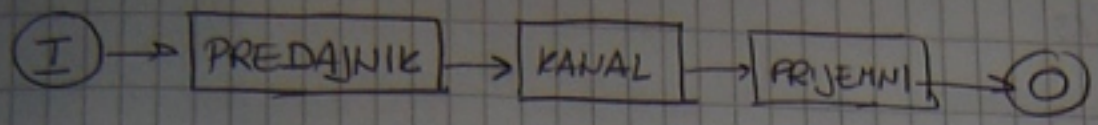
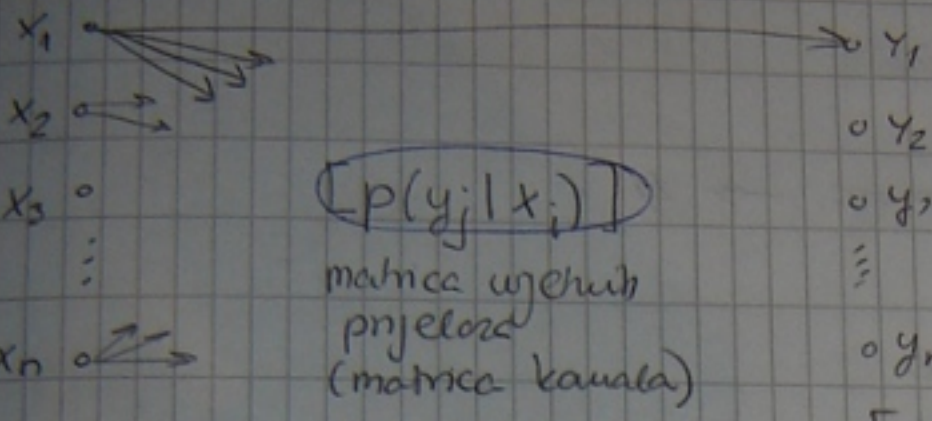
Predavanje 13.09.2010.





izvor generira → ulaze u kom. kanal

$[p(x_i)]$

$[p(y_j)]$ vjerojatnost pojavljivanja

$[p(x_2|y_j)]$

$H(x)$ entropija

$-\log_2 p(x_i)$ → količina informacija

$i = 1, \dots, n$

$$H(x) = -\sum_{i=1}^{n} p(x_i) \log_2 (x_i) \quad [\text{bit/simbol}]$$

SVOJSTVA:

1. $H(x) \geq 0$

   $H(x) = H(x)_{max} = \log_2 n$

ZADATAK

x_i - kuja

f_i - frekvencija

$I(x_i) = -\log_2 p(x_i)$

$I(x) = ?$

$$I(x) = \sum_{i=1}^{n} p(x_i) \log_2 p(x_i)$$

↳ prosječni

$p(A) = \frac{8000}{54400} = 0,1471$

$p(B) = 0,01313$

$\vdots \qquad \vdots$

$p(M) = 0,0639$

vjerojatnosti pojavljivanja koja

$I(x) = 3,5037$ bit/simbol $\rightarrow 4$

dodatak $N = 10^6$ simbol

$M = I(x) \cdot N$

↓veličina memorije

- Komunikacijski sustav

činega: 1) matrica $[p(x_i)]$, $[p(y_j|x_i)]$

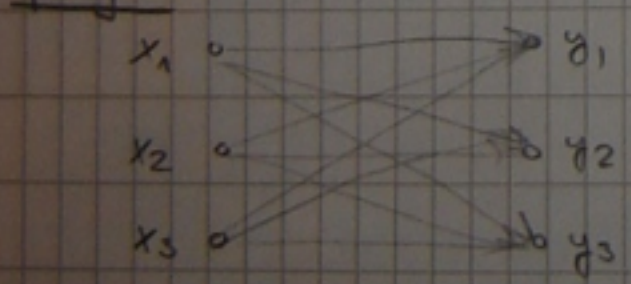
2) $[p(y_j)]$, $[p(x_i|y_j)]$

3) $[p(x_i, y_j)]$

matrica združenih vjerojatnosti

$$p(x_i, y_j) = p(x_i)\, p(y_j|x_i)$$
$$p(y_j)\, p(x_i|y_j)$$

primjer



$p(x_i)$ znamo

$p(y_j|x_i)$ znamo

$p(y_j) = ?$

$$p(y_1) = p(x_1, y_1) + p(x_2, y_1) + p(x_3, y_1)$$
$$= p(x_1)\, p(y_1|x_1) + p(x_2)\, p(y_1|x_2) + p(x_3)\, p(y_1|x_3)$$

bez formula:
Znamo $p(x_i)$, $p(y_j|x_i)$

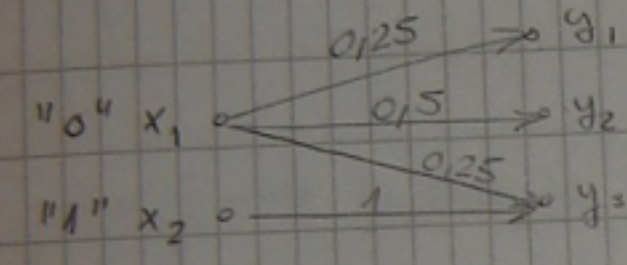
$$[p(y)] = [p(x)] * [p(y|x)]$$ matrica uvjetnih vjerojatnosti u kanalu

$$p(y_j|x_i) = \begin{bmatrix} p(y_1|x_1) & \cdots & p(y_m|x_1) \\ \vdots & \ddots & \vdots \\ & & p(y_m|x_n) \end{bmatrix}$$

primjer

Ⓘ

$X = \{0, 1\}$



a)

$$[p(y_j|x_i)] = \begin{bmatrix} 0,25 & 0,5 & 0,25 \\ 0 & 0 & 1 \end{bmatrix}$$

$$p(x_i, y_j) = p(x_i)\, p(y_j|x_i)$$

$$[p(x_i, y_j)] = \begin{matrix} x_1 \\ x_2 \end{matrix} \begin{bmatrix} 0,125 & 0,25 & 0,125 \\ 0 & 0 & 0,5 \end{bmatrix} \begin{matrix} \\ \\ \end{matrix} \begin{matrix} y_1 & y_2 & y_3 \end{matrix}$$

$[p(y_j)] = [1/8 \quad 1/4 \quad 5/8]$ → ZA FORMULU

FORMULE

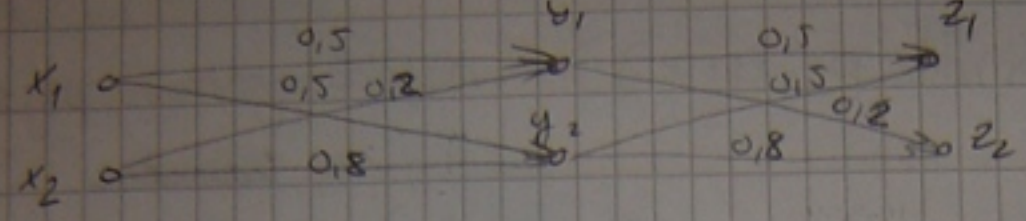
$I(x_i) = -\log_2 p(x_i)$

$H(x)$, $I(x)$

$[p(y)] = [p(x)] * [p(y|x)]$

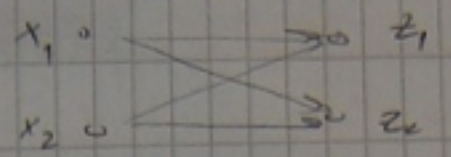
$[p(x_i, y_j)] = \begin{bmatrix} p(x_1, y_1) & \cdots \\ & \ddots \end{bmatrix}$

- kaskoda kanala



→ FORMULE (obvezno!)

- kaskodu možemo svesti na jedan kanal - i želimo!



