$\log_2 x \Rightarrow \log x$

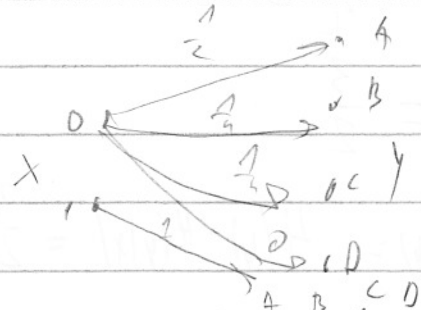
Zadatak

$C = ?$ $\qquad C = \max I(X;Y)$

1) $[P(Y|X)] \wedge [P(X|Y)]$

2) sistem

3) kontekst



$$[P(Y|X)] = \begin{array}{c|cccc} & A & B & C & D \\ \hline 0 & \frac{1}{2} & \frac{1}{4} & \frac{1}{4} & 0 \\ 1 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{2} \end{array}$$

$P(X=0) = p$

$P(X=1) = 1-p$

$[p(X)] = p, (1-p)$

$I(X;Y) = H(Y) - H(Y|X)$

$[p(Y)] = ?$ $\leftarrow p(y_j|x_i)$, $p(x_i)$

$P(x_i, y_j) = p(x_i)\, p(y_j|x_i)$

$$P(X;Y) = \begin{bmatrix} \frac{p}{2} & \frac{p}{4} & \frac{p}{4} & 0 \\ 0 & \frac{1-p}{4} & \frac{1-p}{4} & \frac{1-p}{2} \end{bmatrix}$$

$$P(Y) = \begin{bmatrix} \frac{p}{2} & \frac{1}{4} & \frac{1}{4} & \frac{1-p}{2} \end{bmatrix}$$

$H(Y) = \sum_j p(y_j) \log_2 p(y_j)$

$= -\left\{ \frac{p}{2} \log_2 \frac{p}{2} + \frac{1}{4} \log_2 \frac{1}{4} \cdot 2 + \frac{1-p}{2} \log_2 \frac{1-p}{2} \right\} =$

$= \frac{3}{2} + \frac{1}{2} H(p)$

$H(p) = H(p, 1-p) = -p \log_2 p - (1-p) \log (1-p)$

$$H(y|x) = -\sum_i \sum_j p(x_i, y_j) \log_2 p(y_j|x_i)$$

$$H(y|x) = -\left\{ \frac{p}{2}\log_2\frac{1}{2} + \frac{p}{4}\log_2\frac{1}{4}\cdot 2 + 2\cdot\frac{1-p}{4}\log_2\frac{1}{4} + \frac{1-p}{2}\log_2\frac{1}{2} \right\}$$

$$= \frac{3}{2}$$

$$I(x,y) = H(y) - H(y|x) = \frac{3}{2} + \frac{1}{2}H(p) - \frac{3}{2} = \frac{1}{2}H(p)$$

$$C = \max \frac{1}{2}H(p) = \frac{1}{2}\max_{p(x)} H(p) = \frac{1}{2}\log_2 2 = \frac{1}{2} \text{ bit/simbol}$$

~~NOVI ZADATAK~~

NEMA U ZBIRCI

SIMETRIČNI

DJELOMIČNO SIMETR. — $[p(y|x)] = \begin{bmatrix} \frac{1}{4} & \frac{1}{3} & \frac{1}{4} & \frac{1}{6} \\ \frac{1}{4} & \frac{1}{6} & \frac{1}{4} & \frac{1}{3} \end{bmatrix}$ r

s

$\sum = \sum = \sum = \sum$

pretpostavka max Cap. $\Longrightarrow$ $C = p(x_i) = 1/M$

$C = \log_2 [\text{card } H(y) - H(r)]$

II način

$$C = \log_2(4) - H\left(\frac{1}{2}, \frac{1}{4}, \frac{1}{4}, 0\right)$$

$$= 0{,}5 \ \text{bit/simbol}$$

Zadatak

Diskretno bezmemorijsko izvorište X generira simbole $X = \{0, 1, 2, 3, 4\}$ s vjerojatnostima

$$P_X(x) = \begin{cases} 1/16, & x=0 \text{ ili } x=4 \\ 1/4, & x=1 \text{ ili } x=3 \\ 3/8, & x=2 \end{cases}$$

