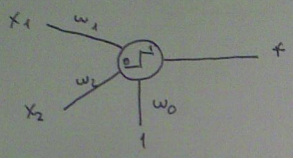
UMJETNA INTELIGENCIJA

Riješenja zadataka sa wikija.

by: Rale

Perceptron

– Odredite izlaz perceptrona



$x_0 = 1$ $\quad w_0 = -3,3$

$x_1 = -0,3$ $\quad w_1 = 3,9$

$x_2 = 2,3$ $\quad w_2 = 0,5$

$f = ?$

$$f = f(net) = f\left(\sum_{i=1}^{n} w_i \cdot x_i\right)$$

$$f(net) = w_0 \cdot x_0 + w_1 \cdot x_1 + w_2 \cdot x_2$$

$$f(net) = -3,3 \cdot (1) + 3,9 \cdot (-0,3) + 0,5 \cdot (2,3)$$

$$f(net) = -3,32$$

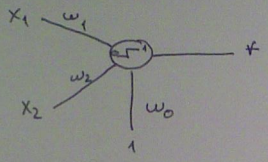
TLU - perceptron

$f(net) \leq 0 \Rightarrow f = 0$

$f(net) > 0 \Rightarrow f = 1$

$-3,32 \leq 0 \Rightarrow f = 0$

- Odredite potreban iznos težine kod perceptrona



$x_0 = 1$ $\quad w_0 = 0,6$ $\quad f = 1$

$x_1 = 1,1$ $\quad w_1 = ?$

$x_2 = -2,1$ $\quad w_2 = -4,5$

$w_1 = ?$

$$f(net) = 1$$

$$\Rightarrow net > 0$$

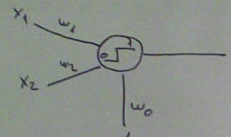
$$w_0 \cdot x_0 + w_1 \cdot x_1 + w_2 \cdot x_2 > 0$$

$$0,6 + 1,1 \cdot w_1 + 2,1 \cdot 4,5 > 0$$

$$1,1\, w_1 > -10,05$$

$$w_1 > -9,1363$$

- Postupak učenja perceptrona



k	x_0	x_1	x_2	t
0	1	3,8	0,4	0
1	1	0,1	-0,7	1
2	1	-0,2	3,9	1
3	1	4,6	-0,2	1

$w_0 = 2,4$
$w_1 = 3,7$
$w_2 = -1,6$

$\eta = 0,5$

$w_0 - w_2 = ?$ nako jedna epoha

$$w_i(k+1) = w_i(k) + \eta\, \varepsilon_i(k) \cdot x_i(k)$$

$$\varepsilon(k) = t_T - O_0$$

$$net = \sum w_i \cdot x_i$$

$net = w_0 \cdot x_0 + w_1 \cdot x_1 + w_2 \cdot x_2$

t - traženi izlaz
O - dobiveni izlaz

1-korak

$net_0 = 2,4 \cdot 1 + 3,7 \cdot 3,8 - 1,6 \cdot 0,4 = 15,82$

$net_0 > \emptyset \Rightarrow O_0 = 1$, $\varepsilon_0(0) = t_0 - O_0 = 0 - 1 = -1$

$w_0(1) = w_0(\emptyset) + \eta \cdot \varepsilon_0(\emptyset) \cdot x_0(\emptyset)$

$w_0(1) = 2,4 - 0,5 \cdot 1 = 1,9$

$w_1(1) = w_1(0) + \eta\, \varepsilon_0(0) \cdot x_1(0)$

$w_1(1) = 3,7 - 0,5 \cdot 3,8 = 1,8$

$w_2(1) = w_2(0) - \eta \cdot \varepsilon(0) \cdot x_2(0)$

$w_2(1) = -1,6 - 0,5 \cdot 0,4 = -1,8$

2-korak

$net_1 = 1,9 \cdot 1 + 1,8 \cdot 0,1 - 1,8 \cdot (-0,7) = 3,34$

$net_1 > 0 \Rightarrow o = 1 \qquad \varepsilon(1) = 1 - 1 = 0$

$t = 0 \Rightarrow$ težine funkcije ostaju nepromijenjene

$w_0 = 1,9$

$w_1 = 1,8$

$w_2 = -1,8$

3-korak

$net_2 = 1,9 \cdot 1 + 1,8 \cdot (-0,2) - 1,8 \cdot (3,9) = -5,48$

$net_2 \leq 0 \Rightarrow o = 0 \qquad \varepsilon(2) = (1 - 0)$

$w_0(3) = 1,9 \cdot 1 + 0,5 \cdot 1 = 2,4$

$w_1(3) = 1,8 + 0,5 \cdot (-0,2) = 1,7$

$w_2(3) = -1,8 + 0,5 \cdot 3,9 = 0,15$

4-korak

$net_3 = 2,4 + 1,7 \cdot 4,6 + 0,15 \cdot (-0,2) = 10,19$

$net_3 > 0 \Rightarrow o = 1 \qquad \varepsilon(3) = 1 - 1 = 0$

$t = 0 \Rightarrow$ tež tje ostaju nepromijenjene

$w_0(4) = 2,4$

$w_1(4) = 1,7$

$w_2(4) = 0,15$

- što može naučiti perceptron

X_1	X_2	Y
3	0,5	0
0	3,8	1
2,4	3,6	1
0	-1,79	0
0,6	4	1
2,4	0	0
3	3,5	1
0,6	-1,6	0

TLU - perceptron

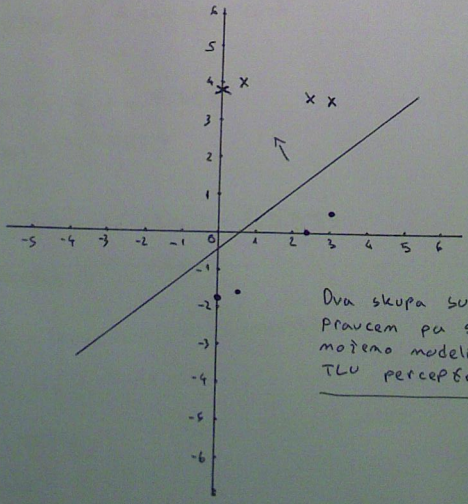
Može li se sustav modelirati TLU - perceptronom?

Rj: Grafičkom metodom provjeravamo linearnu razdvojivost skupova.

X_1, X_2 - koordinate

∅ — •

1 — X

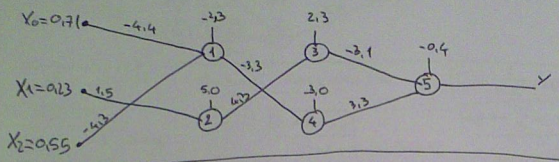


Dva skupa su razdvojiva pravcem pa sustav možemo modelirati TLU perceptronom.

