

## Chapter: Parallelogram

### Exercise: 16A

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**Question 1:** ABCD is a parallelogram in which  $\angle A = 110^\circ$ . Find the measure of each of the angles  $\angle B$ ,  $\angle C$  and  $\angle D$ .

**Solution:**

It is given that ABCD is a parallelogram in which  $\angle A$  is equal to  $110^\circ$ .

Sum of the adjacent angles of a parallelogram is  $180^\circ$ .

$$\therefore \angle A + \angle B = 180^\circ$$

$$110^\circ + \angle B = 180^\circ$$

$$\angle B = 180^\circ - 110^\circ$$

$$\angle B = 70^\circ$$

$$\therefore \angle B = 70^\circ$$

$$\text{Also, } \angle B + \angle C = 180^\circ$$

$$70^\circ + \angle C = 180^\circ$$

$$\angle C = 180^\circ - 70^\circ$$

$$\therefore \angle C = 110^\circ$$

$$\text{Further, } \angle C + \angle D = 180^\circ$$

$$110^\circ + \angle D = 180^\circ$$

$$\angle D = 180^\circ - 110^\circ$$

$$\angle D = 70^\circ$$

$$\therefore \angle D = 70^\circ$$

**Question 2:** Two adjacent angles of a parallelogram are equal. What is the measure of each of these angles?

**Solution:**

Let the required angle be  $x^\circ$

As the adjacent angles are equal, we have:

$$x + x = 180^\circ \quad (\text{since the sum of adjacent angles of a parallelogram is } 180^\circ)$$

$$2x = 180^\circ$$

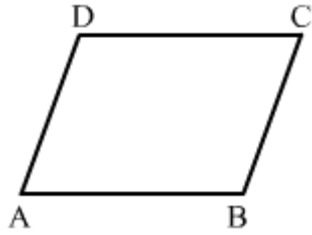
$$x = \frac{180^\circ}{2}$$

$$x = 90^\circ$$

Hence, the measure of each of the angles is  $90^\circ$ .

**Question 3:** Two adjacent angles of a parallelogram are in the ratio 4:5. Find the measure of each of its angles.

**Solution:**



Let ABCD be the parallelogram.

Then,  $\angle A$  and  $\angle B$  are its adjacent angles.

Let  $\angle A = 4x^\circ$  and  $\angle B = 5x^\circ$

$\therefore \angle A + \angle B = 180^\circ$  (since sum of the adjacent angles of a parallelogram is  $180^\circ$ )

$$4x + 5x = 180^\circ$$

$$9x = 180^\circ$$

$$x = \frac{180^\circ}{9}$$

$$x = 20^\circ$$

$$\therefore \angle A = 4 \times 20^\circ = 80^\circ$$

$$\angle B = 5 \times 20^\circ = 100^\circ$$

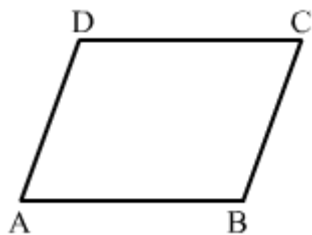
Opposite angles of parallelogram are equal.

$$\therefore \angle C = \angle A = 80^\circ$$

$$\angle D = \angle B = 100^\circ$$

**Question 4: Two adjacent angles of a parallelogram are  $(3x - 4)^\circ$  and  $(3x - 16)^\circ$ . Find the value of  $x$  and find the measure of each of its angles.**

**Solution:**



Let ABCD be a parallelogram.

Let  $\angle A = (3x - 4)^\circ$

$$\angle B = (3x - 16)^\circ$$

$\therefore \angle A + \angle B = 180^\circ$  since the sum of adjacent angles of a parallelogram is  $180^\circ$

$$3x - 4 + 3x - 16 = 180^\circ$$

$$3x - 4 + 3x - 16 = 180^\circ$$

$$6x + 12 = 180^\circ$$

$$6x = 180^\circ - 12^\circ$$

$$6x = 168^\circ$$

$$x = \frac{168^\circ}{6}$$

$$x = 28$$

$$\therefore \angle A = (3 \times 28 - 4)^\circ$$

$$= (84 - 4)^\circ$$

$$= 80^\circ$$

$$\angle B = 3 \times 28 + 16^\circ$$

$$= 84 + 16^\circ$$

$$= 100^\circ$$

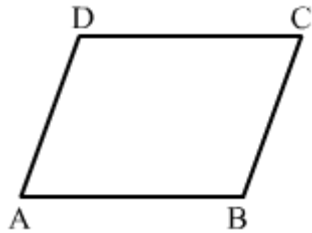
The opposite angles of a parallelogram are equal.

