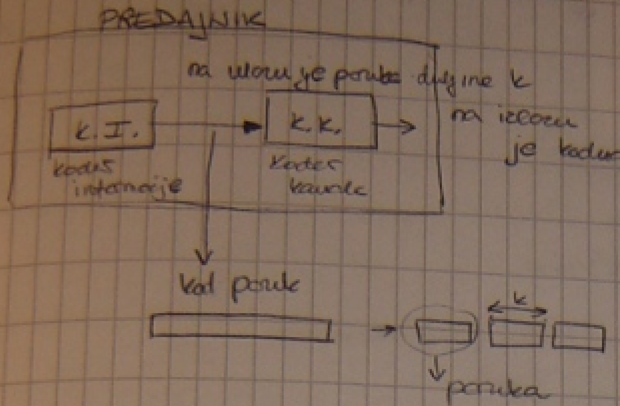
II CIKLUS

ZAŠTITNO KODIRANJE I

25.10.2010.



na izlazu je kodna riječ duljine n

P_{err}

dekoder kanala

- koristi kôd za detekciju pogreške
- može detektirati i ispraviti pogreške

ponovno slati nekim algoritmom

↳ TCP, UDP

Abeceda koda

$F_q \quad , \quad q = 2, 3, \ldots$

$F_2 = \{0, 1\}$

$2^{n-k} = \sum_{i=0}^{t} \binom{n}{i}$

primjer

K_1 000, 011, 101, 110 $d = 2$, $= M$ $n = 3$

K_2 00000, 01101, 10110, 11011 $n = 5$

oznaka blok-koda

$K : (n, M, d)$

otkrivanje (s) — $d(K) = s + 1$

ispravljanje (t) — $d(K) \geq 2t + 1$

oznaka koda K (u literaturi C)

n - duljina kodne riječi

PARITETNO KODIRANJE

najjednostavnije blok kodiranje

P: } xoranjem / mod 2
N:

vjerojatnost neotkrivenih pogrešaka

P_{np}

kk		
00	000	011, 101, 110
01	011	
10	101	
11	110	

$\underbrace{\quad}_{n=3}$

koliko ima ostalih znakova koji se mogu pojaviti

$$P_{np} = \binom{3}{2} p^2 (1-p)$$

p - vjerojatnost pogreške

Linearni binarni blok kodovi

a) $x + y \in K \quad , x, y \in K$

b) $a \cdot x \in K \quad , a = \{0, 1\}$

c) $00\ldots0 \in K$

primjer $K : \{10011, 11101, 01110, 00000\}$

a)
$$\begin{array}{r} 10011 \\ \oplus\ 11101 \\ \hline 01110 \end{array} \in K$$

sve kombinacije tako

b) ✓

c) ✓

vektorski prostor : baza prostora

k 00000	baza
11100	011
00111	101
11011	

$G_{k \times n}$

$M = 4$

$k = 2$

$\boxed{M = 2^k}$

primjer

$\overset{k}{\boxed{}} \cdot G = \overset{\leftarrow n \rightarrow}{\boxed{}}$

00

01

10 → koder kanala

11

$$G = \begin{bmatrix} 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 \end{bmatrix}$$

$$c = \underbrace{[0\,0]}_{\text{poruka}} \cdot \begin{bmatrix} 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 \end{bmatrix} = [0\,0\,0\,0\,0]$$

$$c = [0\,1] \cdot [\;G\;] = [0+1 \quad 0+0 \; \dots]$$

→ primjer generiranja kodnih riječi

(n, M, d)

