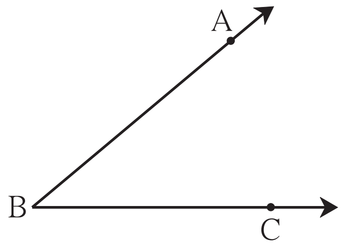
4 Angles and Pairs of Angles

Let's recall.



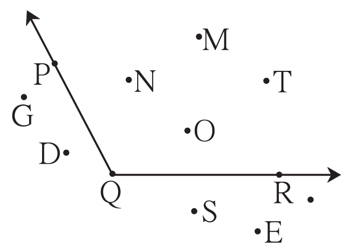
- Write the name of the angle shown alongside ....
- Write the name of its vertex ....
- Write the names of its arms ....
- Write the names of the points marked on its arms ....

Let's learn.

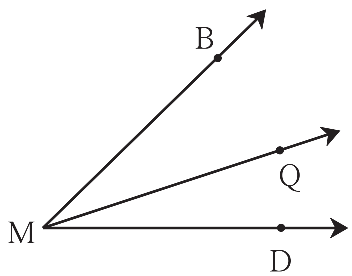
The Interior and Exterior of an Angle

In the plane of the figure alongside, the group of points like point N, point M, point T which are not on the arms of the angle, form the **interior** of the ∠PQR.

The group of points in the plane of the angle like point G, point D, point E, which are neither on the arms of the angle nor in its interior, form the **exterior** of the angle.



Adjacent Angles



Look at the angles in the figure alongside. The ray MQ is a common arm of the angles ∠BMQ and ∠QMD while M is their common vertex. The interiors of these angles do not have a single common point. They may be said to be neighbouring angles. Such angles are called adjacent angles.

Adjacent angles have one common arm and the other arms lie on opposite sides of the common arm. They have a common vertex. Adjacent angles have separate interiors.

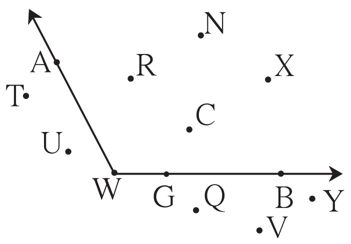
In the given figure, MB is the common arm of the angles ∠BMD and ∠BMQ. However, they are not adjacent angles because they do not have separate interiors.

Now I know!

Two angles which have a common vertex, a common arm and separate interiors are said to be adjacent angles.

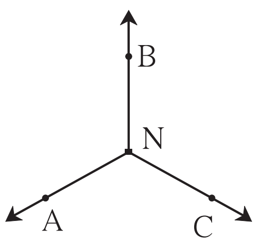
Practice Set 15

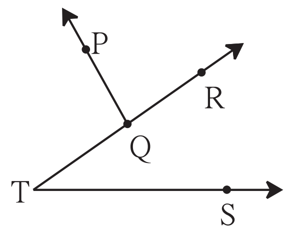
1. Observe the figure and complete the table for ∠AWB.



Points in the interior	
Points in the exterior	
Points on the arms of the angles	

2. Name the pairs of adjacent angles in the figures below.



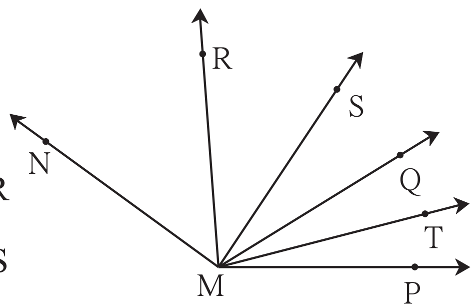


3. Are the following pairs adjacent angles?

   If not, state the reason.