Umjetna neuronska mreža

– Određivanje izlaza mreže

sigmoida
$a = 1$



$y = ?$ izlaz neurona

$$O = \frac{1}{1 + e^{-net}} \qquad net = \sum w_i \cdot x_i$$

1. neuron

$net_1 = 0,71 \cdot -4,4 - 2,3 \cdot 1 - 4,3 \cdot 0,55 = -7,7895$

$O_1 = \frac{1}{1+e^{-(-7,7895)}} = 4,140935 \cdot 10^{-4}$

2. neuron

$net_2 = 0,23 \cdot 1,5 + 5 \cdot 1 = 5,343$

$O_2 = \frac{1}{1+e^{-5,343}} = 0,995524$

3. neuron

$net_3 = 4,3 \cdot 0,995524 + 2,3 \cdot 1 = 6,57735$

$O_3 = \frac{1}{1+e^{-6,57735}} = 0,998861$

4. neuron

$net_4 = -3,3 \cdot 4,140935 \cdot 10^{-4} - 3 = -3,00136$

$O_4 = \frac{1}{1+e^{-3,00136}} = 0,047364$

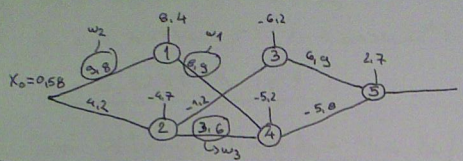
5. neuron

$net_5 = 3,3 \cdot 0,047364 - 3,1 \cdot 0,998861 - 0,4 = -3,33338$

$O_5 = \frac{1}{1+e^{3,33338}} = 0,0342$

$y = O_5 = 0,0342$

- Učenje mreže



$\eta = 0,81$

$y = 0,98268$

$w_1 - w_2 = 2$ nakon korekcije

$$O = \frac{1}{1+e^{-net}}$$

$$net = \sum w_i x_i$$

1. neuron

$net_1 = 14,084$

$O_1 = 0,999999992$

2. neuron

$net_2 = -2,264$

$O_2 = 0,094141486$

3. neuron

$net_3 = -6,312378832$

$O_3 = 1,80934 \cdot 10^{-3}$

4. neuron

$net_4 = 4,038527$

$O_4 = 0,982688$

5. neuron

$net_5 = -2,387105$

$O_5 = 0,048011183$

y - traženi izlaz

O_5 - stvarni izlaz

$y \neq O_5 \Rightarrow$ radimo korekciju

Greška u izlaznom sloju

$$\delta_k \leftarrow O_k (1 - O_k) \cdot (t_k - O_k)$$

$$\delta_5 \leftarrow O_5 (1 - O_5) \cdot (t_5 - O_5)$$

$$\delta_5 \leftarrow 0,048011183 (1 - 0,048011183) \cdot (0,98268 - 0,048011183)$$

$$\delta_5 = 0,042720599$$

Greška u skrivenom sloju

$$\delta_h \leftarrow O_h (1 - O_h) \cdot \sum w_{hs} \cdot \delta_s$$

$$\delta_4 \leftarrow O_4 (1 - O_4) \cdot w_{4,5} \cdot \delta_5$$

$$\delta_4 = 0,982688 (1 - 0,982688) \cdot (-5,8 \cdot 0,042720599)$$

$$\delta_4 = -4,215296 \cdot 10^{-3}$$

$\delta_3 \leftarrow o_3(1-o_3) \cdot w_{3,5} \cdot \delta_5$

$\delta_3 = 1,80934 \cdot 10^{-3} (1 - 1,80934 \cdot 10^{-3}) \cdot (6,9 \cdot 0,047720593)$

$\delta_3 = 5,1323779 \cdot 10^{-4}$

$\delta_2 \leftarrow o_2(1-o_2) \cdot (w_{2,3} \cdot \delta_3 + w_{2,4} \cdot \delta_4)$

$\delta_2 = 0,9382688 \cdot (1-0,9382688) \cdot (-1,2 \cdot 5,1323779 \cdot 10^{-4} + 3,6 \cdot (-4,1245296 \cdot 10^{-3}))$

$\delta_2 = -2,6951557 \cdot 10^{-4}$

$\delta_1 \leftarrow o_1(1-o_1) \cdot w_{1,4} \cdot \delta_4$

$\delta_1 = 0,9999992 \cdot (1-0,9999992) \cdot 8,9 \cdot (-4,1245296 \cdot 10^{-3})$

$\delta_1 = -3,0012868 \cdot 10^{-3}$

[redacted] Usadanje faktora

$w_{ij} \leftarrow w_{ij} + \eta \cdot \delta_j \cdot o_i$

$w_1 \rightarrow w_{14} + 0,81 \cdot \delta_4 \, o_1 = 8,8936585$

$w_2 \rightarrow w_{01} + 0,81 \cdot \delta_1 \cdot x_0 = 9,7990589$

$w_3 \rightarrow w_{24} + 0,81 \cdot \delta_4 \cdot o_2 = 3,5996785 4$

Genetski algoritam

- Rezolucija pretrage

2-varijable

$x_{min}, x_{max} \in [-185, 79]$

$\tilde{x} = 0,05$

n-broj bitova $= ?$

$$2^n \geq \frac{x_{max} - x_{min}}{\tilde{x}}$$

$$2^n \geq \frac{79 - (-185)}{0,05}$$

$$2^n \geq 5280 \quad | \log_2$$

$$n \geq \log_2 5280$$

$n \geq 12,36 \approx 13$-bitova za 1 varijablu

2-varijabla $\Rightarrow 2 \cdot n = 26$ bitova

- Operator križanja

  - S jednom točkom prijeloma

$k_1 = 00\ 10\ 00$

$k_2 = 10\ 01\ 00$

Točka prekida iza 4-bita

Mutacija 3 i 5 bita

$r_1, r_2 = ?$

$k_1 = 0010\ 00$

$k_2 = 1001\ 00$

$r_1 = 0010\ \underline{0}\ 0$

$r_2 = 1001\ \underline{0}\ 0$

Mutiramo 3 i 5 bit

$r_1 = 000010$

$r_2 = 101110$

- S dvije točke prijeloma

$k_1 = 00\ 11\ 00$

$k_2 = 11\ 01\ 10$

Točke prekida iza 3 i 5 bita

Mutacija 2 i 5 bita

r_1, r_2

$k_1 = 001\ 10\ 0$

$k_2 = 110\ 11\ 0$

$r_1 = 0\underline{0}1\ 1\underline{1}0$

$r_2 = 1\underline{1}0\ 1\underline{0}0$

Mutiramo 2 i 5 bit.