$[n, k, d]$

$M = 2^k$

$k : [n, k, d]$

2 kodne riječi $000 \ldots 0$ jednake duljine

$111 \ldots 1$

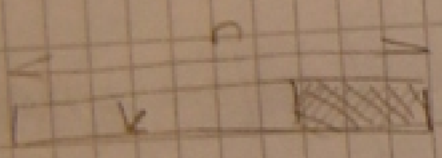
$k = ?$

$d[k] = ?$

$d[k] = n$

$k = 2$ jer je $M = 2^k$, a $M = 2$

obično $[n, k]$ u zadacima



$G = [I \vdots A]$

$G = [m \vdots p]$

000

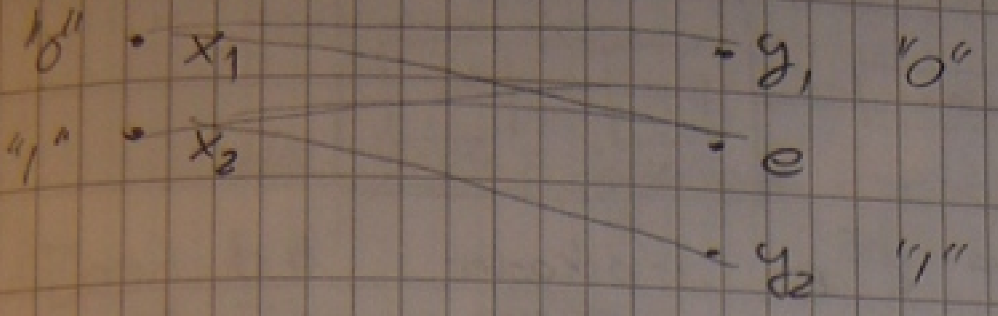
111 $\parallel$ $M = 2$

p_g

vjerojatnost pogrešnog dekodiranja

$$P_{pd} = \binom{3}{2} p_g^2 (1 - p_g) + \binom{3}{3} p_g^3$$

kanal s brisanjem simbola



000 → 0ee   111 → 1ee

00e   11e

eee   ovdje ne znamo   eee

$$P_{pd} = \binom{3}{3} p_e^3 \cdot \frac{1}{2}$$

primjer: koliko pogrešaka može ispraviti

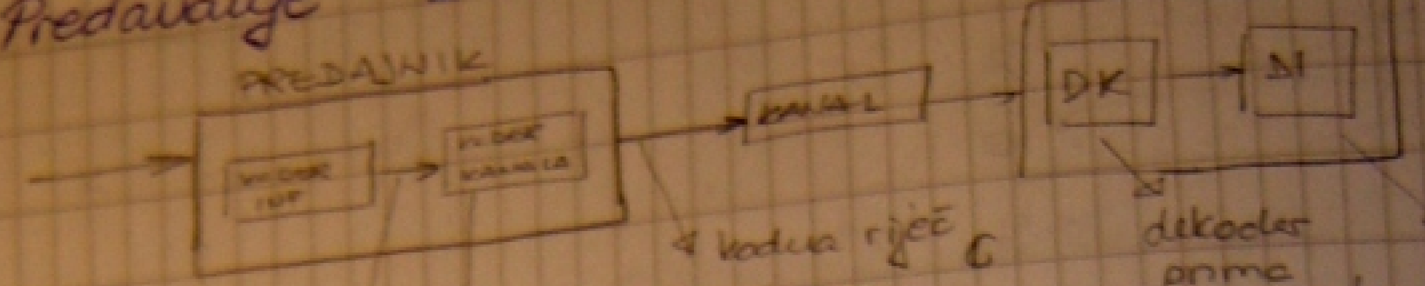
$[n, 4]$, $d_{min}(K) = 3$

$n_{min} = ?$ $t = \left\lfloor \frac{d-1}{2} \right\rfloor = 1$

$$2^{n-4} \geq \sum_{0}^{1} \binom{n}{i} = 1 + n$$

za $n = 7$ vrijedi

Predavanje 2.11.2010.



kodna riječ c

poruka d — zaštitno kodiranje

dekoder prima neki c'

Δd

G generirajuća matrica koda

H matrica provjere pari

$d \xrightarrow{G} c$

$d_{min}(K) \rightarrow$ otkrivanje pogreške, s

$d_{min}(K) \rightarrow$ ispravljanje pogreške, t

$$\boxed{\begin{array}{l} \text{i) } d(K) \geq s+1 \\ \text{ii) } d(K) \geq 2t+1 \end{array}}$$

(n, M, d)

n → duljina koda

M → broj kodnih riječi

d → min distanca koda

PARITETNO KODIRANJE

primjer:

A	00	0
B	01	1
C	10	1
D	11	0

→ 011
101
110

$$P_{np}(K_1) = \binom{3}{2} p_g^2 (1-p_g)^{3-2}$$

vjerojatnost neotkrivenih pogrešaka

n=3 a.5 k=4

primjer:

(vertikalna i horizontalna parni paritet
prijenos zaštićeni

Na koder kanale dolaze dvije poruke

$x_1 = [00]$ → KK

$x_2 = [10]$ → KK

00	0	→ 000	odaslano
10	1	→ 101	
10	1	→ 101	

primljeno 010 101 101

odrediti poziciju na kojoj je došlo do pogreške

0	1	0
1	0	1
1	0	1

sigurno je pogrešan 2. bit

koliko pog. neki kod može otkriti

$[n, k] \to t \qquad 2^{n-k} \geq \sum_{i=0}^{t} \binom{n}{i}$

DEKODIRANJE LINEARNOG BINARNOG KODA

1) PRINCIP NAJBLIŽEG SUSJEDA

2) SINDROMSKO DEKODIRANJE

razlika primljene i poslane riječi

a) vektor pogreške $\vec{e} = \vec{y} - \vec{x} = [e_1, \dots, e_n]$

b) std. niz

c) razred

d) H

e) sindrom (s)

b)

