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CBCS PATTERN

B.B.A.
(Computer Application)
Semester-I

CBCS
3 CREDITS

BUSINESS STATISTICS

Dr. P. G. DIXIT

NIRALI PRAKASHAN
ADVANCEMENT OF KNOWLEDGE

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A Book Of

BUSINESS STATISTICS

(Course Code 105)
For B.B.A. (Computer Application)
Semester – I : Credit-3
As Per Revised Syllabus (CBCS Pattern) Effective from June 2019

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Statistical Thinking will one day be necessary for effective citizenship as the ability to read and write

H.G. Wells

Dedicated to

My Son

Mr. Kalpak Dixit

in his loving memory

Preface ...

I am indeed very happy to place this book is in the hands of **first year 'B.B.A. (C.A.)'** students. This book is written according to new prescribed syllabus (CBCS Pattern) by Pune University which comes into force from the academic year 2019.

The main purpose of the book is to provide foundation as well a comprehensive background of 'Descriptive measures of statistics' to beginners in simple and intersecting manner. In order to make the contents of the book easier to comprehend, I have included a requisite number of illustrations, remarks, figures, diagrams etc. To elucidate statistical concepts, Applications of Statistics in real life situations is emphasized through illustrative examples. We have included the additional features MS-EXCEL commands in to obtain summary statistics. It will give an exposure to statistical computing package. Ample number of graded problems, are provided at the end of each chapter along with hints and answers. A specimen paper is set for student's self assessment.

While writing the book we have borne in mind that many students have not offered mathematics at XIth and XIIth std.

This book will also serve the purpose of reference book for M.B.A., C.A., B.C.A., I.C.W.A., M.P.M., classes.

I am thankful to Mr. D. K. Furia, Mr. Jignesh Furia and the staff of Nirali Prakashan for bringing out this book in short time. Mrs. Anagha Medhekar, Mr. Santosh Bare, Mrs. Anjali Muley and Mr. Pandya deserve special thanks for the co-operation they have extended to us. Finally, our families deserve special thanks for their support, encouragement and tolerance.

We requires our colleagues, teaching Statistics to offer their criticisms and suggestions, for further improvement of the book.

July 2019 **Author**

Syllabus ...

1. **Concept of Statistics** **(12)**

   Role of statistics. In informatics business sciences tabulation, Data condensations and tabulation, Data condensation and graphical method : Raw data, Attributes and variables, Classification, Frequency distributions, Cumulative frequency distributions.

   Graphs : Histogram, Frequency polygon. Diagrams : Multiple bar, Pie, Subdivided bar.

2. **Measures of Central Tendency** **(12)**

   Criteria for good measures of central tendency.

   Arithmetic mean, median and mode for grouped and ungrouped data, combined mean.

3. **Measures of Dispersion** **(12)**

   Concept of dispersion, absolute and relative measures of dispersion, range, variance, standard deviation, coefficient of variation, quartile deviation, coefficient of quartile deviation.

4. **Correlation and Regression (for ungrouped data)** **(12)**

   Concept of correlation, positive and negative correlation.

   Karl Pearson's coefficient of correlation.

   Meaning of regression, two regression equations, regression coefficients and properties.

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Contents ...

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Chapter 1...

Introduction to Statistics

Contents ...

Key Words :

Uses of statistics, Scope of Statistics, Limitations of Statistics, Sample, Population, SRSWOR, SRSWR. Stratified Sampling, Random Sampling.

Objectives :

In this chapter the various aspects of statistics, uses, scope and applications in various fields are discussed. The concept of statistical population and sample is also introduced. Random sample and methods of drawing sample are introduced.

1.1 Introduction

It is believed that Statistics is in use from the time when man began to count and measure. In ancient days kings used to maintain records of land, agricultural yield, wealth, taxes, live stock, soldiers, weapons, deaths and births etc. There are references that Hebrews conducted population census. In ancient days Maurya kings, King Ashoka, Gupta kings had collected Statistics. Kautilya's Arthashastra mentions that the statistics of population, land etc. were collected from time to time. Emperor Akbar gave details of population, land, agriculture etc. in his publication Ain-i-Akbari.

It is considered that the word Statistics seems to be derived from the Italian word 'statista' or the Greek word 'statistika'. Both the words have the same meaning 'political states'.

The word statistics carries several meanings. Many times statistics is considered as statistical data, which contains numerical information of a characteristic under study.

For example : Statistics of a batsman, population statistics etc.

Statistics or statistical methods is treated as a branch of science which deals with **(i) collection, (ii) presentation, (iii) analysis and (iv) interpretation of data.**

Wherever data are generated, the use of statistics becomes inevitable. Statistics performs number of functions such as (i) presentation of facts and figures. This enables to get an overall idea about the phenomenon. (ii) forecasting, (iii) planning, (iv) controlling, (v) exploring etc.

Statistics plays a role in every walk of life, right from simple situation such as finding average marks in examination to a very complex phenomenon such as rainfall prediction or measuring changes in prices.

Statistics helps in decision-making whenever phenomenon contains uncertainities. LIC, banks, defence department, government agencies, industries, business, trade etc. make use of statistics in planning, forecasting, controlling, decision-making. Index numbers are widely used in almost all fields such as economics industry business, import, export etc. Now-a-days ISO 9000 makes use of statistical tools for standardising the quality of industrial production.

1.2 Definition (Oct. 2014)

Statistics can be defined as the science of collection, presentation, analysis and interpretation of data.

Number of statisticians had made an attempt to define statistics. They used statistics for different purpose, with a different view-point. Accordingly they defined statistics emphasizing their view point. Two definitions are given below.

(a) Webster's Definition : Webster defines statistics as "the classified facts representing the conditions of people in the state, especially those facts which can be stated in a table or tables of numbers or in any tabular or classified arrangement."

The above definition gives importance to presentation of facts and figures. Remaining aspects of statistics are not considered in this definition.

(b) Horace Secrist's Definition : Secrist defines statistics as follows : 'By statistics we mean aggregates of facts affected to a marked extent by multiplicity of causes numerically expressed, enumerated or estimated according to resonable standards of accuracy, collected in a systematic manner for a predetermined purpose and placed in relation to each other.

The above definition takes into account almost all functions and aspects of statistics. It covers the fair important aspects viz. (i) collection, (ii) presentation, (iii) analysis and (iv) interpretation of data.

1.3 Importance of Statistics

We know that many phenomena in nature and activities, experiments are subject to measurements, moreover variation in different types of characteristics is inevitable. For example, income of a family, height of a person, sales of a company, electricity consumption of a city etc. This produces voluminous data. It becomes difficult to comprehend. This forces the use of statistical methods. Thus statistics is important from the following view points.

(i) Statistical methods enable to condense the data. It facilitates several functions apart from summerisation.
(ii) Statistical methods give tools of comparison.
(iii) Estimation, prediction is also possible using statistical tools.
(iv) We can get idea about the shape, spread, symmetry of the data.
(v) Inter-relation between two or more variables can be measured using statistical techniques.
(iv) Statistical methods help in planning, controlling, decision-making etc.
(vii) The use of statistical methods is important because considerable amount of time, money, manpower can be saved.
(viii) Uncertainities can be reduced to get reliable results.
(ix) Statistical methods give systematic methods of data collection and investigation.

Thus statistics reveals several aspects of phenomena.

H. G. Wells expresses the importance and need of statistics in the following words.

"Statistical thinking will one day be necessary for effective citizenship as the ability to read and write".

1.4 Scope and Applications of Statistics

The tools and techniques given by statistical methods are used in almost all fields at several phases. Because of diversified applications of statistics, an exhaustive list of fields is difficult to prepare. However, some of them are stated below. We find use of statistics indispensable in the agriculture, business, commerce, demography, economics, education, government agencies, industries, social sciences, biological sciences, medical sciences, management sciences etc. We discuss briefly the scope of statistics in some of the above stated fields.

(a) Statistics in industry : Industry makes use of statistics at several places such as administration, planning, production, growth and development. In many industries 'Statistical Quality Control' division is separately operating. Mainly, whether manufactured goods possess a desirable standard or not is examined using various control charts. These inspections are done at the time of production. On-line process capability study is conducted to set-up the machines to give desired standards. Moreover purchased goods or semifinished goods are inspected using acceptance sampling plans of various types. Now-a-days, ISO 9000 makes use of Statistics to a large extent. Apart from this in some industries the technique known as designs of experiment is also used. Newly installed machinery is tested for its performance using statistical methods. Sampling is required to be used because of its several advantages. Multiple regression planes are used for forecasting, when several factors are interlinked. Efficiency measurement, index number of production, work sampling etc. are very much useful for administration and planning department.

(b) Statistics and Economics : In the field of economics, huge amount of data are needed to be processed and interpreted. Statistics is very much helpful in this field. In order to collect data, various statistical methods of investigations are used. Many a times questionnaires are drafted. A proper representative of a group is selected using sampling methods. Statistical methods are used in this activity to get reliable results. Estimation of national income, per capita income, poverty line, industrial production etc. is done using statistical techniques. Probability distribution of income can be useful in various economic activities. A tool known as index number developed in Statistics is used every now and then in economics. It performs number of functions. It measures average increase in prices, production, income, volume of import, export etc. Index numbers are called as economic barrometers. Index numbers are used in determining real income, deflation, cost of living index numbers. To measure the changes in prices of shares in stock market index number provides the best tool. Several interlinked activities in economics can be studied.

For example, (i) the relation between prices and supply (ii) the relation between demand and prices (iii) the relation between sales and profit.

Demand analysis, time series analysis techniques are mainly developed to study economics. Those are the gifts given by statistics.

Richard Lipsey says " The role of statistical analysis is two fold. First, we wish to use observations from the real world to test our theories. Second, we wish to use such observations to give us measures of the quantitative relations between economic variables.

(c) Statistics and Management Sciences, Business sciences (April 2015, Oct. 2014): Most of the managerial functions make use of statistics. For efficient working of various

sections of management such as sales, production, marketing statistical method are used. Different statistical tools such as forecasting, tests of significance, index numbers, time series analysis, statistical quality control, estimation play vital role in management activities. Apart from this, various optimisation techniques known as linear programming, transportation techniques, job assignment problems, sequencing, CPM and PERT, replacement problems, inventory control are also useful.

Portfolio management makes use of regression analysis. The regression coefficient called beta index in portfolio is used in decision-making. Risk measurement is done using standard deviations, covariance. Various statistical techniques are used in decision-making.

(d) Statistics and Social Sciences : Bowley says "Statistics is the science of measurement of social organism, regarded as a whole in all its manifestation". Research in social sciences need questionnaire. Further analysis is required to be done using statistical tools. In social sciences we need to test association between two variables such as (i) education and criminality (ii) education and marriage adjustment score (iii) sex and education (iv) richness and criminality etc.

(e) Statistics and Informatics : Statistics and computer science both are together useful in providing solutions to the problems in various fields. Particularly, whenever data analysis techniques are employed to large data, use of computers becomes indispensable. Conjectures supported by statistical data have sound ground of approval. Now-a-days several statistical software packages like MINITAB, MATLAB, STATPACK, SAS, SPSS, SYSSTAT, R etc. are used for data analysis. Forecasting, prediction, estimation, curve fittings etc. are the commonly used statistical techniques. The use of software packages provides the unusual opportunities to get the data summarised in appropriate way. The suitability of modal used for analysis can be quickly determined by means of software package, otherwise it is a time consuming and tedious procedure. Although software packages are useful to great extent, it cannot replace totally the necessity of statistician. In order to interpret the output or to decide the suitability of statistical model for analysis, to design the quessionaire, to design the experiment etc. statistician's help is essential.

The other aspect of statistics and computer science may be discussed as follows. Computer is an assembly of several components. The life of each component is a variable having some probability distribution. The average life of each component as well the assembled product can be determined using statistical methods. Reliability of component and system may help the manufacturer to decide the guarantee period of computer as well to user to decide the policy of replacement of spare parts. In general to a computer consultant, theory of queues and optimisation techniques may be useful to plan out his work schedule.

1.5 Types of Data

Collection of data is a very important work and needs to be done carefully. One has to decide the objectives clearly before collecting the data. In order to determine dependable and reliable results, proper data should be collected in a proper way. The data according to the method of collection are of two types viz., (a) Primary data, (b) Secondary data.

Apart from the method of collection the type of data according its nature are also in existence. (viz. time series data, cross-sectional data).

(a) Primary Data :

Primary data means original data (i.e. facts and figures) obtained by an investigator himself. Primary data may be a result of a survey or enquiry conducted. This may be regarded as first-hand information. Population census results, is a classical example of primary data. Primary data are also called as *raw data*. No doubt, primary data are more reliable than any other type, but are expensive and time consuming.

Primary data are collected by the following methods :

1. Direct personal investigation or interview.

In this method, the investigator meets concerned persons known as 'informants' and collects necessary information by the process of interview. Investigator should be thorough in handling problems of investigation. This will result into reliable data. Investigator has to go upto the source of original information. For example, if he wants to know the amount of production, in a particular industry, he should collect the figures by visiting the machine floor, rather than from office or bulletin. This is the best method of collecting primary data. However, the investigator has to take certain precautions.

2. Indirect oral investigation.
3. Investigation through questionnaire.

(b) Secondary Data :

Data taken from sources like office records, bulletins, reports etc. which are already collected by some other agency is called *'secondary data.'*

The data which are already collected may be tabulated, classified, ordered etc. Hence, it is called processed or finished data. Thus, secondary data can also be called *finished product*.

'Secondary data' is a relative term. For example, if 'A' collects original data, then it becomes primary data for him; whereas if the same data is used by B, then it becomes secondary data for B. In this case, the only difference is that the user of secondary data may not have thorough understanding of the background as the user of primary data has.

Difference between Primary and Secondary Data :

(a) The main difference lies in the method of collection.

(b) Primary data are original in nature. Hence those are more accurate than secondary data.

(c) Collection of primary data is expensive as well as time-consuming.

(d) Primary data can be elicited in accordance with the objectives of a study. Secondary data may fail in this regard.

The methods of data collection are (i) surveys, (ii) laboratory experiments, (iii) simulation.

Surveys : With the help of sample surveys or complete enumeration primary or secondary data can be collected.

Laboratory Experiments : The observations generated in laboratory experiments will be a method of data collection.

Simulation : Some experiments cannot be conducted in laboratory. **For example**, genetic experiments, experiments with hazardous material or radioactive material. In such cases, now-a-days the data are generated using simulation techniques with the help of computers. It has tremendous scope in industry, business etc. **For example**, how many counters or salesmen are required in a departmental store can be simulated using queueing theory.

The Other Types of Data :

There is yet another angle of looking at data. Earlier we have considered the way of collection. However, the type of data exists due to the nature of data and some other characteristics. If we consider the data when it was collected. Thus, we introduce the time characteristics. It gives rise to the data specially termed as **time series data**. Sometimes at a fixed time moment we collect data, where time is considered but hold constant. Such data are referred to as **cross-sectional data**. The specific definitions are as follows :

Time Series Data : The data arranged in the chronological order (as per the order of occurrence) are called as time series data.

For example :

(1) Daily sales of a departmental store.

(2) Daily electricity consumption of a town.

(3) Price of gold recorded daily.

Cross-sectional data : The values of variables observed at a particular time at several places or on several objects are called as cross-sectional data.

For example :

(1) Sales on a specific day of several departmental store is a cross-sectional data. However daily sales of a specific departmental store constitutes time series data.

(2) Electricity consumption on a specific day for several towns constitutes cross-sectional data. However, daily electricity consumption of a specific town is a time series data.

Exercise

[A] Theory Questions :

1. Define 'statistics'.
2. Explain the importance of statistics or statistical methods.
3. Describe the scope and utility of statistics in the field of (i) industry, (ii) economics, (iii) management sciences, (iv) social sciences, (v) Informatics, (vi) Business science.
4. Mention the application of statistics in the following fields :
   (i) industry, (ii) economics, (iii) management sciences, (iv) social sciences.
5. distinguish between primary data and secondary data.
6. Explain the terms : time series data, cross-sectional data, failure data.

❐❐❐

Chapter **2**...

Data Condensation

Contents ...

Key Words :

Variable, Attributes, Discrete Variable, Continuous Variable, Raw Data, Primary Data, Secondary Data, Classification, Frequency, Inclusive and Exclusive Method of Classification, Class Limits, Class Boundaries, Class Mark, Open End Class, Relative Frequency, Cumulative Frequency, Histogram, Frequency Polygon, Frequency Curve, Ogive Curves.

Objectives :

This chapter explains the first two aspects of statistics viz. collection and presentation of data. Classification is a tool of data condensation. It becomes easier to analyse the data after classification. Graphical representation has several advantages.

2.1 Variables and Attributes (April 2015)

While studying any phenomenon we come across two types of characteristics : (i) constant and (ii) variable. The characteristic which does not change its value (or nature) is considered as **constant**.

For example : Height of a person after 25 years of age, altitude of a certain place from sea level etc. On the other hand there are many characteristics which are qualitative or quantitative in nature and change their values (or nature). *For example :* Examination result

of a candidate can be recorded as pass or fail which is a qualitative variable characteristics, whereas we can express a candidate's performance as percentage of marks which is a quantitative variable.

Statistics involves the study of variable characteristics. Hence, we include the related and necessary definitions.

Attribute (B.B.M. April 2015) : A qualitative characteristic like sex, nationality, religion, grade in examination, blood group, beauty, defectiveness of an article produced by a machine is called as *attribute*.

Variable (B.B.M. April 2015) : A quantitative characteristic (which changes its value) like weight of person, examination marks, population of a country, profit of a salesman, is called as *variable*.

It can be clearly noticed that variables can be measured by numbers.

Further the variables can be divided into two categories : (i) discrete and (ii) continuous.

Discrete variable : A variable taking only particular values or isolated values is called as *discrete variable*.

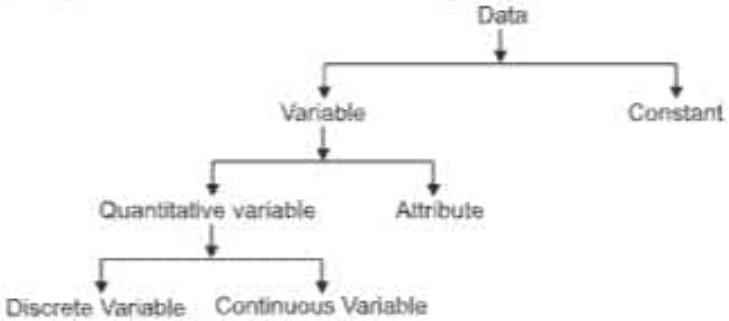
For example : Number of students in a class, number of articles produced by a machine, population of a country, number of workers in a factory etc. are discrete variables. Most of the discrete variables have integral values.

Continuous variable : A variable taking all possible values in a certain range is called as *continuous variable*.

For example : Weight of a person, length of a screw produced by a machine, temperature at a certain place, agricultural production, electricity consumption of a family, speed of a vehicle are the examples of continuous variable.

It is observed that many continuous variables such as marks, income, weight of a person etc. look like discrete variables after the measurement. This is mainly due to the limitations of the measuring instruments. Using better instruments one can have accurate measurement and overcome this difficulty.

The following diagram summarizes the various types of data :



2.2 Classification

In order to study a characteristic or a group of characteristics of any type, the first phase is to collect the data.

Raw data : The unprocessed data in terms of individual observations are called as raw data.

For the sake of further statistical analysis, the data items are arranged in increasing (or decreasing) order. However, if there is a huge amount of observations, merely ordered arrangement is not enough. It does not furnish much useful information nor does it reduce the bulk of data. Data in this form are difficult to comprehend, analyse and interpret.

For example : Income of 5000 individuals is given for analysis.

It becomes quite essential to condense the data in a suitable form. Classification can be used as a tool to condense the data.

Classification : The entire process of making homogeneous and non-overlapping groups of observations according to similarities is called as *classification*.

The groups so formed are called as **class intervals or classes**.

Objectives of Classification : The objectives of classification can be summarised as follows :

1. It condenses the data.
2. It omits unnecessary details.
3. It facilitates the comparison with other data.

For example : In case of classification of income of 5000 individuals, one can find the number of individuals below poverty line or income distribution of two countries can be compared.

4. It reveals prominent features of the data.

For example : We can find the income group in which majority of families lie.

5. It enables further analysis like computation of averages, dispersion etc.

2.3 Frequency Distribution

We proceed to study how the observations are classified and a frequency distribution is formed.

Frequency distribution of continuous variable :

The procedure of classification of continuous variable differs slightly from that of a discrete variable.

Procedure :

1. Find the smallest and the largest observation. Calculate the difference between them. This difference is called as the *range*.
2. Decide the classes, by dividing the range into several intervals. The number of classes be preferably between 7 to 20.
3. Prepare first column of table by entering the class intervals.
4. Classify the observations one-by-one in the appropriate class by putting tally marks in the second column against the corresponding class. Cross the observation from the original data to avoid double counting.
5. Count the tally marks and enter the number in the third column.

Illustration 1 : *The following are the scores in intelligence test conducted for 80 candidates of a certain class.*

112	*77*	*115*	*91*	*137*	*88*	*89*	*71*
100	*93*	*64*	*116*	*95*	*95*	*106*	*92*
84	*86*	*97*	*124*	*84*	*117*	*97*	*80*
103	*114*	*83*	*77*	*94*	*114*	*63*	*61*
120	*126*	*98*	*98*	*116*	*108*	*94*	*105*
108	*99*	*87*	*96*	*88*	*95*	*73*	*92*
91	*129*	*108*	*81*	*82*	*102*	*86*	*111*
119	*90*	*109*	*101*	*107*	*75*	*123*	*104*
106	*84*	*75*	*99*	*72*	*128*	*114*	*93*
83	*82*	*124*	*114*	*130*	*81*	*101*	*91*

Prepare the frequency distribution of the data by taking suitable class intervals.

Solution : In the given problem we note that the highest and the lowest observations are respectively 137 and 61. Hence, the range is 137 – 61 = 76. In this case it is suitable to make 8 classes each of width 10. Since the lowest observation is 61, it is convenient to choose the first class as 60 to 69, the next as 70 to 79 and so on. The last class will be 130 to 139. According to the procedure described above, we classify the observations and prepare a table of three columns. First column includes classes, seconds includes tally marks and the third includes frequencies. The first observation is 112, it lies between 110-119, therefore, we put a tally mark to include the observation in this class. Likewise all the observations are classified and the process gives the following table 2.1.

Table 2.1 : Frequency Distribution of Scores of 80 Candidates

Class Intervals	Tally Marks	Frequency
60 – 69	𝍸	3
70 – 79	𝍸 ||	7
80 – 89	𝍸 𝍸 𝍸 |	16
90 – 99	𝍸 𝍸 𝍸 𝍸	20
100 – 109	𝍸 𝍸 𝍸	14
110 – 119	𝍸 𝍸 |	11
120 – 129	𝍸 ||	7
130 – 139	||	2
Total	–	80

Frequency : The number of observations in a class is called as *frequency* or *class frequency.*

Frequency Distribution : A table containing class intervals along with frequencies is called as *frequency distribution.*

2.4 Methods of Classification

There are two methods of classification : (I) inclusive method (II) exclusive method. We bring out the difference between the two methods.

I. Inclusive Method : In this method the observation equal to upper limit is included in the same class. Therefore, the method is called as *inclusive method*. It can be observed that the upper limit of class is not the same as the lower limit of succeeding class. Therefore, a discontinuity is observed between the classes. *For example*,

Table 2.2

Daily Sales in ₹
2000 – 2999
3000 – 3999
4000 – 4999

II. Exclusive Method : In this method the observation equal to upper limit does not belong to the same class. It is included in the next class. Therefore, the method is called as *exclusive method*. For example, the observation 4000 is included in 4000 – 5000. In other words, the observation equal to upper limit is excluded from the same class.

For example,

Table 2.3

Daily Sales in ₹
2000 – 3000
3000 – 4000
4000 – 5000

In this case upper limit of one class is the lower limit of subsequent class. The classes are observed to be continuous without any gap in between them.

We explain below few more terms related to the frequency distribution.

Class-limits : The two numbers designating the class-interval are called as *class limits*. With reference to table 2.1, the first class interval is 60–69, in this case 60 and 69 are the class limits. The smallest possible observation that can be included in the class is *lower limit* and the largest possible observation that can be included in the class is the *upper limit*. In the above example 60 and 69 are lower and upper limits of the class interval 60–69.

Class boundaries : The class boundaries are the numbers upto which the actual magnitude of observation in the class can extend. The class boundaries are also called as actual limits or extended limits. For the sake of clarity, let us consider the frequency distribution with classes 10–19, 20–29, ... etc. In this case an observation 19.2 will be rounded-off to 19 and placed in 10–19, whereas the observation 19.6 will be rounded-off to 20 and will be placed in 20–29. Therefore, the actual magnitude of the observation in the class 20–29 will be between 19.5–29.5.

Note : If the classes are not continuous then, we need to determine class boundaries. If d is the gap between two classes then

$$\text{lower boundary} = \text{lower limit} - \frac{d}{2}$$

$$\text{upper boundary} = \text{lower limit} + \frac{d}{2}$$

The illustration below will make out the difference between class limits and class boundaries.

Illustration 2 : *Convert the class limits 10-19, 20-29, 30-39 into class boundaries.*

Class limits	*Class boundaries*
10 – 19	*9.5 – 19.5*
20 – 29	*19.5 – 29.5*
30 – 39	*29.5 – 39.5*

Solution : Note that the gap between the first and second class interval is

$$d = 20 - 19 = 1$$

$$\therefore \quad \text{lower boundary} = \text{lower limit} - \frac{d}{2} = 20 - \frac{1}{2} = 19.5$$

$$\text{upper boundary} = \text{upper limit} + \frac{d}{2} = 29 + \frac{1}{2} = 29.5$$

It can be clearly seen that in case of exclusive method of classification, class limits and class boundaries are same.

Using class-boundaries the classes are made continuous however original frequency associated do not alter.

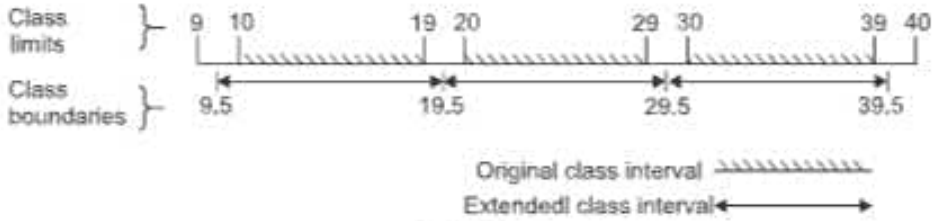


Fig. 2.1

Class-mark or Mid-values : It is the mid-point of class interval and the same can be obtained as follows :

$$\text{Mid-value} = \frac{\text{Upper limit} + \text{Lower limit}}{2}$$

$$= \frac{\text{Upper boundary} + \text{Lower boundary}}{2}$$

Class-width : It is the actual length of the class interval. We can find class width as follows :

$$\text{Class width} = \text{Upper boundary} - \text{Lower boundary}$$

$$= \begin{pmatrix}\text{Lower limit of the}\\ \text{succeeding class}\end{pmatrix} - \begin{pmatrix}\text{Lower limit of the}\\ \text{class under}\\ \text{consideration}\end{pmatrix}$$

$$= \begin{pmatrix}\text{Upper limit of the}\\ \text{class under}\\ \text{consideration}\end{pmatrix} - \begin{pmatrix}\text{Upper limit of the}\\ \text{preceding class}\end{pmatrix}$$

Open end class : A class in which one of the limits is not specified is called an open end class.

For example, in the following frequency distribution there are two open end classes.

Table 2.4

Daily Sales in ₹	
below 2000	Open end classes
2000 – 3000	
3000 – 4000	
4000 and above	

The class 'below 2000' has no lower limit and the class '4000 and above' has no upper limit. Therefore, these classes are open end classes. Whenever the extreme observations are widely spread, open end classes are used. In case of income distribution or the classification of sales of a company, open end classes may be required. Open end classes create some problems in further analysis, therefore, as far as possible the open end classes should be avoided.

Ilustration 3 : Find the mid-point and width of each class given the classes below 10, 10-20, 20-40, 40-60, 60-70, above 70.

Solution :

Class	Mid-point	Width
below 10	Not defined for open end class	
10 – 20	$\frac{10+20}{2} = 15$	10
20 – 40	30	20
40 – 60	50	20
60 – 70	65	10
above 70	Not defined for open end class	

Illustration 4 : Given the classes 0-9, 10-19, 20-29, 30-39 find the mid-point and width of each class.

Solution :

Class	Mid-point	Width
0 – 9	$\frac{0+9}{2} = 4.5$	10
10 – 19	14.5	10
20 – 29	24.5	10
30 – 39	34.5	10

Note : Width = Difference between two successive lower limits.

2.5 Cumulative Frequencies

In many situations it is required to find the number of observations below or above a certain value. *For example* : In case of a frequency distribution of income, the number of persons below poverty line or in case of frequency distribution of examination marks, number of candidates above 60 etc. is required to be found. In this case cumulative frequencies are much useful. There are two types of cumulative frequencies : (i) less than type cumulative frequency (ii) more than type cumulative frequency.

Less than type cumulative frequency of a class is the number of observations less than or equal to the upper limit of the corresponding class. Similarly more than type cumulative frequency is the number of observations more than or equal to the lower limit of the corresponding class.

It is clear from the above explanation that the less than type cumulative frequencies can be obtained by computing cumulative sum of frequencies from the lowest class to highest class. We illustrate the procedure of computing the less than type and more than type cumulative frequencies.

Illustration 5 : *For the following frequency distribution find (i) less than cumulative frequencies (ii) more than cumulative frequencies.*

Marks	*0-10*	*10-20*	*20-30*	*30-40*	*40-50*
Frequency	*5*	*12*	*15*	*4*	*4*

Solution :

Marks	Frequency	Less than cumulative frequency	More than cumulative frequency
0 – 10	5	5	4 + 4 + 15 + 12 + 5 = 40
10 – 20	12	5 + 12 = 17	4 + 4 + 15 + 12 = 35
20 – 30	15	5 + 12 + 15 = 32	4 + 4 + 15 = 23
30 – 40	4	5 + 12 + 15 + 4 = 36	4 + 4 = 8
40 – 50	4	5 + 12 + 15 + 4 + 4 = 40	4
Total	40	–	–

It can be noted that the less than cumulative frequency is increasing in nature. Less than cumulative frequency of the lowest class is same as the usual frequency and the less than type cumulative frequency of highest class is the total number of observations. In case of more than cumulative frequencies exactly reverse observations will be seen.

A table containing upper limits along with less than type cumulative frequency or lower limits along with more than type cumulative frequency is called as *cumulative frequency distribution.*

2.6 Relative Frequencies

Two different frequency distributions may not have the same total frequency, hence for the purpose of comparison and interpretation, sometimes it is better to express the frequency of a class in terms of proportion (or percentage) of the total number of observations. The proportion of number of observations in a class is the *relative frequency*. Therefore,

$$\text{Relative frequency} = \frac{\text{Class frequency}}{\text{Total frequency}}$$

It can be noted that the relative frequency maintains the same pattern which is observed in class frequencies. The total of relative frequencies is 1.

Relative frequencies are widely used in economics, commerce etc. We illustrate how the relative frequency helps comparison.

Illustration 6 : *The following table gives the frequency distribution of marks in accountancy out of 60. Find the relative frequencies.*

Marks	*0-10*	*10-20*	*20-30*	*30-40*	*40-50*	*50-60*
No. of students	*5*	*25*	*27*	*32*	*6*	*5*

Solution :

Marks	Frequency	Relative Frequency
0–10	5	0.05
10–20	25	0.25
20–30	27	0.27
30–40	32	0.32
40–50	6	0.06
50–60	5	0.05
Total	100	1.00

2.7 Guidelines for the Choice of Classes

Classification of data is a sort of compromise, therefore, it becomes important to choose appropriate number of classes. The classes should be chosen, so that it will condense the data and it will also maintain the patterns in the original data.

(1) The number of classes should not be too large, otherwise it will not serve the purpose of condensation.

(2) The number of classes should not be too small. If the number of classes is too small it will not reveal the pattern in the original data. Moreover, due to the small number of classes, each class will be too wide. For further computations it is assumed that the observations in a class are situated at the centre of the class. The assumption will not remain valid for wider classes.

The number of classes should be between 7 to 20. However, according to the needs and requirements of the situation appropriate number of classes is chosen.

If the number of observations is large, naturally the number of classes will be large.

(3) As far as possible, classes should be of uniform width.

Sturge's Rule : If N is the total number of observations to be classified, then according to sturge's rule, the number of classes is approximately $1 + 3.222 \log N$. By the other approach as a thumb rule, the number of classes is approximately $\sqrt{N}$.

(4) As far as possible, open end classes should be avoided.

(5) The class width should be preferably 5 or multiple of 5.

(6) The lower limit of the starting class be preferably multiple of 5.

For example : The classes may be of the type 0–9, 10–19... or 11-20, 21-30 ... etc.

2.8 Graphs

Here we discuss the various graphs associated with frequency distribution. Generally graphs are used to represent mathematical relationship between two variables, otherwise diagrams are used.

(i) Histogram : It is one of the popularly used graphs for the representation of frequency distribution. It is a series of adjacent rectangles erected on X-axis with class interval as base, hence width of rectangle is equal to class width. Height of rectangle is taken as proportional to class frequency. In case of inclusive method of classification, extended class interval is used as base, where extended class interval is an interval designated by class boundaries.

Note :

1. A serious drawback of histogram is that, it cannot be drawn for a frequency distribution with open end class.
2. In case of discrete variable, histogram need not contain adjacent rectangles, those may be separated like bar diagram.
3. Histograms are useful to find mode, which is discussed in the next chapter.

Illustration 7 : *Draw a histogram to represent the following frequency distribution :*

Size of farm in hectares	*1–20*	*21–40*	*41–60*	*61–80*	*81–100*	*101–120*
No. of farms	*12*	*38*	*16*	*5*	*3*	*1*

Solution :

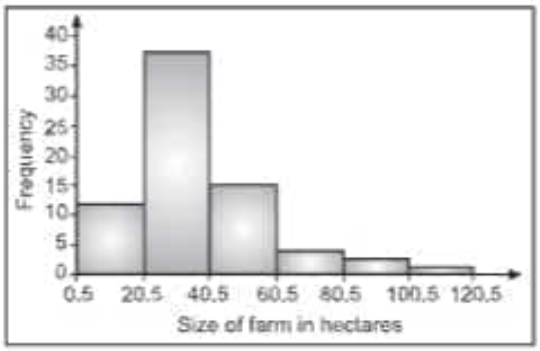


Fig. 2.2 (a) : Histogram

Histogram using MS-Excel : To draw histogram follow steps given below. Take mid-values on X-axis and frequency on Y-axis. Enter mid values in column A and corresponding frequencies in column B on worksheet. Select the frequencies by clicking the mouse then go to insert command on main menu. Select

Insert – – > chart.

Then following windows will appear on the screen one-by-one.

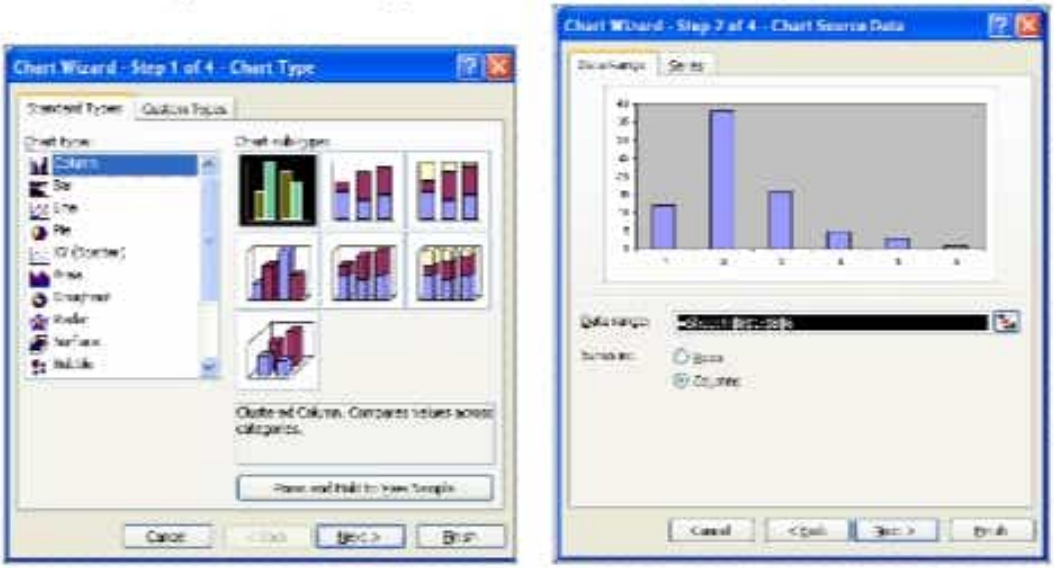


Fig. 2.2 (b)

Select chart type **(column)** and click **next**.

It is at bottom command line.

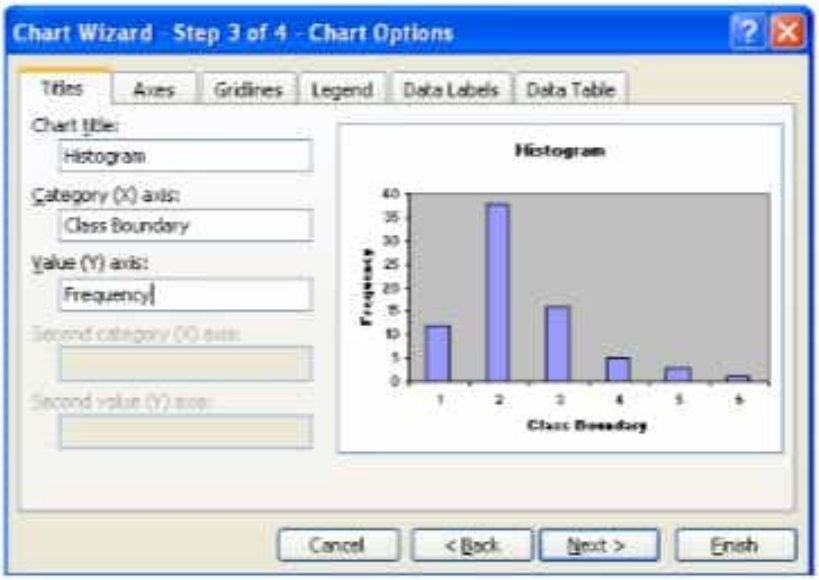
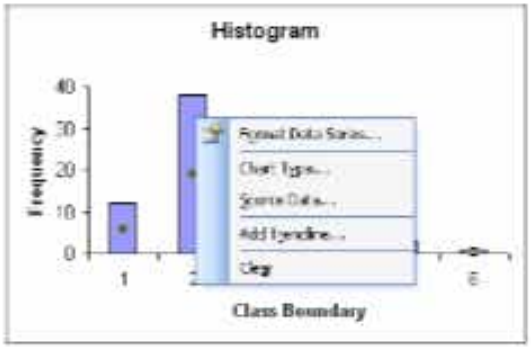
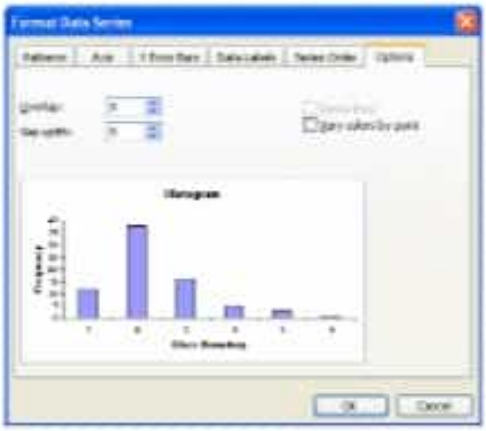


Fig. 2.2 (c)

In the data range, select frequencies and click **Next**.

Give chart title and x, y axis, click Finish, Right click on one of the bar as shown in Fig. 2.2 (d), select sub-menu Format Data Series to get Fig. 2.2 (b) then select Gap width 0 as shown in Fig. 2.2 (d). Click OK to get histogram as shown in Fig. 2.2 (e).





(d) **(e)**

Fig. 2.2

After going through all steps of chart wizard, following histogram will appear on the screen.

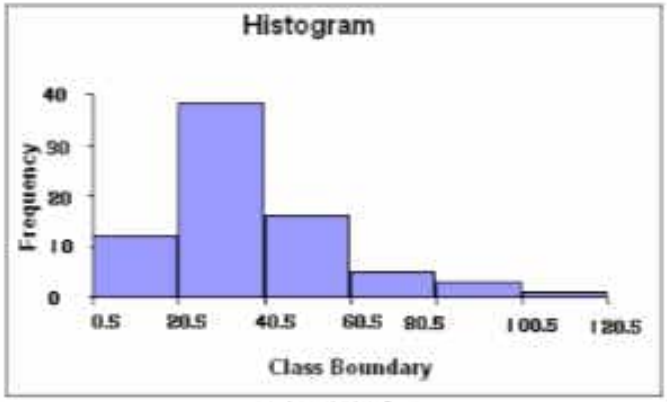
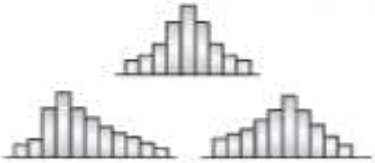


Fig. 2.2 (f)

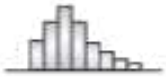
Histogram and ISO 9000 : Now-a-days manufacturing units and industries have to maintain quality of their product as per norms laid down by Indian Standards (IS) or International Standards Organisation (ISO). To achieve quality standards several statistical tools are used. Such tools are known as Quality Control (QC) or Process Control (PC) tools. Histogram is an important tool. It has three fold purpose (i) It displays the pattern of variation, (ii) It gives idea about process behaviour, (iii) It helps to decide where to focus the efforts for improvement. Some interpretations based on histogram are illustrated below :



Symmetric distribution.

