KAPACITET DISKRETNOG KOMUNIKACIJSKOG KANALA

Rješeni zadaci (nerješeni su od str. 9)

log = log 2

1.1

$X = \{x_1, x_2, x_3, x_4\}$

$p(x_1) : p(x_2) : p(x_3) : p(x_4) = 1 : 2 : 2 : 5$

$$[p(y_j|x_i)] = \begin{bmatrix} 0.1 & 0.2 & 0.2 & 0.5 \\ 0.1 & 0.2 & 0.5 & 0.2 \\ 0.1 & 0.5 & 0.2 & 0.2 \\ 0.5 & 0.1 & 0.2 & 0.2 \end{bmatrix}$$

a) Odredite entropiju ulaznog i izlaznog skupa signala
b) Odredite entropiju šuma i transinformaciju u kanalu

a) $\sum_{i=1}^{4} p(x_i) = 1 \Rightarrow [p(x)] = [0.1 \quad 0.2 \quad 0.2 \quad 0.5]$ ≤

$$[p(x_i, y_j)] = [p(x_i) \cdot p(y_j|x_i)] = \begin{bmatrix} 0.01 & 0.02 & 0.02 & 0.05 \\ 0.02 & 0.04 & 0.1 & 0.04 \\ 0.02 & 0.1 & 0.04 & 0.04 \\ 0.25 & 0.05 & 0.1 & 0.1 \end{bmatrix} \begin{matrix} p(y_1) \\ p(y_2) \\ p(y_3) \\ p(y_4) \end{matrix}$$

≤ $p(y_1)\ p(y_2)\ p(y_3)\ p(y_4)$

$[p(y)] = [0.3 \quad 0.21 \quad 0.26 \quad 0.23]$

$H(y) = -\sum_{i=1}^{4} p(y_i) \log p(y_i) = -(0.3 \log 0.3 + 0.21 \log 0.21 + 0.26 \log 0.26 + 0.23 \log 0.23) = 1.987$ bit/simbol

$H(x) = -(0.1 \log 0.1 + 0.2 \log 0.2 \cdot 2 + 0.5 \log 0.5) = 1.761$ bit/simbol

b) transinformacija $I(x;y) = H(x) + H(y) - H(x,y)$
entropija šuma $H(y|x) = H(x,y) - H(x)$
} potreban $H(x,y) = -\sum_{i=1}^{4}\sum_{j=1}^{4} p(x_i, y_j) \log_2 p(x_i, y_j)$

ŽIVIO NAM CASIO MENU → 8 SPREADSHEET

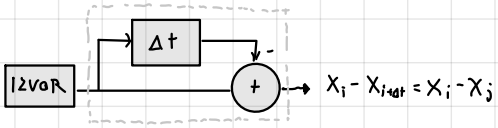
$H(x,y) = 3.5219$

$I(x;y) = H(x) + H(y) - H(x,y) = 0.226$ bit/simbol

1.2

$X = \{4, 5, 6\}$

$$[p(x_i, x_j)] = \begin{matrix} & 4 & 5 & 6 \\ \end{matrix} \begin{bmatrix} 0.1172 & 0.1172 & 0.1563 \\ 0.0713 & 0.2738 & 0.0713 \\ 0.2023 & 0.0253 & 0.0253 \end{bmatrix} \begin{matrix} 4 \\ 5 \\ 6 \end{matrix}$$



Odredite entropiju skupa simbola na izlazu sa slike

$Y = \{x_i - x_j\} = \{-2, -1, 0, 1, 2\}$

4-6 | 4-5, 5-6 | 4-4, 5-5, 6-6 | 5-4, 6-5 | 6-4

$$Y = \begin{pmatrix} -2 & -1 & 0 & 1 & 2 \\ 0.1563 & 0.1985 & 0.3563 & 0.0966 & 0.2023 \end{pmatrix}$$

$H(Y) = -\sum_{i=0}^{4} P(Y_i) \log P(Y_i) = 2.195$ bit/simbol

1.3

$P(x_i) = p_i \quad , i = 1,2,3$

koji uvijet uz $\sum_{i=0} p(x_i) = 1$ mora biti ispunjen da vrijedi $H(X) = H(Y)$

$$[P(Y|X)] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1-p_G & p_G \\ 0 & p_G & 1-p_G \end{bmatrix} \qquad [P(X,Y)] = \begin{bmatrix} p_1 & 0 & 0 \\ 0 & p_2(1-p_G) & p_2 p_G \\ 0 & p_3 p_G & p_3(1-p_G) \end{bmatrix}$$

$P(Y_1)\ P(Y_2)\ P(Y_3)$

$$[P(Y_j)] = [p_1 \quad p_2 + p_G(p_3 - p_2) \quad p_3 + p_G(p_2 - p_3)]$$

$H(X) = H(Y)$

$p_1 \log p_1 + p_2 \log p_2 + p_3 \log p_3 = p_1 \log p_1 + (p_2 + p_G(p_3 - p_2)) \log (p_2 + p_G(p_3 - p_2)) + (p_3 + p_G(p_2 - p_3)) \log (p_3 + p_G(p_2 - p_3))$

$\Rightarrow p_2 = p_2 + p_G(p_3 - p_2)$

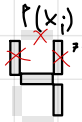
$p_G(p_3 - p_2) = 0 \Rightarrow p_2 = p_3$

$\Rightarrow p_3 = p_3 + p_G(p_3 - p_2)$

$\Rightarrow p_2 = p_3$

1.4

$P(x_i) = p \quad i = 0, \dots, 9$



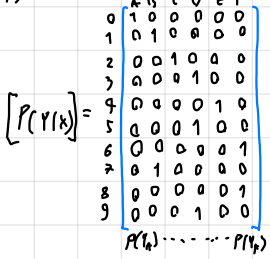
kvar na uređaju, otkazale gornje 3 oznake

a) Izračunajte entropiju po jednom prikazu na 7-segmentnom indikatoru prije kvara
b) Izračunajte entropiju po jednom prikazu na 7-segmentnom indikatoru **nakon** kvara

a) $H(X) = \log 10 = 3.3219 \ \text{bit/simbol}$

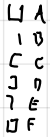
$H_{maks} = \log n$ ako su svi simboli jednako vjerojatni

b)



$[P(Y|X)] =$

$P(Y_1) \dots P(Y_6)$



$$[P(Y)] = \left[\frac{1}{10} \quad \frac{2}{10} \quad \frac{1}{10} \quad \frac{3}{10} \quad \frac{1}{10} \quad \frac{2}{10}\right]$$

$H(Y) = -\sum_{i=0}^{6} P(Y_i) \log P(Y_i) = 2.446 \ \text{bit/simbol}$

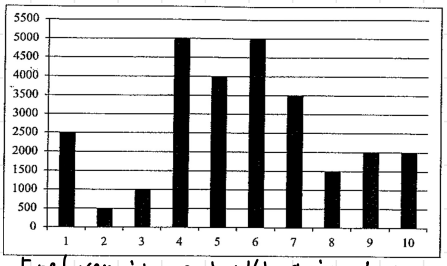
1.5

$X = \{1, 2, \dots 10\}$

$[P(X)] = [2500_p \ 500_p \ 1000_p \ 5000_p \ 4000_p \ 5000_p \ 3500_p \ 1500_p \ 2000_p \ 2000_p]$

$p = \frac{y}{27000}$

a) Izračunajte srednji sadržaj slike
b) Izračunajte minimalno vrijeme potrebno za prijenos dane slike od A do B modemom od 56 kbit/s



Frekvencije pojavljivanja boja

a) $I(x) = \sum_{i=1}^{10} P(x) \log P(x) = 3.0798 \ \text{bit/simbol}$

b) $\frac{H(x) \sum_{n=1}^{10} F_n}{56000 \ \text{bit/s}} = \frac{3.0798 \cdot 27000}{56000} = 1.4849 \ s$

1.6

$X = \{-2, -1, 0, 1, 2\}$

$P(x_i) = p \quad i = 0, 1, \dots, 5$

Na pokazniku se minus ne upali u 30% sl.

