AMS 341 (Fall, 2016) Exam 2 - Solution notes Estie Arkin

Mean 68.9, median 71, top quartile 82, bottom quartile 58, high 100 (3 of them!), low 14.

1. (10 points) Find the dual of the following LP: Min $z = x_1 - 2x_2 + 5x_3 + x_4$

$$\begin{array}{rll} \text{subject to} & x_1 + 3x_2 + 2x_3 - x_4 & \leq 15 \\ & 2x_2 - x_3 + x_4 & \geq 5 \\ & 2x_1 + x_2 - 5x_3 & = 10 \\ & x_1, x_2, x_3 \geq 0, & x_4 \text{ unrestricted} \end{array}$$

$$\begin{array}{lll} \max & z = 15y_1 + 5y_2 + 10y_3 & \\ \text{s.t.} & y_1 + 2y_3 & \leq 1 \\ & 3y_1 + 2y_2 + y_3 & \leq -2 \\ & 2y_1 - y_2 - 5y_3 & \leq 5 \\ & -y_1 + y_2 & = 1 \\ & y_1 & \leq 0 \\ & y_2 & \geq 0 \\ & y_3 & \text{unrestricted} \end{array}$$

2. (5 points) A minimization LP is being solved by the big M method. e_1 is the excess variable in constraint 1 and $a_1 a_2$ are the artificial variables of constraints 1,2 respectively. The optimal is given below:

z	x_1	x_2	e_1	a_1	a_2	RHS
1	0	$(6-M)/3$	$-M$	0	$(3-5M)/3$	$10M/3+4$
0	0	-1/3	-1	1	-2/3	10/3
0	1	2/3	0	0	1/3	4/3

Which one of the following statements is true: (Circle one)

The original LP has no feasible solution.

3. (15 points) A company manufactures and sells dog food of two types. Each bag of type 1 dog food contains 2 pds of lamb and 4 pds of turkey, and sells for \$6. Each bag of type 2 dog food contains 1 pd of turkey and 1 pd of lamb, and sells for \$2. A total of 30 pds of lamb and 50 pds of turkey are available. The company manager requires that at most 20 bags of dog food 2 are produced. Let $D1, D2$ be the number of bags of dog food type 1,2 produced.

$$\begin{array}{lll} \max & 6D_1 + 2D_2 & \\ \text{s.t.} & 2D_1 + D_2 & \leq 30 \\ & 4D_1 + D_2 & \leq 50 \\ & D_2 & \leq 20 \\ & D_1,\ D_2 & \geq 0 \end{array}$$

The optimal solution is $D_1 = 10$, $D_2 = 10$. Answer the following using *graphical* sensitivity analysis. Graph the LP and show your work!

(a). Suppose the price of sale price of food type 1 is subject to change. For what range of prices does the current optimal solution remain optimal?

$-4 \leq -c_1/2 \leq -2$ so $8 \geq c_1 \geq 4$.

(b). What is the range of values of the third right hand side (b_3) for which the current BFS remains optimal?

$10 \leq b_3$

(c). What is the most that the company should be willing to pay for another pound of turkey?

Solve $2D_1 + D_2 = 30$ and $4D_1 + D_2 = 50 + \delta$, get $D_=10 + \delta/2$ and $D_2 = 10 - \delta$, plug into $z = 80 + \delta$ so shadow price is 1.

4. (24 points, 3 points for each part) Answer TRUE or FALSE:

True Every balanced transportation problem has a feasible solution.

True The big M method can end with an unbounded objective.

False At the end of phase 1, if $w = 0$ then all artificial variables must be non basic.

False The basic feasible solutions to a balanced transportation problem are always non degenerate.

False A primal problem (P) and its its dual (D) must have the same number of variables.

True If a primal problem (P) is unbounded, then its dual (D) must be infeasible.

False An LP with degenerate Basic Feasible Solutions may have an infinite number of Basic Feasible Solutions (BFS).

False The cost of a BFS found by Vogel's method (for a minimization BTP) is always $\leq$ the cost of the BFS found by Northwest Corner method.

