

Zad. 3.1.-3.3)

Impuloni odziv i odziv na jedinici skok.

$$y(n) - 0,8y(n-1) + 0,2y(n-2) = u(n)$$

a.)

$$\begin{aligned} y(-n) &= 0 \\ y(-2) &= 0 \end{aligned}$$

$$u(n) = \delta(n)$$

$$y(n) = u(n) + 0,8y(n-1) - 0,2y(n-2)$$

$$h(-2) = h(-1) = 0$$

$$h(n) = \delta(n) + 0,8h(n-1) - 0,2h(n-2)$$

$$h(0) = \delta(0) + 0,8h(-1) - 0,2h(-2)$$

$$h(0) = 1$$

$$\Rightarrow \text{arc tg } \frac{0,2}{0,8} = \text{arc tg } \frac{1}{4} = 26,56^\circ$$

$$g^2 - 0,8g + 0,2 = 0$$

$$g_{1/2} = \frac{0,8 \pm j0,4}{2}$$

$$g_1 = 0,4 + j0,2$$

$$g_2 = 0,4 - j0,2$$

$$h(n) = 0,447 \cdot \left[A \cos(26,56^\circ n) + B \sin(26,56^\circ n) \right]$$

$$h(0) = 0,447 \cdot A = 1$$

$$\rightarrow A = 2,237$$

$$\begin{aligned} h(-1) &= 0,447 \cdot \left[2,237 \cos(-26,56^\circ) - 0,447 \sin(-26,56^\circ) \right] = 0,447 \cdot (2,0009 - 0,447) = 0 \\ &= 0,8944 - 0,1998 = 0 \quad \rightarrow \quad 0,8944 = 0,1998 \end{aligned}$$

$$B = 4,476$$

$$h(n) = 0,447 \cdot \left[2,237 \cdot \cos(26,56^\circ n) + 4,476 \cdot \sin(26,56^\circ n) \right]$$

b

$$u(n) = \mu(n)$$

$$y(n) - 0,8y(n-1) + 0,2y(n-2) = u(n)$$

$$y_{ch} = 0,447 \cdot \left[A \cos(26,56^\circ n) + B \sin(26,56^\circ n) \right]$$

panikkulamno

$$\boxed{U_{dp} = K}$$

$$K = 0,8K + 0,2K = 1$$

$$0,4K = 1$$

$$\boxed{K = 2,5}$$

$$y(n) = 0,447 \cdot A \cos(26,56^\circ n) + 0,447 B \sin(26,56^\circ n) + 2,5$$

$$y(0) = 1$$

$$y(0) = 0,447A + 2,5 = 1$$

$$0,447A = -1,5$$

$$\boxed{A = -3,356}$$

$$y(-1) = 0 = -1,3418 - 0,1998B + 2,5 =$$

$$-0,1998B = -1,1582$$

$$\boxed{B = 5,797}$$

$$y(n) = 0,447 \left[-3,356 \cdot \cos(26,56^\circ n) + 5,797 \sin(26,56^\circ n) \right]$$

$$2) y(n) - \sqrt{3}y(n-1) + y(n-2) = u(n)$$

$$\boxed{a} \quad u(n) = \delta(n)$$

$$y(n) = u(n) + \frac{1}{\sqrt{3}}y(n-1) - \frac{1}{\sqrt{3}}y(n-2)$$

$$h(-1) = h(-2) = 0$$

$$h(n) - \sqrt{3}h(n-1) + h(n-2) = 0$$

$$g^2 - \sqrt{3}g + 1 = 0$$

$$\omega_{1,2} = \frac{\sqrt{3} \pm j}{2} = \frac{\sqrt{3}}{2} \pm j \frac{1}{2}$$

$$\sqrt{\frac{3}{4} + \frac{1}{4}} = 1$$

$$\operatorname{arc} \tan \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \operatorname{arc} \tan \frac{\sqrt{3}}{3} = \frac{\pi}{6}$$