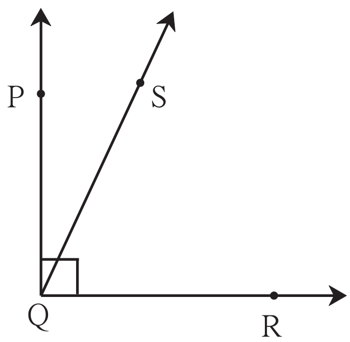
   (i) ∠PMQ and ∠RMQ (ii) ∠RMQ and ∠SMR

   (iii) ∠RMS and ∠RMT (iv) ∠SMT and ∠RMS



Let's learn.

Complementary Angles

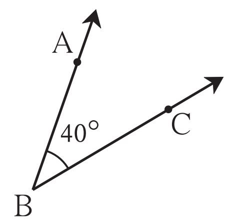


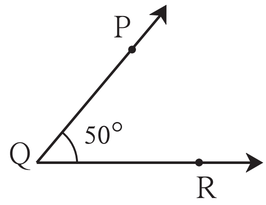
- Draw ∠PQR, a right angle.
- Take any point S in its interior.
- Draw ray QS.
- Add the measures of the angles ∠PQS and ∠SQR. What will be the sum of their measures?

If the sum of the measures of two angles is 90° they are known as complementary angles.

Here, ∠PQS and ∠SQR are mutually complementary angles.

Example Observe the angles in the figure and enter the proper number in the box.





$m\angle ABC = \boxed{}^\circ$

$m\angle PQR = \boxed{}^\circ$

$m\angle ABC + m\angle PQR = \boxed{}^\circ$

The sum of the measures of $\angle ABC$ and $\angle PQR$ is 90°. Therefore, they are complementary angles.

Example Find the measure of the complement of an angle of measure 70°?

Solution : Let the measure of the complementary angle be x.

$70 + x = 90$

$\therefore\ 70 + x - 70 = 90 - 70$

$x = 20^\circ$

The measure of the complement of an angle of measure 70° is 20°.

Example Angles of measures $(a + 15)^\circ$ and $(2a)^\circ$ are complementary. What is the measure of each angle?

Solution : $a + 15 + 2a = 90$

$3a + 15 = 90$

$3a = 75$

$a = 25$

$\therefore\ a + 15 = 25 + 15 = 40^\circ$

and $2a = 2 \times 25 = 50^\circ$

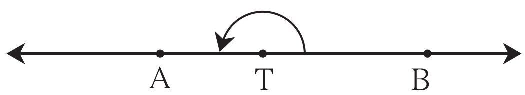
Practice Set 16

1. The measures of some angles are given below. Write the measures of their complementary angles.

   (i) 40° (ii) 63° (iii) 45° (iv) 55° (v) 20° (vi) 90° (vii) x°

2. $(y - 20)^\circ$ and $(y + 30)^\circ$ are the measures of complementary angles. Find the measure of each angle.

Let's recall.

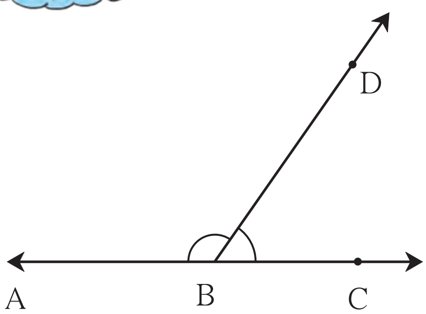


T is a point on line AB.

- What kind of angle is $\angle ATB$?
- What is its measure?

Let's learn.

Supplementary Angles



- A line AC is shown in the figure alongside. A ray BD stands on it. How many angles are formed here?

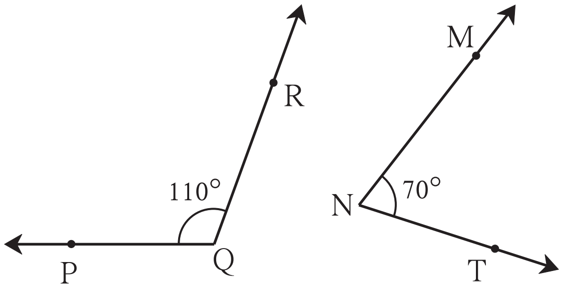
$m\angle ABD = \boxed{}^\circ$, $m\angle DBC = \boxed{}^\circ$

$m\angle ABD + m\angle DBC = \boxed{}^\circ$

If the sum of the measures of two angles is 180° they are known as supplementary angles.