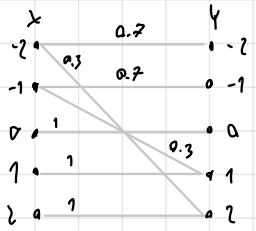
a) Grafički pokazati prijelaz u kanalu

b) Odrediti vjerojatnost pojave pojedine vrijednosti na indikatoru instrumenta

c) Odrediti entropiju ulaznog i izlaznog skupa simbola

d) Izračunati transinformaciju i ekvivokaciju u ovom sustavu

a)



b) $[P(Y)] = [0.7p \quad 0.07p \quad p \quad 1.3p \quad 1.3p]$

$= [0.15 \quad 0.15 \quad 0.2 \quad 0.26 \quad 0.26]$

c) $H(X) = H_{maks} = \log_2 5 = 2.322$ bit/simbol

$H(Y) = -\sum_{i=1}^{5} P(Y) \log_2 P(Y) = 2.269$ bit/simbol

d) $H(X|Y) = H(X) - I(X;Y)$

$I(X;Y) = \sum_{ij} P(x_i, y_j) \log \frac{P(x_i, y_j)}{P(x_i) P(y_j)}$

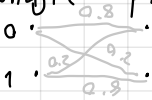
1.7

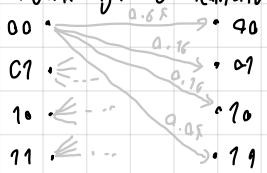
$X = \{x_1, x_2, x_3, x_4\} \quad [P(x_i)] = [p \quad p \quad p \quad p] \quad p = 0.25 \Rightarrow [0.25 \quad 0.25 \quad 0.25 \quad 0.25]$

$x_1 = 00 \quad x_2 = 01 \quad x_3 = 10 \quad x_4 = 11$

$P(\text{pogreška}) = 0.2$

Izračunajte promjenu ekvivakacije u kanalu ako se uvede jedan paritetni bit





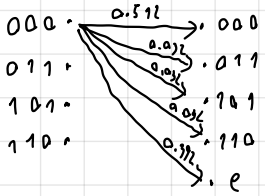
$P(Y|X) = \begin{bmatrix} 0.64 & 0.16 & 0.16 & 0.04 \\ 0.16 & 0.64 & 0.04 & 0.16 \\ 0.04 & 0.16 & 0.64 & 0.16 \\ 0.04 & 0.16 & 0.16 & 0.64 \end{bmatrix}$

$P(X,Y) = P(X) P(Y|X) = \begin{bmatrix} 0.16 & 0.04 & 0.04 & 0.01 \\ 0.04 & 0.16 & 0.01 & 0.04 \\ 0.04 & 0.01 & 0.16 & 0.04 \\ 0.01 & 0.04 & 0.04 & 0.16 \end{bmatrix}$

$H(X,Y) = \sum_{i=1}^{4} \sum_{j=1}^{4} P(x_i, y_j) \log P(x_i, y_j) = 3.322$

$H(X,Y) = H(Y) + H(X|Y)$

$H_1(X,Y) = H(Y) - H(H,Y) = 1.678$ bit/simbol



$P(Y|X) = \begin{bmatrix} 0.512 & 0.032 & 0.032 & 0.032 & 0.032 \\ 0.032 & 0.512 & 0.032 & 0.032 & 0.032 \\ 0.032 & 0.032 & 0.512 & 0.032 & 0.032 \\ 0.032 & 0.032 & 0.032 & 0.512 & 0.032 \end{bmatrix}$

$P(X,Y) = [P(Y_i|X_j) \cdot 0.25]$

$H(X,Y) = 3.5$ bit/simbol

$H_2(X|Y) = H(Y) - H(X|Y) = 1.319$ bit/simbol

$\Delta H(X|Y) = |H_1(X|Y) - H_2(X|Y)| = 0.125$ bit/simbol

1.8

$X = \{a, b, c\}$

$[P(x)] = [2p, 2p, p] = [0.4 \quad 0.4 \quad 0.2]$

$[P(Y|X)] = \begin{bmatrix} 0.7 & 0.1 & 0.2 \\ 0.2 & 0.7 & 0.1 \\ 0.1 & 0.2 & 0.7 \end{bmatrix}$

a) Odredite vjerojatnost pojavljivanja pojedinih simbola na izlazu iz kanala

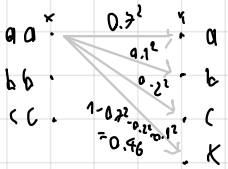
b) Odredite ekvivokaciju i transinformaciju u kanalu

c) Izračunajte promjenu transinformacije ako se svaka poruka jednom ponovi pa se provede većinska odluka o poruci

a)
$$[P(X,Y)] = [P(Y|X)\cdot P(X)] = \begin{bmatrix} 0.2 & 0.04 & 0.08 \\ 0.08 & 0.24 & 0.04 \\ 0.02 & 0.08 & 0.16 \end{bmatrix} \qquad [P(Y)] = [0.38 \quad 0.36 \quad 0.26]$$
$\Sigma = P(x_1)\,P(y_1)\;P(y_2)\;P(y_3)$
0.38 0.36 0.26

b) $H(X|Y) = \sum_i \sum_j p(x,y) \log_2 P(X|Y) = H(X,Y) - H(Y)$
$I(X;Y) = H(X) + H(Y) - H(X,Y)$
$H(X) = -\sum_i P_X \log P_X = 1.5219$ bit/simbol
$H(Y) = 1.5664$ bit/simbol
$H(X;Y) = -\sum_{i=0}^{3}\sum_{j=0}^{3} P(x_i,y_j)\log_2 P(x_i,y_j) = 2.6787$ bit/simbol
$H(X|Y) = 1.1123$ bit/simbol
$I(X;Y) = 0.4096$ bit/simbol

c) $X = \{aa, bb, cc\}$ $[P(X)] = [0.4 \quad 0.4 \quad 0.2]$



$$[P(Y|X)] = \begin{bmatrix} 0.49 & 0.01 & 0.04 & 0.46 \\ 0.04 & 0.49 & 0.01 & 0.46 \\ 0.01 & 0.04 & 0.49 & 0.46 \end{bmatrix} \qquad [P(X,Y)] = [P(X)\cdot P(Y|X)] = \begin{bmatrix} 0.196 & 0.004 & 0.016 & 0.184 \\ 0.016 & 0.196 & 0.004 & 0.184 \\ 0.002 & 0.004 & 0.008 & 0.092 \end{bmatrix} \downarrow \Sigma$$
$P(Y) = 0.214 \quad 0.204 \quad 0.028 \quad 0.46$

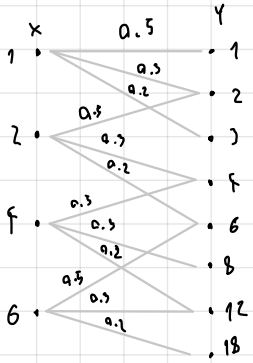
$H(X) = 1.5219$
$H(Y) = 1.8263$
$H(X;Y) = 2.793$
$I(X;Y) = H(X) + H(Y) - H(X;Y) = 0.5552$ ← NETOČNO

1.9

$X \to Z \to Y$
$x = \{1,2,4,6\}$ $[P(X)] = [0.25 \quad 0.25 \quad 0.25 \quad 0.25]$
$z = \{1,2,3\}$ $[P(Z)] = [0.5 \quad 0.3 \quad 0.2]$
$Y = X \cdot Z = \{1,2,3,4,6,8,12,18\}$ $[P(Y)] = ?$

a) Odredite $H(Y)$
b) Odredite $I(X;Y)$

a)



