① a) Familija mora sadržavati Ω i biti zatvorena na operacije unije i komplementiranje.

b) $A=\{2,4,6\}$, $B=\{4,5,6\}$. $\mathcal{F}$ sadrži i:

$A^c=\{1,3,5\}$, $B^c=\{1,2,3\}$, $A\cap B=\{4,6\}$,

$A^c\cap B^c=\{1,3\}$, $A^c\cap B=\{5\}$, $A\cap B^c=\{2\}$.

Skupovi $\{1,3\}$, $\{2\}$, $\{4,6\}$ i $\{5\}$ čine particiju od Ω, prema tome najmanja $\mathcal{F}$ je ona generirana tim skupovima i ima $2^4=16$ elemenata.

c) Elementarni događaji su svi nizovi brojeva s točno jednom jedinicom na zadnjem mjestu.

$C_1=\{$pojavio se paran broj$\}$

$C_1^c=\{$nije se pojavio paran broj$\}$

$C_1^c=\{1, n1, nn1, nnn1, \dots\}$, n-neparan.

$P(\{n\dots n1\})=\left(\frac{1}{3}\right)^k\cdot\frac{1}{6}$

$P(C_1^c)=\frac{1}{6}+\frac{1}{3}\cdot\frac{1}{6}+\left(\frac{1}{3}\right)^2\cdot\frac{1}{6}+\dots=\frac{1}{6}\cdot\frac{1}{1-\frac{1}{3}}=\frac{1}{4}$

$P(C_1)=\frac{3}{4}$. $P(C_2)=1$ jer svaki poteg ima 1.

$C_3^c=\{1, s1, ss1, \dots\}$, $s\in\{2,4,5,6\}$

$P(C_3^c)=\frac{1}{6}\cdot\frac{1}{1-\frac{2}{3}}=\frac{1}{2}$ $\quad P(C_3)=\frac{1}{2}$.

② a) $A=\{$meta je pogođena$\}$

$A^c=\{$meta nije pogođena$\}$

$P(A^c)=(1-.4)(1-.6)(1-.7)(1-.8)$

$P(A)=0.9856$

b) $B_i=\{$promašio je samo i-ti strijelac$\}$

$C=\{$meta je pogođena triput$\}$

Očito je $C=\bigcup B_i$ i B_i disjunktni pa je $P(C)=\sum P(B_i)$

$P(B_4|C)=\frac{P(B_4\cap C)}{P(C)}=\frac{P(B_4)}{P(C)}=$

$=\frac{0.4\cdot0.6\cdot0.7\cdot0.2}{0.2016+0.0896+0.0576+0.0336}=.0879$

③ a) Neka je $Y=\frac{X-a}{\sigma}$, za gustoću od Y, uz $x=a+\sigma y$ vrijedi

$g(y)=\phi(x)\cdot\frac{dx}{dy}=\phi(a+\sigma y)\cdot\sigma=$

$=\frac{1}{\sqrt{2\pi}\sigma}\cdot e^{-\frac{1}{2}\left(\frac{a+\sigma y-a}{\sigma}\right)^2}\cdot\sigma=$

$=\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}y^2}$, tj. $Y\sim\mathcal{N}(0,1)$.

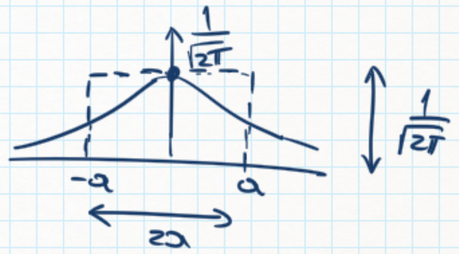
b) $P(-3<X<9)=.9973$

Pravilo $3\sigma\Rightarrow 3\sigma=9-3\Rightarrow\sigma=2$.

$P(X>0)=P\left(\frac{X-3}{2}>-\frac{3}{2}\right)=\frac{1}{2}+\frac{1}{2}\Phi\left(\frac{3}{2}\right)=.9332$

c) $P(|x|<a)=P(-a<X<a)=$

$=\frac{1}{\sqrt{2\pi}}\int_{-a}^{a}e^{-\frac{1}{2}x^2}dx<\frac{1}{\sqrt{2\pi}}\cdot 2a=\sqrt{\frac{2}{\pi}}\cdot a<a$



④

a) $F_Y(y) = P(Y < y) = 1 - P(\min\{X_1, \dots, X_n\} > y)$

$= 1 - P(X_1 > y, \dots, X_n > y) = (\text{nez}) = 1 - \prod_{i=1}^{n} P(X_i > y)$

$= 1 - \prod_{i=1}^{n} e^{-\lambda_i y} = 1 - e^{-(\sum \lambda_i)\cdot y}$. $E(Y) = \frac{1}{\sum \lambda_i}$.

b) Gustoca s.v. (X, Y) je

$f(x,y) = \lambda_1 \lambda_2 e^{-\lambda_1 x - \lambda_2 y}$

Gustoca za $Z = X + Y$, uz $y = z - x$ i $0 < x < z$

$g_Z(z) = \int_{-\infty}^{+\infty} f(x, z-x)\,dx = \int_0^z \lambda_1\lambda_2 e^{-\lambda_1 x - \lambda_2 z + \lambda_2 x}\,dx$

$= \lambda_1\lambda_2 e^{-\lambda_2 z} \cdot \left(\frac{1}{\lambda_2 - \lambda_1} e^{-(\lambda_1 - \lambda_2)x} \right)\Big|_0^z = \frac{\lambda_1\lambda_2}{\lambda_2 - \lambda_1}\left(e^{-\lambda_1 z} - e^{-\lambda_2 z}\right)$.

⑤ X_i – vrijeme ispravljanja i-tog ispita

$\mu = E(X_i) = 6$

$\sigma_{X_i} = 6$

a) $P(\sum X_i < 250) =$

$P\left(\frac{\sum X_i - n\mu}{\sigma\sqrt{n}} < \frac{250 - 240}{6\sqrt{40}} \right) =$

$(CGT) \doteq \frac{1}{2} + \frac{1}{2}\Phi^*\left(\frac{\sqrt{10}}{12}\right) = .6039$

b) $P(\sum X_i > 260) = .2991$

⑥ $E(\hat{\Theta}_1) = \theta$

a) $E(W) = 0$

$E(\hat{\Theta}_2) = E(\hat{\Theta}_1 + W) = E(\hat{\Theta}_1) + E(W) = \theta$

b) $E(\hat{\Theta}_1) = a\theta + b$, $a \neq 0$, $b \in \mathbb{R}$

$E(\hat{\Theta}_2) = E\left(\frac{\hat{\Theta}_1 - b}{a}\right) = \frac{1}{a}\left(E(\hat{\Theta}_1) - b\right)$

$= \frac{1}{a}(a\theta + b - b) = \theta$.