Završni ispit

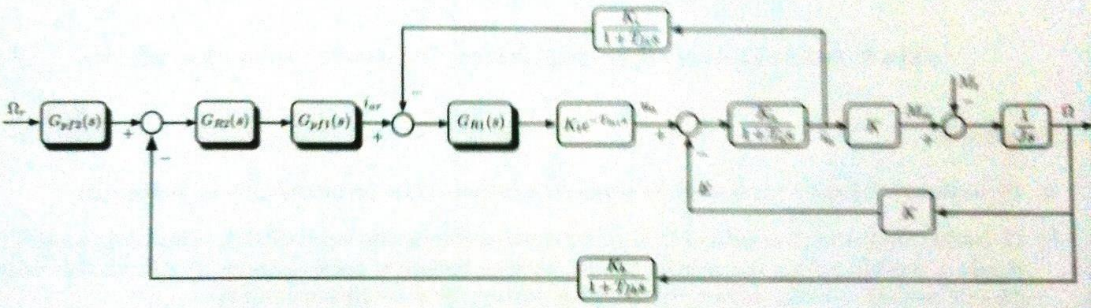
5. veljače 2013.

Ime i Prezime: **Matični broj:**

Napomena: Zadatke obavezno predati s rješenjima nakon završetka testa.

1. zadatak (15 bodova)

Kaskadna struktura upravljanja brzinom istosmjernog motora prikazana je na slici 1, pri čemu pojedini parametri iznose: $K_a = 4.5\,A/V$, $T_a = 0.025\,\mathrm{s}$, $K = 1.33\,\mathrm{Vs/rad}$, $K_t = 44$, $T_{mt} = 1.66\,\mathrm{ms}$, $K_i = 0.1\,\mathrm{V/A}$, $T_{fi} = 2\,\mathrm{ms}$, $K_b = 0.0318$, $T_{fb} = 20\,\mathrm{ms}$, $J = 3.2\mathrm{kg\ m^2}$.



Slika 1: Blokovska shema kaskadnog upravljanja brzinom DC motora s nezavisnom uzbudom

Potrebno je:

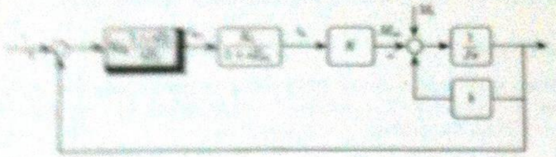
a) Projektirati PI regulator struje armature $G_{R1}(s)$ prema tehničkom optimumu kao i prefiltar referentn vrijednosti struje armature $G_{pf1}(s)$. **(4 boda)**

b) Projektirati regulator brzine vrtnje motora $G_{R2}(s)$ prema simetričnom optimumu uz $a = 2$. Takod je potrebno projektirati prefiltar u referentnoj grani brzine vrtnje $G_{pf2}(s)$. **(4 boda)**

c) Projektirati regulator brzine vrtnje motora $G_{R2}(s)$ prema simetričnom tako da fazno osiguranje izn $\gamma = 42°$ i odrediti maksimalnu dozvoljenu promjenu momenta tromosti, tako da minimalno fa osiguranje iznosi $\gamma = 37°$. Skicirati bodeov dijagram za nominalni moment tromosti i za maksimal dozvoljenu promjenu. **(7 bodova)**

2. zadatak (10 bodova)

Nadređena petlja upravljanja brzinom vrtnje istosmjernog motora s nezavisnom i konstantnom uzbudom prikazana je blokovskom shemom na slici 2. Pritom su: $K_i = 0.5$, $T_{ei} = 5\,\mathrm{ms}$, $K = 1.33\,\mathrm{Vs/rad}$ i $J = 3\,\mathrm{kgm^2}$, $b = 0.1\,\mathrm{Nms}$.



