(10.)

(1) $y(t)=\int_{-\infty}^{t} u(\tau)d\tau$

a) lin.

$u_1(t) \Rightarrow y_1(t)=\int_{-\infty}^{t} u_1(\tau)d\tau$

$u_2(t) \Rightarrow y_2(t)=\int_{-\infty}^{t} u_2(\tau)d\tau$

$y^*(t)=\alpha y_1(t)+\beta y_2(t)=\alpha\int_{-\infty}^{t} u_1(\tau)d\tau+\beta\int_{-\infty}^{t} u_2(\tau)d\tau$

$u(t)=\alpha u_1(t)+\beta u_2(t) \Rightarrow y^{**}(t)=\int_{-\infty}^{t} (\alpha u_1(\tau)+\beta u_2(\tau))d\tau=$

$=\alpha\int_{-\infty}^{t} u_1(\tau)d\tau+\beta\int_{-\infty}^{t} u_2(\tau)d\tau$

$y^*(t)=y^{**}(t) \Rightarrow$ sustav linearan

vrem.

b) $u_1(t)=u(t-M) \Rightarrow y_1(t)=\int_{-\infty}^{t} u(\tau)d\tau$

$y(t-M)=\int_{-\infty}^{t} u(\tau)d\tau$

$\Big\} \; y_1(t)=y(t-M)$ vrem. stalan (ne ovisi o "t")

vrem.

$u_1(t) = u(t-M) \to y_1(t) = \int_{-\infty} u_1(\tau)\, d\tau$

$$y(t-M) = \int_{-\infty}^{t} u(\tau)\, d\tau \quad \Big\} \; y_1(t) = y(t-M)$$ vrem. [illegible]

(ne ovisi o "t")

mem.
c) memorijski
kauz.
d) kauzalan } ovisi o prošlim trenutcima (od $-\infty$)

(2) sve isto kao pod 1., ali nije vremenski stalno, već promjenjivo.

$$y(t) = \int_{0}^{t} u(\tau)\, d\tau$$

pobuda impuls ⇒ početni uvjeti

(3.) $y(n) = \left(\frac{1}{2}\right)^n u(3n+2)$

a) lin. $u(n) = \alpha u_1(n) + \beta u_2(n)$ $\quad y(n)^{**} = \left(\frac{1}{2}\right)^n \left[\alpha u_1(3n+2) + \beta u_2(3n+2)\right]$

$$\left.\begin{array}{l} u_1(n) \Rightarrow y_1(n) = \left(\frac{1}{2}\right)^n u_1(3n+2) \\ u_2(n) \Rightarrow y_2(n) = \left(\frac{1}{2}\right)^n u_2(3n+2) \end{array}\right\} y(n)^{*} = \alpha y_1(n) + \beta y_2(n) =$$

$$= \alpha \left(\frac{1}{2}\right)^n u_1(3n+2) + \beta \left(\frac{1}{2}\right)^n u_2(3n+2)$$

$y^{**}(n) = y^{*}(n) \Rightarrow$ linearno

vrem.

b) $u_1(n) = u(n-M) \Rightarrow y(n) = \left(\frac{1}{2}\right)^n u(3(n-M)+2)$

$y(n) \Rightarrow y(n-M) \Rightarrow y(n-M) = \left(\frac{1}{2}\right)^{n-M} u(3(n-M)+2)$ $\Big\}$ $y(n) \neq y(n-M)$

vrem. promjenjivo

mem.

c) memorijski

kauz.

d) nekauzalno $\Big\}$ ovisi o budućim trenutcima (3n+2)

(5.) a) $u(t) = \ldots$

lin.

$u_1(t) \Rightarrow \ldots$

$y(t)^{*} = \alpha \ldots$

b) $u_1(t) = u(t \ldots$

vrem.

$y(t-M) = \ldots$

mem.

c) bezmem...

d) kauza...

kauz.

(6.) a) $u(n) = \ldots$

lin.

