

7.1: Percentages Introduction

What is Percent?

Cent means 100 and per means divided by.

Discount is 25% means $\frac{25}{100}$

for every 100 you are getting discount 25.

Conversion of % to fraction ($\div 100$)

$$25\% = \frac{25}{100} = \frac{1}{4}$$

$$60\% = \frac{60}{100} = \frac{3}{5}$$

$$16\frac{2}{3}\% = 16.\overline{67}\%$$

$$16\frac{2}{3}\%$$

$$= \frac{50}{3}\% = \frac{50}{3} \times \frac{1}{100} = \frac{1}{6}$$

$$83\frac{1}{3}\% = 83.\overline{33}\%$$

$$= \frac{250}{3}\%$$

$$= \frac{250}{3} \times \frac{1}{100} = \frac{5}{6}$$

Conversion of fraction to % ($\times 100$)

$$\frac{3}{4} = \frac{3}{4} \times 100 = 75\%$$

$$\frac{2}{7} = \frac{2}{7} \times 100 = 28\frac{4}{7}\%$$

$$\text{1) } P\% \times x = \frac{P}{100} \times x \quad \text{2) } 20\% \times 800 = \frac{20}{100} \times 800 = 160$$

$$\text{3) } a\% \times b = b\% \times a$$

$$75 \times 32\%$$

$$75\% \times 32 = \frac{3}{4} \times 32 = 24$$

Few $\frac{1}{n}$'s

$$10\% = \frac{10}{100} = \frac{1}{10}$$

$$20\% = \frac{1}{5}$$

$$30\% = \frac{3}{10}$$

$$25\% = \frac{1}{4}$$

$$50\% = \frac{1}{2}$$

$$75\% = \frac{3}{4}$$

$$100\% = 1$$

$$120\% = \frac{6}{5}$$

$$12\frac{1}{2}\% = \frac{1}{8}$$

$$6\frac{1}{4}\% = \frac{1}{16}$$

$$37\frac{1}{2}\% = \frac{3}{8}$$

$$62\frac{1}{2}\% = \frac{5}{8}$$

$$8\frac{1}{3}\% = \frac{1}{12}$$

$$16\frac{2}{3}\% = \frac{1}{6}$$

$$33\frac{1}{3}\% = \frac{1}{3}$$

$$66\frac{2}{3}\% = \frac{2}{3}$$

$$88\frac{1}{3}\% = \frac{5}{6}$$

$$11\frac{1}{9}\% = \frac{1}{9}$$

$$9\frac{1}{11}\% = \frac{1}{11}$$

$$142\frac{1}{7}\% = \frac{1}{7}$$

$$24\frac{4}{7}\% = \frac{2}{7}$$

Ex

$$2104 \times 12.5\%$$

$$= 2104 \times \frac{1}{8} = 263$$

$$133\frac{1}{3}\% = 100\% + 33\frac{1}{3}\% \\ = 1 + \frac{1}{3} = \frac{4}{3}$$

$$\text{Ex } 45\% \times 210$$

$$\frac{50\% - 5\%}{105\% - 10.5\%} = 45\%$$

10% = unit digit can be eliminated

1% = last 2 digits can be eliminated.

Ex

$$620 \times 10\% = 62$$

$$6234 \times 10\% = 623.4$$

$$5162 \times 1\% = 51.62$$

Ex

$$42\% \times 210$$

1

$$40\%, 2\%$$

$$84 + 4.2 = 88.2$$

⊗ Original value is always $\frac{100\%}{Total}$ (or) $\frac{100}{\downarrow}$
Base Value

⊗ A number itself is always 100% .

Savings is 20% of my Income
 \downarrow
Base Value

Treat Income = 100% (or) 100 Base Value

① In terms of fraction

$$25\% = \frac{1}{4} \rightarrow \text{Result}$$

$\rightarrow \text{original value}$

Savings is 20% of my Income

$$20\% = \frac{1}{5} \rightarrow \text{Savings}$$

$\rightarrow \text{original value}$

↓
Income

To find

- 1) What % of x is y ?
 - 2) y is what % of x ?
 - 3) What % is y of x ?
- anything after of is denominator.

Q.1) 32 is what % of 80?

$$32 = x\% \times 80$$

$$(Or) \quad \frac{y}{n} \times 100$$

$$32 = \frac{x}{100} \times 80$$

$$\Rightarrow \frac{32}{80} \times 100$$

$$\therefore x = 40\%$$

$$\therefore 40\%$$

Q.2) What % of 26 is 52?

$$x\% \times 26 = 52$$

$$(Or) \quad \frac{52}{26} \times 100$$

$$\frac{n}{100} \times 26 = 52$$

$$= 200\%$$

$$x = 200\%$$

Q.3) 18% of a number is 90, then find the number

$$18\% \times N = 90$$

$$\frac{18}{100} \times N = 90$$

$$N = 500$$

$$(0.18) \rightarrow 90$$

$$100\% \rightarrow \frac{90}{18} \times 100 = 500$$

Concept

1) A number is increased by 20%.

$$\text{Original number} = 100\%$$

$$\text{New Number} = 120\%.$$

$$20\% = 1/5$$

$$\text{Original Number} = 5$$

$$\text{New Number} = 5+1 = 6$$

2) A number is decreased by 25%.

$$\text{Original number} = 100\%, \quad (0.1)$$

$$\text{New Number} = 75\%$$

$$25\% = \frac{1}{4}$$

$$\text{Original Number} = 4$$

$$\text{New Number} = 4-1 = 3$$

Q.4) A number is increased by 25%, then it becomes 600. The original number is

Concept 1

Let original number be x .

$$x + 25\% x = 600 \Rightarrow x + \frac{25}{100} x = 600 \Rightarrow \frac{125x}{100} = 600 \Rightarrow x = 480.$$

Concept 2

Original number = 100%

New Number = 125%

$$\therefore \frac{125}{100} \rightarrow \frac{600}{125} \times 100 = 480.$$

Concept 3

25% = $\frac{1}{4}$, Original number = 4

New Number = $4+1=5$

$$5 \rightarrow 600$$

$$4 \rightarrow \frac{600 \times 4}{5} = 480.$$

Q.2) A number is decreased by 20%, it becomes 120. Find the original number.