$$\therefore \angle C = \angle A = 80^\circ$$

$$\angle D = \angle B = 100^\circ$$

**Question 5: The sum of two opposite angles of a parallelogram is  $130^\circ$ . Find the measure of each of its angles.**

**Solution:**



Let ABCD be a parallelogram and let the sum of its opposite angles be  $130^\circ$ .

$$\angle A + \angle C = 130^\circ$$

The opposite angles are equal in a parallelogram.

$$\therefore \angle A = \angle C = x^\circ$$

$$x + x = 130^\circ$$

$$2x = 130^\circ$$

$$x = \frac{130^\circ}{2}$$

$$x = 65^\circ$$

$$\therefore \angle A = 65^\circ \text{ and } \angle C = 65^\circ$$

$$\angle A + \angle B = 180^\circ \quad (\text{since the sum of adjacent angles of a parallelogram is } 180^\circ)$$

$$65^\circ + \angle B = 180^\circ$$

$$\angle B = 180^\circ - 65^\circ$$

$$\angle B = 115^\circ$$

$$\angle D = \angle B = 115^\circ \quad [\text{opposite angles of parallelogram are equal}]$$

**Question 6:** Two sides of a parallelogram are in the ratio 5:3. If its perimeter is 64 cm. find the lengths of its sides.

**Solution:**

Let the lengths of two sides of the parallelogram be  $5x$  cm and  $3x$  cm, respectively.

Then, its perimeter  $= 2(5x + 3x)$  cm

$$= 16x \text{ cm}$$

$$\therefore 16x = 64$$

$$x = \frac{64}{16}$$

$$x = 4$$

$$\therefore \text{one side} \Rightarrow 5 \times 4 \text{ cm} = 20 \text{ cm}$$

$$\text{Other side} \Rightarrow 3 \times 4 \text{ cm} = 12 \text{ cm}$$

**Question 7:** The perimeter of a parallelogram is 140 cm. If one of the sides is longer than the other by 10 cm. find the length of each of its sides.

**Solution:**

Let the lengths of two sides of the parallelogram be  $x$  cm and  $x + 10$  cm, respectively. Then, its

Perimeter  $= 2[x + x + 10]$  cm

$$= 2[2x + 10] \text{ cm}$$

$$= 4x + 20 \text{ cm}$$

$$= 4x + 20 \text{ cm}$$

$$4x + 20 = 140$$

$$4x = 140 - 20$$

$$4x = 120$$

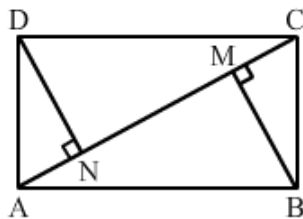
$$x = \frac{120}{4}$$

$$x = 30$$

Length of one side  $= 30$  cm

Length of the other side  $\Rightarrow 30 + 10 \text{ cm} = 40 \text{ cm}$

**Question 8:** In the adjacent figure, ABCD is a rectangle. If BM and DN are perpendiculars from B and D on AC, prove that  $\triangle BMC \cong \triangle DNA$ . Is it true that  $BM = DN$ ?



**Solution:**

In  $\triangle BMC$  and  $\triangle DNA$

$$\angle DNA = \angle BMC = 90^\circ$$

$$\angle BCM = \angle DNA \quad \text{alternate angles}$$

$$BC = DA \quad \text{opposite sides}$$

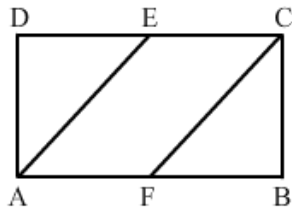
By AAS congruency criteria:

$$\triangle BMC \cong \triangle DNA$$

Yes, it is true that BM is equal to DN.

(by corresponding parts of congruent triangles BMC and DNA )

**Question 9:** In the adjacent figure, ABCD is a parallelogram and line segments AE and CF bisect angles A and C respectively. Show that  $AE \parallel CF$ .