Slika 2: Blokovska shema upravljanja brzinom DC motora s nezavisnom uzbudom

Potrebno je:

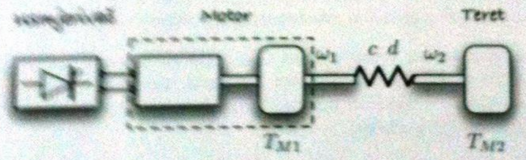
a) **(5 bodova)** Odrediti parametre PI regulatora brzine vrtnje prema modulnom optimumu.

b) **(2 boda)** Osigurava li regulator iz a) dijela zadatka eliminaciju regulacijskog odstupanja u ustaljenom stanju u slučaju referentne veličine oblika funkcije linearnog porasta (rampe). Obrazložiti odgovor. Ako ne osigurava koliko iznosi regulacijskog odstupanje u ustaljenom stanju?

c) **(3 boda)** Odrediti ekvivalentnu vremensku konstantu i karakteristične odnose. Nacrtajte odziv sustava i označite nadvišenje i vrijeme prvog maksimuma.

3. zadatak (15 bodova)

Za elektromehanički sustav s elastičnim prijenosom zadani su sljedeći normirani parametri: $T_{M1} = 1.0\,s$ - motor; $T_{M2} = 4.0\,s$ - teret; $c = 100\,Nm/rad$ - konstanta krutosti; $d = 0.5\,Nms/rad$ - konstanta prigušenja; $T_N = 1\,s$ - normirana vremenska konstanta.



Slika 3: Skica radnog stroja s remenskim prijenosom

a) **(6 bodova)** Potrebno je nacrtati strukturnu blokovsku shemu nadomjesnog kontinuiranog regulacijskog kruga brzine vrtnje s PI regulatorom brzine vrtnje te odrediti parametre regulatora uz korištenje optimuma dvostrukog odnosa uz $D_i = 0.5$, uz nadomjesnu vremensku konstantu podređenog regulacijskog kruga struje $T_{ei} = 0.01\,s$ i vrijeme uzorkovanja $T = 0.001\,s$.

b) **(6 bodova)** Koliko bi iznosili parametri podoptimalnog (u smislu optimuma dvostrukog odnosa) PI regulatora kojim se postiže nadomjesna vremenska konstanta zatvorenog kruga $T_e = 0.4\,s$, uz dominantni karakteristični odnos $D_2 = 0.5$. Koliko u tom slučaju iznosi karakteristični odnos D_3?

c) **(3 boda)** Izvesti jednadžbu za nadomjesnu konstantu zatvorenog kruga uz $D_i = 0.5$.

Napomena: **Nadomjesnu vremensku konstantu zatvorenog kruga u a) dijelu zadatka odredite koristeći približnu relaciju.**

Z1_2013

(1) $K_a = 4.5\ A/V$, $T_a = 25\ ms$

$K = 1.33\ Vs/rad$, $J = 3.2\ kg\,m^2$

$K_t = 44$, $T_{mi} = 1.66\ ms$

$K_i = 0.1\ V/A$, $T_{fi} = 2\ ms$

$K_b = 0.0318$, $T_{fb} = 20\ ms$

---

a) $$G_{P1}(s) = \frac{K_t K_a K_i}{(1+T_{mi}s)(1+T_a s)(1+T_{fi}s)}$$

$$G_{S1}(s) = \frac{K_{S1}}{(1+T_{I1}s)(1+T_{\Sigma 1}s)} \rightarrow K_{S1} = K_t K_a K_i = 19.8$$

$$T_{\Sigma 1} = T_{mi} + T_{fi} = 3.66\ ms$$

$$T_{I1} = T_a \Rightarrow \boxed{T_{I1} = 25\ ms}$$

$$K_{R1} = \frac{1}{2K_{S1}}\frac{T_{I1}}{T_{\Sigma 1}} \rightarrow \boxed{K_{R1} = 0.1725}$$

$$\boxed{G_{PF1}(s) = \frac{K_i}{1+T_{fi}s}}$$

---

b) $a = 2$

$$G_{a1}(s) = \frac{1}{2T_{\Sigma 1}^2 s^2 + 2T_{\Sigma 1}s + 1} \approx \frac{1}{1+2T_{\Sigma 1}s}$$

$$G_{P2}(s) = \frac{K K_b}{Js(1+2T_{\Sigma 1}s)(1+T_{fb}s)} \quad \cancel{\approx} \quad \cancel{\frac{T_M}{K_S}} \ \cancel{K_{S2}}$$

