Zadatak

H_1 ... {osoba ima bolest}

H_0 ... {osoba nema bolest}

A ... {test na bolest je pozitivan}

$$\mathbb{P}(A|H_1) = 0{,}99 \qquad \mathbb{P}(A^c|H_1) = \frac{\mathbb{P}(A^c \cap H_1)}{\mathbb{P}(H_1)} = \frac{\mathbb{P}(H_1) - \mathbb{P}(A \cap H_1)}{\mathbb{P}(H_1)}$$

$$\mathbb{P}(A|H_0) = 0{,}05$$

$$= 1 - \mathbb{P}(A|H_1)$$

$$\mathbb{P}(H_1) = 0{,}02$$

$$= 0{,}01$$

$$\mathbb{P}(H_0) = 0{,}98. \qquad \mathbb{P}(A^c|H_0) = 0{,}95$$

$$\mathbb{P}(H_1|A) = \frac{\mathbb{P}(A|H_1)\cdot\mathbb{P}(H_1)}{\mathbb{P}(H_1)\mathbb{P}(A|H_1) + \mathbb{P}(H_0)\mathbb{P}(A|H_0)} = \frac{0{,}99\cdot 0{,}02}{0{,}99\cdot 0{,}02 + 0{,}98\cdot 0{,}05} = 0{,}2877$$

$$\mathbb{P}(H_0|A^c) = \frac{\mathbb{P}(A^c|H_0)\mathbb{P}(H_0)}{\mathbb{P}(A^c|H_1)\mathbb{P}(H_1) + \mathbb{P}(A^c|H_0)\mathbb{P}(H_0)} = \frac{0{,}95\cdot 0{,}98}{0{,}01\cdot 0{,}02 + 0{,}95\cdot 0{,}98}$$

$$= 0{,}99978$$

Zadatak

12 kuglica.

$$\begin{cases} 5 \text{ s brojem } 1 \\ 4 \text{ s brojem } 2 \\ 3 \text{ s brojem } 3 \end{cases}$$

X - zbroj brojeva na dvije kuglice

$$X \sim \begin{pmatrix} 2 & 3 & 4 & 5 & 6 \\ P_2 & P_3 & P_4 & P_5 & P_6 \end{pmatrix}$$

P_2 ... (dvije jedinice) $\dfrac{\binom{5}{2}}{\binom{12}{2}} = 0,1515$

P_3 ... (dvojka i jedinica) $\dfrac{\binom{5}{1}\binom{4}{1}}{\binom{12}{2}} = 0,303$

P_4 .. (2+2 ili 1+3) $\dfrac{\binom{3}{1}\binom{5}{1} + \binom{4}{2}}{\binom{12}{2}} = 0,318$

P_5 .. (3+2) $\dfrac{\binom{3}{1}\binom{4}{1}}{\binom{12}{2}} = 0,1818$

P_6 .. (3+3) $\dfrac{\binom{3}{2}}{\binom{12}{2}} = 0,0454$

$\mathbb{E}(X) = 3,66666$

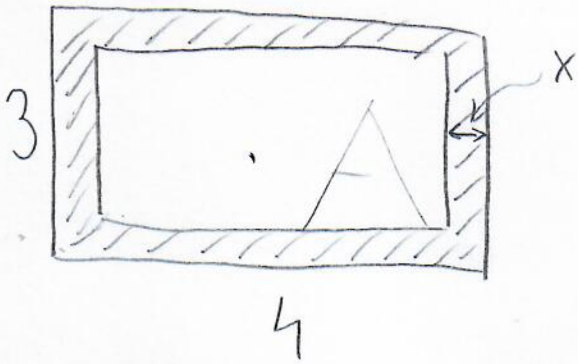
$D(X) = \mathbb{E}(X^2) - \mathbb{E}(X)^2 = 14,606 - 13,4444 = 1,16155$

Zadatak

X - udaljenost točke do najbliže stranice pravokutnika.

