1 a) $A=\{1\}$, $B=\{2\}$

$P(A\cap B)=P(\emptyset)=0$

$P(A)=P(B)=\frac{1}{6}$

$P(A)\cdot P(B)=\frac{1}{36}$

$\Rightarrow P(A\cap B)<P(A)\cdot P(B)$

b) $A=\{1\}$, $B=\{1\}$

$P(A)=P(B)=P(A\cap B)=\frac{1}{6}$

$P(A)\cdot P(B)=\frac{1}{36}$

$\Rightarrow P(A\cap B)>P(A)\cdot P(B)$

c) $A=\{1,2,3,4\}$, $B=\{3,4,5\}$

$P(A)=\frac{2}{3}$, $P(B)=\frac{1}{2}$

$P(A)\cdot P(B)=\frac{1}{3}$

$P(A\cap B)=P(\{3,4\})=\frac{1}{3}$

$\Rightarrow P(A\cap B)=P(A)\cdot P(B)$

A i B su nezavisni

2 $A=\{$pale su dvije šestice$\}$

$H_i=\{$izvukli smo i neispravnih kocki$\}$

$P(H_0)=\dfrac{\binom{2}{2}}{\binom{4}{2}}=\dfrac{1}{6}$ $\qquad P(A|H_0)=\dfrac{1}{36}$

$P(H_1)=\dfrac{\binom{2}{1}\binom{2}{1}}{\binom{4}{2}}=\dfrac{4}{6}$ $\qquad P(A|H_1)=\dfrac{1}{6}$

$P(H_2)=\dfrac{\binom{2}{2}}{\binom{4}{2}}=\dfrac{1}{6}$ $\qquad P(A|H_2)=1$

$$P(A)=\sum_{i=0}^{2}P(A|H_i)P(H_i)=\frac{61}{216}$$

$$P(H_2|A)=\frac{P(A|H_2)P(H_2)}{P(A)}=\frac{36}{61}$$

3 a) $Y=aX+b$, $a<0$, $b\in\mathbb{R}$

b) Mora, zbog nezavisnosti je

$E(XY)=E(X)\cdot E(Y)$

pa je

$$r(X,Y)=\frac{E(XY)-E(X)E(Y)}{\sqrt{D(X)D(Y)}}=0.$$

c) Npr.: $X\sim U(-1,1)$, $Y=X^2$

ili

$X \backslash Y$	-1	1
-1	0	$1/4$
0	$1/2$	0
1	0	$1/4$

d) $r(aX+b,cY+d)=$

$$=\frac{E[(aX+b-E(aX+b))(cY+d-E(cY+d))]}{\sqrt{D(aX+b)D(cY+d)}}$$

$$=\frac{E[(aX+b-aE(X)-b)(cY+d-cE(Y)-d)]}{\sqrt{a^2c^2D(X)D(Y)}}$$

$$=\frac{acE[(X-E(X))(Y-E(Y))]}{ac\sqrt{D(X)D(Y)}}=r(X,Y)$$

④ $X_1, X_2 \sim U(0,1)$ nezavisne

$X = \max\{X_1, X_2\}$ i $f(Y|X) = \frac{1}{X}$

a) $P(X \le x) = P(\max\{X_1, X_2\} \le x)$

$= P(X_1 \le x, X_2 \le x) = \frac{x-0}{1-0} \cdot \frac{x-0}{1-0}$

$= x^2 = F_X(x) \Rightarrow f_X(x) = 2x, \quad 0 \le x \le 1$

$f_Y(y) = \int_{-\infty}^{+\infty} f(x,y)\,dx = \int_y^1 f(y|x) f_X(x)\,dx =$

$= 2(1-y), \quad 0 \le y \le 1$

b) $E(Y) = \int_{-\infty}^{+\infty} y f_Y(y)\,dy = 2\int_0^1 (y - y^2)\,dy = \frac{1}{3}$

c) $P(Y > E(Y)) = P(Y > \frac{1}{3}) =$

$= \int_{1/3}^1 2(1-y)\,dy = \frac{4}{9}$

⑤ $X, Y \sim N(0, \sigma^2)$

$P(X^2 + Y^2 \le z) = \iint f_{XY}(x,y)\,dx\,dy =$

$= \iint f_X(x) f_Y(y)\,dx\,dy = \iint \left(\frac{1}{\sigma\sqrt{2\pi}}\right)^2 e^{-\frac{1}{2}\left(\frac{x}{\sigma}\right)^2 - \frac{1}{2}\left(\frac{y}{\sigma}\right)^2}\,dx\,dy$

$= \left| \begin{array}{l} x = r\cos\varphi \\ y = r\sin\varphi \\ dx\,dy = r\,dr\,d\varphi \end{array} \right| = \frac{1}{2\pi\sigma^2} \int_0^{\sqrt{z}} \int_0^{2\pi} r e^{-\frac{r^2}{2\sigma^2}}\,dr\,d\varphi$

$= \frac{2\pi}{2\pi\sigma^2} \int_0^{\sqrt{z}} r e^{-\frac{r^2}{2\sigma^2}}\,dr = \frac{1}{\sigma^2} \cdot \left(-\frac{1}{2} 2\sigma^2 e^{-\frac{r^2}{2\sigma^2}}\right)\Big|_0^{\sqrt{z}}$

$= 1 - e^{-\frac{1}{2\sigma^2} z} \Rightarrow \lambda = \frac{1}{2\sigma^2}$

⑥ $X_i \sim U(-1, 1)$

$E(X_i) = 0 \quad D(X_i) = \frac{1}{3}$

$P(-c < \bar{X} < c) = 0.99$

$P\left(\frac{-c-0}{\sqrt{\frac{1}{3}}/\sqrt{100}} < \frac{\bar{X}-0}{\sqrt{\frac{1}{3}}/\sqrt{100}} < \frac{c-0}{\sqrt{\frac{1}{3}}/\sqrt{100}}\right) = 0.99$

Po CGT $\frac{\bar{X}-0}{\sqrt{\frac{1}{3}}/\sqrt{100}}$ aproksimiramo s $N(0,1)$.

$\Phi^{*-1}(0.99) = \frac{c}{\sqrt{\frac{1}{3}}/\sqrt{100}}$

$c = \sqrt{\frac{1}{3}}\, \Phi^{*-1}(0.99)/10$

$c = \sqrt{\frac{1}{3}} \cdot 2.5758/10$

$c = 0.1487$