VIS - MEĐUISPIT

(1)

a) $\Omega = \{P, B, CP, CB, CCP, CCB, CCCP, CCCB, \dots\}$

↖ zaustavimo se kad izvučemo plavu ili bijelu.

$\mathcal{F} = \mathcal{P}(\Omega)$

$\mathbb{P}(P) = \frac{8}{24} = \frac{1}{3}$, $\mathbb{P}(C) = \frac{6}{24} = \frac{1}{4}$

$\mathbb{P}(B) = \frac{10}{24} = \frac{5}{12}$

24 kuglice
6 C
8 P
10 B

$\mathbb{P}(\underbrace{C \cdots C}_{k-1} P) = \left(\frac{1}{4}\right)^{k-1} \cdot \frac{1}{3}$

$\mathbb{P}(\underbrace{C \cdots C}_{k-1} B) = \left(\frac{1}{4}\right)^{k-1} \cdot \frac{5}{12}$

$A = \{$izvukli smo plavu prije bijele$\} = \{P, CP, CCP, CCCP, \dots\}$

$\mathbb{P}(A) = \mathbb{P}(\{P\} \cup \{CP\} \cup \{CCP\} \cup \{CCCP\} \cup \dots\}$

$= \sum_{k=0}^{\infty} \mathbb{P}(\underbrace{C \cdots C}_{k} P) = \sum_{k=0}^{\infty} \left(\frac{1}{4}\right)^k \cdot \frac{1}{3} = \frac{1}{3} \cdot \frac{1}{1-\frac{1}{4}} = \frac{1}{3} \cdot \frac{4}{3} = \frac{4}{9}$

b) $\Omega = \{P, B, CP, CB, \dots, CCCCCCP, CCCCCCB\}$ ← konačan prostor.

$\mathcal{F} = \mathcal{P}(\Omega)$

$A = \{P, CP, CCP, \dots, CCCCCCP\}$

$\mathbb{P}(A) = \sum_{k=0}^{6} \mathbb{P}(\underbrace{CC \cdots C}_{k} P) = \frac{8}{24} + \frac{6}{24} \cdot \frac{8}{23} + \frac{6}{24} \cdot \frac{5}{23} \cdot \frac{8}{22} + \frac{6}{24} \cdot \frac{5}{23} \cdot \frac{4}{22} \cdot \frac{8}{21}$

$+ \frac{6}{24} \cdot \frac{5}{23} \cdot \frac{4}{22} \cdot \frac{3}{21} \cdot \frac{8}{20} + \frac{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2}{21 \cdot 23 \cdot 22 \cdot 21 \cdot 20} \cdot \frac{8}{19}$

$+ \frac{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{24 \cdot 23 \cdot 22 \cdot 21 \cdot 20 \cdot 19} \cdot \frac{8}{18} = 0,4444\ldots = \frac{4}{9}$

c) 1P 0B / 2P 0B / 3P 0B / 2P 1B

$\binom{24}{3}$ - načina za odabrati 3 kuglice bez ponavljanja.

$A = \{$ mezi odebrané 3, více je plných než bytelných $\}$

$$P(A) = \frac{\binom{8}{1}\binom{6}{2}\binom{10}{0} + \binom{8}{2}\binom{6}{1}\binom{10}{0} + \binom{8}{3}\binom{6}{0}\binom{10}{0} + \binom{8}{2}\binom{6}{0}\binom{10}{1}}{\binom{24}{3}}$$

$$= \frac{8 \cdot 3 \cdot 5 + 4 \cdot 7 \cdot 6 + 8 \cdot 7 + 4 \cdot 7 \cdot 10}{24 \cdot 23 \cdot 22} \cdot 6 = 0{,}3083$$

d) 1P 0B 2C + 2P 0B 1C + 3P 0B 0C + 2P 1B 0C

$$P(A) = \binom{3}{1}\left(\frac{8}{24}\right)\left(\frac{6}{24}\right)^2 + \binom{3}{2}\left(\frac{8}{24}\right)^2\left(\frac{6}{24}\right) + \left(\frac{8}{24}\right)^3 + \binom{3}{2}\left(\frac{8}{24}\right)^2\left(\frac{10}{24}\right)$$

↑ poredak plavih ↑ poredak plavih ↑ poredak plavih

$$= \frac{3 \cdot 8 \cdot 6^2 + 3 \cdot 8^2 \cdot 6 + 8^3 + 3 \cdot 8^2 \cdot 10}{24^3} = 0{,}32175$$

