1. Labos iz SiS-a 2011/12

FER ☺

Labos napravljen zahvaljujući kolegama:

Redom kako su se pojavljivali u temi....

simpson, luciFER, Djani, Edson, mijauism, jcavric, felle, Iva2204, Sinus, DoctorEvil, **darxsys**, MindReader, Scrat, oiramcro, Chernobylite, filippm, lord of light, starlight, Damir, ninach, Frigi, vido, Nosferatu

Posebna zahvala na trudu: Scrat, DoctorEvil, vido, Nosferatu + svi ostali ☺

IME I PREZIME :)

1. Vremenski kontinuiran signal je periodičan ako za njega vrijedi $x(t)=x(t+T)$, gdje je T period za $\forall t \in \mathbb{R}, T \in \mathbb{R}_+$.

a) $x_1(t)=2$, $x_1(t+T)=2$

Funkcija $x_1(t)$ je kontinuirana i periodična za $\forall T \in \mathbb{R}_+$

$x_2(t)=\sin\left(\frac{2\pi}{T}\cdot t\right)$

$x_2(t+T)=\sin\left(\frac{2\pi}{T}\cdot(t+T)\right)=\sin\left(\frac{2\pi}{T}\cdot t+\frac{2\pi}{T}\cdot T\right)=\sin\left(\frac{2\pi}{T}\cdot t+2\pi\right)=$

$=\sin\left(\frac{2\pi}{T}\cdot t\right)\cdot\cos(2\pi)+\cos\left(\frac{2\pi}{T}\cdot t\right)\cdot\sin(2\pi)$

$=\sin\left(\frac{2\pi}{T}\cdot t\right)\cdot 1+\cos\left(\frac{2\pi}{T}\cdot t\right)\cdot 0=\sin\left(\frac{2\pi}{T}\cdot t\right)=x_2(t)$

$x_2(t)$ je periodičan sa temeljnim periodom T, $\forall T \in \mathbb{R}_+$

$x_3(t)=\sin\left(\frac{2\pi}{T}\cdot t^2\right)$

$x_3(t+T)=\sin\left(\frac{2\pi}{T}\cdot(t+T)^2\right)=\sin\left(\frac{2\pi}{T}\cdot(t^2+2t\cdot T+T^2)\right)=\sin\left(\frac{2\pi}{T}\cdot t^2+4\pi\cdot t+2\pi\cdot T\right)=$

$=x(t)=\sin\left(\frac{2\pi}{T}\cdot t^2\right)$

Iz čega proizlazi jednadžba:

$\frac{2\pi}{T}\cdot t^2+4\pi\cdot t+2\pi\cdot T=\frac{2\pi}{T}\cdot t^2+2k\pi$

$4\pi\cdot t+2\pi\cdot T=2k\pi$

$2t+T=k$ $\Rightarrow$ Kako bi k trebao biti $k\in\mathbb{N}$ za $\forall t\in\mathbb{R}$ što nije zadovoljeno, ova je funkcija aperiodična

$x_3(t)$ nije periodična

2. Vremenski diskretan signal je periodičan ako za njega vrijedi $x(n) = x(n+N)$, gdje je N period za $\forall n \in \mathbb{Z}, N \in \mathbb{N}$.

a) $x_1(n) = 2$, $x_1(n+N) = 2$ $\boxed{x_1(n)}$ je periodičan za $\forall N \in \mathbb{N}$, s osnovnim periodom $N=1$

$x_2(n) = \sin\left(\frac{2\pi}{N} \cdot n\right)$, $x(n+N) = \sin\left(\frac{2\pi}{N} \cdot (n+N)\right) = \sin\left(\frac{2\pi}{N} \cdot n + 2\pi\right) =$

$= \sin\left(\frac{2\pi}{N} \cdot n\right) \cdot \cos(2\pi) + \sin(2\pi) \cdot \cos\left(\frac{2\pi}{N}\right)$

$= \sin\left(\frac{2\pi}{N} \cdot n\right) = x_2(n)$

$\boxed{x_2(n) \text{ je periodičan za } \forall N \in \mathbb{N}, \text{ osnovni } N=1}$

$x_3(n) = \sin\left(\frac{2\pi}{N} \cdot n^2\right)$

$x_3(n+N) = \sin\left(\frac{2\pi}{N}(n+N)^2\right) = \sin\left(\frac{2\pi}{N}(n^2 + 2nN + N^2)\right) = \sin\left(\frac{2\pi}{N} \cdot n^2 + 4\pi n + 2\pi N\right) =$

$= \sin\left(\frac{2\pi}{N} \cdot n^2\right) \cdot \cos(4\pi n + 2\pi N) + \cos\left(\frac{2\pi}{N} \cdot n^2\right) \cdot \sin(4\pi n + 2\pi N)$

Da bi to bilo jednako $\sin\left(\frac{2\pi}{N} \cdot n^2\right)$ treba biti $\cos(4\pi n + 2\pi N) = 1$, $\sin(4\pi n + 2\pi N) = 0$

iz čega slijedi: $4\pi n + 2\pi N = 2k\pi \quad /: 2\pi$

$$2n + N = k$$

Kako mora biti $k \in \mathbb{N}, n \in \mathbb{N}$ i $N \in \mathbb{N}$ ova jednadžba je ispunjenja za svaki $N \in \mathbb{N}$.

$\boxed{x_3(n) \text{ je periodičan za } \forall N \in \mathbb{N}}$

b) Kod:

```
N=5
n=[0:1:30]
stem(n,sin(2*pi*n.^2/N))
grid on
```

Pomoću slike možemo ispitati periodičnost signala $x_3(n)$ jer smo uzeli za $N=5$ interval za n od 0 do 30 pa je iz same slike vidljivo ponavljanje istih segmenata 6 puta, a i također možemo očitati sa grafa vrijednosti $x(n)$ i $x(n+5)$ i primjetiti da će biti jednake za svaki n iz [0:30], međutim izvan tog intervala nemožemo ništa zaključiti sa slike.

c) $f_1(n) = \sin\left(\frac{2\pi}{N_1} \cdot n\right)$ $\quad f_2(n) = \sin\left(\frac{2\pi}{N_2} \cdot n\right)$

Pitanje je hoće li $f_1(n) \cdot f_2(n)$ biti periodičan?