(7) a) $u(t) = \alpha u_1(t) + \beta u_2(t) \Rightarrow$

lin.

$$\Rightarrow {}^{**}y(t) = \frac{\alpha u_1(t) + \beta u_2(t)}{1 + \alpha u_1(t-1) + \beta u_2(t-1)}$$

$$u_1(t) \Rightarrow y_1(t) = \frac{u_1(t)}{1 - u_1(t-1)} \qquad u_2(t) \Rightarrow y_2(t) = \frac{u_2(t)}{1 - u_2(t-1)}$$

$$^{*}y(t) = \alpha y_1(t) + \beta y_2(t) = \frac{\alpha u_1(t)}{1 - u_1(t-1)} + \frac{\beta u_2(t)}{1 - u_2(t-1)}$$

$^{*}y(t) \neq {}^{**}y(t) \Rightarrow$ nelinearno

vrem.

b) $u_1(t) = u(t-n) \Rightarrow y(t) = \dfrac{u(t-n)}{1 - u(t-n-1)}$

$$y(t-n) = \frac{u(t-n)}{1 - u(t-n-1)} \qquad y(t-n) = y(t) \Rightarrow \text{vrem. stalno}$$

mem.

c) memorijski
d) kauzalni

$\Big\}$ ovisnost o prošlom trenutku $(t-1)$

kauz.

5.) a) lin. $u(t) = \alpha u_1(t) + \beta u_2(t) \Rightarrow y(t)^{**} = (\alpha u_1(t) + \beta u_2(t))^2 =$

$= \alpha^2 u_1^2(t) + 2\alpha\beta u_1(t) u_2(t) + \beta^2 u_2^2(t)$

$u_1(t) \Rightarrow y_1(t) = u_1^2(t) \qquad u_2(t) \Rightarrow y_2(t) = u_2^2(t)$

$y(t)^{*} = \alpha y_1(t) + \beta y_2(t) = \alpha u_1^2(t) + \beta u_2^2(t) \qquad y(t)^{*} \neq y(t)^{**} \Rightarrow$ nelinearno

b) vrem. $u_1(t) = u(t-n) \Rightarrow y(t) = u^2(t-n)$

$y(t-n) = u^2(t-n)$

$\Big\}\ y(t) = y(t-n)$ vrem. stalan

mem. c) bezmemorijski
kauz. d) kauzalan

$\Big\}$ ovisi samo o sadašnjem trenutku (t)

e) nema inverz (polinom)

6.) a) $u(n) = \alpha u_1(n) + \beta u_2(n) \Rightarrow y(n)^{**} = \sum_{k=}^{n} (\alpha u_1(n) + \beta u_2(n)) =$

-2)

1)

6. a) $u(n) = \alpha u_1(n) + \beta u_2(n) \Rightarrow {}^{**}y(n) = \sum_{k=-\infty}^{n} (\alpha u_1(n) + \beta u_2(n)) =$

lin.

$$= \alpha \sum_{k=-\infty}^{n} u_1(n) + \beta \sum_{k=-\infty}^{n} u_2(n)$$

$$u_1(n) \Rightarrow y_1(n) = \sum_{k=-\infty}^{n} u_1(n) \qquad u_2(n) \Rightarrow y_2(n) = \sum_{k=-\infty}^{n} u_2(n)$$

$${}^{*}y(n) = \alpha y_1(n) + \beta y_2(n) = \alpha \sum_{k=-\infty}^{n} u_1(n) + \beta \sum_{k=-\infty}^{n} u_2(n)$$

${}^{*}y(n) = {}^{**}y(n) \Rightarrow$ linearan

b) $u_1(n) = u(n-M) \Rightarrow y(n) = \sum_{k=-\infty}^{n} u(n-M)$

$y(n-M) = \sum_{k=-\infty}^{n} u(n-M)$

} $y(n) = y(n-M)$ vremenski stalno

...ki } ovisnost
...ni } o prošlim
trenutcima
(t-1)

mem.
c) memorijski
d) kauzalan
kauz.
} ovisi o prošlim trenutcima (od $-\infty$)

e) invert.: $u(n) = y(n) - y(n-1)$

lin.

7. a) $u(n) = \alpha u_1(n) + \beta u_2(n) \Rightarrow {}^{**}y(n) = n\left(\alpha u_1(n) + \beta u_2(n)\right) =$

$= \alpha n u_1(n) + \beta n u_2(n)$

$u_1(n) \Rightarrow y_1(n) = n u_1(n) \qquad u_2(n) \Rightarrow y_2(n) = n u_2(n)$

${}^{*}y(n) = \alpha y_1(n) + \beta y_2(n) = \alpha n u_1(n) + \beta n u_2(n)$

${}^{*}y(n) = {}^{**}y(n) \Rightarrow$ linearan

vrem.

b) $\left.\begin{array}{l} u_1(n) = u(n-M) \Rightarrow y(n) = n u(n-M) \\ y(n-M) = (n-M) u(n-M) \end{array}\right\} y(n) \neq y(n-M)$ vremenski promjenjiv

mem.

c) bezmemorijski } ovisi o sadašnjem trenutku (n)
d) kauzalan

kauz.

e) nema inverz (polinom)