1 Rational and Irrational numbers

Let's recall.

We are familiar with Natural numbers, Whole numbers, Integers and Rational numbers.

Natural numbers

1,2,3,4,...

Whole numbers

0,1,2,3,4, ...

Integers

...,−4,−3,−2,−1, 0, 1, 2, 3,...

Rational numbers

$\frac{-25}{3}$, $\frac{10}{-7}$, -4, 0, 3, 8, $\frac{32}{3}$, $\frac{67}{5}$, etc.

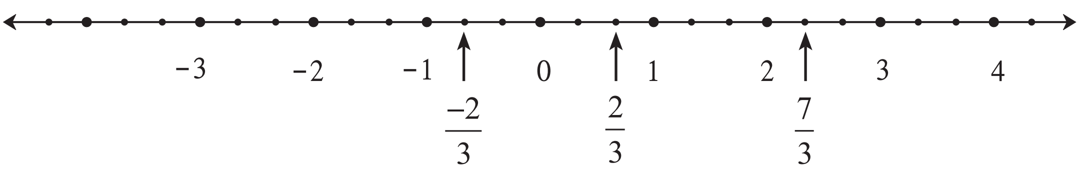
Rational numbers : The numbers of the form $\frac{m}{n}$ are called rational numbers.
Here, m and n are integers but n is not zero.
We have also seen that there are infinite rational numbers between any two rational numbers.

Let's learn.

To show rational numbers on a number line

Let us see how to show $\frac{7}{3}$, 2, $\frac{-2}{3}$ on a number line.

Let us draw a number line.



- We can show the number 2 on a number line.
- $\frac{7}{3} = 7 \times \frac{1}{3}$, therefore each unit on the right side of zero is to be divided in three equal parts. The seventh point from zero shows $\frac{7}{3}$; or $\frac{7}{3} = 2 + \frac{1}{3}$, hence the point at $\frac{1}{3}$ rd distance of unit after 2 shows $\frac{7}{3}$.

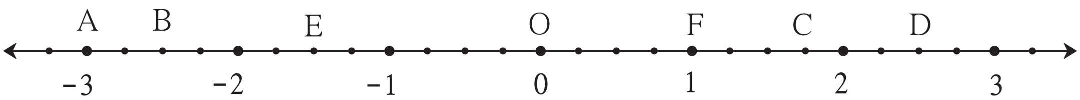
- To show $\frac{-2}{3}$ on the number line, first we show $\frac{2}{3}$ on it. The number to the left of 0 at the same distance will show the number $\frac{-2}{3}$.

Practice set 1.1

1. Show the following numbers on a number line. Draw a separate number line for each example.

(1) $\frac{3}{2}, \frac{5}{2}, -\frac{3}{2}$ (2) $\frac{7}{5}, \frac{-2}{5}, \frac{-4}{5}$ (3) $\frac{-5}{8}, \frac{11}{8}$ (4) $\frac{13}{10}, \frac{-17}{10}$

2. Observe the number line and answer the questions.



(1) Which number is indicated by point B?

(2) Which point indicates the number $1\frac{3}{4}$?

(3) State whether the statement, 'the point D denotes the number $\frac{5}{2}$' is true or false.

Let's learn.

Comparison of rational numbers

We know that, for any pair of numbers on a number line the number to the left is smaller than the other. Also, if the numerator and the denominator of a rational number is multiplied by any non zero number then the value of rational number does not change. It remains the same. That is, $\frac{a}{b} = \frac{ka}{kb}$, $(k \neq 0)$.

Ex. (1) Compare the numbers $\frac{5}{4}$ and $\frac{2}{3}$. Write using the proper symbol of <, =, >.

Solution : $\frac{5}{4} = \frac{5\times3}{4\times3} = \frac{15}{12}$ $\qquad$ $\frac{2}{3} = \frac{2\times4}{3\times4} = \frac{8}{12}$

$\frac{15}{12} > \frac{8}{12}$ $\qquad$ $\therefore \frac{5}{4} > \frac{2}{3}$

Ex. (2) Compare the rational numbers $\frac{-7}{9}$ and $\frac{4}{5}$.

Solution : A negative number is always less than a positive number.

Therefore, $-\frac{7}{9} < \frac{4}{5}$.

To compare two negative numbers,

let us verify that if a and b are positive numbers such that $a < b$, then $-a > -b$.

$$\left.\begin{array}{l} 2 < 3 \text{ but } -2 > -3 \\ \frac{5}{4} < \frac{7}{4} \text{ but } \frac{-5}{4} > \frac{-7}{4} \end{array}\right\} \text{Verify the comparisons using a number line.}$$

Ex. (3) Compare the numbers $\frac{-7}{3}$ and $\frac{-5}{2}$.

