Test2

Friday, January 15, 2021 9:33 AM

1. $$\begin{cases} y' = \dfrac{(x+y)^2}{1+(x+y)^2} \\ y(x=0) = 4 \end{cases}$$

Let $v = x+y$. $v' = 1 + y'$

$$v' - 1 = -\frac{v^2}{1+v^2}$$

$$(1+v^2)\,dv = dx$$

$$v + \frac{1}{3}v^3 = x + c$$

$$v(x=0) = 4 \Rightarrow c = \frac{76}{3}$$

$$x+y + \frac{1}{3}(x+y)^3 = x + \frac{76}{3}$$

P.S. $$y + \frac{1}{3}(x+y)^3 = \frac{76}{3}$$

2. $$xy' + y = 3x^2$$

Method 1:

$$(xy)' = 3x^2$$

$$y = x^2 + \frac{c}{x}. \quad c \text{ is constant}$$

Method 2 :

By inspection, $y_1 = x^2$

Let $y = x^2 + \frac{1}{z}$. $y' = 2x - \frac{z'}{z^2}$

$$y' = 3x - \frac{1}{x}y$$

$$2x - \frac{z'}{z^2} = 3x - x - \frac{1}{xz}$$

$$\frac{dz}{z} = \frac{dx}{x}$$

$$z = cx$$

$\therefore\ y = x^2 + \frac{c}{x}$. c is const.

3. Let $\begin{cases} M = 3xy - y^2 \\ N = x^2 - xy \end{cases}$

$$M_y = 3x - 2y \neq 2x - y = N_x$$

Let $f(x) = \frac{M_y - N_x}{N} = \frac{x-y}{x(x-y)} = \frac{1}{x}$

$$\rho(x) = e^{\int \frac{1}{x} dx} = x$$

$\Rightarrow \begin{cases} \bar{M} = xy(3x - y) \\ \bar{N} = x^2(x-y) \end{cases}$

$$\Rightarrow \begin{cases} \bar{M} = \quad \\ \bar{N} = x^2(x-y) \end{cases}$$

$$F(x,y) = \int \bar{M}\,dx + g(y)$$

$$= x^3y - \frac{1}{2}x^2y^2 + g(y)$$

$$\therefore \bar{N} = x^3 - x^2y + g'(y) = x^2(x-y)$$

$$\therefore g'(y) = 0, \quad g(y) = C$$

G.S. $x^3y - \frac{1}{2}x^2y^2 = C$

4. $$\begin{cases} \dfrac{dT(t)}{dt} = k(A-T) \\ T(t=0) = T_0 \end{cases}$$

$$\Rightarrow T(t) = A + (T_0 - A)e^{-kt}$$

$$\begin{cases} T_{20} = A_1 + (T_0 - A_1)e^{-20k} \\ T_x = A_2 + (T_{20} - A_2)e^{-kx} \end{cases}$$

$$\Rightarrow \begin{cases} 100 + (250 - 100)e^{-20k} = 200 \\ \quad e^{-kx} \end{cases}$$

$$\Rightarrow \begin{cases} [illegible] \\ 70 + (200-70)e^{-kx} = 100 \end{cases}$$

$$\Rightarrow \begin{cases} k = \frac{1}{20} \ln \frac{3}{2} \\ x = 20 \ln \frac{17}{6} \approx 20.83 \end{cases}$$

$\therefore$ I should wait 20.83 minutes at the garage.