AMS 341 Yixuan Lin

Operations Research I: Deterministic Models

Sample Exam

READ THESE INSTRUCTIONS CAREFULLY.

Do not start the exam until told to do so.
Upload your answers before 16:00
This examination is OPEN BOOK and OPEN NOTES.
Remember to write your name (First Last) and ID Number on your answer.

Note that:

Make sure you OPEN the camera. It would be better to show your hand in camera all the time.
You can NOT talk/communicate with others or using computer to find the solution.
You can NOT use cell phone.

If you did not do anything mentioned below, you will be considered cheating!!!

Academic integrity is expected of all students at all times, whether in the presence or absence of members of the faculty. Understanding this, I declare that I shall not give, use, or receive unauthorized aid in this examination. I have been warned that any suspected instance of academic dishonesty will be reported to the Academic Judiciary and that I will be subjected to the maximum possible penalty permitted under University guidelines.

Signature: **(If you upload your answer, you are regarded as signing this.)**

Work carefully, and GOOD LUCK!!!

1. (15 points) A manufacturer of lawn and garden equipment makes two types of lawn mowers: A push type and a self-propelled model. The push type requires 9 minutes to assemble and 2 minutes to package; the self-propelled mower requires 12 minutes to assemble and 6 minutes to package. Each type has an engine. The company has 12 hours of assembly time available, 5 hours of packaging time and 75 engines. Profits are $ 70 for the self-propelled model and $ 45 for the push type mower. The company wishes to maximize its profit.

(a). Define the variables you are using in the formulation.

(b). The objective function is:

(c). The constraints are:

2. (15 points) The following is a tableau for an LP which is a **MIN** problem:

z	x_1	x_2	x_3	s_1	e_2	s_3	RHS	ratio
1	-1	1	-4	0	0	0	0	
0	1	1	2	1	0	0	8	
0	1	1	-1	0	1	0	2	
0	-1	1	1	0	0	1	6	

(a). What are the basic variables, and what are they equal to?

(b). What are the non-basic variables, and what are they equal to?

(c). This is not an optimal BFS. Which variable should be selected to enter the basis?

(d). Do one pivot. What is the next tableau?

z	x_1	x_2	x_3	s_1	e_2	s_3	RHS	ratio

3. (10 points)(a) Consider the feasible region given by the following constraints: Sketch the feasible region.

$$x_1 + x_2 \geq 1 \quad (1)$$
$$x_1 - x_2 \leq 2 \quad (2)$$
$$2x_1 + 3x_2 \leq 6 \quad (3)$$
$$x_1 \geq 0 \quad (4)$$
$$x_2 \geq 0 \quad (5)$$

In the following parts (b), (c), (d) circle the correct answers:
(b). Is the point $x_1 = 0$, $x_2 = 0$ a feasible point? YES NO
Is it a basic solution? YES NO

(c). Is the point $x_1 = 2$, $x_2 = 0$ a feasible point? YES NO
Is it a basic solution? YES NO

(d). Is the point $x_1 = 1$, $x_2 = 1$ a feasible point? YES NO
Is it a basic solution? YES NO

(e). How many feasible solutions does an LP with the above constraints have?

(f). How many basic feasible solutions does an LP with the above constraints have?

(e). If objective function is: max $z = x_1 + x_2$, what is the optimal solution?

4. (10 points) Consider the following LP:

$$\begin{aligned} \min z = {} & 2x_1 - x_2 + x_3 \\ \text{s.t.} \quad & x_1 + 2x_2 + x_3 \geq 8 \\ & x_1 - x_3 \geq -2 \\ & x_2 \quad \text{urs} \\ & x_1, x_3 \geq 0 \end{aligned}$$

Rewrite the LP in standard form.

5. (10 points) Find the dual of the following LP:

$$
\begin{aligned}
\max z = {} & x_1 - 2x_2 - x_3 + 3x_4 \\
\text{s.t.} \quad & x_1 + x_2 - x_3 + 3x_4 \geq 2 \\
& 2x_2 - x_3 + x_4 \leq 30 \\
& x_1 + x_2 - x_3 = 20 \\
& x_1 \geq 0 \\
& x_2 \leq 0 \\
& x_3, x_4 \quad \text{urs}
\end{aligned}
$$

6. (10 points) A maximization LP is being solved by the Simplex method. Here is the current tableau:

z	x_1	x_2	x_3	x_4	x_5	x_6	RHS
1	0	0	0	0	2	3	8
0	1	-1	0	0	0	0	4
0	0	2	1	0	-1	1	3
0	0	1	0	1	0	1	2

Which one of the following statements is true: (Circle one)

(i). This is an optimal tableau, and the LP has a unique optimal solution.

(ii). This is an optimal tableau, but the LP has multiple optimal solutions.

(iii). This is an optimal tableau and the LP is unbounded.

(iv). This is not an optimal tableau for the LP.

If it has multiple optimal solutions, please write down the general form of all optimal solutions:

7. (15 points) Using Big-M to solve the following LP:

$$\begin{aligned} \max z = {} & 4x_1 + 4x_2 + x_3 \\ \text{s.t.} \quad & x_1 + x_2 + x_3 \leq 2 \\ & 2x_1 + x_2 \leq 3 \\ & 2x_1 + x_2 + 3x_3 \geq 3 \\ & x_1, x_2, x_3 \geq 0 \end{aligned}$$

8. (15 points) A company produces two products. Each product can be produced on one of 2 machines. The length of time (in hours) needed to produce each product on each machine is given below:

Product	Machine 1	Machine 2
1	4	3
2	7	4

Each month 500 hours of time are available on each machine. Each month, customers are willing to buy up to the quantities of each of the product at the prices given below. Products can be sold only in the month they are produced (inventory not allowed). The company's goal is to maximize revenue from selling products during the next 2 months.

Product	Demand month 1	demand month 2	price month 1	price month 2
1	100	190	$ 55	$ 12
2	140	130	$ 65	$ 32

(a). Define the variables you are using in the formulation.

(b). The objective function is:

(c). The constraints are: