AMS 341 (Summer, 2021) Yixuan Lin

Operations Research I: Deterministic Models

Final: Thursday, July 1, 2021

READ THESE INSTRUCTIONS CAREFULLY.

Do not start the exam until told to do so.
Upload your answers before 16:40
This examination is OPEN BOOK and OPEN NOTES.
Remember to write your name (First Last) and ID Number on your answer.

Note that:

Make sure you OPEN the camera. It would be better to show your hand in camera all the time.
You can NOT talk/communicate with others or using computer to find the solution.
You can NOT use cell phone.

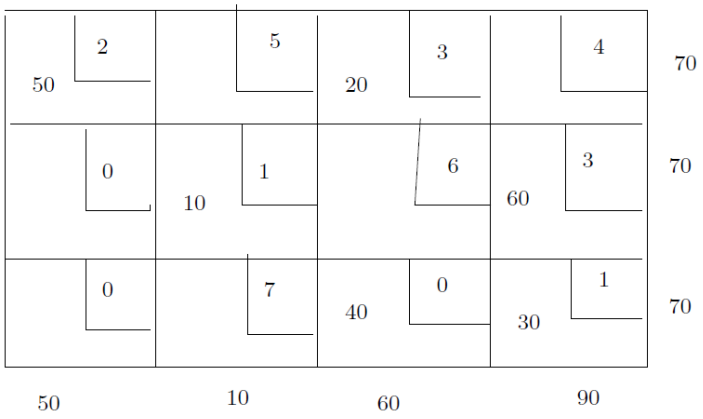
If you did not do anything mentioned below, you will be considered cheating!!!

Academic integrity is expected of all students at all times, whether in the presence or absence of members of the faculty. Understanding this, I declare that I shall not give, use, or receive unauthorized aid in this examination. I have been warned that any suspected instance of academic dishonesty will be reported to the Academic Judiciary and that I will be subjected to the maximum possible penalty permitted under University guidelines.

Signature: **(If you upload your answer, you are regarded as signing this.)**

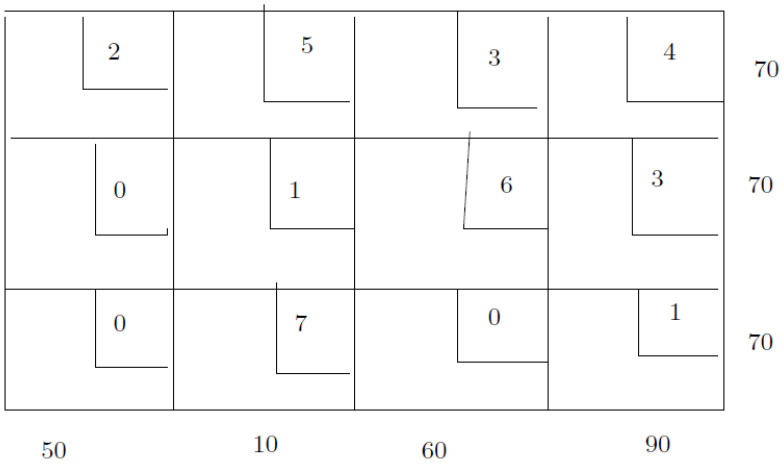
Work carefully, and GOOD LUCK!!!

1. (8 points) Consider the following (minimum) Balanced Transportation problem and current BFS given.



(a). This BFS is not optimal. Which variable enters the basis next? (2 points)

(b). Show the next BFS (list only the basic variables). (6 points)



2. (8 points) Considering the following integer programming problem

$$\begin{aligned} \max z &= 4x_1 + 3x_2 \\ \text{s.t.} \quad & 2x_1 + x_2 \leq 4 \\ & -x_1 + 2x_2 \leq 4 \\ & x_1, x_2 \geq 0, \quad \text{integer} \end{aligned}$$

If the optimal tableau to the LP relaxation is

z	x_1	x_2	s_1	s_2	RHS
1	0	0	2.2	0.4	10.4
0	1	0	0.4	-0.2	0.8
0	0	1	0.2	0.4	2.4

(1). If we want to solve the problem using the cutting plane method, what is the cut that should be added? (Just show the cut) (2 points)

(2). Show your branch and bound tree and state what is the optimal solution you get applying the branch and bound method: (Giving an optimal solution without work shown will receive almost no credit.) (6 points)

3. (10 points) Solving the following upper bound LP.

$$
\begin{array}{lrl}
\max & z = 4x_1 + 3x_2 + 5x_3 & \\
\text{s.t.} & 2x_1 + 2x_2 + x_3 + x_4 & \leq 9 \\
& 4x_1 - x_2 - x_3 \quad + x_5 & \leq 6 \\
& 2x_2 + x_3 & \leq 6 \\
0 \leq & x_1 & \leq 2 \\
0 \leq & x_2 & \leq 3 \\
0 \leq & x_3 & \leq 4 \\
0 \leq & x_4 & \leq 5 \\
0 \leq & x_5 & \leq 7
\end{array}
$$

4. (10 points) Find the optimal solution to the MCNFP in Figure 1 using the bfs in Figure 2 as a starting basis. Note that the number on arc following: $(x_{ij})\ U_{ij} < c_{ij} >$.

