Zadaci za vježbu 3.

ad 3

Zad 3.1

$t = 1$

$n = 9$

$M =$ 52 kodne riječi

$$M \leq \frac{2^n}{\sum_{i=0}^{t} \binom{n}{i}} = \frac{2^9}{\binom{9}{0} + \binom{9}{1}} = 51,2$$

$52 \leq 51,2$? $\Rightarrow$ Ne postoji

Zad 3.2

$K[n, 2]$

$d(K) = 5 \longrightarrow t = \left\lfloor \frac{5-1}{2} \right\rfloor = 2$

$n_{min} = ?$

$$2^k \leq \frac{2^n}{\sum_{i=0}^{t} \binom{n}{i}} = \frac{2^n}{}$$

$$4 \leq \frac{2^n}{\binom{n}{0} + \binom{n}{1} + \binom{n}{2}}$$

$n = 4$ $4 \leq 1.45$ NE

$n = 5$ $4 \leq 2$ NE

$n = 6$ $4 \leq 2.9$ NE

$\boxed{n = 7}$ $4 \leq 4.41$ DA

Zad 3.3

$n = 2$ bita

5 bit brz ot.

a) 

$c' = \underline{100}, \underline{011}, \underline{101},$

1 0 0

0 1 1

1 0 1

b) $c' = 111110011$

greška

1 1 1

1 1 0

0 1 1

c) $\underline{011}, \underline{111}, \underline{110},$

0 1 1

1 1 1

1 1 0

Zad 3.4

$K = \begin{cases} 10011 \\ 11101 \\ 01110 \\ 00000 \end{cases}$ a) $d(K) = 3$ $\quad t = \left\lfloor \frac{3-1}{2} \right\rfloor = 1$

3, 4 $\quad s = 3 - 1 = 2$

b) $x + y \in K$

$10011 + 11101 = 01110$ ✓

$11101 + 01110 = 10011$ ✓

$10011 + 01110 = 11101$ ✓

$00000 + \ldots = \ldots$

$a \cdot x \in K$ ✓ Lin. je

Zad 3.5

$n \geq 1 \quad K[n, k, d]$

$k = ?$

$d = ?$

$d = n$, $k = 1$ jer je $G = [111 \ldots 1] \Rightarrow k = 1$

$K = \begin{cases} 0 \cdot G \\ 1 \cdot G \end{cases}$

Zad 3.6

a) $K = \begin{cases} 00000 \\ 10010 \\ 10100 \\ 00110 \end{cases}$ $\quad G = \begin{bmatrix} 10010 \\ 10100 \end{bmatrix}$ $\quad G = \begin{bmatrix} 10010 \\ 00110 \end{bmatrix}$

$G = \begin{bmatrix} 10010 \\ 00110 \end{bmatrix}$

b) $K' = ?$

$G = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix}$ $\quad K' = \begin{cases} 00000 \\ 10010 \\ 01010 \\ 11000 \end{cases}$

Zad 3.7

$$H = \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 \end{bmatrix} \underset{\substack{R_2 = R_1 + R_2 \\ R_3 = R_1 + R_3}}{\sim} \begin{array}{c} \begin{array}{ccccccc} & & a & & & b & c \end{array} \\ \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix} \end{array} \sim \begin{array}{c} \begin{array}{ccccccc} & & c & & & a & B \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ \underbrace{0 & 0 & 1}_{A} & 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

$$G = \begin{array}{c} \begin{array}{ccccccc} & & c & & & a & b \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

$$G=\left[\begin{array}{ccc|cc|cc} 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 & 0 \end{array}\right]$$

Riječi

0.0.0	0000000 ✓	1100001	1+2
1.	1001100 ✓	1011010	1+3
2.	0101101	0110111	2+3
3.	0011010	1111011 ✓	1+2+3

KRIVO

0111100 1110000

0001011 1000111

$$G=\left[\begin{array}{ccc|cc|cc} 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \end{array}\right]$$

0	0000000	1110000	1+2
1	1001100	1000111	1+3
2.	0111100	0110111	2+3
3	0001011	1111011	1+2+3

→ stupci se zamjenjuju točno kao i kod H, ne trebа se ništa ponovno zbrajati/oduzimati

Zad 3.8

$$H = \begin{bmatrix} 1 & \boxed{1} & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 \end{bmatrix}$$

$$c' \cdot H^T = [110110] \cdot \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} = \underbrace{[1 \ 1 \ 0]}$$ = 6

$c' = [110110]$

$[100110]$

greška na 2. bitu

Zad 3.9

$K = [7,4] = [n,k]$

$\underbrace{0.1111001}_{1.}\,001100\ldots$

K: $G = \begin{bmatrix} 1000 & 111 \\ 0100 & 110 \\ 0010 & 101 \\ 0001 & 011 \end{bmatrix}$

K*: $G = \begin{array}{c} \begin{matrix} a & b & c & d & e & f & g & h \end{matrix} \\ \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix} \end{array} \sim \begin{array}{c} \begin{matrix} b & c & d & e & a & f & g & h \end{matrix} \\ \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 & 1 \end{bmatrix} \end{array}$

A

b c d e z f g h

$H = \begin{array}{c} \begin{matrix} a & b & c & d & e & f & g & h \end{matrix} \\ \begin{bmatrix} 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 & 0 & 1 \end{bmatrix} \end{array} \sim \begin{array}{c} \begin{matrix} b & c & d & e & a & f & g & h \end{matrix} \\ \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix} \end{array}$

$$[01111001] \cdot \begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = [1\ 0\ 0\ 0]$$