$$G_{S2}(s) = \frac{K_{S2}}{T_M s(1+T_{\Sigma 2}s)} \rightarrow \frac{K_{S2}}{T_M} = \frac{K K_b}{J} = \cancel{1.3213} = 0.0132$$

$$T_{\Sigma 2} = 2T_{\Sigma 1} + T_{fb} = 27.32\ ms$$

$$T_{I2} = a^2 T_{\Sigma 2} \Rightarrow \boxed{T_{I2} = 109.28\ ms}$$

$$K_{R2} = \frac{1}{aK_{S2}}\frac{T_M}{T_{\Sigma 2}} \Rightarrow \boxed{K_{R2} = 1384.7154}$$

$$\boxed{G_{PF2}(s) = \frac{K_b}{(1+T_{I2}s)(1+T_{fb}s)}}$$

c) $\gamma = 42^\circ \rightarrow a = \mathrm{tg}\,\gamma + \frac{1}{\cos\gamma} = 2.246$

$$T_{I_2} = a^2 T_{\Sigma 2} \rightarrow \boxed{T_{I_2} = 137.82\ \mathrm{ms}}$$

$$K_{R2} = \frac{1}{a K_{S2}} \frac{T_M}{T_{\Sigma 2}} \rightarrow \boxed{K_{R2} = 1233.03}$$

$\gamma_{min} = 37^\circ \rightarrow \omega_c^* = b\,\omega_c$

$\cancel{J \in [\ ]}$

$$\omega_c = \frac{1}{a T_{\Sigma_2}} \qquad \omega_c^* = \frac{b}{a T_{\Sigma_2}}$$

$$\rightarrow \gamma_{min} = -180^\circ + \mathrm{arctg}(T_{I_2}\omega_c^*) - \mathrm{arctg}(T_{\Sigma_2}\omega_c^*) + 180^\circ$$

$$\frac{T_{I_2}\omega_c^* - T_{\Sigma_2}\omega_c^*}{1 + T_{I_2}\omega_c^* T_{\Sigma_2}\omega_c^*} = \mathrm{tg}\,\gamma_{min}$$

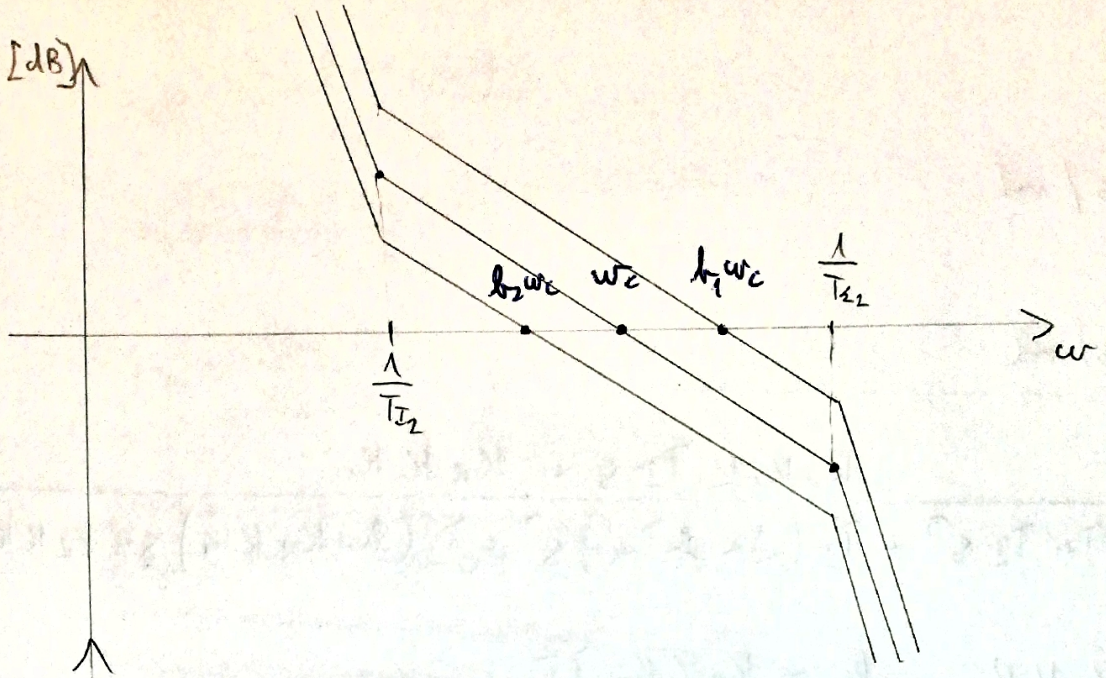
$$\rightarrow T_{I_2}\omega_c^* = ab$$
$$T_\Sigma \omega_c^* = \frac{b}{a}$$