Concept 1

Let original number be x

$$x - 20\% \text{ of } x = 120$$

$$x - \frac{20x}{100} = 120$$

$$\frac{80x}{100} = 120$$

$$x = 150$$

Concept 2 (in terms of %)

Let original number be 100%.

New Number = 80%

$$80\% \rightarrow 120$$

$$100\% \rightarrow \frac{120}{80} \times 100 = 150$$

Concept 3 (in terms of fractions)

$$20\% = \frac{1}{5}$$

$$\text{Original Number} = 5$$

$$\text{New Number} = 5 - 1 = 4$$

$$4 \rightarrow 120$$

$$5 \rightarrow \frac{120}{4} = 5 \Rightarrow 150$$

Q.3) A number is decreased by $33\frac{1}{3}\%$, then it becomes 180. Find the original number.

$$33\frac{1}{3}\% = \frac{1}{3}$$

$$\text{Original number} = 3$$

$$\text{New Number} = 3 - 1 = 2$$

$$2 \rightarrow 180$$

$$3 \rightarrow \frac{180 \times 3}{2} = 270.$$

Concept

<u>A</u>	<u>B</u>
You	Your friend
100	150

How much less you spent = ₹50/-

How much more your friend spent = ₹50/-

How much % less you spent = $\frac{50}{150} \times 100 = 33\frac{1}{3}\%$

How much % more your friend spent = $\frac{50}{100} \times 100 = 50\%$.

1) What % of A is B? $\Rightarrow \frac{B}{A} \times 100$

2) What % is A less than B?

3) What % of B is A? $\Rightarrow \frac{A}{B} \times 100$

4) What % is B more than A?

What % is A less than B? $\frac{B-A}{B} \times 100$

What % is B more than A? $\frac{B-A}{A} \times 100$.

Q.1) A is 25% less than B. By what % is B more than A?

$$\text{let } B = 100$$

$$A = 75\% \text{ of } B \\ = 75$$

$$\frac{25}{75} \times 100 = 33\frac{1}{3}\%$$

Q.2) A is $28\frac{4}{7}\%$ more than B. By what % is B less than A?

$$28\frac{4}{7}\% = \frac{2}{7}$$

$$B = 7 \text{ (assume)}$$

$$A = 7 + \frac{2}{7} \times 7 \\ = 9$$

$$\frac{2}{9} \times 100 = 22\frac{2}{9}\%$$

Concept

Price, consumption and expenditure.

When expenditure is constant

$$\text{Price ratio} \propto \frac{1}{\text{Quantity ratio}}$$

$$\text{If price ratio} = a : b$$

$$\begin{aligned}\text{Quantity Ratio} &= \frac{1}{a} : \frac{1}{b} \\ &= b : a\end{aligned}$$

Ex

Apples cost for today is ₹ 10/-

$$\text{Expenditure} = \sum 100/-$$

$$\therefore \text{Quantity} = 5$$

Apples cost for tomorrow is ₹ 15/-

$$\text{Expenditure} = \sum 100/-$$

$$\therefore \text{Quantity} = 4$$

$$\begin{aligned}\therefore \text{Price Ratio} &= 10 : 15 \\ &\Rightarrow 2 : 3\end{aligned}$$

$$\text{Quantity Ratio} = 5 : 4$$

$$\textcircled{*} \quad \text{Price Ratio} = 4 : 3 : 2$$

$$\text{Quantity Ratio} = \frac{1}{4} : \frac{1}{3} : \frac{1}{2} \quad \text{LCM}(4, 3, 2)$$

$$= 3 : 4 : 6$$

Q8 The price of sugar increased by 10%, in order to keep the expenditure as constant by what % consumption must be reduced?

Let Price of Sugar = 100

Original Price = 100

New Price = 110

$$\text{Price Ratio} = 100 : 110 \\ = 10 : 11$$

$$\text{Quantity Ratio} = 11 : 10$$

$$\text{Required \%} = \frac{1}{11} \times 100 = 9\frac{1}{11}\%$$

Alternative

$$\begin{array}{ccc} 100 & \xrightarrow{\uparrow 10\%} & 110 \\ \rightarrow \text{Price} & & \downarrow \leftarrow \begin{array}{l} \text{Quantity} \\ 10 \\ 100 \end{array} \\ & & \frac{10}{110} \times 100 = 9\frac{1}{11}\% \end{array}$$

(OR)

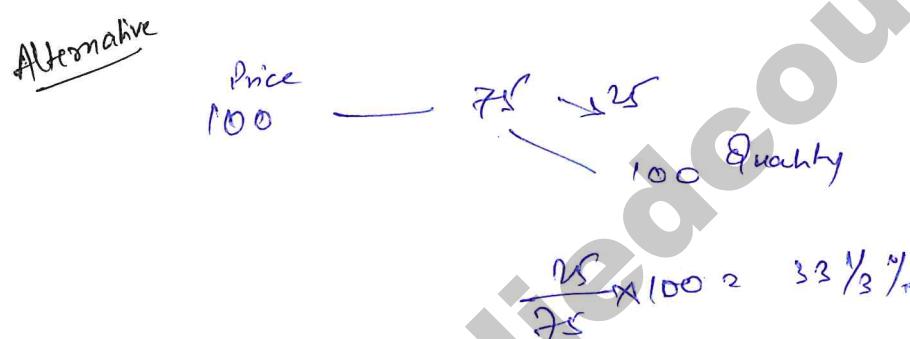
$$\frac{10}{100+10} \times 10 = \frac{10}{110} \times 100 \\ = 9\frac{1}{11}\%$$