$[p(z_j|x_i)]$

$p(x_1) = p(x_2) = 0,5$

a) $[p(z|x)] = ?$

b) $p(z_1), p(z_2) = ?$

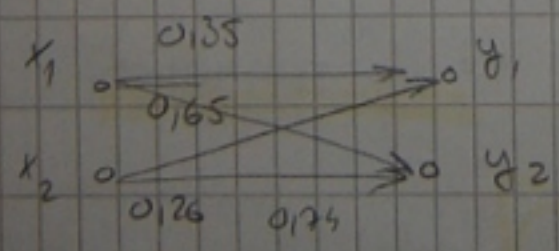
$[p(y)] = [p(x)] * [p(y|x)]$

$[p(z)] = [p(y)] * [p(z|y)]$

$= [p(x)] * [p(y|x)] * [p(z|y)]$

$= [p(x)] * [p(z|x)]$

$$[p(z|x)] = \begin{bmatrix} 0,5 & 0,5 \\ 0,2 & 0,8 \end{bmatrix} * \begin{bmatrix} 0,5 & 0,5 \\ 0,2 & 0,8 \end{bmatrix} = \begin{bmatrix} 0,35 & 0,65 \\ 0,26 & 0,74 \end{bmatrix}$$



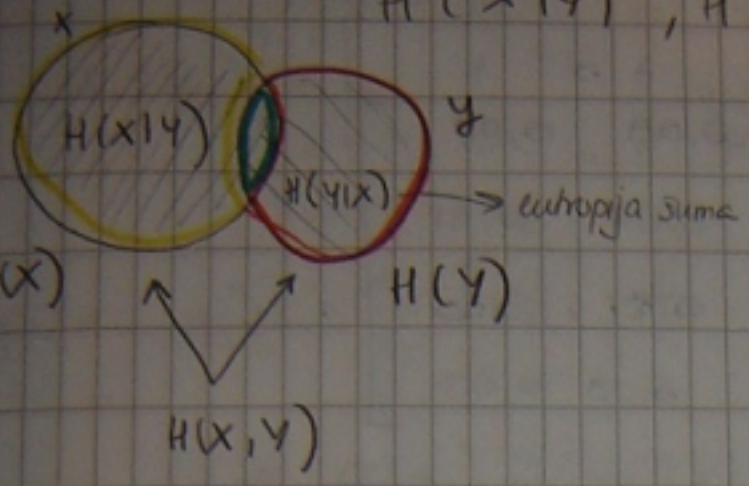
F

$p(x_i)$ $i = 1, \dots, n \rightarrow H(x)$

$p(y_j)$ $j = 1, \dots, n \rightarrow H(y)$

$p(x_i, y_j) \rightarrow H(X,Y)$

$H(X|Y)$, $H(Y|X)$



→ entropija šuma

↘ $I(X;Y)$

TRANSINFORMACIJA

preko nje dobijemo C

presjek to je $I(X;Y)$

↳ količina informacije koja se prenosi neiskrivljena

(I)

$p(x_i) \rightarrow$ kodiranje $\cdot \quad f(p(x_i))$

$q(x_i)$

↓

pretpostavka da ne znamo $p(x_i)$

relativna entropija: $D(p||q) \neq D(q||p)$

$I(X;Y) = H(X) - H(X|Y)$

$= H(Y) - H(X|X)$

$= H(X) + H(Y) - H(X,Y)$

ZADATAK

A	"00"	x_1	y_1	"00"	A
B	"01"	x_2	y_2	"01"	B
C	"10"	x_3	y_3	"10"	C
D	"11"	x_4	y_4	"11"	D

$0,9 \cdot 0,9$

$0,9 \cdot 0,1$

$0,1 \cdot 0,5$

$0,1 \cdot 0,1$

$p(y_j|x_i)$

$I(X;Y) = \sum_i \sum_j p(x_i;y_j) \log_2 \frac{p(y_j|x_i)}{p(x_j)}$

$$[p(y_j|x_i)] = \begin{bmatrix} 0,9 & 0,1 \\ 0,1 & 0,9 \end{bmatrix}$$

$$[p(y_j|x_i)] = \begin{bmatrix} 0,81 & 0,09 & 0,09 & 0,01 \\ 0,09 & 0,81 & 0,01 & 0,81 \\ 0,09 & 0,01 & 0,81 & 0,09 \\ 0,01 & 0,09 & 0,09 & 0,89 \end{bmatrix}$$

$p(x_i, y_j) = p(x_i) \cdot p(y_j|x_i)$ znamo

↓ zbrajamo po stupcima $[p(y_j)] = [\frac{1}{4} \quad \frac{1}{4} \quad \frac{1}{4} \quad \frac{1}{4}]$

$I(x;y) = 1,062$ bit/simbol

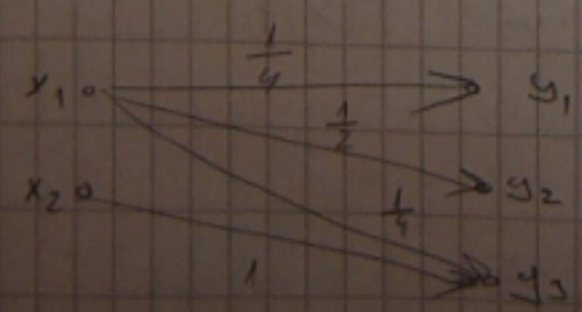
- Zadatak s parnim paritetnim bitom

000	000
011	011
101	101
110	110
	X error

→ $I(x;x)$

$$[p(y_j|x_i)] = \begin{bmatrix} \frac{1}{4} & \frac{1}{2} & \frac{1}{4} \\ 0 & 0 & 1 \end{bmatrix}$$

$p(0) = p(1) = 0,5$



$$H(X|Y) = -\sum_i \sum_j p(x_i, y_j) \log_2 p(x_i | y_j)$$

$$[p(x_i|y_j)] = \begin{matrix} x_1 \\ x_2 \end{matrix} \begin{matrix} y_1 & y_2 & y_3 \end{matrix} \begin{bmatrix} 1 & 1 & 1/5 \\ 0 & 0 & 4/5 \end{bmatrix} \leftarrow \text{iz} \quad p(x_i|y_j) = \frac{p(x_i, y_j)}{p(y_j)}$$

$$[p(x_i, y_j)] = \begin{matrix} x_1 \\ x_2 \end{matrix} \begin{matrix} y_1 & y_2 & y_3 \end{matrix} \begin{bmatrix} 1/8 & 1/4 & 1/8 \\ 0 & 0 & 1/2 \end{bmatrix}$$

$$= p(x_i)\, p(y_j|x_i) \longrightarrow p(y_j) = \begin{bmatrix} 1/8 & 1/4 & 5/8 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 1/5 \\ 0 & 0 & 4/5 \end{bmatrix} \begin{bmatrix} 1/8 \\ 1/4 \\ 5/8 \end{bmatrix}$$

$p(x_i|y_j)$ $p(y_j)$

$$I(X;Y) = I(Y;X)$$

$$C = \max_{\{p(x_i)\}} I(X;Y)$$

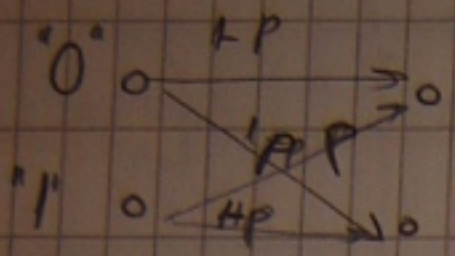
$$C = \max_{\{p(x_i)\}} [H(X) - H(X|Y)]$$

pr. $n=3$

$\frac{1}{2}$ ——— $\frac{1}{2}$

$\frac{2}{3}$ ——— $\frac{2}{3}$

$H(X|Y) \longrightarrow C = \max_{\{p(x_i)\}} [H(X)] = \log_2 3$ bit/simbol



pogreška prijenosa
1 bita

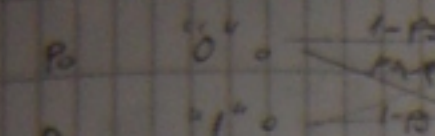
vjerojatnost pojave pogrešnog simbola na izlazu

$$\left\{ \begin{array}{ll} 0 \to 1 & p(0)\,p(1|0) \\ 1 \to 0 & p(1)\,p(0|1) \end{array} \right\} +$$

$$p_0\,p + p_1\,p$$

Predavanje 20.09.2010.

