

3. DZ

11.

Huffman

$A =$

17	17	3	4	5
5	2	3	15	5
8	18	17	23	17
18	17	3	4	6

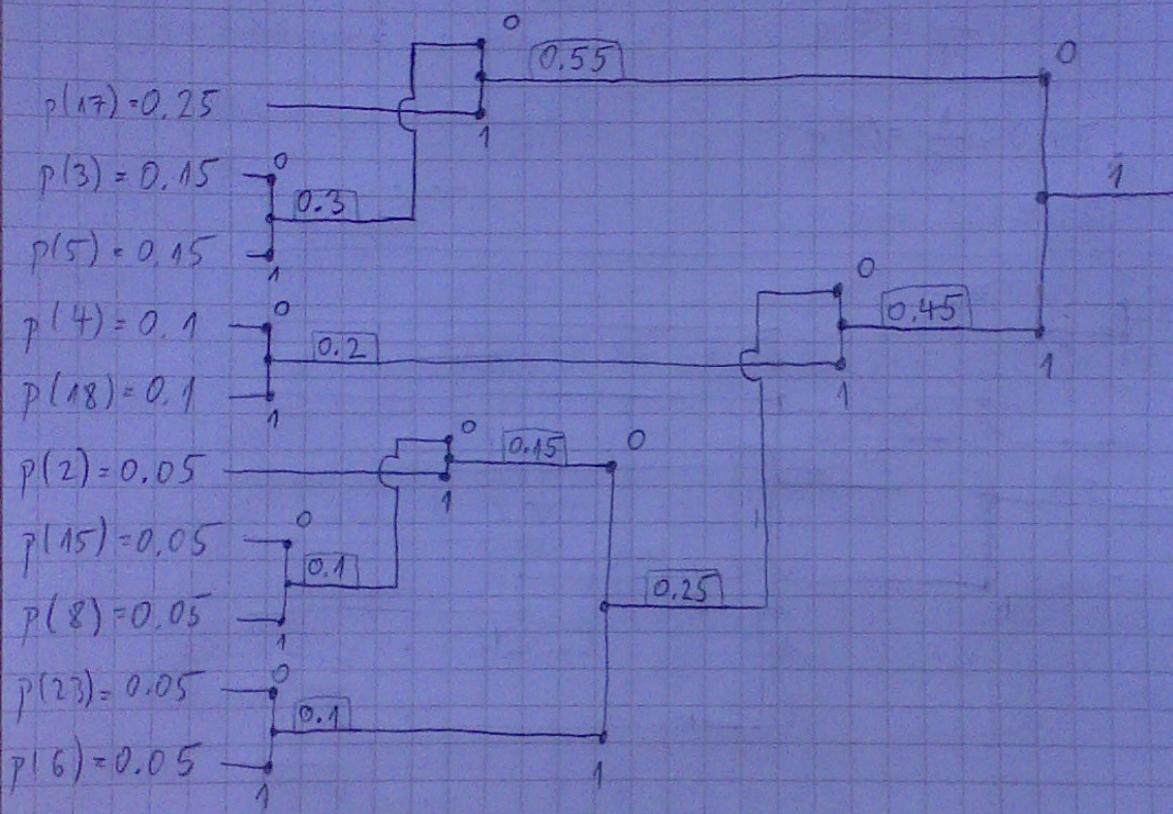
-ukupno 20 simbola

$$p(17) = 0.25$$

$$p(3) = p(5) = 0.15$$

$$p(4) = p(18) = 0.1$$

$$p(2) = p(15) = p(8) = p(23) = p(6) = 0.05$$



simbol

kod

duljina kodne riječi ( $l$ )