1) $H(X) = ?$

2) neka su zadane f-je $g_1(x) = x^2$, $g_2(x) = (x-2)^2$

odredite $H(g_1(x)) = ?$

$H(g_2(x)) = ?$

1)

$$H(X) = -\left(\frac{1}{16}\log_2\frac{1}{16}\cdot 2 + \frac{1}{4}\log_2\frac{1}{4}\cdot 2 + \frac{3}{8}\log_2\frac{3}{8}\right) = \quad \text{bit/simb}$$

$$X \sim \begin{pmatrix} 0 & 1 & 2 & 3 & 4 \\ 1/16 & 1/4 & \frac{3}{8} & 1/4 & 1/16 \end{pmatrix}$$

$$g(X) \sim \begin{pmatrix} 0 & 1 & 4 & 9 & 16 \\ 1/16 & 1/4 & \frac{3}{8} & 1/4 & 1/16 \end{pmatrix}$$

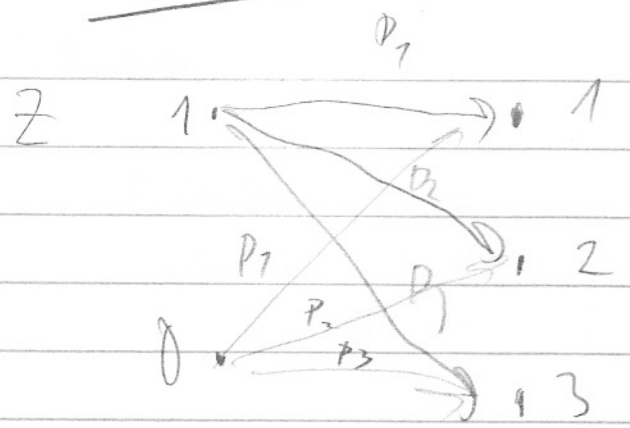
$H(g(X)) = H(X)$

$$g_2(X) \sim \begin{pmatrix} 1 & 4 & 9 \\ \cancel{1/4} + \frac{3}{8} & \frac{1}{16} + \frac{1}{4} & \frac{1}{4} \end{pmatrix}$$

$0 \to 4$
$1 \to 1$
$2 \to 1$
$3 \to 4$
$4 \to 9$

$H(g_2(X)) = H(X) = \frac{5}{8}$

Zadatak 3



$$\sum p_i = 1$$

$$p_i \neq 0, \quad i = 1,2,3$$

$$C = \max I(X,Y)$$

$$[p(x)] = [u \quad (1-u)] \qquad [p(y)] = [p_1 \quad p_2 \quad p_3]$$

$$[p(y|x)] = \begin{bmatrix} p_1 & p_2 & p_3 \\ p_1 & p_2 & p_3 \end{bmatrix}$$

$$[p(x,y)] = \begin{bmatrix} up_1 & up_2 & up_3 \\ (1-u)p_1 & (1-u)p_2 & (1-u)p_3 \end{bmatrix}$$

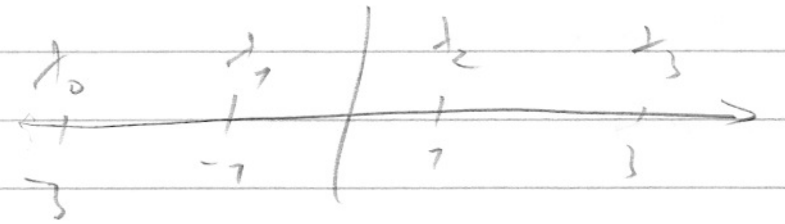
$$H(Y) = -[p_1 \log p_1 + p_2 \log p_2 + p_3 \log p_3]$$

$$H(Y|X) = -[up_1 \log p_1 + up_2 \log p_2 + up_3 \log p_3 + (1-u)p_1 \log p_1$$
$$+ (1-u)p_2 \log p_2 + (1-u)p_3 \log p_3]$$

$= + (1 + (1-1)) H(y) = H(y)$

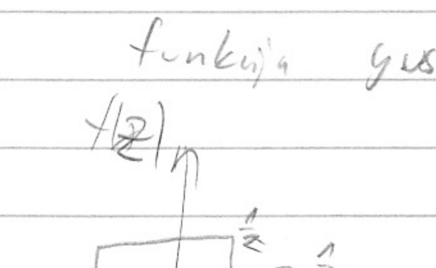
$I(x;y) = H(y) - H(y) = \emptyset$

U prijenos informacije koristi se prijenosni sustav s 4 razine amplituda (-3, -1, 1, 3) kojima su pridruženi simboli x_0 x_1 x_2 x_3



U prijenosu se pojavljuje šum z koji se zbraja na otaslemi x, tj. prijem $y = x + z$

funkcija gustoće razdiobe šuma na slici



Na prijamnoj strani odluka o primljenom simbolu provodi se na način da se gleda vrijednost primljenog signala ako je $y \leq -2 \rightarrow x_0$

$y \in \langle -2, 0]$ $y > -2$; $y \leq 0 \rightarrow x_1$

$y \in \langle 0, 2] \rightarrow x_2$

$y > 2 \rightarrow x_3$

na prednjoj strani svi simboli
imaju jednaku vjerojatnost

$H(x)$
$H(y)$

$H(y|x)$
$* I(x, y)$

$[p(x)] = \left(\frac{1}{4} \quad \frac{1}{4} \quad \frac{1}{4} \quad \frac{1}{4} \right)$

$\cancel{p(z) = \left(-1{,}5 \quad -0{,}5 \quad 0{,}5 \quad 1{,}0 \right)}$

$p(z) \left(\begin{matrix} -1{,}5-0{,}5 & -0{,}5 \ 0{,}5 & 0{,}5-1{,}5 \\ \frac{1}{4} & \frac{1}{2} & \frac{1}{4} \end{matrix} \right)$

$\cancel{p(x) =} \left(\right.$

$y = x + z$

Zadatak

Promatrajmo vezu između tipkovnice i računala kao bezmemorijski kanal.

Na zaslonu računala želimo ispisati br $0 - 9$

Odredite C ako

1) pritisak $0 - 9 \rightarrow 0 - 9$

2) $0 - 9$ rezultira pojavom sa jed. vjer. pripadajućeg broja ili njemu susjednog br

$0 \rightarrow 0$, $0 \rightarrow 1$ $\qquad$ $1 \rightarrow 1$, $1 \rightarrow 2$ $\qquad$ $9 \rightarrow 9$, $9 \rightarrow 0$

1)

$0 \rightarrow 0$

$1 \rightarrow 1$

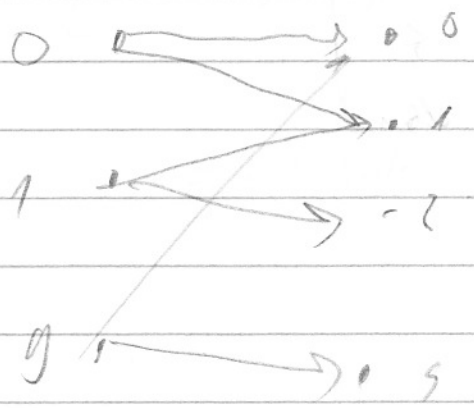
$\vdots$

$9 \rightarrow 9$

$p(x_i) = \frac{1}{10}$ $\quad$ i $0..9$

$C = \log_2 10$ bit/simb

b)



$p(x_i) = \frac{1}{10}$

$$P(Y|X) = \begin{array}{c|cccccc} & 0 & 1 & 2 & \dots & & 9 \\ \hline 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & \dots & \\ 1 & 0 & \frac{1}{2} & \frac{1}{2} & \dots & & \\ \vdots & & & & & & \\ 9 & \frac{1}{2} & \dots & & & & \frac{1}{2} \end{array}$$

potpuno simetričan kanal

$C = \log_2(\text{kard. } Y) - H(\frac{1}{2}, \frac{1}{2})$

$= \log_2 10 - 1$ bit/simb

Zadatak

Otrebite dengsom D Ø C

$$[p(y/x)] = \begin{array}{c|c} & \begin{array}{ccc} D & Ø & C \end{array} \\ \begin{array}{c} A \\ B \\ C \end{array} & \begin{bmatrix} 2/3 & 1/3 & Ø \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ 0 & 1/3 & 2/3 \end{bmatrix} \end{array}$$

$$C = \max_{\{P(x_i)\}} I(X:Y)$$

$$I(X:Y) = H(Y) - H(Y/X)$$

$$[p(x)] = [\,p(A)\ \ p(B)\ \ p(C)\,]$$

$$[p(x,y)] = \begin{pmatrix} \frac{2}{3}p_A & \frac{1}{3}p_A & 0 \\ \frac{1}{3}p_B & \frac{1}{3}p_B & \frac{1}{3}p_B \\ 0 & \frac{1}{3}p_C & \frac{2}{3}p_C \end{pmatrix}$$

$$[P(y)] = \left[\frac{2}{3}p_A + \frac{1}{3}p_B,\ \frac{1}{3},\ \frac{1}{3}p_B + \frac{2}{3}p_C\right] \to H(y)$$

$$H(Y/X) = \dots = \log_2 3 - \frac{2}{3}(p_A + p_C)$$

$$C = \max_{p(x_i)} I(x;y) = \max_{\{p(x_i)\}} \left(H(y) + \frac{2}{3}\underbrace{(p_A + p_C)}_{\to 1} - \log_2 3 \right)$$