$$h(n) = A \cdot \cos \frac{\pi}{6} n + B \sin \frac{\pi}{6} n$$

$$h(0) = \delta(0) + \sqrt{3}h(-1) - h(-2)$$

$$h(-1) = A \cdot \frac{\sqrt{3}}{2} - B \cdot \frac{1}{2} = 0$$

$$\boxed{h(0)=1}$$

$$h(0) = \boxed{A=1}$$

$$\boxed{B=\sqrt{3}}$$

$$\frac{\sqrt{3}}{2} - \sqrt{3} \cdot \frac{1}{2}$$

$$\boxed{h(n) = \cos \frac{\pi}{6} n + \sqrt{3} \sin \frac{\pi}{6} n}$$

b

$$u(n) = \mu(n)$$

homogeno η_1

$$y_H = A \cos \frac{\pi}{6} n + B \sin \frac{\pi}{6} n$$

$$y_p = k$$

$$K - \sqrt{3}K + K = 1$$

$$(2 - \sqrt{3})k = 1$$

$$k = \frac{1}{2 - \sqrt{3}}$$

$$y(n) = A \cos \frac{\pi}{6} n + B \sin \frac{\pi}{6} n + \frac{1}{2 - \sqrt{3}}$$

$$y(-1) = 0 = A \cdot \frac{\sqrt{3}}{2} - B \cdot \frac{1}{2} + \frac{2 + \sqrt{3}}{7}$$

$$y(0) = 1 = A + \frac{2 + \sqrt{3}}{7}$$

$$\frac{(5 - \sqrt{3}) \cdot \sqrt{3}}{2 \cdot 7} + \frac{2(2 + \sqrt{3})}{2 \cdot 7} - \frac{3 \cdot 1}{2}$$

$$5\sqrt{3} - 3 + 4 + 2\sqrt{3} = 7\sqrt{3}$$

$$7\sqrt{3} + 1 = 7\sqrt{3}$$

$$\boxed{B = \frac{7\sqrt{3} + 1}{7}}$$

$$A = \frac{7}{7} - \frac{2 + \sqrt{3}}{7} = \boxed{\frac{5 - \sqrt{3}}{7} = A}$$

$$\boxed{y(n) = \frac{5 - \sqrt{3}}{7} \cdot \cos \frac{\pi}{6} n + \frac{7\sqrt{3} + 1}{7} \cdot \sin \frac{\pi}{6} n + \frac{2 + \sqrt{3}}{7}}$$

$$3) \quad y(m) - 2y(m-1) + y(m-2) = u(m)$$

a) $u(m) = \delta(m)$

$$\frac{q^2}{2} - 2q + q = 0$$

$$\boxed{\frac{q}{2} = q = 1}$$

$$\begin{aligned} h(0) &= 1 \\ h(-1) &= h(-2) = 0 \end{aligned}$$

$$h(m) = (C_1 + C_2 m) \cdot 1^n$$

$$h(0) = C_1 + 0 = 1$$

$$\boxed{C_1 = 1}$$

$$h(m) = (1+m) \cdot 1^n$$

$$h(-1) = C_1 - C_2 = 0$$

$$\boxed{C_2 = 1}$$

b)

$$u(m) = \mu(m)$$

homogen

$$y_h = (C_1 + C_2 m) \cdot 1^n$$

partikular

$$y_p = K \cdot (1)^n \cdot n^2$$

$$K \cdot m^2 - 2 \cdot K(m-1)^2 + K(m-2)^2 = 1$$

$$\begin{aligned} Km^2 - 2K(m^2 - 2m + 1) + K(m^2 - 4m + 4) &= 1 \\ Km^2 - 2Km^2 + 4mK - 2K + m^2K - 4mK + 4K &= 1 \end{aligned}$$

$$2K = 1$$

$$\boxed{K = \frac{1}{2}}$$

$$\boxed{y_p = \frac{1}{2} \cdot (1)^n \cdot m^2}$$

$$y(m) = (C_1 + C_2 \cdot m) \cdot 1^n + \frac{1}{2} \cdot 1^n \cdot m^2$$

$$y(0) = 1 = C_1$$

$$y(-1) = 0 = 1 - C_2 + \frac{1}{2} =$$

$$\begin{aligned} -C_2 &= -\frac{3}{2} \\ \boxed{C_2 = \frac{3}{2}} \end{aligned}$$

$$\boxed{y(m) = \left(1 + \frac{3}{2}m\right) \cdot 1^n + \frac{1}{2} \cdot 1^n \cdot m^2}$$

