TEORIJA
INFORMACIJE VOL. 2

ENTROPIJSKO KODIRANJE

Riješeni zadaci

2.1

a) Postoji li prefiksni kod (baza 2) sa sljedećim duljinama kodnih riječi: $\{5,3,5,2,1,5\}$? Ako postoji, tada napišite sve kodne riječi danog koda. Ako ne postoji, tada zašto?

b) Napišite sve kodne riječi prefiksnog koda čije su duljine kodnih riječi: $2,5,2,3,5,2$.

c) Dan je prefiksni kod s duljinama kodnih riječi kao pod b). Za koje je vjerojatnosti pojavljivanja simbola $X = \{x_1, \dots, x_6\}$ srednja duljina kodne riječi jednaka entropiji $-H(x)$? Odredite $H(x)$.

a) $l_i = \{5, 3, 5, 2, 1, 5\}$ $i = 1, \dots, 6$

kraftova nejednakost

$\sum d^{-l_i} \leq 1 \iff$ postoji prefiksni kod

$2^{-5} + 2^{-3} + 2^{-5} + 2^{-2} + 2^{-1} + 2^{-5} = \frac{33}{32} \geq 1 \implies$ ne postoji

b) $l_i = \{2, 5, 2, 3, 5, 2\}$ $\sum d^{-l_i} = 1 \leq 1 \implies$ postoji prefiksni kod

l_i	2	5	2	3	5	2
kod A	01	0000	10	001	0001	11
kod B	01	0010	10	000	0011	11
kod C	00	0110	10	010	0111	11
kod D	00	1110	01	110	1111	10
kod E	00	1010	01	100	1011	11

c) $l_i = \{2, 5, 2, 3, 5, 2\}$

$L(x_i) = H(x_i)$

$\sum_{i=1}^{n} p(x_i) l_i = -\sum_{i=1}^{n} p(x_i) \log_2 p(x_i)$

$l_i = -\log p(x_i)$

$[p(x_i)] = \left[\frac{1}{4} \quad \frac{1}{16} \quad \frac{1}{4} \quad \frac{1}{8} \quad \frac{1}{16} \quad \frac{1}{4}\right]$

$H(x_i) = \frac{19}{8} = 2.375 \; \text{bit}/\text{simbol}$

2.2

Odredite koji od navedenih kodova su JDK, prefiksni i/ili nesingularni

Simboli	kod A	kod B	kod C	kod D	kod E
x_1	000	0	0	0	0
x_2	001	01	10	10	10
x_3	010	011	110	110	1100
x_4	011	0111	1110	1110	1101
x_5	100	01111	11110	1011	1110
x_6	101	011111	111110	1101	1111

kod A) $S_0 = \{000, 001, 010, 011, 100, 101\}$

$S_1 = \{\varnothing\}$

$\implies$ JDK

$\sum d^{-l_i} = \frac{3}{4} \leq 1$ može biti prefiksni ali nije

nesingularan

B) $S_0 = \{0, 01, 011, 0111, 01111, 011111\}$

$S_1 = \{1, 11, 111, 1111, 11111\}$

$S_2 = \{1, 11, 111, 1111\}$

$\vdots$

$S_5 = \{1\}$

$S_6 = \{\varnothing\}$

$\implies$ JDK

$\sum d^{-l_i} = \frac{63}{64}$ moguće je stvoriti prefiksan kod ovih duljina

kod je nesingularan jer za razl. x daje razl. $C(x)$

C) $S_0 = \{0, 10, 110, 1110, 11110, 111110\}$

$S_1 = \{\varnothing\}$

$\implies$ JDK

kod je prefiksan, a uz iste duljine kao b zadovoljava k. jednakost

d) $S_0 = \{0, 10, 110, 1110, 1011, 1101\}$

$S_1 = \{11, 1\}$

$S_2 = \{1, 0, 10, 01\}$

$\implies$ Nije JDK

nije prefiksni

$\sum_i d^{-l_i} = \frac{17}{16}$ ne zadovoljava kraftovu jednakost

kod je nesingularan

e) $S_0 = \{0, 10, 1100, 1101, 1110, 1111\}$
$S_1 = \{\emptyset\}$
$\Rightarrow$ JDK
prefiksni, nesingularan
$\sum d^{-l_i} = 1 \leq$ može biti prefiksni

2.3

Diskretno bezmemorijsko izvorište generira simbole x_i, $i = 1,...,4$. Provedeno su 4 različita kodiranja.