$\vec{e}$

00000	11100	00111	11011
10000	01100	10111	01011
01000			
00100			
00010			

→ razred

zbrajamo

$K = \begin{cases} 00000 \\ 11100 \\ 00111 \\ 11011 \end{cases}$

<u>primjer</u> Dan je binarni kôd K s gen. matricom G

$$G = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix} \qquad d \longrightarrow \overset{G}{\boxed{K}} \longrightarrow c$$

a) provedite std. niz koda K

b) odredite tablicu sindroma koda K

$$K = \begin{cases} [00]\cdot[G] = [0000] \\ [01]\cdot[G] = [0101] \\ [10]\cdot[G] = [1010] \\ [11]\cdot[G] = [1111] \end{cases} \qquad [4, 2]$$

vč

0000	0101	1010	1111
1000	1101	0010	0111
0100	0001	1110	1011
0010	0110	1000	1101
0001	0100	1011	1110

---

d) H matrica provjere pariteta

a) ortogonalnost	$c \to c'$
b) dualan kôd $\to K^{\perp}$	$x \in K$
c) Lin $K^{\perp}$	$x' \in K^{\perp}$
$K \to G$	$x \cdot x' = 0$
$K^{\perp} \to H$	

c) sindrom je $x' \cdot H^T = 0$

$S(x') = x' \cdot H^T = 0$ ako je $\neq 0.0$ postoji pogreška

$$G = [I \,\vdots\, A]$$

$$H = [-A^T \,\vdots\, I]$$

standardni oblik mat G i H

primjer

a) $H^T = \begin{bmatrix} 110 \\ 101 \\ 100 \\ 010 \\ 001 \end{bmatrix}$ $c' = [1\,1\,0\,1\,1]$

$s' = c' \cdot H^T = [0\,0\,0]$ nema pogreške

$$H = \begin{bmatrix} 1 & 1 & \vdots & 1 & 0 & 0 \\ 1 & 0 & \vdots & 0 & 1 & 0 \\ 0 & 1 & \vdots & 0 & 0 & 1 \end{bmatrix}$$

b) $c' = [1\,0\,0\,1\,1]$

$s' = c' \cdot H^T = [1\,0\,1]$

mjesto pogreške

signira je greške na 2. bitu

poslano je $[1\,1\,0\,1\,1] = c$

VJEROJATNOST ISPRAVNOG DEKODIRANJA

primjer: $[n, k]$, $t = 1$

$$P_{id} = \binom{7}{0} p_g^0 (1-p_g)^7 + \binom{7}{1} p_g (1-p_g)^6$$

$$\boxed{P_{id} = \sum_{i=0}^{t} \binom{n}{i} p_g^i (1-p_g)^{n-i}}$$

$P_{pg.dek.} = 1 - P_{id}$

kodna brzina: $R(K) = \frac{k}{n} \leq 1$
(code rate)

Zadatak: dan je bin. kôd $[7,4]$ s

$$G = \left[\begin{array}{cccc|ccc} 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \end{array}\right]$$

i) $H = ?$
ii) $d(K) = ?$
iii) Je li perfektan?
iv) $c' = [1110100]$ i $c = ?$

$$A^T = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \end{bmatrix}$$

i) $$H = \begin{bmatrix} 1 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}$$

$M = 16$

ii) koliko min. stupaca H treba zbrojiti da se dobije 0
to je isto min. distanca
$d(K) = 3$

iii) $$M = \frac{2^n}{\sum_{i=0}^{t} \binom{n}{i}} = \frac{2^7}{1+7} = 2^4 = 16$$

$t \leq \frac{d-1}{2} = 1$ kôd je perfektan iv)

$$S = c' \cdot H^T = [1\ 1\ 1\ 0\ 1\ 0\ 0] \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = [0\ 0\ 0]$$

$c = c'$

zi 14 Bin. blok kod K [n, 2] ima min. Hammingovu udaljenost $d_{min} = 5$, odredite min. duljinu kodne riječi - n.

$$t = \left\lfloor \frac{5-1}{2} \right\rfloor \quad 2$$

$K: [n, 2]$

$d = 5$

$$2^{n-k} \geq \binom{n}{0} + \binom{n}{1} + \binom{n}{2}$$

$$2^{n-2} \geq 1 + \binom{n}{1} + \binom{n}{2}$$

$$n = 7 \quad 2^5 \geq 1 + 7 + 21 \quad ?!$$

$$32 \geq 29$$

Hammingov kod

(1/2)

HAM ($r = 3$)

r nam definira duljinu kodne riječi

matrica H u hamm. kodu

npr. $[7,4]$

$$H = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{bmatrix} \qquad r \times (2^r - 1)$$

$r = n - k = 3$

$$c = [\ \underset{\uparrow}{}\ \underset{\uparrow}{}\ \underset{\uparrow}{}\ \underset{\uparrow}{}\ ]$$

primjer: $d = [1010]$

$k = 4$

$n = 7$

$r = 3$

zaštitni (kontrolni) bitovi uvijek dolaze na 2^{i-1} mjest...