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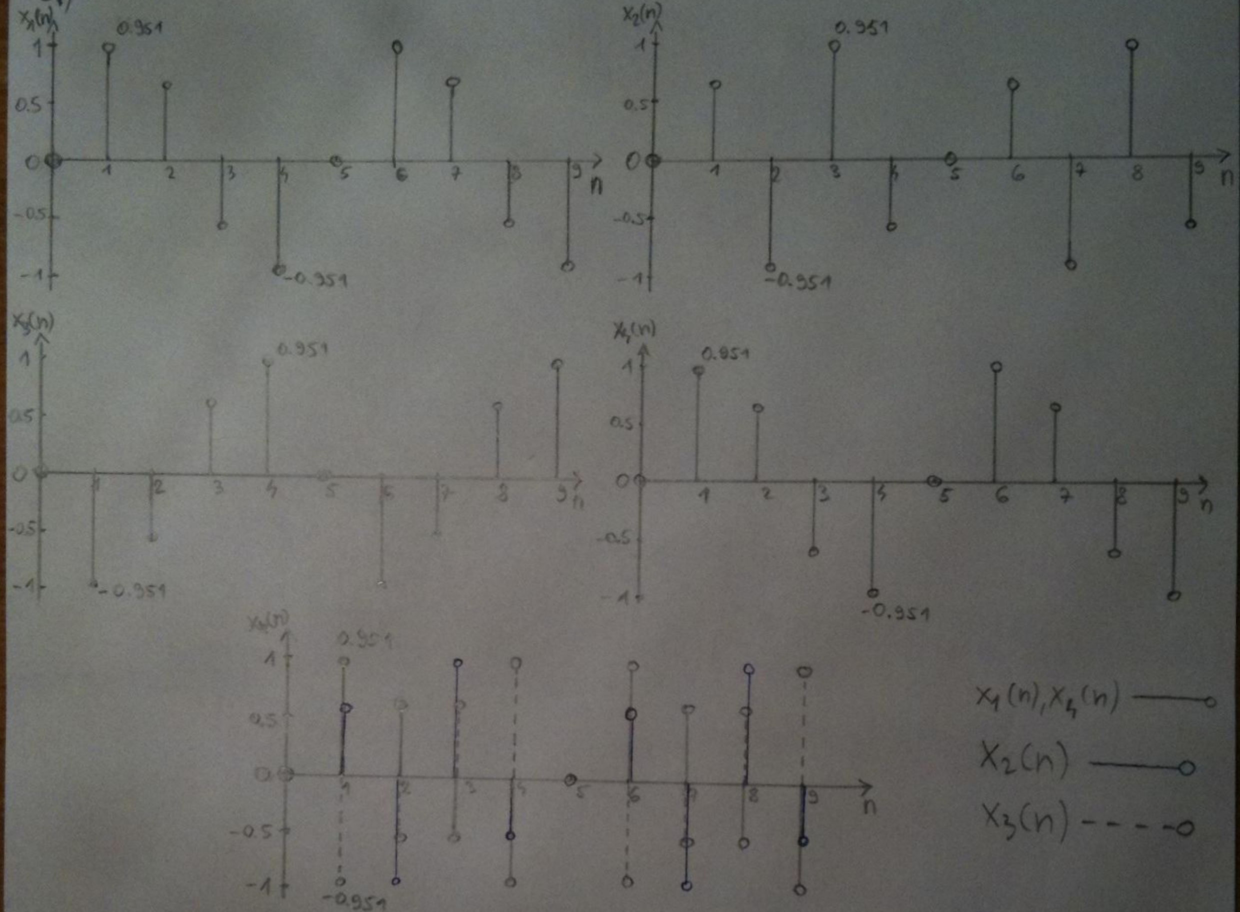
3.

a)



b) Nacrtao sam 3 različita signala.

c) Jednaki signali su $x_1(n) = \sin(\frac{2\pi}{5} n)$ i $x_4(n) = \sin(\frac{12\pi}{5} n)$ što je vidljivo ako signal $x_4(n)$ zapišemo kao $x_4(n) = \sin(\frac{2\pi}{5} \cdot n + \frac{10\pi}{5} \cdot n)$ dalje slijedi $x_4(n) = \sin(\frac{2\pi}{5} \cdot n + 2\pi \cdot n)$, $n \in [0,1,2,\dots,9]$ slijedi $x_4(n) = \sin(\frac{2\pi}{5} \cdot n)$ odnosno ta dva signala su zapravo jednaka.

4. Totalna energija vremenski kontinuiranih je $E_\infty = \int_{-\infty}^{\infty} |x(t)|^2 dt$ odnosno za signal definiran na intervalu $[t_1, t_2] \in \mathbb{R}$ računamo kao $E_{[t_1,t_2]} = \int_{t_1}^{t_2} |x(t)|^2 dt$.

Totalna snaga vremenski kontinuiranih signala je $P = \lim_{L\to\infty} \frac{1}{L} \int_{-L/2}^{L/2} |x(t)|^2 dt$

dok srednju snagu računamo kao $P_{[t_1,t_2]} = \frac{1}{L} \int_{t_1}^{t_2} |x(t)|^2 dt$, $L = t_2 - t_1$.

Totalna energija vremenski diskretnih signala je $E = \sum_{n=-\infty}^{\infty} |x(n)|^2$, $n \in \mathbb{Z}$

odnosno za signal definiran na $[n_1, n_2] \in \mathbb{Z}$ je $E = \sum_{n=n_1}^{n_2} |x(n)|^2$

Totalna snaga vremenski diskretnih signala je $P = \lim_{N\to\infty} \frac{1}{2N+1} \cdot \sum_{n=-N}^{N} |x(n)|^2$, $n \in \mathbb{Z}$

dok srednju snagu računamo sa $P_{[n_1,n_2]} = \frac{1}{n_2 - n_1 + 1} \cdot \sum_{n=n_1}^{n_2} |x(n)|^2$

a) Energija i snaga kontinuiranog signala $x_1(t)$

$$E_{x_1} = \int_{-\infty}^{\infty} |\cos(\tfrac{\pi}{3} t)|^2 dt = \int_{-\infty}^{\infty} \cos^2(\tfrac{\pi}{3} t) dt = \int_{-\infty}^{\infty} \frac{1 + \cos(\frac{2\pi}{3} t)}{2} dt = \frac{1}{2} \int_{-\infty}^{\infty} (1 + \cos(\tfrac{2\pi}{3} t)) dt =$$

$$= \frac{1}{2} \left(t \Big|_{-\infty}^{\infty} + \frac{3}{2\pi} \sin(\tfrac{2\pi}{3} t) \Big|_{-\infty}^{\infty} \right) = \frac{1}{2} \Big(\infty + \infty + \frac{3}{2\pi} \underbrace{\sin(\tfrac{2\pi}{3} t) \Big|_{-\infty}^{\infty}}_{[-1,1]} \Big) = \infty \quad \boxed{E_{x_1(t)} = \infty}$$

$$P_{x_1} = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} |\cos(\tfrac{\pi}{3} t)|^2 dt = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} \cos^2(\tfrac{\pi}{3} t) dt = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} \frac{1 + \cos(\frac{2\pi}{3} t)}{2} dt =$$

$$= \lim_{T\to\infty} \frac{1}{T} \cdot \frac{1}{2} \cdot \left(\int_{-T/2}^{T/2} dt + \int_{-T/2}^{T/2} \cos(\tfrac{2\pi}{3} t) dt \right) = \frac{1}{2} \lim_{T\to\infty} \frac{1}{T} \left(T + \frac{3}{2\pi} \cdot \left[\sin(\tfrac{2\pi}{3} \cdot \tfrac{T}{2}) + \sin(\tfrac{2\pi}{3} \cdot \tfrac{T}{2}) \right] \right)$$

$$= \frac{1}{2} \lim_{T\to\infty} 1 + \frac{3}{2\pi} \cdot \frac{2 \cdot \sin(\frac{\pi}{3} \cdot T)}{T} = \frac{1}{2} \cdot (1 + 0) = \frac{1}{2} \quad \boxed{P_{x_1(t)} = \frac{1}{2}}$$

