AMS 341 (Fall, 2016) Exam 1 - Solution notes Estie Arkin

Mean 80.4, median 83.5, top quartile 91, bottom quartile 70.75, high 100, low 39.

1. (15 points) To supplement my income I plan to bake some cakes to sell at the Farmer's Market. Each chocolate cake sells for \$10 and each vanilla cake for \$7.50. A chocolate cakes requires 4 egg and 40 minutes of baking time. Each vanilla cake requires 1 egg and 50 minutes of baking time. Assume at most 1 cake fits in my oven at any time. I currently have 30 eggs and 8 hours of baking time.

Assuming that I can sell all cakes that I bring to the Farmer's Market, formulate an LP to maximize my profit:

(a). Define the variables you are using in the formulation. $c =$ number of chocolate cakes baked, $v =$ number of vanilla cakes baked.

(b). The objective function is: $\max z = 10c + 7.5v$

(c). The constraints are:

$$4c + 1v \leq 30$$
$$40/60c + 50/60v \leq 8$$
$$c, v \geq 0$$

2. (12 points) The following is a tableau for an LP which is a MAX problem:

z	s_1	x_1	x_2	e_3	x_3	RHS	ratio
1	0	2	-1	0	0	12	
0	0	0	1	0	1	4	
0	0	2	0	1	0	3	
0	1	1	-3	0	0	6	

(a). What are the basic variables, and what are they equal to? $s_1 = 6$, $e_3 = 3$ $x_3 = 4$ ($z = 12$)

(b). What are the non-basic variables, and what are they equal to? $x_1 = x_2 = 0$

(c). This is not an optimal BFS. Which variable should be selected to enter the basis? x_2 enters, x_3 leaves.

(d). Do one pivot. What is the next tableau?

z	s_1	x_1	x_2	e_3	x_3	RHS	ratio
1	0	2	0	0	1	16	
0	0	0	1	0	1	4	
0	0	2	0	1	0	3	
0	1	1	0	0	3	18	

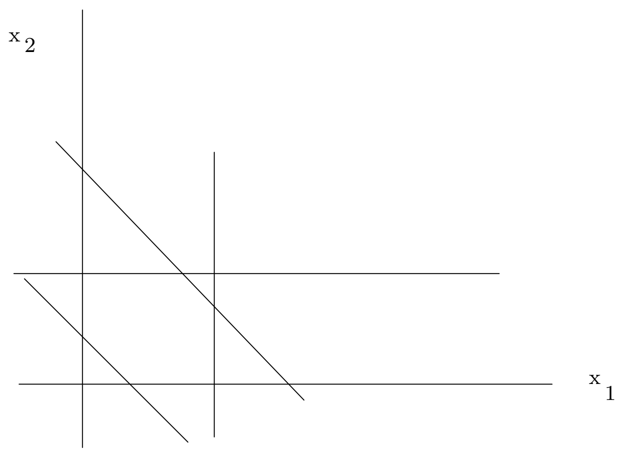
3. (20 points)(a) Consider the feasible region given by the following constraints: Sketch the feasible region.

$$x_1 \leq 5 \quad (1)$$
$$x_2 \leq 4 \quad (2)$$
$$x_1 + x_2 \leq 8 \quad (3)$$

$$
\begin{aligned}
x_1 + x_2 &\geq 2 && (4)\\
x_1 &\geq 0 && (5)\\
x_2 &\geq 0 && (6)
\end{aligned}
$$

In standard form:

$$
\begin{aligned}
x_1 + s_1 &= 5\\
x_2 + s_2 &= 4\\
x_1 + x_2 + s_3 &= 8\\
x_1 + x_2 - e_4 &= 2\\
x_1, x_2, s_1, s_2, s_3, e_4 &\geq 0
\end{aligned}
$$



In parts (b), (c), (d) circle the correct answers:
(b). Is the point $x_1 = 4$, $x_2 = 0$ a feasible point? YES
Is it a basic solution? NO
(c). Is the point $x_1 = 5$, $x_2 = 0$ a feasible point? YES
Is it a basic solution? YES
(d). Is the point $x_1 = 0$, $x_2 = 0$ a feasible point? NO
Is it a basic solution? YES
(e). How many feasible solutions does the LP have? ∞
(f). How many basic feasible solutions does the LP have? 6
4. (10 points) Consider the following LP:

$$
\begin{array}{lcl}
\min & z = 2x - y + 2w & \\
\text{s.t.} & x + y & \geq 10\\
 & -x - y + 3w & = -2\\
 & x & \leq 0\\
 & y & \geq 0\\
 & w & \text{unrestricted}
\end{array}
$$

Rewrite the LP in standard form.

$$
\begin{array}{lll}
\min & z = -2x' - y + 2w_1 - 2w_2 & \\
\text{s.t.} & -x' + y - e_1 & = 10 \\
 & -x' + y - 3w_1 + 3w_2 & = 2 \\
 & x', y, w_1, w_2, e_1 & \geq 0
\end{array}
$$

5. (5 points) A maximization LP is being solved by the Simplex method. Here is the current tableau:

z	x_1	x_2	s_1	e_2	x_3	x_4	RHS
1	0	3	0	0	2	3	10
0	1	1	0	0	0	0	3
0	0	2	1	0	-1	0	3
0	0	1	0	1	0	2	2

Which one of the following statements is true: (Circle one)

(i). This is an optimal tableau, and the LP has a unique optimal solution.

6. (20 points) Steelco manufacter steel by combining Alloy 1 and Alloy 2. The steel must meet the following requirements: 3.2-3.5% carbon (i.e., at least 3.2 but no more than 3.5 percent); 1.8-2.5% silicon; 0.9-1.2% nickel; tensile strength of at least 45,000 pounds per square inch (psi). Assume that the tensile strength of a mixture of the two alloys is determined by averaging that of the two alloys mixed. For example a one ton mixture that is %40 Alloy 1 and %60 Alloy 2 has tensile strength of $0.4(42,000) + 0.6(50,000)$. The cost and properties of the alloys are given below:

	Cost per ton	percent silicon	percent nickel	percent carbon	tensile strength (psi)
Alloy 1	\$ 190	2	1	3	42,000
Alloy 2	\$ 200	2.5	1.5	4	50,000

Formulate an LP to minimize the cost of producing one ton of steel.

(a). Define the variables you are using in the formulation. A_1, A_2 amount of alloy 1,2 bought.

(b). The objective function is: $\min z = 190A_1 + 200A_2$

(c). The constraints are:

$$
\begin{gathered}
0.02A_1 + 0.025A_2 \leq 0.025 \\
0.02A_1 + 0.025A_2 \geq 0.018 \\
0.01A_1 + 0.015A_2 \leq 0.012 \\
0.01A_1 + 0.015A_2 \geq 0.009 \\
0.03A_1 + 0.04A_2 \leq 0.035 \\
0.03A_1 + 0.04A_2 \geq 0.032 \\
42A_1 + 50A_2 \geq 45 \\
A_1 + A_2 = 1 \\
A_1, A_2 \geq 0
\end{gathered}
$$

7. (18 points, 3 points for each part) Answer TRUE or FALSE:

False All optimal solutions to an LP must be basic (a BFS).

True Any BFS can be an optimal solution to an LP depending on the choice of objective function.

True Two different BFS (to the same LP) must have the same number of non basic variables.

False At the end of the Simplex method, all slack variables must equal zero.

False There is only one correct way to formulate an LP.

True An LP in cannonical form is also in standard form.