a) Provjerite kraftovu nejednakost
b) Pravjerite JDK

Simboli	kod A	kod B	kod C	kod D
x_1	00	0	0	0
x_2	01	10	11	100
x_3	10	11	100	110
x_4	11	110	110	111

a) $k = \sum_{i=1}^{4} d^{-l_i} \leq 1$

A: 1 ✓   B: $\frac{9}{8}$ ✗   C: 1 ✓   D: $\frac{7}{8}$ ✓

b) A i D prefiksni
B: $S_0 = \{0, 10, 11, 110\}$ C: $S_0 = \{0, 11, 100, 110\}$
$S_1 = \{0\}$ $S_1 = \{0\}$
$\Rightarrow$ nije JDK $\Rightarrow$ nije JDK

2.4

Diskretno bezmemorijsko izvorište X generira x_i, $i = 1,...,100$. C je prefiksni kod za X. Dokažite da zbroj duljina kodnih riječi mora biti veći od 664.

$\sum_{i=1}^{100} 2^{-l_i} \leq 1$

Aritmetička sredina $\geq$ geometrijska sredina

$\frac{1}{100} \sum_{i=1}^{100} 2^{-l_i} \geq \sqrt[100]{\prod_{i=1}^{100} 2^{-l_i}} = \sqrt[100]{2^{-\sum_{i=1}^{100} l_i}} \quad / \cdot 100$

$100 \cdot \sqrt[100]{2^{-\sum_{i=1}^{100} l_i}} \leq \sum_{i=1}^{100} 2^{-l_i} \leq 1$

$\sqrt[100]{2^{-\sum l_i}} \leq \frac{1}{100}$

$2^{-\sum l_i} \leq \left(\frac{1}{100}\right)^{100}$

$-\sum l_i \leq \log_2 \left(\frac{1}{100}\right)^{100}$

$\sum l_i \geq 100 \cdot \log_2 100 = 664$

2.5

Kodirajte segment slike, A, binarnim kodom takvim da je srednja duljina kodne riječi minimalna (Huffman!). Odredite srednju duljinu kodne riječi kao i veličinu memorije (u kbyte-ima) za sliku.

$A = \begin{bmatrix} 17 & 17 & 3 & 4 & 5 \\ 5 & 2 & 3 & 13 & 5 \\ 8 & 18 & 17 & 23 & 17 \\ 15 & 17 & 3 & 8 & 6 \end{bmatrix}$

$p(17) = 5/20$ $P(2) = 1/20$ $P(23) = 1/20$
$p(3) = 3/20$ $P(13) = 1/20$ $P(6) = 1/20$
$p(4) = 2/20$ $P(8) = 1/20$
$P(5) = 3/20$ $P(18) = 2/20$

$\Rightarrow$

$17: \frac{5}{20}$	$2: \frac{1}{20}$
$3: \frac{3}{20}$	$13: \frac{1}{20}$
$5: \frac{3}{20}$	$8: \frac{1}{20}$
$4: \frac{2}{20}$	$23: \frac{1}{20}$
$18: \frac{2}{20}$	$6: \frac{1}{20}$
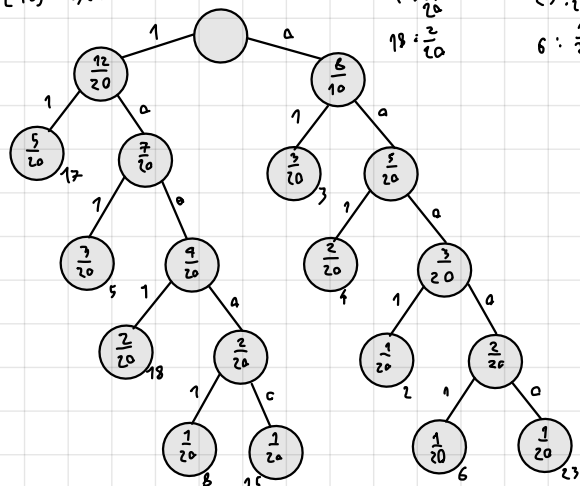
17: 11   3: 01
5: 101   4: 001
18: 1001   2: 0001
8: 10001   6: 00001
15: 10000   23: 00000

$L = \sum_{i=1}^{20} p_i l_i = 3.15 \frac{bit}{simbol}$

$M = 20 \cdot L = 63 \, bit = \frac{63}{1024 \cdot 8} = 7.69 \cdot 10^{-3} \, kB$

2.6

$[P(x_i)] = [0.081 \quad 0.113 \quad 0.086 \quad 0.06 \quad 0.098 \quad 0.085 \quad 0.031 \quad 0.135 \quad 0.063 \quad 0.06 \quad 0.091 \quad 0.097 \quad a] \quad d = 4$