$$\max \Rightarrow \frac{2}{3}p_A + \frac{1}{3}p_B = \frac{1}{3}$$

$$\frac{1}{3}p_B + \frac{2}{3}p_C = \frac{1}{3}$$

$$\dots\ p_C = p_A \Rightarrow p_A + p_C = 1$$

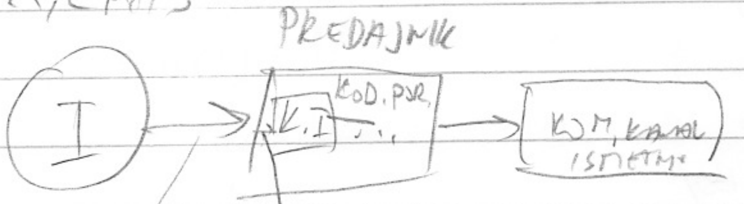
$$p_B = 0$$

$$[P(Y)] = \left[\frac{1}{3} \quad \frac{1}{3} \quad \frac{1}{3} \right]$$

$C = \frac{2}{3}$ bit/simbol

ENTROPIJSKO KODIRANJE

$X, [p(x)]$



$x_1 x_2 x_1 \ldots$ povorka

$x_i \to C(x_i)$

metode bez gubitaka

s gubitkom

1) SHANNON-FANO
2) HUFFMAN
3) ARITMETIČKO KODIRANJE

} $p[p(x)]$

METODE RJEČNIKA

4) LZ77
5) LZW

} $\cancel{p(x)}$

PODJELA KODOVA

1. NESINGULARNI kodovi $\forall x_i \to C(x_i)$ ako $x_p \neq x_j \to C(x_i) \neq C(x_j)$

Primjer

$A \to C(A) = 0$

$B \to C(B) = 01$

$C \to C(C) = 1$

$ABC \to II \to 0\ 011 \to$ AACC / ABC

2. JEDNOZNAČNO DEKODABILNI KODOVI

$X, C(X)$

$X^+, C(X)^+; \; C^+ : X^+ \to C(X)^+$

$C^+(x_1 x_2) \to C(x_1) C(x_2)$

$\forall x, x' \in C^+, \; x_i \neq x_j$

$C(x_i)^+ \neq C(x_j)^+$

Primjer

$C(A) = 0$

$C(B) = 01$

$C(C) = 011$

$ABC \to 001011 \to ABC$

- moramo primit cijelu poruku

SARDINAS-PATERSONOV ALGORITAM

3. PREFIXNI KODOVI

niti jedna kraća riječ nije prefix dužeg

INF. MJERE

a) $L, L(X)$ - srednja duljina kodne riječi

$X, [P(x_i)] = [p_1, p_2 \dots p_n]$

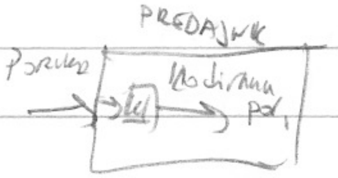
$x_i \rightarrow c(x_i) \rightarrow l_i$

$$L = L(X) = \sum_{i}^{n} p_i \cdot l_i \quad \frac{bit}{simbol}$$

b) EFIKASNOST KODA

$\varepsilon = \frac{H(X)}{L(X)}; \quad H(X) \leq L(X)$

ENTROPIJSKO KODIRANJE



Kodovi

a) nesingularni

b) jednoznačno dekodabilni — Sardinas-Pattersonov test

c) prefiksni

$$L(x) = L = \sum_{i=1}^{n} P_i l_i \text{ bit/simbol} \qquad d=2: \{0,1\}$$

$$\varepsilon = \frac{H(x)}{L(x)} \leq 1$$

Koji je od navedenih kodova jednoznačno dekodabilan

Primer

$k_1 = \{11, 10, 01, 001, 0001, 0000\}$

$k_2 = \{01, 10, 11, 0, 111, 0000, 0001\}$

$k_3 = \{01, 10, 001, 100, 000, 111\}$

$S_0 = K$

$S_0 = K$

$\vdots$

$S_i = \{ \ldots \}$

$\downarrow$

$S_{i+1} = \{ \quad \}$

$c(y)$ ako i samo ako $\exists\, c(x) \in S_0,\ c(x)c(y) \in S_i$

$\exists\, c(z) \in S_i,\ c(z)c(y) \in S_0$

NDK: $S_{i+1} = \{\}$ $i \geq 1$

ili

$S_{i+1} = S_i = \ldots \notin S_0$

niti jedan nije iz S_0

K_1: $S_0 = \{11, 10, 01, 001, 0001, 0000\} = K$

$i = \emptyset$

$S_1 = \{\} \rightarrow K_1$ JDK

$K_3 = \{01, 10, 001, 100, 000, 111\} = S_0$

$i = 0$

$S_1 = \{\emptyset\}$

$i = 1$

$S_2 = \{1, \underline{01}, 00\}$ ~~JDK~~

---

primjer

$K, x = \{a, b, c, d\}$

x_i	$p(x_i)$	$K(x_i)$	$l_i(x_i)$
a	0,5	0	1
b	0,25	01	2
c	0,125	011	3
d	0,125	111	3

1) $H(x) = ?$

2) PREFIKNI ?

3) $L(x) = ? \rightarrow H(x)$.

4) JDK?

