Properties of Determinants:-

Properties 1:- The value of a determinant does not change when rows & columns are interchanged

$$\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

Properties 2:-

If any two rows (or two columns) of a determinants are interchanged, the value of the determinant is multiplied by -1.

$$\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = - \begin{vmatrix} a_2 & b_2 & c_2 \\ a_1 & b_1 & c_1 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

Properties 3:- If all the elements of one row (or one column) of a determinants are multiplied by the same number K, the value of the new determinant is K times the value of the given determinant.

$$\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} ; \begin{vmatrix} Ka_1 & Kb_1 & Kc_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = K\Delta$$

Properties 4 :- If two rows (or two column) of a determinant are identical, the value of the determinant is zero

$$\begin{vmatrix} a_1 & b_1 & a_1 \\ a_2 & b_2 & a_2 \\ a_3 & b_3 & a_3 \end{vmatrix} = 0$$

Properties 5 In a determinant the sum of the product of the elements of any row (column) with the cofactors of the corresponding elements of any other row (column) is zero.

~~Propo~~
$$\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

$$\Delta = a_1 A_1 + b_1 B_1 + c_1 C_1$$

$$0 = a_1 A_2 + b_1 B_2 + c_1 C_2.$$

$$0 = a_1 A_3 + b_1 B_3 + c_1 C_3$$

$$0 = a_2 A_3 + b_2 B_3 + c_2 C_3$$

Properties 6:- If in a determinant each element in any row (or column) consist of the sum of two terms, then the determinant can be expressed as the sum of two determinants of the same order

$$\Delta = \begin{vmatrix} a_1+\alpha_1 & b_1 & c_1 \\ a_2+\alpha_2 & b_2 & c_2 \\ a_3+\alpha_3 & b_3 & c_3 \end{vmatrix} = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} + \begin{vmatrix} \alpha_1 & b_1 & c_1 \\ \alpha_2 & b_2 & c_2 \\ \alpha_3 & b_3 & c_3 \end{vmatrix}$$

Properties 7:-

If to the element of a row (or column) of a determinant are added m times the corresponding elements of another row (or column), the value of the determinant thus obtained is equal to the value of the original determinant

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = \begin{vmatrix} a_1+ma_2 & b_1+mb_2 & c_1+mc_2 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$