Sample Midterm

Operations Research I: Deterministic Models

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1. (a) Denote by x_1 and x_2 the number of push type and self-propelled lawn mowers respectively.

   (b) The objective function is

   $$\max\ 45x_1 + 70x_2$$

   (c) The constraints are

   $$\begin{aligned} 9x_1 + 12x_2 &\leq 720 \\ 2x_1 + 6x_2 &\leq 300 \\ x_1 + x_2 &\leq 75 \\ x_1, x_2 &\geq 0 \end{aligned}$$

2. (a) BV $= \{s_1, e_2, s_3\}$

   $$\begin{cases} s_1 &= 8 \\ e_2 &= 2 \\ s_3 &= 6 \end{cases}$$

   (b) NBV $= \{x_1, x_2, x_3\}$

   $$\begin{cases} x_1 &= 0 \\ x_2 &= 0 \\ x_3 &= 0 \end{cases}$$

   (c) x_2 is the entering variable because it has the most positive coefficient in row 0.

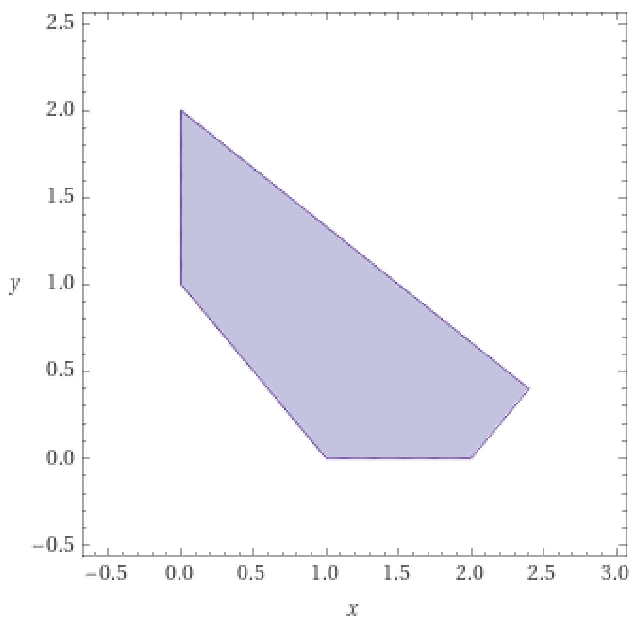
(d) The initial tableau is

z	**x_1**	**x_2**	**x_3**	**s_1**	**e_2**	**s_3**	**RHS**	**ratio**
1	-1	1	4	0	0	0	0	
0	1	1	2	1	0	0	0	8
0	1	1	-1	0	1	0	2	2*
0	-1	1	1	0	0	1	6	-

The next tableau is

z	**x_1**	**x_2**	**x_3**	**s_1**	**e_2**	**s_3**	**RHS**	**ratio**
1	-2	0	-3	0	-1	0	-2	
0	0	0	3	1	-1	0	6	
0	1	1	-1	0	1	0	2	
0	-2	0	2	0	-1	1	4	

3. (a) The feasible region is



(b) $x_1 = x_2 = 0$

It is not a feasible solution but a basic solution.

(c) $x_1 = 2, x_2 = 0$

It is a feasible solution as well as a basic solution.

(d) $x_1 = 1, x_2 = 1$

It is a feasible solution but not a basic solution.

(e) Infinite feasible solutions.

(f) Five bfs

$$\begin{cases} (0,1,0,3,3) \\ (1,0,0,1,4) \\ (2,0,1,0,2) \\ (0,2,1,4,0) \\ (2.4,0.4,1.8,0,0) \end{cases}$$

(g) The optimal solution is

$$\begin{cases} x_1 = 2.4 \\ x_2 = 0.4 \\ z = 2.8 \end{cases}$$

4. The standard form is

$$\begin{aligned} \min \quad & z = 2x_1 - x_2 + x_3 \\ s.t \quad & x_1 + 2x_2' - 2x_2'' + x_3 - e_1 = 8 \\ & x_1 - x_3 - e_2 = -2 \\ & x_1, x_2', x_2'', x_3, e_1, e_2 \geq 0 \end{aligned}$$

5. The dual is

$$\begin{aligned} \min \quad & w = 2y_1 + 30y_2 + 20y_3 \\ s.t \quad & y_1 + y_3 \geq 1 \\ & y_1 + 2y_2 + y_3 \leq -2 \\ & -y_1 - y_2 - y_3 = -1 \\ & 3y_1 + 2y_2 = 3 \\ & y_1 \leq 0 \\ & y_2 \geq 0 \\ & y_3 \; urs \end{aligned}$$