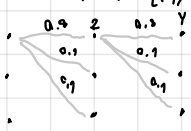
$$[P(Y|X)] = \begin{array}{c} \\ 1 \\ 2 \\ 4 \\ 6 \end{array} \begin{array}{c} \begin{array}{cccccccc} 1 & 2 & 3 & 4 & 6 & 8 & 12 & 18 \end{array} \\ \begin{bmatrix} 0.5 & 0.3 & 0.2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.5 & 0 & 0.3 & 0.2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.5 & 0 & 0.3 & 0.2 & 0 \\ 0 & 0 & 0 & 0 & 0.5 & 0 & 0.3 & 0.2 \end{bmatrix} \end{array} \qquad [P(X;Y)] = [P(X)\,P(Y|X)] = \begin{bmatrix} 0.125 & 0.075 & 0.05 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.125 & 0 & 0.075 & 0.05 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.125 & 0 & 0.075 & 0.05 & 0 \\ 0 & 0 & 0 & 0 & 0.125 & 0 & 0.075 & 0.05 \end{bmatrix}$$
$[P(Y)] = [0.125 \quad 0.2 \quad 0.05 \quad 0.2 \quad 0.175 \quad 0.075 \quad 0.125 \quad 0.05]$

$H(Y) = \sum_{i=1}^{8} P(y_i)\log P(y_i) = 2.8313$

b) $H(X) = 2$
$H(X,Y) = 3.5835$
$I(X;Y) = H(X) + H(Y) - H(X,Y) = 1.3758$

1.10

$X \to Z \to Y$ $x = \{x_1, x_2, x_3\}$ $[P(X)] = [\frac{1}{3} \quad \frac{1}{3} \quad \frac{1}{3}]$



$$[P(X,Z)] = [P(X)\cdot P(Z|X)] = P(X) \begin{bmatrix} 0.8 & 0.1 & 0.1 \\ 0.1 & 0.8 & 0.1 \\ 0.1 & 0.1 & 0.8 \end{bmatrix} = P(Z,Y) \Rightarrow [P(Y)] = [\tfrac{1}{3} \quad \tfrac{1}{3} \quad \tfrac{1}{3}]$$
$P(Z) = \frac{1}{3} \quad \frac{1}{3} \quad \frac{1}{3}$

1.11

$X = \{x_1, x_2\}$ $\quad [P(X)] = [0.2 \;\; 0.8]$

$X \to Z \to Y$

$$[P(Z|X)] = \begin{bmatrix} 0.8 & 0.1 & 0 & 0.1 \\ 0.4 & 0.5 & 0.1 & 0 \end{bmatrix} \qquad [P(Y|Z)] = \begin{bmatrix} 0 & 0.2 & 0.7 & 0 & 0.1 & 0 & 0 & 0 \\ 0.3 & 0 & 0.4 & 0.3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.2 & 0.4 & 0.4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.8 & 0.1 & 0.1 \end{bmatrix}$$

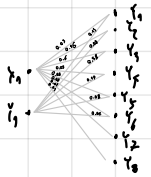
a) Odredite entropiju skupa simbola $H(X)$
b) Izračunajte vjerojatnost pojavljivanja simbola na izlazu iz sustavnog kanala
c) Odredite entropiju izlaznog skupa simbola $H(Y)$
d) Nacrtajte binarni komunikacijski kanal
e) Odredite $I(X;Y)$

a) $H(X) = -\sum_{i=1}^{2} P(x_i) \log P(x_i) = 0,7219 \; \text{bit/simbol}$

b) $[P(Y)] = [P(X)][P(Z|X)][P(Y|Z)] = [0.58 \;\; 0.42 \;\; 0.08 \;\; 0.02][P(Y|Z)] = [0.126 \;\; 0.096 \;\; 0.504 \;\; 0.182 \;\; 0.08 \;\; 0.058 \;\; 0.002 \;\; 0.002]$

c) $H(Y) = 2.1368 \; \text{bit/simbol}$

d) $$[P(Y|X)] = [P(Z|X)][P(Y|Z)] = \begin{bmatrix} 0.03 & 0.16 & 0.6 & 0.03 & 0.08 & 0.08 & 0.01 & 0.01 \\ 0.13 & 0.08 & 0.58 & 0.17 & 0.08 & 0.01 & 0 & 0 \end{bmatrix}$$



e) $$[P(X,Y)] = [P(X)\,P(Y|X)] = \begin{bmatrix} 0.006 & 0.032 & 0.12 & 0.006 & 0.016 & 0.016 & 0.002 & 0.002 \\ 0.12 & 0.064 & 0.384 & 0.136 & 0.064 & 0.032 & 0 & 0 \end{bmatrix}$$

$H(X;Y) = 2.7966 \; \text{bit/simbol}$

$I(X;Y) = H(X) + H(Y) - H(X;Y) = 0.0602 \; \text{bit/simbol}$

1.12

$X = \{x_1, \ldots, x_8\}$

Ispravan prijenos v $k_1 = 0.93$, v $k_2 = 0.86$, ostali jednako vjerojatni

a) $I(X;k_2) = ?$

b) $H(X;k_2)$, $H(X|k_1)$, $H(k_1|k_2) = ?$ $\qquad X \to k_1 \to k_2 \to Y$

a) $$[P(k_2|X)] = [P(k_1|X)][P(k_2|k_1)] = \begin{bmatrix} 0.93 & 0.01 & \cdots & 0.01 \\ 0.01 & 0.93 & & \vdots \\ \vdots & & \ddots & \\ 0.01 & \cdots & & 0.93 \end{bmatrix} \begin{bmatrix} 0.86 & 0.2 & & 0.02 \\ 0.02 & 0.86 & & \vdots \\ \vdots & & \ddots & \\ 0.02 & \cdots & & 0.86 \end{bmatrix} = \begin{bmatrix} 0.0287 & & \\ & \ddots & \\ & & 0.0287 \end{bmatrix}$$

$$I(X;Y) = \sum_{i=1}^{8} \sum_{j=1}^{8} P(x_i)\,P(y_j|x_i) \log_2 \frac{P(y_j|x_i)}{P(y_j)} = 1.7223 \; \text{bit/simbol}$$

b) $H(k|k_1) = -\sum_{i=1}^{8} \sum_{j=1}^{8} P(x_i, k_j) \log_2 P(k_i|k_j) = 0.5824 \; \text{bit/simbol}$

isto za ostale.

1.13

$X = \{x_1, x_2, x_3, x_4\}$ $\quad P(x) \to \Sigma$

$$[P(x_i, y_j)] = \begin{bmatrix} \frac{1}{8} & \frac{1}{16} & \frac{1}{32} & \frac{1}{32} \\ \frac{1}{16} & \frac{1}{8} & \frac{1}{32} & \frac{1}{32} \\ \frac{1}{16} & \frac{1}{16} & \frac{1}{16} & \frac{1}{16} \\ \frac{1}{4} & 0 & 0 & 0 \\ \frac{1}{2} & \frac{1}{4} & \frac{1}{8} & \frac{1}{8} \end{bmatrix} \quad P(y) \downarrow \; \Sigma$$

1) Odredite entropiju ulaznog i izlaznog skupa simbola
2) Odredite združenu entropiju $H(X;Y)$, entropiju šuma $H(Y|X)$ i transinformaciju $I(X;Y)$
3) Procijenite donju granicu za kapacitet danog diskretnog komm. kanala

1) $H(Y) = 1.75 \qquad H(X) = \log(n) = 2$

2) $H(X;Y) = 3.375 \qquad H(Y|X) = 1.375 \qquad I(X;Y) = 0.375$

3) $C = \max_{\{P(x_i)\}} I(X;Y) = 0.375 \; \text{bit/simbol}$

1.14

$X=\{0,1\}$ $[P(X)]=[0.3\ \ 0.5]$

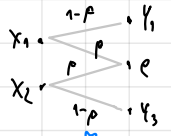
$[P(Y|X)]=\begin{bmatrix}1-\varepsilon & \varepsilon\\ \varepsilon & 1-\varepsilon\end{bmatrix}$ ε - vjerojatnost pogreške

1) Odredite entropiju ulaznog skupa simbola $H(X)$
2) -11- -11- izlaznog -11- -11- $H(Y)$
3) Odredite združenu entropiju $H(X,Y)$
4) Odredite $I(X;Y)$
5) Za koje vrijednosti ε je transinformacija maksimalna
6) Za koje vrijednosti ε je kapacitet minimalan

