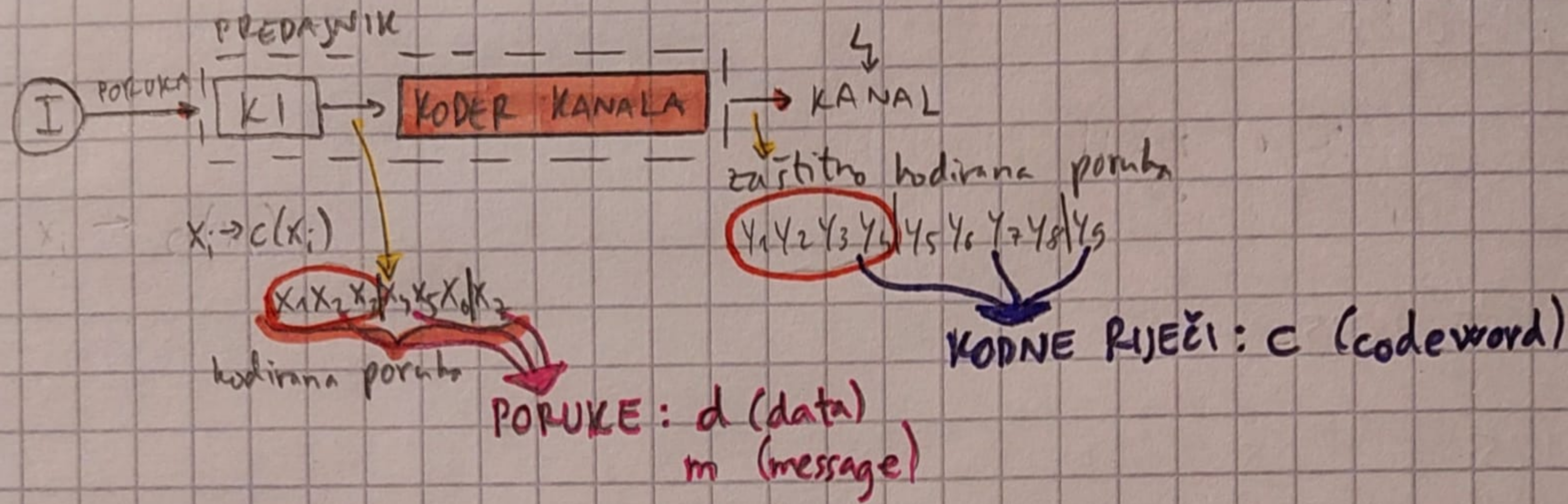
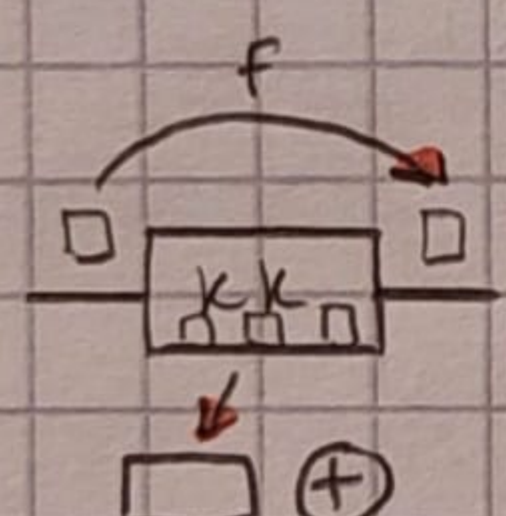
ZAŠTITNO KODIRANJE



- binarni blok kodovi → pojmovi

- linearni bin. blok kodovi → kodiranje, G → generiranje matrica koda
  → dekodiranje, H → matrica provjere pariteta

- kodovi: 1) paritetno kodiranje
  2) Hammingovo kodiranje
  3) ciklični kodovi
  } kodovi bez memorije
  4) konvolucijski kodovi → turbo kodovi } kodovi s memorijom



- kodovi (struktura kodne riječi): 1) linearni
  2) nelinearni

- ispravljanje pogreške (R_x, dekoder) → prijemna strana

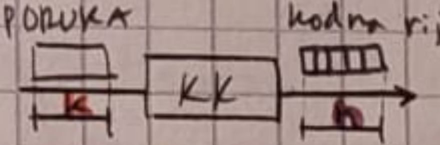
1) FEC (Forward Error Correction)

- kodovi za otkrivanje i ispravljanje pogrešaka
- odrađuje se u dekoderu

2) BEC (Backward Error Correction)

- kodovi za otkrivanje pogrešaka
- odrađuje se na R_x
- ponovno slanje kodne riječi za ispravljanje pogrešaka
- postupci ARQ (Automatic Repeat ReQuest)

- bitni pojmovi:
  a) oznaka koda, K
  b) abeceda koda, F_2
  ↳ $F_2 = \{0, 1\}$ → binarni kodovi
  c) blok kôd, K
  ↳ K je blok kôd ako su sve njegove kodne riječi jednake duljine



k = duljina poruke, n = duljina kodne riječi

$k \leq n$

$\boxed{R = \frac{k}{n}}$ → kodna brzina (code rate)

Pr1) $K_1 = \begin{cases} 000 \\ 011 \\ 101 \\ 110 \end{cases}$

↳ 4 kodne riječi, n = 3 (paritet)

$K_2 = \begin{cases} 00000 \\ 01101 \\ 10110 \\ 11011 \end{cases}$ $d(K_2) = 3$

n = 5

d) Hammingova udaljenost koda, d = br. pozicija u kojima se rekе dvije kodne riječi razlikuju

↳ minimalna Hammingova udaljenost d(K)

↳ $\boxed{d(K) = \min_{x,y \in K} \{ x \neq y \mid d(x,y) \}}$

↳ definira svojstva koda

Pr2) $x = [00000]$, $y = [01101]$ } $d(x,y) = 3$

e) zapis koda (n, M, d(K))

↳ n = duljina kodne riječi

↳ d(K) = min. Hamm. udaljenost

↳ M = ukupni br. kodnih riječi

$\boxed{M = 2^k}$

f) otkrivanje i ispravljanje pogrešaka

↳ otkrivanje (s): $\boxed{d(K) \geq s + 1}$

Pr3) $K_1 = \begin{cases} 000 \\ 011 \\ 101 \\ 110 \end{cases}$

↳ paritet

d(K) = 2

s = 1

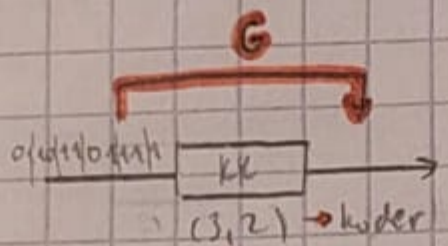
↳ ispravljanje (t): $\boxed{d(K) \geq 2t+1}$

$t = \left\lfloor \frac{d(K)-1}{2} \right\rfloor$

Pr4) $K = \begin{Bmatrix} 000 \\ 011 \\ 101 \\ 110 \end{Bmatrix}$ (sve udaljenosti 2)

$d(K) = 2$
$t = 1$

- $d(K)$ definira s, t
- (n,k) = dimenzija koda
  ↳ definira M



$K_1 = \begin{cases} 00 \to 000 \\ 01 \to 011 \\ 10 \to 101 \\ 11 \to 110 \end{cases}$ (d iznad lijevog stupca, c iznad desnog)

- osnovni zadatak teorije kodiranje: $\boxed{M = f(n,d)}$
  A (u knjizi oznaka)

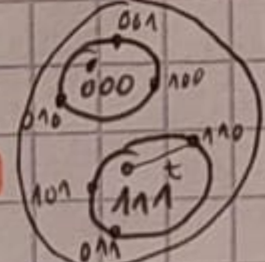
g) kugla kodne riječi

Pr5) Pretp. da kod $K = \{\overbrace{000, 111}^{2}\}$.

$M = 2 = 2^k$
$n = 3,\ k = 1$

* 000	111
001	110
010	101
100	011

$t = 1$



DEKODIRANJE

1) princip najbližeg susjeda *
2) dekodiranje standardnim nizom
3) sindromsko dekodiranje

h) perfektan kod = ako sfere koje čine kodn. riječi pokrivaju sve druge kodne riječi

↳ npr. neperfektan kod:

$$\boxed{M < \frac{2^n}{\sum_{i=0}^{t}\binom{n}{i}}}$$

$$\boxed{M = \frac{2^n}{\sum_{i=0}^{t}\binom{n}{i}}}$$

Pr6) Postoji li bin. kod K koji može ispraviti jednostruku pogrešku i koji pritom ima 52 kodne riječi (svaka duljine 9)?

$K = ?$
$t = 1$
$M = 52$
$n = 9$

$$M \leq \frac{2^9}{\sum_{i=0}^{9} \binom{9}{i}} = \frac{2^9}{1+9} = \frac{2^9}{10} = 51.2 \Rightarrow M = 51 \Rightarrow \text{ne postoji kod}$$

Pr7) $(7,4) \rightarrow t=1$ perf. kod?

$n = 7$
$k = 4$
$M = 2^4 = 16$

$$M \leq \frac{2^7}{1+7} = 2^4 = 16 \Rightarrow \text{perf. kod}$$

i) ekvivalencija kodova: **a)** kod K_2 se može dobiti iz K_1 invertiranjem bitova na jednoj ili više pozicija u kodnim riječima

b) zamjenom bitova u kodnim riječima

$K_1 \longrightarrow K_2 \longrightarrow$ Hamm. udaljenost zadržana

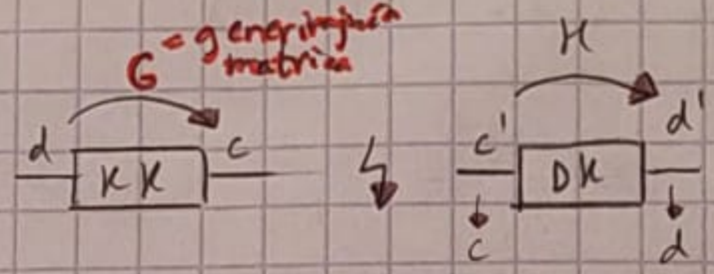
Pr8)

$$K_2 = \begin{array}{c} 00 \rightarrow \\ 01 \rightarrow \\ 10 \rightarrow \\ 11 \rightarrow \end{array} \begin{Bmatrix} 00\ 000 \\ 01\ 101 \\ 10\ 110 \\ 11\ 011 \end{Bmatrix} \xrightarrow{a)\ 0\to 1,\ 1\to 0} K_3 = \begin{Bmatrix} 0\ 0\ 1\ 0\ 0 \\ 0\ 0\ 0\ 1\ 1 \\ 1\ 1\ 0\ 0\ 0 \\ 1\ 1\ 1\ 1\ 1 \end{Bmatrix}$$

b)

- $G \rightarrow H$ ✓
- ~~$H \rightarrow G$~~

PARITETNO KODIRANJE



- $(k+1, k)$
- parni paritet: $d = [1\,0\,1]$
  $c = [1\,0\,1 \vdots 0]$
- neparni paritet: $d = [1\,0\,1]$
  $c = [1\,0\,1 \vdots 1]$
- vertikalna i horizontalna provjera zalihosti

$d_1 = [a\ b\ c]$ prep. $Z = R_1 + R_2 + R_3 = a+d+b+e+c+f = c_1 + c_2$
$d_2 = [d\ e\ f]$

paritet

a	b	c	c_1
d	e	f	c_2
R_1	R_2	R_3	Z

Pr1) Na ulazu kodera kanala koji koristi parit. kodiranje horiz. i vert. zalihosti pojavljuju se $d_1 = [1\,1]$, $d_2 = [0\,1]$. Primljena kodna riječ je 100011101. Koristeći v. i h. provjeru zalihosti odredite na kojem mjestu se nalazi pogr. u primljenoj kodnoj riječi.

$d_1 = [1\,1]$
$d_2 = [0\,1]$
$c' = [1\,0\,0 | 0\,1\,1 | 1\,0\,1]$

greška

1	0	0
0	1	1
1	0	1

Pr2) $K = \{000, 111\}$ **a)** Odredite vjer. pogr. dekodiranja.

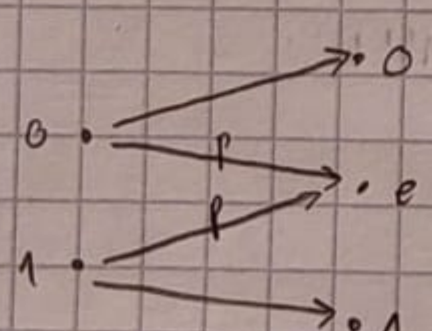
$P_{pd} = ?$

000	111
001	011
010	101
100	110

zadano
$P_{err/bit} = 0.2 = p$

$P_{pd} = \binom{3}{2} \cdot p^2 \cdot (1-p)^1 + \binom{3}{3} \cdot p^3 = 0.104$

b)



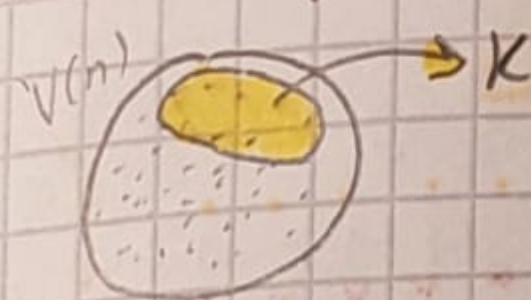
$P_{pd} = ?$

000	111
00e	|
0e0	=
e00	|
ee0	
0ee	
e0e	
eee	eee

$P_{pd} = \binom{3}{3} \cdot p^3$

LINEARNI BINARNI BLOK KODOVI

- vektorski prostor, $V(n)$



a) $X = [x_1\ x_2 \dots x_n]$

b)

x	y	x+y	xy
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

c) neutralni element: "+" $\rightarrow 0$ $\quad \boxed{-1 = 1}$
"·" $\rightarrow 1$

d) $x = [x_1\ x_2 \dots x_n]$
$y = [y_1\ y_2 \dots y_n]$

$a \rightarrow$ skalar

$\boxed{x+y = [x_1+y_1\ \ x_2+y_2\ \dots\ x_n+y_n]}$

$\boxed{a \cdot x = [ax_1\ \ ax_2\ \dots\ ax_n]}$

Linearni binarni blok kod

Za kod K definiran kao podskup v. prostora $V(n)$, kažemo da je linearan ako vrijedi:
1) $\forall x, y \in K \quad x+y \in K$
2) a skalar, $a \cdot x \in K$, $\forall x$, $a \in \{0, 1\}$
3) $[000 \dots 0] \in K$

- $K = \begin{cases} 00 & 000 \\ 01 & 101 \\ 10 & 110 \\ 11 & 011 \end{cases}$ 3) ✓ 2) ✓ 1) ✓

Težina kodne riječi $W(x)$ je br. pozicija u kodnoj riječi gdje su jedinice

Pr 1) $x = [0011]$ $\quad w(x) = 2$ $\qquad d(x, y) = w(1111) = 4$
$y = [1100]$

$\boxed{d \Leftrightarrow w \Rightarrow d(x, y) = w(x+y)}$

$d \xrightarrow{G} \boxed{K} \to c$

G = generirajuća matrica koda K
↳ čine ju bazni vektori

$G = [\ ]_{k \times n}$

$C = d \cdot G$

Pr2) $K = \left\{ \begin{matrix} 00000 \\ 01101 \\ 10110 \\ 11011 \end{matrix} \right.$ $\begin{matrix} \leftarrow 00 \\ \leftarrow 01 \\ \leftarrow 10 \\ \leftarrow 11 \end{matrix}$

$n=4$
$k=2$
$t=1$

baza vektorskog prostora

$d \to G \Rightarrow c$

$x_3 = a x_1 + b x_2$

$$d \left\{ \begin{matrix} 00 \\ 01 \\ 10 \\ 11 \end{matrix} \right. \cdot \overset{G_1}{\begin{bmatrix} 0 & 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 \end{bmatrix}} = \overset{c}{\left\{ \begin{matrix} 00000 \\ 10110 \\ 01101 \\ 11011 \end{matrix} \right.}$$

↳ promijenjen redoslijed kodnih riječi

$G_1 = \begin{bmatrix} 0 & 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 \end{bmatrix}$

$G_2 = \begin{bmatrix} 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 & 1 \end{bmatrix}$ → dobili bi isto

- standardni oblik generirajuće matrice G: $G = [I_k \mid A] \to c = [d \mid p]$

matrica zaštite

zalihosni bitovi

↳ zaštitni bitovi

$\boxed{G \to H}$

Pr2) $G = \left[\begin{array}{cc|ccc} 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 \end{array} \right] = [I \mid A]$

$d \to G \to c$	
00 →	00000
01 →	01101
10 →	10110
11 →	11011

Matrice G_1 i G_2 su **ekvivalentne** ako se G_2 može dobiti iz G_1 na slj. način:

1) zamjena redaka u G_1
2) zamjena stupaca u G_1
3) zbrajanje redaka u G_1

Pr3) Zadan je lin. bin. blok kod kod kojeg je $n \geq 1$. Odredite k i d.

$K = \left\{ \begin{matrix} 00\ldots0 \\ \underbrace{11\ldots1}_{n} \end{matrix} \right\}$ $M = 2 = 2^k$

$k = 1$
$d = n$

Pr4) $(n, k) = (7, 4)$

$G = [\qquad]_{4 \times 7}$

- zapis <u>lin.</u> bin. blok koda: $[n, k, d] \to [n, k]$
↳ inače (n, M, d)

DEKODIRANJE linearnih binarnih blok kodova

1) princip najbližeg susjeda, $d(K)$
2) standardni niz
3) sindromsko dekodiranje → H

- vektor pogreške kodne riječi, $e(c')$
- standardni niz
- razred standardnog niza
- matrica provjere pariteta H, $G \leftrightarrow H$ → ortogonalnost → dualni kod, $K^{\perp}$ → linearni kod
- sindrom kodne riječi $s(c') \leftrightarrow K$

- vektor pogreške

poslana kodna riječ, primljena kodna riječ

$\forall c \in K$, $R_x : c'$ ⇒ $\boxed{e = c \oplus c'}$

Pr1) $c = [1011]$, $c' = [1111]$ } $e = [0100]$

- **standardni niz**
  - tablica → 1. redak čine kodne riječi K
    → 1. stupac su vektori pogreške
    → ostali retci/stupci : $c \oplus e$ ▼

(brojevi iz **Pr2)** s prethodne strane)

$\underline{\underline{e}}$					$s(c)$
00000	01101	10110	11011	←	000
00001	01100	10111	11011	←	001
00010	01111	10111	11001	←	010
00100	01001	10010	11111	←	100
01000	00101	11110	10011	←	101
10000	11101	00110	01011	←	110

na kojem bitu je greška

razred (closet)

011 nema
111 nema

perf. kod uključuje sve sindrome koda ⇒ ovo nije perf. kod

Pr2) $R_x : c' = [10010]$ → std. niz → pozicija → vektor pogreške

$e(c') = [00100]$

▼ $c = e + c'$ ⇒ $c = [10110]$ (iz stupca se može dobiti također)

Pr2.2) $G = \left[\begin{array}{cc|ccc} 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 \end{array}\right]$, A = the right block

$H = \left[\begin{array}{cc|ccc} 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{array}\right]$

$c = c' \rightarrow s(c) = c \cdot H^T = 0$

$c' = c + e \rightarrow s(c') = s(c+e) = \overset{0}{s(c)} + s(e) = s(e)$

⇒ $\boxed{s(c') = s(e)}$

$G \cdot H^T = 0$

$H^T = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

- **matrica provjere pariteta H**

  - ortogonalnost : $x, y \rightarrow x \perp y \Rightarrow x \cdot y = 0$

$K^{\perp} = \{ y \in V(n) \mid \forall x \in K, \underbrace{y \cdot x = 0}_{} \}$ ⇒ dualni kod koda K

→ ako je c=d, vrijedi ovo

  - $K^{\perp}$: generirajuća matrica H = [ ] (→ vektori baze) ⟶ matrica provjere pariteta : G
  - K : generirajuća matrica G ⟶ matrica provjere pariteta : H
  - $\forall c \in K$: $\boxed{c \cdot H^T = 0}$ $\boxed{G \cdot H^T = 0}$
  - $\boxed{G = [I_k | A]} \Rightarrow \boxed{H = [-A^T | I_{n-k}]} = \boxed{[A^T | I_{n-k}]}$ (→ u literaturi; $-1 \equiv 1$)

  - pretp. $G \neq [I_k | A] \equiv$ ZAMJENA REDAKA, ZAMJENA STUPACA, DODAVANJE JEDNOG REDKA DRUGOM → $G = [I_k | A]$

$H \neq [A^T | I_k] \leftarrow H = [A^T | I_k]$

$G \cdot H^T = 0$ ✓

  - G → H ✓
  - H ✗ G

- **sindrom primljene kodne riječi**

T_x : c
R_x : c'

$\boxed{s(c') = c' \cdot H^T}$

→ pozicija sindroma u matrici H daje mjesto pogreške u c'

Pr3) Zadan je lin. bin. kod K.

$[n, k] = [7, 4]$

lin.	nelin.
[n,k],	(n,k)

↓
d(K) → s, t

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ & \ddots & & & 1 & 1 & 0 \\ & & & & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix} = [I_k | A]$$

(A = zadnja tri stupca)

$M = 2^k = 2^4 = 16$ kodnih riječi

a) $H = [A^T | I] = \begin{bmatrix} 1 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}$

b) $H \rightarrow d(K) = 3$ ⊕

→ najm. br stupaca koje trebamo zbrojiti po modulu 2 da bi dobili 0 (zbrajamo po retcima)

a) Odredite H.
b) Odredite d(K).
c) Provjerite je li kod perfektan.
d) Odredite kodnu riječ koja je poslana (c) ako je primljena $c' = [1110101]$

c) $M \leq \frac{2^n}{\sum_{i=0}^{t} \binom{n}{i}} = \frac{2^7}{\binom{7}{0} + \binom{7}{1}} = \frac{2^7}{8} = 2^4 = 16$ ✓ perf. kod

d) $s(c') = ?$

$s(c') = c' \cdot H^T$

$$[1110101] \cdot \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = [001]$$

↓
H → pozicija → mjesto pogr.

$c = [1110100]$

Pr4) Zadan je lin. bin. blok kod $[n,k]=[7,4]$. Jedini mogući v. pogr. koji se mogu pojaviti za zadani kod su e. Odredite matricu provjere pariteta H.

$[n,k]=[7,4]$

$H=[?]$

$* \cdot H^T = ▼$ (množenje matrica)

$$e = \left\{ \begin{array}{lcl} 0000000 & \rightarrow & 000 \\ 1000000 \, * & \rightarrow & 001 \; ▼ \\ 1100000 & \rightarrow & 010 \\ 1110000 & \rightarrow & 011 \\ 1111000 & \rightarrow & 100 \\ 1111100 & \rightarrow & 101 \\ 1111110 & \rightarrow & 110 \\ 1111111 & \rightarrow & 111 \end{array} \right.$$

$$H^T = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

→ gradimo matricu H^T red po red

Pr5) Zadan je kod K čije su kodne riječi oblika $c=[c_1 c_2 c_3 c_4]$ i čija je matrica provjere pariteta H. Odredite sve kodne riječi zadanog koda.

$$H = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{2\times 4}$$

$r=2$

$n=4$

$k=2$

$c \cdot H^T = 0$

DOBAR NAČIN

$$[c_1 c_2 c_3 c_4] \cdot \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 0 & 1 \\ 1 & 1 \end{bmatrix} = 0$$

$$\left\{ \begin{array}{l} c_1 + c_2 + c_4 = 0 \\ c_2 + c_3 + c_4 = 0 \end{array} \right\} \rightarrow c_1 = c_3$$

MOGUĆNOSTI:

c_1	c_2	c_3	c_4
0	0	0	0
0	1	0	1
1	0	1	1
1	1	1	0

} K

POGR. NAČIN

$H = [A^T \mid I_{n-k}]$

↓

$G = [I_k \mid A]$

$$H = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}$$

$$H' = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}$$

$$\underbrace{H'' = \begin{bmatrix} \underbrace{1 \; 1}_{A^T} & | & 1 & 0 \\ 1 \; 1 & | & 0 & 1 \end{bmatrix} \rightarrow G'' \begin{bmatrix} 1 & 0 & | & 1 & 1 \\ 0 & 1 & | & 1 & 1 \end{bmatrix}}_{G \cdot H^T = 0}$$

$$G' = \begin{bmatrix} 1 & 0 & | & 1 & 1 \\ 1 & 1 & | & 0 & 1 \end{bmatrix}$$

$$G = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$$

KRIVO

KRIVO

npr. da smo rekli:

$$G = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix}$$

d		c
00	→	0000
01	→	0101
10	→	1011
11	→	1110

ili

$$G = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$$

d		c
00	→	0000
01	→	1110
10	→	1011
11	→	0101

Pr6) Odredite sve k.r. lin. bin. blok koda sa zadanom H.

$$H = \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

$n=7$

$c=[c_1 c_2 c_3 c_4 c_5 c_6 c_7]$

$c \cdot H^T = 0$

$$\left\{ \begin{array}{l} c_1 + c_2 + c_4 + c_7 = 0 \\ c_4 + c_5 + c_7 = 0 \\ c_1 + c_3 + c_4 + c_7 = 0 \\ c_6 + c_7 = 0 \end{array} \right.$$

$c_2 = c_1 + c_4 + c_7$

$c_5 = c_4 + c_7$

$c_3 = c_1 + c_4 + c_7$

$c_6 = c_7$

$\dot{c}_1$	c_2	c_3	$\dot{c}_4$	c_5	c_6	$\dot{c}_7$
0	0	0	0	0	0	0
0	1	1	0	1	1	1
0	1	1	1	1	0	0
0	0	0	1	0	1	1
1	1	1	0	0	0	0
1	0	0	0	1	1	1
1	0	0	1	1	0	0
1	1	1	1	0	1	1

Pr7) Zadan je kod K s matricom H. Ako je primljena k.r. $c' = [110110]$. Odredite najvjer. poslanu kodnu riječ.