$\boxed{1}\ \boxed{0}\ 1\ \boxed{1}\ 0\ 1\ 0$

$c = [1011010]$

nesistematični kôd

$[7,4]$

$$G = \begin{array}{c} \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix} \\ \begin{array}{ccccccc} k_1 & k_2 & d_1 & k_3 & d_2 & d_3 & d_4 \end{array} \end{array}$$

$k_1 = d_1 \oplus d_2 \oplus d_4 =$

$k_2 = d_1 \oplus d_3 \oplus d_4$

$k_3 = d_2 \oplus d_3 \oplus d_4$

pr. Hamm [15, 11] $k_1\ k_2\ d_1\ k_3\ d_2\ d_3\ d_4\ k_4\ d_5\ d_6\ d_7\ d_8 \ldots$

$$G = \left[\begin{array}{ccccccccc}
1 & 1 & 1 & 0 & 0 & \cdots & 0 & \cdots \\
1 & 0 & 0 & 1 & 1 & & 0 & \cdots \\
0 & 1 & 0 & 1 & 0 & \ddots & 0 & \\
1 & 1 & 0 & 1 & 0 & & 0 & \ddots \\
1 & 0 & 0 & 0 & 0 & & 1 & \\
0 & 1 & 0 & 0 & 0 & & 1 & \\
1 & 1 & 0 & 0 & 0 & & 1 & \\
0 & 0 & 0 & 1 & 0 & & 1 & \\
1 & 0 & 0 & 1 & 0 & & 1 & \\
0 & 1 & 0 & 1 & 0 & & 1 & \\
1 & 1 & 0 & 1 & 0 & \cdots & 1 & \cdots
\end{array}\right]$$
$\quad k_1\quad k_2 \qquad\quad k_3 \qquad\qquad\qquad k_4$

primjer $\quad c \rightarrow c' = [1011000]$

$c = [1011010]$

sindrom primljene kodne riječi

$$s(c') = c' \cdot H^T$$

$$s(c') = [1011000] \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} = [0\ 1\ 1]$$

pogreška na 6. mjestu

1. način

1 0 1 1 0 0 0 / 0

/ 1

/ 1

$(110)_2 = 6_{10}$

Hammingov kod primjer zadatka

$BER = \frac{1}{8}$, $n = ?$ $\frac{k}{n} = max.$

u koder kanala ulaze 111011 ...

odredite prvi blok podataka koji će izaći iz kodera

k_1 k_2 1 k_3 1 1 1 k_4 0 1 1 k_5

i) ako uzmemo da je $n=8$?? ne može jer je k_4 zadnji

ii) $n = 9$??

iii) $n = 7$ ne može nam se dogoditi dvostruka pogreška jer se događa jedan u 8 bitova

primjer [5, 2]

Hamming $\rightarrow M = 2^k = 4$

00	$c_1 = [00000]$
01	$c_2 = [10011]$
10	$c_3 = [11100]$
11	$c_4 = [01111]$

a) $G = ?$

b) $c = ?$

c) linearnost = ?

$$G = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 \end{bmatrix}$$

pr [7, 4]

$$P_{pd} = 1 - P_0 - P_1 = P_2 + P_3 + \dots + P_7$$

koliko kod može ispraviti pogrešaka t

$$2^{n-k} = \sum_i^t \binom{n}{i}$$

$$8 \geq 1 + 7$$

$$\underline{\underline{t = 1}}$$

$$P_{pd} = 1 - \left\{ \binom{7}{0} p_b^0 (1-p_b)^{7-0} + \binom{7}{1} p_b^1 (1-p_b)^{7-1} \right\}$$

$g(x)$ – generirajući polinom koda K
– uvijek je stupnja $n-k$ $\quad x^{n-k} + \ldots + 1$

$[n, k]$

$g(x) \to x^n + 1$

pr $n = 5$

$$G = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

$k \times n$

ne možemo otkriti gdje su zaštitni

$x^n + 1$

$(x+1)(x^4 + x^3 + x^2 + x + 1)$

$g(x) = x + 1$

$n - k = 1$

$k = 4$

standardni oblik gen. matrice G

pr $[7, 4]$

$g(x) = x^3 + x + 1$

$g = [\ 1\ 0\ 1\ 1\ ]$

$$G = \begin{bmatrix} 0 & 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix} \oplus \to \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix} \oplus$$

$$\to \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix}$$ ✓

$g(x) \rightarrow$ shema c.k.