1) $H(X)=\log n=1$ bit/simbol
2) $[P(X,Y)]=[P(X)P(Y|X)]=\begin{bmatrix}\frac{1-\varepsilon}{2} & \frac{\varepsilon}{2}\\ \frac{\varepsilon}{2} & \frac{1-\varepsilon}{2}\end{bmatrix}\begin{matrix}\downarrow\Sigma\\ P(Y)\end{matrix}$ $\quad H(Y)=1$ bit/simbol
   $\frac{1}{2}\ \ \frac{1}{2}$
3) $H(X,Y)=-\sum_{i}^{2}\sum_{j}^{2}P(x_i,y_j)\log P(x_i,y_j)=-\left((1-\varepsilon)\log\frac{1-\varepsilon}{2}+\varepsilon\log\frac{\varepsilon}{2}\right)=-\left((1-\varepsilon)[\log(1-\varepsilon)-1]+\varepsilon[\log\varepsilon-1]\right)=1-(1-\varepsilon)\log(1-\varepsilon)-\varepsilon\log\varepsilon$
4) $I(X;Y)=1+1-H(X,Y)=1+\varepsilon\log_2\varepsilon+(1-\varepsilon)\log_2(1-\varepsilon)$
5) maks za $\varepsilon=0,1$, $I(X;Y)=1$ bit/simbol
6) $C(\varepsilon)=\max_{P(x_i)}I(X;Y)=1+\varepsilon\log_2\varepsilon+(1-\varepsilon)\log_2(1-\varepsilon)\ /\ \frac{d}{d\varepsilon}$
   $\Rightarrow \varepsilon=1/2$ $\quad C_{min}=0$

1.15

$X=\{x_1,x_2\}$ $[P(X)]=[\alpha\ \ 1-\alpha]$



$C=?$

$C=\max_{\{P(x_i)\}}I(X;Y);\quad I(X;Y)=H(Y)-H(Y|X)$

$[P(Y|X)]=\begin{bmatrix}1-p & p & 0\\ 0 & p & 1-p\end{bmatrix}$ $\quad [P(X,Y)]=[P(X)P(Y|X)]=\begin{bmatrix}\alpha(1-p) & \alpha p & 0\\ 0 & (1-\alpha)p & (1-\alpha)(1-p)\end{bmatrix}$

$1-p\ \ 2p\ \ 1-p$ $\qquad \alpha(1-p)\ \ p\ \ (1-\alpha)(1-p)$

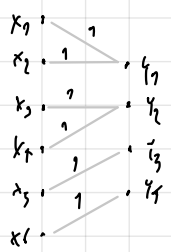
$H(Y)=-\left(\alpha(1-p)[\log\alpha+\log(1-p)]+p\log p+(1-\alpha)(1-p)[\log(1-\alpha)+\log(1-p)]\right)$
$=-\{\log\alpha[\alpha(1-p)]+\log(1-\alpha)[(1-\alpha)(1-p)]+p\log p+(1-p)\log(1-p)\}$

$H(Y|X)=-\alpha(1-p)\log(1-p)-\alpha p\log p-(1-\alpha)p\log p-(1-\alpha)(1-p)\log(1-p)=-p\log p-(1-p)\log(1-p)$

$I(X;Y)=(1-p)\left[-\alpha\log\alpha-(1-\alpha)\log(1-\alpha)\right]=(1-p)H(X)$

$C=\max_{\{P(x_i)\}}I(X;Y)=(1-p)\max_{\{P(x_i)\}}H(X)=(1-p)$ bit/simbol $\quad\leftarrow$ max za $p_{x_i}=p\ \forall i$ $\quad=\log n$

1.16



$[P(Y|X)]=\begin{bmatrix}1 & 0 & 0 & 0\\ 1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{bmatrix}$ $\qquad [P(Y)]=[p_1+p_2\ \ p_3+p_4\ \ p_5\ \ p_6]$

$C=\max_{\{P(x_i)\}}I(X;Y)=\max_{\{P(x_i)\}}[H(Y)-H(Y|X)]$

$H(Y|X)=-\sum_i\sum_j P(x_i,y_j)\log_2 P(y_j|x_i)=0$ $\quad\leftarrow \log_2 1=0$

$C=\max_{\{P(x_i)\}}H(Y)=\log_2 4=2$ bit/simbol

1.17

$V=\{v_1,\dots,v_8\}$ $\quad [P(v)]=[0.05\ \ 0.05\ \ 0.15\ \ 0.25\ \ 0.3\ \ 0.05\ \ 0.05\ \ 0.1]$ $\quad \tau=15ms$

$R=\frac{H(X)}{\tau_s}$ bit/s

$H(A)=-\sum_{i=0}^{8}P(x_i)\log P(x_i)=2.6282$

$R=\frac{H(A)}{0.015}=175$ bit/s

1.18

$X = \{\cdot, -\}$ $\quad t(x_1) = 0.2$ $\quad t(x_2) = 0.6$ $\quad [P(x_i)] = \left[\frac{2}{3} \ \frac{1}{3}\right]$ $\quad t_{pauza} = 0.2\,s$

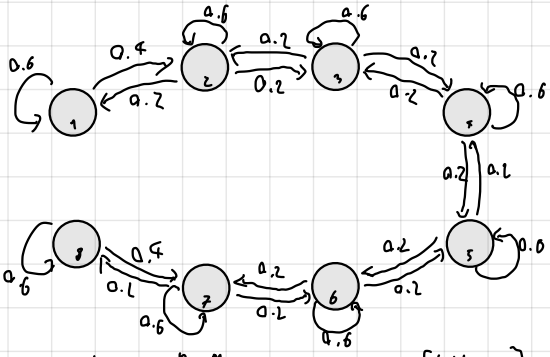
$H(X) = 0.9183$ bit/simbol

$T_s = p_{x1} t_{x1} + p_{x2} t_{x2} + t_{pauza} = 0.533$ s/simbol

$R = \frac{H}{T} = 1.7226$ bit/simbol

1.19

$t = 10\,s$



1) Skicirajte markovljev lanac
2) Izračunajte stacionarne vrijednosti pojave straja u nekom položaju
3) Izračunajte vlastiti sadržaj informacije (entropija!) koji straj generira svojim položajem ako se uzima i ako se ne uzima ovisnost u njegovom kretanju

$$[P(x_j|x_i)] = \begin{bmatrix} 0.6 & 0.4 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.2 & 0.6 & 0.2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.2 & 0.6 & 0.2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.2 & 0.6 & 0.2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.2 & 0.6 & 0.2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.2 & 0.6 & 0.2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.2 & 0.6 & 0.2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.4 & 0.6 \end{bmatrix}$$

$H'(X) = -\sum_{i=1}^{n}\sum_{j=1}^{m} P(x_i x_j) \log_2 P(x_j|x_i) \left[\frac{bit}{simbol}\right]$

$[P(x_j)] = [P(x_1) \dots P(x_8)]\,[P(x_j|x_i)]$

$P(x_1) = 0.6\,P(x_1) + 0.2\,P(x_2) \Rightarrow P(x_2) = 2P(x_1)$

$P(x_2) = 0.4\,P(x_1) + 0.6\,P(x_2) + 0.2\,P(x_3) \Rightarrow P(x_3) = 2P(x_1)$

$P(x_3) = 0.2\,P(x_2) + 0.6\,P(x_3) + 0.2\,P(x_4) \Rightarrow P(x_4) = 2P(x_1)$

$\vdots \Rightarrow P(x_5) = P(x_6) = P(x_7) = 2P(x_1)$

$P(x_7) = 0.2\,P(x_6) + 0.6\,P(x_7) + 0.4\,P(x_8) \Rightarrow P(x_8) = P(x_1)$

$[P(x)] = [p \ 2p \ 2p \ 2p \ 2p \ 2p \ 2p \ p] \quad p = \frac{1}{14}$

$= \left[\frac{1}{14} \ \frac{1}{7} \ \frac{1}{7} \ \frac{1}{7} \ \frac{1}{7} \ \frac{1}{7} \ \frac{1}{7} \ \frac{1}{14}\right]$