Pr: BSK – bin. sim. kanal



$$[P(y_j|x_i)] = \begin{bmatrix} 1-p_s & p_s \\ p_s & 1-p_s \end{bmatrix}$$

$C = \max_{\{P(x_i)\}} I(X;Y)$

$$P(x_i, y_j) = \begin{bmatrix} p_0(1-p_s) & p_0 p_s \\ p_1 p_s & p_1(1-p_s) \end{bmatrix}$$

$C = \max (H(Y) - H(Y|X))$

↳ tražimo derivacijom po P_s

$$[P(y_j)] = [p_0(1-p_s) + p_1 p_s \quad p_0 p_s + p_1(1-p_s)]$$

Zad 2i05

$X = \{0, 1, 2, 3, 4\}$

$(-1) \bmod 5 = 4$!!

$$P(y_j|x_i) = \begin{cases} 0{,}5 & y_j = (x_i \pm 1) \bmod 5 \\ 0, & \text{inače} \end{cases}$$

$P(y_j\|x_i)$	x_i \ y_j	0	1	2	3	4
	0	0	0,5	0	0	0,5
	1	0,5	0	0,5	0	0
	2	0	0,5	0	0,5	0
	3	0	0	0,5	0	0,5
	4	0,5	0	0	0,5	0

za $x_1 = 0$ $P(y_1) = \begin{cases} 0{,}5 & \text{za } 1 \\ 0{,}5 & \text{za } 4 \end{cases}$

$x_2 = 1$ $P(y_2) = \begin{cases} 0{,}5 & \text{za } 2 \\ 0{,}5 & \text{za } 0 \end{cases}$

$x_3 = 2$ $P(y_3) = \begin{cases} 0{,}5 & \text{za } 3 \\ 0{,}5 & \text{za } 1 \end{cases}$

$x_4 = 3$ $P(y_3) = \begin{cases} 0{,}5 & \text{za } 4 \\ 0{,}5 & \text{za } 2 \end{cases}$

$x_5 = 4$ $P(y_5) = \begin{cases} 0{,}5 & \text{za } 0 \\ 0{,}5 & \text{za } 3 \end{cases}$

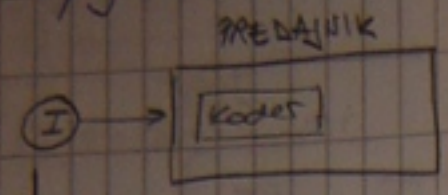
$$P(x_i, y_j) = \begin{bmatrix} 0 & 0{,}01 & 0 & 0 & 0{,}01 \\ 0{,}01 & 0 & 0{,}01 & 0 & 0 \\ 0 & 0{,}01 & 0 & 0{,}01 & 0 \\ 0 & 0 & 0{,}01 & 0 & 0{,}01 \\ 0{,}01 & 0 & 0 & 0{,}01 & 0 \end{bmatrix}$$

$P(y_1) = 0{,}02$ $P(y_2) = 0{,}02$ $P(y_3) = 0{,}02$

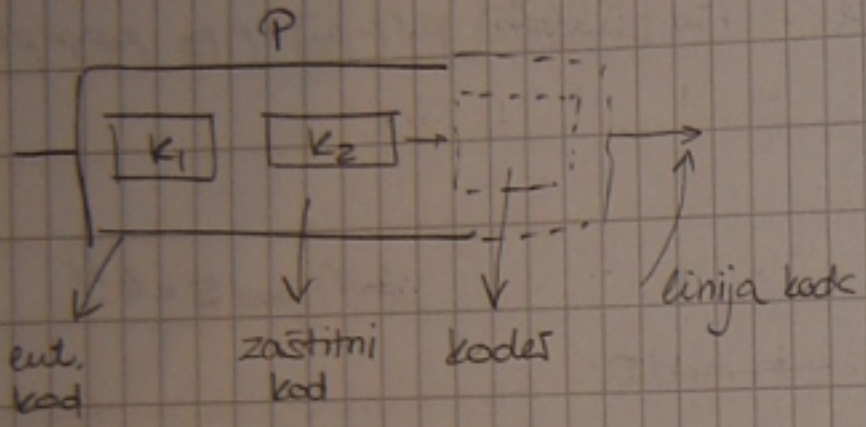
$P(y_4) = 0{,}02$ $P(y_5) = 0{,}02$

↳ iz toga $H(Y)$ pa C (...)

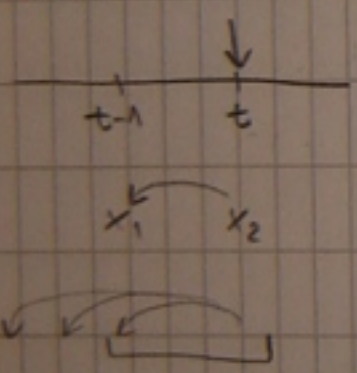
Entropijsko kodiranje



	pomjes
$c(x_1)$	0
$c(x_2)$	01
$c(x_3)$	11



- IZVORIŠTA S MEMORIJOM



Markovljevi procesi (lanci)

$X_{-1}, X_{-2}, X_{-3} \ldots \rightarrow S_1, S_2, \ldots S_n$

poprimaju vrijednosti iz vrlog konačnog skupa

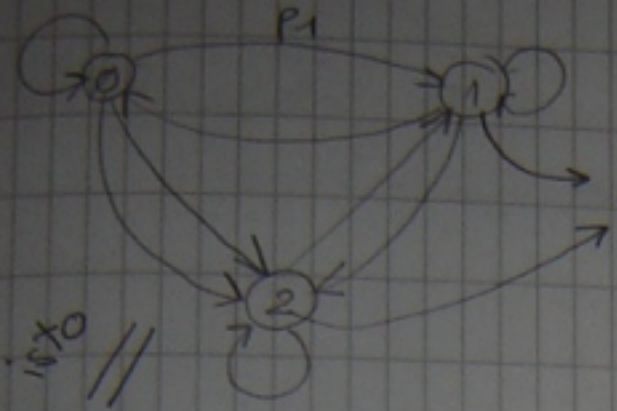
(*) $p\{X_0 = S_j \mid X_{-1}, X_{-2}, \ldots\} = p\{X_0 = S_j \mid X_{-1}\}$

↳ možemo samo na osnovu prethodnog simbola s danom vjerojatnošću iskazati koji će biti sljedeći simbol

$H(X) = -\sum p(x_i) \log_2 p(x_i)$ ne vrijedi za izvore s memorijom

(I)

$[p(x_j|x_i)]$ matrica izvorišta $\neq [p(x_j|x_i)]$ → vezano uz kanal

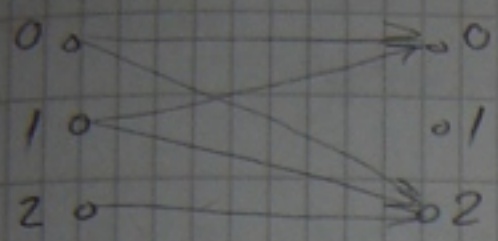


stacionarne vjerojatnosti P_0, P_1, P_2

SVOJSTVA

1) stacionarnost = $P_j = P_i \cdot [p(y_j|x_i)]$

2) ergodičnost - iz svakog u konačnom br. koraka može doći u bilo koje stanje



primjer DHMZ je utvrdio sljedeću zakonitost

DANAS / SUTRA	S	K	Sn
SUNCE	0,3	0,1	0,6
KIŠA	0,1	0,3	0,6
SNIJEG	0,2	0,2	0,6

a) danas ⇒ sunce

→ sutra

→ prekosutra

$P_i = [1 \quad 0 \quad 0]$ (S K Sn) danas

$P_s = [pd][p(y_j|x_i)]$ sutra

$= [0,3 \quad 0,1 \quad 0,6]$

prekosutra:

$P_{PS} = [PS][p(y_j|x_i)]$

$P_{PS} = [0,22 \quad 0,18 \quad 0,6]$

b) vjerojatnost S, K, Sn sutrašnjeg dana

0 1 1 2 1 0 1 0 0 2 2 1 0 2 1 . . .

$$[p(x_j|x_i)] = \begin{matrix} 0 \\ 1 \\ 2 \end{matrix} \begin{bmatrix} & 0 & 1 & 2 \\ & & & \\ & & & \\ & & & \end{bmatrix}$$