$\rightarrow$ cikl. KODER

$g(x) = x^4 + x^3 + 1$

15-11

$x^n + 1 = g(x) \cdot h(x)$

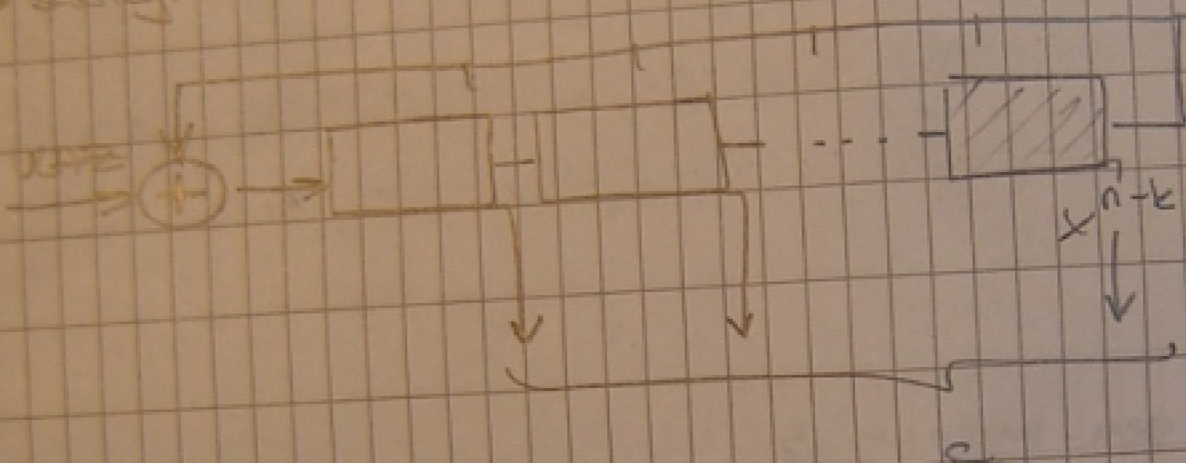
[15, 11]

c. k.

ser.

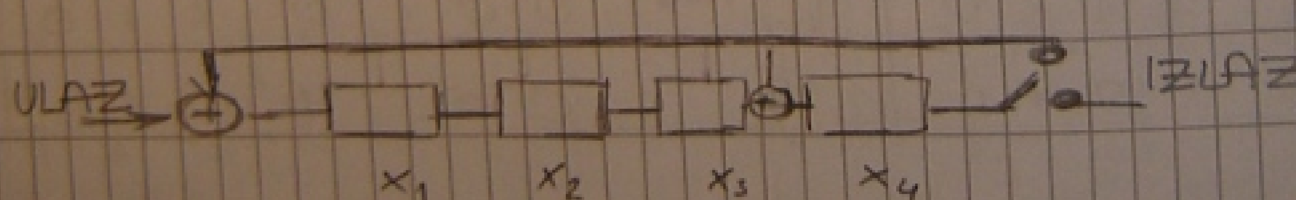
par.

$\rightarrow$ sastoji se od pomoćnih registara i mod 2 zbrajanja

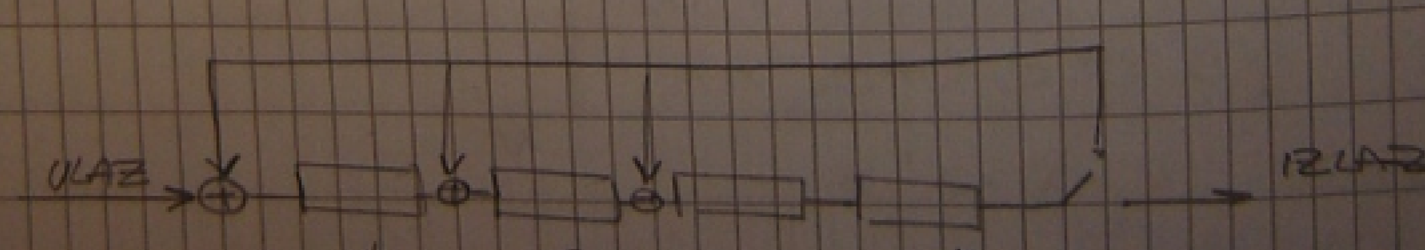


$g(x) = x^4 + x^3 + 1$

broj pomoćnih registara



primjer $\quad g(x) = x^4 + x^2 + x + 1$



$c(x) = d(x) + r(x)$

$1011 \sqcup \sqcup \sqcup$

$d = x^3 + x + 1 \quad / \cdot x^3$

$d = x^6 + x^4 + x^3$

$$r(x) = \frac{x^{n-k}\, d(x)}{g(x)}$$

PRIMJER $[15, 11]$

$g(x) = x^4 + x^3 + 1$

odredite zaštitni dio za prvi kodni riječ

ako se na ulazu pojavljuje $\underbrace{10101010101}_{d}$

$c = ?$ $[d \mid r]$

I iz $g(x) \to G \xrightarrow{d} c$

II

$$r(x) = \frac{x^{n-k} \cdot d(x)}{g(x)}$$

$c = [d \mid r]$

$$r(x) = \frac{x^4 \cdot (x^{10} + x^8 + x^6 + x^4 + x^2 + 1)}{(x^4 + x^3 + 1)}$$

$(x^{14} + x^{12} + x^{10} + x^8 + x^6 + x^4) : (x^4 + x^3 + 1) = x^{10} + x^9 + x^5 + x^2 + 1$

$-x^{14} \pm x^{13} \pm x^{10}$

$+x^{13} + x^{12} + x^8 + x^6 + x^4$

$x^{13} + x^{12} + x^9$

$x^9 + x^8 + x^6 + x^4$

$x^9 + x^8 + x^5$

$x^6 + x^5 + x^4$

$x^6 + x^5 + x^2$

$x^4 + x^2$

$x^4 + x^3 + 1$

$x^3 + x^2 + 1 \quad \overbrace{1101}^{r}$

$$s(d'(x)) = \frac{x^{n-k} d'(x)}{g(x)}$$

ZADATAK $g(x) = x^3 + x^2 + 1$

$[7, k]$ $\rightarrow k = 4$

odredite sindrom za <u>prvu</u> kodnu riječ ako se na ulazu pojavljuje 100 111 0000 11 . . .

