.56^\circ) & , t<0 \\ e^{-t}(0.37\cos(2t) + 0.17\sin(2t)) + \frac{2\sqrt{17}}{17}\sin(2t - 75.96^\circ) & , t>0 \end{cases}$$

- ZA $t \gg 0$ ODZIV SUSTAVA JEDNAK JE SAMO PARTIKULARNOM RJEŠENJU y_{p_2}

4. $y'(t) + 3y(t) = u(t)$

$y(0) = 0$

$u(t) = \sin(t) + 2\sin(2t) + 3\sin(3t) + 4\sin(4t)$, $t \geq 0$

- OPĆA HOMOGENA JEDNADŽBA

$y_h(t) = Ce^{st}$

$Ce^{st}(s+3) = 0$, $Ce^{st} \neq 0$

$s+3=0 \rightarrow s=-3$

$y_h(t) = C_1 e^{-3t}$ SUSTAV JE STABILAN.

- PRIJENOSNA FUNKCIJA

$$H(s) = \frac{1}{s+3}$$

- FREKVENCIJSKA KARAKTERISTIKA

$s = jw$

$$H(jw) = \frac{1}{3+jw}$$

$$|H(jw)| = \frac{1}{\sqrt{9+w^2}}$$

$$\angle H(jw) = -\mathrm{arctg}\,\frac{w}{3}$$

$$\angle H(jw) = \mathrm{arctg}\,\frac{-w}{3}$$

- PARTIKULARNA JEDNADŽBA

$u_1(t) = \sin(t) \rightarrow y_{P1}(t) = K\sin(t+\Theta)$

$K = |H(jw)|\big|_{w=1} = \frac{\sqrt{10}}{10}$ $\Theta = \angle H(jw)\big|_{w=1} = -18.43^\circ$

$y_{P1}(t) = \frac{\sqrt{10}}{10}\sin(t - 18.43^\circ)$, $t \geq 0$

$$u_2(t) = 2\sin(2t) \rightarrow y_{P_2}(t) = K\sin(2t+\Theta)$$

$$K = 2\cdot|H(jw)|\Big|_{w=2} = \frac{2\sqrt{13}}{13} \qquad \Theta = \angle H(jw)\Big|_{w=2} = -33.69^\circ$$

$$y_{P_2}(t) = \frac{2\sqrt{13}}{13}\sin(2t-33.69^\circ), \quad t \geq 0$$

$$u_3(t) = 3\sin(3t) \rightarrow y_{P_3}(t) = K\sin(3t+\Theta)$$

$$K = 3\cdot|H(jw)|\Big|_{w=3} = \frac{\sqrt{2}}{2} \qquad \Theta = \angle H(jw)\Big|_{w=3} = -45^\circ$$

$$y_{P_3}(t) = \frac{\sqrt{2}}{2}\sin(3t-45^\circ), \quad t \geq 0$$

$$u_4(t) = 4\sin(4t) \rightarrow y_{P_4}(t) = K\sin(4t+\Theta)$$

$$K = 4\cdot|H(jw)|\Big|_{w=4} = \frac{4}{5} \qquad \Theta = \angle H(jw)\Big|_{w=4} = -53.13^\circ$$

$$y_{P_4}(t) = \frac{4}{5}\sin(4t-53.13^\circ), \quad t \geq 0$$

$$y_P(t) = y_{P_1}(t) + y_{P_2}(t) + y_{P_3}(t) + y_{P_4}(t)$$

- TOTALNI ODZIV

$$y(t) = y_h(t) + y_P(t)$$

$$y(0) = 0$$

$$y(0) = C_1 - 0.0999 - 0.307 - 0.5 - 0.639 = 0$$

$$C_1 = 1.55$$

$$y(t) = 1.55\,e^{-3t} + \frac{\sqrt{10}}{10}\sin(t-18.43^\circ) + \frac{2\sqrt{13}}{13}\sin(2t-33.69^\circ) + \frac{\sqrt{2}}{2}\sin(3t-45^\circ) + \frac{4}{5}\sin(4t-53.13^\circ), \quad t \geq 0$$

- ZA $t \gg 0$ ODZIV SUSTAVA POSTAJE SAMO PARTIKULARNI DIO RJEŠENJA

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5. $$y(n) + 0.5\,y(n-1) = u(n)$$

$$y(-1) = 1$$

$$u(n) = \cos(0.5\pi n + 0.2\pi) + 2\cos(\pi n) + 3\cos(1.5\pi n) + 4\cos(2\pi n)$$

- OPĆA HOMOGENA JEDNADŽBA

$$y_h(n) = Cz^n$$

$$Cz^{n-1}(z+0.5) = 0, \quad Cz^{n-1} \neq 0$$

$$z + 0.5 = 0 \rightarrow z = -\frac{1}{2}$$

$$y_h(n) = C_1\left(-\frac{1}{2}\right)^n$$ SUSTAV JE STABILAN.

