1) A = " barem jedna jedinica" $\quad |\Omega| = 6^3$

$$\mathbb{P}(A) = 1 - \frac{5^3}{\underbrace{6^3}_{\text{niti jedna jedinica}}}$$

B = " točno jedna šestica"

brojevi koji nisu 6 na preostalim mjestima

$$\mathbb{P}(B) = \frac{3 \cdot 5 \cdot 5}{6^3}$$

mjesto za šesticu

C = " tri različita broja"

moguće znamenke na prvoj kocki
— " — na drugoj
— " — na trećoj

$$\mathbb{P}(C) = \frac{6 \cdot 5 \cdot 4}{6^3}$$

$$\mathbb{P}(A|C) = \frac{\mathbb{P}(A \cap C)}{\mathbb{P}(C)} = \frac{3 \cdot \cancel{4} \cdot \cancel{5}}{\cancel{4} \cdot \cancel{5} \cdot 6} = \frac{1}{2}$$

mjesto za jedinicu

preostale znamenke

$$\mathbb{P}(A \cap C) = \left(\text{jedino je moguća točno jedna jedinica} \right) = \frac{3 \cdot 5 \cdot 4}{6^3}$$

B i C
Nezavisne $\Leftrightarrow$ $\mathbb{P}(B \cap C) = \mathbb{P}(B) \cdot \mathbb{P}(C)$

$$\mathbb{P}(B \cap C) = \left(\text{slično kao i } A \cap C \right) = \frac{3 \cdot 5 \cdot 4}{6^3}$$

$\nparallel$ nisu nezavisni događaji

$$\mathbb{P}(B) \cdot \mathbb{P}(C) = \frac{3 \cdot 5 \cdot 5 \cdot 6 \cdot 5 \cdot 4}{6^6}$$

2) 4C, 3B, 2P

X - broj crvenih   X može poprimiti 0, 1, 2, 3

Y - broj plavih   Y može poprimiti 0, 1, 2

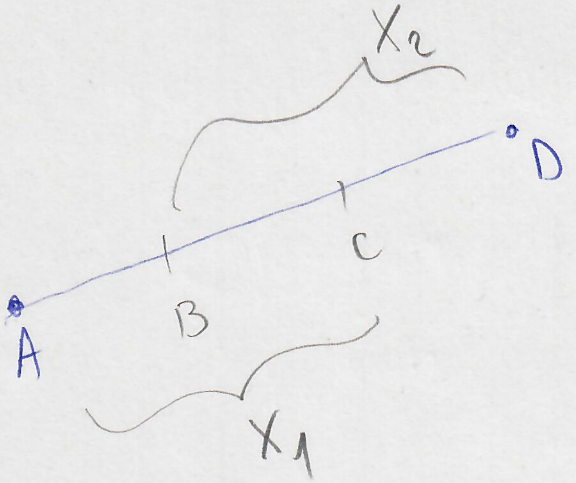
Recimo da izvlačimo sve kuglice: $|\Omega| = \binom{9}{3} = 84$

a)

$X \backslash Y$	0	1	2	
0	$\frac{1}{84}$	$\frac{2\cdot 3}{84}$	$\frac{3}{84}$	$\frac{10}{84}$
1	$\frac{4\cdot 3}{84}$	$\frac{2\cdot 4\cdot 3}{84}$	$\frac{4}{84}$	$\frac{40}{84}$
2	$\frac{2\cdot 3\cdot 3}{84}$	$\frac{2\cdot 2\cdot 3}{84}$	0	$\frac{30}{84}$
3	$\frac{4}{84}$	0	0	$\frac{4}{84}$
	$\frac{35}{84}$	$\frac{42}{84}$	$\frac{7}{84}$	$\frac{84}{84}$

$$\mathbb{P}(Y=1 \mid X=1) = \frac{\mathbb{P}(Y=1 \text{ i } X=1)}{\mathbb{P}(X=1)} = \frac{24}{40}$$

3)

