AMS 341 (Summer, 2021) Yixuan Lin

Operations Research I: Deterministic Models

Midterm: Thursday, June 10, 2020

READ THESE INSTRUCTIONS CAREFULLY.

Do not start the exam until told to do so.
Upload your answers before 16:10.
This examination is OPEN BOOK and OPEN NOTES.
Remember to write your name (First Last) and ID Number on your answer.

Note that:

Make sure you OPEN the camera. It would be better to show your hand in camera all the time.
You can NOT talk/communicate with others or using computer to find the solution.
You can NOT use cell phone.

If you did not do anything mentioned below, you will be considered cheating!!!

Academic integrity is expected of all students at all times, whether in the presence or absence of members of the faculty. Understanding this, I declare that I shall not give, use, or receive unauthorized aid in this examination. I have been warned that any suspected instance of academic dishonesty will be reported to the Academic Judiciary and that I will be subjected to the maximum possible penalty permitted under University guidelines.

Signature: **(If you upload your answer, you are regarded as signing this.)**

Work carefully, and GOOD LUCK!!!

1. (10 points) Consider the feasible region given by the following constraints:
(a). Sketch the feasible region.

$$\begin{aligned} \max z &= 2x_1 - x_2 \\ \text{s.t.} \qquad 2x_1 + 3x_2 &\le 6 \\ -x_1 + x_2 &\ge 1 \\ x_1, x_2 &\ge 0 \end{aligned}$$

(b). Is the point $x_1 = 1/2$, $x_2 = 3/2$ a feasible point? **YES NO**
Is it a basic solution? **YES NO**

(c). Is the point $x_1 = -1$, $x_2 = 0$ a feasible point? **YES NO**
Is it a basic solution? **YES NO**

(d). Is the point $x_1 = 0$, $x_2 = 2$ a feasible point? **YES NO**
Is it a basic solution? **YES NO**

(e). How many **feasible solutions** does an LP with the above constraints have?

(f). How many **basic feasible solutions** does an LP with the above constraints have?

(g). What is the optimal solution?

2. (8 points) Rewrite the following LP in standard form:

$$
\begin{aligned}
\max z = 2x_1 + 4x_2 - 5x_3 \\
\text{s.t.} \qquad x_1 - 3x_2 + x_3 &\leq 5 \\
2x_1 + x_2 - x_3 &\geq -10 \\
4x_1 - 2x_2 + 3x_3 &\geq 3 \\
x_1 & \quad \text{urs} \\
x_2 &\leq 0 \\
x_3 &\geq 0
\end{aligned}
$$

3. (8 points) Find the dual of the following LP:

$$
\begin{aligned}
\min z = 3x_1 + 2x_2 - 4x_3 + x_4 \\
\text{s.t.} \quad 2x_1 + 2x_2 + x_3 - 3x_4 &\leq 20 \\
x_1 + 4x_2 - 2x_3 &= 10 \\
3x_1 - x_3 + x_4 &\geq 5 \\
x_1 &\geq 0 \\
x_2 &\leq 0 \\
x_3, x_4 &\quad \text{urs}
\end{aligned}
$$

4. (10 points) A **minimization** LP is being solved by the Simplex method. Here is the current tableau:

z	x_1	x_2	x_3	x_4	x_5	x_6	RHS
1	-3	0	-1	0	0	0	6
0	2	0	3	1	1	0	5
0	1	1	1	0	-1	0	1
0	0	0	3	0	3	1	3

Which one of the following statements is true: (Circle one)

(i). This is an optimal tableau, and the LP has a unique optimal solution.
(ii). This is an optimal tableau, but the LP has multiple optimal solutions.
(iii). This is an optimal tableau and the LP is unbounded.
(iv). This is not an optimal tableau for the LP.

If it has multiple optimal solutions, please write down the general form of all optimal solutions:

5. (14 points) Using Big-M to solve the following LP:

$$\begin{aligned} \min z &= -x_1 - x_2 \\ \text{s.t.} \qquad -x_1 + x_2 - x_4 &= 2 \\ x_1 - x_2 - x_3 + 2x_4 &= 1 \\ x_1, x_2, x_3, x_4 &\geq 0 \end{aligned}$$

6. (15 points) A Company produces Joy-Con during four shifts each day: 6am – 12 pm, 12 pm – 6 pm, 6pm - midnight, midnight – 6 am. The hourly salary paid to the employees of each Joy-Con made during each shift, the price charged for each Joy-Con made during each shift, and the number of defects in each Joy-Con produced during a shift are given in the table below. Each of the company's 35 workers can be assigned to one of the four shifts. A worker produces 10 Joy-Cons during a shift, but because of machinery limitations, no more than 15 workers can be assigned to each shift. Each day, at most 250 Joy-Cons can be sold, and the average number of defects per Joy-Con for the day's production cannot exceed four. Formulate an LP to maximize the daily profit (sales revenue minus labor cost).

Shift	Hourly salary	Defects (per Joy-Con)	Price
6 am – 12 pm	\$10	5	\$18
12 pm – 6 pm	\$17	4	\$23
6 pm – Midnight	\$25	3	\$27
Midnight – 6 am	\$30	2	\$34

(a). Define the variables you are using in the formulation.

(b). The objective function is:

(c). The constraints are:

7. (15 points) There are 2 cities. Each day, city 1 and city 2 produces 300 tons of waste and 600 tons of waste, respectively. The cities require that waste must be incinerated at incinerator 1 or 2. Each day, each incinerator can process up to 700 tons of waste. The cost to incinerate waste is \$55 per ton at incinerator 1 and \$60 per ton at incinerator 2. Incineration reduces each ton of waste to 0.4 tons of debris, which must be dumped at one of two landfills. Each landfill can receive at most 400 tons of debris per day. It costs \$5 per mile to transport a ton of material (either debris or waste). Distance (in miles) between locations are shown below. Formulate an LP that can be used to minimize the total cost of disposing of the waste of both cities.

	Incinerator 1	Incinerator 2
City 1	30	5
City 2	36	45
Landfill 1	5	9
Landfill 2	8	6

(a). Define the variables you are using in the formulation.

(b). The objective function is:

(c). The constraints are:

8. (20 points) The following is a starting tableau for an LP:

z	x_1	x_2	x_3	x_4	x_5	RHS	ratio
1	a	1	-3	0	0	0	
0	b	c	d	1	0	6	
0	-1	2	e	0	1	1	

(a). What are the basic variables, and what are they equal to?

(b). What are the non-basic variables, and what are they equal to?

(c). The current tableau is given below. **Find the values of the unknowns a trough n.** (Hint: start with finding the basic variables in the following tableau.)

z	x_1	x_2	x_3	x_4	x_5	RHS	ratio
1	0	-1/3	j	k	l	n	
0	g	2/3	2/3	1/3	0	f	
0	h	i	-1/3	1/3	1	m	