Mean 77.6, median 81, high 99 (2 of them!), low 34.

1. (20 points) Consider the following LP:

$$\begin{aligned}
\max \quad & z = 2x_1 - x_2 + x_3 \\
\text{s.t.} \quad & x_1 + 2x_2 - x_3 \geq 1 \\
& 3x_1 - 2x_2 + x_3 \leq 20 \\
& x_1, x_2 \geq 0 \\
& x_3 \quad \text{unrestricted}
\end{aligned}$$

(a). Rewrite the LP in standard form.

$$\begin{aligned}
\max \quad & z = 2x_1 - x_2 + x_3' - x_3'' \\
\text{s.t.} \quad & x_1 + 2x_2 - x_3' + x_3'' - e_1 = 1 \\
& 3x_1 - 2x_2 + x_3' - x_3'' + s_2 = 20 \\
& x_1, x_2, x_3', x_3'', e_1, s_2 \geq 0
\end{aligned}$$

Common mistake: keeping x_3 unrestricted.

(b). What is the dual of the given LP? (You can either state the dual of the original problem or the dual of its standard form.)

$$\begin{aligned}
\min \quad & z = y_1 + 20y_2 \\
\text{s.t.} \quad & y_1 + 3y_2 \geq 2 \\
& 2y_1 - 2y_2 \geq -1 \\
& -y_1 + y_2 = 1 \\
& y_1 \leq 0 \\
& y_2 \geq 0
\end{aligned}$$

2. (20 points) Each day Eastinghouse produces capacitors during three shifts: 8am-4pm, 4pm-midnight, midnight-8am. The hourly salary paid to the employees of each shift, the number of capacitors produced by a worker during the shift, the price charged for each capacitor made during each shift, and the number of defects in each capacitor produced during a shift are given in the table below. Each of the company's 25 workers can be assigned to one of the three shifts. Because of machinary limitations, no more than 10 workers can be assigned to each shift. Each day, at most 250 capacitors can be sold, and the average number of defects per capacitor for the day's production cannot exceed three. Formulate an LP to maximize Eastinghouse's daily profit (sales revenue minus labour cost). (Your formulation does NOT have to be put into standard form. Do NOT solve, just formulate!)

(a). Define the variables you are using in the formulation. x_i number of workers assigned to shift i (8-4am is shift 1 etc).

(b). The objective function is: Max $z = (18 \cdot 10 - 12 \cdot 8)x_1 + (22 \cdot 9 - 16 \cdot 8)x_2 + (24 \cdot 12 - 20 \cdot 8)x_3$

(c). The constraints are:

$$x_1 \leq 10$$

$$x_2 \leq 10$$
$$x_3 \leq 10$$
$$x_1 + x_2 + x_3 \leq 25$$
$$10x_1 + 9x_2 + 12x_3 \leq 250$$
$$4 \cdot 10x1 + 3 \cdot 9x_2 + 2 \cdot 12x_3 \leq 3(10x_1 + 9x_2 + 12x_3)$$
$$x_1, x_2, x_3 \geq 0$$

Common mistakes: defining also variables for number of capacitators produced and number of defects, but not connecting all variables.

3. (25 points) Consider the feasible region given by the following constraints: (It may be helpful to sketch it and/or put it into standard form.)

$$
\begin{aligned}
x_1 + x_2 \;\; &\leq 4 & (1)\\
x_1 \;\; &\geq 1 & (2)\\
x_2 \;\; &\leq 3 & (3)\\
x_1 \;\; &\geq 0 & (4)\\
x_2 \;\; &\geq 0 & (5)
\end{aligned}
$$

In standard form:

$$
\begin{aligned}
x_1 + x_2 + s_1 \;\; &= 4\\
x_1 - e_2 \;\; &= 1\\
x_2 + s_3 \;\; &= 3\\
x_1, x_2, s_1, e_2, s_3 \;\; &\geq 0
\end{aligned}
$$

(a). Is the point $x_1 = 0$, $x_2 = 0$ a feasible point? No, it does not satusfy the second constraint. Is it a basic solution? Yes. Let $x_1 = x_2 = 0$ be the 2 non basic variables, so $s_1 = 4, e_2 = -1, s_3 = 3$ are the basic variables.

Common mistake: saying that because it is not a feasible solution it cannot be basic. This is not true!

(b). Is the point $x_1 = 2$, $x_2 = 2$ a feasible point? Yes, all 5 constraints hold. Is it a basic solution? No. For $x_1 = x_2 = 2$ we get $s_1 = 0$, $e_2 = s_3 = 1$, so there are 4 basic variables (which are not equal to zero) and there should be only 3.

(c). Is there an objective function for which an LP with these constraints is unbounded? If so, give such an objective function. If not, explain (briefly!) why not. No, since the feasible region is bounded.

