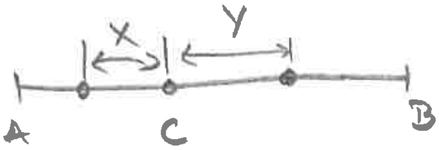
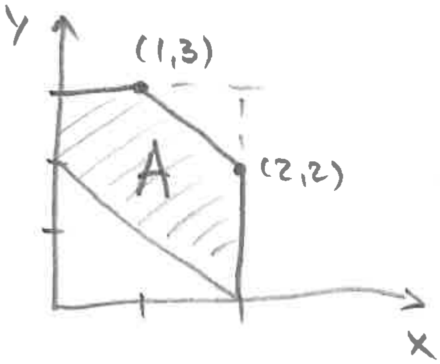
1



$$\Omega = [0,2] \times [0,3]$$

$X \sim U(0,2)$

$Y \sim U(0,3)$



$$P(2 < X+Y < 4) = \frac{m(A)}{m(\Omega)}$$

$$= 1 - \frac{m(\Omega \setminus A)}{m(\Omega)}$$

$$= \frac{7}{12}$$

②

a) Neka je $\{H_1, \dots, H_n\}$ potpun sustav događaja. Za svaki događaj $A \subset \Omega$ vrijedi

$$P(A) = \sum_{i=1}^{n} P(H_i) P(A \mid H_i).$$

Ako je još $P(A) > 0$, tada za svaki $i \in \{1, \dots, n\}$ vrijedi

$$P(H_i \mid A) = \frac{P(H_i) P(A \mid H_i)}{\sum_{j=1}^{n} P(H_j) P(A \mid H_j)}$$

Dokaz:

Po definiciji je $P(A \mid H_i) = \dfrac{P(A \cap H_i)}{P(H_i)}$

$$P(A) = P((A \cap H_1) \cup \dots \cup (A \cap H_n))$$
$$= P(A \cap H_1) + \dots + P(A \cap H_n)$$
$$= \sum_{i=1}^{n} P(H_i) P(A \mid H_i)$$

$$P(H_i \mid A) = \frac{P(H_i \cap A)}{P(A)} = \frac{P(H_i) P(A \mid H_i)}{\sum_{j=1}^{n} P(H_j) P(A \mid H_j)}$$

b) $A = \{$mail sadrži riječ „nagrada"$\}$ $\qquad P(H_1) = 0.5$

$H_1 = \{$mail je spam$\}$ $\qquad P(H_2) = 0.5$

$H_2 = \{$mail nije spam$\}$ $\qquad P(A \mid H_1) = 0.01$

$P(A \mid H_2) = 10^{-5}$

$$P(H_1 \mid A) = \frac{P(H_1) P(A \mid H_1)}{P(H_1) P(A \mid H_1) + P(H_2) P(A \mid H_2)} = \frac{1000}{1001} = 0.9990$$

③

a)

$x \backslash y$	1	2	3	p_i
1	.1	.05	.05	.2
3	.05	.2	.05	.3
5	.1	.2	.2	.5
q_j	.25	.45	.3	1

b) $(Y \mid X=3) \sim \begin{pmatrix} 1 & 2 & 3 \\ \frac{1}{6} & \frac{2}{3} & \frac{1}{6} \end{pmatrix}$

c) X i Y su nezavisne ako za sve x_i i y_j vrijedi: $P(X=x_i, Y=y_j) = P(X=x_i)\,P(Y=y_j)$.
No $P(X=1, Y=1) = 0.1$ dok je $P(X=1)\cdot P(Y=1) = 0{,}125$, prema tome X i Y nisu nezavisne.

d)
$$P(X \geq 3) = P(X=3) + P(X=5) = 0.8$$

$$P(X \geq 3 \mid Y \leq 2) = \frac{P(X \geq 3, Y \leq 2)}{P(Y \leq 2)} =$$

$$= \frac{0.05 + 0.2 + 0.1 + 0.2}{0.25 + 0.45}$$

$$= \frac{11}{14} \doteq 0.7857$$

(4)

a) Neka je X s.v. s eksponencijalnom razdiobom tada za svaki $x, t > 0$ vrijedi

$$P(X < x+t \mid X > t) = P(X < x)$$

Dokaz:

