(1) a) $K = \{1, \dots, 100\}$

$\Omega = \{ (x_1, x_2, x_3, x_4) \mid x_1 \in K, x_2 \in K \setminus \{x_1\}, x_3 \in K \setminus \{x_1, x_2\}, x_4 \in K \setminus \{x_1, x_2, x_3\} \}$

$|\Omega| = 100 \cdot 99 \cdot 98 \cdot 97$

$A = \{ x_1 < x_4 \}$

$x_1 = 1 \quad x_4 \in \{2, \dots, 100\}$

$x_1 = 2 \quad x_4 \in \{3, \dots, 100\}$

$\vdots$

$x_1 = i \quad x_4 \in \{i+1, \dots, 100\}$

$\vdots$

$x_1 = 99 \quad x_4 = 100$

$|A| = 98 \cdot 97 \left(\sum_{i=1}^{99} i \right)$

$|A| = 98 \cdot 97 \cdot 99 \cdot 100 / 2$

$P(A) = 1/2$

b) $AB = \{$ zatvorena je veza od A do B $\}$

$ACD = \{$ —— " —— preko C $\}$ ...

$P(AB) = P(ACB \cup ADB)$

$= 1 - P(\overline{ACB} \cap \overline{ADB})$

$= 1 - P(\overline{ACB}) \cdot P(\overline{ADB})$

$= 1 - (1 - P(ACD))(1 - P(ADB))$

$= 1 - (1 - p_{AC} \cdot P(CEB \cup CFD))(1 - p_{AD} \cdot p_{DB})$

$= 1 - \left(1 - p_{AC} \cdot \left(1 - (1 - p_{CE} p_{ED})(1 - p_{CF} p_{FB})\right)\right)(1 - p_{AD} \cdot p_{DB}) = 0.9573$

(2) $X \sim \begin{pmatrix} 3 & 6 & 9 & 12 & 15 & 18 \\ 1/6 & 1/6 & 1/6 & 1/6 & 1/6 & 1/6 \end{pmatrix}$

$Y \sim \begin{pmatrix} 0 & 3 \\ 1/3 & 2/3 \end{pmatrix}$ $\qquad XY \sim \begin{pmatrix} 0 & 3 \cdot 3^2 & 4 \cdot 3^2 & 5 \cdot 3^2 & 6 \cdot 3^2 \\ 1/3 & 1/6 & 1/6 & 1/6 & 1/6 \end{pmatrix}$