(Q.2) The price of the tea powder decreased by 25%, in order to keep the expenditure as constant by what % consumption must be increased?

$$\text{Price Ratio} = 100 : 75 \\ 4 : 3$$

$$\text{Quantity Ratio} = 3 : 4$$

$$\frac{1}{3} \times 100 = 33\frac{1}{3}\%$$



7.2 : Percentages Solved Problems

Q.1) If 40% of $a = b$, then $b\%$ of 40 is same as

- (a) 4% of a (b) 8% of a (c) 16% of a (d) 8% of b

$$40\% \times a = b$$

$$\frac{2}{5} \times a = b$$

$$b\% \text{ of } 40 = b \times 40\%$$

$$b \times \frac{2}{5} = \frac{2}{5} \times a \times \frac{2}{5}$$

$$= \frac{4}{25} \times a \times 100$$

$$= 16\% \text{ of } a.$$

Q.2) What % is 6 hours in a day?

$$1 \text{ day} = 24 \text{ hours}$$

What % is 6 of 24

$$\frac{6}{24} \times 100 = 25\%$$

(OR)

$$24 \text{ hrs} \quad \underline{\hspace{2cm}} \quad 100\%.$$

$$6 \text{ hrs} \quad \underline{\hspace{2cm}} ?$$

$$\frac{6}{24} \times 100 = 25\%.$$

Q.3) A student has to obtain 40% of the total marks to pass. He got 270 marks and failed by 50 marks. The maximum marks are:

$$40\% \text{ of total Marks} = \text{pass marks}$$

$$(270 + 50) \text{ marks} = \text{pass marks}$$

$$\Rightarrow 320 \text{ marks}$$

$$40\% \rightarrow 320$$

$$100\% \rightarrow \frac{320 \times 100}{40}$$
$$= 800$$

Q.4) A student who secures 20% marks in examination fails by 30 marks. Another student who secures 32% marks gets 42 marks more than those required to pass. The percentage of marks required to pass is

$$\text{let total marks} = x$$

$$\text{1st person pass marks} = 20\% \text{ of } x + 30$$

$$\text{2nd person pass marks} = 32\% \text{ of } x - 42$$

$$20\% \text{ of } x + 30 = 32\% \text{ of } x - 42$$

$$32\% \text{ of } x - 20\% \text{ of } x = 72$$

$$12\% \text{ of } x = 72$$

$$\frac{12}{100} \times x = 72$$

$$x = 600$$

$$\text{Pass Mark} = 20\% \text{ of } 600 + 30 = 120 + 30 = 150$$

$$\text{Pass Marks (\%)} = \frac{150}{600} \times 100 = 25\%$$

(Automotive)

Poss Marks

$$12\% \left[\begin{array}{l} 20\% \\ 32\% \end{array} \right] \quad -42 \quad \left[\begin{array}{l} 30 \\ 72 \end{array} \right] \rightarrow 5\% \quad \rightarrow -7\%$$

$$12\% \rightarrow 72$$

$$1\% \rightarrow \frac{72}{12} = 6$$

$$(co) 20\% + 5\% = 25\%$$

$$(co) 32\% - 7\% = 25\%$$

Q.57 A man multiplying a numbers by $\frac{3}{5}$ instead of $\frac{5}{3}$. Find the % change in the error.

$N \times \frac{5}{3} \rightarrow$ correct way

$N \times \frac{3}{5} \rightarrow$ Error

Let N be LCM (3, 5) = 15
 denominators of $(\frac{5}{3}, \frac{3}{5})$

$$15 \times \frac{5}{3} = 25 \quad] \quad 16$$

$$15 \times \frac{3}{5} = 9 \quad] \quad 16$$

$$\frac{16}{25} \times 100 = 64\%$$

Q.6) If 75% of a number is added to 125, then the result is the number itself. The number is

$$75\% \times N + 125 = N$$

$$\frac{3}{4}N + 125 = N$$

$$125 = N - \frac{3}{4}N$$

$$\frac{1}{4}N = 125$$

$$N = 500$$

(Alternative)

$$75\% + 125 = 100\%$$

$$25\% \rightarrow 125$$

$$100\% \rightarrow \frac{125}{25} \times 100 = 500$$

Q.7) If the numerator of a fraction is increased by 20% and the denominator is decreased by 5%, the value of the fraction becomes $\frac{5}{2}$. The original fraction is

Let the original fraction = $\frac{x}{y}$

$$\text{New fraction} = \frac{120\% \text{ of } x}{95\% \text{ of } y} = \frac{120x}{95y}$$

$$\frac{120x}{95y} = \frac{5}{2}$$

$$\Rightarrow \frac{x}{y} = \frac{95}{48}$$

Q.8) A reduction of 20% in the price of wheat enable to buy 5 kg more wheat for ₹ 320. The original rate of wheat is.

