

Note

SM GROUP

2.4

$$E = \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2 - G \frac{m_1 m_2}{|\vec{r}_2 - \vec{r}_1|}$$

$$\mu \equiv \frac{m_1 m_2}{m_1 + m_2}, \quad \vec{r}_1 = -\frac{\mu}{m_1} \vec{r}, \quad \vec{r}_2 = \frac{\mu}{m_2} \vec{r}$$

$$r = |\vec{r}_2 - \vec{r}_1|$$

$$E = \frac{1}{2} \mu v^2 - G \frac{\mu M}{r} \quad M = m_1 + m_2$$

2.5

$$\vec{L} = m_1 \vec{r}_1 \times \vec{v}_1 + m_2 \vec{r}_2 \times \vec{v}_2$$

$$= m_1 \left( +\frac{\mu}{m_1} \vec{r} \right) \times \left( +\frac{\mu}{m_1} \vec{v} \right)$$

$$+ m_2 \left( -\frac{\mu}{m_2} \vec{r} \right) \times \left( -\frac{\mu}{m_2} \vec{v} \right)$$

$$= \frac{\mu^2}{m_1} (\vec{r} \times \vec{v}) + \frac{\mu^2}{m_2} (\vec{r} \times \vec{v})$$

$$= \mu^2 (\vec{r} \times \vec{v}) \left( \frac{1}{m_1} + \frac{1}{m_2} \right)$$

$$= \mu^2 (\vec{r} \times \vec{v}) \left( \frac{m_1 + m_2}{m_1 m_2} \right)$$

$$= \mu (\vec{r} \times \vec{v})$$

$$= \vec{L}$$

Note

SM GROUP

2.6

total orbital angular momentum

(a)

$$L = \mu \sqrt{A^2 a^2 (1 - e^2)}$$

$$\text{or } A = \frac{L}{\mu} \frac{1}{p}$$

$$\pi a b = \frac{1}{2} \cdot \frac{L}{\mu} p$$

$$\therefore L = \frac{2\pi a b \mu}{p}$$

mvr

$$L^2 = \frac{4\pi^2 a^2 b^2 \mu^2}{p^2} = \frac{4\pi^2 a^4 (1 - e^2) \mu^2}{p^2}$$

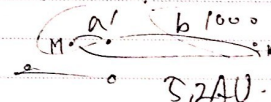
$$a = 5.2 \text{ AU}, \quad e = 0.048, \quad p = 11.86 \text{ yr}$$

$$M_\odot = 1.99 \times 10^{30} \text{ kg}, \quad M_J = 317.83 \times 5.97 \times 10^{27} \text{ kg} = 1.90 \times 10^{27} \text{ kg}$$

$$L = 2.72 \times 10^{28} \text{ AU}^2 / \text{yr}$$

(b)

Center of mass



$$a + b = 5.2 \text{ AU}$$

1000 : 1

$$\frac{M_\odot + M_J}{M_\odot} (3.2 - a')$$

$$a' + b = 5.2$$

$$19 \text{ km}$$

$$15.19 \times 10^3$$

(b)

$$p^2 = \frac{4\pi^2}{GM_{\oplus}} a^3$$

$$a^3 = \frac{p^2}{4\pi^2} GM_{\oplus}$$

$$= \frac{(86400 \text{ sec})^2}{4\pi^2} \cdot (6.673 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^{-2}) \cdot (5.973 \times 10^{24} \text{ kg})$$

$$24 \text{ hr} \times \frac{3600 \text{ sec}}{1 \text{ hr}} = 86400$$

$$a = 42240313.158 \text{ m}$$

$$= 42240 \text{ km}$$

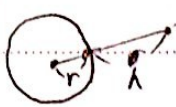
$$= h + r_{\oplus}$$

$$\therefore h = (42240 - 6378) \text{ km}$$

$$= 35862 \text{ km}$$



2.8

 ~~$h = 610 \text{ km}$~~ 

$$r = 6.378 \times 10^6 \text{ m}$$

(A)

$$h = 610 \text{ km} = 610 \times \frac{1000 \text{ m}}{1 \text{ km}} = 6.10 \times 10^5 \text{ m}$$

$$= 610000 \text{ m}$$

$$R = r + h$$

$$= 6.378 \times 10^6 \text{ m} + 6.10 \times 10^5 \text{ m}$$

$$= 6.988 \times 10^6 \text{ m}$$

$$= 6.988 \times 10^6 \text{ m}$$

$$P^2 = \frac{4\pi^2}{GM_{\oplus}} R^3 \quad ; \text{ Kepler's third law}$$

$$P^2 = \frac{4\pi^2}{(6.673 \times 10^{-11} \text{ N} \cdot \text{m}^2 \cdot \text{kg}^{-2}) \cdot (5.9736 \times 10^{24} \text{ kg})} \cdot (6.988 \times 10^6 \text{ m})^3$$

$$\text{kg} \cdot \text{m} / \text{sec}^2$$

$$= 3.3796 \times 10^7 \text{ sec}^2$$

$$P = 5813 \text{ sec}$$