# Math 10C: Algebra & Number

## Notes

### Factorization
Things you need to know related to factorization:
- **prime factors**
  Find the **prime numbers** that multiply together to make the target number, e.g. $2^2$ x 3 = 12
  Also, explain why 0 and 1 have no prime factors.
- **greatest common factor (GCF)**
  To find this using prime factorization, find all the prime factors that the two numbers have in common, then multiply them together.
- **least common multiple (LCM)**
  The LCM will include all the prime factors from both numbers, but make sure to remove duplicates.
- **square root**
  If the prime factors of a number can be grouped in identical pairs, it is a perfect square. This method can also be used to find the square root of a perfect square, by finding the prime factorization and then multiplying one number from each pair of factors.
  e.g. 196 = $2^2$ x $7^2$, so it is a perfect square. The square root of 196 is 2 x 7 = 14.
- **cube root**
  The same process as finding square roots or perfect squares, except this time the prime factors must be grouped into identical groups of three.
  e.g. 1000 = $2^3$ x $5^3$, so it is a perfect cube. The cube root of 1000 is 2 x 5 = 10.

### Polynomial multiplication
Multiply polynomials (monomials, binomials, and trinomials) using tiles, diagrams, and symbols. Know the **distributive property** (each term of one polynomial must be multiplied with each term of the other polynomial - FOIL).
To check your answer, you can **substitute numbers** for variables and see if your answer and the original are the same.

### Factoring Trinomials
Know the concept of common factors and be able to factor trinomials using tiles, diagrams, and symbols. Watch out for special polynomials when factoring! These include:
- **perfect square trinomials**: $a^2 + 2ab + b^2 = (a + b)^2$
- **difference of squares**: $a^2 - b^2 = (a - b)(a + b)$

Remember to look for **common factors** first. To check your answer, you can multiply the factors again to get the original expression.

### Irrational Numbers
Things you need to do related to irrational numbers:
- **sort** numbers into rational and irrational
- **approximate the value** of an irrational number and order them on a number line
- **simplify** radicals and **expand** mixed radicals, e.g. $\sqrt{8} = 2\sqrt{2}$
- know the subsets of the real numbers (natural, whole, integer, rational, irrational)

Roots and Powers

Things you need to do related to roots and powers:
- explain why the negative power of a number becomes the reciprocal of the positive power:
  - this can be done using patterns. $2^3 = 8$, $2^2 = 4$, $2^1 = 2$, $2^0 = 1$. therefore, $2^{-1}$ must equal $1/2$.
- explain why $a^{\frac{1}{n}} = \sqrt[n]{a}, n > 0$.
  - using the exponent laws, $2^{1/2} + 2^{1/2} = 2^1$, which means $2^{1/2}$ must be the square root of 2.
  - you can convert fractional exponents to radical expressions using this rule
- know the **exponent laws**:
  - multiplication (add the exponents)
  - division (subtract the exponents)
  - exponents of exponents $(a^m)^n = a^{mn}$
  - exponents on multiplication expressions: $(a \times b)^m = a^m b^m$
  - exponents on division expressions $(a \div b)^m = a^m \div b^m$

## Practice Questions

Factorization
1. Find the prime factorization of 4200.
2. Find the LCM for the following number sets:
   a)  54, 104
   b)  8, 9, 21
3. Find the GCF for the following number sets:
   a)  27, 1386
   b)  52, 78
4. Stephanie can run one lap around a track in 4 minutes, and Lisa can run the same lap in 6 minutes. If they start running at the same time, how long will it be until they complete a lap together?
5. You are making fruit baskets with apples and bananas. Each basket must have the same number of apples and the same number of bananas. If you have 30 apples and 42 bananas, how many fruit baskets can you make, and how many apples and bananas are in each basket?
6. Determine whether the following numbers are perfect squares, perfect cubes, neither, or both. If the number is a perfect square or cube or both, write out its square/cube root(s).
   
   a) 512                b) 168                c) 225                d) 729

Polynomial multiplication
7. Multiply the following polynomials using algebra tiles:
   a)  $x(x + 4)$
   b)  $(2x + 3)(x + 2)$
8. Multiply the following polynomials using symbols:
   a)  $(2p^3 + 5p^2 - 4) \times (3p + 1)$
   b)  $(x^3 + 2x^2) \times (7x + 4)$
   c)  $(t-1) \times (t^2 - 3t + 5)$
   d)  $(-t^3 + 3t) \times (-3t^2 + 2t)$
9. What is the expression for the area of a square if its side length is $(x + 7)$?
10. If two numbers have a sum of 9, then what polynomial represents their product?
11. If two numbers differ by 5, then what polynomial represents their product?
12. A rectangular swimming pool is twice as long as it is wide.  A small concrete walkway surrounds the pool.  The walkway is a constant 2 feet wide and has an area of 196 square feet.  Find the dimensions of the pool.