$H(X) = \frac{7}{4}$ bit/simbol

optimalan

$\left\{ \begin{matrix} 0 \\ 01 \\ 011 \end{matrix} \right.$ => nije prefiksni

$L(X) = L = \sum_{i=1}^{4} p_i l_i = \left(\frac{1}{2} \cdot 1 + \frac{1}{4} \cdot 2 + \frac{1}{8} \cdot 3 \cdot 2\right) = 7/4$ bit/simbol

$S_0 = \{0, 01, 011, 111\} = C$

$i=0$

$S_1 \{1, 11\}$

$i=1$

$S_2 = \{1, 11\}$

$i=2$

$S_3 = S_2 = S_1$ => JDK

SHANNON-FANO (prefiksni kod)

komunikacija između računala i terminala odvija se preko 8 poruka, vjerojatnost

			l_i
x_7	1/2	0	1
x_6	1/4	10	2
x_5	1/16	1100	4
x_4	1/16	1101	4
x_3	1/32	11100	5
x_2	1/32	11101	5
x_1	1/32	11110	5
x_0	1/32	11111	

Kodirajte navedeni skup poruka S-F metodom kodiranja

1) SORT. ↓ vjerojatnosti x_i

2) podijelite na 2 podskupa tako da je razlika vj. minimalna

3) pridruživanje simbola $\{0,1\}$ $\Rightarrow x_i^0$
$d=2$

NUŽAN I DOVOLJAN UVJET POSTOJANJA PREFIKSNOG KODA

$$\boxed{\sum_{i=1}^{n} d^{-l_i} \leq 1}$$

KRAFTOVA NEJEDNAKOST

$l_i = \{1, 3, 4, 2, 1, 4\}$
$d=2$
prefiksni

$$\sum = 2^{-1} + 2^{-3} + 2^{-4} + 2^{-2} + 2^{-1} + 2^{-4}$$
$$= 1{,}03.. > 1$$
$\Rightarrow$ nije prefiksni kod

NUŽAN I DOVOLJAN UVJET OPTIMALNOSTI KODA

$$H(X) \leq L(X) \leq L(X)+1$$

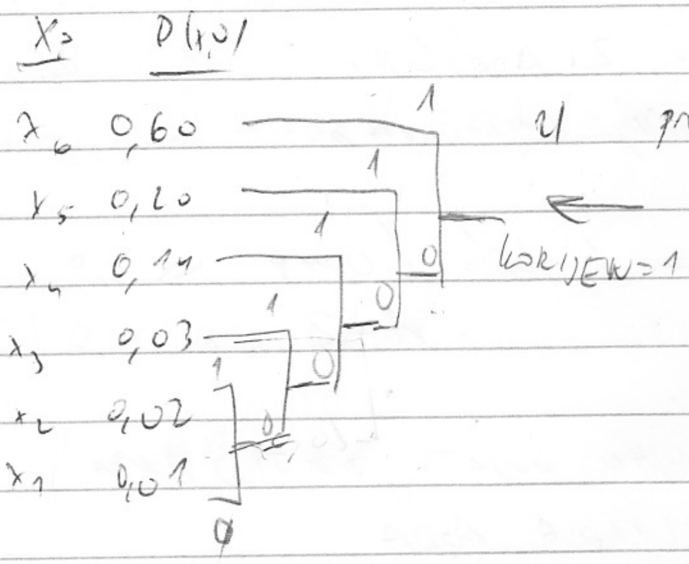
$\min(L(X))$, uz uvjet $\sum_i d^{-l_i} \leq 1$

STROGO OPTIMALAN

$$L(X) = H(X)$$

HUFFMANovo KODIRANJE

primjer $X, [p(i)] = [0{,}6,\ 0{,}2,\ 0{,}14,\ 0{,}03,\ 0{,}02,\ 0{,}01]$ $d=2$



1) SORTIRANJE

2) pronaći 2 simbola s najmanjim vjerojatnostima i udružiš u novi simbol $V = V_1 + V_2$