---

ako stupce bcde pomaknem u desno

$H = \begin{array}{c} \begin{matrix} & & & & b & c & d & e \end{matrix} \\ \begin{bmatrix} 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$

$$[01111001] \cdot \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = [0\ 0\ 0\ 1]$$

ako stupce bcde pomaknem za 1 u desno

$$H = \begin{array}{cccccccc} a & f & b & c & d & e & g & h \end{array}$$
$$\begin{bmatrix} 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}$$

$$[0 1 1 1 1 0 0 1] \cdot \begin{bmatrix} 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = [1 1 0 1]$$

zad 3.10

a) $K = \begin{cases} 0000 \\ 0101 \\ 1010 \\ 1111 \end{cases}$ $d(K) = 2$ $R = \frac{k}{n} = \frac{2}{4} = 0.5$

$k = 2$
$n = 4$

b) $K = \begin{cases} 0000 \\ 1110 \\ 1111 \\ 0101 \\ 1010 \end{cases}$ $n = 4$ $M = 2^k = 5 \quad / \log_2$

$k = \log_2 5$

$R = \dfrac{\log_2 5}{4} = 0.5805$

c) $K = \begin{cases} 0000 \\ 0011 \\ 0101 \\ 0110 \\ 1001 \\ 1010 \\ 1100 \\ 1111 \end{cases}$ $8 = 2^k \Rightarrow k = 3$ $R = \frac{3}{4} = 0.75$

$n = 4$

d) $n = 7$
$M = 16 \Rightarrow k = 4$ $R = \frac{4}{7}$

zad 3.11

a) $H = \left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ \underbrace{1 \quad 1 \quad 0}_{A} & & & 0 & 0 & 1 \end{array}\right]$

$111 \oplus 101 \oplus 010 = 000$ $d = 3$

$010 \oplus 010 = 000$ $d = 2$ $d(K) = 2$

b) $G = \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 \end{array}\right]$ $c = [001010]$

zad 3.12

$K = \begin{cases} 0101 \\ 1010 \\ 1100 \end{cases}$

$K^{\perp} = ?$

$$[x_1 \, x_2 \, x_3 \, x_4] \cdot \overset{H^T}{\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}} = [x_2 + x_4 \quad x_1 + x_3 \quad x_1 + x_2]$$

$x_2 + x_4 = 0$
$x_1 + x_3 = 0$
$x_1 + x_2 = 0$

0000
1111

3.1.4

$$K = \begin{cases} 00000 \\ 11010 \\ 01111 \\ 10101 \end{cases} \qquad p_g \qquad d(K)=3$$

a) $G = \begin{bmatrix} 10\,101 \\ 01\,\underbrace{111}_{A} \end{bmatrix}$ $\qquad H = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 \end{bmatrix}$

b)

00000	11010	01111	10101	000
00001				001
00010				010
00011				011
00100				100
00101				101
00110				110
00111				111

$$[\,-\,-\,-\,-\,-\,] \cdot \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = [\,-\,-\,-\,]$$

c) $n=5$
$k=2$
$n-k=3 \Rightarrow 8$ sindroma
iz H se vidi da se sve jednostr. pogr. uspješno ispr.
jer nema 2 ista stupca $\rightarrow$ sindrom pokriva vidi se u H
situaciju bez greške, 5 sit. jednostruke greške i
2 situacije dvostruke greške

$$P_{isp} = p_g^0(1-p_g)^5 + \binom{5}{1} p_g (1-p_g)^4 + \binom{5}{2} p_g^2 (1-p_g)^3$$

Zad 3.15

a)

$$G = \begin{bmatrix} 1000 & 1110 \\ 0100 & 1101 \\ 0010 & 1011 \\ 0001 & 0111 \end{bmatrix} \quad H = \begin{bmatrix} 1110 & 1000 \\ 1101 & 0100 \\ 1011 & 0010 \\ 0111 & 0001 \end{bmatrix} \quad \underline{d(K)=4}$$

(the right-hand part 0111 etc. of G is marked A)

b)

$$[1\,1\,1\,1\,1\,0\,0\,x] \cdot \begin{bmatrix} 1&1&1&0 \\ 1&1&0&1 \\ 1&0&1&1 \\ 0&1&1&1 \\ 1&0&0&0 \\ 0&1&0&0 \\ 0&0&1&0 \\ 0&0&0&1 \end{bmatrix} = [0+2\cdot 0 \quad 1+0? \quad 1+0? \quad 1+1\cdot ?]$$

$X = 1$ poslano: $[1\,1\,1\,0\,1\,0\,0\,1]$

Zad 3.16

$d_1 = 101$ 100101

$d_2 = 011$ 001011

$d_3 = 111$ 010110

$d_4 = 110$?

$$G = \begin{bmatrix} 1&0&1 \\ 0&1&1 \\ 1&0&0 \end{bmatrix} \quad \begin{bmatrix} 0&0&1 \\ 1&1&1 \\ 1&0&0 \end{bmatrix} \quad \begin{bmatrix} 1&0&1 \\ 1&1&1 \\ 1&0&0 \end{bmatrix}$$

01 11 10

3×1 ili $1\times 1 + 2\times 0$

$2\times 1 + 1\times 0$ ili $0\times 1 + 3\times 0$ ili

$$\begin{matrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 0 & 0 & 1 \end{matrix}$$

$\boxed{10}$/x

Zad 3.17

$$G = \begin{bmatrix} 1010101 \\ 0110011 \\ 0001111 \end{bmatrix} \sim \begin{bmatrix} 100 & 1101 \\ 010 & 1011 \\ 001 & 0111 \end{bmatrix} \quad H = \begin{bmatrix} 110 & 1000 \\ 101 & 0100 \\ 011 & 0010 \\ 111 & 0001 \end{bmatrix}$$

(the part 1101 / 1011 / 0111 is marked A)

a)

$$G^{\perp} = \begin{bmatrix} 1110000 \\ 1001100 \\ 0101010 \\ 1101001 \end{bmatrix}$$