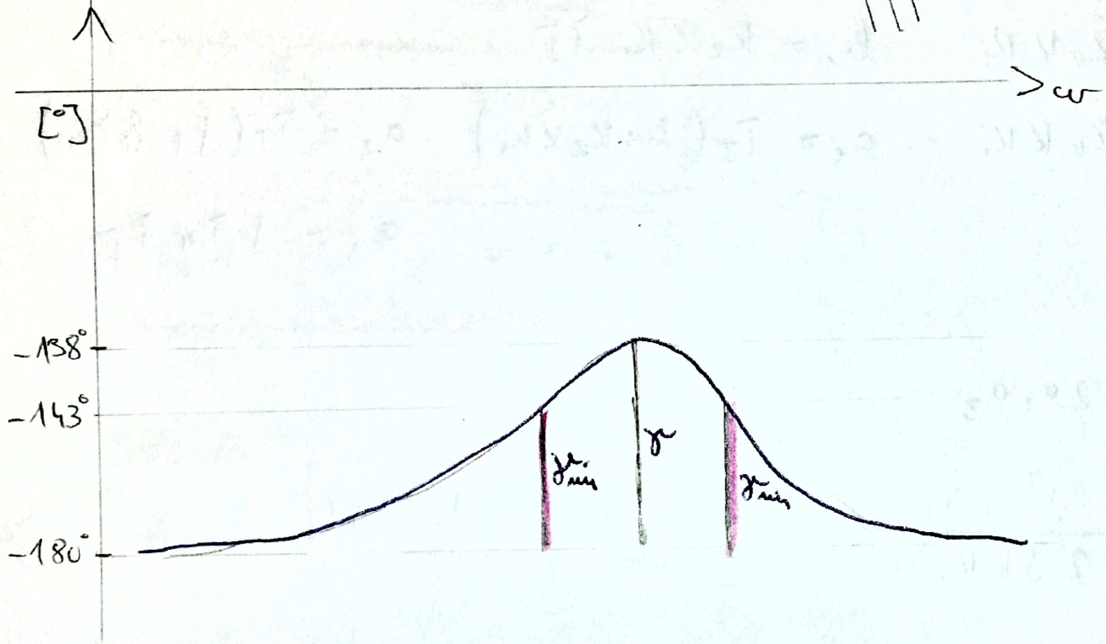
$$\rightarrow \mathrm{tg}\,\gamma_{min} = \frac{b}{1+b^2}\left(a - \frac{1}{a}\right)$$

$$b^2 - \frac{a^2-1}{a\,\mathrm{tg}\,\gamma_{min}} \cdot b + 1 = 0 \rightarrow b_1 = 1.8489$$
$$b_2 = 0.5409$$

$$\boxed{J_* \in [0.54\,J_M,\ 1.85\,J_M]}$$

MAKS. DOZVOLJENA PROMJENA


[dB]
$\frac{1}{T_{I_2}}$
$b_2\omega_c$
ω_c
$b_1\omega_c$
$\frac{1}{T_{z_2}}$
ω


[°]
ω
$-138°$
$-143°$
$-180°$
γ_{min}
γ
γ_{min}

(2) $K_i = 0.5$

$T_{ei} = 5\ ms$

$K = 1.33\ Vs/rad$

$J = 3\ kg\ m^2$

$b = 0.1\ Nm\ s/rad$

a) $$G_r(s) = \frac{K_R K_i K T_I s + K_R K K_i}{J T_{ei} T_I s^3 + T_I (J + b T_{ei}) s^2 + T_I (b + K_R K K_i) s + K_R K K_i}$$

