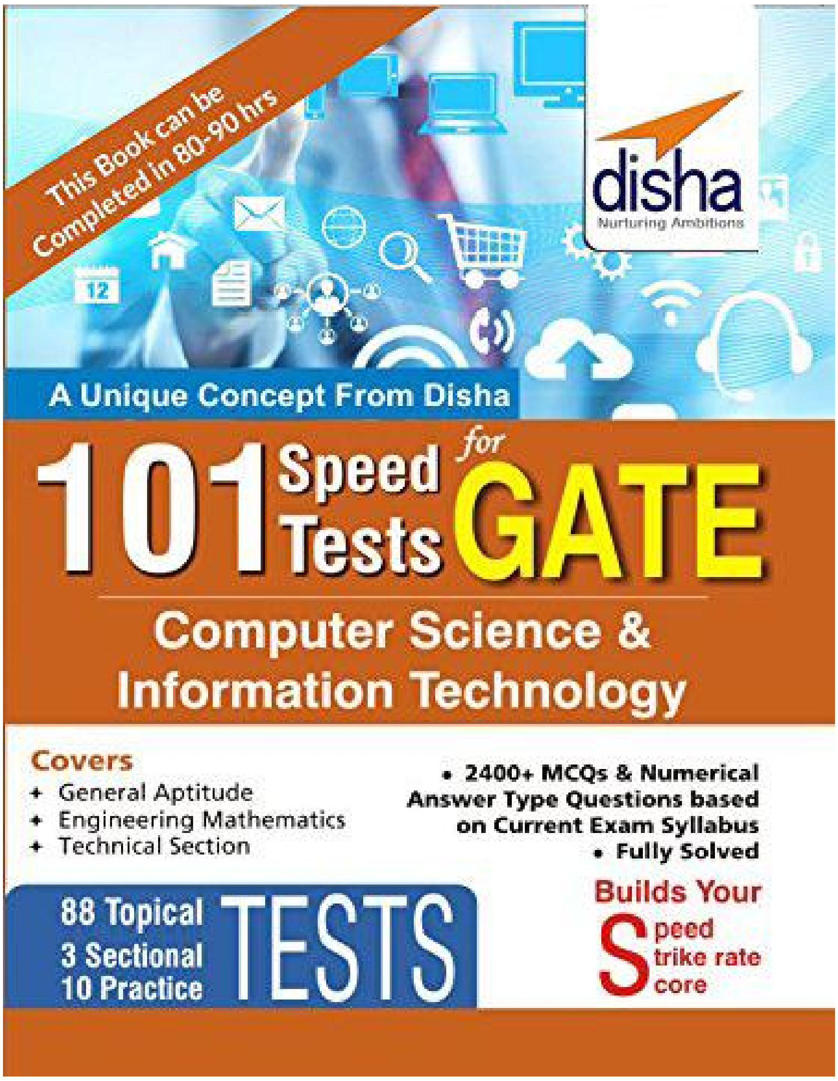
Turing Machines

This Chapter is taken from our Book:



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101 SPEED TEST

60

Turing Machines

Max. Marks : 19 **No. of Qs. 19** **Time : 30 min.** **Date : ..../......../..............**

1. Which of the following statements is/are false?

1. For every non-deterministic Turing machine, there exists an equivalent deterministic Turing machine.
2. Turing recognisable languages are closed under union and complementation.
3. Turing decidable languages are closed under intersection and complementation.
4. Turing recognisable languages are closed under union and intersection.

(a) 1 and 4 only (b) 1 and 3 only
(c) 2 only (d) 3 only

2. Which of the following is/are undecidable?

1. G is a CFG. Is $L(G) = \Phi$?
2. G is a CFG. Is $L(G) = \Sigma^*$?
3. M is a Turing machine. Is L(M) regular?
4. A is a DFA and N is an NFA. Is L(A) = L(N)?

(a) 3 only (b) 3 and 4 only
(c) 1, 2 and 3 only (d) 2 and 3 only

3. Let $< M >$ be the encoding of a Turing machine as a string over $\Sigma = \{0, 1\}$. Let $L = \{<M>|$ *M is a Turing machine that accepts a string of length* $2014\}$. Then, L is

(a) decidable and recursively enumerable
(b) undecidable but recursively enumerable
(c) undecidable and not recursively enumerable
(d) decidable but not recursively enumerable

4. Define languages L_0 and L_1 as follows

$L_0 = \{< M, W, 0 > |M \text{ halts on } W\}$

$L_1 = \{<M, W, 1>| \text{ M does not halts on } W\}$

Here < M, W, i > is a triplet, whose first component, M is an encoding of a turing machine, second component W is a string and third component i is a bit.