**Solution:**

$$\angle A = \angle C$$

(Opposite angles of a parallelogram are equal)

$$\frac{1}{2} \angle A = \frac{1}{2} \angle C$$

$$\angle EAD = \angle FCB$$

(AE and CF bisect the angles A and C, respectively)

In  $\triangle ADE$  and  $\triangle CBF$ :

$$\angle B = \angle D$$

(opposite angles of a parallelogram are equal)

$$\angle EAD = \angle FCB$$

(proved above)

$$AD = BC$$

(opposite sides of a parallelogram are equal)

By AAS concurrency criteria:

$$\triangle ADE \cong \triangle CBF$$

$$DE = BF$$

(corresponding parts of congruent triangles)

$$CD = AB$$

(opposite sides of a parallelogram are equal)

$$\text{Also, } CD - DE = AB - BF$$

$$CE = AF$$

ABCD is a parallelogram.

$$\therefore CD \parallel AB$$

(opposite sides of a parallelogram are parallel)

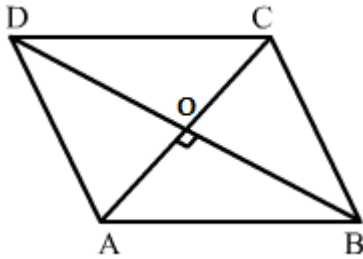
$$CE \parallel AF$$

If one pair of sides of a quadrilateral is parallel and equal, then it is a parallelogram. Therefore, AECF is a parallelogram.

$$\therefore AE \parallel CF$$

**Question 10:** The length of the diagonals of a rhombus are 16 cm and 12 cm respectively. Find the length of each of its sides.

**Solution:**



Let ABCD be a rhombus.

Let AC and BD be the diagonals of the rhombus intersecting at a point O.

Let AC=16 cm

BD=12 cm

We know that the diagonals of a rhombus bisect each other at right angles.

$$\therefore AO = \frac{1}{2} AC$$

$$= \frac{1}{2} \times 16 \text{ cm}$$

$$= 8 \text{ cm}$$

$$BO = \frac{1}{2} BD$$

$$= \frac{1}{2} \times 12 \text{ cm}$$

$$= 6 \text{ cm}$$

From the right  $\triangle AOB$

$$AB^2 = AO^2 + BO^2$$

$$AB^2 = 8^2 + 6^2 \text{ cm}^2$$

$$AB^2 = 64 + 36 \text{ cm}^2$$

$$AB^2 = 100 \text{ cm}^2$$

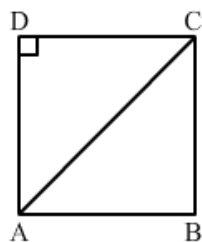
$$AB = \sqrt{100} \text{ cm}^2$$

$$= 10 \text{ cm}$$

Hence, the length of the side AB is 10 cm.

AB=BC=CD=DA=10 cm (all sides of a rhombus are equal)

**Question 11:** In the given figure ABCD is a square. Find the measure of  $\angle CAD$



**Solution:**

In  $\triangle ADC$

$DA=DC$  (all sides of a square are equal)

$\angle ACD = \angle CAD$

Let  $\angle ACD = \angle CAD = x^\circ$  [Angle opposite to the equal sides are equal]

$x + x + 90^\circ = 180^\circ$  since the sum of the angles of a triangle is  $180^\circ$

$2x + 90^\circ = 180^\circ$

$2x = 90^\circ$

$x = \frac{90^\circ}{2}$

$x = 45^\circ$

$\therefore \angle CAD = 45^\circ$

**Question 12: The sides of a rectangle are in the ratio 5:4 and its perimeter is 90 cm. Find its length and breadth.**

**Solution:**

Let the length of two sides of the rectangle be  $5x$  cm and  $4x$  cm, respectively.

Then, its perimeter  $= 2(5x + 4x)$  cm

$= 18x$  cm

$\therefore 18x = 90$

$x = \frac{90}{18}$

$x = 5$

Length of one side  $\Rightarrow 5 \times 5$  cm  $= 25$  cm

Length of the other side  $\Rightarrow 4 \times 5$  cm  $= 20$  cm

$\therefore$  Length of the rectangle  $= 25$  cm

Breadth  $= 20$  cm

**Question 13: Name each of the following parallelogram.**

**(i) The diagonals are equal and the adjacent sides are unequal**

**Solution:**

The diagonals are equal and the adjacent sides are unequal.

Hence, the given parallelogram is a rectangle.

**(ii) The diagonals are equal and the adjacent sides are equal.**

**Solution:**

The diagonals are equal and the adjacent sides are equal.

Hence, the given parallelogram is a square.

**(iii) The diagonals are unequal and the adjacent sides are equal.**

**Solution:** The diagonals are unequal and the adjacent sides are equal.

Hence, the given parallelogram is a rhombus.

