SIS - MASS - 2. CIKLUS - DIO I 10.5.08

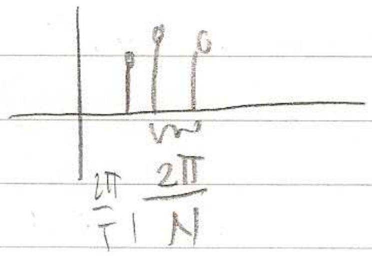
1

Datum

2. CIRKUS

AJMOOOOO

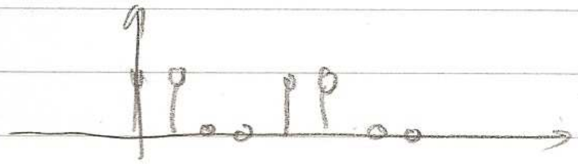
Fourierov red diskretnih signala



Sg. $\forall x \in$ Diskret. Periodič.

$$x(n) = x(n+N)$$

Spektar diskretnih signala je periodičan s $T = 2\pi$



DTFS - Deskret Time Fourier signal (Red)

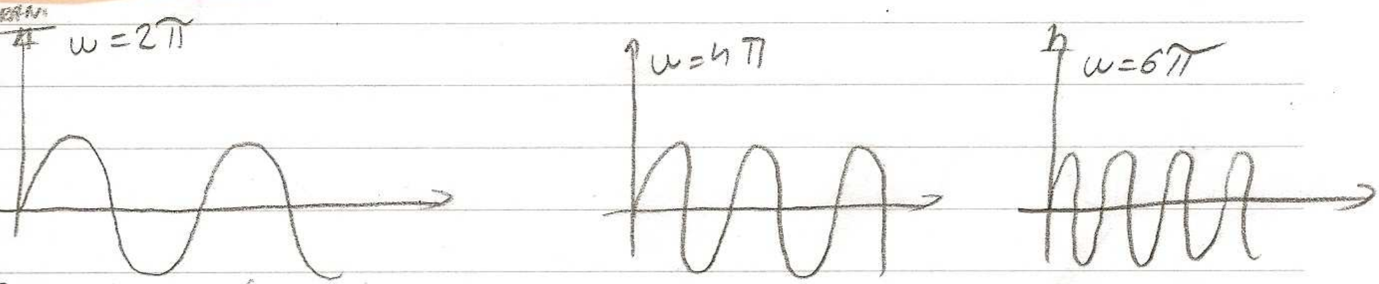
$$X_k = \frac{1}{N} \sum_{k=0}^{N-1} x(n) \cdot e^{-jk\frac{2\pi}{N} \cdot n} \quad , k = 0, 1, \ldots, N-1$$

IDTFS:

$$x(n) = \sum_{k=0}^{N-1} X_k e^{jk\frac{2\pi}{N} \cdot n}$$

Otipkavanje

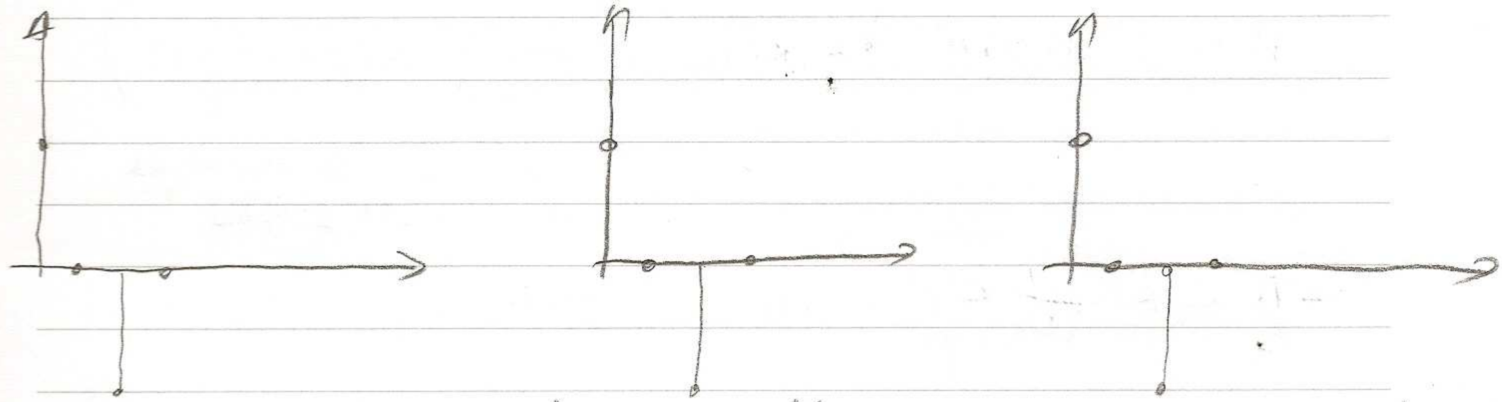
KONTINUIRANI



S jednom kružnom frekv. samo jedan signal

DISKRET.

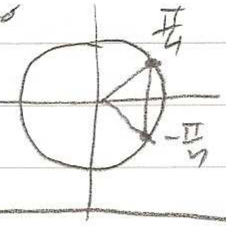
$x_1(n) = \cos\left(n\frac{\pi}{2}\right)$ $x_2(n) = \cos\left(n\frac{3\pi}{2}\right)$ $x_3 = \cos\left(n\frac{5}{2}\pi\right)$



$-\pi \leq \omega < \pi \rightarrow$ jednoznačni oblici

Za različite kružne frekv. diskretni signal može dati iste oblike $\longrightarrow$ ALIASING

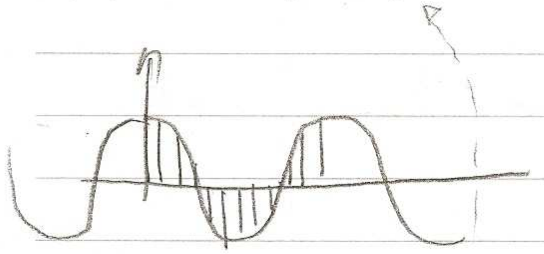
! ALI i cos



$\cos\left(\frac{\pi}{4}n\right) = \cos\left(-\frac{\pi}{4}n\right)$

$$\boxed{\begin{aligned} \omega &= \omega_a + 2k\pi \\ \omega &= (2\pi - \omega_a) + 2k\pi \end{aligned}}$$

$x(t) = \cos(\omega t)$



Koliko često ćemo uzimati uzorke?