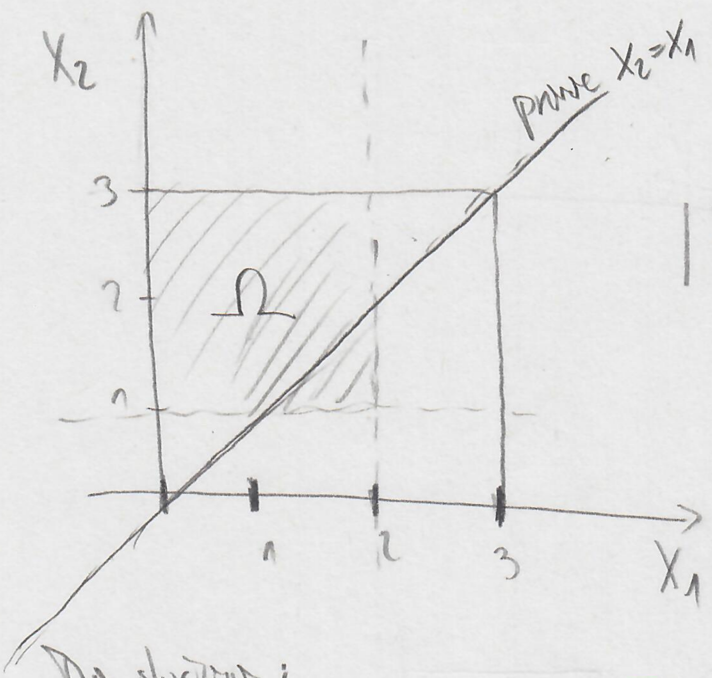


$$Z = |\overline{X_1X_2}|$$

$$F_Z(z) = \mathbb{P}(Z \le z) = \frac{|\{Z \le z\}|}{|\Omega|}$$

$$F_Z(z) = ?$$

Očito je $F(z) = \begin{cases} 0 & z \le 0 \\ ? & z \in [0,3] \\ 1 & z \ge 3 \end{cases}$



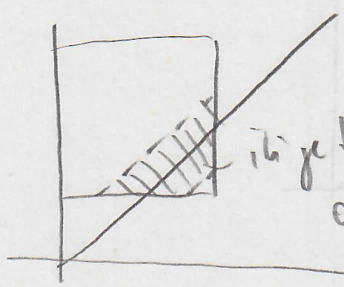
$$|\{Z \le z\}| = |\{(x_1,x_2),\ |x_1-x_2| < z\}|$$

$$= |\{(x_1,x_2),\ X_2 \le X_1 + z \ \text{ i } \ X_2 \ge X_1 - z\}|$$

veliki trokut - mali trokut

$$\Rightarrow F_Z(z) = \frac{\frac{(1+z)^2}{2} - \frac{(1-z)^2}{2}}{4} = \frac{z}{2}, \quad z \le 1$$

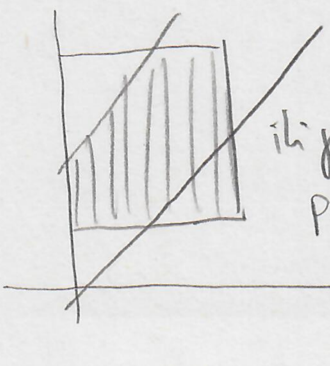
Dva slučaja:



ili je to površina ove pruge (kad je $z \le 1$)

