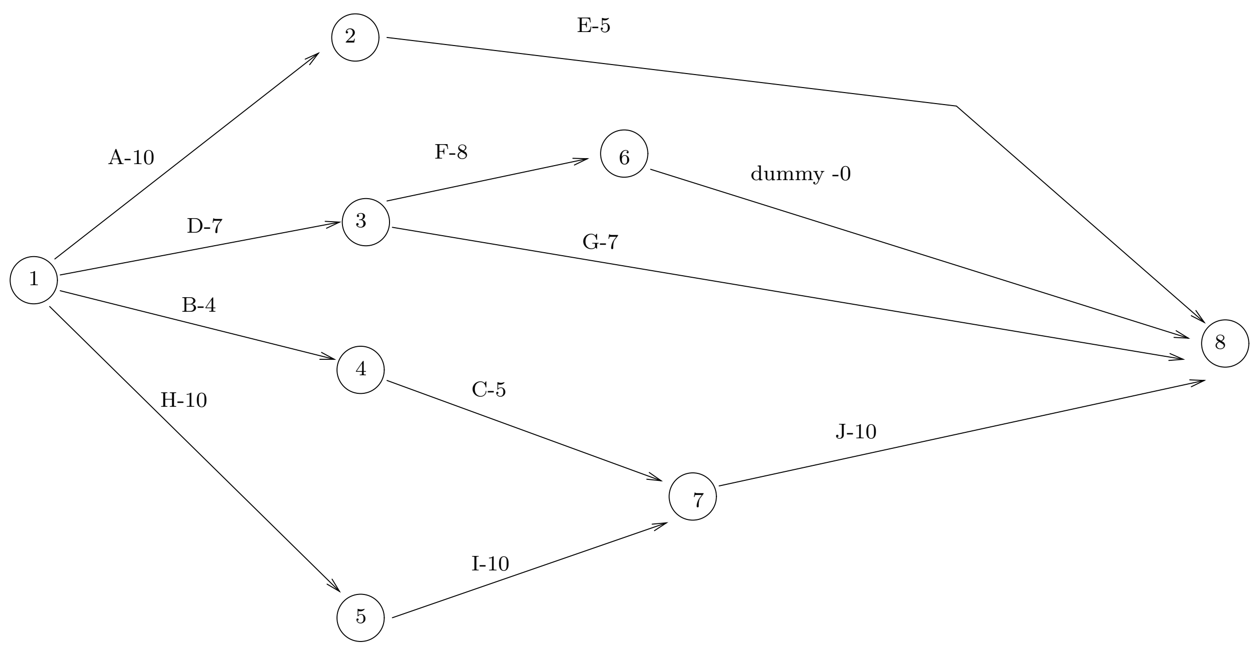
AMS 341 (Fall, 2016) Exam 3 - Solution notes Estie Arkin

Mean 74.11, median 80, top quartile 91, bottom quartile 56, high 100, low 24.

1. (18 points) The following is a list of tasks that have to be completed as soon as possible.

Activity	Predecessors	Time (hours)
A	-	10
B	-	4
C	B	5
D	-	7
E	A	5
F	D	8
G	D	7
H	-	10
I	H	10
J	C,I	6

(a). Draw a project network. Make sure to number the nodes.



Common mistakes: graph undirected, multiple start or end nodes.

	1	2	3	4	5	6	7	8
ET	0	10	7	4	10	15	20	26
LT	0	21	18	15	10	26	20	26

(b). What are the critical activities for this project?

H,I,J

(c). How soon can the project be completed? 26
(d). We are informed that task A has been extended from 10 to 20 hours. How soon can the project be completed?

No change, still 26

2. (15 points, 3 points for each part) Answer TRUE or FALSE:

(a). True The Balanced Transportation problem can also be solved using the Simplex method.

(b). False When using the cutting plane approach to solve an integer programming problem, the objective value after adding a cut might be strictly better than the objective value of the LP relaxation.

(c). True A project network can have more than one critical path.

(d). True A (deterministic) dynamic programming problem may have multiple optimal solutions.

(e). True The Branch and Bound method described in class can be used to solve mixed integer programming problems (a problem where some variables are restricted to be integers and the remaining variables can be fractional.)

3. (10 points) Consider the following Balanced Transportation problem, and given BFS.

					supply
			50	50	100
				100	100
	50	50			100
	100		0		100
demand	150	50	50	150	

Suppose variable x_{21} was chosen as the entering variable. In the table below give the new BFS. Make sure to write in *only* the basic variables:

					supply
				100	100
	50			50	100
	50	50			100
	50		50		100
demand	150	50	50	150	

Common mistakes: Given a BFS that is NOT obtained from the previous one, or a solution that is feasible but not basic.

