ΙΟΥΝ 2015

1) $x(t) = \begin{cases} e^{4t}, & t<0 \\ e^{-2t}, & t\geq 0 \end{cases}$

α) ΕΝΕΡΓΕΙΑ

$$E = \int_{-\infty}^{+\infty} |x(t)|^2 dt = \int_{-\infty}^{0} e^{8t} dt + \int_{0}^{+\infty} e^{-4t} dt =$$

$$= \frac{e^{8t}}{8}\Big|_{-\infty}^{0} + \frac{e^{-4t}}{-4}\Big|_{0}^{+\infty} =$$

$$= \frac{1}{8} + \frac{1}{4} = \frac{3}{8}$$

β) CTFT $X(j\omega) = \int_{-\infty}^{+\infty} x(t) \cdot e^{-j\omega t} dt =$

$$= \int_{-\infty}^{0} e^{4t} \cdot e^{-j\omega t} + \int_{0}^{+\infty} e^{-2t} \cdot e^{-j\omega t} dt = \int_{-\infty}^{0} e^{t(4-j\omega)} dt + \int_{0}^{+\infty} e^{t(-2-j\omega)} dt =$$

$$= \frac{1}{4-j\omega}(1-0) + \frac{-1}{2+j\omega}(-1) = \frac{1}{4-j\omega} + \frac{1}{2+j\omega}$$

$$= \frac{2+j\omega + 4-j\omega}{8+2j\omega+\omega^2} = \frac{6}{8+2j\omega+\omega^2}$$

γ) $|H(j\omega)| = \sqrt{\frac{36}{64+16\omega^2+\omega^4+4\omega^2}} = \frac{6}{\sqrt{\omega^4+20\omega^2+64}}$

$\omega = 0$ max για ω

$$\frac{6}{\sqrt{64}} = \frac{6}{8} = \frac{3}{4}$$ ✓

6

2. $f(n) = \{\underline{1}, 4, 0, 4\}$

$N = 4$

a) DTF u 4 točke

$$X(k) = \sum_{n=0}^{N-1} x(n) \cdot e^{-2\pi j k \frac{n}{N}}$$

$$= \sum_{n=0}^{3} x(n) \cdot e^{-2\pi j k \frac{n}{4}}$$

$$= 1 + 4e^{-2\pi j k \frac{1}{4}} + 0 + 4e^{-2\pi j k \frac{3}{4}}$$

$$= 1 + 4e^{-jk\frac{\pi}{2}} + 4e^{-jk\frac{3\pi}{2}}$$

$X(0) = 1 + 4 + 4 = 9$ $\quad X(1) = 1 + 4e^{-jk\frac{\pi}{2}} + 4e^{-jk\frac{3\pi}{2}} =$

$$= 1 + 4\left(\cos\left(\tfrac{\pi}{2}1\right) - j\sin\left(\tfrac{\pi}{2}1\right)\right) + 4\left(\cos\left(\tfrac{3\pi}{2}1\right) - j\sin\left(\tfrac{3\pi}{2}1\right)\right)$$

$$= 1 + 4(-j) + 4(j) = 1$$

$X(2) = 1 + 4e^{-j\pi} + 4e^{-j3\pi}$ $\quad X(3) = 1 + 4e^{\frac{-j3\pi}{2}} + 4e^{\frac{-j9\pi}{2}}$

b) DTFS kod periodičnog signala

$$X_k = \frac{1}{N} \sum_{n=0}^{N-1} x(n) e^{2\pi j k \frac{n}{N}}$$

Period 4

$0 \to 4$

$$X_k = \frac{1}{N} \sum_{-\infty}^{\infty}$$

$$= \frac{1}{4} \sum_{0}^{4} \sum f(n+4l) \, e^{-2\pi j k \frac{n}{N}} = \frac{1}{4} f(4l) e^{-2\pi j k \frac{n}{4}}$$

$$+ \frac{1}{4} f(4l+1) e^{-2\pi j k \frac{n}{4}}$$

$$+ \frac{1}{4} f(4l+2) e^{2\pi j k \frac{n}{4}} +$$

3. $y(t) = (7+t)u(t)$ JUN 2015.

