final summer 2021

Thursday, July 1, 2021 1:34 PM

1./(a)

$$\begin{cases} u_1 + v_1 - 2 = 0 \\ u_1 + v_3 - 3 = 0 \\ u_2 + v_2 - 1 = 0 \\ u_2 + v_4 - 3 = 0 \\ u_3 + v_3 = 0 \\ u_3 + v_4 = 1 \end{cases} \Rightarrow \begin{cases} u_1 = 0 \\ v_1 = 2 \\ u_2 = 2 \\ v_2 = -1 \\ u_3 = 0 \\ v_3 = 3 \\ v_4 = 1 \end{cases}$$

$\overline{c_{12}} = 2 - 5 = -3$

$\overline{c_{14}} = 1 - 4 = -3$

$\overline{c_{21}} = 4$ ✓

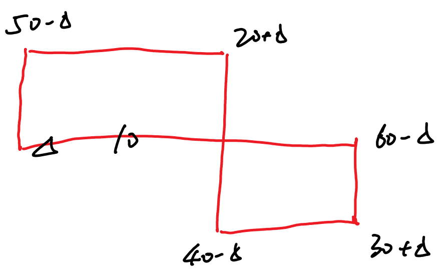
$\overline{c_{23}} = 5 - 6 = -1$

$\overline{c_{31}} = 2$

$\overline{c_{32}} = -1$

X_{21} enters

(b)



$\delta = \min\{50, 40, 60\} = 40$

10		60		70
40	10		20	70
			70	70
50	10	60	90	

X_{33} leaves

2./(1)

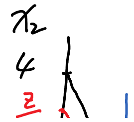
$$\begin{cases} x_1 - s_2 = 0.8 - 0.4 s_1 - 0.8 s_2 \\ x_2 - 2 = 0.4 - 0.2 s_1 - 0.4 s_2 \end{cases}$$

0.4 is closer to 0.5

the cut is $0.4 - 0.2 s_1 - 0.4 s_2 \le 0$

(2) branch and bound tree

Subproblem 1



Subproblem 1

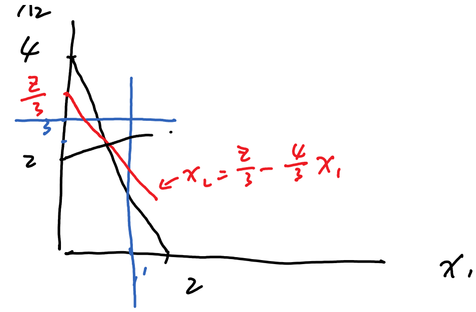
$z = 10.4,\ x_1 = 0.8,\ x_2 = 2.4$

$x_2 \geq 3$ $\qquad$ $x_2 \leq 2$

subproblem 2
infeasible

subproblem 3

$\boxed{z = 10,\ x_1 = 1,\ x_2 = 2}$



the optimal solution is $z = 10,\ x_1 = 1,\ x_2 = 2$

3. Upper bound

	z	x_1	x_2	x_3	x_4	x_5	s_1	s_2	s_3	RHS	ratio
	1	-4	-3	-5	0	0	0	0	0	0	
s_1	0	2	2	1	1	0	1	0	0	9	9
s_2	0	4	-1	-1	0	1	0	1	0	6	—
s_3	0	0	2	1	0	0	0	0	1	6	6*

$\min\{6, 4\} = 4$, Let $x_3' = 4 - x_3$

	z	x_1	x_2	x_3'	x_4	x_5	s_1	s_2	s_3	RHS	ratio
	1	-4	-3	5	0	0	0	0	0	20	
s_1	0	2	2	-1	1	0	1	0	0	5	2.5*
s_2	0	4	-1	1	0	1	0	1	0	10	2.5
s_3	0	0	2	-1	0	0	0	0	1	2	-

$\min\{2.5, 2\} = 2$. Let $x_1 = 2 - x_1'$

	z	x_1'	x_2	x_3'	x_4	x_5	s_1	s_2	s_3	RHS	ratio
	1	4	-3	5	0	0	0	0	0	28	
s_1	0	-2	2	-1	1	0	1	0	0	1	$0.5^* < 3$
s_2	0	-4	-1	1	0	1	0	1	0	2	-

s_3	0	0	2	-1	0	0	0	0	1	2	1

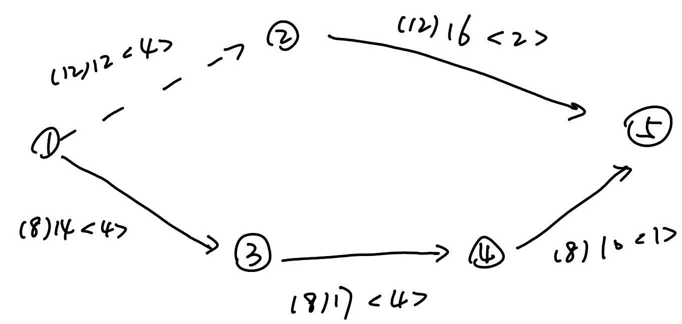
	1	1	0	3.5	1.5	0	1.5	0	0	29.5
x_2	0	-1	1	-0.5	0.5	0	0.5	0	0	0.5
s_2	0	-5	0	0.5	0.5	1	0.5	1	0	2.5
s_3	0	2	0	0	-1	0	-1	0	1	1