Minimalna srednja duljina kodne riječi (Huffman). Izračunajte srednju duljinu L i efikasnost koda. Odredite koliko se poveća efikasnost ovim kodiranjem u odnosu na kodiranje istog skupa binarnim kodom
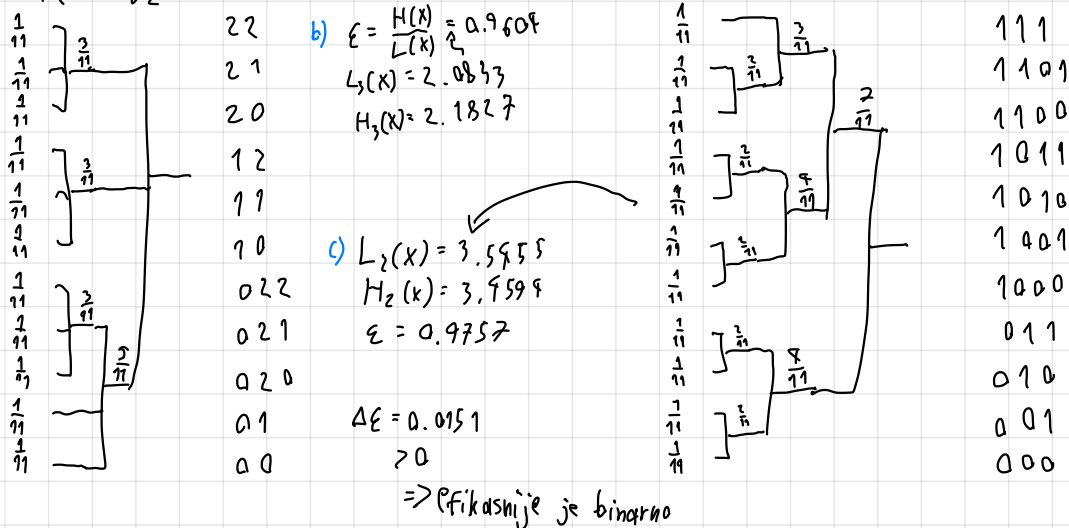

$L_4(x) = \sum p_i l_i = 1.865$ dibit/simbol

$H_4(x) = \sum p_i \log_4 p_i = 1.7546$ dibit/simbol

$L_2(x) = 3.557$ bit/simbol

$H_2(x) = 3.5091$ bit/simbol

$\varepsilon_4 = 0.9408$

$\varepsilon_2 = 0.9865$

$\varepsilon = \varepsilon_4 - \varepsilon_2 = 0.0458$

5 × 3

5 × 4

2 × 5

2.7
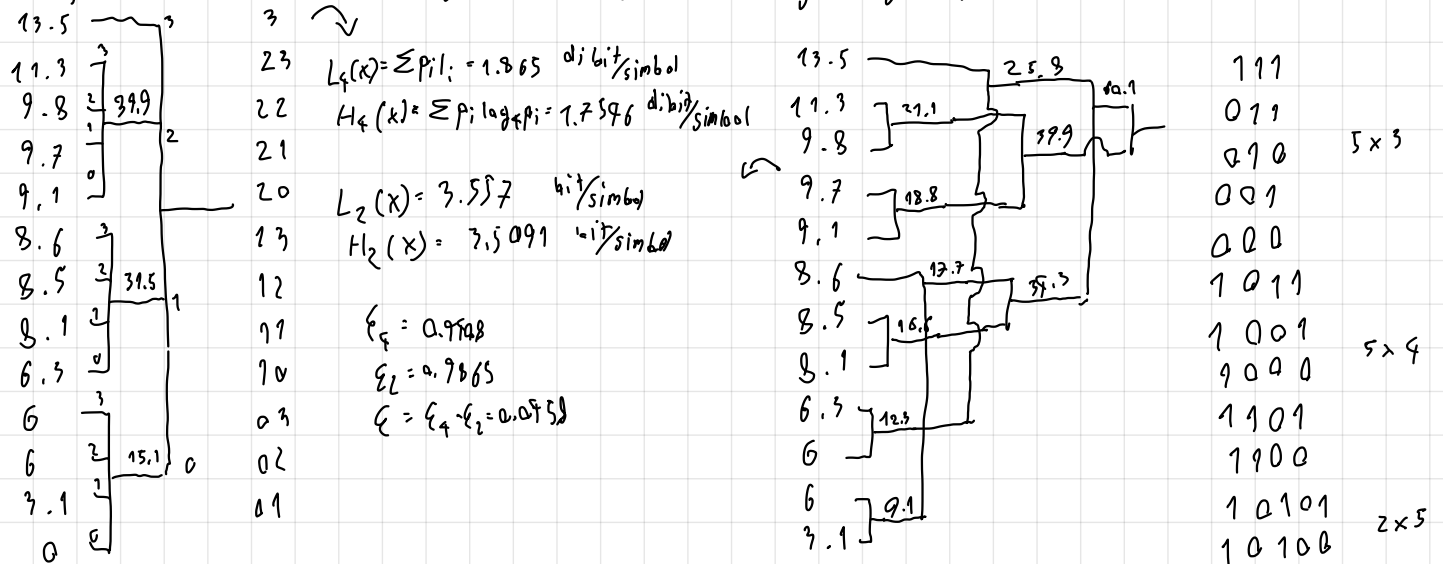
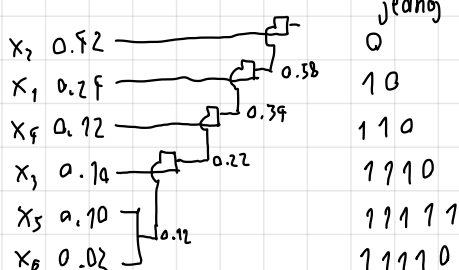$X_i \quad i = 1, \dots, n \qquad P(x_i) = p, \quad H(x) = 3.4599$ bit/simbol