17

0 1

2

3

0 0 0

3

5

0 0 1

3

4

1 1 0

3

18

1 1 1

3

2

1 0 0 1

4

15

1 0 0 0 0

5

8

1 0 0 0 1

5

23

1 0 1 0

4

6

1 0 1 1

4

L =  $\sum_{i=1}^{10} p_i l_i$

$$L = \sum_{i=1}^{10} p_i l_i = 0,25 \cdot 2 + 0,15 \cdot 3 + 0,15 \cdot 3 + 0,1 \cdot 3 + 0,1 \cdot 3 + \\ + 0,05 \cdot 4 + 0,05 \cdot 5 + 0,05 \cdot 5 + 0,05 \cdot 4 + 0,05 \cdot 4$$

$$\underline{L = 3,1 \text{ bit/simbol}}$$

$$H(x) = -\sum_i p(x_i) \cdot \log_2 p(x_i)$$

$$\underline{H(x) = 3,066 \text{ bit/simbol}}$$

$$M = L \cdot 20 = 3,1 \frac{\text{bit}}{\text{simbol}} \cdot 20 \text{ simbola} = 62 \text{ bit}$$

$$1 \text{ byte} = 8 \text{ bit}$$

$$1 \text{ kbyte} = 1024 \text{ byte}$$

$$M = \frac{62}{8} = 7,75 \text{ byte}$$

$$M = \frac{7,75}{1024} \text{ kbyte} = \underline{0,007568359 \text{ kbyte}}$$

12)  $x_i, i = 1, \dots, 10$

$x_i$	1	2	3	4	5	6	7	8	9	10
$p(x_i)$	0.3	0.11	0.06	0.17	0.13	0.07	0.13	0.08	0.05	0.03