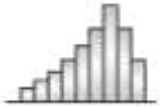
Non-symmetric distribution

Bimodal (having two centres) distribution. It is a mixture of two distributions.



Lower extreme values are not recorded separately.

Left end is not tapered.



Upper extreme values are not recorded separately.

Right end is not tapered.

Narrow spread distribution.

Widely spread distribution.

Fig. 2.3

(ii) Frequency Polygon : Generally, a graph is expected to be in the form of a smooth curve. Histogram does not fulfil this requirement. Therefore, another important way of presentation of frequency distribution is frequency polygon or frequency curve. This type of graph enables us to understand the pattern in the data more clearly. Mid-values are taken on X-axis and frequencies on Y-axis to draw the graph. Successive points are joined by the line segments. Further, to complete polygon we obtain closed figure by taking two more classes. One preceding to first class and the other succeeding to last class. Frequency of each class is taken to be zero. Mid-points of these classes are used to get closed figure. The figure so obtained is called as frequency polygon.

Note :

1. We can draw frequency polygon using histogram. In this case we join the mid-points of upper sides of all the rectangles by line segments. Further to get closed figure we join the mid-values of preceeding class and succeeding class to the frequency distribution.

2. Histogram gives rough idea about the nature of frequency distribution. The border of histogram represents the frequency distribution. the boarder is zigzag, so we need to make it more smooth. Using frequency polygon and frequency curve it is possible to do so. The following figures will demonstrate how to make the border smooth by reducing the class width.

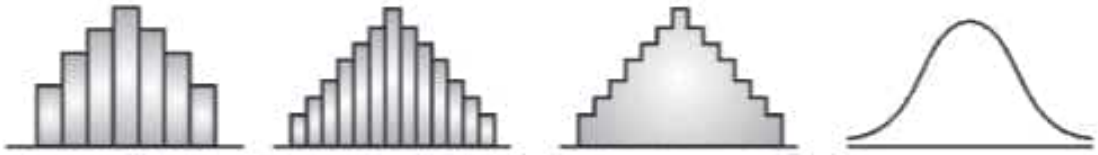


Fig. 2.4

(iii) Frequency Curve : There is little difference in frequency polygon and frequency curve. If the points (or vertices of frequency polygon) are joined by a smooth curves instead of straight lines we get a closed figure called as *frequency curve*. While drawing frequency curve we should take care that the area under the curve is same as that of frequency polygon.

It can also be noticed that, we can draw frequency curve using histogram by the similar procedure which is used in case of frequency polygon.

Illustration 8 : *Draw a frequency polygon and a frequency curve for the following data :*

Monthly house rent	*100-300*	*300-500*	*500-700*	*700-900*	*900-1100*	*1100-1300*
No. of families	*6*	*16*	*24*	*20*	*10*	*4*

Solution : Mid-values of classes are taken on X-axis and frequency is taken on Y-axis. First point we need to plot is (200, 6), second point will be (400, 16) and so on. The last point will be (1200, 4). To get a closed figure we take two more points (0, 0) and (1400, 0). Joining

these points by line segments (or smooth curve) we get frequency polygon (or frequency curve).

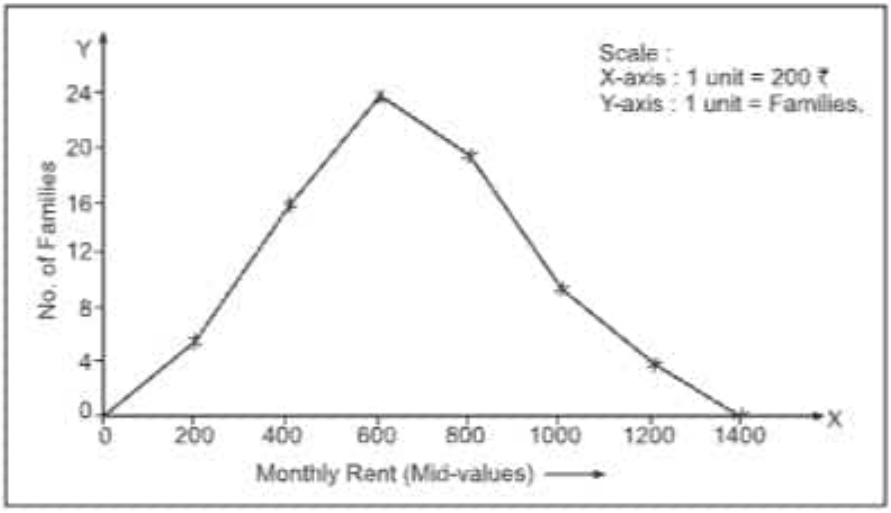


Fig. 2.5 : Frequency Polygon

(iv) Cumulative Frequency Curve or Ogive (B.B.M. April 2015) : Cumulative frequency distribution is represented by cumulative frequency curve (or ogive). There are two types of cumulative frequencies, hence, there are two types of cumulative frequency curves. For less than type cumulative curve upper boundaries of classes are taken on X-axis and less than cumulative frequencies on Y-axis. A preceding class before first class is also taken into consideration for drawing this curve. Cumulative frequency of this class is taken to be zero. Similarly, to draw more than type cumulative frequency curve lower boundaries are taken on X-axis and more than cumulative frequencies on Y-axis. In this case a succeeding class to the last class is taken with cumulative frequency zero. Those points are joined by smooth curve to get the cumulative frequency curve.

This type of curve is useful in finding median which is discussed in the subsequent chapter.

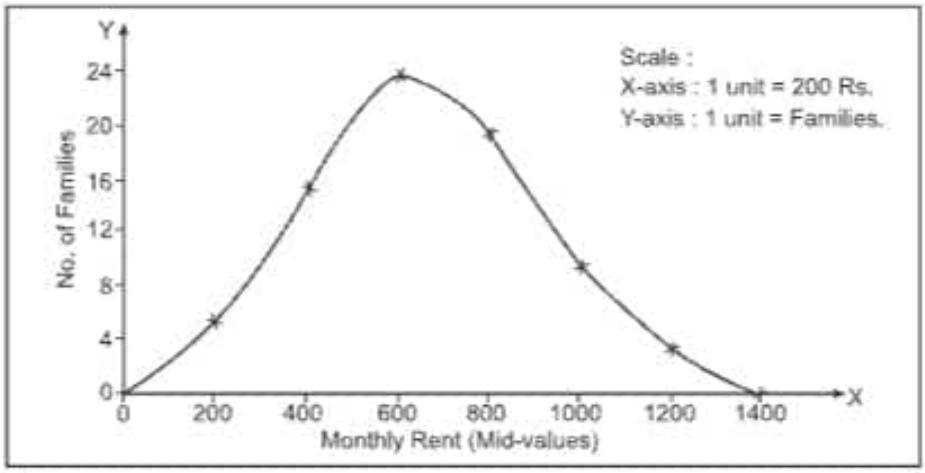


Fig. 2.6 : Frequency Curve

Illustration 9 : *Draw less than cumulative frequency curve and more than cumulative frequency curve for the following frequency distribution :*

Marks	*0–10*	*10–20*	*20–30*	*30–40*	*40–50*
No. of students	*5*	*12*	*43*	*32*	*8*

Solution : To draw less than type cumulative frequency curve we find out the required cumulative frequencies. In this problem class limits and class boundaries are same.

Upper limits	0	10	20	30	40	50
Cumulative frequencies	0	5	17	60	92	100

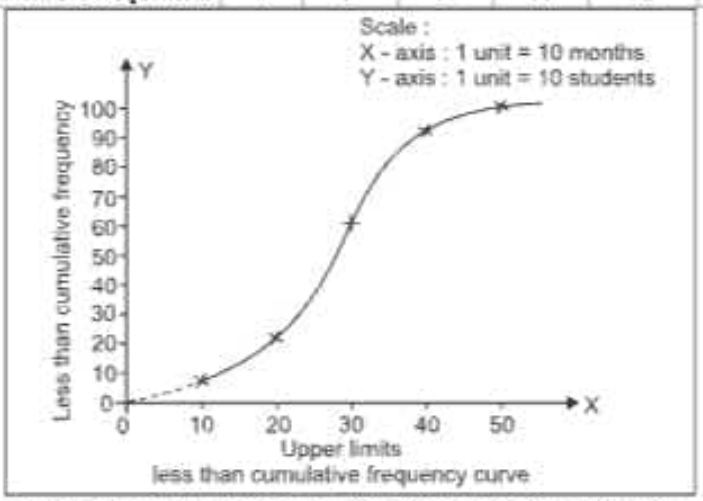


Fig. 2.7 : Less than Cumulative Frequency Curve

In order to draw more than cumulative frequency curve we obtain more than cumulative frequencies.

Lower limits	0	10	20	30	40	50
Cumulative frequencies	100	95	83	40	8	0

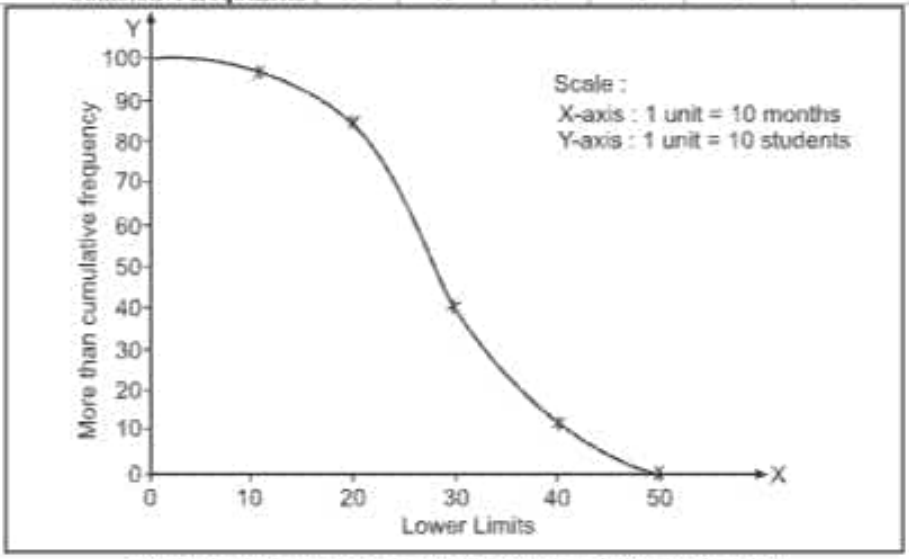


Fig. 2.8 : More than Cumulative Frequency Curve

2.9 Diagrams
(Simple bar, Multiple Bar, Sub-Divided Bar and Pie Diagram)

In the earlier discussion we have studied the methods of summarising voluminous data. Those methods are adopted to serve the purpose of condensation, comparison and for revealing patterns. However, these methods have their own limitations. Especially when table is large in size, comparison becomes difficult. Perhaps a more effective way to serve the purpose of comparison and revealing the patterns, is graphical or diagrammatic representation. Diagrams and graphs are easy to understand and create an effect which lasts for a longer time. They use voluminous, uninteresting, dry data and present the facts in an attractive and impressive manner. They facilitate comparison and hence, conclusions can be drawn quickly, which is not possible with the help of a table or frequency distribution to the same extent. Moreover, patterns present in the data are more clearly exhibited by graphs and diagrams. Due to such several advantages, graphs and diagrams are believed to be powerful tools to convey information to a layman. Therefore, graphs and diagrams are found to be of immense use in several fields to emphasize the facts. LIC, banks, government agencies, industries use graphs to show their growth, development, extension activities etc.

There are several types of diagrams used in practice to represent the information in statistical table viz. simple bar diagram, multiple bar diagram, subdivided bar diagram, percentage bar diagram and pie diagram. Two of which are discussed below,

(i) Simple bar diagram : In order to represent data related to a single variable, simple bar diagram or bar diagram is used. For example : Yearly sales, monthly production, yearly population, countrywise population, yearly inputs etc. In this type of diagram, year, month, country etc. are taken on X-axis and corresponding values of the variable are taken on Y-axis. In this case rectangles of equal width and height proportional to the value of variable are erected on horizontal axis.

Illustration 10 : Represent the following data using simple bar diagram :

Year	1981	1982	1983	1984	1985
Production in (milion tonnes)	45	40	50	52	47

Solution :

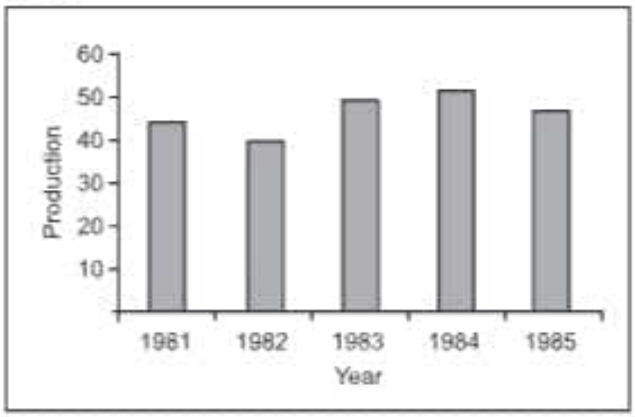


Fig. 2.9 : Bar diagram

Illustration 11 : Use a bar diagram to represent following data :

Year	1983	1984	1985	1986	1987
Profit of a company (in lakhs ₹)	2.5	2.0	– 1.0	2.8	3.0

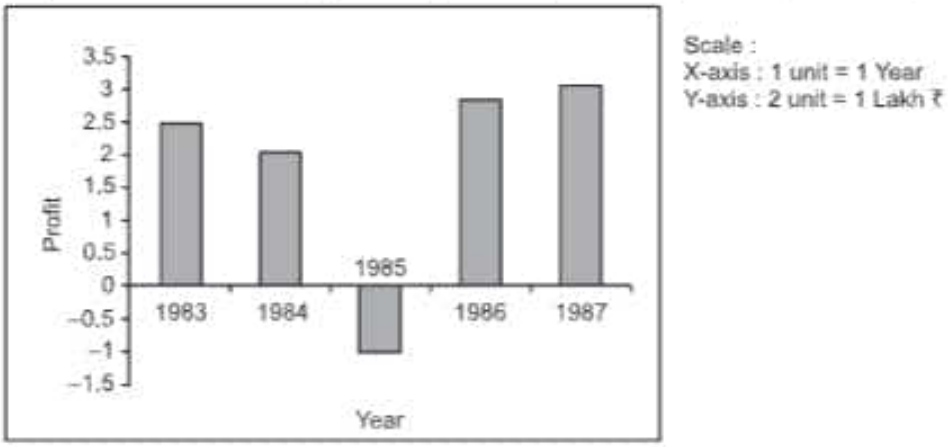


Fig. 2.10 : Bar diagram

Note :

1. Sometimes horizontal bars are used instead of vertical bars.

2. When two or more variables are involved, bar diagram cannot be used. However, in order to overcome this drawback **multiple bar diagram** can be used.

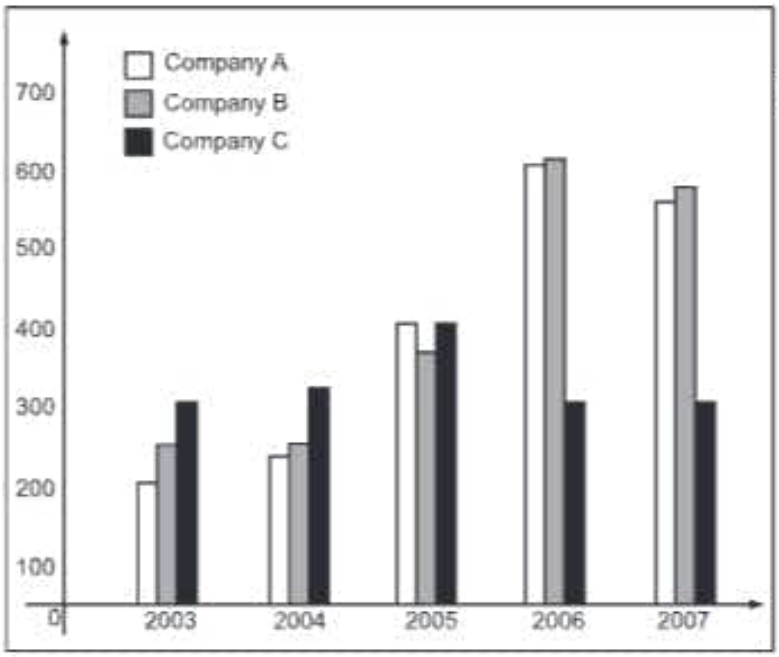
(ii) Multiple bar diagram : In case of two or more variables multiple bar diagram is used. Similarly, whenever there are two or more components associated with a variable, this type of diagram is preferred.

For example : Yearwise strength of a college can be divided into two components, girls and boys. In this diagram, with respect to each variable or component, separate bar is used. Such bars are drawn adjacent to each other for the same year or month etc. (to which change in data is related). Bars associated with different variables or components are displayed in different shades or colours. As usual the bars are of same width and height is maintained proportional to the value.

Illustration 12 : Draw a multiple bar diagram to represent the following data :

Profit of company	**Year**				
(₹ in lakhs)	**2003**	**2004**	**2005**	**2006**	**2007**
Company A	200	250	400	600	570
Company B	250	260	350	610	590
Company C	300	315	415	390	400

Solution :



Scale :
Y-axis : 1 unit = 100 Companies

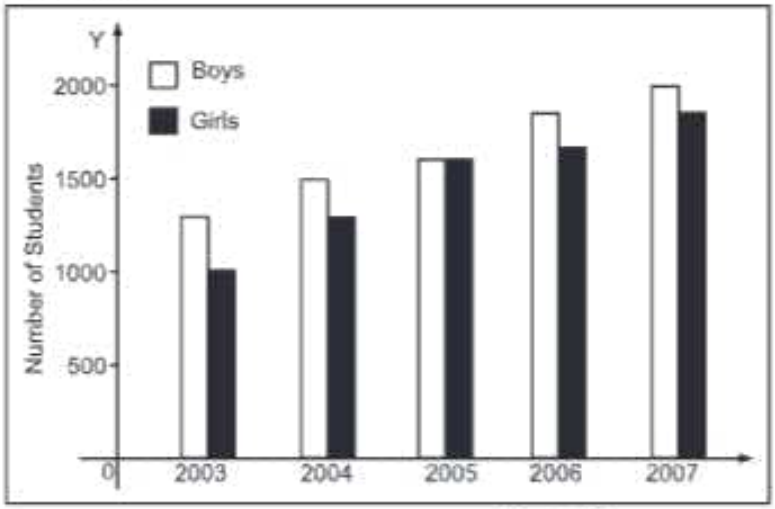
Fig. 2.11

Illustration 13 : Yearwise and sexwise strength of certain college is given below.

Year	2003	2004	2005	2006	2007
Boys	1250	1500	1600	1900	2000
Girls	1000	1300	1600	1800	1900

Represent the data by multiple bar diagram.

Solution :



Scale :
Y-axis : 2 units = 500 students

Fig. 2.12

(iii) Subdivided bar diagram : When a single variable involves two or more components, subdivided, bar diagram is used. A bar representing the total value is divided into several parts. Those parts represent the different components. The parts are chosen such that the height is proportional to the respective component. These parts are displayed in different colours or shades.

Illustration 14 : Following is a table showing faculty wise strength for 4 year :

Year	No. of Students			
	Arts	Science	Commerce	Total
1982 - 83	800	800	1400	3000
1983 - 84	750	1000	1750	3500
1984 - 85	700	1100	1800	3600
1985 - 86	900	1200	1900	4000

Represent the data by subdivided bar diagram.

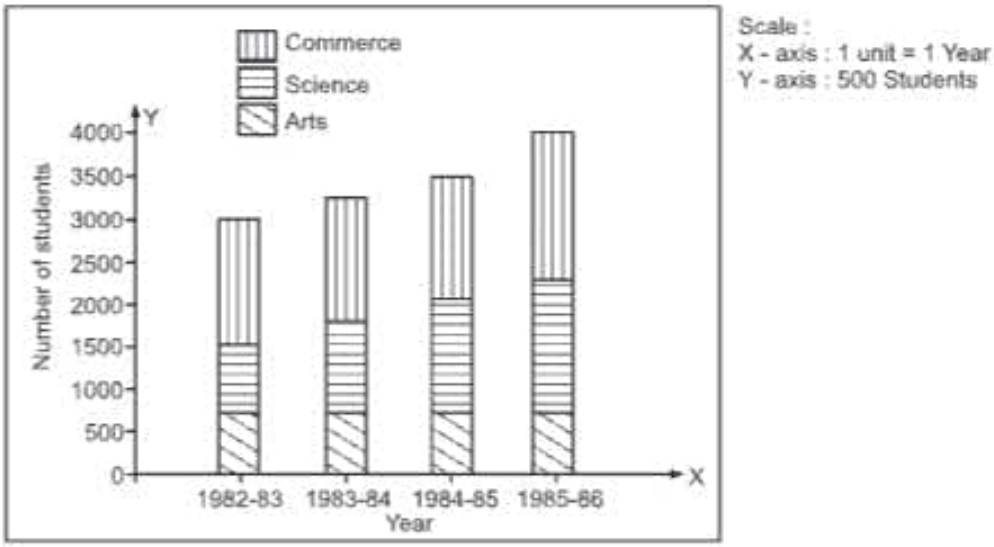


Fig. 2.13 : Sub-divided bar diagram

Illustration 15 : Present the following data using a suitable diagram.

Class	F. Y.	S. Y.	T. Y.
Pass	300	325	210
Fail	100	125	90
Total	400	450	300

Solution : In this case subdivided bar diagram is suitable because along with total components are also known.

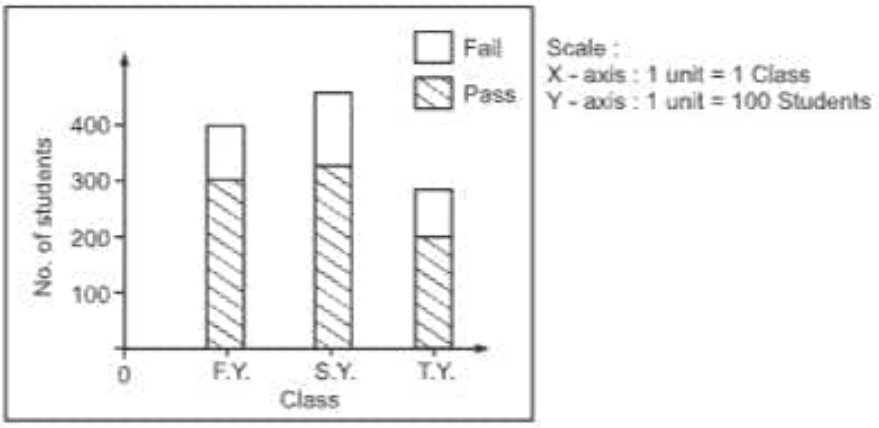


Fig. 2.14 : Sub-divided bar diagram

(iv) Pie diagram : When a variable is expressed as a sum of several components we use subdivided bar diagram. Such data can also be represented by pie diagram. In this diagram a circle is divided into several sectors as shown in illustration (by radial lines). Sectors have area proportional to the value of the component. Clearly number of sectors is same as number of components.

In order to draw pie diagram we express the data component wise in terms of percentage of total. Since 100 % corresponds to angle 360°, we take 3.6° for 1 %. Thus, we obtain angle for each sector and divide the corresponding circle into several sectors. We take angle for the sector proportional to the percentage which keeps area proportional to the value of the component.

$$\text{Percentage for component} = \frac{\text{Component magnitude}}{\text{Total magnitude}} \times 100$$

$$\text{Angle for component} = (\text{Percentage for component}) \times 3.6°$$

Illustration given below will clarify the procedure.

Illustration 16 : Draw a pie diagram to represent the following data :

Group of Item	Average monthly expenses (in ₹) of a family
Food	2400
Clothing	1400
House rent	1600
Fuel and lighting	600
Miscellaneous	2000

Solution : Here the total of expenses is ₹ 8000. We express the values of components in terms of percentage. Then we obtain angle for sector by taking product of percentage and 3.6.

Item	Percentage	Angle in degrees
Food	$\frac{2400}{8000} \times 100 = 30.0$	$30 \times 3.6 = 108°$
Clothing	$\frac{1400}{8000} \times 100 = 17.5$	$17.5 \times 3.6 = 63°$
House rent	$\frac{1600}{8000} \times 100 = 20.0$	$20 \times 3.6 = 72°$
Fuel and lighting	$\frac{600}{8000} \times 100 = 7.5$	$7.5 \times 3.6 = 27°$
Miscellaneous	$\frac{2000}{8000} \times 100 = 25.0$	$25 \times 3.6 = 90°$
Total	100.0	360°

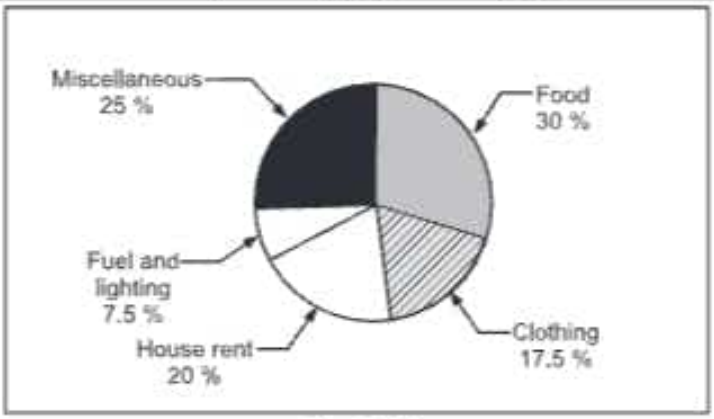


Fig. 2.15

Illustration 17 : The table below gives data relating education, obtained from census 1971 of India. Represent the data by pie diagram and percentage bar diagram.

Education level	Percentage
Illiterate	70.60 %
Literate	10.85 %
Primary	9.26 %
Non-S.S.C.	5.29 %
S.S.C.	2.23 %
Others	01.77 %

Solution : Angle in degrees obtained from percentage for different educational levels are calculated below :

Education	Illiterate	Literate	Primary	Non-S.S.C.	S.S.C.	Others	Total
Angle	$70.6 \times 3.6 =$ 254.16°	$10.85 \times 3.6 =$ 30.06°	$9.26 \times 3.6 =$ 33.34°	$5.29 \times 3.6 =$ 19.04°	$2.23 \times 3.6 =$ 8.03°	$1.77 \times 3.6 =$ 6.37°	360°

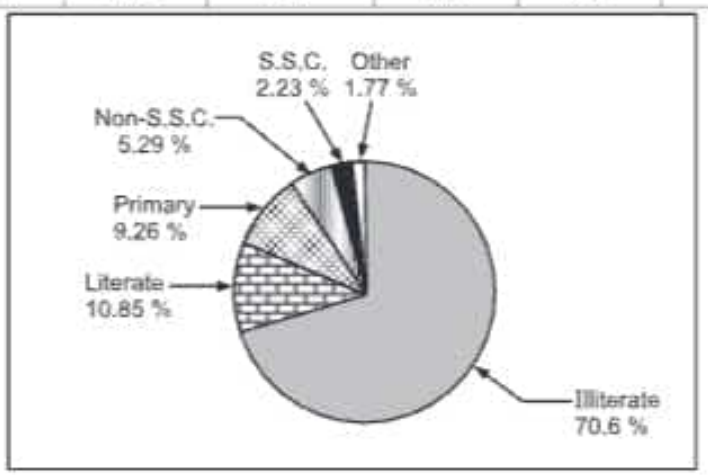


Fig. 2.16

Note :

1. In order to compare two phenomena using pie diagrams, we draw two separate pie diagrams such that their radii are taken proportional to the square root of total value.
2. If there are more components, percentage bar diagram or subdivided bar diagram does not remain effective. In such cases pie diagram is preferable.

2.10 Choice of Diagram

We give below the basis for the choice of suitable diagram.

(i) When we study the changes in totals related to a single variable, bar diagram is used.

(ii) When we study the changes in totals for several variables together, multiple bar diagram is used.

(iii) When we study the changes in the components and changes in totals, subdivided bar diagram or percentage bar diagram or pie diagram is used.

2.11 Advantages and Limitations of Graphs

Advantages :

1. Information is presented in condensed form.
2. Facts are presented in more effective and impressive manner as compared to tables.
3. Easy to understand for a layman.
4. Create effect which lasts for longer time.
5. Facilitate the comparison.
6. Help in revealing patterns.

Limitations :

1. Using graphs we find the values approximately, while, tables give exact values.
2. Graphs give only a general idea about the phenomenon, which is not sufficient for further satistical analysis.

2.12 General Rules for Construction of Graphs

Following are the general rules which should be observed while constructing diagrams.

1. Height and width of bars in histogram should be properly chosen, so that graph looks attractive.
2. A suitable scale should be chosen to occupy the available space properly.
3. Index should be provided, if essential.
4. Graphs should be neat and clean.
5. Scale should be mentioned.

Case Study : (1) The manager of a departmental store would like assign different work at different period of time to salesmen during the day. Particularly salesman required during peak hour is more, where as during slack period, how many will be made free for other work such as to main inventory, attach price bar code, packaging, sorting removing the spoiled material etc. He obtained frequency distribution of customers during every hour. He could make available proportionate and adequate number of salesmen as well he could open the additional counters. He had prepared work shedule based upon the frequency distribution of customer.

(2) The owner of the perfect shoes manufacturing company wants to prepare production schedule according to the various sizes of shoes.

He prepared the sales frequency distribution according to size of shoes. It helped him a lot to prepare the manufacturing schedule.

Solved Examples

Example 2.1 : *Find more than cumulative distribution for the following frequency distribution :*

Class	*11-15*	*16-20*	*21-25*	*26-30*	*31-35*
Frequency	*8*	*12*	*15*	*10*	*5*

Solution :

Class	11-15	16-20	21-25	26-30	31-35
More than cummulative frequency	50	42	30	15	5

Example 2.2 : *The frequency distribution of daily expenditure of 100 college students is given below :*

Daily Expenditure (₹)	*50-59*	*60-69*	*70-79*	*80-89*	*90-99*	*100-109*	*110-119*	*120-129*
Number of Students	*3*	*10*	*18*	*25*	*24*	*10*	*6*	*4*

Obtain :

(i) Class Boundaries of fourth class.

(ii) Class Width of any class.

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Obtain :

(i) Class Boundaries of fourth class.

(ii) Class Width of any class.

(iii) Modal class.

(iv) Class-mark of last class.

(v) Number of students having expenditure less than ₹ 89.

Solution :

(i) Class boundaries of 4th class = Class boundaries of (80 – 89) : (79.5 – 89.5)

(ii) Class width = Upper limit of class under consideration
– Upper limit of preceeding class
= 10

All classes are of equal width 10.

(iii) Modal class = Class with maximum frequency
= 80 – 89

(iv) Class mark of last class = Mid-point of (120 – 129)
= 124.5

(v) Number of students having expenditure
less than ₹ 89 = Less than cumulative frequency of class (80-89)
= 3 + 10 + 18 + 25 = 56

Example 2.3 : *Draw histogram for the following frequency distribution.*

Marks	*0-10*	*10-20*	*20-30*	*30-40*	*40-50*	*50-60*
Number of Students	*15*	*25*	*60*	*40*	*35*	*25*

Solution :

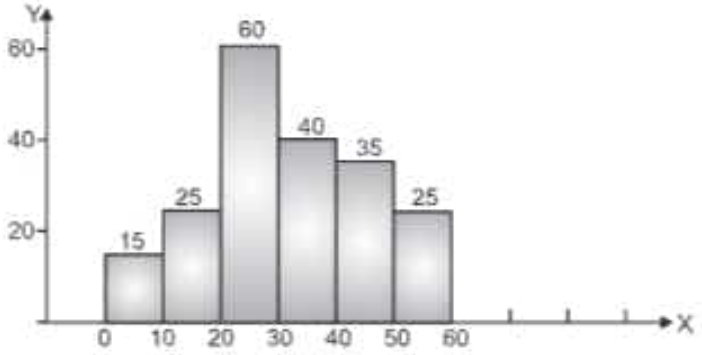


Fig. 2.17

Points to Remember

1. There are two data types : variables and attributes. The attributes are qualitative where as variables are numerical quantities.
2. Inclusive classification : classification with classes which include both the limits. Exclusive classification : classification with classes which exclude the upper limits of classes.

3. Class-mark is the mid-point of class internal.
4. Class frequency is the number of observations in a class.
5. To make the classes continuous, we obtain class boundaries.
6. Histogram gives the idea of symmetry, spread and central value of frequency distribution.
7. Relative frequency = class frequency ÷ Total frequency.
8. Class width = Upper limit of succeeding class – Upper limit of the class.

Exercise

[A] Theory Questions :

1. Explain the need of classification.
2. Explain the different methods of classification briefly.
3. Explain the following terms with illustrations :
   (i) attribute (ii) variable (iii) discrete variable (iv) continuous variable (v) raw data.
4. Explain the following terms :
   (i) class limits (ii) class boundaries (iii) class width (iv) class frequency (v) less than type cumulative frequency (vi) more than type cumulative frequency (vii) relative frequency (viii) open end class.
5. Explain the general guidelines or principles of choosing the classes.
6. What do you mean by classification ?
7. Discuss the importance of classification in statistical analysis.
8. (a) State the advantages of graphical presentation of data.
   (b) State the limitations of graphical presentation of data.
9. Explain the construction of the following graphs along with the rough sketches :
   (i) histogram (ii) frequency polygon (iii) frequency curve (iv) ogives.
10. What are the uses of histogram and ogives ?
11. (A) Discuss the importance of graphs in presentation of statistical data.
12. (B) Distinguish between :
    (a) Inclusive method of classification and exclusive method of classification.
    (b) Variable and attribute.
    (c) Raw data and classified data.
    (d) Discrete variable and continuous variable.

[B] Frequency Distribution :

12. Heights in cm of 50 students in a class are given below :

168.9	163.1	161.5	168.0	167.1	157.5	163.9	168.9
166.7	160.8	161.3	161.5	162.0	166.3	162.6	168.0
170.1	165.8	165.2	164.5	171.3	158.0	158.7	159.6
167.4	162.1	166.7	169.0	167.0	160.3	167.7	157.7
164.9	168.3	164.0	157.6	172.5	171.1	168.2	172.6
169.3	159.2	171.7	163.7	162.3	171.9	169.7	167.7
170.2	169.0						

Classify the above data by using 'exclusive method' of classification. Take the first class interval as 157–160.

13. The marks out of 100 scored by 40 students in the subject statistics are given below :

56	78	62	37	54	39	62	60	47	41
28	82	38	72	62	44	54	42	50	52
42	55	57	65	68	47	42	56	47	48
56	56	55	66	42	52	48	48	53	68

Classify the data by using 'inclusive method' of classification. Take the starting class to be 25 - 29.

14. Following is a frequency distribution of 95 shops according to daily shops in a super market on a particular day.

Daily sales (in '000 ₹)	No. of Shops
10 – 20	12
20 – 30	23
30 – 40	47
40 – 50	*
50 – 60	3
60 and above	2

(i) Find the missing frequency.

(ii) Form the less than type cumulative frequency distribution.

(iii) Is the classification exclusive ?

(iv) How many shops have sales less than or equal to ₹ 50,000 ?

(v) Obtain the more than cumulative frequency distribution.

(vi) How many shops have sales more than ₹ 40,000 ?

(vii) Is there any open end class ? If yes, state those.

(viii) Obtain the width of class and class mark of the classes for which it is possible.

15. Following is a frequency distribution of number of students according to marks scored in a certain examination.

Marks	0-19	20-39	40-59	60-79	89-99
No. of students	8	26	24	12	5

(i) State whether the classification is inclusive.

(ii) Obtain class-boundaries of each class. Are the class boundaries and limits same ?

(iii) Find width and class-mark of each class.

(iv) Obtain the less than cumulative frequency distribution, hence obtain the number of students scoring marks less than or equal to 59.

(v) Obtain the more than cumulative frequency distribution and hence find the number of students scoring marks more than or equal to 60.

16. The following is the distribution of the height of students in a class of secondary school.

Height (in cm)	Number of students
130 – 134	5
135 – 139	15
140 – 144	28
145 – 149	24
150 – 154	17
155 – 159	10
160 – 164	1

Find : (i) class-mark of 3rd class.
(ii) class width of any class.
(iii) class boundaries of 5th class.
(iv) class limits of 6th class.
(v) number of students whose height is less than 149 cm.

17. Answer the following questions for the given frequency distribution :

I.Q.	60-69	70-79	80-89	90-99	100-109	110-119	120-129
Number of students	21	37	51	49	21	13	4

(i) State the type of classification.
(ii) State the class mark of 4th class.
(iii) State the class-boundaries of 5th class.
(iv) How many students have I.Q. less than 99 ?
(v) How many students have I.Q. more than 80 ?

18. The frequency distribution of marks obtained by 100 students in F.Y.B. Com. is given below :

Marks	0-9	10-19	20-29	30-39	40-49
No. of students	10	24	30	20	16

Answer the following questions :
(i) State the type of classification.
(ii) Find the class-mark of 3rd class.
(iii) State the class boundaries of 5th class.
(iv) Find the class width of 2nd class.
(v) Find the number of students getting marks less than 30.

19. Answer the questions using following frequency distribution of age of 50 citizens : **(Oct. 2014)**

Age (years)	Below 30	31-40	41-50	51-60	61-70	Above 71
Frequency	3	7	–	16	8	2

(i) State type of classification.
(ii) Identify open end classes and state them.
(iii) Find missing frequency.
(iv) Find class-mark of fifth class.
(v) Obtain class boundaries of fourth class.

20. Following is the frequency distribution of number of students according to marks scored in a certain examination :

Marks	0-19	20-39	40-59	60-79	80-99
No. of students	8	26	24	12	5

(i) State the type of classification.
(ii) Obtain the class boundaries of the third class.
(iii) Class width of the fourth class.
(iv) Class-mark of second class.
(v) How many students getting the marks less than 79 ?

21. Answer questions using the following frequency distribution of 100 companies :

Profit (00,000) ₹	No. of companies
0-100	09
100-200	15
200-300	18
300-400	21
400-500	–
500-600	14
600-700	05

(i) State type of classification.
(ii) Find missing frequency.
(iii) Find class-mark of fifth class.
(iv) Identify median class.
(v) Find class width of third class.

22. The frequency distribution of daily expenditure of 100 college students is given below :

Daily Expenditure (₹)	50-59	60-69	70-79	80-89	90-99	100-109	110-119	120-129
Number of Students	3	10	18	25	24	10	6	4

Obtain :
(i) Class boundaries of fourth class.
(ii) Class width of any class.
(iii) Modal class.
(iv) Class-mark of last class.
(v) Number of students having expenditure less than ₹ 89.

23. The following data relate to the income of 90 persons :

Income (₹)	500-999	1000-1499	1500-1999	2000-2499
Number of Persons	15	22	45	8

Answer the following questions :
(i) Find class-mark of 3rd class.
(ii) Find class width of 2nd class.
(iii) Find number of persons having income less than ₹ 1,500.
(iv) Find percentage of persons earning more than ₹ 1,500.
(v) State the modal class.

[C] Cumulative Frequency Distribution :

24. Obtain less than cumulative frequency distribution for the following data. Also represent it graphically.

Class	100-150	150-200	200-250	250-300	300-350
Frequency	12	15	08	03	01

25. Find less than cumulative frequencies and more than cumulative frequencies for the frequency distribution given below :

Class	100-150	150-200	200-250	250-300	300-350
Frequency	12	15	30	8	2

Also draw ogive curves.

26. Find more than cumulative distribution for the following frequency distribution and represent it by suitable graph.

Class	11-15	16-20	21-25	26-30	31-35
Frequency	8	12	15	10	5

27. Can the following be a less than type cumulative frequency distribution ?

Upper limit	10	20	30	40
Less than cumulative frequency	2	18	12	50

Justify your answer.

28. Obtain the frequency distribution from the following. cumulative frequency distributions :

(a)

Marks below	Number of stuents
10	1
20	8
30	35
40	46
50	50

(b)

Age in years	No. of persons
Less than 20	15
Less than 30	35
Less than 40	72
Less than 50	108
Less than 60	120
Less than 70	124

Also find greater than cumulative frequencies.

29. Prepare the frequency distribution from the following cumulative frequency distribution :

Income more than ₹	No. of persons
500	100
1000	96
1500	92
2000	59
2500	28
3000	2

30. Convert the following less than cumulative frequency distribution to usual frequency distribution. Also find more than cumulative frequencies.

Waiting time in minutes at octri check post	No. of vehicles
less than 1	12
less than 3	58
less than 5	206
less than 8	372
less than 12	500
less than 16	520

31. Convert the frequency distribution from the following more than cumulative frequency distribution. Also obtain the less than type cumulative frequency distribution.

Height of students	No. of students
More than 145 cm.	130
More than 150 cm.	123
More than 155 cm.	111
More than 160 cm.	89
More than 165 cm.	51
More than 170 cm.	21
More than 175 cm.	0

[D] Graphical Presentation :

32. Draw the histogram, frequency polygon and ogive curves for the following frequency distribution. :

Weight in lb	80-89	90-99	100-109	110-119	120-129	130-139	140-149
Frequency	8	16	20	26	50	13	5

33. Draw a histogram for the following income distribution :

Monthly income	1000-2000	2000-2500	2500-3500	3500-5000
Frequency	120	125	180	150

34. Draw less than cumulative frequency curve for frequency distribution of intelligence quotient given below. Also obtain number of candidates having intelligence quotient between 105 and 125.

