2.E5

3.1-1a

DTFT $x_1(n) = \{\dots, 0, 0, 1, 1, \underline{1}, 1, 1, 0, 0, \dots\}$ (indeksi $-2, -1, 0, 1, 2$)

L=5 različitih od 0

$$X(\omega) = \sum_{n=-\infty}^{+\infty} x(n) e^{-j\omega n} = 1\cdot e^{j2\omega} + 1\cdot e^{j\omega} + 1 + 1\cdot e^{-j\omega} + 1\cdot e^{-2j\omega}$$

$$= \cos 2\omega + \cancel{j\sin 2\omega} + \cos\omega + \cancel{j\sin\omega} + 1 + \cos\omega - \cancel{j\sin\omega} + \cos 2\omega - \cancel{j\sin 2\omega}$$

$$= \underline{1 + 2\cos\omega + 2\cos 2\omega}$$

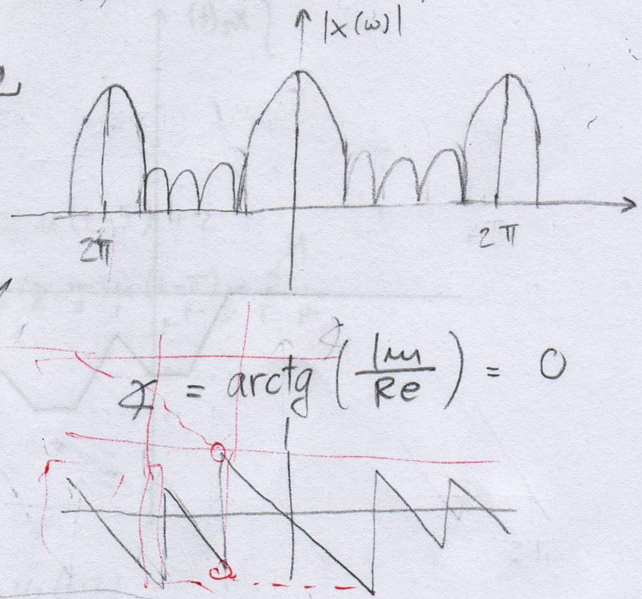
$$= e^{2j\omega}\left(1 + e^{-j\omega} + e^{-2j\omega} + e^{-3j\omega} + e^{-4j\omega}\right) = e^{2j\omega}\frac{e^{-5j\omega} - 1}{e^{-j\omega} - 1}$$

$$x_1(0) = \sum_{n=-2}^{2} 1 = 5$$

$$X(e^{j\omega}) = \begin{cases} 5, & \omega = 0 \\ e^{2j\omega}\dfrac{e^{-5j\omega} - 1}{e^{-j\omega} - 1}, & \omega \neq 0 \end{cases}$$

$$X(e^{j\omega}) = e^{2j\omega}\,\frac{e^{-\frac{5}{2}j\omega}\left(e^{-\frac{5}{2}j\omega} - e^{\frac{5j\omega}{2}}\right)}{e^{-j\omega/2}\left(e^{-j\omega/2} - e^{j\frac{\omega}{2}}\right)}$$

$$= \frac{\sin\left(\frac{5\omega}{2}\right)}{\sin\left(\frac{\omega}{2}\right)}, \ \omega \neq 0 \ \rightarrow \ |X(j\omega)| = \left|\frac{\sin\left(\frac{5\omega}{2}\right)}{\sin\left(\frac{\omega}{2}\right)}\right|$$



$\measuredangle = \operatorname{arctg}\left(\frac{Im}{Re}\right) = 0$

MATLAB ?

3.2.-1a

TEOREM OTIPKAVANJA :
(SHANNONOV TEOREM)

↳ Vremenski kontinuirani signal $x(t)$, s frekvencijama ne većim od F_{max}, može biti egzaktno rekonstruiran iz svojih uzoraka $x(n) \triangleq x(nT)$, ako je otipkavanje provedeno s frekvencijom $F_s = \frac{1}{T}$ koja je veća od $2F_{max}$.

NYQUISTOV KRITERIJ : Minimalna frekvencija otipkavanja za koju je moguća rekonstrukcija signala x iz njegovih uzoraka x_s naziva se Nyquistova frekvencija.

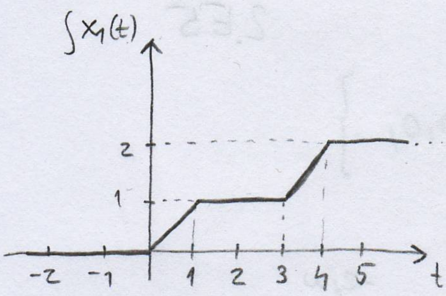
$f = 8kHz$ $\quad T = \frac{1}{f} = 1{,}25\cdot 10^{-4}$ $\quad f_s > 2f_{max} = 16000$ $\quad T_s = \frac{1}{f_s} = 6{,}25\cdot 10^{-5}$

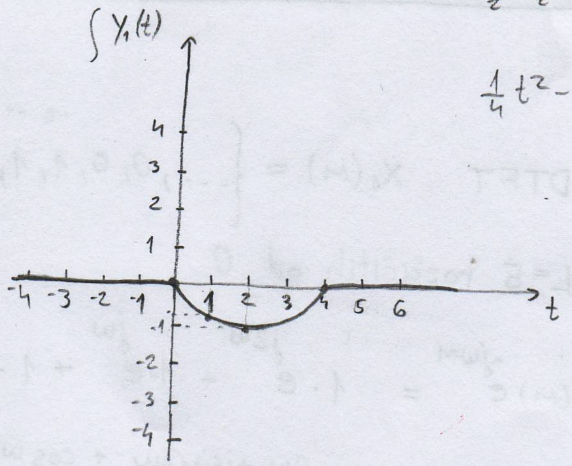
3.3-1 a

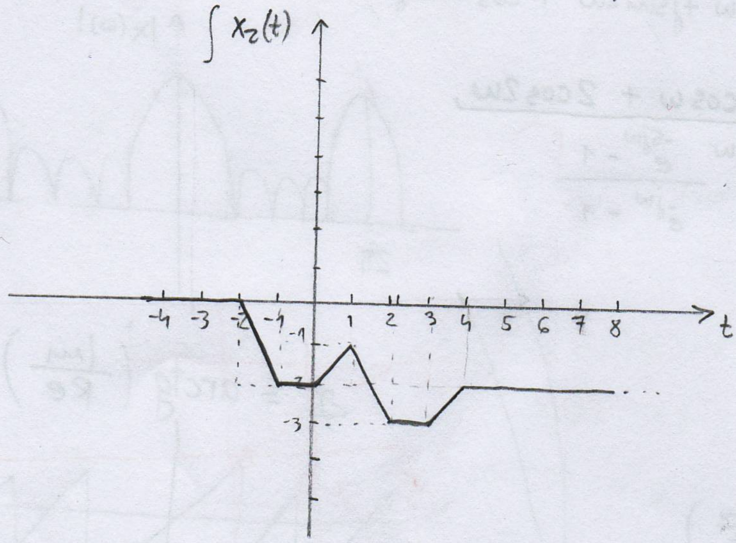
$y = \frac{1}{2}t - 1$

$\frac{1}{2} \cdot \frac{1}{2}t^2 - t$

$\frac{1}{4}t^2 - t$







3.3-3a

1. LINEARAN SUSTAV : $y_1 = S(u_1)$, $y_2 = S(u_2)$

   ↳ homogenost: $S(\alpha u_1) = \alpha S(u_1) = \alpha y_1$, $S(\beta u_2) = \beta S(u_2) = \beta y_2$

   ↳ aditivnost: $S(\alpha u_1) + S(\beta u_2) = \alpha y_1 + \beta y_2$

   ⟹ SUPERPOZICIJA: $S(\alpha u_1 + \beta u_2) = \alpha S(u_1) + \beta S(u_2)$

   Sustav je linearan ako ih zadovoljava.