→ naći parove, prebrojati ih

→ P_0, P_1, P_2 $\quad \sum p_i = 1$

→ (n-1) simbola

$[P_0 \; P_1 \; P_2] = [P_0 \; P_1 \; P_2][p(x_j|x_i)]$

$p_1 + p_2 + p_3 = 1$

$p_1 = 0,3\,p_1 + 0,1\,p_2 + 0,2\,p_3$

$p_2 = 0,1\,p_1 + 0,3\,p_2 + 0,2\,p_3$

$$[p_1 \quad p_2 \quad p_3] \begin{bmatrix} 0,3 \\ 0,1 \\ 0,2 \end{bmatrix}$$

$H(X)$ najmanja moguća vrijednost za kodiranje → ne možemo manje

$H(X) = 3$ bit / simbol za 8 simbola kod su svi jednaki

$p(x_1) = p(x_2) = \dots = p(x_8)$

$H(X) \leq 3$ bit nisi koliko smo dobro odabrali metodu kodiranja

- ENTROPIJSKO KODIRANJE

$X = \{x_1, x_2, \dots x_n\}$

$c(x_1), c(x_2), \dots c(x_n)$

Kodiramo – baza = 2 (u zadacima i 3 i 4)

DULJINA KODNE RIJEČI

$p(x_i)$	(x_i)	$c(x_i)$	e_i
p_1	x_1	0	1
p_2	x_2	1	1
p_3	x_3	01	2
p_4	x_4	10	2

$L = \sum p_i e_i$ [bit]

efikasnost koda

$$\varepsilon = \frac{H(X)}{L(X)}$$

stvarna mjera efikasnosti

$H(X) \leq L(X)$

↳ moraju biti H i L u istoj bazi

NESINGULARAN KÔD

$c(x_i) \neq c(x_j)$ ne jamci jednoznačnost

primjer odredite koji su od navedenih kodova nesingularni jednoznačno dekodirki i/ili prefiksni

1) $K_1 = \{0, 101\}$ N, JDK, P

2) $K_2 = \{1, 101\}$ N, JDK

3) $K_3 = \{0, 10, 110, 111\}$ P, N

4) $K_4 = \{0, 01, 10, 1\}$ P, N

NA ISPITU

DZ zi 03

$X = \{x_1, x_2, x_3, x_4\}$

$P_X = \{1/2, p/2, (1-p)/2, (1-p)/2\}$

$p \in (0, 1)$

$$p(y_j | x_i) = \begin{bmatrix} 1-f & f & 0 & 0 \\ f & 1-f & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad 0 \leq f \leq 1$$

$4 \times 4 \quad 4 \times 4$

$H(f) = \log_2 \frac{1}{f} \cdot \log_2 \frac{1}{1-f}$

$[p(x_i, y_j)] = [p(x_i)] \cdot [p(y_j | x_i)]$

$C = \max I(X;Y) = \max (H(Y) - H(Y|X))$

$(-,-,-)$

$[p(y_j)] = [1/2 \quad p/2 \quad \frac{1-p}{2} \quad \frac{1-p}{2}]$

$(-,-,-)$

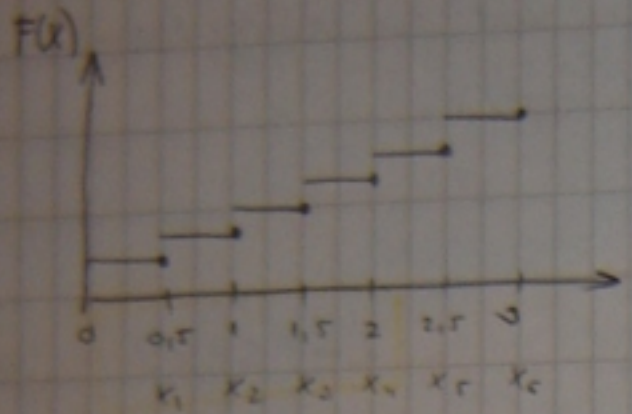
z: 04

$f(u) = a \cdot u \cdot (3-u)$, $u \in [0,3]$

$f(u) = 0$, $u \notin [0,3]$

$a \in R$

$t = 10$ ms



$p(x_i)$ -

$p(x_1) = p(x \leq 0,5)$

$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

$$a\int_0^3 u(3-u)\,du = a\int_0^3 (3u-u^2)\,du =$$

$$= a\left(\frac{3}{2}u - \frac{u^2}{2}\right)\Big|_0^3$$

$$1 = a\frac{9}{2} \rightarrow a = \frac{2}{9}$$

$$p(x_1) = p(x \leq 0,5) = F(0,5) = \int_0^{0,5} f(u)F = \frac{2}{27}$$

$$p(x_2) = p(0,5 < x \leq 1) = F(1) - F(0,5) = \frac{5}{27} \qquad F(u) = \frac{2}{9}\int_0^u (3t - t^2)\,dt$$

$$p(x_3) = p(1 < x \leq 1,5) = F(1,5) - F(1) = \frac{13}{54} \qquad F(u) = \frac{2}{9}\left(\frac{3}{2}u^2 - \frac{u^3}{3}\right)$$

$$p(x_4) = p(1,5 < x \leq 2) = F(2) - F(1,5) = \frac{13}{54} \qquad F(u) = \frac{1}{3}u^2 - \frac{2}{27}u^3$$

$$p(x_5) = p(2 < x \leq 2,5) = F(2,5) - F(2) = \frac{5}{27} \qquad F(x) = p(X \leq x)$$

$$p(x_6) = p(2,5 < x \leq 3) = F(3) - F(2,5) = \frac{2}{27}$$

$$\Sigma = 1$$

$$f(x) = \sum p(x_i) \cdot \text{ens}\, p(x_i)$$

$$N = \frac{\text{co}}{10 \cdot 10^3} \cdot$$

$$I(x) = f(x) \cdot N$$

Predavanje 27.09.2010.

x_i, $p(x_i)$

↓

$c(x_i)$ (ili $k(x_i)$) kodna riječ

L srednja duljina kodne riječi

$H(x) \leq L(x)$

↳ uvjet optimalnosti koda

l_i, $i = 1, \dots, n$

$\sum_{i=1}^{n} d^{-l_i} \leq 1$ uvjet postojanja prefiksnog koda

↓

d je baza $d = 2, 3, \dots$

Primjer SF - kodiranje

8 poruka

vjerojatnosti pojavljivanja

x_i	x_7	x_6	x_5	x_4	x_3	x_2	x_1	x_0
$p(x_i)$	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{32}$	$\frac{1}{32}$	$\frac{1}{32}$	$\frac{1}{32}$

treba pronaći optimalno kodiranje:

a) srednja duljina kodne riječi bude minimalna

b) najveća duljina kodne riječi mora biti minimalna

a) → optimalnim kodom smatrat ćemo kod kojeg je kodna riječ minimalna

→ sortirati simbole (poruke) po padajućim vjerojatnostima

(već' jesmo)

x_7 $\frac{1}{2}$ 0 → podijeliti ih na 2 skupa između kojih je

x_6 $\frac{1}{4}$ 1 0 razlika kod se zbroje vjerojatnosti minimalna

x_5 $\frac{1}{16}$ 1 1 0 najbolje $\frac{1}{2}$ samo i ostalo skupa

x_4 $\frac{1}{16}$ 1 1 1 → opet podjela na podskupe na istu foru

x_3 $\frac{1}{32}$ 1 1 1 0 0 → mora biti razlika

x_2 $\frac{1}{32}$ 1 1 1 0 1

x_1 $\frac{1}{32}$ 1 1 1 1 0

x_0 $\frac{1}{32}$ 1 1 1 1 1

$l_i = \{1, 2, 4, 4, 5, 5, 5, 5\}$

$L = 1,125$ bit

b) Aritmetičko kôd algoritma ovisi
→ znači od 000, 001, ..., 111

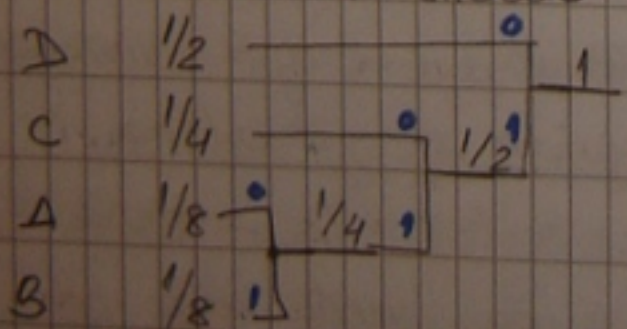
$$\varepsilon_{SF} \leq \varepsilon_{HUFF} \leq \varepsilon_{ARIT}$$
→ efikasnije

primjer Huffman
dan je skup simbola $X = \{A, B, C, D\}$
$p(A) = p(B) = \frac{1}{8}$
$p(C) = 1/4$
$p(D) = 1/2$
kodirajte dani skup simbola binarnim kodom:
a) SF
b) Huffman

a)