$$P(X < x+t \mid X > t) = \frac{P(X < x+t, X > t)}{P(X > t)}$$

$$= \frac{P(t < X < x+t)}{P(X > t)} = \frac{F(x+t) - F(t)}{1 - F(t)}$$

$$= \frac{e^{-\lambda t} - e^{-\lambda(x+t)}}{e^{-\lambda t}} = 1 - e^{-\lambda x} = P(X < x)$$

b) $F_X(x) = 1 - e^{-\lambda x}, \quad x > 0 \qquad E[X] = 2 \Rightarrow \lambda = \frac{1}{2}$

$$P(|X-2| > 1) = P(X < 1) + P(X > 3)$$

$$= 1 - e^{-\frac{1}{2}} + e^{-\frac{3}{2}}$$

$$= 0.617$$

(5)

a) $\vartheta_{X_k}(t) = e^{ia_k t - \frac{1}{2}\sigma_k^2 t^2}$, za $k=1,2$

$\vartheta_{s_k X_k}(t) = \vartheta_{X_k}(s_k t) = e^{ia_k s_k t - \frac{1}{2}\sigma_k^2 s_k^2 t^2}$, za $k=1,2$

$\vartheta_{s_1 X_1 + s_2 X_2}(t) = \vartheta_{s_1 X_1}(t) \cdot \vartheta_{s_2 X_2}(t)$

$= e^{i(a_1 s_1 + a_2 s_2)t - \frac{1}{2}(\sigma_1^2 s_1^2 + \sigma_2^2 s_2^2)t^2}$

što je karakteristična funkcija normalne razdiobe

$N(a_1 s_1 + a_2 s_2, \sigma_1^2 s_1^2 + \sigma_2^2 s_2^2)$

b) Zbog stabilnosti $X-Y \sim N(a, \sigma^2)$

$a = a_X - a_Y = 0$

$\sigma^2 = \sigma_X^2 + \sigma_Y^2 = 25$

$P(-c < X-Y < c) = 0,9$

$\Phi^*\left(\frac{c}{\sigma}\right) = 0,9$

$\frac{c}{\sigma} = \Phi^{*-1}(0,9)$

$\frac{c}{5} = 1.64$

$c = 8.2$

(6)

a) Stavimo $Z = \min\{X_1, \dots, X_n\}$

$$F_Z(x) = P(Z < x) = 1 - P(Z > x) = 1 - P(X > x)^n$$

$$= 1 - \left(\frac{1-x}{1-\alpha}\right)^n$$

$$f_Z(x) = n\left(\frac{1-x}{1-\alpha}\right)^{n-1} \frac{1}{1-\alpha} = \frac{n}{(1-\alpha)^n}(1-x)^{n-1}$$

$$E[Z] = \frac{n}{(1-\alpha)^n} \int_\alpha^1 x(1-x)^{n-1} dx = \left| \begin{matrix} t = 1-x \\ dt = -dx \end{matrix} \right| =$$

$$= -\frac{n}{(1-\alpha)^n} \int_{1-\alpha}^0 (1-t)t^{n-1} dt$$

$$= \frac{1}{n+1} + \frac{n}{n+1}\alpha \implies E[Y] = \frac{n}{n+1}\alpha$$

b) $S \quad \frac{n+1}{n}$.

⑥

a) t-test

$\alpha = 5\%$

$H_0 \ldots a = 30$

$H_1 \ldots a < 30$

$$\hat{t} = \frac{\bar{x} - 30}{s_x/\sqrt{n}} = \frac{29.327 - 30}{\sqrt{7.716/11}} = -0.80355$$

$$t_{10,\ 0.95} = 1.812$$

$$-0.80355 > -1.812$$

uz $\alpha = 5\%$ ne odbacujemo H_0

b)

$H_0 \ldots a_1 = a_2$ $\qquad \alpha = 0.1$ $\qquad \bar{y} = 30.62$ $\quad s_y^2 = 6.993$

$H_1 \ldots a_1 < a_2$

$$\hat{t} = \frac{\bar{x} - \bar{y}}{s_z}\sqrt{\frac{nm}{n+m}} = \frac{29.327 - 30.62}{2.715} \cdot \sqrt{\frac{110}{21}} = -1.11185$$

$t_{19,0.1} = -1.328 < \hat{t}$ $\quad$ ne odbacujemo H_0

Uz $\alpha = 0.1$ ne prihvaćamo hipotezu da je u Y veći prinos nego u X