Let $L = L_0 \cup L_1$. Which of the following is true?

(a) L is recursively enumerable, but $\overline{L}$ is not
(b) $\overline{L}$ is recursively enumerable, but L is not
(c) Both L and $\overline{L}$ are recursive
(d) Neither L nor $\overline{L}$ is recursively enumerable

5. Let M be a turing machine has $Q = \{q_0, q_1, q_2, q_3, q_4\}$ a set of states, input alphabets {0, 1}. The tape alphabets are {0, 1, B, X, Y}. The symbol B is used to represent end of input string. The final state is q_4. The transistions are as follows.

1. $(q_0, 0) = (q_1, X, R)$
2. $(q_0, Y) = (q_3, Y, R)$
3. $(q_1, 0) = (q_1, 0, R)$
4. $(q_1, 1) = (q_2, Y, L)$
5. $(q_1, Y) = (q_1, Y, R)$
6. $(q_2, 0) = (q_2, 0, L)$
7. $(q_2, X) = (q_0, X, R)$
8. $(q_2, Y) = (q_2, Y, L)$
9. $(q_3, Y) = (q_3, Y, R)$
10. $(q_3, B) = (q_4, B, R)$

Which of the following is true about M?

(a) M halts on L having 100 as substring
(b) M halts on L having 101 as substring
(c) M halts on $= 0^n1^n$, $n \geq 0$
(d) M halts on L not having 1100 substring

6. Referring to Turing machine in previous question, what will be the output when the input is $I_1 = 0100$ and $I_2 = 0010$?

(a) $I_1 = 0$ and $I_2 =$ Blank (b) $I_1 =$ Blank and $I_2 = 0$
(c) $I_1 = 1$ and $I_2 = 1$ (d) None of the above

7. Consider the following problem x.

Given a Turing machine M over the input alphabet Σ, any state q of M.

And a word $w \in \Sigma^*$ does the computation of M on w visit the state q?

Which of the following statements about x is correct?

(a) x is decidable
(b) x is undecidable but partially decidable
(c) x is undecidable and not even partially decidable
(d) x is not a decision problem

8. A single tape Turing Machine M has two states q^0 and q^1, of which q^0 is the starting state. The tape alphabet of M is {0, 1, B} and its input alphabet is {0, 1}. The symbol B is the blank symbol used to indicate end of an input string. The transition function of M is described in the following table

	0	1	B
q^0	$q^{1,1,R}$	$Q^{1,1,R}$	Halt
q^1	$q^{1,1R}$	$Q^{0,1,L}$	qH0, B, L

The table is interpreted as illustrated below.

The entry $(q^{1,1,R})$ in row q^0 and column 1 signifies that if M is in state q^0 and reads 1 on the current tape square, then it writes 1 on the same tape square, moves its tape head one position to the right and transitions to state q^1.

Which of the following statements is true about M?

(a) M does not halt on nay string in $(0 + 1)^+$
(b) M dies not halt on any string in $(00 + 1)^*$
(c) M halts on all string ending in a 0
(d) M halts on all string ending in a 1

9. An FSM (Finite State Machine) can be considered to be a TM (Turing Machine) of finite tape length

(a) without rewinding capabilitiy and unidirectional tape movement.
(b) rewinding capacity, and unidirectional tape movement.
(c) without rewinding capability and bidirectional tape movement.
(d) rewinding capability and bidirectional tape movement.

10. A Push Down Machine (PDM) behaves like a Turing Machine (TM) when number of auxiliary memory it has, is __________

11. If nL can be recognized by a multitape TM with time complexity f, then L can be recognized by a one-tape machine with time complexity

(a) $O(f^2)$ (b) $o(f^2)$

(c) $o(h)$ (d) $O(h^2)$

12. If T is a TM recognizing L, and T reads every symbol in the input string, $\tau_T(n) \geq 2n + 2$, then any language that can be accepted by a TM T with $\tau_T(n) = 2n + 2$ is

(a) regular (b) not regular

(c) uncertain (d) none of these

13. Consider an alternate Turing machine model, in which there is an input tape on which the tape head can move in both directions but cannot write, and one or more work tapes, one of which serves as an output tape. For a funciton f, denoted by DSpace(f), the set of languages that can be recognized by a Turing machine of this type which uses no more than $f(n)$ squares on any work tape for any input string of length n. The only restriction we need to make on f is that $f(n) > 0$ for every n. The language of balanced strings of parentheses are in

(a) DSpace $(1 + \lceil \log_2 (n + 1 \rceil)$. ($\lceil x \rceil$) means the smallest integer greater than or equal to x.)

(b) DSpace $(1 + \lceil \log_2 n \rceil)$

(c) DSpace $(1 \lceil \log_2 n^2 \rceil)$

(d) None of these

14. Which of the following is/are undecidable?

1. G is a CFG. Is L(G) = Φ?
2. G is a CFG. Is L(G) = Σ*?
3. M is a Turning machine. Is L(M) regular?
4. A is a DFA and N is an NFA. Is L(A) = L(N)?

(a) 3 only (b) 3 and 4 only

(c) 1, 2 and 3 only (d) 2 and 3 only

15. Next move function δ of a Turing machine M = (Q, Σ, Γ, δ, q_0, B, F) is a mapping

(a) $\delta : Q \times \Sigma \rightarrow Q \times \Gamma$

(b) $\delta : Q \times \Gamma \rightarrow Q \times \Sigma \times \{L, R\}$

(c) $\delta : Q \times \Sigma \rightarrow Q \times \Gamma \times \{L, R\}$

(d) $\delta : Q \times \Gamma \rightarrow Q \times \Gamma \times \{L, R\}$

16. Turing machine with transition table is shown below.

Present state	0	1	b
q_0	$0Rq_0$	$1Rq_1$	bRq_f
q_1	$0Rq_1$	$1Rq_0$	
q_f accept			

In this

(a) set of all even palindromes over {0, 1}

(b) string over {0, 1} containing even number of 1

(c) set of all string with even number of 1 and even number 0

(d) none of these

17. If L can be recognized by a TM T with a doubly infinite tape, and $\tau_t = f$, then L can be recognized by an ordinary TM with time complexity

18. Consider the Turing Machine

M = (Q, Σ, Γ, δ, q_0, B, F) accept language L on {a, b} and δ is defined by

$\delta(q_0, a) = (q_1, B, R)$

$\delta(q_0, b) = (q_1, B, R)$,

$\delta(q_0, B) = (q_3, B, R)$,

$\delta(q_1, b) = (q_2, B, R)$,

$\delta(q_1, a) = (q_2, B, R)$,

$\delta(q_2, a) = (q_0, B, R)$,

$\delta(q_2, b) = (q_0, B, R)$.

The language accepted by Turing machine is defined by

(a) L = {ω : |ω| is even}

(b) L = {ω : |ω| is odd}

(c) L = {ω : |ω| is multiple of 3}

(d) None of these

19. Consider the Turing machine

M = (Q, Σ, Γ, δ, q_0, B, F); with

Q = {q_0, q_1, q_2, q_3, q_4}

F = {q_4}

$\delta(q_0, 1) = (q_0, 1, R)$

$\delta(q_0, 0) = (q_1, 1, R)$,

$\delta(q_1, 1) = (q_1, 1, R)$,

$\delta(q_1, B) = (q_2, B, L)$,

$\delta(q_2, 1) = (q_3, 0, L)$,

$\delta(q_3, 1) = (q_3, 1, L)$,

$\delta(q_3, B) = (q_4, B, R)$.

Suppose x and y are a positive number represented in a unary and 0 is symbol separated them. At the end of computation the function computed by turning machine (ignore, zero) will be

(a) f (X, Y) = X = Y (b) f (x, y) = x – y

(c) f (x, y) = 2x – y (d) None of these

Hints & Solutions

1. (c) 1. Non-deterministic Turing Machine can be simulated by a deterministic Turing Machine with exponential time true.
2. Turing recongnizable language are "not" closed under complementation. For any Turing recognizable language the Turing Machine '*T*' recognizing '*L*' may not terminate on inputs
$x \notin L$ - False
3. Turing decidable languages are CLOSED under union and complementation. It is easy to determine if turing machine is decidable-*True*

So, answer is option (c) only 2.

