2018

GATE Computer Science & Information Technology

Time : 3 hrs **MM :** 100

Read the following instructions carefully

1. In this paper, there are 65 questions carrying 100 marks to be completed in 3 hrs.
2. Questions 1 to 25 carry 1 mark each and questions 26 to 55 carry 2 marks each.
3. Questions 56 to 65 belong to General Aptitude (GA) Type. Questions 56 to 60 carry 1 mark each and questions 61 to 65 carry 2 marks each.
4. Unattempted questions will carry zero marks.
5. For questions 1 to 25 and 56 to 60, 1/3 mark will be deducted for each wrong answer. For questions 26 to 55 and 61 to 65, 2/3 mark will be deducted for each wrong answer.
6. There is no negative marking for numerical answer type questions.
7. No physical calculator will be allowed. Charts, graph sheets and mathematical tables are not allowed in the examination hall.

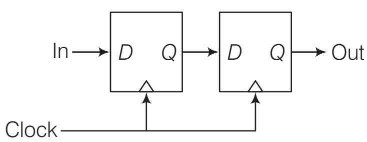
(1 *Mark Questions*)

1. Consider a long-lived TCP session with an end-to-end bandwidth of 1 Gbps ($=10^9$ bits-per-second). The session starts with a sequence number of 1234. The minimum time (in seconds, rounded to the closest integer) before this sequence number can be used again is ________.

Sol. (34) 1 sec $= 10^9$ bits; 1 sec $= \frac{10^9}{8}$ bytes

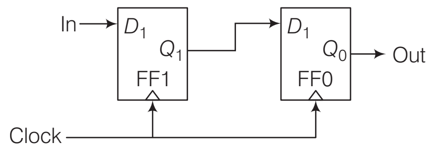
$$\frac{2^{32} \times 8}{10^9} \text{ sec} = 2^{32} \text{ bytes} \Rightarrow 34.35 \text{ sec}$$

2. Consider the sequential circuit shown in the figure, where both flip-flops used are positive edge-triggered D flip-flops.



The number of states in the state transition diagram of the circuit that have a transition back to the same state on some value of "in" is ________.

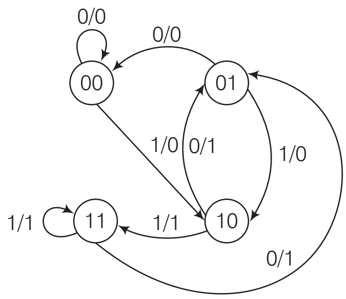
Sol. (2) Given



The state table is a s follows

Present state		Input	Flip-Flop inputs		Next state		
Q_1	Q_0	x	$D_1 = x$	$D_0 = Q_1$	Q_1	Q_0	Output $=Q_0$
0	0	0	0	0	0	0	0
0	0	1	1	0	1	0	0
0	1	0	0	0	0	0	0
0	1	1	1	0	1	0	0
1	0	0	0	1	0	1	1
1	0	1	1	1	1	1	1
1	1	0	0	1	0	1	1
1	1	1	1	1	1	1	1

The state transition diagram is shown below



So, there are 2 states 00 and 11 where transition is happening back to the same state.

Hence, answer is 2.

3. Let $\oplus$ and $\odot$ denote the Exclusive OR and Exclusive NOR operations, respectively. Which one of the following is NOT CORRECT?

(a) $\overline{P \oplus Q} = P \odot Q$ (b) $\overline{P} \oplus Q = P \odot Q$

(c) $\overline{P} \oplus \overline{Q} = P \oplus Q$ (d) $(P \oplus \overline{P}) \oplus Q = (P \odot \overline{P}) \odot \overline{Q}$

Sol. (d) (A) $\overline{P \oplus Q} = P \odot Q$ true

(B) $\overline{P} \oplus Q = (\overline{\overline{P}})Q + \overline{P}\,\overline{Q} = PQ + \overline{P}\,\overline{Q} = P \odot Q$ true

(C) $\overline{P} \oplus \overline{Q} = (\overline{\overline{P}})\overline{Q} + (\overline{P})(\overline{\overline{Q}}) = P\overline{Q} + \overline{P}Q = P \oplus Q$ true

(D) $(P \oplus \overline{P}) \oplus Q = (P \odot \overline{P}) \odot \overline{Q}$ false

$(P \oplus \overline{P}) \oplus Q = 1 \oplus Q = \overline{Q}$

$(P \odot \overline{P}) \odot \overline{Q} = 0 \oplus \overline{Q} = Q$

$\therefore (P \oplus \overline{P}) \oplus Q \neq (P \odot \overline{P}) \odot \overline{Q}$

4. A 32-bit wide main memory unit with a capacity of 1 GB is built using 256 M $\times$ 4-bit DRAM chips. The number of rows of memory cells in the DRAM chip is 2^{14}. The time taken to perform one refresh operation is 50 nanoseconds. The refresh period is 2 milliseconds.

The percentage (rounded to the closest integer) of the time available for performing the memory read/write operations in the main memory unit is ________.

Sol. (59) In order to design memory of 1GB, number of chips required

$$= \frac{1\text{ GB}}{256\text{ M} \times 4} = \frac{2^{33}}{2^{30}} = 2^3 = 8 \text{ chips}$$

So, to make the word size as 32 bits, we can arrange them in 1 row.

Rows : $\boxed{C1}\boxed{C2}\boxed{C3}$ ... $\boxed{CB}$ $\boxed{\text{C:Chip l256M}\times 4}$

Refresh circuit is designed for memory unit so that the entire chip data can be refreshed.

Now, design the chips into a banks, i.e. 4 rows of chips grouped into 1 bank so that the overhead can be minimised. So, refresh occurs in parallel and the refresh overhead can be given as

$$\text{Refresh overhead} = \frac{\text{Time required for refresh}}{\text{Refresh interval}}$$

$$= \frac{2^{14} \times 50\text{ ns}}{2\text{ ms}} = 2^{13} \times 50 \times 10^{-9} \times 10^3 = 0.4096 = 40.96\%$$

So, in order to read/write operations in the main memory unit, the % time available, is

$\Rightarrow$ $1 - 0.4096 = 0.5904 \Rightarrow 59.04\% \simeq 59\%$

5. Which one of the following is a closed form expression for the generating function of the sequence $\{a_n\}$, where $a_n = 2n + 3$ for all $n = 0, 1, 2, ...$?

(a) $\dfrac{3}{(1-x)^2}$ (b) $\dfrac{3x}{(1-x)^2}$ (c) $\dfrac{2-x}{(1-x)^2}$ (d) $\dfrac{3-x}{(1-x)^2}$

Sol. (d) Given, $a_n = 2n + 3$

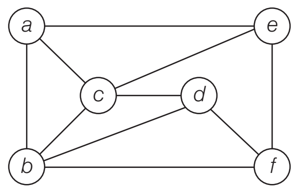
The required series is [3, 5, 7, 9, 11, ...]

Since, generating function for 1 is $\dfrac{1}{1-x}$ and n is $\dfrac{x}{(1-x)^2}$, the generating function for a_n is $A(x) = \dfrac{2x}{(1-x^2)} + \dfrac{3}{1-x}$

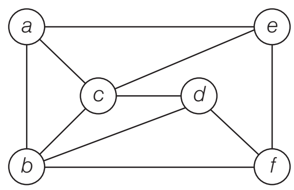
$$= \frac{2x + 3(1-x)}{(1-x)^2} = \frac{2x + 3 - 3x}{(1-x)^2} = \frac{3-x}{(1-x)^2}$$

Hence, correct option is (d).

6. The chromatic number of the following graph is ____.

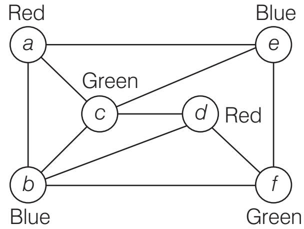


Sol. (3) Given,



$\because$ Largest complete sub graph is K_3, chromatic number is atleast 3.

$\therefore$ We can try for a chromatic number of 3 by using 3 colors, as follows:



Since, we have successfully, properly coloured as vertices with only 3 colors, the chromatic number of this graph is 3.

Alternate Method

Welch Powell's Theorem

b	c	d	e	f	a
c_1	c_2	c_3	c_1	c_2	c_3

$X(c_1) \le 3$ and $X(c_1) \ge 3$

[$\because$ we have 3 mutually adjacent vertices]

$\therefore \quad X(c_1) = 3$

7. In an Entity-Relationship (ER) model, suppose R is a many-to-one relationship from entity set E1 to entity set E2. Assume that E1 and E2 participate totally in R and that the cardinality of E1 is greater than the cardinality of E2.