a) Kodirajte ternarnim kodom (Huffman!) dani skup simbola X
b) Odredite efikasnost danog koda
c) Odredite za koliko se smanji ili poveća efikasnost koda ovim kodiranjem u odnosu na kodiranje binarno

a) $H(x) = \log_2 n = 3.4599 \Rightarrow n = 11$

b) $\varepsilon = \dfrac{H(x)}{L(x)} = 0.9608$

$L_3(x) = 2.0833$

$H_3(x) = 2.1827$

c) $L_2(x) = 3.5455$

$H_2(x) = 3.4599$

$\varepsilon = 0.9757$

$\Delta \varepsilon = 0.0151$
> 0

$\Rightarrow$ efikasnije je binarno

2.8

x_i	X_1	X_2	X_3	X_4	X_5	X_6
$P(x_i)$	0.24	0.52	0.1	0.12	0.1	0.02

Kodirajte huffmanovim kodom d = 2. Odredite duljinu trajanja binarnih simbola "0" i "1" tako da prosječan broj simbola koji se prenesu komm. kanalom u jednoj sekundi 400.
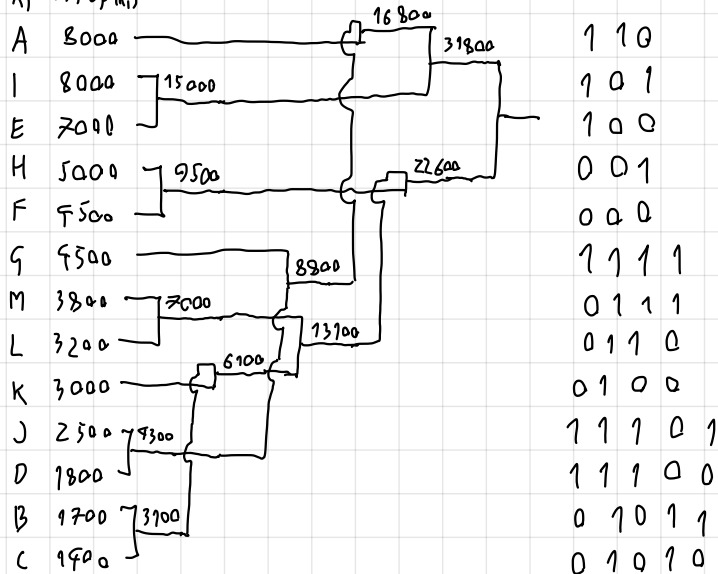

$L_2(x) = 2.26$ bit/simbolu

$t = \dfrac{1}{400 \cdot 2.26} = 1.1 \cdot 10^{-3}$ s

Mirna digitalizirana slika s bojama A,B,C,...,M opisana je u tablici. Odredite hoffmana. Odredite vrijeme potrebno za prijenos kodirane slike između 2 računala koja su povezana modemskom vezom 33600 bit/s

