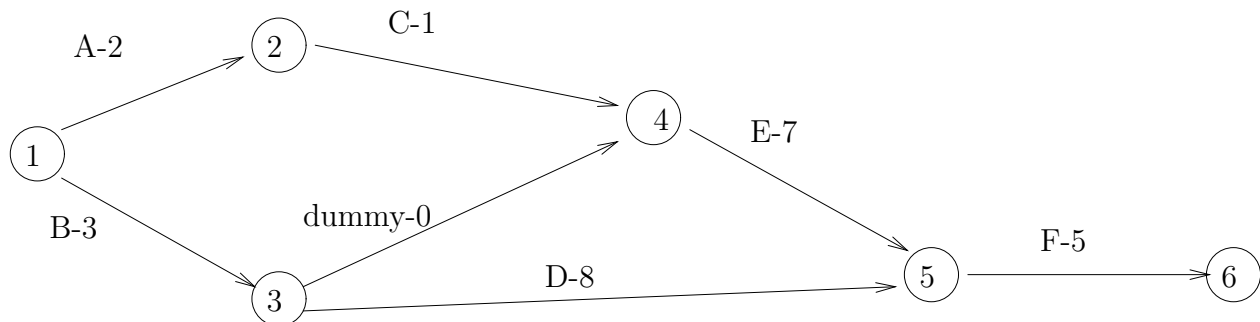


Mean 73.925, median 76, top quartile 87, high 99, low 12.

1. (10 points) I am planning a vacation once the spring semester is over. There are many tasks that have to be completed before I can go. Fortunately, I can get help from family and friends. The following are the tasks that would have to be undertaken before the vacation:

Task	Predecessors	Time (hours)
A	-	2
B	-	3
C	A	1
D	B	8
E	B,C	7
F	D,E	5

(a). Draw a project network.



Common mistakes: several start nodes, undirected edges, wrong predecessors.

(b). What is my critical path? You may find the path either by computing the total float for each node, or by inspection. (Your answer should be a list all critical activities.) Tasks B,D,F (or nodes 1,3,5,6).

2. (10 points) Consider the following (minimum) Balanced Transportation problem: Find an initial BFS for the problem using the min cost method:

75	9		3	25	4		1	100
0	6	75	1		5	25	0	100
	7		8	100	2		1	100
75		75		125		25		

Common mistake: not enough basic variables. You should have $4 + 3 - 1 = 6$.

3. (15 points) Plans are being made for the energy systems for a new building. The three possible sources of energy are electricity, natural gas and a solar heating unit. The energy requirements are: 20 units of electricity, 10 units for water heating and 30 units for space heating. The size of the roof limits the solar heater to 30 units. There is no limit on the amount of electricity or natural gas bought. Electricity needs can only be met by buying electricity at a cost of \$ 50 per unit. Other energy needs can be met by any sources with the following unit costs:

	Electricity	Natural Gas	Solar heater
Water heating	\$ 90	\$ 60	\$ 30
Space heating	\$ 80	\$ 50	\$ 40

Formulate a Balanced Transportation Problem to minimize the *total* cost of meeting the energy needs. Give your formulation in terms of a cost and requirement table.

	Electricity	Water heating	Space heating	dummy	supply
Electricity	\$ 50	\$ 90	\$ 80	\$ 0	60
Natural Gas	\$ M	\$ 60	\$ 50	\$ 0	40
Solar heater	\$ M	\$ 30	\$ 40	\$ 0	30
demand	20	10	30	70	

Common mistakes: Confusing Electricity supply (unlimited) and demand (20), having supply be infinite instead of giving a large number, such as 60, forgetting to balance the problem.

4. (15 points) Consider the following Linear Programming problem:

$$\begin{aligned}
 \max \quad & z = -4x_1 - x_2 \\
 \text{s.t.} \quad & 4x_1 + 3x_2 \geq 6 \\
 & x_1 + 2x_2 \leq 3 \\
 & 3x_1 + x_2 = 3 \\
 & x_1, x_2 \geq 0
 \end{aligned}$$

(a). What is the dual of the LP?

$$\begin{array}{ll}
 \min & w = 6y_1 + 3y_2 + 3y_3 \\
 \text{s.t.} & 4y_1 + y_2 + 3y_3 \geq -4 \\
 & 3y_1 + 2y_2 + y_3 \geq -1 \\
 & y_1 \leq 0, y_2 \geq 0, y_3 \text{ unrestricted}
 \end{array}$$

(b). The final tableau for the given LP is given below. e_1 is the excess variable of the first constraint, s_2 the slack variable of the second constraint, and a_1, a_3 the artificial variables of constraints 1,3. The final tableau was found using the big M method. What is the optimal solution to the dual? Make sure to state the objective value and the value of all dual variables.

z	x_1	x_2	e_1	s_2	a_1	a_3	RHS
1	0	0	0	1/5	M	M-7/5	-18/5
0	0	1	0	3/5	0	-1/5	6/5
0	1	0	0	-1/5	0	2/5	3/5
0	0	0	1	1	-1	1	0

$$w = -18/5, y_1 = 0, y_2 = 1/5, y_3 = -7/5.$$

5. (15 points) I am planning a trip to Disneyworld and will buy a 5 day ticket. Each day I will go to a single park, and I want to visit all 5 parks. The average wait in lines at each park depends on the day of the week, so I want to plan my visit to minimize my average wait time. Solve the problem as an assignment problem. Make sure to state which day is spent at each park.