Which one of the following is true about R?

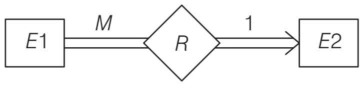
(a) Every entity in E1 is associated with exactly one entity in E2

(b) Some entity in E1 is associated with more than one entity in E2

(c) Every entity in E2 is associated with exactly one entity in E1

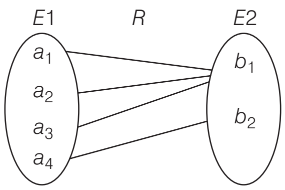
(d) Every entity in E2 is associated with at most one entity in E1

Sol. (a) Since, it is a many-to-one relationship from E1 to E2,



No entity in E1 can be related to more than one entity in E2.

E1 entities > E2 entities



Every entity is E1 is associated with exactly one entity in E2.

8. Consider the following C program:

```
#include < stdio.h>
struct ournode
{
      char x, y, z;
};
int main ()
{
   struct ournode p = {'1', '0', 'a' + 2};
   struct ournode *p = & q;
   printf("%c, %c", *((char*) q+1), *((char*) q+2));
      return 0;
}
```

The output of this program is

(a) 0, c (b) 0, a + 2 (c) '0', 'a + 2' (d) '0', 'c'

Sol. (a) In structure declaration 'a' + 2 will be adding 2 to ASCII value of a, which results in character c.

Structure pointer is printing character for second element and third element by typecasting it to character pointer and printing value pointed by them hence result will print character 0 and c.

9. The value of $\int_0^{\pi/4} x \cos(x^2)dx$ correct to three decimal places (assuming that $\pi = 3.14$) is ________.

Sol. (0.289)

Given, $\int_0^{\pi/4} x \cos(x^2)dx$

Let, $\quad t = x^2$

On differentiating the above equation,

$$dt = 2x\,dx$$

$$\Rightarrow \quad xdx = \frac{dt}{2}$$

when $x = 0, t = 0$ and when $x = \frac{\pi}{4}$,

$$t = \left(\frac{\pi}{4}\right)^2 = \frac{\pi^2}{16}$$

So, the given integral is reduced to

$$\int_0^{\pi^2/16} \cos t\, dt = [\sin t]_0^{\pi^2/16}$$

$$= \sin\left(\frac{\pi^2}{16}\right) - \sin(0) = 0.28898$$

$$\simeq 0.289$$

10. The following are some events that occur after a device controller issues an interrupt while process L is under execution.

(P) The processor pushes the process status of L onto the control stack.

(Q) The processor finishes the execution of the current instruction.

(R) The processor executes the interrupt service routine.

(S) The processor pops the process status of L from the control stack.

(T) The processor loads the new PC value based on the interrupt.

Which one of the following is the correct order in which the events above occur?

(a) QPTRS (b) PTRSQ

(c) TRPQS (d) QTPRS

Sol. (a) Processor required to handle the interrupt:

1. First current instruction is completed.
2. It pushes the status in control stack.
3. Initialize the program counter.
4. Execute the ISR for device controller.
5. Resume the old process.

11. Consider the following statements regarding the slow start phase of the TCP congestion control algorithm. Note the *cwnd* stands for the TCP congestion window and MSS denotes the Maximum Segment Size.

(i) The *cwnd* increases by 2 MSS on every successful acknowledgment.

(ii) The *cwnd* approximately doubles on every successful acknowledgment.

(iii) The *cwnd* increases by 1 MSS every round trip time.

(iv) The *cwnd* approximately doubles every round trip time.

Which one of the following is correct?

(a) Only (ii) and (iii) are true
(b) Only (i) and (iii) are true
(c) Only (iv) is true
(d) Only (i) and (iv) are true

Sol. (c) In slow start phase of TCP congestion control, the congestion window size is exponentially increased, in other words, *cwnd* approximately doubles every round trip time.

12. Two people, P and Q, decide to independently roll two identical dice, each with 6 faces. Numbered 1 to 6. The person with the lower number wins. In case of a tie, they roll the dice repeatedly until there is no tie. Define a trial as a throw of the dice by P and Q. Assume that all 6 numbers on each dice are equi-probable and that all trials are independent. The probability (rounded to 3 decimal places) that one of them wins on the third trial is ________.

Sol. (0.023) Given, there are two identical dices, each having 6 faces. Favorable events for tie = {(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)}. Since, two dices are thrown, total sample space will be $6 \times 6 = 36$.

P(Tie in any trial) = P(P = 1 and Q = 1) + P(P = 2 and Q = 2) + ... + P (P = 6 and Q = 6)

$$= \frac{1}{36} + \frac{1}{36} + \frac{1}{36} + \frac{1}{36} + \frac{1}{36} + \frac{1}{36} = \frac{6}{36} = \frac{1}{6}$$

P(one of them wins) $= 1 -$ P(Tie)

$$= 1 - \frac{1}{6} = \frac{5}{6}$$

So, required probability = P(Ist trial Tie) $\times$ P(IInd trial Tie) $\times$ P(one of them wins) $= \frac{1}{6} \times \frac{1}{6} \times \frac{5}{6} = \frac{5}{216} = 0.023$ (rounded to 3 decimal places)

13. Consider the following two tables and four queries in SQL.

Book (<u>isbn</u>, bname), Stock (<u>isbn</u>, copies)

```
Query 1 : SELECT B.isbn, S.copies
          From Book B INNER JOIN Stock S
          ON B.isbn = S.isbn;
Query 2: SELECT B.isbn, S.copies
          FROM Book B LEFT OUTER JOIN Stock S
          ON B.isbn = S.isbn;
Query 3: SELECT B.isbn, S.copies
          FROM Book B RIGHT OUTER JOIN Stock S
          ON B.isbn = S.isbn;
Query 4: SELECT B.isbn, S.copies
          FROM Book B FULL OUTER JOIN Stock S
          ON B.isbn = S.isbn;
```

Which one of the queries above is certain to have an output that is superset of the outputs of the other three queries?

(a) Query 1
(b) Query 2
(c) Query 3
(d) Query 4

Sol. (d)

Book (<u>isbn</u>, bname)		Stock (<u>isbn</u>, copies)	
1	P	3	100
3	Q	5	150
5	R	9	150
7	S	11	500
9	T		

Query 1 :

isbn	copies
3	100
5	150
9	150

Query 2:

isbn	copies
3	100
5	150
9	150
1	Null
7	Null

Query 3:

isbn	copies
3	100
5	150
9	150
11	500

Query 4:

isbn	copies
3	100
5	150
9	150
1	Null
7	Null
11	500

Query 4 is full outer join so that full outer join record set superset of records compare to inner join, left outer join and right outer join.

14. Consider a process executing on an operating system that uses demand paging. The average time for a memory access in the system is M units if the corresponding memory page is available in memory and D units if the memory access causes a page fault. It has been experimentally measured that the average time taken for a memory access in the process is x units. Which one of the following is the correct expression for the page fault rate experienced by the process?

(a) (D − M)/(X − M) (b) (X − M)/(D − M)
(c) (D − X)/(D − M) (d) (X − M)/(D − X)

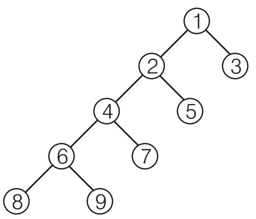
Sol. (b) Experimentally measured average time can be given as

$$\text{EMAT} = P * S + (1 - P) * M$$
$$X = P * D + (1 - P) * M$$
$$X = P * D + M - P * M$$
$$X - M = P\,(D - M),\ P = \frac{X - M}{D - M}$$

where, M = Memory Access Time
D = Page Fault Service Time (s)
P = Page Fault Rate
X = Effective Access Time

15. The postorder traversal of a binary tree is 8, 9, 6, 7, 4, 5, 2, 3, 1. The inorder traversal of the same tree is 8, 6, 9, 4, 7, 2, 5, 1, 3. The height of a tree is the length of the longest path from the root to any leaf. The height of the binary tree above is ________.

Sol. (4) Postorder and inorder traversal tree for the given binary is shown below



So, the height of the tree is 4.

16. Let G be a finite group on 84 elements. The size of a largest possible proper subgroup of G is ________.

Sol. (42) Given, | G | = 84

By Lagrange's theorem any subgroup size is a divisor of 84.

So, the factors of 84 are 1, 2, 3, 4, 6, 7, 12, 14, 21, 28, 42 and 84.

But a proper subgroup cannot have same size as group.

So, largest divisor of 84, other than 84 is 42. Since, we know that order of subgroup (L) ≠ order of Group (G)

So, largest proper subgroup can have in size of 42.