Da bi se dobio $K^{\perp}$ iz K uz zadanu G

1. Svesti G na standarni oblik.
2. kreirati H
3. $G^{\perp}$ se dobije tako da se naprave iste zamjene kao i u G

b) 1001100 $d=3$ c) $R = \frac{k}{n} = \frac{4}{7}$

1011010

Zad 3.18

$K[n,k] = [7,4]$

$$G = \left[\begin{array}{cccc|ccc} - & - & - & - & - & - & - \\ - & - & - & - & - & - & - \\ - & - & - & - & - & - & - \\ - & - & - & - & - & - & - \end{array}\right] \quad (\text{right block: } A)$$

$$H = \left[\begin{array}{cccc|ccc} - & - & - & - & 1 & [illegible] & [illegible] \\ - & - & - & - & [illegible] & [illegible] & [illegible] \\ - & - & - & - & [illegible] & [illegible] & [illegible] \end{array}\right]$$

0000000 → 000
1000000 → 001
1100000 → 010
1110000 → 011
1111000 → 100
1111100 → 101
1111110 → 110
1111111 → 111

$$[1000000]\begin{bmatrix} 0 & 0 & 1 \\ - & - & - \\ - & - & - \\ - & - & - \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = [001]$$

$$[1100000]\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 1 \\ - & - & - \\ - & - & - \\ - & - & - \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = [010]$$

$$[1110000]\cdot\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \\ - & - & - \\ - & - & - \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = [011]$$

$$[1111000]\cdot\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \\ - & - & - \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = [100]$$

$$[1111100]\cdot\underbrace{\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{H^T} = [101]$$

$$H = \left[\begin{array}{ccccc|ccc} 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 0 & 0 & 1 \end{array}\right]$$

Mogu biti bilošto

zad 3.19 $[6,3]$ $\underbrace{d_1 d_2 d_3}_{\text{poruka}} \underbrace{c_4 c_5 c_6}_{\text{zaštita}}$

$c_4 = d_1 \oplus d_2 \oplus d_3$
$c_5 = d_1 \oplus d_3$
$c_6 = d_2 \oplus d_3$

$c' = [\underbrace{010}_{\text{poruka}}\underbrace{111}_{\text{zaš.}}]$ $c = ?$

$d_1 = 0$ $d_2 = 1$ $d_3 = 0$ $c_4 = 1$ $c_5 = 0$ $c_6 = 1$

010101

$$K = \begin{cases} 000\,000 \\ 001\,111 \\ 010\,101 \\ 011\,010 \\ 100\,110 \\ 101\,001 \\ 110\,011 \\ 111\,100 \end{cases}$$

$$G = \left[\begin{array}{ccc|ccc} 1&0&0&1&1&0 \\ 0&1&0&1&0&1 \\ 0&0&1&1&1&1 \end{array}\right] \quad (\text{A} \text{ under the last three columns})$$

$$H = \left[\begin{array}{ccc|ccc} 1&1&1&1&0&0 \\ 1&0&1&0&1&0 \\ 0&1&1&0&0&1 \end{array}\right]$$

5. bit greške

$$[010111] \cdot \begin{bmatrix} 1&1&0 \\ 1&0&1 \\ 1&1&1 \\ 1&0&0 \\ 0&1&0 \\ 0&0&1 \end{bmatrix} = [010]$$

010101

zad 3.20
$K = [n,k] = [3,2]$

$$G = \begin{bmatrix} 1&0&1 \\ 0&1&2 \end{bmatrix}$$

1 2 0 1 2 0 1 2
-2 -1 0 1 2 3 4 5 6 7 8

	K
00	000
01	012
02	021
10	101
11	110
12	122
20	202
21	211
22	220

$$H = \begin{bmatrix} 2 & 1 & 1 \end{bmatrix}$$

$1+2 = 3_{(4)} = \underline{\underline{0_{(3)}}}$

$d_{min} = 2$

$$\begin{array}{r} 101 \\ +\,021 \\ \hline 122 \end{array} \qquad \begin{array}{r} 202 \\ 012 \\ \hline 211 \end{array} \qquad \begin{array}{r} 202 \\ 021 \\ \hline 220 \end{array}$$

zad 3.21 $[7,3]$

$$H = \begin{bmatrix} 1&0&0&1&0&1&1 \\ 0&1&0&1&1&1&0 \\ 0&0&1&0&1&1&1 \\ 1&1&1&1&1&1&1 \end{bmatrix} \begin{matrix} 0 \\ 1 \\ 0 \\ 0 \end{matrix}$$

↑

$c' = [0000011]$
$c = ?$

$$[0000011] \cdot \begin{bmatrix} 1&0&0&1 \\ 0&1&0&1 \\ 0&0&1&1 \\ 1&1&0&1 \\ 0&1&1&1 \\ 1&1&1&1 \\ 1&0&1&1 \end{bmatrix} = [0100]$$

0101
0011

	sindrom
0000000 →	0000
0000001 →	1011
0000010 →	1111
0000100 →	0111
0001000 →	1101
0010000 →	0011
0100000 →	0101
1000000 →	1001

greška nije u jednom bitu (jer sindrom nije iz H)

gleda se za koje 2 greške je pogreška tj. sindrom 0100

0100

$$\begin{array}{r} 1101 \\ \oplus\ 1001 \\ \hline 0100 \end{array} \qquad \begin{array}{r} 0001000 \\ \oplus\ 1000000 \\ \hline 1001000 \end{array}$$ ← NE, jer piše susjedna

$$\begin{array}{rr} 0000001 & 1011 \\ \oplus\ 0000010 & 1111 \\ \hline 0000011 & 0100 \end{array}$$ → pogreška na zadnja 2 bita