$E(Z) = E(X) + E(Y)$

$= 3 \cdot 3.5 + 2 = 12.5$

$E(Z^2) = E(X^2) + E(Y^2) + 2E(XY)$

$= 196.5$

$V(Z) = E(Z^2) - E(Z)^2$

$= 40.25$

③ X - broj grešaka na stranici

$X \sim B(220, 1/200)$

$P(x=0) = (1 - \frac{1}{200})^{220} = .332$

$P(x=1) = 220 \cdot \frac{1}{200} \cdot (1 - \frac{1}{200})^{219} = .367$

$P(x \geq 2) = 1 - P(x=1) - P(x=0) = .301$

ili

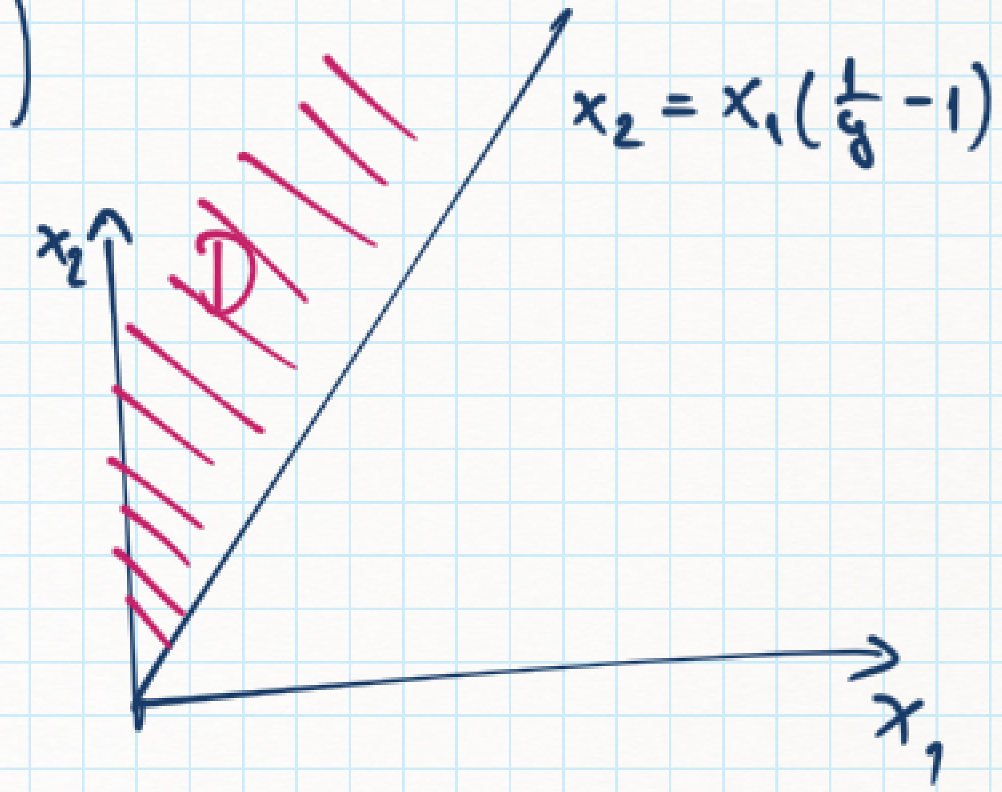
$X \sim P(\frac{220}{200})$

$P(x=0) = .333$

$P(x=1) = .366$

$P(x \geq 2) = .301$

④ $F_Y(y) = P\left(\frac{X_1}{X_1 + X_2} < y\right) = P\left(X_1\left(\frac{1}{y} - 1\right) < X_2\right)$



$$= \int_D f_{X_1}(x_1) \cdot f_{X_2}(x_2)\, dx_1\, dx_2$$

$$= \int_0^{+\infty} \lambda_1 e^{-\lambda_1 x_1} \int_{x_1(\frac{1}{y}-1)}^{+\infty} \lambda_2 e^{-\lambda_2 x_2}\, dx_2\, dx_1$$

$$= \int_0^{+\infty} \lambda_1 e^{-\lambda_1 x_1} \left(-e^{-\lambda_2 x_2}\right)\Big|_{x_1(\frac{1}{y}-1)}^{+\infty} dx_1$$

$$= \int_0^{+\infty} \lambda_1 e^{-\lambda_1 x_1} \cdot e^{-\lambda_2 x_1(\frac{1}{y}-1)}\, dX_1$$

$$= \int_0^{+\infty} \lambda_1\, e^{-(\lambda_1 + \lambda_2(\frac{1}{y}-1))x_1}\, dx_1$$

$$= -\frac{\lambda_1}{\lambda_1 + \lambda_2(\frac{1}{y}-1)}\, e^{-(\lambda_1 + \lambda_2(\frac{1}{y}-1))x_1}\Big|_0^{+\infty}$$

$$= \frac{\lambda_1}{\lambda_1 + \lambda_2(\frac{1}{y}-1)}$$

5 a) X_i – greška zaokruživanja i-tog broja

$X_i \sim U(-\frac{1}{2}, \frac{1}{2})$

$E(X_i) = 0$

$D(X_i) = \frac{1}{12}$

$X = \sum X_i$

$$Z = \frac{X - nE(X_i)}{\sqrt{nD(X_i)}} \sim N(0,1)$$

$P(X > 3) = P(Z > 1) = \frac{1}{2}(1 - \phi^*(1)) = 0.1587$

b) $Y_i = |X_i|$

$E(Y_i) = E(|X_i|) = \frac{1}{4}$

$E(Y_i^2) = E(|X_i|^2) = \frac{1}{12}$

$D(Y_i) = E(Y_i^2) - E(Y_i)^2 = 1/48$

$$Z = \frac{\sum Y_i - nE(Y_i)}{\sqrt{nD(Y_i)}}$$

$P(\sum Y_i < 30) =$

$= P(Z < 2)$

$= \frac{1}{2}(1 + \phi^*(2))$

$= 0.97725$

6 $X_i \sim U(\theta, \theta+1)$

$f_{X_i}(x_i) = \frac{1}{\theta+1-\theta} = 1, \ \forall x \in (\theta, \theta+1)$

$$L(\theta) = \begin{cases} f_{X_1}(x_1) \cdot \ldots \cdot f_{X_n}(x_n) = 1, & x_i \in (\theta, \theta+1), \forall i \\ 0, & \text{inače} \end{cases}$$

L ima maksimum u 1 koji nije jedinstven.

Uvjet $x_i \in (\theta, \theta+1) \ \forall i$ je ekvivalentan sa

$x_i \geq \theta, \forall i$ i $x_i \leq \theta+1, \forall i$

$\iff$

$\min\{x_i \mid i=1 \ldots n\} \geq \theta$ i $\max\{x_i \mid i=1 \ldots n\} \leq \theta+1$

dakle

$$L(\theta) = \begin{cases} 1, & \max\{x_i\} - 1 \leq \theta \leq \min\{x_i\} \\ 0, & \text{inače} \end{cases}$$

tj. svaki $\hat{\theta}$ između $\max\{x_i\} - 1$ i $\min\{x_i\}$ je MLE procjenitelj za θ.