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MATHEMATICAL MODELING OF WHOLE LANDSCAPE EVOLUTION

Garry Willgoose
Earth and Biosphere Institute, School of Geography, University of Leeds, Leeds, United Kingdom, LS2 9JT; email: g.r.willgoose@leeds.ac.uk

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■ **Abstract** The mathematical modeling of landform evolution consists of two components: the processes represented (i.e., considered dominant) in the model and the (typically computer) model representation of these processes. This review discusses the current debates surrounding processes represented in landform evolution. The potential impact on both evolving landforms and computer model structure is discussed. Issues specifically discussed include (*a*) the fundamental nature of mass conservation and the role of detachment- and transport-limited processes in mass conservation equations, (*b*) the interaction between detachment- and transport-limitation in channels, (*c*) the role of hillslope erosion and soil properties and their interaction with channel processes, (*d*) the interactions with tectonics when applying these models at large scale, (*e*) depositional structures and implications for paleo-climatic interpretation, (*f*) engineering applications of these models, and (*g*) numerical issues in the computer implementations. This review is not a model comparison. However, many applications are at the boundaries of computer capabilities so a comparison of existing models is provided.

INTRODUCTION

Conceptual and analytic models linking landform characteristics with process have a long heritage (Gilbert 1909, Kirkby 1971), but computational models of the dynamics of hillslope and basin form date from Ahnert (1976). The availability of digital elevation models (DEMs) in the mid-1980s triggered an interest in understanding what DEMs quantitatively said about hydrology and erosion on landforms. These modern landform evolution models (LEMs) are spatially distributed (not just hillslopes) and involve coupling the hillslope, channel, and tectonic processes. It is the basis of these modern LEMs that is reviewed in this paper.

Geomorphology has historically been replete with qualitative and descriptive models of landform development, many of very limited predictive usefulness and almost all impossible to test in any scientific fashion. LEMS are helping us to highlight process understanding of observed geomorphology and to test postulated long-term process links, and they have forced us to improve the precision of our ideas about landform development. As a result, LEMs are finding a range of novel uses in practical applications in Earth sciences, quite apart from the obvious application of forward modeling of landform evolution using known or simulated climate, hydrology, erosion, and tectonics. These uses are as experimental tools to understand (*a*) the spatial organization of runoff generation and channel networks (Rodriguez-Iturbe et al. 1992, Willgoose & Perera 2001), (*b*) the remote sensing of erosion process parameters from digital elevation models (Willgoose 1994), (*c*) the linking climate and tectonic processes with detailed stratigraphy (Kolterman & Gorelick 1992), (*d*) spatial distribution of soils and vegetation (Saco et al. 2003), and (*e*) the impact of continental-scale erosion and deposition on basin development and large-scale crustal dynamics feedbacks (Braun & Sambridge 1997).

There are now models that address sufficiently complex spatial and temporal feedbacks that allow us to observe novel and unexpected behavior, providing new ideas about landform evolution process feedbacks. These novel insights can be tested against the geologic record. A major developing area is an interest in methodologies that use LEMs to (*a*) quantitatively test competing conceptual models of landform evolution and (*b*) assess the value a priori of field data (fission track, etc.) in differentiating competing conceptualizations. This review discusses these issues.

Even in the relatively straightforward area of forward modeling there are some novel engineering applications in the assessment and design of containment structures for hazardous and nuclear waste, in which the containment structure (essentially a man-made landform) needs to survive for periods up to 1000 years, a period long enough for significant landform evolution to have occurred and for feedbacks between erosion and landform to have occurred (Willgoose & Riley 1998). These engineering applications also highlight some deficiencies in our understanding and ability to quantify soil development/evolution, rock weathering, and vegetation feedbacks in runoff and soil development.

The mathematical modeling of landform evolution consists of two components: the processes represented (i.e., considered dominant) in the model and the (typically computer) model representation of these processes. This review is not intended to be a model comparison. However, many applications are at the boundaries of computer capabilities so some discussion of the models, and how their implementation interacts with the scope of problems that are addressed, is necessary.