so ω_s, $T_s = \frac{2\pi}{\omega_s}$

$x(n) = x(n \cdot T_s)$

$x(n) = x(\omega \cdot n \cdot T_s) = x\left(\omega \cdot n \cdot \frac{2\pi}{\omega_s}\right) = x\left(\boxed{2\pi \frac{\omega}{\omega_s}} \cdot n\right)$

da ne bi bilo Aliasinga to mora bit $[-\pi, \pi]$

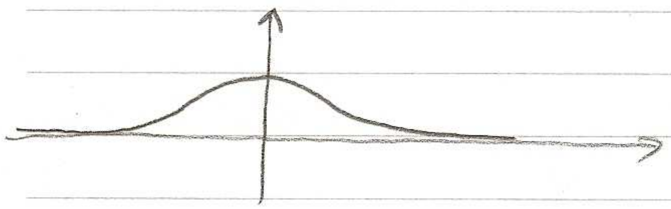
$-\pi < 2\pi \frac{\omega}{\omega_s} < \pi$

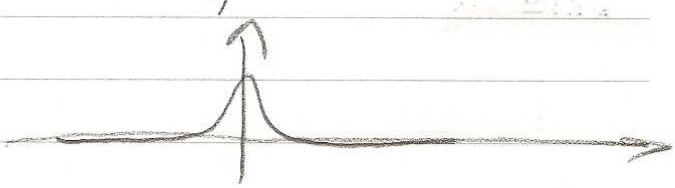
$-\frac{1}{2} < \frac{\omega}{\omega_s} < \frac{1}{2}$

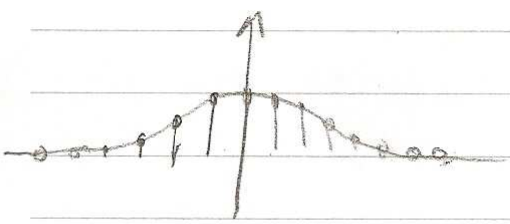
$-\omega_s < 2\omega < \omega_s \rightarrow \boxed{|\omega_s| > 2\omega}$ → da nema Aliasa

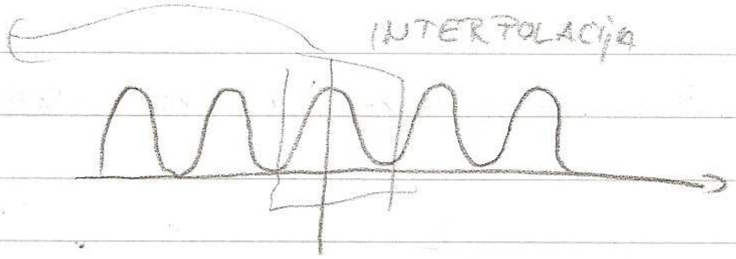
INTERPOLATOR - filtrira središnju komponentu

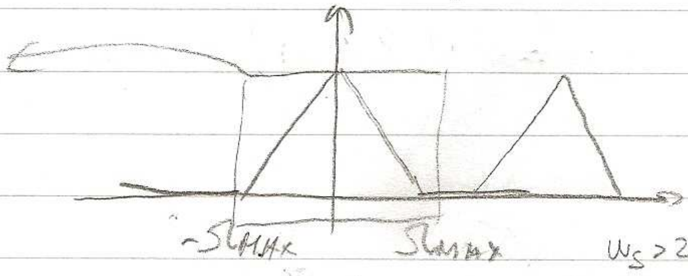
Signal | Spektar



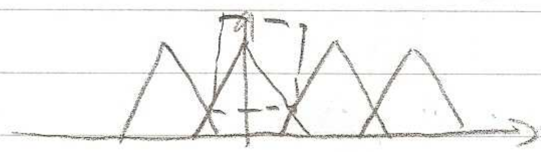








$\omega_s > 2\Omega_{MAX}$



$\omega_s < 2\Omega_{MAX}$

Preklapanje spektra! Nećemo moći vratiti u vremensku domenu! Jer ne znamo koja je središnja komponenta zbog preklapanja

Zadaci

(1) $x(t) = \sin(8000\pi t) + 2\cos(24000\pi t + \frac{\pi}{3}) + \sin(16000\pi t)$

$f_s = 10\,kHz \rightarrow \omega_s = 2\pi f = 20000\pi$

Kak' će izgledati vremenski oblik signala kod ga

1) otipkavanje

$x_1(t) = \sin(8000\pi t)$, $\omega = 8000$ $\qquad \omega_n = 2\pi \frac{\omega}{\omega_s} = 2\pi \cdot \frac{8000\pi}{20000\pi} = 0{,}8\pi$

$x_1(n) = \sin(0{,}8\pi n)$

✓ nema ALIASA

$-\pi < 0{,}8\pi < \pi$ ✓

$x_2(t) = 2\cos(24000\pi t + \frac{\pi}{3})$, $\omega = 24000\pi$

$\omega_n = 2\pi \frac{\omega}{\omega_s} = 2{,}4\pi$ Nije dobro jer $-\pi < 2{,}4\pi < \pi$ (X) Laž!

$\omega_n = \omega_a + 2k\pi$

$\omega_a = \omega_n - 2k\pi$, za $k = 1$

$\omega_a = 2{,}4\pi - 2\pi = 0{,}4\pi$ W OK! $-\pi < 0.4\pi < \pi$ ✓

$x_2(n) = 2\cos(0{,}4\pi n + \frac{\pi}{3})$

$x_3(t) = \sin(16000\pi t)$, $\omega = 16000\pi$

$\omega_n = 2\pi \frac{\omega}{\omega_s} = 1{,}8\pi$ $\qquad \omega_n = \omega_a + 2k\pi$

$\omega_a = \omega_n - 2k\pi$, $k = 1$

$x_3(n) = \sin(-0{,}4\pi n)$ ← NEZGODNO ZA RAČUNATI $\qquad \omega_a = 1{,}8\pi - 2\pi = -0{,}4\pi$ W ok!

$x_3(n) = \sin(0{,}4\pi n + \pi)$

Datum

Otipkani signal:

$$x(n) = \sin(0{,}8\pi n) + 2\cos\left(0{,}4\pi n + \frac{\pi}{3}\right) + \sin\left(0{,}4\pi n + \pi\right)$$

2) Sad vraćamo u vremensku domenu i gledamo gdje se promijenio oblik

1) $\omega_n = 0{,}8\pi = 2\pi \cdot \frac{\omega}{20000\pi}$, $\omega = 8000\pi$ Ok!

2) $\omega_n = 0{,}4\pi \cdot 2\pi \cdot \frac{\omega}{20000\pi}$, $\omega = 4000\pi$ Promjena!

3) $\omega_n = 0{,}4\pi \Rightarrow \omega = 4000\pi$ Promjena!