The optimal solution is $x_2 = 0.5$, $s_2 = 2.5$, $s_3 = 1$

$x_1' = x_3' = 0 \Rightarrow x_1 = 2,\ x_3 = 4$

$z = 29.5$

4



$y_1 = 0$

$y_2 = -7$

$y_3 = -4$

$y_4 = -8$

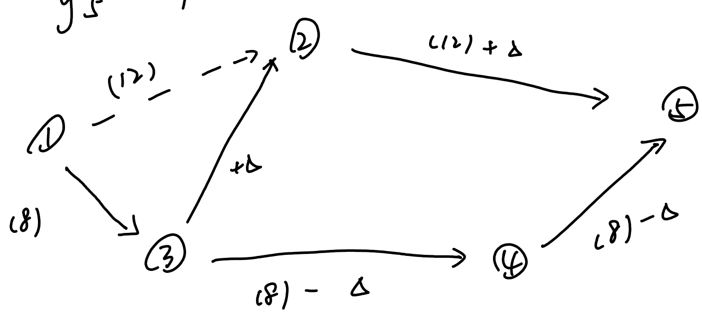
$y_5 = -9$

$\overline{c_{12}} = 7 - 4 = 3 \quad (u)$

$\overline{c_{32}} = 3 - 1 = 2 \quad (l) \quad \times$

$\overline{c_{24}} = 1 - 2 = -1 \quad (l)$

x_{32} enters



$\Delta = 4$

x_{25} leaves

$y_1 = 0$

$y_2 = -5$

..

$\overline{c_{12}} = 5 - 4 = 1 \quad (u)$

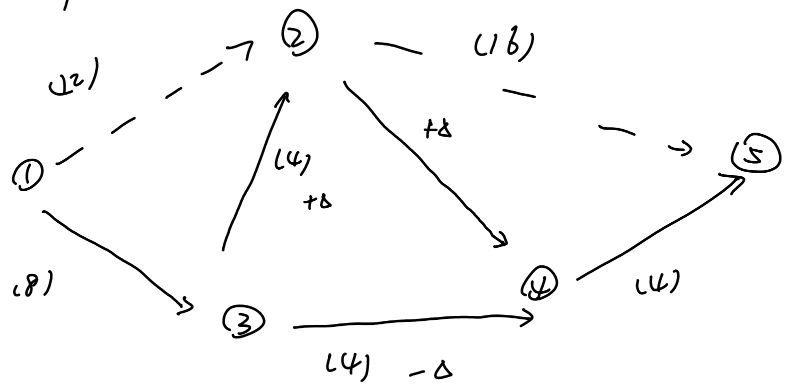
$\overline{c_{25}} = 4 - 2 = 2 \quad (u)$

$y_2 = -5$ $\quad\quad$ $\overline{c_{25}} = 4 - 2 = 2 \quad (u)$

$y_3 = -4$ $\quad\quad$ $\overline{c_{24}} = 3 - 2 = 1 \quad (l) \quad \times$

$y_4 = -8$

$y_5 = -9$



$\Delta = 4$

x_{34} leaves

$y_1 = 0$ $\quad\quad$ $\overline{c_{12}} = 5 - 4 = 1 \quad (u)$

$y_2 = -5$ $\quad\quad$ $\overline{c_{25}} = 3 - 2 = 1 \quad (u)$

$y_3 = -4$ $\quad\quad$ $\overline{c_{34}} = 3 - 4 = -1 \quad (l)$

$y_4 = -7$

$y_5 = -8$

the optimal solution is

$x_{12} = 12,\ x_{13} = 8,\ x_{32} = 8,\ x_{24} = 4,\ x_{25} = 16,\ x_{34} = 0,\ x_{45} = 4$

$Z = 12 \times 4 + 8 \times (4+1) + 16 \times 2 + 4 \times (2+1) = 132$

5/(1)

contractor

value type

12	13	20	18	1
8	20	10	18	1
20	22	30	25	1
0	0	0	0	1
1	1	1	1	

(2)