Expenditure is same, i.e., ₹ 320

$$\text{Price Ratio} \propto \frac{1}{\text{Quantity Ratio}}$$

Let the original price be ₹ 100

∴ New price = ₹ 80

$$\begin{aligned}\text{Price Ratio} &= 100 : 80 \\ &= 5 : 4\end{aligned}$$

$$\text{Quantity Ratio} = \frac{4}{5} : 1$$

$$\begin{aligned}1 &\rightarrow 5 \\ 4 &\rightarrow 5 \times 4 = 20 \text{ kgs} \\ 5 &\rightarrow 5 \times 5 = 25 \text{ kgs}\end{aligned}$$

$$\text{Original Price} = \frac{320}{20} = ₹ 16 \text{ per kg.}$$

(Alternative)

Let original price = ₹ n

New price = ₹ $n - 20\%$, $n = \frac{4}{5}n$

Total Exp. = ₹ 320

$$\frac{320}{n} = y \quad \dots (i)$$

$$\left| \begin{array}{l} \frac{320}{\frac{4}{5}n} = y + 5 \quad \dots (ii) \\ \frac{320}{n} = y \end{array} \right.$$

$$\begin{aligned} \frac{320}{\frac{4}{5}n} - \frac{320}{n} &= 5 \\ \Rightarrow \frac{400}{n} - \frac{320}{n} &= 5 \Rightarrow \frac{80}{n} = 5 \Rightarrow n = ₹ 16 \end{aligned}$$

(Alternative)

$$20\% \text{ } \longrightarrow 5\text{kg}$$

$$100\% \text{ } \longrightarrow \frac{5}{20} \times 100 = 25\text{kg}$$

$$\text{Original Qty} = 25 - 5 = 20\text{kg}$$

$$\frac{320}{20} = \text{₹}16 \text{ per kg.}$$

7.3: Problems on Successive Percentage

Q8 Let Ravi's salary be ₹10,000

It increased by 10% in next month and again decreased by 10%, next to next month. What is the net effect value in %?

$$\begin{array}{r} 10,000 \\ \uparrow 10\% \\ 1,000 \\ \hline 11,000 \\ \downarrow 10\% \\ 1,100 \\ \hline 9,900 \end{array}$$

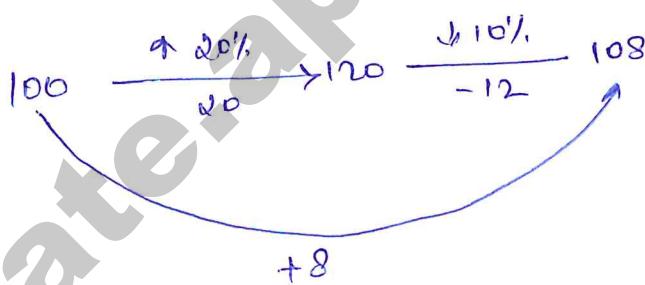
$$10,000 - 9,900 = 100$$

$$\frac{100}{10,000} \times 100 = 1\% \text{ decrease.}$$

$$\boxed{\frac{x+y + \frac{xy}{100}}{100}}$$

increased \uparrow sign
decreased \downarrow sign.

Q.9 A number is increased by 20% and then it is decreased by 10%. Find the net increase or decrease %.



$$\text{Net effect } (\%) = \frac{8}{100} \times 100 = 8\% \text{ increase}$$

(Alternative)

$$\frac{x+y + \frac{xy}{100}}{100}$$

$$= 20\% - 10\% + \frac{(20)(-10)}{100}$$

$$= 10 - 2 = 8\% \text{ increase.}$$

Q.2) The price of TV increased by 20%, and its sales was decreased by 20%. Find the % change in the turnover to the shop keeper?

$$\text{Price } \uparrow = 20\% = 1/5$$

$$\text{Original Price} = 5$$

$$\text{New Price} = 5 + 1 = 6$$

$$\text{Sales } \downarrow = 20\% = 1/5$$

$$\text{Original Sales} = 5$$

$$\text{New Sales} = 5 - 1 = 4$$

$$\text{Total Sales} = 5 \times 5 = 25$$

$$\text{New Sales} = 6 \times 4 = 24$$

$$\text{New change (\%)} = -1/25 \times 100 = -4\%$$

\therefore 4% decrease

(Alternative)

$$x+y + \frac{xy}{100}$$

$$20\% - 20\% + \frac{20 \times (-20)}{100}$$

-4%

\therefore decrease of 4%.

Q.3) The length and breadth of the rectangle are increased by 10% and 20%, respectively. Find the % change in the area of rectangle?

$$\text{Area of a rectangle} = l \times b$$

$$l \uparrow 10\%, \quad b \uparrow 20\%$$

$$= \frac{l}{10} \quad = \frac{l}{5}$$

$$\text{Original length} = 10$$

$$\text{New length} = 10 + 1 = 11$$

$$\text{Original Breadth} = 5$$

$$\text{New Breadth} = 5 + 1 = 6$$

$$\text{Original Area} = 10 \times 5 = 50$$

$$\text{New Area} = 11 \times 6 = 66$$

$$\text{Change of Area} = 66 - 50 = 16$$

$$\text{Change of Area (in %)} = \frac{16}{50} \times 100 \\ = 32\% \text{ increase}$$

Alternative

$$x + y + \frac{xy}{100}$$

$$10\% + 20\% \rightarrow \frac{10 \times 20}{100}$$

$$= 32\% \text{ increase}$$

Q4) Radius of circle is decreased by 50%, then find out % change in area of circle?

$$\text{Area of circle} = \pi r^2$$

$$\text{Radius} \downarrow 50\%$$

$$= \frac{1}{2}$$

$$\text{Original Radius} = 2$$

$$\text{New Radius} = 2 - 1 = 1$$

$$\begin{aligned}\text{Area (original)} &= \pi 2^2 \\ &= 4\pi\end{aligned}$$

$$\begin{aligned}\text{New Area} &= \pi 1^2 \\ &= \pi\end{aligned}$$

$$\begin{aligned}\text{Change in Area} &= \pi - 4\pi \\ &= -3\pi\end{aligned}$$

$$\begin{aligned}\text{Change in Area (\%)} &= \frac{-3\pi}{4\pi} \times 100 \\ &= -75\%\end{aligned}$$

