Hw2

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2.1

$x^2y' + (xy)^2 - 2 = 0$ $\qquad y_1 = -\frac{1}{x}$

$$y' + y^2 = \frac{2}{x^2}$$

$$y = -\frac{1}{x} + \frac{1}{z(x)}$$

$$\Rightarrow \quad z' + \frac{2}{x} z = 1$$

$$z = \frac{x}{3} + \frac{C}{x^2}$$

$$y = -\frac{1}{x} + \frac{3x^2}{x^3 + C}$$

2.2 $\quad y' = 2x - \frac{1}{x} y + \frac{1}{x^3} y^2$

$$y = cx^2 + \frac{1}{z} \qquad c = 1$$

$$y' = 2x - \frac{z'}{z^2}$$

$$z' + \left(-\frac{1}{x} + \frac{2}{x}\right) z = -\frac{1}{x^3}$$

$$\rho(x) = e^{\int \frac{1}{x} dx} = x$$

$$[\rho(x)z]' = \int -\frac{1}{x^2}\,dx$$

$$z = x^{-1}\left(\frac{1}{x} + c\right)$$

$$\Rightarrow y = x^2 + \frac{x^2}{1+cx}$$

2.3 $x^2y' + 6x^2y^2 - 1 = 0$ $\qquad y_1 = \frac{A}{x}$

$A = \frac{1}{2}$ or $-\frac{1}{3}$

$$y' = \frac{1}{x^2} - 6y^2$$

Let $y = \frac{1}{2x} + \frac{1}{z}$

$$\Rightarrow z' + \left(\frac{-6}{x}\right)z = 6$$

$$\rho(x) = e^{-\int \frac{6}{x}dx} = x^{-6}$$

$$[\rho(x)z]' = \int 6x^{-6}\,dx$$

$$z = x^6\left(-\frac{6}{5}x^{-5} + c\right)$$

$$= -\frac{6x - 5cx^6}{5}$$

$$y = \frac{1}{2x} + \frac{5}{5cx^6 - 6x}$$

2.4 $y' = \cos x - \frac{1}{5}\sin x \tan x + \frac{y^2}{\qquad}$

2.4 $$y' = \cos x - \frac{1}{2}\sin x \tan x + \frac{y^2}{2\cos x}$$

Let $y = \sin x + \frac{1}{z}$

$$z' + \tan x \, z = -\frac{1}{2\cos x}$$

$$\rho(x) = e^{\int \tan x \, dx} = \cos x^{-1}$$

$$[\rho(x) z]' = \int -\frac{1}{2}\sec^2 x \, dx$$

$$z = \cos x \left(-\frac{1}{2}\tan x + C\right)$$

$$= \frac{2C\cos x - \sin x}{2}$$

$$y = \sin x + \frac{2}{2C\cos x - \sin x}$$

$$y(x=0) = \frac{1}{C} = 2019$$

P.S. $$y = \frac{4038}{2\cos x - 2019\sin x}$$