3) ponovi $\Sigma = 1$

0,60	0,6	0,6	0,6	1
0,20	0,2	0,2	0,4	
0,14	0,14	0,2		
0,03	0,06			
0,03 → 0,06				

	$C(x_i)$	$l(x_i)$
x_6	1	1
x_5	01	2
x_4	001	3
x_3	0001	4
x_2	00001	5
x_1	00000	5

Primjer X, $d=3$ $\{0,1,2\}$

Kodiranje baza različita od 2
=> provjeriti

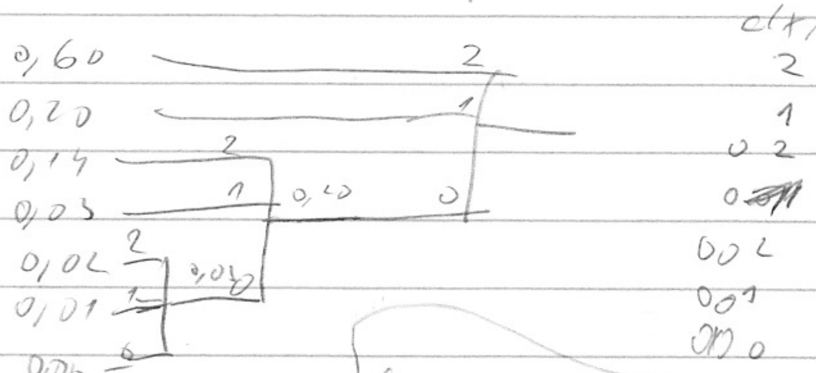
$N \rightarrow$ broj simbola $(N=6)$
$d \rightarrow$ baza $(d=3)$

$k = \left\lceil \frac{N-1}{d-1} \right\rceil$ $\quad k=3$

$N' = (d-1)\cdot k + 1$ $\quad N' = 7$

Ako je $N' = N$ tada je kodiranje moguće provesti u bazi d

Ako je $N' \neq N$ tada u sustav simbola X treba dodati $N'-N$ simbola s vjerojatnosti pojavljivanja 0



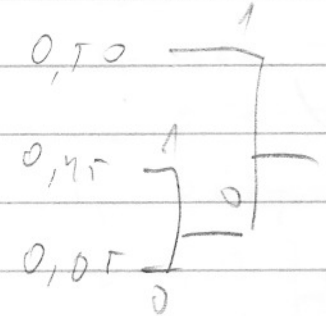
$L(X) = \sum_{i=1}^{6} l_i \cdot p_i = 1\cdot 0{,}6 + 1\cdot 0{,}2 + \dots = 1{,}23$ $\frac{\text{ternarnih simbola}}{\text{simbolu}}$

$\varepsilon(X) = \frac{H(X)}{L(X)}$ $\quad$ ~~$H(X) = \sum p_i \log_2 p_i$~~ $\quad H(X) = \sum p_i \log_3 p_i$

Primjer

$X = \{A_0, A_1, A_2\}$ $\quad d = 2$

$[p(x)] = [0{,}50,\ 0{,}45,\ 0{,}05]$



$\varepsilon = \frac{H(X)}{L(X)} = \frac{1{,}2345}{1{,}5}$ bit/simbol $= 0{,}823$ (82,3%)

Je li moguće poboljšati?