				λ_i
12	13	20	18	12
8	20	10	18	8
20	22	30	25	20
0	0	0	0	0

=>

					λ_i
	0	1	8	6	12
	0	12	2	10	8
	0	2	10	5	20
	0	0	0	0	0
μ_j	0	0	0	0	

$k=1$

					λ_i
	0	0	7	5	13
	0	11	1	9	9
	0	1	9	4	21
	1	0	0	0	0
μ_j	-1	0	0	0	

$k'=1$

					λ_i
	1	0	7	5	13
	0	10	0	9	10
	0	0	8	3	22
	2	0	0	0	0
μ_j	-2	0	0	0	

the optimal solution $X_{12} = X_{23} = X_{31} = X_{44} = 1$

the optimal solution $X_{12} = X_{23} = X_{31} = X_{44} = 1$

$$Z = \Sigma(\lambda_i + u_j) = 43$$

6. (1) $\max\ Z = 4X_1 + X_2$

s.t. $X_1 + 2X_2 = 6$

$X_1 - X_2 - e_2 = 3$

$2X_1 + X_2 + S_3 = 10$

$X_1, X_2, e_2, S_3 \geq 0$

(2) Phase I

	W	X_1	X_2	e_2	S_3	a_1	a_2	RHS	ratio
	1	0	0	0	0	-1	-1	0	
	1	2	1	-1	0	0	0	9	
a_1	0	1	2	0	0	1	0	6	6
a_2	0	1	-1	-1	0	0	1	3	3*
S_3	0	2	1	0	1	0	0	10	5
	1	0	3	1	0	0	-2	3	
a_1	0	0	3	1	0	1	-1	3	1*
X_1	0	1	-1	-1	0	0	1	3	-
S_3	0	0	3	2	1	0	-2	4	$\frac{4}{3}$
	1	0	0	0	0	-1	-1	0	
X_2	0	0	1	$\frac{1}{3}$	0	$\frac{1}{3}$	$-\frac{1}{3}$	1	

x_1	0	1	0	$-\frac{2}{3}$	0	$\frac{1}{3}$	$\frac{2}{3}$	4
s_3	0	0	0	1	1	-1	-1	1

$W=0$. no artificial variables in BV, so drop all $a v_s$.

Phase II

	Z	X_1	X_2	e_2	s_3	RHS	ratio
	1	-4	-1	0	0	0	
X_2	0	0	1	$\frac{1}{3}$	0	1	
x_1	0	1	0	$-\frac{2}{3}$	0	4	
s_3	0	0	0	1	1	1	

	1	0	0	$-\frac{7}{3}$	0	17
X_2	0	0	1	$\frac{1}{3}$	0	1
X_1	0	1	0	$-\frac{2}{3}$	0	4
S_3	0	0	0	1	1	1

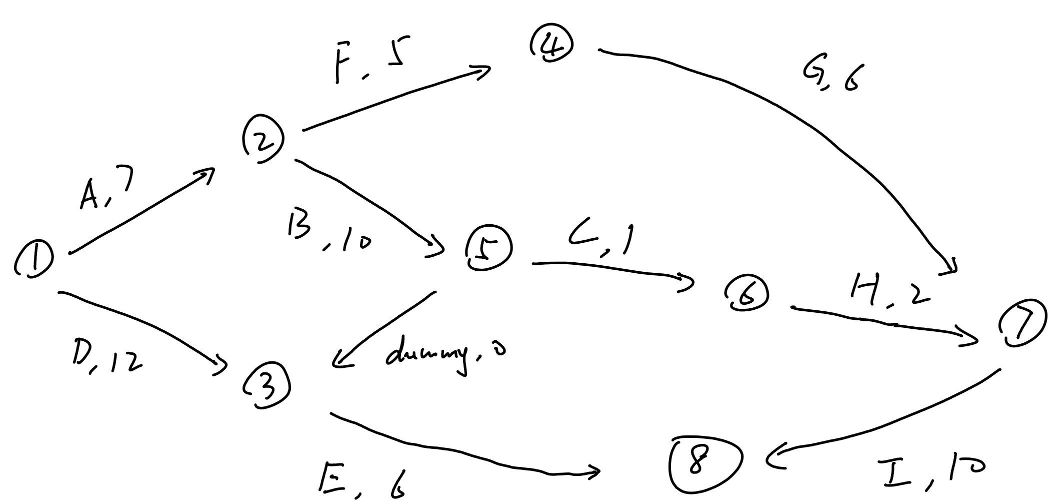
e_2 enters, s_3 leaves

$\Rightarrow$

	1	0	0	0	$\frac{7}{3}$	$\frac{58}{3}$
X_2	0	0	1	0	$-\frac{1}{3}$	$\frac{2}{3}$
X_1	0	1	0	0	$\frac{2}{3}$	$\frac{14}{3}$
e_2	0	0	0	1	1	1

the optimal $X_1=\frac{14}{3}$, $X_2=\frac{2}{3}$, $Z=\frac{58}{3}$

7. (a)



