



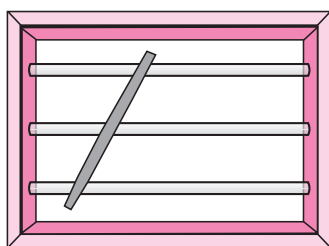
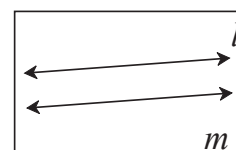
## Let's study.

- Properties of angles formed by parallel lines and its transversal
- Tests of parallelness of two lines
- Use of properties of parallel lines



## Let's recall.

**Parallel lines :** The lines which are coplanar and do not intersect each other are called parallel lines.



Hold a stick across the horizontal parallel bars of a window as shown in the figure.

How many angles are formed ?

- Do you recall the pairs of angles formed by two lines and their transversal ?

In figure 2.1, line  $n$  is a transversal of line  $l$  and line  $m$ .

Here, in all 8 angles are formed. Pairs of angles formed out of these angles are as follows :

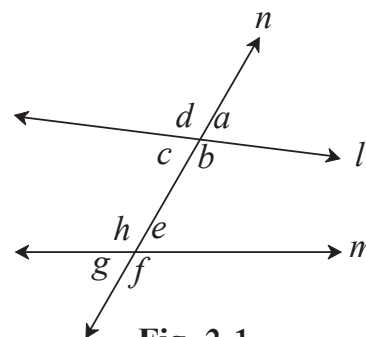


Fig. 2.1

**Pairs of corresponding angles**

- (i)  $\angle d, \angle h$
- (ii)  $\angle a, \square$
- (iii)  $\angle c, \square$
- (iv)  $\angle b, \square$

**Pairs of alternate interior angles**

- (i)  $\angle c, \angle e$  (ii)  $\angle b, \angle h$

**Pairs of alternate exterior angles**

- (i)  $\angle d, \angle f$
- (ii)  $\angle a, \angle g$

**Pairs of interior angles on the same side of the transversal**

- (i)  $\angle c, \angle h$
- (ii)  $\angle b, \angle e$

**Some important properties :**

- (1) When two lines intersect, the pairs of opposite angles formed are congruent.
- (2) The angles in a linear pair are supplementary.

- (3) When one pair of corresponding angles is congruent, then all the remaining pairs of corresponding angles are congruent.
- (4) When one pair of alternate angles is congruent, then all the remaining pairs of alternate angles are congruent.
- (5) When one pair of interior angles on one side of the transversal is supplementary, then the other pair of interior angles is also supplementary.



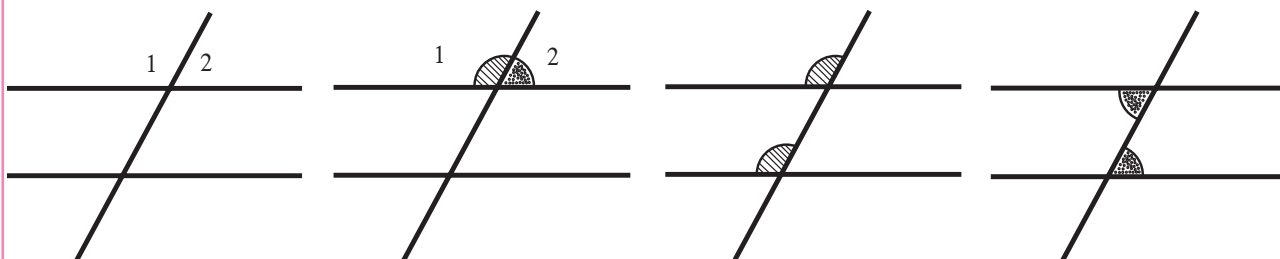
## Let's learn.

## Properties of parallel lines

## Activity

To verify the properties of angles formed by a transversal of two parallel lines.

Take a piece of thick coloured paper. Draw a pair of parallel lines and a transversal on it. Paste straight sticks on the lines. Eight angles will be formed. Cut pieces of coloured paper, as shown in the figure, which will just fit at the corners of  $\angle 1$  and  $\angle 2$ . Place the pieces near different pairs of corresponding angles, alternate angles and interior angles and verify their properties.





## Let's learn.

We have verified the properties of angles formed by a transversal of two parallel lines. Let us now prove the properties using Euclid's famous fifth postulate given below.

If sum of two interior angles formed on one side of a transversal of two lines is less than two right angles then the lines produced in that direction intersect each other.

### Interior angle theorem

**Theorem :** If two parallel lines are intersected by a transversal, the interior angles on either side of the transversal are supplementary.

**Given :** line  $l \parallel$  line  $m$  and line  $n$  is their transversal. Hence as shown in the figure  $\angle a$ ,  $\angle b$  are interior angles formed on one side and  $\angle c$ ,  $\angle d$  are interior angles formed on other side of the transversal.

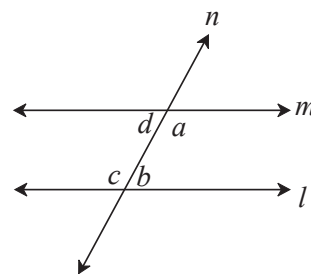


Fig. 2.2

**To prove :**  $\angle a + \angle b = 180^\circ$   
 $\angle d + \angle c = 180^\circ$

**Proof :** Three possibilities arise regarding the sum of measures of  $\angle a$  and  $\angle b$ .

(i)  $\angle a + \angle b < 180^\circ$  (ii)  $\angle a + \angle b > 180^\circ$  (iii)  $\angle a + \angle b = 180^\circ$

Let us assume that the possibility (i)  $\angle a + \angle b < 180^\circ$  is true.

Then according to Euclid's postulate, if the line  $l$  and line  $m$  are produced will intersect each other on the side of the transversal where  $\angle a$  and  $\angle b$  are formed.

But line  $l$  and line  $m$  are parallel lines. ....given

$\therefore \angle a + \angle b < 180^\circ$  impossible .....(I)

Now let us suppose that  $\angle a + \angle b > 180^\circ$  is true.

$\therefore \angle a + \angle b > 180^\circ$

But  $\angle a + \angle d = 180^\circ$

and  $\angle c + \angle b = 180^\circ$  . . . . . angles in linear pairs

$\therefore \angle a + \angle d + \angle b + \angle c = 180^\circ + 180^\circ = 360^\circ$

$\therefore \angle c + \angle d = 360^\circ - (\angle a + \angle b)$

If  $\angle a + \angle b > 180^\circ$  then  $[360^\circ - (\angle a + \angle b)] < 180^\circ$

$\therefore \angle c + \angle d < 180^\circ$

∴ In that case line  $l$  and line  $m$  produced will intersect each other on the same side of the transversal where  $\angle c$  and  $\angle d$  are formed.

∴  $\angle c + \angle d < 180^\circ$  is impossible.

That is  $\angle a + \angle b > 180^\circ$  is impossible..... (II)

∴ the remaining possibility,

$\angle a + \angle b = 180^\circ$  is true.....from (I) and (II)

∴  $\angle a + \angle b = 180^\circ$  Similarly,  $\angle c + \angle d = 180^\circ$

Note that, in this proof, because of the contradictions we have denied the possibilities  $\angle a + \angle b > 180^\circ$  and  $\angle a + \angle b < 180^\circ$ .

Therefore, this proof is an example of indirect proof.

### Corresponding angles and alternate angles theorems

**Theorem :** The corresponding angles formed by a transversal of two parallel lines are of equal measure.

**Given :** line  $l \parallel$  line  $m$   
line  $n$  is a transversal.

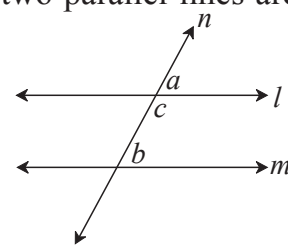
**To prove :**  $\angle a = \angle b$

**Proof :**  $\angle a + \angle c = 180^\circ$  .....(I) angles in linear pair

$\angle b + \angle c = 180^\circ$  .....(II) property of interior angles of parallel lines

$\angle a + \angle c = \angle b + \angle c$  .....from (I) and (II)

∴  $\angle a = \angle b$



**Fig. 2.3**

**Theorem :** The alternate angles formed by a transversal of two parallel lines are of equal measures.

**Given :** line  $l \parallel$  line  $m$   
line  $n$  is a transversal.

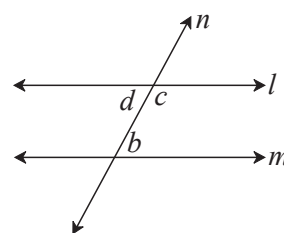
**To prove :**  $\angle d = \angle b$

**Proof :**  $\angle d + \angle c = 180^\circ$  .....(I) angles in linear pair

$\angle c + \angle b = 180^\circ$  .....(II) property of interior angles of parallel line

$\angle d + \angle c = \angle c + \angle b$  .....from (I) and (II)

∴  $\angle d = \angle b$



**Fig. 2.4**

### Practice set 2.1

2. In figure 2.6, line  $p \parallel$  line  $q$  and line  $l$  and line  $m$  are transversals. Measures of some angles are shown. Hence find the measures of  $\angle a$ ,  $\angle b$ ,  $\angle c$ ,  $\angle d$ .



2. In figure 2.6, line  $p \parallel$  line  $q$  and line  $l$  and line  $m$  are transversals. Measures of some angles are shown. Hence find the measures of  $\angle a$ ,  $\angle b$ ,  $\angle c$ ,  $\angle d$ .



3. In figure 2.7, line  $l \parallel$  line  $m$  and line  $n \parallel$  line  $p$ . Find  $\angle a$ ,  $\angle b$ ,  $\angle c$  from the given measure of an angle.



- 4\*. In figure 2.8, sides of  $\angle PQR$  and  $\angle XYZ$  are parallel to each other. Prove that,  
 $\angle PQR \cong \angle XYZ$

5. In figure 2.9, line  $AB \parallel$  line  $CD$  and line  $PQ$  is transversal. Measure of one of the angles is given.

Hence find the measures of the following angles.

- (i)  $\angle ART$       (ii)  $\angle CTQ$   
 (iii)  $\angle DTQ$     (iv)  $\angle PRB$

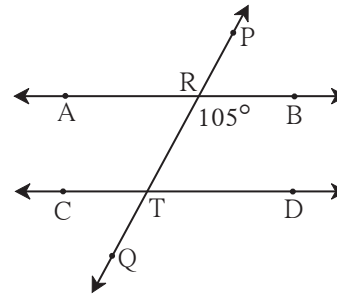


Fig. 2.9



Let's learn.

### Use of properties of parallel lines

Let us prove a property of a triangle using the properties of angles made by a transversal of parallel lines.

**Theorem :** The sum of measures of all angles of a triangle is  $180^\circ$ .

**Given :**  $\triangle ABC$  is any triangle.

**To prove :**  $\angle ABC + \angle ACB + \angle BAC = 180^\circ$ .

**Construction :** Draw a line parallel to seg  $BC$  and passing through  $A$ . On the line take points  $P$  and  $Q$  such that,  $P - A - Q$ .

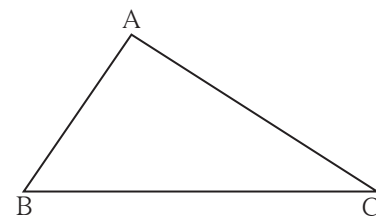


Fig. 2.10

**Proof :** Line  $PQ \parallel$  line  $BC$  and seg  $AB$  is a transversal.

$$\therefore \angle ABC = \angle PAB \dots \text{alternate angles} \dots (I)$$

line  $PQ \parallel$  line  $BC$  and seg  $AC$  is a transversal.

$$\therefore \angle ACB = \angle QAC \dots \text{alternate angles} \dots (II)$$

$\therefore$  From I and II ,

$$\angle ABC + \angle ACB = \angle PAB + \angle QAC \dots (III)$$

Adding  $\angle BAC$  to both sides of (III).

$$\angle ABC + \angle ACB + \angle BAC = \angle PAB + \angle QAC + \angle BAC$$

$$= \angle PAB + \angle BAC + \angle QAC$$

$$= \angle PAC + \angle QAC \dots (\because \angle PAB + \angle BAC = \angle PAC)$$

$$= 180^\circ \dots \text{Angles in linear pair}$$

That is, sum of measures of all three angles of a triangle is  $180^\circ$ .

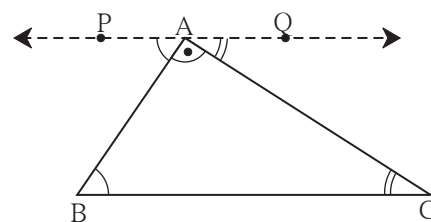
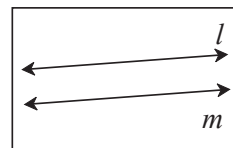


Fig. 2.11



In fig. 2.12, How will you decide whether line  $l$  and line  $m$  are parallel or not ?



**Fig. 2.12**



### Tests for parallel lines

Whether given two lines are parallel or not can be decided by examining the angles formed by a transversal of the lines.

- (1) If the interior angles on the same side of a transversal are supplementary then the lines are parallel.
- (2) If one of the pairs of alternate angles is congruent then the lines are parallel.
- (3) If one of the pairs of corresponding angles is congruent then the lines are parallel.

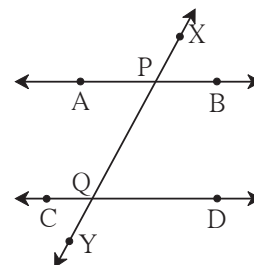
### Interior angles test

**Theorem :** If the interior angles formed by a transversal of two distinct lines are supplementary, then the two lines are parallel.

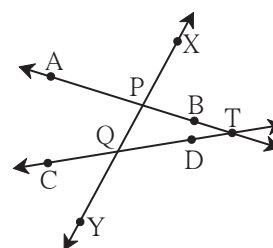
**Given :** Line XY is a transversal of line AB and line CD.  
 $\angle BPQ + \angle P Q D = 180^\circ$

**To prove :** line AB  $\parallel$  line CD

**Proof :** We are going to give an indirect proof.  
 Let us suppose that the statement to be proved is wrong. That is, we assume, line AB and line CD are not parallel, means line AB and CD intersect at point T.  
 So  $\triangle PQT$  is formed.



**Fig. 2.13**



**Fig. 2.14**

$\therefore \angle TPQ + \angle PQT + \angle PTQ = 180^\circ$  .....sum of angles of a triangle  
 but  $\angle TPQ + \angle PQT = 180^\circ$  .....given  
 That is the sum of two angles of the triangle is  $180^\circ$ .  
 But sum of three angles of a triangle is  $180^\circ$ .  
 $\therefore \angle PTQ = 0^\circ$ .

$\therefore$  line PT and line QT means line AB and line CD are not distinct lines.

But, we are given that line AB and line CD are distinct lines.

$\therefore$  we arrive at a contradiction.

$\therefore$  our assumption is wrong. Hence line AB and line CD are parallel.

Thus it is proved that if the interior angles formed by a transversal are supplementary, then the lines are parallel.

This property is called **interior angles test** of parallel lines.

### Alternate angles test

**Theorem :** If a pair of alternate angles formed by a transversal of two lines is congruent then the two lines are parallel.

**Given :** Line  $n$  is a transversal of line  $l$  and line  $m$ .

$\angle a$  and  $\angle b$  is a congruent pair of alternate angles.

That is,  $\angle a = \angle b$

**To prove :** line  $l \parallel$  line  $m$

**Proof :**  $\angle a + \angle c = 180^\circ$  .....angles in linear pair

$\angle a = \angle b$  ..... given

$\therefore \angle b + \angle c = 180^\circ$

But  $\angle b$  and  $\angle c$  are interior angles on the same side of the transversal.

