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Geometrical Constructions





Can you tell?

- (1) When constructing a building, what is the method used to make sure that a wall is exactly upright? What does the mason in the picture have in his hand?
 - What do you think is his purpose for using it?
- (2) Have you looked at lamp posts on the roadside? How do they stand?

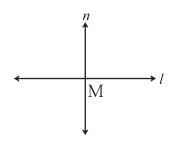


Try this.

The Perpendicular

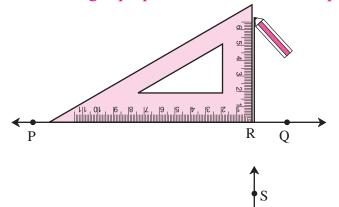
In the figure here, line l and line n intersect at point M. Measure every angle formed at the point M.

If an angle between line l and line n is a right angle, we say that the lines are **perpendicular** to each other. This is written as 'line $l \perp$ line n' in symbols. It is read as 'Line l is perpendicular to line n'.



Drawing a perpendicular to a line at a point on the line.

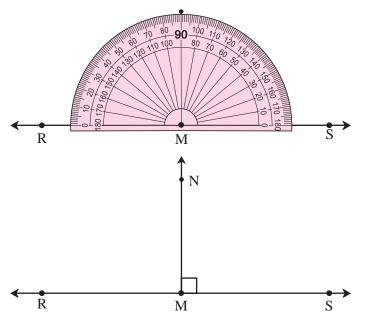
Q



(1) Using a set square

- Draw line PQ. Take point R anywhere on the line.
- Place the set square on the line in such a way that the vertex of its right angle is at point R and one arm of the right angle falls on line PQ.
- Draw a line RS along the other arm of the right angle of the set square.
- The line RS is perpendicular to the line PQ at R.

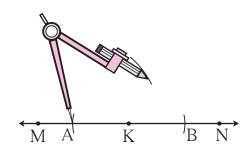
(2) Using a protractor



- Draw line RS. Take point M anywhere on the line.
- In order to draw a perpendicular through M, place the centre of the protractor on point M, as shown.
- Mark a point N at the 90° mark on the protractor.
- Draw a line passing through points M and N.
- The line MN is perpendicular to line RS at M.

Line MN \perp line RS.

(3) Using a compass



T M A K B N

- Draw line MN. Take point K anywhere on the line.
- Place the compass point on point K. Draw two arcs on either side of point K to cut the line MN at equal distances from K. Name the points of intersection A and B respectively.
- Place the compass point at A and, taking a convenient distance greater than half the length of AB, draw an arc on one side of the line.
- Place the compass point at B and using the same distance, draw another arc to intersect the first one at T.
- Draw a line passing through points K and T.
- The line KT is perpendicular to line MN at K. Line KT \perp line MN.



Think about it.

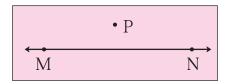
Why must we take a distance greater than half of the length of AB?

What will happen if we take a smaller distance?

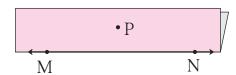
- 1. Draw line l. Take any point P on the line. Using a set square, draw a line perpendicular to line l at the point P.
- 2. Draw a line AB. Using a compass, draw a line perpendicular to AB at the point B.
- 3. Draw line CD. Take any point M on the line. Using a protractor, draw a line perpendicular to line CD at the point M.
- ♦ Drawing a perpendicular to a line from a point outside the line.

(1) By folding the paper

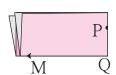
- Draw a line MN on a paper.
 Take a point P anywhere outside the line.
- Keeping the line MN in view, fold the paper along the line MN.



 Now fold the paper through point P in such a way that the part of line MN on one side of the fold falls on the part of line MN on the other side of the fold.



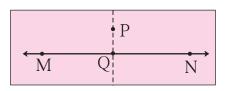
• Unfold the paper. Name the point of intersection of the two folds Q. Draw the line PQ. This line falls on a fold in the paper.



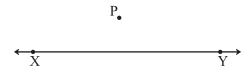
Using a protractor, measure every angle formed at the point \boldsymbol{Q} .

Line PQ is perpendicular to line MN.

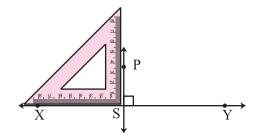
Line PQ \perp line MN.



(2) Using a set square



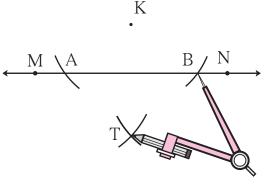
- Draw line XY. Take point P anywhere outside XY.
- Place one of the arms of the right angle of a set square along the line XY.