(d). Consider the objective function max $z = x_1 - x_2$. Let s_1, e_2, s_3 be the slack and excess variables of the constraints. Here is the first tableau of phase II the LP. Pivot until you find the optimal solution. Make sure to state which variable enters the basis and which leaves at each iteration. (If you have to do more than 2 pivots, you are doing something wrong!)

z	x_1	x_2	s_1	e_2	s_3	RHS
1	0	1	0	-1	0	1
0	0	1	1	1	0	3
0	1	0	0	-1	0	1
0	0	1	0	0	1	3

e_2 enters s_1 leaves

z	x_1	x_2	s_1	e_2	s_3	RHS
1	0	2	1	0	0	4
0	0	1	1	1	0	3
0	1	1	1	0	0	4
0	0	1	0	0	1	3

4. (20 points) A bank is attempting to determine where its assets should be invested during the current year. At present $500,000 is available for investment in bonds, home loans, auto loans, and personal loans. The annual return on each type of investement is known to be: bonds 10%; home loans 16%; auto loans 13%; personal loans 20%. To ensure that the bank's portfolio is not too risky, the following three restrictions are placed:

(1) The amount invested in personal loans cannot exceed the amount invested in bonds.

(2) The amount invested in home loans cannot exceed the amount invested in auto loans.

(3) No more than 25% of the total amount invested may be in personal loans.

The bank's objective is to maximize the annual rate of return on its investment. Let B, H, A, P be the amounts invested in bonds, home loans, auto loans and personal loans respecively. Use the Lindo output below to answer each of the following parts, or say that the answer is unkown using the given Lindo output.

$$\text{max} \quad 0.1B + 0.16H + 0.13A + 0.2P$$

$$
\begin{aligned}
\text{s.t.} \quad 2) & \quad B + H + A + P && \leq 500000 \\
3) & \quad P - B && \leq 0 \\
4) & \quad H - A && \leq 0 \\
5) & \quad -0.25B - 0.25H - 0.25A + 0.75P && \leq 0
\end{aligned}
$$

objective function value   73750

variable	value	reduced cost
B	125000	.000000
H	125000	.000000
A	125000	.000000
P	125000	.000000

row	slack or surplus	dual prices
2)	0.000000	0.1475
3)	0.000000	0.0450
4)	0.000000	0.0150
5)	0.000000	0.0100

Range in which basis remains unchanged :

objective coefficient ranges

variable	current coef	allowable increase	allowable decrease
B	0.10	0.045	0.01
H	0.16	0.010	0.03
A	0.13	0.010	0.09
P	0.20	infinity	0.01

righthand side ranges

row	current RHS	allowable increase	allowable decrease
2	500000	infinity	500000
3	0	125000	250000
4	0	250000	250000
5	0	125000	125000

(a). What would be the profit if only \$400,000 can be invested? We decrease the RHS of the first constraint by 100000 which is less than the allowable decrease of 500000, so within range. new $z = 73750 - 0.1475 \cdot 100000 = 59000$.

Common mistake: forgetting to show the range check!

(b). What would be the profit if the interest on home loans is 14% (instead of 16%)? We decrease the cost coefficient of H by 0.02 which is less than the allowable decrease of 0.03, so within range. new $z = 73750 - 0.02 \cdot 125000 = 71250$

(c). What would be the profit if the interest on home loans is 14% (instead of 16%) and only \$400,000 can be invested? The suggested change is to an objective coefficient and RHS, a multiple change, we cannot tell what the new optimal solution will be.

Common mistake: using the 100% rule. It applies only when all changes are of the same type.

(d). What would be the profit if the interest on home loans is 12% (instead of 16%)? We decrease the cost coefficient of H by 0.04 which is greater than the allowable decrease of 0.03, so out of range. A new BFS will be optimal, we need to rerun Lindo.

5. (15 points) The tableau below is for Phase I of the Two Phase Method. a_1 and a_3 are the artificial variables.

w	x_1	a_1	s_2	x_2	x_3	a_3	e_3	RHS
1	0	0	0	-3	0	-2	0	0
0	0	1	1	1	0.5	1	0	2
0	1	2	0	0	0	-2	0	4
0	0	0	0	1	2	1	1	0

(a). At this tableau, we have: $w = 0$, $x_1 = 4$, $a_1 = 0$, $s_2 = 2$, $x_2 = 0$, $x_3 = 0$, $a_3 = 0$, $e_3 = 0$

(b). The basic variables for this tableau are: w, x_1, s_2, e_3

(c). The tableau shows an optimal solution to the Phase I LP. Is the original LP feasible? Yes, since $w = a_1 = a_3 = 0$.

(d). For this part, assume that the original LP had objective function $\max z = x_1 + x_2$. What is the first tableau for Phase II (after "clean-up" if needed).

z	x_1	s_2	x_2	x_3	e_3	RHS
1	-1	0	-1	0	0	0
0	0	1	1	0.5	0	2
0	1	0	0	0	0	4
0	0	0	1	2	1	0

We need to clean-up the objective function to make it canonical, so add the second constraint (third row) from the objective function.

z	x_1	s_2	x_2	x_3	e_3	RHS
1	0	0	-1	0	0	4
0	0	1	1	0.5	0	2
0	1	0	0	0	0	4
0	0	0	1	2	1	0

Common mistake: Not dropping the artificial variables from the tableau, or instead dropping some original variables, forgetting to get the new tableau into canonical form, cleaning-up also x_2 which is not basic, and in the process messing up the other basic variables s_2, e_3.

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