$C = [0000000]$

zad 3.22

a) $H = \begin{bmatrix} 1 & 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 \end{bmatrix} \sim \begin{bmatrix} 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 \end{bmatrix}$ A

$G = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 1 \end{bmatrix}$

$d(K) = 2$

$t = 0$

$s = 1$

00000	000
00010	001
00111	011
11000	010
10100	011
10000	100
00001	101
01000	110
00100	111

$[\quad] \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix} = [\quad]$

i za 11001 se dobije sindrom 111

b) $c' = [11010]$

$e = 10100$ i 01001

$c' \cdot H^T = [11010] \cdot \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix} = [0\ 1\ 1]$

pošto nema stupca u H $\begin{smallmatrix} 0 \\ 1 \\ 1 \end{smallmatrix}$ ⇒ nije 1-struka pogr. onda gledam koje 2 zbrojena daju 011 to su:

1. i 3. i 2. i 5.

$e = 10100$ $\quad e = 01001$

$c = 01110$ $\quad c = 10011$

zd. 3.23 $K = [n,k] = [7,3]$

$$H = \begin{bmatrix} 1 & x_1 & x_2 & 1 & 0 & 0 & 0 \\ 0 & x_3 & x_4 & 0 & 1 & 0 & 0 \\ 1 & x_5 & x_6 & 0 & 0 & 1 & 0 \\ 1 & x_7 & x_8 & 0 & 0 & 0 & 1 \end{bmatrix} \qquad G = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & x_1 & x_3 & x_5 & x_7 \\ 0 & 0 & 1 & x_2 & x_4 & x_6 & x_8 \end{bmatrix}$$

A

a) $d(K) = 4$

$c = [0110011]$ pripada kodu

$$\begin{array}{lcccc} & 010 & x_1 & x_3 & x_5 & x_7 \\ \oplus & 001 & x_2 & x_4 & x_6 & x_8 \\ \hline & 011 & 0 & 0 & 1 & 1 \end{array}$$

$d = 2$

$x_1 = x_2 = x$

$x_3 = x_4 = y$

$x_5 \mathrel{!=} x_6$

$x_7 \mathrel{!=} x_8$

x y x_5 x_7	
0000	NE
0001	NE
0010	NE
0011	NE
0100	NE
0101	NE
0110	NE
0111	NE
1000	NE
1001	NE
1010	NE
1011	NE
1100	NE
1101	DA
1110	DA
1111	NE

$$H = \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

npr.:

$$H_1 = \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_2 = \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

b) $c' = [0110111]$

po H_2:

$$[0110111] \cdot \begin{bmatrix} 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = [0100] \quad 5.\ \text{bit}$$

$c = \boxed{0110011}$ po H_2

H_1:

$$[0110111] \cdot \begin{bmatrix} 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = [0100] \quad \text{po } H_1$$

ali kodna riječ je 011

zad 3.29

Zad. 3.24 $[n,k]=[7,4]$

$S(c')=?$ $\quad 2^r-1 \quad r=3$

$c'=[1101110]$ $\quad$ poruka 0110

$$H=\begin{bmatrix}1&0&1&0&1&0&1\\0&1&1&0&0&1&1\\0&0&0&1&1&1&1\end{bmatrix}$$

$$[1101110]\cdot\begin{bmatrix}1&0&0\\0&1&0\\1&1&0\\0&0&1\\1&0&1\\0&1&1\\1&1&1\end{bmatrix}=[0\,0\,1]$$

$c=[0101110]$

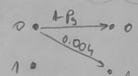
0111

Zad 3.25

$x=[10101\ldots]$ $\quad p_g=0.004$

$[n,k]=[7,4]$

Hamm: ul: 1010 $\longrightarrow$ izl: 1011010 $\quad$ 3 zašt. bita



Parni: 1010 $\longrightarrow$ 01010 $\quad$ 1 zašt. bit

kod hamm. $t=1$

parni $t=0$

$$P_H=\binom{7}{0}\cdot\underbrace{(1-p_g)}_{0.996}^7+\binom{7}{1}p_g(1-p_g)^6=0.999668$$

$$P_P=\binom{5}{0}(1-p_g)^5=0{,}980159$$

$$\Delta p=0.01951$$

Zad 3.26

$P_1=[1011]$

$P_2=[0110]$

$P_3=[1011]$

	1	2	3	4	5	6	7
A	0	1	1	0	0	1	1
B	1	1	0	0	1	1	0
C	0	1	1	0	0	1	1

a) ako ide kolona po kolona znači mogu max 3 bita biti kriva za redom jer se

b) 010,011,1 00,000,010,111,101

zad 3.29

zad 3.27

$$H = \begin{bmatrix} 1&1&0&1&1&0&0 \\ 1&0&1&1&0&1&0 \\ 0&1&1&1&0&0&1 \end{bmatrix}$$

a) $c_1 = [0010011]$. $\qquad c_2 = [0001111]$

$$[0010011] \cdot \begin{bmatrix} 1&1&0 \\ 1&0&1 \\ 0&1&1 \\ 1&1&1 \\ 1&0&0 \\ 0&1&0 \\ 0&0&1 \end{bmatrix} = [000] \qquad [0001111] \begin{bmatrix} 1&1&0 \\ 1&0&1 \\ 0&1&1 \\ 1&1&1 \\ 1&0&0 \\ 0&1&0 \\ 0&0&1 \end{bmatrix} = [000]$$