- PRIJENOSNA FUNKCIJA

$$H(z) = \frac{1}{1+0.5z^{-1}}$$

- FREKVENCIJSKA KARAKTERISTIKA

$$z = e^{jw}$$

$$H(e^{jw}) = \frac{1}{1+0.5e^{-jw}}$$

$$H(e^{jw}) = \frac{1}{1+0.5\cos(w) - 0.5j\sin(w)}$$

$$|H(e^{jw})| = \frac{2}{\sqrt{5+\cos(w)}}$$

$$\angle H(e^{jw}) = -\operatorname{arctg}\frac{-0.5\sin(w)}{1+0.5\cos(w)}$$

$$\angle H(e^{jw}) = \operatorname{arctg}\frac{\sin(w)}{2+\cos(w)}$$

- PARTIKULARNA JEDNADŽBA

$$u_1(n) = \cos(0.5\pi n + 0.2\pi) \rightarrow y_{P1}(n) = A\cos(0.5\pi n + \Theta)$$

$$A = 1 \cdot \left|H(e^{jw})\right|\Big|_{w=\frac{\pi}{2}} = \frac{2\sqrt{5}}{5} \qquad \Theta = 36^\circ + \angle H(e^{jw})\Big|_{w=\frac{\pi}{2}} = 62.57^\circ$$

$$y_{P1}(n) = \frac{2\sqrt{5}}{5}\cos\left(\frac{\pi}{2}n + 62.57^\circ\right)$$

$$u_2(n) = 2\cos(\pi n) \rightarrow y_{P2}(n) = A\cos(\pi n + \Theta)$$

$$A = 2 \cdot \left|H(e^{jw})\right|\Big|_{w=\pi} = 4 \qquad \Theta = \angle H(e^{jw})\Big|_{w=\pi} = 0^\circ$$

$$y_{P2}(n) = 4\cos(\pi n)$$

$$u_3(n) = 3\cos(1.5\pi n) \rightarrow y_{P2}(n) = A\cos(1.5\pi n + \Theta)$$

$$A = 3 \cdot \left|H(e^{jw})\right|\Big|_{w=\frac{3\pi}{2}} = \frac{6\sqrt{5}}{5} \qquad \Theta = \angle H(e^{jw})\Big|_{w=\frac{3\pi}{2}} = -26.57^\circ$$

$$y_{P3}(n) = \frac{6\sqrt{5}}{5}\cos\left(\frac{3\pi}{2}n - 26.57^\circ\right)$$

$$u_4(n) = 4\cos(2\pi n) \rightarrow y_{P4}(n) = A\cos(2\pi n + \Theta)$$

$$A = 4 \cdot \left|H(e^{jw})\right|\Big|_{w=2\pi} = \frac{8}{3} \qquad \Theta = \angle H(e^{jw})\Big|_{w=2\pi} = 0^\circ$$

$$y_{P4}(n) = \frac{8}{3}\cos(2\pi n)$$

$$y_P(n) = y_{P1}(n) + y_{P2}(n) + y_{P3}(n) + y_{P4}(n)$$

$$y_P(n) = \frac{2\sqrt{5}}{5}\cos\left(\frac{\pi}{2}n + 62.57^\circ\right) + 4\cos(\pi n) + \frac{6\sqrt{5}}{5}\cos\left(\frac{3\pi}{2}n - 26.57^\circ\right) + \frac{8}{3}\cos(2\pi n)$$

- TOTALNI ODZIV

$$y(n) = y_h(n) + y_p(n)$$

$$y(-1) = 1 \rightarrow y(0) = 9.3$$

$$y(0) = C_1 + 0.412 + 4 + 2.399 + 2.666 = 9.3$$

$$C_1 = -0.17$$

$$y(n) = (-0.17)\left(-\frac{1}{2}\right)^n + \frac{2\sqrt{5}}{5}\cos(0.5\pi n + 62.57^\circ) + 4\cos(\pi n) + \frac{6\sqrt{5}}{5}\cos(1.5\pi n - 26.57^\circ) + \frac{8}{3}\cos(2\pi n), \quad n \geq 0$$

- ZA $n >> 0$ ODZIV SUSTAVA POSTANE SAMO PARTIKULARNI DIO RJEŠENJA – SUSTAV JE STABILAN.