Auditorne vježbe

10. siječnja 2018.

Optimum dvostrukog odnosa

1. zadatak

Izvesti vezu između karakterističnog odnosa D_2 i faktora prigušenja ζ za sustav drugog reda dan prijenosnom funkcijom:

$$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}. \tag{1}$$

Rješenje: $D_2 = \frac{1}{4\zeta^2}$.

2. zadatak

Za sustav trećeg reda dan prijenosnom funkcijom

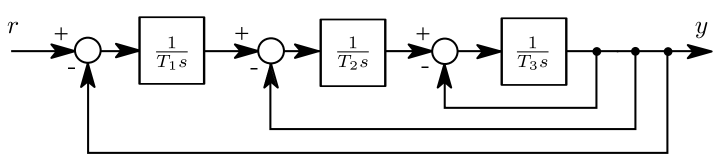
$$G(s) = \frac{1}{a_3 s^3 + a_2 s^2 + a_1 s + a_0} \tag{2}$$

potrebno je:

a) Nacrtati ekvivalentnu kaskadnu strukturu koja se sastoji od odgovarajućih nadomjesnih vremenskih konstanti.

b) Izvesti izraze za optimum dvostrukog odnosa.

Rješenje:

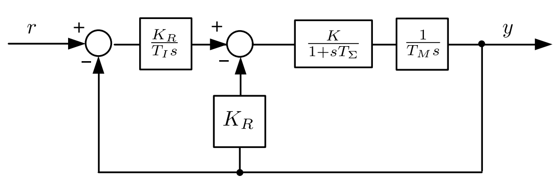


Slika 1: Nadomjesna kaskadna struktura

$$\begin{aligned} a_1^2 - 2a_2a_0 &= 0, \\ a_2^2 - 2a_3a_1 &= 0. \end{aligned} \tag{3}$$

3. zadatak

Za sustav prikazan slikom 2 potrebno je:



Slika 2: Proces s astatizmom prvog reda upravljan modificiranim PI regulatorom

a) Izvesti parametre modificiranog PI regulatora koristeći optimum dvostrukog odnosa.

b) Odrediti može li sustav slijediti referencu u obliku funkcije linearnog porasta. Ukoliko ne može, odrediti pogrešku slijeđenja na referencu

c) Odrediti može li sustav slijediti referencu u obliku funkcije linearnog porasta ukoliko se koristi klasični PI regulator s istim parametrima. Ukoliko ne može, odrediti pogrešku slijeđenja na referencu

Rješenje:

a) $K_R = \frac{1}{2K}\frac{T_M}{T_\Sigma}$, $T_I = 4T_\Sigma$

b) $\lim_{t\to\infty}(r(t) - y(t)) = T_I$.

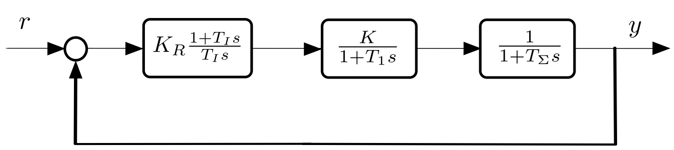
c) $\lim_{t\to\infty}(r(t) - y(t)) = 0$.

Rješenje: $T_e = 0.3$ s, $T_I = 0.296$ s, $K_{\omega_1} = 25.3$, $D_3 = \frac{a_3}{D_2^2 T_e^3} = 0.72$.

Prošireni optimum dvostrukog odnosa

4. zadatak

Zadan je sustav s jednom dominantnom vremenskom konstantom i jednom nedominantnom vremenskom konstantom ($T_1 >> T_\Sigma$) prikazan na Sl. 3.



Slika 3: Sustav s jednom dominantnom i jednom nedominantnom vremenskom konstantom

Potrebno je:

a) Odrediti parametre PI regulatora brzine koristeći modificirani optimum dvostrukog odnosa.

b) Usporediti dobivene parametre regulatora sa parametrima regulatora dobivenim korištenjem tehničkog optimuma.