$c' = [110110]$

$$H = \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 \end{bmatrix}$$

$c = ?$

$$s(c') = [110] = c' \cdot H^T = [110110] \cdot \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

$c = [100110]$

$\left.\begin{array}{l} V: G = [I \mid A] \\ c' = [d \mid p] \\ c = ? \end{array}\right\}$ uvijek provjeriti vrijedi li: $c' \cdot H^T = 0$!

- **Hammingov kôd**

- $r \geq 2$; $r \in \mathbb{N}$
- $V(r)$
- $H = [\;\;]_{r \times 2^r - 1}$
  $1 \ldots 2^r - 1$ (stupci)
- dim.: $[2^r - 1, 2^r - r - 1]$
- perf. kod
- $t = 1$, $s = 1, 2$

Pr8)

$$H^T = \begin{array}{c|c} \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} & \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \end{array} \end{array} \Rightarrow H = \begin{bmatrix} 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 \end{bmatrix}$$

Pr9) $d = [1101]$

Hamm; $n = 7$, $k = 4$

$c \to d$

$c \to p \to 2^i$, $i = 0, 1, 2 \ldots$

BITOVI ZAŠTITE U C (1, 2, 4, 8, 16, ...)

$\hookrightarrow$ paritetni bitovi na pozicijama 2^i

I

$c = [p_1, p_2, d_1, p_3, d_2, d_3, d_4]$

_	_	1	_	1	0	1	
1		1		1		1	$\to p_1$
	0	1			0	1	$\to p_2$
			0	1	0	1	$\to p_3$

$c = [1\,0\,1\,0\,1\,0\,1]$

II

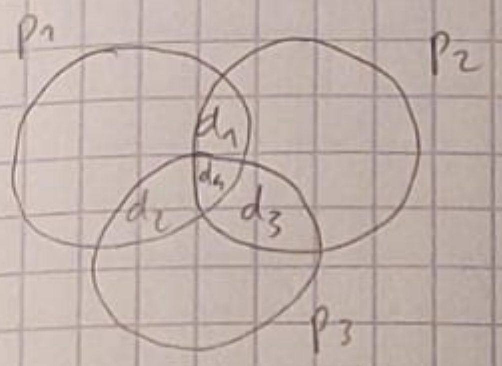
$$\left.\begin{array}{l} p_1 = d_1 + d_2 + d_4 \\ p_2 = d_1 + d_3 + d_4 \\ p_3 = d_2 + d_3 + d_4 \end{array}\right\} *$$

$c = d \cdot G$

$$G = \begin{array}{c} \begin{array}{ccccccc} p_1 & p_2 & d_1 & p_3 & d_2 & d_3 & d_4 \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ \cancel{0} & \cancel{1} & \cancel{0} & \cancel{1} & \cancel{0} & \cancel{1} & \cancel{0} \\ 1 & 1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix} \end{array} \begin{array}{c} \\ d_1 \\ d_2 \\ d_3 \\ d_4 \end{array}$$

$\oplus$

$c = dG = 1010101$

* 

→ nijedan paritetni ne kontrolira neki dr. paritetni bit

Pr6) dekodiranje

I $c' = [1\,0\,1\,1\,1\,0\,1]$

$c = ?$

$$H^T = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$\oplus \oplus \oplus$

$s(c') = c' H^T = [1\,0\,0]$

$c' = [1\,0\,1\,0\,1\,0\,1]$

II 1 0 1 1 1 0 1

1 1 1 1 → 0
0 1 0 1 → 0
1 1 0 1 → 1

[1 0 0]
↳ pogr. na 4. bitu

(možemo ili I ili II)

- svopovite pogreške

$c = [c_1 \boxed{c_2\, c_3\, c_4} \ldots$

- interleaver (preplitanje bitova)

$a = [a_1 a_2 \ldots a_7]$ →
$b = [b_1 b_2 \ldots b_7]$ →
$c = [c_1 c_2 \ldots c_7]$ →

mem →

a_1	~~a_2~~	- - -	a_7
b_1	~~b_2~~	- - -	b_7
~~c_1~~	c_2	- - -	c_7

— | BMM | — | INT | →

$a_1 b_1 \boxed{c_1 a_2 b_2} c_2 a_3 b_3 c_3 \ldots$

CIKLIČNI KODOVI

- def: → linearan kod
  → ciklični posmak bilo koje kodne riječi cikličnog koda ponovno daje kodnu riječ iz K

Pr1)

0 1 1 1 0 ∈ K
1 1 1 0 0
1 1 0 0 1
1 0 0 1 1
0 0 1 1 1
0 0 0 0 0

TABL. FAKTORIZACIJE POLINOMA S MREŽE / IZ KNJIGE ⇒ moći ćemo ju imati na ispitu

- $a = [\overset{3}{1}\overset{2}{0}\overset{1}{1}\overset{0}{1}] \Rightarrow a(x) = x^3 + x + 1$
  ↳ polinomski zapis kodne riječi

* kao dodavanje 0 u zbroj

- $a = [a_{n-1}\ a_{n-2} \dots a_1\ a_0] \Rightarrow a(x) = a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \dots + a_1x + a_0$

$$\Rightarrow xa(x) = a_{n-1}x^n + a_{n-2}x^{n-1} + \dots + a_1x^2 + a_0x$$
$$= a_{n-1}\cdot x^n + \underbrace{a_{n-1}\cdot x^0}_{}{}^{*} + \underbrace{a_{n-1}\cdot x^0}_{} + a_{n-2}\cdot x^{n-1} + \dots + a_1x^2 + a_0x$$
$$= a_{n-1}\cdot(x^n+1) + a_{n-2}\cdot x^{n-1} + \dots + a_1x^2 + a_0x + a_{n-1}$$

$$\boxed{c \in K : c(x) = \text{ostatak dijeljenja } \frac{x \cdot a(x)}{x^n+1}}$$

$$a_{n-1}(x^n+1) + a_{n-2}x^{n-1} + \dots + a_1x^2 + a_0x + a_{n-1} : (x^n+1) = \dots = a_{n-2}x^{n-1} + \dots + a_1x^2 + a_0x + a_{n-1} = c(x)$$

Pr2) $a = [\overset{2}{1}\overset{1}{0}\overset{0}{1}] \to [011]$, $n = 3$

$a(x) = x^2 + 1$

$c(x) = \text{ost. } \frac{x \cdot a(x)}{x^n+1}$

$$x^3 + x : (x^3+1) = 1$$
$$\underline{x^3 + 1}$$
$$x + 1 \to [011]$$

- def: - $\forall\ a(x), b(x) \in K \Rightarrow (a(x) + b(x)) \in K$

  - $\boxed{r(x) \Rightarrow \text{ost. } \frac{r(x)\cdot a(x)}{x^n+1} \in K}$

- cikl. kod određen je u potpunosti **generirajućim polinomom $g(x)$**

- $g(x)$ - jedinstven
  - najmanji stupanj
  - $x^n + 1 = g(x) \cdot h(x)$
  - G
  - hardverski izgled kodera
  - utječe na dekodiranje

- $g(x) = g_{n-k} \cdot x^{n-k} + g_{n-k-1} \cdot x^{n-k-1} + \dots + g_1x + g_0$

- $g(x) = g_r x^r{}^{\star} + g_{r-1}x^{r-1} + \dots + g_1x + g_0{}^{\star}$ $\quad [r = n-k]$

★ → oni stupnjevi uvijek postoje u polinomu

$$\boxed{G = \begin{bmatrix} g_r & g_{r-1} & \dots & g_1 & g_0 & 0 & 0 & \dots & 0 \\ 0 & g_r & \dots & & g_1 & g_0 & 0 & & 0 \\ 0 & \dots & 0 & g_r & \dots & & g_1 & g_0 & 0 \\ 0 & \dots & & 0 & g_r & \dots & & g_1 & g_0 \end{bmatrix}}$$

Pr3) $x^5 + 1 = (x+1)(x^4+x^3+x^2+x+1)$

$g(x) = x+1$

$$G = \underbrace{\begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}}_{n} \Big\} k$$

→ nije matrica u std. obliku

Pr4) Koji od navedenih kodova su ciklični?

a) $K_1 = \{000, 100, 010\}$ ✗

b) $K_2 = \{000, 100, 010, 001\}$ ✗ (nije linearan jer zbrajanjem kodnih r. ne daje k.r.)

c) $K_3 = \{000, 111\}$ ✓

d) $K_4 = \{0000, 1010, 0101, 1111\}$ ✓

$G = [I_k \mid A]$ (svođenje na std. oblik)

Pr5) $g(x) = x^3+x+1 \rightarrow g = [1011]$

$n=7, k=4$

ciklični kod

gen. matr. u std obliku?

$$G = \left[\begin{array}{cccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1^{*} & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \end{array}\right] \oplus \quad \oplus \text{ zbog } * \sim \left[\begin{array}{cccc|ccc} 1 & 0 & 0 & 1^{*} & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \end{array}\right] \oplus \sim \left[\begin{array}{cccc|ccc} 1 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \end{array}\right]$$

I_k

$H = [A^T \mid I_{n-k}]$

- matrica provjere pariteta H
  - $g(x) \rightarrow G$
  - $h(x) \rightarrow H$

$$\boxed{x^n + 1 = g(x) \cdot h(x)}$$

$h(x) = h_k x^k + h_{k-1} x^{k-1} + \dots + h_1 x + h_0$

$$\boxed{H = \begin{bmatrix} h_0 & h_1 & \dots & h_{k-1} & h_k & \dots & 0 \\ & & \ddots & & & & \\ 0 & \dots & h_0 & \dots & h_{k-1} & h_k & \end{bmatrix}}$$

Pr6) Zadan je cikl. bin. kod K s oznakom [n,k]. Kod K sadrži kodnu riječ $c = [10101010] \in K$. Koliko najmanje smije iznositi k, a da bude zadržano svojstvo lin. koda?

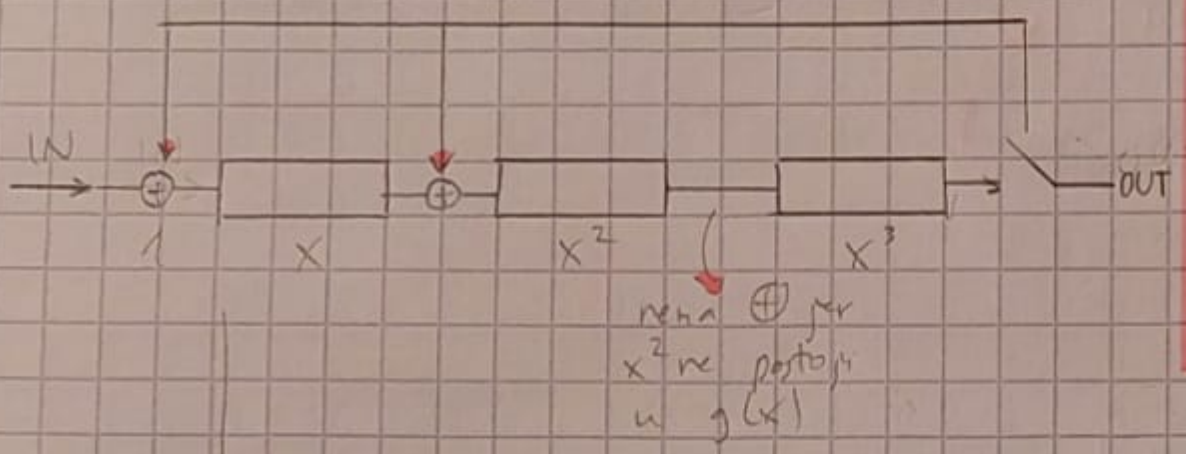
$n = 8$

$$K = \left\{ \begin{matrix} 10101010 \\ 01010101 \\ 00000000 \\ 11111111 \end{matrix} \right\} \; n = 4 = 2^k \Rightarrow k = 2$$

- $g(x) \rightarrow$ hardverska izvedba kodera

Pr7) $g(x) = x^3 + x + 1$

posmični reg. $\rightarrow \times 3$



$\rightarrow$ iščitavamo $g(x)$

Koder cikl. koda sastoji se od posmičnih registara i zbrajala po modulu 2. Broj posm. reg. odg. najvećem stupnju gen. polinoma. Svaki reg. (IN $\rightarrow$ OUT) ima ispred sebe zonu koju kontrolira. Ako u gen. pol. postoji stupanj koji je vezan uz taj reg., u to područje stavi modul po 2 zbrajalo, poveži ga s tim i s povratnom vezom.

Pr8) Odredite sve bin. cikl. kodove gdje je duljina k.r. 7 i koji sadrže k.r. 1111000.

$n = 7$

$c = [1111000] \rightarrow c(x) = x^6 + x^5 + x^4 + x^3$

$g(x) = ?$

$x^7 + 1 = g(x) \cdot h(x)$

$x^7 + 1 = (x+1)(x^3+x+1)(x^3+x^2+1)$

$g_1(x) = x + 1$

$g_2(x) = x^3 + x + 1 \quad \longrightarrow \quad x^{7-4} = x^3 \qquad 2$

$g_3(x) = x^3 + x^2 + 1$

$$\begin{array}{l} x^6 + x^5 + x^4 + x^3 : (x+1) = x^5 + x^3 \\ \underline{-x^6 - x^5} \\ 0 \quad 0 \quad x^4 + x^3 \\ \qquad \underline{-x^4 - x^3} \\ \qquad \quad 0 \end{array}$$

$$\begin{array}{l} x^6 + x^5 + x^4 + x^3 : (x^3 + x^2 + 1) = x^3 + x + 1 \\ \underline{+\cancel{x^6} + \cancel{x^5} \quad + x^3} \\ \qquad x^4 \quad 0 \\ \qquad \cancel{x^4} \quad x^3 + x \\ \qquad \quad \underline{x^3 + x^2 + 1} \\ \qquad \quad x^2 + x + 1 \end{array}$$

$G=[I|A] \rightarrow c=[\underbrace{d}_{\text{data}}|\overbrace{p}^{\text{paritetni bitovi}}]$

$d(x)=d_0+d_1x+d_2x\ldots$

$c=[d|p]$

↳ CRC (Cyclic Redundancy Check)

$c(x)=[d(x) \,\vdots\, r(x)]$

$$\boxed{r(x)=\text{ost.}\ \frac{x^{n-k}\,d(x)}{g(x)}}$$

Pr) Zadan je lin. cikl. kod kojem pripada k.r. $c=[011011]$.

a) Ispišite sve k.r. zadanog koda u bin. i polinomnom obliku.

b) Odredite gen. pol. $g(x)$.

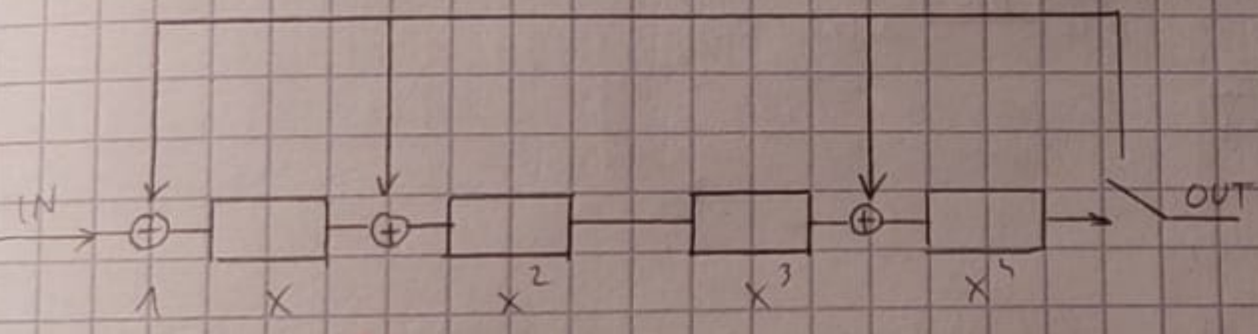
c) Kodirajte poruku $d=[11]$ koristeći CRC.

$c=[011\ 011]$
$n=6$

a)
$$K=\left\{\begin{array}{cccccc} 5&4&3&2&1&0\\ 0&1&1&0&1&1\\ 1&1&0&1&1&0\\ 1&0&1&1&0&1\\ 0&0&0&0&0&0 \end{array}\right. \qquad K=\left\{\begin{array}{l} x^4+x^3+x+1\\ x^5+x^4+x^2+x\\ x^5+x^3+x^2+1\\ 0 \end{array}\right.$$

b) $g(x)=x^{4}+x^3+x+1$; $4 \rightarrow n-k \Rightarrow k=2$

$$\boxed{g(x)=x^{n-k}+\ldots+1}$$



c) $d=[\overset{1}{1}\overset{0}{1}] \rightarrow d(x)=x+1$

$r(x)=\text{ost.}\ \frac{x^{n-k}\,d(x)}{g(x)}=\text{ost}\ \frac{x^4\cdot(x+1)}{x^4+x^3+x+1}$

$k=2$

$(x^5+x^4):(x^4+x^3+x+1)=x$

$\cancel{x^5+x^4}+x^2+x$

$r(x)=x^2+x \rightarrow r=[0110] \Rightarrow c=[11\,\vdots\,\underbrace{0110}_{\text{CRC}}]$

ISPITNI

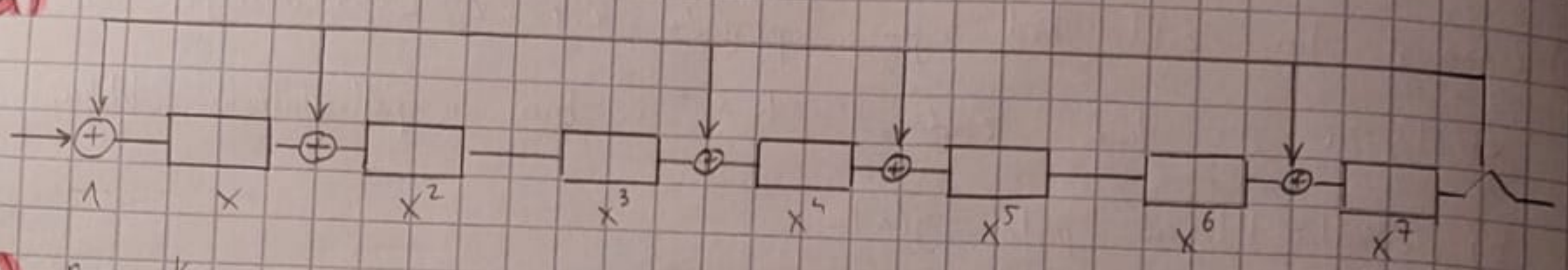
Pr 10) Gen. polinom $g(x)=x^7+x^6+x^4+x^3+x+1$ koristi se u najmanjem mogućem cikl. kodu $[n,k]$.

a) Nacrtajte koder.

b) Odredite kodnu brzinu zadanog koda.

c) Na ulaz kodera kanala ulazi slijed bitova 0110101... Odredite cikl. provjeru zalihosti za k.r. koja se prva pojavljuje na izlazu kodera kanala.

a)



b) $R=\frac{k}{n}$

$n-k=7$

$x^n+1=g(x)\cdot h(x)$

$x^n=g(x)\cdot h(x)$ (s ost. 1) *

$g=[1\,1\,0\,1\,1\,0\,1\,1]$

x^n
9 8 7 6 5 4 3 2 1 0
1 0 0 0 0 0 0 0 0 0
1 1 0 1 1 0 1 1
0 1 0 1 1 0 1 1 1 0
1 1 0 1 1 0 1 1
1 1 0 1 1 0 1 0
1 1 0 1 1 0 1 1
0 0 0 0 0 0 0 1 *

$\Rightarrow x^9+1$

$n=9$
$k=2$

$d=[0\,1]$
$d(x)=1$

c) $d(x) \xrightarrow{d}$ KK $\xrightarrow{c=[d|r]} c(x)$

$c'(x) \xrightarrow{c'}$ DK $\xrightarrow{d'} d'(x)$

CRC: $r(x)=\text{ost.}\ \dfrac{x^{n-k}d(x)}{g(x)}$

$c'(x)=c(x)+e(x)$

$s(c'(x))=s(c(x)+e(x))=\overset{0}{s(c(x))}+s(e(x))=s(e(x)) \Rightarrow s(c'(x))=s(e(x))$

$s(c'(x))=\text{ost.}\ \dfrac{x^{n-k}\cdot c'(x)}{g(x)}$

Pr 11) Zadan je cikl. kod $[7,4]$ s gen. polinomom $g(x)=x^3+x+1$.

a) Je li $c(x)=x^5+x^4+x^3+x$ kodna riječ zadanog koda?

b) Odredite k.r. koja je poslana ako je primljena k.r. $c'(x)=x^4+x+1$.
Napomena: zad. riješite koristeći matr. provjere pariteta i nakon toga provjerite koristeći sindromom primljene kodne riječi.

a) $x^5+x^4+x^3+x : (x^3+x+1) = x^2+x$

$x^5 \quad\ \ + x^3+x^2$

x^4+x^2+x

x^4+x^2+x

$0 \Rightarrow$ DA

- 2. način: $s(c(x)) = \text{ost.} \dfrac{x^{n-k} \cdot c(x)}{g(x)} = \text{ost.} \dfrac{x^3(x^5+x^4+x^3+x)}{x^3+x+1}$

$$\begin{array}{l} x^8+x^7+x^6+x^4 : (x^3+x+1) = x^5+x^4 \\ \cancel{x^8}+x^6+x^5 \\ \hline \quad x^7+x^5+x^4 \\ \quad \cancel{x^7}+x^5+x^4 \\ \hline \qquad 0 \end{array}$$

$0 \rightarrow$ DA

b) $c = [0010011]$

$x^7+1 = g(x) \cdot h(x) \Rightarrow h(x) = x^4+x^2+x+1$

$$H = \begin{bmatrix} 1 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 & 1 \end{bmatrix}$$

$$s(c') = [0010011] \cdot \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = [100] \Rightarrow \text{postoji greška na 1. poziciji}$$

$c = [1010011]$

PROVJERA

$c'(x) = c(x) + e(x)$

$s(c'(x)) = s(e(x))$ cikl. posmak k.r. za n-k bitova ulijevo

$s(c'(x)) = \text{ost.} \dfrac{x^{n-k} \cdot c'(x)}{g(x)} = \dfrac{x^3(x^4+x+1)}{g(x)}$

$$\begin{array}{l} x^7+x^4+x^3 : (x^3+x+1) = x^4+x^2 \\ \cancel{x^7}+x^5+\cancel{x^4} \\ \hline \quad x^5+x^3 \\ \quad \cancel{x^5}+\cancel{x^3}+x^2 \\ \hline \qquad x^2 \end{array}$$