D	1/2	0		
C	1/4	1	0	
A	1/8	1	1	0
B	1/8	1	1	1

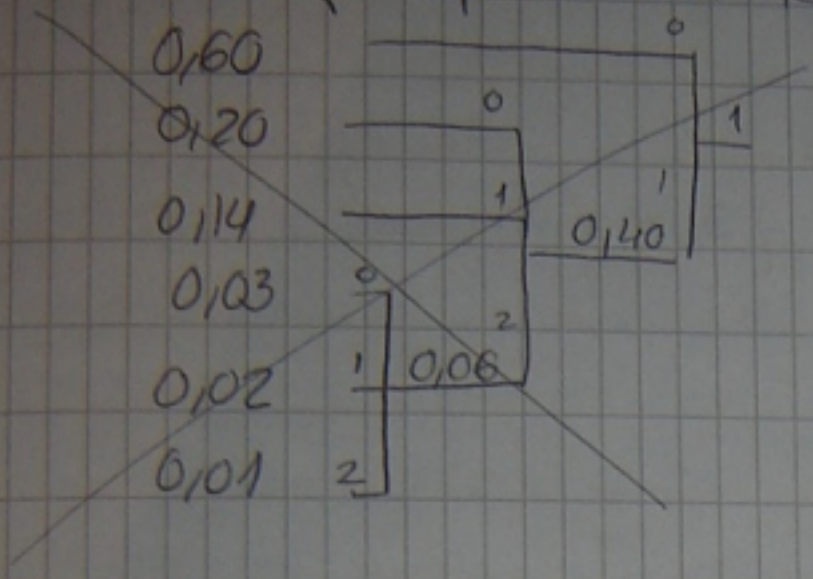
b) → skiciramo i silazno



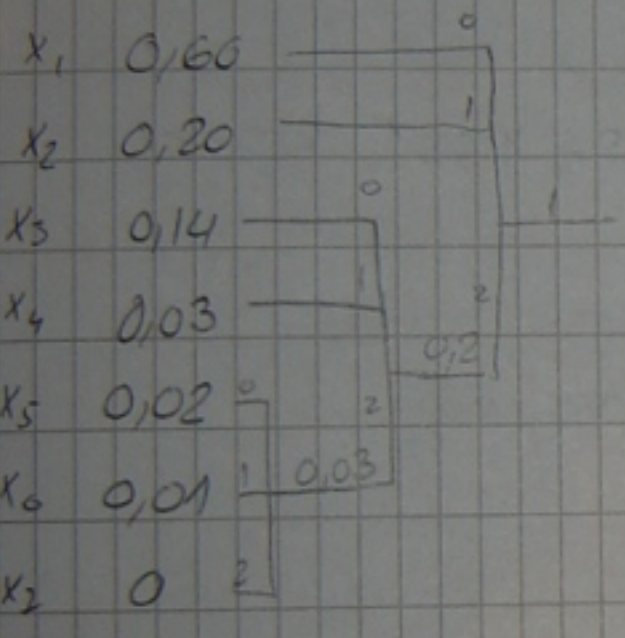
l_i	1	0	slučajno
	2	10	su isti Huffmanov
	3	110	i SF
	3	111	

→ radimo s bocom 2
→ promatramo 2 simbola s najmanjim vjerojatnostima i provodimo ih u jedan novi simbol čija je vjerojatnost pojavljivanja jednaka zbroju

primjer Dan je skup simbola s vjerojatnošću pojavljivanja ... kodirajte optimalnim teksturim kodom



moramo dodati simbol $p = \emptyset$



x_1 0

x_2 1

x_3 20

x_4 21

x_5 220

x_6 221

Za slučaj kodiranja B-ta 2 moram proširiti

N = br simbola

$N = 6$

B = baza

$B = 3$

$k = \left\lceil \frac{N-1}{B-1} \right\rceil = 3$

gornja granica

$N' = (B-1) \cdot k + 1$

$N' = 2 \cdot 3 + 1 = 7$

ako je $N' = N$ tada je kodiranje moguće provesti

imam u skupu simbola

N dodajemo $N' - N$ simbola

SF, H → ovise o $p(x_i)$

primjeri ARITMETIČKO KODIRANJE

Dan je skup simbola $X = \{S, W, I, M, \_\}$

s vjerojatnostima

$P_{\_} = 0,1$

$P_M = 0,1$

$P_I = 0,2$

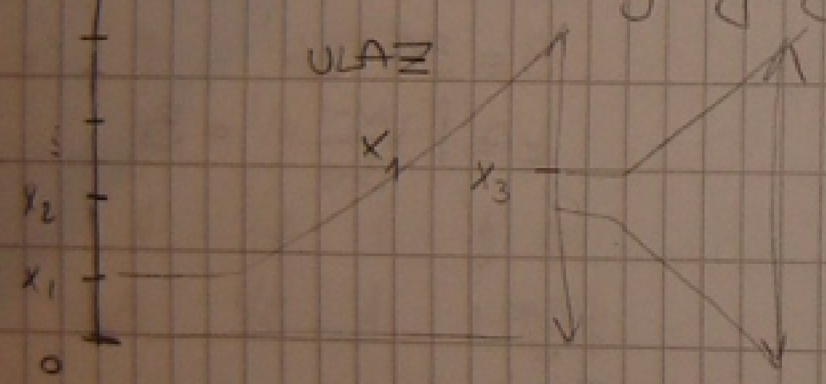
$P_W = 0,1$

$P_S = 0,5$

optimalno kodirajte poruku "SWISS_MISS" koristeći aritmetičko kodiranje

→ $\sum p_i = 1$

kumulativni zbroj vjerojatnosti pojavljivanja



1. formiraj kumulativne skupove (podskupove)
2. složiti ih onim redom kako se pojavljuju u poruci

SIMBOL	f	vjerojatnost pojavljivanja	kum. pod. $[P_S, Q_S)$
S	5	0,5	[0, 0,5)
W	1	0,1	[0,5, 0,6)
I	2	0,2	[0,6, 0,8)
M	1	0,1	[0,8, 0,9)
_	1	0,1	[0,9, 1)

↳ slažemo po pojavi u poruci

KODIRANJE

$D=0$, $G=1$

$$D' = D + (G-D)\cdot DS$$
$$G' = D + (G-D)\cdot GS$$

"S" $D' = 0 + (1-0)\cdot 0 = 0$

$G' = 0 + (1-0)\cdot 0{,}5 = 0{,}5$

$D' \rightarrow D=0$ $G' \rightarrow G=0{,}5$

"W" $D' = 0 + (0{,}5-0)\cdot 0{,}5 = 0{,}25$

$G' = 0 + (0{,}5-0)\cdot 0{,}6 = 0{,}3$

"I" $D' = 0{,}28$

$G' = 0{,}29$

(...)