The main models currently shared in the geomorphology community are SIBERIA (Willgoose et al. 1991a, Willgoose 2004), ARMOUR (Willgoose & Sharmeen 2005), DELIM (Howard 1994), GOLEM (Tucker & Slingerland 1994), CHILD (Tucker et al. 2001), CASCADE (Braun & Sambridge 1997), ZSCAPE (Densmore et al. 1998), and CAESAR (Coulthard et al. 2002). Some of these were recently compared by Coulthard (2001). Table 1 provides a brief feature comparison of the models based on online documentation, inspection of code, and, where possible, personal communication with the developer. The author cautions against reading too much into the capabilities or limitations of these models from papers or Table 1, as all of these models are works in progress (up-to-date manuals are

TABLE 1 Feature comparison of landform evolution models (LEMs)[a]

Model	Transport processes[b]	Grading[c]	Soils[d]	Tectonics[e]	DEM[f]	Code[g]	Other[h]
SIBERIA	T_GDC	1	GP	S	G,MF	F95	OB,PCV
ARMOUR	T_GD	∞	W		G,SF	F77	B,PC
DELIM	TDC	—	—	S	G,SF	F77	—
GOLEM	TC	—	EP	S	G,SF	F77	—
CHILD	T_GDCL[i]	∞	GP	L	T,SF	C++	V
CASCADE	TD	—	—	C	T,SF	C&F77	B,PG
ZSCAPE	TDCL	—	GP	L	G,SF	F77	—
CASEAR	TDC	—	E	—	G,MF	EXE	C

[a]Not all of these capabilities are available simultaneously when running the model.

[b]Transport and erosion mechanisms: T, transport-limited; T_G, transport-limited based on grading; D, detachment-limited; C, soil creep (with or without angle of repose threshold); L, explicit landslides.

[c]No. of grading fractions used for tracking transported/deposited sediment; 1, characteristic diameter only (e.g., d_{50}); ∞, arbitrary number limited by computer memory.

[d]Soils/Regolith model: S, simple input depth; E, S + erosion/deposition balance; G, E + tracking sediment grading; W, G + grading change from weathering/armouring; P, rock-soil production function.

[e]Tectonics model: S, simple vertical time and space varying input; L, S + strike/slip/lateral deformation; C, L + crustal dynamics feedback.

[f]Elevation representation: G, gridded; T, TIN; Drainage Analysis Method; SF, single flow direction (typically D8); MF, multiple flow directions (typically Dinifinity).

[g]Language for source code: EXE, executable only available.

[h]Other capabilities: O, multiprocessor computer (using open MP); B, Beowulf cluster (using either MPI or PVM); P, built in Monte Carlo prediction probability limits and GLUE analysis; C, can use historical climate data; V, vegetation model interacting with landform and climate; G, glacier/ice sheet.

[i]Landsliding, debris flow with runout, and woody debris only in the Oregon State version (S.T. Lancaster, personal communication).

typically available on the Web or from the authors) and are routinely extended by their developers as applications arise.

The fluvial erosion modules are the heart of these models and are all similar in principle but vary in the constitutive equation used to represent the processes. All use an explicit transport capacity model with mass balance for overland and river flow. If the potential sediment transport entering a node is greater than that exiting then deposition occurs. The rate of deposition may be limited by the maximum rate of deposition of sediment (a function of the sediment settling velocity and near bed concentration). If the potential transport rate exiting the node exceeds that entering then erosion occurs. The erosion rate may be limited by the ability to entrain material from the surface (detachment-limited, D in Table 1) or not (transport-limited, T in Table 1). The detachment rate may be limited by the availability of, or ability to move, sediment of the appropriate size grading in the surface (e.g., armoring) or bedrock exposure (e.g., bedrock incision). Both of these detachment-limited processes are subjects of continuing research and are discussed in sections below. The mass balance equation is

$$\frac{\partial z}{\partial t} = U + \underline{\nabla} \cdot Q_s \text{ where} \begin{cases} \nabla \cdot Q_s > 0 & \text{then} \quad \nabla \cdot Q_s = \textit{deposition rate} \\ \nabla \cdot Q_s < 0 & \text{then} \quad \nabla \cdot Q_s = \textit{erosion rate} \end{cases}, \quad (1)$$

where z is the elevation, U is the rate of tectonic uplift, and Q_s is the sediment transport flux/unit width. For detachment-limitation, this equation collapses to a particularly simple form:

$$\frac{\partial z}{\partial t} = U - D, \quad (2)$$

where D is the detachment rate or incision rate for bed rock rivers. Finally, the potential transport rate may vary with the grading of the material eroded and deposited, so that the transport capacity is a function not only of the hydraulic conditions but also the sediment transported from upstream and entrained from the bed (T_G in Table 1) so that Equation 1 must be written for each size fraction i

$$\frac{\partial z}{\partial t} = U + \sum_i \underline{\nabla} \cdot Q_{si} \text{ where} \begin{cases} \nabla \cdot Q_{si} > 0 & \text{then} \quad \nabla \cdot Q_{si} = \textit{deposition rate} \\ \nabla \cdot Q_{si} < 0 & \text{then} \quad \nabla \cdot Q_{si} = \textit{erosion rate} \end{cases}. \quad (3)$$