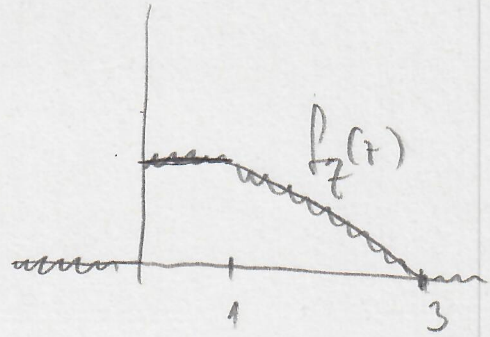
kvadrat - mali trokut

$$F_Z(z) = \frac{4 - \frac{(3-z)^2}{2}}{4} = -\frac{z^2}{8} + \frac{3z}{4} - \frac{1}{8}$$



ili je površina krajeva pravokutnika (kod je $z \ge 1$)

$$f_Z(z) = F'(z) = \begin{cases} \frac{1}{2}, & z \in [0,1] \\ \frac{3}{4} - \frac{z}{4}, & z \in \langle 1, 3] \end{cases}$$



$$\mathbb{E}(Z) = \int_0^1 z \cdot \frac{1}{2} + \int_1^3 \left(\frac{3}{4} - \frac{z}{4}\right) z \, dz = \frac{13}{12}$$

4) a), b) u knjizi.

c) $X_1 \sim \mathcal{P}(\lambda_1)$, $X_2 \sim \mathcal{P}(\lambda_2)$ $\quad \mathbb{P}(X_1 = k) = \frac{\lambda_1^k}{k!} e^{-\lambda_1}$, $\mathbb{P}(X_2 = k) = \frac{\lambda_2^k}{k!} e^{-\lambda_2}$

$$\mathbb{P}(X_1 = k \mid X_1 + X_2 = n) = \frac{\mathbb{P}(X_1 = k \text{ i } X_1 + X_2 = n)}{\mathbb{P}(X_1 + X_2 = n)} = \frac{\mathbb{P}(X_2 = n-k \text{ i } X_1 = k)}{\mathbb{P}(X_1 + X_2 = n)}$$

$X_1 + X_2 \sim \mathcal{P}(\lambda_1 + \lambda_2)$

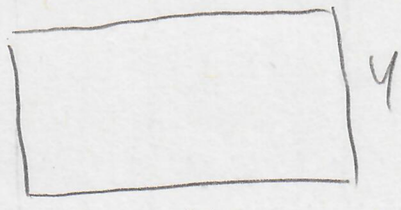
$$\overset{\text{nezavisnost}}{=} \frac{\frac{\lambda_1^k \cancel{e^{-\lambda_1}}}{k!} \frac{\lambda_2^{n-k} \cancel{e^{-\lambda_2}}}{(n-k)!}}{\frac{(\lambda_1+\lambda_2)^n}{n!} \cancel{e^{-\lambda_1-\lambda_2}}}$$

$$= \binom{n}{k} \frac{\lambda_1^k \lambda_2^{n-k}}{(\lambda_1+\lambda_2)^k (\lambda_1+\lambda_2)^{n-k}}$$

$$= \binom{n}{k} \left(\frac{\lambda_1}{\lambda_1+\lambda_2}\right)^k \left(\frac{\lambda_2}{\lambda_1+\lambda_2}\right)^{n-k} = \binom{n}{k} p^k (1-p)^{n-k},$$

$$p = \frac{\lambda_1}{\lambda_1 + \lambda_2}$$

Feladat 5:



$$f_X(x) = \frac{C}{x^2}, \quad x \in [1,4], \qquad f_Y(y) = \frac{1}{2}y + D, \quad y \in [2,3]$$

a) $$1 = \int_1^4 \frac{C}{x^2}\,dx = -\frac{C}{x}\Big|_1^4 = -\frac{C}{4} + C = \frac{3}{4}C \Rightarrow \boxed{C = \frac{4}{3}}$$

$$1 = \int_2^3 \frac{1}{2}y + D\,dy = \frac{1}{4}y^2 + Dy\Big|_2^3 = \frac{9}{4} - 1 + D = \frac{5}{4} + D$$

$$\Rightarrow \boxed{D = -\frac{1}{4}}$$

$$\Rightarrow f_X(x) = \frac{4}{3}\,\frac{1}{x^2}, \; x \in [1,4], \qquad f_Y(y) = \frac{1}{2}y - \frac{1}{4}, \; x \in [2,3]$$

