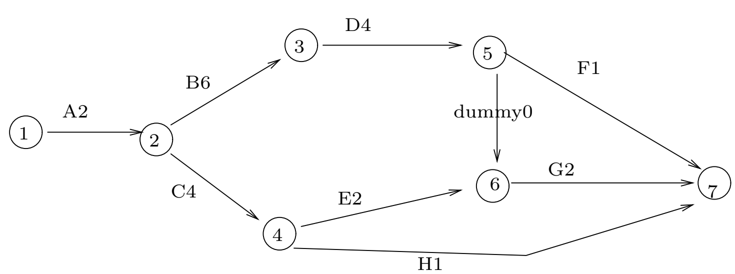
AMS 341 (Spring, 2009) Exam 2 - Solution notes Estie Arkin

Mean 68.56, median 70, high 99, low 13.

1. (12 points) A frazzled student is trying to plan all the work they must complete before graduating. Each of the arrows in the diagram below corresponds to a task to be done. The student carefully drew the project network shown below. The numbers by each arrow indicate the time required for that task (in hours). The letter refers to the name of the task.

Activity	Predecessors	Time (months)
A	-	2
B	A	6
C	A	4
D	B	4
E	C	2
F	B,D	1
G	D,E	2
H	C	1

(a). Draw a project network.



(b). What is the critical path for this student? You may find the path either by computing the total float for each node, or by inspection. (Your answer should be a list all critical activities.)

	1	2	3	4	5	6	7
ET	0	2	8	6	12	12	14
LT	0	2	8	10	12	12	14

Critical nodes are 1-2-3-5-6-7, critical activities (path) A-B-D-dummy-G.

(c). What is the total float of task E? $TF(E) = LT(6) - ET(4) - 2 = 12 - 6 - 2 = 4$.

2. (10 points) We wish to solve an integer programming problem. All variables are restricted to be integer. We began by solving the LP relaxation of the problem and got the final (optimal) tableau for it. Unfortunately, not all the variables are integer.

z	x_1	x_2	x_3	x_4	x_5	RHS
1	0	-1.5	-3	0	0	6.25
0	0	3	0	1	0	10
0	1	2	5	0	0	5
0	0	1.2	-2.6	0	1	5.4

To solve the problem using the cutting plane method, what cut (constraint) would you add? Note: Do not solve the problem, just state the added constraint. $0.4 - 0.2x_2 - 0.4x_3 \leq 0$ which can be rewritten as $x_2 + 2x_3 \geq 2$.

3. (10 points) Consider the following Linear Programming problem:

$$\begin{array}{lll} \max & z = 3x_1 + x_2 - x_3 & \\ \text{s.t.} & 2x_1 + x_2 + x_3 & \leq 8 \\ & 4x_1 + x_2 - x_3 & \leq 10 \\ & x_1, x_2, x_2 & \geq 0 \end{array}$$

The final tableau for the given LP is given below. s_1, s_2 are the slack variable of the constraints What is the optimal solution to the dual? Make sure to state the objective value and the value of all dual variables.

z	x_1	x_2	x_3	s_1	s_2	RHS
1	0	0	1	1/2	1/2	9
0	0	1	3	2	-1	6
0	1	0	-1	-1/2	1/2	1

$y_1 = y_2 = 1/2$, $w = 9$.

4. (15 points) A large baking company bakes cakes at 3 different bakeries. The cost of baking 1 cake at bakery 1 is \$6, at bakery 2 \$4, and at bakery 3 \$5. Each bakery can bake up to 100 cakes. The company then sends these cakes to 3 stores. Store 1 demands least 75 cakes, store 2 at least 120 cakes and store 3 at least 80 cakes. The shipping costs per cake from bakeries to stores are given in the table below: (The "-" means that bakery 1 cannot ship to store 1.)

	store 1	store 2	store 3
bakery 1	-	1.5	3
bakery 2	2.5	4	5
bakery 3	1	2	3

The company's goal is to minimize the total cost. Formulate the problem as a Balanced Transportation problem by giving the the transportation tableau (cost and requirement matrix).

	store 1	store 2	store 3	dummy	supply
bakery 1	M	6+1.5	6+3	0	100
bakery 2	4+2.5	4+4	4+5	0	100
bakery 3	5+1	5+2	5+3	0	100
demand	75	120	80	25	

Common mistakes: not adding in baking costs, not balancing, not making cost big M.

5. (15 points) The AMS department is scheduling faculty to teach its courses next semester. Professors A,B, and C will each be teaching at most one course, and professors D and E will each be teaching at most 2 courses. There are 5 courses to be taught. Based on surveys from previous years, the department knows how succesful each professor is at teaching each course. The data is given in the table below (small numbers are better!) The "-" means that professor C cannot teach course 1.

	Prof A	Prof B	Prof C	Prof D	Prof E
course 1	4	3	-	5	7
course 2	1	3	6	1	7
course 3	3	4	7	2	7
course 4	6	1	5	5	5
course 5	4	5	4	4	3

The department's goal is to assign professors to courses such that the teaching next semester is as "succesful" as possible. Formulate the problem as an Assignment problem by giving the cost matrix. Note: You are asked to formulate an assignment problem, not a Balanced Transportation Problem and not a Linear Program. Do not solve the problem you formulated.