0010011
0001111

$d(K) = 3$

b) $c' = c + e$

$$S = c' \cdot H^T = (c+e) \cdot H^T = \underbrace{c \cdot H^T}_{\phi} + e \cdot H^T = e \cdot H^T$$

c)
0000000 → 000
0000001 → 001
0000010 → 010
0000100 → 100
0001000 → 111
0010000 → 011
0100000 → 101
1000000 → 110

zad 3.28

$X = \{x_0, \dots x_{127}\}$

$k = \log_2 128 = 7$ (broj podatkovnih bitova u poruci), $r = 4$. $n = 11$

poruka 11110110000

$S = 1010 \Rightarrow$ greška 5. bit

poruka ispr. = 11111110000

1111000

- ako ima 128 simbola, svaki simbol je kodiran sa 7 bitova (Shannon - Fano)
- pretpost. da je 1 poruka = 1 simbol = 7 bitova
- hamm 7 bitova poruke se štiti sa 4 bita = 11 bitova uk. za 1 poruku
  $n = 11$
- $r = n - k = 11 - 7 = 4$

Zad 3.29

16 poruka

16 simbola → 1 simbol = 4 bita = k

$n = r + k = r + 4$

Sindrom može adresirati sve pogr. do četverostrukih

$2^k = M \leq \frac{2^n}{\sum_{i=0}^{t} \binom{n}{i}}$ broj riječi našeg koda $\leq \frac{\text{broj riječi koje je moguće generirati s } n \text{ simbola}}{\text{zbroj mogućih kombinacija grešaka u svakoj riječi}}$

$\sum_{i=0}^{4} \binom{n}{i} \leq 2^{n-k} = 2^r$

$\binom{n}{0} + \binom{n}{1} + \binom{n}{2} + \binom{n}{3} + \binom{n}{4} \leq 2^r$

$\binom{r+4}{0} + \binom{r+4}{1} + \binom{r+4}{2} + \binom{r+4}{3} + \binom{r+4}{4} \leq 2^r$

najmanji r za koji ovo vrijedi je

$r = 11 \Rightarrow R = \frac{k}{n} = \frac{4}{15}$

Zad. 3.30

$B = 4\text{ kHz}$

$\frac{S}{N} = 30\text{ dB}$

$10 \log_{10} \frac{S}{N_0} = 30$

$\frac{S}{N_0} = 1000$

128 simbola $= 2^{7}$

7 bitova za 1 simbol

1 simbol = 7 bitova

1 simbol + zaštita = _ _ b _ b b b _ b b b = 11 bitova za 1 simbol

$X_{poruka} = \frac{C}{n} = \frac{B \cdot \log_2 (1 + \frac{S}{N})}{n} = \frac{4 \cdot 10^3 \cdot \log_2 (1 + 1000)}{11} = 3624{,}45 \frac{\text{poruka}}{s}$

zad 3.31   [15, k]   [15, 11]

CRC = ?

$\boxed{10101010101\ldots}$

$g(x) = x^4 + x^3 + 1$

$k = 15 - 4 = 11$

$\overset{10}{1}\overset{9}{0}\overset{8}{1}\overset{7}{0}\overset{6}{1}\overset{5}{0}\overset{4}{1}\overset{3}{0}\overset{2}{1}\overset{1}{0}\overset{0}{1}$

$d(x) = x^{10} + x^8 + x^6 + x^4 + x^2 + 1$

$x^{15-11} = x^4$

$x^4\, d(x) = x^{14} + x^{12} + x^{10} + x^8 + x^6 + x^4$

$(x^{14} + x^{12} + x^{10} + x^8 + x^6 + x^4) : (x^4 + x^3 + 1) = x^{10} + x^9 + x^5 + x^2 + 1$

$-x^{14} \pm x^{13} \pm x^{10}$

$x^{13} + x^{12} + x^8 + x^6 + x^4$

$-x^{13} \pm x^{12} \pm x^9$

$x^9 + x^8 + x^6 + x^4$

$x^9 \pm x^8 \pm x^5$

$x^6 + x^5 + x^4$

$x^6 + x^5 + x^2$

$x^4 + x^2$

$x^4 + x^3 + 1$

$r(x) = x^3 + x^2 + 1$ [1101]

---

zz $g(x) = x^4 + x^3 + 1$ G = ?

- zadnji red G matr. ima 15 bitova od toga je zauzeto $15 - 5^* = 10$
- zbog polinoma $g(x)$ koji sadrži x^4 potenciju ⇒ 5 bitova
- ovih 5^* je zbog polinoma $g(x)$ koji sadrži x^4 potenciju ⇒ 5 bitova "odlazi" na def polinoma unutar matrice
- pošto prvi dio G ima jed. matricu i 1. jedinica polinoma (x^4 ← to označ) je dio jedinič. matr. znači na jediničnu matricu se troši 11 bitova

⇒ $k = 11$

$$G = \left.\begin{bmatrix} & \vdots & \\ & \vdots & \\ \underbrace{0000\ldots0}_{I-11}\underbrace{11001}_{g(x) = x^4 + x^3 + 1} \end{bmatrix}\right\} 11$$

b) $d(x) = x^4 + x + 1$

$[15, 7]$

$$\underbrace{\left[\begin{array}{ccc|c} 1 & 0 & 0 & \\ 0 & 1 & 0 & \\ 0 & 0 & 1 & \\ & & \vdots & \\ 0 & 0 & 0 & 1 \end{array}\right]}_{7} \quad \underbrace{}_{x^{12}+x^9+x^8}$$

$0\ldots0\,1\,0\,0\,1\,1$

$x^{n-k} \cdot d(x) : g(x) - c(x)$

$r(x)$

$$c(x) = \overbrace{d(x) \cdot x^{n-k}} + r(x)$$

$$r(x) = \text{ost}\ \frac{x^{n-k} \cdot d(x)}{g(x)} =$$

$$(x^{12} + x^9 + x^8) : (\underbrace{x^8 + x^7 + x^6 + x^4 + 1}_{g(x)}) = x^4 + x^3$$