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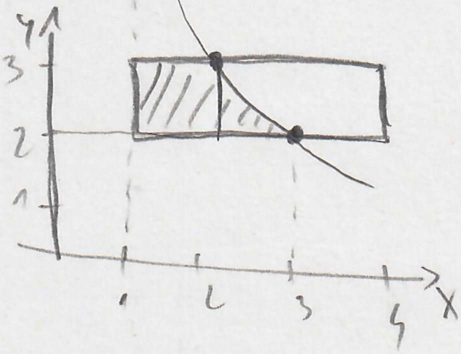
b) $$\mathbb{P}(X \cdot Y \le 6) = \mathbb{P}\left(Y \le \frac{6}{X}\right) =$$



$$= \mathbb{P}((X,Y) \in [1,2] \times [2,3]) + \mathbb{P}((X,Y) \in \triangle)$$

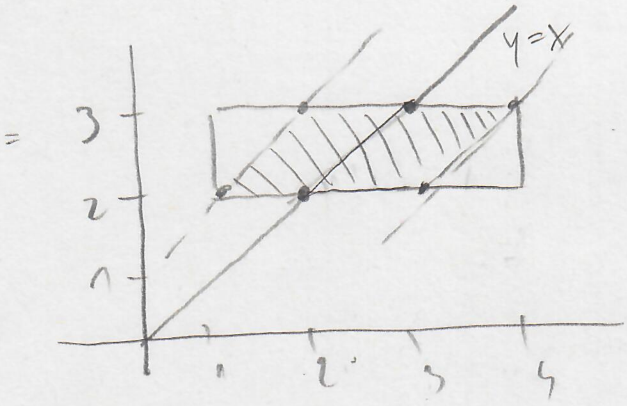
$$= \int_1^2 \int_2^3 \frac{4}{3}\frac{1}{x^2} \cdot \left(\frac{1}{2}y - \frac{1}{4}\right) dy\,dx + \int_2^3 \int_2^{\frac{6}{x}} \frac{4}{3}\,\frac{1}{x^2} \cdot \left(\frac{1}{2}y - \frac{1}{4}\right) dy\,dx$$

$$= \frac{4}{3}\left(-\frac{1}{x}\Big|_1^2\right) \cdot \left(\frac{1}{4}y^2 - \frac{1}{4}y\Big|_2^3\right) + \frac{4}{3}\int_2^3 \frac{1}{x^2} \cdot \left(\frac{1}{4}y^2 - \frac{1}{4}y\Big|_2^{\frac{6}{x}}\right) dx$$

$$= \frac{4}{3}\left(1 - \frac{1}{2}\right) \cdot \left(\frac{9}{4} - 1 - \frac{3}{4} + \frac{1}{2}\right) + \frac{4}{3}\int_2^3 \frac{1}{x^2}\left(\frac{36}{4}\,\frac{1}{x^2} - 1 - \frac{6}{4}\,\frac{1}{x} + \frac{1}{2}\right) dx =$$

$$= \frac{2}{3} + \frac{11}{108} = \frac{83}{108} = 0{,}7685$$

c) $\mathbb{P}(|X-Y| \leq 1) = \mathbb{P}(Y \leq X+1 \;;\; Y \geq X-1)$



$$= \int_1^2 \int_2^{x+1} \frac{4}{3} \frac{1}{x^2}\left(\frac{1}{2}y - \frac{1}{4}\right) dy\,dx + \int_2^3 \int_2^3 \frac{4}{3} \frac{1}{x^2}\left(\frac{1}{2}y - \frac{1}{4}\right) dy\,dx + \int_3^4 \int_{x-1}^3 \frac{4}{3} \frac{1}{x^2}\left(\frac{1}{2}y - \frac{1}{4}\right) dy\,dx$$

$$= \frac{\ln 2}{3} + \cancel{\frac{2}{9}} + \ln\left(\frac{4}{3}\right) - \cancel{\frac{2}{9}} = 0{,}518$$

6) Zadatak u knjizi