# Math 10C Pre-IB Coordinate Geometry Review Package

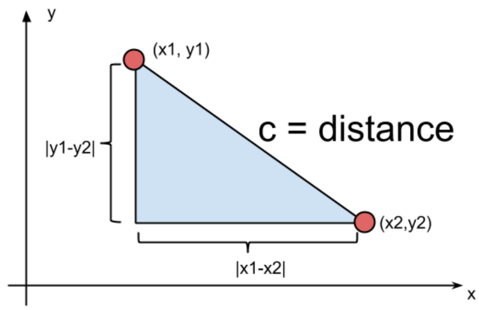
## Line Segments

### Distance

The distance of a line is expressed $d = \sqrt{(y_2 - y_1)^2 + (x_2 - x_1)^2}$

(using the Pythagorean Theorem to find the distance of line segments)

Diagram:

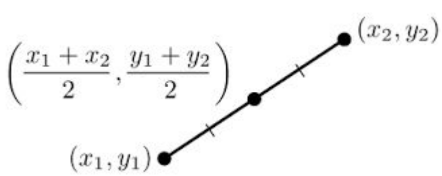



### Midpoint

The midpoint of a line segment is the middle of a line segment. To find the coordinates of the midpoint, $(x,y) \rightarrow (\frac{X1+X2}{2}, \frac{Y1+Y2}{2})$

**Please note that order matters here**

Diagram:




### Slope

The slope of a line is defined to be $m = \frac{\Delta y}{\Delta x}$ (or rise over run) where $m$ is the slope.

### Collinear Points

Collinear points are points that all lay on the same line segment (they can be connected by a straight line all together). Any two points are collinear, so teachers will only test on three or more points.


Collinear points P, Q, and R.

Example:

In this example, points P, Q and R are collinear.

Note that the slopes between P, Q and R are all the same since they are all on the same line segment, and that's how to determine whether the three points are collinear. In short, P, Q and R are collinear if $m_{PQ} = m_{PR}$.

## Lines

### Graphing by Table of Values

First, the interval at which a table of values for a certain linear function is created must be determined. For example, graphing y=2x+3. It would be possible to choose an interval of 2 units for x, and the table would look like the following:

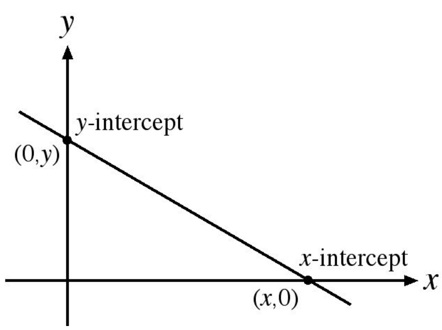
| x | y = 2x + 3 |
|---|---|
| -4 | 2(-4) + 3 = -8 + 3 = -5 |
| -2 | 2(-2) + 3 = -4 + 3 = -1 |
| 0 | 2(0) + 3 = 0 + 3 = 3 |
| 2 | 2(2) + 3 = 4 + 3 = 7 |

Plot each pair of x and y values onto your graph. Here, since it's linear, you should be able to connect all 4 points in a line. In more advanced units, you may use this same method to graph parabolas and other more complicated functions.

### Intercepts

In the function y=2x+3, the y-intercept would be 3,
the x-intercept would be -3/2.

The y-intercept is where the line crosses the y-axis and the x-intercept is where the line crosses the x-axis.




Generally speaking, *to find the y-intercept of an equation, plug in x=0 and solve for y*, and *to find the x-intercept of an equation, plug in y=0 and solve for x*.

i.e. y=0 → y=2x+3, we would have 0=2x+3; x=-3/2.
Therefore, -3/2 is the x-intercept of y=2x+3

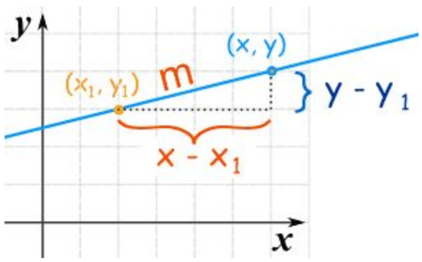
To use a graphing calculator to find the y-intercept, use [2nd CALC value]. Enter x=0
To use a graphing calculator to find the x-intercept, use [2nd CALC zero] and set the boundaries on both sides of where the line crosses the x-axis.

**Slope-point form**
If you draw a straight line with a certain slope from a specific point on a on a cartesian plane, then only one line could be drawn. In this manner, with a point and a slope, we could describe a line.

The line with slope $m$ that passes through point $(x_1, y_1)$ has the equation

$$y - y_1 = m(x - x_1)$$

This is because, by definition, the slope of this line m is always equal to $(y - y_1)/(x - x_1)$.




**Domain and Range**
Domain is all the possible x values for a function (the independent variable); and range is all the possible y values for a function (the dependent variable).

## <u>Linear Relations</u>

**Intercepts**
The point(s) on a graph where two or more specified lines cross each other (they all exist at that point). The x-axis and y-axis can be considered as lines in questions asking for intercepts.

**Equation of a Line**
There are three ways to express the equation of a line:
*general form, slope-intercept form*, and *slope point form*.

<u>*General form*</u>*: Ax+By+C=0* where A, B and C are all integer coefficients and x and y are variables

<u>*Slope-intercept form*</u>*: y=mx+b, where m is the slope and b is the y-intercept of the line*

*Slope-point form*: $y - y_1 = m(x - x_1)$ as mentioned above

Note that these forms are all interchangeable through algebraic manipulations. So if you get the equation of a line in a form that you are not comfortable with, you could change it into one with which you are more comfortable.