CAESAR solves for sediment transport with a probabilistic Lagrangian solver (using probabilities of detachment and travel distance) that eliminates the need to explicitly distinguish between transport- and detachment-limitation but is mathematically equivalent to the Eulerian formulation of Equation 3. For the case of a long hillslope, where the probability of entrainment, mass-entrained, and travel distance are statistically independent (solutions are available for more general problems but are more complex):

$$Q_s = \lambda ML \quad \text{for} \quad L < \text{hillslope length}, \quad (4)$$

where λ is the rate of occurrence of entrainment (detachment) events per unit area per unit time, M is average mass of sediment entrained per entrainment event, and L is the average travel distance of the entrained sediment. Detachment-limitation occurs when either L approaches infinity, indicating that sediment doesn't redeposit, or when λ is small, indicating a constraint on entraining sediment.

Explicit detachment-limitation for other transport processes (e.g., creep) is not normally considered. Modeling of landsliding and debris flows is done by explicitly modeling the probability of the failure event, the mass of the event and the runout distance, and calculating the travel path for the event using the topography. In principle, the geomorphic expression of these processes can be explicitly calculated using the Eulerian model of Equation 4 in the divergence term of Equation 1, but the analytic complexity of the constitutive relationship (e.g., Iverson 1997) currently precludes this.

Soil or regolith depth modeling is performed with another mass balance equation (E in Table 1):

$$\frac{\partial R}{\partial t} = W - \underline{\nabla} \cdot Q_s, \tag{5}$$

where R is the regolith depth and W is the regolith-soil production rate or rate of conversion from bedrock to regolith (typically using Heimsath et al. 1997; P in Table 1). Generally, surface elevation evolution is considered independent of soil depth evolution, but there is a possible feedback if the soil production process alters the bulk density. For instance, if 1 mm of rock generates 3 mm of soil then the landform surface will rise 2 mm. The evidence for this type of feedback is weak and it is generally ignored. It is also possible to model the evolution of the grading of the soil (G and W in Table 1), but our inability to predict the grading of material generated by the soil production process in Equation 5 generally means that a soil production function cannot be used simultaneously.

One of the key outcomes of the mathematical representations above is that it has allowed a deeper understanding of the links between process and landform. For instance, the equations can be nondimensionalized (Daily & Harleman 1966) to explore scaling relationships (e.g., Willgoose et al. 1991b), limits on mountain belt relief (Whipple & Tucker 1999), and adjustment times for landscapes to abrupt change (Whipple 2001). The most notable outcome of this work has been a scaling relationship linking the longitudinal concavity of rivers and hillslopes with transport-limited processes (Willgoose et al. 1991c). In the subsequent 14 years, a considerable body of literature has been published on the area-slope relationship (Figure 1), extending to more general tectonic regimes (Willgoose 1994) and detachment-limited processes (Moglen & Bras 1995, Sinclair & Ball 1996). Willgoose (2001) showed that the area-slope relationship and the planar

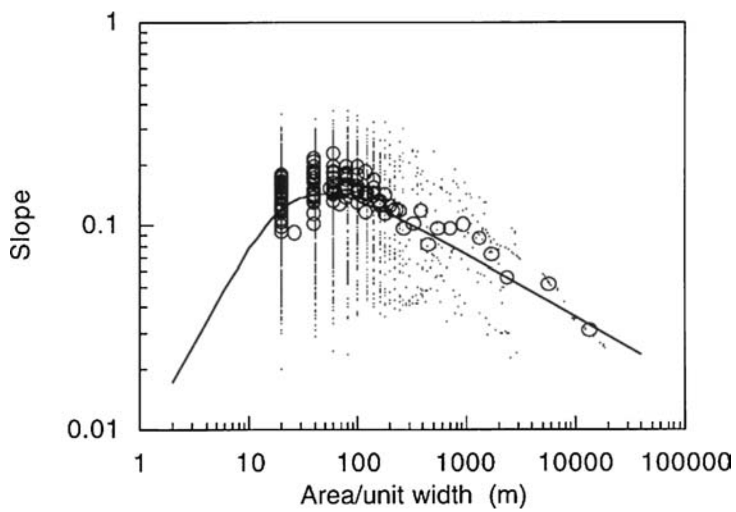

Figure 1 Typical area-slope data (*dots*) and analytic fit to the data based on transport-limited processes (*line*) (from Willgoose 1994).

drainage network are sufficient to completely characterize catchment elevation properties. Tucker & Whipple (2002) summarize the state of the art by comparing the implications of the area-slope relationship for inferring process.