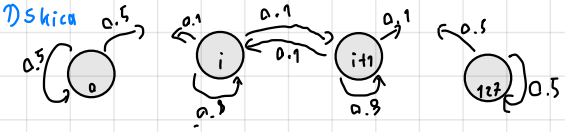
$H(X) = 2.95$ bit/položaj

$$[P(x_i x_j)] = [P(x_i) \cdot P(x_j|x_i)] = \begin{bmatrix} \frac{3}{56} & \frac{1}{56} & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{56} & \frac{3}{56} & \frac{1}{56} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{56} & \frac{3}{56} & \frac{1}{56} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{56} & \frac{3}{56} & \frac{1}{56} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{56} & \frac{3}{56} & \frac{1}{56} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{56} & \frac{3}{56} & \frac{1}{56} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{56} & \frac{3}{56} & \frac{1}{56} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{56} & \frac{3}{56} \end{bmatrix}$$

$H'(X) = 1.3138$ bit/simbol

1.20

Skica



2) Izračunajte stacionarne vjerojatnosti pojave pojedinih razina piksela
3) Izračunajte $H(x)$, $H'(x)$
4) Minimalni kapacitet kanala kojim se prenosi 640x480 25FPS

2)

$$[P(x_j|x_i)] = \begin{bmatrix} 0.5 & 0.5 & 0 & 0 & \dots & 0 \\ 0.1 & 0.8 & 0.1 & 0 & & \\ 0 & 0.1 & 0.8 & 0.1 & & \\ \vdots & & 0.1 & 0.8 & 0.1 & \\ & & & 0.1 & 0.8 & 0.1 \\ 0 & & & & 0.5 & 0.5 \end{bmatrix} \quad \times 128$$

$P(x_0) = 0.5P(x_0) + 0.1P(x_0) \Rightarrow P(x_1) = 5P(x_0)$

$P(x_1) = 0.5P(x_0) + 0.8P(x_1) + 0.1P(x_2) \Rightarrow P(x_2) = 5P(x_0)$

$P(x_2) = 0.1P(x_1) + 0.8P(x_2) + 0.1P(x_3) \Rightarrow P(x_3) = 5P(x_0)$

$\vdots \quad P(x_3) = P(x_4) = \dots = P(x_{126}) = 5P(x_0)$

$P(x_{127}) = 0.1P(x_{126}) + 0.5P(x_{127}) \Rightarrow P(x_{127}) = P(x_0)$

$P(x_0) = \frac{1}{632}$

$[P(x)] = \left[\frac{1}{632} \ \frac{5}{632} \ \dots \ \frac{5}{632} \ \frac{1}{632}\right]$

3) $H(X) = 6.9892$ bit/simbol

$$[P(x_i, x_j)] = [P(x) \cdot P(x_j|x_i)] = \frac{1}{632} \begin{bmatrix} 0.5 & 0.5 & 0 & & \\ 0.5 & 4 & 0.5 & & \\ & \ddots & \ddots & \ddots & \\ & & 0.5 & 4 & 0.5 \\ & & 0 & 0.5 & 0.5 \end{bmatrix}$$

$H'(X) = 0.92$ bit/piksel

4) $N = 640 \cdot 480 = 307\,200$

$R_s = 25$ slika/s

$C = N \cdot R_s \cdot H'(X) = 7\,082\,500$ bit/s

1.21

$X \to k_1 \to k_2$

$X = \{1, 2\}$ Markovljev lanac 25% ostaje u 1, a u 50% u 2

$$[P(k_1|x)] = \begin{bmatrix} 0.8 & 0.1 & 0 & 0.1 \\ 0.4 & 0.5 & 0.1 & 0 \end{bmatrix} \quad [P(k_2|k_1)] = \begin{bmatrix} 0 & 0.2 & 0.7 & 0 & 0.1 & 0 & 0 & 0 \\ 0.3 & 0 & 0.4 & 0.3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.2 & 0.4 & 0.4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.8 & 0.1 & 0.1 \end{bmatrix}$$

a) Izračunajte vjerojatnost pojavljivanja simbola na izlazu iz sustava kanala
b) Odredite transinformaciju u cijelom sustavu kanala ako se izvorište promatra kao bezmemorijsko

a) $[P(k_2|X)] = [P(k_1|X)][P(k_2|k_1)] = \begin{bmatrix} 0.03 & 0.16 & 0.6 & 0.03 & 0.08 & 0.08 & 0.01 & 0.01 \\ 0.15 & 0.08 & 0.58 & 0.17 & 0.08 & 0.05 & 0 & 0 \end{bmatrix} \downarrow \Sigma = P(k_2)$

$[P(x_j|x_i)] = \begin{bmatrix} 0.25 & 0.75 \\ 0.5 & 0.5 \end{bmatrix}$ $[P(X)] = [P(x)][P(x_j|x_i)] = [0.4 \quad 0.6]$

$[P(x_i,k_2)] = \begin{bmatrix} 0.052 & 0.066 & 0.24 & 0.011 & 0.032 & 0.012 & 0.005 & 0.005 \\ 0.09 & 0.053 & 0.138 & 0.102 & 0.053 & 0.025 & 0 & 0 \end{bmatrix} \downarrow \Sigma = P(k_2)$

$[P(k_2)] = [0.102 \quad 0.112 \quad 0.378 \quad 0.113 \quad 0.08 \quad 0.036 \quad 0.005 \quad 0.005]$

b) $H(X) = 0.971$

$H(k_2) = 2.122$

$H(X,k) = 2.9931$ bit/simbol

$I(k:Y) = H(k) + H(Y) - H(X;Y) = 0.0443$ bit/simbol

Nerješeni zadaci

1.1

$X = \{x_1, x_2, x_3, x_4\}$ $[P(x_i)] = [0.4 \quad 0.3 \quad 0.2 \quad 0.1]$

$I(x_1 x_2 x_1 x_3) = ?$

$= \log_2(P(x_1) \cdot P(x_2) P(x_1) P(x_3)) = 6.7$

1.2

$z = \{0, 1\}$ $[P(z)] = [p \quad 1-p]$

$p \sim \begin{pmatrix} 0 & 0.5 & 1 \\ 0.33 & 0.33 & 0.33 \end{pmatrix}$ $H(z) = \begin{pmatrix} 0 & 1 & 0 \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{pmatrix}$

$E(H_2) = 0 + 1 \cdot \frac{1}{3} + 0 = \frac{1}{3}$

1.3

$X_1 X_2 X_3 \sim \begin{pmatrix} 000 & 001 & 011 & 101 & 111 \\ \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} \end{pmatrix}$ $H(X_1 X_2 X_3) = 2.322$ bit/simbol

$X_1 \sim \begin{pmatrix} 0 & 1 \\ \frac{2}{5} & \frac{3}{5} \end{pmatrix}$ $X_2 \sim \begin{pmatrix} 0 & 1 \\ \frac{2}{5} & \frac{3}{5} \end{pmatrix}$ $X_3 \sim \begin{pmatrix} 0 & 1 \\ \frac{1}{5} & \frac{4}{5} \end{pmatrix}$

$H(X_1) = -(0.6 \log 0.6 + 0.4 \log 0.4) = 0.971 = H(X_2)$

$X_1 X_2 \sim \begin{pmatrix} 00 & 01 & 10 & 11 \\ \frac{2}{5} & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} \end{pmatrix}$ $H(X_1 X_2) = 1.922$ bit/simbol

$H(X,Y) = H(Y) + H(X|Y)$ $H(X_2|X_1) = 1.922 - 0.971 = 0.951$ bit/simbol

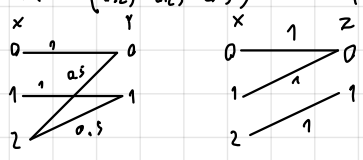
$H(X_2|X_1=0) = -\left(\frac{2}{3} \log_2 \frac{2}{3} + \frac{1}{3} \log_2 \frac{1}{3}\right) = 0.918$ bit/simbol

$H(X_2|X_1=1) = 1$ bit/simbol

$H(X_3|X_1, X_2) = H(X_1 X_2 X_3) - H(X_1 X_2) = 0.4$ bit/simbol

1.4

$X \sim \begin{pmatrix} 0 & 1 & 2 \\ 0.25 & 0.25 & 0.5 \end{pmatrix}$ $Y = \{1,2\}$ $Z = \{1,2\}$