17. Match the following

Field	Length in bits
P. UDP Header's Port Number	I. 48
Q. Ethernet MAC Address	II. 8
R. IPv6 Next Header	III. 32
S. TCP Header's Sequence Number	IV. 16

(a) P-III, Q-IV, R-II, S-I (b) P-II, Q-I, R-IV, S-III
(c) P-IV, Q-I, R-II, S-III (d) P-IV, Q-I, R-III, S-II

Sol. (c) UDP Header's Port Number ⇒ 16 bit

Ethernet MAC Address ⇒ 48 bit

IPv6 Next Header ⇒ 8 bit

TCP Header's Sequence Number ⇒ 32 bit

18. Let N be an NFA with n states. Let k be the number of states of a minimal DFA which is equivalent to N. Which one of the following is necessarily true?

(a) $k \geq 2^n$ (b) $k \geq n$ (c) $k \leq n^2$ (d) $k \leq 2^n$

Sol. (d) n is number of states of given NFA (may not be minimal) and k is number of states of equivalent min DFA. First we convert NFA to DFA using subset construction algorithm and we get an equivalent DFA which will have atmost 2^n states. Hence k, which is number of states in minimal DFA should be less than 2^n.

19. Consider a system with 3 processes that share 4 instances of the same resource type. Each process can request a maximum of K instances. Resource instances can be requested and released only one at a time. The largest value of K that will always avoid deadlock is ________.

Sol. (2) Number of processes = 3

P_1 P_2 P_3
| | | No deadlock
|

Maximum each process can request for 2 resources so that, there will not be any deadlock, because we have only 4 resources available.

∴ Demand < number of processes + number of resources

$$3k < 3 + 4 \Rightarrow 3k < 7$$

$$\Rightarrow \quad k < \frac{7}{3} \Rightarrow k < 2.333$$

So, the value of $k = 2$.

20. Consider the following processor design characteristics

I. Register-to-register arithmetic operations only.
II. Fixed-length instruction format.
III. Hardwired control unit.

Which of the characteristics above are used in the design of a RISC processor?

(a) I and II only (b) II and III only
(c) I and III only (d) I, II and III

Sol. (d) RISC processor characteristics:

1. It supports mode registers, so ALU operations are performed only on a register data. (Also called Register-to-Register Architecture)
2. It supports fixed length instructions.
3. It uses hard-wire control unit.
4. It allows successive for pipeline.

21. The set of all recursively enumerable languages is

(a) closed under complementation
(b) closed under intersection
(c) a subset of the set of all recursive languages
(d) an uncountable set

Sol. (b) The set of RE (Recursive Enumerable) languages is closed under intersection, not closed under complementation, is not a subset of set of REC language. Recursive languages are turing recognizable and number of turing machine is countable.

22. Consider a matrix $A = uv^T$, where $u = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$, $v = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$.

Note that v^T denotes the transpose of v. The largest eigen value of A is ________.

Sol. (3) Given, $u = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ and $v = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

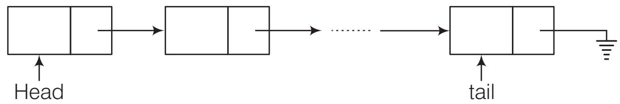
Matrix, $A = uv^T = \begin{bmatrix} 1 \\ 2 \end{bmatrix} [1 \quad 1]$ $\quad (\because v^T = [1 \quad 1])$

$$= \begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}$$

$$(1-\lambda)(2-\lambda) - 2 = 0$$
$$\Rightarrow \quad 2 - \lambda - 2\lambda + \lambda^2 - 2 = 0$$
$$\Rightarrow \quad \lambda^2 - 3\lambda + 2 - 2 = 0$$
$$\Rightarrow \quad \lambda^2 - 3\lambda = 0$$
$$\Rightarrow \quad \lambda(\lambda - 3) = 0$$
$$\Rightarrow \quad \lambda = 0 \text{ or, } \lambda = 3$$

The largest eigenvalue is 3.

23. A queue is implemented using a non-circular singly linked list. The queue has a head pointer and tail pointer, as shown in the figure. Let n denote the number of nodes in the queue. Let enqueue be implemented by inserting a new node at the head and dequeue be implemented by deletion of a node from the tail.



Which one of the following is the time complexity of the most time-efficient implementation of enqueue and dequeue, respectively, for this data structure?

(a) $\theta(1), \theta(1)$ (b) $\theta(1), \theta(n)$ (c) $\theta(n), \theta(1)$ (d) $\theta(n), \theta(n)$

Sol. (b) **Enqueue** In this, operation can be performed by inserting a node at the beginning and modifying the head pointer to point to new node. So, this requires time $\theta(1)$.

Dequeue In this, operation needed to point to previous node of the node pointed by tail pointer that requires traversal of the node till the last node. This operation requires $\theta(n)$ time in the worst case.

24. Consider the following C program:

```
#include <stdio. h>
int counter = 0;
int calc (int a, int b) {
    int c;
    counter ++;
    if (b == 3) return (a*a*a);
    else {
        c = calc (a, b/3);
        return (c*c*c);
    }
}
int main () {
    calc(4, 81);
    printf("%d", counter);
}
```

The output of this program is ________.

Sol. (4) The output of given program is 4.

25. Which one of the following statements is FALSE?

(a) Context-free grammar can be used to specify both lexical and syntax rules
(b) Type checking is done before parsing
(c) High-level language programs can be translated to different intermediate representations
(d) Arguments to a function can be passed using the program stack

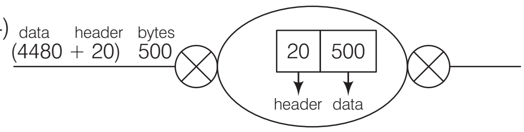
Sol. (b) Type checking is done before parsing is clearly false because in compiler type checking is done in semantic analysis phase which comes after parsing phase.

(2 Marks Questions)

26. Consider an IP packet with a length of 4500 bytes that includes a 20-byte IPv4 header and a 40-byte TCP header. The packet is forwarded to an IPv4 router that supports a Maximum Transmission Unit (MTU) of 600 bytes. Assume that the length of the IP header in all the outgoing fragments of this packet is 20 bytes. Assume that the fragmentation offset value stored in the first fragment is 0.

The fragmentation offset value stored in the third fragment is ________.

Sol. (144)



Total number of fragments $= \frac{4480}{576} = 7.777 \simeq 8$

	(1)	(2)	(3)
Date Length	576 Bytes (0-575)	576 Bytes (576-1151)	576 Bytes (1152-1727)
Offset	$\frac{0}{8} = 0$	$\frac{576}{8} = 72$	$\frac{1152}{8} = 144$

27. Consider the following four relational schemas. For each schema, all non-trivial functional dependencies are listed. The underlined attributes are the respective primary keys.

Schema I Registration (<u>rollno</u>, courses)

Field 'courses' is a set-valued attribute containing the set of courses a student has registered for.

Non-trivial functions dependency:

rollno → courses

Schema II: Registration (<u>rollno, courseid</u>, email)

Non-trivial functional dependencies:

rollno, courseid → email

email → rollno

Schema III: Registration (<u>rollno, courseid</u>, marks, grade)

Non-trivial functional dependencies:

rollno, courseid → marks, grade

marks → grade

Schema IV: Registration (<u>rollno, courseid</u>, credit)

Non-trivial functional dependencies:

rollno, courseid → credit

courseid → credit

Which one of the relational schemas above is in 3NF but not in BCNF?

(a) Schema I (b) Schema II
(c) Schema III (d) Schema IV

Sol. (b)

Schema I Candidate keys: {Rollno} and highest form is BCNF.

Schema II Registration (<u>rollno, courseid</u>, email)

Primary key [<u>rollno, courseid</u>]

Non-trivial functional dependencies:

{ rollno, courseid → email

email → rollno }

candidate keys {<u>rollno, courseid</u>,}

<u>email courseid</u>}

Given relation is in 3NF but not in BCNF.

Schema III Candidate key {<u>rollno, courseid</u>}

Highest normal form is 2NF and it is not in 3NF.

Schema IV Candidate keys (<u>rollno, courseid</u>)

Highest normal form is 1NF and it is not in 2NF.

28. Consider the following problems L(G) denotes the language generated by a grammar G. L(M) denotes the language accepted by a machine M.

I. For an unrestricted grammar G and a string *w*, whether $w \in L(G)$

II. Given a Turing Machine M, whether L(M) is regular.

III. Given two grammars G_1 and G_2, whether $L(G_1) = L(G_2)$

IV. Given an NFAN, whether there is a deterministic PDAP such that N and P accept the same language.

Which one of the following statements is correct?