Solution : Let us first compare $\frac{7}{3}$ and $\frac{5}{2}$.

$$\frac{7}{3} = \frac{7\times 2}{3\times 2} = \frac{14}{6}, \qquad \frac{5}{2} = \frac{5\times 3}{2\times 3} = \frac{15}{6} \qquad \text{and} \qquad \frac{14}{6} < \frac{15}{6}$$

$$\therefore \frac{7}{3} < \frac{5}{2} \qquad \therefore \frac{-7}{3} > \frac{-5}{2}$$

Ex. (4) $\frac{3}{5}$ and $\frac{6}{10}$ are rational numbers. Compare them.

Solution : $\frac{3}{5} = \frac{3\times 2}{5\times 2} = \frac{6}{10}$ $\qquad \therefore \frac{3}{5} = \frac{6}{10}$

The following rules are useful to compare two rational numbers.

If $\frac{a}{b}$ and $\frac{c}{d}$ are rational numbers such that b and d are positive, and

(1) if $a \times d < b \times c$ then $\frac{a}{b} < \frac{c}{d}$

(2) if $a \times d = b \times c$ then $\frac{a}{b} = \frac{c}{d}$

(3) if $a \times d > b \times c$ then $\frac{a}{b} > \frac{c}{d}$

Practice Set 1.2

1. Compare the following numbers.

(1) $-7, -2$ (2) $0, \frac{-9}{5}$ (3) $\frac{8}{7}, 0$ (4) $\frac{-5}{4}, \frac{1}{4}$ (5) $\frac{40}{29}, \frac{141}{29}$

(6) $-\frac{17}{20}, \frac{-13}{20}$ (7) $\frac{15}{12}, \frac{7}{16}$ (8) $\frac{-25}{8}, \frac{-9}{4}$ (9) $\frac{12}{15}, \frac{3}{5}$ (10) $\frac{-7}{11}, \frac{-3}{4}$

Let's learn.

Decimal representation of rational numbers

If we use decimal fractions while dividing the numerator of a rational number by its denominator, we get the decimal representation of a rational number. For example, $\frac{7}{4} = 1.75$. In this case, after dividing 7 by 4, the remainder is zero. Hence the process of division ends.

Such a decimal form of a rational number is called a terminating decimal form.

We know that every rational number can be written in a non-terminating recurring decimal form.

For example, (1) $\frac{7}{6} = 1.1666... = 1.1\dot{6}$ (2) $\frac{5}{6} = 0.8333... = 0.8\dot{3}$

(3) $\frac{-5}{3} = -1.666... = -1.\dot{6}$

(4) $\frac{22}{7} = 3.142857142857... = 3.\overline{142857}$ (5) $\frac{23}{99} = 0.2323... = 0.\overline{23}$

Similarly, a terminating decimal form can be written as a non-terminating recurring decimal form. For example, $\frac{7}{4} = 1.75 = 1.75000... = 1.75\dot{0}$.

Practice Set 1.3

1. Write the following rational numbers in decimal form.

(1) $\frac{9}{37}$ (2) $\frac{18}{42}$ (3) $\frac{9}{14}$ (4) $\frac{-103}{5}$ (5) $-\frac{11}{13}$

Let's learn.

Irrational numbers

In addition to rational numbers, there are many more numbers on a number line. They are not rational numbers, that is, they are irrational numbers. $\sqrt{2}$ is such an irrational number.

We learn how to show the number $\sqrt{2}$ on a number line.

- On a number line, the point A shows the number 1. Draw line l perpendicular to the number line through point A.

  Take point P on line l such that OA = AP = 1 unit.
- Draw seg OP. The Δ OAP formed is a right angled triangle.

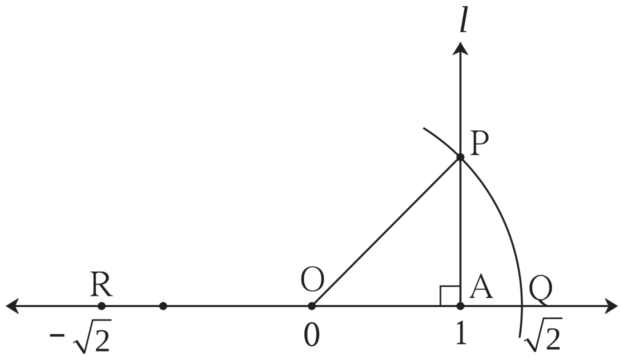
By Pythagoras theorem,

$OP^2 = OA^2 + AP^2$

$= 1^2 + 1^2 = 1 + 1 = 2$

$OP^2 = 2$

$\therefore$ $OP = \sqrt{2}$ ...(taking square roots on both sides)



- Now, draw an arc with centre O and radius OP. Name the point as Q where the arc intersects the number line. Obviously distance OQ is $\sqrt{2}$.

