1 Evaluate the sum $\sum_k \epsilon_{ijk}\epsilon_{lmk}$

(a) $i = j$

Since $\epsilon_{ijk} = 0$ when $i = j$,

$$\sum_k \epsilon_{ijk}\epsilon_{lmk} = 0$$

(b) $i = l$

When $i = 1$,

j \ k	1	2	3
1	0	0	0
2	0	0	1
3	0	-1	0

m \ k	1	2	3
1	0	0	0
2	0	0	1
3	0	-1	0

Thus,

$$\sum_k \epsilon_{1jk}\epsilon_{1mk} = \begin{cases} 1 & \text{if } j = m \\ 0 & \text{if } j \neq m \end{cases}$$

Likewise, when $i = 2$,

j \ k	1	2	3
1	0	0	-1
2	0	0	0
3	1	0	0

m \ k	1	2	3
1	0	0	-1
2	0	0	0
3	1	0	0

Thus,

$$\sum_k \epsilon_{2jk}\epsilon_{2mk} = \begin{cases} 1 & \text{if } j = m \\ 0 & \text{if } j \neq m \end{cases}$$

Last

j \ k	1	2	3
1	0	1	0
2	-1	0	0
3	0	0	0

m \ k	1	2	3
1	0	1	0
2	-1	0	0
3	0	0	0

Thus,

$$\sum_k \epsilon_{3jk}\epsilon_{3mk} = \begin{cases} 1 & \text{if } j = m \\ 0 & \text{if } j \neq m \end{cases}$$

In sum,

$$\sum_k \epsilon_{ijk}\epsilon_{lmk} = \sum_k \epsilon_{ijk}\epsilon_{imk} = \delta_{jm} = \begin{cases} 1 & \text{if } j = m \\ 0 & \text{if } j \neq m \end{cases}$$

(c) $i = m$

(d) $j = l$

(e) $j = m$

(f) $l = m$

(g) $i \neq j$ or m

(h) $j \neq l$ or m

2 Show that $\sum_k \epsilon_{ijk}\epsilon_{lmk} = \delta_{il}\delta_{jm} - \delta_{im}\delta_{jl}$