2. VREMENSKI NEPROMJENJIV : Sustavi koji ne mijenjaju parametre tijekom vremena

   ↳ za bilo koju pobudu $u(n)$ daje odziv $y(n)$, a za zakašnjeli ulaz $D_{nk}(u)(n)$ daje zakašnjeli odziv $D_{nk}(y)(n)$

3. MEMORIJSKI SUSTAV : Sustavi kojima odziv ovisi ne samo o trenutnoj vrijednosti ulaznog signala nego i o prošlim i budućim vrijednostima

3.3 - 3b

1. $y(t) = u^2(t)$

↳ BEZMEMORIJSKI

↳ NELINEARAN $\quad y(t) = \left(\alpha u_1(t) + \beta u_2(t)\right)^2 = \times$

↳ VREM. NEPROMJENJIV
$$y_1(t) = u^2(t-T)$$
$$y(t-T) = u^2(t-T) \quad ||$$

2. $y(t) = t\,u(t) + 2$

↳ BEZMEMORIJSKI

↳ NELINEARAN

↳ VREM. PROMJENJIV
$$y_1(t) = t\,u(t-T) + 2$$
$$y(t-T) = (t-T)\,u(t-T) + 2 \quad \#$$

3. $y(t) = u(t-2)$

↳ MEMORIJSKI

↳ LINEARAN $\quad y(t) = \alpha u_1(t-2) + \beta u_2(t-2)$

↳ VREM. NEPROMJENJIV
$$y_1(t) = u(t-2-T)$$
$$y(t-T) = u(t-T-2) \quad ||$$

$$x_1(t) = \mu(t) - \mu(t-1) + \mu(t-3) - \mu(t-4) \Rightarrow g[0,1] + g[3,4]$$

$$\hookrightarrow y_1(t) = \left(\frac{1}{2}t - 1\right)\left[\mu(t) - \mu(t-4)\right] = \frac{1}{2}t\mu(t) - \frac{1}{2}t\mu(t-4) - \mu(t) + \mu(t-4)$$

$$x_2(t) = -2\left(\mu(t+2) - \mu(t+1)\right) + \mu(t) - \mu(t-1) - 2\left(\mu(t-1) - \mu(t-2)\right) + \mu(t-3) - \mu(t-4)$$

$$x_2(t) = -2x_1(t+2) + x_1(t)$$

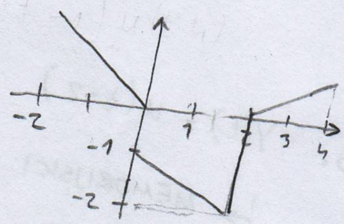
$$y_2(t) = -2y_1(t+2) + y_1(t)$$

$$y_2(t) = -2\left(\underbrace{\frac{1}{2}(t+2) - 1}_{\frac{1}{2}t}\right) \cdot \left[\mu(t+2) - \mu(t-2)\right] + \left(\frac{1}{2}t - 1\right)\left[\mu(t) - \mu(t-4)\right] =$$

$$= -t\left[\mu(t+2) - \mu(t-2)\right] + \left(\frac{1}{2}t - 1\right)\left[\mu(t) - \mu(t-4)\right] =$$

$$= -t\left(\mu(t+2) - \mu(t)\right) + \left(-\frac{1}{2}t - 1\right)\left(\mu(t) - \mu(t-2)\right) + \left(\frac{1}{2}t - 1\right)\left(\mu(t-2) - \mu(t-4)\right)$$

$$y_2(t) = -t\, g[-2,2] + \left(\frac{1}{2}t - 1\right) g[0,4]$$



3.3-5b

$$x(n+1) = 1{,}08 \cdot \left(x(n) + 8000\right)$$

$$x(n+1) = 1{,}08\, x(n) + 8640$$

$$x(0) = 0$$

$$x(15) = ?$$

$$x(n+1) - 1{,}08\, x(n) = 8640 \qquad c \cdot q^{n+1} - 1{,}08\, c q^{n} = 0$$

$$\boxed{x(n) = C q^{n}}$$

$$c q^{n} \left(q - 1{,}08\right) = 0$$

$$q = 1{,}08$$

$$x_h(n) = C (1{,}08)^{n}$$

$$\boxed{x_p(n) = K}$$

$$K - 1{,}08\, K = 8640$$

$$-0{,}08\, K = 8640$$

$$K = -108000$$

$$x(n) = C (1{,}08)^{n} - 108\,000 \qquad x(0) = 0$$

$$C = 108\,000$$

$$x(n) = 108\,000\, (1{,}08)^{n} - 108\,000$$

$$x(15) = 234\,594{,}26$$

Nakon 15g imamo 234 594, 26 kn

dižemo 10 000, ostaje 224 594, 26 kn

Kamata na to je 242 561, 8 kn

↳ uvijek smo u plusu, kamata je veća od onog što dižemo