① a) Događaji A i B su NEZAVISNI ako vrijedi bilo koja od jednakosti $P(A|B)=P(A)$ ili $P(B|A)=P(B)$

b) $\boxed{\Rightarrow}$ $P(A\cap B)=P(B)\cdot P(A|B)=P(B)\cdot P(A)$

$\boxed{\Leftarrow}$ $P(A|B)=\frac{P(A\cap B)}{P(B)}=\frac{P(A)\cdot P(B)}{P(B)}=P(A)$

c) $P(B|A)=\frac{P(A\cap B)}{P(A)}=\frac{P(A)\cdot P(B)}{P(A)}=P(B)$

d) $\frac{P(B\cap A)}{P(A)}=\frac{P(B\cap A^c)}{P(A^c)} \iff \frac{P(B\cap A)}{P(A)}=\frac{P(B)-P(B\cap A)}{1-P(A)}$

$\iff P(B\cap A)=P(A)\cdot P(B)$

② a) 0, b) $(1/3)^8$, c) $(1/3)^8+8\cdot 2/3\cdot(1/3)^7+\binom{8}{2}(2/3)^2(1/3)^6$,

d) $\binom{8}{4}(2/3)^4(1/3)^4+\binom{8}{3}(2/3)^3(1/3)^5$.

③ a) KNJIŽICE

b) $P(Y=n)=\sum_{k=1}^{n-1}P(X_1=k,X_2=n-k)=(\text{nezavisnost})$

$=\sum_{k=1}^{n-1}P(X_1=k)\cdot P(X_2=n-k)=\sum_{k=1}^{n-1}q^{k-1}p\cdot q^{n-k-1}p=$

$=\sum_{k=1}^{n-1}q^{n-2}p^2=(n-1)q^{n-2}p^2$, $n\in\mathbb{N}$

(4) a) $\int_0^2 (ax^2+bx)\,dx = 1$ $\iff$ $\frac{8}{3}a + 2b = 1$ $\iff$ $a = 3/16$

$\int_0^2 x(ax^2+bx)\,dx = 1\frac{25}{60}$ $\qquad$ $4a + \frac{8}{3}b = 1\frac{25}{60}$ $\qquad$ $b = 1/4$

b) $P(X<1) = \int_0^1 f(x)\,dx = a/3 + b/2 = 3/16$

c) $P(X < 1\frac{1}{4} \mid X > 1) = \dfrac{P(1 < X < 1\frac{1}{4})}{P(X>1)} = \dfrac{133}{832}$

(5) a) $f_X(x) = \int_0^1 2x\,dy = 2x$ $\qquad$ $1 = \int_0^1\int_0^1 cx\,dx\,dy \Rightarrow c = 2$

$EX = \int_0^1 x f_X(x)\,dx = \frac{2}{3}$

$f_Y(y) = \int_0^1 2x\,dx = 1$

$EY = \int_0^1 y f_Y(y)\,dy = \frac{1}{2}$

uniformna $\Rightarrow Var Y = \frac{1}{12}$

X i Y nezavisne $\Rightarrow$

$Var Z = Var X + Var Y = \frac{5}{36}$

$EZ = EX - EY = \frac{1}{6}$

$Var X = \int_0^1 x^2 f_X(x)\,dx - (EX)^2 = \frac{1}{18}$

b) $1 = \int_0^1\int_0^x cx\,dx\,dy \Rightarrow c = 3$

$g_Z(z) = \int_{-\infty}^{+\infty} f(x,y)\left|\frac{\partial y}{\partial z}\right| dx = \int_z^1 3x\,dx = \frac{3}{2}(1-z^2)\,,\ z\in[0,1]$

$EZ = \int_0^1 z g(z)\,dz = 3/8$ $\qquad$ $EZ^2 = \int_0^1 z^2 g(z)\,dz = 1/5$

$Var Z = 1/5 - (3/8)^2 = 19/320$

⑥ a)

$$L(x_1,\ldots,x_n,\lambda) = \prod_i f(x_i,\lambda) = \prod_i 2\lambda^2 x_i e^{-\lambda^2 x_i^2}$$

$$= (2\lambda^2)^n e^{-\lambda^2 \sum x_i^2} \cdot \prod x_i \quad / \ln$$

$$\ln L = n \ln(2\lambda^2) + (-\lambda^2 \sum x_i^2) + \sum \ln x_i \quad / \frac{\partial}{\partial \lambda}$$

$$\frac{d \ln L}{\partial \lambda} = \frac{4n\lambda}{2\lambda^2} - 2\lambda \sum x_i^2 = 0 \Rightarrow \lambda = \sqrt{\frac{n}{\sum x_i^2}}$$

b) mora vrijediti

$$E\left(\frac{1}{n}\sum x_i^2\right) = \frac{1}{\lambda^2}$$

$$E\left(\frac{1}{n}\sum x_i^2\right) = \frac{1}{n}\sum E x_i^2 = \frac{1}{n}\sum E X_1^2 = \frac{1}{n} \cdot n E X_1^2 = E X_1^2$$

$$E X_1^2 = \int_0^\infty x^2 f(x)\,dx = 2\lambda^2 \int_0^\infty x^3 e^{-\lambda^2 x^2} dx = \ldots = \frac{1}{\lambda^2}$$