(2)

1.P: 6 B, 4 C

2.P: 5 B, 6 C

$H_0 = \{$ iz prve kutije je izvučeno 0 bijelih $\}$

$H_1 = \{$ iz prve kutije je izvučena 1 bijela $\}$

$H_2 = \{$ —"— 2 bijele $\}$

$H_3 = \{$ —"— 3 bijele $\}$

$A = \{$ ukupno su izvučene 2 bijele kuglice $\}$

$$\mathbb{P}(A) = \mathbb{P}(A|H_0)\mathbb{P}(H_0) + \mathbb{P}(A|H_1)\cdot\mathbb{P}(H_1) + \mathbb{P}(A|H_2)\cdot\mathbb{P}(H_2) + \mathbb{P}(A|H_3)\cdot\mathbb{P}(H_3)$$

$$\mathbb{P}(H_0) = \frac{\binom{4}{3}}{\binom{10}{3}}, \quad \mathbb{P}(H_1) = \frac{\binom{6}{1}\binom{4}{2}}{\binom{10}{3}}, \quad \mathbb{P}(H_2) = \frac{\binom{6}{2}\binom{4}{1}}{\binom{10}{3}}, \quad \mathbb{P}(H_3) = \frac{\binom{6}{3}}{\binom{10}{3}}$$

$$\mathbb{P}(A|H_0) = \mathbb{P}(\text{iz 2. kutije je izvučeno 2 bijele}) = \frac{\binom{5}{2}\binom{6}{1}}{\binom{11}{3}}$$

$$\mathbb{P}(A|H_1) = \mathbb{P}(\text{—"— 1 bijela}) = \frac{\binom{5}{1}\binom{6}{2}}{\binom{11}{3}}$$

$$\mathbb{P}(A|H_2) = \mathbb{P}(\text{—"— 0 bijelih}) = \frac{\binom{6}{3}}{\binom{11}{3}}$$

$$\mathbb{P}(A|H_3) = 0$$

$$\mathbb{P}(A) = \frac{23}{110} \approx 0{,}209$$

$$\mathbb{P}(H_0|A) = \frac{\mathbb{P}(A|H_0)\cdot\mathbb{P}(H_0)}{\mathbb{P}(A)} = \frac{4}{69} \approx 0{,}058$$

$$\mathbb{P}(H_1|A) = \frac{\mathbb{P}(A|H_1)\cdot\mathbb{P}(H_1)}{\mathbb{P}(A)} = \frac{15}{23} \approx 0{,}652$$

$$\mathbb{P}(H_2|A) = \frac{\mathbb{P}(A|H_2)\cdot\mathbb{P}(H_2)}{\mathbb{P}(A)} = \frac{20}{69} = 0{,}290$$

(3) a) $S_n = X_1 + X_2 + \dots + X_n$, $X_k \sim B(3, \frac{1}{2})$

↳ Opet binomna vjerojatnost s parametrima $\underbrace{3+3+\dots 3}_{n \text{ puta}}$, $\frac{1}{2}$

$S_n \sim B(3n, \frac{1}{2})$.

$E(S_n) = \frac{3}{2}n$, $D(S_n) = 3n \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{3n}{4}$

b) $Y_k = 3 - X_k$

$X_1 = 0 \Rightarrow Y_1 = 3$
$X_1 = 1 \Rightarrow Y_1 = 2$
$X_1 = 2 \Rightarrow Y_1 = 1$
$X_1 = 3 \Rightarrow Y_1 = 0$

X_1 \ Y_1	0	1	2	3
0	0	0	0	$\frac{1}{8}$
1	0	0	$\frac{3}{8}$	0
2	0	$\frac{3}{8}$	0	0
3	$\frac{1}{8}$	0	0	0

c) Vrijedi $Y_1 = 3 - X_1 \Rightarrow r(X_1, Y_1) = -1$

d) $P(X_1 = i, Y_2 = j) = P(X_1 = i) P(Y_2 = j) = \binom{3}{i}\left(\frac{1}{2}\right)^3 \cdot \binom{3}{j}\left(\frac{1}{2}\right)^3$

