(a) $\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

From the law of cosines,

$$|\vec{a} - \vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2 - 2|\vec{a}||\vec{b}|\cos(\alpha - \beta) \tag{1}$$

$$\begin{aligned}
|\vec{a} - \vec{b}|^2 &= (a\cos\alpha - b\cos\beta)^2 - (a\sin\alpha - b\sin\beta)^2 \\
&= (a^2\cos^2\alpha - 2ab\cos\alpha\cos\beta + b^2\cos^2\beta) + (a^2\sin\alpha - 2ab\sin\alpha\sin\beta + b^2\sin^2\beta) \\
&= a^2 + b^2 - 2ab(\cos\alpha\cos\beta + \sin\alpha\sin\beta
\end{aligned}$$

$$|\vec{a}|^2 + |\vec{b}|^2 - 2|\vec{a}||\vec{b}|\cos(\alpha - \beta) = a^2 + b^2 - 2ab\cos(\alpha - \beta)$$

$$\therefore\ \cos(\alpha - \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$

(b) $\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$

From the Pythagorean identity,

$$\sin^2(\alpha-\beta)+\cos^2(\alpha-\beta)=1 \tag{2}$$

$$\begin{aligned}
\sin(\alpha-\beta) &= \sqrt{1-\cos^2(\alpha-\beta)} \\
&= \sqrt{1-(\cos\alpha\cos\beta+\sin\alpha\sin\beta)^2} \\
&= \sqrt{1-(\cos^2\alpha\cos^2\beta+2\cos\alpha\cos\beta\sin\alpha\sin\beta+\sin^2\alpha\sin^2\beta)} \\
&= \sqrt{1-\{\cos^2\alpha(1-\sin^2\beta)+2\cos\alpha\cos\beta\sin\alpha\sin\beta+\sin^2\alpha(1-\cos^2\beta)\}} \\
&= \sqrt{1-(\cos^2\alpha-\cos^2\alpha\sin^2\beta+2\cos\alpha\cos\beta\sin\alpha\sin\beta+\sin^2\alpha-\sin^2\alpha\cos^2\beta)} \\
&= \sqrt{\cos^2\alpha\sin^2\beta-2\cos\alpha\cos\beta\sin\alpha\sin\beta+\sin^2\alpha\cos^2\beta} \\
&= \sqrt{(\cos\alpha\sin\beta-\sin\alpha\cos\beta)^2} \\
&= \cos\alpha\sin\beta-\sin\alpha\cos\beta
\end{aligned}$$

$$\therefore\ \sin(\alpha-\beta)=\cos\alpha\sin\beta-\sin\alpha\cos\beta$$