Here $\angle ABD$ and $\angle DBC$ are supplementary angles.

Example Observe the angles in the figure below and enter the proper number in the box.



$m\angle PQR = \boxed{}°$ $\quad m\angle MNT = \boxed{}°$

$m\angle PQR + m\angle MNT = \boxed{}°$

$\angle PQR$ and $\angle MNT$ are supplementary angles.

Example Find the measure of the supplement of an angle of 135°.

Solution: Let the supplementary angle measure p°.

The sum of the measures of two supplementary angles is 180°.

$135 + p = 180$

$\therefore\ 135 + p - 135 = 180 - 135$

$\therefore\ p = 45$

$\therefore$ The measure of the supplement of an angle of 135° is 45°.

Example $(a + 30)°$ and $(2a)°$ are the measures of two supplementary angles. What is the measure of each angle?

Solution: $a + 30 + 2a = 180$

$\therefore\ 3a = 180 - 30$

$\therefore\ 3a = 150$

$\therefore\ a = 50$

$\therefore a + 30 = 50 + 30 = 80°$

$\therefore\ 2a = 2 \times 50 = 100°$

$\therefore$ The measures of the angles are 80° and 100°.

Practice Set 17

1. Write the measures of the supplements of the angles given below.

   (i) 15° (ii) 85° (iii) 120° (iv) 37° (v) 108° (vi) 0° (vii) $a°$

2. The measures of some angles are given below. Use them to make pairs of complementary and supplementary angles.

   $m\angle B = 60°$ $\quad m\angle N = 30°$ $\quad m\angle Y = 90°$ $\quad m\angle J = 150°$

   $m\angle D = 75°$ $\quad m\angle E = 0°$ $\quad m\angle F = 15°$ $\quad m\angle G = 120°$

3. In ΔXYZ, $m\angle Y = 90°$. What kind of a pair do $\angle X$ and $\angle Z$ make?
4. The difference between the measures of the two angles of a complementary pair is 40°. Find the measures of the two angles.
5. □PTNM is a rectangle. Write the names of the pairs of supplementary angles.

6*. If $m\angle A = 70°$, what is the measure of the supplement of the complement of $\angle A$?
7. If $\angle A$ and $\angle B$ are supplementary angles and $m\angle B = (x + 20)°$, then what would be $m\angle A$?

Let's discuss.

Discuss the following statements. If a statement is right, give an example. If it is wrong, state why.

- Two acute angles can make a pair of complementary angles.
- Two right angles can make a pair of complementary angles.
- One acute angle and one obtuse angle can make a pair of complementary angles.
- Two acute angles can form a pair of supplementary angles.
- Two right angles can form a pair of supplementary angles.
- One acute angle and one obtuse angle can form a pair of supplementary angles.

Let's learn.

Opposite Rays

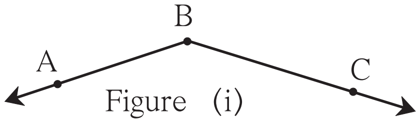


Figure (i)

(1) Name the rays in the figure alongside.
(2) Name the origin of the rays.
(3) Name the angle in figure (i).

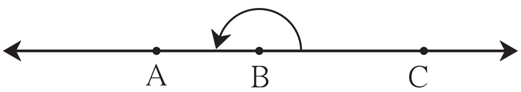


Figure (ii)

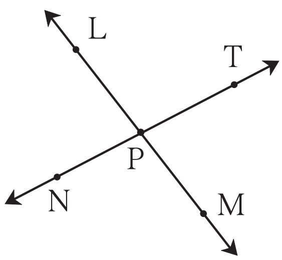
(1) Name the angle in figure (ii) alongside.
(2) Name the rays whose origin is point B.

