NUCLEAR ENGINEERING, UC BERKELEY

HW4 for Numerical Simulation of Radiation Transport

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PROBLEM 1

Calculate an integration $I = \int_{x_{min}}^{x_{max}} f(x)dx$ using rejection Monte Carlo method procedures:

1) Sample random points in the rectangle area [xmin, xmax]*[0, fmax]:

$$\begin{aligned} x_p &= x_{min} + (x_{max} - x_{min}) * \xi_1 \\ y_p &= 0 + fmax * \xi_2 \end{aligned} \tag{1}$$

2) For each random point P (px, py), check if it's under the function curve, i.e. if $y_p \le f(x_p)$. If this is true, accept the point, otherwise reject the point.

3) The probability for a point to be accepted is approximately

$$prob = \frac{N_{accept}}{N_{tot}} \tag{2}$$

4) The integral is I = Area * prob where Area = (xmax- xmin)*fmax

Assuming the true value for π is 3.14159, the result for $\pi = 4\int_0^1 sqrt(1-x^2)dx$ and $\pi = 4\int_0^1 \frac{1}{1+x^2}$ with different numbers of samples is listed in table 1:

nb	f1 relative error	f2 relative error
10	0.036	0.108
100	0.082	0.0196
1000	0.0186	0.0377
10000	0.0025	0.0013

Table 1: Relative error of rejection Monte Carlo method for integral calculation

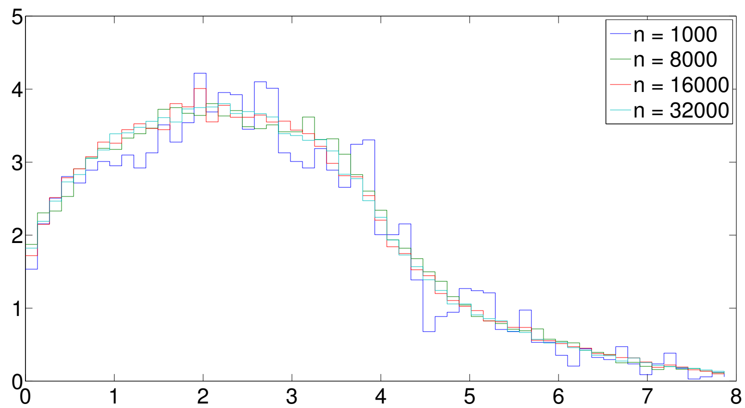

Figure 1: Pb3: Flux distribution in the slab

PROBLEM 3

b) The flux distribution in the slab for different number of source photons are plotted in figure 1. The flux distribution in the slab from SN code and Monte Carlo code are plotted in figure 2.

c) The outgoing partical current at the left courdary is 0.8085, and at the right boundary is 0.0569. The results from SN code was 0.8234 and 0.0582. The absorption rate in the left half of the slab is 2.57, in the right half is 0.56. The absorption rates are 2,54 and 0.56 in SN code.

d) and e) The escape probabilities through the left and right boundaries and the absorption probabilities in the left and right half of the slab from the Monte Carlo code and SN code are compared in table 2. The result from Monte Carlo method and from SN method agree with each other.