a) Huffman  $\rightarrow$  svaka kodna riječ ima paran broj bitova

$$p(x_4) = 0,17$$

$$p(x_{10}) = 0,17$$

$$p(x_5) = 0,13$$

$$p(x_7) = 0,13$$

$$p(x_2) = 0,11$$

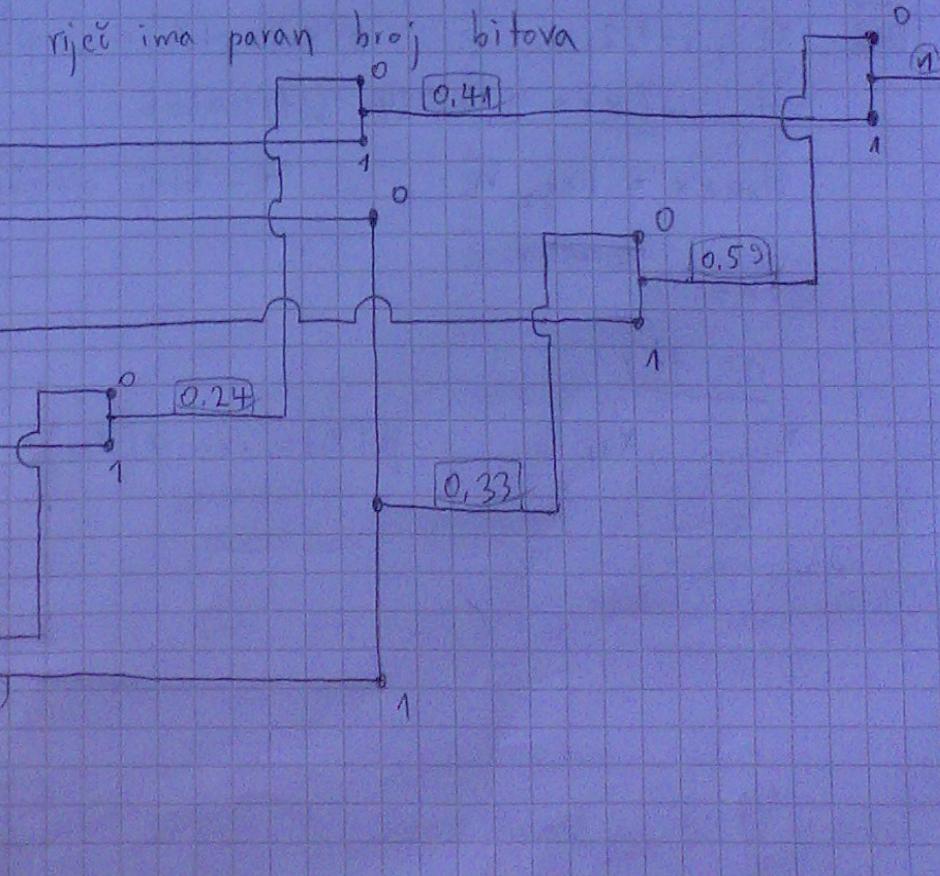
$$p(x_9) = 0,08$$

$$p(x_6) = 0,07$$

$$p(x_3) = 0,06$$

$$p(x_8) = 0,05$$

$$p(x_1) = 0,03$$



<u>simbol</u>	<u>kod</u>	<u>duljina kodne riječi (l)</u>
$x_4$	11	2
$x_{10}$	0000	4
$x_5$	0100	4
$x_7$	0110	4
$x_2$	1010	4
$x_9$	0010	4
$x_6$	1000	4
$x_3$	1001	4
$x_8$	001100	6
$x_1$	001110	6

b)  $H(x) = ?$

$$H(x) = - \sum_i p(x_i) \log_2 p(x_i)$$

$$H(x) = 3,1562 \text{ bit/simbol}$$

c)  $L = ?$

$$L = \sum_i p_i \cdot l_i$$

$$L = 0,17 \cdot 2 + 0,17 \cdot 4 + 0,13 \cdot 4 + 0,13 \cdot 4 + 0,11 \cdot 4 + 0,08 \cdot 4 \\ + 0,07 \cdot 4 + 0,06 \cdot 4 + 0,05 \cdot 6 + 0,03 \cdot 6$$

$$L = 3,82 \text{ bit/simbol}$$

13)  $x_i, i=1,2,\dots \quad p(x_1) = p(x_2) = \dots = p(x_n) = \frac{1}{n}$

$$H_{\max}(x) = 3,4594 \text{ bit/simbol}$$

$$H_{\max} = \log_2 n \quad n - \text{broj simbola}$$

$$n = 2^{3,4594}$$

$$\underline{n = 11 /}$$

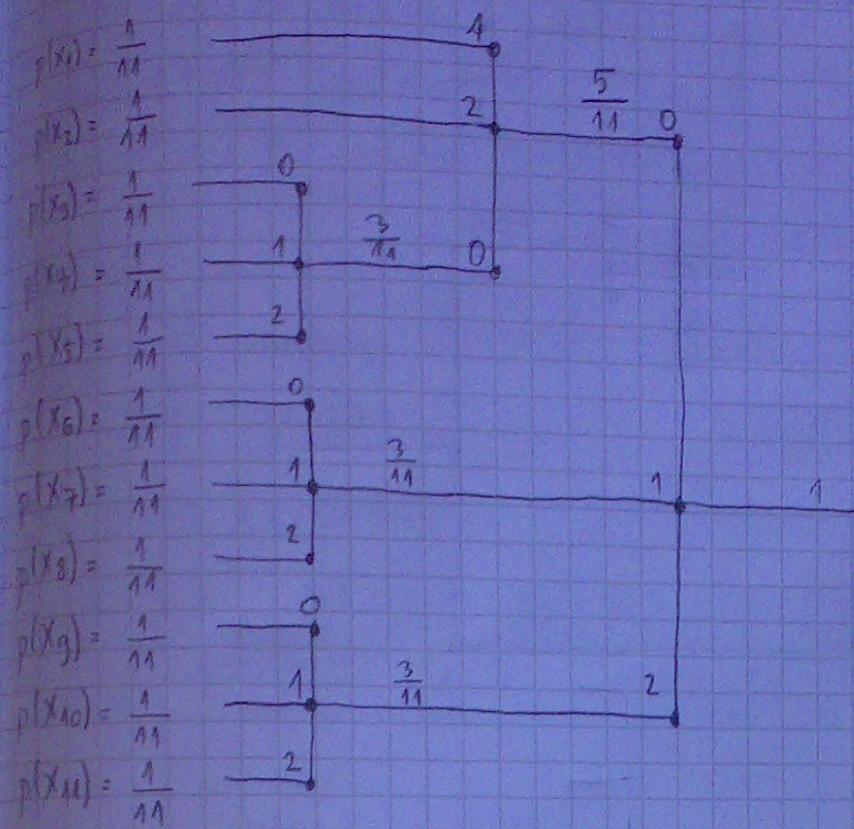
$$x_1, \dots, x_n$$

$$p(x_1) = \dots = p(x_n) = \frac{1}{11} =$$

Huffman

ternarni kod

0, 1, 2

simbolkodduljina kodne riječi (l)

