1

$T \sim N(\mu, \sigma^2)$

(a)

$E(T) = \mu = 36$

$P(T > 42) = 0.2$

$P(Z > \frac{42-36}{\sigma}) = 0.2$

$\frac{1}{2}(1 - \phi^*(\frac{6}{\sigma})) = 0.2$

$\frac{6}{\sigma} = 0.842$

$\sigma = 7.125891$

(b)

$P(t_1 < T < t_2) = 0.85$

$P(36 - z_{0.925} \cdot \sigma < T < 36 + z_{0.925} \sigma) = 0.85$

$\langle 25.74, 46.26 \rangle$

(c)

$P(\mu - 3\sigma < T < \mu + 3\sigma) = 0.997$

(d)

$T_1 \sim N(\mu, \sigma^2)$

$T_2 \sim N(\mu, \sigma^2)$

$(T_1 + T_2) \sim N(\mu + \mu, \sigma^2 + \sigma^2)$

$P(T_1 + T_2 < 70) = P(Z < \frac{70 - 2\mu}{\sqrt{2}\sigma}) = \frac{1}{2} - \frac{1}{2}\phi^*(0.1984613)$

$= 0.421525$

(2)

(a)

$$c\int_0^1\int_0^2 y\,dx\,dy = 1$$

$$c = \frac{1}{2}$$

(b)

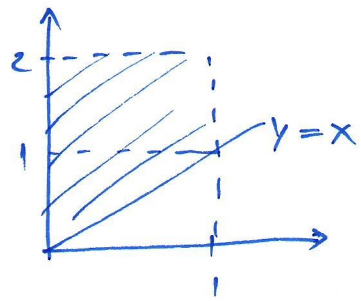
$$f_X(x) = \frac{1}{2}\int_0^2 y\,dy = 1$$

$$f_Y(y) = \frac{1}{2}y\int_0^1 dx = \frac{1}{2}y$$

(d)

$$P(X<Y) = 1-\frac{1}{2}\int_0^1\int_0^x y\,dy\,dx$$

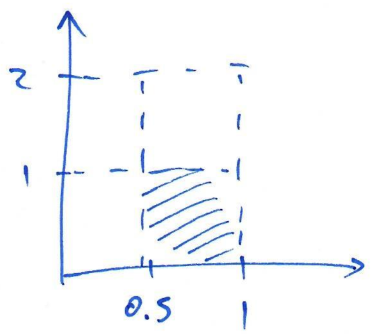
$$= 1-\frac{1}{12} = \frac{11}{12}$$



(c)

Nezavisne jer je $f_X \cdot f_Y = f$, $\forall x,y$

(e)



$$P(Y<1 \mid X>0.5) =$$

$$= \frac{P(\{Y<1\}\cap\{X>0.5\})}{P(X>0.5)}$$

$$= \frac{\int_0^1\int_{0.5}^1 \frac{1}{2}y\,dx\,dy}{1/2} = \frac{1}{4}$$

③

(a)

$$E(X_i-\bar{X})=0 \quad \Rightarrow \quad D(X_i-\bar{X})=E(X_i-\bar{X})^2$$

$$E(S^2)=\frac{1}{n-1}\sum_{i=1}^{n}E(X_i-\bar{X})^2$$

$$=\frac{1}{n-1}\sum_{i=1}^{n}D(X_i-\frac{1}{n}\sum_{j=1}^{n}X_j)$$

$$=\frac{1}{n-1}\sum_{i=1}^{n}D\left(\frac{n-1}{n}X_i-\frac{1}{n}\sum_{j\neq i}X_j\right)$$

$$=\frac{1}{n-1}\sum_{i=1}^{n}\left(\left(\frac{n-1}{n}\right)^2\sigma^2+\left(\frac{1}{n}\right)^2\sigma^2(n-1)\right)$$

$$=\sigma^2$$

(b)

$\bar{x}=998.7667$

$\hat{s}^2=4.5989$

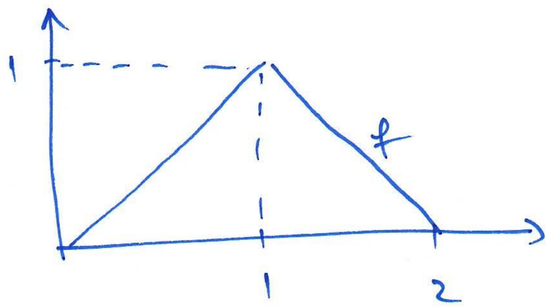
$1-\alpha=95\%$

$t_{29,0.975}=2.045$

$\langle 997.9660, 999.5673\rangle$

(4)

(a)



$E(X_i) = 1$

$$D(X_i) = \int_{-\infty}^{\infty} (x - E(x))^2 f(x)\,dx$$

$$= 2\int_0^1 (x-1)^2 x\,dx = \frac{1}{6}$$

(b)

Prema CGT $\quad Z = \dfrac{\overline{X} - 1}{\sqrt{\frac{1}{6}}/10}\quad$ možemo aproksimirati jediničnom normalnom razdiobom.

$$P(0.90 < \overline{X} < 1.02) =$$

$$P(-0.10 \cdot 10\sqrt{6} < Z < 0.02 \cdot 10\sqrt{6}) =$$

$$= \frac{1}{2}\Phi^*(2.44949) + \frac{1}{2}\Phi^*(0.4898979)$$

$$= 0.68079$$