I.Q.	60-69	70-79	80-89	90-99	100-109	110-119	120-129
Frequency	21	37	51	49	21	13	4

35. Draw a frequency curve, frequency polygon and histogram for the following data :

Mid-values	25	35	45	55	65
Frequencies	5	12	33	13	7

36. Draw less than cumulative frequency curve and more than cumulative frequency curve for the following frequency distribution of marks in statistics :

Marks	0-20	20-40	40-60	60-80	80-100
No. of students	2	18	42	28	5

37. Draw histogram for the following data :

Weight (kg)	30 - 40	40 - 50	50 - 60	60 - 70	70 - 80
Number of Students	40	50	70	30	10

38. Draw histogram, frequency polygon and ogives for the following frequency distribution.

Class	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50
Frequency	4	12	18	16	3

39. From the following frequency distribution of weights of 50 students, draw less than ogive curve :

Weight (kg)	10-15	15-20	20-25	25-30	30-35
Number of Students	5	12	15	10	8

[E] Miscellaneous Problems :

40. Among a group of students, 10% scored marks below 20, 20% scored marks between 20 and 40, 35% scored marks between 40 and 60, 20% scored marks between 60 and 80, and the remaining 30 students scored marks between 80 and 100.
    (a) Using the information prepare a frequency distribution of marks of students.
    (b) If minimum 40 marks are required for passing, how many students have passed the examination ?
    (c) If maximum 60 marks are required for getting first class, how many students secured first class ?

41. Prepare a frequency distribution for each of the following :

(a) Mid-value	:	47.5	52.5	57.5	62.5	
Frequency	:	4	9	17	10	
(b) Class-mark	:	4	8	12	16	20
Frequency	:	24	45	20	10	1

42. Following is a frequency distribution of heights in cm.

Classes	150-154	55-159	160-164	165-169	170-174
Frequency	2	17	29	21	1

    (a) Obtain class boundaries of each of the classes.
    (b) Determine the class width.

43. Present the following information in a frequency distribution.
    In a branch of a certain co-operative bank, 50 % fixed deposits are less than ₹ 5000. Thirty percent fixed deposits are of the amount ₹ 5,000 to ₹ 10,000. The number of fixed deposits of amount in between ₹ 10,000 to ₹ 20,000 is 150. It is 15 % of total deposits. The remaining 5 % deposits are of amount more than ₹ 20,000.

44. Find the frequencies a, b, c, d in the following frequency distribution.

Class	0 – 10	10 – 20	20 – 30	30 – 40	Total
Frequency	a	b	c	d	100

    Given that : (i) $d = 3a$ (ii) $b : c = 7 : 3$ (iii) $c : d = 3 : 5$

F : Diagrams :

45. Represent the following data by a suitable diagram :

Year	No. of students admitted
1991	1200
1992	1500
1993	1800

46. Represent the following data by a suitable diagram.

Country	India	Shri Lanka	U. S. A.	U. K.	Mexico
Population growth rate	2.5 %	1.8 %	2.0 %	1.8 %	3.2 %

47. Using a suitable diagram represent the following data :

Year	Birth rate (per thousand)	Death rate (per thousand)
1921 - 30	46.4	36.3
1931 - 40	45.2	31.2
1941 - 50	39.9	27.4
1951 - 60	41.7	22.8
1961 - 70	41.1	18.9
1971 - 80	37.0	14.0
1981 - 90	32.5	11.4
1991 - 00	26.0	9.0

48. Draw a pie diagram and percentage bar diagram to represent the following data :

Components	Cost of construction of a house
Labour	25 %
Bricks	15 %
Cement	20 %
Steel	15 %
Timber	10 %
Supervision	15 %

49. Draw a suitable diagram to represent the following data :

Year	Exports (crores ₹)	Imports (crores ₹)
1983 - 84	430	260
1984 - 85	350	300
1985 - 86	360	290
1986 - 87	400	300

50. Draw a bar diagram and pie diagram to represent following data :

Gas	Oxygen	Nitrogen	Carbon dioxide	Others
Percentage in atmosphere	21	78	0.03	0.97

51. Draw a pie diagram and subdivided bar diagram to represent the following data :

Country	Percentage of population in the world in 1980 - 81
India	15.53
China	21.72
Russia	6.05
U.S.A.	5.04
Others	23.69

52. Represent the following data by a suitable diagram :

Census year	Urban population in India
1931	12.18 %
1941	14.10 %
1951	17.62 %
1961	18.26 %
1971	20.22 %
1981	23.73 %

53. Draw a bar diagram to represent the following data related to the capacity of production of electricity (in crores kilowatt).

Year	Total
1975 - 76	7920
1976 - 77	8850
1977 - 78	9130
1978 - 79	9790
1979 - 80	10560

54. Draw a pie diagram for the following data :

Items	Food	House rent	Clothing	Education	Saving	Miscellaneous
Expenditure	300	200	125	110	90	75

55. Draw a bar diagram for the data in problem No. 34.

56. Present the following information using suitable diagram : **(B.B.A. April 2015)**

Mode of transport	Bus	Train	Aeroplane	Private vehicle	Own vehicle	Total
No. of passengers	1250	2250	100	600	500	5000

64. The following is information regarding port traffic at Mumbai Port Trust, represent it by a suitable diagram. (Figures are in million tonnes)

Year	Total
1995 – 96	34
1996 – 97	34
1997 – 98	32
1998 – 99	31

65. Represent the following data expressing yearly values in thousand ₹ by suitable diagram.

Year	Expenditure	Income
1980	63	70
1985	84	96
1990	105	125

66. Represent the cost of per article by pie diagram and percent diagram.

Manufacturing cost : 85 %
Taxes : 8 %
Packing and transportation expenses : 7 %

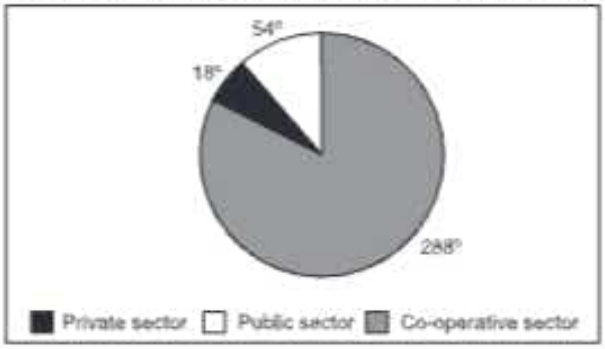
67. Represent the following information by suitable diagram.

Age group	Urban population	Rural population
0 - 5	13 %	10 %
5 - 15	25 %	22 %
15 - 35	32 %	38 %
35 - 65	20 %	20 %
above 65	10 %	10 %

68. Represent the following data by suitable diagram :

Country	Death rate per 1000 persons	Birth rate per 1000 persons
India	10.3	20.9
Pakistan	10.7	39.1
China	6.7	21.1
Sri Lanka	5.8	21.2
Japan	6.7	9.9

69. Answer the questions based on the following diagram



Sectorwise direct loans given by a nationalised bank in 1997–98.

Index

Fig. 2.18

(a) What is the type of diagram ?

(b) State the sector taking maximum loan amount.

(c) State the sector taking minimum loan amount among all the sectors.

70. Following diagram shows industrywise direct loans given by a industrial development bank, using the diagram answer the questions given below the diagram.

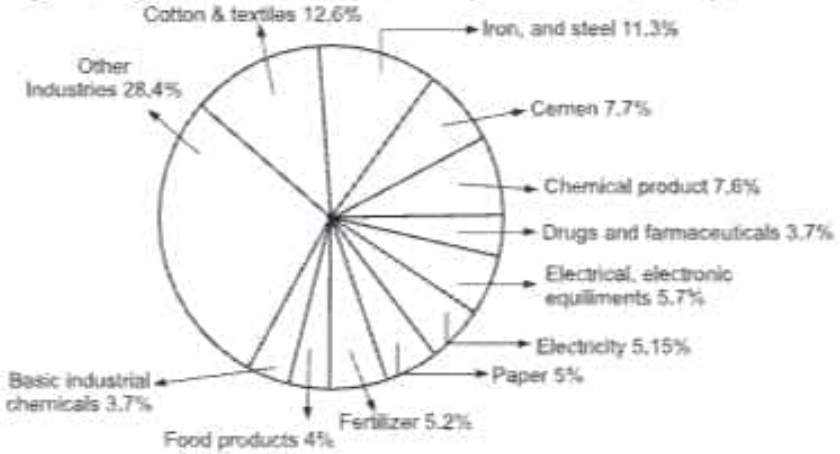


Fig. 2.19

(a) State the type of diagram.

(b) State the sector which is allotted maximum loan amount.

(c) State the industrial sectors receiving loan amount less than 5 %.

71. composition of port folio of a industrial development bank is given by the following data draw a suitable diagram.

Rupee loans : 56 %	Refinance : 6 %
Foreign currency loans : 11 %	SIDBI : 5 %
Investment in Industry : 8 %	Investment in financial institutions : 5 %
Bills finance : 7 %	Equipment leasing : 2 %

72. Marks scored by Sunil in the annual examination are given below :

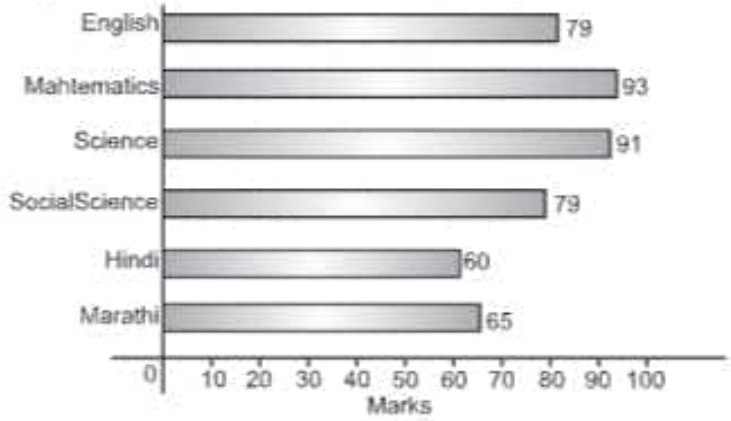


Fig. 2.20

(a) Which is the type of diagram ?
(b) State the subject in which he has scored maximum marks.
(c) State the subject in which he has scored least marks.

Answers

[B]

12.

Class	157-160	160-163	163-166	166-169	169-172	172-174
Frequency	7	9	8	14	10	2

13.

Class	25-29	30-34	35-39	30-44	45-49	50-54	55-59
Frequency	1	0	3	6	6	6	7

Class	60-64	65-69	70-74	75-79	80-84
Frequency	4	4	1	1	1

14. (i) 8
(ii) Less than cumulative frequencies 12, 35, 82, 90, 93, 95.
(iii) yes (iv) 90 (v) more than cumulative frequencies 95, 83, 60, 13, 5, 2
(vi) 13 (vii) 60 and above (viii) except the last class all have same width, which is 10
Class marks : 15, 25, 35, 45, 55, not defined.

15. (i) yes (ii) class boundaries 0 – 19.5, 19.5 – 39.5, 39.5 – 59.5, 59.5 – 79.5, 79.5 – 99.5.
Class boundaries are not same as class limits.
(iii) Class marks 9.5, 29.5, 49.5, 69.5, 89.5.
All classes have same width which is 20.
(iv) Less than cumulative frequencies 8, 34, 58, 70, 75.
No. of students having marks less than or equal to 59 is 58.
(v) More than cumulative frequencies 75, 67, 41, 17, 5.
No. of students having marks more than or equal to 60 is 17.

16. (i) 142 (ii) 5 (iii) 149.5 – 154.5 (iv) 155 – 159 (v) 72

17. (i) inclusive (ii) 94.5 (iii) 99.5 – 105.5 (iv) 158 (v) 138.

18. (i) inclusive (ii) 24.5 (iii) 39.5 – 49.5 (iv) 90 (v) 64.

19. (i) inclusive (ii) Below 30, Above 71 (iii) 14 (iv) 65.5 (v) 50.5 – 60.5.

20. (i) inclusive (i) 39.5 – 59.5 (ii) 20 (iv) 29.5 (v) 75.

21. (i) exclusive (ii) 18 (iii) 450 (iv) 300 – 400 (v) 100.

22. (i) 79.5 – 89.5 (ii) 10 (iii) 80 – 89 (iv) 56.

23. (i) 1749.5 (ii) 500 (iii) 37 (iv) 53 (iv) 1500 – 1999.

[C]

24. 12, 27, 35, 38, 39.

25. Less than cumulative frequencies : 12, 27, 57, 65, 67.

More than cumulative frequencies : 67, 55, 40, 10, 2.

26. 50, 42, 30, 15, 5.

27. No, less than cumulative frequency cannot be decreasing.

28. (a)

Class	0-10	10-20	20-30	30-40	40-50
Frequency	1	7	27	11	4

(b)

Class	01-20	20-30	30-40	40-50	50-60	60-70
Frequency	15	20	37	36	12	4

29.

Class	Frequency
500 – 1000	4
1000 – 1500	4
1500 – 2000	33
2000 – 2500	31
2500 – 3000	26
above 3000	2

30.

Class	0-1	1-3	3-5	5-8	8-12	12-16
Frequency	12	46	148	166	128	20
More than cum. frequency	520	508	462	314	148	20

31.

Class	145-150	150-155	155-160	160-165	165-170	170-175
Frequency	7	12	22	38	30	21
More than cum. frequency	7	19	41	79	109	130

[E]

40. (a)

Class	0-20	20-40	40-60	60-80	80-100
Frequency	20	40	70	40	30

(b) 140 (c) 70.

41.

(a)

Class	Frequency
45-50	4
50-55	9
55-60	17
60-65	10

(b)

Class	Frequency
2-6	24
6-10	45
10-14	20
14-18	10

42. (a) 149.5 – 154.5, 154.5 – 159.5, 159.5 – 164.5, 164.5 – 169.5, 169.5 – 174.5

(b) Classes are of same width. class width = 5.

43.

Class	Below 5000	5000-1000	10000-20000	Above 20000
Frequency	500	300	150	50

44.

Class	0-10	10-20	20-30	30-40
Frequency	10	42	18	30

45. Bar diagram
46. Bar diagram
47. Multiple bar diagram or several bar diagrams
49. Multiple Bar diagram or several bar diagrams
52. Bar diagram
56. Pie diagram
57. Multiple bar diagram or several bar diagrams
58. Multiple bar diagram or several bar diagrams
59. Bar diagram
61. Bar diagram
62. Multiple bar diagram or several bar diagrams
63. Bar diagram
64. Bar diagram.
65. Several bar diagram or multiple bar diagram
67. Bar diagram
68. Multiple bar diagram or several bar diagrams.
69. (a) Pie diagram, (b) private, (c) co-operative.
70. (a) Pie, (b) other industries, (c) chemical, food, drug.
71. Pie diagram.
72. (a) Bar diagram, (b) Mathematics, (c) Hindi.

❑❑❑

Chapter 3...

Tabulation

Contents ...

Key Words :

Tabulation, stub, caption, head note, foot note, source note, one-way table, two-way table, three way table, manifold table.

Objectives :

A tabulation as a tool of data condensation and presentation is explained in this chapter. It is a effective way of data presentation. It is similar to classification.

3.1 Introduction

In the previous chapter we have studied classification of data. Tabulation can be considered as the next operation to classification. We use in day-to-day life various types of tables for example, marklist, electricity bill, balance sheet, time-table of a class, railway time-table. Presenting the information in tabular form has many advantages. Statistical table gives an orderly arrangement of data in columns and rows. Tabulation is one of the simplest way of summarizing data. It is an important way to convey the information in a meaningful fashion. Statistical table helps in many respects like locating desired information, getting overall view of the phenomenon under study. Tabulation is found to be widely useful in several fields.

3.2 Classification and Tabulation

Definition : A statistical table is the logical listing of quantitative data in various rows and columns with self explanatory title, row headings, column headings and notes regarding source of data, context etc.

Tabulation is a classification of qualitative characteristic.

For example : Sex of 100 students is recorded, which is divided into two groups viz. boys and girls and frequency is placed against the respective group. One can easily notice that classification and tabulation serve the same purpose of presenting data in neat and compact form. Both the processes simplify the complexities and facilitate comparison. Whenever there are two qualitative characteristics, two way table is used; which is discussed later in the same chapter.

3.3 Parts of Table

We discuss below the different parts of a table.

1. **Table number :** At the top of every table, a number should be mentioned for ready reference. *For example* : Socio-economic condition of a certain city is given in table 31.
2. **Title :** A table should have suitable, brief and self-explanatory title. It should give an idea about the contents of the table, when the data are collected and to which it relates. Preferably title is placed at the top of the table in bold face type lettering.
3. **Stub :** Row titles in a table are referred as stub.
4. **Caption :** Column headings in a table are called as caption.
5. **Body :** This is the main and most important part of a table. This includes numerical information.
6. **Head note :** Usually the units of numerical data are specified in the head note. Similarly, it gives information which is not covered in the title, stub or caption. It is placed just below the title.
7. **Foot note :** It includes the information regarding numerical data such as explanation of symbols, signs, abbreviations etc. The rounding off rules those are used in the formation of a table are also mentioned here. It is placed directly below the table.
8. **Source note :** It is observed just below foot note. It gives particulars of the source data such as publication, page number, table number etc.

The following diagram is a sketch of a table showing its various parts.

Table No.

Title : .....

Head Note .....

	Caption
Stub	Body

Foot Note : .....

Source Note : .....

3.4 Objectives of Tabulation

The objectives of classification and tabulation are more or less same. We list below the objectives of tabulation :

1. It simplifies the complex data.
2. It omits unnecessary details.
3. It facilitates comparison with the other data.
4. It reveals prominent features and the patterns present within the data.
5. It helps in further analysis and interpretation.

3.5 Requisites of a Good Table

In order to make a table effective, attractive and intelligible some general guidelines are given below.

1. A table should bear a number for ready reference.
2. Each table should have a brief and self-explanatory title.
3. Stub and caption should be clear enough.
4. The use of short forms in stub and caption should be avoided except those which are very commonly used like Rs., cm., cc. etc.
5. To distinguish main classes, thick lines should be used and to distinguish sub-classes, thin lines should be used.
6. Use of dash (–) instead of zero should be avoided. No entry to be kept blank.
7. Use of ditto marks (---) should be avoided, because there is a possibility of taking it to be 11.
8. Explanation regarding signs, symbols, abbreviations, rounding off rules etc. should be mentioned in foot note.
9. Sub-totals should be obtained as and when required.
10. Units should be mentioned in head note.
11. Columns (or rows) to be compared should be placed adjacent to each other.
12. Larger figures should be abbreviated. For example, if all values are of the type 16000, 20000, ... then those could be written as 16, 20 ..... However, it should be mentioned in the foot note that the figures are in thousands.

3.6 Types of Tables

Tables are classified on the basis of number of characteristics involved in it. We discuss the following types of tables.

1. One way table : This is a simple type of table which considers single characteristics. Stub or caption is subdivided to include the group of characteristics under study.

Illustration 1 :

Table 3.1

Classwise distribution of number of students in a college ABC

Class	Number of students
F.Y. S.Y. T.Y.	
Total	

The above table uses only one characteristic viz. class. In this case stub is subdivided into three groups to include the three classes viz. F.Y., S.Y., T.Y.

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2. Two-way table : In this type of table two characteristics are considered simultaneously. Stub and caption are subdivided to include the two characteristics under consideration. One characteristic is taken in stub and the other in caption.

Illustration 2 :

Table 3.2 : Classwise and sexwise distribution of number of students in a college ABC

Sex / Class	Male	Female	Total
F.Y. S.Y. T.Y.			
Total			

The above table uses two characteristics viz. class and sex.

3. Three-way table : Such a table considers three characteristics simultaneously. In the construction of such tables an order of precedence among the characteristics should be first fixed on the basis of their importance.

Illustration 3 :

Table 3.3

Classwise, facultywise and sexwise distribution of no. of students in a college ABC.

Class	Faculty									Total		
	Arts			Science			Commerce					
	M	F	T	M	F	T	M	F	T	M	F	T
F.Y. S.Y. T.Y.												
Total												

Foot Note : M = Male, F = Female, T = Total

The above table includes three characteristics viz. class, faculty, sex.

4. Higher order or manifold table : This type of table contains more than three characteristics. However, as the number of characteristics increases, table becomes large, complicated and confusing. In such cases it is advisable to construct several two-way or three-way tables.

Example 1 : *Information obtained from a college register is described below. Represent the same in a form of neat table.*

"The number of students in a college in the year 1961 was 1100, of those 980 were boys and rest girls.

In 1971, the number of boys increased by 100% and that of girls increased by 300% as compared to their strength in 1961. In 1981 the total number of students in a college was 3600, the number of boys being double the number of girls."

From the table also determine the percent increase in

(i) the total strength

(ii) the number of girls

(iii) the number of boys in 1981 as compared to 1961.

Solution : In the problem we can see that there are two-characteristics viz. sex and year. Hence, we have to make a two-way table.

In 1961, total strength and number of boys is given. Hence the number of girls will be $1100 - 980 = 120$.

In 1971 boys are increased by 100% means, the boys increased by the same number exactly. Hence the number of boys will be 980 + 980 = 1960. The number of girls is increased by 300% means the increase will be $\frac{120}{100} \times 300 = 360$ and hence the number of girls will be 120 + 360 = 480. In the year 1981 total strength 3600 is to be divided in the ratio 2 : 1. Therefore, we get 2400 boys and 1200 girls. The table containing the above calculated entries will be as follows :

Table 3.4

Sexwise and yearwise strength of a college

Sex / Class	Boys	Girls	Total
1961	980	120	1100
1971	1960	480	2440
1981	2400	1200	3600
Total	5340	1800	7140

Source Note : Data are taken from college records

(i) Net increase in total strength for 1981 as compared to 1961

$$= 3600 - 1100 = 2500$$

$$\text{Percent increase} = \frac{2500}{1100} \times 100 = 227.27$$

(ii) Net increase in the strength of girls for 1981 as compared to 1961

$$= 1200 - 120 = 1080$$

$$\text{Percent increase} = \frac{1080}{120} \times 100 = 900$$

(iii) Net increase in the strength of boys for 1981 as compared to 1961

$$= 2400 - 980 = 1420$$

$$\text{Percent increase} = \frac{1420}{980} \times 100 = 144.90$$

Example 2 : *Out of the total number of 1807 women who were interviewed for employment in a textile mill at Ahmedabad, 512 were from textile areas and the rest from the non-textile areas. Amongst the married women who belonged to textile areas, 247 were experienced and 73 inexperienced, while for non-textile areas, the corresponding figures were 49 and 520. The total number of inexperienced women was 1341 of whom 111 resided in textile areas. Of the total number of women, 918 were unmarried and of these, the number of experienced women in the textile and non-textile areas was 154 and 16 respectively. Present the information in tabular form.*

Also obtain the percentage of experienced women, percentage of married women amongst those who were interviewed.

Solution : The above information is regarding three characteristics : experience, residential status and marital status. Hence, we prepare three-way table. From the given figures, remaining figures can be easily obtained just by addition or subtraction of related figures.

Table 3.5

Distribution of number of women according to experience, marital status and residential status

	Textile area			Non-textile area			Total		
	E	N-E	T	E	N-E	T	E	N-E	T
Married	247	73	320	49	520	569	296	593	889
Unmarried	154	38	192	16	710	726	170	748	918
Total	401	111	512	65	1230	1295	466	1341	1807

Foot Note : E = Experienced, N - E = Non-experienced, T = Total

Source Note : .....

Percentage of experienced women amongst interviewed women

$$= \frac{466}{1807} \times 100 = 25.79$$

Percentage of married women amongst interviewed women

$$= \frac{889}{1807} \times 100 = 49.20$$

Exercise

[A] Theory Questions :

1. Define the term Tabulation.
2. Explain the purpose of tabulation.
3. Explain the different parts of statistical table.
4. What are the types of statistical tables ? Explain each type with illustration.
5. What are requirements of a good table ?
6. Distinguish between classification and tabulation.
7. (a) State the objectives of tabulation.
   (b) What is tabulation ? What are the advantages of tabulation ?

[B] Numerical Problems :

8. Prepare a blank table giving the following information about workers in a certain industry.
   (a) Sex : male, female
   (b) Age group : 20-30, 30-40, 40 and above
   (c) Skill : skilled, unskilled.
9. Draw a blank table to summarise examination result of a college.
   (a) Class : F.Y. B.Com., S.Y. B.Com., T.Y. B.Com.
   (b) Examination grades : Fail, pass, second class, first class, first class with distinction.
   (c) Sex : male, female.

10. Represent the following information in the tabular form giving a suitable title.

    A supermarket divided into main sections viz. grocery, vegetables, medicines, textiles and novelties recorded the following sales in 1981, 1982 and 1983.

    In 1981, sales in grocery, vegetables, medicines and novelties were ₹ 6,25,000, ₹ 2,20,000, ₹ 1,88,000 and ₹ 94,000 respectively. Sale of textile was 30% of the total sales during the year. In 1982, the total sales showed 10% increase, while grocery and vegetables showed respectively 8% and 10% increase over their corresponding figure in 1981. Medicine sale was dropped by 13,000 while sale of textiles was ₹ 5,36,000.

    In 1983, though the total sale remained the same as in 1982, grocery fell by ₹ 22,000, vegetables by ₹ 32,000, medicine by Rs.10,000 and novelties by ₹ 12,000.

11. Represent the following data in the tabular form :

    The chairman of a group of three companies A, B and C in his annual statement, gave the following analysis of the profit for the year ending on 31st Dec. 1969, from the trading in the various parts of the world : "For company 'A', the total profit was ₹ 1,30,000 of which ₹ 1,00,000 came from U.K., ₹ 10,000 from trade with countries in Asia, ₹ 3,000 from African countries, ₹ 15,000 from countries in Europe and only ₹ 2,000 from U.S.A. As for company 'B' it made ₹ 67,000 profit from U.K., but had no trade with U.S.A., profit from countries in Asia and Africa were respectively ₹ 2,500 and ₹ 1,500 while that from European countries was ₹ 5,000 making the total profit of ₹ 76,000. Finally, company 'C' made the lowest total profit of ₹ 52,800 of which ₹ 40,000 was made in U.K., profit from countries in Asia was ₹ 5,700 compared with ₹ 2,100 from American countries and ₹ 5000 from Europe.

12. Production of wheat during 1972-73 in a certain state was 1.02 million tonnes. It was considerably increased in the next year. In 1973-74 it was increased by 0.172 million tonnes. The production of rice and jowar was increased by 0.02 million tonnes and 0.123 million tonnes respectively in 1973-74. The production of pulses was 0.122 million tonnes in 1973-74, which was less by 32 thousand tonnes than the previous year. The production of jowar was 1.79 million tonnes and that of rice was 0.83 million tonnes in 1973-74.

    Tabulate the information given above.

13. Prepare a complete table from the following information :

    "In the year 1980 the total strength of students of three colleges X, Y, Z in a city were in the ratio 4 : 2 : 5. The strength of the college Y was 1000. The proportion of girls and boys in all colleges was in the ratio 2 : 3. The facultywise distribution of boys and girls in Arts, Science and Commerce was in the ratio 1 : 2 : 2 in all the three colleges."

14. Present the following information in a tabular from. Obtain the quantities which are not directly supplied.

    In the annual report of a XYZ Oil Company it is stated that the company drilled in all 882 and 487 wells during the year 1987 and 1988 respectively. Company constructed two types of drilling machines viz. wild cat and developmental. During the year 1987 total of wild cat wells and a total of developmental wells were 40 and 842 respectively; the corresponding figures in 1988 were 46 and 441. Wells were further divided into three categories viz. oil, gas and dry hole.

    Among the wild cat wells drilled in 1987, 6 resulted in oil, 4 in gas and 30 in dry holes, whereas the corresponding figures in 1988 were 6, 4 and 36. Out of the developmental wells drilled in 1987, 660 resulted in oil, 77 in gas and 105 in dry holes, the comparable figures for 1988 were 300, 77 and 64.

15. The total number of accidents in Southern Railway in 1960 was 3500 and it decreased by 300 in 1961 and by 700 in 1962. The total number of accidents in meter gauge section showed an increase from 1960 to 1962. It was 284 in 1960, 346 in 1961 and 428 in 1962. In meter gauge section 'Not compensated' cases were 49 in 1960, 77 in 1961 and 108 in 1962. 'Compensated' cases in broad gauge section were 2867, 2587 and 2152 in these three years respectively.

    Prepare a neat table from the above report.

16. Total strength of a certain college is 2000. Exactly 60% of this belong to rural areas and the rest from urban areas. The total strength of Arts faculty and Science faculty are in the ratio 4 : 1. The total number of students of Arts faculty residing in rural areas is 1000, while that of Science is 200. There are 900 male students of Arts faculty who stay in rural areas and 500 male students staying in urban areas, the corresponding figures from Science faculty are 175 and 125.

    Present the information in a suitable tabular form. Obtain figures which are not directly provided.

17. Present the following information in a tabular form, by computing the figures which are not directly given.

    Exactly 20% of the number of students in a university of strength 20,000 are ladies, 33 out of every 40 students are Maharashtrian, 13 out of every 16 gents are Maharashtrian, 40% other than Maharashtrain gents and 55% of Maharashtrian gents have taken Arts subjects, whereas 40% of ladies from Maharashtra and equal percentage of ladies from other states have taken Science subjects.

18. Prepare a neat table and present the following information.

    In 1984, exports and imports of a certain country in crores of ₹ were 320 and 250 respectively. In 1985 export was increased by 20 crores and import by 4%. In 1986, export did not change in its value. However, import decreased by 20. In 1987, export was further decreased by 30 crores and import decreased by 15%.

19. Present the following information in a tabular form.

Company	Data
A	Prices increased, demand decreased
B	Prices increased, demand not changed
C	Prices decreased, demand decreased
D	Prices decreased, demand decreased
E	Prices not changed, demand decreased
F	Prices increased, demand not changed
G	Prices decreased, demand increased
H	Prices decreased, demand increased
I	Prices increased, demand increased
J	Prices not changed, demand decreased
K	Prices doubled, demand decreased
L	Prices increased, demand decreased
M	Prices not changed, demand not changed
N	Prices increased, demand decreased
O	Prices decreased, demand decreased
P	Prices increased, demand decreased
Q	Prices increased, demand increased
R	Prices increased, demand decreased
S	Prices increased, demand decreased
T	Prices increased, demand decreased

20. Present the following information in a tabular form. In a college there are 60% boys. In an examination 70% students passed. Among the boys 360 passed, which is 75% of the boys.

21. Present the following information in a tabular form by computing the figures which are not directly given.

    In a certain interview, there were 150 candidates of which 56% were males. 36 candidates were successful in the interview. The proportion of males to females in the successful candidates is 5 : 4.

22. Present the following information in a tabular form determining the figures which are not given.

    Out of 800 employees appeared for a promotion test, 320 were married. Among 240 who were unsuccessful, 96 were married.

23. Present the following information in a table after computing figures those are not given.

    A morbidity survey revealed the following information. Out of 240 persons exposed to small-pox, 112 were attacked. Out of 240 persons, 152 had been vaccinated and of those only 48 were attacked. **(April 2006)**

24. Prepare a statistical table using for the following information. Also find the figures which are not given directly.

Employed graduates	= 286
Unemployed graduates	= 48
Employed undergraduates	= 450
Unemployed undergraduates	= 216

25. Present the following data in a statistical table by computing the figures which are not directly supplied.

No. of fathers with dark eyes and sons with dark eyes = 50

No. of fathers without dark eyes and sons with dark eyes = 90

No. of fathers with dark eyes and sons without dark eyes = 80

No. of fathers without dark eyes and sons without dark eyes = 780.

26. A social worker conducted a survey which revealed the information as follows.

In 1980 the number of readers in literature, fiction and other type of books in a city library were 10000, 50000 and 10000 respectively.

In 1990 the corresponding figures were 12000, 52000 and 9000.

Present the above data in a tabular form.

27. A survey on musical entertainments gave the following results.

Total number of citizens interviewed was 6000, of which 40 % were females.

Among males 5 % liked classical music, 10 % liked light music, 8 % liked western music and the remaining preferred film songs. The corresponding figures for females were 10 %, 12 %, 6 % respectively.

Present the above information in a suitable table.

28. A locality was divided in three areas : administrative, main city and suburbs. A survey of housing conditions gave the following information.

There were 70,00,000 building of which 20,00,000 were in suburbs and 1,50,000 were in administrative area. In main city, 80 % buildings were inhabitated and remaining were under construction. The corresponding figures for suburbs were 75 % and 25% respectively. In administrative area, 4000 buildings were under construction.

Present the following information in a statistical table by computing the figures which are not directly given.

29. A census report gave the following data regarding a certain locality.

Among 35 crores of population, 24 crores persons were belonged to agricultural category. Among the agricultural category 7 crores were self supporting whereas 3 crores were earning dependents and remaining were non-earning. In non- agricultural category the number of self supporting and non-earning dependents were respectively 3.5 crores and 6 crores. The rest of the persons were earning dependents.

Present the above information in a suitable statistical table by computing the figures which are not directly supplied.

24. Prepare a statistical table using for the following information. Also find the figures which are not given directly.

Employed graduates	= 286
Unemployed graduates	= 48
Employed undergraduates	= 450
Unemployed undergraduates	= 216

25. Present the following data in a statistical table by computing the figures which are not directly supplied.

No. of fathers with dark eyes and sons with dark eyes = 50

No. of fathers without dark eyes and sons with dark eyes = 90

No. of fathers with dark eyes and sons without dark eyes = 80

No. of fathers without dark eyes and sons without dark eyes = 780.

26. A social worker conducted a survey which revealed the information as follows.

In 1980 the number of readers in literature, fiction and other type of books in a city library were 10000, 50000 and 10000 respectively.

In 1990 the corresponding figures were 12000, 52000 and 9000.

Present the above data in a tabular form.

27. A survey on musical entertainments gave the following results.

Total number of citizens interviewed was 6000, of which 40 % were females.

Among males 5 % liked classical music, 10 % liked light music, 8 % liked western music and the remaining preferred film songs. The corresponding figures for females were 10 %, 12 %, 6 % respectively.

Present the above information in a suitable table.

28. A locality was divided in three areas : administrative, main city and suburbs. A survey of housing conditions gave the following information.

There were 70,00,000 building of which 20,00,000 were in suburbs and 1,50,000 were in administrative area. In main city, 80 % buildings were inhabitated and remaining were under construction. The corresponding figures for suburbs were 75 % and 25% respectively. In administrative area, 4000 buildings were under construction.

Present the following information in a statistical table by computing the figures which are not directly given.

29. A census report gave the following data regarding a certain locality.

Among 35 crores of population, 24 crores persons were belonged to agricultural category. Among the agricultural category 7 crores were self supporting whereas 3 crores were earning dependents and remaining were non-earning. In non- agricultural category the number of self supporting and non-earning dependents were respectively 3.5 crores and 6 crores. The rest of the persons were earning dependents.

Present the above information in a suitable statistical table by computing the figures which are not directly supplied.

30. A manufacturing company found the scraps purchased, was of the following types.
Scrap A contains 65 % aluminium, 20 % iron, 2 % copper, 2 % manganese, 3 % magnesium and 8 % silicon.
Scrap B contains aluminium, iron, copper, manganese, and magnesium respectively 70 %, 15 %, 3 %, 2 % and 4 %.
Compute the figures which are not directly given and present the information in tabular form.

31. Complete the following table showing data related to examination result.

Year / Class	F.Y.	S.Y.	T.Y.	Total
First class	148	82	–	–
Second class	192	95	38	–
Pass class	–	108	–	210
Fail	20	–	90	150
Total	–	325	275	1000

32. Complete the following table by finding values of a, b, c, d, e, f, g, h regarding the number of passengers travelling from city A to B.

Mode of transport	Male	Female	Total
S.T. Bus	800	a	1250
Train	b	750	2250
Aeroplane	4 c	c	d
Private vehicle	e	100	6 d
Own vehicle	350	150	f
Total	g	h	5000

33. In a sample study about tea drinking habits in towns A and B following data were obtained :

Town A	Town B
52% of the people were male	50% of the people were males
65% of the people were tea drinkers	75% of the people were tea drinkers
40% of the people were male tea drinkers	42% of the people were male tea drinkers

Answers

[B] Note : Draw a neat table for each problem as shown in illustration. We provide only numerical figures in the answer.

10.

	Grocery	Vegetables	Medicine	Textile	Novelty
1981	625	220	188	483	64
1982	675	242	175	536	143
1983	653	210	165	612	131

Foot note : Figures are in thousand ₹.

11.

	A	B	C
U.K.	100	6.7	40
Asia	10	2.5	5.7
Africa	3	1.5	2.1
Europe	15	5	5
U.S.A	2	0	0

Foot note : Figures are in thousand ₹.

12.

	Wheat	Rice	Jowar	Pulses
1972 - 73	1.020	0.81	1.667	0.1188
1973 -74	1.192	0.83	1.790	0.1220

Foot note : Figures are in million tonnes.

13.

	X		Y		Z	
	Boys	Girls	Boys	Girls	Boys	Girls
Arts	240	160	120	80	300	200
Science	480	320	240	160	600	400
Commerce	480	320	240	160	600	400

14.

	Wild cat			Development		
	Oil	Gas	Dry	Oil	Gas	Dry
1987	6	4	30	660	77	105
1988	6	4	36	300	77	64

15.

	1960		1961		1962	
	BG	MG	BG	MG	BG	MG
Compensated	2867	235	2587	269	2152	320
Non-compensated	349	235	267	77	220	108

Foot Note : BG = Broad gauge, MG = Meter gauge

16.

	Arts		Science	
	Male	Female	Male	Female
Rural	900	100	175	25
Urban	500	100	125	75

17.

	Arts		Science	
	Gents	Ladies	Gents	Ladies
Maharashtrian	7150	2100	5850	1400
Non-Maharashtrain	1200	300	1800	200

18.

	Export	Import
1984	320	250
1985	340	260
1986	340	240
1987	310	204

Note : Figures are in crore ₹.

19.

	Prices		
Demand	**Decreased**	**Not changed**	**Increased**
Decreased	3	2	8
Not changed	0	1	2
Increased	2	0	2

20.

	Passed	Failed
Boys	360	120
Girls	200	120

21.

	Successful	Unsuccessful
Male	20	64
Female	16	50

22.

	Successful	Unsuccessful
Married	224	96
Unmarried	336	144

23.

	Vaccinated	Non-vaccinated
Attacked	48	64
Not-attacked	104	24

24.

	Graduates	Undergraduates
Employed	286	450
Unemployed	48	216

25.

Father / Son	Dark eyes	Without dark eyes
Dark eyes	50	90
Without dark eyes	80	780

26.

Year	Literature	Fiction	Other
1980	10	59	10
1990	12	52	9

Note : Figures are in thousands.

27.

	Classical	Ligth	Western	Film song
Female	240	288	144	1728
Male	180	360	288	2772

28.

	Administrative	Main city	Suburbs
Inhabited	146	3880	1500
Other	4	970	500

Note : Figures are in thousands.

29.

	Self supporting	Earning dependents	Non-earning
Agricultural	7.0	3.0	14.0
Non-agricultural	3.5	1.5	6.0

Note : Figures are in crores.

30.

Scrap	Aluminium	Iron	Copper	Manganese	Magnesium	Silicon
A	65	20	2	2	3	8
B	70	15	3	2	4	6

31.

	F.Y.	S.Y.	T.Y.	Total
First class	148	82	85	315
Second class	192	95	38	325
Pass class	40	108	62	210
Fail	20	40	90	150
Total	400	325	275	1000

32.

	Male	Female	Total
S.T. Bus	800	450	1250
Train	1800	750	2250
Aeroplane	80	20	100
Private vehicle	500	100	600
Own vehicle	350	150	500
Total	3530	1470	5000

❐❐❐

Chapter 4...

Measures of Central Tendency

Contents ...

Key Words :

Central Tendency, Average, Arithmetic Mean, Deviation, Combined Mean, Median, Deciles, Percentiles, Box Plot, Cumulative Frequency, Mode, Empirical Relation.

Objectives :

Averages are tools of summarizing data, finding representative. It also facilitates the comparison. The methods of determining averages are illustrated in this chapter. The third and fourth aspects of statistics are analysis and interpretation. Averages help in both analysis and interpretation.

4.1 Introduction

We have studied in the previous chapters the various methods of summarizing data and its graphical representation. However it becomes essential to condense the data into a single value. Such a single value is treated as a representative of data and it is referred to an **average** or **central value** or measure of **central tendency.** It is desired that all the important properties of the observations in the data should be represented in the average. The word average is very commonly used in day-to-day life,

For example : Average marks, average profit, average run-rate of a team in one day. A single value is suitable for comparison. Therefore, average is essential quantity. Average is a value around which most of the observations are clustered, hence this single value itself gives clear idea regarding phenomenon under study.

There are several types of averages used in practice according to the type of data and purpose. In this chapter we study three important averages viz. mean, median and mode.

4.2 Objectives or Requisites of Ideal Average

The following are the objectives of average :

1. To obtain a single representative quantity for the entire data.
2. To facilitate comparison.

There are several averages in use, hence it is necessary to discuss the requisites of good or ideal average. **The following are requisites of good average :** **(April 2015)**

1. It should be simple to understand and easy to calculate.
2. It should be rigidly defined.
4. It should be based on all observations in the data.
4. It should be capable of further mathematical treatment.
5. It should be least affected by extreme observations.