That is, the number shown by the point Q is $\sqrt{2}$.

If we mark point R on the number line to the left of O, at the same distance as OQ, then it will indicate the number $-\sqrt{2}$.

We will prove that $\sqrt{2}$ is an irrational number in the next standard. We will also see that the decimal form of an irrational number is non-terminating and non-recurring.

Note that -

In the previous standard we have learnt that π is not a rational number. It means it is irrational. For calculation purpose we take its value as $\frac{22}{7}$ or 3.14 which are very close to π ; but $\frac{22}{7}$ and 3.14 are rational numbers.

The numbers which can be shown by points of a number line are called real numbers. We have seen that all rational numbers can be shown by points of a number line. Therefore, all rational numbers are real numbrs. There are infinitely many irrational numbers on the number line.

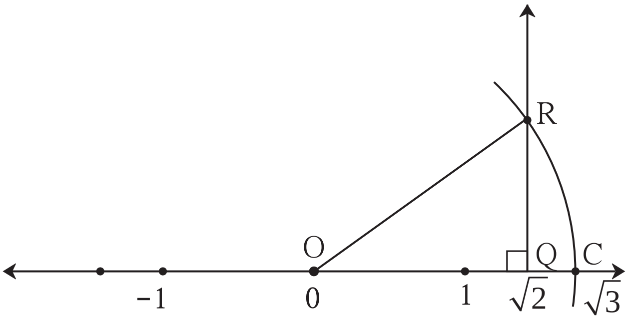
$\sqrt{2}$ is an irrational number. Note that the numbers like $3\sqrt{2}$, $7 + \sqrt{2}$, $3 - \sqrt{2}$ etc. are also irrational numbers; because if $3\sqrt{2}$ is rational then $\frac{3\sqrt{2}}{3}$ should also be a rational number, which is not true.

We learnt to show rational numbers on a number line. We have shown the irrational number $\sqrt{2}$ on a number line. Similarly we can show irrational numbers like $\sqrt{3}$, $\sqrt{5}$ . . . on a number line.

Practice Set 1.4

1. The number $\sqrt{2}$ is shown on a number line. Steps are given to show $\sqrt{3}$ on the number line using $\sqrt{2}$. Fill in the boxes properly and complete the activity.

Activity :



- The point Q on the number line shows the number ....
- A line perpendicular to the number line is drawn through the point Q. Point R is at unit distance from Q on the line.
- Right angled Δ ORQ is obtained by drawing seg OR.
- l (OQ) = $\sqrt{2}$, l(QR) = 1

  $\therefore$ by Pythagoras theorem,

  $$[l(OR)]^2 = [l(OQ)]^2 + [l(QR)]^2$$
  $$= \boxed{}^2 + \boxed{}^2 = \boxed{} + \boxed{}$$
  $$= \boxed{} \qquad \therefore l(OR) = \boxed{}$$

  Draw an arc with centre O and radius OR. Mark the point of intersection of the line and the arc as C. The point C shows the number $\sqrt{3}$.

2. Show the number $\sqrt{5}$ on the number line.

3★. Show the number $\sqrt{7}$ on the number line.

Answers

Practice Set 1.1

2. (1) $\frac{-10}{4}$ (2) C (3) True

Practice Set 1.2

1. (1) -7 < -2 (2) $0 > \frac{-9}{5}$ (3) $\frac{8}{7} > 0$ (4) $\frac{-5}{4} < \frac{1}{4}$ (5) $\frac{40}{29} < \frac{141}{29}$

(6) $\frac{-17}{20} < \frac{-13}{20}$ (7) $\frac{15}{12} > \frac{7}{16}$ (8) $\frac{-25}{8} < \frac{-9}{4}$ (9) $\frac{12}{15} > \frac{3}{5}$

(10) $\frac{-7}{11} > \frac{-3}{4}$

Practice Set 1.3

(1) $0.\overline{243}$ (2) $0.\overline{428571}$ (3) $0.6\overline{428571}$ (4) -20.6 (5) $-0.\overline{846153}$