1) $\underline{a_1}\ \underline{a_2}\ \underline{a_3}\ \underline{a_4}\ \underline{a_5}\ \underline{a_6}$ $\quad a_i \in \{0,1,\dots,9\}$ $\quad |\Omega| = 10^6$ $\quad P(A) = \frac{|A|}{|\Omega|}$

a) $A = \{$sve znamenke različite$\}$

$$|A| = 10\cdot 9\cdot 8\cdot 7\cdot 6\cdot 5 = \frac{10!}{4!} \qquad P(A) = \frac{\frac{10!}{4!}}{10^6}$$

b) $A = \{$prve tri znamenke su jednake, a prve 4 nisu$\}$

$$|A| = \underbrace{\binom{10}{1}}_{\text{prve 3 znam}}\underbrace{\binom{9}{1}}_{\text{4 znamenka}}\underbrace{10\cdot 10}_{\text{zadnje dvije}}$$

c) $\{$dvije skupine s po 3 jednake znamenke$\} = A$

$$|A| = \underbrace{\binom{6}{3}}_{\text{pozicije za prvu znamenku}}\binom{10}{2} \leftarrow \text{znamenke}$$

d) $A = \{$broj se sastoji od točno 2 znamenke$\}$

$$|A| = \underbrace{\binom{10}{2}}_{\text{odabir znamenke}}(2^6-2)$$

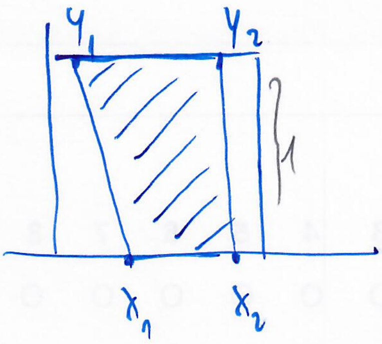
jedan takav broj se može postaviti s binarnim brojem od 6 znamenaka koji nema sve nule ili sve jedinice $(0\,1\,1\,0\,1\,0)$

Takvih ima 2^6-2

e)

$$|A| = 10^6 - 530436$$

2.)



$\frac{a+c}{2} \cdot v$

$$z = \frac{|x_2 - x_1| + |y_1 - y_2|}{2} \cdot 1$$

Definiramo slučajne varijable $X = |x_2 - x_1|$, $Y = |y_1 - y_2|$

$$\Rightarrow \quad z = \frac{1}{2}(X+Y)$$

$$f_X(x) = 2 - 2x \,, \quad x \in [0,1]$$

$$f_Y(y) = 2 - 2y \,, \quad y \in [0,1]$$

$$f_{XY}(x,y) = f_X(x) \cdot f_Y(y) = 4(1-x)(1-y) \quad , x,y \in [0,1]$$

$$f_Z(z) = \int_0^1 f_{XY}(x,y) \left| \frac{\partial y}{\partial z} \right| dx$$

$$\frac{\partial y}{\partial z} = \frac{\partial}{\partial z}(2z - x) = 2$$

$$= \int_0^1 4(1-x)(1-(2z-x)) \cdot 2 \, dx$$

$$= 8\int_0^1 (1-x)(1-2z+x) dx = 8 \int_0^1 1 \cancel{-x} - 2z + 2zx \cancel{+x} - x^2 dx$$

$$= 8\left(x \cancel{-\frac{1}{2}x^2} - 2zx + zx^2 \cancel{+\frac{1}{2}x^2} - \frac{1}{3}x^3\right)\Big|_0^1$$

$$= 8\left(1 - 2z + z - \frac{1}{3}\right) = 8\left(\frac{2}{3} - z\right) \quad ??$$

3)

a) $X \sim \mathcal{G}(p) \Leftrightarrow \mathbb{P}(X=k) = p(1-p)^{k-1}, \ k \in \mathbb{N}$

$$\mathbb{P}(X>k) = (1-p)^k, \ k \in \mathbb{N}$$

b) $X \sim \mathcal{E}(\lambda)_{\lambda>0} \Leftrightarrow \mathbb{P}(X \leq x) = 1-e^{-\lambda x}, \ x>0$

$$\Updownarrow \quad f_X(x) = \lambda e^{-\lambda x}, \ x>0.$$

$$\mathbb{P}(\cancel{X}>x) = e^{-\lambda x}, \ x>0$$

c) $X_n \sim \mathcal{G}\left(\frac{p}{n}\right) \rightarrow \tilde{p}$

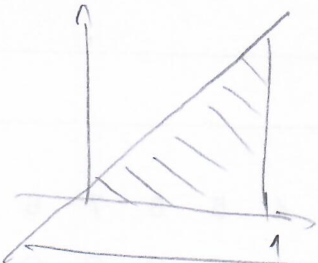
$x \in \mathbb{N}$

$$\mathbb{P}\left(\frac{X_n}{n}>x\right) = \mathbb{P}(X_n > nx) = (1-\tilde{p})^{nx}$$

$$= \left(1-\frac{p}{n}\right)^{nx}$$

$$= \left[\left(1-\frac{1}{\left(\frac{n}{p}\right)}\right)^{\frac{n}{p}x}\right]^p = \left(e^{-x}\right)^p = e^{-px}.$$

