Math 30-1 Polynomial, Radial and Rational Functions
General Overview of Major Concepts

Key Concepts

Function: A relationship between two variables (usually shown as x and y) in which for every x value (more precisely the input variable), only one y (the output variable) value will be given. One can use the vertical line test to determine if the graph of an equation is a function.

Translation: Moving the function up or down, left or right on a graph.

- vertical translations of k units **up**, replace the y [or $f(x)$] in the function equation with (y-k);
- for translation of k units **down**, replace y with (y+k)
- for translations to the **left**, replace x with (x+k)
- for translations to the **right**, replace x with (x-k)

Stretches: Rescaling the function bigger or smaller and can be done in the x or y-axis direction

- Stretching by 2 in the x-axis is done through the transformation $g(x) = f(\frac{x}{2})$
- Stretching by 2 in the y-axis is done through the transformation $g(x) = 2f(x)$

Reflection: Creating a mirror image across a vertical or horizontal line on the graph that is usually the x or y axis.

- Flipping the image on the y-axis can be done through the transformation $f(x) \rightarrow f(-x)$
- Flipping the image on the x-axis can be done through the transformation $f(x) \rightarrow -f(x)$

Radical Functions: A function with a radical in the form of $f(x) = a\sqrt{b(x-h)} + d$. The part under the radical sign CANNOT be negative (for math 30).

Rational Functions: A function with the x-variable in the denominator of a fraction. Note: f(x)=x/3 is not a rational function because it can be written as $f(x) = x \div 3$ which is simply a straight line.

- Horizontal asymptotes are y-values that the graph approaches when the x value gets infinitely large or infinitely small
- For a rational function in the form of $f(x) = \frac{ax+b}{cx+d}$, horizontal asymptotes can be found by -a/c

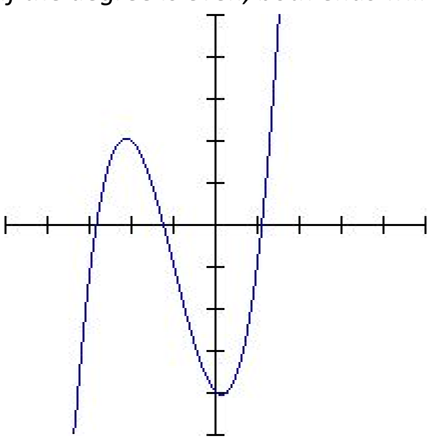
- Vertical asymptotes are x-values that the graph approaches when the y value gets infinitely large or infinitely small
- For a rational function in the form of $f(x) = \frac{ax+b}{cx+d}$, vertical asymptotes can be found by -d/c

- It will take some thoughts to find the asymptotes of the rational function with x variables that have a power greater than 1. Ask a study space volunteer if you want to know more about this :)

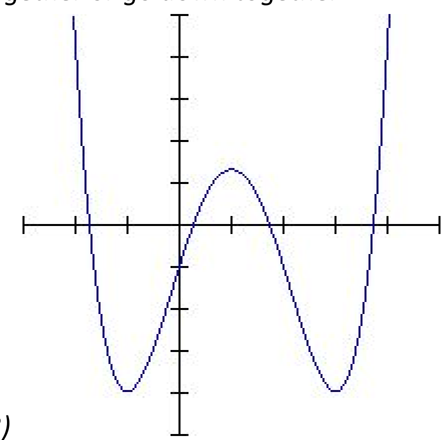
Polynomials:

Leading Coefficient: The coefficient of the variable with the highest power

Degree of Polynomial: The highest power that the variables in the polynomial has is its degree
Degree of the polynomial can dictate its shape
If the degree is odd, one end of the polynomial function on the graph goes down and the other end goes up
If the degree is even, both ends will either go up together or go down together



Odd (degree 3)



Even (d4)

Usually, the number of changes in direction in a polynomial function graph is 1 less than its degree, this would only be wrong if the polynomial has two or more zeros that are the same value

Zeros/Roots of Polynomial: solutions to P(x) = 0; they are numerical values that can be real or imaginary

X-intercepts of Polynomial: values at which P(x) = 0 on a graph; they cannot be imaginary

Factors of Polynomial: polynomial expressions in the form of (ax-b) that make up the P(x)

Questions

1. $y = f(x)$ is transformed to $y = f(\frac{x}{5}) + 5$, what transformation is it?