$\Rightarrow s(c'(x)) = x^2$

$s(e(x)) = x^2$

$e(x)$	$s(e(x))$
—	0 0 0
1	$x+1$
x	⋮
x^2	⋮
x^3	⋮
x^4	
x^5	
x^6	x^2

$e(x) = 1 \rightarrow s(e(x)) = \text{ost.} \dfrac{x^3 \cdot 1}{x^3+x+1} = x+1$

$$\begin{array}{l} x^3 : (x^3+x+1) = 1 \\ \cancel{x^3}+x+1 \\ \hline \quad x+1 \end{array}$$

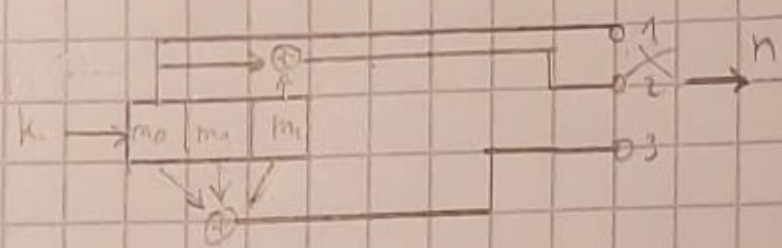
$c(x) = c'(x) + e(x)$

$c(x) = x^4+x+1+x^6$

$\quad = x^6+x^4+x+1 \rightarrow c = [1010011]$

KONVOLUCIJSKI KODOVI

- memorijski kodovi
- posmični registri (m stanja)
- mod 2 zbrajalo ⊕

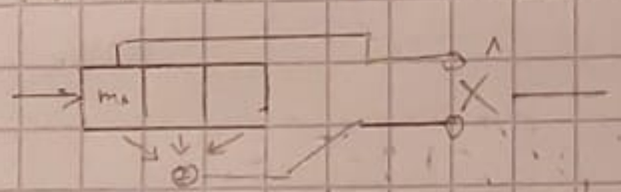


(n, k, L)

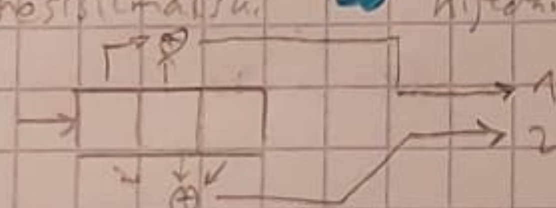
↳ granična duljina kodera: $\boxed{L = m + 1}$

- kodovi po izvedbi:

a) sistematski ➡ ulazni bit je ujedno i izlazni bit



b) nesistematski ➡ nijedan ulazni bit ne ide izravno na izlaz

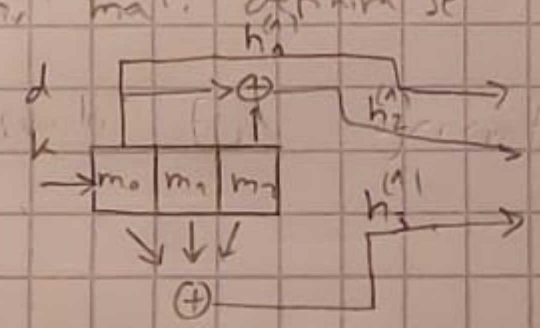


- gen. mat. definira se preko f-jskih generatora

$$\boxed{h_i^{(j)} = [h_{i0}^{(j)} \; h_{i1}^{(j)} \; \ldots]}$$

j → izlaz, i → ulaz, m

*



m_0 aktivan

$$h_1^{(1)} = [1\,0\,0]$$
$$h_1^{(2)} = [1\,0\,1]$$
$$h_1^{(3)} = [1\,1\,1]$$

$$\boxed{G = \begin{bmatrix} G_0 & G_1 \ldots G_m & \ldots & 0 \\ 0 & G_0 & G_1 \ldots G_m & 0 \\ 0 & \ldots & G_0 \; G_1 \ldots G_m \end{bmatrix}}$$

$$\boxed{G_l = \begin{bmatrix} h_1^{(1)} & h_1^{(2)} & \ldots & h_1^{(k)} \\ h_2^{(1)} & h_2^{(2)} & \ldots & h_2^{(k)} \\ & \vdots & & \end{bmatrix}, \; l = 0, \ldots, m}$$

$$\begin{array}{cccccc} & 0 & 1 & & & \\ [0\,1] & \downarrow & \downarrow & & & \end{array}$$

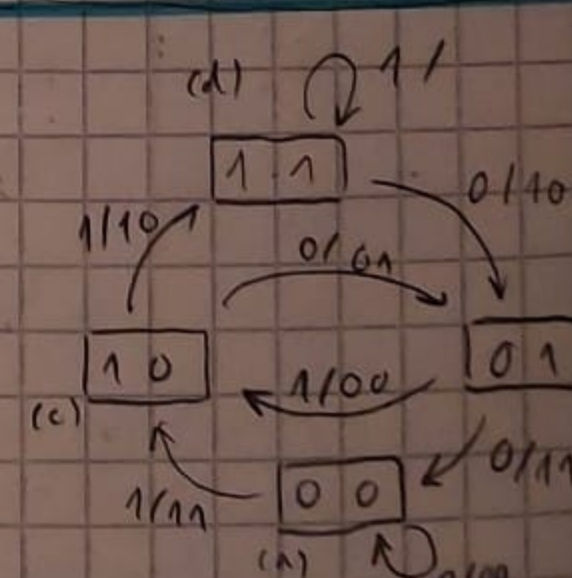
$$G = \begin{bmatrix} 111 & 001 & 011 & 000 & 000 \\ 000 & \boxed{111} & 001 & 011 & 000 \\ 000 & 000 & 111 & 001 & 011 \end{bmatrix}$$

$$c = d \cdot G = [1\,0\,1] \cdot G = [\underbrace{111}_{1 \cdot 111 + 0 \cdot 000 + 1 \cdot 000} \; 001 \; 100 \; 001 \; 011]$$

★ $d = [0\,1\,1]$ (ulaz), $c = d \cdot G$

$c = [000 \; 111 \; 110 \; 010\,011]$

- prikaz konvolucijskih kodova
  - → ② stablasti dijagram
  - → ③ rešetkasti dijagram ➡ dekodiranje (Viterbi)
  - → ① dijagram stanja ➡ T(D) → Hammingova udaljenost



$$G = \begin{bmatrix} 11 & 01 & 11 & 00 & 00 & 00 \\ 00 & 11 & 01 & 11 & 00 & 00 \\ 00 & 00 & 11 & 01 & 11 & 00 \\ 00 & 00 & 00 & 11 & 01 & 11 \end{bmatrix}$$

$d = [0\,1\,0]$

$c = [00\,11\,01 \,\vdots\, 11\,00]$

$d = [0\,1\,0\,1]$

$c = [00\,11\,01\,00\,01\,11]$

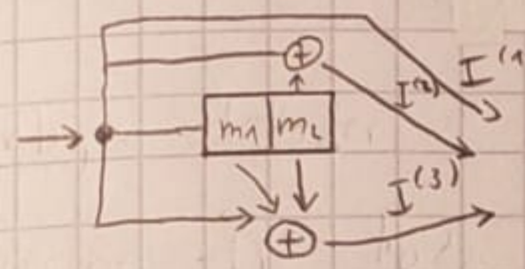
$$\begin{array}{c} \;\;a\;c\;d \\ d = [0\,1\,1] \end{array}$$

$c = [00\,11\,10 \,\vdots\, 01\,11]$

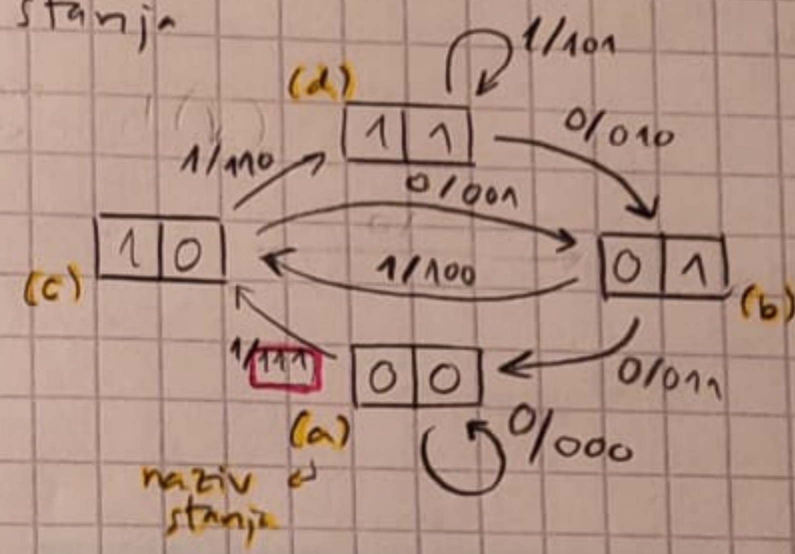
$d = [0\,1\,1\,0]$

$c = [00\,11\,10 \,\vdots\, 10\,11\,00]$

① *



— 4 stanja



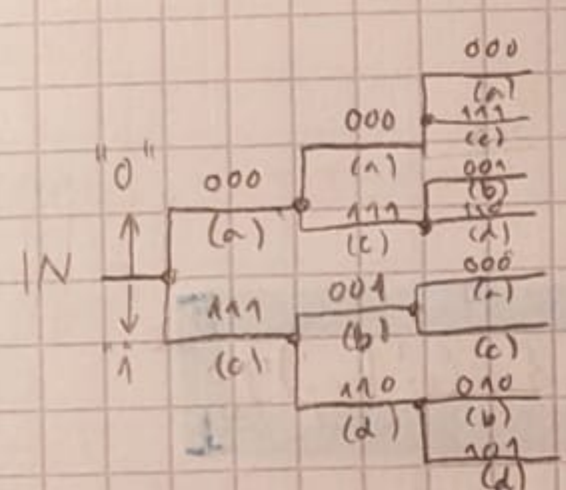
— za svaki ulazni takt generiram 3 izlaza

↳ ULAZ IZLAZ

x / yz

---

②



— Hammingova udaljenost

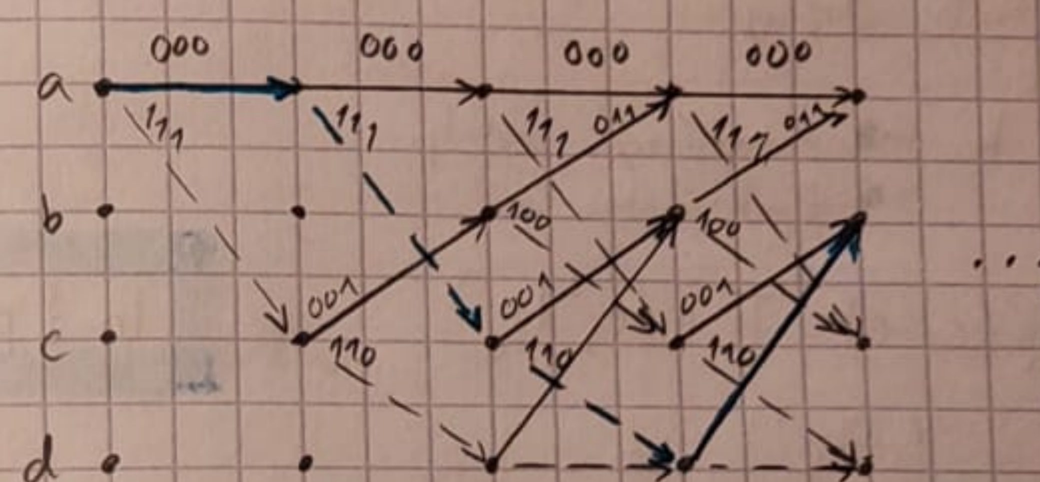
— Euklidska

$L = m + 1 = 3$

2^L

— Viterbi: nakon 3. koraka, put s najboljom metrikom preživljava

③ — spojimo sva stanja koja imaju isti izlaz i isto stanje



$\longrightarrow$ "0"

$\dashrightarrow$ "1"

$c' = [000 \ 111 \ 110 \ 010 \ \ldots]$

↳ dekodiranje

- prijenosna funkcija konvolucijskog koda
  - performanse koda → min. udaljenost koda

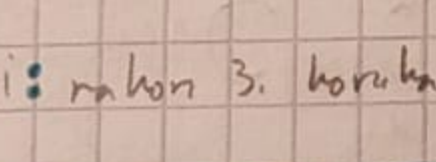
OZNAKA ZA P.F. — N na one grane gdje ulaz 1

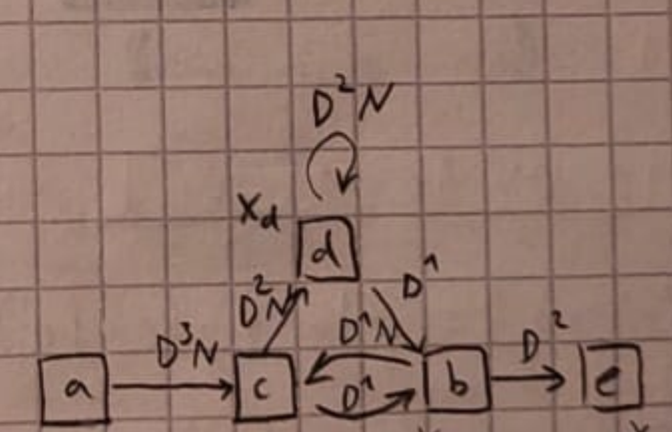
→ $T(D)$, $T(D, N)$

- dijagram stanja → $T(D)$
  - ↳ stanje (a) podijeliti na 2 stanja:
    - → a (ulaz), e (izlaz)
  - ↳ stanjima a, b, c, d, e pridodajemo odr. var.

x_a, x_b, x_c, x_d, x_e

↳ na prijelazima između pojedinih stanja s var. D^i označiti težinu izlaza





$$\boxed{T(D) = \frac{x_e}{x_a} = f(D)}$$

$x_c = D^3 x_a + D x_b$ (1)

$x_b = D x_c + D x_d$ (2)

$x_d = D^2 x_c + D^2 x_d$ (3)

$x_e = D^2 x_b$ (4)

$(4) \to T(D) = \frac{D^2 x_b}{x_a} = \frac{D^2 \cdot D^4}{1 - 2D^2} = D^6 \cdot \frac{1}{1 - 2D^2} = *$

$(1) \to x_a = \frac{x_c - D x_b}{D^3}$

$(2) \to x_d = \frac{x_b - D x_c}{D}$ $(3) \to x_d = \frac{D^2 x_c}{1 - D^2}$

$x_c = \frac{x_b (1 - D^2)}{D}$

(1) $x_a = \frac{x_b (1 - 2D^2)}{D^4}$

$\frac{x_b}{x_a} = \frac{D^4}{1 - 2D^2}$

$\frac{1}{1 - x} = \sum_{n=0}^{\infty} x^n, \ |x| < 1$

$* = D^6 (1 + 2D^2 + 4D^4 + \ldots)$

$$T(D,N) = \frac{x_e}{x_a} = \frac{D^6 \cdot N}{1-2D^2N} = D^6N + 2D^8N^2 + \dots$$

$x_c = D^3 N x_a + D N x_b$ (1)

$x_b = D x_c + D x_d$ (2)

$x_d = D^2 N x_c + D^2 N x_d$ (3)

$x_e = D^2 x_b$ (4)

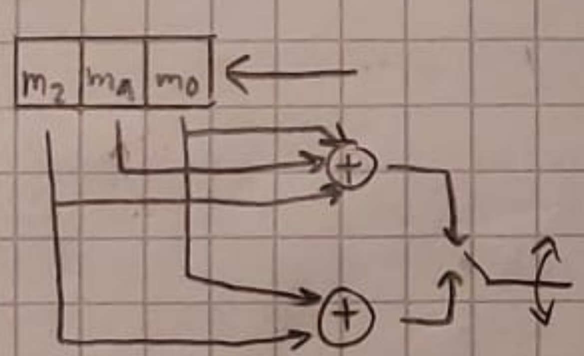
DEKODIRANJE K.K.

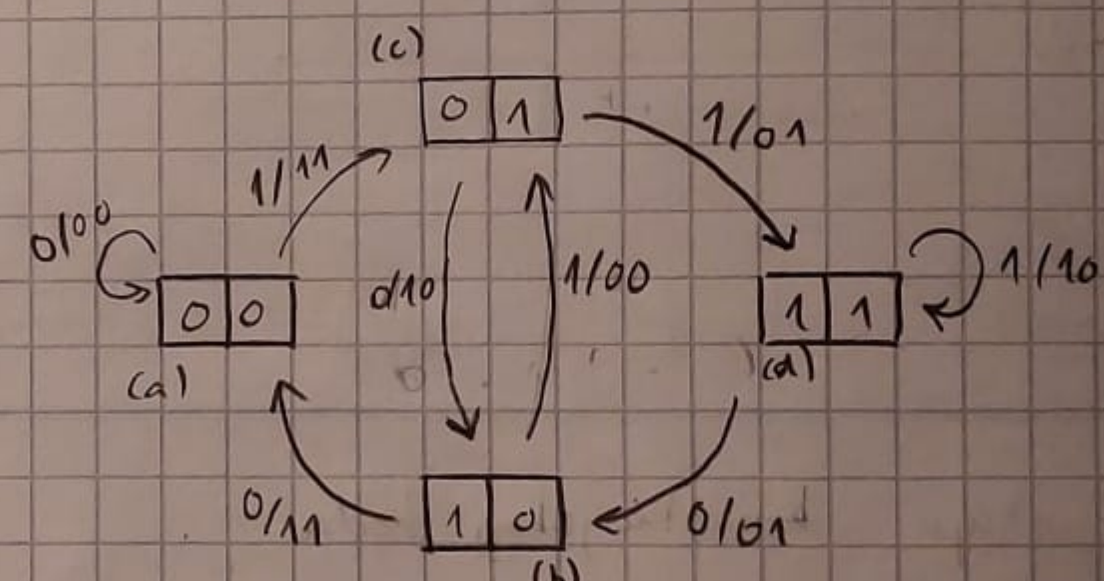
- rešetkasti dijagram
- metrika
  - Hamming. udalj.
  - Euklidska

$$p(c', c) = \prod_j p(c_j' \mid c_j)$$

$$\boxed{\ln\{p(c' \mid c)\} = \sum_j \ln\{p(c_j' \mid c_j)\}}$$

ZAD 1 Zadan je konv. koder (2, 1, 3) (izlaz ulaz stanje+1). Neka se na prijemnoj str. pojavljuje c'. Odredite najvj. poslanu kodnu riječ c.





$\boxed{n_2\ n_1}$

zadano $\begin{cases} P_b = 0.1 & \text{(vjer. pogr. bita)} \\ c' = [10\ 01\ 10\ 11\ 00] \end{cases}$

$c = ?$

① $\ln\{p(c', c)\} = \sum_j \ln\{p(c_j' \mid c)\}$

$\ln(p(0|0)) = \ln(p(1|1)) = \ln 0.9 = -0.11$

$\ln(p(0|1)) = \ln(p(1|0)) = \ln 0.1 = -2.3$

②

put	metrika	Hamm. udalj.
00 00 00 00 00	-12.05	5
00 00 11 10 11	-14.24	6
⋮		
00 11 10 11 00	-5.46	2

↳ najvjv. put

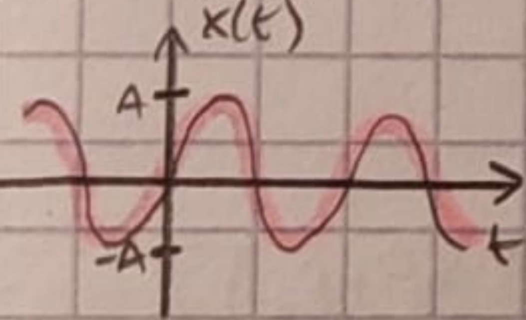
TURBO KODOVI

- 2 konv. kodera
- interleaver
- 4G

KOMUNIKACIJSKI KANALI U KONT. VREMENU

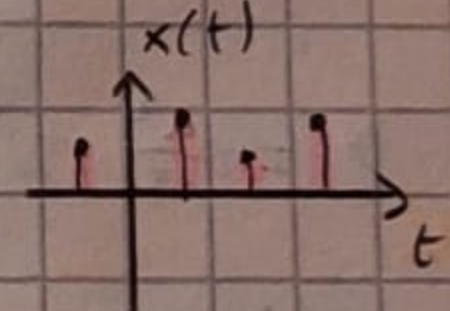
- signali
- linearni i vremenski komunikacijski sustavi
- AD pretvorba
- kapacitet komunikacijskih kanala u kont. vremena

- signal → pojava koja opisuje neku fiz. veličinu ($u(t), i(t)$)
  - → $x(t)$
    - ↳ $t \in \mathbb{R}$ periodični   neperiodični
  - → podjela: a) kontinuirani i diskretni u vremenu



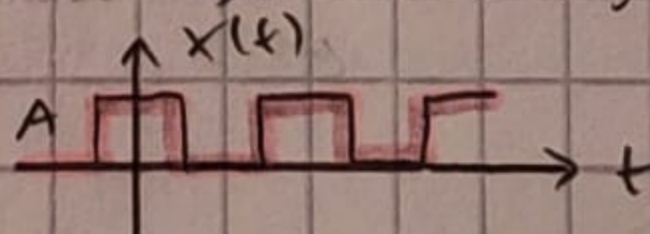
$x(t) \in (a, b)$
$x(t) = A \sin 2\pi f t \ [V]$
$x(t) \in [-A, A]$

$x[k]$, $x[kT]$
$t = kT$, $k \in \mathbb{Z}$



b) analogni i digitalni signali

$x(t) = A \cos 2\pi f t$
$x(t) \in (a, b)$



c) deterministički i slučajni
↳ sluč. procesi

$$E = \int_{-\infty}^{+\infty} R i^2(t)\, dt = \int_{-\infty}^{+\infty} \frac{u^2(t)}{R}\, dt, \quad [Ws]$$

$$P = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} R i^2(t)\, dt \quad [W]$$

PERIODIČNI SIGNALI

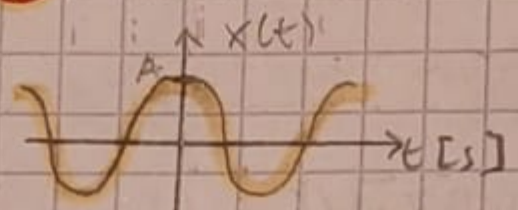
- $x(t) = x(t+T)$, $t \in \mathbb{R}$, $T = \text{konst}$ (period)

- spektar signala

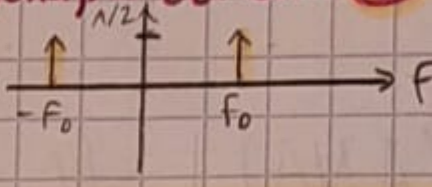
vrem. domena

t: $x(t) = A\cos 2\pi f t$, $\omega_0 = 2\pi f_0$ ⇒ kružna frekvencija

frekvencijska domena f $x(f)$



$\rightleftarrows$



$$x(t) = \sum_{k=-\infty}^{+\infty} c_k e^{jk2\pi f_0 t} \rightleftarrows x(f) = \sum_{k=-\infty}^{+\infty} c_k \delta(f - k f_0)$$

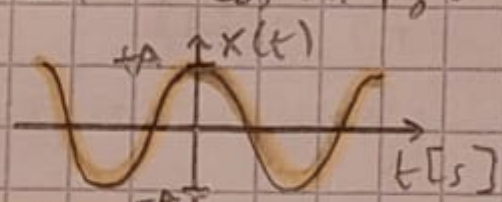
$$c_k = \frac{1}{T}\int_{-T/2}^{T/2} x(t) e^{-jk2\pi f_0 t} dt$$

DIRAC $\delta(t) = \begin{cases} \neq 0, & t = 0 \\ = 0, & t \neq 0 \end{cases}$

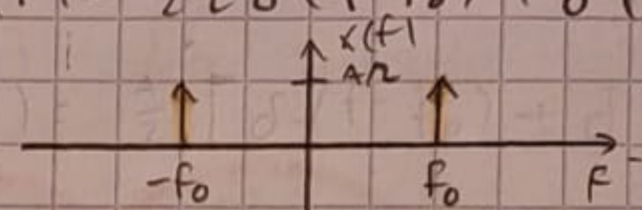
$$\int_{-\infty}^{+\infty} \delta(t) dt = 1$$

$$\int_{-\infty}^{+\infty} \delta(t - t_0) x(t) = x(t_0)$$

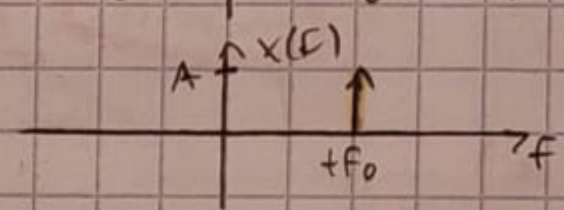
1) $x(t) = A\cos 2\pi f_0 t$, [V] $\qquad x(f) = \frac{A}{2}[\delta(f - f_0) + \delta(f + f_0)]$



$\rightleftarrows$



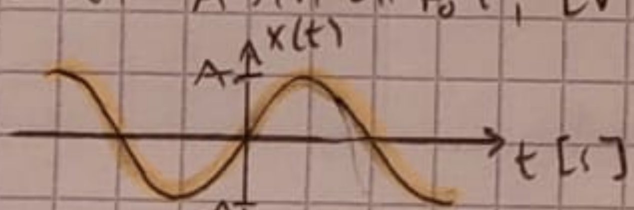
– dvostrani spektar

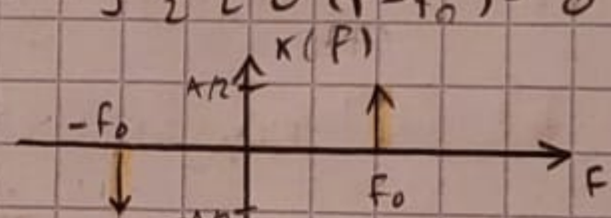


$R = 1\Omega$ → osim ako nije zadano drugačije

$P = \frac{A^2}{2}$

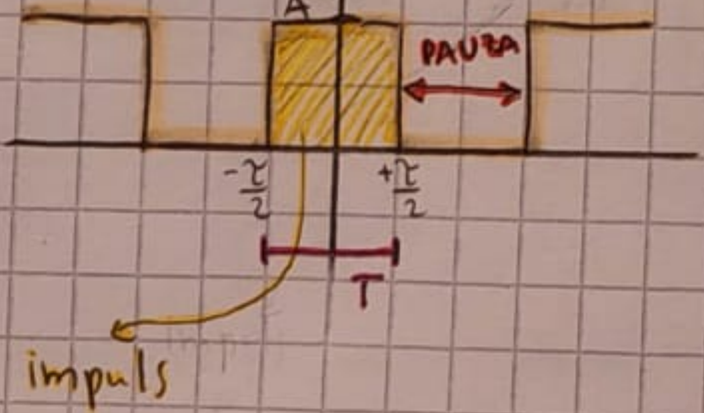
2) $x(t) = A\sin 2\pi f_0 t$, [V] $\qquad x(f) = -j\frac{A}{2}[\delta(f - f_0) - \delta(f + f_0)]$





$P = \frac{A^2}{2}$

- periodičan slijed pravokutnih impulsa



$$P = \frac{1}{T}\int_{-\tau/2}^{+\tau/2} |x(t)|^2 dt = \frac{1}{T}\int_{-\tau/2}^{+\tau/2} A^2 dt = \frac{A^2}{T} t \Big|_{-\tau/2}^{\tau/2}$$

$P = \frac{A^2}{T} \cdot \tau$ ⇒ srednja snaga periodičnog slijeda pravokutnih impulsa

$c_k = \frac{A\tau}{T} \frac{\left(\sin \frac{k\pi\tau}{T}\right)}{\frac{k\pi\tau}{T}}$ → osnovni član u spektru per. slijeda prav. impulsa

$c_0 = \frac{A\tau}{T}$

$T_0 = \frac{1}{f_0}$, $T = T_0$

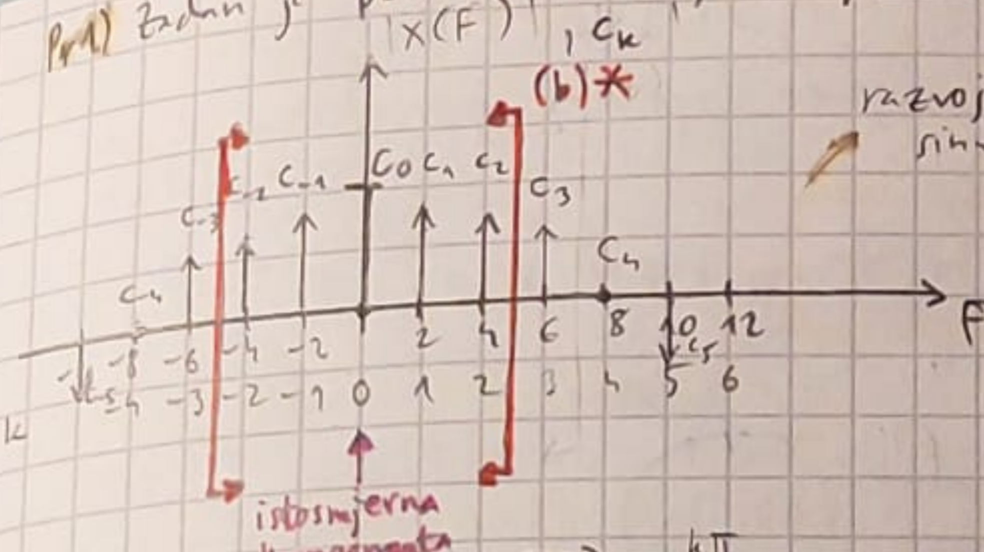
$\frac{\tau}{T - \tau} = \frac{1}{4} \to 4\tau = T - \tau$

$5\tau = T$

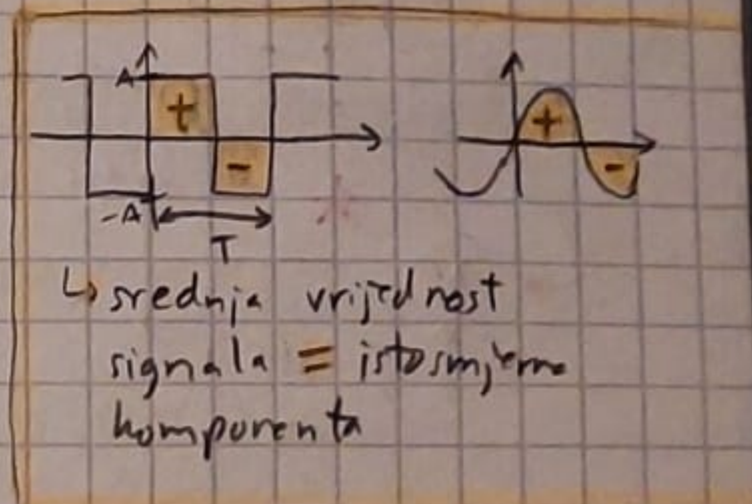
$\frac{\tau}{T} = \frac{1}{5}$

↳ $\frac{\tau}{T}$

Pr1) Zadan je periodičan slijed prav. impulsa amplitude $A = 1\,V$, $f = 2\,kHz$, $\frac{\tau}{T} = \frac{1}{4}$. Nacrtajte spektar zadanog signala.



- komponente spektra nalaze se na višekratnicima osnovne frekvencije signala
- $c_0 = \frac{1}{4}\,V$
- spektar per. slijeda prav. impulsa je beskonačan



↳ srednja vrijednost signala = istosmjerna komponenta

$$c_k = \frac{A\tau}{T} \cdot \frac{\sin\left(\frac{k\pi\tau}{T}\right)}{\frac{k\pi\tau}{T}} \quad \frac{k\pi\tau}{T} \rightarrow \frac{k\pi}{4}$$

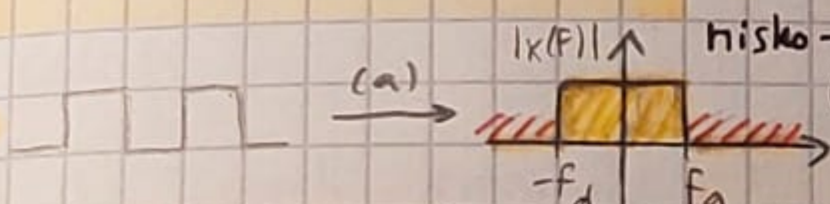
$c_2 = 1 \neq 0 \Rightarrow$ postoji

$c_5 < 0$

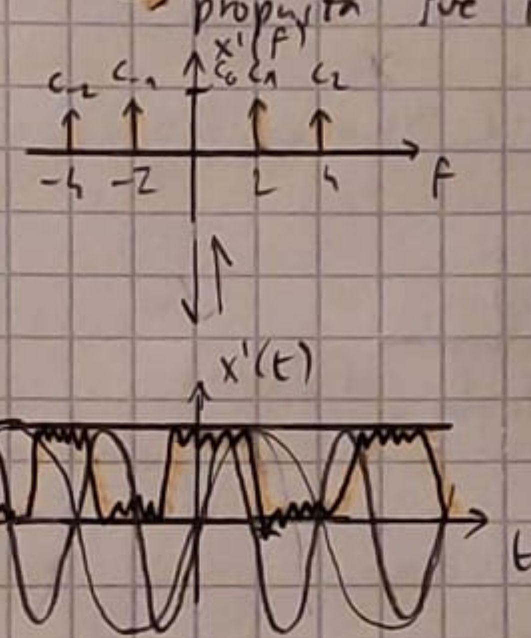
$$P = |c_0|^2 + 2 \cdot \sum_{k=1}^{\infty} |c_k|^2 \quad \rightarrow \text{za } R = 1\,\Omega \; *$$

niskopropusni filtar = put kanala, $|X(f)|$ → uklanja određene dijelove signala

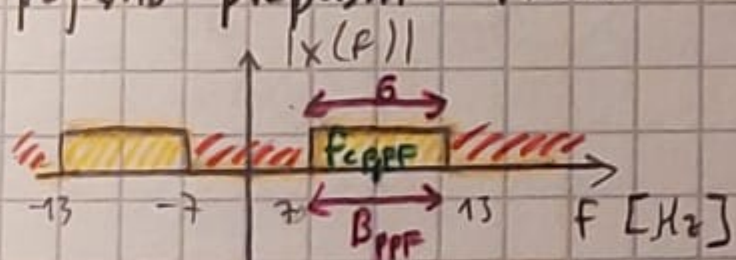
↳ propušta sve između donje i gornje gran. frekv.



$f_d = f_g = 5\,kHz$ *



pojasno-propusni filtar

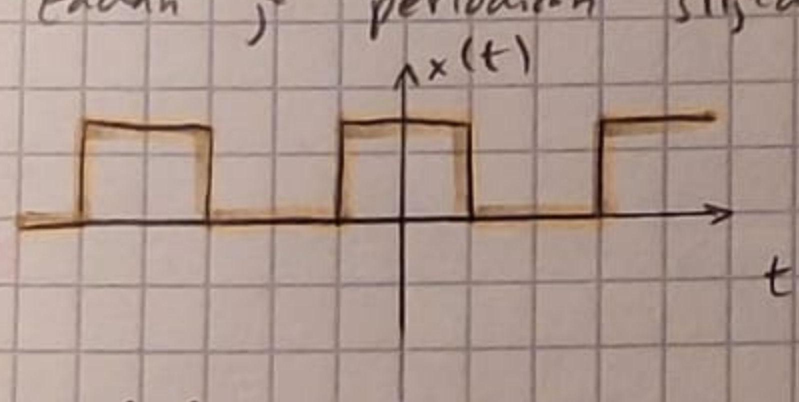


$B_{PPF} = 6\,Hz$ = širina pojasa

$f_{cPPF} = 10\,Hz$ = centralna frekvencija propuštanja

Pr2) Zadan je periodičan slijed prav. impulsa $x(t)$, amplitude 1V i ima te slike. Omjer snage sadržane u istosmjernoj komponenti tog signala prema srednjoj snazi cijelog signala iznosi $\frac{1}{4}$. Odredite sr. snagu signala.

ISPITNI



$x(t)$, $A = 1\,V$

$\frac{P_0}{P} = \frac{1}{4}$, $P = ?$

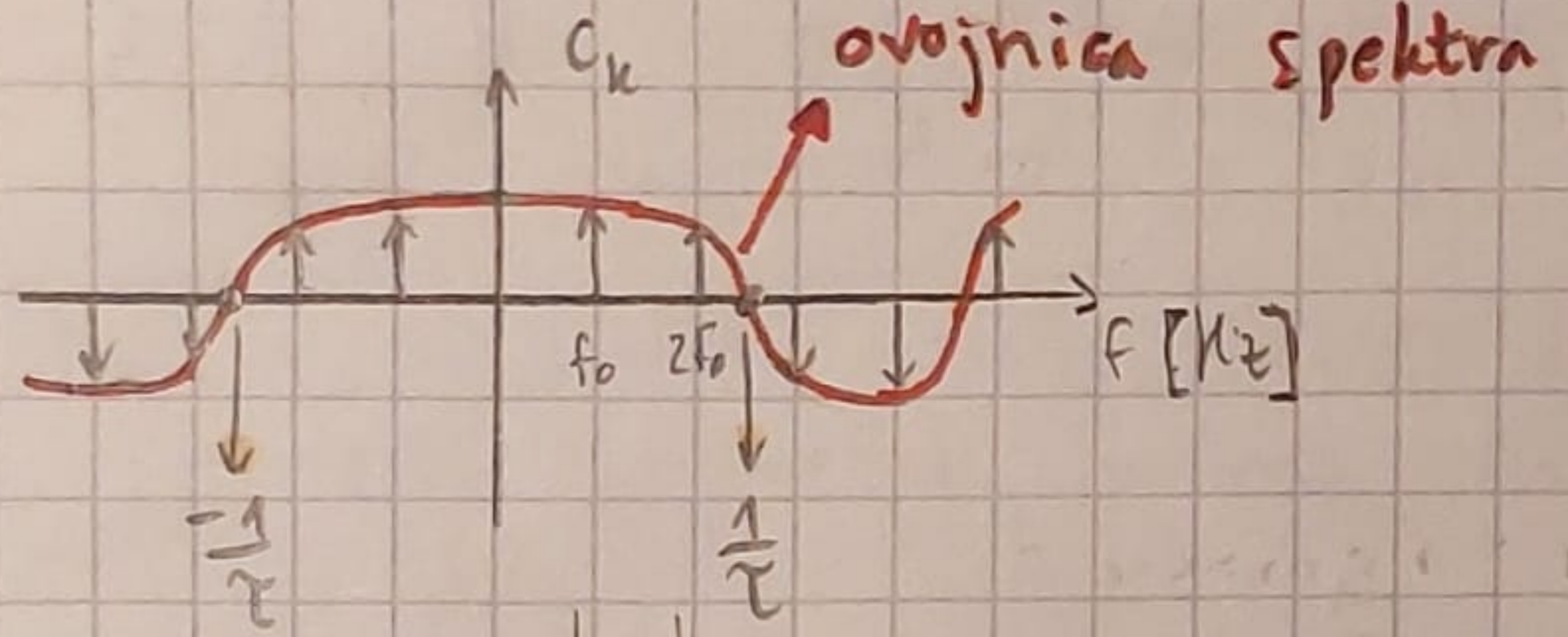
$$P_0 = \frac{A^2\tau^2}{T^2}$$

$$P = \frac{A^2\tau}{T}$$

$$\frac{\frac{A^2\tau^2}{T^2}}{\frac{A^2\tau}{T}} = \frac{1}{4} \Rightarrow \frac{\tau}{T} = \frac{1}{4}$$

$$P = \frac{A^2\tau}{T} = \frac{1}{4}\,W$$

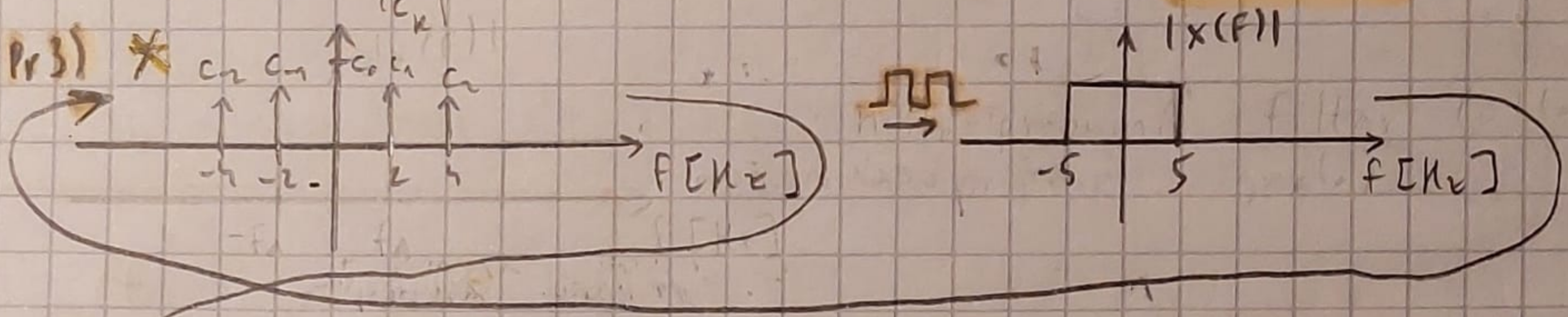
↓ sr. snaga

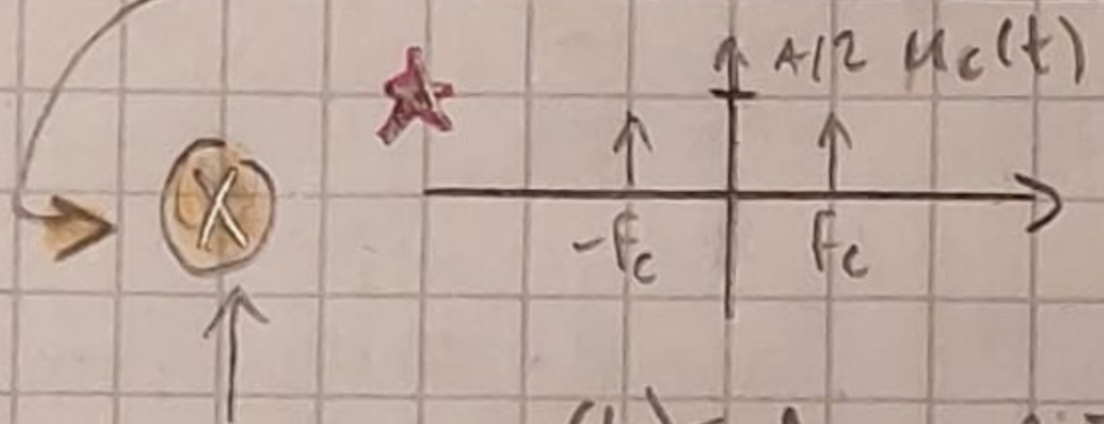


- nultočke ovojnice spektra periodičkog slijeda pravo. impulsa

$$\hookrightarrow \boxed{t_0 = \frac{k}{\tau}}, \; k \in \mathbb{Z} \setminus \{0\}$$