	Monday	Tuesday	Wednesday	Thursday	Friday
Magic Kingdom	24	33	30	35	37
Epcot	18	32	36	20	27
Hollywood Studios	30	40	27	25	27
Animal Kingdom	26	31	35	25	15
Typhoon Lagoon	24	25	34	28	32

Subtract the smallest element from each row:

	Monday	Tuesday	Wednesday	Thursday	Friday
Magic Kingdom	0	9	6	11	13
Epcot	0	14	18	2	9
Hollywood Studios	5	15	2	0	2
Animal Kingdom	11	16	20	10	0
Typhoon Lagoon	0	1	10	4	8

Subtract the smallest element from each col:

	Monday	Tuesday	Wednesday	Thursday	Friday
Magic Kingdom	0	8	4	11	13
Epcot	0	13	16	2	9
Hollywood Studios	5	14	0	0	2
Animal Kingdom	11	15	18	10	0
Typhoon Lagoon	0	0	8	4	8

The zero's can be covered by 4 lines, for example rows 3,4,5 and col1. The smallest uncovered element is 2, we subtract 2 from uncovered elements and add 2 to elements that are double covered:

	Monday	Tuesday	Wednesday	Thursday	Friday
Magic Kingdom	0	6	2	9	11
Epcot	0	11	14	0	7
Hollywood Studios	7	14	0	0	2
Animal Kingdom	13	15	18	10	0
Typhoon Lagoon	2	0	8	4	8

Now we get 5 zero's each in a different row and col: Magic Kingdom-Monday, Epcot-Thursday, Hollywood Studios-Wednesday, Animal Kingdom-Friday and Typhoon Lagoon-Tuesday.

6. (20 points) Tiffany is planning her summer vacation, in which she will visit several different exciting cities. She is trying to choose which cities to visit. The table below shows the number of days she feels she must devote to each city, the cost of visiting the city and the "enjoyment" she will get out of visiting the city:

city	1	2	3	4	5	6	7
enjoyment	17	10	15	19	7	13	9
days to visit	4	1	3	2	2	1	4
cost	\$170	\$210	\$320	\$240	\$500	\$120	\$90

Cities 2 and 3 are mutually exclusive (she should not visit both). Cities 2,4,5 each have terrific zoo, so Tiffany insists on visiting at least one of them. City 6 is on her way to city 7 so if she visits city 7 she should also visit city 6.

Tiffany wishes to maximize her enjoyment, however she has only 10 days and \$750 to spend. Formulate an integer programming problem. (Do NOT solve - just formulate!)

(a). Define the variables: $x_i = 1$ if Tiffany goes to city i and 0 if not. Common mistake: Let x_i be city i .

(b). What is the objective function? (Max or Min?)

$$\max z = 17x_1 + 10x_2 + 15x_3 + 19x_4 + 7x_5 + 13x_6 + 9x_7$$

(c). What are the constraints?

$$\begin{aligned}
 4x_1 + 1x_2 + 3x_3 + 2x_4 + 2x_5 + 1x_6 + 4x_7 &\leq 10 \\
 170x_1 + 210x_2 + 320x_3 + 240x_4 + 50x_5 + 120x_6 + 90x_7 &\leq 750 \\
 x_2 + x_3 &\leq 1 \\
 x_2 + x_4 + x_5 &\geq 1 \\
 x_7 &\leq x_6 \\
 x_1, x_2, x_3, x_4, x_5, x_6, x_7 &\text{ binary}
 \end{aligned}$$

Common mistakes: forgetting to state variables are binary, $x_6 = x_7$, $x_2 \neq x_3$. 7. (15 points) My family is planning a vacation in Disneyworld. We have a total of 4 days, and we are trying to decide how many days to spend in the Magic Kingdom, Epcot, Hollywood studios and Animal Kingdom. We have decided to spend at most 2 days in any one park, and we do not have to visit all 4 parks. We estimate the "fun value" we would get for spending some days at each park, and want to maximize our total fun:

	0 days	1 day	2 days
Magic Kingdom	0	3	5
Epcot	0	1	2
Hollywood studios	0	3	4
Animal Kingdom	0	2	2

To solve the problem using Dynamic Programming define $f_i(s)$ = the maximum fun achievable in stages i and above and number of days s . We let Magic Kingdom be stage 1, Epcot stage 2 etc.

$f_4(0) = 0, f_4(1) = 2, f_4(2) = f_4(3) = f_4(4) = 2, f_3(0) = 0, f_3(1) = \max\{0 + 2, 3 + 0\} = 3,$
 $f_3(2) = \max\{0 + 2, 3 + 3, 4 + 0\} = 5, f_3(3) = f_3(4) = \max\{0 + 2, 3 + 2, 4 + 2\} = 6, f_2(0) = 0,$
 $f_2(1) = \max\{0 + 3, 1 + 0\} = 3, f_2(2) = \max\{0 + 5, 1 + 3, 2 + 0\} = 5, f_2(3) = \max\{0 + 6, 1 + 5, 2 + 3\} = 6,$
 $f_2(4) = \max\{0 + 6, 1 + 6, 2 + 5\} = 7,$

$f_1(4) = \max\{0 + 7, 3 + 6, 5 + 5, \} = 10,$

2 days in Magic Kingdom, 0 days in Epcot and 1 day each in Hollywood Studios and Animal Kingdom.

Common mistake: Calculating $f_1(2)$ instead of 4. Giving an answer with no (or incorrect) work.