9. Decimal Fractions

Soumitra : Sir, today I saw MRP ₹ 24.50 printed on a box of medicine. What does it mean?

Teacher : MRP means maximum retail price – the seller can sell that medicine for a maximum of 24 rupees and 50 paise.

Rekha : But how does '₹ 24.50' mean 'twenty-four rupees fifty paise'?

Teacher : '24.50' has been written in decimal form. To understand the answer to your question, you will first have to learn about decimal fractions and the way they are written.

❒ Decimal fractions

A fraction whose denominator is 10, 100 or 1000 or any other ten times multiple of 10 is called a decimal fraction. For example, $\frac{5}{10}$, $\frac{68}{100}$, $\frac{285}{100}$. These fractions are written in the numerator and denominator form.

It is convenient to write these fractions in another way. To use this new method, let us look at our usual method of writing numbers. In this method, we make new places for tens, hundreds, thousands and so on. The place value of each of these is 10 times that of the previous place. For example, one ten equals 10 units, one hundred equals10 tens and so on.

Now let us think in the opposite direction. If we divide one hundred into 10 equal parts, each part is one ten. The tens place is to the right of the hundred. One ten is divided into ten parts. Each part is one unit. The units place is to the right of the tens place.

Similarly, if one unit is divided into ten equal parts, each part becomes $\frac{1}{10}$. For this, a place is made to the right of the units place. $\frac{1}{10}$ means 'one-tenth'. This place is called the tenths place or the first decimal place.

❒ The decimal point

The decimal place is created for writing a fraction. When writing numbers, a dot (.) is written after the last digit of the whole part of a number to indicate the end of that part. This symbol is called a **decimal point**. The decimal point is used to write $8\frac{5}{10}$ as 8.5. This is read as 'eight point five'.

'$20\frac{3}{10}$' is written as '20.3' .

'Seven tenths' can be written as '$\frac{7}{10}$' or '0.7'.

'$\frac{7}{10}$' is the usual way of writing the number and '0.7' is the decimal way.

Problem Set 36

Write the following mixed fractions in decimal form and read them aloud.

(1) $3\ \frac{9}{10}$ (2) $1\ \frac{4}{10}$ (3) $5\ \frac{3}{10}$ (4) $\frac{8}{10}$ (5) $\frac{7}{10}$

❒ Hundredths

If $\frac{1}{10}$ is divided into 10 equal parts, each part becomes $\frac{1}{100}$ or one hundredth. Therefore, note that 1 tenth =10 hundredths, or 0.1=0.10. By multiplying $\frac{1}{100}$ by 10 we get $\frac{10}{100}=\frac{1}{10}$. Therefore, it is possible to create a hundredths place next to the tenths place. After creating a hundredths place we can write $\frac{14}{100}$ as 0.14.

$\frac{14}{100} = \frac{10+4}{100} = \frac{10}{100} + \frac{4}{100} = \frac{1}{10} + \frac{4}{100}$ meaning that when writing $\frac{14}{100}$ in decimal form, 1 is written in the tenths place and 4 is written in the hundredths place. This fraction is written as 0.14 and is read as 'zero point one four'. Similarly, $6\ \frac{57}{100}$ is written as 6.57 and $50\ \frac{71}{100}$ is written as 50.71.

While writing $\frac{3}{100}$ in decimal form, we must remember that there is **no number in the tenths place** and so, we put 0 in that place, which means that $\frac{3}{100}$ is written as 0.03.

Study how the decimal fractions in the table below are written and read.

Fractions	Tens	Units	Tenths	Hundredths	Decimal fractions in figures	Decimal fractions in words
$7\ \frac{5}{10}$		7	5		7.5	Seven point five
$7\ \frac{5}{100}$		7	0	5	7.05	Seven point zero five
$\frac{82}{100}$		0	8	2	0.82	Zero point eight two
$25\ \frac{6}{100}$	2	5	0	6	25.06	Twenty-five point zero six

Problem Set 37

Write the following mixed fractions in decimal form and read them aloud.

(1) $9\ \frac{1}{10}$ (2) $9\ \frac{1}{100}$ (3) $4\ \frac{53}{100}$ (4) $\frac{78}{100}$ (5) $\frac{5}{100}$ (6) $\frac{5}{10}$ (7) $\frac{2}{10}$ (8) $\frac{20}{100}$

❒ Place value of the digits in decimal fractions

We can determine the place value of the digits in decimal fractions in the same way that we determine the place values of digits in whole numbers.