Pr3) *

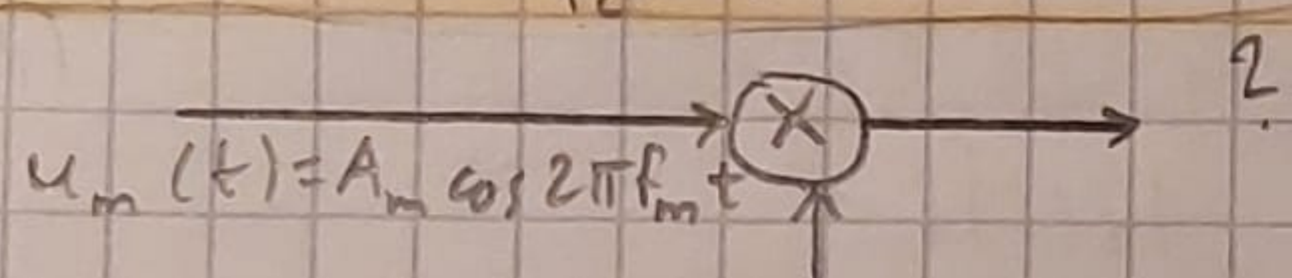




→ množimo * i ★

$u_c(t) = A_c \cos 2\pi f_c t$

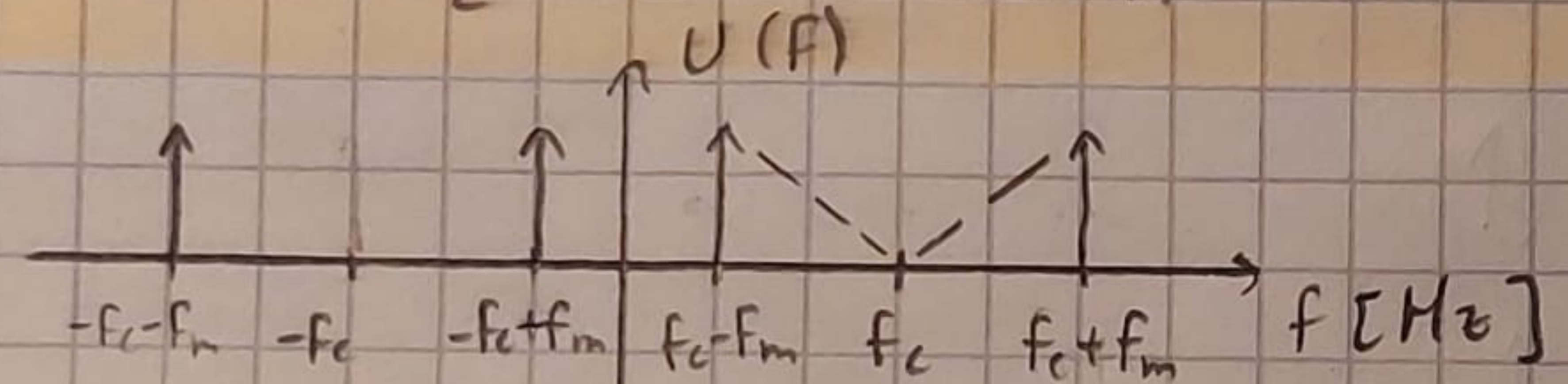
$f_c = 20$ Hz

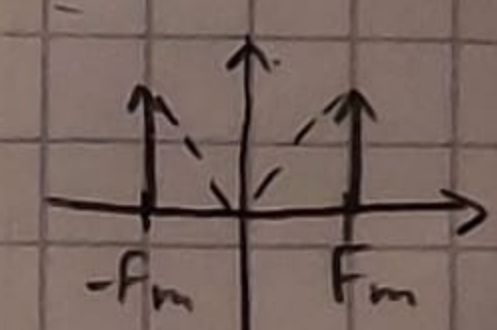


$f_c >> f_m \Rightarrow$ pretpostavka

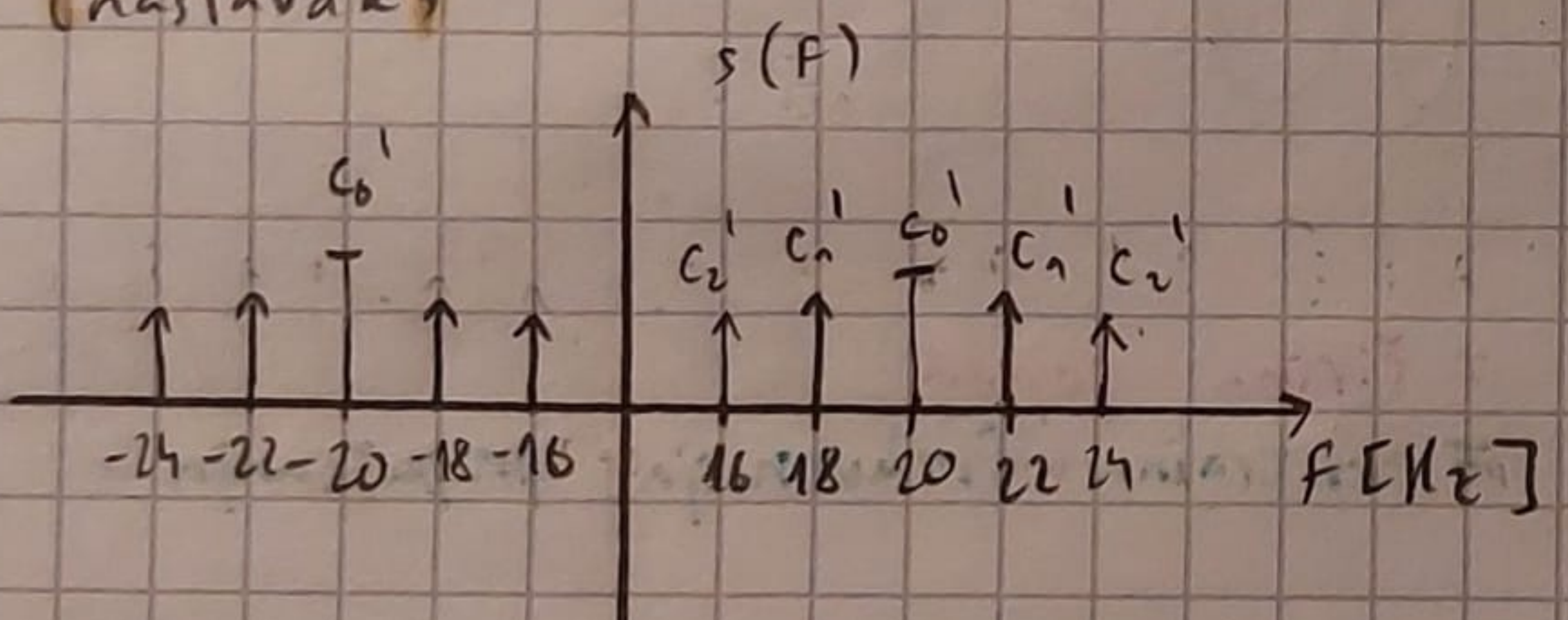
$u_c(t) = A_c \cos 2\pi f_c t$

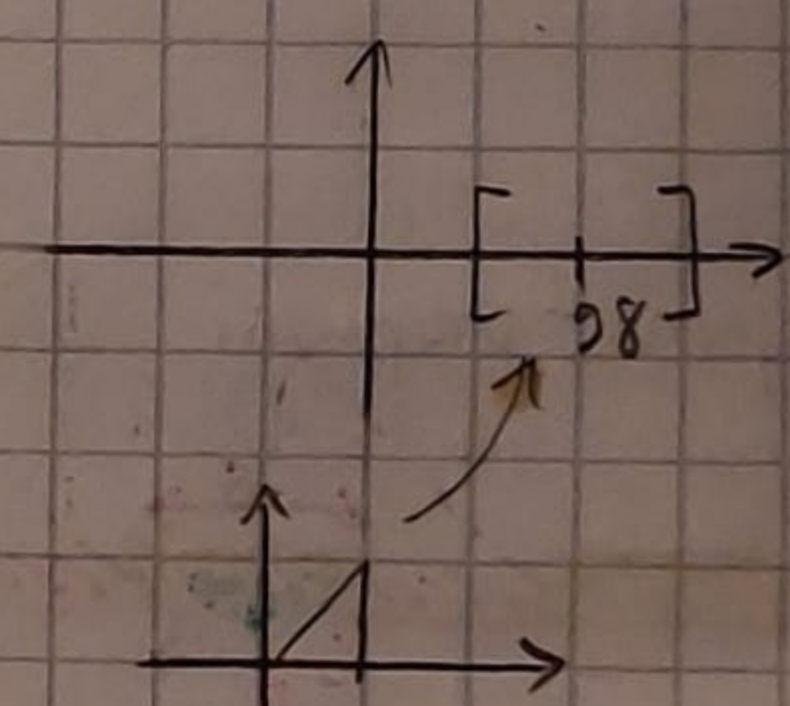
$$u(t) = \frac{A_m A_c}{2}\left[\cos 2\pi (f_c - f_m)t + \cos 2\pi (f_c + f_m)t\right]$$





Pr3) (nastavak)



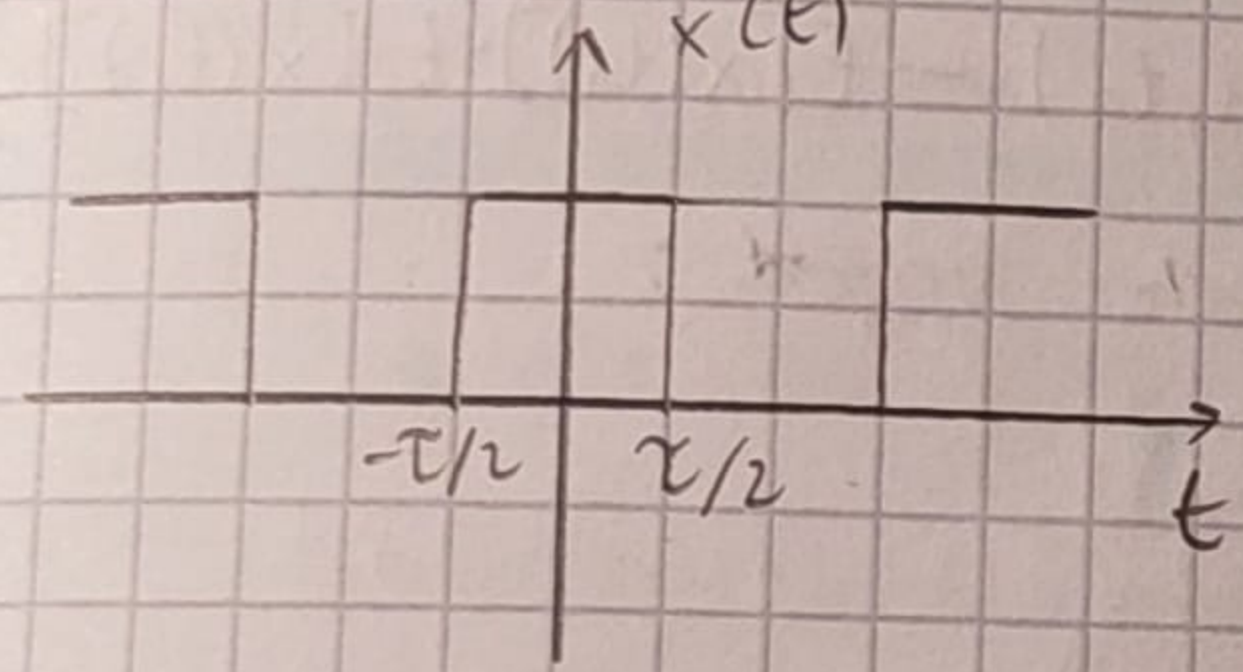


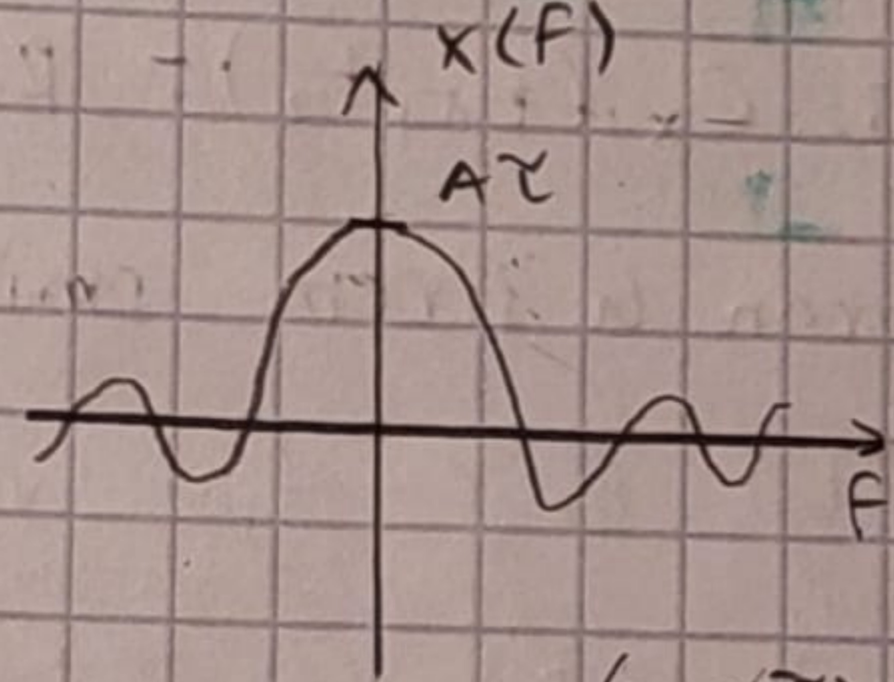
NEPERIODIČNI SIGNALI

- $x(t) \neq x(t+T_0)$
- $t \rightleftarrows f$ → veza preko Fourierove transformacije

$$X(f) = \int_{-\infty}^{+\infty} x(t)\, e^{-j2\pi f t}\, dt$$

$$x(t) = \int_{-\infty}^{+\infty} X(f)\, e^{j2\pi f t}\, df$$





→ spektar usamljen i beskonačan

→ $P=0$

→ $E = A^2\tau$

$X(f) = A\tau \; \sin(2\pi f(\frac{\tau}{2})$

- podjela

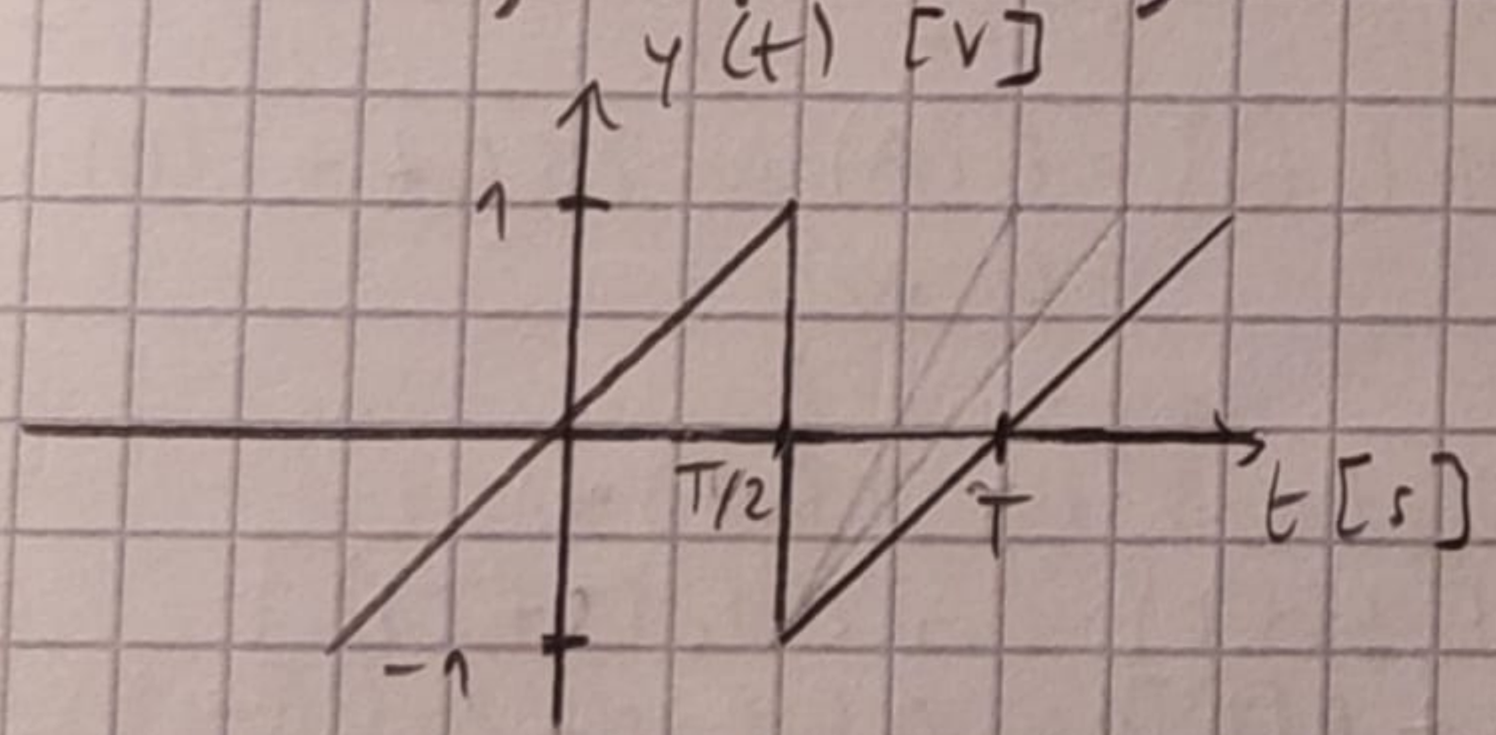
konačna energija

1) signali energije → $E<\infty,\ P=0$

2) signali snage → $P<\infty,\ E=\infty$

3) niti signal energije niti signal snage → $E=0,\ P=\infty$

ZAD1) Zadan je per. signal $y(t)$ kao na slici.



$E=\infty$ (jer je signal periodičan)

$P = \frac{2}{T}\int_0^{T/2} \left|\frac{2A}{T}\cdot t\right|^2 dt = \frac{A^2}{3}$ ⇒ signal snage

(periodični signali su uvijek signali snage)

SLUČAJNI SIGNALI

- slučajni procesi
- 3 veličine koje ih karakteriziraju
  1) očekivanje: $\boxed{\mu_x(t) = E[x(t)]}$
  2) autokorelacijska funkcija: $\boxed{R_x(t_1, t_2) = E[x(t_1) \cdot x(t_2)]}$
  3) funkcija kovarijance: $\boxed{C_x(t_1, t_2) = R_x(t_1, t_2) - E[x(t_1)] \cdot E[x(t_2)]}$
- slučajni proces je **stacionaran u širem smislu** ako vrijedi:
  1) $\mu_x(t) = \text{konst.} \neq f(t)$ (nije fja vremena)
  2) $R_x(t_1, t_2) = R_x(\tau) = K_x(\tau)$, $\tau = |t_2 - t_1|$
     (ovisi o razlici vremena)

ZAD1) Zadan je sluč. signal $x(t) = A\cos(2\pi f_c t) + B\sin(2\pi f_c t)$ gdje su A i B nezavisne Gaussove varijable s očekivanjem 0 i varijancom 1, dok je f konst. Je li $x(t)$ stacionaran u širem smislu?

$x(t) = A\cos(2\pi f_c t) + B\sin 2\pi f_c t$
A, B nez.
$E(A) = E(B) = 0$
$E(A^2) = E(B^2) = 1$
$f = \text{konst.}$

$$\mu_x(t) = E[x(t)] = E[A\cos(2\pi f_c t) + B\sin(2\pi f_c t)] = E(A)\cos 2\pi f_c t + E(B)\sin 2\pi f_c t = 0 \ ✓$$

$$R_x(\tau) = E[x(t) \cdot x(t+\tau)] = E[(A \cdot \cos 2\pi f_c t + B \cdot \sin 2\pi f_c t) \cdot (A\cos 2\pi f_c(t+\tau) \cdot \sin 2\pi f_c(t+\tau))]$$
$$= E[A^2] \cdot \cos 2\pi f_c t \cdot \cos 2\pi f_c(t+\tau) + E[B^2] \cdot \sin 2\pi f_c t \cdot \sin 2\pi f_c(t+\tau)$$
$$= \ldots = \cos 2\pi f_c \tau \ ✓$$
(fja je ovisna samo o razlici vremena)

$\Rightarrow x(t)$ je stac. u širem smislu

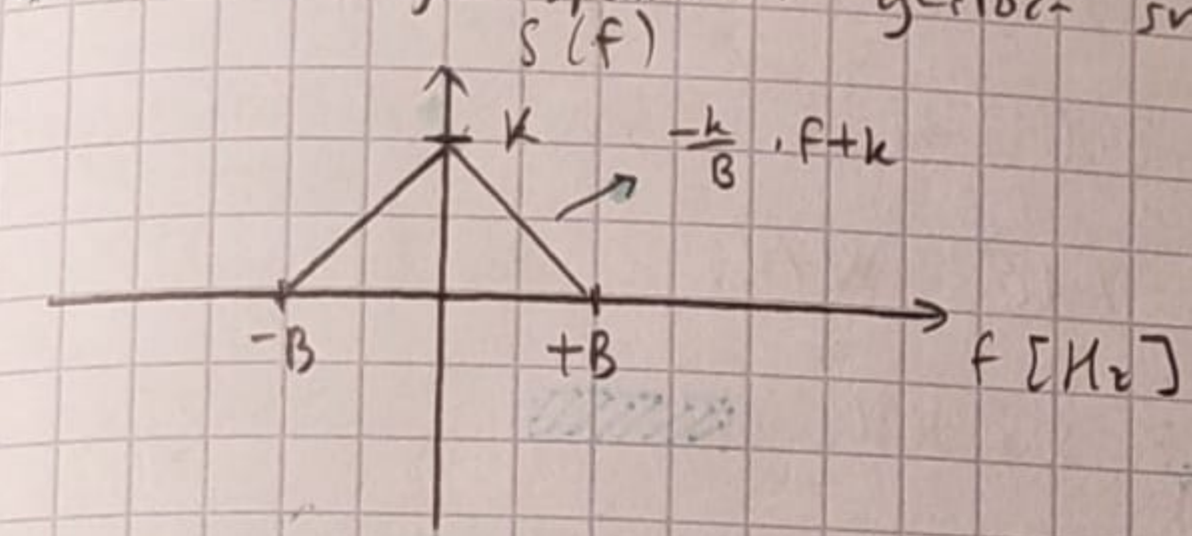
- spektralna gustoća snage $\rightarrow$ razdioba snage po jedinici pojasa prijenosa po Hz

$$\boxed{S_x(f) = \int_{-\infty}^{+\infty} R_x(\tau) e^{-j2\pi f\tau} d\tau \ \left[\frac{W}{Hz}\right]} \Rightarrow \text{snaga}$$

$$\boxed{R_x(\tau) = \int_{-\infty}^{+\infty} S_x(f) e^{j2\pi f\tau} df}$$

$$\boxed{P = R_x(\tau = 0)}$$

Pr1) Na slici je spektralna gustoća snage $S(f)$. Odredite srednju snagu signala.



$$P_s = \int_{-B}^{+B} S(f)\cdot df = 2\int_0^B \left(-\frac{k}{B}\cdot f + k\right)\cdot df$$
$$= 2\left\{ -\frac{k}{B}\frac{f^2}{2}\Big|_0^B + kf\Big|_0^B \right\}$$
$$= k\cdot B \ [W]$$

- $w(t)$ (slučajni proces) $\rightarrow$ bijeli šum $\Rightarrow$ ako su njegove varijable nekorelirane
  $\hookrightarrow C_x(t_1, t_2) = 0, \quad t_1 \neq t_2$

  $\rightarrow$ striktno bijeli šum $\Rightarrow$ ako su njegove var. nekorelirane i nezavisne

  $\rightarrow$ aditivni bijeli Gaussov šum $\Rightarrow$ ako su njegove var. nekorelirane, nezavisne i ako se ravnaju po Gaussovoj razdiobi

  additive white Gaussian noise **(AWGN)**

$$f_x(x) = \frac{1}{\sigma_x\sqrt{2\pi}}\, e^{\frac{-(x-\mu_x)^2}{2\sigma^2}}$$

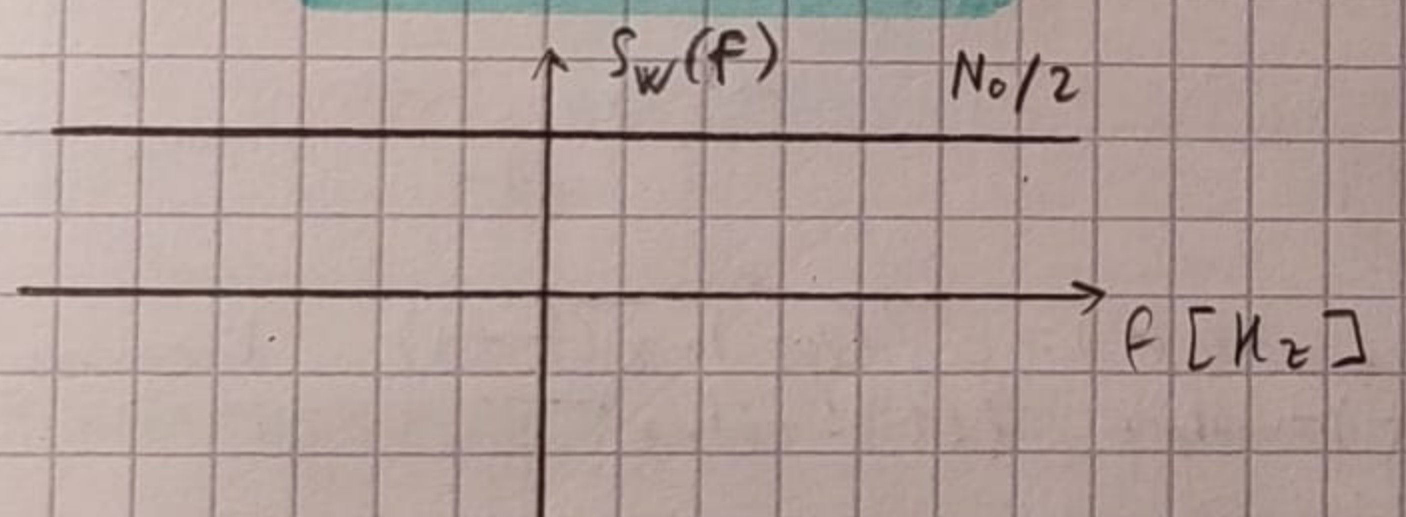
- AWGN $\rightarrow$ od -120 do $\div 140$ dB

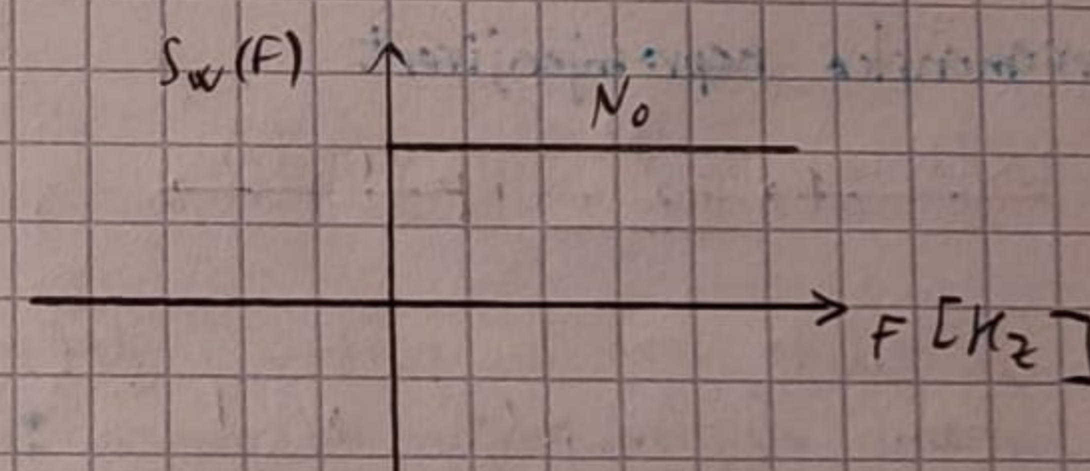
$$\boxed{\int_{-\infty}^{+\infty} \delta(t-t_0)\cdot x(t)\,dt = x(t_0)}$$