$r_1 = 011100$

$r_2 = 100110$

- Uniformni

$K_1 = 101010$

$K_2 = 110000$

$R = 100111 \quad \overline{R} = 011000$

Mutirano 3 i 6 bit

$r_1, r_2 = ?$

$$D_1 = R_1 \cdot R_2 + R \cdot (R_1 \oplus R_2)$$
$$D_2 = R_1 \cdot R_2 + \overline{R} \cdot (R_1 \oplus R_2)$$

$R_1 \cdot R_2$

```
  101010
  110000  AND
  100000
```

$R_1 \oplus R_2$

```
  101010
  110000  XOR
  011010
```

$R \cdot (R_1 \oplus R_2)$

```
  100111
  011010  AND
  000010
```

$\overline{R} (R_1 \oplus R_2)$

```
  011000
  011010  AND
  011000
```

$R_1 R_2 + R \cdot (R_1 \oplus R_2)$

```
  100000
  000010  OR      => r1
  100010
```

$R_1 \cdot R_2 + \overline{R} (R_1 \oplus R_2)$

```
  100000
  011000  OR      => r2
  111000
```

Mutirano 3 i 6 bit

$r_1 = 101011$

$r_2 = 110001$

– Jedan korak algoritma

$f(x,y,z) = (x-4)^2 + (y-3)^2 + (z-3)^2 + 4xy$ → traži se minimum

3-turnirska selekcija

kromosom | xx yy zz |

$x \in [0,3]$

$y \in [-1,2]$

$z \in [-1,2]$

$K_1 = 00\ 01\ 01$

$K_2 = 00\ 00\ 00$

$K_3 = 10\ 01\ 01$

funkcija dobrote $= -f$

$R = 10\ 00\ 10$

$\bar{D} = 01\ 11\ 01$

$*1, *2, *3, N, d_1, d_2, *d_1, *d_2, M = ?$

$$X = X_{min} + \frac{K}{2^n - 1}(X_{max} - X_{min})$$

	x	y	z
00	0	-1	-1
01	1	0	0
10	2	1	1
11	3	2	2

	x	y	z		x	y	z
$K_1 =$	00	01	01		0	0	0
$K_2 =$	00	00	00	=	0	-1	-1
$K_3 =$	10	01	01		2	0	0

$ff1 = -f(0,0,0) = (-4)^2 + (-3)^2 + (-3)^2 + 4 \cdot 0 = -34$

$ff2 = -f(0,-1,-1) = (-4)^2 + (-4)^2 + (-4)^2 + 4 \cdot 1 = -48$

$ff3 = -f(2,0,0) = (-2)^2 + (-3)^2 + (-3)^2 + 4 \cdot 2 = -22$

Najmanji fitnes ima jedinka 2 pa ju odbacujemo

$\boxed{N=2}$

Ulimamo kromosome

$k_1 = 00\ 01\ 01$

$k_3 = 10\ 01\ 01$

$$\boxed{\begin{aligned} D_1 &= R_1 \cdot R_2 + R \cdot (R_1 \oplus R_2) \\ D_2 &= R_1 \cdot R_2 + \overline{R} \cdot (R_1 \oplus R_2) \end{aligned}}$$