∴ 75% decrease

(Alternative)

$$\begin{aligned}r &\downarrow 50\% \\ R &= 50\% \\ r &= R\end{aligned}$$

$$\begin{aligned}\frac{\pi r^2 - \pi R^2}{\pi R^2} &= (-50) + (-50) + \frac{(-50) \times (-50)}{100} \\ &= -100 + 25\end{aligned}$$

$$= -75\%$$

∴ 75% decrease

Q57: Price of sugar is increased by 25% and consumption is decreased by 16%, then find out % change in expenditure?

$$\text{Expenditure} = \text{Price} \times \text{Quantity}$$

$$\text{Price} \uparrow 25\% = 1/4$$

$$\text{Consumption} \downarrow 16\% = 4/25$$

$$\text{Original Price} = 4$$

$$\text{New Price} = 4+1 = 5$$

$$\text{Original Consumption} = 25$$

$$\text{New Consumption} = 25 - 4 = 21$$

$$\text{Expenditure} = 4 \times 25 = 100$$

$$\text{New Expenditure} = 21 \times 5 = 105$$

$$\text{Change in Expenditure} = \frac{105 - 100}{100} = 5$$

$$\text{Change } (\%) = \frac{5}{100} \times 100 = 5\% \text{ increase}$$

(Alternative)

$$x+y = \frac{xy}{100}$$

$$= 25\% - 16\% + \frac{(25) \times (-16)}{100}$$

$$= 9 - 4$$

$$= 5\% \text{ increase}$$

Q.6) The present population of a city is 1,80,000. If it increases at the rate of 10% p.a., its population after 2 years will be

$$\begin{aligned}\text{population after } n \text{ years} &= \text{original population} \times \left(1 + \frac{\gamma}{100}\right)^n \\ &= 1,80,000 \times \left(1 + \frac{10}{100}\right)^2 \\ &= 1,80,000 \times \left(\frac{11}{10}\right)^2 \\ &= 1,80,000 \times \frac{11}{10} \times \frac{11}{10} \\ &= 2,17,800\end{aligned}$$

(Alternative)

$$\begin{array}{r} 1,80,000 \\ (+) 10\%. \quad 18,000 \\ \hline 1,98,000 \\ + (10\%) \quad 19,800 \\ \hline 2,17,800 \end{array}$$

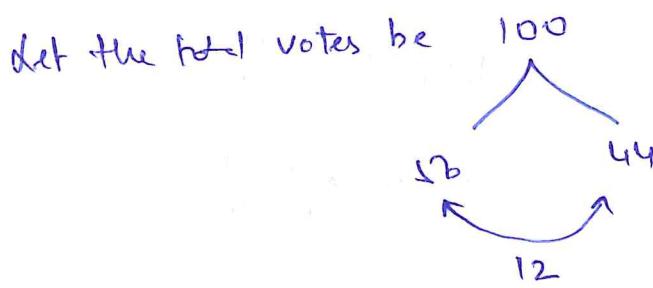
(Alternative)

$$\begin{aligned}n+1 &\rightarrow \frac{n+1}{100} \\ &= \frac{10+10 + 10 \times 10}{100} \\ &= 21\% \\ 1,80,000 \times 21\% &= 37,800\end{aligned}$$

$$\begin{aligned}\therefore \text{New population} &= 1,80,000 + 37,800 \\ &= 2,17,800\end{aligned}$$

problem on votes

Q.7) There were only two candidates in an election, one get 56% votes and was won by margin of 144 votes. The total number of votes are :



$$\begin{aligned} 12 &\rightarrow 144 \\ 100 &\rightarrow \frac{144}{12} \times 100 \\ &= 1200 \text{ votes in total.} \end{aligned}$$

Q.8) 8% of the voters in an election did not cast their votes. In this election, there were only two candidates. The winner by obtaining 48% of the total voter defeated his contestant by 1200 votes. The total number of voters are

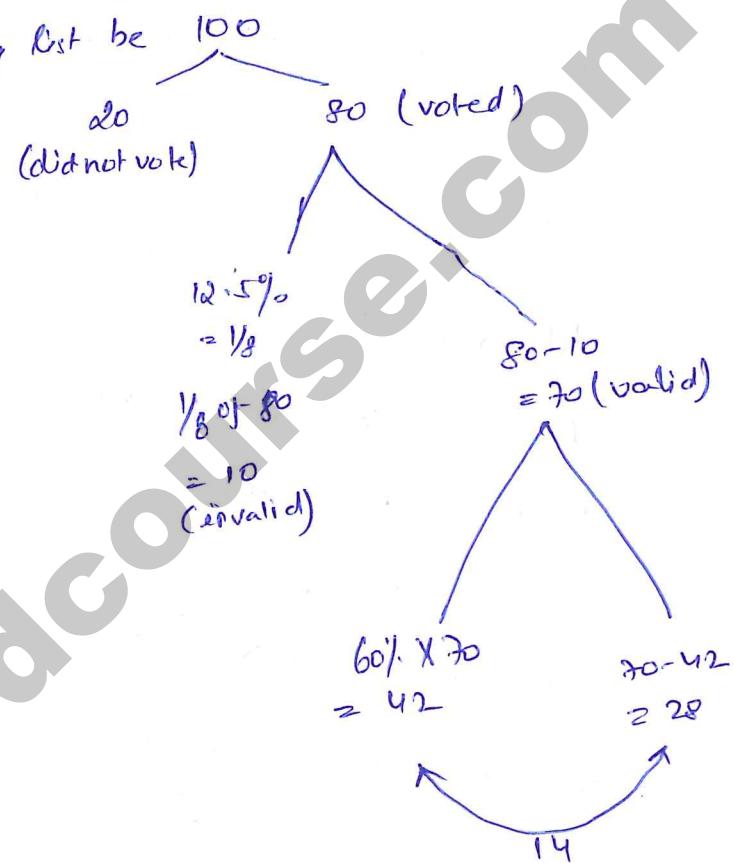
Let total no. of voters be 100

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graph TD; A[100] --> B[8  
(did not vote)]; A --> C[92  
(voted)]; B --> C;  
  