$P(Y|X) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0.5 & 0.5 \end{bmatrix}$ $P(Z|X) = \begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$ $P(X,Y) = \begin{bmatrix} 0.25 & 0 \\ 0 & 0.25 \\ 0.25 & 0.25 \end{bmatrix}$ $P(X,Z) = \begin{bmatrix} 0.25 & 0 \\ 0.25 & 0 \\ 0 & 0.5 \end{bmatrix}$

$P(X|Y) = \frac{P(XY)}{P(Y)} = \begin{bmatrix} \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & \frac{1}{2} & \frac{1}{2} \end{bmatrix}$

$P(Y) = [0.5 \quad 0.5]$ $P(Z) = [0.5 \quad 0.5]$

$H(X) = -(0.25 \log 0.25 \cdot 2 + 0.5 \log 0.5) = 1.5$ bit/simb. $H(Y) = \log_2 2 = H(Z) = 1$ bit/simbol

$$[P(YZ)] = P(Y) \cdot [P(X|Y)][P(Z|X)] = [0.5 \quad 0.5] \cdot \begin{bmatrix} 0.25 & 0.25 \\ 0.25 & 0.25 \end{bmatrix}$$

$H(YZ) = 2$

$I(XY) = H(X) + H(Y) - H(XY) = 1.5 + 1 - 2 = 0.5$ bit/simbol

$I(XZ) = H(X) + H(Z) - H(XZ) = 1.5 + 1 - 1.5 = 1$ bit/simbol

1.5

$X = \{0, 1\}$ $Y = \{0, 1, 2\}$ $P(x,y) = k(x+y)$

$P(x,y) = \begin{bmatrix} 0 & k & 2k \\ k & 2k & 3k \end{bmatrix}$ $k(1+2+1+2+3) = 1$, $k = \frac{1}{9}$

$P(X|Y) = \begin{bmatrix} 0 & \frac{1}{9} & \frac{2}{9} \\ \frac{1}{9} & \frac{2}{9} & \frac{3}{9} \\ \frac{1}{9} & \frac{2}{9} & \frac{2}{9} \end{bmatrix}$ $\begin{matrix} \frac{3}{9} \\ \frac{6}{9} \\ \frac{5}{9} \end{matrix}$

$H(X) = -\frac{3}{9}\log\frac{3}{9} - \frac{6}{9}\log\frac{6}{9} = 0.918$ bit/simbol

$H(Y) = 1.352$ bit/simbol

$H(XY) = 2.197$

$I(XY) = H(X) + H(Y) - H(XY) = 0.073$ bit/simbol

1.6

$X \sim \begin{pmatrix} a & 1 & 2 \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{2} \end{pmatrix}$ $H(X) = 1.5$ bit/simbol

$P(Y|X) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix} \Rightarrow P(X,Y) = \begin{bmatrix} \frac{1}{4} & 0 \\ 0 & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} \end{bmatrix}$ $P(Y) = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \end{bmatrix}$

$H(Y) = 1$ bit/simbol

$P(Z|X) = \begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \Rightarrow P(X,Z) = \begin{bmatrix} \frac{1}{4} & 0 \\ \frac{1}{4} & 0 \\ 0 & \frac{1}{2} \end{bmatrix}$ $P(Z) = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \end{bmatrix}$

$H(Z) = 1$ bit/simbol

$H(XY) = -\left(\sum p_{ij} \log p_{ij}\right) = 2$

$= H(X) + H(Y|X) = H(Y) + H(X|Y)$

$\Rightarrow H(Y|X) = H(XY) - H(X) = 0.5$

$\Rightarrow H(X|Y) = H(XY) - H(Y) = 1$

$H(XZ) = \frac{3}{2}$ bit/simbol

$H(X|Z) = H(XZ) - H(Z) = 0.5$ bit/simbol

$P(Z|Y) = [P(X|Y)][P(Z|X)]$

$P(X|Y) = \frac{P(XY)}{P(Y)} = \begin{bmatrix} \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & \frac{1}{2} & \frac{1}{2} \end{bmatrix}$

$P(Z|Y) = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix}$ $P(YZ) = \begin{bmatrix} \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} \end{bmatrix}$

$H(YZ) = 2$ $H(Y|Z) = H(YZ) - H(Z) = 1$ bit/simbol

1.7

$X \sim \mathcal{X}$ $Y \sim \mathcal{Y}$

$H(X) = 11$ bit/simbol

$H(Y|X) = H(X|Y)$

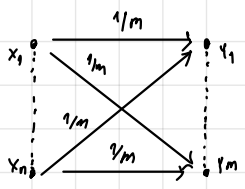
$|Y| = ?$

$\Rightarrow H(Y) = 11$ bit/simbol

$\log_2 n = 11$

$n = 2^{11} = 2048$

1.8



$p(y_j|x_i) = 1/m$

$P(X,Y) = \begin{bmatrix} p_1 & p_1 & \cdots & p_1 \\ p_2 & p_2 & \cdots & p_2 \\ \vdots & & & \vdots \\ p_n & p_n & \cdots & p_n \end{bmatrix}$ $\begin{matrix} P(x) \\ mp_1 \\ mp_2 \\ \vdots \\ mp_n \end{matrix}$

$\underbrace{\sum p_i \quad \cdots \quad \sum p_i}_{\text{svi isti}}$

$H(X) = -\sum_{i=1}^{n} p_{xi} \log p_{xi} = -\sum_{i=1}^{n} mp_i \log mp_i$

$H(Y) = H_{max}(Y) = \log_2 m$

$H(X|Y) = H(XY) - H(Y)$

$H(XY) = \sum_{i=1}^{n}\sum_{j=1}^{m} p_{ij} \log p_{ij} = \sum_{i=1}^{n}\sum_{j=1}^{m} p_i \log p_j = \sum_{i=1}^{n} mp_i \log p_i$

$H(X|Y) = \sum_{i=1}^{n} mp_i \log p_i - \log m$

$H(Y|X) = H(XY) - H(X)$

1.9

$|X| = m \quad [P(X)] = [p_1 \; p_2 \; \dots \; p_m]$

$q \sim \begin{pmatrix} 1 & \dots & m-2 & m-1 \\ p_1 & \dots & p_{m-2} & p_{m-1}+p_m \end{pmatrix}$

$H(X)$ preko $H(Y), p_m, p_{m-1}$ i $H\left(\frac{p_{m-1}}{p_{m-1}+p_m}, \frac{p_m}{p_{m-1}+p_m}\right) = ?$ $\quad H(a,b) = -(a\log_2 a + b\log_2 b)$

$H(X) = -(p_1\log p_1 + \dots + p_{m-2}\log p_{m-2} + \dots + p_m \log p_m) = H(q) - (p_{m-1}+p_m)\log(p_{m-1}+p_m) + p_{m-1}\log p_{m-1} + p_m \log p_m$

$H(a,b) = \frac{-p_{m-1}}{p_{m-1}+p_m}\left[\log p_{m-1} - \log(p_{m-1}+p_m)\right] - \frac{p_m}{p_{m-1}+p_m}\left(\log p_m - \log(p_{m-1}+p_m)\right)$

$= -\frac{p_{m-1}\log p_{m-1} + p_m \log p_m}{p_{m-1}+p_m} + \frac{(p_{m-1}+p_m)\log(p_{m-1}+p_m)}{p_{m-1}+p_m} = -\frac{p_{m-1}\log p_{m-1} + p_m \log p_m}{p_{m-1}+p_m} + \log(p_{m-1}+p_m)$

$= H(q) + (p_{m-1}+p_m) H\left(\frac{p_{m-1}}{p_{m-1}+p_m}, \frac{p_m}{p_{m-1}+p_m}\right)$

