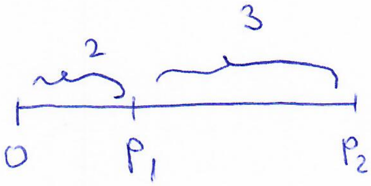
Zadatak 1



a) Točke A i B nalaze se u intervalu $[0,2]$, $\Omega = [0,2]^2$,

$m(\Omega) = 4$

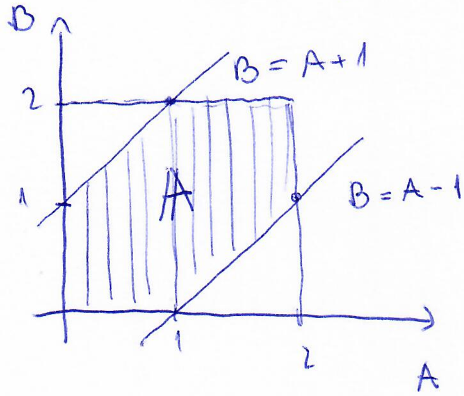
Tražimo $P(|A-B|<1)$

Riješimo nejednadžbu $|A-B|<1$:

$$-1 < A-B < 1$$

$-1 < A-B$, tj. $B < A+1$

$A-B<1$, tj. $B > A-1$



$$P(|A-B|<1) = P(\mathcal{A}) = 1 - P(\mathcal{A}^c)$$

$$= 1 - \frac{2 \cdot \frac{1 \cdot 1}{2}}{4} = 1 - \frac{1}{4} = \frac{3}{4}$$

b) A je u intervalu $[0,2]$, a C u $[2,5]$, $\Omega = [0,2] \times [2,5]$,

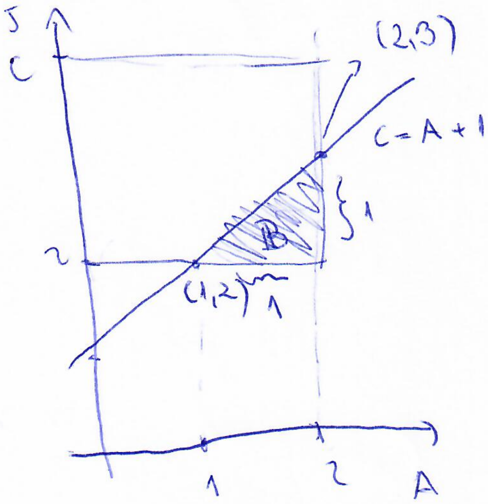
$m(\Omega) = 2 \cdot 3 = 6$

Kako znamo da je $C \geq A$, tada je $\overline{AC} = C - A$.

Tražimo $P(\cancel{C} C - A < 1)$

$C - A < 1$, tj. $C < A+1$

$$P(C-A<1) = P(\mathcal{B}) = \frac{\frac{1 \cdot 1}{2}}{6} = \frac{1}{12}$$



Zadatak 2

a) Pretpostavimo da skup elementarnih događaja možemo rastaviti na n međusobno disjunktnih događaja:

$$\Omega = H_1 \cup H_2 \cup \dots \cup H_n,$$

pri čemu su događaji H_i i H_j disjunktni za $i \neq j$ i vrijedi $P(H_i) > 0$ za svaki i. Tada $H_1, H_2, \dots, H_n$ čini potpun sustav događaja

Bayesova formula:

$$P(H_i|A) = \frac{P(H_i)P(A|H_i)}{\sum_{j=1}^{n} P(H_j) \cdot P(A|H_j)}$$

b)

$A = \{$ izvučene obje crvene iz prvog snopa $\}$

$B = \{$ izvučene 2 crvene i 2 plave karte $\}$

Tražimo $P(A|B)$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{\dfrac{\overbrace{\binom{6}{2}}^{\text{biramo 2 crvene iz 1. snopa}} \cdot \overbrace{\binom{6}{2}}^{\text{biramo 2 plave iz 2. snopa}}}{\binom{10}{2} \cdot \binom{10}{2}}}{\underbrace{\dfrac{\binom{6}{2} \cdot \binom{6}{2}}{\binom{10}{2} \cdot \binom{10}{2}}}_{\text{obje crvene su iz 1. snopa}} + \underbrace{\dfrac{6 \cdot 4 \cdot 4 \cdot 6}{\binom{10}{2} \cdot \binom{10}{2}}}_{\text{1 crvena je u 1. snopu, a jedna u 2.}} + \underbrace{\dfrac{\binom{4}{2} \cdot \binom{4}{2}}{\binom{10}{2} \cdot \binom{10}{2}}}_{\text{obje crvene su u drugom snopu}}}$$

$$= \frac{15^2}{15^2 + 24^2 + 6^2} = \frac{225}{837} \approx 0.2688$$

Zadatak 3

a) $D(X+y) = E[(x+y)^2] - [E(x+y)]^2$

$= E(x^2+2xy+y^2) - (E(X)+E(y))^2$

$= E(x^2) + 2E(xy) + E(y^2) - (E(x))^2 - 2E(x)\cdot E(y) - (E(y))^2$

$= \left(E(x^2) - (E(x))^2 + E(y^2) - (E(y))^2 + 2\cdot(E(xy) - E(x)\cdot E(y))\right)$