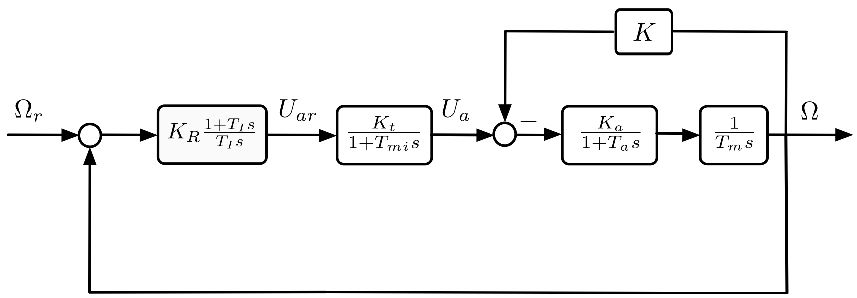
Rješenje:

a) $K_R = \frac{1}{2K}(\frac{T_1}{T_\Sigma} + \frac{T_\Sigma}{T_1})$, $T_I = \frac{(\frac{T_1}{T_\Sigma} + \frac{T_\Sigma}{T_1})}{1+(\frac{T_1}{T_\Sigma} + \frac{T_\Sigma}{T_1})}(T_1 + T_\Sigma)$.

b) $K_R \approx \frac{1}{2K}\frac{T_1}{T_\Sigma}$, $T_I \approx T_1$.

5. zadatak

Nadređena petlja upravljanja brzinom vrtnje istosmjernog motora s nezavisnom i konstantnom uzbudom prikazana je blokovskom shemom na slici 4. Pritom su: $K_t = 1.5$ V/V, $T_{mi} = 5\,\mathrm{ms}$, $K_a = 5\,\mathrm{A/V}$, $T_a = 15\,\mathrm{ms}$, $T_m = 0.4\,\mathrm{s}$ i $K = 1.33\,\mathrm{Vs/rad}$. Pretpostavlja se idealno mjerenje brzine vrtnje.



Slika 4: Blokovska shema upravljanja brzinom DC motora s nezavisnom uzbudom

Potrebno je odrediti parametre PI regulatora brzine koristeći prošireni optimum dvostrukog odnosa.
Rješenje: $K_R = 6.6775, T_I = 0.0396$

AUDITORNE XI

(1.) $$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$= \frac{1}{\frac{1}{\omega_n^2}s^2 + \frac{2\zeta}{\omega_n}s + 1} = \frac{1}{a_2 s^2 + a_1 s + a_0}$$

$\rightarrow$ $$a_0 = 1$$
$$a_1 = \frac{2\zeta}{\omega_n}$$
$$a_2 = \frac{1}{\omega_n^2}$$

$\rightarrow$ $$a_i = \prod_{j=1}^{i} T_j = T_1 \cdot T_2 \cdot \ldots \cdot T_i$$

$$a_1 = T_1 \qquad\qquad T_1 = a_1$$
$$\Rightarrow$$
$$a_2 = T_2 T_1 \qquad\qquad T_2 = \frac{a_2}{a_1}$$

$\rightarrow$ $$D_i = \frac{T_i}{T_{i-1}}$$

$$D_2 = \frac{T_2}{T_1} = \frac{a_2}{a_1^2} = \frac{\frac{1}{\cancel{\omega_n^2}}}{\frac{4\zeta^2}{\cancel{\omega_n^2}}}$$