Much of the ongoing debate about LEMs is about the processes represented rather than the mathematics of the modeling, so the following discussion concentrates on process understanding. However, given that most models operate near the limits of available computer technology, the paper concludes with some observations on unresolved computational problems.

CHANNEL PROCESSES

The main debate on river processes is whether and/or where river incision rates are detachment- or transport-limited and how this is modeled in LEMs. The modeling of meandering on floodplains is also an active interest, but only CHILD attempts to model this challenging problem (Lancaster & Bras 2002).

Detachment-limited erosion is best explained using the example of a bed-rock channel. The lowering rate of the channel bed is determined by the ability of the flow to detach particles from the exposed bedrock surface, which is generally assumed to be a function of the bed shear stress (Howard 1994). When a particle is entrained into the flow it is assumed that the flow has enough power to be able to transport it without depositing it further downstream.

On the other hand, transport-limited erosion assumes that the lowering of the bed (normally sand or gravel) is constrained by the ability to transport that particle once it is entrained, not by the ability to detach the particle from the surface. Three cases occur: (*a*) the actual transport is less than the potential transport so a particle can be entrained, (*b*) the actual and potential transport are equal so no entrainment can occur, and (*c*) actual transport exceeds potential so excess sediment is deposited.

This simplistic distinction will suffice for the moment. Some subtleties are discussed in the hillslope section, below.

The key consequence is that a detachment-limited channel doesn't deposit sediment. This should not be confused with disallowing sediment to cover the bed as it passes through the channel as bed load, only that it cannot accumulate in the channel. In fact, recent work (Sklar & Dietrich 2004) has enhanced the traditional shear stress detachment-limited model to allow for an enhanced detachment as a result of an interaction of sediment bouncing along the bed (the so called tools effect) and the protection provided by the bed load (the cover effect). They postulate that the detachment rate is greatest for some intermediate sediment load when there are sufficient tools to abrade the surface but limited sediment cover protecting the bed. The main difficulty is that the tools effect is dependent on the grading of the sediment load, which is largely determined by the transport off the hillslope (and presumably its interaction with channel incision at the base of the hillslope), which requires coupling with a hillslope model that tracks the grading of eroded sediment.

In a bedrock channel, if the potential detachment rate is higher than the capacity to detach particles from the bed then the actual detachment is determined by this detachment limitation; otherwise, it is limited by the transport capacity and is determined by the divergence of the sediment flux capacity. Generally, upstream reaches are dominated by bedrock channels (and are thus detachment-limited) and downstream reaches by alluvial channels (transport-limited) (Howard 1994). The point at which the channel switches from the detachment- to transport-limited is determined by the relative rates of detachment capacity and transport capacity, with high detachment rates relative to transport meaning that the channel will be transport-limited and high relative transport rates meaning the channel will be detachment-limited. Recent work by the author (G. Willgoose & M.J. Kirkby, manuscript in preparation) shows that the catchment relief is a function of this interaction, rather than being determined solely by the detachment regime, as generally implicitly assumed (Howard 1997). Thus the correct modeling of both regimes and the transition point is crucial to the correct modeling of the distribution of catchment elevations (as shown, using the area-slope relation, for transport limitation by Willgoose 2001).

Characterizing this transition point will require modeling of the interaction of the sediment grading dependence of the tools effect and transport capacity of rivers, and the grading of source material from the hillslope (which only some LEMs can do) unless empirical means can be determined to characterize this point from the field or DEMs. The area-slope relationship is of little use in determining regions of detachment- and transport-limited erosion. It is an unfortunate coincidence of the process parameters that the accepted models of detachment and transport erosion both yield area-slope relationships that are nearly identical and are both close to that observed in nature (concavity approximately 0.35–0.5).

HILLSLOPE FLUVIAL PROCESSES

Fluvial erosion of hillslopes is normally considered a scaled-down version of a channel, either as sheetflow or a series of parallel rills. This ignores the highly episodic nature of overland flow; the effect on erosion rate of permanent vegetation; and the potential for coupling between vegetation cover, soil depth and grading, runoff generation, and erosion (by, for instance, water limitation on vegetation growth).

A vegetation-climate-erosion link has long been recognized, with erosion greatest for an annual precipitation of approximately 200–300 mm, when runoff is significant but rainfall is too low to support complete vegetation cover (Langbein & Schumm 1958). Moglen et al. (1998) showed, using a time-varying cyclic climate, that for either very dry ($\ll$200 mm) or very wet climates ($\gg$200 mm) the time-varying drainage density had a frequency equal to, and synchronous with, the climate. For intermediate climates approximately 200 mm/year, a bimodal distribution of drainage density was generated with frequency twice that of the applied climate.

