1°) 52 karte $= 4 \cdot 13$

Izvučimo 10 karata.

a) Nakon izvlačenja karte vraćamo u špil.

$$\Omega = \{(a_1, a_2, \dots, a_{10}),\ a_i \in \{P, H, K, T\}\}$$

$$A = \{(a_1, a_2, \dots, a_{10}),\ a_i \in \{P, H, K, T\},\ \text{barem } 8 \text{ je istih}\}$$

$$|\Omega| = 4^{10}$$

$$|A| = \{8 \text{ je istih}\} + \{9 \text{ je istih}\} + \{10 \text{ je istih}\}$$

$$= \underbrace{\binom{4}{1}}_{\text{boja}} \underbrace{\binom{10}{8}}_{\text{mjesta}} \underbrace{3^2}_{\text{preostale}} + \binom{4}{1}\binom{10}{9} 3 + \binom{4}{1}$$

$$\mathbb{P}(A) = \frac{4 \cdot 45 \cdot 9 + 4 \cdot 10 \cdot 3 + 4}{1048576} = \frac{1744}{1048576} = 0{,}00166$$

b) Nakon izvlačenja karte ne vraćamo u špil

$$\Omega = \{\{a_1, a_2, \dots, a_{10}\},\ a_i \in \{P_1, P_2, \dots, P_{13}, H_1, H_2, \dots, H_{13}, K_1, \dots K_{13}, T_1, \dots T_{13}\}\}$$

a_i su različiti

$$A = \{\{a_1, a_2, \dots, a_{10}\} \in \Omega,\ \text{barem } 8\ a_i \text{ ima isto slovo}\}$$

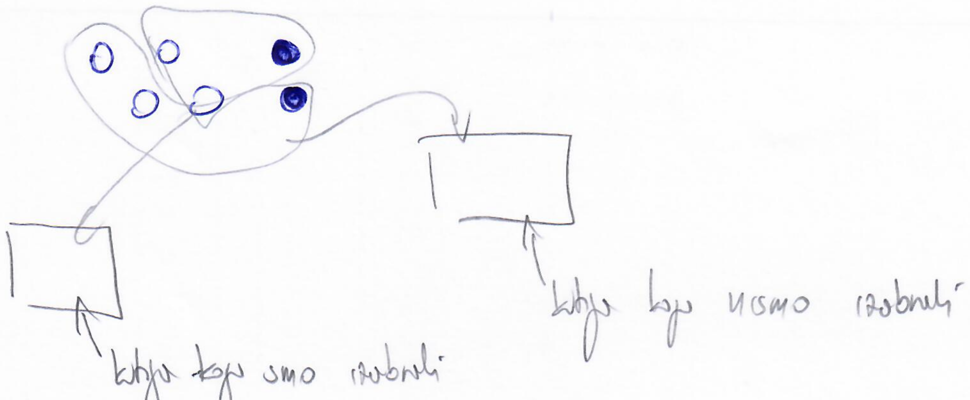
$$|\Omega| = \binom{52}{10}$$

$$|A| = \{8 \text{ je istih}\} + \{9 \text{ je istih}\} + \{10 \text{ je istih}\}$$

$$= \underbrace{\binom{4}{1}}_{\text{boja}}\binom{13}{8} \cdot \binom{39}{2} + \binom{4}{1}\binom{13}{9} 39 + \binom{4}{1}\binom{13}{10}$$

JMBAG:	Ime i prezime:

2) 4b i 2C



$H_0 = \{$ u kutiju koju smo izabrali, stavili smo 0 crnih kuglica $\}$

$H_1 = \{$ ——"—— , ——"— 1 —"— $\}$

$H_2 = \{$ ——"—— ——"— 2 —"— $\}$

$$\mathbb{P}(H_0) = \frac{\binom{4}{3}}{\binom{6}{3}} = \frac{1}{2} \cdot \frac{4}{20} = \frac{1}{5}$$

$$\mathbb{P}(H_1) = \frac{\binom{4}{2}\binom{2}{1}}{\binom{6}{3}} = \frac{3}{5}, \quad \mathbb{P}(H_2) = \frac{\binom{4}{1}\binom{2}{2}}{\binom{6}{3}} = \frac{4 \cdot 1}{20} = \frac{1}{5}.$$

a) A - {izvukli smo dvije kuglice iste boje}

$$\mathbb{P}(A) = \mathbb{P}(A|H_0)\mathbb{P}(H_0) + \mathbb{P}(A|H_1)\mathbb{P}(H_1) + \mathbb{P}(A|H_2)\mathbb{P}(H_2)$$
$$= 1 \cdot \frac{1}{5} + \frac{1}{3} \cdot \frac{3}{5} + \frac{1}{3} \cdot \frac{1}{5} = \frac{1}{5}\left(1 + 1 + \frac{1}{3}\right) = \frac{1}{5}\left(\frac{7}{3}\right) = 0{,}466$$

b) B - {drugu izvučenu kuglicu je bijela}, C - {prvu izvučenu kuglicu je bijela}

$$\mathbb{P}(B|C) = \frac{\mathbb{P}(B \cap C)}{\mathbb{P}(C)} = \frac{\sum_{i=0}^{2} \mathbb{P}(B \cap C | H_i) \cdot H_i}{\sum_{i=0}^{2} \mathbb{P}(C|H_i) H_i} = \frac{1 \cdot \frac{1}{5} + \frac{1}{3} \cdot \frac{3}{5} + 0}{1 \cdot \frac{1}{5} + \frac{2}{3} \cdot \frac{3}{5} + \frac{1}{3} \cdot \frac{1}{5}} =$$
$$= \frac{2}{1 + 2 + \frac{1}{3}} = \frac{2 \cdot 3}{10} = \frac{6}{10}$$

