Hw3

Tuesday, January 19, 2021 7:45 PM

1. $$\begin{cases} P'(t) = -0.001\,P(100-P) \\ P(t=0) = 1000 \end{cases}$$

$$\frac{100\,dP}{P(100-P)} = -0.1\,dt$$

$$\ln\left|\frac{P}{100-P}\right| = -0.1t + C$$

$$P(0) = 1000 \Rightarrow \quad C = \ln\frac{10}{9}$$

$$P \to \infty, \quad 0.1T_p = \ln\frac{10}{9}$$

$$T_p = 10\ln\frac{10}{9}$$

2. $$r^2 - r - 2 = 0$$

$$r_1 = 2, \quad r_2 = -1$$

$$y_c = C_1 e^{2x} + C_2 e^{-x}$$

$$\begin{cases} y(0) = C_1 + C_2 = 1 \\ y'(0) = 2C_1 - C_2 = 2 \end{cases} \Rightarrow \begin{cases} C_1 = 1 \\ C_2 = 0 \end{cases}$$

$$y = e^{2x}$$

$$\therefore \quad y = e^{2x}$$

3. $$x^2y'' - 3xy' + 4y = 0$$

$$xy' = \frac{dy}{dx}$$

$$x^2y'' = \frac{d^2y}{dt^2} - \frac{dy}{dt}$$

$$\frac{d^2y}{dt^2} - 4\frac{dy}{dt} + 4y = 0$$

$$r_{1,2} = 2$$

$$y = (c_1 + c_2 t)e^{2t} = (c_1 + c_2 \ln x)x^2$$

4. $$(4x^2 + 3y^2)\,dx + 2xy\,dy = 0$$

Let $\begin{cases} M = 4x^2 + 3y^2 \\ N = 2xy \end{cases}$

$$M_y = 6y \neq 2y = N_x$$

$$f(x) = \frac{M_y - N_x}{N} = \frac{2}{x}$$

$$\rho(x) = e^{\int f(x)\,dx} = x^2$$

$$\begin{cases} \overline{M} = x^2(4x^2 + 3y^2) \\ \overline{N} = 2x^3y \end{cases}$$

$$F(x,y) = \int \overline{N}\,dy + g(x)$$

$$= x^3y^2 + g(x)$$

$$\overline{M} = 3x^2y^2 + g'(x) = x^2(4x^2 + 3y^2)$$

$$g'(x) = 4x^4$$

$$g(x) = \frac{4}{5}x^5 + C$$

$$\therefore F(x,y) = x^3y^2 + \frac{4}{5}x^5 + C = 0$$