**Parallel and Perpendicular Lines**

Let's say we have two lines which have respectively equations

$$y = m_1 x + b \text{ and } y = m_2 x + b$$

If they are parallel, then $m_1 = m_2$. If they are perpendicular, then $m_1 m_2 = -1$.

To put this into words, **if two lines are parallel, they have the same slope** and **if two lines are perpendicular, they have slopes that are negative reciprocals**.

## System of Equations

**Solving 2x2 Systems (algebraically and graphically)**

To solve a 2 by 2 linear systems algebraically, usually either elimination or substitution is used.

For example: solve the linear system of equations for x and y

(1) *3x + y= -2*

(2) *2y - x= -6*

To use elimination: we could multiply equation (1) by a factor of two and then subtract the second equation, which would first eliminate the y

*[2*(3x + y= -2)]*

\- *(2y - x= -6)*

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7x = 2

After elimination of one of the two variables, there is 1 by 1 linear equation that is easily solved algebraically for x where x=2/7. Then this can be repeated for the other variable

To use substitution: isolate y in equation (1), which will give y=-3x-2. The key to substitution is to **plug this expression of the isolated variable (which in this case is y) into the other equation**, which is equation (2).

This gives: **2(-3x-2) - x = -6**;

After expanding and solving algebraically for x, one of the solutions is x=2/7

After finding the solution to one variable, plug this into one of the equations and solve for the other variable. Here, we can directly plug x = 2/7 into y=-3x-2 which has y already isolated and ready to go as **y = -3(2/7)-2** and gives y=-20/7.

If you are solving systems of equations by hand, plug your solutions into the original equations, and see if the equations hold true. If they do not, then you know that you have made a mistake. **Re-plugging solutions back into original equation(s) is always a good strategy for checking a solution.**
For 2 by 2 linear equations, to save time, you could check your solution by only plugging in your solutions to the equation which was not used to arrive at the solution for the second variable. It is extremely likely that if an error was made in solving the first equation then the solution will not work for the second equation.

To graph out a solution:

1. Isolate y for both linear equations in the system; enter the two equations into a graphing calculator under Y1 and Y2
2. Use *intersect* (2nd CALC 5) to find the intersection of the two lines; be sure you select the correct lines on your graph
3. The x and y values at the intersection, which is the solution, is then given.

There are three categories of solutions that are possible from a 2 by 2 system of linear equations:

If and only if the **slope of the two linear equations are different**, the system will have only **one solution**.

If the **slope of the two linear equations are the same**, the system can **either have infinite or no solutions** (they are the same line for infinite solutions or they run parallel to each other and never cross for no solutions).

It is also important that you practice how to set up system of linear equations. For the questions below, what's important is not that you get the correct answer (although that is the purpose), but being confident that you can set up such a system of linear equations correctly.

The rest of the work could be relayed to the calculator if the teacher does not specify to solve algebraically.

**<u>Practice Questions: (With Answers)</u>**

1. Find the midpoint of the segment joining the points (4, -2) and (-8,6). [ (-2, 2) ]

2. Find the distance between the points (3, -2) and (6,4). [ $3\sqrt{5}$ ]

3. What is the slope of the line passing through the points (4,6) and (-1,-2)? [8/5]

4. The point (-4,-2) lies on a circle. What is the length of the radius of this circle if the center is located at (-8,-10)? [ $\sqrt{80}$ ]

5. Find the slope of a line perpendicular to the line whose equation is $2y + 6x = 24$. [⅓]

6. Find the midpoint of the segment connecting the points $(a, b)$ and $(5a, -7b)$.
[ $(3a, -3b)$ ]

7. Find the equation of the line parallel to the line whose equation is $y = 6x + 7$ and whose $y$-intercept is 8. [$y = 6x + 8$]

8. *In a coordinate system, P = (2, 7) and Q = (2, −3). What is the y-coordinate of R if PQR is an isosceles triangle? [2]

9. Line $l$ is perpendicular to $y = 3x - 4$ and it also passes through point (3, 7). What is the y-intercept of line $l$? [8]

10. Line $l$ is parallel to $y = -4x + 2$ and it also has a y-intercept of 9. What is the equation of $l$? [y=-4x+9]

11. The equation of line $l_1 : y = 5x + 5$. Line $l_1$ is parallel to line $l_2$ and is perpendicular to line $l_3$. If line $l_1$ and line $l_3$ intersects at x=4, and $l_2$ has a y-intercept of -4, then what is the intersection between $l_2$ and $l_3$? [Hint: First find the equation of $l_2$ and $l_3$.
[149/26, 641/26]

12. A certain team game has two types of scoring: major scoring and minor scoring. In a particular tournament, team A scored 15 major scores and 14 minor scores for a total of 103 points. In the same tournament, team B scored 11 major scores and 12 minor scores for a total of 79 points. How many points are awarded for a major score, how many for a minor score? [5 points for major, 2 points for minor]

13. When he applied for a job at the Super Snowboard Shop, Terry had his choice of two pay options:
1. 300 dollars/week plus 5% of sales
2. 200 dollars/week plus 13% of sales
Determine the amount of sales required to make both options equally advantageous for Terry. [1250 dollars]

14. Tally bought 50 juice boxes, consisting of apple juice boxes and orange juice boxes. She spends 50.4 dollars. If an apple juice box costs 0.9 dollars, and an orange juice box costs 1.1 dollars, how many of each juice box did she buy? [23 apple; 27 orange]