Test1

Friday, January 8, 2021 9:33 AM

1. $(4y^3 - 2y)\,\underline{dy} = (4x - x^3)\,dx$

$$\begin{cases} y^4 - y^2 = 2x^2 - \frac{1}{4}x^4 + C \\ y(x=0) = 4 \end{cases}$$

$\Rightarrow \quad C = 4^2(4^2 - 1) = 240$

P.S. : $y^4 - y^2 = 2x^2 - \frac{1}{4}x^4 + 240$

2. $u = 3x - y \quad \Rightarrow \quad y = 3x - u$

$y' = 3 - u'$

$3 - u' + 6x(3x - u) = 2 + 9x^2 + (3x - u)^2$

$1 - u' = u^2$

$$\frac{du}{1-u^2} = dx \qquad\qquad \frac{1}{1-u} + \frac{1}{1+u}$$

$$\int \frac{1}{2}\left(\frac{1}{1+u} + \frac{1}{1-u}\right) du = dx$$

$$\frac{1}{2} \ln\left|\frac{1+u}{1-u}\right| = x + C$$

G.S. $\quad e^{-2x} \cdot \dfrac{3x-y+1}{3x-y-1} = C$

3. $\quad y' + \dfrac{4x^3}{x^2+1} y = \dfrac{2x e^{-2x^2}}{x^2+1}$

$$\rho(x) = e^{\int 4x - \frac{4x}{x^2+1} dx}$$

$$= e^{2x^2} (x^2+1)^{-2}$$

$$[\rho(x) y]' = 2x (x^2+1)^{-3}$$

$$y = e^{-2x^2} (x^2+1)^2 \left(\int 2x (x^2+1)^{-3} dx + C \right)$$

$$= e^{-2x^2} (x^2+1)^2 \left[\left(-\tfrac{1}{2}\right) (x^2+1)^{-2} + C \right] \quad \text{(G.S.)}$$

$\because \; y(0) = 4$

$\therefore \quad C = \dfrac{9}{2}$

P.S. $\quad y = e^{-2x^2} \left[-\tfrac{1}{2} + \tfrac{9}{2} (x^2+1)^2 \right]$

4. $\quad y' + xy = xy^5$

Let $\quad u = y^{-4} \quad \Rightarrow \quad y = u^{-\frac{1}{4}}$

$$y' = -\tfrac{1}{4} u^{-\frac{5}{4}} u'$$

Back substitution gives

$$-\frac{1}{4}u^{-\frac{5}{4}}u' + x \cdot u^{-\frac{1}{4}} = x\,u^{-\frac{5}{4}}$$

$$u' - 4x\,u = -4x$$

$$\rho(x) = e^{\int -4x\,dx} = e^{-2x^2}$$

$$[\rho(x)\,u]' = -4x\,e^{-2x^2}$$

$$u = e^{2x^2}\left(\int -4x\,e^{-2x^2}\,dx + c\right)$$

$$= e^{2x^2}\left(e^{-2x^2} + c\right)$$

$$\therefore\ y^{-4} = 1 + c\,e^{2x^2}$$

G.S. $y^4 + c\,e^{2x^2}y^4 - 1 = 0$