Example (1) In 73.82, the place value of 7 is $7 \times 10 = 70$, and of 3, it is $3 \times 1 = 3$.

Similarly, the place value of 8 is $8 \times \frac{1}{10} = \frac{8}{10} = 0.8$ and

the place value of 2 is $2 \times \frac{1}{100} = \frac{2}{100} = 0.02$

Example (2) Place values of the digits in 210.86.

Digits	2	1	0	8	6
Place	Hundreds	Tens	Units	Tenths	Hundredths
Place value	$2 \times 100 = 200$	$1 \times 10 = 10$	0 0	$8 \times \frac{1}{10} = 0.8$	$6 \times \frac{1}{100} = 0.06$

Problem Set 38

Read the decimal fraction and write down the place value of each digit.

(1) 6.13 (2) 48.84 (3) 72.05 (4) 3.4 (5) 0.59

❒ Use of decimal fractions

Sir : Now we will see how 24.50 equals 24 rupees and 50 paise. How many rupees is one paisa ?

Sumit : 100 paise make one rupee, therefore, 1 paisa is one hundredth of a rupee or 0.01 rupee.

Sir : And 50 paise are ?

Sumit : 50 hundredths of a rupee, or 0.50 rupees, so 24.50 rupees is 24 rupees and 50 paise.

Sir : When a large unit of a certain quantity is divided into 10 or 100 parts to make smaller units, it is more convenient to write them in decimal form. As we just saw, 100 paise = 1 rupee. Similarly, 100 cm = 1 metre, so 75 cm = 0.75 m. 10 mm = 1 cm, so 1 mm = 0.1cm. 3 mm are 0.3 cm. 6.3 cm are 6 cm and 3 mm.

Now study the following table.

100 paise = 1 rupee	100 cm = 1 m
1 paisa $= \frac{1}{100}$ rupee = 0.01 rupee	1 cm $= \frac{1}{100}$ m = 0.01 m
50 paise $= \frac{50}{100}$ rupee = 0.50 rupee	25 cm $= \frac{25}{100}$ m = 0.25 m
75 paise $= \frac{75}{100}$ rupee = 0.75 rupee	60 cm $= \frac{60}{100}$ m = 0.60 m = 0.6 m

1 rupee	= 100 paise	1 m	= 100 cm
5 rupees	= 500 paise	8 m	= 800 cm
0.50 rupee	= 0.5 rupee = 50 paise	0.3 m	= 30 cm
0.07 rupee	= 7 paise	0.40 m	= 40 cm
4.5 rupees	= 4 rupees 50 paise	2.65 m	= 2 m 65 cm
17.65 rupees	= 17 rupees 65 paise	14.9 m	= 14 m 90 cm

Problem Set 39

1. Write how many rupees and how many paise.

(1) ₹ 58.43 (2) ₹ 9.30 (3) ₹ 2.30 (4) ₹ 2.3

2. Write how many rupees in decimal form.

(1) 6 rupees 25 paise (2) 15 rupees 70 paise (3) 8 rupees 5 paise

(4) 22 rupees 4 paise (5) 720 paise

3. Write how many metres and how many centimetres.

(1) 58.75 m (2) 9.30 m (3) 0.30 m (4) 0.3 m

(5) 1.62 m (6) 91.4 m (7) 7.02 m (8) 0.09 m

4. Write how many metres in decimal form.

(1) 1 m 50 cm (2) 50 m 40 cm (3) 50 m 4 cm (4) 734 cm

(5) 10 cm (6) 2 cm

5. Write how many centimetres and how many millimetres.

(1) 6.9 cm (2) 20.4 cm (3) 0.8 cm (4) 0.5 cm

6. Write how many centimetres in decimal form.

(1) 7 cm 1 mm (2) 16 mm (3) 144 mm (4) 8 mm

❒ Writing half, quarter, three-quarters and one and a quarter in decimal form

'Half' is usually written as $\frac{1}{2}$. To convert this fraction into decimal form, the denominator of $\frac{1}{2}$ must be converted into an equivalent fraction with denominator 10.