There is growing recognition that to model the evolution of landforms we must model soil processes. For instance, as noted above, the tools model of Sklar requires a grading source term for the hillslope erosion. Moreover, for models of the evolution of anthropogenic landforms with unnatural soils (Willgoose & Riley 1998) the hillslope erosion processes are strongly modified by rapidly evolving soil armors. Recent work using the ARMOUR model (Willgoose & Sharmeen 2005, Sharmeen & Willgoose 2005a) has shown that the soil erodibility can change by several orders of magnitude over a few years solely from armoring. By coupling their armoring model with one for weathering of the armor particles, a surface where the transport- and detachment-limitation processes balance is created (Sharmeen & Willgoose 2005b). There is a balance between coarsening of the surface by armoring and fining of the surface by weathering. The transport-detachment balance arises because the transport capacity is limited by the ability to carry the large-diameter particles entrained from the armor. The detachment limitation arises because there is scarcity of fine material for entrainment into the flow with the generation of entrainable fines being controlled by armor weathering. This suggests that for undisturbed hillslopes, the erosion equation is a combination of transport and weathering processes and that the grading of the soil is crucial. These results are consistent with the ubiquitous finding of stone armors on undisturbed hillslopes but suggest that it is the fines generation and entrainment processes that provide control rather than a Shields shear stress threshold.

A number of other applications also require a soil grading representation:

- Soil grading as the source material for downstream fining in alluvial (i.e., transport-limited) rivers (Gasporini et al. 1999).
- Erosion and subsequent armoring of soils recovering after agricultural abandonment.
- Long-term modeling of the interaction between vegetation and erosion after drought in semiarid zones. Modeling plant response to water stress (Fernandez-Illescas et al. 2001) and water holding capacity (a function of the finest fraction of the soil and soil depth) determines drought recovery.
- Slope stability and postlandslide/debris flow modeling requires an understanding of regolith build up (e.g., Densmore et al. 1998).
- Paleo-climate interpretation of the grading and facies structure of depositional structures. This is discussed further below.

TECTONIC INTERACTIONS

The original LEMs were largely developed for small catchments so the tectonic interactions were simple, with no feedback from the erosion to the tectonics. Moreover, the area-slope analytic solutions outlined above generally make simple temporal assumptions (constant, cyclic, episodic, or no uplift); however, few researchers have recognized that they make no assumptions about the spatial variation of uplift.

Recent interest in the effect of depositional loading and erosional unloading on crustal dynamics (e.g., mountain building in New Zealand, Taiwan, and the

Himalayas, passive margin evolution, thrust-fold belts, subsidence) by Braun & Sambridge (1997) and Kooi & Beaumont (1994) has led to the coupling of simple landform models with geophysics models of crustal dynamics (C in Table 1). These crustal models have involved thin-plate models for the crust and strain- and temperature-dependent softening of plate collision zones. The role of the surface LEM is to transfer eroded material from one location to another. The crustal plate lateral stiffness means that the enhanced uplift and subsidence caused by the buoyancy of the crust floating on the mantle (isostasy) is transmitted laterally (with a characteristic distance) from the point at which the erosion or deposition occurs. This characteristic distance increases with the thickness and rigidity of the crust. That the interaction is solely driven by the mass of sediment means that only mass rates of sediment transfer need be modeled (which are generally assumed not to interact with sediment characteristics such as grading). Accordingly, grading and other stratigraphic properties in depositional zones are generally not modeled in these crustal plate scale models.

One aspect of these crustal models influences the design of the LEMs (Sambridge & Braun 1997). Modeling lateral movement (e.g., thrust faults) and compression and tension deformation is much easier when the grid can be laterally deformed across the fault boundary to naturally follow the movement of the topography. These grid deformations are more easily done with a TIN (triangulated irregular network) than gridded elevations. Gridded elevations require interpolation of the lateral movements of the elevations of the nodes at every step. The TIN nodes are simply moved laterally and the nodes are periodically remeshed. The extra overhead of the TIN representation and remeshing is small relative to the complexity of the crustal dynamics modeling.

PALEOCLIMATOLOGY AND DEPOSITIONAL AND METASTABLE LANDFORMS

Sedimentary deposits are a window on past climates and tectonics. To date, LEMs have primarily focused on erosional landforms. Depositional structures have, until recently, received little focused attention. We are still in the early stages of being able to quantitatively link observed stratigraphy to past depositional environments, and by implication to past climate and tectonics.