Energija i snaga diskretnog signala $x_2(t)$

$$E_{x_2} = \sum_{n=-\infty}^{\infty} |\sin(\tfrac{\pi}{7} \cdot n)|^2 = \sum_{n=-\infty}^{\infty} \sin^2(\tfrac{\pi}{7} \cdot n) = \sum_{n=-\infty}^{\infty} \frac{1 - \cos(\frac{2\pi}{7} \cdot n)}{2} = \frac{1}{2} \cdot \sum_{n=-\infty}^{\infty} (1 - \cos(\tfrac{2\pi}{7} \cdot n)) =$$

$$= \frac{1}{2} \cdot 2 \cdot \sum_{n=0}^{\infty} (1 - \cos(\tfrac{2\pi}{7} \cdot n)) = \sum_{n=0}^{\infty} (1 - \cos(\tfrac{2\pi}{7} \cdot n))$$

Iz čega slijedi da suma divergira jer: $\lim_{n\to\infty} (1 - \cos(\frac{2\pi}{7} \cdot n)) = (1 - (-1, 1)) = (0, 2) \neq 0$

pa pošto $\sum a_n$, a_n ne teži u 0 slijedi $S = \sum a_n = \infty$ $\boxed{E_{x_2} = \infty}$

Pošto je $x_2(n) = \sin(\frac{\pi}{7} \cdot n)$ periodičan i $N = 14$ slijedi:

$$P_{x_2} = \frac{1}{N} \cdot \sum_{n=0}^{N-1} |\tilde{x}_2(n)|^2 = \frac{1}{14} \cdot \sum_{n=0}^{13} \sin^2(\tfrac{\pi}{7} \cdot n) = \frac{1}{14} \cdot \sum_{n=0}^{13} \frac{1 - \cos(\frac{2\pi}{7} \cdot n)}{2} =$$

$$= \frac{1}{14} \cdot \frac{1}{2} \left(\sum_{n=0}^{13} 1 - \sum_{n=0}^{13} \cos(\tfrac{2\pi}{7} \cdot n) \right) = \frac{1}{14} \cdot \frac{1}{2} \cdot ((13+1) - (0)) = \frac{1}{2} \quad \boxed{P_{x_2} = \frac{1}{2}}$$

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b) Kod za $x_2(n)$:

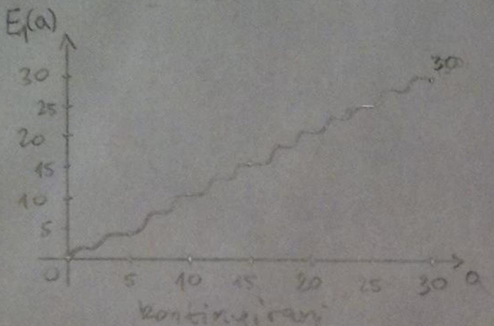
```
function x2 = zad4x2(n)
x2 = sin(n*pi/7);
```

d) $$E_1 = a + \frac{3 \cdot \sin\left(\frac{2\pi \cdot a}{3}\right)}{2\pi}$$

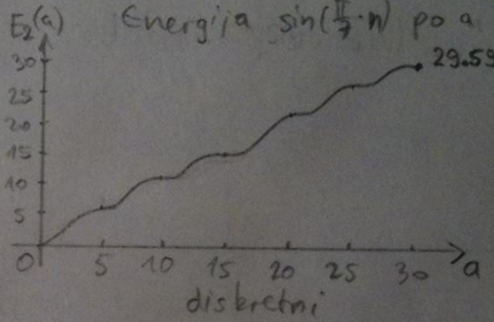
Kod za E_2:

```
function energija = zad4E2(a)
 suma = 0;
 for i=-a:a
     suma = suma + abs(sin(i*pi/7))^2;
   end
energija = suma;
```

f) Energija $\cos(\frac{\pi}{3}t)$ po a

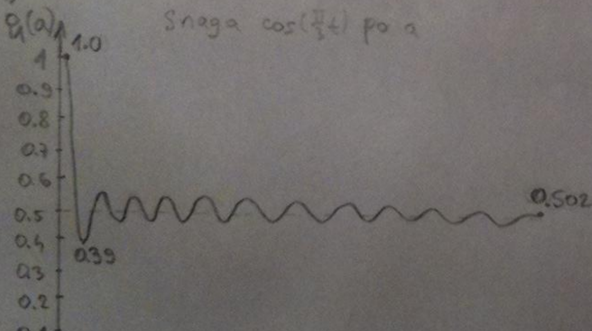


Energija $\sin(\frac{\pi}{7} \cdot n)$ po a

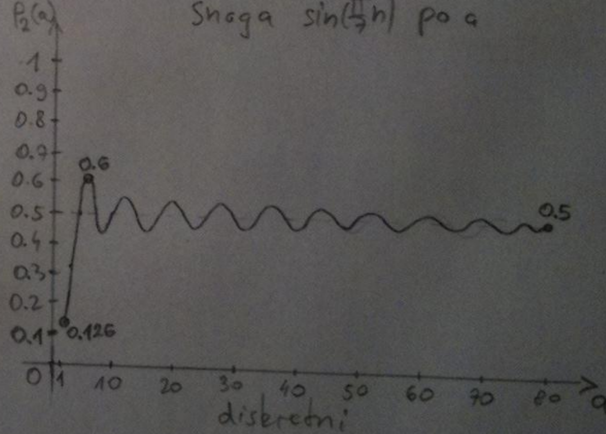


- Sa povećanjem parametra a energija E_1 i E_2 raste.
- Kada $a \to \infty$ očekujem da i $E \to \infty$

h) Snaga $\cos(\frac{\pi}{3}t)$ po a



Snaga $\sin(\frac{\pi}{7}n)$ po a



- Sa povećanjem parametra a vidljivo je da se snaga P_1 i P_2 stabiliziraju oko 0.5
- Kada $a \to \infty$ očekujem da su snage $P_1 = 0.5$ i $P_2 = 0.5$

(5.) Fourierova analiza: $CTFS_{T_0}(f(t)) = F_k = \frac{1}{T_0}\int_{T_0} f(t)\cdot e^{-jk\omega_0 t}\,dt$, $\forall k \in \mathbb{Z}$

Fourierova sinteza: $ICTFS_{T_0}(F_k) = f(t) = \sum_{k=-\infty}^{\infty} F_k \cdot e^{j\omega_0 k t}$, $\forall k \in \mathbb{Z}$

t – vrijeme, T_0 – period signala, $f(t)$ – signal,
F_k – spektar, k – red harmonika

a) $x(t) = 110\cdot\sin(120\pi\cdot t) + 50\cdot\cos(360\pi\cdot t + \frac{\pi}{3})$

$P_1 = \frac{2\pi}{120\pi} = \frac{1}{60}$ $\qquad P_2 = \frac{2\pi}{360\pi} = \frac{1}{180}$ $\qquad P = V(P_1, P_2) = \frac{1}{60}$

$T_0 = 2\cdot P = 2\cdot\frac{1}{60} \Rightarrow T_0 = \frac{1}{30} \Rightarrow \omega_0 = 60\pi$

$110\cdot\sin(120\pi\cdot t) \rightarrow T_1 = \frac{1}{60} = \frac{T_0}{2}$, $\omega_1 = 2\omega_0$

$50\cdot\cos(360\pi t + \frac{\pi}{3}) \rightarrow T_2 = \frac{1}{180} = \frac{T_0}{6}$, $\omega_2 = 6\omega_0$