X_i	A	B	C	D	E	F	G	H	I	J	k	L	M
$P(K_i)$	8000	1700	1900	1800	7000	9500	9500	5000	8000	2500	3000	3200	3800

$\Sigma = 54900$

X_i 54900 $p(k_i)$

A	8000		16800		1 1 0
I	8000	15000	31800		1 0 1
E	7000				1 0 0
H	5000	9500	22600		0 0 1
F	9500				0 0 0
G	9500		8800		1 1 1 1
M	3800	7000			0 1 1 1
L	3200		13700		0 1 1 0
K	3000	6100			0 1 0 0
J	2500	4300			1 1 1 0 1
D	1800				1 1 1 0 0
B	1700	3700			0 1 0 1 1
C	1900				0 1 0 1 0

$L_z(x) = 3.5386 \dfrac{bit}{simbol}$

$\dfrac{L(x) \cdot 54900}{33600} = 5.7292 \ s$

Nerješeni zadaci

2.1

Nesingularni - ni jedna dva simbola nemaju isti kod

JDK - niti jedan simbol se ne može dobiti kombinacijom drugih (sardinass-patterson)

Prefiksni - ni jedan simbol nije prefiks drugog

a) $K_1 = \{0, 101\}$

nesingularan

JDK: $S_0 = \{0, 101\}$ $S_1 = \{\emptyset\}$ JDK √

Prefiksni : je (i zadovoljava kraftovu jednakost $\sum d^{-l_i} = 2^{-1} + 2^{-3} = \frac{5}{8}$)

b) $k_2 = \{1, 101\}$

nesingularan

JDK: $S_0 = \{1, 101\}$ $S_1 = \{01\}$ $S_2 = \{\emptyset\}$ JDK √

Prefiksni nije

c) $k_3 = \{0, 10, 110, 111\}$

nesingularan

JDK: $S_0 = \{0, 10, 110, 111\}$ $S_1 = \{\emptyset\}$ √

Prefiksni : je

d) $K_4 = \{0, 01, 10, 11\}$

nesingularan

JDK: $S_0 = \{0, 01, 10, 11\}$ $S_1 = \{1\}$ $S_2 = \{1, 0\}$ X

Prefiksni: nije

2.2

X	P(X)	K
a	0.5	0
b	0.25	01
c	0.125	011
d	0.125	111

1) $H(X) = -\sum p_i \log p_i = \frac{7}{4}$ bit/simbol

2) k nije prefikshi

3) $L(X) = \frac{7}{4}$ kad je optimalan za L=H

4) Sard-Patt: $S_0 = \{0, 01, 011, 111\}$ $S_1 = \{1, 11\}$ $S_3 = \{11, 1\}$ ... JDK √

2.3

	k_1	k_2	k_3	k_4	JDK: ?
a	1	10	00	110	
b	10	00	01	111	
c	100	11	10	11	
d	1000	110	11	01	

k_1: $S_0 = \{1, 10, 100, 1000\}$ $S_1 = \{0, 00, 000\}$ $S_2 = \{\emptyset\}$ JDK

k_2: $S_0 = \{10, 00, 11, 110\}$ $S_1 = \{0\}$ $S_2 = \{0\}$ JDK

k_3: $S_0 = \{00, 01, 10, 11\}$ $S_1 = \{\emptyset\}$ JDK

k_4: $S_0 = \{110, 111, 11, 01\}$ $S_1 = \{0, 1\}$ $S_2 = \{1, 10, 11, 1\}$ X nije JDK

2.4

$S_0 = \{00, 10, 001, 00101, 10101, 1101, 011, 111\}$

$S_1 = \{1, 01, 101\}$

$S_1 = \{0, 0101, 101, 11, 1, 01\}$

$S_3 = \{0, 01, 0101, 01, 01, 1, 0, 0101, 101, 11, ...\}$ DA

Craftova jednakost $\sum_i d^{-l_i} = 2^{-2} \cdot 2 + 2^{-3} \cdot 3 + 2^{-4} + 2^{-5} \cdot 2 = 1 \leq 1$ može biti paritetni?

2.5

$$S = \begin{pmatrix} S_1 & S_2 & \cdots & S_m \\ P_1 & P_2 & & P_m \end{pmatrix} \quad m=6 \quad l_i = \{1,1,2,3,2,3\} \quad \text{craft} \Rightarrow \sum d^{-l_i} \leq 1 \quad d=2: \frac{7}{7} \quad d=3: \frac{26}{27}$$

2.6

x	P_i	C
1	0.4	0
2	P	10
3	0.6-P	11

$H(X) \leq L(X) < H(X)+1$

$0.4 + 1.2 = -(0.4\log 0.4 + p\log p + (0.6-p)\log(0.6-p))$

UPISIVANJE U TABLE MODE DAJE RASPON: $\langle 0, 0.6 \rangle$

2.7

$$X \sim \begin{pmatrix} a & b & c & d & e & f & g \\ 0.22 & 0.35 & 0.15 & 0.09 & 0.09 & 0.05 & 0.05 \end{pmatrix} \quad L = 2.53$$