$d = [1001110]$

$$s = \frac{x^3 (x^6 + x^3 + x^2 + x)}{x^3 + x^2 + 1}$$

$$(x^9 + x^6 + x^5 + x^4) : (x^3 + x^2 + 1) = x^6 + x^5 + x^4 + x^3 + x^2 + x$$

$$\begin{array}{l}
x^9 + x^8 + x^6 \\ \hline
x^8 + x^5 + x^4 \\
x^8 + x^7 + x^5 \\ \hline
x^7 + x^4 \\
x^7 + x^6 + x^4 \\ \hline
x^6 \\
x^6 + x^5 + x^3 \\ \hline
x^5 + x^3 \\
x^5 + x^4 + 1 \\ \hline
x^4 + x^2 + 1 \\
x^4 + x^3 + x \\ \hline
x + 1
\end{array}$$

(11)

zi11 $x_1 = [1\,1]$

$x_2 = [0\,1]$

i) $c' = [1\,0\,0\,0\,1\,1\,1\,0\,1]$

ii) $[1\,1\,1\,1\,1\,0\,0\,1\,1$

iii) $[0\,1\,1\,1\,1\,1\,1\,1\,0]$

i)
$$\begin{array}{c|c|c} 1 & 0 & 0 \\ \hline 0 & 1 & 1 \\ \hline 1 & 0 & 1 \end{array}$$

pogreška je u 1. redu i 2. stupcu $x_{12} = 1$

ii)
$$\begin{array}{c|c|c} 1 & 1 & 1 \\ \hline 1 & 1 & 0 \\ \hline 0 & 1 & 1 \end{array}$$

$x_{12} = 0$

iii)
$$\begin{array}{c|c|c} 0 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline 1 & 1 & 0 \end{array}$$

$x_{22} = 0$

Zi 15

$$H = \begin{bmatrix} 1 & 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 \end{bmatrix}$$

i) $n = 5$

$n - k = 3$

$k = 2$

$M = 2^k = 4$

$\Big\} d = 3$ $\quad t = \left\lfloor \frac{d-1}{2} \right\rfloor = 1$

e	S
00000	000
00001	101
00010	001
00100	111
01000	110
10000	100

može ispraviti

$s(y) = e \cdot H^T$

00101	010
01001	011
10100	

dvostruke

ii) $c' = [11010]$

$s(c') = c' \cdot H^T = [011]$

ne može znati što je poslano

zi 10 $\quad K: [n,k] = [7,3]$

i)

$$H = \begin{bmatrix} 1 & x_1 & y_1 & \vdots & 1 & 0 & 0 & 0 \\ 0 & x_2 & y_2 & \vdots & 0 & 1 & 0 & 0 \\ 1 & x_3 & y_3 & \vdots & 0 & 0 & 1 & 0 \\ 1 & x_4 & y_4 & \vdots & 0 & 0 & 0 & 1 \end{bmatrix} = [A^T \vdots I_4]$$

$c = [0110011]$

$d(K) = 4$

$$s = c \cdot H^T = [0110011] \begin{bmatrix} 1 & 0 & 1 & 1 \\ x_1 & x_2 & x_3 & x_4 \\ y_1 & y_2 & y_3 & y_4 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = [0000]$$

$x_1 + y_1 + 0 + 0 = 0$

$x_2 + y_2 + 0 + 0 = 0$

$x_3 + y_3 + 1 + 0 = 0$

$x_4 + y_4 + 0 + 1 = 0$

0,0
1,1
0,0
1,1
1,0
0,1
1,0
0,1

$$G = [I \vdots A] = \begin{bmatrix} 1 & 0 & 0 & \vdots & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & \vdots & 1 & 1 & 0/1 & 1/0 \\ 0 & 0 & 1 & \vdots & 1 & 1 & 1/0 & 0/1 \end{bmatrix}$$