graph TD; D[92] --> E["48% of 100  
= 48"]; E --> F[44];
```

$$\begin{aligned} 4 &\rightarrow 1200 \\ 100 &\rightarrow \frac{1200}{4} \times 100 \\ &= 30,000 \text{ voters.} \end{aligned}$$

Q.9) In an election two candidates participated. 20% voters did not vote. 12.5% votes declared invalid and winner get 60% of the valid votes and won by 7000 voter. find the total no. of valid votes.

Let the total voters in voting list be



$$\begin{aligned} 14 &\rightarrow 7000 \\ 70 &\rightarrow \frac{7000 \times 70}{14} \\ &= 35000 \text{ valid votes.} \end{aligned}$$

Q.10) Radha spends 20% of her salary on food, 10% on house rent, 15% on entertainment and 15% on transport. If her savings at the end of the month are ₹1,800, then her salary per month (in ₹).

$$\text{Income} - \text{Expenditure} = \text{Savings}$$

Let income be ₹ 100.

$$\text{Food} = 20\% \times 100 = 20$$

$$\text{Rent} = 10\% \times 100 = 10$$

$$\text{Entertainment} = 15\% \times 100 = 15$$

$$\text{Transport} = 15\% \times 100 = 15$$

$$\begin{aligned}\text{Total Expenditure} &= 20 + 10 + 15 + 15 \\ &= ₹ 60\end{aligned}$$

$$\text{Savings} = 100 - 60 \\ = ₹ 40$$

$$40 \rightarrow 1800$$

$$100 \rightarrow \frac{1800}{40} \times 100$$

$$= ₹ 4500/- (\text{Salary})$$

Alternative

$$\text{Expenditure (\%)} = 20\% + 10\% + 15\% + 15\% = 60\%$$

$$\text{Savings (\%)} = 100\% - 60\% = 40\%$$

$$40\% \rightarrow 1800$$

$$100\% \rightarrow \frac{1800}{40} \times 100 = ₹ 4500/-$$

Q.117 Chiru spends 30% of his monthly income on food articles, 40% of the remaining on transport and saves 50% of the remaining. If he saves ₹ 1,680, what will be his monthly income in ₹?

Let monthly income be ₹ 100

$$\text{Food } 30\% \quad \frac{30}{\Sigma 70}$$

$$\begin{array}{r} \text{Transport } 40\% \\ (\text{Food } 40\%) \end{array} \quad \frac{28}{\Sigma 42}$$

$$\begin{array}{r} \text{Savings } 50\% \\ (\Sigma 42 \times 50\%) \end{array} \quad \frac{21}{\Sigma 21}$$

$$\begin{array}{r} 21 \rightarrow 1680 \\ 100 \rightarrow \frac{1680}{21} \times 100 \\ = ₹ 8,000/- \quad (\text{Monthly Income}) \end{array}$$

(Alternative)

Let monthly income be n.

$$n \times (100 - 30) \times (100 - 40) \times (100 - 50) = 1680$$

$$\Rightarrow n \times 70\% \times 60\% \times 50\% = 1680$$

$$\Rightarrow n \times \frac{7}{10} \times \frac{3}{5} \times \frac{1}{2} = 1680$$

$$\Rightarrow n = ₹ 8,000/-$$

7.4 Previous Year Gate Questions

Q.1) There are two candidates P and Q in an election. During the campaign 40% of the voters promised to vote for P, and rest for Q. However, on the day of election 15% of the voters went back on their promise to vote for P and instead voted for Q. 25% of the voters went back on their promise to vote for Q and instead voted for P. Suppose P lost by 2 votes, then what was the total number of voters?

(A) 100 (B) 110 (C) 90 (D) 95 (GATE 2011)

Solution

Let total no. of voters = 100

The diagram shows the distribution of 100 voters between candidates P and Q. P initially has 40 voters. 15% of P's voters switch to Q, leaving P with 34 voters and adding 6 to Q. 25% of Q's voters switch to P, adding 15 to P and leaving Q with 66 voters. The final count is P: 51 and Q: 49, with a difference of 2.

$$\begin{array}{r} \text{P} \\ 40 \\ 15\% \text{ of } 40 = \frac{-6}{34} \\ +15 \\ \hline 51 \end{array}$$
$$\begin{array}{r} \text{Q} \\ 60 \\ +6 \\ 25\% \text{ of } 60 \\ -15 \\ \hline 49 \end{array}$$

$\frac{2}{2} \times 100 = 100 \text{ (Ans)}$

- Q.27 An average household spent his monthly budget on various items, viz., on food - ₹ 4,000, clothing - ₹ 1,200, Rent - ₹ 2,000, Savings - ₹ 1,500 and others - ₹ 1,800. The approximate % of the monthly budget not spent on savings is (GATE 2012)
- (a) 10%. (b) 14%. (c) 81%. (d) 86%.

Solution

$$\begin{array}{rcl} \text{food} & \rightarrow & 4000 \\ \text{cloth} & \rightarrow & 1200 \\ \text{Rent} & \rightarrow & 2000 \\ \text{Savings} & \rightarrow & 1500 \\ \text{Others} & \rightarrow & 1800 \\ \hline & & 10500 \end{array}$$

$$\begin{aligned} \% \text{ spent on savings} &= \frac{1500}{10500} \times 100 \\ &= \frac{1}{7} \times 100 \\ &= 14\frac{2}{7}\% \approx 14\%. \end{aligned}$$

$$\begin{aligned} \therefore \% \text{ not spent on savings} &= 100 - 14 \\ &= 86\%. \text{ (Ans)} \end{aligned}$$

Q.3) Round trip tickets to a tourist destination are eligible for a discount of 10% on the total fare. In addition, groups of 4 or more get a discount of 5% on the total fare, if the one way single person fare is ₹100, a group of 5 tourist purchasing round-trip tickets will be charged ₹ _____ (GATE 2014)

Solution

$$\text{one way trip cost is ₹100/- per person}$$

$$\text{Then two way trip} = ₹200/- \text{ per person}$$

$$\text{Cost of 5 person} = 5 \times 200 = \frac{1000}{100}$$

$$\text{Discount @ 10\%} = \frac{100}{900}$$

$$\text{Discount @ 5\%} = \frac{50}{850} \text{ (Ans)}$$

Q.4) One percentage of the people of country X are taller than 6 feet. Two percentage of the people of country Y are taller than 6 feet. There are thrice as many people in country X as in country Y. Taking both countries together, what is the percentage of people taller than 6 feet _____ (GATE 2014)

No. of people	<u>X</u>	<u>Y</u>
	300	100 (let)
taller than 6 feet	3	2

$$\frac{3}{300} \times 100 = 1.25\% \text{ (Ans)}$$

$$\frac{2}{100} \times 100 = 2\% \text{ (Ans)}$$

Q.5) The given question is followed by two statements; select the most appropriate option that solves the question. Capacity of a solution tank A is 70% of the capacity of tank B. How many gallons of solution are in tank A and tank B?

Statements

- I. Tank A is 80% full and Tank B is 40% full.
II. Tank A is full contains 14,000 gallons of solution.

- (a) Statement - I alone is sufficient
(b) Statement - II alone is sufficient
(c) Either statement I or II alone is sufficient
(d) Both the statements I and II together are sufficient

Solution

Let the capacity of tank B be 100 gallons
Then the capacity of tank A = 70 gallons

From Statement I

% of Gallon in Tank A = 80%
Tank B = 40%

Answer is not possible from Statement I alone.

Statement II

Tank A = 14,000 gallons. $\frac{A}{70} : \frac{B}{100}$
14,000 20,000.

Statement II alone is not possible.

Both Statement I and II together (Ans).

I-I
I-II

Yes, we can get the solution.

Q.67 In a huge pile of apples and oranges, both ripe and unripe mixed together, 15% are unripe fruits. Of the unripe fruits, 45% are apples. Of the ripe ones, 66% are oranges. If the pile contains a total of 5692000 fruits, how many of them are apples?

(GATE 2016)

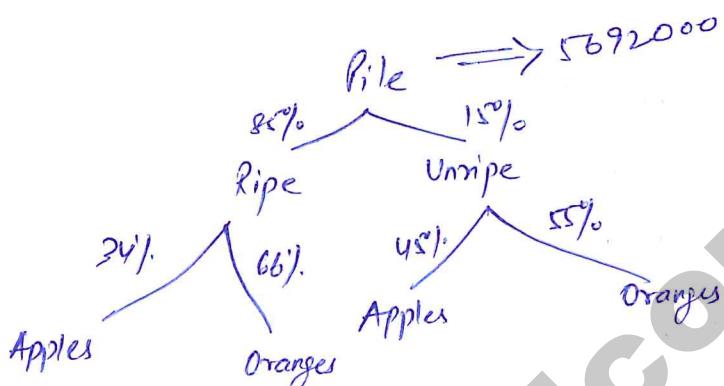
(a) 2029198

(b) 2467482

(c) 2789080

(d) 3577422

Solution



$$\text{No. of apples} =$$

$$\frac{\text{Ripe}}{34\%} \times \frac{85\%}{100} + \frac{\text{Unripe}}{15\%} \times \frac{45\%}{100}$$

$$\left(\frac{34}{100} \times \frac{85}{100} + \frac{15}{100} \times \frac{45}{100} \right) \times 5692000$$

$$\left(\frac{2890}{10000} + \frac{675}{10000} \right) \times 5692000$$

$$\frac{3565}{10000} \times 5692000$$

$$= 2029198 \quad (\text{ans})$$

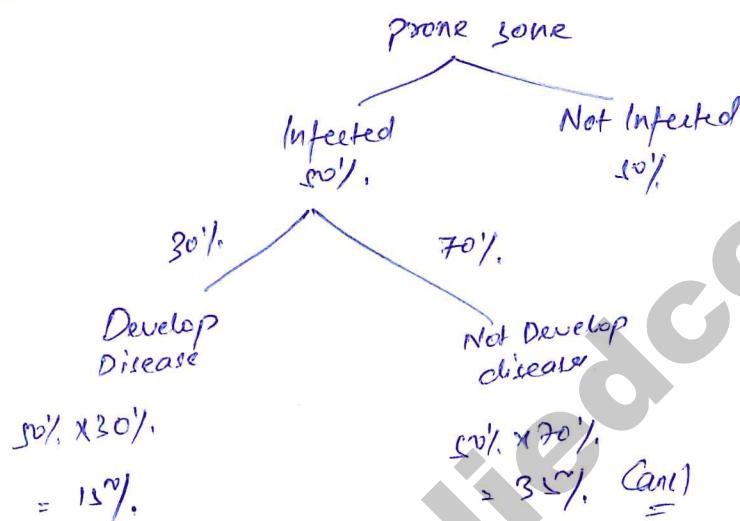