$L = [2 \quad 2 \quad 3 \quad 3 \quad 3 \quad 4 \quad 4]$ $\qquad H = 2.579$

b	0.35	0	0	
a	0.22	0	1	
c	0.15	1	0	0
d	0.09	1	0	1
e	0.09	1	1	0
f	0.05	1	1	1	0
g	0.05	1	1	1	1

PO POLA

$\epsilon \cdot \frac{H}{C} = 0.98$

2.8

$$f = \{15, 7, 6, 6, 5\} \Rightarrow p = \begin{Bmatrix} a & b & c & d & e \\ 0.39 & 0.18 & 0.15 & 0.15 & 0.12 \end{Bmatrix}$$

d	0.39	0	0	
b	0.18	0	1	
c	0.15	1	0	
d	0.15	1	1	0
e	0.12	1	1	1

0 0 0	1	1 1 1	1 0	1 1 1	1 1 0	0 0
a	b	c	c	e	d	a

2.9

X_4 0.3
X_3 0.21
X_6 0.15
X_5 0.14
X_1 0.1
X_2 0.1
X_7 0
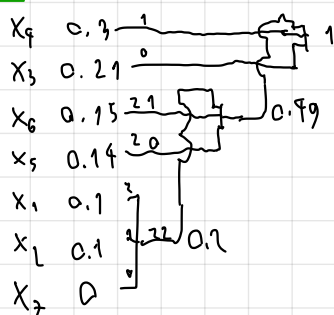
$X_i = \{1, 0, 21, 20, 222, 221, 220\}$

$L_i = \{1/1, 2, 2, 3, 3, 3\}$

$H(X) = 2.47$

$L(X) = 1.69$

$H(x \cdot \inf)$???
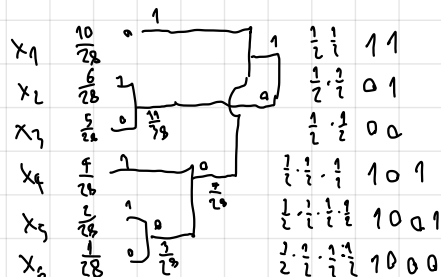
2.10

X_1 $\frac{10}{28}$
X_2 $\frac{6}{28}$
X_3 $\frac{5}{28}$
X_4 $\frac{4}{28}$
X_5 $\frac{2}{28}$
X_6 $\frac{1}{28}$

$\frac{1}{2}\frac{1}{2}$	1 1	
$\frac{1}{2}\frac{2}{2}$	0 1	
$\frac{1}{2}\cdot\frac{1}{2}$	0 0	
$\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}$	1 0 1	
$\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}$	1 0 0 1	
$\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}$	1 0 0 0	

d) $L_i = [2 \ 2 \ 2 \ 3 \ 4 \ 4]$

b) $[P(X_i)] = \begin{bmatrix} \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{8} & \frac{1}{16} & \frac{1}{16} \end{bmatrix}$

2.11

000	001	010	011	100
$$X_i \sim \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ \frac{2}{13} & \frac{3}{13} & \frac{3}{13} & \frac{5}{13} & \frac{11}{65} \\ & & \frac{13}{65} & & \frac{7\cdot1}{13} \end{pmatrix}$$

$L(x) = 3$

5	10
3	01
5	00
2	111
2	110

$L(x) = 2.292$

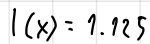$6500 (L(N-L(x_2)) = 4600$

2.12

$X \sim \begin{pmatrix} 1 & 2 & \cdots & 100 \\ 0.01 & 0.01 & & 0.01 \end{pmatrix}$ $2^k \leq m = 100 < 2^7$ $p = \frac{1}{m}$

$k = 100 - 2^6 = 36$

kodnih riječi duljine $6 = 2^6 - 36 = 28$ kodnih riječi duljine 7 ima $2 \cdot 36 = 72$