$\therefore$  line  $l \parallel$  line  $m$  ..... interior angles test

This property is called the **alternate angles test** of parallel lines.

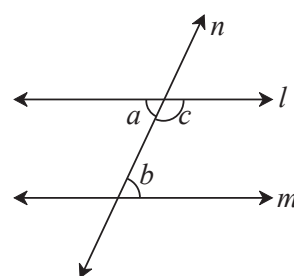


Fig. 2.15

### Corresponding angles Test

**Theorem :** If a pair of corresponding angles formed by a transversal of two lines is congruent then the two lines are parallel.

**Given :** Line  $n$  is a transversal of line  $l$  and line  $m$ .

$\angle a$  and  $\angle b$  is a congruent pair of corresponding angles.

That is,  $\angle a = \angle b$

**To prove :** line  $l \parallel$  line  $m$

**Proof :**  $\angle a + \angle c = 180^\circ$  .....angles in linear pair

$\angle a = \angle b$  ..... given

$\therefore \angle b + \angle c = 180^\circ$

That is a pair of interior angles on the same side of the transversal is congruent.

$\therefore$  line  $l \parallel$  line  $m$  .....interior angles test

This property is called the **corresponding angles test** of parallel lines.

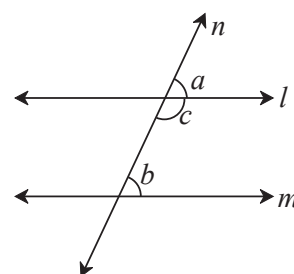


Fig. 2.16



**Corollary I :** If a line is perpendicular to two lines in a plane, then the two lines are parallel to each other.

**Given :** Line  $n \perp$  line  $l$  and line  $n \perp$  line  $m$

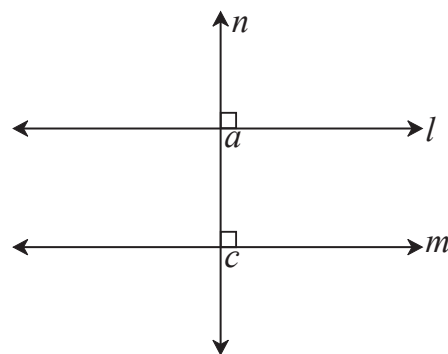
**To prove :** line  $l \parallel$  line  $m$

**Proof :** line  $n \perp$  line  $l$  and line  $n \perp$  line  $m$  ...given

$$\therefore \angle a = \angle c = 90^\circ$$

$\angle a$  and  $\angle c$  are corresponding angles formed by transversal  $n$  of line  $l$  and line  $m$ .

$\therefore$  line  $l \parallel$  line  $m$  ....corresponding angles test

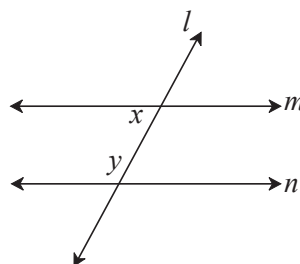


**Fig. 2.17**

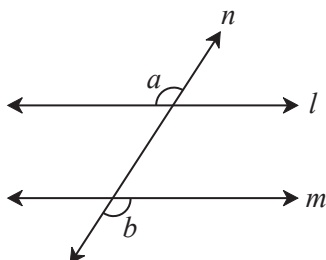
**Corollary II :** If two lines in a plane are parallel to a third line in the plane then those two lines are parallel to each other. Write the proof of the corollary.