Dogodio se gubitak informacija

Signal u vremenskoj domeni nakon vraćanja

Rj: $x(t) = \sin(8000\pi t) + 2\cos\left(4000\pi t + \frac{\pi}{3}\right) + \sin(4000\pi t + \pi)$

Datum

DFT:

$$X(k) = \sum_{n=0}^{N-1} x(n)\, e^{-jk\frac{2\pi}{N}n} \quad , \quad k\, 0, 1, \dots N-1$$

IDFT:

$$x(n) = \frac{1}{N} \sum_{n=0}^{N-1} X(k)\, e^{-jk\frac{2\pi}{N}n} \quad , \quad n = 0, 1, \dots N-1$$

• 2 a) $x(n) = \delta(n)$ za N točaka

$$X(k) = \sum_{n=0}^{N-1} x(n)\, e^{-jk\frac{2\pi}{N}n}$$

$$X(k) = \underset{1}{\delta(0)} \cdot \underset{1}{e^{0}} + \underset{0}{\delta(1)\, e^{\dots}} + \underset{0}{\delta(2)^{-\dots}} + \dots \underset{0}{\delta(N-1)\, e^{\dots}}$$

$$X(k) = 1 \quad , \quad k = 0, 1, \dots N-1$$

• 4 a) $x(n) = \cos\left(\frac{\pi}{2} n\right) \quad , \quad n = 0, 1, 2, 3, \qquad 3 = N-1 \quad \boxed{N=4}$

$$X(k) = \sum_{n=0}^{3} x(n)\, e^{-jk\frac{\pi}{2}n} \quad , \quad k = 0, 1, 2, 3$$

$$X_k = \underset{1}{x(0)}\, e^{0} + \underbrace{x(1)\, e^{-jk\frac{\pi}{2}}}_{0} + \underbrace{x(2)\, e^{-jk\pi}}_{-1} + \underbrace{x(3)\, e^{-jk\frac{3\pi}{2}}}_{0} \quad , \quad k = 0, 1, 2, 3$$

$$X_k = 1 - 1 \cdot e^{-jk\pi} \quad , \quad k = 0, 1, 2, 3$$

$$X_k = 1 - \cos(k\pi)$$

$X(0) = 0 \qquad X(2) = 0$

$X(1) = 2 \qquad X(3) = 2$

$$X(k) = \{0, 2, 0, 2\}$$

Datum

b) $x(n) = \left(\frac{1}{2}\right)^n$, $n = 0, 1, 2, 3,$ $N = 4$

$$X(k) = \sum_{n=0}^{3} x(n)\, e^{-jk\frac{2\pi}{4}n}$$

$$X(k) = \sum_{n=0}^{3} x(n)\, e^{-jk\frac{\pi}{2}n}$$

$$X(k) = \underset{\substack{\downarrow \\ 1}}{x(0)} + \underset{\substack{\downarrow \\ \frac{1}{2}}}{x(1)} \cdot e^{-jk\frac{\pi}{2}} + \underset{\substack{\downarrow \\ \frac{1}{4}}}{x(2)} \cdot e^{-jk\pi} + \underset{\substack{\downarrow \\ \frac{1}{8}}}{x(3)} \cdot e^{-jk\frac{3}{2}\pi}$$

$$X(0) = 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} = \frac{15}{8}$$

$$X(1) = 1 + \frac{1}{2}(-j) + \frac{1}{4}(-1) + \frac{1}{8}j = \frac{3}{4} - \frac{3}{8}j$$

$$X(2) = 1 + \frac{1}{2}(-1) + \frac{1}{4}(1) + \frac{1}{8}(-1) = \frac{5}{8}$$

$$X(3) = 1 + \frac{1}{2}j + \frac{1}{4}(-1) + \frac{1}{8}(-j) = \frac{3}{4} + \frac{3}{8}j$$

TRIK ZADATAK!

5) $X(k) = \frac{3}{4} + \frac{3}{8}e^{-jk\frac{\pi}{2}} - \frac{3}{4}e^{-j\pi k} - \frac{3}{8}e^{-j\frac{3}{2}\pi k}$, $k = 0, 1, 2, 3$, $N = 4$

$$X(k) = \sum_{n=0}^{3} x(n)\, e^{-jk\frac{\pi}{2}n}$$

$$X(k) = x(0) + x(1)\, e^{-jk\frac{\pi}{2}} + x(2)\, e^{-jk\pi} + x(3)\, e^{-jk\frac{3}{2}\pi}$$

Samo usporedujemo! I prepišemo koeficijente!

$$x(n) = \left\{\frac{3}{4}, \frac{3}{8}, -\frac{3}{4}, -\frac{3}{8}\right\}$$

Datum

Ovaj li mogo doći?

$$e^{jx} = \cos(x) + j\sin(x)$$

$$e^{-jx} = \cos(-x) + j\sin(-x)$$

$$e^{-jx} = \cos(x) - j\sin(x)$$

$\boxed{171.\ \text{iz dz2}}$

$N=4$

vrijednost u nuli

$$x(n) = \{\underline{0}, 0, 1, 0\}$$

$$X(k) = \sum_{n=0}^{4} x(n)\, e^{-jk\frac{\pi}{2}n} \rightarrow X(k) = x(2)\, e^{-jk\pi}$$

$$x(k) = \cos(k\pi)$$

$$x(0) = 1,\ x(1) = -1,\ x(2) = 1,\ x(3) = 1$$

$$x(k) = \{\underline{1}, -1, 1, -1\}$$

ili

$N=4$

$$x(n) = \{1, \underline{1}, 0, 1\}$$

$$x(k) = x(-1)\, e^{jk\frac{\pi}{2}} + x(0) + x(2)\, e^{-jk\pi}$$

$$x(k) = 1 \cdot e^{jk\frac{\pi}{2}} + 1 + 1 \cdot e^{-jk\pi}$$

uvrštava se k i lijepo dalje

Datum

$$\boxed{\text{Trajanje signala} \cdot \text{Širina spektra} = 2\pi}$$

$$D \cdot B = 2\pi$$

* Odredi F-red tj. koeficijente F-reda

$$x(n) = \cos\left(\frac{\pi n}{3}\right) + \sin\left(\frac{\pi n}{4}\right)$$

1) Odredit jel periodičan i period

$$\cos\left(\frac{\pi}{3}n\right) \Rightarrow \omega = \frac{\pi}{3} \Rightarrow \omega = \frac{2\pi}{N} \Rightarrow N_1 = 6$$

$$\sin\left(\frac{\pi}{4}n\right) \Rightarrow \omega = \frac{\pi}{4}, \quad N_2 = 8$$

Tražimo najmanji zajednički višekratnik

6, 8	2
3, 4	3
1, 4	4
1	

$$4 \cdot 2 \cdot 3 = 24 \qquad \boxed{N = 24} \qquad \boxed{\omega = \frac{2\pi}{N} = \frac{\pi}{12}}$$