<u>Factoring Trinomials</u>

13. Factor the following expressions completely:
    a) $x^3 - 5x^2 - 24x$
    b) $3p^2 - 2p - 5$
    c) $25x^2 + 10x + 1$
    d) $9x^2 + 42x - 15$
    e) $x^2 + 6xy + 9y^2$
    f) $2x^2 + 16x + 32$
    g) $8x^2 - 98$
    h) $45x^2 - 20y^2$

14. Factor the following expressions using algebra tiles:
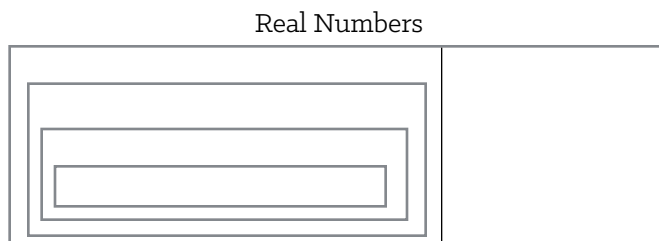    a) $x^2 + 6x + 5$
    b) $x^2 - 8x - 20$

<u>Irrational Numbers</u>

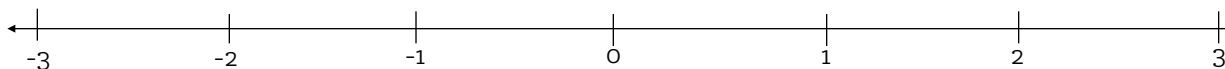15. Classify the following numbers as rational or irrational:
    a) $\frac{1}{3}$            e) $\sqrt[5]{-0.03125}$
    b) $-\sqrt{2}$        f) $8.4723\ldots$
    c) $\sqrt{-2}$        g) $\sqrt{49}$
    d) $\frac{3}{0}$          h) $(-2)^{\frac{1}{2}}$

16. Fill in the diagram of the subsets of the real numbers below:

Real Numbers



17. Order the following numbers on the number line below:
    a) $-0.\overline{75}$      b) $-\frac{1}{4}$      c) $1.3572\ldots$      d) $-2\sqrt{2}$      e) $\sqrt[3]{-6}$      f) $32^{\frac{1}{5}}$



18. Expand the following mixed radicals:
    a) $6\sqrt{5}$
    b) $2\sqrt[3]{3}$
    c) $5\sqrt{6}$

19. Simplify the following radicals:
    a) $\sqrt{75}$
    b) $\sqrt{30}$
    c) $\sqrt{150}$

Roots and Powers

20. $\sqrt{32} + \sqrt{72} - \sqrt{98} = ?$

21. If $2^A = 3$ and $2^B = 5$, then $2^{2A+B} = ?$

22. If $27^2 = 81^{b+2}$, what is b?

23. Simplify the following expressions, making sure that the variables in your answer are written with positive exponents:

a) $(-4)^{-2}$

d) $\dfrac{-3a^{-7}b^{-11}}{12a^4b^{-3}}$

b) $(3a^3)^{-2}$

e) $\left(\frac{2}{5}a^{-3}b^{-1}\right)^{-3}$

c) $\dfrac{5(-4)^0}{2^{-1}}$

f) $\dfrac{10a^7b^9c^6}{5a^6b^{10}c^8}$

24. Simplify the following exponents, and convert any fractional exponents to radicals:

a) $\left(27a^{\frac{1}{2}}\right)^{\frac{2}{3}}$

b) $\left(2^{-\frac{5}{4}}\right)\left(2^{-\frac{4}{3}}\right)$

c) $\left(\dfrac{9a^{-2}}{16b^{-4}}\right)^{\frac{3}{2}}$

25. A bacterial culture currently contains 5000 bacterium cells. This particular type of bacteria doubles every 8 hours. If the amount of bacteria is represented by the letter A, and the elapsed time (in hours) is represented by the letter $t$, the formula used to find the amount of bacteria as time passes is:

$$A = 5000\,(2)^{\frac{t}{8}}$$

a) How many bacteria will be in the culture in 8 hours?
b) How many bacteria were in the sample 8 hours ago?
c) How many hours will it take for sample to grow to 160 000 cells?