$$\boxed{D_2 = \frac{1}{4\zeta^2}}$$

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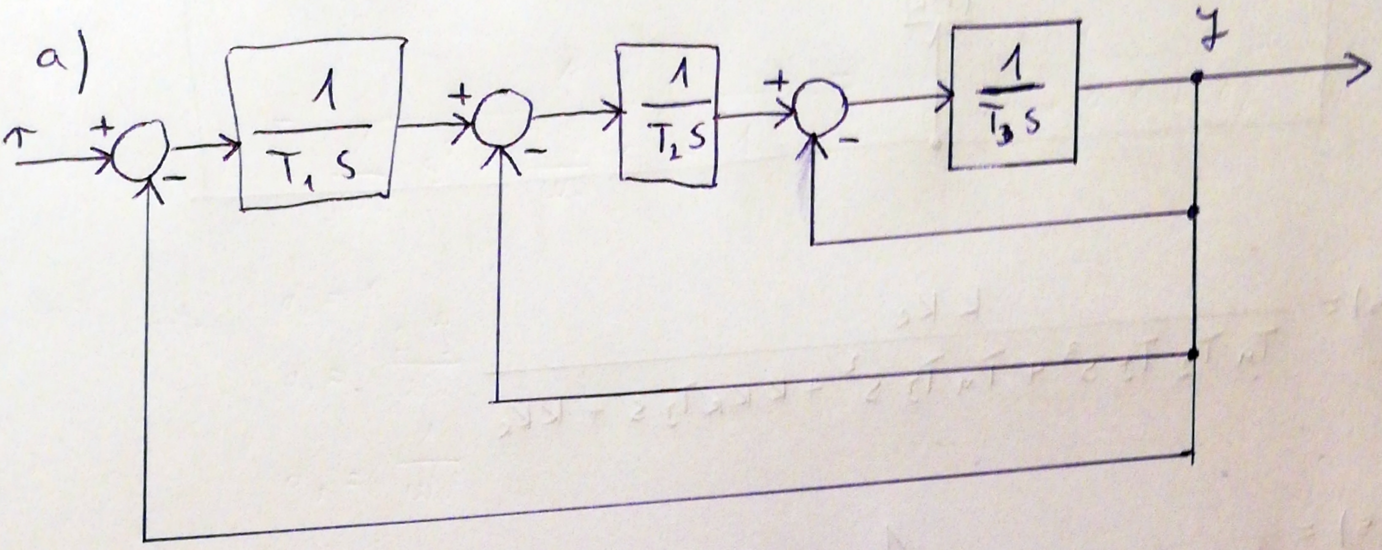
2

$$G(s) = \frac{1}{a_3 s^3 + a_2 s^2 + a_1 s + a_0}$$

a)



b) $$G(s) = \frac{1}{T_3 T_2 T_1 s^3 + T_2 T_1 s^2 + T_1 s + 1}$$

$\rightarrow$ $a_0 = 1$

$a_1 = T_1$

$a_2 = T_2 T_1$

$a_3 = T_3 T_2 T_1$

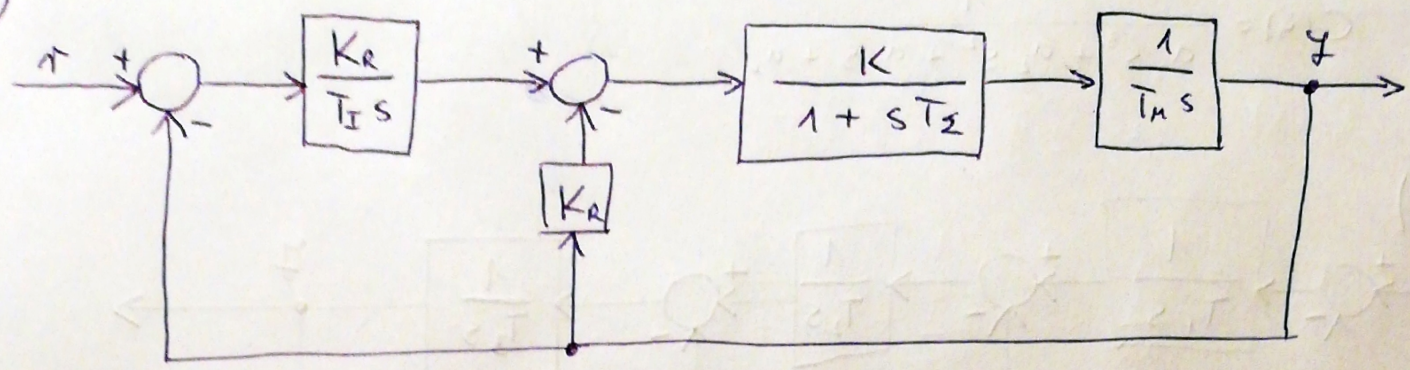
$\rightarrow$ $D_1 = 0.5$

$D_2 = \frac{T_2}{T_1} = \frac{a_2}{a_1^2}$ $\rightarrow$ $a_2 = 0.5\, a_1^2$

$D_3 = \frac{T_3}{T_2} = \frac{a_3 a_1}{a_2^2}$ $\rightarrow$ $a_3 a_1 = 0.5\, a_2^2$

$$\boxed{\begin{aligned} a_1^2 - 2 a_2 &= 0 \\ a_2^2 - 2 a_1 a_3 &= 0 \end{aligned}}$$