Ime i prezime:	JMBAG:

3) $X \sim$ broj izvučenih asova

$Y \sim$ broj izvučenih pikova

$Y \backslash X$	0	1	2	
0	$\frac{\binom{12\cdot 3}{2}}{\binom{52}{2}}$	$\frac{3\cdot 12\cdot 3}{\binom{52}{2}}$	$\frac{\binom{3}{2}}{\binom{52}{2}}$	$\frac{\binom{13\cdot 3}{2}}{\binom{52}{2}}$
1	$\frac{12\cdot 12\cdot 3}{\binom{52}{2}}$	$\frac{3\cdot 12+12\cdot 3}{\binom{52}{2}}$	$\frac{3}{\binom{52}{2}}$	$\frac{13\cdot 3\cdot 13}{\binom{52}{2}}$
2	$\frac{\binom{12}{2}}{\binom{52}{2}}$	$\frac{12}{\binom{52}{2}}$	$\frac{0}{\binom{52}{2}}$	$\frac{\binom{13}{2}}{\binom{52}{2}}$
	$\frac{\binom{12\cdot 4}{2}}{\binom{52}{2}}$	$\frac{4\cdot 12\cdot 4}{\binom{52}{2}}$	$\frac{\binom{4}{2}}{\binom{52}{2}}$	1

2

$Y \backslash X$	0	1	2	
0	0,4751	0,081	0,002	0,558
1	0,3257	0,0542	0,002	0,382
2	0,0497	0,009	0	0,058
	0,850	0,144	0,004	1

Varijable nisu nezavisne jer je

$\mathbb{P}(X=2, Y=2)=0$

$\mathbb{P}(X=2), \mathbb{P}(Y=2) \neq 0$

$\mathbb{E}X = \frac{204}{1326} \sim 0,152$

$\mathbb{E}Y = \frac{1}{2} \sim 0,498$

$\mathbb{E}(XY) = \frac{102}{1326} \sim 0,0762$

$\operatorname{cov}(X,Y) = \mathbb{E}(XY) - \mathbb{E}X\mathbb{E}Y = \frac{102}{1326} - \frac{1}{2}\cdot\frac{204}{1326} = 0$

Ime i prezime:

JMBAG:

4.) a), b) iz knjižice

c) $\boxed{X \sim \mathcal{G}(p), \quad P(X=k) = p(1-p)^{k-1}, \quad EX = \frac{1}{p}, \quad P(X>k) = (1-p)^k}$

$p = 0.2.$

$P(X>7) = (1-p)^7 = 0{,}8^7 = 0{,}209.$

d) $P(X>7 \mid X>3)$ $\quad \boxed{P(X>k+m \mid X>k) = P(X=m)}$

$= P(X>4) = (1-p)^4 = 0{,}8^4 = 0{,}4096$

e) $P(\text{petak sunčano} \mid \text{subota kišilo}) = P(\text{petak sunčano}) = 0{,}8.$

Ime i prezime:

JMBAG:

5.

$X \sim \mathcal{E}xp(\lambda), \quad f(x) = \lambda e^{-\lambda x}, \quad x > 0$

$\mathbb{E}(X) = \frac{1}{\lambda} = 15. \Rightarrow \lambda = \frac{1}{15}.$

a) $\mathbb{P}(X < 10) = \int_0^{10} \lambda e^{-\lambda x} dx = 1 - e^{-\lambda 10} = 1 - e^{-\frac{10}{15}}$

b) Tražimo T takav da je

$0{,}9 = 1 - e^{-\lambda T} \Rightarrow e^{-\lambda T} = 0{,}1$

$-\lambda T = \ln(0{,}1)$

$T = \frac{1}{-\lambda} \ln(0{,}1) = 34{,}5$ min.

c) $T_1, T_2, T_3 \sim \mathcal{E}xp(\lambda)$

$S = T_1 + T_2 + T_3, \quad \mathbb{E}(S) = 3 \cdot \mathbb{E}(X) = 45$ minuta.

d) $Tram \sim Exp(\mu), \quad Bus \sim Exp(\lambda)$

$\min(Tram, Bus) \sim Exp(\lambda + \mu)$

$\mathbb{E}(\min(Tram, Bus)) = \frac{1}{\lambda + \mu}.$

Ime i prezime:	JMBAG: