13 Pythagoras’ Theorem

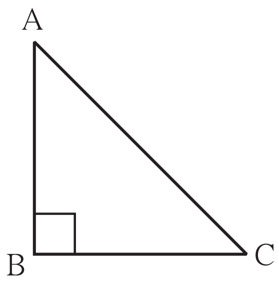
ZBPQZL

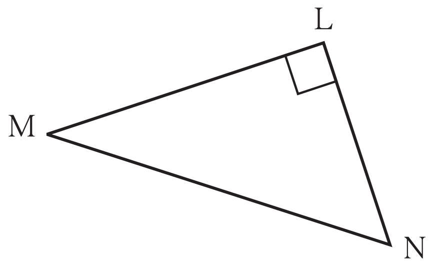
Let’s recall.

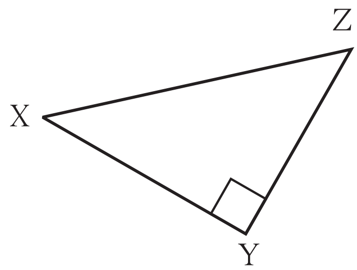
Right-angled Triangle

We know that a triangle with one right angle is called a right-angled triangle and the side opposite to the right angle is called the hypotenuse.

- Write the name of the hypotenuse of each of the right-angled triangles shown below.







The hypotenuse of ΔABC ☐

The hypotenuse of ΔLMN ☐

The hypotenuse of ΔXYZ ☐

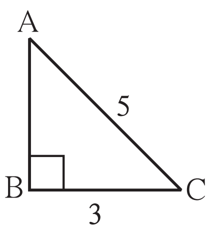
Pythagoras’ Theorem

Pythagoras was a great Greek mathematician of the 6th century BCE. He made important contributions to mathematics. His method of teaching mathematics was very popular. He trained several mathematicians.

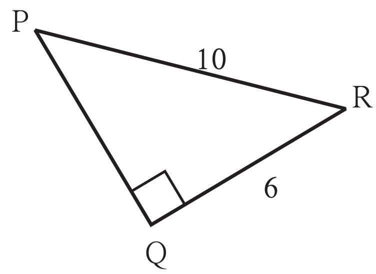
People of many countries had long known of a certain principle related to the right-angled triangle. It is also given in the book called Shulvasutra, of ancient India. As Pythagoras was the first to prove the theorem, it is named after him. This theorem of Pythagoras states that **in a right-angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides**.

Activity : Draw right-angled triangles given the hypotenuse and one side as shown in the rough figures below. Measure the third side. Verify Pythagoras’ theorem.

(i)



(ii)



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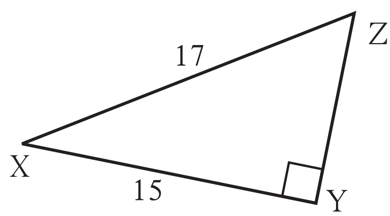
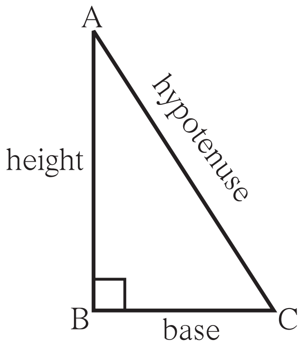


Let's learn.



With reference to the figure alongside, the theorem of Pythagoras can be written as follows :

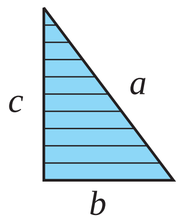
In ΔABC, if $\angle B$ is a right angle, then
$[l(AC)]^2 = [l(AB)]^2 + [l(BC)]^2$

Generally, in a right-angled triangle, one of the sides forming the right angle is taken as the base and the other as the height.

Then, the theorem can be stated as
$(\text{hypotenuse})^2 = (\text{base})^2 + (\text{height})^2$.

Follow the steps given below to **verify Pythagoras' theorem.**

Activity : From a cardsheet, cut out eight identical right-angled triangles. Let us say the length of the hypotenuse of these triangles is 'a' units, and sides forming the right angle are 'b' and 'c' units. Note that the area of this triangle is $\frac{bc}{2}$. Next, on another cardsheet, use a pencil to draw two squares ABCD and PQRS each of side (b + c) units. Now, place 4 of the triangle cut-outs in the square ABCD and the remaining 4 in the square PQRS as shown in the figures below. Mark by lines drawn across them, the parts of the squares covered by the triangles.



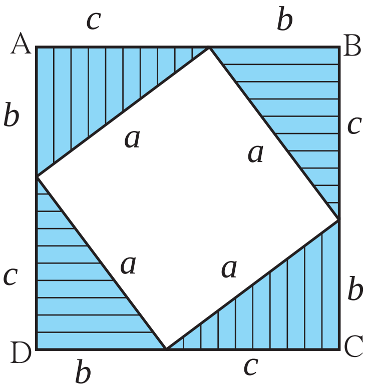


Figure (i)

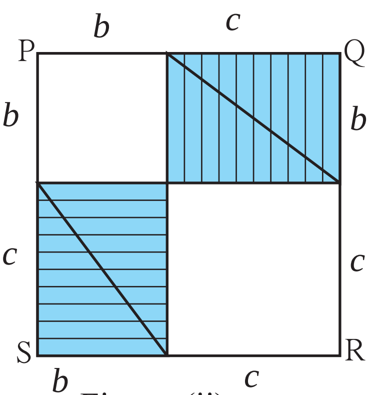


Figure (ii)

Observe the figures. In figure (i) we can see a square of side *a* units in the uncovered portion of square ABCD. In figure (ii) we see a square of side b and another of side c in the uncovered portion of the square PQRS.

In figure (i), area of square ABCD = $a^2 + 4 \times$ area of right-angled triangle

$$= a^2 + 4 \times \frac{1}{2}\, bc$$

$$= a^2 + 2bc$$

In figure (ii), area of square PQRS $= b^2 + c^2 + 4 \times$ area of right-angled triangle

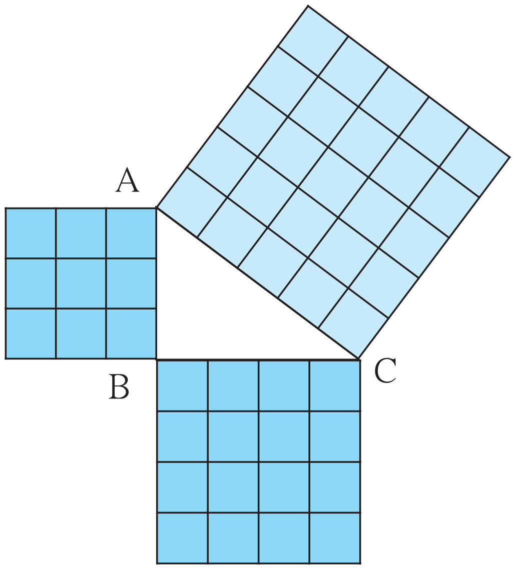
$$= b^2 + c^2 + 4 \times \frac{1}{2} bc$$
$$= b^2 + c^2 + 2bc$$

Area of square ABCD = Area of square PQRS

$$\therefore\ a^2 + 2bc = b^2 + c^2 + 2bc$$
$$\therefore\ a^2 = b^2 + c^2$$

- Without using a protractor, can you verify that every angle of the vacant quadrilateral in figure (i) is a right angle ?

Activity : On a sheet of card paper, draw a right-angled triangle of sides 3 cm, 4 cm and 5 cm. Construct a square on each of the sides. Find the area of each of the squares and verify Pythagoras' theorem.

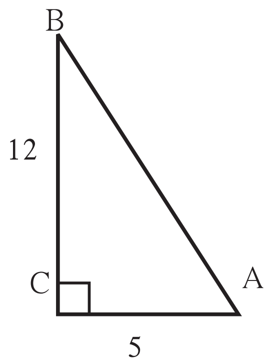


Given two sides of a right-angled triangle, you can find the third side, using Pythagoras' theorem.

Example In ΔABC, $\angle$C = 90°, l(AC) = 5 cm and l(BC) = 12 cm. What is the length of seg (AB)?

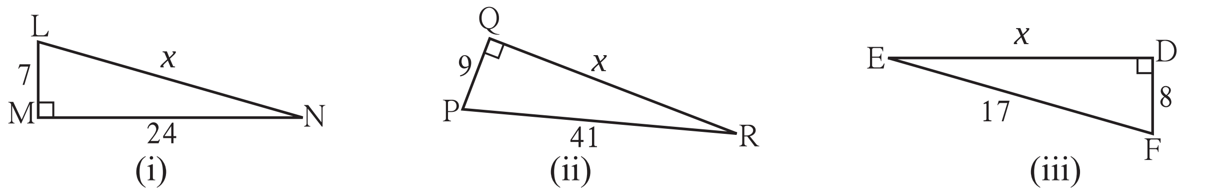
Solution : In the right-angled triangle ABC, $\angle$C = 90°.
Hence, side AB is the hypotenuse.
According to Pythagoras' theorem,

$l(AB)^2 = l(AC)^2 + l(BC)^2$
$= 5^2 + 12^2$
$= 25 + 144$
$l(AB)^2 = 169$
$l(AB)^2 = 13^2$
$l(AB) = 13$
$\therefore$ Length of seg AB = 13 cm.



Practice Set 48

1. In the figures below, find the value of 'x'.



(i) (ii) (iii)

2. In the right-angled ΔPQR, $\angle P = 90°$. If $l(PQ) = 24$ cm and $l(PR) = 10$ cm, find the length of seg QR.

3. In the right-angled ΔLMN, $\angle M = 90°$. If $l(LM) = 12$ cm and $l(LN) = 20$ cm, find the length of seg MN.

4. The top of a ladder of length 15 m reaches a window 9 m above the ground. What is the distance between the base of the wall and that of the ladder ?

Let's learn.

If, in a triplet of natural numbers, the square of the biggest number is equal to the sum of the squares of the other two numbers, then the three numbers form **a Pythagorean triplet.** If the lengths of the sides of a triangle form such a triplet, then the triangle is a right-angled triangle.

Example Do the following numbers form a Pythagorean triplet : (7, 24, 25) ?

Solution : $7^2 = 49$, $24^2 = 576$, $25^2 = 625$

$\therefore$ $49 + 576 = 625$

$\therefore$ $7^2 + 24^2 = 25^2$

7, 24 and 25 is a Pythagorean triplet.

Activity : From the numbers 1 to 50, pick out Pythagorean triplets.

Practice Set 49

1. Find the Pythagorean triplets from among the following sets of numbers.

(i) 3, 4, 5 (ii) 2, 4, 5

(iii) 4, 5, 6 (iv) 2, 6, 7

(v) 9, 40, 41 (vi) 4, 7, 8

2. The sides of some triangles are given below. Find out which ones are right-angled triangles?

(i) 8, 15, 17 (ii) 11, 12, 15 (iii) 11, 60, 61 (iv) 1.5, 1.6, 1.7

(v) 40, 20, 30