	Prof A	Prof B	Prof C	Prof D	prof D'	Prof E	Prof E'
course 1	4	3	M	5	5	7	7
course 2	1	3	6	1	1	7	7
course 3	3	4	7	2	2	7	7
course 4	6	1	5	5	5	5	5
course 5	4	5	4	4	4	3	3
dummy	0	0	0	0	0	0	0
dummy	0	0	0	0	0	0	0

Common mistakes: formulating as a Balanced TP (supplies and demand not 1), ignoring the fact that professors D,E can teach 2 courses each, solving some other assignment problem, no M.

6. (20 points) A consulting company has 10 employees, each of whom can work on at most two team projects. Six projects are under consideration. Each project requires 4 (specific) employees. The required employees and the revenue of each project is given in the table below:

	Project 1	Project 2	Project 3	Project 4	Project 5	Project 6
Required Employees	1,4,5,8	2,3,7,10	1,6,8,9	2,3,5,10	1,6,7,9	2,4,8,10
Revenue($)	10,000	15,000	6,000	8,000	12,000	9,000

Each worker who is used on any project must be paid the retainer in the table below:

Worker	1	2	3	4	5	6	7	8	9	10
Retainer($)	800	500	600	700	800	600	400	500	400	500

Formulate an integer programming problem to maximize the company's profit. (Do NOT solve - just formulate!)

(a). Define the variables: $x_i = 1$ if worker i is used, $x_i = 0$ if worker i is not used. Similarly, $y_j = 1$ if project j is undertaken, and $y_j = 0$ otherwise.

(b). What is the objective function? (Max or Min?)

$$\begin{aligned} \max \quad & 10,000y_1 + 15,000y_2 + 6,000y_3 + 8,000y_4 + 12,000y_5 + 9,000y_6 - 800x_1 \\ & -500x_2 - 600x_3 - 700x_4 - 800x_5 - 600x_6 - 400x_7 - 500x_8 - 400x_9 - 500x_{10} \end{aligned}$$

(c). What are the constraints? There are 2 types of constraints in addition to the variables being binary). First, each worker can work on at most 2 projects. So for each worker, we add up the projects that the worker is required for:

$$
\begin{aligned}
y_1 + y_3 + y_5 &\leq 2 \\
y_2 + y_4 + y_6 &\leq 2 \\
y_2 + y_4 &\leq 2 \\
y_1 + y_6 &\leq 2 \\
y_1 + y_4 &\leq 2 \\
y_3 + y_5 &\leq 2 \\
y_2 + y_5 &\leq 2 \\
y_1 + y_3 + y_6 &\leq 2 \\
y_3 + y_5 &\leq 2 \\
y_2 + y_4 + y_6 &\leq 2
\end{aligned}
$$

The second type of constraint is to ensure that if a project j is undertaken, then all 4 workers required are "used". So if worker i is required for project j, we write a constrain $y_j \leq x_i$. This gives us $4 \cdot 6$ constraints, such as $y_1 \leq x_1$. Or we can use the following 6 constraints:

$$
\begin{aligned}
4y_1 &\leq x_1 + x_4 + x_5 + x_8 \\
4y_2 &\leq x_2 + x_3 + x_7 + x_{10} \\
4y_3 &\leq x_1 + x_6 + x_8 + x_9 \\
4y_4 &\leq x_2 + x_3 + x_5 + x_{10} \\
4y_5 &\leq x_1 + x_6 + x_7 + x_9 \\
4y_6 &\leq x_2 + x_4 + x_8 + x_{10}
\end{aligned}
$$

Finally x_i, y_j are all binary variables.

Common mistakes: Paying workers retainer for *each* project instead of once per worker if they are used. Defining x_{ij} if a worker i works on project j.

7. (18 points) A family is planning a summer vacation in Italy. It has a total of 5 days, and it is trying to decide how many days to spend in Rome, Florence and Venice. It estimates the enjoyment it would get for spending some days in each city, and wants to maximize its total enjoyment:

	0 days	1 day	2 days	3 days	4 days and above
Rome	0	3	5	6	7
Florence	0	1	2	6	6
Venice	0	4	5	5	6

To solve the problem using Dynamic Programming define $f_i(s)$ = the maximum enjoyment achievable in stages i and above and state s.

Solve the problem. Make sure to state at the end how many days are spent in each city. (A solution by guessing will get no credit, I want to see your computations using $f_i(s)$ with the stages and states you defined.) To solve the problem using Dynamic Programming define $f_i(s)$ = the maximum enjoyment achievable in cities i and above with s days.

$f_3(0) = 0,\ f_3(1) = 4,\ f_3(2) = 5, f_3(3) = 5,\ f_3(4) = f_3(5) = 6,\ f_2(0) = 0,\ f_2(1) = \max\{0 + 1, 4 + 0\} = 4,\ f_2(2) = \max\{0 + 2, 4 + 1, 5 + 0\} = 5,\ f_2(3) = \max\{0 + 6, 4 + 2, 5 + 1, 5 + 0\} = 6$, $f_2(4) = \max\{0+6, 4+6, 5+2, 5+1, 6+0\} = 10$, $f_2(5) = \max\{0+6, 4+6, 5+6, 5+2, 6+1, 6+0\} = 11$,

$f_1(5) = \max\{0 + 7, 4 + 7, 5 + 6, 6 + 5, 10 + 3, 11 + 0\} = 13$,

1 day in Rome, 3 days in Florence and 1 day in Venice.

Common mistake: using only 4 days (calculating $f_1(4)$) instead of 5 days.