$$4) \quad y(m) - 2y(m-1) + 5y(m-2) = u(m)$$

B

a) $u(m) = \delta(m)$

$$z^2 - 2z + 5 = 0$$

$$\frac{z^2 - 2z + 5}{2} = 1 + 2j$$

$$\sqrt{1+4} = \sqrt{5}$$

$$\text{arc tg } 2 = 63,43^\circ$$

$$h(m) = \sqrt{5} \cdot [A \cos(63,43^\circ m) + B \sin(63,43^\circ m)]$$

$$h(0) = 1$$

$$h(-1) = 0$$

$$h(0) = \sqrt{5} \cdot A = 1 \rightarrow A = \frac{\sqrt{5}}{5}$$

$$h(-1) = 0,4473 - B \cdot 1,9999 = 0$$

$$0,4473 - B \cdot 1,9999 \rightarrow B = 0,224$$

$$h(m) = \cos(63,43^\circ m) + 0,501 \sin(63,43^\circ m)$$

b) $u(m) = \mu(m)$

homogeno \Rightarrow

$$y_H = \sqrt{5} \cdot [A \cos(63,43^\circ m) + B \sin(63,43^\circ m)]$$

partikularno \Rightarrow

$$y_p = K$$

$$K - 2K + 5K = 1$$

$$K = \frac{1}{4}$$

$$y(m) = \sqrt{5} \cdot [A \cos(63,43^\circ m) + B \sin(63,43^\circ m)] + \frac{1}{4}$$

$$y(0) = 1$$

$$y(-1) = 0$$

$$y(-1) = 0,3355 - B \cdot 1,9999 + \frac{1}{4} = 0$$

$$y(0) = 1 = \sqrt{5} \cdot A + \frac{1}{4} \rightarrow A = \frac{3\sqrt{5}}{20}$$

$$-B \cdot 1,9999 = -0,5855$$

$$B = 0,2928$$

$$y(m) = 0,75 \cos(63,43^\circ m) + 0,655 \sin(63,43^\circ m)$$

Zad. 3.1-4.

- kada se fiksna potoka poljopi u vlastitom fiksnom ustoru, sustav postaje nestabilan
- potak je zau rezonancija

Zad 3.1-5

a) $y(m) - 0,8y(m-1) + 0,2y(m-2) = u(m)$

$$g_1 = 0,4 + j0,2$$

$$g_2 = 0,4 - j0,2$$

Sustav je stabilan, kerak faktor se velaze unutar kružnice

b) $y(m) - \sqrt{3}y(m-1) + y(m-2) = u(m)$

$$g_1 = \frac{\sqrt{3}}{2} + j\frac{1}{2}$$

$$g_2 = \frac{\sqrt{3}}{2} - j\frac{1}{2}$$

činke su velaze ne rubnuju kružnicu pa je sustav marginalno stabilan

c) $y(m) - 2y(m-1) + y(m-2) = u(m)$

$$g_1 = g_2 = 1$$

Sustav je nestabilen

d) $y(m) - 2y(m-1) + 5y(m-2) = u(m)$

$$g_1 = 1 + 2j$$

$$g_2 = 1 - 2j$$

nestabilan sustav jer su izvan kružnice.

$$1) \quad y(t) = \int_{-\infty}^t u(\tau) d\tau \quad (\text{integrator})$$

Iz izlaza u trenutku t potrebno je poznati "predat" ulazne peje u sustav kojeg

→ linearost

$$u(\tau) = \alpha u_1(\tau) + \beta u_2(\tau)$$

$$y(t) = \int_{-\infty}^t (\alpha u_1(\tau) + \beta u_2(\tau)) d\tau$$

$$y(t) = \int_{-\infty}^t \alpha u_1(\tau) d\tau + \int_{-\infty}^t \beta u_2(\tau) d\tau$$

$$y(t) = \alpha y_1(t) + \beta y_2(t) \quad - \text{sustav je linearan}$$

$$y_1(t) = \int_{-\infty}^t u_1(\tau) d\tau$$

$$y_2(t) = \int_{-\infty}^t u_2(\tau) d\tau$$

→ memorije austore

→ integrator je memorijski sustav

$$y(t) = \int_{-\infty}^t u(\tau) d\tau = \int_{-\infty}^0 u(\tau) d\tau + \int_0^t u(\tau) d\tau$$