In figure (i), ray BC and ray BA meet to form an obtuse angle while in figure (ii) ray BC and ray BA meet to form a straight angle and we get a straight line. Here, ray BC and ray BA are opposite rays.

Now I know !

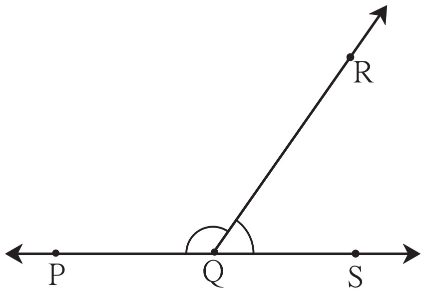
> Two rays which have a common origin and form a straight line are said to be opposite rays.

Practice Set 18



1. Name the pairs of opposite rays in the figure alongside.
2. Are the ray PM and PT opposite rays? Give reasons for your answer.

Let's learn. Angles in a Linear Pair



- Write the names of the angles in the figure alongside.
- What type of a pair of angles is it?
- Which arms of the angles are not the common arms?
- $m\angle PQR = \boxed{\quad}^\circ$ • $m\angle RQS = \boxed{\quad}^\circ$
- $m\angle PQR + m\angle RQS = 180^\circ$

The angles $\angle PQR$ and $\angle RQS$ in the figure above are adjacent angles and are also supplementary angles. The arms that are not common to both angles form a pair of opposite rays i.e. these arms form a straight line. We say that these angles form a linear pair. **The sum of the measures of the angles in a linear pair is 180°.**

Now I know!

Angles which have a common arm and whose other arms form a straight line are said to be angles in a linear pair. Angles in a linear pair are supplementary angles.

Activity : Use straws or sticks to make all the kinds of angles that you have learnt about.

Practice Set 19

Draw the pairs of angles as described below. If that is not possible, say why.

(i) Complementary angles that are not adjacent.

(ii) Angles in a linear pair which are not supplementary.

(iii) Complementary angles that do not form a linear pair.

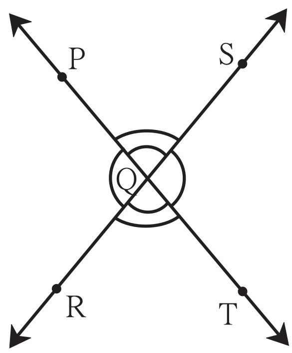
(iv) Adjacent angles which are not in a linear pair.

(v) Angles which are neither complementary nor adjacent.

(vi) Angles in a linear pair which are complementary.

Let's learn. Vertically Opposite Angles

In the figure alongside, line PT and line RS intersect each other at point Q. Thus, four angles are formed. $\angle PQR$ is formed by the rays QP and QR. The rays opposite to ray QP and QR are QT and QS respectively. These opposite rays form the angle $\angle SQT$. Hence, $\angle SQT$ is called the opposite angle of $\angle PQR$.



Now I know!

The angle formed by the opposite rays of the arms of an angle is said to be its opposite angle.

Let's learn. **The Property of Vertically Opposite Angles**

- Name the angle opposite to ∠PQS in the figure.

  As shown in the figure, $m\angle PQS = a$, $m\angle SQT = b$, $m\angle TQR = c$, $m\angle PQR = d$.

  ∠PQS and ∠SQT are the angles in a linear pair.

  $\therefore\ a + b = 180°$

  Also $m\angle SQT$ and $m\angle TQR$ are two angles in a linear pair.

  $\therefore\ b + c = 180°$

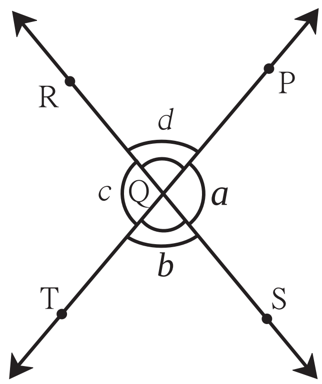
  $\therefore\ a + b = b + c$

  $\therefore\ a = c$ (Subtracting b from both sides.)