**(iv) All the sides are equal and one angle is  $60^\circ$**

**Solution:** All the sides are equal and one angle is  $60^\circ$

Hence, the given parallelogram is a rhombus.

**(v) All the sides are equal and one angle is  $90^\circ$**

**Solution:** All the sides are equal and one angle is  $90^\circ$

Hence, the given parallelogram is a square.

**(vi) All the angles are equal and the adjacent sides are unequal.**

**Solution:** All the angles are equal and the adjacent sides are unequal.

Hence, the given parallelogram is a rectangle.

**Question 14: Which of the following statements are true and which are false?**

**(i) The diagonals of a parallelogram are equal.**

**Solution:** The given statement is false.

The diagonals of a parallelogram bisect each other, but they are not equal in length

**(ii) The diagonals of a rectangle are perpendicular to each other.**

**Solution:** The given statement is false.

The diagonals of a rectangle are equal and bisect each other, but they are not perpendicular.

**(iii) The diagonals of a rhombus are equal.**

**Solution:** The given statement is false.

All the sides of a rhombus are equal,

but the diagonals are not equal.

**(iv) Every rhombus is a kite.**

**Solution:** The given statement is true.

**(v) Every rectangle is a square.**

**Solution:** The given statement is false.

Every square is a rectangle, but every rectangle is not a square.



**(vi) Every square is a parallelogram.**

**Solution:** The given statement is true.

**(vii) Every square is a rhombus.**

**Solution:** The given statement is true.

**(viii) Every rectangle is a parallelogram.**

**Solution:** The given statement is true.

**(ix) Every parallelogram is a rectangle.**

**Solution:** The given statement is false.

A rectangle is a special type of parallelogram, but every parallelogram is not a rectangle.

**(x) Every rhombus is a parallelogram.**

**Solution:** The given statement is true.

### Exercise – 16 B

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#### Objective Questions

**Tick ( ) the correct answer in each of the following:**

**Question 1: The two diagonals are not necessarily equal in a**

- (a) Rectangle      (b) Square      (c) Rhombus      (d) isosceles trapezium

**Solution:**

- (c) Rhombus

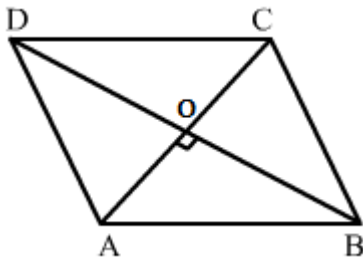
In a rhombus, the two diagonals are not necessarily equal.

**Question 2: The lengths of the diagonals of a rhombus are 16 cm and 12 cm. The length of each side of the rhombus is**

- (a) 8 cm      (b) 9 cm      (c) 10 cm      (d) 12 cm

**Solution:**

- (c) 10 cm



Let ABCD be a rhombus

Let AC and BD be the diagonals of the rhombus

Intersecting at a point O.

$$AC=16 \text{ cm}$$

$$BD=12 \text{ cm}$$

We know that the diagonals of a rhombus bisect each other at right angles.

$$\therefore AO = \frac{1}{2} AC$$

$$= \frac{1}{2} \times 16 \text{ cm}$$

$$= 8 \text{ cm}$$

$$BO = \frac{1}{2} BD$$

$$= \frac{1}{2} \times 12 \text{ cm}$$

$$= 6 \text{ cm}$$

From the right  $\triangle AOB$ :

$$AB^2 = AO^2 + BO^2$$

$$AB^2 = 8^2 + 6^2 \text{ cm}^2$$

$$AB^2 = 64 + 36 \text{ cm}^2$$

$$AB^2 = 100 \text{ cm}^2$$

$$AB = \sqrt{100} \text{ cm}$$

$$AB = 10 \text{ cm}$$

Hence, the length of the side AB is 10 cm.

Therefore, the length of each side of the rhombus is 10 cm because all the sides of a rhombus are equal.

**Question 3:** Two adjacent angles of a parallelogram are  $(2x + 25)^\circ$  and  $(3x - 5)^\circ$ . The value of x is.

(a) 28 cm

(b) 32 cm

(c) 36 cm

(d) 42 cm

**Solution:**

(b) 32

We know that the sum of adjacent angles of a parallelogram is  $180^\circ$

$$2x + 25 + 3x - 5 = 180^\circ$$

$$5x + 20 = 180^\circ$$

$$5x = 180^\circ - 20^\circ$$

$$5x = 160^\circ$$

$$x = \frac{160^\circ}{5}$$

$$x = 32^\circ$$

Therefore, the value of x is  $32^\circ$ .