"S!" $D' = 0{,}282465$

$G' = 0{,}28266625$

rezultat → $[D', G')$

primjer Imamo skup simbola $A = \{1, 2, 3\}$

ARIT. DEK. $p_1 = 0{,}8$

$p_2 = 0{,}02$

$p_3 = 0{,}18$

dekodirajte primljenu poruku

$L = 0{,}772352$

$d = 4$ simbola

1) KUM. PODSKUPOVI

SIMBOL	P(x)	KUM. PODSK
1	0,8	[0;0,8)
2	0,02	[0,8; 0,82)
3	0,18	[0,82; 1)

a) L → [0, 0,8) → "1"

D = 0

G = 0,8

"1" D' = 0

G' = 0,64

"2" D' = 0,64

G' = 0,656

→ "3" D' = 0,656 "3"

G'' = 0,8

D = 0,656

G = 0,8

"1" D' = 0,656

G' = 0,7712

"2" D' = 0,7712 "2"

G' = 0,7741

D = 0,7712

G = 0,7741

"1" D' = 0,7712 "1"

G'' = 0,7735

L = 1321

Metode rječnika

D- Rječnik → Tx
Rx } mogu imati poseb rječnik

LZ77 - primjer
↳ kodiramo poruku

D: A B A A B B A B A A C A

A B A A B B A B A A C A B B

1) pomicni prozor 6 simb
2) prozor za kodiranje } + DULJINA 5 simb

(pomak, duljina, sljedeći simbol)

PP PZK

(0, 0, A)
(0, 0, B)
(2, 1, A)
ABAA (3, 1, B)
ABAABB A C A A C (6, 4, C)

DEK. A B A A B B A A C

BABAAC AACAB (3, 4, B)
(1, 1, kraj)

LZW primjer

D(0) = A
D(1) = B
D(2) = C
D(3) = D

BABAABAAA ⇒ $(104508)_{10} \rightarrow (\quad)_2$

ne postoji
D(4) = BA
D(5) = AB
D(6) = BAA
D(7) = ABA
D(8) = AA

DEKODIRANJE

L = 1 2 2 4 7 3

A B B AB A B AC

7

D(1) = A
D(2) = B
D(3) = C

D(4) = AB
D(5) = BB
D(6) = BA
D(7) = ABA
D(8) = ABC

primjer 0 0 140 250

A A B ABACBAA

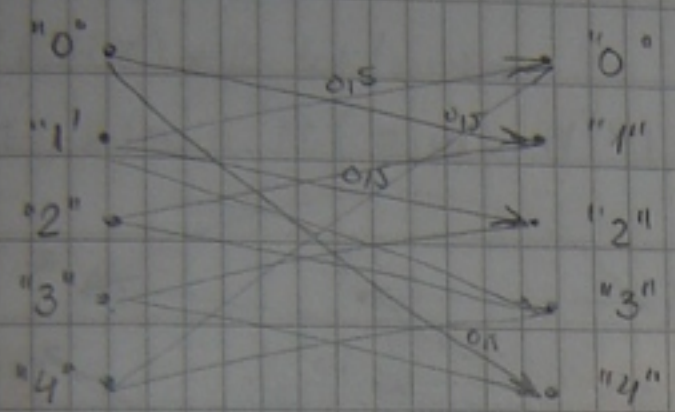
D(0) = A
D(1) = B
D(2) = C
D(3) = AA
D(4) = AB
D(5) = BA

D(6) ABA

Predavanje 4.10.2010.

Zi05 $X = \{0, 1, 2, 3, 4\}$

$$p(y_j|x_i) = \begin{cases} 0{,}5, & y_j = (x \pm 1) \bmod 5 \\ 0, & \text{inače} \end{cases}$$



$p(x_j|x_i) = 0{,}5$

$\max I(X;Y)$

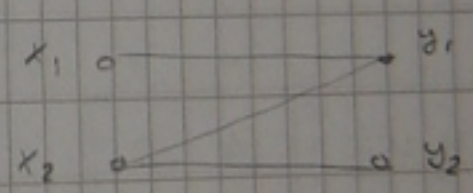
$(\dots)$

$H(Y) = 2{,}3219$ bit/simbol

$H(Y|X) = 1$ bit/simbol

$C_1 = 1{,}3215$ bit/simbol

Zi06 $X = \{x_1, x_2\}$



$$p(y_j|x_i) = \begin{bmatrix} 1 & 0 \\ 0{,}5 & 0{,}5 \end{bmatrix}$$

$C = ?$

$$p(x_i, y_j) = \begin{bmatrix} 1-p & 0 \\ p/2 & p/2 \end{bmatrix}$$

$p(x_1) = 1-p$

$p(x_2) = p$

$p(y_1) = \frac{2-p}{2}$

$p(y_2) = p/2$

$$I(X;Y) = \sum_i \sum_j p(x_i, y_j) \log_2 \frac{p(x_i, y_j)}{p(x_i)\, p(y_j)}$$

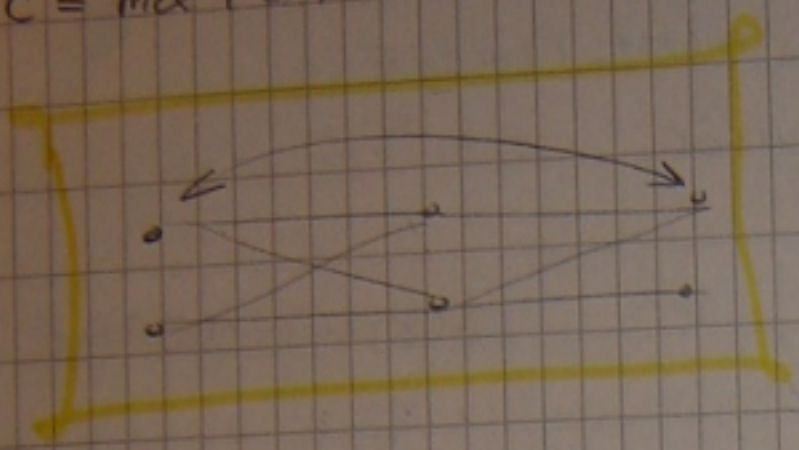
$$= (1-p) \log\left(\frac{2}{2-p}\right) + \frac{p}{2} \log\left(\frac{1}{p(2-p)}\right)$$

$$\frac{dI(X;Y)}{dp} = \frac{1}{\ln 2} \ln\left(\frac{2}{2-p}\right) + \frac{1-p}{\ln 2 (2-p)} + \frac{1}{2 \ln 2} \ln\left(\frac{1}{p/2-p}\right) - \frac{1}{\ln 2} \cdot \frac{1-p}{2-p} = 0$$

$p(x_1) = 0,4$

$p(x_2) = 0,6$

$C = \max I(X;Y) = 0,322$ bit/simbol



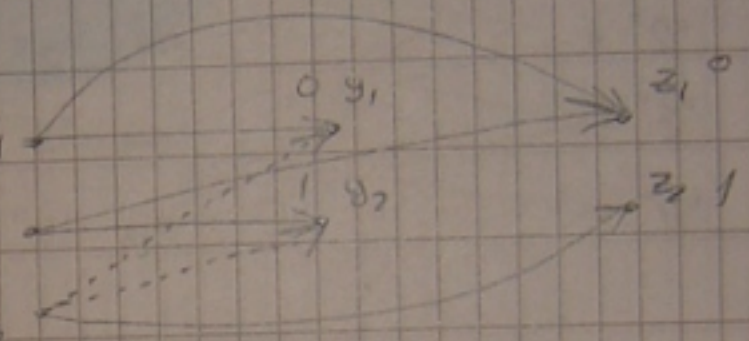
u ispitu !!