4.3 Arithmetic Mean (A.M.)

This is very commonly used and widely applicable average.

Definition : Arithmetic mean (A.M.) or mean is a sum of observations divided by number of observations i.e.

$$\text{A.M.} = \frac{\text{Sum of the observations}}{\text{Number of observations}}$$

According to the different types of data calculation of A.M. differs slightly. We consider these cases as given below :

Case (i) Individual Observations or Ungrouped Data :

Suppose $x_1, x_2, \ldots, x_n$ is a set of n observations by definition, arithmetic mean will be

$$\text{A.M.} = \frac{x_1 + x_2 + \ldots + x_n}{n} \qquad \ldots (4.1)$$

Numerator of right side of (4.1) can be symbolically written as $\sum x$ i.e. $x_1 + x_2 + \ldots + x_n$.

Symbol $\sum$ (sigma) represents the sum. Further it is a customary to denote A.M. by $\bar{x}$. Hence

$$\text{A.M.} = \bar{x} = \frac{\sum x}{n}$$

Case (ii) Discrete Frequency Distribution :

Suppose $x_1, x_2, \ldots, x_n$ are values with $f_1, f_2, \ldots, f_n$ as the corresponding frequencies. Clearly to find the sum of observations we need to add observation x_1, f_1 times, observation

x_2, f_2 times and so on. Hence sum of observations will be $f_1x_1 + f_2x_2 + \ldots + f_nx_n$ and total number of observations will be $f_1 + f_2 + \ldots + f_n$. Hence,

$$\bar{x} = \frac{f_1x_1 + f_2x_2 + \ldots + f_nx_n}{f_1 + f_2 + \ldots + f_n}$$

Using $\sum$ notation we get

$$\bar{x} = \frac{\sum fx}{\sum f}$$

Case (iii) Continuous Frequency Distribution :

In this case, frequency is associated to the entire class and not to any specific single value. This creates difficulty in choosing $x_1, x_2, \ldots, x_n$.

For calculation purpose we make a reasonable assumption that the frequency is associated with mid-point of class or equivalently the frequency is distributed over the respective class uniformly. Thus, taking $x_1, x_2, \ldots, x_n$ as the mid-values of class intervals we calculate mean by the same formula discussed in case (ii), i.e.

$$\bar{x} = \frac{\sum f \cdot x}{\sum f} = \frac{\sum f \cdot x}{N}$$

Illustration 1 : *Calculate the arithmetic mean of marks scored by a student in 7 subjects given below : 61, 68, 69, 63, 70, 60, 78.*

Solution :

$$\bar{x} = \frac{\text{Total marks scored}}{\text{Number of subjects}}$$

$$\bar{x} = \frac{61 + 68 + 69 + 63 + 70 + 60 + 78}{7} = \frac{469}{7} = 67$$

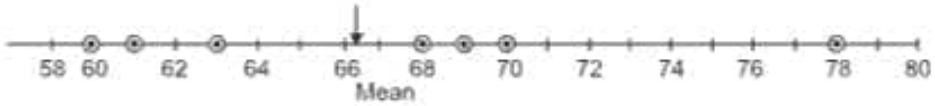

Fig. 4.1

It can be noticed in the above illustration that the observations are nearer to 60, so for convenience we assume the mean to be 60 and obtain the sum of excess of marks. It will be 1 + 8 + 9 + 3 + 10 + 10 + 18 = 49. We find the average of excess and add in the assumed mean. Thus mean will be $60 + \frac{49}{7} = 67$.

The above discussion leads to a short-cut method of finding arithmetic mean.

Short-cut Method or Derivation Method or Assumed Mean Method :

This method reduces the calculations involved in finding mean. Following are the steps in the computational procedure of mean.

(1) Decide a suitable figure 'a' which is referred as assumed mean.

(2) Subtract 'a' from each observation, the difference so calculated is called deviation from 'a', we denote deviation by 'd'.

(3) Find sum of deviations

$\sum d$ in case of individual observations.

$\sum fd$ in case of frequency distribution.

(4) Use the following formula and find the mean :

$$\bar{x} = a + \frac{\sum d}{n}$$ in case of individual observations

and

$$\bar{x} = a + \frac{\sum fd}{N}$$ in case of frequency distribution.

Illustration 2 : *Calculate arithmetic mean for the following frequency distribution :*

Observation (x)	*103*	*110*	*112*	*118*	*95*
Frequency (f)	*4*	*6*	*10*	*12*	*3*

Solution : We solve the problem by both the methods.

1. Direct method :

x	f	f.x
103	4	$103 \times 4 = 412$
110	6	$110 \times 6 = 660$
112	10	$112 \times 10 = 1120$
118	12	$118 \times 12 = 1416$
95	3	$95 \times 3 = 285$
Total	**N = 35**	**$\sum fx = 3893$**

$$\therefore \quad \bar{x} = \frac{\sum fx}{\sum f} = \frac{3893}{35} = 111.2286$$

2. Deviation method :

Taking assumed mean a = 100, we prepare the following table and use deviation method.

x	**Deviations $d = x - a$ $d = x - 100$**	f	f·d
103	3	4	12
110	10	6	60
112	12	10	120
118	18	12	216
95	– 5	3	– 15
	Total	**N = 35**	**$\sum fd = 393$**

Thus $\bar{x} = a + \frac{\sum fd}{N} = 100 + \frac{393}{35} = 100 + 11.2286 = 111.2286$

Step-deviation method : We have seen that deviation method reduces the calculations when the observations are large in magnitude. Sometimes the observations or deviations are multiples of some number. Especially when we deal with frequency distribution of continuous variables, deviations are found to be multiple of class width. In this situation step-deviation method is advisable.

Steps in the computational procedure are given below :

(1) Decide a suitable figure 'a'. (assumed mean a).

(2) Subtract 'a' from each observation and find deviation d (or class-mark) i.e.

$$d = x - a.$$

(3) Divide d, obtained in (2) by convenient figure 'h' (or by class width).

This figure is called as step-deviation.

i.e. $d' = \frac{d}{h}$

(4) Find sum of step deviations

$\sum d'$ in case of individual observations

$\sum fd'$ in case of frequency distribution.

(5) Use the following formula to find the mean

$$\bar{x} = a + \left(\frac{\sum d'}{n} \times h\right)$$ in case of individual observations

and

$$\bar{x} = a + \left(\frac{\sum fd'}{N} \times h\right)$$ in case of frequency distribution.

Illustration 3 : The following is a distribution of monthly salaries of the employees of a firm.

Salaries in ₹	No. of employees
0 – 500	2
500 – 1000	8
1000 – 1500	12
1500 – 2000	23
2000 – 2500	25
2500 – 3000	20
3000 – 3500	9
3500 – 4000	1

Compute arithmetic mean of salaries.

Solution : We use step-deviation method to find the mean.

Class	Mid-values	$d = x - 1750$	$d' = \frac{d}{500}$	f	fd'
0 – 500	250	– 1500	– 3	2	– 6
500 – 1000	750	– 1000	– 2	8	– 16
1000 – 1500	1250	– 500	– 1	12	– 12
1500 – 2000	1750	0	0	23	0
2000 – 2500	2250	500	1	25	25
2500 – 3000	2750	1000	2	20	40
3000 – 3500	3250	1500	3	9	27
3500 – 4000	3750	2000	4	1	4
Total	–	–	–	100	62

$$\bar{x} = a + \left(\frac{\sum fd'}{N} \times h\right)$$

Note that $a = 1750$, $\sum fd' = 62$, $N = 100$ and $h = 500$.

Hence, $$\bar{x} = 1750 + \frac{62}{100} \times 500$$

$$\therefore \quad \bar{x} = 1750 + 310 = 2060$$

Thus average salary is ₹ 2,060.

Effect of change of origin and Scale on Arithmetic mean :

Change of origin means to add or to subtract a constant from each observation. Thus, if the original variable is denoted by x than x – a or x + a is a variable obtained by shifting the origin (where a is a constant). The new variable x – a is also referred as deviation. In this situation arithmetic mean need not be obtained again however from the earlier mean we can determine the mean after the change of origin.

(1) If $y = x - a$ then $\bar{y} = \bar{x} - a$.

(2) If $y = x + a$ then $\bar{y} = \bar{x} + a$.

Similarly, changing of scale means to multiply or to divide the observations by a constant. Thus, if x is the variable $\frac{x}{c}$ or cx is a variable obtained by changing the scale, c being constant. In this case also we need not find the arithmetic mean once again due to change in scale. The change of scale is similar to step deviation. However the same relation is observed between old variable and the variable after changing the scale. We summarize the rules below :

(3) If $y = \frac{x}{c}$ then $\bar{y} = \frac{\bar{x}}{c}$.

(4) If $y = cx$ then $\bar{y} = c\bar{x}$.

(5) If $y = ax + b$ then $\bar{y} = a\bar{x} + b$.

(6) If $y = \frac{x-a}{c}$ then $\bar{y} = \frac{\bar{x}-a}{c}$.

Illustration 4 : Suppose the arithmetic mean of 50 observations is 120. Find the arithmetic mean if each observation is

(i) increased by 10

(ii) decreased by 5

(iii) doubled

(iv) reduced to one third

(v) doubled and then increased by 5

(vi) increased by 5 and then doubled.

Solution : This illustration explains the change of origin and scale (or linear transformations). Let x = Original variable = y = New variable.

(i) $y = x + 10$, $\bar{y} = \bar{x} + 10 = 120 + 10 = 130$

(ii) $y = x - 5$, $\bar{y} = \bar{x} - 5 = 120 - 5 = 115$

(iii) $y = 2x$, $\bar{y} = 2\bar{x} = 2 \times 120 = 240$

(iv) $y = \frac{x}{3}$, $\bar{y} = \frac{\bar{x}}{3} = \frac{120}{3} = 40$

(v) $y = 2x + 5$, $\bar{y} = 2\bar{x} + 5 = 2 \times 120 + 5 = 245$

(vi) $y = 2(x + 5)$, $\bar{y} = 2(\bar{x} + 5) = 2(120 + 5) = 250$.

4.4 Merits and Demerits of Arithmetic Mean

Arithmetic mean possesses most of the requisites of good average. Hence it is widely used. We state below its merits and demerits :

Merits : **(April 2015)**

1. It is easy to calculate and simple to follow.
2. It is based on all observations.
4. It is rigidly defined.
4. It possesses sampling stability.
5. It is capable of further mathematical treatment. Given the means and sizes of two or more groups we can find mean of combined group. We can find the total given the mean and number of observations.

Demerits : **(April 2015)**

1. It is applicable only for quantitative data.
2. It is unduly affected by extreme observations.
3. It cannot be computed for frequency distribution with open end class.
4. It cannot be determined graphically.
5. Sometimes arithmetic mean may not be an observation in a data.

   For example, arithmetic mean of number of T.V. sets sold daily is 5.25.

4.5 Mean of Combined Groups

Many times it is required to compute mean of two groups combined together. If means and sizes of groups are known we can determine the combined mean i.e. mean of combined group.

Let $\bar{x}_1$ be the arithmetic mean of first group of size n_1. Similarly $\bar{x}_2$ be mean of second group of size n_2, then the combined mean is derived as follows :

$$\bar{x}_1 = \frac{\text{(Sum of observations in first group)}}{n_1}$$

hence, $n_1 \bar{x}_1 =$ Sum of observations in first group

Similarly $n_2 \bar{x}_2 =$ Sum of observations in second group.

Thus, the combined mean $\bar{x}_c$ is

$$\bar{x}_c = \frac{\left(\begin{matrix}\text{Sum of the observations in} \\ \text{first group}\end{matrix}\right) + \left(\begin{matrix}\text{Sum of the observations} \\ \text{in second group}\end{matrix}\right)}{\text{(Size of first group) + (Size of second group)}}$$

$$\bar{x}_c = \frac{n_1 \bar{x}_1 + n_2 \bar{x}_2}{n_1 + n_2}$$

Illustration 5 : *Arithmetic mean of weight of 100 boys is 50 kg and the arithmetic mean of 50 girls is 45 kg. Calculate the arithmetic mean of combined group of boys and girls.*

Solution : Let $\bar{x}_1$ and n_1 be the mean and size of group of boys and $\bar{x}_2$ and n_2 be the mean and size of group of girls. So that $n_1 = 100$, $\bar{x}_1 = 50$, $n_2 = 50$, $\bar{x}_2 = 45$. Hence, combined mean is

$$\bar{x}_c = \frac{n_1 \bar{x}_1 + n_2 \bar{x}_2}{n_1 + n_2} = \frac{(100 \times 50) + (50 \times 45)}{100 + 50}$$

$$= \frac{7250}{150} = 48.3333$$

Illustration 6 : *The mean weekly salary paid to 300 employees of a firm is ₹ 1,470 There are 200 male employees and the remaining are females. If mean salary of males is ₹ 1,505 Obtain the mean salary of females.*

Solution : Suppose $\bar{x}_1$ and n_1 are mean and group size of males. $\bar{x}_2$ and n_2 are mean and size of group of females, $\bar{x}_c$ is mean of all the employees considered together.

Now, $$\bar{x}_c = \frac{n_1 \bar{x}_1 + n_2 \bar{x}_2}{n_1 + n_2}$$

$\therefore$ $$1470 = \frac{(200 \times 1505) + (100 \times \bar{x}_2)}{200 + 100}$$

$\therefore$ $$1470 = \frac{301000 + 100\, x_2}{300}$$

$\therefore$ $$441000 = 301000 + 100\ \bar{x}_2$$

$\therefore$ $$4410 = 3010 + \bar{x}_2$$

$\therefore$ $$\bar{x}_2 = ₹\ 1.400$$

4.6 Median

We have seen that arithmetic mean cannot be calculated for qualitative observations like beauty, debating skill, honesty, blindness. Moreover if a frequency distribution includes open end class, mean does not exist and it is unduly affected by extreme observations. In order to overcome these drawbacks, other measures of central tendency, median or mode are used.

Illustration : The arithmetic mean of 38, 43, 41, 39, 52, 48, 60, 167 is 61. This cannot be the representative value of the data, because among 8 observations, 7 are smaller than arithmetic mean. Thus incase extreme observations are widely separated from most of the observations, arithmetic mean does not remain suitable, whereas median is suitable.

Definition : Median is the value of middle most observation in the data when the observations are arranged in increasing (or decreasing) order of their values.

Thus, median is the central observation. It divides the data into two equal parts. There are equal number of observation above as well as below the median. It is also called as **positional average**.

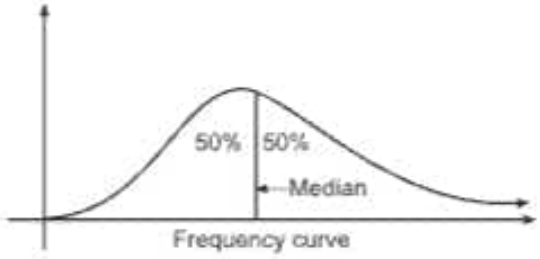

Fig. 4.2

(i) Computation of Median for Ungrouped data :

It may be noticed that in case of individual observations or ungrouped data computation of median does not require any formula. It can be determined by inspection.

Suppose n is the number of observations in the data. If n is odd then there is only one middle most observation which is $\frac{(n+1)^{th}}{2}$ observation. On the other hand if n is even then there are two middle most observations which are $\left(\frac{n}{2}\right)^{th}$ and $\left(\frac{n}{2}+1\right)^{th}$. In this case we take median to be mean of these two middle most observations. We follow the procedure described below for calculating median.

Step 1 : Arrange the observations in increasing (or decreasing) order.

Step 2 : Compute the median by the following criteria :

Median = The value of $(n+1)/2$ th observation if n is odd.

$$\text{Median} = \frac{\left(\begin{matrix}\text{The value of } (n/2)^{th} \\ \text{observation}\end{matrix}\right) + \left(\begin{matrix}\text{The value of } (n/2+1)^{th} \\ \text{observation}\end{matrix}\right)}{2} \text{ if n is even}$$

Illustration 7 : *Following are the temperatures recorded in a certain city, observed in a certain week.*

35, 38, 40, 39, 35, 36, 37

Obtain the median temperature.

Solution : The ordered arrangement of 7 observations is

35, 35, 36, $\boxed{37}$, 38, 39, 40

Since, n = 7 is odd we get,

Median = The value of $(n+1)/2^{th}$ observation

= The value of 4^{th} observation = 37.

Illustration 8 : *The following are the sales in ₹ for 6 days in a certain week.*

3020, 4120, 3600, 3250, 3830, 4000

Obtain the median sale.

Solution : The ordered arrangement of 6 observations is

3020, 3250, $\boxed{3600, 3830}$, 4000, 4120

Since n = 6 is even we get two middle observations. Hence

$$\text{Median} = \frac{\left(\begin{matrix}\text{The value of } (n/2)^{th} \\ \text{observation}\end{matrix}\right) + \left(\begin{matrix}\text{The value of } (n/2+1)^{th} \\ \text{observation}\end{matrix}\right)}{2}$$

$$\text{Median} = \frac{\left(\begin{matrix}\text{The value of } 3^{rd} \\ \text{observation}\end{matrix}\right) + \left(\begin{matrix}\text{The value of } 4^{th} \\ \text{observation}\end{matrix}\right)}{2} = \frac{3600+3830}{2} = 3715 ₹$$

(ii) Computation of Median for Continuous frequency distribution : Suppose N is the total frequency. Since the variable under consideration is continuous we can estimate the value of $\left(\frac{N}{2}\right)^{th}$ observation. Hence regardless of N whether it is even or odd in continuous frequency distribution we take median to be the value of $\left(\frac{N}{2}\right)^{th}$ observation.

Computational procedure :

Step 1 : Obtain the class boundaries.

Step 2 : Obtain less than cumulative frequencies.

Step 3 : Locate the median class. Where median class is the class in which median i.e. $\left(\frac{N}{2}\right)^{th}$ observation falls. In other words, it is in a class where less than cumulative frequency is equal to or exceeds $\frac{N}{2}$ for the first time.

Step 4 : Apply the formula and find the median.

$$\text{Median} = l + \left(\frac{N/2 - \text{c.f.}}{f} \times h\right)$$

where, l = Lower boundary (extended class limit) of the median class

N = Total frequency

c.f. = Less than cumulative frequency of the class just **preceding** to median class.

f = Frequency of median class

h = Class width

Illustration 9 : *Calculate median for the following frequency distribution :* **(April 2015)**

Marks	*below 20*	*21-40*	*41-60*	*61-80*	*81-100*
No. of students	*1*	*9*	*32*	*16*	*7*

Solution :

Class boundaries	**Frequency**	**Less than cumulative frequency**
0 – 20.5	1	1 < N/2
20.5 – 40.5	9	c.f. = 10 < N/2
40.5 – 60.5 Median class	f = 32	42 > N/2
60.5 – 80.5	16	58
80.5 – 100	7	65 = N

Median = The value of N/2 i.e. 32.5th observation.

Median class : 40.5 – 60.5. because N/2 exceeds less than cumulative frequency for the first time in this class.

Therefore, l = 40.5, N/2 = 32.5, c.f. = 10, f = 32, h = 20.

Hence,

$$\text{Median} = l + \left(\frac{N/2 - \text{c.f.}}{f}\right) \times h$$

$$= 40.5 + \frac{32.5 - 10}{32} \times 20$$

$$= 54.5625$$

4.7 Median – by Graphical Method

Median can be obtained graphically by means of ogive curve. Plot less than cumulative frequency curve taking upper boundaries on x-axis, and less than cumulative frequency on y-axis. Draw a line parallel to x-axis passing through point $\frac{N}{2}$ on y-axis. From the point of intersection of the line and ogive curve, draw a perpendicular to x-axis. The value at the foot of perpendicular is the median.

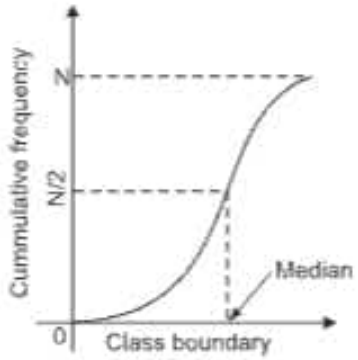

Fig. 4.3

Illustration 10 : *Obtain the median, from the following frequency distribution using formula and also graphically.*

Monthly Salary (₹)	*1400-1600*	*1600-1800*	*1800-2000*	*2000-2200*	*2200-2400*	*2400-2600*
Frequency	*12*	*30*	*55*	*40*	*35*	*28*

Solution : Here the classes are continuous, hence they can be used as they are :

Class	Frequency	Less than type cumulative frequency
1400 – 1600	12	12
1600 – 1800	30	42
1800 – 2000	55	97
2000 – 2200	40	137
2200 – 2400	35	172
2400 – 2600	28	200 = N

$$\text{Median} = \left(\frac{N}{2}\right)^{th} \text{ observation}$$

$$= \left(\frac{200}{2} = 100\right)^{th} \text{ observation}$$

Median lies in the (2000 – 2200) class, since 100 lies between less than cumulative frequencies 97 and 137,

$$\text{Median} = l + \left(\frac{N/2 - \text{c.f.}}{f}\right) \times h$$

$$= 2000 + \left(\frac{100 - 97}{40}\right) \times 200$$

$$= 2015$$

To obtain median graphically we use less than type cumulative frequency curve.

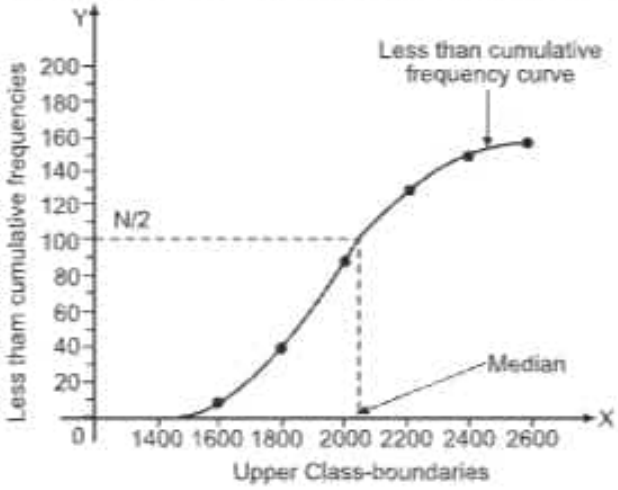

Fig. 4.4

4.8 Merits and Demerits of Median

Merits :

1. It is easy to understand and easy to calculate.
2. It is not affected due to extreme observations.
3. It can be computed for a distribution with open end classes.
4. It can be determined graphically.
5. It is applicable to qualitative data also. In this case observations are arranged in order according to the quality and the middle most observation can be obtained. The quality of this item is taken to be average quality or median quality.

Demerits :

1. It is not based on all the observations, hence it is not proper representative.
2. It is not capable of further mathematical treatment.
3. It is not as rigidly defined as the arithmetic mean.

4.9 Mode

It is yet another measure of central tendency developed to overcome the drawbacks of arithmetic mean. Apart from this, in some situations mode is the proper average.

Definition : The observation with maximum frequency or the most repeated observation is called as mode.

It is clear from earlier discussion that the general nature of frequency curve is bell shaped in majority of situations. Thus initially frequency is small, it increases and reaches the maximum and then it declines. The value on x-axis at which the maxima or the peak of the frequency curve appears is a mode.

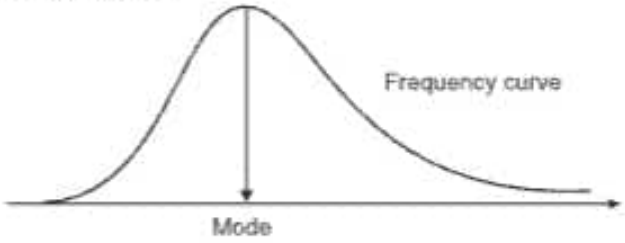

Fig. 4.5

In case of election results, a political party with largest votes (i.e. maximum frequency) is considered as representative. Thus, it is mode or modal opinion. In this situation, mode is the appropriate average. Similarly, to estimate the crop yield, too good quality or too poor quality crop is not considered. A quality of crop most commonly found is taken into account, which is nothing but mode. In titration experiment, out of three readings a repeated reading is taken to be final reading. It is mode and not the arithmetic mean. Thus in number of situations mode is appropriate.

(i) Computation of mode for Individual observations and Discrete frequency distribution : In this case we can find the observation with the largest frequency just by inspection. If the largest frequency occurs twice (or more), then we say there are two (or many) modes.

Illustration 11 : *Find the mode of the following frequency distribution :*

x	10	11	12	13	14	15
f	2	5	10	21	12	13

Solution : Since maximum frequency is associated with observation 13, the mode is 13.

(ii) Computation of mode for Continuous frequency distribution :

Step 1 : Obtain the class – boundaries.

Step 2 : Locate the modal class. Modal class is class in which mode lies or a class with the largest frequency.

Step 3 : Apply the formula and find the mode.

$$\text{Mode} = l + \left(\frac{f_m - f_1}{2f_m - f_1 - f_2}\right) \times h$$

where, l = Lower boundary (or extended class limit) of modal class

f_m = Frequency of (or extended class limit) modal class

f_1 = Frequency of pre-modal class

f_2 = Frequency of post-modal class

h = Width of modal class

Illustration 12 : *Calculate modal income from the following income distribution :*

Daily income (₹)	*30 and below*	*31-60*	*61-90*	*91-120*	*121-150*	*above 150*
No. of Persons	*22*	*198*	*110*	*95*	*42*	*33*

Solution :

Class boundaries	Frequency
below 30.5	$f_1 = 22$
30.5 – 60.5	$f_m = 198$ Modal class
60.5 – 90.5	$f_2 = 110$
90.5 – 120.5	95
120.5 – 150.5	42
above 150.5	33

Modal class is 31–60. Since the corresponding frequency is the highest.

Here we get $l = 30.5$, $f_m = 198$, $f_1 = 22$, $f_2 = 110$, $h = 30$

$$\text{Mode} = l + \left(\frac{f_m - f_1}{2f_m - f_1 - f_2}\right) \times h$$

$$= 30.5 + \frac{198 - 22}{2 \times 198 - 22 - 110} \times 30 = 50.5$$

Note :

1. If the maximum frequency is repeated, to find the mode uniquely, a method of grouping is adopted and a modal class is determined. The method of grouping is beyond the scope of book.
2. Mode cannot be determined if modal class is at the extreme. (i.e. the maximum frequency occurs at the beginning or at the end of the frequency distribution.)
3. Modal, pre-modal and post-modal classes should be of the same width.
4. If $f_1 = f_2$ then mode is the class-mark of modal class.

(iii) Computation of mode – by Empirical relation : Arithmetic mean, mode and median are averages, hence we expect that those should be identical in value. However, this is true only in ideal situation. It is true whenever the frequency curve is perfectly symmetric and bell-shaped. For a moderately asymmetric unimodal frequency distribution the following empirical relationship holds approximately.

Mean – Mode ≈ 3 (Mean – Median) ... (4.2)

In some situations mode is ill-defined (see notes 1, 2 stated above). To overcome this difficulty in computing mode, the empirical relation (1) is used. If any two averages included in (4.2) are known, the remaining third can be computed. Therefore, if mean and median are known, then mode can be determined.

The empirical relation cannot be theoretically proved. Karl Pearson has stated it on the basis of vast experience. This relationship is observed to be valid for number of data sets after actual computations.

(iv) Computation of mode – by graphical method : Mode can be obtained graphically with the help of histogram. Mode is the x-co-ordinate of point P or the value at foot of perpendicular from P to x-axis, shown in Fig. 4.6.

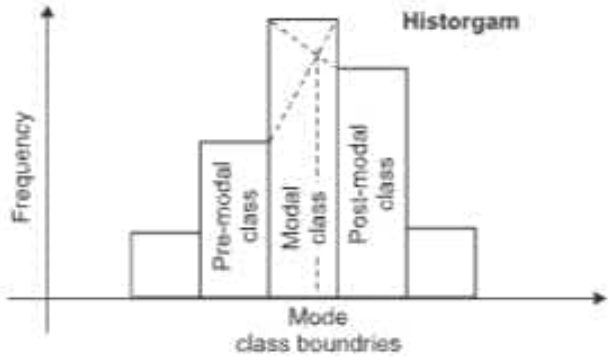

Fig. 4.6 : Histogram

Merits and Demerits of mode :

Merits :

1. It is simple to understand and easy to compute.
2. It is applicable for qualitative and quantitative data.
3. It is not affected by extreme observations.
4. It can be computed for distribution with open end classes.
5. It can be determined graphically.

Demerits :

1. It is not based on all the observations.
2. It is not capable of further mathematical treatment.
3. It is not rigidly defined like arithmetic mean.
4. It is indeterminate if the modal class is at the extreme of the distribution.

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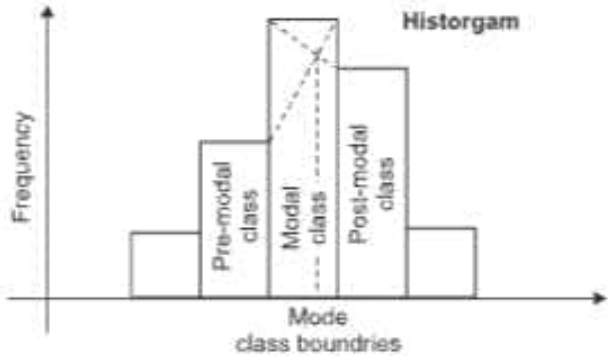

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1. It is not based on all the observations.
2. It is not capable of further mathematical treatment.
3. It is not rigidly defined like arithmetic mean.
4. It is indeterminate if the modal class is at the extreme of the distribution.

Note : It is possible to have two modes, such frequency distribution is called as bimodal frequency distribution. Sometimes bimodal frequency distribution is an indication of mixture of two frequency distributions.

For example, operator or machine is changed in manufacturing process. In medical sciences, two types of anaemia viz. microcytic and macrocytic are found in same population which give bimodal frequency curve.

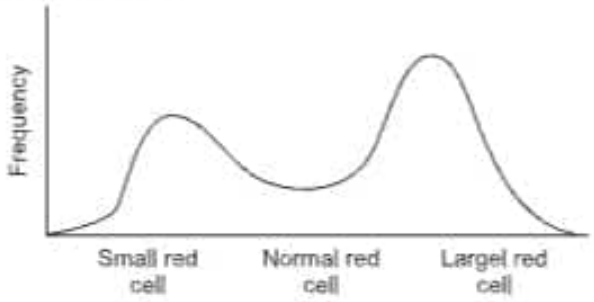

Fig. 4.7

Illustration 13 : *Calculate arithmetic mean and mode for the following data :*

Monthly salary (₹)	*Number of workers*
Below 400	*0*
Below 600	*4*
Below 800	*14*
Below 1000	*33*
Below 1200	*45*
Below 1400	*49*
Below 1600	*50*

Solution : We need to prepare frequency distribution from the given cumulative frequency distribution.

Class	**Frequency**	**Mid-values x**	$u = \frac{x - 900}{200}$	**fu**
400 - 600	4 – 0 = 4	500	– 2	– 8
600 - 800	14 – 4 = 10	700	– 1	– 10
800 - 1000	33 – 14 = 19	900	0	0
1000 - 1200	45 – 33 = 12	1100	1	12
1200 - 1400	49 – 45 = 4	1300	2	8
1400 - 1600	50 – 49 = 1	1500	3	3
Total	50	–	–	5

$$\text{Mean} = a + \frac{\sum fu}{N} \times h, \text{ where, } a = 900, \sum fu = 5, N = 50, h = 200$$

$$= 900 + \frac{5}{50} \times 200 = ₹\ 920$$

Modal class : 800 – 1000

$$\text{Mode} = l + \left(\frac{f_m - f_1}{2f_m - f_1 - f_2}\right) \times h$$

Here $l = 800$, $f_m = 19$, $f_1 = 10$, $f_2 = 12$, $h = 200$

$$\therefore \text{Mode} = 900 + \left(\frac{19 - 10}{38 - 10 - 12}\right) \times 200 = 912.5$$

Solved Examples

Example 4.1 : *From the following data find the missing frequencies, it is given that mean is 15.3818 and total frequency is 55.*

Class	*9-11*	*11-13*	*13-15*	*15-17*	*17-19*	*19-21*
Frequency	*3*	*7*	–	*20*	–	*5*

Solution : Let the missing frequencies be a and b

Class	Mid-value x	Frequency f	f · x
9 - 11	10	3	30
11 - 13	12	7	84
13 - 15	14	a	14a
15 - 17	16	20	320
17 - 19	18	b	18b
19 - 21	20	5	100
Total	–	35 + a + b = N = 55	534 + 14a + 18 b = $\sum$ fx

We get two equations from the given information

i.e. $35 + a + b = 55$ ($\because$ Total frequency N = 55)

$\therefore$ $a + b = 20$ ... (1)

$$\bar{X} = \frac{\sum fx}{N} \text{ gives}$$

$$15.3818 = \frac{534 + 14a + 18b}{55}$$

$\therefore$ $845.999 = 534 + 14a + 18b$

$\therefore$ $14a + 18b = 311.999$ ... (2)

Solving (1) and (2) we get, a = 12.0002, b = 7.9998.

After rounding-off the values, a = 12 and b = 8.

Thus, frequency of the class 11-13 is 12 and that of 17-19 is 8.

Example 4.2 : *Find the arithmetic mean given that* $\sum (x - 10) = 230$ *and* $n = 50$.

Solution : Let $d = x - 10$, $a = 10$, hence $\sum d = 230$

$$\therefore \quad \text{Mean} = a + \frac{\sum d}{n} = 10 + \frac{230}{50} = 14.6$$

Example 4.3 : *Arithmetic mean of 50 items is 104. While checking, it was noticed that observation 98 was misread as 89. Find the correct value of mean.*

Solution :

$$\text{Incorrect mean} = 104 = \frac{\text{Incorrect sum}}{n}$$

$$\therefore \quad \text{Incorrect sum} = 104 \times 50 = 5200$$

$$\text{Correct sum} = \text{Incorrect sum} + \text{Correct observation} - \text{Incorrect observation}$$

$$= 5200 + 98 - 89 = 5209$$

$$\therefore \quad \text{Correct mean} = \frac{\text{Correct sum}}{n}$$

$$= \frac{5209}{50} = 104.18.$$

Example 4.4 : *The number of washing machines sold in a shop per day are distributed as follows. Find median*

No. of machines sold	*0*	*1*	*2*	*3*	*4*	*5*
No. of days	*6*	*10*	*4*	*3*	*3*	*1*

Solution : Let X = No. of machines sold, f = No. of days.

X	f	Less than type cumulative frequency
0	6	6
1	10	16
2	4	20
3	3	23
4	3	26
5	1	27 = n

$$\text{Median} = \text{The value of } \left(\frac{n+1}{2} = \frac{27+1}{2} = 14\right)^{th} \text{ observation in the ordered arrangement}$$

$$= 1$$

Example 4.5 : *A salesman has given a target to complete average daily sales of ₹ 1000. In a particular week, average sales of first 6 days is ₹ 980. What should be his sales on seventh day in order to make-up the target ?*

Solution : Here we use average as arithmetic mean

$$\bar{X} = \frac{\sum x}{n} = \frac{\sum x}{7} = 1000$$

$\therefore$ Total sales for 7 days $= \sum x = n\bar{X} = 7 \times 1000 = ₹\ 7000$

The average of first 6 days $= \frac{\sum x}{6} = 980.$

Total sales for 6 days $= 6 \times 980 = ₹\ 5880$

Sales required on 7th day $= 7000 - 5880 = ₹\ 1120$

Example 4.6 : *The median of a group of 100 observations is computed to be 70. While verifying, it was found that the observation 13 was misread as 31. Find the correct median.*

Solution : Note that the median is 70. The observation 31 is to be replaced by correct observation as 13. This change does not affect the middle most observation in the ordered arrangement, hence median will remain same. Thus the median after correction is 70.

Note : *However, arithmetic mean will change.*

Example 4.7 : *Calculate mode of the following frequency distribution*

Class	50–100	100–150	150–200	200–250	250–300	300–350	350–400
Frequency	5	15	25	18	12	3	2

Solution : Modal class = (150–200)

$$\text{Mode} = l + \left(\frac{f_m - f_1}{2f_m - f_1 - f_0}\right) \times h = 150 + \left(\frac{25 - 18}{50 - 18 - 15}\right) \times 50$$

$$= 150 + \left(\frac{7}{17}\right) \times 50 = 170.5882$$

Example 4.8 : *Following is a frequency distribution regarding the number of family members, number of earning members in a certain locality.*

Income per month	No. of families	No. of family members	
		Earners	Non-earners
0 – 2000	22	25	40
2000 – 3000	59	75	143
3000 – 4000	70	91	179
4000 – 6000	25	57	136
6000 – 10000	15	42	85
10000 – 14000	9	30	17
Total	200	320	600

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$$\bar{X} = \frac{\sum x}{n} = \frac{\sum x}{7} = 1000$$

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Solution : Modal class = (150–200)

$$\text{Mode} = l + \left(\frac{f_m - f_1}{2f_m - f_1 - f_0}\right) \times h = 150 + \left(\frac{25 - 18}{50 - 18 - 15}\right) \times 50$$

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		Earners	Non-earners
0 – 2000	22	25	40
2000 – 3000	59	75	143
3000 – 4000	70	91	179
4000 – 6000	25	57	136
6000 – 10000	15	42	85
10000 – 14000	9	30	17
Total	200	320	600

Calculate :

1. Average monthly income per family
2. Average monthly income per earning member
3. Per capita income
4. Average family size
5. The median family income.

Solution :

Income	Mid-point (x)	No. of families (f)	f.x	Less than cumulative frequency
0 – 2000	1000	22	22000	22
2000 – 3000	2500	59	147500	81
3000 – 4000	3500	70	245000	151
4000 – 6000	5000	25	125000	176
6000 – 10000	8000	15	120000	191
10000 – 14000	12000	9	108000	200
	Total	200	767500	–

1. Average monthly income per family $= \frac{\sum fx}{\sum f} = \frac{767500}{200} = ₹\ 3837.5$

2. Average monthly income per earning member $= \frac{\text{Total income}}{\text{No. of earning members}}$

$$= \frac{767500}{320} = ₹\ 2398.44$$

3. Per capita income $= \frac{\text{Total income}}{\text{Total population}} = \frac{7675000}{320 + 600} = ₹\ 834.24$

(Total population = No. of earners + No. of non-earners.)

4. Average family size $= \frac{\text{Total number of earners and non-earners}}{\text{Total number of families}}$

$$= \frac{320 + 600}{200} = \frac{920}{200} = 4.6$$

5. The median of family income.

$$\text{Median} = \text{The value of } \left(\frac{N}{2} = \frac{200}{2} = 100\right)^{th} \text{ observation}$$

$$= l + \left(\frac{\frac{N}{4} - \text{C.F.}}{f}\right) h = 3000 + \left(\frac{100 - 81}{70}\right) \times 1000 = ₹\ 3271.43$$

Example 4.9 : *The monthly income (₹) of 10 families in a village is as follows :*

1200, 1000, 1100, 1250, 950, 1300, 1350, 1150, 1200, 1050.

Find Mean, Median and Mode of this Income Distribution.

Solution : $\text{Mean} = \frac{\sum x}{n} = \frac{11550}{10} = 1155$

The ordered arrangement to find the median is as follows :

950, 1000, 1050, 1100, 1150, 1200, 1200, 1250, 1300, 1350.

$$\text{Median} = \text{The value of } \left(\frac{n+1}{2} = \frac{11}{2} = 5.5\right)^{th} \text{ observation}$$

$$= \frac{5^{th} \text{ observation} + 6^{th} \text{ observation}}{2}$$

$$= \frac{1150 + 1200}{2}$$

$$= ₹\ 1175$$

$$\text{Mode} = \text{Observation with maximum frequency}$$

$$= ₹\ 1200$$

Thus, Mean = ₹ 1155, Median = ₹ 1175, mode = ₹ 1200

Example 4.10 : *The following data relates to age distribution of 50 persons :*

Age (years)	*Frequency*
20-30	*3*
30-40	*7*
40-50	*14*
50-60	*16*
60-70	*8*
70-80	*2*

Find mode of above distribution

Solution : Modal class : 50–60

$$\text{Mode} = l + \frac{f_m - f_0}{2f_m - f_0 - f_1} \times h$$

$l = 50,\ f_m = 16,\ f_0 = 14,\ f_1 = 8,\ h = 10.$

$\therefore$ $$\text{Mode} = 50 + \left(\frac{16-14}{32-14-8}\right) \times 10$$

$$= 52 \text{ years}$$

Case Study

Shriram Oxygen Ltd. is a company in a manufacturing of industrial oxygen based in a industrial area of Washi, Navi Mumbai. There are in all about 1000 employees in this company. They are of various grades.

For example, there is a managing director, about 10 directors, 30 senior general managers, about 200 managers, 150 officers and rest are workers of different grades. Company's monthly salary budget is about ₹ 30 lac.

Management of this company is of the opinion to increase the productivity by not increasing the man power but through increasing the salary of existing employees.

Existing salary of managing director is approximately ₹ 1 lacs per month, directors get around ₹ 75,000/- per month, general manager gets around ₹ 50,000. Whereas workers salary varies from ₹ 20,000 to ₹ 50,000 as per their grades.

Company has a revised budget of ₹ 40 lac per month. Company would like to know about what is the average salary per month. Whether to find mean would be appropriate or should median be used. What would be average revised salary per month ?

Points to Remember

1. Arithmetic mean $(\bar{X}) = \frac{\sum x}{n}$ for ungrouped data

   $= \frac{\sum fx}{\sum f}$ for frequency distribution.