Iz formula: $\sin(\omega t) = \frac{e^{j\omega t} - e^{-j\omega t}}{2j}$; $\cos(\omega t) = \frac{e^{j\omega t} + e^{-j\omega t}}{2}$ slijedi:

$$x(t) = \frac{110}{2j}\cdot\left(e^{j2\omega_0 t} - e^{j\cdot(-2)\cdot\omega_0 t}\right) + \frac{55}{2}\cdot\left(e^{j\frac{\pi}{3}}\cdot e^{j6\omega_0 t} + e^{-j\frac{\pi}{3}}\cdot e^{j\cdot(-6)\omega_0 t}\right)$$

kad usporedimo sa $x(t) = \sum_{k=-\infty}^{\infty} X_k e^{jk\omega_0 t}$ i sredimo:

$$x(t) = 55\cdot e^{-j\frac{\pi}{2}}\cdot e^{j2\omega_0 t} + 55\cdot e^{j\frac{\pi}{2}}\cdot e^{j(-2)\omega_0 t} + 25\cdot e^{j\frac{\pi}{3}}\cdot e^{j6\omega_0 t} + 25\cdot e^{-j\frac{\pi}{3}}\cdot e^{j(-6)\omega_0 t}$$

dobijemo:

$$\boxed{\begin{array}{lll} X_2 = 55\cdot e^{-j\frac{\pi}{2}} & X_{-2} = 55\cdot e^{j\frac{\pi}{2}} & \text{za sve ostale } k \\ X_6 = 25\cdot e^{j\frac{\pi}{3}} & X_{-6} = 25\cdot e^{-j\frac{\pi}{3}} & X_{\text{ostali } k} = 0 \end{array}}$$

b) Kod za CTFS(x(t)):

```
syms k t T0;
f1 = 110*sin(120*pi*t) + 50*cos(360*pi*t + pi/3);
FR = simplify(int(f1*exp(-1i*k*t*2*pi/T0),t,-T0/2,T0/2)/T0);
FR = subs(FR,T0,1/30);
pretty(FR)
```

Rezultat:

$$FR = \frac{220 i \sin(\pi k)}{4\pi - \pi k^2} + \frac{25(\cos(\pi k) - i\sin(\pi k))\cdot\left(-3\sqrt{3} + \frac{ik}{2}\right)}{\pi\cdot(k^2 - 36)} -$$

$$- \frac{25\,(\cos(\pi k) + i\sin(\pi k))\,\left(-3\sqrt{3} + \frac{ik}{2}\right)}{\pi\,(k^2 - 36)}$$

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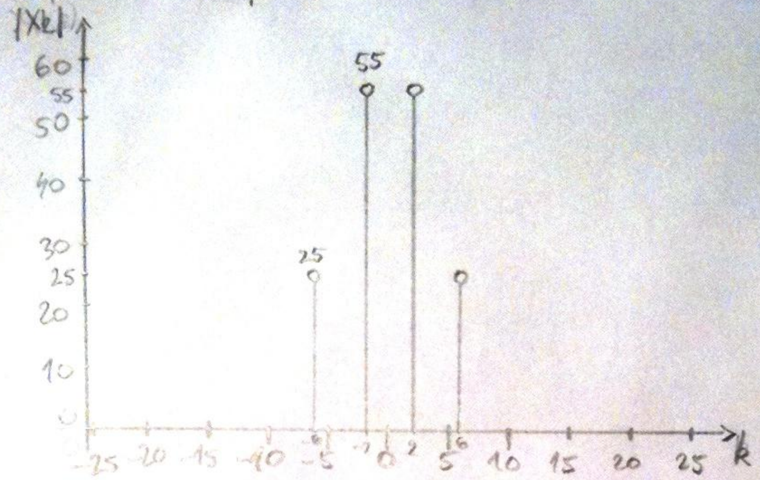
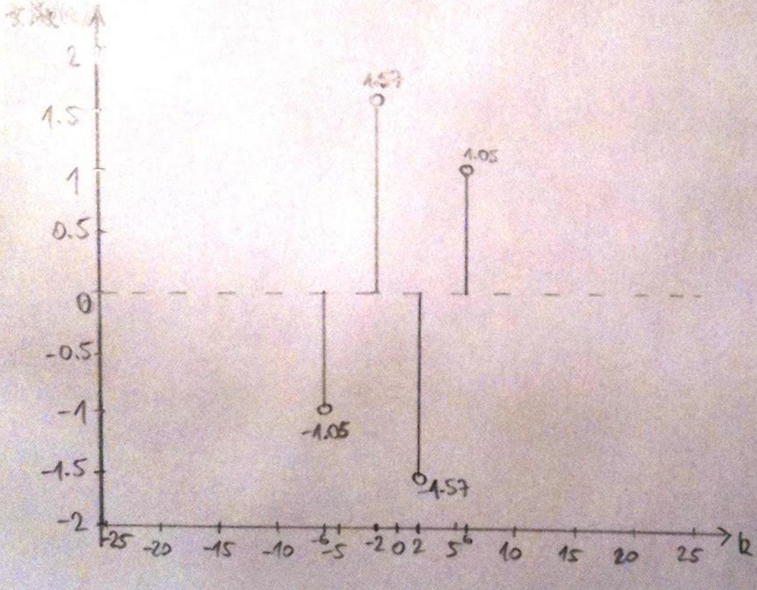
c) amplitudni spektar |Xk|



fazni spektar ∡Xk



⑥ Parsevalova relacija za CTFS: $P_f = \frac{1}{T_0}\int_{T_0} |f(t)|^2 dt = \sum_{k=-\infty}^{\infty} |F_k|^2$

$f(t)$ - signal
F_k - spektar
T_0 - period signala
t - vrijeme
k - red harmonika

a) $x(t)_{T_0} = 1$ u trajanju T $\quad w_0 = \frac{2\pi}{T_0}$

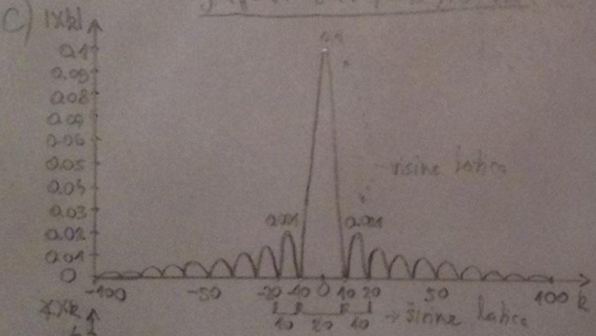
$$X_k = \frac{1}{T_0}\int_{-T/2}^{T/2} 1\cdot e^{-jkw_0t}\,dt = \frac{1}{T_0}\int_{-T/2}^{T/2} e^{-jk\frac{2\pi}{T_0}t}\,dt =$$

