

HW4 for Numerical Simulation of Radiation Transport

Xin Wang

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PROBLEM 1

Calculate an integration $I = \int_{x_{min}}^{x_{max}} f(x) dx$ using rejection Monte Carlo method procedures:

- 1) Sample random points in the rectangle area $[x_{min}, x_{max}] \times [0, f_{max}]$:

$$\begin{aligned}x_p &= x_{min} + (x_{max} - x_{min}) * \xi_1 \\y_p &= 0 + f_{max} * \xi_2\end{aligned}\tag{1}$$

- 2) For each random point P (px, py), check if it's under the function curve, i.e. if $y_p \leq f(x_p)$. If this is true, accept the point, otherwise reject the point.

- 3) The probability for a point to be accepted is approximately

$$prob = \frac{N_{accept}}{N_{tot}}\tag{2}$$

- 4) The integral is $I = Area * prob$ where $Area = (x_{max} - x_{min}) * f_{max}$

Assuming the true value for π is 3.14159, the result for $\pi = 4 \int_0^1 \sqrt{1-x^2} dx$ and $\pi = 4 \int_0^1 \frac{1}{1+x^2}$ with different numbers of samples is listed in table 2:

| nb | f1 relative error | f2 relative error |
|-------|-------------------|-------------------|
| 10 | 0.036 | 0.108 |
| 100 | 0.082 | 0.0196 |
| 1000 | 0.0186 | 0.0377 |
| 10000 | 0.0025 | 0.0013 |

Table 1: Relative error of rejection Monte Carlo method for integral calculation

PROBLEM 3

| N_{TOT} | P_{escR} | P_{escL} | f2 relative error |
|-----------|------------|------------|-------------------|
| 125 | 10 | 0.036 | 0.108 |
| 250 | 100 | 0.082 | 0.0196 |
| 500 | 1000 | 0.0186 | 0.0377 |
| 1000 | 10000 | 0.0025 | 0.0013 |

Table 2: Relative error of rejection Monte Carlo method for integral calculation

PROBLEM 4

a) The photon from the beam incident to the face of the slab, at $z_0=0$ with the angle $\mu = 1$.

The total collision cross section can be found online as attenuation coefficient:

$$\begin{aligned}\rho_{concrete} &= 2.3(g/cm^3) \\ \mu_{tot} &= 4.557 * 0.01 * \rho(cm)\end{aligned}\tag{3}$$

Sample the distance to collision:

$$dist = -1/\mu_{tot} \ln(\xi_3)\tag{4}$$

b) So the first interaction would be at $z_1 = z_0 + dist = dist$ if z_1 is within the slab $[0, 5]cm$.

c) Determine the collision type: absorption or scattering: To do so, we need to at first calculate the scattering cross section. The microscopic total Compton scattering

| isotope Z | weight fraction(%) |
|-----------|--------------------|
| 1 | 0.022100 |
| 6 | 0.002484 |
| 8 | 0.574930 |
| 11 | 0.015208 |
| 12 | 0.001266 |
| 13 | 0.019953 |
| 14 | 0.304627 |
| 19 | 0.010045 |

Table 3: Ordinary concrete composition

cross section can be obtain as:

$$\begin{aligned}
\sigma_{cs} &= 2\pi r_0 \left\{ \frac{1+\alpha}{\alpha^2} \left[\frac{2(1+\alpha)}{1+2\alpha} - \frac{1}{\alpha} \ln(1+2\alpha) \right] + \left[\frac{1}{2\alpha} \ln(1+2\alpha) - \frac{1+3\alpha}{(1+2\alpha)^2} \right] \right\} [cm^2] \\
\alpha &= \frac{E_{in}}{m_0 c^2} \\
r_0 &= \frac{e^4}{(m_0 c^2)^2}
\end{aligned} \tag{5}$$

The macroscopic Compton scattering cross section is:

$$\mu_{cs} = \sigma_{cs} * N \tag{6}$$

The atomic number density N can be calculated from :

$$N = \rho * Av / A \tag{7}$$

The average atomic mass A for ordinary concrete can be calculated from the material composition in table 3

the Z/A ratio for ordinary concrete is 0.50932.