AMS 341 (Spring, 2010) Estie Arkin

Operations Research I: Deterministic Models

Exam 1: Thursday, March 11, 2010

READ THESE INSTRUCTIONS CAREFULLY. Do not start the exam until told to do so. Make certain that you have all 6 pages of the exam. You will be held responsible for any missing pages.

Write your answers on this examination, using the backs of pages if needed. (Use back of pages also for scratch paper if you need it.)

This examination is CLOSED BOOK and CLOSED NOTES. You may not use any books, papers, or materials other than your pen or pencil. You may use a 4 by 6 “cheat sheet”, which should be turned in with your exam.

The following items should NOT be on your desk - put them INSIDE your bag!

- calculator
- cell phone
- pager

If I see any of these items, even turned off, this will be considered cheating!!!

Work carefully, and GOOD LUCK!!!

Last (Family) Name (PRINT CLEARLY): ______________________________

First Name (PRINT CLEARLY): ______________________________

ID Number: ______________________________

Academic integrity is expected of all students at all times, whether in the presence or absence of members of the faculty. Understanding this, I declare that I shall not give, use, or receive unauthorized aid in this examination. I have been warned that any suspected instance of academic dishonesty will be reported to the Academic Judiciary and that I will be subjected to the maximum possible penalty permitted under University guidelines.

Signature:

____

1. (9 points) Consider the following LP:

$$\begin{array}{lll}
\min & z = -2x_1 + x_3 & \\
\text{s.t.} & -x_1 + 5x_2 + x_3 & \geq 1 \\
 & x_1 - 2x_2 + x_3 & = 10 \\
 & x_1, x_3 & \geq 0 \\
 & x_2 & \text{unrestricted}
\end{array}$$

Rewrite the LP in standard form.

2. (15 points) Consider the feasible region given by the following constraints: (It may be helpful to sketch it and/or put it into standard form.)

$$\begin{aligned}
x_1 + x_2 &\leq 6 \qquad (1)\\
x_1 &\leq 2 \qquad (2)\\
x_2 &\leq 4 \qquad (3)\\
x_1 &\geq 0 \qquad (4)\\
x_2 &\geq 0 \qquad (5)
\end{aligned}$$

(a). Is the point $x_1 = 0$, $x_2 = 4$ a feasible point? Is it a basic solution?

(b). Is the point $x_1 = 2$, $x_2 = 2$ a feasible point? Is it a basic solution?

(c). The point $x_1 = 2$, $x_2 = 4$ is a basic feasible solution. Is it a degenerate basic feasible solution?

3. (15 points) You are given the tableau for a max problem. Give conditions on the unkowns a_1, a_2, a_3, b, c that make the following true. Your conditions should be as general as possible (don't just give an example, such as $a_1 = 3$.)

z	x_1	x_2	x_3	x_4	x_5	RHS
1	c	2	0	0	0	10
0	-1	a_1	1	0	0	4
0	a_2	-4	0	1	0	1
0	a_3	3	0	0	1	b

(a). The current BFS is optimal.

(b). The current BFS is optimal and there are multiple optimal solutions.

(c). The LP is unbounded.

4. (8 points) Consider the following LP:

$$
\begin{array}{lll}
\min & z = 3x_1 + x_2 & \\
\text{s.t.} & x_1 - x_2 + x_3 & = 1 \\
 & x_2 + x_4 & = 2 \\
 & x_1 + x_2 - x_5 & = 6 \\
 & x_1, x_2, x_3, x_4, x_5 & \geq 0
\end{array}
$$

We wish to solve this problem using the big M method. Set up the *first* tableau (in canonical form!) we should use.

5. (27 points) A bank is open Monday-Friday from 9am to 5pm. From past experiance, the bank knows that it needs (at least) the following number of tellers:

Time period	9-10	10-11	11-noon	noon-1	1-2	2-3	3-4	4-5
Tellers required	4	3	4	6	5	6	8	8

The bank hires two types of tellers. Full time tellers work 9-5 every day, except for 1 hour off for lunch. (The bank determines when a full time teller takes lunch hour, but it must be either noon-1 or 1-2.) Full time employees are paid \$8 per hour (this includes payment for the lunch hour). The bank can also hire part time tellers. Each part time teller must work exactly 3 consecutive hours each day, and gets paid \$5 per hour. To maintain quality of service, at most 5 part time tellers can be hired. Formulate an LP to minimize the cost of the bank to meet teller requirements. (Your formulation does NOT have to be put into standard form.)

(a). Define the variables you are using in the formulation.

(b). The objective function is:

(c). The constraints are:

6. (16 points) A company produces and sells wooden soldiers and wooden trains. Each soldier requires 3 board feet of lumber and 2 hours of labor. Each train requires 5 board feet of lumber and 4 hours of labor. A total of 145 board feet of lumber and 90 hours of labor are availble. Upto 50 soldiers and 50 trains can be sold. Trains sell for \$55, and soldiers for \$32. In addition to producing trains and soldiers itself, the company can buy (from an outside supplier) extra soldiers at \$27 each and extra trains at \$50 each. Let SM, TM be the number of soldiers and trains made by the company, and SB, TB the number of soldiers and trains bought from the supplier. Use the Lindo output below to answer each of the following parts.

$$
\begin{array}{lcr}
\text{max} & 32SM + 55TM + 5SB + 5TB & \\
\text{s.t. } 2) & 3SM + 5TM & \le 145 \\
3) & 2SM + 4TM & \le 90 \\
4) & SM + SB & \le 50 \\
5) & TM + TB & \le 50
\end{array}
$$

objective function value 1715.00000

variable	value	reduced cost
SM	45.000000	.000000
TM	.000000	4.000000
SB	5.000000	.000000
TB	50.000000	.000000

row	slack or surplus	dual prices
2)	10.000000	0.000000
3)	.000000	13.500000
4)	.000000	5.000000
5)	.000000	5.000000

Range in which basis remains unchanged :

OBJ coefficient ranges

variable	current coef	allowable increase	allowable decrease
SM	32.000000	infinity	2.00000
TM	55.000000	4.00000	infinity
SB	5.00000	2.00000	5.00000
TB	5.00000	infinity	4.00000

righthand side ranges

row	current RHS	allowable increase	allowable decrease
2	145.00000	infinity	10.000000
3	90.00000	6.66667	90.00000
4	50.00000	infinity	5.00000
5	50.00000	infinity	50.00000

(a). If the company can purchase trains for \$48, what would be the new optimal profit?

(b). What is the most that the company should be willing to pay to for another board foot of lumber?

(c). If only 40 trains could be sold, what would be the new optimal solution (the z)?

(d). If only 40 trains could be sold, and 91 hours of labor are available, what would be the new optimal solution (the z)?

7. (10 points) The tableau below is for Phase I of the Two Phase Method. a_1 and a_2 are the artificial variables or constraints 1,2, e_1 e_2 are the excess variables of constraints 1 and 2, and s_3 is the slack variable of the third constraint.

w	x_1	x_2	e_1	e_2	s_3	a_1	a_2	RHS
1	0	0	-1/2	-1	-1/2	-1/2	0	1/2
0	1	0	-3/4	0	-1/4	1	0	9/4
0	0	0	-1/2	-1	-1/2	1/2	1	1/2
0	0	1	1/2	0	1/2	-1/2	0	2

(a). At this tableau, we have:

$w =$

$x_1 =$

$x_2 =$

$e_1 =$

$e_2 =$

$s_3 =$

$a_1 =$

$a_2 =$

(b). The basic variables for this tableau are:

(c). The tableau shows an optimal solution to the Phase I LP. Is the original LP feasible? Explain briefly.