2) Zapišemo preko komplex

$$x(n) = \frac{1}{2}\left(\underbrace{e^{j\frac{\pi}{3}n}}_{k=4} + \underbrace{e^{-j\frac{\pi}{3}n}}_{k=-4}\right) + \frac{1}{2j}\left(\underbrace{e^{j\frac{\pi}{4}n}}_{k=3} - \underbrace{e^{-j\frac{\pi}{4}n}}_{k=-3}\right)$$

3) Računamo red

$$X_k = \frac{1}{N}\sum_{n=0}^{N-1} x(n) \cdot e^{-j\frac{2\pi}{N}n} \rightarrow k \cdot \frac{2\pi}{N} = k \cdot \frac{\pi}{12}$$

Sad gledamo gore

$$k\frac{\pi}{12} = \frac{\pi}{3} \Rightarrow k = 4$$

$$k\frac{\pi}{12} = -\frac{\pi}{3} \Rightarrow k = -4$$

$$k\frac{\pi}{12} = \frac{\pi}{4} \Rightarrow k = 3$$

$$k\frac{\pi}{12} = -\frac{\pi}{4} \Rightarrow k = -3$$

$$X_4 = \frac{1}{2} \qquad X_{-4} = \frac{1}{2}$$

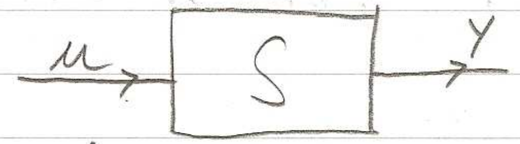
$$X_3 = \frac{1}{2j} \qquad X_{-3} = -\frac{1}{2j}$$

$$x(n) = \sum_{n=-\infty}^{\infty} X_k e^{jk\frac{2\pi}{N}n} \qquad x(n) = X_{-5} e^{j\frac{-5\pi}{12}n} + X_{-4} e^{j\frac{-4\pi}{12}n} + \dots$$

Datum

Uvod u Sustave

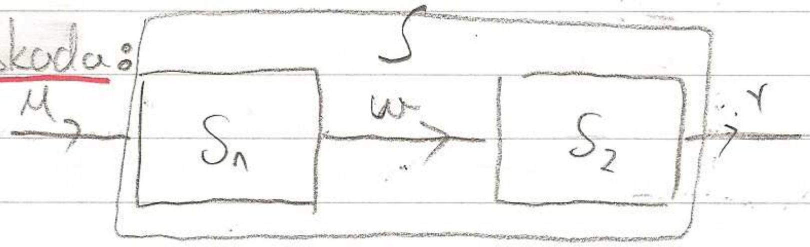
Oznaka sustava



Uspoređujemo izlaz i ulaz i zaključujemo što sustav radi

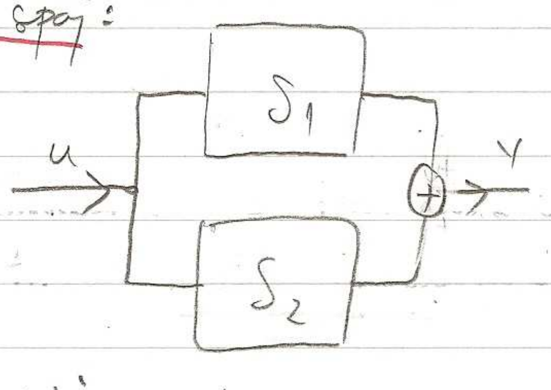
a) DISKRETNI sustavi
b) KONTINUIRANI sustavi

* Kaskada:

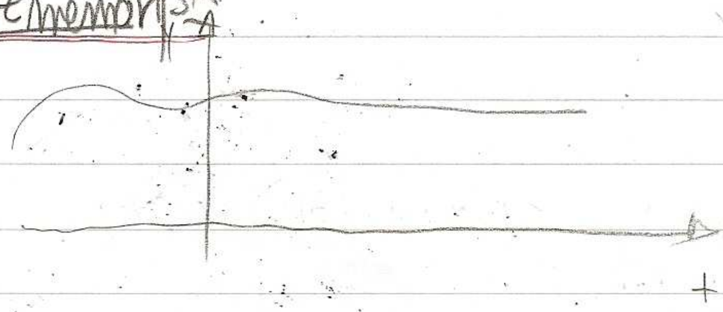


$$\left.\begin{array}{l} w = S_1(u) \\ y = S_2(w) \end{array}\right\} \quad y = S_2(S_1(u))$$

* Paralelni spoj:



* Bezmemorijski



$$\boxed{y(t) = u(t)}$$

Datum

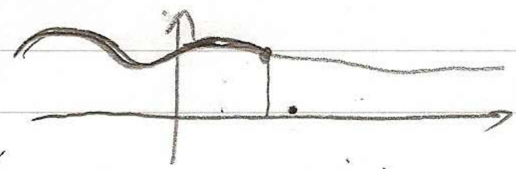
* **Memorijski** $y(t) = u(t-4)$

$y(t) = u(t+4)$ - Pustimo ga cijelog i zapamtimo i onda kad je sve u memoriji onda možemo "gledati u budućnost"

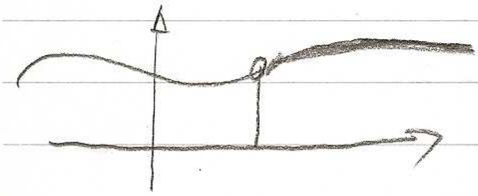
- integral je memorijski
- i derivacija isto → $\lim\limits_{\Delta t \to} \frac{x(t+\Delta t) - x(t)}{\Delta t}$
- sve što NIJE $y(t) = u(t)$ je MEMORIJSKI s.
- Bilo koje čeprkanje po vremenskoj domeni → MEM

pazi! $y(n) = u\left(n + \underbrace{\frac{3 - 3e^{-j4\pi n}}{8}}_{0}\right) \Rightarrow y(n) = u(n)$ → Bez MEM

* **KAUZALNI** sustav - sustavi koji ovise o trenutnoj ili prošloj vrijednosti ili obje