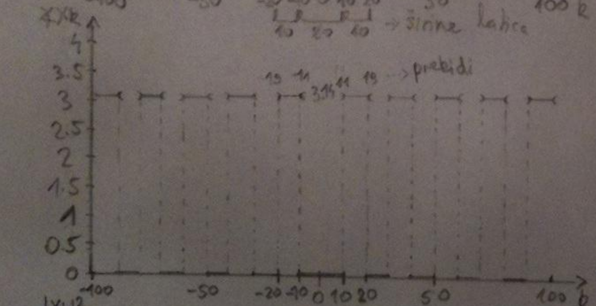
$$= \frac{1}{T_0}\cdot\frac{1}{-j\frac{2\pi}{T_0}\cdot k}\left(e^{-jk\frac{2\pi}{T_0}t}\Big|_{-T/2}^{T/2}\right) = \frac{1}{\pi\cdot k}\cdot\frac{-1}{2j}\cdot\left(e^{-jk\cdot\pi\cdot\frac{T}{T_0}} - e^{jk\pi\cdot\frac{T}{T_0}}\right) =$$

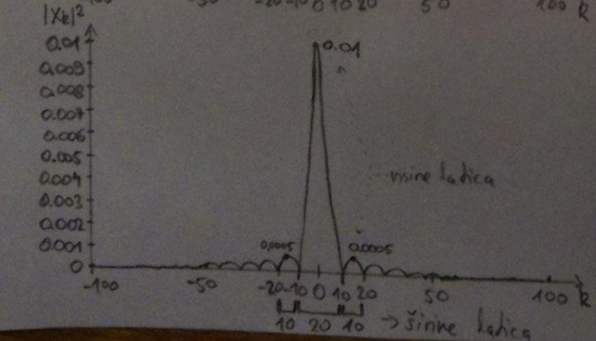
$$= \frac{1}{\pi\cdot k}\cdot\frac{e^{jk\pi\frac{T}{T_0}} - e^{-jk\pi\cdot\frac{T}{T_0}}}{2j} = \frac{\sin\left(\frac{k\pi\cdot T}{T_0}\right)}{\pi\cdot k} \qquad \boxed{X_k = \frac{\sin\left(\frac{k\cdot\pi\cdot T}{T_0}\right)}{k\cdot\pi}}$$

b) $\boxed{\text{Sin((pi*T*k)/T0)/(pi*k)}}$

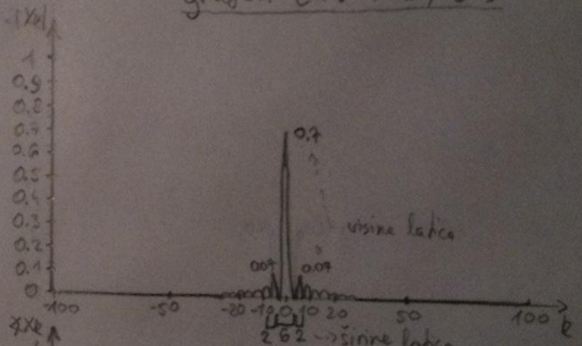
c) grafovi za: T=2, T0=20

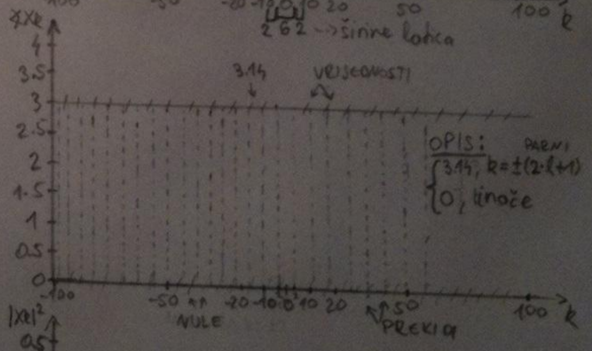


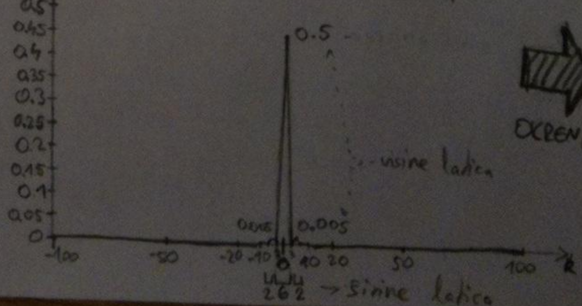




grafovi za: T=2, T0=3







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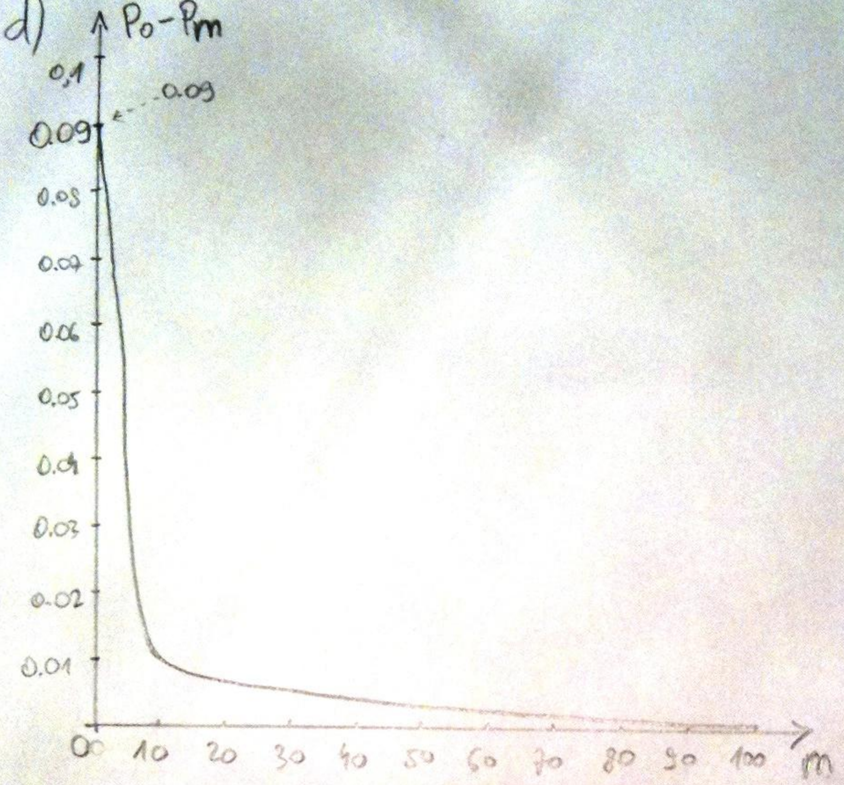
d)



graf za:

$T=2$

$T_0=20$

$P_0 = \frac{2}{20} = \frac{1}{10}$

$P_0 = 0.1$

- Kada $m \to \infty$ možemo reći da će razlika snaga $P_0 - P_m \to 0$

7. $CTFT(f(t)) = F(j\omega) = \int_{-\infty}^{\infty} f(t)\cdot e^{-j\omega t}\,dt$, analiza

$ICTFT(F(j\omega)) = f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(j\omega)\, e^{j\omega t}\,d\omega$, sinteza