(X_1 i Y_2 su nezavisne

X_1 \ Y_2	0	1	2	3
0	$\frac{1}{64}$	$\frac{3}{64}$	$\frac{3}{64}$	$\frac{1}{64}$
1	$\frac{3}{64}$	$\frac{9}{64}$	$\frac{9}{64}$	$\frac{3}{64}$
2	$\frac{3}{64}$	$\frac{9}{64}$	$\frac{9}{64}$	$\frac{3}{64}$
3	$\frac{1}{64}$	$\frac{3}{64}$	$\frac{3}{64}$	$\frac{1}{64}$

e) X_1 i Y_2 su nezavisne
pa je $r(X_1, Y_2) = 0$

f) Pogledati dokaz u knjizi

(4)

a) $Y = \min(X_1, \dots, X_n)$

$X_i \sim \mathcal{G}(p_i)$, $\mathbb{P}(X_i = k) = (1-p_i)^{k-1} p_i = q_i^{k-1} p_i$

$$\mathbb{P}(Y=k) = \mathbb{P}(Y > k-1) - \mathbb{P}(Y > k)$$

$$= \mathbb{P}(X_1 > k-1, \dots, X_n > k-1) - \mathbb{P}(X_1 > k, \dots, X_n > k)$$

$$= \mathbb{P}(X_1 > k-1)\mathbb{P}(X_2 > k-1) \cdots \mathbb{P}(X_n > k-1) - \mathbb{P}(X_1 > k)\mathbb{P}(X_2 > k) \cdots \mathbb{P}(X_n > k)$$

$$\left(\mathbb{P}(X_i > k) = \sum_{j=k+1}^{\infty} \mathbb{P}(X_i = j) = \sum_{j=k+1}^{\infty} q_i^{j-1} p_i = \sum_{j=0}^{\infty} q_i^{j+k} p_i = p_i q_i^k \frac{1}{1-q_i} = q_i^k\right)$$

$$= q_1^{k-1} \cdot q_2^{k-1} \cdots q_n^{k-1} - q_1^k \cdot q_2^k \cdots q_n^k = (q_1 \cdots q_n)^{k-1}(1 - q_1 \cdots q_n)$$

Označimo $p = (1 - q_1 \cdots q_n) \Rightarrow Y \sim \mathcal{G}(1 - q_1 \cdots q_n) = \mathcal{G}(p)$

(5)

a) Neka je (x_n) niz koji zdesna konvergira u x.

$x_n \searrow x$. Tada je $\{X \le x\} = \bigcap_{n \in \mathbb{N}} \{X < x_n\}$

Neprekidnost vjerojatnosti: $\mathbb{P}(\{X \le x\}) = \lim_{n \to \infty} \mathbb{P}(\{X < x_n\})$

$$\mathbb{P}(X = x) = \mathbb{P}(X \le x) - \mathbb{P}(X < x)$$

$$= \lim_{n \to \infty} \mathbb{P}(X \le x_n) - \mathbb{P}(X < x) = \lim_{n \to \infty} F(x_n) - F(x) = F(x^+) - F(x)$$

b)

$$f_X = \begin{cases} 0 & x \le -1 \\ C_1(x-1) & -1 < x \le 1 \\ C_2(x-1) & 1 < x < 4 \\ 0 & x \ge 4 \end{cases}$$

po lijevom pravcu.

$$-2C_1 = f_X(-1) = f_X(4) = C_2 \cdot 3$$

Powstało ispod grafu y 1:  $= 1$

$$-2C_1 \cdot 2 + 3 \cdot 3 \cdot C_2 = 1$$

$$C_1 = -\frac{3}{2} C_2 \Rightarrow 6C_2 + 9C_2 = 1$$

$$C_2 = \frac{1}{15}$$

$$C_1 = -\frac{3}{2} \cdot \frac{1}{15} = -\frac{1}{5}$$

$$F_X(x) = \int_{-\infty}^{x} f(t)\,dt = \begin{cases} 0 & x < -1 \\ -\frac{3}{2} \cdot \frac{1}{15} \int_{-1}^{x} (t-1)\,dt = -\frac{3}{4} \cdot \frac{1}{15} (t-1)^2 \Big|_{-1}^{x} = -\frac{3}{4} \cdot \frac{1}{15} (x-1)^2 + \frac{6}{15} & -1 < x \leq 1 \\ \frac{1}{15} \int_{1}^{x} (t-1)\,dt + \frac{6}{15} = \frac{1}{2} \cdot \frac{1}{15} (x-1)^2 + \frac{6}{15} & 1 < x < 4 \\ 1 & x > 4 \end{cases}$$