### Practice set 2.2

1. In figure 2.18,  $y = 108^\circ$  and  $x = 71^\circ$   
Are the lines  $m$  and  $n$  parallel ? Justify ?



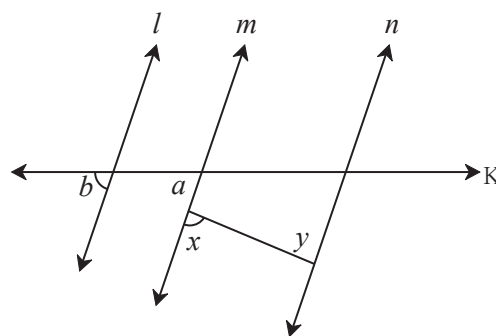
**Fig. 2.18**



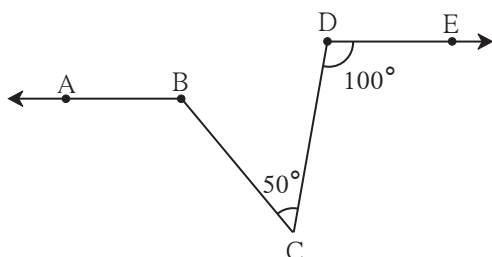
**Fig. 2.19**

2. In figure 2.19, if  $\angle a \cong \angle b$  then prove that line  $l \parallel$  line  $m$ .

3. In figure 2.20, if  $\angle a \cong \angle b$  and  $\angle x \cong \angle y$  then prove that line  $l \parallel$  line  $n$ .



**Fig. 2.20**



**Fig. 2.21**

4. In figure 2.21, if ray  $BA \parallel$  ray  $DE$ ,  $\angle C = 50^\circ$  and  $\angle D = 100^\circ$ . Find the measure of  $\angle ABC$ .

(Hint : Draw a line passing through point C and parallel to line AB.)

5.

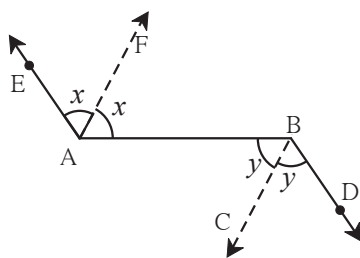


Fig. 2.22

In figure 2.22, ray  $AE \parallel$  ray  $BD$ , ray  $AF$  is the bisector of  $\angle EAB$  and ray  $BC$  is the bisector of  $\angle ABD$ . Prove that line  $AF \parallel$  line  $BC$ .

6. A transversal  $EF$  of line  $AB$  and line  $CD$  intersects the lines at point  $P$  and  $Q$  respectively. Ray  $PR$  and ray  $QS$  are parallel and bisectors of  $\angle BPQ$  and  $\angle PQC$  respectively. Prove that line  $AB \parallel$  line  $CD$ .