6. This is an optimal tableau, but the LP has multiple optimal solutions.

   Another optimal tableau is

z	$\mathbf{x_1}$	$\mathbf{x_2}$	$\mathbf{x_3}$	$\mathbf{x_4}$	$\mathbf{x_5}$	$\mathbf{x_6}$	**RHS**
1	0	0	0	0	2	3	8
0	1	0	0	1	0	1	6
0	0	0	1	-2	-1	-1	-1
0	0	1	0	1	0	1	2

   The general form of all optimal solutions is

$$x = cb_1 + (1-c)b_2$$

   where $c \in [0,1], b_1 = (4,0,3,2,0,0)^T, b_2 = (6,2,-1,0,0,0)$

7. The initial tableau is

z	$\mathbf{x_1}$	$\mathbf{x_2}$	$\mathbf{x_3}$	$\mathbf{s_1}$	$\mathbf{s_2}$	$\mathbf{e_3}$	$\mathbf{a_3}$	**RHS**	**ratio**
1	-4	-4	-1	0	0	0	M	0	
1	-4-2M	-4-M	-1-3M	0	0	M	0	-3M	
0	1	1	1	1	0	0	0	2	2
0	2	1	0	0	1	0	0	3	-
0	2	1	3	0	0	-1	1	3	1*

   The first tableau is

z	$\mathbf{x_1}$	$\mathbf{x_2}$	$\mathbf{x_3}$	$\mathbf{s_1}$	$\mathbf{s_2}$	$\mathbf{e_3}$	$\mathbf{a_3}$	**RHS**	**ratio**
1	$-\frac{10}{3}$	$-\frac{11}{3}$	0	0	0	$-\frac{1}{3}$	$M+\frac{1}{3}$	1	
0	$\frac{1}{3}$	$\frac{2}{3}$	0	1	0	$\frac{1}{3}$	$-\frac{1}{3}$	1	$\frac{2}{3}$*
0	2	1	0	0	1	0	0	3	3
0	$\frac{2}{3}$	$\frac{1}{3}$	1	0	0	$-\frac{1}{3}$	$\frac{1}{3}$	1	3

   The second tableau is

z	**x₁**	**x₂**	**x₃**	**s₁**	**s₂**	**e₃**	**a₃**	**RHS**	**ratio**
1	$-\frac{3}{2}$	0	0	$\frac{11}{2}$	0	$\frac{11}{6}$	$M-\frac{3}{2}$	$\frac{11}{2}$	
0	$\frac{1}{2}$	1	0	$\frac{3}{2}$	0	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{3}{2}$	3
0	$\frac{3}{2}$	0	0	$-\frac{3}{2}$	1	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{3}{2}$	1^*
0	$\frac{1}{2}$	0	1	$-\frac{1}{2}$	0	$-\frac{1}{6}$	$\frac{1}{2}$	$\frac{1}{2}$	1

The optimal tableau is

z	**x₁**	**x₂**	**x₃**	**s₁**	**s₂**	**e₃**	**a₃**	**RHS**
1	0	0	0	4	1	$\frac{7}{3}$	$M-1$	7
0	0	1	0	2	$-\frac{1}{3}$	$\frac{2}{3}$	$-\frac{2}{3}$	1
0	1	0	0	-1	$\frac{2}{3}$	$-\frac{1}{3}$	$\frac{1}{3}$	1
0	0	0	1	0	$-\frac{1}{3}$	0	$\frac{1}{3}$	0

The optimal solution is $(1,1,0,0,0,0,0)$, $z=8$.

8. (a) Denote by x_{ijk} the quantities of product j produced by machine k at month i for $i,j,k=1,2$.

   (b) The objective function is

$$\max 55\sum_k x_{11k} + 12\sum_k x_{21k} + 65\sum_k x_{12k} + 12\sum_k x_{22k}$$

   (c) The constraints are

$$\begin{aligned} 4x_{i11} + 7x_{i21} &\le 500, \quad \forall i \\ 3x_{i12} + 4x_{i22} &\le 500, \quad \forall i \\ \sum_k x_{11k} &\le 100 \\ \sum_k x_{12k} &\le 140 \\ \sum_k x_{21k} &\le 190 \\ \sum_k x_{22k} &\le 130 \\ x_{ijk} &\ge 0, \quad \forall i,j,k \end{aligned}$$