$R_1 \cdot R_2$

$00\ 01\ 01$

$10\ 01\ 01$ AND

$00\ 01\ 01$

$R_1 \oplus R_2$

$00\ 01\ 01$

$10\ 01\ 01$ XOR

$10\ 00\ 00$

$R \cdot (R_1 \oplus R_2)$

$10\ 00\ 10$

$10\ 00\ 00$ AND

$10\ 00\ 00$

$\overline{R} \cdot (R_1 \oplus R_2)$

$01\ 11\ 01$

$10\ 00\ 00$ AND

$00\ 00\ 00$

$R_1 R_2 + R \cdot (R_1 \oplus R_2)$

$00\ 01\ 01$

$10\ 00\ 00$ OR

$10\ 01\ 01$ $\Rightarrow d_1$

$R_1 \cdot R_2 + \overline{R}\,(R_1 \oplus R_2)$

$00\ 01\ 01$

$00\ 00\ 00$ OR

$00\ 01\ 01$ $\Rightarrow d_2$

	x	y	z			
$d_1 = 10$	0 1	0 1	=	2	0	0
$d_2 = 00$	0 1	0 1		0	0	0

$ffd_1 = -f(2,0,0) = -22$

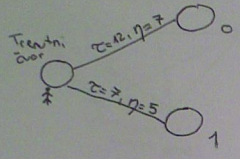
$ffd_2 = -f(0,0,0) = -34$

veći fitnes ima ffd_1 pa on ulazi u populaciju.

$\boxed{M=1}$

Mravlji algoritmi

– Vjerojatnost prelaska u sljedeći čvor



$\alpha = 1$

$\beta = 1$

$p_0, p_1 = ?$

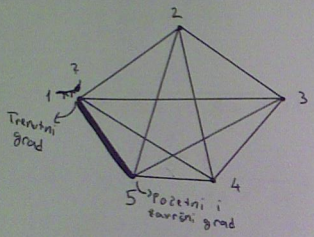
$$p_{ij}^k = \begin{cases} \dfrac{\tau_{ij}^{\alpha} \cdot \eta_{ij}^{\beta}}{\sum_{l \in N_i^k} (\tau_{il}^{\alpha} \cdot \eta_{il}^{\beta})}, & \text{ako je } j \in N_i^k \\ 0, & \text{ako } j \notin N_i^k \end{cases}$$

Naš $N_{Tč}^k = \{0, 1\}$

$$p_{Tč,0}^k = \frac{12 \cdot 7}{12 \cdot 7 + 7 \cdot 5} = 0{,}70588$$

$$p_{Tč,1}^k = \frac{7 \cdot 5}{12 \cdot 7 + 7 \cdot 5} = 0{,}29411$$

- Konstrukcija puta kod TSP-a



$\alpha = 2$

$\beta = 1$

4 - prijelaza

τ	1	2	3	4	5
1	0	9	13	7	15
2	9	0	14	16	1
3	13	14	0	10	3
4	7	16	10	0	16
5	15	1	3	16	0

	1	2	3	4	5
1	0	19	8	11	12
2	19	0	8	10	4
3	8	8	0	1	5
4	11	10	1	0	13
5	12	4	5	13	0

$$p_{ij}^{k} = \begin{cases} \dfrac{\tau_{ij}^{\alpha} \cdot \eta_{ij}^{\beta}}{\sum_{l \in N_i^k} (\tau_{il}^{\alpha} \cdot \eta_{il}^{\beta})} & , j \in N_i^k \\ 0 & , j \notin N_i^k \end{cases}$$

N_i^k - u koje sve čvorove možemo otići iz trenutnog čvora

1-korak

$N_1^k = \{2, 3, 4\}$

$$P_{12} = \frac{9^2 \cdot 19}{9^2 \cdot 19 + 13^2 \cdot 8 + 7^2 \cdot 11} = 0,4486$$

$$P_{13} = \frac{13^2 \cdot 8}{9^2 \cdot 19 + 13^2 \cdot 8 + 7^2 \cdot 11} = 0,3941$$

$$P_{14} = \frac{7^2 \cdot 11}{9^2 \cdot 19 + 13^2 \cdot 8 + 7^2 \cdot 11} = 0,1571$$

Prelazimo u čvor (2)

2-korak

$N_2^k = \{3, 4\}$

$$P_{23} = \frac{14^2 \cdot 8}{14^2 \cdot 8 + 16^2 \cdot 10} = 0,37984$$

$$P_{24} = \frac{16^2 \cdot 10}{14^2 \cdot 8 + 16^2 \cdot 10} = 0,6201$$

Prelazimo u čvor (4)

3-korak

$N_4^k = \{3\}$

$$P_{43} = \frac{10^2 \cdot 1}{10^2 \cdot 1} = 1$$

Prelazimo u čvor (3)

4-korak

$N_4^k = \{5\}$ -> povratak u početni čvor (5)

Put = 2 -> 4 -> 3 -> 5