Hw1

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$$y'' + y' = \sin 20x$$

1.

$y_1 = \cos x + \sin x$ $\quad$ $LHS = -2\sin x \neq \sin 20x$

$y_1' = -\sin x + \cos x$

$y_1'' = -\cos x - \sin x$

$y_2 = \cos 20x + \sin x$. $\quad y_2' = -20\sin 20x + \cos x$

$y_2'' = -20^2\cos 20x - \sin x$

$LHS \neq RHS$

$y_3 = \cos x + \sin 20x$. $\quad y_3' = -\sin x + 20\cos 20x$

$y_3'' = -\cos x - 20^2\sin 20x$

$LHS \neq RHS$

2. $y(0) = C = 2020$

$y' = 2020 \cdot 7x^6 e^{x^7} = 7x^6 y$

$LHS = RHS$

3. $y' + \cot x\, y = 0$

$$-\frac{dy}{y} = \cot x\, dx$$

$$-\ln|y| = \ln|\sin x| + C$$

$$\frac{1}{y} = C\sin x$$

$$y\left(\frac{\pi}{2}\right) = \frac{1}{C} = 2020$$

$$\quad " = 2020 \quad \frac{1}{\;}$$

$$y = 2020 \frac{1}{\sin x}$$

4. $$y' + \frac{1}{7+x} y = \sec x \cdot \tan x / 7+x$$

$$\rho(x) = e^{\int \frac{1}{7+x} dx} = 7+x$$

$$(\rho(x) y)' = \sec x \cdot \tan x$$

$$y = \frac{1}{7+x} \left(\int \sec x \cdot \tan x \, dx + c \right)$$

$$= \frac{1}{7+x} (\sec x + c)$$

$$y(x=0) = \frac{c+1}{7} = \pi \Rightarrow c = 7\pi - 1$$

$$y = \frac{1}{7+x} (\sec x + 7\pi - 1)$$

5 $$y' = x y^{2020} y' + {}^{2021}$$

$$(1 - x y^{2020}) y' = y^{2021}$$

$$\frac{dx}{dy} + \frac{1}{y} x = y^{-2021}$$

$$\rho(y) = e^{\int \frac{1}{y} dy} = y$$

$$[\rho(y) x]' = y^{-2020}$$

$$[e(y)x]' = y^{-2020}$$

$$x = \frac{1}{y}\left(\int y^{-2020}\,dy + C\right)$$

$$= \frac{1}{y}\left(\frac{y^{-2019}}{-2019} + C\right)$$

$$= -\frac{y^{-2020}}{2019} + \frac{C}{y}$$