AMS 341 (Fall, 2016) Estie Arkin

Operations Research I: Deterministic Models

Exam 1: Thursday, September 29, 2016

READ THESE INSTRUCTIONS CAREFULLY. Do not start the exam until told to do so. Make certain that you have all 7 pages of the exam. You will be held responsible for any missing pages.

Write your answers on this examination, using the backs of pages if needed. (Use back of pages also for scratch paper if you need it.)

This examination is CLOSED BOOK and CLOSED NOTES. You may not use any books, papers, or materials other than your pen or pencil. You may use a 4 by 6 "cheat sheet", which should be turned in with your exam.

The following items should NOT be on your desk - put them INSIDE your bag!

- calculator
- cell phone

If I see any of these items, even turned off, this will be considered cheating!!!

Work carefully, and GOOD LUCK!!!

Last (Family) Name (PRINT CLEARLY): ______________________________

First Name (PRINT CLEARLY): ______________________________

ID Number: ______________________________

Academic integrity is expected of all students at all times, whether in the presence or absence of members of the faculty. Understanding this, I declare that I shall not give, use, or receive unauthorized aid in this examination. I have been warned that any suspected instance of academic dishonesty will be reported to the Academic Judiciary and that I will be subjected to the maximum possible penalty permitted under University guidelines.

Signature:

____

1. (15 points) To supplement my income I plan to back some cakes to sell at the Farmer's Market. Each chocolate cake sells for \$10 and each vanilla cake for \$7.50. A chocolate cakes requires 4 egg and 40 minutes of baking time. Each vanilla cake requires 1 egg and 50 minutes of baking time. Assume at most 1 cake fits in my oven at any time. I currently have 30 eggs and 8 hours of baking time.

Assuming that I can sell all cakes that I bring to the Farmer's Market, formulate an LP to maximize my profit:

(a). Define the variables you are using in the formulation.

(b). The objective function is:

(c). The constraints are:

2. (12 points) The following is a tableau for an LP which is a MAX problem:

z	s_1	x_1	x_2	e_3	x_3	RHS	ratio
1	0	2	-1	0	0	12	
0	0	0	1	0	1	4	
0	0	2	0	1	0	3	
0	1	1	-3	0	0	6	

(a). What are the basic variables, and what are they equal to?

(b). What are the non-basic variables, and what are they equal to?

(c). This is not an optimal BFS. Which variable should be selected to enter the basis?

(d). Do one pivot. What is the next tableau?

z	s_1	x_1	x_2	e_3	x_3	RHS

3. (20 points)(a) Consider the feasible region given by the following constraints: Sketch the feasible region.

$$
\begin{aligned}
x_1 &\le 5 && (1)\\
x_2 &\le 4 && (2)\\
x_1 + x_2 &\le 8 && (3)\\
x_1 + x_2 &\ge 2 && (4)\\
x_1 &\ge 0 && (5)\\
x_2 &\ge 0 && (6)
\end{aligned}
$$

In parts (b), (c), (d) circle the correct answers:
(b). Is the point $x_1 = 4$, $x_2 = 0$ a feasible point? YES NO
Is it a basic solution? YES NO

(c). Is the point $x_1 = 5$, $x_2 = 0$ a feasible point? YES NO
Is it a basic solution? YES NO

(d). Is the point $x_1 = 0$, $x_2 = 0$ a feasible point? YES NO
Is it a basic solution? YES NO

(e). How many feasible solutions does the LP have?

(f). How many basic feasible solutions does the LP have?

4. (10 points) Consider the following LP:

$$
\begin{array}{lcl}
\min & z = 2x - y + 2w & \\
\text{s.t.} & x + y & \geq 10 \\
& -x - y + 3w & = -2 \\
& x & \leq 0 \\
& y & \geq 0 \\
& w & \text{unrestricted}
\end{array}
$$

Rewrite the LP in standard form.

5. (5 points) A maximization LP is being solved by the Simplex method. Here is the current tableau:

z	x_1	x_2	s_1	e_2	x_3	x_4	RHS
1	0	3	0	0	2	3	10
0	1	1	0	0	0	0	3
0	0	2	1	0	-1	0	3
0	0	1	0	1	0	2	2

Which one of the following statements is true: (Circle one)

(i). This is an optimal tableau, and the LP has a unique optimal solution.

(ii). This is an optimal tableau but the LP has multiple optimal solutions.

(iii). This is an optimal tableau and the LP is unbounded.

(iv). This is not an optimal tableau for the LP.

6. (20 points) Steelco manufacter steel by combining Alloy 1 and Alloy 2. The steel must meet the following requirements: 3.2-3.5% carbon (i.e., at least 3.2 but no more than 3.5 percent); 1.8-2.5% silicon; 0.9-1.2% nickel; tensile strength of at least 45,000 pounds per square inch (psi). Assume that the tensile strength of a mixture of the two alloys is determined by averaging that of the two alloys mixed. For example a one ton mixture that is %40 Alloy 1 and %60 Alloy 2 has tensile strength of $0.4(42,000) + 0.6(50,000)$. The cost and properties of the alloys are given below:

	Cost per ton	percent silicon	percent nickel	percent carbon	tensile strength (psi)
Alloy 1	\$ 190	2	1	3	42,000
Alloy 2	\$ 200	2.5	1.5	4	50,000

Formulate an LP to minimize the cost of producing one ton of steel.

(a). Define the variables you are using in the formulation.

(b). The objective function is:

(c). The constraints are:

7. (18 points, 3 points for each part) Answer TRUE or FALSE:

(a). __________ All optimal solutions to an LP must be basic (a BFS).

(b). __________ Any BFS can be an optimal solution to an LP depending on the choice of objective function.

(c). __________ Two different BFS (to the same LP) must have the same number of non basic variables.

(d). __________ At the end of the Simplex method, all slack variables must equal zero.

(e). __________ There is only one correct way to formulate an LP.

(f). __________ An LP in cannonical form is also in standard form.