(a) Only I and II are undecidable
(b) Only III is undecidable
(c) Only II and IV are undecidable
(d) Only I, II and III are undecidable

Sol. (d) The given statements, I, II and III are undecidable.

29. Consider Guwahati (G) and Delhi (D) whose temperatures can be classified as high (H), medium (M) and low (L). Let $P(H_G)$ denotes the probability that Guwahati has high temperature. Similarly, $P(M_G)$ and $P(L_G)$ denotes the probability of Guwahati having medium and low temperatures, respectively. Similarly, we use $P(H_D)$, $P(M_D)$ and $P(L_D)$ for Delhi.

The following table gives the conditional probabilities for Delhi's temperature given Guwahati's temperature.

	H_D	M_D	L_D
H_G	0.40	0.48	0.12
M_G	0.10	0.65	0.25
L_G	0.01	0.50	0.49

Consider the first row in the table above.

The first entry denotes that if Guwahati has high temperature (H_G), then the probability of Delhi also having a high temperature (H_D) is 0.40; i.e., $P(H_D | H_G) = 0.40$. Similarly, the next two entries are $P(M_D | H_G) = 0.48$ and $P(L_D | H_G) = 0.12$. Similarly for the other rows.

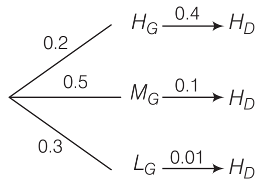
If it is known that $P(H_G) = 0.2$, $P(M_G) = 0.5$ and $P(L_G) = 0.3$, then the probability (correct to two decimal places) that Guwahati has high temperature given that Delhi has high temperature is ________.

Sol. (0.60) The condition probability table given is

	H_D	M_D	L_D
H_G	0.40	0.48	0.12
M_G	0.10	0.65	0.25
L_G	0.01	0.50	0.49

$$P(H_G) = 0.2$$
$$P(M_G) = 0.5$$
$$P(L_G) = 0.3$$

Drawing the tree diagram for H_D we get,



$$P(H_G | H_D) = \frac{P(H_G \cap H_D)}{P(H_D)}$$

$$P(H_G \cap H_D) = P(H_G) \times P(H_D | H_G)$$

From diagram, $P(H_G \cap H_D) = 0.2 \times 0.4 = 0.08$

$$P(H_D) = P(H_G) \,.\, P(H_D | H_G) + P(M_G)$$
$$P(H_D | M_G) + P(L_G)\, P(H_D | L_G)$$
$$P(H_D) = 0.2 \times 0.4 + 0.5 \times 0.1 + 0.3 \times 0.01$$
$$= 0.08 + 0.05 + 0.003 = 0.133$$

Required probability, $P(H_G | H_D) = \dfrac{0.2 \times 0.4}{0.133} = 0.60$

[rounding upto 2 decimal place]

30. The size of the physical address space of a processor is 2^P bytes. The word length is 2^W bytes. The capacity of cache memory is 2^N bytes, the size of each cache block is 2^M words. For a K-way set-associative cache memory, the length (in number of bits) of the tag field is

(a) $P - N - \log_2 K$
(b) $P - N + \log_2 K$
(c) $P - N - M - W - \log_2 K$
(d) $P - N - M - W + \log_2 K$

Sol. (b) Given, MM space $= 2^P$ bytes

Physical Address (PA) size $= P$ bits

CM size $= 2^N$ bytes

Block size $= 2^M$ words

Size of cache block $= 2^M$ words * 2^W bytes/word

$= 2^{(M+W)}$ bytes

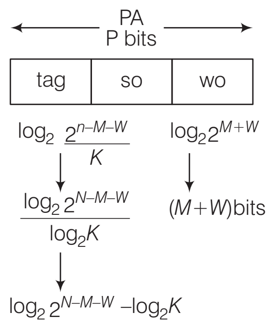
$$\text{Number of lines} = \frac{\text{CM size}}{\text{Cache block size}}$$

$$\Rightarrow \frac{2^N}{2^{M+W}} \Rightarrow 2^{N-W-M}$$

$$\text{Number of sets} = \frac{\text{Number in cm}}{\text{P - way}}$$

$$= \frac{2^{N-M-W}}{K}$$

Address format



$$\text{Number of bits for set offset} = \log_2 \frac{2^{N-M-W}}{K}$$

$\Rightarrow \log_2 2^{N-M-W} - \log_2 K \Rightarrow (N - M - W - \log_2 K)$

$\therefore$ Tag size

$\Rightarrow P - (N - M - W - \log_2 K + M + W)$

$\Rightarrow P - N + M + W + \log_2 K - M - W \quad \Rightarrow P - N + \log_2 K$

31. Consider the following program written in pseudo-code. Assume that x and y are integers.

```
count (x, y){
        if (y != 1) {
            if (x != 1){
                print ('*');
                count (x / 2, y);
            }
            else {
                y = y - 1;
                count (1024, y);
            }
        }
}
```

The number of times that the print statement is executed by the call count (1024, 1024) is ________.

Sol. (10230)

32. Consider the weights and values of items listed below. Note that there is only one unit of each item.

Item Number	Weight in (kgs)	Value in (Rupees)
1	10	60
2	7	28
3	4	20
4	2	24

The task is to pick a subset of these items such that their total weight is not more than 11 kgs and their total value is maximized. Moreover, no item may be split. The total value of items picked by an optimal algorithm is denoted by V_{opt}.

A greedy algorithm sorts the items by their value-to-weight ratios in descending order and packs them greedily, starting from the first item in the ordered list. The total value of items picked by the greedy algorithm is denoted by $V_{\text{greedy.}}$ The value of $V_{\text{opt}} - V_{\text{greedy}}$ is ________.

Sol. (16)

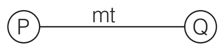
33. Consider a simple communication system where multiple nodes are connected by a shared broadcast medium (like Ethernet or wireless). The nodes in the system use the following carrier-sense based medium access protocol. A node that receives a packet to transmit will carrier-sense the medium for 5 units of time. If the node does not detect any other transmission in this duration, it starts transmitting its packet in the next time unit. If the node detects another transmission, it waits until this other transmission finishes, and then begins to carrier-sense for 5 time units again. Once they start to transmit, nodes do not perform any collision detection and continue transmission even if a collision occurs. All transmissions last for 20 units of time. Assume that the transmission signal travels at the speed of 10 meters per unit time in the medium.

Assume that the system has two nodes P and Q, located at a distance d meters from each other. P starts transmitting a packet at time $t = 0$ after successfully completing its carrier-sense phase. Node Q has a packet to transmit at time $t = 0$ and begins to carrier-sense the medium.

The maximum distance d (in meters, rounded to the closest integer) that allows Q to successfully avoid a collision between its proposed transmission and P's ongoing transmission is ________.

Sol. (50) A node that receives a packet to transmit with carrier sense the medium for 5 units of time which implies that

$\rightarrow$ Channel sensing time = 5 units of the propagation speed
= 10 mt/unit time



In order to collision avoidance, P's packet have to be travelled a distance of 50 m in 5 units of time.

[$\because$ P starts transmitting a packet at time $t = 0$ after successfully completing its carrier sense phase.
Node Q has a packet to transmit at time $r = 0$ and begins to carrier sense the medium].

34. Let G be a simple undirected graph. Let T_D be a depth first search tree of G. Let T_B be a breadth first search tree of G. Consider the following statements.

I. No edge of G is a cross edge with respect to T_D. (A cross edge in G is between two nodes neither of which is an ancestor of the other in T_D)

II. For every edge (u,v) of G, if u is at depth i and v is at depth j in T_B, then $|i - j| = 1$.

Which of the statements above must necessarily be true?

(a) I only
(b) II only
(c) Both I and II
(d) Neither I nor II

Sol. (a)

35. Consider the first-order logic sentence

$\phi \equiv \exists s \exists t \exists u \forall v \forall w \forall x \forall y\, \psi(s,t,u,v,w,x,y)$

where, $\psi(s,t,u,v,w,x,y)$ is a quantifier-free first-order logic formula using only predicate symbols and possibly equality, but no function symbols.

Suppose ϕ has a model with a universe containing 7 elements.

Which one of the following statements is necessarily true?

(a) There exists at least one model of ϕ with universe of size less than or equal to 3
(b) There exists no model of ϕ with universe of size less than or equal to 3
(c) There exists no model of ϕ with universe of size greater than 7
(d) Every model of ϕ has a universe of size equal to 7

Sol. (b) There exists no model of ϕ with universe of size less than or equal to 3.

36. Consider the following C program

```
#include <stdio.h>
void fun1 (char *s1, char *s2) {
        char *tmp;
        tmp = s1;
        s1 = s2;
        s2 = tmp;
}
void fun2 (char **s1, char **s2) {
      char *tmp;
      tmp = *s1;
      *s1 = *s2;
      *s2 = tmp;
}
int main ( ) {
      char *str1 = "Hi", *str2 = "Bye";
      fun1 (str1, str2) ; printf("% s % &", str1, str2);
      fun2 (&str1, &str2); printf("%s %s", str1, str2);
      return 0;
}
```

The output of the program above is

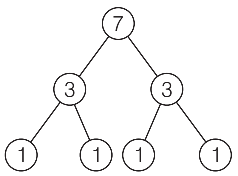
(a) Hi Bye Bye Hi (b) Hi Bye Hi Bye
(c) Bye Hi Hi Bye (d) Bye Hi Bye Hi

Sol. (a) The first call to the function 'fun1 (str1, str2);' is **Call by Value**. Hence, any change in the formal parameters are NOT reflected in actual parameters. Hence, str1 points at "Hi" and str2 points at "Bye".

The second call to the function 'fun2(&str1, &str2);' is **Call by Reference**. Hence, any change in the formal parameters are reflected in actual parameters.Hence, str1, now points at "Bye" and str2 points at "Hi". Hence, Answer is Hi Bye Bye Hi.

37. The number of possible min-heaps containing each value from {1, 2, 3, 4, 5, 6, 7} exactly once is ________.

Sol. (80)



Now do direct $= \dfrac{7!}{7 \times 3 \times 3} = 80$. Hence, 7! because 8 items to be filled, now 7 because root has 7 nodes as its decedent including itself, same as 3 and 3 and 1 and 1.

38. Let N be the set of natural numbers. Consider the following sets.

P. Set of rational numbers (positive and negative).
Q. Set of functions from {0, 1} to N.
R. Set of functions from N to {0, 1}
S. Set of finite subsets of N.

Which of the sets above are countable?

(a) Q and S only (b) P and S only
(c) P and R only (d) P, Q and S only

Sol. (d)

P : Set of Rational numbers are **countable**. Rational numbers are of the form pq where p,q are integers. Enumeration procedure, take $p+q$ and write down all possible values (positive and negative).

Q : Set of functions from {0, 1} to N. There are N_2 such functions. Hence **countable**.

R : Set of functions from N to {0, 1}. There are $2N$ such functions. Important theorem ⇒ If a set S is countable, then $P(S)$, i.e. $2S$ is uncountable.

Hence, statement R is uncountable.

S : Set of finite subsets of N. They are countable. Important theorem ⇒

Every subset of a countable set is either countable or finite.

39. Consider the unsigned 8-bit fixed point binary number representation below $b_7\, b_6\, b_5\, b_4\, b_3\, .\, b_2\, b_1\, b_0$

where, the position of the binary point is between b_3 and b_2. Assume b_7 is the most significant bit. Some of the decimal numbers listed below cannot be represented exactly in the above representation.

(i) 31.500
(ii) 0.875
(ii) 12.100
(iv) 3.001

Which one of the following statements is true?

(a) None of (i), (ii), (iii), (iv) can be exactly represented.
(b) Only (ii) cannot be exactly represented.
(c) Only (iii) and (iv) cannot be exactly represented.
(d) Only (i) and (ii) cannot be exactly represented.

Sol. (c) Binary code : $(b_7\, b_6\, b_5\, b_4\, b_3\, .\, b_2\, b_1\, b_0)_2$

$$(31.5)_{10} = (11111.1)_2$$
$$(0.875)_{10} = (0.111)_2$$
$$(12.100)_{10} = (01100.000110\ldots)_2$$

↓

Only 3 bits of fraction space available.
So cannot be stored.

So, we can say we cannot represent the decimal number 12.100 in its equivalent binary form, since the fractional part needs more than 3 bits.

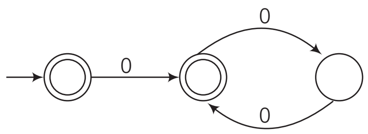
$$(3.001)_{10} = (00011.000000\ldots\ldots)_2$$

So, we can say we cannot represent the decimal number 3.001 in its exact binary form, because the fractional part needs more than 3 bits. It is also not accurate storage.

40. Given a language L, define L^i as follows

$$L^0 = \{\varepsilon\},\ L^i = L^{i-1} \cdot L \text{ for all } i > 0$$

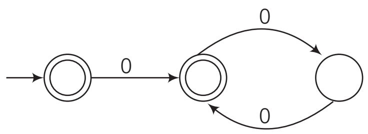
The order of language L is defined as the smallest k such that $L^k = L^{k+1}$. Consider the language L_1 (over alphabet 0) accepted by the following automaton.



The order of L_1 is ________.

Sol. (2) In order to find the order of L_1 first we need to find smallest value of k which satisfies

$$L_1^k = L_1^{k+1}$$



$$L_1 = \varepsilon + (00)^*$$

Now, put the values of $k = 0$, 1 and 2 and find the order k as follows

Try $k = 0$: $L_1^0 = L_1^1$

$\Rightarrow$ $\varepsilon = L_1$ which is false

So order is not 0.

Try $k = 1$: $L_1^1 = L_1^2$

$\Rightarrow$ $L_1^2 = L_1$

Now, $L_1^2 = (\varepsilon + 0\,(00)^*)\,(\varepsilon + 0\,(00)^*)$

$= \varepsilon + 0\,(00)* + 00\,(00)^* = 0^*$

Clearly, $L_1^2 \neq L_1$

So order is not 1.

Try $k = 2$: $L_1^2 = L_1^3$

Now, $L_1^3 = L_1^2 . L_1$

$= 0^*\ (\varepsilon + 0\,(00)^*) = 0^*$

Clearly, $L_1^3 = L_1^2 = 0^*$.

So order of L_1 is 2

41. A lexical analyzer uses the following patterns to recognize three tokens T_1, T_2 and T_3 over the alphabet $\{a, b, c\}$.

$$T_1 : a?\ (b|c)*a$$
$$T_2 : b?\ (a|c)*b$$
$$T_3 : c?\ (b|a)*c$$

Note that 'x?' means 0 or 1 occurrence of the symbol x. Note also that the analyzer outputs the token that matches the longest possible prefix.

If the string bbaacabc is processed by the analyzer, which one of the following is the sequence of tokens it outputs?

(a) $T_1T_2T_3$ (b) $T_1T_1T_3$
(c) $T_2T_1T_3$ (d) T_3T_3

Sol. (d) Answer is T_3T_3 which given string 'bbaacabc' because from first T_3 bbaac is taken from second T_3 abc is taken, longest possible prefix.

Hence, T_3T_3 token output.

42. Let G be a graph with 100! vertices, with each vertex labelled by a distinct permutation of the numbers 1, 2, ... 100. There is an edge between vertices u and v if and only if the label of u can be obtained by swapping two adjacent numbers in the label of v. Let y denote the degree of a vertex in G and z denote the number of connected components in G.

Then, $y + 10z =$ ________.

Sol. (109) If there are n numbers then we will have $(n - 1)$ adjacent pairs. Therefore according to the given data every vertex adjacent to 99 vertices and moreover we will have only one component in the graph.

So, the graph is a regular graph with each vertex connected to 99 other vertices.

So, $y = 99$

The number of connected components $= z = 1$

By swapping only 2 adjacent numbers at a time, many times we can go from any vertex to any other vertex

Graph is connected.

So, $z = 1$

So, $y + 10z = 99 + 10 \times 1 = 109$

43. The instruction pipeline of a RISC processor has the following stages. Instruction Fetch (IF), Instruction Decode (ID), Operand Fetch (OF), Perform Operation (PO) and Writeback (WB). The IF, ID, OF and WB stages take 1 clock cycle each for every instruction. Consider a sequence of 100 instructions. In the PO state, 40 instructions take 3 clock cycles each, 35 instructions take 2 clock cycles each, and the remaining 25 instructions take 1 clock cycle each. Assume that there are no data hazards and no control harzards. The number of clock cycles required for completion of execution of the sequence of instruction is ________.

Sol. (219) Total instructions = 100

Number of stages = 5

In normal case, total cycles = 100 + 5 – 1 = 104 cycles

Now, for PO stage 40 instruction takes 3 cycles, 35 takes 2 cycles and rest of the 25 takes 1 cycle.