First, the LEMs are not yet sufficiently physically defensible to be able to do this with any certainty. For instance, temporary sediment storage within a basin (e.g., terraces, floodplains) means that temporal synchronicity between erosion of sediment and its deposition downstream does not occur, and our ability to model storage structures within basins is poorly developed. For instance, in the erosion community it is well known that fine particles tend not to redeposit once eroded. On the other hand, coarse particles from the hillslopes typically deposit on the floodplain, with those in the channel flow typically being sourced from bank erosion and meandering. Recent data from luminescence studies of floodplain sediments (e.g., Wallinga et al. 2001) indicate ages in the range of 10,000 to

200,000 years, although the older dates may be for inactive deposits rather than that part of the floodplain being actively reworked by river meandering and overbank deposition. These ages suggest the possibility of very significant lags for those grading fractions found in floodplain sediments (i.e., silt and coarser).

Second, scientific hypothesis testing is difficult because the mathematics means we cannot simply run the models backward from today. Equifinality (e.g., the same area-slope relation can be generated by a broad range of initial conditions and processes) means that different histories lead to the same observed data, even without unconformities, so that the inversion from observation to paleo-data will be, as a consequence, nonunique.

Koltermann & Gorelick (1992) showed that with the empirical sediment source terms, the models can be linked to past climate. Coulthard et al. (2002) was able to link changes in the deposition rate on an alluvial fan to gross changes in the management of the catchment, but quantitative links to known climate dynamics were less successful. To date, these are the only works that have compared a model with field data quantitatively, although the sedimentology community is increasingly interested in modeling these processes, particularly off-shore and in the laboratory (e.g., Paola 2000). These depositional models have been linked to simple erosion source terms, not erosional LEMs, with empirical models for sediment grading, so the long-term interactions between the evolving erosional and depositional regimes are ignored, and climate-grading interactions are applied rather than arising out of the evolving landform.

Willgoose et al. (2003) and van der Beek & Braun (2003) have recently proposed a method to constrain paleo-histories using field data. This technique adapts a technique from hydrology called GLUE (Beven & Binley 1992; P in Table 1) and involves Monte Carlo simulation over the range of feasible scenarios, probabilistically assessing how well they fit the data and determining probability bounds for the model predictions and/or climate scenarios. It is then possible to objectively assess the value of field data in improving the reliability of model predictions (i.e., determine the value of field data in improving understanding) or constrain the range of paleo-data that can generate the observed data. Monte Carlo simulation potentially requires large computing resources, and is only currently feasible with Beowulf clusters of computers or similar high-performance computers (O and B in Table 1).

ANTHROPOGENIC LANDFORMS AND ENGINEERING APPLICATIONS

Most of the applications for LEMs have been in the scientific understanding of the evolution of natural landforms. As LEMs have matured, their application to environmental management problems has increased and includes long-term stability of rehabilitated mine landforms (Willgoose & Riley 1998), the fate of tailings in river systems (Hudson-Edwards et al. 2003), and agricultural erosion (Gyasi-Agyei & Willgoose 1996, Mitas & Mitasova 1998). For mine rehabilitation, the anthropogenic landforms have features that are not normally encountered in natural landscapes, so specific validation of the temporal dynamics of these features is required. Moreover, the models must be used to predict behavior, rather than to develop posterior scientific understanding. Accordingly, a range of validation studies for these models have been carried out aimed specifically at validating the models for abandoned minesites (Hancock et al. 2000) and model tailings facilities (Hancock & Willgoose 2004) looking specifically at gully development and short-term (10–100 years) landform evolution. These complement earlier work by the author and his colleagues (Hancock & Willgoose 2001) and others (Schumm et al. 1987) on more natural experimental landforms.

There is considerable merit in using LEMs for these 10–100 year timescales because aspects of long-term behavior appear that are not apparent when using traditional agricultural erosion models like WEPP and CREAMS. For instance, gully development can be the critical failure mechanism for containment structures. It is common to design containment structures so that the spatially average erosion (predicted by models like WEPP) is less than the depth of the capping layer. However, when the landform is able to evolve and as a result dynamically develops gullies and valleys, the depth of gullies can be much greater than the depth of capping material, potentially releasing the encapsulated waste. Willgoose & Riley (1998) predicted an average erosion of 300 mm/1000 years, but gully depths up to 7 m. Gully development can be constrained by bands of rock or armor development some distance from critical locations (e.g., hazardous and nuclear waste). This leads to cost-effective rehabilitation strategies that do not require continuous rock cover to maintain containment integrity. Features of the surrounding natural landscape (e.g., rock outcrops) may also be a critical constraint on containment failure.