$x^{12} + x^{11} + x^{10} + x^8 + x^4$

$x^{11} + x^{10} + x^9 + x^4$

$x^{11} + x^{10} + x^9 + x^7 + x^3$

$x^7 + x^4 + x^3$

$$\underline{r(x) = x^7 + x^4 + x^3}$$

$$c(x) = x^{12} + x^9 + x^8 + x^7 + x^4 + x^3$$

$[0001, 0011, 1001, 1000]$

c) $c(x) = x^{14} + x^5 + x + 1$

$S(c'(x)) = S(c(x)) \cdot S(e(x))$

$S(c'(x)) = x^r \cdot c'(x) \mod (g(x))$

$$\text{ost}\ \frac{x^r \cdot c'(x)}{g(x)}$$

$$(x^{22} + x^{13} + x^9 + x^8) : (x^8 + x^7 + x^6 + x^4 + 1) = x^{14} + x^{13} + x^{11} + x^7 + x^5 + x^4 + x^2$$

$x^{22} + x^{21} + x^{20} + x^{18} + x^{14}$

$x^{21} + x^{20} + x^{18} + x^{14} + x^{13} + x^9 + x^8$

$x^{21} + x^{20} + x^{19} + x^{17} + x^{13}$

$x^{19} + x^{18} + x^{17} + x^{14} + x^9 + x^8$

$x^{19} + x^{18} + x^{17} + x^{15} + x^{11}$

$x^{15} + x^{14} + x^{11} + x^9 + x^8$

$x^{13} + x^9 + x^8 + x^7$

$x^{13} + x^{12} + x^{11} + x^9 + x^5$

$x^{12} + x^{11} + x^8 + x^7 + x^5$

$x^{12} + x^{11} + x^{10} + x^8 + x^4$

$x^{10} + x^7 + x^5 + x^4$

$x^{10} + x^9 + x^8 + x^6 + x^2$

$x^9 + x^8 + x^7 + x^6 + x^5 + x^4 + x^2 \ldots$

x^2

- za 1. poruku se od niza uzima 11 bitova jer se to jedino množeću s G može ostvariti
- definira se $d(x)$ od tih simbola
- dijeli se i ostatak je $r(x)$, tj. CRC, duljina CRC-a je $n-k$, jer je uk. duljina poruke 15, od toga korisnih bitova ima 11, $15-11=4$

zad 3.32

$g(x) = x^3+x^2+1$ $\quad [7,k] \Rightarrow k = 7-3 = \underline{\underline{4}}$ ← štiti se 4 bita poruke

← uk. ima 7 bitova

7 bitova $\underbrace{1001110}$0000110110 $= x^6+x^3+x^2+x$

$x^{7-4=3}\cdot(x^6+x^3+x^2+x) = x^9+x^6+x^5+x^4$

NE jer se množi s x^{n-k} samo kad imam poruku, a ovdje već imam kodnu riječ

$(x^9+x^6+x^5+x^4):(x^3+x^2+1) = x^6+x^5+x^4+x^3+x^2+x$

$x^9+x^8+x^6$

$x^8+x^5+x^4$

$x^8+x^7+x^5$

x^7+x^4

$x^7+x^6+x^4$

x^6

$x^6+x^5+x^3$

x^5+x^3

$x^5+x^4+x^2$

$x^4+x^3+x^2$

x^4+x^3+x

x^2+x ✓

zad 3.33 K [15, 7]

$g(x) = x^8+x^7+x^6+x^4+1$

a) ako je $x^{15}+1$ djeljivo s $g(x)$ bez ost. onda je $g(x)$ gen. polinom koda K

$(x^{15}+1):(x^8+x^7+x^6+x^4+1) = x^7+x^6+x^4+1$

$x^{15}+x^{14}+x^{13}+x^{11}+x^7$

$x^{14}+x^{13}+x^{11}+x^7+1$

$x^{14}+x^{13}+x^{12}+x^{10}+x^6$

$x^{12}+x^{11}+x^{10}+x^7+x^6+1$

$x^{12}+x^{11}+x^{10}+x^8+x^4$

$x^8+x^7+x^6+x^4+1$

0 //

x je

Zad 3.34 [8,3] $t = 1$

a)

e	s
00000000	00000
10000000	10100
01000000	01010
00100000	00101
00010000	10000
00001000	01000
00000100	00100
00000010	00010
00000001	00001

$n-k=5$

bez ϕ → 8 sindroma = 8 bitova uk., jer ispr. 1 gr. ⟹ $n=8$

$k=3$ $8-k=5$

$e \cdot H^T = s$

H^T $[5 \times 8]^T$ H^T_{8} $\underbrace{[\;]}_{5}$

H $[8 \times 5]$

— ako je sindrom od 5 bitova ($n-k$ = br. bit. sindroma) +0. koji je ako je sve ispr.

— u zad. je zadan br. sindroma 8 i rečeno je da ispravlja jednostruku pogr. ⟹ $n=8$ bitova ima poruka

— broj korisnih bitova (inform.) n − br. bit sindr. $= 3$ ⟹

G ima 3 retka i 8 stupaca

H^T ima 8 redaka i 5 stupaca ⟹ H ima 5 redaka i 8 stup

$$e \cdot H^T = e \cdot \begin{bmatrix} 1&0&1&0&0 \\ 0&1&0&1&0 \\ 0&0&1&0&1 \\ 1&0&0&0&0 \\ 0&1&0&0&0 \\ 0&0&1&0&0 \\ 0&0&0&1&0 \\ 0&0&0&0&1 \end{bmatrix} \Rightarrow H = \begin{bmatrix} 1&0&0&1&0&0&0&0 \\ 0&1&0&0&1&0&0&0 \\ 1&0&1&0&0&1&0&0 \\ 0&1&0&0&0&0&1&0 \\ 0&0&1&0&0&0&0&1 \end{bmatrix}$$

A^T I

$$G = \begin{bmatrix} 1&0&0&1&0&1&0&0 \\ 0&1&0&0&1&0&1&0 \\ 0&0&1&0&0&1&0&1 \end{bmatrix}$$

5 2 0

$g(x) = x^5 + x^2 + 1$

b) $[n,k] = [8,3]$

c) $g(x) = x^5 + x^2 + 1$

e) $c' = [01101011] \rightarrow s = [0\,0\,1\,0\,0]$ 6. bit greške

$c = [01101111]$

Zad 3.35 $[n, k]$

$g(x) = x^4 + x^2 + 1$

011010...