Zi 12 $\quad X = \{0, 1, 2\}$

$p(x_1) = 1/4$

$p(x_2) = 1/4$

$p(x_3) = 1/2$



$$H(X) = -\frac{1}{4}\log_2\frac{1}{4} - \frac{1}{4}\log_2\frac{1}{4} - \frac{1}{2}\log_2\frac{1}{2} = 3/2 \text{ bit/simbol}$$

$$p(y_j|x_i) = \begin{matrix} \\ 0 \\ 1 \\ 2 \end{matrix}\begin{matrix} \begin{matrix} 0 & 1 \end{matrix} \\ \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0,5 & 0,5 \end{bmatrix} \end{matrix}$$

$$p(z_k|x_i) = \begin{matrix} \\ 0 \\ 1 \\ 2 \end{matrix}\begin{matrix} \begin{matrix} 0 \end{matrix} \\ \begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \end{matrix}$$

$$[p(x_i, y_j)] = [p(x_i)][p(y_j|x_i)]$$

$$[p(x_i, y_j)] = \begin{bmatrix} 1/4 & 0 \\ 0 & 1/4 \\ 1/4 & 1/4 \end{bmatrix}$$

$$[p(x_i, z_k)] = \begin{bmatrix} \frac{1}{4} & 0 \\ 1/4 & 0 \\ 0 & 1/2 \end{bmatrix}$$

$[p(y_j)] = [1/2 \quad 1/2]$ bit/simbol

$[p(z_k)] = [1/2 \quad 1/2]$

$H(Y) = 1$ bit/simbol

$H(X,Y) = 2$ bit/simbol

$H(Y|X) = H(X,Y) - H(X) = 2 - 3/2 = 0,5$ bit/simbol

$H(X|Y) = H(X,Y) - H(Y) = 2 - 1 = 1$ bit/simbol

$H(Z) = 1$ bit/simbol

$H(X|Z) = H(X,Z) - H(Z) = 3/2 - 1 = 0{,}5$ bit/simbol

$$[p(z_k|y_j)] = \begin{bmatrix} 0{,}5 & 0{,}5 \\ 0{,}5 & 0{,}5 \end{bmatrix} \qquad p(y_j, z_k) = \begin{bmatrix} 1/4 & 1/4 \\ 1/4 & 1/4 \end{bmatrix}$$

$H(Y|Z) = 2$ bit/simbol

$H(Z|Y) = H(Z,Y) - H(Y) = 2 - 1 = 1$ bit/simbol

Zi13

simbol	p(x)	q(x)
A	0,5	1/3
B	0,25	1/3
C	0,25	1/3

a) $H(p) = -\sum_{i=1}^{3} p(x_i) \log_2 p(x_i) = 1{,}5$ bit/simbol

$H(q) = 1{,}585$ bit/simbol

$D(p\|q) = \sum_i p_i \log \frac{p_i}{q_i} = 0{,}0849$ bit/simbol

$D(q\|p) = 0{,}08365$ bit/simbol

b) $D(p\|q) = D(q\|p)$

$p \log \frac{p}{q} + (1-p) \log \frac{1-p}{1-q} = q \log \frac{q}{p} + (1-q) \log \frac{1-q}{1-p}$

$x = \{0, 1\}$

$\log \frac{p}{q} \cdot (p+q) + \log \frac{1-p}{1-q} (2-p-q) = 0$

p	q
p "1"	"1" q
1-p "0"	"0" 1-q

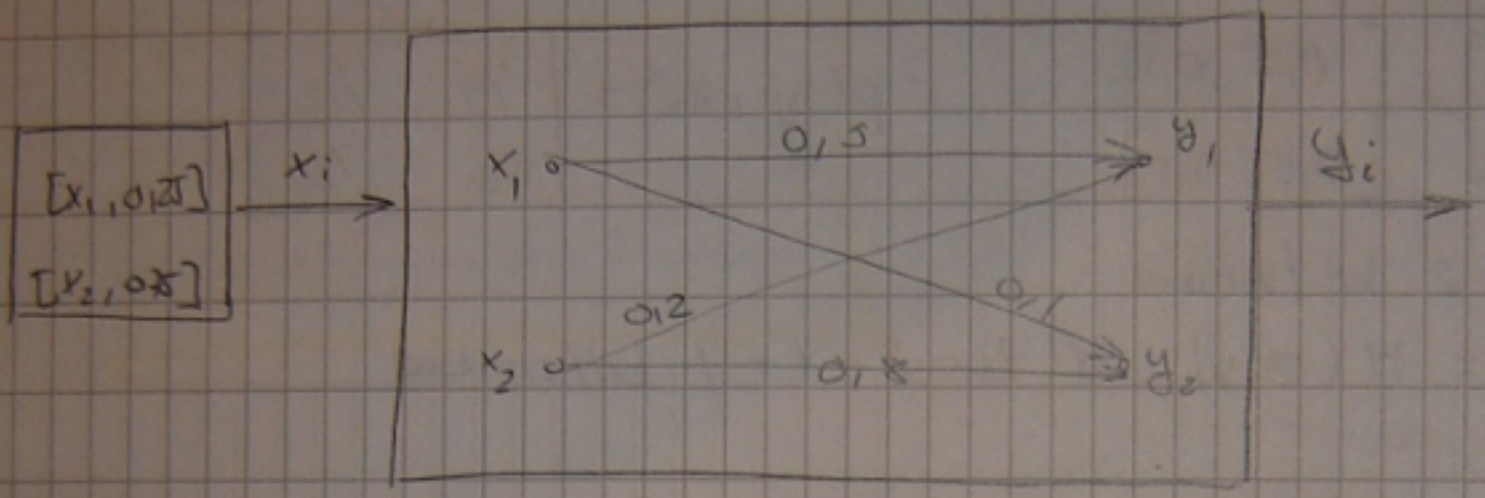
$p = 1 - q$

Zi 20

$X = \{x_1, x_2\}$

$p(x_1) = 0,25$

$p(x_2) = 0,75$



$$p(y_j|x_i) = \begin{bmatrix} 0,3 & 0,1 \\ 0,2 & 0,8 \end{bmatrix}$$

a) vjerojatnost pojave pogrešnog simbola

$p = p(x_1)\,p(y_2|x_1) + p(x_2)\,p(y_1|x_2) = 0,175$

b) matrica združenih vjerojatnosti

$$[p(x_i, y_j)] = \begin{bmatrix} 0,225 & 0,025 \\ 0,15 & 0,6 \end{bmatrix}$$

$$[p(y_j)] = [0,375 \quad 0,625]$$

c) iznos korisne informacije koja se prijenosi po kanalu

$I(X;Y) = H(Y) - H(Y|X) = 0,2058$ bit/simbol

$H(Y) = 0,9544$ bit/simbol

$H(Y|X) = 0,6586$ bit/simbol

zi 21 promatramo vezu između tipkovnice i računala kao diskretni bezmemorijski kom. kanal
Na zaslonu želimo ispisati brojeve 0-9

a) $[p(x_i)] = \left[\frac{1}{10} \quad \frac{1}{10} \quad \frac{1}{10} \quad \cdots \quad , \quad \frac{1}{10}\right]$

$$p(y_j|x_i) = \begin{bmatrix} 1 & 0 & 0 & \cdots \\ 0 & 1 & 0 & \\ 0 & 0 & 1 & \\ 0 & 0 & 0 & \\ \vdots & & & \ddots \end{bmatrix}$$

$$p(x_i, y_j) = \begin{bmatrix} 1/10 & 0 & 0 & \cdots \\ 0 & 1/10 & 0 & \\ 0 & 0 & 1/10 & \\ \vdots & & & \ddots \end{bmatrix}$$

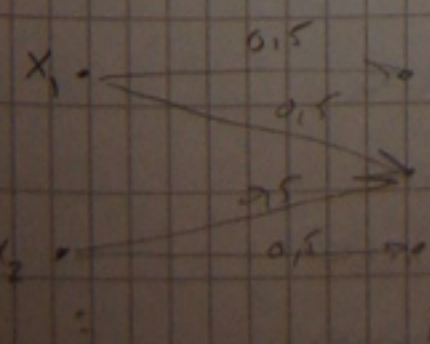
$H(Y) = \log_2 10$

$H(X,Y) = \log_2 10$

nema entropije šuma $H(Y|X) = H(X,Y) - H(Y) = 0$

$C = \max I(X;Y) = \log_2 10 = 3{,}32$ bit/simbol

b) može poći u sljedeća



$H(Y|X) = 1$

$C = \max I(X;Y) = \log_2 10 = 1$

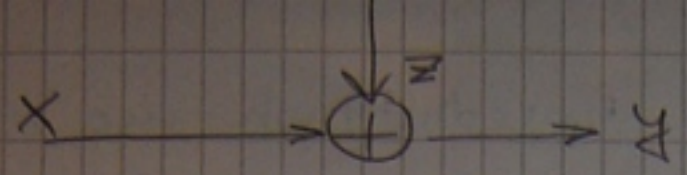
$p(y_j) = [0{,}1 \quad \cdots \quad 0{,}1]$

$$p(y_j|x_i) = \begin{bmatrix} 0{,}5 & 0{,}5 & 0 & 0 & \cdots \\ 0 & 0{,}5 & 0{,}5 & 0 & \cdots \\ \vdots & & & & \ddots \\ 0{,}5 & & & & 0{,}5 \end{bmatrix}$$