$= D(x) + D(y) + 2\operatorname{cov}(X,y)$

b) $D(X=1)$

$D(Y)=4$

$D(X+y) = D(X) + D(y) + 2r(X,y)\cdot\sqrt{D(X)\cdot D(y)}$

$= 1+4+4r(x,y)$

$= 5 + 4\underbrace{r(x,y)}_{\leq 1} \leq 9$

$D(X+y) = 9$, za $r(X,y)=1$, tj. za $y = aX+b$, $a>0$, $b\in\mathbb{R}$

$(a=2)$

Zadatak 4

Vjerojatnost da na tri kocke padnu tri uzastopna broja je

$p = \frac{6 \cdot 4}{6^3} = \frac{1}{9}$, jer na 4 načina možemo odabrati najmanji broj na kockama i za svaki taj odabir kocke možemo poredati na 6 načina

a) $X_1 \sim B(10, \frac{1}{9})$

$$P(X_1 = k) = \binom{10}{k} \left(\frac{1}{9}\right)^k \left(\frac{8}{9}\right)^{10-k}, \quad k = 0, 1, \dots, 10$$

$$P(X_1 \geq 2) = 1 - P(X_1 < 2)$$

$$= 1 - P(X_1 = 0) - P(X_1 = 1)$$

$$= 1 - \binom{10}{0} \left(\frac{1}{9}\right)^0 \left(\frac{8}{9}\right)^{10} - \binom{10}{1} \left(\frac{1}{9}\right)^1 \left(\frac{8}{9}\right)^9$$

$$\approx 0,30712$$

b) $X_2 \sim G(\frac{1}{9})$

$$P(X_2 = k) = \left(\frac{8}{9}\right)^{k-1} \cdot \frac{1}{9}, \quad k = 1, 2, 3, \dots$$

$$P(X_2 \geq 10) = \sum_{k=10}^{+\infty} P(X_2 = k)$$

$$= \sum_{k=10}^{+\infty} \left(\frac{8}{9}\right)^{k-1} \cdot \frac{1}{9}$$

$$= \frac{1}{9} \cdot \sum_{k=9}^{+\infty} \left(\frac{8}{9}\right)^k$$

$$= \frac{1}{9} \cdot \left(\frac{8}{9}\right)^9 \cdot \sum_{k=0}^{+\infty} \left(\frac{8}{9}\right)^k$$

$$= \frac{1}{9} \cdot \left(\frac{8}{9}\right)^9 \cdot \frac{1}{1 - \frac{8}{9}} = \left(\frac{8}{9}\right)^9$$

Zadatak 5

$$f(x) = \frac{C}{x^3}, \quad x \geq 1$$

a) $$1 = \int_1^{+\infty} \frac{C}{x^3}\,dx = -\frac{C}{2}\cdot\frac{1}{x^2}\Big|_1^{+\infty} = \frac{C}{2} \Rightarrow \boxed{C=2}$$

$$E(X) = \int_1^{+\infty} x f(x)\,dx = \int_1^{+\infty} \frac{2}{x^2}\,dx = -\frac{2}{x}\Big|_1^{+\infty} = 2$$

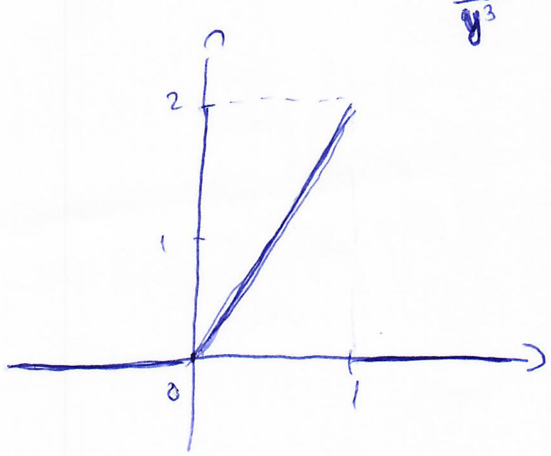
$$P(X > E(X)) = P(X > 2) = \int_2^{+\infty} \frac{2}{x^3}\,dx = -\frac{1}{x^2}\Big|_2^{+\infty} = \frac{1}{4}$$

b) $Y = \frac{1}{X}$, $\quad X \in [1, +\infty\rangle \Rightarrow Y \in [0,1]$

$X = \frac{1}{y} = \psi^{-1}(y)$

Tada je $g_Y(y) = f(\psi^{-1}(y)) \cdot \left|\frac{d\psi^{-1}(y)}{dy}\right|$

$$= \frac{2}{\frac{1}{y^3}} \cdot \left|\frac{-1}{y^2}\right| = 2y$$



2. način: Odrediti funkciju razdiobe varijable Y iz definicije:

$$F_Y(z) = P(Y < z) = P\left(\frac{1}{X} < z\right) = P\left(\frac{1}{z} < X\right) = 1 - P\left(X \leq \frac{1}{z}\right)$$

$$= 1 - \int_1^{\frac{1}{z}} \frac{2}{x^3}\,dx = 1 + \frac{1}{x^2}\Big|_1^{\frac{1}{z}} = 1 + z^2 - 1 = z^2$$

pa je $f_Y(z) = F_Y'(z) = 2z$