AMS 341 (Spring, 2009) Estie Arkin

Operations Research I: Deterministic Models

Exam 2: Thursday, May 14, 2009, 5-6:30pm

READ THESE INSTRUCTIONS CAREFULLY. Do not start the exam until told to do so. The exam time is one and a half hours. Make certain that you have all 7 pages of the exam. You will be held responsible for any missing pages.

Write your answers on this examination, using the backs of pages if needed. (Use back of pages also for scratch paper if you need it.)

This examination is CLOSED BOOK and CLOSED NOTES. You may not use any books, papers, or materials other than your pen or pencil. You may use a 4 by 6 "cheat sheet", which should be turned in with your exam.

The following items should NOT be on your desk - put them INSIDE your bag!

- calculator
- cell phone
- pager

If I see any of these items, even turned off, this will be considered cheating!!!
Work carefully, and GOOD LUCK!!!

Last (Family) Name (PRINT CLEARLY): _____

First Name (PRINT CLEARLY): _____

ID Number: _____

Academic integrity is expected of all students at all times, whether in the presence or absence of members of the faculty. Understanding this, I declare that I shall not give, use, or receive unauthorized aid in this examination. I have been warned that any suspected instance of academic dishonesty will be reported to the Academic Judiciary and that I will be subjected to the maximum possible penalty permitted under University guidelines.

Signature:

_____

1. (12 points) A frazzled student is trying to plan all the work to be completed before moving out from his dorm room. The student has a large family, and many friends that can be enlisted to help with the various tasks (packing, moving, etc.). The following are the activities that would have to be undertaken before move out:

Task	Predecessors	Time (days)
A	-	2
B	A	6
C	A	4
D	B	4
E	C	2
F	B,D	1
G	D,E	2
H	C	1

(a). Draw a project network.

(b). What is the critical path for this student? You may find the path either by computing the total float for each node, or by inspection. (Your answer should be a list all critical activities.)

(c). What is the total float of task E?

2. (10 points) We wish to solve an integer programming problem. All variables are restricted to be integer. We began by solving the LP relaxation of the problem and got the final (optimal) tableau for it. Unfortunately, not all the variables are integer.

z	x_1	x_2	x_3	x_4	x_5	RHS
1	0	-1.5	-3	0	0	6.25
0	0	3	0	1	0	10
0	1	2	5	0	0	5
0	0	1.2	-2.6	0	1	5.4

To solve the problem using the cutting plane method, what cut (constraint) would you add? Note: Do not solve the new LP, just state the added constraint.

3. (10 points) Consider the following Linear Programming problem:

$$\begin{aligned} \max \quad & z = 3x_1 + x_2 - x_3 \\ \text{s.t.} \quad & 2x_1 + x_2 + x_3 \quad \leq 8 \\ & 4x_1 + x_2 - x_3 \quad \leq 10 \\ & x_1, x_2, x_2 \quad \geq 0 \end{aligned}$$

The final tableau for the given LP is given below. s_1, s_2 are the slack variables of the constraints What is the optimal solution to the dual? Make sure to state the objective value and the value of all dual variables.

z	x_1	x_2	x_3	s_1	s_2	RHS
1	0	0	1	1/2	1/2	9
0	0	1	3	2	-1	6
0	1	0	-1	-1/2	1/2	1

4. (15 points) A large baking company bakes cakes at 3 different bakeries. The cost of baking 1 cake at bakery 1 is \$6, at bakery 2 \$4, and at bakery 3 \$5. Each bakery can bake up to 100 cakes. The company then sends these cakes to 3 stores. Store 1 demands least 75 cakes, store 2 at least 120 cakes and store 3 at least 80 cakes. The shipping costs per cake from bakeries to stores are given in the table below: (The "-" means that bakery 1 cannot ship to store 1.)

	store 1	store 2	store 3
bakery 1	-	1.5	3
bakery 2	2.5	4	5
bakery 3	1	2	3

The company's goal is to minimize the total cost. Formulate the problem as a Balanced Transportation problem by giving the the transportation tableau (cost and requirement matrix).

5. (15 points) The AMS department is scheduling faculty to teach its courses next semester. Professors A,B, and C will each be teaching at most one course, and professors D and E will each be teaching at most 2 courses. There are 5 courses to be taught. Based on surveys from previous years, the department knows how succesful each professor is at teaching each course. The data is given in the table below (small numbers are better!) The "-" means that professor C cannot teach course 1.

	Prof A	Prof B	Prof C	Prof D	Prof E
course 1	4	3	-	5	7
course 2	1	3	6	1	7
course 3	3	4	7	2	7
course 4	6	1	5	5	5
course 5	4	5	4	4	3

The department's goal is to assign professors to courses such that the teaching next semester is as "succesful" as possible. Formulate the problem as an Assignment problem by giving the cost matrix. Note: You are asked to formulate an assignment problem, not a Balanced Transportation Problem and not a Linear Program. Do not solve the problem you formulated.

6. (20 points) A consulting company has 10 employees, each of whom can work on at most two team projects. Six projects are under consideration. Each project requires 4 (specific) employees. The required employees and the revenue of each project is given in the table below:

	Project 1	Project 2	Project 3	Project 4	Project 5	Project 6
Required Employees	1,4,5,8	2,3,7,10	1,6,8,9	2,3,5,10	1,6,7,9	2,4,8,10
Revenue($)	10,000	15,000	6,000	8,000	12,000	9,000

Each worker who is used on any project must be paid the retainer in the table below:

Worker	1	2	3	4	5	6	7	8	9	10
Retainer($)	800	500	600	700	800	600	400	500	400	500

Formulate an integer programming problem to maximize the company's profit.

(a). Define the variables:

(b). What is the objective function? (Max or Min?)

(c). What are the constraints?

7. (18 points) A family is planning a summer vacation in Italy. It has a total of 5 days, and it is trying to decide how many days to spend in Rome, Florence and Venice. It estimates the enjoyment it would get for spending some days in each city, and wants to maximize its total enjoyment:

	0 days	1 day	2 days	3 days	4 days and above
Rome	0	3	5	6	7
Florence	0	1	2	6	6
Venice	0	4	5	5	6

To solve the problem using Dynamic Programming define $f_i(s)$ = the maximum enjoyment achievable in stages i and above and state s.

Solve the problem. Make sure to state at the end how many days are spent in each city. (A solution by guessing will get no credit, I want to see your computations using $f_i(s)$ with the stages and states you defined.)