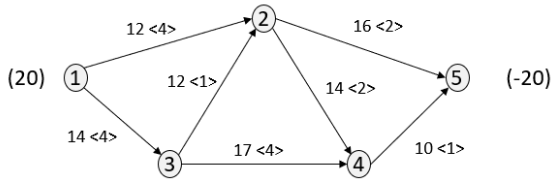


Figure 1.

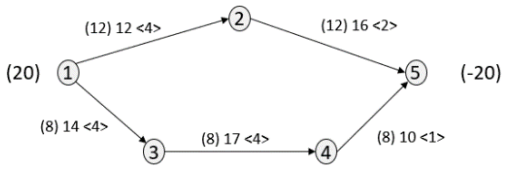


Figure 2.

5. (14 points) A company intends to subcontract three special values for its nuclear power plants. Four bids have been received, and, according to government restriction, no subcontractor may be permitted to produce more than one valve type. The bides, in terms of thousands of dollars per valve type, and the potential subcontractor information are given in table.

	Contractor 1	Contractor 2	Contractor 3	Contractor 4
Valve type A	12	13	20	18
Valve type B	8	20	10	18
Valve type C	20	22	30	25

(1). Formulate the problem as an assignment problem by giving the cost matrix. (4 points)

(2). Using Hungarian Method to solve it. (10 points)

6 . (20 points) Consider the following LP:

$$\begin{aligned} \max z = {} & 4x_1 + x_2 \\ \text{s.t.} \quad & x_1 + 2x_2 = 6 \\ & x_1 - x_2 \geq 3 \\ & 2x_1 + x_2 \leq 10 \\ & x_1, x_2 \geq 0 \end{aligned}$$

(1). Rewrite the LP in standard form (5 points).

(2). Using Two-Phase to solve it (15 points).

7.(20 points) The following is a list of tasks that have to be completed as soon as possible.

Activity	Predecessors	Time (hours)
A	-	7
B	A	10
C	B	1
D	-	12
E	B,D	6
F	A	5
G	F	6
H	C	2
I	G,H	10

(a). Draw a project network (activity on arc). Make sure to number the nodes. (6 points)

(b). What are the critical activities for this project? (6 points)

(c). How soon can the project be completed? (2 points)

(d). What is the total float of activity D? (2 points)

(e). If activity D is made shorter by 1 hour, would the project take less time to complete? Circle YES or NO, and give a brief explanation. (2 points)

(f). If activity B is made shorter by 1 hour, would the project take less to complete? Circle YES or NO, and give a brief explanation. (2 points)

8.(30 points) A company manufactures two products (1 and 2). Each unit of product 1 can be sold for $15, and each unit of product 2 for $25. Each product requires raw material and two types of labor (skilled and unskilled). Currently, the company has available 100 hours of skilled labor, 70 hours of unskilled labor, and 30 units of raw material. Because of marketing considerations, at least 3 units of product 2 must be produced. The relevant LP is:

$$\max z = 15x_1 + 25x_2$$

$$\begin{aligned} \text{s.t.} \quad & 3x_1 + 4x_2 \leq 100 && \text{(Skilled labor constraint)} \\ & 2x_1 + 3x_2 \leq 70 && \text{(Unskilled labor constraint)} \\ & x_1 + 2x_2 \leq 30 && \text{(Raw material constraint)} \\ & x_2 \geq 3 && \text{(Product 2 constraint)} \\ & x_1, x_2 \geq 0 \end{aligned}$$

(1). Find the dual of this LP (5 points):

(2). If the optimal tableau to the LP is

z	x_1	x_2	s_1	s_2	s_3	e_4	RHS
1	0	0	0	0	15	5	435
	0	0	1	0	-3	-2	16
	0	0	0	1	-2	-1	13
	1	0	0	0	1	2	24
	0	1	0	0	0	-1	3

Using the complementary slackness to find the solution of the dual LP (5 points). (remember to check the slack variables' value first)

(3). Based on the original LP solve the following questions (20 points):

(a) What are the shadow price for these constraints? (2 points)

(b). Assuming the current basis remains optimal (it does), what would the company's revenue be if 35 units of raw material were available? (5 points)

(c) If we want to keep the current basic variable optimal, how can we change the cost for each unit of product 2? (5 points)

(d) A new constraint is added: $x_1 + 2\,x_2 \leq 26$. What is the optimal solution? (Show the detail of you work.) (8 points)