2. Median $= l + \left(\frac{\frac{N}{2} - Cf}{f}\right) \times h$.

3. Mode $= l + \left(\frac{f_m - f_1}{2f_m - f_1 - f_2}\right) \times h$

4. If $y = ax + b$ then $\bar{y} = a\bar{x} + b$, $y = \frac{x - c}{d}$ then $\bar{y} = \frac{\bar{x} - c}{d}$

5. Combined arithmetic mean $= \frac{n_1 \bar{x} + n_2 \bar{y}}{n_1 + n_2}$

6. Median can be obtained graphical using ogive curves.
7. Mode can be obtained graphically using histogram.
8. Arithmetic mean is the best average.
9. Arithmetic mean cannot be determined by graph.

Exercise

[A] Theory Questions :

1. What do you mean by central tendency ? Explain the purpose of measures of central tendency.
2. State the requisites of an ideal average.
3. Define mean, median, mode and state the formula for each, in case of (i) individual observations (ii) frequency distributions.
4. Discuss merits and demerits of (i) mean, (ii) median, (iii) mode.
5. Explain graphical method of determination of (i) median (ii) mode.

[B] Discrete Series :

6. Monthly consumption of electricity in units of a certain family in a year is given below :

   210, 207, 315, 250, 240, 232, 216, 208, 209, 215, 300, 290.

   Compute the mean, median and modal consumption of electricity.

7. The marks obtained by 12 students are given below :

   30, 55, 50, 40, 50, 60, 55, 62, 55, 45, 61, 65

   Calculate mean, median and mode for the above data.

8. Compute the mean, mode and median for the following data : **(April 2011)**

   68, 49, 38, 41, 49, 54, 89, 99, 67

9. Find the mean, median and mode of the following observations : **(Oct. 2014)**

   61, 62, 63, 62, 63, 62, 64, 64, 60, 65.

10. In a set of 50 items, arranged in ascending order of magnitude the values of 24^{th}, 25^{th} and 26^{th} items are 40, 42 and 45 respectively. Find the median. Also find the median if the number of observations was 51.

11. Calculate mean and median weight of the group of students with weights (in kg) given below :

    51, 52, 53, 51, 53, 54, 54, 50, 55, 53.

    If a new group of students with weights in kg as 50, 56, 58, 57, 60 is added to the original group, find mean and median of combined group.

12. Compute median of the following series

    5, 20, 18, 12, 0, 21, 18, 26, 5, 15, 20.

13. The following figures represent the number of books issued at the counter of commerce college library on 8 different days.

    96, 98, 75, 80, 102, 100, 94, 75.

    Calculate the median and mode of the data.

14. Compare the average runs scored by cricters A and B using arithmetic mean.

Cricketer	Runs scored				
A	5	20	90	75	100
B	40	35	60	65	50

15. The weekly income of 10 families in a village is as follows :

1200, 1000, 1100, 1250, 950, 1300, 1350, 1150, 1200, 1050.

Find the mean, mode, median of the income distribution.

[C] Frequency Distribution :

16. Find the mean, mode and median of the following data.

X	5	6	7	8	9	10	11	12
Frequency	8	10	9	6	5	4	4	1

17. Find the mean, median and mode of the following frequency distribution.

Marks	0-20	20-40	40-60	60-80	80-100
No. of frequency	5	12	32	40	11

18. Find arithmetic mean, mode and median of following frequency distribution.

Marks	0-20	20-40	40-60	60-80	80-100
No. of students	4	8	9	20	9

19. Compute arithmetic mean, mode and median of the following frequency distribution.

Weight in kg.	30-40	40-50	50-60	60-70	70-80
No. of students	3	5	12	20	10

20. Determine arithmetic mean, mode and median of marks from the data given below :

Marks	0-10	10-20	20-30	30-40	40-50
No. of students	1	3	10	4	2

21. The monthly profit in rupees of 100 shops are distributed as follows :

Profit (in ₹) per shop	0-100	100-200	200-300	300-400	400-500	500-600
No. of shops	12	18	27	20	17	6

(i) Calculate the mode for above data. (ii) Find mode graphically.

22. A study of a certain operation shows the following distribution for 180 workers. Calculate the median. Also find it graphically.

Class interval (in seconds)	10-30	30-50	50-70	70-90	90-110
Frequency	10	40	80	35	15

23. Find the mean. mode and median for the following data :

Class	100-200	200-300	300-400	400-500
Frequency	15	20	10	5

24. Compute the median for the following frequency distribution. Also find it graphically.

Dividend (%)	0-20	20-40	40-60	60-80	80-100
No. of companies	20	35	15	8	2

25. Find the mean, mode and median of the following frequency distribution.

Weight (kg)	30-40	40-50	50-60	60-70	70-80
No. of students	4	5	7	3	1

26. Find mode for the following frequency distribution of income of 70 workers :

Income (₹)	Less than 1000	1000-2000	2000-3000	3000-4000	4000-5000	Above 5000
No. of Workers	08	14	13	25	07	03

27. The following data relates to age distribution of 50 persons :

Age (Years)	Frequency
20-30	3
30-40	7
40-50	14
50-60	16
60-70	8
70-80	2

Find mode of above distribution.

28. Following is the frequency distribution of sales of companies :

Sale (00,000 ₹)	0-20	20-40	40-60	60-80	80-100
No. of companies	05	18	20	12	05

Find the mode.

29. Following is the frequency distribution of percentage of dividend declared by companies :

Dividend %	10-15	15-20	20-25	25-30	30-35
No. of companies	15	20	35	10	5

Find the mode.

30. Calculate median for the following distribution :

Class	5-15	15-25	25-35	35-45	45-55
Frequency	5	15	20	15	5

31. Draw the histogram for the following frequency distribution :

Sales (in thousand ₹)	0-20	20-40	40-60	60-80	80-100
No. of companies	5	18	20	12	5

Hence locate the mode.

[D] Missing Values :

32. If mean of the following frequency distribution is 15.82 find the missing value of *.

X	10	12	13	17	*	25	18	30
Frequency	25	17	13	15	14	8	6	2

33. Find the missing frequency of the following frequency distribution if the arithmetic mean is 26.90.

Class	10-15	15-20	20-25	25-30	30-35	35-40	40-45
Frequency	5	6	8	*	7	5	4

34. You are given the following complete frequency distribution. It is known that the total frequency is 100 and the median is 44. Find the missing frequencies. Also compute the mean after finding missing frequencies.

Class	Frequency	Class	Frequency
10-20	5	50-60	–
20-30	12	60-70	10
30-40	–	70-80	4
40-50	20		

35. Mean daily salary of 50 employees in a firm is ₹ 188.40. Frequency distribution of salaries of these employees in which some frequencies are missing is given below :

Salary	140-160	160-180	180-200	200-220	220-240
Frequency	6	–	17	–	5

Find the missing frequencies.

36. The daily expenditure of 100 families is given below :

Expenditure	20-29	30-39	40-49	50-59	60-69
No. of families	14	–	27	–	15

If the mode of the distribution is 43.5, find the missing frequencies.

[E] Combined Mean :

37. Find the combined mean of the following data : **(April 2015)**

Group I	$\bar{x}_1 = 2100$	$n_1 = 100$
Group II	$\bar{x}_2 = 1500$	$n_2 = 200$

38. Average monthly sale of certain departmental store for first 11 months was ₹ 56000. Due to repairs and renewal of shop in the last month the sales dropped down to ₹ 8000. Find the average monthly sales in the year.

39. Obtain the combined mean profit per salesman from the following data

	Mean profit per salesman	**No. of salesman**
Shop 1	2000	5
Shop 2	3000	12
Shop	5000	3

40. Find the combined arithmetic mean and salary given that :

Group	**Male**	**Female**
No. of employees	100	50
Arithmetic mean of salary	₹ 6000	₹ 5100

41. Given

Group 1	Group 2
$n_1 = 100$	$n_2 = 100$
$\sum x = 600$	$\sum y = 800$

Find $\bar{x}$, $\bar{y}$ and combined mean of the two groups.

[F] Miscellaneous Problems :

42. A set of 10 values has arithmetic mean 20. Find the arithmetic mean if, (i) each value is doubled and then increased by 2 (ii) each value is increased by 5 and then doubled. (iii) each value is decreased by 5 (iv) each value is increased by 3.

43. The arithmetic mean of 10 items is 30. What will be mean, if each item is doubled ?

44. If $n = 10$ and $\sum (x - 5) = 90$ find the mean.

45. Obtain the average bonus per employee for the following frequency distribution.

Salary Group (₹)	1000-2000	2000-4000	4000-6000	above 6000
Bonus (₹)	300	400	450	500
Frequency	5	12	5	3

46. Calculate median and mode wage from the following data : (i) by using the formula (ii) by graphical method :

Wages in ₹	No. of workers
above 130	520
above 140	470
above 150	399
above 160	210
above 170	105
above 180	45
above 190	7

47. Find the median and mode of the following data by computational method and graphical method :

No. of days absent	No. of students
less than 5	29
less than 10	224
less than 15	465
less than 20	582
less than 25	634
less than 30	644
less than 35	650
less than 40	653
less than 45	655

48. Obtain the mean, median and mode from following data :

Monthly Rent (in ₹)	No. of families
221-240	6
241-260	9
261-280	11
281-300	14
301-320	20
321-340	15
341-360	10
361-380	8
381-400	7

49. Average of marks of 30 candidates was 40. Later on it was found that a score of 47 was misread as 74. Find the correct average. **(April 2015) (April 2011)**

50. The mean weight of 98 students as calculated from a frequency distribution is 50 kg. It was later found that the frequency of the class 30-40 was wrongly taken as 8 instead of 10. Calculate the correct arithmetic mean. **(April 2011)**

51. A salesman has given a target to complete average daily sales of ₹ 5000. In a particular week average of first 6 days is ₹ 4990. What should be his sales on seventh day in order to make-up the target ?

Answers

[B] 6. Mean = 241, Median = 224, No mode.

7. Mean = 52.33, Mode = Median = 55.

8. Mean = 61.56, Mode = 49, Median = 54.

9. Mean = 62.6, Median = 62.5, Mode = 62

10. 43.5, 45

11. Original data : Mean = 52.6, Median = 53, Combined data : Mean = 53.8, Median = 53

12. 18

13. Mode = 75, Median = 95

14. $\bar{X}_A = 58 > \bar{X}_B = 50$.

15. Mean = 1155, mode = 1200, Median = 1175.

[C] 16. Mean = 7.4894, Mode = 6, median = 7.

17. Mean = 58, Median = 60.2857, Mode = 64.32

18. Mean = 58.8, Median = 64, Mode = 65,

19. Mean = 60.8, Mode = 64.44, Median = 62.5,

20. Mean = 26.8, Mode = 25.3846, Median = 26,

21. 256.25

22. 60

23. Mean = 260, Mode = 233.33, Median = 250.

24. Median = 25.7142

25. Mean = 51, Mode = 53.3333, Median = 53.333.

26. 3400

27. 52

28. 42.

29. 21.875.

30. 30

31. By histogram 42

[D] 32. 24

33. 15

34. Missing frequencies 25, 24, Mean = 44.2

35. 12, 20

36. 23, 21

[E] 37. 1700

38. 52,000

39. 3050

40. ₹ 5700

41. $\bar{X} = 6$, $\bar{Y} = 8$, Combined mean = 7.

[F] 42. (i) 45 (ii) 50 (iii) 15 (iv) 23

43. 60

44. 14

45. 402

46. Median = 157.3545, Mode = 155.8416

47. Median = 12.1473, Mode = 11.35

48. Mean = Median = 310.5, Mode = 311.409

49. 39.5

50. 49.7

51. ₹ 5600

Objective Type Questions

1. Arithmetic mean of a group is 20. If each observation is increased by 5, find the mean of new observations.
2. State the imperical relation between mean, mode and median.
3. If $n = 10$, $\sum (x - 6) = 30$, find $\bar{x}$.
4. State the mode of following frequency distribution :

Class	0-10	10-20	20-30	30-40	40-50
Frequency	7	10	22	10	8

5. If each frequency is doubled, then what will happen to the arithmetic mean.

6. If frequency distribution has open end class, which average will be possible to compute.
7. Individual observations are not known but the total of 10 observations is known. Suggest the average which can be computed.
8. Suggest the average which you can compute if all the observations except the largest and smallest are known.

Answers

1. 25
3. 9
4. 25
5. Will not change
6. Mode, Median
7. Mean
8. Median

□□□

Chapter **5**

Measures of Dispersion

Contents ...

Key Words :

Dispersion, Deviation, Relative Dispersion Absolute Dispersion, Maximum, Minimum, Range, Coefficient Of Range, Standard Deviation (S.D.), Coefficient of Variation (C.V.), Variance.

Objectives :

The reliability of average is more if dispersion is less. Measures of dispersion is a tool which summerizes the internal variation or variation within the observations. The techniques of measurement of dispersion are discussed in this chapter. Statistics is in existence because of variation. Statistician has to talk in terms of S.D. and C.V. There are some situations such as genetics, biodiversity etc. where larger S.D. or C.V. has its importance.

5.1 Introduction

We have seen that, average condenses information into a single value. However, average alone is not sufficient to describe the frequency distribution completely. There may be two frequency distributions or data sets with same means but those may not be identical.

Illustration : Marks of students A, B, C in 5 subjects are as follows :

Student	Marks					A.M.
A	51	52	50	48	49	50
B	30	35	50	65	70	50
C	0	15	45	95	95	50

Notice that the average marks of all students are the same but they differ in variation. Clearly we can say that A is more consistent than B and B is more consistent than C.

For further study and analysis it becomes essential to measure the extent of variation. Observations are scattered or dispersed from central value. This variation is called as *dispersion*. Thus, next important aspect of comparison or study of frequency distribution or data sets is dispersion. Moreover it plays very important role in further analysis.

Average remains good representative, if dispersion is less (i.e. if the observations are close to it). Thus, dispersion decides the reliability of average.

5.2 Measures of Dispersion (B.B.A. April 2015)

In this chapter we study the following measures of dispersion : (i) range and (ii) standard deviation. These measures have the same units as that of the observation, for example, ₹, cm., hours, etc., and the measures are called as **absolute measures of dispersion.**

Absolute and Relative Measures of Dispersion (B.B.A., B.B.M. April 2015)

It can be very well seen that absolute measures possess units and hence create difficulty in comparison of dispersion for two or more frequency distributions or data sets.

For example : For a group of persons, variation in height and variation in weight is to be compared. Height may be in cm and weight may be in kg. Therefore, comparison is not possible until a unitless quantity is available. Therefore, with respect to every absolute measure of dispersion, relative measure of dispersion is defined. Relative measure can be obtained by dividing the absolute measure by corresponding average. Such a relative measure is called as coefficient of the respective absolute measure.

5.3 Range and Coefficient of Range

Range is a crude measure of dispersion. However, it is the simplest measure and suitable if the extent of variation is small.

Definition : If L is the largest observation and S is the smallest observation then range is the difference between L and S. Thus,

$$\text{Range} = L - S$$

and the corresponding relative measure is

$$\text{Coefficient of range} = \frac{L - S}{L + S}$$

In case of frequency distribution lower limit of first and upper limit of last class intervals are taken to be the smallest and the largest observations respectively.

Note : Requisites of good measures of dispersion are same as those of average.

Merits of Range : (1) It is simple to understand and easy to calculate.

(2) It is rigidly defined.

Demerits of Range : (1) It is not based on all observations. It does not give proper idea regarding variation between the extreme observations.

For example : Range of 0, 3, 5, 200 is same as that of 0, 50, 100, 150, 200, however, variation patterns are different.

(2) It cannot be determined for frequency distribution with open end class.

Applications of Range :

Range is suitable measure of dispersion in case of small group with less variation. (i) It is widely used in the branch of statistics known as Statistical Quality Control. (ii) The changes in prices of shares lowest and highest observations are used. (iii) Temperature at a certain place is recorded using maximum and minimum value. (iv) Range used in medical sciences to check whether blood pressure, hemoglobin count etc. is normal.

Illustration 1 : *Compute range and coefficient of range for the following data :*
100, 24, 14, 105, 21, 35, 106.

Solution : Here,

$$\text{Smallest observation (S)} = 14$$
$$\text{Largest observation (L)} = 106$$
$$\text{Range} = L - S = 106 - 14 = 92$$
$$\text{Coefficient of range} = \frac{L-S}{L+S} = \frac{92}{106+14}$$
$$= \frac{92}{120} = 0.7667$$

Illustration 2 : *Determine the range and the coefficient of range for the following data :*

Electricity consumption per month	*100-150*	*150-300*	*300-450*	*450-600*
No. of families	*28*	*56*	*43*	*23*

Solution :

$$\text{Range} = \text{Largest observation (L)} - \text{Smallest observation (S)}$$
$$= 600 - 100 = 500$$
$$\text{Coefficient of range} = \frac{L-S}{L+S} = \frac{500}{700} = \frac{5}{7}.$$

5.4 Quartile Deviation or Semi-interquartile Range

The range uses only two extreme items. Hence, any change in the inbetween observations is not going to affect the range. This is a main drawback of range. Moreover in many situations extreme items are widely separated from remaining items. In this situation range will overestimate the dispersion. Thus, range fails to give true picture of dispersion. In order to overcome these drawbacks range of middle 50% items is computed.

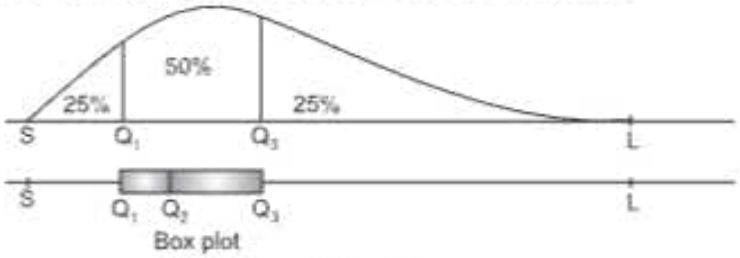

Fig. 5.1

Partition Values : Quartiles

Quartiles, Deciles and Percentiles : Earlier we have seen that median divides the total number of observations into two equal parts. Similarly in order to make four equal parts we use quartiles, for making 10 equal parts we use deciles and for making 100 equal parts we use percentiles, when the observations are ordered.

Definitions : The observations Q_1, Q_2, Q_3 which divide the total number of observations into 4 equal parts are called *quartiles.*

Median, quartiles, deciles and percentiles are called **partition values** in common. The procedure of obtaining median is used to compute other partition values with appropriate changes. To obtain the partition values of series of individual observations, tedious calculations or formulae are not required. However, to compute partition values of a continuous frequency distribution, corresponding formula of median is suitably modified. In this case, first of all less than cumulative frequency is determined. Using these cumulative frequencies a class in which partition value lies is decided and then using the formula, partition value is determined.

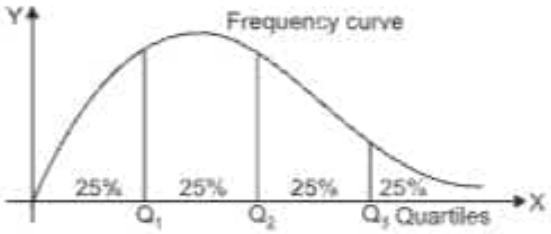

Fig. 5.2

$$\text{First quartile } (Q_1) = l + \left(\frac{\frac{N}{4} - \text{C.F.}}{f}\right) \times h$$

$$\text{Second quartile } (Q_2) = \text{Median} = l + \left(\frac{\frac{N}{2} - \text{C.F.}}{f}\right) \times h$$

$$\text{Third quartile } (Q_3) = l + \left(\frac{\frac{3N}{4} - \text{C.F.}}{f}\right) \times h$$

Note :

1. Median $= Q_2$.
2. The area between any two successive quartiles in 25% of the total area under the frequency curve.
3. Quartiles can be determinal graphically using less than cumulative frequency curve.
4. Minimum $< Q_1 < Q_2 < Q_3 <$ Maximum.

Illustration 3 : Compute the quartiles for the following series of observations.

26, 30, 35, 5, 6, 7, 9, 20, 40, 45, 11, 18, 15, 49, 60. **(April 2015)**

Solution : To find the quartiles first we arrange the observations in increasing (or decreasing) order of their magnitudes. Ordered arrangement will be

5, 6, 7, $\boxed{9}$, 11, 15, 18, $\boxed{20}$, 26, 30, 35, $\boxed{40}$, 45, 49, 60.

First quartile or lower quartile Q_1

$$= \left(\frac{(n+1)}{4}\right)^{th} \text{ observation} = \left(\frac{15+1}{4} = 4\right)^{th} \text{ observation} = 9$$

Second quartile or median Q_2

$$= \left(\frac{n+1}{2}\right)^{th} \text{ observation } = \left(\frac{15+1}{2} = 8\right)^{th} \text{ observation} = 20$$

Third quartile or upper quartile Q_3

$$= \left(\frac{3\,(n+1)}{4}\right)^{th} \text{ observation}$$

$$= \left(\frac{3\,(15+1)}{4} = 12\right)^{th} \text{ observation} = 40.$$

Illustration 4 : Obtain the quartiles from the following frequency distribution using formula and also graphically.

Weekly Salary (₹)	1400-1600	1600-1800	1800-2000	2000-2200	2200-2400	2400-2600
Frequency	12	30	55	40	35	28

Solution : Here the classes are continuous, hence they can be used as they are :

Class	Frequency	Less than type cumulative frequency
1400–1600	12	12
1600–1800	30	42
1800–2000	55	97
2000–2200	40	137
2200–2400	35	172
2400–2600	28	200 = N

$$Q_1 = \left(\frac{N}{4}\right)^{th} \text{ observation} = \left(\frac{200}{4} = 50\right)^{th} \text{ observation.}$$

$$\text{First quartile } (Q_1) \;=\; l + \left(\frac{\frac{N}{4} - \text{C.F.}}{f}\right) \times h$$

Since we have to consider 50^{th} observation, and from less than cumulative frequencies we obsere that $42 < 50 < 97$, we have to consider the class of less than cumulative frequency in which partition value lies. Therefore, 1800 – 2000 is the first quartile class.

$\therefore$ Q_1 lies in (1800 – 2000) class

$$\therefore \qquad Q_1 \;=\; 1800 + \frac{50-42}{55} \times 200$$

$$= 1800 + 29.0909 \;=\; ₹\ 1829.0909$$

$$Q_2 \;=\; \left(\frac{N}{2}\right)^{th} \text{ observation } = \left(\frac{200}{2} \;=\; 100\right)^{th} \text{ observation}$$

$$= l + \left(\frac{N/2 - \text{C.F.}}{f}\right) \times h$$

Note that 100th observation lies in class (2000–2200).

Since we get from less than cumulative frequency that $97 < 100 < 137$. Hence, (2000–2200) is a Q_2 class.

$$\therefore \quad Q_2 = 2000 + \left(\frac{100 - 97}{40}\right) \times 200 = 2015$$

Similarly, $Q_3 = \left(\frac{3N}{4}\right)^{th}$ observation $= \left(\frac{3 \times 200}{4} = 150\right)^{th}$ observation.

Since 150th observation lies in class 2200–2400, it is Q_3 class $(137 < 150 < 172)$.

$$Q_3 = l + \left(\frac{3N/4 - C.F.}{f}\right) \times h$$

$$= 2200 + \left(\frac{150 - 137}{35}\right) \times 200 = ₹\ 2274.2857$$

To obtain Q_1, Q_2, Q_3 graphically we use less than type cumulative frequency curve.

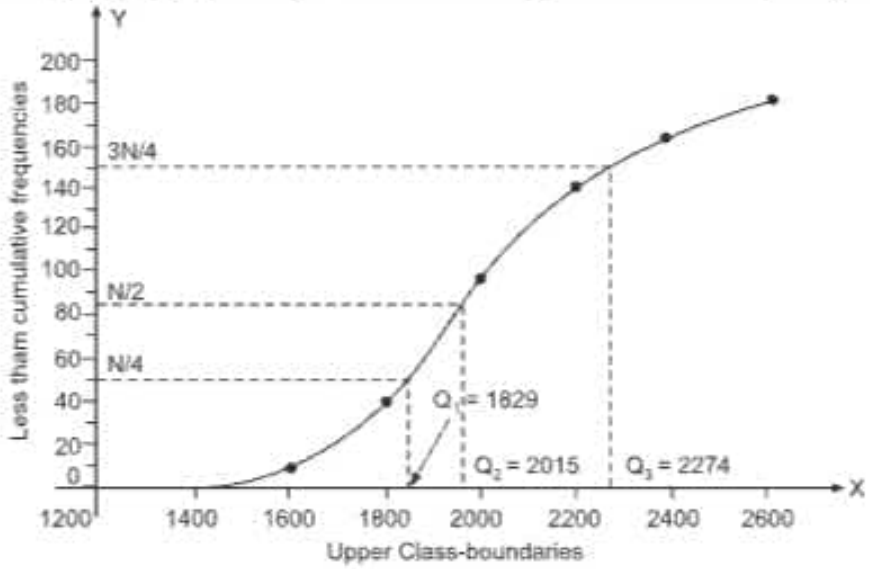

Fig. 5.3

BOX AND WHISKER PLOT

There is one more way of graphical representation of data known as box and whisker plot.

To draw box plot we find the three quartiles and the extreme observations. We illustrate the procedure by the following example.

Illustration 5 : Construct box plot to represent the data given below

26, 30, 35, 5, 6, 7, 9, 20, 40, 45, 11, 18, 15, 49, 60.

Solution : Clearly the ordered arrangement is

5, 6, 7, [9], 11, 15, 18, [20], 26, 30, 35, [40], 45, 49, 60.

Note that the minimum is 5, maximum is 60 and the three quartiles are respectively 9, 20, 40. We take observations from minimum to maximum on line and put the rectangular box to include the first quartile and the third quartile. Thus the length of box is $Q_3 - Q_1$. In this case it is $40 - 9 = 31$. We divide the box in two boxes by putting horizontal line at median. The box plot is drawn below :

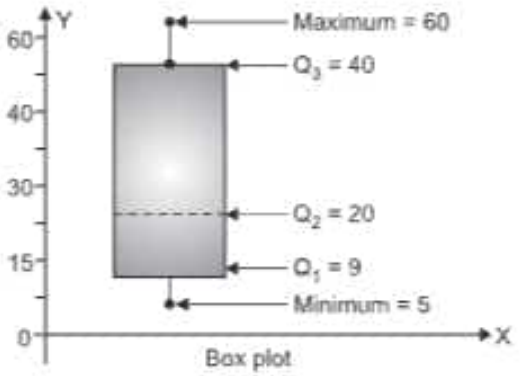

Fig. 5.4

Uses of box plot :

1. It gives the idea about the spread of data.
2. The box represents the interquartile range $Q_3 - Q_1$ of the data. In other words it gives the range in which middle 50% observations lie.
3. It gives the idea about the symmetry of the data around the median.
4. Median divides the data in two equal parts, box plot gives idea about how the observations are clustered or spread in each part of data.
5. The box plot facilitates the comparison of the aspects (i) central tendency, (ii) spread, (iii) symmetry.

Note : Box plot can also be drawn horizontally.

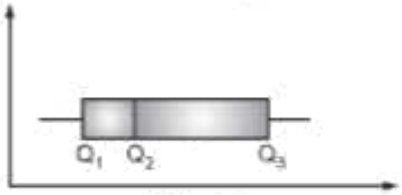

Fig. 5.5

Clearly the middle 50% items lie inbetween the two quartiles Q_1 and Q_3. The measure of dispersion based on these quartiles is given below :

Quartile Deviation (Q.D.) or Semi-Interquartile Range $= \dfrac{Q_3 - Q_1}{2}$.

And the corresponding measure of comparison is

Coefficient of Quartile Deviation $= \dfrac{Q_3 - Q_1}{Q_3 + Q_1}$

Illustration 6 : Compute (i) range and coefficient of range (ii) quartile deviation and coefficient of quartile deviation for the following data :

100, 24, 14, 105, 21, 35, 106, 16, 100, 72, 68, 103, 61, 90, 20.

(April 2010, Oct. 2010)

Solution : (i) Here, Smallest observation (S) = 14

Largest observation (L) = 106

$$\text{Range} = L - S = 106 - 14 = 92$$

$$\text{Coefficient of range} = \frac{L-S}{L+S} = \frac{92}{106+14} = \frac{92}{120} = 0.7667$$

(ii) To find quartile deviation, we arrange the observations in ascending order as follows :

14, 16, 20, [21] , 24, 35, 61, 68, 72, 90, 100, [100] , 103, 105, 106

Q_1 = The value of $\left(\frac{n+1}{4} = \frac{15+1}{4} = 4\right)^{th}$ item in the ordered arrangement

= 21

Q_3 = The value of $\left(\frac{3(n+1)}{4} = \frac{3 \times 16}{4} = 12\right)^{th}$ item in the ordered arrangement

= 100

$$\therefore \quad \text{Q.D.} = \frac{Q_3 - Q_1}{2} = \frac{100-21}{2} = 39.5$$

$$\text{Coefficient of Q.D.} = \frac{Q_3 - Q_1}{Q_3 + Q_1} = 0.6529$$

Illustration 7 : Compute Q.D. and Coefficient of Q.D. for the following frequency distribution.

Daily Wages (in ₹)	below 35	35–40	40–45	45–50	50–55	55–60	60–65	above 65
No. of workers	12	18	22	26	36	23	19	8

Solution :

Class	**Frequency**	**Less than type cumulative frequency**	
below 35	12	12	
35–40	18	30	
40–45	22	52	→ Q_1 class
45–50	26	78	
50–55	36	114	
55–60	23	137	→ Q_3 class
60–65	19	156	
above 65	8	164 = N	

$$Q_1 = \text{The value of } \left(\frac{N}{4} = \frac{164}{4} = 41\right)^{st} \text{ observation}$$

Therefore, (40–45) is Q_1 class

$$\therefore \quad Q_1 = l + \frac{N/4 - C.F.}{f} \times h = 40 + \frac{41 - 30}{22} \times 5 = 42.5$$

$$Q_3 = \text{The value of } \left(\frac{3N}{4} = \frac{3 \times 164}{4} = 123\right)^{rd} \text{ observation}$$

Therefore, Q_3 lies in (55–60)

$$\therefore \quad Q_3 = l + \frac{3N/4 - C.F.}{f} \times h = 55 + \frac{123 - 114}{23} \times 5 = 56.9565$$

$$\therefore \quad Q.D. = \frac{Q_3 - Q_1}{2} = \frac{56.9565 - 42.5}{2} = 7.2283$$

$$\text{Coefficient of Q.D.} = \frac{Q_3 - Q_1}{Q_3 + Q_1} = \frac{55.9565 - 42.5}{55.9565 + 42.5} = 0.1454$$

Remark : One of the requisites of a good measure is that, it should be based on all the observations. However, Q.D. depends upon only two partition values. Therefore, it is not affected by any changes except the upper and lower quartile.

5.5 Standard Deviation and Coefficient of Variation

Here we discuss a measure of dispersion which satisfies most of the requisites of good measure and free from the drawbacks present in the other measures of dispersion.

Definition : The positive square root of mean of squares of the deviations taken from arithmetic mean is called as **standard deviation** (S.D.)

It is denoted by σ (read as sigma, a lower case Greek letter).

Therefore, $\sigma = \sqrt{\frac{\sum (x - \bar{x})^2}{n}}$ for individual observations

$= \sqrt{\frac{\sum f (x - \bar{x})^2}{N}}$ for frequency distributions

After simplification we can have computational formula for σ in more suitable form as follows :

$$\sigma = \sqrt{\frac{\sum x^2}{n} - \bar{x}^2} \quad \text{for individual observations}$$

$$= \sqrt{\frac{\sum fx^2}{N} - \bar{x}^2} \quad \text{for frequency distribution.}$$

where, $\bar{x}$ is a arithmetic mean.

Note : The quantity σ^2 is called as **variance.** Prof. R. A. Fisher has suggested the term variance.

Relative measure of S.D. is called coefficient of variation.

Coefficient of Variation : Prof. Karl Pearson suggested the relative measure of standard deviation. It is called as coefficient of variation (C.V.)

$$\text{It is given by C.V.} = \frac{\text{S.D}}{|\text{A.M.}|} \times 100 = \frac{\sigma}{|\bar{x}|} \times 100\% \quad \ldots (5.1)$$

Coefficient of variation is always expressed in percentage.

Remarks : (1) R.H.S. of (5.1) includes the multiplier 100, because $\frac{\sigma}{|\bar{x}|}$ is too small in many cases. Thus, for convenience it is multiplied by 100.

2. Frequently we need to compare dispersions of two or more groups. If the values in data set are large in magnitude, naturally variation among them will be proportionately larger.

For example, S.D. of weights of a group of elephants will be larger than that of a group of human beings. Suppose S.D. of weights of a group of elephants is 15 kg and that of human beings is also 15 kg. In this case we cannot say, both the groups have identical variation. This is because average weight of a group of elephants is larger than that of the average weight of a group of persons. Therefore for comparing variations between two different data sets, a measure based on the ratio of σ and $\bar{x}$ would be appropriate. This is achieved in coefficient of variation. It measures variation in all data sets using a common yard stick; moreover it is free from units.

3. According to Prof. Karl Pearson, C.V. is the percentage variation in mean whereas S.D. gives the total variation in the mean.

Uses of Coefficient of Variation :

It is already discussed that for comparison of variability, homogeneity, stability, uniformity, consistency, a unitless measure of dispersion is coefficient of variation (C.V.).

In manufacturing process C.V. is very important quantity. Larger the C.V., larger is the variation and poorer is the quality. In quality control section every effort is made to improve upon the quality, which means the items to be manufactured as per specifications. The extent of deviation from specifications can be measured by C.V. Thus, C.V. is unit of measurement of variation.

Almost all industries reduced the C.V. of their goods to considerable extent in last 50 years. This was due to competition. In pharmaceutical industries C.V. is as low as 1 or less than 1. The variation in weight of tablets is almost negligible.

Earlier the Japanese industrial product and American industrial product have same average quality, however, there was considerable difference in C.V. C.V. of Japanese goods was less than 5 times than that of American goods.

As a result of low C.V. the Japanese goods were more popular.

If C.V. is increased how it affects is explained below with the following example. Suppose we purchase a bag or pauch of edible oil packed by a automatic filling machine. Suppose the volume of oil is expected to be 1 litre. If the machine is set for C.V. = 1, (since

C.V. = 0 is impossible). Using statistical laws we can conclude that approximately 99.73% of the bags filled by machine will contain oil in the range 970 ml to 1030 ml. This range is reasonable for user. Instead if the machine is set to C.V. = 5, then 14% bags will found to contain 900 ml to 950 ml oil, another 2.1% will found to contain oil between 850 ml to 900 ml. Approximately 16% bags will contain 900 ml or less oil. Thus alongwith average one have take extreme care to reduce C.V.

Let us discuss an example from automobile industry. Suppose company A and B manufacture scooters which give 50 km per litre. Suppose C.V. of company A is 1 and that of company B is 5.

Among the scooters manufactured by company A, 99.73% will run 48.5 to 51.5 per litre. On the other hand among the scooters manufactured by company B, 14% will run 45 to 47.5 km per litre and 2.1% will run 42.5 to 45 km per litre. Thus, in case of C.V. = 5 about 16.1% customers will be unhappy. Although averages are same, they differ in C.V. which has considerable effect.

C.V. of industrial product depends upon raw material. Hence, a good quality of raw material ultimately give homogeneous end product.

In chemical and pharmaceutical industries C.V. is reduced by thorough mixing, pounding to convert raw material into homogeneous end product.

C.V. and Least Count :

Use of proper measuring instrument is also a way to check whether C.V. is maintained properly. If appropriate instrument is not used, C.V. will be inflated. As a thumb rule in industry.

Least count = $\frac{1}{10}$ specified range.

For example, if the inner diameter of cylinder is required to be between 0.95 cm and least count of the instrument should be $\left(\frac{1}{10}\right)^{th}$ of the specified range which $\frac{1}{10}(1.05 - 0.95)$ = 0.01 cm = 0.1 mm.

Illustration 8 : *Compute S.D. and C.V. for the following data :*

36, 15, 25, 10, 14.

Solution :

						Total
x	36	15	25	10	14	100
x^2	1296	225	625	100	196	2442

$$\bar{x} = \frac{\sum x}{n}$$

$$= \frac{100}{5} = 20$$

$$\sigma = \sqrt{\frac{\sum x^2}{n} - \bar{x}^2}$$

$$= \sqrt{\frac{2442}{5} - 20^2}$$

$$= \sqrt{88.4} = 9.4021$$

$$\text{C.V.} = \frac{\sigma}{|\bar{x}|} \times 100$$

$$= 47.0106\ \%$$

In order to reduce the bulk of calculation similar to mean we can use 'deviation method' and 'step deviation method' to calculate S.D.

S.D. by Deviation Method :

Step 1 : Decide assumed mean 'a'.

Step 2 : Let $d = x - a$. Compute deviation 'd'.

Step 3 : Find sum of deviations and sum of squares of deviations

$\sum d$, $\sum d^2$. for individual observations.

$\sum fd$, $\sum fd^2$, for frequency distribution.

Step 4 : Apply formula and find S.D. as follows :

$$\sigma = \sqrt{\frac{\sum d^2}{n} - \left(\frac{\sum d}{n}\right)^2} \quad ; \quad \text{for individual observations}$$

$$\sigma = \sqrt{\frac{\sum fd^2}{N} - \left(\frac{\sum fd}{N}\right)^2} \quad ; \quad \text{for frequency distribution}$$

Illustration 9 : *Compute S.D. and C.V. of marks scored by 10 candidates given below : 54, 61, 64, 69, 58, 56, 49, 57, 55, 50.* **(April 2015)**

Solution : Let $a = 57$, $d = x - 57$

x	54	61	64	69	58	56	49	57	55	50	Total
d	−3	4	7	12	1	−1	−8	0	−2	−7	3
d^2	9	16	49	144	1	1	64	0	4	49	337

$$\sigma = \sqrt{\frac{\sum d^2}{n} - \left(\frac{\sum d}{n}\right)^2} = \sqrt{\frac{337}{10} - \left(\frac{3}{10}\right)^2}$$

$$= \sqrt{33.61} = 5.7974$$

C.V. requires $\bar{x}$, hence $\bar{x} = a + \frac{\sum d}{n} = 57.3$

$$\text{C.V.} = \frac{\sigma}{|\bar{x}|} \times 100 = \frac{5.7974}{57.3} \times 100 = 10.1176\ \%$$

S.D. by Step Deviation Method :

Step 1 : Decide assumed mean 'a'.

Step 2 : Find the deviations, $d = x - a$.

Step 3 : Find the step deviations, $d' = \frac{d}{h}$.

Step 4 : Find the sum of d' and d'^2.

$\sum d', \sum d'^2$; for individual observations

$\sum fd', \sum fd'^2$; for frequency distribution

Step 5 : Apply the formula.

$$\sigma = \sqrt{\frac{\sum d'^2}{n} - \left(\frac{\sum d'}{n}\right)^2} \times h \quad ; \quad \text{for individual observations.}$$

$$\sigma = \sqrt{\frac{\sum fd'^2}{N} - \left(\frac{\sum fd}{N}\right)^2} \times h \; ; \quad \text{for frequency distribution.}$$

Illustration 10 : *Calculate the standard deviation and coefficient of variation for the frequency distribution of marks of 100 candidates given below :*

Marks	*0–20*	*20–40*	*40–60*	*60–80*	*80–100*
Frequency	*5*	*12*	*32*	*40*	*11*

Solution : We use step-deviation method to find σ.

Class	**Mid-values x**	**Freq. f**	$\mathbf{d' = \frac{x-50}{20}}$	**f × d'**	**f × d'²**
00-20	10	5	– 2	– 10	– 10 × – 2 = 20
20-40	30	12	– 1	– 12	– 12 × – 1 = 12
40-60	50	32	0	0	0
60-80	70	40	1	40	40 × 1 = 40
80-100	90	11	2	22	22 × 2 = 44
Total	–	100	–	40	$\sum fd'^2 = 116$

Here, a = 50, h = 20, N = 100

$$\text{Mean} = a + \frac{\sum fd'}{N} \times h$$

$$= 50 + \frac{40}{100} \times 20 = 58$$

$$\text{S.D.} = \sqrt{\frac{\sum fd'^2}{N} - \left(\frac{\sum fd'}{N}\right)^2} \times h$$

$$\sigma = \sqrt{\frac{116}{100} - \left(\frac{40}{100}\right)^2} \times 20 = 20$$

$$C.V. = \frac{\sigma}{|\bar{x}|} \times 100$$

$$= \frac{20}{58} \times 100 = 34.4828\ \%$$

Merits of S.D. :

1. It is based on all observations.
2. It is rigidly defined.
3. It is capable of further mathematical treatment.
4. It does not ignore algebraic signs of deviations.
5. It is not much affected by sampling fluctuations.

Demerits of S.D. :

1. It is difficult to understand and to calculate.
2. It cannot be computed for a distribution with open end class.
3. It is unduly affected due to extreme deviations.
4. It cannot be calculated for qualitative data.

Important Notes :

1. If all the observations are increased (or decreased) by a constant, S.D. remains the same.
2. If each of the observation is multiplied by constant K. then S.D. is K times the original S.D.
3. If all the observations are equal, S.D. is zero (why ?).
4. If data contains only one observation, S.D. is zero (why ?)

As far as variance is concerned smaller variance is better in many situations. However there are some situations in genetical sciences where larger variance is better.

Variance and standard deviation are used in number of situations. Some of them are discussed below :

(a) Precision of an instrument is inversely proportional to variance. Therefore precision $= \frac{k}{\text{variance}}$.

(b) In portfolio analysis, risk is described in terms of variance of prices of shares.