   (Stretching by 5 in the x-axis, and translating 5 units up, in any order)

2. What is the vertical asymptote of the function $f(x) = \frac{5x+3}{2x-3}$?

   ($x = \frac{3}{2}$)

3. If the function $f(x) = x^2 + 1$ is transformed into $g(x) = f(2x)$ what is the equation of the new function?

($g(x) = 4x^2 + 1$)

4. What are the zeros of the polynomial function $p(x) = x(4 - x)(3x + 2)(x - 1)$?

($0,\ 4\ -\frac{2}{3}, 1$)

5. What are the potential zeros of the polynomial function $p(x) = 3x^3 - 6x^2 - 4x + 8$?

($\pm 1,\ \pm \frac{1}{3},\ \pm 2,\ \pm \frac{2}{3},\ \pm 4,\ \pm \frac{4}{3},\ \pm 8,\ \pm \frac{8}{3}$)

6. Factor completely the polynomial function $p(x) = x^4 - 1$.

($p(x) = (x^2 + 1)(x + 1)(x - 1)$)

7. If $\frac{3x^3 - 6x^2 + 2x + k}{x - 2}$ has a remainder of -3, then the value of k is?

(-7)

8. A polynomial function has the zeros -4, 0, 0, and 1 and no nonreal roots. The graph of the polynomial passes through point (-1, -3). What is the polynomial function?

($p(x) = \frac{x^2(x+4)(x-1)}{2}$)

9. For the function $y = 4^x$, state another single transformation that would result in the identical image as a vertical stretch by 16.

(Horizontal translation left 2)

10. For the function $2y = f(\frac{x}{4} - 1) - 6$, state the mapping.

($4x + 4,\ \frac{y}{2} - 3$)

11. For a function $y = f(x)$, a vertical stretch and a horizontal stretch yield equivalent results. What can be concluded about the x and y intercepts of $y = f(x)$?

(The x and y intercepts are at the origin.)

12. For the function f(x)= $\frac{x+3}{x^3+3x^2}$, state the domain and range.

($x \varepsilon (-\infty, -3) \cup (-3, 0) \cup (0, \infty)$ and $y \varepsilon (0, \infty)$)

13. State the number of distinct potential rational zeros of $6x^4 + 3x^3 - 2x + 8$.

(20 distinct potential zeros)

CHALLENGE: For the function $h(x) = \frac{x^4 - 7x^3 + 9x^2 + 27x + 54}{2}$, what are the potential zeroes?

(±1,±2,±3,±6,±27,±54)

CHALLENGE: A parabola y=f(x) with its vertex at (3,k) undergoes the following transformation in the given order:

1. Reflection over the x-axis
2. Vertical translation n units up

The transformed function, y=g(x), is graphed along with y=f(x). The function y=g(x) has its new vertex at (h, 4), and y=f(x)and y=g(x) intersect at point (1,3). Determine the value of k and n.

(k=2; n=6)

CHALLENGE: A degree 1 polynomial function $y = f(x)$ has only one single zero at x=2 and the equation of this function has a leading coefficient of 1. The function is transformed to $x = af(0.5y - 4) - 4$, which is also expressed as $y = g(x)$. The function $y = g(x)$ is transformed again to $x = g(y) + k$, given that $k = f(-4)$ in the function $y = f(x)$. The functions $x = g(y) + k$ and $x = f(y)$ become expressions expressed as $(bx + c)/(dx + e)$ and $(px + q)/(rx + s)$, respectively, where b, d, p, r are all coefficients, and c, e, q, s, are all constants. Given that the remainder when $(bx + c)/(dx + e)$ is divided by $(px + q)/(rx + s)$ is 12, determine the value of a .

(a = -5/2)