That means all other stages are perfectly fine and working with CPI (Clock Cycle Per Instruction) = 1

PO stage

40 instruction takes 3 cycles, i.e. these instructions suffering from 2 stall cycle

35 instruction takes 2 cycles, i.e. these instructions suffering from1 stall cycle.

25 instruction takes 1 cycle, i.e. these instructions suffering from 0 stall cycle.

So, extra cycle would be

$40 * 2 + 35 * 1 + 20 * 0 = 80 + 35 = 115$ cycle.

Total cycles $= 104 + 115 = 219$

44. Consider the following solution to the producer-consumer synchronization problem. The shared buffer size is N. Three semaphores empty, full and mutex are defined with respective initial values of 0, N and 1. Semaphore empty denotes the number of available slots in the buffer, for the consumer to read from. Semaphore full denotes the number of available slots in the buffer, for the producer to write to. The placeholder variables, denoted by P, Q, R and S, in the code below can be assigned either empty or full. The valid semaphore operations are : wait ( ) and signal ( ).

Producer	Consumer
do {	do {
wait (P);	wait (R);
wait (mutex);	wait (mutex);
//Add item to buffer	//Consume item from buffer
signal (mutex);	signal (mutex);
signal (Q);	signal (S);
} while (1);	} while (1);

Which one of the following assignment to P, Q, R and S will yield the correct solution?

(a) P : full, Q : full, R : empty, S : empty
(b) P : empty, Q : empty, R : full, S : full
(c) P : full, Q : empty, R : empty, S : full
(d) P : empty, Q : full, R : full, S : empty

Sol. (c) Full = N, empty = 0, mutex = 1

- Initially buffer will be empty, so consumer should not start first, so option B, D are eliminated.
- With option A consumer will never consume the item, so it is wrong.
- Option 'C' is correct answer which proper functionality of produce and consumer.

Given, N = Buffer size

Number of semaphores = 3 (empty, full, mutex)

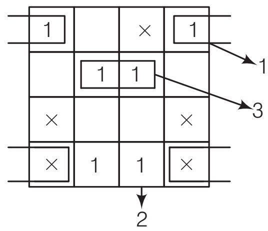
$$\begin{array}{cccl} \downarrow & \downarrow & \downarrow & \leftarrow \text{Initial values} \\ 0 & N & 1 & \end{array}$$

⇒ Semaphore "empty" denotes the number of available slots in the buffer for the consumer to read from.

⇒ Semaphore "full" denotes the number of available slots in the buffer for the producer to write to.

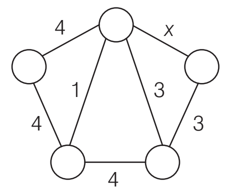
45. Consider the minterm list form of Boolean function F given below.
$F(P, Q, R, S) = \Sigma m\ (0, 2, 5, 7, 9, 11) + d\,(3, 8, 10, 12, 14)$.
Here, m denotes a minterm and d denotes a do not care term. The number of essential prime implicants to the function F is ________.

Sol. (3) $F(P, Q, R, S) = \Sigma m\ (0, 2, 5, 7, 9, 11) + d\ (3, 8, 10, 12, 14)$



Number of EPI= 3

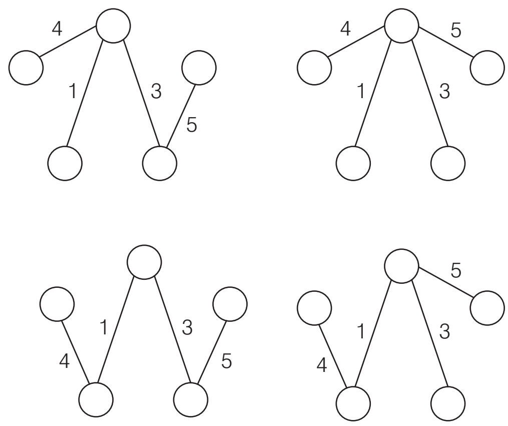
46. Consider the following undirected graph G.



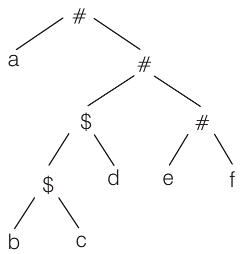
Choose a value of x that will maximize the number of minimum weight spanning trees (MWSTs) of G. The number of MWSTs of G for this value of x is ________.

Sol. (4) For $x = 1, 2, 3, 4$ only 2 spanning trees possible.

For $x = 5$ total 4 spanning trees possible.



47. Consider the following parse tree for the expression a#b\$c\$d#e#f, involving two binary operators \$ and #.



Which one of the following is correct for the given parse tree?

(a) \$ has higher precedence and is left associative; # is right associative

(b) # has higher precedence and is left associative; \$ is right associative

(c) \$ has higher precedence and is left associative; # is left associative

(d) # has higher precedence and is right associative; \$ is left associative

Sol. (a) If any given parse tree or syntax free low level operators having higher precedence than upper level operators. The given expression a#b\$c\$d#e#f

Hence, \$ is higher precedence than #.

\$ is left associative because in the sub expression b \$ c \$ d, b \$ c will be evaluated first as per given tree. As per the given tree structure right # is higher precedence than left #. e.g. e#f will be executed first hence right associative.

Hence, \$ has higher precedence and left associative; # is right associative.

48. Assume that multiplying a matrix G_1 of dimension $p \times q$ with another matrix G_2 of dimension $q \times r$ requires pqr scalar multiplications. Computing the product of n matrices $G_1\, G_2\, G_3 \ldots G_n$ can be done by parenthesizing in different ways. Define $G_i G_{i+1}$ as an explicitly computed pair for a given parenthesization if they are directly multiplied. For example, in the matrix multiplication chain $G_1 G_2 G_3 G_4 G_5 G_6$ using parenthesization $(G_1 (G_2 G_3))\,(G_4 (G_5 G_6))$. $G_2 G_3$ and $G_5 G_6$ are the only explicitly computed pairs.

Consider a matrix multiplication chain $F_1 F_2 F_3 F_4 F_5$, where matrices F_1, F_2, F_3, F_4 and F_5 are of dimensions $2 \times 25, 25 \times 3, 3 \times 16, 16 \times 1$ and 1×1000, respectively. In the parenthesization of $F_1 F_2 F_3 F_4 F_5$ that minimizes the total number of scalar multiplications, the explicitly computed pairs is/are

(a) $F_1 F_2$ and $F_3 F_4$ only (b) $F_2 F_3$ only

(c) $F_3 F_4$ only (d) $F_1 F_2$ and $F_4 F_5$ only

Sol. (c) $F_1 F_2 F_3 F_4 F_5$

Cost of the multiplication is its explicit pairs are $F_1 F_2$ AND $F_3 F_4$ is $(((F_1 F_2)(F_3 F_4))F_5)$

$$F_1 F_2 = 150$$
$$F_3 F_4 = 48$$
$$((F_1 F_2)(F_3 F_4)) = 6$$
$$(((F_1 F_2)(F_3 F_4)))F_5) = 2000$$
$$\text{Total} = 2204$$

Cost of the multiplication is if its explicit pairs are $F_1 F_2$ AND $F_3 F_4$ is $((F_1 (F_2 (F_3 F_4)))F_5)$

$$F_3 F_4 = 48$$
$$(F_2 (F_3 F_4)) = 75$$
$$(F_1 (F_2 (F_3 F_4))) = 50$$
$$((F_1 (F_2 (F_3 F_4)))F_5) = 2000$$
$$\text{Total} = 2173$$

49. In a system, there are three types of resources : E, F and G. Four processes P_0, P_1, P_2 and P_3 execute concurrently. At the outset, the processes have declared their maximum resource requirements using a matrix named Max as given below.

For example Max$[P_2, F]$ is the maximum number of instances of F that P_2 would required. The number of instances of the resources allocated to the various processes at any given state is given by a matrix named Allocation.

Consider a state of the system with the Allocation matrix as shown below and in which 3 instances of E and 3 instances of F are the only resources available.

Allocation

	E	F	G
P_0	1	0	1
P_1	1	1	2
P_2	1	0	3
P_3	2	0	0

Max

	E	F	G
P_0	4	3	1
P_1	2	1	4
P_2	1	3	3
P_3	5	4	1

From the perspective of deadlock avoidance, which one of the following is true?

(a) The system is in safe state.

(b) The system is not in safe, state but would be safe if one more instance of E were available.

(c) The system is not in safe state, but would be safe if one more instance of F were available.

(d) The system is not in safe state, but would be safe if one more instance of G were available.