**Question 4: The diagonals do not necessarily intersect at right angles in a**

- (a) parallelogram      (b) rectangle      (c) rhombus      (d) kite

**Solution:**

(a) parallelogram

In a parallelogram, the diagonals do not necessarily intersect at right angles.

**Question 5: The length and breadth of a rectangle are in the ratio 4:3. If the diagonal measures 25 cm then the perimeter of the rectangle is**

- (a) 56 cm      (b) 60 cm      (c) 70 cm      (d) 80 cm

**Solution:**

(c) 70 cm

Let ABCD be a rectangle and let the diagonal AC be 25 cm,

Length AB be 4x cm and breadth BC be 3x cm.

Each angle of a rectangle is a right angle

$$\therefore \angle ABC = 90^\circ$$

From the right  $\triangle ABC$ :

$$AC^2 = AB^2 + BC^2$$

$$25^2 = 4x^2 + 3x^2$$

$$625 = 16x^2 + 9x^2$$

$$625 = 25x^2$$

$$x^2 = \frac{625}{25}$$

$$x^2 = 25$$

$$x = \sqrt{25}$$

$$x = 5$$

$$\text{Length} = 4 \times 5 = 20 \text{ cm}$$

$$\text{Breadth} = 3 \times 5 = 15 \text{ cm}$$

$$\therefore \text{Perimeter of the rectangle} = 2(20 + 15) \text{ cm}$$

$$= 70 \text{ cm}$$

**Question 6: The bisectors of any two adjacent angles of a parallelogram intersect at.**

- (a)  $30^\circ$       (b)  $45^\circ$       (c)  $60^\circ$       (d)  $90^\circ$

**Solution:**

- (d)  $90^\circ$

The bisectors of any two adjacent angles of a parallelogram intersect at  $90^\circ$ .

**Question 7: If an angle of a parallelogram is two-thirds of its adjacent angle, the smallest angle of the parallelogram is**

- (a)  $54^\circ$       (b)  $72^\circ$       (c)  $81^\circ$       (d)  $108^\circ$

**Solution:**

- (b)  $72^\circ$

Let  $x^\circ$  be the angle of the parallelogram.

Sum of the adjacent angles of a parallelogram is  $180^\circ$

$$\therefore x + \left(\frac{2}{3} \times x\right) = 180^\circ$$

$$x + \frac{2x}{3} = 180^\circ$$

$$x + \frac{2x}{3} = 180^\circ$$

$$\frac{5x}{3} = 180^\circ$$

$$x = 180^\circ \times \frac{3}{5}$$

$$x = 108^\circ$$

Hence, one angle of the parallelogram is  $180^\circ$ .

Its adjacent angle =  $180^\circ - 108^\circ = 72^\circ$

Therefore, the smallest angle of the parallelogram is  $72^\circ$

**Question 8: The diagonals do not necessarily bisect the interior angles at the vertices in a**

- (a) Rectangle      (b) Square      (c) Rhombus      (d) all of these

**Solution:**

- (a) rectangle

In a rectangle, the diagonals do not necessarily bisect the interior angles at the vertices.

**Question 9: In a square ABCD,  $AB = (2x + 3)$  cm and  $BC = (3x - 5)$  cm. Then the value of  $x$  is**

- (a) 4      (b) 5      (c) 6      (d) 8

**Solution:**

- (d) 8

All the sides of a square are equal.

$$\therefore AB = BC$$

$$2x+3=3x-5$$

$$3+5=3x-2x$$

$$8=x$$

Therefore, the value of x is 8.

**Question 10: If one angle of parallelogram is  $24^\circ$  less than twice the smallest angle then the largest angle of the parallelogram is**

(a)  $68^\circ$

(b)  $102^\circ$

(c)  $112^\circ$

(d)  $176^\circ$

**Solution:**

(c)  $112^\circ$

Let  $x^\circ$  be the smallest angle of the parallelogram.

The sum of adjacent angles of a parallelogram is  $180^\circ$

$$\therefore x+2x-24^\circ=180^\circ$$

$$3x-24^\circ=180^\circ$$

$$3x=180^\circ+24^\circ$$

$$3x=204^\circ$$

$$x=\frac{204^\circ}{3}$$

$$x=68^\circ$$

$$\therefore \text{smallest angle} = 68^\circ$$

$$\text{Largest angle} = 180^\circ - 68^\circ = 112^\circ$$