$$y(t_0) + \int_{t_0}^t u(\tau) d\tau$$

$$2) \quad y(t) = \frac{d}{dt} u(t) \quad \text{derivator}$$

→ kozalnost

ako derivator otkao definisava
onda je mekozalan

$$y(t) = \lim_{\Delta t \rightarrow 0} \frac{f(t + \Delta t) - f(t)}{\Delta t}$$

ako go definisava kao

onda je kozalan

$$y(t) = \lim_{\Delta t \rightarrow 0} \frac{f(t) - f(t - \Delta t)}{\Delta t}$$

→ linearost

$$u(t) = \alpha u_1(t) + \beta u_2(t)$$

$$y(t) = \frac{d}{dt} \left[\alpha u_1(t) + \beta u_2(t) \right]$$

$$y(t) = \alpha \frac{d}{dt} u_1(t) + \beta \frac{d}{dt} u_2(t)$$

→ sustav je linearan

$$y_1(t) = \frac{d}{dt} u_1(t)$$

$$y_2(t) = \frac{d}{dt} u_2(t)$$

→ derivator je memorijski austav

Zad. 3.2.-2(a))

1) $y''(t) + 2y'(t) + 25y(t) = u(t)$

$$s^2 + 2s + 25 = 0$$

$$s_{1,2} = \frac{-2 \pm \sqrt{4-100}}{2} = -1 \pm j\sqrt{24}$$

$$y_H = e^{-t} (A \cos 2\sqrt{6}t + B \sin 2\sqrt{6}t)$$

$$h_A(t) = e^{-t} (A \cos 2\sqrt{6}t + B \sin 2\sqrt{6}t)$$

$$\alpha_1 = 2$$

$$\alpha_2 = 25$$

$$\beta_0 = 0$$

$$\beta_1 = 0$$

$$\beta_2 = 1$$

$$h_A(0^+) = 0$$

$$h_A'(0^+) = 1$$

$$h_A'(t) = -e^{-t} A \cos(2\sqrt{6}t) + 2\sqrt{6} \cdot e^{-t} \cdot \sin(2\sqrt{6}t) \cdot A + e^{-t} \cdot B \sin(2\sqrt{6}t) + e^{-t} \cdot 2\sqrt{6} \cdot B \cos(2\sqrt{6}t)$$

$$h_A'(0^+) = 1 \cdot A - 2\sqrt{6} \cdot 0 = 1$$

$$h_A(0^+) = 1 \cdot A = 0$$

$$B = -\frac{1}{2\sqrt{6}} \cdot \frac{\sqrt{6}}{\sqrt{6}} = -\frac{\sqrt{6}}{12}$$

$$A = 0$$

$$h_A(t) = -e^{-t} \frac{\sqrt{6}}{12} \sin 2\sqrt{6}t$$

$$M=1$$

$$N=2$$

N>M & N

$$h(t) = b_0 \cdot \delta(t) + \sum_{m=0}^M (b_{N-m} D^m) h_A(t)$$

$$= \sum_{m=0}^0 (b_{2-N} D^m) \cdot h_A(t) = b_2 D^0 h_A(t)$$

$$h(t) = b_2 \cdot h_A(t)$$

$$h(t) = 1 \cdot h_A(t) \\ = -e^{-t} \frac{\sqrt{6}}{12} \cdot \sin 2\sqrt{6}t \quad t > 0$$

$$2) \quad y''(t) + 23y(t) = u(t)$$

$$s^2 + 23s = 0$$

$$s(s+23)=0$$

$$s_1=0$$

$$s_2=-23$$

$$a_1=0$$

$$a_2=23$$

$$b_0=b_1=0$$

$$b_2=1$$

$$y_H = C_1 \cdot e^{0t} + C_2 \cdot e^{-23t}$$

$$y_H = C_1 + C_2 \cdot e^{-23t} \quad t>0$$

$$h_A(t) = C_1 + C_2 \cdot e^{-23t}$$

$$h_A'(0^+) = 1$$

$$h_A(0^+) = 0 = C_1 + C_2$$

$$h_A'(0^+) = -23C_2 = 1$$

$$\boxed{C_2 = -\frac{1}{23}}$$

$$\boxed{C_1 = \frac{1}{23}}$$

$$M=0 \\ N=2$$

$$\boxed{h_A(t) = \frac{1}{23} - \frac{1}{23} \cdot e^{-23t} \quad t>0}$$

$$h(t) = b_0 \cdot \delta(t) + \sum_{m=0}^M (b_{N-m} D^m) h_A(t) = b_2 \cdot 0^{\circ} h_A(t)$$

