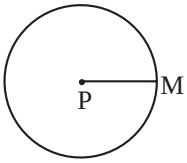
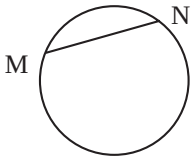
7. Circles

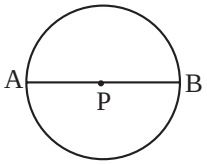
❒ Radius, chord and diameter



1. The line joining the centre of the circle to any point on the circle is called a **radius** of the circle.
   In the diagram, P is the centre of the circle while M is a point on the circle. PM is a radius of the circle.
   A circle has many radii. They are all of the same length.

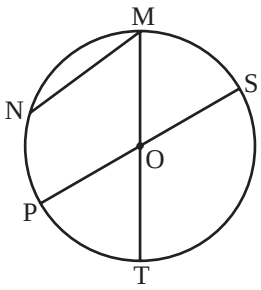


2. A line joining any two points on a circle is called a **chord**.
   In the diagram, M and N are two points on the circle.
   Line MN is a chord of the circle.



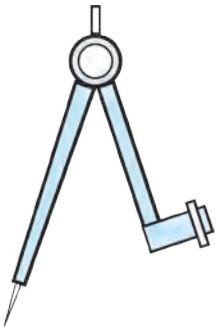
3. A chord passing through the centre of the circle is called a **diameter**. In the diagram, chord AB passes through the centre P. Therefore, chord AB is also a diameter of the circle.
   A circle has many chords and diameters.

- The centre of the circle below is O. There are other points and lines given in the diagram. Find the radii, chords and diameters in the diagram and write their names in the box provided.



Radius	
Diameter	
Chord	

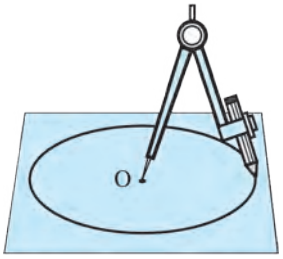
❒ Drawing a circle



We use a compass to draw a circle. One arm of the compass has a metal point while the other arm has a place to fix a pencil. A pencil of a suitable length is fixed to the compass.

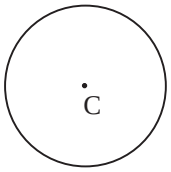
❒ How to draw a circle using a compass

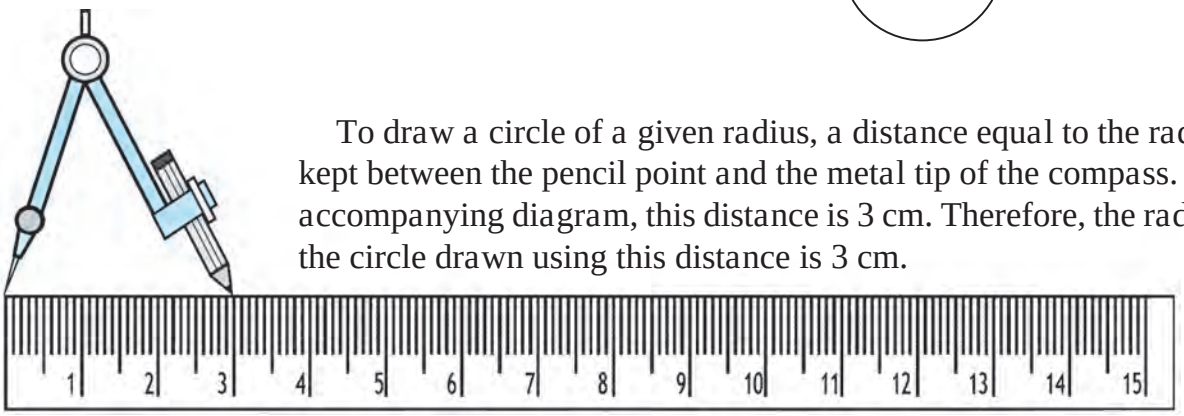
- First fix the pencil to the compass. Align the metal tip of the compass with the pencil point as shown in the picture on the right.