$\rightarrow \boxed{R_w(\tau) = \sigma^2\delta(\tau)}$

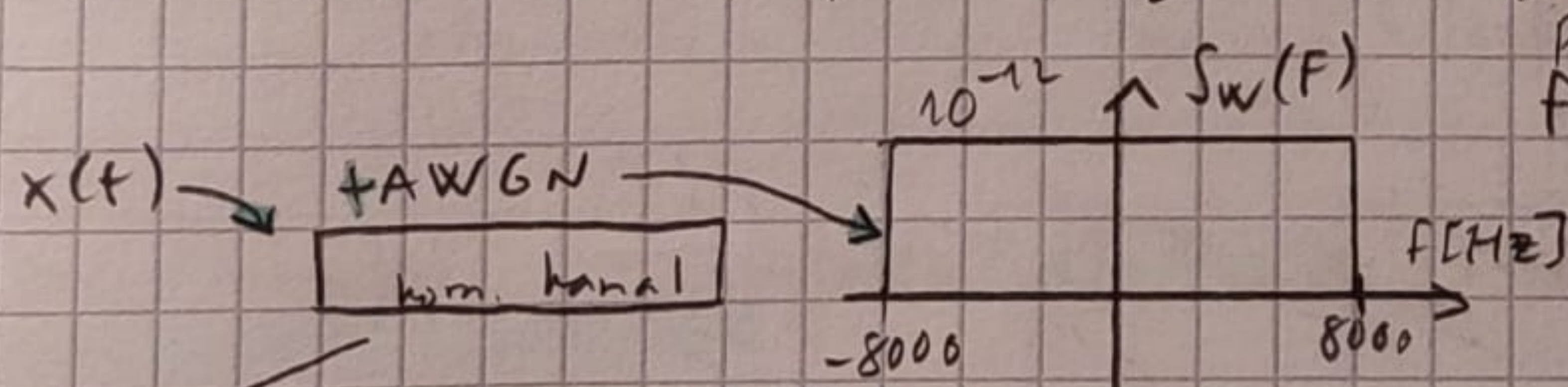
$\rightarrow S_w(f) = \int_{-\infty}^{+\infty} R_w(\tau) e^{-j2\pi f\tau}\, d\tau = \sigma^2 \int_{-\infty}^{+\infty} \delta(\tau)\, \underbrace{e^{-j2\pi f\tau}}_{\tau=0,\ =1}\, d\tau$ ($t_0 = 0$)

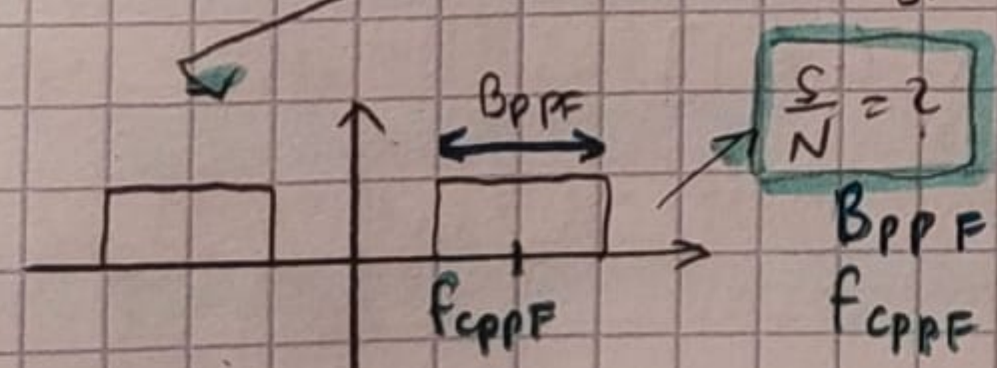
$\boxed{S_w(f) = \sigma^2 = \frac{N_0}{2}}$ $\rightarrow$ spektralna gustoća





Pr2) Signal $x(t) = 3\cos(12\,000\pi t) + \cos(5000\pi t) + \cos(10000\pi t)$ [V] (6 kHz, 2.5 kHz, 5 kHz) prisutan je u kom. kanalu s AWGN spektralne gustoće snage kao na slici. Signal i aditivni šum propušteni su kroz idealni pojasno-propusni filtar sa širinom pojasa 4 kHz i centralnom frekv. propuštanja 6.5 kHz. Odredite srednji omjer signala na izlazu i šuma.

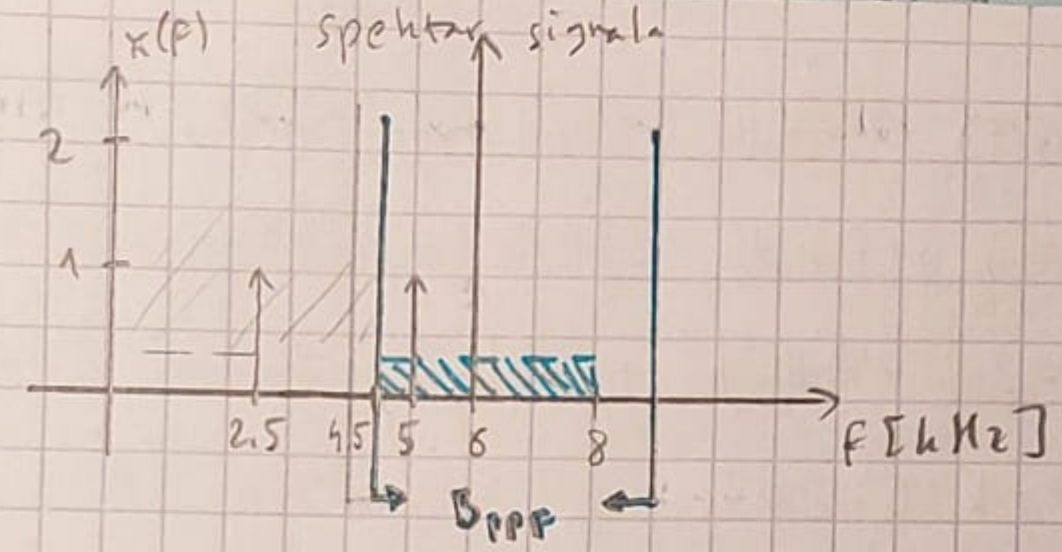




$\boxed{\frac{S}{N} = ?}$

$B_{PPF} = 4$ kHz
$f_{cPPF} = 6.5$ kHz

$$\frac{N_0}{2} = \begin{cases} 10^{-12}\ W/Hz, & [-8000 \div 8000]\ Hz \\ 0, & \text{inače} \end{cases}$$



$S = \frac{1^2}{2} + \frac{1}{2} \cdot 3^2 = 5\ W$ → jednostrani spektar

$S = 2 \cdot \left(\left(\frac{3}{2}\right)^2 + \left(\frac{1}{2}\right)^2\right) = 5W$ → dvostrani spektar

$N_0 = 2 \cdot 10^{-12}\ W/Hz$

$B_W = 3.5\ kHz$

$N = N_0 \cdot B_W = 7 \cdot 10^{-9}\ W$

$\frac{S}{N}\Big|_{dB} = 10 \log \frac{S}{N} = 88.54\ dB$

LINEARNI I VREMENSKI NEPROMJENJIVI SUSTAVI

- LTI → linear and time invariant systems
- linearnost

$x_1(t) \rightarrow$ kom. kanal $\rightarrow y_1(t)$

$x_2(t) \rightarrow$ kom. kanal $\rightarrow y_2(t)$

$ax_1(t) + bx_2(t) \rightarrow$ kom. kanal $\rightarrow ay_1(t) + by_2(t)$

→ LTI

21.1. 7:30h

- vremenska nepromjenjivost

$x(t-t_0) \rightarrow$ kom. kanal $\rightarrow y(t-t_0)$

Pr1) Odredite je li sustav def. kao $y(t) = \cos(2t) \cdot x(t+1)$ linearan i vrem. nepromjenjiv. Napomena: $x(t)$ = ulaz, $y(t)$ = izlaz

$y(t) = \cos(2t) \cdot x(t+1)$
LTI = ?

– lin: $y_1(t) = \cos(2t) \cdot x_1(t+1)$
$y_2(t) = \cos(2t) \cdot x_2(t+1)$

$x(t) = a \cdot x_1(t) + b \cdot x_2(t)$

$$y(t) = \cos(2t) \cdot x(t+1) = \cos 2t \cdot [ax_1(t+1) + b \cdot x_2(t+1)]$$
$$= a \cos 2t \cdot x_1(t+1) + b \cdot \cos 2t \cdot x_2(t+1)$$
$$= y_1(t) + y_2(t)\ ✓$$

– vrem. nepr.: $y(t) = \cos(2t) \cdot x(t+1)$

$y_1(t) = \cos(2t) \cdot x_1(t+1)$
$t \rightarrow t - t_0$

$y_2(t) = \cos(2t) \cdot x_2(t+1)$
$x_2(t) \rightarrow x_2(t-t_0)$

$y_1(t-t_0) = \cos(2(t-t_0)) \cdot x_1(t-t_0+1) \neq y_2(t) = \cos(2t) \cdot x_2(t-t_0+1)$

→ nije vrem. nepr.

Pr2)

$h(t)$ → impulsni odziv → kako kanal pokazuje izlaz kad je nešto na ulazu

$x(t)$ → kom. kanal → $y(t) = x(t) \cdot h(t)$

$X(f)$ → $H(f)$ → $Y(f) = X(f) \cdot H(f)$

$h(t) \rightleftharpoons H(f)$

$\mu_x(t)$, $S_x(f)$

$H(f)$ → prijenosna f-ja kanala → odnos (omjer) izlaz-ulaz kanala

$$H(f) = \int_{-\infty}^{+\infty} h(t) e^{-j2\pi ft} dt$$ → za poznati impulsni odziv kanala

$$h(t) = \int_{-\infty}^{+\infty} H(f) e^{j2\pi ft} df$$ → za poznatu prijenosnu f-ju kanala

$$H(f) = \underbrace{|H(f)|}_{\text{amplitudni dio prijenosne f-je}} \cdot \underbrace{e^{-j\theta(f)}}_{\text{fazni dio prijenosne f-je}}$$

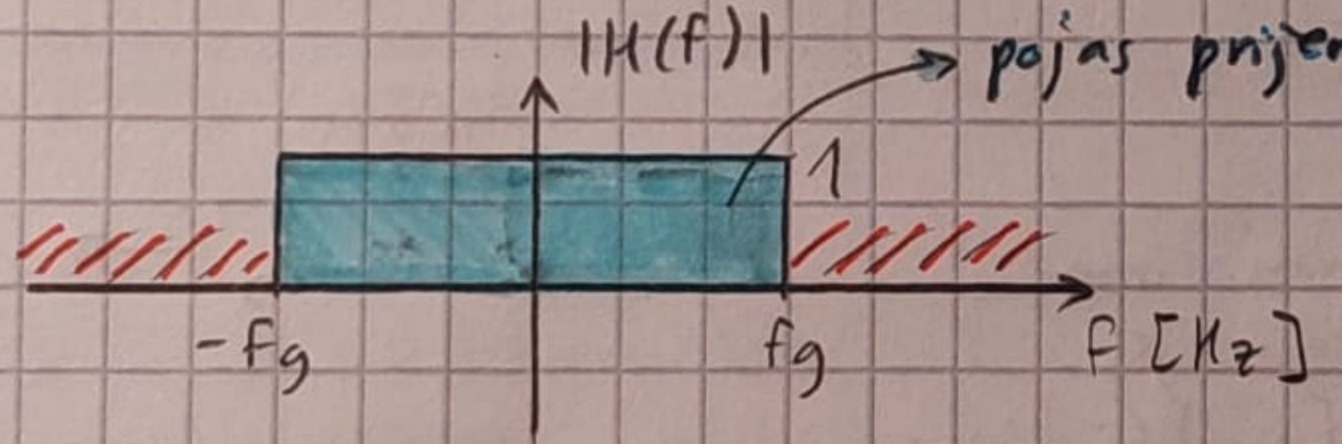
* = amplitudni spektar

★ = fazni spektar

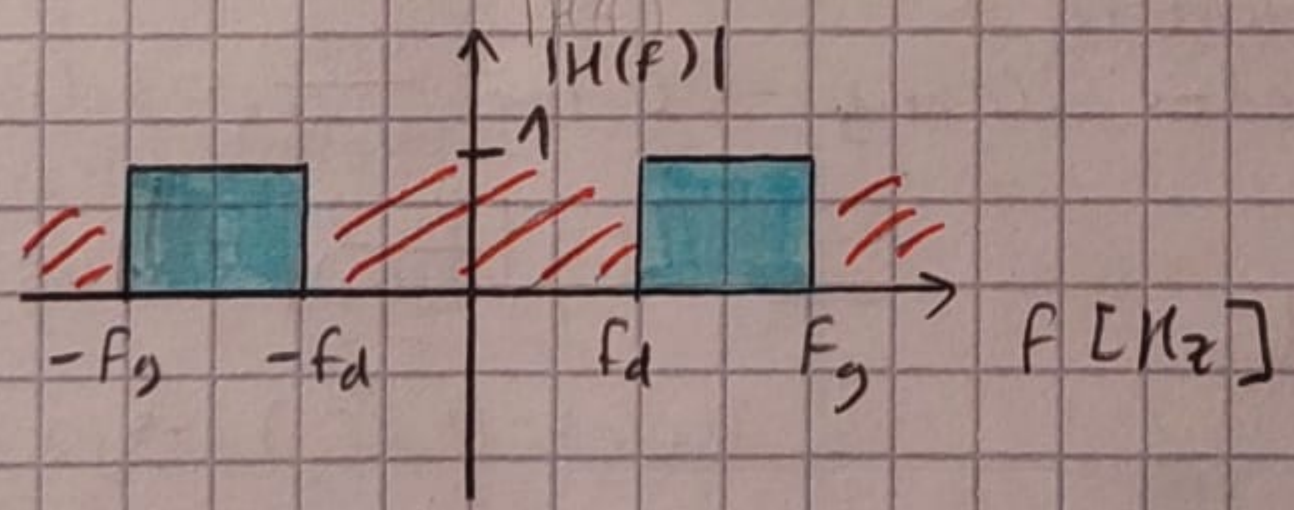
$$H(-f) = H(f) \qquad -\theta(f) = \theta(-f)$$

- kom. kanali:

→ niskopropusni: $|H(f)| = 1$, $|f| \leq f_g$



→ pojasnopropusni:



kanal je filtar koji ograničava signal
$x(t)$, $t \in \mathbb{R}$; $f_x(x)$
$var(x) = E((x - E(x))^2) = E(x^2) - (E(x))^2 = \sigma_x^2$
$E(x) = 0$ $E(x)$ = srednja snaga

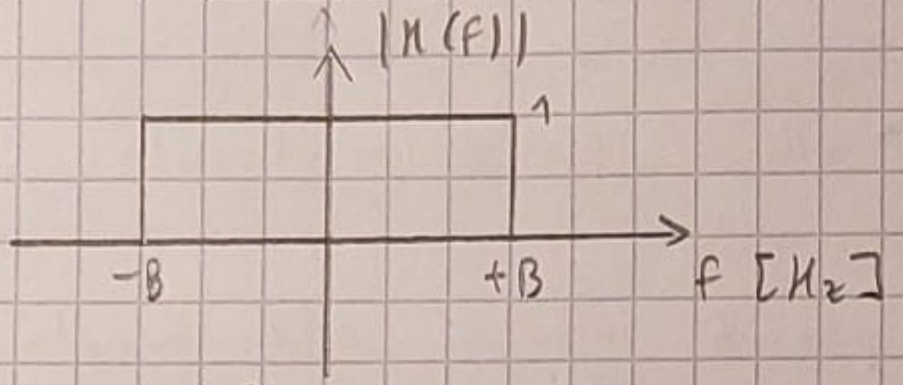
- odziv kanala na pobudu slučajnim signalom

$$\mu_y = \mu_x \cdot |H(0)|$$

$$S_y(f) = S_x(f) \cdot |H(f)|^2$$

Pr3) Na ulaz ovakvog kanala dolazi signal $x(t)$ čija je autokor. f-ja $R_x(\tau)=\delta(\tau)$. Odredite autokor. f-ju na izlazu.

$x(t) \rightarrow$ kom. kanal $\rightarrow R_y(\tau)=?$



$R_y(\tau) \rightleftarrows S_y(t)$

$R_x(\tau) \rightleftarrows S_x(t)$

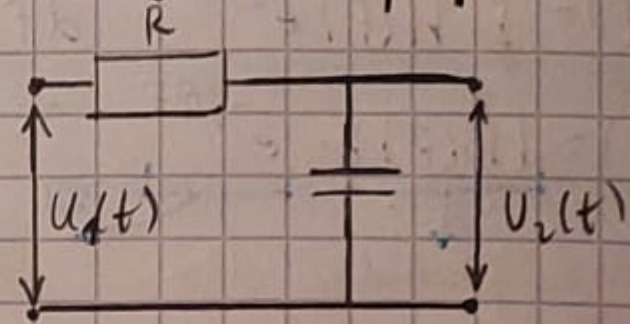
$|H(f)|$

$$S_x(f)=\int_{-\infty}^{+\infty}\delta(\tau)\cdot e^{-j2\pi f\tau}\cdot d\tau$$
$$=1$$
$$\int_{-\infty}^{+\infty}\delta(t-t_0)\cdot x(t)\,dt=x(t_0)$$

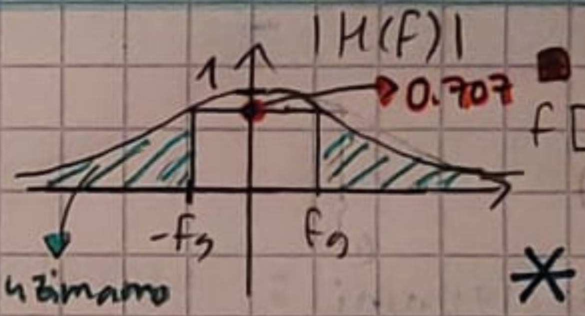
$$S_y(f)=S_x(f)\cdot|H(f)|^2=1\cdot 1^2=1$$
$$R_y(\tau)=\int_{-\infty}^{+\infty}S_y(f)\cdot e^{j2\pi f\tau}df$$
$$=\int_{-\infty}^{+\infty}e^{j2\pi f\tau}df$$
$$=\frac{1}{j2\pi f\tau}\cdot e^{j2\pi f\tau}\Big|_{-B}^{B}$$
$$=2B\frac{\sin(2\pi\tau B)}{2\pi\tau B}$$

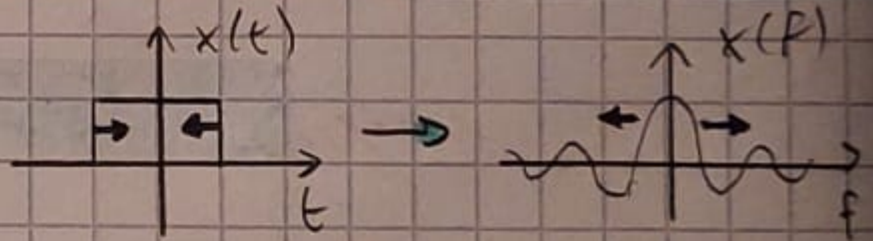
spektr. f-ja def. kako se signal guši od ulaza prema izlazu

RC - krug (niskopropusni filtar)



$$\boxed{|H(f)|=\left|\frac{U_2(f)}{U_1(f)}\right|=\frac{1}{\sqrt{1+(2\pi fRC)^2}}}$$





Pr4) Neka kom. kanal u kont. vremenu ima karakteristiku RC-kruga pri čemu je $R=100\,\Omega$, $C=50\,nF$. Prijenosna f-ja RC-kruga odr. je izrazom $H(f)=\left|\frac{U_2(f)}{U_1(f)}\right|$. Odredite gran. frekv. tog kanala f_g ako se prilikom njenog određivanja primjenjuje kriterij da je na toj frekv. amplitudni odziv RC-kruga 100 puta manji od $H(0)$.

amplitudni odziv = amplitudni dio

$R=100\,\Omega$
$C=50\,nF$
$$|H(f)|=\left|\frac{U_2(f)}{U_1(f)}\right|$$
$$|H(f)|=\frac{|H(0)|}{100}$$
$f_g=?$

$$|H(0)|=\frac{1}{\sqrt{1+(2\pi fRC)^2}}=1 \quad \text{(jer je } f=0\text{, točka na y-osi)}$$
$$|H(f)|=10^{-2}$$

$$* \quad 10\log\frac{P(0)}{P(f_g)}=3dB \Rightarrow \frac{P(0)}{P(f_g)}=10^{3/10}$$
$$20\log\frac{U_2(0)}{U_2(f_g)}=3dB \Rightarrow \frac{U_2(0)}{U_2(f_g)}=10^{3/20}$$
$$20\log\left|\frac{U_2(0)}{U_2(f_g)}\right|=20\log\left|\frac{U_2(0)/U_1(0)}{U_2(f_g)/U_1(f_g)}\right|=\bigstar$$

$U_1(0)=U_1(f_g)$ → jer su na ulazu

$$\bigstar=20\log\left|\frac{H(0)}{H(f_g)}\right|=3\,dB$$
$$20\log|H(0)|-20\log|H(f_g)|=3dB$$
$$-20\log|H(f_g)|=3\,dB$$
$$|H(f_g)|=10^{-3/20}=0.707$$

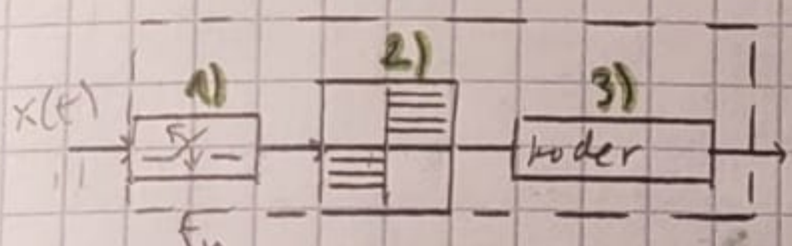
$$\frac{1}{\sqrt{1+(2\pi f_g RC)^2}} = 10^{-2}$$

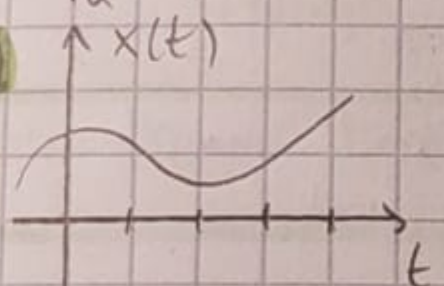
$f_g = 3.18 \cdot 10^{6}$ Hz

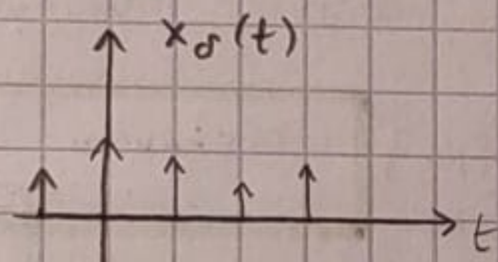
AD PRETVORBA

- 3 faze kroz koje prolazi:

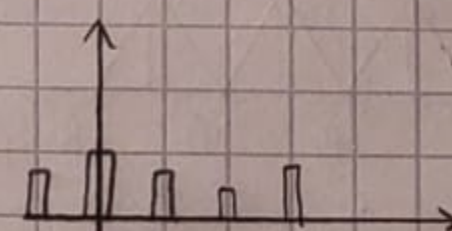
1) uzorkovanje signala → uzimanje vrijednosti signala u vrem. razmacima
2) kvantiziranje uzoraka → određivanje u koji dio (kvant?) ulazi uzorak
3) kodiranje



1) 



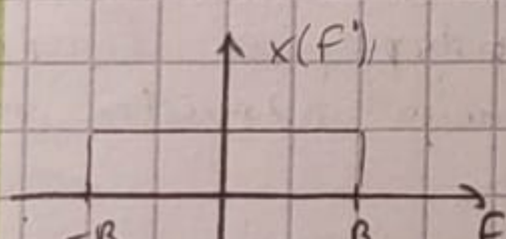
↳ idealno



↳ realno

f_u = potencija uzorkovanja

a) T_x (predajnik) → uzorkovani signal jednoznačno je određen svojim uzorcima uzetim u trenucima $T_n = \frac{n}{2B}$, $n \in \mathbb{Z}$



$x(f) = 0$ → uzorkovanje u osnovnom pojasu signala (to radimo)
$|f| > B$ → signali ograničenog spektra
B → pojas prijenosa

b) R_x (prijamnik) → signal temeljem njegovih uzoraka moguće je jednoznačno rekonstruirati ako su oni razmaknuti za $\Delta = \frac{1}{2B}$ [s]

$$\boxed{x_\delta(t) = \sum_{n=-\infty}^{+\infty} x(nT_u) \cdot \delta(t - nT_u)}$$ *