$L = 0.28 \cdot 6 + 0.72 \cdot 7$

2.13

$p = \begin{bmatrix} \frac{1}{16} & \frac{3}{16} & \frac{1}{16} & \frac{4}{16} & \frac{1}{16} & \frac{2}{16} & \frac{2}{16} & \frac{1}{16} & \frac{1}{32} & \frac{1}{32} \end{bmatrix}$
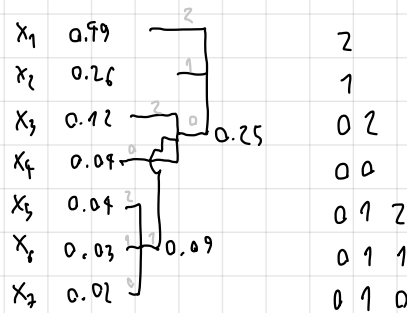

$l(x) = 1.125$

3 bita po znaku

slika $= 1024 \cdot 768 \cdot 1.125 \cdot 3 = 2654208$ bit

$t = \frac{slika}{R} = \frac{slika}{384 \cdot 1000} = 6.912$

2.14

X_1 0.99

X_2 0.26

X_3 0.12

X_4 0.09

X_5 0.04

X_6 0.03

X_7 0.02

0

1 0

1 1 0

1 1 1   1 1

1 1 1   1 0

1 1 1   0 1

1 1 1   0 0

$L(x) = 2.02$ bit/simbol

$H(x) = 2.013$

X_1 0.99

X_2 0.26

X_3 0.12

X_4 0.09

X_5 0.04

X_6 0.03

X_7 0.02

2

1

0 2

0 0

0 1 2

0 1 1

0 1 0

$L(x) = 1.34$ tbit/simbol

$H(x) = 1.27$

2.16

$P_A \geq P_B \geq P_C \geq P_D$

P_A
P_B
P_C
P_D

$\geq P_B \Rightarrow \geq P_A$
$\geq P_A$

2.21

$X = \{a, b, c\}$

$[P(x_j | x_i)] = \begin{bmatrix} 0.8 & 0.1 & 0.1 \\ 0.1 & 0.7 & 0.2 \\ 0 & 0.2 & 0.8 \end{bmatrix}$

$[P(k_i, x_j)] = [P(x)] \cdot [P(x_j | k_i)] = \begin{bmatrix} \frac{8}{55} & \frac{2}{55} & \frac{1}{22} \\ \frac{1}{55} & \frac{14}{55} & \frac{1}{11} \\ 0 & \frac{4}{55} & \frac{4}{11} \end{bmatrix}$

$H(x_j | x_i) = 0.92$

$H(x) = 1.49$

$P(x_1) = 0.8 \, P(x_1) + 0.1 \, P(x_2)$

$P(x_2) = 0.1 \, P(x_1) + 0.7 \, P(x_2) + 0.2 \, P(x_3)$

$P(x_3) = 0.1 \cdot P(x_1) + 0.2 \, P(x_2) + 0.8 \, P(x_3)$

$[P(x)] = \begin{bmatrix} \frac{2}{11} & \frac{4}{11} & \frac{5}{11} \end{bmatrix}$

b) NEDA MI SE VIŠE HUFF

ITD ITD HUFF 22, 23

$R_s = "RIBA\_RIBI"$

$R = [0, \frac{2}{9})$ $D = 0$ $G = 1$

$I = [\frac{2}{9}, \frac{5}{9})$ $R: L=1$ $D = D + 0 \cdot 1 = 0$ $G = G - (1 - \frac{2}{9}) \cdot 1 = \frac{2}{9}$

$B = [\frac{5}{9}, \frac{7}{9})$ $I: L = \frac{2}{9}$ $D = D + \frac{2}{9} \cdot \frac{2}{9} = \frac{4}{81}$ $G = G - (1 \cdot \frac{5}{9}) \cdot \frac{2}{9} = \frac{10}{81}$

$A = [\frac{7}{9}, \frac{9}{9})$ $B: L = \frac{6}{81}$ $D = \frac{22}{243}$ $G = \frac{26}{243}$

$\_ = [\frac{9}{9}, \frac{9}{9})$ $A: L = \frac{9}{243}$ $D = \frac{226}{2187}$ $G = \frac{230}{2187}$