a) LINEARAN

$u(t) \to \alpha u_1(t) + \beta u_2(t) \overset{?}{\Rightarrow}$

$\to \alpha(7+t)u_1(t) + \beta(7+t)u_2(t)$

$y_1(t) = (7+t)u_1(t) \to \alpha(7+t)u_1(t)$

$+ \to$ LINEARAN

$y_2(t) = (7+t)u_2(t) \qquad \beta(7+t)u_2(t)$

---

b) VREMENSKI PROMENLJIV

$y(t) = (7+t)u(t)$

$u(t) = u(t+T) \overset{?}{\Rightarrow} y(t) = (7+t)u(t+T)$

$u_1 \overset{?}{\to} y(t) \overset{ako}{\to} y(t+T) = (7+t+T)u(t)$ NIJE VREMENSKI STALAN

c) KAUZALAN MEMORIJA SADAŠNJOST I PROŠLOST

KAUZALAN, ima memoriju, (sadašnjost i prošlost)

d) BIBO

$u(t) = M$

$y(t) = (7+t)M$

NESTABILAN

4. $y(n) - \frac{11}{30} y(n-1) + \frac{1}{30} y(n-2) = u(n)$

1) [illegible] [illegible]

$u(n) = \delta(n)$

$a_1 = +\frac{1}{6}$

$y(n) = c \cdot a^n$

$a_2 = +\frac{1}{5}$

$a^2 - \frac{11}{30} a + \frac{1}{30} a = 0$

~~$a_1 = \frac{11}{60} + \frac{\sqrt{114}}{60} i$ $\quad a_2 = \frac{1}{60} - \frac{\sqrt{114}}{60} i$~~

~~$y_h(n) = c_1 \left(\frac{1}{60} + \frac{\sqrt{114}}{60} i\right)^n + c_2 \left(\frac{1}{60} - \frac{\sqrt{114}}{60} i\right)^n$~~

$h_A(t) = c_1 \left(+\frac{1}{5}\right)^n + c_2 \left(+\frac{1}{6}\right)^n$ h(n)

~~$h(0) = c_1 + c_2 = 1$~~

h(0) u pot

h(1) u pot

$h(1) = $

$h(1) - \frac{11}{30} h(0) + \frac{1}{30} \cancel{h(-1)} = \cancel{\delta(1)}$

$h(1) - \frac{11}{30} = 0$

$h(1) = \frac{11}{30}$

$c_1 + c_2 = 1$ $\qquad c_1 = 1 - c_2$

$\frac{1}{6} c_1 + \frac{1}{5} c_2 = \frac{11}{30}$

$c_1 = -5$

$\frac{1}{6} - \frac{1}{6} c_2 + \frac{1}{5} c_2 = \frac{11}{30}$

$h_A(t) = +6 \left(\frac{1}{5}\right)^n - 5 \left(\frac{1}{6}\right)^n$

$\frac{1}{30} c_2 = \frac{1}{5}$

$c_2 = 6$

$h(n) = 6 \left(\frac{1}{5}\right)^n - 5 \left(\frac{1}{6}\right)^n$

h) $v(n)\, h(n)$ won ZBROJ

PO $\boxed{1}$

$$\sum_{n=-\infty}^{+\infty} v(n)\, h(n-m) = \sum_{m=-\infty}^{+\infty} h(m)\, v(n-m)$$

$$= \sum_{m=-\infty}^{+\infty} \left[6\left(\frac{1}{5}\right)^{n} - 5\left(\frac{1}{6}\right)^{n}\right] u(n-m)$$

$\downarrow$
0

$$= \sum_{m=0}^{+\infty} \left(6\left(\frac{1}{5}\right)^{n-m} - 5\left(\frac{1}{6}\right)^{n-m}\right) = \sum_{m=0}^{\infty} 6\left(\frac{1}{5}\right)^{n}\left(\frac{1}{5}\right)^{-m} - 5\left(\frac{1}{6}\right)^{n}\left(\frac{1}{6}\right)^{-m} =$$

$$= 6\cdot\left(\frac{1}{5}\right)^{n} \sum_{m=0}^{+\infty} (5)^{m} - 5\cdot\left(\frac{1}{6}\right)^{n} \sum_{m=0}^{+\infty} (6)^{m} =$$

$$= 6\left(\frac{1}{5}\right)^{n} \cdot \frac{5^{n+1}-1}{4} - 5\cdot\left(\frac{1}{6}\right)^{n} \cdot \frac{6^{n+1}-1}{5} =$$

$=$

- 4 - $h(n)$

$\mu(n)$ z dom.