However, practical applications require further refinement in the science of soil armor development, rock particle weathering (Wells et al. 2004), gully development, and vegetation ground cover interactions. There is also a need for robust methods to assess the reliability of predictions (whether statistically or otherwise). Geomorphic indicators are needed that can reliably show that a rehabilitated site 20–30 years after rehabilitation is safe for up to 1000 years in the future (to allow regulatory handover of rehabilitated waste sites). Moreover, with a couple of exceptions, the current lack of easy-to-use user interfaces inhibits the use of these models by the broader engineering community.

NUMERICS

Computer run times constrain the development of more sophisticated landform evolution models and practical applications that use Monte Carlo simulation. Accordingly, significant advances in the internal numerics and applications occur in parallel. The author sees two main areas needing attention.

The first is simply the speed at which the internal algorithms work. Many of the existing models can only be run on high-performance computers (e.g., see Coulthard 2001). The heart of the problem is that the transport-limited evolution problem is "stiff" with a short timestep required for the largest drainage areas near the outlet, whereas the evolution of the landform is controlled by the slow rate of evolution of the hillslopes. Thus the model must use small timesteps (either because of numerical stability or mass balance constraints) over long periods of time. One numerical analysis approach is to use an implicit solver (e.g., Fagherazzi et al. 2002). An alternative and faster solver used in SIBERIA is an approximate analytic solution to the stiff component of the sediment balance equations so that the limiting constraints on timestep size are then mass balance errors. The determination of drainage areas using drainage directions (e.g., D8, Dinfinity, etc.) cannot be made implicit. The author has found that if an implicit solver is used, assuming given drainage directions, then the maximum size timestep is limited by changes in drainage directions from step to step. Precipiton models (Chase 1992) appear to circumvent this problem but amount to implicitly assuming a large timestep and introduce noise in the solution.

CPU efficient ways of eliminating explicit drainage direction calculation are needed. This would allow the use of state-of-the-art solvers for hyperbolic equations. For grids, D8 drains all flow to the steepest downslope neighboring node. For TINs (CHILD, CASCADE), an analogous algorithm is used with all flow draining down the triangle edge with steepest downslope direction. In both cases, for a change of direction to occur, a discrete amount of relative erosion must occur at an adjacent point before the drainage direction can switch (Figure 2). This threshold inhibits convergence of flow, which is one of the main drivers in valley erosion, so that valley incision and landform evolution is slowed. Multiple flow direction models (e.g., Dinfinity; Tarboton 1997) address this by gradually changing flow direction with erosion. Figure 3 shows two simulations using SIBERIA, one using D8 and the other using Dinfinity. The Dinfinity run evolves approximately

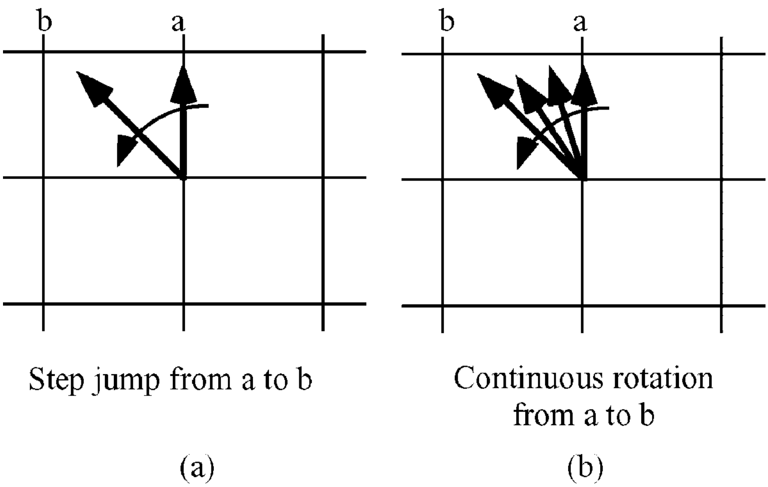

Figure 2 Threshold on flow convergence when point b is eroding faster than point a in (*a*) D8 and (*b*) Dinfinity.