CRC = ?

$k = 2$

$d \in [0, 1)$

$d(x) = 1$

$d(x) \cdot x^4 = x^4 \quad x^{n-k}$

$x^4 : (x^4 + x^2 + 1) = 1$

$\underline{x^4 + x^2 + 1}$

$x^2 + 1 \qquad [0101]$

Zad 3.36

$g_1(x) = x^3 + x + 1$

$g_2(x) = x^3 + x^2 + 1$

Kodovi su ekvivalentni ako se jedan iz drugog mogu dobiti uzastopnom primj. slj. postupaka

(1) permutacijom

(2) zamjenom dvije pozicije

$K: Hamm[n, k] = [7, 4]$

$$H_{H} = \begin{bmatrix} 0&0&0&1&1&1&1 \\ 0&1&1&0&0&1&1 \\ 1&0&1&0&1&0&1 \end{bmatrix}$$

$$G_1 = \begin{bmatrix} & & & & & & \\ 0&1&0&1&1&0&0 \\ 0&0&1&0&1&1&0 \\ 0&0&0&1&0&1&1 \end{bmatrix} \underset{R_2 = R_2 - R_4}{\sim} \begin{bmatrix} 1&0&0&1&1&1&0 \\ 0&1&0&0&1&1&1 \\ 0&0&1&0&1&1&0 \\ 0&0&0&1&0&1&1 \end{bmatrix} \underset{R_1 = R_1 - R_4}{\sim} \begin{bmatrix} 1&0&0&0&1&0&1 \\ 0&1&0&0&1&1&1 \\ 0&0&1&0&1&1&0 \\ 0&0&0&1&\underbrace{0&1&1}_{A} \end{bmatrix}$$

$$H_1 = \begin{bmatrix} 1&1&1&0&1&0&0 \\ 0&1&1&1&0&1&0 \\ 1&1&0&1&0&0&1 \end{bmatrix} \quad \text{usporediti s } H_H = \begin{bmatrix} 0&0&0&1&1&1&1 \\ 0&1&1&0&0&1&1 \\ 1&0&1&0&1&0&1 \end{bmatrix}$$

$$H_1 \sim \begin{bmatrix} 0&0&0&1&1&1&1 \\ 0&1&1&1&0&1&0 \\ 1&0&1&0&0&1&1 \end{bmatrix} \sim \begin{bmatrix} 0&0&0&1&1&1&1 \\ 0&1&1&0&1&0&1 \\ 1&0&1&0&0&1&1 \end{bmatrix} \sim \begin{bmatrix} 0&0&0&1&1&1&1 \\ 0&1&1&0&0&1&1 \\ 1&0&1&0&1&0&1 \end{bmatrix}$$

G_1 i Hamm su ekvivel.

$$G_2 = \begin{bmatrix} & & & & & & \\ & & & & & & \\ 0&0&1&1&0&1&0 \\ 0&0&0&1&1&0&1 \end{bmatrix} \sim \begin{bmatrix} 0&1&0&1&1&1&0 \\ 0&0&1&0&1&1&1 \\ 0&0&0&1&1&0&1 \end{bmatrix} \sim \begin{bmatrix} 1&0&0&0&1&1&0 \\ 0&1&0&0&0&1&1 \\ 0&0&1&0&1&1&1 \\ 0&0&0&1&1&0&1 \end{bmatrix}$$

$$\underbrace{}_{A}$$

$$H_2 = \begin{bmatrix} 1&0&1&1&1&0&0 \\ 1&1&1&0&0&1&0 \\ 0&1&1&1&0&0&1 \end{bmatrix} \quad ; \quad H = \begin{bmatrix} 0&0&0&1&1&1&1 \\ 0&1&1&0&0&1&1 \\ 1&0&1&0&1&0&1 \end{bmatrix}$$

$$H_2 \sim \begin{bmatrix} 0&0&1&1&1&0&1 \\ 0&1&1&0&0&1&1 \\ 1&0&1&1&0&1&0 \end{bmatrix} \sim \begin{bmatrix} 0&0&0&1&1&1&1 \\ 0&1&1&0&0&1&1 \\ 1&0&1&0&1&0&1 \end{bmatrix}$$ devin

zad 3.37

$g(x) = x^{10} + x^7 + x^6 + x^4 + x^2 + 1$

$[21, 11]$

da bi bio gen. polinom mora biti djelitelj $\underbrace{x^{21}+1}$ to se dobije kao x^7+1