$X_1$	0 1	2
$X_2$	0 2	2
$X_3$	0 0 0	3
$X_4$	0 0 1	3
$X_5$	0 0 2	3
$X_6$	1 0	2
$X_7$	1 1	2
$X_8$	1 2	2
$X_9$	2 0	2
$X_{10}$	2 1	2
$X_{11}$	2 2	2

b)

$$\mathcal{E} = ?$$

$$\mathcal{E} = \frac{H(X)}{L}$$

$$L = \sum p_i l_i$$

$$L = \frac{1}{11} \cdot (2 + 2 + 3 + 3 + 3 + 2 + 2 + 2 + 2 + 2)$$

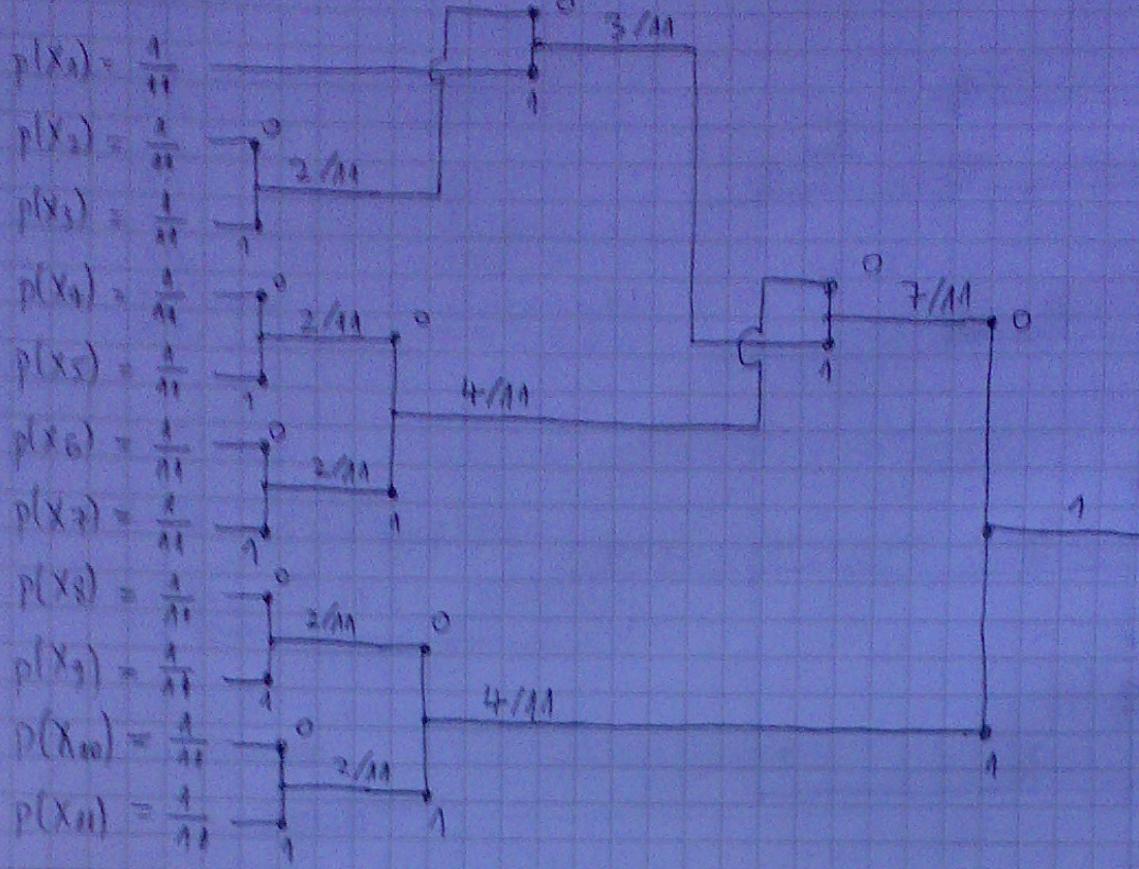
$$L = 2.272 \text{ bit/simbol}$$

$$H(X) = \log_3 11$$

$$|\mathcal{E} = 0.96|$$

$$H(X) = \log_3 11 = 2.182 \text{ bit/simbol}$$

c) Huffman - binarni kod 0,9



<u>symbol</u>	<u>kod</u>	<u>duljina kodne rječi (l)</u>
$x_1$	011	3
$x_2$	0100	4
$x_3$	0101	4
$x_4$	0000	4
$x_5$	0001	4
$x_6$	0010	4
$x_7$	0011	4
$x_8$	100	3
$x_9$	101	3
$x_{10}$	110	3
$x_{11}$	111	3

$$L = \sum p_i l_i$$

$$L = \frac{1}{11} \cdot (3 + 4 + 4 + 4 + 4 + 4 + 4 + 3 + 3 + 3 + 3) = \frac{39}{11} = 3,5454$$

$$\varepsilon = \frac{H(x)}{L} = \frac{3,4594}{3,5454} = 0,9757$$

Binarni kod je efikasniji od tvarnica!

14.  $\{a, b, c, d\}$

$$p_a : p_b : p_c : p_d = 1 : 2 : 3 : 4$$

aritmetičko kodiranje

- sljed od 10 simbola  $\rightarrow$  kodirana poruka

$$L = (0,0011 \overset{\checkmark}{0} 110001001011101)_2$$

$\begin{matrix} 3 & 4 & 6 & 7 & 11 & 14 & 16 & 17 & 18 & 19 \end{matrix}$