$c = [0000000]$

$$H = \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0/1 & 1/0 & 0 & 0 & 1 & 0 \\ 1 & 1/0 & 0/1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$a = [011]$

ii) $c' = [0110111]$

$s = [0100]$

na 5. mjestu

$c' = [0110011]$

zi16

$$H = \begin{bmatrix} 1 & 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}$$

i) $c_1 = [0010011]$ $\qquad d_{min} = 3$

$c_2 = [0001111]$

$s_1 = c_1 \cdot H^T = [000]$

$s_2 = c_2 \cdot H^T = [000]$

ii) $c' = c + e$

$s = c' \cdot H^T = (c+e) \cdot H^T = c \cdot H^T + e \cdot H^T = e \cdot H^T$

$c \cdot H^T = 0$

iii)

e	$s = e \cdot H^T$
1000000	110
0100000	101
0010000	011
0001000	111
0000100	100
0000010	010
0000001	001

zi17 $[15,7]$

$$g(x) = x^8 + x^7 + x^6 + x^4 + 1$$

i) $(x^{15}+1) : (x^8 + x^7 + x^6 + x^4 + 1) = x^7 + x^6 + x^4 + 1$

$$\cancel{x^{15}} + x^{14} + x^{13} + x^{11} + x^7$$

$$\cancel{x^{14}} + \cancel{x^{13}} + x^{11} + x^7 + 1$$

$$\cancel{x^{14}} + \cancel{x^{13}} + x^{12} + x^{10} + x^6$$

$$x^{12} + x^{11} + x^{10} + x^7 + x^6 + 1$$

$$\cancel{x^{12}} + x^{11} + x^{10} + x^8 + x^4$$

$$x^8 + x^7 + x^6 + x^4 + 1$$

$$0$$

ii) $d(x) = x^4 + x + 1 \quad = [0010011]$

kodna riječ u sistematičnom obliku

$$[d(x) \cdot x^{n-k} \cdot p(x)]$$

$$p(x) = \text{ost} \frac{x^{n-k} d(x)}{g(x)} = x^8 (x^4 + x + 1) : (x^8 + x^7 + x^6 + x^4 + 1)$$

$(x^{12} + x^9 + \cancel{x^8}) : (x^8 + x^7 + x^6 + x^4 + 1) = x^4 + x^3$

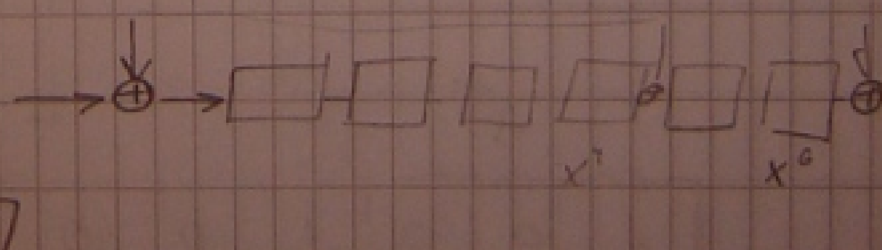
$$\cancel{x^{12}} + x^{11} + x^{10} + \cancel{x^8} + x^4$$

$$x^{11} + x^{10} + x^9 + x^4$$

$$x^{11} + x^{10} + x^9 + x^7 + x^3$$

$$x^7 + x^4 + x^3$$

10011000



$c = [001001110011000]$

iii) $c = [100000 \cdot 0010001 1]$

$x^{n-k} \cdot d(x)$ ost $\cdot$ 111001000 zi18

$s_1 = [10100]$

$s_2 = [01010]$

$s_3 = [00101]$

$s_4 = [10000]$

$s_5 = [01000]$

$s_6 = [00100]$

$s_7 = [00010]$

$s_8 = [00001]$

$k \quad [n, k]$

$t = 1$

i) $s = c' \cdot H^T = (c + e) \cdot H^T = c \cdot H^T + e \cdot H^T = e \cdot H^T$

$s = e \cdot H^T \quad / e^{-1}$

$e^{-1} \cdot s = H^T \longrightarrow s = H^T$

$\downarrow$

jedniowa macierz jest [illegible] jednostkowa

$$H^T = \begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H = \left[\begin{array}{ccc|ccccc} \multicolumn{3}{c|}{\overbrace{\hphantom{1 \quad 0 \quad 0}}^{A^T}} & & & & & \\ 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \end{array}\right]$$

$$G = \left[\begin{array}{ccccc:ccc} 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 \end{array}\right]$$

ii) $n = 8$

$k = 3$

iii) $g(x) = ?$

$g(x) = x^5 + x^2 + 1$

iv) bla bla . ...

v) $c' = [0 1 1 0 1 0 1 1]$

$s = c' \cdot H^T = [0 0 1 0 0]$ na 6. je greška

$c' = [0 1 1 0 1 1 1 1]$

zi 19

$M = 128$ $\quad [11, 7]$

$k = 7$

$p(x_1) = p(x_2) = \dots = p(x_{128})$

$x_0 = [0000000]$

$\vdots$

$x_{127} = [1111111]$

1 greška

1 1 1 1 0 1 1 0 0 0 0

1 1 0 1 0 0 | 1

1 1 1 1 0 0 | 0

1 0 1 1 | 1

0 0 0 0 | 0

$\uparrow (0101)_2 = 5_{10}$

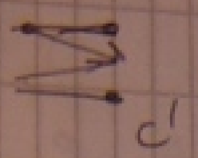
$d = [1111000]$

2. MI 2009/2010 A

1) Hammingov

$K: [n,k] = [7,4]$

$K \rightarrow$



c

$c = [1100\ abc]$

$a,b,c \in \{0,1\}$

$c' = [1100???]$

$$H = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{bmatrix}$$

$c' \cdot H^T \Rightarrow \quad 1 + a + c = 0 \quad \rightarrow a = c + 1 = 1$

$1 + b + c = 0 \quad \rightarrow b = c + 1 = 1$

$a + b + c = 0 \quad \rightarrow c + 1 + c + 1 + c = 0$

$c = 0$

c)