$(x^{21}+1) : (x^{10}+x^7+x^6+x^4+x^2+1) = x^{11} + x^8 + x^7 + x^2 + 1$

$x^{21}+x^{18}+x^{17}+x^{15}+x^{13}+x^{11}$

$x^{18}+x^{17}+x^{15}+x^{13}+x^{11}+1$

$x^{18}+x^{15}+x^{14}+x^{12}+x^{10}+x^{8}$

$x^{17}+x^{14}+x^{13}+x^{12}+x^{11}+x^{10}+x^{8}+1$

$x^{17}+x^{14}+x^{13}+x^{11}+x^{9}+x^{7}$

$\cancel{x^{12}}+x^{10}+\cancel{x^{9}}+\cancel{x^{8}}+x^{7}+x^{2}+1$

$\cancel{x^{12}}+x^{9}+x^{8}+x^{6}+x^{4}+x^{2}$

$x^{10}+x^{7}+x^{6}+x^{4}+x^{2}+1$

0

$h(x) = ?$

$x^n - 1 = g(x) \cdot h(x)$

$h(x) = (x^n - 1) : g(x) = x^{11} + x^8 + x^7 + x^2 + 1$

zad 3.38

$K, [n,k] = [15, 11]$

i) $C_1 = [111111111111111]$

npr. $C_2 \in K^{\perp}$; $C_2 = [110000000000000]$ ← riječ parne težine

$C_1 \cdot C_2 = 0$

ii) Zbrojiti riječi C_1 i C_2, zarotirati i dobije se polinom

$$\begin{array}{r} 111111111111111 \\ \oplus\ 001111111111000 \\ \hline 110000000000111 \end{array}$$

2 mjesta

$$\begin{array}{ccccc} 1 & 1 & 1 & 1 & 1 \\ x^4 & x^3 & x^2 & x^1 & x^0 \end{array}$$

$$g(x) = x^4 + x^3 + x^2 + x^1 + 1$$

provjera je li to doista polinom toga koda tako da je $(x^{15}+1) : g(x)$ bez ostatka (onda je $g(x)$ polinom)

$$(x^{15}+1) : (x^4+x^3+x^2+x+1) = x^{11} + x^{10} + x^6 + x^5 + x + 1$$

$$\begin{array}{l}
\cancel{x^{15}} + x^{14} + x^{13} + x^{12} + x^{11} \\ \hline
x^{14} + x^{13} + x^{12} + x^{11} + 1 \\
\cancel{x^{14}} + x^{13} + x^{12} + x^{11} + x^{10} \\ \hline
x^{10} + 1 \\
\cancel{x^{10}} + x^9 + x^8 + x^7 + x^6 \\ \hline
x^9 + x^8 + x^7 + x^6 + 1 \\
x^9 + x^8 + x^7 + x^6 + x^5 \\ \hline
x^5 + 1 \\
\cancel{x^5} + x^4 + x^3 + x^2 + x \\ \hline
x^4 + x^3 + x^2 + x + 1 \\
x^4 + x^3 + x^2 + x + 1 \\ \hline
0
\end{array}$$

Zad 3.39

$[n,k] = [?, 11]$

a) $n = 15$ $\quad [15, 11]$ $\quad R = \frac{11}{15} = 0.7\dot{3}\dot{3}$

b) $g(x) = ?$ $\quad g(x)$ sadrži $x^4 + 1$

$c_1 = [001111111111000]$

$c_2 = [111111111111111]$

$$x^{15} + 1 = (x+1)(x^2+x+1)(x^4+x+1)(x^4+x^3+1)\cdot(x^4+x^3+x^2+x+1)$$

$$
\begin{array}{ll}
c_1 & 001111111111000 \\
\otimes & 111111111111111 \\
c_2 & \\
\hline
 & \underbrace{11}_{2}0000000000\overset{\xleftarrow{}}{11}\!\!\!\!{}_{2}\,1
\end{array}
$$

$$\overset{11111}{g(x) = x^4 + x^3 + x^2 + x + 1}$$

jos treba $(x^{15}+1) : g(x)$ pa ostatak mora biti ϕ, no neda mi se to sada rješavati

c) $C = [00000000111]$

$$\left.\begin{array}{l} d(x) = x^2 + x + 1 \\ x^{n-k} = x^4 \end{array}\right\} x^6 + x^5 + x^4$$

$$(x^6 + x^5 + x^4) : (x^4 + x^3 + x^2 + x + 1) = x^2 +$$

$$\frac{x^6 + x^5 + x^4 + x^3 + x^2}{x^3 + x^2}$$

$CRC = [1100]$

d) K linearan $[n,k] \rightarrow K^\perp\ [n, n-k]$

$$h(x) = (x^{15}+1) : (x^4 + x^3 + x^2 + x + 1) = x^{11} + x^{10} + x^6 + x^5 + x + 1$$

$\cancel{x^{15}} + x^{14} + x^{13} + x^{12} + x^{11}$

$x^{14} + x^{13} + x^{12} + x^{11} + 1$

$x^{14} + x^{13} + x^{12} + \cancel{x^{11}} + x^{10}$

$x^{10} + 1$

$x^{10} + x^9 + x^8 + x^7 + x^6$

$x^9 + x^8 + x^7 + x^6$

$x^9 + x^8 + x^7 + x^6 + x^5$

x^5

$x^5 + x^4 + x^3 + x^2 + x$

$x^4 + x^3 + x^2 + x$

$x^4 + x^3 + x^2 + x + 1$

1 ostatak

$h(x) = x^{11} + x^{10} + x^6 + x^5 + x + 1$

$K^\perp = [15, 4]$

$d^* = [0001] = 1$

zad 3.39

d) $r(x) = \text{ost}\left[\frac{x^{11} \cdot d(x)}{h(x)}\right]$ jer je $K^{\perp}$

$x^{11} : (x^{11} + x^{10} + x^{6} + x^{5} + x + 1) = 1$

$r(x) = x^{10} + x^{6} + x^{5} + x + 1$

$= [10001100011]$

Zad 3.40

$n=7$

$c=[1111000]$

$x^7+1 = (x+1)(x^6+x^5+x^4+x^3+x^2+x+1)$.

$g(x)=1$ je generirajući polinom za svaki ciklički kod $[n,n]$

$\hookrightarrow$ dovršiti još