$$y(n) - \frac{11}{30}y(n-1) + \frac{1}{30}y(n-2) = u(n) \qquad h(n)$$

$$\frac{Y(z)}{z^0} - \frac{11}{30}\frac{Y(z)}{z^1} + \frac{1}{30}\frac{Y(z)}{z^2} = U(z)$$

$$Y(z)\left(1 - \frac{11}{30}z^{-1} + \frac{1}{30}z^{-2}\right) = U(z)$$

$$H(z) = \frac{Y(z)}{U(z)} = \frac{1}{1 - \frac{11}{30}z^{-1} + \frac{1}{30}z^{-2}} = \frac{z^2}{z^2 - \frac{11}{30}z + \frac{1}{30}}$$

$$\frac{H(z)}{z} = \frac{z}{z^2 - \frac{11}{30}z + \frac{1}{30}} = \frac{z}{\left(z-\frac{1}{5}\right)\left(z-\frac{1}{6}\right)} = \frac{A}{z-\frac{1}{5}} + \frac{B}{z-\frac{1}{6}}$$

HEAVISIDEOV RAZVOJ

$$A = \lim_{z\to\frac{1}{5}} \cancel{\left(z-\frac{1}{5}\right)} \cdot \frac{z}{\cancel{\left(z-\frac{1}{5}\right)}\left(z-\frac{1}{6}\right)} = \frac{\frac{1}{5}}{\frac{1}{30}} = 6$$

$$B = \lim_{z\to\frac{1}{6}} \frac{z \cdot U(z)}{\left(z-\frac{1}{5}\right)} = \frac{\frac{1}{6}}{-\frac{1}{30}} = -5$$

$$\frac{H(z)}{z} = \frac{6}{z-\frac{1}{5}} + \frac{-5}{z-\frac{1}{6}} \Rightarrow H(z) = 6\frac{z}{z-\frac{1}{5}} + (-5)\frac{z}{z-\frac{1}{6}}$$

$$= \left(6\cdot\left(\frac{1}{5}\right)^n - 5\left(\frac{1}{6}\right)^n\right)\mu(n)$$

5ο ζητούμενο

$h_1(t) = \delta(t) - e^{-3t} u(t)$

$H_2(s) = \frac{6}{s+4}$

$H_1(s) = 1 - \frac{1}{s+3}$

$u(t) = \mu(t) \xrightarrow{\mathcal{L}} U(s) = \frac{1}{s}$

$y(0^-) = \frac{-5}{6}$

$y'(0^-) = -2$

$Y(s) = H(s) \cdot U(s) = (H_1(s) + H_2(s)) \cdot U(s) =$

$= \frac{s+2}{s+3} + \frac{6}{s+4} = \frac{(s+2)(s+4) + 6s + 18}{(s+3)(s+4)} = \frac{s^2 + 6s + 8 + 6s + 18}{(s+3)(s+4)} =$

$= \left(\frac{s^2 + 12s + 26}{(s+3)(s+4)} \right) \cdot \frac{1}{s} = \frac{s^2 + 12s + 26}{s(s+3)(s+4)}$

$= \frac{A}{s} + \frac{B}{s+3} + \frac{C}{s+4}$

$A = \lim_{s \to 0} \frac{s^2 + 12s + 26}{(s+3)(s+4)} = \frac{26}{12} = \frac{13}{6}$

$B = \lim_{s \to -3} \frac{s^2 + 12s + 26}{(s+4)s} = \frac{1}{3}$

$C = \lim_{s \to -4} \frac{s^2 + 12s + 26}{(s+3)s} = \frac{-3}{2}$

$Y(s) = \frac{13}{6} \frac{1}{s} + \frac{1}{3} \frac{1}{s+3} + \frac{-3}{2} \frac{1}{s+4}$

$y(t) = \left(\frac{13}{6} + \frac{1}{3} e^{-3t} - \frac{3}{2} e^{-4t} \right) \mu(t)$

λάθος (δεν βρήκα την y(t) αλλά [illegible])

$$Y(s) = \frac{s^2+12s+26}{(s+3)(s+4)}$$

$$y''(t) + 7y'(t) + 12y(t) = u''(t) + 12u'(t) + 26u(t)$$

$a_0 = 1 \quad a_1 = 7 \quad a_2 = 12 \quad b_0 = 1 \quad b_1 = 12 \quad b_2 = 26$

$$y(0^+) - y(0^-) = 1\cdot(u(0^+) - u(0^-))$$

$$y(0^+) - y(0^-) = 0 \qquad y(0^+) = \frac{-5}{6}$$

$$y'(0^-) = y'(0^+) = -2$$

lzr tu posnbu

$$s^2Y(s) - sy(0^-) - y'(0^-) + 7sY(s) - 7y(0^-) + 12Y(s) = s^2U(s) - sU(0^-) - U'(0^-) + 12sU(s) - 12U(0^-) + 26U(s)$$