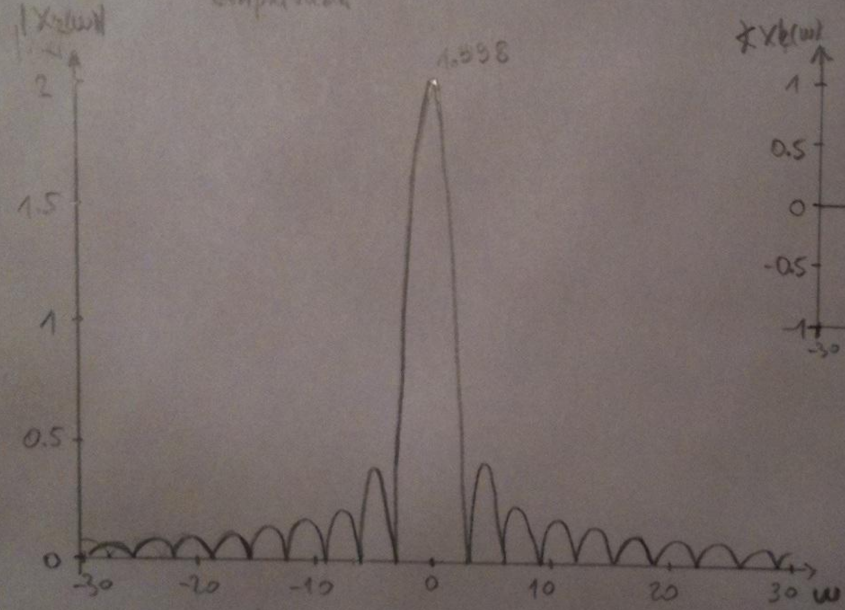
$f(t)$ - signal
$F(j\omega)$ - spektar
t - vrijeme
ω - kružna frekvencija
$\forall \omega \in R, \forall t \in R$

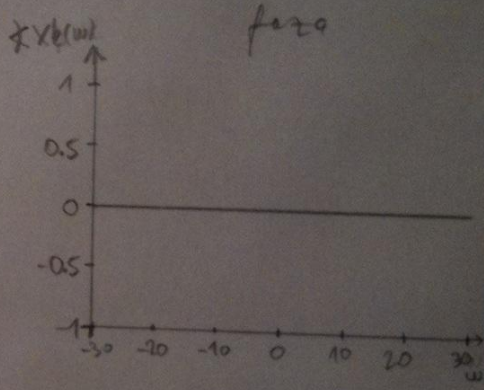
a) $T = 2$ $f(t)=1$, trajanja 2, od -1 do 1

$$F(j\omega) = \int_{-T/2}^{T/2} f(t)\cdot e^{-j\omega t}\,dt = \int_{-T/2}^{T/2} 1\cdot e^{-j\omega t}\,dt = \int_{-1}^{1} e^{-j\omega t}\,dt = -\frac{1}{j\omega}\cdot\left(e^{-j\omega t}\right)\Big|_{-1}^{1} =$$

$$= -\frac{1}{j\omega}\cdot\left(e^{-j\omega} - e^{j\omega}\right) = \frac{2}{\omega}\cdot\left(\frac{e^{j\omega}-e^{-j\omega}}{2j}\right) = \boxed{\frac{2\cdot\sin(\omega)}{\omega} = F(j\omega)}$$

b) $-(-\sin(w)+\cos(w)*i)/w + (\sin(w)+\cos(w)*i)/w$

c)




- Razlika ovog grafa i grafa u 6. c) dijelu je prvenstveno što ovaj graf prikazuje kontinuirani spektar dok drugi graf prikazuje diskretnu razdiobu amplitude. Oni su naizgled slični, ali razlikuju se i u minimumima odnosno maksimumima, također u širini latica međutim oba teže u 0 kad se w udaljava od nule.

⑧ Dirichletove uvjete zadovoljava:

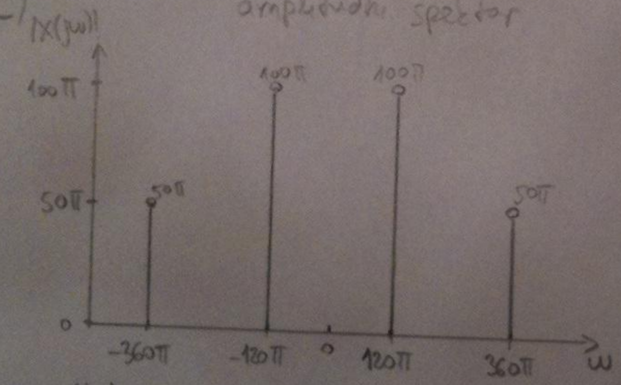
1. signal koji je apsolutno integrabilan $\int_{-\infty}^{\infty} |f(t)|\,dt < \infty$
2. signal koji ima konačan broj minimuma i maksimuma na bilo kojem konačnom intervalu domene
3. ima konačni broj diskontinuiteta koji su konačni na bilo kojem konačnom intervalu domene
4. na mjestu diskontinuiteta $f(t_d^+) * f(t_d^-)$ signal konvergira u $\frac{f(t_d^+)+f(t_d^-)}{2}$

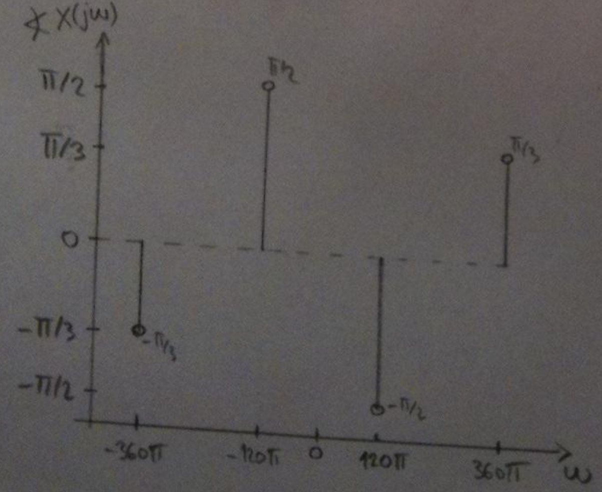
b) <u>Kod:</u>

```
syms t;
f1 = 110*sin(120*pi*t) + 50*cos(360*pi*t + pi/3);
FT = fourier(y)
```

```
FT = -110*pi*(dirac(w-120*pi) - dirac(120*pi+w))*i +
   + 50*pi*(dirac(w-360*pi)*(1/2 + (3^(1/2)*i)/2) -
   - dirac(360*pi+w)*(-1/2 + (3^(1/2)*i)/2))
```

c) amplitudni spektar





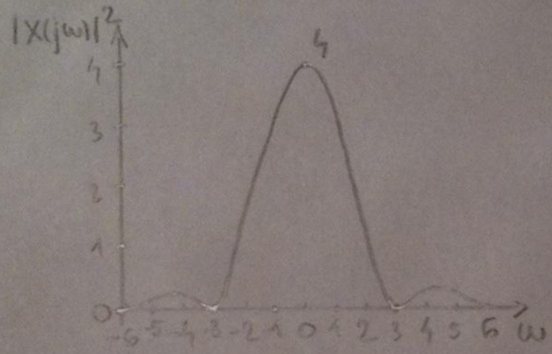
- Razlika ovog spektra i spektra u 5.c) dijelu je u amplitudnom spektru pošto ovdje imamo Diracovu delta funkciju koja ima beskonačnu amplitudu dok je tamo bio spektar konačne amplitude.
Faza se razlikuje samo u domeni, a vrijednosti su iste.