$\rightarrow$ $b_0 = K_R K K_i$ $\quad b_1 = K_R K K_i T_I$

$a_0 = K_R K K_i$ $\quad a_1 = T_I (b + K_R K K_i)$ $\quad a_2 = T_I (J + b T_{ei})$

$a_3 = J T_{ei} T_I$

$\rightarrow$ $a_2^2 = 2 a_1 a_3$

$$K_R = \frac{J^2 + T_{ei}^2}{2 J K K_i T_{ei}} \Rightarrow \boxed{K_R = 451.1291}$$

$\rightarrow$ $a_1^2 - 2 a_0 a_2 = b_1^2$

$$T_I = \frac{2 K_R K K_i (J + b T_{ei})}{b^2 + 2 b K_R K K_i} \Rightarrow \boxed{T_I = 30\ s}$$

b) $$e_\infty = \lim_{s \to 0} \left\{ s \cdot [G_r(s) - 1] \frac{1}{s^2} \right\} = \frac{T_I b}{K_R K K_i}$$

$\rightarrow$ NE OSIGURAVA ZBOG MOMENTA TERETA TJ. ON STVARA REGULACIJSKO ODSTUPANJE U STAC. STANJU.

c)

$$a_1 = \frac{T_I(b + K_a K K_i)}{K_a K K_i}$$

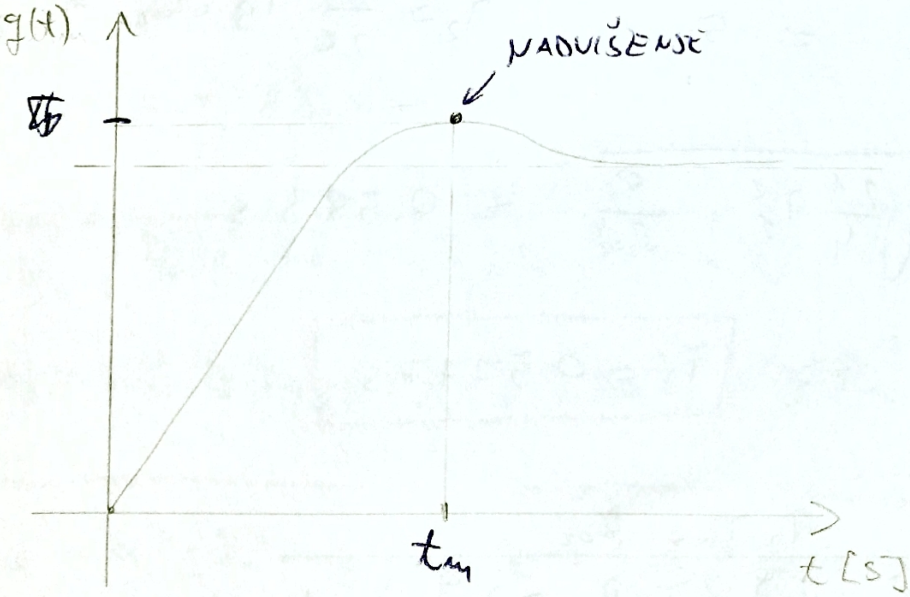
$$a_2 = \frac{T_I(J + b T_{ei})}{K_a K K_i}$$

