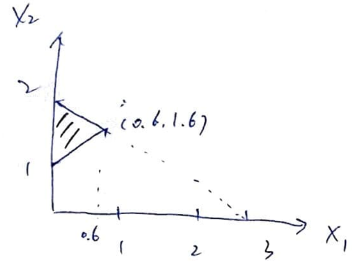
Midterm AMS 341

summer 2021

1./(a)




(b) $(\frac{1}{2}, \frac{3}{2})$ is a feasible point but not a basic solution.

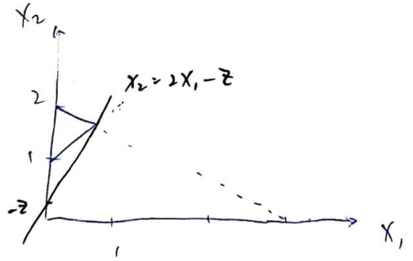
(c) $(-1, 0)$ is not a feasible point but a basic solution.

(d) $(0, 2)$ is a feasible point as well as a basic solution.

(e) Infinite

~~(d)~~ (f) 3 bfs

(g)




the optimal solution is $(0.6, 1.6)$. $Z = -0.4$

2. Let $y_2 = -x_2$

$$\max \quad z = 2x_1 - 4y_2 - 5x_3$$

$$\text{s.t} \quad x_1' - x_1'' + 3y_2 + x_3 + s_1 = 5$$

$$2x_1' - 2x_1'' - y_2 - x_3 - e_2 = -10$$

$$4x_1' - 4x_1'' + 2y_2 + 3x_3 - e_3 = 3$$

$$x_1', x_1'', y_2, x_3, s_1, e_2, e_3 \geq 0$$

3.

$$\max \quad w = 20y_1 + 10y_2 + 5y_3$$

$$\text{s.t} \quad 2y_1 + y_2 + 3y_3 \leq 3$$

$$2y_1 + 4y_2 \geq 2$$

$$y_1 - 2y_2 - y_3 = -4$$

$$-3y_1 + y_3 = 1$$

$$y_1 \leq 0$$

$$y_2 \text{ urs}$$

$$y_3 \geq 0$$

4. (iii) is true.

Another optimal tableau is

| $z$ | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ | $x_6$ | RHS |
|---|---|---|---|---|---|---|---|
| 1 | -3 | 0 | -1 | 0 | 0 | 0 | 6 |
| 0 | 2 | 0 | 3 | 1 | 1 | 0 | 5 |
| 0 | 3 | 1 | 4 | 1 | 0 | 0 | 6 |
| 0 | -6 | 0 | -6 | -3 | 0 | 1 | -12 |

The general form of all optimal solutions is

$$x = cb_1 + (1-c)b_2$$

where $c \in [0,1]$, $b_1 = (0,1,0,5,0,3)^T$, $b_2 = (0,6,0,0,5,-12)^T$

5. The initial tableau is

| $Z$ | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $a_1$ | $a_2$ | rhs | ratio |
|---|---|---|---|---|---|---|---|---|
| 1 | +1 | +1 | 0 | 0 | $-M$ | $-M$ | 0 | |
| 1 | +1 | +1 | $-M$ | $M$ | 0 | 0 | $3M$ | |
| 0 | -1 | 1 | 0 | -1 | 1 | 0 | 2 | – |
| 0 | 1 | -1 | -1 | 2 | 0 | 1 | 1 | 0.5* |

First tableau is

| $Z$ | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $a_1$ | $a_2$ | rhs | ratio |
|---|---|---|---|---|---|---|---|---|
| 1 | $+1-\frac{M}{2}$ | $+1+\frac{M}{2}$ | $-\frac{M}{2}$ | 0 | 0 | $-\frac{M}{2}$ | $\frac{5}{2}M$ | |
| 0 | $-\frac{1}{2}$ | $\frac{1}{2}$ | $-\frac{1}{2}$ | 0 | 1 | $\frac{1}{2}$ | $\frac{5}{2}$ | 5* |
| 0 | $\frac{1}{2}$ | $-\frac{1}{2}$ | $-\frac{1}{2}$ | 1 | 0 | $\frac{1}{2}$ | $\frac{1}{2}$ | – |

Second tableau is

| $z$ | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $a_1$ | $a_2$ | rhs | ratio |
|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 0 | 1 | 0 | $-2-M$ | $-1-M$ | $-5$ | |
| 0 | $-1$ | 1 | $-1$ | 0 | 2 | 1 | 5 | |
| 0 | 0 | 0 | $-1$ | 1 | 1 | 1 | 3 | |

$x_1$ is the entering variable now. but it has a nonpositive coefficient in each constraint.

Therefore. the LP is unbounded.

6. (a) Denote by $X_i$ the number of workers assigned to shift $i$. (6am - 12pm etc.)

(b) The objective function is

$$\max \quad z = (18\times10 - 10\times6)X_1 + (23\times10 - 17\times6)X_2 + (27\times10 - 25\times6)X_3 + (34\times10 - 30\times6)X_4$$

(c) The constraints are

$$X_i \le 15$$
$$\sum_{i=1}^{4} X_i \le 35 \qquad \forall\, i = 1, 2, 3, 4$$
$$10\sum_{i=1}^{4} X_i \le 250$$

$$5X_1 + 4X_2 + 3X_3 + 2X_4 \le 4(X_1 + X_2 + X_3 + X_4)$$

$$X_1, X_2, X_3, X_4 \ge 0$$

7. (a) Denote by $x_{ij}$ the quantities of waste processed by incinerator j from city i.

Denote by $y_{ij}$ the quantities of debris dumped at landfill j from incinerator i.

(b) The objective function is

$$\min\ Z = (30\times5+55)X_{11} + (36\times5+55)X_{21} + (5\times5+60)X_{12}$$
$$+ (45\times5+60)X_{22} + 25y_{11} + 40y_{12} + 45y_{21} + 30y_{22}$$

(c) The constraints are

$$X_{11} + X_{12} \le 300$$
$$X_{21} + X_{22} \le 600$$
$$X_{11} + X_{21} \le 700$$
$$X_{12} + X_{22} \le 700$$
$$y_{11} + y_{21} \le 400$$
$$y_{12} + y_{22} \le 400$$

$$y_{11} + y_{12} + y_{21} + y_{22} = 0.4\,(X_{11} + X_{12} + X_{21} + X_{22})$$
$$X_{ij},\ y_{ij} \ge 0, \quad \forall\, i = 1,2;\ j = 1,2$$

8. (a) $BV = \{X_4, X_5\}$

$X_4 = 6, \quad X_5 = 1$

(b) $NBV = \{X_1, X_2, X_3\}$

$X_1 = X_2 = X_3 = 0$

(c) $g = 1, h = 0$, because entering variable is $X_1$ and pivot row is row 1.

$f = 2, \quad c = d = 2, \quad b = 3 \quad (R_2' = \frac{1}{3} R_2)$

$i = \frac{8}{3}, \quad m = 3, \quad e = -1 \quad (R_3' = R_3 + R_2')$

$a = 2, j = -\frac{13}{3}, k = -\frac{2}{3}, l = 0, n = -4 \quad (R_1' = R_1 + (-2) R_2')$

$\therefore \begin{cases} a = 2 \\ b = 3 \\ c = 2 \\ d = 2 \\ e = -1 \\ f = 2 \\ g = 1 \\ h = 0 \end{cases} \qquad \begin{cases} i = \frac{8}{3} \\ j = -\frac{13}{3} \\ k = -\frac{2}{3} \\ l = 0 \\ m = 3 \\ n = -4 \end{cases}$