4. (12 points) We wish to solve an integer programming problem. All variables are restricted to be integer. We began by solving the LP relaxation of the problem and got the final (optimal) tableau for it. Unfortunately, not all the variables are integer.

z	x_1	x_2	x_3	s_1	s_2	s_3	RHS
1	0	1.5	3	0	0	0	6.25
0	0	3	0	1	0	1	10
0	1	-1.2	5.2	0	0	1.1	5
0	0	1.3	-0.6	0	1	2.9	4.5

(a). To solve the problem using branch and bound, what constraint would you add to get subproblem 2 and what constraint would you add to get subproblem 3 (from the LP relaxation which is subproblem 1)? Note: Do not solve the problem, just state the variable and the 2 constraints.

$s_2 \leq 4$ and $s_2 \geq 5$. Note: This is the only fractional variable in the LP relaxation, so the only possible variable to branch on.

(b). To solve the problem using the cutting plane method, what is the cut that should be added?

$0.5 - 0.3x_2 - 0.4x_3 - 0.9s_3 \leq 0$ (or $5 \leq 3x_2 + 4x_3 + 9s_3$)

5. (20 points) A family is planning its gift purchases for their very spoiled dog, Fido. It has a budget of at most 500$ to spend, and is considering various toys. The table below gives the costs, as well as the number of hours they estimate Fido will play with each toy before destroying it. Being very spoiled also means that Fido would get bored by getting the same toy more than once. The family's goal is to keep Fido busy with his toys for as long as possible.

toy	1	2	3	4	5	6	7	8
cost	60	20	25	130	50	200	80	90
hours of play	3	2	2	5	4	8	4	5

(a). Define the variables: x_i is a binary variable that is equal to 1 if toy i is bought, 0 if not.

(b). What is the objective function? $\max 3x_1 + 2x_2 + 2x_3 + 5x_4 + 4x_5 + 8x_6 + 4x_7 + 5x_8$

(c). What are the constraints?

$60x_1 + 20x_2 + 25x_3 + 130x_4 + 50x_5 + 20x_6 + 80x_7 + 90x_8 \leq 500$, x_i binary for $i = 1, \ldots, 8$

(d). The family wishes to add a constraint that if toy numbered 3 is bought, then they must also purchase toy number 5 or 8 (it is ok to buy both 5 and 8). Formulate such a constraint(s).

$x_3 \leq x_5 + x_8$

(e). The family wishes to add a constraint that if toy numbered 6 is bought they should not buy toy number 7. Formulate such a constraint(s).

$x_6 + x_7 \leq 1$

6. (15 points) A bus company has a total of 4 busses it can use on its daily routes. There are 3 possible round trip routes (NY-Atlantic City, NY-Boston, NY-Stony Brook). The table below shows the profit given how many busses are assigned to each route: (Note, if no busses are sent on a particular route, the profit is zero for that route.)

	1 bus	2 busses	3 busses	4 busses
NY-Atlantic City	100	150	180	200
NY-Boston	80	160	200	210
NY-Stony Brook	120	140	160	170

Assume that each bus is assigned to a single route. The company wishes to maximize its profit. To solve the problem using Dynamic Programming define $f_i(s)$ = the maximum profit stages i and above and state s. Define route 1 NY-Atlantic city, route 2 NY-Boston, route 3 NY-Stony Brook.

$f_3(0) = 0$, $f_3(1) = 120$, $f_3(2) = 140$, $f_3(3) = 160$ $f_3(4) = 170$, $f_2(0) = 0$, $f_2(1) = 120$, $f_2(2) = 200$, $f_2(3) = 280$, $f_2(4) = 320$, $f_1(4) = 380$, assign 1 bus to route 1, 2 to route 2 and 1 to route 3.

7. (10 points) The AMS department is scheduling instructors to teach its courses in the summer session. Instructors A,B, and D will each be teaching at most one course, and instructors C and E will each be teaching at most 2 courses. There are 6 courses to be taught. Based on surveys from previous years, the department knows how succesful each instructor is at teaching each course. The data is given in the table below (small numbers are better!) The "-" means that a professor cannot teach that course.

	A	B	C	D	E
course 1	4	3	-	5	7
course 2	1	3	6	1	7
course 3	3	4	7	2	7
course 4	6	1	5	5	5
course 5	4	5	4	4	3
course 6	4	-	3	1	3

The department's goal is to assign professors to courses such that the teaching in summer is as "succesful" as possible. Formulate the problem as an Assignment problem by giving the cost matrix. Note: You are asked to formulate an assignment problem, not a Balanced Transportation Problem and not a Linear Program. Do not solve the problem you formulated.

	A	B	C	C'	D	E	E'
course 1	4	3	M	M	5	7	7
course 2	1	3	6	6	1	7	7
course 3	3	4	7	7	2	7	7
course 4	6	1	5	5	5	5	5
course 5	4	5	4	4	4	3	3
course 6	4	M	3	3	1	3	3
dummy	0	0	0	0	0	0	0

Common mistakes: Leaving "-" instead of M, forgetting that some teachers can teach 2 courses, not balancing the problem, formulating LP or BTP instead of assignment, or trying to solve a problem using the Hungarian method (before a correct formulation is given).