  $\therefore$ ∠PQS and ∠TQR are congruent angles.

  Also, $m\angle PQR = m\angle SQT$

  i.e. ∠PQR and ∠SQT are congruent angles.

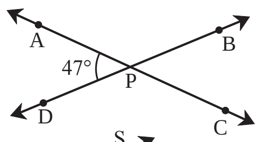


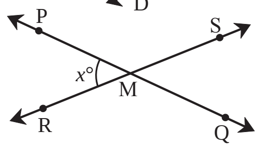
Now I know!

The vertically opposite angles formed when two lines intersect, are of equal measure.

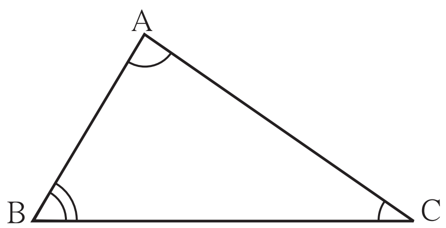
Practice Set 20

1. Lines AC and BD intersect at point P. $m\angle APD = 47°$
   Find the measures of ∠APB, ∠BPC, ∠CPD.
2. Lines PQ and RS intersect at point M. $m\angle PMR = x°$
   What are the measures of ∠PMS, ∠SMQ and ∠QMR?





Let's learn. **Interior Angles of a Polygon**

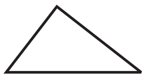
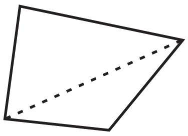
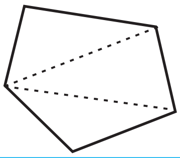
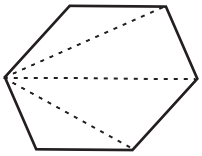
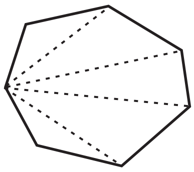


Interior Angles of a Triangle

∠A, ∠B, ∠C are the interior angles of ΔABC.

$m\angle ABC + m\angle BAC + m\angle ACB = \boxed{}°$

Observe the table given below and draw your conclusions.

Number of sides	**Name of the polygon**	**Polygon**	**Number of triangles**	**Sum of interior angles**
3	Triangle		1	$180^\circ \times 1 = \boxed{}$
4	Quadrilateral		2	$180^\circ \times 2 = \boxed{}$
5	Pentagon		3	$180^\circ \times 3 = \boxed{}$
6	Hexagon		4	$180^\circ \times \boxed{} = \boxed{}$
7	Heptagon		5	
8	Octagon		6	
⋮	⋮	⋮	⋮	⋮
n	A figure with n sides		(n–2)	$180^\circ \times (n-2)$

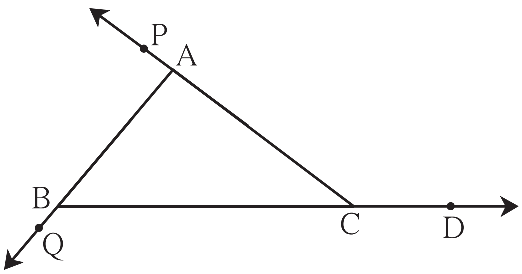
Note that the number of triangles formed in a polygon as shown above is two less than the number of sides the polygon has.

Now I know!

The sum of the measures of the interior angles of a polygon is $= 180^\circ \times (n-2)$

Let's learn.

The Exterior Angle of a Triangle



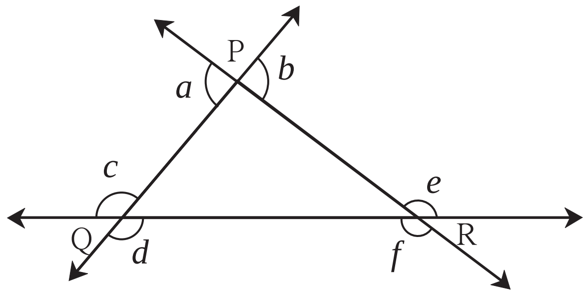
If the side BC of ΔABC is extended as shown in the figure, an angle $\angle ACD$ is formed which lies outside the triangle.