$\frac{1}{2} = \frac{1\times5}{2\times5} = \frac{5}{10}$ so the decimal form of $\frac{1}{2}$ will be $\frac{5}{10}$ or 0.5

Just as $\frac{1}{2} = \frac{1\times5}{2\times5} = \frac{5}{10}$ = 0.5, note that $\frac{1}{2} = \frac{1\times50}{2\times50} = \frac{50}{100}$ = 0.50

Therefore, 'half' is written as '0.5' or 0.50'. 'Quarter' and 'three quarters' are written in fractions as $\frac{1}{4}$ and $\frac{3}{4}$ respectively. Let us convert them into decimal fractions. 10 is not divisible by 4. Therefore, the denominators of $\frac{1}{4}$ and $\frac{3}{4}$ cannot be made into fractions with multiples of 10. However, 4 × 25 = 100, so the denominator can be 100.

A quarter $= \frac{1}{4} = \frac{1\times25}{4\times25} = \frac{25}{100} = 0.25$ and Three quarters $= \frac{3}{4} = \frac{3\times25}{4\times25} = \frac{75}{100} = 0.75$

One and a quarter $= 1\frac{1}{4} = 1.25$ One and a half $= 1\frac{1}{2} = 1.50 = 1.5$

One and three quarters $= 1\frac{3}{4} = 1.75$ Seventeen and a half $= 17\frac{1}{2} = 17.50 = 17.5$

Problem Set 40

Write the following fractions as decimal fractions.

(1) Two and a half (2) Two and a quarter (3) Two and three quarters

(4) Ten and a half (5) Fourteen and three quarters (6) Sixteen and a quarter

(7) Twenty-eight and a half

❒ Adding decimal fractions

Sir : If the cost of one pencil is two and a half rupees and the cost of a pen is four and half rupees, what is the total cost?

Sumit : Two and a half rupees means two rupees and one half rupee. Similarly, four and a half rupees means four rupees and one half rupee. 4 rupees and 2 rupees make 6 rupees and two half rupees make one rupee, so both objects together cost 6+1=7 rupees.

Sir : Correct! Now, see how this is done using decimals.

The sum of the 0's in the hundredths place is 0.

0.5 + 0.5 is the same as $\frac{5}{10} + \frac{5}{10} = \frac{5+5}{10} = \frac{10}{10} = \frac{1}{1} = 1$

$$\begin{array}{r} {\scriptstyle 1} \\ 2.50 \\ + \quad 4.50 \\ \hline 7.00 \end{array}$$

This 1 is carried over to the units place. There is nothing in the tenths place, so we put a zero there. In the units place, 2 + 4 = 6 plus the carried over 1 makes 7.

So 2.50 rupees and 4.50 rupees add up to 7 rupees.

We use the decimal system to write whole numbers. We extend the same method to write fractions; therefore, we can add in the same way as we add whole numbers.

I will now show some more additions. Watch carefully.

(1) $\begin{array}{r} 3.7 \\ + \quad 12.2 \\ \hline 15.9 \end{array}$ (2) $\begin{array}{r} 6.8 \\ + \quad 5.5 \\ \hline 12.3 \end{array}$ (3) $\begin{array}{r} 16.9 \\ + \quad 7.5 \\ \hline 24.4 \end{array}$

Sumit : There is no carried over number in the first sum, but there are carried over numbers in the second and third sums.

Rekha : While adding whole numbers, we add units first. Similarly, here, tenths are added first. In the second example, the sum of the tenths place is 13. 13 tenths are 10 tenths + 3 tenths = 1 unit + 3 tenths.

Sumit : That is why, in the sum, 3 stayed in the tenths place and 1 was carried over to the units place. 6 + 5 plus 1 carried over makes 12.

Sir : Your observations are absolutely correct. We write digits one below the other according to their place values while adding whole numbers. We do the same thing here. Remember that while writing down an addition problem and the total, the decimal points should always be written one below the other.

Study the following additions. (Note that: 10 tenths = 1 unit. 10 hundredths = 1 tenth)

Example (1) Add : 7.09 + 54.93

First, add the digits in the 100ths place. 9 + 3 = 12.

	1	1		1
+		7	.	0 9
	5	4	.	9 3
	6	2	.	0 2

The 1 from the sum 12 in the hundredths place is carried over to the tenths place and 2 is written in the hundredths place. Adding 1 + 9 gives 10 tenths or 1 unit. This 1 is carried over to the units place. 0 is left in the tenths place. Then, the addition is completed in the usual way.