$$\mathbb{P}(X \leq x) = \frac{m(A_x)}{m(A)} = \frac{12-(3-2x)(4-2x)}{12}$$

$$= \frac{\cancel{12}-\cancel{12}+8x+6x-4x^2}{12}$$



$$= \frac{7x-2x^2}{6}$$

za $x \in [0, \frac{3}{2}]$, inače je 0.

$$F_X(x) = \begin{cases} \frac{7x-2x^2}{6} & x \in [0, \frac{3}{2}] \\ 0 & \text{inače} \end{cases}$$

$$f_X(x) = F_X'(x) = \begin{cases} \frac{7}{6} - \frac{2}{3}x, & x \in [0, \frac{3}{2}] \\ 0 & \text{inače} \end{cases}$$

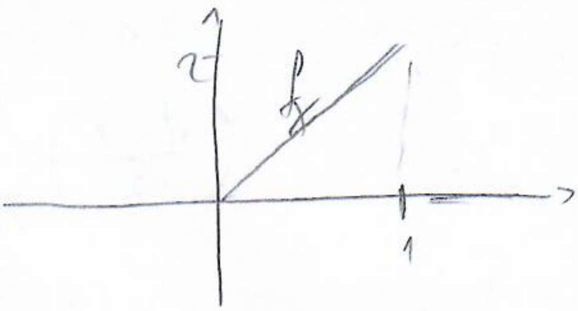
$$EX = \int_0^{\frac{3}{2}} x f_X(x) dx = \int_0^{\frac{3}{2}} \left(\frac{7}{6}x - \frac{2}{3}x^2\right) dx = \frac{7}{12}x^2 - \frac{2}{9}x^3 \Big/_0^{\frac{3}{2}}$$

$$= 0{,}5625$$

Dodatak

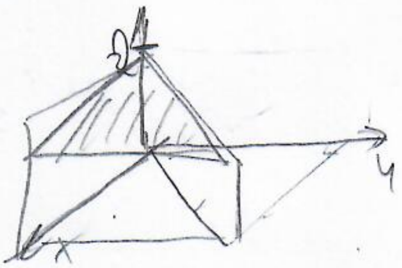
a)
$$f(y|x) = f_{Y|X=x}(y) = \frac{f(x,y)}{f_X(x)} = \frac{f(x,y)}{\int_{-\infty}^{+\infty} f(x,y)\,dx}$$

$X \ldots \; f_X(x) = 2x \;,\; 0 \leq x \leq 1$



b)
$$f_{Y|X=x}(y) = \frac{1}{x} \;,\; y \in [0,x]$$

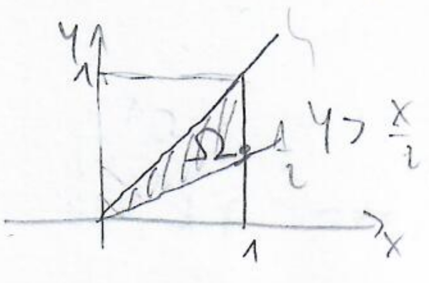
c)
$$f_{X,Y}(x,y) = f_{Y|X=x}(y) \cdot f_X(x) = \begin{cases} 2 \;, & 0 \leq y \leq x \leq 1 \\ 0 \;, & \text{inače} \end{cases}$$



$$\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} f_{XY}(x,y) = \frac{1 \cdot 1}{2} \cdot 2 = 1$$

d)
$$f_Y(y) = \int_{-\infty}^{+\infty} f_{X,Y}(x,y)\,dx = \int_y^1 2\,dx = \begin{cases} 2(1-y) \;, & y \in [0,1] \\ 0 \;, & \text{inače} \end{cases}$$

e)
$$P(X < 2Y) = \iint_{\Delta} 2\,dx\,dy = 2\left(\frac{1}{2} - \frac{1}{2}\cdot\frac{1}{2}\right)$$
$$= 2 \cdot \frac{1}{4} = \frac{1}{2}$$