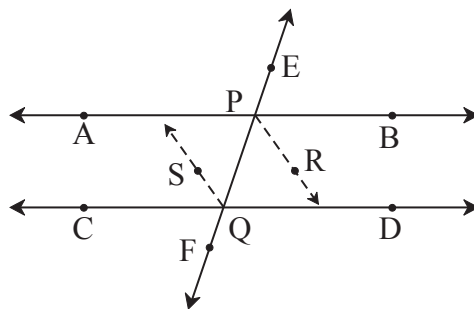

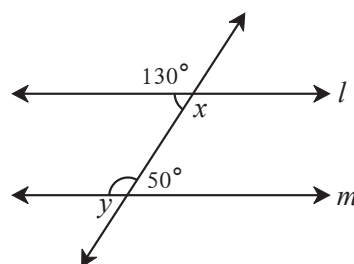


Fig. 2.23

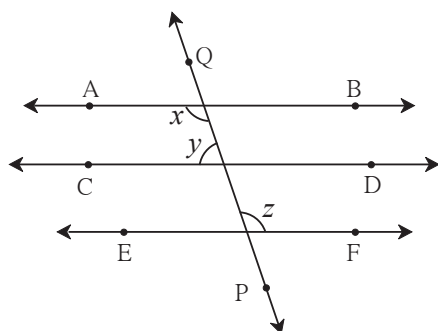
### ◆◆◆◆◆◆◆◆◆◆ Problem set 2 ◆◆◆◆◆◆◆◆◆◆

1. Select the correct alternative and fill in the blanks in the following statements.
  - (i) If a transversal intersects two parallel lines then the sum of interior angles on the same side of the transversal is .....  
 (A)  $0^\circ$  (B)  $90^\circ$  (C)  $180^\circ$  (D)  $360^\circ$
  - (ii) The number of angles formed by a transversal of two lines is .....  
 (A) 2 (B) 4 (C) 8 (D) 16
  - (iii) A transversal intersects two parallel lines. If the measure of one of the angles is  $40^\circ$  then the measure of its corresponding angle is .....  
 (A)  $40^\circ$  (B)  $140^\circ$  (C)  $50^\circ$  (D)  $180^\circ$
  - (iv) In  $\Delta ABC$ ,  $\angle A = 76^\circ$ ,  $\angle B = 48^\circ$ ,  $\therefore \angle C =$  .....  
 (A)  $66^\circ$  (B)  $56^\circ$  (C)  $124^\circ$  (D)  $28^\circ$
  - (v) Two parallel lines are intersected by a transversal. If measure of one of the alternate interior angles is  $75^\circ$  then the measure of the other angle is .....  
 (A)  $105^\circ$  (B)  $15^\circ$  (C)  $75^\circ$  (D)  $45^\circ$
- 2\*. Ray  $PQ$  and ray  $PR$  are perpendicular to each other. Points  $B$  and  $A$  are in the interior and exterior of  $\angle QPR$  respectively. Ray  $PB$  and ray  $PA$  are perpendicular to each other. Draw a figure showing all these rays and write -
  - (i) A pair of complementary angles (ii) A pair of supplementary angles.
  - (iii) A pair of congruent angles.

3. Prove that, if a line is perpendicular to one of the two parallel lines, then it is perpendicular to the other line also.
4. In figure 2.24, measures of some angles are shown. Using the measures find the measures of  $\angle x$  and  $\angle y$  and hence show that line  $l \parallel$  line  $m$ .
- 
- The diagram shows two horizontal lines,  $l$  and  $m$ , intersected by a transversal line. At the intersection with line  $l$ , the top-right angle is labeled  $130^\circ$  and the bottom-right angle is labeled  $x$ . At the intersection with line  $m$ , the bottom-right angle is labeled  $50^\circ$  and the bottom-left angle is labeled  $y$ .

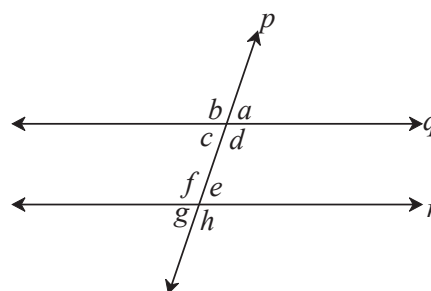


**Fig. 2.24**

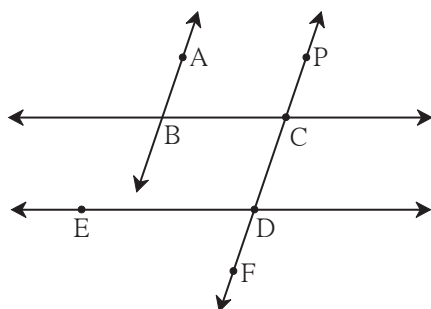


**Fig. 2.25**

6. In figure 2.26, if line  $q \parallel$  line  $r$ , line  $p$  is their transversal and if  $a = 80^\circ$  find the values of  $f$  and  $g$ .

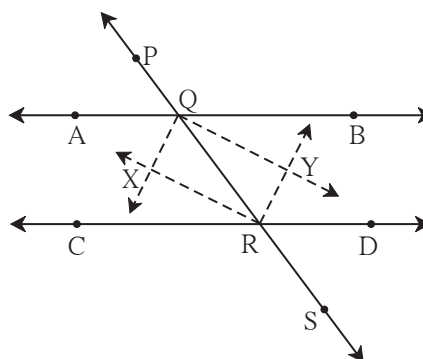


**Fig. 2.26**



**Fig. 2.27**

8. In figure 2.28, line PS is a transversal of parallel line AB and line CD. If Ray QX, ray QY, ray RX, ray RY are angle bisectors, then prove that  $\square$  QXRY is a rectangle.



**Fig. 2.28**