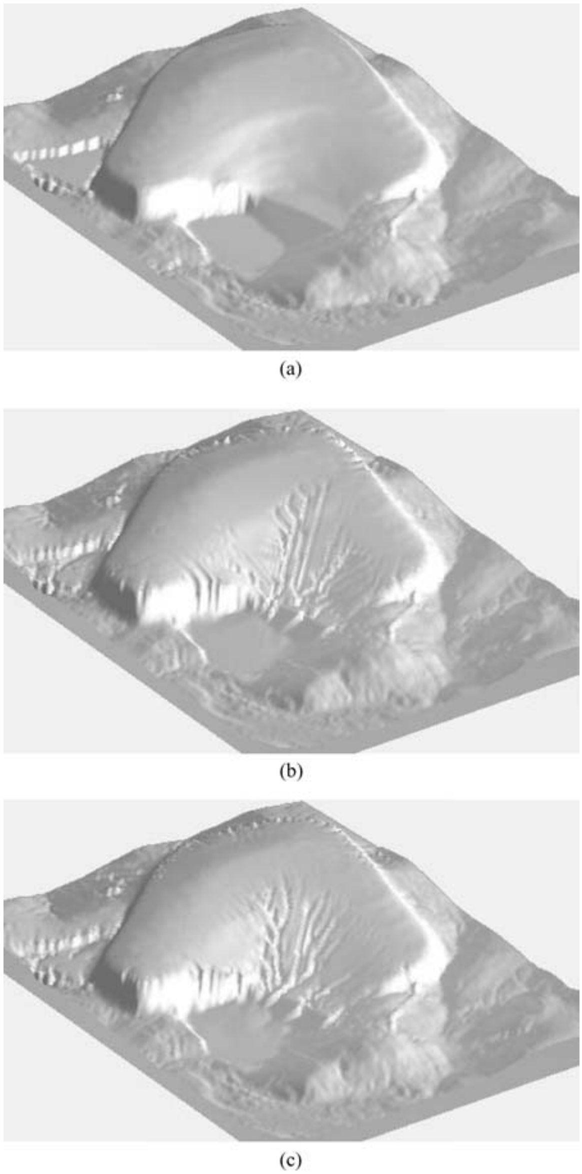

Figure 3 SIBERIA simulations showing the effect of the different drainage analysis algorithms (*a*) initial condition, (*b*) 1000 years evolution using D8, and (*c*) 1000 years evolution using Dinfinity. Elevations are vertically exaggerated (horizontal dimensions 1×1.5 km, elevation relief 38 m). Note the more curved and less linear valleys in (*c*).

20% faster, and grid artifacts are less obvious. These grid artifacts also occur in the TIN models, although the random spacing of the nodes makes them less obvious. Alternative methods based on the solution of PDE for overland flow are possible (Mitas & Mitasova 1998). Although no comparison was carried out, the results appear superior to the D8 technique, albeit at substantially increased computational cost.

CONCLUSIONS

The history of all modeling exercises is to (*a*) develop models, (*b*) show that models can, in principle, match observations (model calibration), (*c*) show that the models' processes lead to the observed behavior (model validation), and (*d*) enhancement and/or modification of model to improve its performance. LEM development for the past 15 years has concentrated on (*a*) and (*b*). Recent years have seen a more focused interest on model validation. If the models are to be useful for understanding the processes that created the geomorphology we see today then we must validate that link. Moreover, the engineering applications require a guarantee that predicted geomorphic behavior in the future will follow from processes that the model is calibrated to. This requires quantitative testing of modeling predictions. The earth sciences community has not been good at model validation, mainly because testing requires an ability to (*a*) make predictions for case studies that are independent of where the model was developed/calibrated; (*b*) determine error bands on those predictions (and potentially on observed data), which are required so that it is possible to objectively assert that an incorrect model should be rejected; and (*c*) repeat experiments. All three of these requirements are difficult to achieve in earth sciences where the site specificity of nature suggests insurmountable problems. A key development is the ability to use repeatable Monte Carlo simulations with the model as a surrogate for field-scale repeatability (Willgoose et al. 2003). Together with field observations of process (e.g., detachment- versus transport-limited channels) this should allow us to identify when models are unsatisfactory.

From the discussion above it is also clear that LEMs are more than just erosion models driving evolving landforms, but potentially involve a range of physical and ecological processes over a variety of time- and space scales. A number of communities are involved, including geomorphology, engineering, geology, and physics. To facilitate this collaboration, a recent initiative in the United States (Community Surface Dynamics Modeling System; Syvitski et al. 2004) has proposed a 10-year plan to develop a generic LEM modeling framework that can link the process and application components discussed in this review. Syvitski compares this initiative to the historical development of global climate models.

The author concludes with a word of caution. We are still some way from having a complete and comprehensive model of landform evolution, although the models currently available work for a range of useful problems. The science challenge is to identify under what conditions they do not work (by comparing with data) and addressing these deficiencies. Models are by their nature simplifications of the real world, and herein lay their primary science role. Model simplicity allows the interactions and casual links to be easily understood. Models that incorporate every possible process are unlikely to be useful because they are no longer easier than the real world to understand, and equifinality will make process interpretation impossible.

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Annual Review of Earth and Planetary Sciences
Volume 33, 2005

CONTENTS

INDEXES

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