2) 0000

1011

1110

0101

$\rightarrow \boxed{KK} \rightarrow K^\perp$

$M = 4 \rightarrow k = 2$

$n = 4$

$$G = [I \,\vdots\, A] = \begin{bmatrix} 10 \vdots 11 \\ 01 \vdots 01 \end{bmatrix}$$

$K \quad G \rightarrow H$

$K^\perp \quad G^\perp \rightarrow H^\perp$

$$H = \begin{bmatrix} 10 & 10 \\ 11 & 01 \end{bmatrix}$$

$G^\perp =$ | [00] $\cdot H = [0000]$
[01] $\cdot H = [1101]$
[10] $\cdot H = [1010]$
[11] $\cdot H = [0111]$

3) zi 19

4) $c-[1001110]$

$K: c_i \rightarrow g_i(x)$

kodirati $d=[101]$

$$G=\begin{bmatrix} 1110100 \\ 0111010 \\ 0011101 \end{bmatrix}$$

1001110

0011101 $\quad x^4+x^3+x^2+1=g(x) \quad g=[11101]$

0111010

1110100

1101001

1010011

0100111

$c=[101]\cdot G=[1101001]$ c)

5) [6,3]

$c' = [101100]$

$Pe = 0{,}01$

$t = 1$

$G = [\, I \;\vdots\; A \,]$

$$G = \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{array}\right]$$

$H = [\, A^T \;\vdots\; I \,]$

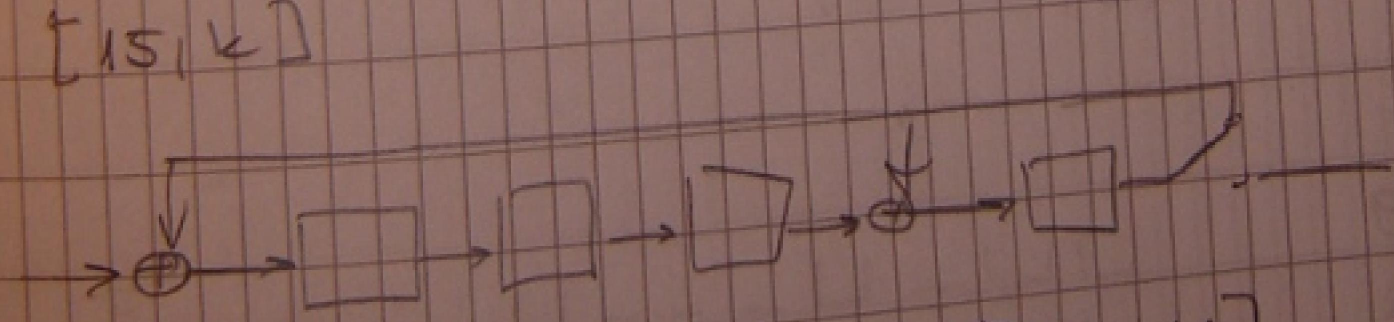
$s(c') = c' \cdot H^T = [011]$ na 6. mj je greška

$c = [101001]$

$d = [101]$

$$P_{pd} = 1 - \binom{6}{0} pe^0 (1-pe)^6 - \binom{6}{1} pe^1 (1-pe)^5 = 1{,}46 \cdot 10^{-3}$$

6) [15, k]



$\rightarrow [15, 11]$

$g(x) = x^4 + x^3 + 1$

(...)

7) $[7,4]$

$K \qquad H$

$K^{\perp} \quad G$

$$c^{\perp} = [1\,0\,1\,0] \cdot H = [0\,1\,1\,0\,0\,0\,0]$$

8) $[7,4]$

9) $[7,4]$

$P_y = m$

$$\frac{P_{isp}(H)}{P_{isp}(P)} = \frac{\binom{7}{0} m^0 (1-m)^7 + \binom{7}{1} m^1 (1-m)^6}{\binom{5}{0} m^0 (1-m)^5} = (1-m)(1-6m)$$

10) $n = 9$ $\qquad \sqcup\ \sqcup\ \_\ \sqcup\ \_\ \_\ \_\ \sqcup\ \_$ $\qquad \varepsilon = \frac{5}{9}$

$n = 8$

$n = 7$ $\qquad \boxed{0}\ \boxed{0}\ 1\ \boxed{0}\ 1\ 1\ 0$ $\qquad \varepsilon = \frac{4}{7}$