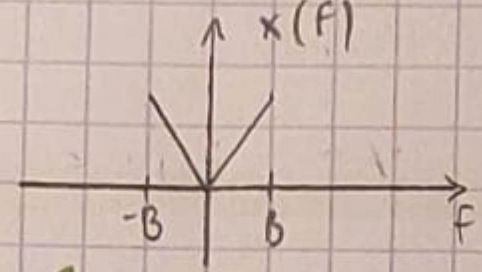
- Frekvencija uzorkovanja f_u, $T_u = \frac{1}{f_u}$

Teorem
Da bismo signal mogli rekonstruirati temeljem njegovih uzoraka mora vrijediti:

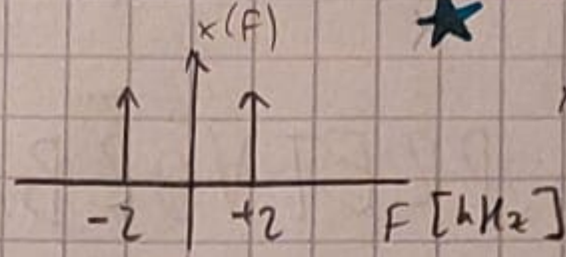
$f_u \geq 2B$ (Frekv. uzork. barem 2 puta veća od najv. frekv. u spektru signala)

$$\sum_{n=-\infty}^{+\infty} \delta(t - nT_u) \rightleftharpoons \frac{1}{T_u} \sum_{n=-\infty}^{+\infty} \delta\left(f - \frac{n}{T_u}\right)$$

$$\boxed{x_\delta(t)} \rightleftharpoons X(f) \cdot \left[f_u \sum_{n=-\infty}^{+\infty} \delta(f - nf_u) \right] = \ldots = \boxed{f_n \cdot \sum_{n=-\infty}^{+\infty} X(f - nf_u)}$$

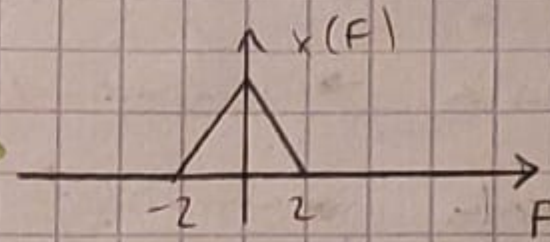


$\equiv$



$x(t) = A \cos 2\pi \underbrace{2 \cos t}_{f}$ [V]

|||



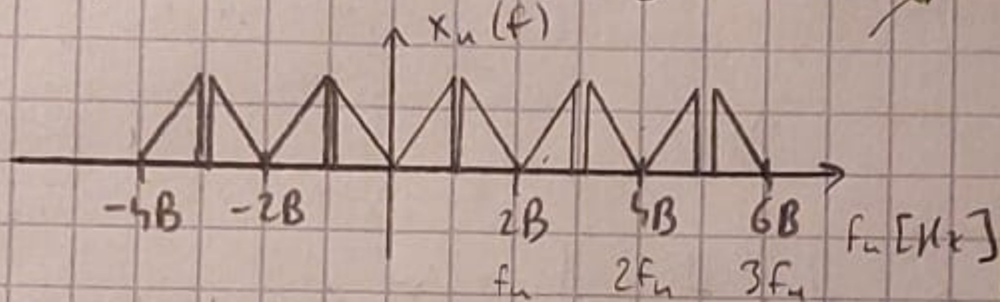
spektar

B = beskonačan

↳ bandwidth = pojas prijenosa

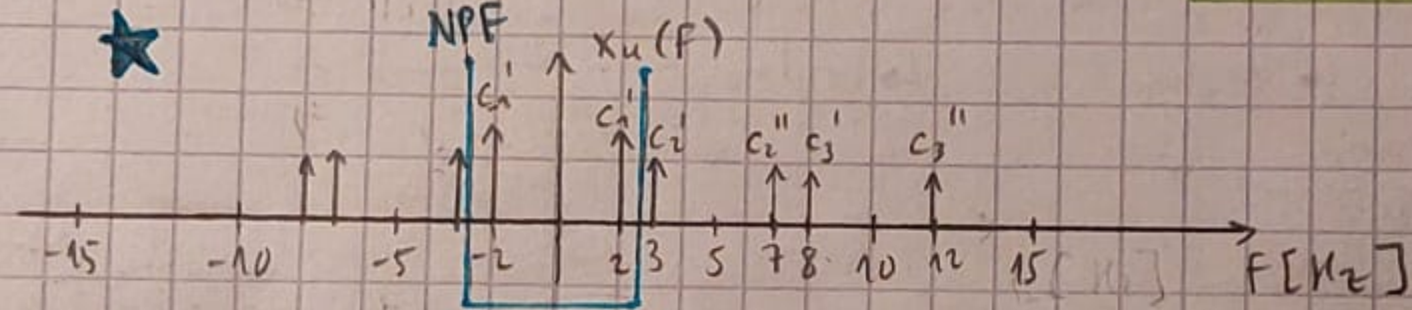
spektar uzorkovanog signala

za kosinus

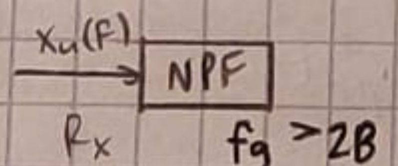


$n = 0$ → osnovni spektar

uzorkovani signal uvijek u sebi ima osnovni signal

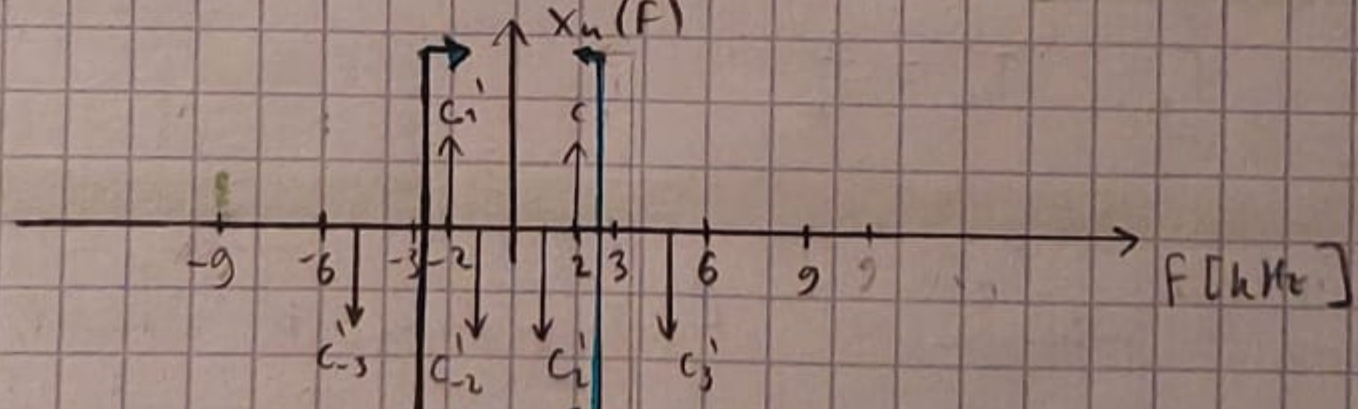


- rekonstrukcija signala temeljem uzoraka na prijamnoj strani
  ↳ signal se propusti kroz niskopropusni filtar koji ima graničnu frekvenciju neznatno veću od $2B$



- poduzorkovanje

$f_m = 2$ kHz
$f_u = 3$ kHz
} teorem o uzimanju uzoraka nije zadovoljen



$6-2,\ 6+2$

$\cancel{f_n > 2B}$

→ preklapanje imamo

→ sve kvadrate i takve fje srediti na sin ili cos

Pr1) Signal $x(t) = 10\cos(600\pi t)\cdot\cos^2(1600\pi t)$ [V] uzorkovan je frekv. 4 kHz

a) Odredite srednju snagu koja se troši na jed. otporu.

b) Skicirajte ampl. spektar u području [−9, +9] kHz,

c) Odredite interval za gornju graničnu frekv. NPF koji se koristi za rekonstrukciju signala.

a) $\cos^2 x = \frac{1+\cos(2x)}{2}$

$$\boxed{P = \frac{A^2}{2}\ [W]}$$

$P = ?$, $R = 1\Omega$

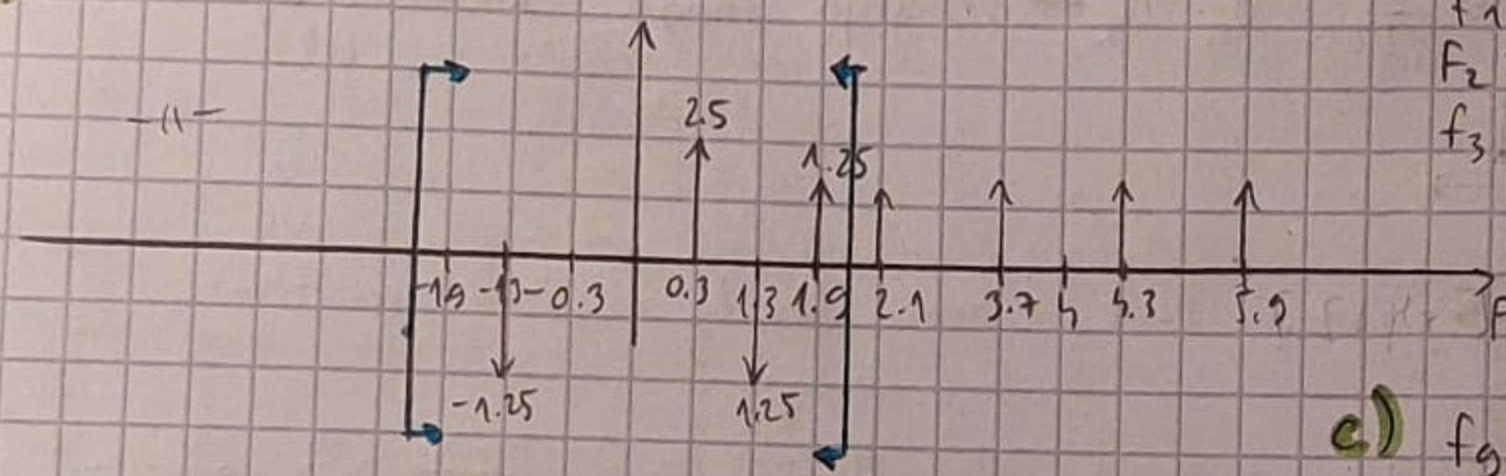
$\frac{1}{2} + \frac{\cos(3800\pi t)}{2}$

$x(t) = 10\cos(2\cdot 300\pi t)\cdot\left[\frac{1+\cos(3800\pi t)}{2}\right]$

$= 5\cos(600\pi t) + 2.5\cos(3800\pi t) + 2.5\cos(2600\pi t)$

$P = \frac{1}{2}\left[5^2 + (2.5)^2\cdot 2\right] = 18.75$ W

b)



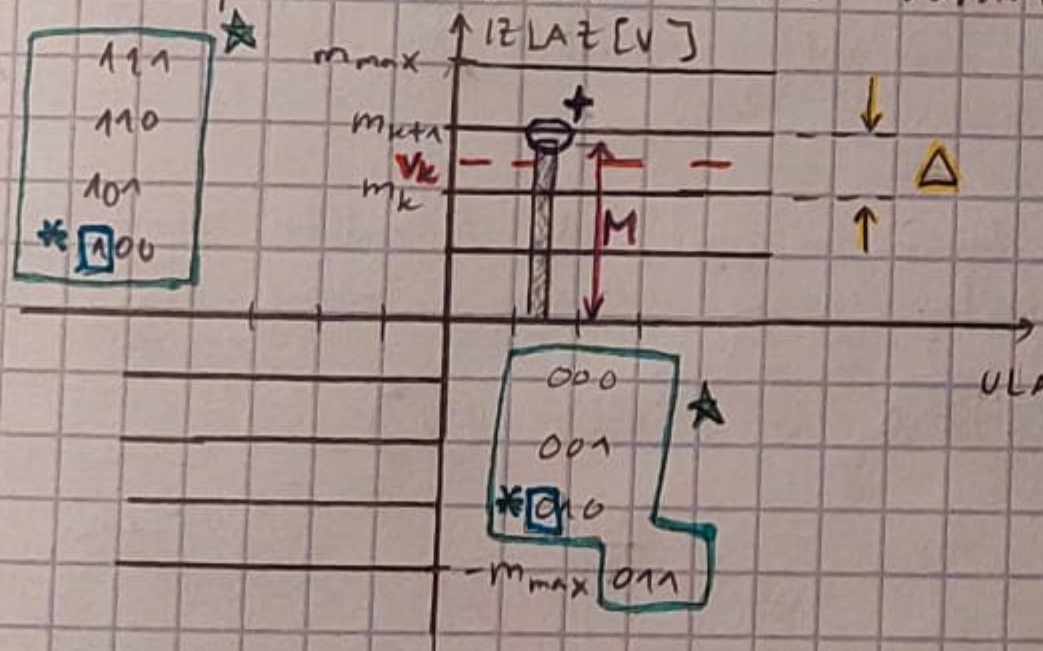
$f_1 = 300$ Hz
$F_2 = 1.9$ kHz
$f_3 = 1.3$ kHz

→ da bi ispravno izvukli osnovni spektar, trebamo stati iza najdalje komponente

4 ± 0.3
4 ± 1.9

c) $f_g = (1.9 \div 2.1)$ kHz

2) sklop: **kvantizator** → ima karakteristiku kvantizacije (odnos ulaz - izlaz)



↳ linearan (kvantizator)

↳ nelinearan (kvantizator)

⇒ A - zakon

⇒ μ - zakon

★ KODER 3)

- kvantizacijske stepenice → L = br. kvant. step. → $\boxed{L = 2^r}$
- raspon amplituda: $-m_{max}$, m_{max}
- r bitova → svaku stepenicu kodiramo u r bitova

↳ vodeća znamenka govori radi li se o +/− stepenici *

- kvantizacijski šum N_q ✦

$$v_k = \frac{m_k + m_{k+1}}{2}$$

↳ gubitak kod AD pretvorbe

↳ $Q = \overset{\text{ulaz}}{M} - V$

↳ $E[M] = E[V] = 0 \Rightarrow E[Q] = 0$

↳ razdioba kvant. šuma se ravna po uniformnoj razdiobi kad je razmak između kvant. stepenica mali

$$f_Q(q) = \begin{cases} \frac{1}{\Delta}, & -\frac{\Delta}{2} \le q \le \frac{\Delta}{2} \\ 0, & \text{inače} \end{cases}$$

↳ $\text{var}(Q) = \int_{-\Delta/2}^{\Delta/2} q^2 f_Q(q) \cdot dq = \frac{1}{\Delta} \int_{-\frac{\Delta}{2}}^{\frac{\Delta}{2}} q^2 dq = \frac{1}{\Delta} \frac{q^3}{3} \Big|_{-\frac{\Delta}{2}}^{\frac{\Delta}{2}} = \dots$

$= \frac{\Delta^2}{12} = \sigma_Q^2 = N_q \Rightarrow$ srednja snaga kvantizacijskog šuma

$\xrightarrow{\text{KOM. KANAL}}$

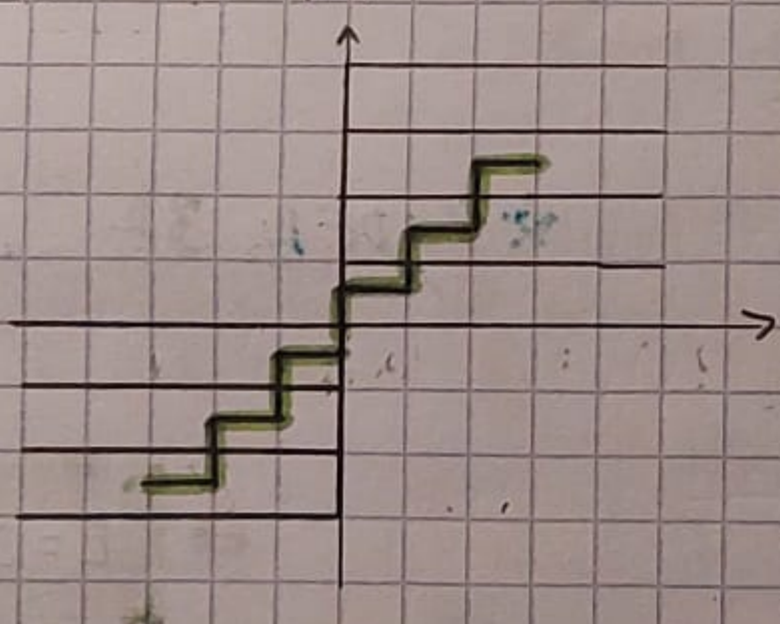
AWGN

$\boxed{N \neq N_q}$ → postoji šum u kom. kanalu i šum kvantizacije

↳ $N_q = \frac{\Delta^2}{12} = \left| \Delta = \frac{2m_{max}}{L} \right| = \frac{1}{12} \cdot \frac{4m_{max}^2}{L^2} = \frac{1}{3} \cdot \frac{m_{max}^2}{2^{2r}} = \frac{1}{3} m_{max}^2 \cdot 2^{-2r}$

$$\boxed{N_q = \frac{1}{3} m_{max}^2 \cdot 2^{-2r}}$$

Pr2) Na ulaz lin. kvantizatora dolazi kosinusni signal. Odredite omjer sr. snage signala prema srednjoj snazi kvant. šuma.



$x(t) = A_c \cos 2\pi f t, \; [V]$

$\frac{S}{N_q} = ? \qquad S = \frac{A_c^2}{2}$

$N_q = \frac{1}{3} \cdot A_c^2 \cdot 2^{-2r}$

$$\boxed{\frac{S}{N_q} = \frac{\frac{A_c^2}{2}}{\frac{1}{3} A_c^2 \cdot 2^{-2r}} = \frac{3}{2} 2^{2r}} \Rightarrow \text{za sinusni / kosinusni signal}$$

$$\boxed{\left.\frac{S}{N_q}\right|_{dB} = 1.76 + 6.02r \; [dB]}$$

Pr3) Na ulaz kvant. koji koristi 256 kvant. stup. dovodi se slj. signal. Odredite omjer sr. snage signala prema sr. snazi kvant. šuma

$x(t) = 5\cos 4\cos \pi t$ [V]

$L = 256$

$\left.\frac{S}{N_Q}\right|_{dB} = ?$

$L = 2^r \Rightarrow r = 8$

$\left.\frac{S}{N_Q}\right|_{dB} = 1.76 + 6.02 \cdot 8 \text{ dB} = 49.92 \text{ dB}$

Pr4) Signal u kont. vremenu $x(t)$ može se jednozn. rekonstr. temeljem njegovih uzoraka uzetih u trenucima t_u u ms. Odredite max frekv.

$x(t)$

$x[nT_u]$

$T_u = 1\,ms$

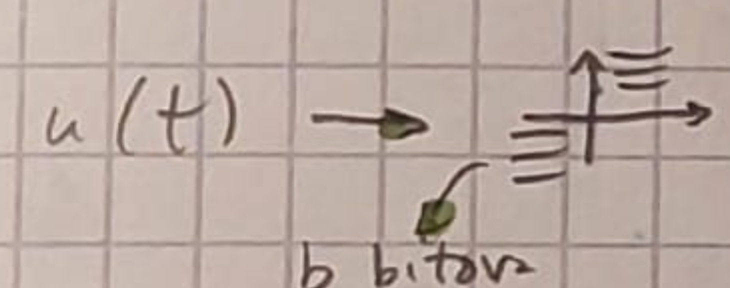
$f_{max} = ?$

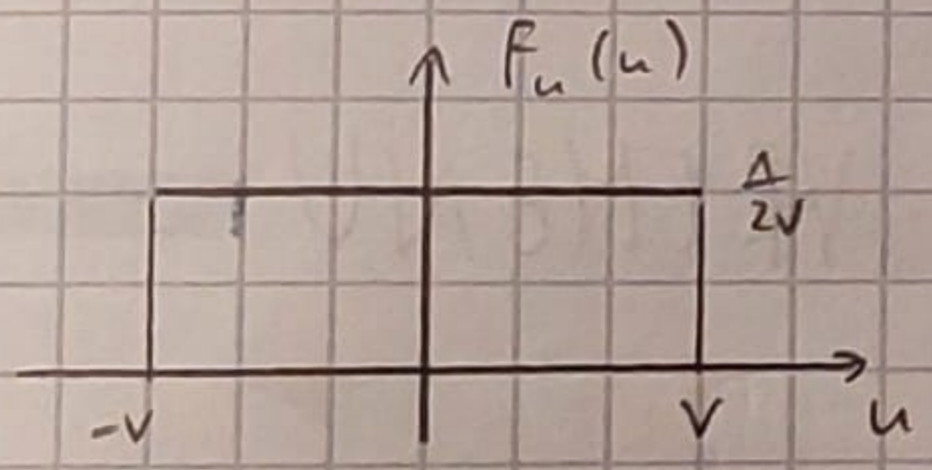
$f_u = \frac{1}{T_u} = 1000$ Hz

$f_{max} = 500$ Hz

Pr5) Signal $u(t)$ se dovodi na ulaz lin. kvantizatora koji svaki uzorak signala kodira s b bitova. Vrijednosti razine signala ravnaju se po U razdiobi na $[-V, +V]$. Odredite potreban br. bitova b za koji je omjer sr. snage signala i sr. snage kv. šuma na izl. kvant. veći od 40dB.

NI ZBIRKA
NI ISPIT





$\frac{S}{N_Q} > 40$ dB

$b = ?$

$N_Q = \frac{1}{3} \cdot V^2 \cdot 2^{-2b}$

$E(u(t)) = \int_{-V}^{V} u f_u(u)\,du = \frac{1}{2V}\int_{-V}^{V} u\,du = \frac{1}{2V} \cdot \left.\frac{u^2}{2}\right|_{-V}^{V}$

↳ sr. snaga signala $= \frac{1}{4V} \cdot \left. u^2 \right|_{-V}^{V} = 0$

$var(X) = E[x^2] - E[x] = E[x^2]$

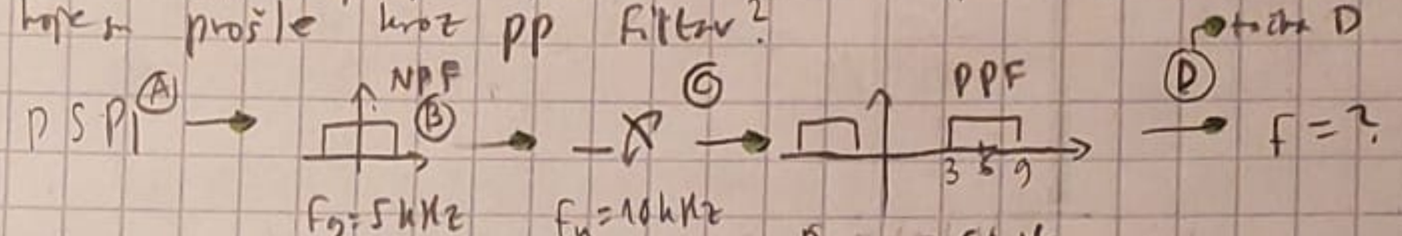
$S = \int_{-V}^{V} u^2 f_u(u)\,du = \frac{1}{2V} \cdot \left.\frac{u^3}{3}\right|_{-V}^{+V} = \frac{1}{6V} \cdot \left. u^3 \right|_{-V}^{V}$

$= \frac{1}{6V}(2V^3) = \frac{1}{3}V^2$

$\frac{S}{N_Q} = \frac{\frac{V^2}{3}}{\frac{1}{3}V^2 \cdot 2^{-2b}} = 2^{2b} \Rightarrow \left.\frac{S}{N}\right|_{dB} = 10\log_{10} 2^{2b} = 20b\log_{10}2 > 40$

$b \geq 7$ ✓

Pr6) Per. slijed prav. impulsa amplitude 1V, frekv. 2kHz i omjera $\frac{\tau}{T} = \frac{1}{6}$ propušten je kroz idealni NPF granične frekv. 5kHz, a potom idealno uzorkovan s frekv. uzorkovanja 10 kHz. Uzorkovani signal je potom propušten na idealni pojasnopropusni filter sa širinom pojasa propuštanja 6 kHz i centr. frekv. pojasa propuštanja 6 kHz. Na kojim frekv. se nalaze komp. signala koje su prošle kroz PP filter?