$$\boxed{h(t) = \frac{1}{23} - \frac{1}{23} e^{-23t} \quad t>0}$$

$$3) \quad y''(t) = u(t)$$

$$\boxed{s^2=0}$$

$$\boxed{S_1=S_2=0}$$

$$a_1=a_2=0$$

$$b_0=b_1=0$$

$$b_2=1$$

$$h'(0^+) = 1$$

$$h(0^+) = 0$$

$$y_H = (C_1 + C_2 t) \cdot e^{0 \cdot t}$$

$$\boxed{h_A(t) = C_1 + C_2 t \quad t>0}$$

$$h_A'(t) = C_2 = 1$$

$$\boxed{h_A(t) = t, \quad t>0}$$

$$h_A(t) = C_1 = 0$$

$$h(t) = b_2 \cdot h_A(t) =$$

$$\boxed{h(t) = t, \quad t>0}$$

$$4) \quad y''(t) - 2y'(t) + 17y(t) = u(t)$$

$$\begin{aligned}a_1 &= -2 \\a_2 &= 17\end{aligned}$$

$$b_0 - b_1 = 0$$

$$b_2 = 1$$

$$s^2 - 2s + 17 = 0$$

$$s_{1,2} = \frac{2 \pm j\sqrt{8}}{2}$$

$$s_{1,2} = 1 \pm j\sqrt{4}$$

$$y_H = e^t (A \cos 4t + B \sin 4t)$$

$$h_A(0^+) = 0$$

$$h_A'(0^+) = 1$$

$$h_A(t) = e^t (A \cos 4t + B \sin 4t) \quad t > 0$$

$$h_A(0^+) = A = 0$$

$$h_A(t) = e^t \cdot A \cos 4t - A e^t \cdot 4 \sin 4t + e^t \cdot B \sin 4t + e^t \cdot 4 \cdot B \cos 4t$$

$$h_A'(0^+) = 1 = A + 4B$$

$$B = \frac{1}{4}$$

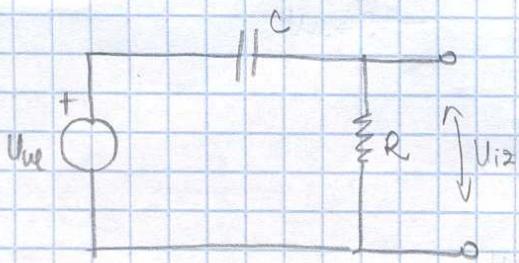
$$h_A(t) = e^t \cdot \frac{1}{4} \sin 4t \quad t > 0$$

$$h(t) = b_2 \cdot h_A(t) = e^t \cdot \frac{1}{4} \sin 4t, \quad t > 0$$

Zadanie 3.2.-5.

$$R = 1 \text{ k}\Omega$$

$$C = 10 \mu\text{F}$$



$$U_{ul} = U_C + U_R$$

$$U_{i2} = U_R$$

$$U_{i2} = i \cdot R$$

$$\boxed{i = \frac{U_{i2}}{R}}$$

$$U_{ul} = \frac{1}{C} \int i \, d\tau + U_{i2}$$

$$U_{ul} = \frac{1}{C} \cdot \int \frac{U_{i2}}{R} \, d\tau + U_{i2} / \frac{d}{dt}$$