$\angle ACD$ is an exterior angle of ΔABC. $\angle ACD$ and $\angle ACB$ are angles in a linear pair. $\angle PAB$ and $\angle QBC$ are also exterior angles of ΔABC.

Now I know !

On extending one side of a triangle, the angle obtained which forms a linear pair with the adjacent interior angle of the triangle is called an exterior angle of that triangle.

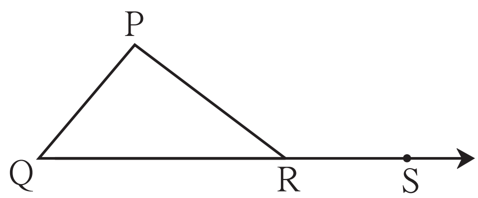
Example In the figure alongside, all exterior angles of a triangle are shown. *a*, *b*, *c*, *d*, *e*, *f* are the exterior angles of ΔPQR. In the same way, every triangle has six exterior angles.



Let's learn.

The Property of an Exterior Angle of a Triangle

In the figure alongside, $\angle PRS$ is an exterior angle of ΔPQR. The interior angle adjacent to it is $\angle PRQ$. The other two interior angles, $\angle P$ and $\angle Q$ are further away from $\angle PRS$. They are called the remote interior angles of $\angle PRS$.



$m\angle P + m\angle Q + m\angle PRQ = \boxed{}\,^\circ$...........(sum of the three angles of a triangle)

$m\angle PRS + m\angle PRQ = \boxed{}\,^\circ$...........(angles in a linear pair)

$\therefore\ m\angle P + m\angle Q + m\angle PRQ = m\angle PRS + m\angle PRQ$

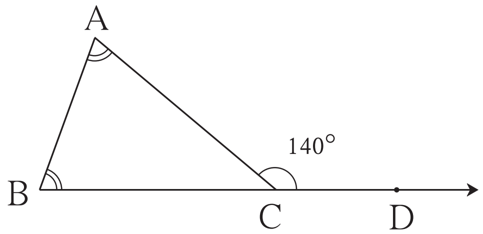
$\therefore\ m\angle P + m\angle Q = m\angle PRS$ .... (subtracting $m\angle PRQ$ from both sides)

Now I know!

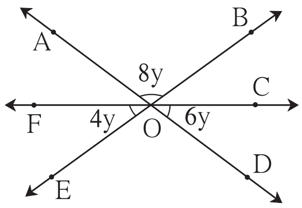
The measure of an exterior angle of a triangle is equal to the sum of the measures of its remote interior angles.

Practice Set 21

1. $\angle ACD$ is an exterior angle of ΔABC. The measures of $\angle A$ and $\angle B$ are equal. If $m\angle ACD = 140°$, find the measures of the angles $\angle A$ and $\angle B$.



2. Using the measures of the angles given in the figure alongside, find the measures of the remaining three angles.



3*. In the isosceles triangle ABC, $\angle A$ and $\angle B$ are equal. $\angle ACD$ is an exterior angle of ΔABC. The measures of $\angle ACB$ and $\angle ACD$ are $(3x - 17)°$ and $(8x + 10)°$ respectively. Find the measures of $\angle ACB$ and $\angle ACD$.
Also find the measures of $\angle A$ and $\angle B$.

ICT Tools or Links

- With the help of Geogebra, draw two rays with a common origin. Using the 'Move' option, turn one ray and observe the position where the two rays become opposite rays.
- Form angles in a linear pair. By 'moving' the common arm, form many different pairs of angles all of which are linear pairs.
- Using Polygon Tools from Geogebra, draw many polygons. Verify the property of the interior angles of a polygon.

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