- prvih 5 simbola = ?

$$L = ( \quad )_{10} = ?$$

$$L = 0,211514473$$

$$p_a = 0,1 \quad p_b = 0,2 \quad p_c = 0,3 \quad p_d = 0,4$$

verovatnost:

a      0,1

kumulativni skupovi  $[D_S, G_S]$

[0, 0,1)

b      0,2

[0,1, 0,3)

c      0,3

[0,3, 0,6)

d      0,4

[0,6, 1)

$$D' = D + (G - D) \cdot D_S$$

$$D = 0,16 \quad G = 0,22$$

$$G' = D + (G - D) \cdot G_S$$

a:  $D' = 0,16$

$G' = 0,166$

prvi simbol je sigurno b

b:  $D' = 0,166$

$G' = 0,178$

$$D = 0,1 \quad G = 0,3$$

c:  $D' = 0,178$

$G' = 0,196$

$$A: D' = 0,1 + (0,3 - 0,1) \cdot 0 = 0,1$$

treći  
simbol

$$G' = 0,1 + 0,2 \cdot 0,1 = 0,12$$

d:  $D' = 0,196$

$G' = 0,22$

b:  $D' = 0,12$       drugi simbol



c:  $D' = 0,16$        $\leftarrow$  "c"

$$D = 0,196 \quad G = 0,22$$

d:  $D' = 0,22$   
 $G' = 0,3$

četvrti simbol

"d"  
↑

a:  $D' = 0,196 \quad G' = 0,1984$

b:  $D' = 0,1984 \quad G' = 0,2032$

c:  $D' = 0,2032 \quad G' = 0,2104$

d:  $D' = 0,2104 \quad G' = 0,22$

$$D = 0,2104 \quad G = 0,22$$

a:  $D' = 0,2104 \quad G' = 0,21136 \rightarrow$  peti simbol

b:  $D' = 0,21136 \quad G' = 0,211328 \rightarrow "b"$

c:  $D' = 0,211328 \quad G' = 0,211616$

d:  $D' = 0,211616 \quad G' = 0,22$

prvi pet simbola je "bcd db"