$$\frac{dU_{ul}}{dt} = \frac{1}{CR} U_{i2} + \frac{dU_{i2}}{dt}$$

→ jedna zde je konec

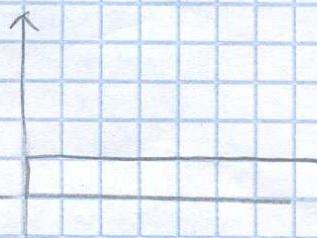
$$U = U_{ul}$$

$$y = U_{i2}$$

$$U = \frac{1}{CR} y + y'$$

$$\boxed{b_0 = 1}$$

$$U(t) = \begin{cases} 8mt & -\infty < t < 0 \\ 1 & 0 < t < \infty \end{cases}$$



$$U(0^-) = 0$$

$$U(0^+) = 1$$

* Zdroj

ještě návazka

$$y_p = K_1 \cos(t) + K_2 \sin(t)$$

$$y_p' = -K_1 \sin(t) + K_2 \cos(t)$$

$$-cost = \frac{1}{CR} [K_1 \cos(t) + K_2 \sin(t)] - K_1 \sin(t) + K_2 \cos(t)$$

$$\frac{1}{CR} = \frac{1}{10^3 \cdot 10^{-6}} = 100$$

$$-cost = (100K_1 + K_2) cost + \sin(t)(100K_2 - K_1)$$

$$100K_1 + K_2 = -1 \quad | \cdot 100$$

$$100K_2 - K_1 = 0 \quad | \cdot 100$$

$$100K_1 + K_2 = -1 \quad | -100$$

$$-100K_1 + 100K_2 = 0$$

$$10001K_2 = -1$$

$$\boxed{K_2 = -0,0001}$$

Zad. 3.2-7.a)

a) $y''(t) + 2\zeta \omega_n y'(t) + \omega_n^2 y(t) = u(t)$

$$\omega_n = 0,4$$

$$\zeta = \frac{1}{\omega_n}$$

$$\zeta = -0,125$$

$$y(0) = 2$$

$$y'(0) = -2$$

$$y''(t) - 0,1y'(t) + 0,16y(t) = u(t)$$

$$H(s) = \frac{1}{s^2 - 0,1s + 0,16}$$

$$H(j\omega) = \frac{1}{-\omega^2 - 0,1j\omega + 0,16}$$

$$|H(j\omega)| = \frac{1}{\sqrt{(0,16 - \omega^2)^2 + (0,1\omega)^2}}$$

b) $y''(t) + 0,2y'(t) + 0,16y(t) = u(t)$

$$\zeta = 0,25$$

$$H(s) = \frac{1}{s^2 + 0,2s + 0,16}$$

$$H(j\omega) = \frac{1}{-\omega^2 + j0,2\omega + 0,16}$$

$$|H(j\omega)| = \frac{1}{\sqrt{(0,16 - \omega^2)^2 + 0,4\omega^2}}$$

c) $y''(t) + 0,8y'(t) + 0,16y(t) = u(t)$

$$\zeta = 1$$

$$H(s) = \frac{1}{s^2 + 0,8s + 0,16}$$

$$H(j\omega) = \frac{1}{-\omega^2 + j0,8\omega + 0,16}$$

$$|H(j\omega)| = \frac{1}{\sqrt{(0,16 - \omega^2)^2 + 0,64\omega^2}}$$

nášlesek 3.2-5.

$$100k_1 - 0,0001 = -1$$

$$100k_1 = -0,9999$$

$$k_1 = -0,01$$

$$y_p = -0,01 \cos(t) - 0,0001 \sin(t)$$

$$y(0^-) = -0,01$$

$$y(0^+) = b_0 u(0^+)$$

$$y(0^+) - y(0^-) = 1 \cdot u(0^+)$$

$$y(0^+) = 1 + y(0^-)$$

$$y(0^+) = 0,99$$

Zadanie 3.2-7.c

homogený signal

$$\sqrt{3} \cos(4t)$$

a)

$$H(j_4) = \frac{1}{-16 - 0,4j + 0,16} = \frac{1}{-15,84 - 0,4j}$$

non signal $A_1 \cos(4t + \varphi)$

$$|H(j_4)| = 0,0631$$

$$\text{angle } -\frac{0,4}{-15,84} = 91,44^\circ$$



$$A = \sqrt{3} \cdot 0,0631 = 0,1093$$

$$\rightarrow 0,1093 \cos(4t + 91,44^\circ)$$

b)

$$H(j_4) = \frac{1}{-16 + j0,8 + 0,16} = \frac{1}{0,8j - 15,84} \cdot \frac{-15,84 - 0,8j}{-15,84 - 0,8j}$$

$$A = \sqrt{3} \cdot 0,06305 = 0,1092$$

$$|H(j_4)| = 0,06305$$



$$\arctg \frac{-0,8}{-15,84} = 182,89^\circ$$

$$\rightarrow 0,1092 \cos(4t + 182,89^\circ)$$



c)

$$H(j\omega) = \frac{1}{-16 + 32j + 0,16} = \frac{1}{3,2j - 15,84}$$

$$A = \sqrt{3 \cdot 0,06188} = 0,1072$$



$$\rightarrow 0,1072 \cos(4t + 191,42^\circ)$$

$$\text{Korc } \rightarrow \frac{-3,2}{-15,84} = 191,42^\circ$$

$$|H(j\omega)| = 0,06188$$