2. (d) 1. G is a CFG. Is L(G) = ϕ?
Decidable
2. G is a CFG. Is L(G) = Σ * ?
Undecidable
3. M is a Turing Machine. Is L (M) regular?
Undecidable.
4. A is a DFA and N is an NFA. Is L(A) = L(N)?
Decidable.

Hence, the correct answer is (d) 2 and 3 only.

3. (b) The language is recursive enumerable and it is undecidable

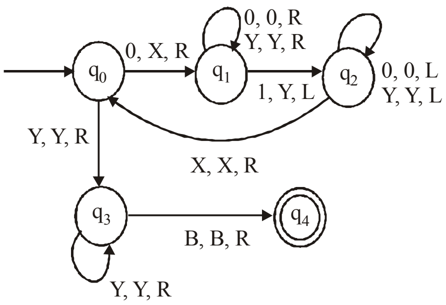
4. (a) If $L_0 \cup L_1$ is recursively enumerable, it means we can find out for all W $\in \Sigma^*$, where M halts or does not halt. This means that if $L_0 \cup L_1$ is recursively enumerable, the halting problem would be decidable. But we know, the halting problem is undecidable. Therefore $L = L_0 \cup L_1$ is not RE.

Since, $\overline{L} = (L_0 \cup L_1)^c = L_0^c \cap L_1^c = \phi$ which is regular language and hence is RE.

Therefore, $\overline{L}$ is RE.

So, correct option is (b), which is $\overline{L}$ is RE but *L* is not.

5. (c) Drawing the equivalent turing machine we have



6. (a) The TM carries out the functions of (m – n) in ($0^m\ 10^n$)
$\rightarrow$ when m $\geq$ n, the output is difference between m and n.
$\rightarrow$ when m < n, the output is blank.

7. (a) Since it is possible to create a turing machine for the problem, 20 this problem is decidable.

8. (a) This turning machine starts at 90 if it doesn't get any input symbol but *B* then it halts.
So if (00 + 1) *is chosen then the *M*/*C* can halt. Option (b) is wrong.
Option (c) & (d) are possible but not necessary.
Option (a) (0 + 1) * 1 or more occurrence of 0 or 1.
So 0, 1, 00, 01, 10, 11........are valid strings & the machine doesn't halt for them.

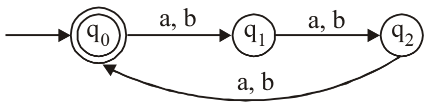
9. (a) **10. (b)** **11. (a)** **12. (c)** **13. (a)**

14. (d) Which are/is undecidable
1. G is CFG. Is L(G) = ϕ ?
– Decidable
2. G is a CFG, Is L(G) = Σ^*?
– Undecidable
3. M is a turing machine. Is L(M) regular?
– Undecidable (Preseint in all options)
4. A is a DFA and N is a NFA.
Is L(A) = L(N) ?
– Decidable

Ans. (d) 2 and 3 only.
PS. Refer clsere properties (wiki)

15. (d) **16. (b)** **17. (a)**

18. (c) Since, Turing machine is moving only in rith direction we can construct a finite automaton according to the transitions givne as follows



$\Rightarrow$ Strings are of form : {(a + b) (a + b) (a + b))}*
Hence, length of string is multiple of 3.

19. (a)

	0	1	B
q_0	$(q_1, 1, R)$	$(q_0, 1 R)$	–
q_1	–	$(q_1, 1, R)$	(q_2, B, L)
q_2	–	$(q_3, 0, L)$	–
q_3	–	$(q_3, 1, L)$	(q_4, B, R)

Strings are of the from : $1^X 0 1^Y$
Let the string be 1011

$$q_0\,1011 \vdash 1q_0\,011 \vdash 11q_1 11$$
$$\vdash 111q_1 1 \vdash 1111q_1 B$$
$$\vdash 111q_2\,1 \vdash 11q_3 10$$
$$\vdash 1q_3 110 \vdash q_3 1110$$
$$\vdash q_3 B1110 \vdash q_4 1110$$

Hence we are computing addition function.