- Take a convenient distance between the pencil point and the metal tip.
- Take any point on a piece of paper.
- Hold the metal tip steady on the chosen point and turn the pencil point around it on the sheet of paper. The shape created by the pencil point will be a circle.

The point at which the metal tip of the compass is held is the **centre** of the circle. In this diagram, C is the centre of the circle.





To draw a circle of a given radius, a distance equal to the radius is kept between the pencil point and the metal tip of the compass. In the accompanying diagram, this distance is 3 cm. Therefore, the radius of the circle drawn using this distance is 3 cm.

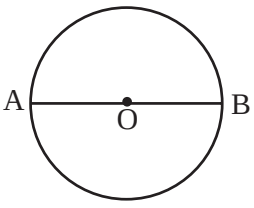
Problem Set 28

1. Draw circles with the radii given below.

(1) 2 cm (2) 4 cm (3) 3 cm

2. Draw a circle of any radius. Show one diameter, one radius and one chord on that circle.

❒ Relationship between radius and diameter



Study the circle given alongside. Think over the following questions.

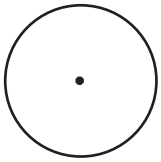
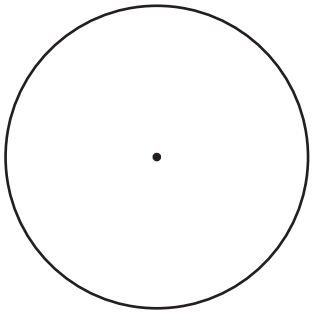
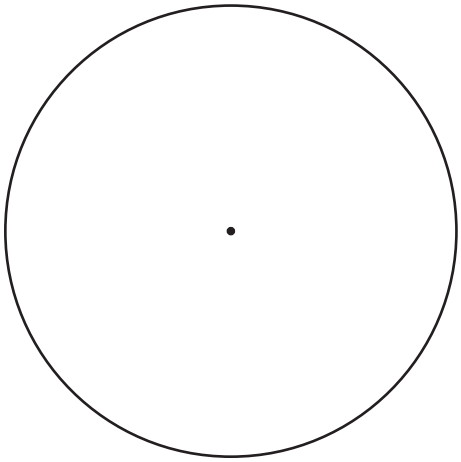
- Which are the radii in the circle?
- How many radii make up diameter AB?
- If the length of one radius is 3 cm, what is the length of the diameter?
- How long is the diameter as compared to the radius?

The diameter of a circle is twice the length of its radius.

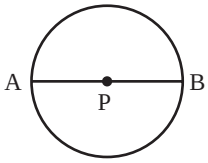
- If another diameter CD is drawn on the same circle, will its length be the same as that of AB?

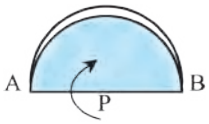
All the diameters of a circle are of the same length.

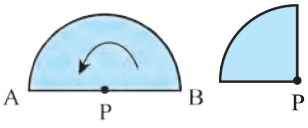
Test 1 : Measure the diameters and radii of the circles given below with a ruler and verify the relationship between their lengths.

Diagram			
Radius	1 cm		
Diameter	2 cm		

Test 2 :







1. Draw a circle on a piece of paper and cut it out.
2. Name the centre of the circle P.
3. Draw the diameter of the circle and name it AB. Note that PA and PB are radii of the circle.
4. Fold the circular paper along AB as shown in the picture.

Fold the paper at P in such a way that point B will fall on point A. Radius PB falls exactly on radius PA. In other words, they coincide.

From this, we can see that every radius of a circle is half the length of its diameter.

Problem Set 29

1. If the radius of a circle is 5 cm, what will its diameter be?

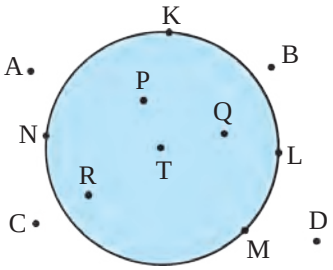
2. If the diameter of a circle is 6 cm, what will its radius be?

3. Complete the following table by filling in the blanks.

Radius	4 cm		9 cm	
Diameter		16 cm		22 cm

❒ The interior and the exterior of a circle

We play 'Land and Sea' inside a circle on the playground. In this game, the children inside the circle are in the 'sea', while the children outside the circle are on 'land'.



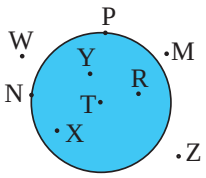
In the picture alongside, K, L, M and N are points on a circle with centre T.

The coloured area inside the circle in the picture is the interior of the circle. P, Q, R and T are points in the interior of the circle.

A, B, C and D are points in the exterior of the circle.

Problem Set 30

In the table below, write the names of the points in the interior and exterior of the circle and those on the circle.

Diagram	Points in the interior of the circle	Points in the exterior of the circle	Points on the circle
			

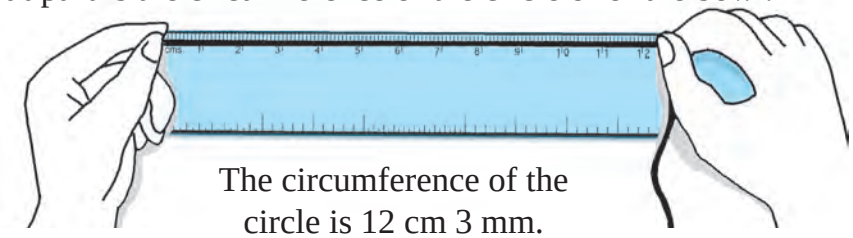
❒ The circumference of a circle

Take a bowl with a circular edge.

Wind a string once around the bowl and make a full circle around it.

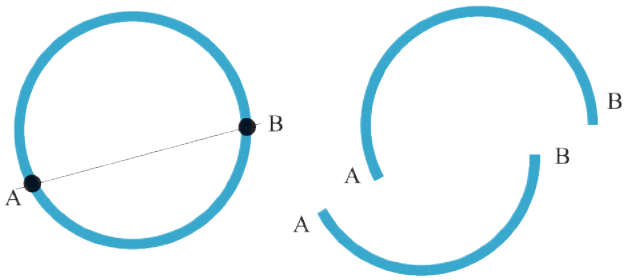
Unwind this circle and straighten it out as shown.

Measure the straightened part with a ruler. The length of that part is the circumference of the circle or of the bowl.



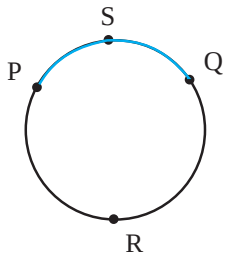
The circumference of the circle is 12 cm 3 mm.

❒ An arc of a circle



Shown alongside is a plastic bangle. If the bangle breaks at points A and B, it will split into two parts as shown in the picture.

Each of these parts is an **arc of a circle.**



On the given circle, there are two points P and Q. These two points have divided the circle into two parts. Each of these parts is an **arc of the circle.**

This means that P and Q have created two arcs. P and Q are the end points of both arcs.

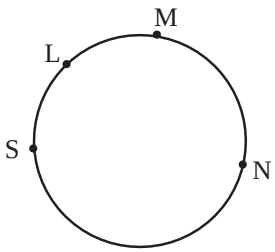
From the name 'arc PQ', we cannot say which of the two arcs we are speaking of. So, an additional point is taken on each arc. This point is used to give each arc a three-letter name. In the figure, there are two arcs, arc PSQ and arc PRQ.

Problem Set 31

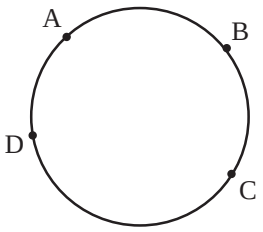
1. In the figure given alongside, points S, L, M and N are on the circle. Answer the questions with the help of the diagram.

(1) Write the names of the arcs with end-points S and M.

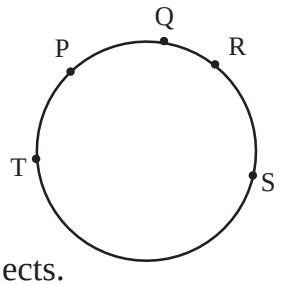
(2) Write the names of the arcs with the end-points L and N.



2. Write the names of arcs that points A, B, C and D in the given circle give rise to.



3. Give the names of the arcs that are made by points P, Q, R, S and T in the figure.



4. Measure and note down the circumference of different circular objects. (It is convenient to use a measuring tape for this purpose.)

Part Two

8. Multiples and Factors

Dada : I have 12 *pedhas*. I have to put them in equal groups so that no *pedha* is left over. How many *pedhas* must I put in each group ?

Sanju : Putting into groups means dividing. None left over means that the remainder will be 0.

Anju : 12 is divisible by 2, so we can make groups of two.

Manju : 12 is divisible by 3, so we can make groups of 3.

Sagar : We can also make groups of four.

Anita : Can we make groups of five?

Manju : No, because 12 is not divisible by 5.

Anju : We can divide 12 by six, so groups of six can be made.

Manju : We cannot make groups of 7, 8, 9, 10 or 11 because we cannot divide 12 by any of these numbers.

Sanju : We could make one group of twelve and give it to one person. Or, we could give 12 people 1 *pedha* each.

Dada : Very good. 12 is exactly divisible by 1, 2, 3, 4, 6 and 12, which means the remainder is 0. These numbers are called **divisors** or **factors** of 12.
Similarly, 1, 2, 4, 8 and 16 are factors of 16.

Problem Set 32

Write the factors of the following numbers.

(1) 8 (2) 5 (3) 14 (4) 10 (5) 7 (6) 22 (7) 25 (8) 32 (9) 33

❒ Multiples

Dada : You know what a divisor and a dividend is. Do you know what a multiple is?

Anju : I don't know what a multiple is, but I think it must be related to multiplication.

Dada : Right! Let me give you an example. You can solve 20 ÷ 5, can't you?

Anju : Yes. When we divide the dividend 20 by the divisor 5, the quotient is 4 and the remainder is 0.

Dada : When the division of a dividend leaves no remainder, the dividend is said to be a **multiple** of the divisor. In such a case, the dividend is the product of the divisor and the quotient. Here, 20 is a multiple of 5, but 21 is not.

Now tell me, can we divide 84 chalksticks into groups of six?

Suraj : Let me divide by 6. 84 can be divided exactly by 6 and the quotient is 14. Thus, we can make 14 groups of 6. So, 84 is the multiple of 6 and 6 is a factor of 84.

Dada : If the number of chalksticks is 6, 12, 18, 36 or 84, then we can make exact groups of 6 with none left over. It means that 6, 12, 18, 36 and 84 are multiples of 6, or that they are exactly divisible by 6. To see whether the number of chalksticks is a multiple of 6, divide that number by 6. If the remainder is 0, the number is a multiple of 6.

Each number in the 3 times table is exactly divisible by 3 or is a multiple of 3. Similarly, the numbers in the 7 times table are multiples of 7. Numbers in the 9 times table are multiples of 9.

We use this idea all the time. Let me ask you a few questions so as to make it clear.

I have a 200 *ml* measure. Will I be able to measure out 1 litre of milk with it?

Suraj : I litre is 1000 *ml*. 1000 = 200 × 5, which means that 1000 is a multiple of 200. So we can measure out 1 litre of milk with the 200 millilitre measure. 5 measures of 200 *ml* make 1 litre.

Dada : Can we measure out one and a half litres of milk with the 200 *ml* measure?

Anju : One and a half litres is 1500 *ml*. 1500 is not divisible by 200. So, it is not a multiple of 200. So the 200 *ml* measure cannot be used to measure out one and a half litres of milk.

Dada : I have 400 grams of *chana*. I have to make pouches of 60 grams each. Is that possible, if I don't want any left overs ?

Anju : No. 400 is not a multiple of 60.

Dada : How much more *chana* will I need to make those pouches of 60 grams each?

Anju : We will have to find the multiple of 60 that comes directly after 400.

60 × 6 = 360, 60 × 7 = 420. So, we need 20 grams more of *chana*.

❒ Tests for divisibility

Study the 2 times table and see which numbers appear in the units place. Similarly, divide 52, 74, 80, 96 and 98 by 2 to see if they are exactly divisible by 2. What rule do we get for determining whether a number is a multiple of 2?

Now study the 5 and 10 times tables.

See what rules you get for finding multiples of 5 and 10,
that is, numbers divisible by 5 and 10.

Test for divisibility by 2	:	If there is 0, 2, 4, 6 or 8 in the units place, the number is a multiple of 2, or is exactly divisible by 2.
Test for divisibility by 5	:	Any number with 5 or 0 in the units place is a multiple of 5 or, is divisible by 5.
Test for divisibility by 10	:	Any number that has 0 in the units place is a multiple of 10.

Problem Set 33

1. (1) Write five three-digit numbers that are multiples of 2.
(2) Write five three-digit numbers that are multiples of 5.
(3) Write five three-digit numbers that are multiples of 10.

2. Write 5 numbers that are multiples of 2 as well as of 3.

3. A ribbon is 3 metres long. Can we cut it into 50 cm pieces and have nothing left over ? Write the reason why or why not.

4. A ribbon is 3 metres long. I need 8 pieces of ribbon each 40 cm long. How many centimetres shorter is the ribbon than the length I need?

5. If the number given in the table is divisible by the given divisor, put ✓ in the box. If it is not divisible by the divisor, put × in the box.

Divisor / Number	2	5	10
15	×	✓	×
30			
34			
46			

Divisor / Number	2	5	10
55			
63			
70			
84			

❒ Prime and composite numbers

Some numbers are given in the tables below. Write all of their factors.

Numbers	Factors
2	1, 2
3	1, 3
4	1, 2, 4
5	
6	

Numbers	Factors
11	
12	
16	
19	
25	

Dada : What do you notice on studying the table?

Ajay : The number 1 is a factor of every number. Some numbers have only 1 and the number itself as factors. For example, the only factors of 3 are 1 and 3. Similarly, the factors of 2 are only 1 and 2 and the factors of 19 are only 1 and 19. Some numbers have more than two factors.

Dada : Numbers like 2, 3, 19 which have only two factors are called prime numbers.

A number which has only two factors, 1 and the number itself, is called a prime number.

Ajay : What do we call numbers like 4, 6 and 16 which have more than two factors?

Dada : Numbers like 4, 6 and 16 are called composite numbers.

A number which has more than two factors is called a composite number.

Dada : Think carefully and tell me whether 1 is a prime or composite number.

Ajay : The number 1 has only one factor, 1 itself, so I can't answer your question.

Dada : You're right. 1 is considered neither a prime number nor a composite number.

1 is a number which is neither prime nor composite.

Problem Set 34

1. Write all the prime numbers between 1 and 20.

2. Write all the composite numbers between 21 and 50.

3. Circle the prime numbers in the list given below.
22, 37, 43, 48, 53, 60, 91, 57, 59, 77, 79, 97, 100

4. Which of the prime numbers are even numbers?

❒ Co-prime numbers

Dada : Tell me all the factors of 12 and 18.

Anju : I'll tell the factors of 12: 1, 2, 3, 4, 6, 12.

Manju : I'll give the factors of 18: 1, 2, 3, 6, 9, 18.

Dada : Now find the common factors of 12 and 18.

Anju : Common factors?

Dada : 1, 2, 3 and 6 are in both groups, which means that they are common factors. Now tell me the factors of 10 and 21.

Sanju : Factors of 10 : 1, 2, 5, 10.

Manju : Factors of 21: 1, 3, 7, 21.

Dada : Which of the factors in these two groups are common?

Sanju : 1 is the only common factor.

Dada : Numbers which have only 1 as a common factor are called **co-prime numbers**, so 10 and 21 are co-prime numbers. The common factors of 12 and 18 are 1, 2, 3 and 6; which means that the common factors are more than one. Therefore, 12 and 18 are not co-prime numbers. Now tell me whether 8 and 10 are co-prime numbers.

Manju : The factors of 8 are 1, 2, 4 and 8 and the factors of 10 are 1, 2, 5 and 10. These numbers have two factors, 1 and 2, in common, so 8 and 10 are not co-prime numbers.

Problem Set 35

Determine whether the pairs of numbers given below are co-prime numbers.

(1) 22, 24 (2) 14, 21 (3) 10, 33 (4) 11, 30

(5) 5, 7 (6) 15, 16 (7) 50, 52 (8) 17, 18

Activity 1 :

- Write numbers from 1 to 60.
- Draw a blue circle around multiples of 2.
- Draw a red circle around multiples of 4.
- Do all numbers with a blue circle also have a red circle around them?
- Do all the numbers with a red circle have a blue circle around them?
- Are all multiples of 2 also multiples of 4?
- Are all multiples of 4 also multiples of 2?.

Activity 2 :

- Write numbers from 1 to 60.
- Draw a triangle around multiples of 2.
- Draw a circle around multiples of 3.
- Now find numbers divisible by 6. Can you find a property that they share?

❒ Eratosthenes' method of finding prime numbers

Eratosthenes was a mathematician who lived in Greece about 250 BC. He discovered a method to find prime numbers. It is called Eratosthenes' Sieve. Let us see how to find prime numbers between 1 and 100 with this method.

1	11	21	31	41	51	61	71	81	91
2	12	22	32	42	52	62	72	82	92
3	13	23	33	43	53	63	73	83	93
4	14	24	34	44	54	64	74	84	94
5	15	25	35	45	55	65	75	85	95
6	16	26	36	46	56	66	76	86	96
7	17	27	37	47	57	67	77	87	97
8	18	28	38	48	58	68	78	88	98
9	19	29	39	49	59	69	79	89	99
10	20	30	40	50	60	70	80	90	100

- 1 is neither a prime nor a composite number. Put a square ☐ around it
- 2 is a prime number, so put a circle around it.
- Next, strike out all the multiples of 2. This tells us that of these 100 numbers more than half numbers are not prime numbers.

- The first number after 2 not yet struck off is 3. So, 3 is a prime number.
- Draw a circle around 3. Strike out all the multiples of 3.
- The next number after 3 not struck off yet is 5. So, 5 is a prime number.
- Draw a circle around 5. Put a line through all the multiples of 5.
- The next number after 5 without a line through it is 7. So, 7 is a prime number.
- Draw a circle around 7. Put a line through all the multiples of 7.

In this way, every number between 1 and 100 will have either a circle or a line through it. The circled numbers are prime numbers. The numbers with a line through them are composite numbers.

❒ One more method to find prime numbers

1	2	3	4	5	6
7	8	9	10	11	12
13	14	15	16	17	18
19	20	21	22	23	24
25	26	27	28	29	30
31	32	33	34	35	36
–	–	–	–	–	–
–	–	–	–	–	–

- See how numbers from 1 to 36 have been arranged in six columns in the table alongside.

  Continue in the same way and write numbers up to 102 in these six columns.

- You will see that, in the columns for 2, 3, 4, and 6, all the numbers are composite numbers except for the prime numbers 2 and 3. This means that all the remaining prime numbers will be in the columns for 1 and 5. Now isn't it easier to find them? So, go ahead, find the prime numbers !

Something more

- Prime numbers with a difference of two are called twin prime numbers.
  Some twin prime number pairs are 3 and 5, 5 and 7, 29 and 31 and 71 and 73.
  5347421 and 5347423 are also a pair of twin prime numbers.
- There are eight pairs of twin prime numbers between 1 and 100. Find them.
- Euclid the mathematician lived in Greece about 300 BC. He proved that if prime numbers, 2, 3, 5, 7, ..., are written in serial order, the list will never end, meaning that the number of prime numbers is infinite.

✦✦✦