Sol. (a)

	Max need			Current Allocation			Current Available			Remaining need = (Max − Alloc)		
	E	F	G	E	F	G	E(3)	F(3)	G(0)	E	F	G
P_0	4	3	1	1	0	1	4	3	1	3	3	0
P_1	2	1	4	1	1	2	5	3	4	1	0	2
P_2	1	3	3	1	0	3	6	4	6	0	3	0
P_3	5	4	1	2	0	0	8	4	6	3	4	1

Safe sequence : P_0, P_2, P_1, P_3. Since, we are able to generate safe sequence; we can say the system is in safe state. Hence, A option is correct.

50. Consider the relation $r(A, B)$ and $s(B, C)$, where $s.B$ is a primary key and $r.B$ is a foreign key referencing $s.B$. Consider the query Q : $r \bowtie . (\sigma B < 5^{(s)})$

Let LOJ denote the natural left outer-join operation. Assume that r and s contain no null values.

Which one of the following queries is NOT equivalent to Q?

(a) $\sigma_{B<5}(r \bowtie s)$ (b) $(\sigma_{B<5}(r \text{ LOJ } s)$

(c) r LOJ $(\sigma_{B<5}(s))$ (d) $\sigma_{B<5}(r)$ LOJ s

Sol. (c)

r

A	B
a_1	1
a_2	3
a_3	3
a_4	5
a_5	5

Foreign key

s

B	C
1	c_1
3	c_2
3	c_2
7	c_2

Primary key

Given query : $r \bowtie (\sigma_{B<5}(s))$

A	B	C
a_1	1	c_1
a_2	3	c_1
a_3	3	c_1

A : $\sigma_{B<5}(r \bowtie s)$

A	B	C
a_1	1	c_1
a_2	3	c_1
a_3	3	c_1

B : $\sigma_{B<5}(r \bowtie s)$

A	B	C
a_1	1	c_1
a_2	3	c_1
a_3	3	c_1

C : $r \bowtie (\sigma_{B<5}(s)$

A	B	C
a_1	1	c_1
a_2	3	c_1
a_3	3	c_1
a_4	5	Null
a_5	5	Null

D : $\sigma_{B<5}(r) \bowtie s$

A	B	C
a_1	1	c_1
a_2	3	c_1
a_3	3	c_1

Option "c" query result not equal to given query.

51. Consider the following C code. Assume that unsigned long int type length is 64 bits.

```
     unsigned long int fun (unsigned long int n) {
          unsigned long int i, j=0, sum =0;
          for (i= n; i> 1; i= i/2) j++;
          for (; j> 1; j= j/2) sum ++;
          return (sum);
}
```

The value returned when we call fun with the input 2^{40} is

(a) 4 (b) 5 (c) 6 (d) 40

Sol. (b) for (i= n; i> 1; i= i/2) j+ + ;

Condition	Increment of j
$2^{40} > 1$	1
$2^{39} > 1$	2
$2^{38} > 1$	3
:	
:	
$2^1 > 1$	40
$1 > 1$	break

Final value of j = 40,

For (; j > 1; j = j/2) sum + + ;

Condition	Increment of sum
40 > 1	1
20 > 1	2
10 > 1	3
5 > 1	4
2 > 1	5
1 > 1	break

Final value of sum is 5.

52. Consider the following languages.

I. $\{a^m b^n c^p d^q \mid m + p = n + q, \text{ where } m, n, p, q \geq 0\}$

II. $\{a^m b^n c^p d^q \mid m = n \text{ and } p = q, \text{ where } m, n, p, q \geq 0\}$

III. $\{a^m b^n c^p d^q \mid m = n = p \text{ and } p \neq q, \text{ where } m, n, p, q \geq 0\}$

IV. $\{a^m b^n c^p d^q \mid mn = p + q, \text{ where } m, n, p, q \geq 0\}$

Which of the language above are context-free?

(a) I and IV only

(b) I and II only

(c) II and III only

(d) II and IV only

Sol. (c)

I. $\{a^m b^n c^p d^q \mid m+p=n+q\}$ since, the equation can be rearranged as $m-n+p-q=0$ by push, pop, push and pop and check if stack is empty at end so it is clearly a CFL.

II. $\{a^m b^n c^p d^q \mid m=n \text{ and } p=q\}$ since, one comparison at a time can be done by PDA solit is clearly a CFL.

III. $\{a^m b^n c^p d^q \mid m=n=p \text{ and } p \neq q\}$ since, $m=n=p$ is a double comparison which cannot be done by PDA, it is not CFL.

IV. $\{a^m b^n c^p d^q \mid mn=p+q\}$, since mn involves multiplying number of a's and number b's which cannot be done by a PDA is not a CFL.

So, only I and II are CFL's

53. A processor has 16 integer registers ($R0, R1, \ldots, R15$) and 64 floating point registers ($F0, F1, \ldots, F63$). It uses a 2-byte instruction format. There are four categories of instructions : Type-1, Type-2, Type-3 and Type-4. Type-1 category consists of four instructions, each with 3 integer register operands (3Rs). Type-2 category of eight instructions, each with 2 floating point register operands (2Fs).

Type-3 category consists of fourteen instructions, each with one integer register operand and one floating point register operand ($1R+1F$). Type-4 category consists of N instructions, each with a floating point register operand (IF).

The maximum value of N is ______.

Sol. (32) Instruction size = 16 bit

Type **I instruction design**

16 bit

opcode	IR	IR	IR
4 bit	4 bit	4 bit	4 bit

Number of operations $= 2^4$
$= 16$
$\downarrow$
Free opcodes $= 16 - 4$
$= 12$

Type **II instruction design**

16 bit

opcode	FR	FR
4 bit	6 bit	6 bit

Free opcodes after type 1 instruction = 12
Free opcodes after type 2 instruction = 12 – 8 = 4

Type **III instruction design**

16 bit

opcode	IR	FR
6 bit	4 bit	4 bit

Expand opcode size = 6 bit
$\downarrow$
Number of opcodes $= 4 \times 2^2 = 16$
$\therefore$ Number of free opcodes after type 3 instruction
$= 16 - 14$
$= 2$

Type **IV instruction design**

16 bit

opcode	FR
10 bit	6 bit

Expand opcode size = 10 bit
$\downarrow$
Number of opcodes $= 2 \times 2^4$
$= 2^5 = 32$

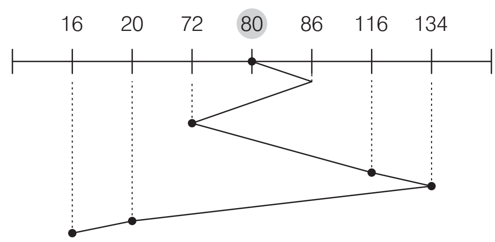
54. Consider a storage disk with 4 platters (numbered as 0, 1, 2 and 3), 200 cylinders (numbered as 0, 1, ..., 199) and 256 sectors per track (numbered as 0, 1, ,......, 255). The following 6 disk requests of the form [sector number, cylinder number, platter number] are received by the disk controller at the same time.

[120, 72, 2], [180, 134, 1], [60, 20,0], [212, 86, 3], [56, 116, 2], [118, 16, 1]

Currently the head is positioned at sector number 100 of cylinder 80 and is moving towards higher cylinder numbers.The average power dissipation in moving the head over 100 cylinders is 20 milliwatts and for reversing the direction of the head movement once is 15 milliwatts. Power dissipation associated with rotational latency and switching of head between different platters is negligible.

The total power consumption in milliwatts to satisfy all of the above disk requests using the Shortest Seek Time First disk scheduling algorithm is ______.

Sol. (85)



Head starts at : 80
Total head movements in SSTF
$= (86-80)+(86-72)+(134-72)+(134-16)$

$= 6 + 14 + 62 + 118 = 200$

Power dissipated in moving 100 cylinders $= 20$ mW

Power dissipated by 200 movements (say P_1)

$= 0.2 * 200 = 40$ mW

Power dissipated in reversing head direction once $= 15$ mW

Number of times head changes its direction = 3

Power dissipated in reversing head direction (say P_2)

$= 3 * 15 = 45$ mW

Total power consumption is

$$P_1 + P_2 = 40 + 45 = 85 \text{ mW}$$

55. Consider a matrix P whose only eigenvectors are the multiples of $\begin{bmatrix} 1 \\ 4 \end{bmatrix}$. Consider the following statements.

I. P does not have an inverse

II. P has a repeated eigenvalue

III. P cannot be diagonalized

Which one of the following option is correct?

(a) Only I and III are necessarily true

(b) Only II is necessarily true

(c) Only I and II are necessarily true

(d) Only II and III are necessarily true

Sol. (d) Only eigenvector is $\begin{bmatrix} 1 \\ 4 \end{bmatrix}$ multiples means that eigenvalue is repeated since if eigenvalues were distinct we will get one more independent eigen vector.

