Variance, Effect Size, Pearson Correlation

Stu Field

1 The t-test Variance Calculations

The general formula for the t-statistic is:

$$t = \frac{\hat{\mu_1} - \hat{\mu_2}}{SE_{\mu_1 - \mu_2}} \tag{1}$$

1.1 Equal-n; Equal Variances

$$t = \frac{\hat{\mu_1} - \hat{\mu_2}}{\sqrt{\frac{1}{2}(\hat{\sigma}_1^2 + \hat{\sigma}_2^2)} \times \sqrt{\frac{2}{n}}} \tag{2}$$

1.2 Unequal-n; Equal Variances

$$t = \frac{\hat{\mu_1} - \hat{\mu_2}}{\sqrt{\frac{\hat{\sigma}_1^2(n_1-1) + \hat{\sigma}_2^2(n_2-1)}{n_1+n_2-2}} \times \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} \tag{3}$$

1.3 Unequal-n; Unequal Variances

$$t = \frac{\hat{\mu_1} - \hat{\mu_2}}{\sqrt{\frac{\hat{\sigma}_1^2}{n_1} + \frac{\hat{\sigma}_2^2}{n_2}}} \tag{4}$$

2 Effect Size

$$\delta = \frac{\hat{\mu_1} - \hat{\mu_2}}{\sqrt{\hat{\sigma}^2_{pooled}}} \tag{5}$$

where $\hat{\sigma}^2$ is the pooled variance of the two means, or for k groups:

$$\hat{\sigma}^2_{pooled} = \frac{\sum_{i=1}^{k}(n_i - 1)\hat{\sigma}_i^2}{\sum_{i=1}^{k}(n_i - 1)}, \qquad i = 1, \ldots, k. \tag{6}$$

3 Pearson Correlation

The correlation between 2 random variables is described by the parametric Pearson's product-moment correlation coefficient (ρ):

$$\rho_{xy} = \frac{cov(x, y)}{\sigma_x \cdot \sigma_y} \tag{7}$$

where,

$$cov(x, y) = \frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{(n-1)} \tag{8}$$

$$\sigma_x^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{(n-1)} \tag{9}$$

$$cov(x, x) = var(x) = \sigma_x^2 \tag{10}$$

3.1 Significance of ρ_{xy}

Statistical significance of the estimate of ρ can be evaluated via the calculation of a t-statistic via the following:

$$t = \rho\sqrt{\frac{n-2}{1-\rho^2}} \tag{11}$$

The same t-statistic can also be used for the non-parametric Spearman correlation coefficient.