5. (6 points) Suppose an LP is solved twice. Once using the Big M method and a second time using the 2 phase method. Is it possible that the optimal solution to the big M method has all artificial variables equal to zero, and that the optimal solution to the 2 phase method $w > 0$? NO. If the solution to the big M problem has all artificial variables equal to zero then we know the original LP is feasible. If phase 1 ends with $w > 0$ we know that the original LP is not feasible. These can not both be true at the same time.

6. (20 points) A company produces widgets at 2 factories, A and B. They have orders from 3 customers for May and June as in the table below.

	company 1	company 2	company 3
May	200	310	400
June	450	520	350

The per unit shipping costs are:

	company 1	company 2	company 3
A	\$40	\$28	\$32
B	\$36	\$38	\$24

The production capacity at each factory is 400 a month. The company has 200 widgets in inventory at factory A and 250 at facotry B. Widgets can be delivered early but not late. In case it cannot meet demand, it has a contract to buy widgets from another company at a cost of \$700 per widget (cost includes delivery).

Formulate a Balanced Transportation Problem to minimize the costs by giving the cost and requirement table.

	co 1 May	co 2 May	co 3 May	co 1 June	co 2 June	co 3 June	supply
A inv	40	28	32	40	28	32	200
A May	40	28	32	40	28	32	400
A June	M	M	M	40	28	32	400
B inv	36	38	24	36	38	24	250
B May	36	38	24	36	38	24	400
B June	M	M	M	36	38	24	400
buy	700	700	700	700	700	700	180
demand	200	310	400	450	520	350	

Common mistakes: Having one supply per factory (ignoring the months), one demand for companies (ignoring months), setting up 2 separate BTPs one for each month (no possible, since we do not know how much inventory will carry over).

7. (12 points) A company produces and sells chairs and desks. Each chair requires 3 board feet of lumber and 2 hours of labor. Each desk requires 5 board feet of lumber and 4 hours of labor. A total of 145 board feet of lumber and 90 hours of labor are availble. Upto 50 chairsrs and 50 desks can be sold. Chairs sell for \$55, and desks for \$32. In addition to producing chairs and desks itself, the company can buy (from an outside supplier) extra chairs at \$27 each and extra desks at \$50 each. Let CM, DM be the number of chairs and desks made by the company, and CB, DB the number of chairrs and desks bought from the supplier. Use the Lindo output below to answer each of the following parts.

$$
\begin{array}{llll}
\max & 32CM + 55DM + 5CB + 5DB & \\
\text{s.t.}\ 2) & 3CM + 5DM & \leq 145 \\
3) & 2CM + 4DM & \leq 90 \\
4) & CM + CB & \leq 50 \\
5) & DM + DB & \leq 50
\end{array}
$$

objective function value 1715.00000

variable	value	reduced cost
CM	45.000000	.000000
DM	.000000	4.000000
CB	5.000000	.000000
DB	50.000000	.000000

row	slack or surplus	dual prices
2)	10.000000	0.000000
3)	.000000	13.500000
4)	.000000	5.000000
5)	.000000	5.000000

Range in which basis remains unchanged :

OBJ coefficient ranges

variable	current coef	allowable increase	allowable decrease
CM	32.000000	infinity	2.00000
DM	55.000000	4.00000	infinity
CB	5.00000	2.00000	5.00000
DB	5.00000	infinity	4.00000

righthand side ranges

row	current RHS	allowable increase	allowable decrease
2	145.00000	infinity	10.000000
3	90.00000	6.66667	90.00000
4	50.00000	infinity	5.00000
5	50.00000	infinity	50.00000

(a). If the company can purchase desks for \$48, what would be the new optimal profit? Profit from DB increases from 5 to 7, an increase of $2 \leq$ allowable increase ∞ yes. Same BFS is optimal. new $z = 1715 + 2 \cdot 50 = 1815$.
(b). What is the most that the company should be willing to pay to for another board foot of lumber? 0 dual price of the lumber constraint.
(c). If only 40 desks could be sold, what would be the new optimal solution (the z)? Deacrease $10 \leq$ the allowable decrease 50, so same BFS will be optimal. new $z =$ old $z + (-10)5 = 1715 - 50 = 1665$.
8. (8 points) Consider the following (minimum) Balanced Transportation problem: Find an initial BFS for the problem using the min cost method:

4	3	3	3 100	100
3 0	1 50	2 50	3	100
6 100	55	5	5	100
7 50	5	5	4 50	100
150	50	50	150	