4) $\int_0^1 \int_0^x \frac{1}{x}\, dy\, dx = C \Rightarrow C = 1$



$f_Y(y) = \int_y^1 \frac{1}{x} dx = \ln|x| \Big|_y^1 = -\ln y \,,\; y \in \langle 0,1 \rangle$

$g_Z(z) = \int_{-\infty}^{+\infty} f(x,y) \left|\frac{\partial y}{\partial z}\right| dx$

$\cancel{z} = x+y \quad y = z-x \quad \frac{\partial y}{\partial z} = 1$

$\circ\; z \in \langle 0,1 \rangle \quad g_Z(z) = \int_{\frac{z}{2}}^{z} \frac{1}{x} dx = \ln 2$

$\circ\; z \in \langle 1,2 \rangle \quad g_Z(z) = \int_{\frac{z}{2}}^{1} \frac{1}{x} dx = \ln\frac{2}{z}$

$x, y \in [0,1] \Rightarrow z \in [0,2]$

$z \in \langle 0,1 \rangle \Rightarrow$

$F_Z(z) = \mathbb{P}(Z<z) = \int_0^{\frac{z}{2}} \frac{1}{x} \int_0^{x} dy\, dx + \int_{\frac{z}{2}}^{z} \frac{1}{x} \int_0^{z-x} dy\, dx$

$= \cancel{\frac{z}{2}} + z\ln x \Big|_{\frac{z}{2}}^{z} - \cancel{\frac{z}{2}} = z(\ln z - \ln z + \ln 2)$

$= z\ln 2$

$z \in \langle 1,2 \rangle$

$F_Z(z) = \int_0^{\frac{z}{2}} \frac{1}{x} \int_0^{x} dy\, dx + \int_{\frac{z}{2}}^{1} \frac{1}{x} \int_0^{z-x} dy\, dx$

$= \frac{z}{2} + z\left[\ln 1 - \ln\frac{z}{2}\right] - 1 + \frac{z}{2}$

$= z - 1 - z\ln\frac{z}{2} = z\left[1 - \ln\frac{z}{2}\right] - 1 \quad , z \in \langle 1,2 \rangle$

$f_Z(z) = \begin{cases} \ln 2 & z \in \langle 0,1 \rangle, \\ -\ln\left(\frac{z}{2}\right) & z \in \langle 1,2 \rangle \end{cases}$

5) $\mathbb{P}(g-0.1 \leq M_n \leq g+0{,}1) \geq 0{,}95$?

$$M_n = \frac{Y_1+Y_2+\dots+Y_n}{n} = \frac{\sum_{i=1}^{n}(g+X_i)}{n} = \frac{ng + \sum_{i=1}^{n} X_i}{n} = g + \frac{1}{n}\sum_{i=1}^{n} X_i$$

$$\mathbb{P}(g-0{,}1 \leq M_n \leq g+0{,}1)$$

$$= \mathbb{P}\left(g-0{,}1 \leq g + \frac{1}{n}\sum_{i=1}^{n} X_i \leq g+0{,}1\right)$$

$$= \mathbb{P}\left(-0{,}1 \leq \frac{1}{n}\sum_{i=1}^{n} X_i \leq 0{,}1\right) = (*)$$

Prema CGT je $Y := \dfrac{\frac{1}{n}\sum_{i=1}^{n} X_i}{\frac{\sigma}{\sqrt{n}}} \sim \mathcal{N}(0,1)$, $\sigma = 2$

$$\Rightarrow (*) = \mathbb{P}\left(\frac{-0{,}1}{\frac{2}{\sqrt{n}}} \leq Y \leq \frac{0{,}1}{\frac{2}{\sqrt{n}}}\right)$$

$$= \mathbb{P}\left(\frac{-0{,}1\cdot\sqrt{n}}{2} \leq Y \leq \frac{0{,}1\cdot\sqrt{n}}{2}\right) = \Phi^*\left(\frac{0{,}1\cdot\sqrt{n}}{2}\right) \geq 0{,}95$$

$$\Leftrightarrow \quad \frac{0{,}1\sqrt{n}}{2} \geq 1{,}96$$

Odnosno $\sqrt{n} \geq 39{,}2 \Rightarrow \boxed{n \geq 1537}$

6.) $f(\lambda, x_1, \dots, x_n) = \prod_{i=1}^{n} f(\lambda, x_i) = \frac{\lambda^n}{2^n \sqrt{x_1 \cdot \dots \cdot x_n}} e^{-\lambda \sum_{i=1}^{n} \sqrt{x_i}}$

$\ln f(\lambda, x_1, \dots, x_n) = n \ln \lambda - n \ln 2 - \frac{1}{2} \sum_{i=1}^{n} \ln x_i - \lambda \sum_{i=1}^{n} \sqrt{x_i}$

$\frac{\partial}{\partial \lambda} \left(\ln f(\lambda, x_1, \dots, x_n) \right) = \frac{n}{\lambda} - \sum_{i=1}^{n} \sqrt{x_i}$

$\Rightarrow \boxed{\lambda = \frac{n}{\sum_{i=1}^{n} \sqrt{x_i}}}$ MLE