$$a_3 = \frac{J T_{ei} T_I}{K_a K K_i}$$

$$\boxed{T_e = a_1 = 30.01 \text{ s}}$$

$$\boxed{D_2 = \frac{a_2}{T_e^2} = 3.33 \cdot 10^{-4}}$$

$$\boxed{D_3 = \frac{a_3}{D_2^2 T_e^3} = 0.5}$$



(3) $T_{m_1} = 1\,s$

$T_{m_2} = 4\,s$ $\rightarrow T_{m\Sigma} = 5\,s$

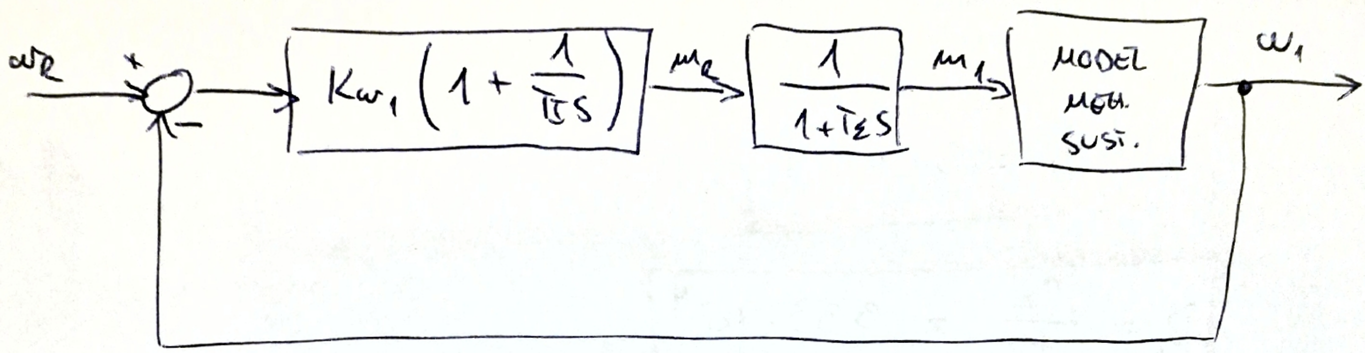
$c = 100\ Nm/rad$

$d = 0.5\ Nms/rad$

$T_B = 1\,s$

---

a)



$T_{ei} = 10\,ms$ $\rightarrow$ $T_\Sigma = 11\,ms$

$T = 1\,ms$

$D_i = 0.5$

$$\Omega_{02} = \sqrt{\frac{c}{T_B T_{m_2}}} = 5 \qquad \xi_2 = \frac{d}{2c} T_B \Omega_{02} = \frac{1}{80}$$

$$T_e = \frac{3}{2} T_\Sigma + \sqrt{\frac{21}{4} T_\Sigma^2 + \frac{8}{\Omega_{02}^2}} = 0.583\,s$$

$$\rightarrow T_I = T_e - \frac{d}{c} \Rightarrow \boxed{T_I = 0.5777\,s}$$

$$\rightarrow K_{\omega i} = \frac{T_I T_{m\Sigma} \Omega_{02}^2}{D_2 T_e^2 \Omega_{02}^2 - 2\xi_2 (T_e \Omega_{02} - 2\xi_2) - 1}$$

$$\boxed{K_{\omega i} = 22.7624}$$

b) $T_e = 0.4\ s$

$D_2 = 0.5$

$\rightarrow T_I = T_e - \frac{d}{c} \Rightarrow \boxed{T_I = 0.395\ s}$

$\rightarrow \boxed{K_{w_1} = 51.9395}$

$\rightarrow D_3 D_2^2 T_e^3 = a_3$

$a_3 = \frac{T_I T_{M\Sigma}}{K_{w_1}}\left(T_\Sigma + \underbrace{\frac{2g}{\Omega_0}}_{\frac{d}{c}}\right) + \frac{T_I}{\Omega_{02}^2}$

$$D_3 = \frac{a_3}{D_2^2 T_e^3}$$

$a_3 = 0.0164$

$\boxed{D_3 = 1.03}$

---

c) $D_i = 0.5$

$\rightarrow T_I = T_e - 2g_2 \Omega_{02}^{-1}$

$$\rightarrow K_{w_1} = \frac{T_I T_{M\Sigma} \Omega_{02}^2}{D_2 T_e^2 \Omega_{02}^2 - 2g_2(T_e \Omega_{02} - 2g_2) - 1}$$

$\rightarrow a_3 = K_{w_1}^{-1} T_I T_{M\Sigma}(T_\Sigma + 2g\Omega_0^{-1}) + T_I \Omega_{02}^{-2} = D_3 D_2^2 T_e^3$

$$\boxed{T_e^3 - 4T_\Sigma T_e^2 - 8\Omega_{02}^{-2} T_e + 8\Omega_{02}^{-2} T_\Sigma = 0}$$