9. Parsevalov teorem za CTFT: $\int_{-\infty}^{\infty} |x(t)|^2 dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} |X(j\omega)|^2 d\omega$

$x(t)$ – signal, $X(j\omega)$ – spektar, t – vrijeme, ω – kružna frekvencija

a) $X(j\omega) = \frac{2 \cdot \sin(\omega)}{\omega}$, za $x(t)$ jedinični impuls trajanja $T=2$ pa je: $|X(j\omega)|^2 = \frac{4 \sin^2(\omega)}{\omega^2}$

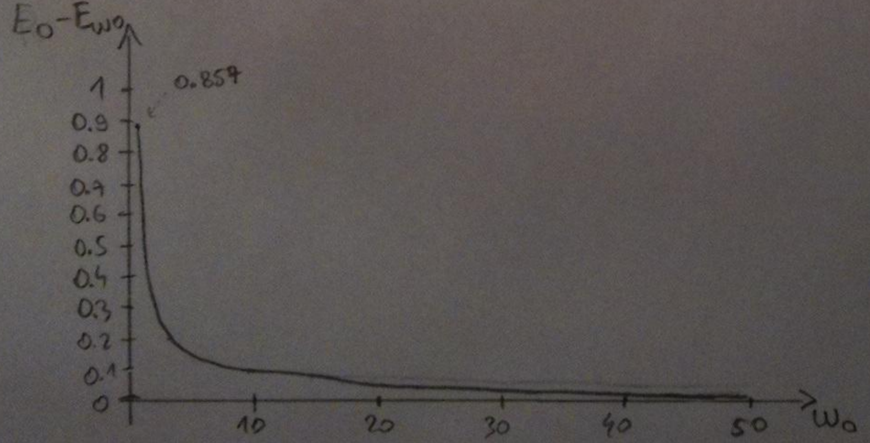
X_{sq} = (4 * abs(sin(w))^2) / abs(w)^2



b) $\omega_0 = \frac{2\pi}{T_0} = \frac{2\pi}{T} = \frac{2\pi}{2} = \pi$

$E_{\omega_0} = \frac{1}{2\pi} \int_{-\pi}^{\pi} \left| \frac{2 \cdot \sin(\omega)}{\omega} \right|^2 d\omega = 1.8057$

$E_0 = \frac{1}{2\pi} \int_{-\infty}^{\infty} \left| \frac{2\sin(\omega)}{\omega} \right|^2 d\omega = 2$



– Kada $\omega_0 \to \infty$ onda razlika $(E_0 - E_{\omega_0}) \to 0$

10.

$$DTFT(x(n)) = X(e^{j\Omega}) = \sum_{n=-\infty}^{+\infty} x(n) \cdot e^{-j\Omega n}$$

$$IDTFT(X(e^{j\Omega})) = x(n) = \frac{1}{2\pi} \int_{-\pi}^{+\pi} X(e^{j\Omega}) \cdot e^{j\Omega n} d\Omega$$

$x(n)$ – signal

$X(e^{j\Omega})$ – spektor

n – vrijeme (korak)

Ω – kružna frekvencija

$\forall \Omega \in \mathbb{R}, \ \forall n \in \mathbb{Z}$

a) $x(n) = \{\underline{1}, 1, 1, 1\}$, $N = 4$

$$X(e^{j\Omega}) = \sum_{n=-\infty}^{\infty} x(n) \cdot e^{-j\Omega n} = \sum_{n=0}^{3} e^{-j\Omega n} = 1 + e^{-j\Omega} + e^{-j2\Omega} + e^{-j3\Omega} =$$

$$= \frac{1 - e^{-j4\Omega}}{1 - e^{-j\Omega}} = \frac{e^{-j2\Omega}(e^{j2\Omega} - e^{-j2\Omega})}{e^{-j\frac{\Omega}{2}}(e^{j\frac{\Omega}{2}} - e^{-j\frac{\Omega}{2}})} = e^{-j\frac{3}{2}\Omega} \cdot \frac{\sin(2\Omega)}{\sin(\frac{\Omega}{2})}$$

$$\boxed{X(e^{j\Omega}) = \frac{\sin(2\Omega)}{\sin(\frac{\Omega}{2})} \cdot e^{-j\frac{3}{2}\Omega}}$$

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$$|X(e^{j\Omega})| = \left| \frac{\sin(2\Omega)}{\sin(\frac{\Omega}{2})} \cdot e^{-j\frac{3}{2}\Omega} \right| = \left| \frac{\sin(2\Omega)}{\sin(\frac{\Omega}{2})} \right| \cdot \left| \cos(\tfrac{3}{2}\Omega) - j\sin(\tfrac{3}{2}\Omega) \right| =$$

$$= \left| \frac{\sin(2\Omega)}{\sin(\frac{\Omega}{2})} \right| \cdot \sqrt{\cos^2(\tfrac{3}{2}\Omega) + \sin^2(\tfrac{3}{2}\Omega)} = \left| \frac{\sin(2\Omega)}{\sin(\frac{\Omega}{2})} \right|, \ \Omega \neq 0$$

Za $\Omega = 0$ slijedi:

$$\lim_{\Omega \to 0} \left| \frac{\sin(2\Omega)}{\sin(\frac{\Omega}{2})} \right| = \left(\begin{matrix} \text{L'Hospitalovo} \\ \text{pravilo} \end{matrix} \right) = \lim_{\Omega \to 0} \left| \frac{2 \cdot \cos(2\Omega)}{\frac{1}{2} \cdot \cos(\frac{\Omega}{2})} \right| = \frac{2}{\frac{1}{2}} = 4, \ \Omega = 0$$

$$\boxed{|X(e^{j\Omega})| = \begin{cases} 4, & \Omega = 0 \\ \left| \frac{\sin(2\Omega)}{\sin(\frac{\Omega}{2})} \right|, & \text{inače} \end{cases}}$$

$$\measuredangle X(e^{j\Omega}) = \text{arctg}\left(\frac{Im(X(e^{j\Omega}))}{Re(X(e^{j\Omega}))} \right) = \text{arctg}\left(\frac{Im\left(\frac{\sin(2\Omega)}{\sin(\frac{\Omega}{2})} \cdot e^{-j\frac{3}{2}\Omega} \right)}{Re\left(\frac{\sin(2\Omega)}{\sin(\frac{\Omega}{2})} \cdot e^{-j\frac{3}{2}\Omega} \right)} \right) =$$

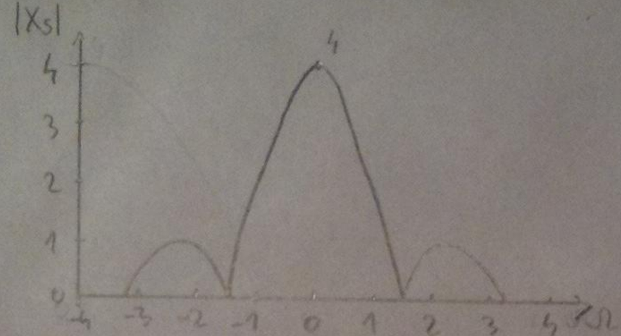
$$= \measuredangle\left(1 - \frac{3}{2}\Omega\right) + \measuredangle \frac{\sin(\Omega \cdot 2)}{\sin(\frac{\Omega}{2})}$$