<u>Zadatak</u>

a) F_X razdioba od X. slučaj $a \geq 0$.

$$F_X(x) = \begin{cases} F_X\left(\frac{x-b}{a}\right), & a \geq 0 \\ 1 - F_X\left(\frac{x-b}{a}\right), & a < 0 \\ \begin{cases} 0, & x < b \\ 1, & x \geq b \end{cases} \end{cases}$$

$$\mathbb{P}(aX + b \leq x) = \mathbb{P}(aX \leq x - b) = \mathbb{P}\left(X \leq \frac{x-b}{a}\right)$$
$$= F_X\left(\frac{x-b}{a}\right)$$

b)
$$\Phi^*(u) = \frac{1}{\sqrt{2\pi}} \int_{-u}^{u} e^{-\frac{1}{2}\tau^2} d\tau, \quad \Phi(u) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{u} e^{-\frac{1}{2}\tau^2} d\tau$$

$$\Phi(0) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{0} e^{-\frac{1}{2}\tau^2} d\tau = \frac{1}{2} \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} e^{-\frac{1}{2}\tau^2} d\tau = \frac{1}{2} \quad \left(e^{-\frac{1}{2}\tau^2} \text{ je parna}\right.$$

$$\Phi(u) = \Phi(0) + \frac{1}{\sqrt{2\pi}} \int_{0}^{u} e^{-\frac{1}{2}\tau^2} d\tau$$

$$\Phi^*(u) = 2 \frac{1}{\sqrt{2\pi}} \int_{0}^{u} e^{-\frac{1}{2}\tau^2} d\tau = 2 \cdot \left(\Phi(u) - \Phi(0)\right) = 2\left(\Phi(u) - \frac{1}{2}\right)$$
$$= 2\Phi(u) - 1$$
$$\Phi(u) = \frac{1}{2}\left(1 + \Phi^*(u)\right)$$

c)
$\mu = 40$ kg $\quad n = 60 \quad np = 2400$

$\sigma = 5$ kg

$$\mathbb{P}\left(\sum_{i=1}^{n} X_i \leq 2500\right) = \mathbb{P}\left(\frac{\sum X_i - 2400}{5\sqrt{60}} \leq \frac{2500 - 2400}{5\sqrt{60}}\right)$$
$$= \frac{1}{2}\left(1 + \Phi^*(2{,}582)\right) = 0{,}99506$$

Zadatak

$X_1, \dots, X_n \sim Exp(\lambda)$, $\lambda > 0$, μ očekivanje X_i.

$M_n = \min\{X_1, \dots, X_n\}$

$T = c M_n$

$\mu = \frac{1}{\lambda}$

T nepristrani procjenitelj: $\mathbb{E}(T) = \frac{1}{\lambda}$

$f_X(x) = \lambda e^{-\lambda x}, \ x > 0$

$F_X(x) = 1 - e^{-\lambda x}, \ x > 0$

$$\mathbb{P}(M_n < x) = 1 - \mathbb{P}(M_n \geq x) = 1 - \mathbb{P}(X_1 \geq x, \dots, X_n \geq x)$$

$$= 1 - \mathbb{P}(X_1 \geq x) \cdots \mathbb{P}(X_n \geq x) = 1 - e^{-\lambda x} \cdots e^{-\lambda x}$$

$$= 1 - e^{-\lambda n x}$$

$\Rightarrow M_n \sim Exp(\lambda n)$; $\mathbb{E}(M_n) = \frac{1}{\lambda n}$

$$\Rightarrow \frac{1}{\lambda} = \mathbb{E}(T) = \mathbb{E}(c M_n) = c \frac{1}{\lambda n} \Rightarrow \boxed{c = n}$$

b) standardno

$$\hat{\mu} = \frac{1}{n} \sum x_i = \bar{x}$$