1.10

$X_1 \sim \begin{pmatrix} 1 & 2 & \dots & m \\ p_{x_{1_1}} & p_{x_{1_2}} & \dots & p_{x_{1_m}} \end{pmatrix} \quad X_2 \sim \begin{pmatrix} m+1 & \dots & m+n \\ p_{x_{2_1}} & \dots & p_{x_{2_n}} \end{pmatrix}$

$X = \begin{cases} X_1 & p = \alpha \\ X_2 & p = 1-\alpha \end{cases}$

d) $H(X)$ preko $\alpha, H(X_1), H(X_2)$

$\log \alpha p_i = \log\alpha + \log p_i$

$-H(X) = \alpha p_{x_{1_1}} \log \alpha p_{x_{1_1}} + \dots \alpha p_{x_{1_m}} \log \alpha p_{x_{1_m}} + (1-\alpha) p_{x_{2_1}} \log((1-\alpha)p_{x_{2_1}}) + \dots [1-\alpha] p_{x_{2_m}} \log (1-\alpha) p_{x_{2_m}}$

$= \underbrace{(\alpha p_{x_{1_1}} + \alpha p_{x_{1_2}} + \alpha p_{x_{1_m}})}_{\alpha(p_{x_{1_1}} + \dots p_{x_{1_m}}) = \alpha} \log\alpha + \underbrace{\alpha p_{x_{1_1}} \log p_{x_{1_1}} + \dots \alpha p_{x_{1_m}} \log p_{x_{1_m}}}_{\alpha H(X_1)} + (1-\alpha)\log(1-\alpha) + (1-\alpha) p_{x_{2_i}} \log(p_{x_{2_i}})$

$= -[\alpha \log\alpha + \alpha H(X_1) + (1-\alpha)\log(1-\alpha) + (1-\alpha) H(X_2)]$

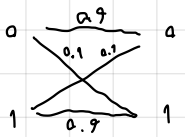
b) $\max_\alpha H(X_1) = \log_2 m \Rightarrow m = 2^{H(X_1)}$

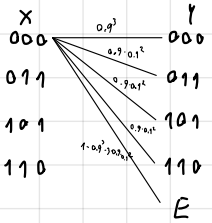
$\max_\alpha H(X_2) = \log_2 n \qquad n = 2^{H(X_2)}$

$\max_\alpha H(X) = \log_2 m + n > \log_2 2^{H(X_1)} + 2^{H(X_2)}$

1.11

$X = \{x_1, x_2, x_3, x_4\} = \{00, 01, 10, 11\} \quad [P(x_i)] = \left[\frac{1}{4} \; \frac{1}{4} \; \frac{1}{4} \; \frac{1}{4}\right]$





$P(Y|X) = \begin{bmatrix} & 000 & 011 & 101 & 110 & E \\ 000 & 0.9^3 & 0.9\cdot0.1^2 & 0.9\cdot0.1^2 & 0.9\cdot0.1^2 & \frac{61}{250} \\ 011 & 0.9\cdot0.1^2 & 0.9^3 & 0.9\cdot0.1^2 & 0.9\cdot0.1^2 & \frac{61}{250} \\ 101 & 0.9\cdot0.1^2 & 0.9\cdot0.1^2 & 0.9^3 & 0.9\cdot0.1^2 & \frac{61}{250} \\ 110 & 0.9\cdot0.1^2 & 0.9\cdot0.1^2 & 0.9\cdot0.1^2 & 0.9^3 & \frac{61}{250} \end{bmatrix}$

$P(X,Y) = \begin{bmatrix} & 000 & 011 & 101 & 110 & E & P(x) \\ 000 & \frac{0.9^3}{4} & \frac{0.9\cdot0.1^2}{4} & \frac{0.9\cdot0.1^2}{4} & \frac{0.9\cdot0.1^2}{4} & \frac{61}{1000} & \frac{1}{4} \\ 011 & \frac{0.9\cdot0.1^2}{4} & \frac{0.9^3}{4} & \frac{0.9\cdot0.1^2}{4} & \frac{0.9\cdot0.1^2}{4} & \frac{61}{1000} & \frac{1}{4} \\ 101 & \frac{0.9\cdot0.1^2}{4} & \frac{0.9\cdot0.1^2}{4} & \frac{0.9^3}{4} & \frac{0.9\cdot0.1^2}{4} & \frac{61}{1000} & \frac{1}{4} \\ 110 & \frac{0.9\cdot0.1^2}{4} & \frac{0.9\cdot0.1^2}{4} & \frac{0.9\cdot0.1^2}{4} & \frac{0.9^3}{4} & \frac{61}{1000} & \frac{1}{4} \\ P(Y) = & \frac{189}{1000} & \frac{189}{1000} & \frac{189}{1000} & \frac{189}{1000} & \frac{61}{250} & \end{bmatrix}$

$H(X) = 2$

$H(Y) = 2.314$

$H(XY) = 3.012$

$I(XY) = H(X) + H(Y) - H(XY) = 1.302$

1.12

$[P(Y|X)] = \begin{bmatrix} 1 & 0 \\ f & g \end{bmatrix} \quad f+g=1 \qquad X \sim \begin{pmatrix} x_1 & x_2 \\ 1-\mu & \mu \end{pmatrix} \qquad H(X) = -\sum_i p_i \log p_i$

Odredite $\mu = F(f,g)$ za minimalni $I(X;Y)$

$[P(X,Y)] = \begin{bmatrix} 1-\mu & 0 \\ \mu f & \mu g \end{bmatrix}$

$P(Y) = [1-\mu+\mu f \quad \mu g]$

$I(X,Y) = H(X) + H(Y) - H(XY) = [(1-\mu)\log(1-\mu) + \mu\log\mu] + [(1-\mu+\mu f)\log(1-\mu+\mu f) + \mu g \log \mu g] - [(1-\mu)\log(1-\mu) + \mu f \log \mu f + \mu g \log \mu g]$

$= \mu(1-f)\log\mu - \mu f \log f + (1-\mu+\mu f)\log(1-\mu+\mu f) = \mu g \log\mu - \mu f \log f + (1-\mu g)\log(1-\mu g)$

$\log\mu + \log g$ $\qquad \log(1-\mu+\mu f)$

1.13

$$f = \begin{cases} 0 & x<0 \\ aU(3-U) & 0 \le x \le 3 \\ 0 & x>3 \end{cases} \qquad \int_0^3 f(u)du = 1 \Rightarrow a = \frac{2}{9}$$

$P(0) = 0$ $\quad P(0.5) = \int_0^{0.5} f(x)dx = \frac{2}{27}$ $\quad P(1) = \frac{5}{27}$ $\quad P(1.5) = \frac{13}{54}$

$P(2) = \frac{13}{54}$ $\quad P(2.5) = \frac{5}{27}$ $\quad P(3) = \frac{2}{27}$

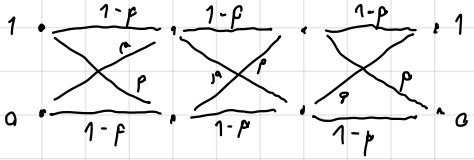
$H(X) = \sum_i p_i \log p_i = 2.457$

$R = \frac{H(X)}{T} = 249.655$

$R_{bas} = 15.679$ kbit $= 1.835$ kbyte

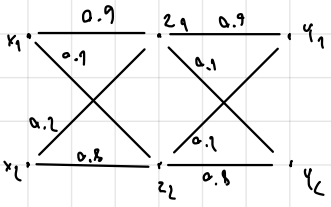
1.14

K binarnih simetričnih kanala. Vjerojatnost greške svakog p. Vezani u seriju. Kaja je vjerojatnost greške.



$$P(Y|X) = \begin{bmatrix} 1-p & p \\ p & 1-p \end{bmatrix}^k = \begin{bmatrix} (1-p)^2+p^2 & 2(p-p^2) \\ 2(p-p^2) & 1-2p \end{bmatrix}^{k=2} = \begin{bmatrix} (1-2p)^3 & \cdots \\ \cdots & (1-2p)^3 \end{bmatrix}^{k=3} = 1-(1-2p)^k$$ idk. negdje sam fulao