3.



$$G_r(s) = \frac{K K_R}{T_M T_\Sigma T_I s^3 + T_M T_I s^2 + K K_R T_I s + K K_R}$$

$$G_r(s) = \frac{1}{\frac{T_M T_\Sigma T_I}{K K_R} s^3 + \frac{T_M T_I}{K K_R} s^2 + T_I s + 1}$$

$\longrightarrow$ $a_0 = 1$ $\quad a_1 = T_I$ $\quad a_2 = \frac{T_M T_I}{K K_R}$ $\quad a_3 = \frac{T_M T_\Sigma T_I}{K K_R}$

$\rightarrow$ $a_1 = T_1$ $\qquad T_1 = T_I$

$a_2 = T_2 T_1$ $\quad \Rightarrow \quad$ $T_2 = \frac{T_M}{K K_R}$

$a_3 = T_3 T_2 T_1$ $\qquad T_3 = T_\Sigma$

$\rightarrow$ $D_i = 0.5$

$D_3 = \frac{T_3}{T_2}$ $\rightarrow$ $\boxed{K_R = \frac{1}{2K} \frac{T_M}{T_\Sigma}}$

$D_2 = \frac{T_2}{T_1}$ $\rightarrow$ $\boxed{T_I = 4 T_\Sigma}$

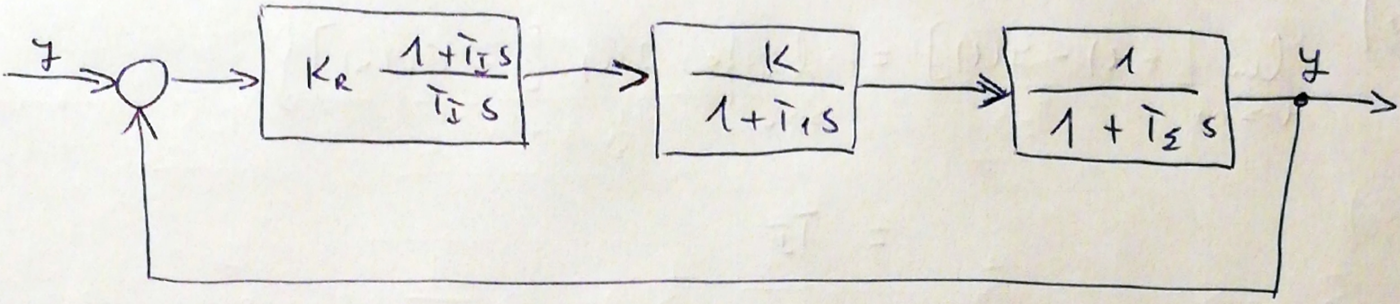
b)

$$\lim_{t \to \infty} \left[r(t) - y(t) \right] = \lim_{s \to 0} \left\{ s \cdot \underbrace{R(s)}_{\frac{1}{s^2}} \left[1 - G_r(s) \right] \right\}$$

$$= T_I$$

c) $$\lim_{t \to \infty} \left[r(t) - y(t) \right] = 0$$

4



a)

$$G_r(s) = \frac{T_I s + 1}{\frac{T_1 T_\Sigma T_I}{K K_R} s^3 + \frac{T_I (T_1 + T_\Sigma)}{K K_R} s^2 + \frac{T_I (1 + K K_R)}{K K_R} s + 1}$$

$\rightarrow \quad b_1 = T_I \qquad b_0 = 1$

$$a_3 = \frac{T_1 T_\Sigma T_I}{K K_R} \qquad a_2 = \frac{T_I (T_1 + T_\Sigma)}{K K_R} \qquad a_1 = \frac{T_I (1 + K K_R)}{K K_R} \qquad a_0 = 1$$

