

Operations Research I: Deterministic Models

Exam 2: Thursday, May 13, 2010, 5:15-6:45pm

READ THESE INSTRUCTIONS CAREFULLY. Do not start the exam until told to do so. The exam time is one and a half hours. Make certain that you have all 7 pages of the exam. You will be held responsible for any missing pages.

Write your answers on this examination, using the backs of pages if needed. (Use back of pages also for scratch paper if you need it.)

This examination is CLOSED BOOK and CLOSED NOTES. You may not use any books, papers, or materials other than your pen or pencil. You may use a 4 by 6 "cheat sheet", which should be turned in with your exam.

The following items should NOT be on your desk - put them INSIDE your bag!

- calculator
- cell phone
- pager

If I see any of these items, even turned off, this will be considered cheating!!!
Work carefully, and GOOD LUCK!!!

Last (Family) Name (PRINT CLEARLY): _____

First Name (PRINT CLEARLY): _____

ID Number: _____

Academic integrity is expected of all students at all times, whether in the presence or absence of members of the faculty. Understanding this, I declare that I shall not give, use, or receive unauthorized aid in this examination. I have been warned that any suspected instance of academic dishonesty will be reported to the Academic Judiciary and that I will be subjected to the maximum possible penalty permitted under University guidelines.

Signature:

1. (10 points) I am planning a vacation once the spring semester is over. There are many tasks that have to be completed before I can go. Fortunately, I can get help from family and friends. The following are the tasks that would have to be undertaken before the vacation:

| Task | Predecessors | Time (hours) |
|------|--------------|--------------|
| A | - | 2 |
| B | - | 3 |
| C | A | 1 |
| D | B | 8 |
| E | B,C | 7 |
| F | D,E | 5 |

(a). Draw a project network.

(b). What is my critical path? You may find the path either by computing the total float for each node, or by inspection. (Your answer should be a list all critical activities.)

2. (10 points) Consider the following (minimum) Balanced Transportation problem: Find an initial BFS for the problem using the min cost method:

| | | | | | | | | |
|----|---|----|---|-----|---|----|---|-----|
| | 9 | | 3 | | 4 | | 1 | 100 |
| | 6 | | 1 | | 5 | | 0 | 100 |
| | 7 | | 8 | | 2 | | 1 | 100 |
| 75 | | 75 | | 125 | | 25 | | |

3. (15 points) Plans are being made for the energy systems for a new building. The three possible sources of energy are electricity, natural gas and a solar heating unit. The energy requirements are: 20 units of electricity, 10 units for water heating and 30 units for space heating. The size of the roof limits the solar heater to 30 units. There is no limit on the amount of electricity or natural gas bought. Electricity needs can only be met by buying electricity at a cost of \$ 50 per unit. Other energy needs can be met by any sources with the following unit costs:

| | Electricity | Natural Gas | Solar heater |
|---------------|-------------|-------------|--------------|
| Water heating | \$ 90 | \$ 60 | \$ 30 |
| Space heating | \$ 80 | \$ 50 | \$ 40 |

Formulate a Balanced Transportation Problem to minimize the *total* cost of meeting the energy needs. Give your formulation in terms of a cost and requirement table.

4. (15 points) Consider the following Linear Programming problem:

$$\begin{aligned}
 \max \quad & z = -4x_1 - x_2 \\
 \text{s.t.} \quad & 4x_1 + 3x_2 \geq 6 \\
 & x_1 + 2x_2 \leq 3 \\
 & 3x_1 + x_2 = 3 \\
 & x_1, x_2 \geq 0
 \end{aligned}$$

(a). What is the dual of the LP?

(b). The final tableau for the given LP is given below. e_1 is the excess variable of the first constraint, s_2 the slack variable of the second constraint, and a_1, a_3 the artificial variables of constraints 1,3. The final tableau was found using the big M method. What is the optimal solution to the dual? Make sure to state the objective value and the value of all dual variables.

| z | x_1 | x_2 | e_1 | s_2 | a_1 | a_3 | RHS |
|-----|-------|-------|-------|-------|-------|-------|-------|
| 1 | 0 | 0 | 0 | 1/5 | M | M-7/5 | -18/5 |
| 0 | 0 | 1 | 0 | 3/5 | 0 | -1/5 | 6/5 |
| 0 | 1 | 0 | 0 | -1/5 | 0 | 2/5 | 3/5 |
| 0 | 0 | 0 | 1 | 1 | -1 | 1 | 0 |

5. (15 points) I am planning a trip to Disneyworld and will buy a 5 day ticket. Each day I will go to a single park, and I want to visit all 5 parks. The average wait in lines at each park depends on the day of the week, so I want to plan my visit to minimize my average wait time. Solve the problem as an assignment problem. Make sure to state which day is spent at each park.

| | Monday | Tuesday | Wednesday | Thursday | Friday |
|-------------------|--------|---------|-----------|----------|--------|
| Magic Kingdom | 24 | 33 | 30 | 35 | 37 |
| Epcot | 18 | 32 | 36 | 20 | 27 |
| Hollywood Studios | 30 | 40 | 27 | 25 | 27 |
| Animal Kingdom | 26 | 31 | 35 | 25 | 15 |
| Typhoon Lagoon | 24 | 25 | 34 | 28 | 32 |

6. (20 points) Tiffany is planning her summer vacation, in which she will visit several different exciting cities. She is trying to choose which cities to visit. The table below shows the number of days she feels she must devote to each city, the cost of visiting the city and the “enjoyment” she will get out of visiting the city:

| city | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---------------|-------|-------|-------|-------|-------|-------|------|
| enjoyment | 17 | 10 | 15 | 19 | 7 | 13 | 9 |
| days to visit | 4 | 1 | 3 | 2 | 2 | 1 | 4 |
| cost | \$170 | \$210 | \$320 | \$240 | \$500 | \$120 | \$90 |

Cities 2 and 3 are mutually exclusive (she should not visit both). Cities 2,4,5 each have terrific zoo, so Tiffany insists on visiting at least one of them. City 6 is on her way to city 7 so if she visits city 7 she should also visit city 6.

Tiffany wishes to maximize her enjoyment, however she has only 10 days and \$750 to spend. Formulate an integer programming problem. (Do NOT solve - just formulate!)

(a). Define the variables:

(b). What is the objective function? (Max or Min?)

(c). What are the constraints?

7. (15 points) My family is planning a vacation in Disneyworld. We have a total of 4 days, and we are trying to decide how many days to spend in the Magic Kingdom, Epcot, Hollywood studios and Animal Kingdom. We have decided to spend at most 2 days in any one park, and we do not have to visit all 4 parks. We estimate the “fun value” we would get for spending some days at each park, and want to maximize our total fun:

| | 0 days | 1 day | 2 days |
|-------------------|--------|-------|--------|
| Magic Kingdom | 0 | 3 | 5 |
| Epcot | 0 | 1 | 2 |
| Hollywood studios | 0 | 3 | 4 |
| Animal Kingdom | 0 | 2 | 2 |

To solve the problem using Dynamic Programming define $f_i(s)$ = the maximum fun achievable in stages i and above and state s .

Solve the problem. Make sure to state how many days are spent at each park. (A solution by guessing will get no credit, I want to see your computations using $f_i(s)$ with the stages and states you defined.)