Q7) A person moving through a tb prone zone has a 50% probability of becoming infected. However, only 30% of infected people develop the disease. What percentage of people moving through a tb prone zone remains infected but does not show symptoms of disease? (GATE 2016)

(a) 15

(b) 33

(c) 35

(d) 37

Solution

Q8) $(x\% \text{ of } y) + (y\% \text{ of } x)$ is equivalent to

(GATE 2016)

- (a) $2\% \text{ of } xy$ (b) $2\% \text{ of } (xy/100)$ (c) $ny\% \text{ of } 100$ (d) $100\% \text{ of } ny$

$$\frac{x}{100} \times y + \frac{y}{100} \times x$$

$$= \frac{xy}{100} + \frac{xy}{100}$$

$$= \frac{2xy}{100}$$

$$= 2\% \text{ of } xy \text{ (Ans)}$$

- Q.9) Rajiv spends 40% of his salary on food, 20% on house rent, 10% on entertainment and 10% on conveyance. If his savings at the month end are ₹ 2,000, then his monthly salary is - (GATE 2017)
- (a) ₹ 6,000 (b) ₹ 8,000 (c) ₹ 10,000 (d) ₹ 12,000

Solution
Let the monthly salary be ₹ 100

Expenses

$$\begin{array}{r}
 40\% \times 100 = 40 \\
 20\% \times 100 = 20 \\
 10\% \times 100 = 10 \\
 10\% \times 100 = 10 \\
 \hline
 & ₹ 80
 \end{array}$$

$$\text{Savings} = 100 - 80 = ₹ 20.$$

$$\begin{array}{r}
 20 \\
 100 \\
 \hline
 \xrightarrow{\quad} 2000 \\
 \xrightarrow{\quad} 2000 \times 100 \\
 \hline
 20 \\
 = ₹ 10,000
 \end{array}$$

- Q.10) If the radius of a right circular cone is increased by 50%, its volume increase by
 (a) 75%. (b) 100%. (c) 125%. (d) 237.5%. (GATE 2017)

Solution
Volume of cone = $\frac{1}{3}\pi r^2 h$ $50\% \uparrow = \frac{1}{2}$

Original Radius = r , New Radius = $r+1 = 3r$
 Original Volume = $\frac{1}{3}\pi r^2 h$, New Volume = $\frac{1}{3}\pi (3r)^2 h$

$$\text{Original Volume : New Volume} = \frac{4}{9} : 1 \quad \therefore \frac{4}{9} \times 100 = 44.44\% \text{ (Ans)}$$

Q.11) A designer uses marbles of four different colours for his designs. The cost of each marble is the same, irrespective of the color. The cost of each marble increased by 25%. Therefore, the designer decided to reduce equal number of marbles of each color to keep the total cost unchanged. What is the percentage of blue marbles in the new design?

(GATE 2018)

<u>Blue</u>	<u>Black</u>	<u>Red</u>	<u>Yellow</u>
40%	25%	20%	15%

(a) 35.75%

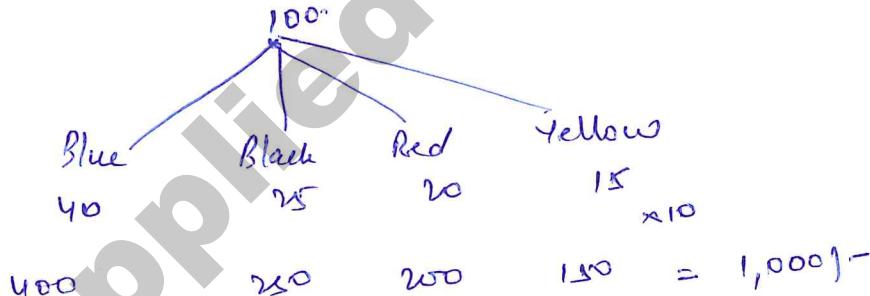
(b) 40.25%

(c) 43.75%

(d) 46.25%

Let the no. of marbles = ₹ 100

Let price of each marble be ₹ 10.



Price increased to ₹ 12.5/-

Let No. of marbles reduced is n.

$$(40-n + 25-n + 20-n + 15-n) \times 12.5 = 1000$$

$$(100-4n) 12.5 = 1000$$

$$1250 - 50n = 1000$$

$$n = 25$$

<u>Blue</u>	<u>Total</u>
40	100
-25	-20
15	(5x4)
	80

$$\frac{35}{80} \times 100 = \frac{175}{4} = 43.75\% \text{ (Ans)}$$

1. $\frac{d}{dx} \int_{\sin x}^{\cos x} f(t) dt = f(\cos x) \cdot (-\sin x) - f(\sin x) \cdot (\cos x)$

2. $\frac{d}{dx} \int_{x^2}^{x^3} f(t) dt = f(x^3) \cdot 3x^2 - f(x^2) \cdot 2x$

3. $\frac{d}{dx} \int_{\sqrt{x}}^{x^2} f(t) dt = f(x^2) \cdot 2x - f(\sqrt{x}) \cdot \frac{1}{2\sqrt{x}}$

4. $\frac{d}{dx} \int_{\ln x}^{\ln x} f(t) dt = f(\ln x) \cdot \frac{1}{x} - f(\ln x) \cdot \frac{1}{x}$

5. $\frac{d}{dx} \int_{\tan x}^{\cot x} f(t) dt = f(\cot x) \cdot (-\sec^2 x) - f(\tan x) \cdot \sec^2 x$

6. $\frac{d}{dx} \int_{\sin x}^{\cos x} f(t) dt = f(\cos x) \cdot (-\sin x) - f(\sin x) \cdot (\cos x)$

7. $\frac{d}{dx} \int_{x^2}^{x^3} f(t) dt = f(x^3) \cdot 3x^2 - f(x^2) \cdot 2x$

8. $\frac{d}{dx} \int_{\sqrt{x}}^{x^2} f(t) dt = f(x^2) \cdot 2x - f(\sqrt{x}) \cdot \frac{1}{2\sqrt{x}}$

9. $\frac{d}{dx} \int_{\ln x}^{\ln x} f(t) dt = f(\ln x) \cdot \frac{1}{x} - f(\ln x) \cdot \frac{1}{x}$

10. $\frac{d}{dx} \int_{\tan x}^{\cot x} f(t) dt = f(\cot x) \cdot (-\sec^2 x) - f(\tan x) \cdot \sec^2 x$