Example (2) Add : 45.83 + 167.4

$$\begin{array}{r} 45.83 \\ +\ 167.4 \\ \hline \end{array}$$

We arrange the numbers so that the places and decimal points come one below the other.

	1	1	1		
		4	5	. 8	3
+	1	6	7	. 4	0
	2	1	3	. 2	3

$\frac{4}{10} = \frac{4\times10}{10\times10} = \frac{40}{100}$ Therefore, to make the denominators of the fractions equal, 167.4 is written as 167.40 and then the fractions are added.

As usual, the digits with the smallest place values are added first and then those with bigger place values are added serially.

Example (3)

$$\begin{array}{r} 10.46 \text{ Rupees} \\ +\ 35.92 \text{ Rupees} \\ \hline 46.38 \text{ Rupees} \end{array}$$

Example (4)

$$\begin{array}{r} 48.80 \text{ m} \\ +\ 2.57 \text{ m} \\ \hline 51.37 \text{ m.} \end{array}$$

Example (5)

$$\begin{array}{r} 7.5 \text{ cm} \\ +\ 14.2 \text{ cm} \\ +\ 9.6 \text{ cm} \\ \hline 31.3 \text{ cm} \end{array}$$

Problem Set 41

1. Convert the following into decimal fractions and add them.
(1) 'One and a half metre' and 'two and a half metres'
(2) 'Four and three quarter rupees' and 'seven and a quarter rupees'
(3) 'Six and a half metres' and 'three and three quarter metres'.

2. (1) 23.4 + 87.9 (2) 35.74 + 816.6 (3) 6.95 + 74.88 (4) 41.03 + 9.98

3. (1) 51.4 cm + 68.5 cm (2) 94.7 m + 1738.45 m (3) ₹ 5158.75 + ₹ 841.25

❒ Subtraction of decimal fractions

Study the subtraction of decimal fractions given below.

$$\begin{array}{r} 15.41 \\ -\ \ 8.58 \\ \hline 6.83 \end{array}$$

T	U		Tenths	Hundredths
	14		13	
0	4		3	11
–	~~1~~ ~~5~~ 8	.	~~4~~ 5	1 8
0	6	.	8	3

8 hundredths cannot be subtracted from 1 hundredth, so 1 tenth (or 10 hundredths) from 4 tenths are borrowed. The borrowed 10 hundredths and the original one hundredth make 11 hundredths. 11 hundredths minus 8 hundredths are 3 hundredths. They are written in the hundredths place under the line. The rest of the subtraction is carried out using the same method.

Problem Set 42

1. Subtract the following :

(1) 25.74 – 13.42 (2) 206.35 – 168.22 (3) 63.4 – 31.8 (4) 63.43 – 31.8

(5) 63.4 – 31.83 (6) 8.23 – 5.45 (7) 18.23 – 9.45 (8) 78.03 – 41.65

2. Vrinda was 1.48 m tall. After a year, her height became 1.53 m. How many centimetres did her height increase in a year?

Something more

❒ Decimals used for measurement

We need to measure distance, mass (weight) and volume every day. We use suitable units for these measurements. Kilometre, metre and centimetre for distance; litre, millilitre for volume and kilogram and gram for mass are the units that are used most of the time.

All these units are decimal units. In this method, gram, metre and litre are taken as the basic units for mass, distance and volume respectively. Units larger than these increase 10 times at every step and smaller units become $\frac{1}{10}$ of the previous unit at each step.

Look at the table of these units given below.

Quantity	Kilo (Th) (Thousand)	Hecto (H) (Hundred)	Deca (Ten)	The basic unit of measurement	Deci (Tenth)	Centi (Hundredth)	Milli (Thousandth)
Distance	1 kilometre = 1000 m	1 hectometre = 100 m	1 decametre = 10 m	metre	1 decimetre = $\frac{1}{10}$ m	1 centimetre = $\frac{1}{100}$ m	1 millimetre = $\frac{1}{1000}$ m
Mass				gram			
Volume				litre			

The origin of the terms kilo, hecto… milli is in the Greek or Latin language. Their English equivalents are given in brackets along with the terms.