(c) For the comparison of performance of two or more instruments, machines, coefficient of variation is used.

(d) The spread of variable is approximately taken as $(\bar{x} - 3\sigma, \bar{x} + 3\sigma)$.

Thus standard deviation helps in estimating lower limit and upper limit of the items.

5.6 Standard Deviation of Combined Group

Suppose there are two groups with sizes n_1, n_2 having arithmetic means $\bar{x}_1$, $\bar{x}_2$; standard deviations σ_1, σ_2 respectively. Then the mean of combined group is

$$\bar{x}_c = \frac{n_1 \bar{x}_1 + n_2 \bar{x}_2}{n_1 + n_2}$$

Let $d_1 = \bar{x}_1 - \bar{x}_c$ and $d_2 = \bar{x}_2 - \bar{x}_c$. Then S.D. of combined group is given by.

$$\sigma_c = \sqrt{\frac{n_1(\sigma_1^2 + d_1^2) + n_2(\sigma_2^2 + d_2^2)}{n_1 + n_2}}$$

Illustration 11 : *A group of 50 items have mean and standard deviation 61 and 8 respectively. Another group of 100 observations has mean and standard deviation 70 and 9 respectively. Find mean and standard deviation of combined group.*

Solution : We are given that : $n_1 = 50$, $\bar{x}_1 = 61$, $\sigma_1 = 8$, $n_2 = 100$, $\bar{x}_2 = 70$ and $\sigma_2 = 9$. Therefore combined mean

$$\bar{x}_c = \frac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2}$$

$$= \frac{(50 \times 61) + (100 \times 70)}{50 + 100} = 67$$

$\therefore$ $d_1 = \bar{x}_1 - \bar{x}_c = 61 - 67 = -6$ and $d_2 = \bar{x}_2 - \bar{x}_c = 70 - 67 = 3$.

$\therefore$ Combined S.D. is

$$\sigma_c = \sqrt{\frac{n_1(\sigma_1^2 + d_1^2) + n_2(\sigma_2^2 + d_2^2)}{n_1 + n_2}}$$

$$\sigma_c = \sqrt{\frac{50(64 + 36) + 100(81 + 9)}{150}}$$

$$= 9.6609$$

Illustration 12 : *The mean weight of 150 students is 60 kg. The mean weight of boys is 70 kg, with standard deviation of 10 kg. For girls the mean weight is 55 kg with standard deviation of 15 kg. Find the number of boys and combined standard deviation.*

Solution : Let there be n_1 boys with mean $\bar{x}_1$ and S.D. σ_1. Similarly, there be n_2 girls with mean $\bar{x}_2$ and standard deviation σ_2. Hence, we get : $n_1 + n_2 = 150$, $\bar{x}_c = 60$, $\bar{x}_1 = 70$, $\bar{x}_2 = 55$, $\sigma_1 = 10$, $\sigma_2 = 15$.

$$\bar{x}_c = \frac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2}$$

$\therefore$ $$60 = \frac{70n_1 + 55n_2}{n_1 + n_2}$$

$$60n_1 + 60n_2 = 70n_1 + 55n_2$$

$$n_2 = 2n_1 \quad \ldots (1)$$

Note that

$$n_1 + n_2 = 150$$

$$\therefore \quad n_1 + 2n_1 = 150 \qquad \text{... from (1)}$$

$$n_1 = 50$$

$\therefore$ Number of boys = 50.

We get $d_1 = \bar{x}_1 - \bar{x}_c = 70 - 60 = 10$ and

$$d_2 = \bar{x}_2 - \bar{x}_c = 55 - 60 = -5$$

$\therefore$ Combined standard deviation

$$\sigma = \sqrt{\frac{n_1\left(\sigma_1^2 + d_1^2\right) + n_2\left(\sigma_2^2 + d_2^2\right)}{n_1 + n_2}}$$

$$\therefore \quad \sigma = \sqrt{\frac{50\,(100 + 100) + 100\,(225 + 25)}{150}}$$

$$= 15.2753 \text{ kg.}$$

Illustration 13 : *The mean and standard deviation of 10 observations were 9.5 and 2.5 respectively. If one more observation with value 15 is included in the group, obtain the mean and standard deviation of these 11 observations.*

Solution : Let there be two groups, first group of original 10 observations and second group of new single observation. Hence,

$$n_1 = 10, \qquad n_2 = 1$$

$$\bar{x}_1 = 9.5, \qquad \bar{x}_2 = 15 \text{ (why ?)}$$

$$\sigma_1 = 2.5, \qquad \sigma_2 = 0 \text{ (why ?)}$$

Combined mean

$$\bar{x}_c = \frac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2}$$

$$= \frac{10 \times 9.5 + 15}{11} = 10$$

$\therefore$ $d_1 = \bar{x}_1 - \bar{x}_c = -0.5$ and $d_2 = \bar{x}_2 - \bar{x}_c = 5$

$$\therefore \quad \sigma_c = \sqrt{\frac{10\,(6.25 + 0.25) + (25 + 0)}{11}}$$

$$= 2.8604$$

Solved Examples

Example 5.1 : *The number of runs scored by cricketers A and B in 5 test matches are shown below :*

A	*5*	*20*	*90*	*76*	*102*	*90*	*6*	*108*	*20*	*16*
B	*40*	*35*	*60*	*62*	*58*	*76*	*42*	*30*	*30*	*20*

Find (i) which cricketer is better in average ? (ii) which cricketer is more consistent ?

Solution :

$$\text{Mean of A} = \frac{\sum x}{n} = \frac{533}{10} = 53.3$$

$$\text{S.D. of A} = \sqrt{\frac{\sum x^2}{n} - \left(\frac{\sum x}{n}\right)^2}$$

$$= \sqrt{\frac{45161}{10} - (53.3)^2}$$

$$= 40.9293$$

$$\therefore \quad \text{C.V. of A} = 76.79\%$$

$$\text{Mean of B} = \frac{\sum y}{n} = \frac{453}{10} = 45.3$$

$$\text{S.D. of B} = \sqrt{\frac{\sum y^2}{n} - \left(\frac{\sum y}{n}\right)^2} = \sqrt{\frac{23373}{10} - (45.3)^2}$$

$$= 16.8882$$

$$\therefore \quad \text{C.V. of B} = 37.28\%$$

(i) A gives better average runs (mean A > mean B).

(ii) B is more consistent (C.V. of B < C.V. of A).

Example 5.2 : *Arithmetic mean and standard deviation of 12 items are 22 and 3 respectively. Later on it was observed that the item 32 was wrongly taken as 23. Compute correct mean, standard deviation and coefficient of variation.*

Solution :

$$\text{Incorrect sum } (\sum x) = n \times \text{Incorrect mean} = 12 \times 22 = 264$$

$$\text{Correct } \sum x = \text{Incorrect } \sum x + \text{Correct item} - \text{Incorrect item}$$

$$\sum x = 264 - 23 + 32 = 273$$

$$\text{Correct mean} = \frac{273}{12} = 22.75$$

$$\sigma^2 = \frac{\sum x^2}{n} - (\bar{x})^2$$

$$\therefore \quad n\left[\sigma^2 + (\bar{x})^2\right] = \sum x^2$$

$$\therefore \quad \text{Incorrect } \sum x^2 = n\left[\sigma^2 + (\bar{x})^2\right] \qquad \text{with } \sigma \text{ and } \bar{x} \text{ incorrect.}$$

$$= 12\,(9 + 484) = 5916$$

$$\text{Correct } \sum x^2 = \text{Incorrect } \sum x^2 + (\text{Correct item})^2 - (\text{Incorrect item})^2$$

$$= 5916 + 32^2 - 23^2 = 6411$$

$$\text{Correct } \sigma = \sqrt{\frac{\sum x^2}{n} - \left(\frac{\sum x}{n}\right)^2} \qquad \text{with correct } \sum x^2 \text{ and } \sum x$$

$$= \sqrt{\frac{6411}{12} - (22.75)^2}$$

$$= \sqrt{16.6875} = 4.0850$$

$$\text{Correct C.V.} = \frac{\sigma}{|\bar{x}|} \times 100 = 17.9562\%$$

Example 5.3 : *For a set of 90 items the mean and standard deviation are 59 and 9 respectively. For 40 items selected from those 90 items the mean and standard deviation are 54 and 6 respectively. Find the mean and standard deviation of the remaining items.*

Solution : We have

Group 1	**Group 2**	**Combined Group**
$n_1 = 40$	$n_2 = 50$	$n = 90$
$\bar{x}_1 = 54$	$\bar{x}_2 = ?$	$\bar{x}_c = 59$
$\sigma_1 = 6$	$\sigma_2 = ?$	$\sigma_c = 9$

To find $\bar{x}_2$ we use $\bar{x}_c$.

$$\bar{x}_c = \frac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2} \text{ gives}$$

$$59 = \frac{40 \times 54 + 50\,\bar{x}_2}{90}$$

$$\therefore \quad \bar{x}_2 = 63.$$

$$\therefore \quad d_1 = \bar{x}_1 - \bar{x}_c = -5,\ d_2 = \bar{x}_2 - \bar{x}_c = 4$$

$$\sigma_c^2 = \frac{n_1(\sigma_1^2 + d_1^2) + n_2(\sigma_2^2 + d_2^2)}{n_1 + n_2}$$

$$81 = \frac{40\,(36 + 25) + 50\,(\sigma_2^2 + 16)}{90}$$

$$\therefore \quad \sigma_2 = 9.$$

Example 5.4 : *Given that : n = 10, $\sum(x-20) = 8$, $\sum(x-20)^2 = 762$. Find mean and S.D.*

Solution : Let $d = x - 20$, Hence $\bar{x} = 20 + \frac{\sum d}{n} = 20.8$

$$\text{S.D.} = \sqrt{\frac{\sum d^2}{n} - \left(\frac{\sum d}{n}\right)^2} = \sqrt{\frac{762}{10} - \left(\frac{8}{10}\right)^2} = 8.6925$$

Example 5.5 : *Compute standard deviation of the following frequency distribution :*

Weight (in Kg)	30–40	40–50	50–60	60–70	70–80
No. of standards	3	5	12	20	10

Solution :

Class	mid-point (x)	frequency (f)	$d' = \frac{x-55}{10}$	fd'	fd'²
30–40	35	3	−2	−6	12
40–50	45	5	−1	−5	5
50–60	55	12	0	0	0
60–70	65	20	1	20	20
70–80	75	10	2	20	40
Total		50	–	29	77

$$\sigma = h \cdot \sqrt{\frac{\sum fd'^2}{\sum f} - \left(\frac{\sum fd'}{\sum f}\right)^2} = 10\sqrt{\frac{77}{50} - \left(\frac{29}{50}\right)^2} = 10.9709$$

Example 5.6 : *The following data represents the goals scored by two teams in foot ball matches.* **(April 2015)**

Number of goals scored	0	1	2	3	4
No. of matches by Team A	20	12	8	3	2
No. of matches by Team B	18	10	7	6	4

Which team scores more goal in a average ? Which team is more consistent ?

Solution : In order to test the consistency, we have to determine coefficient of variation.

Let X = Number of goals scored.

f = Number of matches played.

Team A			
X	**f**	**fx**	**fx²**
0	20	0	0
1	12	12	12
2	8	16	32
3	3	9	27
4	2	8	32
Total	45	45	103

Team B			
X	**f**	**fx**	**fx²**
0	18	0	0
1	10	10	10
2	7	14	28
3	6	18	54
4	4	16	64
Total	45	58	156

$$\bar{X} = \frac{\sum fx}{\sum f} = \frac{45}{45} = 1$$

$$\sigma = \sqrt{\frac{\sum fx^2}{\sum f} - \bar{X}^2}$$

$$= \sqrt{\frac{103}{45} - \left(\frac{45}{45}\right)^2}$$

$$= 1.13529$$

$$\text{C.V. (A)} = \frac{\sigma}{\bar{X}} \times 100$$

$$= 113.529\ \%$$

$$\bar{X} = \frac{\sum fx}{\sum f} = \frac{58}{45} = 1.2889$$

$$\sigma = \sqrt{\frac{\sum fx^2}{\sum f} - \bar{X}^2}$$

$$= \sqrt{\frac{156}{45} - 1.2889^2}$$

$$= 1.3437$$

$$\text{C.V. (B)} = \frac{\sigma}{\bar{X}} \times 100$$

$$= 104.25\ \%$$

Conclusion :

1. Since $\bar{X}_B > \bar{X}_A$, team B is better in average performance.
2. Since C.V. (B) < C.V. (A), team B is more consistent than A.

Example 5.7 : The following is information regarding portfolios A and B.

Portfolio	Average return	Risk (variance)
A	10 % ($\bar{X}$)	15 (σ_1^2)
B	20 % ($\bar{Y}$)	30 (σ_2^2)

We assume that the portfolios are independent, find the average return and combined risk if (i) equal investment in both portfolios is considered (ii) 25% of the shares are from portfolio A and the remaining from portfolio B.

Case (i) : If we invest 50 % amount of total in each portfolio then

Average return (R) $= 0.5\,\bar{X} + 0.5\,\bar{Y}$ (**Result :** If $Z = ax + by$, then $\bar{Z} = a\,\bar{x} + b\,\bar{y}$)

$= 0.5 \times 10 + 0.5 \times 20$ Here $a = b = 0.5$

$= 15$

Combined risk $= 0.5^2\,\sigma_1^2 + 0.5^2\,\sigma_2^2$ (**Result :** $\sigma^2_{ax+by} = a^2\sigma_x^2 + b^2\,\sigma_y^2$)

$= 11.25$

Thus the combined risk reduces.

Case (ii) : If we invest 25 % in portfolio A and 75 % in portfolio B then

Average return $= 0.25\,\bar{X} + 0.75\,\bar{Y}$ ($a = 0.25,\ b = 0.75,\ \bar{z} = a\,\bar{x} + b\,\bar{y}$)

$= 17.5\ \%$

Combined risk $= 0.25^2\,\sigma_1^2 + 0.75^2\,\sigma_2^2$ ($a^2\sigma_1^2 + b^2\,\sigma_2^2$)

$= 17.81$

Note : One can determine the percentage of investment in each portfolio so that the total risk is minimum, similarly one can find investment pattern that will maximise the total return. The details are beyond the scope of book.

Example 5.8 : *Compute Range and Coefficient of Range for the daily wages (₹) of 8 workers in a factory : 90, 120, 150, 80, 120, 125, 105, 75.*

Solution : Largest observation (L) = 150

Smallest observation (S) = 75

Range = L – S = 150 – 75 = 75

$$\text{Coefficient of range} = \frac{L-S}{L+S} = \frac{75}{150+75} = \frac{75}{225} = \frac{1}{3}$$

Example 5.9 : *Compute Standard Deviation for the following data :*
15, 18, 22, 25, 10.

Solution :

						Total
x	15	18	22	25	10	90
x^2	225	324	484	625	100	1758

$$\text{Standard deviation } (\sigma) = \sqrt{\frac{\sum x^2}{n} - \bar{X}^2}$$

$n = 5, \ \bar{X} = \frac{\sum x}{n} = \frac{90}{5} = 18$

$$\therefore \quad \sigma = \sqrt{\frac{1758}{5} - 18^2} = \sqrt{351.6 - 324} = \sqrt{27.6}$$

$$= 5.2536$$

Example 5.10 : *Two workers on the same job show the following results over long period of time :*

	Worker 'A'	***Worker 'B'***
Mean time of completing the job (in minutes)	*30*	*25*
Standard deviation	*6*	*4*

(i) Which worker appears to be more consistent in the time he requires to complete the job ? Why ?

(ii) Which worker is faster in completing the job ? Why ?

Solution : (i) Consistency is compared by coefficient of variation (C.V.).

$$\text{C.V. (A)} = \frac{\sigma_A}{\bar{X}_A} \times 100 = \frac{6}{30} \times 100 = 20\%$$

$$\text{C.V. (B)} = \frac{\sigma_B}{\bar{X}_B} \times 100 = \frac{4}{25} \times 100 = 16\%$$

Since C.V. (B) < C.V. (A), where, B is more consistent.

(ii) Worker is faster if the mean time required is smaller, since $\bar{X}_B = 25 < \bar{X}_A = 30$, Worker B is more faster.

Case Study

Parag Infotech Pvt. Ltd. is a company to provide a software solutions. Directors of the company have taken a decision to double the capital and expand it in a big way. In view of this company decides to recruit at least 50 computer engineers. Company invited applications from fresh computer engineering graduates having at least 70% marks at their final examination. Company also expected furnish details of marks obtained from their SSC examination onwards.

Company received 200 applications. Most of the applications have secured marks between 70% to 73% in their final examination. Due to short time to recruit, company is not interested to conduct personal interview of all the applicants but to select 70 of the best applicants for personal interview of the final selection. Company feels that 2% to 3% variation in final examination marks may be due to chance and has no effect in the performance.

Statisticians have advised to company to use the concept of measures of dispersion. Discuss the use of range and standard deviation in this regard to take the proper decision.

Points to Remember

1. Range = Largest observations – Smallest observation.

   $$\text{Coefficient of range} = \frac{\text{Largest observation} - \text{Smallest observation}}{\text{Largest observation} + \text{Smallest observation}}$$

2. $$\text{Standard deviation (S.D.)} = \sigma = \sqrt{\frac{\sum x^2}{n} - \bar{X}^2} \quad \text{for discrete series}$$

   $$= \sqrt{\frac{\sum fx^2}{\sum f} - \bar{X}^2} \quad \text{for frequency distribution}$$

3. $$\text{Coefficient of variation (C.V.)} = \frac{\sigma}{\bar{X}} \times 100\%.$$

4. C.V. is used for the comparison of variation.

Exercise

[A] Theory Questions :

1. What is dispersion ? What purpose does it serve in the study of distribution ?
2. What type of measures will you use for comparison of dispersion in different distributions ? Mention any two of such measures.
3. Explain relative measure of dispersion and state its utility.
4. Define : Range, quartile deviation and standard deviation. State the formula for each incase of ungrouped data and frequency distribution.
5. Compare critically the two measures of dispersion : range and standard deviation.

6. State the merits and demerits of each of the following measures of dispersion : range and S.D.
7. Explain why S.D. is the best measure of dispersion.
8. What is utility of C.V. ?
9. Write a note on dispersion.
10. Write a note on measures of dispersion.

[B] Discrete Series :

1. Find the standard deviation of the following data : 2, 3, 5, 2, 7, 5, 7, 6, 11, 12.
2. Find the arithmetic mean and standard deviation of the following series

   14, 8, 11, 10, 13, 16, 5, 9, 12, 2.
3. Monthly consumption of electricity in units of a certain family in a year is given below :

   210, 207, 315, 250, 240, 232, 216, 208, 209, 315, 300, 200.

   Compute : (i) range and coefficient of range

   (ii) standard deviation and coefficient variation.
4. Calculate the range and coefficient of range for the following data :

   88, 52, 67, 38, 59, 46. Also compute standard deviation.
5. Monthly consumption of electricity in units of six families in a city is given below. Compute coefficient of variation.

   210, 207, 315, 320, 250, 240.
6. Compute the (i) range and the coefficient of range (ii) quartile deviation and coefficient of quartile deviation for the following data :

   8, 12, 10, 18, 28, 17, 20, 22, 12, 9, 16.

   Also find the new range and coefficient of range in which each observation is doubled.
7. Calculate the range and the coefficient of range for the following data :

   125, 140, 110, 105, 130, 95, 115, 125, 80.
8. Compute the range and coefficient of range for the data given below :

   52, 45, 60, 53, 48, 65, 42, 45, 60.
9. The prices of shares of a company from Monday to Friday are as follows :

Days	Mon.	Tues.	Wed.	Thur.	Fri.
Price (₹)	524	502	544	519	558

   Calculate the range and the coefficient of range.
10. Compute the standard deviation for the following data :

    15, 18, 22, 25, 10.
11. Calculate the coefficient of variation for the following series :

    12, 18, 15, 20, 16.

12. Find the standard deviation and coefficient of variation for the following data :

6, 4, 5, 3, 12, 10.

13. Which of the following two series A and B is more stable ? Why ?

A	4	4	2	3	6	8	2	0	1	– 1
B	8	7	5	5	6	7	4	3	4	1

14. Using coefficient of variation find which of the following batsman is more consistent in scoring :

Score of A	42	115	6	73	7	19	119	36	84	29
Score of B	47	12	76	42	4	51	37	48	13	0

15. Compare the variation between the weight and the height of a group of 10 persons using coefficient of variation.

Sr. No.	1	2	3	4	5	6	7	8	9	10
Weight (kg)	70	65	65	64	69	63	65	70	71	62
Height (cm)	170	140	151	145	165	167	156	160	153	168

[C] Frequency Distribution :

16. A survey conducted to determine distance travelled (in kms) per litre of petrol by newly introduced motorcycle gives the following distribution :

Distance (km)	40-45	45-50	50-55	55-60	60-65
No. of Motorcycles	10	17	23	40	10

Find the (i) standard deviation (ii) quartile deviation, (iii) coefficient of quartile deviation.

17. Find the variance for the following frequency distribution :

Class	5-15	15-25	25-35	35-45	45-55
Frequency	05	15	12	18	08

18. Compute the standard deviation for the following data :

Marks	0-10	10-20	20-30	30-40	40-50
No. of Students	3	7	25	20	5

19. Calculate the coefficient of variation (C.V.) for the following data :

Class	0-10	10-20	20-30	30-40	40-50	50-60
Frequency	5	9	15	21	6	4

20. Calculate the standard deviation and coefficient of variation for the following frequency distribution :

X	2	4	6	8	10
Frequency	2	4	14	8	2

21. Find the standard deviation and quartile deviation from the following data :

Marks	0 - 10	10 - 20	20 - 30	30 - 40	40 - 50
Frequency	10	16	30	32	12

22. Find the standard deviation and coefficient of variation and quartile deviation of distribution of daily wages.

Daily wages	1 - 20	21 - 40	41 - 60	61 - 80	81 - 100
Frequency	5	32	45	17	1

23. Compute the coefficient of variation and coefficient of quartile deviation for the following data :

Class	0 - 20	20 - 40	40 - 60	60 - 80	80 - 100
Frequency	6	32	45	17	0

24. Obtain the standard deviation for the following data :

Class	20 - 40	40 - 60	60 - 80	80 - 100
Frequency	6	8	4	2

25. Compute the standard deviation of the following frequency distributions :

Marks	0 - 10	10 - 20	20 - 30	30 - 40	40 - 50
No. of students	1	3	10	4	2

26. Find the coefficient of variation, quartile deviation and coefficient of quartile deviation for the following data :

Size of item	2	4	6	8	10	12
No. of items	6	10	20	24	12	8

27. A share broker studied 100 companies and obtained the following data for the year 2012-13.

Divident declared (%)	0 - 8	8 - 16	16 - 24	24 - 32	32 - 40
No. of companies	15	30	40	10	5

Calculate the mean and the standard deviation of the above data and obtain the coefficient of variation.

28. (a) Two automatic tea filling machines A and B tested for the performance. Machines are supposed to fill 500 gm. tea in each packet. A random sample of 100 filled packets on each machine showed the following distribution.

Weight in gm.	Frequency A	Frequency B
485–490	12	10
490–495	18	15
495–500	20	24
500–505	22	20
505–510	24	18
510–515	4	13

Which machine is more consistent ? Why ?

(b) Find the quartile deviation and its coefficient for the following frequency distribution. **(P.U. 2011)**

Class	0-20	20-40	40-60	60-80	80-100
Frequency	3	12	20	10	5

[D] Combined Standard Deviation :

29. Find combined deviation from the following data :

Workers	Number	Average Salary	Standard Deviation
Male	80	1520	06
Female	20	1420	05

30. Two workers on the same job show the following results over long period of time :

	Worker 'A'	Worker 'B'
Mean time of completing the job (in minutes)	30	24
Standard Deviation	6	4
Number of jobs	10	10

(i) Which worker appears to be more consistent in the time he requires to complete the job ? Why ?

(ii) Which worker is faster in completing the job ? Why ?

(iii) Find the combined mean and standard deviation of the two workers together.

31. For a set of 50 items, the mean and standard deviation are 60 and 3 respectively. For another set of 100 items, the mean and standard deviation are 63 and 4 respectively. Find the mean and the standard deviation of combined group.

32. Information about the daily salaries of employees in firms A and B is stated below :

Firm	No. of employees	Mean Salary	S.D. of Salary
A	60	₹ 400	₹ 10
B	40	₹ 500	₹ 11

(i) Which firm gives more amount as salary ?

(ii) Which firm has smaller variation in salary ?

(iii) Find the combined mean and S.D. of two firms.

33. Information regarding daily salaries of two companies A and B is given below :

	Company A	Company B
No. of workers	600	400
Mean salary	₹ 180	₹ 200
S.D. of salary	₹ 9	₹ 10

(i) Which company pays larger salary ?

(ii) Which company has less variation in salaries ?

(iii) Find combined mean and S.D. of two firms A and B.

34. Find the combined standard deviation of groups A and B taken together given that :

Group	Size	Arithmetic mean	Standard deviation
A	100	60	6
B	200	63	4

35. Find the combined mean and standard deviation from the following data.

Group	Arithmetic mean	S.D.	Size
A	50	10	100
B	55	11	150

36. Find the arithmetic means of each group from the following data :

Group	S.D.	C. V.
1	16	40%
2	20	50%

37. The arithmetic mean and standard deviation of a group of 50 items are 61 and 8 respectively. In a second group of 100 items they are 70 and 9 respectively. Find the combined mean and S.D. of the two groups.

38. The means of two samples of sizes 50 and 100 are 40 and 25 respectively. The standard deviations of those samples are 10 and 8 respectively. Obtain the combined standard deviation.

39. The arithmetic mean and the standard deviation of the values of 100 items in a group are 80 and 5 respectively. In a second group of 25 items, each item has a value equal to 60. Find the combined standard deviation of two groups taken together.

40. Calculate the combined variance of the two groups of items.

	Group I	Group II
No. of observations	40	60
Arithmetic mean	25	30
Standard deviation	6	4

[E] Miscellaneous Problems :

41. The arithmetic mean and standard deviation of 20 observations are 10 and 2 respectively. Later on it was noticed that item 8 taken was incorrect. Calculate arithmetic mean and standard deviation if

    (i) the wrong item is omitted.

    (ii) the wrong item is replaced by 12.

42. The mean and standard deviation of 100 observations are 40 and 5.1 respectively. It was later discovered that an observation 40 was misread as 50. Calculate correct mean and standard deviation.
43. If $n = 10$, $\sum (x - 120) = 20$, $\sum (x - 120)^2 = 200$. Find the mean and the standard deviation.
44. If $n = 100$, $\sum x = -20$, $\sum x^2 = 220$, find standard deviation and coefficient of variation.
45. Find the standard deviation of set A, Set B, Set C and Set D and comment on findings.

Set A :	1	2	3	4	5
Set B :	11	12	13	14	15
Set C :	10	20	30	40	50
Set D :	4	4	4	4	4

46. The range, arithmetic mean and standard deviation of 10 items are 20, 62, 10 respectively. If each observation is increased by 5, what will be the range, arithmetic mean and standard deviation.

Answers

[B]

1. 3.2558.
2. $\bar{X} = 10$, $\sigma = 4$
3. (i) 115, 0.2233 (ii) 41.95, 17.35%
4. Range = 50, Coefficient of range = 0.3968, $\sigma = 16.1314$
5. 45.4239%
6. Range = 20, Coefficient of range = 0.5555
   New range = 40, $Q_1 = 10$, $Q_3 = 20$, Q.D. = 5,
   Coefficient of quartile deviation = 0.3333. New coefficient of range = 0.5555
7. Range = 60, Coefficient of range = 0.2727
8. Range = 23, Coefficient of range = 0.215.
9. Range = 56, Coefficient of range = 0.0528
10. 5.2536
11. 16.75%
12. $\sigma = 3.2489$, C.V. = 48.73%
13. Series B more stable, C.V. (A) = 89.1898% C.V. (B) = 40%
14. B is more consistent C.V. (A) = 75.54 %, C.V. (B) = 70.82 %
15. C.V. (weight) = 4.67 % < C.V. (height) = 6.18 %

[C]

16. C.V. = 5.7383, $Q_1 = 19.375$, $Q_3 = 35.9375$, Q.D. = 16.5625,
    Coefficient of Q.D. = 0.2994.
17. 144.14

18. 9.5029
19. $\sigma = 12.8279$, C.V. 43.73%
20. $\sigma = 1.9137$, C.V. = 30.5378
21. S.D. = 11.4891, $Q_1 = 33$, $Q_3 = 57.3889$, Q.D. = 12.19445, Coefficient of Q.D. = 0.2698.
22. S.D. = 16.4572, C.V. = 35.85 %
23. C.V. = 36.3505, $Q_1 = 31.875$, $Q_3 = 56.444$, Q.D. = 12.293, Coefficient of Q.D. = 0.2782
24. S.D. = 18.8680, $Q_1 = 26$, $Q_3 = 12$, Q.D. = 6, Coefficient of Q.D. = 1/3
25. 9.6307
26. 37.3625 %
27. $\bar{x} = 16.8$, $\sigma = 8.1584$, C.V. = 48.56 %
28. (a) C.V. (A) = 1.4294 % C.V. (B) = 1.5084 %, Machine A is more consistent.
    (b) $Q_1 = 35.8333$, $Q_2 = 65$, Q.D. = 14.5834, Coefficient of Q.D. = 0.2893.

[D]

29. 40.42029
30. (i) C.V. (A) = 20% > C.V. (B) = 16.6667%, B is more consistent
    (ii) B (iii) Combined mean = 27, Combined S.D. = 5.9161
31. Combined mean = 62, Combined S.D. = 3.9581
32. (i) B (ii) C.V. (A) = 2.5% > C.V. (B) = 2.2%, B has smaller variation
    (iii) Combined mean = 440, Combined S.D. = 50.0839
33. (i) B (ii) Both name same C.V. = 5%, both are equal in variation
    (iii) Combined mean = 188, Combined S.D. = 14.2969%
34. 4.97
35. $\bar{x}_c = 53$, $\sigma_c = 10.8904$
36. 40, 40
37. $\bar{x}_c = 67$, $\sigma_c = 9.6605$
38. 11.225
39. $\sigma_c = 9.1651$
40. $\sigma_c = 5.477$

[E]

41. (i) Mean = 10.1053, S.D. = 1.9922 (ii) Mean = 10.2, S.D. = 1.99
42. Mean = 39.9, S.D. = 5
43. Mean = 116, S.D. = 22.9783
44. S.D. = 1.4697, C.V. = 734.8469%
45. $\sigma_A = \sigma_B = \sqrt{2}$, $\sigma_C = 10\sqrt{2}$, $\sigma_D = 0$.
46. Range = 20, $\bar{x} = 67$, $\sigma = 10$.

Objective Type Questions

1. Find the standard deviation of 2, 2, 2, 2, 2.
2. If the standard deviation of 1, 2, 3, 4, 5 is $\sqrt{2}$ then state the standard deviation of
   (a) 11, 12, 13, 14, 15 (b) 10, 20, 30, 40, 50 (c) $-1, -2, -3, -4, -5$.
3. If each observation is doubled what will be the standard deviation ?
4. If each observation is increased by 5, what will be the standard deviation ?
5. Suppose there are two groups with following details :

Group	A	B
Size	10	10
Arithmetic mean	50	50
Standard deviation	4	6

Find the standard deviation of the combined groups.

Answers

1. 0 2. (a) $\sqrt{2}$ (b) $10\sqrt{2}$ (c) $\sqrt{2}$
2. S.D. will be doubled.
3. S.D. will be change.
5. $\sigma_c = \sqrt{26}$.

❑❑❑

Chapter 6...

Correlation and Regression

Contents ...

Key Words :

Bivariate Data, Correlation, Scatter Diagram, Covariance, Karl Pearson's Coefficient of Correlation, Ranks, Rank Correlation, Regression Lines, Regression Coefficients, Coefficient of Determination, Standard Error of Regression Estimate.

Objectives :

In this chapter we study the technique to bivariate data to know whether there is any interrelationship between them. A particular type of relationship viz. the extent of linear relationship is being measured using correlation. Such measure is developed for quantitative and qualitative data.

The relation between correlated variables can be established using regression analysis. It is useful in forecasting or prediction of one variable when the value of other variable is known. The estimates are more reliable if the r^2 is larger.

6.1 Introduction

Many a times we come across situations where two variables are interrelated.

For example : (i) Marks and intelligence quotient of students. (ii) Rainfall and agricultural production. (iii) Demand and price of a certain commodity. (iv) Income and expenditure of a family. (v) Height of son and that of father. In these situations we may be interested in examining the relation between the two variables. Such interrelated variables are called as *correlated variables.* The extent of linear relation between the two variables is called as *correlation.*

Bivariate Data :

In order to determine correlation, we require data regarding two concerned variables. These data are called as *bivariate data.* Suppose X and Y are the variables under consideration.

Whenever the variables X and Y are the variables measured on the same item, they are likely to be correlated.

For example, the income of family (X) and the expenditure of family (Y). We record the values of X and Y for each of the families under study. Suppose it gives a set of n pairs (x_1, y_1); (x_2, y_2); ... ; (x_n, y_n) where x is income and y is the expenditure of the family. This set of n pairs is a **bivariate data.** When n is large, for convenience the data are expressed in bivariate frequency distribution or two-way frequency distribution. In this case we make m classes of X and n classes of Y. Like univariate classification, pairs (x, y) are classified by using tally marks $(i, j)^{th}$ class. Number of tally marks is denoted by f_{ij}.

Remark : Note that (x_i, y_i) is an ordered pair. First component in every pair is observation on variable X and second component is on variable Y. In the further analysis the components X_i and Y_i are inseparable, i.e. we cannot rearrange the pairs as (x_1, y_{10}) or (x_3, y_4) etc.

6.2 Types of Correlation (April 2015, Oct. 2014)

Positive Correlation, Negative Correlation, No Correlation

It may be noticed that in some cases, increase in value of one variable is associated with increase in value of other variable or decrease in value of one variable is associated with decrease in value of other variable. Correlation between these variables is said to be **positive**.

For example : Marks and intelligence quotient. In this case, there is a positive correlation between these variables.

On the other hand in some other situations increase in value of one variable is accompanied by decrease in value of other variable and vice-versa. Here the changes in values of two variables are in opposite direction. Correlation between these variables is said to be **negative**. **(April 2010)**

For example : Consider supply and price of commodity. Clearly if supply of commodity is more, price falls down and if there is a scarcity of a commodity, then price goes up. Hence, there is a negative correlation between supply and price of a commodity.

Soimetimes, change in one variable is not related to change in other variable then we say that there is **no correlation**.

For example : Height of student and his examination score.

There are several measures of correlation of which three are in general used :
(i) Scatter diagram, (ii) Product moment correlation coefficient and (iii) Rank correlation.

6.3 Scatter Diagram

In order to visualise the correlation between two variables, the first step is scatter diagram.

Suppose $\{(x_i, y_i); i = 1, 2, \ldots, n\}$ are bivariate data on two variables x and y.

If these n pairs are plotted on a graph paper, taking one of the variable on X axis and other on Y axis, we get a diagram called as *Scatter diagram.* With the help of scatter diagram we get a general idea about the existence of correlation and the type of correlation. However, it fails to give correct numerical value of correlation. It is easy but crude and approximate method of measuring correlation. In this method we need to find out correlation by visual judgement only. We classify scatter diagrams broadly into 5 categories which are depicted below in Fig. 6.1 to Fig. 6.8.

Fig. 6.1 : Positive Perfect Correlation

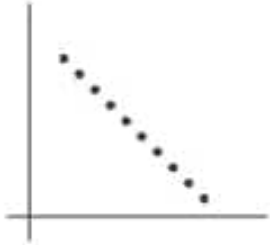

Fig. 6.2 : Negative Perfect Correlation

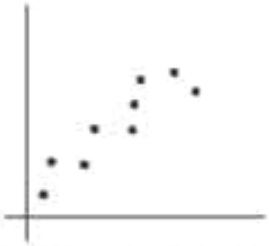

Fig. 6.3 : Positive Correlation

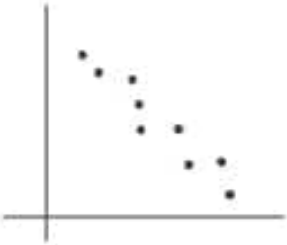

Fig. 6.4 : Negative Correlation

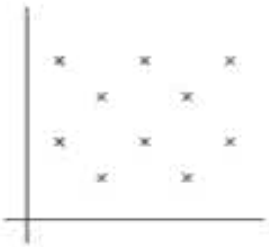

Fig. 6.5 : No Correlation

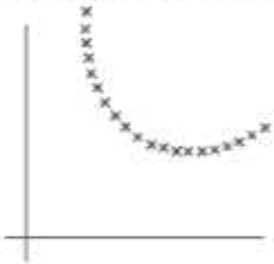

Fig. 6.6 : Non-linear Correlation

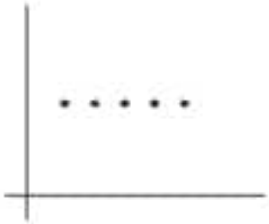

Fig. 6.7 : No Correlation

Fig. 6.8 : No Correlation

In Fig. 6.1 and Fig. 6.3 we see that the changes in value of one variable and changes in value of other variable are in the same direction. Hence, the correlation is positive or direct. Moreover in Fig. 6.1 all the points lie on the same line, hence correlation is perfect positive.

In Fig. 6.2 and Fig. 6.4 we see that changes in values of one variable and those of other variable are in opposite direction. Hence, the correlation is negative or inverse. Specifically in Fig. 6.2 we observe that points fall on the same line. This is an indication of negative perfect correlation. In Fig. 6.5 we see that the points are scattered in a haphazard manner without showing any particular pattern. This is an indication of almost no correlation. In Fig. 6.6 points show non-linear pattern.

In Fig. 6.7 and 6.8 one of the variables is not really a variable. It is a constant. It does not increase or decrease for any type of change in the other variable. Thus, change in one variable is not at all associated with that of in the other variable. Hence, in this situation, there is no correlation between the two variables. This type of scatter diagram will be observed in the following situations.

For example : Suppose X is Interest on debenture. Y is Dividend paid on shares. X is fixed, whereas Y depends upon company's profit. Clearly there is no correlation between X and Y.

Thus, we can draw conclusions regarding correlation between two variables by means of scatter diagram.

6.4 Merits and Demerits of Scatter Diagram

Merits :

1. Scatter diagram is the simplest method of studying correlation.
2. It is easy to understand.
3. It is not influenced by extreme values.

Demerits :

1. It does not give a numerical measure of correlation.
2. It is a subjective method.
3. It cannot be applied to qualitative data.

Illustration 1 : *Following table gives aptitude score (X) and creativity (Y).*

X	*63*	*61*	*62*	*52*	*69*	*72*	*55*	*67*	*80*	*73*
Y	*69*	*65*	*67*	*60*	*72*	*86*	*62*	*75*	*82*	*83*

Draw scatter diagram and comment on the type of correlation between X and Y.

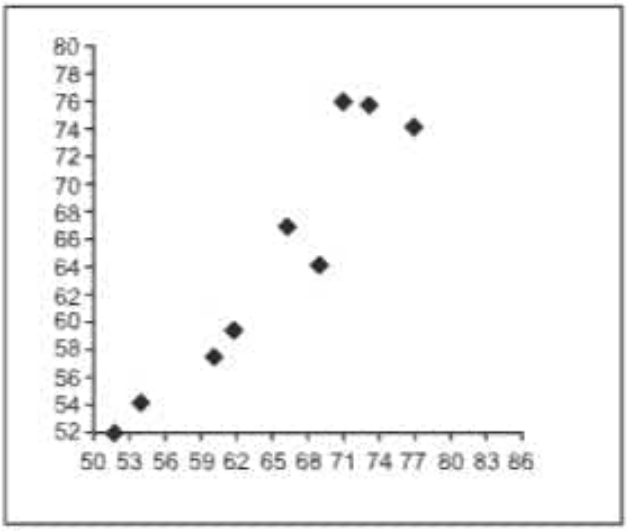

Fig. 6.9

Interpretation : There exist positive correlation of high degree between X and Y.

6.5 Covariance

We introduce the concept of covariance. It will be required to study correlation and regression critically. The drawbacks of scatter diagram as a measure of correlation can be overcome by covariance. The covariance is the joint mutual variation between two variables.

Covariance : The covariance between X and Y is denoted by Cov (X, Y) and is defined as

$$\text{Cov}(X, Y) = \frac{\sum (x - \bar{x})(y - \bar{y})}{n}$$

The computational formula after simplification will be

$$\text{Cov}(X, Y) = \frac{\sum xy}{n} - \bar{x}\,\bar{y}$$

Remark : (1) Cov (X, Y) is similar to variance.

Note that $\text{Var}(X) = \frac{\sum x^2}{n} - \bar{x}^2$ can be expressed as $\text{Var}(X) = \frac{\sum x \cdot x}{n} - \bar{x} \cdot \bar{x}$. Here, replacing the second x by y and second $\bar{x}$ by $\bar{y}$ we get, $\frac{\sum xy}{n} - \bar{x}\,\bar{y}$.

(2) Cov (X, Y) = Cov (Y, X).

(3) Cov (X, X) = Var (X).

(4) Cov (X, constant) = 0.

(5) Covariance may be negative, positive, zero whereas variance is non-negative.

(6) If a, b, h, k are constants then

$$\text{Cov}(X - a, Y - b) = \text{Cov}(X, Y)$$

$$\text{Cov}\left(\frac{X - a}{h}, \frac{Y - b}{k}\right) = \frac{1}{hk}\,\text{Cov}(X, Y), \quad h \neq 0,\ k \neq 0$$

6.6 Karl Pearson's Coefficient of Correlation (Or Product Moment Correlation Coefficient)

If X and Y are correlated, then we get scatter diagrams of the following types. Here we plot the deviations $(x - \bar{x}, y - \bar{y})$.