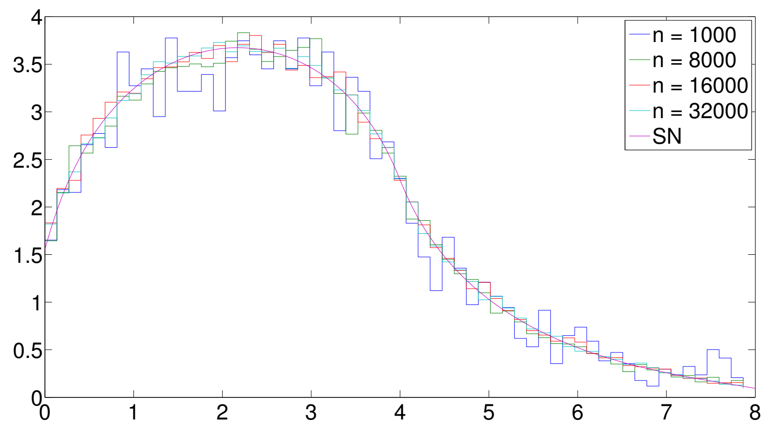

Figure 2: Pb3: Comparison of flux distribution in the slab with SN code

N_{TOT}	P_{escR}	P_{escL}	P_{absR}	P_{absL}
125	0.0080	0.2160	0.6080	0.1680
250	0.0200	0.1600	0.6520	0.1680
500	0.0120	0.1820	0.6840	0.1220
1000	0.0120	0.2070	0.6430	0.1470
2000	0.0135	0.1985	0.6450	0.1430
4000	0.0173	0.2070	0.6318	0.1440
8000	0.0174	0.2086	0.6310	0.1417
16000	0.0158	0.2048	0.6378	0.1393
32000	0.0137	0.2064	0.6407	0.1432
64000	0.0147	0.2064	0.6358	0.1430
SN	0.01455	0.2058	0.6367	0.1420

Table 2: Pb3: Comparison of escape probabilities and absorption probabilities in the slab

PROBLEM 4

The procedures to calculate the energy reflection factor(albedo) for a beam of 2Mev photons normally incident on a face of a slab of ordinary concrete, 5cm thick is the following:

a) The photon from the beam incident to the face of the slab, at z0=0 with the angle $\mu = 1$.

The total collision cross section can be found online as attenuation coefficient:

$$\rho_{concrete} = 2.3(g/cm^3)$$
$$\Sigma_{tot} = 4.557 * 0.01 * \rho(cm^{-1}) \tag{3}$$

Sample the distance to collision:

$$dist = -\mu/\Sigma_{tot} ln(\xi_3) \tag{4}$$

b) So the first interaction would be at $z1 = z0 + dist = dist$ if z1 is within the slab [0, 5]cm. If Z1 is not within [0, 5]cm, then score zero

c) If Z1 is within [0, 5]cm, determine the collision type (absorption or scattering): To do so, we need to at first calculate the scattering cross section. The microscopic total Compton scattering cross section can be obtain as:

$$\begin{aligned}
\sigma_{cs} &= 2\pi r_0 \{\frac{1+\alpha}{\alpha^2}[\frac{2(1+\alpha)}{1+2\alpha} - \frac{1}{\alpha} ln(1+2\alpha)] + [\frac{1}{2\alpha} ln(1+2\alpha) - \frac{1+3\alpha}{(1+2\alpha)^2}]\}[cm^2] \\
\alpha &= \frac{E_{in}}{m_0 c^2} \\
r_0 &= \frac{e^4}{(m_0 c^2)^2}
\end{aligned} \tag{5}$$

The macroscopic Compton scattering cross section is:

$$\Sigma_{cs} = \sigma cs * N \tag{6}$$

The atomic number density N can be calculated from :

$$N = \rho * Av/A \tag{7}$$

The average atomic mass A for ordinary concrete can be calculated from the material composition in table 3 and the Z/A ratio for ordinary concrete is 0.50932.

d) If absorbed, i.e. $\xi > Pcs = \frac{sigma_cs}{sigma_tot}$, score zero. Otherwise the photon is scattered. Sample the scattered photon energy E1 as shown in the flow chart and calculate the scattered angle $cos\theta_1$.

isotope Z	weight fraction(%)
1	0.022100
6	0.002484
8	0.574930
11	0.015208
12	0.001266
13	0.019953
14	0.304627
19	0.010045

Table 3: Ordinary concrete composition

$$cos(\theta_s) = 1 - \lambda_1 + \lambda_0 \tag{8}$$

$$m_0c^2 = 0.511Mev \tag{9}$$

$$\lambda_0 = \frac{m_0c^2}{E_0} cos(\theta_1) = cos(\theta_0)cos(\theta_s) + sin(\theta_0)sin(\theta_s)cos(\beta) \tag{10}$$

e) If scattering anble is smaller or equal to 90, then the photon will not go back without another scattering. It doens't contribute significantly to the reflection process.

f) If the scattering angle is larger than 90, calculate the second interaction distance s2.

g) If $s2 < \|x_1/cos(\theta_1)\|$, the photon has the second interaction in the slab. It doesn't contribute significantly to the reflection process.

h) If $s2geq\|x_1/cos(\theta_1)\|$, the photon reflects back with a single scattering. Score $E_1/cos(\theta_1)$

The energy reflection factor (albedo) is $Score/N_{source} = 0.17\%$.