1.15



$P(x_1) = P(x_2) = 0.5$

$P(Y|X) \begin{bmatrix} 0.9 & 0.1 \\ 0.2 & 0.8 \end{bmatrix}^2 = \begin{bmatrix} 0.83 & 0.17 \\ 0.34 & 0.66 \end{bmatrix}$ $\quad P(X,Y) = \begin{bmatrix} 0.415 & 0.085 \\ 0.17 & 0.33 \end{bmatrix}$ $\quad P(Y) = \begin{bmatrix} \frac{117}{200} & \frac{83}{200} \end{bmatrix}$

$I(X,Y) = H(X) + H(Y) - H(X,Y) = 1 + 0.979 - 1.791 = 0.188$

1.16

$Z \sim \begin{pmatrix} 0 & 1 \\ 1-p & p \end{pmatrix}$ $\quad X \sim \begin{pmatrix} 1 & 2 & \cdots & n \\ q_1 & q_2 & \cdots & q_n \end{pmatrix}$ $\quad Y = XZ \sim \begin{pmatrix} 0 & 1 & 2 & \cdots & n \\ 1-p & pq_1 & pq_2 & \cdots & pq_n \end{pmatrix}$

1) $H(Y)$ preko $H(X)$ i $H(Z)$

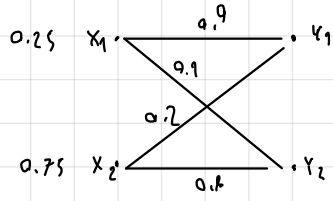
$H(Z) = (1-p)\log(1-p) + p\log p$

$H(X) = q_1 \log q_1 + q_2 \log q_2 + \ldots$

$H(Y) = (1-p)\log(1-p) + pq_1 \log p + pq_1 \log q_1 + \ldots pq_n \log p + pq_n \log q_n = H(Z) + pH(X)$

2) p i q, $H(Y) = H_{max} \Rightarrow 1-p = pq$ $\quad q = \frac{1-p}{p}$ $\quad 1-p+(1-p)n = 1 \Rightarrow p = \frac{n}{n+1}$

1.17



$P(Y|X) = \begin{bmatrix} 0.9 & 0.1 \\ 0.2 & 0.8 \end{bmatrix}$ $\quad P(X,Y) = \begin{bmatrix} 0.225 & 0.025 \\ 0.15 & 0.6 \end{bmatrix}$

$P(\text{pogreška}) = 0.15 + 0.025 = 0.175$

$I(X,Y) = H(X) + H(Y) - H(XY) = 0.811 + 0.954 - 1.77 = 0.295$

1.18

$X = \{x_1, x_2, x_3\}$ $\quad Y = \{y_1, y_2, y_3\}$

$$[P(X,Y)] = \begin{bmatrix} \frac{1}{6} & \frac{1}{12} & \frac{1}{4} & \frac{1}{2} \\ \frac{1}{12} & \frac{1}{24} & \frac{1}{8} & \frac{1}{4} \\ \frac{1}{12} & \frac{1}{24} & \frac{1}{8} & \frac{1}{4} \\ \frac{1}{3} & \frac{1}{8} & \frac{1}{2} & \end{bmatrix}$$

$P(XY) = P(X)P(Y)$

1.19

1) $p \sim \begin{pmatrix} a & b & c \\ 0.5 & 0.25 & 0.25 \end{pmatrix}$ $\quad H(p) = 1.5$ $\quad D(p\|q) = \sum p \log \frac{p}{q} = 0.085$

$q \sim \begin{pmatrix} a & b & c \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{pmatrix}$ $\quad H(q) = 1.585$ $\quad D(q\|p) = 0.082$

2) $\sum p \log \frac{p}{q} = \sum q \log \frac{q}{p}$ $\quad p = 1-q$

1.20

$X \sim \begin{pmatrix} x_1 & x_2 & x_3 & x_4 \\ \frac{p}{2} & \frac{p}{2} & \frac{(1-p)}{2} & \frac{(1-p)}{2} \end{pmatrix}$

$$[P(Y|X)] = \begin{bmatrix} 1-f & f & 0 & 0 \\ f & 1-f & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Odrec|i p za max I(X,Y)

$$[P(X,Y)] = \begin{bmatrix} \frac{p(1-f)}{2} & \frac{pf}{2} & 0 & 0 \\ \frac{pf}{2} & \frac{p(1-f)}{2} & 0 & 0 \\ 0 & 0 & \frac{(1-p)}{2} & 0 \\ 0 & 0 & 0 & \frac{(1-p)}{2} \end{bmatrix}$$

$$P(Y) = \begin{bmatrix} p & p & \frac{(1-p)}{2} & \frac{(1-p)}{2} \end{bmatrix}$$

$H(f) = f \log \frac{1}{f} + (1-f) \log \frac{1}{1-f}$

? ? ?

1.21

$$[P(Y|X)] = \begin{bmatrix} 1 & 0 \\ 0.5 & 0.5 \end{bmatrix} \quad [P(X,Y)] = \begin{bmatrix} x_1 & 0 \\ 0.5(1-x_1) & 0.5(1-x_2) \end{bmatrix}$$

$$\begin{bmatrix} 0.5(1-x_1) & 0.5(1-x_2) \end{bmatrix}$$

$C = \max_{\{P\}} I(X;Y) = \max_{\{P\}} \left(H(X) + H(Y) - H(X,Y) \right)$

$I(X;Y) = H(Y) - H(Y|X) = 0.5(1+x_1)\log 0.5(1+x_1) + 0.5(1-x_1)\log 0.5(1-x_1) - (1-x_1)\log 0.5$

$= 0.5(1+x_1)\log 0.5(1+x_1) + 0.5(1-x)\log 0.5(1-x) + (1-x) \; /'$

$0.5\log 0.5(1+x) + \frac{0.5}{\ln 2} - 0.5\log 0.5(1-x) - \frac{0.5}{\ln 2} - 1 = 0 \Rightarrow x =$

$x_1 = \frac{3}{5} \quad x_2 = \frac{2}{5} \qquad C = 0.322$

1.22

$$[P(Y|X)] = \begin{bmatrix} 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \end{bmatrix}$$

$P(Y) = \frac{1}{5} \; \frac{1}{5} \; \frac{1}{5} \; \frac{1}{5} \; \frac{1}{5}$

$P(X) = \frac{1}{n} = \frac{1}{5}$

$C = \max_{P(X)} I = \log 5 - \left(-\log \frac{1}{2}\right) = 1.322$

1.23

$$[P(Y|X)] = \begin{bmatrix} 1-\beta & \beta & 0 & 0 \\ \beta & 1-\beta & 0 & 0 \\ 0 & 0 & 1-\gamma & \gamma \\ 0 & 0 & \gamma & 1-\gamma \end{bmatrix}$$

$\gamma = 0, \; \beta = 1$

$P(Y) = \begin{bmatrix} \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \end{bmatrix}$

$H(X) = 2$

$$P(X,Y) = \begin{bmatrix} \frac{1-\beta}{4} & \frac{\beta}{4} & 0 & 0 \\ \frac{\beta}{4} & \frac{1-\beta}{4} & 0 & 0 \\ 0 & 0 & \frac{1-\gamma}{4} & \frac{\gamma}{4} \\ 0 & 0 & \frac{\gamma}{4} & \frac{1-\gamma}{4} \end{bmatrix}$$

a)

$$= \begin{bmatrix} 0 & \frac{1}{4} & 0 & 0 \\ \frac{1}{4} & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{4} & 0 \\ 0 & 0 & 0 & \frac{1}{4} \end{bmatrix} = \begin{bmatrix} \frac{1-\beta}{4} & \frac{\beta}{4} & 0 & 0 \\ \frac{\beta}{4} & \frac{1-\beta}{4} & 0 & 0 \\ 0 & 0 & \frac{1-\beta}{4} & \frac{\beta}{4} \\ 0 & 0 & \frac{\beta}{4} & \frac{1-\beta}{4} \end{bmatrix}$$

$I = H(Y) - H(Y|X) = 2$

$I = 2 + (1-\beta)\log\frac{1}{4}(1-\beta) + \beta \log \frac{1}{4}\beta = 2 - H(\beta)$