AMS 341 Yixuan Lin

Operations Research I: Deterministic Models

Final Sample

READ THESE INSTRUCTIONS CAREFULLY.

Do not start the exam until told to do so.
Upload your answers before 16:40
This examination is OPEN BOOK and OPEN NOTES.
Remember to write your name (First Last) and ID Number on your answer.

Note that:

Make sure you OPEN the camera. It would be better to show your hand in camera all the time.
You can NOT talk/communicate with others or using computer to find the solution.
You can NOT use cell phone.

If you did not do anything mentioned below, you will be considered cheating!!!

Academic integrity is expected of all students at all times, whether in the presence or absence of members of the faculty. Understanding this, I declare that I shall not give, use, or receive unauthorized aid in this examination. I have been warned that any suspected instance of academic dishonesty will be reported to the Academic Judiciary and that I will be subjected to the maximum possible penalty permitted under University guidelines.

Signature: **(If you upload your answer, you are regarded as signing this.)**

Work carefully, and GOOD LUCK!!!

1. (15 points) Consider the following LP:

$$\begin{aligned} \max z = {} & 4x_1 + 4x_2 + x_3 \\ \text{s.t.} \quad & x_1 + x_2 + x_3 \leq 2 \\ & 2x_1 + x_2 \leq 3 \\ & 2x_1 + x_2 + 3x_3 \geq 3 \\ & x_1, x_2, x_3 \geq 0 \end{aligned}$$

(1). Rewrite the LP in standard form (5 points).

(2). Using Two-Phase to solve it (10 points).

2. (20 points) A company manufactures and sells items of two types. Let x_1, x_2 be the number of each type of item made. The company faces the following LP to maximize its profits:

$$\begin{aligned} \max\ z &= 2x_1 + x_2 - x_3 \\ \text{s.t.} \quad & x_1 + 2x_2 + x_3 \leq 8 \\ & -x_1 + x_2 - 2x_3 \leq 4 \\ & x_1\ ,\ x_2\ ,\ x_3 \geq 0 \end{aligned}$$

Final Tableau:

z	X_1	X_2	X_3	X_4	X_5	RHS
1	0	3	3	2	0	16
0	1	2	1	1	0	8
0	0	3	-1	1	1	12

Answer the following using graphical sensitivity analysis.

(a) Find the dual of this LP (5 points)

(b) Using the complementary slackness to find the solution of the dual LP (5 points). (remember to check the slack variables' value first)

(c). Suppose the price of sale price of type 1 item is subject to change. For what range of prices does the current optimal solution remain optimal?

(d). What is the range of values of the first right hand side (b_1) for which the current BFS remains optimal?

(e). What is the dual (shadow) price of the second constraint?

(f) Suppose that a new activity X_6 is proposed with unit return 6 and consumption vector $\mathbf{a}_6 = (2,1)'$. Find a new optimal solution. Is the new activity worth considering?

3. (10 points) Solving the following upper bound LP.

$$
\begin{array}{lcrcl}
\max & z & = & 2x_1 + 4x_2 + 7x_3 + x_4 + 5x_5 & \\
\text{s.t.} & & & 3x_1 + 5x_2 + 11x_3 + 2x_4 + 7x_5 & \leq \quad 10 \\
0 & \leq & & x_1 \ , \ x_2 \ , \ x_3 \ , \ x_4 \ , \ x_5 & \leq \quad 1
\end{array}
$$

4. (12 points) Considering the following integer programming problem

$$\begin{aligned} \max\ z &= 7x_1 + 3x_2 \\ \text{s.t.}\quad & 2x_1 + x_2 \le 9 \\ & 3x_1 + 2x_2 \le 13 \\ & x_1, x_2 \ge 0;\ x_1, x_2 \text{ integer} \end{aligned}$$

If the optimal tableau to the LP relaxation is

z	x_1	x_2	s_1	s_2	RHS
1	0	5/3	0	7/3	91/3
0	0	-1/3	1	-2/3	1/3
0	1	2/3	0	1/3	13/3

(1). If we want to solve the problem using the cutting plane method, what is the cut that should be added? (Just show the cut) (2 points)

(2). Show your branch and bound tree and state what is the optimal solution you get applying the branch and bound method: (Giving an optimal solution without work shown will receive almost no credit.) (10 points)

5. (8 points) Consider the following (minimum) Balanced Transportation problem and current BFS given.

35	8	6	10	9
10	9	12	13	7
	14	9	16	5
45		10		30
	20		30	
				30

(a). This BFS is not optimal. Which variable enters the basis next? (2 points)

(b). Show the next BFS (list only the basic variables). (6 points)

	8	6	10	9
	9	12	13	7
	14	9	16	5
45		20		30
	30		40	
				40

6. (15 points) A company intends to use 4 machines to satisfy 4 jobs. Each machine must be assigned to complete one job and each job can be assigned at most one machine. The cost is given as follows.

	job 1	job 2	job 3	job 4
Machine 1	14	5	8	7
Machine 2	2	12	6	5
Machine 3	7	8	3	9
Machine 4	2	4	6	10

(1).(5 points) Formulate the problem as an assignment problem by giving the cost matrix.

(2).(10 points) Using Hungarian Method to solve it.

7. (15 points) Find the optimal solution to the MCNFP in Figure 1 using the bfs in Figure 2 as a starting basis. Note that the number on arc following: $(x_{ij})\ U_{ij} < c_{ij} >$.

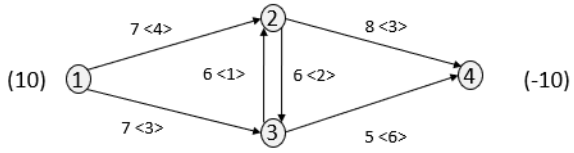

Figure 1.

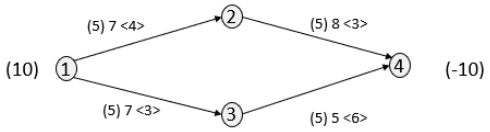

Figure 2.

8.(25 points) The following is a list of tasks that have to be completed as soon as possible.

Activity	Predecessors	Time(hours)
A	-	6
B	-	9
C	AB	8
D	AB	7
E	D	10
F	CE	12

(a). (7 points) Draw a project network (activity on arc). Make sure to number the nodes.

(b). (6 points) What are the critical activities for this project?

(c). (3 points) How soon can the project be completed?

(d). (3 points) If activity D is made shorter by 1 hour, would the project take less time to complete? Circle YES or NO, and give a brief explanation.

(e). (3 points) If activity B is made shorter by 1 hour, would the project take less to complete? Circle YES or NO, and give a brief explanation.

(f). (3 points) What is the total float of activity E?