- Kodirati sklijed od 4 simbola "acda"

$$D' = D + (G - D) \cdot D_s$$

$$G' = D + (G - D) \cdot G_s$$

a:  $D = 0 \quad G = 1$

$$D' = 0 + (1 - 0) \cdot 0 = 0$$

$$\underline{G' = 0 + (1 - 0) \cdot 0,1 = 0,1}$$

$$D = D' = 0 \quad G = G' = 0,1$$

c:

$$D' = 0 + (0,1 - 0) \cdot 0,3 = 0,03$$

$$\underline{G' = 0 + (0,1 - 0) \cdot 0,6 = 0,06}$$

$$D = 0,03 \quad G = 0,06$$

d:

$$D' = 0,03 + (0,06 - 0,03) \cdot 0,6 = 0,048$$

$$\underline{G' = 0,03 + (0,06 - 0,03) \cdot 1 = 0,06}$$

$$D = 0,048 \quad G = 0,06$$

a:

$$D' = 0,048 + (0,06 - 0,048) \cdot 0 = 0,048$$

$$G' = 0,048 + (0,06 - 0,048) \cdot 0,1 = 0,0492 \rightarrow$$
 kodirana poruka je

bilo koji broj iz intervala  $[0,048, 0,0492]$

Vizimimo broj 0,0482 i pretvorimo ga u binarni  
zapis.

$$0,0482_{10} = 0.00001100010$$

LZ77 algoritam

kodirati paruku      a a a a b b b b b b c c c c c d \*

$$\text{pp} = G \quad \text{pk} = 5$$

a a a a b b b b b b c c c c c d	(0, 0, a)
a a a a b b b b b b c c c c c d	(1, 1, a)
a a a a b b b b b b c c c c c d	

NISAM URADIO OVAJ ZADATAK :-)  
IMAJU NEKA RJEŠENJA NA forumu

16.

$$LZW$$

$$D[1] = a$$

$$D[2] = b$$

kodirati poruku

a b b b b b b a b a b a b a a a b

1 2 3 4 5 6 7 8 9 10 11 12  
v v v v v v v v v v v v  
2 3 4 5 6 7 8 9 10 11 12

$$D[3] = ab$$

$$D[4] = bb$$

$$D[5] = bba$$

$$D[6] = bbb$$

$$D[7] = aba$$

$$D[8] = abab$$

$$D[9] = ba$$

$$D[10] = aa$$

$$D[11] = aa$$

1, 2, 4, 5, 3, 7, 2, 1, 10, 2

17. LZW

$$D[0] = \# \quad D[1] = A \quad D[2] = B \quad \dots \quad D[26] = Z$$

dekodirati poruku

20 15 2 5 15 18 14 15 20 27 29 31 36 30 32 34 0

T O B E O R N O T T O B E O R T O B E O R N O T #

$$D[27] = TO$$

$$D[34] = OT$$

$$D[28] = OB$$

$$D[35] = TT$$

$$D[29] = BE$$

$$D[36] = TOB$$

$$D[30] = EO$$

$$D[37] = BEO$$

$$D[31] = OR$$

$$D[38] = OBT$$

$$D[32] = RN$$

$$D[39] = TOBE$$

$$D[33] = NO$$

$$D[40] = EOR$$

$$D[41] = RNO$$

T O B E O R T O B E O

T O B E O R N O T T O B E O R N O T #

TM NT =

$$D[42] = OT \#$$

18.)

$$x_i, i = 1, 2, \dots, 100$$

bezmemorijska izvoriste  $\rightarrow$  svi simboli jednako verjetni

$C(x)$  - prefiksni kod

$$\sum l_i > 664$$

↓

za optimalnost koda

$$L = \sum p_i l_i \quad H(x) \leq L \leq H(x) + 1$$

$$H(x) = -\sum p_i \log_2 p_i = \log_2 100 = 6,64$$

$$p(x_i) = \frac{1}{100}$$

$$H(x) \leq L \leq H(x) + 1$$

$$L = \sum p_i l_i = p(x_i) \cdot \sum l_i$$

$$\sum l_i = \frac{L}{p(x_i)} = \frac{L}{\frac{1}{100}} = 100 \cdot L$$

$$\text{ako je } L = H(x)$$

$$\sum l_i = 100 \cdot H(x) = 664$$

$$\text{ako je } L = H(x) + 1$$

$$\sum l_i = 100 \cdot (H(x) + 1)$$

$$\sum l_i = 100 \cdot 6,64 + 100$$

$$\sum l_i = 664 + 100 = 764$$

$\Rightarrow$  ako kod nije optimalno kodiran, znači

ako nije  $L = H(x)$

$\sum l_i$  može biti veći od 664 /