* **ANTIKAUZALNI** sustav - sustav koji ovisi samo o budućoj vrijednosti



Pr. $y(t) = u(-t)$

$y(4) = y(-4)$ kauz. $\quad y(-4) = y(4)$ → ANTIKAUZALAN

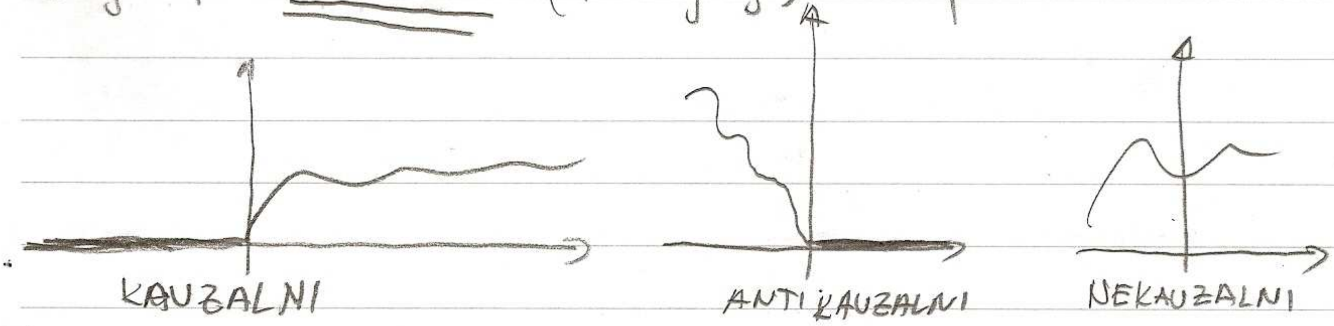
$y(0) = y(0)$ kauz $\quad$ Rj: NEKAUZALAN ???

Datum

- $y(t) = u(t)$ KAUZ.
- $y(t) = u(t-4)$ KAUZ.
- $y(t) = u(t+4)$ ANTI KAUZ

* BEZMEMORIJSKI $\xrightarrow{\text{isključivo}}$ KAUZALAN
-ali ne i obratno.

Digresija: **SIGNALI** (Ponavljanje) - Ne miješati sa SUSTAVIMA



* NEKAUZALNI - ovisi i o trenutnom i o prošlom ali i o budućem odzivu

$y(t) = \mu(t-1) + \mu(t+1)$

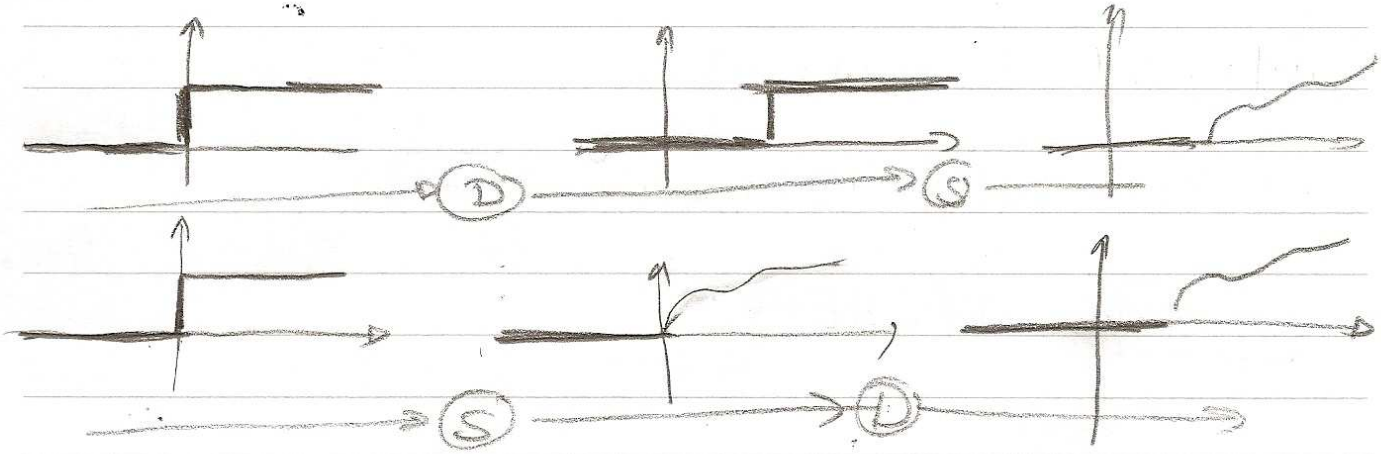
$y(t) = \mu(t) + \mu(t+2)$

$-\infty$	0	$+\infty$
KAUZALNI		ANTIKAUZALNI
NEKAUZALNI		
	KAUZAL.	

$y(t) = u(t^2)$ NEKAUZALAN

Datum

* VREMENSKA NEPROMJENJIVOST



Ak' su slike iste
VR. NEPROMJ.

- $y(t) = t \cdot u(t^2)$

$y_1(t) = t \cdot u(t^2 - T)$ 1) zakasnimo signal

$y(t-T) = (t-T)\, u((t-T)^2)$ 2) zakasnimo ulaz

$y(t-T) = (t-T)\, u(t^2 - 2tT + T^2)$

3) usporedimo: $y_1 \neq y$

VREMENSKI PROMJENJIV

- $y(t) = 3\,u(t+4)$

$y_1(t) = 3\,u(t+4-T)$

$y(t-T) = 3\,u((t-T)+4) \Rightarrow y(t-T) = 3\,u(t+4-T)$

$y_1 = y$

VR. NEPR.

Datum

- $y(t) = u(t) + u(t-1)$

$y_1(t) = u(t-T) + u(t-1-T)$

$y(t-T) = u(t-T) + u((t-T)-1)$

$\boxed{y_1 = y}$ VR. NEPR.

- $y_1(t) = u(t) + t \cdot u(t-1)$

$y_1(t) = u(t-T) + t \cdot u(t-1-T)$

$y(t-T) = u(t-T) + (t-T) \cdot u((t-T)-1)$

$y_1 \neq y$

VR. PROMJ

* LINEARNOST

- $y(t) = t\,u(t)$

$y_1(t) = t\,u_1(t)$
$y_2(t) = t\,u_2(t)$

$u(t) = \alpha u_1(t) + \beta u_2(t)$

$y(t) = t(\alpha u_1(t) + \beta u_2(t)) = t\alpha u_1(t) + t\beta u_2(t)$

$y(t) = \alpha y_1(t) + \beta y_2(t)$ SUSTAV JE

LINEARAN

Datum

- $y(t) = u(t) + 4$

$y_1(t) = u_1(t) + 4$
$y_2(t) = u_2(t) + 4$

$u(t) = \alpha u_1(t) + \beta u_2(t)$

$y(t) = \alpha u_1(t) + \beta u_2(t) + 4$

NE LINEARAN

- ako sustav ima slobodni koeficijent uvijek je NELINEARAN

- $y(t) = u(t) + 5u(t+1)$

$y_1(t) = u_1(t) + 5u_1(t+1)$
$y_2(t) = u_2(t) + 5u_2(t+1)$

$u(t) = \alpha u_1(t) + \beta u_2(t)$
$u(t+1) = \alpha u_1(t+1) + \beta u_2(t+1)$

$y(t) = \alpha u_1(t) + \beta u_2(t) + 5(\alpha u_1(t+1) + \beta u_2(t+1))$

$y(t) = \alpha(u_1(t) + 5u_1(t+1)) + \beta(u_2(t) + 5u_2(t+1))$

$y(t) = \alpha y_1(t) + \beta y_2(t)$ LINEARAN!