**1.** 2 x 2 x 2 x 3 x 5 x 5 x7

**2. a)** 2808. **b)** 504.

**3. a)** 9. **b)** 26.

**4.** 12 minutes.

**5.** 6 baskets. 5 apples and 7 bananas in each basket.

**6. a)** Perfect cube: 8^3. **b)** Neither. **c)** Perfect square: 15^2. **d)** Both: 9^3 or 27^2.

**7. a)** x^2 +4x. **b)** 2x^2 + 7x +6.

**8.**
**a)** 6p^4 + 17p^3 + 5p^2 -12p -4
**b)** 7x^4 + 18x^3 + 8x^2.
**c)** t^3  - 4t^2 + 8t -5
**d)** 3t^5 – 2t^4 – 9t^3 + 6t^2.

**9.** Area = x^2 + 14x + 49.

**10.** 81 – 9x – 9y + xy.

**11.** 5x – 5y – 25 + xy.

**12.** Length: 30feet. Width: 15feet.

**13.**
**a)** x(x+3)(x-8)
**b)** 3p^2 – 2p – 5.
**c)** (5x+1)^2
**d)** 3(x+5)(3x-1)
**e)** (x+3y)^2
**f)** 2(x+4)^2
**g)** 2(2x+7)(2x-7)
**h)** 5(3x+2y)(3x-2y)

**14.**
**a)** (x+1)(x+5) **b)** (x-10)(x+2)

**15.**
**a)** Rational.
**b)** Irrational.
**c)** Neither (It's a complex number).
**d)** Neither (It's undefined).
**e)** Rational.
**f)** Irrational.
**g)** Rational.
**h)** Nether (It's a complex number).

**16.**

## Real Numbers

Rational: -3/2, 5.5, 11.23

Integers: … -3, -2, -1, 0, 1, 2, 3…

Whole: 0, 1, 2, 3…

Natural: 1, 2, 3…

Irrational: 3.1415982…
(2)^1/2
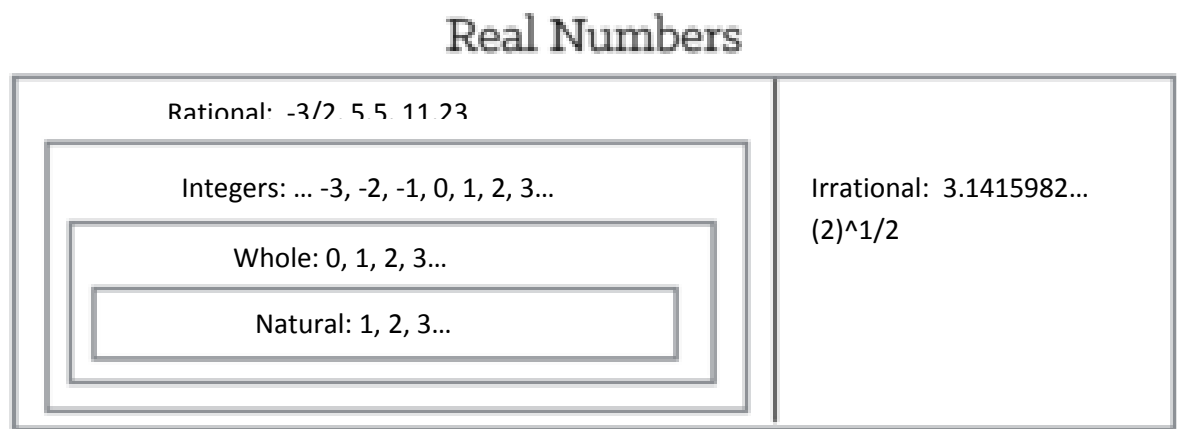
**17.** From left to right: $-2\sqrt{2}$, $\sqrt[3]{-6}$, $-0.\overline{75}$, $-1/4$, 1.3572…, 32^(1/5)

**18.**
**a)** $\sqrt{180}$
**b)** $\sqrt{24}$
**c)** $\sqrt{150}$

**19.**
**a)** $5\sqrt{3}$
**b)** $\sqrt{30}$
**c)** $5\sqrt{6}$

**20.** $3\sqrt{2}$

**21.** 45.

**22.** b = -1/2.

**23.**

**a)** 1/16.

**b)** $\dfrac{1}{9a^6}$

**c)** 10

**d)** $\dfrac{-1}{4a^{11}b^8}$

**e)** $\dfrac{125a^9b^3}{8}$

**f)** $\dfrac{2a}{bc^2}$

**24.**

**a)** $9\sqrt[3]{a}$

**b)** 2^(-31/12)

**c)** $\dfrac{27b^6}{64a^3}$

**25.**

**a)** 10 000. **b)** 2500. **c)** 40 hours.