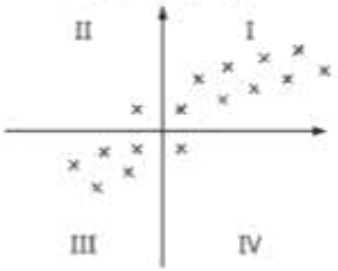

Fig. 6.10 : Positive Correlation

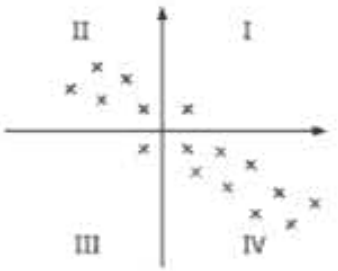

Fig. 6.11 : Negative Correlation

Let us examine the two situations independently. In Fig. 6.10 of positive correlation, observe that both the co-ordinates have same sign, either positive or negative. Thus, $\sum (x - \bar{x})(y - \bar{y}) > 0$. In other words Cov $(x, y) > 0$ for positively correlated variables. On the other hand in Fig. 6.11, one of the co-ordinates is always negative. Hence, $\sum (x - \bar{x})(y - \bar{y}) < 0$. That is Cov $(x, y) < 0$ for negatively correlated variables. Also, covariance measures the extent of joint variation between x and y. Due to these properties of covariance, a relative measure of correlation is defined using covariance. It is discussed below :

Karl Pearson's coefficient of correlation : Karl Pearson's coefficient of correlation is denoted by r and it is defined as follows :

$$r = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sqrt{\sum (x - \bar{x})^2 \times \sum (y - \bar{y})^2}} \quad \ldots (6.1)$$

where $\bar{x}$ and $\bar{y}$ are arithmetic means of x and y respectively. Formula given by (6.1) can be put in simplified way for calculation purpose.

$$r = \frac{\sum xy - n\bar{x}\bar{y}}{\sqrt{(\sum x^2 - n\bar{x}^2)(\sum y^2 - n\bar{y}^2)}} \quad \ldots (6.2)$$

or

$$r = \frac{\frac{1}{n}\sum xy - \bar{x}\bar{y}}{\sigma_x \sigma_y} \quad \ldots (6.3)$$

where, σ_x = standard deviation of x, σ_y = standard deviation of y.

Using covariance, correlation will be

$$r = \frac{\text{Cov}(X, Y)}{\sigma_x \sigma_y} \quad \ldots (6.4)$$

The above (6.1), (6.2), (6.3), (6.4) formulae give one and the same numerical value; however according to convenience and type of data available we choose formula.

In most of the cases where raw or unsummerised data are given we use formula (b).

To study the algebraic properties the formulae (d) or (a) are most suitable.

Properties of correlation coefficient 'r' :

1. Correlation coefficient 'r' lies between –1 and 1 (i.e. $-1 < r < 1$).

Interpretation : If $r > 0$ the correlation is positive and if $r < 0$. The correlation is negative. If $r = 0$ we say the variables are uncorrelated. Larger the numerical value of r more close is the extent of relationship between the variables. In general for $|r| > 0.8$, we consider high correlation. If |r| is between 0.3 to 0.8 we say that correlation is considerable. If $|r| < 0.3$ we say that correlation is negligible. If $r = 1$ we say that there is perfect positive correlation whereas if $r = -1$ we say that there is perfect negative correlation. The above interpretation is general. For more valid interpretation one has to take into account value of n also. Details are beyond the scope of book.

2. Correlation coefficient does not change due to change of origin. In other words if a constant is added or subtracted from each observation, correlation coefficient remains same. Corr $(x \pm a, y \pm b) = \text{Corr}(x, y)$.

3. Correlation remains numerically same under the change of scale. In other words if we divide or multiply each observation by constant correlation remains same numerically.

$$\text{Corr}(ax, by) = \text{Corr}(x, y) \quad ; \text{ if a and b have same algebraic signs.}$$
$$= -\text{Corr}(x, y) \quad ; \text{ if a and b have opposite algebraic signs.}$$

4. Correlation coefficient between X and Y is same as that of between Y and X.

i.e. $\text{Corr}(x, y) = \text{Corr}(y, x)$.

5. $\text{Corr}(x, x) = 1$, $\text{Corr}(x, -x) = -1$.

6.7 Computational Procedure of Correlation Coefficient

We propose two methods for computing correlation coefficient.

(i) Direct method.

(ii) Deviation method.

(i) Direct method : Following are the steps involved in the calculations of Karl Pearson's correlation coefficients.

Step 1 : Obtain sum of x values i.e. $\sum x$ and hence $(\bar{x})$.

Step 2 : Obtain sum of y values, i.e. $\sum y$ and hence $(\bar{y})$.

Step 3 : Obtain sum of squares of x, i.e. $\sum x^2$.

Step 4 : Obtain sum of squares of y, i.e. $\sum y^2$

Step 5 : Obtain sum of products of x and y, i.e. $\sum xy$.

Step 6 : Find r by applying formula

$$r = \frac{\sum xy - n\bar{x}\,\bar{y}}{\sqrt{\sum (x^2 - n\,\bar{x}^2) \times \sum (y^2 - n\,\bar{y}^2)}}$$

Illustration 2 : *Following are the values of import of raw material and export of finished products in suitable units.*

Export	*10*	*11*	*14*	*14*	*20*	*22*	*16*	*12*	*15*	*13*
Import	*12*	*14*	*15*	*16*	*21*	*26*	*21*	*15*	*16*	*14*

Calculate the coefficient of correlation between the import values and export values.

Solution : Let x : Quantity exported, y : Quantity imported.

Preparing table as follows calculations can be made simple. Here we use direct method

x	y	x^2	y^2	xy
10	12	100	144	120
11	14	121	196	154
14	15	196	225	210
14	16	196	256	224
20	21	400	441	420
22	26	484	676	572
16	21	256	441	336
12	15	144	225	180
15	16	225	256	240
13	14	169	196	182
Total = 147	**170**	**2291**	**3056**	**2638**

Here n = 10, hence $\bar{x} = \frac{\sum x}{n} = \frac{147}{10} = 14.7$ and $\bar{y} = \frac{\sum y}{n} = \frac{170}{10} = 17$.

$$r = \frac{\sum xy - n\bar{x} \cdot \bar{y}}{\sqrt{(\sum x^2 - n\bar{x}^2) \times \sum (y^2 - n\bar{y}^2)}}$$

$$= \frac{2638 - 10 \times 14.7 \times 17}{\sqrt{(2291 - 10 \times 14.7^2)(3056 - 10 \times 17^2)}}$$

$$= \frac{139}{\sqrt{130.1 \times 166}}$$

$$= 0.9458$$

Interpretation : There is a high positive correlation between import of raw material and export of finished product.

(ii) Deviation method : Sometimes original values are large. In order to reduce the bulk of calculation we use deviation method. Here we subtract a convenient number from x observations, similarly we subtract some other number from y observations. Due to the

property of correlation coefficient it does not affect the correlation coefficient. Procedural steps involved in this method are as follows :

Step 1 : Obtain deviations $u = x - a$ (a being constant)

Step 2 : Obtain deviations $v = y - b$ (b being constant)

Step 3 : Obtain sum of u and v. i.e. $\sum u$ and $\sum v$.

Step 4 : Obtain sum of squares of u and v. i.e. $\sum u^2$ and $\sum v^2$.

Step 5 : Obtain sum of products of u and v' i.e. $\sum uv$.

Step 6 : Find r by applying the formula.

$$r = \frac{\sum uv - n\,\bar{u}\,\bar{v}}{\sqrt{\sum (u^2 - n\,\bar{u}^2) \times \sum (v^2 - n\,\bar{v}^2)}}$$

where, $\bar{u} = \frac{\sum u}{n}$ and $\bar{v} = \frac{\sum v}{n}$.

Illustration 3 : Compare correlation between the heights of father and son from the following data.

Height of father (in inches)	65	63	67	64	68	70	68	71
Height of son (in inches)	68	65	68	65	69	68	71	70

Solution : Let x = Height of father, y = Height of son.

We use deviation method by taking $u = x - 60$ and $v = y - 65$.

x	y	u	v	u^2	v^2	uv
65	68	5	3	25	9	18
63	65	3	0	9	0	0
67	68	7	3	49	9	21
64	65	4	0	16	0	0
68	69	8	4	64	16	32
70	68	10	3	100	9	30
68	71	8	6	64	36	48
71	70	11	5	121	25	55
Total	–	56	24	448	104	183

$$\bar{u} = \frac{\sum u}{n} = \frac{56}{8} = 7, \quad \bar{v} = \frac{\sum v}{n} = \frac{24}{8} = 3$$

$$r = \frac{\sum uv - n\bar{u}\,\bar{v}}{\sqrt{\sum (u^2 - n\bar{u}^2) \times (\sum v^2 - n\,\bar{v}^2)}}$$

$$= \frac{183 - 8 \times 7 \times 3}{\sqrt{(448 - 8 \times 7^2)\,(104 - 8 \times 3^2)}}$$

$$= \frac{15}{\sqrt{56 \times 32}} = 0.3543$$

Illustration 4 : *Compute correlation coefficient between supply and price of commodity using following data.*

Supply	*152*	*158*	*169*	*182*	*160*	*166*	*182*
Price	*198*	*178*	*167*	*152*	*180*	*170*	*162*

Solution : Here we use deviation method to find r.

Let x = supply, u = x – 150, y = price, v = y – 160.

x	y	u	v	u^2	v^2	uv
152	198	2	38	4	1444	76
158	178	8	18	64	324	144
169	167	19	7	361	49	133
182	152	32	– 8	1024	64	– 256
160	180	10	20	100	400	200
166	170	16	10	256	100	160
182	162	32	2	1024	4	64
Total	–	**87**	**87**	**2833**	**2385**	**521**

Here $n = 7, \sum u = 119, \sum v = 87, \sum u^2 = 2833 \sum v^2 = 2385, \sum uv = 521$

$\therefore \quad \bar{u} = 17, \ \bar{v} = 12.4286$

$$r = \frac{\sum uv - n\bar{u}\,\bar{v}}{\sqrt{(\sum u^2 - n\bar{u}^2) \times (\sum v^2 - n\bar{v}^2)}}$$

$$r = \frac{521 - 7 \times 17 \times 12.4286}{\sqrt{(2833 - 7 \times 17^2)\,(2.385 - 7 \times 12.4286^2)}}$$

$$r = \frac{-958}{\sqrt{810 \times 1303.7142}} = \frac{-958}{1027.6227}$$

$$= -0.9322$$

Interpretation : There is high negative correlation between supply and price.

Illustration 5 : *Find correlation coefficient between X and Y, given that,*

n = 25, $\sum x = 75, \sum y = 100, \sum x^2 = 250, \sum y^2 = 500, \sum xy = 325$.

Solution : Here $\bar{x} = \frac{75}{25} = 3, \ \bar{y} = \frac{100}{25} = 4$

$\therefore$

$$r = \frac{\sum xy - n\,\bar{x}\,\bar{y}}{\sqrt{(\sum x^2 - n\,\bar{x}^2) \times \sum (y^2 - n\,\bar{y}^2)}}$$

$$r = \frac{325 - 25 \times 3 \times 4}{\sqrt{(250 - 25 \times 9)\,(500 - 25 \times 16)}}$$

$$= \frac{25}{\sqrt{25 \times 100}} = \frac{25}{50} = 0.5$$

Illustration 6 : *Compute the product moment coefficient of correlation for the following data : $n = 100$, $\bar{x} = 62$, $\bar{y} = 53$, $\sigma_x = 10$, $\sigma_y = 12$, $\sum (x - \bar{x})(y - \bar{y}) = 8000$.*

Solution :
$$r = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sqrt{\sum (x - \bar{x})^2 \sum (y - \bar{y})^2}}$$

Dividing numerator and denominator by n we get,

$$r = \frac{\sum (x - \bar{x})(y - \bar{y})/n}{\sqrt{\frac{\sum (x - \bar{x})^2}{n} \frac{\sum (y - \bar{y})^2}{n}}} = \frac{\sum (x - \bar{x})(y - \bar{y})/n}{\sigma_x \sigma_y}$$

$$= \frac{8000/100}{10 \times 12} = 0.6667$$

Illustration 7 : *Compute correlation coefficient between X and Y given that :*

$n = 100$, $\sum (x - 35) = 25$, $\sum (y - 19) = 68$, $\sum (x - 35)^2 = 167$,
$\sum (y - 19)^2 = 162$, $\sum (x - 35)(y - 19) = 130$

Solution : Let, $u = x - 35$ and $v = y - 19$

$\therefore$ $\bar{u} = 0.25$ $\bar{v} = 0.68$

$$r = \frac{\sum uv - n\bar{u}\bar{v}}{(\sum u^2 - n\bar{u}^2) \times (\sum v^2 - n\bar{v}^2)} = \frac{113}{\sqrt{160.75 \times 115.76}}$$

$$= 0.8283$$

6.8 Merits and Demerits of Karl Pearson's Coefficient of Correlation

Merits :

1. Karl Pearson's coefficient of correlation determines a single value which summarises the extent of linear relationship. It also indicates type of correlation.
2. It depends upon all observations.

Demerits :

1. It cannot be computed for qualitative data such as honesty and intelligence, beauty and intelligence.
2. It is unduly affected by extreme values.
3. It measures only linear relationship.

For example : Suppose

X	– 2	– 1	0	1	2
Y	4	1	0	1	4

Here $\sum x = 0$, $\sum y = 10$, $\sum xy = 0$. Hence, Cov (X, Y) $= \frac{\sum xy}{n} - \bar{x}\bar{y} = 0$.

Therefore, Corr (X, Y) = 0. However $Y = X^2$, which is non-linear. Hence, correlation fails to measure non-linear relationship. Details are beyond the scope of book.

Note : To overcome the demerit (1) Spearman's rank correlation is used which is discussed below.

6.9 Regression As Prediction Model

In earlier discussions we have studied correlation. It gives extent of linear relationship between two variables. If two variables are correlated, we can use this correlation for prediction of variable given the other variable.

Regression : Technique of prediction on the basis of correlation is called as *regression.*

Since correlation measures the linear relation between two variables, we find a linear equation in these variables. In otherwords, we state the relation in terms of equation of straight line. Using scatter diagram we get an idea of correlation. One can obtain a line passing through these points. However, if correlation is not perfect (i.e. $r \neq \pm 1$) then several lines can be drawn through these points. Out of those lines, how to choose the best line is a problem. So a line which minimizes the total of sum of squares of differences between true value and the value given by straight line is chosen. The principle is called as ***least square principle***. The equation so obtained is called as ***least square regression line***.

Using regression equation one can find relation between advertising expenses and increase in sales, similarly the relation between sales and profit.

Suppose (x_1, y_1), (x_2, y_2), ..., (x_n, y_n) are n pairs of observations on variable X and Y. Since there are two variables, there will be two regression lines.

1. Regression line of Y on X : In this case we assume y as dependent variable or ***response variable*** and x as independent variable or ***explanatory variable***. Therefore this line can be used to predict values of y for known values of x. Suppose the equation of such a line is $y = a + bx$. Mainly we need to fix the constants a and b. This can be done by least square of differences between actual value of y and its estimate $(\hat{y})$ ($\hat{y}$ is read as y hat) obtained from equation.

$$\text{Error in estimation} = \text{True value} - \text{Estimate using the line } y = a + bx$$
$$= y - \hat{y} = y - (a + bx) = y - a - bx$$

Sum of squares of errors is denoted by,

$$S = \sum (y - a - bx)^2$$

Using mathematical methods we choose the constants a and b so that S is minimum. These methods gives rise the following two equations in a and b

$$\sum y = na + b \sum x$$
$$\sum xy = a \sum x + b \sum x^2$$

The above equations are called as *normal equations*. Solving the normal equations simultaneously we get, a and b

$$b = \frac{n \sum xy - \sum x \cdot \sum y}{n \sum x^2 - (\sum x)^2} \quad \text{and} \quad a = b \frac{\sum x}{n} - \frac{\sum y}{n}$$

Hence,
$$b = \frac{\frac{\sum xy}{n} - \bar{x}\,\bar{y}}{\sigma_x^2} \quad \text{and} \quad a = b\,\bar{x} - \bar{y}$$

$\therefore$
$$b = \frac{\text{Cov}(x, y)}{\sigma_x^2} \quad \text{and} \quad a = b\,\bar{x} - \bar{y}$$

The constant b involved in the equation is called as ***regression coefficient of y on x***. Hence instead of writing it as b, henceforth we write it as b_{yx}.

$$\therefore \qquad b_{yx} = \frac{Cov\,(x, y)}{\sigma_x^2}$$

Substituting these values of a and b in $y = a + bx$ and simplifying the same we get,

$$y - \bar{y} = b_{yx}(x - \bar{x})$$

as least square regression equation of y on x.

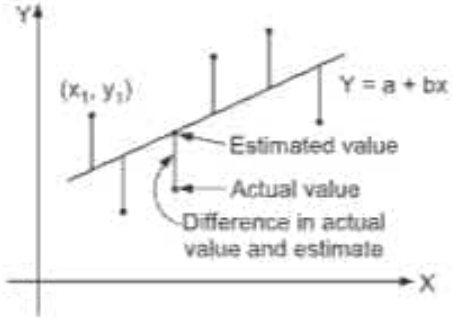

Fig. 6.12

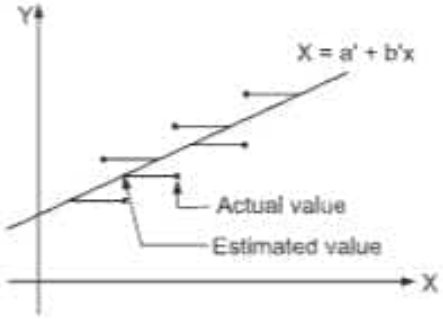

Fig. 6.13

2. Regression line of X on Y : In this case we assume x as dependent variable and y as independent variable. This line is used to predict values of x for known values of y. Its least square equation is obtained using same technique which is used for obtaining regression equation of y on x.

Thus the equation of line will be

$$x - \bar{x} = b_{xy}(y - \bar{y})$$

Coefficient involved in the above equation is known as *regression coefficient of X on Y.*

$$\therefore \qquad b_{xy} = \frac{\sum (x - \bar{x})(y - \bar{y})/n}{\sigma_y^2} = \frac{\frac{\sum xy}{n} - \bar{x}\,\bar{y}}{\sigma_y^2}$$

$$= \frac{Cov\,(x, y)}{\sigma^2 x}$$

Illustration 8 : *Following data gives expenditure incurred on advertisement and the sales for 10 years.*

Advertisement expenses in thousand ₹(X)	*10*	*12*	*15*	*14*	*16*	*20*	*19*	*24*	*26*	*30*
Sales in lakh ₹(Y)	*5.0*	*5.1*	*5.4*	*5.5*	*5.7*	*5.9*	*6.0*	*7.3*	*7.5*	*7.8*

(i) Find the appropriate line of regression to estimate sales for given advertisement. Also estimate sales if advertisement expenses is ₹ 35,000.

(ii) To achieve sales target of ₹ 10 lakhs how much you need to invest in advertisement.

(iii) If company does not invest any amount in advertisement what will be the sales ?

(iv) Find the increase in sales per thousand ₹ advertisement expenses.

Solution : Let X = Advertisement expenses

Y = Sales.

Here to estimate sales we need to find the regression line of Y on X, similarly to estimate expenditure required to achieve the target sales we need the regression line of X on Y.

Procedure :

(1) Prepare the table to find $\sum x$, $\sum y$, $\sum x^2$, $\sum y^2$, $\sum xy$.

(2) Find $\bar{X}$, $\bar{Y}$, σ_x^2, σ_y^2, Cov (X, Y), b_{xy}, b_{yx}.

(3) Determine the regression lines.

X	Y	X^2	Y^2	XY
10	5.0	100	25.00	50.0
12	5.1	144	26.01	61.2
15	5.4	225	29.16	81.0
14	5.5	196	30.25	77.0
16	5.7	256	32.49	91.2
20	5.9	400	34.81	118.0
19	6.0	361	36.00	114.0
24	7.3	576	53.29	175.2
26	7.5	676	56.25	195.0
30	7.8	900	60.84	234.0
Total = 186	61.2	3834	384.10	1196.6

$n = 10$, $\bar{X} = \frac{\sum x}{n} = \frac{186}{10} = 18.6$, $\bar{Y} = \frac{\sum y}{n} = \frac{61.2}{10} = 6.12$

$$\sigma_x^2 = \frac{\sum x^2}{n} - \bar{X}^2 = \frac{3834}{10} - 18.6^2 = 383.40 - 345.96 = 37.44$$

$$\sigma_y^2 = \frac{\sum y^2}{n} - \bar{y}^2 = \frac{384.10}{10} - 6.12^2 = 38.4100 - 37.4544 = 0.9556$$

$$\text{Cov}(x, y) = \frac{\sum xy}{n} - \bar{X}\bar{Y} = \frac{1196.6}{10} - 18.5 \times 6.12 = 119.660 - 113.832$$

$$= 5.828$$

$$b_{yx} = \frac{\text{Cov}(x, y)}{\sigma_x^2} = \frac{5.828}{37.44} = 0.1557$$

$$b_{xy} = \frac{\text{Cov}(x, y)}{\sigma_y^2} = \frac{5.828}{0.9556} = 6.0988$$

(i) To estimate sales (y) for given advertisement express (x), we use regression line of y on x

$$y - \bar{y} = b_{yx}(x - \bar{x})$$

$$y - 6.12 = 0.1557(x - 18.6)$$

$$y - 6.12 = 0.1557x - 2.89602$$

$$y = 0.1557x + 3.2240$$

Estimate of y for x = 35 we substitute x = 35 in the above equation

$$y = 0.1557 \times 35 + 3.2240 = 8.6735$$

Interpretation : If we spend ₹ 35,000 on advertisement then sales will be approximately ₹ 8.6735 lakhs.

(ii) To estimate advertisement expenses (X) for achieving sales target (Y) we use regression line of X on Y.

$$x - \bar{x} = b_{xy}(y - \bar{y})$$

$$x - 18.6 = 6.0988(y - 6.12)$$

$$x - 18.6 = 6.0988y - 37.3247$$

$$x = 6.0988y - 18.7247$$

To estimate x for y = 10, substitute y = 10 in the above equation

$$\therefore \quad x = 6.0988 \times 10 - 18.7247$$

$$= 42.2633 \text{ thousand } ₹$$

To achieve sales target of ₹ 10 lakhs. We have to spend ₹ 42,263.30.

(iii) To find sales when advertisement expenses is zero, we put x = 0 in the regression line of y on x.

$$y = 0.1557(0) + 3.224 = 3.224$$

$$y = 3.224 \text{ lakhs } ₹$$

Interpretation : If y = a + bx is equation of regression line then the intercept a is the value of y for x = 0.

(iv) y = 0.1557 x + 3.224 is equation in which slope is 0.1557. Thus for unit increase in x, y increases by 0.1557 lakhs of ₹. Hence if we increase advertisement expenses by ₹ one thousand, sales will approximately increase by 0.1557 lakhs of ₹ or ₹ 15,570.

Geometric Interpretation of Regression line :

We can visualise the line, slope intercept geometrically in the following figure.

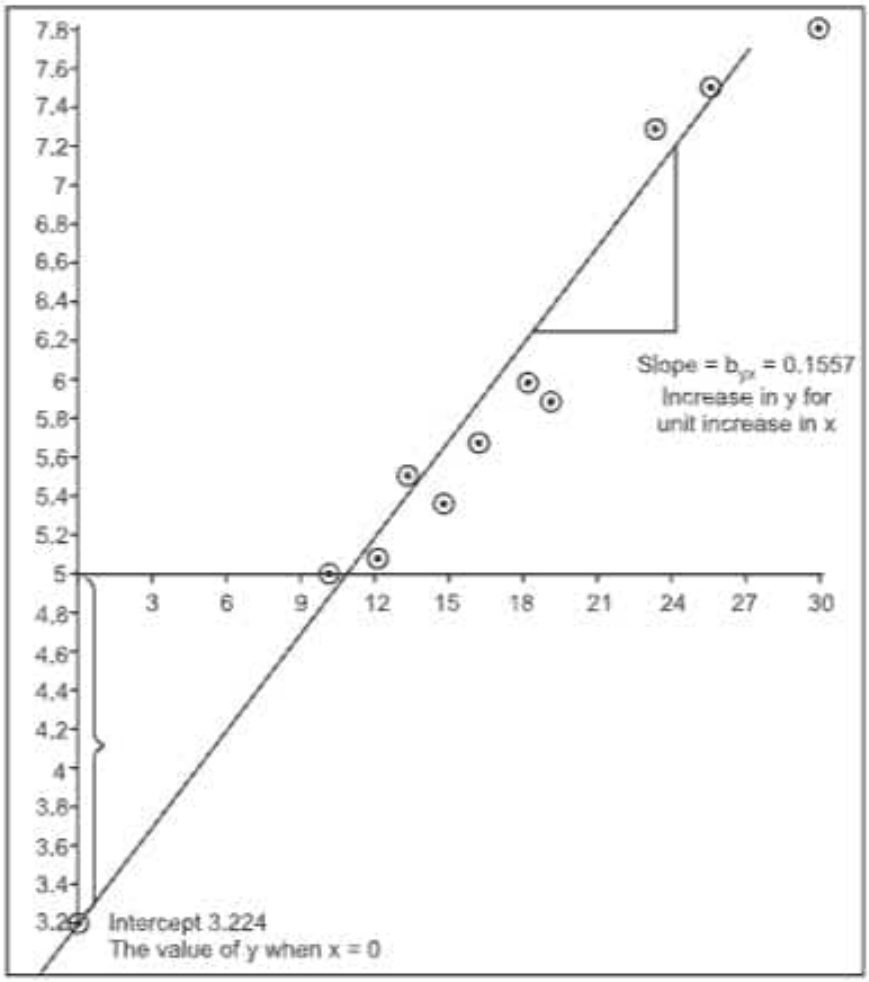

Fig. 6.14

6.10 Interpretation of Regression Coefficient

The regression line of Y on X is $y - \bar{y} = b_{yx}(x - \bar{x})$, we can write it as $Y = b_{yx} X + C$. Clearly, unit change in X will make change of b_{yx} units in Y. If b_{yx} is positive, then increase (or decrease) in X by one unit will be associated with increase (or decrease) in Y by b_{yx} units. On the other hand, if b_{yx} is negative, then increase (or decrease) in X by one unit is going to cause decrease (or increase) in Y by b_{yx} units.

For example, suppose X = supply and Y = price (₹) and the regression line is $Y = -1.2 X + 5$. Here we interpret the regression coefficient as follows. If supply (X) increases by one unit, the price is going to decrease by ₹ 1.20, or unit decrease in supply is

going to cause increase in price by ₹ 1.20. Thus b_{yx} is the amount of change in Y per unit change in X.

Let X = expenditure on advertisement in ₹ and Y = annual profit in ₹. Suppose Y = 12 X + 19 is the regression line, then we interpret it as follows. For every rupee spent on advertisement, profit is estimated to rise by ₹ 12.

Coefficient of determination :

Definition : If r is the correlation coefficient then r^2 is called as coefficient of determination.

The coefficient of determination measures the **strength** of regression of Y on X. Larger the value of r^2, more powerful is the regression model. In other words, reliability in determining the Y values on the basis of X is more. Hence, it is called as the **coefficient of determination**. Also note that $|r| > r^2$; since $0 < |r| < 1$. Thus correlation coefficient overestimates the actual extent of linear relationship. Hence, in advanced studies, use of r^2 is recommended than that of r. However, a drawback of coefficient of determination is that it is always positive, hence fails to give the idea about the type of relationship between X and Y.

6.11 Applications of Correlation and Regression

There are number of fields where correlation and regression are used as tools in the analysis.

(a) In agricultural experiments, one can use regression line to estimate change in agricultural production due to various factors viz. fertilizers, irrigation facility, fertility of soil etc.

Similarly, one can study the relation between two variables such as germination time (Y) and temperature of soil (X).

Relation between alkalinity of water in a river (or pond) and growth of fungi can be studied using regression and correlation.

(b) In business and trade, correlation and regression helps in planning and forecasting to a great extent.

In portfolio analysis, β (beta) index is used quite often. Suppose Y is return on a security (a share of particular company) and X is the return on all other remaining securities measured in terms of index, then regression coefficient b_{yx} is called as beta index of a security. If $\beta > 1$, the share is treated as aggressive otherwise defensive.

(c) In medical sciences, we can find the regression line between age of a person (X) and blood pressure (Y). This helps in preparing the scale for blood pressure agewise. Further it may be used in finding abnormalities. Similarly using regression analysis one can find growth chart for a normal baby. It may be about age in months and weight or age and height.

(d) An economist may be interested to know the relationship between age and productivity index. Effect of training or education on change in total turnover can be measured using similar techniques.

(e) In portfolio analysis, risk is measured in terms of variances and covariances. If Y = Return of a security, X = Market return then,

Systematic risk of Y = $b_{yx}^2 \ \sigma_x^2$

Unsystematic risk of Y = $\sigma_y^2 - b_{yx}^2 \ \sigma_x^2$.

Thus the total risk σ_y^2 is partitioned into systematic and unsystematic risks.

An optimal way of investing in two or more portfolios can be obtained using linear combination, which has the least variance.

(f) We see a wonderful application of correlation in the field of physical education and sports.

Several aptitude tests are given for players. There are certain events or activities in the sport with performances correlated to each other.

For example, the performance in athletics events discuss (disc-throw), shot-put (iron ball throw) and javelin (spear throw) are correlated. Among three events correlation between any two is high positive. This indicates that the aptitude required for any of these there is same. Thus one can reduce the number of tests. Performance in one event can be used to estimate the performance in other.

6.12 Linear Regression : Cause and Effect

By means of correlation analysis we get an idea about the type of correlation and the extent of correlation between two variables. However, this does not tell us anything about cause and effect relationship. If X and Y are correlated, then we cannot say X is the cause and Y is the effect or vice versa. Even in case of perfect correlation also, we cannot conclude that one of the variable is the cause and the other is effect. If X is the cause and Y is effect, then X and Y are correlated (or dependent) but not vice-versa. It is possible that some common factors influence both X and Y, due to which they turn out to be correlated. For example, price of a commodity and demand.

Sometimes correlation between two variables is found just due to pure chance. It is called as ***'spurious or non-sense correlation'***.

For example, suppose X = Number of literates in a country and Y = Number of criminals in a country. Clearly as X increases, in majority cases Y also increases; thus there is a high positive correlation between X and Y. However, we cannot say that X is cause and Y is effect. In other words, we cannot conclude that literacy is the cause of crime. Both X and Y show similar trend because of third common variable that the population. Increase (or decrease) in population will have effect on X and Y of similar kind. Hence there will be positive correlation between X and Y.

We have seen in the above discussion that the correlated variables cannot be sorted out as cause and effect. However, regression does this job. This demands two separate prediction equations for the two variables.

For example, suppose X is the intelligence quotient (I.Q.) and Y is the marks in an examination. Then we can have regression line of Y on X to estimate marks in examination, given the I.Q. Here we assume that I.Q. is the cause and marks is the effect. Similarly using regression line of X on Y, we can estimate I.Q. whenever marks are available, reversing the role of cause and effect.

Suppose X is the total rainfall in a certain period and Y is the agricultural yield. In this case, there is no point in finding regression line of X on Y, as yield cannot be cause to rainfall. Regression line of Y on X makes sense and hence is of much use.

Thus we note the importance of two regression lines for prediction purposes.

Illustration 9 : *Obtain regression lines for following data :*

X	2	3	5	7	8	10	12	15
Y	2	5	8	10	12	14	15	16

Find estimate of :

(i) Y when X = 6.

(ii) X when Y = 20.

Solution : To find regression lines we require to calculate regression coefficients b_{xy} and b_{yx}. These coefficients depend upon $\sum x$, $\sum y$, $\sum x^2$, $\sum y^2$, $\sum xy$. So we prepare the following table and simplify the calculations :

	x	y	x^2	y^2	xy
	2	2	4	4	4
	3	5	9	25	15
	5	8	25	64	40
	7	10	49	100	70
	9	12	81	144	108
	10	14	100	196	140
	12	15	144	225	180
	15	16	225	256	240
Total	63	82	637	1014	797

n = number of pairs of observations = 8

$$\bar{x} = \frac{\sum x}{n} = \frac{63}{8} = 7.875$$

$$\sigma^2 x = \frac{\sum x^2}{n} - \bar{x}^2 = \frac{637}{8} - (7.875)^2 = 17.6094$$

$$\bar{y} = \frac{82}{8} = 10.25$$

$$\sigma^2 y = \frac{\sum y^2}{n} - \bar{y}^2 = \frac{1014}{8} - (10.25)^2 = 21.6875$$

$$\text{Cov}(x, y) = \frac{\sum xy}{n} - \bar{x}\,\bar{y} = \frac{797}{8} - 7.875 \times 10.25 = 18.9063$$

$$b_{yx} = \frac{\text{Cov}(x, y)}{\sigma_y^2} = \frac{18.9063}{17.6094} = 1.0736$$

$$b_{xy} = \frac{\text{Cov}(x, y)}{\sigma_y^2} = \frac{18.9063}{21.6875} = 0.8718$$

Regression line of Y on X :

$$y - \bar{y} = b_{yx}(x - \bar{x})$$
$$y - 10.25 = 1.0736(x - 7.875)$$
$$y = 1.0736x + 1.7954$$

(i) Estimate of y for x = 6, can be obtained by substituting x = 6 in the above regression equation.

$$\therefore \quad y = 1.0736 \times 6 + 1.7954$$
$$y = 8.237$$

Regression line of X on Y :

$$x - \bar{x} = b_{xy}(y - \bar{y})$$
$$x - 7.875 = 0.8718(y - 10.25)$$
$$x - 7.875 = 0.8718y - 8.93595$$
$$x = 0.8718y - 1.06095$$

(ii) Estimate of x can be obtained by substituting y = 20 in the above equation.

$$\therefore \quad x = 16.37505$$

Note : For estimation of x and estimation of y separate equations are to be used.

The above problem using MS-EXCEL is solved as follows :

Enter the data in columns A and B (X and Y respectively). Then use following path to obtain regression lines directly. Click on **Tools** then following Fig. 6.15 appears on screen.

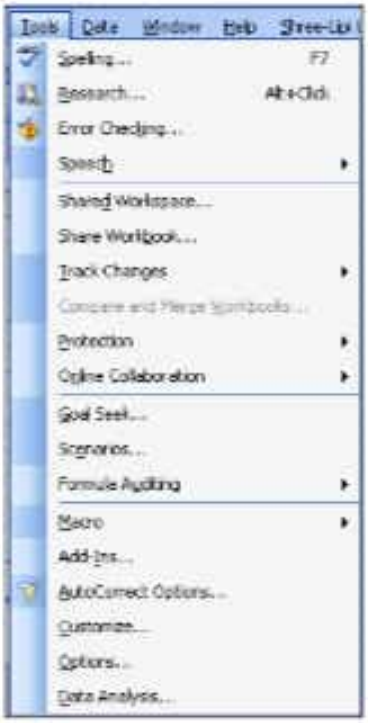

Fig. 6.15

Then click on **Data Analysis**, following window appears.

Fig. 6.16

Then click on **Regression** function.

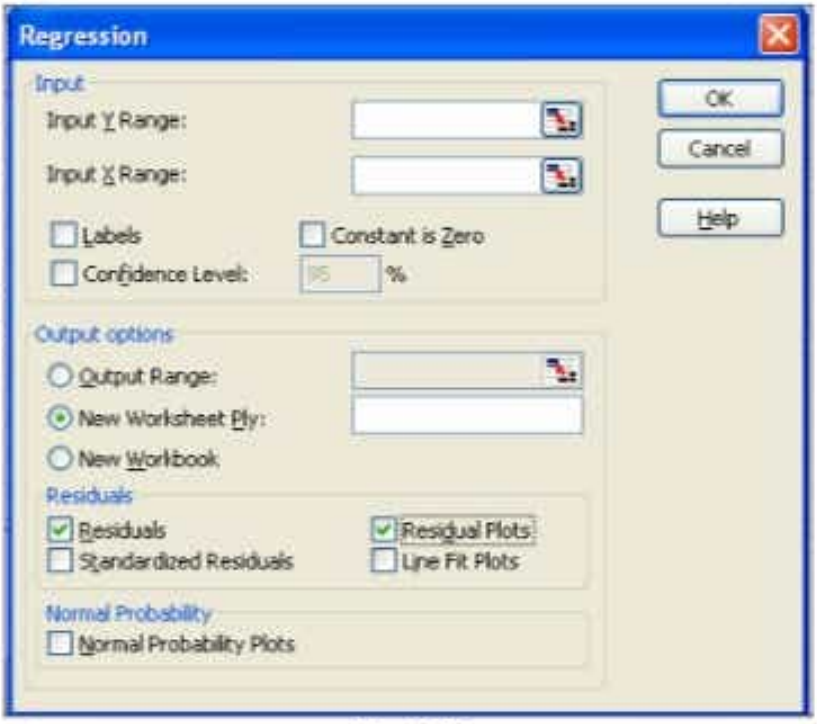

Fig. 6.17

Now, if we want to find regression line of Y on X, then enter the data range of Y in input Y range and X in input X range and click on OK. Then it will create new worksheet having following summary output i.e. dependent variable in Y range and independent variable in input X range.

	A	B	C	D	E	F	G	H	I
1	SUMMARY OUTPUT								
2									
3	*Regression Statistics*								
4	Multiple R	0.96745							
5	R Square	0.93596							
6	Adjusted F	0.925287							
7	Standard E	1.360816							
8	Observatio	8							
9									
10	ANOVA								
11		*df*	*SS*	*MS*	*F*	*ignificance F*			
12	Regressior	1	162.3891	162.3891	87.69166	8.41E-05			
13	Residual	6	11.11091	1.851819					
14	Total	7	173.5						
15									
16		*Coefficient*	*tandard Err*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*ower 95.0%*	*pper 95.0%*
17	Intercept	1.795031	1.023074	1.754547	0.129871	-0.70834	4.298403	-0.70834	4.298403
18	X Variable	1.073647	0.114652	9.364383	8.41E-05	0.793103	1.354191	0.793103	1.354191
19									

Fig. 6.18

It returns many values but we are interested only in highlighted values. We know that equation of line is given by $Y = mX + c$.

where, m is slope (i.e. b_{yx}) and intercept (i.e. $\bar{X} - b_{yx}\bar{Y}$)

Hence line of regression of Y on X is given by

Y = (slope) X + intercept i.e. here $Y = 1.073647\ X + 1.795031$

For estimating Y when X = 6, put X = 6 in above equation.

$Y = 1.073647 \cdot 6 + 1.795031,$

$Y = 8.237$

Similarly, for finding regression line of X on Y by changing the data in input Y range and input X range.

	A	B	C	D	E	F	G	H	I
1	SUMMARY OUTPUT								
2									
3	*Regression Statistics*								
4	Multiple R	0.96745							
5	R Square	0.93596							
6	Adjusted F	0.925287							
7	Standard E	1.226215							
8	Observatio	8							
9									
10	ANOVA								
11		*df*	*SS*	*MS*	*F*	*ignificance F*			
12	Regressior	1	131.8534	131.8534	87.69166	8.41E-05			
13	Residual	6	9.021614	1.503602					
14	Total	7	140.875						
15									
16		*Coefficient*	*tandard Err*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*ower 95.0%*	*pper 95.0%*
17	Intercept	-1.06052	1.048071	-1.01188	0.350649	-3.62506	1.504019	-3.62506	1.504019
18	X Variable	0.871758	0.093093	9.364383	8.41E-05	0.643968	1.099548	0.643968	1.099548
19									

Fig. 6.19

Hence line of regression of X on Y is $X = 0.871758 \cdot Y - 1.06052$ and estimated value of $X = 16.37505$ for $y = 20$.

6.13 Properties of Regression Coefficient (Oct. 2014)

1. Correlation coefficient and regression coefficients have same algebraic signs.

Proof : Note that $b_{xy} = \frac{Cov(x, y)}{\sigma_y^2}$, $b_{yx} = \frac{Cov(x, y)}{\sigma_x^2}$

and $r = \frac{Cov(x, y)}{\sigma_x \sigma_y}$

Clearly, numerator of each coefficient is same and denominator of each coefficient is positive. Hence, numerator decides algebraic sign. Thus, all coefficients have same algebraic sign. Hence, if $r > 0$, then $b_{yx} > 0$ and $b_{xy} > 0$. If $r = 0$, $b_{yx} = b_{xy} = 0$. If $r < 0$ then, $b_{xy} < 0$ and $b_{yx} < 0$.

2. Correlation coefficient is a square root of product of regression coefficients. $\left(\text{i.e. } r = \sqrt{b_{yx} \cdot b_{xy}}\right)$ or correlation coefficient is geometric mean of regression coefficients.

Proof : $b_{yx} \cdot b_{xy} = \frac{Cov(x, y)}{\sigma_x^2} \times \frac{Cov(x, y)}{\sigma_y^2} = \left(\frac{Cov(x, y)}{\sigma_x \sigma_y}\right)^2 = r^2$

$\therefore \quad r = \sqrt{b_{yx} b_{xy}}$

Note : Choose positive square root if regression coefficients are positive, otherwise, negative.