Datum

- $y(t) = \frac{1}{u(t)}$

$y_1(t) = \frac{1}{u_1(t)}$

$y_2(t) = \frac{1}{u_2(t)}$

$u(t) = \alpha u_1(t) + \beta u_2(t)$

$y(t) = \frac{1}{\alpha u_1(t) + \beta u_2(t)}$ Nemremo rastavit nazivnik → NELINEARAN!

- $y(t) = 2^{u(t)}$

$y_1(t) = 2^{u_1(t)}$
$y_2(t) = 2^{u_2(t)}$

$u = \alpha u_1(t) + \beta u_2(t)$

$y(t) = 2^{\alpha u_1(t)} \cdot 2^{\beta u_2(t)}$ - α i β moraju ić dole
- mora biti +, ali •
NE LINEARAN

$y_1(t) = 2^{u(t-T)}$
$y(t-T) = 2^{u(t-T)}$ $y_1 = y$ ⇒ VREM. NEPROMJE.

- $y(t) = u(t^2)$

$y_1 = u_1(t^2)$
$y_2 = u_2(t^2)$

$u(t)^2 = \alpha u_1 t^2 \beta u_2(t^2)$

$y = \alpha u_1(t)^2 + \alpha u_2(t^2)$

$y(t) = \alpha y_1 + \beta y_2$

LINEARAN

Datum

$y_1(t) = u(t^2 - T)$

$y(t-T) = u((t-T)^2)$ $\quad y_1 \neq y$ VR. PROMJ.

• $y(t) = \dfrac{u(t)}{1 + u(t-1)}$

$y_1(t) = \dfrac{u_1(t)}{1 + u_1(t-1)}$

$y_2(t) = \dfrac{u_2(t)}{1 + u_2(t-1)}$

$u(t) = \alpha u_1(t) + \beta u_2(t)$

$u(t-1) = \alpha u_1(t-1) + \beta u_2(t-1)$

$y(t) = \dfrac{\alpha u_1(t) + \beta u_2(t)}{1 + \alpha u_1(t-1) + \beta u_2(t-1)}$ Nelinearan

i Nepromjenjivi

• $y(t) = \int_0^t u(\tau)\, d\tau$ Linearan 

$y_1(t) = \int_0^t u(\tau - T)\, d\tau \;\left|\begin{matrix} \tau = t & a = \tau - T \Rightarrow \tau = a + T \\ \tau = 0 & da = d\tau \end{matrix}\right|$

$y(t-T) = \int_0^{t-T} u(\tau)\, d\tau \;\left|\begin{matrix} \tau = a \\ d\tau = da \end{matrix}\right|$

$y_1 = \int_{a=-T}^{t-T} u(a)\, da$ $\quad y_1 \neq y$ Vremenski promjenjiv

$y(t-T) = \int_0^{t-T} u(a)\, da$

Datum

- $y(t) = \int_{-\infty}^{t} u(\tau)\, d\tau$

$$y_1(t) = \int_{\tau=-\infty}^{\tau=t} u(\tau - T)\, d\tau \quad \left| \begin{array}{ll} a = \tau - T & \tau = a+T \\ da = d\tau & \\ & \end{array} \right.$$

$\tau = t$

$a + T = t$

$a = t - T$

$$y_1(t) = \int_{-\infty}^{t-T} u(a)\, da$$

$$y(t-T) = \int_{-\infty}^{t-T} u(\tau)\, d\tau \quad \left| \begin{array}{l} a = \tau \\ da = d\tau \end{array} \right|$$

$$y(t-T) = \int_{-\infty}^{t-T} u(a)\, da$$

$y = y_1$ VR. NEPR.

- $y(n) = u(\overbrace{3 + \cos(n\pi)}^{S})$ ¿

$y_1(n) = u_1(3 + \cos(n\pi))$
$y_2(n) = u_2(3 + \cos(n\pi))$

$u(3 + \cos(n\pi)) = \alpha u_1(3 + \cos(n\pi)) + \beta u_2(3 + \cos(n\pi))$

$y(n) = \alpha u_1(3 + \cos(n\pi)) + \beta u_2(3 + \cos(n\pi))$

$\underbrace{y(n) = \alpha y_1(n) + \beta y_2(n)}$ LINEARNI

Datum

- $y(n) = \left(\frac{1}{2}\right)^n u(3n+2)$

$y_1(n) = \left(\frac{1}{2}\right)^n u(3n+2-N)$

$y(n-N) = \left(\frac{1}{2}\right)^{n-N} u(3(n-N)+2)$ $\quad y_1 \neq y$ VR. PROMJ.

$y_1(n) = \left(\frac{1}{2}\right)^n u_1(3n+2)$

$y_2(n) = \left(\frac{1}{2}\right)^n u_2(3n+2)$

$u(3n+2) = \alpha u_1(3n+2) + \beta u_2(3n+2)$

$y = \left(\frac{1}{2}\right)^n (\alpha u_1(3n+2) + \beta u_2(3n+2))$

$y = \left(\frac{1}{2}\right)^n \alpha u_1(3n+2) + \left(\frac{1}{2}\right)^n \beta u_2(3n+2)$

$y = \left(\frac{1}{2}\right)^n \alpha \cdot y_1(n) + \left(\frac{1}{2}\right)^n \beta \cdot y_2(n)$

Datum

AUTOMATI

- imaju unutarnja stanja

Def:

- ULAZI – "ulazna abeceda" = {0, 1}

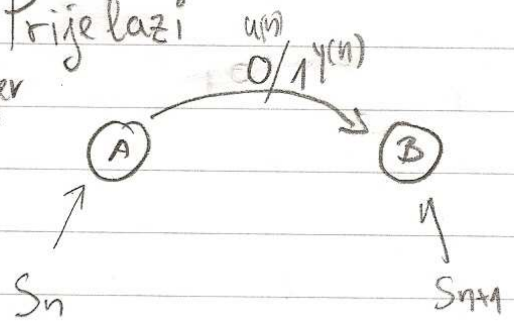
- IZLAZI "izlazna abeceda" = {0, 1}
- STANJA

↳ oznaka S_n, S_0, A.

