AMS 341 (Spring, 2009) Estie Arkin

Operations Research I: Deterministic Models

Exam 1: Thursday, March 12, 2009

READ THESE INSTRUCTIONS CAREFULLY. Do not start the exam until told to do so. Make certain that you have all 7 pages of the exam. You will be held responsible for any missing pages.

Write your answers on this examination, using the backs of pages if needed. (Use back of pages also for scratch paper if you need it.)

This examination is CLOSED BOOK and CLOSED NOTES. You may not use any books, papers, or materials other than your pen or pencil. You may use a 4 by 6 "cheat sheet", which should be turned in with your exam.

The following items should NOT be on your desk - put them INSIDE your bag!

- calculator
- cell phone
- pager

If I see any of these items, even turned off, this will be considered cheating!!!

Work carefully, and GOOD LUCK!!!

Last (Family) Name (PRINT CLEARLY): ______________________________

First Name (PRINT CLEARLY): ______________________________

ID Number: ______________________________

Academic integrity is expected of all students at all times, whether in the presence or absence of members of the faculty. Understanding this, I declare that I shall not give, use, or receive unauthorized aid in this examination. I have been warned that any suspected instance of academic dishonesty will be reported to the Academic Judiciary and that I will be subjected to the maximum possible penalty permitted under University guidelines.

Signature:

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1. (20 points) Consider the following LP:

$$\begin{array}{lll} \max & z = 2x_1 - x_2 + x_3 & \\ \text{s.t.} & x_1 + 2x_2 - x_3 & \geq 1 \\ & 3x_1 - 2x_2 + x_3 & \leq 20 \\ & x_1, x_2 & \geq 0 \\ & x_3 & \text{unrestricted} \end{array}$$

(a). Rewrite the LP in standard form.

(b). What is the dual of the given LP? (You can either state the dual of the original problem or the dual of its standard form.)

2. (20 points) Each day Eastinghouse produces capacitors during three shifts: 8am-4pm, 4pm-midnight, midnight-8am. The hourly salary paid to the employees of each shift, the number of capacitors produced by a worker during the shift, the price charged for each capacitor made during each shift, and the number of defects in each capacitor produced during a shift are given in the table below. Each of the company's 25 workers can be assigned to one of the three shifts. Because of machinary limitations, no more than 10 workers can be assigned to each shift. Each day, at most 250 capacitors can be sold, and the average number of defects per capacitor for the day's production cannot exceed three. Formulate an LP to maximize Eastinghouse's daily profit (sales revenue minus labour cost). (Your formulation does NOT have to be put into standard form. Do NOT solve, just formulate!)

shift	hourly salary	capacitors produced (per worker)	defects (per capacitor)	price
8am-4pm	\$12	10	4	\$18
4pm-midnight	\$16	9	3	\$22
midnight-8am	\$20	12	2	\$24

(a). Define the variables you are using in the formulation.

(b). The objective function is:

(c). The constraints are:

3. (25 points) Consider the feasible region given by the following constraints: (It may be helpful to sketch it and/or put it into standard form.)

$$x_1 + x_2 \leq 4 \quad (1)$$
$$x_1 \geq 1 \quad (2)$$
$$x_2 \leq 3 \quad (3)$$
$$x_1 \geq 0 \quad (4)$$
$$x_2 \geq 0 \quad (5)$$

(a). Is the point $x_1 = 0$, $x_2 = 0$ a feasible point? Is it a basic solution?

(b). Is the point $x_1 = 2$, $x_2 = 2$ a feasible point? Is it a basic solution?

(c). Is there an objective function for which an LP with these constraints is unbounded? If so, give such an objective function. If not, explain (briefly!) why not.

(d). Consider the objective function $\max z = x_1 - x_2$. Let s_1, e_2, s_3 be the slack and excess variables of the constraints. Here is the first tableau of phase II the LP. Pivot until you find the optimal solution. Make sure to state which variable enters the basis and which leaves at each iteration. (If you have to do more than 2 pivots, you are doing something wrong!)

z	x_1	x_2	s_1	e_2	s_3	RHS
1	0	1	0	-1	0	1
0	0	1	1	1	0	3
0	1	0	0	-1	0	1
0	0	1	0	0	1	3

4. (20 points) A bank is attempting to determine where its assets should be invested during the current year. At present \$500,000 is available for investment in bonds, home loans, auto loans, and personal loans. The annual return on each type of investement is known to be: bonds 10%; home loans 16%; auto loans 13%; personal loans 20%. To ensure that the bank's portfolio is not too risky, the following three restrictions are placed:
(1) The amount invested in personal loans cannot exceed the amount invested in bonds.
(2) The amount invested in home loans cannot exceed the amount invested in auto loans.
(3) No more than 25% of the total amount invested may be in personal loans.

The bank's objective is to maximize the annual rate of return on its investment. Let B, H, A, P be the amounts invested in bonds, home loans, auto loans and personal loans respecively. Use the Lindo output below to answer each of the following parts, or say that the answer is unkown using the given Lindo output.

$$
\begin{array}{lcr}
\text{max} & 0.1B + 0.16H + 0.13A + 0.2P & \\
\text{s.t. } 2) & B + H + A + P & \leq 500000 \\
3) & P - B & \leq 0 \\
4) & H - A & \leq 0 \\
5) & -0.25B - 0.25H - 0.25A + 0.75P & \leq 0
\end{array}
$$

objective function value 73750

variable	value	reduced cost
B	125000	.000000
H	125000	.000000
A	125000	.000000
P	125000	.000000

row	slack or surplus	dual prices
2)	0.000000	0.1475
3)	0.000000	0.0450
4)	0.000000	0.0150
5)	0.000000	0.0100

Range in which basis remains unchanged :

objective coefficient ranges

variable	current coef	allowable increase	allowable decrease
B	0.10	0.045	0.01
H	0.16	0.010	0.03
A	0.13	0.010	0.09
P	0.20	infinity	0.01

righthand side ranges

row	current RHS	allowable increase	allowable decrease
2	500000	infinity	500000
3	0	125000	250000
4	0	250000	250000
5	0	125000	125000

(a). What would be the profit if only \$400,000 can be invested?

(b). What would be the profit if the interest on home loans is 14% (instead of 16%)?

(c). What would be the profit if the interest on home loans is 14% (instead of 16%) and only \$400,000 can be invested?

(d). What would be the profit if the interest on home loans is 12% (instead of 16%)?

5. (15 points) The tableau below is for Phase I of the Two Phase Method. a_1 and a_3 are the artificial variables.

w	x_1	a_1	s_2	x_2	x_3	a_3	e_3	RHS
1	0	0	0	-3	0	-2	0	0
0	0	1	1	1	0.5	1	0	2
0	1	2	0	0	0	-2	0	4
0	0	0	0	1	2	1	1	0

(a). At this tableau, we have:

$w =$

$x_1 =$

$a_1 =$

$s_2 =$

$x_2 =$

$x_3 =$

$a_3 =$

$e_3 =$

(b). The basic variables for this tableau are:

(c). The tableau shows an optimal solution to the Phase I LP. Is the original LP feasible?

(d). For this part, assume that the original LP had objective function $\max z = x_1 + x_2$. What is the first tableau for Phase II (after "clean-up" if needed).

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