(a) $\sum_{i,j} \epsilon_{ijk}\ \delta_{ij} = 0$

From the definition of the **Kronecker delta symbol** (Equation (1.14))

$$\delta_{ik} = \begin{cases} 0 & \text{if } i \neq k \\ 1 & \text{if } i = k \end{cases} \tag{1.14}$$

and **permutation symbol** (or **Levi-Civita density**) (Equation (1.67))

$$\epsilon_{ijk} = \begin{cases} 0 & \text{if any index is equal to any other index} \\ 1 & \text{if } i,j,k \text{ form and } \textit{even} \text{ permutation of 1,2,3}, \\ -1 & \text{if } i,j,k \text{ form and } \textit{odd} \text{ permutation of 1,2,3} \end{cases} \tag{1.67}$$

When $i = j$,

$$\begin{aligned} \delta_{ij} &= 1 \\ \epsilon_{ijk} &= 0 \end{aligned}$$

$$\therefore \sum_{ij} \delta_{ij}\ \epsilon_{ijk} = 0,$$

and when $i \neq j$,

$$\begin{aligned} \delta_{ij} &= 0 \\ \epsilon_{ijk} &= 1 \end{aligned}$$

$$\therefore \sum_{ij} \delta_{ij}\ \epsilon_{ijk} = 0.$$

Therfore,

$$\therefore \sum_{ij} \delta_{ij}\ \epsilon_{ijk} = 0.$$

(b) $\sum_{j,k} \epsilon_{ijk}\ \epsilon_{ljk} = 2\delta_{il}$

$$\begin{aligned} \sum_{j,k} \epsilon_{ijk}\ \epsilon_{ljk} &= \cancel{\epsilon_{i11}\ \epsilon_{l11}} + \epsilon_{i12}\ \epsilon_{l12} + \epsilon_{i13}\ \epsilon_{l13} + \epsilon_{i21}\ \epsilon_{l21} + \cancel{\epsilon_{i22}\ \epsilon_{l22}} + \epsilon_{i23}\ \epsilon_{l23} \\ &\quad + \epsilon_{i31}\ \epsilon_{l31} + \epsilon_{i32}\ \epsilon_{l32} + \cancel{\epsilon_{i33}\ \epsilon_{l33}} \\ &= \epsilon_{i12}\ \epsilon_{l12} + \epsilon_{i13}\ \epsilon_{l13} + \epsilon_{i21}\ \epsilon_{l21} + \epsilon_{i23}\ \epsilon_{l23} + \epsilon_{i31}\ \epsilon_{l31} + \epsilon_{i32}\ \epsilon_{l32} \end{aligned}$$

If $i = l$,

$$\begin{aligned} \sum_{j,k} \epsilon_{ijk}\ \epsilon_{ljk} &= (\epsilon_{i12})^2 + (\epsilon_{i13})^2 + (\epsilon_{i21})^2 + (\epsilon_{i23})^2 + (\epsilon_{i31})^2 + (\epsilon_{i32})^2 \\ &= 0^2 + 0^2 + 0^2 + 1^2 + 0^2 + (-1)^2 \quad = 2 \qquad \text{if } i = 1 \\ &= 0^2 + (-1)^2 + 0^2 + 0^2 + 1^2 + 0^2 \quad = 2 \qquad \text{if } i = 2 \\ &= 1^2 + 0^2 + (-1)^2 + 0^2 + 0^2 + 0^2 \quad = 2 \qquad \text{if } i = 3. \end{aligned}$$

$$\therefore \sum_{j,k} \epsilon_{ijk}\ \epsilon_{ljk} = 2 \qquad \text{when } i = l$$

And if $i \neq l$,

$$\begin{aligned} \sum_{j,k} \epsilon_{ijk}\ \epsilon_{ljk} &= \epsilon_{i12}\ \epsilon_{l12} + \epsilon_{i13}\ \epsilon_{l13} + \epsilon_{i21}\ \epsilon_{l21} + \epsilon_{i23}\ \epsilon_{l23} + \epsilon_{i31}\ \epsilon_{l31} + \epsilon_{i32}\ \epsilon_{l32} \\ &= 0 \cdot 0 + 0 \cdot (-1) + 0 \cdot 0 + 1 \cdot 0 + 0 \cdot 1 + (-1) \cdot 0 \quad = 0 \qquad \text{when } i = 1,\ l = 2 \\ &= 0 \cdot (-1) + 0 \cdot 0 + 0 \cdot (-1) + 1 \cdot 0 + 0 \cdot 0 + (-1) \cdot 0 \quad = 0 \qquad \text{when } i = 1,\ l = 3 \\ &= 0 \cdot 0 + (-1) \cdot 0 + 0 \cdot 0 + 0 \cdot 1 + 1 \cdot 0 + 0 \cdot (-1) \quad = 0 \qquad \text{when } i = 2,\ l = 1 \\ &= 0 \cdot 1 + (-1) \cdot 0 + 0 \cdot (-1) + 0 \cdot 0 + 1 \cdot 0 + 1 \cdot 0 \quad = 0 \qquad \text{when } i = 2,\ l = 3 \\ &= (-1) \cdot 0 + 0 \cdot 0 + (-1) \cdot 0 + 0 \cdot 1 + 0 \cdot 0 + 0 \cdot (-1) \quad = 0 \qquad \text{when } i = 3,\ l = 1 \\ &= (-1) \cdot 0 + 0 \cdot (-1) + (-1) \cdot 0 + 0 \cdot 0 + 0 \cdot 1 + 0 \cdot 0 \quad = 0 \qquad \text{when } i = 3,\ l = 2 \end{aligned}$$

$$\therefore \sum_{j,k} \epsilon_{ijk}\ \epsilon_{ljk} = 0 \qquad \text{when } i \neq l$$

Therefore,

$$\sum_{j,k} \epsilon_{ijk}\ \epsilon_{ljk} = 2\delta_{il} = \begin{cases} 0 & \text{if } i \neq l \\ 2 & \text{if } i = l \end{cases}$$

(c) $\sum_{i,j,k} \epsilon_{ijk}\ \epsilon_{ijk} = 6$

When $i = 1$, then the ϵ_{ijk} is

j \ k	1	2	3
1	0	0	0
2	0	0	1
3	0	-1	0

When $i = 2$, then the ϵ_{ijk} is

j \ k	1	2	3
1	0	0	-1
2	0	0	0
3	1	0	0

When $i = 3$, then the ϵ_{ijk} is

j \ k	1	2	3
1	0	1	0
2	-1	0	0
3	0	0	0

Therefore,

$$\sum_{i,j,k} \epsilon_{ijk}\ \epsilon_{ijk} = 1^2 + (-1)^2 + (-1)^2 + 1^2 + 1^2 + (-1)^2 = 6$$