- poč. stanja i funkcije prijelaza

Prijelazi

Primjer



S_n – trenutno stanje

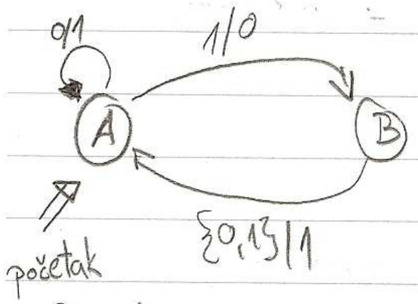
S_{n+1} – novo stanje

$S_{n+1} = f(S_n, u(n))$

$f(A, 0) = B$

$y(n) = f(S_n, u(n))$

$f(S_n, u(n)) = (S_{novo}, y(n))$



$S_0 = A$

$\{0,1\}/1 = 0/1$

$1/1$

TABLICA PRIJELAZA

stanja \ ulaz	0	1
A	A/1	B/0
B	A/1	A/1

$u(n) = \{0\ 1\ 1\ 0\ 0\ 1\ 0\}$

$y(n)\ \{1\ 0\ 1\ 1\ 1\ 0\ 1\}$

$S_n\ \{A\ A\ B\ A\ A\ A\ B\ A\}$

$u = 101^a 0^b$

$1^a = \underbrace{111 \ldots 1}_{a}$

$1^1 = 1$

$1^2 = 11$

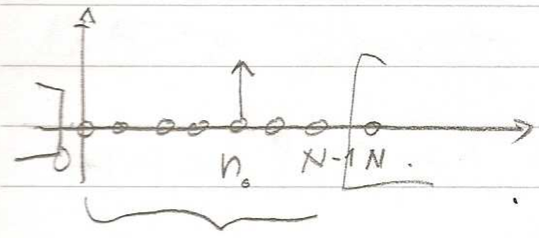
$x(n+1) = f_1(x(n), u(n))$
$y(n) = f_2(\ ? \ )$

Zad za AKT. 8. tj.

• 2 b)

DFT u N točaka

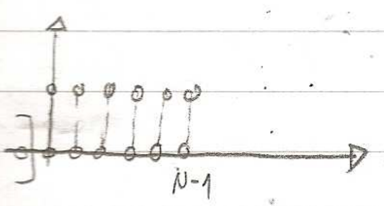
$$x(n) = \delta(n-n_0), \quad 0 < n_0 < N$$



$$X(k) = \sum_{n=0}^{N-1} x(n) e^{-j\frac{2\pi}{N}kn} = \sum_{n=0}^{N-1} \delta(n-n_0) e^{-j\frac{2\pi}{N}kn} = \delta(n-n_0) e^{-j\frac{2\pi}{N}kn_0}$$

Rj: $X(k) = e^{-j\frac{2\pi}{N}kn_0}$

• 3 a) $x(n) = \mu(n) - \mu(n-N)$ u N točaka



DFT: $X(k) = \sum_{n=0}^{N-1} x(n) e^{-j\frac{2\pi}{N}kn} = \sum_{n=0}^{N-1} e^{-j\frac{2\pi}{N}kn}$

$$= \sum_{n=0}^{N-1} \left(e^{-j\frac{2\pi}{N}k}\right)^n = \frac{1 - e^{-j\frac{2\pi}{N}k \cdot N}}{1 - e^{-j\frac{2\pi}{N}k}} = \frac{1 - e^{-j2\pi k}}{1 - e^{-j\frac{2\pi}{N}k}} = 0$$

$e^{-j2\pi k} = 1$

za $k \neq 0$

formula za sumu g. reda

$$\sum_{n=0}^{N} a^n = \frac{1 - a^{N+1}}{1 - a}$$ na službenom šalabahteru!

Datum

2\. način

$$k=0 \Rightarrow X(0)=\sum_{n=0}^{N-1} e^{j0} = \sum_{n=0}^{N-1} 1 = N$$

$$x(k) = \begin{cases} N, & k=0 \\ 0, & k\neq 0 \end{cases}$$

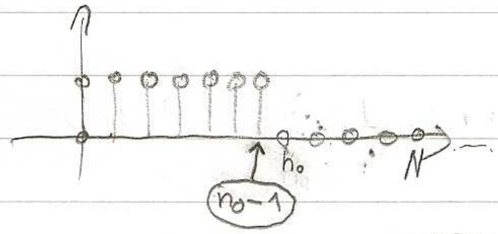
$$x(k) = N\,\delta(k), \quad 0 \le k \le N$$

→ kod kože u N točaka ili u točkama 0,1,... ⇒ <u>DFT</u>

→ kod kože odredi fourierov red → <u>DTFS</u>

• 3\. b) DFT, u N-točaka

$$x(n) = \mu(n) - \mu(n-n_0), \quad 0 < n_0 < N$$



$$X(k) = \sum_{n=0}^{N-1} x(n)\, e^{-j\frac{2\pi}{N}kn} = \sum_{n=0}^{n_0-1} 1\cdot e^{-j\frac{2\pi}{N}kn} =$$

(objašnjenje granica) $$= \sum_{n=0}^{N-1} (\mu(n) - \mu(n-n_0))\, e^{-j\frac{2\pi}{N}kn}$$

$$= \frac{1-e^{-j\frac{2\pi}{N}kn_0}}{1-e^{-j\frac{2\pi}{N}k}} = \left\{ 1-a = \left(a^{\frac{1}{2}}\cdot a^{-\frac{1}{2}} - a^{\frac{1}{2}}\cdot a^{\frac{1}{2}}\right) = a^{\frac{1}{2}}\left(a^{-\frac{1}{2}} - a^{\frac{1}{2}}\right) \right.$$

$$a = e^{-j\frac{2\pi}{N}k}$$

$$e^{-j\frac{\pi}{N}k}\left(e^{j\frac{\pi}{N}k} - e^{-j\frac{\pi}{N}k}\right)\cdot\frac{2j}{2j} = 2j\, e^{-j\frac{\pi}{N}k}\cdot \sin\left(\frac{\pi}{N}k\right)$$

$$1 - a^{n_0} = 2j\, e^{-j\frac{\pi}{N}kn_0} \sin\left(\frac{\pi}{N}kn_0\right) \Bigg\} =$$

$$= e^{-j\frac{\pi}{N}k(n_0-1)} \cdot \frac{\sin\left(\frac{\pi}{N}kn_0\right)}{\sin\left(\frac{\pi}{N}k\right)}$$

2. način

$$\sum_{i=0}^{n} e^{j(\theta + i\varphi)} = \frac{\sin\left[(n+1)\cdot\frac{\varphi}{2}\right]}{\sin\frac{\varphi}{2}} \cdot e^{j(\theta + \frac{\varphi}{2}n)}$$

↑ ↑ ↑ ↑ ↑ ↑ ↑ ↑

IMA NA SLUŽBENOM ŠALIĆU

- 10.3 iz Jerenove zbirke

$x(n) = \{\underline{2}, 0, -1, 1\}$ Odredi DFT, $N=4$

$$X(k) = \sum_{n=0}^{3} x(n)\, e^{-j\frac{2\pi}{4}kn}$$

sad za pojedine n-ove uvrstiti vrijednosti !