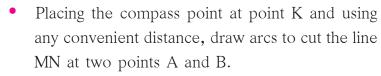
• Slide the set square along the line in such a way that the other arm of its right angle touches point P. Draw a line along this side, passing through point P. Name the line PS.

Measure the angles to verify that the line is a perpendicular.

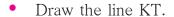
(3) Using a compass and ruler



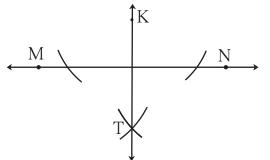
• Draw line MN. Take any point K outside the line.



- Place the compass point at A and taking a distance greater than half of AB, draw an arc on the lower side of line MN.
- Place the compass point at B and using the same distance, draw an arc to cut the previous arc at T.



• Line KT is perpendicular to line MN. Verify.



Think about it.

In the above construction, why must the distance in the compass be kept constant?

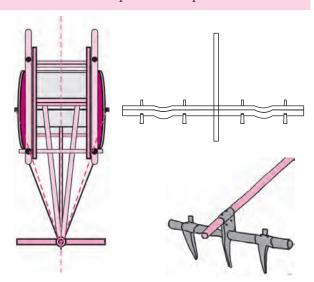
The Perpendicular Bisector

A wooden 'yoke' is used for pulling a bullock cart.

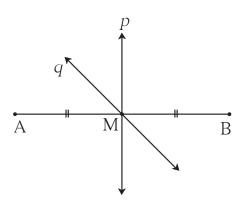
How is the position of the yoke determined?

To do that, a rope is used to measure equal distances from the spine/ midline of the bullock cart. Which geometrical property is used here?

Find out from the craftsmen or from other experienced persons, why this is done.



The Perpendicular Bisector of a Line Segment



Line p and line q pass through the point M on seg AB. Line p and line q are bisectors of the segment AB.

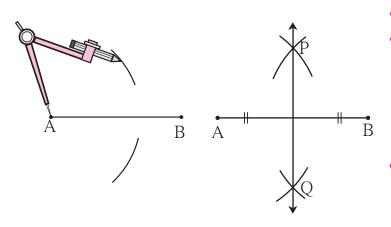
Measure the angle between line p and seg AB.

Of the two lines p and q, line p is a bisector and also perpendicular to seg AB.

Hence, line p is called the **perpendicular bisector** of seg AB.

Why is line q not a perpendicular bisector of seg AB?

♦ Drawing the perpendicular bisector of a segment, using a compass.

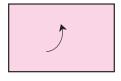


- Draw seg AB.
 - Place the compass point at A and taking a distance greater than half the length of seg AB, draw two arcs, one below and one above seg AB.
- Place the compass point at B and using the same distance draw arcs to intersect the previous arcs at P and Q. Draw line PQ.
- The line PQ is the perpendicular bisector of seg AB. Verify.



Try this.

Activity: Take a rectangular sheet of paper. Fold the paper so that the lower edge of the paper falls on its top edge and fold it over again from right to left. Observe the two folds that have formed on the paper. Verify that each fold is a perpendicular bisector of the other. Then measure the distances to fill in the blanks below.



$$l(XP) = \dots cm$$

 $l(YP) = \dots cm$

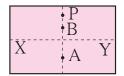
Fold 1

$$l(XA) = \dots cm$$

 $l(YA) = \dots cm$



Fold 2



$$l(XB) = \dots cm$$

$$l(YB) = \dots cm$$

You will see that all points on the vertical fold are equidistant from the endpoints of the horizontal fold.

- 1. Draw line l. Take point P anywhere outside the line. Using a set square, draw a line PQ perpendicular to line l.
- 2. Draw line AB. Take point M anywhere outside the line. Using a compass and ruler, draw a line MN perpendicular to line AB.
- 3. Draw a line segment AB of length 5.5 cm. Bisect it using a compass and ruler.
- 4. Take a point R on line XY. Draw a line perpendicular to XY at R, using a set square.

BBB

Carl Gauss's Clever Trick

This is a story from the childhood of the great mathematician Carl Friedrich Gauss. The boys in Carl's class were making a lot of noise. To keep them occupied, their teacher set them the task of adding up all the numbers from 1 to 100. Carl completed the task in two or three minutes and sat quietly with arms crossed. Other children, for fear of the teacher, kept on with their calculations.

'Don't be idle! Do what I told you,' shouted the teacher angrily.

Carl showed the teacher his addition. The teacher was astonished to see that he had the correct answer.

How had Carl carried out the addition?

That would be 101×100 .

But this is the sum of numbers from 1 to 100, taken twice.

Therefore, the sum of all the numbers from 1 to 100 would be

$$\frac{101 \times 100}{2} = 101 \times 50 = 5050$$

You could try using Carl's method to find the sum of numbers from 1 to 50.