(b)

	1	2	3	4	5	6	7	8
ET	0	7	17	12	17	18	20	30
LT	0	7	17	14	17	18	20	30

$TF(1,2) = 7-7=0$, $TF(1,3) = 17-12 = 5$

$TF(2,4) = 7-5=2$, $TF(2,5) = 10-10=0$

$TF(5,3) = 0$ $TF(5,6) = 1-1=0$

$TF(4,7) = 8-6=2$ $TF(6,7) = 2-2=0$

$TF(3,8) = 13-6=7$ $TF(7,8) = 10-10=0$

critical activities: A, B, C, H, I

(c) $ET(8) = 30$

(d) $TF(D) = TF(1,3) = 5$

(e) No, D is not a critical activity.

(f) Yes. B is a critical activity.

8. (1)

		x_1	x_2	
max		$\geq$	$\geq$	
	$y_1 \geq$	3	4	≤ 100
min	$y_2 \geq$	2	3	≤ 70
	$y_3 \geq$	1	2	≤ 30
	$y_4 \leq$	0	1	≥ 3
		$\geq$	$\geq$	
		15	25	

The dual is

$$\min w = 100y_1 + 70y_2 + 30y_3 + 3y_4$$

s.t. $3y_1 + 2y_2 + y_3 \geq 15$

$4y_1 + 3y_2 + 2y_3 + y_4 \geq 25$

$y_1, y_2, y_3 \geq 0, \quad y_4 \leq 0$

(2) $x_1 = 24, \quad x_2 = 3, \quad s_1 = 16, \quad s_2 = 13, \quad Z = 435$

$$\begin{cases} x_1 e_1 = 0 \\ x_2 e_2 = 0 \end{cases} \Rightarrow e_1 = e_2 = 0$$

$$\begin{cases} y_1 s_1 = 0 \\ y_2 s_2 = 0 \end{cases} \Rightarrow y_1 = y_2 = 0$$

$$\begin{cases} y_3 = 15 \\ 2y_3 + y_4 = 25 \end{cases} \Rightarrow \begin{cases} y_3 = 15 \\ y_4 = -5 \end{cases}$$

the solution of dual is $y_1 = y_2 = 0, \ y_3 = 15, \ y_4 = -5$

$w = 435$

(3) (a) $y = C_{BV}^T B^{-1} = (0, 0, 15, -5)$

(b) $b_3' = 35$

$$b' = \begin{pmatrix} 100 \\ 70 \\ 35 \\ 3 \end{pmatrix}$$

RHS of optimal row 0 $= C_{BV}^T B^{-1} b' = 15 \times 35 - 15 = 510$

the company's revenue becomes 510.

(c) $c_2 \in BV$, $c_2' = c_2 + \Delta c_2$

$$c_{BV} = \begin{pmatrix} 15 \\ 25 + \Delta c_2 \\ 0 \\ 0 \end{pmatrix}, \quad B = \begin{pmatrix} 3 & 4 & 1 & 0 \\ 2 & 3 & 0 & 1 \\ 1 & 2 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

$$B^{-1} = \begin{pmatrix} 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & -3 & 2 \\ 0 & 1 & -2 & 1 \end{pmatrix}$$

$$c_{BV}^T B^{-1} = \begin{pmatrix} 0 & 0 & 15 & \Delta c_2 - 5 \end{pmatrix}$$

$$\Delta c_2 - 5 \geq 0 \Rightarrow \Delta c_2 \geq 5$$

(d) $x_1 = 24$, $x_2 = 3$ does not satisfy $x_1 + 2x_2 \leq 26$

	z	x_1	x_2	s_1	s_2	s_3	e_4	s_4	RHS	ratio
	1	0	0	0	0	15	5	0	435	
s_1	0	0	0	1	0	-3	-2	0	16	
s_2	0	0	0	0	1	-2	-1	0	13	
x_1	0	1	0	0	0	1	2	0	24	
x_2	0	0	1	0	0	0	-1	0	3	
s_4	0	1	2	0	0	0	0	1	26	
	0	0	0	0	0	-1	0	1	-4	s_3 enters
	1	0	0	0	0	0	5	15	375	s_4 leaves
s_1	0	0	0	1	0	0	-2	-3	28	
s_2	0	0	0	0	1	0	-1	-2	21	
x_1	0	1	0	0	0	0	2	1	20	
x_2	0	0	1	0	0	0	-1	0	3	

s_3	0	0	0	0	0	1	0	-1	4

the optimal solution is $x_1 = 20$, $x_2 = 3$, $z = 375$