$p(x_i, y_j) = [p(x_i)][p(y_j|x_i)]$

$H(Y) = \log_2 10$ $\quad H(X,Y) = \log_2 20$

Ex 22



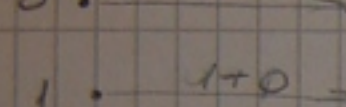
$p(z=0) = p(z=a) = 0,5 \quad a \in R$

$X = \{0, 1\}$

$p(0) + p(1) = 1$

$z = 0, 1, -1, a$

a) $z_1 = 0$

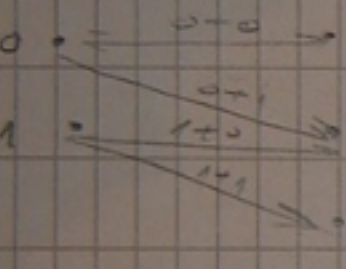


specifični šumovi

$\Rightarrow$ idealan kanal $\quad C = 1$ bit/simbol

$$p(x_i, y_j) = \begin{bmatrix} 0,5 & 0 \\ 0 & 0,5 \end{bmatrix}$$

b) $z_2 = 1$

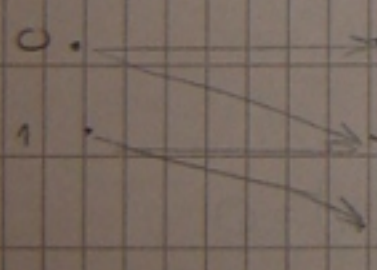


$$p(x_i, y_j) = \begin{bmatrix} 0,25 & 0,25 & 0 \\ 0 & 0,25 & 0,25 \end{bmatrix}$$

$H(x_i, y_j) = 2$ bit/simbol $\quad H(x) = 1$ bit/simbol

$C = I(x; y) = 0,5$ bit/simbol

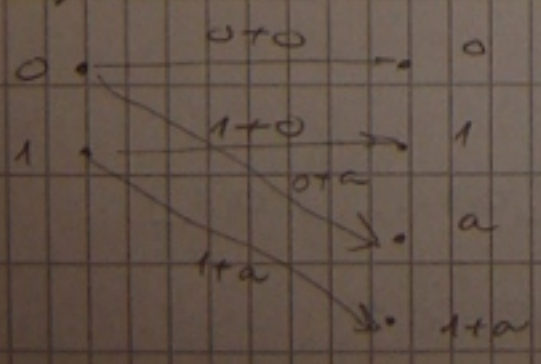
c) $z_3 = -1$



$$p(x_i, y_j) = \begin{bmatrix} 0,25 & 0,25 & 0 \\ 0 & 0,25 & 0,25 \end{bmatrix}$$

$C = 0,5$ bit/simbol

d) $z_4 = a$



$$p(x_i, y_j) = \begin{bmatrix} 0,25 & 0 & 0,25 & 0,25 \\ 0 & 0,25 & 0 & 0,25 \end{bmatrix}$$

$H(X,Y) = 2$ bit/simbol

$H(X) = 1$ bit/simbol

$H(Y) = 2$ bit/simbol

$C = I = 1$ bit/simbol

zi01

x_i	1	2	3	4	5
f_i	1000	1500	900	2000	1100

$$p_i = \frac{f_i}{\sum f_i} \rightarrow$$

$p_1 = 0{,}154$

$p_2 = 0{,}23$

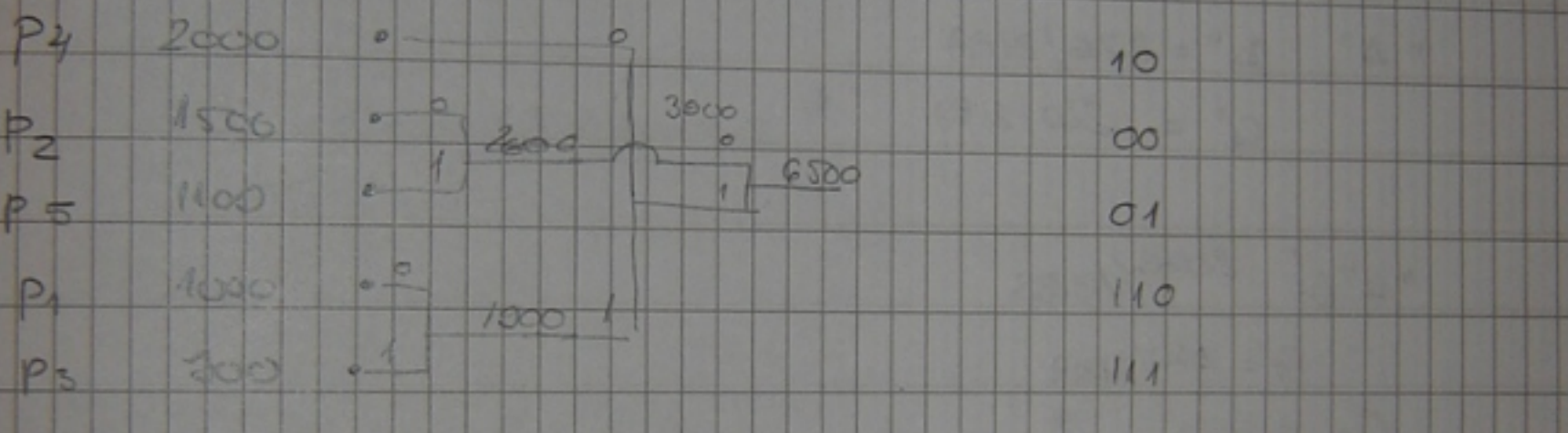
$p_3 = 0{,}138$

$p_4 = 0{,}308$

$p_5 = 0{,}169$

$\Delta M = M_{RAV} - M_{OPT}$

/6500



$M_{opt} = c_1 \cdot f_1 + c_2 \cdot f_2 + \dots + c_5 \cdot f_5 = 14900$ bit

$M_{samomjerno} = 3 \cdot \sum f_i = 3 \cdot 6500 = 19500$ bit

$\Delta M = 19500 - 14900 = 4600$ bit

zi02 R I B A _ R I B I

aritmetičkim kodom

SIMBOL	p	$[D_s, G_s)$
R	2/9	$[0, 2/9)$
I	3/9	$[2/9, 5/9)$
B	2/9	$[5/9, 7/9]$
A	1/9	$[7/9, 8/9)$
_	1/9	$[8/9, 1)$

"R" $A = 0 \quad G = 1$

$A' = 0$

$G' = 0 \cdot (1-0) - \frac{2}{9} = \frac{2}{9}$

"I" $A' = \frac{4}{81}$

$G' = 14/81$

"B" $A' = 22/243$

$G' = 26/245$

"A" $A'' = 226/2187$

$G' = 230/2187$

"-" $D = 2066/10883$

$G' = 230/2187$

(:)

$[0,1049828 \; ; \; 0,1049839]$

DZ Mirnu dig. sliku s vjerojatnostima pojavljivanja nijansama sivoga dane su vjerojatnosti

$p = \{ 1/16, 3/16, 1/16, 4/16, 1/16, 2/16, 2/16, 1/16, 1/32, 1/32$

rezolucije 1024 x 768 potrebno je optimalno kodirati Huffmanom s bazom 8.
Izračunajte minimalno vrijeme potrebno za prijenos slike od računala A do B, modemom 384 kbit/s

zi07 LZ77

aacaacabcabaaac*

PP 4

PZK 6

a a c a a c a b c a b a a a c *

(0, 0, a)
(1, 1, c)
(3, 4, b)
(3, 3, a)
(1, 2, c)
(0, 0, *)

dekodirati 0 1 1 0 2 4 6

zi09 D[0] = a
D[1] = b
D[2] = ab
D[3] = bb
D[4] = ba
D[5] = aa
D[6] = abb

abbaabbaabb