P is a 2×2 matrix with repeated eigenvalue and P cannot be diagonalized since P has dependent eigenvectors.

$\therefore$ I and III are true.

General Aptitude (GA) Questions

(1 *Mark Questions*)

56. What is the missing number in the following sequence?

2, 12, 60, 240, 720, 1440, ..., 0

(a) 2880 (b) 1440 (c) 720 (d) 0

Sol. (b)

$$2 \times 6 = 12$$
$$12 \times 5 = 60$$
$$60 \times 4 = 240$$
$$240 \times 3 = 720$$
$$720 \times 2 = 1440$$
$$1440 \times 1 = 1440$$
$$1440 \times 0 = 0$$

So, 1440 must be answer.

57. "From where are they bringing their books? .... bringing .... books from ....?"

The words that best fill the blanks in the above sentence are

(a) Their, they're, there (b) They're, their, there

(c) There, their, they're (d) They're, there, there

Sol. (b) They're used for pointing group. Their is possessive pronoun which pointing people. There is used for place.

58. What would be the smallest natural number which when divided either by 20 or by 42 or by 76 leaves a remainder of 7 in each case?

(a) 3047 (b) 6047 (c) 7987 (d) 63847

Sol. (c) We need a number that can be written as $20x + 7$, $42y + 7$, $76z + 7$ for three integers x, y and z.

We basically need to find least common multiple of $20(2 * 2 * 5)$, $42(2 * 3 * 7)$ and $76(2 * 2 * 19)$ which is

$2 * 2 * 5 * 3 * 7 * 19 = 20 * 21 * 19 = 420 * 19 = 7980$

So, number $= 7980 + 7 = 7987$

59. "A .... investigation can sometimes yield new facts, but typically organized ones are more successful."

The word that best fills the blank in the above sentence is

(a) meandering (b) timely

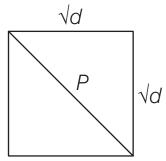
(c) consistent (d) systematic

Sol. (a) Meandering means proceeding in a convoluted or undirected fashion.

60. The area of a square is d. What is the area of the circle which has the diagonal of the square as its diameter?

(a) πd (d) πd^2 (c) $\frac{1}{4}\pi d^2$ (d) $\frac{1}{2}\pi d$

Sol. (d)



Area of square $= d$

$(\text{side})^2 = d$

Side of square $= \sqrt{d}$

Diagonal of square $= \sqrt{d + d} = \sqrt{2d}$

Diagonal of square = diameter of circle

So, diameter of circle $= \sqrt{2d}$

$$\text{Radius of circle} = \frac{\text{diameter}}{2} = \frac{\sqrt{2d}}{2}$$

Area of circle $= \pi r^2$

$$= \pi\left(\frac{\sqrt{2}d}{2}\right)^2 = \pi\left(\frac{2d}{4}\right)$$

$$= \pi\frac{d}{2} = \frac{1}{2}\pi d$$

(2 Marks Questions)

61. In $pqr \neq 0$ and $p^{-x} = \frac{1}{q}$, $q^{-y} = \frac{1}{r}$, $r^{-z} = \frac{1}{p}$, what is the value of the product xyz?

(a) -1 (b) $\frac{1}{pqr}$

(c) 1 (d) pqr

Sol. (c)

$$\frac{1}{q} = p^{-x} \quad \text{...(i)}$$

$$\frac{1}{r} = q^{-y} \quad \text{...(ii)}$$

$$\frac{1}{p} = r^{-z} \quad \text{...(iii)}$$

From Eq. (i)

$$\frac{1}{q} = p^{-x} = \left(\frac{1}{p}\right)^x$$

$$= r^{-zx} \quad \text{[From Eq. (iii)]}$$

$$= \left(\frac{1}{r}\right)^{zx} = q^{-yzx} \quad \text{[From Eq. (ii)]}$$

$$\frac{1}{q} = \left(\frac{1}{q}\right)^{yxz}$$

So, $xyz = 1$

62. In appreciation of the social improvements completed in a town, a wealthy philanthropist decided to gift ₹ 750 to each male senior citizen in the town and ₹ 1000 to each female senior citizen. Altogether, there were 300 senior citizens eligible for this gift. However, only 8/9th of the eligible men and 2/3rd of the eligible women claimed the gift. How much money (in Rupees) did the philanthropist give away in total?

(a) 1,50,000 (b) 2,00,000
(c) 1,75,000 (d) 1,51,000

Sol. (b) Number of senior male $= M$

Number of senior female $= F$

Male + Female $(M + F) = 300$ ...(i)

$$\text{Total money} = \frac{8}{9}M \times 750 + \frac{2}{3}F \times 1000$$

$$= \frac{8}{3} \times 250\,M + \frac{2000}{3}F$$

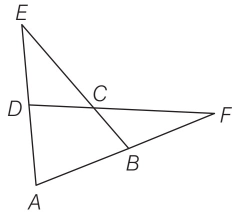
$$= \frac{2000}{3}M + \frac{2000}{3}F$$

$$= \frac{2000}{3}(M + F)$$

Put the value of $(M + F)$ from Eq. (i), we get

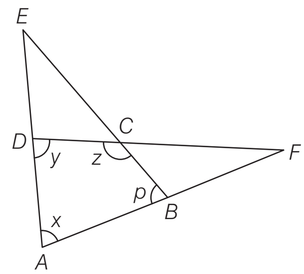
$$\text{Total money} = \frac{6000}{9}(300) = 2{,}00{,}000$$

63. In the figure below, $\angle DEC + \angle BFC$ is equal to ______.



(a) $\angle BCD - \angle BAD$ (b) $\angle BAD + \angle BCF$
(c) $\angle BAD + \angle BCD$ (d) $\angle CBA + \angle ADC$

Sol. (a)



$$\angle E + \angle F = ?$$

In ΔEAB,

$$x + p + E = 180 \quad \text{...(i)}$$

In ΔAFD,

$$x + y + F = 180 \quad \text{...(ii)}$$

In $\square$ $ABCD$,

$$x + y + z + p = 360 \quad \text{...(iii)}$$

Eq. (i) and Eq. (ii) = Eq. (iii)

$x + p + E + x + y + F = x + y + z + p$, $E + F = z - x$

$$\angle DEC + \angle BFC = \angle BCD - \angle BAD$$

64. A six sided unbiased die with four green faces and two red faces is rolled seven times. Which of the following combinations is the most likely outcome of the experiment?

(a) Three green faces and four red faces
(b) Four green faces and three red faces
(c) Five green faces and two red faces
(d) Six green faces and one red face

Sol. (c) Four green, two red faces

$$P(G) = \frac{4}{6} = \frac{2}{3}$$

$$q(R) = \frac{1}{3}, n = 7$$

A: option (1), $P(3G,\ 4R) = {}^7C_3\left(\frac{4}{6}\right)^3\left(\frac{2}{6}\right)^4$

B: (2), $P(4G,\ 3R) = {}^7C_4 \times\left(\frac{4}{6}\right)^4 \times\left(\frac{2}{6}\right)^3$

C: (3), $P(5G,\ 2R) = {}^7C_5 \times\left(\frac{4}{6}\right)^5 \times\left(\frac{2}{6}\right)^2$

D: (4), $P(6G,\ 1R) = {}^7C_6 \times\left(\frac{4}{6}\right)^6 \times\left(\frac{2}{6}\right)$

Option A is clearly smaller and hence eliminated.

$$\frac{C}{B} = \frac{{}^7C_5}{{}^7C_4}\cdot\frac{4}{6}\cdot\frac{6}{2} = \frac{21}{35}\cdot 2 > 1$$

So, $C > B$

$$\frac{C}{D} = \frac{{}^7C_5}{{}^7C_6}\cdot\frac{6}{4}\cdot\frac{2}{6} = \frac{21}{7}\cdot 0\cdot 5 > 1$$

So, $C > D$

Option C is maximum value.

So, five green faces and two red faces.

65. In a party, 60% of the invited guests are male and 40% are female. If 80% of the invited guests attended the party and if all the invited female guests attended, what would be the ratio of males to females among the attendees in the party?

(a) 2 : 3 (b) 1 : 1

(c) 3 : 2 (d) 2 : 1

Sol. (b) Let total males $= 60x$

Total females $= 40x$

Total number of people $= 60x + 40x = 100x$

Total number of people who attended the party

$= 0.8(60x + 40x)$

$= 80x$

Let y males attended. It is given all females attended.

$$40x + y = 80x,$$
$$y = 40x$$

which is same as females.