$$X(k) = 2\cdot \cancel{e^{-j\frac{\pi}{2}k\cdot 0}} + \cancel{0\cdot e^{-j\frac{\pi}{2}k\cdot 1}} + (-1)\cdot e^{-j\frac{\pi}{2}\cdot k\cdot 2} + 1\cdot e^{-j\frac{\pi}{2}k\cdot 3}$$

$$X(k) = 2 - \underbrace{e}_{-1}{}^{-j\pi k} + e^{-j\frac{3\pi}{2}k}$$

$X(k) = 2 - (-1)^k + e^{-j\frac{3\pi}{2}k}$, sad uvrstiti k-ove

$X(0) = 2 - 1 + 1 = 2$

$X(1) = 2 + 1 + j = 3 + j$

$X(2) = 2 - 1 + \underbrace{e^{-j3\pi}}_{-1} = 0$

$X(3) = 2 + 1 + e^{-j\frac{9\pi}{2}} = 2 + 1 - j = 3 - j$

$$\boxed{X(k) = \{\underline{2}, 3+j, 0, 3-j\}}$$

- 10.6. a) iz Zbirke → Težak zadatak

– u N-točaka

$$x_1(n) = x(n)\cdot\cos\left(\frac{2\pi}{N}n\cdot m\right), \quad 0\le m\le N$$

$$x(n) \xrightarrow{DFT} X(k)$$

$$X_1(k) = \sum_{n=0}^{N-1} x_1(n)\, e^{-j\frac{2\pi}{N}k\cdot n} = \sum_{n=0}^{N-1} x(n)\cdot\cos\left(\frac{2\pi}{N}n\cdot m\right) e^{-j\frac{2\pi}{N}k\cdot n} =$$

$$= \sum_{n=0}^{N-1} x(n)\left[\frac{1}{2}\left(e^{j\frac{2\pi}{N}n\cdot m} + e^{-j\frac{2\pi}{N}n\cdot m}\right)\right] e^{-j\frac{2\pi}{N}k\cdot n} =$$

$$= \frac{1}{2}\left[\sum_{n=0}^{N-1} x(n)\, e^{j\frac{2\pi}{N}n\cdot m}\cdot e^{-j\frac{2\pi}{N}kn} + \sum_{n=0}^{N-1} x(n)\, e^{-j\frac{2\pi}{N}n\cdot m}\cdot e^{-j\frac{2\pi}{N}kn}\right] =$$

$$= \frac{1}{2}\left[\underbrace{\sum_{n=0}^{N-1} x(n)\, e^{j\frac{2\pi}{N}n(m-k)}}_{X(k-m)} + \sum_{n=0}^{N-1} x(n)\cdot e^{-j\frac{2\pi}{N}n(m+k)}\right]$$

$$= \frac{1}{2}\left[X(k-m) + X(k+m)\right]$$

→ Zad. iz Sustava

- 9. TJ. 3

VR. NEPR.? LINEARAN?

$$y(n) = \sum_{k=-\infty}^{n} \frac{u(k)}{n-k}$$

$$y_1(n) = \sum_{k=-\infty}^{n} \frac{u(k-N)}{n-k}$$

$$y(n-N) = \sum_{k=-\infty}^{n-N} \frac{u(k)}{n-(N+k)} = \left| \begin{array}{l} \text{supstitucija} \\ N+k=m \\ k=m-N \end{array} \right| = \sum_{m-N=-\infty}^{m-N} \frac{u(m-N)}{n-m}$$

⇨

Datum

$$= \sum_{m=-\infty+N}^{n-N+N} \frac{u(m-N)}{n-m}$$

$Y_1 = Y$ ✓ VR. NEPROMJ.

Nema veze kaj su druge varijable sumiranja!
Bitno da je isti oblik !!

$$Y(n) = \sum_{k=-\infty}^{n} \frac{u(k)}{n-k} = \sum_{k=-\infty}^{n} \frac{\alpha u_1(k) + \beta u_2(k)}{n-k} =$$

$$u(n) = \alpha u_1(n) + \beta u_2(n)$$

$$= \sum_{k=-\infty}^{n} \frac{\alpha u_1(k)}{n-k} + \sum_{k=-\infty}^{n} \frac{\beta u_2(k)}{n-k} =$$

$$= \alpha \sum_{k=-\infty}^{n} \frac{u_1(k)}{n-k} + \beta \sum_{k=-\infty}^{n} \frac{u_2(k)}{n-k} = \alpha y_1(n) + \beta y_2(n)$$

LINEARAN!

• 9.ti. 4

$$y(t) = \int_{-\infty}^{\infty} e^{-\tau} \mu(-\tau)\, u(t-\tau)\, d\tau$$

$$y_1(t) = \int_{-\infty}^{\infty} e^{-\tau} \mu(-\tau)\, u(t-T-\tau)\, d\tau$$

$$y(t-T) = \int_{-\infty}^{\infty} e^{-\tau} \mu(-\tau)\, u(t-T-\tau)\, d\tau$$

$= $ VR. NEPROMJ.

Da li je memorijski??

Da!

Datum

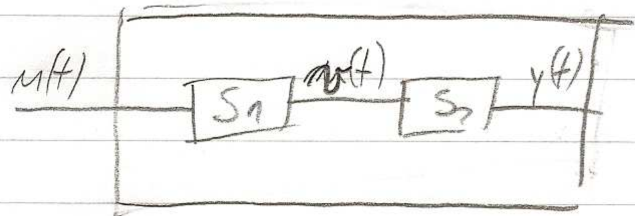
9. tj. 5

Kaskadni spoj 2 linearna sustava.
Da li je i taj spoj linearan?

$S_1 \to LIN$

$S_2 \to LIN$

$S = S_1 + S_2 \Rightarrow LIN$??



$$S_1(\alpha u_1(t) + \beta u_2(t)) = \alpha S_1 u_1(t) + \beta S_2 u_2(t)$$

$$v(t) = S_1(u(t))$$

$$y(t) = S_2(v(t)) = S_2(S_1(\alpha u_1(t) + \beta u_2(t))) = S u(t)$$

$$S(u(t)) = S_2(S_1(u(t))) = S_2(\underbrace{S_1(\alpha u_1(t) + \beta u_2(t))}_{LIN}) =$$

$$= \underbrace{\alpha S_2(S_1(u_1(t))) + \beta S_2(S_1(u_2(t)))}_{LIN!}$$

$\boxed{\text{iz DZ}}$

$$y(n) = \sum_{k=-\infty}^{\infty} x(k) \cdot \underbrace{\delta(k-n)}$$ Bezmemorijski ??

↳ ova će biti ≠ 0 samo kad $k = n$

$$y = x(n) \underbrace{\delta(n-n)}_{1} = \underline{\underline{x(n)}}$$

Bezmemorijski!