3. Both regression coefficients cannot exceed unity simultaneously.

Proof : If possible let us assume $b_{yx} > 1$ and $b_{xy} > 1$.

Hence, $b_{xy} \cdot b_{yx} > 1$

$\therefore \quad r^2 > 1$

which is impossible ($\because r < 1$). Thus our assumption is incorrect.

We state few more properties without proof.

4. Regression coefficient can be expressed in terms of correlation coefficient as follows :

$$b_{yx} = r \cdot \frac{\sigma_y}{\sigma_x} \text{ and } b_{xy} = r \cdot \frac{\sigma_x}{\sigma_y}$$

5. Correlation coefficient lies between two regression coefficients.

6. Regression coefficients remain unchanged due to change of origin. In otherwords if a constant is substracted or added from each observation, regression coefficients remain same.

7. Regression coefficients are affected by change of scale as follows

If $u = \frac{x - a}{h}$ and $v = \frac{y - b}{k}$

then $b_{uv} = \frac{k}{h} b_{xy}$ and $b_{vu} = \frac{h}{k} b_{yx}$

8. If $r = \pm 1$ then, regression coefficients are reciprocals of each other.

9. Regression coefficients are equal if $\sigma_x = \sigma_y$.

6.14 Properties of Regression Lines

(1) Regression lines coincide if $r = \pm 1$. Thus if there exists perfect correlation, points in scatter diagram lie on the same straight line.

(2) Regression lines are perpendicular to each other, if $r = 0$. Thus if the variables are uncorrelated, points on scatter diagram will exhibit maximum spread.

(3) The point of intersection of regression lines is $(\bar{x}, \bar{y})$.

(4) Larger the value of correlation coefficient smaller is the acute angle between regression lines.

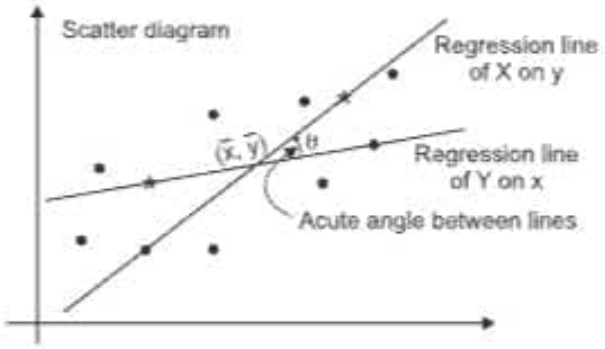

Fig. 6.20

6.15 Standard Error of Regression Estimate

With usual notation r as correlation coefficient σ_x and σ_y as standard deviations of x and y respectively; we state standard error of regression estimate.

(1) Standard error of regression estimate of y on x is $\sigma_y \sqrt{1 - r^2} / \sqrt{n - 2}$.

(2) Standard error of regression estimate of x on y is $\sigma_x \sqrt{1 - r^2} / \sqrt{n - 2}$.

Hence estimates are reliable for larger value of r^2.

Note : The above discussion leads to conclusion that rather than r we should consider r^2 for testing reliability of regression estimates. Therefore, regression analysis claims validity if r^2 is sufficiently large. The quantity r^2 is called as the coefficient of determination.

6.16 Correlation and Regression Analysis

Correlation coefficient gives the extent of linear relationship between two variables. Whereas regression analysis establishes the functional relationship between two correlated variables. Regression analysis helps in estimation, prediction etc. however correlation coefficient is an indicator of linear relationship.

Correlation coefficient is symmetric i.e. Corr (x, y) = Corr (y, x), however regression coefficients are not symmetric i.e. $b_{xy} \neq b_{yx}$.

Correlation coefficient is unitless quantity, however regression coefficients posses units of measurement.

Computation of regression coefficients by deviation method : Note that by one of the property of regression coefficients we can subtract a suitable constant and make computations easy.

Procedure :

(1) Substract a constant 'a' from x values and constant 'b' from y values.
Denote $u = x - a$ and $v = y - b$.

(2) Prepare table containing columns u, v, u^2, v^2 and uv.

(3) Use the following formulae and compute regression coefficients.

$$b_{xy} = b_{uv} = \frac{\frac{\sum uv}{n} - \bar{u}\,\bar{v}}{\sigma_v^2}$$

and

$$b_{yx} = b_{vu} = \frac{\frac{\sum uv}{n} - \bar{u}\,\bar{v}}{\sigma_u^2}$$

where,

$$\bar{u} = \frac{\sum u}{n} \qquad \bar{v} = \frac{\sum v}{n}$$

$$\sigma_u^2 = \frac{\sum u^2}{n} - \bar{u}^2 \quad \text{and} \quad \sigma_v^2 = \frac{\sum v^2}{n} - \bar{v}^2$$

Illustration 10 : *The table below gives the respective heights x and y of a sample of 10 fathers and their sons :*

(i) Find regression line of y on x.
(ii) Find regression line of x on y.
(iii) Estimate son's height if father's height is 65 inches.
(iv) Estimate father's height if son's height is 60 inches.
(v) Compute correlation coefficient between x and y.

Height of father x (inches)	*65*	*63*	*67*	*64*	*68*	*62*	*70*	*66*	*68*	*67*
Height of son y (inches)	*68*	*66*	*68*	*65*	*69*	*66*	*68*	*65*	*71*	*67*

Solution : Let $u = x - 62$, $v = y - 65$. We prepare the table to simplify the computations.

x	y	u	v	u^2	v^2	uv
65	68	3	3	9	9	9
63	66	1	1	1	1	1
67	68	5	3	25	9	15
64	65	2	0	4	0	0
68	69	6	4	36	16	24
62	66	0	1	0	1	0
70	68	8	3	64	9	24
66	65	4	0	16	0	0
68	71	6	6	36	36	36
67	67	5	2	25	4	10
Total		40	23	216	85	119

$$n = \text{number of pairs} = 10$$

$$\bar{u} = \frac{40}{10} = 4, \quad \sigma_u^2 = \frac{216}{10} - 4^2 = 5.6$$

$$\bar{v} = \frac{23}{10} = 2.3 \quad \sigma_v^2 = \frac{85}{10} - (2.3)^2 = 3.21$$

$$\text{Cov}(u, v) = \frac{119}{10} - 4 \times 2.3 = 2.7$$

$$\therefore \quad b_{xy} = b_{uv} = \frac{2.7}{3.21} = 0.8411, \quad \text{and} \quad b_{yx} = b_{vu} = \frac{2.7}{5.6} = 0.4821$$

$$\bar{x} = \bar{u} + 62 = 66, \quad \bar{y} = \bar{v} + 65 = 67.3$$

(i) Regression line of Y on X is $y - \bar{y} = b_{yx}(x - \bar{x})$

$$\therefore \quad y - 67.3 = 0.4821(x - 66)$$

$$\therefore \quad y = 0.4821x + 35.4814$$

(ii) Regression line of X on Y is $x - \bar{x} = b_{xy}(y - \bar{y})$

$$\therefore \quad x - 66 = 0.8411(y - 67.3)$$

$$\therefore \quad x = 0.8411y + 9.3940$$

(iii) Estimate of son's height y for x = 65

$$y = 0.4821 \times 65 + 35.4814 = 66.8179 \text{ inches}$$

(iv) Estimate of father's height x for y = 60

$$x = 0.8411 \times 60 + 9.394 = 59.86 \text{ inches}$$

(v) Correlation coefficient,

$$r = \sqrt{b_{xy} \cdot b_{yx}} = \sqrt{0.8411 \times 0.4821} = 0.63678$$

We choose positive square root because regression coefficients are positive.

Illustration 11 : *Revenue department is trying to estimate the monthly amount of unpaid taxes. Suppose x denote field audit labour hours and y denote unpaid taxes. Using last 10 months data the following summary is obtained.*

$\sum x = 441$, $\sum y = 272$, $\sum x^2 = 19461$, $\sum y^2 = 7428$, $\sum xy = 12{,}005$.

Determine regression line of y on x. Also obtain standard error of regression estimate.

Solution : Here we require to find b_{yx}.

$$\bar{x} = \frac{441}{10} = 44.1 \qquad \bar{y} = \frac{272}{10} = 27.2$$

$$\sigma_x^2 = \frac{19461}{10} - (44.1)^2 = 1.29$$

$$\sigma_y^2 = \frac{7428}{10} - (27.2)^2 = 2.96$$

$$\text{Cov}(x, y) = \frac{12005}{10} - 44.1 \times 27.2 = 0.98$$

$$b_{yx} = \frac{\text{Cov}(x, y)}{\sigma_x^2} = \frac{0.98}{1.29} = 0.7597$$

Regression line of y on x is :

$$y - \bar{y} = b_{yx}(x - \bar{x})$$

$$y - 27.2 = 0.7597\,(x - 44.1)$$

$$y = 0.7597\,x - 6.3023$$

S.E. of regression estimate of y on x $= \sigma_y = \sqrt{1 - r^2} \,/\, \sqrt{n - 2}$.

Note that : $r^2 = \dfrac{\text{Cov}(x, y)}{\sigma_x^2 \sigma_y^2}$

$$= \frac{0.98}{1.29 \times 2.96} = 0.256652$$

$\therefore$ S.E. $= \sqrt{2.96 \times (1 - 0.256652)} \,/\, \sqrt{8}$

$$= \frac{1.4833}{2.8284} = 0.5244$$

Illustration 12 : *Determine regression line for price given the supply, hence estimate price when supply is 180 units, from the following information.*

x = Supply, y = Price in ₹ per unit, n = 7

$\sum (x - 150) = 119$, $\sum (y - 160) = 84$

$\sum (x - 150)^2 = 2835$, $\sum (y - 160)^2 = 2387$

$\sum (x - 150)(y - 160) = 525$.

Also find correlation coefficient between price and supply.

Solution : Let, $u = x - 150$, $v = y - 160$

$\therefore$ $\bar{u} = \dfrac{119}{7} = 17$, $\bar{v} = \dfrac{84}{7} = 12$

$$\sigma_u^2 = \frac{2835}{7} - (17)^2 = 405 - 289 = 116$$

$$\sigma_v^2 = \frac{2387}{7} - (12)^2 = 341 - 144 = 197$$

$$\text{Cov}(x, y) = \text{Cov}(u, v) = \frac{525}{7} - 17 \times 12 = -129$$

$$\bar{x} = 150 + \bar{u} = 167 \quad \text{and} \quad \bar{y} = 160 + \bar{v} = 172$$

$$b_{yx} = b_{vu} = \frac{\text{Cov}(u, v)}{\sigma_u^2} = \frac{-129}{116} = -1.1121$$

Equation of regression line of y on x is,

$$y - \bar{y} = b_{yx}(x - \bar{x})$$
$$y - 172 = -1.1121\,(x - 167)$$
$$y = -1.1121\,x + 357.7207$$

Estimate of y for x = 180

$$y = -1.1121 \times 180 + 357.7207$$
$$= 157.54$$

$$r = \text{Corr}\,(x, y) = \frac{\text{Cov}\,(u, v)}{\sigma_u\,\sigma_v}$$

$$= \frac{-129}{\sqrt{116 \times 197}} = -0.8534$$

Illustration 13 : *Compute regression coefficients and hence verify that correlation coefficient lies between them.*

n = 100, $\bar{x}$ = 60, $\bar{y}$ = 50, σ_x = 10, σ_y = 12, $\sum (x - \bar{x})(y - \bar{y})$ = 8400.

Solution : $$\text{Cov}\,(x, y) = \frac{\sum (x - \bar{x}) - (y - \bar{y})}{n}$$

$$= \frac{8400}{100} = 84$$

$\therefore$ $$b_{xy} = \frac{\text{Cov}\,(x, y)}{\sigma_y^2}$$

$$= \frac{84}{144} = 0.5833$$

$$b_{yx} = \frac{\text{Cov}\,(x, y)}{\sigma_x^2}$$

$$= \frac{84}{100} = 0.84$$

$$r = \frac{\text{Cov}\,(x, y)}{\sigma_x\,\sigma_y} = \frac{84}{120} = 0.7$$

Clearly r lies between the two regression coefficients.

Illustration 14 : *A study of wheat prices at Mumbai and Kanpur yield the following data :*

	Mumbai	***Kanpur***
Arithmetic mean	*₹20*	*₹21*
Standard deviation	*₹0.326*	*₹0.207*

Correlation coefficient between the prices at Mumbai and Kanpur is 0.774. Estimate the price at Kanpur if the price at Mumbai is ₹25 using the above data.

Solution : Let y : Price of wheat at Kanpur, x : Price of wheat at Mumbai.

We obtain regression line of y on x from estimation of price at Kanpur.

$$y - \bar{y} = r \cdot \frac{\sigma_y}{\sigma_x} (x - \bar{x}) \qquad \left(\because \ b_{yx} = r \frac{\sigma_y}{\sigma_x} \right)$$

$$y - 21 = 0.774 \times \frac{0.207}{0.326} (25 - 20)$$

$$y - 21 = 2.457$$

$$y = ₹ 23.46$$

Therefore price at Kanpur is ₹ 23.46.

Illustration 15 : *Given $x - 4y = 5$ and $x - 16y = -64$ are the regression lines, find (i) regression coefficient of x on y, (ii) regression coefficient of y on x, (iii) Corr (x, y), (iv) $\bar{x}, \bar{y}$, (v) σ_y if $\sigma_x = 8$.*

Solution : Here by looking at the equations we cannot decide which of the equation is regression equation of x on y and which is of y on x. We arbitrarily decide one of the line as regression line of y on x, and find regression coefficients. Then we verify whether these values are admissible.

Suppose the equation $x - 16y = -64$ represent regression line of x on y. We write it in usual form as

$$x - \bar{x} = b_{xy} (y - \bar{y}) \qquad \ldots (1)$$

Therefore the equation can be reformed as

$$x = 16y - 64. \qquad \ldots (2)$$

Comparing coefficients of y in equations (1) and (2) we get $b_{xy} = 16$.

On the other hand, $x - 4y = 5$ will be regression line of y on x. Writing it in usual form we get $4y = x - 5$.

$$\therefore \qquad y = \frac{1}{4} x - \frac{5}{4} \qquad \ldots (3)$$

Theoretically, equation of y on x is

$$y - \bar{y} = b_{yx} (x - \bar{x}) \qquad \ldots (4)$$

Comparing coefficients of x in equations (3) and (4) we get, $b_{yx} = \frac{1}{4}$

We know that, $b_{yx} \cdot b_{xy} \leq 1$

However here, $b_{xy} \cdot b_{yx} = 16 \times \frac{1}{4} = 4 < 1$

Hence, our choice of regression lines is incorrect. Exchanging the choice we get $x - 16y = -64$ as regression line of y on x. Writing it in usual manner we get :

$$y = \frac{1}{16} x + 4 \qquad \ldots (5)$$

Comparing equations (4) and (5) we get, $b_{yx} = \frac{1}{16}$. Similarly, $x - 4y = 5$ will be the regression line of x on y. Writing it in usual form we get,

$$x = 4y + 5 \quad \ldots (6)$$

Comparing equations (1) and (6) we get, $b_{xy} = 4$.

$\therefore$ Correlation coefficient $= r^2 = b_{yx} \cdot b_{xy} = \frac{1}{16} \times 4 = \frac{1}{4}$

$\therefore$ $$r = \sqrt{\frac{1}{4}} = \frac{1}{2}$$

(We choose positive square root because regression coefficients are positive).

(iv) Note that $(\bar{x}, \bar{y})$ is the point of intersection of regression lines. Thus $(\bar{x}, \bar{y})$ will satisfy both the equations.

Therefore, we get,

$$\bar{x} - 4\bar{y} = 5 \quad \ldots (7)$$

and $$\bar{x} - 16\bar{y} = -64 \quad \ldots (8)$$

Solving equations (7) and (8), we get,

$$\bar{x} = 28, \quad \bar{y} = \frac{23}{4}$$

(v) To find σ_y we use $b_{xy} = r\frac{\sigma_x}{\sigma_y} = 4$.

$$\frac{1}{2} \times \frac{\sigma_x}{\sigma_y} = 4 \qquad \sigma_y = \frac{\sigma_x}{8} = 1$$

Illustrative 16 : *Find correlation coefficient between heights of fathers and their sons from the following data : (heights in inches).*

Height of Fathers	*65*	*66*	*67*	*68*	*69*	*70*	*72*	*67*
Height of Sons	*67*	*68*	*66*	*68*	*72*	*72*	*69*	*70*

Solution : Let Father's height (X), Son's height (Y). $U = X - 65$, $V = Y - 65$

X	Y	U	V	U^2	V^2	UV
65	67	0	2	0	4	0
66	68	1	3	1	9	3
67	66	2	1	4	1	2
68	68	3	3	9	9	9
69	72	4	7	16	49	28
70	72	5	7	25	49	35
72	69	7	4	49	16	28
67	70	2	5	4	25	10
Total	–	**24**	**32**	**108**	**162**	**115**

$$n = 8, \quad \bar{U} = \frac{\sum U}{n} = \frac{24}{8} = 3, \quad \bar{V} = \frac{\sum V}{n} = \frac{32}{8} = 4$$

$$\sigma_u = \sqrt{\frac{\sum U^2}{n} - \bar{U}^2} = \sqrt{\frac{108}{8} - 3^2} = \sqrt{4.5} = 2.1213$$

$$\sigma_v = \sqrt{\frac{\sum V^2}{n} - \bar{V}^2} = \sqrt{\frac{162}{8} - 4^2} = \sqrt{4.25} = 2.0616$$

$$\text{Corr} = \frac{\frac{\sum UV}{n} - \bar{U}\,\bar{V}}{\sigma_u\,\sigma_v} = \frac{\frac{115}{8} - 3 \times 4}{2.1213 \times 2.0616} = 0.5431$$

Illustrative 17 : *The correlation coefficient between two variables X and Y is 0.6. If the means of two series are 13 and 27 respectively and standard deviations are 1.5 and 2 respectively, find the regression line of Y on X.*

Solution : Given : $\bar{X} = 13$, $\bar{Y} = 27$, $\sigma_x = 1.5$, $\sigma_y = 2$, $r = 0.6$.

Regression line of Y on X

$$Y - \bar{Y} = b_{yx}\,(X - \bar{X})$$

$$Y - 27 = 0.6 \times \frac{2}{1.5} \times (X - 13)$$

$$Y = 0.8\,(X - 13) + 27$$

$$Y = 0.8\,X + 16.6$$

Case Study

Alto Pharmaceuticals Ltd. is a company in manufacturing various life saving drugs. It has a manufacturing unit in Anand (Gujrat) and India wide distribution network. Many sales executives, sales representatives and medical representatives are working throughout the country.

It has been observed by the company that since last six months sales have gone down and had a adverse effect on the company's profit.

Senior market executives had a meeting to discuss the problem and concluded that a incentive scheme is to be introduced to promote the sale. Company collected the data for last six months regarding actual incentive given the sales representative and the sales. Suggest the appropriate statistical tools to know whether the incentive scheme has a effect on the company's sale.

Points to Remember

1. Correlation coefficient (r) lies between – 1 and 1.
2. $r = \dfrac{\frac{1}{n}\sum (x-\bar{x})(y-\bar{y})}{\sigma_x \sigma_y} = \dfrac{\frac{1}{n}\sum xy - \bar{x}\,\bar{y}}{\sigma_x \sigma_y}$.
3. Corr (ax + b, cy + d) = Corr (x, y), if a and b have same signs
   = – Corr (x, y), if a and b have opposite signs
4. Regression coefficient of y and x = $b_{yx} = r\dfrac{\sigma_y}{\sigma_x} = \dfrac{\frac{1}{n}\sum xy - \bar{x}\,\bar{y}}{\sigma_x^2}$.

   Regression coefficient of x on y = $b_{xy} = r\dfrac{\sigma_x}{\sigma_y} = \dfrac{\frac{1}{n}\sum xy - \bar{x}\,\bar{y}}{\sigma_y^2}$.
5. $r = \sqrt{b_{xy}\, b_{yx}}$.
6. r, b_{xy}, b_{yx} have same signs.
7. If $r = \pm 1$ regression lines coinside.
8. If $r = \pm 1$ and $\sigma_x = \sigma_y$ then $b_{xy} = b_{yx}$.
9. The two regression lines intersect at $(\bar{x}\ \bar{y})$.
10. Regression line of y on x is $y - \bar{y} = b_{yx}(x - \bar{x})$. It is used to predict y.

    Regression line of x on y is $x - \bar{x} = b_{xy}(y - \bar{y})$. It is used to predict x.

Exercise

[A] Theory Questions :

1. Explain the terms : Bivariate data, covariance, correlation, regression.
2. State the different measures of correlation and describe each of the measures in detail.
3. Describe scatter diagram and explain how it is used to measure correlation.
4. State merits and limitations of scatter diagram as a measure of correlation.
5. Define Karl Pearson's coefficient of correlation or product moment correlation coefficient 'r'. State its merits and demerits. How will you interpret the cases (i) r = + 1, (ii) r = – 1, (iii) r = 0 ?
6. State the properties of Karl Person's correlation coefficient.
7. State the limitations of Karl Pearson's coefficient of correlation.
8. State the merits and demerits of Karl Pearson's correlation coefficient.
9. Explain the term 'regression analysis'.

10. Write a note on 'correlation'.
11. State the equations for regression lines of (i) y on x, (ii) x on y. Discuss the nature of the regression equations in case of $r = -1$, $r = \pm 1$, $r = 0$.
12. Why there are two regression lines ?
13. State utility of regression lines.
14. Explain the least square principle for obtaining regression lines.
15. Define regression coefficients and state the properties.
16. Distinguish between regression and correlation.
17. Can any two lines be regression lines ? Give reasons in support of your answer.
18. How would you interpret regression coefficients ?
19. State the situations where regression analysis is used.
20. State the properties of regression (i) lines, (ii) coefficients.
21. With usual notation, prove that

    (a) $b_{yx} \cdot b_{xy} = r^2$, (b) b_{yx} and b_{xy} cannot exceed unity simultaneously.
22. Define coefficient of determination and state its utility.
23. Show that r, b_{yx} and b_{xy} have same algebraic sign.

[B] Karl Pearson's Coefficient of Correlation from Raw Data :

1. Find the Karl Pearson's correlation coefficient between sales (X) and expenses (Y) from the following data and interpret your results :

Firms	1	2	3	4	5	6	7	8	9	10
Sales (X) (Lakhs ₹)	50	50	56	60	64	65	65	60	60	50
Expenses (Y) (Lakhs ₹)	11	13	14	15	14	15	15	14	16	13

2. Calculate the Karl-Pearson's coefficient of correlation from following data :

Price	22	24	26	28	30	32	34	36	38	40
Demand	60	58	58	50	48	48	48	42	36	32

3. Calculate the Karl-Pearson's coefficient of correlation from the following data :

Demand in Tonnes	9	11	13	15	17	19	21	23
Supply in Tonnes	6	8	10	12	14	16	18	20

4. Compute product moment correlation coefficient between income and expenditure from the following data.

Year	1981	1982	1983	1884	1885	1886	1887	1888
Daily income (₹)	100	110	115	120	125	130	132	140
Average daily expenditure (₹)	85	90	92	100	110	125	125	130

5. From the following data of marks in Mathematics and Statistics, calculate product moment correlation coefficient and interpret the result.

Marks in Statistics	60	70	80	90	10	20	30	40	50
Marks in Mathematics	65	70	80	75	45	40	50	60	55

6. Daily income and savings in ₹ for 10 employees in a certain company are given below :

Income	250	750	820	900	780	360	980	390	650	620
Savings	60	68	62	86	84	51	91	47	53	58

Compute the Karl Pearson's coefficient of correlation between income and savings.

7. Calculate the Karl Pearson's correlation coefficient between advertisement cost and sales from the following data :

Advertisement cost (in thousand ₹)	41	67	65	92	84	77	27	100	38	80
Sales in lakh ₹	46	52	57	85	61	67	59	90	50	83

8. Obtain correlation coefficient between population density (per square miles) and death rate (per thousand persons) from data related to 5 cities.

Population density	200	500	400	700	300
Death rate	12	18	16	21	10

9. The following table gives frequency distribution of 50 clerks in a certain office according to age and pay. Find Karl Pearson's correlation, if any, between age and pay.

Age (in years)	20-30	30-40	40-50	50-60	60-70
Pay (in ₹)	4000	6000	5500	5000	4500

Hint : Take x = the age in years with values as mid-points of class interval.

10. Find the Karl-Pearson's coefficient of correlation between population and pollution.

Population in lakhs (X)	11	12	13	14	15
Pollution in suitable units (Y)	50	52	60	68	80

[C] Karl Pearson's Coefficient of Correlation (Summarised Data) :

11. Given that : $r = 0.4$, $\sum (x - \bar{x})(y - \bar{y}) = 108$, $\sigma_y = 3$ and $\sum (x - \bar{x})^2 = 900$. Find number of pairs of observations viz. n.
12. Find number of pairs of observations from the following data.
    $r = -0.4$, $\sum x = 100$, $\sum x^2 = 2250$, $\sum y = 100$, $\sum y^2 = 2250$, $\sum xy = 1900$.
13. Find correlation coefficient between x and y given that : $n = 8$,
    $\sum (x - \bar{x})^2 = 36$, $(y - \bar{y})^2 = 44$, $\sum (x - \bar{x})(y - \bar{y}) = 24$.
14. Find coefficient of correlation from the following information.
    $n = 10$, $\sum (x - 30) = 11$, $\sum (y - 25) = 7$, $\sum (x - 30)^2 = 215$, $\sum (y - 25)^2 = 163$,
    $\sum (x - \bar{x})(y - \bar{y}) = 186$.
15. Given : $n = 6$, $\sum (x - 18.5) = -3$, $\sum (y - 50) = 20$, $\sum (x - 18.5)^2 = 19$,
    $\sum (y - 50)^2 = 850$, $\sum (x - 18.5)(y - 50) = -120$. Calculate coefficient of correlation.
16. Calculate coefficient of correlation from the following information.
    $n = 5$, $\sum x = 20$, $\sum x^2 = 90$, $\sum y = 20$, $\sum y^2 = 90$, $\sum xy = 73$.
17. Given :

Number of pairs of X and Y series	=	15
Arithmetic mean of X	=	25
Arithmetic mean of Y	=	18
Standard deviation of X	=	3
Standard deviation of Y	=	3
Sum of products of X and Y ($\sum XY$)	=	6870

    Find correlation coefficient between X and Y.

18. From the following data compute the coefficient of correlation :

Number of pairs of observations	=	10
Sum of X series	=	9
Sum of Y series	=	5
Sum of squares of X series	=	653
Sum of squares of Y series	=	595
Sum of product of X and Y series	=	534

19. From the following data compute the coefficient of correlation between X and Y.

Number of pairs of observations	=	10
Sum of deviations of X series	=	– 170
Sum of deviations of Y series	=	– 20
Sum of squares of deviations of X series	=	8000
Sum of squares of deviations of Y series	=	2000
Sum of products of deviations of X and Y series	=	2500

20. Coefficient of correlation between variables X and Y is 0.3 and their covariance is 12. The variance of X is 9, find the standard deviation of Y.

21. If correlation coefficient between X and Y is 0.8 find that between :

(i) X and – Y (ii) 2X and 3Y **(April 2015)**

(iii) X – 10 and Y + 15 (iv) $\frac{X}{2}$ and $\frac{Y}{5}$

(v) $\frac{X-10}{3}$ and $\frac{10-Y}{5}$

[D] Spearman's Rank Correlation Coefficient :

22. Obtain 'Rank Correlation Coefficient' for the results of beauty contest :

Ranks by Judge A	1	5	6	7	8	2	4	3
Ranks by Judge B	1	7	6	2	8	4	5	3

23. Eight contestants in a musical contest were ranked by two judges A and B, in the following manner :

Sr. No.	1	2	3	4	5	6	7	8
Ranks by Judge A	7	6	2	4	5	3	1	8
Ranks by Judge B	5	4	6	3	8	2	1	7

Compute rank correlation coefficient between the two judges and comment on it.

24. Ranks obtained by 6 students in Statistics and Accountancy are given below :

Ranks in Statistics	5	6	4	3	2	1
Ranks in Accountancy	6	2	1	4	3	5

Compute Spearman's Rank Correlation Coefficient.

25. Obtain the Rank Correlation Coefficient for the ranks given by two judges in a contest :

Rank by Judge 'A'	3	6	2	4	5	1
Rank by Judge 'B'	4	5	2	3	6	1

26. The following data relates to the ranks given by judges in a contest :

Sr. No. of Candidate	1	2	3	4	5	6	7	8	9	10
Rank by Judge A	1	5	6	1	2	3	4	7	9	8
Rank by Judge B	5	6	9	2	8	7	3	4	10	1

Compute the rank correlation between the ranks given by judge A and that of judge B. Interprete.

27. The scores obtained by 6 candidates in drawing (X) and in music (Y) are given below :

Candidate	1	2	3	4	5	6
X	24	29	19	14	30	19
Y	37	35	16	26	23	27

Allot the ranks to X and Y and compute Spearman's rank correlation coefficient.

[E] Regression (Raw Data) :

28. Obtain line of regression of y on x for the data given below :

x	06	02	10	04	08
y	09	11	05	08	07

Also estimate y when x = 5.

29. The following data given the sales and expenses of 10 firms.

Firm No.	1	2	3	4	5	6	7	8	9	10
Sales (in '000 ₹)	45	70	65	30	90	40	50	75	85	60
Expenses (in '000 ₹)	35	90	70	40	95	40	60	80	80	50

Obtain the least square regression line of expenses on sales. Estimate expenses if sales are ₹ 75000.

30. A panel of examiners A and B assessed 7 candidates independently and awarded the following marks.

Candidate	1	2	3	4	5	6	7
Marks By A	40	34	28	30	44	38	31
Marks by B	32	39	26	30	38	34	28

Eighth candidate was awarded 36 marks by examiner A. Using appropriate regression line, estimate the marks awarded by the examiner B.

31. The failure of a certain electronic device is suspected to increase linearly with its temperature. Fit a least square regression line through the following data to predict failure rate.

Temperature °F	55	65	75	85	95	105
Failure rate	0	3	7	10	11	11

Also predict the failure rate at 70°C.

32. Samples of soils are collected from various depths below ground level and tested in the laboratory to determine their shear strength. The collected field data are given below :

Depth (m)	2	3	4	5	6	7
Shear strength	14	20	32	39	42	56

Find the Karl Pearson's coefficient of correlation between depth and shear strength. Interpret the result. Also predict shear strength at depth 10 m.

33. A departmental store gives in-service training to its salesmen followed by a test to consider whether it should terminate the services of any of the salesman who does not qualify in the test. The following data give the test scores and sales made by ten salesmen during a certain period.

Test score	14	19	24	21	28	22	15	20	19	20
Sales ('00 ₹)	31	36	48	37	50	45	33	41	39	40

Calculate the coefficient of correlation between the test scores and sales. Does it indicate that the termination of services of the low test scores is justified ? If the firm wants a minimum sales volume of ₹ 3000, what is the minimum test score that will ensure continuation of the services ? Also obtain the standard error of regression estimate.

34. Suppose x is rainfall in suitable units and y is level of rusting of iron material used for construction measured in suitable units ?

x	43	45	59	21	80
y	6	7	8	1	10

Estimate y if x = 30 using regression analysis.

[F] Regression (Summarized Data) :

35. Given the regression equations : $3x + 2y - 26 = 0$ and $6x + y - 31 = 0$.
find : (i) means of x and y. (ii) correlation between x and y.

36. If the regression equation of Y on X is $2X + 3Y = 1$, obtain the regression coefficient of Y on X.

37. Given : $\bar{X} = 80$, $\bar{Y} = 50$, $\sigma_x = 15$, $\sigma_y = 10$ and $r = -0.4$. Find line of regression of X on Y. Also estimate X when Y = 60.

38. The correlation coefficient between two variables X and Y is 0.6. If the means of two series and 13 and 27 respectively and standard deviations are 1.5 and 2 respectively, find the regression line of Y on X.

39. Given the following data : **(April 2011)**

	Rainfall (in inches)	**Yield (in quintals)**
Mean	27	**40**
Standard Deviation	3	**6**

Correlation coefficient = 0.8. Estimate the yield when rainfall is 29 inches.

40. Following is the information about the bivariate frequency distribution :
$n = 20$, $\sum x = 80$, $\sum y = 40$, $\sum x^2 = 1680$, $\sum y^2 = 320$, $\sum xy = 480$.
(i) Obtain the regression lines.
(ii) Estimate y for x = 3 and estimate x for y = 3.

41. You are given the following information about two variables x and y.
$n = 10$, $\sum x^2 = 385$, $\sum y^2 = 192$, $\bar{x} = 5.5$, $\bar{y} = 4$, $\sum xy = 185$.
Find (i) Regression line of y on x. (ii) regression line of x on y.
(iii) Standard error of regression estimate of y on x.

42. Compute regression coefficients from the following data :
$n = 8$, $\sum(x - 45) = -40$, $\sum(x - 45)^2 = 4400$, $\sum(y - 150) = 280$,
$\sum(y - 150)^2 = 167432$, $\sum(x - 45)(y - 150) = 21680$.

43. For a bivariate data we have $\bar{X} = 53$, $\bar{Y} = 28$, $b_{yx} = -1.5$, $b_{xy} = -0.2$.
Find (i) correlation coefficient between X and Y.
(ii) estimate of y for x = 60.
(iii) estimate of x for y = 30.

44. The regression equations are $3x - y - 5 = 0$ and $4x - 3y = 0$. Find
(i) Arithmetic mean of x and y.
(ii) Coefficient variations of x and y, if $\sigma_x = 2$.
(iii) Correlation coefficient between x and y.

45. The following results were obtained from records of age (X) and systolic blood pressure (Y), of a group of 10 men :

	X	Y
Mean	53	142
Variance	130	165

$\sum (x - \bar{x})(y - \bar{y}) = 1220$

Find the appropriate regression equation and use it to estimate the blood pressure of a man with age 45 years.

46. The two regression equations of variables x and y are $x = 19.13 - 0.87\ y$ and $y = 11.64 - 0.5\ x$. Find $\bar{x}$, $\bar{y}$ and Corr (x, y).

47. The regression equations are given by $8x - 10y + 66 = 0$ and $40x - 18y - 214 = 0$. Find $\bar{x}$, $\bar{y}$, Corr (x, y). Also find σ_y given that $\sigma_x = 3$.

48. Given the following data :

	Marks in Mathematics	Marks in English
Mean	80	50
Standard Deviation	15	10

The correlation coefficient between marks in Mathematics and English is – 0.4. Estimate the marks in Mathematics obtained by student who scored 60 marks in English.

Answers

[B] 1. 0.7647 2. – 0.9673 3. 1
4. 0.9593 5. 0.95 6. 0.7804
7. 0.7784 8. 0.9207 9. 0
10. 0.9747

[C] 11. 9 12. 5
13. 0.6030 14. 0.9955 15. – 0.9395
16. – 0.7 17. 0.8888 18. 0.8566
19. 0.6825 20. 13.3333
21. (i) and (iv) – 0.8, (ii), (iii), (iv) 0.8

[D] 22. 0.5952 23. 0.5714 24. – 0.2571
25. 0.8857 26. 0.1030 27. 0.1857

[E] 28. $y = -0.65x + 11.9$, Estimate of $y = 8.65$
29. $y = 1.01289x + 2.2135$, Estimated expenses = ₹ 78180
30. 33
31. $y = 0.2343x - 11.7423$, Estimated failure rate = 4.6571
32. $r = 0.9863$, $y = 8.2286x - 3.0286$, Estimated shear strength = 79.2571
33. $r = 0.9425$, Justified, $x = 0.6156 - 4.4241$, Estimate of score = 14.04
Standard error of estimate = 0.454.
34. $y = 0.1471x - 0.8966$, $\hat{y} = 3.5167$

[F] 35. $\bar{x} = 4$, $\bar{y} = 7$, $r = -0.5$ 36. – 2/3
37. $x = -0.6y + 110$, Estimate of $x = 74$ 38. $y = 0.8x + 16.6$
39. 43.2 quintals
40. (i) $3x = 4y + 4$, $17y = 4x + 18$, (ii) $x = 5.3333$, $y = 1.7647$

41. (i) $y = -0.4242x + 6.3331$, (ii) $x = -1.09375y + 9.875$, (iii) 0.4630.
42. $b_{yx} = 5.4952$, $b_{xy} = 0.1484$.
43. $r = -0.5477$, $x = 52.6$, $y = 17.5$.
44. (i) $\bar{x} = 3$, $\bar{y} = 4$, (ii) C.V. (X) = 66.6667%, C.V. (Y) = 100%, (iii) $r = 0.6667$.
45. $y = 0.833x + 97.851$. 135.336.
46. $\bar{x} = 15.9335$, $\bar{y} = 3.6726$, $r = -0.6593$
47. $\bar{x} = 13$, $\bar{y} = 17$, $\rho = 0.6$, $\sigma_y = 2$.
48. 74

Objective Type Questions

1. If X + Y = constant then state the Corr (X, Y) giving reasons.
2. If $X \propto Y$ state the Corr (X, Y) giving reasons.
3. If $X \propto \frac{1}{Y}$ state the Corr (X, Y) giving reasons.
4. If Corr (X, Y) $\pm$ 1 state the nature of regression lines.
5. If Corr (X, Y) = 0 state the nature of regression lines.
6. If Corr (X, Y) = 0.8 then find the Corr (2X, 2Y), Corr (X, –Y), Corr $\left(\frac{X}{2}, \frac{Y}{3}\right)$.
7. State the Corr (X, X).
8. State the Corr (X, – X).
9. If $\sigma_x = \sigma_y = 2$, Cov (X, Y) = 0.8, find Corr (X, Y).
10. Give examples of : (i) uncorrelated variables (ii) positively correlated variables (iii) negatively correlated variables.
11. If Corr (X, Y) = 0 then find regression coefficients.
12. If Corr (X, Y) = 1, $b_{yx} = 2$ find b_{xy}.
13. If Corr (X, Y) = 1, $\sigma_x = \sigma_y$ then show that $b_{yx} = b_{xy}$.
14. Explain why regression coefficients have same algebraic signs.
15. State the point of intersection of regression lines.
16. Find the correlation between X and Y if :

X	1	2	3	4	5
Y	1	2	3	4	5

17. Find the correlation if :

X	1	2	3	4	5
Y	5	4	3	2	1

Answers

1. Corr (X, Y) = – 1
2. Cor (X, Y) = $\pm$ 1
3. Corr (X, Y) = 0
4. Lines will coincide
5. Lines will be parallel
6. Corr (2X, 2Y) = 0.8, Corr (X, – Y) = – 0.8, Corr (X/2, Y/3) = 0.8
7. Corr (X, X) = 1
8. Corr (X, – X) = – 1
9. Corr (X, Y) = 0.4
10. $b_{yx} = b_{xy} = 0$
12. $b_{xy} = \frac{1}{2}$
15. $(\bar{X}, \bar{Y})$
16. r = 1
17. r = – 1

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SPECIMEN QUESTION PAPER

Time : 3 Hours **Maximum Marks : 70**

Instructions to the candidates :

1. All questions are compulsory.
2. Figures to right indicate full marks.
3. Use of calculator is allowed.

1. Attempt any two of the following : **(7 each)**

(a) State the importance and applications of statistics in the field of business.

(b) Represent the following data by appropriate diagram :

Year	2013	2014	2015	2016	2017	2018
Total sale (in crores ₹)	200	189	207	300	304	310
Profit (in lakh ₹)	15	14	19	22	25	23

(c) Compute the quartiles of the following data :

Marks	0-20	20-40	40-60	60-80	80-100
No. of students	10	90	320	160	70

2. Attempt any two of the following : **(7 each)**

(a) State the requisites of good average. Also discuss the merits and demerits of arithmetic mean.

(b) Find the standard deviation and coefficient of variation for the following frequency distribution.

Class	0-10	10-20	20-30	30-40	40-50	50-60
Frequency	4	10	19	12	4	1

(c) Find the combine mean and standard deviation given that :

$n_1 = 50$ $\bar{X}_1 = 70$ $\sigma_1 = 10$

$n_2 = 100$ $\bar{X}_2 = 55$ $\sigma_2 = 15$

3. Attempt any two of the following : **(7 each)**

(a) Define correlation, state its limits. Also state the difference between correlation and regression.

(b) Find the Karl Pearson's coefficient of correlation between X and Y given the following :

X	65	63	67	64	68	62	70	66	68	67
Y	68	66	68	65	69	66	68	65	71	67

(c) The regression equations are :

$$3x - y - 5 = 0$$

$$4x - 3y = 0$$

Find : (i) $\bar{X}$, $\bar{Y}$. (ii) Coefficient variation of X given $\sigma_x = 2$, (iii) Corr (X, Y).

4. Attempt any two of the following : **(7 each)**

(a) Which player is more consistent ? Why ?

	Runs scored									
Player 1	42	115	6	73	7	19	119	36	84	29
Player 2	47	12	76	42	4	51	37	48	13	0

(b) State the merits and demerits of Karl Pearon's coefficient of correlation.

(c) Represent the following data using pie diagram.

Age group	0-10	10-20	20-40	40-60	60 and above
Population %	13	25	32	20	10

5. Attempt any two of the following : **(7 each)**

(a) Draw the histogram and hence find the mode of the following data :

Income (in lakh ₹)	0-5	5-10	10-50	15-20	20-25
No. of persons	12	40	32	15	4

(b) Compute the quartile deviation for the following frequency distribution.

Class	20-40	40-60	60-80	80-100
Frequency	60	80	40	20

(c) Find the regression line of Y n X for the following data :

X	2	3	5	7	8	10	12	15
Y	2	5	8	10	12	14	15	16

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