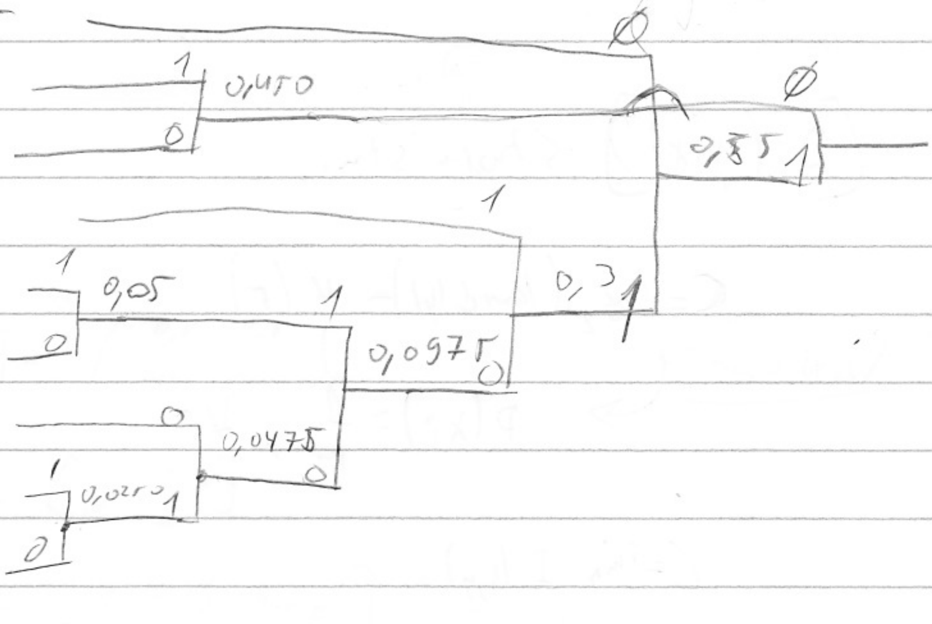
$L(X) \to \frac{bit}{simbol}$

Ako biramo sve parove simbola efikasnost će se popraviti

A_0A_0	0,25
A_0A_1	0,225
A_0A_2	0,025
A_1A_0	0
A_1A_1	0,
A_1A_2	0,
A_2A_0	0,
A_2A_1	0,
A_2A_2	0,

Veća vjerovatnost 1

A_0A_0 = 0,25
A_0A_1 0,225
A_1A_0 0,225
A_1A_1 0,2025
A_0A_2 0,025
A_2A_0 0,025
A_1A_2 0,0225
A_2A_1 0,0225
A_2A_2 0,0025



$H(X) = 2,469$ bit/par simbola

$L(X) = 2,52$

$\mathcal{E} = 0,98$

$\boxed{\mathcal{E}_{SF} \leq \mathcal{E}_{HUFF}}$

Kap. stałego sim. kanału

$\boxed{p(y|x)}$ stałego sim.

$C = \log_2(\text{kard}(y)) - H(r)$

$\hookrightarrow \quad p(x_i) = \frac{1}{m}, \ \forall i$

$C = \max I(x, y)$

$\forall p(x_i)$

$C = \max \begin{cases} C_1 & C_2 \\ \uparrow & \uparrow \\ p(x)^1 & p(x)^2 \end{cases}$

P7

zad 1

$$[p(y/x)] = \begin{bmatrix} 0,5 & 0,5 \\ 0 & 1 \end{bmatrix}$$

$C = \max I(X;Y) = ?$

$$[p(x)] = [p \quad 1-p]$$

$$[p(x,y)] = \begin{bmatrix} 0,5p & 0,5p \\ 0 & 1-p \end{bmatrix}$$

$$I(x;y) = H(y) - H(y/x)$$

$$[p(y)] = [0,5p \quad 1-0,5p]$$

$$H(y) = -\{0,5p \log_2(0,5p) + (1-0,5p)\log_2(1-0,5p)\}$$

$$H(y/x) = -\{0,5p \log_2 \frac{1}{2} + 0,5p \log_2 \frac{1}{2}\} = p$$

$\vdots$

$$H(y) = -0,5p \log_2 \frac{0,5p}{1-0,5p} - \log_2(1-0,5p)$$

$$I(x;y) = 0,5p \log_2 \frac{1-0,5p}{0,5p} - \log_2(1-0,5p) - p$$

$$\frac{dI}{dp} = 0,5 \log_2 \frac{1-0,5p}{0,5p} + \left[\frac{0,5p}{(1-0,5p)\log 2} \cdot \frac{-0,5 \cdot 0,5p - 0,5(1-0,5p)}{(0,5p)^2}\right] - \frac{-0,5}{(1-0,5p)\log_2} - 1$$

$$\frac{dI}{dp} = 0,5 \log_2 \frac{1-0,5p}{0,5p} - 1 = 0$$

$\boxed{p = 0,4}$ $\quad C = 0,322$