① a) $P(A_i) = \dfrac{\binom{4}{1}\binom{48}{12}}{\binom{52}{13}}$

$P(A_1 \cap A_2 \cap A_3 \cap A_4) =$

$= P(A_1 | A_2 \cap A_3 \cap A_4) \cdot P(A_3 | A_1 \cap A_2) \cdot P(A_2 | A_1) \cdot P(A_1)$

$= 1 \cdot \dfrac{\binom{2}{1}\binom{24}{12}}{\binom{26}{13}} \cdot \dfrac{\binom{3}{1}\binom{36}{12}}{\binom{39}{13}} \cdot \dfrac{\binom{4}{1}\binom{48}{12}}{\binom{52}{13}}$

$= \dfrac{13^4}{\binom{52}{4}}$

b) Neka su $A, B \in \mathcal{F}$ i $P(A), P(B) \in \langle 0, 1 \rangle$

Tvrdimo $P(A \cap B) = P(A)P(B) \Rightarrow P(A^c \cap B^c) = P(A^c)P(B^c)$

$P(A^c \cap B^c) = P((A \cup B)^c) = 1 - P(A \cup B) = 1 - P(A) - P(B) + P(A \cap B)$

$= 1 - P(A) - P(B) + P(A)P(B) = (1 - P(A))(1 - P(B)) =$

$= P(A^c)P(B^c)$

② Označimo događaje

H_A = poslana je AAAA

H_B = — " — BBBB

H_C = — " — CCCC

A = primljen je znak A

B = — " — B

C = — " — C

$P(H_A) = P(H_C) = 0.3$

$P(H_B) = 0.4$

$P(A | H_A) = 0.6$

$P(B | H_A) = P(C | H_A) = 0.2$

$$P(H_A | \{ACAB\}) = \frac{P(H_A) P(\{ACAB\} | H_A)}{\sum_{x \in \{A,B,C\}} P(H_x) P(\{ACAB\} | H_x)}$$

$$= \frac{0.3 \cdot 0.6^2 \cdot 0.2^2}{0.3 \cdot 0.6^2 \cdot 0.2^2 + 0.4 \cdot 0.6 \cdot 0.2^3 + 0.3 \cdot 0.6 \cdot 0.2^3}$$

$= 9/16$

③ Iz distribucije imamo $b = 1 - 7a$, iz varijance

a) je $2a + 16b - (4b - 2a)^2 = 4$, pa je $a = 1/10$ i $b = 3/10$.

$$P(X > EX) = P(X > 1) = 3/10.$$

b)
$$X_1 + X_2 \sim \begin{pmatrix} -2 & -1 & 0 & 3 & 4 & 8 \\ .04 & .2 & .25 & .12 & .3 & .09 \end{pmatrix}$$

Kako su X_1 i X_2 nezavisne to su i X_1 i X_2^2 pa je i $r(X_1, X_2^2) = 0$.

④ $\varphi_X(t) = E(e^{itX}) = \dfrac{pe^{it}}{1 - qe^{it}}$

a) $EX = -i\varphi_X'(0) = \dfrac{1}{p}$

$$\mathrm{Var}\,X = -\varphi_X''(0) + (\varphi_X'(0))^2 = \frac{1-p}{p^2}$$

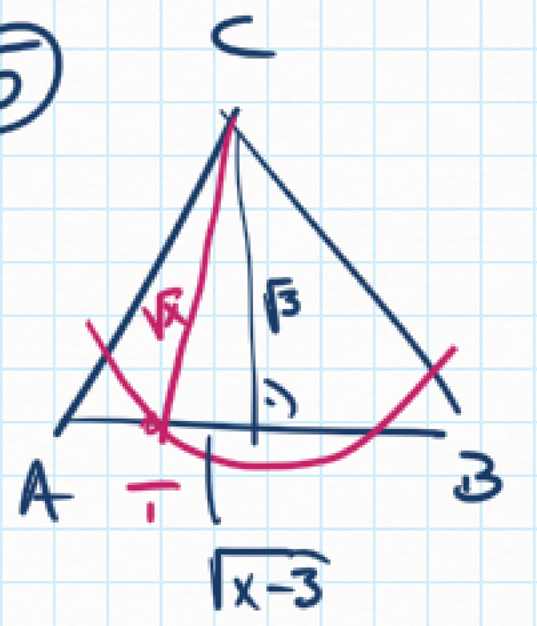
b) Stavimo $p = P(A \cup B) = p_A + p_B$, označimo sa p_n vjerojatnost da je pokus završio u n-tom ponavljanju realizacijom događaja A.

$p_n = (1-p)^{n-1} \cdot p_A$

Sada je vjerojatnost da se A dogodio prije B

$$\sum_{n=1}^{\infty} (1-p)^{n-1} p_A = \frac{p_A}{p}.$$

⑤



$X = |TC|^2$

$F(x) = P(X < x) = \sqrt{x-3}, \quad x \in [3, 4]$

$f(x) = \dfrac{1}{2\sqrt{x-3}}, \quad x \in [3, 4]$

$EX = \displaystyle\int_{-\infty}^{\infty} x f(x)\,dx = 10/3$

$\mathrm{Var}\,X = EX^2 - (EX)^2$

$= \dfrac{56}{5} - \left(\dfrac{10}{3}\right)^2$

$= 4/45$