$\rightarrow \quad a_2^2 - 2 a_1 a_3 = \frac{a_1^2}{b_1^2} \left(b_2^2 - b_1 b_3 \right)$

$$K_R = \frac{1}{2K} \left(\frac{T_1}{T_\Sigma} + \frac{T_\Sigma}{T_1} \right)$$

$\rightarrow \quad a_1^2 - 2 a_0 a_2 = \frac{a_0^2}{b_0^2} \left(b_1^2 - b_0 b_2 \right)$

$$T_I = \frac{\frac{T_1}{T_\Sigma} + \frac{T_\Sigma}{T_1}}{1 + \frac{T_1}{T_\Sigma} + \frac{T_\Sigma}{T_1}} \left(T_1 + T_\Sigma \right)$$

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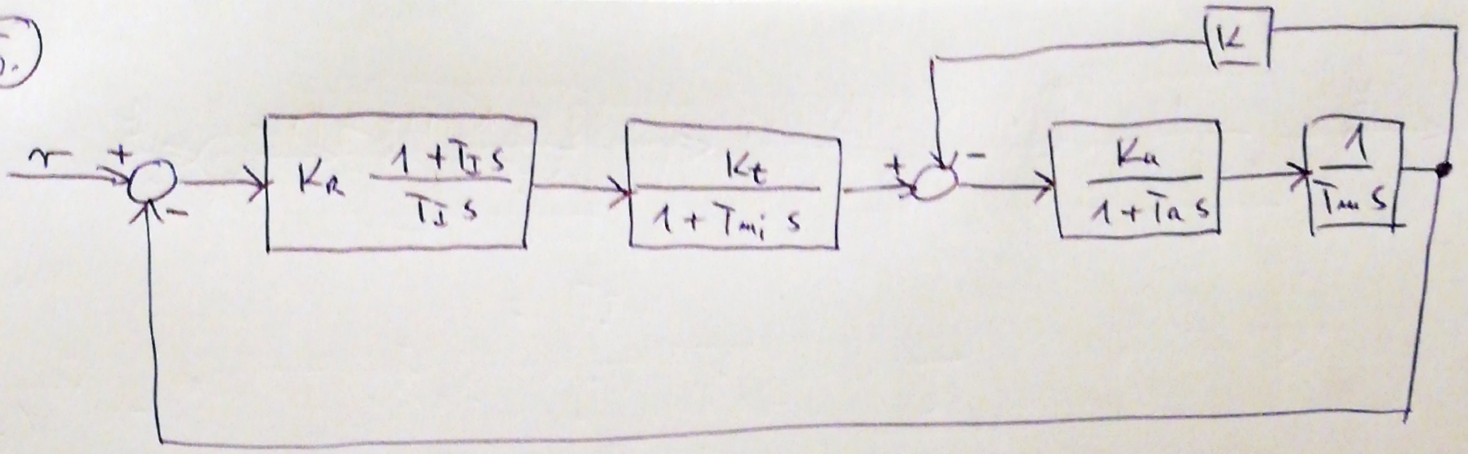
b) $K_R \approx \frac{1}{2K} \frac{T_1}{T_\Sigma}$

$T_I \approx T_1$

5.



$$G_r(s) = \frac{K_R K_t K_a (1 + T_I s)}{T_I T_{mi} T_m T_a s^4 + T_I T_m (T_{mi} + T_a) s^3 + T_I (T_{mi} K K_a + T_m) s^2 + T_I K_a (K + K_R K_t) s + K_R K_t K_a}$$

$\longrightarrow$ $b_0 = K_R K_t K_a \qquad b_1 = K_R K_t K_a T_I$

$a_0 = K_R K_t K_a \qquad a_1 = T_I K_a (K + K_R K_t) \qquad a_2 = T_I (T_{mi} K K_a + T_m)$

$a_3 = T_I T_m (T_{mi} + T_a) \qquad a_4 = T_I T_{mi} T_m T_a$

$\longrightarrow$ $a_2^2 - 2 a_1 a_3 = \frac{a_1^2}{b_1^2} \left(b_2^2 - 2 b_1 b_3 \right)$

$$\boxed{K_R = 0.6759}$$

$\longrightarrow$ $a_1^2 - 2 a_0 a_2 = \frac{a_0^2}{b_0^2} \left(b_1^2 - 2 b_0 b_2 \right)$

$$\boxed{T_I = 0.0435\ s}$$