           $\_: L = \frac{9}{2187}$ $D = \frac{2066}{19683}$ $G = \frac{2090}{19683}$

           $R: L = \frac{5}{19683}$ $D = \frac{2066}{19683}$ $G = 0.10500883 \, 45$

           $I: L = 9.516 \cdot 10^{-5}$ $D = 0.10497379098$ $G = 0.10498879$

           $B:$ $D = 0.10985820556$ $G = 0.10498853978$

           $I:$ $D = 0.10498279983$ $G = 0.10985879124$

SVI ARITM NA ISTU FORU

| a | a | a | a | a | a | a | a | a | a | * |

$(0, 0, a) \quad (1, 8, a) , (0, 0, *)$

a a a a a a a a a d *

$PP = 6 \quad PZK = 5$

| a | a | a | a | b | b | b | c | c | d | * | $(0, 0, a)$ |

a a a a b b b c c d $(1, 3, b)$

a a a a b b b c c d $(1, 2, c)$

a a b b b c c d * $(1, 1, d)$

          $(0, 0, *)$

$PP = 9 \quad PZK = 6$

| a | a | c | a | a | c | a | b | c | a | b | a | a | a | c | $(0, 0, a)$ |

a a c a a c $(1, 1, c)$

a a c a a c a b c a $(3, 9, b)$

a c a b c a b a a d c $(3, 3, a)$

c a b a a a c $(1, 2, c)$

          $(0, 0, *)$

$PP = 7 \quad PZK = 5$

| 0 | 5 | 6 | 6 | 6 | 5 | 1 | 1 | 2 | 2 | 1 | 1 | 0 | 1 | 2 | 2 | 2 | 2 | 1 | 0 | 0 | 5 | 5 | 0 | 1 | 1 | 3 | 1 | 5 | 5 | 6 | 6 | 0 | 2 |

                    $(0, 0, 0)$ 3 3 9 3 1 0 *

| 0 | 5 | 6 | 6 | 6 | 5 | 1 | 1 | 2 | 2 | 1 | 1 | $(0, 0, 5)$ | | 1 | 0 | 1 | 2 | 2 | 2 | 2 | 1 | 0 | 0 | 5 | 5 | 0 | $(7, 1, 0)$ |

a 5 6 6 6 5 1 1 2 2 1 1 $(0, 0, 6)$ 1 2 2 2 2 1 0 0 5 5 0 1 1 $(0, 9, 5)$

a 5 6 6 6 5 1 1 2 2 1 1 $(1, 2, 5)$ 2 2 2 2 1 0 0 5 5 0 1 1 3 $(1, 1, 0)$

0 5 6 6 6 5 1 1 2 2 1 1 $(0, 0, 1)$ 2 2 1 0 0 5 5 0 1 1 3 1 5 $(6, 1, 1)$

0 5 6 6 6 5 1 1 2 2 1 1 $(1, 1, 2)$ 1 0 0 5 5 0 1 1 3 1 5 5 6 $(0, 0, 3)$

5 6 6 6 5 1 1 2 2 1 1 0 1 $(1, 1, 1)$ 0 0 5 5 0 1 1 3 1 5 5 6 6 $(2, 9, 5)$

6 6 5 1 1 2 2 1 1 0 1 2 2 $(1, 1, 0)$ 5 5 0 1 1 3 1 5 5 6 6 0 2 $(3, 1, 6)$

5 1 1 2 2 1 1 0 1 2 2 2 2 $(6, 3, 2)$ 0 1 1 3 1 5 5 6 6 0 2 3 3 $(1, 1, 0)$

1 1 0 1 2 2 2 2 1 0 0 5 $(1, 1, 1)$ 1 3 1 5 5 6 6 0 2 3 3 9 3 $(0, 0, 2)$

2.34

D: max = D[15]   a b b a b b a b b a b b d c c a

0 = a   9 = bab

1 = b   10 = bba

2 = c   11 = abbd   OUTPUT:

3 = d   12 = de

4 = e   13 = cc   0, 1, 1, 5, 7, 6, 8, 3, 4, 2, 2, 0

5 = ab   14 = cc

6 = bb   15 = ca   d) $D = [0000, 0001, 0010, \ldots, 1111]$

7 = ba

8 = abb   b) ?

2.35

D:

0 = a   0   1   1   0   2   4   6

1 = b   a   b   b   a   ab ba abb

2 = ab

3 = bb

4 = ba

5 = aa

6 = abb

7 = bad

2.36

D: 0   1   a   4   5

0 = a   a   b   d   aa aad

1 = b

2 = ab

3 = ba

4 = aa

5 = aaa

2.37

D: a b b a b a b a d c c c c d

0   a   5 bb   10 cc   0, 1, 1, 4,   7,   3, 2, 10,   2, 3

1   b   6 ba   11 ccc

2   c   7 abd   12 cd   3 2 5 6 0 1 7 8

3   d   8 abad   d c ab ba a b aba abad

4   ab   9 dc

2.38

D   1   0   5   6   0   3   5   8   10   2   9

0   a   4 ba   8 ad   12 abac   b   a   ba bab a   d   ab ad aba c da

1   b   5 ab   9 dy   13 cd

2   c   6 bab   10 aba

3   d   7 babq   11 ada