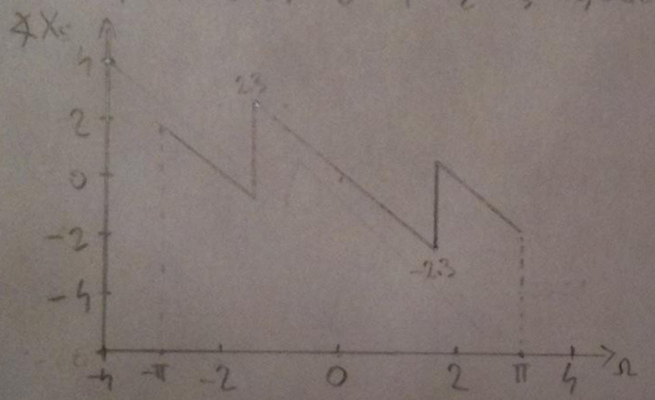
$$\boxed{\measuredangle X(e^{j\Omega}) = \measuredangle 1 - \frac{3}{2}\Omega + \measuredangle \frac{\sin(2\Omega)}{\sin(\frac{\Omega}{2})}}$$

OKRENI

b) Kod:

```
x=[0 0 0 1 1 1 1 0 0 0 0]
[Xs,w]=freqz(x,1);
Xs=Xs.*exp(-1j*w*4);
subplot(2,1,1), plot(w,abs(Xs));
subplot(2,1,2), plot(w,phase(Xs));
```





d) Što je parametar N veći to će amplituda (ekstremi) imati veću vrijednost i bit će gušći graf odnosno širina latica će biti manja.
Faza će također za veći N postizati veće pozitivne i negativne vrijednosti (ekstreme) te će graf također biti gušći, manjih šiljaka.

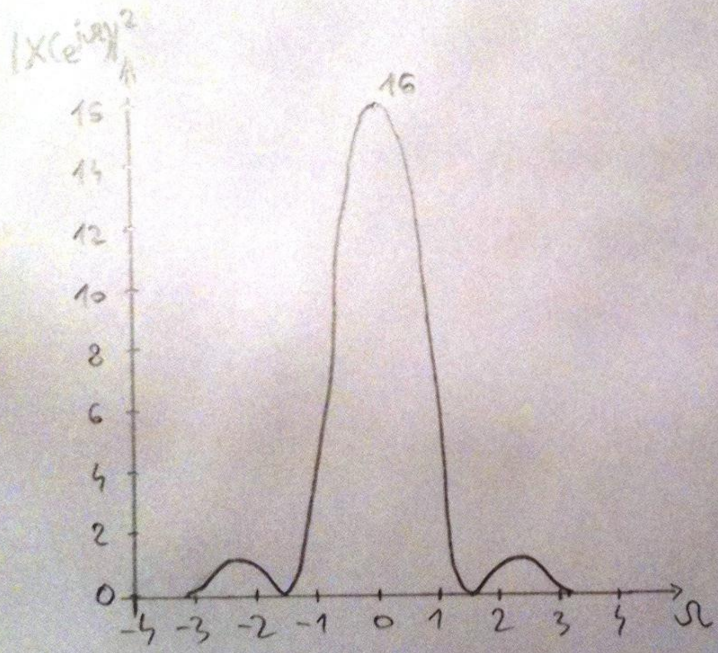
e) Kada mijenjamo početni korak u kojem signal počinje amplitudni spektar se neće promjeniti dok se fazni mijenja već pri svakoj promjeni koraka jako naglo čak kaotično.

(11.) Parsevalova relacija za DTFT: $$\sum_{n=-\infty}^{\infty} |x(n)|^2 = \frac{1}{2\pi}\int_{-\pi}^{\pi} |X(e^{j\Omega})|^2 \, d\Omega$$

$x(n)$ – signal, $X(e^{j\Omega})$ – spektar, n – vrijeme (korak), Ω – kružna frekvencija

a) Kod:

```
x=[1 1 1 1];
w=[-pi:0.01:pi];
Xs=freqz(x,1,w);
plot(w, abs(X).^2)
```



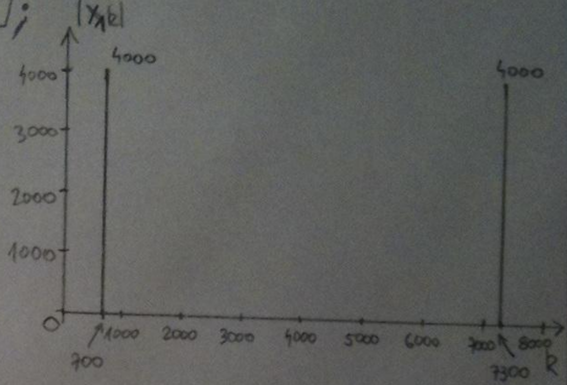
12. $DTFS_N(x(n)) = X_k = \frac{1}{N}\sum_{n=0}^{N-1} x(n)\cdot e^{-2\pi jk\frac{n}{N}}$

$IDTFS_N(X_k) = x(n) = \sum_{k=0}^{N-1} X_k \cdot e^{2\pi jk\frac{n}{N}}$

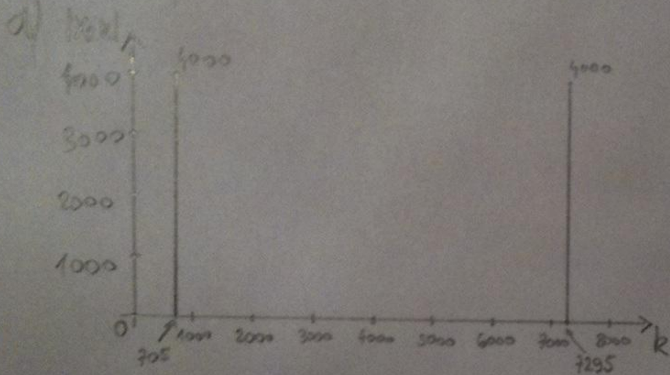
$x(n)$ - signal
X_k - spektar
n - vrijeme (korak)
k - red harmonika

b) Kod:

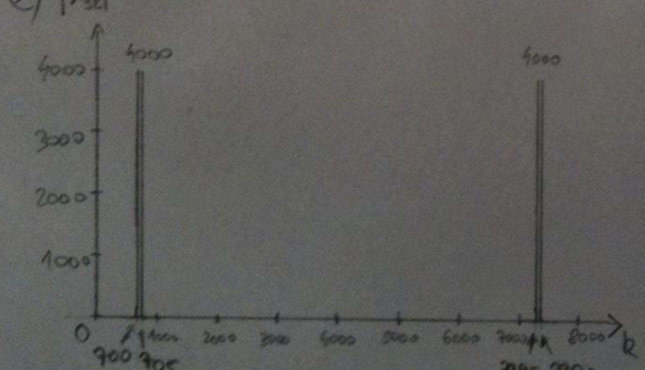
```
t=[0:(1/8000):(1-1/8000)];
x=sin(2*pi*700*t);
Xd=fft(x);
A=abs(Xd);
N=length(Xd);
w=([0:N-1])*8000/N;
plot(w,A)
```



d) |X2k|



e) |X3k|



f) Spektar u podzadatku e) je zbroj amplitudnih spektara u podzadacima b) i d) odnosno

$$|X_{3k}| = |X_{1k}| + |X_{2k}|$$