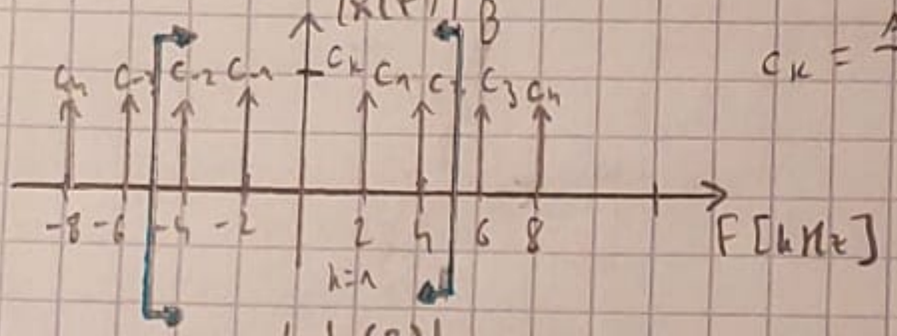
$f_g = 5\,kHz$ $\quad f_u = 10\,kHz$

$B_{PPF} = 6\,kHz$

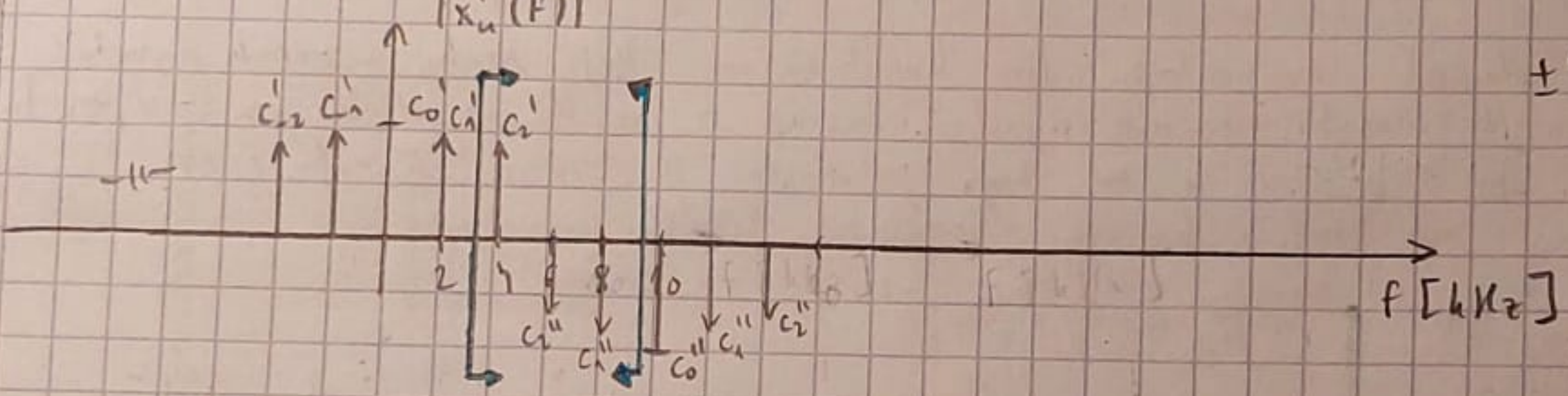
$f_{cPPF} = 6\,kHz$

$f = 2\,kHz$

$\frac{\tau}{T} = \frac{1}{6}$

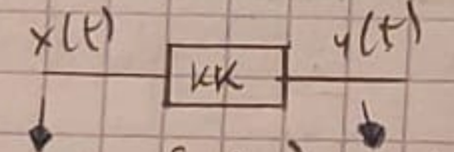


$$c_k = \frac{A\tau}{T} \cdot \frac{\sin\left(k\pi \frac{\tau}{T}\right)}{k\pi\frac{\tau}{T}} = \frac{A}{6} \cdot \frac{\sin\left(\frac{k\pi}{6}\right)}{\frac{k\pi}{6}}$$



$\pm 4, \pm 6, \pm 8$

KAPACITET KOM. KANALA U KONT. VREMENU



$f_X(x)$ $\quad f_{XY}(x,y)$ $\quad f_Y(y)$

$(f_1(x))$ $\quad (f_2(x))$

$c = F(I(x;y))$

$$\boxed{H(x) = -\int_{-\infty}^{+\infty} f_X(x)\log_2 f_X(x)\,dx}$$

$$\boxed{H(y) = -\int_{-\infty}^{+\infty} f_Y(y)\log_2 f_Y(y)\,dy}$$

$$\boxed{f_X(x) = \int_{-\infty}^{+\infty} f_{XY}(x,y)\,dy} \qquad \boxed{f_Y(y) = \int_{-\infty}^{+\infty} f_{XY}(x,y)\,dx}$$

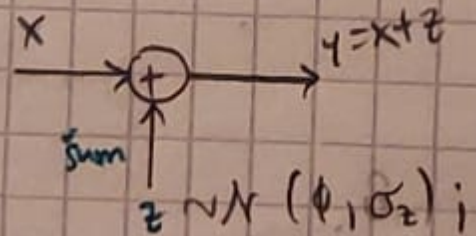
$$\boxed{H(y|x) = -\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} f_{XY}(x,y)\log_2 \frac{f(x,y)}{f_X(x)}\,dx\,dy}$$

$$\boxed{I(x;y) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} f_{XY}(x,y)\log_2 \frac{f_{XY}(x,y)}{f_X(x)f_Y(y)}\,dx\,dy}$$

$$\boxed{I(x;y) = H(y) - H(y|x)}$$

$$\boxed{\int x \ln x\,dx = ?}$$

pomaže pri rješavanju integrala



$z \sim N(0, \sigma_z)$; $\phi(z)$

θ, $x(t)$, $\underset{1D}{f_X(x)} \Rightarrow H(x) \rightarrow H(x)_{max}$

Q: Koja 1D razdioba pretvara $H(x)$ u $H(x)_{max}$?

A: $f_X(x) = \frac{1}{\sigma_X\sqrt{2\pi}} e^{-\frac{x^2}{2\sigma_X^2}}$, $E(x) = 0$

$$\boxed{H(x) = \ln\left(\sqrt{2\pi e}\,\sigma_X\right)}$$

$$\boxed{I(x;y) = H(y) - H(x+z \mid x) = H(y) - H(z)}$$

$x:\ E(x) = \phi$, $E(x^2) = \delta_x^2$ | $z:\ E(z) = \phi$, $E(z^2) = \delta_z^2$

$y = x + z \Rightarrow E(y) = 0$

$$E(y^2) = E((x+z)^2) = E(x^2) + E(z^2) = \sigma_x^2 + \sigma_z^2 = \sigma_y^2$$

$$I(x;y) = \ln\left(\sqrt{2\pi e}\sqrt{\sigma_y^2}\right) - \ln\left(\sqrt{2\pi e}\sqrt{\sigma_z^2}\right) = \frac{1}{2}\ln\left(\frac{\sigma_x^2 + \sigma_z^2}{\sigma_z^2}\right) = \frac{1}{2}\ln\left(1 + \frac{\sigma_x^2}{\sigma_z^2}\right)$$

$$\left.\begin{matrix} \sigma_x^2 = S \\ \sigma_z^2 = N \end{matrix}\right\} = \frac{1}{2}\ln\left(1 + \frac{S}{N}\right) \quad \left[\frac{nat}{simb}\right]$$

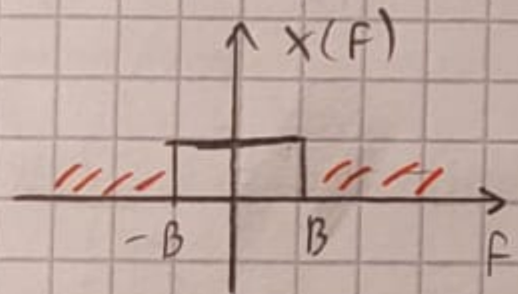
$$\boxed{C = \frac{1}{2}\log_2\left(1 + \frac{S}{N}\right) \quad [bit/simb]}$$

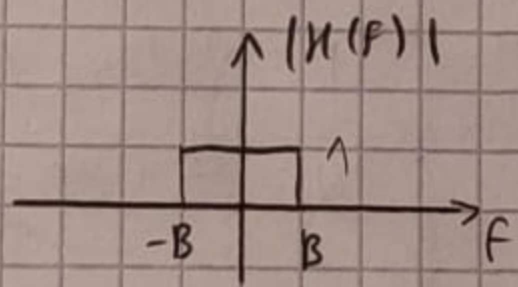
- kontinuiranu veličinu predočavamo diskretnom

$$\begin{aligned} x &= [x_1\ x_2\ \dots\ x_n], & E[x_k] &= 0, & E[x_k^2] &= \sigma_{x_k}^2 \\ y &= [y_1\ y_2\ \dots\ y_n], & E[y_k] &= 0, & E[y_k^2] &= \sigma_{y_k}^2 \\ z &= [z_1\ z_2\ \dots\ z_n], & E[z_k] &= 0, & E[z_k^2] &= \sigma_{z_k}^2 \end{aligned}$$

$$I(x;y) = \sum_{k=1}^{n}\log_2\left(\sqrt{2\pi e}\sqrt{\sigma_{x_k}^2 + \sigma_{z_k}^2}\right) - \sum_{k=1}^{n}\log_2\left(\sqrt{2\pi e} - \sqrt{\sigma_{z_k}^2}\right)$$

↳ pretp: $\begin{cases} \sigma_{x_k}^2 = \sigma_x^2 \\ \sigma_{y_k}^2 = \sigma_y^2 \\ \sigma_{z_k}^2 = \sigma_z^2 \end{cases}$ $= \frac{n}{2}\log_2\left(1 + \frac{\sigma_x^2}{\sigma_z^2}\right)$ [bit/simb.]





$Y(f) = X(f) \cdot H(f)$

$$\boxed{\begin{gathered} C = 2 \cdot B \cdot D \\ \downarrow \\ D = \text{dinamika} = \text{br. bitova kojim kodiramo svaki uzorak} \\ ?\ D = \frac{1}{2}\log_2\left(1 + \frac{S}{N}\right) \end{gathered}}$$

$f_u = 2B$

$n = 2B\ \frac{uzorak}{s}$

$$\boxed{I(x;y) = B \cdot \log_2\left(1 + \frac{S}{N}\right) \quad [bit/s]}$$

$$\boxed{C = B\log_2\left(1 + \frac{S \cdot |H(f)|^2}{N}\right)}$$ → realnije formula

! $$\boxed{C = B \cdot \log_2\left(1 + \frac{S \cdot |H(f)|^2}{\Gamma \cdot N}\right)}$$

$\Gamma = 1$ (osim ako je drugačije navedeno), $|H(f)| = 1$

↳ kodno pojačanje

$$\boxed{E = \frac{C}{B} = \log_2\left(1 + \frac{S}{N}\right) \quad [b/s/Hz]}$$ → efikasnost

$\frac{E_b}{N_0}$ → jamči ispravan prijenos (E_b = energija po bitu)

$$S = \frac{E_b}{T_b} = E_b \cdot R_b = E_b \cdot C$$

$$\frac{C}{B} = \log_2\left(1 + \frac{E_b \cdot C}{N_0 \cdot B}\right) \Rightarrow \frac{E_b}{N_0} = \frac{2^{\frac{C}{B}} - 1}{\frac{C}{B}}$$ ⇒ min. E po bitu koja mi osigurava ispravan prijenos jednog bita

$$B \to \infty \Rightarrow C_\infty = \frac{S}{N_0}\log_2 e$$

Pr1) U realnom kanalu u kont. vremenu omjer sr. snage signala prema sr. sn. šuma je 10^5. Odredite koliko će se smanjiti prijenosna brzina u tom kanalu u odnosu na kapacitet uslijed korištenja neoptimalnog kodnog sustava Γ koji unosi smanjenje sr. sn. signala prema sr. sn. šuma u iznosu od 20 db.

C = max moguća prijenosna brzina

$\frac{S}{N} = 10^5$

$\frac{C}{R} = ?$

$\Gamma = 20$ dB

$|H(f)| = 1$

$$\frac{C}{R} = \frac{B \log_2 \left(1 + \frac{S}{N}\right)}{B \cdot \log_2 \left(1 + \frac{S/N}{\Gamma}\right)} = 1.67$$

$$10 \log_{10} \Gamma = 20 \Rightarrow \Gamma = 100$$

Pr2)

$\rightarrow \boxed{-R-} \rightarrow R_{izl} = 64$ kb/s

$f_u = 8$ kHz $= 8000 \frac{\text{uzorak}}{s}$

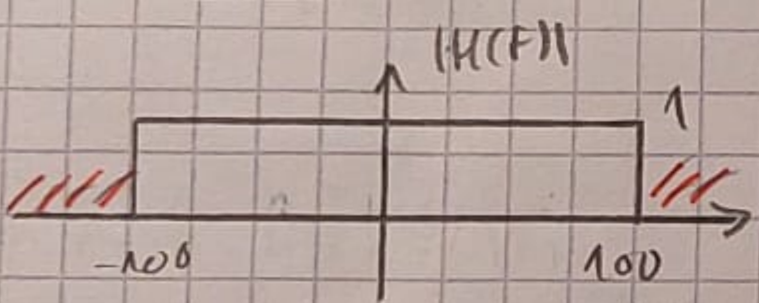
$r = 8 \frac{\text{bit}}{\text{uzorak}} \rightarrow D$

Pr3) U AWGN kanalu djeluje bijeli Gaussov šum spektr. gustoće snage 5 nW/Hz, $\forall f \in \mathbb{R}$. Kanal je ograničen na pojas frekv. $[-100, 100]$. Koliko iznosi sr. snaga signala na ulazu AWGN kanala ako dinamika u tom kanalu iznosi 5 bit/uz.?

(ispitni)

AWGN kanal

$\rightarrow \boxed{\quad} \rightarrow$



$D = 5$ bit/uz

$S = ?$

$$S_N(f) = \begin{cases} 5 \text{ nW/Hz} & , -100 \div 100 \text{ Hz} \\ 0 & , \text{inače} \end{cases}$$

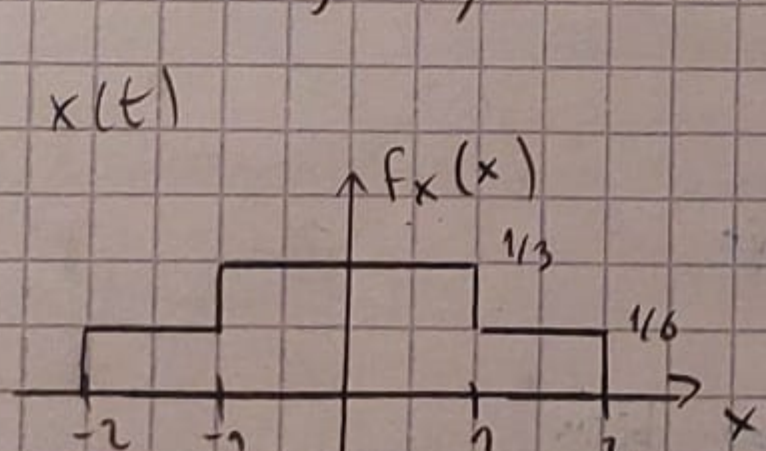
$$5 \text{ nW/Hz} = \frac{N_0}{2}$$

$$D = \frac{1}{2} \log_2 \left(1 + \frac{S}{N}\right)$$

$$N = \frac{N_0}{2} \cdot 2B = \ldots = 5 \cdot 10^{-9} \cdot 200 \cdot 10^3 = 10^{-3} \text{ W}$$

Pr4) Zadan je signal $x(t)$ i njegova distribucija $f_X(x)$

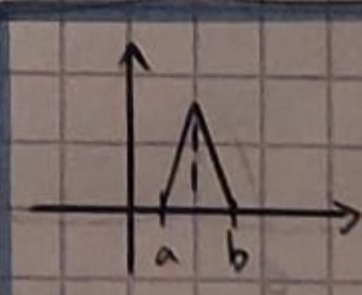
$x(t)$



$$H(x) = -\int_{-\infty}^{+\infty} f_X(x) \log_2 f_X(x) \, dx$$

$$= -2 \cdot \left\{ \int_{-2}^{-1} \frac{1}{6} \log_2 \frac{1}{6} dx + \int_{1}^{0} \frac{1}{3} \log_2 \frac{1}{3} \right\}$$

$$= 1.9$$



$\int x \ln x \, dx = ?$

ZADACI

ZAD1) Zadan je bin. blok kod K. Na ulaz zadanog koda kanala dolaze 3 poruke $d_1 = 101$, $d_2 = 011$, $d_3 = 111$ i kodne riječi na izlazu: c_1, c_2, c_3. Odredite kodnu riječ koja odg. poruci 110.

$\rightarrow \boxed{kk} \rightarrow$

$d_1 = [101]$ $\quad c_1 = [101\ 101]$
$d_2 = [011]$ $\quad c_2 = [110\ 100]$
$d_3 = [111]$ $\quad c_3 = [101\ 011]$

$d_4 = [110]$ $\quad c_4 = ?$

(1) $n = 6,\ k = 3$

$c_i = d_i \cdot G$

$$G = \begin{bmatrix} g_{11} & g_{12} & g_{13} & \cdots & g_{16} \\ g_{21} & g_{22} & g_{23} & \cdots & g_{26} \\ g_{31} & g_{32} & g_{33} & \cdots & g_{36} \end{bmatrix}$$

(2)
$c_1 = [g_{11}+g_{31}\quad g_{12}+g_{32}\quad g_{13}+g_{33}\ \cdots\ g_{16}+g_{36}]$
$c_2 = [g_{21}+g_{31}\quad g_{22}+g_{32}\quad g_{23}+g_{33}\ \cdots\ g_{26}+g_{36}]$
$c_3 = [g_{11}+g_{21}+g_{31}\quad g_{12}+g_{22}+g_{32}\ \cdots\ g_{16}+g_{26}+g_{36}]$

$$\left.\begin{aligned} g_{11}+g_{31} &= 1 \\ g_{21}+g_{31} &= 1 \\ g_{11}+g_{21}+g_{31} &= 1 \end{aligned}\right\} \quad \begin{aligned} & g_{11} = 1+g_{31} \\ & g_{21} = 1+g_{31} \\ & 1+g_{31}+1+g_{31}+g_{31} = 1 \\ & \hookrightarrow 1+1 = 0 \\ & \hookrightarrow g_{31}+g_{31} = 0 \end{aligned}$$

ostale ...

$$G = \begin{bmatrix} 0 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 \end{bmatrix}$$

$c_4 = [110]\,G = [011001]$

ZAD2) Zadan je LBCK (lin. bin. cikl. kod) K, dim $[n,k]$. Poznato je da $\overbrace{\text{kodne riječi}}^{*}$ pripadaju zadanom kodu.

a) kodna brzina?
b) gen. polinom $g(x) = ?$
c) koder?
d) Na ulazu kodera pojavljuje se poruka [00000000111], odredite CRC (r) u bin. i polinomskom obliku.
e) Odredite cikl. provjeru zalihosti za 1. kodnu riječ koja se pojavljuje na izlazu kodera kanala dualnog koda.

K, $[n,k] = [?, 11]$

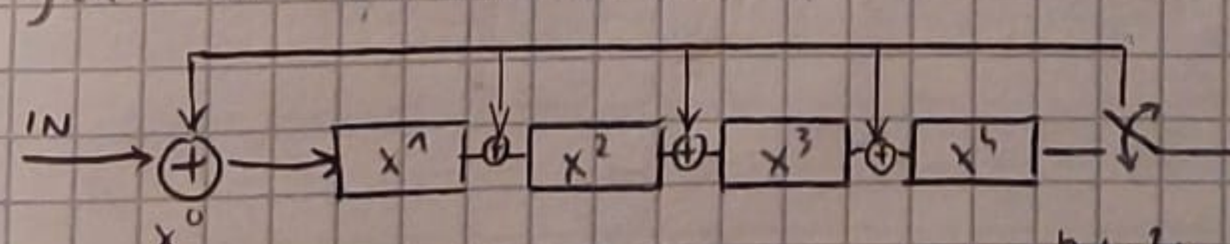
$$K = \left\{\begin{matrix} 001111111111000 \\ 111111111111111 \end{matrix}\right\} * \\ \begin{matrix} 110000000000111 \\ 000000000000000 \\ 100000000001111 \\ 000000111111111 \leftarrow \end{matrix}$$

a) $R = \frac{k}{n} = \frac{11}{15}$

b) $g(x) = x^{n-k} + \ldots + 1$

$g(x) = x^4 + x^3 + x^2 + x + 1$

c)



d) $d = [00000000111]$

$c = [d \mid r]$

$r(x) = \text{ost}\ \frac{x^{n-k}\, d(x)}{g(x)}$

$r(x) = \text{ost}\ \frac{x^4 (x^2+x+1)}{g(x)} \Rightarrow r(x) = x^3 + x^2$

$r = [1100]$

e) $CRC = ?$

$$\overbrace{[0001]}^{d}0001000 \longrightarrow \boxed{k^{\perp}} \longrightarrow c^{\perp} = [\underset{0001}{d^{\perp}} | r^{\perp}]$$

$[n^{\perp}, k^{\perp}] = [15, 11]$

$x^{15} + 1 = g(x) \cdot h(x)$

$g(x) \to G$, $h(x) \to G^{\perp}$

$h(x) = x^{11} + x^{10} + x^{6} + x^{5} + x + 1$

$r^{\perp}(x) = x^{10} + x^{6} + x^{5} + x + 1$

ZAD3) Razmatrajte sust. u kojem se svaki bit štiti Hammingovim kodom Hamm(r). Zadan kod je LBK (lin. bin. kod).

a) minimalni r potrebno za zaštitu poruke duljine 1 bit i matrica provjere pariteta u stand. obliku

b) $P_{b} = 0.1$, kad koder ne reagira na grešku?

a) $r_{min} = ?$

$d = 1$ bit

$d = [\underbrace{\quad}_{1\ bit}]$

$H = [\qquad]$

$[2^{r}-1, 2^{r}-1-r] = [n, k] = [3, 1]$

$$\left.\begin{array}{l} 2^{r}-1-r = 1 \\ 2^{r}-r = 2 \end{array}\right\} r = 2$$

ili

$c = [\_\ \_\ d_1]$

$H = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix} = [A^{T} | I]$

$H = \left[\begin{array}{c|cc} 1 & 1 & 0 \\ 1 & 0 & 1 \end{array}\right] \to G = [1 1 1] = [I | A]$

$A^{T} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

b) $P_e = 0.1^{3}$

ZAD4.) Zadan je lin. bin. cikl. kod K za koji je $n=4$, $M=4$.

a) Sve kodne riječi, udaljenost $t=?$

b) gen. polinom?

c) matrica G u std. obliku

a) $M=4 \rightarrow 4$ kodne riječi
duljina $= 4$ bita

$$K = \begin{cases} 0000 \\ 1111 \\ 1010 \\ 0101 \end{cases}$$

$d(K) = ?$

$v(4)$

b) $g(x) = x^2+1$

ZAD5.) Signal: $x(t) = 10\cos(2000\pi t)\cdot\cos(6000\pi t)$ [V]. Odredite:

a) min. frekv. uzorkovanja

b) srednja snaga signala

c) Signal $x(t)$ dovodimo na kvantizator koji provodi jedn. kvantizaciju, svaki s 8 bit/uz. Koliko max može iznositi raspon ampl. na ulazu da vrijedi $\frac{S}{N_q} \geq 40$ dB.

a) $f_u = ?$

$$x(t) = \ldots 5\cos(4000\pi t) + 5\cos(8000\pi t)$$

$f = 2000$ Hz $\qquad f = 4000$ Hz

$f_u = 8000$ Hz

b) $P = \frac{5^2}{2} + \frac{5^2}{2} = 25$ W

c) $N_q = \frac{1}{3} A_n^2\, 2^{-2r}$ $\qquad \frac{S}{N_q} \geq 40$ dB

$P = 25$ W $\qquad \rightarrow 2\cdot m_{max} = 44.34$ V